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Predicting market power in wholesale electricity markets∗

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Abstract

The traditional measure of market power is the HHI, which gives implausible results given the low elasticity of demand in electricity spot markets, unless it is adapted to take account of contracting. In its place the Residual Supply Index has been proposed as a more suitable index to measure potential market power in electricity markets, notably in California and more recently in the EU Sector Inquiry. The paper investigates its value in identifying the ability of firms to raise prices in an electricity market with contracts and capacity constraints and find that it is most useful for the case of a single dominant supplier, or with a natural extension, for the case of a symmetric oligopoly. Estimates from the Sector Inquiry seem to fit this case better than might be expected, but suggests an alternative definition of the RSI defined over flexible output that should give a more reliable relationship.

Key words: Residual Supply Index, Cournot equilibrium, Lerner Index, electricity markets, market power

JEL classification D43, K21, L94

1 Introduction

Electricity wholesale markets in Europe are typically very concentrated, and in most Continental countries the two largest generation companies provide more than 50% of domestic supply (EC, 2006). Where internal transmission constraints restrict the number of generators that can compete to supply consumers in a particular area, levels of concentration can be even higher. Thus in each of the two separated parts of Denmark the Herfindahl Hirshman Index (HHI) exceeded

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5,000 in 2004 (EC, 2006). In Italy, which fragments into separate price zones if interzonal transmission links bind, Calabria had an HHI averaging just below 5,000 in the first two months of 2007, although other zones had lower values. The EU Sector Inquiry examined market power in the electricity and gas markets, publishing its findings in 2007. It found values for the HHI in Germany ranging from 1,795 to 2,665 (based on total generation). The values are rather less for in-merit generation capacity, but rather more allowing for exports over interconnectors (London Economics, 2007, ¶6.2). The values for HHI in The Netherlands ranged from 1,861 to 3,397 (based on total generation and ignoring interconnector capacity) or from 997 to 3,388, allowing the largest generator to obtain the maximum allowed capacity of 400 MW on the interconnector (London Economics, 2007, ¶8.2).

Electricity has a very low elasticity of demand, particularly in the short time periods over which spot markets clear. The standard Cournot oligopoly model that might seem suitable for describing electricity wholesale market equilibria when markets are tight has the well-known property that the Lerner Index (the proportional price-cost margin for a firm) is directly proportional to the market share of that firm and inversely proportional to the elasticity of demand. High market shares and low elasticities should therefore lead to very high price-cost mark-ups – considerably higher than are observed. We therefore have an apparent inconsistency between theory and evidence.

2 Modelling market power

In the simple Cournot oligopoly model, firm $i$ maximizes profit $\pi_i = pq_i - C_i(q_i)$, for which the f.o.c. is

$$\frac{d\pi_i}{dq_i} = 0 = p - C_i' = p \left( \frac{q_i}{Q} \right) \left( -\frac{Qdp}{pdQ} \right),$$

where demand, $Q = \sum q_j$, is a function of the price, $p$, and $C_i'(q_i)$ is the marginal cost of firm $i$. The Lerner Index, (LI) $L_i$, for firm $i$ (a standard measure of the ability to raise prices) is then given by

$$L_i \equiv \frac{p - C_i'}{p} = \frac{s_i}{\varepsilon},$$

where $s_i$ is the market share of the firm $q_i/Q$, and $\varepsilon$ is the elasticity of market demand, as a positive number. Estimates for the value of the short-run demand elasticity for electricity are quite low, and over periods of months possibly below 0.25 for the “domestic and other industry” sector, judging by the response to extraordinarily sharp price increases in Norway over the period November 2002 to May 2003 (von der Fehr, Amundsen and Bergman, 2005).

$^1$The HHI is defined as the sum of the squared market shares measured in percentages, with 10,000 corresponding to a monopoly, and 5,000 a symmetric duopoly.
The attractiveness of the HHI as a suitable measure of market power follows from various manipulations of the Cournot oligopoly solutions to (1) and (2). If there are constant returns to scale, \( C_i' = c_i \), then total industry profits \( \Pi \) are given by

\[
\Pi = \sum (p - c)q_i = \sum (pL_i)(s_iQ)
\]

\[
= \frac{pQ}{\varepsilon} (\sum s_i^2), \quad \text{and so from (2)}
\]

\[
\frac{\Pi}{pQ} = \frac{H}{\varepsilon} = \sum s_iL_i, \quad (3)
\]

thus the ratio of industry profits to revenue is the HHI divided by the market demand elasticity, \(^2\) which is also equal to the weighted average LI, \( \sum s_iL_i \).

This immediately poses a puzzle for conventional Cournot oligopoly analysis, for the combination of low demand elasticities with small numbers of competing firms (high HHI) would normally suggest a very high Lerner Index, in the cases cited above approaching 100%, and an improbably high ratio of profits to revenue - approaching 100%. \(^3\) Of course, elasticities in the longer run are higher, and short-run profit maximization that induces excessive entry would be imprudent, quite apart from attracting the attention of competition authorities.

Nevertheless, economists analyzing the electricity market, either in the course of market surveillance or in a merger inquiry, need tractable analytical models of price setting if they are to propose behavioural or structural remedies for the threat of market power. Although Supply Function Equilibrium models (for example, as deployed by Green and Newbery, 1992 and more recently by Hortacsu and Puller, 2008) are theoretically attractive, they pose formidable practical and conceptual problems if they are to be used for market monitoring, and even more so in quasi-judicial investigations of the kind conducted by competition authorities. Although it is possible to test whether firm behaviour is profit maximizing, given the bids of other firms (Sweeting, 2007; Hortacsu and Puller, 2008), as there may be many such equilibria it is hard to make firm predictions about price setting under a different industry structure (e.g. post-merger, or after increasing transmission capacity into a constrained zone). More to the point, Newbery (2008) shows that during any period in which the largest firm is pivotal (as described below), it will behave as a Cournot oligopolist (and more generally, supply functions approach the Cournot solution at peak demand), \(^4\) and so we are still left with the Lerner mark-up problem.

\(^2\) The HHI is conventionally computed by taking the shares as percentages rather than fractions, e.g. in a symmetric 5-firm case with \( s_i = 20\% \), HHI = \( 5 \times 20^2 = 2,000 \). To interpret \( H/\varepsilon \), \( H \) must be measured as a fraction - in this case \( H = 0.2 \).

\(^3\) Note that the Lerner index cannot exceed 100%, so in (2) the elasticity is either a perceived elasticity (allowing for longer-run responses or exit threats) or the equilibrium is found at a point on the demand schedule where the elasticity is higher (as in the linear case).

\(^4\) Willems, Rumiantseva and Weigt (2005) found that if they calibrated Cournot and supply function models...
A part of the solution is to note that forward contracts greatly reduce the incentive to exercise market power in the spot market, as Allaz and Vila (1993) noted for Cournot competition and Newbery (1998) confirmed for Supply Function Equilibria. Forward contracts are extremely important in liberalized electricity supply industries, given the considerable volatility of prices over periods of hours, days, seasons, and, in the case of storage hydro systems like Norway, even over years, combined with the need of retailers to secure their supplies ahead of time when the spot market or power exchange is only a relatively thin residual market.

2.1 The Residual Supply Index

Given the apparent potential to raise prices above the competitive level, electricity wholesale markets are typically subject to market surveillance, in many case by a Market Monitoring Unit. Such units collect data to assess whether there are incentives to raise prices appreciably, and to investigate cases in which prices appear to be unreasonably high. One of the more attractive indices of market power is the Residual Supply Index, RSI, which was initially developed by the California Independent System Operator. The RSI for company $i$ measures the percent of supply capacity remaining in the market after subtracting company $i$’s capacity to supply to the prompt market (after allowing for contractual commitments to supply on terms unrelated to the spot price). Smaller values of the RSI imply greater market power. The RSI measures the extent to which a firm is pivotal, that is, its uncommitted capacity is essential if demand is to be met (at an acceptable price).

$$\text{RSI}_i = \left( \frac{\text{Total Capacity} \text{ less } \text{Company } i\text{'s Relevant Capacity}}{\text{Total Demand}} = \frac{\sum_{j \neq i} k_j + x_i}{D} \right)$$

where:

- **Total Capacity** is the total regional supply capacity plus total net imports, with identical cost and demands to the German electricity market, each model explained the same fraction of observed price variations, suggesting that Cournot models may be as suitable as supply function models for short-run analysis.

5But note that Murphy and Smeers (2005) show that if the choice of capacity is chosen knowing that in the subsequent periods contracting will make competition more intense and profits lower, they will invest less in order to keep capacity tight and prices higher in compensation, just as Kreps and Wilson (1982) showed that intense price (Bertrand) competition in the post investment period would lead to Cournot choices of capacity that would constrain output and support Cournot equilibria. In this paper we assume that capacity decisions have already been made and do not respond to any changes in the market.


7See Sheffrin (2001, 2002a,b) and Sheffrin and Chen (2002).
Company \( i \)'s Relevant Capacity is company's \( i \)'s capacity, \( k_i \), less company \( i \)'s contract obligations, taken as \( x_i \), and

Total Demand, \( D \), is metered load plus purchased ancillary services.

When the RSI is greater than 100 percent, the suppliers other than company \( i \) have enough capacity to meet the demand of the market, and company \( i \) might be expected to have less influence on the market clearing price.\(^8\) On the other hand if residual supply is less than 100 percent of demand, company \( i \)'s uncommitted capacity is needed to meet demand, and is, therefore a pivotal player in the market. In such periods Cournot behaviour is to be expected (at best, collusion might lead to even less competitive outcomes). Most Market Monitoring Units take as a screen an RSI of less than 110%, as this provides for a minimal level of reserves, below which the Loss of Load Probability rises sharply, and with it the scarcity value of power. Figure 1 provides evidence from the California market that the RSI might be a useful indicator of market power and the ability to raise the price-cost margin. When used as a merger screen, the RSI is sometimes qualified by defining capacity to be economic capacity, defined as the capacity that would cover its costs at some price not greater than a predetermined mark-up (say 50%) above the market clearing price. Brunekreeft (2008) has used the RSI in his innovative social cost-benefit analysis of unbundling vertically integrated German electricity companies.

\(^8\)In a supply function equilibrium the price-cost margin can remain high even when no firm is pivotal, and, of course, collusion can also increase the price-cost margin.
3 Analysis

Consider an isolated region (or one in which net imports are constrained and have been subtracted from domestic demand) in which all but one of the generators are non-pivotal in a given period (e.g. one hour), and suppose that they produce at full capacity (which requires that their marginal cost is sufficiently below the market price), so that the only generator $i$ with market power produces $q_i$, sells $x_i$ forward contracts at price $f$ at the expected spot price, $Ep$, and thus has profits

$$\pi_i = pq_i + x_i(f - p) - C_i(q_i). \quad (4)$$

The f.o.c. for the choice of output and hence spot sales is

$$\frac{\partial \pi_i}{\partial q_i} = 0 = p - C_i'(q_i) - \frac{p}{\varepsilon} \left( \frac{q_i - x_i}{Q} \right).$$

where $Q = D(p)$. The firm’s RSI is $r_i = (\sum_{j \neq i} k_j + x_i)/D(p)$ (at the prevailing price, $p$) so $q_i - x_i = D(p) - (\sum_{j \neq i} k_j + x_i) = Q(1 - r_i)$, from which we derive the Lerner condition:

$$L_i \equiv \frac{p - C_i'}{p} = \frac{1 - r_i}{\varepsilon}. \quad (5)$$

The f.o.c. give exactly the same expression for the Lerner Index as (2), although the effect of more contract cover is to increase the RSI for the firm, possibly very substantially. This simple model suggests that the RSI is potentially useful as an index of market power (see the Appendix for details of the model).

The case for the HHI as an index of market power in the presence of contracts is less clear. If $s_j \equiv (q_j - x_j)/Q$, the shares of uncommitted output (where $\sum s_j < 1$ if there is any contracting) then it is still the case that

$$\frac{H}{\varepsilon} = \sum s_i L_i,$$

as in (3), although the $s_i$ are typically unobservable. There is no longer any simple relationship between industry profits and the HHI.

3.1 The Lerner Index and RSI in wholesale electricity markets

Many electricity wholesale markets operate either as pools or power exchanges, in which generators submit offers to supply varying amounts at successively higher prices, and the demand side specifies the level of demand it would take at successively lower prices.\(^9\) Such markets are best described by the intersection of supply and demand functions, with generators submitting supply

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\(^9\)In a pool, demand typically represents all uncontracted consumption demand, while in a power exchange some of the demand bids come from suppliers, including generators, who have precommitted more sales than they have contracted production, and wish to purchase the shortfall.
functions, as in figure 2, which shows the Market Clearing Price (MCP) in the Amsterdam Power Exchange.10

The two extremes considered here are the benchmark competitive case, in which the supply function is the marginal cost schedule, and the Cournot equilibrium, in which the firm inelastically offers a fixed supply at whatever MCP is determined from the demand side. In an electricity wholesale market, the market might be considered competitive if the MCP were set at the System Marginal Cost, SMC, $m_S$, which would be the marginal cost of the most expensive plant called upon to operate. One would normally expect generating companies to have a variety of plant with differing variable costs, and that they would dispatch them in order of increasing variable costs, lowest variable cost first (the merit order). If the variable costs of each plant type is constant, the cost function will now be $C_i(q_i) = \sum_{j}^{k-1} g_{ji} b_j + (q_i - \sum_{j}^{k-1} g_{ji}) m_i$, $b_j < b_{j+1} < m_i$, where $g_{ji}$ is the capacity of plant type $j$ held by firm $i$, whose constant marginal cost is $b_j$, and $m_i$ is the marginal cost of the $k^{th}$-least expensive plant that firm $i$ finds it profitable to commit, where $k$ is defined by $\text{Arg max}_k \sum_{j}^{k-1} g_{ji} \leq q_i$. Apart from calculating actual profits, the only interest of the cost function lies in its marginal cost, and the previous formulae will continue to

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10 European power exchanges and electricity pools typically operate under the fiction that there are no transmission constraints, which are then handled by the System Operator calling for bids and offers in a balancing market or mechanism (see e.g. Newbery, 2005). In many regions in the US, nodal pricing or locational marginal prices are determined at each node by supply and demand there, and these typically differ, possibly substantially, in the presence of transmission constraints. Transmission constraints effectively fragment the market into submarkets, and the analysis of this paper properly refers to those constrained sub-markets.
work with $C_i^t = m_i$.

London Economics, in its analysis of six European electricity markets, defined the Lerner Index as \((\text{price}-\text{SMC})/\text{price}\), which we can term $L_S$

$$L_S = \frac{p - m_S}{p}, \quad \text{(6)}$$

$$L_i = L_S + \frac{m_S - m_i}{p}. \quad \text{(7)}$$

The EU Sector Inquiry (London Economics 2007) has explored the extent to which various electricity companies exercise market power, using a variety of indices, including this version of the Lerner Index, $L_S$, and the closely related Price-cost mark-up (PCMU) $P = (p - m_S)/m_S$, where $P = L_S/(1 - L_S)$. Of the various markets studied, the German market is one of the more interesting, and a statistically highly significant relation is found between the RSI and both the LI and PCMU.\(^{11}\) London Economics regressed $L_S$ for various companies $i$ on its RSI for each hour:

$$L_S = \alpha - \beta r_i. \quad \text{(8)}$$

If equation (5) holds then $\alpha = \beta$ if $m_i = m_S$ and otherwise $\alpha < \beta$ (although $\frac{m_S - m_i}{p}$ is likely to be small if the dominant firm is sufficiently diversified and has marginal plant similar to those of other generators). The (robust Huber-White) estimated values for company CO2 were $\alpha = 3.56 \pm 0.26$ and $\beta = 3.13 \pm 0.24$ (London Economics, 2007, p352), consistent with (5) although correcting for auto-regression increased the values of $\alpha$ and $\beta$ and slightly increased the value of their difference. The results for other companies were similar, e.g. for company CO10 the values were $\alpha = 3.64 \pm 0.1$ and $\beta = 3.56 \pm 0.1$ (not correcting for auto-regression, which for CO2 raised the standard errors from 0.1 to 0.26). The other two companies (CO3 and CO17) had similar values for $\alpha$ but rather lower values for $\beta$.

The results of this empirical estimation seem surprisingly good,\(^{12}\) and are consistent with a demand elasticity of $\varepsilon = 1/\alpha = 0.26$, which might seem rather high but is consistent with estimates derived from the impact of the large price rise in Scandinavia following the drought of 2002, which for the domestic sector were about 0.23 (von der Fehr et al, 2005). Note that if imports and other capacity-constrained production is subtracted from total demand to give the residual demand facing the oligopolists, then the elasticity of residual demand will typically be higher than for total demand.

\(^{11}\)The significance of the relationship is given by the high $t$-value, although the value for $R^2$ was very low at 2%.

\(^{12}\)Surprising in the sense that there have been extensive criticisms of the Sector Inquiry, so that one might not expect a good relationship between the simple theory and messy reality. For one such set of criticisms see the report from NERA Economic Consulting (available on the NERA website at http://www.nera.com/NewsletterIssue/NL_ERI_EN454_Issue33-final3.pdf)
London Economics also estimated this equation for other countries, usually finding very significant parameter values, although low values for $R^2$. Thus for Spain $\alpha = 4.1 \pm 1.9$ and $\beta = 3.5 \pm 1.7$ (semi-robust estimates for CO1) and similar estimates for the other large company, CO4. These two companies on average accounted for 70% of demand and had RSIs below 110% for over 40% of the hours. Thus Spain has similar values to Germany, but for the largest company in The Netherlands (with 27% of capacity), $\alpha = 45.2 \pm 7.5$, $\beta = 43.9 \pm 6.2$ (correcting for serial correlation), suggesting again that they are equal, but also suggesting remarkably small values for the demand elasticity of 0.02. The largest company is the only one with an RSI below 110% for any significant fraction of the time (20% in its case). In contrast the two largest German firms had an RSI less than 110% for over 55% of the time.

Curiously, the simple model seems to match the Dutch market with only one pivotal generator most of the time, better than the German market, which has several (up to four) pivotal generators or Spain (almost a duopoly), but empirically the parameter estimates are rather good for Germany and less good for The Netherlands and Spain (with large standard errors and a very poor $R^2$). That suggests extending the model to several generators with market power as more descriptive of Spain and Germany.

### 3.2 Symmetric oligopoly with contracting

Suppose next that there is a symmetric $n$-firm oligopoly, where each firm has capacity $k_i$ and has identical cost function $C(q_i)$. Each firm’s profit is given by (4) and the f.o.c. again give the Lerner index (2) with $s_i = (q_i - x_i)/D$ is the share of uncommitted output available to the spot market as a share of total demand, and $L_i = L_S = s_i/\varepsilon$. In the perfectly symmetric case where $x_i = x$, then $q_i = q = D/n$. If the ratio of demand to capacity is $q/k = \lambda = (1 + \delta)^{-1}$ (the load factor), then the RSI $r$ is

$$r = \frac{(n-1)k + x}{D} = \left(\frac{n-1}{n\lambda} + \frac{x}{nq}\right). \quad (9)$$

This gives a relation between $x$ and $r$ that combined with (2) gives

$$L = \frac{s}{\varepsilon} = \frac{1 - x/q}{n\varepsilon} = \frac{\lambda + n - 1}{n\lambda} - r \approx \frac{1 + \delta(1 - \frac{1}{n}) - r}{\varepsilon} \approx \frac{1 - r}{\varepsilon}, \quad (10)$$

for small $\delta$, i.e. when demand is tight, just as in (5). Note that in the estimated equation (8) $\alpha = (1 + \delta(1 - \frac{1}{n}))/\varepsilon > \beta = 1/\varepsilon$, consistent with the econometric estimates for German and Spanish companies that suggest that $\alpha > \beta$.

This model might work for Spain provided the fringe companies’ output (and imports) is subtracted from total demand, in which case $\varepsilon$ is the elasticity of the residual demand, which might still be 70-90% of total demand. The implied elasticity of total demand will be lower than
that estimated. Given that $\beta = 3.5$, the elasticity of the residual demand would be 0.29 and for total demand might be 0.23.

One should be cautious about the linearity implied by the approximation (10) as $\varepsilon$ is not constant for linear demand, and the approximation is only valid for $q/k$ near 1. To make further progress in relating the Lerner index to the RSI we need a theory of forward contracting to determine the equilibrium $x/q$.

4 Determination of the equilibrium level of contracting

**Proposition 1** For an oligopoly of $n$ capacity-constrained firms with possibly varying marginal costs and capacities, facing a capacity-constrained fringe of firms and a linear demand schedule, the contract cover (i.e. the fraction of output contracted) of each oligopolist will be the same fraction of output, $1 - \frac{1}{n}$.

**Proof.** See Appendix. ■

Contract coverage (measured by $x/q$) increases from 0 (under monopoly) to 50% (under a duopoly) to 80% (if there are five firms), and converges on full coverage if there are sufficiently many firms.

In the special case of symmetric firms facing the linear demand schedule $D(p) = a - p$, where supply equals demand in equilibrium (i.e. no net imports) $D(p) = \sum q_i = Q$, with the same constant marginal costs, $C_i' = m = m_S$, $A \equiv a - m$, the formulae in the appendix give

$$x = \frac{(n - 1)A}{n^2 + 1}, \quad q = \frac{nA}{n^2 + 1}, \quad \frac{x}{q} = 1 - \frac{1}{n}, \quad (11)$$

$$p = m + \frac{A}{(n^2 + 1)}, \quad Q = \frac{n^2 A}{n^2 + 1}. \quad (12)$$

The share of uncommitted output, $1 - x/q = 1/n$, so each firm’s ratio of uncommitted output to market demand is $1/n^2$, and contracting has the same effect on market power as squaring the number of firms. Output $q = A/(n + \frac{1}{n})$ is greater than the output without contracting, $q = A/(n + 1)$ for $n > 1$, so contracting reduces market power in oligopolistic markets.

We can now return to the equilibrium RSI, where from (9)

$$r = \frac{(n - 1)k}{Q} + \frac{x}{nq} = \frac{(n - 1)k}{n^2} \left( \frac{t}{t} + 1 \right),$$

$$L_S = \frac{t}{m + t}, \quad P = \frac{p - m}{m} = \frac{t}{m},$$

where $t = \frac{A}{n^2 + 1}$ varies with the level of demand (which drives prices, and hence both the LI and RSI). Clearly, neither $L_S$ nor $P$ are any longer simple linear functions of the RSI. Nevertheless,

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13 This covers the apparently more general case, $Q = a - bp$ by defining units of output suitably.
Figure 3: Relation between Lerner Index, Price-cost mark-up and RSI

Figure 3, which shows the resulting relationships where \( m = 1, \ n = 3 = k, \) and where \( a \) varies linearly from 6 to 13 (roughly corresponding to the shape of the British load duration curve, and for which the RSI is less than 1 for 50% of the hours), suggests that \( L_S \) is roughly linear in the RSI, and this can be explored by substituting for \( z \) in the equations above. Note that the PCMU looks more like a quadratic, perhaps explaining why London Economics found the quadratic a better fit for the PCMU.

If \( r \) is close to 1, then let \( \gamma \equiv k(n - 1) + m(n^2 - n + 1) \) and \( r = 1 + z \) and expand \( L_S \) as a power series in \( z \) where is \( L_S \) of the form \( L_S = C(1 + Dz)^{-1} \):

\[
L_S = \alpha - \beta r, \\
\alpha = \frac{k(n - 1)}{\gamma}(1 + \frac{mn^2}{\gamma}), \quad \beta = \frac{k(n - 1)mn^2}{\gamma^2}.
\]

For the case above, \( \alpha = 132/69 = 0.78, \ \beta = 54/169 = 0.32 \). Regressing LI on the RSI in figure 3 gives \( \alpha = 0.75, \ \beta = 0.28 \). This time \( \alpha > \beta \) consistent with the empirical evidence (although the differences here are larger than observed).

As before, the Lerner Index is, from (6)

\[
L_S = \frac{1}{n^2\varepsilon},
\]

where again \( \varepsilon \) is the elasticity of market demand at the equilibrium price.\(^{14}\) This corresponds to (2) as \( s = 1/n^2 \).

\(^{14}\)For the linear demand \( 1/\varepsilon = Q/p = (A + c)/p - 1 \). Define \( \theta = c(n^2 + 1)/A \), then from (12) \( 1/\varepsilon = n^2/(1 + \theta) \), while \( L = 1/(1 + \theta) \).
4.1 Asymmetric oligopolists

The general case considered in the appendix had a set of $n$ capacity-unconstrained oligopolists with marginal costs $m_i$, with the remaining fringe of capacity-constrained firms producing constant output equal to their total capacity $K$ at the prevailing market price determined by linear demand, $p = a - Q$. It is straightforward to demonstrate that there is no simple linear relationship between the LI and RSI in this case, but if we redefine a modified RSI, $\rho$, in terms of flexible demand, $R$, defined as total demand less the output of capacity-constrained firms and imports, then it is possible to derive more appealing relationships. Define $B = a - m - K$, where $m = \frac{1}{n} \sum m_i$, average marginal cost, and substitute into the equations in the appendix to give the equilibrium outputs and the price:

$$p = m + \frac{B}{n^2 + 1}, \quad R = \frac{n^2 B}{n^2 + 1},$$

$$q_i = n(m - m_i) + \frac{R}{n}, \quad x_i = (1 - \frac{1}{n})q_i,$$

The modified RSI is $\rho_i = \frac{\sum_{j \neq i} k_j + x_i}{R}$ and replacing $x_i = (1 - \frac{1}{n})q_i = \frac{(n-1)}{n}R + (n-1)m_\sigma_i$ where $m - m_i = m_\sigma_i$, gives

$$\rho_i = \frac{\sum_{j \neq i} k_j + (n-1)m_\sigma_i}{R} + \frac{(n-1)}{n^2}.$$

Note that for linear demand, the elasticity of residual demand $\varepsilon_{RD} = p/R$. If $m_S = m(1+\sigma_S)$ the relevant LI is

$$L_S = \frac{p - m_S}{p} = 1 - \frac{1 + \sigma_S m}{\varepsilon_{RD} R},$$

$$= 1 - \frac{(1 + \sigma_S) m}{\varepsilon_{RD}} \rho_i - \frac{(n-1)}{n^2} \sum_{j \neq i} k_j + \frac{(n-1)m_\sigma_i}{m_\sigma_i},$$

$$= \alpha - \beta \rho_i, \quad \alpha = 1 + \frac{(1 + \sigma_S)(n-1)m}{\sum_{j \neq i} k_j + (n-1)m_\sigma_i \varepsilon_{RD}},$$

$$\beta = \frac{(1 + \sigma_S) m}{\sum_{j \neq i} k_j + (n-1)m_\sigma_i \varepsilon_{RD}} = \frac{n^2}{n-1}(\alpha - 1).$$

Thus with this modified RSI, the LI is again linearly dependent on the RSI, although there again no obvious reason why the coefficients should have very similar values. Thus if $n = 2$ (the Spanish case), $\beta = 4(\alpha - 1)$. Note that as estimated in Spain $\beta = 3.5$ and $\alpha - 1 = 3.1$, but the relationship was estimated on the traditional, not the modified RSI.

The final appealing relationship that survives translation to an asymmetric oligopoly with linear demand is the unweighted average LI, $\overline{L}$:

$$\overline{L} = \frac{1}{n} \sum_{i=1}^{n} (1 - m_i) \frac{p - m_i}{p} = \frac{p - m_i R}{p} = \frac{1}{n^2 \varepsilon_{RD}}.$$

which has the same form as (10) but with a different marginal cost:
5 Assessment of the suitability of the RSI

In very simple models, the Lerner index is negatively linearly related to the RSI with equal and opposite coefficients, as widely observed in the econometric estimations in the Sector Inquiry (London Economics, 2007). For more complex market structures it seems desirable to define the RSI over flexible output if one is to derive linear relationships between the Lerner index and the RSI. One final point to note (and discussed in the Sector Inquiry) is that the LI and the analysis here relates to short-run marginal costs (SRMC), ignoring the long-run marginal cost (LRMC) that included the cost of capacity, which must be covered if the firms are to make positive profits. A full analysis would need to take account of stochastic features of electricity markets (that determine the reserve margin) and investment decisions, which would determine the level of capacity relative to demand at various periods, and hence the equilibrium LI (measured on the SRMC) needed to cover the LRMC.

References


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Appendix

The theory of forward contracting below follows Allaz and Vila (1993) under the assumption of full information about demand and costs, and that contract positions are private information and only revealed when the spot market opens, which means that there will only be one round of contracting (Ferreira, 2006). Traders are assumed to be risk neutral when arbitraging the expected spot and forward contract prices, ensuring their equality in expectation. Liski and Montero (2006) have argued that market power can be sustained with contracting under the (realistic) assumption that the game in which players contract and then sell into a spot market is repeated, and the same would also be true with a repeated spot market without contracting. However, the problem here is not to give reasons why mark-ups are high given the apparent considerable market power, rather why they are so low. The following proposition extends the model of Allaz and Vila to account for more than two firms and for differing variable costs.

**Proof of Proposition 1**

Linear demand can be taken as \( D(p) = a - p \), where supply equals demand in equilibrium (i.e. no net imports): \( D(p) = \sum q_i = Q \). The set of firms operating at full capacity (because their marginal costs are sufficiently below the market price) has total capacity \( K \), and the \( n \) asymmetric Cournot oligopolists produce output \( q_i \) with capacities \( k_i \), at constant marginal costs \( m_i \),\(^{15} \) and contract cover \( x_i \). By definition their capacity constraints do not bind (otherwise they are part of the capacity constrained set). The problem facing the oligopolists is to maximize profits given by (4), and as before, the f.o.c.’s w.r.t. \( q_i \) are

\[
q_i = x_i + p - m_i.
\]

Adding all outputs gives

\[
K + \sum q_i = K + \sum x_i + n(p - \overline{m}) = Q = a - p,
\]

\[
p = \frac{a - S + n\overline{m}}{n + 1}, \quad Q = \frac{n(a - \overline{m}) + S}{n + 1},
\]

\[
q_i = x_i + \frac{a - S + M_i}{n + 1} = \frac{a - S_{-i} + M_i + nx_i}{n + 1},
\]

where \( S = K + X, \quad X = \sum x_i \) is committed sales, \( M_i = \sum_{j \neq i} m_j - nm_i, \quad S_{-i} = S - x_i \), and \( \overline{m} = \frac{1}{n} \sum j m_j \).

Solving for the equilibrium level of contract cover as before, the first stage (marginal) profit function from (4) is \( \pi_i = (p - m_i)q_i \) (eliminating the second term through arbitrage). Substitute

\(^{15}\)Marginal costs need only be constant for the marginal generating plant - any inframarginal plant can have any convex cost function provided its marginal cost at full capacity is less than \( m_i \).
for \( p \) and \( q_i \) to give

\[
\pi_i(x_i) = \frac{1}{(n+1)^2} (a - S_{-i} + M_i - x_i)(a - S_{-i} + M_i + nx_i).
\]

The f.o.c.’s are (setting \( a - \overline{m} \equiv A \) and noting that residual demand facing the oligopoly is \( R = Q - K \))

\[
2nx_i = (n-1)(a - S_{-i} + M_i), \text{ which, summed gives}
\]

\[
2nX = n(n-1)(A - K) - (n-1)^2 X,
\]

\[
X = \frac{n(n-1)(A - K)}{n^2 + 1}, \quad S = \frac{n(n-1)A + (n+1)K}{n^2 + 1},
\]

\[
p = \overline{m} + \frac{A - K}{n^2 + 1}, \quad Q = \frac{n^2A + K}{n^2 + 1}, \quad R = \frac{n^2(A - K)}{n^2 + 1},
\]

\[
x_i = (n-1) \left( (\overline{m} - m_i) + \frac{A - K}{n^2 + 1} \right),
\]

\[
q_i = n \left( (\overline{m} - m_i) + \frac{A - K}{n^2 + 1} \right), \quad \frac{x_i}{q_i} = 1 - \frac{1}{n}.
\]

That completes the proof of the Proposition.