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Dynamic Factor Models in Estimation and Forecasting

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Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence
March 2008

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Contents

Introduction

The growing amount of information available to policy-makers and researchers has made possible the application of dynamic factor models to the forecasting of macroeconomic indicators. While traditional VARs and VECMs include only a few variables from the large data sets available to policy makers, dynamic factor models allow the incorporating of information extracted from many variables into a small-scale econometric model.

Dynamic factor models as developed by Stock and Watson (1998), have recently become a subject of intensive research. They were used for the forecasting of macroeconomic variables in the US, UK and Euro-area (Stock and Watson (2002), Marcellino, Stock and Watson (2003), Artis, Banerjee and Marcellino (2003)). Factor-augmented VARs have also become an instrument of monetary policy analysis (Favero and Marcellino (2001), Bernanke, Boivin and Eliasz (2004), Belviso and Milani (2005), Stock and Watson (2005)) and forecasting (Hansson, Jansson, and Lof (2003), Monch (2005)).

Another growing stream of research concerns the measurement of common stochastic trends in large data sets, using a dynamic factor structure. It was initiated by Bai and Ng (2004) and Bai (2004). The factor structure allows for the evaluation of the number of common trends and the testing of cointegration in large panels. Despite theoretical developments there are still very few empirical applications of this approach. Flad (2007) uses PANIC as developed in Bai and Ng (2004), to measure common trends of the money market interest rates in the euro area, Bai (2004) measures common trends of the sectoral employment in the USA.

Banerjee and Marcellino (2007) consider implications of common stochastic trends in large panels for modelling the short run dynamics of small systems of variables. They introduce the factor-augmented error-correction model (FECM) which incorporates non-stationary factors extracted from a large panel of data into a small system of variables. Banerjee and Marcellino (2007) argue that such a model may overcome the problem of omitted variables in a small system.

Though the stream of research about factor models is growing, little is known about the performance of the factor models in small samples and in the presence of different types of structural breaks. Except for Banerjee and Marcellino (2007), there are no empirical applications of factor-augmented models which evaluate implications of cointegration in large panels for small systems of variables.

This thesis addresses the issue of the relative performance of dynamic factor models in finite samples in the presence of structural breaks. It extends an existing literature by considering new data sets and evaluating finite sample properties of dynamic factor models and factor-augmented VARs and VECMs in Monte Carlo exercises.

In the first paper, the relative forecasting performance of dynamic factor models and small-scale VARs is evaluated on the basis of data available for two large emerging market economies, Brazil and Russia. As these two economies can be characterized by the small time spans of available data and structural breaks, an empirical exercise conducted for these countries, allows us to explore the relative performance of factor models in small samples in the presence of structural changes.

The results of this forecasting exercise show that both VARs and factor models are useful in forecasting inflation and output growth, but their relative performance differs for different forecast variables. It allows us to suggest that the relative efficiency of forecasting models depends on the statistical properties of the series under consideration, in particular, on the persistence of the series and on the type and size of the structural changes in the series.

The findings of the first paper provide the motivation for the second paper, in which the relative performance of autoregressive models and dynamic factor models is explored in the Monte Carlo exercise. The data are generated with structural breaks and the role of intercept correction and differencing for robustifying forecasts in the presence of breaks is also evaluated. Factor models work well if there is a break in the middle of estimation period. However, their relative performance deteriorates if there is a break in the end of the estimation period. Intercept correction and differencing are also found to be useful in several cases.

It can be an explanation of the results of the empirical exercise conducted for Brazil and Russia, where factor forecasts perform well for the series with a break in the middle of estimation period (output growth in Russia), but the relative performance of factor forecasts deteriorates if there is a break in the forecasting period (inflation in Brazil).

In the third paper the performance of factor-augmented error-correction models is evaluated in a Monte Carlo exercise and a detailed empirical application is conducted for the forecasting of bank retail rates in the euro-area. The hypothesis considered is that the inclusion of the common stochastic trends extracted from the large data set can improve the in-sample and the out-of-sample performance of the small-scale ECMs.

It is found both in the Monte Carlo exercise and the empirical application that in-sample the FECM performs well relative to the models which are not augmented by factors, but the forecasting performance of the FECM is no better than the forecasting performance of other models for the one-period horizon. However, the results of the empirical exercise indicate that the forecasting performance of the FECM can improve for the longer horizons where the long run relations can dominate the short run cycles which are better approximated by simple VAR models.

Chapter 1
Forecasting Emerging Market Indicators: Brazil and Russia

Abstract

The adoption of inflation targeting in emerging market economies makes accurate forecasting of inflation and output growth of primary importance for these economies. Since only short spans of data are available for emerging markets, autoregressive and small-scale vector autoregressive models can be suggested as forecasting tools. However, these models include only few time series from the whole variety of data available to forecasters. Therefore dynamic factor models extracting information from a large number of time series, can be suggested as a reasonable alternative. In this paper two approaches are evaluated on the basis of data available for Brazil and Russia. The results allow us to suggest that the forecasting performance of the models considered depends on the statistical properties of the series to be forecast, which are affected by structural changes and changes in operating regime. This interaction between the statistical properties of the series and the forecasting performance of models requires more detailed investigation.

1 Introduction: Monetary Policy and Forecasting

Forecasts of inflation and output growth provide the basis for the development of monetary policy within an inflation targeting framework. According to Svensson (1999) an inflation targeting framework is characterized by (1) an explicit quantitative inflation target; (2) an operating procedure that can be described as inflation-forecast targeting, namely the use of an internal conditional inflation forecast as an intermediate target variable; and (3) a high degree of transparency and accountability.

The operating procedure can be described as inflation-forecast targeting in the following sense: the central bank's internal conditional inflation forecast is used as an intermediate target variable. An instrument path is selected which results in a conditional inflation forecast in line with a target for the inflation forecast. This instrument path then constitutes the basis for the current instrument setting.

In the theoretical literature (Svensson (1999), Woodford (2003)) this procedure is referred to as a targeting rule as opposed to an instrumental (Taylor) rule that expresses an interest rate as a prescribed function of predetermined or forward-looking variables, or both. The targeting rule does not specify a formula for the central bank's interest-rate operating target. Rather, an interest rate is set at whatever level may turn out to be required in order for the bank's conditional forecast to be in line with an inflation target.

During the 1990s several advanced industrial countries (United Kingdom, Sweden, Norway, Canada, Australia, and New Zealand) introduced inflation targeting as a framework for the conduct of monetary policy. Towards the end of the 1990s a few post-Soviet countries (Czech Republic (1997), Poland (1998), and Hungary (2001)) also shifted to inflation targeting. Brazil adopted an inflation targeting framework in 1999 and the Central Bank of Russian Federation started announcing inflation targets in 2003.

A classical example of inflation-forecast targeting is the procedure used by the Bank of England. The Bank of England adopts a given operating target i_t for the overnight interest rate at date t if and only if the Bank's forecast of the evolution of inflation over the next two years, conditional upon the interest rate remaining at the level i_t, implies an inflation rate of 2.5 percent per annum (the Bank's current inflation target) two years after date t (Vickers (1998)). In the development of the conditional inflation forecast the Bank of England uses a suite of models rather then a single model (Hatch (2001)). The Bank's large-scale core model of the UK economy is supplemented by small-scale macroeconometric models, Phillips-curve models, vector autoregressive models, and survey data. The final inflation projection published in the Inflation Report is the result of the collective judgement of the Monetary Policy Committee.

The experience of the Bank of England and the central banks of other industrial countries has been used by central banks of emerging market economies. In the second half of the 1990s the central banks in many emerging markets have abandoned fixed exchange rate regimes and replaced them with more flexible exchange rate arrangements. The fixed exchange rate was used as a nominal anchor to achieve a rapid stabilization of the price level. However, while inflation did decline significantly, it did not decline enough to prevent a large real appreciation of national currencies. This real appreciation eroded relative competitiveness of emerging market economies and ultimately created significant current account deficits. Under these conditions the central banks of these economies were forced to abandon fixed exchange rates. When abandoning the exchange rate peg, the central banks had to decide which nominal anchor to use instead of a fixed exchange rate. The successful experience of advanced industrial countries suggested the adoption of inflation targeting.

The most serious objection raised against the adoption of inflation targeting in emerging market economies is the limited ability to forecast inflation in these economies (Jonas and Mishkin (2003)). This is partly the result of the relatively frequent occurrence of shocks and the large degree of openness of emerging markets. Mainly due to an inability to forecast inflation and economic growth accurately, the countries that opted for the inflation targeting regime had significant deviations from their chosen targets. The central banks of these countries (Czech Republic, Poland) responded by the widening of target bands and the introduction of exceptional events into their monetary programs. But they also tried to improve their conditional inflation forecasts by the development of forecasting tools and the incorporation of a growing amount of information.

In this paper we look at the experience of Brazil and Russia, two of the largest emerging market economies. The IMF and the World Bank include them in the ten largest economies in the world with respect to the dollar estimates of GDP, which are computed using purchasing power parity (PPP). Therefore the investigation of these economies is of particular interest.

We focus on forecasting CPI inflation and GDP growth in Brazil and Russia.

Forecasts from autoregressive (AR) models and small-scale vector autoregressive (VAR) models are compared with those from dynamic factor models. Given the small time span of reliable data for Brazil and Russia, AR and small-scale VAR models, including only few variables and few parameters, can be considered as a reasonable forecasting tool. On the other hand, dynamic factor models extract information from a large number of time series, despite the small time span of data. We provide evidence on the relative forecasting performance of AR, VAR, and dynamic factor models in small sample in the presence of structural changes.

The presence of structural changes in forecast variables and many predictors raises the important question about the correction of models for these non- stationarities. Since the complexity of the structural changes and lack of observations complicate the modeling of these changes explicitly, forecasts can be robustified by application of methods proposed by Clements and Hendry (1998, 1999). Among these methods are intercept correction of the forecast and additional differencing of the variable to be forecast. Their efficiency is going to be evaluated in application to autoregressive models.

The paper is organized as follows. In section 2 we briefly consider economic developments and monetary policy in Brazil and Russia over the last ten years, and evaluate the role of forecasting in implementation of monetary policy. Section 3 describes the forecasting models, data sets, and criteria for forecast comparison. In Section 4 the results of forecast comparison are reported. In Section 5 we propose some general conclusions and suggestions for further research.

2 Inflation Targeting in Brazil and Russia

2.1 Brazil

The crawling peg regime in Brazil, initiated in mid-1994, successfully brought annual inflation to one-digit figures in less than three years. However, it led to the overvaluation of the national currency and a growing current account deficit. Trade imbalances and accumulated public debt left Brazil vulnerable to a confidence crisis, which became a reality with the international financial turmoil of 1997-1998 culminating with the Russian moratorium in August 1998. The Russian crisis generated a capital flight from Brazil, and the Central Bank of Brazil was forced to abandon the crawling peg regime: the real was forced to float on January 1999.

The new exchange rate regime required a new anchor for monetary policy and in July 1999 Brazil adopted inflation targeting as the monetary policy framework. The Broad Consumer Price Index (IPCA) was chosen for measuring inflation. The targets were set at 8% for 1999, 6% for 2000 and 4% for 2001. Tolerance intervals of 2% for each year were also defined.

In order to support the monetary policy decision process, the Research Department of the Central Bank of Brazil has developed a set of tools which include a structural model of the transmission mechanism of monetary policy to prices, short-term inflation forecasting models, leading inflation indicators, and surveys of market expectations (Bogdanski, Tombini and Werlang (2000)). The structural model includes an IS-type equation, a Phillips curve, an uncovered interest parity condition, and monetary policy rules. This model is complemented by a set of short-term models including Autoregressive Moving Average (ARMA) and Vector Autoregressive (VAR) models. The forecasts of the structural and time-series models are complemented by survey data-based forecasts and used for the projection of CPI inflation and GDP growth.

Bogdanski, Tombini and Werlang (2000) emphasize that monetary policy decisions in the Bank of Brazil are taken on the basis of the widest information set available. This information includes dynamics of production, investment, and consumption; developments in the labour market; state of public finance; dynamics of disaggregated price indices; exports, imports, and exchange rate dynamics; changes in the international economy; and market expectations. Using this data, the Monetary Policy Committee of the Bank of Brazil develops the baseline scenario and decides on the inflation target and the interest rate path.

Implementing inflation targeting, the Central Bank of Brazil succeeded in keeping the inflation rate within the tolerance intervals in 1999 and 2000 (Table 1). However, the Argentine crisis and the terrorist attacks to the United States in September 2001 generated large capital outflows from the Brazilian economy and rapid depreciation of the real. Together with the accelerated growth of administered prices it implied an increase of the CPI inflation rate above the tolerance interval. In 2002 the confidence crisis continued. It was triggered by concerns that the new president, who had been elected that year, would default on the national debt. Therefore the depreciation of the real continued and inflation accelerated. As a result, despite the upward shift of the inflation targets and expanding of the tolerance intervals (up to 2.5%) the Central Bank of Brazil failed to hit the inflation targets in 2002 and 2003: inflation reached levels well above the tolerance intervals. Only in 2004 did the Central Bank of Brazil succeed in decelerating inflation and bringing the inflation rate within the tolerance interval.

Table 1 Forecast and actual inflation in Brazil and Russia

Year	Brazil		Russia	
	Target	Actual	Forecast/Target	Actual
1999	8 (6 -10)	8.9	30	35.5
2000	6 (4 - 8)	6	18.6	20.2
2001	4 (2 - 6)	7.7	12 - 14	18.6
2002	3.5 (1.5 - 5.5)	12.5	12 - 14	15.1
2003	4 (1.5 - 6.5)	9.3	10-12	12
2004	5.5 (3 - 8)	5.7	8-10	11.7

2.2 Russia

The Central Bank of Russia has started announcing inflation targets much later than the Bank of Brazil. From 1995 onwards Russia had the crawling band regime. As in Brazil, the introduction of the crawling band allowed inflation to decrease significantly but it did not decrease sufficiently to prevent the real appreciation of the national currency. In 1998, the Asian crisis and decrease of oil prices in the international market led to large capital outflows from the Russian economy. The adverse external factors combined with the growing public debt led to a currency crisis and default on national obligations in August 1998. The crawling band regime was abandoned, the exchange rate of rouble devaluated more than 3 times and the inflation rate reached 84.4% per annum at the end of 1998.

In the aftermath of the currency crisis the Central Bank of the Russian Federation applied a discretionary, "just-do-it" approach to monetary policy without an explicit nominal anchor. The Central Bank of Russia tried to slow down inflation and protect the exchange rate of the rouble from sharp changes by making significant interventions in the foreign exchange market.

Inflation forecasts, produced in 1999-2002 by the Central Bank of the Russian Federation, systematically underestimated actual inflation (Table 1). These inflation forecasts were conditioned by expectations of relatively low oil prices in the international markets and moderate economic growth in Russia. However, high oil prices together with improved relative competitiveness of domestic producers after the devaluation of the rouble implied higher than expected rates of economic growth. In addition, large interventions of the Bank of Russia in the foreign exchange market under the conditions of growing capital inflows led to significant an increase of inflation rates well above forecast levels.

In 2002 the Central Bank of the Russian Federation announced for the first time an inflation target for the next year. The Bank of Russia decided to target the CPI. The inflation target for 2003 was set by the Bank of Russia at 10-12 %. This target was met as the inflation rate amounted to 12 %.

In 2003 the Bank of Russia announced inflation targets for the next three years. According to Monetary Policy Guidelines for 2004 the rate of inflation had to be reduced to 8-10 % in 2004, 6.5-8.5 % in 2005, and 5.5-7.5 % in 2006. However, in 2004 the inflation rate amounted to 11.7 % well above the target range. This overshoot was conditioned by a level of economic activity higher than the level that was supposed in any scenario of economic development for 2004.

From 2002 onwards two principal scenarios of economic development have been considered by the Bank of Russia when setting an inflation target and selecting instruments for the following year. These two scenarios differ in their different prospects for global economic development, including oil price dynamics in the international markets, world economic growth rates, world interest rates and exchange rates of major world currencies. The first (pessimistic) scenario is based on assumptions of relatively low oil prices and high dollar-denominated interest rates. In the second (basic) scenario stable oil prices and low interest rates are assumed. The main internal factors taken into account in the development of monetary program are labour market dynamics, consumer and investment demands, the state of public finance. On the basis of these two scenarios variants of monetary program for the next year are developed.

According to the basic scenario for 2004 the growth rate of national product would amount to 5.2 % while according to the pessimistic scenario the growth rate would amount to 3.8 %. However, the growth rate of national product has in fact amounted to 7.1 %. This high growth was associated with good external prospects and growing consumer and investment demands inside of country, which were not assumed in any scenario. Consequently, the inflation rate was pushed above the target range.

This early inflation targeting experience indicates that the success or failure of inflation targeting in Brazil and Russia in the coming years will depend in large degree on the ability to produce accurate forecasts of economic developments inside of these countries and abroad. It raises the issue of development of accurate forecasting tools.

The Brazilian and Russian economies have passed through large transformations and structural changes. In particular, the currency crisis in 1998 - 1999 and the following change in the policy regime have affected significantly the dynamics of many macroeconomic time series in these economies.

The 1998 - 1999 crisis implied a change in the slope of inflation both in Brazil and Russia (Figures 1 and 3, Appendix C). In both countries inflation was declining over 1995 - 1997, but in 1998 the trend was broken. The currency crisis in August 1998 implied an explosion of inflation in Russia over the last two quarters of 1998 and the first quarter of 1999 with the following slow adjustment to the lower levels, while in Brazil the abandoning of the crawling peg in January 1999 did not lead to a large one-time shock but implied a shift to a higher level of inflation.

Turning to output growth, the 1998 crisis affected it significantly in Russia and led to a sharp fall in the rate of output growth in 1998. In the aftermath of the crisis, the rate of output growth shifted to a higher level (Figure 4, Appendix C). In Brazil the dynamics of output growth was similar to that of output growth in Russia, but the effect of the currency crisis on output growth was not as large as in Russia (Figure 2, Appendix C).

In the aftermath of the crisis, inflation and output growth stabilized in Russia. However, in Brazil a new confidence crisis in 2002-2003 provoked a large shock to inflation with the following slow adjustment to a lower level.

This preliminary analysis suggests that the dynamics of CPI inflation and GDP growth in Brazil and Russia is not only subject to one-time shocks and shifts but also to nonlinear adjustment processes. This raises the issue about the ability of different forecasting models to accommodate structural changes and fit the non-linear dynamics of the series of interest. Lack of data does not allow us to estimate efficiently non-linear models which include many parameters. On the other hand, presence of structural changes can imply instability of estimated parameters for linear models and failure in forecasting.

In this paper we evaluate the forecasting performance of different linear models in the small sample in the presence of structural changes. We also evaluate efficiency of some methods which were proposed by Clements and Hendry (1999) in order to robustify forecasts from linear models in the presence of structural changes.

3 Methodology

In this section forecasting approaches and criteria for the evaluation of their relative merits are represented briefly. Given the small time span of data available, small-scale linear models (AR, VAR) can be suggested as forecasting tools, because of their parsimonious specification and good performance. However, small-scale models include only few economic time series of the whole variety of data available to policy makers.

Another approach, combining information from a large number of time series with parsimonious specification has been the topic of investigation in the last years. Dynamic factor models, as developed by Stock and Watson (1998), have been successfully used to forecast macroeconomic variables in the US, UK and Euro-area, (Stock and Watson (2002), Marcellino, Stock, and Watson (2003), Artis, Banerjee and Marcellino (2003)). Some evidence in favour of dynamic factor models was found for transition economies (Banerjee, Marcellino and Masten (2004)). There have also been attempts to incorporate the information extracted by factor models into traditional small-scale models with the purpose of forecasting and policy analysis (Stock and Watson (1999), Favero and Marcellino (2001), Bernanke and Boivin (2003)).

The primary justification for the use of factors models in data sets for emerging economies, as described in Banerjee, Marcellino and Masten (2004), is their usefulness as a particular efficient means of extracting information from a large number of time series, albeit of short time span. Forecasts of key macroeconomic variables may be improved significantly, not least because in a rapidly changing economy the ranking of variables as good leading indicators for inflation or output growth is not clear a priori. Therefore factor models provide a methodology that remains "agnostic" about the structure of economy, by employing as much information as possible in the construction of forecasting exercise.

The design of this forecasting exercise replicates one developed in Artis, Banerjee and Marcellino (2003). All forecasting models are specified and estimated as a linear projection of an one-step ahead forecast variable, y_{t+1}, onto t-dated predictors. More precisely, the forecasting models all have the form,

$$y_{t+1} = \mu + \alpha(L)y_t + \beta(L)'Z_t + \varepsilon_{t+1}, \tag{1}$$

where μ is a constant, $\alpha(L)$ is a scalar lag polynomial, $\beta(L)$ is a vector lag polynomial, and Z_t is a vector of predictor variables.

The construction of the forecast variable y_t depends on whether the original series is modelled as *I(0), I(1)* or *I(2)*. Recall that series integrated of order d, denoted *I(d)* are those for which the d-th difference (Δ^d) is stationary. Denoting by x the original series (usually in logs) in the *I(0)* case the forecast series $y_{t+1} = x_{t+1}$. In the *I(1)* case, the forecast series y is the growth in the original series x between time period t and $t+1$: $y_{t+1} = \Delta x_{t+1} = x_{t+1} - x_t$. In the *I(2)* case, y is the difference of growth in x between t and $t+1$: $y_{t+1} = \Delta^2 x_{t+1} = \Delta x_{t+1} - \Delta x_t = x_{t+1} - 2x_t + x_{t-1}$. This is a convenient formulation because, given that x_t and its lags are known when forecasting, the unknown component of y_{t+1} conditional on the available information is equal to x_{t+1} independently of the choice of the order of integration. This makes the mean square forecast error (MSFE) from models for a twice differenced variable directly comparable with that from models for first differences.

3.1 Forecasting Models

Various forecasting models, which are compared, differ in their choice of Z_t. Let us list the forecasting models and briefly discuss their main characteristics.

Autoregressive forecast (ar_aic). A univariate autoregressive forecast is taken as a benchmark. It is based on (1) excluding Z_t. The lag length is chosen by the Akaike Information Criteria (AIC) with a maximum of 4 lags.

Autoregressive forecast with second differencing (ar_i2_aic). Clements and Hendry (1999) showed that the second differencing of the forecast variable can improve the forecasting performance of autoregressive models in the presence of structural changes, even in the case of over-differencing. Hence, this model corresponds to (1), excluding Z_t and treating the variable of interest as *I(2)*.

Autoregressive forecast with intercept correction (ar_ic_aic). An alternative remedy in the presence of structural changes is to put the forecast back on track by adding past forecast errors to the forecast. Clements and Hendry (1999) show that simple addition of the forecast error can be useful. Hence, the forecast is given by $\hat{y}_{t+1} + \varepsilon_t$, where $\hat{y}_{t+1}$ is the AR forecast and ε_t is the forecast error made when forecasting y_t in period *t-1.* However, both intercept correction and second differencing increase the MSFE, when not needed, by adding a moving average component to the forecast error, and thus are not costless.

Random walk forecast (rw). Since random walk forecast is found to be a robust benchmark in many forecasting exercises, it is also included in this exercise. This model correspond to (1), excluding Z_t and setting $\alpha(L)$ to be equal to 1.

VAR forecast (var_aic). VAR forecasts are constructed using equation (1) with different regressors Z_t . In particular, for GDP growth Z_t includes the money market interest rate and for CPI inflation Z_t includes the nominal exchange rate and GDP growth. The lag length is chosen by the Akaike Information Criteria (AIC) with the maximum of 4 lags.

Factor-based forecasts. These forecasts are based on setting Z_t in (1) to be estimated factors from a dynamic factor model. Stock and Watson (1998) show that, if the set of predictor variables can be described by an approximate dynamic factor model, then under certain assumptions (restrictions on moments and stationarity conditions) the space spanned by the latent factors can be estimated consistently by the principal components of the covariance matrix of the predictor time series. Stock and Watson (1998) also provide conditions under which these estimated factors can be used to construct asymptotically efficient forecasts. The dynamic factor model is briefly reviewed in Appendix A.

For each of the factor-based models, factors can be extracted from the unbalanced panel (prefix *fnbp*), or from the balanced panel (prefix *fbp*). The former contains more variables than the latter, and therefore more information. The only drawback is that missing observations have to be estimated in a first stage, which can introduce noise in the factor estimation.

Two types of factor-based forecasts are considered. First, we consider the model which includes both factors and lags of forecast variable (*fnbp_ar_aic* and *fbp_ar_aic*). The selection of a number of factors and lags is based on AIC. The maximum number of factors is equal to 6 and the maximum number of lags of dependent variable is equal to 4. Second, we consider the model where only up to 6 factors appear as regressors, but not lags of dependent variable (*fnbp_aic* and *fbp_aic*).

In order to evaluate the role of each factor in forecasting, for the unbalanced panel we also consider forecasts using a fixed number of factors, from 1 to 4 (*fnbp_ar_1* to *fnbp_ar_4* and *fnbp_1* to *fnbp_4*).

3.2 Forecast Comparison

The forecast comparison is performed in a simulated out-of-sample framework where all statistical calculation are done using a fully recursive methodology. The models are first estimated using data from 1995:1 to 2002:2, and one-quarter ahead forecasts are computed. Then the estimation sample is augmented by one quarter and the corresponding one-quarter ahead forecasts are computed again. The forecast period for one-quarter ahead forecasts is 2002:3 - 2004:4 for a total of 10 quarters, and the final estimation sample for one-quarter ahead forecasts is therefore 1995:1 - 2004:3.

Every quarter (i. e. every augmentation of the sample) all standardization of data and model estimation are repeated. A simulated out of sample MSFE is then computed as an average of the sum of squares of all comparisons between an actual value of the variable and its forecast (under any methods given in section 3.1 above).

The forecasting performance of the described methods is examined by comparing their simulated out-of-sample MSFE relative to the benchmark AR forecast. West (1996) standard errors are computed around the relative MSFE.

It is worth noting that the reported comparison criteria are based on averaging forecast errors, whose magnitude can differ substantially over forecasting period. They also do not provide information about the directional accuracy of forecasts which can be of particular importance.

The choice of the forecast horizon is conditioned by the availability of the data and small sample size, and the chosen forecast horizon, one quarter, is of rather limited relevance for the decisions about the monetary policy. Since the inflation target is set one year in advance, it requires one-year ahead forecasts. However, the Monetary Policy Committee meets every month in order to adjust its forecasts and decide on interest rate path, and every quarter it issues inflation report and produce forecasts for the next quarter. Thus, the one-quarter ahead forecasting is relevant for the monitoring economy and adjusting monetary policy over the year.

3.3 Data

The data sets for Brazil and Russia include respectively 41 and 47 quarterly series over the period 1995:1 - 2004:4. These series are extracted from the OECD database (Main Economic Indicators), the IMF database (International Financial Statistics), the database of the Central Bank of Brazil, and the database of the Russian Statistical Agency. They include series characterizing real output and income (GDP and its main components, production indices), labour market indicators (employment, unemployment, vacancies); interest rates (money market rates, lending and deposit rates); stock price indices; producer and consumer price indices; money aggregates; survey data; miscellaneous (exports, imports, exchange rates, international oil prices etc.). A complete list of series for both countries is reported in the Appendix B.

Following Banerjee, Marcellino and Masten (2004) the data are pre-processed in four stages before being modelled with a factor representation. First, all series excluding financial (interest rates, stock prices and exchange rates) are seasonally adjusted using original X-11 ARIMA procedure.

Second, logarithms are taken of all nonnegative series that are not already in rates or percentage points, and the series are transformed to account for stochastic or deterministic trends. The same transformation is applied to all the series of the same type.

The main choice is whether prices and nominal variables are *I(1)* or *I(2)*. Given the small time span of the sample and adjustment processes ongoing over the period under consideration it is hard to rely upon formal tests in deciding whether prices and other nominal series are *I(1)* or *I(2)*. Even if the price series are not generated by *I(2)* processes, second differencing can robustify the forecasts in the presence of structural breaks (see Clements and Hendry(1999)). In order to evaluate the role of second differencing in the forecasting performance of the factor models, this exercise is performed both under the assumption of *I(1)* prices and under the assumption of *I(2)* prices. In the first case all prices are treated as series generated by *I(1)* processes, and differenced only once. In the second case all prices and other nominal series are treated as series generated by *I(2)* processes, and differenced twice.

Third, all series are standardized before being used for factors estimation, e. g. they are transformed to series with zero mean and with the standard deviation equal to one.

Finally, the transformed seasonally adjusted series are screened for large outliers (outliers exceeding six times the interquartile range). Each outlying observation is recorded as missing data, and the EM algorithm (Stock and Watson (1998)) is used to estimate the factor model for the resulting unbalanced panel.

This procedure implies that the factors, which are estimated using differenced series, do not have large outliers. Large outliers in differenced series are generated by shifts in mean in original series. This type of structural break is excluded from the estimated factors, which are then used in forecasting.

Using the cumulative trace R^2 from the regressions of individual series on the estimated factors we find that the estimated factors fit the data quite well both for nominal series treated as *I(1)* and for nominal series treated as *I(2)* (Tables 1 and 2, Appendix D). If nominal series are differenced once, the first six factors explain 56% of the variability of the 41 series for Brazil and 63% of the variability of the 47 series for Russia. If nominal series are differenced twice, the first six factors explain 54% of the total variability of the data for Brazil and 62% of the total variability of the data for Russia.

For Brazil the first estimated factor explains real variables including production, consumption, and labour market indicators, while the second and the third factors explain interest rates and prices. This result for Brazil does not depend on the order of differencing of nominal series. For Russia, if nominal series are differenced once, the first factor explains consumer prices and exchange rates, the second factor loads on production series and producer prices, while the third factor explains interest rates. If nominal series are differenced twice, the first factor explains production variables as well as consumer prices and exchange rates, the second factor explains some production series and producer prices, and the third factor loads on the interest rates and money aggregates.

In Tables 1–2 (Appendix D) we report the R^2 in the regression of each variable to be forecast on the estimated factors. The first 3–4 estimated factors explain most of the variability of CPI inflation and GDP growth in Brazil in Russia. If nominal series are differenced once, the first three factors explain 50% of the variability of inflation in Brazil and 88% of the variability of inflation in Russia, and they also explain 76% of the variability of output growth in Brazil and 82% of the variability of output growth in Russia. This result does not change significantly, if nominal series are differenced twice.

Therefore the estimated factors are found to be informative about the data sets as whole, and about the variables to be forecast in particular. Let us now turn to their forecasting efficacy.

4 Forecasting Results

In this section the results of the forecast comparison for the Brazilian and Russian GDP growth and CPI inflation are reported. Forecasting is performed for one-quarter horizon for a total of 10 quarters. Relative MSFE are reported in Tables 3 – 4 in Appendix D.

4.1 Brazil

The results for Brazil are reported in Table 3 (Appendix D). In Figures 5 and 6 (Appendix C) we report actual values and one-quarter ahead forecasts from the best non-factor and factor models.

Let us consider the case when the price series are treated as *I(1)*. Both for GDP growth and CPI inflation, most of the factor forecasts do outperform the benchmark autoregressive forecast. On the other hand, most of the factor forecasts are outperformed by the VAR forecast. The VAR forecast is best for inflation, while for output growth there is a factor forecast (*fnbp_3*) that outperforms the VAR, but the gain provided by this factor forecast comparing to the VAR forecast is not large.

The random walk forecast, the intercept corrected AR forecast, and the AR forecast for the price series differenced twice outperform the benchmark for CPI, but they do not provide gains in the forecasting of GDP. This result corresponds to the evidence provided by the analysis of the dynamics of these series: while inflation was the subject of several structural changes, there is no certain evidence of non-stationarities in the dynamics of output growth in Brazil. Therefore the methods robustifying for structural changes appear to be efficient for inflation but not for output growth.

Figure 6 (Appendix C) shows that the VAR and the best factor model provide poor forecasts for GDP growth although they outperform the benchmark. The visual analysis of the graph of these forecasts allows us to suggest that they are biased downwards. This result requires further investigation and explanation.

The VAR and the best factor forecasts of CPI inflation (Figure 4, Appendix C) are biased downwards in the first three quarters of forecasting, but then they converge to the actual values of the series and perform well. In the case of CPI the forecast failure in the first quarters of the forecasting is conditioned by the large outlier in the inflation rate in 2002 triggered by the confidence crisis.

In order to evaluate the effect of additional differencing of the price series on the forecasting performance of the factor models the exercise is repeated under the assumption that all prices, money aggregates, wages, and exchange rates are generated by *I(2)* processes, and all these series are differenced twice. The AR forecast of the GDP growth and the AR forecast of the twice differenced CPI are compared with the factor forecasts (other non-factor forecasts are not considered in this case). Accordingly, the forecasting results for GDP can be compared directly with the results of the exercise performed under the assumption of *I(1)* prices, while this direct comparison with the *I(1)* case is not possible for CPI, since the forecast variable and the benchmark forecast are different in this case.

There is no obvious ranking of the factor forecasts performed under the assumption of *I(1)* prices and the factor forecasts performed under the assumption of *I(2)* prices: some factor models perform better under the assumption of *I(1)* prices while others perform better when prices are treated as *I(2)*. However, most of the factor forecasts do improve their performance for the GDP series under the assumption of the *I(2)* prices. This can be explained by the fact that the variance of the price series decreases after second differencing and the twice differenced prices do not dominate the dynamics of estimated factors, which are used for forecasting. Thus, the estimated factors become more informative about output series rather than about prices and provide additional gains in forecasting GDP growth.

4.2 Russia

The results for Russia are reported in Table 4 (Appendix D). In Figures 7 and 8 (Appendix C) we report actual values and one-quarter ahead forecasts from the best non-factor and factor models.

The graphs of GDP growth and CPI inflation (Figures 3 - 4, Appendix C) provide ample evidence of structural changes in these series. While output growth shifted to a higher mean in the aftermath of the 1998 crisis, inflation, which exploded in 1998, converged to a lower level in the following years.

High levels of inflation before the currency crisis and the explosion of inflation in 1998 implied the upward bias of the benchmark AR forecast for CPI. This forecast is outperformed by the random walk forecast, the intercept corrected AR forecast, and the AR forecast for the twice differenced series. The gains provided by the random walk forecasts and the corrected AR forecasts are large. They reach 84% for the random walk forecast and 82% for the AR forecast of second differences. It can mean that the CPI is better described as generated by *I(2)* process. On the contrary, there is no evidence that GDP is better treated as *I(2)* series: the random walk forecast and the corrected AR forecasts do not provide large gains in the forecasting of GDP comparing to the benchmark.

These differences in the efficiency of intercept correction and second differencing can be explained by differences in size and direction of structural changes in output and inflation as well as different persistence of these series.

There is at least one factor forecast for each forecast variable that provides gains comparing to the AR benchmark. These gains are not large for GDP, but they reach 76% for CPI (*fnbp_ar_1*). The VAR forecasts outperform the benchmark both for GDP and CPI. For GDP growth the VAR forecast is the best with the relative gain of 29% comparing to the AR benchmark.

Figure 8 (Appendix C) shows that, as in the case of Brazil, both VAR and factor models provide poor forecasts for output growth: both of them have lower volatility than actual values of the series and the factor forecasts appear to be biased downwards. On the contrary, the random walk forecast, which is the best forecast for the CPI inflation, and the best factor forecast follow closely the actual inflation (Figure 7, Appendix C).

Since intercept correction and second differencing appear to be so efficient for CPI, it is reasonable to consider the factor forecasts performed under the assumption of *I(2)* prices. The results of comparison of the AR forecast with the factor forecasts computed with the use of twice differenced price series are shown in Table 4 (Appendix D).

Because prices are differenced twice in this case, the benchmark forecast for CPI is the AR forecast of the second differences. This is a more robust benchmark than the AR forecast of the first differences and not one factor model outperforms it.

The benchmark forecast of GDP does not change under the assumption of the *I(2)* prices and the factor forecasts for output growth, evaluated under the assumptions of *I(1)* prices and *I(2)* prices, are directly comparable. Most of the factor forecasts of output growth do improve their performance significantly under the assumption of *I(2)* prices and provide significant gains compared to the benchmark. As in the case of Brazil this result can be explained by the decrease of the variance of price series after second differencing, which do not dominate the factor dynamics, and factors become more informative about output series.

5 Conclusions

In this paper the relative forecasting performance of autoregressive, vector autoregressive, and factor models was compared on the basis of data sets which are available for the Brazilian and Russian economies.

Both Brazil and Russia have passed through large transformations and structural changes. In particular, the currency crisis in 1998-1999 implied structural changes in CPI inflation, GDP growth and other macroeconomic variables in these countries. It raises the issue about the ability of different forecasting models to accommodate these structural changes.

Since only short spans of reliable time series are available for Brazil and Russia, AR and simple VAR models can be expected to perform comparatively well. On the other hand, the availability of the large set of macroeconomic indicators suggests factor models. The results of our forecasting exercise show that both VAR and factor models are useful in forecasting inflation and output growth, but their relative performance differs for different forecast series and different series treatment.

Because of the complexity of ongoing changes and short time spans of data, structural changes are not modelled explicitly. However, two types of corrections for structural changes are considered: intercept correction and second differencing as proposed by Clements and Hendry (1999). These methods, applied to AR forecasts, produce certain gains in forecasting inflation, but they are not efficient in forecasting output growth. The outcome may be explained by a higher persistence of inflation or larger breaks in its dynamics comparing to output growth.

The results of the exercise allow us to suggest that the efficiency of different forecasts models and the efficiency of their corrections depend on the statistical properties of the series under consideration, in particular, on the persistence of the series and on the type and size of the structural changes in the series. It also points the direction for future research which can be detailed Monte Carlo simulations in order to evaluate the effect of different structural breaks on the relative forecasting performance of the models under consideration.

Another interesting direction of research would be the evaluation of different forecast combinations in order to bring our forecasting exercise closer to the decision making process ongoing in the Central Banks. There decisions are not based on one best model, but the whole set of models is used to produce the final projection of output and inflation.

Appendix A. Dynamic Factor Model

This appendix briefly reviews a dynamic factor model. The material draws on Stock and Watson (1998). Let y_t denote the scalar series to be forecast and let X_t be a N-dimensional multiple time-series of predictor variables, observed for $t = 1, ..., T$, where y_t and X_t are both taken to have mean 0. Suppose that (X_t, y_t) admit a dynamic factor model representation with $\bar{r}$ common dynamic factors f_t,

$$y_{t+1} = \beta(L)f_t + \gamma(L)y_t + \epsilon_{t+1,} \tag{A1}$$

$$X_{it} = \lambda_i(L)f_t + e_{it}, \tag{A2}$$

for $i = 1, ..., N$, where $e_t = (e_{1t}, ..., e_{Nt})'$ is the $N \times 1$ idiosyncratic disturbance, and $\lambda_i(L)$ and $\beta(L)$ are lag polynomials in nonnegative powers of L. It is assumed that $E(\epsilon_{t+1}|f_t, y_t, X_t, f_{t-1}, y_{t-1}, X_{t-1}, ...) = 0$. If the lag polynomials $\lambda_i(L)$, $\beta(L)$, and $\gamma(L)$ have finite orders of at most q, A1 and A2 can be rewritten as,

$$y_{t+1} = \beta' F_t + \gamma(L)y_t + \epsilon_{t+1}, \tag{A3}$$

$$X_t = \Lambda F_t + e_t, \tag{A4}$$

where $F_t = (f_t', ..., f_{t-q}')'$ is $r \times 1$, $r \leq (q+1)\bar{r}$, the ith row of Λ in A3 is $(\lambda_{i0}, ..., \lambda_{iq})$, and $\beta = (\beta_0, ..., \beta_q)'$.

Stock and Watson (1998) show that, under this finite lag assumption and some additional assumptions (restrictions on moments and stationarity), the column space spanned by the dynamic factors f_t can be estimated consistently by the principal components of the $T \times T$ covariance matrix of the *X's*.

The principal component estimator is computationally convenient, even for very large N. It can be generalized to handle data irregularities such as missing observations using the EM algorithm. The consistency of the estimated factors implies that they can be used to construct asymptotically efficient forecasts for the series y_{t+1}.

Appendix B Data Description

This appendix lists time series used to construct factor-based forecasts. The transformation codes are: 1 = no transformation; 2 = first differences; 3 = second differences; 4 = levels of logarithms; 5 = first differences of logarithms; and 6 = second differences of logarithms.

Brazil

No	Mnemonic	Code	Description
Output and income			
1.	gdp	5	gross domestic product, index, 1995=100, sa
2.	manuf	5	manufacturing, index, 1995=100, sa
3.	constr	5	construction, index, 1995=100, sa
4.	mining	5	mining, index, 1995=100, sa
5.	prodsteel	5	production of manufactured crude steel, index, 1995=100, sa
6.	publutil	5	public utilities, index, 1995=100, sa
7.	agr	5	agriculture, index, 1995=100, sa
8.	serv	5	services, index, 1995=100, sa
9.	transp	5	transport, index, 1995=100, sa
10.	commun	5	communication, index, 1995=100, sa
11.	trade	5	trade, index, 1995=100, sa
12.	conspriv	5	private consumption, index, 1995=100, sa
13.	consgov	5	government consumption, index, 1995=100, sa
14.	invest	5	gross investment, index, 1995=100, sa
Labour market			
15.	earning	5/6	real monthly earnings: all activities, index,1995, sa
16.	hours	5	monthly hours of work, index, 1995=100, sa
17.	unempl	2	unemployment rate, %, sa
Interest rates			
18.	irmm	2	money market rate, % pa
19.	irtb	2	treasury bill rate, % pa
20.	irdep	2	deposit rate, % pa
Stock prices			
21.	bovespa	5/6	BOVESPA stock price index, 1995=100

Prices

22.	ppagr	5/6	producer price index, agriculture, 1995=100, sa
23.	ppconstr	5/6	producer price index, construction, 1995=100, sa
24.	pws	5/6	whosale price index, 1995=100, sa
25.	cpi	5/6	cpi, total, 1995=100, sa

Money aggregates

26.	m0	5/6	monetary base M0, mln BRL, sa
27.	m2	5/6	monetary aggregate M2, mln BRL, sa

Survey data

28.	utiliz	2	manufacturing: rate of capacity utilization, %, sa
29.	utilicons	2	production: future tendency, % balance,sa
30.	utilcap	2	producer prices, future tendency, % balance, sa
31.	stock	2	manufacturing: finished good stock, % balance, sa
32.	ftprod	2	manufacturing: production, future tendency, % balance, sa
33.	ftprice	2/3	manufacturing: selling prices, future tendency, % balance, sa

Miscellaneous

34.	exp	5	exports, index, 1995=100, sa
35.	imp	5	imports, index, 1995=100, sa
36.	intprpetr	5	average price of crude petroleum, USD/barrel
37.	nomexr	5/6	nominal effective exchange rate, index,1995=100
38.	realexr	5/6	real effective exchange rate, index, 1995=100
39.	gdpus	5	gdp, USA, index, 1995=100, sa
40.	cpus	5/6	cpi, USA, index, 1995=100, sa
41.	irus	2	treasury bill rate, USA, % pa

Russia

No	Mnemonic	Code	Description
Output and income			
1.	gdp	5	gross domestic product, index, 1995=100, sa
2.	indtotal	5	industrial production, index, 1995=100, sa
3.	agr	5	agriculture, index 1995=100, sa
4.	constr	5	construction, index, 1995=100, sa
5.	servm	5	market services, index, 1995=100, sa
6.	transp	5	transport and communication, index 1995=100, sa
7.	trade	5	trade, index 1995=100, sa
8.	servnm	5	nonmarket services, index, 1995=100, sa
9.	conspriv	5	private consumption, index, 1995=100, sa
10.	consgov	5	government consumption, index, 1995=100, sa
11.	sav	5	gross savements, index, 1995=100, sa
12.	capital	5	gross fixed capital formation, index, 1995=100, sa
13.	indmain	5	industrial production, main industries, index, 1995=100, sa
14.	prodpetr	5	production, crude petroleum, mln tonnes, sa
15.	prodgas	5	production, natural gas, mln cub m, sa
16.	retail	5	retail sales, index, 1995=100, sa
17.	realinc	5	real income, index 1995=100, sa
18.	realdinc	5	real disposable income, index, 1995=100, sa
Labour Market			
19.	wage	5/6	real wage, index, 1995=100, sa
20.	empl	5	employment, mln persons, sa
21.	unempl	2	unemployment rate, %, sa
22.	vacan	5	unfilled vacancies, th persons, sa
Interest rates			
23.	irmm	2	money market rate, % pa
24.	irdep	2	deposit rate, % pa
25.	irlend	2	lending rate, % pa
Stock prices			
26.	rts	5/6	RTS stock price index, 1995=100

Prices

27.	pp	5/6	producer price index, industrial production, total, 1995=100, sa
28.	ppoil	5/6	produce price, crude petroleum, RUR/tonne, sa
29.	ppgas	5/6	produce price, natural gas, RUR/th cub m, sa
30.	ppconstr	5/6	producer price index, construction, 1995=100, sa
31.	trcost	5/6	transportation costs, index, 1995=100, sa
32.	cpiserv	5/6	cpi, services, 1995=100, sa
33.	cpifood	5/6	cpi, food, 1995=100, sa
34.	cpi	5/6	cpi, total, 1995=100, sa

Money aggregates

35.	money	5/6	money, mln RUR, sa
36.	qmoney	5/6	money + quasi money, mln RUR, sa

Survey data

37.	utiliz	2	manufacturing: rate of capacity utilization, %, sa
38.	ftprod	2	production: future tendency, % balance, sa
39.	ftconstr	2	construction: business situation, future tendency, %balance, sa
40.	ftprice	2	producer prices, future tendency, % balance, sa

Miscellaneous

41.	exp	5	exports, index, 1995=100, sa
42.	imp	5	imports, index, 1995=100, sa
43.	intprpetr	5	average price of crude petroleum, USD/barrel
44.	intprgas	5	price of russian natural gas, USD/ th cub m
45.	ofexr	5/6	official exchange rate, RUR/USD
46.	nomexr	5/6	nominal effective exchange rate, index, 1995=100
47.	realexr	5/6	real effective exchange rate, index, 1995=100

Appendix C Figures

Figure 1 Brazil: CPI inflation

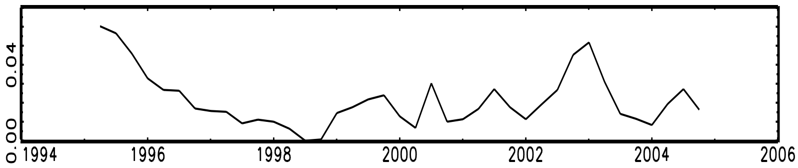


Figure 2 Brazil: GDP growth

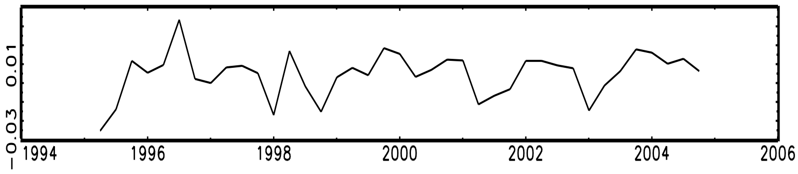


Figure 3 Russia: CPI inflation

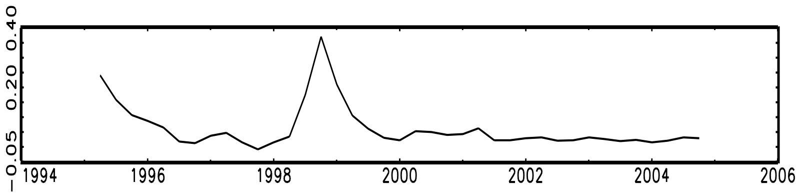


Figure 4 Russia: GDP growth

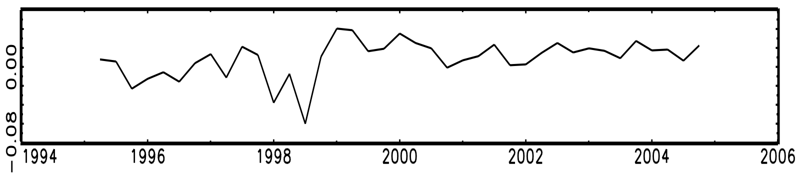


Figure 5 Brazil: CPI inflation forecasts

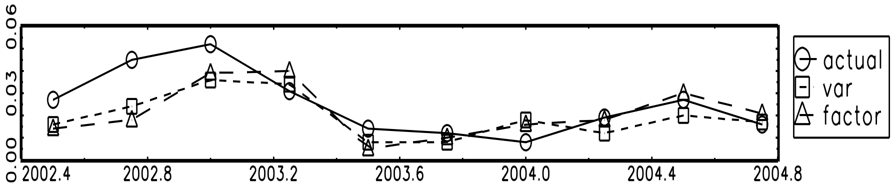


Figure 6 Brazil: GDP growth forecasts

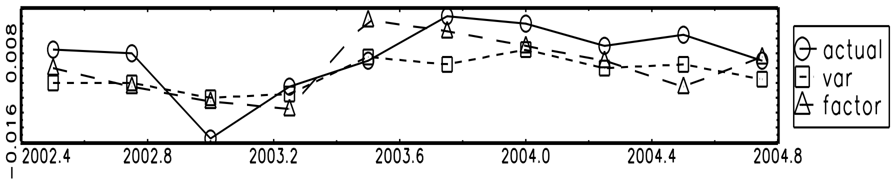


Figure 7 Russia: CPI inflation forecasts

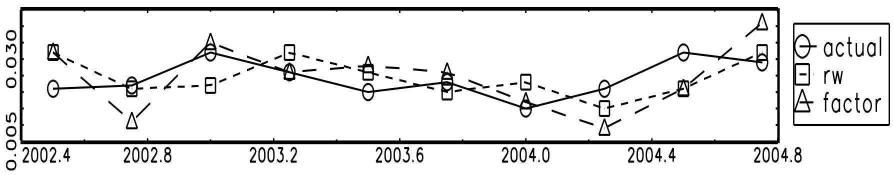
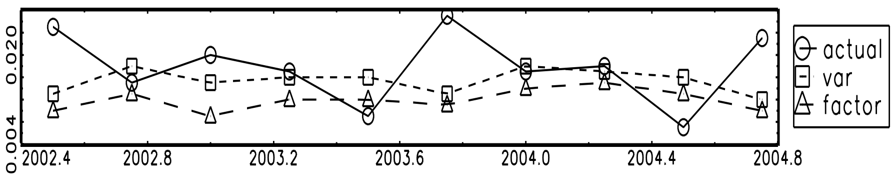


Figure 8 Russia: GDP growth forecasts



Appendix D Tables

Table 1: Brazil, cumulative R^2 from regressions of variables on factors

Factor	I(1) Prices			I(2)Prices		
	Average	CPI inflation	GDP growth	Average	CPI inflation	GDP growth
1	0.19	0.03	0.74	0.17	0.00	0.75
2	0.29	0.06	0.74	0.28	0.52	0.75
3	0.39	0.50	0.76	0.36	0.64	0.78
4	0.45	0.68	0.81	0.44	0.64	0.79
5	0.51	0.69	0.81	0.49	0.71	0.79
6	0.56	0.69	0.82	0.54	0.73	0.79

Table 2: Russia, cumulative R^2 from regressions of variables on factors

Factor	I(1) Prices			I(2)Prices		
	Average	CPI inflation	GDP growth	Average	CPI inflation	GDP growth
1	0.21	0.79	0.07	0.24	0.58	0.64
2	0.36	0.86	0.77	0.31	0.67	0.75
3	0.46	0.88	0.82	0.43	0.77	0.85
4	0.54	0.91	0.86	0.52	0.81	0.88
5	0.59	0.92	0.88	0.57	0.81	0.88
6	0.63	0.92	0.89	0.62	0.87	0.89

Notes:

I(1) Prices Prices and money series are differenced once
I(2) Prices Prices and money series are differenced twice

Table 3: Brazil, relative MSFE, 1-step-ahead forecasts, quarterly data

Forecast Method	I(1) Prices GDP growth	I(1) Prices CPI inflation	I(2)Prices GDP growth	I(2)Prices CPI inflation
ar_aic	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
ar_ic_aic	1.19 (0.36)	1.10 (0.67)	-	-
ar_i2_aic	1.55 (0.88)	0.90 (0.32)	-	-
rw	1.14 (0.32)	0.85 (0.30)	-	-
var_aic	0.73 (0.38)	0.64 (0.21)	-	-
fbp_ar_aic	1.32 (0.53)	0.97 (0.03)	0.92 (0.58)	1.13 (0.22)
fbp_aic	0.76 (0.23)	1.38 (0.81)	0.92 (0.58)	0.98 (0.02)
fnbp_ar_aic	1.26 (0.62)	0.83 (0.09)	0.76 (0.57)	0.92 (0.24)
fnbp_aic	0.83 (0.24)	0.79 (0.15)	0.76 (0.57)	0.88 (0.06)
fnbp_ar_1	1.11 (0.22)	1.04 (0.05)	0.94 (0.21)	1.28 (0.26)
fnbp_ar_2	0.90 (0.26)	0.85 (0.08)	1.09 (0.61)	1.01 (0.12)
fnbp_ar_3	0.84 (0.52)	0.76 (0.13)	1.12 (0.60)	1.08 (0.12)
fnbp_ar_4	1.21 (0.47)	0.78 (0.13)	0.81 (0.61)	0.67 (0.25)
fnbp_1	0.88 (0.08)	1.57 (0.91)	0.84 (0.10)	1.06 (0.07)
fnbp_2	0.88 (0.08)	1.09 (0.29)	0.72 (0.39)	0.96 (0.09)
fnbp_3	0.71 (0.47)	0.87 (0.13)	0.67 (0.42)	1.08 (0.23)
fnbp_4	1.09 (0.33)	0.74 (0.15)	0.81 (0.61)	0.68 (0.21)
RMSE for ar_aic	0.009	0.13	0.009	0.012

Notes:

The forecasts in the rows of tables are (see section 3.1 for details):

ar_aic	AR model (AIC selection), benchmark
ar_ic_aic	AR model (AIC selection) with intercept correction
ar_i2_aic	AR model (AIC selection) for second-differenced variable
rw	random walk
var_aic	VAR model (AIC selection)
fbp_ar_aic	Factors from balanced panel (AIC selection) and AR terms
fbp_aic	Factors from balanced panel (AIC selection)
fnbp_ar_aic	Factors from unbalanced panel (AIC selection) and AR terms
fnbp_aic	Factors from unbalanced panel (AIC selection)
fnbp_ar_n	n factors from unbalanced panel and AR terms
fnbp_n	n factors from unbalanced panel

Table 4: Russia, Relative MSFE, 1-step-ahead forecasts, quarterly data

Forecast Method	I(1) Prices GDP growth	I(1) Prices CPI inflation	I(2)Prices GDP growth	I(2)Prices CPI inflation
ar_aic	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
ar_ic_aic	1.80 (0.80)	0.38 (0.25)	-	-
ar_i2_aic	0.97 (0.26)	0.18 (0.26)	-	-
rw	1.23 (0.28)	0.16 (0.25)	-	-
var_aic	0.71 (0.22)	0.29 (0.26)	-	-
fbp_ar_aic	2.21 (1.07)	2.41 (1.33)	0.76 (0.34)	7.07 (15.33)
fbp_aic	2.19 (1.05)	2.67 (2.12)	0.76 (0.34)	5.76 (10.83)
fnbp_ar_aic	1.30 (0.46)	1.59 (1.01)	2.31 (1.88)	2.15 (1.39)
fnbp_aic	1.32 (0.25)	2.10 (1.43)	0.58 (0.31)	2.15 (1.39)
fnbp_ar_1	0.92 (0.10)	0.24 (0.26)	1.00 (0.05)	3.57 (3.61)
fnbp_ar_2	1.69 (0.60)	0.86 (0.20)	0.59 (0.27)	4.25 (6.51)
fnbp_ar_3	1.87 (0.66)	1.58 (0.68)	0.53 (0.31)	3.03 (4.96)
fnbp_ar_4	1.30 (0.46)	2.38 (1.62)	0.69 (0.41)	2.15 (1.39)
fnbp_1	0.92 (0.10)	0.78 (0.26)	1.14 (0.13)	1.86 (0.96)
fnbp_2	1.52 (0.43)	0.44 (0.24)	0.59 (0.27)	3.01 (3.07)
fnbp_3	1.87 (0.66)	0.79 (0.25)	0.53 (0.31)	2.07 (1.58)
fnbp_4	1.16 (0.18)	1.66 (0.81)	0.54 (0.32)	2.15 (1.39)
RMSE for ar_aic	0.009	0.18	0.009	0.007

Chapter 2

Co-Breaking and Forecasting Performance of Factor Models

Abstract

Macroeconomic data available for emerging market economies are characterized by short time spans and sharp shocks. Short spans of data suggest the adoption of simple models as forecasting tools. However, the availability of a large number of variables makes the class of dynamic factor models a reasonable alternative. In this paper the relative performance of two approaches is explored in the Monte Carlo exercise performed for the data spans which are available for emerging market economies. The data are generated with structural breaks and the role of intercept correction and differencing for robustifying forecasts in the presence of breaks is also evaluated. Factor models work well if there is a break in the middle of estimation period. However, their relative performance deteriorates if there is a break in the end of the estimation period. Intercept correction and differencing are also found to be useful in several cases.

1 Introduction

Macroeconomic forecasts form an important component for the development of monetary policy. Because monetary policy has delayed effects on output and inflation, a forward-looking approach is essential to inflation targeting, and indeed to monetary policy in general. The choice of appropriate forecasting tools and the accuracy of forecasting are of key relevance for the choice of policy instruments and success of monetary policy.

For many economies only short spans of reliable data are available. This is the case of transition economies of Central and Eastern Europe, as well as emerging market economies of Latin America. Most of the aggregate data available for the European Union are also of short time spans.

Lack of data may lead to high estimation uncertainty and forecast failure. Failures in the forecasting of inflation may be one of the reasons why transition countries that have recently opted the inflation targeting (the Czech Republic, Poland, and Hungary), failed to hit their targets. The Central Banks of these countries have attempted to improve their forecasts by development of forecasting tools and incorporation of a growing amount of information.

When the reliable data are of short time spans, small-scale models can be suggested as forecasting tools, because of their parsimonious specification and good performance. However, small-scale models include only few economic time series of the whole variety of data available to policy makers.

Another approach combining information from a large number of time series with parsimonious specification, has become the subject of intensive research recently. Dynamic factor models as developed by Stock and Watson (1998), were successfully used to forecast macroeconomic variables in the US, the UK, and the Euro-area (Stock and Watson, 2002, Marcellino, Stock, and Watson, 2003, Artis, Banerjee and Marcellino, 2003). Some evidence in favour of dynamic factor models was found for transition economies (Banerjee, Marcellino and Masten, 2006).

There have been attempts to incorporate information extracted by factor models into traditional small-scale models with the purpose of forecasting and policy analysis (Stock and Watson, 1999, Favero and Marcellino, 2001, Bernanke and Boivin, 2003). Banerjee, Marcellino and Masten (2007) performed a Monte Carlo simulation in order to investigate forecasting performance of different models for the time dimensions and the number of variables corresponding to those which are available for transition economies.

The primary justification for the use of factor models in large data sets as suggested in Banerjee, Marcellino and Masten (2006), is that they are particularly efficient means of extracting information from a large number of time series, despite the short time span. Forecasts of key macroeconomic variables may be improved significantly, because in a rapidly changing economy the ranking of variables as good leading indicators for, say, inflation or industrial production growth, is not clear a priori. Therefore, factor models provide a methodology that remains "agnostic" about the structure of economy, by employing as much information as possible in the construction of the forecasting exercise.

Banerjee, Marcellino and Masten (2006) evaluate the relative performance of different methods in forecasting gross domestic products (GDP), consumer prices and interest rates for five transition countries (the Czech Republic, Poland, Hungary, Slovakia, and Slovenia). They use quarterly data and produced one-quarter-ahead forecasts. Forecasting models are compared on the basis of relative mean square forecast errors.

Factor models perform relatively well in forecasting GDP and treasury bill rates for the Czech Republic, Poland, and Hungary. Non-factor methods perform better in forecasting GDP and interest rates for Slovakia and Slovenia. However, factor models provide some gains in forecasting CPI in Slovakia, Slovenia, and the Czech Republic. Thus, no strong evidence of better performance of any forecasting method was found, although factor models were found useful in many cases.

In addition, the empirical performance of two methods for robustifying forecasts in the presence of structural changes was evaluated. These methods are second-differencing and intercept correction as proposed by Clements and Hendry (1999). However, second differencing and intercept correction provided forecasting gains only in a few cases.

In the chapter 1 of the thesis the forecasting exercise is conducted for output growth and inflation in Brazil and Russia. As in Banerjee, Marcellino and Masten (2006) quarterly data are used and one-quarter-ahead forecasts are evaluated. Forecasts of standard small-scale models (univariate and vector autoregressive models) are compared with those of dynamic factor models.

The findings are mixed. Relative performance of forecasting models (in a sense of relative mean square forecast errors) differs over forecast variables. Some factor models outperform other models for output growth both in Brazil and Russia.

However, for inflation all factor forecasts are outperformed by the VAR forecasts in Brazil and by the random walk forecast in Russia.

Models are robustified for structural breaks by second-differencing and intercept correction. These methods appear to be efficient in the forecasting of inflation, but they provide no gains in the forecasting of output growth.

The evidence given suggests that close attention must be paid to statistical properties of forecast variables. The different statistical properties of the variables being forecast may be a reason for different relative performance of forecasting models for these variables. Differences in the relative performance of forecasting models for different types of variables (production, prices, and financial variables) is also found in other research (Stock and Watson, 1998, Artis, Banerjee and Marcellino, 2004).

It is of interest to perform a Monte Carlo exercise to evaluate the relative efficiency of different models in the time and cross-section dimensions corresponding to those which are available in the empirical exercises for emerging market economies. Such Monte Carlo exercise can provide some evidence or explanation for the relative performance of different models for variables generated by different stochastic processes.

Emerging market economies have passed through large transformations and structural changes. Policy-makers in these countries have often failed to forecast accurately future economic developments, because they were not able to take account of structural changes which happened in their countries. It raises the issue about the ability of different forecasting models to accommodate structural changes.

As structural change is an event which usually affects the whole economy or a sector of the economy, one can expect co-breaking between a range of economic indicators. Co-breaking can be picked up by the principal component estimator which measures co-movement of series. In this case one obtains estimates of factors with a break in their dynamics. It is of interest to conduct a simulation exercise to evaluate the relative performance of factor forecasts with factors extracted from the series subject to co-breaking.

Stock and Watson (2007) argue that the factor space can be consistently estimated even in the presence of structural instability in factor loadings for individual series. However, instability in forecasting equations for individual series may lead to forecast failure.

In the Monte Carlo experiment performed under the assumption of validity of the factor representation, Banerjee, Marcellino and Masten (2007) explored sensitivity of forecasting methods to various characteristics of generated data: persistence of factors, amount of autocorrelation, and time-varying parameters. They investigated the performance of forecasting models for small time and cross-section dimensions of data and found that changes in persistence of factors and factor loadings are important for the relative performance of factor models.

Stock and Watson (2007) and Banerjee, Marcellino and Masten (2007) deal with in-sample instability of forecasting models. This structural instability can be taken into account before forecasting. However, structural changes may happen in the forecasting period. It might be impossible to accommodate such changes into a model. A Monte Carlo exercise, which evaluates the robustness of different forecasting models in the presence of unknown structural changes, can be instructive. Efficacy of intercept correction and second differencing in the presence of different types of structural changes and shocks can also be evaluated in such experiment.

In this paper we present the results of the experiment conducted for the time and cross-section dimensions corresponding to those, which are used in few empirical exercises conducted for transition and emerging market economies. Assuming validity of the factor representation we investigate sensitivity of forecasting models to the choice of forecasting horizon, degree of factor persistence, and two simple types of non-stationarity in the middle of the estimation sample and at the forecasting period: a shift in the mean of factors and a shift in the intercept of the forecasting equation. These two types of structural changes allow us to investigate the relative performance of forecasting models for data with co-breaking.

The paper is organized as follows. In section 2 we briefly describe the experiment design. Section 3 presents the results of the experiment. In Section 3 we draw some general conclusions and offer some suggestions for further research.

2 Methodology

The basic design of the simulation exercise is taken from Stock and Watson (1998) and adapted for the purposes of this paper. The data are generated by a dynamic factor model. Unlike Banerjee, Marcellino and Masten (2007), we do not investigate the effects of correlation in idiosyncratic errors or time-varying factor loadings on the forecasting performance of models. Idiosyncratic errors are assumed to be uncorrelated and factor loadings are constant.

The simplest dynamic factor model is

$$F_t = \Pi F_{t-1} + u_t, \quad \Pi = \pi I_r, \tag{1}$$

$$X_t = \Lambda F_t + e_t, \tag{2}$$

$$y_{t+1} = \tau' F_t + \varepsilon_{t+1}, \tag{3}$$

where t=1,...,T; y_t is a scalar variable to be forecast; τ is a $r \times 1$ vector of parameters; F_t is an $r \times 1$ vector of factors, generating data; π is a parameter, measuring the persistence of factors (π is assumed to be less then one in absolute value); X_t is a $N \times 1$ vector of leading indicators for y_t; Λ is a $N \times r$ matrix of factor loadings; u_t is $N(0, I_r)$, e_t is $N(0, I_N)$, and ε_t is $N(0, 1)$; u_t, e_t and ε_t are independent.

Under the given assumptions, the column space spanned by dynamic factors can be estimated consistently by the principal components of the $(T \times T)$ covariance matrix of the $X's$ (Stock and Watson, 1998). The consistency of the estimated factors implies that they can be used to construct asymptotically efficient forecasts of y_t.

However, uncertainty induced by estimation of factors, may imply that factor forecasts are less efficient than forecasts produced by other models in small samples. Bai and Ng (2006) show that estimation of parameters of forecasting models adds $O(T^{-1})$ uncertainty to the forecast, while estimation of factors adds $O(N^{-1})$ uncertainty. Hence, when N is small relative to T, factor forecasts can be outperformed by other forecasting models.

The amount of data available to forecasters is growing over time. The number of available data series (N) grows as well the number of observations (T). In applications of factor models it generates a trade-off between time spans and cross-section spans of data, as existing indicators are of larger time spans than new indicators. One solution for this problem is suggested by Stock and Watson (1998). They propose use of the EM algorithm to estimate missing values of variables in unbalanced panels. However, this approach works only for a small number of missing values, as estimation of a large number of missing values induces high uncertainty.

The trade-off between a long data set with a limited number of data series and a shorter data set with a larger number of data series can be the subject of a simulation exercise. In this simulation exercise we consider few combinations of time and cross-section spans. It allows us to evaluate the impact of N/T ratio on the forecasting performance of factor models.

In applications factors are extracted from standardized series. It means that series are demeaned before the estimation of factors and estimated factors have zero mean as well. However, if the mean of series changes over sample, and this change cannot be modelled explicitly and extracted from series before the estimation of factors, one obtains factor estimates with a break in their dynamics.

The break in the mean of observable series may be generated by the break in the mean of factors or the break in the intercept of the forecasting equation. The robustness of factor models to these two types of break in mean is the subject of this simulation exercise. Efficiency of intercept correction and second differencing in the presence of these two types of break is also a subject of investigation.

If structural break affects the whole economy or a sector of the economy, there may be co-breaking between the forecast variable and other variables which are used as leading indicators for the forecast variable. Then the leading indicators contain information about the structural break in the forecast variable. In this exercise we consider data generating processes which lead to a break both in the leading indicators and the forecast variable.

In order to evaluate the relative performance of forecasting models in the presence of structural breaks, we consider two basic data generating processes which represent modifications of the model (1)-(3). For the first data generating process (DGP 1) we modify the equation (1) by the introduction of a deterministic component:

$$F_t = D_t^F + \Pi F_{t-1} + u_t, \quad D_t^F = \mu_t \iota$$

where D_t^F is the deterministic component of factor process, $\iota_r = (1, 1, ..., 1)'$ is an $(r \times 1)$ vector, and μ_t is a scalar function such that

$$\mu_t = \{ \begin{array}{ll} 0, & t < t_b \\ \mu, & t \geq t_b \end{array} ,$$

where t_b is the date of break. Then the h-step conditional expectation of y_t (for $h > 1$) is

$$y_{t+h|t} = \tau' \sum_{i=1}^{h-1} \Pi^{h-i-1} D_{t+i}^F + \tau' \Pi^{h-1} F_t.$$

For the second data generating process (DGP 2) we modify the equations (2) and (3) into

$$X_t = D_t^X + \Lambda F_t + e_t, \quad D_t^X = \eta_t \iota$$

$$y_{t+1} = D_{t+1}^y + \tau' F_t + \varepsilon_{t+1},$$

where D_t^X is the deterministic component of leading indicators, η_t is a scalar function, $\iota_N = (1, 1, ..., 1)'$ is a $(N \times 1)$ vector, and D_t^y is the deterministic component of the forecast variable. For simplicity assume that $D_t^y = \eta_t$ and η_t is

$$\eta_t = \{ \begin{array}{l} 0, t < t_b \\ \eta, t \geq t_b \end{array} .$$

The assumption of homogeneity of the structural break over variables is a restrictive one. In the real data structural changes can have different size and timing over variables. Only for well-selected homogeneous set of economic indicators one can expect that structural changes will have the same size and timing over variables.

The h-step conditional expectation of y_t is

$$y_{t+h|t} = D^y_{t+h} + \tau' \Pi^{h-1} F_t,$$

The size of μ and η are changing parameters in the simulations.

In empirical forecasting exercises original non-stationary series are transformed to stationary series before estimation of factors. The shift in mean of transformed series is equivalent to the change in slope of original series. This is a simple type of structural change. Perhaps, more complex functions can describe better the adjustment processes in transition economies.

The forecasting performance of dynamic factor models is compared with the performance of autoregressive models. All forecasting models are specified and estimated as a linear projection of an h-step-ahead variable, y^h_{t+h} onto t-dated predictors. More precisely, the forecasting models all have the form

$$y^h_{t+h} = \alpha + \beta(L) y_t + \gamma(L)' Z_t + v^h_{t+h}, \tag{4}$$

where α is a constant, $\beta(L)$ is a scalar lag polynomial, $\gamma(L)$ is a vector lag polynomial, and Z_t is a vector of predictor variables.

The "h-step ahead projection" approach in (4) (Clements and Hendry, 1999) differs from the standard approach of estimating a *1*-step-ahead model and then iterating that model forward to obtain h-step-ahead predictions. The h-step-ahead projection approach has two main advantages. First, additional equations for the forecasting of leading indicators are not needed. Second, the potential impact of specification error in the *1*-step-ahead model can be reduced by using the same horizon for estimation as for forecasting. However, forecasts based on h-step-ahead projection omit some information about the dynamics of the forecast variable, which is available to a forecaster at the date of forecasting (except for $h = 1$).

Four autoregressive forecasts are produced. They all correspond to (4) with Z_t excluded. Let us list them and briefly discuss their main characteristics.

The autoregressive forecast with one lag (*ar1*) is chosen as a benchmark. This choice is conditioned by the results of the empirical exercise conducted for Brazil and Russia. In that exercise we find that in many cases Bayesian Information Criteria chooses the lag length for AR model to be equal to zero. It implies that the AR forecast degenerates to a simple average of past values which seems to be a very weak benchmark. In this case it makes sense to consider the model with a fixed lag length as a benchmark.

In order to evaluate the role of increasing lag length in forecasting performance, we also produce the autoregressive forecast with three lags (*ar3*). The autoregressive forecast with the lag length chosen by BIC (*arbic*) is computed in order to evaluate the relative forecasting performance of the model chosen by the asymptotic criteria in small sample.

Two types of autoregressive forecasts are also considered: the intercept-corrected autoregressive forecast (*aric*) and the autoregressive forecast of the differenced forecast variable (*ardif*). Clements and Hendry (1999) argue that in the presence of structural breaks over the forecasting period the forecast can be put back on track by adding past forecast errors to the forecast. They show that simple addition of the h-period ahead forecast error can be useful. Hence, the forecast is given by $\hat{y}^h_{t+h} + \hat{v}^h_t$, where $\hat{y}^h_{t+h}$ is the AR forecast and $\hat{v}^h_t$ is the forecast error made when forecasting y^h_t in period (t-h).

An alternative remedy in the presence of structural breaks over the forecasting period is additional differencing of the forecast variable. In this case one estimates the following model:

$$x^h_{t+h} = \mu + \beta(L)x_t + v^h_{t+h},$$

where $x^h_{t+h} = y_{t+h} - y_t$ and $x_t = y_t - y_{t-1}$. The forecast of x^h_{t+h} is then used to construct the forecast of y^h_{t+h}. Clements and Hendry (1999) show that the differencing of the forecast variable can improve the forecasting performance of autoregressive models in the presence of structural breaks, even in the case of over-differencing.

However, if there is no break, both intercept correction and additional differencing increase the forecast error variance by adding a moving average component to the forecast error, and thus these techniques are not costless.

Four factor forecasts are produced in the simulations. All these forecasts correspond to (4), where y_t is excluded and Z_t is composed of true or estimated factors. We compare performance of forecasting models with true factors and estimated factors in order to evaluate the impact of uncertainty induced by the estimation of factors on forecast accuracy.

We evaluate the forecast produced by the data generating process (*facdgp*), the forecast produced by (4) where Z_t is substituted by the true factors (*factr*), the forecast produced by fully estimated factor model Z_t substituted by the estimated factors and the number of factor chosen by BIC (*fest*), and the intercept-corrected forecast produced by the fully estimated factor model (*festic*).

Forecasting models are compared on the basis of the mean square forecast error criteria (MSFE). It is computed by averaging square forecast errors $(\hat{y}^h_{t+h|t} - y^h_{t+h})^2$ over replications. Then the MSFE for each model is divided by the MSFE of the benchmark model. The standard errors around these relative MSFE are also computed. The MSFE of the benchmark (ar1) is normalized to one.

The MSFE is a standard criteria in forecasting exercises and by using it we can refer to previous research. However, it may not be the best criteria with the given small time dimensions and admitting the adjustment processes in the data.

In the empirical exercise performed for Brazil and Russia we used accordingly 41 and 47 quarterly series over period 1995:1 - 2004:4 of total 40 observations. On the other hand, in the empirical exercise performed for the euro-area we used 44 monthly series over period 1999:1 - 2007:2 of total 98 observations. These are data spans which are quite often met in empirical exercises based on data available for emerging market economies. We usually can find 40 to 50 quarterly observations and 100 to 120 observations for 40 to 50 variables.

In this simulation exercise we consider three combinations of T and N: $T = 50$ and $N = 50$; $T = 100$ and $N = 50$; $T = 50$ and $N = 100$. It allows us to evaluate the impact of N/T ratio on estimation of factors and forecast accuracy.

In order to evaluate the adjustment of forecasting models to a structural break, we consider two dates of a break: one is in the middle of the estimation period ($t_b = T/2$), and another is in the end of the estimation period ($t_b = T$). Then the h-step-ahead forecast is evaluated. So that when $t_b = T$, the break happens in the period when forecast is produced.

As we want to make possible references to the empirical research (Banerjee, Marcellino and Masten, 2006, and Bystrov, 2007) we produced 1-step-ahead forecasts. However, in the most of empirical exercise one is interested in few-period-ahead forecasts. In order to evaluate the impact of the longer forecasting horizon on the relative performance of forecasting models, we also evaluate 3-step-ahead forecasts.

The effect of the number of factors and factor persistence is also explored in the experiment. The number of factors generating data varies from 1 to 3. The factor persistence parameter π takes values 0.3, 0.6, and 0.9.

3 Results

Relative MSFEs of forecasting models for the basic data generating process without break are shown in Figures 1 and 2 in Appendix. We present the results for a few parameter combinations of the whole set of parameter combinations. The tables with the whole set of results can be presented upon request.

Figure 1 shows that the relative performance of factor models for 1-step-ahead forecasts deteriorates with an increase in the number of factors and an increase in the persistence of factors. Figure 2 indicates that the relative performance of factor forecasts for 3-step-ahead forecasts deteriorates as the number of factors grows and improves as the persistence of factors grows. Comparison of Figures 1 and 2 allows us to conclude that the relative performance of factor forecast deteriorates as the forecasting horizon increases.

As in Banerjee, Marcellino and Masten (2007) *ar3* and *arbic* forecasts have performance which is very close to the performance of the benchmark *ar1*. Intercept correction and differencing provide no gains as there are no structural breaks.

In Figure 3 we report relative MSFE of 1-step-ahead forecasts for the data generating process with no break for different N/T ratios. Differences in the relative performance of forecasting models over different N/T are not large. The generated data are informative about factors dynamics and factor models perform well even in samples of relatively small size.

If there is a break in factors in the middle of estimation period ($t_b = T/2$), factor models perform better than in the case of no break both for 1-step-ahead forecasts (Figure 5) and 3-step-ahead forecasts (Figure 8). For 1-step-ahead forecasts gains grow with an increase in the size of break and the number of factors, but decrease with an increase in the persistence of factors. For 3-step-ahead forecasts gains grow as the size of break and the persistence of factors grow. The gains may be better explained by the specification of the data generating process rather than by the estimation effect, as the ratio of MSFE of the true factor model and the estimated factor model does not change.

The relative performance of 1-step-ahead factor forecasts improves if there is a break in factors in the end of the estimation sample ($t_b = T$), at the period when the forecast is produced (see Figure 5). Gains increase as the size of break grows and the number of factors grows, but decrease as the persistence of factors grows. However, the relative performance of estimated 3-step-ahead forecasts deteriorates in the case of break at the period when forecasts are produced (Figure 9). This is an estimation effect, since the factor forecasts produced by the true model perform well in this case.

If there is a break in the forecasting equation in the middle of estimation period, we find that the fully estimated factor model, *fest*, performs better than the factor model with true factors, *factr* (Figures 6 and 10). These effect increases with an increase in the size of break and an increase in the number of factors.

A possible explanation of the better performance of the fully estimated factor model relative to the model with true factors is that estimated factors being biased estimates of true factors, pick up a break in the series and thus allow the accommodation of the break in the estimated forecasting equation.

A break in the forecasting equation at the period when forecasts are produced, implies deterioration of the performance of the estimated factor model (Figures 7 and 11). Losses increase with an increase in the size of break, the number of factors, and the persistence of factors. This effect does not depend on the forecasting period. In this case estimated factors do not accommodate the structural break and factor forecasts fail relative to autoregressive forecasts.

Intercept corrected forecasts and forecasts of differenced series perform better for 3-step-ahead forecasts. These methods are found to be useful when there is a break in factors in the end of estimation sample (Figures 5 and 9). In particular, intercept-corrected 3-step-ahead factor forecasts are found to be the best for the process with a break in factors at period T, when the size of break is equal to two.

4 Conclusions

In this paper we present the results of the Monte Carlo exercise, in which we have investigated the relative performance of different forecasting models for data of the time and cross-section dimensions corresponding to those, which are available for transition economies and emerging market economies. We have explored the efficacy of forecasting models in the presence of the breaks of two types: a shift in mean of factors generating data and a shift in intercept of the forecasting equation.

Several general observations follow. The relative performance of factor forecasts improves if there is a shift in mean of data in the middle of estimation sample. This result holds whether the break is in factor dynamics or in the forecasting equation.

However, if there is a break in the forecasting equation, fully estimated factor models outperform factor models with true factors. One possible explanation of this phenomenon is that estimated factors pick up a break, which is common both for leading indicators and forecast variable. Then the break is accommodated in the fully estimated factor model. On the other hand, there is no break in the dynamics of true factors and the model with true factors do not accommodate the break in data.

In the case of break in the end of estimation sample factor forecasts outperform other models only if there is a break in the factor dynamics and the forecasting horizon is 1-step-ahead. However, the relative performance of factor forecasts deteriorates if there is a break in the forecasting equation, or the forecasting horizon is 3-step-ahead. In this case, intercept correction and differencing appear to be efficient techniques which allow us to improve both autoregressive forecasts and factor forecasts.

A break in the end of the estimation sample might be an explanation of the relatively bad performance of factor forecasts in empirical exercises. As the change happens at the period when the forecast is produced, factor forecasts fail to outperform other models. However, as the amount of information about break accumulates over time, one can expect that the relative performance of factor forecasts will improve.

Interesting directions for future research in this context are mostly related to the careful investigation of statistical properties of real data and empirical factors extracted from these data, and performing Monte Carlo simulations with data generating processes which may be a good approximation of the data used in empirical exercises. More complex functions can be used for the description of the adjustment processes in emerging market economies. The performance of factor models can also be evaluated in comparison with more elaborated benchmarks than simple autoregressive models.

Appendix. Figures

Notes:

ar1 - AR (1) model (benchmark)

ar3 - AR (3) model

arbic - AR model chosen by BIC

aric - intercept-corrected AR (BIC) model

ardif - AR (BIC) model for differenced series

facdgp - forecast produced by DGP

factr - factor forecast, true factors

fest - factor forecast, fully estimated model

festic - intercept-corrected factor forecast, fully estimated model

FIGURE 1. RELATIVE MSFE: NO BREAK, 1-STEP-AHEAD FORECASTS, T=50, N=50

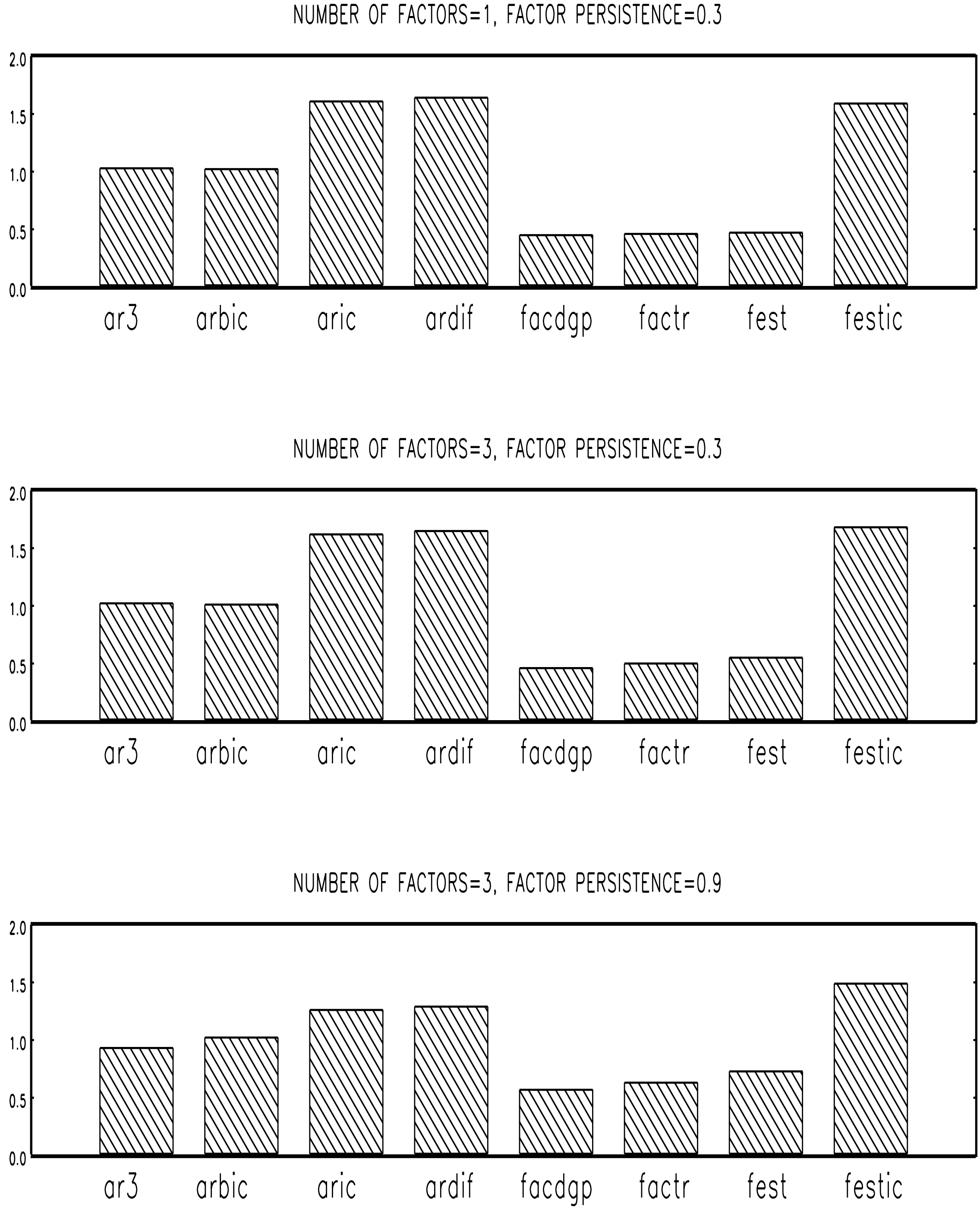


FIGURE 2. RELATIVE MSFE: NO BREAK, 3-STEP-AHEAD FORECASTS, T=50, N=50

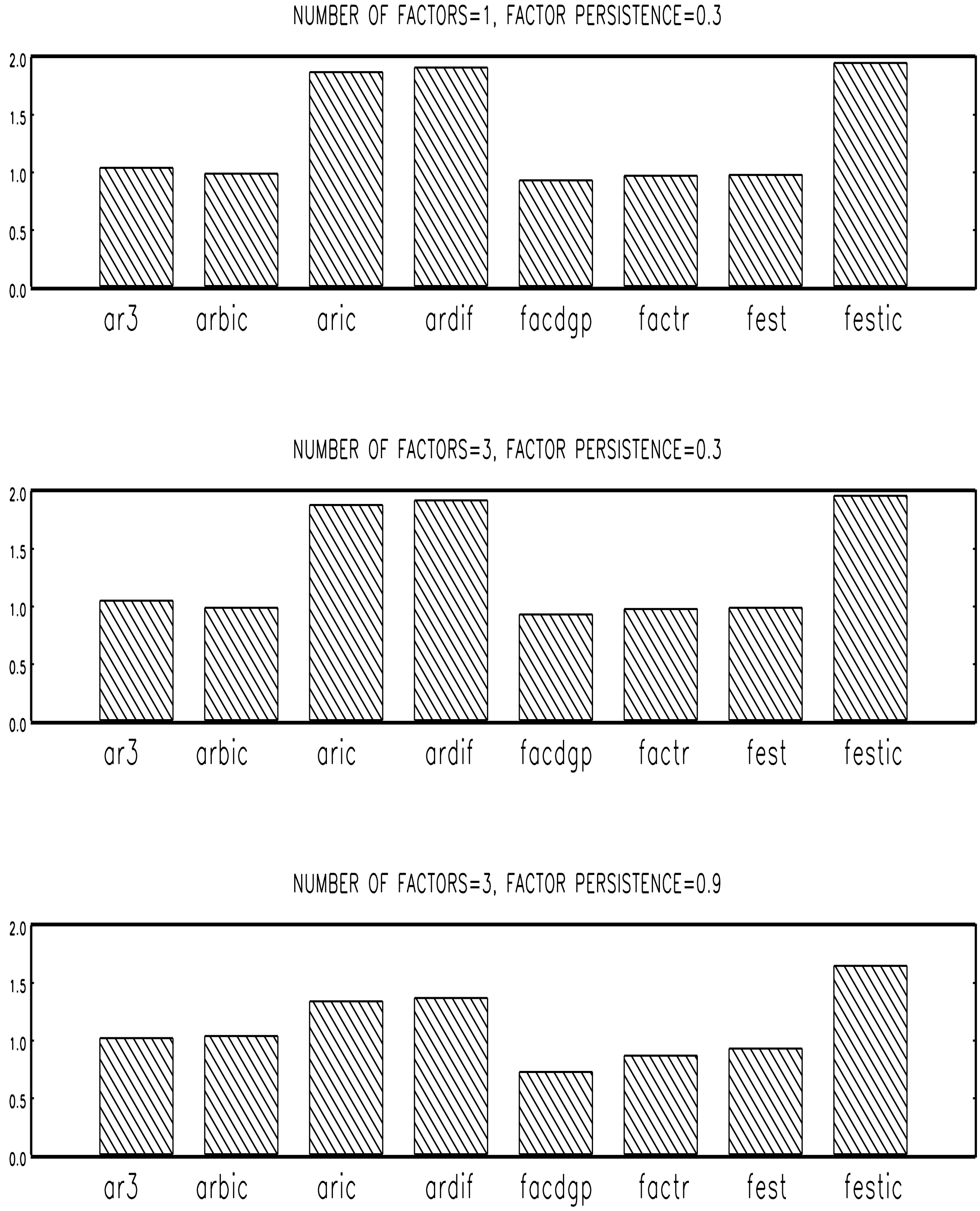


FIGURE 3. RELATIVE MSFE: NO BREAK, 1-STEP-AHEAD FORECASTS,
NUMBER OF FACTORS=2, FACTOR PERSISTENCE=0.6

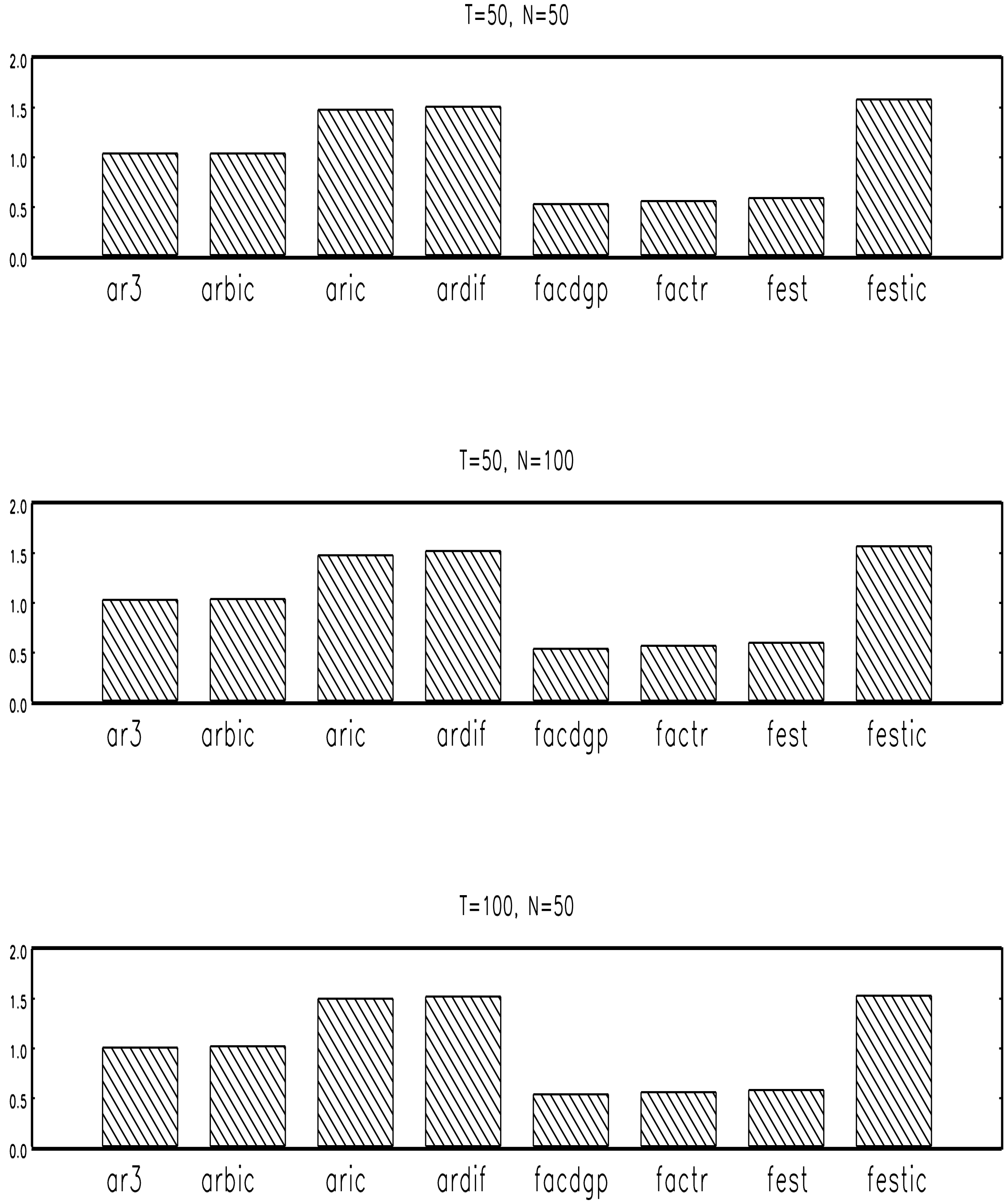


FIGURE 4. RELATIVE MSFE: BREAK IN FACTORS, DATE OF BREAK=T/2,
T=50, N=50, 1-STEP-AHEAD FORECASTS

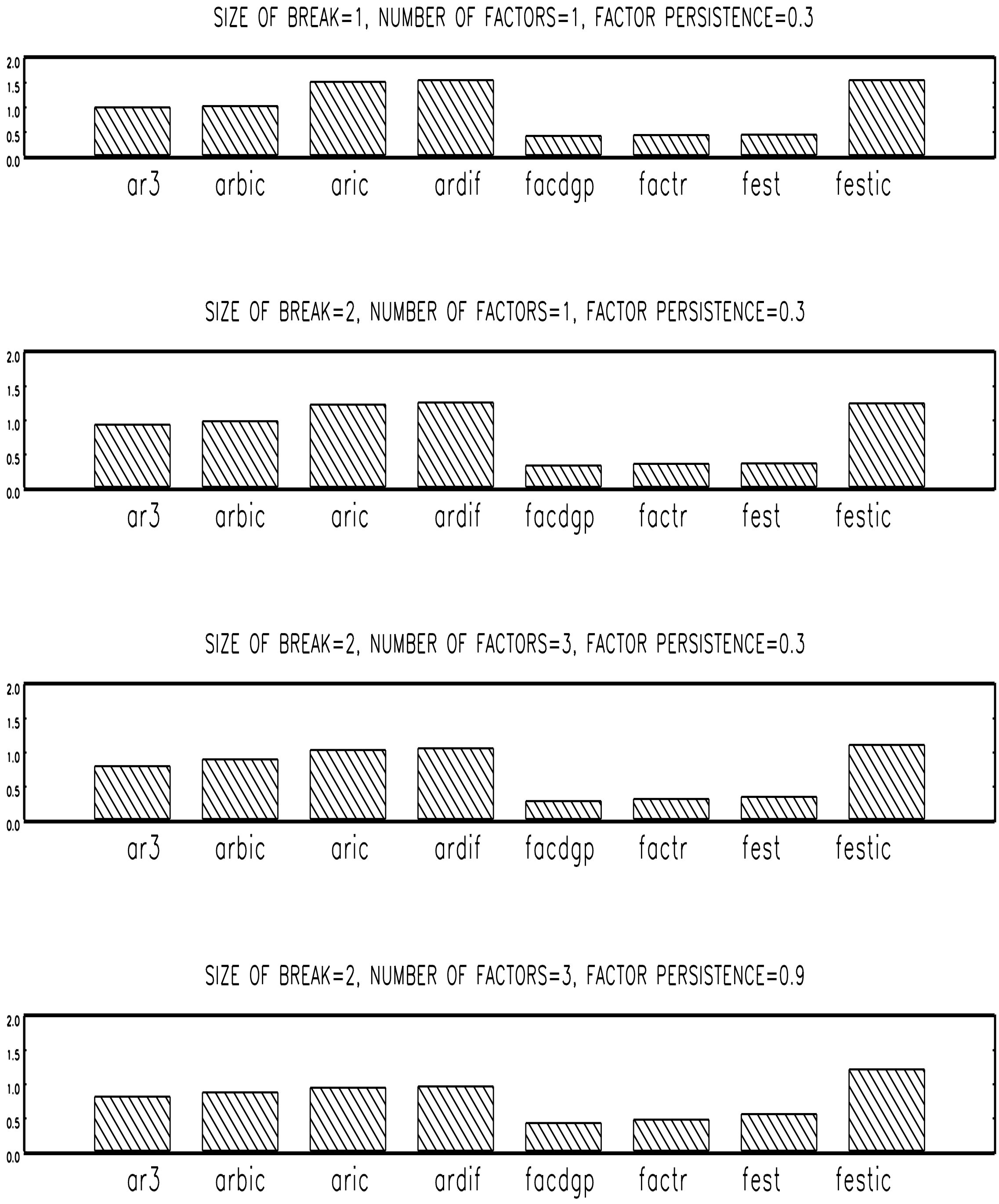


FIGURE 5. RELATIVE MSFE: BREAK IN FACTORS, DATE OF BREAK=T,
T=50, N=50, 1-STEP-AHEAD FORECASTS

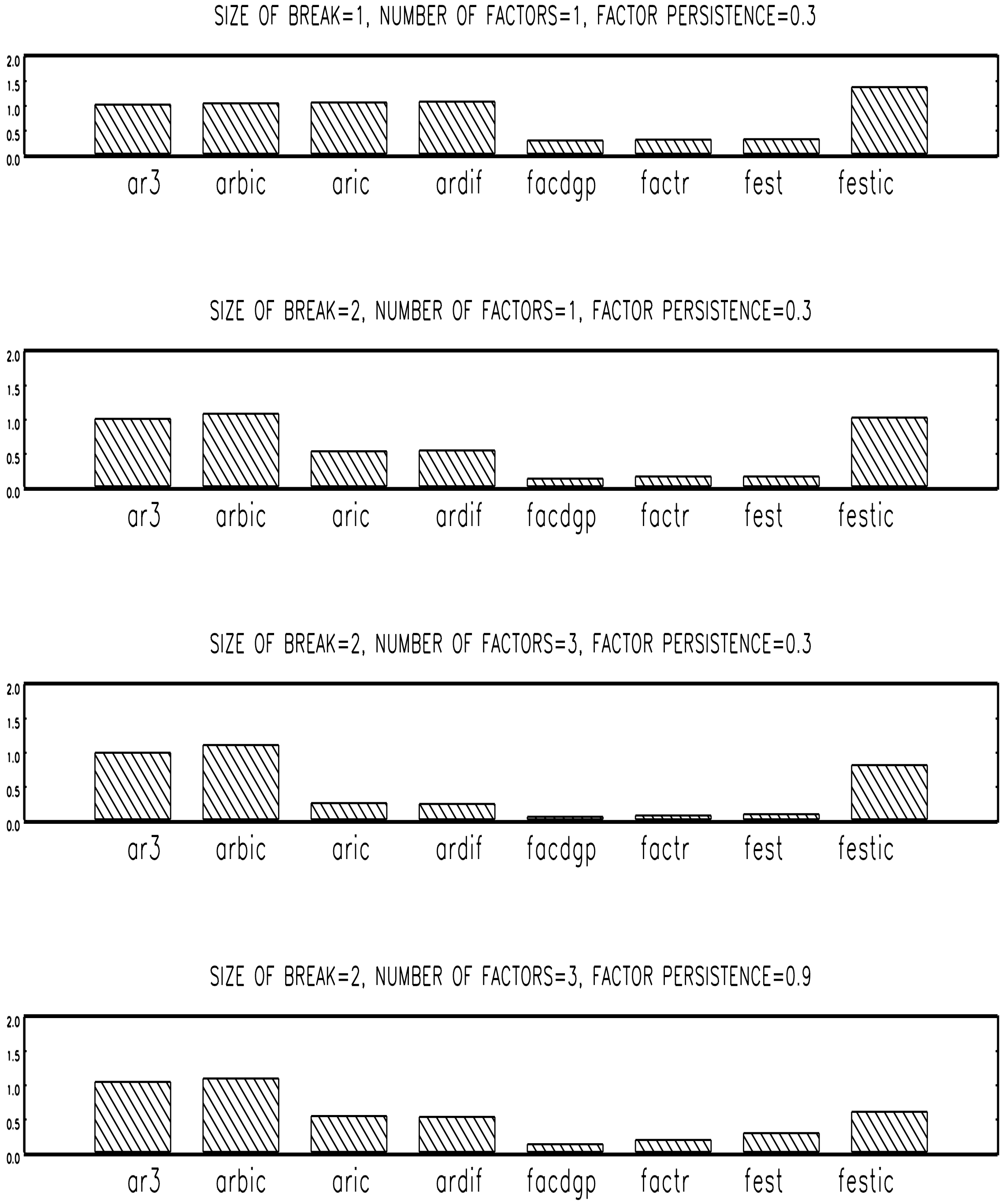


FIGURE 6. RELATIVE MSFE: BREAK IN FORECASING EQUATION, DATE OF BREAK=T/2,
T=50, N=50, 1-STEP-AHEAD FORECASTS

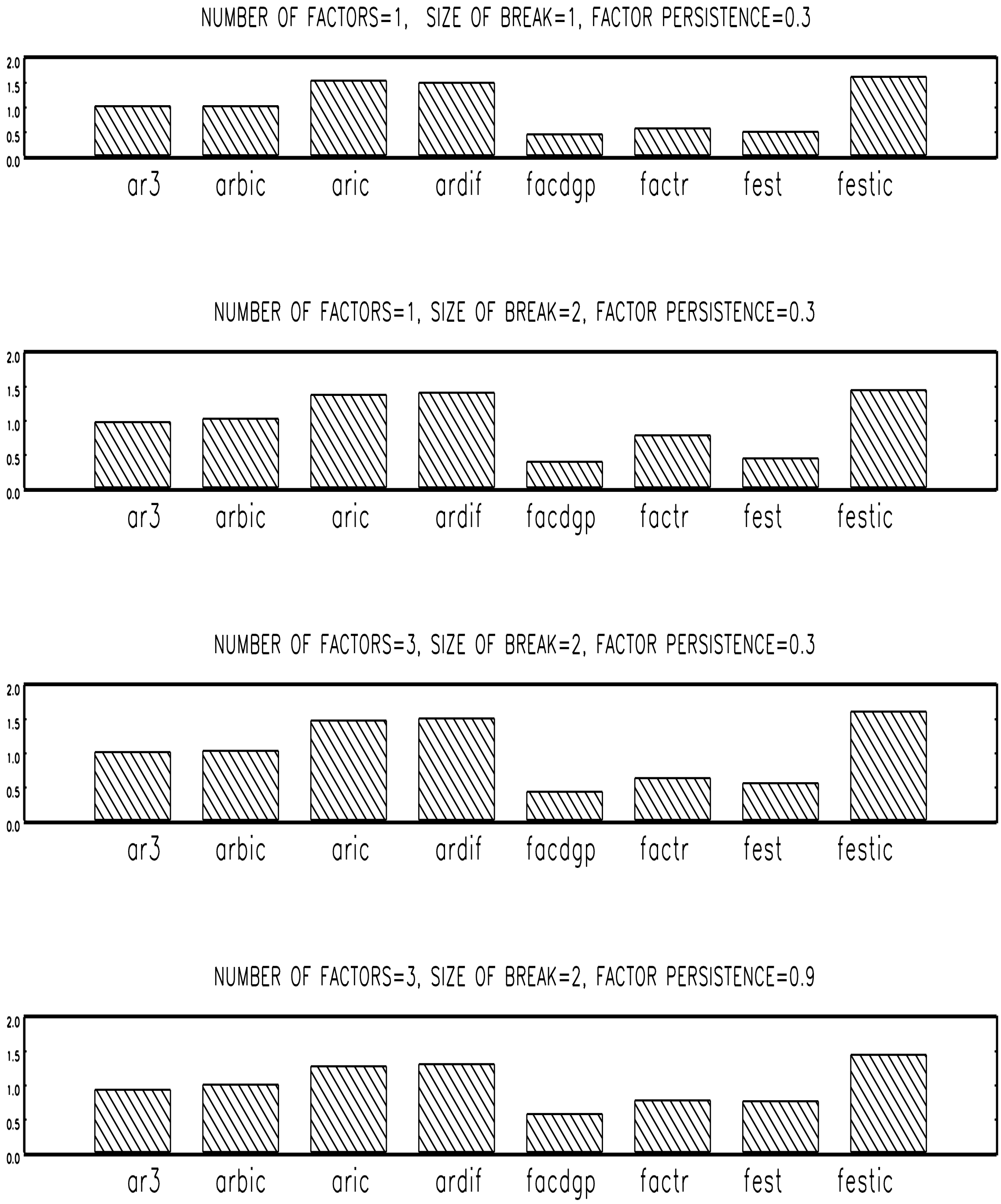


FIGURE 7. RELATIVE MSFE: BREAK IN FORECASING EQUATION, DATE OF BREAK=T,
T=50, N=50, 1-STEP-AHEAD FORECASTS

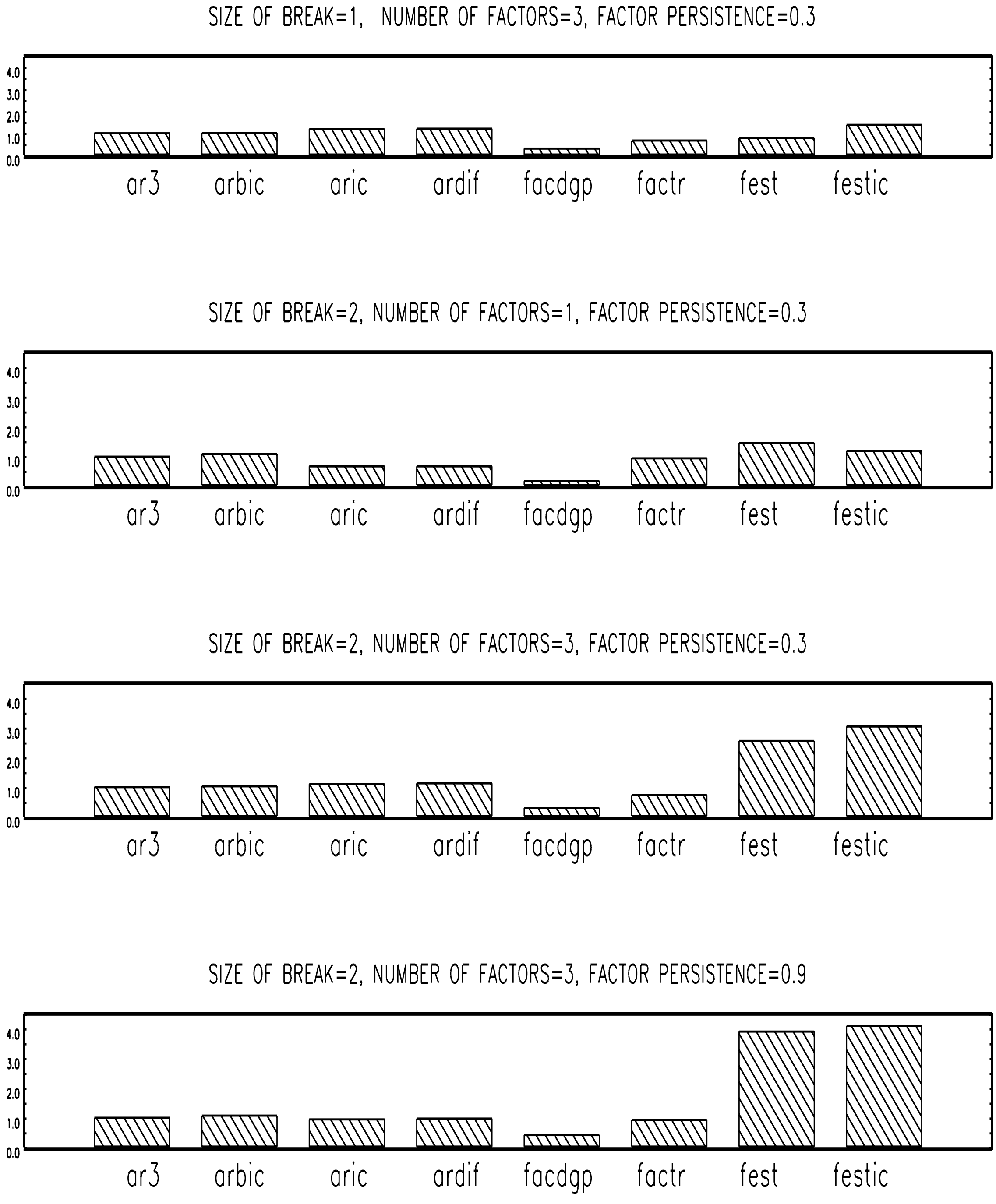


FIGURE 8. RELATIVE MSFE: BREAK IN FACTORS, DATE OF BREAK=T/2,
T=50, N=50, 3-STEP-AHEAD FORECASTS

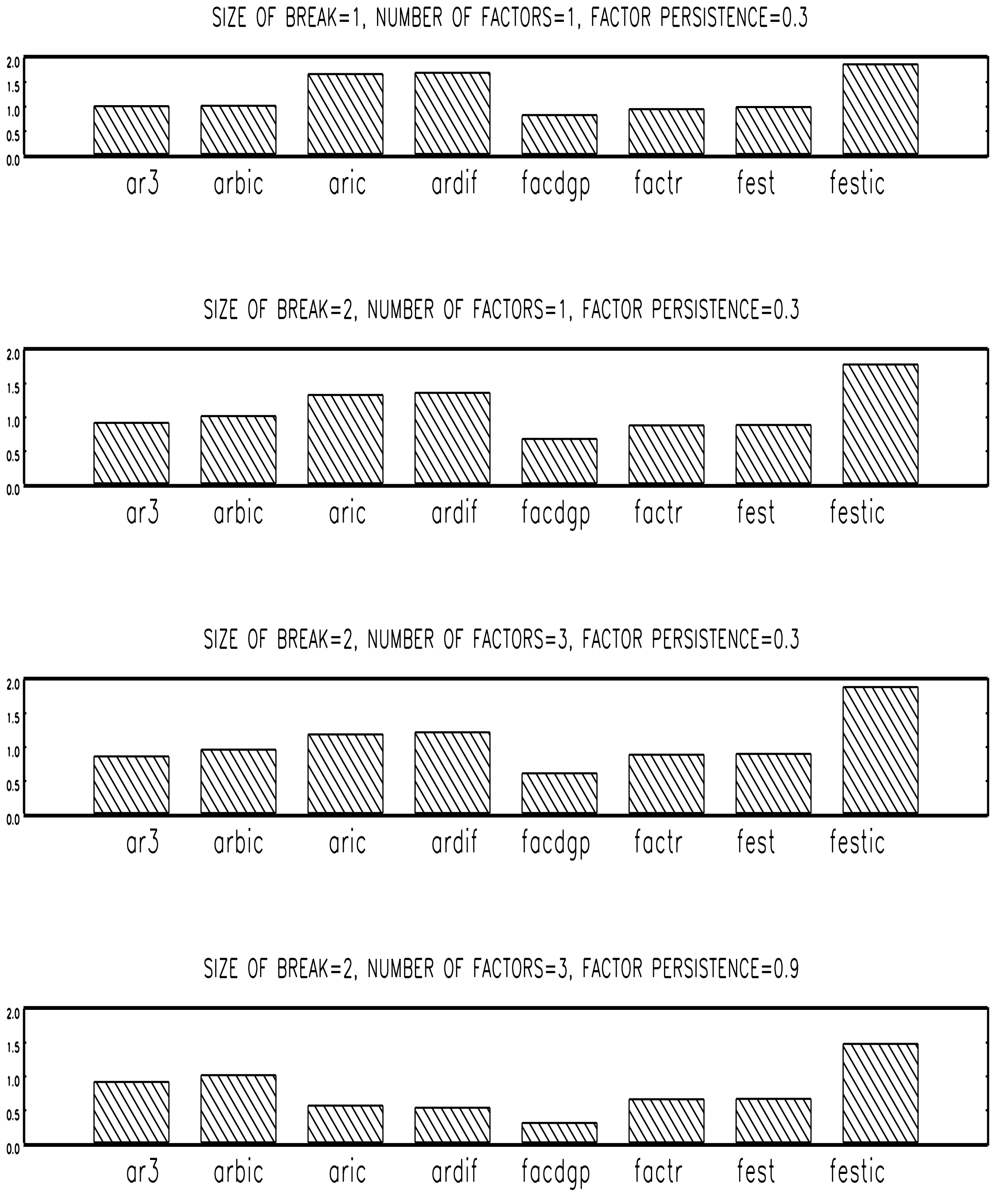


FIGURE 9. RELATIVE MSFE: BREAK IN FACTORS, DATE OF BREAK=T,
T=50, N=50, 3-STEP-AHEAD FORECASTS

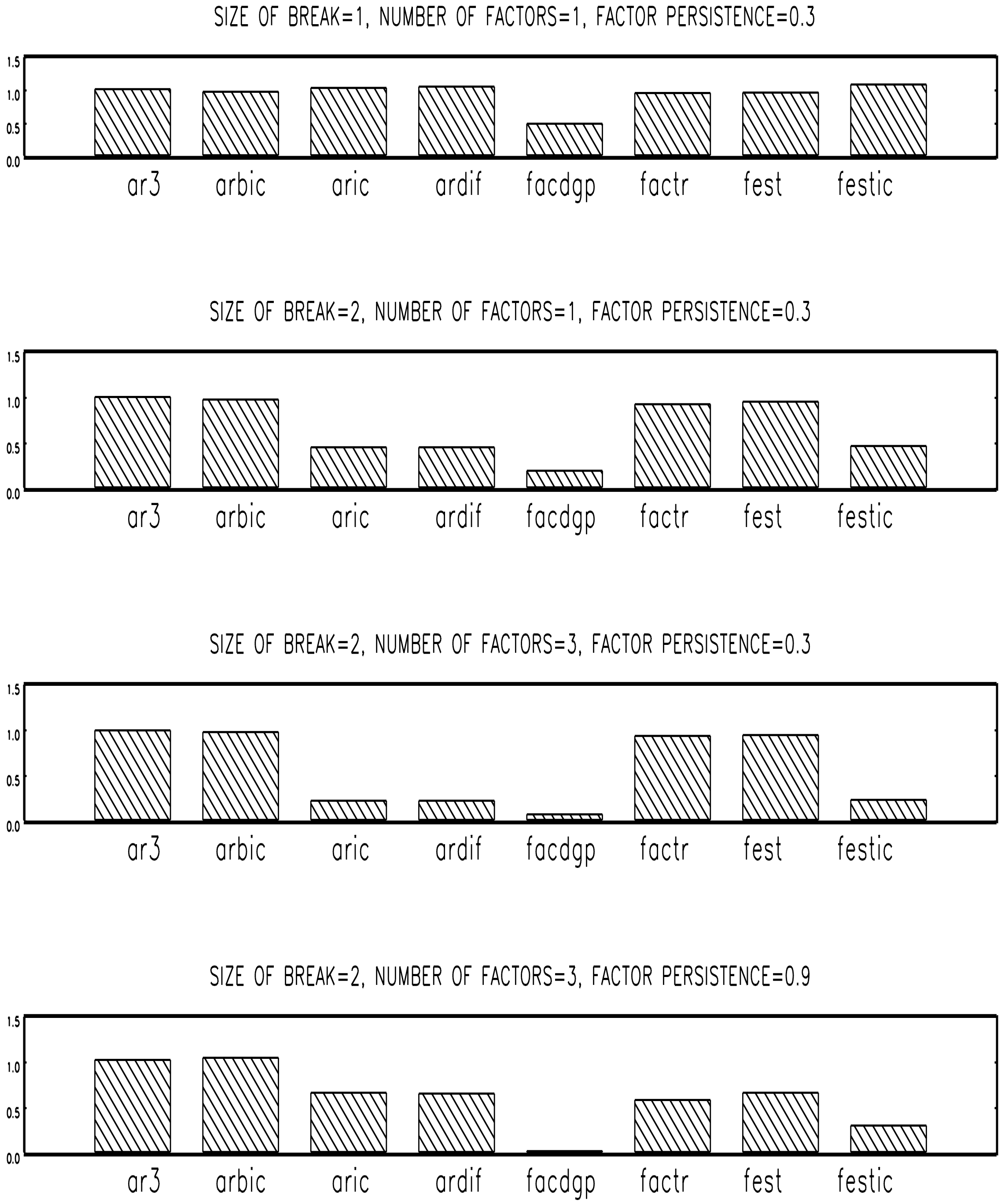
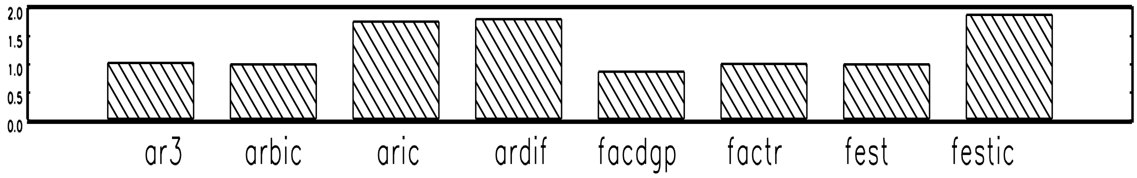
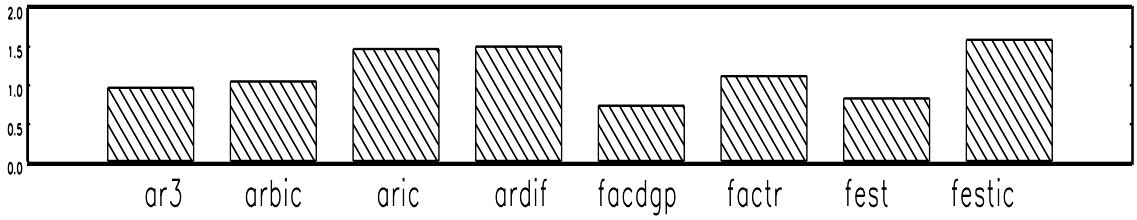


FIGURE 10. RELATIVE MSFE: BREAK IN FORECASTING EQUATION, DATE OF BREAK=T/2, T=50, N=50, 3-STEP-AHEAD FORECASTS

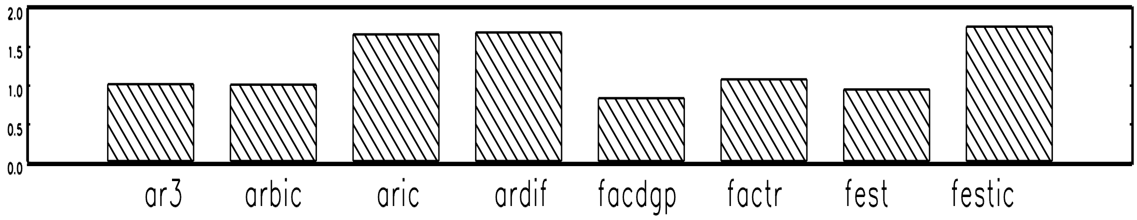
SIZE OF BREAK=1, NUMBER OF FACTORS=1, FACTOR PERSISTENCE=0.3



SIZE OF BREAK=2, NUMBER OF FACTORS=1, FACTOR PERSISTENCE=0.3



SIZE OF BREAK=2, NUMBER OF FACTORS=3, FACTOR PERSISTENCE=0.3



SIZE OF BREAK=2, NUMBER OF FACTORS=3, FACTOR PERSISTENCE=0.9

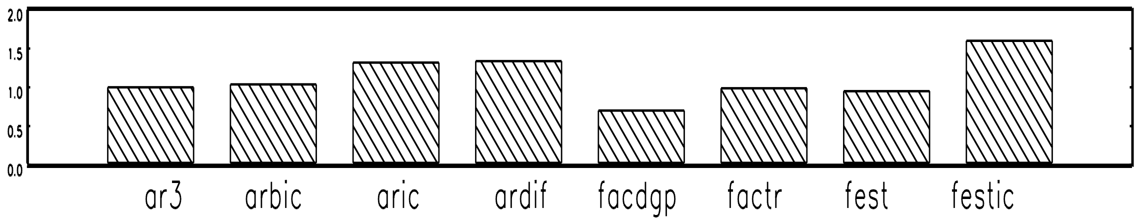
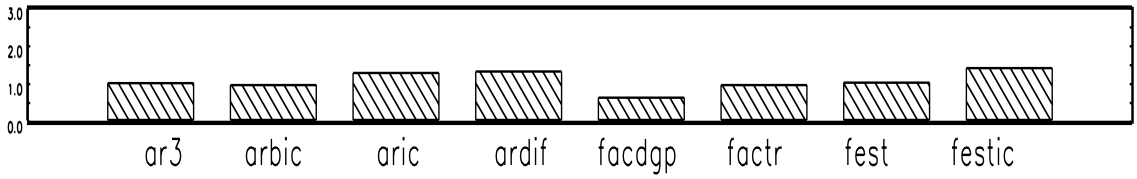
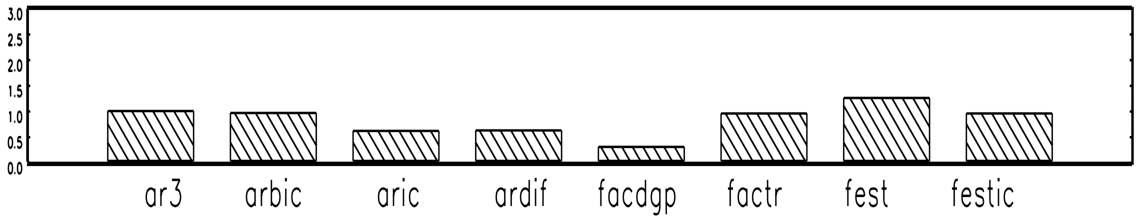


FIGURE 11. RELATIVE MSFE: BREAK IN FORECASING EQUATION, DATE OF BREAK=T,
T=50, N=50, 3-STEP-AHEAD FORECASTS

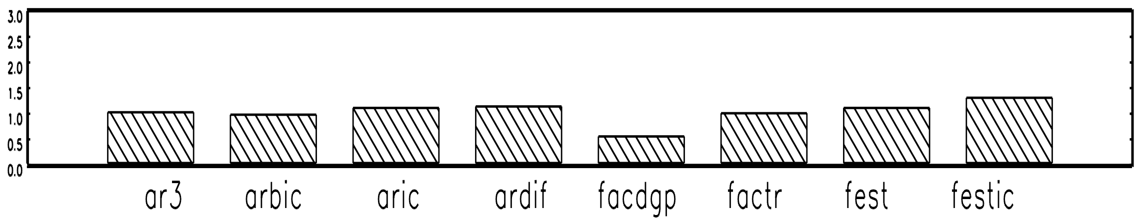
SIZE OF BREAK=1, NUMBER OF FACTORS=3, FACTOR PERSISTENCE=0.3



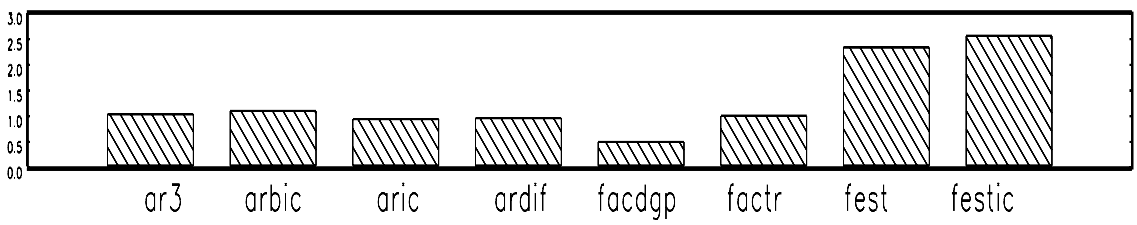
SIZE OF BREAK=2, NUMBER OF FACTORS=1, FACTOR PERSISTENCE=0.3



SIZE OF BREAK=2, NUMBER OF FACTORS=3, FACTOR PERSISTENCE=0.3



SIZE OF BREAK=2, NUMBER OF FACTORS=3, FACTOR PERSISTENCE=0.9



Chapter 3

Factor Augmented Error Correction Models

Abstract

In this paper the performance of factor-augmented error-correction models (FECM) is evaluated in the Monte Carlo exercise and in the application to the interest rate pass-through model for the euro area. The hypothesis considered is that the inclusion of common stochastic trends extracted from a large data set can improve in-sample and out-of-sample performance of a small-scale ECM. It is found both in the Monte Carlo exercise and the empirical exercise that in-sample the FECM performs well relative to the models which are not augmented by factors, but the forecasting performance of the FECM is no better than the forecasting performance of other models in most cases.

1 Introduction

This paper addresses the issue of the estimation and the forecasting of cointegrated systems, using information extracted from the common stochastic trends representation. Few common stochastic trends is a parsimonious representation of co-movement in a large system of non-stationary variables with many cointegrating relations. This parsimonious representation of co-movement in a large system may be of use in the estimation and the forecasting of small-scale cointegrated subsystem.

Any cointegrated system of *I(1)* variables can be written in the common stochastic trends representation or the error correction representation. If the number of variables in the system is large, estimation of the error correction model (ECM) for the whole system may be unfeasible or associated with large estimation errors. Instead, a small-scale ECM including only few variables of interest, is usually estimated. However, such small-scale system can be misspecified due to omitted variables.

Under certain assumptions, it is possible to interpret the common stochastic trends representation of the cointegrated system as a dynamic factor model, and estimate common stochastic trends by principal components as in Bai (2004). Common factors extracted from the whole data set can be used to augment the small-scale ECM.

In recent work, factor-augmented VARs have become the subject of the intensive research. Stock and Watson (1999) estimate the Phillips curve, using common factors as measures of aggregate real activity. Favero and Marcellino (2001), Bernanke and Boivin (2003) evaluate policy rules using estimated factors as additional regressors in VARs. Bernanke, Boivin, and Eliasz (2004), Belviso and Milani (2005), and Stock and Watson (2005) evaluate factor-augmented structural VARs. Hansson, Jansson, and Lof (2003) forecast GDP, and Monch (2005) forecasts the yield curve in the US using factor-augmented VARs.

All these applications ignore possible cointegration between variables and evaluate factor-augmented VARs in differences. In this paper we are going to evaluate factor-augmented error correction models (FECM), as introduced by Banerjee and Marcellino (2007). Performance of these models is going to be evaluated on the basis of their in-sample fit and forecasting accuracy.

We consider both a simulation exercise, in which we evaluate performance of FECM for data generated by a large-scale error-correction model, and an application of FECM to the modelling of interest rate pass-through in the euro-area and the forecasting of bank interest rates.

The paper is organized as follows. The second section introduces the FECM. In the third section we report the results of the simulation exercise. In the fourth section the results of the empirical exercise are reported. The fifth section presents the conclusions.

2 Cointegration and Dynamic Factor Models

Assume that an N-dimensional stochastic process $\{X_t\}$, where N is large, is generated by the VAR(p) model:

$$X_t = \Pi_1 X_{t-1} + ... + \Pi_p X_p + \epsilon_t, \quad t = 1, 2, ..., T, \tag{1}$$

where ϵ_t is $(0, \Omega)$. The VAR(p) can be reparametrized into the error correction representation (Johansen, 1995):

$$\Delta X_t = \alpha\beta' X_{t-1} + \sum_{j=1}^{p-1} \Gamma_j \Delta X_{t-j} + \epsilon_t, \tag{2}$$

where α is a $N \times (N - r)$ matrix of loadings of cointegration relations and β is a $N \times (N - r)$ matrix of cointegration vectors, and $\Gamma_j = -\sum_{s=j+1}^{p} \Pi_s$ is a $N \times N$ matrix of lag loadings.

We assume that $rank(\alpha) = rank(\beta) = (N - r) < N$, where $N - r$ is a number of cointegration vectors and r is a number of common trends. Then the common trends representation of (1) is

$$X_t = \Psi F_t + u_t, \tag{3}$$

where $F_t = \alpha'_\perp \sum_{s=1}^{t} \epsilon_s$ is a $r \times 1$ vector of common stochastic trends, $\Psi \beta_\perp (\alpha'_\perp \Gamma \beta_\perp)^{-1}$, is a $N \times r$ matrix of loadings of common trends into the variables, and $u_t = C(L)\epsilon_t$ is a $N \times 1$ stationary moving average process.

When N is large, estimation of the model (1) or (2) may be associated with high estimation uncertainty or unfeasible, as for large N the number of parameters in the model may be close or larger than the number of available observations. This is why small-scale VARs or ECMs including only few components of the vector X_t, are estimated in the most of applications.

However, such small-scale models are generally misspecified: they omit many variables which are of potential use, in particular, in the forecasting. We are interested in the augmentation of these models in such a way that improves their forecasting performance.

Let us divide the vector X_t into a sub-vector $Y_t = \{X_{it}\}_{i=1}^{k}$ of the first k components ($k > r$) and a sub-vector $Z_t = \{X_{it}\}_{i=k+1}^{N}$ of the last $N - k$ components of the vector X_t. Assume that we are interested in the forecasting of the components of the vector Y_t. The dynamics of this vector conditional on Z_t can be described by the following system:

$$\Delta Y_t = \alpha_Y \beta' \begin{pmatrix} Y_{t-1} \\ Z_{t-1} \end{pmatrix} + \sum_{j=1}^{p-1} [\Gamma_{Yj} \Delta Y_{t-j} + \Gamma_{ZYj} \Delta Z_{t-j}] + \epsilon_{Yt}, \tag{4}$$

where $\alpha_Y = \{\alpha_{ij}\}_{i,j=1}^{k,r}$, $\epsilon_{Yt} = \{\epsilon_{it}\}_{i=1}^{k}$.

The standard ECM representation of Y_t,

$$\Delta Y_t = \alpha_Y \beta_Y' Y_{t-1} + \sum_{j=1}^{p-1} \Gamma_{Yj} \Delta Y_{t-j} + \epsilon_{Yt}, \tag{5}$$

where α_Y and β_Y are of rank no higher than $k - r$, omits Z_{t-1} and lags of ΔZ_t and is generally misspecified. This misspecification can lead to a forecast failure. We are interested in finding other proxies for (4) which allow us to incorporate information about Z_t and ΔZ_t but do not require estimation of the whole system (2). For this purpose we are going to use the common trends representation of X_t.

The common trends representation (3) can be interpreted as a dynamic factor model presented in Bai (2004):

$$X_t = \Psi F_t + u_t, \tag{6}$$

$$F_t = F_{t-1} + e_t, \tag{7}$$

where $e_t = \alpha_\perp' \epsilon_t$. Bai (2004) proves that under certain conditions the common stochastic trends F_t can be consistently estimated by the principal components estimator. However, the representation (6)-(7) is only valid, when e_t and u_{it} ($i = 1, 2, ..., N$) are orthogonal, which is only a special case, as $e_t = \alpha_\perp' \epsilon_t$ and $u_t = C(L)\epsilon_t$

are generated by the same stochastic process. Bai (2004) shows that when u_t and e_{it} are correlated, one can get the generalized dynamic factor representation:

$$X_{it} = \psi_i(L)' F_t + v_{it}, \quad i = 1, 2, ..., N, \tag{8}$$

where $\psi_i(L)$ is a vector of polynomials of the lag operator. The common factors can be consistently estimated for the generalized model (Bai, 2004).

The representation (3) implies that Y_t and F_t are cointegrated. Therefore, from the Granger representation theorem there must exist an ECM representation of the type

$$\begin{pmatrix} \Delta F_t \\ \Delta Y_t \end{pmatrix} = \begin{pmatrix} 0 \\ \gamma_Y \end{pmatrix} \delta' \begin{pmatrix} F_{t-1} \\ Y_{t-1} \end{pmatrix} + \sum_{j=1}^{q} \begin{pmatrix} 0 & 0 \\ A_{Yj} & A_{YFj} \end{pmatrix} \begin{pmatrix} \Delta F_{t-j} \\ \Delta Y_{t-j} \end{pmatrix} + \begin{pmatrix} e_t \\ e_{Yt} \end{pmatrix} \tag{9}$$

The specification (9) is labeled by Banerjee and Marcellino (2007) as a Factor Augmented Error Correction Model (FECM).

Since the FECM (9) includes $k + r$ variables but the r factors F_t are random walks, there can be at most k cointegrating relations in (9). There could be at most $k - r$ cointegrating relations in the ECM including Y_t only. However, in addition to these $k - r$ cointegrating relations, in the FECM there are r cointegrating relations between X_t and F_t, that proxy the potentially omitted cointegrating relations in (5). Moreover, the lags of ΔF_t proxy the potentially omitted lags of ΔZ_t in the standard ECM (5). Therefore, the FECM includes additional information, which is omitted in the standard ECM (5), and can provide a better approximation to the data generating process.

The FECM can be of particular interest, because it allows identifying long run relationships between the variables of interests and it can provide gains in forecasting by the incorporation of additional information extracted from the whole data set.

The FECM (9) can be estimated using two-stage procedure:

1)estimation of the common factors F_t by principal components of the covariance matrix of integrated series;

2)estimation of the FECM (9), where F_t and ΔF_t are substituted by their estimates.

In order to evaluate the in-sample and the forecasting performance of the FECM relative to that of the ECM in finite samples, we are going to perform a set of simulation experiments. In these experiments the in-sample fit and forecasts of ECM and FECM are going to be compared, using a differenced VAR model as a benchmark. The performance of the factor-augmented VAR is also evaluated.

The in-sample performance of the models is evaluated on the basis of their relative squared standard errors. The forecasting performance of the models is going to be evaluated on the basis of the relative mean squared forecast errors. The squared standard errors differ from the mean squared errors as the standard errors are adjusted for degrees of freedom used in the model fitting.

3 Monte Carlo Exercise

3.1 Experiment Design

The basic data-generating process is an error - correction model:

$$\Delta X_t = \alpha\beta' X_{t-1} + \epsilon_t, \tag{10}$$

where X_t is a $N \times 1$ vector, α and β are $N \times (N-r)$ matrices, r is the number of common stochastic trends, and $\epsilon_t \sim N(0, \Sigma)$ where Σ is a diagonal matrix.

Let us fix r=1 and set the cointegration vector equal to

$$\beta' = \begin{pmatrix} -1 & 1 & 0 & 0 & ... & 0 \\ -1 & 0 & 1 & 0 & ... & 0 \\ -1 & 0 & 0 & 1 & ... & 0 \\ . & . & . & . & ... & 0 \\ -1 & 0 & 0 & 0 & ... & 1 \end{pmatrix}$$

For a given β two versions of the basic data generating process are considered. These two versions have different matrix α. For the data generating process 1 (DGP 1) it is

$$\alpha = \begin{pmatrix} 0 & 0 & 0 & ... & 0 \\ -a_1 & 0 & 0 & ... & 0 \\ 0 & -a_1 & 0 & ... & 0 \\ 0 & 0 & -a_1 & ... & 0 \\ . & . & . & ... & 0 \\ 0 & 0 & 0 & ... & -a_1 \end{pmatrix}$$

where $a_1 \in (0,1]$. There is no common cointegration in this process: each cointegration relation affects a single variable.

The error-correction representation of the DGP 1 is

$$\begin{array}{lclcl} \Delta x_{1t} & = & & & \epsilon_{1t} \\ \Delta x_{2t} & = & -a_1(x_{2t-1} - x_{1t-1}) & + & \epsilon_{2t} \\ \Delta x_{3t} & = & -a_1(x_{3t-1} - x_{1t-1}) & + & \epsilon_{3t} \\ & ... & & & \\ \Delta x_{Nt} & = & -a_1(x_{Nt-1} - x_{1t-1}) & + & \epsilon_{Nt} \end{array}$$

Let us assume that $X_0 = 0$. Then we can write the stochastic trend representation of the DGP 1 as

$$\begin{array}{lllllllll} x_{1t} = & & \Sigma_{s=1}^{t-1}\epsilon_{1s} & & & & & + & \epsilon_{1t} \\ x_{2t} = & a_1 & \Sigma_{s=1}^{t-1}\epsilon_{1s} & + & a_1 & \Sigma_{j=1}^{t-2}(1-a_1)^j\Sigma_{s=1}^{t-j-1}\epsilon_{1s} & + \ \Sigma_{j=1}^{t-2}(1-a_1)^j\epsilon_{2t-j} & + & \epsilon_{2t} \\ x_{3t} = & a_1 & \Sigma_{s=1}^{t-1}\epsilon_{1s} & + & a_1 & \Sigma_{j=1}^{t-2}(1-a_1)^j\Sigma_{s=1}^{t-j-1}\epsilon_{1s} & + \ \Sigma_{j=1}^{t-1}(1-a_1)^j\epsilon_{3t-j} & + & \epsilon_{3t} \\ & ... & & & & & & & \\ x_{Nt} = & a_1 & \Sigma_{s=1}^{t-1}\epsilon_{1s} & + & a_1 & \Sigma_{j=1}^{t-2}(1-a_1)^j\Sigma_{s=1}^{t-j-1}\epsilon_{1s} & + \ \Sigma_{j=1}^{t-1}(1-a_1)^j\epsilon_{Nt-j} & + & \epsilon_{Nt} \end{array}$$

The data are generated by one common stochastic trend process, lags of this common stochastic trend, and an autocorrelated idiosyncratic process.

The moving average representation of the differenced process is

$$\begin{array}{lllll} \Delta x_{1t} = & & & + & \epsilon_{1t} \\ \Delta x_{2t} = & -a_1(\epsilon_{2t-1} - \epsilon_{1t-1}) & -a_1\Sigma_{s=2}^{t-1}(1-a_1)^{s-1}(\epsilon_{2t-s} - \epsilon_{1t-s}) & + & \epsilon_{2t} \\ \Delta x_{3t} = & -a_1(\epsilon_{3t-1} - \epsilon_{1t-1}) & -a_1\Sigma_{s=2}^{t-1}(1-a_1)^{s-1}(\epsilon_{3t-s} - \epsilon_{1t-s}) & + & \epsilon_{3t} \\ & ... & & & \\ \Delta x_{Nt} = & -a_1(\epsilon_{Nt-1} - \epsilon_{1t-1}) & -a_1\Sigma_{s=2}^{t-1}(1-a_1)^{s-1}(\epsilon_{Nt-s} - \epsilon_{1t-s}) & + & \epsilon_{Nt} \end{array}$$

By changing the parameter a_1 we change the speed of the adjustment of the series to the long run level. In the Monte Carlo exercise we consider two values of a_1: $a_1 = 0.5$ and $a_1 = 1$.

For the data generating process 2 (DGP 2) we choose

$$\alpha = \begin{pmatrix} 0 & 0 & 0 & ... & 0 \\ -1 & 0 & 0 & ... & 0 \\ -a_2 & -1 & 0 & ... & 0 \\ -a_2 & 0 & -1 & ... & 0 \\ . & . & . & ... & 0 \\ -a_2 & 0 & 0 & ... & -1 \end{pmatrix}$$

where $a_2 \in (0, 1]$. There is one common cointegration relation and $N-2$ idiosyncratic cointegration relations.

The error-correction representation of the DGP 2 is

$$\begin{array}{lcrcrcl} \Delta x_{1t} & = & & & & & \epsilon_{1t} \\ \Delta x_{2t} & = & & - & (x_{2t-1} - x_{1t-1}) & + & \epsilon_{2t} \\ \Delta x_{3t} & = & -a_2(x_{2t-1} - x_{1t-1}) & - & (x_{3t-1} - x_{1t-1}) & + & \epsilon_{3t} \\ & ... & & & & & \\ \Delta x_{Nt} & = & -a_2(x_{2t-1} - x_{1t-1}) & - & (x_{Nt-1} - x_{1t-1}) & + & \epsilon_{Nt} \end{array}$$

The stochastic trend representation can be written as

$$\begin{array}{lclllcl} x_{1t} & = & \Sigma_{s=1}^{t-1}\epsilon_{1t-s} & & & + & \epsilon_{1t} \\ x_{2t} & = & \Sigma_{s=1}^{t-1}\epsilon_{1t-s} & & & + & \epsilon_{2t} \\ x_{3t} & = & \Sigma_{s=1}^{t-1}\epsilon_{1t-s} & +a_2\epsilon_{1t-1} & -a_2\epsilon_{2t-1} & + & \epsilon_{3t} \\ & ... & & & & & \\ x_{Nt} & = & \Sigma_{s=1}^{t-1}\epsilon_{1t-s} & +a_2\epsilon_{1t-1} & -a_2\epsilon_{Nt-1} & + & \epsilon_{2t} \end{array}$$

Each variable starting from x_3, can be decomposed into one common stochastic trend, one common cycle, and an autocorrelated idiosyncratic component.

The moving average representation for the differenced variables is

$$
\begin{array}{lclllcl}
\Delta x_{1t} & = & & & & & \epsilon_{1t} \\
\Delta x_{2t} & = & -a_2(\epsilon_{2t-1}-\epsilon_{1t-1}) & & & + & \epsilon_{2t} \\
\Delta x_{3t} & = & -a_2(\epsilon_{2t-1}-\epsilon_{1t-1}) & -a_2(\epsilon_{2t-2}-\epsilon_{1t-2}) & +\epsilon_{1t-1}-\epsilon_{3t-1} & + & \epsilon_{3t} \\
 & ... & & & & & \\
\Delta x_{Nt} & = & -a_2(\epsilon_{2t-1}-\epsilon_{1t-1}) & -a_2(\epsilon_{2t-2}-\epsilon_{1t-2}) & +\epsilon_{1t-1}-\epsilon_{Nt-1} & + & \epsilon_{Nt}
\end{array}
$$

Changing the parameter a_2 we change the speed of adjustment to the common cointegration relation. In the Monte Carlo exercise we consider two values: $a_2 = 0.5$ and $a_2 = 1$.

Assume that we are interested in the dynamics of the sub-vector $Y_t = (x_{2t}, x_{3t})'$. For all data generating processes the sub-vector Y_t has one cointegration vector:

$$\beta_Y' = \begin{pmatrix} -1 & 1 \end{pmatrix}$$

The vector $(X_{1t}, Y_t')' = (X_{1t}, X_{2t}, X_{3t})'$ has two linear independent cointegration vectors:

$$\gamma_Y' = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

We evaluate the relative performance of four models of the dynamics of the sub-vector $(x_{2t}, x_{3t})'$: the baseline VAR, the factor-augmented VAR (FVAR), the VECM, and the factor-augmented ECM (FECM). Their in-sample performance is compared on the basis of the relative squared standard errors in the equation for x_3, and their forecasting performance is compared on the basis of the relative mean squared forecast error (MSFE) for x_3.

The covariance matrix Σ is parameterized as follows:

$$\Sigma = \begin{pmatrix} \sigma_1^2 & 0 & 0 & ... & 0 \\ 0 & \sigma_2^2 & 0 & ... & 0 \\ 0 & 0 & \sigma_2^2 & ... & 0 \\ . & . & . & ... & . \\ 0 & 0 & 0 & ... & \sigma_2^2 \end{pmatrix},$$

where σ_1^2 and σ_2^2 are chosen in such a way that the variance of the differenced common factor Δx_{1t} explains 80% of the variance of the differenced series Δx_{it} $(i = 2, 3, ..., N)$, as in the empirical exercise common factors explain more than 80% of the variance of differenced series.

Six combinations of the time dimension T and space dimension N are considered. It allows us to evaluate possible changes in the relative performance of the models when T changes keeping N fixed and N changes keeping T fixed.

3.2 Forecasting Models

Vector Autoregressive Model, VAR:

$$\Delta Y_t = \mu + \Sigma_{s=1}^{p} \Phi_{ys} \Delta Y_{t-s} + \varepsilon_{1t},$$

where $Y_t = (x_{2t}, x_{3t})'$.

Factor-Augmented Vector Autoregressive Model, FVAR:

$$\begin{pmatrix} \Delta Y_t \\ \Delta F_t \end{pmatrix} = \mu + \Sigma_{s=1}^{p} \begin{pmatrix} \Phi_{ys} & \Phi_{fs} \end{pmatrix} \begin{pmatrix} \Delta Y_{t-s} \\ \Delta F_{t-s} \end{pmatrix} + \varepsilon_t,$$

where F_t is a vector of factors extracted from the covariance matrix of X_t.

Vector Error-Correction Model (VECM):

$$\Delta Y_t = \mu + \alpha\beta' Y_{t-1} + \Sigma_{s=1}^{p} \Phi_{ys} \Delta Y_{t-s} + \varepsilon_t,$$

where β is a (2×1) cointegration vector and α is a (2×1) vector of adjustment coefficients.

Factor - Augmented (Vector) Error-Correction Model (FECM):

$$\begin{pmatrix} \Delta Y_t \\ \Delta F_t \end{pmatrix} = \mu + \delta\gamma' \begin{pmatrix} Y_{t-1} \\ F_{t-1} \end{pmatrix} + \Sigma_{s=1}^{p} \begin{pmatrix} \Phi_{ys} & \Phi_{fs} \end{pmatrix} \begin{pmatrix} \Delta Y_{t-s} \\ \Delta F_{t-s} \end{pmatrix} + \varepsilon_t,$$

where γ is a matrix of cointegration vectors, and δ is a matrix of adjustment coefficients.

3.3 Results of Experiment

Figures 1 and 2 (Appendix 2) show the dynamics of the true common trend, x_{1t}, and the fitted common trend obtained by regressing x_{1t} on its principal component estimate, for DGP 1 and DGP 2 accordingly (T=100, N=100). In the case of DGP 1 the fitted common trend follows closely the dynamics of x_{1t}, while for the DGP 2 the fitted values deviate significantly from the true stochastic trend. This deviations are conditioned by the noise component in the stochastic trends representation of the DGP 2 due to the presence of the common cointegration relation in this process.

Tables 1-8 (Appendix 3) show the results of the simulation exercise. The measure of the in-sample fit is the squared standard errors and the measure of the forecast accuracy is the mean squared forecast errors.

For DGP 1 factor-augmented models have smaller standard errors in-sample than the models with no factors included, although gains are not large. The FECM has the smallest standard errors for all combinations of time and cross-section dimensions. For $a_1 = 0.5$ the relative in-sample performance of the factor-augmented models improves comparing with $a_1 = 1$.

The relative forecasting performance of the factor-augmented models is not as good as their in-sample fit. Both for $a_1 = 0.5$ and $a_1 = 1$ factor-augmented models do not perform better than models with no factors included. For $a_1 = 1$ the baseline VAR model is the best for all combinations of time and cross-section dimensions. For $a_1 = 1$ the VAR is outperformed both by the VECM and the FECM. However, the forecasts produced by VECM are better than the forecasts produced by FECM for all combinations of time and cross-section dimensions.

The relative forecasting performance of the FECM improves as the cross-section dimension N grows given fixed time dimension T. This result supports the Proposition 1 in Bai (2004) which states that the estimated common stochastic trends are uniformly consistent when the cross-section dimension N is sufficiently large relative to the time dimension T. Increasing N we obtain more precise estimates of factors which are used in the FECM.

For DGP 2 as for DGP 1 the factor-augmented models perform better in-sample with the FECM having the smallest standard errors for all time and cross-section dimensions. However, the forecasting performance of the factor-augmented models is worse than the forecasting performance of the models with no factors. Both for $a_2 = 0.5$ and $a_2 = 1$ the best forecasting model is the baseline VAR. The relative forecasting performance of VECM and FECM improves for $a_2 = 0.5$ relative to $a_2 = 1$. The performance of the FECM improves with increase of the cross-section dimension of the data.

Overall, the factor-augmented models perform better than the models with no factors in-sample. However, out-of-sample VAR and VECM perform better than FVAR and FECM. The relative forecasting performance of the FECM improves slowly with an increase of the cross-section dimension of the process.

One caution is necessary. The results obtained for two simple types of data generating processes. Another results may be found for more general data generating processes with more complex relations between variables.

4 Empirical Example: Interest Rate Pass-Through in the Euro Area

The relative performance of the factor-augmented models is also evaluated in the forecasting exercise for bank retail rates in the euro-area based on the interest rate pass-through model.

The size and the speed of adjustment of bank retail rates to money market rates is important for the success of monetary policy. Central Banks set their official rates and steer money market interest rates. Changes in money market rates, in turn, affect bank retail rates, albeit to varying degrees. The efficacy of monetary policy depends on how much and how fast bank retail rates adjust to changes in money market rates.

Interest rate pass-through is particularly important for euro-area countries. The introduction of common monetary policy in the euro area in January 1999 may have affected bank behavior and interest rate pass-through, since the monetary policy of the European Central Bank reacts to conditions at the euro are and not to country-specific developments. Therefore, the interest rate pass-through for individual countries and the euro area as whole has become a subject of a number of empirical studies recently (de Bondt, 2005, Sørensen and Werner, 2006, and Flad, 2006).

Sørensen and Werner (2006) evaluate interest rate pass-through for ten euro-area countries using country-specific data in a panel error-correction model. De Bondt (2005) evaluates interest rate pass-through for the whole euro area using aggregated data in a univariate ECM, a VAR, and a VECM. Flad (2000) applies PANIC analysis and structural factor models to measure the transmission of the monetary policy shocks in the euro area and other EU countries. However, these studies do not evaluate the forecasting performance of the models under consideration.

Sørensen and Werner (2006), and De Bondt (2005) choose a market rate corresponding to a given bank retail rate on the basis of the correlation analysis. The chosen market rate is the only one of the range of rates of comparable maturity which could be used to model interest rate pass-through. Using only one rate of the band of rates which can represent the marginal costs of lending or borrowing, may lead to the problem of omitted variables. Flad (2006) identifies the money market stance by the factors extracted from the money market interest rates. The factors can also be used to augment the error-correction models measuring long-run pass-through.

In this paper we perform the forecasting exercise for four bank lending rates in the euro area using two empirical interest rate pass-through models: VAR and VECM. We augment these models by the factors extracted from the money market rates in the euro area, the United Kingdom, and the United States. Then we compare the forecasting performance of the VAR, VECM and the factor- augmented VAR and VECM.

Changes in the pound and the dollar market rates approximate marginal pricing costs for the European banks, since they are the market-based sources for banks to attract deposits or loans as well as the changes in the euro market rates. However, differences in the currency of denomination requires an additional assumption that uncovered interest rate parity holds.

4.1 Data Set and Factors

The data set which is used to estimate the common stochastic trends of the market interest rates, includes 15 euro rates, 15 pound rates, and 14 dollar rates of a total of 44 interest rates. These are monthly data from January 1999 to February 2007. They are taken from the website of the European Central Bank, the Bank of England, and the Datastream.

Factors are extracted from the levels of interest rates. The IPC1 and IPC2 criteria proposed by Bai (2004) for the selection of the number of common trends, suggest four and three factors accordingly.

Given the differences in the number of factors suggested by different criteria, the PANIC tests $MQ^c_{c,f}$ and $MQ^\tau_{c,f}$ as proposed by Bai and Ng (2004), were also performed. The results of the testing suggest four factors in the case of the $MQ^c_{c,f}$ test and six factors in the case of the $MQ^\tau_{c,f}$ test.

Since the results, suggested by information criteria and testing, are not coherent, the number of factors is chosen to be equal to four using an informal criteria proposed by Forni et al (2000). This criteria suggests choosing only those factors which explain more than 5% of the variation in series. The fourth factor explains 8.5% of the variance of the differenced series with the total share of the variance explained by the four factors amounting to 82%. Any additional factor explains less than 5% of the variation in the series. The dynamics of the four extracted factors is shown at Figure 3, Appendix 2.

Four bank lending rates are considered. A corresponding market rate is selected for each of these bank rates. Selection is based on the correlation between the bank rates and the market rates of comparable maturity. The bank rates and the corresponding market rates are listed in the Table 1 below.

Table 1

Bank Retail Rates	**Market Rates**
Loans to Enterprises up to 1 year	3 Month EURIBOR
Loans to Enterprises over 1 year	3 Year Euro Bond Rate
Consumer Loans	1 Year EURIBOR
Mortgage Loans	7 Year Euro Bond Rate

The dynamics of the bank retail rates is shown at Figure 4, Appendix 2. This figure provides an evidence of the structural break in November 2005 for three bank rates: the rates on short term and long term loans to enterprises and the rate on mortgage loans. This break within forecasting period can affect the results of the forecasting exercise.

4.2 Methodology

Four models are evaluated. These are the same as those that were evaluated in the Monte Carlo exercise. We set $Y_t = (mr_t, bt_t)'$, where br_t is a bank retail rate and mr_t is the corresponding market rate.

The in-sample performance of the models is evaluated on the basis of the relative squared standard errors of the equation explaining a bank retail rate. The VAR model is taken as a benchmark.

The forecasting performance of the models is evaluated on the basis of the relative root mean square forecast error for a bank retail rate. The forecasting is performed recursively using Stock and Watson (1998) methodology. Three forecasting horizons are considered: 1, 3, and 6 months. The first forecast produced in February 2004.

The last forecast value is evaluated for February 2007. It implies 36 forecasts evaluated for 1-month horizon, 34 forecasts for 3-month horizon, and 31 forecasts for 6-month horizon.
The presence of structural breaks in the bank retail rates over the considered period could lead to the forecast failure. In order to deal with this problem, the intercept corrected forecasts (Clements and Hendry, 1999) are evaluated for all models.

4.3 Results of Empirical Exercise

The results of the performed exercise are represented in Tables 9-12 (Appendix 3). Most of the models have smaller standard errors than the baseline VAR model in-sample. However, the gains are not large. The FECM model is the best in-sample model for the rates on long-term loans to enterprises, consumer loans, and mortgage loans. The VECM is the best in-sample model for the rate on short-term loans to enterprizes.

Since the number of observations over which the forecast evaluation is performed is quite small, the standard errors of the estimated relative forecast errors are large. For this reason there are very few cases when we can talk about significant differences in the forecasting performance of the models.

For the one-month horizon, most of the models do not outperform the benchmark VAR model. For the rates on loans to enterprizes the FVAR forecast appears to be the best. For the rate on consumer loans the VECM has the lowest relative forecast error. Factor-augmented forecasts do not outperform models with no factors included in most of cases.

Turning to the three-month horizon, there are a few forecasts outperforming the benchmark for all bank lending rates. The intercept correction improves the performance of all models relative to the benchmark for the rates on loans to enterprises and the rate on mortgage loans. The best forecast for the rate on short-term loans to enterprises is the intercept-corrected VECM, the best forecast for the rate on long-term loans to enterprises is the intercept-corrected VAR, and the best forecast for the rate on mortgage loans is the intercept-corrected FVAR. However, the intercept correction is inefficient for the rate on consumer loans. The FECM forecast is the best for this rate. For the three-month horizon the FECM outperforms the VECM in three out of eight cases including intercept-corrected models.

For the six-month horizon, the FECM outperforms the VECM in four out of eight cases. The intercept-corrected FECM is the best model for the rate on short-term loans to enterprises for the six-month horizon. As in the case of the three-month forecast the intercept-correction is efficient for all bank rates but the rate on consumer loans. The intercept corrected FVAR is the best forecast for the rate on long-term loans to enterprises and the intercept-corrected VAR model is the best for the rate on mortgage loans.

Significant gains in the forecasting all bank rates but the rate on the consumer loans are provided by the intercept correction for 3 and 6-month forecasting horizons. However, the intercept correction provide no gains for 1-month forecasts. These results can be explained by the presence of the structural break in the interest rates on loans to enterprises and mortgage loans over forecasting period, while there is no such break in the rate on consumer loans.

The efficacy of the intercept correction for the long forecasting horizons is conditioned by the methodology: projection method used in the forecasting, increases the size of the structural break for the long horizons and makes the intercept correction more efficient. When the intercept correction is efficient, the corrected benchmark model is often outperformed by other models.

The in-sample performance of the models and the forecasting results for 1-month horizon are very much in-line with the results of the Monte Carlo exercise. Both in the empirical application and in the Monte Carlo exercise factor-augmented models provide better in-sample fit than models with no factors. However, factor-augmented forecasts do not outperform forecasts produced by the VAR and the VECM.

Some gains provided by the FVAR and FECM can be explained by the better performance of the factor-augmented models for longer forecasting horizons and in the presence of structural breaks. However, this requires further investigation in Monte Carlo exercises and other empirical applications.

5 Conclusions

In this paper the performance of the factor-augmented error-correction models is evaluated in the Monte Carlo exercise and the empirical application. The hypothesis considered is that the inclusion of the common stochastic trends extracted from the large data set can improve the in-sample and the out-of-sample performance of the small-scale ECM.

It is found both in the Monte Carlo exercise and the empirical application that although in-sample the FECM performs well relative to the models which are not augmented by factors, the forecasting performance of the FECM is no better than the forecasting performance of other models for the one-period horizon. One possible explanation of this evidence can be that the estimated common stochastic trends approximate the long run dynamics of the variables while the short-term forecasting requires information about common stationary cycles.

Another reason of the relatively bad forecasting performance of the FECM can be estimation uncertainty induced by the estimation of factors. This problem can be resolved by the increasing the cross-section dimension of the data.

The results of the empirical exercise indicate that the forecasting performance of the FECM may improve for the longer horizons. This requires further investigation in Monte Carlo exercises and empirical applications.

Appendix 1. Data Description

No	Mnemonic	Description
euro rates		
1.	eonia	eonia
2.	eu1m	1 month euribor
3.	eu3m	3 month euribor
4.	eu6m	6 month euribor
5.	eu1y	1 year euribor
6.	eu2y	2 year bond rate
7.	eu3y	3 year bond rate
8.	eu5y	5 year bond rate
9.	eu7y	7 year bond rate
10.	eu10y	10 year bond rate
11.	eusw2y	euro vs euribor swap rate, 2 years
12.	eusw3y	euro vs euribor swap rate, 3 years
13.	eusw5y	euro vs euribor swap rate, 5 years
14.	eusw7y	euro vs euribor swap rate, 7 years
15.	eusw10y	euro vs euribor swap rate, 10 years
uk rates		
16.	sonia	uk sonia
17.	uk1w	1 week libor
18.	uk1m	1 month libor
19.	uk3m	3 month libor
20.	uk6m	6 month libor
21.	uk1y	1 year libor
22.	uk5y	zero coupon yield, 5 years
23.	uk10y	zero coupon yield, 7 years
24.	uk10y	zero coupon yield, 10 years
25.	uk20y	zero coupon yield, 20 years
26.	uksw2y	uk swap, 2 years
27.	uksw3y	uk swap, 3 years
28.	uksw5y	uk swap, 5 years
29.	uksw7y	uk swap, 7 years
30.	uksw10y	uk swap, 10years

us rates

31.	us3m	us treasury bill, 3 months
32.	us6m	us treasury bill, 6 months
33.	us1y	us treasury bill, 1 year
34.	us2y	us treasury bill, 2 years
35.	us3y	us treasury bill, 3 years
36.	us5y	us treasury bill, 5 years
37.	us7y	us treasury bill, 7 years
38.	us10y	us treasury bill, 10 years
39.	us20y	us treasury bill, 20 years
40.	usw2y	us swap, 2 years
41.	usw3y	us swap, 3 years
42.	usw5y	us swap, 5 years
43.	usw7y	us swap, 7 years
44.	us10y	us swap, 10 years

Appendix 2. Figures

FIGURE 1. DGP 1: TRUE FACTOR AND FITTED FACTOR

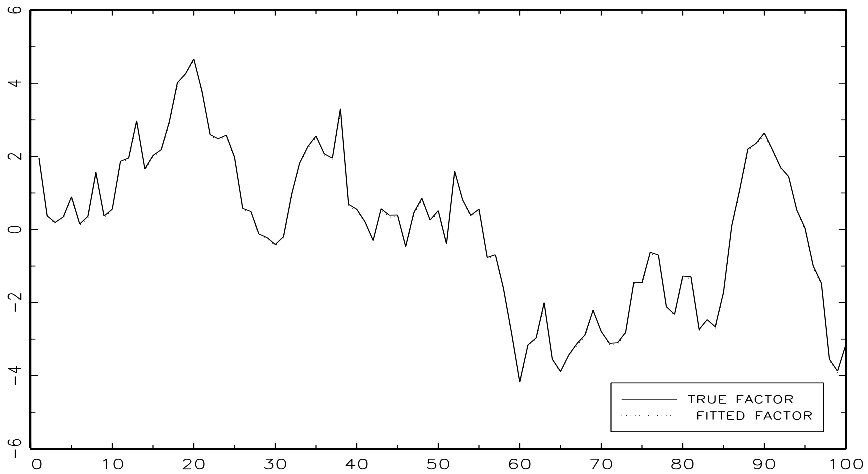


FIGURE 2. DGP 2: TRUE FACTOR AND FITTED FACTOR

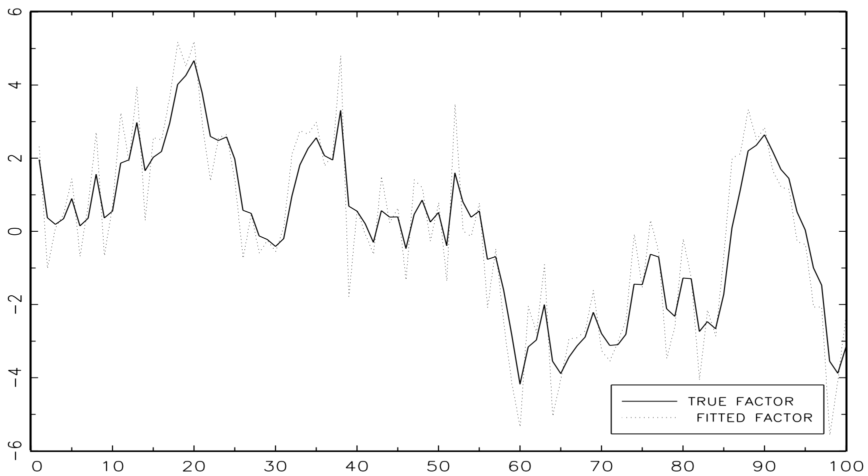


FIGURE 3. EMPIRICAL EXERCISE: FACTORS DYNAMICS

FIGURE 4. BANK RETAIL RATES

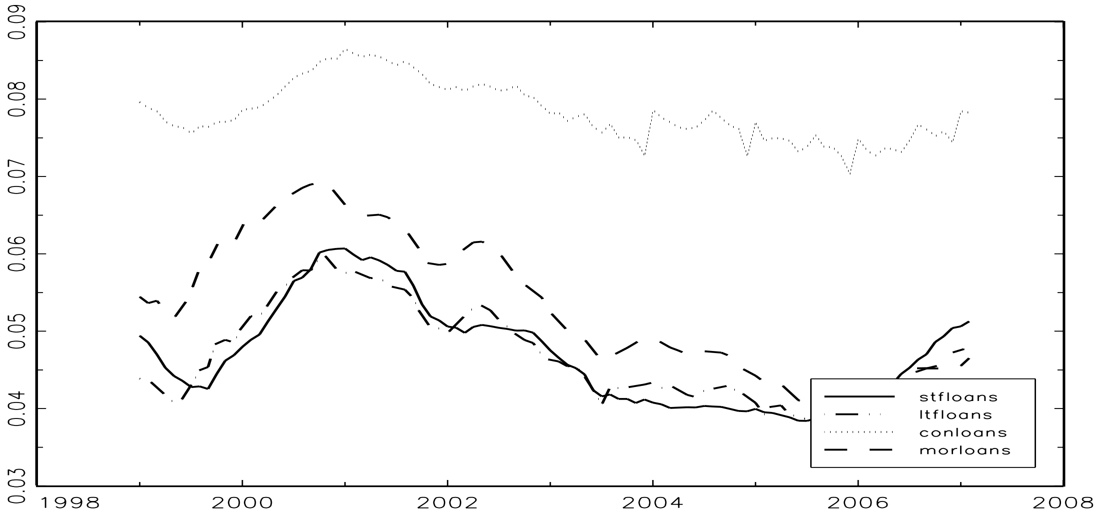
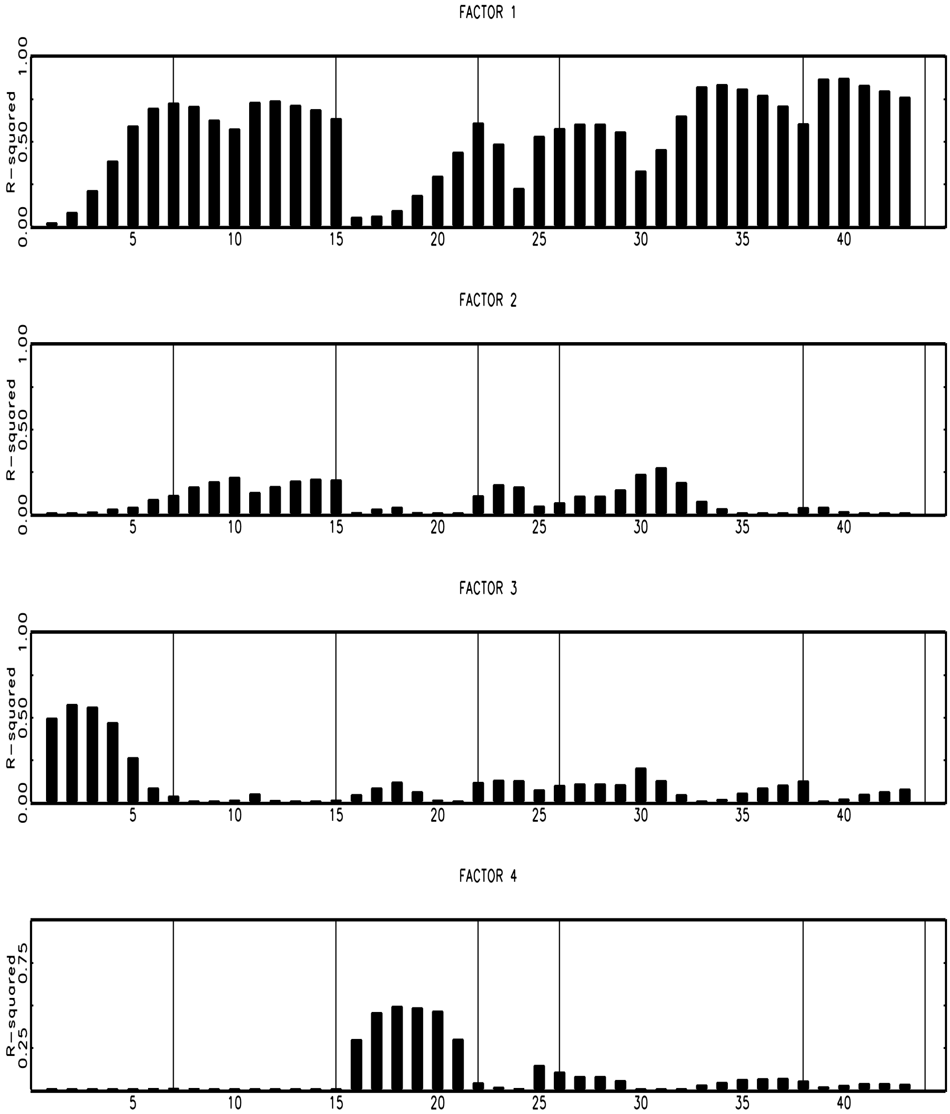


FIGURE 5. EMPIRICAL EXERCISE: FACTOR LOADINGS



Appendix 3. Tables

Table 1. Monte Carlo Experiment:
DGP 1, $a_1 = 1$, in-sample performance

Parameters		Relative SE			
T	N	VAR	FVAR	ECM	FECM
50	50	1.00 (.)	0.96 (0.21)	0.98 (0.22)	0.92 (0.23)
50	100	1.00 (.)	0.97 (0.20)	0.98 (0.22)	0.94 (0.22)
50	200	1.00 (.)	0.98 (0.21)	0.99 (0.14)	0.96 (0.22)
100	100	1.00 (.)	0.97 (0.14)	0.99 (0.15)	0.94 (0.14)
100	200	1.00 (.)	0.98 (0.15)	0.99 (0.15)	0.94 (0.15)
100	300	1.00 (.)	0.96 (0.14)	0.99 (0.15)	0.91 (0.15)

Table 2. Monte Carlo Experiment:
DGP 1, Forecasting Results, $a_1 = 1$, 1 period forecasts

Parameters		Relative MSFE			
T	N	VAR	FVAR	ECM	FECM
50	50	1.00 (.)	1.04 (1.43)	1.27 (2.15)	1.55 (3.39)
50	100	1.00 (.)	1.07 (1.52)	1.30 (2.24)	1.53 (2.75)
50	200	1.00 (.)	1.05 (1.46)	1.21 (1.90)	1.43 (2.30)
100	100	1.00 (.)	1.00 (1.44)	1.25 (1.90)	1.62 (2.91)
100	200	1.00 (.)	1.02 (1.38)	1.31 (2.35)	1.56 (2.53)
100	300	1.00 (.)	1.05 (1.44)	1.25 (2.02)	1.56 (2.40)

Table 3. Monte Carlo Experiment:
DGP 1, $a_1 = 0.5$, in-sample performance

Parameters			Relative SE		
T	N	VAR	FVAR	ECM	FECM
50	50	1.00 (.)	0.90 (0.20)	0.97 (0.23)	0.83 (0.21)
50	100	1.00 (.)	0.93 (0.19)	0.97 (0.22)	0.87 (0.22)
50	200	1.00 (.)	0.94 (0.13)	0.97 (0.22)	0.89 (0.22)
100	100	1.00 (.)	0.93 (0.14)	0.98 (0.14)	0.87 (0.13)
100	200	1.00 (.)	0.93 (0.14)	0.98 (0.15)	0.88 (0.14)
100	300	1.00 (.)	0.94 (0.14)	0.97 (0.15)	0.88 (0.13)

Table 4. Monte Carlo Experiment:
DGP 1, Forecasting Results, $a_1 = 0.5$, 1-step forecasts

Parameters			Relative MSFE		
T	N	VAR	FVAR	ECM	FECM
50	50	1.00 (.)	1.02 (1.35)	0.80 (1.40)	0.96 (1.52)
50	100	1.00 (.)	1.06 (1.53)	0.76 (1.19)	0.93 (1.46)
50	200	1.00 (.)	1.05 (1.47)	0.71 (1.08)	0.86 (1.22)
100	100	1.00 (.)	1.00 (1.42)	0.55 (0.90)	0.73 (1.14)
100	200	1.00 (.)	1.02 (1.44)	0.59 (1.01)	0.72 (1.14)
100	300	1.00 (.)	1.06 (1.50)	0.57 (0.90)	0.68 (0.93)

Table 5. Monte Carlo Experiment:
DGP 2, $a_2 = 1$, in-sample performance

Parameters		Relative SE			
T	N	VAR	FVAR	ECM	FECM
50	50	1.00 (.)	0.96 (0.21)	0.97 (0.23)	0.91 (0.22)
50	100	1.00 (.)	0.97 (0.20)	0.96 (0.22)	0.92 (0.21)
50	200	1.00 (.)	0.97 (0.20)	0.98 (0.22)	0.93 (0.22)
100	100	1.00 (.)	0.97 (0.14)	0.99 (0.14)	0.93 (0.13)
100	200	1.00 (.)	0.97 (0.14)	0.98 (0.15)	0.93 (0.14)
100	300	1.00 (.)	0.97 (0.14)	0.98 (0.15)	0.93 (0.14)

Table 6. Monte Carlo Experiment:
DGP 2, Forecasting Results, $a_2 = 1$, 1-step forecasts

Parameters		Relative MSFE			
T	N	VAR	FVAR	ECM	FECM
50	50	1.00 (.)	1.02 (1.42)	2.03 (3.26)	2.19 (3.52)
50	100	1.00 (.)	1.04 (1.54)	1.91 (2.73)	2.11 (3.08)
50	200	1.00 (.)	1.04 (1.46)	1.89 (2.72)	2.05 (3.07)
100	100	1.00 (.)	1.01 (1.44)	1.82 (2.68)	2.01 (2.99)
100	200	1.00 (.)	1.01 (1.48)	1.77 (2.60)	1.98 (2.92)
100	300	1.00 (.)	1.02 (1.40)	1.78 (2.47)	1.97 (2.74)

Table 7. Monte Carlo Experiment:
DGP 2, $a_2 = 0.5$, in-sample performance

Parameters			**Relative SE**		
T	N	VAR	FVAR	ECM	FECM
50	50	1.00 (.)	0.96 (0.20)	0.98 (0.23)	0.91 (0.22)
50	100	1.00 (.)	0.97 (0.20)	0.98 (0.22)	0.93 (0.22)
50	200	1.00 (.)	0.97 (0.22)	0.98 (0.22)	0.94 (0.23)
100	100	1.00 (.)	0.97 (0.14)	0.98 (0.14)	0.92 (0.14)
100	200	1.00 (.)	0.97 (0.14)	0.98 (0.15)	0.92 (0.14)
100	300	1.00 (.)	0.97 (0.14)	0.98 (0.15)	0.92 (0.14)

Table 8. Monte Carlo Experiment:
DGP 1, Forecasting Results, $a_2 = 0.5$, 1-step forecasts

Parameters			**Relative MSFE**		
T	N	VAR	FVAR	ECM	FECM
50	50	1.00 (.)	1.04 (1.54)	1.70 (3.04)	2.11 (3.70)
50	100	1.00 (.)	1.05 (1.49)	1.55 (2.32)	1.83 (1.41)
50	200	1.00 (.)	1.03 (1.53)	1.61 (2.51)	1.80 (1.94)
100	100	1.00 (.)	1.04 (1.60)	1.70 (2.62)	2.05 (2.97)
100	200	1.00 (.)	1.04 (1.49)	1.73 (2.57)	1.99 (2.94)
100	300	1.00 (.)	1.05 (1.59)	1.64 (2.65)	1.92 (3.13)

Table 9. Empirical Exercise: Relative Standard Errors

Model	Relative Standard Errors: Loans to Enterprizes Short-term	Loans to Enterprizes Long-term	Consumer Loans	Mortgage Loans
VAR	1.00	1.00	1.00	1.00
FVAR	0.95	1.02	0.97	0.97
VECM	0.90	0.89	0.82	0.98
FECM	1.03	0.81	0.78	0.94
SE for VAR	0.003	0.009	0.017	0.005

Table 10. Empirical Exercise: Forecasting Results, 1-month forecasts

Model	Relative MSFE: Loans to Enterprizes Short-term	Loans to Enterprizes Long-term	Consumer Loans	Mortgage Loans
VAR	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
FVAR	0.92 (0.12)	0.99 (0.03)	1.03 (0.06)	1.13 (0.36)
VECM	1.09 (0.18)	1.17 (0.22)	0.85 (0.27)	1.28 (0.22)
FECM	1.41 (0.31)	1.06 (0.13)	0.90 (0.21)	2.92 (1.36)
VAR IC	2.27 (0.95)	2.13 (1.07)	2.55 (1.96)	2.18 (0.99)
FVAR IC	2.23 (1.08)	2.19 (1.09)	2.77 (2.15)	1.58 (0.56)
VECM IC	1.76 (0.55)	1.68 (0.56)	1.47 (0.37)	2.79 (1.57)
FECM IC	1.25 (0.24)	2.07 (0.82)	1.85 (0.88)	2.80 (1.28)
RMSE for VAR	0.035	0.078	0.174	0.049

Table 11. Empirical Exercise:
Forecasting Results, 3-month forecasts

Model	**Relative MSFE** Loans to Enterprizes Short-term	Loans to Enterprizes Long-term	Consumer Loans	Mortgage Loans
VAR	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
FVAR	1.25 (0.25)	1.03 (0.04)	0.97 (0.10)	1.23 (0.23)
VECM	0.92 (0.39)	2.48 (2.24)	1.08 (0.26)	1.06 (0.04)
FECM	2.22 (1.00)	1.17 (0.25)	0.88 (0.19)	5.09 (7.98)
VAR IC	0.71 (0.24)	0.94 (0.17)	1.64 (0.45)	0.99 (0.25)
FVAR IC	1.23 (0.33)	0.97 (0.17)	1.54 (0.55)	0.82 (0.21)
VECM IC	0.31 (0.27)	1.64 (0.68)	1.41 (0.40)	0.98 (0.26)
FECM IC	0.40 (0.26)	1.09 (0.47)	1.55 (0.54)	1.83 (0.65)
RMSE for VAR	0.092	0.122	0.195	0.102

Table 12. Empirical Exercise:
Forecasting Results, 6-month forecasts

Model	**Relative MSFE** Loans to Enterprizes Short-term	Loans to Enterprizes Long-term	Consumer Loans	Mortgage Loans
VAR	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
FVAR	1.10 (0.18)	1.35 (0.42)	1.09 (0.10)	1.16 (0.16)
VECM	0.62 (0.45)	1.61 (1.04)	1.77 (0.54)	2.91 (3.37)
FECM	2.33 (1.72)	1.78 (1.07)	1.25 (0.30)	4.37 (7.31)
VAR IC	0.31 (0.38)	0.41 (0.19)	1.43 (0.28)	0.41 (0.16)
FVAR IC	0.63 (0.41)	0.18 (0.17)	1.27 (0.20)	0.26 (0.15)
VECM IC	0.20 (0.41)	1.41 (0.75)	1.87 (0.37)	0.72 (0.19)
FECM IC	0.13 (0.41)	1.14 (0.71)	1.84 (0.50)	2.43 (2.77)
RMSE for VAR	0.232	0.220	0.196	0.239

Conclusions

In this work we investigate the relative performance of dynamic factor models in finite samples and in the presence of structural changes. We extend an existing literature by the exploration of new data sets and the evaluation of finite sample properties of factor models in the simulation exercises.

In Chapter 1 we evaluate the relative performance of factor forecasts in the empirical exercise conducted for Brazil and Russia. This exercise does not provide a uniform evidence of the better performance of any considered model. However, it allows us to suggest that the relative performance of factor models depends on the presence of structural breaks, their timing, and their size.

These properties of data are explored in the simulation exercise in Chapter 2. In particular, the timing and sources of structural breaks are the subject of investigation. We consider deterministic breaks in mean of factors and in intercept of the forecasting equation and find that for both types of break, factor forecasts perform well, if there is a break in the middle of the estimation period. However, if there is a break in the end of estimation period, estimated factor models fail to accommodator it and perform worse than other methods.

In-sample and out-of-sample performance of factor-augmented error correction models (FECM) is the subject of investigation in Chapter 3. The performance of FECMs is explored both in the simulation exercise and in the application to the interest rate pass-through in the euro area. Although it is found that FECMs perform well in-sample, their forecasting performance is no better than the forecasting performance of other methods in most cases. As the found evidence is still very limited, another exercise with more complex data-generating processes and a new set of real data may be instructive.

References

Artis, M. J., A. Banerjee, and M. Marcellino (2004), "Factor Forecasts for the UK", Journal of Forecasting, 24, 279-98.

Bai, J., 2004, Estimating Cross-Section Common Stochastic Trends in Non-Stationary Panel Data, Journal of Econometrics 122, 137-183.

Bai, J. and S. Ng (2002), Determining the Number of Factors in Approximate Factor Models, Econometrica, 70, 191221.

Bai, J. and S. Ng (2004), A PANIC Attack on Unit Roots and Cointegration, Econometrica, 72, 1127 - 1177.

Bai, J. and S. Ng (2006), Confidence Intervals for Diffusion Index Forecasts and Inference for Factor-Augmented Regressions, Econometrica, 74, 1133 - 1150.

Banerjee, A. and M. Marcellino (2007) Dynamic Factor Error Correction Model and Factor Augmented Error Correction Models, mimeo.

Banerjee, A., M. Marcellino, and I. Masten (2006), "Forecasting Macroeconomic Variables for the New Member States", in M.J. Artis, A. Banerjee and M. Marcellino (eds.) The Central and Eastern European Countries and the European Union, Cambridge University Press, Cambridge, Chapter 4, 108-134.

Banerjee, A., M. Marcellino, and I. Masten (2007), "Forecasting Macroeconomic Variables Using Diffusion Indexes in Short Samples with Structural Change, mimeo

Bernanke, B. and Boivin (2003), Monetary Policy in Data-Rich Environment, Journal of Monetary Economics, 50, 525-546.

Bernanke, B., J. Boivin, and P. Eliasz, 2004, A Factor - Augmented Vector Autoregressive (FAVAR) Approach, NBER Working Paper No. 10220

Bogdanski, J., A. A. Tombini, and S. R. Werlang (2000) "Implementing Inflation Targeting in Brazil", Central Bank of Brazil, Working Paper.

The Central Bank of the Russian Federation (2002), Monetary Policy Guidelines for year 2003.

The Central Bank of the Russian Federation (2003), Monetary Policy Guidelines for year 2004.

Clements, M. P. and D. F. Hendry (1998), Forecasting Economic Time Series, Cambridge University Press, Cambridge.

Clements, M. P. and D. F. Hendry (1999), Forecasting Non-Stationary Economic Time Series, MIT Press, Cambridge, Massachusetts.

Favero, C. A. and M. Marcellino (2001), Large Data Sets, Small Models and Monetary Policy in Europe, CEPR Working Paper #208.

Flad, M. (2006), A PANIC-report on Monetary Policy Transmission and Financial Integration in Europe, mimeo

Hansson, J., Jansson, P. and Lf, M. (2003), "Business Survey Data: Do They Help in Forecasting the Macro Economy?," Working Paper Series 151, Sveriges Riksbank.

Hatch, N. (2001), Modelling and Forecasting at the Bank of England, in Understanding Economic Forecasts, edited by D. F. Hendry and N. R. Ericsson, MIT Press, Cambridge, Massachusetts.

Johansen (1995), Likelihood-Based Inference in Cointegrated Vector Autoregressive Models, Oxford University Press, Oxford

Jonas, J. and F. Mishkin (2003), Inflation Targeting in Transition Countries: Experience and Prospects, NBER Working Paper #9667.

Marcellino, M., Stock, J. H., and M. W. Watson (2003), Macroeconomic Forecasting in Euro Area: Country-Specific versus Euro Wide Information, European Economic Review, 47, 1-18.

Monch, E.(2005), Forecasting the Yield Curve in Data-Rich Environment: a Non Arbitrage Factor-Augmented VAR Approach, ECB WP 544

Stock, J. H. and M. W. Watson (1998), Diffusion Indexes, NBER Working Paper #6702.

Stock, J. H. and M. W. Watson (1999), Forecasting Inflation, Journal of Monetary Economics, 44, 293-335.

Stock, J. H. and M. W. Watson (2002), Macroeconomic Forecasting using Diffusion Indexes, Journal of Business and Economic Statistics, 20, 147-162.

Stock, J. H. and M. W. Watson (2005), Implication of Dynamic Factor Models for VAR Analysis, NBER WP 11467.

Stock, J. H. and M. W. Watson (2007), "Forecasting in Dynamic Factor Models Subject to Structural Instability", mimeo

Svensson, L. O. E. (1999), Inflation Targeting as Monetary Policy Rule, Journal of Monetary Economics, 43, 607-654.

Vickers, J. (1998), Inflation Targeting in Practice: The UK Experience, Bank of England Quarterly Bulletin, November 1998.

West, K. D. (1996), Asymptotic Inference about Predictive Ability, Econometrica, 64, 1067-1084

Woodford, M. (2003), Interest and Prices, Princeton University Press, Princeton.