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AND THE DISTRIBUTIVE EFFECTS OF THE U.S.
GREAT RECESSION

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Financial Intermediation, House Prices, and the Distributive Effects of the U.S. Great Recession*

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Abstract

This paper quantifies the effects of credit spread and income shocks on aggregate house prices and households' welfare. We address this issue within a stochastic dynamic general equilibrium model with heterogeneous households and occasionally binding collateral constraints. Credit spread shocks arise as innovations to the financial intermediation technology of stylized banks. We calibrate the model to the U.S. economy and simulate the Great Recession as a contemporaneous negative shock to financial intermediation and aggregate income. We find that (i) in the Great recession constrained agents (borrowers) lose more than unconstrained agents (savers) from the aggregate house prices drop; (ii) credit spread shocks have, by their nature, re-distributive effects and - when coupled with a negative income shock as in the Great Recession - give rise to larger (smaller) welfare losses for borrowers (savers); (iii) imposing an always binding collateral constraint, the non-linearity coming from the combination of the two shocks vanishes, and the re-distributive effects between agents' types are smaller.

Keywords: Housing Wealth, Mortgage Debt, Borrowing Constraints, Heterogeneous Agents, Welfare, Aggregate Credit Risk

JEL Classification Numbers: E21, E32, E43, E44, I31.

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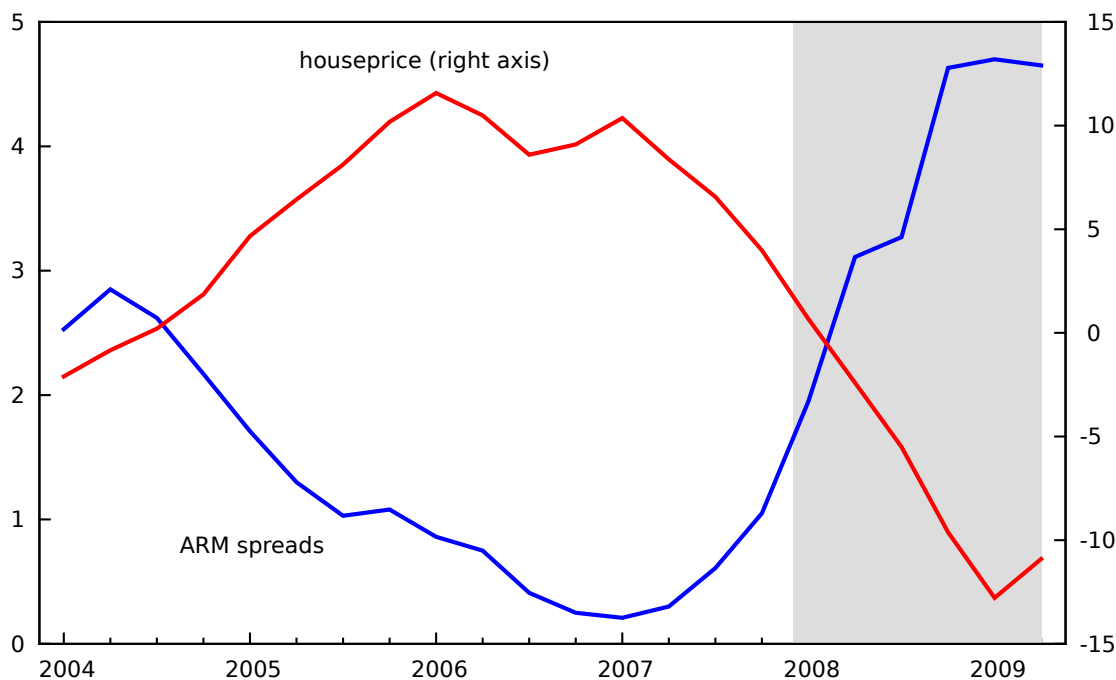
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1 Introduction

The U.S. Great Recession has been linked to turbulences within the financial markets and, in particular, the banking system. This widely accepted fact triggered a debate among economists and policy-makers about the welfare consequences of the financial innovation process that preceded the crisis and that possibly fuelled the economic collapse. The debate also highlights that the crisis deeply involved all U.S. households, especially through the collapse of the housing market with an unprecedented drop in aggregate house prices of 23.5% between 2007:IV and 2009:II (the NBER recession dates). These concerns are summarized by the strong negative co-movement of mortgage spreads and aggregate house prices at the onset and during the recent recession, as shown in figure 1. The blue line shows the spread between the one-year amortizing

Figure 1: Spreads vs house prices



Notes: Shaded areas are NBER recession dates. Spreads of ARM over Fed Funds rate are shown in levels (percent p.a.). House prices is the cyclical component of the Case-Shiller aggregate houseprice index using a HP-filter with smoothing parameter 1600. For a detailed data description see appendix A.

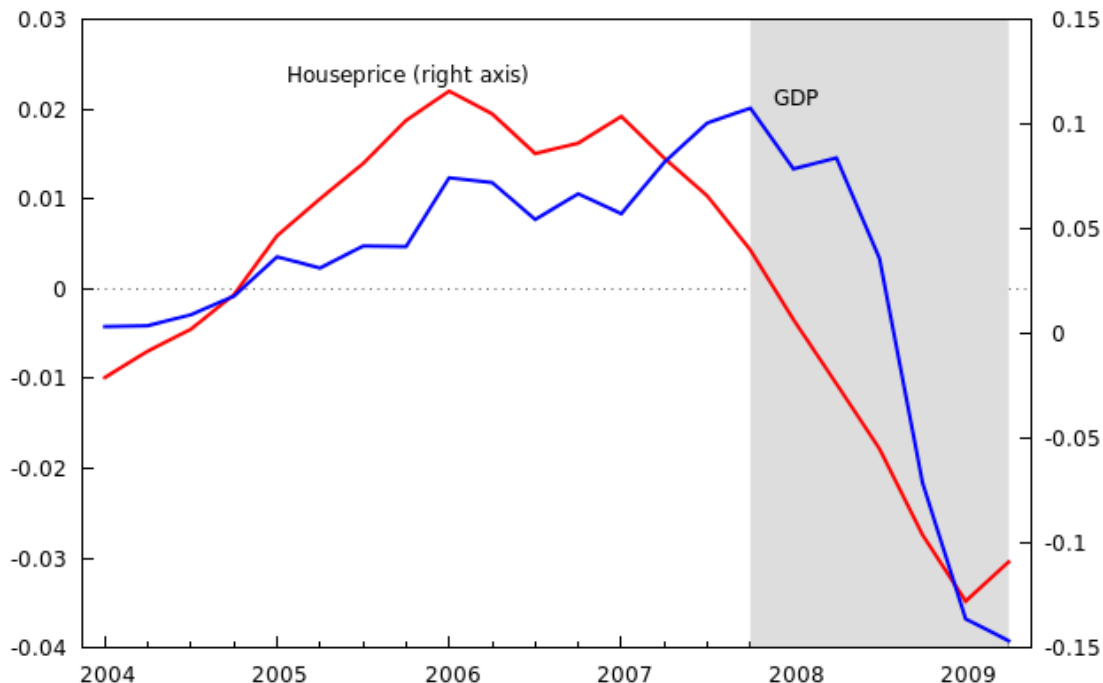
adjustable mortgage rate (ARM) and the federal funds rate from 2004:I to 2009:II. The red line is the de-trended series of aggregate house prices (for a detailed data description see appendix A). During the ten quarters preceding the crisis, spreads were low (between 0% and 1%) and house prices were about 10% above trend. In late 2007, spreads jumped to 4-5% when house

prices collapsed by about 20%.

While existence and direction of the causality between these two series is part of the debate itself, in this paper we examine the effects of exogenous changes in credit spreads on endogenous aggregate house prices. In this respect, we share the view that fluctuations in spreads largely reflect disturbances in the financial markets' assessment of credit risk (Bordo, 2008). Accordingly, Gilchrist and Zakrajšek (2012) finds that variation in the price of default risk rather than variation in the risk of default is the main predictive factor for corporate bond spreads. Furthermore, we share the view of Adrian and Shin (2010) that variations in the price of default risk reflects variations in the effective risk-bearing capacity of the financial sector because of aggregate portfolio losses.

Most importantly, beside the disruptions in the financial markets, the US Great Recession has been characterized by a large drop in GDP (5.4% between the NBER recession dates). Figure 2 shows the de-trended quarterly series of US GDP and aggregate house prices. We observe a strong positive co-movement of aggregate income and house prices before and during the Great Recession.

Figure 2: GDP vs House prices



Notes: Shaded areas are NBER recession dates. The blue line is the cyclical component of the Case-Shiller aggregate house price index; the red line is real GDP. Both series are logged and HP-filtered with Parameter 1600. For a detailed data description see appendix A.

The stylized facts highlighted in figures 1 and 2 motivate our interest in quantifying the impact of financial and income shocks on aggregate house prices. Given that housing represents a major share in the average US household portfolio (Iacoviello, 2011b), we quantify the welfare consequences of the Great Recession. In particular, we study the distributive effects between borrowers and savers in a stochastic dynamic general equilibrium model with endogenous collateral constraints. In our economy households differ in their level of patience; this modelling choice is not new to the recent quantitative literature on the Great Recession (Iacoviello (2011a) and Justiniano, Primiceri, and Tambalotti (2013)) and allows us to quantify the welfare effects of changes in household indebtedness, house prices and leverage. In our model, agents are fully rational and derive utility from both the consumption of perishable goods and housing services coming from housing stock. Housing is the only physical asset in the economy and it is fixed in supply. Agents can borrow using one-period financial securities but have to collateralize short positions by a fraction of the expected value of their available housing stock. In this otherwise standard model, we introduce a financial intermediation sector; debt and savings need to be intermediated by competitive financial intermediaries and their intermediation technology is subject to exogenous shocks. These shocks give rise to a spread between the borrowing and lending.¹ A second source of aggregate disturbance comes from standard aggregate income shocks that directly affect the households' endowment of the perishable good; this is a reduced form way to capture the cyclical behavior of productivity shocks.

We calibrate the model to the US economy and simulate the Great Recession as a contemporaneous negative income and financial shock that follows a period of economic expansion; in fact, we rely on the empirical observation that both income and financial intermediation were above (below) the long run trend before (after) the recession. To calibrate our key parameters we consider moments from both micro and macro data. In particular, from the Survey of Consumer Finances (waves 1998 - 2007), we are able to match the leverage and the wealth share of borrowers relative to savers. This calibration strategy, although different from previous literature that targets macro moments only, results in calibrated parameters that are compatible with recent contributions (Iacoviello and Guerrieri (2012)).

¹We consider a simple modelling choice for the financial intermediation in the spirit of Cooper and Ejarque (2000) and Cúrdia and Woodford (2010). Otherwise, the link to these studies is only limited as the former looks at the business cycle properties of financial shocks within a representative agent framework, while the latter studies the implications of spread shocks for the optimal conduct of monetary policy.

A very delicate issue for the calibration exercise is what time frame to use, that is, whether to incorporate a recession or not. We take the following stance. Our main goal is to keep coherence between the modelling choice and the research question. We want to study the Great Recession as a state-contingent exogenous event, that hit the US economy in late 2007, following a period characterized by banking innovation and increasing aggregate leverage.² Therefore, we consider the Great Recession as a low probabilistic event embedded in a business cycle framework. For this reason, to calibrate the model, we include the quarters of the recession until 2009:II.³ The structural nature of our exercise allows us to conduct counter-factual experiments in order to disentangle the quantitative effects of income and intermediation shocks on aggregate house prices and agents' welfare.

We have three major findings. First, we find that our benchmark model quantitatively explains about 60% of the observed drop of house prices during the Great Recession. The major share is attributed to real income shocks, with financial intermediation shocks explaining about 5% of the explained drop. This finding confirms that the observed behavior of aggregate house prices, before and after the Recession, could be partially related to changes in fully expected shocks; more importantly, we find that, as opposite to the widespread view, financial related shocks have very limited quantitative effects on aggregate house prices.

Second, we find a significant difference in the distributive effects of income and financial shocks. In the Great Recession, the negative income shock is quantitatively the main driver behind the absolute drop in house prices and the absolute level of agents' welfare losses; a negative financial intermediation shock acts thoroughly distributive, with savers gaining at the expense of borrowers. The quantitative size of this effect non-linearly depends on the borrowers' leverage just before entering the recession. The higher the borrowers' leverage the more sensitive is the wealth distribution to price changes. In addition, changes in the financial intermediation efficiency are the main determinants of changes in leverage; it is also the main driver of the occasionally binding collateral constraint⁴. Therefore, in the Great Recession, when a negative

²Between 2001 and 2007 aggregate mortgage debt expanded by 59%, despite the 19% increase in housing value.

³For the micro data, SCF is run every three years. We decided to not include the 2010 wave of the survey in the analysis in order to be coherent with respect to other calibrated parameters in the model; however, even when including the 2010 wave, the targeted values are very similar.

⁴This mechanism is in line with the microeconomic evidence of Mian and Sufi (2010) who found that an increase in credit supply, coupled with the effect of collateralized debt on increasing house prices, created an unprecedented increase in household leverage in the quarters preceding the crisis.

intermediation shock coupled with a negative income shock hits the economy, the borrowers' (savers') welfare loss is larger (smaller) than the sum of the implied welfare losses of each shock in isolation. This non-linearity depends on the interaction between the dynamics of the aggregate wealth, the change in households' leverage and the binding/not binding state of the collateral constraint.

Third, we find that if we restrict the collateral constraint to be always binding, this non-linearity is virtually shut down and the distributive effects from borrowers to savers vanish. This is an important finding as previous studies (notably, Iacoviello (2005)) usually assume that the constraints are always binding. The intuition for this result is that when the growth rate of the borrowers' debt is forced to be proportional to the changes in expected housing wealth, borrowers leverage up slower in expansions and de-leverage slower in contractions when compared to our benchmark model. This implies that when the crisis hits, borrowers have more outstanding debt in the benchmark model that they need to roll-over. Very recently, Iacoviello and Guerrieri (2012) are exploring the quantitative properties of occasionally binding collateral constraints and the relative non-linear effects deriving from changes in demand of housing; with respect to this contribution, we highlight a mechanism of non-linearity that arises from changes in leverage and aggregate debt driven by financial intermediation shocks.

The mechanism behind the three findings is the following. Negative realization of one of the exogenous shocks (or both) lead to credit contractions. In a credit contraction - given that it is more costly to roll-over the existing debt - borrowers choose optimally to reduce indebtedness. If the reduction in debt is sufficiently large, borrowers need to reduce their housing stock; for a given supply then, house prices decrease. This makes borrowers suffer in terms of both wealth and expected life-time utility. On the other hand - because of the lower demand for debt - savers potentially face a lower interest rate on savings; this potentially hurts them because the consumption tomorrow is relatively more expensive. However, they can undo this effect by buying houses to smooth consumption when housing is relatively cheap (and savers expect the house price to rise again tomorrow). So, the savers gain in terms of wealth and do not suffer much in terms of expected life-time utility. The size of this distributive effect depends crucially on how the interest rates move. In this paper we quantitatively show what exactly distinguishes financial shocks from income shocks. Another important remark concerns the non-linearity generated by

the collateral constraint. In states of the world where borrowers choose optimally to move away from the constraint it becomes slack; that is, the borrowers are not restricted at which pace to reduce their debt as it is the case in models with an always binding constraint. This implies a change in the elasticity of the demand for debt and housing with respect to changes in house prices that have non negligible quantitative effects.

The present study is related to two important strands of literature. First, we relate to the recent literature that studies the financial sector as an important source of macroeconomic fluctuations (Quadrini and Urban, 2012) and that financial frictions played a pre-eminent role in explaining the observed drop in US aggregate economic activity (Hall, 2011). Recently, Guerrieri and Lorenzoni (2011) find that a shock to the spread between the interest rate on borrowings and the interest rate on savings - in presence of a collateral constraint that links debt to the level of durables - generates a decrease in borrowers' demand for durables that increases as agents are close to the credit constraint. While their analysis abstracts from aggregate house prices and endogenous changes in wealth, we explicitly emphasize the channel that goes through the endogenous change in house prices.

Second, our analysis relates to recent studies on the distributive effects of the Great Recession. Compared to Glover, Heathcote, Krueger, and Ríos-Rull (2011) - a study on the intergenerational re-distribution during the Great Recession - we focus on a different dimension of agent heterogeneity and welfare, namely on the distribution between constrained agents (borrowers) and unconstrained agents (savers). Similar to Hur (2012), we find that the constrained agents always lose more than unconstrained agents.⁵ Both aforementioned studies are silent about the inherent distributive nature of financial shocks, that is the focus of the present paper.⁶

The rest of the paper is structured as follows: In the following section we present the model. Section 3 presents the quantitative analysis. In section 4 we compare the predictions of the benchmark model to alternative specifications including the case of an always binding constraint. Section 5 concludes.

⁵Hur (2012) considers an overlapping generations model with collateral constraints; he finds the constrained agents are mostly from the young cohort, and those agent suffer the most from the a recession.

⁶Another distinguishing element of our analysis to Hur (2012) and Guerrieri and Lorenzoni (2011), is that they consider the recession as an unanticipated event while in our economy agents take into account the probability of negative aggregate shocks when making decisions about the future.

2 Model

2.1 The physical economy

Uncertainty. Time is discrete and denoted by $t = 0, 1, \dots$. In each period t , the world experiences one of Z possible exogenous events $z \in \mathcal{Z} = \{1, \dots, Z\}$. The resolution of uncertainty is represented by an event tree Σ with root σ_0 that is given by a fixed event z_0 in which the economy starts at time 0. Each node is characterized by a history of events, denoted by $\sigma^t = (\sigma_0, \dots, \sigma_t) \in \Sigma^t = \times_{k=0}^t \Sigma_k$. Each node has Z immediate successors ($\sigma_t z^+$) and a unique predecessor (σ_t^-). The exogenous events follow a Markov process with transition matrix Π .

Agents and Endowments At each node σ_t there are two types of agents, borrowers (denoted by a subscript b) and savers (denoted by a subscript s). Borrowers and savers differ in their rates of time preference, in the sense that borrowers discount future more than savers. Formally, we have $\beta_s > \beta_b$, where $\beta_i \in (0, 1)$ for $i = s, b$. Each group consists of infinitely many agents but the group size differs: denote by n_b and n_s the relative size of borrowers and savers, respectively. Note that we choose the normalization $n_b + n_s = 1$.

At each node σ_t , there is a perishable consumption good (henceforth: non-durable consumption good). The total endowment of the perishable good is stochastic and depends on the realization of the shock alone, that is $y(\sigma_t^-) = y(z)$, where $y : \mathcal{Z} \rightarrow R_{++}$ is a time-invariant function. Given the assumption on the relative population size, at each node, borrowers receive $y_b(z) = n_b \cdot y(z)$ and savers receive $y_s(z) = n_s \cdot y(z)$. In addition to the non-durable consumption good, agents trade houses. Houses are the only physical asset in the economy. At period 0, agent $i = b, s$ owns a stock $h_i(\sigma_0^-) \geq 0$ of houses and we normalize $\sum_{i=b,s} h_i(\sigma_0^-) = 1$.

At node σ_t let $h_i(\sigma_t)$ agent i 's end-of-period stock of houses. We assume that houses are traded cum services. That is, buying a house allows the agent to enjoy the housing services in the same period: if agent i owns $h_i(\sigma_t)$ houses then he receives a service stream of $1 \cdot h_i(\sigma_t)$. Other than the service stream, houses do not yield any dividend payments.⁷

Markets. At each node, spot markets open and agents trade the perishable consumption good. We choose the perishable good as the numeraire and - without loss of generality - normalize its

⁷These assumptions are for simplicity. We could allow the service stream of houses to depend on the realization of the shock z or on the identity of the agent.

price to be equal to 1. Agents can trade housing in every period; that is, agents $i = s, b$ can buy a unit of housing at node σ_t at price $q(\sigma_t)$. As long as $h_i \geq 0$, there is no possibility of default since no promises are made when agents hold a positive amount of the physical asset. In addition to houses, there are two financial assets, debt and savings, both one-period securities. We denote agent i 's end-of-period debt holdings by $d_i(\sigma_t)$ and end-of-period savings by $s_i(\sigma_t)$, respectively. Denote the prices of the respective securities by $p_j(\sigma_t)$ for $j = d, s$. For reasons that will be clear below, we distinguish these two assets because their effective returns differ. Debt is assumed to be a security for which only negative (short) positions are allowed, that is, $d_i(\sigma_t) \leq 0$; in savings, agents can only take positive (long) positions, or $s_i(\sigma_t) \geq 0$, for $i = b, s$ and all σ_t . Asset $j = d, s$ traded at σ_t promises a nominal pay-off $b_j(\sigma_t z)$ at any successor node $\sigma_t z$. We normalize $b_j(\sigma_t z) = 1$ for all $\sigma_t, \sigma_t z$. For the remainder of the paper, we will introduce the notation in terms of real interest rates: denote by $R_D(\sigma_t) = \frac{1}{p_d(\sigma_t)}$ the real interest rate on debt and $R(\sigma_t) = \frac{1}{p_s(\sigma_t)}$ the real interest rate on savings. We also restrict borrowers to hold zero savings and savers to hold zero debt (this motivates also the labels of the two agents); formally, for all nodes σ_t , we have $d_b(\sigma_t) \leq 0$, $s_b(\sigma_t) = 0$, $d_s(\sigma_t) = 0$, and $s_s(\sigma_t) \geq 0$.⁸

Collateral Requirements and Default. Similar to Kiyotaki and Moore (1997) we assume limits on debt obligations. We thereby model one key feature that distinguishes houses from other assets: houses are widely used as collateral for debt obligations (mortgages). As in Iacoviello (2005), the theoretical justification for collateral constraints is the ability of borrowers to default on their debt promises. If the borrowers default in some successor node $\sigma_t z^+$, lenders can seize the borrowers' assets, $q(\sigma_t z^+)h_b(\sigma_t)$ by paying a proportional transaction cost of (in expected terms) of $(1 - m)E[q(\sigma_t z^+)|\sigma_t]h_b(\sigma_t)$ that is not redistributed. This transaction cost can be thought of a loss associated with bankruptcy. Lenders will therefore never accept a debt contract where the borrowers' promises exceed the expected collateral value of housing. Formally, in each node σ_t , promises made by the borrower have to satisfy

$$R_D(\sigma_t)d(\sigma_t) + mE[q(\sigma_t z^+)|\sigma_t]h_b(\sigma_t) \geq 0. \quad (1)$$

⁸This is only for the ease of exposition. When computing the equilibrium policy functions, we allow borrowers and savers to trade both assets, debt and savings. Borrowers will only want to take long positions in savings for high relative wealth shares. In the calibrated economy, along the equilibrium path, this never occurs, unless the initial wealth share of the borrowers is very high.

Note that in some successor node $\tilde{z} \in \sigma_t z^+$ it might be still optimal for the borrowers to default ex-post. We assume throughout the analysis, however, that m is small enough, so that there will be never default in equilibrium:

Assumption 1

$$m \leq \frac{\min(q(\sigma_t z^+))}{E[q(\sigma_t z^+)|\sigma_t]} \quad \text{for all } \sigma_t.$$

There is no default in equilibrium if and only if this condition is satisfied.⁹ When solving the model equilibrium numerically, we assume that the condition holds and verify ex post that it is indeed satisfied for all prices along the equilibrium path. This allows us to treat debt as risk free.¹⁰

Utilities and budget constraints Agents $i = s, b$ maximize a time-separable utility function

$$U_i(c_i, h_i) = E_0 \sum_{t=0}^{\infty} \beta_i^t u(c_{s,t}, h_{s,t}) \quad (2)$$

where E_0 is the expectation operator at the the starting date $t = 0$. We consider period-by-period utility functions $u(c, h) : \mathbb{R}_{++} \times [0, 1] \rightarrow \mathbb{R}$ characterized by constant elasticity of substitution

$$u(c, h) = \frac{\Psi(c, h)^{1-\gamma}}{1-\gamma}, \quad \text{and} \quad \Psi(c, h) = [\phi c^\rho + (1-\phi)h^\rho]^{\frac{1}{\rho}}$$

Note that this class of preferences is strictly monotone, continuously differentiable, strictly concave, and satisfies Inada conditions for both c_i and h_i .

⁹Assuming default costs equal to zero, borrowers default in some successor node $\tilde{z} \in \sigma_t z^+$ iff

$$-mE[q(\sigma_t z^+)|\sigma_t]h_b(\sigma_t) + q(\tilde{z})h_b(\sigma_t) < 0,$$

that is, whenever the realized value of housing is smaller than the maximum amount promised. Since in any financial markets equilibrium house prices and - by the Inada conditions - h_b are strictly positive, for m being small enough, this condition does not hold. As an alternative to a condition on m , we could just assume default costs high enough, so that it is never optimal for the borrowers to default.

¹⁰We evaluated the robustness of our results when replacing equation (1) by the following collateral requirement:

$$R_D(\sigma_t)d(\sigma_t) + m \cdot \min(q(\sigma_t z^+)) h_b(\sigma_t) \geq 0.$$

This is a tighter constraint and ensures that there is no default in equilibrium, independent of the value of m . While the qualitative implications remain unaffected, this specification implied slightly smaller quantitative effects on house prices and welfare. The intuition for the smaller quantitative effects is that leverage in states of high intermediation is lower compared to the benchmark model and the wealth distribution is therefore less sensitive to price changes. We stick to the collateral constraint as outlined in the main text because it became standard in macroeconomic models with mortgage debt and it puts our results better in context.

At each node, the savers' budget constraint is given by

$$c_s(\sigma_t) + q(\sigma_t)h_s(\sigma_t) + s_s(\sigma_t) \leq n_s y(\sigma_t) + s_s(\sigma_t^-)R(\sigma_t^-) + q(\sigma_t)h_s(\sigma_t^-) + n_s \Upsilon(s^t). \quad (3)$$

The right hand-side is the savers' available income; it consists of the endowment of the perishable good $n_s y(\sigma_t)$, gross return on savings, and housing stock carried over from the previous period. Finally, $\Upsilon(s^t)$ are resources that are redistributed in a lump-sum fashion from the financial sector to the households, of which savers receive a share n_s representing their share in the population. The reason why we need this re-distribution will be explained in detail below.

Analogously, the borrowers' budget constraint reads as

$$c_b(\sigma_t) + q(\sigma_t)h_b(\sigma_t) + d_b(\sigma_t) \leq n_b y(\sigma_t) + d(\sigma_t^-)R_D(\sigma_t^-) + q(\sigma_t)h_b(\sigma_t^-) + n_b \Upsilon(s^t). \quad (4)$$

The right hand-side is the borrowers' available income; it consists of the endowment of the perishable good $n_b y(\sigma_t)$, the value of housing stock net of the debt burden from the previous period plus resources being redistributed from the financial sector to the households, of which borrowers receive the amount $n_b \Upsilon(s^t)$.

Financial Intermediaries. Intermediaries demand aggregate deposits $S(\sigma_t)$ and supply aggregate debt $D(\sigma_t)$. The real pay-offs for each unit lend are given by the real interest rates, $R_D(\sigma_t)$ and $R(\sigma_t)$, respectively. The collateral constraints and assumption 1 make sure that debt is risk free. The key distortion in the intermediation sector is modelled similar to Cooper and Ejarque (2000).¹¹ We assume that in each node σ_t only a fraction of savings can be transformed into debt. This fraction is stochastic and depends on the realization of the current shock only, that is, $\theta(\sigma_t^- z) = \theta(z)$ and $\theta(z) : \mathcal{Z} \rightarrow (0, 1]$ is a time-invariant function.

This exogenous financial shock represents a reduced form way to model the risk-bearing capacity of the financial sector; in particular changes in the intermediation technology θ potentially reflects changes in the value of equity associated with a risky asset portfolio or changes in monitoring by the bank managers (as a consequence of changes in risk aversion). Consequently, while we remain agnostic about the exact foundation of the θ , we point that the observed variations

¹¹Another example for the inclusion of a supply-sided friction in the banking sector into an international macro model are Kalemli-Ozcan, Papaioannou, and Perri (2012).

in the spread series in the period 2005-2009 mainly reflects changes in the households' price for risk rather than changes in the default risk.¹²

Financial intermediaries are otherwise risk neutral and maximize expected profits on their portfolio, that is,

$$\max_{D(\sigma_t), S(\sigma_t) \geq 0} R_D(\sigma_t)D(\sigma_t) - R(\sigma_t)S(\sigma_t) \quad (5)$$

subject to the constraint

$$D(\sigma_t) \leq \theta(\sigma_t)S(\sigma_t). \quad (6)$$

Because intermediaries act on competitive markets with free entry, equilibrium interest rates are such that intermediaries make zero profits:

$$R_D(\sigma_t)\theta(\sigma_t) - R(\sigma_t) = 0. \quad (7)$$

This last relation implies that in this economy, generically, there is a spread between loan and deposit rates. In particular, we have that the interest rate on debt is always at least as big as the interest rate on savings, or $R_D(\sigma_t) \geq R(\sigma_t)$.

Transfers from the Banking sector to the Household sector. In order to complete the description of the model, we are left to specify the re-distribution function $\Upsilon(s^t)$. The intermediation process as outlined above implies an aggregate intermediation loss in terms of real resources that, in equilibrium, is given by $(1 - \theta(\sigma_t))S(\sigma_t)$. This can be easily verified by combining the households budget constraints, using market clearing conditions in the debt and savings markets, respectively, and the zero profit condition of financial intermediaries. The aggregate resource constraint, then, reads as:

$$\sum_{b,s} c_i(\sigma_t) + (1 - \theta(\sigma_t))S(\sigma_t) = y(\sigma_t) + \Upsilon(s^t)$$

On the left hand side, we have the borrowers' and savers' consumption plus the resources 'eaten up' by the financial sector. On the right hand side we have aggregate income plus total transfers. In order to have the intermediation process as a purely redistributive distortion, we choose $\Upsilon(s^t)$

¹²The inclusion of a more detailed micro-founded banking sector is an interesting avenue that we leave for future research.

such that all resources ‘lost’ in the intermediation sector are redistributed back to the agents, so that aggregate consumption is a function of aggregate income only. Therefore, aggregate transfers are defined as follows:

$$\Upsilon(s^t) \equiv (1 - \theta(\sigma_t))S(\sigma_t) \quad (8)$$

We interpret this transfer as income generated by the intermediation sector that is redistributed back to the households because they are either the managers of the bank or the residual claimants on the portfolio revenues of the bank. The inclusion of the transfer function has two advantages. The first one is that any effect of a θ shock on house prices and welfare comes through the model and is not generated artificially by an aggregate loss of resources. The second advantage is computational, as the re-distribution of resources makes sure that aggregate consumption is a function of aggregate endowment only, which is an essential requirement in order to apply the concept of wealth recursive equilibria proposed by Kubler and Schmedders (2003) to our framework.

2.2 Financial Market Equilibrium with Intermediation and Houses as Collateral

The economy is a collection of period-by-period utility functions, impatience parameters, state-dependent endowments and state-dependent financial intermediation efficiency, aggregate Transfers, transition probabilities, and the bankruptcy cost in case of default,

$$\mathcal{E} = \left(u, (\beta_i, y_i, h_i(\sigma_0^-))_{i=b,s}, \theta, \Upsilon, \Pi, m \right).$$

Definition 1 *A financial markets equilibrium for an economy \mathcal{E} , initial housing stocks $(h_i(\sigma_0^-))_{i=b,s}$ and initial shock z_0 is a collection*

$$\left((\bar{h}_b(\sigma_t), \bar{d}_b(\sigma_t), \bar{c}_b(\sigma_t)), (\bar{h}_s(\sigma_t), \bar{d}_s(\sigma_t), \bar{c}_s(\sigma_t)), (\bar{D}(\sigma_t), \bar{S}(\sigma_t)), \bar{q}(\sigma_t), \bar{R}_D(\sigma_t), \bar{R}(\sigma_t), \bar{\Upsilon}(\sigma_t)) \right)_{\sigma_t \in \Sigma}$$

satisfying the following conditions:

(1) Markets clear for all $\sigma_t \in \Sigma$:

$$\bar{h}_b(\sigma_t) + \bar{h}_s(\sigma_t) = 1$$

$$\bar{D}(\sigma_t) + \bar{d}_b(\sigma_t) = 0$$

$$\bar{S}(\sigma_t) - \bar{s}_s(\sigma_t) = 0$$

(2) For borrowers,

$$(\bar{h}_b(\sigma_t), \bar{d}_b(\sigma_t), \bar{c}_b(\sigma_t)) \in \arg \max_{c_b \geq 0, h_b \geq 0, d_b \leq 0} U_b(c_b, h_b)$$

such that for all $\sigma_t \in \Sigma$

$$c_b(\sigma_t) + \bar{q}(\sigma_t)h_b(\sigma_t) + d_b(\sigma_t) \leq n_b y(\sigma_t) + d_b(\sigma_t^-) \bar{R}_D(\sigma_t^-) + \bar{q}(\sigma_t)h_b(\sigma_t^-) + n_b \bar{\Upsilon}(\sigma_t)$$

$$\bar{R}_D(\sigma_t)d_b(\sigma_t) + m \cdot E[\bar{q}(\sigma_t z) | \sigma_t] h_b(\sigma_t) \geq 0$$

(3) For savers,

$$(\bar{h}_s(\sigma_t), \bar{s}_s(\sigma_t), \bar{c}_s(\sigma_t)) \in \arg \max_{c_s \geq 0, h_s \geq 0, s_s \geq 0} U_s(c_s, h_s)$$

such that for all $\sigma_t \in \Sigma$

$$c_s(\sigma_t) + \bar{q}(\sigma_t)h_s(\sigma_t) + s_s(\sigma_t) \leq n_s y(\sigma_t) + s_s(\sigma_t^-) \bar{R}(\sigma_t^-) + \bar{q}(\sigma_t)h_s(\sigma_t^-) + n_s \bar{\Upsilon}(\sigma_t)$$

(4) For financial intermediaries

$$(\bar{D}(\sigma_t), \bar{S}(\sigma_t)) \in \arg \max_{D \geq 0, S \geq 0} \bar{R}_D(\sigma_t)D(\sigma_t) - \bar{R}(\sigma_t)S(\sigma_t)$$

such that for all $\sigma_t \in \Sigma$

$$D(\sigma_t) \leq \theta(\sigma_t)S(\sigma_t)$$

(5) Aggregate transfers are given by

$$\bar{\Upsilon}(\sigma_t) = (1 - \theta(\sigma_t))\bar{S}(\sigma_t)$$

2.3 Wealth Recursive Equilibria

We conjecture that the arguments by Kubler and Schmedders (2003) for existence of Markov equilibria carry over to the present model. We outline briefly in which dimensions our setup differs from theirs and why the arguments by Kubler and Schmedders (2003) nevertheless carry over. The following three features distinguishes our setup from theirs:

- (a) the physical asset pays off in terms of a service stream from which agents derive utility; this service stream of housing and non-durable consumption are aggregated assuming a constant elasticity of substitution that is strictly lower than infinity;
- (b) the exogenous shocks to financial intermediation associated with aggregate intermediation cost Υ_t ;
- (c) the collateral constraint involves the conditional expectation of the relative price of the housing stock tomorrow and the no-default condition on m in assumption 1.

Feature (a) distinguishes our setup from Araujo, Páscoa, and Torres-Martínez (2002) where durable and non-durable consumption goods are perfect substitutes. In contrast, we allow for imperfect substitutability; recall that we assume the period-by-period utility function to be of CES type. As this utility function satisfies standard conditions, the presence of durable consumption does not constitute a problem in establishing existence.

Feature (b) is a somewhat non-standard feature; because $\theta(z)$ is a time-invariant function, the Markovian structure is preserved. Furthermore, it is strictly bounded away from zero and bounded above by 1, so both financial assets, savings and debt, will have strictly positive prices, that is, there will be an upper bound on real interest rates that is strictly smaller than infinity. Most importantly, the real costs associated with intermediation are redistributed back to the agents, so that aggregate consumption is still a function of aggregate endowment only. Aggregate endowment, in turn, is a time-invariant function of the exogenous process, so that this assumption ensures homotheticity, that is, the distribution of aggregate endowment does not depend in some non-linear way on the distribution of financial wealth across agents. We therefore conclude that the inclusion of the intermediation shock does not pose a threat to existence.

The presence of social default costs that are proportional to the collateral value as described in (c) was recently also introduced by Brumm, Grill, Kubler, and Schmedders (2011). Since

it is proportional to the collateral value, we are still left with a convex problem and standard arguments of best-response should go through.

For the quantitative part, it is useful to understand the recursive formulation. Since we have only two agents, the wealth distribution is represented by a single value on the unit interval, the borrowers' beginning-of-period wealth-share:¹³

$$\omega_b(\sigma_t) = \frac{q(\sigma_t)h_b(\sigma_t^-) + R_D(\sigma_t^-)d(\sigma_t^-)}{q(\sigma_t)} \quad (9)$$

Note that the collateral constraints, the constraints on asset holdings, and the utility functions satisfying Inada-conditions, together with assumption 1, imply that the wealth share lies in the unit interval, $\omega_b \in [0, 1]$; by definition, $\omega_s = 1 - \omega_b$. The equilibrium policy function is then a function of the discrete exogenous state variable z and the financial wealth distribution is $\Omega = (\omega_b, 1 - \omega_b)$.

As we are interested to solve for an equilibrium numerically, we follow Kubler and Schmedders (2003) and compute ϵ -equilibria.¹⁴ For the approximation of the equilibrium policy functions we adopt the time-iteration algorithm with linear interpolation proposed by Grill and Brumm (2010); that is, we approximate the equilibrium policy on a fine grid for the borrowers' wealth share; for points outside the grid we use linear piecewise interpolation. See appendix B for a detailed description of the algorithm.

3 Quantitative Analysis

This section studies the quantitative effects of the Great Recession on house prices and welfare. The purpose is to assess the dynamic effects of each shock in isolation compared to a Great Recession; the Great Recession is modelled as a contemporaneous negative shock to both aggregate income and financial intermediation, resembling the empirical facts that motivates our research question.

The next subsection outlines our calibration strategy. We then have a short section on the long-run stationary wealth distribution. We finally present impulse responses for three different

¹³Here, we used the market clearing conditions for the housing, debt, and savings markets and the fact that financial intermediaries make zero-profits in equilibrium, so that $h_b(\sigma_t^-) + h_s(\sigma_t^-) = 1$ and $R_D(\sigma_t^-)d_b(\sigma_t^-) + R(\sigma_t^-)s_s(\sigma_t^-) = 0$.

¹⁴For a definition and interpretation of ϵ -equilibria, we refer to the original text.

scenarios: first, we show the dynamic responses of quantities and prices of a negative financial intermediation shock only; second, for a negative income shock only; and third, for the simulated Great Recession. In each scenario, we assume that the economy was running for a long time and converged to the mean of the stationary distribution. We then enforce a sequence of economic expansion (high income, high intermediation, or both) for 10 quarters preceding the crisis. In Period 0 then, we hit the economy with negative shocks that last for seven quarters (as in the data, the NBER dates 2007:IV-2009:II). After the imposed sequences, the system is free to re-adjust towards the long run mean. Note that - in each scenario - we report the dynamic responses averaging over 10000 artificial economies.

3.1 Calibration

In the benchmark calibration, we assume an elasticity of substitution between houses and consumption equal to 1, so that $\rho = 0$. The risk aversion is set equal to $\gamma = 2$. These are standard values used in the literature. In general, it is not straightforward to calibrate these parameters as macro and micro evidence span a relatively large sets with parameter estimates. Similar to (Glover, Heathcote, Krueger, and Ríos-Rull, 2011), the risk aversion γ is the crucial parameter for the elasticity of house prices with respect to aggregate shocks, whereas the elasticity of substitution between consumption and savings plays an important role for the elasticity of welfare gains/losses to changes in the wealth distribution. Therefore, in section 4, we provide a sensitivity analysis for different values for the risk aversion and allow for some substitutability between housing and non-durable consumption as recently found by Bajari, Chan, Krueger, and Miller ((forthcoming)). Notice that one period in the model corresponds to one quarter in the data.

The parameter ϕ is the expenditure share of non-durable consumption. We pick the value to match the average housing wealth over GDP in the data during the period 1998-2007. For aggregate housing wealth, we considered the sum of the value of owner occupied real estate of private households plus the residential housing wealth of non-financial non-corporate private business. The savers' discount factor β_s is set so that the average interest rate on savings in the model matches the average return on savings equal to 1.5% during 1998 - 2007 (at annualized level). The borrowers' discount factor β_b and m are jointly calibrated to match the average

Table 1: calibration

| Parameter | Value | Model | Data | Source/Target |
|--------------------------|---------|--------|--------|----------------------------------------------------------------|
| Preferences | | | | |
| γ | 2 | | | Benchmark value from literature |
| ρ | 0 | | | Benchmark value from literature |
| Targeted parameters | | | | |
| ϕ | 0.97 | 196% | 196% | Average housing value over GDP (annualized) 1998 - 2007 |
| β_s | 0.996 | 1.5% | 1.5% | Average return on savings (annualized) |
| β_b | 0.988 | 11.7% | 11.3% | Borrowers' financial wealth share (SCF average 1998-2007) |
| m | 0.5 | 45% | 44.4% | Borrowers' leverage ratio (SCF average between 1998-2007) |
| Relative population size | | | | |
| n_b | 0.42 | 42% | 42% | Share of borrowers (SCF average 1998-2007) |
| Intermediation shock | | | | |
| π_H^θ | 0.565 | | 56.5% | Probability of low spreads during 1998-2009:II |
| ρ_θ | 0.868 | 0.868 | 0.868 | Autocorrelation of spreads during 1998-2009:II |
| θ_L | 0.9985 | 1.8 % | 1.75 % | average spread during 1998-2009:II (annualized) |
| θ_H | 0.99207 | 1.27 % | 1.27 % | standard deviation of spread during 1998-2009:II (annualized) |
| Income shock | | | | |
| π_H^y | 0.85 | | 15% | Probability of recession between 1980-2009:II (NBER dates) |
| ρ_y | 0.93 | | 0.93 | autocorrelation of linearly de-trended log-GDP 1980-2009:II |
| y_L | 0.99325 | 3% | 3% | standard deviation of linearly de-trended log-GDP 1980-2009:II |
| y_H | 1.01 | 1 | | we normalized $E(y)=1$ |

wealth share of the borrowers relative to the savers and leverage ratio of the borrowers. Since there is not necessarily a one-to-one mapping between the parameters and their targets, we follow an iterative procedure to find the values for β_s , β_b , m and ϕ ; that is, we first guess values for the parameters and then compare the computed moments match the data counterparts. If not, we change the values. We repeat until the model moments match the data counterparts. The procedure leads to a quite satisfactory match between model and data moments.¹⁵

The relative population size of borrowers is set to 42%, corresponding to the fraction of borrowers in the SCF when using the weighted average share of households with a negative net asset position as defined in the appendix A. The estimate is in line with the calibration in

¹⁵The variable definitions used to calculate the data moments are as close as possible to the definition of the model counterparts. For a detailed description on how we compute the relative wealth share and the leverage ratio in the data, see appendix A.

Iacoviello (2008).

The stochastic processes for the exogenous state variables y_t and θ_t are assumed to be independent; this is in line with the correlation in the data and it is coherent with the modelling decision.¹⁶ We choose the state space and the transition probabilities for aggregate income and intermediation shocks by applying the simple persistence rule, as in Backus, Gregory, and Zin (1989) and Mendoza (1991). First, we assume that both aggregate income and the credit spread shock take two values each, that is $y_t = \{y_L, y_H\}$ and $\theta_t = \{\theta_L, \theta_H\}$. For both shocks, we assume that transition probabilities are given by:

$$\pi_{ij} = (1 - \rho)\pi_j + \delta_{ij}\rho \quad \text{for } i, j = H, L$$

where $\delta_{ij} = 1$ if $i = j$ and 0 otherwise; $\pi_j > 0$ is the unconditional probability of being in state j , and by definition we have $\sum_j \pi_j = 1$. The parameter ρ governs the persistence of the shock.

The unconditional probability of a high intermediation efficiency, $P(\theta = \theta_H)$, is set to 0.565, the fraction of quarters in which the U.S. experienced low spreads between 1998:I and 2009:II. We set $\theta_L = 0.99207$, $\theta_H = 0.9985$, and $\rho_\theta = 0.868$ so that we match the mean, standard deviation and the autocorrelation of the spreads in the data (data counterparts see table; for a data description see appendix A). For the income process, we choose the y_H and y_L and ρ_y to match the mean, normalized to $E(y) = 1$, the standard deviation and the autocorrelation of linearly de-trended log-GDP in the data. We assume the unconditional probability of a recession is 15%, so that $\pi_H = 0.85$, the probability of recessions in the data during 1980-2009:II using the NBER recession dates.

To summarize, the exogenous state space is then given by $\Sigma = \{1, \dots, 4\}$ and - given that income and intermediation shock are uncorrelated - the transition matrix for the exogenous process is just the Kronecker product of the individual transition probability matrices for the income shock and the intermediation shock. Table 1 summarizes the calibrated parameter values and the targets.

¹⁶We also conducted a VAR analysis for GDP growth and spreads for different lag-lengths and orderings and found no evidence for significant spillover terms and no contemporaneous correlations between GDP and mortgage spreads. Only in one specification (VAR of order two) Granger-causality of output growth on spreads could not be rejected, even though the coefficients for individual lag of output were not significant different from zero.

3.2 Stationary wealth distribution

Figure 3: Wealth distribution

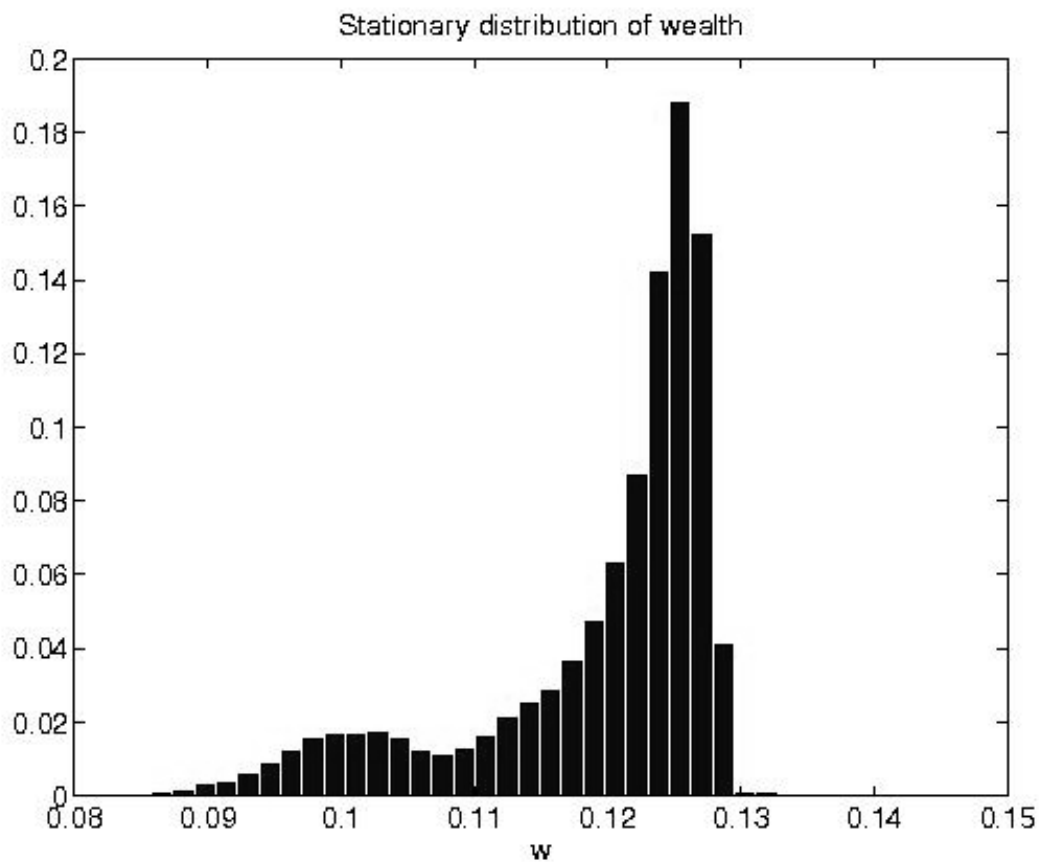


Figure 3 shows the long-run stationary wealth distribution simulated over 100,000 time periods.¹⁷ Recall that the wealth distribution across agents is entirely summarized by the borrowers' fraction of wealth ω_b . On average, the borrowers hold 11.7% of the total wealth of the economy, given by q . The distribution of borrowers' wealth share is very concentrated around the mean and has a spike to the right at around 12.6% which correspond to states of the world when there is a sufficiently long period of credit and income expansions. In these states, the borrowers' collateral constraint is binding and the interest rate on borrowing is relatively low; demand for housing is high and therefore expected house prices are high. This marginally relaxes the constraint, so that aggregate debt and savings are high. Because house prices are rising and borrowers are accumulating housing, their wealth share increases. Conversely, negative

¹⁷Because of the simple persistence rule used to discretize the exogenous processes, the high number of simulation periods makes sure that the exogenous processes have the same stochastic properties as the data counterpart.

realizations of aggregate shocks make the borrowers' wealth share drop. We will explain in details these mechanisms in the following section(s).

3.3 Financial Intermediation Shock

The numerical experiment is the following. We assume that - previous to date -10, the economy was running for a long time it converged to its long-run mean of the stationary distribution. At date -10, we simulate ten periods of credit expansion by enforcing a sequence of positive realizations for financial intermediation efficiency (θ_H). This resembles the observed pattern in the data previous to the Great Recession (see figure 1). In date 0, we hit the economy with a negative intermediation shock (θ_L) and keep it low for seven quarters. We then plot the average dynamic impulse responses averaged over 10,000 simulated economies in order to average out income shocks, so that $y_t = E(y)$ for all t .

Figure 4: Negative theta shock

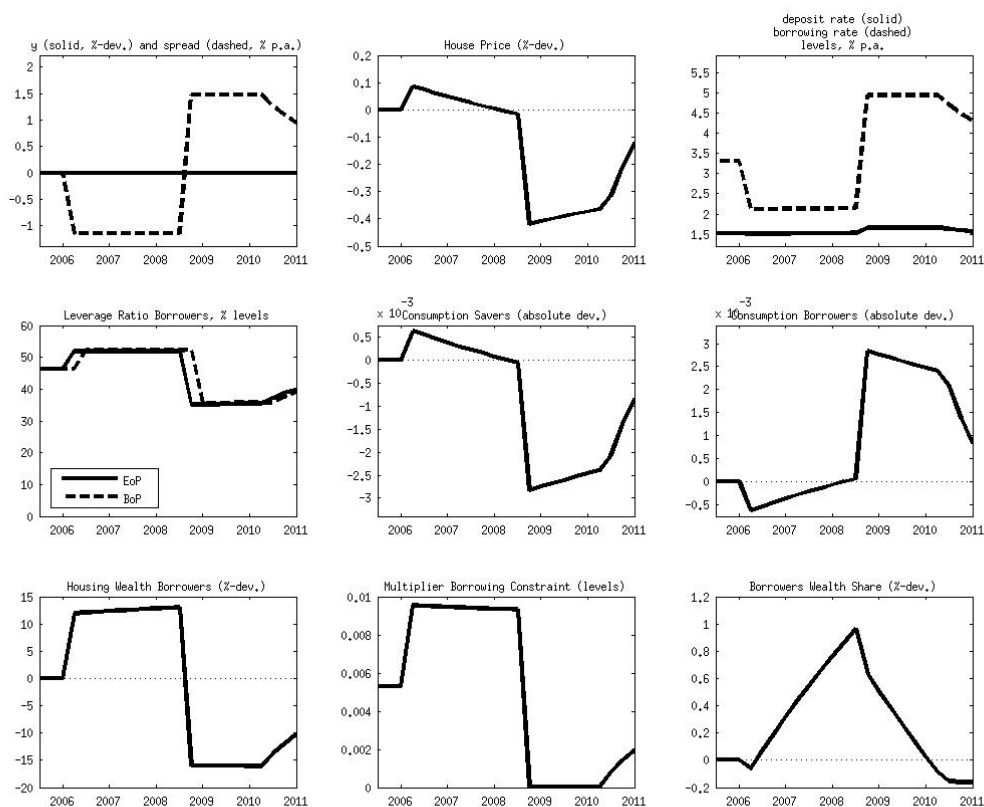


Figure 4 plots the average impulse responses of quantities and prices as percentage deviations

from the pre-date-0 averages. Notice that interest rates are plotted in percent p.a., the leverage ratios are in percent and the multiplier on the borrowing constraint is expressed in levels. We both plot what we call the beginning-of-period leverage (defined below) and the end-of-period leverage ratio. The beginning-of-period leverage ratio is defined as the borrowers' mortgage debt carried over from last period over total housing wealth, evaluated at house prices today, that is:

$$L_t^{BoP} = -\frac{R_{D,t-1}d_{b,t-1}}{q_t h_{b,t-1}}. \quad (10)$$

This represents the debt outstanding carried over from the previous period, evaluated at the equilibrium house price of the current period; it measures the effective debt burden the borrower has to roll over in the current period, just before she chooses consumption, housing, and debt level today (given the previous period expectation about house prices today). The end-of-period leverage is defined as the amount of debt promised to repay tomorrow, relative to end of period wealth:

$$L_t^{EoP} = -\frac{d_{b,t}}{q_t h_{b,t}}. \quad (11)$$

From the graphical inspection of figure 4, the model qualitatively captures the documented behavior of aggregate house prices. In particular, as a result of the credit spread shock, the interest rate on borrowing increases and borrowers are forced to reduce their level of debt in next period. The increase in interest rate, in fact, lowers the present discounted value of housing and induces a negative credit demand effect.¹⁸ Given that they cannot roll-over the existing debt, they are forced to de-leverage; in order to pay back the existing debt, they have to reduce their housing stock. As the sequence of negative spread shock occurs, the deflationary pressure on house prices leads to an equilibrium house price drop of about 0.4%. The borrowers end up in an unconstrained equilibrium (where the collateral constraint is not binding), given that the drop in next period debt is bigger than the drop in next period housing wealth. In such a state of the world, the deflationary pressure on house prices comes from two effects: the de-leveraging effect and the lost of the additional value that the borrowers give to housing due to binding collateral constraint. These effects result to be quantitative small. This comes from

¹⁸The increase in the interest rate on borrowing qualitatively corresponds to the behavior of targeted interest rate in the data counterpart since the end of 2008. Real interest rate on mortgages were in fact declining in late 2007; given the decrease in the inflation rate during the Great Recession, interest rate started to increase, reaching a peak of about 7% in late 2008.

the fact that savers offset the deflationary pressure. Accordingly, given that, in equilibrium, they have to reduce their savings, they optimally postpone non-durable consumption and invest in housing at a depreciated value. As a result, they quite completely offset the deflation on house prices and their share of aggregate wealth increases relative to borrowers. In the quarters subsequent the shock, as the exogenous variables move towards the long run state of the economy, the debt exposure of the borrowers increases because the economy expects house prices to rise, and the collateral constraint returns to be binding. Notice that, when the collateral constraint is binding, the borrowers attach an additional value to housing and the inflationary pressure is such that the relative wealth share starts recovering.

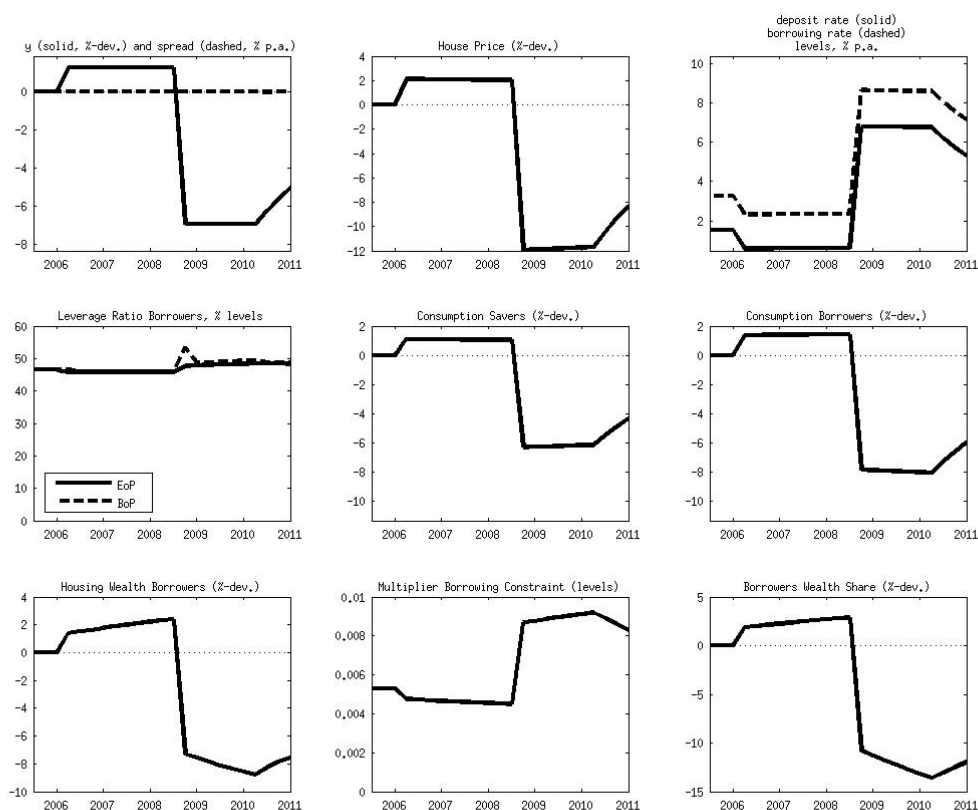
3.4 Aggregate Income Shock

In this section we undertake a similar numerical experiment. In this case we assume that - previous to date 0 - the economy was running for a long time and the model variables converged to their long run distribution. At date -10, we force a sequence of realization of positive income shock (y_H) that resembles the observed pattern of the de-trended series of GDP preceding the crisis (see Quadrini and Urban (2012)). In date 0, we hit the economy with a negative realization of the shock (y_L) for seven quarters. During this experiment we keep the intermediation efficiency fixed at $\theta(\sigma_0^-) = E(\theta)$ and hit the economy with a negative income shock, that is, $y(\sigma_0) = y_L$. Figure 5 plots the aggregate dynamics of allocation and prices.

The response of the economy highlights the difference between an income and a credit spread shock. Quantitatively, we see that the effect of a negative income shock on aggregate house prices is significantly bigger with respect to a financial intermediation shock (13.5%). In presence of this kind of shock, savers are not able to offset the deflationary pressure on house prices. In fact, with an aggregate drop in income, both type of agents reduce their demand for normal goods. In equilibrium borrowers have to reduce their optimal level of debt. Moreover, given that the collateral constraint is binding, and that they also face a negative income shock, they reduce their housing stock. Savers require a lower house price to clear the market given that they are hit by a negative shock as well. In terms of welfare, the increase in the interest rate on savings, and the acquired long position on deflated housing, allow savers to buffer the negative wealth effect coming from the aggregate house price drop. ¹⁹ The graphical inspection furthermore

¹⁹In the data, as for the interest rate on borrowing, the interest rate on savings increased in mid-2008. Real

Figure 5: Negative income shock



reveals that the leverage is quite constant along the cycle because the collateral constraint is always binding. In the period the recession hits, the beginning-of-period debt shoots up, because of the house prices drop is bigger than expected. In addition, borrowers keep up to borrow so that also the end-of-period leverage ratio rises slightly. This pattern seems to find evidence in the data; as showed by Boz and Mendoza (2010), the crisis resulted in an on-impact increase in leverage, as land prices fell faster than the reduction in debt.

3.5 The Great Recession

The numerical experiment is the following. We assume that - previous to date 0 - the economy was running for a long time, so that it converged to its long-run stationary distribution. At date -10, we simulate ten periods of credit expansion due to both a high financial intermediation

interest rate on savings declined in late 2007 as an on-impact effect of the recession. However, given the decrease in the inflation rate during the Great Recession, interest rate started to increase in late 2008. In particular, real interest rates on saving accounts, certificate of deposits and the federal funds rate reached a level above 2.5%; interest rate on less liquid accounts, such money market accounts, reached a level of around 4%.

efficiency (θ_H) and high aggregate income (y_H). In date 0, we hit the economy with a contemporaneous negative shock to aggregate income (y_L) and financial intermediation (θ_L) and keep it low for seven quarters. As we argued in the introduction, this resembles the observed pattern in the data previous to the Great Recession (see figure 1).

Figure 6: The Simulated Great Recession

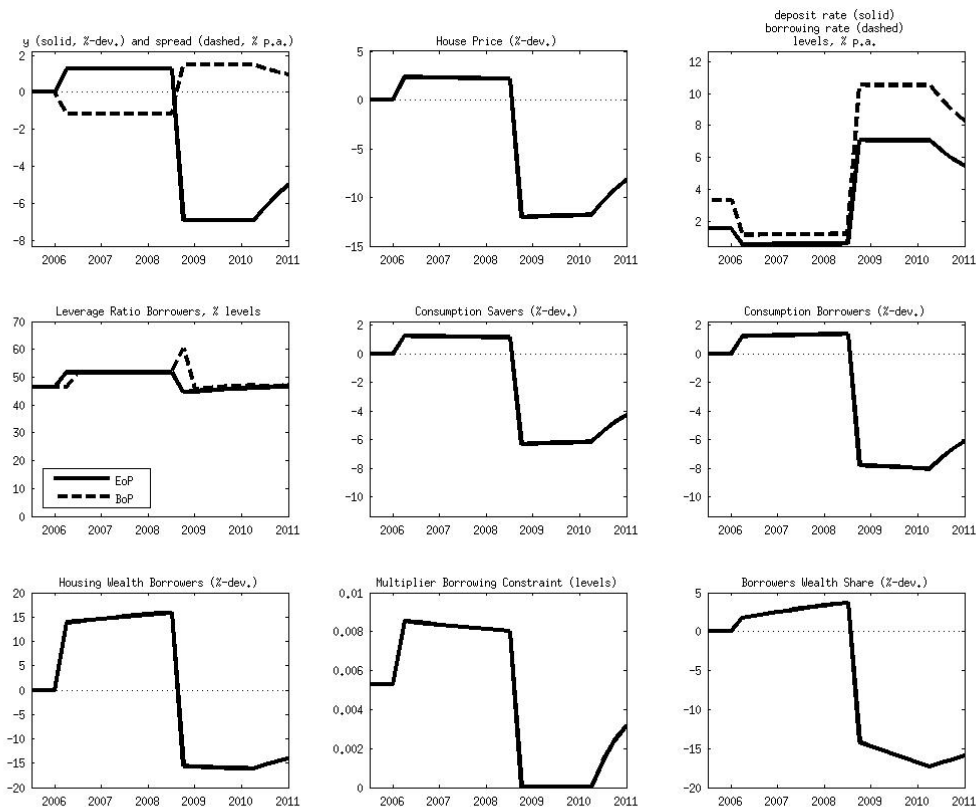


Figure 6 shows the average responses of the simulated Great Recession. Three observations stand out. First, house prices rise in the expansion previous to the recession and drops sharply when the recession hits. The model is consistent with the dynamics observed in the data and explains slightly about 60% of the observed drop in house prices during the Great Recession (minus 13.6% in the simulated Great Recession versus minus 22.5% in the data during the NBER recession dates).

Second, the leverage increases in the credit expansion previous to the recession as the borrowing interest rate is low. In the period the recession hits, the beginning-of-period debt shoots up, because of the house prices drop. Recall that this is the effective debt burden that has to

be rolled-over in the current period, so this makes the borrowers - on impact - relatively poorer in terms of wealth. The question is now whether the borrowers are able to roll over this high debt burden. Because negative financial intermediation shock hits at the same time, they find it too expensive to do so and de-leverage. This can be seen when looking at the end-of-period leverage ratio which drop to more than 10% with respect to the initial level.

Third, because of the high debt burden, the borrowers' wealth share drops sharply. This needs not necessarily translate into larger decreases of consumption and housing, as we will show below. However, when looking at the consumption and housing responses, we see that the borrowers sharply reduces their housing stock; this is because they cannot roll over their debt burden. This implies larger (smaller) welfare costs for borrowers (savers) compared to the case in which financial intermediation would not deteriorate together with aggregate income. Because of the reduction in debt, the savers substitute the savings by buying the housing stock sold by the borrowers when the price fell.

3.6 Welfare consequences of the Great Recession

Table 2: Welfare gains, leverage, and distribution of wealth

| Exogenous Shock(s) | Δq | L_b^{BoP} | Δw | L_b^{EoP} | λ_b | λ_s |
|--------------------------|------------|-------------|------------|-------------|-------------|-------------|
| Financial Intermediation | -0.35 | 52.39 | - 1.04 | 35.43 | -0.06 | 0.00 |
| Income | -13.46 | 53.38 | -16.03 | 48.47 | -1.10 | -0.17 |
| Great Recession | -13.66 | 60.24 | -20.31 | 46.25 | -1.24 | -0.15 |

Notes: Column two is the change in house price between period -1 (the period just before the shock occurs) and period 7 (following the recession hits). Columns three shows borrowers' beginning-of-period leverage ratio defined as $L_{b,t}^{BoP}$ in the period of the shock $t = 0$; note that this leverage ratio is a function of the price today only (variables with subscript $t - 1$ are given numbers). Column four shows the corresponding change in borrowers' wealth share between period -1 and period 7. Column five reports the borrowers' end-of-period leverage ratio defined as L_t^{EoP} , after the Great Recession - in period $t = 7$. Columns six and seven show welfare gains/losses of borrowers and savers, respectively. All numbers are in percent.

We now turn to the distributive effects of the simulated Great Recession and compare it to two counter-factual scenarios in which we compute the welfare effects of just a negative income shock (fixing the level of financial intermediation efficiency) and just a financial intermediation shock keeping income fixed, respectively. We define the welfare gain of a shock sequence as the compensation (in terms of total consumption equivalents) that is needed to make agent $i = b, s$ indifferent between the expected life-time utility when aggregate income, intermediation and wealth are equal to their long-run averages (as it is the case in period $t = -11$) and the agents'

expected life-time utility after the economy experienced the shock sequence, so that negative numbers reflect losses. We denote by λ_b and λ_s the welfare gains of borrowers and savers, respectively, expressed in total consumption equivalents. We also report both the beginning- and end-of-period leverage ratios of the economy in the period the shock realizes. Based on the simulated equilibrium dynamics of the calibrated economy we summarize two key findings from table 2:

1. High leverage makes the wealth distribution more sensitive to price changes.
2. In a Recession, a contemporaneous negative intermediation shock causes the wealth distribution to translate into higher (smaller) welfare losses for borrowers (savers).

Result 1 says that the higher the leverage ratio in the economy when entering a recession, the more the wealth gets distributed away from borrowers to savers. In other words, a given house price drop due to an aggregate income shock leads to more redistribution of wealth from borrowers to savers, the higher the leverage previous to the shock. If the economy is experiencing high intermediation efficiency previous to a recession, the leverage ratio of the borrower is high; the borrowers' wealth share is then very sensitive to price changes and there is substantial more re-distribution of wealth with respect to normal times.

On the other hand, result 2 deals with the second important question: whether larger redistribution of wealth translates into more inequality in terms of welfare. We find this crucially depends on the binding/non binding state of the collateral constraint, so whether borrowers want to stay on the constraint or move away from it. This result implies that only if financial intermediation deteriorates in a recession, the wealth loss translates into larger (smaller) welfare losses of borrowers (savers). The intuition for both results is summarized in the following paragraphs.

Intuition for Key Result 1 Let us now show the intuition behind these results graphically. Recall equation (10). To see the effects on wealth distribution, we can rewrite the borrowers' wealth share in terms of the leverage ratio:

$$w_{b,t} = h_{b,t-1}(1 - L^{BoP}(q_t)) \quad (12)$$

When financial intermediation is high previous to the recession, then collateral constraint binds and beginning of period leverage is directly connected to the expectation of house prices, as can be seen by substituting the collateral constraint of the previous period in the definition of the leverage ratio in (10):

$$L^{BoP}(q_t) = m \frac{E_{t-1}[q_t]}{q_t}$$

where, by assumption 1, $L^{BoP}(q_t)$ is strictly smaller than one. Moreover, $L(q_t)$ is decreasing in q_t , implying that if the price falls below expectation, leverage will increase (taking previous period expectations as given). Applying total derivatives around $q = q_t = E_{t-1}q_t$, one obtains the growth rate of borrowers' wealth share being proportional to the growth rate of house prices:

$$\left. \frac{dw_{b,t}}{w_{b,t}} \right|_{q=E_{t-1}q_t} = \frac{L(q)}{1-L(q)} \frac{dq_t}{q_t} = \frac{m}{1-m} \frac{dq_t}{q_t}.$$

In case the collateral is not binding, beginning of period leverage is strictly smaller than $m \frac{E_{t-1}}{q_t}$ and therefore $\left. \frac{dw_{b,t}}{w_{b,t}} \right|_{q=E_{t-1}q_t} < \frac{m}{1-m} \frac{dq_t}{q_t}$. That is, when the borrowers' leverage is high previous to the shock, so that the collateral constraint is binding, any aggregate price drop makes the borrower - on impact - relatively poorer in terms of financial wealth.

Of course, the price today is not just a number but an equilibrium outcome; that is, the pricing function depends on the state of the economy. We have no closed form solution for this pricing function but from the simulated economy, we can plot the equilibrium house prices as a function of the wealth share; This function is - for any realization of the exogenous shock $z \in \mathcal{Z}$ - decreasing in w_b , or

$$q = Q(w_b, z) \quad \frac{\partial Q}{\partial w_b} < 0. \quad (13)$$

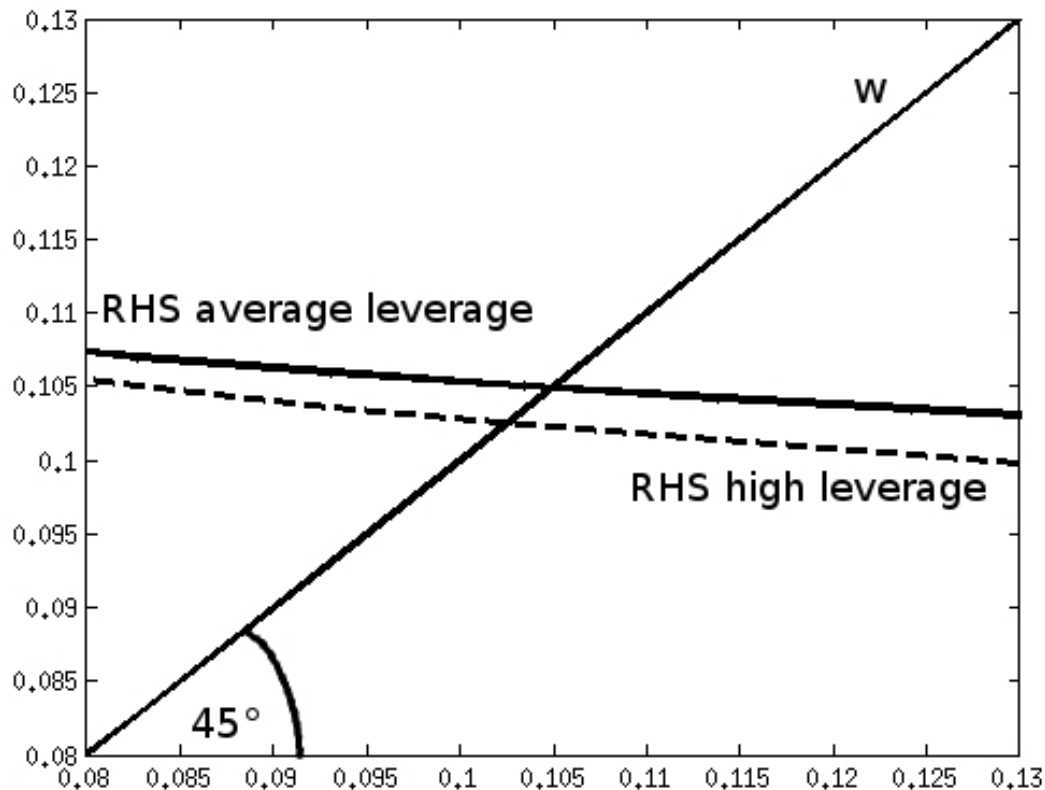
Given the promised value of previous-period debt, $R_{D,t-1}d_{b,t-1}$, and given the housing stock carried over from last period, $h_{b,t-1}$, the equilibrium wealth share in period t is implicitly defined by the solution to (12) and (13), or

$$w_{b,t} = h_{b,t-1} \left(1 - \frac{R_{D,t-1}d_{b,t-1}}{Q(w_{b,t}, z_t)} \right) \quad (14)$$

Figure 7 plots the left-hand side and right hand side of equation (14) as a function of the borrowers' wealth share w_b assuming that the realization of the exogenous shock z_t is the Great

Recession (low income and low intermediation).

Figure 7: Response of equilibrium wealth share to a negative income shock depends on the beginning-of-period leverage

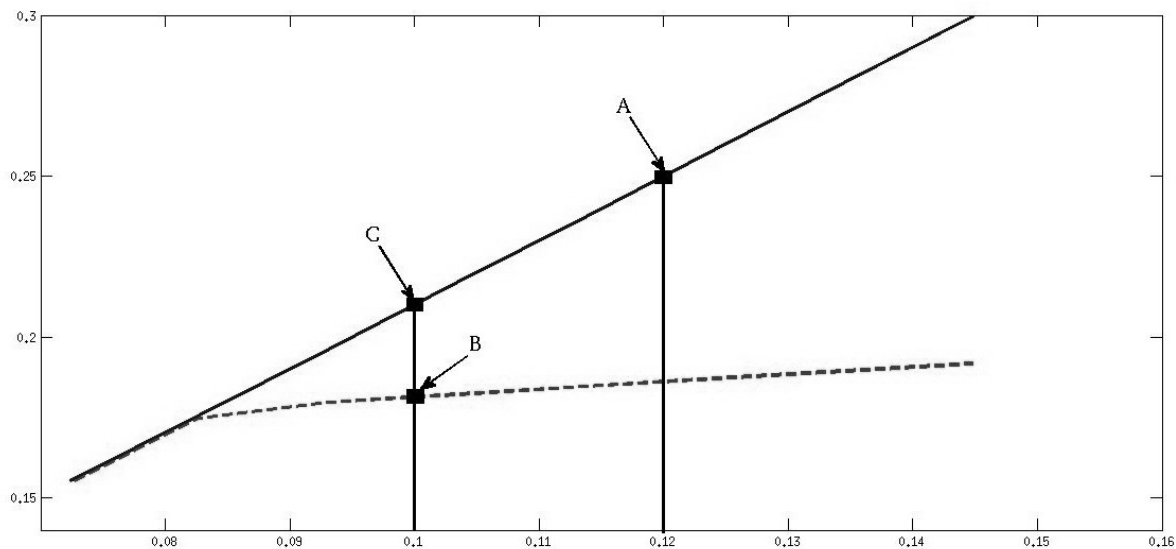


Notes: The figure plots the left-hand side and right hand side (RHS) of equation (14) as a function of the borrowers' wealth share w_b , assuming that the realization of the exogenous shock z_t is the Great Recession (low income and low intermediation). The solid line shows the right-hand side under the assumption that $h_{b,t-1}$ and $R_{D,t-1}d_{b,t-1}$ are equal to the respective long-run averages. The dashed line shows the right-hand side under the assumption that $h_{b,t-1}$ and $R_{D,t-1}d_{b,t-1}$ are equal to the respective values when intermediation and income was high for 10 periods, so that the collateral constraint was binding and leverage was high.

The figure illustrates key finding 1 so that when previous period debt is high (dashed line), the wealth share is more sensitive to exogenous shocks to the house price, compared to the case when debt carried over from last period is equal to the average level. This illustrates the relation between leverage and wealth dynamics during a recession; the effect comes from a different elasticity of wealth with respect to changes in price which in turns depends on the aggregate state of financial intermediation.

Intuition for Key Result 2. Result (2) is concerned with the combination of an income and negative intermediation shock. In fact, when the house price falls and there is a contemporaneous negative intermediation shock, borrowers face a higher interest rate that prevents them from rolling over the debt and moving away from the constraint. This forces the borrowers to substantially decrease housing stock.

Figure 8: Equilibrium housing policy depends non-linearly on wealth when financial intermediation efficiency changes from high to low



Notes: Solid line: housing policy as a function of the borrowers' wealth share conditional on high financial intermediation efficiency. Dashed line: housing policy as a function of wealth conditional on low financial efficiency. The vertical line intersecting at A is the borrowers' wealth share after 10 periods of expansion and the vertical line intersecting at B is borrowers' wealth share in the period when the Great Recession hits the economy.

Figure 8 plots the policy function of the borrowers' housing stock for high (solid line) and low efficiency (dashed line), respectively. Following the Great Recession, relative wealth of the borrower drops. Because financial intermediation also switches from high to low, the housing stock drops from A to B. This is substantially more than if intermediation would stay high; in this last case, for a same drop in wealth, the decrease of the housing stock would have been less sharp (C). In other words, the elasticity of demand for housing with respect to wealth depends on the efficiency of the financial intermediation sector.

Summary of the welfare effects. First, both agents lose in response to an aggregate negative income shock, and borrowers always lose more than savers because they are financially constrained and unable to buffer negative shocks. Second, while borrowers experience a welfare

loss in case of negative financial intermediation shock, savers are virtually not affected. Third, in the simulated recession, we observe that the borrowers' welfare loss is larger than the algebraic sum of the welfare losses in response to a negative income and intermediation shocks in isolation. The opposite is true for the savers. This comes from a non-linearity in the reaction of consumption when borrowers are forced to de-leverage and move away from the constraint. In such a scenario savers even gain from the joint income and intermediation shock (relative to an income shock alone) because they become relatively richer.

This set of results leads to the conclusion that, following the Great Recession, while both types of agents experienced a welfare loss, savers could buffer the negative impact of the negative aggregate shocks by substituting savings with depreciated houses. This conclusion, while qualitatively comparable with the recent finding of Hur (2012), highlights a different mechanism. In this model savers are able to buffer the negative effects of the Great Recession because of the asymmetric effects of financial intermediation shocks and the high leverage previous to the shock.

Regarding the size of the welfare losses, in the benchmark model, in the Great Recession, borrowers lose on average -1.24% every quarter in terms of total consumption compared to the expected utility facing average realizations of the shocks and holding average wealth. These numbers are in the same order of magnitude as in Glover, Heathcote, Krueger, and Ríos-Rull (2011) who have found welfare losses for different age cohorts ranging from -1.6% to -8.3% .²⁰

Another important remark related to the magnitudes of the obtained numbers concerns the error analysis of our numerical algorithm. That is, if the mistakes agents make using our algorithm is bigger (in consumption equivalents) than the calculated welfare gains/losses, these numbers would have no quantitative validity. We find that the maximum relative Euler Error of our approximation is $3e-5$ (or -4.5 in $\log(10)$ -scale). This implies that an agent, using our approximation of the equilibrium policy functions, would lose 30 Dollars for each million spent. For details see appendix B.4. We therefore conclude that our quantitative findings are valid and quantitatively meaningful.

²⁰These numbers are bigger the higher the parametrized risk aversion; we will show a similar result in the section 4. Also notice that the fraction of borrowers in our model represent the 42% of total US population; consequently the macroeconomic relevance of the welfare loss we found is perfectly compatible with recent findings.

3.7 Always binding collateral constraint

We solve the model employing a global solution method; compared to the widely used log-linearization, this is necessary to take into account the fact that the collateral constraint is not always binding, but comes at the cost of a more complex numerical implementation. In this section we show how big the mistake would be if one would assume that the borrowers' constraint is always binding.

Table 3: Always binding collateral constraint

| Exogenous Shock(s) | Δq | L_b^{BoP} | Δw | L_b^{EoP} | λ_b | λ_s |
|--------------------------|------------|-------------|------------|-------------|-------------|-------------|
| Financial Intermediation | -0.67 | 45.52 | - 2.91 | 44.61 | -0.05 | 0.01 |
| Income | -13.32 | 51.89 | -14.03 | 44.69 | -1.07 | -0.20 |
| Great Recession | -13.85 | 52.31 | -16.91 | 44.53 | -1.13 | -0.19 |

Notes: Column two is the change in house price between period -1 (the period just before the shock occurs) and period 7 (following the recession hits). Columns three shows borrowers' beginning-of-period leverage ratio defined as $L_{b,t}^{BoP}$ in the period of the shock $t = 0$; note that this leverage ratio is a function of the price today only (variables with subscript $t - 1$ are given numbers). Column four shows the corresponding change in borrowers' financial wealth share between period -1 and period 7. Column five reports the borrowers' end-of-period leverage ratio defined as L_t^{EoP} , after the Great Recession - in period $t = 7$. Columns six and seven show welfare gains/losses of borrowers and savers, respectively. All numbers are in percent.

To this end, we solve an alternative specification of the model by forcing the borrowers to have an always binding constraint. In this case, the leverage ratio of the economy is always equal to $m \frac{E_{t-1} q_t}{q_t}$ which therefore needs to be re-calibrated for this specification in order to match the leverage ratio we find in data. The results are summarized in table 3. Compared to the benchmark model, we find that in a version of the model with always binding collateral constraint: (i) the quantitative effects on house prices are bigger relative to the benchmark model for a negative financial intermediation shock; ii) in the Great Recession, the welfare losses by borrowers (savers) are smaller (higher) in absolute terms. To summarize, the borrowers' welfare loss is lower by 0.07 percentage points (in absolute terms), while the savers' lose 0.04 percentage points more, when compared to the benchmark model. In percentage terms, this amounts to 5.6% less for the borrowers and to 26.7% more for the savers when compared to the losses in the benchmark model. Most importantly, the non-linearity of previous-period leverage completely vanishes, as the borrowers' wealth losses and the agents' welfare gains are just the algebraic sum of the effects when the economy is hit with each shock separately.

The reason is that models with always binding constraint have the peculiarity of a constant

elasticity of demand for debt with respect to changes in interest rate; in other words, following a spread shock, the borrowers' change in next period debt has to be strictly proportional to the present discounted value drop of next period housing wealth. When debt is costly, borrowers are prevented to move away from the constraint. Aggregate debt moves less with respect to the benchmark case and this, in equilibrium, reduces savers' capability of switching from savings to housing. This is the reason why house prices drop more in response to a negative intermediation shock. In sum, however, the elasticity of borrowers' wealth share to any given drop in house prices is always constant and given by $\frac{m^{AB}}{1-m^{AB}}$, where the superscript stands for 'always binding'. Note that in order to match the average leverage ratio in the data, $m^{AB} = 0.45$ which is lower than $m = 0.5$ in the benchmark calibration. The elasticity of the borrowers' wealth share is therefore constant and strictly less than one. This result suggest that the assumption of always binding collateral constraints is not innocuous when making welfare analysis.

4 Sensitivity Analysis

In this section we compare the quantitative implications when changing the elasticity of substitution between housing and non-durable consumption and the risk aversion. Notice that, for all changes in these parameters, we re-calibrate the rest of the parameters that are chosen to match the targeted data moments. This allows us to evaluate the relative performance with respect to the benchmark case.

4.1 Elasticity of substitution between housing and non-durable consumption

Here we conduct a sensitivity analysis for one of the two parameters that we fixed in the benchmark calibration to unity: the elasticity of substitution between housing and non-durable consumption. Table 4 summarizes the quantitative findings for an increasing substitutability between housing and non-durable consumption, setting $\rho = 1.25$.²¹

Table 4 shows that with increasing substitutability between housing and non-durable consumption, house prices (and therefore wealth) react more to an intermediation shock when compared to the benchmark case. This, similarly to the case with always binding constraint,

²¹This parameter value is taken from Piazzesi, Schneider, and Tuzel (2007) who consider a representative agent framework with housing; As mentioned earlier in the paper, an elasticity of substitution bigger than one between housing and non-durable consumption has also recently been found by Bajari, Chan, Krueger, and Miller ((forthcoming)).

Table 4: Elasticity of substitution

| Exogenous Shock(s) | Δq | L_b^{BoP} | Δw | L_b^{EoP} | λ_b | λ_s |
|--------------------------|------------|-------------|------------|-------------|-------------|-------------|
| Financial Intermediation | -0.41 | 55.45 | -1.14 | 32.31 | -0.05 | 0.00 |
| Income | -13.47 | 55.03 | -16.72 | 49.70 | -1.06 | -0.20 |
| Great Recession | -13.71 | 63.69 | -22.06 | 45.76 | -1.20 | -0.17 |

relates to a decreased elasticity of demand for debt with respect to changes in interest rate on borrowing. In addition, in this calibration, the Great Recession leads to smaller (bigger) welfare losses for borrowers' (savers'); borrowers are less worse off because they substitute housing by non-durable consumption which is less painful when these goods are substitutes. This is also the reason why there is less redistribution in terms of welfare from borrowers to savers, so that in absolute terms savers lose more. Nevertheless, the key findings of the role of leverage for the wealth dynamics and the role of the intermediation shock in a recession still remain.

4.2 Risk aversion

In this section we show quantitative analysis from Great Recession episodes for different values of the risk aversion taken from the related literature. In particular, while the business cycle literature usually features a log-separable utility function with an elasticity of substitution and risk aversion equal to unity, the macro-finance literature and recent contributions on the distributive effects of Great Recession focus on broader risk aversion parameter set.²² Table 5 summarizes the effects of the simulated Great Recession for the benchmark and other model specifications for different values of the risk aversion parameter.

Table 5: Welfare Effects of the Great Recession: Comparison of different risk aversion

| Model | $\Delta q/q$ | L_b^{BoP} | $\Delta w_b/w_b$ | L_b^{EoP} | λ_b | λ_s |
|------------------------|--------------|-------------|------------------|-------------|-------------|-------------|
| $\gamma=1$ | -7.80 | 56.35 | -10.17 | 40.17 | -1.01 | -0.20 |
| $\gamma=2$ (benchmark) | -13.66 | 60.24 | -20.31 | 46.25 | -1.24 | -0.15 |
| $\gamma=3$ | -18.63 | 64.02 | -29.45 | 49.89 | -1.50 | -0.12 |
| $\gamma=5$ | -26.49 | 69.36 | -37.82 | 50.45 | -2.23 | -0.07 |

Results confirm that, the higher the risk aversion, the higher the negative impact of the Great

²²Glover, Heathcote, Krueger, and Ríos-Rull (2011) set the risk aversion equal to 3 in the benchmark case, and then conduct sensitivity analysis; they find that the magnitude of equilibrium price responses increase non monotonically as risk aversion increases; Piazzesi, Schneider, and Tuzel (2007), in a capital asset pricing model with housing, find that higher level of risk aversion better perform in matching the moments of housing returns.

Recession on equilibrium aggregate house prices; this relates to higher elasticity of demand of housing with respect to income shocks. In particular, the model explains about 85% of the observed drop in house prices during the Great Recession for a risk aversion parameter equal to 3. The welfare analysis also confirms that bigger wealth shocks (due to house price drop) translate into more negative welfare effects for borrowers; this effect is again amplified by the financial intermediation shocks that prevented them to access the financial market to smooth negative income shocks. By opposite, savers, are more able to smooth the negative effects of the Great Recession. This relates to drop in aggregate debt that, in equilibrium, allow savers to reallocate the portfolio from savings towards cheaper housing. Consequently, the higher the risk aversion, the smaller the overall welfare losses for savers.

5 Conclusions

Using a dynamic general equilibrium model calibrated to the US economy, we evaluate the quantitative effects of (i) aggregate income shocks and (ii) shocks to financial intermediation on house prices and on welfare of two types of agents: leveraged agents (borrowers) and non-leveraged agents (savers).

The quantification of welfare costs related to the US Great Recession along this cross sectional heterogeneity complements recent contributions (Glover, Heathcote, Krueger, and Ríos-Rull, 2011; Hur, 2012) and adds a new mechanism stemming from shocks to the capital market. Our set-up is in fact suitable to evaluate the welfare consequences of credit supply shocks in a recession, and complements other recent studies by exploring the effect of financial intermediation shocks in a model with endogenous collateral constraints.

We find that, following a the Great Recession, all the agents in the economy experience a welfare loss, and borrowers lose always more than savers. This finding comes from the fact that savers, being unconstrained, change the portfolio allocation and benefit from buying the deflated asset (housing). We find that a financial intermediation shock that occurs in a recession, forces borrowers to de-leverage, and amplifies the re-distribution from savers to borrowers, both in terms of financial wealth and in terms of welfare. Finally, we find that a model where borrowers are always borrowing constrained the non-linearity in the amplification mechanism coming from the financial intermediation shock vanishes and the effects on wealth and welfare

are quantitatively smaller.

We provide a number of sensitivity checks. While the distributive effects (both in terms of financial wealth and welfare) between borrowers and savers are decreasing in the substitutability between housing and non-durable consumption, the drop in house prices is bigger the higher the risk aversion, leading to a proportional increase in redistribution.

Although the paper focuses on the distributive effects of the Great Recession on borrowers and savers, we do not explicitly consider the possibility that borrowers can default on their debt obligations. While this could potentially be beneficial for borrowers at the expenses of the creditors, empirical evidence suggests that such a feature of the U.S. Great Recession was related to a subset of borrowers, the sub-primers, which are not explicitly modelled here. Adding this third heterogeneity to the analysis is, in our opinion, an interesting avenue for future research.

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Appendix

A Data

The series of federal funds rate, one year mortgage interest rate in Figure 1 are from Federal Reserve Economic Data, St. Louis (the release for the interest rate series is the Primary Mortgage Market Survey by Freddie Mac); all series are at quarterly frequency and seasonally adjusted. The series for house prices is the National Composite Home Price Index for the United States

(the release is by S&P/Case-Shiller Home Price Indices). The spread has been calculated as the algebraic difference between one year mortgage interest rate and the federal funds rate each quarter.

We calculate housing wealth as percentage of US nominal GDP (yearly) by using historical data of the flows of funds tables from the Board of Governors. US nominal GDP is from the Bureau of Economic Analysis. Our definition of housing wealth includes real estate at market value of households, non-profit and non-financial non-corporate business.

The micro-data used for the calibration of the relative wealth distribution of borrowers and the leverage ratio are provided by the 1998 to 2007 waves of Survey of Consumer Finances (SCF). Unfortunately, the SCF does not provide information on the precise date when households were interviewed; consequently, we assume that the observed portfolios in 2007 reflect the distribution of household net worth in the end of 2007. Averaging for all the waves between 1998 and 2007 helps in targeting data moments that are not strongly influenced by the years preceding the Great Recession. Surveyed households have been partitioned between borrowers and savers depending on their net asset position; the net asset position is defined as the sum of savings bonds, directly held bonds, cash value of life insurances, certificates of deposits, quasi-liquid retirement accounts and all other types of transaction accounts (we consider this aggregated values as deposits in to the model) minus the debt secured by primary residence (mortgages, home equity loans, etc.) and the debt secured by other residential property, credit card debt and other forms of debt (we refer to this aggregated values as debt into the model). If the net asset position is positive we consider the household as saver type of agent in our model economy, otherwise we consider her as a borrower. The reason to use a broad definition of aggregate deposits and debt on the data counterpart is that it is difficult to target borrowers and savers by strictly restricting the attention to some forms of debt rather than others. We moreover define the net wealth per capita as the sum of the net asset position and the value of primary residence and other residential properties, for both leveraged and net savers. We finally aggregate the net wealth for both groups (borrowers and savers) and we calculate the relative net wealth of borrowers as the ratio between their net wealth over the total net wealth of the economy. The leverage ratio of the borrowers is instead obtained as the weighted average mean (using SCF sample weights) of the net asset position over the value of primary and secondary residences. For

both the calculation of the net wealth position and the leverage ratio we pre-multiply the value of aggregate deposits by the average value the theta shock. This is done to keep the targeted values for the calibration coherent with the model specification. For these values we determine a range that could be matched by the model. The reference bound is obtained by cutting the 5% tails of the distribution of the net worth as defined by each waves of SCF. This is done to cut the extreme observations that may bias the average values of the net worth of the US economy. We want in fact to avoid the possibility of including in the range of borrowers households that feature big positions in the stock market and housing market and little savings. With such a definition we obtain values for the relative wealth share and leverage of respectively 11.3% and 44%). The other bound of the range is represented by the value obtained using the whole sample of households. With such a definition we obtain a value of relative wealth share of the borrowers of 13.6% and a leverage ratio of 49%.

B Numerical Details

The employed algorithm is our adoption of the time-iteration procedure with linear interpolation as in Grill and Brumm (2010). As we have only two agents, a fine enough grid for wealth is enough to deliver satisfactory small Euler errors, so we do not adapt the grid around the points where the collateral constraint is binding, as proposed by Grill and Brumm (2010).

B.1 Equilibrium conditions

We want to describe the equilibrium in our economy in terms of policy functions that map the current state into current policies. Furthermore, we want to focus on recursive mappings, that is, time-invariant functions that satisfy the period-by-period first-order equilibrium conditions. In what follows, we characterize these equilibrium conditions in every detail. For each agent $i = b, s$, denote by $\nu_i(w, z)$ the Lagrange multiplier with respect to her budget constraint and by $\phi_i(w, z)$ the Kuhn-Tucker multiplier attached to her collateral constraint, respectively. In addition, we treat saving and debt as two separate assets: saving is an asset in which the agent only can take long positions, $s_i \geq 0$, and debt is an asset with return R_D in which agents can only take short positions, $d_i \leq 0$. Denote the Kuhn-Tucker multipliers attached to these inequalities by χ_i and μ_i , respectively. Then, for each tuple consisting of wealth and exogenous state today,

$\sigma = (w, z)$, (time-invariant) policy and pricing functions have to satisfy the following system of equations (we will show below how to solve for these time-invariant functions):

- Agent's first order conditions

$$\begin{aligned}
u_1(c_i(\sigma), h_i(\sigma)) - \nu_i(\sigma) &= 0 \\
u_2(c_i(\sigma), h_i(\sigma)) - q(\sigma)\nu_i(\sigma) &= 0 \\
-\nu_i(\sigma) + \beta^i E[\nu_i(\sigma^+) | \sigma] R(\sigma) + \chi_i(\sigma) &= 0, \quad i = s, b \\
-\nu_i(\sigma) + \beta^i E[\nu_i(\sigma^+) | \sigma] R_D + \phi_i(\sigma) R_D(w, z) - \mu_i(\sigma) &= 0 \\
-\nu_i(\sigma)q(\sigma) + u_2(c_i(\sigma), h_i(\sigma)) + \\
+\beta^i E[\nu_i(\sigma^+)q(\sigma^+) | \sigma] + \phi_i(\sigma)mE[q(\sigma^+) | \sigma] &= 0
\end{aligned}$$

- Agent's budget constraints

$$\begin{aligned}
n_b y(s) + n_b \Upsilon(\sigma) + w \cdot q(\sigma) - d_b(\sigma) - s_b(\sigma) - q(\sigma)h_b(\sigma) - c_b(\sigma) &= 0 \\
n_s y(s) + n_s \Upsilon(\sigma) + (1 - w) \cdot q(\sigma) - d_s(\sigma) - s_s(\sigma) - q(\sigma)h_s(\sigma) - c_s(\sigma) &= 0
\end{aligned}$$

NB: Here we used already the definition for the borrower's wealth share and rewrote the budget constraints in these terms (see the law of motion for wealth below as a reminder for how we defined the wealth share).

- Zero profits in the financial sector

$$\theta(s) \cdot R_D(\sigma) - R(\sigma) = 0$$

- Market clearing in housing and financial sector

$$\begin{aligned}
h_s(\sigma) + h_b(\sigma) - 1 &= 0 \\
d_b(\sigma) + d_s(\sigma) + \theta(s) \cdot (s_b(\sigma) + s_s(\sigma)) &= 0
\end{aligned}$$

- Transfers

$$\Upsilon(\sigma) - (1 - \theta(s))(s_b(\sigma) + s_s(\sigma)) = 0$$

- Complementary slackness conditions

$$\begin{aligned} \mu_i(\sigma) &\geq 0, d_i(\sigma) \geq 0, & \mu_i(\sigma) \perp d_i(\sigma) \\ \chi_i(\sigma) &\geq 0, s_i(\sigma) \geq 0, & \chi(\sigma) \perp s_i(\sigma), \quad i = s, b \\ \phi_i(\sigma) &\geq 0, CC_i(\sigma) \geq 0, & \phi_i(\sigma) \perp CC_i(\sigma) \end{aligned}$$

where $CC_i(\cdot)$ is the collateral constraint of agent i , that is,

$$CC_i(\sigma) \equiv R_D(\sigma)d_i(\sigma) + mE[q(\sigma^+)|\sigma]h_i(\sigma) \geq 0$$

- Implicit “Law of motion” for borrower’s wealth share

$$w^+(\sigma, z^+) \equiv \frac{R_D(\sigma)d_b(\sigma) + R(\sigma)s_b(\sigma) + q(w^+(\sigma, z^+), z^+)h_b(\sigma)}{q(w^+(\sigma, z^+), z^+)}.$$

B.2 Algorithm

The structure of the above period-by-period equilibrium conditions can be summarized as follows: Given a guess for the policy and pricing functions tomorrow - let’s denote them by f^{prime} - we can compute the expectations in the agents’ first order conditions; the functions that map current states in current policies - denoted by f - then are obtained by solving the static system of non-linear given in the previous subsection. More formally, the structure of the problem can be summarized as follows. For all tuples $\sigma = (w, z)$, we have

$$\psi(f^{prime})(\sigma, f(\sigma), \mu(\sigma)) = 0, \quad \zeta(\sigma, f(\sigma)) \geq 0 \perp \mu(\sigma) \geq 0.$$

The system of equations $\psi[f^{prime}](\cdot)$ contains first order conditions of agents and the financial sector and market clearing conditions. The function $\zeta(\cdot)$ contains the sign restrictions and collateral constraints and $\mu(\cdot)$ denote the respective Kuhn-Tucker multipliers. A recursive policy

function f then solves $\psi[f](\sigma, f(\sigma)\mu(\sigma)) = 0$ such that complementary slackness conditions are satisfied. The time iteration algorithm defined below finds the approximate recursive policy function iteratively.

In each iteration, taken as given a guess for f^{prime} , we obtain f by solving the above system of equations and then update our guess by interpolating the obtained policy function on the implicitly defined wealth tomorrow. The following box summarizes our algorithm in form of Pseudo-code:

1. Select a grid \mathcal{W} , an initial guess f^{init} and an error tolerance ϵ . Set $f^{prime} = f^{init}$.
2. Make one time-iteration step:
 - (a) For all $\sigma = (w, z)$, where $w \in \mathcal{W}$, find the function $f(\sigma)$ that solves

$$\psi(f^{prime})(\sigma, f(\sigma), \mu(\sigma)) = 0, \quad \zeta(\sigma, f(\sigma)) \geq 0 \perp \mu(\sigma) \geq 0.$$

- (b) Use the solution f and the guess f^{prime} to update wealth tomorrow and interpolate f on the obtained values for wealth tomorrow.
3. If $\|f - f^{prime}\| < \epsilon$, go to step 4. Else set $f^{prime} = f$ and repeat step 2.
4. Set numerical solution \tilde{f} equal to the solution of the infinite horizon problem, $\tilde{f} = f$.

B.3 Kuhn-Tucker equations (Garcia-Zangwill trick)

In each grid point - given the guesses of the policy functions next period - we have to solve a system of nonlinear equations, containing both inequalities and equalities. The period-by-period equilibrium conditions are basically standard Kuhn-Tucker (K-T) conditions. In order to employ standard non-linear equation solvers like `fsolve` in Matlab or Ziena's `Knitro`, it is computationally more stable to eliminate the inequalities and recast the problem as a system consisting of equations only. In this section we describe how to do this. In general, we can write Kuhn-Tucker conditions to any convex NLP problem as:

$$\begin{aligned}
\Delta f(x)' + \sum_{j=1}^r \lambda_j \Delta g_j(x)' + \sum_{j=1}^s \mu_j \Delta h_j(x)' &= 0 \\
\lambda_j \geq 0, g_j(x) \geq 0, & \quad j = 1, \dots, r \\
\lambda_j g_j(x) = 0, & \quad j = 1, \dots, r \\
h_j(x) = 0, & \quad j = 1, \dots, s
\end{aligned} \tag{15}$$

plus a constraint qualification restriction (CQ). The system in (15) are mixtures of equalities and inequalities. Since inequalities tend to be cumbersome and potentially prevent numerical software to solve the NLP via path-following, we will rewrite the K-T conditions so that it is a system consisting solely of equations (Zangwill and Garcia, 1981). The reformulation is as follows. Let k be a positive integer, and given $\alpha \in \mathbb{R}^1$, define:

$$\begin{aligned}
\alpha^+ &= [\max\{0, \alpha\}]^k \\
\alpha^- &= [\max\{0, -\alpha\}]^k.
\end{aligned}$$

Hence, we always have $\alpha^+ \geq 0$, $\alpha^- \geq 0$, and $\alpha^+ \alpha^- = 0$. Note also that both variables, α^+ and α^- , are $(k-1)$ -continuously differentiable. Using this transformation, we can recast the K-T conditions and create the Kuhn-Tucker *equations* (Zangwill and Garcia, 1981):

$$\begin{aligned}
\Delta f(x)' + \sum_{j=1}^r \alpha_j^+ \Delta g_j(x)' + \sum_{j=1}^s \mu_j \Delta h_j(x)' &= 0 \\
\alpha_j^- - g_j(x) &= 0, \quad j = 1, \dots, r \\
h_j(x) &= 0, \quad j = 1, \dots, s
\end{aligned} \tag{16}$$

where $\alpha = (\alpha_1, \dots, \alpha_r) \in \mathbb{R}^r$ and (α^+, α^-) are defined as above. Note that the (K-T) equations defined here are precisely equivalent to the K-T conditions in (15). In particular, if (x^*, α^*, μ^*) satisfies the K-T equations, then (x^*, λ^*, μ^*) satisfies the the K-T conditions with $\lambda_j^* \equiv (\alpha_j^*)^+$, $j = 1, \dots, r$. Conversely, if (x^*, λ^*, μ^*) satisfies the K-T conditions in (15), then

(x^*, α^*, μ^*) satisfies the K-T equations in (16) with

$$\alpha_j^* \equiv \begin{cases} (\lambda_j^*)^{1/k} & \text{if } g_j(x^*) = 0 \\ -(g(x^*)_j)^{1/k} & \text{if } g_j(x^*) > 0 \end{cases} \quad j = 1, \dots, r.$$

B.4 Numerical Accuracy

In order to measure the accuracy of our approximation procedure, we calculate two statistics: first, we compute the relative Euler errors along the equilibrium path for very long time series. Second, for each exogenous shock, we randomly draw 3000 points from the wealth grid and compute the relative Euler Errors. To summarize the findings, in all simulated models, the maximum relative Euler Error is 3e-5 (or -4.5 in log(10)-scale). This implies that an agent, using our approximation of the equilibrium policy functions, would lose 30 Dollars for each million spent. It is important to compare this number to the welfare gains we obtain in the benchmark model. The borrowers' welfare loss on impact of an financial intermediation shock is 0.07 percentage points, that is, in log(10) scale equal to -3.15. This number is one order of magnitude bigger, so even when netting these numbers by the mistake agents make, we conclude that our quantitative findings are still valid.

