INTERNATIONAL TRADE AND GROWTH:
THE IMPACT OF SELECTION AND IMITATION

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Sarah Stölting *

European University Institute

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Abstract

This paper develops an endogenous growth model with heterogeneous firms to analyze the impact of intra-industry trade on productivity growth. Growth is generated by selection, and sustained by entrants imitating successful incumbents. Firms are subject to idiosyncratic productivity shocks and some firms, mostly those with relatively low productivity levels, are forced to exit. This results in an increase in average productivity of the economy. The intra-industry effect of trade works through self-selection of the most productive firms into the export market. It leads to a reallocation of resources towards more efficient firms. Since the effect of selection and imitation on growth is amplified by the trade-induced selection process, opening up to trade increases the growth rate of productivity.

Keywords: Endogenous growth, Intra-industry trade, Heterogeneous firms, Selection

JEL-Codes: F10, L11, O40

1 Introduction

This paper analyzes the impact on productivity growth of opening up an economy to costly trade. For this purpose an endogenous growth model with heterogeneous firms and intra industry-trade is developed. Growth is driven by a mechanism of selection and sustained by entrants imitating successful incumbents. International trade makes selection tougher, and leads thus to a permanent increase in the productivity growth rate.

* Mailing address: Villa San Paolo, Via della Piazzuola 43, 50133 Florence, Italy; E-mail: sarah.stolting@eui.eu
In recent years there has been an increasing number of empirical and theoretical research papers analyzing the effects of trade on productivity. Bernard and Jensen (1995) published one of the first papers using firm-level data to investigate productivity differences between exporting and non-exporting firms. Since then, there has been a number of papers based on firm-level data from different countries. The two most important results of these studies are the following: First, there are large differences within industries in the export behavior of firms. Even in the so-called export-sectors, a large part of firms sell their products only in the domestic market. Secondly, exporting firms have higher performance characteristics than non-exporting firms, i.e. their productivity tends to be significantly higher, they are larger, more capital intensive and pay higher wages. Bernard and Jensen (1995) find that labor productivity for exporters is approximately a third greater than for non-exporters in the US in 1987. Concerning total factor productivity, Bernard et al. (2007) show that exporters are more productive by 3%. Their study is based on US data from the year 2000. The question of causality, i.e. whether more efficient firms become exporters or whether firms improve their performance after entering the export market, has been addressed by Bernard and Jensen (1999). They find clear evidence for more efficient firms becoming exporters, since performance measures are higher ex-ante for exporters. These differences, related to the export status among firms within industries, suggest that there is a self-selection of more productive firms into export markets. Similar evidence exists for different countries over different time periods (e.g. Baldwin and Gu (2004) for Canada, Eaton et al. (2004) for France and Van Biesebroeck (2005) for selected Sub-Saharan countries, among others).

Both ‘old’ trade theory and ‘new’ trade theory fail to consider firm level differences within sectors. New models have been developed in the last years in order to take into account intra-industry heterogeneity in terms of productivity. Important contributions are the models developed by Melitz (2003), Bernard et al. (2003) and Eaton and Kortum (2002). The focus here will be on Melitz (2003), which is a combination of the trade model of Krugman (1980) and the dynamic industry model of Hopenhayn (1992). As in Krugman (1980) the underlying assumptions of the model are CES preferences, monopolistic competition, increasing returns to scale and variable iceberg-type costs to trade. Melitz (2003) introduces some additional assumptions on heterogeneity of firms and on trade barriers: firms have different levels of labor productivity, the productivity of each firm is drawn randomly and firms face fixed costs of trade when exporting. This departure from the Krugman (1980) model yields the following result: exposing a country to costly trade makes only the more productive firms being involved in export activities, i.e. their profits and their market shares increase, and forces the least productive ones to exit the market. This means that opening up to costly trade leads to an increase in productivity by reallocating resources to more efficient firms, i.e. through a mechanism of selection.

An important issue missing in trade models with intra-industry heterogeneity is productivity

\footnote{Wagner (2007) provides a detailed overview of existing studies on productivity characteristics of exporting firms.}
growth over time. The approach in Melitz (2003) assumes zero-growth in the steady state. There have been very few papers which introduce growth in this framework, among them Baldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2006). In both papers endogenous growth comes from innovation of new product varieties, but there are differences in the assumptions concerning R&D. Baldwin and Robert-Nicoud (2008) find that openness can either lead to slower or faster growth, depending on the impact of a reduction in trade costs on marginal costs of innovation in different R&D specifications. The main result of Gustafsson and Segerstrom (2006) is dependent on the size of intertemporal knowledge spillovers in R&D. Trade liberalization with weak spillovers leads to an increase in productivity growth, and with strong spillovers to a decrease productivity growth. In contrast to Baldwin and Robert-Nicoud (2008), the effect on the productivity growth rate is only temporary.

The ambiguous result of both papers is similar to the empirical evidence on the effect of trade on growth. Lopez (2005) and Berg and Krueger (2003) provide surveys on empirical studies analyzing whether trade has a positive impact on the growth rate of the economy, and they show that there is a large divergence in the evidence. While some papers find that the relationship is positive (mostly without being able to establish causality due to endogeneity problems), other papers find no significant correlation. On the other hand, as mentioned above, there is very clear and strong evidence for self-selection of highly productive firms into the export market. This mechanism of self-selection leads to a reallocation of resources from low-productivity to high-productivity firms. Reallocation of resources can be of great importance to the evolution of productivity growth. For example, Pavcnik (2002) shows that about one third of aggregate productivity growth of Chilean plants over the period 1979 to 1986 can be explained by this type of reallocation of resources. Similarly, Bernard and Jensen (2004a) find that about 40% of total factor productivity growth can be attributed to a redistribution of resources across firms in the US manufacturing sector during the late 80s and early 90s.

Despite the fact that there is clear evidence for selection playing an important role in explaining economic growth, the growth literature based on selection is quite limited. The first papers incorporating selection as a growth mechanism were developed in the early 80s. Being based on evolutionary economics literature, most of these contributions are focused on bounded rationality. Gabler and Licandro (2007) and Luttmer (2007) are the first to provide models of endogenous growth through selection of successful firms and imitation by entrants based on rational expectations. When calibrated to US data, both papers find that a significant part of output growth can be attributed to selection and imitation, about one-fifth in the former and one-half in the latter. Even though the two papers are similar, they differ in the setup: Luttmer (2007) works in a framework of monopolistic competition, and emphasizes on matching the observed size distribution of firms, while the model of Gabler and Licandro (2007) is based on an environment of perfect competition.

This paper develops a model of endogenous growth with intra-industry trade and firm hetero-
geneity. Endogenous growth is generated by idiosyncratic firm productivity improvements, selection of existing firms and imitation of surviving firms by entrants, as in Gabler and Licandro (2007) and Poschke (2007). Hence, in this model, both the mechanism through which the economy is affected by opening up to costly trade and the mechanism generating growth work through a channel of selection, i.e. high productivity firms expand their market share and low-productivity firms either loose market share or exit the market. Concerning the trade component, the model is based on Melitz (2003). The aim of the paper is to analyze how trade affects growth through the specific channel of selection. Moving from a closed economy to an economy with costly trade makes the growth rate permanently increase, because the effect of selection and imitation on growth is amplified by the selection process that is due to trade.

The following mechanism underlies the result. The existence of fixed costs of production makes it impossible for firms with low productivity to generate positive profits. This implies a cutoff productivity level below which exit is optimal. The idiosyncratic productivity shock hitting incumbent firms is more likely to push firms with already low productivity levels below the cutoff. This means that the average productivity of the whole economy, and also the distribution of incumbents, shift to the right. To ensure that there are always new firms replacing the exiting ones, entry takes place. In order to always have entrants above the cutoff productivity level, the distribution of entrants has to follow the distribution of incumbents in its movement to the right. This is achieved by allowing entrants to imitate imperfectly successful incumbents. Therefore growth is sustainable.

If the economy opens up to trade and hence gives firms the opportunity to export their product, which is assumed to require a payment of a fixed cost, the cutoff productivity level and aggregate productivity increase. This comes from the fact that only the most productive firms will be able to afford paying this fixed fee for exporting, while less productive firms serve only the domestic market. This leads to the following effect: the demand on the domestic factor market for the only factor of production, which is labor, increases. Two reasons are underlying this. First exporters need more labor in order to pay the fixed costs of exporting and to serve the foreign market, and second more entry takes place due to higher potential returns. The increased labor demand leads to higher real wages, and forces the least productive firms to exit the market. In other words, the cutoff productivity level for production is higher in an economy where international trade is possible but requires a fixed initial investment, than in an economy where no inter-country exchange is possible. For the growth mechanism, this means that selection is tougher, and hence the average productivity increases at a faster rate than in a closed economy.

The remaining of the paper is organized as follows: in Section 2 the setup for the closed economy is presented, and in Section 3 the model is extended to the open economy case with trade between two symmetric countries. Section 4 provides a calibration, numerical solution and results of the model. Section 5 concludes.
2 Closed Economy

2.1 Demand

There is a continuum of households in the economy. Each household lives forever and inelastically supplies labor. The population does not grow, and aggregate labor supply is normalized to one. Preferences of the representative household are given by

\[ U = \sum_{t=0}^{\infty} \beta^t \ln(C_t), \]

where

\[ C_t = \left( \int_{\omega \in \Omega} q_t(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}. \]

Households consume different varieties \( \omega \), and the total set of varieties is given by \( \Omega \). Different varieties are substitutes, and the elasticity of substitution between any two varieties is given by \( \theta > 1 \). The discount factor is \( \beta \), with \( \beta \in (0,1) \). Aggregate expenditure in the economy is given by \( E_t = C_t P_t \), where \( P_t \) is the aggregate price level:

\[ P_t = \left( \int_{\omega \in \Omega} p_t(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}. \]

The static consumers problem is given by maximizing consumption of each variety, taking into account aggregate expenditure. Solving this maximization problem yields the households demand for each variety \( \omega \)

\[ q_t(\omega) = \left( \frac{p_t(\omega)}{P_t} \right)^{-\theta} C_t. \]

Hence, optimal expenditure for variety \( \omega \) is

\[ e_t(\omega) = \left( \frac{p_t(\omega)}{P_t} \right)^{1-\theta} E_t. \]

Households also choose the optimal intertemporal allocation of consumption by maximizing the lifetime utility, taking into account their budget constraint. They can consume or invest in shares of a mutual fund, that pays a return \( r_t \), which is the real interest rate. Firms in the economy generate aggregate profits, and since firms are owned by households, profits are transferred as dividends, allowing consumers to shift consumption over time. This eliminates any liquidity constraints of firms. Solving the dynamic optimization problem yields the standard Euler Equation, which defines the growth rate of consumption

\[ g_t \equiv \frac{C_{t+1}}{C_t} = \beta(1 + r_t). \]
This implies that the gross real interest rate in the economy is given by $1 + r_t = (1 + g_t)/\beta$.

### 2.2 Supply

There is a continuum of firms, each choosing to produce a different variety $\omega$. Technology for a firm with productivity $\varphi$ is given by

$$q_{\omega,t}(\varphi) = \varphi_{\omega,t}(l_{\omega,t} - f^p). \quad (3)$$

Marginal costs are constant and $f^p$ is the fixed cost of production. Firms are heterogeneous in their productivity levels $\varphi$. Every period each firm receives a shock to its productivity. This idiosyncratic shock follows a random walk

$$\ln(\varphi_{\omega,t+1}) = \ln(\varphi_{\omega,t}) + \eta_{\omega,t+1}. \quad (4)$$

The idiosyncratic productivity shock is assumed to be normally distributed, $\eta_{\omega,t} \sim N(0, \sigma^2_\eta)$, i.e. the expected growth rate of firm specific productivities is zero for each firm. The subscript $\omega$ is dropped from now on, because each firm produces a different variety, even if two firms have the same productivity. Firms which have the same $\varphi$ charge the same price, hire the same amount of labor and hence make the same profits, even if they supply different varieties. Profits of a firm in period $t$ are given by:

$$\pi_t(\varphi) = q_t(\varphi)p_t(\varphi) - w_t l_t(\varphi).$$

It follows from the profit maximization problem that a firm with productivity $\varphi$ will charge a price

$$p_t(\varphi) = \frac{\theta}{\theta - 1} \frac{w_t}{\varphi_t}. \quad (5)$$

Plugging the optimal price into the optimal expenditure from the household problem yields the firms revenue:

$$e_t(\varphi) = E_t \left( \frac{\theta}{\theta - 1} \frac{\varphi_t}{w_t} P_t \right)^{\theta-1}. \quad (6)$$

It follows that profits can be rewritten as

$$\pi_t(\varphi) = \frac{1}{\theta} e_t(\varphi) - f^p. \quad (7)$$

From now on nominal wages are normalized to one, i.e. $w_t = 1$ for all periods.

### 2.3 Firm Entry and Exit

The firm dynamics are based on the model of Hopenhayn (1992). Every existing firm receives an idiosyncratic shock in each period as is specified in equation (4). This means that some firms
will decide to exit the market because their productivity is lower than a certain threshold $\varphi^*$, below which producing would yield a negative firm value. The probability density function of incumbent firms is given by $\mu_t(\varphi)$. No specific distributional form is assumed since it is determined endogenously in equilibrium. The mean and and the variance are denoted by $x_t^i$ and $\sigma_t^2$ respectively.

Entering firms have to pay a sunk entry cost $f_e$, and are less productive on average than incumbent firms even though they try to imitate successful incumbents. They start with a productivity level which they draw from a log-normal distribution $\gamma_t(\varphi)$ with a mean $x_t^e$ and variance $\sigma_t^2$. The imitation process is modeled as in Poschke (2007): The mean of the entrants productivity distribution follows the productivity of the best incumbent, $\varphi_t^{\text{max}}$, with a constant distance $\kappa > 0$:

$$x_t^e = \varphi_t^{\text{max}} - \kappa,$$

where $\varphi_t^{\text{max}}$ is defined as being the average of the best 5 percent of all producing firms. Figure 1 provides an graphical illustration of the imitation process.

![Productivity Density Function](image)

Figure 1: Productivity Density and the Imitation Parameter

The timing is defined as follows: A firm takes the decision to exit at the beginning of period $t$. The relevant threshold for the decision to produce in $t$ is given by $\varphi_t^*$. If the decision has been taken to produce in a given period, then an incumbent gets a new productivity draw, pays the fixed costs of production $f_p$ and produces. The entry decision of new firms is also taken at the beginning of period $t$. If entry occurs, then the entrant has to pay the fixed entry costs $f_e$, gets its initial productivity draw out of the distribution $\gamma_t(\varphi)$, pays the fixed costs of production $f_p$ and produces.
Both fixed costs $f^p$ and $f^e$, are payed in labor units. See Appendix A for a graphical illustration of the timing assumptions in the economy.

The value function of a firm with productivity level $\varphi$ is given by

$$V(\varphi) = \max_p \left\{ \pi(\varphi) + \frac{1}{1 + r} \max \left\{ \int_0^\infty V(\varphi') \nu^\eta(\varphi'/\varphi) d\varphi', 0 \right\} \right\},$$

where $\nu^\eta$ is the probability density function of the exponential of the idiosyncratic productivity shock $e^{\eta_{t-1}}$. This means that $\nu^\eta(\varphi'/\varphi)$ is the probability that a firm with productivity $\varphi$ today receives a shock such that it has a productivity $\varphi'$ tomorrow.

**Free exit**: Some firms decide to exit the market because their productivity does not ensure them a positive expected future value. Firms with a productivity level $\varphi_t < \varphi^*_t$ exit the market. The free exit condition is given by:

$$\int_0^\infty V(\varphi') \nu^\eta(\varphi'/\varphi^*) d\varphi' = 0 \quad (10)$$

**Free entry**: A fixed sunk cost $f^e$ has to be payed by each firm which wants to start production. New firms will enter the market until the net value of entering is driven to zero. It follows that the free entry condition is given by

$$V^e = \int_0^\infty V(\varphi) \gamma(\varphi) d\varphi = f^e \quad (11)$$

**Transition function**: In every period there are incumbent firms with distribution $\mu_t(\varphi)$ and entrants with distribution $\gamma_t(\varphi)$. In the following period the 'new' PDF of incumbents will be the one of the old surviving incumbents (i.e. those firms that have a productivity level higher than the cutoff), plus the new entrants. Hence the transition function for the distribution of incumbent firms is given by

$$N' \mu'(\varphi') = N \int_{\varphi^*}^\infty \nu^\eta(\varphi'/\varphi) \mu(\varphi) d\varphi + N^e \gamma(\varphi'),$$

where $N$ is the number of incumbents, and $N^e$ the number of firms entering the market.

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2Fixed costs here are not constant, but evolve over time. Since they are given in terms of labor, and nominal wages are normalized to one, this means that they are increasing in terms of consumption.
2.4 Aggregation

The aggregate productivity level is denoted as $\bar{\phi}$. It is also the average productivity weighted by relative output shares and is given by:

$$\bar{\phi}_t = \left( \int_0^\infty \phi^{\theta-1} \mu_t(\phi) d\phi \right)^{\frac{1}{\theta-1}}.$$  \hspace{1cm} (13)

Using the definition of the aggregate price level given above, the optimal price chosen by firms and the definition of the cutoff level, the aggregate price level can be expressed as

$$P_t = \left( \int_0^\infty p_t(\phi)^{1-\theta} N_t \mu_t(\phi) d\phi \right)^{\frac{1}{\theta-1}} = \frac{\theta}{\theta-1} N_t^{\frac{1}{\theta-1}} \bar{\phi}^{-1}. \hspace{1cm} (14)$$

Aggregate output is

$$Q_t = \left( \int_0^\infty q_t(\phi)^{\frac{\theta}{\theta-1}} N_t \mu_t(\phi) d\phi \right)^{\frac{\theta}{\theta-1}} = q(\bar{\phi}_t) N_t^{\theta} \bar{\phi}_t, \hspace{1cm} (15)$$

and aggregate profits are

$$\Pi_t = \int_0^\infty \pi_t(\phi) N_t \mu_t(\phi) d\phi = N_t \pi_t(\bar{\phi}_t). \hspace{1cm} (16)$$

2.5 Equilibrium

The stationary equilibrium is defined as sequences of prices $\{p_t\}_{t=0}^\infty$, $\{P_t\}_{t=0}^\infty$, sequences of real numbers $\{N_t\}_{t=0}^\infty$, $\{Q_t\}_{t=0}^\infty$, $\{\bar{\phi}_t\}_{t=0}^\infty$, functions $l(\phi; \mu)$, $v(\phi; \mu)$, and sequences of probability density functions $\{\mu_t\}_{t=0}^\infty$, such that:

- **consumers** choose optimally consumption according to (1) and asset holdings to satisfy the Euler equation (2),
- **firms** set prices optimally according to (5), yielding the value function (9),
- **exit** is optimal and given by the free exit condition (10),
- **entry** is optimal and given by the free entry condition (11),
- the **labor market clears**: $L = L^p_t + L^e_t$, where $L^p_t = E_t - \Pi_t$ is the amount of labor used in production and $L^e_t = N^e_t f^e$ the amount of labor used for paying the entry costs,
- the **stationary distribution** of firms $\mu_t(\phi)$ evolves according to the transition function (12).
2.6 Balanced Growth Path

The balanced growth path (BGP) is defined as a state of the economy in which aggregate productivity, consumption and output grow at a constant rate $g$, aggregate prices decrease at the same constant rate, the distribution of firm productivities shifts up at steps of $g$, its shape is invariant\footnote{Even though the evolution of firm-specific productivities follows a random walk, the distribution of firms is stationary. Its variance remains finite over time since exit takes place mostly in the lower part of the distribution and since the probability of surviving decreases with the age of the firm. For more details see Poschke (2007).}, and aggregate expenditures, aggregate profits, the number of firms, the number of entrants and the interest rate are constant. The economy can then be stationarized, and to distinguish it from the growing economy, stationarized variables are denoted with a hat. The relevant equations for the BGP, i.e. the equations that have to be rewritten in stable terms, are the law of motion of productivity (4), the value function (9), the transitions function for the distribution of productivities (12), the free exit condition (10), and the free entry condition (11).

The random walk of productivities (4) gets a downward drift in the stationarized economy. The distribution shifts to the right every period by a step of size $g$, but the idiosyncratic productivity shock is such that it has a zero mean, i.e. a firm does not expect its productivity to change. Hence in expectations each firm has a decreasing productivity relative to the overall distribution:

$$\ln(\hat{\phi}_{\omega,t+1}) = \ln(\hat{\phi}_{\omega,t}) - g + \eta_{\omega,t+1}. \tag{17}$$

Combining equation (7) and (6), firm specific profits can be rewritten as

$$\pi(\hat{\phi}) = \frac{1}{\theta} \left( \frac{\theta - 1}{\theta} \right)^{\theta - 1} \hat{\phi}^{\theta - 1} k - f^p, \tag{18}$$

where $k = EP^{\theta - 1}$. Substituting this expression into the value function (9), and using the Euler Equation (2) yields a stationary expression for the value function of a firm

$$v(\hat{\phi}) = \pi(\hat{\phi}) + \beta \max \left\{ \int_0^\infty v(\hat{\phi}')\nu(\hat{\phi}')d\hat{\phi}', 0 \right\}. \tag{19}$$

Applying the same method, the free exit condition (10) in the balanced growth path is

$$\int_0^\infty v(\hat{\phi}')\nu(\hat{\phi}' / \hat{\phi}_*)d\hat{\phi}' = 0, \tag{20}$$

the free entry condition (11) is

$$v^e = \int_0^\infty v(\hat{\phi})\gamma(\hat{\phi})d\hat{\phi} = f^e, \tag{21}$$
and the transition function (12) can be rewritten as

\[ \mu(\hat{\phi}') = \int_{\hat{\phi}'}^\infty \nu^r(\hat{\phi}'/\hat{\phi}) \mu(\hat{\phi}) d\hat{\phi} + \frac{N^c}{N} \gamma(\hat{\phi}') \]

(22)

Given these equations (17)-(22), the balanced growth path of the closed economy can be solved numerically.

3 Open Economy

In order to analyze the impact of trade on economic growth, the previous setup is adjusted to an open economy framework. Only trade between two symmetric countries is considered for simplicity. An extension to a larger number of countries trading with each other does not alter the main results.\(^4\) The assumption of symmetry implies that both countries have the same wage, which is normalized to one, and that the aggregate variables of both countries are the same. Another assumption that is made, is that exporting firms face an additional fixed cost \(f^x\) for exporting in every period they serve the foreign market, and also variable, iceberg type, trade costs \(\tau\). The existence of fixed costs to exporting is crucial. Otherwise the only effect of trade is an increase of consumers welfare due to a rise in the number of varieties available for consumption as in Krugman (1980). There exist several empirical studies which find that firms face fixed costs to enter the export market, for example Bernard and Jensen (2004b) for the US.

On the demand side there are no changes in the setup due to opening up the economy. Consumers still face the same maximization problem subject to the same constraints, which means that the demand for each variety is determined as in the closed economy and is given by equation (1). On the other hand firms now also have to make an additional decision: after receiving their productivity draw firms have to evaluate whether they want to pay the fixed investment to export, or only serve the domestic market.

3.1 Supply

The production function is the same as before, and firms that sell only in the domestic market pay the fixed costs \(f^p\), but firms which also enter the export market now pay additionally the fixed cost \(f^x\). The profit function changes because now profits can be generated from local and from foreign sales. Production, prices, the amount of labor used and profits for the local market are denoted by \(q^d\), \(p^d\), \(l^d\), \(\pi^d\) and for the exporting market by \(q^x\), \(p^x\), \(l^x\), \(\pi^x\).

\[ \pi^d(\varphi) = q^d(\varphi)p^d(\varphi) - l^d(\varphi) \]

\[ \pi^x(\varphi) = q^x(\varphi)p^x(\varphi) - l^x(\varphi) \]

\(^4\)See Melitz (2003) for trade between \(n\) number of symmetric countries
\[ \pi_t^e(\varphi) = q_t^e(\varphi)p_t^e(\varphi) - l_t^e(\varphi) \]

The total amount of labor spent in production by a firm with productivity \( \varphi \) is \( l_t^e(\varphi) = l_t^d(\varphi) + l_t^x(\varphi) \), where \( l_t^x = 0 \) if the firm sells only in the domestic market.

The price for domestic sales \( p_t^d \) is the same as in closed economy and given by equation (5), but a firm that exports will set higher prices in the export market because of the per unit trade costs:

\[ p_t^x(\varphi) = \tau \left( \frac{\theta}{\theta - 1} \right) \frac{w_t}{\bar{\varphi}} \]  

(23)

Overall profits of a firm with productivity \( \varphi \) in period \( t \) are given by

\[ \pi_t(\varphi) = \pi_t^d(\varphi) + \max \{0, \pi_t^x(\varphi)\} \]  

(24)

where \( \pi_t^d(\varphi_t) \) is given by equation (7), and \( \pi_t^x(\varphi) = \frac{1}{\theta} c_t^x(\varphi) - f^x \), with \( c_t^x(\varphi) = \tau^{1-\theta} c_t^d(\varphi) \).

### 3.2 Firm Entry and Exit

The value function of a firm with productivity \( \varphi \) is given by equation (9). Notice that profits that enter the value function are not the same as in closed economy, because they now consist of domestic and export sales. In the open economy there are two cutoff levels, one for producing \( \varphi^*_t \) (which is given by equation (10)) and one for exporting \( \varphi^*_x \). The productivity cutoff level for entering the export market is \( \varphi^*_x = \inf \{ \varphi_t : \varphi_t > \varphi^*_x \text{ and } \pi_t^x(\varphi_t) \geq 0 \} \), and can be determined by the following equation:

\[ \pi_t^x(\varphi^*_x) = 0 \]  

(25)

The free entry condition is again given by equation (11), and the transition function of the distribution of incumbents by equation (12).

The timing is the same than in the closed economy, except of the decision to enter the export market. Once the firms, incumbents and entrants, got their productivity draw for a given period, they decide whether to export or not. Entering the export market takes place if \( \varphi_t \geq \varphi^*_x \).

### 3.3 Aggregation

Aggregate productivity is as before given by the weighted average productivity, with the weight being relative output shares. It can not be defined in the same way as in the closed economy, because equation (13) does not take into account the higher market share of exporting firms. In order to do so, it has to be considered that some firms export, and some firms serve only the domestic market. Hence there are two aggregate productivity levels, \( \bar{\varphi}_t^d \) for all firms (but taking into account only
domestic market shares), and $\tilde{\phi}_t^x$ for exporting firms only (including only exporting market shares):

$$\tilde{\phi}_t^d = \left( \int_0^\infty \varphi_t^{\theta-1} \mu_t(\varphi_t) d\varphi_t \right)^{\frac{1}{\theta-1}} \quad \tilde{\phi}_t^x = \left( \frac{1}{1 - M(\varphi_t^{*x})} \int_{\varphi_t^{*x}}^\infty \varphi_t^{\theta-1} \mu_t(\varphi_t) d\varphi_t \right)^{\frac{1}{\theta-1}},$$

where $1 - M(\varphi_t^{*x})$ is the ex-ante probability for each firm to draw a productivity level higher than the exporting cutoff. The total aggregate productivity level, which also reflects the relative market shares, is then given by:

$$\tilde{\varphi}_t = \left( \frac{N_t}{N_t + N_t^x} \varphi_t^{d (\theta-1)} + \frac{N_t^x}{N_t + N_t^x} \left( \frac{1}{\tau} \varphi_t^{x (\theta-1)} \right) \right)^{\frac{1}{\theta-1}},$$

where $N_t^x$ is the number of firms exporting, or the number of varieties exported to the other country.\(^5\) The variable trade costs $\tau$ reflect the output shrinkage linked to exporting. Since every exporting firm is also producing for the domestic market, the total number of firms producing in the economy is $N_t$. Since additionally to domestic varieties, the consumers also have access to imported varieties, the total mass of different varieties available to a consumer is $N_t + N_t^x$.

The aggregate price level is now given by

$$P_t = \frac{\theta}{\theta-1} (N_t + N_t^x)^{\frac{1}{\theta-1}} \tilde{\varphi}_t^{-1},$$

and aggregate profits by

$$\Pi_t = \frac{1}{\theta} E_t - f^p N_t - f^x N_t^x.$$

### 3.4 Equilibrium

The stationary equilibrium is defined as sequences of prices $\{p_t^d\}_{t=0}^\infty$, $\{p_t^x\}_{t=0}^\infty$, $\{P_t\}_{t=0}^\infty$, sequences of real numbers $\{N_t\}_{t=0}^\infty$, $\{N_t^x\}_{t=0}^\infty$, $\{Q_t\}_{t=0}^\infty$, $\{\varphi_t^\ast\}_{t=0}^\infty$, $\{\varphi_t^{*x}\}_{t=0}^\infty$, functions $l(\varphi; \mu)$, $v(\varphi; \mu)$, and sequences of probability density functions $\{\mu_t\}_{t=0}^\infty$, such that:

- **consumers** choose optimally consumption according to (1) and asset holdings to satisfy the Euler equation (2),

- **firms** set prices optimally according to (5) in the domestic market and (23) in the foreign market, yielding the value function (9),

- the **export decision** is taken optimally and given by equation (25): only firms with $\varphi_t > \varphi_t^{*x}$ export,

\(^5\)Note that by the assumption of symmetry, this is also equal to the number of varieties imported to the domestic country.
• **exit** is optimal and given by the free exit condition (10),

• **entry** is optimal and given by the free entry condition (11),

• the labor market clears: \( L = L^p_t + L^e_t \), where \( L^p_t = E_t - \Pi_t \) is the amount of labor used in production including the labor needed to pay the fixed costs of exporting and \( L^e_t = N^e_t f^e \) the amount of labor used for paying the entry costs,

• the stationary distribution of firms \( \mu_t(\varphi) \) evolves according to the transition function (12).

### 3.5 Balanced Growth Path

The balanced growth path is defined in the same way as in the closed economy, except for the expression for profits, now given by (24). Rewriting this equation yields:

\[
\pi(\hat{\varphi}) = \frac{1}{\theta} \left( \frac{\theta - 1}{\theta} \right)^{\theta - 1} \hat{\varphi}^{\theta - 1} k - f^p + \max \left\{ \frac{1}{\theta} \left( \frac{\theta - 1}{\theta} \right)^{\theta - 1} \tau^{1 - \sigma} \hat{\varphi}^{\theta - 1} k - f^x, 0 \right\}
\]  

(27)

The value function, the free exit and entry conditions and the transition function are still given by equation (19), (20), (21) and (22) respectively. Note that profits entering the equations are not the same as in the closed economy. The export decision is taken according to (25), and hence:

\[
\frac{1}{\theta} \left( \frac{\theta - 1}{\theta} \right)^{\theta - 1} \tau^{1 - \sigma} (\hat{\varphi}^* x)^{\theta - 1} k - f^x = 0.
\]  

(28)

Given equations (19), (20), (21), (22), (27) and (28) the balanced growth path can be solved numerically.

### 4 Solution

#### 4.1 Calibration

In this section, parameter values for the open economy model are calibrated to the U.S. manufacturing sector in order to derive quantitative conclusions of the selection and imitation mechanism on the growth rate of productivity. The parameters that need to be calibrated are the discount factor \( \beta \), the elasticity of substitution \( \theta \), the fixed costs of production \( f^p \), entry \( f^e \) and exporting \( f^x \), the variable exporting cost \( \tau \), the variance of the productivity distribution of entrants \( \sigma^2 e \), the variance of the idiosyncratic productivity shock \( \sigma^2 \eta \), and the imitation parameter \( \kappa \). Common values from the literature are assigned to \( \beta \) and \( \theta \). All other parameters are jointly chosen by minimizing the distance between some moments observed in the data and the equivalent moment of the model by using a genetic algorithm as described by Dorsey and Mayer (1995).
The moments observed in the data used for the calibration are the following: the proportion of exporters, the size advantage of exporters, the size of entrants relative to incumbents, the seven-year survival rate of entrants, the exit rate, the average firm size and the annual growth rate. The first two observations help to determine the trade costs $f_x$ and $\tau$. The size of entrants relative to incumbents allows me to find the imitation parameter $\kappa$, since it establishes a relationship between the distribution of incumbents and entrants. The seven-year survival rate of entrants, the exit rate and the average firm size give some good indications about the firm dynamics and scale, and thus help to calibrate the parameters $f^e$, $f^p$ and $\sigma^2_e$. Finally, the growth rate of output determines the variance of the idiosyncratic productivity shock $\sigma^2_\eta$.

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Target (U.S.)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion of exporters</td>
<td>21%</td>
<td>23.85%</td>
</tr>
<tr>
<td>Size advantage of exporters (Ratio domestic sales)</td>
<td>4.8</td>
<td>4.70</td>
</tr>
<tr>
<td>Size of entrants relative to incumbents</td>
<td>18%</td>
<td>17.58%</td>
</tr>
<tr>
<td>7-year survival rate of entrants</td>
<td>48%</td>
<td>44.22%</td>
</tr>
<tr>
<td>Exit rate</td>
<td>8%</td>
<td>6.7%</td>
</tr>
<tr>
<td>Average firm size (employment)</td>
<td>80.3</td>
<td>82.81</td>
</tr>
<tr>
<td>Growth rate</td>
<td>3%</td>
<td>2.99%</td>
</tr>
</tbody>
</table>

Analyzing the 1992 Census of Manufacturers Bernard et al. (2003) report that the proportion of exporters is 21 percent for the U.S. They also show that exporting firms have a size advantage of 4.8 for the ratio of average U.S. sales. This measure is the ratio of average output of exporting plants to the average for non-exporting plants. The survival rate of firms seven years after entry in the market is 48 percent according to Bartelsman et al. (2004) for the U.S. manufacturing sector. They also find that the exit rate averaged over the time period 1989 to 1997 is approximately 8 percent, and that the size in terms of employment of new firms is 18 percent of incumbents size. Using the same dataset, Bartelsman et al. (2003) show that the average size of manufacturing firms in the U.S. is 80.3 in terms of employment. The annual growth rate is set to 3 percent, which is the average output growth rate in the 1990s according to the NIPA tables. The calibration targets and the values generated by the model are given in Table 1. All targets are reasonably well matched by the model statistics.

The parameter values resulting from the calibration are summarized in Table 2. Fixed costs of production and entry are given in percentage of output of the average producing firm. Fixed costs of exporting are given as percentage of the average output of exporting firms. Note that the fact that $f^e$ as percentage of average output of all producing firms is 33%. This figure is seems quite high. However, the average producing firm in this case is not an exporter, but produces only domestically.
the variance of the entrants distribution is substantially higher than the variance of incumbents shocks is consistent with evidence provided by Bartelsman and Dhrymes (1998): they find that young plants face more uncertainty about their productivity than older plants. The variable trade costs \( \tau \) take the value 1.14. The imitation parameter is given as the relative distance between the mean productivity level of entrants and the average productivity level of producing firms: new firms have a productivity of approximately 72% of the average productivity of incumbents. This matches closely the empirical finding of Jensen et al. (2001).8 The parameter values taken from the literature are the following: The discount factor \( \beta \) is set to 0.95, which implies an annual interest rate of approximately 5 percent. For \( \theta \), the elasticity of substitution between any two varieties, the value adopted form the literature is 3.8. It is taken from Bernard et al. (2003), who obtain this value by calibrating their model to fit U.S. plant and macroeconomic trade data.9

### 4.2 Results

In this section the solution of the model is discussed. Appendix B describes the algorithm used to obtain this solution. The aim of the paper is to analyze how trade affects growth.

Unambiguously, opening up to trade yields a higher growth rate: As can be seen in Table 3 it increases by 15 basis points from 2.84% to 2.99%, meaning that the growth rate is more than 5 percent higher in the open economy. Considering that the effect on the growth rate is due only to an increase in selection, disregarding any other source of variation, this is a substantial change.

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8Jensen et al. (2001) find that in their panel the average productivity of entrants in 1992 is 45 in terms of value-added per hours worked in 1987 Dollars, while the average productivity in the industry is 54. This yields a relative distance of 83.33% between the two groups. However, only data form census years is considered, hence entrants in 1992 are firms that entered between 1987 and 1992 and are still alive in 1992. Recalculating the relative distance generated in my model, taking the mean of entrants which entered in the last 5 years and are still alive in year 5, yields a relative distance of 83.72%.

9This value is lower than usually in the literature, hence the resulting markup is higher. However, the presence of fixed costs in the model justifies this choice of \( \theta \). See Ghironi and Melitz (2005) for a more detailed discussion.
So far the analysis consisted of comparing the steady states of autarky and open economy. More realistically I will now analyze trade liberalization in the open economy, i.e. the decrease in trade costs. How does a cheaper access to export markets affect the growth rate of the economy? As can be seen in Figure 2, a decrease in the variable costs of trade in the model leads to higher gain in aggregate productivity growth compared to the autarky case.10

![Figure 2: Growth Effect of a Change in Variable Costs of Exporting](image)

The intuition behind this effect is simple. When variable costs of trade decrease, the productivity level necessary to derive positive profits form export markets is lower. Hence, the export cutoff level $\phi^x$ decreases and more firms have access to foreign markets. Now more firms serve the market abroad. Hence the demand for labor necessary for the additional production and for paying the fixed costs of exporting increases. This drives real wages up, which means that it is no longer

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10Note that the growth differential for the baseline value from the calibration is higher than 15 basis points in this graph. This is because the number of grip points is higher in this exercise than in the baseline case. Also, the steps in the figure come from the discretization necessary to compute the numerical solution. The results here are computed for 1000 grid points, and a substantially larger number would be needed in order to obtain a smooth line.
profitable for the least productive firms to stay in the market. The production cutoff level $\varphi^*$ hence increases: higher exposure to international trade has as consequence that firms with the lowest productivity levels in the economy have to exit the market. The implications of the higher cutoff level is that each remaining firm has now a higher probability to be hit by a bad shock which forces it out of the market. This tougher selection makes the economy profit form an increase in its growth rate. The same is true for a decrease of fixed trade costs.

The distributions of producing firms and entrants is shown in Figure 3. The firm distribution is skewed to the right. This is consistent with well established empirical evidence. Note that the entrants distribution is lagging behind the incumbents distribution. This due to the imperfect imitation process characterizing entry.

![Stationary Distribution – Open Economy](image)

Figure 3: Stationary Distribution Open Economy - Entrants and all Firms

### 4.3 Competition and the Growth Effect of Trade

In the previous discussion it has become clear that trade-induced selection is an important factor which influences the growth rate of the economy. How this is related to competition aspects is explained in more detail in this section.

The quite restrictive assumption of CES preferences has some important implications for the competition effects in this model. Since the elasticity of substitution is constant, it does not adjust to a change in the number of competing firms or prices. The markups charged by firms are constant, hence their prices do not vary with the increase in competition. Thus, there is no competition effect in the sense of price adjustments by firms.
However, the elasticity of substitution directly affects the growth rate. It is an indicator of how competitive the economy is. When $\theta$ is low, different varieties are only very imperfect substitutes, which implies a low degree of competitiveness in the market. Opposite to this, high values of $\theta$ stand for higher competitiveness. Thus different levels of elasticities imply different growth rates because selection plays a more or less important role. Since the markup depends negatively on the demand elasticity, a low $\theta$ yields a high markup. In this case, since firms can charge a high markup, less productive firms can make profits which are high enough to stay in the market. This means that for low elasticities, selection does not play a big role. Hence, the higher the elasticity of substitution, the more competitive the market and the higher the growth rate.

Figure 4: Effect of the Elasticity of Substitution on the Growth Differential

Figure 4 shows how the growth rate difference between the open and closed economy varies with $\theta$. That this difference in growth rates is not constant but hump shaped comes from the fact that the trade-induced additional selection impacts the economy differently for different levels of competitiveness. The explanation for the increase in the range of relatively low values of $\theta$ is as follows. Allowing for trade leads to an increase of the aggregate productivity level, and hence to a decrease in the aggregate price level. The demand for each variety depends on the relative price charged for the specific variety: The higher the relative price, the lower the demand for this good, and a decrease of the aggregate price level directly implies an increase in relative prices. How consumers react to a change in relative prices depends on the elasticity of substitution $\theta$. For low values of $\theta$, consumers do not react a lot to this change in relative prices. Thus the loss in market
shares of low productivity firms is limited, and some less efficient firms can continue to survive in the market. This means that for low values of $\theta$ the trade-induced selection effect has a relatively weak effect on the domestic economy in terms. When $\theta$ increases, the additional selection effect coming from trade plays an increasingly important role in the economy. The intuition behind the decrease in the growth differential in the range of relatively high values of the elasticity of substitution is similar: larger values of $\theta$ stand for more competition, different varieties are close substitutes. In this case, competition in the economy is very important. Additional selection induced by trade is then marginal and has a small, or no effect on the economy. Thus the growth differential decreases with an increased substitutability between goods.

5 Conclusion

This paper has analyzed the impact of opening up an economy to costly trade on the productivity growth rate. For this purpose an endogenous growth model with firm heterogeneity and intra-industry trade has been developed. Growth is generated by selection of more productive firms into the market. The least productive firms are forced to exit. Incumbent firms are hit every period by an idiosyncratic productivity shock and entrants are able to partly imitate successful incumbents. Exposure to international trade has the effect to increase the minimum productivity level required for production. This makes selection tougher, i.e. forces more low-productivity firms to give up their position in the market, and hence increases the growth rate of aggregate productivity.

For the last years there has been an ongoing debate about the benefits and shortcomings of globalization. One of the main fears is that opening up to trade could force some firms to close down. The model developed in this paper does not allow for a general statement about the relation between trade and growth. However one very important conclusion can be drawn: considering the channel of the selection effect of trade on growth, countries that open up to trade will face closure of firms, but will gain in aggregate productivity and grow at a faster rate. It follows that in the short run, a protectionist policy could preserve some job opportunities. The long run consequences are however likely to be lower average productivity levels, higher prices and lower growth rates.
Appendix A. Timing

Appendix B. Algorithm

The algorithm used to obtain the numerical solution of the balanced growth path constructed in the following way.

First the state space of productivities is discretized, which means that a grid of productivities $\hat{\varphi}$ is created. The number of grid points is set to 200. A higher number of grid points does not have an implication on the main results of the model. Then the variable $k$ and the growth rate $g$ are guessed. For a given $k$ and $g$, the transition probability matrix $\nu'(\hat{\varphi}'/\hat{\varphi})$, denoted $T$, can be computed, taking into account the downward drift according to equation (17). The next step is to create the distribution of entrants $\gamma(\hat{\varphi})$, which is assumed to be lognormal. Then the variable $k$ can be determined using the free entry condition, i.e. the $k$ is computed for which the free entry condition (21) holds, given $g$. This allows then to compute the value function (19) by value function iteration. Firms which get a negative value from production choose to exit, hence the cutoff productivity level $\hat{\varphi}^*$ is known. The cutoff level allows then to create a transition probability matrix $T_x$ which includes exit. Using this, the stationary firm distribution, for given $g$, can be
obtained directly by $\mu = (I - T_x)^{-1}\gamma$. In the case of the open economy the decision of entering the export market has to be included. This is done by evaluating the profits for exporting, $\pi^e$, for every existing productivity level. Firms with $\pi^e > 0$ decide to export, and other firms only serve the domestic market. This also delivers the export market cutoff $\hat{\varphi}^e_x$. Profits made from exporting enter the overall profits which are used to compute the value function. The last step is to obtain the growth rate $g$. This is done via the imitation mechanism. The mean of the entrants distribution is normalized to zero, and the equilibrium growth rate is the one fulfilling equation (8).
References


