EUI Working Papers
ECO 2007/15

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EUI Working Paper **ECO** 2007/15

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July 27, 2007

Abstract. This paper studies a model of optimal redistribution policies in which agents face unemployment risk and in which savings may provide partial self-insurance. Moral hazard arises as job search effort is unobservable. The optimal redistribution policies provide new insights into how an unemployment insurance scheme should be designed: *First*, the unemployment insurance policy is recursive in an agent's wealth level, and thus independent of the duration of the unemployment spell. *Second*, the level of benefit payments is negatively related to the agent's asset position. The reason behind the latter result is twofold; in addition to the first-order insurance effect of wealth, an increase in non-labor income (wealth) amplifies the opportunity cost of employment and thus reduces the agent's incentive to search for a job.

During unemployment the agent decumulates assets and the sequence of benefit payments is observationally *increasing* - a result that stands in sharp contrast with previous studies.

JEL-classification: D82, H21, J64 and J65.

Keywords: Unemployment insurance; Moral hazard; Self-insurance; Decentralized taxes.

1. INTRODUCTION

Between the ages of 18 and 40, an American worker can expect to be unemployed on five different occasions.[1] An average spell of unemployment lasts for approximately three months. Unsurprisingly then, unemployment is perceived as one of the greatest economic risks an individual faces during her working life, and insurance against such shortfalls in labor income is of high importance. Whereas most modern economies provide unemployment insurance through a governmentally sponsored unemployment benefits programme,[2] several empirical studies suggest that this is not the only source of insurance available to the unemployed. Of the total fraction of unemployed eligible for benefits, Blank and Card (1991) estimate that only 67% take up unemployment insurance, indicating that many of the unemployed find insurance elsewhere. Among the group of participating individuals, Gruber (1997) finds that the consumption smoothing effect of insurance is particularly high at late stages of the unemployment spell, arguing that this occurs when financial wealth is depleted. Lastly, Gruber (1998) shows that unemployment benefits have a significant crowding-out effect on savings, not only suggesting that unemployment benefits and wealth act as close substitutes, but also that savings is an important factor to consider when designing an unemployment benefits programme.

Motivated by these issues, this paper is develops a theoretical model in order to characterize an optimal unemployment benefit programme in the presence of *moral hazard* and partial *self-insurance*. An infinitely lived individual can at any date either be employed or unemployed. While working she faces an idiosyncratic exogenous risk of losing her job, and while unemployed she can devote time and effort to search for a new job. The agent enjoys consumption and leisure, and she may reallocate resources intertemporally by means of a riskless asset. A utilitarian government provides unemployment insurance. It has information on the agents' consumption level and preferences, but not on their search effort. The government's redistribution policy must therefore be *incentive compatible*.

In this setting, the government has full control over the agent's consumption and search effort allocations, and may thus choose these directly. Allowing the government to choose allocations, rather than policies, simplifies the problem considerably. However, it also forces

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[1]According to the Bureau of Labor Statistics' National Longitudinal Survey of Youth, 1979 (NLSY79).

[2]As unemployment insurance reduces the opportunity cost of employment, it evokes substantial moral hazard effects in the labor market (Meyer, 1990; Moffitt, 1985). Private insurance solutions are thus unlikely to function efficiently, and may even fail to exist. As a consequence, most modern economies relies exclusively on a governmentally funded unemployment insurance programme (Oswald, 1986; Chiu and Karni, 1998).

the analysis to proceed in two separate steps: The first step characterizes the optimal allocations while the second implements these allocations through a tax system in a decentralized economy.

I show that the government's intertemporal first order condition must observe an inverse Euler equation (Rogerson, 1985). By Jensen's inequality, this optimality condition implies a wedge between the agent's intertemporal marginal rate of substitution and the economy-wide interest rate (the marginal rate of transformation). Said differently, in relation to a frictionless economy, the agent is saving constrained. The reason behind this result is straightforward: In order to provide incentives to exert search effort, the government wishes to generate a positive correlation between consumption and employment. When the agent's utility function is concave, higher savings weakens this correlation and thus decreases search effort. Thus, at an optimal programme, a crowding-out effect of unemployment insurance on savings is indeed desired.

Following recent developments in the dynamic public finance literature, I construct tax (or policy-) functions that implement the optimal allocations in a decentralized economy (cf. Kocherlakota (2005); Albanesi and Sleet (2006); and Golosov and Tsyvinski (2006)). By *implement*, I mean a tax system such that the solution to a decentralized maximization problem faced by an individual agent that takes the tax system as given, coincides with the government's optimal solution. The resulting tax functions are *simple*: Current taxes depend solely on the agent's current and previous employment state, and on her level of assets. These tax functions provide new insights into how an optimal unemployment insurance scheme should be designed: *First*, the unemployment insurance policy is time-invariant, and thus independent of the duration of the unemployment spell. *Second*, unemployment benefit payments relate negatively to the agent's asset position: In addition to the first-order insurance effect of wealth, a ceteris paribus increase in non-labor income (wealth) amplifies the opportunity cost of employment and thus reduces the agent's incentive to search for a job. Moreover, during unemployment the agent decumulates assets and the sequence of benefit payments is observationally *increasing* - a result that stands in sharp contrast with previous studies (e.g. Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997; Pavoni, 2007; Pavoni and Violante, 2007).

The essential economic mechanisms in this paper are closest related to those in Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), and Pavoni (2007). In their seminal study, Shavell and Weiss (1979) show that consumption ought to be decreasing with respect to the duration of the unemployment spell, a result further confirmed and strengthened in

Hopenhayn and Nicolini (1997) and Pavoni (2007).[3] Since these studies abstract from savings, the policy recommendation is immediate; unemployment benefits are given as the difference between consumption and labor income, and should therefore decrease along the duration of the unemployment spell. I deviate from this literature by relaxing two assumptions: Firstly, I model employment and unemployment as *re*current states, while previous studies have assumed that employment is an absorbing state. Secondly - and more importantly - I allow for partial *self-insurance* by means of a riskless asset. This has salient implications for the optimal unemployment benefit policy. While the consumption pattern largely remains unaltered, the benefit policy does not.

In order identify the effect of savings and benefit payments on consumption, I rely on recent developments in the dynamic public finance literature. Following Kocherlakota (2005) and Albanesi and Sleet (2006), I consider tax systems that resemble modern economies' combined usage of taxes and markets to reallocate resources in the economy. Kocherlakota (2005) and Albanesi and Sleet (2006) consider dynamic versions of *Mirrleesian taxation* (Mirrlees, 1971); concisely, a utilitarian government wishes to allocate resources in an economy where skills are unobservable, but labor income is not. Although the economy explored in this paper functions under fundamentally different informational frictions, the proximity of some results should be noted. As in both Kocherlakota (2005) and Albanesi and Sleet (2006), (wealth-) taxes and marginal taxes are period-by-period expected to be zero. Moreover, whereas Kocherlakota (2005) puts no restrictions on the process governing the evolution of agents' skills, the resulting tax system admits a complex structure in which the tax in any period depends upon the full history of past labor income reports. In contrast, Albanesi and Sleet (2006) assume that the evolution of agents' skills are identically and independently distributed over time, and show that the tax system lends itself to a recursive representation in the agents' wealth. Although the evolution of employment status in this paper is endogenous and exhibits high persistence, the tax system admits a simple recursive representation in the agents' wealth and current employment status transition.

In a recent paper, Shimer and Werning (2005) consider a problem closely related to the question explored in this paper. Similar to this paper, Shimer and Werning (2005) first consider the optimal allocations, and then, by proving an equivalence result, derive the decentralized policy that implements these allocation . However, the two papers show considerable differences: Shimer and Werning (2005) consider a version of McCall's (1970) search model with hidden reservation wages. This paper considers hidden search effort

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[3]In fact, Pavoni (2007) finds that consumption should be *non-increasing*: By exogenously imposing a minimum lower bound on the agent's present value utility - a constraint that may be interpreted as a minimum subsistence level - the consumption sequence embeds a flat profile whenever this constraint is binding.

decisions. More importantly, all qualitative properties explored in Shimer and Werning (2005) hinges on the assumption of CARA utility, and thus on potentially negative consumption levels.[4] Abstracting from some standard regulatory conditions, this paper puts no restrictions on the specific functional form of the agents' momentary utility function.

2. Structure of the economy

The economy is populated by a utilitarian government and a continuum of risk-averse agents. The planning horizon is infinite. Time is discrete and denoted by $t = 0, 1, \ldots$ In any given period t, an agent can either be employed or unemployed and the agent's employment status is publicly observable.

When an agent is employed, she earns a gross wage, w. There is no on-the-job search and the probability of losing the job is exogenously given at the constant hazard rate $1 - \gamma$.

When unemployed, the agent receives unemployment benefits and searches for a job with effort e. The probability of finding a job, conditional on search effort, is denoted $p(e)$. Search effort - and thus the probability of finding a job - is considered private information, not observable by the government or by any other agent in the economy.[5] The wage distribution is degenerate, and a job offer is, consequently, always accepted. The agents can save using a riskless bond that pays net pre-tax return equal to $r > 0$. The intertemporal price of consumption, $1/(1 + r)$, is denoted by q. Savings are publicly observable.

2.1. **Model.** Formally, employment status in any period t is given by $\theta_t \in \Theta = \{0, 1\}$. Let $\theta_t = 1$ denote employment. The history of employment status up to period t is given by $\theta^t = (\theta_0, \ldots, \theta_t) \in \Theta^t$, where $\Theta^t = \{0, 1\} \times \{0, 1\} \times \ldots \times \{0, 1\}$, represent all possible histories up to period t.

At time zero, each agent is born as either employed or unemployed, and she is entitled some level of initial cash-on-hand, b_0. The initial entitlement/employment status-pair, (b_0, θ_0), is taken as given by each agent in the economy (the government included). The joint distribution of (b_0, θ_0) is given by $\psi(b_0, \theta_0)$, with support on $B \times \Theta$, where B is some subset of the real numbers, $B \subseteq \mathbb{R}$. Thus, at every date, t, each agent is distinguished by her initial entitlements and history of employment status, (b_0, θ^t).

Without any loss of generality, I will henceforth formulate the problem such the agents choose p - the probability of finding a job -, rather than effort e, directly. The agent then ranks contemporaneous consumption and search effort allocations according an additively

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[4]In Shimer and Werning (2005) it is shown that their results do not extend to a setting with CRRA utility.

[5]This is the source of moral hazard in the model; if benefit payments would be made contingent upon search effort, the economy would reach its first best allocation.

separable felicity function, $\{u(c) - (1-\theta)v(p)\}$. There is no disutility from working.[6] The function u and v are strictly increasing and once continuously differentiable. In addition, u is strictly concave and v is strictly convex. The standard Inada conditions apply for u; $u'(0) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$.

An *allocation* in this economy is denoted $\boldsymbol{\sigma} = \{c_t, p_t\}_{t=0}^{\infty}$, where

$$c_t : B \times \Theta^t \to \mathbb{R}_+$$

$$p_t : B \times \Theta^t \to [0, 1]$$

Here, $c_t(b_0, \theta^t)$ is the amount of consumption an (b_0, θ_0)-agent is assigned under history θ^t. The contemporaneous probability of finding a job, $p_t(b_0, \theta^t)$, is defined equivalently. Let $\lambda(b_0, \theta^{t+1})$ denote the probability measure for history θ^{t+1}, conditional on (b_0, θ_0). For notational convenience let $p_t(b_0, \theta^t)$ be defined as γ if and only if $\theta_t = 1$. $\lambda(b_0, \theta^{t+1})$ is then recursively given by

$$\lambda(b_0, \theta^{t+1}) = \begin{cases} p_t(b_0, \theta^t)\lambda(b_0, \theta^t), & \theta_{t+1} = 1 \\ (1 - p_t(b_0, \theta^t))\lambda(b_0, \theta^t), & \theta_{t+1} = 0 \end{cases}$$

An agent's net present value utility of an allocation $\boldsymbol{\sigma}$ is given as

$$V(\boldsymbol{\sigma}, b_0, \theta_0) = \sum_{t=0}^{\infty} \beta^t \int_{\Theta^t} \{u(c_t(b_0, \theta^t)) - (1 - \theta_t)v(p_t(b_0, \theta^t))\}\lambda(b_0, \theta^t)d\theta^t \qquad (1)$$

The utilitarian government wishes to find $\boldsymbol{\sigma}$ that maximizes the sum of net present value utilities

$$\hat{V}(\psi) = \max_{\boldsymbol{\sigma}} \int_{B \times \Theta} \{V(\boldsymbol{\sigma}, b_0, \theta_0)\}d\psi \qquad (2)$$

subject to each agent's present value budget constraint

$$b_0 \geq \sum_{t=0}^{\infty} q^t \int_{\Theta^t} \{c_t(b_0, \theta^t) - \theta_t w\}\lambda(b_0, \theta^t)d\theta^t, \quad \forall\, (b_0, \theta_0) \in B \times \Theta \qquad (3)$$

Furthermore, since the search effort allocation is private information, the optimal allocation must also respect incentive compatibility

$$\{p_t\}_{t=0}^{\infty} = \text{argmax}\{V(\boldsymbol{\sigma}, b_0, \theta_0)\}, \quad \forall\, (b_0, \theta_0) \in B \times \Theta \qquad (4)$$

The motivation behind the *incentive compatibility constraint* is simple: Each agent takes the consumption allocation as given and chooses search effort to maximize her private utility. Without any loss of generality, the problem is organized such that the government directly proposes a search effort allocation that coincides with the agent's private optimal choice.

Constraint (3) ensures feasibility. It should be noted that this constraint will always hold as an equality; if it did not, the government could simply increase the agent's period

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[6]Including disutility from working would not change any of the results in the paper.

zero consumption without inflicting with incentive compatibility. An allocation that is both incentive compatible and feasible will be referred to as *incentive feasible*.

Note that in (3), q is the constant intertemporal price equal to $1/(1+r)$. Implicitly, this assumes that there exist an exogenous financial sector, willing to borrow and lend at the intertemporal price q.

The following lemma states that maximizing (1) subject to individual incentive compatibility and feasibility, is equal to solving the more complicated problem given in (2). The result is standard and the proof is merely included for completeness.

Lemma 1. *Define $\boldsymbol{\sigma}^*$ as the allocation that maximizes (1) for each $(b_0, \theta_0) \in B \times \Theta$, subject to individual incentive compatibility and feasibility. Define $\hat{\boldsymbol{\sigma}}^*$ as the allocation that solves (2). Then*

$$\hat{V}(\psi) = \int_{B \times \Theta} V(\boldsymbol{\sigma}^*, b_0, \theta_0) d\psi$$

Proof. By construction, $\hat{V}(\psi) \geq \int_{B \times \Theta} V(\boldsymbol{\sigma}^*, b_0, \theta_0) d\psi$. If the inequality was strict, then there exist some (b_0, θ_0) such that $V(\hat{\boldsymbol{\sigma}}^*, b_0, \theta_0) > V(\boldsymbol{\sigma}^*, b_0, \theta_0)$. Since $\hat{\boldsymbol{\sigma}}^*$ is incentive compatible and delivers b_0, $\boldsymbol{\sigma}^*$ could not have attained the maximum in (1). $\square$

2.2. **A recursive formulation.** Following the insights provided by Lemma 1, the problem of interest is given by

$$V(b_0, \theta_0) = \max_{\boldsymbol{\sigma}} \sum_{t=0}^{\infty} \beta^t \int_{\Theta^t} \{u(c_t(b_0, \theta^t)) - (1 - \theta_t)v(p_t(b_0, \theta^t))\} \lambda(b_0, \theta^t) d\theta^t \qquad (5)$$

$$\text{s.t.} \quad \{p_t\}_{t=0}^{\infty} = \operatorname{argmax}\{V(\boldsymbol{\sigma}, b_0, \theta_0)\} \qquad (6)$$

$$b_0 = \sum_{t=0}^{\infty} q^t \int_{\Theta^t} \{c_t(b_0, \theta^t) - \theta_t w\} \lambda(b_0, \theta^t) d\theta^t \qquad (7)$$

Under an optimal allocation, $\boldsymbol{\sigma}^*$, equations (5) and (7) can be written as

$$V(b_0, \theta_0) = u(c_0^*(b_0, \theta_0)) - (1 - \theta_0)v(p_0^*(b_0, \theta_0)) + \beta \int_{\Theta^1} V(\boldsymbol{\sigma}^*, b^*(\theta_1), \theta_1) \lambda(b_0, \theta^1) d\theta^1 \qquad (8)$$

$$b_0 = c_0^*(b_0, \theta_0) - \theta_0 w + q \int_{\Theta^1} b^*(\theta_1) \lambda(b_0, \theta^1) d\theta^1 \qquad (9)$$

The following lemma asserts that, given the budget $b^*(\theta_1)$, re-optimizing the problem in period one, does not alter period zero present value utility.

Lemma 2. *$V(\boldsymbol{\sigma}^*, b^*(\theta_1), \theta_1)$ maximizes the agent's utility subject to the budget $b^*(\theta_1)$ and incentive compatibility. That is, $V(\boldsymbol{\sigma}^*, b^*(\theta_1), \theta_1) = V(b^*(\theta_1), \theta_1)$.*

Proof. See Appendix A. $\square$

The result is not trivial. If $V(b^*(\theta_1), \theta_1) > V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1)$ for at least one θ_1, period zero incentive compatibility is violated. The idea behind the proof lies in the fact that $V(b_0, \theta_0)$ is strictly increasing in b_0, and that $b^*(\theta_1)$ must therefore be resource minimizing given utility $V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1)$. The Inada conditions on u then guarantees that duality holds: If $b^*(\theta_1)$ is resource minimizing under utility $V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1)$, $V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1)$ must be utility maximizing under the budget $b^*(\theta_1)$.

Let b_e and b_u denote period $t + 1$ contingent claims in the employed and unemployed state, respectively. Then - by exploiting the insights provided by Lemma 2 and following the arguments outlined in Spear and Srivastava (1987) - problem (5) can be made recursive as

$$V(b, \theta) = \max_{c, p, b_e, b_u} \{u(c) - (1 - \theta)v(p) + \beta(pV(b_e) + (1 - p)V(b_u))\} \tag{10}$$

subject to

$$p = \text{argmax}_p \{u(c) - \theta v(p) + \beta(pV(b_e) + (1 - p)V(b_u))\} \tag{11}$$

and

$$b = c - \theta w + q(pb_e + (1 - p)b_u) \tag{12}$$

Since the function v is differentiable and strictly convex, the incentive compatibility constraint (11) can be replaced by its first order condition

$$v'(p) = \beta(V(b_e) - V(b_u))$$

The solution to (10)-(12) yields a value function, $V(b, \theta)$, associated with policy functions $c(b, \theta)$, $p(b, \theta)$, $b_e(b, \theta)$ and $b_u(b, \theta)$. When there is no confusion regarding the agent's employment status, the policy functions will be addressed by their respective initial letter, and reliance on b will be left implicit.

Previous studies on optimal unemployment insurance adopt a dual formulation to the problem in (10)-(12). Specifically, the literature has, without exception, followed the cost-minimization framework commonly employed in the repeated-agency literature. Fundamentally, this approach amounts to minimize (3) such that the agent receives a pre-specified level of present value utility, and subject to incentive compatibility. Due to Spear and Srivastava (1987), this dual formulation lends itself straightforwardly to a recursive representation. In contrast, this paper adopts a *primal* approach. The reason for this is twofold: First, the primal formulation simplifies the subsequent analysis and provides an intuitive recursive representation in terms of (non-labor) cash-on-hand, b. Second, this way of formulating the problem has a quite appealing and natural interpretation: Akin to a *social planner*, the government maximizes the agent's utility by choosing current consumption, search effort, and one period ahead Arrow securities at prices qp and $q(1 - p)$. By respecting incentive

compatibility, moral hazard is internalized through *individually* and *quantity contingently* priced assets.

3. ANALYSIS

Consistent with the formulation of the problem in (10), the government chooses *allocations* rather than *policies*. While it facilitates the analysis of the governments optimal policy problem, it also restricts the subsequent analysis to proceed in two separate steps. The first step concerns the optimal allocations. The second step considers the tax functions that implement these allocations in a decentralized *bond economy*.

Although the two steps presented above may appear distinctly separate, they are, in effect, intimately related. Thus, as a third step, Section 3.3 will show how the shape of the derived tax functions are closely tied to the incentive compatibility constraint, and how a quite esoteric optimality condition, commonly known as the inverse Euler equation, relate to a more familiar form of the standard Euler equation.

3.1. **Allocations.** Analogous to the definition of b_e and b_u, let c_e and c_u denote period $t+1$ consumption at the associated employment states. During employment, moral hazard is absent and the first order necessary conditions from (10) (together with the envelope condition) gives

$$u'(c) = \frac{\beta}{q}u'(c_e) = \frac{\beta}{q}u'(c_u) \tag{14}$$

When $\beta = q$, condition (14) implies that consumption is constant for any two consecutive periods; on a period-by-period basis, the agent is fully insured.

The equivalent optimality conditions for an unemployed agent gives

$$\frac{1}{u'(c)} = \frac{q}{\beta}\left(p\frac{1}{u'(c_e)} + (1-p)\frac{1}{u'(c_u)}\right) \tag{15}$$

$$\mu v''(p) = \lambda q(b_e - b_u) \tag{16}$$

$$\frac{\mu}{\lambda} = p(1-p)\left(\frac{1}{u'(c_u)} - \frac{1}{u'(c_e)}\right) \tag{17}$$

Where λ and μ are the Lagrange multipliers on the budget- and the incentive compatibility constraint, respectively.

Equation (15) is commonly known as the "inverse Euler equation" (Rogerson, 1985). When $c_e \neq c_u$, Jensen's inequality implies

$$u'(c) < \frac{\beta}{q}(pu'(c_e) + (1-p)u'(c_u)) \tag{18}$$

Rearranging terms, equation (18) infers that there is a wedge between the agent's marginal rate of substitution and the economy's marginal rate of transformation. In particular, (18) implies that current marginal utility of consumption is lower than the expected future

marginal utility. In other words, the agent is savings constrained relative to an economy with no private information. Golosov, Kocherlakota and Tsyvinski (2003) interpret this wedge as an "implicit tax".

According to the *standard* Euler equation, an optimal intertemporal plan has the property that any marginal, temporary and feasible change in behavior equates marginal benefits to marginal costs in the present and in the future. The inverse Euler equation *appears* to violate this logic. For a given value of p, consider the choice of reallocating resources from period t to period $t+1$. If an increase in savings would bring about a proportional increase in b_e as well as b_u, equation (18) reveals that, at least on the margin, such a policy would increase overall utility. However, the incentive compatibility constraint in (11) does generally not permit a proportional increase in b_e and b_u. To keep the choice of p unaltered, the incentive compatibility constraint forces the increase in resources to be relatively low in future states where the marginal utility of resources is relatively high, and vice versa. Period $t+1$ marginal utilities will thus be "weighted" by their respective incentive compatible inflow of state contingent resources. In contrast, utility maximization implies relatively high weights of resource inflow to states in which the marginal benefit of resources is relatively high. Since incentive compatibility inflicts with period $t+1$ resources only, it is thus optimal to relegate a high degree of resources to period t consumption. As a result, the agent appears savings constrained. The inverse Euler equation is simply the resulting expression when these conflicting forces are internalized. Section 3.3 will more algebraically confirm the validity of this interpretation of the inverse Euler equation.

Lemma 3. *If $V(b, \theta)$ is concave and $q = \beta$, then*

  (i) $c_e(b, 0) > c(b, 0) > c_u(b, 0)$.
  (ii) $c(b, 1) > c(b, 0)$.
  (iii) $b > b_u(b, 0) > b_e(b, 0)$ and $b_u(b, 1) > b = b_e(b, 1)$.

Proof. (i) Assume that $c_u(b, 0) \geq c_e(b, 0)$. Then from equation (16), $b_e(b, 0) \geq b_u(b, 0)$. From (15) it is immediate that $c \in (c_e, c_u)$ and thus that $b_u(b, 0) \geq b$. By concavity of V, $c(b, \theta)$ is non-decreasing, and thus $c(b, 0) \geq c_e(b, 0) \geq c(b, 1)$, where the last inequality follows from $b_e(b, 0) \geq b_u(b, 0) \geq b$. When $\theta = 1$, we have that $b = b_e(b, 1)$. Moreover, since $c(b, 0) \geq c(b, 1) = c_u(b, 1)$, $b \geq b_u(b, 1)$. Collecting inequalities yield

$$b_e(b, 0) \geq b_u(b, 0) \geq b = b_e(b, 1) \geq b_u(b, 1)$$

From the budget constraint, and using the fact that $w > 0$, this implies that $c(b, 1) > c(b, 0)$, which contradicts $c(b, 1) \leq c(b, 0)$. Since $c(b, 1) \leq c(b, 0)$ was a corollary of $c_u(b, 0) \geq c_e(b, 0)$, we must have $c_u(b, 0) < c_e(b, 0)$.

Claims *(ii)* and *(iii)* are immediate consequences of the proof of *(i)*.   □

The mechanisms underlying the proof can be seen from equation (16), in which the utility gain/cost from a marginal increase in p is equalized. If $c_u > c_e$, the left-hand side in equation (16) states the utility *gained* through a marginal increase in p. It is a gain since a small increase in c_e, accompanied with a decrease in c_u, attains the marginal change in the right-hand side of the incentive compatibility constraint (11) necessary to accompany the change in p. Such a change provides more insurance and thus *increases* utility. However, due to interiority, there is an associated utility *cost*; b_e must be larger than b_u, and an increase in p thus increase the share of the budget spent on period $t+1$ resources. The proof then proceeds by showing that $c_u > c_e$ together with $b_u < b_e$, cannot be budget feasible since the wage when employed is strictly positive.

In a two period setting, the terms b_e and b_u in equation (16) may be replaced by $c_e - w$ and c_u, respectively. The intuition behind the result in Lemma 3 is then straightforward: To provide incentives to exert search effort, the government generates a positive correlation between employment and consumption, $c_e > c_u$. Insurance is provided by a low intertemporal variance, $c_e > c > c_u$. Concavity then ensures that this logic extends to a setting with an infinite planning horizon.

Remarks. The notion of Lemma 3 is equivalent to Proposition 1 in Hopenhayn and Nicolini (1997). The proof is however substantially different: Here, employment is not an absorbing state and the problem is primal rather than dual.

In Lemma 3, concavity of $V(b, \theta)$ is assumed.[7] The assumption is common in the literature and is indispensable for the analysis (Hopenhayn and Nicolini, 1997; Ljungqvist and Sargent, 2004). The difficulty in proving concavity lies in the fact that the choice set in (10) is not necessarily convex, and that (functions of) some choice variables does not enter the Bellman equation additively.[8]

Previous studies on optimal unemployment insurance abstract from self-insurance (e.g. Shavell and Weiss (1979), Hopenhayn and Nicolini (1997) and Pavoni (2007)). In the absence of savings, the policy implication from Lemma 3 is lucid; the tax/subsidy policy is defined as the difference between consumption and labor income, and benefit payments should therefore decrease along the duration of an unemployment spell. While Lemma 3

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[7]Indeed, conditions (14)-(16) are derived using Benveniste and Scheinkman's (1979) envelope theorem - a theorem that requires concavity.

[8]Note that these are sufficient, but not necessary conditions for concavity. All numerical solutions in, for instance, Hopenhayn and Nicolini (1997) and Ljungqvist and Sargent (2004) display a strictly concave value function (or, equivalently, a strictly convex cost function).

reveals that the consumption pattern remains unaltered in the current setting with self-insurance, the unemployment benefit policy does not: Most theoretical models of self-insurance (e.g. Aiyagari (1994)) display a decreasing consumption profile even in the absence of *any* unemployment benefit programme. It is thus the aim of the subsequent section to characterize the policy that can implement the optimal allocations in an economy with self-insurance.

3.2. Decentralization.

3.2.1. *A fiscal implementation.* The previous section characterized the constrained Pareto-optimal allocations attainable in the economy. This section will demonstrate *how* these allocations may be attained in a setting in which *the agents* choose consumption, search effort, and savings, taking the government's policy as given. The ultimate task of this section is thus to find the tax policy such that the agents' private choices corresponds to the optimal allocations derived above.

The agents in the decentralized economy have access to a riskless bond, a, that pays net (pre-tax) return equal to r. At time zero, the agents enter a market economy with a given level of cash-on-hand equal to b_0. For a given tax policy, the agents maximize their utility by choosing consumption, savings, and search processes that fulfill their intertemporal budget constraint. If there is a one-to-one correspondence between the chosen processes and the optimal allocation, $\boldsymbol{\sigma}^*$, the tax allocation is called a *fiscal implementation* of $\boldsymbol{\sigma}^*$.

Formally,

Definition 1. *Let $b_0 = a_0 - T_0$ be given. If there exist a tax allocation $\hat{T} = \{T_t\}_{t=0}^{\infty}$, $T_t : \Theta^t \times \mathbb{R}^t \to \mathbb{R}$, such that $\{c_t, a_{t+1}, p_t\}_{t=0}^{\infty}$ solves*

$$V(b_0, \theta_0) = \max_{\{c_t, a_{t+1}, p_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \int_{\Theta^{t+1}} \{u(c_t(b_0, \theta^t)) - (1 - \theta_t)v(p_t(b_0, \theta^t))\}\lambda(b_0, \theta^t)d\theta^t \quad (19)$$

subject to

$$w\theta_t + a_t(b_0, \theta^{t-1}) - T_t(\theta^t, a(b_0, \theta^t)) = c_t(b_0, \theta^t) + qa_{t+1}(b_0, \theta^t) \quad \text{for } t = 0, 1, \ldots \quad (20)$$

*and $\{c_t, p_t\}_{t=0}^{\infty}$ equals the optimal allocation $\boldsymbol{\sigma}^*$, then $\hat{T}$ is said to be a **fiscal implementation** of $\boldsymbol{\sigma}^*$.*

Note that the tax allocation has a very general form. Taxes in any period t may depend on the full history of employment as well on the full history of asset positions. To the extent that an optimal allocation depend on the complete employment history, the reason for the former is straightforward. The motivation behind the latter is less obvious; since the agents choose $t + 1$ assets using information available up to period t, it is plausible to conjecture that taxes in $t + 1$ will themselves only depend on information available up to period t.

However, as shown by Kocherlakota (2005), this intuition may fail; when actions are hidden there might not exist a fiscal implementation limited to this information set.

The following proposition shows that a fiscal implementation exists and that the resulting tax functions are *simple*: The tax level is recursive and contingent on the agent's current transition and her level of wealth.

Proposition 1. *There exist a time invariant tax function, $T_t = T(a_t, \theta_t, \theta_{t-1})$, that implements $\boldsymbol{\sigma^*}$.*

Proof. The proof is direct and establishes a one-to-one relationship between the government's and the agent's problem.

By Bellman's Principle of Optimality, the government's problem in (10)-(12) can be split up as

$$V(b, \theta) = \max_{c, \zeta}\{u(c) + X(\zeta, \theta)\}$$

$$\text{s.t.} \quad b = c - \theta w + q\zeta$$

$$X(\zeta, \theta) = \max_{p, b_e, b_u}\{-(1 - \theta)v(p) + \beta(pV(b_e) + (1 - p)V(b_u))\}$$

$$\text{s.t.} \quad v'(p) = \beta(V(b_e) - V(b_u))$$

$$\zeta = pb_e + (1 - p)b_u$$

Define functions T_e and T_u as $T_e(\zeta, \theta) = \zeta - b_e(\zeta, \theta)$ and $T_u(\zeta, \theta) = \zeta - b_u(\zeta, \theta)$, respectively. By definition,

$$X(\zeta, \theta) = \max_{p}\{-(1 - \theta)v(p) + \beta(pV(\zeta - T_e(\zeta, \theta)) + (1 - p)V(\zeta - T_u(\zeta, \theta)))\}$$

Thus,

$$V(b, \theta) = \max_{c, \zeta}\{u(c) + \max_{p}\{-(1 - \theta)v(p) + \beta(pV(\zeta - T_e(\zeta, \theta)) + (1 - p)V(\zeta - T_u(\zeta, \theta)))\}\}$$

$$= \max_{c, \zeta, p}\{u(c) - (1 - \theta)v(p) + \beta(pV(\zeta - T_e(\zeta, \theta)) + (1 - p)V(\zeta - T_u(\zeta, \theta)))\}$$

$$\text{s.t.} \quad b = c - \theta w + q\zeta$$

Where the last equality follows, again, from the Principle of Optimality. By construction, if $a' = \zeta$, the above Bellman equation is the recursive formulation of the decentralized problem given in Definition 1. $\qquad \square$

The above proposition hinges upon an important assumption: As in Kocherlakota (2005) and Albanesi and Sleet (2006), I assume that the fiscal implementation is such that the

optimal allocation is *"affordable"*. Affordability means that if the agent had the possibility to buy the optimal allocation, she would period-by-period afford it. That is,

$$w\theta_t + a_t - T_t = c_t + q(p_t b_{e,t+1} + (1 - p_t)b_{u,t+1})$$

This restriction is crucial for separating the effect of savings and taxes on consumption. Affordability implies that the government's state variable, b_t, must equal the agent's non-labor cash-on-hand $a_t - T_t$. As a consequence, taxes are strictly redistributive

$$a_{t+1} = (p_t(a_{t+1} - T_{e,t+1}) + (1 - p_t)(a_{t+1} - T_{u,t+1})) \tag{21}$$

By Lemma 3, it is thus immediate that $b_{u,t+1} > a_{t+1} > b_{e,t+1}$. The agent is consequently positively taxed when employed and negatively taxed when unemployed (or equivalently, receiving an unemployment benefit).

When savings and taxes are identified as above, the intuition underlying Proposition 1 is quite straightforward. Bellman's Principle of Optimality reveals that savings, a', is a sufficient state variable for the choice of b_e, b_u and p. The tax functions are then defined as the difference between savings and the optimal $t + 1$ non-labor cash-on-hand, b_e and b_u. By the design of the tax function, the agent can always choose the assigned allocation. Any other feasible choice amounts to imitating the $t + 1$ allocation of some other agent. By construction, imitating someone else is incentive compatible and budget feasible. Thus, since the allocation is optimal under incentive compatibility and budget feasibility, imitation cannot be optimal.

The tax functions in Proposition 1 are recursive in an agent's wealth, her current and previous employment state. Akin to the tax functions that map *savings* to state contingent cash-on-hand, functions $b_e(b, \theta)$ and $b_u(b, \theta)$ map period t resources to period $t + 1$ state contingent cash-on-hand. Why, then, could the tax functions not be recursive in (b, θ)? Inasmuch the optimal allocation still would be attainable for an agent operating in the decentralized economy, choosing the allocation would no longer be optimal: Imitating someone else is feasible, but *not* incentive compatible. By the same logic underlying the inverse Euler equation, the agent would, then, increase savings to equalize equation (18), violating the incentive compatibility of the optimal allocation.

Remarks. There is a continuum of tax systems that may implement any incentive feasible allocation. To appreciate this, consider an arbitrary incentive feasible allocation at time t. The agent consumes c and she exerted search effort in the previous period inducing p_{-1}. Her asset position and unemployment benefit handouts equal a and τ, respectively. Then another allocation with $a' = a + \epsilon$, $\tau' = \tau - \epsilon$ and $c' = c$, is still incentive compatible, feasible, and generates the same level of utility to the agent *for any real value of ε*. At

one extreme, 100% wealth- and labor taxes with lump-sum transfers equal to consumption, would indeed implement *any* allocation. Arguably, such a tax system is quite draconian and does not resemble the combined usage of taxes and markets to reallocate resources observed in most current economies. At another extreme, zero taxes and individually and quantitative-contingently priced Arrow securities could be designed to exactly mimic the problem in (10)-(12). While perhaps elegant, and by construction optimal, such a market arrangement requires an elaborate pricing system relying on common knowledge of individual asset positions and preferences.

Ruling out such elaborate asset structures and focusing on the one bond scenario, one may, alternatively, view the problem of indeterminacy as a question regarding savings. Specifically, it is a question regarding whether it is the government, or the agent (or any combination of the two), that carries out the intertemporal allocations of resources. Of course, inasmuch there really are a continuum of possible arrangement of storage, one may legitimately wonder on what basis one can rationally chose between those arrangements.

As in Kocherlakota (2005) and Albanesi and Sleet (2006), this paper imposes two assumptions in order to identify the effect of self-insurance from taxes/benefits on consumption. First, agents save using a riskless bond. The presence of a riskless bond can be thought of as a parsimonious representation of a more elaborate underlying diversified portfolio choice (at the intertemporal price q). Second, the optimal allocation is assumed to be period-by-period affordable. Fundamentally this assumes that all intertemporal transfers of resources are actualized by the agents' savings. This identification scheme guarantees to attain the optimal allocation with minimal governmental interference.

3.2.2. *Characterization.* While taxes has been shown to have a simple recursive representation, so far little has been shown regarding their properties. Examining the qualitative properties of the tax function T corresponds to examine how $T = a - b$ responds to a change in a. To this end, I will derive and exploit the properties of the *marginal tax functions*.

This section will state the main results, supported by brief comments. In the subsequent section, I will relate the results presented here to properties of a "weighted" Euler equation, and, in turn, relate this equation to the inverse Euler equation. For clarity of exposition, focus is put on the case (of interest) at $\theta = 0$. To facilitate notation, let $T_e(a')$ and $T_u(a')$ denote period $t+1$ taxes at the associated employment states at $\theta = 0$.

Proposition 2. *If $V(b, \theta)$ is concave, there exist marginal tax functions given by*

$$T_e'(a') = 1 - \frac{u'(c_u)}{pu'(c_u) + (1-p)u'(c_e)}, \quad T_u'(a') = 1 - \frac{u'(c_e)}{pu'(c_u) + (1-p)u'(c_e)}$$

Proof. See Appendix A. □

The idea behind the proof is to consider an infinitesimal change in a'. The resulting marginal change in taxes must be such that the government's first order conditions hold, incentive compatibility is preserved and the budget balances. In addition, the agent's decentralized first order condition must hold

$$u'(c) = \frac{\beta}{q}(pu'(c_e)(1 - T'_e(a')) + (1 - p)u'(c_u)(1 - T'_u(a')))$$

Combining the marginal taxes in Proposition 2 with the inverse Euler equation in (15) gives

$$T'_e(a') = 1 - \frac{qu'(c)}{\beta u'(c_e)}, \quad T'_u(a') = 1 - \frac{qu'(c)}{\beta u'(c_u)}$$

If $\beta = q$, and since $c_e > c > c_u$, it is evident that $T'_e(a') < 0$ and $1 > T'_u(a') > 0$. Thus, *both unemployment benefits and "reemployment taxes" are decreasing with the agents asset position.*

Corollary 1. *Marginal taxes are expected to be zero.*

Proof. When the agent is unemployed Proposition 2 together with the inverse Euler equation (15), gives the result.

When the agent is employed, taxes satisfies $a' = \gamma(a' - T_e(a')) + (1 - \gamma)(a' - T_u(a'))$. If taxes are differentiable, the derivative of this expression with respect to a' gives the result. $\qquad\square$

Zero expected marginal taxes are not particularly surprising in this setting; by the construction of the tax functions, taxes are always expected to be zero. A ceteris paribus change in savings mimics the action taken by some other agent and taxes respond accordingly.

The main part of the literature on optimal unemployment insurance has concluded that benefit payments ought to decrease along the duration of unemployment. The result is intuitive; in the absence of savings, a decreasing benefit profile induces a decreasing consumption profile, providing both insurance as well as sufficient search effort incentives. Abstracting from savings, Lemma 3 confirms this result. Nevertheless, Proposition 1 shows that this result does not immediately generalize to a setting in which partial self-insurance is present: The tax policy is time-invariant and thus independent of the duration of the unemployment spell. In addition, the following proposition reveals that the intuition supporting a decreasing benefit profile fails in the current setting. Indeed, along the duration of the unemployment spell, the agent will decumulate assets and the sequence of unemployment benefits will observationally be *increasing*.

Proposition 3. *If $V(b, \theta)$ is concave and $\beta = q$, then (i) $a > a'$, (ii) $T_u(a) > T_u(a')$, and (iii) $T_e(a) < T_e(a')$.*

Proof. By Proposition 2, $1 > T_u'(a') > 0$. Thus for any a_1 and a_2, such that $a_1 > a_2$, $T_u(a_1) > T_u(a_2)$. If $a' \geq a$, $1 > T_u'(a')$ implies that $b_u \geq b$, which contradicts Lemma 3, part *(iii)*. Thus $a > a'$, $T_u(a) > T_u(a')$ and, by Proposition 2, $T_e(a) < T_e(a')$. □

The result is intuitive. During unemployment, the agent exploits the insurance effect of savings by decumulating assets. Proposition 2 infers that unemployment taxes are positively related to the agent's asset position. Thus, as the agent's level of assets decline, so does the level of the tax. Since unemployment taxes are negative this implies that unemployment benefits will increase.

Accompanied with the inverse Euler equation, Proposition 3 has an intuitive explanation. First, wealth has a first order insurance effect. The higher is an agent's wealth, the less she needs to worry about loss of consumption if she loses her job. Second, in order to provide incentives to exert search effort, the government wishes to generate a positive correlation between consumption and employment. When the agent's utility function is concave, a higher level of savings makes it costlier for the government to induce such a correlation and the agent's search effort decreases. By generating a negative correlation between savings and unemployment benefits, the government manages to mitigate the distortionary effect of savings on search.

3.3. **The Euler equation, taxes, and the inverse Euler equation.** I now provide a deeper intuition underlying some of the results presented in the preceding sections. To this end I will consider an equivalent version of the government's problem in which the sole choice is strictly intertemporal, and not state contingent. It will be shown how this problem formulation leads to a "weighted Euler equation", and further how these weights relate to marginal taxes. At the optimum, the weighted Euler equation implies the inverse Euler equation.

The inverse Euler equation can be thought of as the outcome when savings are chosen to balance two conflicting forces: To maximize utility, resources should be allocated to where the marginal benefit of resources is relatively high. For incentive compatibility, resources should be allocated to states in which the marginal benefit of resources is relatively low. Since incentive compatibility inflicts with period $t + 1$ resources only, it is thus optimal relegate a relatively high degree of resources to period t consumption. As a result, the agent *appears* savings constrained.

For a given value of savings, it is instructive to think of the optimal division of period $t+1$ resources across employment states as *functions* fulfilling two restrictions: The incentive compatibility constraint and the budget constraint. Similar to the tax functions explored in the previous section, these functions then allocate, for a given level of savings, resources to the different employment states. Let the government choose savings, a', and let the functions

$\delta_e(a')$ and $\delta_u(a')$ allocate resources between employment states such that the budget is balanced and incentive compatibility holds. That is, for a given p, $a' = p\delta_e(a') + (1-p)\delta_u(a')$ and $v'(p) = \beta(V(\delta_e(a')) - V(\delta_u(a')))$.

The government then faces the following intertemporal maximization problem

$$V(b) = \max_{a'}\{u(b - qa') + \beta(pV(\delta_e(a')) + (1 - p)V(\delta_u(a')))\}$$

The first order condition to the above problem, evaluated at the optimal solution, is given by

$$u'(c) = \frac{\beta}{q}(pV'(b_e)\delta_e'(a') + (1 - p)V'(b_u)\delta_u'(a')) \tag{22}$$

Equation (22) resembles a standard Euler equation, and has an interpretation in terms of marginal intertemporal trade-offs: The utility cost of an marginal increase in savings equals its feasible marginal utility gain. As with standard intertemporal problems, the $t+1$ feasible marginal utility gain is determined by the feasible inflow of resources in period $t+1$ - a marginal decrease of period t consumption is accompanied by a proportional marginal increase of period $t + 1$ resources, weighted by the interest rate: $1 = p\delta_e'(a') + (1 - p)\delta_u'(a')$. In addition, however, there is a further restriction on how the period $t + 1$ resources must be divided between employment states. In order to leave p unaltered, a *marginal* incentive compatibility constraint must hold

$$V'(\delta_e(a'))\delta_e'(a') = V'(\delta_u(a'))\delta_u'(a')$$

One can combine this marginal incentive compatibility constraint with the "marginal budget constraint" above, to solve for the weights $\delta'(a')$

$$\delta_e'(a') = \frac{V'(b_u)}{pV'(b_u) + (1 - p)V'(b_e)}, \quad \delta_u'(a') = \frac{V'(b_e)}{pV'(b_u) + (1 - p)V'(b_e)} \tag{23}$$

The expressions above reveals an important feature: Whenever $V'(b_u) > V'(b_e)$, $\delta_e'(a') > \delta_u'(a')$, and vice versa. That is, for states in which the marginal value of resources is relatively high, the marginal inflow of resources should be relatively low. Substituting the relationship in (23) into (22) gives the inverse Euler equation.

It is important to note that the functions in (23) are directly related to the marginal taxes derived in Proposition 2. Specifically, $\delta'(a') = 1 - T'(a')$. The intuition underlying the shape of the tax function then becomes evident: For a certain choice of p to remain incentive compatible, an increase in savings must be divided between employment states such that the incentive compatibility constraint holds. That is, the inflow of resources must be relatively high at states in which the marginal value of resources is relatively low. By Lemma 3, the marginal value of resources is high in the unemployed state, and the

additional inflow must therefore be low. Since the optimal policy is recursive in an agent's wealth, a higher level of assets must induce a lower level of unemployment benefits.

4. Concluding Remarks

This paper has studied a model of optimal redistribution policies in which the foremost risk in an agent's life is unemployment. Moral hazard arises as job search effort is unobservable. The model permits agents to self-insure by means of a riskless bond.

In contrast with previous studies in the literature, it is shown that the optimal unemployment insurance *policy* does not display any duration dependence. Whereas wealth encodes the agents' relevant employment status history, the insurance policy is time-invariant and, instead, contingent on the agents' asset position. In order to induce job search effort, the government wishes to provide a positive correlation between consumption and employment status. Since a higher level of savings reduces the correlation, unemployment benefits relate negatively to wealth. The agents decumulates assets over the unemployment spell in order to exploit the intrinsic insurance effect of wealth. Thus, the sequence of benefit payments is, observationally, increasing with the duration of unemployment.

The policy implications from the analysis are stark; unemployment benefits should be asset based and relate negatively to wealth. As wealth itself encodes insurance possibilities, the negative relation between wealth and unemployment benefits is intuitive. However, asset based approaches have commonly been criticized for its distortive, and negative, effect on savings (e.g. Hubbard, Skinner and Zeldes (1995), Gruber (1998)). Although undesirable per se, this paper has revealed an additional effect of wealth; a higher level of savings reduces the opportunity cost of being employed and thus increases the unemployment duration. Together, the net distortive effect of an asset based scheme appears to be favorable.

There are several ways in which an asset based unemployment insurance programme could be accomplished. As with Medicaid, food stamps, and until recently, Aid to Families with Dependent Children (AFDC), to mention a few social policies in the United States, unemployment benefits may be asset based means tested; that is, unemployment benefits are paid only if an agent has assets below a specified maximum amount. Alternatively, and obviously, schemes may be more elaborate with a continuous decline in benefit payments as assets increases.

References

Aiyagari, R. S.: 1994, Uninsured idiosyncratic risk and aggregate saving, *Quarterly Journal of Economics* **109**(3).

Albanesi, S. and Sleet, C.: 2006, Dynamic optimal taxation with private information, *The Review of Economic Studies* **73**(1), 1–30.

Aliprantis, C. D. and Border, K. C.: 1999, *Infinite Dimensional Analysis: A Hitchhiker's Guide*, 2 edn, Springer Verlag.

Benveniste, L. M. and Scheinkman, J. A.: 1979, On the differentiability of the value function in dynamic models of economics, *Econometrica* **47**(3), 727–732.

Blank, R. and Card, D.: 1991, Recent trends in insured and uninsured unemployment: Is there an explanation, *Quarterly Journal of Economics* **106**(4), 1157–1190.

Chiu, H. W. and Karni, E.: 1998, Endogenous adverse selection and unemployment insurance, *Journal of Political Economy* **106**(4), 806–827.

Golosov, M., Kocherlakota, N. and Tsyvinski, A.: 2003, Optimal indirect and capital taxation, *The Review of Economic Studies* **70**(3), 569–587.

Golosov, M. and Tsyvinski, A.: 2006, Designing optimal disability insurance: A case for asset testing, *Journal of Political Economy* **114**(2), 257–269.

Gruber, J.: 1997, The consumption smoothing benefits of unemployment insurance, *American Economic Review* **87**(1), 192–205.

Gruber, J.: 1998, Unemployment insurance, consumption smoothing, and private insurance: Evidence from the psid and cex, *Research in Employment Policy* **1**, 3–32.

Hopenhayn, H. A. and Nicolini, J. P.: 1997, Optimal unemployment insurance, *Journal of Political Economy* **105**(2), 412–438.

Hubbard, G. R., Skinner, J. and Zeldes, S. P.: 1995, Precautionary saving and social insurance, *Journal of Political Economy* **103**(2), 360–399.

Kocherlakota, N.: 2005, Zero expected wealth taxes: A mirrlees approach to dynamic optimal taxation, *Econometrica* **73**(5), 1587–1621.

Ljungqvist, L. and Sargent, T. J.: 2004, *Recursive Macroeconomic Theory*, 2 edn, The MIT Press, Cambridge, Massachusetts.

McCall, J.: 1970, Economics of information and job search, *Quarterly Journal of Economics* **84**(1), 757–782.

Meyer, B. D.: 1990, Unemployment insurance and unemployment spells, *Econometrica* **58**(4), 757–782.

Mirrlees, J. A.: 1971, An exploration in the theory of optimum income taxation, *The Review of Economic Studies* **38**(2), 175–208.

Moffitt, R.: 1985, Unemployment insurance and the distribution of unemployment spells, *Journal of Econometrics* **28**(1), 85–101.

Oswald, A. J.: 1986, Unemployment insurance and labor contracts under assymetric information: Theory and facts, *American Economic Review* **76**(3), 365–377.

Pavoni, N.: 2007, On optimal unemployment compensation, *Journal of Monetary Economics* .

Pavoni, N. and Violante, G. L.: 2007, Optimal welfare-to-work programs, *The Review of Economic Studies* **74**(1), 283–318.

Rogerson, W.: 1985, Repeated moral hazard, *Econometrica* **53**(1), 69–76.

Shavell, S. and Weiss, L.: 1979, The optimal payment of unemployment insurance benefits over time, *Journal of Political Economy* **87**(6), 1347–1362.

Shimer, R. and Werning, I.: 2005, Liquidity and insurance for the unemployed, *Federal Reserve Bank of Minneapolis, Research Department Staff Report* **366**.

Spear, S. E. and Srivastava, S.: 1987, On repeated moral hazard with discounting, *The Review of Economic Studies* **54**(4), 599–617.

APPENDIX A. PROOFS

A.1. **Lemma 2.**

Proof. Equations (8) and (9) are repeated for convenience:

$$V(b_0, \theta_0) = u(c_0^*(b_0, \theta_0)) - (1 - \theta_0)v(p_0^*(b_0, \theta_0)) + \beta \int_{\Theta^1} V(\boldsymbol{\sigma}^*, b^*(\theta_1), \theta_1)\lambda(b_0, \theta^1)d\theta^1 \qquad \text{(A1)}$$

$$b_0 = c_0^*(b_0, \theta_0) - \theta_0 w + q \int_{\Theta^1} b^*(\theta_1)\lambda(b_0, \theta^1)d\theta^1 \qquad \text{(A2)}$$

The proof proceeds in three steps: *First* it will be shown that for any *utility maximizing* or *resource minimizing* allocation, the Inada-conditions on $u(c)$ implies that if $c_t(b_0, \theta^t) = 0$, then $c_{t+s}(b_0, \theta^{t+s}) = 0$, for $s > 0$, almost surely ($\lambda(b_0, \theta^{t+s})$-a.s.). *Second*, focusing on the interior case, it will then be shown that $b^*(\theta_1)$, as given in equation (A2), is resource minimizing under the value $V(\boldsymbol{\sigma}^*, b^*(\theta_1), \theta_1)$. *Third* it will be shown that *duality* holds; that is if $b^*(\theta_1)$ is resource minimizing under $V(\boldsymbol{\sigma}^*, b^*(\theta_1), \theta_1)$, then $V(\boldsymbol{\sigma}^*, b^*(\theta_1), \theta_1)$ is utility maximizing under $b^*(\theta_1)$ - that is, $V(\boldsymbol{\sigma}^*, b^*(\theta_1), \theta_1) = V(b^*(\theta_1), \theta_1)$.

Step 1. For *any* utility maximizing or resource minimizing allocation, define $\delta(b_0, \theta^t)$ as

$$\delta(b_0, \theta^t) = u(c_t) - v(p_t) + \beta(p_t u(c_{t+1}^1) + (1 - p_t)u(c_{t+1}^0))$$

The dependency of c_t, p_t and c_{t+1}, on (b_0, θ^t) and $(b_0, (\theta^t, \theta_{t+1}))$ is here left implicit. Assume that $\lambda(b_0, \theta^t) > 0$. Consider the following problem

$$\max_{x,z,y}\{y - q(px + (1-p)z)\}$$
$$\text{s.t.} \quad \delta(b_0, \theta^t) = u(c_t - y) - v(p_t) + \beta(p_t u(c_{t+1}^1 + x) + (1 - p_t)u(c_{t+1}^0 + z))$$
$$u(c_{t+1}^1 + x) - u(c_{t+1}^0 + z) = u(c_{t+1}^1) - u(c_{t+1}^0)$$
$$c_t \geq y, \quad c_{t+1}^1 \geq -x, \quad c_{t+1}^0 \geq -z$$

where the allocation $\{c_t, p_t\}_{t=0}^{\infty}$ is incentive feasible. At the optimal allocation, the solution to the above problem is given by $x = y = z = 0$. To see why, notice that any deviation of x, y, and z from zero, fulfilling the above restrictions, is feasible and incentive compatible. Moreover, such a perturbation frees up period t resources equal to $y - q(px + (1-p)z)$. These additional resources may, if properly discounted, be allocated as period zero consumption - or, in a resource minimizing setting, as less period zero resources - without inflicting with incentive compatibility.

Assume that $c_t = 0$. Then the first order necessary conditions to the above problem with respect to x, y and z, evaluated at zero, must observe

$$\frac{1}{u'(0)} \geq \frac{\beta}{q}\left(p_t \frac{1}{u'(c_{t+1}^1)} + (1 - p_t)\frac{1}{u'(c_{t+1}^0)}\right) \qquad \text{(A3)}$$

Since $u'(0) = \infty$, c_{t+1}^1 must also equal zero whenever $p_t > 0$. The same holds for c_{t+1}^1 whenever $(1 - p_t) > 0$. Thus if $c_t(b_0, \theta^t) = 0$ for any θ^t with $\lambda(b_0, \theta^t) > 0$, then $c_{t+s}(b_0, \theta^{t+s}) = 0$, $\lambda(b_0, \theta^{t+s})$-a.s.

Step 2. Consider the problem of choosing sequences $c_t : V \times \Theta^t \to \mathbb{R}_+$ and $p_t : V \times \Theta^t \to [0,1]$ in order to solve

$$b'(V_0, \theta_0) = \min_{c,p} \{ \sum_{t=0}^{\infty} q^t \int_{\Theta^t} \{ c_t(b_0, \theta^t) - \theta_t w \} \lambda(b_0, \theta^t) d\theta^t \} \tag{A4}$$

$$\text{s.t.} \quad V(\boldsymbol{\sigma}, V_0, \theta_0) \geq V_0 \tag{A5}$$

$$\{p_t\}_{t=0}^{\infty} = \text{argmax} \{ V(\boldsymbol{\sigma}, V_0, \theta_0) \}, \quad \forall \, (b_0, \theta_0) \in B \times \Theta \tag{A6}$$

Where $V_0 = V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1)$ from the utility maximizing solution in (A1). If the constraint in (A5) is non-binding, then $c_0 = 0$ and, by *Step 1.* above, $c_t(V_0, \theta^t) = 0 \, \forall \, \theta^t$. I will henceforth refer to this solution as the *zero solution*. It is important to note that a non-zero solution attains at least as high utility as the zero solution; at any non-zero solution, the agent could exert the same search effort as at the zero solution (which is zero), and attain a strictly higher level of utility. Thus, independently of c_0 being interior, constraint (A5) must hold as an equality.

Could b' in (A4) take on a smaller value than $b^*(\theta_1)$ in (A2)? If so, there exist a $b'(\theta_1)$ such that $V(b'(\theta_1), \theta_1) = V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1) \, \forall \, \theta_1 \in \Theta$ and $b'(\theta_1) < b^*(\theta_1)$ for at least one value of θ_1. At this alternative allocation, p_0^* is still incentive compatible and

$$V(b_0, \theta_0) = u(c_0^*(b_0, \theta^0)) - (1 - \theta_0)v(p_0^*(b_0, \theta^0)) + \beta \int_{\Theta^1} V(b'(\theta_1), \theta_1) \lambda(b_0, \theta^1) d\theta^1$$

$$b_0 > c_0^*(b_0, \theta^0) - \theta_0 w + q \int_{\Theta^1} b'(\theta_1) \lambda(b_0, \theta^1) d\theta^1$$

Where the last inequality together with monotonicity of $V(b_0, \theta_0)$ implies thus that $\boldsymbol{\sigma^*}$ could not have attained the maximum in (5).

Step 3. In order to complete the proof, it must be shown that $V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1)$ attains the maximum value under resources $b^*(\theta_1)$.

Assume that $V(b^*(\theta_1), \theta_1) > V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1)$. By Berge's Maximum Theorem (Aliprantis and Border, 1999), $V(b^*(\theta_1), \theta_1)$ is continuous in b. Since any non-zero solution renders greater utility than the zero solution, $c_1(b^*(\theta_1), \theta_1) > 0$, and there exist a $b^{**}(\theta_1)$ arbitrarily close to $b^*(\theta_1)$ such that $b^*(\theta_1) > b^{**}(\theta_1)$ and $V(b^{**}(\theta_1), \theta_1) > V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1)$. This contradicts that $b^*(\theta_1)$ was resource minimizing for $V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1)$. Thus $V(b^*(\theta_1), \theta_1) = V(\boldsymbol{\sigma^*}, b^*(\theta_1), \theta_1)$. $\qquad \square$

A.2. **Proposition 2.**

Proof. The proof is direct and derives the implied marginal taxes from an infinitesimal change in assets.

By construction, the equilibrium tax functions satisfies

$$a' = p(a')(a' - T_e(a')) + (1 - p(a))(a' - T_u(a'))$$

Thus, if the tax functions are differentiable, the following must hold for the marginal tax

$$p'(a')(T_u(a') - T_e(a')) = pT_e'(a') + (1 - p)T_u'(a') \tag{A7}$$

From the incentive compatibility constraint we have

$$v''(p)p'(a') = \beta(V_e'(a')(1 - T_e'(a')) - V_u'(a')(1 - T_u'(a'))) \tag{A8}$$

Substituting the relationships $b_e = a' - T_e(a')$ and $b_u = a' - T_u(a')$ into (16) (the government's first order condition for p) gives

$$q(T_u(a') - T_e(a')) = \frac{\mu}{\lambda}v''(p) \tag{A9}$$

Where λ and μ are the multipliers on the budget and incentive compatibility constraint, respectively. Substituting (A9) into (A7)

$$p'(a')v''(p)\frac{\mu}{\lambda q} = pT_e'(a') + (1 - p)T_u'(a') \tag{A7'}$$

Substituting (A8) into (A7$'$)

$$\beta(V_e'(a')(1 - T_e'(a')) - V_u'(a')(1 - T_u'(a')))\frac{\mu}{\lambda q} = pT_e'(a') + (1 - p)T_u'(a') \tag{A7''}$$

In addition, the agent's decentralized first order condition must hold:

$$u'(c) = \frac{\beta}{q}(pu'(c_e)(1 - T_e'(a')) + (1 - p)u'(c_u)(1 - T_u'(a'))) \tag{A10}$$

Using equation (15) and solving equations (A7$''$) and (A10) yields

$$T_e'(a') = 1 - \frac{u'(c_u)}{pu'(c_u) + (1 - p)u'(c_e)}, \quad T_u'(a') = 1 - \frac{u'(c_e)}{pu'(c_u) + (1 - p)u'(c_e)} \qquad \Box$$