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Asymmetric Awareness and Moral Hazard*

Sarah Auster†

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Abstract

This paper introduces asymmetric awareness into the classical principal-agent model and discusses the optimal contract between a fully aware principal and an unaware agent. The principal enlarges the agent's awareness strategically when proposing the contract. He faces a trade off between participation and incentives. Leaving the agent unaware allows him to exploit the agent's incomplete understanding of the world. Making the agent aware enables the principal to use the revealed contingencies as signals about the agent's action choice. The optimal contract reveals contingencies that have low probability but are highly informative about the agent's effort.

Keywords: : Unawareness, Moral Hazard, Incomplete Contracts.

1 Introduction

The standard moral hazard model analyzes the optimal contract between a principal and an agent in the presence of privately observable effort. As in most economic models the underlying assumption is that all decision makers are fully aware. That is, both the principal and the agent know every possible outcome realization and its distribution conditional on the agent's effort. In reality there are contracting situations where one party has a better understanding of the underlying uncertainties than the other. The question we want to address in this paper is whether the party with superior awareness can use his better understanding of the world strategically in the presence of moral hazard.

Consider the owner of a firm who wants to hire a manager. It is likely that the firm owner is aware of more eventualities concerning his firm than the manager. Suppose for example that there

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is the possibility of a lawsuit that may significantly diminish the firm's profits and suppose further that the possibility of a lawsuit never crossed the manager's mind. The question is under which conditions it is profitable for the firm owner to disclose the potential lawsuit to the manager when offering the contract. We find that this depends generally on two factors. First, it depends on how likely the event is: if a costly lawsuit is highly probable, it may be easier to hire the manager when the possibility of a lawsuit is not disclosed. Second, it depends on how much the agent's effort affects the probability of the event: if the manager can reduce the probability of the lawsuit significantly, it may be optimal to make his payment contingent on its realization.

This paper proposes a theoretical model, which introduces asymmetric awareness in the canonical moral hazard model. It analyzes the optimal contract between a fully aware principal and an unaware agent. A decision maker is called unaware when there exist contingencies that he does not know, and he does not know that he does not know, and so on *ad infinitum* (Li, 2009). In the proposed model, the agent is assumed to be unaware of some relevant events, which means there are contingencies that affect the agent's payoff in some states but that have never crossed his mind. Further, the agent is assumed to be unaware of his unawareness, so he believes that his description of the world is correct and complete. This implies that the agent is oblivious to the possibility that the principal is aware of contingencies that he is unaware of. The principal on the other hand is assumed to be fully aware. Moreover, the principal knows that the agent is unaware and he knows what the agent is unaware of. When writing the contract the principal can make the agent aware of some or all relevant contingencies.

Conceptually unawareness is quite different from assigning probability zero to an event because it captures the idea of ignorance of the existence of an event. An agent is considered unaware if and only if he assigns probability zero to an event and its negation, which cannot be captured by the standard state space model (Heifetz, Meier, Schipper, 2009). However, in our application unawareness is descriptively the same as assigning probability zero to certain propositions being true. Hence, the proposed model can be regarded as a special case of heterogeneous priors, where the principal can credibly inform the agent about the true probability.

In line with the standard principal-agent model, the risk-neutral principal proposes a contract to the risk-averse agent. The principal is the owner of a risky project, whose outcome is a function of the realization of a finite number of elementary contingencies of which the agent only knows a subset. These contingencies can be thought of as elementary propositions that can be either true or false, e.g. whether there is a costly lawsuit or whether there is not. The probability of a contingency to be realized depends on the agent's privately taken action. The agent's effort can be high or low and it is assumed that implementing high effort is always optimal. Since the principal cannot observe the agent's effort, the terms of the contract have to be such that it is in the agent's best interest to exert the level of effort the principal wishes to implement. The compensation scheme is made contingent on the observable and verifiable outcome, rewarding the agent for outcome realizations that are relatively likely under high effort. The agent is assumed to have limited liability, thus transfers have to be non-negative in each state of the world. If the principal leaves the agent unaware of some contingencies, there is a non-empty set of possible outcomes that the agent does not take into account. It is optimal for the principal to construct the contract such that the agent receives the minimal payment whenever an unforeseen outcome is realized. In our model the minimal payment is zero, which facilitates notation considerably. The results hold for any other minimal payment as long as it is low enough.[1] Thus, the optimal contract in our environment can be interpreted as a contract that promises a fixed payment and that rewards the agent with boni for certain outcomes. Whenever a contingency is realized that is not anticipated by the agent, the bonus is not paid.

The main question this paper addresses is whether and how the principal makes the agent aware. The rationale for leaving the agent unaware is what we refer to as the participation effect. If the agent is unaware, he is deluded about the true world and the principal can exploit the agent's wrong beliefs. Since there is a positive probability that the principal pays zero and since the agent does not take this into account the principal can pay the agent in expectation less than his reservation utility. The rationale for making the agent aware is what we refer to as the incentive effect. Since the probability of a contingency to be realized depends on the effort of the agent, its realization is informative about the agent's action choice. Disclosing a contingency in the contract allows the principal to use its realization as a signal, which implies that the information structure is richer and providing incentives is less costly.

Thus, when contemplating the announcement of a contingency the principal faces the following trade off: the cost of announcement is the payment to the agent in the states where the respective contingency is realized. The cost increases with the probability of the unforeseen contingency given the optimal level of effort. The gain of enhancing the agent's awareness is a richer information structure. The gain increases with the informativeness of the signal. The principal includes contingencies in the contract, for which the gains outweigh the losses. Roughly speaking, these are contingencies that are very unlikely but highly informative.

If the agent is unaware after reading the contract, his perception of the world differs from the principal's perception of the world. The question naturally arises as to whether the agent can extract any information about the objective world upon reading the contract. Put differently, can

[1] Low enough means that the minimal payment constraint is not binding for outcomes within the agent's awareness. Otherwise the trade off for the respective outcomes changes.

the agent rationalize the proposed contract given his beliefs or should he get suspicious? We show that whenever the principal's expected profit evaluated at the agent's beliefs is non-negative and the optimal effort choice is the same given both beliefs, the proposed contract maximizes the principal's payoff also from the viewpoint of the agent. The reason for this is simply that the principal uses the signals within the agent's awareness optimally. Since the agent believes that there exist no other relevant contingencies the proposed contract optimizes the principal's expected payoff given the agent's beliefs.

Next, we allow for competition among principals. In the benchmark model without unawareness, principals engage in a Bertrand competition over the compensation scheme. In equilibrium principals make zero profits and the whole surplus goes to the agent. We call this equilibrium full awareness equilibrium. Without requiring the contract to be rationalizable for the agent, the full awareness equilibrium is the unique symmetric Nash equilibrium in our environment. The rationale behind this result is that principals engage in a Bertrand competition over the compensation scheme for every level of awareness. Once profits are small enough it is optimal to deviate and reveal another contingency to the agent in order to capture the whole market. If the contract is required to be rationalizable for the agent, this equilibrium still exists and it is unique when the number of competing principals is large enough.

Finally, some generalizations are discussed. There is more than one valid way to model unawareness. We discuss an alternative concept of unawareness and its implication on the basic model. If we allow the agent to be on average right despite being unaware, the participation loss is zero and full revelation is optimal. Next, throughout the main part of the analysis we assume that contingencies are independent of each other. Giving up this assumption the optimal compensation scheme depends on the ratio of the probability perceived by the agent to the objective probability. The optimal contract can no longer necessarily be rationalized by the agent. Further, we assume that output is a discrete one-to-one mapping from the contingencies to real numbers. It is discussed how results change when different forms of output functions are considered. Next, the analysis abstracts from the optimal action choice. We show that whenever the principal wishes to implement low effort, it is optimal to not reveal any contingencies. The reason is simply that under low effort there is no incentive effect and hence only the participation effect matters. Lastly, we discuss the optimal contract when the agent is the residual claimant. There is an additional effect on the participation constraint because the agent's valuation of the project generally depends on his level of awareness. It is favorable to the principal to disclose negative outcome shocks, because their revelation lowers the agent's outside option.

Section 2 gives an overview of the related literature. Section 3 introduces the theoretical model.

In Section 4 the optimal contract with observable effort is characterized as a benchmark. The main part of the paper is devoted to the analysis of the optimal contract with unobservable effort in Section 5. We proceed in two steps: First, the optimal compensation scheme is characterized for a given revelation strategy. Second, the optimal revelation strategy is analyzed. Finally, justifiability of the optimal contract is examined. Section 6 introduces competition among principals and Section 7 discusses generalizations of the basic model. Section 8 offers some concluding remarks. All proofs can be found in the appendix.

2 Related Literature

It is not possible to incorporate non-trivial unawareness in the standard state space model. This has been shown in the seminal paper by Dekel, Lipman and Rustichini (1998). In response, Heifetz, Meier and Schipper (2006), Li (2009) and Board and Chung (2009) have proposed general state space models that allow for non-trivial unawareness. Our model adopts the structure of Heifetz, Meier and Schipper (2006). Their unawareness structure consists of a complete lattice of spaces, where each space captures a particular horizon of propositions. In a companion work Heifetz, Meier and Schipper (2009) introduce probabilistic beliefs to the model.

Filiz-Ozbay (2008) is one of the first attempts to incorporate unawareness into contracting problems. She considers a contracting situation between a fully aware insurer and an unaware insuree. The key difference between our work and her paper is the presence of moral hazard and the assumption on beliefs. In Filiz-Ozbay (2008) there is no hidden action, which implies that the revelation of new states only involves a participation effect. In contrast to our set up the participation effect in her environment is ambiguous due to the assumptions on equilibrium beliefs. We assume that the agent assigns correct beliefs to the contingencies he is aware of, whereas Filiz-Ozbay (2008) assumes that the agent assigns an arbitrary probability belief to a newly revealed state with the restriction that the principal's payoff evaluated at the agent's belief is non-negative and that relative probability beliefs previously held are unchanged. By allowing for a wider range of equilibrium beliefs, it is possible that, upon the revelation of a new state, the agent's beliefs deviate further from the objective probabilities than before. This is favorable to the principal because he profits from the agent's misperception of the world. For this reason the participation effect of revealing new states can be positive or negative in the set up of Filiz-Ozbay (2008). If our consistency notion of beliefs was applied to her model, the insurer would always stay silent because the participation effect is strictly negative. Also Ozbay (2008) analyzes a setting where the decision maker is unaware of certain contingencies and a fully aware announcer strategically mentions contingencies before the decision maker chooses an action. Both Filiz-Ozbay (2008) and Ozbay (2008) explore the possibility that the unaware agent is able to reason why the other agent proposed the observed contract.

In a second strand of the literature contracting problems with unawareness of actions are analyzed. In these models agents are aware of all of nature's moves but can be unaware of their own action space. Von Thadden and Zhao (2010) propose a moral hazard model where the agent is unaware of his own action space but has a default action. The principal is fully aware and chooses between making the agent aware or leaving him unaware. When the default action is close enough to the first best effort level, it is optimal to leave the agent unaware. Making the agent aware involves a trade off between enlarging the agent's action space and adding costly incentive constraints. Hence, compared to our model participation and incentive effect are reversed. Also Zhao (2008) considers a moral hazard problem with unawareness of actions and default actions. In his set up both principal and agent can be unaware of their action space.

Unawareness in our model is descriptively the same as a special case of heterogeneous priors. Santos-Pinto (2008) analyzes a principal-agent model with an agent that holds wrong beliefs over the impact of his effort. He defines positive self-image as first-order stochastic dominance of the agent's perceived distribution over the actual distribution for any action. He defines effort and self-image to be complements if first-order stochastic dominance is stronger for high effort than for low effort. He shows that whenever the compensation scheme for an accurate agent is monotone and whenever effort and positive self-image are complements, the impact of positive self-image is always favorable to the principal. When effort and self-image are substitutes on the other hand, positive self-image relaxes the participation constraint and tightens the incentive constraint. In our framework the agent implicitly has a positive self-image if he is unaware of outcome decreasing contingencies. If we assume that the true distribution conditional on high effort first-order stochastically dominates the true distribution conditional on low effort and that the agent is unaware of negative outcome shocks only, unawareness implies that effort and positive self-image are substitutes.[2] Hence, our results are consistent with the findings of Santos-Pinto (2008).

3 The Model

There are two individuals involved in the contracting problem: A principal and an agent. The principal is risk neutral and the agent is risk averse. The agent receives utility from monetary transfers C and disutility from effort e. We assume that the utility function is separable in money and effort: $U(C, e) = v(C) - e$, where v satisfies the Inada conditions. Effort can take two possible values $e \in \{e^L, e^H\}$, where $e^L < e^H$.

The uncertainty of the environment is captured by a finite set of elementary contingencies,

[2] First-order stochastic dominance implies that the probability of low outcomes is more likely under low than high effort. Unawareness implicitly implies that the agent assign probability zero to some of these outcomes given high and low effort. Thus, the agent's beliefs deviate stronger for low than high effort.

denoted by Θ. A contingency $\theta \in \Theta$ is a random variable with realizations 0 and 1. It can be thought of as an elementary proposition that can be either true or false. The probability of $\theta = 1$ depends on the effort of the agent, denoted by e. Throughout the main part of the analysis it will be assumed that the contingencies in Θ are independent of each other.

Assumption 1 *The random variables θ and θ' are independent, for any $\theta, \theta' \in \Theta$.*

Awareness Structure: Different from the standard moral hazard problem the agent is unaware of some contingencies. The subset the agent is aware of is denoted by $\Theta_A \subset \Theta$. The principal is aware of the entire set Θ. Further he knows that the agent is unaware and he knows which contingencies the agent is unaware of. The agent is unaware of his unawareness and is unaware of the principal's superior awareness. When the principal writes the contract he can enlarge the agent's awareness by mentioning contingencies in the contract, denoted by $X \subseteq \Theta \backslash \Theta_A$. The agent updates his awareness and considers henceforth the contingencies in the set $\widehat{\Theta} = \Theta_A \cup X$.

State Spaces: A state of the world in this environment can be thought of as a sequence of 0's and 1's of length $|\Theta|$. Since the agent is unaware of some contingencies, he does not perceive the actual state space but a less expressive one. A state in the agent's subjective state space can be thought of as a sequence of 0's and 1's of length $|\Theta_A| < |\Theta|$. For example, let $\Theta = \{\theta_1, \theta_2\}$ and $\Theta_A = \{\theta_1\}$. Objectively there are four states of the world $\{(0,0),(0,1),(1,0),(1,1)\}$, but the agent only perceives two $\{(0),(1)\}$.

Outcomes: There is a project with stochastic outcome Y, which is observable and verifiable. The outcome is defined by the function:

$$y : \{0,1\}^{|\Theta|} \longrightarrow \mathbb{R}.$$

Let $\mathcal{Y}$ denote the range of function y. Since the agent does not know the objective state space, he cannot know $\mathcal{Y}$. Instead he perceives a mapping from his subjective state space to $\mathbb{R}$. In line with Heifetz, Meier and Schipper (2009) we assume that the agent's perceived outcome is equivalent to the objective outcome when the contingencies the agent is unaware of are not realized, which is denoted by $\theta = 0, \theta \in \widehat{\Theta}$.[3] The assumption on the agent's perception of the world is crucial in the analysis. It is discussed in more detail in Section 7.1. The agent's outcome function is

$$\widehat{y} : \{0,1\}^{|\widehat{\Theta}|} \longrightarrow \mathbb{R},$$

where $\widehat{y}(\widehat{\Theta}) = y(\widehat{\Theta}, \theta = 0, \forall \theta \notin \widehat{\Theta})$. Let $\widehat{\mathcal{Y}}$ denote the range of function $\widehat{y}$.

[3] $\theta = 0$ can refer to an elementary proposition being true or false, but it means that the world is as the agent knows it.

Assumption 2 $|\mathcal{Y}| = 2^{|\Theta|}$.

A2 imposes that the outcome differs across every state of the world, which implies that the agent knows a subset of possible outcomes whenever he is not fully aware, $\widehat{\mathcal{Y}} \subseteq \mathcal{Y}$. The assumption that y is one-to-one is important for the characterization of the trade off. More general outcome functions are discussed in Section 7.3.

Probability Measures: Let $\pi(y|e)$ denote the probability of $y \in \mathcal{Y}$ given effort e and assume $\pi(y|e) > 0, \forall y \in \mathcal{Y}$. The distribution over $\mathcal{Y}$ is known to the principal. The agent is assumed to have correct beliefs over the distribution of contingencies within his awareness. So if the agent is aware of $\theta \in \Theta$, but unaware of $\theta' \in \Theta$, he assesses the probability of contingency θ to be realized correctly. This implies that he assigns the correct conditional probability to every $y \in \widehat{\mathcal{Y}}$ given that none of the unforeseen contingencies are realized. Under the assumption of independence the probability that $\theta = 0$ for all $\theta \notin \widehat{\Theta}$ is constant across all $y \in \widehat{\mathcal{Y}}$. Let $\Pi(\widehat{\Theta}|e) := \prod_{\theta \notin \widehat{\Theta}} \Pr[\theta = 0|e]$ denote the probability that $y \in \widehat{\mathcal{Y}}$. Then the agent assigns probability

$$\widehat{\pi}(y|e) := \frac{\pi(y|e)}{\Pi(\widehat{\Theta}|e)}, \quad y \in \widehat{\mathcal{Y}},$$

to outcome $y \in \widehat{\mathcal{Y}}$ conditional on effort e. Assumption A3 summarizes the agent's view of the world.

Assumption 3 *The agent is aware of outcomes* $y \in \widehat{\mathcal{Y}}$ *and assigns probability belief* $\widehat{\pi}(y|e)$ *to them.*

A3 imposes that even when the agent cannot describe the world exhaustively, he understands his restricted world correctly. For a detailed discussion see Section 7.1.

The Contract: As in the standard principal agent problem effort is assumed to be non-observable, thus the principal offers a contract based on the observable and verifiable Y. The distribution of Y depends on the effort of the agent. It is assumed that $E[Y|e^H] > E[Y|e^L]$ and that $E[Y|e^H] - E[Y|e^L]$ is large enough, such that it is always optimal to induce high effort. This allows us to abstract the analysis from the choice of effort. In Section 7.4 the optimal action choice will be discussed. The agent is assumed to have limited liability, thus the outcome contingent compensation $C : \mathcal{Y} \longrightarrow \mathbb{R}_+$ is non-negative for all $y \in \mathcal{Y}$. Now suppose $\widehat{\Theta} \neq \Theta$, such that $\mathcal{Y} \backslash \widehat{\mathcal{Y}}$ is non-empty. The principal can construct the contract such that he pays zero to the agent when an unforeseen level of outcome is realized, e.g. by finding a functional form of the compensation scheme on $\mathcal{Y}$ satisfying zero payments for all $y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$ or by including a "zero payment otherwise" clause in the contract. The agent cannot rationalize such a clause, but it does not change his expected utility. We will abstract from the question of how the principal can implement zero payments in the unforeseen states, but analyze a reduced form of this model. Zero payments at $y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$ are optimal because any positive payment in these states will leave the agent's expected utility unaffected, but make the principal strictly worse off. We can now define the contract proposed by the principal:

Definition 1 *A contract is a pair* $(\widehat{\Theta}, C)$ *with* $\Theta_A \subseteq \widehat{\Theta} \subseteq \Theta$ *and* $C : \mathcal{Y} \longrightarrow \mathbb{R}_0^+$.

Let $(\widehat{\Theta}^*, \widehat{C}^*)$ denote the contract that maximizes the principal's expected payoff. Following Filiz-Ozbay (2008) we can introduce a notion of incompleteness.

Definition 2 *A contract* $(\widehat{\Theta}, C)$ *is incomplete if* $\widehat{\Theta} \neq \Theta$. *Otherwise it is complete.*

The principal's outside option in the case of rejection is assumed to be zero. His expected payoff is given by

$$EU_P = \begin{cases} \sum_{y \in \mathcal{Y}} \pi(y|e) \left[y - C(y)\right] & \text{if the agent accepts,} \\ 0 & \text{if the agent rejects.} \end{cases}$$

The agent assesses his expected utility with respect to his restricted state space. The outside option of rejecting the contract is $\bar{U}$:

$$EU_A = \begin{cases} \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e) \upsilon\left(C\left(\widehat{y}\right)\right) - e & \text{if the agent accepts,} \\ \overline{U} & \text{if the agent rejects.} \end{cases}$$

4 The Optimal Contract with Observable Effort

In order to have a benchmark it is useful to first characterize the contract when effort is observable. If effort is observable and verifiable the contract can be made directly contingent on the action of the agent. The principal solves the problem:

$$\max_{\widehat{\Theta}, C(\cdot)} \quad \sum_{y \in \mathcal{Y}} \pi(y|e^H) \left[y - C(y)\right]$$

subject to

$$\sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H) \upsilon\left(C(y)\right) - e^H \geq \overline{U}$$

$$C(y) \geq 0, \quad \forall y \in \mathcal{Y}.$$

We know that when the contract is complete it is optimal to give the agent full insurance. This can be seen from the first order conditions

$$\frac{1}{\upsilon'(C(y))} = \lambda, \qquad \forall y \in \mathcal{Y}.$$

The agent receives $C^{FB} = \upsilon^{-1}\left(\bar{U} + e^H\right)$ independent of the realization of Y. The first best is achieved. Now suppose the principal leaves the agent unaware of some contingencies. We know that $C(y) = 0, \forall y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$. The first order conditions for $C(y), y \in \widehat{\mathcal{Y}}$ are

$$\frac{1}{\upsilon'(C(y))} = \lambda \frac{1}{\Pi(\widehat{\Theta}|e^H)}.$$

This implies that the transfer across $y \in \widehat{\mathcal{Y}}$ is constant. The optimal compensation scheme is simply $\widehat{C}^*(y) = C^{FB}, \forall y \in \widehat{\mathcal{Y}}$ and $\widehat{C}^*(y) = 0, \forall y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$. Having characterized the optimal compensation scheme for a given $\widehat{\Theta}$ we can now turn to the optimal announcement $\widehat{\Theta}^*$.

Proposition 4.1 *Under A1, A2, A3 and observable effort,* $\widehat{\Theta}^* = \Theta_A$.

The reason for result 4.1 is that the probability to pay zero is highest when the agent's awareness level is lowest. Thus, whenever effort is observable it is optimal to disclose nothing.

5 The Optimal Contract with Unobservable Effort

Under unobservable effort the principal maximizes his expected profit subject to a participation constraint and an incentive constraint. The participation constraint assures that the agent accepts the contract. The incentive constraint leads the agent to exert high effort e^H. The principal solves:

$$\max_{\widehat{\Theta}, C(\cdot)} \quad \sum_{y \in \mathcal{Y}} \pi(y|e^H)\,[y - C(y)] \tag{1}$$

subject to

$$\sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H)\upsilon\left(C\left(y\right)\right) - e^H \geq \overline{U} \tag{2}$$

$$e^H \in \arg\max_e \left\{ \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e)\upsilon\left(C\left(y\right)\right) - e \right\} \tag{3}$$

$$C(y) \geq 0, \qquad \forall y \in \mathcal{Y}, \tag{4}$$

where 2 is the participation constraint, 3 is the incentive constraint and 4 is the limited liability constraint. We assume that the solution to the problem exists.[4] The analysis of the optimal contract can be divided into two steps. In step one the principal chooses the optimal compensation scheme given announcement $\widehat{\Theta}$. In step two he chooses the optimal level of awareness.

[4] See Grossman and Hart (1983) for sufficient conditions.

5.1 Step 1: Optimal Compensation Scheme given Awareness $\widehat{\Theta}$

The optimal compensation scheme given awareness $\widehat{\Theta}$ is characterized by 2, 3 and the necessary condition

$$\frac{1}{\upsilon'(C(y))} = \frac{1}{\Pi(\widehat{\Theta}|e^H)}\left(\lambda + \gamma\left[1 - \frac{\widehat{\pi}\left(y|e^L\right)}{\widehat{\pi}\left(y|e^H\right)}\right]\right), \quad \forall y \in \widehat{\mathcal{Y}}, \tag{5}$$

as well as $C(y) = 0, \forall y \in \mathcal{Y}\backslash\widehat{\mathcal{Y}}$.[5] Let $\widehat{C}^*$ denote the solution to this system of equations.

Lemma 5.1 *Assume A1, A2 and A3. Under* $\widehat{C}^*$*, both* $\lambda > 0$ *and* $\gamma > 0$.

Proof See Appendix A.2.

First suppose $\widehat{\Theta} = \Theta$. Then 5 is simply

$$\frac{1}{\upsilon'(C(y))} = \lambda + \gamma\left[1 - \frac{\pi(y|e^L)}{\pi(y|e^H)}\right], \quad \forall y \in \mathcal{Y},$$

which is the familiar first order condition of the standard principal agent problem. Note that under full awareness the optimal compensation scheme varies with the likelihood ratio $\frac{\pi(y|e^L)}{\pi(y|e^H)}$.

Now suppose $\widehat{\Theta} \neq \Theta$. The first order conditions show that the optimal compensation varies with the likelihood ratio of the restricted information structure $\widehat{\Theta}$ instead of Θ. Since the agent is unaware of the contingencies in $\Theta\backslash\widehat{\Theta}$, these signals cannot be used to induce e^H. Further it can be shown that the optimal transfer rule at $y \in \widehat{\mathcal{Y}}$ is equivalent to the standard optimal contract under full awareness and information structure $\widehat{\Theta}$. To see this, set $C(y) = 0, \forall y \in \mathcal{Y}\backslash\widehat{\mathcal{Y}}$ and rewrite the optimization problem of the principal as

$$\min_{\widehat{\Theta}, C(\cdot)} \Pi(\widehat{\Theta}|e^H)\sum_{y\in\widehat{\mathcal{Y}}}\widehat{\pi}(y|e^H)C(y)$$

subject to 2 and 3. Since $\Pi(\widehat{\Theta}|e^H)$ is simply a constant for a given $\widehat{\Theta}$, this optimization problem is equivalent to the standard complete contract optimization problem when only the signals in $\widehat{\Theta}$ are available to the principal. Let $C^C_{\widehat{\Theta}}$ denote the optimal compensation scheme given full awareness and restricted information structure $\widehat{\Theta}$. Then the expected profit of the incomplete contract is simply the expected payment of $C^C_{\widehat{\Theta}}$ weighted by the probability that none of the unforeseen contingencies are realized

$$E[\widehat{C}^*(Y)|e^H] = \Pi(\widehat{\Theta}|e^H)E[C^C_{\widehat{\Theta}}(Y)|e^H].$$

[5] Inada conditions assure that 4 is not binding for $C(y), y \in \widehat{\mathcal{Y}}$.

5.2 Step 2: Optimal Revelation $\widehat{\Theta}^*$

To characterize $\widehat{\Theta}^*$ it is useful to compare the expected payment of complete contracts under different information structures.

Lemma 5.2 *Let Z be a non-empty subset of $\Theta \backslash \widehat{\Theta}$. Then*

$$\Delta C_{\widehat{\Theta}}^{Z} := E\left[C_{\widehat{\Theta}}^{C}(Y)|e^H\right] - E\left[C_{\widehat{\Theta}\cup Z}^{C}(Y)|e^H\right] \geq 0,$$

with strict inequality if and only if $\exists \theta \in Z$ such that $\Pr[\theta = 1|e^H] \neq \Pr[\theta = 1|e^L]$.

Proof See Appendix A.3.

This result is in line with Holmström's *Sufficient Statistic Theorem* (1979), which states that a signal θ is valuable if and only if it is informative.[6] Valuable means that both principal and agent can be made better off by including θ or simply that agency costs are reduced. Under independence θ is informative if and only if $\Pr[\theta = 1|e^H] \neq \Pr[\theta = 1|e^L]$.

We are now ready to identify the trade off faced by the principal when contemplating the announcement of set $\widehat{\Theta}$. We know that $\widehat{\Theta}^* \neq \Theta_A$ whenever there exist a $\widehat{\Theta}$ such that the expected payment is lower under $\widehat{\Theta}$ than under awareness Θ_A. Formally:

$$\Pi(\widehat{\Theta}|e^H) E\left[C_{\widehat{\Theta}}^{C}(Y)|e^H\right] < \Pi(\Theta_A|e^H) E\left[C_{\Theta_A}^{C}(Y)|e^H\right],$$

or simply

$$E\left[C_{\widehat{\Theta}}^{C}(Y)|e^H\right] < \prod_{\theta \in X} \Pr\left[\theta = 0 \mid e^H\right] E\left[C_{\Theta_A}^{C}(Y)|e^H\right].$$

Using $\Delta C_{\Theta_A}^{X} = E\left[C_{\Theta_A}^{C}(Y)|e^H\right] - E\left[C_{\widehat{\Theta}}^{C}(Y)|e^H\right]$ we can restate the inequality in terms of gains and losses of revealing set X:

$$\left(1 - \prod_{\theta \in X} \Pr\left[\theta = 0|e^H\right]\right) E\left[C_{\Theta_A}^{C}(Y)|e^H\right] < \Delta C_{\Theta_A}^{X}. \tag{6}$$

The LHS of 6 shows the loss of revelation. With probability $1 - \prod_{\theta \in X} \Pr\left[\theta = 0|e^H\right]$ one of the contingencies in X is realized and the principal can no longer pay zero to the agent. The RHS shows the efficiency gain due to the richer information structure. This trade off arises because the announcement of a contingency has opposing effects on participation and incentive constraint. On the one hand the inclusion of a contingency increases the probability that the agent receives

[6] Holmström (1979) shows this for continuous outcome and continuous effort.

a positive transfer. This tightens the participation constraint. On the other hand the announcement of an informative signal relaxes the incentive constraint due to a richer information structure. Under observable effort there is no incentive effect; that is why it is optimal to keep the agent unaware. Under unobservable effort the principal chooses $\widehat{\Theta}$ such that the net gain of revelation is maximized. Since Θ is finite he compares a finite number of announcement strategies and their respective expected payoffs. Whenever there exists a $\widehat{\Theta}$ such that the incentive effect outweighs the participation effect, the principal enlarges the agent's awareness. Whether this is the case or not generally depends on the exogenous parameters of the model.

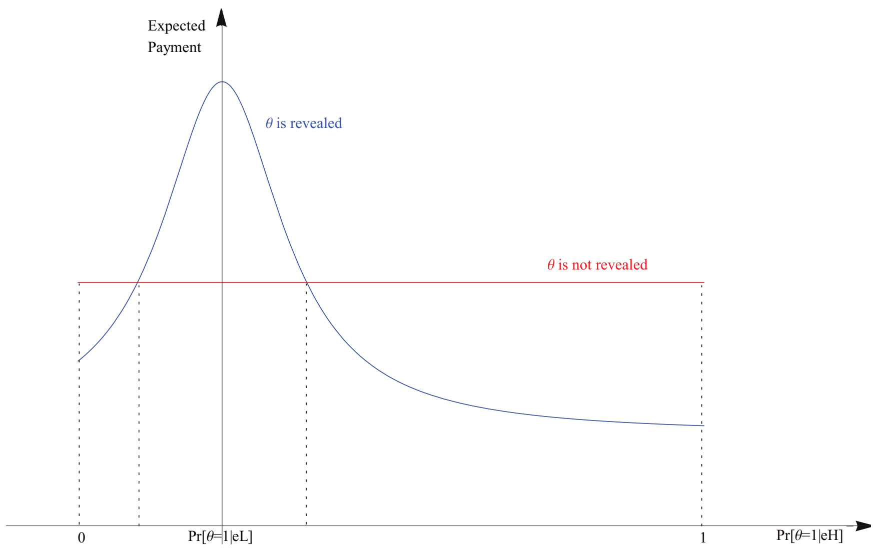
5.3 Single Announcements

To find some conditions on the trade off in 6 it useful to first analyze one step deviations from the set Θ_A. Consider the announcement of $\theta \notin \Theta_A$ and define $\Delta^\theta := \left|\Pr[\theta = 1|e^H] - \Pr[\theta = 1|e^L]\right|$. Δ^θ can be interpreted as a measure of informativeness of the signal θ. The larger Δ^θ the more informative is the realization of θ about the effort of the agent. Let $V_{\widehat{\Theta}} := E[C^C_{\widehat{\Theta}}(Y)|e^H]$ denote the principal's expected payment to the agent given full awareness, restricted information structure $\widehat{\Theta}$ and optimal compensation scheme $C^C_{\widehat{\Theta}}$.

Lemma 5.3 *Assume A1, A2 and A3. $V_{\widehat{\Theta}}$ is monotonically decreasing in $\Delta^\theta, \forall\theta \in \widehat{\Theta}$.*

Proof See Appendix A.4

Figure 1: Expected Compensation under $\theta \in \widehat{\Theta}$ and $\theta \notin \widehat{\Theta}$



The significance of Δ^θ is illustrated in Figure 1.[7] It shows the expected payment to the agent as a function of $\Pr[\theta = 1|e^L]$. If the principal does not include θ in the contract, the expected payment is independent of $\Pr[\theta = 1|e^L]$, which is illustrated by the straight red line. The expected payment is decreasing in $\Pr[\theta = 1|e^H]$, because whenever $\theta \notin \widehat{\Theta}$ and $\theta = 1$ the principal pays zero. Thus, the higher $\Pr[\theta = 1|e^H]$ the lower is the expected payment when θ is not revealed. For $\Pr[\theta = 1|e^H]$ large enough, the two curves do not intersect and the principal prefers to leave the agent unaware independent of Δ^θ.

If the principal reveals θ, he uses θ as a signal about the agent's effort. The larger Δ^θ the lower is the expected payment. The signal is uninformative if $\Pr[\theta = 1|e^H] = \Pr[\theta = 1|e^L]$, which implies that the expected payment to the agent under $\theta \in \widehat{\Theta}$ as a function of $\Pr[\theta = 1|e^L]$ has its maximum at $\Pr[\theta = 1|e^H]$. Note that at this point the expected transfer to the agent under $\theta \in \widehat{\Theta}$ is strictly greater than under $\theta \notin \widehat{\Theta}$ due to the assumption of full support. That is, whenever a signal is uninformative the gain of including it in the contract is equal to zero whereas the loss is strictly positive due to the positive probability of zero payments.

The principal discloses θ if the expected payment to the agent is lower under $\theta \in \widehat{\Theta}$ than under $\theta \notin \widehat{\Theta}$. Thus, the intuition for Figure 1 is that if there is a contingency θ outside the agent's awareness with $\Pr[\theta = 1|e^H]$ sufficiently low, the principal will enlarge the agent's awareness if the contingency is sufficiently informative.

Proposition 5.4 *Assume A1, A2 and A3. For every $\theta \in \Theta \backslash \Theta_A$ with $\Delta^\theta > 0$ there exists a unique $\pi^\theta \in (0, 1)$ such that for all $\Pr[\theta = 1|e^H] < \pi^\theta$, the expected payment under $\Theta_A \cup \theta$ is strictly lower than under Θ_A. If there exists a $\theta \in \Theta \backslash \Theta_A$ for which $\Pr[\theta = 1|e^H] < \pi^\theta$, then $\widehat{\Theta}^* \neq \Theta_A$.*

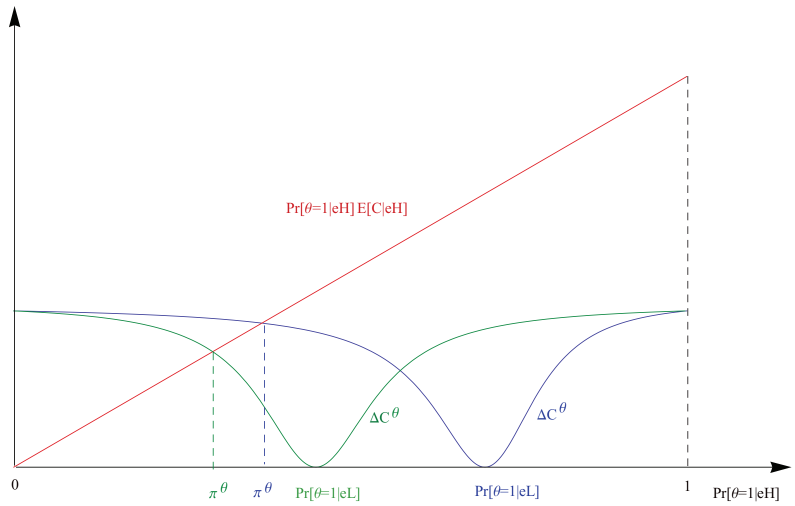
Proof Omitted.

Proposition 5.4 states the following: For any informative signal θ there exists a critical probability π^θ given high effort, such that when the actual probability is lower than this threshold, the incentive gain of disclosing θ outweighs the participation loss. To see this, consider the trade off faced by the principal:

$$\Pr\left[\theta = 1|e^H\right] E\left[C^C_{\Theta_A}(Y)|e^H\right] \quad \text{vs.} \quad \Delta C^\theta_{\Theta_A}.$$

[7] This figure shows the expected payment for the following specification: $v(C) = \frac{C^{1-\sigma}}{1-\sigma}, \sigma = 0.5, e^H = 1, e^L = 0, \bar{U} = 5$. There are two contingencies. Contingency $\theta' \in \Theta_A$ with $\Pr[\theta' = 1|e^H] = 0.65$ and $\Pr[\theta' = 1|e^L] = 0.5$ and contingency $\theta \notin \Theta_A$ with $\Pr[\theta = 1|e^H] = 0.2$.

Figure 2: Trade Off



It is easy to see that

$$\lim_{\Pr[\theta=1|e^H]\to 0} \Pr\left[\theta=1|e^H\right] E\left[C^C_{\Theta_A}(Y)|e^H\right]=0 \quad \text{and} \quad \lim_{\Pr[\theta=1|e^H]\to 0} \Delta C^{\theta}_{\Theta_A}>0.$$

From Lemma 5.3 we know that $\Delta C^{\theta}_{\Theta_A}$ has its minimum at $\Pr[\theta=1|e^H]=\Pr[\theta=1|e^L]$. At this point $\Delta C^{\theta}_{\Theta_A}=0$, which is smaller than $\Pr\left[\theta=1|e^H\right] E\left[C^C_{\Theta_A}(Y)|e^H\right]$. As we know that $\Delta C^{\theta}_{\Theta_A}$ is monotonically decreasing on the interval $\left(0,\Pr[\theta=1|e^L]\right)$, we know that there is exactly one π^{θ} on this interval such that

$$\Pr[\theta=1|e^H] E\left[C^C_{\Theta_A(Y)}|e^H\right]<\Delta C^{\theta}_{\Theta_A},$$

for all $\Pr[\theta=1|e^H]<\pi^{\theta}$, illustrated in Figure 2.

5.4 Combined Announcements

In the previous section we characterized the gains and losses of revealing a contingency θ for a given set Θ_A. When the agent is unaware of more than one contingency the trade off associated with the revelation of one of these contingencies generally depends on the revelation of other contingencies. To see how the trade off changes as other contingencies are revealed we will consider the special case of symmetric contingencies. We will see that, under some conditions, the net gain of making the agent aware worsens as more and more contingencies are revealed. This implies that whenever the revelation of any single contingency yields a negative net gain, a combined revelation cannot be profitable. Moreover, if the number of contingencies outside the agent's awareness is large enough, full revelation can never be optimal.

To see this, suppose the agent is unaware of N symmetric contingencies, $\Theta = \{\Theta_A, \theta_1, ..., \theta_N\}$, and let $\alpha := \Pr[\theta_i = 0|e^H], i = 1, ...N$ and $\widehat{\Theta}_n := \Theta_A \cup \theta_1 \cup ... \cup \theta_n$.[8] The net gain of revealing the nth contingency is:

$$NG(\theta_n) := \alpha^{N-n} \left(\Delta C^{\theta_n}_{\widehat{\Theta}_{n-1}} - (1-\alpha) E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H] \right).$$

The value of $NG(\theta_n)$ is determined by two factors. First, there is the difference between incentive gain and participation loss as we saw in the previous section. This difference is weighted by the probability that the losses and gains are realized, α^{N-n}. Note that both $\Delta C^{\theta_n}_{\widehat{\Theta}_{n-1}}$ and $(1-\alpha)E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H]$ are decreasing in n, whereas α^{N-n} is exponentially increasing in n. Figure 3(a) shows $\alpha^{N-n}\Delta C^{\theta_n}_{\widehat{\Theta}_{n-1}}$ and $\alpha^{N-n}(1-\alpha)E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H]$ as a function of n. Generally $NG(\theta_n)$ can decrease or increase in n.

Lemma 5.5 *Assume A1, A2, A3 and symmetric contingencies. $NG(\theta_n)$ is decreasing in n if*

$$E[C^C_{\widehat{\Theta}_n}(Y)|e^H]^2 < E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H] E[C^C_{\widehat{\Theta}_{n+1}}(Y)|e^H] \quad \textit{for all} \quad n \in \{1, ..., N-1\}.$$

Proof See Appendix A.5

Lemma 5.5 states that the revelation of contingency θ_{n+1} yields a lower net gain than θ_n if $E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H]$ is sufficiently convex in n, depicted in Figure 3(b).

If $E[C^C_{\widehat{\Theta}_n}(Y)|e^H]^2 < E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H]E[C^C_{\widehat{\Theta}_{n+1}}(Y)|e^H]$ is satisfied on some interval, it allows us to make predictions about the combined announcement of contingencies. It is easy to see that whenever $NG(\theta_n)$ is decreasing in n for all $n \in \{\bar{n}, \bar{n}+1, ..., N\}$ and $NG(\theta_{\bar{n}}) < 0$ then $NG(Z) < 0$ for all $Z =\subseteq \{\theta_{\bar{n}}, \theta_{\bar{n}+1}, ..., \theta_N\}$; that is $NG(\theta_{n+1}) < NG(\theta_n)$ for all $n \geq \bar{n}$ and $NG(\theta_{\bar{n}}) < 0$ imply $NG(\theta_n) < 0$ for all $n \geq \bar{n}$ and hence

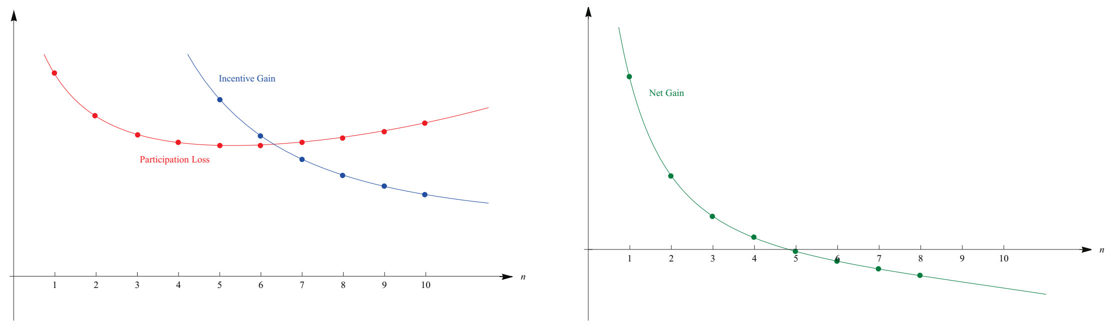
$$NG(Z) = \alpha^{N-(\bar{n}-1)} E[C^C_{\widehat{\Theta}_{\bar{n}-1}}(Y)|e^H] - \alpha^{N-z} E[C^C_{\widehat{\Theta}_z}(Y)|e^H] = \sum_{n=\bar{n}}^{z} NG(\theta_n) < 0,$$

where $z = \bar{n} - 1 + |Z|$.

If $E[C^C_{\widehat{\Theta}_n}(Y)|e^H]^2 < E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H]E[C^C_{\widehat{\Theta}_{n+1}}(Y)|e^H]$ is not satisfied, $NG(\theta_n)$ can be increasing on some intervals. Nevertheless, we can derive some intuition for the limiting case. Suppose N is

[8] For notational convenience let $\widehat{\Theta}_0 = \Theta_A$.

Figure 3: Trade Off θ_n



(a) Weighted Incentive Gain and Participation Loss of Revealing θ_n

(b) Net Gain of Revealing θ_n

large. Then $E[C^C_{\widehat{\Theta}_n}(Y)|e^H]$ converges to C^{FB} as $n \longrightarrow N$, which implies that the incentive gain converges to zero, the participation loss converges to $(1-\alpha)C^{FB}$ and their difference converges to $-(1-\alpha)C^{FB}$. Consequently, we can always find an N large enough such that full revelation is not optimal.

5.5 Justifiability of the Contract

If the contract is incomplete, the agent's perception of the world differ's from the principal's. An important question is whether the proposed contract can elicit suspicion on the side of the agent. Filiz-Ozbay (2008) introduces an equilibrium refinement, which requires that the equilibrium contract maximizes the principal's expected payoff from the viewpoint of the agent. In line with her equilibrium refinement, we define justifiability of a contract as follows.

Definition 3 *A contract $(\widehat{\Theta}, C)$ is called justifiable if it is a solution to the optimization problem*

$$\max_{\tilde{\Theta} \subseteq \widehat{\Theta}, C(\cdot), e} EU_P^A := \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e) \left[y - C(y) \right]$$

subject to 2, 3 and $C(y) \geq 0, \forall y \in \widehat{\mathcal{Y}}$.

Proposition 5.6 *Assume A1, A2 and A3. $(\widehat{\Theta}^*, \widehat{C}^*)$ is justifiable according to definition 3 if and only if $EU_P^A(\widehat{\Theta}^*, \widehat{C}^*, e^H) \geq 0$ and $EU_P^A(\widehat{\Theta}^*, \widehat{C}^*, e^H) \geq EU_P^A(\widehat{\Theta}^*, \widehat{C}^*, e^L)$.*

Proof See Appendix A.6.

A necessary and sufficient condition for the optimal contract to be justifiable is that the principal's expected utility is non-negative and that e^H is the optimal action choice from the viewpoint of the agent. Up to now the optimization problem was disconnected from the actual outcome levels in $\mathcal{Y}$

because only the likelihood ratio associated with each outcome level is relevant for the induction of e^H. If we require the contract to be justifiable, this is no longer necessarily the case. Whenever the optimal contract is incomplete, the agent perceives only a subset of possible outcomes. It is possible that the optimal contract leaves the agent unaware of high outcomes and that $EU_P^A(\widehat{\Theta}^*, \widehat{C}^*, e^H) < 0$ whereas $EU_P(\widehat{\Theta}^*, \widehat{C}^*, e^H) \geq 0$. If $EU_P^A(\widehat{\Theta}^*, \widehat{C}^*, e^H) < 0$ the refinement introduces another dimension in the trade off. The principal is no longer only concerned with the distributional properties of the contingencies in $\Theta \backslash \Theta_A$, but also with the outcomes an announcement reveals. In this case disclosing a contingency is most profitable when it is very unlikely, very informative and when it reveals high outcomes.

Given that the expected profit evaluated at the agent's beliefs is non-negative and higher at e^H than at e^L, the optimal contract $(\widehat{\Theta}^*, \widehat{C}^*)$ is justifiable. To see this, we proceed in two steps as before. First, we discuss justifiability of $\widehat{C}^*$ for a given $\widehat{\Theta}$ and second we show that $\widehat{\Theta}^*$ is consistent with the agent's beliefs.

If the optimal contract is complete, principal and agent share the same belief. Hence, $\widehat{C}^*$ maximizes the principal's expected payoff from the agent's perspective. If the optimal contract is incomplete, we know that the transfer rule for outcomes within the agent's awareness coincides with the optimal compensation scheme of the complete contract given information structure $\widehat{\Theta}$. Since the agent thinks that the contract is complete, $\widehat{C}^*$ solves the principal's optimization problem given the agent's beliefs.

When contemplating possible revelation strategies, the agent can only take into account contracts within his awareness. For $\widehat{\Theta}^*$ to be justifiable it has to be true that

$$E\left[C^C_{\widehat{\Theta}^*}(Y)|e^H\right] \leq \prod_{\theta \in \widehat{\Theta}^* \backslash Z} \Pr[\theta = 0|e^H] E\left[C^C_{\widehat{\Theta}^* \backslash Z}(Y)|e^H\right], \tag{7}$$

for any $Z \subseteq \widehat{\Theta}^* \backslash \Theta_A$. This coincides with the optimality condition of the principal. Consequently, 7 is always fulfilled and $\widehat{\Theta}^*$ can be rationalized by the agent. Hence, whenever the principal's expected payoff evaluated at the agent's beliefs is non-negative and e^H is the optimal effort choice under both beliefs, the optimal contract is justifiable according to definition 3.

6 Competing Principals

In our set up we analyze the optimization problem of a monopolistic principal. This section addresses the question of how these results change when principals compete against each other. Suppose there are N principals that are all aware of Θ. They make simultaneous offers, denoted by $(\widehat{\Theta}_i, C_i), i = 1, ..., N$. The agent updates his awareness after hearing all of the offers and accepts at most one. He considers henceforth every contingency in $\widehat{\Theta}_1 \cup ... \cup \widehat{\Theta}_N$. If the agent is indifferent between two or more contracts he accepts each contract with equal probability. We focus on symmetric equilibria $(\widehat{\Theta}_i, C_i) = (\widehat{\Theta}, C), i = 1, ..., N$. Principal $i's$ payoff function is

$$EU_i = \frac{1}{N}\left(E[Y|e^H] - E[C(Y)|e^H]\right), \qquad i = 1, ..., N.$$

In the absence of asymmetric awareness, principals engage in a Bertrand competition over transfer rule C. The second-best allocation is achieved, where principals make zero profits and the surplus goes to the agent. We assume that the second best surplus is strictly positive.[9] Let C^* denote the equilibrium compensation scheme under full awareness.

Proposition 6.1 *Assume A1, A2 and A3. There is a unique symmetric Nash equilibrium in which the agent is fully aware and each principal offers the complete zero profit contract* (Θ, C^*).

Proof See Appendix A.7.

The intuition for proposition 6.1 is that whenever the announcements of the other principals promote full awareness the own announcement is not payoff relevant. Further, any change in the compensation scheme yields either negative or zero expected profits for the standard Bertrand competition argument. Thus, the equilibrium exist. The reason for the equilibrium to be unique is simply that for any level of awareness principals engage in a Bertrand competition over the compensation scheme. As profits become small enough it is profitable to deviate and enlarge the agent's awareness. This deviation allows the deviator to capture the whole market and make positive profits, because the agent recognizes the zero payments of the other contracts in the newly revealed states.

When we require the contract to be justifiable according to definition 3, the symmetric zero profits Nash equilibrium is no longer necessarily unique. Suppose each principal announces $\widehat{\Theta}_i = \widehat{\Theta} \subset \Theta, i = 1, ...N$ and offers compensation $C_i = \widehat{C}, i = 1, ..., N$ such that the agent's utility is maximized across the states the agent is aware of, the incentive constraint is satisfied and

$$\sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H)[y - \widehat{C}(y)] = 0,$$

[9]Generally it is possible that the project yields a negative second best surplus and principals offer an incentive compatible contract nevertheless. In that case proposition 6.1 does not necessarily hold. To see this, suppose there are 2 principals and $\Theta = \{\theta_1, \theta_2\}$ and $\Theta_A = \{\theta_1\}$. Assume further that under $\widehat{\Theta}_1 = \widehat{\Theta}_2 = \theta_1$ after Bertrand competition the participation constraints of both principals and the agent hold with equality. Then the revelation of θ_2 yields a zero or negative payoff for both principals and hence deviating is not profitable.

which implies

$$EU_i = \frac{1}{N} \sum_{y \in \mathcal{Y}} \pi(y|e)[y - \widehat{C}(y)] > 0,$$

due to the zero payments at $y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$. Any deviation in the compensation scheme C_i either makes the agent worse off or makes the contract non-justifiable according to definition 3. Hence, principals can make positive profits in equilibrium. A joint deviation in $\widehat{\Theta}_i$ and C_i, allows principal i to capture the whole market. Whether this deviation is profitable or not depends on the number of principals among other parameters of the model. The larger the number of principals the smaller is the expected profit when no one is deviating. This implies that we can find an N large enough such that deviating is always profitable until the agent is fully aware.

Proposition 6.2 *Assume A1, A2 and A3 and assume the agent rejects any non-justifiable contract according to definition 3. There exists a critical $\bar{N} \in \mathbb{N}$, such that for every $N > \bar{N}$ in equilibrium the agent is fully aware.*

Proof See Appendix A.8.

The result that competition promotes awareness if the number of competitors is large enough was first shown by Filiz-Ozbay (2008). The line of argument is essentially the same. The profit of each principal in a symmetric equilibrium with unawareness decreases with the number of principals. Enlarging the agent's awareness in her environment implies changing the beliefs of the agent, which allows the deviator to capture the whole market. Hence, whenever the number of principals is large enough revealing new states to the agent is profitable and in equilibrium the agent is fully aware.

7 Discussion

7.1 Beliefs under Unawareness

Generally there is more than one natural way to think about unawareness. One possible interpretation of unawareness is that the agent observes outcome repeatedly and anticipates frequencies correctly, but is unaware of the data generating process. In our framework this would imply that the agent knows the true distribution of Y but is unaware of some contingencies in Θ. He associates part of the variation in outcome to the realizations of $\theta \in \Theta_A$ and interprets the rest as noise.

A second way to think about unawareness is that there are possible events that have never happened to the agent before and that have never crossed his mind.[10] He signs the contract given

[10]This concept of unawareness is prevalent in the literature of unawareness. See for example Modica, Rustichini and Tallon (1998) and Heifetz, Meier and Schipper (2009).

the world as he knows it. In his status quo world elementary propositions outside his awareness can be true or false. Since he cannot spell out these propositions he cannot imagine a world in which a proposition that used to be true (false) turns out to be false (true). Consequently, it is natural to assume that the agent perceives a world in which propositions outside his awareness are always true or false respectively.[11] The agent's beliefs are systematically biased.

Both concepts of unawareness are interesting and relevant in reality. The basic model captures the latter. If the first concept of unawareness was applied to our set up and only outcome was observable and verifiable, full revelation would be optimal. Since the agent predicts the frequency of each outcome correctly, the participation loss of revelation is zero and only the incentive effect matters.

We assume further that the agent assigns correct probabilities to the contingencies within his awareness. This implies that even if the agent cannot describe the world exhaustively, he understands his restricted world correctly. Generally one can depart from this assumption and allow for a wider range of beliefs. This adds effects of heterogeneous beliefs or ambiguity to the problem, which generally affect both participation loss and incentive gain.[12]

7.2 Dependence between Contingencies

Throughout the analysis we assumed that the realization of contingencies is independent of each other. Giving up this assumption unawareness has an additional effect on the optimal compensation scheme. To see this, consider first the case of observable effort. When $\widehat{\Theta} \neq \Theta$, the necessary condition is

$$\frac{1}{\upsilon'(C(y))} = \lambda \frac{\widehat{\pi}(y|e^H)}{\pi(y|e^H)}, \qquad \forall y \in \widehat{\mathcal{Y}}.$$

$\frac{\widehat{\pi}(y|e^H)}{\pi(y|e^H)}$ is the ratio of the probability the agent perceives to the objective probability. Under dependence this ratio depends on y, because the probability that none of the unforeseen contingencies are realized varies across $y \in \widehat{\mathcal{Y}}$. It is optimal to promise a high payment at y if the probability that one of the unforeseen contingencies is realized is relatively high, because the probability that the principal has to keep his promise is relatively low. This makes a deviation from the optimal risk sharing rule profitable. Since the agent cannot rationalize a non-constant compensation scheme, such a contract is not justifiable according to definition 3.

[11]For example, if the agent is unaware of the potential lawsuit he anticipates a world in which no lawsuit is realized. In contrast, if he was assumed to be on average right he would perceive outcomes in between outcomes when the lawsuit is realized and when it is not. This would imply that he has some idea (even if vague) about the potential lawsuit, which implies that he is not unaware.

[12]Filiz-Ozbay (2008) provides a detailed analysis of the participation effect.

When effort is not observable, the first order condition is

$$\frac{1}{v'(C(y))} = \frac{\widehat{\pi}(y|e^H)}{\pi(y|e^H)} \left\{ \lambda + \gamma \left[1 - \frac{\widehat{\pi}(y|e^L)}{\widehat{\pi}(y|e^H)} \right] \right\}, \quad \forall y \in \widehat{\mathcal{Y}}.$$

It shows that there are two sources of variation in the optimal compensation scheme $\widehat{C}^*$ across $y \in \widehat{\mathcal{Y}}$. First, $\widehat{C}^*$ varies with the ratio of perceived to objective probability just as in the case of observable effort. The second source of variation comes from the agent's perceived likelihood ratio as in the basic model. Again, the agent cannot rationalize the first source of variation and the optimal contract is not necessarily justifiable according to definition 3. If we require the contract to be justifiable, the optimal compensation scheme ignores the variation in the probability of $y \in \widehat{\mathcal{Y}}$ and is equivalent to $(\widehat{\Theta}^*, \widehat{C}^*)$ under independence.

7.3 The Output Function

In the basic model output is discrete and differs across every state of the world. If y is not one-to-one and contingencies are not observable, both participation loss an incentive gain of revelation are affected. The participation loss is generally diminished, the incentive gain is no longer unambiguous. To see this, suppose there are only two possible realizations of outcome. The project can be either a success or a failure, $\mathcal{Y} = \{s, f\}$ with $s > f$. Whether the project is a success or a failure depends on the realization of Θ. If $\widehat{\Theta} \subset \Theta$ the agent is aware of outcomes $\{s, f\}$ but beliefs probability distribution $\widehat{\pi}(.|e)$.[13]

Now consider the trade off the principal faces upon the announcement of $\theta \notin \Theta_A$. Since the revelation of θ does not reveal any new outcomes the participation loss of announcement is zero. The effect on incentives is no longer unambiguous but depends on how the revealed contingency affects the perceived distribution of the agent. To see this, suppose $\Theta = \{\theta_1, \theta_2\}$, $\Theta_A = \{\theta_1\}$ and assume θ_2 is not informative, i.e. $\Pr[\theta_2 = 1|e^H] = \Pr[\theta_2 = 1|e^L]$. In the basic model $\Pr[\theta = 1|e^H] = \Pr[\theta = 1|e^L]$ implies $\Delta C^{\theta}_{\Theta_A} = 0$, because the principal has the choice to ignore the realization of θ_2. Under $\mathcal{Y} = \{s, f\}$ this is no longer the case. Suppose that $y = s$ whenever $\theta_1 = \theta_2 = 0$ and $y = f$ otherwise. Solving for the optimal compensation scheme, we find that the expected payment under $\widehat{\Theta}^* = \Theta_A$ is smaller than the expected payment under $\widehat{\Theta}^* = \Theta$.[14] There is an incentive loss of revealing θ_2 because the perceived distribution of the unaware agent is more informative about the

[13] Generally it is possible that the agent is only aware of one outcome. In that case the principal has to make the agent aware of the other outcome, otherwise it is impossible to induce e^H.

[14] If the principal leaves the agent unaware, the optimal compensation scheme is

$$C^u(s) = v^{-1}\left(\bar{U} + \frac{\Pr[\theta_1 = 1|e^L]e^H - \Pr[\theta_1 = 1|e^H]e^L}{\Pr[\theta_1 = 1|e^L] - \Pr[\theta_1 = 1|e^H]}\right), \; C^u(f) = v^{-1}\left(\bar{U} - \frac{\Pr[\theta_1 = 0|e^L]e^H - \Pr[\theta_1 = 0|e^H]e^L}{\Pr[\theta_1 = 1|e^L] - \Pr[\theta_1 = 1|e^H]}\right).$$

action choice than the true distribution.

The same intuition holds when we consider a continuous output function. Suppose output is a function

$$x = y + \varepsilon,$$

where $\varepsilon \sim N(0, \sigma^2)$ is an error term with variance σ^2. Suppose further that the agent is aware of the error term and perceives outcome

$$\widehat{x} = \widehat{y} + \varepsilon.$$

This implies that the agent is aware of the full range of x, $(-\infty, +\infty)$. Nevertheless, if $\widehat{\Theta} \subset \Theta$ his perceived distribution on the range of possible outcomes differs from the actual distribution. When making the agent aware, his perceived distribution is not necessarily more informative after updating his awareness than before.[15] A further discussion on heterogeneous priors and moral hazard can be found in Santos-Pinto (2008) and De la Rosa (2011).

7.4 Optimal Action Choice

Throughout the analysis we assumed that $E[Y|e^H] - E[Y|e^L]$ is large enough such that the principal always finds it optimal to induce e^H. Unawareness makes incentives more costly, hence it is generally possible that for different levels of awareness different levels of effort are optimal. Since effort can only be high or low, the analysis of the optimal contract under low effort is straight forward.

Proposition 7.1 *Assume A1, A2 and A3. If e^L is the action choice, the optimal contract is (Θ_A, C^L) with $C^L(y) = v^{-1}(\bar{U} - e^L), \forall y \in \widehat{\mathcal{Y}}$ and $C^L(y) = 0, \forall y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$.*

If the principal reveals contingency θ_2, the optimal compensation scheme is

$$\begin{aligned} C^a(s) &= v^{-1}\left(\bar{U} + \frac{\left(1 - \Pr[\theta_1 = 0|e^L]\Pr[\theta_2 = 0|e^L]\right)e^H - \left(1 - \Pr[\theta_1 = 0|e^H]\Pr[\theta_2 = 0|e^H]\right)e^L}{\Pr[\theta_1 = 0|e^H]\Pr[\theta_2 = 0|e^H] - \Pr[\theta_1 = 0|e^L]\Pr[\theta_2 = 0|e^L]}\right) \\ C^a(f) &= v^{-1}\left(\bar{U} - \frac{\Pr[\theta_1 = 0|e^L]e^H - \Pr[\theta_1 = 0|e^H]e^L}{\Pr[\theta_1 = 1|e^L] - \Pr[\theta_1 = 1|e^H]}\right). \end{aligned}$$

So $C^u(f) = C^a(f)$ and $v(C^u(s)) - v(C^a(s)) = -\frac{\Pr[\theta_2=1|e^H]}{\Pr[\theta_2=0|e^H]}\frac{e^H - e^L}{\Pr[\theta_1=0|e^H] - \Pr[\theta_1=0|e^L]} < 0$, which implies that $E[C^u(Y)|e^H] < E[C^a(Y)|e^H]$.

[15] It is possible to find environments with continuous outcome functions for which Holmström's *Sufficient Statistic Theorem* (1979) and consequently our results hold. For example: Let output be a function $y = \sum_{\theta \in \Theta} y^\theta$ with $y^\theta = f^\theta(\theta) + \varepsilon^\theta$ and $y^\theta(0) = 0, \forall \theta \in \Theta$, where y^θ is observable and verifiable. The agent perceives $\widehat{y} = \sum_{\theta \in \widehat{\Theta}} y^\theta$. If the principal is allowed to write the compensation on subsets of $\{y^1, ..., y^{|\Theta|}\}$ without revealing every y^θ to the agent (one can think of different projects contributing to the overall outcome), he faces the same trade off as in the basic model of this paper. Revealing a $\theta \notin \Theta_A$ to the agent allows him to use the realization of y^θ as a signal, which makes the induction of incentives less costly but tightens the participation constraint.

Proof See Appendix A.9.

The optimal contract inducing low effort leaves the agent unaware because there is no incentive effect. The principal induces low effort in equilibrium if

$$E[Y|e^L] - \prod_{\theta \in \Theta \setminus \Theta_A} \Pr[\theta = 0|e^L] \upsilon^{-1}(\bar{U} - e^L) > E[Y|e^H] - \prod_{\theta \in \Theta \setminus \widehat{\Theta}^*} \Pr[\theta = 0|e^L] E[C^C_{\widehat{\Theta}^*}(Y)|e^H].$$

Whether this is the case or not depends on the distributional properties of the random variable Y, but we can easily find examples where e^H is the optimal action choice under full awareness and e^L is the optimal action choice under asymmetric awareness.

7.5 The Agent as the Residual Claimant

The basic model assumes that the principal is the residual claimant. In the presence of asymmetric awareness ownership of the project matters, because the agent's valuation of the project depends on his level of awareness. Thus, if the agent is the residual claimant, unawareness generally affects his perceived outside option $\sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H)\upsilon(y)$. The principal solves:

$$\max_{\widehat{\Theta}, C(\cdot)} \sum_{y \in \mathcal{Y}} \pi(y|e^H)[P - C(y)]$$

subject to

$$\sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H)\upsilon\left(y + C(y) - P\right) \geq \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H)\upsilon(y)$$

$$e^H \in \arg\max_e \left\{ \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e)\upsilon(y + C(y) - P) - e \right\}$$

$$C(y) \geq 0, \qquad \forall y \in \mathcal{Y},$$

where P is the premium paid by the agent and $C(y)$ is the outcome contingent transfer.[16] As in in the basic model, revealing a contingency $\theta \notin \Theta_A$ involves a trade off between participation and incentives. In addition, enlarging the agent's awareness affects his perceived outside option. This effect is favorable to the principal if

$$\sum_{y \in \tilde{\mathcal{Y}}} \tilde{\pi}(y|e^H)\upsilon(y) < \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H)\upsilon(y),$$

[16]Due to the limited liability constraint the principal would like to scale up both P and C. In order to have a solution, one has to assume that such a contract elicits suspicion on the side of the agent.

where $\tilde{\Theta} = \widehat{\Theta} \cup \theta, \theta \in \Theta \backslash \widehat{\Theta}$. This is the case if $E[Y|\theta = 1] < E[Y|\theta = 0]$, i.e. if $\theta = 1$ is a negative outcome shock. Consequently, if the agent is the residual claimant, not only do the distributional properties of $\theta \notin \Theta_A$ matter but also the outcome its announcement reveals. Roughly speaking, when the agent is the residual claimant the principal includes contingencies in the contract that are very unlikely, highly informative and that reveal "bad" outcomes.

8 Conclusion

This paper incorporates asymmetric awareness in the classical principal-agent model. It shows that the principal makes the agent strategically aware and that the optimal contract can be incomplete. Enlarging the agent's awareness involves a trade off between participation and incentives. The cost of disclosing contingencies to the agent is the payment in the states that the agent is initially unaware of. The gain of announcing is the richer information structure that is used to induce incentives. Hence, it is profitable to announce contingencies that have a low probability but are highly correlated with the effort of the agent. Under relatively mild assumptions the optimal contract is justifiable for the agent, which means that the optimal contract maximizes the principal's expected profit evaluated at the beliefs of the agent.

If we allow for competition among principals, in the unique symmetric Nash equilibrium the agent is fully aware and the principals make zero profits. If the contract is required to be justifiable for the agent, this equilibrium is no longer necessarily unique but when the number of principals is large enough uniqueness is restored.

In the proposed model, the principal is able to implement zero payments whenever there is an event the agent is initially unaware of. This may not be feasible in real life contracting situations. Restricting the set of feasible contracts allows us to generate results that are closer to observed contracts, but the basic trade off prevails. If, for example, the compensation scheme is restricted to be monotone in the outcome, there are three effects driving the optimal revelation strategy: In addition to the participation and incentive effect we saw in the basic model also the revealed outcomes matter, where it is most costly to disclose low outcomes to the contracting partner. Restricting the set of feasible contracts adds interesting features to the optimal compensation scheme and revelation strategy, but as long as the contracting partner with superior awareness is able to profit from the other's limited understanding of the underlying uncertainties the basic trade off prevails.

A Appendix

A.1 Proof of Proposition 4.1

Suppose $\Theta_A \subset \widehat{\Theta}^*$. The optimal compensation scheme is $\widehat{C}^*(y) = \upsilon^{-1}(\bar{U} + e^H)$ for all $y \in \widehat{\mathcal{Y}}$ and $\widehat{C}^*(y) = 0$ for all $y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$. Then the expected payment is

$$\prod_{\theta \in \Theta \backslash \widehat{\Theta}^*} \Pr[\theta = 0|e^H]\upsilon^{-1}(\bar{U} + e^H),$$

which is clearly greater than $\prod_{\theta \in \Theta \backslash \Theta_A} \Pr[\theta = 0|e^H]\upsilon^{-1}(\bar{U} + e^H)$, the expected payment under $\widehat{\Theta}^* = \Theta_A$, due to the assumption $\pi(y|e) > 0, \forall y \in \mathcal{Y}$. Hence $\widehat{\Theta}^*$ cannot be optimal. ∎

A.2 Proof of Lemma 5.1

Suppose $\lambda = 0$. Since $\sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H) = \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^L) = 1$ and $\widehat{\pi}(.|e^H) \neq \widehat{\pi}(.|e^L)$ there must exist some $y \in \widehat{\mathcal{Y}}$ such that $\widehat{\pi}(y|e^H) - \widehat{\pi}(y|e^L) < 0$. But since $\gamma \geq 0$, $\lambda = 0$ would imply that $\frac{1}{\upsilon'(C(y))} \leq 0$ for some $y \in \widehat{\mathcal{Y}}$, which violates the assumption $\upsilon'(\cdot) > 0$. Hence $\lambda > 0$.
Now suppose $\gamma = 0$. Then the first order conditions of the optimization problem imply that compensation is fixed across outcomes within the agent's awareness. But this implies, that the incentive constraint is no longer satisfied. Hence, $\gamma > 0$. ∎

A.3 Proof of Lemma 5.2

Let $\tilde{y} : \widehat{\Theta} \cup Z \longrightarrow \mathbb{R}$ denote the output function, let $\tilde{\mathcal{Y}}$ denote the range of y and let $\tilde{\pi}(.|e)$ denote the probability belief given awareness $\widehat{\Theta} \cup Z$. Further, let $\rho : \tilde{\mathcal{Y}} \longrightarrow \widehat{\mathcal{Y}}$ be the mapping from set $\tilde{\mathcal{Y}}$ to set $\widehat{\mathcal{Y}}$, where $\rho(\tilde{y}(\widehat{\Theta} \cup Z)) = \widehat{y}(\widehat{\Theta})$. Now consider the compensation scheme $\tilde{C}$ with $\tilde{C}(y) = C^C_{\widehat{\Theta}}(\rho(y))$ for all $y \in \tilde{\mathcal{Y}}$. Note that $\tilde{C}$ satisfies both participation and incentive constraint with equality.

Now suppose $\Pr[\theta = 1|e^H] = \Pr[\theta = 1|e^L], \forall \theta \in Z$. Then for any $y \in \tilde{\mathcal{Y}}$ we have

$$\frac{\tilde{\pi}(y|e^L)}{\tilde{\pi}(y|e^H)} = \frac{\widehat{\pi}(y|e^L)}{\widehat{\pi}(y|e^H)}.$$

Hence $C^C_{\widehat{\Theta}}$ satisfies the first order conditions and consequently solves the optimization problem. $\Delta C^Z_{\widehat{\Theta}} = 0$.

Now suppose $\Pr[\theta = 1|e^H] \neq \Pr[\theta = 1|e^L]$ for some $\theta \in Z$. Then there must exist some $y, y' \in \rho^{-1}(y), y \in \widehat{\mathcal{Y}}$ such that

$$\frac{\tilde{\pi}(y|e^L)}{\tilde{\pi}(y|e^H)} \neq \frac{\tilde{\pi}(y'|e^L)}{\tilde{\pi}(y'|e^H)}.$$

Consequently, $C^C_{\widehat{\Theta}}$ does not satisfy the first order conditions. Hence, $C^C_{\widehat{\Theta}}$ is feasible but not optimal, which implies that $E[C^C_{\widehat{\Theta}\cup Z}(Y|e^H)] < E[C^C_{\widehat{\Theta}}(Y|e^H)]$ and $\Delta C^Z_{\widehat{\Theta}} > 0$. ∎

A.4 Proof of Lemma 5.3

W.l.o.g. we can assume $\Pr[\theta = 1|e^H] > \Pr[\theta = 1|e^L], \theta \in \widehat{\Theta}$. $C^C_{\widehat{\Theta}}$ is the compensation scheme that solves the principal's optimization problem given full awareness and information structure $\widehat{\Theta}$. Let $E_{\widehat{\Theta}}$ denote the expectation operator with respect to awareness $\widehat{\Theta}$. Then we have:

$$E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e, \theta = 1\right] > E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e, \theta = 0\right], \quad e = e^L, e^H,$$

which follows directly from the first-order conditions. The incentive constraint can be rewritten as

$$\begin{aligned} & \Pr[\theta = 1|e^H]E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e^H, \theta = 1\right] + \Pr[\theta = 0|e^H]E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e^H, \theta = 0\right] - e^H \\ = \quad & \Pr[\theta = 1|e^L]E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e^L, \theta = 1\right] + \Pr[\theta = 0|e^L]E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e^L, \theta = 0\right] - e^L. \end{aligned}$$

Now consider probability $\Pr[\theta = 1|e^L] - \varepsilon$ with $\varepsilon > 0$. Under $C^C_{\widehat{\Theta}}$ and $\Pr[\theta = 1|e^L] - \varepsilon$ the participation constraint is clearly satisfied with equality. Looking at the incentive constraint it is easy to see that

$$\begin{aligned} & \Pr[\theta = 1|e^H]E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e^H, \theta = 1\right] + \Pr[\theta = 0|e^H]E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e^H, \theta = 0\right] - e^H \\ > \quad & \Pr[\theta = 1|e^L]E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e^L, \theta = 1\right] + \Pr[\theta = 0|e^L]E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e^L, \theta = 0\right] - e^L \\ & -\varepsilon\left(E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e^L, \theta = 1\right] - E_{\widehat{\Theta}}\left[\upsilon(C^C_{\widehat{\Theta}}(Y))|e^L, \theta = 0\right]\right). \end{aligned}$$

We know that under the optimal transfer rule both constraints are satisfied with equality. Consequently, given $\Pr[\theta = 1|e^L] - \varepsilon$, $C^C_{\widehat{\Theta}}$ is feasible but not optimal, so we have $V_{\widehat{\Theta}}(\Pr[\theta = 1|e^L]) > V_{\widehat{\Theta}}(\Pr[\theta = 1|e^L] - \varepsilon)$. The same line of reasoning applies to $\Pr[\theta = 1|e^H]$. In that case both constraints are slack. Hence, the $V_{\widehat{\Theta}}$ is decreasing in Δ^θ for all $\theta \in \widehat{\Theta}$. ∎

A.5 Proof of Lemma 5.5

$$NG(\theta_n) - NG(\theta_{n+1}) = \alpha^{N-n-1}\left(\alpha^2 E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H] - 2\alpha E[C^C_{\widehat{\Theta}_n}(Y)|e^H] + E[C^C_{\widehat{\Theta}_{n+1}}(Y)|e^H]\right)$$

This function is continuous in α. Moreover, as α goes to zero $NG(\theta_n) - NG(\theta_{n+1})$ is strictly positive. Now if $NG(\theta_n) - NG(\theta_{n+1}) < 0$ for some α, there has to be an α such that $NG(\theta_n) - NG(\theta_{n+1}) = 0$

by the *Intermediate Value Theorem.* Solving for α we have

$$\alpha_{1/2} = \frac{2E[C^C_{\widehat{\Theta}_n}(Y)|e^H] \pm \sqrt{4E[C^C_{\widehat{\Theta}_n}(Y)|e^H]^2 - 4E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H]E[C^C_{\widehat{\Theta}_{n+1}}(Y)|e^H]}}{2E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H]}.$$

If the discriminant $4(E[C^C_{\widehat{\Theta}_n}(Y)|e^H]^2 - E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H]E[C^C_{\widehat{\Theta}_{n+1}}(Y)|e^H]) < 0$ there are no real roots. Hence, whenever $E[C^C_{\widehat{\Theta}_n}(Y)|e^H]^2 < E[C^C_{\widehat{\Theta}_{n-1}}(Y)|e^H]E[C^C_{\widehat{\Theta}_{n+1}}(Y)|e^H]$, $NG(\theta_n) - NG(\theta_{n+1}) > 0$ for all $\alpha \in (0, 1)$. ∎

A.6 Proof of Proposition 5.6

It is clear that whenever $EU^A_P(\widehat{\Theta}^*, \widehat{C}^*, e^H) < 0$, the agent thinks that the principal would be strictly better off by not offering the contract. Similarly if $EU^A_P(\widehat{\Theta}^*, \widehat{C}^*, e^H) \geq EU^A_P(\widehat{\Theta}^*, \widehat{C}^*, e^L)$ the agent cannot rationalize why the principal proposes an incentive compatible contract. Hence, $(\widehat{\Theta}^*, \widehat{C}^*)$ cannot be justifiable.

Now suppose $EU^A_P(\widehat{\Theta}^*, \widehat{C}^*, e^H) \geq 0$ and $EU^A_P(\widehat{\Theta}^*, \widehat{C}^*, e^H) \geq EU^A_P(\widehat{\Theta}^*, \widehat{C}^*, e^L)$.

Justifiability of $\widehat{C}^*$*:* If $\widehat{\Theta}^* = \Theta$, principal and agent share the same belief. Hence, $\widehat{C}^*$ maximizes the principal's expected payoff from the agent's perspective. If $\widehat{\Theta}^* \neq \Theta$, $\widehat{C}^*(y) = C^C_{\widehat{\Theta}^*}(y)$ for all $y \in \widehat{\mathcal{Y}}$. Since the agent thinks that the contract is complete, $\widehat{C}^*$ maximizes the principal's payoff according to the beliefs of the agent.

Justifiability of $\widehat{\Theta}^*$*:* $\widehat{\Theta}^*$ is optimal for the principal given the agent's beliefs if

$$E\left[C^C_{\widehat{\Theta}^*}(Y|e^H)\right] \leq \prod_{\theta \in \widehat{\Theta}^* \setminus Z} \Pr[\theta = 0|e^H] E\left[C^C_{\widehat{\Theta}^* \setminus Z}(Y)|e^H\right],$$

for any $Z \subseteq \widehat{\Theta}^* \setminus \Theta_A$. This coincides with the optimality condition of the principal. Hence, whenever $EU^A_P(\widehat{\Theta}^*, \widehat{C}^*, e^H) \geq 0$ and $EU^A_P(\widehat{\Theta}^*, \widehat{C}^*, e^H) \geq EU^A_P(\widehat{\Theta}^*, \widehat{C}^*, e^L)$, $(\widehat{\Theta}^*, \widehat{C}^*)$ is justifiable. ∎

A.7 Proof of Proposition 6.1

Existence: Consider a deviation of principal i. The strategies are $\widehat{\Theta}_j = \Theta$ and $C_j = C^*$ for all $j = 1, ..., i-1, i+1, ..., N$. A deviation in $\widehat{\Theta}_i$ clearly leaves the expected payoff unaffected because the agent is aware of $\widehat{\Theta}_1 \cup ... \cup \widehat{\Theta}_N = \Theta$ for all $\widehat{\Theta}_i \subseteq \Theta$. A deviation in C_i is not profitable for the standard Bertrand argument. C^* maximizes the agent's expected utility given the zero outside option of the principal. A deviation $C_i \neq C^*$ must make either the agent or the principal worse off. If the agent is worse off, he rejects the contract and the expected payoff is zero. If the principal is worse off, he has a negative expected payoff. Hence, deviating in $(\widehat{\Theta}_i, C_i)$ is not profitable.

Uniqueness: It is easy to see that this is the only symmetric Nash equilibrium. If $\widehat{\Theta}_i = \Theta, i = 1, ..., N$ principal's engage in a standard Bertrand competition. The unique symmetric equilibrium is $C_i = C^*$ for all $i = 1, ..., N$. Now suppose $\widehat{\Theta}_i = \tilde{\Theta} \subset \Theta, i = 1, ..., N$. Given $\tilde{\Theta}$ principals engage in a Bertrand competition over the compensation rule. But as profits become small enough it is profitable to announce a contingency in $\Theta \backslash \tilde{\Theta}$. This deviation allows principal i to capture the whole market, because the agent realizes that he receives zero in some states if he accepts one of the other contracts. Hence, $\tilde{\Theta}$ cannot be an equilibrium announcement and the unique symmetric Nash equilibrium is $(\widehat{\Theta}_i, C_i) = (\Theta, C^*), i = 1, ..., N$. ∎

A.8 Proof of Proposition 6.2

Suppose $\widehat{\Theta}_i = \widehat{\Theta} \subset \Theta, i = 1, ..., N$. Let $\bar{C}$ denote the solution to the problem

$$\max_C \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^H) \upsilon \left(C\left(y\right)\right) - e^H$$

subject to

$$\sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e) \left[y - C(y)\right] \geq 0$$

$$e^H \in \arg\max_e \left\{ \sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e) \upsilon \left(C\left(y\right)\right) - e \right\},$$

and let $\widehat{C}(y) = \bar{C}(y), \forall y \in \widehat{\mathcal{Y}}$ and $\widehat{C}(y) = 0, \forall y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$. Then principal i's expected profit is $EU_i(\widehat{\Theta}, \widehat{C}) = \frac{1}{N} \sum_{y \in \mathcal{Y}} \pi(y|e) \left[y - \widehat{C}(y)\right]$. Since $\widehat{C}$ is independent of N, the expected profit is decreasing in N. If i announces $\theta \in \Theta \backslash \widehat{\Theta}$, he offers a compensation scheme $\tilde{C}$ which solves the following optimization problem

$$\max_C \sum_{y \in \mathcal{Y}} \pi(y|e) \left[y - C(y)\right]$$

subject to

$$\sum_{y \in \tilde{\mathcal{Y}}} \tilde{\pi}(y|e^H) \upsilon \left(C(y)\right) \geq \sum_{y \in \tilde{\mathcal{Y}}} \tilde{\pi}(y|e^H) \upsilon \left(\widehat{C}(y)\right)$$

$$e^H \in \arg\max_e \left\{ \sum_{y \in \tilde{\mathcal{Y}}} \tilde{\pi}(y|e) \upsilon \left(C\left(y\right)\right) - e \right\},$$

where $\tilde{y}$ is the agent's perceived outcome function with range $\tilde{\mathcal{Y}}$ and probability belief $\tilde{\pi}(.|e)$ associated to set $\tilde{\Theta} := \widehat{\Theta} \cup \theta$. Principal i's expected profit is $EU_i(\tilde{\Theta}, \tilde{C}) = \sum_{y \in \mathcal{Y}} \pi(y|e) \left[y - \tilde{C}(y)\right]$, which is independent of N. Hence there always exists an $\bar{N}$ such that

$$\frac{1}{N} \sum_{y \in \mathcal{Y}} \pi(y|e) \left[y - \widehat{C}(y)\right] < \sum_{y \in \mathcal{Y}} \pi(y|e) \left[y - \tilde{C}(y)\right]$$

for all $N \geq \bar{N}$. ∎

A.9 Proof of Proposition 7.1

When e^L is optimal, the principal solves

$$\max_{\widehat{\Theta}, C(\cdot)} \quad \sum_{y \in \mathcal{Y}} \pi(y|e^L) \left[y - C(y)\right]$$

subject to

$$\sum_{y \in \widehat{\mathcal{Y}}} \widehat{\pi}(y|e^L) v\left(C\left(y\right)\right) - e^L \geq \overline{U}$$

$$C(y) \geq 0, \quad \forall y \in \mathcal{Y}.$$

The optimal compensation scheme for a given $\widehat{\Theta}$ is $C^L(y) = v^{-1}(\bar{U} - e^L), \forall y \in \widehat{\mathcal{Y}}$ and $C^L(y) = 0, \forall y \in \mathcal{Y} \backslash \widehat{\mathcal{Y}}$. The expected payment is

$$E[C^L(Y)] = \prod_{\theta \in \Theta \backslash \widehat{\Theta}} \Pr[\theta = 0|e^L] v^{-1}(\bar{U} - e^L),$$

which is clearly minimized for $\widehat{\Theta} = \Theta_A$. ∎

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