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Christoph Bertsch, Claudio Calcagno and Mark Le Quement

**EUROPEAN UNIVERSITY INSTITUTE, FLORENCE**
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**CHRISTOPH BERTSCH,**
**CLAUDIO CALCAGNO**
**and**
**MARK LE QUEMENT**

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# State aid and tacit collusion*


Christoph Bertsch Claudio Calcagno

Mark Le Quement

European University Institute, Economics Department

via della Piazzuola 43, 50133 Firenze, Italy



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**Abstract**

Both literature and policy debate on State aid or government subsidies have focused on the trade-off between the potential inefficiencies caused by state intervention (inefficient allocation of resources, moral hazard) and the potential gains from intervention (whether related to the resolution of market failures or to the achievement of some dimension of social equity).

The debate however has ignored another important negative effect of State aid: governments, by setting up aid schemes to ailing firms, may increase the likelihood of (tacit) collusion in an industry characterised by idiosyncratic shocks. Indeed, in a repeated-game setting, a systematic bailout regime increases the expected profits of a firm from cooperation and simultaneously raises the probability that competitors will still be in business to carry out punishment against cheaters.

Despite the generality of the model and of its key insight, we study this problem through an application to the banking sector, as it has recently been subject of much attention within the context of the ongoing economic crisis.



Keywords: Subsidies, dynamic oligopoly, government policy, banking



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# 1 Introduction and motivation

The financial crisis affecting the global economy since 2007 is of stunning proportions and has involved (or is expected to involve) the bankruptcies of a large number of global firms, including major financial institutions. The rapid deterioration of the financial and economic situation has prompted massive interventions from Treasuries (and Central Banks) the world over. State intervention, and in particular State aid, has been playing a major role, with governments rushing to rescue large fractions of their industries. Apart from the fiscal burden that it represents, State aid entails some thorny legal considerations, especially in the European Union, where Art. 87 of the Rome Treaty explicitly forbids measures that confer (through public resources) economic advantages to selected entities, affecting trade. The European Commission has therefore been under pressure to soften its stance and owing to the gravity of the situation, in October 2008, it published a Communication on the application of State aid rules to measures taken in relation to financial institutions in the context of the current global financial crisis.[1]

To understand the economic magnitude of the aid involved in recent months, it is worth noting that the number of cases in the European Union where aid was legally granted grew from about 600 per year over the period from 2004 to 2006 to over 1,000 in 2008. Moreover, the amount of aid to be recovered (i.e. illegal aid, or subsidies that should not have been granted by the various EU countries), grew from under €30m in 2005 to over €900m in 2008.[2]

From a theoretical perspective, most of the debate on State aid or government subsidies has focused on the trade-off between inefficiencies caused by intervention (inefficient allocation of resources, moral hazard) and potential gains following from aid measures. Arguments in favour of State aid range from equity considerations to the potential resolution of existing market failures. In the financial sector, for example, intervention (including that of a Central Bank) is often justified on the grounds of gains in the stability of the financial system. Bankruptcies of individual banks may trigger contagion effects across the sector (through the interbank and asset markets), and may also harm consumers directly through the loss of private deposits (subject to national deposit insurance schemes). On

[1] Official Journal C 270, 25.10.2008, pages 8–14. The accompanying press release explained that aid would be approved if: "Non-discriminatory access in order to protect the functioning of the Single Market by making sure that eligibility for a support scheme is not based on nationality; State commitments to be limited in time in such a way that it is ensured that support can be provided as long as it is necessary to cope with the current turmoil in financial markets but will be reviewed and adjusted or terminated as soon as improved market conditions so permit; State support to be clearly defined and limited in scope to what is necessary to address the acute crisis in financial markets while excluding unjustified benefits for shareholders of financial institutions at the taxpayer's expense; [a]n appropriate contribution of the private sector by way of an adequate remuneration for the introduction of general support schemes (such as a guarantee scheme) and the coverage by the private sector of at least a significant part of the cost of assistance granted; [s]ufficient behavioural rules for beneficiaries that prevent an abuse of state support, like for example expansion and aggressive market strategies on the back of a state guarantee; [a]n appropriate follow-up by structural adjustment measures for the financial sector as a whole and/or by restructuring individual financial institutions that had to rely on state intervention."

[2] Notice that this series is not cumulative, that is, the amount known to be recovered in 2008 only includes new cases disputed in 2008. Admittedly, these sums were also high before 2005. Source: European Commission, DG Competition, http://ec.europa.eu/competition/state_aid/studies_reports/measures.html.

the other hand, State aid can distort the competitive process (prompting a misallocation of resources), as well as create moral hazard: if firms expect that the Government will intervene to help them in case of failure (or in adverse circumstances more generally), these may have the incentive to take excessive risks.

The economic effects studied have always impinged on firms' *unilateral* behaviour. In this article, in contrast, we develop a simple (infinite-horizon) model that sheds light on aresult that to our knowledge had not been stressed before: a government policy aimed at systematically rescuing firms in the presence of negative idiosyncratic shocks facilitates (tacit) collusion. Collusion is easier to sustain because the punishment threat faced by a firm is more efficient, which is in turn due to the increased survival probability of firms. Indeed, when firms are guaranteed to be in business in the next periods, expected future cooperative profits increase, so that foregoing such profits implies an increased opportunity cost. Furthermore, the guaranteed presence of competitors in the future makes the expected punishment harsher than in an environment where competitors may exit the market due to an exogenous shock, which would leave the deviant firm unpunished.

In this paper we investigate the impact of State aid on firms' incentives to (tacitly) collude and its overall impact on consumer welfare. Although the model and its implications are general and would match any industry, we study this problem through an application to the banking sector, as it has recently received much attention due to large bailouts. This article proceeds as follows: in Section 2 we briefly consider the relevant literature; in Section 3, we present the model. Section 4 shows and discusses the results, including the welfare analysis. Section 5 concludes and suggests some extensions.

## 2 Related literature

There is not a vast literature on the economics of State aid as such[3], possibly reflecting the richness of the literature on subsidies and trade. State aid is typically criticised by economists, as it leads to a variety of inefficiencies. Besley and Seabright (1999) discuss two broad classes of effects: externalities arising from aid (strategic trade policy, tax competition and economic geography considerations) and inefficient competition between governments. Dewatripont and Seabright (2006) go beyond intergovernmental issues and build a model where local politicians invest in wasteful projects purely to show their diligence and win votes. Collie (2000) instead proposes an economic explanation of why individual states may have an incentive to subsidise firms with the aim of reducing oligopolistic distortions. He shows that a multilateral institution responsible for prohibiting subsidies can increase welfare.

[3] An extensive review of the role and the effects of State aid can be found in Nitsche and Heidhues (2006). For an equally policy-oriented approach based on economic theory, the reader is also directed to OFT (2004) and Buelens et al (2007).

Friederiszick et al (2008) review the efficiency rationales for aid (tackling market failures such as externalities, public goods, asymmetric information and lack of coordination) as well as equity considerations. They also point towards cross-border (positive) externalities in the case of EU State aid. Their paper then highlights the potential costs of State aid (beyond the direct cost of intervention) such as anti-competitive effects, "picking wrong winners" and international spillover effects. Among the potential distortions of competition, they list the support of inefficient production; the distortion of dynamic (inter-temporal) incentives; the potential increase in market power; and the distortion of production and location decisions across EU countries. Finally, they propose an actual effects-based framework to assess whether particular State aid measures should be approved. Martin and Valbonesi (2008) develop a model of the impact of State aid on market structure and performance in an integrating market (i.e. a common market with increasing trade flows) and find that in equilibrium governments grant State aid, reducing common market welfare. However, they only focus on non-cooperative equilibria.

As discussed above, in the case of banking, it may be beneficial to sacrifice some level of competition in the interest of financial stability[4]; however this relationship is rather complex and results are far from being clear-cut as shown by Allen and Gale (2004). This is nevertheless beyond the scope of our paper, since we only take banking as an example and our model is not sector-specific. Rather, we simply point towards some work on imperfect competition in banking in dynamic setups. Hellman et al (2000) show that in a dynamic model deposit rate controls can be superior to costly regulation.[5] They explore the interaction between financial liberalisation and prudential regulation and find that capital requirements (though costly) can prevent banks from excessive risk-taking in a static model. By contrast, this positive effect comes at a very high cost in a dynamic model because of the business-stealing effect that induces banks to opt for riskier strategies than their competitors and offer better interest rates to depositors in order to collect more deposits. Perotti and Suarez (2002) investigate the impact of competition on the riskiness of banks' portfolio choices. In particular, they examine the relationship between the optimal riskiness of portfolio choice and banking regulation (merger policy and market entry regulation) in an oligopoly context. The main mechanism in their model is a strategic substitutability between portfolio decisions of duopolistic banks. In particular a given duopolist has an incentive to invest in the prudent asset if the competitor chooses a risky strategy (since she can expect high monopoly benefits if the competitor fails).

In industrial economics, models of tacit collusion have been heavily applied, but not in the context of State aid. We take a standard framework of tacit collusion for repeated oligopoly interaction (originally due to Stigler

[4] See Carletti (2008) for an excellent survey.

[5] Hellman et. al. (2000) do not model imperfect competition explicitly but make assumptions on derivatives of the deposit demand function. See Repullo (2004) for an explicit modelling of imperfect competition that delivers some additional insights.

(1964))[6] and analyse the effect of State aid in an application to the banking industry.

## 3 Model

We develop a model on the relationship of State aid and tacit collusion through an application to the banking industry. The mechanism we describe is not specific to the financial industry and can be readily transposed to other industries, and we indicate over the course of the analysis how the general setup can be translated to other sectors.

The starting block of our model is Freixas and Rochet's (2008) extension of the Klein-Monti model to Cournot oligopoly. The original monopolistic model features a single bank facing an upward-sloping demand for deposit and a downward-sloping demand for loans.[7]

To shed light on the impact of State aid on tacit collusion and welfare, our model will make simplifications with respect to Freixas and Rochet (2008). In our economy, the banking industry is characterised by a duopoly competing in the deposit market over an infinite horizon. We consider time in a discrete fashion. Production (management) costs are normalised to *nil*. Banks simultaneously set interest rates $r_1$ and $r_2$, where $r_1, r_2 > 0$. The (linear) demand function is given by:

$$Q(r_1, r_2) = \min\left\{\overline{Q}, \max\left\{r_1, r_2\right\}\right\} \tag{1}$$

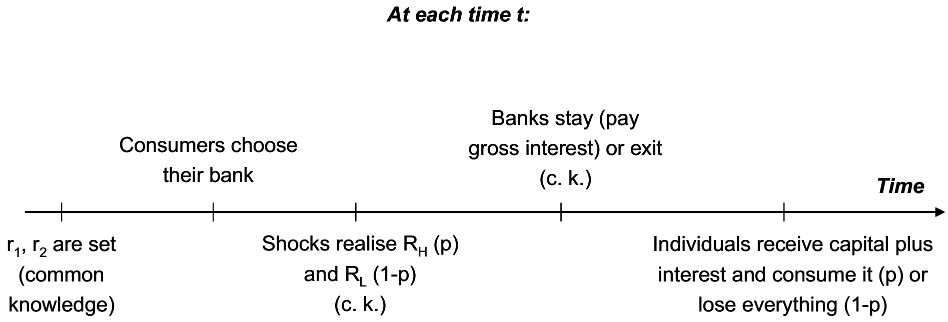
The environment is stochastic in the sense that there is a single asset (project) in which banks invest their funds. This asset (project) is subject to the following shocks: it yields net return $R_H$ (with prob. $p$) and $R_L$ (with prob. $(1-p)$).[8] We assume that $R_H < \overline{Q}$ (this simply normalises static competitive profits to *nil*).

For simplicity, and without loss of generality, we normalise $R_L$ to $-1$. This means that all funds are lost in the presence of a bad shock and nothing is returned to depositors.

Banks have discount factor $0 < \delta < 1$ and the timeline of our game, for each period $t \in [1, \infty)$, is given in Figure 1.[9]

[6] Clear, and sound economic reasoning (presented in a suitable manner for policy-making) on the economics of tacit collusion can be found in Ivaldi et al. (2003).
[7] See Klein (1971) and Monti (1972).
[8] In another industry one could argue that firms can be hit with a certain probability in each period by a cost shock or a shock on their production technology that forces them to leave the market.
[9] Discount factors over infinite-horizon games are often interpreted as the probability that the game will actually be played in a given period, so as to implicitly relax the infinite-horizon interpretation. Note that this is different from the adverse shocks that we introduce (with prob. $p$) as these are *idiosyncratic*.

Figure 1: Timeline of the game




Every period, bank $i = 1, 2$ maximises profits $((R_i - r_i) * Q(r_i, r_j))$ by choosing $r_i$. We assume that the market (deposits and profits) is equally shared by the banks when they set the same interest rate. Importantly, when a bank receives a bad shock ($R_L$), it is forced out of the market (as in Perotti and Suarez (2002)). It goes bankrupt because it cannot repay depositors and its authorisation to operate is not renewed. We make the following assumption:

**Condition 1** $\quad R_H > 4\frac{1-p}{p}$.

This condition is necessary to ensure positive levels of welfare.[10] Notably banks will set interest rates only taking the good state of the world ($R_H$) into account. If a bank is alone in the market, profit maximisation leads to $r^M = \frac{R_H}{2}$, $Q(r^M) = \frac{R_H}{2}$ and $\pi^M = \left(\frac{R_H}{2}\right)^2$. Competition, by contrast, leads to $r^C = R_H$, and $Q(r^M) = R_H$ and $\pi^M = 0$. Let us define $W^M = \frac{1}{2}\left(\frac{R_H}{2}\right)^2$ as the consumer surplus at the monopoly (or collusive) price level and $W^C = \frac{R_H^2}{2}$ as the consumer surplus under perfect competition.

The next element in our economy is a national deposit insurance scheme (NDIS), covering 100% of deposits. Hence, when a bank goes bankrupt, depositors are returned their initial investments (without interest). The NDIS is funded by flat-rate taxes. The size of the insurance scheme is: $\Phi_{C1} = -\frac{R_H}{4}$ whenever one collusive duopolist bank fails; $\Phi_{C2} = -\frac{R_H}{2}$ whenever a monopolist bank fails or both banks in a collusive duopoly fail; $\Phi_{NC1} = -\frac{R_H}{2}$ whenever banks compete and one receives a bad shock; $\Phi_{NC2} = -R_H$ whenever banks compete and both receive a bad shock.

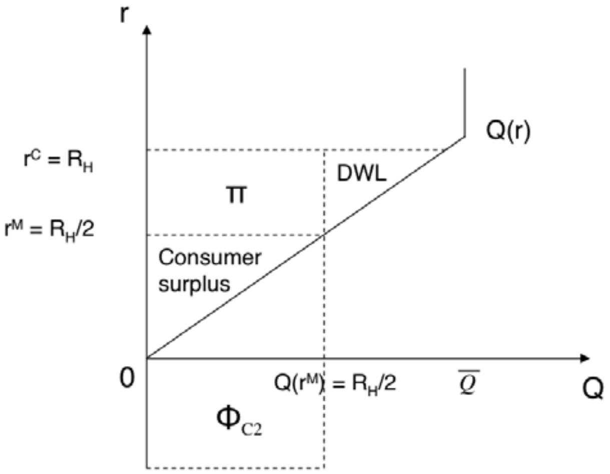
Notice that the introduction of the deposit insurance scheme is not only crucial within the context of a banking model, but it also generates two important features. First, it enables us to isolate the effect of State aid on collective

[10] Under reasonable parameter assumptions we have $R_H > 4\frac{(1-p)}{p}$, e.g. for $p = 0.9$ and $R_H = 0.6$ we have $0.6 > 0.\overline{4}$ (recall $R_L = -1$, so $R_H = 0.6$ is not particularly high).

competitive behavior when we compare social welfare under a State aid and a No State aid regime. This follows from the fact that depositors get their deposits fully refunded under both regimes whenever a bank fails, while the financing of the scheme is done by a non-distortionary tax in both regimes. Second, the introduction of deposit insurance makes our model readily comparable to applications to other industries where the failure of a firm does not cause an immediate loss in wealth to its customers.

Figure 2 summarises, graphically, the above discussion. Notice that as the demand schedule is upward sloping, the various areas in the graph (profit $\pi$, consumer surplus and deadweight loss, or DWL) are inverted (horizontally) with respect to a traditional diagrammatic analysis of linear demands. For simplicity, we have only depicted the potential loss from bankruptcy ($\Phi_{C2}$) in the case of monopoly. Notice the analogy to a model of duopolistic competition in a non-banking industry with a downward sloping demand curve.

Figure 2: A simple model of demand for deposits




Finally, we define State aid (also financed through lump-sum taxes) as the following (cumulative) actions. First, the State renews the bank's authorisation to operate even when the bank has gone bankrupt. Second, the State incurs a sunk cost $\gamma$ per bank rescue. We do not explicitly model how this cost arises but it could for instance be justified as the cost of resources devoted by the financial regulator to examine the books of failing bank and facilitate its rescue. In a non-financial industry this cost could for instance arise from costly restructuring. Notice that we do not impose any assumption on $\gamma$. Instead, in the welfare analysis section, we determine upper bounds on its value such that the aid policy is welfare-enhancing to consumers.

As mentioned earlier, the NDIS is independent of whether there is a State aid policy (however, in total terms, the NDIS will be costlier in the presence of a State aid regime since banks are always rescued and can fail several times, whereas in absence of State aid there can be two bankruptcies at most in the economy). The NDIS is a banking-specific concept, but as the NDIS is exactly matched in size by a lump-sum tax, our framework is general and goes beyond the presented application to the banking sector.

# 4 Results

In this section, we first demonstrate that the introduction of State aid decreases the minimal discount factor for which tacit collusion is possible. This implies that there is a range of values of the discount factor for which the implementation of State aid causes a change in the competitive state of the economy from competition to collusion. Next, we examine the implications of this result for the welfare effects of a State aid policy. We demonstrate that for this range of values of the discount factor, State aid always affects welfare negatively, while it has a positive effect for all other values of the discount factor if the intrinsic cost of rescuing ($\gamma$) is not too high.

## 4.1 Critical discount factors

We suppose a collusive regime in which firms adopt a simple trigger strategy by charging the collusive price as long as no one in the cartel defects and reverting to the competitive price as soon as one firm has defected from the tacit collusive agreement.

The first step is to derive the critical discount factors above which tacit collusion is sustainable under each policy regime. In the case of no State aid, the collusive profits need to embed the probability that a firm becomes a monopolist, if the competitor receives a negative shock, as well as the probability that the firm itself goes bankrupt. The profits in the period of deviation also need to account for the probability that the deviating firm goes bankrupt, since the shock occurs after market conduct is chosen. Likewise, punishment profits (from next period onwards) need to encompass the possibility that the deviating firm will actually be a monopolist for some time, as well as the possibility that the deviating firm goes out of business.

Without State aid the incentive compatibility constraint (ICC) for tacit collusion - which we fully derive in

Appendix A.1 - looks as follows:

$$\overbrace{\sum_{t=1}^{\infty}\delta^{t-1}\left(p^{2t-1}\frac{\pi^M}{2}+p^t(1-p^{t-1})\pi^M\right)}^{\text{Present value of expected profits under collusion}} \tag{2}$$

$$\geq \underbrace{p\pi^M}_{\text{Expected instantaneous profit from deviating}} + \underbrace{\sum_{t=2}^{\infty}\delta^{t-1}\left(p^{2t-1}\pi^C+p^t(1-p^{t-1})\pi^M\right)}_{\text{Present value of expected profit under punishment}}$$

Noting that $\pi^C = 0$, tacit collusion can be sustained for all $\delta \geq \delta^{NA} = \frac{1}{2p^2}$.

With State aid, by contrast, a bank that has received a bad shock is rescued at no cost to it. There is no profit (nor loss) in the period of failure. Noting once again that the competitive profit is zero every period, we set up the following ICC:

$$\sum_{t=1}^{\infty}\delta^{t-1}(p\frac{\pi^M}{2}+(1-p)*0) \geq p\pi^M + \sum_{t=2}^{\infty}\delta^{t-1}(p\delta^{t-1}*\pi^C+(1-p)\delta^{t-1}*0) \tag{3}$$

So tacit collusion can be sustained for all $\delta \geq \delta^A = \frac{1}{2}$. This is the traditional result obtained in a supergame where symmetric duopolists compete on price. The only difference is that both the collusive profit and the deviation profit have to be scaled by the probability of receiving a good shock; however the actual probability $p$ cancels out.

Figure 3 provides a graphical representation of this result. When banks place little value on the future (left half of the chart) tacit collusion cannot be sustained, regardless of whether there is State aid. When banks care much about the future relative to the likelihood of a good shock ($\delta \geq \frac{1}{2p^2}$, i.e. the area at the top-right corner) tacit collusion can be sustained in either regime. Finally, for intermediate values of the discount factor ($\frac{1}{2} \leq \delta < \frac{1}{2p^2}$, i.e. the area at the bottom-right) only the competitive outcome is sustainable in the absence of an aid policy while tacit collusion is made possible by a State aid policy.

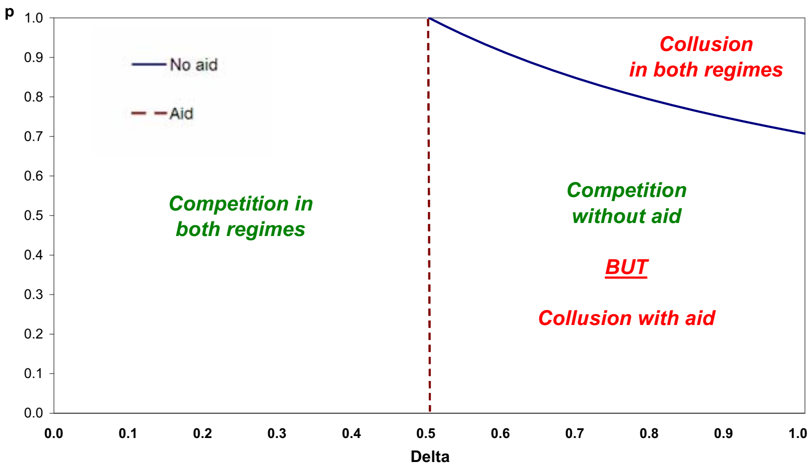
The results are summarised in Proposition 1:

**Proposition 1** ***State aid and collusion***

*In the presence of idiosyncratic shocks, a State aid policy that keeps banks (firms) in business after a shock facilitates tacit collusion; that is, tacit collusion can be sustained for lower values of discount factors.*

**Proof.** The proof of is trivial, as $\frac{1}{2} = \delta^A < \delta^{NA} = \frac{1}{2p^2}$, $\forall p < 1$. ■

Figure 3: Probability of good shock occurring and discount factors determing collusive vs. competitive outcomes




## 4.2 Welfare

In the last section we examined the relation between competitive behaviour and the exogenous discount factor for the two regimes, State aid and no State aid. We derived two critical discount factors that defined three regions in the $[p \times \delta]$ parameter space, among which one is characterised by a difference in collusive behavior under the two policy regimes. We now perform a comparison of social welfare under the two regimes and summarise our results in Proposition 2 at the end of the section.

### 4.2.1 No State aid

**Under collusion ($\delta \geq \frac{1}{2p^2}$):** The exercise here is to capture all possible scenarios, namely collusive duopoly and monopoly, considering all the possible combinations of events (good and bad shocks). Each scenario will be carried out with a certain probability and will entail a certain welfare level (including the tax level too) which are fully derived in Appendix A.2. Expected consumer welfare under a high enough discount factor to sustain tacit collusion and in the absence of State aid is computed to be:

$$E(W_C^{NA}) = \frac{R_H\left((1-p) - \frac{p}{4}R_H\right)}{2\left(1-\delta p^2\right)} + \frac{R_H\left(\frac{p}{2}R_H - 2(1-p)\right)}{2\left(1-\delta p\right)} \tag{4}$$

Note that Condition 1 assures positive welfare levels.[11]

**Under no collusion ($\delta < \frac{1}{2p^2}$):** Here, we carry out the analogous exercise; however, due to the lower discount factor, there can only be either a monopoly (if the competitor has received a bad shock) or a competitive duopoly. In either case there can be good or bad shocks (in the latter it could be that only one bank receives the bad shock in the last period). The expected consumer welfare in the absence of aid, under a low enough discount factor to guarantee competition and in the presence of both banks on the market, is given as follows :

$$E(W_{NC}^{NA}) = \frac{p\,(R_H)^2}{4\,(1-\delta p^2)} + \frac{R_H\left(\frac{p}{2}R_H - 2(1-p)\right)}{2\,(1-\delta p)} \tag{5}$$

See in Appendix A.2 for the derivation. Again Condition 1 ensures positive welfare levels.

### 4.2.2 State aid

We proceed in the same fashion as in subsection 4.2.1. However, here, banks never exit. When consumers' deposits are lost due to the bad shocks, the Government refunds them the original capital, as well as paying the direct rescuing costs $\gamma$. To close the model, we need to compute the total expected stream of aid and set up a corresponding lump-sum tax (which includes the financing of both the NDIS and the direct rescuing costs $\gamma$) on consumers, thus reducing their welfare.

**Under collusion ($\delta \geq \frac{1}{2}$):** In the presence of State aid, the expected consumer welfare under a high enough discount factor to sustain tacit collusion is:

$$E(W_{C}^{A}) = \frac{R_H\left(\frac{p}{4}R_H - (1-p)\right)}{2\,(1-\delta)} - \frac{2(1-p)\gamma}{1-\delta} \tag{6}$$

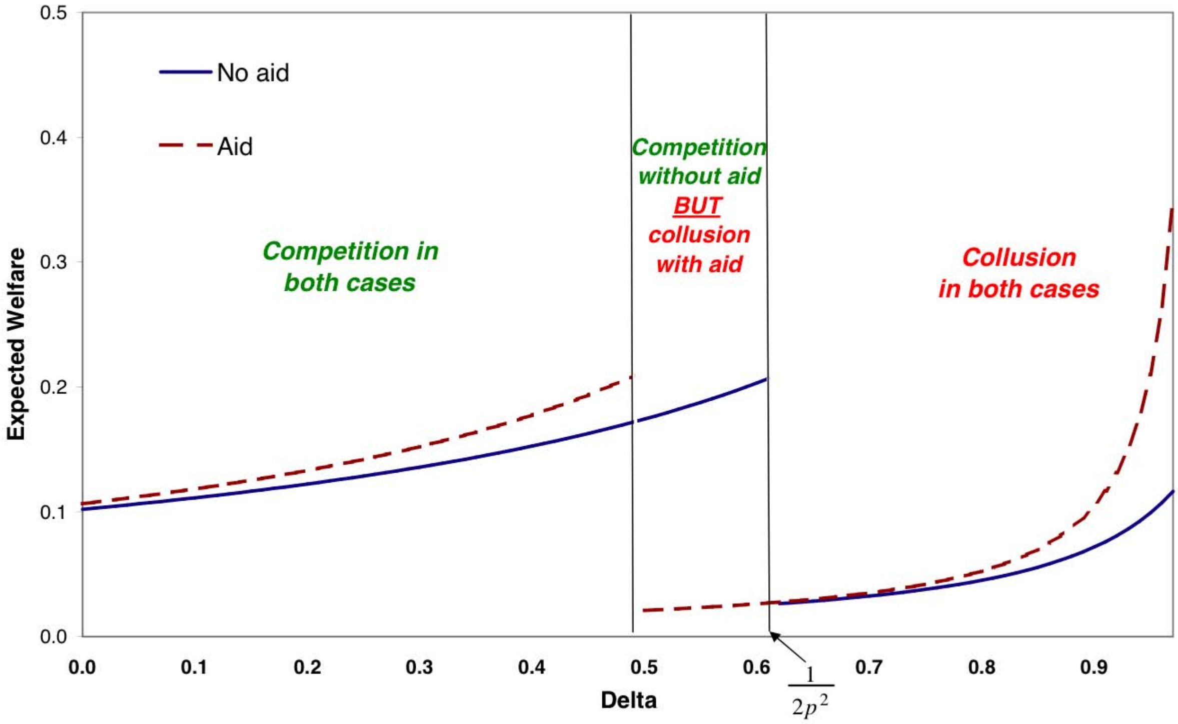
**Under no collusion ($\delta < \frac{1}{2}$):** In the presence of State aid, the expected consumer welfare under a discount factor low enough to guarantee competition (when both banks are in the market) is:

$$E(W_{NC}^{A}) = \frac{R_H\,(pR_H - (1-p))}{2\,(1-\delta)} - \frac{2(1-p)\gamma}{1-\delta} \tag{7}$$

[11] To see this notice that $(1-p) - \frac{p}{4}R_H < 0$ if $R_H > 4\frac{(1-p)}{p}$ and $\frac{p}{2}R_H - 2(1-p) > 0$ if $R_H > 4\frac{(1-p)}{p}$. Moreover $\frac{R_H/2}{1-\delta p^2} < \frac{R_H/2}{1-\delta p}$. For expected welfare to be positive we need to have $\frac{R_H/2}{1-\delta p^2}[(1-p) - \frac{p}{4}R_H] > -\frac{R_H/2}{1-\delta p}[\frac{p}{2}R_H - 2(1-p)]$ which is assured since $\frac{1-\delta p}{1-\delta p^2} < 2$.

### 4.3 Summary and discussion of the results

Having derived these four benchmark levels of consumer welfare corresponding to the two possible policy regimes and the two possible competitive regimes, it is instructive to plot them jointly in a diagram. We do so in Figure 4.3. The horizontal axis corresponds to the discount factor $\delta$. Three relevant regions can be identified. In the left region ($\delta < \frac{1}{2}$) banks compete with each other regardless of whether there is State aid. In the right region ($\delta \geq \frac{1}{2p^2}$) tacit collusion can be sustained in either regime. However, in the central region, tacit collusion can *only* be sustained in the presence of State aid. In the left and in the right regions, State aid *can* be consumer welfare-enhancing and this depends on whether the vertical distance between the broken and the solid line (for any given discount factor) exceeds the (expected tax bill due to the) direct costs of rescuing banks $\gamma$. In the central region, we see that State aid is in contrast welfare decreasing.




Effect of State aid on consumer welfare, by discount factor

The above figure abstracts from the cost $\gamma$ of rescuing a bank. In what follows, we compute the upper limits on $\gamma$ such that the expected welfare under State aid is larger than under no State aid, in the cases where public intervention does not affect the competitive state of the economy. Call $\widehat{\gamma}_{NC}$ this limit value of $\gamma$ for the case of no collusion in both policy regimes, i.e. for $\delta < \frac{1}{2}$:

$$E(W_{NC}^{A}) \geq E(W_{NC}^{NA}) \Longleftrightarrow$$

$$\gamma \leq \widehat{\gamma}_{NC} = \frac{1-\delta}{2(1-p)} \left( \frac{R_H \left(pR_H - 2(1-p)\right)}{2\left(1-\delta\right)} - E(W_{NC}^{NA}) \right) \tag{8}$$

In Appendix A.3 we show that $\widehat{\gamma}_{NC} \geq 0$. We now compute the upper limit $\widehat{\gamma}_{C}$ corresponding to the case of collusion in both regimes, i.e. for $\delta \geq \frac{1}{2p^2}$:

$$E(W_{C}^{A}) \geq E(W_{C}^{NA}) \Longleftrightarrow$$

$$\gamma \leq \widehat{\gamma}_{C} = \frac{1-\delta}{2(1-p)} \left( \frac{R_H \left(\frac{p}{4}R_H - (1-p)\right)}{2\left(1-\delta\right)} - E(W_{C}^{NA}) \right) \tag{9}$$

Again we show in Appendix A.3 that $\widehat{\gamma}_{C} \geq 0$.

The results on the welfare effects of State intervention can be summarised by Proposition 2.

**Proposition 2** ***State aid and social welfare***

*(i) In the range of discount factors such that there is competition in absence of State aid but tacit collusion with State aid, such a policy reduces consumer welfare. That is,* $E(W_{C}^{A}) < E(W_{NC}^{NA})$, $\forall\ \frac{1}{2} \leq \delta < \frac{1}{2p^2}$*. This is true even in the absence of direct rescuing costs.*

*(ii) In an environment where duopolistic banks (firms) compete regardless of whether there is State aid* ($\delta < \frac{1}{2}$)*, such a policy reduces consumer welfare if and only if the direct costs of rescuing a bank (firm) exceed* $\widehat{\gamma}_{C}$*.*

*(iii) In an environment where duopolistic banks (firms) can sustain tacit collusion regardless of whether there is State aid* ($\delta \geq \frac{1}{2p^2}$)*, such a policy reduces consumer welfare if and only if the direct costs of rescuing a bank (firm) exceed* $\widehat{\gamma}_{NC}$*.*

**Proof.** The proof of (i) is in Appendix A.4. For the proof of (ii) and (iii) see above inequalities. ■

Proposition 2 introduces a simple dichotomy between two scenarios that summarises the effect of State Aid on social welfare. In one scenario (cases ii) and iii) above), State aid does not influence the competitive state of the economy, and is welfare-improving as long as its intrinsic cost is not too large. Indeed, in such a case, the only effect of State aid is to beneficially preserve the existence of the market, which is advantageous to both firms and consumers. In the other scenario (case i)), where State aid affects the competitive state of the economy by triggering collusion, we find that its overall effect is *always* negative. This means that in such a scenario, the adverse collusion-creating effect of State aid dominates its beneficial market-preserving effect. The dominance of the first force over the second force does not appear like an a priori necessity. We attribute this feature to the fact that this scenario corresponds to relatively low values of the discount factor. For such values of the discount factor, the adverse price effect of collusion thus dominates the long run positive benefits related to the preservation of the market.

# 5 Conclusion

The literature on State aid and the related policy debate have typically focused on the adverse efficiency effects of such policies (misallocation of resources, moral hazard) and on countervailing arguments based on equity, cohesion and social policy.

Here we developed a very simple infinite-horizon model that sheds light on a separate result that to our knowledge had not been stressed before: a government policy aimed at systematically rescuing firms in the presence of negative idiosyncratic shocks facilitates (tacit) collusion. This is because expected future co-operative profits increase (since firms are ensured to be in business in the subsequent periods); and because the guaranteed presence of competitors in the next periods makes the (expected) punishment phase harsher than under an environment where competitors may exit the market due to an exogenous shock, which would leave the deviant firm unpunished.

Examining the implications of this result for the welfare effects of State aid policy, our main result is the identification of a range of discount factors for which an aid policy is always welfare-detrimental because of its collusion-creating effect.

The analysis in this paper has focused on the basics of the mechanism to highlight the key insight. However, this paper can set the stage for interesting extensions. One possible direction is to devise more complex aid policies (stochastic, with repayments, limited to the last failing firm) or to consider a more complex competitive setup (introducing asymmetries, entry, a richer menu of contracts). Another possibility would be to embed a true banking model within our framework, for instance by endogenising portfolio choice and modelling an interbank (wholesale) market.

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# A Appendix

## A.1 Derivation of the critical discount factors

### A.1.1 No State aid

We start by deriving the expected collusive profit (LHS of the ICC). At each period $t$, a bank gets collusive profits $\delta^t \frac{\pi^M}{2}$ with prob $p_1(t)$; monopoly profits $\delta^t \pi^M$ with prob $p_2(t)$; and 0 with prob. $p_3(t)$. The respective probabilities can be written as follows:[12]

$$
\begin{aligned}
p_1(t) &= p^t(p^{t-1}) = p^{2t-1} \\
p_2(t) &= p^t\left[(1-p) + p(1-p) + p^2(1-p) + ... + p^{t-2}(1-p)\right] = p^t(1-p)\frac{1-p^{t-1}}{1-p} = p^t(1-p^{t-1}) \\
p_3(t) &= 1 - p^{2t-1} - p^t(1-p^{t-1}) = 1 - p^t
\end{aligned}
$$

[12] Notably the probability of the competitor being in the market in period $t$ can be computed as $p^{t-1}$. The fact that the competitor might have to leave the market in period $t$ does not affect the profits of the other bank in period $t$.

This yields:

$$
\begin{aligned}
LHS &= \sum_{t=1}^{\infty} \delta^{t-1} \left( p^{2t-1} \frac{\pi^M}{2} + p^t (1-p^{t-1}) \pi^M \right) \\
&= \left( \frac{p}{2(1-\delta p^2)} + \frac{p}{1-\delta p} - \frac{p}{(1-\delta p^2)} \right) \pi^M \\
&= \left( \frac{p}{1-\delta p} - \frac{p}{2(1-\delta p^2)} \right) \pi^M
\end{aligned}
$$

Next, we turn to the right-hand side of ICC, i.e. the deviation profit this period (obtained with probability $p$ since the shock occurs after the interest rate decision) plus the expected punishment stream from next period onwards. The former profit is simply $p\pi^M$. As for the latter, there are four possible events at each time $t \geq 2$:

1. Both banks are in the market at the beginning of the period and the deviating bank has a good shock in that period: $p_4(t) = p^{2(t-1)} * p$
2. Both banks are in the market at the beginning of the period and the deviating bank has a bad shock in that period: $p_5(t) = p^{2(t-1)} * (1-p)$
3. The deviating bank will be in the market at time $t \geq 2$ and earn monopoly profit alone: $p_6(t) = p^t \left[ (1-p) + p(1-p) + p^2(1-p) + ... + p^{t-2}(1-p) \right] = p^t(1-p)\frac{1-p^{t-1}}{1-p} = p^t(1-p^{t-1})$
4. The deviating bank will not be in the market: $p_7(t) = 1 - p^{2(t-1)} - p^t(1-p^{t-1})$.

However, it is only $p_6(t)$ that is associated to a non-zero payoff ($p_4(t)$ and $p_5(t)$ are associated to $\pi^C = 0$ and $p_7(t)$ to previous exit). Thus, adding over $t$:

$$
\begin{aligned}
RHS &= p\pi^M + \sum_{t=2}^{\infty} \delta^{t-1} p^t (1-p^{t-1}) \pi^M \\
&= p\pi^M + \frac{\delta p^2}{1-\delta p}\pi^M - \frac{\delta p^3}{1-\delta p^2}\pi^M
\end{aligned}
$$

Constructing the overall ICC by comparing LHS against RHS (i.e. (2)), solving for $\delta$ and noticing that $\pi^M$ falls through, one gets that tacit collusion is sustainable if:

$$
\underbrace{(\frac{1}{1-\delta p} - \frac{1}{2(1-\delta p^2)})p\pi^M}_{\text{expected profit of collusion}} \geq \underbrace{(1 + \frac{\delta p}{1-\delta p} - \frac{\delta p^2}{1-\delta p^2})p\pi^M}_{\text{expected profit of deviation}}
$$

$$
\delta \geq \frac{1}{2p^2}
$$

### A.1.2 State aid

With State aid a bank that has received a bad shock is rescued and allowed to operate in the following period. There is no profit (nor actual loss) in the period of failure. Setting up the ICC and solving for the critical discount factor, we obtain the traditional supergame result in a symmetric price-setting duopoly:

$$\begin{aligned}
\underbrace{p\sum_{t=1}\delta^{t-1}\frac{\pi^M}{2}+(1-p)\sum_{t=1}\delta^{t-1}*0}_{\text{expected profit of collusion}} &\geq \underbrace{p\pi^M+(1-p)*0+p\sum_{t=2}\delta^{t-1}*\underbrace{\pi^C}_{=0}+(1-p)\sum_{t=2}\delta^{t-1}*0}_{\text{expected profit of deviation}} \\
\frac{p\pi^M}{2(1-\delta)} &\geq p\pi^M
\end{aligned}$$

That is, collusion is sustainable for $\delta \geq \frac{1}{2}$.

## A.2 Derivation of the welfare equations

### A.2.1 No State aid

**Under collusion ($\delta \geq \frac{1}{2p^2}$):**

There are six possible states of a consumer in terms of welfare at time $t$: (I) there is a collusive duopoly and the deposits are returned with interest by both banks; (II) there is a collusive duopoly and all deposits are lost because of the bad shocks to both duopolists; (III) there is a collusive duopoly and only one bank receives a bad shock; (IV) there is a monopoly and the deposits are returned with interest; (V) there is a monopoly and the deposits are lost because of a bad shock; (VI) there is no market at all (banks have exited and consumer welfare is *nil*).

$$p_I(t)=p^{2t}$$

$$p_{II}(t)=p^{2(t-1)}(1-p)^2$$

$$p_{III}(t)=2\left(p^{2(t-1)}p(1-p)\right)=2\left(p^{2t-1}(1-p)\right)$$

$$\begin{aligned}
p_{IV}(t) &= 2*\left[p^{t-1}\left((1-p)+p(1-p)+p^2(1-p)+...+p^{t-2}(1-p)\right)\right]*p \\
&= 2p^t(1-p^{t-1})
\end{aligned}$$

$$
\begin{aligned}
p_V(t) &= 2*\left[p^{t-1}\left((1-p)+p(1-p)+p^2(1-p)+...+p^{t-2}(1-p)\right)\right]*(1-p) \\
&= 2p^{t-1}(1-p)(1-p^{t-1})
\end{aligned}
$$

$$
p_{VI}(t) = 1-(p_I(t)+p_{II}(t)+p_{III}(t)+p_{IV}(t)+p_V(t))
$$

The next step is to associate consumer welfare values to each state:

$$
\begin{aligned}
p_I(t) &: \quad W^C = \frac{1}{2}\left(\frac{R_H}{2}\right)^2 \\
p_{II}(t) &: \quad \Phi_{C2} = -\frac{R_H}{2} \\
p_{III}(t) &: \quad \frac{W^M}{2} + \Phi_{C1} = \frac{1}{4}\left(\frac{R_H}{2}\right)^2 - \frac{R_H}{4} \\
p_{IV}(t) &: \quad W^M = \frac{1}{2}\left(\frac{R_H}{2}\right)^2 \\
p_V(t) &: \quad \Phi_{C2} = -\frac{R_H}{2} \\
p_{VI}(t) &: \quad 0
\end{aligned}
$$

Next, we simply sum up these welfare levels (adjusted by the probabilities) over time, accounting for the discount factors. Notice that the sum of the losses is the same as the total size of the NDIS and thus the expected present discounted value of total taxes in the economy, which thus enter as negative terms ($\Phi < 0$):

$$
\begin{aligned}
E(W_C^{NA}) &= \sum_{t=1}^{\infty} \delta^{t-1}\{p^{2t}*W^C + p^{2(t-1)}(1-p)^2*\Phi_{C2} + 2\left(p^{2t-1}(1-p)\right)*\left(\frac{W^M}{2}+\Phi_{C1}\right) \\
&\quad +2p^t(1-p^{t-1})*W^M + 2p^{t-1}(1-p)(1-p^{t-1})*\Phi_{C2}\} \\
&= \frac{R_H/2}{1-\delta p^2}[(1-p)-\frac{p}{4}R_H] + \frac{R_H/2}{1-\delta p}[\frac{p}{2}R_H - 2(1-p)]
\end{aligned}
$$

**Under no collusion ($\delta < \frac{1}{2p^2}$):**

There are six possible states of a consumer in terms of welfare at time $t$: (I) there is a competitive duopoly and the deposits are returned with interest by both banks; (II) there is a competitive duopoly and all deposits are lost because of the bad shocks to both duopolists; (III) there is a competitive duopoly and only one bank receives a bad shock; (IV) there is a monopoly and the deposits are returned with interest; (V) there is a monopoly and the deposits are lost because of a bad shock; (VI) there is no market at all (banks have exited and consumer welfare is *nil*). These events occur, respectively, with the same probabilities $p_I(t)$ through $p_{VI}(t)$ that we discussed above; it is just that "collusive duopoly" has to be replaced with "competitive duopoly". However, the welfare levels associated to each probability are slightly different:

$$\begin{aligned}
p_I(t) &: \quad W^C = \frac{R_H^2}{2} \\
p_{II}(t) &: \quad \Phi_{NC2} = -R_H \\
p_{III}(t) &: \quad \frac{W^C}{2} + \Phi_{NC1} = \frac{R_H^2}{4} - \frac{R_H}{2} \\
p_{IV}(t) &: \quad W^M = \frac{1}{2}\left(\frac{R_H}{2}\right)^2 \\
p_V(t) &: \quad \Phi_{C2} = -\frac{R_H}{2} \\
p_{VI}(t) &: \quad 0
\end{aligned}$$

We sum again these probability-adjusted welfare levels over time, to obtain:

$$\begin{aligned}
E(W_{NC}^{NA}) &= \sum_{t=1}^{\infty} \delta^{t-1}\{p^{2t} * W^C + p^{2(t-1)}(1-p)^2 * \Phi_{NC2} + 2\left(p^{2t-1}(1-p)\right) * \left(\frac{W^M}{2} + \Phi_{NC1}\right) \\
&\quad +2p^t(1-p^{t-1}) * W^M + 2p^{t-1}(1-p)(1-p^{t-1}) * \Phi_{C2}\} \\
&= \frac{R_H/2}{1-\delta p^2}[\frac{p}{2}R_H] + \frac{R_H/2}{1-\delta p}[\frac{p}{2}R_H - 2(1-p)]
\end{aligned}$$

### A.2.2 State aid

We proceed in the same fashion as before. However, here, banks never exit. When consumers' deposits are lost due to the bad shocks, the Government refunds them the original capital, as well as paying the direct rescuing costs $\gamma$.

**Under collusion ($\delta \geq \frac{1}{2}$):**

There are three scenarios that can characterise the economy every period: (I) both banks have a good shock and return deposits with interest (which occurs with probability $\widehat{p}_I(t) = p^2$); (II) only one bank receives a bad shock (probability $\widehat{p}_{II}(t) = 2p(1-p)$); (III) both banks receive a bad shock ($\widehat{p}_{III}(t) = (1-p)^2$). This is true every period and each probability is associated to the following welfare levels:

$$\begin{aligned}
\widehat{p}_I(t) &: \quad W^M = \frac{1}{2}\left(\frac{R_H}{2}\right)^2 \\
\widehat{p}_{II}(t) &: \quad \frac{W^M}{2} + \Phi_{C1} - \gamma = \frac{1}{4}\left(\frac{R_H}{2}\right)^2 - \frac{R_H}{4} - \gamma \\
\widehat{p}_{III}(t) &: \quad \Phi_{C2} - 2\gamma = -\frac{R_H}{2} - 2\gamma
\end{aligned}$$

We can therefore sum this stream of expected payoffs and then subtract the present discounted value of the total tax bill (NDIS and direct aid):

$$
\begin{aligned}
E(W_C^A) &= \sum_{t=1}^{\infty 2} \delta^{t-1}\{p^2 W^M + 2p(1-p)\left(\frac{W^M}{2} + \Phi_{C1} - \gamma\right) + (1-p)^2\left(\Phi_{C2} - 2\gamma\right)\} \\
&= \frac{R_H/2}{1-\delta}[\frac{p}{4}R_H + p - 1] - \frac{2(1-p)\gamma}{1-\delta}
\end{aligned}
$$

**Under no collusion ($\delta < \frac{1}{2}$):**

We proceed exactly as in the case of collusion. The probabilities are the same as those derived above but the associated welfare levels are slightly different:

$$
\begin{aligned}
\widehat{p}_I(t) &: \qquad W^C = \frac{R_H^2}{2} \\
\widehat{p}_{II}(t) &: \qquad \frac{W^C}{2} + \Phi_{NC1} - \gamma = \frac{R_H^2}{4} - \frac{R_H}{2} - \gamma \\
\widehat{p}_{III}(t) &: \qquad \Phi_{NC2} - 2\gamma = -R_H - 2\gamma
\end{aligned}
$$

Summing up over time:

$$
\begin{aligned}
E(W_{NC}^A) &= \sum_{t=1}^{\infty} \delta^{t-1}\{p^2 W^C + 2p(1-p)\left(\frac{W^C}{2} + \Phi_{NC1} - \gamma\right) + (1-p)^2\left(\Phi_{NC2} - 2\gamma\right)\} \\
&= \frac{R_H/2}{1-\delta}[pR_H - 2(1-p)] - \frac{2(1-p)\gamma}{1-\delta}
\end{aligned}
$$

## A.3 On the direct cost of rescuing

In this section we show the positivity of gammas. For the threshold $\widehat{\gamma}_{NC}$:

$$
\widehat{\gamma}_{NC} = \frac{1-\delta}{2(1-p)}\{\frac{R_H/2}{1-\delta}[pR_H - 2(1-p)] - \frac{R_H/2}{1-\delta p^2}[\frac{p}{2}R_H] - \frac{R_H/2}{1-\delta p}[\frac{p}{2}R_H - 2(1-p)]\} \geq 0
$$

or

$$
\underbrace{pR_H(\frac{1}{1-\delta} - \frac{1/2}{1-\delta p^2} - \frac{1/2}{1-\delta p})}_{>0} + \underbrace{2(1-p)(-\frac{1}{1-\delta} + \frac{1}{1-\delta p})}_{<0} \geq 0
$$

It can be shown that the first term is larger in absolute terms. As a result the threshold is positive. To see this remember from condition 1 that $R_H > 4\frac{1-p}{p}$. Consequently we have that $pR_H > 2(1-p)$. Further notice that

$$
\frac{1}{1-\delta} - \frac{1/2}{1-\delta p^2} - \frac{1/2}{1-\delta p} > -[-\frac{1}{1-\delta} + \frac{1}{1-\delta p}]
$$

which gives us the result that $\widehat{\gamma}_{NC}$ is positive. For the threshold $\widehat{\gamma}_C$:

$$\widehat{\gamma}_C = \frac{1-\delta}{2(1-p)}\{\frac{R_H/2}{1-\delta}[\frac{p}{4}R_H - (1-p)] - \frac{R_H/2}{1-\delta p^2}[(1-p) - \frac{p}{4}R_H] - \frac{R_H/2}{1-\delta p}[\frac{p}{2}R_H - 2(1-p)]\} \geq 0$$

or

$$\underbrace{\frac{p}{4}R_H[\frac{1}{1-\delta} + \frac{1}{1-\delta p^2} - \frac{1}{1-\delta p}]}_{>0} + (1-p)[-\frac{1}{1-\delta} - \frac{1}{1-\delta p^2} + \frac{2}{1-\delta p}] \geq 0$$

For the same argument as before we have $\frac{p}{4}R_H > (1-p)$. Moreover

$$\begin{aligned} [\frac{1}{1-\delta} + \frac{1}{1-\delta p^2} - \frac{1}{1-\delta p}] &> -[-\frac{1}{1-\delta} - \frac{1}{1-\delta p^2} + \frac{2}{1-\delta p}] \\ \frac{1}{1-\delta p} &> 0 \end{aligned}$$

which is true since $\delta p < 1$, hence $\widehat{\gamma}_C$ is positive.

## A.4 Proof of (i) in Proposition 2

**Proof.** By contradiction,

$$\begin{aligned} E(W_C^A) &= \frac{R_H/2}{1-\delta}[\frac{p}{4}R_H + p - 1] - \frac{2(1-p)\gamma}{1-\delta} \geq \\ E(W_{NC}^{NA}) &= \frac{R_H/2}{1-\delta p^2}[\frac{p}{2}R_H] + \frac{R_H/2}{1-\delta p}[\frac{p}{2}R_H - 2(1-p)] \end{aligned}$$

$$[\frac{p}{4}R_H - (1-p)] \geq \frac{1-\delta}{1-\delta p^2}[\frac{p}{2}R_H] + \frac{1-\delta}{1-\delta p}[\frac{p}{2}R_H - 2(1-p)] \tag{10}$$

Note that RHS in equation (10) is continuous and decreasing in delta if $(\frac{p}{4}R_H + p - 1) > 0$ which is equivalent to $R_H > 4\frac{1-p}{p}$:[13]

$$\frac{\partial}{\partial\delta}(\frac{1-\delta}{1-\delta p^2}) = \frac{-(1-\delta p^2) + (1-\delta)p^2}{(1-\delta p^2)^2} = \frac{p^2-1}{(1-\delta p^2)^2} < 0$$

$$\frac{\partial}{\partial\delta}(\frac{1-\delta}{1-\delta p}) = \frac{-(1-\delta p) + (1-\delta)p}{(1-\delta p)^2} = \frac{p-1}{(1-\delta p)^2} < 0$$

[13] Which is the relation introduced in the assumption above.

Yielding:

$$\frac{\partial RHS}{\partial \delta} = \underbrace{\frac{p^2-1}{(1-\delta p^2)^2}}_{<0}\underbrace{[\frac{p}{2}R_H]}_{>0} + \underbrace{\frac{p-1}{(1-\delta p)^2}}_{<0}\underbrace{[\frac{p}{2}R_H + 2p - 2]}_{>0} < 0$$

Moreover:

$$\frac{1-\delta}{1-\delta p^2} < \frac{1-\delta}{1-\delta p}$$

Consequently the RHS of equation (10) is biggest for the smallest value of delta in the given range, which is $\delta = \frac{1}{2}$. After plugging in we arrive at:

$$\begin{aligned} & \frac{1/2}{1-p^2/2}[\frac{p}{2}R_H] + \frac{1/2}{1-p/2}[\frac{p}{2}R_H - 2(1-p)] \\ = \quad & \frac{1}{1-p^2/2}[\frac{p}{4}R_H] + \frac{1}{1-p/2}[\frac{p}{4}R_H - (1-p)] \end{aligned}$$

Thus we have:

$$\frac{1}{1-p^2/2}[\frac{p}{4}R_H] + \frac{1}{1-p/2}[\frac{p}{4}R_H + p - 1] \leq [\frac{p}{4}R_H + p - 1]$$

$$R_H \leq 4\frac{(1-p)}{p}\frac{(\frac{1}{2}-\frac{p^2}{4})}{(\frac{1}{p}-\frac{p^2}{4})}$$

Leading to a contradiction since $\frac{(\frac{1}{2}-\frac{p^2}{4})}{(\frac{1}{p}-\frac{p^2}{4})} < 1$ and we assumed that $R_H > 4\frac{(1-p)}{p}$.[14] Hence we have that: $E(W_C^A) < E(W_{NC}^{NA})$ , $\forall\ p \in (0,1) \wedge \frac{1}{2} \leq \delta < \frac{1}{2p^2}$. ■

[14] $\frac{(\frac{1}{2}-\frac{p^2}{4})}{(\frac{1}{p}-\frac{p^2}{4})} < 1$ leads to $p < 2$. Which is always true, knowing that $p \in [0,1]$.