EUI WORKING PAPERS

Identifying Monetary Policy Shocks via Changes in Volatility

MARKKU LANNE
and
HELMUT LÜTKEPOHL

EUROPEAN UNIVERSITY INSTITUTE

Department of Economics

Identifying Monetary Policy Shocks via Changes in Volatility

MARKKU LANNE

and

HELMUT LÜTKEPOHL

Identifying Monetary Policy Shocks via Changes in Volatility[1]

Markku Lanne

European University Institute,
University of Jyväskylä, RUESG and HECER
School of Business and Economics,
P.O. Box 35, FIN-40014 University of Jyväskylä, FINLAND

and

Helmut Lütkepohl

Department of Economics, European University Institute
Via della Piazzuola 43, I-50133 Firenze, ITALY

Abstract. A central issue of monetary policy analysis is the specification of monetary policy shocks. In a structural vector autoregressive setting there has been some controversy about which restrictions to use for identifying the shocks because standard theories do not provide enough information to fully identify monetary policy shocks. In fact, to compare different theories it would even be desirable to have over-identifying restrictions which would make statistical tests of different theories possible. It is pointed out that some progress towards over-identifying monetary policy shocks can be made by using specific data properties. In particular, it is shown that changes in the volatility of the shocks can be used for identification. Based on monthly US data from 1965-1996 different theories are tested and it is found that associating monetary policy shocks with shocks to nonborrowed reserves leads to a particularly strong rejection of the model whereas assuming that the Fed accommodates demand shocks to total reserves cannot be rejected.

Key Words: Monetary policy, structural vector autoregressive analysis, vector autoregressive process, impulse responses

JEL classification: C32

---

1 Introduction

Over the last two decades, a large literature has developed which evaluates monetary policy within a structural vector autoregressive (SVAR) framework (see, e.g., Christiano, Eichenbaum and Evans (1999), henceforth CEE). A central question in evaluating monetary policy is how to identify the monetary policy shocks. Various competing economic theories have been used to formulate restrictions which help in identifying the shocks. Unfortunately, the implied restrictions do not suffice for a full identification of the shocks in some of these models. Hence, additional restrictions have to be formulated which are often ad hoc and do not have a convincing theoretical foundation. Even if theoretical considerations suffice to identify the monetary policy shocks, there may be no over-identifying information which could be used to test different theories against the data.

In this paper we will argue that sometimes the statistical properties of the data can be used to identify the shocks. In particular, using an idea of Klein and Vella (2000), Rigobon (2003) has shown that a change in volatility in the shocks can be used as identifying information. We will adapt his result to our needs. Our general model setup is that of CEE, that is, we use an SVAR model. These authors also argue that there may have been changes in the volatility of the US monetary policy shocks over their sample period from 1965-1996 but that the remaining structure of the model is found to be time invariant. Thus we will also assume that the DGP is a VAR with constant parameters apart from changes in the volatility of shocks. This assumption will be used to identify the shocks and thereby we can test theoretical assumptions that cannot be checked by formal statistical tests in the CEE framework.

More specifically, we will consider a monthly VAR model for the US with six variables, real GDP, the GDP deflator, a spot commodity prices index, the federal funds rate, nonborrowed reserves and total reserves. Such a model was also considered by Bernanke and Mihov (1998b) (henceforth BM). The first three variables are viewed as nonpolicy variables whereas the monetary policy shocks are determined from the last three variables. BM consider a model for the federal funds market to find identifying restrictions for the monetary policy shocks. Unfortunately, this model does not fully identify the shocks and CEE question the additional restrictions imposed. CEE also find evidence for a change in the volatility of the monetary policy shocks. We will confirm this finding with further statistical tests and then use these data properties to over-identify the shocks. Thereby the assumptions of different models which can be embedded in this framework become testable. Our setup will enable us to perform such tests and we find that the data are at odds

with some identifying schemes which have been used in previous publications whereas other identification schemes cannot be rejected. Thus, we are able to use statistical tools for discriminating between competing models.

The structure of the paper is as follows. In the next section the model setup is presented and identification issues are discussed. In Section 3 the empirical analysis is considered. Conclusions are drawn in Section 4. A mathematical result concerning the identification of shocks via changes in the volatility and details of our estimation method are presented in the Appendix.

2 Model Setup

2.1 The Statistical Model

The general setup is an SVAR model. More precisely, an AB-model in the terminology of Amisano and Giannini (1997) is used (see also Lütkepohl (2005, Chapter 9)):

$$\mathsf{A}y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + \mathsf{B}\varepsilon_t, \tag{2.1}$$

where y_t is a K-dimensional vector of observable variables, ε_t is a K-dimensional vector of structural innovations with mean zero and identity covariance matrix, i.e., $\varepsilon_t \sim (0, I_K)$, and A, B and A_i $(i = 1, \ldots, p)$ are $(K \times K)$ parameter matrices. The model in (2.1) is a structural form with corresponding reduced form error term $u_t = \mathsf{A}^{-1}\mathsf{B}\varepsilon_t \sim (0, \mathsf{A}^{-1}\mathsf{B}\mathsf{B}'\mathsf{A}^{-1'})$. The reduced form error terms can be estimated from the data. To obtain estimators of the structural errors ε_t, a one-to-one mapping from the reduced form error covariance matrix to A and B is required. Identifying restrictions have to be imposed on A and B to obtain a unique relation.

2.2 Economic Setup

In our empirical model the observable variables will be divided in two groups. The first one contains variables whose current values are in the monetary authority's information set and are not influenced instantaneously by policy decisions. The second group contains variables which are determined within the money market. The first set of variables is $y_{1t} = (gdp_t, p_t, pcom_t)'$, where gdp_t, p_t and $pcom_t$ denote logs of real GDP, the log implicit GDP deflator and an index of commodity prices, respectively. The money market variables are collected in $y_{2t} = (TR_t, NBR_t, FF_t)'$, where TR_t, NBR_t and FF_t denote total reserves, nonborrowed reserves and the federal funds rate, respectively. Thus, $K = 6$ and y_{1t} and y_{2t} are both three-dimensional, as in BM.

Our partitioning of $y_t = (y'_{1t}, y'_{2t})'$ implies that we can choose

$$\mathsf{A} = \begin{bmatrix} I_3 & 0 \\ -\mathsf{A}_{21} & I_3 \end{bmatrix}, \tag{2.2}$$

and $v_t = (v'_{1t}, v'_{2t})' = \mathsf{B}\varepsilon_t$ has a block-diagonal covariance matrix. Hence, B is also block-diagonal,

$$\mathsf{B} = \begin{bmatrix} \mathsf{B}_{11} & 0 \\ 0 & \mathsf{B}_{22} \end{bmatrix}, \tag{2.3}$$

where the B_{ii}'s $(i = 1, 2)$ are both (3×3). In this model setup the matrix A_{21} can be estimated by OLS from

$$y_{2t} = \mathsf{A}_{21}y_{1t} + A_{2,1}y_{t-1} + \cdots + A_{2,p}y_{t-p} + v_{2t}, \tag{2.4}$$

where $A_{2,i}$ consists of the last three rows of A_i $(i = 1, \ldots, p)$. Moreover, since we are just interested in identifying the monetary shocks, we just need to recover the money market innovations ε_{2t}. In other words, we need restrictions which ensure a one-to-one mapping from $E(v_{2t}v'_{2t}) = \mathsf{B}_{22}\mathsf{B}'_{22}$ to B_{22}.

Following BM and CEE the demand for total reserves is specified as

$$TR_t = -\alpha FF_t + f_{TR}(\text{policy information}) + \sigma_d \varepsilon_t^d,$$

the demand for borrowed reserves is given by

$$BR_t = \beta FF_t - \gamma NBR_t + f_{BR}(\text{policy information}) + \sigma_b \varepsilon_t^b$$

and the Fed policy rule for setting nonborrowed reserves is

$$NBR_t = f_{NBR}(\text{policy information}) + \phi^d \sigma_d \varepsilon_t^d + \phi^b \sigma_b \varepsilon_t^b + \sigma_s \varepsilon_t^s,$$

where ε_t^s is the exogenous monetary policy shock. The policy information consists of all lagged variables and the current values of gdp_t, p_t and $pcom_t$. The functions $f_*(\cdot)$ are all linear functions and α, β, γ, ϕ^d, ϕ^b, σ_d, σ_b and σ_s are parameters.

Using $TR = NBR + BR$, CEE derive from these relations that

$$\mathsf{B}_{22} = \begin{bmatrix} \sigma_d \dfrac{\beta - \phi^d \alpha \gamma + \phi^d \alpha}{\beta + \alpha} & -\alpha \sigma_s \dfrac{\gamma - 1}{\beta + \alpha} & -\alpha \sigma_b \dfrac{-1 + \phi^b \gamma - \phi^b}{\beta + \alpha} \\[3ex] \sigma_d \phi^d & \sigma_s & \sigma_b \phi^b \\[3ex] \sigma_d \dfrac{\phi^d \gamma - \phi^d + 1}{\beta + \alpha} & \sigma_s \dfrac{\gamma - 1}{\beta + \alpha} & \sigma_b \dfrac{-1 + \phi^b \gamma - \phi^b}{\beta + \alpha} \end{bmatrix}. \tag{2.5}$$

Thus, the nine elements of B_{22} are determined by eight free parameters $\psi = (\alpha, \beta, \gamma, \phi^d, \phi^b, \sigma_d^2, \sigma_b^2, \sigma_s^2)'$. These parameters are not identified in a model with time invariant covariance matrix $\Sigma_{v2} = E(v_{2t}v_{2t}')$ because this matrix is symmetric and, thus, has six distinct elements only. Hence, further restrictions are needed. CEE consider different specifications of policy shocks and derive the following restrictions (see also BM):

- *FF policy shock*: $\phi^d = 1/(1 - \gamma)$ and $\phi^b = -\phi^d$. These restrictions mean that the monetary shocks are induced through the federal funds rate and correspond to the assumption of Bernanke and Blinder (1992) that the Fed targets the federal funds rate.

- *NBR policy shock*: $\phi^d = \phi^b = 0$. The assumption that policy shocks can be associated with the errors in the equation for nonborrowed reserves was made by Christiano and Eichenbaum (1992).

- *NBR/TR policy shock*: $\alpha = \phi^b = 0$. BM derived this restriction from the assumption made by Strongin (1995) that shocks to total reserves are demand shocks which are accommodated by the Fed.

- *BR policy shock*: $\phi^d = 1$, $\phi^b = \alpha/\beta$ and $\gamma = 0$. These restrictions are obtained if the Fed is assumed to target borrowed reserves, as e.g. in Cosimano and Sheehan (1994).

Unfortunately, these restrictions still do not over-identify the shocks. Consequently, they are not sufficient to actually test the underlying assumptions against the data in CEE's framework. Therefore BM assume in addition that $\gamma = 0$ to obtain over-identified models. As CEE pointed out, such an approach is unsatisfactory because rejection of a particular set of restrictions may then be caused by the ad hoc assumption rather than false restrictions derived from theory.

In our empirical analysis there is, however, a way out of this dilemma. Both BM and CEE find that over the sample period considered there is some change in the structure of the relations. BM actually fit models to different sample periods while CEE find that there may have been a change in the volatility of the shocks whereas the remaining structure is unaffected. Even with the minimal changes diagnosed by CEE we may be able to identify B_{22} as we will argue now.

2.3 Identification of Shocks via a Change in Volatility

Suppose there is just one change in the volatility of the shocks during the sample period, say in period T_B, so that

$$E(v_{2t}v'_{2t}) = \begin{cases} \mathsf{B}_{22}\mathsf{B}'_{22} & \text{for } t = 1, \ldots, T_B - 1, \\ \mathsf{B}_{22}\Omega\mathsf{B}'_{22} & \text{for } t = T_B, \ldots, T, \end{cases} \tag{2.6}$$

where $\Omega = \text{diag}(\omega_1, \omega_2, \omega_3)$ is a (3×3) diagonal matrix with positive diagonal elements ω_i and T is the sample size. Here the diagonal elements of Ω represent the changes in the variances of the shocks after the possible change in volatility has occurred. If the ω_i's are different from one, there is a change in volatility. Proposition A in the Appendix implies that B_{22} is (locally) identified if all ω_i's are distinct. It generalizes a result by Rigobon (2003) for bivariate systems. Thus, all we need to know is whether the volatility changes in different shocks are proportional. If they are not, then B_{22} is identified. In fact, the volatility in one of the shocks may not change at all, that is, one of the ω_i's may be unity. The crucial condition is that they are all distinct. If there are other restrictions on B_{22}, as in the present analysis, identification is already obtained if there are enough distinct ω_i's. The advantage of this setup is that changes in the variances can be investigated with statistical means, as we will see in Section 3.2, and, hence, we do not have to rely exclusively on information from economic theory to ensure identification.

Local rather than global identification is obtained only in this case because it is always possible to reverse the signs of all elements in a single column of B_{22} without affecting the likelihood. For practical purposes this is no problem, of course, because it just means that, to obtain identification, we have to specify whether a shock is positive or negative. For estimation and deriving asymptotic results local identification is sufficient.

In the empirical analysis we will actually allow for the possibility of various changes in volatility. Suppose there are $n + 1$ different regimes and the covariances in the different regimes are $\mathsf{B}_{22}\mathsf{B}'_{22}$, $\mathsf{B}_{22}\Omega_1\mathsf{B}'_{22}$, $\ldots$, $\mathsf{B}_{22}\Omega_n\mathsf{B}'_{22}$, where the Ω_i's are all diagonal matrices. Then local identification is ensured, for example, if the diagonal elements in only one of the Ω_i matrices are all distinct. Again this result is analogous to a bivariate result of Rigobon (2003).

One may argue that the assumption of a time invariant B_{22} is a strong one because this matrix represents the instantaneous effects of shocks and these may change as well if the volatility changes. Clearly, there may even be changes in some or all of the other VAR parameters. Such changes can be checked by formal statistical tests, however. Of course, our model is useful only if it is consistent with the data. We have used the rather restrictive

change in volatility here because even such a small change suffices to get identification and it was argued by CEE and Bernanke and Mihov (1998a) that structural changes found by other authors in the data set underlying our empirical study may have been due to just this kind of change rather than a change in the whole dynamic structure. We will address this issue in the empirical analysis. Of course, identification of the shocks can also be obtained if more substantial structural changes have occurred. In that case the impulse responses may be affected, however, and this fact has to be taken into account in the evaluation of the model.

If a change in the volatility of shocks is diagnosed and identification of B_{22} is ensured by the data properties, then all the restrictions from the economic theories are over-identifying and, hence, can be tested. Since there are only eight elements in the vector of economic parameters ψ while B_{22} has nine elements, there is in fact already one restriction implied by the overall general model which nests the others, provided the ω_i's of at least one Ω_j matrix are distinct. If there are only two different ω_i's, then an over-identifying restriction may not be available in the general model while the additional restrictions implied by the different theories can still be tested under suitable conditions. In the next section we present the empirical analysis and discuss these issues in the context of our model.

3   Empirical Analysis

3.1   The Data

Monthly US data from BM for the period 1965M1-1996M12 are used in our empirical analysis. The monthly data for gdp and p are constructed from lower frequency data. These data were also used by CEE and BM. Hence, our results are directly comparable to those of the earlier studies. Using the same sample period, although longer time series are available, has the advantage that the results are not driven by the extended sample period but differences to the other studies are a direct consequence of the alternative methods used.

3.2   Estimation and Testing

Estimation under our assumption of a change in volatility is done by a multi-step iterative procedure. In the first step equation wise OLS is applied to a model such as (2.4) with an additional constant term. We denote the

residuals by $\tilde{v}_{2t}$ and define

$$\widetilde{\Sigma}_1 = \frac{1}{T_B - 1} \sum_{t=1}^{T_B - 1} \tilde{v}_{2t}\tilde{v}_{2t}' \quad \text{and} \quad \widetilde{\Sigma}_2 = \frac{1}{T - T_B + 1} \sum_{t=T_B}^{T} \tilde{v}_{2t}\tilde{v}_{2t}'.$$

Then the following concentrated log likelihood type function is maximized with respect to ψ, ω_1, ω_2 and ω_3:

$$\log l = -\frac{T_B - 1}{2} \left(\log |\mathsf{B}_{22}\mathsf{B}_{22}'| + \text{tr} \left\{ \widetilde{\Sigma}_1 (\mathsf{B}_{22}\mathsf{B}_{22}')^{-1} \right\} \right)$$

$$-\frac{T - T_B + 1}{2} \left(\log |\mathsf{B}_{22}\Omega\mathsf{B}_{22}'| + \text{tr} \left\{ \widetilde{\Sigma}_2 (\mathsf{B}_{22}\Omega\mathsf{B}_{22}')^{-1} \right\} \right) \qquad (3.1)$$

and thereby we obtain estimators $\widetilde{\mathsf{B}}_{22}$ and $\widetilde{\Omega}$ of B_{22} and Ω, respectively.

Although (3.1) looks like a Gaussian log likelihood function, the OLS estimators of the VAR coefficients from (2.4) are not ML estimators due to the assumed heteroskedasticity. Therefore, in the next step the estimators $\widetilde{\mathsf{B}}_{22}$ and $\widetilde{\Omega}$ obtained in this way are used to perform a feasible multivariate GLS estimation of the VAR coefficients in (2.4). These are then used again in (3.1) to obtain new estimates of the structural parameters and this procedure is iterated. Gaussian ML estimators are obtained upon convergence. Details of this estimation procedure are provided in the Appendix.

Although we have presented the estimation method for two different regimes only for convenience, it is straightforward to apply it when there are more than two regimes. In our empirical analysis we have used models with up to three different regimes. Moreover, Rigobon (2003) shows that a slight misspecification of the times where the regimes change, does not affect the identification so that the time invariant parameters can be estimated consistently under usual assumptions even in this case.

Having the ML estimators opens up the possibility to perform likelihood ratio (LR) tests. Some tests are of particular importance in the present context. Assuming again two different regimes for illustrative purposes and denoting the reduced form residual covariance matrices in the two regimes by Σ_1 and Σ_2, respectively, a test of interest is, for example,

$$H_0 : \Sigma_1 = \Sigma_2 \quad \text{vs.} \quad H_1 : \Sigma_1 \neq \Sigma_2. \qquad (3.2)$$

In other words, the null hypothesis specifies that there is no regime change. Since we consider reduced form parameters here, there is no identification problem. For our three-equation model (2.4) the asymptotic null distribution of the corresponding LR statistic is $\chi^2(6)$, provided that LR tests have standard asymptotic properties. Given that the data generation process may

have unit roots and may be cointegrated, the asymptotic properties of LR tests are in general not necessarily standard. For the present case, standard asymptotic properties are obtained, however, because the cointegration properties do not affect the estimator of the residual covariance matrix asymptotically (see, e.g., Lütkepohl (2005, Chapter 7)). The test is in fact a Chow type test. Its small sample properties may not be ideal, as pointed out by Candelon and Lütkepohl (2001). According to these results the test may reject a true null hypothesis too often in small samples. This property may be useful to keep in mind in our empirical analysis.

If the null hypothesis in (3.2) is rejected, a further hypothesis of interest will be that only the variances have changed while the correlation structure and hence the B_{22} matrix is constant across regimes. Recall that some previous authors have indicated that only the volatility of the shocks and not the impulse responses of the system may have changed. To check that hypothesis we may use a principle components decomposition $\Sigma_i = P_i \Omega_i P_i'$ $(i = 1, 2)$, where $\Omega_i = \mathrm{diag}(\omega_{i1}, \omega_{i2}, \omega_{i3})$ with ω_{ik} being the kth largest eigenvalue of Σ_i and P_i is the corresponding matrix of eigenvectors. Note that P_i is an orthogonal matrix. The principal components decomposition is locally unique if all ω_{ik}'s are distinct, that is, P_i is unique apart from a possible reversal of signs of its columns (e.g., Magnus and Neudecker (1988, Chapter 17)). Thus, we can test

$$H_0 : P_1 = P_2 \quad \text{vs.} \quad H_1 : P_1 \neq P_2, \tag{3.3}$$

provided Ω_1 and Ω_2 both have distinct diagonal elements. Because the P_i's are orthogonal (3×3) matrices, the corresponding LR statistic has an asymptotic $\chi^2(3)$ distribution under H_0. Since the value of the likelihood function does not change if any other decomposition of the covariance matrices is used, it is clear that a test of (3.3) is effectively a test of a time invariant B_{22} matrix.

3.3 Results

We have estimated a set of different VAR models for the levels variables by the ML procedure described in the previous subsection. All models have 13 lags as in BM's study. In Table 1 LR tests for the number of regime changes in the volatility are provided. Different authors have expressed a range of views and presented corresponding evidence on where regime shifts may have occurred. There seems to be some consensus in the literature that the Volcker era differs from the pre- and post-Volcker periods, at least as far as monetary policy is concerned. Therefore we consider structural breaks in 1979M10 and 1984M2. These breaks were also considered by BM.

Table 1: LR tests for regime changes

H_0 (type (3.2))	Break(s)	Test statistic	p-value
$\Sigma_1 = \Sigma_2 = \Sigma_3$	1979M10, 1984M2	334.617	2.457e-64
$\Sigma_1 = \Sigma_2$	1979M10	26.026	0.0002
$\Sigma_2 = \Sigma_3$	1984M2	220.535	8.003e-45
$\Sigma_1 = \Sigma_3$	1979M10, 1984M2	260.596	2.227e-53

H_0 (type (3.3))	Break(s)	Test statistic	p-value
$P_1 = P_2 = P_3$	1979M10, 1984M2	39.068	6.941e-7
$P_1 = P_2$	1979M10	10.103	0.0177
$P_2 = P_3$	1984M2	3.243	0.3557
$P_1 = P_3$	1979M10, 1984M2	33.773	2.212e-7

We have checked the break dates and present the results in Table 1. They confirm that using models with regime changes in 1979M10 and 1984M2 is reasonable. On the basis of the p-values we clearly reject constant reduced form covariance matrices throughout the full sample period at any common significance level. Notice that in the table, Σ_1, Σ_2 and Σ_3 denote the residual covariance matrices corresponding to the periods until 1979M9, 1979M10 – 1984M1 and from 1984M2 – 1996M12, respectively. Even though the tests may be biased in small samples and reject too often, the p-values are too small to defend constant covariance matrices.

The tests in the lower half of Table 1 check whether the correlation structure associated with the residual covariance matrices is constant through time so that the nonconstancy is due only to changes in volatility. In other words, hypotheses of the type (3.3) are tested. It turns out that there may in fact be a change in the correlation structure in 1979M10 whereas there is little evidence for such a change in 1984M2. This result is in line with CEE's view that the crucial difference between the monetary shocks in the Volcker- and post-Volcker-periods is in the higher volatility in the former regime. Thus, there is some evidence that the 1984M2 break is consistent with our model assumptions while the pre-Volcker break may have induced more substantial changes in the reduced form error term.

To identify the shocks it is, of course, enough that there is one break point of the sort discussed in Section 2.3. Therefore the following analysis is based on a model where all three Σ_i's ($i = 1, 2, 3$) are distinct and $P_2 = P_3$. Thus, we consider a model where $\Sigma_2 = \mathsf{B}_{22}\mathsf{B}'_{22}$ and $\Sigma_3 = \mathsf{B}_{22}\Omega\mathsf{B}'_{22}$ with $\Omega =$

Table 2: Estimates of structural parameters with standard errors in parentheses

Parameter	unrestricted		FF		NBR		NBR/TR		BR	
α	0.3165	(0.1639)	0.1967	(0.1539)	68.4312	(72.8482)	0.0000		0.1912	(0.1504)
β	3.8785	(1.9464)	0.3730	(0.1828)	0.4403	(0.1786)	14.2856	(6.2248)	13.1717	(5.6372)
γ	−0.0966	(0.0821)	−0.1819	(0.0425)	0.0812	(0.0369)	−0.0375	(0.2039)	0.0000	
ϕ^b	−0.1674	(0.1999)	−0.8461	(0.0304)	0.0000		0.0000		0.0145	(0.0132)
ϕ^d	0.8313	(0.0283)	0.8461	(0.0304)	0.0000		0.8420	(0.0275)	1.0000	
σ_b	0.0266	(0.0102)	0.0184	(0.0019)	0.0134	(0.0014)	0.0926	(0.0420)	0.0856	(0.0382)
σ_d	0.0077	(0.0008)	0.0078	(0.0008)	0.4651	(0.4970)	0.0080	(0.0008)	0.0077	(0.0008)
σ_s	0.0164	(0.0023)	0.0032	(0.0014)	0.0163	(0.0016)	0.0161	(0.0016)	0.0166	(0.0016)
ω_1	1.7395	(0.3965)	1.6743	(0.3819)	0.0559	(0.0128)	1.5969	(0.3638)	1.7377	(0.3962)
ω_2	0.0375	(0.0085)	0.0566	(0.0129)	0.2930	(0.0668)	0.0425	(0.0097)	0.0458	(0.0104)
ω_3	0.0673	(0.0153)	0.0455	(0.0104)	0.0804	(0.0195)	0.0597	(0.0137)	0.0603	(0.0138)

diag($\omega_1, \omega_2, \omega_3$) while Σ_1 is left unrestricted. The estimates of the parameters of primary interest for our purposes for the unrestricted and several restricted models are shown in Table 2.

One question of particular interest is whether the Ω matrix has distinct diagonal elements because this identifies the shocks and opens up the possibility to test the alternative structural restrictions from the economic models discussed in Section 2.2. Clearly, the estimates and their standard errors in the last subperiod are such that one may suspect that they are different. Notice that one-standard error intervals around the estimates for the unrestricted model do not overlap. Clearly, one may feel that this criterion is not strong enough to conclude that all ω_i's are distinct. After all, this assumption is the basis for our parameter identification and, thus, the validity of our subsequent tests rests on it. Therefore it may be worth pointing out that the evidence for at least two different diagonal elements of Ω is quite strong in all models. In the following we will also consider the possibility that only two of the three ω_i's may be distinct. Even then we have over-identifying restrictions which can be tested.

Since our previous results suggest that all the identification schemes presented in Section 2.2 can be tested against the data, we present the corresponding LR tests in Table 3. In the table p-values for two alternative degrees of freedom (d.f.) of the corresponding χ^2 distributions are reported. The first one is obtained under the assumption that all ω_i's are distinct. For example, for the FF scheme we have two d.f. in this case. The second column of p-values for a χ^2 distribution with one d.f. represents the worst case situation if only two ω_i's differ. In this case, there is at least one restriction and possibly more. Thus, the first column of p-values in Table 3 considers the most favorable case for the models whereas the last column of p-values considers the most difficult scenario for the models to conform with the data.

Based on the asymptotic p-values in Table 3 it turns out that both the NBR and BR schemes can be strongly rejected at common significance levels even in the most favorable situation for the models (d.f. $= 2$ and 3 for NBR and BR, respectively). In contrast, the FF and NBR/TR schemes cannot be rejected at the 5% level even under the least favorable scenario for the models (d.f. $= 1$).

In Table 2 also the estimates of the structural parameters obtained under the different sets of restrictions are given. Clearly, the NBR scheme produces some very different parameter estimates from the other identification schemes even if sampling uncertainty is taken into account. In particular, restricting the parameter ϕ^d to zero seems to have a strong effect. This parameter is clearly different from zero in all the other identification schemes. In other words, eliminating the innovations ε_t^d from the equation for nonborrowed

11

Table 3: LR tests of over-identifying restrictions

Identification scheme	H_0	LR statistic	p-values d.f.= no. of restr.	d.f.=1
FF	$\phi^d = 1/(1-\gamma)$ and $\phi^b = -\phi^d$	3.116	0.211	0.078
NBR	$\phi^d = \phi^b = 0$	62.476	2.713e-14	2.697e-15
NBR/TR	$\alpha = \phi^b = 0$	3.191	0.203	0.074
BR	$\phi^d = 1$, $\phi^b = \alpha/\beta$ and $\gamma = 0$	18.476	0.0004	1.721e-5

reserves and thereby imposing that the ε_t^d shocks have a delayed impact on nonborrowed reserves only is problematic.

It may also be worth pointing out that in the unrestricted model the parameter γ is not significantly different from zero judged on the basis of its t-ratio. On the other hand, it becomes significant in the FF scheme where other restrictions are imposed. Recall that BM used the restriction $\gamma = 0$ to obtain over-identified models and thus a possibility for statistical model comparison. Given that this parameter becomes significant in the FF scheme sheds doubt either on the restriction or on the identification scheme. Other reasons why the restriction $\gamma = 0$ may be problematic were discussed by CEE. We estimated an additional FF model with $\gamma = 0$ (hence, $\phi^d = 1$ and $\phi^b = -1$). This restriction produced a p-value of 0.0003 and, hence, was clearly rejected by the LR test. This result reinforces the conclusion that ϕ^d to some extent drives the results. The parameter ϕ^d is freely estimated only in the NBR/TR scheme (with an estimate very close to the estimated value in the unrestricted model), and this scheme cannot be rejected. Likewise, in the FF scheme the estimated value is close to the unrestricted estimate, but once γ is forced to equal zero and, hence, ϕ^d is set to unity, this model is rejected. In the BR model ϕ^d also equals unity and this model is rejected. These results suggest that the assumption that the Fed fully accommodates reserves demand shocks ($\phi^d = 1$) is not supported by the data. On the other hand, the NBR identification scheme with $\phi^d = 0$ is also rejected. Hence, the results seem to be most sensitive to the value of ϕ^d which is also estimated with a very small standard error in the unrestricted model. The FF scheme has the drawback that γ has to be smaller than zero for ϕ^d to be smaller than one. Thus, in the FF scheme γ has the wrong sign if $\phi^d < 1$ as indicated by our estimation results. In this respect the NBR/TR scheme is preferable because it is acceptable even without taking the effect of nonborrowed reserves on banks' borrowing into account (i.e., γ is not significantly different from zero).

In Figure 1 we present the monetary policy shocks implied by the different models. Notice that these shocks are only identified from 1979M10 on-

wards because our identification scheme applies only after the first subperiod. Therefore only the shocks from 1979M10 − 1996M12 are displayed in Figure 1. The shocks associated with the NBR scheme are quite different from the monetary shocks implied by the other identification schemes. In particular, the NBR shocks are considerably more volatile after the mid-1980s. On the other hand, the unrestricted and the NBR/TR and BR shocks appear to be quite similar at first glance. To some extent also the FF shocks fall roughly in this group although they do not display the spikes in 1984 which can be seen in the NBR/TR and BR shocks.

We have also determined the impulse responses induced by a monetary policy shock and present the graphs in Figures 2 and 3. These are the responses to a 25 basis points reduction in the federal funds rate on impact. Thus, an expansionary monetary policy shock is considered. On the left-hand side of Figure 2 the impulse responses from a model where the parameter vector ψ is unrestricted are shown with bootstrapped 95% confidence intervals.[2] Given the estimation uncertainty reflected in these intervals, the impulse responses of the FF, NBR/TR and BR schemes which are shown on the right-hand side of Figure 2 are quite similar. The FF impulse responses are overall closest to those obtained from the unrestricted model. The NBR/TR and BR impulse responses are almost identical because both models restrict the first two elements in the last column of the B_{22} matrix to zero (see (2.5)). In Figure 2 they are so close together that they are almost indistinguishable. All these impulse responses are plausible reactions to an expansionary monetary policy shock. In particular, there is a significant increase in GDP and the commodity price index. Moreover, the GDP deflator increases, although not significantly in the unrestricted model. All other effects are generally insignificant or become insignificant after a few months.

The impulse responses obtained from the NBR scheme are quite different from the other ones. Therefore they are shown separately in Figure 3 together with the 95% confidence intervals obtained for the unrestricted model. This way the substantial differences of the NBR scheme to the other identification schemes becomes apparent (notice the change in the scales of the graphs).

The overall message from our analysis is that the data resist both the NBR and BR schemes proposed by Christiano and Eichenbaum (1992) and Cosimano and Sheehan (1994), respectively. While the monetary policy shocks implied by the BR scheme are still close to those from models which are not rejected by the data, using the NBR scheme for monetary policy analysis is clearly problematic. In some respects, the preferred specification

---

[2]The intervals are determined by Hall's percentile method as proposed by Benkwitz, Lütkepohl and Wolters (2001) using 2000 bootstrap replications.

is Strongin's (1995) NBR/TR identification scheme. It is not rejected by the data, produces sensible estimates of the structural parameters and delivers plausible impulse responses.

4   Conclusions

A large body of literature has discussed the question how to identify monetary policy shocks for the US. Clearly, this is an important problem for assessing monetary policy. Different money market models have been proposed and the corresponding shocks have been derived and estimated. The fact that there is still disagreement as to which shocks actually reflect the effects of monetary policy is a consequence of the problem that the different theories do not provide sufficient restrictions for an empirical model to be able to check them by statistical tests.

Given this state of affairs we have proposed a setup where identifying information from changes in the volatility of the shocks can be used to obtain unique specifications of the shocks. Using monthly data for the US from 1965 to 1996 as in BM and CEE we find with statistical tools that the Volcker period displays larger volatility of the shocks and we have used this statistical information in specifying monetary policy shocks. The fact that there was a decrease in volatility after the Volcker period was also found by other authors and actually seems to be a widely accepted view in the related literature.

Using the statistical information on the volatility of the shocks opens up the possibility to test different theoretical assumptions against the data. In particular, we have tested four different identification schemes proposed by Bernanke and Blinder (1992) (FF), Christiano and Eichenbaum (1992) (NBR), Strongin (1995) (NBR/TR) and Cosimano and Sheehan (1994) (BR) which have also been considered and further investigated by other authors. In these identification schemes monetary policy shocks enter via the federal funds rate, nonborrowed reserves, total reserves or borrowed reserves, respectively. So far the empirical results have been inconclusive or otherwise not fully satisfactory. In our framework it turns out that the NBR and BR schemes are clearly rejected by the data whereas FF and NBR/TR cannot be rejected at common significance levels. Even though BR is overall rejected by our formal statistical test, the implied impulse responses associated with a monetary policy shock are very similar to those implied by the NBR/TR identification scheme. In contrast, the FF scheme results in slightly different impulse responses and it also produces an implausible value for at least one of the structural parameters.

In summary, the NBR scheme is clearly problematic from the point of

view of monetary policy analysis. At least for the time period under consideration in this study, the NBR scheme cannot be recommended for policy analysis. On the other hand, the NBR/TR scheme is overall the most plausible one. It is not rejected by the data, produces structural parameters of expected sign and results in plausible responses to monetary shocks.

Appendix

A.1 An Identification Result

In this appendix we prove that a change in volatility can be used to identify shocks in a structural VAR. The crucial result for this purpose is stated in the following proposition.

Proposition A. Let Σ_1 and Σ_2 be two symmetric positive definite $(n \times n)$ matrices and let $\Omega = \text{diag}(\omega_1, \ldots, \omega_n)$ be an $(n \times n)$ diagonal matrix. If there exists an $(n \times n)$ matrix B such that $\Sigma_1 = BB'$ and $\Sigma_2 = B\Omega B'$, then B is locally unique (i.e., B is unique apart from possible sign reversal of its columns), if all ω_i's $(i = 1, \ldots, n)$ are distinct. $\qquad\square$

Proof: Let the $(n \times n)$ matrix Q be such that $BB' = BQQ'B'$ and $B\Omega B' = BQ\Omega Q'B'$. The first relation implies that Q is orthogonal and the second relation implies $\Omega = Q\Omega Q'$ and, hence, $Q\Omega = \Omega Q$ or, denoting the ijth element of Q by q_{ij}, $\omega_i q_{ij} = \omega_j q_{ij}$ $(i, j = 1, \ldots, n)$. Thus, $q_{ij} = 0$ for $i \neq j$ if $\omega_i \neq \omega_j$. In other words, if all diagonal elements of Ω are distinct, Q is a diagonal matrix with ± 1 on the diagonal because the diagonal elements of a diagonal matrix are its eigenvalues and the eigenvalues of an orthogonal matrix are all ± 1. This proves Proposition A.

A.2 ML Estimation with a Change in Volatility

In this section we provide details on our estimation procedure. The point of departure is the Gaussian log likelihood function (apart from additive constants)

$$\log l = -\frac{1}{2}\sum_{t=1}^{T} \log |\Sigma_t| - \frac{1}{2}\sum_{t=1}^{T} \text{tr}(v_{2t}v_{2t}'\Sigma_t^{-1}),$$

where

$$\Sigma_t = E(v_{2t}v_{2t}') = \begin{cases} \Sigma_1 = \mathsf{B}_{22}\mathsf{B}_{22}' & \text{for } t = 1, \ldots, T_B - 1, \\ \Sigma_2 = \mathsf{B}_{22}\Omega\mathsf{B}_{22}' & \text{for } t = T_B, \ldots, T, \end{cases} \qquad (A.1)$$

15

and $v_{2t} = y_{2t} - CZ_t$. Here C is the matrix of all VAR parameters in (2.4) and a constant and Z_t contains all the regressors from (2.4) plus the deterministic term used in our empirical analysis. The normal equations for C are

$$\sum_{t=1}^{T} \Sigma_t^{-1}(y_{2t} - CZ_t)Z_t' = 0 \qquad (A.2)$$

(see Lütkepohl (2005, Sec. 17.2.2)). Using $(A.1)$ and standard rules for the column vectorization operator vec, it follows that the ML estimator for C satisfies

$$\text{vec}(\widetilde{C}) = \left[\sum_{t=1}^{T_B-1}(Z_t Z_t' \otimes \widetilde{\Sigma}_1^{-1}) + \sum_{t=T_B}^{T}(Z_t Z_t' \otimes \widetilde{\Sigma}_2^{-1})\right]^{-1}$$

$$\times \left[\sum_{t=1}^{T_B-1}(Z_t \otimes \widetilde{\Sigma}_1^{-1})y_{2t} + \sum_{t=T_B}^{T}(Z_t \otimes \widetilde{\Sigma}_2^{-1})y_{2t}\right], \qquad (A.3)$$

where $\widetilde{\Sigma}_i$ denotes the ML estimator of Σ_i ($i = 1, 2$). If some other estimators $\widetilde{\Sigma}_i$ are used instead, $\widetilde{C}$ is a feasible multivariate GLS estimator of C. Using such an estimator of C and plugging the resulting $\tilde{v}_{2t} = y_{2t} - \widetilde{C}Z_t$ into (3.1), we obtain estimators of the structural parameters in B_{22} and Ω by maximizing the resulting "concentrated" log likelihood (3.1) in the usual way. From these estimators new estimators of Σ_1 and Σ_2 may be obtained and used again in $(A.3)$ and so on. The procedure can be used to compute the Gaussian ML estimators by continuing the iterations until convergence. This method was used in Section 3.

References

Amisano, G. and Giannini, C. (1997). *Topics in Structural VAR Econometrics*, 2nd edn, Springer, Berlin.

Benkwitz, A., Lütkepohl, H. and Wolters, J. (2001). Comparison of bootstrap confidence intervals for impulse responses of German monetary systems, *Macroeconomic Dynamics* **5**: 81–100.

Bernanke, B. S. and Blinder, A. (1992). The federal funds rate and the channels of monetary transmission, *American Economic Review* **82**: 901–921.

Bernanke, B. S. and Mihov, I. (1998a). The liquidity effect and long-run neutrality, *Carnegie-Rochester Conference Series on Public Policy* **49**: 149–194.

Bernanke, B. S. and Mihov, I. (1998b). Measuring monetary policy, *Quarterly Journal of Economics* **113**: 869–902.

Candelon, B. and Lütkepohl, H. (2001). On the reliability of Chow-type tests for parameter constancy in multivariate dynamic models, *Economics Letters* **73**: 155–160.

Christiano, L. J. and Eichenbaum, M. (1992). Identification and the liquidity effect of a monetary policy shock, *in* A. Cukierman, Z. Hercowitz and L. Leiderman (eds), *Political Economy, Growth, and Business Cycles*, MIT Press, Cambridge, MA, pp. 335–370.

Christiano, L. J., Eichenbaum, M. and Evans, C. (1999). Monetary policy shocks: What have we learned and to what end?, *in* J. B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics*, Vol. 1A, Elsevier, Amsterdam, pp. 65–148.

Cosimano, T. and Sheehan, R. (1994). The federal reserve operating procedure, 1984-1990: An empirical analysis, *Journal of Macroeconomics* **16**: 573–588.

Klein, R. and Vella, F. (2000). Employing heteroskedasticity to identify and estimate triangular semiparametric models, *mimeograph*, Rutgers University.

Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis*, Springer-Verlag, Berlin.

Magnus, J. R. and Neudecker, H. (1988). *Matrix Differential Calculus with Applications in Statistics and Econometrics*, John Wiley, Chichester.

Rigobon, R. (2003). Identification through heteroskedasticity, *Review of Economics and Statistics* **85**: 777–792.

Strongin, S. (1995). The identification of monetary policy disturbances: Explaining the liquidity puzzle, *Journal of Monetary Economics* **35**: 463–498.
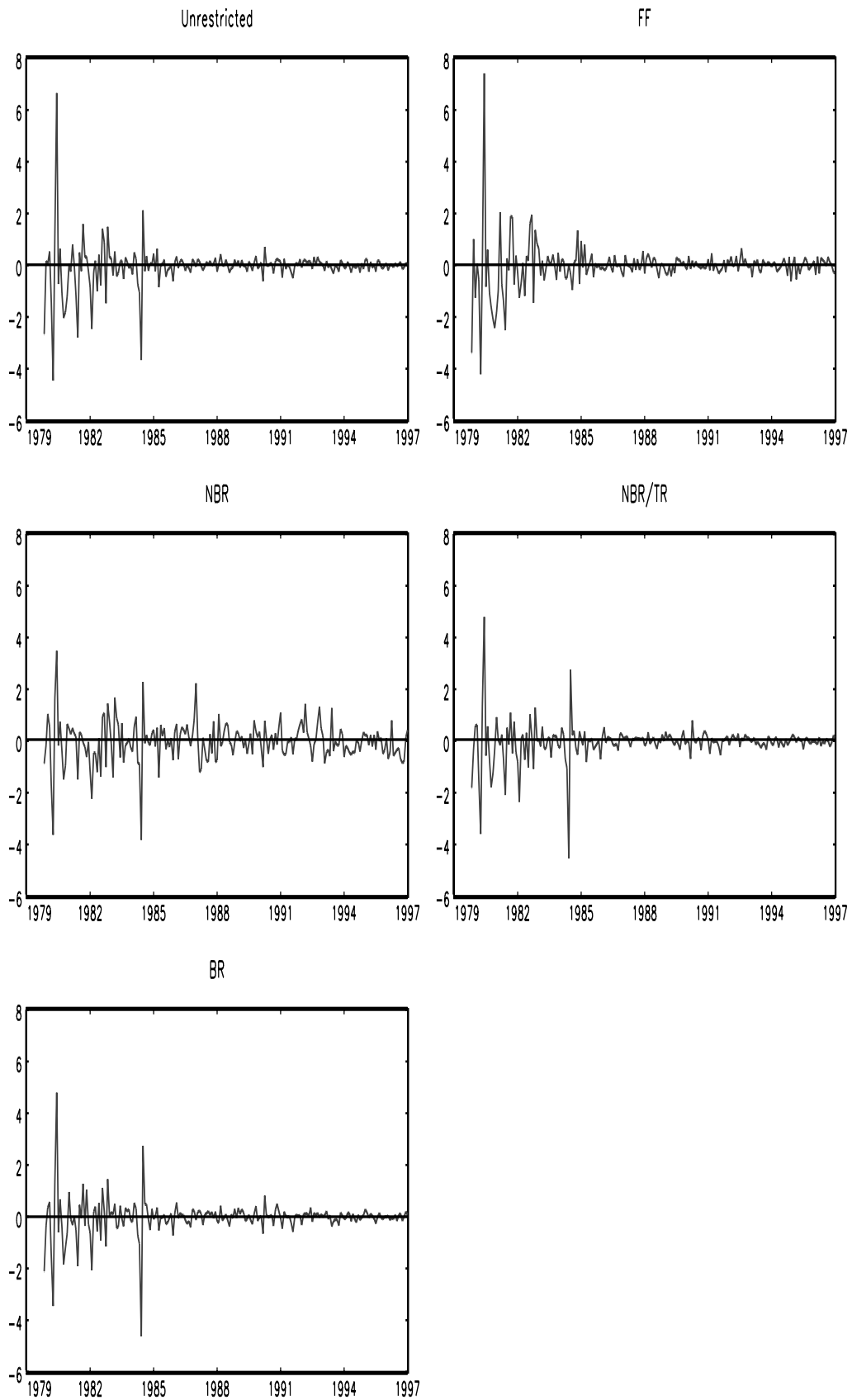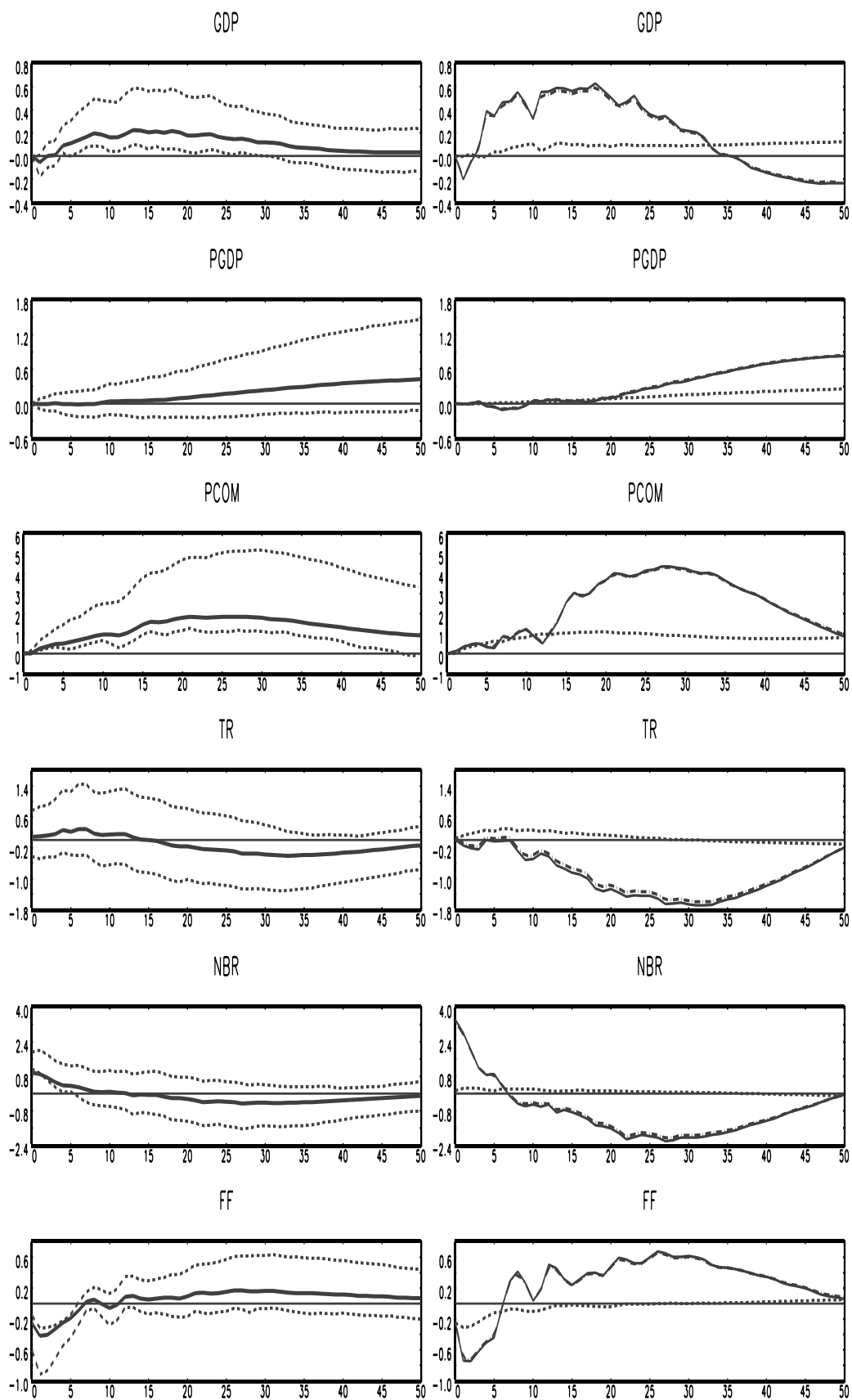
Figure 1: Estimated monetary policy shocks.

Figure 2: Responses to different monetary policy shocks (left-hand column: impulse responses from unrestricted model with 95% confidence intervals; right-hand column: — NBR/TR, $-\cdot-$ BR, $\cdots\cdots$ FF impulse responses).
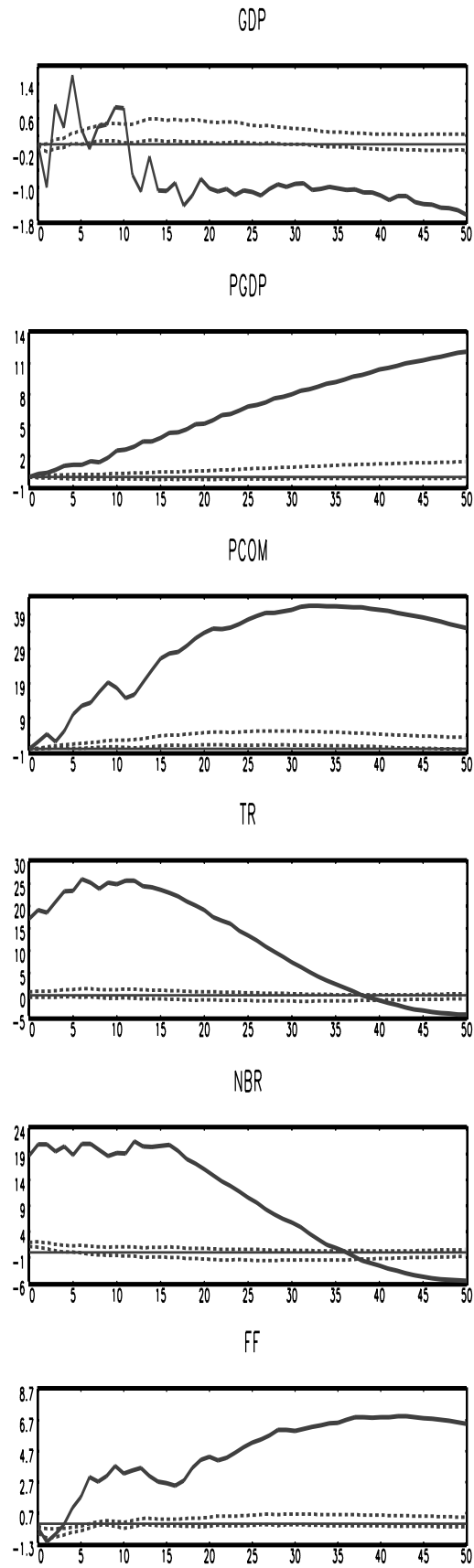
19

Figure 3: Responses to NBR monetary policy shocks with 95% confidence bands from unrestricted model.