European University Institute

EUI Working Papers

PRE-ANNOUNCEMENT AND TIMING – THE EFFECTS
OF A GOVERNMENT EXPENDITURE SHOCK

Alexander Kriwoluzky

EUROPEAN UNIVERSITY INSTITUTE, FLORENCE

DEPARTMENT OF ECONOMICS

*Pre-anouncement and Timing – The Effects
of a Government Expenditure Shock*

ALEXANDER KRIWOLUZKY

Pre-announcement and Timing – The Effects of a Government Expenditure Shock[*]

Alexander Kriwoluzky[¶]

European University Institute Florence

October 12, 2009

Abstract

This paper investigates the effect of a government expenditure shock on consumption and real wages. I identify the shock by exploiting its pre-announced nature, i.e. different signs of the responses in investment, hours worked and output during the announcement and after the realization of the shock. Since pre-announcement leads to a non-stationary moving average representation, I estimate and identify a VMA model. The identifying restrictions are derived from a DSGE model, which is estimated by matching the impulse response functions of the VMA model. Private consumption is found to respond negatively during the announcement period and positively after the realization. The reaction of real wages is significantly positive on impact, decreases during the announcement horizon, and is again significantly positive for two quarters after the realization.

JEL classification: C32, E62, H0.

Keywords: Fiscal Policy shock, Bayesian Estimation, DSGE model, Vector Autoregression.

1   Introduction

What are the effects of an innovation in government expenditure on the economy? It still remains disputed whether private consumption and real wages rise or fall in response. While economic theory supports both outcomes, empiricists have not yet been able to discriminate between different explanations, as the literature is divided over the appropriate methodological approach. On the one hand, the narrative approach identifying changes in government spending by military buildup dates in the US, predicts a decrease in consumption and real wages. On the other hand, the structural Vector Autoregression (SVAR) approach identifying the shocks either with timing or sign restriction, predicts the opposite.

In this paper I investigate the effects of a government expenditure shock by employing an SVAR model approach and identifying it via sign restriction as in Mountford and Uhlig (2005) and Pappa (2005). Moreover, I explicitly model pre-announcement of the fiscal policy shock and its consequences: the behavior of investment, GDP and hours worked differs during the announcement period and after the realization of the shock[1]. By doing so, I first avoid both criticism of the narrative approach - the potential small sample problems, be it with one shock or a combination of many - and also the ongoing discussion whether 'abnormal' fiscal policy episodes lead to 'abnormal' or 'normal' behavior of the economy.

Second, the approach set out in the paper also sidesteps the potential pitfalls of the SVAR approach. In a recent paper, Ramey (2008) pointed out that the different results obtained by using this approach are simply due to a faulty timing assumption in the SVAR literature which neglects or misses that changes in the government budget are often pre-announced or known to the public beforehand. One recent example is The American Recovery and Reinvestment Act of 2009, which announces government expenditures from 1009 to 2012 with most spending taken place in 2010 and 2011. As I will demonstrate in this paper, identifying the government expenditure shock by exploiting the pre-announcement effects explicitly, i.e. considering qualitative differences in the response of investment during the pre-announcement period and after the realization of the shock, overcomes the critique by Ramey.

When pursuing this approach another issue arises: the moving average representation of the data generated by a pre-announced policy is potentially non-stable so that it cannot be approximated by a VAR model[2]. Non-stability of the moving average representation has two consequences: First, the information set of the agent in the economy is larger than the information set of the econometrician: the agent has news about future, pre-announced changes, which are not contained in the information set observed by

---

[1] The existence of those differences were found by House and Shapiro (2006) and Trabandt and Uhlig (2006) among others.

[2] As pointed out first by Hansen and Sargent (1991) and more recently by Leeper, Walker, and Yang (2008).

the econometrician. This news is discounted by the agent in a different way than by the econometrician. The econometrician estimates innovations as a discounted sum of past news, where older news receives less weight than more recent. But, for an agent yesterday's news on pre-announced policy can be more relevant than today's. One challenge is therefore to align the information set of the econometrician with that of the agent. In this paper I achieve this by using additional information taken from the DSGE model. A second difficulty arises when attempting to estimate the non-stable moving average process. One possibility, as laid out by Lippi and Reichlin (1993) and Lippi and Reichlin (1994) is to flip the unstable roots of the moving average process into a stable region via Blaschke factors. Another way to proceed is to estimate the moving average process with a Kalman Filter[3]. In this paper I therefore estimate the Vector moving average representation.

Since there exists no common knowledge about the behavior of the macroeconomic variables, I formally derive the sign restrictions from a dynamic stochastic general equilibrium (DSGE) model. As DSGE model, I employ a model laid out by Galí, López-Salido, and Vallés (2007). It is well suited to resolving the debate because it addresses the arguments why the classic results should hold as well as the typical arguments why the classic results fail: households which cannot smooth consumption, imperfect labor markets and a certain degree of price stickiness. Depending on its parametrization, for example the fraction of rule-of-thumb consumers and the degree of price stickiness, the model features several classic or Keynesian characteristics. In the limit, i.e. with no rule-of-thumb consumers and firms allowed to reset prices each period, it boils down to a neoclassical model. To what extend those components influence the variables of interest and the resulting sign restrictions for the VMA model depends on a small number of structural parameters. The parameters of the DSGE model are estimated jointly.

The results for a three-quarter pre-announced increase in government expenditures show strong qualitative differences during the announcement period and after the realization of the shock: output and hours worked respond negatively during the announcement period and positively afterwards; investment responds negatively one additional quarter before responding positively. Private consumption mimics this behavior and shows a stable, slightly negative response during the announcement period followed by a significant positive response after the realization of the shock. Real wages react significantly positively on impact, decrease (and even become negative) during the announcement horizon and react significantly positively for two quarters after the realization.

The paper is organized in the following way: the next section summarizes the related literature. In section 3 I lay out the econometric strategy. The DSGE model used to derive the sign restrictions is described in section 4. The results are summarized in section 5. Section 6 concludes.

---

[3]This is shown by Eric Leeper on slides available on his webpage. Appendix A.5 summarizes his insights to keep the paper self-explanatory.

2 Related literature

In this section I first shortly review the theoretical work on the effects of a government expenditure shock before I discuss the existing empirical approaches and findings in more detail.

The evolution of the literature on the effects of a government expenditure shock can be summarized in the following way: starting out from a neoclassic growth model, which is step by step extended with market imperfections and nominal rigidities, an ultra New Keynesian model (as Ramey called it) evolved. The first neoclassic attempts to study the effects of fiscal policy date back to Hall (1980) and Barro (Barro (1981) and Barro (1987)). Building upon this work, Aiyagari, Christiano, and Eichenbaum (1990) and Baxter and King (1993) expand a neoclassical growth model of a government sector. In these models, an increase in government expenditure creates a negative wealth effect for the household, which will reduce consumption and increase labor supply. The increased labor supply induces real wages to decrease and interest rates to increase.

Rotemberg and Woodford (1992) and Devereux, Head, and Lapham (1996) introduced market imperfections, increasing returns to scale as well as monopolistic and oligopolistic competition respectively into the neoclassic growth model. In their models, a government spending shock, or a general demand shock, increases demand for goods, thereby labor demand and thus real wages. In a recent paper, Galí, López-Salido, and Vallés (2007) extend the New Keynesian model with rule-of-thumb consumers, who neither borrow nor save, only consume the disposable income each period. Since those households do not feel intertemporally poorer, they do not decrease consumption as a response to a positive government expenditure shock. Confronted with this large number of competing models, empiricists have tried to discriminate between them by investigating the response of real wages and private consumption after a change in government spending.

Findings in existing empirical studies are twofold depending on the identification scheme of the government expenditure shock. Ramey and Shapiro (1998) use a narrative approach to identify the VAR model. They interpret times of large military buildups in the US, the Korean war, the Vietnam war and the Carter-Reagan buildup, as sudden and unforseen increases in government expenditure. The resulting reactions of macroeconomic variables to these events are thus interpreted as deviations from normal behavior. They find that output and hours rise, while consumption and real wages fall. Burnside, Eichenbaum, and Fisher (2004) employ a similar methodology to estimate the impulse responses of macroeconomic variables to a government expenditure shock and compare them to impulse responses implied by a standard neoclassic model. The results indicate that hours worked rise and investment briefly increases, while real wages and consumption decrease. Thus they conclude that the standard neoclassic model can account reasonably well for the effects of fiscal policy shocks. A similar conclusion is drawn by Edelberg, Eichenbaum, and Fisher (1999), who modify a neoclassic growth model distinguishing two types of capital, nonresidential and residential.

A structural VAR approach is chosen by Blanchard and Perotti (2002) to identify a government expenditure shock. They require fiscal policy variables not to respond immediately to other innovations in the economy, i.e. they employ the recursiveness assumption. Their findings corroborate the results of Ramey and Shapiro (1998) concerning output and hours worked, but contradict their findings for consumption and real wages. Mountford and Uhlig (2005) also use a structural VAR, but do not consider any timing restriction. Instead, they employ sign restrictions to restrict the responses of fiscal variables, while the responses of other macroeconomic variables are left open. Besides the different methodology, they additionally allow for a pre-announcement of fiscal policy shocks. Indeed, as has been widely acknowledged and mentioned, most fiscal policy shocks are pre-announced. Their findings, however, confirm the results of Blanchard and Perotti (2002) except for consumption, which only shows a weak positive response.

The debate about the empirical evidence was reopened by Ramey (2008)[4]. Her paper takes up two issues. First, she stresses the importance of the composition of government expenditures. The dataset used by Blanchard and Perotti (2002) includes government consumption as well as government investment expenditure. An increase in the latter can be productive and potentially complement private consumption and investment so that it might lead to a positive response in those variables. For these reasons Ramey advocates using defense spending as a proxy for government expenditures in the VAR. Second, she states that the findings of the studies differ due to pre-announcement effects, implying that Blanchard and Perotti (2002) employ faulty timing to identify the fiscal policy shock. In her paper, a neoclassic DSGE model including a pre-announced government expenditure shock is set up and used to simulate artificial data. It is then demonstrated that, if the pre-announcement of the shock is taken into account, a negative response in consumption is estimated. If not, consumption appears to react positively, a clearly misleading result.

In his summary and discussion of the recent literature, Perotti (2007) acknowledges the concerns with respect to the structural VAR methodology. As a possible way of overcoming its weaknesses he suggests employing annual data and distinguishing between shocks to defense spending and to civilian government spending. However, using annual data, the recursive assumption that the fiscal sector does not react contemporarily to the state of the economy might not hold anymore. But, as Perotti mentions, the narrative approach has considerable weaknesses of its own: First, it suffers from a small sample size, second, it is not entirely clear whether the whole change in government expenditure was announced at once or whether it was a combination of small changes, i.e. whether there were numerous revisions of the military budget, occurring one after the other, causing private consumption to respond multiple times.

In Ravn, Schmitt-Grohé, and Uribe (2007), the authors dismiss Ramey's critique of the usage of structural VAR models. They point out that shocks are by assumption

---

[4]The first version dates back to 2006.

orthogonal to the information set and consequently identify a structural VAR as in Blanchard and Perotti (2002).

Pre-announced fiscal policy is considered in Mertens and Ravn (2009) and Tenhofen and Wolff (2007). The former authors address the issue of non-invertibility of the moving average representation by using a Vector Error Correction Model. The government expenditure shock is then identified by a combination of long run and zero restrictions. The latter authors augment the original VAR model by Blanchard and Perotti (2002) with expectations and consider a one period pre-announcement only. This paper differs in two dimensions: first by using sign restriction to identify the government expenditure, putting less structure and fewer restrictions on the VAR model, and second by estimating a VMA model to circumvent the issue of non-invertibility.

Besides the problem of non-invertibility of the moving average representation, Chung and Leeper (2007) discuss the importance of the intertemporal government budget constraint for a structural VAR analysis. In order to estimate reduced form shocks that can be mapped into structural innovations, government debt and private investment should be included in a VAR model.

Another important issue to take into account, as pointed out by Ramey (2008), is the composition of government expenditure and what part is used in the estimation. Abstracting from government transfers, government expenditure is defined as the sum of government investment expenditure and government consumption expenditure. Both types have different implications for the variables of interest. As described in Turnovsky and Fisher (1995), an increase in government investment expenditure increases productivity and therefore private consumption and real wages. Including government investment expenditure in the analysis would therefore favor a Keynesian outcome, i.e. an increase in both variables of interest and an adulteration of the analysis. Furthermore, government consumption expenditure is quantitatively much more relevant (since the 1970s it has been about five times as large as government investment expenditure). I will therefore employ government consumption expenditure when estimating the effects of an innovation in government expenditure.

3 Econometric strategy

Before describing the estimation methodology, I will discuss one note of caution when estimating fiscal policy set out by Leeper, Walker, and Yang (2008).

3.1 The non-invertibility of the VMA model representation and its consequences

Leeper, Walker, and Yang (2008) discuss the problem that private agents have foresight about future fiscal policy, which the investigating econometrician does not have, i.e. the information set of the private agent is larger than the information set of the econometrician. This leads to the effect, that agents discount news differently compared to the econometrician. More precisely, the econometrician estimates innovations as a discounted sum of old news where former news receives less weight. For the agent former news has a larger effect on today's variables due to pre-announcement. Secondly, they show that whenever foresight about future fiscal variables is present the resulting moving average representation of the data exhibits roots inside the unit circle. Consequently, the moving average process is not invertible and cannot be approximated by a VAR model. Furthermore, estimation of a moving average process with this kind of long memory property proves to be very difficult.

There exist two possibilities to address the latter issue: flipping the roots of the moving average process outside the unit circle by either applying a Blaschke-factor or a Kalman Filter.[5] I will employ the latter and thus obtain correct estimates of the reduced form moving average coefficients[6]. In order to recover the information set of the private agent I will apply sign restrictions.

These restrictions are derived from impulse response functions of the DSGE model. Two issues become crucial: the choice of the DSGE model and its parametrization. As DSGE model I employ a model laid out by Galí, López-Salido, and Vallés (2007). It is well suited to navigating through the debate because it addresses the arguments why the classic results should hold as well as the typical arguments why the classic results fail: households which cannot smooth consumption, imperfect labor markets and a certain degree of price stickiness. Depending on its parametrization, for example the fraction of rule-of-thumb consumers and the degree of price stickiness, the model features several classic or Keynesian characteristics. In the limit, i.e. with no rule-of-thumb consumers and firms allowed to reset prices each period, it boils down to a neoclassical model. Therefore, as Figure 3 indicates, this DSGE model allows for positive as well as negative responses in consumption and real wages. To what extend these components influence the variables of interest and the resulting sign restrictions for the VMA model depends on a small number of structural parameters, which have to be estimated.

---

[5]Leeper, Walker, and Yang (2008) discuss both possibilities extensively. Blaschke factors are used by Mertens and Ravn (2009).

[6]More algebraic details are given in appendix A.5.

3.2 Choice of the estimation methodology

The estimation methodology for the structural parameters of the DSGE model is chosen on the following grounds: due to advances in computational power there are various ways to estimate a DSGE model nowadays. However, procedures and results differ substantially depending on the econometric interpretation of the DSGE model. Geweke (1999) distinguishes between a strong and a weak interpretation. The former requires the DSGE model to provide a full description of the data generating process. Formally, it has to exhibit as many structural shocks as the observable variables to be explained. The DSGE model described above is clearly not intended to be a proper representation of the data generating process[7]. One estimation strategy therefore is to extend the DSGE model with additional structural shocks and several additional features and frictions such as for example capacity utilization and a habit in consumption. This is no such thing as a free lunch. This comes at the cost of a diluted analysis. The result cannot be traced back to certain DSGE model components such as for example the share of rule-of-thumb households and the degree of price stickiness, whose effects are to be investigated. Attempting to apply the strong econometric interpretation to DSGE model without extensions results in biased estimation results.[8] I therefore follow the weak econometric interpretation of the DSGE model and do not assume that it is a proper representation of the data generating process. It is estimated by matching the impulse response functions of the DSGE model and a VMA model to a government expenditure shock.

Thus, on the one hand the parameters of the DSGE model are estimated by matching the corresponding impulse response functions of the VMA model. On the other hand, the structural impulse response functions of the VMA model are identified by applying sign restrictions which are derived from the DSGE model. Both distributions are therefore conditional distributions: they depend on a realization of the impulse response function of the VMA model and on restrictions from the DSGE model, i.e. a realization of a vector of structural parameters of the DSGE model and a realization of the coefficients of the VMA model, respectively. To take those dependencies into account it is necessary to consider both, the impulse response functions of the VMA model and of the DSGE model as stochastic and to characterize their joint distribution.

After I have set up the necessary notation, I will decompose the joint distribution of the parameters of the DSGE model and the structural parameters of the VMA model, show how they are connected and how they can be evaluated. The distribution of the VMA model coefficients conditional on restrictions derived from the DSGE model is laid out in Appendix A.2 and the distribution of the parameters of the DSGE model conditional on φ^V is described in Appendix A.3. The estimation methodology is an application of the methodology laid out in Kriwoluzky (2009) and additional details

---

[7]As is pointed out by Galí, López-Salido, and Vallés (2007).

[8]In that case it is not possible to identify all structural parameters.

and information can be found there.

3.3   Notation of the VMA model and the DSGE model

Let Y_t be a $m \times 1$ vector at date $t = 1+l, \ldots, T$, φ_i be coefficient matrices of size $m \times m$ and ϵ an $i.i.d.$ one-step ahead forecast error, distributed: $\epsilon \sim \mathcal{N}(0, I_{m \times m})$. The VMA model containing m variables is then defined by:

$$Y_t = \sum_{i=0}^{\infty} \varphi_i^V \epsilon_{t-i}. \tag{1}$$

The impulse response function of a VMA model to an innovation in variable j at horizon i φ_{ik}^V is given by the j-th column of the i-th coefficient matrix.

Due to the assumption that $\Sigma_\epsilon = I_{m \times m}$, this structural moving average representation cannot be estimated directly. Instead, the reduced form moving average representation with error term $u_t = A\epsilon_t$, where $u \sim \mathcal{N}(0, \Sigma)$, is estimated. The reduced form moving average coefficients are defined as $\Phi_i = \varphi_i^V A^{-1}$:

$$Y_t = \sum_{i=0}^{\infty} \Phi_i u_{t-i}, \tag{2}$$

The factorization $\Sigma = A'A$ does not have a unique solution, which leads to an identification problem of A. The impulse matrix $\breve{A}$ is defined as a sub matrix of A of size $m \times n$ where n is the number of structural shocks under consideration, i.e. the structural shock of interest as well as other shocks necessary to distinguish this shock.

The fundamental solution of the DSGE model is given by[9]:

$$\hat{x}_t = T(\tilde{\theta})\hat{x}_{t-1} + R(\tilde{\theta})z_t, \tag{3}$$

where $T(\tilde{\theta})$ and $R(\tilde{\theta})$ are matrices one obtains after solving the DSGE model with standard solution techniques. z is a vector collecting the structural shocks of the DSGE model. z is assumed to be normally distributed with $z \sim \mathcal{N}(0, \Sigma_{DSGE})$.

The endogenous state variables of the DSGE model are related to the set of observable variables y via an observation equation:

$$\hat{y}_t = G\hat{x}_t + Hw_t \tag{4}$$

where G and H denote matrices picking the corresponding endogenous states and measurement errors w. w is assumed to be distributed $w \sim \mathcal{N}(0, \Sigma_{ME})$.

---

[9]$\widehat{x}_t$ denotes the percentage deviation of the generic variable x_t from a deterministic steady state x chosen as approximation point.

The impulse response functions of the variables in x to a structural shock j at horizon i $\varphi_{i,j}^D$ are given by:

$$\varphi_{i,j}^D(\tilde{\theta}) = T(\tilde{\theta})^i R(\tilde{\theta}) z_j, \quad i = 0, 1, 2, ...K. \tag{5}$$

The DSGE model is employed to estimate the structural impulse response functions of the VMA model. This induces matching a given realization of the impulse response function of the VMA model to the i-th shock at horizon k, $\varphi_{i,k}^V$:

$$\varphi_{i,k}^V = \varphi_{i,k}^D(\tilde{\theta}) + \omega_{i,k}. \tag{6}$$

Stacking the impulse response functions over $1, .., K$ periods together yields:

$$\varphi_i^V = \varphi_i^D(\tilde{\theta}) + \omega_i \tag{7}$$

with all vectors of dimension $m*k\times1$. The error term ω_i has the property $E[\omega_i\omega_i'] = \Omega_{\omega_i}$.

The vector comprising the vector of deep parameters $\tilde{\theta}$, the vectorized variance covariance matrices Σ_{ME} and Σ_{DSGE} as well as the vectorized Ω is defined as $\theta = [\ \tilde{\theta} \quad \Sigma_{DSGE} \quad vec(\Omega) \quad vec(\Sigma_{ME}) \ ]'$.

3.4 The joint posterior distribution

I describe the joint posterior distribution of the parameters of the DSGE model and of the impulse response function of a VAR model based on the following considerations: On the one hand, the parameters of the DSGE model are estimated by matching the corresponding impulse response function of the VMA model. On the other hand, the structural impulse response functions of the VMA model are identified by applying zero and sign restrictions which are derived from the DSGE model. Both distributions are therefore conditional distributions: they depend on a realization of the impulse response function of the VMA model and on restrictions from the DSGE model, i.e. a realization of a vector of structural parameters of the DSGE model, respectively. This section sets out, how the conditional distributions can be combined to derive the joint distribution.

The joint posterior distribution of the vector of parameters of the DSGE model (θ) and the coefficients of the VMA model, φ^V, given a matrix with time series observations Y, $p(\theta, \varphi^V|Y)$, can be decomposed in different ways. In the case that the likelihood of the VMA model impulse response functions depends on restrictions from the DSGE model, $p(\theta, \varphi^V|Y)$ is given as:

$$p(\varphi^V, \theta|Y) = p(\varphi^V|\theta, Y)p(\theta|Y). \tag{8}$$

Since the DSGE model is estimated by matching corresponding impulse response functions, the coefficients of the VMA model constitute a statistic sufficient for estimating the parameters of the DSGE model, i.e. $p(\theta|\varphi^V, Y) = p(\theta|\varphi^V)$. The joint posterior distribution can then be written as:

$$p(\varphi^V, \theta|Y) = p(\varphi^V|Y)p(\theta|\varphi^V). \tag{9}$$

The framework presented in Kriwoluzky (2009) is based on the argument that both distributions are at least proportionally equal:

$$p(\varphi^V|Y)p(\theta|\varphi^V) \propto p(\varphi^V|\theta, Y)p(\theta|Y), \tag{10}$$

and can be approximated sufficiently well by Monte Carlo Markov Chain Methods [10].

4 The DSGE model

The DSGE model employed here was originally laid out by Galí, López-Salido, and Vallés (2007). I relax the assumption of no initial debt in order to contrast the DSGE model with the data. The DSGE model consists of two types of households, households with access to government bond and capital markets and households without. Goods are produced by perfectly competitive firms, using goods produced by intermediate firms as inputs. Intermediate good firms are monopolistic competitors and subject to a Calvo pricing mechanism, i.e. with a certain probability they receive a signal allowing them to reset their price which they are not allowed to change otherwise. Intermediate good firms have access to a production technology combining capital and labor. Labor is supplied by labor unions.

The government consists of a monetary authority setting the nominal interest rate and a fiscal authority issuing bonds and raising taxes. Government expenditure is modeled as an exogenous process.

4.1 Households

The DSGE model exhibits a continuum of infinitively lived households indexed by $i \in [0, 1]$. There exist two types of households, households optimizing intertemporally and households which are not allowed to save. The latter are called rule-of-thumb households. More precisely, they are not allowed to participate in the capital and bonds goods market, an assumption made to reflect the fact that some households have indeed only limited access to the credit market. The share of rule-of-thumb households in the economy is given by λ.

Intertemporally optimizing households maximize utility depending on consumption c^o and labor n^o:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(log(c_t^o) - \frac{n_t^{o1+\nu}}{1+\nu} \right), \tag{11}$$

---

[10]Note that the conditional distributions of interest ($p(\theta|\varphi^V)$ and $p(\varphi^V|\theta, Y)$) are on different sides of the proportionality sign in (10). It is therefore possible to employ a Gibbs sampling algorithm. The sampling algorithm is described in Appendix A.4.

where β is the discount factor and ν a parameter measuring the disutility of labor. The household buys government bonds B yielding the return R, invest i^o units in capital goods k^o yielding return R^k and receives dividends d from the ownership of firms. With real wages denoted by w and lump sum taxes by t^o the budget constraint is given by:

$$c_t^o + i_t^o + \frac{B_t}{p_t R_t^b} = w_t n_t^o + R_t^k k_{t-1}^o + \frac{B_{t-1}}{p_t} + t_t^o + d_t. \tag{12}$$

Capital accumulates according to:

$$k_t^o = (1-\delta)k_{t-1}^o + \Phi\left(\frac{i_t^o}{k_{t-1}^o}\right)k_{t-1}^o, \tag{13}$$

where Φ denotes investment to capital adjustment costs. With respect to the adjustment cost function the following assumptions are made: $\Phi(\delta) = \delta$, $\Phi'(\delta) = 1$, $\Phi' > 0$, $\Phi'' \leq 0$. The elasticity of the investment to capital ratio with respect to Tobin's $q_t = \frac{1}{\Phi'\left(\frac{i_t^o}{k_{t-1}^o}\right)}$ is defined as: $\eta \equiv -\frac{1}{\Phi''(\delta)\delta}$.

Rule-of-thumb households maximize each period's utility:

$$U = log(c_t^r) - \frac{n_t^{r\,1+\nu}}{1+\nu} \tag{14}$$

subject to the budget constraint:

$$c_t^r = w_t n_t^r - t_t^r \tag{15}$$

Both types of households are assumed to consume the same amount of goods in the steady state. This can be achieved through the appropriate choices of the lump sum taxes t^r and t^o. Aggregate variables, i.e. aggregate consumption c, labor n, capital k and investment i are defined as the weighted average of the corresponding variables of the rule-of-thumb household and the intertemporally optimizing household:

$$
\begin{aligned}
c_t &= \lambda c_t^r + (1-\lambda)c_t^o &(16)\\
n_t &= \lambda n_t^r + (1-\lambda)n_t^o &(17)\\
i_t &= (1-\lambda)i_t^o &(18)\\
k_t &= (1-\lambda)k_t^o. &(19)
\end{aligned}
$$

4.2 Firms

The economy contains two sectors. In one sector perfectly competitive firms produce the final good y using as inputs intermediate goods produced by monopolistically competitive firms.

4.2.1 Final good firms

Final good firms have access to the following production function:

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\varpi_p-1}{\varpi_p}} dj \right)^{\frac{\varpi_p-1}{\varpi_p}}, \tag{20}$$

where $y(j)$ denotes the intermediate good produced by firm j and $\varpi_p > 1$ the elasticity of substitution between different intermediate goods. Demand for each good $y(j)$ is given by

$$y_t(j) = \left(\frac{p_t(j)}{p_t} \right)^{-\varpi_p} y_t. \tag{21}$$

Cost minimization under perfect competition yields

$$p_t^{1-\varpi_p} = \left(\int_0^1 p_t(j)^{1-\varpi_p} dj \right). \tag{22}$$

4.2.2 Intermediate good firms

An intermediate good firm j, $j \in [0,1]$ produces the good $y(j)$ using the production function:

$$y_t(j) = k_{t-1}(j)^\alpha n_t(j)^{1-\alpha}, \tag{23}$$

where α denotes the capital share in the production. Taking the real wage and capital as given, cost minimization implies:

$$\frac{k_t(j)}{n_t(j)} = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{w_t}{R_t^k} \right). \tag{24}$$

Real marginal costs MC common to all firms can be derived as

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} R_t^{k\alpha} w_t^{1-\alpha}. \tag{25}$$

Intermediate good firms are subject to a Calvo pricing mechanism. Each period the firm receives a price signal with probability ϑ. The intermediate firm is allowed to set the price $p_t(j)$ and maximizes

$$E_t \sum_{k=0}^{\infty} \vartheta^k \beta^k \left(y_{t+k}(j)(\frac{p_t(j)}{p_t} - mc_{t+k}) \right) \tag{26}$$

subject to the demand function:

$$y_t(j) = \left(\frac{p_t(j)}{p_t} \right)^{-\varpi_p} y_t. \tag{27}$$

13

4.3 Government sector

The government sector consists of a monetary authority setting nominal interest rates R and a fiscal authority setting lump sum taxes t and issuing nominal government bonds B. The monetary authority follows the simple interest rate rule:

$$R_t = \phi_p i \pi_t. \tag{28}$$

The fiscal authority has to balance the government budget constraint given by:

$$t_t + \frac{B_t}{p_t R_t} = g_t + \frac{B_{t-1}}{p_t}. \tag{29}$$

Since the fiscal authority can adjust bonds and lump sum taxes, an additional fiscal policy rule is needed to determine both variables. The fiscal policy rule is of the form[11]

$$\frac{t_t}{\bar{t}} = \phi_b \frac{b_{t-1}}{\bar{b}} + \phi_g \frac{g_t}{\bar{g}}. \tag{30}$$

4.4 Labor unions

As an additional friction, an imperfect labor market is assumed. The introduction of wage-setting by unions leads to the outcome that the amount of labor supplied by the households is equal across households, i.e. $n_t^o = n_t^r$. A continuum of unions is assumed representing a certain type of labor. Let ϖ_w denote the elasticity of substitution across different types of labor. Effective labor input hired by firm j is then given by:

$$n_t(j) = \left(\int_0^1 n_t(j,i) di \right)^{\frac{\varpi_w}{\varpi_w - 1}}. \tag{31}$$

Assuming furthermore that the proportion of rule-of-thumb households is uniformly distributed across households and therefore across unions, a typical union (z) sets the wage in order to maximize the following expression:

$$\lambda \left[\frac{w_t(z) n_t(z)}{c_t^r(z)} - \frac{n_t(z)^{1+\nu}}{1+\nu} \right] + (1-\lambda) \left[\frac{w_t(z) n_t(z)}{c_t^o(z)} - \frac{n_t(z)^{1+\nu}}{1+\nu} \right] \tag{32}$$

subject to:

$$n_t(z) = \left(\frac{w_t(z)}{w_t} \right)^{-\varpi_w} n_t. \tag{33}$$

---

[11]$\bar{x}$ denote steady state values.

4.5 Market clearing and equilibrium

The goods market clearing condition is given by:

$$y_t = c_t + i_t + g_t. \tag{34}$$

An equilibrium is defined in the following way:

Definition 1 *An equilibrium is an allocation $\{c_t, n_t, i_t, k_t, b_t, y_t\}$ and a price system $\{p_t(j), p_t, w_t(z), w_t, R_t^k, R_t^b\}$ and an inflation rate π_t such that for a monetary policy R_t and fiscal policy t_t, an initial price level p_{-1}, initial values for k_{-1} and b_{-1} and given an exogenous processes for government expenditure $\{g_t\}$:*

i *for each intertemporally maximizing household an allocation $\{c_t^o, n_t^o, k_t^o, x_t^o\}$ maximizes (11) subject to the budget constraint (12) and the capital accumulation equation (13), given prices $\{w_t, R_t^k, R_t^b\}$ and profits $\{d_t\}$;*

ii *for each rule-of-thumb household an allocation $\{c_t^r, n_t^r\}$ maximizes (14) subject to the budget constraint (15), given prices $\{w_t\}$;*

iii *the definitions*

$$
\begin{align}
c_t &= \lambda c_t^r + (1-\lambda)c_t^o \tag{35} \\
n_t &= \lambda n_t^r + (1-\lambda)n_t^o \tag{36} \\
i_t &= (1-\lambda)i_t^o \tag{37} \\
k_t &= (1-\lambda)k_t^o \tag{38}
\end{align}
$$

hold;

iv *the production allocation $\{y_t(j), y_t\}$ and prices $\{p_t, p_t(j), R_t^k, w_t\}$ solve the cost minimization problem of the final good firms and the profit maximization problem of each intermediate firm j subject to the demand function (27) and technology (23);*

v *for each labor union z the allocation $\{n_t, n_t(z), c_t^r(z), c_t^o(z)\}$ and prices $\{w_t(z), w_t\}$ maximize the pay-off function (32) subject to the demand for each labor union (33);*

vi *the government budget constraint (29) is fulfilled;*

vii *markets are clear.*

5 Results

5.1 Pre-announcement and timing – A Monte Carlo Study

In this section the DSGE model is calibrated to mimic the DSGE model employed by Ramey (2008). I simulate artificial data from it and estimate a structural VAR model with this dataset taking pre-announcement effects into account.

In order to mimic the DSGE model laid out by Ramey, which is a neoclassical growth model with government spending and non-distortionary taxes, the DSGE model from section 4 is calibrated in the following way. The fraction of rule-of-thumb households is set to 0 ($\lambda = 0$), the elasticity of the investment to capital ratio with respect to Tobin's q is set fairly high ($\eta = 10$), the probability of not optimizing prices is set very low to mirror almost flexible prices ($\vartheta = 0.05$) and the elasticity of substitution between intermediate goods is set to 1 ($\mu_p = 1$). Remaining parameters are chosen similar to Ramey, i.e. the discount factor $\beta = 0.99$, the depreciation rate $\delta = 0.025$, the capital share in production $\alpha = 0.33$ and the parameter measuring the disutility of labor $\nu = 1$. The policy parameters of the fiscal and monetary authorities are set such as to ensure uniqueness of the solution: $\phi_b = 0.3$, $\phi_g = 0.1$ and $\phi_\pi = 1.5$. The processes for actual government expenditures g_t and the forecast of government expenditure g_t^f are taken from Ramey:

$$\hat{g}_t = \hat{g}_{t-2}^f \tag{39}$$

$$\hat{g}_t^f = 1.4\hat{g}_{t-1}^f - 0.18\hat{g}_{t-2}^f - 0.25\hat{g}_{t-3}^f + \epsilon_t^g \tag{40}$$

Since I am going to estimate a VAR model with four variables, the DSGE model from which the data is simulated has to incorporate at least this number of shocks. I therefore augment the model with a preference shock to labor, a shock to total factor productivity (as Ramey does) and a monetary policy shock (i.e. a shock to equation (28)).

The resulting impulse response function of the DSGE model to a shock in government expenditure is plotted in Figure 1. It is very similar to the figure in Ramey (2008). Note that investment first reacts positively to the shock and becomes negative afterwards. In order to provide evidence that making use of the pre-announced nature of a government expenditure shock, i.e. restricting variables to respond differently during the announcement of the shock and after realization ,can resolve the problem of flawed estimation approaches, I identify the government expenditure shock employing the following sign restrictions: government expenditure are assumed to be zero during the first period (pre-announcement period) and to be positive afterwards, hours worked are assumed to respond positively and investment is restricted to respond positively during the first quarter and to respond negatively afterwards.[12]

---

[12]Note that I do not describe the joint distribution of the impulse response function of the DSGE model and the VAR model. This is due to the fact that the sign restrictions are identical across a reasonable parameter space of the DSGE model.

The result of the Monte Carlo Experiment is shown in Figure 2. The response of consumption is significantly negative. This finding is robust with respect to a faulty pre-announced period (one only instead of two) and whether the government expenditure shock is ordered first or second. Given this encouraging result I will apply the methodology to the data. This time, however, the sign restrictions will not be identical across the parameter space of the DSGE model.

5.2   Data

I now apply the methodology to quarterly US data ranging from the first quarter of 1948 to the third of 2007. The VMA model consists of seven variables: government consumption expenditure, real GDP, private consumption, hours worked, private investment, real wages and real federal debt. The data was obtained from the internet, mostly from NIPA and FRED. In appendix A.1 a detailed description of the exact source can be found. In order to remove long term trends from the data I employ a HP filter with a smoothing parameter equal to 1600.[13]

5.3   Specification of the identifying restriction and the prior distribution

I order the government expenditure shock similarly as in Mountford and Uhlig (2005) to come second after a business cycle shock; output, real wages, private investment, private consumption and hours worked are restricted to respond positively. By doing so the business cycle shock is assumed to explain most of the variance of the variables of the VMA model. The government expenditure shock is constructed to be orthogonal to the business cycle shock, i.e. most of the potential co-movements will be removed. The pre-announcement horizon is set to three quarters following the number suggested by Yang (2007). All variables except private consumption and real wages are restricted to exhibit the signs derived from the impulse response function of the DSGE model. The sign is allowed to switch, i.e. it can differ during the pre-announcement of the shock and after its realization.

The mean of the prior distributions of the parameters of the DSGE model is specified very closely to values used for calibration by Galí, López-Salido, and Vallés (2007). One exception is the choice of η. In order to allow for a wider range of impulse responses, the mean is set to 7. The standard deviations of the prior distribution are chosen to ensure that the impulse response functions of the DSGE model cover a wide range of possibilities as depicted in Figure 3. Not all parameters are estimated. The values used to calibrate those parameters are taken entirely from Galí, López-Salido, and Vallés

---

[13]As suggested by Hodrick and Prescott (1997).

(2007), except for the ratio of real debt to GDP which is set to $\frac{\bar{b}}{\bar{y}} = 0.6$. Calibrated parameters include: $\beta = 0.99$, $\delta = 0.025$, $\alpha = 1/3$, $\frac{\bar{g}}{\bar{y}} = 0.2$, $\phi_\pi = 1.5$ and $\mu_\pi = 1.2$.

5.4 Estimation Results

The smoothed Kalman filter maximum likelihood estimates of the VMA model variables are depicted in Figure 4. The plot indicates that the time series is very well described by the VMA model.

The impulse response functions of the VMA model are shown in Figure 5. The first subplot displays the announcement and the realization of the government consumption expenditure shock: the first three periods are restricted to zero followed by one positive period. Afterwards, government expenditures fall and even become negative. Given the fact that real debt responds positively over five quarters (even though the response is restricted for four quarters only), the decline in government expenditure gives rise to a policy rule in which government consumption expenditure reacts negatively on an increase in real debt - as suggested among others by Leeper and Yang (2008), Bohn (1991) and Corsetti, Meier, and Müller (2009).

The responses of output and hours worked in Figure 5 display a significantly different qualitative behavior before and after the realization of the government consumption expenditure shock. Both react negatively during the announcement horizon and strongly positively after the realization of the shock. The response of private investment shows similar behavior, but the sign of the response changes after four (not after the announcement horizon of three quarters) from negative to positive. This implies that the restrictions derived from the DSGE model are negative over the complete restriction horizon.

Summarizing the estimated sign restrictions from the DSGE model, it can be stated that real debt is restricted to react positively over four periods, private investment to react negatively, while output and hours worked are restricted to respond negatively during the announcement period and positively after the realization.

These identifying restrictions imply the following results for private consumption and real wages. Private consumption displays significantly negative behavior throughout the announcement period and a very strong positive response afterwards. Real wages react significantly positively on impact, decrease throughout the announcement period and increase after the realization. During the fifth and sixth period the response of real wages is positive.

What is driving this result? One advantage of the methodology employed is that the estimated impulse response functions of the VMA model can be interpreted in economic terms by means of the DSGE model. Key for the interpretation are the estimation results of the structural parameters of the DSGE model. Before I interpret

18

the results, I present and discuss the estimation results of the structural parameters and the corresponding impulse response function of the DSGE model.

The prior and posterior distribution is plotted in Figure 7. Table 1 gives an overview of the characteristics of the prior and posterior distribution. The plot and the table indicate that all parameters are well identified and that the estimates are within a very reasonable range. The estimated share of rule-of-thumb consumers (0.36 at the posterior mean) is in the range of the estimates obtained by Forni, Monteforte, and Sessa (2009) in studies using European data: $0.34 - 0.37$. The posterior mean of the calvo parameter ($\vartheta = 0.74$) is estimated slightly higher than in a recent study by Smets and Wouters (2007), who estimated it at about ($\vartheta = 0.66$). However, the relatively high estimate of the price stickiness parameter is consistent with a characteristic of the DSGE model pointed out by Furlanetto and Seneca (2009): the DSGE model employed obeys many real frictions, which are present in the DSGE model of Smets and Wouters (2007). Once those real rigidities were included, the resulting nominal frictions would have been estimated at lower values. The posterior mean estimate of the elasticity of labor with respect to wages ($\nu = 0.14$) is in line with, though slightly lower than the estimates obtained by Rotemberg and Woodford (1997) and Rotemberg and Woodford (1999). The value for the estimate of the elasticity of the investment to capital-capital ratio with respect to Tobins's Q ($\eta = 4.2$) is higher than the value calibrated by Galí, López-Salido, and Vallés (2007) ($\eta = 1$), implying less investment adjustment costs. Given the response of government expenditure in Figure 5, the persistence of government expenditure ($\rho_g = 0.74$) is lower than that calibrated by Galí, López-Salido, and Vallés (2007) ($\rho_g = 0.9$). The elasticity of taxes with respect to government expenditure ($\phi_g = 0.08$) and the elasticity of taxes with respect to government debt ($\phi_b = 0.32$) are closely estimated to the values set by Galí, López-Salido, and Vallés (2007) (($\phi_g = 0.1$) and $\phi_= 0.33$).

A comparison of the impulse response functions of the DSGE model and the VMA model is plotted in Figure 6. The impulse response functions of the VMA model are well matched by those of the DSGE model. In particular, the movement of output, hours worked, private consumption, and real wages (for some periods) are very well matched by the DSGE model. Impulse response functions not matched very well include private investment, government consumption expenditure and real debt after they become negative.

The impulse response of government consumption expenditure of the VMA model is very well matched by the DSGE model up to the point where government consumption expenditure becomes negative. Since the DSGE model employed in this analysis does not feature a policy rule with spending reversals, the behavior of government consumption expenditure after the shock cannot be matched by the impulse response function of the DSGE model. A policy rule featuring government spending reversals would be straightforward to implement. On the other hand, a modified policy rule would also affect other endogenous variables differently compared to the original DSGE model I

19

want to consider. In order to be as transparent as possible on the choice of the DSGE model and its consequences, I choose to abstain from modifying the DSGE model in any dimension. A similar reasoning, i.e. a lack in the richness of the specification of the fiscal policy rules, applies to the response of real debt. Initially, the DSGE model has a lower response of real debt, which increases over time, while the impulse response function of the VMA model displays a strong initial positive response, which becomes negative after five periods.

The impulse response of private investment displays the largest difference between the VMA model and the DSGE model: the VMA model displays a strong negative response, which becomes positive after the restriction horizon of four periods. Even though, the adjustment costs of inflation are estimated more flexibly than in the original calibration by Galí, López-Salido, and Vallés (2007), the response of investment is less volatile in the DSGE model than in the VMA model.

Most importantly, the impulse response functions of the DSGE model and the VMA model, which are at the center of the analysis, correspond very well: firstly the variables which display a qualitative difference between the announcement and the realization of the shock, i.e. output and hours worked; and secondly the variables under inspection, private consumption over the complete horizon and real wages except for the first period. Since the impulse responses correspond so well, the DSGE model can be used to recover the mechanism behind the impulse responses of private consumption and real wages. Key for the mechanism are the expectations formed by the firm sector. Figure 8 shows the impulse response functions of the nominal interest rate and inflation in the DSGE model. Both responses rise after the pre-announcement of the shock. Firms anticipate higher inflation after the realization of the shock and increase prices after the announcement immediately. Monetary authorities increase nominal interest rates as a response to the rise in inflation. The increase in prices leads to an initial drop in GDP, hours worked, private consumption and investment. The result is stagflation during the announcement. The increase in government expenditure leads then to a rise in output, hours worked and private consumption for two reasons: a rise in income for the share of rule-of-thumb households and a drop in prices due to the negative response of output so far - prices overshoot.

5.5   Fiscal Multiplier and Variance decomposition

I compute two kinds of fiscal multipliers. The first multiplier is defined as the ratio of the response of output divided by the change in government expenditure and scaled by the average share of government consumption expenditure in GDP over the sample[14]. The result is shown in Figure 9. Due to the negative response of output after the announcement, the fiscal multiplier is negative. After two periods it is strongly rising,

---

[14]This definition is also used in Mountford and Uhlig (2008).

becomes positive after four periods and significantly larger than 1 after five periods. Eventually it is decreasing again.

The second multiplier is defined as the cumulative change in output divided by the change in government consumption expenditure and scaled by the average share of government consumption expenditure in GDP over the sample. The result is depicted in Figure 10. The negative response of output in the beginning causes the multiplier to not be positive until five periods after the announcement.

The share of the variance of the variables explained by the two shocks is shown in Figure 11 and Figure 12 for the business cycle and the government consumption expenditure shock respectively. As assumed, the business cycle shock explains most of the variance in all variables. For all variables except government expenditure this is up to 40% with a median around 10%. The government expenditure shock explains up to 10% of the variance during the announcement period for the variables except real wages. Here up to 40% is explained during the impact period, but much less, below 15%, in the following periods. After the realization of the shock, up to 20% of the variance in output and private consumption is explained.

5.6   Comparison with other studies

To the best of my knowledge there are three studies considering the effects of a pre-announced government expenditure shock using a SVAR model approach.

The results presented in this paper are most similar to those obtained by Mertens and Ravn (2009). In contrast to the other studies they also address the issue of the non-invertibility of the VMA representation. Even though they do not find qualitative differences in the response of private consumption and output after the announcement and the realization of a shock, they find a very strong announcement effect: both variables increase strongly when the shock takes place. During the announcement private consumption reacts negatively.

The results obtained are also in line with Tenhofen and Wolff (2007). They consider a one quarter announcement horizon and find a negative response of private consumption to a pre-announced government expenditure shock. After the shock, private consumption increases steadily, but, in contrast to the result in this paper, does not become positive.

Mountford and Uhlig (2008) consider different policy scenarios for a four quarter announcement horizon without explicitly modeling the pre-announcement. For an announced increase in government expenditure they find an immediate rise in private consumption and output as the effect of the announcement. Despite a more persistent response in output and private consumption they do not find any other announcement effects.

Putting the results into the context of the debate between Ramey (2008) and Blanchard and Perotti (2002), I find partial support for both views: while there are pre-announcement effects that cause private consumption to respond negatively during the first periods as pointed out by Ramey (2008), the realization of the shock leads to a strong positive response in private consumption as found by Blanchard and Perotti (2002).

6 Conclusion

This paper has investigated the effect of a government expenditure shock on private consumption and real wages by employing a structural VMA model. The identification key has been to model the pre-announcement of a government expenditure shock and its consequences on other economic variables explicitly.

The application of this idea is not straightforward for two reasons: first, when assuming that policy is pre-announced, the moving average representation of the data generated by this policy is potentially non-stable so that it cannot be approximated by a VAR model. I have therefore estimated a VMA model directly. Second, since the restrictions are not common knowledge I have employed a DSGE model, laid out initially by Galí, López-Salido, and Vallés (2007), from which to derive the sign restrictions. The DSGE model is well suited to the problem because it addresses the typical arguments of the Keynesian as well as the classic view of the economy. On the one hand it features households which cannot smooth consumption, imperfect labor markets and a certain degree of price stickiness. How strong these features influence the result and the restrictions depend on its parametrization, for example the proportion of rule-of-thumb consumers and the degree of price stickiness. In the limit, i.e. with no rule-of-thumb consumers and firms allowed to reset prices each period, it boils down to a neoclassical model. Therefore, as Figure 3, indicates this DSGE model allows for positive as well as negative responses in consumption and real wages. The parametrization of the DSGE model and the corresponding identifying assumptions for the VMA model are estimated by matching the corresponding impulse response functions of the VMA model. Thus the parameters of the VMA model and the DSGE model are estimated jointly.

The results for a three quarter pre-announced increase in government expenditures show strong qualitative differences during the announcement period and after the realization of the shock: output and hours worked respond negatively during the announcement period and positively afterwards, investment responds negatively one additional quarter before responding positively. Private consumption mimics this behavior and shows a stable, slightly negative response during the announcement period followed by a significant positive response after the realization of the shock. Real wages react significantly positively on impact, decrease (and even become negative) during the announcement horizon and react significantly positively for two quarters after the realization.

References

AIYAGARI, S. R., L. J. CHRISTIANO, AND M. EICHENBAUM (1990): "The output, employment, and interest rate effects of government consumption," Discussion Paper / Institute for Empirical Macroeconomics 25, Federal Reserve Bank of Minneapolis.

BARRO, R. J. (1981): "Output Effects of Government Purchases," *Journal of Political Economy*, 89(6), 1086–1121.

BARRO, R. J. (1987): "Government spending, interest rates, prices, and budget deficits in the United Kingdom, 1701-1918," *Journal of Monetary Economics*, 20(2), 221–247.

BAXTER, M., AND R. G. KING (1993): "Fiscal Policy in General Equilibrium," *American Economic Review*, 83(3), 315–34.

BLANCHARD, O., AND R. PEROTTI (2002): "An Empirical Characterization Of The Dynamic Effects Of Changes In Government Spending And Taxes On Output," *The Quarterly Journal of Economics*, 117(4), 1329–1368.

BOHN, H. (1991): "Budget balance through revenue or spending adjustments? : Some historical evidence for the United States," *Journal of Monetary Economics*, 27(3), 333–359.

BURNSIDE, C., M. EICHENBAUM, AND J. D. M. FISHER (2004): "Fiscal shocks and their consequences," *Journal of Economic Theory*, 115(1), 89–117.

CHUNG, H., AND E. M. LEEPER (2007): "What Has Financed Government Debt?," NBER Working Papers 13425, National Bureau of Economic Research, Inc.

CORSETTI, G., A. MEIER, AND G. J. MÜLLER (2009): "Fiscal Stimulus with spending reversals," CEPR Discussion Papers 7302, C.E.P.R. Discussion Papers.

DEVEREUX, M. B., A. C. HEAD, AND B. J. LAPHAM (1996): "Monopolistic Competition, Increasing Returns, and the Effects of Government Spending," *Journal of Money, Credit and Banking*, 28(2), 233–54.

EDELBERG, W., M. EICHENBAUM, AND J. D. FISHER (1999): "Understanding the Effects of a Shock to Government Purchases," *Review of Economic Dynamics*, 2(1), 166–206.

FORNI, L., L. MONTEFORTE, AND L. SESSA (2009): "The general equilibrium effects of fiscal policy: Estimates for the Euro area," *Journal of Public Economics*, 93(3-4), 559–585.

FURLANETTO, F., AND M. SENECA (2009): "Fiscal Shocks and Real Rigidities," *Topics in Macroeconomics*, 9(1), 1781–1781.

GALÍ, J., J. LÓPEZ-SALIDO, AND J. VALLÉS (2007): "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association*, 5(1), 227–270.

GEWEKE, J. (1999): "Computational experiments and reality," Discussion paper, University of Minnesota and Federal Reserve Bank of Minneapolis.

HALL, R. E. (1980): "Labor Supply and Aggregate Fluctuations," NBER Working Papers 0385, National Bureau of Economic Research, Inc.

HANSEN, L. P., AND T. J. SARGENT (1991): "Two Difficulties in Interpreting Vector Autoregressions," in *Rational Expectations Econometrics*, ed. by L. P. Hansen, and T. J. Sargent. Westview Press.

HODRICK, R. J., AND E. C. PRESCOTT (1997): "Postwar U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit and Banking*, 29(1), 1–16.

HOUSE, C. L., AND M. D. SHAPIRO (2006): "Phased-In Tax Cuts and Economic Activity," *American Economic Review*, 96(5), 1835–1849.

KRIWOLUZKY, A. (2009): "Matching Theory and Data: Bayesian Vector Autoregression and Dynamic Stochastic General Equilibrium Models," Economics Working Papers ECO2009/29, European University Institute.

LEEPER, E. M., T. B. WALKER, AND S.-C. S. YANG (2008): "Fiscal Foresight: Analytics and Econometrics," NBER Working Papers 14028, National Bureau of Economic Research, Inc.

LEEPER, E. M., AND S.-C. S. YANG (2008): "Dynamic scoring: Alternative financing schemes," *Journal of Public Economics*, 92(1-2), 159–182.

LIPPI, M., AND L. REICHLIN (1993): "The Dynamic Effects of Aggregate Demand and Supply Disturbances: Comment," *American Economic Review*, 83(3), 644–52.

——— (1994): "VAR analysis, nonfundamental representations, blaschke matrices," *Journal of Econometrics*, 63(1), 307–325.

MERTENS, K., AND M. O. RAVN (2009): "Measuring The Impact Of Fiscal Policy In The Face Of Anticipation: A Structural VAR Approach," manuscript, European University Institute.

MOUNTFORD, A., AND H. UHLIG (2005): "What are the Effects of Fiscal Policy Shocks?," SFB 649 Discussion Papers SFB649DP2005-039, Sonderforschungsbereich 649, Humboldt University, Berlin, Germany.

——— (2008): "What are the Effects of Fiscal Policy Shocks?," NBER Working Papers 14551, National Bureau of Economic Research, Inc.

PAPPA, E. (2005): "New-Keynesian or RBC Transmission? The Effects of Fiscal Shocks in Labour Markets," CEPR Discussion Papers 5313, C.E.P.R. Discussion Papers.

PEROTTI, R. (2007): "In Search of the Transmission Mechanism of Fiscal Policy," NBER Working Papers 13143, National Bureau of Economic Research, Inc.

RAMEY, V. A. (2008): "Identifying Government Spending Shocks: It's All in the Timing," Discussion paper, University of California, San Diego.

RAMEY, V. A., AND M. D. SHAPIRO (1998): "Costly capital reallocation and the effects of government spending," *Carnegie-Rochester Conference Series on Public Policy*, 48, 145–194.

RAVN, M. O., S. SCHMITT-GROHÉ, AND M. URIBE (2007): "Explaining the Effects of Government Spending Shocks on Consumption and the Real Exchange Rate," NBER Working Papers 13328, National Bureau of Economic Research, Inc.

ROTEMBERG, J., AND M. WOODFORD (1997): "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," in *NBER Macroeconomics Annual 1997, Volume 12*, NBER Chapters, pp. 297–361. National Bureau of Economic Research, Inc.

ROTEMBERG, J., AND M. WOODFORD (1999): "Interest-Rate-Rules in an Estimated Sticky-Price-Model," in *Monetary Policy Rules*, ed. by J. Taylor. University of Chicago Press, Chicago.

ROTEMBERG, J. J., AND M. WOODFORD (1992): "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity," *Journal of Political Economy*, 100(6), 1153–1207.

SMETS, F., AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

TENHOFEN, J., AND G. B. WOLFF (2007): "Does anticipation of government spending matter? Evidence from an expectation augmented VAR," Discussion Paper Series 1: Economic Studies 2007,14, Deutsche Bundesbank, Research Centre.

TRABANDT, M., AND H. UHLIG (2006): "How Far Are We From The Slippery Slope? The Laffer Curve Revisited," SFB 649 Discussion Papers SFB649DP2006-023, Sonderforschungsbereich 649, Humboldt University, Berlin, Germany.

TURNOVSKY, S. J., AND W. H. FISHER (1995): "The composition of government expenditure and its consequences for macroeconomic performance," *Journal of Economic Dynamics and Control*, 19(4), 747–786.

YANG, S.-C. S. (2007): "A Chronology Of Postwar U.S. Federal Income Tax Policy," Caepr Working Papers 2007-021, Center for Applied Economics and Policy Research, Economics Department, Indiana University Bloomington.

A Appendix

A.1 Data description

The frequency of all data used is quarterly.

Real GDP: This series is *BEA NIPA table 1.1.6 line 1 (A191RX1)*.

Nominal GDP: This is a measure of nominal GDP given by the series *GDP, Gross Domestic Product* at the Federal Reserve Board of St. Louis' website *http://research.stlouisfed.org/fred2/*. It is measured in billions of dollars.

Implicit GDP Deflator: The implicit GDP deflator is calculated as the ratio of **Nominal GDP** to **Real GDP**.

Private Consumption: Nominal consumption expenditures for non-durables and services is the sum of the respective values of the series *PCND, Personal Consumption Expenditures: Nondurable Goods* and *PCESV, Personal Consumption Expenditures: Services* at the Federal Reserve Board of St. Louis' website *http://research.stlouisfed.org/fred2/*. Both series are measured in billions of dollars.

Private Investment: Total real private investment is the sum of the respective values of the series *BEA NIPA table 1.1.6 line 6 (A006RX1)* and *PCDG, Personal Consumption Expenditures: Durable Goods* at the Federal Reserve Board of St. Louis' website *http://research.stlouisfed.org/fred2/* (billions of dollars) in real terms.

Government Expenditure: Current government expenditure is the series in *BEA NIPA table 3.1 line 15 (W022RC1)*.

Government Debt: The annual government debt is the historical series that can be copied from TreasuryDirect at *http://www.treasurydirect.gov/govt/reports/pd/histdebt/histdebt.htm*. The quarterly data is generated by a linear interpolation. The values have been converted from dollars to billions of dollars. Note that the beginning of the fiscal year changed from July 1 to October 1 in 1977.

Hours worked: This series is downloadable from the website of the Bureau of Labor Statistics at *http://data.bls.gov/cgi-bin/srgate*. The series' identification number is: *PRS84006033*. It is an index (1992=100).

Wage: The wage rate is the series *COMPNFB, Nonfarm Business Sector: Compensation Per Hour* at the Federal Reserve Board of St. Louis' website *http://research.stlouisfed.org/fred2/*.

Government Consumption Expenditures: Government consumption expenditures is the series *BEA NIPA table 3.1 line 16 (A955RC1)*.

Civilian Population: This is a quarterly measure for the population given by the respective average of the monthly values of the series *CNP16OV, Civilian Non-institutional Population* at the Federal Reserve Board of St. Louis' website *http://research.stlouisfed.org/fred2/*. The numbers have been converted from thousands to billions.

A.2 The posterior distribution of the VMA model

The conditional distribution described in this section is $p(\varphi^V|\theta, Y)$ from the right-hand side of (10). It is conditional since the prior distribution for the impulse response functions $p(\varphi^V) = p(\varphi^V|\theta)$ is derived from the DSGE model.[15] The posterior distribution of the structural impulse responses φ^V is obtained by combining the coefficient estimates of the reduced form VMA model Φ with an impulse matrix A.

$$p(\varphi^V|\theta, Y) = p(A, \Phi|\theta, Y)J(\varphi^V \to A, \Phi) \tag{A-1}$$

Therefore, the distribution in focus is the right-hand side of equation (A-1). The Jacobian is derived in appendix A.2.1. The posterior distribution $p(A, \Phi|\theta, Y)$ is derived by maximizing the likelihood function $p(Y|\phi, A)$ with respect to the reduced VMA model coefficients and its variance covariance matrix $\Sigma = A'A$ first[16]. The likelihood is multiplied by a prior distribution containing the identifying restrictions $p(A, \Phi|\theta)$ to yield the posterior distribution. Since the identifying restrictions are sign restrictions they put zero prior weight on the parameter regions of Φ and A whenever the sign restrictions are not satisfied.

The posterior distribution is evaluated in the following way. Denote the likelihood estimates as $vec(\tilde{\Phi})$ and $vec(\tilde{\Sigma})$[17]. $vec(\Phi)$ and $vec(\Sigma)$ are normally distributed with

$$[vec(\Sigma)'vec(\Phi)']' \sim \mathcal{N}([vec(\tilde{\Sigma})'vec(\tilde{\Phi})']', \Sigma_l), \tag{A-2}$$

where Σ_l is the inverse of the Hessian computed at $[vec(\tilde{\Sigma})'vec(\tilde{\Phi})]'$. Every realization of the vector of parameters of the DSGE model θ is associated with an impulse response function of the DSGE model and a realization of the impulse matrix $\breve{A}(\theta)$. A sequence of realizations of θ yields a sequence of restrictions and therefore a related prior probability distribution.

---

[15] The impulse response functions of the DSGE model define a probability distribution of impulse response functions dependent on θ.

[16] See Appendix A.2.2 for a detailed description.

[17] $vec(\Sigma)$ summarizes only the unique entries in Σ.

A.2.1 Setup of the prior distribution $p(\varphi^V|\theta)$

The prior distribution for the reduced form coefficients of the VMA model and the impulse matrix A conditional on restrictions derived from the DSGE model is derived in the following steps. First, a prior distribution for φ^V given impulse response functions of the DSGE model is formulated. The distribution is decomposed into conditional normal distributions. The conditional normal distributions are written in terms of the reduced form coefficients and the impulse matrix. To do so, the distribution has to be scaled by the Jacobian.

Denote the impulse response functions in period k as φ_k^V. If all shocks are included, the matrix is of size $m \times m$, where the entry i, j corresponds to the response of variable i to an innovation in variable j. The prior for the impulse responses has to be specified for as many periods as there are impulse response functions to be estimated. The vectorized impulse responses are assumed to be normally distributed:

$$
\begin{bmatrix}
vec(\varphi_0) \\
vec(\varphi_1) \\
vec(\varphi_2) \\
\vdots \\
vec(\varphi_l)
\end{bmatrix}
\sim \mathcal{N}
\left(
\begin{bmatrix}
vec(\bar\varphi_0) \\
vec(\bar\varphi_1) \\
vec(\bar\varphi_2) \\
\vdots \\
vec(\bar\varphi_k)
\end{bmatrix}
,
\begin{bmatrix}
\bar V_{00} & \bar V_{01} & \bar V_{02} & \cdots & \bar V_{0l} \\
\bar V_{10} & \bar V_{11} & \bar V_{12} & \cdots & \bar V_{1k} \\
\bar V_{20} & \bar V_{21} & \bar V_{22} & \cdots & \bar V_{2k} \\
\vdots & & & & \\
\bar V_{k0} & \bar V_{k1} & \bar V_{k2} & \cdots & \bar V_{kk}
\end{bmatrix}
\right).
\tag{A-3}
$$

The probability distribution $p(\varphi_0, \varphi_1, ..., \varphi_k)$ can be decomposed into a marginal distribution of $p(\varphi_0)$ and succeeding conditional distributions:

$$
p(\varphi_0, \varphi_1, ..., \varphi_k) = p(\varphi_k|\varphi_{k-1} \cdots \varphi_0) p(\varphi_{k-1}|\varphi_{k-2} \cdots \varphi_0) \cdots p(\varphi_1|\varphi_0) p(\varphi_0),
\tag{A-4}
$$

with

$$
p(vec(\varphi_0)) = \mathcal{N}(vec(\bar\varphi_0), \bar{\bar V}_{00})
\tag{A-5}
$$

$$
p(vec(\varphi_i|vec(\varphi_{i-1}) \cdots vec(\varphi_0)) = \mathcal{N}(\chi_i, \Delta_{ii}), i = 1 \cdots k,
\tag{A-6}
$$

and χ_i and Δ_{ii} abbreviate the usual definitions for conditional distributions:

$$
\chi_i = vec(\bar\varphi) + \begin{bmatrix} \bar V_{i0} \cdots \bar V_{ii-1} \end{bmatrix}
\begin{bmatrix}
\bar V_{00} & \cdots & \bar V_{0i-1} \\
\vdots & \ddots & \vdots \\
\bar V_{i-1,0} & \cdots & \bar V_{i-1,i-1}
\end{bmatrix}^{-1}
\begin{bmatrix}
vec(\varphi_0 - \bar\varphi_0) \\
\vdots \\
vec(\varphi_{i-1} - \bar\varphi_{i-1})
\end{bmatrix}
$$

$$
\Delta_{ii} = \bar V_{ii} - \begin{bmatrix} \bar V_{i0} & \cdots & \bar V_{ii-1} \end{bmatrix}
\begin{bmatrix}
\bar V_{00} & \cdots & \bar V_{0i-1} \\
\vdots & \ddots & \vdots \\
\bar V_{i-1,0} & \cdots & \bar V_{i-1,i-1}
\end{bmatrix}^{-1}
\begin{bmatrix}
\bar V_{0i} \\
\vdots \\
\bar V_{i-1,i}
\end{bmatrix}
$$

In order to write the prior distribution in terms of the reduced form coefficients it is necessary to scale the probability distribution with the Jacobian:

$$
p(\varphi) = p(f(\Phi)) J(\varphi \Rightarrow \Phi).
\tag{A-7}
$$

The relationship between structural and reduced form moving average coefficients is given by:

$$\varphi_0 = A \tag{A-8}$$
$$\varphi_i = \Phi_i A, i = 1 \cdots k.$$

Note that Φ_0 is omitted since this matrix is normalized to an identity matrix by assumption. This also indicates that it is not possible to infer on φ_0 from the estimated reduced VMA model.

The Jacobian is calculated in the following way. Applying the vec-operator yields:[18]

$$vec(\varphi_i) = (A' \otimes I_{m \times m})vec(\Phi_i). \tag{A-9}$$

The Jacobian matrix is defined as:

$$J(\varphi \to \Phi) = det \begin{bmatrix} \frac{\partial vec(\varphi_1)}{\partial vec(\Phi_0)} & \frac{\partial vec(\varphi_1)}{\partial vec(\Phi_1)} & \cdots & \frac{\partial vec(\varphi_1)}{\partial vec(\Phi_k)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial vec(\varphi_k)}{\partial vec(\Phi_0)} & \frac{\partial vec(\varphi_k)}{\partial vec(\Phi_1)} & \cdots & \frac{\partial vec(\varphi_k)}{\partial vec(\Phi_k)} \end{bmatrix}. \tag{A-10}$$

Due to the fact that $\frac{\partial vec(\varphi_i)}{\partial vec(\Phi_j)} = 0$ for $j > i$, the matrix becomes a block triangular matrix and the determinant is given by:

$$J(\varphi \to \Phi) = |\frac{\partial vec(\varphi_0)}{\partial vec(\Phi_0)}| \times |\frac{\partial vec(\varphi_1)}{\partial vec(\Phi_1)}| \cdots |\frac{\partial vec(\varphi_k)}{\partial vec(\Phi_k)}|$$
$$J(\varphi \to \Phi) = |(A' \otimes I_{m \times m})|^k = |A|^{mk} \tag{A-11}$$

Given (A-3), its decomposition defined in (A-4) and the Jacobian (A-11) a prior distribution for the reduced form coefficients conditional on $\varphi_0 = A$ is formulated as:

$$p(A, \Phi_1, ..., \Phi_k | \theta) = p(\Phi_k | \Phi_{k-1} \cdots A, \theta) \cdots p(\varphi_1 | A, \theta) p(A | \theta) J(\varphi \to \Phi), \tag{A-12}$$

where

$$p(vec(A)) = \mathcal{N}(vec(\bar{\varphi}_0), \bar{\bar{V}}_{00}) \tag{A-13}$$
$$p(vec(\Phi_i) | vec(\Phi_{i-1}) \cdots vec(A)) = \mathcal{N}(\bar{\Phi}_i, \bar{\bar{V}}_i i) \tag{A-14}$$

with

$$\bar{\Phi}_i = (A' \otimes I_{m \times m})\theta_i \tag{A-15}$$
$$\bar{\bar{V}}_{ii} = (A^{-1'} \otimes I_{m \times m})\Delta_{ii}(A^{-1'} \otimes I_{m \times m}). \tag{A-16}$$

The prior distribution (A-12) consists of conditional normal distributions (A-16), which can be easily evaluated, and a non-standard distribution for the impulse matrix. The non-standard distribution arises because (A-15) is scaled by (A-11).

---

[18]Note that $vec(AB) = (I \otimes A)vec(B) = (B' \otimes I)vec(A)$

A.2.2 The likelihood for the reduced form coefficients

Consider the VMA(k) process

$$Y_t = u_t + \Phi_1 u_{t-1} + \Phi_2 u_{t-2} + \cdots + \Phi_k u_{t-k}. \tag{A-17}$$

This can be written in state space form:

$$\xi_{t+1} = F\xi_t + U_{t+1} \tag{A-18}$$
$$y_t = H\xi_t, \tag{A-19}$$

where

$$\xi_t = \underset{m*k \times 1}{\left[\begin{array}{ccc} u_t & \cdots & u_{t-k} \end{array}\right]'}$$

$$F = \underset{m*k \times m*k}{\left[\begin{array}{ccccc} 0 & 0 & \cdots & 0 & 0 \\ I_m & 0 & \cdots & 0 & 0 \\ 0 & I_m & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_m & 0 \end{array}\right]}$$

$$U_{t+1} = \underset{m*k \times 1}{\left[\begin{array}{cccc} u_{t+1} & 0 & \cdots & 0 \end{array}\right]'}$$

$$H = \underset{m* \times m*k}{\left[\begin{array}{cccc} I_m & \Phi_1 & \cdots & \Phi_k \end{array}\right]}$$

Given an initial condition for y_0 and Σ_0 the likelihood can then be written as:

$$p(y_T, \ldots, y_0 | \Phi_1, ..., \Phi_k, \Sigma) = p(y_T | y_{T-1} \ldots y_0, \Phi_1, ..., \Phi_k, \Sigma) \cdots p(y_0 | \Phi_1, ..., \Phi_k, \Sigma), \tag{A-20}$$

where:

$$p(y_t | y_{t-1} \ldots y_0, \Phi_1, ..., \Phi_k) = \mathcal{N}(y_{t|t-1}, \Sigma_{t|t-1}) \tag{A-21}$$

and $y_{t|t-1}$ and $\Sigma_{t|t-1}$ denote the optimal forecast at time t, which is a function of the coefficient matrices. The impulse matrix A is not part of the likelihood function; instead, the variance covariance matrix $\Sigma = A'A$.

A.3 The conditional distribution of the DSGE model parameters

Since the DSGE model is not assumed to be a proper representation of the data generating process, the structural parameters are not estimated by matching the data Y. Instead, the DSGE model is assumed to replicate the implied dynamics of the data, i.e. the impulse response functions of the VAR model. This induces matching a given

realization of the impulse response function of the VAR model to the j-th shock at horizon i, $\varphi_{j,i}^V$:

$$\varphi_{j,i}^V = \varphi_{j,i}^D(\tilde{\theta}) + \omega_{j,i}. \tag{A-22}$$

Stacking the impulse response functions over $1,..,K$ periods together yields:

$$\varphi_j^V = \varphi_j^D(\tilde{\theta}) + \omega_j, \tag{A-23}$$

with all vectors of dimension $m * K \times 1$. The error term ω_i has the property $E[\omega_i \omega_i'] = \Omega_{\omega_i}$, which was part of the vector θ. Since the structural shocks are assumed to be independent, the probability of $p(\theta|\varphi^V)$ can be written as:

$$p(\theta|\varphi^V) = p(\theta|\varphi_1^V, \varphi_2^V, \cdots \varphi_i^V) = p(\theta|\varphi_1^V)p(\theta|\varphi_2^V).\cdots p(\theta|\varphi_i^V) \tag{A-24}$$

The vector θ is estimated in two steps: first Ω_{ω_i} is estimated, and afterwards the vector of deep parameters $\tilde{\theta}$. The variance covariance is estimated by making use of the relationship:

$$\omega_i = \varphi_i^V - \varphi_i^D(\tilde{\theta}). \tag{A-25}$$

For every realization of φ_i^V a reasonable number of draws from $p(\theta)$ are taken, and the corresponding impulse response function $\varphi_i^D(\tilde{\theta})$ and the error terms are computed. $\tilde{\Omega}_{\omega_i}$ is then estimated as the covariance matrix of these error terms. For each shock i the likelihood $l_i\left(\tilde{\theta}|\varphi_i^V, \tilde{\Omega}_{\omega_i}\right)$ is given by:

$$l_i\left(\tilde{\theta}|\varphi_i^V, \tilde{\Omega}_{\omega_i}\right) = -\frac{Km}{2}ln(2\pi) - \frac{1}{2}ln(|\tilde{\Omega}_{\omega_i} \otimes I_K|) - \frac{1}{2}(\omega_i)'\left(\tilde{\Omega}_{\omega_i} \otimes I_K\right)^{-1}(\omega_i) \tag{A-26}$$

Combining this likelihood with a prior distribution for θ yields a posterior distribution.

A.4   Sampling algorithm

In order to evaluate the joint posterior distribution of the parameters of the DSGE model and the VAR model I use a Gibbs sampling algorithm combined with a Metropolis-Hastings step. The Gibbs sampling algorithm allows me to draw from the conditional distributions laid out in detail in sections **??** and A.3. The Metropolis-Hastings step is an acceptance/rejection sampling algorithm that determines the probability space where the implied impulse response functions of the DSGE model and those of the VAR model coincide. It is carried out 20 times.

To initialize the algorithm, candidate distributions for the conditional distributions $p(\varphi^V|\theta, Y)$ and $p(\theta|\varphi^V)$ are derived for $j = 1 \dots d$:

1. Draw a θ_j from $p(\theta)$.

2. For every realization of the vector of deep parameters of the DSGE model derive the corresponding sign restriction.

3. Draw a Φ from $vec(\Phi) \sim \mathcal{N}(vec(\tilde{\Phi}), \tilde{\Sigma}_\Phi)$ and an orthonormal matrix Q so that φ^V satisfies the restrictions derived from φ^D.

4. For every realization of φ^V derived from step 3 find the θ that maximizes (A-26) combined with the prior $p(\theta)$.

5. Repeat this d times.

The d vectors of deep structural parameters define the candidate distribution $p(\theta|\varphi^V)^c$ and a corresponding distribution of $p(\varphi^V|\theta)^c$ for the following algorithm. At each iteration $i = 1, \ldots, I$ conduct the following steps:

1. Draw n times from $p(\theta|\varphi^V)^c$.

2. For every realization of the vector of deep parameters of the DSGE model derive the corresponding sign restrictions.

3. Given the sign restrictions, draw a Φ from $vec(\Phi) \sim \mathcal{N}(vec(\tilde{\Phi}), \tilde{\Sigma}_\Phi)$. Compute the lower Cholesky decomposition and find an $\breve{A} = \tilde{A}Q$ fulfilling the sign restrictions in 2. Compute the corresponding φ^V, yielding $p(\varphi^V|\theta)^{\tilde{i}}$.

4. For every realization of φ^V derived from step 3 find the θ that maximizes (A-26) combined with the prior $p(\theta)$. This yields $p(\theta|\varphi^V)^{\tilde{i}}$.

5. Carry out acceptance-rejection by comparing $p(\theta|\varphi^V)^{\tilde{i}}$ with $p(\theta|\varphi^V)^{i-1}$. Keep the corresponding vectors from $p(\varphi^V|\theta)^{\tilde{i}}$. This yields $p(\theta|\varphi^V)^{i-1}$ and $p(\varphi^V|\theta)^i$.

6. Start again at 1.

The chain converges if $p(\theta|\varphi^V)^i$ and $p(\theta|\varphi^V)^{i-1}$ and also $p(\varphi^V|\theta)^i$ and $p(\varphi^V|\theta)^{i-1}$ are similar, i.e. the acceptance rate is low. It is important to note that the candidate $p(\theta|\varphi^V)^c$ is not adjusting over the algorithm in order to avoid the algorithm becoming stuck and to allow for a continuous wide range of the parameters of the DSGE model. Thus, d should be chosen high enough. I chose $d = 200$.

A.5 Kalman Filter and root-flipping

This section provides insights into how the Kalman Filter flips the roots of a non-stable process similar to a Blaschke factor and therefore recovers the correct econometric

estimates. The calculations and the example in this section are mostly taken from slides by Eric Leeper[19].

To keep this section self-explanatory, I first set out briefly the DSGE model used as an example. The DSGE model, as in Leeper, Walker, and Yang (2008), is a standard growth model with log preferences, inelastic labor supply, and complete depreciation of capital. The equilibrium equations for consumption c_g, output v_g and capital k_g are given by:

$$\frac{1}{c_{g,t}} = \alpha_g \beta_g E_t (1 - \tau_{g,t+1}) \frac{1}{c_{g,t+1}} \frac{v_{g,t+1}}{k_{g,t}}$$
$$c_{g,t} + k_{g,t} = v_{g,t}$$
$$v_{g,t} = a_{g,t} k_{g,t-1}^{\alpha_g},$$

where β_g is the discount factor, α_g the share of capital in the production function and a_g an exogenous technology shock. The government sets taxes according to:

$$t_{g,t} = \tau_{g,t} v_{g,t}.$$

After log linearizing the system of equations, the equilibrium is characterized by a second-order difference equation in capital, which is solved by

$$\hat{k}_{g,t} = \alpha_g \hat{k}_{g,t-1} + \hat{a}_{g,t} - (1 - \theta_g) \left(\frac{\bar{\tau}_g}{1 - \bar{\tau}_g} \right) \sum_{i=0}^{\infty} \theta_g^i E_t \hat{\tau}_{g,t+i+1}, \tag{A-27}$$

where $\theta_g = \alpha_g \beta_g (1 - \bar{\tau}_g)$. In a DSGE model with pre-announced taxes the tax rate $\hat{\tau}_t$ is a function of news shocks $\epsilon_{g,\tau,t-q}$, where q denotes the pre-announcement horizon. Define $\kappa_g = (1 - \theta_g) \left(\frac{\bar{\tau}_g}{1 - \bar{\tau}_g} \right)$. For $q = 2$ (A-27) can be written as:

$$\hat{k}_{g,t} = \alpha_g \hat{k}_{g,t-1} + \hat{a}_{g,t} - \kappa_g \left(\epsilon_{g,\tau,t-1} + \theta \epsilon_{g,\tau,t} \right), \tag{A-28}$$

equation (A-28) can be written as

$$(1 - \alpha_g L) \hat{k}_{g,t} = -\kappa_g (L + \theta_g) \epsilon_{g,\tau,t}.$$

Invertibility of this stochastic process requires $|\theta_g| > 1$, which is not the case. One way to achieve invertibility is to employ a Blaschke factor [20] $(L + \theta_g)/(1 + \theta_g L)$. Define a new error term $\epsilon_{g,\tau,t}^{\star} = (L + \theta_g)/(1 + \theta_g L) \epsilon_{g,\tau,t}$. Then,

$$(1 - \alpha_g L) \hat{k}_{g,t} = -\kappa (L + \theta_g) \frac{(1 + \theta_g L)}{(L + \theta_g)} (L + \theta_g)/(1 + \theta_g L) \epsilon_{g,\tau,t}$$
$$(1 - \alpha_g L) \hat{k}_{g,t} = -\kappa (1 + \theta_g L) \epsilon_{g,\tau,t}^{\star}$$
$$(1 - \alpha_g L) \hat{k}_{g,t} = -(1 - \theta) \left(\frac{\bar{\tau}_g}{1 - \bar{\tau}_g} \left(\epsilon_{g,\tau,t}^{\star} + \theta_g \epsilon_{g,\tau,t-1}^{\star} \right) \right). \tag{A-29}$$

---

[19]From ZEI summer school 2008.

[20]See Lippi and Reichlin (1993), Lippi and Reichlin (1994) and Mertens and Ravn (2009) for further information.

Equation (A-29) is now an invertible stochastic process for $|\theta_g| < 1$.

Another way to achieve invertibility is to write equation (A-28) as a state space system in the innovation representation:

$$\begin{aligned}
x_{g,t+1} &= A_g x_{g,t} + K_g a_{g,t} \\
y_{g,t} &= C_g x_{g,t} + a_{g,t},
\end{aligned}$$

with $x_{g,t} = -(\kappa_g \theta_g)^{-1} \hat{k}_{g,t} - \epsilon_{g,\tau,t}$, $y_{g,t} = -(\kappa_g \theta_g)^{-1} \hat{k}_{g,t}$, $a_{g,t} = \epsilon_{g,\tau,t-1}$, $A_g = \alpha_g$, $C_g = 1$ and $K_g = (\alpha_g + \theta_g^{-1})$. The condition for invertibility of the system is found by using $a_{g,t} = y_{g,t} - C_g x_{g,t}$ and rewriting the state space system as:

$$\begin{aligned}
x_{g,t+1} &= (A_g - C_g K_g) x_{g,t} + K_g y_{g,t} \\
a_{g,t} &= y_{g,t} - C_g x_{g,t}.
\end{aligned}$$

The system is stable if the eigenvalues of $(A_g - C_g K_g)$ are inside the unit circle. Since $A_g - C_g K_g = \alpha_g - \alpha_g - \theta_g^{-1} = -\theta_g^{-1}$ and $\theta_g^{-1} > 1$, the system is not stable. But, it can be shown that there exists a K_g so that the system is stable. This involves the following assumptions concerning time invariance of the Kalman Filter:

1. The pair (A_g', C_g') is stabilizable. A pair (A_g', C_g') is stabilizable if $y_g' C_g = 0$ and $y_g' A_g = \lambda_g y_g'$ for some complex number λ_g and some complex vector y_g implies that $|\lambda_g| < 1$ or $y_g = 0$.

2. The pair (A_g, G_g) is detectable. The pair (A_g, G_g) is detectable if $G_g' y_g = 0$ and $A_g y_g = \lambda_g y_g$ for some complex number λ_g and some complex vector y_g implies that $|\lambda_g| < 1$ or $y_g = 0$.

Both assumptions are fulfilled for the process and the conditions yield $\alpha_g = \lambda_g$ or $y = 0$. The Riccati equation for the covariance matrix of the innovation $a_{g,t}$ can be solved for a time invariant solution:

$$\sigma_\infty = \frac{1 - \theta_g^2}{\theta_g^2},$$

which yields a corresponding Kalman gain:

$$K_g = \alpha_g + \theta_g$$

The condition for stability now reads $A_g - K_g C_g = -\theta_g$. Since $|\theta_g| < 1$ the process is now invertible.

A.6 Loglinearized Equations of the DSGE model

The loglinearized DSGE model consists of the following equations:

$$\hat{c}_t^o = \hat{c}_{t+1}^o - \hat{R}_t + \hat{\pi}_{t+1} \tag{A-30}$$

$$\hat{q}_t = -\hat{c}_{t+1}^o + \hat{c}_t^o + (1 - \beta(1 - \delta))\hat{R}_{t+1}^k + \beta\hat{q}_{t+1} \tag{A-31}$$

$$\hat{i}_t^o - \hat{k}_{t-1}^o = \eta\hat{q}_t \tag{A-32}$$

$$\hat{k}_t^o = (1 - \delta)\hat{k}_{t-1}^o + \delta\hat{i}_t^o \tag{A-33}$$

$$\hat{c}_t^r = \frac{\bar{y}}{\bar{c}}\frac{(1 - \alpha)}{\mu_p}(\hat{w}_t + \hat{n}_t) - \frac{\bar{t}^r}{\bar{c}}(\phi_b\hat{b}_{t-1} + \phi_g\hat{g}_t) \tag{A-34}$$

$$\hat{c}_t + \varphi\hat{n}_t = \hat{w}_t \tag{A-35}$$

$$\hat{c}_t = \lambda\hat{c}_t^r + (1 - \lambda)\hat{c}_t^o \tag{A-36}$$

$$\hat{k}_t = \hat{k}_t^o \tag{A-37}$$

$$\hat{i}_t = \hat{i}_t^o \tag{A-38}$$

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1 - \alpha)\hat{n}_t \tag{A-39}$$

$$\hat{y}_t = \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{i}}{\bar{y}}\hat{i}_t + \frac{\bar{g}}{\bar{y}}\hat{g}_t \tag{A-40}$$

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] - \lambda_p\hat{mc}_t \tag{A-41}$$

$$\hat{mc}_t = \hat{y}_t - \hat{n}_t - \hat{w}_t \tag{A-42}$$

$$\hat{R}_t^k = \hat{c}_t + (1 + \varphi)\hat{n}_t - \hat{k}_{t-1} \tag{A-43}$$

$$\frac{\bar{b}}{\bar{y}\beta}\hat{b}_t - \frac{\bar{b}}{\bar{y}\beta}\hat{R}_t + \frac{\bar{b}}{\bar{y}}\hat{\pi}_t = (\frac{\bar{b}}{\bar{y}} - \frac{\bar{t}}{\bar{y}}\phi_b)\hat{b}_{t-1} + (\frac{\bar{g}}{\bar{y}} - \frac{\bar{t}}{\bar{y}}\phi_g)\hat{g}_t \qquad \text{(A-44)}$$

$$\hat{R}_t = \phi_\pi \hat{\pi}_t \qquad \text{(A-45)}$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t-3} \qquad \text{(A-46)}$$

A.7   Tables and Figures

Table 1: Prior and posterior distribution of the structural parameters of the DSGE model

Parameter	Prior distribution			Posterior distribution	
	distribution	mean	std	mean	std
ρ_g	beta	0.8	0.1	0.74	0.02
ϕ_g	gamma	0.1	0.1	0.08	0.01
ϕ_b	gamma	0.33	0.1	0.32	0.02
λ	beta	0.4	0.1	0.36	0.02
ϑ	beta	0.7	0.15	0.74	0.03
η	normal	7	5	4.2	0.81
ν	gamma	0.2	0.1	0.14	0.01
σ_g	invgamma	0.02	0.026	0.01	0.001
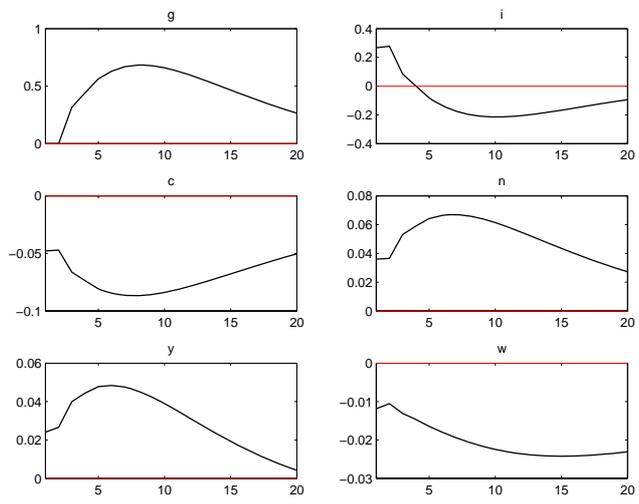
Figure 1: Government expenditure shock two periods preannounced DSGE model calibrated to redo Ramey
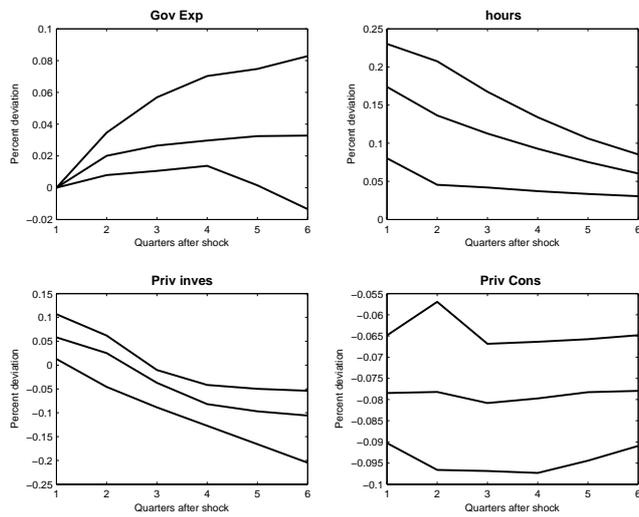
Figure 2: Result Monte Carlo Experiment
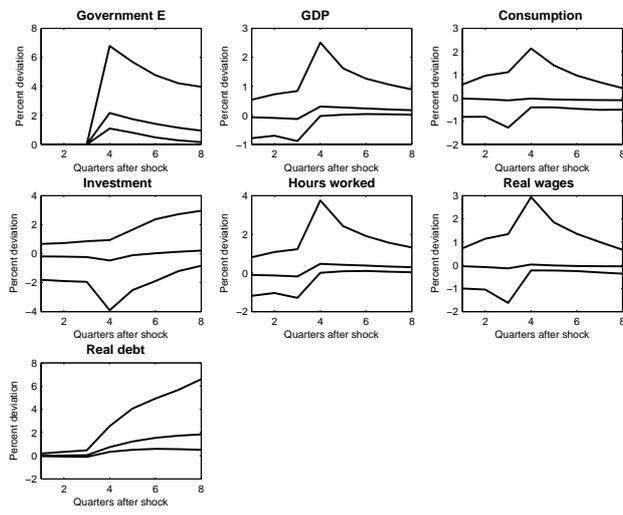
Figure 3: Impulse response function from prior distribution of the DSGE model. All impulse responses included (99 % probability bands)
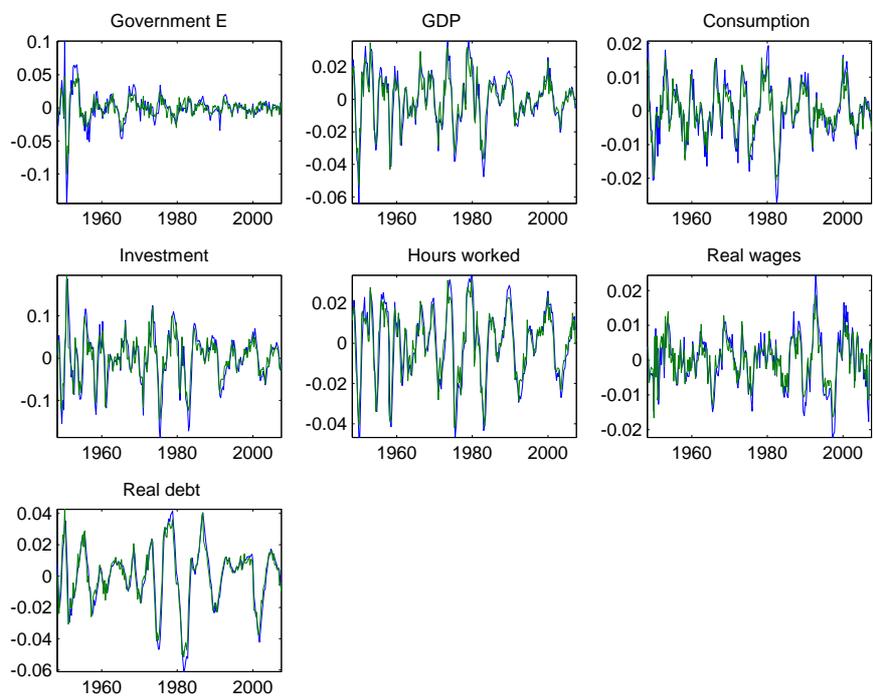
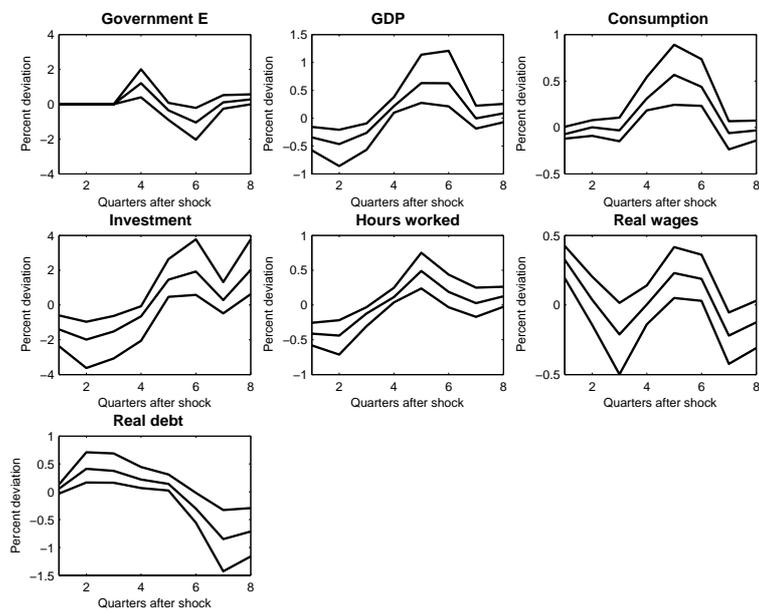Figure 4: Data (blue) vs. likelihood estimates VMA (green)

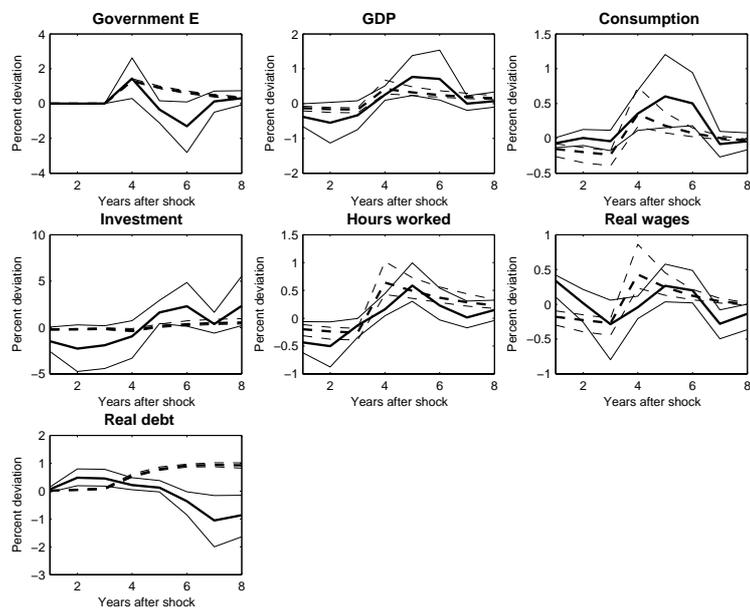Figure 5: Impulse Responses after a pre-announced increase in government expenditures. Pre-announcement three quarters. (68% probability bands)

Figure 6: Impulse Responses of the DSGE model (dashed) and the VMA model after a pre-announced increase in government expenditures. Pre-announcement three quarters. (68% probability bands)
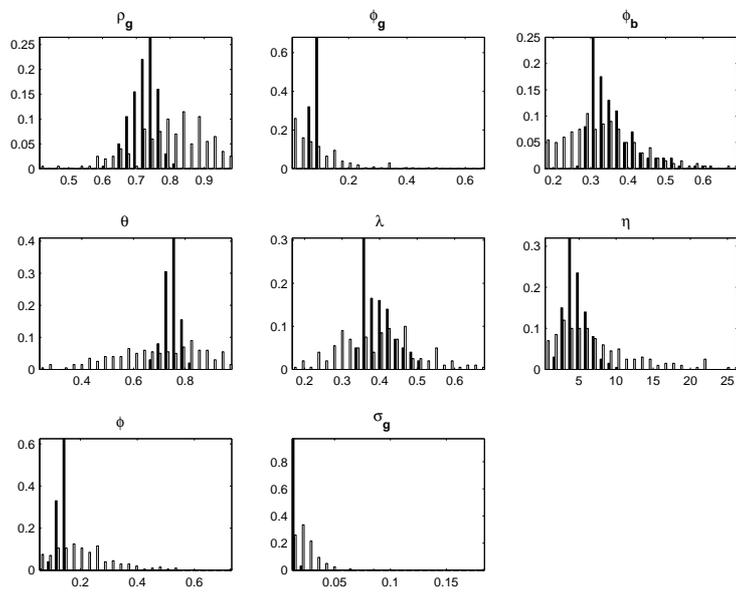
Figure 7: Prior (white) vs. Posterior (black) distribution of the DSGE model with 3 quarters pre-announcement.
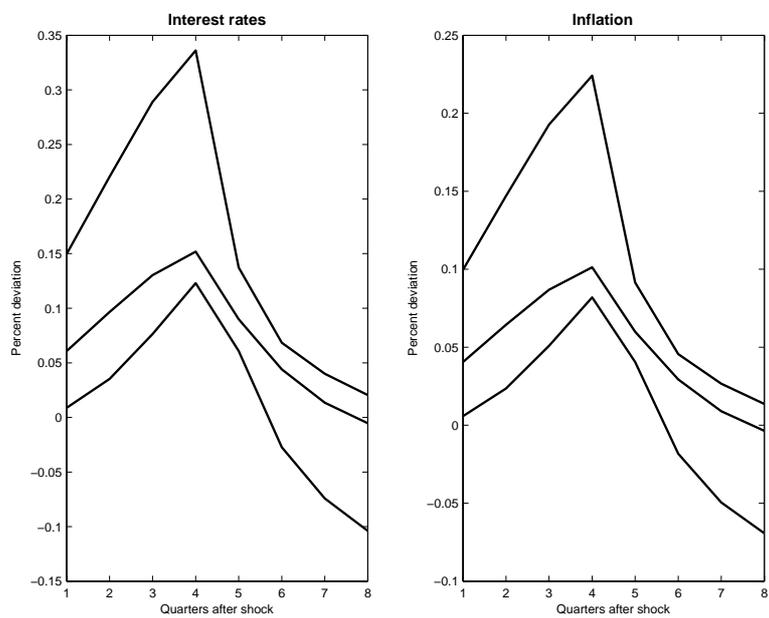
Figure 8: Impulse responses of the nominal interest rate and inflation in the DSGE model to a government expenditure shock with 3 quarters pre-announcement.
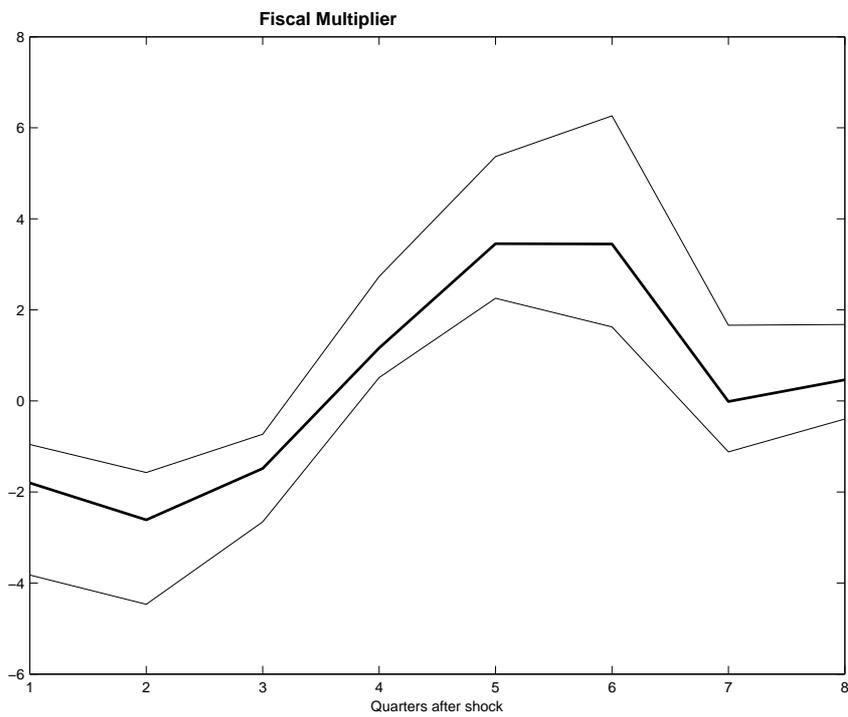
Figure 9: Fiscal multiplier defined as fm=(response of gdp/change in government consumption expenditures)/average share of government consumption expenditures in gdp. 68% probability bands.
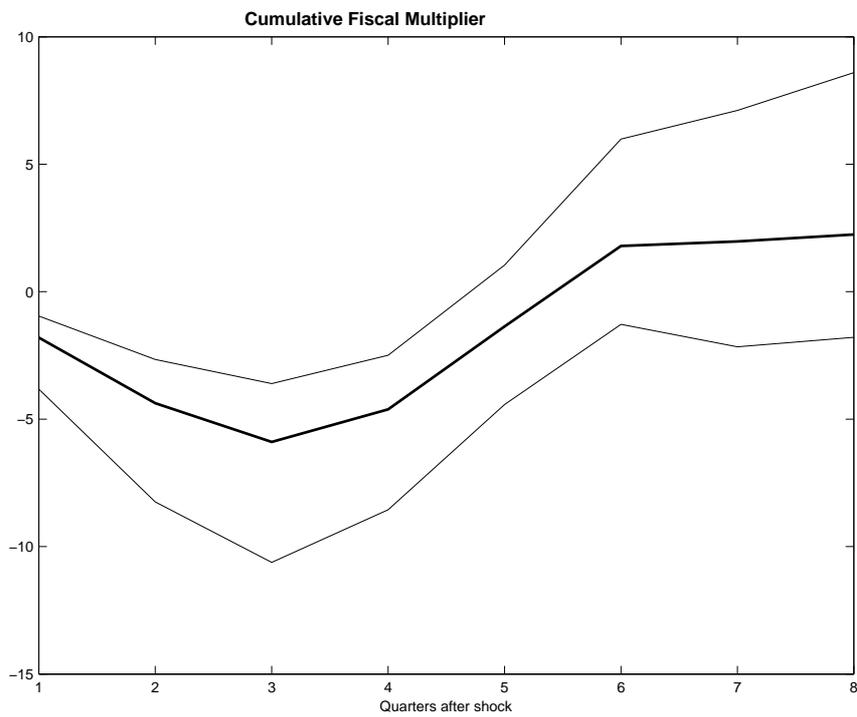
Figure 10: Fiscal multiplier defined as fm=(cumulative response of gdp/change in government consumption expenditures)/average share of government consumption expenditures in gdp. 68% probability bands.
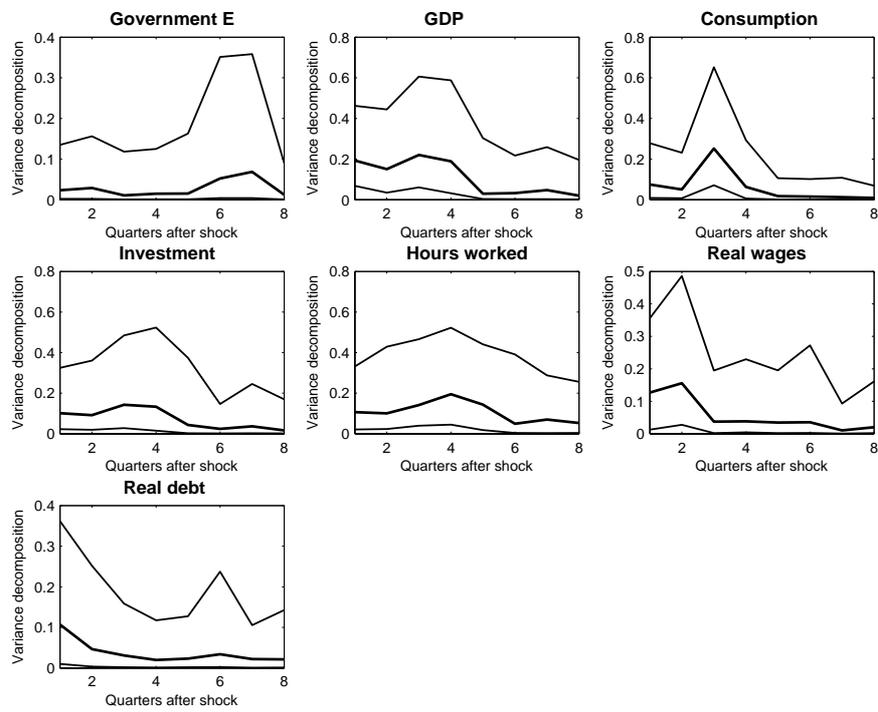
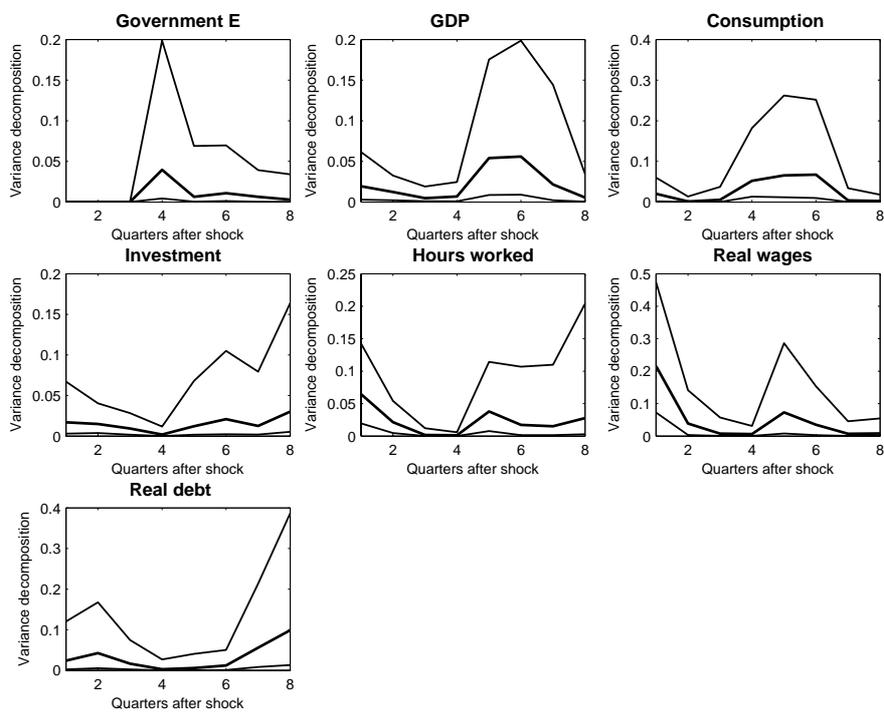Figure 11: Variance decomposition: Business cycle shock. 68% probability bands.

Figure 12: Variance decomposition: Government expenditure shock. 68% probability bands.