European University Institute

Department of Economics

Heterogeneous Individuals in the International Economy

Tobias Broer

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, December 2009

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"Heterogeneity is the essence of a modern economy."

Robert M. Solow

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Introduction

Does financial globalisation increase inequality? Should households with little financial wealth still hold foreign assets? Are consumption patterns of low income earners more dispersed than those of the rich? Do aggregate savings rise or fall when a society becomes more unequal?

These are some of the questions I raise in this thesis. Its central theme is the role of individual heterogeneity in an international economy, with a focus on idiosyncratic risks and income inequality within countries. This allows me to look at some well-recognised policy issues from a new angle, such as the fall in the US current account, which I argue could be influenced by changes in the structure of domestic income inequality. But the combination of the individual with the international level of analysis also enables me to ask new questions, for example whether wealthier households are less biased towards domestic assets in their portfolio decisions than poorer households, and why this could be. The thesis can thus be understood as an attempt to link two literatures that have largely remained separate in the past: that on imperfect domestic risk-sharing, on the one hand, and the international economics literature on the other. This is especially true for chapters 1 and 2. They build on the analysis in chapters 3 and 4, which concentrate on an economic environment where risk-sharing is imperfect because agents cannot commit to honour financial contracts, and analyse the resulting equilibrium and its characteristics in a closed economy.

Chapter 1 looks at global imbalances. It is motivated by the fact that, since the 1980s, the US has not only seen a significant fall in its net foreign asset position, but also a strong rise in domestic inequality and the volatility of incomes. I show how the second fact might help explain the first, against the intuition from simple bufferstock savings models. The key to the result is what I call "endogenous financial deepening": more volatile income makes individuals less inclined to default on financial contracts as this triggers exclusion from future financial trade. The consequences of this improvement in individual incentives after an increase in risk are similar to an increase in the economy's

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aggregate borrowing capacity. But interestingly, higher risk has very little impact on consumption inequality, which is determined mainly by international interest rates. The chapter shows these results both analytically and in a quantitative example. For the analytical part, it uses a small open economy version of the standard limited commitment model. For the quantitative results, it builds on the model by Krueger and Perri (2006) calibrated to the US economy, to show the effect of changes in income risk on net foreign asset positions in partial equilibrium. But I also analyse the general equilibrium of a two country economy, calibrated to the US and China. In both environments, the rise in income risk observed in the US since the early 1980s causes a strong fall in its net foreign asset position.

Chapter 2 is about home bias in portfolios. Rather than looking at aggregate country portfolios, however, it focuses on portfolios of individual households, and how they differ along the wealth distribution. That wealthier households hold a larger fraction of their portfolio in risky assets has been well-documented in the household finance literature. The chapter uses data from the US Survey of Consumer Finances to document that wealthier households also hold on average a higher share of their wealth in foreign assets. This relative home bias of the poor does not seem to be explained by fixed participation costs alone, as the portfolio share of foreign assets increases with financial wealth even among participants in foreign asset markets. The chapter then shows how both biases of poorer agents' portfolios, towards safe and home assets, can arise in a simple two country economy with income and portfolio heterogeneity. Poor investors are naturally biased against domestic equity when wages and capital returns are positively correlated, making equity a bad hedge against fluctuations in labour income relative to bonds. Home bias in consumption, on the other hand, leads to a bias against foreign assets in the bond portfolio.

Chapter 3 takes a more detailed look at the structure of heterogeneity in the limited commitment model, where financial contracts can only be enforced by the threat of exclusion from future financial trade. Its main theoretical contribution is to prove existence and uniqueness of a closed economy stationary equilibrium when incomes follow a standard markov process, and to solve analytically for the joint distribution of consumption, income and wealth. I show how the asymmetric nature of partial insurance under limited commitment, where negative income shocks are pooled but positive shocks can lead to large idiosyncratic jumps in consumption, implies a characteristic form of non-linearity and heteroscedasticity, with declining conditional variances as income increases. In a quantitative part, the paper compares the exact joint distributions in the Krueger and Perri (2006) model to non-parametric estimates of their counterparts in US micro-data, and in a simple Ayagari economy.

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The final chapter of the thesis starts from new evidence on the response of consumption to permanent and transitory income shocks in US micro-data, presented recently by Blundell et al (2008). They discuss their main finding, excess smoothness in the reaction of consumption to permanent income shocks that has declined with rising income volatility, in the context of limited commitment models and their financial deepening effect of rising income risk. The chapter analyses this link formally. In a simple version of the model, I derive the response of consumption to income shocks in closed form, including an expression for the upward bias of Blundell et al's estimator in this environment, where their identifying assumption of no history dependence in consumption is violated. I then compute the response of consumption to income shocks in the calibrated limited commitment economy presented by Krueger and Perri (2006). In their original calibration to the US economy, consumption responses to permanent shocks are an order of magnitude smaller than in the data. But the introduction of a limited amount of heterogeneity in discount factors brings the model roughly in line with the data. In both calibrations, however, the upward bias of Blundell et al's identification scheme leads to estimates about twice as large as the true value of the coefficients.

The chapters are self-contained and can be read as individual papers, in any order. This leads to some repetition in the description of the environments, and comes with some differences in notation across chapters, to which I would like to alert the reader.

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Chapter 1

Domestic or global imbalances? Rising inequality and the fall in the US current account

Abstract[1]

This chapter shows how the rise in individual income risk in the US since the 1980s might help explain the fall in its foreign asset position. The key to this result is endogenous financial deepening in an open economy with participation-constrained domestic financial markets. More volatile income makes individuals less inclined to default on financial contracts as this triggers exclusion from future financial trade. Lower incentives to default, in turn, increase the insurability of income shocks, thus lowering the need for precautionary savings. My theoretical results show that, contrary to the case of unconstrained complete markets, individual participation-constraints guarantee a well-defined stationary equilibrium at a given world interest rate. Based on an analytical solution to the stationary consumption distribution, I show that higher income risk can lower mean consumption and aggregate asset holdings. Consumption inequality, on the other hand, is almost entirely determined by the level of world interest rates, and remains largely unaffected by changes in income risk. A quantitative exercise shows that the observed rise in individual income risk in the US since the 1980s can explain a significant fall in net foreign assets.

JEL Classification Codes: D31, D52, E21, F21, F41

Keywords: Current Account, Global Imbalances, Heterogeneous Agents, Inequality, Incomplete Markets, Participation Constraints, Default

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[1] I would like to thank Dirk Krueger, Assaf Razin, and seminar participants at the Econometric Society's European Winter Meetings 2008, the 2009 EER Young Talented Economists Clinic, Normac 2009, as well as at the Board of Governors of the Federal Reserve, the ECB, Ente Einaudi, IIES, Queen Mary University of London, the Swiss National Bank, and the Universities of Bern, Bonn, Cambridge, Carlos III and Warwick, for helpful comments.

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1.1 Introduction

Over the past 25 years, the US has experienced a significant rise in both cross-sectional income inequality and the uncertainty of individual incomes. Simple economic models suggest this should have increased individual savings at the same time as consumption inequality. But instead, during the same period, US savings fell, current account deficits accumulated to about 40 percent of 2004 GDP, while consumption inequality increased only little. Since 2007, while current account deficits narrowed, the declining value of the relatively risky US foreign investments increased the US net liability position further, thus reinforcing concerns about its sustainability. This chapter shows how, in an open economy, a rise in individual income risk can actually lower the aggregate foreign asset position, while leaving consumption inequality largely unchanged. The crucial assumption is that individuals have access to complete domestic insurance markets, but also the option to default on contracts, at the price of permanent exclusion from financial trade. This restricts transfers under the insurance scheme to amounts that individuals find optimal to pay, rather than choose the outside option of default. Higher income risk increases individuals' incentives to remain insured and thus to honour contracts, which is equivalent to a financial deepening in the economy. Under these "debt-constraints" to complete domestic risk-sharing, I analyse the effect of changes in income risk on consumption volatility and aggregate savings in an open economy. I analytically show that, for a given world interest rate, an increase in income risk can lower the mean of the stationary consumption distribution, thus decreasing the amount of stationary assets, while leaving relative consumption inequality unaffected. Also, I develop a new algorithm based on the associated planner's problem as in Marcet and Marimon (2009), to show quantitatively that the observed rise in individual income risk in the US between 1980 and 2003 can explain a significant fall in net foreign assets.

Figure 1.1 shows the large and, until recently, increasing US current account deficit since 1980. Understanding the reasons for the corresponding rise in foreign indebtedness is important, mainly because different explanations have different implications for its sustainability. For example, it has been argued that the fall in US net assets is a necessarily temporary phenomenon, linked to a strong rise in US house prices, that will eventually have to unwind (see e.g. Roubini et al 2004, Roubini 2005). Other authors, however, have attributed at least a part of this fall to changes in the structure of the world economy that imply a permanently lower US net asset position. Thus, Mendoza et al (2007) have focused on the impact of capital account liberalization in countries whose domestic financial markets are less developed relative to the US. In their model, once capital markets are liberalized, higher precautionary savings and lower appetite for risk in the rest of the world result in capital flows to the US concentrated in bonds,

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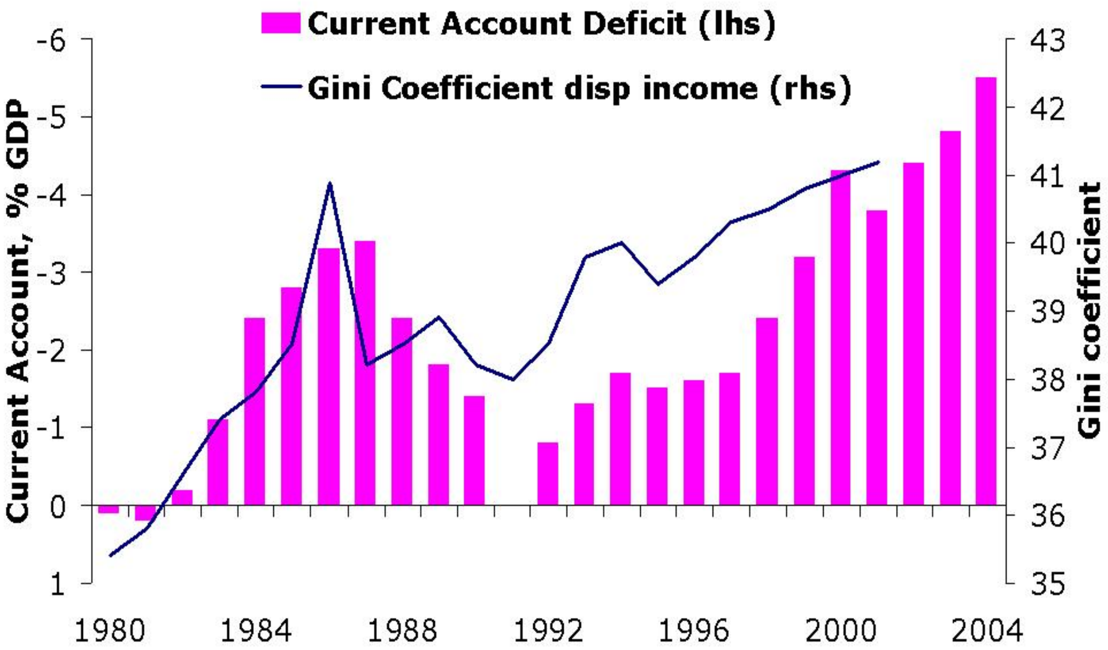

FIGURE 1.1: US current account and Gini coefficients. Source: IMF and Brandolini et al (2007)

in line with the evidence. However, the underlying comparative advantage of deeper domestic financial markets in the US is exogenous to the model. In another contribution, Fogli and Perri (2006) show how the relatively more important reduction in US macro-volatility since 1980 implies a stronger reduction in the bufferstock savings of a representative US consumer than in other countries. But crucially, while international asset trade is limited to non-contingent bonds in their model, they assume domestic trade of a set of complete state-contingent assets that warrants the focus on representative national agents. This assumption, however, has been largely rejected by the data (see for example Zeldes (1989)). Moreover, as figure 1.1 shows, while US debt increased, cross-sectional domestic income inequality rose strongly, partly attributable to a rise in the uncertainty of individual incomes (see Krueger and Perri (2006), and more recently Heathcote et al (2008b)). And in the absence of perfect domestic risk-sharing, these changes in income risk will affect aggregate debt dynamics.[2]

This chapter analyses net asset positions in a simple open economy model that relaxes the assumption of a representative agent, and does not assume exogenous comparative financial advantage. Instead, it makes the depth of domestic financial markets depend endogenously on the riskyness of individual income. This allows me to look at the impact of changes in idiosyncratic income and consumption risk on aggregate savings and

[2]Cabellero et al (2006) also have a model of global imbalances, based on a lower capacity to generate financial assets from real investments in the rest of the world, relative to the US.

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asset positions. But importantly, it also allows me to analyse the effect of international variables, such as interest rates, on individuals' decisions and, ultimately, the domestic consumption distribution.

If non-contingent debt was the main savings vehicle of the economy, as in Fogli et al (2006), an increase in individual income risk would yield a rise, not a fall, in equilibrium savings, together with higher consumption volatility. On the other hand, in an economy where domestic markets are complete, but individuals can default on contracts at the price of permanent exclusion from financial trade, the relationship between income risk and consumption volatility is known to be less simple. Krueger and Perri (2006) show that under this assumption of participation-constrained complete markets, a rise in income risk has two offsetting effects: first, it raises the income realizations of individuals who receive positive shocks, and thus, for a given upper limit to redistribution, increases the volatility of consumption. But higher income risk also makes the outside option of financial autarky, where it translates one-to-one into higher consumption volatility, less appealing. This second effect acts to increase the insurability of income shocks, and thus deepens financial markets and reduces consumption volatility. Krueger and Perri (2006) show that the latter, financial deepening effect becomes more important for high levels of income risk, causing consumption volatility to first rise and then fall as income risk increases. Aggregate savings mainly act as a precaution against this consumption volatility.

This chapter shows analytically that the open economy setting breaks the closed economy link between consumption risk and precautionary savings. Particularly, relaxing individual debt constraints leaves relative consumption inequality largely unchanged. Rather, it can be interpreted as an increase in the country-wide borrowing capacity that leads to an increase in stationary debt holdings, or a fall in the net asset position. To derive these results, I first show that, unlike with unconstrained complete markets, a debt-constrained economy that faces a given world interest rate has a unique stationary equilibrium that does not depend on initial conditions. So individual participation-constraints "close small open economies" (Schmidt-Grohé et al 2003). As shown in Broer(2009b), the optimality conditions of an associated planner's problem, as in Marcet and Marimon (2009), allow me to solve analytically for the stationary consumption distribution even with standard, independent Markov processes for the incomes of a large number of individuals. The stationary equilibrium has the interesting feature that consumption follows a geometric distribution whose shape depends largely on the world interest rate, while its position is determined by participation constraints. Thus, looser participation

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constraints increase aggregate debt holdings and decrease aggregate consumption in stationary equilibrium. However, as mentioned above, the effect of higher income risk on participation-constraints depends on the initial level of income risk, and therefore the particular economy under analysis. A second part of the chapter thus looks at the US example, and evaluates the effect of the observed rise in US income volatility on its net foreign asset position and the consumption distribution quantitatively. The analysis is comparative static in nature, as I abstract from transitions and focus on steady states associated with the level of individual income risk in the early 1980s, on the one hand, and the higher volatility of incomes observed more recently, on the other. The exercise should ideally account for changes in income heterogeneity in both the US and its main economic partners during this period. Unfortunately, comprehensive cross-country data on the evolution of income risk are as yet unavailable, and in some cases unfeasible.[3] Comparative studies of simpler inequality measures have found that, apart from the United Kingdom, other OECD countries have experienced less important increases in income inequality since 1980 than the US (see e.g. Brandolini et al 2007). To focus on the open economy effect of the relatively large changes in income heterogeneity in the US, I first analyse their effect at an exogenously given world interest rate.[4] In a second exercise I analyse a two country general equilibrium model where the US trades bonds with a large developing country with less sophisticated domestic financial markets. To capture the change in income risk, I use the stochastic process of individual incomes in the US estimated by Krueger and Perri (2006) for the years 1980 and 2003. For the second country I choose a process in line with the observed change in inequality in China. To solve the model, I develop a new algorithm based on Marcet and Marimon (2009) to compute the stationary consumption distributions and net asset positions. The results show that the increase in income risk in the US can indeed explain a significant part of the fall in the net foreign asset position, both at a given interest rate as well as in a general equilibrium exercise.

The rest of the chapter is structured as follows: Section 1.2 describes the environment of an open economy with debt-constrained domestic financial markets. Section 1.3 derives the analytical results on the basis of the associated planner's problem. Section 1.4 reports the computational algorithm and quantitative results. An appendix contains most proofs.

[3]Thus, in the UK, for example, household panel data have been collected only since the beginning of the 1990s. However, Heathcote et al (2008b) is one paper in a recent project to compare measures of individual inquality and income risk across countries. See http://www.econ.umn.edu/ fperri/Cross.html.

[4]The assumption of an exogenous interest rate has also been made in contributions concentrating entirely on the domestic consequences of increases in individual income volatility in the US. See for example Heathcote et al (2008a).

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1.2 An open economy with debt-constrained domestic financial markets

This section presents a simple model of an open economy where domestic financial markets are constrained by individual default, and defines the competitive equilibrium.

1.2.1 Agents, countries, time

The economy consists of an individual country and a rest of the world. The theoretical analysis focuses on the individual country and assumes that it takes prices of goods and assets traded with the rest of the world as given. A later quantitative section, however, also looks at the example of a 2 region world economy.
The small country is populated by a large number of individuals of unit mass. Individuals are indexed by i, located on a unit-interval $i \in \mathbb{I} = [0, 1]$. Time is discrete $t \in \{0, 1, 2, ..., \infty\}$ and a unique perishable endowment good is used for consumption.

1.2.2 The endowment process

The consumption endowment of agent i in period t, $z_{i,t}$, takes values in a finite set Z: $z_{i,t} \in Z = \{z_1 > z_2 > ... > z_N\}, N \geq 2$. Endowments follow a stochastic process described by a Markov transition matrix F. F has strictly positive entries, is identical across agents, monotone (in the sense that the conditional expectation of an increasing function of tomorrow's income is itself an increasing function of today's income), and has a unique ergodic distribution $\Phi_Z : \mathbb{Z} \to [0, 1]$, where $\mathbb{Z}$ is the power set of Z. Thus, in the long-run, aggregate income $Y = \int_{\mathbb{I}} z_i$ is constant, while individual income fluctuates. Let s_t denote the state of the economy in period t, a vector containing individual incomes and asset holdings of all agents.

1.2.3 Preferences

Agents live forever and order consumption sequences according to the utility function

$$U = E_{s_0} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \tag{1.1}$$

where E_{s_0} is the mathematical expectation conditional on s_0, $0 < \beta < 1$ discounts future utility, $c_{i,t}$ is consumption by agent i in period t, and $u : R^+ \to R$ is an increasing,

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strictly concave, continuously differentiable function that satifies Inada conditions and is identical for all agents in the economy.

1.2.4 Asset markets

I choose a specification of the economy similar to that by Alvarez and Jermann (2000), amended for the international setting. Agents engage in sequential trade of a complete set of state-contingent bonds domestically, but international asset trade is limited to non-contingent bonds.[5]

Individual endowment realisations are verifiable and contractable, but asset contracts are not completelely enforceable: at any point, individuals can default on their contractual payments at the price of eternal exclusion from financial markets. Thus the total amount an agent can borrow today against any income state z_j tomorrow is bounded by the option to default into financial autarky. There, consumption is forever equal to income. Given the markov structure of income, the value of default as a function of the vector of current income z can be written as

$$W(z) = \sum_{t=0}^{\infty} (\beta F)^t U(z) = (I - \beta F)^{-1} U(z) \tag{1.2}$$

I denote holdings of bonds and Arrow-Debreu securities paying off in state s_t by b and $a(s_t)$ respectively. In any state s_t, $V(z(s_t), a(s_t), b_t)$ is the contract value as a function of income $z(s_t)$ and current asset holdings $\{a(s_t), b_t\}$.

As in Alvarez and Jermann (2000) individual i's participation constraint for any state s_{t+1} tomorrow can be written as a constraint on the claims she can issue against s_{t+1} income. This borrowing constraint is "not too tight" in the words of Alvarez and Jermann (2000) if it assures participation but does not constrain contracts otherwise

$$a_i(s_{t+1}) + Rb_{i,t+1} \geq A_i(s_{t+1}) = min\{\alpha(s_{t+1}) : V(z_i(s_{t+1}), \alpha(s_{t+1}, 0)) \geq W(z_i(s_{t+1}))\} \tag{1.3}$$

Note that bonds are redundant in this setting, although including them facilitates the setup of the planner's problem in an open economy where aggregate bond holdings, denoted B, are potentially non-zero.

Importantly, the portfolio constraint (1.3) limits the issuance of assets that demand net repayments in high income periods, when the outside option of default is most attractive. On the one hand, this reduces transfers from high to low income individuals under insurance contracts. But on the other, it defines a maximum level of debt that

[5]This is non-restrictive as there is no aggregate risk and the law of large numbers holds. It requires, however, no default on foreign debt on a country level. In a previous version of this chapter I show that Broner and Ventura's (2006) result applies to my setting. Thus, perfect secondary markets prevent governments from defaulting on agents' foreign liabilities.

individuals, and thus the country on aggregate, can sustain. The attractiveness of default during periods of high individual income, determined by the value of the outside option of financial autarky W, is thus the main determinant of the aggregate net asset position in stationary equilibrium. The next section briefly considers how W is affected by changes in income risk.

1.2.5 Income risk and the value of default

Under the assumption that default leads to exclusion from all financial transactions, the value of default equals the expected utility of individual income streams given by (1.2). The assumption of monotonicity of both utility and transitions ensures that these autarky values are increasing in the level of current income. However, the relationship between autarky values and income risk is more difficult to characterise. Particularly, a change in risk can come via changes in transition probabilities F, via a change in the support of endowments Z, or both. In this chapter, I follow Kehoe and Levine (2001) and define a rise in risk as a mean-preserving spread to the income support Z. This, however, does not imply mean-preserving spreads to the conditional income distribution for all individuals. Rather, given persistence, it raises (lowers) current and expected future income for today's high (low) income earners. So for low levels of uncertainty, higher risk increases both expected income and autarky values for the income-rich. However, although their expected income continues to rise, as a consequence of concave utility the prospect of negative shocks weighs more heavily on expected utility as higher risk decreases income, and thus consumption, in low income states. Given Inada conditions, this effect necessarily outweighs the gain in expected income at some point. Thus, autarky values of high income individuals roughly follow an inverse U-shape relation with income risk. So we would expect portfolio constraints to first become tighter, and then loosen, as income risk rises. The analytical part of this chapter shows that this is indeed the case. The quantitive section shows the location of a model calibrated to the US economy on this "Laffer curve" of default incentives.

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1.2.6 The household's problem

In every period, households maximise their expected utility by choosing current consumption and assets subject to budget and borrowing constraints

$$V(z(s_t), a(s_t), b_t) = max_{c_t, \{a(s_{t+1})\}, b_{t+1}} \sum_{s=0}^{\infty} \beta^s u(c_{t+s})$$

$$s.t.\ c_t + \sum_{s_{t+1}} a(s_{t+1}) q(s_{t+1}) + b_{t+1} \leq Rb_t + a(s_t) + z(s_t) \tag{1.4}$$

$$a(s_{t+1}) + Rb_{t+1} \geq A(s_{t+1}) \tag{1.5}$$

As shown in Alvarez and Jermann (2000) this problem has a recursive representation as

$$V(z(s), a(s), b) = max_{c, \{a(s')\}, b'} \{u(c) + \beta E_s V(z', a(s'), b')\}$$

$$s.t.\ c + \sum_{s'} a(s') q(s') + b' \leq Rb + a(s) + z(s)$$

$$a(s') + Rb' \geq A(s')$$

$$A(s') = min\{\alpha(s') : V(z(s'), \alpha(s'), 0) \geq W(z(s'))\}$$

where c, b', a' are policy functions of the state variables $(z(s), a(s), b)$.

1.2.7 Definition of competitive equilibrium

The competitive equilibrium in this economy is a set of asset prices $q(s'), R$, a set of individual decision rules $c, b', a'(s')$ with associated value functions $V(z, a, b)$
such that

1. $V(z, a, b)$ is the households maximum value function associated to the household problem given $q(s'), R$

2. $V(z, a, b)$ is attained by $c, b', a'(s')$

3. Markets for state-contingent assets clear
   $\int_{\mathbb{I}} a_i(s_t) = 0,\ \forall s_t, t$

4. The interest rate on bonds is equal to the world interest rate R.

The competitive equilibrium is called "stationary" if prices and aggregate bond holdings are constant, and the distribution of individual consumption is stationary through time.

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1.3 Analytical properties of the consumption distribution and aggregate savings in stationary equilibrium

In this section I show analytically how, unlike with unconstrained complete markets, individual participation constraints ensure the existence of a stationary equilibrium in an economy that faces a given world interest rate smaller than its agents' rate of time preference. I show how across stationary equilibria, a rise in income risk can leave consumption inequality unchanged, but decreases aggregate asset holdings if the initial level of income risk is high enough. Also, I show that market completeness does not help the most unfortunate individuals in this economy: both their current consumption and expected value from future consumption are the same as without any financial markets. Insurance, however, reduces the number of individuals in this situation significantly. To derive these results I exploit the constrained efficient nature of the economy that allows me to solve the associated planner's problem as in Marcet and Marimon (2009). In chapter 3 I use this method to show existence of equilibrium, and to solve for the joint distribution of consumption, income and wealth, in a closed economy version of the model. Here I build on these results to show how, in an open economy, changes in income risk affect mainly the position of the consumption distribution, while its shape is a function of world interest rates. An increase in income risk can decrease net foreign assets by making default less attractive and thus relaxing, effectively, the economy-wide borrowing limit. To illustrate this, I then focus on a version of the economy with two income values. This is a case previously analysed in Krueger and Perri (2005), Krueger and Uhlig (2006), and Thomas and Worrall (2007), under the additional assumption of i.i.d. transitions for income. Here I use the results in chapter 3, where I derive a closed form solution to the distribution in the case with persistence and CRRA preferences, to show analytically the effect of changes in individual risks on aggregate assets in an open economy.

1.3.1 The planner's problem and first order conditions

Alvarez and Jermann (2000) show that a version of the first welfare theorem applies to the closed economy version of this environment. The small open economy assumption changes aggregate feasibility constraints but, together with an appropriate No-Ponzi condition, leaves this result intact. This allows me to focus on participation-constrained efficient allocations. More particularly, I exploit the results in Marcet and Marimon (2009), and focus on the solution to the participation-constrained social planner's problem.

Marcet and Marimon (2009) show how the efficient competitive equilibrium allocation

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solves the following planner's problem. For a given bounded measurable weighting function $\mu_{i,0} : \mathbb{I} \rightarrow R^+$ in a linear social welfare function $\Omega = \int_{\mathbb{I}} \mu_{i,0} E_0 \sum_0^\infty \beta^t u(c_{i,t})$ the problem of the planner is to distribute resources optimally subject to individuals' participation constraints and the aggregate resources of the economy

$$\begin{aligned} \mathbb{VV}(\Phi_{\mu_{i,0}}, B_0) = max_{\{c_{i,t}\}} \int_{\mathbb{I}} \mu_{i,0} \sum_{t=0}^{\infty} \beta^t u(c_{i,t}) \\ s.t. \int_{\mathbb{I}} c_{i,t} + B_{t+1} = \int_{\mathbb{I}} z_{i,t} + R_t B_t, \ \forall t \\ V_{i,t} \geq W(z_{i,t}), \ \forall t, i \\ B_t \geq -\frac{Y}{R-1}, \ \forall t \end{aligned} \tag{1.6}$$

where the planner's maximum value $\mathbb{VV}$ is a function of $\Phi_{\mu_{i,0}}$, the initial distribution of multipliers induced by $\mu_{i,0}$, and aggregate bond holdings B_0. $V_{i,t}$ denotes the expected value of the consumption sequence the planner gives to agent i starting in period t, and the last line is a No-Ponzi condition on aggregate bonds B, which I assume to be 0 in period 0. Also, I assume that $\mu_{i,0}$ only takes a finite number of values.

Note that the problem in (1.6) is not recursive in the cross-sectional distribution of income. Intuitively, the planner optimally provides an increase in value $V_{i,t}$ to participation-constrained individual i by an increase in both current and future consumption. But this requires the planner to keep her consumption promise even if individual i receives a negative income shock tomorrow. The solution thus has potentially infinite history dependence. But Marcet and Marimon (2009) show how, based on the Lagrangian associated to the sequential planner's problem, this history-dependence can be encoded in a time varying value of individual welfare weights $\mu_{i,t}$. In particular, the assumptions on $\Phi_{\mu_{i,0}}$, utility and transition probabilities ensure that the problem is sufficiently well-behaved to have a saddle-point representation that is recursive in a time-varying distribution of weights $\Phi_{\mu_{i,t}}$ and aggregate bond holdings[6]

$$\mathbb{VV}(\Phi_{\mu_i}, B) = \inf_{\gamma_i \geq 0} \max_{\{c_i\}} \int_{\mathbb{I}} [(\mu_i + \gamma_i)u(c_i) - \gamma_i W_i] + \beta E[\mathbb{VV}((\Phi_{\mu_i'}, B')] \tag{1.7}$$

$$s.t. \int_{\mathbb{I}} c_i + B' = \int_{\mathbb{I}} z_i + RB$$

$$\mu_i' = \mu_i + \gamma_i \tag{1.8}$$

$$B_t \geq -\frac{Y}{R-1} \; \forall t$$

where γ_i corresponds to the multiplier on i's participation constraint in the sequential problem (1.6). Note that the weights of individuals in the social welfare function are now updated every period to meet participation constraints.[7] And when γ_i is zero, so i is unconstrained, (1.8) ensures promise-keeping by the planner. Intuitively, by increasing multipliers the planner allocates a higher than expected consumption path to constrained individuals with positive income shocks, to keep them "happy" with the contract. The absolute weights of the remaining, unconstrained individuals are constant, but decline relative to those for individuals with positive income shocks. This leads to a gradual decline in consumption for these individuals until they either receive a positive income shock, or reach the level of constant consumption that, given prospects for future shocks, just meets the participation constraint corresponding to their income level. The solution of the planner's problem is a sharing rule $\Gamma : Z \times R^+ \to R^{+2}$ that maps current weights μ_i and income shocks z_i into consumption c_i and new weights $\mu_i' = \mu_i + \gamma_i$.
The first order conditions[8] for individual consumption imply

$$\frac{U'(c_{i,t})}{U'(c_{j,t})} = \frac{\mu_{j,t} + \gamma_{j,t}}{\mu_{i,t} + \gamma_{i,t}} \tag{1.9}$$

Thus, since $U'(c)$ is decreasing, individuals with a higher weight receive higher consumption. Also, from the first order condition for aggregate bond holdings, the interest rate is tied to the ratio of the multipliers λ, associated to the aggregate feasibility constraint

[6]To see this, note that the initial weighting function $\mu_{i,0}$ only takes a finite number of values, and that for every $t < \infty$ the set of possible income histories Z^t is finite and bounded. So the exogenous state space is the Euclidian Product of a countable number of compact sets, and thus, according to Tychonoff's theorem, compact. Also, given the No-Ponzi condition, aggregate bond holdings are bounded and thus lie in a convex compact set, implying that feasible consumption allocations are just a simplex, and thus a convex set, every period. With concave utility, the constraint set is therefore compact and convex, and non-empty since autarky is feasible and incentive-compatible. The problem thus fulfills conditions A1 to A5 in Marcet and Marimon (2009), and therefore has a recursive saddle-point representation. For further detail, see the proof of uniqueness and existence in the Appendix.

[7]Again, despite the continuum of agents, the values of multipliers remain countable, since $\mu_i' = \mu_i + \gamma_i$ is a function of current income and the past value of μ_i only. So, given my assumption of a finite support of $\Phi_{\mu_{i,0}}$, the number of individual multipliers remains countable.

[8]Note that continuously differentiable utility and a convex constraint set imply that the value function is differentiable. Also, Inada conditions and the concavity of the utility function imply that the first order conditions, together with participation constraints, are sufficient to characterise the optimum.

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in (3.8)

$$R = \frac{\lambda}{\beta E[\lambda']} = \frac{\beta\lambda}{\lambda'} = \frac{U'(c_i)(\mu_i)}{\beta U'(c_i')(\mu_i + \gamma_i)} \tag{1.10}$$

where the second equality exploits the absence of aggregate uncertainty and the law of large numbers,[9] and the third uses the intratemporal optimality conditions for consumption. Importantly, the interest rate determines the slope of marginal utility for those consumers who are unconstrained tomorrow ($\gamma_j = 0$)

$$U'(c_i) = \beta R U'(c_i') \tag{1.11}$$

Given monotonicity of U', this provides a law of motion for the consumption of unconstrained agents. With CRRA preferences $u = \frac{c^{1-\sigma}}{1-\sigma}$, we can solve for c_i' as

$$c_i' = (\beta R)^{\frac{1}{\sigma}} c_i \tag{1.12}$$

So the lower R, the faster falls consumption of unconstrained agents. With CRRA preferences we can simplify equation (1.10) further by solving for c_i in terms of the multipliers, and integrating across agents, to get

$$R = \frac{1}{\beta}[\frac{C'}{C}\frac{\int_{\mathbb{I}}(\mu_i^{1/\sigma})}{\int_{\mathbb{I}}(\mu_i + \gamma')^{1/\sigma}}]^{\sigma} \tag{1.13}$$

Thus, a fall in the world interest rate either lowers aggregate consumption growth, or increases average growth in individual multipliers, or both. The first effect is standard and leads to non-existence of a stationary equilibrium in small open economies with unconstrained complete markets. The second effect comes from the participation-constrained nature of risk-sharing. It implies, for example, that unless there is perfect insurance ($\gamma_i = 0, \forall i$), the equilibrium closed economy interest rate is below the time preference rate, a result well-known from Alvarez and Jermann (2000). More generally, binding participation constraints increase the shadow value of future resources relative to today's. This is because current consumption only relaxes today's participation constraints. Future consumption relaxes all previous participation constraints, including today's, via the increase in continuation utility under the contract. So when more agents hit their participation-constraints every period, or when a given set of binding

[9] Since the state space is finite every period, the assumption of independent shocks over a continuum of agents ensures that the law of large numbers applies. Formally, $\int_{\mathbb{I}} x(i,t) = \sum_{Z\times\{\mu_{i,t}\}} \int_{\mathbb{I}} I_{\mu,z} = \sum_{Z\times\{\mu_{i,t}\}} \mathbb{III}_{\mu_t,z}$ where $I_{\mu,z}$ is the indicator function of the set $\{i : \mu_i = \mu, z_i = z\}$ and $\mathbb{III}_{\mu_t,z} \in [0,1]$ is the mass of individuals with weight μ and income z in period t. So we can replace integrals with summation over countable sets. Given the continuum of agents $i \in \mathbb{I}$, this ensures that the law of large numbers applies. So the joint distribution of income and weights μ tomorrow is known today. On the law of large numbers in economies with a continuum of agents and independent idiosyncratic risk, see Uhlig (1996).

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constraints becomes more binding, the planner reallocates aggregate consumption to the future. Below I show that this second effect ensures the existence of a stationary equilibrium in this economy.

Note that if $\frac{U'(z_1)}{\beta U'(z_N)} > 1$, (1.10) immediately yields a minimum interest rate $R_{min} > 1$ below which all individuals simply consume their endowments. This is because, whenever $1 < R < R_{min} = \frac{U'(z_1)}{\beta U'(z_N)}$, there are no participation-compatible unconstrained transitions in (1.11). So individual consumption is simply equal to individual income.

1.3.2 Existence, uniqueness and stationarity of equilibrium

The closed economy version of this economy is one of the examples discussed in Marcet and Marimon (2009). An appendix proves that the planner's problem has a unique solution also at a given interest rate $\frac{1}{\beta} > R > 1$. However, in both cases, we do not know if this solution is stationary in terms of the long-run behaviour of aggregate consumption and its distribution across individuals.

For example, in a standard small open economy with complete domestic markets that are not participation-constrained, $R < 1/\beta$ implies that consumption levels are forever declining. So no stationary solution exists. With participation constraints, however, this is not an equilibrium, as the total value that the planner can distribute to individuals declines with the level of aggregate resources. A permanently downward sloping path of aggregate consumption thus necessarily violates individual participation constraints at some point in the future. Instead, in an equilibrium with participation constraints, the aggregate consumption decline slows down as participation constraints become more binding. This is because for given weights $\mu_i + \gamma_i$, individual contract values decline with aggregate resources. This requires stronger increases in relative weights of participation-constrained indivdiuals γ_i. But more binding participation constraints increase the marginal value of future resources according to equation (1.10). This slows the decline in aggregate consumption until it settles down at a stationary level, with a corresponding stationary distribution of individual consumption and aggregate debt holdings. Equation (1.13) shows how the individual consumption volatility, expressed there as growth in average individual planner weights, effectively replaces the non-stationarity of aggregate consumption. In this way, individual participation constraints provide an additional way of "closing small open economies" (Schmidt-Grohé et al 2003).

In the resulting unique stationary equilibrium, consumption in all states is pinned down by participation constraints and the law of motion of unconstrained agents (1.11) given the exogenous interest rate R. The following section uses a closed form example to illustrate the characteristics of the stationary distribution of consumption, and to show how aggregate foreign assets in this stationary equilibrium are effectively determined

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by individual income risks. To do this, I first show how, for a given interest rate R, the position of the consumption distribution moves up and down with autarky values. Then I show how the latter follow an inverse U-shaped relationship with income risk, and what this implies for foreign asset holdings.

1.3.3 A closed form example

Consider an economy in which the income process described in the previous section takes only two values $\{z_h, z_l\} = \{y_0 + \frac{1}{\nu}\epsilon, y_0 - \frac{1}{1-\nu}\epsilon\}$, $\epsilon \geq 0$, where $\nu = \frac{1-q}{2-q-p}$ is the stationary mass of high-income individuals, for transitions given by $F = [p, 1-p; 1-q, q]$. Monotonicity and absolute continuity require $0 < 1-q < p < 1$. Also, I assume income has persistence which is not too different in high and low income states:

$$p, q > 1/2 \tag{1.14}$$

$$\frac{\beta - 1}{\beta} < p - q < \frac{1-\beta}{\beta} \tag{1.15}$$

I define a "**marginal rise in income risk**" as a small widening of the income support $d\epsilon > 0$. The specification of Z ensures that this is a mean-preserving spread for all values of p, q, and thus leaves aggregate resources unchanged.

This example is a generalisation of that considered, in an economy with capital, by Kehoe and Levine (2001), or more recently by Krueger and Perri (2006), who, however, assume independent transitions.

1.3.3.1 The stationary consumption distribution

Remark 1.1. There exists a unique stationary equilibrium with a distribution of consumption $\Phi_C : \mathbb{C} \subseteq R^+ \longrightarrow [0,1]$. If $1 < R < R_{min}$, the stationary distribution of consumption is equal to that of income, so $\Phi_C = \Phi_Z, \mathbb{C} = Z$. If $R_{min} < R < 1/\beta$, Φ_C is

$$\Phi(c_1) = \frac{1-q}{2-q-p} = \nu \tag{1.16}$$

$$\Phi(c_i|_{1<i<m}) = \nu(1-p)q^{i-1} \tag{1.17}$$

$$\Phi(c_m) = (1-\nu)q^{m-1} \tag{1.18}$$

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for

$$
\begin{aligned}
c_1 &= \{\frac{(1-\sigma)(1-\beta q(\beta R)^{\frac{1-\sigma}{\sigma}})}{1+\beta(1-p-q)(\beta R)^{\frac{1-\sigma}{\sigma}}-(1-p)\beta^m q^{m-1}(\beta R)^{\frac{m(1-\sigma)}{\sigma}}}\\
&[\frac{1-\beta(p+q)-\beta^2(1-p-q)}{1-\beta q}W_h-(1-p)\beta^m q^{m-2}(qW_l-\frac{(1-q)W_h}{1-\beta q})]\}^{\frac{1}{1-\sigma}}\\
c_i &= c_1(\beta R)^{\frac{i}{\sigma}}, 1<i<m\\
c_m &= y_0-\frac{1}{1-\nu}\epsilon \qquad (1.19)\\
m &= min\{x\in N : x > \frac{\sigma[ln(y_0-\frac{1}{1-\nu}\epsilon)-ln(c_1)]}{ln(\beta R)}\} \qquad (1.20)
\end{aligned}
$$

Proof

This closed form of the consumption distribution is proved in detail in chapter 3. To see that it is bounded below by z_l, note that an individual at minimum consumption c_m is necessarily constrained today and tomorrow (from stationarity and minimality of c_m). So c_m is determined from her participation constraint

$$W_l = U(c_m) + \beta[(1-q)W_h + qW_l] \qquad (1.21)$$

which is solved by $c_m = z_l = y_0 - \frac{1}{1-\nu}\epsilon$ by the definition of W_l.

An individual in the high income state is always constrained. To derive her consumption c_1, express the expected value of her consumption stream under the contract as an infinite sum of lotteries with two outcomes: either, she receives value W_h. Or, in case of a low income realisation, she receives $(\beta R)^{\frac{i-1}{\sigma}} c_1,\ i=1$, plus participation in the next lottery for $i=2$, and so forth until hitting $c_m = z_l$, where she remains until a high income shock. The discounted sum of the values of these lotteries must be equal to $W_h - u(c_1)$ which defines c_1, and thus, by (1.12) the rest of the support.[10]

The stationary mass at c_1 is equal to that of high income individuals ν. The remaining mass function $\Phi(c_{1+i})$ is simply ν times the probability to move to low income and stay there for $i<m$ periods, which yields a geometric distribution with parameter q. The lower bound c_m has the remaining mass $\Phi(c_m) = \Phi(c_{m-1})\frac{q}{1-q} = \nu\frac{(1-p)q^{m-1}}{1-q} = (1-\nu)q^{m-1}$.■

[10] The corresponding equation is

$$
\begin{aligned}
&W_h = \frac{c_1^{1-\sigma}}{1-\sigma} + p\beta W_h +\\
&(1-p)\{\sum_{i=1}^{\infty}\beta^i q^{i-1}[\ max\{(\beta R)^{\frac{i(1-\sigma)}{\sigma}}\frac{c_1^{1-\sigma}}{1-\sigma}, \frac{(y_0-\frac{1}{1-\nu}\epsilon)^{1-\sigma}}{1-\sigma}\} + (1-q)\beta W_h]\}
\end{aligned}
$$

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This closed form solution of the distribution is a useful building block for characterise the relationship between aggregate debt and income risk in the following section.

1.3.3.2 Income risk and aggregate debt in stationary equilibrium

This section shows how an increase in the riskyness of incomes lowers aggregate assets in this economy, as long as the initial level of risk is high enough. Remark 1.1 shows that changes in income risk $d\epsilon$ affect the stationary consumption distribution only via shifts in its upper and lower bounds, through changes in autarky values W_h, W_l. Stationary assets, which finance the difference between the constant aggregate endowment and aggregate consumption, inherit these comparative statics of consumption with respect to ϵ. This yields the following proposition

Proposition 1.2. There is a value $\epsilon*$, such that for higher initial levels of income risk $\epsilon > \epsilon*$, a marginal increase $d\epsilon > 0$ decreases stationary asset holdings.

Proof

By summing over the distribution in remark 1.1, we can write aggregate consumption as[11]

$$C = \nu c_1[1 + (1-p)\sum_{i=1}^{m-1}(\beta R)^{\frac{i}{\sigma}} q^{i-1}] + (1-\nu)q^{m-1}(y_0 - \frac{1}{1-\nu}\epsilon) \tag{1.24}$$

Thus aggregate consumption is affected by income risk only via changes in the bounds of the consumption distribution. In particular, C is decreasing in income risk ϵ whenever c_1 is, which in turn, from remark 1.1 depends on autarky values W_h and W_l. These are

$$W_h = \frac{(1-\beta q)u(y_0 + \frac{1}{\nu}\epsilon) + \beta(1-p)u(y_0 - \frac{1}{1-\nu}\epsilon)}{1 - \beta(q+p) - \beta^2(1-(q+p))} \tag{1.25}$$

$$W_l = \frac{\beta(1-q)u(y_0 + \frac{1}{\nu}\epsilon) + (1-\beta p)u(y_0 - \frac{1}{1-\nu}\epsilon)}{1 - \beta(q+p) - \beta^2(1-(q+p))} \tag{1.26}$$

Given the assumptions on transition probabilities, W_l is always declining in ϵ, while the high income-autarky value W_h is concave in ϵ with a maximum at some $\epsilon* > 0$.

[11] If $\Phi(c_m) \approx 0$, such that truncation of the geometric distribution is negligible (which is true necessarily as $R \longrightarrow 1/\beta$), we have

$$c_1 = \{\frac{(1-\beta(p+q) - \beta^2(1-p-q))(1-\beta q(\beta R)^{\frac{1-\sigma}{\sigma}})}{(1+\beta(1-p-q)(\beta R)^{\frac{1-\sigma}{\sigma}})(1-\beta q)}(1-\sigma)W_h\}^{\frac{1}{1-\sigma}} \tag{1.22}$$

and aggregate consumption equals

$$C = \nu c_1[1 + \frac{(1-p)(\beta R)^{\frac{1}{\sigma}}}{1-(\beta R)^{\frac{1}{\sigma}} q}] \tag{1.23}$$

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It increases for $\epsilon < \epsilon*$, decreases for $\epsilon > \epsilon*$ and crosses the perfect insurance value at $\bar{\epsilon} > \epsilon*$.[12] Note that this result does not depend on CRRA preferences. So for $\epsilon > \epsilon*$ aggregate consumption declines with income risk ϵ. Stationary aggregate assets are monotonously increasing in aggregate consumption, so the result follows.■

1.3.3.3 The decoupling of income and consumption inequality in open economy

The following result shows that in an open economy facing a given world interest rate, the inequality of consumption can become completely independent from that of income.

Corollary 2: Variance of log-consumption

If $\Phi(c_m) \approx 0$, the variance of log-consumption is

$$Var_c = \Lambda[\frac{\log(\beta R)}{\sigma}]^2 \tag{1.29}$$

where $\Lambda > 0$ is a function of transition probabilities only. So (log) consumption inequality is entirely determined by world interest rates R, where a higher R lowers domestic consumption inequality. If there is a non-negligible mass at the truncation point, $\Phi(c_m) > 0$, this is an upper bound for the cross-sectional variance of individual consumption.

For the simple algebra that leads to the result see chapter 3. The intuition is straightforward: Income risk affects the stationary distribution of consumption mainly via the participation constraint at high income that determines its upper bound, and thus the position of the distribution. Apart from the truncation at z_l, the shape of this distribution, however, depends entirely on the value of interest rates R, via the law of motion (1.12). Therefore, international interest rates determine consumption inequality, while income risk determines mean consumption, and thus asset holdings.

[12] To see this, take the first derivative of autarky values with respect to ϵ

$$\frac{dW}{d\epsilon} = (I - \beta F)^{-1}[\frac{1}{\nu}u'(y_0 + \frac{1}{\nu}\epsilon), -\frac{1}{1-\nu}u'(y_0 - \frac{1}{1-\nu}\epsilon)] \tag{1.27}$$

The persistence assumptions assures that for $\epsilon = 0$ the rise in current utility dominates the fall in future expected utility for high income agents. With strictly positive entries of F, however, Inada conditions on u translate to W_h, so marginal utility goes to infinity as the low income realisation goes to zero: as $\epsilon \longrightarrow y_0$, $\frac{dW_l}{d\epsilon} \longrightarrow -\infty$. By the intermediate value theorem and continuity, there exists an $\epsilon*$ with $\frac{dW_h(\epsilon*)}{d\epsilon} = 0$, and $\bar{\epsilon} > \epsilon*$ with $W_h(\bar{\epsilon}) = 0$. Also, for twice continuously differentiable u the concavity of the utility function translates to the concavity of autarky values as a function of ϵ

$$\frac{dW^2}{d\epsilon^2} = (I - \beta F)^{-1}[(\frac{1}{\nu})^2 u''(y_0 + \frac{1}{\nu}\epsilon), (\frac{1}{1-\nu})^2 u''(y_0 - \frac{1}{1-\nu}\epsilon)] < 0 \tag{1.28}$$

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1.3.4 Income risk, aggregate debt and consumption inequality with general uncertainty and preferences

Proposition 3.6 naturally generalises to the case $N > 2$ with well-behaved, non-CRRA preferences. To see this, note that in this case, the consumption distribution can be characterised by N minimum participation-compatible consumption levels, associated to N autarky values, that provide the upper bounds for geometric sub-distributions. Within these subdistributions, the support is entirely determined by the law of motion (1.11), and monotonously increasing in the upper bounds. So when a rise in income risk reduces all autarky values, the whole support of consumption declines, reducing aggregate consumption and asset holdings in stationary equilibrium (for detail see chapter 3). The shape of the n sub-distributions is again independent of the upper bound, with variance that decreases in R. However, changes in income risk now change relative autarky values and thus do not move the subdistributions in parallel. So the shape of the overall consumption distribution is not independent of income risk. But it is easy to show that the width of the support $\mathbb{C}$ decreases with R.
Chapter 3 also proves existence and uniqueness of stationary equilibrium in a closed economy version of the model. There, the results on the shape of the consumption distribution continue to hold, while the comparative static effect of changes in income risk does not. Consumption thus follows a geometric distribution, implying a significant left skew. Equilibrium interest rates are relatively low in the endowment version of the model, at about 2.5 percent.

1.3.5 Saving after default

Although the characterisation of the consumption distribution and its implications for aggregate debt hold conditional on any value of default, proposition 3.6 depends crucially on the Laffer curve-type relationship between income risk and autarky values, implying that, for high enough levels of risk, autarky values fall and debt constraints are relaxed when income risk rises further. The analytical characterisation of the relation between income risk and autarky values at the basis of this Laffer-curve, in turn, required the assumption that agents simply consume their income forever after default. This assumption, however, may be viewed as too strong, as individuals should have access to some storage, or savings technology to transfer resources between periods even after exclusion from complete financial markets.
The possibility to save in high income periods makes the outside option of default significantly more attractive, as individuals can guard against the risk of very low consumption implied by temporary negative income shocks without saving. Rather than

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with the level of default values, however, this study is concerned with their behaviour after changes in income risk that determines the evolution of the economy's aggregate borrowing capacity. Trivially, for low enough interest rates, the above Laffer-curve relationship holds also when agents can save after default, as for low R, savings become increasingly unattractive and agents thus simply consume their income after default. For intermediate interest rates, however, the impact of increasing risk on autarky values is less clear. It is easy to see that, for unchanged expected income, a rise in risk lowers the value of the outside option even with saving at a given interest rate. Since, however, a rise in risk, in this paper, is defined as a mean-preserving spread to the support of the income distribution, high-income individuals, who are those with binding participation constraints, experience a rise in expected income. With saving, this rise in expected income can continue to dominate the negative effect of increasing risk that holds at any given amount of lifetime resources. The following, quantitative section therefore discusses the sensitivity of its results to allowing individuals to smooth consumption by saving even after default.

1.4 Individual risk and global imbalances: income uncertainty and the US net foreign asset position 1980-2003

The previous section showed that in an open, debt-constrained economy, rises in income risk can lower aggregate savings and asset positions. But importantly, this only holds for an initial level of income risk that is sufficiently high. The sign and importance of the effect of changes in income risk on asset positions thus depends on the particular economy under analysis. Also, the independence of stationary consumption inequality from income risk only holds for the special case with two income values, at a given exogenous interest rate. Thus, this section first analyses a partial equilibrium version of the model that is calibrated to match some stylised features of the US economy in the years 1980 and 2003. Specifically, I use the stochastic process for US individual incomes estimated by Krueger and Perri (2006), and compare debt holdings and consumption inequality in stationary equilibria corresponding to the two endpoints of their sample, respectively 1980 and 2003. A second exercise analyses the General Equilibrium of a stylised 2-country economy, where the US trades bonds with a large developing country, calibrated to capture the evolution of individual income inequality in China. There, I assume domestic asset trade is limited to uncontingent assets, resulting in a rise of precautionary savings in response to an increase in individual income risk. Before turning to the results I briefly describe the calibration of the model parameters.

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1.4.1 Calibration

I calibrate the income process following Krueger and Perri (2006), using their estimates for the years 1980 and 2003, the endpoints of their sample. The authors assume the log of post tax labour income plus transfers (LEA+) $log(z_t)$ to be the sum of a group specific component α_t and an idiosyncratic part y_t. The latter, in turn, is the sum of a persistent AR(1) process m_t, with persistence parameter ρ and variance σ_m^2, plus a completely transitory component ε_t which has mean zero and variance σ_ε^2.
The process for LEA+ is thus of the form

$$\begin{aligned} log(z_t) &= \alpha_t + y_t \\ y_t &= m_t + \varepsilon_t \\ m_t &= \rho m_{t-1} + \nu_t \\ \varepsilon &\sim N(0, \sigma_\varepsilon^2) \\ \nu_t &\sim N(0, \sigma_\nu^2) \end{aligned} \tag{1.30}$$

Using data from the Consumer Expenditure Survey (CEX), the authors first partial out the group-specific component α_t as a function of education and other variables, identifying the variance of the idiosyncratic part of income y_t, as well as (from the short panel dimension of the CEX) its first order autocorrelation. They then fix $\rho = 0.9989$, the value estimated by Storesletten et al (2004), which allows them to identify σ_ν^2 and σ_ε^2.
The results show an increase in the variance of labour income of 18 percentage points between 1980 and 2003, the two periods I focus on. 11 percentage points are due to an increase in within-group inequality, out of which roughly two thirds are accounted for by an increase in the importance of persistent shocks, and one third by that of transitory shocks.
In my exercise I abstract from changes in the common wage rate and differences in the group specific component, which, in the present model as in that of Krueger and Perri, translate fully into consumption differences by construction.
As a baseline calibration, I choose a CRRA utility function with coefficient of relative risk aversion of 1 (log-preferences), a discount factor of 0.96, and a constant interest rate equal to the initial closed economy equilibrium rate of 3.4 percent. I then look at the sensitivity of the results to changes in parameters, and the world interest rate. And I look at the case when agents who default are excluded from all financial transactions in the current period, but allowed to invest in non-contingent bonds in the future to smooth income shocks over time. This reduces the impact of higher income risk on the value under default.

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1.4.2 Model Solution

To solve the model, I first approximate the persistent process for m_t with a 7-state Markov chain using the standard Tauchen and Hussey (1991) method.[13] Following Krueger and Perri (2006) I choose a binary process for the transitory shock. The computational algorithm then follows chapter 3, which describes the recursions to derive the stationary consumption distribution in the general case. I amend this for the fact that, with purely transitory shocks ν_t, the monotonicity condition for F does not hold.

1.4.3 Partial equilibrium results

1.4.3.1 Income risk and net foreign assets

Table 1.1 shows the equilibrium asset positions for different specifications of the economy. In the baseline calibration (I), the rise in income risk between 1980 and 2003 leads to a fall in the stationary level of net foreign assets of more than 50 percent of annual GDP.

TABLE 1.1: Stationary assets and consumption inequality - different calibrations

I	**Baseline**						
	year	R	β	σ	Assets/GDP	Var(log(c))	Save in default?
	1980	1.034	0.96	1	0	0.034	No
	2003	1.034	0.96	1	-0.56	0.04	No
II	**Save in default**						
	year	R	β	σ	Assetss/GDP	Var(log(c))	Save in default?
	1980	1.025	0.96	1	-0.04	0.07	At 2.5%, not in t=0
	2003	1.025	0.96	1	-0.15	0.09	At 2.5%, not in t=0
III	**Save in default**	$\sigma = 2$					
	year	R	β	σ	Assetss/GDP	Var(log(c))	Save in default?
	1980	1.025	0.96	2	-0.79	0.04	At 2.5%, not in t=0
	2003	1.025	0.96	2	-1.23	0.05	At 2.5%, not in t=0

The table reports the results from 3 calibrations of the model, that differ in their punishment of default, risk aversion and the exogenous interest rate. In calibration 1, agents who default face complete financial autarky, while in calibrations II and III they can smooth consumption through saving, although not in the period of default.

However, this calibration features a relatively high world real interest rate, and very strong effects of income risk on the value of default, due to the assumption of permanent exclusion from all financial trade. Thus, a second calibration allows saving in non-contingent bonds starting from the period following default, and reduces the world

[13] Note that this method accords with my assumption of widening the support Z to increase risk, but leaving the transition probabilities unchanged.

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interest rate to 2.5 percent.[14] The results are reported as calibration II in table 1.1. The fall in stationary assets from the observed rise in US income risk is now smaller, at 11 percent of GDP. This is because with saving after default, higher income risk has a smaller impact on autarky values. Calibration III in table 1.1 increases risk aversion in this second calibration to $\sigma = 2$. With more risk averse individuals, the income volatility under financial autarky provides stronger disincentives to default, even when agents are allowed to save in autarky. For a given level of income risk, this translates to lower stationary asset holdings. But as before, the increase in income risk between 1980 and 2003 decreases stationary assets further, by about 40 percent of GDP. Figure

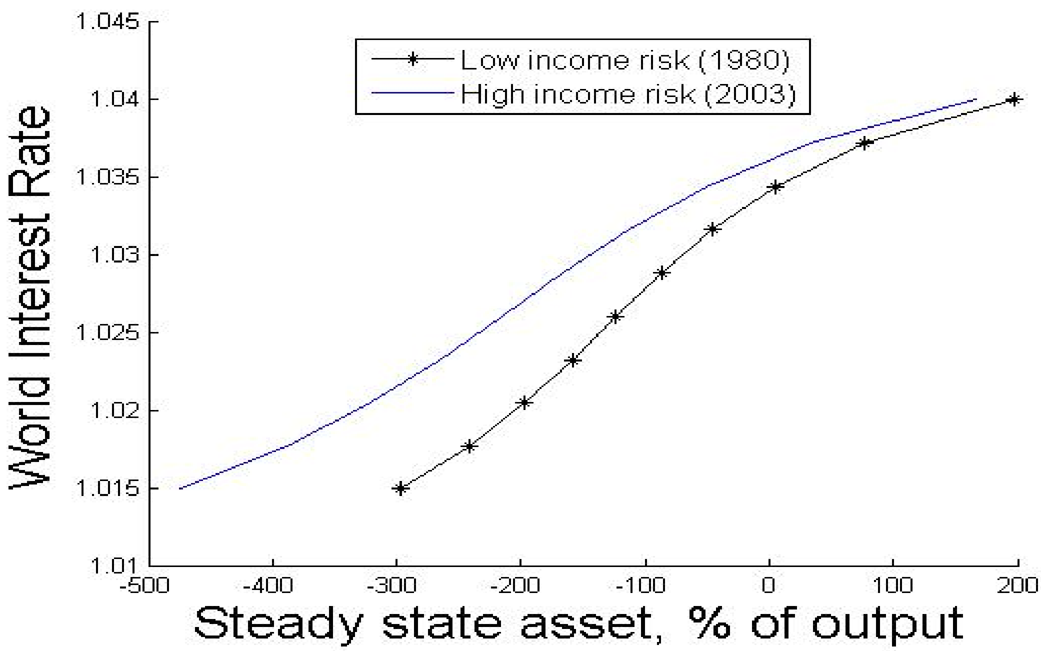

FIGURE 1.2: Asset demand function, baseline calibration.

1.2 shows that this reduction in assets from a rise in income risk holds for all values of world interest rates in the base line calibration. But this monotonicity of stationary foreign assets in risk gets lost when agents are allowed to save under autarky, as figure 1.3 shows. For high interest rates, the additional increase in risk now increases aggregate assets in stationary equilibrium.

1.4.3.2 Income and consumption risk

Figure 1.4 shows the consumption distributions in the baseline case, for low (1980) and high income risk (2003). The sub-distributions, of different colour in the graph,

[14] I choose an interest rate on savings in autarky of 2.5 percent, which is close to the average ex-ante annualised real rate of 2.6 percent on 6 month US treasury bills between 1980 and 2003, deflated using University of Michigan 12 month inflation expectations.

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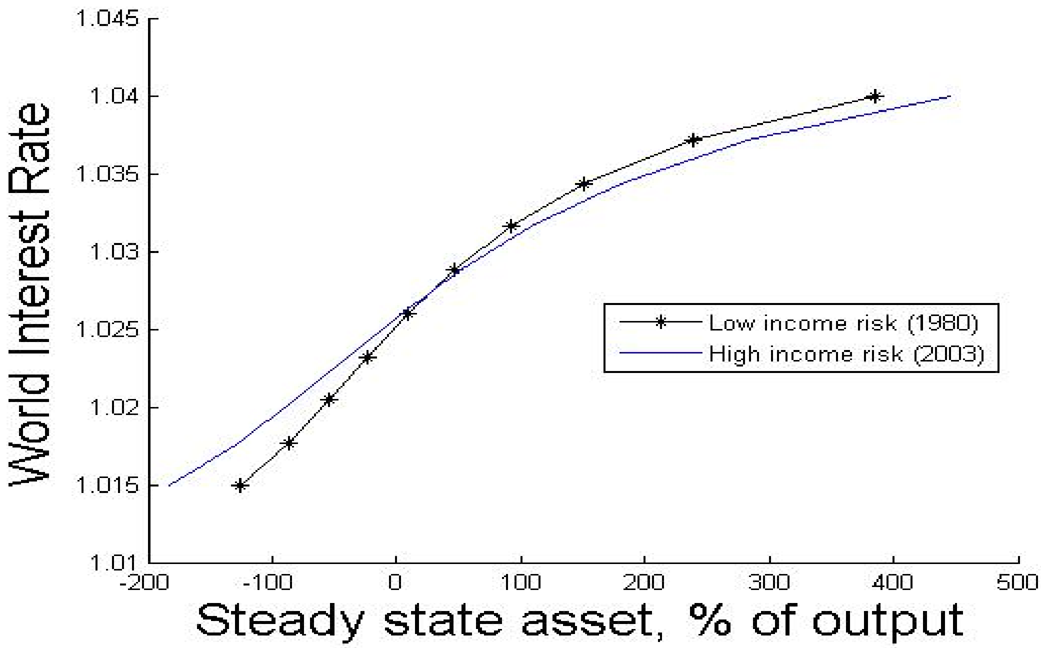

FIGURE 1.3: Asset demand function, log-preferences, saving after default at world interest rate but not in t=0.

correspond to individuals that were last constrained in the same income state, and thus have a common starting value for their declining paths before the next positive shock. Importantly, these sub-distributions are geometric and their shape remains constant between 1980 and 2003 - this is because the interest rate is unchanged in the baseline case. Their positions, however, decline with the fall in autarky values caused by higher income risk. This fall is less pronounced in states that correspond to positive realisations of the binary transitory shock, such as state 1, as there, higher variance translates to an increase in current income, if not value. From table 1.1 we see that the corresponding change in the variance of log consumption is small.

Figure 1.5 illustrates the relationship between interest rates and the consumption distribution. For the income process estimated for 2003, the figure shows how a lower interest rate widens the consumption distribution significantly, as analytically shown for the special case above. Figure 1.6 confirms this finding: the change in consumption volatility due to a change in income risk is an order of magnitude smaller than the changes caused by movements in the world interest rate.

The rise in individual income risk observed in the US since the 1980s can thus potentially explain at least part of the fall in its net foreign asset position. And interestingly, for a

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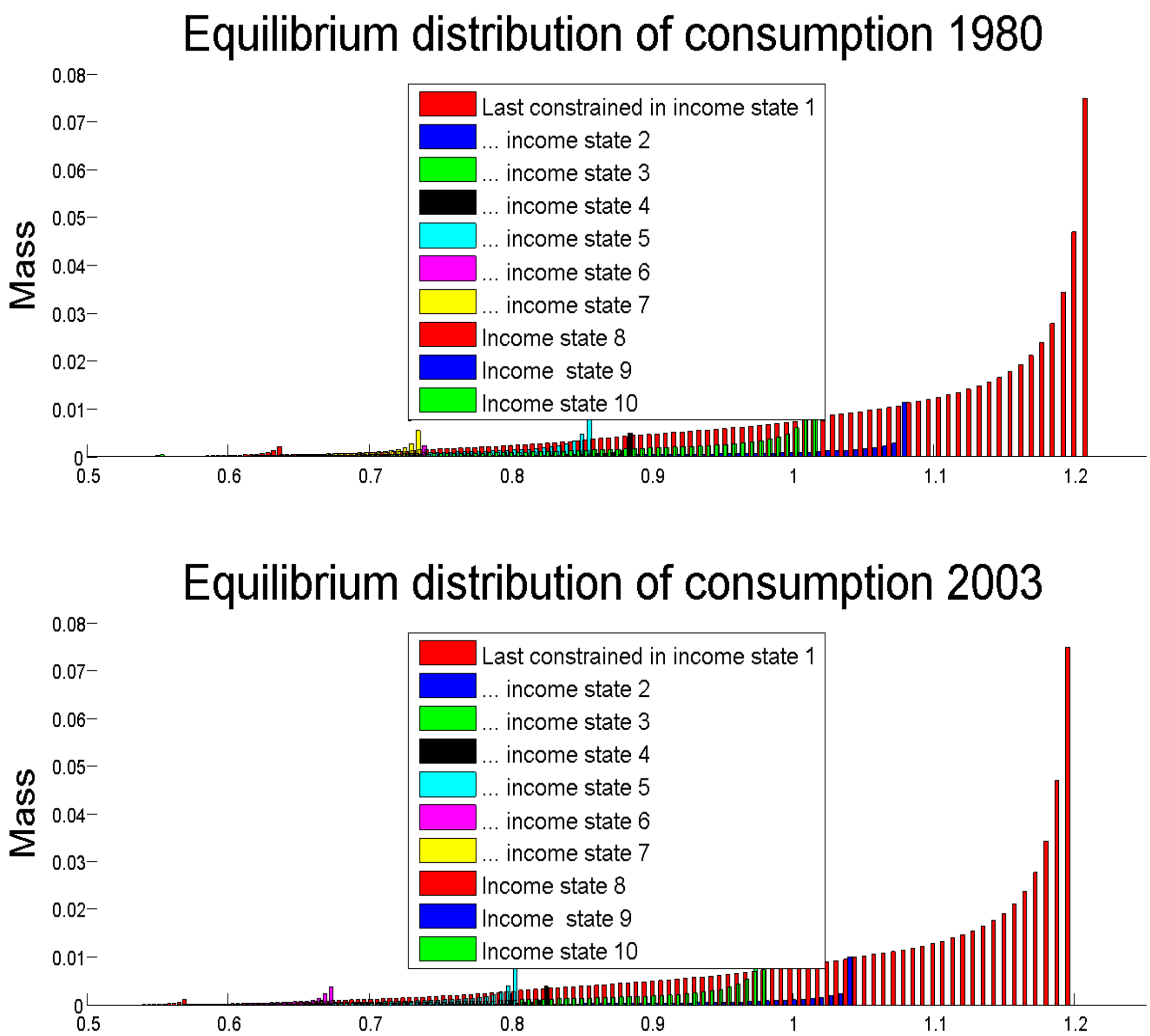

FIGURE 1.4: The consumption distribution in 1980 and 2003, baseline calibration (log-preferences, no savings in autarky).

given interest rate this rise in income risk leaves the distribution of consumption almost unaffected. But changes in world interest rates have an important effect on consumption inequality.

1.4.4 Endogenous financial deepening meets the savings glut: A world economy with rising idiosyncratic risk and differences in financial development

So far, the analysis was agnostic about the determinants of savings outside the US, taking as given a world interest rate. But of course, in a closed world economy, the fall in US savings caused by increased idiosyncratic risk affects the equilibrium interest rate. This section thus looks at the general equilibrium in a simple economy consisting of 2

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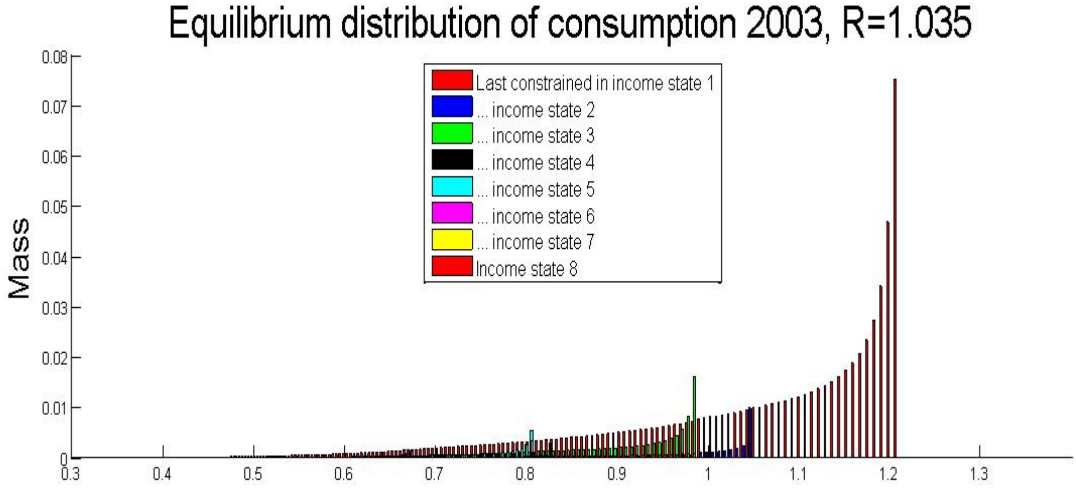

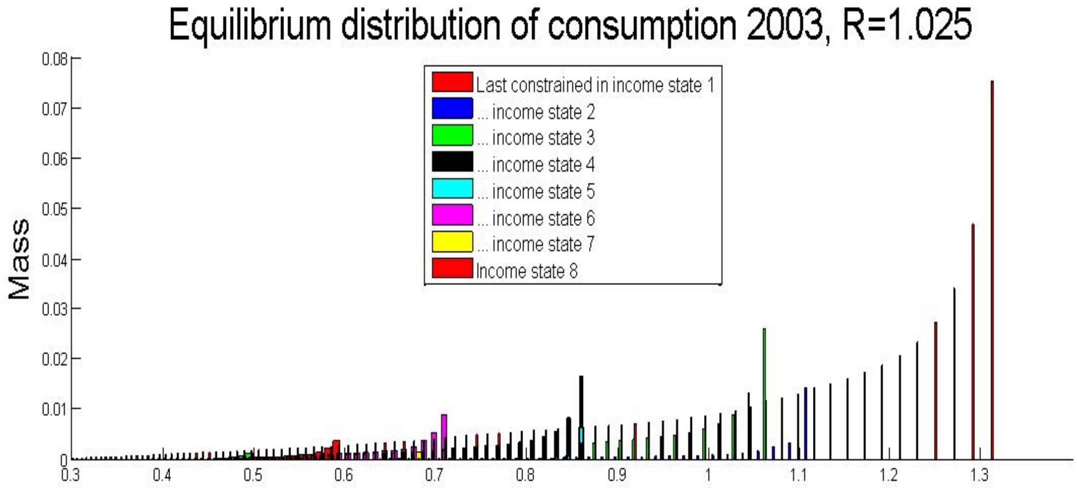

FIGURE 1.5: The consumption distribution in 2003 (high income risk), with high and lower interest rates.

countries that differ both in their domestic financial market structures and the evolution of idiosyncratic risk that their agents experience over time. In particular, I present a stylised world economy consisting of China and the US. Both countries experience a rise in idiosyncratic income uncertainty in line with their historical experience, but differ in their ability to insure against this risk through domestic financial trade. Specifically, US financial markets are assumed to be complete but subject to participation constraints as before, allowing individuals to save at the world interest rate after they default on contracts. Chinese consumers, on the other hand, do not have access to complete domestic financial markets. Rather, I assume that individuals there can only engage in self-insurance through trade in bonds subject to a borrowing limit. As before, I abstract from aggregate risk. International asset trade is limited to non-contingent bonds, whose prices all agents take as given. A stationary equilibrium of the world economy is thus

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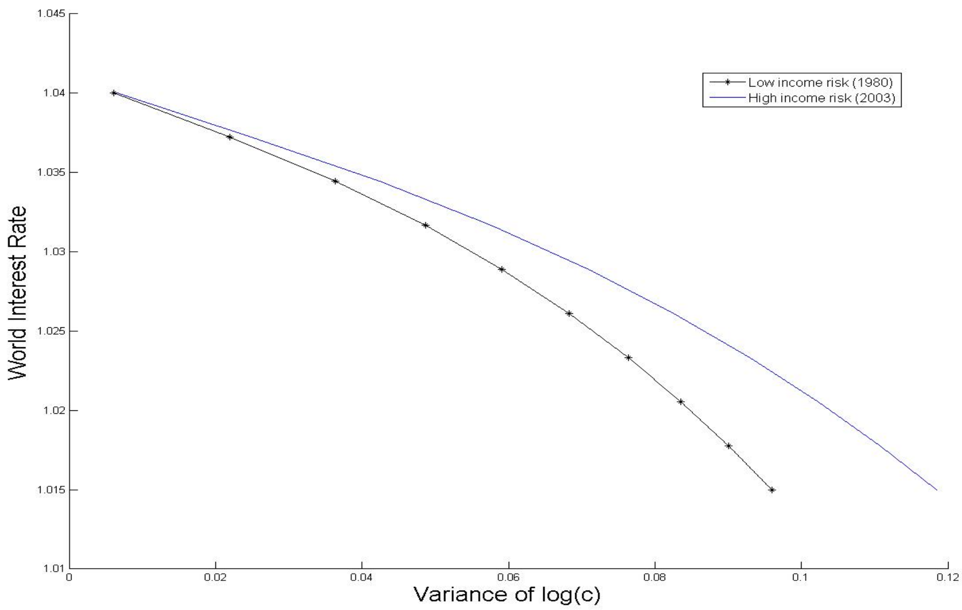

FIGURE 1.6: Variance of log(c), baseline calibration.

a process for individual consumption in both countries, an aggregate net asset position between the two countries and a market clearing interest rate.

The analysis concentrates on the effect of changes in idiosyncratic risk on equilibrium net foreign asset positions over the last 25 years. The process of idiosyncratic risk in the US is unchanged from the previous section. Unfortunately, equivalent estimates of an income process with group-specific heterogeneity, as well as persistent and transitory within-group risk, is infeasible for China, where the necessary household panel survey is not available for the period of interest. We are thus left to estimates of cross-sectional income inequality. This is a problem, as we cannot identify the different components of individual income risk from cross-sectional data alone. But the calibrated model provides a mapping from a specific income process to the cross-sectional consumption inequality and a savings demand schedule. I thus calibrate the components of the income process to capture the Gini coefficients of consumption and income for Chinese urban regions reported in Perloff and Wu (2005) in 1985, plus a zero initial foreign asset position. Assuming that the income process in China has the same permanent-persistent-transitory structure as in the US, including the persistence parameter of 0.9989, this provides three targets for three parameters, namely the variances of the permanent, persistent and transitory component of the income process in (4.13).[15] The increase in idiosyncratic risk in China is then calibrated to capture the observed rise in both Gini coefficients until

[15] For the permanent part of income risk, I choose a uniform distribution of log-income values with 5 support points, and calibrate the support width to capture the moments of the data. Also, for both countries the results reported below are based on a discretisation of the AR(1) component of the income process into a 5-state markov process.

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2001. For this, I assume that the change in permanent income differences in China is entirely captured by the rise of Urban-Rural inequality. But I look at the sensitivity of the results to this assumption below. The results assume a relatively tight borrowing limit corresponding to average quarterly income. As country weights, I use relative GDP of both countries from the Penn World tables in 1980 and 2003.

Table 1.2: Calibration of the income process for China

	permanent	persistent	transitory	Gini income	Gini consumption
1985	0.08	0.0038	0.03	0.19	0.17
2001	0.08	0.057	0.10	0.27	0.21

The table reports the variances of components of an income process for Chinese urban regions that has the same structure as that reported in the text for the US: in the absence of information on group-specific attributes, (between-group) permanent income differences are modelled as a log-uniform distribution with 5 support points, while within-group income risk is the sum of a an AR(1) process with persistence parameter 0.9989 (discretized as a 5-state Markov process), plus a purely transitory binary shock (see the text for details). The parameters are chosen to target the Gini coefficients for consumption and income from Perlach and Wu (2005) for urban regions, and a zero net foreign asset position in 1980.

Table 1.2 reports the implied estimates of the income process in China. In line with the similar Gini coefficients for consumption and income, inequality in the 1980s is estimated to be mainly determined by permanent income differences: both the variance of persistent and transitory income shocks are small. But the observed rise in consumption and income inequality until the early 2000s, stronger for income than for consumption, is in line with a strong increase in both the variance of persistent and transitory shocks, by 5.4 and 7.0 percentage points respectively.

Figure 1.7 plots the resulting equilibria for the early 1980s and the early 2000s. Chinese assets are plotted with a negative sign, such that the intersections of the demand and supply schedules give equilibrium asset positions and interest rates. The initial net interest rate of 2.5 percent is low relative to the discount factor of 0.96, as in many models of imperfect insurance. The increase in risk in the US results in the familiar fall in the savings demand schedule as a result of financial deepening. But in China, the strong rise in idiosyncratic risk after the early 1980s results in a strong rise in precautionary savings. This is exactly as we would expect in a self-insurance economy, where the financial deepening effect of higher income risk is absent, and the precautionary savings effect is relatively strong. The corresponding net effect is a fall in the US net foreign asset position to minus 32 percent of GDP, and a fall in the world interest rate of about 25 basis points.

As it is impossible to distinguish the effect on cross-sectional inequality of increases in permanent income differences from those of the very persistent shocks in the model,

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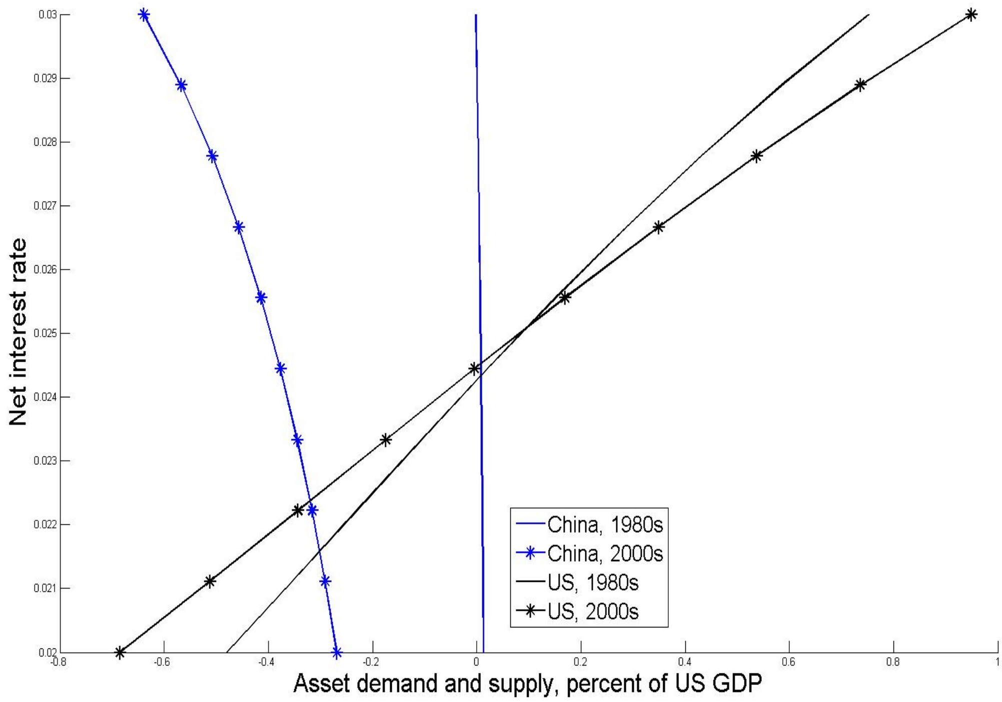

FIGURE 1.7: Asset demand and supply in a two country world economy. The picture depicts US asset supply together with asset demand by China, which has a negative sign.

Figure 1.8 was based on the assumption that increases in permanent income differences are entirely captured by the difference between urban and rural regions. Since precautionary savings are largely unaffected by changes in permanent inequality but rise with persistent shocks to income, this may overstate the equilibrium savings. Therefore, Figure 1.8 shows how the results change when I make the opposite assumption of unchanged persistent shocks (which requires some recalibration also of the variance for transitory shocks, to match both Gini coefficients). As expected, the rise in equilibrium US liabilities is lower, but at 20 percent is still sizeable.

1.5 Conclusion

This chapter has looked at the link between domestic income uncertainty, consumption inequality and net foreign asset positions in an economy where financial markets suffer

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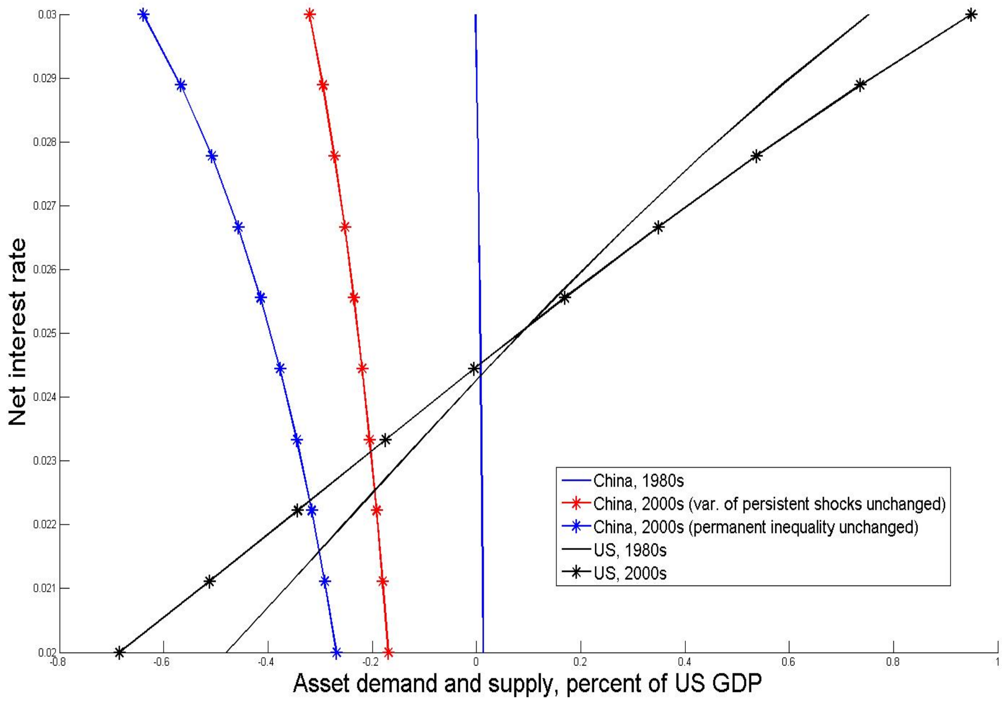

FIGURE 1.8: Asset demand and supply in a two country world economy: sensitivity. The picture depicts US asset supply together with asset demand by China, which has a negative sign. The two scenarios for China correspond to different ways of splitting the increase in income inequality between permanent inequality and near-permanent shocks.

from enforcement constraints. Domestic financial markets were assumed to be complete, but constrained by individuals' option to default on contracts, at the price of permanent exclusion from insurance markets. I showed that, contrary to economies with unconstrained complete markets, this economy has a well-defined stationary equilibrium for any given world interest rate. An analytical solution to the cross-sectional consumption distribution showed that higher income risk can indeed lower aggregate savings by making the punishment of default, financial autarky, less attractive, thus endogenously "deepening" financial markets. However, changes in income risk have only a small effect on consumption inequality, which depends mainly on the international interest rate. A calibration of the model to the US case showed that the changes in income risk observed between 1980 and 2003 might indeed explain an important part of the fall in the net foreign asset position. This holds not only at a constant world interest rate, but also in the general equilibrium of a simple world economy where the US trades bonds with a country that has less sophisticated markets and experiences a strong increase in idiosyncratic risk similar to that seen in China. The "glut" in precautionary savings there and the endogenous financial deepening in the US, both caused by rising idiosyncratic risks, result in a significant deterioration of the US net foreign asset position, and a small fall in the world interest rates.

Future research should generalise this analysis in at least two directions: first, one should also take account of the change in aggregate macroeconomic risk, which declined over the period of analysis. And second, an adequate equilibrium of the world economy should not only take into account advanced countries with deficits and emerging surplus economies, but also countries like Germany or Japan, that experienced surpluses yet have relatively developed domestic financial markets. In this context, the model's prediction of an inverse U-shape relationship between net foreign asset positions and individual income risk is especially interesting.

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Chapter 2

The home bias of the poor: terms of trade effects and portfolios across the wealth distribution

Abstract[1]

Wealthier people generally hold a larger part of their savings in risky assets. Using the US Survey of Consumer Finances, I show that wealthier households also have a higher portfolio share of foreign assets. This relative home bias of the poor does not seem to be explained by fixed participation costs alone, as the portfolio share of foreign assets increases with financial wealth even among participants in foreign asset markets. This chapter shows how both biases of poorer agents' portfolios, towards safe and home assets, can arise in a simple two country economy with income and portfolio heterogeneity. Poor investors are naturally biased against domestic equity when wages and capital returns are positively correlated, making equity a bad hedge against fluctuations in labour income relative to bonds. Home bias in consumption, on the other hand, leads to a bias against foreign assets in the bond portfolio.

JEL Classification Codes: F36, G11, E21, D11, D31

Keywords: Heterogeneous Agents, Home Bias, Terms of Trade, Inequality, International Asset Diversification, Portfolio Choice

[1] I would like to thank David Backus, Charles Engel, John Leahy, and Gianluca Violante for helpful comments, as well as seminar participants at the Bank of England, the European University Institute, Cornell University, and the 2008 La Pietra Mondragone Workshop. Also, I am indebted to Frederico Cepeda at Morningstar for his generous help with US mutual fund data.

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2.1 Introduction

It is well-documented that household portfolios become more diversified as wealth increases. Campbell (2006) and Guiso et al. (2003), for example, show that poor households are less likely to invest in risky assets. Equally, many authors have found that aggregate country-portfolios have surprisingly low shares of foreign assets, the so-called "home bias in portfolios puzzle" (see Lewis, 1997, for a summary of this literature). But little attention has been devoted to the composition of individual household portfolios between domestic and foreign assets, and its relationship to individual wealth.[2] In the empirical part of this chapter, I study the US survey of consumer finances (SCF) and show that wealthier households also seem to invest on average a higher share of their portfolio in foreign assets than those with lower financial wealth.

A prominent explanation for this bias of poorer investors towards safe and home assets relates to fixed costs of participating in the markets for risky and foreign assets. Fixed costs, however, cannot explain the relative home bias of the poor among participants in foreign asset markets, for whom the fixed cost is sunk. In the theoretical part of the chapter, I show that without fixed costs, agents with lower financial wealth optimally have a higher portfolio share of assets that hedge against fluctuations in their future income. Assuming returns to capital and labour are positively correlated, this leads to a bias of poorer investors against equity. With home bias in consumption, however, investors are also biased against foreign bonds, as these are a bad hedge against aggregate productivity shocks at home. Wealthy investors, whose future consumption is less dependent on income, care less about this hedging property than poor investors. Therefore, equilibrium portfolios vary across the wealth distribution and poorer investors tend to have a stronger home bias than rich investors.

The intuition for these results has similarities to Baxter and Jermann (1997) who show that with income from non-marketable human capital, the optimal portfolio of assets consists of two sub-portfolios, one completely diversified, the other designed to hedge against volatility of human capital returns. I show that the hedging portfolio can be dominated by safe domestic assets. And its importance relative to the diversified part of the portfolio declines as total investor wealth rises. To derive the results, I consider a two country model with incomplete markets and heterogeneous consumers that receive an uncertain amount of a country-specific endowment good every period. I derive analytical portfolio shares by assuming (as Cole and Obstfeld, 1991) that preferences over domestic and foreign goods are unit-elastic, but allow for symmetric home bias in consumption.

[2]Hau and Rey (2008a, 2008b) take, in some sense, an intermediate step of looking at individual mutual fund portfolios. They find average home bias which is, at least in the US case, in line with that found using aggregate data, but also report an important degree of heterogeneity between portfolios of different mutual funds.

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The chapter combines three strands of literature. First, from studies of household finances, such as Campbell (2006) or Guiso et al. (2003), I take the stylised fact that wealthier individuals have riskier and more diversified portfolios. Using the 2004 wave of the survey of consumer finances, I illustrate how this is also true for the holdings of foreign assets, whose portfolio share I show to increase with investor wealth. Second, from the international macroeconomics literature I take the idea that general equilibrium terms of trade movements can be important determinants of optimal portfolios, and show how this can lead to variation in portfolios across individuals within the same country. And third, by including uninsurable idiosyncratic income risk within the two countries of an otherwise standard model, I take a first step to extend heterogeneous agents models to the open economy.

My theoretical model is most related to previous contributions trying to explain the home bias of country portfolios. While some authors have focused on the costs of diversification, where high turnover in foreign assets points against important formal investment barriers (Tesar et al 1995, Stulz 2005) but informational asymmetries may play a role (Ahearne et al 2004), most studies have looked at the benefits of diversification, and questioned the usefulness of identical, completely diversified portfolios as a benchmark. For example, non-tradable goods, or a bias in consumption baskets towards locally produced goods, introduce asymmetry in the standard 2-country model and imply variation in portfolios across countries (see e.g. Stockman et al 1995). Similarly, non-tradable risks, for example in returns to human capital, introduce country-specific hedgeing terms in optimal portfolios (Baxter and Jermann 1997) that can lead to home or foreign bias depending on the covariance of returns to labour and capital (Bottazzi et al 1996).

Traditionally, the literature has focused on home bias in equities of aggregate country portfolios. However, empirically, there is also strong home bias in bond portfolios (Tesar and Werner 1995, Burger and Warnock 2004). Moreover, recently Coeurdacier and Gourinchas (2009) have pointed out the importance of bonds for hedging real exchange rate movements. They show that portfolios that also include bonds have very different equity shares, which effectively become hedges against non-financial income risk. Almost no study, however, has looked at home bias in individual, as opposed to aggregate country-level, portfolios. An exception is the work by Harald Hau and Helene Rey (2008a,b), who analyse the equity portfolios of individual mutual funds in developed economies. I go a step further and consider portfolios at the household level.

The model I consider takes three elements from the recent literature on aggregate home bias - non-diversifiable income risks, consumption baskets that are biased towards domestic goods, and portfolios consisting, potentially, of both bonds and equity - and adds another, idiosyncratic income risk and wealth differences between agents of the same country. This environment is rich enough to look at individual portfolios of bonds

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and equity, but also sufficiently simple to allow, together with a particular structure of preferences, approximate closed form solutions to portfolio shares. It allows me to show that the implications for the composition of individual portfolios across the wealth distribution are consistent with the observed facts.

Section 2.2 analyses portfolio shares of foreign assets across the wealth distribution in the 2004 wave of the SCF. Section 2.3 presents a simple two country two good economy, defines the competitive equilibrium and derives the equilibrium terms of trade movements. Section 2.4 contains the results on optimal portfolios and how they vary across the wealth distribution.

2.2 Portfolios across the wealth distribution: evidence from the 2004 SCF

Wealthier and more educated people are more likely to invest in risky assets. This is well-documented for the US (see for example Campbell, 2006, for a review and an illustration using the 2001 SCF data) and a number of European countries (see Guiso et al, 2003, and Carroll, 2002).
Equally, it is well-known that average country portfolios have surprisingly low shares of foreign assets - the “home bias in portfolios puzzle”. This has been interpreted as a consequence of a more general “local bias” of household portfolios, which overweigh local, regional, and national assets (see e.g. Campbell 2006). But compared to the portfolio shares of risky assets in general, or of domestic equity more in particular, there is very little evidence on the home bias of individual households and its determinants. Campbell et al (2006) conclude for the case of Sweden that international diversification possibilities exist, but are usually exploited only by wealthier individuals, who have a higher share of investments in mutual funds (with an average portfolio share of 25 percent for foreign assets). However, they provide no evidence on direct holdings of foreign assets.

To document the evolution of foreign asset holdings across the wealth distribution, I examine the 2004 wave of the US survey of consumer finances (SCF). This survey includes information on the US dollar value of households’ holdings of “bonds issued by foreign governments or companies” and “stock in a company headquartered outside of

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the United States".[3] In order to control for indirect holdings of foreign assets, I include a measure of foreign assets held via mutual funds.[4] I derive a measure of total foreign asset holdings by summing to individuals' direct investments in foreign equity and bonds the reported value of their mutual fund shares in US equity, bond and combination funds multiplied by the average portfolio weight of foreign bonds and equity in each type of fund.[5] Figure 2.1 plots the resulting foreign asset portfolio shares (averaged within every decile of the financial wealth distribution to reduce noise) as a function of individual financial wealth.[6] The figure shows that the portfolio shares of foreign assets are monotonically increasing across deciles of the financial wealth distribution. Richer households thus seem to have lower home bias on average.[7]

The evidence presented in Figure 2.1, however, raises several questions. First, there are at least two potential sources of error in the way I measures individual portfolios. One arises from households under- or misreporting their foreign asset holdings. But since there is evidence of variation in foreign asset shares across mutual funds at least for equities (Hau and Rey 2008a,b), another source of error is the use of average mutual fund portfolios, which could lead to a bias in the results. An appendix argues that both kinds of measurement error are likely to bias any positive relationship between portfolio shares and financial wealth towards zero. This is because off-shore investments for tax evasion are likely to make underreporting more severe for foreign assets, and average mutual fund portfolio shares are likely to under-represent the foreign asset holdings by wealthy

[3]Question codes x7638 and x7641. An obvious problem of this measure is that it does not refer to non-dollar assets, but to assets issued by foreign issuers, in foreign currency and US dollars.

[4]In other words, I do not consider pension funds. One reason for this is that individuals' decisions on pension fund investments are taken under a very different set of constraints compared to other investment decisions. Also, most shares in pension funds are not actively managed as a part of regular portfolio decisions. However, both these arguments do not apply to individual mutual fund investments.

[5]To my knowledge, these average portfolio shares of mutual funds are not readily available from published sources. But Morningstar kindly provided data on portfolio shares of non-US assets for more than 4700 US mutual funds, not including funds of funds. From this I calculated weighted averages for portfolio shares of foreign bonds and equity for the three categories of funds for the year 2003. Since equity (bond) funds seem to often not report zero foreign bond (equity) holdings, I made an adjustment by setting missing observations to zero for all funds that reported portfolio shares summing to at least 99.5 percent. The resulting sample included around 2800 observations for shares of international equity and slightly less for bonds. Using this sample, the average US equity mutual fund invested 17.1 percent in foreign shares, while the average bond fund (disregarding funds of government / municipal bonds) invested 3.6 percent abroad. Combination funds invested on average 10.7 percent in non-US assets.

[6]Both the deciles and the averages take account of the fact that the SCF oversamples parts of the population, by applying the weights suggested by Kennickell (1999), and the multiple imputation procedure used for the SCF. This is because, to eliminate inconsistencies and missing values, the SCF imputes some values from the other information provided by a household. However, rather than simply reporting one best guess for the imputed values, the SCF provides 5 draws per observation from the distribution of the missing values conditional on observables.

[7]The portfolio shares of foreign assets are low relative to those calculated from aggregate US data. Yet it should be kept in mind that the SCF measure of financial wealth, the denominator of the ratio, includes a large range of assets such as insurance contracts, liquid retirement funds, etc., while the numerator only considers bonds and stocks held directly and via mutual funds. Also, the aggregate shares of foreign assets in the country portfolio cannot directly be read from the graph. The ratios of foreign to total assets of the implied weighted aggregate portfolio are 2.75, 4.08, 3.99 percent for bonds, equities and their total respectively.

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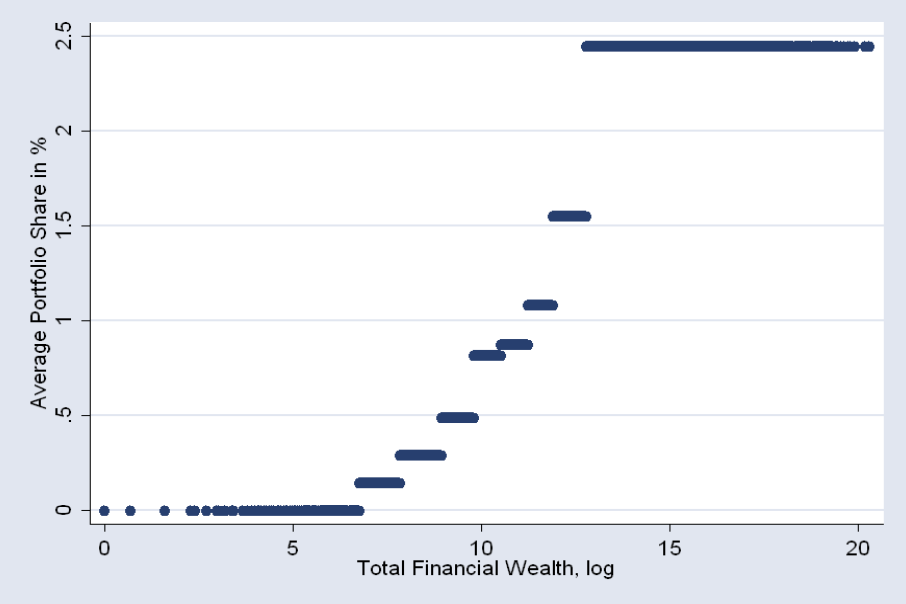

FIGURE 2.1: Portfolio share of total foreign assets (decile average) across the financial wealth distribution

households' if these systematically choose mutual funds with higher foreign exposure. A second question is whether the rise in average portfolio shares across the wealth distribution could merely be due to a higher participation rate of wealthy individuals in the foreign asset market, rather than a rise in individual portfolio shares of participants as they become richer. One factor that could cause such a pattern is fixed costs of entering sophisticated financial markets. An appendix presents a simple model that shows that this implies a non-linear relationship between financial wealth and participation, in the form of a threshold value of assets below which individuals do not hold any foreign assets. Optimal portfolios above the threshold value, however, would not be affected by sunk fixed costs. Thus, any variation in portfolio shares above the threshold value has to be attributed to other factors.

Finally, one might suspect that financial wealth simply captures the effects of other important variables, such as education, age, or income, on portfolios. In this case we would expect an analysis that controls for these variables to yield significantly different results.

In response to this, I perform a more formal econometric analysis. I estimate jointly the probability of participation and the optimal portfolio share of participants with the Heckman (1979) method, conditioning on other variables that were found to be important for portfolio decisions of individuals in previous research. To be precise, I estimate

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the parameters of the following 2 equation system

$$SHARE = \begin{cases} \alpha + \beta_1 ln(FIN) + \beta_2 ln(INCOME) + \epsilon_1 & \text{if } H > 0 \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

with

$$H = a + b_1 AGE + b_2 COL + b_3 FIN_2 + b_4 FIN_3 + b_5 FIN_4 + \epsilon_2 \tag{2.2}$$

Here, SHARE is the portfolio share of foreign assets, FIN is the SCF definition of gross financial wealth and INCOME is the sum of salaries, wages and income or losses from a professional practice, business, limited partnership, or farming. H is an indicator variable that captures the probability of participation in foreign asset markets. This probability is a function of age, a dummy variable "COL" that equals 1 when the household head holds a college degree, and a set of dummies FIN_x that capture financial wealth, taking the value 1 when total financial assets of the household fall in the (weight-adjusted) xth quartile. Only when H is above a threshold, normalised to 0, do agents participate in foreign asset markets and we observe the variable SHARE, their portfolio share of foreign assets. Conditional on participation the portfolio share is a function of income and financial wealth. The errors ϵ_1 and ϵ_2 are assumed to follow a joint normal distribution. The equations are estimated jointly with full maximum-likelihood adjusted for sampling weights. Identification is achieved by restricting the effects of financial wealth to be linear in logs in (2.1), and constant within quartiles in (2.2), which I take to be a proxy for different possible participation thresholds.[8] Results are reported in table 2.1, where numbers in italics are standard errors.[9]

The effect of financial wealth is significant (at the 1 percent level) in both equations. Ceteris paribus, individuals in the bottom quartile of the financial wealth distribution are least likely to invest in foreign assets. But after a jump in the likelihood of participation between the first and second quartile, moving further up the wealth distribution has much smaller, and non-monotonous effects. This is in line with a threshold value of assets beyond which a rise in wealth does not systematically raise the probability of participation. However, higher financial wealth increases significantly the portfolio share of participants in equation (2.1), which cannot be attributed to fixed costs. The effect

[8] I also estimated an alternative specification that included income quartiles in the participation equation. While in the presence of fixed costs of entering foreign asset markets we would expect financial wealth to determine the participation threshold and not income, current income could act as a proxy for future financial wealth. However, the income quartile dummies turned out to be insignificant, so I excluded them from the final specification.

[9] Again, an additional complication is the use in the SCF of multiple imputations for missing values. To account for this, I estimate the same model for each of the 5 implicates separately and then aggregate the estimation results. For the coefficients and standard errors reported in table 2.1, I use the formulae suggested in the SCF codebook (http://www.federalreserve.gov/PUBS/oss/oss2/2004/codebk2004.txt). For the χ^2 value I report a simple average of the following individual values: 75.66, 70.18, 81.42, 69.49, 43.22.

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TABLE 2.1: Heckman model for participation and portfolio share of foreign assets

Equation (1)					
const	ln(FIN)	ln(INCOME)			
-15.16	0.92	-0.12			
2.28	*0.14*	*0.13*			
Equation(2)					
const	AGE	COL	FIN_2	FIN_3	FIN_4
-2.65	0.000	0.11	1.73	1.20	2.15
0.25	*0.0016*	*0.045*	*0.27*	*0.29*	*0.26*
No of obs	4519	Censored:	3378		
$\chi^2(2)$	68.00				

FIN is the SCF measure of total gross financial wealth; INCOME the sum of salaries, wages and income or losses from a professional practice, business, limited partnership, or farming; AGE the age of the household head in years; FIN_x a dummy variable that takes the value 1 when financial wealth falls in the (weight-adjusted) xth quartile of the cumulative distribution; and COL a dummy variable that equals 1 if the head of the household has a college degree. Numbers in italics are standard errors.

of age on the probability of participation is insignificant, but college graduates have on average a higher probability of investing in foreign assets. Finally, for participants the effect of rising income on the portfolio share of foreign assets is insignificant.

This section has shown that individual portfolio shares of foreign assets increase with financial wealth. There is a significant jump in the probability of participation in foreign asset markets between the first and second financial wealth quartiles, consistent with fixed participation costs. But fixed costs cannot explain the significant positive relationship between portfolio shares and financial wealth for participants. The next section presents a simple model of the international economy, where general equilibrium movements in the relative price of home and foreign goods can make home assets better hedges against income fluctuations, and thus lead to the observed pattern of portfolios: poor individuals have a stronger taste for home bonds as in general equilibrium their real payoffs hedge against volatile endowments, which are their dominant source of income.

2.3 A two country heterogeneous agents endowment economy

I consider an economy with two countries, home (H) and foreign (F). In each country there is a large number of agents with unit mass. Individual agents are indexed by h, f at home and abroad respectively. They live for two periods, and receive endowments of a country-specific perishable good H or F.

Agents' preferences are described by a von Neumann-Morgenstern utility function with constant relative risk aversion γ over a Cobb-Douglas aggregate, as for example in Cole and Obstfeld (1991)

$$\mathbb{U}_k = U(c_k)) + \beta E[U(c'_k)] \tag{2.3}$$

$$U(c_k) = \frac{c_k^{1-\gamma} - 1}{1-\gamma} \tag{2.4}$$

$$c_k = c_{k,H}^{\theta} c_{k,F}^{1-\theta} \tag{2.5}$$

$$\theta > \frac{1}{2},\ \gamma > 1 \tag{2.6}$$

where $c_{k,I}$ denotes consumption by agent k of good I and $k \in \{h, f\}$. The assumption $\gamma > 1$ is in line with many studies on home bias. The assumption of identical Cobb-Douglas preferences, on the other hand, is borrowed from Cole and Obstfeld (1991). With $\theta > \frac{1}{2}$ it implies identical bias in consumption towards home goods, and is necessary for an approximate analytical solution to the model, as shown in detail below. More generally, notation is as follows: capital letters H,F denote country-specific variables or goods, small letters h,f denote individual variables that can vary across agents of country H,F. First subscripts denote agents or countries, second subscripts goods. Second period values of a variable x are denoted as x', its distribution as Ψ^x.

2.3.1 Heterogeneity and uncertainty

Heterogeneity of agents within the same country comes from differences in endowments. More precisely, agents in country K receive individual endowments ϵ_k, ϵ'_k of their specific good in period 1 and 2 respectively. Initial endowments ϵ_k are known at the beginning of period 1 before agents choose consumption and portfolios. Income inequality in country K is summarised by the distribution of period 1 endowments across agents Ψ^{ϵ}_K, which is common knowledge.

ϵ'_k, the endowment of individual k in period 2, is the product of two terms: an "individual endowment share" e'_k, and a country-specific "aggregate endowment" Y'_K

$$\epsilon'_k = e'_k * Y'_K \tag{2.7}$$

"Idiosyncratic risk" is given by the probability distribution of e'_k, the period 2 endowment shares of individual k, which I denote $\Psi^{e'}_k$. For simplicity I assume that second period endowment shares are i.i.d. across agents within the same country and independent of all aggregate variables. Also I normalise expected period 2 individual endowment to 1, $\int e'_k \Psi^{e'}_k = 1$. By the iid assumption and the law of large numbers this means the sum of realised endowment shares is always 1 and aggregate period 2 output in country

K simply equals Y'_K.[10]

"Aggregate risk" is summarized by the probability distribution of Y'_H and Y'_F, the aggregate endowments in period 2, denoted $\Psi^{Y'}_H, \Psi^{Y'}_F$. I assume that these are identically distributed across countries and independent of individual random variables and each other.

I assume that all period 2 random variables are log-normally distributed: $(\widehat{e}'_h, \widehat{e}'_f, \widehat{Y'}_H, \widehat{Y'}_F)' \sim N(\ (\overline{e'_h}, \overline{e'_f}, \overline{Y'}, \overline{Y'})',\ \Sigma)$, where a hat denotes natural logarithms $\widehat{z} = ln(z)$ and Σ is a diagonal matrix with entries V_{e_h}, V_{e_f}, V, V.

2.3.2 Incomplete asset markets and borrowing constraints

I impose the simplest structure of asset markets that allows me to analyse two kinds of trade-offs in optimal portfolios: the choice between safe and risky assets on the one hand, and between home and foreign assets on the other.

Like Huggett (1993), agents trade "IOUs" that are in zero net supply and denominated in domestic goods. These are "safe" assets in the sense that for 1 unit of H goods invested today, IOUs in H always pay R^b_H units of good H next period (where "b" stands for "bonds"). Equivalently, foreign IOUs pay R^b_F units of F goods.

In contrast to Huggett's (1993) economy, however, agents can also trade shares in national mutual funds, and are allowed to buy shares and IOUs from foreigners. Shares are also in zero net supply, and risky in the sense that their payoffs are proportional to the stochastic <u>aggregate</u> endowment. Thus the return on home shares is $R^s_H Y'_H$ per unit of H goods invested, equivalently for F. One obvious implication of the exogenous incompleteness of asset markets is that individual claims to future endowments are non-tradable, and that the resulting risk thus is non-diversifiable.

I denote h's holdings of home and foreign IOUs by $a^b_{h,H}$ and $a^b_{h,F}$ respectively, and her holdings of shares by $a^s_{h,H}$ and $a^s_{h,F}$. Asset quantities are denoted in endowment goods of the owner. So if h holds a portfolio $a^b_{h,H}, a^b_{h,F}$, she owns $a^b_{h,H}$ units of H IOUs and $\frac{a^b_{h,F}}{p}$ units of F IOUs. I denote the vector of returns as $\overline{R}$, the vectors of assets held by individuals in H, F as $\overline{a_h}, \overline{a_f}$, and the total value, in terms of their domestic good, of their assets at the end of period 1 as a_h, a_f.

I assume both IOUs and shares have zero default probability. Consistent with this, agents can credibly promise to repay only in units of their income - so borrowing contracts are always written in the endowment good of the issuer. This means agents can issue only domestic assets, but invest both at home and abroad. One consequence of the no-default assumption are individual borrowing constraints: agents in country K

[10] For the derivation of a law of large numbers for continuum economies, see Uhlig (1996).

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can only issue IOUs and mutual fund shares up to maximum amounts B_K^b, B_K^s. In particular, and similar to for example Coeurdacier and Gourinchas (2009), I assume that agents can only sell claims amounting up to a fraction δ^i of their expected period two endowment[11]

$$a_{k,K}^s \geq B_K^s = -\delta_K^s \frac{E[\epsilon_k']}{R_K^s} \tag{2.8}$$

$$a_{k,K}^b \geq B_K^b = -\delta_K^b \frac{E[\epsilon_k']}{R_K^b} \tag{2.9}$$

$$\tag{2.10}$$

2.3.3 The household's problem

A typical home household h maximises expected lifetime utility by choosing in period 1 consumption and a vector of assets $\overline{a_h}$ subject to her budget constraint, borrowing constraints for domestic assets and the non-negativity of foreign asset holdings, taking as given the relative price of foreign goods (in units of the home good) p this period and the vector of returns $\overline{R}$. h's problem is thus given as:

$$max_{c_h, c_h', \overline{a_h}} \frac{c_h^{1-\sigma} - 1}{1-\sigma} + \beta E\{\frac{c_h'^{1-\sigma} - 1}{1-\sigma}\} \tag{2.11}$$

Subject to the constraints

$$\begin{aligned} c_h &= \frac{\epsilon_h - \sum_{i \in \{b,s\}} a_{h,H}^i - \sum_{j \in \{b,s\}} a_{h,F}^j}{p_H} \\ c_h' &= \frac{\epsilon_h' + R_H^b a_{h,H}^b + R_H^s Y_H' a_{h,H}^s + (R_F^b a_{h,F}^b + R_F^s Y_F' a_{h,F}^s)\frac{p'}{p}}{p_H'} \\ a_{h,H}^i &\geq B_H^i, \ for\ i \in \{b,s\} \\ a_{h,F}^j &\geq 0, \ for\ j \in \{b,s\} \\ \epsilon_h' &= e' Y_H' \end{aligned}$$

where $p_H = \theta^{-\theta}(1-\theta)^{-(1-\theta)} p^{1-\theta}$ is the home consumption price index. The problem of a typical foreign household is symmetric.

[11] The "natural" limit to total borrowing in riskless assets would equal the present discounted value of minimum future income $B_K = \frac{\epsilon_{K,min}'}{R}$, which is the highest amount agents can repay for sure. But with log-normal endowments there is a positive probability of having endowment realisations arbitrarily close to 0, such that this formulation does not lead to a non-zero borrowing limit. The problem can be avoided by introducing a positive non-stochastic minimum endowment level for all agents in a country. This can be chosen such that the resulting natural borrowing limit equals the sum of B_K^b and B_K^s above.

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2.3.4 Definition of competitive equilibrium

A competitive equilibrium is

1. A **Consumption Allocation**:
   For every agent k, a consumption sequence of both goods for both periods: $c_{k,H}, c_{k,F}, c'_{k,H}, c'_{k,F}$, where $c'_{k,J}$ is a random variable depending on the realisation of period 2 uncertainty.

2. A set of **Portfolios**:
   For every agent k, a vector $\overline{a_k}$ specifying holdings of all assets in the economy at the end of period 1.[12]

3. A **Price System**, consisting of

   - p, p', the relative prices of F goods in terms of H goods in period 1 and 2, where p' is a random variable with distribution $\Psi^{p'}$.
   - $\overline{R}$, the vector of asset returns.

such that

1. Agents allocate their funds optimally across goods in period 2 given a particular realisation p'.

2. the allocation solves every household's problem (2.11) in period 1 given a relative price p, a distribution $\Psi^{p'}$, and rates of return $\overline{R}$.

3. markets clear:

   - for goods: $\int c_{h,H} dh + \int c_{f,H} df = Y_H$, $\int c_{h,F} dh + \int c_{f,F} df = Y_F$ in both periods
   - and assets: $\int a^i_{h,J} dh + \int p a^i_{f,J} df = 0$, $\forall\ i \in \{b, s\}$, $J \in \{H, F\}$ (each asset is in zero net supply)

4. The distribution of the future relative price $\Psi^{p'}$ is consistent with the joint distribution of random variables e'_h, e'_f, Y'_H, Y'_F, and individual asset holdings at the end of period 1.

[12] Summed across all agents individual quantities imply an **aggregate consumption allocation** for consumption of good K in country J $C_{J,K} = \int c_{j,K} d\Psi^e_J, C'_{J,K} = \int c'_{j,K} d\Psi^{e'}_J$, as well as a **country portfolio** of gross and net asset holdings, and a **net asset position** once net holdings of all assets in a country are summed at period 1 prices.

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Note that optimal portfolios in this environment depend on the distribution of future relative prices $\Psi^{p'}$. But the latter depends on expenditure patterns tomorrow, and thus on savings and portfolio decisions today. In other words, the model has a complicated circular relationship between savings and portfolio decisions on the one hand, and the process for market clearing relative prices $\Psi^{p'}$ on the other.[13] As the next section shows, the assumption of identical preferences across home and foreign agents breaks the link between individual portfolios decisions and equilibrium price dynamics.

2.3.5 Equilibrium terms of trade movements

A well-known consequence of identical homothetic preferences across goods is that the optimal expenditure shares are identical for all agents. Since assets are in zero net supply, any claim of one country on another thus nets out in the excess demand functions for home and foreign goods. Their market clearing relative price is thus independent of the distributions of relative endowments, <u>and</u> of savings decisions in period 1. Taken together, this implies that the equilibrium terms of trade p are independent of the within-country heterogeneity in the economy, given by

$$p = \frac{1-\theta}{\theta}\frac{Y_H}{Y_F} \quad \forall \Psi^{e\prime}_F, \Psi^{e\prime}_H, \Psi^{\epsilon}_F, \Psi^{\epsilon}_H \tag{2.12}$$

Importantly, it is the assumption of identical preferences for all agents in the economy that separates the equilibrium terms of trade from individual heterogeneity. Individuals thus take their portfolio decisions conditional on the equilibrium terms of trade as a function of aggregate uncertainty, which allows a closed form solution for the optimal heterogeneous portfolios despite the incompleteness of asset markets.[14] Since $\theta > \frac{1}{2}$, the assumption of identical preferences implies that home consumers have a preference for domestic goods, while foreign consumers have a relative preference for goods from abroad. Since this study aims to explain stylised facts on individual portfolio decisions in the US economy, whose goods feature strongly in consumption baskets of many other countries, this asymmetry does not seem too restrictive.

Another well-known feature of unit-elastic demand for goods is that claims to country-endowments, or national mutual fund shares, must have equal stochastic consumption

[13] This is similar to the recursive framework with capital accumulation presented by Krusell and Smith (1998), where agents need to know the law of motion for the joint distribution of individual asset holdings and (aggregate and idiosyncratic) shocks, as this determines aggregate savings and thus the returns to capital tomorrow.

[14] This is in contrast to the complete markets framework of most studies on aggregate, or country-level, home bias, which allows more general preferences, for example with symmetric bias towards domestic goods.

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payoffs in equilibrium. For h agents these are

$$\frac{R_H^s Y_H' p_H'}{p_H} = \frac{R_F^s Y_F' p_H'}{p_H} \frac{p'}{p} = R^s Y_H'^{\theta} Y_F'^{(1-\theta)} \tag{2.13}$$

where I set the period 1 relative price of goods to 1 for simplicity and impose $R_H^s = R_F^s \doteq R^s$ from symmetry. So agents are always indifferent between home and foreign mutual fund shares. In this sense, the equilibrium portfolio is never unique with international trade in shares. An appendix discusses conditions for uniqueness and existence of equilibrium.

2.4 Optimal portfolios

Asset holdings differ across individuals for two reasons: first, although the distributions of their future endowment income are the same (due to the i.i.d. assumption), agents differ in wealth due to differences in period 1 income. To smooth consumption, richer agents, with higher current income, save more than poorer agents. Second, poor agents, with low or negative savings, have tomorrow's consumption determined largely by tomorrow's endowment income. Thus, they prefer assets that are good hedges against fluctuations of endowment income, to limit consumption volatility. Aggregate home supply shocks reduce the relative price of home goods, and thus the real returns to home bonds. This makes home bonds better hedges against aggregate home endowment risk than foreign assets or mutual fund shares. Richer agents, whose consumption is mainly determined by asset returns, care relatively less about this hedging, and thus have a lower portfolio share of home bonds.

Note that we can define portfolio shares in two ways, namely as a share of financial wealth, or of total wealth including the present value of claims to future endowments (see also Campbell 2007, section 2.4). Since consumers are indifferent as to the source of claims, the derivations consider wealth portfolio shares. Proposition 1, however, like the empirical section of this chapter, considers financial portfolio shares, defined as a proportion of gross assets.

I show how wealth portfolios are the sum of 2 sub-portfolios: first, a "hedge portfolio", which is the same for all individuals, designed to optimally sell off individual income risk. And second, a "diversified portfolio", determined only by relative returns and preferences, independent of the level of wealth.

Since real payoffs to home and foreign mutual fund shares are always equalised by equilibrium terms of trade movements, in this section I call both of them shares in an "international mutual fund". This allows me simplify notation by denoting returns on home and foreign IOUs as $R_H = R_F$ and those on shares as R_S. Similarly, write h's

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corresponding holdings of bonds and shares as $a_{h,H}, a_{h,F}, a_{h,S}$. I concentrate on the portfolios of home agents. Note that most of the analysis of this section will be conditional on asset returns, since the model is not designed to yield realistic description of asset prices. Particularly, I assume that returns on bonds are similar across countries, in a sense defined below. Within the model, differences in borrowing limits across countries $\delta^s_K, \delta^b_K,\ K \in \{H, F\}$ provide the necessary degree of freedom for this.

2.4.1 Unconstrained portfolios and the bias of the poor against risky assets

Consider first an individual in the home country with non-binding borrowing and short-selling constraints. Imposing the equilibrium relative price as a function of output, we can write the four elements of her portfolio as follows

$$\begin{aligned}
\textit{Real endowment} &: \quad e'_h Y'^{\theta}_H Y'^{1-\theta}_F \\
\textit{Real share return} &: \quad a_{h,S} R_S Y'^{\theta}_H Y'^{1-\theta}_F \\
\textit{Real return to foreign IOUs} &: \quad a_{h,F} R_B Y'^{\theta}_H Y'^{-\theta}_F \\
\textit{Real return to home IOUs} &: \quad a_{h,H} R_B Y'^{\theta-1}_H Y'^{1-\theta}_F
\end{aligned}$$

The first thing to note is that share returns co-move perfectly with endowments. So consumers can short-sell shares to hedge against endowment risk. Furthermore, as $\theta > \frac{1}{2}$ rises to 1, the consumption value of home IOU returns becomes less and less volatile for home agents. This is why home bias in consumption leads to home bias in bonds. To see this more in detail, I take a log-approximation to marginal utility and use the log-normality of random variables to solve the consumer's arbitrage conditions for wealth portfolio shares as a function of the parameters of the model

$$\widetilde{a_{h,F}} = \frac{r_f - y - r_S}{\sigma V} + \frac{\frac{1}{2} + (1-\theta)(\sigma - 1)}{\sigma}) \tag{2.14}$$

$$\widetilde{a_{h,S}} = 2\frac{r_S + y - \frac{1}{2}(r_b + r_f)}{\sigma V} - \widetilde{e'_h} \tag{2.15}$$

$$\widetilde{a_{h,H}} = \frac{r_h - y - r_S}{\sigma V} + \frac{\frac{1}{2} + \theta(\sigma - 1)}{\sigma} \tag{2.16}$$

where a tilde denotes ratios with respect to total wealth $w = \overline{e'_h} Y_H + a_h$, V is the variance of aggregate log-output at home and abroad, and y, r_k are expected growth rates and log returns respectively. Note that, since expected endowments are equal across individuals, the portfolio share of endowment wealth $\widetilde{e'_h}$ falls as total wealth rises. In line with the intuition, the portfolio thus consists of two parts: a hedge portfolio that takes a negative position in shares to guard against endowment risk, equal to $-\widetilde{e'_h}$.

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This part of the portfolio is thus proportional to the relative weight of endowments in total wealth. The second part is a diversified portfolio, independent of endowments, that depends on preferences and relative returns. Since $\theta > \frac{1}{2}$ and $\gamma > 1$, this diversified portfolio has home bias as long as there is not a large excess return on foreign bonds. The following assumption imposes conditions on the exogenous borrowing limits for this to hold.

Assumption 2.1. *Borrowing limits $\delta^s_K, \delta^b_K,\ K \in \{H, F\}$ are such that returns satisfy the following condition*

$$\Delta_r = r_h - r_f > -(2\theta - 1)(\sigma - 1)V < 0 \tag{2.17}$$

Note that there is no closed form solution to equilibrium prices in this model. But it is easy to see that in equilibrium we have to have that

$$r_S + y - \frac{1}{2}(r_b + r_f) > 0 \tag{2.18}$$

In other words, rich agents, with small endowment weights $\widetilde{e'_h}$, have to have incentives to hold positive equity positions, as otherwise there would be an oversupply of shares sold to hedge against endowment risk. This immediately implies that $\widetilde{a_{h,S}}$ rises with wealth.

2.4.2 Constrained portfolios

Individuals can only sell off a fraction $\delta^s_H < 1$ of their endowment wealth. Since the optimal portfolio position in shares falls to $-\widetilde{e'_h}$ as we move down the wealth distribution, there is a strictly positive cutoff value of wealth below which the borrowing constraint on shares is binding. The portfolio shares of investors with $w < w*$ that are constrained in their share position but hold both home and foreign bonds can be derived from their arbitrage condition as before, yielding

$$\begin{aligned}
\widetilde{a_{h,S}} &= -\delta^s_H \widetilde{e'_h} && (2.19)\\
\widetilde{a_{h,F}} &= \frac{-\Delta_r}{2\sigma V} + \frac{\frac{1}{2} + (1-\theta)(\sigma-1)}{\sigma} - \frac{1-\delta_H}{2}\widetilde{e'_h} && (2.20)\\
\widetilde{a_{h,H}} &= \frac{\Delta_r}{2\sigma V} + \frac{\frac{1}{2} + \theta(\sigma-1)}{\sigma} - \frac{1-\delta_H}{2}\widetilde{e'_h} && (2.21)
\end{aligned}$$

Agents with low wealth therefore have a constant negative position in shares. The bond portfolio consists, again, of a hedging and diversified subportfolio. The former consists

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of negative positions in home and foreign bonds, equal to half of the endowment risk that remained after short-selling the maximum amount of shares. The diversified portfolio overweighs home bonds, which are good hedges against volatility of aggregate home endowments that play a stronger role in consumption baskets. Thus $\widetilde{a_{h,H}} > \widetilde{a_{h,F}}$, and total portfolios are biased towards home bonds. Also, both the shares of home and foreign bonds decrease as total wealth w falls, with a slope of $\frac{1}{2}$. Taken together, this implies that, when moving down the wealth distribution, at some positive wealth level $w^{\star\star} < w*$ the short-selling constraint on foreign bonds will start to bind. Individuals with wealth $w < w^{\star\star}$ therefore only have positive investments in home bonds, plus a constant short position in shares.

2.4.3 The home bias of the poor

Up to this point I have considered portfolio shares as a fraction of total net wealth, including endowment wealth. This section maps the results into financial portfolio shares, in order to compare them with the stylised facts of section 2. Note that financial portfolio shares, denominated as a fraction of gross assets, do not sum to one when agents have positive and negative asset positions.

Proposition 2.2. *The poorest investors with positive gross assets hold home bonds only. Across the wealth distribution, the financial portfolio share of home bonds falls, while those of shares and foreign bonds rise, converging to the portfolio shares of the diversified portfolio. So poorer agents have both stronger home bias, and a stronger bias in favour of safe assets.*

Proof

Agents with low wealth $w < w^{\star\star}$ are constrained by both the short-selling constrained for foreign bonds, and the borrowing limit for shares. But from (2.21), there are investors with $w < w^{\star\star}$ that hold positive amounts of home bonds. Their portfolio share of home bonds is thus 1.

Multiplying all portfolio shares in (2.19) - (2.21) by the ratio of total to gross financial assets $\frac{\widetilde{e'_h} Y_H + a_{h,F} + a_{h,H} + a_{h,S}}{a_{h,F} + a_{h,H}}$, and imposing $\widetilde{a_{h,S}} = -\delta_H \widetilde{e'_h}$, we get an expression for the share of IOUs in the financial portfolio of investors with $w : w^{\star\star} \leq w \leq w^{\star}$, who have unconstrained holdings of home and foreign bonds but are borrowing constrained in shares.

$$\widetilde{a_{h,F}}^{fin} = \frac{-\Delta_r}{2\sigma V} + \frac{\frac{1}{2} + (1-\theta)(\sigma-1)}{\sigma} + [\frac{-\Delta_r}{2\sigma V} + \frac{\frac{1}{2} + (1-\theta)(\sigma-1)}{\sigma} - \frac{1}{2}](1-\delta_H)\widetilde{e'_h}^{fin} \quad (2.22)$$

$$\widetilde{a_{h,H}}^{fin} = \frac{\Delta_r}{2\sigma V} + \frac{\frac{1}{2} + \theta(\sigma-1)}{\sigma} + [\frac{\Delta_r}{2\sigma V} + \frac{\frac{1}{2} + \theta(\sigma-1)}{\sigma} - \frac{1}{2}](1-\delta_H)\widetilde{e'_h}^{fin} \quad (2.23)$$

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where $\widetilde{e'_h}^{fin} = \frac{\overline{e'_h} Y_H}{a_{h,F}+a_{h,H}+a_{h,S}}$ falls as gross asset holdings rise. From $\theta > \frac{1}{2}$ and assumption 2.1, the portfolio share of home bonds thus falls, while that of foreign bonds rises, with gross assets.

Equivalently, for $w > w^{\star}$, the financial portfolio shares of unconstrained investors are

$$\widetilde{a_{h,F}}^{fin} = \frac{r_f - r_S}{\sigma V} + \frac{\frac{1}{2} + (1-\theta)(\sigma - 1)}{\sigma} \tag{2.24}$$

$$\widetilde{a_{h,S}}^{fin} = 2\frac{r_S + y - \frac{1}{2}(r_b + r_f)}{\sigma V} - \widetilde{e'_h}^{fin} \tag{2.25}$$

$$\widetilde{a_{h,H}}^{fin} = \frac{r_h - r_S}{\sigma V} + \frac{\frac{1}{2} + \theta(\sigma - 1)}{\sigma} \tag{2.26}$$

So for unconstrained investors, the portfolio share of shares rises, while the others are constant. For large wealth levels, as $\widetilde{e'_h}^{fin}$ goes to zero, all portfolio shares are thus equal to those in the diversified portfolio. ■

Proposition 1 shows that this simple economic environment is able to replicate the observed structure of individual asset holdings across the wealth distribution: Poor individuals do not participate in the markets for foreign or risky assets. And even beyond the value of wealth that makes participation worthwhile, the portfolio shares of foreign and risky assets continue to increase as wealth rises.

2.5 Conclusion

In this chapter I have shown that, according to the Survey of Consumer Finances, wealthier US Households invest a higher share of their portfolio in international assets. This result continues to hold when I take account of the fact that poorer households are less likely to participate in more sophisticated financial markets.

Fixed costs of participating in foreign asset markets do not explain the rising portfolio shares for participants. So I constructed a simple two country model with incomplete markets and income heterogeneity that can account for this finding. Agents in the model receive stochastic endowments of a country-specific tradable good which are affected by idiosyncratic and country-specific shocks. Agents are prevented from access to a complete set of asset markets but can trade in riskless assets and in equity. Assuming log-normal returns, I derived asset portfolios as a function of total investor wealth. Poorer individuals' consumption is mainly determined by endowment income. Relative to richer individuals, they therefore have a bias against equity, which has real payoffs that co-move strongly with individual endowments. But poorer home agents also have a relative bias in favour of home vs. foreign bonds, since home bonds are a good hedge

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against aggregate volatility in the supply of home goods, which have a stronger weight in their consumption.

With regards to policy this study implies that the welfare loss from poorer households' non-participation in sophisticated financial markets may be less important than thought. In future research it would be interesting if this result also holds in different environments. Particularly, one should try to relax the assumptions of unit-elastic preferences, and explore how the model deals with shocks to demand, rather than the supply shocks to endowments this study has looked at.

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Chapter 3

Stationary equilibrium distributions in economies with limited commitment

Abstract[1]

Limited commitment to contracts can explain imperfect risk sharing even when individuals have access to complete insurance markets. Past contributions have focused on the resulting cross-sectional distribution of consumption (Cordoba 2008, Krueger and Perri 2006). In contrast, this paper looks at the joint dynamics of income, consumption and wealth implied by the asymmetric nature of partial insurance under limited commitment, where negative income shocks are largely insured but positive shocks can lead to large rises in consumption. A theoretical section proves the existence and uniqueness of equilibrium in a limited commitment continuum economy where incomes follow a standard markov process, and solves analytically for the joint equilibrium distribution of consumption, income and wealth. Building on Krueger and Perri (2005), I show that individual consumption follows, at least locally, a left-skewed geometric distribution. Also, the conditional distributions of consumption and wealth are highly non-linear and have a characteristic form of heteroscedasticity, with declining conditional variances as income increases. In a quantitative part, the paper compares the exact distributions in the Krueger and Perri (2006) model to non-parametric estimates of their counterparts in US micro-data, and in a simple Ayagari economy.

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3.1 Introduction

The economist's toolbox has two classical ways of modelling the relation between individual incomes and consumption: on the one hand, the assumption of complete insurance markets is especially convenient for macro-economists, as it provides a rationale for their customary focus on a "representative" consumer. On the other, the permanent income hypothesis, that individuals smooth consumption of their expected lifetime resources by simple saving and borrowing, is appealing as it puts minimal requirements on the assets and information available to individuals. However, empirically, there is evidence against both perfect risk-sharing (see e.g. Attanasio and Davis 1996) and simple self-insurance (see e.g. Hall and Mishkin 1982). Moreover, conceptually, the permanent income hypothesis lacks a micro-foundation for the absence of assets other than non-contingent bonds, while the complete markets model requires enforcement of very large and persistent net transfers between individuals, as well as detailed public information on individual contingencies. More recent alternatives to the classical benchmarks, on the other hand, do not restrict asset markets a priori, but take seriously the information and enforcement problems of the complete markets model. Particularly, a growing literature has looked at economies with "limited commitment", where individuals have the option to "default" on contracts. As long as default is unattractive, for example because it leads to exclusion from financial trade in the future, this setup allows for some, but not perfect, risk-sharing even against very persistent shocks to income.

Two recent papers analyse the implications of limited commitment for the cross-sectional distribution of agents in an economy with many agents. Krueger and Perri (2006) show that the model can help reconcile the substantial rise in US income inequality over the last 25 years with the more stable inequality of consumption. Cordoba (2008) concludes, however, that the model captures the concentration of wealth at the top of the distribution less well than a simple Ayagari self-insurance economy. This paper takes a different strategy. Rather than concentrating on particular moments of marginal distributions, it analyses, both theoretically and in a calibrated version of the model, the non-parametric characteristics of the joint distribution of consumption, wealth and income under limited commitment. Particularly, I show how the asymmetry of insurance under limited commitment, where negative income risks are pooled but positive shocks lead to idiosyncratic rises in consumption if participation constraints bind, implies a characteristic form of non-linearity and heteroscedasticity of the joint distributions. The main theoretical contribution of the paper is to prove existence and uniquencess of a stationary equilibrium in a continuum economy with limited commitment to contracts, and to provide an analytical characterisation of the distribution of consumption, income and financial wealth, including a closed form solution for an example with two income states and CRRA preferences. The theory shows how the asymmetric nature of insurance implies

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declining conditional variances of wealth and consumption along the income distribution, and a negative relationship between wealth and income on average. The quantitative part of the paper looks at an economy with capital and a more general income process, to confront the joint equilibrium distribution, and its characteristic form of non-linearity and heteroscedasiticity, with the data. For this, I calculate the exact joint distributions in the Krueger and Perri (2006) calibration of the model, and compare them to non-parametric estimates of their counterparts from US micro-data, and to those from a simple Ayagari economy. The results show that, even with a more realistic income process featuring both near-permanent and transitory shocks, the limited commitment economy still produces very asymmetric joint distributions: consumption growth has a floor slightly below zero, but an upward tail that becomes more important for stronger positive income shocks. And both the mean and variance of wealth fall with income. Both the data and the Ayagari model produce less heteroscedastic distributions, and mean wealth that rises with income.

This work contributes to a large literature that analyses insurance contracts with limited commitment. In early work, Thomas and Worrall (1988) looked at self-enforcing long-term contracts between a firm and a risk-neutral worker, when both can costlessly renege on past commitments to take advantage of random fluctuations in the price of labour. In equilibrium, wages can fluctuate, but only to remain within a time-varying interval of values that satisfies participation constraints of both parties. Kehoe et al (1993) prove the first welfare theorem in an endowment economy with complete markets where participation-constraints on consumption sets prevent default. Competitive equilibria are thus constrained efficient, but may feature less than perfect risk sharing unless discount factors are high enough. Kocherlakota (1996) shows that, with a finite number of agents, relative marginal utilities are a sufficient description of the state of the economy, and equilibrium contracts have "amnesia": constrained agents' consumption is independent of past income realisations. Ligan, Thomas and Worrall (1998) show how this implies asymmetry in the consumption paths of participation-constrained and unconstrained individuals: all unconstrained agents share (in a marginal utility sense) the same drop in consumption, while constrained agents experience relative consumption increases depending on their individual income realisations. Alvarez and Jermann (2000) prove the second welfare theorem and consider asset pricing.
In a similar manner to the present paper, Krueger and Perri (2005) are interested in participation constrained risk sharing in large western economies, and thus look at a setting with a continuum of agents who receive finite income realisations according to an identical Markov process. They use a dual method *à* la Atkeson and Lucas (1992, 1995) to show that, for any given interest rate, there exists a unique stationary consumption distribution, and that aggregate excess demand for consumption increases in interest

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rates. And, based on a conjecture about the existence of a market clearing interest rate, they characterise the consumption distribution for the special case with 2 iid income values. Krueger and Uhlig (2006) analyse a similar economy with the difference that agents can costlessly switch between competitive insurance providers that are risk-neutral and at least as patient as the agents themselves. Rather than autarky, the outside option in this setting thus consists of contracts that break even in expectation over their lifetime, and which any insurance provider is ready to offer. In equilibrium, however, agents never switch as they make initial net payments in exchange for insurance transfers in the later life of the contract. Despite this difference in the outside option, the authors show that, with i.i.d. transitions on two income states, the structure of the joint consumption and income distribution is the same as with exogenous outside options. Finally, Thomas and Worrall (2007) analyse the same setup but interpret the two i.i.d. income states as working vs. unemployment. They give an identical characterisation of the steady-state consumption distribution relative to Krueger and Perri (2005) or Krueger and Uhlig (2006), but provide an example where they can prove existence of a stationary equilibrium, and another one where they show convergence.
Relative to this literature, the theoretical contribution of the present paper is three-fold: First, I am able to show the existence of a unique stationary equilibrium in a limited commitment continuum economy with standard markov uncertainty, under standard assumptions. Second, I provide a closed form for the stationary distribution of consumption, income and wealth with two persistent income values and CRRA preferences. The marginal distribution of consumption is a left-skewed geometric, and the conditional variances of both consumption and wealth decline with income. Third, I characterise analytically the joint distribution for an N-state markov income process. The geometric nature of consumption continues to hold, but only locally. And with i.i.d. uncertainty, both consumption and wealth still see their conditional variances decline with income.

The empirical literature has tested the implications of limited commitment models using, for example, data on consumption and income in rural villages (Townsend 1994, Ligan et al 1998, Eozenou 2008), or from experimental settings (Barr 2008, Albarran 2003). More directly relevant to this paper is the work of Krueger and Perri (2006), who analyse the performance of the limited commitment model, relative to more standard incomplete markets models, in explaining why consumption volatility has increased much less than income risk in the United States over the last 30 years. They find that incomplete market models have too limited risk sharing, while the limited commitment model slightly underpredicts the change in consumption volatility implied by the observed rise in income risk. However, they focus mainly on the relative change in inequality measures. Cordoba (2008) uses numerical simulations to argue that models with - in his case - exogenous, debt-constraints can potentially reproduce key features of the cross-sectional

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distribution of consumption, but capture the wealth distribution much less well than simple incomplete markets models.

In its quantitative section, this paper looks at the shape of joint, rather than marginal, distributions. This is because the non-linear, heteroscedastic shape of the distributions results directly from the asymmetric nature of insurance under limited commitment. It is thus more robust to changes in the calibration or specification of the model than, for example, the shape of right hand tails of marginal distributions. Particularly, I compare the joint densities of consumption, wealth and income in the Krueger and Perri (2006) limited commitment economy to, on the one hand, non-parametric estimates of its counterparts in US micro-data, and, on the other, the distributions in a simple self-insurance economy. The results show that the data does not reproduce the floor in consumption growth or the declining conditional variances of consumption and wealth at higher income values that the limited commitment model predicts. Rather, the shape of the distributions in the data, where mean consumption inceases more or less linearly with income, and wealth increases, rather than falls, as income rises, seem more in line with the distributions from the simple Ayagari self-insurance economy.

Section 3.2 describes the environment of a continuum economy with debt-constrained domestic financial markets. Section 3.3 derives some characteristics of dynamic equilibria on the basis of the associated planner's problem. Section IV gives the analytical characterisation of the stationary joint distribution of consumption, income and wealth, and proves the existence and uniqueness of equilibrium. Section 3.4 reports the results from a calibration of the model to the US economy and compares them to those from a simple self-insurance economy, and US micro-data. An appendix contains most proofs.

3.2 A continuum economy with debt-constrained complete financial markets

This section presents a simple economy with complete asset markets where insurance against idiosyncratic income shocks is constrained by individual default, and defines the competitive equilibrium.

3.2.1 Agents, countries, time

The economy consists of a large number of individuals of unit mass. Individuals are indexed by i, located on a unit-interval $i \in I = [0, 1]$ with Sigma-Algebra $\mathbb{I}$. Denote as $\Phi_I : \mathbb{I} \to [0,1]$ the (constant) non-atomic measure of individuals. Time is discrete $t \in \{0,1,2,...,\infty\}$ and a unique perishable endowment good is used for consumption.

3.2.2 The endowment process

The consumption endowment of agent i in period t, $z_{i,t}$, takes values in a finite set Z: $z_{i,t} \in Z = \{z^1 > z^2 > ... > z^N\}, N \geq 2$. Let $\mathbb{Z}$ be the power set of Z, and denote as $\Phi_{Z,t} : \mathbb{Z} \to [0,1]$ the measure of agents at all (subsets of) income realisations in period t. Endowments follow a Markov process that is independent of i, and $\mathbb{I}$-measurable (i.e. $\{i : z_{i,t+1} = z^k | z_{i,t} = z^j\} \in \mathbb{I},\ \forall z^j, z^k$). Specifically, it is described by a Markov transition matrix F that has strictly positive entries $f_{i,j} > 0, \forall i, j$, is monotone (in the sense that the conditional expectation of an increasing function of tomorrow's income is itself an increasing function of today's income), and has a unique ergodic distribution $\Phi_Z : \mathbb{Z} \to [0,1]$. Thus, in the long-run, aggregate (or average) income $Y = \int z_i d\Phi_I$ is constant, while individual income fluctuates. Let $Z_0 : I \to Z$ be a measurable function that assigns all individuals an initial income value. Also, let s_t denote the state of the economy in period t, a vector containing individual incomes and asset holdings of all agents.

3.2.3 Preferences

Agents live forever and order consumption sequences according to the utility function

$$U = E_{s_0} \sum_{0}^{\infty} \beta^t u(c_{i,t}) \tag{3.1}$$

where E_{s_0} is the mathematical expectation conditional on s_0, $0 < \beta < 1$ discounts future utility, $c_{i,t}$ is consumption by agent i in period t, and $u : R^+ \to R$ is an increasing, strictly concave, twice-continuously differentiable function that satifies Inada conditions and is identical for all agents in the economy.

3.2.4 Asset markets

Agents engage in sequential trade of a complete set of state-contingent bonds. Individual endowment realisations are verifiable and contractable, but asset contracts are not completelely enforceable: at any point, individuals can default on their contractual payments at the price of eternal exclusion from financial markets. Thus the total amount an agent can borrow today against any income state tomorrow is bounded by the option to default into financial autarky. There, consumption is forever equal to income. Given the

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markov structure of income, the value of default as a function of the vector of current income z can be written as

$$W(z) = \sum_{t=0}^{\infty} (\beta F)^t U(z) = (I - \beta F)^{-1} U(z) \tag{3.2}$$

Note that the monotonicity of F implies monotonicity of $W(z)$ (Dardanoni 1995).
I denote holdings of Arrow-Debreu securities paying off in state s_t by $a(s_t)$. In any state s_t, $V(z(s_t), a(s_t))$ is the contract value as a function of income $z(s_t)$ and current asset holdings $a(s_t)$. As in Alvarez and Jermann (2000), individual i's participation constraint for any state s_{t+1} tomorrow can be written as a portfolio constraint on the claims she can issue against s_{t+1} income.[2] This borrowing constraint is "not too tight" in the words of Alvarez and Jermann (2000) if it assures participation but does not constrain contracts otherwise

$$a_i(s_{t+1}) \geq A_i(s_{t+1}) = min\{\alpha(s_{t+1}) : V(z_i(s_{t+1}), \alpha(s_{t+1})) \geq W(z_i(s_{t+1}))\} \tag{3.3}$$

3.2.5 Limited insurance

To focus on the interesting case of limited insurance, I make the following assumptions about the endowment process and preferences:

Assumption 3.1.

$$W(z^1) > \sum_{0}^{\infty} \beta^t u(Y) \tag{3.4}$$

Assumption 3.2.

$$\frac{u'(z^1)}{\beta u'(z^N)} < 1 \tag{3.5}$$

Assumption 1 assures that full insurance is not possible, since the autarky value at high income exceeds that of consuming average income in the economy forever. Assumption 2 implies that there is no positive net interest rate that would implement the autarky equilibrium, as the marginal rate of substitution between the highest and lowest income state is too low. Alvarez and Jermann (2000) show that this is sufficient to rule out autarky as an equilibrium.

[2] An alternative is to restrict choices directly, by requiring that the chosen consumption sequence fulfill participation constraints, as in Kehoe and Levine (1993).

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3.2.6 The household's problem

Every period, households maximise their expected utility by choosing current consumption and assets subject to budget and participation constraints

$$\begin{aligned} V(z(s), a(s)) = max_{c,\{a(s')\}}\{u(c) + \beta E_s V(z', a(s'))\} \\ s.t.\ c + \sum_{s'} a(s')q(s') \leq a(s) + z(s) \\ a(s') \geq A(s') \\ A(s') = min\{\alpha(s') : V(z(s'), \alpha(s')) \geq W(z(s'))\} \end{aligned} \tag{3.6}$$

where c, a' are policy functions of the state variables $(z(s), a(s))$.

3.2.7 Definition of competitive equilibrium

The competitive equilibrium in this economy is a set of asset prices $q(s')$, a set of individual decision rules $c(z, a), a'(z, a)$ with associated value functions $V(z, a)$ such that

1. $V(z, a)$ are the households maximum value functions associated with the household problem given $q(s')$
2. $V(z, a)$ is attained by $c(z, a), a'(z, a)$
3. Markets for state-contingent assets clear
   $\int a_i(s')d\Phi_I = 0, \forall s'$

The competitive equilibrium is called "stationary" if the distribution of individual consumption is stationary through time.

3.3 Efficient allocations

Alvarez and Jermann (2000) show that a version of the first welfare theorem applies to this economy as long as interest rates are "high", in the sense that today's market value of total future resources is finite.[3] This allows me to focus on participation-constrained efficient allocations, where the assumption of some risk sharing assures that

[3]An additional technical condition requires that for all i, there is a constant ζ_i such that for all z_t, $|u(c_{i,t}(s_t))| < \zeta_i(u'(c_{i,t}(s_t)))c_{i,t}(z_t))$. Note that this is a joint condition on utility and the equilibrium allocation. It is met in most relevant cases, for example if relative risk aversion is different from 1 at zero, or if consumption is uniformly bounded away from zero, which is the case in the setting of this paper.

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the interest rate condition is met. More particularly, I exploit the results in Marcet and Marimon (2009), and focus on the solution to the participation-constrained social planner's problem.

3.3.1 The planner's problem

Marcet and Marimon (2009) show how the efficient allocation solves the following planner's problem. For a given measurable assignment of welfare weights to individuals $\mu_0 : I \rightarrow R^+$ in a linear social welfare function $\Omega = \int \mu_i \sum_0^\infty \beta^t u(c_{i,t}) d\Phi_I$ the problem of the planner is to distribute resources optimally subject to individuals' participation constraints and the aggregate resources of the economy

$$\begin{aligned} \mathbb{V}\mathbb{V}(\mu_0, Z_0) = max_{\{c_i(s_t)\}} E_0 \int \mu_i \sum_{t=0}^{\infty} \beta^t u(c_i(s_t)) d\Phi_I \\ s.t. \int c_i(s_t) d\Phi_I \leq \int z_i(s_t) d\Phi_I, \ \forall s_t \\ V_i(s_t) \geq W(z_i(s_t)), \ \forall s_t, i \end{aligned} \tag{3.7}$$

where the planner's maximum value $\mathbb{V}\mathbb{V}$ is a function of the initial measure of weights and income induced by μ_0, Z_0. I assume that the initial weighting function μ_0 is measurable and takes a finite number of finite, positive values $\mu_1, ..., \mu_k$ with $\Phi_I(i : \mu_{i,0} = \mu_k) > 0, \ for \ k = 1, ..., K$ and $\Phi_I(\{i : \mu_i \not\subseteq \{\mu_1, ..., \mu_K\}\}) = 0$.
Note that this problem is non-standard, because the participation-constraints in (3.7) introduce history dependence. Intuitively, the planner provides value to individuals who have attractive outside options by promising them high consumption today and in the future. But this requires him to honour promises made in the past, making the problem non-recursive. As a solution, this section applies a technique proposed by Marcet and Marimon (2009) that makes the problem recursive. Their results, however, do not apply to continuum economies in general, as they focus on an environment with a finite number of agents. But with a finite number of income values and a discrete initial distribution of planner weights, we can always replace integration over an infinity of indivuals i by summation over a countable number of sets of individuals that share all relevant characteristics. In particular, in any period t, we can split the uncountable set I into KN^t sets of individuals $I_{\mu_0,\{z\}}$ that share initial weight μ_k and income history $\{z_0, z_1, ..., z_t\}$. This ensures the countability of the planner's state space. A later section shows that this space remains, in fact, strictly finite.
Marcet and Marimon (2009) show how to capture the history dependence of the problem by an individual-specific summary variable. Particularly, they show that, denoting γ_i the Lagrange multiplier on i's participation constraint in the sequential problem (3.7),

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we can write the latter as

$$\mathbb{VV}(\Phi_{\mu,z}) =$$
$$\min_{\gamma_{jl}\geq 0}\max_{\{c_{jl}\}} \sum_{\mu_j\in\bigvee^t, z_l\in Z} \Phi_{\mu,z}(\mu_j,z_l)[(\mu_j+\gamma_{jl})u(c_{jl})-\gamma_{jl}W_z]+\beta E[\mathbb{VV}(\Phi_{\mu',z'})] \tag{3.8}$$
$$s.t. \sum_{\mu_j\in\bigvee^t, z_l\in Z} \Phi_{\mu,z}(\mu_j,z_l)[c_{jl}-z_l]\leq 0 \tag{3.9}$$
$$\mu_i' = \mu_i+\gamma_i\ \forall i \tag{3.10}$$
$$\Phi_{\mu,z}: \bigvee^t \times Z \to [0,1] \tag{3.11}$$
$$\bigvee^t = \{\mu:\Phi_I(i:\mu_{i,t}=\mu)>0\} \tag{3.12}$$

where I write x_{jl} for the function $x(\mu_j,z_l)$. Note that the weights of individuals in the social welfare function are now updated every period to meet participation constraints, according to the law of motion (3.10). Intuitively, by increasing individual weights μ_i the planner allocates a higher than expected consumption path to individuals with binding participation constraints, to keep them "happy" with the contract. Policies c_{jl},γ_{jl} are a function of planner weights at the beginning of the period and current income realisations only, so do not depend on past state variables. In other words, the time-varying individual weights now summarise history-dependence of the problem. Importantly, the cardinality of the set of individual planner weights with positive mass $\bigvee^t$ increases by a factor of at most N every period, and therefore remains countable. Equivalently, the integration across individuals along measure Φ_I is replaced by the weighted summation over (the Euclidean product of) the set of current income realisations Z and the time-varying set of planner weights with positive mass $\bigvee^t$, where the weights have discrete measure $\Phi_{\mu,z}$.

With discrete $\bigvee^t$ and Z, the state space is finite and bounded, and thus compact, for all t. And Tychonoff's theorem ensures that it remains compact even for a countably infinite number of periods. With concave utility and finite resources, and in the absence of aggregate state variables entering the participation constraints, the constraint set is therefore compact and convex. It is also non-empty since autarky is trivially feasible and incentive-compatible. Marcet and Marimon (2009) show how this is sufficient to ensure the equivalence of the sequential problem (3.7) and the transformed problem (3.8).[4] In particular, the planner's value function is single valued and, given continuously differentiable utility, differentiable. And finally, Inada conditions and concavity of the utility function imply that, to characterise the optimum, participation constraints and the first order conditions suffice.

[4] In other words, the problem fulfills conditions A1 to A5 in Marcet and Marimon (2009). For further detail, see also the proof of existence and uniqueness of a solution to this problem in Broer(2009a), who considers an economy facing a given world interest rate.

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3.3.2 Properties of efficient allocations

Although this paper is mainly concerned with the stationary joint distribution of consumption, income and wealth, the rest of this section shows two features of any consumption allocation with limited commitment: first, there is asymmetry in insurance, as the planner insures consumers against drops in income, while accomodating rises in income with potentially strong consumption increases. Thomas et al (1998) show this in an environment with a finite number of agents, while I analyse the implications for the stationary joint distributions in a continuum economy. Relatedly, contracts feature "amnesia" (Kocherlakota 1996), as history dependence of individual consumption is cut off once participation constraints bind. Throughout, I denote as "continuation value" $V(\mu_t, z_t)$ the utility that an individual with current weight μ_t and income z_t can expect under the planners consumption allocation, as opposed to the autarky value $W(z_t)$ she gets from consuming her income stream from today onwards.

It is easy to see that the solution of the planner's problem defines an operator Γ that maps today's distribution of individual weights and current income into a distribution of weights and income tomorrow.[5] The next Lemma summarises some old and new results that characterise Γ.

Lemma 3.3. *The planner's decision rule Γ has the form*

$$\mu_{i,t+1} = max\{\mu_{t+1}(z_{i,t+1}), \mu_{i,t}\}$$

For every t, $\mu_t(z)$ is strictly increasing in z, and for every z, the sequence $\mu_t(z)$ increases strictly over time. Also, the set of individual planner weights with positive mass is strictly finite: $|\{\mu_j : \Phi_I(\{i : \mu_{i,t} = \mu_j\}) > 0\}| < \infty, \forall t$.

That individual weights increase when participation constraints bind but are constant otherwise is well-known from Marcet and Marimon (2009), and follows directly from the equivalence of γ and the Lagrange multipliers of the untransformed planner's problem. Also, since the outside option of autarky only depends on current income, planner weights of individuals with binding participation constraints $\mu_t(z_{i,t})$ are, for any t, equally a function only of their current income $z_{i,t}$. This lack of history dependence in consumption of constrained individuals is well-known as the "amnesia" property of

[5]Or formally $\Gamma : (Z \times R_+^{KN^t}, \mathbb{Z} \times \mathbb{B}^{KN^t}) \rightarrow [0, 1]$, where B^n is the Borel algebra of the n-dimensional positive Euclidean space, and the cardinality of the set of welfare weights, equal to K in period 0, increases by N every period.

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consumption allocations with limited commitment (since Kocherlakota 1996).[6]
On the other hand, that $\mu_t(z_{i,t})$, the minimum planner weight that ensures participation of individuals with income $z_{i,t}$, is strictly increasing in both income and time has not been shown before. But this result is very useful for showing existence and uniqueness of a stationary solution to (3.8), and to compute it efficiently using first order conditions. It is proved in the appendix, along with its implication that $\bigvee^t$, the set of planner weights $\mu_{i,t}$, is not only countable but strictly finite.

Lemma 1 has immediate consequences for the dynamics of the joint distribution of consumption and income. To see this, note that, for λ the Lagrange multiplier associated with the ressource constraint (3.9), the planner's intratemporal optimality condition equates weighted marginal utilities across agents, $\lambda = (\mu_i + \gamma_i)U'(c_i)\forall i$. From this, relative consumption is monotonously increasing in planner weights

$$\frac{U'(c_{i,t})}{U'(c_{j,t})} = \frac{\mu_{j,t} + \gamma_{j,t}}{\mu_{i,t} + \gamma_{i,t}} \tag{3.13}$$

So the current distribution of planner weights maps monotonously into current consumption. There are thus N minimum participation-compatible consumption values $c^i_{0,t}, i = 1, ..., N$ that correspond to the minimum planner weights $\mu_t(z)$ and are increasing in income. From this, it is easy to see that the highest income earners have highest consumption, while those with lowest consumption have necessarily the lowest income level. This lowest consumption level, since it solves the participation constraint at minimum income with equality, is easily seen to be constant through time, and equal to z^N. So there is a constant lower bound of consumption equal to minimum income.
Intratemporal optimality on the other hand requires growth rates of marginal utility to equal relative growth rates of planner weights, discounted and adjusted for changes in the planner's marginal value of resources λ

$$\frac{U'(c_i)}{U'(c'_i)} = \frac{\mu'_i + \gamma'_i}{\mu'_i}\frac{\lambda}{\lambda'} \ \forall i \tag{3.14}$$

This immediately implies that all unconstrained agents, who have constant planner weights, share the same growth rate of marginal utility, equal to the change in the discounted marginal value of resources to the planner, which can be used to define the interest rate prevailing in competitive equilibrium as $\frac{\lambda}{\beta\lambda'} = R$. The result is a convenient law of motion for consumption of unconstrained agents as a function of equilibrium

[6] To see this formally, consider two agents i, j with different weights $\mu_{i,t} \neq \mu_{j,t}$ who receive a same income shock $z_{i,t+1} = z_{i,t+1}$ that implies autarky values higher than their continuation utility at current weights. With equal income today, they face the same conditional measures over future income realisations. So if $\mu_{i,t+1}$ is the minimum weight that meets i's participation constraint, it is also the minimum weight that meets j's participation constraint. And since continuation values $V(\mu_t, z_t)$ are strictly increasing in μ_t, the cutoff $\mu_t(z_{i,t})$ is unique.

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interest rate R

$$U'(c_i) = \beta R U'(c_i') \tag{3.15}$$

Equations (3.15) and (3.14) show two important characteristics of consumption transitions in limited commitment economies: discreteness and asymmetry. This is because, unless $R\beta = 1$ and insurance is perfect, all unconstrained agents share common, discrete falls in marginal utility over time, independent of their current level of income. Agents with binding participation constraints after a positive income shock, on the other hand, experience jumps in consumption to a level that is specific to their current income.

3.4 Existence and uniqueness of a stationary equilibrium and its distributional characteristics

This section provides an analytical characterisation of the joint distribution of consumption and income. As in Krueger and Perri (2005), I concentrate on stationary consumption distributions.[7] The fact that stationarity of the consumption distribution implies a constant interest rate in the economy and vice versa conveniently means that we can index different stationary distributions by the value of R.[8] Krueger and Perri (2005) show that excess demand is increasing in R for $R > 1$, and conjecture the existence of a market clearing value $R^\star$. Using a different method, I am able to prove the existence of a unique market clearing interest rate. To do this it turns out to be convenient to first characterise the stationary consumption allocation for a given R, and then to exploit its characteristics to show market-clearing at a particular unique value $R^\star$.

3.4.1 The stationary distribution of consumption and income

For the case of two income values and i.i.d. transitions, Krueger and Perri (2005) show that the stationary efficient allocation under participation constraints features a consumption distribution with a discrete number of support points and derive the corresponding frequency mass function. Krueger and Uhlig (2006) show similar results in an environment where risk-averse agents can choose between risk-neutral insurance

[7]Note that limited commitment economies also admit non-stationary pareto-inefficient equilibria, where a path of decreasing interest rates confirms expectations of ever tighter borrowing limits, leading to convergence to autarky. See Bloise et al (2009).

[8]To see this, look at any minimum participation compatible consumption value c_0^i and that corresponding to the first unconstrained transition away from it c_1^i. We have $R\beta = \frac{u'(c_0^i)}{u'(c_1^i)}$. Stationarity implies that this is a constant. The converse is proved by the construction of the stationary distribution in the appendix.

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providers. Thomas and Worrall (2007), moreover, provide examples where they can show existence and convergence to this stationary distribution. This section generalises the previous contributions in several directions: first, it considers the general case of N income values with persistent, rather than i.i.d., transitions. Second, it derives a closed form for both the frequency mass and the support for the joint distributions of consumption, income and financial wealth in the case of two persistent income states when agents have constant relative risk aversion, which allows me to express the variance of log-consumption as a function of the interest rate R. And third, I focus explicitly on the joint distribution of consumption and income, which also allows me to derive the distribution of wealth and financial income.[9]

Proposition 3.4. *For $1 < R < \frac{1}{\beta}$ the interest rate in stationary equilibrium, the joint distribution of income and consumption $\Phi_C : \mathbb{C} \times \mathbb{Z} \longrightarrow [0, 1]$ has the following features:*

1. *Φ_C is discrete, with positive mass at consumption values between minimum income and some upper bound c_0^1 smaller than the highest income level: $\mathbb{C} \subseteq [y_N, c_0^1]$, $c_0^1 < z^1$.*

2. *There are N minimum levels of consumption c_0^i, $i = 1, ..., N$ under which consumption of agents with income i never falls and where participation constraints at income z^i hold with equality. These threshold levels are constant through time and increasing in income $c_0^1 < c_0^2 < .... < c_0^N$. The lower bound of the distribution is minimum income $c_0^N = z^N$.*

3. *Every consumption threshold c_0^i is an upper bound to a geometric subdistribution of consumption Φ_C^i, with support $\{c_j^i\}$ recursively defined by the law of motion $U'(c_{j+1}^i) = (\beta R)^{-1} U'(c_j^i), j = 0, 1, 2, ...,$ and bounded below by z_N. Φ_C is thus a mixture of $N - 1$ geometric distributions. The appendix contains an analytical expression for the frequencies in this distribution.*

4. *Individuals at the highest income level z^1 all have maximum consumption level c_0^1. The support of consumption conditional on income $z^i < z^1, i > 1$ is $[c_0^i, c_1^1]$. So the support of consumption narrows as income rises. For i.i.d. transitions (identical rows in F), this implies that the conditional variance of consumption falls monotonously in income.*

The proof of proposition 3.4 is by construction of the stationary distribution, and can be found in the appendix. The joint distribution of financial returns and income follows as a corollary.

[9]For the two income i.i.d. case, the joint dynamics of consumption and income are also contained in Krueger and Perri (2005) and Krueger and Uhlig (2006).

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Corollary 3.5. *The joint distribution of net financial returns and income* $\Phi_{y_{fin},z}$: $\mathbb{B}([c_0^1 - z^1, c_1^1 - z^N]) \times \mathbb{Z} \longrightarrow [0,1]$ *has the following features:*

- $\Phi_{y_{fin},z}$ *is discrete, and transfers are bounded above and below by* $c_1^1 - z^N, c_0^1 - z^1$ *respectively.*
- *Individuals at minimum income have positive financial returns* $y^N_{fin,0} \geq 0$. *All individuals at the highest income level* z^1, *and participation-constrained individuals at income* $z^i > z^N$ *have negative financial returns* $y^i_{fin,0} \leq 0$, *with strict inequality for* $i = 1$.
- *To the geometric consumption distribution with upper bound* c_0^i *corresponds a distribution that consists of a mass point at* $y^i_{fin,0} \leq 0$, *plus a support* $c_j^i - y_k, k = 1, ...i-1; j = 1, 2, ...$ *The frequency distribution follows from that of the joint distribution of consumption and income, which can be found in the appendix.*

Proposition 3.4 and its corollary show how the asymmetric nature of partial insurance under limited commitment affects the joint cross-sectional distribution: High income individuals have a narrow distribution of consumption, as their minimum participation-compatible consumption level is binding. They also have low financial returns, as they are making net contributions into the insurance scheme. Low income earners, on the other hand, receive net payments from insurance claims, but have a variety of consumption values that decline with the length of their low income spell.

This section has provided a general characterisation of joint distributions under limited commitment. Previous contributions, on the other hand, have focused on a particular example, with 2 income values and i.i.d. transitions. I now turn to a similar example with 2 incomes, but assume persistence in income and preferences that have constant relative risk aversion (CRRA). This allows me to describe the joint distributions in closed form, as an illustration of the more general results above.

3.4.2 A closed form example

A simplified version of the economy, with CRRA preferences $u = \frac{c^{1-\sigma}}{1-\sigma}$, two income values $\{z^h, z^l\}$ and transition matrix $F = [p, 1-p; 1-q, q]$, yields a closed form solution.

Proposition 3.6. *With* $N = 2$ *and CRRA preferences, and for* $1 < R < \frac{1}{\beta}$ *the interest rate in stationary equilibrium, denote the joint distribution of income and consumption*

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$\Phi_C : \mathbb{C} \times \{z^l, z^h\} \longrightarrow [0, 1]$. *The discrete support of consumption* $\mathbb{C}$ *is*

$$c_1 = f(p, q, m, W_h, W_l) \tag{3.16}$$

$$c_i = c_1(\beta R)^{\frac{i}{\sigma}}, 1 < i < m$$

$$c_m = z^l \tag{3.17}$$

$$for\ m = min\{x \in N : x > \frac{\sigma[ln(z^l) - ln(c_1)]}{ln(\beta R)}\} \tag{3.18}$$

The frequency mass function is geometric, given by

$$\Phi_C(c_1, z_h) = \frac{1-q}{2-q-p} \doteq \nu \tag{3.19}$$

$$\Phi_C(c_i, z_l|_{1<i<m}) = \nu(1-p)q^{i-1} \tag{3.20}$$

$$\Phi_C(c_m, z_l) = \nu\frac{(1-p)q^{m-1}}{1-q} \tag{3.21}$$

$$\Phi_C(\cdot, \cdot) = 0\ otherwise \tag{3.22}$$

Here, W_h, W_l are the autarky values at z_h, z_l given by

$$W_h = \frac{(1-\beta q)u(y_0 + \frac{1}{\nu}\epsilon) + \beta(1-p)u(y_0 - \frac{1}{1-\nu}\epsilon)}{1 - \beta(q+p) - \beta^2(1-(q+p))} \tag{3.23}$$

$$W_l = \frac{\beta(1-q)u(y_0 + \frac{1}{\nu}\epsilon) + (1-\beta p)u(y_0 - \frac{1}{1-\nu}\epsilon)}{1 - \beta(q+p) - \beta^2(1-(q+p))} \tag{3.24}$$

and $f() = \{\frac{(1-\sigma)(1-\beta q(\beta R)^{\frac{1-\sigma}{\sigma}})}{1+\beta(1-p-q)(\beta R)^{\frac{1-\sigma}{\sigma}} - (1-p)\beta^m q^{m-1}(\beta R)^{\frac{m(1-\sigma)}{\sigma}}}$ $[\frac{1-\beta(p+q)-\beta^2(1-p-q)}{1-\beta q}W_h - (1-p)\beta^m q^{m-2}(qW_l - \frac{(1-q)W_h}{1-\beta q})]\}^{\frac{1}{1-\sigma}}$. Note that the frequency mass function Φ_C is the same with general, non-CRRA, preferences.

Proof

To obtain the discrete support of consumption $\mathbb{C}$, define c_m as the minimum participation-compatible consumption for an individual in the low income state z^l. As she cannot move further down in consumption, she is necessarily participation-constrained in both income states tomorrow, receiving values W_h and W_l respectively. Thus c_m is determined from her participation constraint as

$$W_l = U(c_m) + \beta[(1-q)W_h + qW_l]$$

which is solved by $c_m = z^l$ from the definition of W_l. So minimum consumption is equal to minimum income.

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The strict monotonicity of the sequence $\mu_t(z^h)$ and the finiteness of initial weights together imply that for any $\mu_{i,0}$, we have $\mu_t(z^h) > \mu_{i,0}$ for some finite t. So in the stationary allocation, an individual in the high income state is always constrained, receiving minimum participation-compatible consumption c_1, whose value we need to determine. Tomorrow she either remains at high income, receiving W_h, or gets a negative income shock and moves down in consumption according to (3.15), which for CRRA preferences becomes $c_i' = (\beta R)^{\frac{1}{\sigma}} c_1$. Thus, the expected value of her consumption stream under the contract can be expressed as the sum of m lotteries with two outcomes: either, in case of a positive income shock z^h, she receives value W_h. Or, in case she moves to low income z^l, she gets current utility $\frac{[(\beta R)^{\frac{i}{\sigma}} c_1]^{1-\sigma}}{1-\sigma}$, $i = 1$ plus participation in the next lottery for $i = 2$, and so forth. If she has not received a positive shock after m-1 periods, her consumption cannot fall by another whole step without violating her participation-constraint at low income. So there is a final lottery between receiving W_h and W_l. This means c_1 is uniquely determined by her participation constraint

$$\begin{aligned} W_h &= \frac{c_1^{1-\sigma}}{1-\sigma} + p\beta W_h + (1-p)\beta \sum_{i=1}^{m-1} \{(\beta q)^{i-1} \frac{[(\beta R)^{\frac{i}{\sigma}} c_1]^{1-\sigma}}{1-\sigma} \\ &\quad + \beta^i q^{i-1}(1-q) W_h\} + (1-p)\beta^m q^{m-1} W_l \end{aligned} \tag{3.25}$$

To derive the mass function Φ_C, note that the stationary mass at c_1 is that at income state z^h, equal to the first entry of the normalised left eigenvector of transition matrix F associated with a unit eigenvalue $\nu = \frac{1-q}{2-q-p}$. $\Phi_C(c_2, z_l)$ is simply ν times transition probability to low income $(1-p)$, and $\Phi_C(c_i, z_l) = \nu(1-p)q^{i-1}$, $i = 2...m-1$ declines geometrically with survival probability q, the probability of remaining in low income state z^l. Finally, the lower bound c_m has mass $\Phi_C(c_m) = \Phi_C(c_{m-1})\frac{q}{1-q}$. ■

The next corollary summarises the shape of the joint distribution of consumption, income and financial wealth, defined as the present discounted value of financial income, and derives some of its second moments. The proof, including closed forms for the joint distributions of both financial income and financial wealth with endowment income, is in the appendix.

Corollary 3.7. *With CRRA preferences and 2 income values, the following is true:*

1. *The covariance between income and consumption is positive. The covariances between income and both financial returns and wealth are negative.*

2. *The mean of consumption increases in income. Its conditional variance decreases.*

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3. *If $\Phi_C(c_m, z_l) \approx 0$, the cross-sectional variance of log-consumption in stationary equilibrium is*

$$Var_c = \tau[\frac{\log(\beta R)}{\sigma}]^2 \tag{3.26}$$

where $\tau > 0$ is a function of transition probabilities only. If there is a non-negligible mass at the truncation point, $\Phi_C(c_m, z_l) > 0$, this is an upper bound for the cross-sectional variance of individual consumption.

With 2 income values, the asymmetric nature of insurance under limited commitment thus implies a geometric cross-sectional distribution of consumption. Negative income shocks lead to a sequence of equal small steps down the distribution, while positive income shocks lead to a variety of consumption responses. And insurance becomes more efficient at higher interest rates, as illustrated by the negative relationship between the cross-sectional variance of consumption and R in corollary 3.7. Finally, the negative correlation between financial wealth and income results because financial markets provide some, if not complete, insurance to individuals.

This section has generalised previous characterisations of limited commitment economies with two income values in several ways. Krueger and Perri (2005), and similarly Krueger and Uhlig (2006), show for the i.i.d. case that the stationary consumption distribution under limited commitment is discrete with geometrically declining mass, for a given constant interest rate. For the CRRA case, I solve for the whole distribution including the support of consumption, wealth and financial income (see appendix) in closed form, analysing the more general case with persistent income. Moreover, the corollaries to proposition 3.6 characterise conditional and second moments of the distribution, including a closed form for the variance of log consumption in the case of negligible truncation, showing how lower interest rates are associated with higher consumption variance in stationary equilibrium.

3.4.3 Existence and uniqueness of a market-clearing interest rate

The previous sections characterised the equilibrium distribution of consumption and income for a given level of interest rates R. This section proves the existence of a unique stationary market-clearing interest rate $R^\star > 1$.

Proposition 3.8. *If agents have non-increasing relative risk aversion*

$$\frac{u'(c_1)c_1}{u''(c_1)} \geq \frac{u'(c_2)c_2}{u''(c_2)} \ \forall c_1 < c_2 \tag{3.27}$$

then there exists a unique stationary market-clearing interest rate $R^\star > 1$.

Proof The algorithm used in Proposition 3.4 defines a mapping $\odot$ from the interval $I_R = [\frac{u'(z^1)}{\beta u'(z^N)}, \frac{1}{\beta}[$ of interest rates to the space of stationary consumption distributions. By summing over the distribution and subtracting constant aggregate income Y, this yields excess demand as a function of interest rates $\overline{\odot} = \int \odot \, d\Phi_I - Y$. Note that this mapping is single-valued, as the algorithm has a unique solution for any $R \in I_R$, and that $\odot(R)$ coincides for $R > 1$ with the stationary solution to the planners problem given interest rate R. Note also that consumption equals income in autarky, so $\overline{\odot}(R_{aut}) = 0$ for $R_{aut} = \frac{u'(z^1)}{\beta u'(z^N)}$. The proof shows that $\overline{\odot}(R)$ is decreasing for $R_{aut} < R < 1$ and increasing for $1 < R < \frac{1}{\beta}$. This implies that for some $R^\star > 1$ excess demand is negative. Existence then follows from the fact that excess demand must be positive for $R = \frac{1}{\beta}$ as perfect insurance is unfeasible by assumption. Uniqueness follows from the monotonicity of $\overline{\odot}(R)$ for $R > 1$.

From Proposition 3.4, the consumption distribution Φ_C splits naturally into N subdistributions Φ_C^m bounded above by c_0^m, the minimum participation-compatible consumption at income $z^m, m = 1, ..., N$. For any m, consider Φ_C^m as a function of the interest rate R. For an individual who is constrained at income z^m we can write

$$V(c_0^m, z^m) - W(z^m) = \sum_{i=0}^{n} \beta^i [\pi_{i|m} u(c_i^m) - \sum_j \pi_{ij|m} u(z_{ij})] = 0 \tag{3.28}$$

where the last equality follows from the fact that the participation constraint is binding. Here, i is the index for unconstrained transitions of consumption starting from the constrained level c_0^m, $i = 0, 1, ..., n$. $z_{ij}, j = N, N-1, ...$ are the possible income states for an individual who has remained unconstrained for i periods, with associated conditional probabilities $\pi_{ij|m}$, while $\pi_{i|m} = \sum_j \pi_{ij|m}$ is the marginal probability that an individual at income z^m remains unconstrained for i periods, and $\pi_0 = 1$. Note that in (3.28), only unconstrained states appear, as continuation and autarky values cancel in the participation constraint for all constrained future states. Differentiating (3.28) totally with respect to c_i yields a condition for any participation-compatible perturbation to the planner's allocation

$$0 = \sum_{i=0}^{n} \beta^i \pi_{i|m} u'(c_i) dc_i = u'(c_0^m) \sum_{i=0}^{n} \pi_{i|m} R^{-i} dc_i \tag{3.29}$$

where the second equality follows from the law of motion (3.15). Since R^{-i} is a positive sequence, dc_i has to take both negative and positive values.

Differentiating the law of motion (3.15) totally yields a recursive definition of $\frac{dc_i}{dR}$

$$\frac{dc_i}{dR} = \frac{\frac{u''(c_{i-1})}{u'(c_{i-1})}}{\frac{u''(c_i)}{u'(c_i)}} \frac{dc_{i-1}}{dR} - \frac{u'(c_i)}{u''(c_i)} \frac{1}{R} \doteq \alpha_1 \frac{dc_{i-1}}{dR} + \alpha_2 \tag{3.30}$$

The constant term α_2 is strictly positive, while $0 < \alpha_1 \leq 1$ for any utility function satisfying non-increasing relative risk aversion. This implies that for any $\frac{dc_{i-1}}{dR} < 0$, $\frac{dc_i}{dR} > \frac{dc_{i-1}}{dR}$, while if $\frac{dc_{i-1}}{dR} > 0$, $\frac{dc_i}{dR} > 0$. In other words, the sequence dc_i crosses the zero line exactly once from below. The change of aggregate consumption by individuals on the mth subdistribution, denoted C^m, is therefore simply

$$\frac{dC^m}{dR} = \nu \sum_{i=0}^{m} \pi_i dc_i < (>) \nu \sum_{i=0}^{m} \pi_i R^{-i} dci = 0 \; for \; R < 1 \; (R > 1) \tag{3.31}$$

where the inequality (inverse inequality) follows from the fact that R^{-i} overweighs (underweighs) the latter, positive elements of the sequence dc_i when interest rates are below (above) 1. As this holds for all m, $\overline{\odot}$ is decreasing in interest rates at levels $R < 1$, reaching a minimum at $R = 1$, and rises with R from thereon. Since for $R = 1/\beta$ insurance is perfect, which is unfeasible by assumption 3.1, excess demand crosses the zero line exactly once at some $1 < R^* < \frac{1}{\beta}$. ■

3.5 The distribution of consumption and wealth compared to the data

This section looks at the stationary joint distribution of consumption, income and wealth, characterised theoretically in the previous section, for a calibrated version of the US economy. I compare these to the distributions in a standard self-insurance economy on the one hand, and in US micro-data on the other.

Previous studies on consumption insurance in calibrated economies usually have not looked at the shape of the implied joint distributions, but focused on particular moments of marginal distributions. This is true also for studies of limited commitment economies, such as Krueger and Perri (2006) who analyse changes in cross-sectional variances of income and consumption over time, or Cordoba (2008), who concentrates on variances and the upper tails of marginal distributions. Studies of the empirical distribution of consumption and income, on the other hand, have pointed out asymmetries. Battistin et al (2007), for example, conclude that the marginal distribution of consumption is close to a log-normal, i.e. has significant right-hand skew. Dynan et al (2006) show that in PSID data, while consumption responds more strongly to negative

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income shocks, this asymmetry has fallen over time, which they take as evidence of declining liquidity constraints. Krueger and Perri (2008), on the other hand, show that in the Italian Household Survey the relation between nondurable consumption and income changes unexplained by a first stage regression on household characteristics is largely linear, with a slightly stronger response of consumption to positive income changes. This section looks at asymmetries in joint distributions both in theory and US micro-data.

3.5.1 A quantitative model calibrated to the US economy

This section briefly describes the Krueger and Perri (2006) calibration of a limited commitment economy with production. For the income process, the authors assume the log of post tax labour income plus transfers (LEA+) $log(z_t)$ to be the sum of a group specific component α_t and an idiosyncratic part y_t. The latter, in turn, is the sum of a persistent AR(1) process m_t, with persistence parameter ρ and variance σ_m^2, plus a completely transitory component ε_t which has mean zero and variance σ_ε^2.
The process for LEA+ is thus of the form

$$\begin{aligned} log(z_t) &= \alpha_t + y_t \\ y_t &= m_t + \varepsilon_t \\ m_t &= \rho m_{t-1} + \nu_t \\ \varepsilon &\sim N(0, \sigma_\varepsilon^2) \\ \nu_t &\sim N(0, \sigma_\nu^2) \end{aligned} \tag{3.32}$$

Using data from the Consumer Expenditure Survey (CEX), the authors first partial out the group-specific component α_t as a function of education and other variables, identifying the variance of the idiosyncratic part of income y_t, as well as (from the short panel dimension of the CEX) its first order autocorrelation. Setting $\rho = 0.09989$, the value estimated by Storesletten et al (2004), then allows the identification of σ_ν^2 and σ_ε^2. In this study, I use $\sigma_\nu^2 = 0.26$ and $\sigma_\varepsilon^2 = 0.12$, the estimate for the year 2003, the endpoint of the Krueger and Perri (2006) sample. I then use the standard Tauchen and Hussey (1999) method to approximate the resulting process using a 7-state Markov chain for m_t, and a binary process for ν_t. It is important to note that the resulting 14 state Markov process does not fulfil the monotonicity assumption of the theory section, as transitions are identical across transitory shocks. The income process thus belongs to a more general class than that analysed in the previous sections.
For preferences, I choose a CRRA utility function with coefficient of relative risk aversion of 1 (log-preferences) and a discount factor of 0.96. In order to capture the features of the US economy more accurately than in the simple theoretical model, I allow agents

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to save at the equilibrium interest rate after default, and introduce production in the economy. In particular, I assume that competitive firms hire capital and labour from households to operate a Cobb-Douglas technology

$$Y = AK^{\alpha}L^{1-\alpha} \tag{3.33}$$

and set the labour share α to 0.3. Again, the calibration follows Krueger and Perri (2006), who choose the depreciation rate of capital δ and total factor productvity A to target a capital-output ratio of 2.6 and an interest rate of 4 percent in their benchmark period. The corresponding values of A and δ are 0.9637 and 0.0754 respectively. The computational algorithm first solves for the stationary equilibrium for a given interest rate, following the appendix that describes the recursions that derive the stationary consumption distribution in the general case.[10] I then use the bisection method to find the market clearing interest rate $R^{\star}$.

3.5.2 Joint distributions of c,y,w - Theory and non-parametric estimates from US micro-data

This section presents the joint distributions of consumption, wealth and income. In particular, I compare the distributions in Krueger and Perri's (2006) limited commitment economy to non-parametric estimates of their counterparts in US-microdata, as well as those in a simple self-insurance Ayagari economy. The latter has the same income process, technology and preferences described before, but agents can only save and borrow in uncontingent bonds subject to a borrowing limit equal to annual income. I calculate the joint distributions by applying a simple histogramm density estimator to the exact theoretical distribution of the limited commitment model, and to a simulation of the Ayagari economy.[11] To compare the theoretical densities to the data, I then estimate bivariate kernel densities for US data on consumption and wealth, based on an optimal choice of the bandwith as in Botev et al (2009).

[10] I amend this for the fact that, with purely transitory shocks ν_t, the monotonicity condition for F does not hold. So I need to reshuffle income states occasionally in order to have decreasing minimum-participation-compatible consumption values $c_0^1 > c_0^2 > ... > c_0^N$ during the algorithm. The solution is facilitated by the fact that, if this monotonicity condition holds, c_0^i can be found quickly using bisections on an interval $[z_i, c_0^{i+1}]$. This yields an algorithm that is extremely efficient when solving for the stationary consumption distribution.

[11] The histogramm density estimation for the Ayagari economy is based on an individual simulated income and consumption path of 100.000 periods, of which I discard the first 1000 for the estimation.

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3.5.2.1 The distribution of consumption and income

Figures 3 and 4 use consumption and income data from the 2003 wave of the US Consumer Expenditure Survey (CEX) to confront their estimated joint density with that from the models. Particularly, I use the dataset constructed by Krueger and Perri (2006), and their definition of income and consumption. Their income measure corresponds to the CEX measure of after-tax labour earnings plus transfers (the sum of wages and salaries of all household members, plus a fixed fraction of self-employment farm and nonfarm income, minus reported federal, state, and local taxes (net of refunds) and Social Security contributions). Importantly, the consumption series includes an imputed measure of services from durables (for details see Krueger and Perri 2006). From both of these series I partial out the effect of a vector of observable individual characteristics, to control for ex-ante differences or predictable changes in life-time wealth.[12]

Figure 3.1 and 3.2 show that the results from the theory continue to hold with the more general income process: the marginal distribution of consumption in the Krueger and Perri (2006) calibration, presented in figure 3.1 where equal colours correspond to individuals who were last constrained in the same income state, is a mixture of geometric subdistributions. And figure 3.2 shows that consumption rises on average with current income, but is highly heteroscedastic. In particular, the conditional variance of consumption declines as we move up the income distribution. The Ayagari economy, interestingly, also has some decline in conditional variances, although less so than the limited commitment economy. The data has a roughly homoscedastic, increasing shape of the conditional distribution. Figure 3.3 presents the joint distribution of consumption and income growth. Its first striking features are the important differences between the 2 model densities: in the limited commitment economy, as suggested by theory, income declines are perfectly shared, resulting in a floor to the distribution slightly below zero. Positive income shocks are followed by a variety of positive consumption responses, leading to a strong rise in the conditional variance of the distribution for larger shocks. The Ayagari model on the other hand has a much more homoscedastic shape around a roughly linear mean response of consumption to income growth. To compare these distributions to the data, figure 3.3 uses log-differences of the raw data, not the residuals from the first stage regression. The resulting estimate of the distribution shows neither the downward cap, nor the heteroscedasticity of the limited commitment model. Rather, the cloud character of the picture suggest important measurement error in the CEX

[12]Particularly, unless otherwise mentioned, I use residuals from a regression of income and consumption on a cubic in the household head's age, and dummies that equal 1 if the household head has a unversity degree, a college degree, a high school degree, is male, is black, is asian, or of some other non-white race. I concentrate on households where the head is between 16 and 64 years of age.

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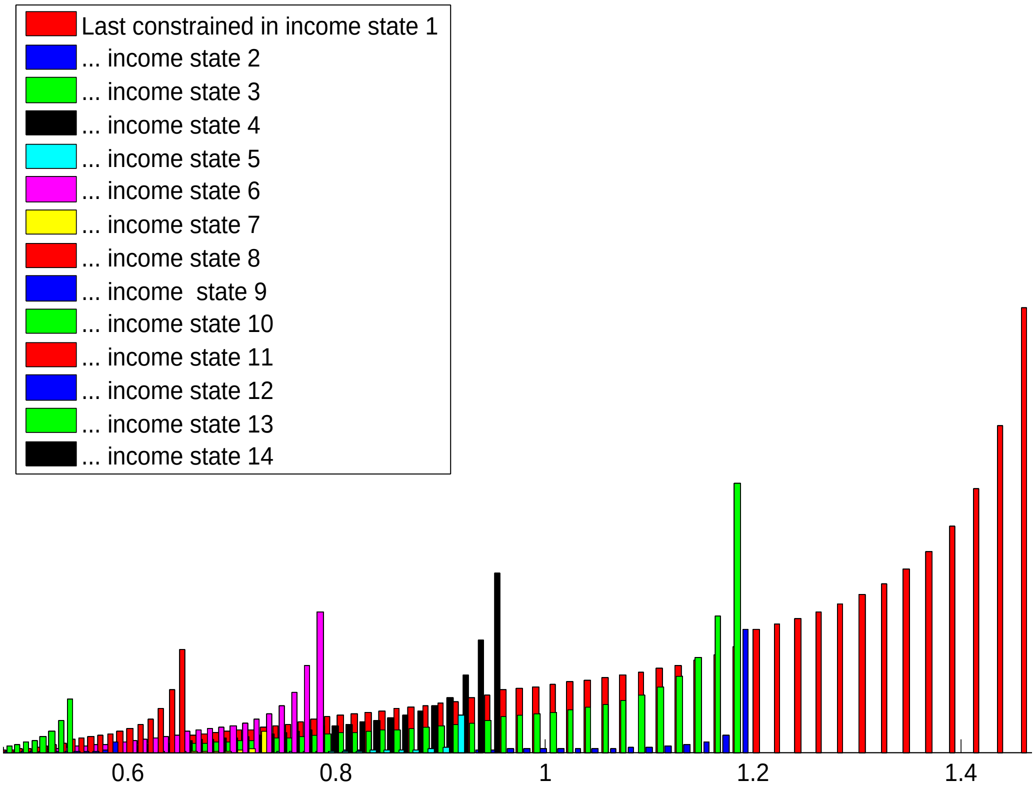

FIGURE 3.1: The marginal distribution of consumption
The figure shows the marginal distribution of consumption in the Krueger and Perri (2006) calibration. Equal colours denote individuals that were last constrained at equal income values and are thus located on the same geometric subdistribution of consumption.

data. The picture is practically unchanged if we use the residuals from the first stage regression.

3.5.2.2 The distribution of wealth and income

Figure 3.4 performs a similar exercise for the joint distribution of wealth and income, using the net worth variable of the 2004 wave of the Survey of Consumer Finances (SCF), and chopping off the upper 1 percent of all distributions to control for outliers and top-coding.

In the limited commitment economy insurance lowers the financial wealth of high income households. So, even with the more general income process, the income rich have minimum wealth. Since individuals slowly deplete their wealth levels after a negative income shock, the income poor have a variety of positive wealth levels, including the

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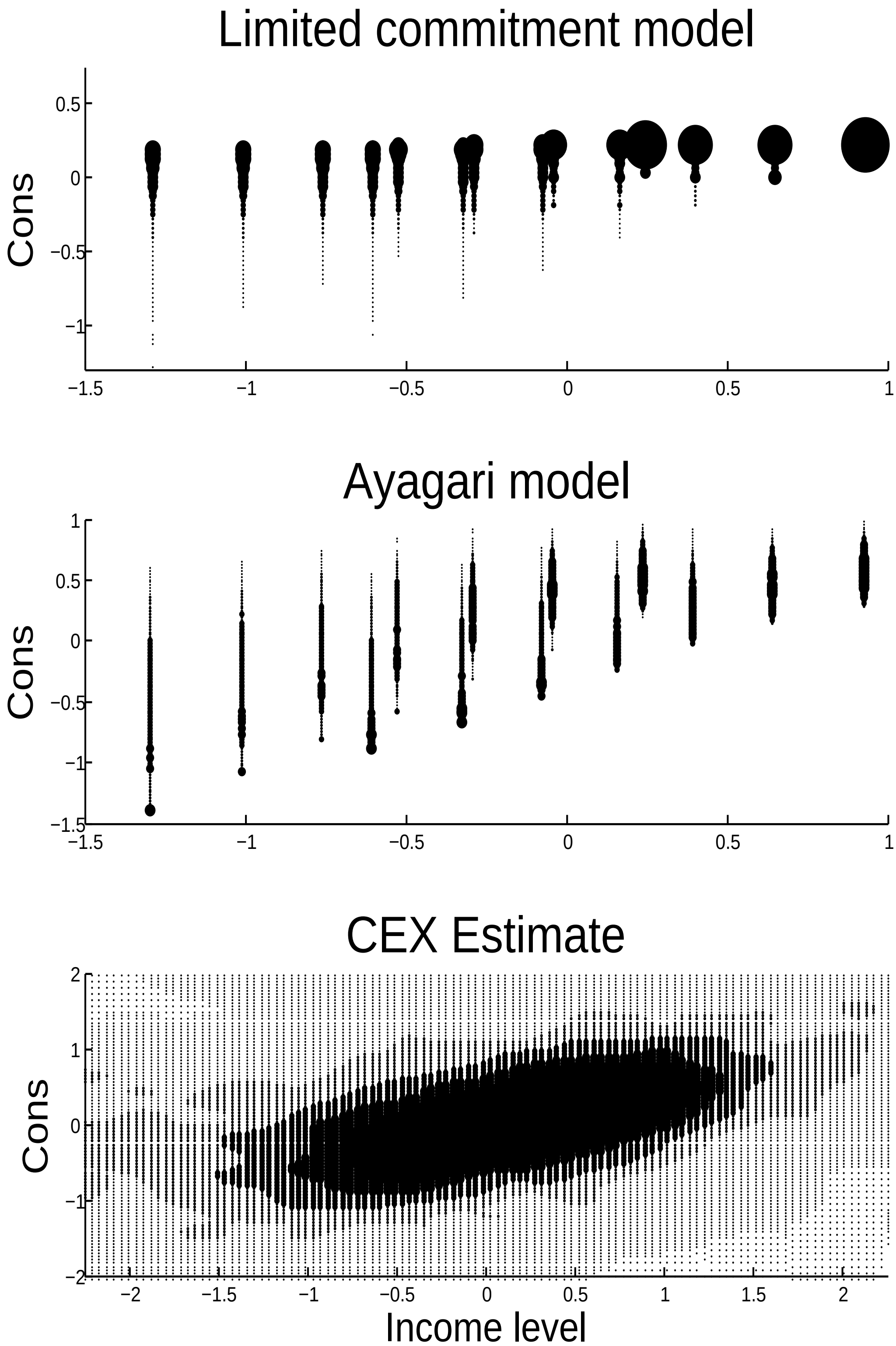

FIGURE 3.2: The joint distribution of consumption and income
The figure shows the joint densities of consumption and income in the limited commitment economy, a simple Ayagari economy, and in CEX data. The size of dots is proportional to the frequency mass at that point. The kernel density estimate of the empirical distribution uses an optimal bandwith (Botev et al 2008), and is based on residuals from a first-stage regression of the variables on observable individual characteristics as described in the main text.

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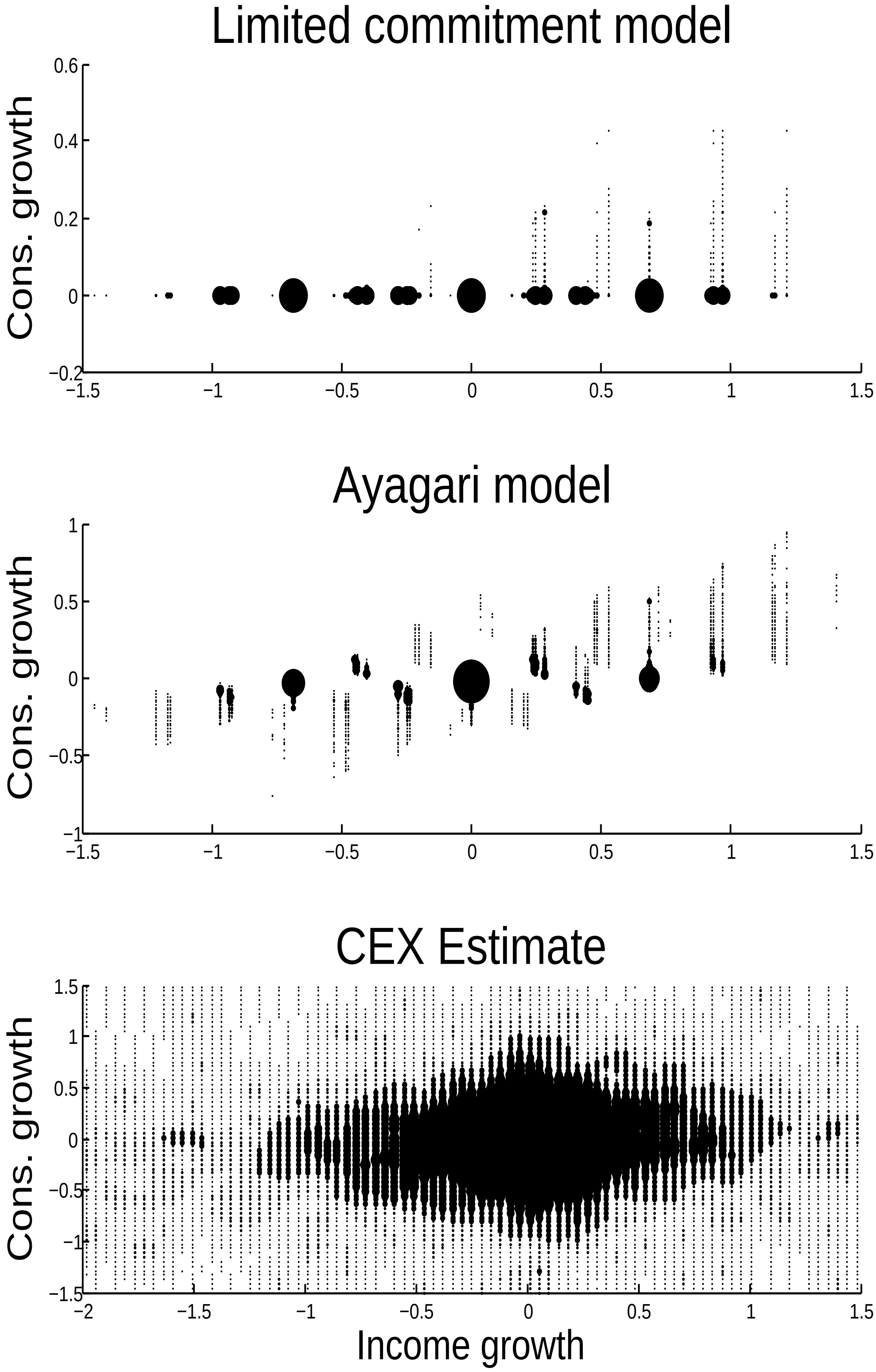

FIGURE 3.3: The joint distribution of consumption and income changes
The figure shows the joint densities of consumption and income growth in the limited commitment economy, a simple Ayagari economy, and in CEX data. The size of dots is proportional to the frequency mass at that point. The kernel density estimate of the empirical distribution uses an optimal bandwith (Botev et al 2008), and is based on differences in the raw data.

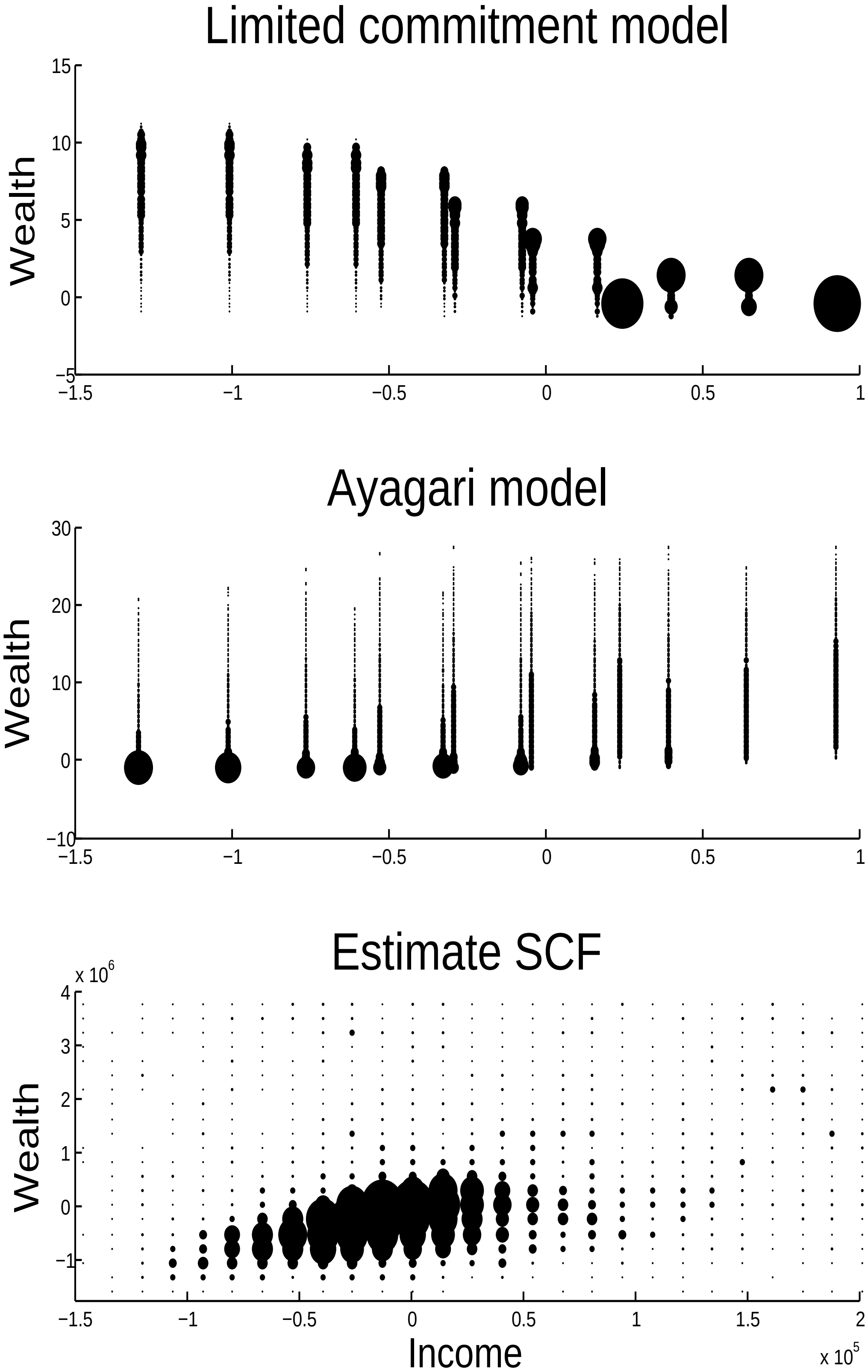

FIGURE 3.4: The joint distribution of financial wealth and income

The figure shows the joint densities of wealth and income in the limited commitment economy, a simple Ayagari economy, and in SCF data. The size of dots is proportional to the frequency mass at that point. The kernel density estimate of the empirical distribution uses an optimal bandwith (Botev et al 2008), and is based on residuals from a first-stage regression of the variables on observable individual characteristics as described in the main text.

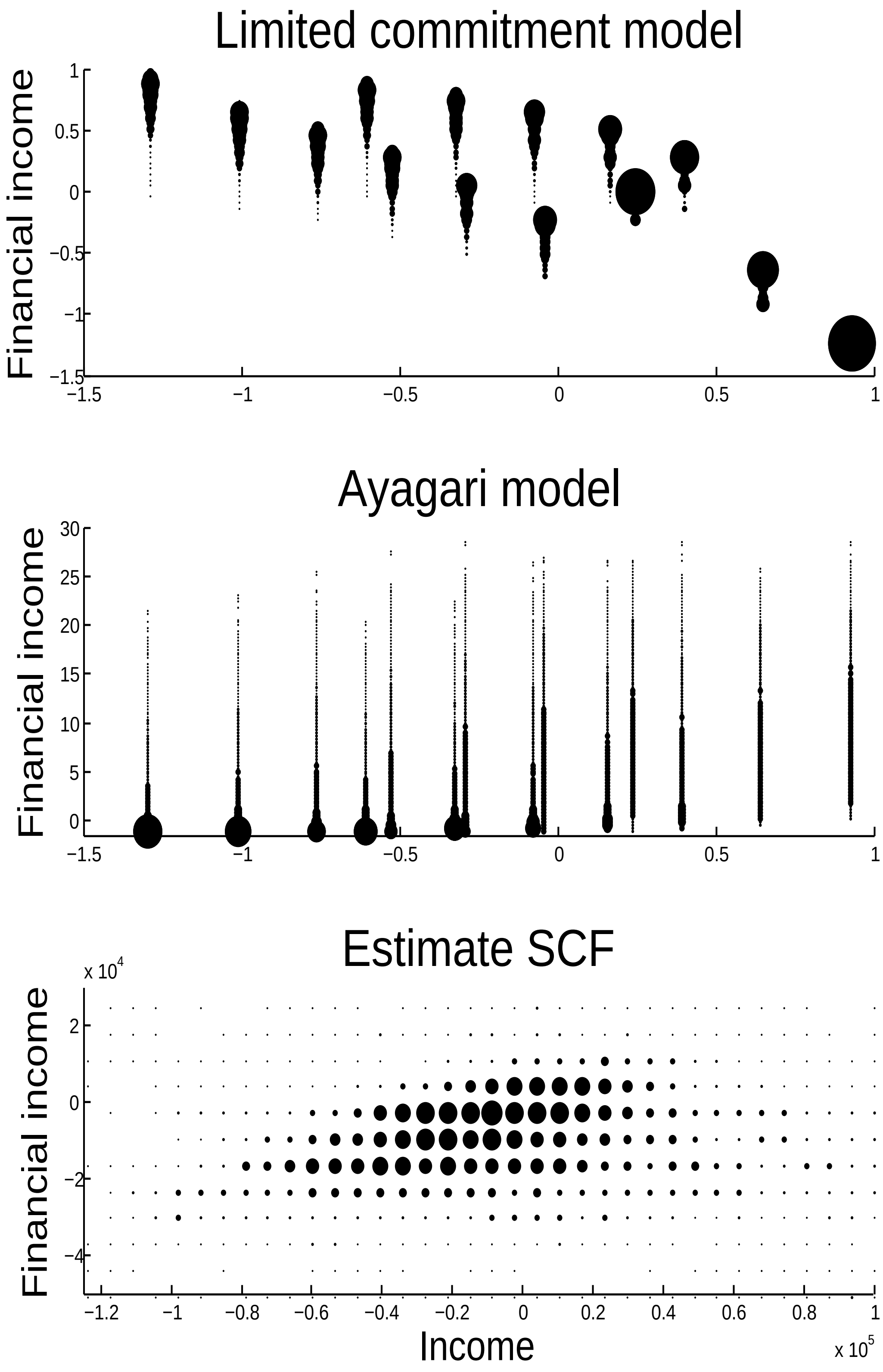

FIGURE 3.5: The joint distribution of net financial returns and income
The figure shows the joint densities of financial returns and income in the limited commitment economy, a simple Ayagari economy, and in SCF data. The size of dots is proportional to the frequency mass at that point. The kernel density estimate of the empirical distribution uses an optimal bandwith (Botev et al 2008), and is based on residuals from a first-stage regression of the variables on observable individual characteristics as described in the main text.

highest in the economy. In the Ayagari economy, on the other hand, the bufferstock nature of wealth leads on average to a positive relationship between income and wealth levels. But there is large variation around the mean, as individuals slowly build up, or draw down, their wealth after income changes. The mass of individuals at the borrowing constraint clearly rises as income falls. Comparing this to the SCF data, we see both an increase in mean wealth, as well as in its variance, as income, measured as salaries plus a proportion of business income, rises.

The SCF is a cross-section, so does not allow us to look at changes in wealth. But figure 3.5 compares the joint distributions of financial income and earned income in the model and the data. Again, the insurance mechanism leads to a strong negative correlation between income and financial returns in the limited commitment model, with the expected declining conditional variances. In the data, we find a positive relationship between financial and other income, as in the Ayagari model.

3.6 Conclusion

This study has looked at the equilibrium distribution of agents in an economy where limited commitment to contracts constrains risk-sharing. The theoretical contribution was to prove existence and uniquencess of a stationary equilibrium in a continuum limited commitment economy, and to provide an analytical characterisation of the distribution of consumption and income, including a closed form solution for an example with two income states and CRRA preferences. The theory showed how the asymmetric nature of insurance in the model, where negative shocks are shared but positive shocks lead to idiosyncratic consumption growth, implies declining conditional variances of wealth and consumption along the income distribution, and a negative relationship between wealth and income on average. The quantitative part of the paper looked at a limited commitment economy with capital and a more general income process, to compare the joint equilibrium distributions, and their characteristic non-linearity and heteroscedasiticity, with non-parametric estimates of the counterparts in US micro data, and those in a simple Ayagari economy. The results showed that, even with a more realistic income process featuring both near-permanent and transitory shocks, the limited commitment economy still produces very asymmetric joint distributions: consumption growth has a floor slightly below zero, but an upward tail that becomes more important for stronger positive income shocks. And both the mean and variance of wealth fall with income. Importantly, both the data and the Ayagari model have less heteroscedastic distributions, and mean wealth that rises with income.

The approach of this paper, to focus on the shape of joint distributions in order to test economic models with heterogeneous agents against the empiricial evidence, provides

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plenty of room for further research. One direction would be to generalise the model economies analysed here, to see if their characteristics are robust. For example, chapter 4 shows that amending the calibration used in this paper to include some heterogeneity in discount factors can largely reconcile the model-impact of near-permanent income shocks on current consumption growth with the data. On the other hand, a more thorough description of the joint distributions in micro-data is needed. Here, the new dataset provided by Blundell et al (2008), who have imputed a series of non-durable consumption for the PSID on the basis of its food expenditure information and a consumption demand function estimated on CEX data, seems very promising. And finally, the equality of model distributions and data should be tested more rigorously, accounting appropriately for the important role of measurement error in the data.

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Chapter 4

Partial insurance with limited commitment

Abstract

Blundell et al (2008) have recently presented new evidence on the response of consumption to permanent and transitory income shocks in US micro-data. I analyse this relationship in an economy with limited commitment to contracts. For a simple version of the model, I derive the response of consumption to income shocks in closed form, including an expression for the upward bias of Blundell et al's estimator in this environment, where their identifying assumption of no history dependence in consumption is violated. I then compute the response of consumption to income shocks in the calibrated limited commitment economy presented by Krueger and Perri (2006). In their original calibration to the US economy, consumption responses to permanent shocks are an order or magnitude smaller than in the data. But the introduction of a limited amount of heterogeneity in discount factors brings the model roughly in line with the data. In both calibrations, however, the upward bias of Blundell et al's identification scheme leads to estimates about twice as large as the true value of the coefficients.

JEL Classification Codes: D52, D31, D91, E44

Keywords: Risk Sharing, Limited Commitment, Insurance, Wealth and Consumption Distribution, Participation Constraints, Default

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4.1 Introduction

Understanding the response of household consumption to income changes is crucial not only as a test of economic theory, but also, for example, in order to evaluate the welfare consequences of inequality or the effectiveness of fiscal and other policies. Recently, Blundell, Pistaferri and Preston (2008, BPP) have presented important new evidence on the effect of income shocks on consumption, based on a novel US data set they constructed. In particular, the authors show that the consumption response to permanent income shocks is much less than one for one. This "partial insurance" not only contradicts the simple permanent income hypothesis, but also exceeds the level we could expect from a more realistic life-cycle model with self-insurance (Kaplan and Violante 2009).[1] BPP discuss their results in the context of recent work on models with superior, but limited, risk-sharing, due for example to limited commitment to contracts. However, rather than using a specific model, they provide stylised facts for others to match. This paper compares their evidence to the degree of partial consumption insurance in a standard limited commitment economy, where insurance is limited because individuals can default on contracts. I find that in the Krueger and Perri (2006) calibration of this environment, consumption responses to permanent shocks are an order or magnitude smaller than in the data. But the introduction of a limited amount of heterogeneity in discount factors brings the model much closer to the coefficients estimated by Blundell et al (2008). However, I also show, both quantitatively and in a simple analytical example, that their estimates have a strong upward bias in the particular environment of a limited commitment economy, where their identifying assumption of no history dependence in consumption is violated.

Tests for models of consumption insurance often face data problems. Particularly, the identification of shocks to individual incomes and their effect on consumption requires longitudinal panel data on income and consumption that for many countries is not available, including the US and the UK. There, authors have either used the information on food consumption in the Panel Study of Income Dynamics (PSID, Hall and Mishkin 1982), relied on synthetic cohorts of groups of individuals with similar characteristics (Attanasio and Davis 1996, Attanasio and Pavoni 2007), or used data from the US Consumer Expenditure Survey (CEX) with its, however, very limited panel dimension (Krueger and Perri 2006). Recently, in a seminal contribution, BPP have imputed a series of non-durable consumption for the PSID by using its food expenditure information and a consumption demand function estimated on CEX data. This allowed them,

[1] Krueger and Perri (2009), however, find that a realistic mix of permanent vs. transitory shocks in a simple permanent income model could explain the observed co-movement of income and consumption growth in the Italian Household survey.

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under some assumptions, to identify the variances of permanent and transitory income shocks, as well as their impact on current consumption. Importantly, while they cannot reject perfect insurance against transitory shocks to income, they find evidence of excess smoothness, with only 2/3 of permanent income shocks translating into current consumption.

Although BPP do not assume a particular economic environment, they discuss their results in the context of recent models where information asymmetries or limited commitment to contracts lead to "partial" insurance against income shocks. Attanasio and Pavoni (2007), for example, argue that an economy where insurance is constrained because individuals can hide both their productivity and their savings can achieve risk-sharing that is not perfect, but superior to self-insurance. A number of other contributions have looked at economies where agents cannot commit to honour financial contracts, which are assumed to be enforceable only by the threat of exclusion from future financial trade. Risk-sharing is generally not perfect in this setting, as fortunate individuals with high income realisations would prefer the outside option of financial autarky, rather than make large net payments into the insurance scheme. Alvarez and Jermann (2000) and Kehoe and Levine (1993) derive conditions for partial consumption insurance in this setting and prove the welfare theorems. And Krueger and Perri (2006) show how a calibrated limited commitment economy can reconcile the small rise in cross-sectional consumption inequality in the US over the past 25 years with a much larger observed rise in income volatility, as the latter strengthens the threat of market exclusion and thus improves insurance. In line with this previous evidence, BPP find a slightly smaller reaction of consumption to permanent income shocks in the second half of their sample, where income is more volatile. They take this as evidence in line with the financial deepening effect of higher income volatility predicted by limited commitment models.

This paper provides a formal comparison between the degree of partial insurance in a limited commitment economy and that estimated by BPP for the US. It is related to Kaplan and Violante (2009), who perform a similar exercise for a life-cycle version of the Ayagari (1993) self-insurance economy. The first contribution of this paper is to calculate the BPP partial insurance coefficients in a standard limited commitment model calibrated to the US economy. Relative to previous contributions that concentrated on moments of the cross-sectional consumption distribution (Krueger and Perri 2006, Cordoba 2008), this paper thus provides a new test of limited commitment theory based on the dynamic comovement of consumption with income shocks. But importantly, the fact that the theoretical model specifies both permanent and transitory income shocks exactly also allows me to test the assumptions underlying BPP's empirical model. Particularly, I calculate, both in a simple analyitical example and the calibrated model, the bias in the BPP estimates relative to the true population values identified in the model.

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And finally, I analyse the evolution of the BPP coefficients when feeding the observed changes in US income volatility since the 1980s into the model, to test the BPP conjecture of a financial deepening effect of increased income volatility on the size of their coefficients.
The results show that in the benchmark Krueger and Perri (2006) calibration of the limited commitment economy, consumption insurance is much stonger than in the data. But the introduction of a limited amount of heterogeneity in discount factors yields coefficient estimates roughly in line with the data. In both calibrations, however, the upward bias of Blundell et al's identification scheme leads to estimates about twice as large as the true value of the coefficients.

Section 4.2 describes the environment of a continuum economy with a limited commitment problem in financial markets, and provides an analytical solution to the BPP coefficients in a simple version of the model. Section 4.3 analyses a version of the model with capital, calibrated to the US economy, and presents the main results on the size of the BPP coefficients in this context, their evolution over time, and the bias in the BPP estimates.

4.2 Linear insurance coefficients in a simple limited commitment economy

4.2.1 The BPP method: linear partial insurance coefficients

To analyse the dynamic properties of individual income, we need data that records the income of particular households over several periods. In the US, the Panel Study of Income Dynamics (PSID) has traditionally been used for this. In order to analyse the joint dynamics of income and consumption, however, researchers there traditionally faced the problem that the PSID only contains information on food consumption and a few other items. Detailed US consumption data, on the other hand, can be found in the consumer expenditure survey (CEX). The latter, however, has only a very short panel component. BPP's first contribution is to use the PSID's information on food expenditure, together with a consumption demand function estimated from CEX data, to impute a comprehensive PSID consumption series.
In order to analyse the degree of insurance against income fluctuations in this new dataset, the authors make a set of assumptions rich enough to allow identification of income shocks, and their impact on current consumption. Particularly, BPP assume the

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following income process

$$y_{i,t} = z_{i,t} + \varepsilon_{i,t} \tag{4.1}$$

where $z_{i,t}$ is a permanent component that follows a random walk with shocks $\zeta_{i,t}$, and $\varepsilon_{i,t}$ is a transitory shock that has an MA(q) structure. BPP then estimate the response of current consumption to permanent and transitory income shocks in the following linear equation

$$\Delta c_{i,t} = \phi_{i,t}\zeta_{i,t} + \varphi_{i,t}\varepsilon_{i,t} + \xi_{i,t} \tag{4.2}$$

where $\Delta c_{i,t}$ denotes the log-difference of consumption of individual i in period t, and $\xi_{i,t}$ is an error term that captures consumption growth unexplained by, and assumed to be uncorrelated with, income movements. The "BPP coefficients" $\phi_{i,t}$ and $\varphi_{i,t}$ thus measure the linear association between income shocks and current consumption growth. A value of 1 would correspond to perfect comovement of consumption and income shocks, while perfect insurance would require both to be zero. In other words, any value of $\phi_{i,t}$ and $\varphi_{i,t}$ between 0 and 1 indicates "partial insurance" of consumption against income shocks.

Without further assumptions, it is generally impossible to identify the variance of the income shocks or the coefficients $\phi_{i,t}$ and $\varphi_{i,t}$ from data on consumption and total individual disposable income alone. But the authors show that they can identify the coefficients in (4.2) using data only on consumption growth and various leads and lags of income growth as long as the following orthogonality conditions hold

$$cov(\Delta c_{i,t}, \zeta_{i,t-1}) = cov(\Delta c_{i,t}, \zeta_{i,t+1}) = 0 \tag{4.3}$$

$$cov(\Delta c_{i,t}, \varepsilon_{i,t-2}) = cov(\Delta c_{i,t}, \varepsilon_{i,t+1}) = 0 \tag{4.4}$$

Under this assumption, and abstracting from the MA structure of $\varepsilon_{i,t}$, BPP show how $\phi_{i,t}$ and $\varphi_{i,t}$ are identified as follows

$$\begin{aligned}\phi_t^{BPP} &= \frac{E[\Delta c_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})]}{E[\Delta y_t(\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})]} \\ \varphi_t^{BPP} &= \frac{E[\Delta c_t \Delta y_{t+1}]}{E[\Delta y_t \Delta y_{t+1}]}\end{aligned} \tag{4.5}$$

Using this identification scheme, BPP's preferred estimate for ϕ_t^{BPP} is 0.65, significantly different from both 0 and 1 and thus providing evidence for partial insurance against permanent income shocks. The coefficient is smaller, however, for the subsample of households with college-educated heads, and is estimated to be "slightly lower" in the later part of the 1980s. Together with an estimated increase in variances of $\zeta_{i,t}$ and $\varepsilon_{i,t}$, BPP take this as evidence in line with the intuition from limited commitment models, that more volatile income increases the benefits of access to financial markets, and thus mitigates the problem of default. Insurance of consumption against transitory shocks on the other hand is estimated to be full, as φ_t^{BPP} is not significantly different from zero. Finally, the life-cycle profile of the coefficients, when estimated separately for different cohorts, turns out to be relatively flat.

4.2.2 Income and consumption comovement under limited commitment to contracts

BPP do not specify a particular economic environment for their analysis. Rather than a "specific structural interpretation" the authors want to provide " "structured facts" " (sic, p. 1889). Their estimates thus give us a set of stylised facts that models of consumption insurance in the US economy should be able to replicate. One goal of this paper is to perform this test for a standard complete markets economy where limited commitment to contracts restricts risk sharing between individuals. In addition, however, the assumption of a particular economic environment also allows me to assess wether or not the BPP identification assumptions (4.3) and (4.4) hold in the model, and thus wether the estimates can be interpreted as partial insurance coefficients.

4.2.2.1 The economy

This paper analyses partial insurance in an economy where asset markets are complete, so agents can in principle insure each other against income risk by writing contracts conditional on any possible contingency ex ante. However, individuals cannot commit to honouring contracts ex post, as they have the opportunity to deny payment at any time, at the price of exclusion from all financial trades in the future. After declaring default, agents thus simply consume their income forever. The resulting higher volatility of consumption gives individuals incentives to honour some contractual net payments. But they may not find it optimal to honour contracts requiring larger transfers. Insurance can thus be partial, as anticipation of default inhibits some but not all risk-sharing contracts.

Particularly, the economy I analyse consists of a large number of individuals of unit mass. Time is discrete and a unique perishable endowment good is used for consumption. Agents live forever and order consumption sequences according to the utility function

$$U = E \sum_{0}^{\infty} \beta_i^t u(c_{i,t}) \tag{4.6}$$

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where E is the mathematical expectation operator, $c_{i,t}$ is consumption by agent i in period t, and $u : R^+ \rightarrow R$ is an increasing, strictly concave, continuously differentiable function that satifies Inada conditions. Note that I index the discount factor β by a subscript i to allow for some heterogeneity in preferences, on top of that in incomes.

I amend the BPP framework by assuming, for tractability, that individual income endowments lie in a finite set Z, $z_{i,t} \in Z = \{z_1 > z_2 > ... > z_N\}, N \geq 2$. Thus, while incomes can be very persistent, income shocks to the infinitely lived agents cannot be permanent. Rather, they follow a Markov process that is identical for all agents and described by a monotone transition matrix F with strictly positive entries F_{ij} and a unique ergodic distribution. Thus, in the long-run, aggregate (or average) income is constant, while individual income fluctuates. In the calibrated model, I will use a finite-state approximation to an AR(I) process with a persistence parameter of 0.9989, the value estimated by Storesletten et al (2004).

Agents engage in sequential trade of a complete set of state-contingent bonds. Individual endowment realisations are verifiable and contractable, but asset contracts are not enforceable: at any point, individuals can default on their contractual payments at the price of eternal exclusion from financial markets. Thus, the total amount an agent can borrow today against any income state z_j tomorrow is bounded by the option to default into financial autarky. There, consumption is forever equal to income. Given the markov structure of income, the value of default as a function of the vector of current income z can be written as

$$W(z) = \sum_{t=0}^{\infty}(\beta F)^t U(z) = (I - \beta F)^{-1} U(z) \tag{4.7}$$

In any state s_t, $V(z(s_t), a(s_t))$ is the contract value as a function of income $z(s_t)$ and holdings of state-specific Arrow Debreu securities $a(s_t)$. As in Alvarez and Jermann (2000) individual i's participation constraint for any state s_{t+1} can be written as a portfolio constraint on the claims she can issue against s_{t+1} income.[2] This borrowing constraint is "not too tight" in the words of Alvarez and Jermann (2000) if it assures participation but does not constrain contracts otherwise

$$a_i(s_{t+1}) \geq A_i(s_{t+1}) = min\{\alpha(s_{t+1}) : V(z_i(s_{t+1}), \alpha(s_{t+1}, 0)) \geq W(z_i(s_{t+1}))\} \tag{4.8}$$

[2] An alternative is to restrict choices directly, by requiring that the chosen consumption sequence fulfill participation constraints, as in Kehoe and Levine (1993).

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The household's problem is to maximise their expected utility by choosing current consumption and assets subject to budget and participation constraints

$$\begin{aligned}
V(z(s), a(s)) = max_{c,\{a(s')\}}\{u(c) + \beta E_s V(z', a(s'))\} \\
s.t.\ c + \sum_{s'} a(s')q(s') \leq a(s) + z(s) \\
a(s') \geq A(s') \\
A(s') = min\{\alpha(s') : V(z(s'), \alpha(s')) \geq W(z(s'))\}
\end{aligned} \tag{4.9}$$

where c and a' are policy functions of the state variables $z(s), a(s)$.

4.2.2.2 Equilibrium comovements of income and consumption with limited commitment

In chapter 3 I use the method proposed by Marcet and Marimon (2009) to analytically characterise the stationary joint distribution of consumption and income in this economy, and show conditions for existence and uniqueness of equilibrium. Like in standard incomplete market models, unless perfect insurance is feasible, the equilibrium interest rate R in this economy is smaller than the rate of time preference $\frac{1}{\beta}$. This is because portfolio constraints lower the supply of state-contingent bonds, which bids up their average price, and thus reduces the interest rate. With low interest rates, agents are relatively impatient, and would like to substitute future consumption in favour of consumption today. This front-loading of consumption, however, is limited by the portfolio constraints on debt issuance (4.8), which define a minimum consumption level for every income realisation. In stationary equilibrium, there are N such minimum levels $c^i, i = 1, ...N$, where participation constraints of individuals with income z_i are binding. It is easy to show that these consumption levels for constrained agents are increasing in income, and that the lowest equals minimum income, $c^N = z^N$, providing a floor of the consumption distribution that just ensures participation of individuals at the lowest income level.[3] On the other hand, agents that are not constrained by their outside option of default, for example after a negative income shock, optimally choose a smooth downward sloping path of consumption, according to the law of motion

$$U'(c_i) = \beta R U'(c_i') \tag{4.10}$$

Figure 4.1 shows the resulting consumption path for a simple example where preferences are homogeneous with constant relative risk aversion σ, and individual income only takes

[3] This is because constrained individuals at minimum income will be constrained in all states of the world tomorrow. So their participation constraint can be written $W(Z^N) = U(c^N) + \beta[\sum_{i=1}^{N} F_{Ni} W(Z^i)]$, which from the definition of W^N is solved by $c^N = z^N$.

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two values $\{z_h, z_l\}$. There is a constant consumption level for all high-income periods, defined by participation constraints, while consumption during low-income spells follows a downward sloping path.

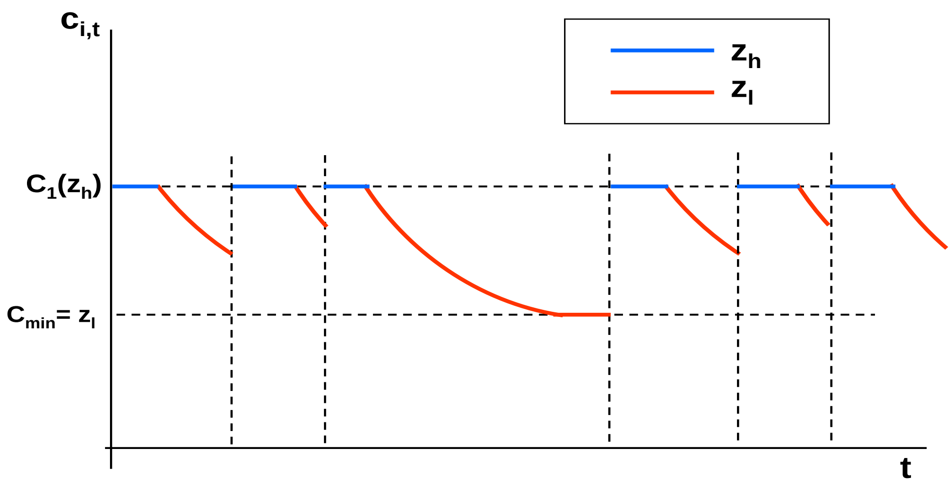

FIGURE 4.1: The consumption path under limited commitment with two income values.

Figure 4.1 and the preceding discussion demonstrate how BPP's model (4.2) fails to capture two key features of consumption insurance with limited commitment. First, the linear specification ignores the asymmetry of consumption responses in the model, where positive income shocks are followed by a variety of sometimes large rises in consumption, while negative income shocks always reduce consumption by small amounts independent of current income. Second, the focus on current income and consumption growth masks the history dependence in consumption of unconstrained agents, whose consumption today is a function of their consumption value yesterday, according to (4.10). Only for constrained agents, there is "amnesia" (Kocherlakota 1996) in consumption, which becomes a function of their current income only. In other words, since unconstrained indivdiuals use asset markets to spread the consumption response to income shocks beyond the period when they occur, contemporaneous covariances ignore a part of the consumption and welfare effect of income fluctuations in limited commitment economies.

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4.2.2.3 An analytical solution to linear insurance coefficients

Although the BPP model does not fully capture the nature of insurance in the limited commitment model, we can use their linear regression coefficients as an indicator for the average degree of insurance in the economy that any theoretical model should match. It turns out that, conditional on an interest rate R, we can easily calculate the regression coefficient of log consumption growth on log income growth in our example with CRRA preferences, 2 income states $\{z_h, z_l\}$, and transitions given by $F = [p, 1-p; 1-q, q]$, as

$$b_{\Delta c \Delta y} = -\frac{1}{2\sigma}(1 + \frac{1}{1-q})\frac{\widehat{\beta} + \widehat{R}}{\widehat{z_h} - \widehat{z_l}} \tag{4.11}$$

where hats denote logarithms.[4] Thus, the degree of partial insurance falls, or the regression coefficient rises, when interest rates fall or the persistence of low income states rises. This is intuitive, as it is mainly the large rises in consumption of individuals who receive a positive shock after a long number of low-income periods that determines the covariance of consumption and income. Higher persistence q lengthens low-income spells, and lower R increases the down-ward slope of consumption for unconstrained low-income earners. Both thus increase the upward jumps in consumption that come with positive income shocks. Negative income shocks, on the other hand, never lead to large falls in consumption as shown above, so their contribution to the contemporaneous comovement of consumption and income is limited.

The simple economy is useful to gain intuition on consumption insurance with limited commitment, and allows us to arrive at closed-form partial equilibrium expressions. In order to see wether limited commitment to complete contracts is able to deliver consumption-income comovements roughly in line with the data, however, we have to compute the general equilibrium of a model properly calibrated to the US economy.

[4] I assume that the truncation of the geometric consumption distribution at minimum income is negligible. With truncation, the true coefficient $\widetilde{b}_{\Delta c \Delta y}$ has bounds that depend on the size of the last, constrained step, and can be expressed as a function of the untruncated coefficient $b_{\Delta c \Delta y}$ as

$$b_{\Delta c \Delta_y}(1 - \frac{q^m(1 + (m + \frac{1}{q})(1-q))}{2-q}) < \widetilde{b}_{\Delta c \Delta_y} =< b_{\Delta c \Delta_y}(1 - \frac{q^m(1 + (m)(1-q))}{2-q}) \tag{4.12}$$

where m denotes the maximum number of steps on a decreasing consumption path until an individual becomes constrained at low income.

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4.3 Partial insurance in a limited commitment model calibrated to the US economy

This section calculates the partial insurance coefficients from equation (4.2) in a version of the limited commitment model with capital that is calibrated to the US economy, including a realistic income process with both near-permanent and transitory shocks. My first aim is to assess wether the limited commitment economy can deliver realistic degrees of income-consumption comovement. But since the model identifies all shocks, which in the data are unobserved, the quantitative analysis also allows me to compare the true population coefficients to those that result from the BPP identification scheme in (4.5). And finally, it enables me to assess the BPP conjecture that an increase in income variability should, via a stronger threat of financial exclusion, result in stronger insurance coefficients.

4.3.1 A calibration of the model to the US economy

My calibration is largely based on that by Krueger and Perri (2006), although I allow for some heterogeneity in preferences, where individuals differ in their degree of patience, as for example in Krusell and Smith (1998). For incomes, I use the Krueger and Perri (2006) specification of the individual income process, which includes deterministic heterogeneity plus both very persistent and completely transitory shocks, of time-varying variance. In particular, they assume the log of post tax labour income plus transfers (LEA+) $m_{i,t}$ to be the sum of a group specific component α_t and an idiosyncratic part $y_{i,t}$. The latter, in turn, is the sum of a persistent AR(1) process $z_{i,t}$, with persistence parameter ρ, plus a completely transitory component $\varepsilon_{i,t}$.
The process for LEA+ is thus of the form

$$\begin{aligned} m_{i,t} &= \alpha_t + y_{i,t} \\ y_{i,t} &= z_{i,t} + \varepsilon_{i,t} \\ z_{i,t} &= \rho z_{i,t-1} + \nu_{i,t} \end{aligned} \tag{4.13}$$

where ε and ν are normal, mean-zero shocks with time-varying variances $\sigma^2_{\varepsilon,t}$ and $\sigma^2_{\nu,t}$ respectively. Using data from the CEX, the authors first partial out the group-specific component α_t as a function of education and other variables, identifying the variance of the idiosyncratic part of income $y_{i,t}$, as well as (from the short panel dimension of the CES) its first order autocorrelation.
To identify σ^2_ν and σ^2_ε, Krueger and Perri (2006) assume $\rho = 0.9989$, the value estimated by Storesletten et al (2004). The estimates show an increase in the variance of both

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persistent and transitory shocks over their sample period 1980 to 2003, with $\sigma^2_{\nu,t}$ rising from 0.19 to 0.26, and $\sigma^2_{\varepsilon,t}$ from 0.074 to 0.12.

For preferences, I choose a CRRA utility function with coefficient of relative risk aversion of 1 (log-preferences). In a baseline scenario I choose a common discount factor for all agents, equal to 0.96. But I also analyse a case with some heterogeneity in βs. Rather than calibrating the distribution of βs to BPP's estimates, I show that introduction of a limited degree of preference heterogeneity can deliver partial insurance coefficients in line with BPP's estimates. Particularly, I assume that discount factors are uniformly distributed on $\{0.90, 0.92, 0.94, 0.96, 0.98\}$.

In order to capture the features of the US economy adequatly, I allow agents to save at the equilibrium interest rate after default, and introduce production in the economy. In particular, I assume that competitive firms hire capital and labour from households to operate a Cobb-Douglas technology

$$Y = AK^{\alpha}L^{1-\alpha} \tag{4.14}$$

and set the labour share α to 0.3. The rest of the calibration follows Krueger and Perri (2006), who choose the depreciation rate of capital δ and total factor productvity A to target a capital-output ratio of 2.6 and an interest rate of 4 percent in the baseline calibration without preference heterogeneity for the year 1980. The corresponding values of A and δ are 0.9637 and 0.0754 respectively.[5]

4.3.2 The BPP coefficients

Chapter 3 derives the joint distribution of consumption and persistent as well as transitory shocks in the calibrated limited commitment economy described in the previous section. On the basis of this, we can calculate the linear association in (4.2) directly from the population covariances

$$\begin{aligned} \phi_t^{pop} &= \frac{E[\Delta c_t \zeta_t]}{E[\zeta_t \zeta_t]} \\ \varphi_t^{pop} &= \frac{E[\Delta c_t \varepsilon_t]}{E[\varepsilon_t \varepsilon_t]} \end{aligned} \tag{4.15}$$

In turn, this allows us to compare the population coefficients $\phi_t^{pop}, \varphi_t^{pop}$ with their estimates on the basis of the BPP identification scheme, $\phi_t^{BPP}, \varphi_t^{BPP}$, which are only identical under assumptions (4.3) and (4.4) and with truely permanent shocks.

[5] To solve the model, I first approximate the persistent process for m_t with a 7-state Markov chain using the standard Tauchen and Hussey (1999) method. Following Krueger and Perri (2006) I choose a binary process for the transitory shock. The computational algorithm then follows chapter 3, where I describe a very simple recursion that derives the stationary consumption distribution using only participation constraints and the law of motion (4.10).

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Table 4.1: The BPP coefficients

Variances of income shocks				
	1980	2003		
$Var(m_t)$	0.19	0.26		
$Var(\varepsilon_t)$	0.074	0.12		
BPP coefficients				
	Baseline		Preference	Heterogeneity
	1980	2003	1980	2003
ϕ_t^{BPP}	0.021	0.028	0.41	0.36
φ_t^{BPP}	0.0078	0.0093	0.131	0.109
ϕ_t^{pop}	0.0125	0.016	0.20	0.18
φ_t^{pop}	0.0063	0.0078	0.126	0.104

The table presents the coefficients of equation (4.2) calculated from the exact stationary distributions of consumption and shocks ($\phi_t^{pop}, \varphi_t^{pop}$), and those estimated using the BPP identification scheme based on covariances between consumption and total income ($\phi_t^{BPP}, \varphi_t^{BPP}$), for both the baseline economy and that with preference heterogeneity. For reference, it also presents the variances of persistent (m_t) and transitory shocks (ε_t) in the Krueger and Perri (2006) calibration.

Table 1.1 presents the results. It reports the true values $\phi_t^{pop}, \varphi_t^{pop}$ together with the estimates from BPP's identification scheme $\phi_t^{BPP}, \varphi_t^{BPP}$, for both calibrations, and the years 1980 and 2003, the endpoints of the Krueger and Perri (2006) sample. The first important thing to note is that, in the baseline case without preference heterogeneity, all coefficients are small, at values below 0.05. Particularly, the values of ϕ_t^{BPP}, the coefficient measuring partial insurance against near-permanent shocks, are much smaller than the preferred BPP estimate of 0.65. In line with the BPP estimates and economic intuition, insurance against transitory shocks is even stronger than that against permanent shocks. However, there is a substantial upward bias in the BPP estimate of responses to permanent income shocks, as ϕ_t^{BPP} exceeds ϕ_t^{pop} by about 75 percent. A later section examines the source of this bias more closely.

Figure 4.2 plots the fitted values from the linear model, using $\phi_t^{BPP}, \varphi_t^{BPP}$, alongside the joint distribution of consumption and income growth . Particularly, the figure presents the histogramme density estimate of the joint distribution of consumption and income growth by plotting black dots proportional in size to frequency mass. It is obvious that negative income shocks always come with small consumption declines, but positive shocks occasionally cause large upward jumps in consumption. The fitted values from the BPP model, as red dots, although only approximately linear in total income growth, understate the response of consumption to positive income shocks, and overstate that to negative shocks.

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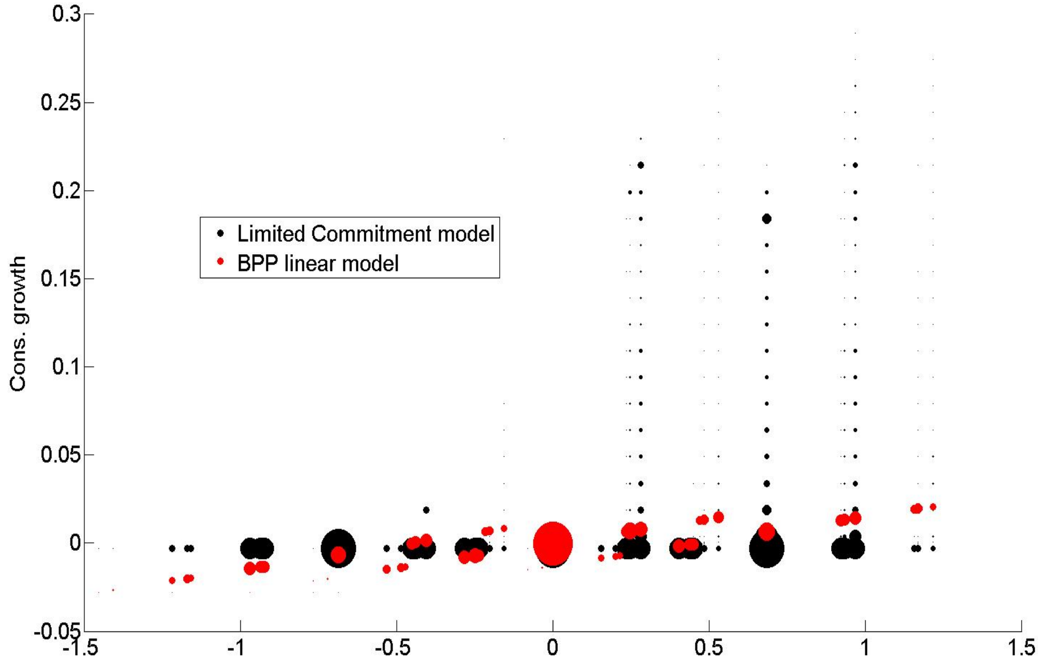

FIGURE 4.2: The joint distribution of consumption and income in the baseline economy, and that predicted by the linear BPP model.
The black dots are proportional to the frequency mass in the joint density of income growth and consumption growth in the baseline calibration of the limited commitment model. The values predicted by BPP's linear model are depicted as red dots.

4.3.3 The role of preference heterogeneity: micro-founding the spenders-savers theory of consumption

The baseline results were derived assuming a common discount factor for all individuals. Interestingly, once we introduce a small degree of heterogeneity in discounting, the results change substantially: assuming a uniform distribution of betas on $\{0.90, 0.92, 0.94, 0.96, 0.98\}$ increases the estimated coefficients by a factor of between 10 and 20. Particularly, using the BPP identification scheme, the coefficient measuring partial insurance against near-permanent shocks, ϕ_t^{BPP}, is estimated to be 0.36 in 2003. This is still lower than BPP's preferred value of 0.65. But, incidentally, it coincides with their estimate on the sub-sample of households with college-educated heads of 0.37. Importantly, the bias in the BPP identification remains unchanged, with estimates about twice as high as the true value.

The intuition behind this important role of preference heterogeneity in a limited commitment economy is straightforward. From the previous section, it is clear that the volatility of consumption, and its comovement with income, crucially depend on the difference between individual rates of time preference and interest rates. Impatient

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individuals, with low discount factors, prefer consumption today over future consumption. Participation constraints, however, put a limit on this front-loading by requiring a minimum amount of wealth in certain states of the world. But with low interest rates, previously constrained individuals reduce consumption relatively quickly after a negative income shock as the intertemporal terms of trade make future consumption relatively expensive. This leads to quick falls of consumption in low-income periods and relatively steep rises once participation constraints bind, resulting in larger consumption volatility for more impatient individuals at any given interest rate. In stationary equilibrium, the few very patient individuals in the economy, however, are not only perfectly insured against shocks, but they also hold a large fraction of aggregate wealth. They are thus the "savers" that practically set the interest rate, at a level $R = \frac{1}{0.98}$. This reduction in the interest rate raises the volatility in consumption that the remaining "spenders" are willing to accept further, thus increasing the average comovement between income and consumption in the economy. Therefore, the limited commitment model with preference heterogeneity can be seen, in some sense, as a possible micro-foundation to Gregory Mankiw's "Saver-Spender Theory of Consumption" (Mankiw 2000).

4.3.4 The bias in the BPP idenfication scheme

Table 1.1 reveals a striking difference between the true population coefficients and their counterparts estimated by the BPP identification for permanent shocks. The BPP identification systematically overstates the coefficients, or understates the extent of insurance against permanent shocks. In 2003, for example, ϕ^{BPP} is 75 percent bigger than ϕ^{pop} in the baseline case.

It turns out that there is a simple intuition behind this upward bias of the BPP coefficients in a limited commitment economy. This is because, with history dependence of consumption, the first of the BPP orthorgonality assumptions (4.3) fails, and $cov(\Delta c_{i,t}, \zeta_{i,t-1})$ is usually positive. This is because history dependence spreads negative income shocks across a number of periods of declining consumption. So there is positive comovement between income declines yesterday and consumption declines today. Positive income shocks, on the other hand, lead to upward jumps in consumption in the same period, and imply no history dependence. The net effect is for $cov(\Delta c_{i,t}, \zeta_{i,t-1})$ to be positive, and stronger for more persistent shocks, where history dependence plays a more prominent role.

When shocks are less than permanent, it turns out that this positive covariance between past shocks and current consumption movements due to history dependence is at least partly offset by mean reversion in income. This is because unless shocks are permanent, a negative shock yesterday increases the likelihood of positive income growth today.

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Since current income and consumption growth are correlated positively this leads to a negative bias in the coefficients.

In our simple example with a 2 state Markov process, both the positive bias from history dependence and the negative bias due to mean reversion, are easily quantifiable. Particularly, the covariance of income growth yesterday with today's consumption growth is given by

$$cov(\Delta c_t, \Delta z_{t-1}) = \tau\nu q(1-p) \ - \ \tau(1-p)(1-q) \tag{4.16}$$

where hats denote logarithms and $\tau = -\frac{\widehat{\beta}+\widehat{R}}{\sigma}(\widehat{z}_h - \widehat{z}_l) > 0$. The second term, $\tau(1-p)(1-q)$, captures mean reversion and vanishes when persistence goes to 1. The first term, $\tau v q(1-p)$, is equal to the mass of individuals who remain at low income today after a negative income shock yesterday. It is exactly for these individuals that history dependence introduces positive comovement between yesterday's income growth and consumption growth today. If $q + \frac{1}{2}p > 1$, i.e. for high enough persistence in income, the history dependence effect dominates, and the aggregate bias is positive. This is true, for example, whenever $p, q > \frac{2}{3}$.

4.3.5 More or less insurance? Rising income risk and the degree of insurance

When they allow the size of coefficients to vary across two sub-periods, BPP estimate a small rise in both the partial insurance against permanent income shocks and their variance. They take this as evidence in line with the intuition from limited commitment models, where higher income variability reduces the attractiveness of financial autarky and thus acts as a threat to default. This intuition turns out to be true, but only for the economy with preference heterogeneity, where the higher variance of income in 2003 induces a fall in the BPP coefficients. The reason for this is fairly simple: Positive comovement of current income and consumption arises from two sources: first, when a positive income shock leads to a binding participation-constraint, increasing consumption. And second from the fact that all individuals with negative income shocks have negative drift in consumption. In the baseline case, the fall in interest rates from 4 to 3.85 percent strengthens the second effect sufficiently to cause a net increase in the comovement of consumption and income. So the BPP coefficients rise. With preference heterogeneity, the patient consumers pin down the interest rates. So the interest rate effect is absent, leaving the rise in income volatility to improve insurance by an increased threat of exclusion, and thus less binding constraints for high income individuals.

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4.4 Conclusion

This chapter has looked at insurance against income shocks in a limited commitment economy. Contrary to previous contributions that focused on the stationary cross-sectional distribution, I compared the comovement of consumption growth with near-permanent and transitory income shocks in the model to recent evidence on this relationship in US micro-data provided by Blundell et al (2008). A standard calibration of the limited commitment model with capital to the US economy implies much stronger insurance against permanent shocks than observed in the data. But the introduction of heterogeneity in discount factors increases the comovement of consumption and shocks to values close to those reported by Blundell et al. In both versions of the model, however, their identification scheme introduces a strong upward bias in the estimates relative to the true values, arising from the history dependence of consumption that is a key feature of the limited commitment model. The model confirmed Blundell et al (2008)'s intuition, that rising income variance should have increased the insurability of permanent income shocks, although this depends on the general equilibrium reaction of interest rates.

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Appendix A

Appendix to Chapter 1

A.1 Existence and uniqueness of a solution to the planner's problem for a given interest rate R

Result:*For every given world interest rate $R_{min} < R < \frac{1}{\beta}$, there exists a unique allocation that solves the planner's problem for a particular weighting function μ in the social welfare function.*

Proof

I prove the existence of a unique solution to the planner's problem by checking that the conditions for a simplified version of Proposition 3 in Marcet and Marimon (2009) hold in this economy.

Given the finite space of individual endowments Z we can apply a version of Tychonoff's theorem to see that the Euclidian product Z^T is compact for countable T. So the exogenous vector of individual states lies in a compact (Borel) subset of the Euclidian Space R^T. And of course, the discrete transition function satisfies the Feller property (Assumption A1 in Marcet and Marimon (2009)). Second, given the No-Ponzi condition, for any given B_t, R, Y the set of feasible consumption allocations $c_{i,t} : \int_{\mathbb{I}} c_i, t \leq \frac{Y}{R-1} + B_t, \forall t$ is just a simplex, so the choice vector lies in a compact and convex set (Assumption A2 in Marcet and Marimon (2009)). Third, note that our objective function is continuous, but unbounded. However, since aggregate resources are bounded each period, so is $\int_{\mathbb{I}} U(c)$ (Assumption B1 in Marcet and Marimon (2009)). Finally, individual autarky is incentive compatible and resource feasible. So the constraint set is convex, compact,

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and non-empty.[1]

Given the continuous objective function, the original sequential problem (1.6) therefore has a solution. Also, Marcet and Marimon (2009) show that, given any initial weighting function μ, these conditions suffice to show that an allocation $\{c_{i,t}\}, i \in \mathbb{I}, t \geq 0$ solves the original problem if and only if there is a sequence of multipliers $\gamma_{i,t},\ i \in \mathbb{I}, t \geq 0$ such that $\{c_{i,t}, \gamma_{i,t}\},\ i \in \mathbb{I}, t \geq 0$ solves the saddle-point functional equation (3.8).

Uniqueness of the solution is assured by the strict concavity of the utility function u.■

[1]Strictly, we have to show that the constraint set has a non-empty interior, or that there is a real number $\varepsilon > 0$, such that $\int_{\mathbb{I}} c_i, t - Y \geq \varepsilon$ and $\int_{\mathbb{I}} [E[\sum_{t=0}^{\infty} (\mu_{i,t} + \gamma_{i,t}) U(c_{i,t}) - W(z_i)] > \varepsilon$.
In fact, without knowing the solution of the problem, the existence of $\varepsilon > 0$ is not trivial to prove. However, once we have the solution, the condition is easy to check. For now, I show the existence of ε for the i.i.d. version of the special case, with $p = q = 1/2$ and $B_{t+1} = B_t = 0$. For this case it is easy to see that as long as the income uncertainty is large enough, or $\epsilon > \nu : \frac{U'(y_0+\nu)}{U'(y_0-\nu)} = \frac{2-\beta}{\beta}$, there are numbers $\xi, \widehat{\varepsilon} > 0$ such that a programme of the form $c(y_h) = y_h - \xi, c(y_l) = y_h + \xi - \widehat{\varepsilon}$ fulfills the conditions above. Intuitively, the expected discounted gain from higher consumption in future low-income states is large enough to allow a ressource-feasible reallocation of current consumption from high to low income agents. Thus the interior of the constraint set is strictly non-empty (Assumption B2 in Marcet and Marimon (2009)). But, as we will see, this history independent sharing rule is not optimal.

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Appendix B

Appendix to Chapter 2

B.1 Measurement error in foreign assets

The measure of total foreign asset holdings used in the empirical part of this study potentially suffers from at least two kinds of measurement error. First, the responses of households to questions on their asset holdings are accurate only insofar individuals both know the accurate dollar value of their assets, and truthfully report it. Since I only look at portfolio investments (in other words I disregard directly owned foreign companies), market values of investments are in principle available, and individuals should report their dollar values at current exchange rates. This may be a strong assumption not only as individuals might not be aware of up-to-date market values for long-term investments or exchange rates, but also, for example, if some of them underreport systematically off-shore investments used to evade tax payments. In the latter case, however, the resulting measurement error would tend to dilute the correlation between wealth and the foreign asset share of the portfolio. So a rejection of the Null hypothesis of no relation would be less likely in the presence of this kind of measurement error. To see this, suppose all individuals were to invest x percent of their foreign asset holdings in unreported offshore vehicles. In this case, the difference between true portfolio shares $\widetilde{a_{true}}$ and those calculated from reported asset holdings can easily be shown to be $\frac{x}{1-x}(\widetilde{a_{true}})(1-\widetilde{a_{true}})$ Thus portfolio shares of foreign assets calculated from individual reports are always smaller than the true shares, and the difference is greatest for intermediate portfolios. As we see foreign asset shares rising from zero to single-digit percentages in Figure 1, the bias will increase along the wealth distribution.

A second source of measurement error results from the use of average portfolio shares in the imputation of households' indirect foreign asset holdings via mutual funds. If rich individuals systematically invest in funds with different exposures to foreign assets,

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this might distort the observed wealth effect on total foreign assets. But again, this error is likely to dampen the observed relationship between wealth and the portfolio share of foreign assets. To see this, suppose all individuals have the same portfolio share of mutual funds, but richer individuals choose funds with a higher (lower) share of foreign assets. Using average mutual fund portfolios introduces measurement error that is negative for rich (poor) individuals, positive for poor (rich) individuals. This biases the wealth effect estimated from observed data towards zero. The bias will be even stronger when richer individuals also have a higher portfolio share of mutual funds. So again, we would be less likely to reject the null of no wealth effect on portfolios in the presence of measurement error, than we would be without it.

B.2 Fixed costs and home bias

This section shows that higher costs of investing in foreign assets alone cannot explain the relative home bias of poorer market participants found in the data. Consider the 2 period problem of a home investor that receives a stochastic share e of aggregate home endowment Y_H, and can invest in home bonds at a return R_H, or foreign assets, yielding R_F units of foreign currency for bonds and $R_S Y_F$ for shares. Assume e, Y_H, Y_F are independent log-normal random variables. To abstract from the general equilibrium terms of trade movements at the basis of the results in the main text, assume that the exchange rate S, defined as the price of foreign currency in units of the home currency, is simply also a mean zero independent log-normal variable. In addition, assume that to buy $a^\star$ units of foreign assets, the investor has to pay a cost of $K = k_0 + k_1 a^\star$, i.e. there are fixed and proportional costs of investing abroad. The investor's problem can be expressed as

$$max_{c_h, c'_h, \overline{a_h}} \frac{c_h^{1-\sigma} - 1}{1-\sigma} + \beta E_{\lambda'}\{\frac{c_h'^{1-\sigma} - 1}{1-\sigma}\} \tag{B.1}$$

Subject to the constraints

$$\begin{aligned} c_h &= eY_H - a^b_{h,H} - \sum_{j\in\{b,s\}} a^j_{h,F} \\ c'_h &= e'Y'_H + R^b_H a^b_{h,H} + \frac{S'}{S}(R^b_F a^b_{h,F} + R^s_F Y'_F a^s_{h,F}) - K \end{aligned} \tag{B.2}$$

where $K = 0$ if the investor has zero foreign asset holdings.

The problem can be seen as a two-stage decision. First, the investor determines the optimal portfolios with and without foreign assets; then she compares expected utility

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for both and decides wether or not to hold foreign assets.
For simplicity consider binary portfolios where the investor either invests in shares or bonds. Given log-normality and independence, and approximating marginal utility from investing in foreign assets at zero real returns ($R_F^s = R_F^b = \frac{S'}{S} = 1$), the approximate share of foreign bonds in a diversified portfolio is

$$\widetilde{a_f} = \frac{1}{2} + \frac{r_F^b - k_1 - r_H^b}{\sigma Var_s} \tag{B.3}$$

where lower case letters denote logs and Var_s is the variance of the log exchange rate. Equivalently, the portfolio share of foreign shares is

$$\widetilde{a_s} = \frac{1}{2} + \frac{r_F^s - k_1 - r_H^b}{\sigma(Var_F + Var_s)} \tag{B.4}$$

So optimal portfolios are a function of risk aversion, excess returns and the variances of payoffs. But importantly, they do not include individual wealth. Also, proportional costs simply show up as a proportionate reduction of returns that affects all portfolios equally. So portfolio do not change with wealth among participants. Fixed costs on the other hand mean that only investors with a large enough portfolio diversify into foreign assets, where for investment in foreign bonds say, the threshold value of total assets is defined as that for which losses from fixed costs exactly offset those from sub-optimal portfolios

$$E[u(e' - aR_H^b)] = E[u(e' - a_h^\star R_H^b - a_f^\star(\frac{S'}{S}R_F^b - k_1) - k_0)] \tag{B.5}$$

Note that the cost structure assumed in this appendix comprises the case of costly acquisition of a fixed amount of information on any given asset. As information, once acquired, is non-exclusive in its use, the corresponding cost structure has different values of k_0 for different foreign assets, and $k_1 = 0$ for all of them. However, the assumption that costs are linearly affine in the size of investment does not capture a scenario where agents can acquire additional information on a given asset at non-zero marginal cost. As the marginal returns to information are increasing in investment size, wealthier individuals might find it optimal in this setting to acquire more information. This can induce variation of portfolios across individuals with different wealth levels. However, this result does not survive when poor agents can pool their foreign investments in a mutual fund, or if they can copy wealthier individuals' investment strategies.

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B.3 Existence and uniqueness of equilibrium

In section 3, I showed that the equilibrium relative price of goods is independent of heterogeneity and the allocation of assets. As long as agents have some preference for both goods ($0 < \theta < 1$), (2.12) thus describes a non-empty, single-valued mapping from the two-dimensional space of aggregate endowments into a market-clearing price. In other words a market-clearing price of goods always exists and is unique for any combination of Y_H, Y_F.

The excess demands for assets are the sum of the quantities solving (2.11), integrated across the distribution of unconstrained agents in both countries, plus maximum borrowing multiplied by the measure of constrained agents. For example, for Home IOUs, remembering that these can only be issued by home agents and that asset quantities are denoted in terms of domestic goods for home and foreign agents, we get

$$a_H = \int a_{h,H} d\Psi^{\epsilon}_{H} + p \int a_{f,H} d\Psi^{\epsilon}_{F} \tag{B.6}$$

$$= \int_{-\infty}^{\epsilon_h *} a_{h,H} B^b_H \Psi^{\epsilon}_{H} + \int_{e_h *}^{\infty} a_{h,H} d\Psi^{\epsilon}_{H} + p \int_{0}^{\infty} a_{f,H} d\Psi^{\epsilon}_{F} \tag{B.7}$$

where $e*$ denotes the level of current home endowment share that solves the first order condition for borrowing at the maximum level B^b_H.

Under financial autarky, existence of an equilibrium price vector $R = (R^b, R^s_H)$ is easy to prove by a fixed point argument. Local uniqueness of both consumption allocation and portfolios can also be shown.

However, global uniqueness is more difficult to prove as individual asset demands are not necessarily monotone in relative returns. Two special cases where the equilibrium can be shown to be globally unique are when agents only trade in either bonds, or shares, but not both, and either $\sigma \leq 1$ (substitution effects dominate income effects) or $B^j_K = \infty$ (unconstrained issuance of assets). This is because with one asset only, total excess demand shows no inter-asset substitution effects. Then, for $\sigma < 1$, all individual asset demands, and therefore total excess demand for assets, are monotone in returns as the substitution effect dominates. For $\sigma > 1$ savers may have decreasing asset demand (as the income effect dominates). But borrowers' asset demand is always increasing in returns, with an elasticity higher than that of savers at optimal borrowing levels as long as everybody faces the same period 2 uncertainty. So if all borrowers are unconstrained the total excess demand is again upward sloping in returns, and the equilibrium globally unique. However, even with only one asset, when a lot of borrowers are constrained, there may be multiple equilibria, as the non-monotonous asset demands of savers can dominate total excess demand.

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With more than 1 asset, possibly traded across countries, the equilibrium is not generally globally unique. But conditions for global uniqueness can be derived for example by imposing the gross substitution property on the system of individuals' arbitrage equations. For the analysis here this is not a problem, however, as I only look at interior portfolios, given an equilibrium vector of returns $\overline{R}$. I do not solve for the equilibrium explicitly, which will be a function of the particular specification of distributions and borrowing constraints in both countries.

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Appendix C

Appendix to Chapter 3

C.1 Proof of Lemma 3.3

The planner's decision rule Γ *has the form*

$$\mu_{i,t+1} = max\{\mu_{t+1}(z_{i,t+1}), \mu_{i,t}\}$$

For every t, $\mu_t(z)$ *is strictly increasing in* z, *and for every* z, *the sequence* $\mu_t(z)$ *increases strictly over time. Also, the set of individual planner weights with positive mass is strictly finite:* $|\{\mu_j : \Phi_I(\{i : \mu_{i,t} = \mu_j\}) > 0\}| < \infty, \forall t$.

Proof

To prove the first statement write the difference between continuation values $V(\mu_i, z^i)$ and autarky values $W(z^i)$ as

$$\Delta_t(\mu, z^i) \doteq V(\mu, z^i) - W(z^i)$$
$$u(\mu) - z^i + F_i \sum_{s=1}^{\infty} \beta^s F^{s-1} max\{(\overline{u_{t+s}(\mu)} - u(z)), \overline{0}\} \tag{C.1}$$

where F_i is the ith row of F, $\overline{u(z)}$ the $Nx1$ vector of utilities from consuming income, $\overline{u_t(\mu)}$ the constant vector of utility from having planner weight μ in period t, and $\overline{0}$ the zero vector. Δ is thus the discounted sum of "utility in excess of autarky" across states where individuals are unconstrained, as in all constrained states autarky and continuation values cancel. It can be interpreted as a measure of insurance benefits promised to individual i. Note that $\Delta_t(\mu_t(z^i), z^i) = 0$ defines the minimum participation compatible planner weight $\mu_t(z^i)$.

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I first show $\Delta(\mu, z^j)$ is strictly increasing in z^j, for all μ. For any μ

$$\begin{aligned}
\Delta_t(\mu, z^j) &= u_t(\mu) - u(z^j) + F_j \sum_{s=1}^{\infty} \beta^s F^{s-1} max\{(\overline{u_{t+s}(\mu)} - \overline{u(z)}), \overline{0}\} \\
&< u_t(\mu) - u(z^{j+1}) + F_j \sum_{s=1}^{\infty} \beta^s F^{s-1} max\{(\overline{u_{t+s}(\mu)} - \overline{u(z)}), \overline{0}\} \\
&\leq u_t(\mu) - u(z^{j+1}) + F_{j+1} \sum_{s=1}^{\infty} \beta^s F^{s-1} max\{(\overline{u_{t+s}(\mu)} - \overline{u(z)}), \overline{0}\} \qquad (C.2) \\
&= \Delta_t(\mu, z^{j+1}) \qquad (C.3)
\end{aligned}$$

where the second inequality follows from monotonicity of F, since the vector $max\{(u_{t+s}(\mu) - \overline{u(z)}), \overline{0}\}$ is decreasing in income values. Since $\Delta_t(\mu_t(z^j), z^j) = \Delta_t(\mu_t(z^{j+1}), z^{j+1}) = 0$ it follows that $\mu_t(z^j) > \mu_t(z^{j+1})$.

To see that the sequence $\mu_t(z)$ is strictly increasing for every income level z, note first that Assumption 1 implies a positive mass of agents with binding participation constraints every period, who experience an increase in their planner weights. Given constant resources, any constant $\mu_{i,t+1} = \mu_{i,t}$ then implies strictly declining consumption of individual i according to (3.13). Thus, since the autarky value $W(z^j)$ is constant through time, constant or declining cutoff values $\mu_{t+1}(z^j) \leq \mu_t(z^j)$ violate participation constraints. So $\mu_{t+1}(z^j) > \mu_t(z^j)$.

Finally, to see that the set of individual planner weights with positive mass is strictly finite, note first that the minimum participation compatible planner weights $\mu_t(z)$ lie in a finite interval defined by $1 < \frac{\mu_t(z^1)}{\mu_t(z^N)} \leq \frac{u'(z^N)}{u'(z^1)}$ $\forall t$. Since initial planner weights are strictly positive and finite, the ratio of maximum and minimum planner weights is thus bounded in all periods. As the sequence of $\mu_t(z^N)$ is strictly increasing there is an $\epsilon > 0$ such that $\frac{\mu_{t+1}(z^N)}{\mu_t(z^N)} > 1+\epsilon$, $\forall t$. But then the number of periods an individual can remain unconstrained is strictly bounded by $T = min(x \in \mathbb{N} : x > \frac{ln(u'(z^N)) - ln(u'(z^1))}{ln(1+\epsilon)})$. Since in every period there are at most N new weights, the number of planner weights is bounded by N^T plus the number of initial weights K. ■

C.2 Proof of Proposition 3.4: The consumption distribution in the general case

For $1 < R < \frac{1}{\beta}$ the interest rate in stationary equilibrium, the joint distribution of income and consumption $\Phi_C : \mathbb{C} \times \mathbb{Z} \longrightarrow [0, 1]$ has the following features:

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1. Φ_C is discrete, with positive mass at consumption values between minimum income and some upper bound c_0^1 smaller than the highest income level: $\mathbb{C} \subseteq [y_N, c_0^1]$, $c_0^1 < z^1$.

2. There are N minimum levels of consumption c_0^i, $i = 1, ..., N$ under which consumption of agents with income i never falls and where participation constraints at income z^i hold with equality. These threshold levels are increasing in income $c_0^1 < c_0^2 < .... < c_0^N$. The lower bound of the distribution is minimum income $c_0^N = z^N$.

3. Every consumption threshold c_0^i is an upper bound to a geometric subdistribution of consumption Φ_C^i, with support $\{c_j^i\}$ recursively defined by the law of motion $U'(c_{j+1}^i = \beta R U'(c_j^i), j = 0, 1, 2, ...$, and bounded below by z_N. Φ_C is thus a mixture of $N-1$ geometric distributions $\Phi_{C,i}, i = 1, ..., N-1$. The appendix contains an analytical expression for the frequencies in this distribution.

4. Individuals at the higest income level z^1 all have maximum consumption level c_0^1. The support of consumption conditional on income $z^i, i > 1$ is $[c_0^i, c_1^1]$. So the support of consumption narrows as income rises. For i.i.d. transitions (identical rows in F), this implies that the conditional variance of consumption falls monotonously in income.

Proof

The proof is by construction of the stationary consumption distribution.

Ad 1-3: The support $\mathbb{C}$

I construct $\mathbb{C}$ “bottom-up”, starting from its lower bound, which we know to be minimum income. Also, from Lemma 1, we know that minimum participation-compatible levels of consumption c_0^i increase in income z^i. Since c_0^i solves the participation constrained of individuals at income z^i with equality, this allows me to recursively determine c_0^i by substituting into the ith participation constraint the autarky values at incomes $z^j > z^i, j = i, i-1, ..., 1$ for future states with non-negative income shocks, and the consumption values given by the law of motion (1.11) for unconstrained states. Starting at $i = N-1$ and moving up income levels assures that this procedure can keep account of binding participation constraints as individuals move down in consumption from c_0^i to Z^N.

To see this in detail, denote as $c^i(c, R)$ the result of applying the law of motion for unconstrained transitions (1.11) i times starting from level c at interest rate R.

We know $c_0^N = Z^N$. Consider minimum participation-compatible consumption in the second lowest income state $N-1$. There, individuals receive c_0^{N-1} today, the value of which we want to determine. They face the “danger” of moving, with probability

$f_{N-1,N}$, to state N, and thus down to $c^1(c_0^{N-1}, R)$ tomorrow. With probability $f_{N-1,i}$, however, they move to income $z^i > z^N$ receiving $W(i)$. So c_0^{N-1} is uniquely determined from the participation constraint

$$W(N-1) =$$

$$U(c_0^{N-1}) + f_{N-1,N} \sum_{s=0}^{\infty} \beta^{s+1} f_{NN}^s max\{U(c^s(c_0^{N-1}, R), N), U(z^N)\} + \tag{C.4}$$

$$\beta \sum_{i=1}^{N-1} f_{N-1,i} W(i) + f_{N-1,N} \frac{\beta^2}{1 - f_{N,N}\beta} \sum_{1}^{N-1} f_{N,i} W(i) \tag{C.5}$$

Here, the second term on the right-hand side of the equation is the value from the declining consumption path starting at c_0^{N-1} and truncated at minimum level c_0^N, weighted by the probability to remain in income state N. The third term is the continuation value when not receiving a negative income shock tomorrow, the fourth from moving down in income tomorrow and then receiving positive income shocks at a later date. Note that the right hand side is increasing in c_0^{N-1} while the left hand side is constant. So the solution is unique.

3. Analogously, one can determine the other values c_0^i from repeated application of this algorithm.

The support of the consumption distribution $\mathbb{C}$ is simply the union of downward-sloping paths starting at minimum participation-compatible consumption
c_0^i $\mathbb{C} = \cup_{i=1}^N \{max[c^j(c_0^i, R), z^N], j = 0, 1, 2, ...\}$. Note that the highest level of consumption c_0^1 is strictly lower than the highest income level z^h from assumption A2, which implies that there is at least one unconstrained transition of individuals at z^1 that receive a shock z^N, which happens with positive probability. With $c_0^1 \geq z^1$ the participation constraint would thus be slack, as continuation utilities under insurance are strictly greater than in autarky in at least 1 state of the world. This however cannot be optimal for the planner, so we have $c_0^1 < z^1$.

Ad 3: The frequency distribution on $\mathbb{C}$

I construct the frequency distribution "top-down". From Lemma 1, I know that all high income individuals are constrained, at the minimum participation-compatible consumption for individuals with the highest income c_0^1. Thus, its mass is equal to the stationary mass of individuals at z^1. The rest of the frequency distribution is then based on the transition probabilities as follows:

Define Φ_C^i to be the subdistribution of consumption that contains all indivduals that were last constrained at income z^i. Out of individuals with highest income last period, all but those that remain at z^1 move down in consumption to $c^1(c_0^1, R)$, according to the law of motion (1.11). Denoting the ith row of F as F_i, and defining the Matrix $\overline{F}_i$ as F

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with the first i columns and rows replaced by zeros, and disregarding other thresholds for now, this would yield a geometric distribution on the downward sloping path from $c^1(c_0^1, R)$ equal to $\coprod_{1,n} = \nu F_1 \overline{F}_1^n$ on support given by $(c^n(c_0^1, R), z)$. However, since after $T_2 = \lfloor \frac{ln(U'(c_0^1)) - ln(U'(c_0^2))}{ln(\beta) ln(R)} \rfloor$ periods individuals at income 2 hit their participation constraint on the downward-sloping path $c^n(c_0^1, R)$, they drop out of this distribution, equivalent to $\overline{F}_1$ shrinking to $\overline{F}_2$. Equivalent reasoning for lower values of income yields the following vector valued sequence of joint frequencies on $c_j^1, z_k, j = 1, 2, ..., k = 1, ..., N$ [1]

$$\coprod_{1,n=\sum_{l=1}^{j-1} T_l + t_j} = \nu F_1 \, \Pi_{l=1}^{j-1} \overline{F}_l^{T_l} \; \overline{F}_j^{t_j}, j = 1, ...N, t_j = 1, ...T_j \tag{C.6}$$

for $T(i) = 1, T_{k>i} = \lfloor \frac{ln(U'(c_0^k)) - ln(U'(c_0^{k+1}))}{ln(\beta) ln(R)} \rfloor$ the integer number of unconstrained transitions between threshold levels of consumption c_0^k and c_0^{k+1}. The marginal subdistribution Φ_C^1 is simply the the row sum of the expression.

More generally, the joint subdistribution of income and consumption starting at consumption threshold c_0^i with support $c_j^i, z_k, j = 1, 2, ..., k = 1, ..., N$ has the vector valued sequence of frequencies

$$\coprod_{i,n=\sum_{l=i}^{j-1} T_l + t_j} = \nu_i \; F_i \Pi_{l=i}^{j-1} \overline{F}_l^{T_l} \; \overline{F}_j^{t_j}, j = i, ...N, t_j = 1, ...T_j \tag{C.7}$$

where $\nu_i = \Phi_Z(z^i) - \sum_{n=0}^{i-1} \sum_t \coprod_{n,t}(i, i)$ is the stationary mass of individuals at income level z^i minus those with income z^i and consumption above the treashold c_0^i.

Ad 4: The conditional distribution of consumption

The strictly positive entries of F ensure that the least upper bound of consumption by individuals at income $z^i, i = 2, ..., N$ is the first downward step from the threshold level for z^1, $c^1(c_0^1, R) = c_1^1$. The greatest lower bound of consumption for indivdiuals at income z^i is of course threshold value c_0^i, so the minimum interval covering the discrete support of consumption conditional on income $z^i, i > 1$ is $[c_1^1, c_0^i]$. Since c_0^i increases with income, the width of the interval decreases.

Monotonicity of transitions ensures that individuals at lower incomes are concentrated in lower parts of subdistributions, which can be shown to lead to conditional means that increase in income. Conditional variances on the other hand are non-monotononic. Assuming i.i.d. uncertainty, however, or identical rows in F, it is evident that $\Phi_{C|z^i}$ is simply $\Phi_{C|z^{i+1}}$ with a truncated tail, and the tail-mass moved to the truncation point. This implies conditional variances that decrease monotononously in income values.

[1] To keep notation concise I take $\Pi_{i=1}^0 x_i = 1, \Sigma_{i=1}^0 x_i = 0, \forall x_i$.

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C.3 Proof of Corollary 3.7

With CRRA preferences and 2 income values, the following is true:

1. *The covariance between income and consumption is positive. The covariances between income and both financial returns and wealth are negative.*
2. *The mean of consumption increases in income. Its conditional variance decreases.*
3. *If $\Phi_C(c_m) \approx 0$, the cross-sectional variance of log-consumption in stationary equilibrium is*

$$Var_c = \tau[\frac{\log(\beta R)}{\sigma}]^2 \tag{C.8}$$

where $\tau > 0$ is a function of transition probabilities only. If there is a non-negligible mass at the truncation point, $\Phi_C(c_m) > 0$, this is an upper bound for the cross-sectional variance of individual consumption.

Proof

Ad 1: The covariance of income and consumption is given by

$$E(c - \mu_c)(z - \overline{z}) \tag{C.9}$$

$$= \nu(c_1 - \mu_c)(z^h - \overline{z}) + \sum_2^m \nu(1-p)q^{i-1}(c_i - \mu_c)(z^l - \overline{z}) \tag{C.10}$$

$$= \nu(c_1 - \mu_c)[(z^h - \overline{z}) - (z^l - \overline{z})] \tag{C.11}$$

$$= \nu(c_1 - \mu_c)[z^h - z^l] > 0 \tag{C.12}$$

where μ_c is the mean of consumption, and the second equality imposes market clearing $\sum_2^m \nu(1-p)q^{i-1}(c_i - \overline{c}) = -\nu(c_1 - \overline{c})$.

To see the second statement, note that the joint distribution of financial returns and income is $\Phi_{y_{fin},z} : \mathbb{B}([c_1 - z^h, c_1 - z^l]) \times \{z^l, z^h\} \longrightarrow [0,1]$ is given by

$$\Phi(y_{fin,1}, z^h) = \frac{1-q}{2-q-p} \doteq \nu \tag{C.13}$$

$$\Phi(y_{fin,i}|_{1<i<m}, z^l) = \nu(1-p)q^{i-1} \tag{C.14}$$

$$\Phi(y_{fin,m}, z^l) = \nu\frac{(1-p)q^{m-1}}{1-q} \tag{C.15}$$

$$\Phi(\cdot,\cdot) = 0 \ otherwise \tag{C.16}$$

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for

$$\begin{aligned} y_{fin,1} &= c_1 - z_h < 0 \\ y_{fin,i} &= z_l - c_1(\beta R)^{\frac{i}{\sigma}} > 0, 1 < i < m \\ y_{fin,i} &= 0 \end{aligned} \tag{C.17}$$

where $\mathbb{B}(I)$ denotes the Borel sets on interval I. So high income agents have strictly negative financial income, while individuals with low income have non-negative financial income. This of course implies negative covariance, equal to

$$E(y_{fin} - \overline{y_{fin}})(z - \overline{z}) \tag{C.18}$$

$$\nu(c_1 - z^h)(z^h - \overline{z}) + \sum_2^m \nu(1-p)q^{i-1}(c_i - z^l)(z^l - \overline{z}) \tag{C.19}$$

$$\leq \nu(c_1 - z^h)(z^h - \overline{z}) < 0 \tag{C.20}$$

where the last line follows from $(c_i - z^l)(z^l - \overline{z}) \leq 0,\ \forall i > 1$ The joint distribution of financial wealth and income $\Phi_W : \mathbb{R} \times \{z^l, z^h\} \longrightarrow [0,1]$ has the same frequencies as $\Phi_{y_{fin},z}$, on a support defined by the recursion

$$A_m = \frac{1-p}{R} A_1 \tag{C.21}$$

$$\begin{aligned} A_i &= y_{fin,i} + \frac{1-p}{R}A_1 + \frac{p}{R}A_{i+1} > A_{i+1}, 1 < i \leq m-1 \\ A_1 &= \frac{1}{1-\frac{q}{R}} y_{fin,1} + \frac{1-q}{R}A_2 < 0 \\ &\mathit{and} \end{aligned} \tag{C.22}$$

$$A_1 = \frac{R(R-p)}{R(R-q-p)-(1-q-p)}[c_1 \frac{R + ((1-q)(1-(\frac{p}{R}(\beta R)^{\frac{1}{\sigma}})^{m-2} - p)(\beta R)^{\frac{m-2}{\sigma}})}{R - p(\beta R)^{\frac{1}{\sigma}}} \tag{C.23}$$

$$-\frac{(1-q)(1-(\frac{p}{R})^{m-2})}{R-p} z_l - z_h] \tag{C.24}$$

The covariance of income and financial wealth is given by

$$E(A - \overline{A})(z - \overline{z}) \tag{C.25}$$

$$= \nu(A_1)(z^h - \overline{z}) + \sum_2^m \nu(1-p)q^{i-1}(A_i)(z^l - \overline{z}) \tag{C.26}$$

$$= (z^h - \overline{z})\nu(A_1) - (z^l - \overline{z})\nu(A_1) \tag{C.27}$$

$$= (z^h - z^l)\nu(A_1) \tag{C.28}$$

$$< 0 \tag{C.29}$$

where the third line exploits the fact that financial wealth sums to zero across individuals. **Ad 2:** Both statements follow immediately from the fact that high-income individuals

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are located at a mass point on the upper bound of the consumption support.

Ad 3:

1. Denote the first entry of the normalised left eigenvector of transition matrix F associated with a unit eigenvalue as $\nu = \frac{(1-q)}{(2-q-p)}$, and the log of x as $\widehat{x}$.

2. The mean of $\log c$ is

$$\mu_c = \nu\{\widehat{c_h} + (1-p)\sum_{i=1}^{\infty}[\frac{\widehat{\beta R}}{\sigma}i + \widehat{c_h}]q^{i-1}\} \tag{C.30}$$

$$= \widehat{c_h} + \frac{1-p}{(1-q)(2-q-p)}\frac{\widehat{\beta R}}{\sigma} \tag{C.31}$$

3. The variance is

$$VAR_c = \quad \nu\frac{(1-p)^2}{(1-q)^2(2-q-p)^2}[\frac{\widehat{\beta R}}{\sigma}]^2 \tag{C.32}$$

$$+\nu(1-p)[\frac{\widehat{\beta R}}{\sigma}]^2\sum_{i=1}^{\infty}[i - \frac{(1-p)}{(1-q)(2-q-p)}]^2 q^{i-1} \tag{C.33}$$

$$= \quad \nu[\frac{\widehat{\beta R}}{\sigma}]^2\{\frac{(1-p)^2}{(1-q)^2(2-q-p)^2} \tag{C.34}$$

$$+(1-p)[\frac{(1+q)}{(1-q)^3} - 2\frac{(1-p)}{(1-q)^3(2-q-p)} + \frac{(1-p)^2}{(1-q)^3(2-q-p)^2}]\} \tag{C.35}$$

$$= \quad \nu[\frac{\widehat{\beta R}}{\sigma}]^2[-\frac{(1-p)^2}{(1-q)^3(2-q-p)} + \frac{(1-p)(1+q)}{(1-q)^3}] \tag{C.36}$$

$$= \quad [\frac{\widehat{\beta R}}{\sigma}]^2\frac{(1-p)(1+q(1-p-q))}{(1-q)^2(2-p-q)^2} \tag{C.37}$$

The more general result for the truncated case with $\Phi_C(c_m) > 0$ is not difficult, but algebraically messy, to compute. But note that the variance of an truncated geometric distribution is stricly lower, and that for the i.i.d. case $1-p=q$ both the mean and the variance reduce to those for an ordinary geometric distribution. ■

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References

1. Ahearne, Alan G., William L. Griever, and Francis E. Warnock (2004). "Information Costs and Home Bias: an Analysis of US Holdings of Foreign Equities", Journal of International Economics Vol. 62, 313-336.

2. Albarran, Pedro and Orazio P. Attanasio (2003). "Limited Commitment and Crowding out of Private Transfers: Evidence from a Randomised Experiment", Economic Journal Vol. 113 No. 486, C77-C85.

3. Alvarez, Fernando and Urban J. Jermann (2000). "Efficiency, Equilibrium, and Asset Pricing with Risk of Default", Econometrica, Vol. 68 No. 4, 775-798.

4. Atkeson, Andrew and Robert E. Lucas (1992). "On Efficient Distribution with Private Information", Review of Economic Studies, Vol. 59 No. 3, 427-53.

5. Atkeson, Andrew and Robert E. Lucas (1995). "Efficiency and Equality in a Simple Model of Unemployment Insurance", Journal of Economic Theory, Vol. 66 No. 1, 64-88.

6. Attanasio, Orazio and Steven J. Davis (1996). "Relative Wage Movements and the Distribution of Consumption", Journal of Political Economy, Vol. 104 No. 6, 1227-62.

7. Attanasio, Orazio and Nicola Pavoni (2007). "Risk Sharing in Private Information Models with Asset Accumulation: Explaining the Excess Smoothness of Consumption", Mimeo, UCL.

8. Ayagari, S. Rao (1993). "Uninsured idiosyncratic Risk and Aggregate Saving", Federal Reserve Bank of Minneapolis Working Paper 502.

9. Baxter, Marianne and Urban J. Jermann (1997). "The International Diversification Puzzle Is Worse Than You Think", American Economic Review, Vol. 87 No. 1, 170-80.

Broer, Tobias (2009), Heterogeneous Individuals in the International Economy
European University Institute DOI: 10.2870/13714

10. Bloise, Gaetano, Pietro Reichlin, and Marco Tirelli (2009). "Indeterminacy of Competitive Equilibrium with Risk of Default". CEPR Discussion Paper No. 7477.

11. Blundell, Richard, Luigi Pistaferri and Ian Preston (2008). "Consumption Inequality and Partial Insurance", American Economic Review, Vol. 98 No. 5, 1887-1921.

12. Barr, Abigail and Garance Genicot (2008). "Risk Sharing, Commitment, and Information: An Experimental Analysis", Journal of the European Economic Association, Vol. 6 No. 6, 1151-1185.

13. Battistin, Erich, Richard Blundell and Arthur Lewbel (2007). "Why is consumption more log normal than income? Gibrat's law revisited", IFS Working Papers W07/08, Institute for Fiscal Studies.

14. Botev, Z. I., J. F. Grotowski and D. P. Kroese (2008), "Kernel density estimation via diffusion", University of Queensland, mimeo.

15. Bottazi, Laura, Paolo Pesenti, and Eric van Wincoop (1996). "Wages, Profits and the International Portfolio Puzzle", European Economic Review, Vol. 40 No. 2, 219-54.

16. Brandolini, Andrea and Timothy M. Smeeding (2007). "Inequality Patterns in Western-Type Democracies: Cross-Country Differences and Time Changes", Luxembourg Income Study Working Paper No. 458.

17. Broner, Fernando A., Alberto Martin and Jaume Ventura (2006). "Sovereign Risk and Secondary Markets", mimeo.

18. Burger, John D. and Francis E. Warnock (2004). "Foreign participation in local-currency bond markets", Review of Financial Economics, Vol. 16 No. 3, 291-304.

19. Campbell, John Y. (2006). "Household Finance", Journal of Finance, Vol. 61 No. 4, 1553-1604.

20. Campbell, John Y. (2007). Portfolio Theory, mimeo.

21. Carroll, Christopher D. (2002). "Portfolios of the rich", in: Luigi Guiso, Michael Haliassos and Tullio Jappelli (eds.), Household Portfolios, MIT Press, Cambridge, MA.

22. Coeurdacier, Nicolas and Pierre-Olivier Gourinchas (2009). "When Bonds Matter: Home Bias in Goods and Assets", mimeo.

Broer, Tobias (2009), Heterogeneous Individuals in the International Economy
European University Institute DOI: 10.2870/13714

23. Cole Harold L. and Maurice Obstfeld (1991). "Commodity Trade and International Risk Sharing", Journal of Monetary Economics Vol. 28 , 3-24.

24. Cordoba, Juan-Carlos (2008). "U.S. inequality: Debt constraints or incomplete asset markets?", Journal of Monetary Economics, Vol. 55 No.2, 350-364.

25. Dardanoni, Valentino (1995). "Income distribution dynamics: monotone Markov chains make light work", Social Choice and Welfare, Vol. 12, 181-192.

26. Dynan, Karen E., Douglas W. Elmendorf and Daniel E. Sichel (2006). "Financial innovation and the Great Moderation: what do household data say?", Proceedings, Federal Reserve Bank of San Francisco.

27. Fogli, Alessandra and Fabrizio Perri (2006). "The "Great Moderation" and the US External Imbalance", Monetary and Economic Studies, Vol. 26, 209-225.

28. Guiso, Luigi, Michael Haliassos and Tullio Jappelli (2003). "Household Stockholding in Europe: Where do we stand, and where do we go?", CEPR Discussion Paper Series No. 3694.

29. Hall, Robert E. and Mishkin, Frederic S. (1982). "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households," Econometrica Vol. 50 No. 2, 461-81.

30. Heathcote, Jonathan, Kjetil Storesletten and Giovanni L. Violante (2008a). "The Macroeconomic Implications of Rising Wage Inequality in the United States", NBER Working Paper 14052.

31. Heathcote, Jonathan, Fabrizio Perri and Giovanni L. Violante (2008b). "Cross-Sectional Facts for Macroeconomists: United States (1967-2006)", mimeo.

32. Heathcote, Jonathan and Fabrizio Perri (2007). The international diversification puzzle is not as bad as you think, NBER Working Paper 13483.

33. Heckman, James J (1979). "Sample Selection Bias as a Specification Error", Econometrica, Vol. 47 No. 1, 153-161.

34. Huggett, Mark (1993). "The risk-free rate in heterogeneous-agent incomplete-insurance economies", Journal of Economic Dynamics and Control, Vol. 17 No. 5-6, 953-969.

35. Kaplan, Greg and Giovanni L. Violante (2009). "How much insurance in Bewley models?", mimeo, New York University.

Broer, Tobias (2009), Heterogeneous Individuals in the International Economy
European University Institute DOI: 10.2870/13714

36. Karolyi, G. Andrew and Rene M. Stulz (2003). “Are financial assets priced locally or globally?”, Handbook of the Economics of Finance, in: G.M. Constantinides, M. Harris and R. M. Stulz (ed.). Handbook of the Economics of Finance, Edition 1, Vol. 1, Chapter 16, 975-1020.

37. Kehoe, Timothy J. and David K. Levine (2001). “Liquidity constrained markets versus debt constrained markets”, Econometrica, Vol. 69 No. 3, 575-598.

38. Kehoe, Timothy J. and David K. Levine (1993). “Debt-Constrained Asset Markets”, Review of Economic Studies, Vol. 60 No. 4, 865-888.

39. Kennickell, Arthur B. (1999). “Revisions to the SCF Weighting Methodology: Accounting for Race/Ethnicity and Homeownership”, Federal Reserve Board, mimeo.

40. Kocherlakota, Narayana R. (1996). “Implications of Efficient Risk Sharing without Commitment”, Review of Economic Studies, Vol. 63 No. 4, 595-609.

41. Krueger, Dirk and Fabrizio Perri (2009). “How do Households Respond to Income Shocks?”, mimeo.

42. Krueger, Dirk and Fabrizio Perri (2006). “Does Income Inequality Lead to Consumption Inequality? Evidence and Theory”, Review of Economic Studies, Vol. 73 No. 1, 169-99.

43. Krueger, Dirk and Fabrizio Perri (2005). “Public or Private Risk Sharing”, mimeo.

44. Krueger, Dirk und Harald Uhlig (2006). “Competitive risk sharing contracts with one-sided commitment”, Journal of Monetary Economics, Vol. 53 No. 7, 1661-1691.

45. Krusell, Per and Anthony A. Smith Jr. (1998). “Income and Wealth Heterogeneity in the Macroeconomy”, Journal of Political Economy, Vol. 106 No. 5, 867-896.

46. Lane, Philip R. and Gian Maria Millesi-Ferreti (2007). “The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, $1970-2004$”. Journal of International Economics, Vol. 73 No. 2, 223-250.

47. Lewis, Karen K. (1997). Trying to Explain Home Bias in Equities and Consumption, Journal of Economic Literature Vol. 37, 571-608.

48. Ligon, Ethan, Jonathan P. Thomas and Tim Worrall (2002). “Informal Insurance Arrangements with Limited Commitment: Theory and Evidence from Village Economies”, Review of Economic Studies, Vol. 69 No. 1, 209-44.

49. Marcet, Albert and Ramon Marimon (2009). “Recursive Contracts”, mimeo.

Broer, Tobias (2009), Heterogeneous Individuals in the International Economy
European University Institute DOI: 10.2870/13714

50. Mendoza Enrique G., Vincenzo Quadrini and Jose-Victor Rios-Rull (2007). "Financial Integration, Financial Deepness and Global Imbalances", NBER Working Paper 12909.

51. Obstfeld, Maurice and Kenneth Rogoff (2000). "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?", NBER Macroeconomics Annual.

52. Perloff, Jeffrey M. and Ximing Wu (2005). "China's Income Distribution, 1985-2001", The Review of Economics and Statistics, Vol. 87 No. 4, 763-775.

53. Roubini, Nouriel and Brad Setser (2004). "The US as a Net Debtor: The Sustainability of the US External Imbalances", mimeo 2004.

54. Roubini, Nouriel (2005). Discussion of "Current Account Fact and Fiction" by David Backus and Frederic Lambert, mimeo.
At https://nber15.nber.org/c/2005/si2005/ifm/roubini.ppt.

55. Schmitt-Grohé, Stephanie and Martin Uribe (2003). "Closing small open economy models", Journal of International Economics, Vol. 61 No. 1, 163-185.

56. Storesletten, Kjetil, C. Telmer, and A. Yaron (2004). "Consumption and Risk Sharing over the Life-Cycle", Journal of Monetary Economics, Vol. 51 No. 3, 609633.

57. Stulz, René M. (2005). "The Limits of Financial Globalization", Journal of Finance, Vol. 60 No. 4, 1595-1638.

58. Tauchen, George and Robert Hussey (1991). "Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models", Econometrica, Vol. 59 No. 2, 371-396.

59. Tesar, Linda L. and Ingrid M. Werner (1995). "Home Bias and the High Turnover", The Journal of International Money and Finance, Vol. 14, 467-492.

60. Thomas, Jonathan and Tim Worrall (1988). "Self-enforcing Wage Contracts", Review of Economic Studies, Vol. 55 No. 4, 541-54.

61. Townsend, Robert M (1994). "Risk and Insurance in Village India", Econometrica, Vol. 62 No. 3, 539-91.

62. Uhlig, Harald (1996). "A Law of Large Numbers for Large Economies", Economic Theory, Vol. 8 No. 1, 41-50.

63. Van Wincoop, Eric and Francis E. Warnock (2006). "Is Home Bias in Assets related to Home Bias in Goods?" NBER Working Paper No. 12728.

Broer, Tobias (2009), Heterogeneous Individuals in the International Economy
European University Institute DOI: 10.2870/13714

64. Zeldes, Stephen P. (1989). "Consumption and liquidity constraints: an empirical investigation", Journal of Political Economy, Vol. 97 No. 2, 305-346.

Broer, Tobias (2009), Heterogeneous Individuals in the International Economy
European University Institute DOI: 10.2870/13714