EUI Working Papers

ECO 2009/34
DEPARTMENT OF ECONOMICS

FLEXIBLE CONTRACTS

Piero Gottardi, Jean Marc Tallon and Paolo Ghirardato

EUROPEAN UNIVERSITY INSTITUTE, FLORENCE
DEPARTMENT OF ECONOMICS

Flexible Contracts

PIERO GOTTARDI,
JEAN MARC TALLON
and
PAOLO GHIRARDATO

EUI Working Paper **ECO** 2009/34

This text may be downloaded for personal research purposes only. Any additional reproduction for other purposes, whether in hard copy or electronically, requires the consent of the author(s), editor(s). If cited or quoted, reference should be made to the full name of the author(s), editor(s), the title, the working paper or other series, the year, and the publisher.

The author(s)/editor(s) should inform the Economics Department of the EUI if the paper is to be published elsewhere, and should also assume responsibility for any consequent obligation(s).

ISSN 1725-6704

© 2009 Piero Gottardi, Jean Marc Tallon and Paolo Ghirardato

Printed in Italy
European University Institute
Badia Fiesolana
I – 50014 San Domenico di Fiesole (FI)
Italy
www.eui.eu
cadmus.eui.eu

Flexible contracts*

Piero Gottardi
European University Institute

Jean Marc Tallon
Paris School of Economics, Université Paris I Panthéon-Sorbonne, CNRS

Paolo Ghirardato
Collegio Carlo Alberto

September 3, 2009

Abstract

This paper studies the costs and benefits of delegating decisions to superiorly informed agents relative to the use of rigid, non discretionary contracts. Delegation grants some flexibility in the choice of the action by the agent, but also requires the use of an appropriate incentive contract so as to realign his interests with those of the principal. The parties' understanding of the possible circumstances in which actions will have to be chosen and their attitude towards risk and uncertainty play then an important role in determining the costs of delegation. The main focus of the paper lies indeed in the analysis of these costs and the consequences for whether or not delegation is optimal.

We determine and characterize the properties of the optimal flexible contract both when the parties have sharp probabilistic beliefs over the possible events in which the agent will have to act and when they only have a set of such beliefs. We show that the higher the agent's degree of risk aversion, the higher the agency costs for delegation and hence the less profitable is a flexible contract versus a rigid one. The agent's imprecision aversion in the case of multiple priors introduces another, additional agency costs; it again implies that the higher the degree of imprecision aversion the less profitable flexible contracts versus rigid ones. Even though, with multiple priors, the contract may be designed in such a way that principal and agent end up using 'different beliefs' and hence engage in speculative trade, this is never optimal, in contrast with the case where the parties have sharp heterogeneous beliefs.

JEL Classification:D86, D82, D81.
Keywords: Delegation, Flexibility, Agency Costs, Multiple Priors, Imprecision Aversion.

*We wish to thank B. Lipman, B. Salanié for helpful discussions. Gottardi: Department of Economics, European University Institute, Villa San Paolo, Via della Piazzuola 43, 50133 Firenze. Email: piero.gottardi at EUI.eu. Tallon: Centre d'Economie de la Sorbonne, 106 bld de l'Hôpital, 75013 Paris. Email jmtallon at univ-paris1.fr. Ghirardato: Collegio Carlo Alberto, Via Real Collegio 30, 10024 Moncalieri. Email: ghirardato at econ.unito.it.

1 Introduction

Motivation. A central problem in organizations is the fact that agents assigned a given task may end up having, at the time they have to act, some superior information on the suitability of the various actions which can be taken to perform the assigned task. As a consequence, it may be desirable, in order to enhance the performance of the organization, to grant agents some degree of discretion in their choice of which action to undertake, or to ask them to report their information before specifying which action should be carried out. The obvious difficulty in doing this is that the interests of such agents may not be aligned with those of the organization. This difficulty can be mitigated and possibly eliminated with the use of appropriate monetary transfers to the agents, that is of appropriate compensation contracts. For such contracts to work, some risk must be typically shifted to the agents. If agents are risk averse, doing this is costly. Moreover, if the nature of the possible realizations of the uncertainty, that is of the possible circumstances in which the actions might have to be taken and of their consequences, is not clearly understood a priori, either because some unforeseen contingencies may arise or because the probabilities of the possible events are not commonly agreed among the parties or may be 'ambiguous', some further difficulties and costs arise.

The presence of these costs implies that, in the decision of whether or not and to which extent to delegate to an agent the choice of which action to undertake, a trade-off is faced. On the one hand, the wider the uncertainty concerning the environment in which the agent will have to take his action and the more important is for the organization the fact that the 'right' action is taken in each possible circumstance, the higher are the benefits of delegating the choice to the agent, that is of offering him a flexible contract granting some flexibility in his choice. On the other hand, the extent and nature of this uncertainty also affect the costs of delegation, in a way which depends on the risk aversion of the agent, as well as on the degree of 'ambiguity' of such uncertainty and the attitude towards it exhibited by the agent. The issue is important as this trade-off naturally arises when the architecture of organizations is evaluated. The main focus of this paper is on the analysis of this trade-off, and in particular of how the cost of delegating decisions to superiorly informed agents varies with the structure of the uncertainty and the agents' attitude towards risk and uncertainty.

Model and results. To this end, we will consider a simple contracting situation between a principal and an agent. The agent must take a costly action which generates some revenue for the principal. Before taking his action, but after signing the contract, the agent receives a private signal over the productivity of the various actions. More precisely, we assume the agent privately learns the realization of a variable which, together with the action chosen by the agent, affects the probability of the different realizations of the principal's revenue. The action chosen by the agent is not observable by the principal but we suppose that, at the time of contracting, the principal has the ability to predefine the set of actions, or possible tasks, available to the agent to choose. Thus the principal could specify a determinate action that the agent must undertake in all the possible circumstances he may have to act - what we will call a *rigid*, or non discretionary, contract. Alternatively, the principal could leave the agent some discretion in his behavior, so that the action the agent undertakes may vary with the information received - a *flexible* contract.

Also, the cost for the agent of undertaking the various actions is deterministic. Hence in the absence of monetary transfers contingent on the realization of the principal's revenue the interests of the principal and the agent are not aligned as the latter would always choose the least costly action among the ones available to him. A flexible contract must then include a suitably designed compensation scheme, which might also vary with the agent's report over the signal received, so as to induce him to take the revenue maximizing action for each realization of the signal. But such variability in the compensation generates possible agency costs. In contrast, a rigid contract is simpler, does not need to rely on high-powered incentives and never incurs any agency cost.

Consider first the case where principal and agent have common and sharp probabilistic beliefs over the possible events in which the agent will have to act. In this environment, if the agent is risk neutral[1], agency costs are zero and the optimal flexible contract always dominates, at least weakly, the rigid contract. The benefits of the flexible contract are larger the greater is the variance of the productivity of the various actions the agent may undertake across the different realizations of the signal, that is the greater the relevance of the information received by the agent. This is no longer true if the agent is risk averse, as agency costs are positive in that case. We characterize the optimal flexible contract when the agent has CARA preferences so as to be able to isolate the effects of changes in the agent's risk aversion. We find that at the

[1]We assume the principal is always risk neutral.

optimal flexible contract the agent's compensation also depends on the agent's report over the signal received and that the agent's utility is not equalized across different realizations of the signal. Also, an increase in the agent's degree of (absolute) risk aversion implies a larger agency cost, and hence a lower profitability for the principal of the optimal flexible contract relative to the rigid contracts. Thus, for a sufficiently high degree of risk aversion, a rigid contract always dominates the flexible one. On the other hand, the effects of increasing risk aversion on the form of the incentive contract, for instance on the variability in the compensation paid to the agent across different realizations of the output, prove more sensitive to changes in the parameters of the environment.

We turn then our attention to situations where the information available to the parties concerning the possible events in which the agent will have to act is not precise enough to pin down a single probability distribution. This might be for instance because the circumstances under which the agent finds himself to operate are totally new, with almost no information available. Or it might capture the fact that these events are hard to describe precisely in full details. We model this fact by assuming that principal and agent have a common set of probabilistic beliefs over the likelihood of these events and allowing them to have possibly different degrees of imprecision aversion.[2] We find that if the agent is risk neutral the optimal flexible contract still always dominates the rigid contracts, though the principal's revenue decreases in either party's degree of imprecision aversion. Even though, with multiple priors the compensation contract may be designed in such a way that principal and agent end up "using different beliefs", and hence possibly engage in mutually beneficial speculative trade, this is never optimal. This stands in contrast with the case in which both principal and agent have sharp, but different prior beliefs, where the surplus generated by the contractual relationship is actually enhanced by the possibility of exploiting the benefits of speculative trade (as in Eliaz and Spiegler (2007)).

If, on the other hand, the agent is risk averse, the degree of his imprecision aversion affects the profitability of the optimal flexible contract relative to the rigid ones. This is because the variability of the agent's utility across realizations of the (ambiguous) signal is costly for the agent. At the same time, as noticed above, such variability allows to enhance the agent's incentives. It then follows that imprecision aversion introduces another agency cost which is

[2] We then follow the contraction model of Gajdos, Hayashi, Tallon, and Vergnaud (2008). An alternative approach would have been to assume smooth ambiguity aversion à la Klibanoff, Marinacci, Mukerji (2005).

distinct from and adds to the cost induced by risk aversion. We find in fact that, as the degree of the agent's imprecision aversion increases, the profitability of the optimal flexible contract decreases relative to the rigid contracts. Also, the features of the contract are properly modified, by reducing the utility differential across signal realizations, the more so the higher the degree of imprecision aversion, so as to minimize such agency costs. The pattern of compensations across states is thus different with imprecision aversion and can even lead, when the degree of imprecision aversion is sufficiently high, to fully insure the agent across the different states in which he may have to act, something that is not possible with mere risk aversion. In contrast, with heterogenous but sharp beliefs such utility differential increases with the heterogeneity of beliefs, so as to exploit the opportunity of speculative trades, as the profit differential in favor of flexible contracts.

Thus, the interaction of imprecision aversion with risk aversion has the effect of further increasing the cost of delegation and hence the relative cost of a flexible contract.

Literature.

The choice in organizations between flexible and rigid contracts has been examined in various other papers. Most of them however focused on the case where, in contrast to the setup considered here, monetary transfers are not allowed and the objectives of principal and agent are at least partly aligned. In such environments the agent may be willing to freely transmit some of his private information to the principal. Dessein (2002) investigates the trade-off between contracts where the choice of the action is delegated to the agent and contracts where the principal retains the control over such choice, but uses the information that is reported to him by the agent. He examines in particular how such trade-off varies with the degree of congruence between the objectives of the principal and the agent. Both Aghion and Tirole (1997) and Szalay (2005) study the consequences of delegating - possibly only in part - control over the decision of the action on the agent's incentives to invest in acquiring information.

Probably the closest paper to ours in this literature is Prendergast (2002). He considers an environment where, like in ours, monetary transfers are allowed, the structure of information is given and the agent has superior information. He also examines how the relative benefits of flexible and rigid contracts vary, but with respect to the magnitude of the uncertainty facing the agent, that is the variability in the possible situations in which he may find himself to act.

Prendergast considers the case where the agent is risk neutral and agency costs are exogenously given (as fixed 'monitoring costs'). On the other hand our main focus here, as argued above, is on the endogenous determination of such costs and the analysis of how they vary with the agent's attitude to uncertainty and the precision of the information of principal and agent concerning the uncertainty they face in the contractual design.

A rather different characterization of the trade-off between rigidity and flexibility is provided by Hart and Moore (2008), where the main cost of delegation lies in the variability of the outcome prescribed by the contract and the deadweight losses this generates.

The effects of ambiguity or imprecision in the probabilistic beliefs concerning the possible realizations of the environment faced by parties in contractual situations have been first examined by Mukerji (1998) and Ghirardato (1994). Mukerji (1998) studies a vertical relationship problem, using the Choquet expected utility model of Schmeidler (1989). He shows that, as a result of ambiguity aversion, the optimal contract might be incomplete and, differently from our setup, exhibit low powered incentives. Ghirardato (1994) looks at a standard moral hazard problem but where parties' "beliefs" are non-additive, reflecting uncertainty aversion: each action taken by the agent induces a non-additive distribution on outcomes. His results are not directly comparable with ours, in particular because of the use of different underlying decision models. He can show in some very particular case that a decrease in the degree of non additivity (i.e., imprecision in our setup) will not decrease the principal's profits.

The paper is organized as follows. The next section describes the environment while Section 3 presents the contracting problem, studies its solutions and characterizes them. Section 4 then studies the trade-off between flexible and rigid contracts, how the choice of delegation varies with different features of the environment, in particular the agent's attitude towards risk. In the final non probabilizable uncertainty over the possible circumstances in which decisions have to be made is introduced and its consequences examined.

2 The set-up

We consider a contractual relationship between a principal, say a firm, and an agent, say a worker. The worker has two possible actions, x and y. The output generated by each action is uncertain: it can be either high ($\bar{R}$) or low ($\underline{R}$). The probability of the different output realizations with action x (resp. y) is also uncertain and depends on some event $\theta \in \{\theta_1, \theta_2\}$: it is $\pi(x, \theta)$ (resp. $\pi(y, \theta)$) for $R = \bar{R}$.

We assume the contract is written before the realization of any source of uncertainty (i.e., before the output and θ are realized). In addition, the realization of the output is publicly observable while the action chosen by the agent is only privately known by him. Furthermore θ, describing some events affecting the execution/profitability of the different possible tasks/actions, is only privately observed by the agent, not by the principal (or any third party). To begin with, we examine the case where both principal and agent have sufficient information over the generating process of this uncertainty to come up with a sharp probabilistic belief over it: let p denote their common belief concerning the occurrence of θ_1.

Although the action undertaken by the agent is not observable, we assume that the principal can a priori impose some restrictions over the set of actions available to the agent. To understand the nature of this restriction we can think, for instance, at a situation where the principal can leave the agent free to choose among different types of software (in which case both x and y are available to the agent) or can decide to install only one software on the agent's computer (in which case only one action is available to the agent). In this framework, therefore a compensation contract is a specification of a set of admissible actions[3] $A \subseteq \{x, y\}$ together with a wage payment w from the principal to the agent, where w can depend on the realized level of the output and the agent's announcement about the realization of the event θ. Let $\bar{w}_i$ (resp. $\underline{w}_i$) denote the compensation paid to the agent when the output is $\bar{R}$ (resp. $\underline{R}$) and the (declared) state is $\theta_i, i = 1, 2$.

In particular, we would like to distinguish the case where the full menu of possible actions is available to the agent, $A = \{x, y\}$, from the cases where only action x - or only action y - is available to the agent. We refer to the contract in the first case as a *flexible contract*, since the agent has the flexibility and the discretion to choose the action he thinks is more appropriate for him (and suitable incentives should be specified in the contract to induce the agent to make a choice also in the principal's interest). In the second case we say on the other hand the contract is *rigid*, as it prescribes the agents to always undertake a given action. The contract can then be of type x or of type y according to which action is specified.

[3]The possibility of imposing such restrictions was earlier considered in various papers starting with Holmstrom (1984) (see Alonso and Matouschek (2008), Armstrong and Vickers (2009) for some recent contributions), typically however in the absence of monetary transfers.

It may help to think of the following time-line:

$t=0$ The contract is signed, specifying the payments due to the agent for each possible realization of the output and each announcement of the agent regarding θ. In addition the contract specifies the set $A \subseteq \{x, y\}$ of possible actions available to the agent.

$t=1$ θ is observed by the agent who announces then its value to the principal.

$t=2$ The agent undertakes an action $z \in A$, not observable by the principal.

$t=3$ Output is revealed (i.e., uncertainty about output is resolved and output is observed)

$t=4$ Compensation is paid to the agent, according to the realized output level and the agent's announcement.

Observe that at the time in which the contract is signed there is symmetric information among the parties, the agent does not know the realization of the uncertainty. Asymmetric information will arise at a later stage, when the agent learns some information about the profitability of the different actions, and chooses then which action to take.

Remark 1 *We ignore here the possibility of renegotiation, in particular at the time in which the realization of θ is learnt by the agent* ($t = 1$)*.*

The principal is the residual claimant of the output and is risk neutral. His payoff, when action z_i, $i = 1, 2$, is implemented in state θ_i, is then given by the expected profit:

$$p[\pi(z_1,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(z_1,\theta_1))(\underline{R}-\underline{w}_1)] \\ +(1-p)[\pi(z_2,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(z_2,\theta_2))(\underline{R}-\underline{w}_2)]$$

The agent has a non separable[4] utility function over the compensation received and the cost c_z of undertaking the action $z \in \{x, y\}$ that is chosen. In particular, in most of the paper we will assume the agent is risk averse and exhibits the following preferences:

Assumption 1 *The agent has a CARA utility function:* $u(w, z) = -\frac{e^{-a(w-c_z)}}{a}$*, with* $a > 0$*.*

[4] A nonseparable utility function in the wage received and the cost incurred allows us to study the comparative statics properties of the optimal contract with respect to the agent's level of risk aversion - one of our objectives. With such specification in fact the rate of substitution between actions and wages is constant and changes in the curvature of the agent's utility function only capture changes in the agent's attitude towards risk in the compensation.

The agent's risk attitude is then described by the single parameter a. It is then convenient to normalize the agent's reservation utility as $-\frac{e^{-a\bar{u}}}{a}$.

Our main goal is to investigate in this set-up the relative profitability of flexible and rigid contracts. While the flexible contract offers the agent the opportunity to choose the best action in each possible contingency, delegating the choice to the agent creates an agency problem, since the action is not observable. Hence the wage schedule will be constrained to satisfy a set of appropriate incentive compatibility constraints. On the other hand, in a rigid contract no agency problem arises, since the agent has no discretion, but the action implemented cannot be adjusted to the different contingencies.

We will assume that action x is both more costly and more productive than action y. At the same time, the additional productivity of action x, relative to action y, is uncertain: it is larger in state θ_1 than in state θ_2.

Assumption 2

- $c_x > c_y$, *i.e.*, $\Delta c \equiv c_x - c_y > 0$.
- $\pi(x, \theta_1) > \pi(x, \theta_2) > \pi(y, \theta_2) > \pi(y, \theta_1)$

Moreover, we will focus our attention on the case where the effect of θ on the anticipated profitability of the different actions is sufficiently important that in state θ_1 the expected revenue net of the cost is higher for action x and in state θ_2 it is for action y:

Assumption 3 $(\pi(x, \theta_1) - \pi(y, \theta_1))(\bar{R} - \underline{R}) > \Delta c > (\pi(x, \theta_2) - \pi(y, \theta_2))(\bar{R} - \underline{R})$

Thus, if there were no agency problems (that is, if both θ and the agent's action were publicly observed), the optimal contract would implement action x in θ_1 and y in θ_2.

3 Contracts

3.1 Optimal flexible contract

In our setup a different action in each of the two states can only be implemented with a flexible contract, and is then subject to appropriate incentive constraints. Under Assumptions 2 and 3 it is clear that the optimal action profile to be implemented at a flexible contract is also given by x in state θ_1 and y in θ_2. Hence the optimal flexible contract is obtained as a solution of the following programme:

$$\begin{array}{ll}\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} & p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ & \quad +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)] \\ \text{s.t.} & \end{array}$$

$$\begin{cases} (IC1) & \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(x,\theta_1)e^{-a(\bar{w}_2-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_2-c_x)} \\ (IC2) & \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \\ (IC3) & \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)} \\ (IC4) & \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(x,\theta_2)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_2))e^{-a(\underline{w}_1-c_x)} \\ (IC5) & \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(x,\theta_2)e^{-a(\bar{w}_2-c_x)}+(1-\pi(x,\theta_2))e^{-a(\underline{w}_2-c_x)} \\ (IC6) & \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)} \\ & \\ (PC) & p[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}]+ \\ & \qquad (1-p)[\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}] \leq e^{-a\bar{u}} \end{cases} \tag{P^{flex}}$$

where constraints (IC1)-(IC6) are the incentive compatibility constraints associated to the possible deviations of the agent, given by misreporting one or both the signals received and/or undertaking a different action than the one specified in one or both of the states; (PC) is then the participation constraint.

We show in the next proposition that, at a solution of this problem, only constraints (IC2), (IC6) and (PC) bind and we also derive some properties of the optimal compensation scheme of the agent.

Proposition 1 *Under Assumption 1-3, a necessary and sufficient condition for the existence of the optimal flexible contract is* $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$. *At such contract action x is implemented in state θ_1 and y in θ_2 and the agent's compensation is obtained as solution of the following simplified problem:*

$$\begin{array}{ll}\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} & p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ & \quad +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)] \\ \textit{s.t. } (IC2),(IC6),(PC) & \textit{holding as equalities and} \quad \bar{w}_1 \geq \bar{w}_2,\ \bar{w}_2 \geq \underline{w}_2 \end{array}$$

and exhibits the following properties:

$$\bar{w}_1 \geq \bar{w}_2 > \underline{w}_2 \geq \underline{w}_1.$$

Let $u(\theta_1) = -\pi(x,\theta_1)e^{-a(\bar{w}_1 - c_x)} - (1-\pi(x,\theta_1))e^{-a(\underline{w}_1 - c_x)}]$ denote the agent's expected utility when state θ_1 occurs; similarly, $u(\theta_2) = -\pi(y,\theta_2)e^{-a(\bar{w}_2 - c_y)} - (1-\pi(y,\theta_2))e^{-a(\underline{w}_2 - c_y)}$ is the utility when θ_2 occurs. The properties we showed that (IC2) is binding at an optimum and $\bar{w}_2 > \underline{w}_2$, together with the fact that $\pi(y,\theta_1) < \pi(y,\theta_2)$, have the following important implication:

Corollary 1 *At the optimal flexible contract, $u(\theta_2) > u(\theta_1)$.*

Thus even though the less costly action y is implemented in state θ_2 the optimal contract is still characterized in that state by a wage that varies with the output realizations. At the same time, the utility of the compensation paid to the manager is higher in state θ_2 than in θ_1. The variability in w_2 and the lack of smoothing in the agent's utility across the realizations of θ can both be justified as a way to reduce the variability in the compensation of the wage paid in θ_1: it can in fact be verified (see also the following sections) that (IC2), (IC6) and (PC) can all be satisfied as equality also with a constant level of w_2 - and hence with the same utility for the agent in state θ_2 as in θ_1 - but this is suboptimal.

Remark 2 *To further understand the determinants of these properties of the optimal flexible contract, it is useful to compare them with those of the optimal contract obtained when the realization of θ is publicly observable. In that case actions x and y are still implemented in states θ_1 and θ_2, but we can show[5] that we have $\bar{w}_2 = \underline{w}_2$ as well as $\bar{w}_1 > \underline{w}_1$, and the agent's expected utility is the same in state θ_1 as in θ_2. Thus the variability in w_2 and the agent's utility we found in the optimal flexible contract in Proposition 1 is due to the need to address the incentive problems arising from the agent's private information over θ (a lower variability in w_2 can only be achieved, as we already argued, at the cost of a higher variability of w_1).*

3.2 Rigid contracts

The optimal rigid contract implementing action z, $z = x, y$, in every state is obtained as a solution of the following programme (note that the only constraint is given by PC, no incentive compatibility constraint appears here as the agent has no discretion over the choice of his action):

[5] When θ is observable the only incentive constraints which need to be considered are (IC3), (IC6), the problem is clearly simpler and an explicit solution for the optimal compensation scheme can be derived. See Appendix B, available online at http://www.eui.eu/Personal/Gottardi/ for a formal derivation.

$$\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad p[\pi(z,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(z,\theta_1))(\underline{R}-\underline{w}_1)] \\ +(1-p)[\pi(z,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(z,\theta_2))(\underline{R}-\underline{w}_2)]$$

$$(PC) \quad p[\pi(z,\theta_1)e^{-a(\bar{w}_1-c_z)}+(1-\pi(z,\theta_1))e^{-a(\underline{w}_1-c_z)}]+ \\ (1-p)[\pi(z,\theta_2)e^{-a(\bar{w}_2-c_z)}+(1-\pi(z,\theta_2))e^{-a(\underline{w}_2-c_z)}]=e^{-a\bar{u}} \qquad (P^{rig})$$

Its solution is very simple in the present framework: the wage should be constant ($\bar{w}_1 = \underline{w}_1 = \bar{w}_2 = \underline{w}_2 = w_z$), at the level determined by the participation constraint, thus equal to the expected cost of undertaking action z. In particular:

i) Fixed x contract: the compensation is $w_x = \bar{u} + c_x$, and profits are:

$$[p\pi(x,\theta_1)+(1-p)\pi(x,\theta_2)]\bar{R}+[p(1-\pi(x,\theta_1))+(1-p)(1-\pi(x,\theta_2))]\underline{R}-\bar{u}-c_x$$

ii) Fixed y contract: the compensation is $w_y = \bar{u} + c_y$, and profits are:

$$[p\pi(y,\theta_1)+(1-p)\pi(y,\theta_2)]\bar{R}+[p(1-\pi(y,\theta_1))+(1-p)(1-\pi(y,\theta_2))]\underline{R}-\bar{u}-c_y$$

4 The choice between flexible and rigid contracts

We are now ready to compare the expected profits of the principal at the optimal flexible contract, which we can find by substituting the values of the agent's compensation obtained by solving the problem in Proposition 1 into the principal's objective function, with the expected profits of the rigid contracts found in the previous section. We can then determine which type of contract is preferable. In particular, we intend to analyze how the choice of the type of contract depends on the level of various parameters of the environment (the cost of undertaking the different types of actions, their probabilities of success, describing both the relative productivity of the different actions as well as the relevance of the uncertainty affecting them, preferences and in particular the agent's risk attitude).

As we said, in the optimal flexible contract the agent's action can be better tailored to the different circumstances under which the agent may find himself to operate. However there is also an agency cost in delegating the choice of the action to the agent since the action is not observable and the agent's objectives are not aligned to those of the principal. We should expect therefore that the advantages of flexibility will be higher the bigger is the difference between the productivity of the two types of actions in state θ_1 relative to the other state θ_2 as well as the smaller is the 'agency cost' which has to be paid to implement the action profile x, y.

Since Proposition 1 only provides, as we noticed, an implicit form solution for the values of the agent's compensation at the optimal flexible contract, in the analysis of this section we rely on the consideration of a numerical example, for which the optimal payment schedule can be solved numerically. The parameters describing the environment exhibit the following values:

a	p	$\bar{u}$	$\bar{R}$	$\underline{R}$	c_x	c_y	$\pi(x,\theta_1)$	$\pi(x,\theta_2)$	$\pi(y,\theta_1)$	$\pi(y,\theta_2)$
1	.5	1	10	5	1.5	1	.8	.45	.2	.4

Table 1: Parameter values for the comparative static exercise

The results obtained below prove however to be robust to changes in the parameter chosen.

4.1 Comparative statics with respect to actions' productivity and cost

Our findings for the comparative statics properties with respect to the levels of the probability of success for each action and event in which it is undertaken and to the cost of the different types of actions c_z are summarized in the following table :

Parameter Range	$\pi(x,\theta_1)$ [.75,.9]	$\pi(x,\theta_2)$ [.35,.55]	$\pi(y,\theta_1)$ [.15,.25]	$\pi(y,\theta_2)$ [.3,.5]	c_x [1.2,1.8]	c_y [.7,1.25]
Profit flexible - profit x	+	−	−	+	?	?
Profit flexible - profit y	+	=	−	−	−	+
$\bar{w}_1 - \underline{w}_1$	−	=	+	?	+	−
$\bar{w}_2 - \underline{w}_2$	−	=	−	+	+	−
$u(\theta_2) - u(\theta_1)$	−	=	−	+	+	−

Table 2: Comparative statics with respect to probabilities and costs

For instance, the first column reports the sign of the effects of increasing $\pi(x, \theta_1)$, within the interval indicated, $[.75, .9]$, while keeping the other parameters fixed at the values indicated in Table 1 on the following variables: (i) the differential between the expected profits at the optimal flexible contract and those at the x rigid contract in the first row and at the y one in the second row; (ii) the spread between the compensation paid for the high and low realization of the output when state θ_1 occurs in the third row and when θ_2 occurs in the fourth one; (iii) the difference in expected utility in the two states. A + (resp. -) sign indicates the increase in the parameter value always increases (decreases) the corresponding variable, while a ? indicates the effect is ambiguous, not always of the same sign.

In particular, we find that the profitability of the flexible contract, relative to both rigid contracts, increases if $\pi(x, \theta_1)$ (probability of success with action x in state 1) increases, or $\pi(y, \theta_1)$ decreases. Such changes increase the productivity of the costlier action (x) relative to the less costly one in state θ_1 as well as relative to state θ_2. The same effects are obtained with a decrease in $\pi(x, \theta_2)$, reducing the difference between the productivity of actions x and y in state θ_2. On the other hand, a change in $\pi(y, \theta_2)$ has opposite effects of the profitability of the flexible contract relative to the two rigid ones, while the effect of increasing the costs c_x and c_y of the two actions on the same profit difference is non monotonic.

We also see that the variability in the compensation paid in the state θ_2, where the less costly action is implemented, always moves in the same direction as the utility differential $u(\theta_2) - u(\theta_1)$, suggesting these two are complementary instruments to address the incentive problems generated by the private information over θ, as already mentioned in Remark 2.

4.2 Comparative statics with respect to risk aversion

Another important determinant of the agency costs of implementing a variable action profile and hence of the trade-off between flexible and rigid contracts is the agent's risk attitude. As shown above, the compensation paid at the rigid contracts is independent of the agent's degree of risk aversion (as described, in the case of CARA preferences, by the single parameter a); this on the other hand, matters for the optimal flexible contract.

To see the consequences of the agent's risk attitude it is useful to consider first the extreme case where the agent is also risk neutral. In that case, agency costs are zero as the first best can be implemented, that is the principal can attain the same level of profits as when all incentive compatibility constraints are ignored.

Proposition 2 *When the agent is risk neutral the optimal flexible contract is first best optimal. The expected profit level is* $p[\pi(x, \theta_1)\bar{R} + (1 - \pi(x, \theta_1))\underline{R}] + (1 - p)[\pi(y, \theta_2)\bar{R} + (1 - \pi(y, \theta_2))\underline{R}] -$

$\bar{u} - pc_x - (1-p)c_y$ *and an optimal compensation*[6] *is given by*

$$\begin{aligned} \bar{w}_1 &= \bar{u} + c_x + \frac{1-\pi(x,\theta_1)}{\pi(x,\theta_1)-\pi(y,\theta_2)}\Delta c \\ \underline{w}_1 &= \bar{u} + c_x - \frac{\pi(x,\theta_1)}{\pi(x,\theta_1)-\pi(y,\theta_2)}\Delta c \\ \bar{w}_2 &= \underline{w}_2 = \bar{u} + c_y \end{aligned} \tag{1}$$

Under Assumption 3, therefore, with risk neutrality the flexible contract is always better than the rigid ones. On the other hand, when the agent is risk averse ($a > 0$) agency costs are positive, as in order to satisfy the incentive constraints the principal's profits have to be reduced from their first best level. This clearly implies the flexible contract may no longer dominate the rigid contracts.

Still based on the parameterization described in Table 1, we provide below a characterization of the comparative statics effects of varying the agent's degree of risk aversion a.

Figure 1 shows how the difference between the expected profits at the optimal flexible contract and the two rigid contracts changes with a. We see this relationship is monotonically decreasing. For low levels of risk aversion, the flexible contract is preferable to the two rigid contracts, but as a increases the profit differential becomes progressively smaller and eventually, from $a \sim 1.6$ onwards in the situation considered, the fixed contract specifying the task x to the agent becomes optimal. This pattern appears to be robust to changes in the value of the other parameters and shows that agency costs are increasing with the agent's risk aversion[7]. Hence we can say that agency costs are increasing and the advantages of delegation decreasing in the agent's degree of risk aversion.

The next two figures illustrate then the implications of the level of the agent's degree of risk aversion for the specific properties of the optimal flexible contract. In particular, Figure 2 describes the effect of varying a on the spread between the compensation paid for the high and low output realizations at the optimal flexible contract respectively in state θ_1 (i.e. $\bar{w}_1 - \underline{w}_1$) and θ_2. It shows that the spread in state θ_1 is first decreasing and then increasing in a while the spread in θ_2 is always increasing in a. Figure 3 shows that the utility differential also varies non monotonically with a, first increasing and then decreasing.

[6]Note that this compensation scheme yields $u(\theta_2) = u(\theta_1)$.

[7]A similar pattern also obtains when the realization of θ is commonly observed: increasing risk aversion makes the rigid contracts more attractive. The profits of the flexible contract when θ is observable are strictly higher than when θ is only privately observed, and we find the difference is increasing in risk aversion.

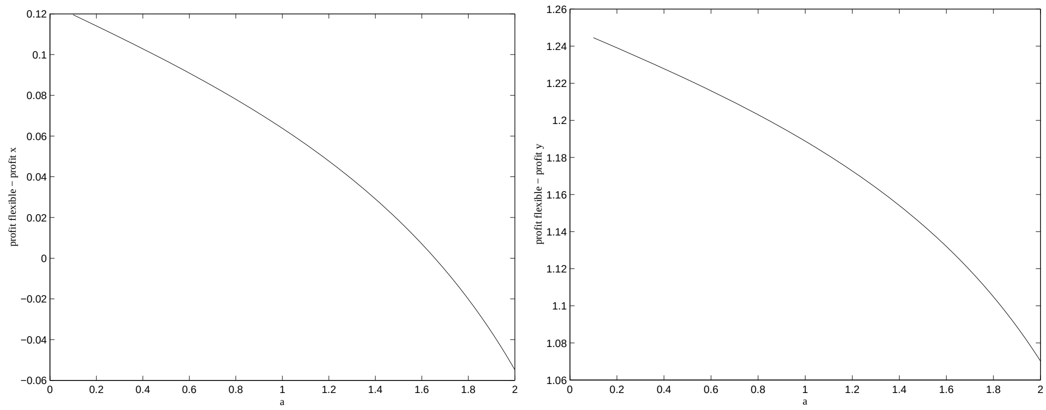


Figure 1: Profit differential between the flexible and rigid contracts as a function of risk aversion

We should point out however that the properties found in Figures 2 and 3, unlike those of Figure 1, are not quite robust to changes in the values of the parameters considered in Table 1[8]. The effects of risk aversion for the properties of optimal incentive contracts prove then to be rather complex, as also found by Jullien, Salanié and Salanié (1999).

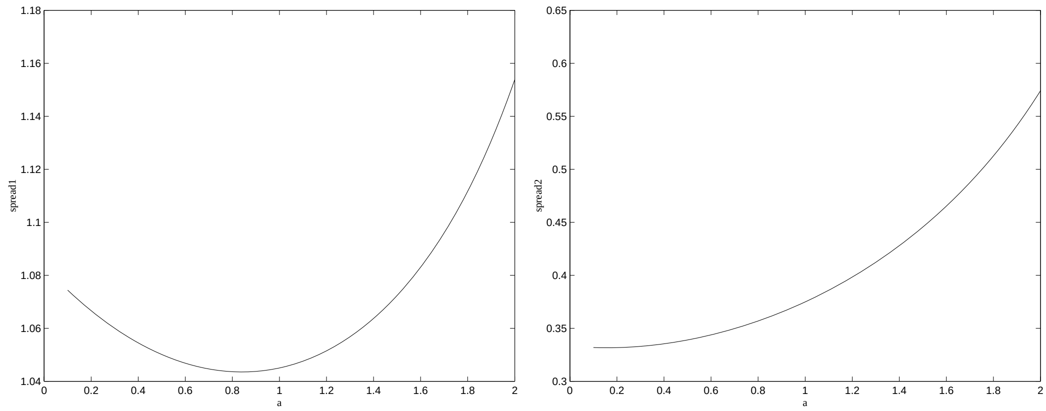


Figure 2: Wage differentials at the optimal flexible contract as a function of risk aversion

Trying to disentangle the various effects of risk aversion, we can first observe that increasing a makes the participation constraint, ceteris paribus, harder to satisfy: such constraint requires the certainty equivalent of the lottery with outcomes $\bar{w}_1 - c_x$,$\underline{w}_1 - c_x$, $\bar{w}_2 - c_y, \underline{w}_2 - c_y$ to equal

[8] Even when θ is observable we find for instance that the spread of the compensation paid in state θ_1 can be non monotonic or monotonically decreasing depending on the values of the parameters.

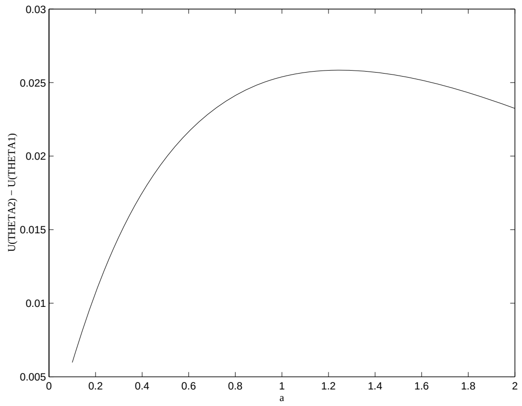


Figure 3: Utility differential $u(\theta_2) - u(\theta_1)$ at the optimal flexible contract as a function of a

$\bar{u}$, but this certainty equivalent decreases with risk aversion.

Each of the two incentive constraints which are binding at an optimum solution, (IC2) and (IC6), then requires a pair of distinct lotteries to have the same expected utility. In the case of (IC2) we cannot rank the two lotteries that are compared, $(\bar{w}_1 - c_x, \underline{w}_1 - c_x)$ (with probabilities $\pi(x, \theta_1), 1 - \pi(x, \theta_1)$) and $(\bar{w}_2 - c_y, \underline{w}_2 - c_y)$ (with probabilities $\pi(y, \theta_1), 1 - \pi(y, \theta_1)$), in terms of riskiness. We know $\underline{w}_1 - c_x$ is the smallest outcome but we do not know how to rank $\bar{w}_1 - c_x$ versus $\bar{w}_2 - c_y$ for instance. Furthermore, the attached probabilities are not the same. Thus, the effect of risk aversion on this constraint is difficult to assess. On the other hand, for (IC6) we can unambiguously say that one of the two lotteries compared is riskier than the other and hence that increasing risk aversion loosens this constraint (i.e., that if the compensation is kept constant when a the constraint becomes slack). Hence, we can say that when a increases, (PC) is harder to satisfy while (IC6) is easier, and the effect on (IC2) is unclear.

5 The choice of delegation with ambiguity

We examine now the case where, at the time the contract is written the information available to the parties concerning the likelihood of the events θ_1, θ_2 is not precise enough for them to have a sharp probability belief over them. This appears rather natural if we are to think of such events as possible contingencies not clearly defined at the contracting stage.

For this purpose, we need a tractable model of decision under uncertainty in such situations, that allows for a simple parameterization of individuals' attitude towards uncertainty, and in particular their ambiguity aversion. We use the model developed by Gajdos, Hayashi, Tallon, Vergnaud (2008) (GHTV henceforth), to which we refer the reader for further details.

In this approach, the decision maker is given some information in the form of a set of ("objective") distributions over the state space. He then uses a maxmin expected utility criterion à la Gilboa and Schmeidler (1989), taking as the set of "priors" a subset of the objective distributions.[9] The smaller the subset, the less ambiguity averse the decision maker. In the limit, if he uses only one distribution in his computation, he is ambiguity neutral (i.e., Bayesian). The model thus gives some structure to the set of priors "used" by the decision maker (contrary to Gilboa and Schmeidler's approach, where the set could be anything since there is no modeled objective set of distributions representing the information available).

Formally, we assume here the set of probability distributions over θ_1 occurring is described by the values $p \in [\underline{p}, \bar{p}]$, which is thus the available information to both parties.[10] In the particular set-up we are considering, with only two "imprecise" states θ_1, θ_2, such criterion amounts to the following: the agent (respectively, the principal) evaluates each possible action according to the minimal expected utility with respect to all $p \in [\underline{p} + \alpha, \bar{p} - \alpha]$ (resp. $p \in [\underline{p} + \psi, \bar{p} - \psi]$). The parameter α describes the agent's imprecision aversion, and ψ that of the principal. The case $\alpha = 0$ is one of extreme imprecision aversion from the agent: he considers all the possible probability distributions and focuses, for each decision, on the one that yields him the worst possible expected utility. The other extreme case is $\alpha = \frac{\bar{p}-\underline{p}}{2}$ which corresponds to expected utility (imprecision neutrality) from the agent: he acts in the same way with imprecise information as in the problem with precise information given by the central probability distribution. Thus the lower is α, the higher the agent's imprecision aversion; in what follows it will thus be convenient to take $-\alpha$ as a measure of the agent's imprecision aversion (and similarly $-\psi$ for the principal).

We will analyze the consequences of changes in the degree of imprecision aversion of both the principal and the agent for the form of the optimal flexible contract and the choice between flexible and rigid contracts. We will compare them with the consequences of changes in the

[9] Note that in the very simple case we look at, this criterion is the same as the one in the α-MEU model of Jaffray (1989) and Ghirardato, Maccheroni and Marinacci (2004). Although formally identical to GHTV's model when there are only two states of nature, in a setup with different agents these models do not provide a clear link between the set of priors of the various decision makers.

[10] The center of this interval, $\hat{p} \equiv \frac{\underline{p}+\bar{p}}{2}$, is then the "central" probability in GHTV's decision criterion.

agent's degree of risk aversion we derived in the previous section. In this regard, it is important to point out that while the degree of risk aversion concerns both the uncertainty over the environment in which decisions will be made (described by θ) as well as the uncertainty over the output realizations, the degree of imprecision aversion only concerns the first source of uncertainty (θ). This is to reflect the fact that, at the contracting stage there is uncertainty concerning the circumstances in which actions will have to be taken in the future, but once θ is realized, the probability distribution of output realizations is not ambiguous.

5.1 Risk neutral but imprecision averse agent and principal

We start with the case of a risk neutral agent and introduce imprecision aversion, possibly for both parties. The optimal flexible contract is now obtained as solution of the problem of maximizing, with respect to $\bar{w}_1, \underline{w}_1, \bar{w}_2, \underline{w}_2$, the following expression of the principal's utility:

$$\begin{aligned}\min_{p\in[\underline{p}+\psi,\bar{p}-\psi]} \{&p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ &+(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)]\}\end{aligned}$$

subject to the agent's participation constraint, given by

$$\begin{aligned}\min_{p\in[\underline{p}+\alpha,\bar{p}-\alpha]} \{&p[\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1-c_x]+ \\ &(1-p)[\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2-c_y]\}\geq\bar{u},\end{aligned}$$

and his incentive compatibility constraints. These are however not affected by the presence of imprecision, which bears only on the θ-state; their expression is then the same as in the previous section (see (8)) and is so omitted.

We can again show that under risk neutrality the principal can attain the same utility as when the incentive constraints are ignored, that is the optimal flexible contract is first best optimal. Moreover, it has the feature that one party is always fully insured across the θ-states. Which one depends obviously on the relative imprecision aversion of principal and agent.

Proposition 3 *When the agent is risk neutral but both principal and agent may be imprecision averse, the optimal flexible contract is still first best optimal. Moreover:,*

- *If $\psi>\alpha$ (i.e., the principal is less imprecision averse than the agent), the agent is fully insured:* $\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1-c_x=\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2-c_y$.

- *If $\alpha > \psi$ (i.e., the agent is less imprecision averse than the principal), the principal is fully insured:* $\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1) = \pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)$.[11]

In the proof we first show that at the optimal contract the expected utility of principal and agent vary comonotonically at the θ states and then that the more imprecision averse party is fully insured. The idea of the argument for the second part of the proof is as follows. Suppose $\psi > \alpha$ and take for instance a situation in which the agent has a higher expected wage in state θ_1 than in state θ_2. This means that, when computing his expected utility, he is actually using the probabilistic belief that puts the least weight on state θ_1, namely $\underline{p}+\alpha$. The net gain from transferring one euro from state θ_1 to θ_2 is hence equal to $1-2(\underline{p}+\alpha)$. For the principal, the net gain of transferring one euro from θ_2 to θ_1 is either $-1+2(\underline{p}+\psi)$ or $-1+2(\bar{p}-\psi)$. The sum of the net gains from this operation is always positive and hence it is always beneficial to make such transfer. A perfectly symmetric reasoning holds when the agent has a lower expected wage in state θ_1 than in state θ_2. Hence, when $\psi > \alpha$ it is optimal for the principal to provide full insurance.

We obtain a similar result for the rigid contracts. When the agent is more imprecision averse than the principal ($\psi > \alpha$), the agent is fully insured, that is the wage is constant, equal to $\bar{u}+c_x$ at the x contract, as in the case of a single prior (or imprecision neutrality). On the other hand, the agent fully insures the principal when the latter is more imprecision averse than the agent ($\alpha > \psi$). Note that in such case rigid contracts are characterized by variable wages, unlike when there is a single prior, as wages will be designed so that the principal's profits are the same for the two θ realizations: $\pi(x,\theta_1)(\bar{R}-\bar{w}_x)-(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_x) = \pi(x,\theta_2)(\bar{R}-\bar{w}_x)-(1-\pi(x,\theta_2))(\underline{R}-\underline{w}_x)$ and similarly for $\bar{w}_y$ and $\underline{w}_y$. Using the participation constraint yields the optimal wage schedule.

Comparing the principal's utility at the optimal flexible contract with that at the rigid contracts yields the following:[12]

Proposition 4 *Under Assumption 3, with risk neutrality and possibly imprecision aversion of both principal and agent, the optimal flexible contract always dominates the rigid ones. Also, the expected utility of the principal at this contract is*

[11] We leave aside the limit case $\psi = \alpha$, similar in all respects to the risk neutrality and imprecision neutrality case studied in the previous sections.

[12] The profits for the two fixed contracts are reported in table 4 in the Appendix.

(i) decreasing in the agent's degree of imprecision aversion $-\alpha$ and does not depend on ψ when $\alpha > \psi$,

(ii) decreasing in the principal's degree of imprecision aversion $-\psi$ and does not depend on α when $\alpha < \psi$.

The first part of the claim can be better understood by observing that in each of these cases the beliefs used by principal and agent are the same both in the flexible and the rigid contracts and we know that with single priors, under Assumption 3, the optimal flexible contract dominates the rigid ones when there are no agency costs. Thus, the choice between flexible and rigid contracts is not affected, under risk neutrality, by the presence of imprecision aversion. On the other hand, the second part of the claim shows that the principal's utility is itself not independent of imprecision aversion, but is rather (weakly) decreasing with imprecision aversion (of either party.) It can be checked that the same is true at the rigid contracts: at both the x and y contracts the principal's utility is (weakly) decreasing with imprecision aversion of either party.

Overall, this shows that even though with imprecision aversion a suitably designed contract could induce the parties to use different "beliefs", this is never optimal, even at the first best where incentive constraints (including those concerning reporting over θ) are ignored. To see this, it is useful to contrast the situation we examined with the one where the two parties have single, but different prior beliefs over θ, say $p^A < p^P$, where p^A (resp. p^P) is the agent's (resp. principal's) belief over θ_1 occurring. With heterogeneous beliefs we find that at the first best contract such difference in beliefs is always exploited: when $p^A < p^P$, adding x to the wage payments in state θ_1 and subtracting y to those in state θ_2 such that $\frac{1-p^P}{p^P} > \frac{x}{y} > \frac{1-p^A}{p^A}$ increases both the principal's profit and the agent's utility. Thus expected wages and expected profits vary anti-comonotonically at the θ states, to exploit the mutually beneficial (speculative) trades allowed by the difference in beliefs, in contrast with what we found with imprecision aversion. The fact that θ is agent's private information over θ limits the extent by which such gains from speculation can be exploited, but still we find the principal's utility is increasing in the beliefs' difference. This is again in contrast with the effect found in Proposition 4 of increasing the parties' imprecision aversion, even though this too augments the difference between the beliefs which may end up being used by them.

To understand such diverse findings, we should point out that with imprecision aversion the "beliefs" used by each party adapt to the payments he receives in a conservative way. Hence to induce the parties to use different beliefs, the payments should be modified in the opposite direction to the one needed to exploit gains from speculative trades, which is clearly non optimal.

To conclude, imprecision aversion of the parties simply decreases the overall surplus available from the relationship when the agent is risk neutral but does not affect the efficiency of contracting. First best is still achievable, the trade off between fixed and flexible contracts is not affected, but the surplus to be split is smaller with imprecision aversion than it is with imprecision neutrality.

5.2 Risk aversion and imprecision aversion

We now examine the effects of imprecision aversion of the two parties in the original setup, where the agent is risk averse and his preferences are as in Assumption 1. We will show that, in contrast to the case of risk neutrality, the trade off between flexible and rigid contracts is affected by the presence of imprecision aversion.

As in the previous section, the degree of imprecision aversion of the agent only matters for the specification of the participation constraint; the incentive compatibility constraints are independent of p and the same as in the specification of the problem with a single prior, (P^{flex}) of Section 3.1. The characterization of the optimal flexible contract proves to be similar to that of the case with a single prior (given in Proposition 1 and Corollary 1), provided the principal is less imprecision averse than the agent (i.e., that risk aversion and imprecision aversion go in the same direction):[13]

Proposition 5 *Suppose Assumptions 1-3 hold, both principal and agent are imprecision averse and the principal is less imprecision averse than the agent (i.e. $\psi > \alpha$). Then at the optimal flexible contract the only binding constraints are (IC2), (IC6), (PC) and payments are such that $\bar{w}_1 \geq \bar{w}_2 \geq \underline{w}_2 \geq \underline{w}_1$ (with $\bar{w}_1 > \underline{w}_1$).*

Since $\bar{w}_2 \geq \underline{w}_2$, the fact that (IC2) and (IC6) hold with equality together with the fact that $\pi(y, \theta_2) > \pi(y, \theta_1)$ imply, as we saw in Section 3.1, that $u(\theta_2) \geq u(\theta_1)$ (with the equality holding

[13] When such condition is violated (the agent is less imprecision averse than the principal) we do not have general characterization results and the analysis of some numerical examples suggests the properties of the optimal contract may be rather different.

if and only if $\bar{w}_2 = \underline{w}_2$). Hence the agent will use beliefs $\bar{p} - \alpha$ (if $u(\theta_2) > u(\theta_1)$) or any p in $[\underline{p} + \alpha, \bar{p} - \alpha]$ (when $u(\theta_2) = u(\theta_1)$) in his evaluation of minimal expected utility. The main difference with the result in Proposition 1 is that we cannot rule out the case $\bar{w}_2 = \underline{w}_2$, in which case the agent is fully insured across θ-state.

Actually we will show that, if imprecision aversion is sufficiently larger for the agent than for the principal, there are parameter configurations such that, at the optimal contract the principal fully insures the agent across the θ states, $u(\theta_2) = u(\theta_1)$, and across the output realizations in the θ_2 state by setting a constant wage $\bar{w}_2 = \underline{w}_2$.

We will illustrate this property for the same environment described in Table 1. Assume, in addition, that the principal is imprecision neutral and the information available over θ is minimal, that is $\bar{p} = 1$ and $\underline{p} = 0$.

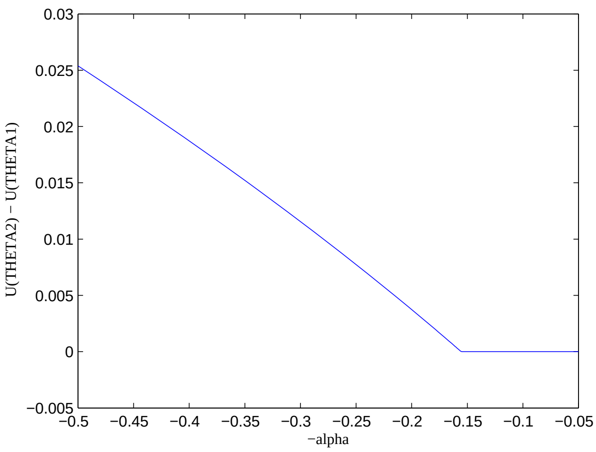
Figure 4 describes how the difference between $u(\theta_2)$ and $u(\theta_1)$, at the optimal flexible contract varies with the degree of the imprecision aversion of the agent (measured as we said by $-\alpha$). The way to read Figure 4 is as follows: as $-\alpha$ goes from $-.5$ (in which case the agent is imprecision neutral) to its maximal level ($-.05$), $u(\theta_2) - u(\theta_1)$ decreases and eventually (at around $\alpha = .15$) becomes zero. Figure 5 shows the spread between the compensation paid for high and low output realization respectively in state θ_1 and θ_2.

How can we understand such findings? Note first that the property we found when parties have a single prior that incentives are helped by a variable wage in state θ_2, and hence $u(\theta_2) > u(\theta_1)$, is still valid. However, imprecision aversion introduces an additional cost for this variability since at the first best, as we saw in the previous section, the agent is fully insured across θ states (when $\psi > \alpha$). This cost is increasing in the agent's degree of imprecision aversion and constitutes an additional agency cost (besides risk aversion) due to imprecision aversion. Hence we face a trade-off between enhancing incentives and minimizing agency costs. When $-\alpha$ is sufficiently high (above $-.15$ in the example) the latter effect prevails over the first one and the agent is fully insured across θ states at the optimal contract.

We consider next the effect of imprecision aversion on the choice between the optimal flexible and the rigid contracts in this same environment. With risk aversion this effect proves again rather complex.

For this we need first to determine the properties of the rigid contracts. It is immediate to see that they are the same as in the case where parties have a single prior studied in Section

Figure 4: Utility differential at the optimal contract as a function of imprecision aversion



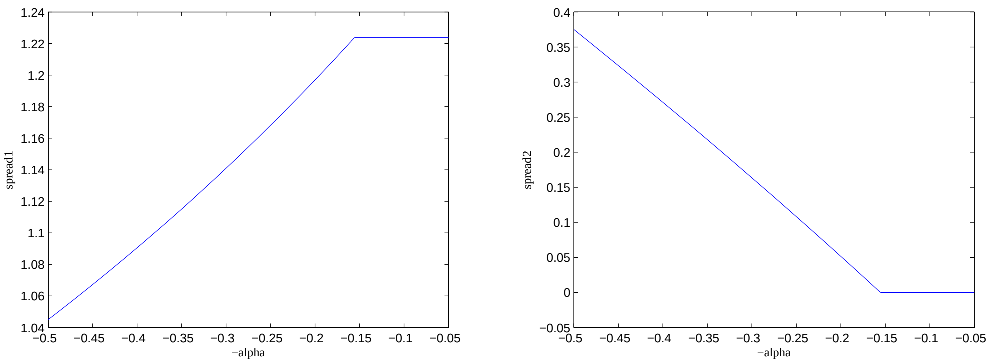


Figure 5: Wage differentials at the optimal contract as a function of imprecision aversion

3.1: the principal fully insures the agent both across θ-states and within each θ state. This is achieved by setting a constant wage $\bar{w} = \underline{w} = w$, which is obtained by solving the participation constraint, yielding $w = \bar{u} + c_x$ (for the x contract, and similarly for the y one).

Figure 6 portrays the difference between the principal's utility at the optimal flexible contract and at the two rigid contracts as a function of the agent's degree of imprecision aversion.

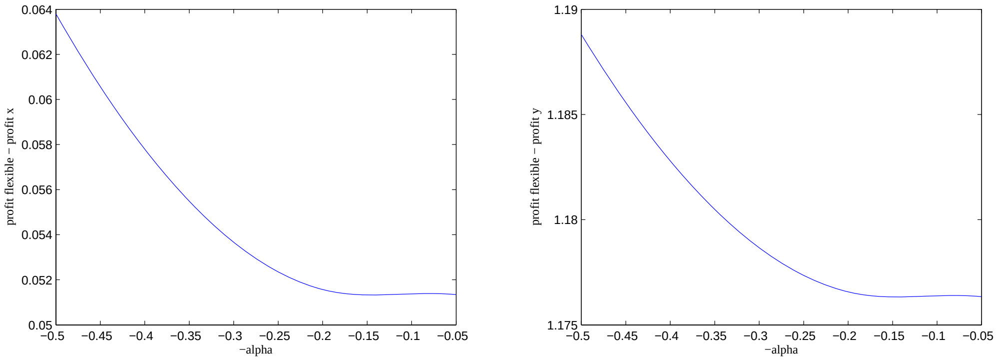


Figure 6: Profit differential as a function of imprecision aversion

The general decreasing shape is similar to the one found with respect to risk aversion (see Figure 1) and reflects the fact that, as noticed above, agency costs are increasing in the agent's degree of imprecision aversion. On the other hand, the mechanism behind and the nature of such costs are rather different, as attested by the diversity in the pattern of the wage spread and utility differentials (contrast Figures 4 and 5 with Figures 2 and 3 respectively.) We see in particular that the profit differential decreases until $-\alpha$ reaches a level of around $-.15$ and is then constant beyond that; as we noticed above, for $-\alpha > -.15$ the agent is fully insured across the θ states, thus the agency costs due to further increasing the agent's imprecision aversion are zero.

We end this section by contrasting again these findings with those obtained when parties are imprecision neutral but have different beliefs p^P and p^A over the realization of state θ_1. In particular we consider the case where p^P and p^A are exactly the same as the "beliefs used" by the parties at the optimal flexible contracts for varying degrees of imprecision aversion of the agent displayed in Figures 4- 6. Hence we always have $p^P < p^A$ since by Proposition 5 we know that at the optimal flexible contract the exhibit this property, $p^P = .5$ and p^A varies between .5

and .95.

We find[14] that when the beliefs' difference is not too high (that is, $.5 \leq p^A \lesssim .85$) the optimal flexible contract is exactly the same as the one found above under imprecision aversion. This can be understood as follows. Note first that, for the parameter configurations of Table 1 expected surplus (output net of costs) is higher in θ_1 than in θ_2. Thus when $u(\theta_2) > u(\theta_1)$ the principal's expected profits are higher in θ_1. The payoffs of principal and agent vary anti-comonotonically and in the opposite direction of what speculative trades would dictate. Hence we can say that in this region the incentive enhancing role of $\bar{w}_2 > \underline{w}_2$ and $u(\theta_2) > u(\theta_1)$ prevails over the benefits of exploiting speculative trades and none of them occurs (analogously to what we saw before happens for the benefits of reducing the agency costs of imprecision aversion).

On the other hand, when the difference in beliefs is high, $p^A \gtrsim .85$, the optimal flexible contract with heterogeneous priors becomes rather different from the one with imprecision aversion: $u(\theta_2) - u(\theta_1)$ is negative and not zero, and $\bar{w}_1 - \underline{w}_1$ keeps increasing instead of staying constant. This can be explained by the fact that in this region the beliefs' difference is so high that the benefits of exploiting it by engaging in speculative trades between principal and agent now outweigh the incentive costs this implies. Hence we have a shift in the agent's payoffs from state θ_2 to θ_1 and viceversa for the principal, as the difference in beliefs would suggest. The "cost" of this is a larger variability of the compensation in θ_1, needed to sustain incentives. In contrast, with imprecision aversion in this region the agent is fully insured, to minimize the agency costs due to it, but it never pays as we noted to engage in speculative trades.

These properties also reveal themselves when we look at the profit differential between flexible and rigid contracts. With imprecision aversion we saw such difference stays constant when $-\alpha$ is increased beyond $-.35$, while with heterogenous priors we find the difference is actually increasing in p^A when p^A is increased beyond .85. Thus in such region increasing the beliefs' difference makes the flexible contract more profitable compared to the rigid ones. This is due to the fact that, as observed above, the optimal flexible contract exhibits the presence of speculative trades, and these are more profitable the larger the difference in beliefs.

[14]The precise results are reported in Figures 7-9 in Appendix B. Notice that the characterization provided in Proposition 1 does not always hold when $p^P < p^A$ (while it does in the opposite case, $p^P > p^A$) and we have therefore to resort to numerical methods.

References

[1] Aghion, P. and J. Tirole (1997): Formal and Real Authority in Organizations, *Journal of Political Economy* 105, 1-29.

[2] Armstrong, M. and J. Vickers (2009): A Model of Delegated Project Choice, mimeo.

[3] Alonso R. and N. Matouschek (2008): Optimal Delegation, *Review of Economic Studies*, 75(1), 259-294.

[4] Dessein, W. (2002): Authority and Communication in Organizations, *Review of Economic Studies* 69, 811-838.

[5] Eliaz, K. and R. Spiegler (2007): A Mechanism Design Approach to Speculative Trade, *Econometrica* 75(3), 875-884.

[6] Gajdos, T., T. Hayashi, J.-M. Tallon and J.-C. Vergnaud (2008): Attitude toward Imprecise Information, *Journal of Economic Theory* 140(1), 23-56.

[7] Ghirardato, P. (1994): Agency Theory with Non-Additive Uncertainty, mimeo, http://web.econ.unito.it/gma/paolo/age.pdf

[8] Ghirardato, P., F. Maccheroni and M. Marinacci. (2004): Differentiating Ambiguity and Ambiguity Attitude, *Journal of Economic Theory* 118, 133–173.

[9] Gilboa, I., and D. Schmeidler (1989): Maxmin Expected Utility with a Non-Unique Prior, *Journal of Mathematical Economics* 18, 141-153.

[10] Hart, O. and J. Moore (2008): Contracts as Reference Points, *Quarterly Journal of Economics* 123(1), 1-48.

[11] Holmstrom, B. (1984): On the Theory of Delegation, in *Bayesian Models in Economic Theory*, ed. by M. Boyer and R. Kihlstrom. Elsevier, Amsterdam.

[12] Jaffray, J.-Y. (1989): Linear utility for belief functions, *Operations Research Letters* 8, 107-112.

[13] Jullien, B., B. Salanié and F. Salanié (1999): Should More Risk-Averse Agents Exert More Effort?, *The Geneva Papers on Risk and Insurance Theory* 24, 19–28.

[14] Klibanoff, P., M. Marinacci and S. Mukerji, S. (2005): A Smooth Model of Decision Making under Ambiguity, *Econometrica* 73(6), 1849-1892.

[15] Mukerji, S. (1998): Ambiguity Aversion and Incompleteness of Contractual Form, *American Economic Review* 88(5), 1207-3.

[16] Prendergast, C. (2002): The Tenuous Trade-Off between Risk and Incentives, *Journal of Political Economy* 110(5), 1071-1102.

[17] Schmeidler, D. (1989): Subjective probability and expected utility without additivity, *Econometrica* 57, 571-587.

[18] Szalay, D. (2005): The Economics of Extreme Options and Clear Advice, *Review of Economic Studies* 72, 1173-1198.

Appendix

Proof of Proposition 1

The proof is decomposed into three Propositions (A.1 to A.3)

Proposition A.1: *At an optimal flexible contract the compensation exhibits the following properties:* $\bar{w}_1 \geq \bar{w}_2 \geq \underline{w}_2 \geq \underline{w}_1$*, and* $\bar{w}_1 > \underline{w}_1$*. Furthermore:*

(i) if $\underline{w}_2 > \underline{w}_1$*, then* $\bar{w}_1 > \bar{w}_2$ *and (IC2) and (IC6) are binding, while (IC1), (IC3), (IC4) and (IC5) are slack.*

(ii) if $\underline{w}_2 = \underline{w}_1$*, then* $\bar{w}_1 = \bar{w}_2$ *and (IC2) binds, while (IC1), (IC3), and (IC6) are automatically satisfied ((IC1) and (IC6) as equalities), and (IC4) and (IC5) are slack.*[15]

Proof.

Step 1: At an optimal solution $\bar{w}_2 \geq \underline{w}_2$.

Proof. Suppose not, that is, $\bar{w}_2 < \underline{w}_2$.

Then, it is immediate to show, given that $c_y < c_x$, $\pi(y,\theta_1) < \pi(x,\theta_1)$, and $\pi(y,\theta_2) < \pi(x,\theta_2)$, that both (IC1) and (IC5) are slack. Start with (IC1): the right hand side of (IC1) is strictly greater than the right hand side of (IC2) and hence, (IC1) is slack. For (IC5), rewrite the constraint as:

$$[\pi(y,\theta_2)e^{-a\bar{w}_2} + (1-\pi(y,\theta_2))e^{-a\underline{w}_2}]e^{ac_y} \leq [\pi(x,\theta_2)e^{-a\bar{w}_2} + (1-\pi(x,\theta_2))e^{-a\underline{w}_2}]e^{ac_x}$$

Then, under the assumption, the expression in bracket in the left hand side is strictly smaller than the one in the right hand side, which implies, together with the order on the cost, that (IC5) is slack.

We now show that if $\bar{w}_2 < \underline{w}_2$, then it is possible to find an improvement for the principal by pushing $\bar{w}_2$ and $\underline{w}_2$ closer. Consider $\Delta\bar{w}_2 > 0$ and $\Delta\underline{w}_2 < 0$ (i.e. a discrete change in $\bar{w}_2, \underline{w}_2$) such that:

(i) $\pi(y,\theta_2)e^{-a(\bar{w}_2+\Delta\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2+\Delta\underline{w}_2-c_y)} = \pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}$

and,

(ii) $\pi(y,\theta_2)\Delta\bar{w}_2 + (1-\pi(y,\theta_2))\Delta\underline{w}_2 < 0$

[15] The argument shows that the stated result holds whenever the agent's utility function can be decomposed as $u(w-c) = u(w)u(-c)$ with u (strictly) concave and increasing (i.e. not only for , CARA).

Note that it is possible to find such a $\Delta\bar{w}_2$ and $\Delta\underline{w}_2$ by concavity of the utility function. By condition (ii), we can conclude that this change improves the principal's profit. It remains to show that it is feasible and satisfies the remaining incentive and the participation constraints.

(IC3) is trivially satisfied since it does not depend on $\Delta\bar{w}_2$ and $\Delta\underline{w}_2$. (IC4) and (IC6) are satisfied by construction, given condition (i) and the same is true for (PC). Thus, it remains to show that (IC2) holds. Given that the left hand side of (IC2) remains unchanged, it is enough to show that:

$$\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_2+\Delta\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2+\Delta\underline{w}_2-c_y)}$$

This follows from condition (i) and the fact that $\pi(y,\theta_1) < \pi(y,\theta_2)$. Indeed, (i) is equivalent to $\pi(y,\theta_2)[e^{-a(\bar{w}_2+\Delta\bar{w}_2)} - e^{-a\bar{w}_2}] + (1-\pi(y,\theta_2))[e^{-a(\underline{w}_2+\Delta\underline{w}_2)} - e^{-a\underline{w}_2}] = 0$. The first term is negative while the second is positive, so we have, given that $\pi(y,\theta_1) < \pi(y,\theta_2)$,

$\pi(y,\theta_1)[e^{-a(\bar{w}_2+\Delta\bar{w}_2)} - e^{-a\bar{w}_2}] + (1-\pi(y,\theta_1))[e^{-a(\underline{w}_2+\Delta\underline{w}_2)} - e^{-a\underline{w}_2}] > 0$, which yields the desired result. □

Step 2: At an optimal solution $\bar{w}_1 > \underline{w}_1$.

Proof. This is a direct consequence of (IC3). □

Step 3: At an optimal solution (IC2) binds.

Proof. We distinguish two cases, according to whether $\underline{w}_2 = \bar{w}_2$ or $\underline{w}_2 < \bar{w}_2$.

Case 1.: $\underline{w}_2 = \bar{w}_2 \equiv w_2$.

In that event, (IC5) is automatically satisfied and therefore can be dropped. Furthermore, (IC2) implies (IC1) which can so also be dropped. Now, by Step 2 $\underline{w}_1 < \bar{w}_1$. Hence, given that $\pi(y,\theta_2) > \pi(y,\theta_1)$, it is possible to show that (IC2) and (IC6) imply (IC3), which can be dropped.

Obviously, (IC2) and (IC4) cannot be simultaneously binding. We show next that (IC2) has to bind and therefore (IC4) is slack. Assume not, i.e., (IC2) is slack and consider (an infinitesimal change) $d\bar{w}_1 < 0$, $d\underline{w}_1 = 0$ and $dw_2 > 0$. Since (IC2) is slack, for sufficiently small such quantities it continues to hold. (IC4) and (IC6) remain satisfied. Choosing $d\bar{w}_1 = -\frac{(1-p)e^{-a(w_2-c_y)}}{p\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}}dw_2$ ensures that the participation constraint continues to hold. By construction, the change in the objective function is equal to $(1-p)\left[\frac{e^{-a(w_2-c_y)}}{e^{-a(\bar{w}_1-c_x)}} - 1\right] dw_2$. Given that $dw_2 > 0$, this quantity is positive (hence leading to an increase in the objective function) if $e^{-a(w_2-c_y)} > e^{-a(\bar{w}_1-c_x)}$, that is if $\bar{w}_1 > w_2 + \Delta c$. This property always holds in the case under consideration ($\underline{w}_2 = \bar{w}_2$): (IC2) can in fact be rewritten as follows:

$$\pi(x,\theta_1)e^{-a\bar{w}_1} + (1-\pi(x,\theta_1))e^{-a\underline{w}_1} \leq e^{-a(w_2+\Delta c)},$$

which in turn implies, together with the property $\bar{w}_1 > \underline{w}_1$ established in Step 2, that $e^{-a\bar{w}_1} < e^{-a(w_2+\Delta c)}$, and therefore $\bar{w}_1 > w_2 + \Delta c$.

Hence, whenever (IC2) is slack we can find a perturbation of the wage bill that increases the Principal's profit, contradicting optimality of the contract. Therefore (IC2) has to bind (and hence (IC4) is slack).

Case 2.: $\underline{w}_2 < \bar{w}_2$.

Assume (IC2) is slack and consider a discrete change $\Delta\underline{w}_2 > 0$ and $\Delta\bar{w}_2 < 0$ such that: (i) $\pi(y,\theta_2)\Delta\bar{w}_2 + (1-\pi(y,\theta_2)\Delta\underline{w}_2 < 0$ and (ii) $\pi(y,\theta_2)e^{-a(\bar{w}_2+\Delta\bar{w}_2)} + (1-\pi(y,\theta_2))e^{-a(\bar{w}_2+\Delta\underline{w}_2)} = \pi(y,\theta_2)e^{-a\bar{w}_2} + (1-\pi(y,\theta_2))e^{-a\bar{w}_2}$. Such numbers exist by strict concavity of u.

Notice that (IC3), (IC4), (IC6) and (PC) are unaffected by these changes and thus continue to hold. We now check (IC1). The left hand side is unchanged and we therefore need to show that: $\pi(x,\theta_1)e^{-a(\bar{w}_2-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_2-c_x)} \leq \pi(x,\theta_1)e^{-a(\bar{w}_2+\Delta\bar{w}_2-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_2+\Delta\underline{w}_2-c_x)}$, which is equivalent to

$$\pi(x,\theta_1)[e^{-a\bar{w}_2} - e^{-a(\bar{w}_2+\Delta\bar{w}_2)}] + (1-\pi(x,\theta_1))[e^{-a\underline{w}_2} - e^{-a(\underline{w}_2+\Delta\underline{w}_2)}] \leq 0$$

But this holds as a consequence of (ii), given that $\Delta\underline{w}_2 > 0$ and $\Delta\bar{w}_2 < 0$ and $\pi(x,\theta_1) > \pi(y,\theta_2)$. Thus, (IC1) continues to hold.

It remains to check (IC5). By construction, the left hand side is unaffected by the change. Given that $\pi(x,\theta_2) > \pi(y,\theta_2)$, one can replicate the argument showing that (IC1) holds to prove that (IC5) holds as well. □

Step 4: At an optimal solution (IC4) is slack.

Proof. Given that $\bar{w}_2 \geq \underline{w}_2$ and $\pi(y,\theta_2) \geq \pi(y,\theta_1)$, we have

$\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)}$.

From the previous step, we know (IC2) is binding, and hence

$$\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}$$

Given that $\bar{w}_1 > \underline{w}_1$ and $\pi(x,\theta_1) \geq \pi(x,\theta_2)$, this establishes that (IC4) is slack, i.e.

$$\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} < \pi(x,\theta_2)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_2))e^{-a(\underline{w}_1-c_x)}$$

□

Step 5: At an optimal solution (IC5) is slack.

Proof. If $\bar{w}_2 = \underline{w}_2$, this is obvious. Consider next the case $\bar{w}_2 > \underline{w}_2$. Then, $\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)}$. From Step 3 we know that (IC2) binds, i.e., $\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} = \pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}$.

Now, by (IC1), $\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(x,\theta_1)e^{-a(\bar{w}_2-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_2-c_x)}$ and hence, since $\bar{w}_2 > \underline{w}_2$ and $\pi(x,\theta_1) > \pi(x,\theta_2)$, $\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} < \pi(x,\theta_2)e^{-a(\bar{w}_2-c_x)} + (1-\pi(x,\theta_2))e^{-a(\underline{w}_2-c_x)}$. As a consequence,

$$\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)} < \pi(x,\theta_2)e^{-a(\bar{w}_2-c_x)} + (1-\pi(x,\theta_2))e^{-a(\underline{w}_2-c_x)}$$

showing that (IC5) is slack. □

Step 6: At an optimal solution, $\bar{w}_1 \geq \bar{w}_2$ and $\underline{w}_1 \leq \underline{w}_2$. Furthermore, if $\underline{w}_1 = \underline{w}_2$, then it must be the case that $\bar{w}_1 = \bar{w}_2$.

Proof. Rewrite (IC1) and (IC6) as follows:

$$\pi(x,\theta_1)\left[e^{-a\bar{w}_1} - e^{-a\bar{w}_2}\right] \leq (1-\pi(x,\theta_1))\left[e^{-a\underline{w}_2} - e^{-a\underline{w}_1}\right] \tag{2}$$

$$\pi(y,\theta_2)\left[e^{-a\bar{w}_2} - e^{-a\bar{w}_1}\right] \leq (1-\pi(y,\theta_2))\left[e^{-a\underline{w}_1} - e^{-a\underline{w}_2}\right] \tag{3}$$

Assume $\bar{w}_1 < \bar{w}_2$, then (2) implies that $\underline{w}_1 > \underline{w}_2$ and (2) and (3) yield that:

$$\frac{\pi(x,\theta_1)}{1-\pi(x,\theta_1)} \leq \frac{e^{-a\underline{w}_2} - e^{-a\underline{w}_1}}{e^{-a\bar{w}_1} - e^{-a\bar{w}_2}} \leq \frac{\pi(y,\theta_2)}{1-\pi(y,\theta_2)}$$

But this is not possible given that $\pi(y,\theta_2) < \pi(x,\theta_1)$. Hence, $\bar{w}_1 \geq \bar{w}_2$. A similar argument establishes that $\underline{w}_1 \leq \underline{w}_2$.

Finally, suppose that $\underline{w}_1 = \underline{w}_2$. Then, using the fact that (IC2) is binding, one can rewrite (IC3) as follows:

$$\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)}$$

which yields $\bar{w}_1 = \bar{w}_2$, since we assumed that $\underline{w}_1 = \underline{w}_2$ and we proved above that $\bar{w}_1 \geq \bar{w}_2$. . □

Step 7: At an optimal solution (IC3) is slack if $\underline{w}_1 < \underline{w}_2$. If $\underline{w}_1 = \underline{w}_2$, (IC3) is automatically satisfied as equality.

Proof. Use (IC2), which is binding, to rewrite (IC3) as follows:

$$\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)} + (1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)} \quad (4)$$

If $\underline{w}_1 < \underline{w}_2$, (4) is equivalent, given that $\bar{w}_1 \geq \bar{w}_2$, to

$$\frac{e^{-a\bar{w}_2} - e^{-a\bar{w}_1}}{e^{-a\underline{w}_1} - e^{-a\underline{w}_2}} \leq \frac{1-\pi(y,\theta_1)}{\pi(y,\theta_1)}$$

But we know by (IC6) that

$$\frac{e^{-a\bar{w}_2} - e^{-a\bar{w}_1}}{e^{-a\underline{w}_1} - e^{-a\underline{w}_2}} \leq \frac{1-\pi(y,\theta_2)}{\pi(y,\theta_2)}$$

and hence, since $\pi(y,\theta_1) < \pi(y,\theta_2)$, (IC3) is slack.

If $\underline{w}_1 = \underline{w}_2$, then we know that $\bar{w}_1 = \bar{w}_2$ and (4) - hence (IC3) - is automatically satisfied. □

Step 8: At an optimal solution (IC1) and (IC6) cannot be simultaneously binding if $\underline{w}_1 < \underline{w}_2$. If $\underline{w}_1 = \underline{w}_2$ they are both automatically satisfied (as equalities).

Proof. Assume $\underline{w}_1 < \underline{w}_2$ and observe that if (IC2) and (IC6) were binding, one would have

$$\frac{1-\pi(x,\theta_1)}{\pi(x,\theta_1)} = \frac{e^{-a\bar{w}_2} - e^{-a\bar{w}_1}}{e^{-a\underline{w}_1} - e^{-a\underline{w}_2}} = \frac{1-\pi(y,\theta_2)}{\pi(y,\theta_2)}$$

a contradiction. □

Step 9: At an optimal solution, if $\underline{w}_1 < \underline{w}_2$ (IC6) binds.

Proof. Assume $\underline{w}_1 < \underline{w}_2$ and (IC6) is slack and consider changing $\bar{w}_1$ and $\underline{w}_1$ by respectively $\Delta\bar{w}_1 < 0$ and $\Delta\underline{w}_1 > 0$ such that, (i) $\pi(x,\theta_1)\Delta\bar{w}_1 + (1-\pi(x,\theta_1))\Delta\underline{w}_1 < 0$ and (ii), $\pi(x,\theta_1)e^{-a(\bar{w}_1+\Delta\bar{w}_1)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1+\Delta\underline{w}_1)} = \pi(x,\theta_1)e^{-a\bar{w}_1} + (1-\pi(x,\theta_1))e^{-a\underline{w}_1}$. Such a change exists by strict concavity of the utility function and provides higher profit to the principal.

Furthermore, this change does not affect (IC1), (IC2), and (PC) and is feasible given that (IC3), (IC4), (IC5) and (IC6) are slack. Hence, (IC6) has to be binding at an optimal solution whenever $\underline{w}_1 < \underline{w}_2$. □

Steps 1-9 complete the proof of Proposition A.1. From this result it then immediately follows:

Corollary A.1: *The optimal flexible contract can be obtained as a solution to the simpler programme below:*

$$\begin{array}{ll} \max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} & p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ & \quad +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)] \\ \text{s.t.} & \\ \left\{ \begin{array}{l} (IC2),(IC6),(PC) \text{ (as stated in } (P^{flex})) \text{ and} \\ (WI) \quad \bar{w}_1 \geq \bar{w}_2 \\ (WII) \quad \bar{w}_2 \geq \underline{w}_2 \end{array} \right. & \end{array} \qquad (P^{flex,R})$$

Observe the constraint $\underline{w}_2 \geq \underline{w}_1$ is implied by $(W1)$ and $(IC6)$.

Proposition A.2: *A necessary and sufficient condition for the existence of a solution to problem $(P^{flex,R})$ (and hence also to (P^{flex})) is that $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$.*

Proof. (*Necessity*) Assume that (IC2) and (IC6) hold as equalities and wages are ordered $\bar{w}_1 \geq \bar{w}_2 \geq \underline{w}_2 \geq \underline{w}_1$.

¿From (IC6) holding as equality, we get $\pi(y,\theta_2)(e^{-a\bar{w}_2}-e^{-a\bar{w}_1}) = (1-\pi(y,\theta_2))(e^{-a\underline{w}_1}-e^{-a\underline{w}_2})$ and, given that the two terms are non-negative and $\pi(y,\theta_2) > \pi(y,\theta_1)$:

$$\pi(y,\theta_1)(e^{-a\bar{w}_2}-e^{-a\bar{w}_1}) \leq (1-\pi(y,\theta_1))(e^{-a\underline{w}_1}-e^{-a\underline{w}_2})$$

i.e., $\pi(y,\theta_1)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_2-c_y)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)}$.

Using (IC2) this implies that

$$\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \leq \pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)}+(1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)}$$

or

$$(\pi(x,\theta_1)e^{a\Delta c}-\pi(y,\theta_1))e^{-a(\bar{w}_1-c_y)} \leq ((1-\pi(y,\theta_1))-(1-\pi(x,\theta_1)e^{a\Delta c}))e^{-a(\underline{w}_1-c_y)}$$

A necessary condition for the above inequality to hold is that $((1-\pi(y,\theta_1))-(1-\pi(x,\theta_1)e^{a\Delta c})) \geq 0$, i.e., $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$.

(*Sufficiency*)

The two binding constraints (IC2) and (IC6) enable one to solve for $\bar{z}_1 = e^{-a\bar{w}_1}$ and $\underline{z}_1 \equiv e^{-a\underline{w}_1}$ as a function of $\bar{z}_2 \equiv e^{-a\bar{w}_2}$ and $\underline{z}_2 \equiv e^{-a\underline{w}_2}$, yielding:

$$\bar{z}_1 = \frac{\left((1-\pi(y,\theta_2))[\pi(y,\theta_1)\bar{z}_2+(1-\pi(y,\theta_1))\underline{z}_2]e^{-a\Delta c}-(1-\pi(x,\theta_1))[\pi(y,\theta_2)\bar{z}_2+(1-\pi(y,\theta_2))\underline{z}_2]\right)}{\pi(x,\theta_1)-\pi(y,\theta_2)}$$

$$\underline{z}_1 = \frac{\left(\pi(x\theta_1)[\pi(y,\theta_2)\bar{z}_2+(1-\pi(y,\theta_2))\underline{z}_2]-\pi(y,\theta_2)[\pi(y,\theta_1)\bar{z}_2+(1-\pi(y,\theta_1))\underline{z}_2]e^{-a\Delta c}\right)}{\pi(x,\theta_1)-\pi(y,\theta_2)}$$

We now want to establish that under the condition $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$, it is possible to find $0 \leq \bar{z}_2 \leq \underline{z}_2$ such that:

$$\begin{aligned} \bar{z}_1 &> 0 \\ \bar{z}_1 &\leq \bar{z}_2 \\ \underline{z}_2 &\leq \underline{z}_1 \\ \bar{z}_2 &\leq \underline{z}_2 \end{aligned}$$

These inequalities ensure that values of the wages satisfying $\bar{w}_1 \geq \bar{w}_2 \geq \underline{w}_2 \geq \underline{w}_1$ can be found. The first inequality is equivalent, under the condition $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$, to

$$\frac{(1-\pi(x,\theta_1))\pi(y,\theta_2)-(1-\pi(y,\theta_2))\pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_2))[(1-\pi(y,\theta_1))e^{-a\Delta c}-(1-\pi(x,\theta_1))]} < \frac{\underline{z}_2}{\bar{z}_2} \tag{5}$$

The next two inequalities are actually equivalent (again under the condition $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$) to the same inequality:

$$\frac{\pi(x,\theta_1)-\pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_1))e^{-a\Delta c}-(1-\pi(x,\theta_1))} \geq \frac{\underline{z}_2}{\bar{z}_2} \tag{6}$$

Thus, to show that we can find some values $\bar{z}_2, \underline{z}_2$ satisfying the last inequality, $\bar{z}_2 \leq \underline{z}_2$, and such that (5) and (6) hold, we need to establish that the following holds:

$$\max\left(1, \frac{(1-\pi(x,\theta_1))\pi(y,\theta_2)-(1-\pi(y,\theta_2))\pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_2))[(1-\pi(y,\theta_1))e^{-a\Delta c}-(1-\pi(x,\theta_1))]}\right) < \frac{\pi(x,\theta_1)-\pi(y,\theta_1)e^{-a\Delta c}}{(1-\pi(y,\theta_1))e^{-a\Delta c}-(1-\pi(x,\theta_1))}$$

Straightforward computation shows that, under the assumption that $\frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)} \geq e^{a\Delta c}$, this is indeed the case. ■

Before solving problem ($P^{flex,R}$), observe that one can rewrite it, with the following change of variables $z = e^{-aw}$, as a problem with a (strictly) concave objective and linear constraints:

$$\max_{\bar{z}_1,\underline{z}_1,\bar{z}_2,\underline{z}_2} \quad \begin{array}{l} p[\pi(x,\theta_1)(\bar{R}+\frac{\log \bar{z}_1}{a})+(1-\pi(x,\theta_1))(\underline{R}+\frac{\log \underline{z}_1}{a})] \\ +(1-p)[\pi(y,\theta_2)(\bar{R}+\frac{\log \bar{z}_2}{a})+(1-\pi(y,\theta_2))(\underline{R}+\frac{\log \underline{z}_2}{a})] \end{array}$$

$$\begin{cases} (IC2') \;\; \pi(x,\theta_1)e^{ac_x}\bar{z}_1+(1-\pi(x,\theta_1))e^{ac_x}\underline{z}_1 = \pi(y,\theta_1)e^{ac_y}\bar{z}_2+(1-\pi(y,\theta_1))e^{ac_y}\underline{z}_2 \\ (IC6') \;\; \pi(y,\theta_2)e^{ac_y}\bar{z}_2+(1-\pi(y,\theta_2))e^{ac_y}\underline{z}_2 = \pi(y,\theta_2)e^{ac_y}\bar{z}_1+(1-\pi(y,\theta_2))e^{ac_y}\underline{z}_1 \\ \\ (PC') \;\; p[\pi(x,\theta_1)e^{ac_x}\bar{z}_1+(1-\pi(x,\theta_1))e^{ac_x}\underline{z}_1]+ \\ \qquad\qquad (1-p)[\pi(y,\theta_2)e^{ac_y}\bar{z}_2+(1-\pi(y,\theta_2))e^{ac_y}\underline{z}_2] \leq e^{-a\bar{u}} \\ \\ (WI') \;\; \bar{z}_1 \leq \bar{z}_2 \\ (WII') \;\; \bar{z}_2 \leq \underline{z}_2 \end{cases}$$

$$(\widetilde{P}^{flex,R})$$

Proposition A.3: *At a solution to the program $(\widetilde{P}^{flex,R})$, (PC') binds. Furthermore, we have that $\bar{w}_2 > \underline{w}_2$.*

Proof. Consider the program $(\widetilde{P}^{flex,R})$. Let λ_2, λ_6, λ_{PC}, λ_I, and λ_{II} denote the Lagrange multipliers associated to the constraints of this problem. The first order conditions obtained by differentiating the Lagrangean with respect to $\bar{z}_1, \bar{z}_2, \underline{z}_2, \underline{z}_1$ are then:

$$\begin{cases} (i) & \frac{p\pi(x,\theta_1)}{a\bar{z}_1} = \lambda_2\pi(x,\theta_1)e^{ac_x} - \lambda_6\pi(y,\theta_2)e^{ac_y} + \lambda_{PC}p\pi(x,\theta_1)e^{ac_x} + \lambda_I \\ (ii) & \frac{p(1-\pi(x,\theta_1))}{a\underline{z}_1} = \lambda_2(1-\pi(x,\theta_1))e^{ac_x} - \lambda_6(1-\pi(y,\theta_2))e^{ac_y} \\ & \qquad +\lambda_{PC}p(1-\pi(x,\theta_1))e^{ac_x} \\ (iii) & \frac{(1-p)\pi(y,\theta_2)}{a\bar{z}_2} = -\lambda_2\pi(y,\theta_1)e^{ac_y} + \lambda_6\pi(y,\theta_2)e^{ac_y} \\ & \qquad +\lambda_{PC}(1-p)\pi(y,\theta_2)e^{ac_y} - \lambda_I + \lambda_{II} \\ (iv) & \frac{(1-p)(1-\pi(y,\theta_2))}{a\underline{z}_2} = -\lambda_2(1-\pi(y,\theta_1))e^{ac_y} + \lambda_6(1-\pi(y,\theta_2))e^{ac_y} \\ & \qquad +\lambda_{PC}(1-p)(1-\pi(y,\theta_2))e^{ac_y} - \lambda_{II} \end{cases}$$

Multiplying each equation by the appropriate z variable, adding the four equations of the above system and using the fact that $(IC2')$ and $(IC6)'$, in the above specification of the optimization problem, are written as equalities, yields the following:

$$\begin{aligned} \frac{1}{a} = \; & \lambda_{PC}[p\pi(x,\theta_1)e^{ac_x}\bar{z}_1 + p(1-\pi(x,\theta_1))e^{ac_x}\underline{z}_1 \\ & +(1-p)\pi(y,\theta_2)e^{ac_y}\bar{z}_2 + (1-p)(1-\pi(y,\theta_2))e^{ac_y}\underline{z}_2] + \lambda_I[\bar{z}_1-\bar{z}_2] + \lambda_{II}[\bar{z}_2-\underline{z}_2] \end{aligned}$$

Using the complementarity slackness condition, we get that $\lambda_I[\bar{z}_1-\bar{z}_2] = \lambda_{II}[\bar{z}_2-\underline{z}_2] = 0$. Hence $\lambda_{PC} > 0$, which establishes that (PC') binds. Hence, we can conclude from the expression above that $\lambda_{PC} = \frac{e^{a\bar{u}}}{a}$.

Next we want to show that $\bar{w}_2 > \underline{w}_2$ or equivalently $\underline{z}_2 > \bar{z}_2$. Assume to the contrary that $\bar{z}_2 = \underline{z}_2 \equiv z_2$. We know in that case that (WI) is slack (otherwise by $(IC6')$ all wages would have to be equal, but this would contradict the fact that $(IC2')$ binds) and hence $\lambda_I = 0$. Rewrite now FOC's (iii) and (iv) as:

$$\begin{cases} (iii) \quad \frac{(1-p)}{az_2} = -\lambda_2 \frac{\pi(y,\theta_1)}{\pi(y,\theta_2)} e^{ac_y} + \lambda_6 e^{ac_y} + \lambda_{PC}(1-p)e^{ac_y} + \frac{\lambda_{II}}{\pi(y,\theta_2)} \\ (iv) \quad \frac{(1-p)}{az_2} = -\lambda_2 \frac{1-\pi(y,\theta_1)}{1-\pi(y,\theta_2)} e^{ac_y} + \lambda_6 e^{ac_y} + \lambda_{PC}(1-p)e^{ac_y} - \frac{\lambda_{II}}{1-\pi(y,\theta_2)} \end{cases}$$

This implies that

$$-\lambda_2 \frac{\pi(y,\theta_1)}{\pi(y,\theta_2)} e^{ac_y} + \frac{\lambda_{II}}{\pi(y,\theta_2)} = -\lambda_2 \frac{1-\pi(y,\theta_1)}{1-\pi(y,\theta_2)} e^{ac_y} - \frac{\lambda_{II}}{1-\pi(y,\theta_2)}$$

or, after some simplification,

$$\lambda_{II} = (\pi(y,\theta_1) - \pi(y,\theta_2))\lambda_2 e^{ac_y}$$

Note that $(\pi(y,\theta_1) - \pi(y,\theta_2)) < 0$ and hence $\lambda_{II} \geq 0$ iff $\lambda_2 \leq 0$. Next observe that (PC') as an equality together with $(IC2')$ imply, if $\bar{z}_2 = \underline{z}_2 \equiv z_2$, that $z_2 = e^{-a(c_y+\bar{u})}$. Plug now the values of λ_{PC} and z_2 into equations (iii) and (iv) and use the expression for λ_{II} obtained above. The two equations are identical and yield $\lambda_6 = \lambda_2 \equiv \lambda$.

We have so a system of four equations – FOC's (i) and (ii), $(IC2')$ and $(IC6')$ – to determine three variables: λ, $\bar{z}_1$ and $\underline{z}_1$. $(IC2')$ and $(IC6')$ can be used to solve directly for $\bar{z}_1$ and $\underline{z}_1$. Now, the two FOC's can be rewritten:

$$\begin{aligned} p\pi(x,\theta_1) &= a\lambda\bar{z}_1(\pi(x,\theta_1)e^{ac_x} - \pi(y,\theta_2)e^{ac_y}) + e^{a\bar{u}}\bar{z}_1 p\pi(x,\theta_1)e^{ac_x} \\ p(1-\pi(x,\theta_1)) &= a\lambda\underline{z}_1((1-\pi(x,\theta_1))e^{ac_x} - (1-\pi(y,\theta_2))e^{ac_y}) + e^{a\bar{u}}\underline{z}_1 p(1-\pi(x,\theta_1))e^{ac_x} \end{aligned}$$

Adding these two equations yields an equation

$$\begin{aligned} p &= a\lambda\left[\bar{z}_1(\pi(x,\theta_1)e^{ac_x} + \underline{z}_1(1-\pi(x,\theta_1))e^{ac_x} - \pi(y,\theta_2)\bar{z}_1 e^{ac_y} - \underline{z}_1(1-\pi(y,\theta_2))e^{ac_y}\right] + \\ &\quad + pe^{a\bar{u}}\left[\bar{z}_1\pi(x,\theta_1)e^{ac_x} + \underline{z}_1(1-\pi(x,\theta_1))e^{ac_x}\right] \end{aligned}$$

which, using $(IC2')$ and $(IC6')$ can be rewritten as:

$$p = a\lambda\left[e^{ac_y}e^{-a(c_y+\bar{u})} - e^{ac_y}e^{-a(c_y+\bar{u})}\right] + pe^{a\bar{u}}\left[e^{ac_y}e^{-a(c_y+\bar{u})}\right] = p\left[e^{ac_y}e^{-ac_y}\right] = p$$

always satisfied, so that one of the two above equations can be dropped. The remaining one can be used to solve for λ. Recall that $\lambda \leq 0$ is needed to ensure that $\lambda_{II} \geq 0$.

Solving then $(IC2')$ and $(IC6')$ with respect to $\bar{z}_1$ and $\underline{z}_1$ we get:

$$\begin{aligned} \underline{z}_1 &= \frac{\pi(x,\theta_1)e^{-a(c_y+\bar{u})} - \pi(y,\theta_2)e^{-a(c_x+\bar{u})}}{\pi(x,\theta_1)-\pi(y,\theta_2)} \\ \bar{z}_1 &= \frac{(1-\pi(y,\theta_2))e^{-a(c_x+\bar{u})} - (1-\pi(x,\theta_1))e^{-a(c_y+\bar{u})}}{\pi(x,\theta_1)-\pi(y,\theta_2)}. \end{aligned}$$

Substituting into the first of the two FOC's above yields:

$$\begin{aligned} p\pi(x,\theta_1) =& [a\lambda(\pi(x,\theta_1)e^{ac_x} - \pi(y,\theta_2)e^{ac_y}) + e^{a\bar{u}}p\pi(x,\theta_1)e^{ac_x}]\cdot \\ & \cdot \frac{(1-\pi(y,\theta_2))e^{-a(c_x+\bar{u})} - (1-\pi(x,\theta_1))e^{-a(c_y+\bar{u})}}{\pi(x,\theta_1)-\pi(y,\theta_2)} \end{aligned}$$

and hence

$$\begin{aligned} & a\lambda(\pi(x,\theta_1)e^{ac_x} - \pi(y,\theta_2)e^{ac_y})\frac{(1-\pi(y,\theta_2))e^{-a(c_x+\bar{u})} - (1-\pi(x,\theta_1))e^{-a(c_y+\bar{u})}}{\pi(x,\theta_1)-\pi(y,\theta_2)} \quad (7) \\ =\ & p\pi(x,\theta_1) - p\pi(x,\theta_1)e^{ac_x}\frac{(1-\pi(y,\theta_2))e^{-ac_x} - (1-\pi(x,\theta_1))e^{-ac_y}}{\pi(x,\theta_1)-\pi(y,\theta_2)} = \\ =\ & p\pi(x,\theta_1)\left[\frac{(1-\pi(x,\theta_1))e^{a\Delta c} - (1-\pi(y,\theta_2)) + \pi(x,\theta_1) - \pi(y,\theta_2)}{\pi(x,\theta_1)-\pi(y,\theta_2)}\right] = \\ =\ & p\pi(x,\theta_1)\left[\frac{(1-\pi(x,\theta_1))(e^{a\Delta c}-1)}{\pi(x,\theta_1)-\pi(y,\theta_2)}\right] > 0 \end{aligned}$$

Since the coefficient of λ in the first term is positive, it follows that the solution for λ of such equation is > 0, a contradiction. Hence, it cannot be that $\bar{z}_2 = \underline{z}_2$. ■

This completes the proof of Proposition **1.■**

Proof of Proposition 2.

The first best optimal contract is obtained as solution of the problem of maximizing the principal's expected profits subject to the agent's participation constraint, which under risk neutrality takes the following form:

$$\begin{aligned} \max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad & p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1) + (1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\ & +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2) + (1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)] \\ \text{s.t.} \quad & p[\pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 - c_x] + \\ & (1-p)[\pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 - c_y] \geq \bar{u} \end{aligned}$$

The maximal level of the principal's expected profits that can be attained at a solution of this problem is then clearly the one stated in the proposition and it is immediate to verify that the compensation profile given in (1) yields such level of expected profits and is then a first best optimum. It remains thus to verify the values in (1) satisfy all the incentive compatibility constraints, which under risk neutrality take the following form:

$$\left\{\begin{array}{l} \pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 \geq \pi(x,\theta_1)\bar{w}_2 + (1-\pi(x,\theta_1))\underline{w}_2 \\ \pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 - c_x \geq \pi(y,\theta_1)\bar{w}_2 + (1-\pi(y,\theta_1))\underline{w}_2 - c_y \\ \pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 - c_x \geq \pi(y,\theta_1)\bar{w}_1 + (1-\pi(y,\theta_1))\underline{w}_1 - c_y \\ \\ \pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 - c_y \geq \pi(x,\theta_2)\bar{w}_1 + (1-\pi(x,\theta_2))\underline{w}_1 - c_x \\ \pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 - c_y \geq \pi(x,\theta_2)\bar{w}_2 + (1-\pi(x,\theta_2))\underline{w}_2 - c_x \\ \pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 \geq \pi(y,\theta_2)\bar{w}_1 + (1-\pi(y,\theta_2))\underline{w}_1 \end{array}\right. \tag{8}$$

This is immediate by direct substitution. □

Proof of Proposition 3.

Let us consider first the solution of the principal's problem when the incentive constraints are ignored, that is the first best contract. We first show that at such contract expected profits and wages net of cost across the θ states are comonotonic, i.e.,

$$\{\pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 - c_x - (\pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 - c_y)\}\times$$
$$\{\pi(x,\theta_1)(\bar{R}-\bar{w}_1) + (1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1) - (\pi(y,\theta_2)(\bar{R}-\bar{w}_2) + (1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2))\} \geq 0$$

Assume, by contradiction, that $\pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 - c_x < \pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 - c_y$ while $\pi(x,\theta_1)(\bar{R}-\bar{w}_1) + (1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1) > \pi(y,\theta_2)(\bar{R}-\bar{w}_2) + (1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)$. Then, change the wage schedule by adding $\varepsilon > 0$ to the expected wage in θ_1 and subtracting $\frac{\bar{p}-\alpha}{1-\bar{p}+\alpha}\varepsilon$ to the expected wage in state θ_2. Straightforward computation shows that this improves the principal's profit while leaving the agent's utility constant. This constitutes a contradiction to the fact that we were at a solution to the first best.

The other case, namely $\pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 - c_x > \pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 - c_y$ while $\pi(x,\theta_1)(\bar{R}-\bar{w}_1) + (1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1) < \pi(y,\theta_2)(\bar{R}-\bar{w}_2) + (1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)$ can be treated in a symmetric manner.

Hence, utility and profit are comonotonic. We proceed now to show that the full insurance property stated in the proposition hold at the first best. Examine first the case where $(\pi(x,\theta_1)-\pi(y,\theta_2))(\bar{R}-\underline{R}) > \Delta c$. In this case, comonotonicity implies that profit and utility are higher in state θ_1 than in state θ_2. Suppose they are both strictly higher in θ_1 than in θ_2: $\pi(x,\theta_1)\bar{w}_1 + (1-\pi(x,\theta_1))\underline{w}_1 - c_x > \pi(y,\theta_2)\bar{w}_2 + (1-\pi(y,\theta_2))\underline{w}_2 - c_y$ and $\pi(x,\theta_1)(\bar{R}-\bar{w}_1) + (1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1) >$

$\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)$. As a consequence, the principal uses beliefs $\underline{p}+\alpha$ and the agent $\underline{p}+\psi$. We show next this cannot happen at an optimum.

Consider a change in the expected wage in θ_1 by an amount ε together with a variation of the expected wage in θ_2 by $-\frac{\underline{p}+\alpha}{1-\underline{p}-\alpha}\varepsilon$. This leaves the agent's utility unchanged while the change in the expected wage bill for the principal is $\frac{(\psi-\alpha)}{1-\underline{p}-\alpha}\varepsilon$. It follows that, when $\psi>\alpha$ the principal's expected wage bill decreases, and hence profits increase, as a result of such change, as long as $\varepsilon<0$. Such change implies then that the expected wage payments in θ_1 decrease, while those in θ_2 increase (i.e., the agent is offered more insurance across the states θ_1 and θ_2). Since the argument applies as long as $\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1-c_x>\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2-c_y$ and $\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)>\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)$, it establishes that the solution when $\psi>\alpha$ is such that the agent is fully insured: $\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1-c_x=\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2-c_y$.

When $\alpha>\psi$ the reverse is true, the principal's profits increase if $\varepsilon>0$, i.e. the expected wage payments in θ_1 increase, while those in θ_2 decrease. The optimum obtains then when the principal is fully insured: $\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)=\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)$.

A reasoning along the same lines allows us to establish that also in the other possible parameter configuration, where $(\pi(x,\theta_1)-\pi(y,\theta_2))(\bar{R}-\underline{R})<\Delta c$, the agent is fully insured when he is more imprecision averse than the principal and vice versa.

It remains then to show that we can always find a first best optimum contract at which the incentive constraints are satisfied. When $\psi>\alpha$ the first best contract is the same as when principal and agent have a single prior belief over θ, for which we showed in the proof of Proposition 2 the incentive constraints are satisfied. We show below that the same is true also when $\psi<\alpha$, that is when the first best optimum prescribes the principal is fully insured. Hence,

$$\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1-[\pi(y,\theta_2)\bar{w}_2+(1-\pi(y,\theta_2))\underline{w}_2]=(\pi(x,\theta_1)-\pi(y,\theta_2))(\bar{R}-\underline{R})$$

Using this equality to simplify the incentive constraints yields the following system of inequalities:

$$\begin{cases} \min\left\{\bar{R}-\underline{R}, \frac{\Delta c}{\pi(x,\theta_2)-\pi(y,\theta_2)}\right\} \geq \bar{w}_2-\underline{w}_2 \geq \frac{\Delta c-(\pi(x,\theta_1)-\pi(y,\theta_2))(\bar{R}-\underline{R})}{\pi(y,\theta_2)-\pi(y,\theta_1)} \\ \bar{w}_1-\underline{w}_1 \geq \min\left\{\frac{\Delta c}{\pi(x,\theta_1)-\pi(y,\theta_1)}, \frac{(\pi(x,\theta_1)-\pi(y,\theta_2))(\bar{R}-\underline{R})-\Delta c}{\pi(x,\theta_1)-\pi(x,\theta_2)}, \bar{R}-\underline{R}\right\} \end{cases}$$

The condition on $\bar{w}_1-\underline{w}_1$ can always be satisfied. Hence a necessary and sufficient condition for the first best to be implementable is that

$$\min\left\{\bar{R}-\underline{R}, \frac{\Delta c}{\pi(x,\theta_2)-\pi(y,\theta_2)}\right\} \geq \frac{\Delta c-(\pi(x,\theta_1)-\pi(y,\theta_2))(\bar{R}-\underline{R})}{\pi(y,\theta_2)-\pi(y,\theta_1)}$$

This is equivalent to $(\pi(x,\theta_1)-\pi(y,\theta_1))(\bar{R}-\underline{R}) > \Delta c$ and $(\pi(x,\theta_1)-\pi(y,\theta_2))(\bar{R}-\underline{R}) > \left(1-\frac{\pi(y,\theta_2)-\pi(y,\theta_1)}{\pi(x,\theta_2)-\pi(y,\theta_2)}\right)\Delta c$. The first inequality holds by Assumption 3. The second one can be rewritten as follows: $(\pi(x,\theta_1)-\pi(y,\theta_1))(\bar{R}-\underline{R})-(\pi(y,\theta_2)-\pi(y,\theta_1))(\bar{R}-\underline{R}) > \Delta c - \frac{\pi(y,\theta_2)-\pi(y,\theta_1)}{\pi(x,\theta_2)-\pi(y,\theta_2)}\Delta c$. Using again Assumption 3, we see that this condition is met whenever $-(\pi(y,\theta_2)-\pi(y,\theta_1))(\bar{R}-\underline{R}) > -\frac{\pi(y,\theta_2)-\pi(y,\theta_1)}{\pi(x,\theta_2)-\pi(y,\theta_2)}\Delta c$ always satisfied under Assumption 3. ■

Proof of Proposition 4.

To establish the first part of the proposition, we compare the principal's expected utility at the first best and at the two rigid contracts. The utility at the first best, for the various possible parameter configurations, are reported in Table 3:

	$(\pi(x,\theta_1)-\pi(y,\theta_2))(\bar{R}-\underline{R}) > \Delta c$	$(\pi(x,\theta_1)-\pi(y,\theta_2))(\bar{R}-\underline{R}) < \Delta c$
$\psi > \alpha$	$(\underline{p}+\psi)[\pi(x,\theta_1)\bar{R}+(1-\pi(x,\theta_1))\underline{R}-c_x]+$ $(1-\underline{p}-\psi)[\pi(y,\theta_2)\bar{R}+(1-\pi(y,\theta_2)\underline{R})-c_y]$ $-\bar{u}$	$(\bar{p}-\psi)[\pi(x,\theta_1)\bar{R}+(1-\pi(x,\theta_1)\underline{R})-c_x]+$ $(1-\bar{p}+\psi)[\pi(y,\theta_2)\bar{R}+(1-\pi(y,\theta_2)\underline{R})-c_y]$ $-\bar{u}$
$\psi < \alpha$	$(\underline{p}+\alpha)[\pi(x,\theta_1)\bar{R}+(1-\pi(x,\theta_1))\underline{R}-c_x]+$ $(1-\underline{p}-\alpha)[\pi(y,\theta_2)\bar{R}+(1-\pi(y,\theta_2)\underline{R})-c_y]$ $-\bar{u}$	$(\bar{p}-\alpha)[\pi(x,\theta_1)\bar{R}+(1-\pi(x,\theta_1)\underline{R})-c_x]+$ $(1-\bar{p}+\alpha)[\pi(y,\theta_2)\bar{R}+(1-\pi(y,\theta_2)\underline{R})-c_y]$ $-\bar{u}$

Table 3: Principal's utility at the first best

Tedious but straightforward computations yield the utility for the two rigid contracts reported in Table 4.

	x-contract	y-contract
$\psi > \alpha$	$(\underline{p}+\psi)[\pi(x,\theta_1)\bar{R}+(1-\pi(x,\theta_1))\underline{R}]+$ $(1-\underline{p}-\psi)[\pi(x,\theta_2)\bar{R}+(1-\pi(x,\theta_2))\underline{R}]$ $-\bar{u}-c_x$	$(\bar{p}-\psi)[\pi(y,\theta_1)\bar{R}+(1-\pi(y,\theta_1))\underline{R}]+$ $(1-\bar{p}+\psi)[\pi(y,\theta_2)\bar{R}+(1-\pi(y,\theta_2))\underline{R}]$ $-\bar{u}-c_y$
$\psi < \alpha$	$(\underline{p}+\alpha)[\pi(x,\theta_1)\bar{R}+(1-\pi(x,\theta_1))\underline{R}]+$ $(1-\underline{p}-\alpha)[\pi(x,\theta_2)\bar{R}+(1-\pi(x,\theta_2))\underline{R}]$ $-\bar{u}-c_x$	$(\bar{p}-\alpha)[\pi(y,\theta_1)\bar{R}+(1-\pi(y,\theta_1))\underline{R}]+$ $(1-\bar{p}+\alpha)[\pi(y,\theta_2)\bar{R}+(1-\pi(y,\theta_2))\underline{R}]$ $-\bar{u}-c_y$

Table 4: Principal's utility at the rigid contracts

Considering each of the eight possible cases, we find that the first best flexible contract is preferred to the two rigid contracts if and only if the condition stated in Assumption 3 is satisfied.

The second part of the claim in the proposition then derives directly from the expressions for the principal's expected utility derived above.■

Proof of Proposition 5.

Careful inspection of steps 1 to 9 of the proof of Proposition A.1, for the case where the parties have sharp probabilistic beliefs, reveals that the same argument goes through also with ambiguity. Note in fact that such steps rely either on a manipulation of the incentive constraints, which are unchanged as we argued, or on some optimality arguments. Two types of optimality arguments in particular are used. The first one concerns changes of the compensation within a given state; this does not depend on the beliefs over θ and only relies on the concavity of the agent's utility function. The second type of argument considers changes of wages - and hence utility - across states and clearly depends on the beliefs over θ. This only appears in establishing case 1 of Step 3. We show here that the same line of reasoning holds with imprecision aversion, provided the principal is less imprecision averse than the agent.

Let's establish that (IC2) has to be binding when w_2 is constant (as in case 1 of step 3 of Proposition A.1). Suppose (IC2) is slack and w_2 constant. This implies that the utility of the agent is higher in state θ_1 than in state θ_2. He thus uses $\underline{p}+\alpha$ in his computation of the expected utility. Consider again (an infinitesimal change) $d\bar{w}_1 < 0$, $d\underline{w}_1 = 0$ and $dw_2 > 0$ but now defined using the belief $\underline{p}+\alpha : d\bar{w}_1 = -\frac{(1-\underline{p}-\alpha)}{(\underline{p}+\alpha)}\frac{e^{-a(w_2-c_y)}}{e^{-a(\bar{w}_1-c_x)}}dw_2$, so that the participation constraint still holds. We will show that this change increases the principal's profit. Let $p \in [\underline{p}+\psi, \bar{p}-\psi]$ be the probability used by the principal in his evaluation of the expected profit. The change in profit is then equal to

$$\left[p\frac{(1-\underline{p}-\alpha)}{(\underline{p}+\alpha)}\frac{e^{-a(w_2-c_y)}}{e^{-a(\overline{w}_1-c_x)}}-(1-p)\right]dw_2$$

Given that $dw_2 > 0$ this term is positive whenever $\frac{e^{-a(w_2-c_y)}}{e^{-a(\overline{w}_1-c_x)}} > \frac{(\underline{p}+\alpha)}{(1-\underline{p}-\alpha)}\frac{(1-p)}{p}$. Observe now that (IC2) can be rewritten as follows:

$$\pi(x,\theta_1)e^{-a\bar{w}_1}+(1-\pi(x,\theta_1))e^{-a\underline{w}_1}\leq e^{-a(w_2+\Delta c)},$$

which in turn implies, by convexity of e^{-ax}, that $e^{-a(\pi(x,\theta_1)\bar{w}_1+(1-\pi(x,\theta_1))\underline{w}_1)} < e^{-a(w_2+\Delta c)}$. Hence a sufficient condition for the changes considered to be beneficial to the principal is that: $e^{a(1-\pi(x,\theta_1))(\bar{w}_1-\underline{w}_1)} > \frac{\underline{p}+\alpha}{1-\underline{p}-\alpha}\frac{1-p}{p}$. Since we established that $\bar{w}_1 > \underline{w}_1$, and hence $e^{a(1-\pi(x,\theta_1))(\bar{w}_1-\underline{w}_1)} > 1$, this condition is always satisfied when $\frac{\underline{p}+\alpha}{1-\underline{p}-\alpha}\frac{1-p}{p} < 1$ or equivalently $p > \underline{p}+\alpha$, always true when $\psi \geq \alpha$, i.e., under the assumption that the principal is less (or equally) imprecision averse than the agent.

Hence, the optimal contract is the solution to:

$$\begin{aligned}
&\max_{\bar{w}_1,\underline{w}_1,\bar{w}_2,\underline{w}_2} \quad \min_{p\in[\underline{p}+\psi,\bar{p}-\psi]} p[\pi(x,\theta_1)(\bar{R}-\bar{w}_1)+(1-\pi(x,\theta_1))(\underline{R}-\underline{w}_1)] \\
&\qquad\qquad\qquad +(1-p)[\pi(y,\theta_2)(\bar{R}-\bar{w}_2)+(1-\pi(y,\theta_2))(\underline{R}-\underline{w}_2)] \\
&\text{s.t.} \quad (IC2),(IC6) \text{ (as stated in } (P^{flex})) \text{ and} \\
&\quad (PC) \quad \max_{p\in[\underline{p}+\alpha,\bar{p}-\alpha]}\left\{p[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}+(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}]+\right. \\
&\qquad\qquad \left.(1-p)[\pi(y,\theta_2)e^{-a(\bar{w}_2-c_y)}+(1-\pi(y,\theta_2))e^{-a(\underline{w}_2-c_y)}]\right\}\leq e^{-a\bar{u}} \\
&\quad (WI) \quad \bar{w}_1 \geq \bar{w}_2 \\
&\quad (WII) \quad \bar{w}_2 \geq \underline{w}_2
\end{aligned}$$

■