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Financial Connections and Systemic Risk*

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Abstract

An important source of systemic risk is overlapping portfolio exposures among financial institutions. We develop a model where institutions form connections through swaps of projects in order to diversify their individual risk. These connections lead to two different network structures. In a clustered network groups of financial institutions within a cluster hold identical portfolios. Defaults occur together but the number of states where this happens is small. In an unclustered network defaults are more dispersed but they occur in more states. With long term finance there is no difference between the two structures in terms of total defaults and welfare. In contrast, when short term finance is used, the network structure matters. Upon the arrival of a signal about banks' future defaults, investors update their expectations of the ability of financial institutions to repay them. If their updated expectations are low, they do not to roll over the debt and there is systemic risk in that all institutions are early liquidated. We compare the clustered and unclustered networks and analyze which is better in welfare terms.

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1 Introduction

Understanding the nature of systemic risk is key to understanding the occurrence and propagation of financial crises. The term usually refers to a situation where many (if not all) financial institutions fail as a result of a common shock or a contagion process. Herring and Wachter (2001) and Reinhart and Rogoff (2009) find evidence that a collapse of commercial real estate values is the main cause for system wide failures of financial institutions during many financial crises. Allen and Gale (2000), Freixas, Parigi and Rochet (2000) and numerous other subsequent papers (see Allen, Babus and Carletti, 2009, for a survey) analyze the risk of contagion where the failure of one financial institution leads to the default of other financial institutions through a domino effect. This type of systemic risk is often used by central banks as the justification for intervening and bailing out institutions that are "too big to fail".

The recent developments in financial markets and the crisis that started in 2007 have highlighted the importance of another type of systemic risk related to the overlap of portfolios and the structure of connections among financial institutions. The emergence of financial instruments in the form of credit default swaps and other credit derivative products, loan sales and collateralized loan obligations have increased the possibility for financial institutions to diversify risk. However, they have also led to more overlap and more similarities among their portfolios. Greater individual diversification may therefore imply greater systemic risk in that the failure of one institution is more likely to coincide with the failure of other institutions having similar portfolios.

In this paper we focus on this latter kind of systemic risk. We analyze a situation where financial institutions - which we call banks - choose individually to exchange projects with others in order to diversify their default risk. This reduces the default probability of individual financial institutions and the cost of raising outside capital from investors because bankruptcy costs are lowered. However, the exchange of projects with other financial institutions also leads to an overlap of intermediaries' portfolios and to a multiplicity of connections among them, which may generate systemic risk. The extent to which this occurs depends on the number and shape of connections among financial institutions as well as on their funding structure.

We develop a simple two-period model, where initially each bank invests in a risky project and needs external funds to finance it. Investors provide the funds to the banks in exchange for a debt contract. We initially consider the case of long term debt and subsequently that of short term debt. As projects are risky, banks may default at the final date. When this occurs, investors recover the return of the bank's project net of bankruptcy costs, while the bank does not receive anything. When default does not occur, investors obtain the repayment specified in the debt contract and the bank retains any surplus. As project returns are independently distributed, each bank has an incentive to diversify by exchanging shares of its own project with other banks. This lowers banks' individual default probabilities and allows them to promise investors a lower repayment. However, exchanging projects is costly. Banks incur a cost for each additional project they exchange, which can be interpreted as a due-diligence effort banks exert when they acquire new projects from other banks. In equilibrium, the number of project exchanges is determined by the trade-off between the advantages of diversification and the due-diligence costs.

The exchange of project shares creates connections - or links - among banks and overlaps in their portfolios. The degree of overlap depends on the particular form of banks' connections. For ease of exposition, we focus on the case of six banks with each of them forming two connections with other banks. This generates two different shapes of connections or, more precisely, network structures. In one, which we call clustered, banks are connected in two clusters of three banks each. Within each cluster all banks hold the same portfolio, but the two clusters are independent of each other. In the second network, which we call unclustered, banks are connected in a circle. Each of them exchanges projects only with the two neighboring banks so that none of the banks holds identical portfolios. In this sense, risk is less concentrated in the unclustered than in the clustered network.

We show that with long term debt the structure of the network does not matter for systemic risk and welfare. Although the two networks entail different pairwise correlations among banks' portfolios, in either network each bank's portfolio is formed by three independently distributed projects with the same distribution of returns. Thus, the number of bank defaults and the expected costs of default are the same in the two structures and so is total welfare.

In contrast, the structure of the network plays an important role in determining systemic risk and welfare when banks use short term debt. The main difference is that investors must recover their opportunity cost each period. Thus at the intermediate date they reconsider their investment and decide whether to roll it over. Investors condition their roll over decision on the arrival of a signal on banks' future solvency. The signal indicates whether all banks will be solvent in the final period (good news) or whether at least one of them will default and will not be able to repay investors the repayment promised in the debt contract (bad news). Upon observing the signal, investors update the probability that their bank will be solvent at the final period and roll over the debt if they expect to be able to recover their opportunity cost for another period. Thus, they always leave the funds at the bank when the good signal is realized, as this indicates that no bank will default. However, when the bad signal arrives, they may not roll over the debt, thus forcing the banks into early liquidation. This source of systemic risk is the focus of our analysis. The decision of the investors to roll over the debt upon the arrival of the bad signal depends on the structure of the network and on the amount that investors obtain after bankruptcy costs in case the bank defaults.

We show that, upon the arrival of bad news, roll over occurs less often in the clustered network than in the unclustered network. When investors receive a high enough proportion of banks' portfolio returns in the case of default −so that bankruptcy costs are low−, debt is rolled over in both networks. As the amount accruing to investors in case of default decreases and investors' opportunity cost increases, debt is still rolled over in the unclustered network but not in the clustered one. The reason is that risk is more concentrated in the clustered network than in the unclustered network. Given that banks have identical portfolios within each cluster, the arrival of a bad signal in the clustered network indicates that at least three banks will default in the final period. Investors infer that the probability of default conditional on the bad signal is high and thus decide not to roll over if they can recover little in case of default. In contrast, in the unclustered network risk is less concentrated as the portfolios held by banks are diverse. Thus, the arrival of the bad signal indicates a lower probability of a rash of bank defaults and banks are more often able to roll over their debt and continue till the final period. When investors obtain little in case of bank default because of high bankruptcy costs and have high opportunity cost, banks are early liquidated in both networks.

The welfare properties of the two network structures with short term finance depend on the investors' roll over decisions and on the proceeds obtainable when banks are early liquidated. When banks are continued and offer investors a repayment of the same magnitude in either network, total welfare is the same in the network structures. When the debt renewal in the intermediate period requires a higher promised repayment in the clustered network relative to the unclustered, then welfare is higher in the latter network. Although the debt is rolled over in both networks, banks face a higher default probability in the clustered network due to the higher promised repayment when the bad signal arrives. This entails higher bankruptcy costs and thus lower welfare. As debt starts not being rolled over and banks are early liquidated in the clustered network only, the comparison of total welfare becomes ambiguous. Initially, when neither the bankruptcy costs nor the proceeds from early liquidation are too high, total welfare remains higher in the unclustered network than in the clustered one. However, as investors start recuperating little in the case of default and more in the case of early liquidation, welfare becomes higher in the clustered network and remains so even when early liquidation occurs in both network structures.

Our paper is related to several strands of literature. Concerning the effects of diversification on banks' portfolio risk, Shaffer (1994) argues that while diversification is good for each bank individually, it can lead to greater systemic risk as banks' investments become more similar. Wagner (2009) shows in a model with two banks that diversification can increase the likelihood of systemic crises and thus be undesirable. Ibragimov, Jaffee and Walden (2010) identify conditions under which it may be socially optimal to have financial intermediaries hold less diversified portfolios in order to have a lower probability of widespread collapses. In these papers, banks always have the same portfolios and social welfare is non-linearly decreasing in the number of bank failures in the system. We consider a more general framework where the degree of diversification, the network structure and the funding structure of financial institutions interact in determining systemic risk and welfare.

In terms of the roll over risk entailed by short term finance, Acharya, Gale and Yorulmazer (2009) explain market freezes in the presence of roll over risk based on incoming information and transaction costs. He and Xiong (2009) show that roll over risk leads to dynamic bank runs. Concerning liquidity risk more generally, Diamond and Rajan (2009) find that liquidity dry-ups can arise from the fear of fire sales; while Bolton, Santos and Scheinkman (2009) look at maturity mismatch and its impact on liquidity demand. All these studies use a representative bank/agent framework. By contrast, we analyze how different network structures affect the roll over risk resulting from short term finance.

More generally, our paper is also related to a strand of literature stressing the importance of externalities among banks as a source of systemic risk (see Allen and Babus, 2009, for a survey on contagion in financial networks). For example, Boyson, Stahel and Stulz (2008) provide evidence of such externalities within the hedge fund sector, while Billio et al. (2010) measure the interconnectedness among hedge funds, banks, brokers, and insurance companies and their impact on systemic risk. Adrian and Brunnermeier (2009) and Danielsson, Shin and Zigrand (2009) point out that designing regulation on individually optimal risk management may not be appropriate. Our paper relates to this literature in that it analyzes how the individual choice of the optimal degree of diversification may lead to multiple network structures with very different properties in terms of systemic risk and welfare.

Some other papers study the extent to which banks internalize the negative externalities that arise from contagion. For instance, Babus (2009) proposes a model where banks share the risk that the failure of one bank propagates through contagion to the entire system. Castiglionesi and Navarro (2010) show that an agency problem between shareholders and debt holders of a bank leads to fragile financial networks. Zawadowski (2010) takes a different approach to show that banks that are connected in a network of hedging contracts fail to internalize the negative effect of their own failure. Banks funded with short-term debt hold insufficient capital to prevent lenders from running. All these papers rely on a domino effect as a source of systemic risk. By contrast, we focus on diversification and overlaps in banks' portfolios as a source of systemic risk in the presence of information externalities.

The rest of the paper proceeds as follows. Section 2 lays out the basic model when banks use long term debt. Section 3 describes the equilibrium that emerges in this case in terms of the individually optimal degree of diversification and the multiple network structures that can arise from it. Section 4 introduces short term debt and analyzes investors' decision to roll over the debt in response to information about banks' future solvency as well as the welfare properties of the different network structures. Finally, Section 5 concludes.

2 The basic model with long term finance

Consider a three-date ($t = 0, 1, 2$) economy with six banks, denoted by $i = 1, ..., 6$, and a continuum of small, risk-neutral investors. Each bank i has access at date 0 to an investment project that yields a stochastic return $\theta_i = \{R_H, R_L\}$ at date 2 with probability p and $1-p$, respectively, with $R_H > R_L > 0$. The returns of the projects are independently distributed across banks.

Each bank raises one unit of funds from investors at date 0 and offers them, in exchange, a long term debt contract that specifies an interest rate r to be paid at date 2. Investors provide finance to one bank only and are willing to do so if they expect to recover at least their opportunity cost deriving from a long term risk free asset with a per period return

r_F and a two period return $r_F^2 < E(\theta_i)$.

We assume that $R_H > r_F^2 > R_L$ so that a bank can pay r only when the project yields a high return. When the project yields a low return R_L, the bank defaults and distributes the returns of the project among the investors. They recover a fraction $\alpha \in [0, 1]$ of the project return and the remaining fraction $(1 - \alpha)$ is lost as bankruptcy costs. Then, investors will finance the banks only if their participation constraint as given by

$$pr + (1 - p)\alpha R_L \geq r_F^2$$

is satisfied. When the project returns R_H, banks acquire the surplus $(R_H - r)$. Otherwise, they receive 0.

Given that projects are risky and returns are independently distributed, banks can reduce their default risk through diversification. In particular, we suppose that each bank exchanges shares of its own project with ℓ_i other banks and that connections are bilateral. That is, bank i exchanges a share of its project with bank j if and only if bank j exchanges a share of its project with bank i. When this happens, there is a link between banks i and j denoted as ℓ_{ij}. Then each bank i ends up with a portfolio of $\ell_i + 1$ projects with a return equal to

$$X_i = \frac{\theta_{i1} + \theta_{i2} + ... + \theta_{i\ell_i+1}}{\ell_i + 1}.$$

Exchanging shares of projects with other banks entails a cost c per link. This can be interpreted as a due diligence cost. The idea is that banks know their own project, but they do not know that of the other banks. Thus they need to exert costly effort to check that the projects of the banks they want to form links with are bona fide as well.

The exchange of project shares creates linkages among banks. The collection of all linkages can be described as a network g of overlapping portfolios. In a network g, each bank has shares of $\ell_i + 1$ independently distributed projects in its portfolio, but banks' portfolios now overlap. The degree of overlap depends on the number of project exchanges or links ℓ_i that each bank has with other banks and on the structure of interconnections among banks for a given number of links.

3 Long term finance

When banks exchange shares of their own projects with other banks, they diversify risk. Since diversification creates links among banks, we model how banks make portfolio investment decisions as a network game. To find the equilibrium network structures, we first derive the participation constraint of the investors and banks' profits when each bank i has ℓ_i links with other banks and holds a portfolio of $\ell_i + 1$ projects.

We denote as $r \equiv r(g, \ell_i)$ the interest rate that bank i promises investors in a network g where banks have ℓ_i links and $\ell_i + 1$ projects. Investors receive r at date 2 when the return of bank i's portfolio, X_i, is $X_i \geq r$, while they receive the return of the portfolio net of bankruptcy costs when $X_i < r$. The participation constraint of the investors is then given by

$$\Pr(X_i \geq r)r + \alpha E(X_i < r) \geq r_F^2, \tag{1}$$

where $E(X_i < r) = \sum_{x<r} x \Pr(X_i = x)$. The first term represents the expected return of the investors when the bank is solvent at date 2 and repays them the promised interest rate. The second term is instead investors' expected return when the bank defaults and investors only receive the bank's expected portfolio return net of the bankruptcy costs. The equilibrium r is the lowest interest rate that satisfies (1) with equality. Diversification increases the probability $\Pr(X_i \geq r)$ that investors receive their promised return so that banks can offer a lower rate of return when they exchange projects.

Banks receive the surplus $X_i - r$ whenever $X_i \geq r$. The expected profit of a bank i in a network g is

$$\pi_i(g) = E(X_i \geq r) - \Pr(X_i \geq r)r - \ell_i c, \tag{2}$$

where $E(X_i \geq r) = \sum_{x \geq r} x \Pr(X_i = x)$. Substituting the equilibrium interest rate r into

(2), the expected profit of bank i becomes

$$\pi_i(g) = E(X_i) - r_F^2 - (1-\alpha)E(X_i < r) - \ell_i c. \tag{3}$$

Thus, the bank's expected profit is given by the expected return of its portfolio net of the bankruptcy costs minus the opportunity cost of the investors and the total due diligence costs to form links. Greater diversification increases the bank's expected profit as it reduces the default probability and thus the promised interest rate to investors, but it also entails greater total due diligence costs.

Banks choose to form connections in order to maximize their expected profits. In particular, they choose the number of banks ℓ_i with which to form links. The equilibrium choice of links determines the structure of the network g. The formation of a link ℓ_{ij} requires the consent of both banks i and j, while any bank i has the discretion to unilaterally terminate links in which it is involved. A network g is an equilibrium if it satisfies the notion of *pairwise stability* introduced by Jackson and Wolinsky (1996). This is defined as follows.

Definition 1 *A network g is pairwise stable if*

(i) for any pair of banks i and j that are linked in the network g, none of them has an incentive to unilaterally sever their link ℓ_{ij}. That is, the expected profit each of them receives from deviating to the network $(g - \ell_{ij})$ is not larger than the expected profit that each of them obtains in the network g ($\pi_i(g - \ell_{ij}) \leq \pi_i(g)$ and $\pi_j(g - \ell_{ij}) \leq \pi_j(g)$);

(ii) for any two banks i and j that are not linked in the network g, at least one of them has no incentive to form the link ℓ_{ij}. That is, the expected profit that at least one of them receives from deviating to the network $(g + \ell_{ij})$ is not larger than the expected profit that it obtains in the network g ($\pi_i(g + \ell_{ij}) \leq \pi_i(g)$ and/or $\pi_j(g + \ell_{ij}) \leq \pi_j(g)$).

In what follows we will concentrate on the case where in equilibrium banks find it optimal to choose $\ell = 2$ links and only symmetric networks are formed. The reason is that this is the minimum number of links such that there are multiple network structures.

To make the analysis more tractable, we impose conditions that restrict our attention to a range of parameters that ensures that for all $\ell = 0, ..., 5$ bankruptcy only occurs when all projects in a bank's portfolio pay off R_L. This requires that investors' participation constraint (1) is satisfied for a value of interest rate $r \in [r_F^2, \frac{\ell R_L + R_H}{\ell+1}]$, where $\frac{\ell R_L + R_H}{\ell+1}$ is the next lowest possible return realization of a bank's portfolio.

We focus on the case where the bank's profit function in (3) is concave in the number of links ℓ. Given $\Pr(X_i \geq r) = 1 - (1-p)^{\ell+1}$ and $\Pr(X_i < r) = (1-p)^{\ell+1}$, from (1) it is then also necessary that

$$(1 - (1-p)^{\ell+1}) \frac{\ell R_L + R_H}{\ell + 1} + (1-p)^{\ell+1} \alpha R_L \geq r_F^2 > R_L \tag{4}$$

for $\ell = 0, .., 5$. Sufficient conditions for this are

$$R_H > 3R_L \tag{5}$$

$$(1 - (1-p)^{\ell+1}) \frac{5R_L + R_H}{6} + (1-p)^{\ell+1} \alpha R_L \geq r_F^2. \tag{6}$$

Condition (5) guarantees that the left hand side of (4) decreases as ℓ increases from 0 to 5. This also implies that (6) is sufficient to guarantee $r < \frac{\ell R_L + R_H}{\ell+1}$ for $\ell = 0, .., 5$.

Given these conditions, (3) can be written as

$$\pi_i(g) = E(X_i) - r_F^2 - (1-p)^{\ell+1}(1-\alpha)R_L - \ell c \tag{7}$$

and it is possible to show the following.

Proposition 1 *For any $c \in [p(1-p)^3(1-\alpha)R_L, p(1-p)^2(1-\alpha)R_L]$ a network g^* where all banks have $\ell^* = 2$ links is pairwise stable and Pareto dominates equilibria with $\ell^* \neq 2$.*

Proof. See Appendix. ■

The choice banks make in equilibrium results from the trade off between the benefit of greater diversification in terms of lower expected bankruptcy costs with higher total due diligence cost.

For $\ell^* = 2$, there are two symmetric equilibrium networks g^* as shown in Figure 1. In the first network, that we define as clustered ($g = C$), banks are connected in two clusters of three banks each. Within each cluster, banks hold identical portfolios but the two clusters are independent of each other. In the second network, denoted as unclustered ($g = U$), banks are all connected in a circle. Each of them exchanges projects only with the two neighboring banks so that none of the banks holds identical portfolios. In this sense, risk is more concentrated in the clustered than in the unclustered network.

Both networks are pairwise stable if the due diligence cost c is in the interval $[p(1-p)^3(1-\alpha)R_L, p(1-p)^2(1-\alpha)R_L]$. No bank has an incentive to deviate by severing or adding a link as it obtains higher expected profit in equilibrium. Given that the bank's expected profit function is concave in ℓ and that investors' only recover their opportunity cost, the restriction on c guarantees also that the equilibrium with $\ell^* = 2$ is the best achievable.

We next consider welfare in the two networks. For either of them, the welfare per bank is the sum of a representative bank i's expected profit and its investors' expected returns. Given that the investors always recover their opportunity cost, this is simply given by

$$W(g) = E(X_i) - (1-\alpha)E(X_i < r) - \ell c. \tag{8}$$

Expression (8) indicates that in the case of long term financing total welfare per bank is just equal to the sum of each bank's expected portfolio return net of expected bankruptcy costs and due diligence costs. Although the two equilibrium networks entail different pairwise correlations among banks' portfolios, in either network each bank's portfolio is formed by $\ell + 1$ independently distributed projects with the same distribution of returns. This implies that in both networks all banks offer the same interest rate to investors and have the same bankruptcy probability. This gives the following result.

Proposition 2 *Total welfare is the same in the clustered and the unclustered networks.*

4 Short term finance

In the previous sections we have assumed that the maturity of the financing matches the maturity of the assets. In practice, particularly for financial institutions, this is usually not the case. Many banks use large amounts of short term debt. There are many reasons for this (see, e.g., Brunnermeier and Oehmke, 2009). For example, short term debt is usually cheaper than long term debt. In this section we analyze the case where banks use short term finance with per period short term risk free rate r_f. As with long term finance, we continue focusing on the clustered and unclustered networks with $\ell^* = 2$, and analyze whether the structure of the network matters for systemic risk and total welfare.

One important feature of short term finance is that it needs to be rolled over every period. If adverse information arrives, investors may refuse to roll over the debt thus forcing the bank into early liquidation. To capture this, we assume that a signal on the future banks' portfolio returns arrives at date 1. The signal can either indicate the good news that no banks will default at date 2 ($S = G$) or the bad news that at least one bank will default ($S = B$). The signal does not reveal any information about any individual bank. As far as individual investors are concerned, all banks look alike and have an equal probability of default once the signal arrives.

For tractability, we continue focusing on the parameter space where the only case where the opportunity cost of short term finance over the two periods r_f^2 cannot be covered is when all projects in a bank's portfolio return R_L so that we consider the range $R_L < r_f^2 < \frac{5R_L+R_H}{6}$. Thus, the good signal arrives when all banks' portfolios return at least $(2R_L + R_H)/3$ and investors are able to obtain the opportunity cost r_f^2 at date 2. The bad news arrives when at least one of the banks is unable to do so at date 2 as its portfolio returns R_L. This means that the probability of $S = G$

$$q(g) = \Pr(\textstyle\bigcap_i (X_i \geq r_f^2),$$

where $\Pr(\bigcap_i (X_i \geq r_f^2) = \Pr(X_1 \geq r_f^2, X_2 \geq r_f^2, ..., X_6 \geq r_f^2)$ represents the probability that none of the six banks defaults. The probability of $S = B$ is then $1 - q(g)$.

Figure 2 shows the timing of the model with short term finance. At date 0 each bank in network $g = C, U$ raises one unit of funds. Investors know the network structure, but do not know the position of any particular bank in the network. The short term debt contract promises investors an interest rate of $r_{01}(g)$ at date 1. This must be such that investors obtain their per period short term risk free rate r_f with $r_f^2 > R_L$. At the beginning of date 1, before investors are repaid $r_{01}(g)$, the signal S arrives. With probability $q(g)$ the signal $S = G$ reveals the good news that no banks will default at date 2. With probability $1 - q(g)$ the signal $S = B$ reveals the bad news that at least one bank will default at date 2. Upon observing the signal, investors decide whether to retain $r_{01}(g)$ or roll it over for a total promised repayment of $\rho_{12}^S(g)$ at date 2. Roll over occurs if $\rho_{12}^S(g)$ allows the investors to recover $r_{01}(g)r_f$ at date 2. Otherwise, the debt is not rolled over and the bank is forced into early liquidation at date 1. In this case, investors receive the risk free rate r_f and the bank receives zero.

Between date 0 and date 1 investors always recover their opportunity since they always recover their opportunity cost r_f at date 1, irrespective of whether the bank is continued or liquidated at date 1. This implies that they are always willing to finance the bank initially and that $r_{01}(g) = r_f$.

At date 1, investors decide whether to roll over the debt conditional on the realization of the signal. When $S = G$ no banks will default at date 2. Investors infer that they will always be able to receive the promised repayment $\rho_{12}^G(g) = r_{01}(g)r_f = r_f^2$ at date 2 so that they will be willing to roll over. Banks retain the return of the portfolio net of what they owe to investors, $X_i - r_f^2$.

When $S = B$, investors update the probability $\Pr(X_i \geq \rho_{12}^B(g)|B)$ that their bank will be able to repay them the total promised repayment $\rho_{12}^B(g)$ at date 2 and roll over their debt if they are able to recover their total opportunity cost r_f^2. Formally, roll over occurs if and only if there exists a total repayment $\rho_{12}^B(g)$ that satisfies investors' date 1 participation constraint

$$\Pr(X_i \geq \rho_{12}^B(g)|B)\rho_{12}^B(g) + \alpha E(X_i < \rho_{12}^B(g)|B) \geq r_f^2, \tag{9}$$

where $E(X_i < \rho_{12}^B(g)|B) = \sum_{x<\rho_{12}^B(g)} x \Pr(X_i = x|B)$ is the expected return of a bank's portfolio conditional on the bad signal $S = B$ when $X_i < \rho_{12}^B(g)$ and the bank defaults at date 2. Expression (9) indicates that investors obtain the promised total repayment $\rho_{12}^B(g)$ whenever $X_i \geq \rho_{12}^B(g)$ and obtain the expected return of bank portfolio $\alpha E(X_i < \rho_{12}^B(g)|B)$ net of the bankruptcy costs for $X_i < \rho_{12}^B(g)$.The equilibrium value of $\rho_{12}^B(g)$ if it exists, is the minimum promised repayment that satisfies (9) with equality and it minimizes the probability $\Pr(X_i < \rho_{12}^B(g)|B)$ of bank default conditional on $S = B$.

The expected profits of bank i at date 0 depend on the realization of the signal and investors' roll over decision at date 1. When the bank continues at date 1, the expected profit is given by

$$\pi_i(g) = q(g)\left[E(X_i \geq r_f^2|G) - r_f^2\right] + (1-q(g))\left[E(X_i \geq \rho_{12}^B(g)|B) - \Pr(X_i \geq \rho_{12}^B(g)|B)\rho_{12}^B(g)\right] - 2c.$$

where $E(X_i \geq \rho_{12}^B(g)|B) = \sum_{x\geq\rho_{12}^B(g)} x \Pr(X_i = x|B)$ and $E(X_i \geq r_f^2|G) = \sum_{x\geq r_f^2} x \Pr(X_i = x|G)$. The first term represents the expected profit when $S = G$. Investors receive r_f^2 at date 2 and the bank retains the expected surplus $E(X_i \geq r_f^2|G) - r_f^2$. The second term is the expected profit when $S = B$. Investors receive $\rho_{12}^B(g)$ with probability $\Pr(X_i \geq \rho_{12}^B(g)|B)$ and the bank obtains the remaining $E(X_i \geq \rho_{12}^B(g)|B) - \Pr(X_i \geq \rho_{12}^B(g)|B)\rho_{12}^B(g)$. Using (9), this simplifies to

$$\pi_i(g) = E(X_i) - r_f^2 - (1 - q(g))(1 - \alpha)E(X_i < \rho_{12}^B(g)|B) - 2c. \tag{10}$$

When the bank is early liquidated after $S = B$, it expects to make positive profits only when $S = G$. Thus its expected profit is only given by

$$\pi_i(g) = q(g)\left[E(X_i \geq r_f^2 | G) - r_f^2\right] - 2c. \tag{11}$$

Note that (10) and (11) imply that, in a given network g, the bank's expected profit is higher with continuation than with early liquidation.

The crucial difference between long and short term financing is that in the latter case the network structure matters for equilibrium interest rates, bank profits and ultimately total welfare whereas it does not in the former case. The reason is that the probability distribution of the signal and the associated conditional probabilities of bank default at date 2 differ in the two networks.

Tables 1 and 2 show all banks' portfolio return realizations and the number of banks defaulting. For simplicity, in what follows we assume that the probability of a project i returning R_H is $p = \frac{1}{2}$ so that all states are equally likely. Since there are 6 projects and each of them can have two possible returns, there are 64 states numbered in the first column describing the possible project return realizations at date 2.

Table 1 is for the clustered network. The first set of columns shows the return realizations of the six projects. The second set of columns shows each bank's portfolio returns in the two clusters. The last column shows the total number of bank defaults. There are 15 default states shaded in gray in the table. In 14 of these there are 3 banks defaulting and in 1 of them all 6 banks default since banks hold identical portfolios within a cluster. There are 48 bank defaults across all states.

Table 2 is for the unclustered network. The first set of columns shows the return realizations of the six projects, while the second set shows each bank's portfolio returns. The last column shows the total number of defaults. There are now 25 default states shaded in gray in the table. In 12 of these there is 1 bank defaulting, in 6 states 2 banks default, in 6 other states 3 banks default and in 1 state all 6 banks default. Again, there are 48 total bank defaults across all states, but they are now more spread out across the states so that there are more default states but with less banks defaulting on average. The reason is that in the unclustered network banks are all connected but none holds identical portfolios. Thus risk is less concentrated in the unclustered than in the clustered network.

These arguments imply that the probability of receiving the good signal $S = G$ is higher in the clustered network than in the unclustered network, that is

$$q(C) > q(U). \tag{12}$$

However, upon the realization of the bad signal, the probability of default is higher in the clustered than in the unclustered network. That is,

$$\Pr(X_i < \rho_{12}^B(C)|B) > \Pr(X_i < \rho_{12}^B(U)|B). \tag{13}$$

Tables 3 and 4 show the conditional distribution of banks' portfolio returns in the clustered and unclustered networks, respectively. In the clustered network there are 15 states where bankruptcy occurs. The total number of defaults is 48 and the number of banks in each state is 6. So the probability of default conditional on the bad signal is $\frac{48}{6\times 15} = \frac{8}{15}$ as shown in Table 3. From Table 1 it can be seen that there are 18 banks across all the default states with $X_i = \frac{2R_L+R_H}{3}$ (3 banks in cluster 1 in each of states 58, 59 and 63, and 3 banks in cluster 2 in each of states 60, 61, and 62). So the probability of $X_i = \frac{2R_L+R_H}{3}$ conditional on the bad signal is $\frac{18}{6\times 15} = \frac{3}{15}$. The other conditional probabilities in Table 3 can be calculated in a similar way.

The difference in the unclustered network is that there are 25 states where bankruptcy occurs. The total number of defaults remains 48 so that the probability of default conditional on the bad signal is $\frac{48}{6\times 25} = \frac{8}{25}$ as shown in Table 4. From Table 2 it can be checked that there are 66 banks across all the default states with $X_i = \frac{2R_L+R_H}{3}$ so that the probability of $X_i = \frac{2R_L+R_H}{3}$ conditional on the bad signal is $\frac{66}{6\times 25} = \frac{11}{25}$. Similarly for the other entries.

Given that the probability distribution of the signal and the associated conditional probabilities of bank default at date 2 differ in the two networks, investors' roll over decisions may also differ in the clustered and unclustered network. The following result then holds for the clustered network.

Proposition 3 *When the bad signal ($S = B$) is realized in the clustered network and $R_H > \frac{13}{12}R_L$,*

A. For $\alpha \geq \alpha_{LOW}(C)$, the bank continues at date 1 and promises investors a repayment $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$, where $\alpha_{LOW}(C) = \frac{45r_f^2-7(R_H+2R_L)}{24R_L}$.

B. For $\alpha_{MID}(C) \leq \alpha < \alpha_{LOW}(C)$, the bank continues at date 1 and promises investors a repayment $\rho_{12}^B(C) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$, where $\alpha_{MID}(C) = \frac{45r_f^2-8R_H-4R_L}{3(R_H+10R_L)}$.

C+D. For $\alpha < \alpha_{MID}(C)$, the bank is liquidated at date 1.

Proof. See the Appendix. ■

The proposition is illustrated in Figure 3. (Note that both Regions C and D lie below the function $\alpha_{MID}(C)$.) The result follows immediately from the investors' participation constraint at date 1. When the bad signal is realized, the bank continues at date 1 whenever investors can be promised a repayment that satisfies (9). Whether this is possible depends on the fraction α of the bank's portfolio return that the investors receive at date 2 when the bank defaults and on the opportunity cost r_f^2 they require over the two periods. When α is high or r_f^2 is low, there exists a repayment $\rho_{12}^B(C)$ that satisfies (9). Investors roll over the debt and the bank continues. The promised repayment compensates the investors for the possibility that they obtain only αX_i in case of default. Given α is high, $\rho_{12}^B(C)$ does not need to be high. Thus, the equilibrium $\rho_{12}^B(C)$ lies in the lowest interval of the bank's portfolio return, $[r_f^2, \frac{2R_L+R_H}{3}]$. As α decreases or r_f^2 increases ($\alpha_{MID}(C) \leq \alpha < \alpha_{LOW}(C)$), investors still roll over the debt but require a higher promised repayment to compensate them for the greater losses in the case of bank default. Thus, $\rho_{12}^B(C)$ is higher and lies in the interval $[\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$. This also implies that, conditional on the realization of the bad signal, bankruptcy does not occur at date 2 only when all projects in a bank's portfolio pay off R_L but also when they pay $\frac{2R_L+R_H}{3}$. As α decreases further –or r_f^2

increases further– and falls below $\alpha_{MID}(C)$, it is no longer possible to satisfy (9) for any $\rho_{12}^B(g) \leq X_i$. Then, investors do not roll over the debt and the bank is early liquidated at date 1.

A similar result holds for the unclustered network.

Proposition 4 *When the bad signal ($S = B$) is realized in the unclustered network,*

A+B+C. For $\alpha \geq \alpha_{LOW}(U)$, the bank continues at date 1 and promises investors a repayment $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$, where $\alpha_{LOW}(U) = \frac{75r_f^2-17(R_H+2R_L)}{24R_L}$.

D. For $\alpha < \alpha_{LOW}(U)$, the bank is liquidated at date 1.

Proof. See the Appendix. ■

Proposition 4 is also illustrated in Figure 3. (Note that all the Regions A, B and C lie above the function $\alpha_{LOW}(U)$). As in the clustered network, the bank continues its operations till date 2 when the fraction α of the bank's portfolio return accruing to investors in case of bank default is sufficiently high. Differently from before though, in case of continuation the bank always offers investors a promised repayment $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$. The reason is that, as stated in (12), the probability of default conditional on the bad signal $S = B$ is greater in the clustered network than in the unclustered network. This implies that investors are more likely to roll over the debt in the unclustered network and also require a lower promised repayment $\rho_{12}^B(C)$ to do so.

We next consider welfare in the two networks with short term finance. As with long term finance, in both networks we can focus on the total welfare per bank as defined by the sum of a representative bank i's expected profit and its investors' expected returns. Differently from before though, with short term finance the total welfare depends on investors' roll over decision, since this affects the bank's expected profit. Using (10) and (11), total welfare per bank is then given by

$$W(g) = E(X_i) - (1 - q(g))(1 - \alpha)E(X_i < \rho_{12}^B(g)|B) - 2c, \tag{14}$$

when the bank is continued till date 2, and

$$W(g) = q(g)\left[E(X_i \geq r_f^2|G)\right] + (1 - q(g))r_f^2 - 2c, \tag{15}$$

when the bank is liquidated at date 1.

As investors' rollover decision at date 1 differs in the clustered and unclustered network depending on the parameter α, so will the welfare. We have the following result.

Proposition 5 *The comparison of total welfare in the two networks is as follows:*

A. For $\alpha \geq \alpha_{LOW}(C)$, total welfare is the same in the clustered and unclustered network ($W(C) = W(U)$).

B+C1. For $\alpha_W < \alpha < \alpha_{LOW}(C)$, total welfare is higher in the unclustered network than in the clustered network ($W(U) > W(C)$), where $\alpha_W = \frac{15r_f^2 - 4R_H - 3R_L}{8R_L}$.

C2+D. For $\alpha < \alpha_W$, total welfare is higher in the clustered network than in the unclustered network ($W(C) > W(U)$).

Proof. See the Appendix. ■

The proposition is illustrated in Figure 4. The result indicates that with short term finance total welfare depends on the network structure. Which structure is better depends crucially on the investors' rollover decision as determined by the parameter α as well as on the opportunity cost r_f^2 accruing to the investors in the case of early liquidation.

In Region A, where $\alpha \geq \alpha_{LOW}(C)$, investors roll over the debt for a promised total repayment $\rho_{12}^B(g) \in [r_f^2, \frac{2R_L + R_H}{3}]$ in both networks. This implies that banks make positive profits for any realization of the signal S and default occurs in 8 states out of 64 in either the clustered or the unclustered network. As with long term finance, total welfare is then the same in both networks.

In Region B, where α lies in between $\alpha_{MID}(C)$ and $\alpha_{LOW}(C)$, investors still roll over the debt in both networks, but in the clustered network they now require a higher total promised repayment $\rho_{12}^B(g) \in [\frac{2R_L + R_H}{3}, \frac{R_L + 2R_H}{3}]$. This implies a higher default probability at date 2 and thus lower total welfare in the clustered network relative to the unclustered network because expected bankruptcy costs are higher.

In Regions C1 and C2 in Figure 4 debt is rolled over when the bad signal is realized in the unclustered network but not in the clustered one. Investors always recover their opportunity cost, but in the clustered network banks now make positive profits only when the good signal is realized. Total welfare is then given by (14) and (15) in the unclustered and clustered networks, respectively. In the former, welfare is decreasing in the bankruptcy costs, $1-\alpha$. Thus, it decreases as α falls. In the latter, welfare is increasing with r_f^2 as investors recover their opportunity cost with early liquidation and there are no bankruptcy costs. As α falls and r_f^2 increases, the total welfare in the unclustered network becomes equal to the one in the clustered network, and it then drops below.

Finally, in Region D, where $\alpha \leq \alpha_{LOW}(U)$, banks are liquidated in both networks when the bad signal is realized and the clustered network leads to higher total welfare. The reason is that the expected return of the bank's portfolio conditional on the good signal, $E(X_i \geq r_f^2|G)$, is higher in the clustered network also as from (12) the good news are more likely to arrive when risk is concentrated.

5 Concluding remarks

Understanding connections among financial institutions is important for understanding systemic risk. In this paper we have developed a model where the number and shape of financial connections interact with the funding structure of financial institutions in determining systemic risk.

We have shown that the structure of financial networks matters for systemic risk and total welfare when banks use short term finance, but not when they use long term finance. The reason is that short term finance entails roll over risk, which is absent with a longer maturity of debt. Investors base the decision to roll over the debt on any interim information about banks' future solvency. When negative information indicating future bank defaults arrives, investors may infer that they will not to be able to recover the opportunity cost associated with the renewal of the debt. When this occurs, they do not roll over the debt thus forcing banks into early liquidation. This roll over risk entailed in the nature of short term finance differs depending on the structure of connections among banks as this affects the degree of overlap of banks' portfolios induced by diversification and thus investors' inference on banks' future solvency.

The key trade off between the clustered and the unclustered structure in our framework derives from the different overlaps and risk concentration among banks' portfolio in the two networks. Banks have identical portfolios in each of the two groups when they are clustered, while they have diverse portfolios when they are unclustered. This implies a higher probability of receiving a bad signal in the unclustered network and, vice versa, a higher probability of default conditional on receiving the bad signal in the clustered network. The consequence is that the clustered network more often entails early liquidation and thus systemic risk than the unclustered network but the latter one can also be more inefficient when bankruptcy costs are high.

These results provide some insights on the desirability of risk concentration. Risk should be dispersed when bankruptcy costs are intermediate, but rather concentrated when bankruptcy costs are high. For low bankruptcy costs it does not matter. The main intuition is that when bankruptcy is inefficient but early liquidation is not, it is optimal to have fewer instances with more banks defaulting as in the clustered network rather than more frequent instances with less banks defaulting as in the unclustered network.

An important topic for future research concerns the implication of our analysis for financial regulation. Our results suggest there are no simple conclusions concerning the desirability of particular patterns of links. The desirability of clustered and unclustered networks depends on the bankruptcy and early liquidation costs. In addition it is not immediately clear what policies central banks and governments should adopt to influence financial links. One possibility is that they are directly able to regulate the network of linkages. However, this would require a great deal of information. One measure to ensure clustered networks rather than unclustered networks if this was optimal might be to limit financial institutions to their home countries rather than allowing them to pursue opportunities in other countries. Much work clearly remains to be done on this.

A Appendix

Proof of Proposition 1. Given (5) and (6), from (3) a bank's expected profit with $\ell = 2$ simplifies to

$$\pi_i(g) = E(X_i) - r_F^2 - (1-p)^3(1-\alpha)R_L - 2c.$$

To show pairwise stability, we first consider severing a link. Suppose that bank 1 severs the link with bank 3 so that its portfolio is now $\frac{2}{3}\theta_1 + \frac{1}{3}\theta_2$ and its profit is

$$\pi_1(g - \ell_{13}) = E(X_i) - r_F^2 - (1-p)^2(1-\alpha)R_L - c.$$

Bank 1 does not deviate if $\pi_i(g) \geq \pi_1(g - \ell_{13})$, which is satisfied for $c \leq p(1-p)^2\beta R_L$.

Suppose now that bank 1 adds a link with bank 4 so that its portfolio is now $\frac{1}{6}\theta_1 + \frac{1}{3}\theta_2 + \frac{1}{3}\theta_3 + \frac{1}{6}\theta_4$ and its profit is

$$\pi_1(g + \ell_{14}) = E(X_i) - r_F^2 - (1-p)^4(1-\alpha)R_L - 3c$$

when bankruptcy occurs when all projects pay off R_L. If bankruptcy occurs more often than this, the expected profit from the deviation will be lower. Thus, it is sufficient for the deviation not to be profitable that $\pi_i(g) \geq \pi_1(g+\ell_{14})$ which requires $c \geq p(1-p)^3(1-\alpha)R_L$. Since all banks are symmetric, this shows that $\ell^* = 2$ is a pairwise stable equilibrium for the range of c given in the proposition.

To see that $\ell^* = 2$ is the Pareto dominant equilibrium it is sufficient to show that bank's expected profit is highest in this case since the investors always obtain their opportunity cost. First note that (7) is concave in ℓ. Combining this with the condition that c lies in the range given in the proposition, it follows that a bank's expected profit in the equilibrium with $\ell^* = 2$ is greater than in either the equilibrium with $\ell^* = 1$ or $\ell^* = 3$ or any other equilibrium. □

Proof of Proposition 3. We proceed in two steps. First, we find the minimum value of α as a function of the short term risk free rate r_f^2 in each interval of the bank's portfolio return X_i such that investors' participation constraint (9) is satisfied for a feasible promised repayment $\rho_{12}^B(C)$. Second, we compare the functions representing the minimum values of α found in the first step to find the equilibrium value of $\rho_{12}^B(g)$.

Step 1. We start by determining the minimum value of α such that (9) is satisfied for $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$. Substituting $\rho_{12}^B(C) = \frac{2R_L+R_H}{3}$ in (9) and using the distribution probability $\Pr(X_i = x|B)$ as in Table 3, we obtain

$$\frac{7}{15}\frac{2R_L + R_H}{3} + \alpha\frac{8}{15}R_L = r_f^2,$$

from which

$$\alpha_{LOW}(C) = \frac{45r_f^2 - 7(R_H + 2R_L)}{24R_L}.$$

This implies that for any $\alpha \geq \alpha_{LOW}(C)$, there exists a value of $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$ such that investors roll over their debt. Analogously, for $\rho_{12}^B(C) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$, we obtain

$$\frac{4}{15}\frac{R_L + 2R_H}{3} + \alpha(\frac{8}{15}R_L + \frac{3}{15}\frac{2R_L + R_H}{3}) = r_f^2$$

from which

$$\alpha_{MID}(C) = \frac{45r_f^2 - 8R_H - 4R_L}{3(R_H + 10R_L)}.$$

Finally, for $\rho_{12}^B(C) \in [\frac{R_L+2R_H}{3}, R_H]$ we obtain

$$\frac{1}{15}R_H + \alpha(\frac{8}{15}R_L + \frac{3}{15}\frac{2R_L + R_H}{3} + \frac{3}{15}\frac{R_L + 2R_H}{3}) = r_f^2$$

from which

$$\alpha_{HIGH}(C) = \frac{15r_f^2 - R_H}{3R_H + 11R_L}.$$

The interpretation of $\alpha_{MID}(C)$ and $\alpha_{HIGH}(C)$ is the same as the one for $\alpha_{LOW}(C)$.

Step 2. To find the equilibrium value of $\rho_{12}^B(C)$ defined as the minimum promised repayment that satisfies (9), we now compare the functions $\alpha_{LOW}(C)$, $\alpha_{MID}(C)$ and $\alpha_{HIGH}(C)$. We then obtain:

$$\alpha_{MID}(C) - \alpha_{LOW}(C) = \frac{7R_H^2 + 20R_H R_L + 108R_L^2 - 45r_f^2(R_H + 2R_L)}{24R_L(R_H + 10R_L)}.$$

We note that $\alpha_{MID}(C) - \alpha_{LOW}(C)$ is positive for $r_f^2 < \overline{r}_f^2 = \frac{7R_H^2+20R_H R_L+108R_L^2}{45(R_H+2R_L)} < \frac{5R_L+R_H}{6}$, and negative otherwise. Similarly, it can be shown that $\alpha_{HIGH}(C) - \alpha_{MID}(C) > 0$ for any $r_f^2 \in [\overline{r}_f^2, \frac{5R_L+R_H}{6}]$ and $R_H > \frac{13}{12}R_L$, while $\alpha_{HIGH}(C) - \alpha_{LOW}(C) > 0$ for any $r_f^2 \in [R_L, \overline{r}_f^2]$. Given that in equilibrium the bank offers the minimum level of $\rho_{12}^B(C)$ that satisfies (9), the proposition follows. □

Proof of Proposition 4. We proceed in two steps as in the proof of Proposition 3.

Step 1. We determine first the minimum value of α such that (9) is satisfied for $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$. Substituting $\rho_{12}^B(U) = \frac{2R_L+R_H}{3}$ in (9) and using the distribution probability $\Pr(X_i = x|B)$ as in Table 4, we obtain

$$\frac{17}{25}\frac{2R_L + R_H}{3} + \alpha\frac{8}{25}R_L = r_f^2,$$

from which

$$\alpha_{LOW}(U) = \frac{75r_f^2 - 17(R_H + 2R_L)}{24R_L}.$$

As before, this implies that for any $\alpha \geq \alpha_{LOW}(U)$, there exists a value of $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$ such that investors roll over their debt. Analogously, for $\rho_{12}^B(U) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$ and $\rho_{12}^B(U) \in [\frac{R_L+2R_H}{3}, R_H]$, respectively, we obtain

$$\frac{6}{25}\frac{R_L + 2R_H}{3} + \alpha(\frac{8}{25}R_L + \frac{11}{25}\frac{2R_L + R_H}{3}) = r_f^2$$

from which

$$\alpha_{MID}(U) = \frac{75r_f^2 - 6(2R_H + R_L)}{11R_H + 46R_L};$$

and

$$\frac{1}{25}R_H + \alpha(\frac{8}{25}R_L + \frac{11}{25}\frac{2R_L + R_H}{3} + \frac{5}{25}\frac{R_L + 2R_H}{3}) = r_f^2$$

from which

$$\alpha_{HIGH}(U) = \frac{25r_f^2 - R_H}{7R_H + 17R_L}.$$

Step 2. We now compare the functions $\alpha_{LOW}(U)$, $\alpha_{MID}(U)$ and $\alpha_{HIGH}(U)$ to find equilibrium value of $\rho_{12}^B(C)$. After some algebraic manipulation it is easy to see that $\alpha_{LOW}(U) < \alpha_{MID}(U) < \alpha_{HIGH}(U)$ for any $r_f^2 \in [R_L, \frac{5R_L+R_H}{6}]$. Thus, the proposition follows given that the bank always offers investors the minimum total repayment that satisfies (9). $\square$

Proof of Proposition 5. The proposition follows immediately from the comparison of total welfare in the two networks in the different regions. We analyze each region in turn.

Region A. For $\alpha \geq \alpha_{LOW}(C) > \alpha_{LOW}(U)$, (9) is satisfied for $\rho_{12}^B(g) \in [r_f^2, \frac{2R_L+R_H}{3}]$ and investors roll over the debt in both networks. Given this, from (14) total welfare is given by

$$W(g) = \frac{R_L + R_H}{2} - \frac{8}{64}(1-\alpha)R_L - 2c \tag{16}$$

for $g = U, C$ as a bank's expected probability of default at date 2 is the same in two structures.

Region B. For $\alpha_{LOW}(C) > \alpha \geq \alpha_{MID}(C) > \alpha_{LOW}(U)$, (9) is satisfied for $\rho_{12}^B(C) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$ in the clustered network and for $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$ in the unclustered network. Investors roll over the debt in both networks but the bank default probabilities now differ in the two structures. From (14) and Table 3, total welfare in the clustered network is given by

$$W(C) = \frac{R_L + R_H}{2} - \frac{15}{64}(1-\alpha)[\frac{8}{15}R_L + \frac{3}{15}\frac{2R_L + R_H}{3}] - 2c, \tag{17}$$

and by (16) in the unclustered network. It follows immediately that $W(U) > W(C)$.

Regions C1 and C2. For $\alpha_{MID}(C) > \alpha \geq \alpha_{LOW}(U)$, (9) cannot be satisfied for any $\rho_{12}^B(C) \leq X_i$ in the clustered network, whereas it is still satisfied for $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$ in the unclustered network. Thus, the bank is liquidated and, from (15), total welfare in the clustered network is now equal to

$$W(C) = \frac{49}{64}\left[\frac{21}{49}\frac{2R_L+R_H}{3} + \frac{21}{49}\frac{R_L+2R_H}{3} + \frac{7}{49}R_H\right] + \frac{15}{64}r_f^2 - 2c,$$

whereas $W(U)$ is still given by (16) in the unclustered network.

Comparing $W(C)$ and $W(U)$ gives

$$W(U) - W(C) = \frac{1}{64}[4R_H + (3+8\alpha)R_L - 15r_f^2].$$

Equating this to zero and solving for α as a function of r_f^2 gives the boundary between Regions C1 and C2:

$$\alpha_W = \frac{15r_f^2 - 4R_H - 3R_L}{8R_L}.$$

It can be easily seen that $W(U) > W(C)$ for $\alpha > \alpha_W$ and $W(U) < W(C)$ for $\alpha < \alpha_W$.

Region D. For $\alpha < \alpha_{LOW}(U)$, (9) cannot be satisfied for any $\rho_{12}^B(g) \leq X_i$ so that banks are early liquidated in both networks. Total welfare is still as in (17) in the clustered network, while, from (15), it equals

$$W(U) = \frac{39}{64}\left[\frac{13}{39}\frac{2R_L+R_H}{3} + \frac{19}{39}\frac{R_L+2R_H}{3} + \frac{7}{39}R_H\right] + \frac{25}{64}r_f^2 - 2c$$

in the unclustered network. The difference between the two expressions is given by

$$W(C) - W(U) = \frac{1}{32}(2R_H + 3R_L - 5r_f^2),$$

which is positive for any $r_f^2 \in [R_L, \frac{5R_L+R_H}{6}]$. □

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Table 1: Clustered network.

State	*States of the world*						*Banks' portfolio returns*						Total defaults
							Cluster 1			*Cluster* 2			
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	X_1	X_2	X_3	X_4	X_5	X_6	
1	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	0
2	R_H	R_H	R_H	R_L	R_H	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
3	R_H	R_H	R_H	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
4	R_H	R_H	R_H	R_H	R_H	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
5	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	R_H	0
6	R_H	R_H	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	R_H	0
7	R_L	R_H	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	R_H	0
8	R_H	R_H	R_H	R_L	R_L	R_H	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
9	R_H	R_H	R_H	R_H	R_L	R_L	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
10	R_H	R_H	R_H	R_L	R_H	R_L	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
11	R_H	R_L	R_H	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
12	R_H	R_L	R_H	R_H	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
13	R_H	R_L	R_H	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
14	R_H	R_L	R_L	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_H	R_H	R_H	0
15	R_H	R_H	R_L	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
16	R_H	R_H	R_L	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
17	R_H	R_H	R_L	R_H	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
18	R_L	R_L	R_H	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_H	R_H	R_H	0
19	R_L	R_H	R_H	R_H	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
20	R_L	R_H	R_H	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
21	R_L	R_H	R_H	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
22	R_L	R_H	R_L	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_H	R_H	R_H	0
23	R_H	R_H	R_H	R_L	R_L	R_L	R_H	R_H	R_H	R_L	R_L	R_L	3
24	R_H	R_L	R_H	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
25	R_H	R_L	R_H	R_L	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
26	R_H	R_L	R_H	R_H	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
27	R_H	R_H	R_L	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
28	R_H	R_L	R_L	R_L	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
29	R_H	R_H	R_L	R_L	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
30	R_H	R_H	R_L	R_H	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
31	R_H	R_L	R_L	R_H	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
32	R_H	R_L	R_L	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
33	R_L	R_H	R_H	R_H	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
34	R_L	R_L	R_H	R_H	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
35	R_L	R_H	R_H	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
36	R_L	R_H	R_H	R_L	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
37	R_L	R_L	R_H	R_L	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
38	R_L	R_L	R_H	R_H	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
39	R_L	R_L	R_L	R_H	R_H	R_H	R_L	R_L	R_L	R_H	R_H	R_H	3
40	R_L	R_H	R_L	R_L	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
41	R_L	R_H	R_L	R_H	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
42	R_L	R_H	R_L	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
43	R_H	R_L	R_H	R_L	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_L	R_L	R_L	3
44	R_H	R_H	R_L	R_L	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_L	R_L	R_L	3
45	R_H	R_L	R_L	R_L	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
46	R_H	R_L	R_L	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
47	R_H	R_L	R_L	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
48	R_L	R_H	R_H	R_L	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_L	R_L	R_L	3
49	R_L	R_L	R_H	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
50	R_L	R_L	R_H	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
51	R_L	R_L	R_H	R_L	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
52	R_L	R_L	R_L	R_L	R_H	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
53	R_L	R_L	R_L	R_H	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
54	R_L	R_L	R_L	R_H	R_L	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
55	R_L	R_H	R_L	R_L	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
56	R_L	R_H	R_L	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
57	R_L	R_H	R_L	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
58	R_H	R_L	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	R_L	3
59	R_L	R_L	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	R_L	3
60	R_L	R_L	R_L	R_L	R_L	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
61	R_L	R_L	R_L	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
62	R_L	R_L	R_L	R_H	R_L	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
63	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	R_L	3
64	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	6

Table 2: Unclustered network.

State	States of the world						Banks' portfolio returns						Total defaults
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	X_1	X_2	X_3	X_4	X_5	X_6	
1	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	0
2	R_H	R_H	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	0
3	R_H	R_H	R_H	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
4	R_H	R_H	R_H	R_H	R_H	R_L	$(R_L+2R_H)/3$	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
5	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	R_H	0
6	R_H	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	0
7	R_L	R_H	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	R_H	$(R_L+2R_H)/3$	0
8	R_H	R_H	R_H	R_L	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	0
9	R_H	R_H	R_H	R_H	R_L	R_L	$(R_L+2R_H)/3$	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
10	R_H	R_H	R_H	R_L	R_H	R_L	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	0
11	R_H	R_L	R_H	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	0
12	R_H	R_L	R_H	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
13	R_H	R_L	R_H	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
14	R_H	R_L	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	0
15	R_H	R_H	R_L	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	0
16	R_H	R_H	R_L	R_H	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
17	R_H	R_H	R_L	R_H	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
18	R_L	R_L	R_H	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	$(R_L+2R_H)/3$	0
19	R_L	R_H	R_H	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	0
20	R_L	R_H	R_H	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	0
21	R_L	R_H	R_H	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
22	R_L	R_H	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	0
23	R_H	R_H	R_H	R_L	R_L	R_L	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	1
24	R_H	R_L	R_H	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	0
25	R_H	R_L	R_H	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	0
26	R_H	R_L	R_H	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
27	R_H	R_H	R_L	R_L	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	1
28	R_H	R_L	R_L	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	1
29	R_H	R_H	R_L	R_L	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	0
30	R_H	R_H	R_L	R_H	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
31	R_H	R_L	R_L	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
32	R_H	R_L	R_L	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
33	R_L	R_H	R_H	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	1
34	R_L	R_L	R_H	R_H	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	1
35	R_L	R_H	R_H	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
36	R_L	R_H	R_H	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
37	R_L	R_L	R_H	R_L	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
38	R_L	R_L	R_H	R_H	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	0
39	R_L	R_L	R_L	R_H	R_H	R_H	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	1
40	R_L	R_H	R_L	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
41	R_L	R_H	R_L	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	0
42	R_L	R_H	R_L	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	0
43	R_H	R_L	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	1
44	R_H	R_H	R_L	R_L	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	$(2R_L+R_H)/3$	2
45	R_H	R_L	R_L	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	2
46	R_H	R_L	R_L	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	1
47	R_H	R_L	R_L	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
48	R_L	R_H	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	2
49	R_L	R_L	R_H	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	2
50	R_L	R_L	R_H	R_L	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	1
51	R_L	R_L	R_H	R_L	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
52	R_L	R_L	R_L	R_L	R_H	R_H	$(2R_L+R_H)/3$	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	2
53	R_L	R_L	R_L	R_H	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	2
54	R_L	R_L	R_L	R_H	R_L	R_H	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	1
55	R_L	R_H	R_L	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	1
56	R_L	R_H	R_L	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	1
57	R_L	R_H	R_L	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
58	R_H	R_L	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	R_L	$(2R_L+R_H)/3$	3
59	R_L	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	3
60	R_L	R_L	R_L	R_L	R_L	R_H	$(2R_L+R_H)/3$	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
61	R_L	R_L	R_L	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
62	R_L	R_L	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	3
63	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	R_L	3
64	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	6

	$X_i = R_L$	$X_i = \frac{2R_L+R_H}{3}$	$X_i = \frac{R_L+2R_H}{3}$	$X_i = R_H$
$\Pr(X_i = x \vert B)$	$\frac{8}{15}$	$\frac{3}{15}$	$\frac{3}{15}$	$\frac{1}{15}$
$\Pr(X_i = x \vert G)$	0	$\frac{21}{49}$	$\frac{21}{49}$	$\frac{7}{49}$

Table 3: Conditional distribution of returns in the *clustered* network

	$X_i = R_L$	$X_i = \frac{2R_L+R_H}{3}$	$X_i = \frac{R_L+2R_H}{3}$	$X_i = R_H$
$\Pr(X_i = x \vert B)$	$\frac{8}{25}$	$\frac{11}{25}$	$\frac{5}{25}$	$\frac{1}{25}$
$\Pr(X_i = x \vert G)$	0	$\frac{13}{39}$	$\frac{19}{39}$	$\frac{7}{39}$

Table 4: Conditional distribution of returns in the *unclustered* network

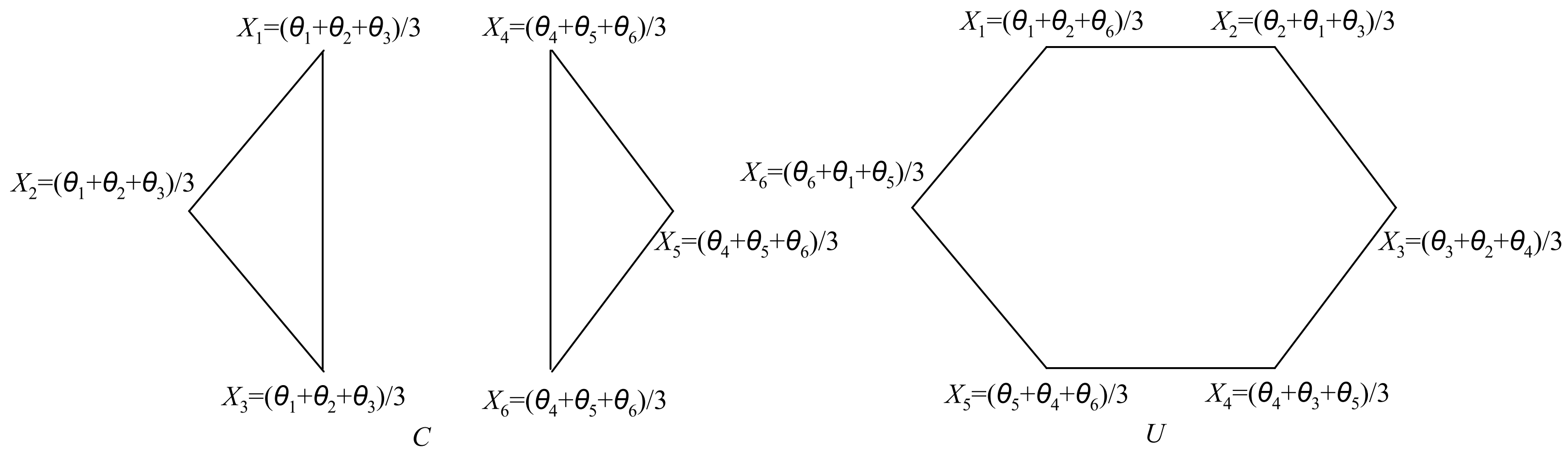

Figure 1: *C* - clustered network; *U* – unclustered network.

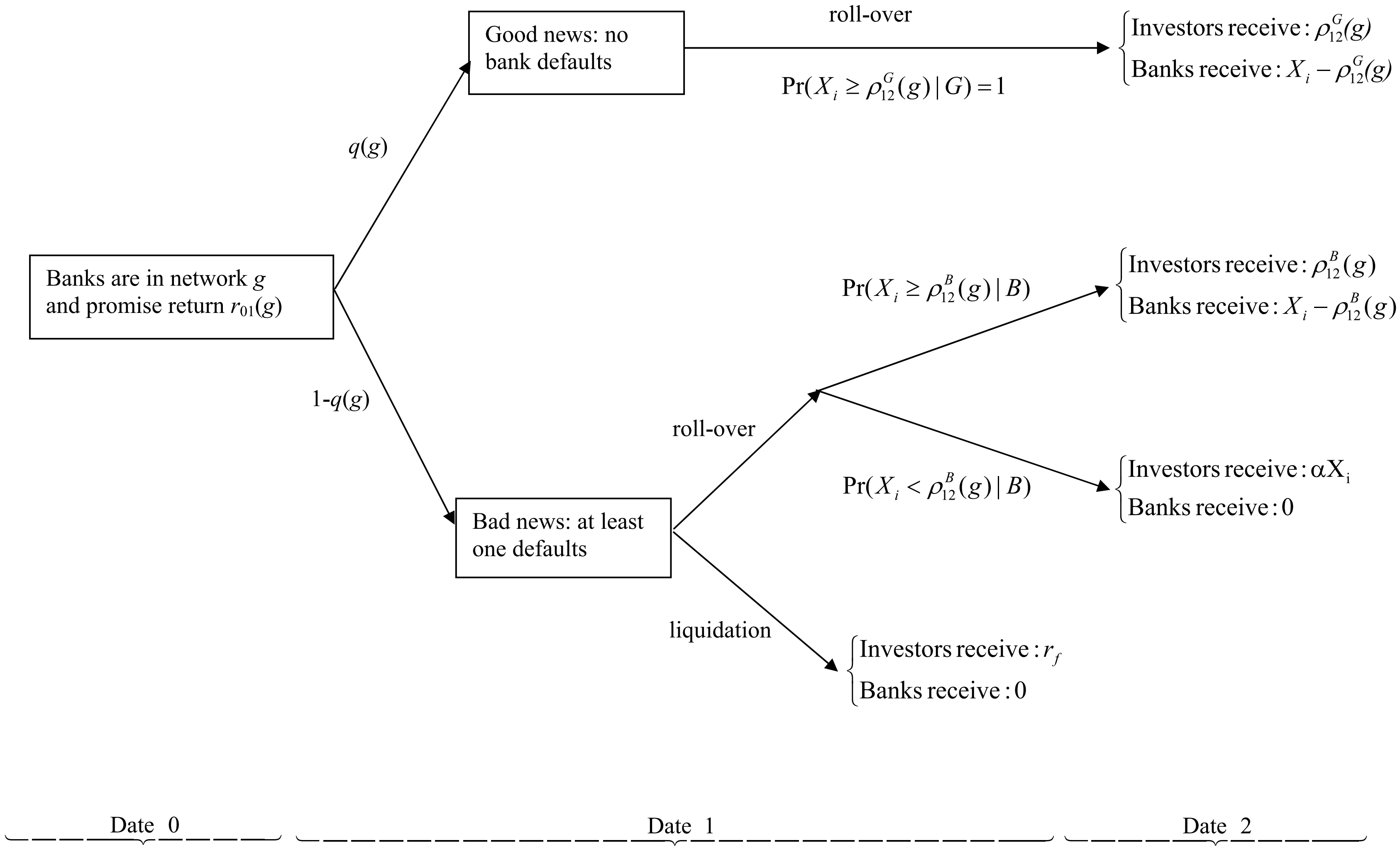

Fig. 2: Timing Sequence

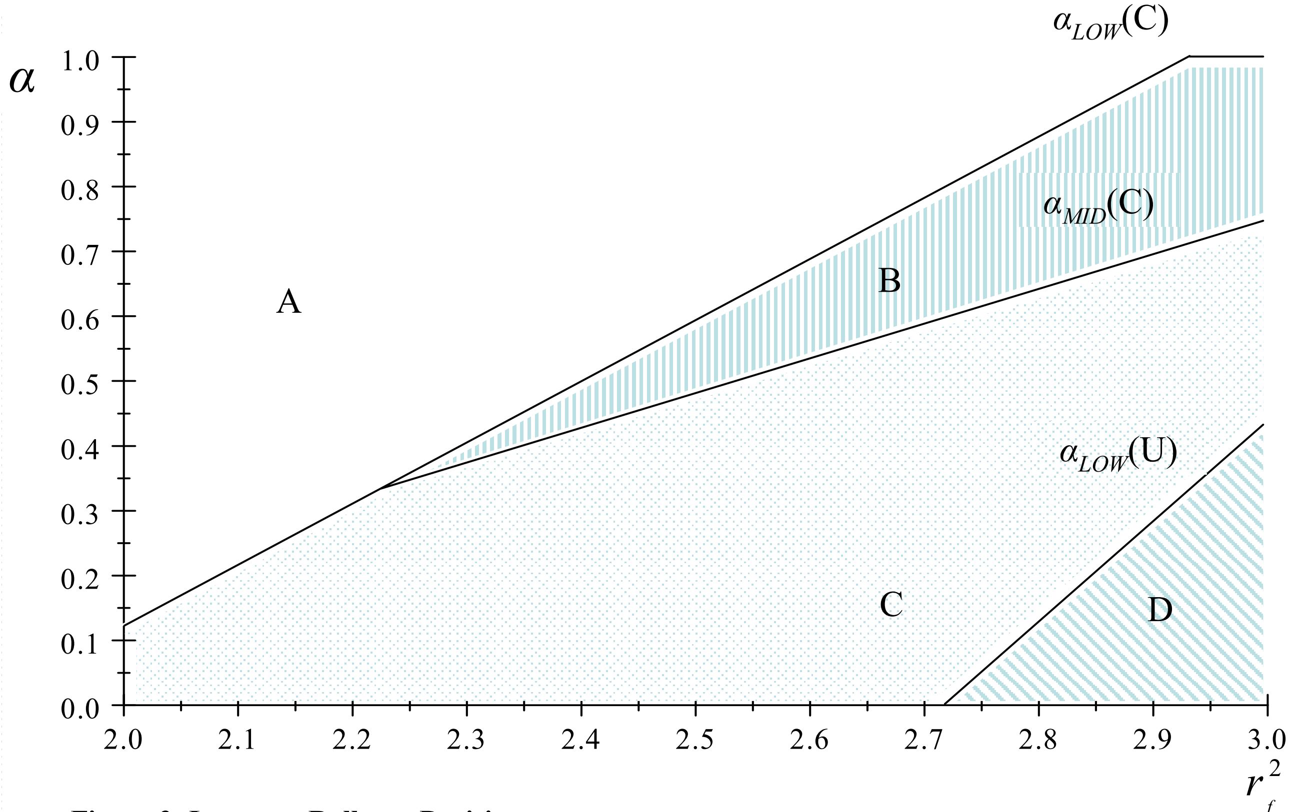

Figure 3: Investors Rollover Decision

This figure depicts how investors decision to rollover their debt when the bad signal arrives changes depending on the fraction α they receive from the project, as the risk free interest rate r_f^2 increases from R_L to $(5R_L+R_H)/6$. In region A, investors in both the clustered and the unclustered network rollover their debt if promised a return ρ_{12} in the interval $[r_f^2,(2R_L+R_H)/3]$. In region B investors rollover their debt in the clustered network only if promised a higher return in the interval $((2R_L+R_H)/3, (R_L+2R_H)/3]$, while investors in the clustered network rollover their debt as before. In region C investors early liquidate the banks in the clustered network, but rollover their debt in the unclustered network. In region D investors early liquidate the banks in both networks.

The plot uses the following parameter values: R_L=2 and R_H=8.

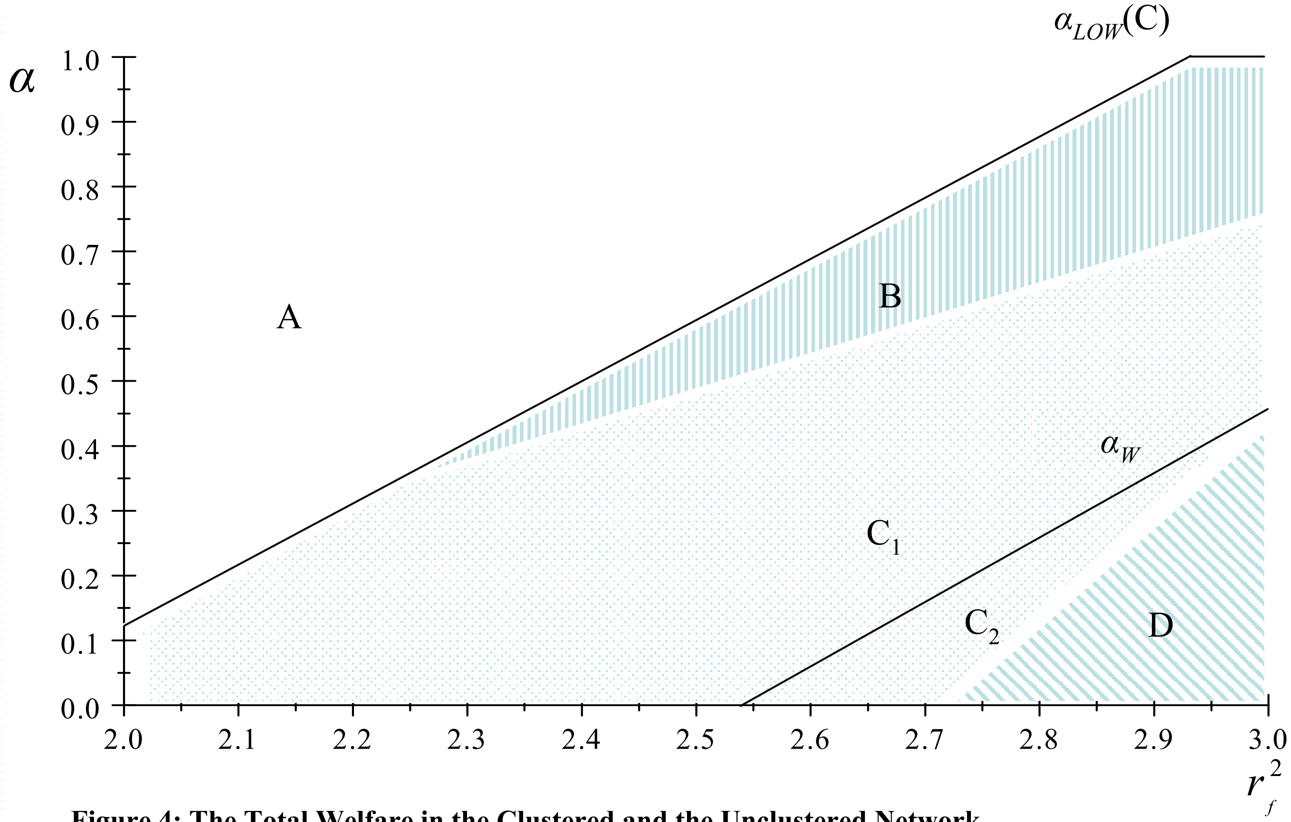

Figure 4: The Total Welfare in the Clustered and the Unclustered Network

This figure depicts the welfare in the clustered network compared with the welfare in the unclustered network depending on the fraction α they receive from the project, as the risk free interest rate r_f^2 increases from R_L to $(5R_L+R_H)/6$. In region A, the welfare is the same in both network. In region B+C_1, the welfare is higher in the unclustered network. In region C_2+D, the welfare is higher in the clustered network.

The plot uses the following parameter values: R_L=2 and R_H=8.

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