International Financial Markets, Cross-Border Transmission of Shocks and Risk-Sharing

Francesca Viani

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute
International Financial Markets, Cross-Border Transmission of Shocks and Risk-Sharing

Francesca Viani

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Jury Members:
Prof. Árpád Ábrahám, EUI
Prof. Giancarlo Corsetti, EUI, Supervisor
Prof. Charles Engel, University of Wisconsin
Prof. Fabrizio Perri, University of Minnesota

© 2010, Francesca Viani
No part of this thesis may be copied, reproduced or transmitted without prior permission of the author
ACKNOWLEDGMENTS

I would like to thank in particular my supervisor, Giancarlo Corsetti, for his support and invaluable help, and for the many comments and suggestions on any chapter of this thesis.

I am also indebted to my second advisors, Arpad Abraham and Morten Ravn, for their precious advices and guidance. It was a pleasure to work with Filipa Sà on our joint paper, one of the chapters of this thesis. For their useful comments, I would also like to thank Andrew Bernard, Russell Cooper, Luca Dedola, Charles Engel, Anna Lipinska, Massimiliano Marcellino, Ramon Marimon, Fabrizio Perri, Saverio Simonelli and Alan Sutherland.

Thanks to Georg, Bertrand, Klemens, Afroditì, Massimiliano, Tim and all my classmates at the EUI for their comments and, especially, for their friendship.

I dedicate this thesis to Federico and to my parents, for their patience, unconditional love, and for having always believed in me.
CONTENTS

I Introduction vi
  0.1 Introduction ............................................................... vii

II Chapters ................................................................. 1

1 INTERNATIONAL FINANCIAL FLOWS AND REAL EXCHANGE RATES 1
  1.1 Introduction ............................................................... 1
  1.2 A measure of cross-border insurance ................................ 5
  1.3 A two-country two-good model ....................................... 7
  1.4 Financial autarky and insurance from relative prices .......... 10
    1.4.1 Changes in relative wealth under Financial autarky ........ 11
    1.4.2 Insurance from relative prices ................................ 12
    1.4.3 The interaction between prices and flows .................... 17
  1.5 Efficient financial flows from external borrowing .............. 19
    1.5.1 External borrowing and equilibrium wealth imbalances ...... 22
    1.5.2 External borrowing and welfare ................................ 25
  1.6 Efficient financial flows from portfolio returns ............... 26
    1.6.1 The direction of optimal flows ................................ 29
    1.6.2 The composition of optimal portfolios ......................... 31
    1.6.3 Insurance from portfolio flows ................................ 32
  1.7 Some remarks on complementarity versus substitutability .... 32
  1.8 Concluding remarks .................................................... 34
  1.9 Appendix A: Standard efficiency condition and empirical methods ... 35
  1.10 Appendix B: Measure of insurance and welfare ................. 37
  1.11 Appendix C: Proofs to the Propositions ......................... 39
    1.11.1 Proposition 1.1 .................................................. 39
    1.11.2 Proposition 1.2 .................................................. 40
    1.11.3 Proposition 1.3 .................................................. 40
    1.11.4 Proposition 1.4 .................................................. 41
    1.11.5 Proposition 1.5 .................................................. 41
    1.11.6 Proposition 1.6 .................................................. 42
    1.11.7 Propositions 1.7 and 1.8 ...................................... 44
  1.12 Appendix D: Wealth effects with endogenous portfolios ....... 45
## CONTENTS

1.13 Appendix E: Portfolio flows under incomplete markets  

2 ASSESSING THE PERFORMANCE OF EMPIRICAL METHODS TO MEASURE CROSS-BORDER INSURANCE  

2.1 Introduction  

2.2 The model  

2.2.1 Asset structure 1: Financial autarky  

2.2.2 Asset structure 2: Trade in uncontingent bonds  

2.2.3 Asset structure 3: Trade in two equities (effectively complete markets)  

2.2.4 Parametrization  

2.3 The performance of empirical methods: Asdrubali, Sorensen and Yosha (1996)  

2.4 Other methods  

2.4.1 Obstfeld (1993)  

2.4.2 Brandt, Cochrane and Santa Clara (2006)  

2.4.3 Flood, Marion and Matsumoto (2008)  

2.5 Concluding remarks  

2.6 Appendix: A note on the use of GDP deflators  

3 MEASURING INTERNATIONAL RISK-SHARING: EMPIRICAL EVIDENCE FOR THE US AND OECD COUNTRIES  

3.1 Introduction  

3.2 An insurance benchmark from macro theory  

3.2.1 A measure of cross-border insurance  

3.2.2 A new empirical method  

3.2.3 The new method in simulations  

3.3 Empirical strategy  

3.3.1 Estimating Stochastic Discount Factors  

3.3.1.1 Stochastic Discount Factors from the traditional open-macro model  

3.3.1.2 Stochastic Discount Factors with time-varying risk-aversion  

3.3.2 Estimating the risk-sharing inefficiency  

3.3.3 Detecting significant changes in the inefficiency over time  

3.3.4 Macroeconomic risk versus degree of insurance  

3.4 Risk-sharing among the US and OECD countries
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4.1</td>
<td>SDFs from the traditional model: nominal exchange rate volatility as</td>
<td>92</td>
</tr>
<tr>
<td></td>
<td>a source of risk</td>
<td></td>
</tr>
<tr>
<td>3.4.2</td>
<td>SDFs with time-varying risk-aversion: a higher degree of insurance and</td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>the effects of the Great Moderation</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>Conclusions</td>
<td>97</td>
</tr>
<tr>
<td>3.6</td>
<td>Appendix A: Insurance measure and changes in relative wealth</td>
<td>99</td>
</tr>
<tr>
<td>3.6.1</td>
<td>Changes in relative wealth under Financial autarky</td>
<td>99</td>
</tr>
<tr>
<td>3.6.2</td>
<td>Changes in relative wealth with trade in uncontingent bonds</td>
<td>100</td>
</tr>
<tr>
<td>3.7</td>
<td>Appendix B: Insurance measure and welfare</td>
<td>101</td>
</tr>
<tr>
<td>3.8</td>
<td>Appendix C: Decomposing changes in the variance of the gap</td>
<td>105</td>
</tr>
<tr>
<td>3.9</td>
<td>Appendix D: Figures and Tables</td>
<td>106</td>
</tr>
<tr>
<td>4</td>
<td>THE MACROECONOMIC IMPLICATIONS OF SOVEREIGN WEALTH FUNDS</td>
<td>128</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>128</td>
</tr>
<tr>
<td>4.2</td>
<td>The Model</td>
<td>131</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Consumers</td>
<td>132</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Firms</td>
<td>134</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Government</td>
<td>136</td>
</tr>
<tr>
<td>4.2.4</td>
<td>Portfolio Allocation</td>
<td>136</td>
</tr>
<tr>
<td>4.2.5</td>
<td>Equilibrium and Balance of Payments Dynamics</td>
<td>139</td>
</tr>
<tr>
<td>4.3</td>
<td>Calibration</td>
<td>141</td>
</tr>
<tr>
<td>4.4</td>
<td>Shocks to Portfolio Preferences</td>
<td>142</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Path 1. Shock to Currency Composition</td>
<td>143</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Path 2. Shock to Asset Composition</td>
<td>143</td>
</tr>
<tr>
<td>4.5</td>
<td>Baseline Results</td>
<td>144</td>
</tr>
<tr>
<td>4.5.1</td>
<td>Path 1. Shock to Currency Composition</td>
<td>144</td>
</tr>
<tr>
<td>4.5.2</td>
<td>Path 2. Shock to Asset Composition</td>
<td>147</td>
</tr>
<tr>
<td>4.6</td>
<td>Robustness checks</td>
<td>148</td>
</tr>
<tr>
<td>4.6.1</td>
<td>Degree of substitutability between assets</td>
<td>148</td>
</tr>
<tr>
<td>4.6.2</td>
<td>Elasticity of substitution between goods</td>
<td>149</td>
</tr>
<tr>
<td>4.7</td>
<td>Conclusions</td>
<td>150</td>
</tr>
<tr>
<td>4.8</td>
<td>Appendix A: Construction of portfolio shares</td>
<td>151</td>
</tr>
<tr>
<td>4.9</td>
<td>Appendix B: Figures and Tables</td>
<td>153</td>
</tr>
<tr>
<td>4.9.1</td>
<td>Path 1: Shock to Currency Composition</td>
<td>153</td>
</tr>
<tr>
<td>4.9.2</td>
<td>Path 2: Shock to Asset Composition</td>
<td>156</td>
</tr>
</tbody>
</table>
4.9.3 Robustness checks: Degree of substitutability between assets (b) . . . 160
4.9.4 Robustness checks: Elasticity of substitution between goods ($\theta$) . . . 161
Part I

Introduction
0.1 Introduction

One of the most relevant events of the last decades has been the fast integration of international financial markets. As most restrictions to cross-border asset trade were removed, the volume of activity in international financial markets increased dramatically, and so did the cross-country flows of resources due to external borrowing and portfolio returns. Many of the macroeconomic consequences of these stronger financial linkages still have to be fully evaluated. This thesis investigates two facets of the strengthening of cross-border financial linkages. First, have developments in international financial markets raised global insurance against national income fluctuations? Second, what would be the macroeconomic effects of changes in the portfolio preferences of big international investors?

Chapters 1 to 3 deal with the first issue, the insurance role of international financial markets, both from a theoretical and an empirical perspective. Chapter 4 is focused on the second topic, and is devoted to studying the macroeconomic implications of the growth of large international investors, Sovereign Wealth Funds.

More specifically, Chapter 1 deals with the theoretical conditions that characterize well-functioning financial markets. Whether financial market integration has raised global insurance, is still a controversial issue in the literature. If this is so, what should we observe in the data? The insurance literature emphasizes that risk-sharing requires financial markets to channel resources to countries that have been made temporarily poorer by some negative conjuncture, net of physical capital accumulation. This standard condition, which provides the basis for virtually every test of international insurance, is however derived focusing on only one of the two channels of cross-border insurance, the financial flows channel, implicitly assuming no interaction between this and the other channel, international relative price fluctuations. In this Chapter I show that testable conditions can only be derived theoretically placing the interaction between prices and financial flows centerstage in the analysis. More specifically, using a two-country general equilibrium model with endogenous portfolio diversification, I show that financial flows and relative prices can be either complements or substitutes in providing insurance. In the case of complementarity, financial inflows raise the international price of a country’s output. This implies the standard condition. In the case of substitutability prices and flows transfer purchasing power in opposite directions. This implies a different condition: financial markets should channel resources “upstream”, from relatively poorer to relatively richer countries. The conditions for substitutability appear to be quantitatively and empirically plausible.

In Chapter 2 I investigate the implications of these findings for empirical methods that
were designed to measure cross-border risk-sharing. Virtually all these methodologies are based on the implicit assumption of no international price fluctuations. Which problems can the large and sustained movements in relative prices observed in the data cause to these methodologies? I assess their performance in a simulated two-country DSGE model with endogenous portfolio diversification, in which price fluctuations and financial markets provide insurance jointly against national income shocks. Results show that standard methods may be completely misleading if relative price movements (realistically) transfer purchasing power across countries, as the benchmarks of risk-sharing on which they are based are invalidated.

In Chapter 3 I propose a new empirical method to quantify cross-border insurance against national income shocks, which is immune from these issues. The method consists in estimating the risk-sharing inefficiency between two countries (contingent on the shocks that hit them during a certain period) using the wedge between their Stochastic Discount Factors (SDFs). This measure quantifies the distance of the observed cross-country allocation of resources from the one that would be created by complete financial markets, which would isolate the relative utility value of wealth of households living in different countries from idiosyncratic income fluctuations. The contingent risk-sharing inefficiency can be attributed either to the strength of uninsurable shocks (the extent of risk to be pooled) or to the degree of insurance against different sources of risk. I apply this method to study the evolution of risk-sharing between the US and OECD countries over the last forty years. When countries’ SDFs are estimated relying on the assumptions of traditional open-macro models (constant risk-aversion), I find that nominal exchange rate volatility represented an important source of macroeconomic risk. When SDFs are estimated assuming time-varying risk-aversion, I find that financial markets integration was partially effective in raising the degree of insurance against macro risk, and that a lower GDP volatility (the so-called Great Moderation) reduced substantially contingent risk-sharing inefficiencies.

Chapter 4 is joint work with Filipa Sà and is devoted to studying the macroeconomic implications of the future growth of large international investors, Sovereign Wealth Funds (SWFs). SWFs are expected to manage an increasing share of foreign exchange reserves in the near future. Compared to central banks, they have higher risk tolerance and invest less in US assets. Their growth might have implications for the level of US net debt and the dollar exchange rate. To study these implications, we use a dynamic general equilibrium model with two countries (the US and the rest of the world), and two asset classes (equities and bonds). The model is characterized by imperfect substitutability between assets and allows for endogenous adjustments in interest rates and asset prices. Therefore, it accounts for capital gains arising from equity price movements, in addition to valuation effects caused
by fluctuations in the exchange rate. The model is used to simulate what will happen if “excess reserves” held by Emerging Markets are transferred from central banks to SWFs. We look separately at two diversification paths: one in which SWFs keep the same allocation across bonds and equities as central banks, but move away from dollar assets (path 1); and another in which they choose the same currency composition as central banks, but shift from US bonds to US equities (path 2). In path 1, the dollar depreciates and US net debt falls on impact and increases in the long run. In path 2, the dollar depreciates and US net debt increases in the long run. In both cases, there is a reduction in the “exorbitant privilege”, the excess return the US receives on its assets over what it pays on its liabilities.
Part II

Chapters
1.1 Introduction

Over the last decades, most restrictions to cross-border asset trade were removed. Financial integration increased dramatically the volume of activity in international financial markets. Lane and Milesi-Ferretti (2003) document that foreign assets and liabilities as a share of GDP increased by 250% among industrialized countries between 1970 and 2001. Cross-border financial flows due to external borrowing and portfolio returns rose by 50-100% during the same period. Many of the macroeconomic consequences of these stronger financial linkages still have to be fully evaluated. Have developments in international financial markets raised global insurance?

As it is well-known, international financial markets may insure residents in different countries against the effects of national business cycle shocks. Local disturbances (affecting productivity, investment, government spending) may temporarily lower national income. If international financial markets are well-functioning, residents in different countries can shed their consumption from national shocks by borrowing from abroad or by getting returns on their portfolio of foreign assets. A large body of literature has investigated whether financial integration actually allowed households and firms in different countries to hedge more efficiently against national disturbances. This insurance literature has confronted with a basic theoretical issue: if international financial markets have become more efficient in providing insurance, what should we observe in the data? Several contributions proposed theoretical efficiency conditions for financial markets and compared them with the data – with mixed results.

What this literature has typically ignored is that international trade in assets is not the only channel of cross-border insurance. Also relative price fluctuations (terms of trade movements) may hedge against national income shocks. For instance, consider a country hit by a fall in productivity that lowers the income of its residents. If the international price of their output rises, residents can sell it at a higher price in international good markets and recover some purchasing power. Price fluctuations can transfer purchasing power across
1.1. INTRODUCTION

countries.\(^1\)

The idea behind this Chapter is that global insurance results from the joint operations of two channels – international financial markets (cross-border financial flows) and relative price fluctuations. Its purpose is to reconsider the theoretical issue in the insurance literature focusing on the interaction between these two channels: if financial flows and relative price fluctuations provide insurance jointly, how should we identify efficient financial markets?

The literature has suggested that efficient financial markets should channel resources to countries that (absent asset trade) would have been made temporarily poorer by national disturbances, net of physical capital accumulation. This condition provides the basis for virtually every test of international insurance (Appendix A describes how it has been brought to the data by existing empirical methods). Yet, this “standard efficiency condition” for financial markets has been derived focusing on only one of the two channels of cross-border insurance, financial flows. Relative price fluctuations have been typically ignored or treated as exogenous, at best they have been credited for giving rise to valuation effects on foreign asset positions.\(^2\) Is the standard condition still valid if prices and flows provide insurance jointly? This Chapter gives a negative answer. It shows that endogenous feedbacks between prices and flows may require efficient financial markets to channel resources “upstream”, from relatively poorer to relatively richer countries (net of physical capital accumulation). Thus the condition of insurance that has been used so far by the empirical literature on risk-sharing measurement, provides a partial and potentially misleading picture of the optimal pattern of financial flows. Testable conditions for cross-border insurance can only be derived placing the interaction between prices and flows centerstage in the analysis.

To derive these results, I write a two-country two-good DSGE model. Countries are inhabited by representative consumers and are specialized in the production of different varieties. National supply shocks create income risk for the representative households. The assumptions of product specialization and home bias in consumption guarantee that relative price fluctuations transfer purchasing power across countries. I assume that financial markets are efficient in providing cross-border insurance and analyze the DSGE model under two dif-

\(^1\)This idea has been formalized by Cole and Obstfeld (1991).

\(^2\)The efficiency benchmark for financial markets has been derived in frameworks with an infinite elasticity of substitution between commodities or with exogenous price fluctuations. One example is the so-called intertemporal approach to the current account, typically based on one-good models or setups in which terms of trade movements are treated as exogenous (see Razin (1993) and Obstfeld and Rogoff (1994)). The same applies to more recent approaches to the current account theory (Ventura (2002)). The few extensions of this theory to setups with traded and non-traded goods (in which price fluctuations are partially endogenized) have not been used as efficiency benchmarks by the insurance literature. See Chapter 2 for a survey of empirical methods.
1.1. INTRODUCTION

Different asset trade regimes, trade in uncontingent bonds and trade in two equities, representing respectively the two functions through which financial markets may foster cross-border risk-sharing, consumption smoothing (through external borrowing) and ex-ante hedging (through portfolio diversification). In both cases, I assume no frictions in international asset trade: agents optimize over their purchase of securities and have rational expectations; there are no costs associated to trade in assets. I use the model to study to which country efficient financial markets channel resources after national shocks. The first asset structure (trade in uncontingent bonds) allows to analyze the optimal pattern of flows from external borrowing. The second one (trade in two equities) makes asset markets effectively complete. I solve for optimal portfolios using the method of Devereux and Sutherland (2006), and study the pattern of financial flows from portfolio returns and the composition of optimal portfolios.

The results show that the optimal pattern of financial flows depends crucially on the interaction between the two channels of cross-border insurance.

Consider a country that is hit by a negative conjuncture and gets temporarily poorer with respect to the rest of the world. The “standard efficiency condition” for financial markets states that, once asset trade is introduced, efficient financial flows should channel resources to this country, either through external borrowing or via returns on international portfolios.

However, receiving an inflow from abroad typically changes the international price of a country’s output. I show that price fluctuations and financial flows can be either complements or substitutes in providing insurance. If an inflow from abroad raises the relative price of a country’s output, the economy benefits twice from the transfer as both financial flows and price fluctuations channel purchasing power to its residents – prices and flows are complements in providing insurance. Complementarity implies the “standard condition”: financial markets are efficient in providing insurance if they channel resources to the relatively poorer country (absent asset trade).

Yet, receiving an inflow from abroad may lower the international price of a country’s output. In this case, as financial flows tend to transfer purchasing power to this economy, relative price fluctuations channel purchasing power abroad – the two channels of insurance are substitutes. Price swings are so strong that the net effect of an inflow is to lower the purchasing power of the receiving country. In this case financial markets are efficient in providing insurance if they transfer resources to the country that (absent asset trade) would have been made relatively richer by the shock – “upstream”. Upstream flows raise global

---

3 As noted by Cole (1991), well-functioning financial markets play two roles. First, frictionless trade in uncontingent bonds allows consumers to smooth the effects of national disturbances on their consumption by borrowing and lending abroad. Second, frictionless trade in contingent assets allows households to form portfolios that hedge them ex-ante against shocks.
1.1. INTRODUCTION

insurance. Yet, they run against the standard condition of efficiency.

More specifically, if agents can trade only in uncontingent bonds, flows and prices being
complements or substitutes determines to which economy efficient financial markets should
channel resources through external borrowing, and the optimal cyclicality of the current
account. If agents can trade in two equities, the same relationship determines the direction
of flows through portfolio returns, and the composition of optimal portfolios. I also analyze
the implications of complementarity and substitutability for equity home bias.

It is important to notice that the effects of upstream flows and the conditions that
generate them are not new to the international literature. In our model the different price
response to financial flows depends on the relative strength of substitution and income effect
of price changes, which, in turn, hinges crucially on the value of the trade elasticity. In
particular, optimal upstream flows arise for a relatively low trade elasticity, around 0.5. This
value is in the range of macro estimates, as documented by Hooper et al. (2000). Corsetti et
al. (2008) use US data to estimate the trade elasticity to be slightly below 0.5. This low trade
elasticity is essential for their open-macro model to match the empirical evidence on the so-
called Backus-Smith puzzle, the low correlation between relative cross-country consumption
and the real exchange rate observed in the data. Corsetti et al. (2008) show that a trade
elasticity around 1/2 is precisely what allows to reconcile standard international models,
driven by business-cycle supply shocks, with the empirical evidence on consumption-real
exchange rate correlations.\footnote{An alternative would be a much higher trade elasticity (around 4, in line with estimates from the trade
literature) and very persistent productivity shocks in an economy with capital accumulation. See Corsetti et al. (2008).}

On the other hand, the literature on the so-called “transfer problem” has studied the
impact of international transfers of wealth on the terms of trade and the real exchange rate.
While most works have estimated the impact of net foreign assets variations on relative prices
in the long-run, Gagnon (1996) quantifies the short-run impact, the one that is relevant for
the issue at handle. He finds that a deterioration in net foreign assets (corresponding in the
short-run to an inflow of resources from abroad) is associated with an impact depreciation
of the terms of trade. This evidence seems to be in favour of substitutability between flows
and prices, necessary for upstream financial flows to be optimal.

This Chapter is related to three strands of literature. First, contributions developing
theories of consumption smoothing and portfolio diversification (as Obstfeld and Rogoff
(1994) and Ventura (2000) on, respectively, the intertemporal and the new theory of the
current account). These approaches – mostly grounded in one-good or partial equilibrium
1.2. A MEASURE OF CROSS-BORDER INSURANCE

frameworks – abstract from any interaction between financial flows and price fluctuations, and provide the theoretical basis for the “standard efficiency condition”.

Second, contributions studying the insurance properties of relative price fluctuations. Cole and Obstfeld (1991), the first ones to analyze this issue, abstract from financial markets. Corsetti et al. (2008) reconcile standard open-macro models with the Backus-Smith evidence. They study how the structure of international financial markets affects relative price fluctuations in a model with international trade in uncontingent bonds.

Finally, the empirical literature on risk-sharing measurement. Obstfeld (1993), Sorensen and Yosha (1998), and Flood et al. (2008) are only some examples of contributions focusing on the direction of financial flows along the business cycle, and adopting empirical methods based on the “standard condition”.

To my knowledge, this Chapter is the first paper that studies the interaction between financial flows and price fluctuations and its relevance for the mechanism of cross-border insurance. It is also the first one to relate endogenous feedbacks between the two insurance channels to optimal portfolio composition.

The rest of the Chapter is organized as follows. The next section presents a measure of cross-border insurance from macro theory, which will be useful to trace the effects of national shocks and the direction of optimal financial flows. Section 1.3 describes the two-country two-good DSGE model. Section 1.4 analyzes the model under Financial autarky, to study the impact of national shocks on the cross-border allocation absent financial markets. Section 1.5 and 1.6 analyze the same model, respectively, under trade in uncontingent bonds and trade in two equities, to study optimal financial flows from external borrowing and portfolio returns. Section 1.7 presents some considerations on complementarity and substitutability, while 1.8 draws some conclusions.5

1.2 A measure of cross-border insurance

Assume the world consists of several countries, each inhabited by a representative agent. Asset markets are complete as agents can trade internationally in a full set of Arrow-Debreu securities. Focus on two countries, Home and Foreign. In what follows $W(s'|s)$ denotes the price in state-of-the-world $s$ of an Arrow-Debreu security paying 1 unit of some numéraire good in the following period if state-of-the-world $s'$ realizes. $U_C(s)$ and $U_C(s'|s)$ are the marginal utilities of consumption of the H consumer, respectively, in state $s$ and in state $s'$.

5Appendix E provides an example of portfolio formation under incomplete markets. It shows that if agents can trade in multiple securities and asset markets are effectively incomplete, “upstream” financial flows can also be the manifestation of poor international insurance.
$P(s)$ and $P(st|s)$ are the prices of the H consumption good in state $s$ and $st$. $\beta$ denotes the discount factor of the H consumer, $\pi(st|s)$ the conditional probability of state $st$ given $s$. The Euler Equation of the H consumer, regulating her purchase in state $s$ of Arrow-Debreu securities paying 1 unit of the numéraire in state $st$, reads

$$U_C(s) \frac{W(st|s)}{P(st|s)} = \beta \cdot \pi(st|s) \cdot U_C(st|s) \frac{1}{P(st|s)}, \forall s, st \quad (1.1)$$

The H consumer buys securities until the marginal cost of purchasing one more asset (left hand side in equation (1.1)) equals its expected marginal benefit (right hand side).

The analogous condition for the F agent is given by

$$U^*_C(s) \frac{W(st|s)}{P^*(st|s) \cdot \omega(s)} = \beta^* \cdot \pi(st|s) \cdot U^*_C(st|s) \frac{1}{P^*(st|s) \cdot \omega(st|s)}, \forall s, st \quad (1.2)$$

where starred variables denote Foreign aggregates, and $\omega$ is the nominal exchange rate between the H and the F currency.

Combining (1.1) and (1.2) gives

$$\beta \frac{U_C(st|s)}{U_C(s)} \frac{P(st|s)}{P(s)} = \beta^* \frac{U^*_C(st|s)}{U^*_C(s)} \frac{P^*(st|s)}{P^*(s)} \frac{\varepsilon(s)}{\varepsilon(st|s)} = \frac{W(st|s)}{\pi(st|s)}, \forall s, st$$

where $m(st|s)$ and $m^*(st|s)$ are the Stochastic Discount Factors (SDFs) of the H and the F consumer. Perfect cross-border insurance against national income fluctuations (given by construction by complete asset markets) equates the SDFs of the two agents, for every states-of-the-world $s$ and $st$.$^6$ Equivalently, in terms of time-dependent notation, full insurance implies

$$m_t = m^*_t, \forall t \quad (1.3)$$

On the contrary, if asset markets are not complete, Stochastic Discount Factors need not be equalized. Uninsurable shocks will drive a wedge between them in some states-of-the-world

$$m_t = \eta_t \cdot m^*_t, \exists t \quad (1.4)$$

where $\eta_t$ represents the wedge between H and F SDFs created by uninsurable disturbances. Taking logarithms on both sides of (1.4) I define the measure of cross-border insurance, the gap, as

$^6$See also Backus and Smith (1993), Gravelle and Rees (2004), and Cochrane (2005).
\[ gap_t \equiv \log (\eta_t) - \log (1) = \log (m_t) - \log (m_t^*) \]  

\( gap \), measures the percentual deviation of the wedge between H and F SDFs observed at time \( t \) from the wedge that would be observed if income risk was perfectly shared (\( \eta = 1 \)). In any model, the \( gap \) arising in response to national shocks measures by construction the percentual deviation from the allocation corresponding to complete financial markets and full insurance against national income shocks.\(^7\)

In the next sections I will prove that the \( gap \) quantifies changes in countries’ relative wealth created by uninsurable shocks. If countries are not perfectly insured against idiosyncratic disturbances, any shock modifies their relative wealth. These asymmetric wealth effects are reflected in the insurance measure: the sign of the \( gap \) indicates the new ranking of wealth, i.e. which country has been made relatively richer (poorer) by the disturbance; its magnitude quantifies the size of the change in countries’ wealth.

Appendix B proves that in the DSGE model used in this Chapter, the \( gap \) is a proxy for the social welfare losses due to the fraction of country-specific risk that cannot be traded in financial markets. The higher the \( gap \) that arises in response to shocks, the further the distance from the optimum of a social welfare function that weights the two countries according to their initial wealth, that is the higher the reduction in social welfare caused by the disturbance.\(^8\)

These properties make the \( gap \) a useful instrument for the analysis of optimal financial flows. In particular, the \( gap \) will be used to study the effects of national income shocks and the direction of financial flows in the model detailed below.

### 1.3 A two-country two-good model

The model is a two-country two-good DSGE framework with endowment shocks and home bias in consumption. Its core structure is the simplest one that allows to analyze, under different asset trade regimes, the interaction between real and financial channels of shocks transmission.\(^9\) Its simplicity allows to get analytical closed form solutions when trade

---

\(^7\)The validity of the \( gap \) as an insurance measure against relative wealth fluctuations depends only on the existence of a representative household in each country. The interpretation of the \( gap \) as a measure of relative wealth imbalances, outlined below, relies on the same assumption. Clearly, the specific form of the Stochastic Discount Factors will depend on the particular model analysed.

\(^8\)This property of the \( gap \) is valid in the DSGE model used here, but might not hold in general. In particular, in the presence of deviations from the law of one price, complete markets would still eliminate divergences in the utility value of wealth across countries (providing full insurance against income fluctuations), but at the expenses of creating inefficient divergencies in consumption levels.

\(^9\)The model is closely related to frameworks employed in Cole and Obstfeld (1991), Kollmann (2006), and Corsetti et al (2008).
1.3. A TWO-COUNTRY TWO-GOOD MODEL

in more than one asset is introduced.

The model consists of two countries, Home and Foreign, each inhabited by a representative household. Countries are specialized in the production of different goods. Households in country H receive utility from consuming a bundle made up of the foreign and the domestic good, according to a CES aggregator

\[ C_t = \left( (\delta)^{1/\theta} (C_{ht})^{\frac{\theta-1}{\theta}} + (1-\delta)^{1/\theta} (C_{ft})^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} ; \theta > 0, \delta > 1/2 \]  

(1.6)

where \( C_{jt} \) denotes consumption of the good produced in country \( j \) and \( \delta \) is a parameter capturing home bias in consumption. \( \theta \) is the elasticity of substitution between H and F-produced goods; in this model it coincides with the trade elasticity. The F consumption bundle is analogously defined. Starred variables denote the corresponding quantities consumed by Foreign agents. Goods are freely tradable but not storable. The period utility of both agents depends on current consumption only and is a Constant Relative Risk Aversion function with risk aversion coefficient \( \rho \).

In each period H and F households receive a stochastic endowment according to the process

\[ \log (Y_{jt}) = \zeta \log (Y_{jt-1}) + \varepsilon_{jt} \]

where \( Y_{jt} \) denotes the endowment received by consumer \( j \) and \( \varepsilon_{jt} \sim iid (0, \sigma^2) \). For simplicity I assume \( Cov(\varepsilon_{it}, \varepsilon_{jt}) = 0 \) for \( i \neq j \).

I assume that the law of one price holds and that the nominal exchange rate is constant and equal to one for simplicity. Due to home bias in consumption purchasing power parity does not hold outside a symmetric Steady State, and the two price indexes \( P_t \) and \( P_t^* \) are tied by the following condition defining the real exchange rate \( Q_t \)

\[ Q_t \equiv \frac{P_t^*}{P_t} \]

Terms of trade are defined as the ratio of the price of H imports and exports

\[ tot_t \equiv \frac{P_{ft}}{P_{ht}} \]

where \( P_{ht} \), and \( P_{ft} \) denote respectively the price of H and F-produced goods.

The relative value of F and H endowments can be written as

\[ V_t^* = \frac{P_{ft}Y_{ft}P_t}{P_{ht}Y_{ht}P_t^*} \]  

(1.7)
1.3. A TWO-COUNTRY TWO-GOOD MODEL

Standard cost minimization delivers H households’ demand for the H and the F variety

\[ C_{ht} = \delta \left( \frac{P_{ht}}{P_t} \right)^{-\theta} C_t \]

\[ C_{ft} = (1 - \delta) \left( \frac{P_{ft}}{P_t} \right)^{-\theta} C_t \]

Analogous conditions hold for F agents. Good market clearing conditions read

\[ Y_{ht} = C_{ht} + C_{ht}^* \]

\[ Y_{ft} = C_{ft} + C_{ft}^* \]

The model is loglinearized around a symmetric Steady State in which endogenous variables are constant and exogenous ones are equal to their mean values.

In the following sections, I will use the model to answer our question. If financial markets are efficient in providing cross-border insurance, what should we observe in the data? More specifically, if financial flows and price fluctuations provide insurance jointly, should financial resources flow to poorer countries, as suggested by the “standard efficiency condition” for financial markets?

To answer this question I will analyze the model under different asset trade regimes. Households’ budget constraints will be pinned down by the specific asset structure assumed. More specifically, I will assume the existence of efficient financial markets that allow agents to borrow from abroad or to diversify their portfolio. Both financial flows and relative price fluctuations will transfer purchasing power across borders. In these frameworks I will simulate a national income shock and study the direction of optimal financial flows. In all these exercises, the gap arising in the model in response to national disturbances will be a useful instrument to analyze the effect of the shocks and the direction of financial flows. For any possible asset structure, if the system is in Steady State up to time \((t - 1)\), the gap arising at time \(t\) in response to shocks is given by

\[ \text{gap}_t = \hat{q}_t - \rho (\hat{c}_t - \hat{c}_t^*) \]

where \(\rho\) is the common risk-aversion coefficient and lowercase hatted variables denote percentual deviations from the Steady State.
1.4. FINANCIAL AUTARKY AND INSURANCE FROM RELATIVE PRICES

As a first step, the next section analyzes the same model in Financial autarky to study the effect of national income shocks on the cross-country allocation if international financial markets were shut down.

1.4 Financial autarky and insurance from relative prices

In this section I study the model under Financial autarky, with two purposes. First, to trace the effects of national disturbances on the cross-country allocation if there were no financial markets. Second, to study cross-border insurance from relative price fluctuations.

As noted by Cole and Obstfeld (1991), if countries are specialized in the production of heterogenous goods, they may be compensated for negative shocks reducing their output by increases in the international price of their exports. Terms of trade volatility may transfer purchasing power across countries and provide some automatic insurance against national shocks.

Under Financial autarky relative price fluctuations represent the only channel of cross-border insurance. Households have no means of smoothing consumption over time and in each period their consumption must equal the value of their income. H and F budget constraints read

\[ C_t P_t = P_{ht} Y_{ht} \]

\[ C^*_t P^*_t = P_{ft} Y_{ft} \]

The relative value of F and H endowments, as defined in equation (1.7), can be written in linearized form as

\[ \hat{\nu}^*_t = (\hat{y}_{ft} - \hat{y}_{ht}) + 2 (1 - \delta) t o l_{t} \]

An exogenous rise in the H endowment represents (in relative terms) a negative income shock for F consumers as it tends to decrease the relative value of the Foreign endowment, \( \hat{\nu}^*_t \). Relative price fluctuations can lessen or magnify this effect. If the terms of trade depreciate, F consumers are partially compensated by a rise in the international price of their output. Price movements provide some automatic insurance to F agents by transferring them some purchasing power, and limit the fall in \( \hat{\nu}^*_t \). On the contrary, a terms of trade appreciation would contribute to hurt F consumers, by reducing the international price of their exports. In this case terms of trade fluctuations would magnify the fall in \( \hat{\nu}^*_t \) that would have occurred at constant prices.
1.4. FINANCIAL AUTARKY AND INSURANCE FROM RELATIVE PRICES

Following the shock, deviations of relative prices from the Steady State map into deviations in relative consumption through their impact on relative income. The gap arising in the model in response to the disturbance measures the deviation from full risk-sharing created by the shock and quantifies the insurance properties of relative prices. The response of the gap reflects also changes in the relative wealth of the two countries created by the disturbance, as the following subsection shows.

1.4.1 Changes in relative wealth under Financial autarky

Agents’ intra-temporal decisions can be solved through standard expenditure minimization. The inter-temporal problem of H consumers may be solved by setting up the Lagrangian

\[
\max_{C_t, \lambda_t} : L = \frac{(C_t)^{1-\rho}}{1-\rho} + \lambda_t (P_{ht}Y_{ht} - P_tC_t), \forall t
\]

where \(\lambda_t\) is the multiplier attached to H agents’ budget constraint, and represents the shadow value of current income, \(P_{ht}Y_{ht}\). The first order condition with respect to consumption reads

\[
(C_t)^{-\rho} = \lambda_t P_t
\]

Its analogue for F consumers is given by

\[
(C_t^*)^{-\rho} = \lambda_t^* P_t^*
\]

where \(\lambda_t^*\) is the shadow value of F household’s current income.

Taking the ratio of (1.8) and (1.9) gives

\[
\left(\frac{C_t}{C_t^*}\right)^{-\rho} = Q_t \left(\frac{\lambda_t}{\lambda_t^*}\right)
\]

Loglinearizing around the symmetric Steady State yields

\[
\hat{q}_t - \rho (\hat{\epsilon}_t - \hat{\epsilon}_t^*) \equiv \text{gap}_t = (\hat{\lambda}_t - \hat{\lambda}_t^*)
\]

The gap arising at time \(t\) in response to shocks reflects asymmetric responses of the shadow value of current income in the two countries. Its sign indicates the new ranking of wealth: if a shock raises more the shadow value of the F consumer’s income (makes him relatively poorer), the gap is negative; positive if the disturbance makes the F agent relatively richer. Thus the gap reflects the change with respect to the Steady State of the relative wealth of F consumers.

European University Institute
DOI: 10.2870/19601
1.4.2 Insurance from relative prices

From the expenditure minimization problem of F and H consumers, we get their demand for, respectively, H and F-produced goods, which read (in loglinearized form)

\[
\frac{1}{\theta} \dot{c}_{ht} = (1 - \delta) t\dot{\omega}_t + \frac{1}{\theta} \dot{c}_t + \dot{g}_t \quad (1.10)
\]

\[
\frac{1}{\theta} \dot{c}_{ft} = -\delta t\dot{\omega}_t + \frac{1}{\theta} \dot{c}
\]

Linearizing the Financial autarky budget constraints we can express Home and Foreign consumption as

\[
\dot{c}_t = (\delta - 1) t\dot{\omega}_t + \dot{y}_{ht} \quad (1.12)
\]

\[
\dot{c}_t^* = (1 - \delta) t\dot{\omega}_t + \dot{y}_{ft} \quad (1.13)
\]

Using the price indexes \(P_t\) and \(P_t^*\) derived from expenditure minimization, the real exchange rate can be written as

\[
\dot{q}_t = (2\delta - 1) t\dot{\omega}_t \quad (1.14)
\]

From the balance trade condition, the terms of trade are given by

\[
t\dot{\omega}_t = \ddot{c}_t - \ddot{c}_t
\]

Substitute for (1.10), (1.11), (1.12), (1.13), and (1.14) to get the response of the terms of trade to shocks to the H and F endowments

\[
t\dot{\omega}_t = \frac{1}{1 - 2\delta (1 - \theta)} : (\dot{y}_{ht} - \dot{y}_{ft}) \quad (1.15)
\]

Assume the system is in Steady State up to time \((t - 1)\) and assume a national shock at time \(t\), an exogenous increase in the H endowment.

Combining the solution for the response to the shock of Home and Foreign consumption and the real exchange rate, the gap arising at time \(t\) can be written as a function of the deep parameters of the model as

\[
gap_t^{FA} = \dot{q}_t - \rho (\dot{c}_t - \dot{c}_t^*) = \frac{(2\delta \rho \theta - \rho - 2\delta + 1)}{(2\delta \theta - 2\delta + 1)} \cdot \varepsilon_{Ht} \quad (1.16)
\]
where \( \varepsilon_{Ht} \) is the i.i.d. shock to the H endowment and the second line follows from substituting (1.15) in (1.12), (1.13), and (1.14). There are two values of the trade elasticity \( \theta \) for which the terms in parentheses in equation (1.16) are equal to zero. These values define two thresholds for the trade elasticity that play a crucial role in shaping the wealth imbalances created by the shock.\(^{10}\)

The first threshold is the value of \( \theta \) for which the denominator in (1.16) is equal to zero

\[
\tilde{\tau} \equiv 1 - \frac{1}{2\delta}
\]

I will refer to \( \tilde{\tau} \) as the transmission threshold, relating to the findings of Corsetti et al. (2008) who analyze the behaviour of the same model under Financial autarky. If the trade elasticity is higher than \( \tilde{\tau} \), an increase in the H endowment depreciates the terms of trade. Foreign consumers benefit from the rise in the international price of their output and the bonanza in the H country raises consumption abroad – the international transmission of shocks is positive. For an elasticity lower than \( \tilde{\tau} \) the same shock appreciates the terms of trade. Foreign consumers are hurt by a fall in the relative price of their output and the windfall in the H country reduces consumption abroad – international transmission is negative.\(^{11}\)

The second threshold is the value of \( \theta \) for which the numerator in (1.16) is equal to zero

\[
\tilde{\eta} \equiv \rho + 2\delta - 1
\]

Since the gap is zero when \( \theta = \tilde{\eta} \), I call this value the efficiency threshold. If the trade elasticity matches \( \tilde{\eta} \), full risk-sharing is attained in response to the shock even in the absence of financial markets.

I simulate a 1% positive shock to the H endowment at time \( t \). The following graphs show the time-\( t \) response of the terms of trade and the gap, both as a function of the trade elasticity.

---

\(^{10}\)The value of the trade elasticity is key in determining the equilibrium volatility of the terms of trade. Therefore, analyzing transmission through relative price fluctuations in terms of the trade elasticity seems a natural choice.

\(^{11}\)The different response of relative prices along the elasticity domain can be explained as follows. After the rise in the H endowment, world demand for the H good must increase. Relative prices must adjust to raise this demand. Corsetti et al. (2008) show that a fall in the relative price of the H good tends, on the one hand, to switch world expenditure towards the H variety, raising its demand (Pareto-Edgeworth substitution effect). On the other hand, the same fall in the H price lowers the value of the endowment of H consumers, the biggest buyers of the H variety. The fall in their purchasing power tends to lower world demand for the H commodity (Pareto-Edgeworth income effect). When \( \theta < \tilde{\tau} \) the degree of substitutability between goods is so low that the income effect prevails. Increasing world demand for the H good requires raising the income of its biggest buyers, H consumers. The relative price of the H variety must increase in equilibrium.

As in Corsetti et al. (2008) strong income effects might give rise to multiple equilibria. It should be noted that this feature would not affect the main results of the analysis (see also the discussion in Corsetti et al. (2002) and (2008)). A low trade elasticity per se does not imply the violation of any stability condition for the model.
elasticity $\theta$.\footnote{I assume $\delta = 0.9, \beta = (1/1.01), \rho = 2$. The persistence of the shock does not influence the response of the gap and the terms of trade under Financial autarky.}
1.4. FINANCIAL AUTARKY AND INSURANCE FROM RELATIVE PRICES

FIGURE 1
Terms of trade response as a function of $\theta$ (Financial autarky)

FIGURE 2
Gap response as a function of $\theta$ (Financial autarky)

If the two varieties were perfect substitutes (if the trade elasticity $\theta$ was infinite), relative prices would not react to the shock and the rise in the H endowment would make Foreign
consumers relatively poorer. The gap arising in response to the disturbance – the change in
the relative wealth of F consumers with respect to the Steady State – would be negative
\[
\lim_{\theta \to -\infty} gap_{t}^{FA} F A = \lim_{\theta \to -\infty} - \frac{(2\delta \rho \theta - \rho - 2\delta + 1)}{(2\delta \theta - 2\delta + 1)} = -\rho
\]

As long as the two goods are not perfect substitutes, however, relative price fluctuations
transfer automatically purchasing power across countries. This can lead to a variety of
outcomes.

If the trade elasticity coincides with the efficiency threshold \((\theta = \tilde{\eta})\), the shock raises
the international price of the F good (the terms of trade depreciate). F consumers benefit
from higher export prices. The extent of the depreciation is exactly sufficient to pool risk
efficiently and relative wealth is not affected by the shock: the gap is zero. As in Cole and
Obstfeld (1991), price fluctuations guarantee full risk-sharing even in the absence of financial
markets. \(^{13}\)

\[
gap_{t}^{FA}(\theta = \tilde{\eta}) = - \frac{(2\delta \rho \tilde{\eta} - \rho - 2\delta + 1)}{(2\delta \tilde{\eta} - 2\delta + 1)} \cdot \varepsilon_{Ht} = 0 \iff \hat{x}_t = \hat{x}_t^*
\]

If the trade elasticity is higher then the efficiency threshold \((\theta > \tilde{\eta})\), the relative price of
the Foreign output rises in response to the shock, but due to the high degree of substitutability
between commodities, only a small depreciation is needed to clear the good markets after
the rise in the supply of the H good. Due to the insufficient terms of trade volatility, price
fluctuations do not transfer enough purchasing power to F consumers. Their relative wealth
falls with respect to the Steady State, as signaled by the negative gap. From (1.16)

\[
gap_{t}^{FA} < 0 \iff \hat{x}_t < \hat{x}_t^*
\]

For a trade elasticity lower than the efficiency threshold, still higher than the transmission
one \((\tilde{\tau} < \theta < \tilde{\eta})\), the relatively low degree of substitutability between varieties requires a
strong depreciation of the terms of trade for good markets to absorb the excess supply of the
H good. Due to the strong rise in the relative price of the Foreign output, the shock makes
F consumers relatively richer (positive gap).

Finally, for a trade elasticity lower than the transmission threshold \((\theta < \tilde{\tau})\), the terms
of trade appreciate in equilibrium. Price movements enlarge the wedge between the relative
value of endowments that would have aroused at given prices. F consumers are hurt both

\(^{13}\)The efficiency threshold generalizes specific models used in other contributions. Cole and Obstfeld (1991)
assume a Cobb-Douglas consumption aggregator and no home bias: \(\theta = 1\), \(\delta = (1/2)\). The core building
block of the monetary model of Corsetti and Pesenti (2005) is based on log utility and unitary trade elasticity,
namely \(\rho = 1\) and \(\theta = 1\). In both papers asset markets are shown to be redundant.
1.4. **FINANCIAL AUTARKY AND INSURANCE FROM RELATIVE PRICES**

by the relative fall in the quantity of their endowment and by the fall in the price of their output, and are made relatively poorer by the disturbance.

1.4.3 The interaction between prices and flows

Absent financial markets, national shocks typically make the real allocation diverge from the one corresponding to full risk-sharing. Local disturbances alter countries’ relative wealth and cause aggregate welfare losses.\(^{14}\) If asset trade was introduced, to which country should financial resources optimally flow after a national shock? As a first step to answer this question, it is useful to study the interaction between cross-border flows and price fluctuations. Proposition 1.1 describes how relative prices react to an exogenous marginal flow of resources across countries in Financial autarky.\(^ {15}\)

**Proposition 1.1** A marginal flow of resources from the H to the F country raises the price of the F output (depreciates the terms of trade) if the trade elasticity is above the transmission threshold \(\tau\) – prices and flows are **complements** in providing insurance; lowers the price of the F output otherwise – prices and flows are **substitutes**.

Figure 3 shows the response of the terms of trade to a marginal flow of resources to Foreign consumers, as a function of the trade elasticity \(\theta\).

---

\(^{14}\) Appendix B proves that in this model the gap reflects aggregate welfare losses due to imperfect insurance.

\(^{15}\) Appendix C shows the Proofs to all the Propositions.
1.4. FINANCIAL AUTARKY AND INSURANCE FROM RELATIVE PRICES

The different response of relative prices along the elasticity domain can be explained as follows. Due to home bias in consumption, a flow of resources to F consumers raises relative world demand for the Foreign good. Relative prices must adjust to reduce this demand. As shown by Corsetti et al. (2008), a rise in the price of the F good has a twofold effect on its demand. The Pareto-Edgeworth substitution effect tends to shift expenditure away from Foreign goods, lowering their demand. The Pareto-Edgeworth income effect raises the value of the endowment of Foreign consumers, the biggest buyers of the F good. The increase in their purchasing power tends to raise world demand for the F variety. For $\theta < \tilde{\tau}$, the income effect prevails. In this case, reducing the demand for the Foreign good requires lowering the income of its biggest buyers, F consumers. The relative price of the F variety must fall in equilibrium.

Thus the threshold $\tilde{\tau}$ defines two areas of the trade elasticity domain, which correspond to different kinds of interactions between prices and flows. For $\theta > \tilde{\tau}$ receiving resources from abroad raises the price of a country’s output on impact: both flows of resources and price fluctuations transfer purchasing power to F consumers – the insurance channels are complements. For $\theta < \tilde{\tau}$, when resources flow to the F country, price fluctuations tend to transfer purchasing power abroad – prices and flows are substitutes in providing insurance.

Proposition 1.2 describes how relative price fluctuations created by cross-border flows affect, in turn, the relative wealth of the two countries.

**Proposition 1.2** A marginal flow of resources from the H to the F country raises the relative wealth of Foreign consumers if the trade elasticity is above the transmission threshold $\tilde{\tau}$ (if prices and flows are complements); lowers the relative wealth of F consumers otherwise (if prices and flows are substitutes).

The first statement is not surprising. When flows and prices are complements, F consumers benefit twice from an inflow of resources from abroad as the relative price of their exports rises. Thus the inflow increases their relative wealth.

The second statement is less obvious. When flows and prices are substitutes, they channel purchasing power in opposite directions and the net effect of a transfer on countries’ wealth is in principle ambiguous. Proposition 2 shows that the low trade elasticity associated with substitutability, generates a high volatility of the terms of trade in response to a marginal transfer. The negative effect of price fluctuations on Foreign consumers’ wealth prevails over

---

16See Corsetti et al. (2008) for a formal analysis of the relative strength of Pareto-Edgeworth substitution and income effect in the same model.
the positive impact of the flow itself. Thus a transfer from abroad lowers the relative wealth of the receiving country.

1.5 Efficient financial flows from external borrowing

Given the interaction between cross-border flows of resources and price fluctuations, how should efficient financial markets be identified? As noted by Cole (1991), well-functioning financial markets play two roles. First, frictionless trade in uncontingent bonds allows consumers to smooth the effects of national disturbances on their consumption by borrowing and lending abroad. Second, frictionless trade in contingent assets allows households to form a portfolio that can hedge them ex-ante against shocks.

In this and the next section I analyze the model under two different asset trade regimes, representing respectively the two functions of efficient financial markets, consumption smoothing (through external borrowing) and ex-ante hedging (through portfolio diversification). In both cases, I assume no frictions in international asset trade: agents are allowed to optimize over their purchase of securities and have rational expectations; there are no costs associated to asset trade. In both setups, price fluctuations and financial flows will transfer purchasing power across countries. Given the interaction between flows and prices, I will study to which economy efficient financial markets – allowing consumers to borrow from abroad or to diversify their portfolio – channel resources after national shocks.

This section studies optimal financial flows from external borrowing. Consider the model presented in section 1.3 under the following asset trade regime: agents can trade in uncontingent bonds paying 1 unit of the H consumption bundle in every state-of-the-world.\textsuperscript{17} \textsuperscript{18}The budget constraint of H consumers reads...

\textsuperscript{17}Self-insurance is typically defined as saving and borrowing activities carried out through a risk-free asset. In a two-good model that allows for deviations from purchasing power parity every asset, even an uncontingent one, bears some riskiness as endogenous price movements tend to alter its real returns. However when the model is approximated to the first order, the expected returns on all assets are equalized. Choosing a symmetric Steady State with a zero bond position as the approximation point, is sufficient to ensure that asset returns do not transfer resources across countries in response to shocks. In terms of allocation and dynamics of the model, it does not matter which particular uncontingent asset we assume to be used to self-insure. Hence, the assumption that the bond pays in unit of the H consumption bundle is unconsequential for the results.

\textsuperscript{18}In this section, I use trade in one bond to analyze pure consumption smoothing activities, abstracting from any valuation effect that could be induced by portfolio holdings. It should be noted however that trade in two bonds denominated in different currencies would create powerful valuation effects that would effectively complete asset markets (up to a first order approximation), as Engel and Matsumoto (2009) show. In the context of our model, our results about the direction of optimal financial flows would not be altered, as this asset structure would yield the same efficient financial flows as those described in the case of trade in two equities (section 1.6).
1.5. EFFICIENT FINANCIAL FLOWS FROM EXTERNAL BORROWING

\[ P_t^B B_{t+1} = \frac{P_{ht} Y_{ht}}{P_t} - C_t + B_t \]  

where \( B_{t+1} \) denotes bonds purchased in period \( t \), \( P_t^B \) their unitary price in terms of the H consumption bundle. Bonds are assumed to be in zero net supply. H households’ inter-temporal problem is given by

\[
\max_{\{C_t, B_{t+1}, \nu_t\}} : L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t)^{1-\rho}}{1 - \rho} + \nu_t \left( P_{ht} Y_{ht} + P_t B_t - C_t P_t - P_t^B P_{t+1} \right) \right] \right\} 
\]

H agent’s first order conditions read

\[ (C_t)^{-\rho} = \nu_t P_t \]  

\[ P_t^B = \beta E_t \left\{ \frac{\nu_{t+1} P_{t+1}}{\nu_t P_t} \right\} \]  

Combining (1.19) and (1.20) with their analogues for the F consumer and loglinearizing gives

\[ -\rho E_t \{ \hat{c}_{t+1} \} + \rho \hat{c}_t = -\rho E_t \{ \hat{c}_{t+1}^* \} - E_t \{ \hat{q}_{t+1}^* \} + \rho \hat{c}^* + \hat{q}_t^* \]

Assuming that at time \( t \) the system is in a symmetric Steady State with zero bond holdings yields

\[ E_t \{ \hat{q}_{t+1} - \rho (\hat{c}_{t+1} - \hat{c}_{t+1}^*) \} = E_t \{ gap_{t+1} \} = 0 \]

The possibility of self-insuring drives to zero the expected risk-sharing inefficiency by equating the expected Stochastic Discount Factors of the representative agents.

The following Proposition describes optimal financial flows from external borrowing

**Proposition 1.3** If flows and prices are complements, in response to national shocks efficient financial markets channel resources through external borrowing to the country that (absent asset trade) would have been made relatively poorer by the disturbance (“standard efficiency condition”). If flows and prices are substitutes, efficient financial markets transfer resources to the country that (absent asset trade) would have been made relatively richer by the shock.
1.5. EFFICIENT FINANCIAL FLOWS FROM EXTERNAL BORROWING

Figure 4 gives a graphical representation. I simulate a 1% increase in the H endowment at time $t$. The graph plots the time-$t$ response of the Financial autarky gap (the relative wealth of F consumers absent asset trade) and F agents borrowing, $-\dot{b}_t^*$ (the inflow of financial resources received by F consumers), both as a function of the trade elasticity $\theta$.\footnote{I assume a persistent but temporary shock with autoregressive parameter $\zeta = 0.95$. The model with trade in uncontigent bonds is non-stationary because of the unit root in wealth accumulation. This feature does not have any bearings for the main results of the analysis. Indeed, it is easy to verify that Proposition 1.3 holds in the same model with stationarity-inducing features à la Schmitt-Grohé and Uribe (2003) (e.g. an endogenous discount factor).}

![Graph showing the response of Financial autarky gap and F current account deficit as a function of $\theta$.](image)

**FIGURE 4**
Response of the Financial autarky gap and F current account deficit as a function of $\theta$

If the trade elasticity is higher than the efficiency threshold $\bar{\eta}$, absent financial markets a temporary bonanza in the H country would have made F consumers relatively poorer; once we introduce efficient external borrowing, F households import resources from abroad. If the trade elasticity is in between the two thresholds ($\bar{\tau} < \theta < \bar{\eta}$), due to the high terms of trade volatility F consumers would have been relatively richer under Financial autarky; once we allow for asset trade, financial flows channel resources to the H country. Thus in the region of the elasticity domain in which prices and flows are complements, in response to the shock efficient financial markets channel resources to the country that, absent asset trade, would...
have been made relatively poorer by the disturbance. Flows from external borrowing are consistent with the predictions of the “standard efficiency condition” for financial markets.\footnote{Notice that the current account is procyclical.}

On the contrary, in the area of the elasticity domain in which prices and flows are substitutes, efficient financial markets transfer resources “upstream”, to the country that (absent asset trade) would have been relatively richer. Under Financial autarky, F consumers would have been hurt both by the relative fall in the quantity of their endowment and by the fall in the price of their output; once we introduce asset trade, F households export resources.\footnote{The current account is countercyclical.} These flows from external borrowing are optimal by construction: they are the best possible transactions occurring in a decentralized equilibrium with borrowing and lending, since I assumed no restrictions, frictions or costs for international asset trade. Yet, these flows runs against the predictions of the “standard efficiency condition”.

Upstream flows can be explained as follows. Focus on the region of the trade elasticity domain in which prices and flows are substitutes. Proposition 1.1 states that for $\theta < \bar{\tau}$ a marginal flow of resources to the F country would lower the price of the Foreign commodity, moving the terms of trade against F consumers. Proposition 1.2 shows that if the F country – the one that would be relatively poorer absent asset trade – imported resources from abroad, relative prices would move substantially against it due to the low trade elasticity. Importing resources from abroad would lower so much the relative price of the F commodity that it would reduce the relative wealth of Foreign consumers. Therefore, efficient financial markets must transfer resources to the H country – upstream.\footnote{Propositions 1.1 and 1.2 defining complementarity and substitutability are proved in the model under Financial autarky. It is important to notice that the relationship tying marginal flows of resources and price fluctuations, is preserved in the model with trade in uncontingent bonds. Propositions 1.7 and 1.8 in Appendix C give a formal proof.}

1.5.1 External borrowing and equilibrium wealth imbalances

The previous section has derived a simple rule relating the financial flows we should observe if financial markets were efficient and countries’ relative wealth absent asset trade. Here I study the relationship between efficient financial flows and realized wealth imbalances. If financial markets are efficient can we observe a temporarily richer country borrowing?

First, notice that the gap arising \textit{in equilibrium} in the model with trade in uncontingent bonds, reflects the change in countries’ relative wealth created by national shocks. To see this consider the inter-temporal problem described in equation (1.18). The Lagrange multiplier
1.5. EFFICIENT FINANCIAL FLOWS FROM EXTERNAL BORROWING

\( v_t \) represents the shadow value of time-\( t \) income. The income received at time \( t \) (minus current consumption), in turn, equals the present discounted value of wealth

\[
(P_h Y_t + B_t P_t - P_t C_t) = \sum_{j=1}^{\infty} \prod_{k=0}^{j-1} P_{t+j}^R [P_{t+j} C_{t+j} - P_{h,t+j} Y_{t+j}] \equiv PDV \text{ wealth}
\]

Hence, \( v_t \) is the shadow value of wealth at time \( t \). Combining H agent’s first order condition with respect to consumption (equation (1.19)) with its analogue for the F consumer, and loglinearizing around the symmetric Steady State with zero bond holdings gives

\[
\dot{q}_t - \rho (\hat{c}_t - \hat{c}_t^*) \equiv \text{gap}_t^{SM} = (\hat{v}_t - \hat{v}_t^*)
\]  

(1.21)

The \text{gap} arising in the smoothing economy, \( \text{gap}_t^{SM} \), reflects asymmetric responses in the shadow value of wealth in the two countries.

In terms of the deep parameters of the model, \( \text{gap}_t^{SM} \) arising in response to an increase in the H endowment can be written as

\[
\text{gap}_t^{SM} = - \left( \frac{\beta - 1}{\zeta \beta - 1} \right) \cdot \left( \frac{2\delta \rho \theta - 2\delta - \rho + 1}{2\delta \theta - 2\delta + 1} \right) \cdot \varepsilon_{Ht} = \left( \frac{\beta - 1}{\zeta \beta - 1} \right) \cdot \varepsilon_{Ht} \cdot \text{gap}_t^{FA}
\]  

(1.22)

where \( \zeta \) and \( \beta \) denote respectively the persistence of the disturbance and households’ discount factor. Setting the risk-sharing inefficiency to zero in expectations, which is the result of self-insurance, leaves it exposed to unexpected \text{i.i.d.} shocks. The \text{gap} arising in equilibrium in the smoothing economy has the same sign as the one that would have aroused after the same shock under Financial autarky.\(^{23}\)

Proposition 1.3 then implies that we could \textit{observe} a relatively richer country \textit{optimally} borrowing from a relatively poorer one – financial resources optimally flowing upstream. The following figure illustrates this point graphically by plotting the response to a 1% temporary rise in the H endowment, of \( \text{gap}_t^{SM} \) (the change in the relative wealth of Foreign consumers created by the shock) and F agents’ current account deficit, both as a function of the trade elasticity \( \theta \)

\(^{23}\)Exceptions may arise if households’ discount factor \( \beta \) is endogenized or if the shock process is non-stationary. See Corsetti et al. (2008) for an example with trade in bonds in which the shock increases the endowment over time, reaching a higher permanent long-run level.
If the trade elasticity is lower than the transmission threshold $\tilde{\tau}$, we could observe a relatively poorer country, $F$, lending abroad.

Notice that in this setup the optimality of upstream financial flows and the countercyclicality of the current account are not due to a higher productivity in the temporarily richer country, as happens, for instance, in the standard Backus-Kehoe-Kydland type of models.\textsuperscript{24} As explained in the previous subsection, the optimality of upstream flows is rather due to the particular kind of interaction between financial flows and price fluctuations.

Notice also that the so-called intertemporal approach to the current account (one of the theories at the basis of the “standard efficiency condition” for financial markets) developed traditionally in one-good or partial equilibrium settings, can be recasted as a special case within our framework. It is characterized by an infinite trade elasticity, which rules out any interaction between terms of trade adjustments and financial flows. The allocation it delivers would be on the far right in Figure 5: the country going through a temporary negative conjuncture would be poorer and borrowing from abroad in equilibrium.

\textsuperscript{24}See Backus et al. (1995).
1.5. EFFICIENT FINANCIAL FLOWS FROM EXTERNAL BORROWING

1.5.2 External borrowing and welfare

Appendix B proves that in our DSGE model, under any asset trade regime, the gap is a measure of the social welfare losses due to imperfect insurance. The higher the welfare losses created by uninsurable shocks, the higher the gap arising in response to the disturbances.

From equation (1.22) it is easy to verify that the equilibrium smoothing gap is lower than the gap that would have aroused after the same shock under Financial autarky.25 Thus the cross-border flows created by external borrowing provide insurance against shocks, containing cross-country wealth divergences and reducing the welfare losses created by national disturbances.

Figure 6 compares the response to the same shock (a 1% increase in the H endowment) of the gap arising under Financial autarky and under trade in uncontingent bonds. Financial flows from external borrowing are beneficial even if they run upstream, as they increase cross-border insurance and reduce the welfare losses created by the shock, with respect to the ones that would have aroused absent asset trade ($gap_{SM} < gap_{FA}$ also for $\theta < \tilde{\tau}$, corresponding to upstream flows).

\[ Viani, Francesca (2010), International Financial Markets, Cross-Border Transmission of Shocks and Risk-Sharing \\
European University Institute \\
DOI: 10.2870/19601 \]

\[ ^{25}\text{Again, exceptions may arise if households’ discount factor } \beta \text{ is endogenized or if the shock process is non-stationary.} \]
1.6 Efficient financial flows from portfolio returns

In this section I analyze the model presented in section 1.3 under a different asset trade regime, representing the second function of efficient financial markets – ex-ante hedging through portfolio diversification. In particular, I choose an asset structure that allows agents to be fully insured against national shocks: assets markets are effectively complete by construction. As for the smoothing case, I assume no frictions in international asset trade. Given the interaction between flows and prices, I study to which economy efficient financial flows from portfolio returns channel resources in response to shocks.

Assume households can trade in two equities, H and F, representing respectively a claim to the H and F country’s endowment. As we will verify at the end of this section, asset markets are effectively complete. The total quantity of each equity is normalized to 1. $S_h, S_f, S_h^*,$ and $S_f^*$ denote the fraction of H and F equities owned respectively by H and F consumers. Due to the normalization, the owner of $S_h$ equities receives a share $S_h$ of the H endowment. Equities real returns, expressed in terms of the H consumption good, are given by

$$R_t = \frac{(P_{ht}/P_t)Y_{ht} + Z_t}{Z_{t-1}}$$

$$R_t^* = \frac{(P_{ft}/P_t)Y_{ft} + Z_t^*}{Z_{t-1}^*}$$

where $Z$ and $Z^*$ denote the real price (in terms of the H consumption bundle) of the two assets.

The budget constraint of H households reads

$$NFA_t = NFA_{t-1} + \frac{Y_{ht}}{P_t} - C_t + R_t \cdot (S_{ht} - 1) + R_t^* S_{ft}$$

where $NFA_t \equiv [(S_{h,t+1} - 1) \cdot Z_t + S_{f,t+1} \cdot Z_t^*]$ denotes H net foreign assets.

Euler equations and asset market clearing conditions are given by

$$\left(\frac{C_t}{Q_t}\right)^{-\rho} = \beta E_t \left\{ (\frac{C_{t+1}}{Q_{t+1}})^{-\rho} \cdot R_{t+1} \right\} = \beta E_t \left\{ (S_{ht} - 1) - (S_{ft} - 1) \right\}$$

$$\left(\frac{C_{t}^*}{Q_{t}^*}\right)^{-\rho} = \beta E_t \left\{ (\frac{C_{t+1}^*}{Q_{t+1}^*})^{-\rho} \cdot R_{t+1}^* \right\} = \beta E_t \left\{ (S_{h}^* - 1) - (S_{f}^* - 1) \right\}$$

26 Also Kollmann (2006) proves that trade in two equities makes asset markets effectively complete in this model.
1.6. EFFICIENT FINANCIAL FLOWS FROM PORTFOLIO RETURNS

\[ S_{h,t} + S_{h,t}^* = 1 \]

\[ S_{f,t} + S_{f,t}^* = 1 \]

The model is loglinearized around a symmetric Steady State with \( NFA = NFA^* = 0 \).\(^{27}\)

H consumers’ budget constraint can be written as

\[ \hat{d}_{nfa_t} = \left( \frac{1}{\beta} \right) \hat{d}_{nfa_{t-1}} + (\delta - 1) \hat{tot}_t + \hat{y}_{ht} - \hat{c}_t + \hat{r}_x t \alpha \]

(1.25)

where \( \hat{r}_x t = \hat{r}_t - \hat{r}_t^* \) is the percentual deviation of the realized return differential between H and F equities from its Steady State value. \( \alpha \) denotes H agents’ gross holdings of the H equity in Steady State. In response to shocks, excess returns \( \hat{r}_x \) deviate from their Steady State value and, combined with asset holdings \( \alpha \), deliver a transfer of resources in consumers’ budget constraints. \( \xi_t \) in equation (1.25) denotes the financial flows from portfolio returns received by H consumers.

I find it convenient to derive a solution for the model using the method developed by Devereux and Sutherland (2006) – DS henceforth – to solve DSGE models with endogenous portfolio choice. Applying this method allows to study in detail how the interaction between flows and prices determines (a) the direction of optimal financial flows via portfolio returns and (b) the composition of optimal portfolios.\(^{28}\)

DS show that in a first-order approximation optimal portfolio choice can be decomposed in two steps.\(^{29}\) The first step consists in solving the system linearized to the first order treating realized portfolio resource transfers (\( \xi \)), as exogenous disturbances.\(^{30}\) Combine the Euler equations (1.23) and (1.24) linearized to the first order and assume that at time \( t \) the system is in a symmetric Steady State to get

\(^{27}\)Even if Steady State net foreign assets are zero, agents may hold a non-zero gross position. Therefore, they can receive flows of resources from the returns on their gross holdings in response to shocks.

\(^{28}\)Since asset markets are effectively complete and the Welfare Theorems hold, in principle I could solve for the Pareto optimal real allocation independently of asset holdings, and retrieve portfolios ex-post, as the ones that replicate a Social Planner solution to the model. However, by using the DS method we can provide an explanation for the direction of optimal flows (in Proposition 1.5). We are also able to provide a rationale for the composition of optimal portfolios (detailed in the Proof to Proposition 1.6).

\(^{29}\)Although applying the DS algorithm to a first order approximation of the model allows to retrieve only the Steady State portfolio (and not its dynamic response to shocks), characterizing the time-invariant asset holdings that arise in a world of infinitesimally small noise is sufficient to analyse the first order dynamics of the macro variables of the system.

\(^{30}\)In a first order approximation the expected returns on all assets are equalized: the expected return differential is zero. Therefore, \( \hat{r}_x \alpha \), the transfer delivered by the Steady State portfolio in response to shocks, can be treated as a mean-zero i.i.d. random variable (\( \xi \)) and included in the system as an exogenous disturbance with these characteristics.
1.6. EFFICIENT FINANCIAL FLOWS FROM PORTFOLIO RETURNS

\[ E_t \{ \hat{q}_{t+1} - \rho (\hat{c}_{t+1} - \hat{c}^*_t) \} = 0 \Rightarrow E_t \{ \text{gap}_{t+1} \} = 0 \]

The solution to this step delivers the same allocation that would arise if agents could only trade in uncontingent bonds.\(^{31}\)

The second step is based on the macro dynamics of the first-order solution, and amounts to recover optimal asset holdings \( \alpha \) as those satisfying second-order optimality conditions for portfolio choice.\(^{32}\) Taking a second-order approximation of the H and F Euler equations and combining them, DS show that the optimal portfolio must satisfy the following condition

\[ \text{Cov}_t \{ [\hat{q}_{t+1} - \rho (\hat{c}_{t+1} - \hat{c}^*_t)], r \hat{x}_{t+1} \} = 0 \]

Translating this equation in our framework, it is easy to see that it implies

\[ \text{Cov}_t \{ \text{gap}_{t+1}, r \hat{x}_{t+1} \} = 0 \]

The wealth asymmetry residual from smoothing decisions can be lowered exploiting movements in asset excess returns. Agents adjust the composition of their portfolio until comovements between excess returns and (endogenous) wealth asymmetries are driven to zero in equilibrium.

Having recovered the optimal portfolio through the second step, the final solution to the system is

\[
\begin{bmatrix}
\hat{q}_t \\
\hat{c}_t \\ \hat{c}_t^*
\end{bmatrix} =
\begin{bmatrix}
\hat{y}_{ht} \\
\hat{y}_{ft}
\end{bmatrix} +
\begin{bmatrix}
A \\
B
\end{bmatrix}
\begin{bmatrix}
\varepsilon_t \\ \varepsilon_t^*
\end{bmatrix}
\]

(1.26)

Matrix \( A \) is the solution to the first step. It describes what would be the impact of national shocks on endogenous variables if agents could only trade in uncontingent bonds, pinning down dynamics that coincide with the smoothing ones. Matrix \( B \), instead, results from the solution to the second step of the DS method. It traces the dynamics given only by resource flows generated by the optimal portfolios. Analogously we can write

\(^{31}\)Crucial to this identification is the fact that the system in this section and in the previous one is linearized around the same approximation point, corresponding to zero net foreign assets.

Notice that in the allocation resulting from this step (which is only a partial solution to the system), households can invest in assets but do not receive any transfer from their returns after national disturbances (\( \xi \) is exogenous and not responding to endowment shocks). Thus portfolio composition cannot be exploited to hedge against shocks. Intuitively, what drives households decisions is the desire to smooth over time the effect of national shocks on consumption.

\(^{32}\)A second-order approximation is needed because assets yield equal expected returns in a first order approximation. Optimal portfolios would then be indeterminate.
1.6. EFFICIENT FINANCIAL FLOWS FROM PORTFOLIO RETURNS

\[ \text{gap}_t = \left[ C \right] \cdot \left[ \hat{y}_{ht} \right] + \left[ D \right] \cdot \left[ \varepsilon_t \right] \]  

(1.27)

where matrices C and D result from combining the appropriate rows of A and B in (1.26). The gap arising in equilibrium in response to shocks can be decomposed in two parts. The first one corresponds to \( \text{gap}^{SM} \), the gap that would arise if agents could only smooth consumption by varying their net foreign assets. The second part reflects the variations in relative wealth due to optimal portfolio returns. The DS method shows that we can rationalize optimal portfolio returns as those able to counterbalance the smoothing gap (the one resulting from the first part of the system), so as to reduce the equilibrium gap to zero.

1.6.1 The direction of optimal flows

From the solution to the full system (1.26), it is easy to prove the following Proposition

**Proposition 1.4** If flows and prices are complements, in response to national shocks efficient financial markets channel resources through portfolio returns to the country that (absent asset trade) would have been made relatively poorer by the disturbance (“standard efficiency condition”). If flows and prices are substitutes, efficient financial markets transfer resources to the country that (absent asset trade) would have been made relatively richer by the shock.

Figure 8 illustrates this result. I simulate a 1% increase in the H endowment at time \( t \). The graph plots the time-\( t \) response of the Financial autarky gap (the relative wealth of F consumers absent asset trade) and the portfolio returns received by F households, \( \xi_t^F \) (the inflow of financial resources received by F consumers), both as a function of the trade elasticity \( \theta \).\(^{33}\)

\(^{33}\)I assume the shock process has an autoregressive parameter \( \zeta = 0.95 \).
1.6. EFFICIENT FINANCIAL FLOWS FROM PORTFOLIO RETURNS

As it happens in the case of external borrowing, in the area of the elasticity domain in which prices and flows are substitutes, efficient financial markets transfer resources upstream through portfolio returns, to the country that (absent asset trade) would have been relatively richer. Notice that the flows from portfolio returns are optimal by construction: they are the best possible transfers arising in a decentralized equilibrium with portfolio diversification, since I assumed no restrictions, frictions or costs for international asset trade, and an asset structure that delivers full risk-sharing. But clearly these flows run against the predictions of the “standard efficiency condition” for financial markets.

The following Proposition, based on the first step of the DS method, shows that the intuition for optimal upstream flows is the same outlined for the smoothing case. The decomposition in (1.27) suggests that we can rationalize optimal portfolio flows as those able to counterbalance the smoothing gap (the one resulting from the first part of the system), so as to reduce the equilibrium gap to zero. Proposition 1.5 shows that a marginal portfolio flow has a different impact on the smoothing gap depending on whether flows and prices are complements or substitutes.

**Proposition 1.5** Receiving a marginal transfer of resources through portfolio returns raises the relative wealth of Foreign consumers absent portfolio flows (the smoothing gap) if and
only if the trade elasticity is above the transmission threshold \( \tilde{\tau} \), that is if and only if prices and flows are complements.

As it happens for the smoothing case, the different response of relative prices to international transfers (here, the position of the trade elasticity relative to the transmission threshold) is what determines the direction of optimal financial flows.

1.6.2 The composition of optimal portfolios

Proposition 1.6 shows that the interaction between price fluctuations and financial flows has also important implications for the composition of optimal portfolios.

**Proposition 1.6** If flows and prices are substitutes, optimal portfolios exhibit home bias; if flows and prices are complements, optimal portfolios may exhibit home bias, “excessive” home bias, or foreign bias.

Figure 7 represents the share of Home equities held in Steady State by H consumers as a function of the trade elasticity \( \theta \). A degree of home bias in equities consistent with the empirical evidence, is compatible with efficient financial markets if prices and flows are substitutes. Complementarity instead is not necessarily consistent with optimal home bias.

---

**FIGURE 7**

H consumers’ share of the H equity, as a function of \( \theta \)

---

34 The Proof to Proposition 1.6 uses the two steps of the DS method to provide a rationale for the composition of optimal portfolios in this model.
1.6.3 Insurance from portfolio flows

Appendix D proves that when consumers can trade in contingent assets, the gap arising in equilibrium in response to shocks reflects changes in relative countries’ wealth created by the disturbances. Clearly, these wealth imbalances are residual after portfolio returns have transferred resources across countries.

If asset markets are effectively complete, national shocks should not create any risk-sharing inefficiency, wealth asymmetry, and, in this model, welfare losses: the equilibrium gap should always be zero. The following figure shows that the asset structure we have assumed in this section effectively completes the market. The picture compares the gap arising after a 1% increase in the H endowment in Financial autarky, in the one-bond economy, and under trade in two equities. In the latter case, financial flows from portfolio holdings provide full insurance against national shocks. Clearly, this happens also in the region of the elasticity domain in which portfolio returns channel resources upstream ($\theta < \tilde{\eta}$).

![FIGURE 9](Image)

1.7 Some remarks on complementarity versus substitutability

This Chapter has shown that efficient financial markets can make resources flow from relatively poorer to relatively richer countries. These flows reduce the risk-sharing inefficiency created by national shocks. Even though upstream flows run against the “standard efficiency
condition” for financial markets, their effects and the conditions that generate them are not new to the international literature.

In our model the interaction between flows and prices, which determines the optimal direction of financial flows, has been shown to depend crucially on the trade elasticity. In particular, optimal upstream flows arise for a relatively low trade elasticity, around 0.5. In the empirical literature, there is uncertainty regarding the value of this parameter. For G-7 countries estimates based on aggregate time series data range between 0.1 to 2, as documented by Hooper et al. (2000). Corsetti et al. (2008) use US data to estimate the trade elasticity to be slightly below 0.5 – a value consistent with the existence of optimal upstream flows. This low trade elasticity is essential for their open-macro model to match the empirical evidence on the so-called Backus-Smith puzzle, the low correlation between relative cross-country consumption and the real exchange rate observed in the data. Corsetti et al. (2008) show that a trade elasticity around 1/2 is precisely what allows to reconcile standard international models, driven by business-cycle supply shocks, with the empirical evidence on consumption-real exchange rate correlations.  

Several contributions have investigated directly the interaction between international wealth transfers and price fluctuations. The literature on the so-called “transfer problem” studies the impact of international transfers of wealth on the terms of trade and the real exchange rate. While most works have estimated the impact of net foreign assets variations on relative prices in the long-run, Gagnon (1996) quantifies the short-run impact, the one that is relevant for the issue at handle. Using panel data for 20 industrial countries for the period 1973-1995, this paper finds that a deterioration in net foreign assets (corresponding in the short-run to a worsening in the trade deficit which implies an inflow of resources from abroad) is associated with an impact depreciation of the terms of trade. This evidence seems to be in favour of substitutability between flows and prices, which is necessary for optimal upstream flows.

---

35 An alternative would be a much higher trade elasticity (around 4, in line with estimates from the trade literature) and very persistent productivity shocks in an economy with capital accumulation. See Corsetti et al. (2008).

36 The definition of the transfer problem dates back to the 1920s when Keynes argued that the war reparations imposed on Germany after the First World War would have represented an excessive burden for the country. At the roots of the argument, there was the worry that transferring resources abroad would have moved international prices against Germany, raising the real cost of reparations.

37 Contributions focusing on long-run effects comprise Obstfeld and Rogoff (1994), Broner et al. (1998), and Lane and Milesi-Ferretti (2008). They typically find that a net foreign asset accumulation is associated with a long-run appreciation of the real exchange rate.
1.8 Concluding remarks

A large body of literature has investigated the consequences of financial market integration for international insurance. These tests are based on the presumption that risk-sharing requires financial markets to channel resources to countries that (absent asset trade) would be made relatively poorer by national shocks, net of physical capital accumulation. This condition of efficiency for financial markets is derived focusing only on one of the two channels of international insurance, cross-border financial flows, and assuming no interaction between this and the second channel, relative price fluctuations.

Against this literature, this Chapter shows that risk-sharing conditions can only be derived focusing on both channels and accounting for their interaction. In particular, financial flows and price fluctuations can be complements or substitutes in providing insurance, depending on the response of the price of a country’s output to financial inflows from abroad. While complementarity implies the standard condition of efficiency, substitutability requires well-functioning financial markets to channel resources “upstream”, from poorer to richer countries.

These results seem to question the validity of existing empirical methods based on the “standard efficiency condition”, and open new challenges for the empirical literature. The next Chapter investigates the implications of these theoretical findings for empirical methods. In particular, it shows the problems these frameworks could be subject to due to relative price fluctuations. Chapter 3 proposes a new empirical method that avoids these issues, hinging on insurance benchmarks that take into account the interaction between price fluctuations and financial flows.
1.9 Appendix A: Standard efficiency condition and empirical methods

According to the “standard efficiency condition”, international financial markets are efficient if, in response to a national income shock, they channel resources to countries that (absent asset trade) would be made temporarily poorer by the disturbance. This section shows that this condition is at the basis of virtually every empirical framework measuring international insurance.\(^\text{38}\)

Sorensen and Yosha (1998) (SY henceforth) assume no relative prices and rely on the following identity:\(^\text{39}\)

\[
C = GDP + NFI - NSAV
\]  
(1.28)

where \(NFI\) and \(NSAV\) denote respectively net financial income and net savings. From (1.28), full insurance – \(C\) and \(Y\) being uncorrelated, according to SY – requires a country experiencing a negative output shock (getting poorer absent asset trade) to receive income from abroad (either from portfolio returns or from external borrowing – net of physical capital accumulation). SY show that the decomposition in (1.28) allows to quantify the fraction of output shocks that is absorbed through different risk-sharing channels, by estimating

\[
-\Delta \log NFI_t \equiv (\Delta \log GDP_t - \Delta \log GNP_t) = \alpha_f + \beta_f \cdot \Delta \log GDP_t + \varepsilon_{ft} 
\]  
(1.29)

\[
\Delta \log NSAV_t \equiv (\Delta \log GNP_t - \Delta \log C_t) = \alpha_s + \beta_s \cdot \Delta \log GDP_t + \varepsilon_{st} 
\]  
(1.30)

\[
\Delta \log C_t = \alpha_u + \beta_u \cdot \Delta \log GDP_t + \varepsilon_{uf} 
\]  
(1.31)

where \(\beta_f\) is interpreted as the fraction of output shock that is absorbed through factor income flows, \(\beta_s\) is the share absorbed through consumption smoothing, and \(\beta_u\) the fraction left uninsured. In a two-country world (1.29) – (1.31) are equivalent to\(^\text{40}\)

\[
(\Delta \log NFI_i^t - \Delta \log NFI_j^t) = \alpha_f + \beta_f \cdot (\Delta \log GDP_i^t - \Delta \log GDP_j^t) + \varepsilon_{ft} 
\]  
(1.32)

\(^\text{38}\) Some tests aim only at verifying the null of full risk-sharing or perfectly integrated capital markets. In these cases, once the null is rejected, it is not clear how to derive implications for the actual degree of insurance. Most of these empirical frameworks test the equality of expected Stochastic Discount Factors across countries (Kollmann (1995), Ravn (2001)), and do not rely directly on the standard efficiency condition.

\(^\text{39}\) Equation (1.28) implies that consumption and output have the same composition, hence the same price.

\(^\text{40}\) See Dedola and De Fiore (2005).
\[
(\Delta \log NSAV_i^t - \Delta \log NSAV_j^t) = \alpha_s + \beta_s \cdot (\Delta \log GDP_i^t - \Delta \log GDP_j^t) + \varepsilon_{st} \quad (1.33)
\]

\[
(\Delta \log C_i^t - \Delta \log C_j^t) = \alpha_u + \beta_u \cdot (\Delta \log GDP_i^t - \Delta \log GDP_j^t) + \varepsilon_{ut} \quad (1.34)
\]

where superscripts denote different countries (or country aggregates). Under the maintained assumption of no relative prices, equations (1.32) – (1.34) imply the same view of optimal financial flows underlying (1.28) and (1.29) – (1.31): efficient financial markets should channel resources to the countries that (absent asset trade) would have been made relatively poorer by national shocks. Estimating slopes in the interval \([0, 1]\) signals a relatively high degree of insurance. Instead, \(\beta_u > 1, \beta_f < 0, \) and \(\beta_s < 0\) indicate that financial markets are highly inefficient, as they channel resources to relatively richer countries. Kalemli-Ozcan et al. (2004) estimate \(\beta_f < 0\) among European countries during the Nineties, and interpret this finding as signaling bad insurance from cross-border ownership of securities.

Under the assumption of no relative price fluctuations Obstfeld (1993) also derive an empirical framework in the form\(^{41}\)

\[
\Delta \log C_i^t = \alpha + \beta \cdot \Delta \log C_j^t + \gamma \cdot \Delta \log GDP_j^t + \varepsilon_t
\]

which underlies the same view of efficient financial markets as the methodology of SY. Both methods – adopted subsequently by a large body of followers – are based on the optimal pattern of financial flows described by the “standard efficiency condition”.

This Chapter questions the validity of the “standard efficiency condition” in a setup in which relative prices (realistically) fluctuate in response to cross-border financial flows. It finds that this condition gives only a partial representation of the optimal pattern of financial flows. The implications for existing empirical frameworks relying on the “standard efficiency condition” are discussed in length in Chapter 2 of this thesis. The main implications are the following. First, if relative prices are well-defined, identity (1.28) should be modified to take into account price fluctuations. Several refinements to the SY and Obstfeld approach have implemented this change, mainly deflating national output with the local CPI, so as to express all variables in terms of the country’s consumption.\(^{42}\) However – the second

---

\(^{41}\)This regression is analogous to (1.31) in SY, but does not impose a unitary coefficient on country \(j\) consumption.

\(^{42}\)See for example Sorensen and Yoshia (1998), Del Negro (2000), and Kalemli-Ozcan et al. (2004).

If relative prices are well-defined, identity (1.28) becomes
implication – once output is properly deflated, the theoretical results of the present Chapter apply: efficient financial markets can make resources flow “upstream”, from relatively poorer to relatively richer countries. That is, estimating \( \beta_f < 0 \) in (1.32) (with output properly deflated, as found by Kalemli-Ozcan et al. (2004)), or \( \beta_u > 1 \) in (1.34) can indicate an optimal outcome and may signal a high degree of insurance. Intuitively, a negative GDP shock may optimally lead to an even larger fall in relative consumption, provided it is compensated by an appreciation of the real exchange rate. In other words, a country’s wealth may be insulated from GDP fluctuations if a fall in its consumption is compensated by a strengthening of its currency. In this sense, the proper regression to run would be (see Chapter 3 for further details and an empirical application of this method)

\[
(\Delta \log C^i_t - \Delta \log C^j_t) - \frac{\Delta \log Q_t}{\rho} = \frac{gap^i_j}{\rho} = \alpha + \beta \cdot (\Delta \log GDP^i_t - \Delta \log GDP^j_t) + \varepsilon_t
\]

1.10 Appendix B: Measure of insurance and welfare

This section is devoted to prove that in the DSGE model used in this Chapter, there exists a monotone relationship between the gap and the social welfare losses due to imperfect insurance. I will show that the higher the gap that opens up at time \( t \), the higher the deviation of the time-\( t \) allocation from the optimum of a social welfare function that weights the two countries according to their previous wealth.

Consider the model outlined in section 1.3. The first-best plan that delivers the maximum social welfare attainable in this model can be characterized as the outcome of a Social Planner maximization. The Social Planner must allocate consumption between the two countries and weights them symmetrically. She solves the following problem

\[
\max E_0 \left\{ \sum_{t=0}^{\infty} \Omega^t (C^t)\frac{1-\rho}{1-\rho} + (1 - \Omega) \beta^t (C^* t)\frac{1-\rho}{1-\rho} \right\} 
\]

s.t. \( C_t = \left[ (\delta)^{1/\theta} (C^{ht}_t)^{\frac{\theta-1}{\theta}} + (1 - \delta)^{1/\theta} (C^{ft}_t)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \)

\[ C^* t = \left[ (\delta)^{1/\theta} (C^*^{ht}_t)^{\frac{\theta-1}{\theta}} + (1 - \delta)^{1/\theta} (C^*^{ft}_t)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \]  

\[
C = \frac{P_y \cdot GDP}{P} + NFI - NSAV
\]

where \( P \) and \( P_y \) are respectively the prices of consumption and output, and I have assumed that net financial income and net savings are expressed in terms of real consumption. Kalemli-Ozcan et al. (2004) motivate the need for this adjustment in footnote (36).
where $\Omega = (1 - \Omega)$. In what follows $\mu_t$ and $\mu^*_t$ denote the multipliers attached to the CES bundle constraints (equations (1.36) and (1.37)), that is the shadow value of consumption, respectively, in the H and in the F country. In the decentralized problem whose solution coincides with the Social Planner allocation, the ratio $\left(\frac{\mu^*_t}{\mu_t}\right)$ corresponds to the real exchange rate $Q_t$. The first order conditions with respect to $C_t$ and $C^*_t$ read

$$\Omega \cdot (C_t)^{-\rho} = \mu_t$$  \hspace{1cm} (1.38)$$

$$\left(1 - \Omega\right) \cdot (C^*_t)^{-\rho} = \mu^*_t$$  \hspace{1cm} (1.39)$$

The sequence of equations (1.38) and (1.39) evaluated at $t = 0, \ldots, \infty$ characterizes the first-best plan that (by construction) delivers the maximum social welfare attainable in this economy, that is the sequence of allocations that maximizes the Social Welfare function in (1.35). In particular, equations (1.38) and (1.39) identify the time-$t$ allocation that is part of this plan. The first-best allocations may coincide with the ones attained in a decentralized model with complete asset markets. In particular, in order for the Planner’s solution to correspond to a complete-markets decentralized allocation, the Planner’s weights $\Omega$ and $(1 - \Omega)$ should represent countries’ relative wealth at time $t = -1$ determined by the initial conditions $C_{-1}$, $C^*_{-1}$, $\mu_{-1}$, and $\mu^*_{-1}$.

In economies in which asset markets are incomplete (or effectively incomplete) the Planner’s first order conditions (1.38) and (1.39) need not hold with equality. Assume that the incomplete market allocations coincide with the first-best plan from $t = 0$ up to time $(t - 1)$, but at time $t$ an idiosyncratic shock makes the incomplete market allocation deviate from the benchmark one. In this case

$$\Omega \cdot (C^{IM}_t)^{-\rho} \cdot \varphi_t = \mu^{IM}_t$$  \hspace{1cm} (1.40)$$

$$\left(1 - \Omega\right) \cdot (C^{*IM}_t)^{-\rho} \cdot \varphi^*_t = \mu^{*IM}_t$$  \hspace{1cm} (1.41)$$

---

43See Ljungqvist and Sargent (2004).
where $\varphi$ and $\varphi^*$ represent the wedge between marginal utility of consumption and CPI created by the shock, and the IM superscript denotes the incomplete markets allocation. Loglinearizing (1.40) and (1.41) around a symmetric Steady State with full risk-sharing gives

$$\hat{\varphi}^*_t = \hat{\mu}^*_{IM} + \rho \hat{\sigma}^*_{IM}$$

$$\hat{\varphi}_t = \hat{\mu}^*_{IM} + \rho \hat{\sigma}^*_{IM}$$

Notice that if the shock is purely idiosyncratic, $\hat{\varphi}^*_t$ and $\hat{\varphi}_t$ must have opposite signs (if $\hat{\varphi}_t$ is positive, $\hat{\varphi}^*_t$ must be negative and vice versa). The incomplete markets gap coincides with the difference between $\hat{\varphi}^*_t$ and $\hat{\varphi}_t$

$$gap_{IM}^t = \hat{q}^*_{IM} - \rho (\hat{\sigma}^*_{IM} - \hat{\sigma}^*_{IM^*}) = \hat{\varphi}^*_t - \hat{\varphi}_t$$

The opposite sign of $\hat{\varphi}^*_t$ and $\hat{\varphi}_t$ is sufficient to ensure that a higher gap (in absolute value) must be generated by an incomplete markets allocation implying higher wedges between marginal utilities of consumption and CPIs, therefore a higher deviation from the Social Planner’s (logged) optimality conditions (1.38) and (1.39). Due to the concavity of the Social Welfare function in (1.35) as a sum of concave functions, a higher deviation from the Social Planner’s foci maps into a lower social welfare. Therefore, the higher the gap that arises in response to shocks in any incomplete markets setup, the higher the loss in social welfare caused by the disturbance with respect to the full risk-sharing benchmark.

1.11 Appendix C: Proofs to the Propositions

1.11.1 Proposition 1.1

**Proof.** Introduce an exogenous transfer of resources in H and F budget constraints

$$C_t^H P_t = P_{ht} Y_{ht} - P_t T_t$$

$$C_t^{*H} P_t^{*H} = P_{ft} Y_{ft} + \frac{P_t}{P^{*H}} T_t$$

(1.42)

The linearized model is solved around the symmetric Steady State in which $T = 0$.\(^{45}\)

The response of the terms of trade, the relative price of the F good, to $\vartheta$, to a marginal flow

\(^{44}\)If this was not the case, one of the resource constraints would be violated.

\(^{45}\)In equation (1.42) I have assumed that the resources transferred are marginal units of the H consumption bundle. However, this assumption is unconsequential for the results. Since at the approximation point $T = 0$, it does not matter which good is transferred across countries.
\( \hat{T}_t \) is given by

\[
tot_t = \frac{2\delta}{(1 - \delta)(2\delta\theta - 2\delta + 1)} \cdot \hat{T}_t
\]

This implies

\[
\frac{\partial tot}{\partial T} > 0 \iff \theta > \tilde{\tau}
\]

1.11.2 Proposition 1.2

**Proof.** Introduce an exogenous transfer of resources in H and F budget constraints. From the Proof to Proposition 1.1

\[
tot_t = \frac{2\delta}{(1 - \delta)(2\delta\theta - 2\delta + 1)} \cdot \hat{T}_t
\]

From the solution to the system linearized around the symmetric Steady State with \( T = 0 \)

\[
\hat{c}_t = (\delta - 1) \cdot tot_t - \hat{T}_t = \left(-2\delta \frac{\delta}{2\delta\theta - 2\delta + 1} - 1\right) \cdot \hat{T}_t
\]

\[
\hat{c}_t^* = (1 - \delta) \cdot tot_t + \hat{T}_t = \left(2\delta \frac{\delta}{2\delta\theta - 2\delta + 1} + 1\right) \cdot \hat{T}_t
\]

\[
\hat{q}_t = (2\delta - 1) \cdot tot_t = -\left(2\delta \frac{2\delta - 1}{(\delta - 1)(2\delta\theta - 2\delta + 1)}\right) \cdot \hat{T}_t
\]

Combining the solution for H and F consumption and the real exchange rate yields

\[
gap_t = \hat{q}_t - \rho (\hat{c}_t - \hat{c}_t^*) = \frac{2\rho \delta (\delta - 1) + \rho (2\delta\theta - 2\delta + 1)(\delta - 1) - \delta(2\delta - 1)}{(2\delta\theta - 2\delta + 1)(\delta - 1)} \cdot \hat{T}_t \tag{1.43}
\]

For \( \theta > \tilde{\tau}, (2\delta\theta - 2\delta + 1) > 0 \). Both numerator and denominator in equation (1.43) are negative for \( 0.5 < \delta < 1 \). A marginal transfer to F consumers raises the gap, their relative wealth.

For \( \theta < \tilde{\tau}, \) the denominator in (1.43) is positive. The numerator is negative for \( \rho > 0, \delta > 0, \theta > 0 \). A flow to the F country lowers the relative wealth of its inhabitants. \( \blacksquare \)
1.11. APPENDIX C: PROOFS TO THE PROPOSITIONS

1.11.3 Proposition 1.3

Proof. Assume the system is in Steady State up to time \((t-1)\) and assume an increase in the supply of the H good at time \(t\). The bonds purchased by H consumers in response to the disturbance (coinciding with the borrowing of F agents) are given by

\[
\hat{b}_t = -\hat{b}_t^* = \left[ \frac{(\delta - 1) (\zeta - 1)}{(4\delta - 4\delta^2 + 4\delta^2 \rho \theta - 4\delta \rho \theta - 1) (\zeta \beta - 1)} \right] \cdot \left\{ \frac{2\delta \rho \theta - 2\delta - \rho + 1}{\rho \theta - 1} \right\} \cdot \varepsilon_{Ht} \quad (1.44)
\]

It is easy to verify that the term in square brackets is positive for all \(\delta < 1, \theta > 0, \zeta < 1\). What determines whether F consumers borrow or lend is the position of the trade elasticity relative to the efficiency threshold \(\tilde{\eta}\) – which pins down the sign of the term in braces.

For \(\theta > \tilde{\eta}\), \(\text{gap}^F < 0\) (from equation (1.16) – the relative wealth of Foreign consumers would fall in Financial autarky) and \(\hat{b}_t^* < 0\) (from (1.44)). For \(\tilde{\tau} < \theta < \tilde{\eta}\), \(\text{gap}^F > 0\) and \(\hat{b}_t^* > 0\). Thus, if prices and flows are complements (\(\theta > \tilde{\tau}\)) the F country imports resources through external borrowing if and only if the shock would have made it relatively poorer absent asset trade.

For \(\theta < \tilde{\tau}\), \(\text{gap}^F < 0\) and \(\hat{b}_t^* > 0\). If prices and flows are substitutes, F consumers (the relatively poorer ones absent asset trade) export resources through financial markets.

1.11.4 Proposition 1.4

Proof. In the full solution to the system (1.26), the portfolio flow received by Foreign consumers (\(\xi^*\)) after an exogenous increase in the H endowment is given by

\[
\xi_t^* = -\xi_t = \left[ \frac{(\delta - 1)}{(\zeta \beta - 1) (4\delta (\delta - 1)(\theta \rho - 1) - 1)} \right] \cdot \left\{ \frac{2\delta \rho \theta - \rho - 2\delta + 1}{\rho \theta - 1} \right\} \cdot \varepsilon_{ht} \quad (1.45)
\]

It is easy to verify that the term in brackets is positive \(\forall \delta < 1, \zeta < 1, \beta < 1\). What determines if Foreign consumers receive an inflow from optimal portfolio returns after a windfall in the Home country, is the position of the trade elasticity relative to the efficiency threshold \(\tilde{\eta}\) – which pins down the sign of the term in braces.

For \(\theta > \tilde{\eta}\), \(\text{gap}^F < 0\) (from equation (1.16) the relative wealth of Foreign consumers would fall in Financial autarky) and \(\xi_t^* > 0\) (from equation (1.45)). For \(\tilde{\tau} < \theta < \tilde{\eta}\), \(\text{gap}^F > 0\) and \(\xi_t^* < 0\). Thus, if prices and flows are complements (\(\theta > \tilde{\tau}\)) the F country receives resources through portfolio returns if and only if the shock would have made it relatively poorer absent asset trade.
1.11. APPENDIX C: PROOFS TO THE PROPOSITIONS

For $\theta < \hat{\tau}$, $gap^{FA} < 0$ and $\xi^*_t < 0$. If prices and flows are substitutes, F consumers (the relatively poorer ones absent asset trade) export resources through financial markets. ■

1.11.5 Proposition 1.5

Proof. From the solution to the first step of the DS method we can retrieve the impact on the smoothing gap ($gap^{SM}$, the first element of the right hand side of equation (1.27)) of a marginal flow of resources to Foreign consumers via portfolio returns ($\xi_t^*$, equivalent to $-\xi_t$ in equation (1.25)).

$$\frac{\partial gap^{SM}}{\partial \xi^*} = \left[\frac{4\delta(\delta - 1)(1 - \theta \rho) + 1}{(\delta - 1)}\right] \left\{ \frac{1}{2\delta \theta - 2\delta + 1} \right\} > 0 \iff (2\delta \theta - 2\delta + 1) > 0 \iff \theta < \hat{\tau}$$

Receiving a marginal transfer of resources through portfolio returns raises the relative wealth of Foreign consumers (the gap) if and only if the trade elasticity is above the transmission threshold $\hat{\tau}$, that is if and only if prices and flows are complements.

Optimal portfolio returns should counterbalance $gap^{SM}$, in order to reduce the equilibrium gap to zero (from (1.27)).

For $\theta > \bar{\eta}$, $gap^{SM} < 0$ (from equation (1.22)) and $\frac{\partial gap^{SM}}{\partial \xi^*} > 0$. For $\hat{\tau} < \theta < \bar{\eta}$, $gap^{SM} > 0$ and $\frac{\partial gap^{SM}}{\partial \xi^*} > 0$. Thus, if prices and flows are complements ($\theta > \hat{\tau}$) the F country should receive resources through portfolio returns if and only if the shock would have made it relatively poorer absent portfolios (in the smoothing allocation).

For $\theta < \hat{\tau}$, $gap^{SM} < 0$ and $\frac{\partial gap^{SM}}{\partial \xi^*} < 0$. If prices and flows are substitutes, portfolio returns should subtract resources to F consumers (the relatively poorer ones in the smoothing allocation). ■

1.11.6 Proposition 1.6

Proof. Consider the second step of the DS method. In equilibrium, optimal cross-border flows arise endogenously from variations in assets excess returns and Steady State gross portfolio positions. Namely

$$\xi_t = -\xi_t^* = \hat{\tau}x_t \alpha$$

where $\alpha$ are the gross Steady State holdings of H consumers of the H equity ($\alpha \equiv Z \cdot (1 - S_h)$, where $Z$ is the H security’s price in Steady State). Notice that, by definition, $\alpha$ is the share

---

46 In the first step of the DS method $\xi^*$ is an i.i.d. shock. It is sufficient to combine the appropriate vectors of the solution to get the impact of a marginal transfer to F consumers on the gap.
of the H commodity that H consumers buy in excess of what they receive as endowment. Linearized excess returns of the H equity with respect to the F stock read

\[ \bar{r}_x t = (\bar{r}_1 - \bar{r}_1^*) = (1 - \beta) (\bar{y}_{ht} - \bar{y}_{ft}) - (1 - \beta) \bar{f}_{ot} t + \beta (\bar{z}_t - \bar{z}_1^*) - (\bar{z}_{t-1} - \bar{z}_1^{*}) \]

A depreciation of the terms of trade in response to an increase the H endowment, tends to lower the relative returns from the H equity. If the trade elasticity is low, relative price volatility is high and offsets the positive impact on excess returns of an increase in the relative quantity of the H good. In this case excess returns from the H equity tend to fall after a rise in the H endowment. From the analytical solution for \( \bar{r}_x t \), it is possible to show that an increase in the H endowment raises the excess returns on the H equity only for

\[ \theta > \tilde{\phi} = \frac{4\delta(\delta - 1) - \rho + 1}{4\delta \rho (\delta - 1)} > \tilde{\eta} \]

The optimal gross position (\( \alpha \)) is the one that, combined with the endogenous response to shocks of asset excess returns (\( r_x t \)), generates the optimal cross-border flows described in Proposition 4. The following graph represents along the domain of the trade elasticity, the response to an increase in the H endowment of the excess returns on the H equity (\( \bar{r}_x \)); the direction of optimal resource transfers (from Proposition 1.4); and H agents’ gross holdings of the H equity (\( \alpha \), computed through the DS method).

**FIGURE 10**
Response of H excess returns, optimal direction of portfolio flows, and optimal gross holdings (\( \alpha \)), as a function of \( \theta \)
1.11. APPENDIX C: PROOFS TO THE PROPOSITIONS

For \( \theta < \tilde{\eta} \), following a positive shock to the H endowment, portfolio returns should channel resources to H consumers. Due to the low degree of substitutability between varieties, the strong depreciation of the terms of trade lowers the relative return on the H stock. By selling claims to the H endowment (\( \alpha < 0 \)) H consumers receive an inflow of resources if their own endowment increases. For \( \tilde{\eta} < \theta < \tilde{\phi} \), portfolio returns should transfer resources to Foreign agents. Thus, H households should go long on their own equity, whose relative return falls: \( \alpha > 0 \). For \( \theta > \tilde{\phi} \) F consumers should still receive a transfer of wealth. The high trade elasticity requires only a small depreciation of the terms of trade and the relative returns on the H equity rise. H consumers should then hold claims to the Foreign endowment: \( \alpha < 0 \).^{47}

The share of the Home equity optimally held by H consumers (\( S_h \)) can be easily computed from their gross holdings \( \alpha \).^{48}

\[
S_h = 1 + (1 - \delta) \left[ \frac{1}{4\delta(\delta - 1)(\rho \theta - 1) + \rho - 1} \cdot \frac{2\rho \theta - \rho - 2\delta + 1}{\theta > \tilde{\phi} \iff \theta > \tilde{\phi}} \right. 
\left. < 0 \iff \theta > \tilde{\phi} \right] \tag{1.46}
\]

For \( \theta < \tilde{\eta} \), H households sell part of the claims to their own endowment and their portfolio exhibits home bias (0.5 < \( S_h < 1 \)).^{49} For \( \tilde{\eta} < \theta < \tilde{\phi} \), H agents go long on the Home equity (\( S_h > 1 \)), a counterfactual "excessive" home bias. For \( \theta > \tilde{\phi} \), H consumers hold less than a half of the claims to their own endowment (\( S_h < 0.5 \)), which gives rise to foreign bias.\(^{50}\)

Thus \( \tilde{\phi} \) represents a foreign bias threshold.\(^{51}\)

---

47 Notice that \( \tilde{\phi} \) represents an asymptot for \( \theta \) gross holdings. When the trade elasticity matches \( \tilde{\phi} \) Home and Foreign equities yield the same returns in response to any shock. The closer the elasticity to this threshold, the larger the amount of assets agents need to hold.

48 Namely, \( S_h = 1 + \alpha (1 - \beta) \).

49 The low degree of substitutability between goods implies a high volatility of excess returns. The share of Foreign equities held by H households does not need to exceed (1/2).

50 Intuitively, \( S_h \) is smaller than (1/2) because the high trade elasticity implies a small depreciation of the terms of trade and a low volatility of excess returns. H consumers must hold a large share of the Foreign stock in order to hedge optimally.

51 This model is very similar to the one analysed in Kollmann (2006). As Kollmann (2006) and Coeurdacier (2005) notice, in order for this framework – and in general all models reconducing equity home bias only to biased consumption expenditures – to generate a realistic degree of equity home bias, the elasticity of substitution between H and F goods should be sufficiently low. Kollmann finds that when the elasticity is high enough, still below unity in his calibration, the model delivers a counterfactual "excessive" home bias in equities (\( S_h > 1 \)). For an elasticity sufficiently above unity, he finds that this kind of models generates an equally counterfactual foreign bias (\( S_h < 0.5 \)).

The interpretation of optimal portfolio formation detailed in the text, provides a rationale for foreign bias and excessive home bias. It shows that only for an elasticity lower than the efficiency threshold, the model can generate a realistic degree of equity home bias. Thus when prices and flows are substitutes, equity home bias is consistent with efficient financial markets. Instead complementarity between flows and prices may well imply "excessive" home bias or foreign bias.
1.11.7 Propositions 1.7 and 1.8

**Proposition 1.7** Under trade in uncontingent bonds, a marginal flow of resources from the H to the F country raises the price of the F output (depreciates the terms of trade) if the trade elasticity is above the transmission threshold \( \tilde{\tau} \); lowers the price of the F output otherwise.

**Proof.** Introduce an exogenous transfer of resources in H and F budget constraints

\[
P^H_t B_{t+1} = \frac{P_{ht} Y_{ht}}{P_t} - C_t + B_t - T_t
\]

\[
\frac{P^H_t B^*_t}{Q_t} = \frac{P_{ft} Y_{ft}}{P^*_t} - C^*_t + \frac{B^*_t}{Q_t} + \frac{T_t}{Q_t}
\]

The linearized model is solved around the symmetric Steady State in which \( B = B^* = T = 0 \). The response of the terms of trade, the relative price of the F good, to a marginal flow \( \hat{T}_t \) is given by

\[
\text{tot}_t = (1 - 2\delta) (\beta - 1) \cdot \frac{2\delta - \beta \delta + 1}{2\delta - 2\delta \theta - 1}
\]

This implies

\[
\frac{\partial \text{tot}_t}{\partial T} > 0 \iff \theta > \tilde{\tau}
\]

Proposition 1.8 A marginal flow of resources from the H to the F country raises the relative wealth of Foreign consumers if the trade elasticity is above the transmission threshold \( \tilde{\tau} \); lowers the relative wealth of F consumers otherwise.

**Proof.** Combining the solution for the response of \( C_t, C^*_t \), and \( Q_t \) to a marginal transfer to the Foreign country, yields

\[
\frac{\partial \text{gap}}{\partial \xi^*} = \left[ \frac{(4\delta(\delta - 1)(1 - \theta \rho) + 1)(\beta - 1)}{(\delta - 1)} \right] \cdot \left\{ \frac{1}{2\delta \theta - 2\delta + 1} \right\} > 0 \iff
\]

\[
(2\delta \theta - 2\delta + 1) > 0 \iff \theta < \tilde{\tau}
\]

Receiving a marginal transfer of resources through external borrowing raises the relative wealth of Foreign consumers (the gap) if and only if the trade elasticity is above the transmission threshold \( \tilde{\tau} \).
1.12 Appendix D: Wealth effects with endogenous portfolios

I write consumers’ utility maximization for a general case in which agents can trade in multiple contingent securities and form a portfolio that hedges them ex-ante – to some extent – against national shocks. The formulation of the problem abstracts from the degree of completeness of asset markets. I leave the characterization of portfolios indeterminate. This allows for the greatest degree of generality, and is sufficient to show that the gap maps into changes in countries’ relative wealth. It should be clear that the purpose of this section is not to find optimal portfolios, nor to derive a full solution to the model, but rather to show that also in the case of trade in multiple contingent assets, the insurance measure reflects cross-border wealth imbalances.

Assume agents can trade in \( n \) assets with state-contingent payoffs \( r_i^t, i = 1, ..., n, \) expressed in \( H \) consumption units. For our purposes, payoffs can be generally characterized as a function of the realization of national shocks at time \( t \). Define \( \alpha_t \) as the vector of gross holdings of the first \((n-1)\) assets purchased by Home consumers at time \( t \). Optimal portfolios \( \alpha \) are not specified, and can be thought of as a function of shock realizations.

H households’ budget constraint reads

\[
NFA_t = r^n_t NFA_{t-1} + \frac{Y_{ht} P_{ht}}{P_t} - C_t + \alpha_{t-1} r_{xt} \xi_t
\]

where \( NFA \) are net foreign assets, \( \alpha_{t-1} \) is a function of the realization of shocks \( Y \) and \( Y^* \) at time \((t-1)\), \( r_{xt} = [r^1_{xt}, ..., r^{n-1}_{xt}] \) is the vector of realized excess returns, and \( r^n_t \) is the \( n \)-th asset payoff. \( \xi_t \) is the transfer of resources delivered at time \( t \) by portfolio returns in response to shocks. Since we are not interested in finding the optimal portfolio, we can abstract from its composition, and write the inter-temporal problem of \( H \) consumers as

\[
\max_{\{C_t, NFA_t, \nu_t\}} : L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t)^{1-\rho}}{1-\rho} + \nu_t (P_t r^n_t NFA_{t-1} + P_{ht} Y_t + P_t \xi_t - P_t C_t - P_t NFA_t) \right] \right\}
\]

where \( \nu_t \) is the shadow value of income at time \( t \), including endowment income, returns from previous period net foreign assets, and flows from portfolio holdings. As for the smoothing case analyzed in the main text, it is easy to show that \( \nu_t \) represents the shadow value of the present discounted value of lifetime income. Following the same steps as in the smoothing case yields

\(^{52}\)In a full solution to the model and up to a second-order approximation, portfolio composition would depend both on exogenous shocks and on second and third-order moments. See Devereux and Sutherland (2006).

\(^{53}\)\( \xi_t \) depends on the realization of the shocks at time \( t \) (through asset returns) and \((t-1)\) (through \( \alpha_{t-1} \)).
\( q_t - \rho (\hat{c}_t - \hat{c}_t^*) \equiv gap_t = (\hat{v}_t - \hat{v}_t^*) \)

The wealth imbalances reflected in \((\hat{v}_t - \hat{v}_t^*)\) are residual after portfolio returns have delivered contingent transfers of resources. When asset markets are effectively complete, the wealth distribution stays constant in response to any shock, and the multipliers \( \nu \) and \( \nu^* \) do not vary over time and states.\(^54\)

1.13 Appendix E: Portfolio flows under incomplete markets

This section shows that if asset markets are effectively incomplete, “upstream” flows can also be the manifestation of bad insurance. I assume that consumers can trade internationally in two assets, both issued by the Home country. In response to shocks originated in the Foreign country, financial flows from portfolio holdings channel resources to the country that absent asset trade would have been made relatively richer by the disturbances. These flows however reflect the fact that the assets available for trade are not sufficient to hedge consumers effectively against Foreign shocks. “Upstream” flows open up higher risk-sharing inefficiencies and welfare losses then those that would have been attained in the same model if only trade in uncontingent bonds was available. Notice that this finding generalizes to a general equilibrium setup Svensson’s (1988) claim that trade in more assets is not necessarily better – in welfare terms – than in fewer. Increasing the share of country-specific risk that can be traded in financial markets beyond one security without achieving market completeness, may yield a lower social welfare in response to some shocks.

Households can trade in one equity and in one bond, both issued by country H that has therefore the monopoly on asset issuance.\(^55\) The H equity is a claim to the endowment of the H country. Its total quantity is normalized to 1. The H bond pays out one unit of the endowment of the Home country in every state. \( S, S^*, B, \) and \( B^* \) denote the fraction of the equity and the holdings of bonds held respectively by H and F households. Both assets are assumed to be in zero net supply.

Equities and bonds real returns, expressed in terms of the H consumption good, are given by

\(^{54}\)This is clearly the case with a full set of Arrow-Debreu securities. See Ljungqvist and Sargent (2004).

\(^{55}\)Positing that all the internationally marketable assets can only be issued by one country is a rather extreme assumption, which could be justified by the presence of limited enforcement in international contracts. Kehoe and Perri (2002) show that this informational problem can lead to severe restrictions to international asset trade.
1.13. APPENDIX E: PORTFOLIO FLOWS UNDER INCOMPLETE MARKETS

\[ R_t^E = \frac{(P_{ht}/P_t)Y_{ht} + Z_t}{Z_{t-1}} \quad ; \quad R_t^B = \frac{P_{ht}}{P_t} \frac{1}{P_{t-1}^{B}} \]

where \( Z \) and \( P^B \) denote the real price (in terms of the \( H \) consumption bundle) of the two assets.

The budget constraint of \( H \) households reads

\[ NFA_t = NFA_{t-1} + \frac{Y_{ht}P_{ht}}{P_t} - C_t + R_t^E(S_t - 1) + R_t^B B_t, \]

where \( NFA_t \equiv [(S_{t+1} - 1)Z_t + B_{t+1}P^B_t] \) denotes net foreign assets.

The Euler equations for asset holdings and asset market clearing conditions are

\[ (C_t)^{-\rho} = \beta E_t \left\{ (C_{t+1})^{-\rho} \cdot R_{t+1}^E \right\} = \beta E_t \left\{ (C_{t+1})^{-\rho} \cdot R_{t+1}^B \right\} \]

\[ \frac{(C_t^*)^{-\rho}}{Q_t} = \beta E_t \left\{ \frac{(C_{t+1}^*)^{-\rho}}{Q_{t+1}} \cdot R_{t+1}^E \right\} = \beta E_t \left\{ \frac{(C_{t+1}^*)^{-\rho}}{Q_{t+1}} \cdot R_{t+1}^B \right\} \]

\[ S_t + S^*_t = 1 \quad ; \quad B_t + B^*_t = 0 \]

Applying the Devereux and Sutherland (2006) method to the system linearized around a symmetric Steady State with zero net foreign assets, yields the analytical solution for \( H \) agents’ holdings of the \( H \) equity

\[ S = 1 - \pi (\beta - 1)(\delta - 1) \cdot \{2\delta \rho \theta - \rho - 2\delta + 1\} \quad (1.47) \]

where \( \pi \) is a positive constant.\(^{56}\) Whether \( H \) consumers go long or short on equities depends on the trade elasticity \( \theta \) being above or below the efficiency threshold, which pins down the sign of the term in braces in (1.47). Asset markets are effectively incomplete.\(^{57}\)

Although agents can trade in multiple assets, returns from the best portfolio possible will not completely wipe out the gap that opens up is response to shocks.

In Figure 11 I simulate a 1% increase in the \( H \) endowment at time \( t \). The graph plots the time-\( t \) response of the Financial autarky gap (the relative wealth of \( F \) consumers absent

---

\(^{56}\)Namely,

\[ \pi = \left( \frac{(\beta - 1)}{\rho} - \frac{\beta(\zeta - 1)}{(4\delta(\zeta - 1)(\rho - 1)^{1 - 1})} \right) \cdot \left( \frac{(4\delta(\zeta - 1)(\rho - 1)^{1 - 1})}{\rho} \right)^{-1} \]

\[ \left( \beta - 1 + \beta \rho (\zeta - 1) \right) \cdot \left( \frac{(4\delta(\zeta - 1)(\rho - 1)^{1 - 1})}{\rho} \right)^{-1} \]

\(^{57}\)Throughout this section I will refer to "optimal portfolios" as the ones optimally chosen by the agents. It should be clear that the adjective optimal does not refer here to Pareto optimality.
asset trade) and the portfolio returns received by F households, $\xi_t^*$ (the inflow of financial resources received by F consumers), both as a function of the trade elasticity $\theta$. The optimal portfolio characterized by (1.47) transfers resources upstream, in response to an H-originated shock for $\theta < \bar{\tau}$. Figure 12 shows that the resulting equilibrium gap (IM gap in the picture) is always smaller in absolute value than the one that would have aroused in the same model if agents could only borrow and lend.

The asset structure assumed delivers good insurance, even though not perfect, against shocks that originate in the H country. However, the same is not true for disturbances that arise in the country that cannot issue securities. In Figure 13 I simulate a 1% increase in the Foreign endowment at time $t$. The graph plots the time-$t$ response of the Financial autarky gap (the relative wealth of F consumers absent asset trade) and the portfolio returns received by F households, $\xi_t^*$ (the inflow of financial resources received by F consumers), both as a function of the trade elasticity $\theta$. In response to Foreign shocks, the best portfolio possible generates “upstream” flows also for $\theta > \bar{\tau}$. Figure 14 shows that these flows enlarge the risk-sharing inefficiency with respect to the one that would have aroused in the same model under trade in uncontingent bonds.

![Figure 11](image-url)

**FIGURE 11**
Response of the Financial autarky gap and F portfolio returns as a function of $\theta$ (H shock)
FIGURE 12
Response of the Financial autarky gap and the IM gap as a function of $\theta$ (H shock)

FIGURE 13
Response of the Financial autarky gap and F portfolio returns as a function of $\theta$ (F shock)
FIGURE 14
Response of the Financial autarky gap and the IM gap as a function of $\theta$ (F shock)
BIBLIOGRAPHY


CHAPTER 2

ASSESSING THE PERFORMANCE OF EMPIRICAL METHODS TO MEASURE CROSS-BORDER INSURANCE

2.1 Introduction

In the last years real and financial linkages across countries got strengthened. The fast integration in financial markets led to the abandonment of most restrictions to cross-border trade in assets. The creation of trade and currency unions spurred international exchanges and stimulated financial markets activity.

One of the dimensions under which these developments have been evaluated is their impact on global insurance. Did stronger international linkages allowed households and firms in different countries to insure more effectively against national business cycle shocks? A vast body of literature has developed empirical methods to quantify the extent of cross-border risk-sharing and to track its evolution over time. Virtually all these methodologies have been developed under the implicit assumption of no international price fluctuations.\(^1\) Under this assumption, the presumption is that international financial markets are efficient in fostering insurance if they channel resources to countries that (absent asset trade) would be made relatively poorer by national shocks (net of physical capital accumulation). Chapter 1 questions the validity of this standard paradigm. It shows that the interaction between relative price fluctuations and cross-border financial flows is crucial to derive testable conditions for financial markets efficiency. In particular, if receiving an inflow from abroad lowers the international price of a country’s output, cross-border risk-sharing requires financial markets to channel resources “upstream”, from poorer to richer countries. These financial flows increase international insurance, yet they run against the condition of efficiency on which standard empirical methods are based.

From an empirical perspective, if the assumption of no relative price fluctuations – at the basis of virtually all empirical tests – might be a relatively innocuous approximation when measuring insurance among different regions within a country (a tasks for which some of these methods were originally designed), it could be instead problematic when quantifying

---

\(^1\)Among these, the methods designed by Asdrubali, Sorensen and Yosha (1996), Obstfeld (1993), Brandt, Cochrane and Santa Clara (2006), and Flood, Marion and Matsumoto (2008).
cross-border risk-sharing. Indeed, the literature has estimated deviations from Purchasing Power Parity across countries to be large and persistent. Which problems can be caused to these empirical methods by the presence of relative price fluctuations? In this Chapter I assess their performance using a simulated two-country DSGE model with endogenous portfolio diversification. Results show that standard methods may be completely misleading if relative price movements (realistically) transfer purchasing power across countries. The reason is that the presence of price fluctuations invalidates the benchmarks of efficiency on which these methodologies are based.

In order to study the issues to which existing empirical methods may be subject, I write a simple two-country two-good DSGE model in which both relative price fluctuations and financial flows can transfer purchasing power across countries, and simulate it under three different asset structures – Financial autarky, trade in uncontingent bonds, and trade in two equities (implying effectively complete asset markets in this model). The risk-sharing properties of these asset trade regimes are known and have been studied in Chapter 1. I apply different empirical methods (respectively, from Asdrubali, Sorensen and Yoshia (1996), Obstfeld (1993), Brandt, Cochrane and Santa Clara (2006), and Flood, Marion and Matsumoto (2008)) to the simulated model in order to check if they identify correctly full risk-sharing in the two-equities case, and if they capture the correct ranking of insurance among asset structures.

The results unveil three typical issues that may affect standard methodologies. First, the relative value of GDPs commonly used as the independent variable in risk-sharing regressions does not necessarily identify which country is made relatively richer/poorer by national shocks, which makes existing methods potentially misleading. Second, the interaction between relative prices and financial flows may require efficient financial markets to channel resources “upstream”, to relatively richer countries – a finding that is typically interpreted as low insurance by existing methods. Finally, the full risk-sharing consumption allocation can be characterized only in conjuncture with relative price movements: with full insurance national consumption can fall after a negative income shock, provided consumers are compensated by an appreciation of their currency – a condition that is not considered by standard benchmarks of risk-sharing. These problems can have severe consequences. Not only they lead to reject full insurance also when the “true” model features effectively complete asset markets, but they can also lead to estimate a higher degree of insurance under, say, Financial autarky than in the case of complete markets. In a world in which relative price fluctuations are realistically large and sustained, the benchmarks of efficiency derived in one-good frameworks and on which standard methods are based, are not valid anymore.
2.2. **THE MODEL**

Most existing empirical methods may then be easily misleading.

This Chapter is closely related to Chapter 1 in this thesis, which questions the validity of the theoretical efficiency condition for financial markets at the basis of most empirical methods. The present paper is the natural follow up of that analysis, as it investigates the consequences of those findings for empirical methods.

This Chapter is also related to the vast literature on empirical risk-sharing measurement. Asdrubali, Sorensen and Yosha (1996), Obstfeld (1993), Brandt, Cochrane and Santa Clara (2006), and Flood, Marion and Matsumoto (2008) are only a non-exhaustive list.

To my knowledge this paper is the first one to assess the performance of standard empirical methods using a simulated model, and to investigate the problems that can be caused by the presence of relative price fluctuations.

The rest of the Chapter is organized as follows. The next section describes the DSGE model used for simulations. Section 3 applies the method of Asdrubali, Sorensen and Yosha (1996) to the simulated model, and uses the results to illustrate the problems that can be caused to standard empirical frameworks by the presence of relative price fluctuations and by their interaction with cross-border financial flows. Section 4 presents the results relative to other methodologies. Section 5 draws some conclusions.

### 2.2 The model

The model used for simulations is the two-country two-good DSGE framework with endowment shocks and home bias in consumption analyzed in Chapter 1. Its core structure is the simplest one that generates a general equilibrium interaction between the two channels of international insurance, relative price fluctuations and cross-border financial flows. The model is simulated under three different asset structures – Financial autarky, trade in uncontingent bonds, and trade in two equities. The risk-sharing properties of these asset trade regimes have been studied in Chapter 1. Financial autarky gives less or equal insurance against national supply shocks than the other asset structures. Trade in uncontingent bonds ranks second, while trade in two equities gives perfect risk-sharing against national disturbances. In order to focus squarely on the problems caused to empirical methods by the presence of relative price fluctuations, we will abstract from the following issues, well-known to the insurance literature: taste shocks, measurement errors, lack of capital gains/losses in National Accounts data, econometric issues due to panel estimation.

For convenience, I report here the main features of the model. It consists of two countries,
2.2. THE MODEL

Home and Foreign, each inhabited by a representative household. Countries are specialized in the production of different goods. Households in country H receive utility from consuming a bundle made up of the foreign and the domestic good, according to a CES aggregator

\[ C_t = \left[ (\delta)^{1/\theta} \left( \frac{C_{ht}}{\sigma} \right)^{\frac{\theta-1}{\theta}} + (1-\delta)^{1/\theta} \left( \frac{C_{ft}}{\sigma} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\sigma}{\theta}}; \theta > 0, \delta > 1/2 \]

where \( C_{jt} \) denotes consumption of the good produced in country \( j \) and \( \delta \) is a parameter capturing home bias in consumption. \( \theta \) is the elasticity of substitution between H and F-produced goods; in this model it coincides with the trade elasticity. The F consumption bundle is analogously defined. Starred variables denote the corresponding quantities consumed by Foreign agents. Goods are freely tradable but not storable. The period utility of both agents depends on current consumption only and is a Constant Relative Risk Aversion function with risk aversion coefficient \( \rho \).

In each period H and F households receive a stochastic endowment according to the process

\[ \log (Y_{jt}) = \zeta \log (Y_{jt-1}) + \varepsilon_{jt} \]

where \( Y_{jt} \) denotes the endowment received by consumer \( j \) and \( \varepsilon_{jt} \sim iid (0, \sigma^e) \).

I assume that the law of one price holds and that the nominal exchange rate is constant and equal to one for simplicity. Due to home bias in consumption Purchasing Power Parity does not hold outside a symmetric Steady State, and the two price indexes \( P_t \) and \( P_t^* \) are tied by the following condition defining the real exchange rate \( Q_t \)

\[ Q_t \equiv \frac{P_t^*}{P_t} \]

Terms of trade are defined as the ratio of the price of H imports and exports

\[ tot_t \equiv \frac{P_{ft}}{P_{ht}} \]

where \( P_{ht} \) and \( P_{ft} \) denote respectively the price of H and F-produced goods.

Good market clearing conditions read

\[ Y_{ht} = C_{ht} + C_{ht}^* \]

\[ Y_{ft} = C_{ft} + C_{ft}^* \]
2.2. THE MODEL

The model is loglinearized around a symmetric Steady State in which countries’ wealth is assumed to be equalized, endogenous variables are constant and exogenous ones are equal to their mean values.

I simulate the model under three different asset trade regimes. Households’ budget constraints are pinned down by the specific asset structure assumed.

2.2.1 Asset structure 1: Financial autarky

Under Financial autarky relative price fluctuations represent the only channel of cross-border insurance. Households have no means of smoothing consumption over time and in each period their consumption must equal the value of their income. H and F budget constraints read

\[ C_t P_t = P_{ht} Y_{ht} \]

\[ C^*_t P^*_t = P_{ft} Y_{ft} \]

Chapter 1 shows that in this model Financial autarky gives less or equal insurance against national shocks than the other two asset trade regimes.

2.2.2 Asset structure 2: Trade in uncontingent bonds

If agents can trade in uncontingent bonds paying 1 unit of the H consumption bundle in every state-of-the-world, the budget constraint of H consumers reads

\[ P^B_t B_{t+1} = \frac{P_{ht} Y_{ht}}{P_t} - C_t + B_t \]

where \( B_{t+1} \) denotes bonds purchased in period \( t \), \( P^B_t \) their unitary price in terms of the H consumption bundle. Bonds are assumed to be in zero net supply. H households’ inter-temporal problem is given by

\[
\max_{\{C_t, B_{t+1}, \nu_t\}} : L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t)^{1-\rho}}{1-\rho} + \nu_t \left( P_{ht} Y_{ht} + P_t B_t - C_t P_t - P^B_t P_{t+1} B_{t+1} \right) \right] \right\}
\]

H agent’s first order conditions read

\[ (C_t)^{-\rho} = \nu_t P_t \]
\[ P_t^B = \beta E_t \left\{ \frac{\nu_{t+1} P_{t+1}}{\nu_t P_t} \right\} \]

In the symmetric Steady State \( B = B^* = 0 \).

Chapter 1 shows that trade in uncontingent bonds gives the second-best degree of insurance among the three asset structures considered.

### 2.2.3 Asset structure 3: Trade in two equities (effectively complete markets)

Assume households can trade in two equities, \( H \) and \( F \), representing respectively a claim to the \( H \) and \( F \) country’s endowment. The total quantity of each equity is normalized to 1. \( S_h, S_f, S'_h, \) and \( S'_f \) denote the fraction of \( H \) and \( F \) equities owned respectively by \( H \) and \( F \) consumers. Due to the normalization, the owner of \( S_h \) equities receives a share \( S_h \) of the \( H \) endowment. Equities real returns, expressed in terms of the \( H \) consumption good, are given by

\[ R_t = \frac{(P_{ht}/P_t)Y_{ht} + Z_t}{Z_{t-1}} \]

\[ R^*_t = \frac{(P_{ft}/P_t)Y_{ft} + Z^*_t}{Z_{t-1}} \]

where \( Z \) and \( Z^* \) denote the real price (in terms of the \( H \) consumption bundle) of the two assets.

The budget constraint of \( H \) households reads

\[ NFA_t = NFA_{t-1} + \frac{Y_{ht} P_{ht}}{P_t} - C_t + R_t \cdot (S_{ht} - 1) + R^*_t S_{ft} \]

where \( NFA_t \equiv [(S_{h,t+1} - 1) \cdot Z_t + S_{f,t+1} \cdot Z^*_t] \) denotes \( H \) net foreign assets.

Euler equations and asset market clearing conditions are given by

\[ (C_t)^{-\rho} = \beta \mathcal{E}_t \{ (C_{t+1})^{-\rho} \cdot R_{t+1} \} = \beta \mathcal{E}_t \{ (C_{t+1})^{-\rho} \cdot R^*_t \} \]

\[ \frac{(C_t^*)^{-\rho}}{Q_t} = \beta \mathcal{E}_t \left\{ \frac{(C_{t+1}^*)^{-\rho}}{Q_{t+1}} \cdot R_{t+1} \right\} = \beta \mathcal{E}_t \left\{ \frac{(C_{t+1}^*)^{-\rho}}{Q_{t+1}} \cdot R^*_t \right\} \]

In the Steady State \( NFA = NFA^* = 0 \).

As shown in Kollmann (2006) and Chapter 1, trade in two equities makes asset markets effectively complete and provides perfect risk-sharing.
2.3. THE PERFORMANCE OF EMPIRICAL METHODS: ASDRUBALI, SORENSEN AND YOSHA

2.2.4 Parametrization

I simulate the behaviour of the model for 1000 periods assuming the following parameter values

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>H consumption home bias</td>
<td>$\delta$ 0.9</td>
</tr>
<tr>
<td>F consumption home bias</td>
<td>$\delta^*$ 0.9</td>
</tr>
<tr>
<td>H discount factor</td>
<td>$\beta$ 1/1.01</td>
</tr>
<tr>
<td>F discount factor</td>
<td>$\beta^*$ 1/1.01</td>
</tr>
<tr>
<td>Risk-aversion coefficient</td>
<td>$\rho$ 2</td>
</tr>
<tr>
<td>H shocks persistence</td>
<td>$\zeta$ 0.95</td>
</tr>
<tr>
<td>F shocks persistence</td>
<td>$\zeta^*$ 0.95</td>
</tr>
</tbody>
</table>

I also assume $\varepsilon_t \sim N(0, 1)$, $\varepsilon_t^* \sim N(0, 1)$, and $\text{Cov} (\varepsilon_t, \varepsilon_t) = 0$.

2.3 The performance of empirical methods: Asdrubali, Sorensen and Yosha (1996)

2.3.1 The methodology

The method of Asdrubali, Sorensen and Yosha (1996) (ASY henceforth) was originally developed to estimate risk-sharing among US states. Yet most subsequent applications used it to estimate cross-border insurance. The methodology assumes no relative prices and relies on the following identity\(^2\)

\(^2\)Equation (2.1) implies that consumption and output have the same composition, hence the same price.
2.3. THE PERFORMANCE OF EMPIRICAL METHODS: ASDRUBALI, SORENSEN AND YOSHA (1996)

\[ C = GDP + NFI - NSAV \]  

(2.1)

where \( NFI \) and \( NSAV \) denote respectively net financial income and net savings. From (2.1), full insurance – \( C \) and \( Y \) being uncorrelated, according to ASY – requires a country experiencing a negative output shock (getting poorer absent asset trade) to receive income from abroad (either from portfolio returns or from external borrowing – net of physical capital accumulation). The method consists in estimating

\[ \Delta \log C_t - \Delta \log C_t^* = \alpha + \beta \cdot (\log Y_t - \log Y_t^*) + \varepsilon_t \]  

(2.2)

where \( \Delta \) denotes the first difference of a variable and \( \beta \) is interpreted as a measure of the uninsured effects on consumption of GDP fluctuations.

Moreover ASY show that the decomposition in (2.1) allows to quantify the fraction of output shocks that is absorbed through different channels of insurance by panel estimating

\[ -\Delta \log NFI_t \equiv (\Delta \log GDP_t - \Delta \log GNP_t) = \alpha_{ft} + \beta_f \cdot \Delta \log GDP_t + \varepsilon_{ft} \]  

(2.3)

\[ \Delta \log NSAV_t \equiv (\Delta \log GNP_t - \Delta \log C_t) = \alpha_{st} + \beta_s \cdot \Delta \log GDP_t + \varepsilon_{st} \]

\[ \Delta \log C_t = \alpha_{ut} + \beta_u \cdot \Delta \log GDP_t + \varepsilon_{ut} \]

where \( \beta_f \) is interpreted as the fraction of output shock that is absorbed through factor income flows, \( \beta_s \) is the share absorbed through consumption smoothing, and \( \beta_u \) the fraction left uninsured. Time fixed effects in (2.3) are meant to capture the effects of aggregate shocks. In an OLS framework, we can replace time fixed effects by adding Foreign GDP as a regressor, and estimate

\[ (\Delta \log GDP_t - \Delta \log GNP_t) = \alpha_f + \beta_f \cdot \Delta \log GDP_t + \gamma_f \cdot \log GDP_t^* + \varepsilon_{ft} \]  

(2.4)

\[ (\Delta \log GNP_t - \Delta \log C_t) = \alpha_s + \beta_s \cdot \Delta \log GDP_t + \gamma_s \cdot \log GDP_t^* + \varepsilon_{st} \]

\[ \Delta \log C_t = \alpha_u + \beta_u \cdot \Delta \log GDP_t + \gamma_u \cdot \log GDP_t^* + \varepsilon_{ut} \]
2.3. **THE PERFORMANCE OF EMPIRICAL METHODS: ASDRUBALI, SORENSEN AND YOSHA**

Notice that under the maintained assumption of no relative prices, equations (2.4) imply the same view of optimal financial flows underlying (2.1): efficient financial markets should channel resources to the countries that (absent asset trade) would have been made relatively poorer by national shocks. Slopes in the interval \([0, 1]\) are interpreted as a relatively high degree of insurance. Instead, \(\beta_u > 1\), \(\beta_f < 0\), and \(\beta_s < 0\) indicate that financial markets are highly inefficient. Kalemli-Ozcan et al. (2004) estimate \(\beta_f < 0\) among European countries during the Nineties, and interpret this finding as signaling bad insurance from cross-border ownership of securities.

**A first adjustment to account for price fluctuations** Since this method had been originally developed to estimate risk-sharing among different regions within a country and abstracted completely from relative price movements, most studies estimating (2.2) and (2.4) in an international setup implicitly modified identity (2.1) to account for international price fluctuations. If relative prices are well-defined, (2.1) becomes

\[
C = \frac{P_y \cdot GDP}{P} + NFI - NSAV
\]  

(2.5)

where \(P\) and \(P_y\) are respectively the prices of consumption and output, and I have assumed that net financial income and net savings are expressed in terms of real consumption. Thus most studies deflate national output with the local CPI, so as to express all variables in terms of the country’s consumption.\(^3\) In all the estimations below I will assume this correction, and deflate output using national CPIs. The Appendix discusses the consequences of not implementing this change and shows the corresponding results.

Notice that by subtracting the national identities (2.5) relative to countries H and F we can write their consumption differential as\(^4\)

\[
(\Delta \log C_t - \Delta \log C^*) = \left(\Delta \log GDP_t - \Delta \log GDP^*_t\right) + \omega \Delta \log TOT_t
\]

\[+ \phi \Delta \log NSAV_t + \kappa \Delta \log NFI_t\]

where \(TOT_t\) are country H terms of trade, \(\omega\), \(\phi\), and \(\kappa\) are positive constants, and \(\nu_t\) is the real relative value of the H and the F endowments. Thus, running (2.2) with properly deflated output is equivalent to estimating

---


\(^4\)All aggregates are expressed in terms of Home consumption.
2.3. **THE PERFORMANCE OF EMPIRICAL METHODS: ASDRUBALI, SORENSEN AND YOSHA**

\[(\Delta \log C_t - \Delta \log C^*) = \alpha + \beta \cdot \Delta \log v_t + \varepsilon_t\]

The impact on consumption differentials of relative price fluctuations is included in the independent variable, \(v_t\). Therefore, according to the ASY method, we can interpret \(\beta\) as the fraction of shocks that is not insured through financial markets.

### 2.3.2 Estimation results: The problems caused by relative price fluctuations

<table>
<thead>
<tr>
<th>(\beta) (fraction uninsured)</th>
<th>(\theta = 0.3)</th>
<th>(\theta = 0.6)</th>
<th>(\theta = 1.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bond</td>
<td>1,74</td>
<td>1,190</td>
<td>0,73</td>
</tr>
<tr>
<td>Complete markets</td>
<td>1,75</td>
<td>1,191</td>
<td>0,72</td>
</tr>
</tbody>
</table>

Table 2.2 reports the estimation results for specification (2.2). Standard errors are negligible and are not reported. The estimation is carried out for three different values of the trade elasticity \(\theta\). In Chapter 1 I show that in the present model these values correspond to different kinds of interactions between price fluctuations and financial flows. For \(\theta = 0.3\) (\(\theta < \hat{\tau}\) in Chapter 1), receiving a financial inflow from abroad lowers the relative price of a country’s output (prices and flows are substitutes). For \(\theta > 0.3\) the same transfer raises the international price of a country’s output (the two insurance channels are complements).

The estimations identify correctly Financial autarky as the case in which financial markets do not absorb any fraction of national disturbances (\(\beta = 1\)). Notice however that in this model the absence of asset trade has a very different impact on consumers’ welfare for different values of the trade elasticity. Chapter 1 shows that under Financial autarky cross-border insurance from relative price fluctuations is very high for \(\theta = 1.2\), while terms of trade...
movements enlarge the risk-sharing inefficiency that would have aroused at constant prices for \( \theta = 0.3 \). The \( \beta \) coefficients estimated by the ASY method for the Financial autarky specification do not give any indication of the welfare losses due to the absence of financial markets.

For \( \theta = 1.2 \) the estimated coefficients identify the correct ranking of insurance across asset trade regimes (\( \beta^{CM} < \beta^B < \beta^{FA} \)). However, full insurance is rejected in the specification with complete markets – more than 70% of the shocks is estimated to be left uninsured. For \( \theta = 0.6 \) and \( \theta = 0.3 \) estimating (2.2) leads to reject the null of full insurance in the complete markets case. Moreover the coefficients do not even identify the correct ranking of insurance across asset structures. The coefficients estimated for the bond and the complete markets case exceed unity, signaling worse insurance than under Financial autarky.

Notice that from an econometric point of view all the coefficients are estimated correctly. The failure to identify full risk-sharing and the correct ranking of insurance reflects the following issues, which concern the interpretation of regression (2.2)

**Issue 1: The consumption allocation corresponding to full risk-sharing depends on relative prices**  According to the ASY method, an allocation delivers full insurance if consumption differentials and GDP differentials are uncorrelated. However from a theoretical point of view, when relative prices are well-defined, consumption and GDP differential may well be correlated in the full risk-sharing allocation. Chapter 1 shows that the allocation achieved in an equilibrium with complete asset markets, should be characterized both by relative consumption and relative prices, according to

\[
\frac{U_{C,t}}{U_{C,t-1}} \frac{RER_t}{RER_{t-1}} = \frac{U^*_{C,t}}{U^*_{C,t-1}}, \forall t
\]

where \( U_C \) and \( U^*_C \) denote respectively the marginal utility of consumption in the H and the F country, and \( RER \) is the real exchange rate between the two currencies. In terms of growth rates, and assuming CRRA utility function with risk-aversion parameter \( \rho \)

\[
\Delta \log RER = \rho \cdot (\Delta \log C_t - \Delta \log C^*_t)
\]

In the full insurance allocation consumption in the Home country can fall in response to a negative income shock, provided H consumers are compensated by an appreciation of their currency. The benchmark of risk-sharing underlying equation (2.2) does not take into account the role played by relative prices in characterizing the full insurance allocation. This is why the ASY method estimates \( \beta \neq 0 \) even in the model with complete asset markets.
2.3. THE PERFORMANCE OF EMPIRICAL METHODS: ASDRUBALI, SORENSEN AND YOSHA

**Issue 2: The relative value of endowments does not reflect relative wealth effects**  The ASY method regresses consumption differentials on the relative value of endowments. The presumption is that an increase in the relative value of the H endowment (the Home country getting relatively richer in equilibrium) should be followed by a financial transfer to the Foreign country if financial markets foster insurance. However, the relative value of endowments does not necessarily indicate which country is made relatively poorer by national shocks. As shown in Chapter 1, for \( \theta = 0.6 \) an increase in the Home endowment raises its relative value under any asset trade regime. But due to a strong terms of trade depreciation, the shock would make Home consumers relatively poorer absent asset trade.\(^5\)

Thus, when trade in assets is introduced, a rise in the Home endowment is optimally followed by a transfer of financial resources to this country, both in the model with trade in two equities and with trade in bonds. For \( \theta = 0.6 \) estimating \( \beta > 1 \) (and a larger \( \beta \) in the case of complete markets) reflects resources flowing to the Home country after an increase in the value of its GDP. Contrary to the interpretation of ASY, these flows are optimal and signal a high degree of insurance. What the method does not consider is that the relative value of endowments does not reflect relative wealth effects.

**Issue 3: Efficient financial markets may channel resources “upstream”**  As shown in Chapter 1, when flows and prices are substitutes (when they channel purchasing power in opposite directions) risk-sharing requires financial markets to channel resources to countries that (absent asset trade) would be made relatively richer by national shocks. This happens in our model for \( \theta = 0.3 \).\(^6\) An increase in the relative value of the H endowment would make the Home country richer absent asset trade. Since flows and prices are substitutes, after the shock efficient financial flows channel resources “upstream”, to the Home country, both in the model with trade in two equities and with trade in bonds. Estimating \( \beta > 1 \) in (2.2) indicates precisely “upstream” flows. Contrary to the standard interpretation of ASY, however, \( \beta > 1 \) corresponds to a high degree of insurance – perfect in the case of trade in two equities.

Table 2.3 reports the results from estimating specification (2.4). In the case of trade in two equities, full insurance is achieved through net factor income flows. The estimations should then find \( \beta_f = 1 \) for all the values of the trade elasticity. For \( \theta = 1.2 \) the estimation rejects full insurance through net factor income flows, but signals that a (small) share of

---

\(^5\)\( \theta = 0.6 \) corresponds to \( \tilde{\tau} < \theta < \tilde{\eta} \) in the notation of that paper. In this region of the trade elasticity domain, under Financial autarky an increase in the H endowment raises the shadow value of Home current income above the shadow value of Foreign current income – the shock makes the H consumer relatively poorer.

\(^6\)\( \theta = 0.3 \) corresponds to \( \theta < \tilde{\tau} \) in Chapter 1.
supply shocks is absorbed through this insurance channel \((\beta > 0)\). For \(\theta = 0.6\) and \(\theta = 0.3\), the estimation signals negative insurance from cross-border ownership of securities. We find similar results when estimating the share of shocks absorbed through consumption smoothing in the model with trade in uncontingent bonds.\(^7\) The issues behind these results are the same discussed above.

2.4 Other methods

2.4.1 Obstfeld (1993)

Given any two countries, assume representative agents, no taste shocks, and CRRA utility function. Complete markets imply

\[
\frac{\beta^t (C_t)^{-\rho}}{(C_0)^{-\rho}} = \frac{\beta^* (C_t^*)^{-\rho}}{(C_0^*)^{-\rho}}
\]

In a two-country environment, the method of Obstfeld (1993) consists in estimating

\[
\log (C_t) = \log (C_t^*) + \log (C_0/C_0^*) + \log (\beta/\beta^*) (t/\rho)
\]

Assuming equal discount factors (as in our model) and differencing, we can estimate\(^8\)

\[
\Delta \log (C_t) = \alpha + \beta \cdot \Delta \log (C_t^*) + \gamma \cdot \Delta \log (Y_t) + \varepsilon_t \tag{2.6}
\]

Markets are complete if \(\beta = 1\) and \(\gamma = 0\).

Using the simulated model described in section 2.2, I test equation (2.6) deflating national output using local CPIs. The results are reported in Table 2.4. They are qualitatively similar to those derived using the method of Asdrubali, Sorensen and Yosha (1996), and unveil that Obstfeld’s framework is subject to the same issues due to the presence of relative price fluctuations.

2.4.2 Brandt, Cochrane and Santa Clara (2006)

Given any two countries, Brandt, Cochrane and Santa Clara (2006) propose the following index of international risk-sharing

\(^7\)The estimation signals (correctly) that no fraction of the shocks is absorbed through smoothing in the complete markets case. Clearly, estimating the share of shocks that is left uninsured yields the same results as for specification (2.2).

\(^8\)See Obstfeld (1993) for the need to carry out an estimation using first-differenced data.
2.4. **OTHER METHODS**

<table>
<thead>
<tr>
<th></th>
<th>( \beta_f ) (share absorbed via NFI)</th>
<th>( \theta = 0.3 )</th>
<th>( \theta = 0.6 )</th>
<th>( \theta = 1.2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>Bond</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>Complete markets</td>
<td>-0.38</td>
<td>-0.09</td>
<td>0.14</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( \beta_s ) (share absorbed via NSAV)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Bond</td>
<td>-0.37</td>
<td>-0.09</td>
</tr>
<tr>
<td>Complete markets</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( \beta_u ) (share uninsured)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bond</td>
<td>1.37</td>
<td>1.095</td>
</tr>
<tr>
<td>Complete markets</td>
<td>1.38</td>
<td>1.0952</td>
</tr>
</tbody>
</table>
Table 2.4: OLS estimation of $\beta$ and $\gamma$ in Obstfeld regression

<table>
<thead>
<tr>
<th>$\beta, \gamma$</th>
<th>$\theta = 0.3$</th>
<th>$\theta = 0.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>$\beta = 0, \gamma = 1$</td>
<td>$\beta = 0, \gamma = 1$</td>
</tr>
<tr>
<td>Bond</td>
<td>$\beta = -0.27, \gamma = 1.27$</td>
<td>$\beta = -0.086, \gamma = 1.086$</td>
</tr>
<tr>
<td>Complete markets</td>
<td>$\beta = -0.273, \gamma = 1.273$</td>
<td>$\beta = -0.087, \gamma = 1.087$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\beta, \gamma$</th>
<th>$\theta = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>$\beta = 0, \gamma = 1$</td>
</tr>
<tr>
<td>Bond</td>
<td>$\beta = 0.158, \gamma = 0.841$</td>
</tr>
<tr>
<td>Complete markets</td>
<td>$\beta = 0.159, \gamma = 0.84$</td>
</tr>
</tbody>
</table>

European University Institute
DOI: 10.2870/19601
2.4. OTHER METHODS

\[
\mu = 1 - \frac{\sigma^2 \left( \log \frac{RER_{t+1}}{RER_t} \right)}{\sigma^2 (\log m_{t+1}) + \sigma^2 (\log m^*_{t+1})}
\]

where

\[
m_{t+1} = -\rho (\log \Delta C_{t+1})
\]

\[
m^*_{t+1} = -\rho (\log \Delta C^*_{t+1})
\]

\(\mu = 1\) is interpreted as perfect risk-sharing.

Table 2.5 reports the results from applying this method to our model. Full insurance is rejected even in the model specification in which asset markets are effectively complete. The reason is the following. According to this methodology the variance of Stochastic Discount Factors expressed in local currencies (\(m\) and \(m^*\)) represents the extent of risk to be shared, and every deviation from Purchasing Power Parity is considered a manifestation of the risk-sharing inefficiency. Chapter 1 shows however that an efficient cross-border allocation can be characterized only by relative consumption in conjuncture with relative prices. In this sense, from a theoretical point of view not all price fluctuations reflect a departure from full risk-sharing.

Table 2.5: Risk-sharing index of Brandt, Cochrane and Santa Clara

<table>
<thead>
<tr>
<th>(\mu) (risk-sharing index)</th>
<th>(\theta = 0.3)</th>
<th>(\theta = 0.6)</th>
<th>(\theta = 1.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>-0.13</td>
<td>-2.74</td>
<td>0.89</td>
</tr>
<tr>
<td>Bond</td>
<td>0.12</td>
<td>0.27</td>
<td>0.5</td>
</tr>
<tr>
<td>Complete markets</td>
<td>0.09</td>
<td>0.28</td>
<td>0.5</td>
</tr>
</tbody>
</table>
2.4.3 Flood, Marion and Matsumoto (2008)

Flood, Marion and Matsumoto (2008) measure the risk-sharing inefficiency using the variance of a country’s consumption on world consumption. In our setup this translates to

\[ \lambda = \text{var} (\log (C_t) - \log (C_t + C_t^*)) \]

\( \lambda = 0 \) is interpreted as perfect risk-sharing.

Table 2.6 shows the results from applying this method to our simulated model. Clearly, the benchmark of efficiency on which the methodology is based does not take into account relative price fluctuations, leading both to a rejection of full risk-sharing in the complete markets case and to problems in capturing the correct ranking of insurance across asset structures.

<table>
<thead>
<tr>
<th>( \lambda ) (inefficiency index)</th>
<th>( \theta = 0.3 )</th>
<th>( \theta = 0.6 )</th>
<th>( \theta = 1.2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>2</td>
<td>0.04</td>
<td>0.39</td>
</tr>
<tr>
<td>Bond</td>
<td>0.61</td>
<td>0.33</td>
<td>0.15</td>
</tr>
<tr>
<td>Complete markets</td>
<td>0.5</td>
<td>0.28</td>
<td>0.14</td>
</tr>
</tbody>
</table>

2.5 Concluding remarks

This Chapter investigates in simulations the robustness of empirical methods designed to measure cross-border risk-sharing, to the presence of international price fluctuations. Checking the performance of the ASY method in a DSGE model with relative price movements has unveiled the three typical issues that may affect existing methodologies. First, the relative value of endowments commonly used as a regressor does not necessarily identify which country is made relatively richer/poorer by national shocks. Second, the interaction between
2.5 CONCLUDING REMARKS

relative prices and financial flows may require efficient financial markets to channel resources “upstream”, to relatively richer countries. Finally, the full risk-sharing consumption allocation can be characterized only in conjuncture with relative price movements: with full insurance national consumption can fall after a negative income shock, provided consumers are compensated by an appreciation of their currency — a condition that is not considered by standard benchmarks of risk-sharing. These problems can have severe consequences. Not only they lead to reject full insurance also when the “true” model features effectively complete asset markets, but they can also lead to estimate a higher degree of insurance under, say, Financial autarky than in the case of complete markets.

In a world in which relative price fluctuations are large and sustained and contribute to transfer purchasing power across countries, the benchmarks of efficiency derived in one-good frameworks and on which standard methods are based, do not appear to be valid anymore. Focusing on the pattern of financial flows across countries as required by most existing empirical methods, may then be easily misleading.

These findings point to the need to find a benchmark of global insurance that is immune from these issues. This is the aim of the next Chapter, which proposes a theoretically-founded benchmark of cross-border risk-sharing. This benchmark constitutes the theoretical foundations for a new empirical method to measure international insurance, which takes into account relative price fluctuations.
2.6 Appendix: A note on the use of GDP deflators

I apply the ASY method to our simulated model without implementing any correction for the presence of relative price fluctuations. Namely, I deflate output using national GDP deflators. Results are shown in the following Table. In this case, the ASY method is subject only to two of the three issues described in the main text.

The coefficients estimated signal a higher degree of insurance under Financial Autarky than under complete markets for $\theta = 0.6$ because the relative value of endowments does not reflect asymmetric wealth effects. Full insurance is always rejected in the two-equity model because the benchmark of risk-sharing on which the method is based does not consider that the efficient consumption allocation depends also on relative prices.

On the other hand, the method captures the correct ranking of insurance across asset trade regimes for $\theta = 0.3$. Contrary to the implementation that uses national CPIs to deflate output, the present version of the method is not affected by the third, typical issue due to relative price fluctuations – the fact that efficient financial markets may channel resources “upstream”. This happens because the coefficients estimated in this version capture also the effects of relative price fluctuations on the relative value of endowments.

Table 2.7: ASY method with GDP-deflated output

<table>
<thead>
<tr>
<th>$\beta$ (fraction uninsured)</th>
<th>$\theta = 0.3$</th>
<th>$\theta = 0.6$</th>
<th>$\theta = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>1.76</td>
<td>0.29</td>
<td>0.85</td>
</tr>
<tr>
<td>Bond</td>
<td>0.938</td>
<td>0.744</td>
<td>0.534</td>
</tr>
<tr>
<td>Complete markets</td>
<td>0.934</td>
<td>0.746</td>
<td>0.532</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


CHAPTER 3

MEASURING INTERNATIONAL RISK-SHARING: EMPIRICAL EVIDENCE FOR THE US AND OECD COUNTRIES

3.1 Introduction

One of the most relevant events of the last decades has been the fast integration of international financial markets. There is quite a generalized agreement that abandoning barriers to cross-border asset trade increased the volume of financial resources flowing across borders. Yet whether financial integration effectively raised global insurance, is still an open issue. The empirical open-macro literature has rejected soundly the hypothesis that international asset trade provides full insurance against national income shocks. However, results about the actual extent of international risk-sharing and its evolution over time are still not conclusive. One reason is that several empirical methods to study cross-border insurance are well-designed to test the null hypothesis of full risk-sharing, but once this null is rejected they cannot (by construction) quantify the actual lack of insurance or the welfare losses it creates.

Moreover the empirical methods that were explicitly designed to gauge the extent of insurance, are based on the implicit assumption of no international price fluctuations. From a theoretical point of view, Chapter 1 shows that the interaction between relative price fluctuations and cross-border financial flows is instead crucial to derive testable conditions for financial markets efficiency. From an empirical perspective, Chapter 2 shows that standard empirical methods to measure international insurance may be completely misleading if relative price movements (realistically) transfer purchasing power across countries. The reason is that the presence of price fluctuations invalidates the benchmarks of risk-sharing on which these methodologies are based.

In this Chapter I propose a new empirical method to quantify cross-border insurance against shocks to national income, which is immune from these issues. The method consists in estimating the risk-sharing inefficiency between two countries (contingent on the shocks that hit them during a certain period) using the wedge between their Stochastic Discount Factors (SDFs). This measure quantifies the distance of the observed cross-country alloca-
tion of resources from the one that would be created by complete financial markets, which would isolate the relative utility value of wealth of households living in different countries from idiosyncratic income fluctuations. This contingent risk-sharing inefficiency can then be attributed either to the strength of uninsurable shocks (the extent of risk to be shared internationally) or to the degree of insurance against different sources of macro risk.\footnote{“Contingent risk-sharing inefficiency” refers to the distance of the observed cross-country allocation from the one that would be achieved through complete asset markets, delivering perfect insurance against national income fluctuations. It should be noted that full income insurance might not necessarily imply that the resulting cross-country allocation is Pareto optimal.}

I apply this method to study the evolution of risk-sharing between the US and industrialized OECD economies over the last forty years, in the period in which financial markets integration took place. When countries’ SDFs are estimated relying on the assumptions of traditional open-macro models (constant risk-aversion), I find that nominal exchange rate volatility represented an important source of macroeconomic risk. Its decrease led to a significant reduction in risk-sharing inefficiencies. When SDFs are estimated assuming time-varying risk-aversion, I find that financial markets integration was partially effective in raising the degree of insurance against macro risk, and that for some countries a lower GDP volatility (the so-called Great Moderation) reduced substantially risk-sharing inefficiencies.

Macroeconomic theory suggests that full insurance against national income fluctuations equalizes the Stochastic Discount Factors (SDFs) of any two economies in any state of the world. Therefore the risk-sharing inefficiency between countries (or country aggregates) can be measured by the wedge between their SDFs. Using a simple theoretical framework I prove that this measure of cross-border risk-sharing, the gap, reflects changes in countries’ relative wealth created by uninsurable shocks. Under certain conditions, it also maps into aggregate welfare losses due to imperfect insurance. I show how its variance can be used to quantify the contingent risk-sharing inefficiency between two countries over a certain time horizon. By regressing the time-series of the gap estimated from the data on different sources of macroeconomic risk (national GDP volatility, government spending, nominal exchange rate fluctuations), we can test whether households in different countries are fully insured against these risks. We can also attribute falls in the contingent inefficiency over time to reductions in the strength of uninsurable shocks (the extent of risk to be shared) or to improvements in the degree of insurance against different sources of risk. I assess the performance of this method using a simulated two-country two-good DSGE model, in which both price fluctuations and financial flows can provide insurance against national income shocks. Contrary to existing empirical methods, the present methodology identifies properly full-risk sharing and captures the correct ranking of insurance across different asset trade regimes.
3.1. **INTRODUCTION**

Finally, I apply this method to study the evolution of risk-sharing between the US and industrialized OECD countries. I adopt two strategies for estimating Stochastic Discount Factors from the data. First, I rely on the assumptions of standard open-macro models and postulate a constant risk-aversion; using non-linear GMM I estimate from the data the deep parameters characterizing the SDFs, and verify that the functional form assumed is not rejected by the data. Second, given the uncertainty on the specific form of the SDFs, I follow Campbell and Cochrane (1999) and assume a utility function with time-varying risk-aversion; I adopt the parametrization that was used by these authors to replicate the equity-premium puzzle and to capture the history of US stock prices.

When SDFs are estimated relying on the assumptions of the traditional macro model (constant risk-aversion), I find that nominal exchange rate volatility represented an important source of macroeconomic risk, whose decrease led to a significant reduction in insurance inefficiencies among the US and OECD countries. Thus constant risk-aversion implies that nominal exchange rate fluctuations might not foster an efficient allocation of resources across countries. Instead, a reduction in nominal exchange rate volatility can lessen substantially the risk to be shared internationally and increase global insurance. This result bears interesting implications for the literature on optimal exchange rate regimes. While in a well-know analysis Baxter and Stockman (1988) found that the only change in the behaviour of macro aggregates caused by the adoption of a fixed exchange rates regime, is a reduction in exchange rate volatility, our results suggest that a fall in this volatility can indeed reduce substantially cross-border risk-sharing inefficiencies. From a policy perspective, these findings might support a rationale for the adoption of fixed exchange regimes and the establishment of currency unions. Indeed, one of the motives behind the creation of the European Monetary Union was the willingness to reduce the uncertainty arising from nominal exchange rate fluctuations among its members.

When SDFs are estimated assuming time-varying risk-aversion, I find that financial markets integration was partially effective in raising the degree of insurance against macro risk. Also the fall in the volatility of macro aggregates – mainly GDP of both the US and OECD countries, and its components – is found to have reduced significantly contingent risk-sharing inefficiencies. Thus time-varying risk-aversion implies that a reduction in the extent of risk to be shared internationally, requires a decrease in the volatility of macro fundamentals, as the one that occurred for several countries during the so-called Great Moderation.

This paper is related to the first two Chapters of this thesis. The first Chapter questions the validity of the theoretical benchmarks of insurance at the basis of standard empirical
methods. Chapter 2 assesses their performance in a simulated DSGE model. The present paper complements those contributions by proposing a new methodology that addresses the issues they unveil.

This Chapter is also related to the literature on empirical risk-sharing measurement. Asdrubali, Sorensen and Yosha (1996), Obstfeld (1993), Brandt, Cochrane and Santa Clara (2006), and Flood, Marion and Matsumoto (2008) are only some representative examples. All these methods are implicitly based on the assumption of no relative price fluctuations.

This paper is the first one to propose a theoretically-consistent measure of cross-border risk-sharing inefficiencies that does not rely on restrictive assumptions about real exchange rate movements.

The rest of the Chapter is organized as follows. The next section describes the new method and its theoretical foundations. Section 3.3 assesses its performance in simulation exercises. Section 3.4 discusses the empirical strategy. Section 3.5 shows the results from applying this method to the US and OECD countries. Section 3.6 concludes.

3.2 An insurance benchmark from macro theory

Following Chapter 1, in this section I derive a theoretical benchmark of asset market completeness, and a measure of cross-border insurance against income risk valid also in presence of relative price fluctuations. I show how this measure can represent the theoretical foundations of a new empirical method to estimate cross-border risk-sharing.

3.2.1 A measure of cross-border insurance

Assume the world consists of several countries, each inhabited by a representative agent. Asset markets are complete as agents can trade internationally in a full set of Arrow-Debreu securities. Focus on two countries, Home and Foreign. In what follows \( W(s|s) \) denotes the price in state-of-the-world \( s \) of an Arrow-Debreu security paying 1 unit of some numéraire good in the following period if state-of-the-world \( s \) realizes. \( U_C(s) \) and \( U_C(s|s) \) are the marginal utilities of consumption of the H consumer, respectively, in state \( s \) and in state \( s^{t} \). \( P(s) \) and \( P(s|s) \) are the prices of the H consumption good in state \( s \) and \( s^{t} \). \( \beta \) denotes the discount factor of the H consumer, \( \pi(s|s) \) the conditional probability of state \( s^{t} \) given \( s \). The Euler Equation of the H consumer, regulating her purchase in state \( s \) of Arrow-Debreu securities paying 1 unit of the numéraire in state \( s^{t} \), reads

\[
U_C(s) \frac{W(s|s)}{P(s)} = \beta \cdot \pi(s|s) \cdot U_C(s|s) \frac{1}{P(s|s)}, \forall s,s^{t} \tag{3.1}
\]
3.2. AN INSURANCE BENCHMARK FROM MACRO THEORY

The H consumer buys securities until the marginal cost of purchasing one more asset (left hand side in equation (3.1)) equals its expected marginal benefit (right hand side).

The analogous condition for the F agent is given by

\[ U^*_C(s) \frac{W(st|s)}{P^*(st|s) \cdot \omega(s)} = \beta^* \cdot \pi(st|s) \cdot U^*_C(st|s) \frac{1}{P^*(st|s) \cdot \omega(st|s)}, \forall s, st \]  

(3.2)

where starred variables denote Foreign aggregates, and \( \omega \) is the nominal exchange rate between H and F currencies.

Combining (3.1) and (3.2) gives

\[ \beta \frac{U_C(st|s)}{U_C(s)} \frac{P(s)}{P(st|s) m(st|s)} = \beta^* \frac{U_C^*(st|s)}{U_C^*(s)} \frac{P^*(s)}{P^*(st|s) m^*(st|s)} = \frac{W(st|s)}{\pi(st|s)} \cdot \forall s, st \]

where \( m(st|s) \) and \( m^*(st|s) \) are the Stochastic Discount Factors (SDFs) of the H and the F consumer. Perfect cross-border risk-sharing against national income shocks (given by construction by complete asset markets) equates the SDFs of the two agents, for every states-of-the-world \( s \) and \( st \). Equivalently, in terms of time-dependent notation, perfect risk-sharing implies

\[ m_t = m^*_t, \forall t \]  

(3.3)

If countries cannot pool risk efficiently the SDFs \( m \) and \( m^* \) need not be equalized: uninsurable shocks will drive a wedge between them in some states of the world. Therefore

\[ m_t = \eta_t \cdot m^*_t \]  

(3.4)

where \( \eta_t \) represents the wedge between H and F SDFs created by uninsurable disturbances. Taking logarithms on both sides of (3.4) I define the measure of cross-border insurance, the gap, as

\[ \text{gap}_t \equiv (\log \eta_t - \log 1) = \log m_t - \log m^*_t \]  

(3.5)

gap\(_t\) measures the percentual deviation of the wedge between H and F SDFs observed at time \( t \) from the wedge that would be observed if income risk was perfectly shared (i.e. if \( \eta \) was equal to 1). Therefore, by construction gap\(_t\) measures the percentual deviation from full insurance against income fluctuations observed at time \( t \).

Appendix B shows that assuming frictionless trade in uncontingent bonds equalizes expected SDFs. In other words, self-insurance drives the conditional mean of the gap to zero.
3.2. AN INSURANCE BENCHMARK FROM MACRO THEORY

\[ E_t (m_{t+1}) = E_t (m^*_t) \Rightarrow E_t (\text{gap}_{t+1}) = 0 \] (3.6)

It is well-known that if income risk is perfectly shared countries’ relative wealth should stay constant over time. Using a simple two-country two-good DSGE model and following closely Chapter 1, Appendix A proves that the gap measures changes in countries’ relative wealth created by uninsurable shocks. If consumers in different countries cannot fully insure against national shocks, idiosyncratic disturbances change their relative wealth. The sign of the gap arising in response to shocks indicates the new ranking of wealth, that is which country has been made relatively richer (poorer) by the shock. Its magnitude quantifies the change in countries’ relative wealth.

Moreover, there exists a monotone relationship between the gap and the social welfare losses due to lack of insurance. Following Chapter 1 I prove in Appendix B that the higher the gap that opens up at time \( t \) the higher the deviation of the time-\( t \) allocation from the optimum of a social welfare function that weights the two countries according to their previous wealth. More specifically, the first-best plan is an (infinite) sequence of allocations engineered by a Social Planner who aims at keeping fixed relative countries’ wealth from \( t = 0 \) up to \( t = \infty \), weighting countries according to their \( t = -1 \) relative wealth. If countries’ relative wealth has been constant until time \( (t-1) \), the gap that opens up at time \( t \) reflects deviations from this first-best allocation. If countries’ relative wealth has changed between \( t = -1 \) and \( (t-1) \) the gap that opens up at time \( t \) reflects deviations from the second-best plan that would be implemented by a Social Planner who, given the previous deviation from the first-best plan, re-optimizes taking into account the new level of relative wealth. In both cases, the gap maps monotonically into social welfare losses due to imperfect risk-sharing.\(^2\)

3.2.2 A new empirical method

Every deviation of the gap from zero measures a percentual deviation from full risk-sharing that may result in welfare losses, and is contingent on the shocks that hit the economies at each point in time. This measure of insurance provides the theoretical foundations for a new empirical method to estimate cross-border risk-sharing. The method consists in estimating the contingent risk-sharing inefficiency between two countries by using some statistic that quantifies the volatility of the gap computed on their Stochastic Discount Factors around zero, within a certain time horizon. The next section shows in detail how this method can be brought to the data, by estimating the deep parameters that characterize the

\(^2\)This holds under the maintained assumption that deviations from the law of one price are contained.
3.2. AN INSURANCE BENCHMARK FROM MACRO THEORY

SDFs and by testing the validity in the data of important assumptions. For the moment, we will assume that we know the “true” functional form of the SDFs of the countries under consideration. We will also assume that the data do not reject the equalization of expected SDFs

\[ E_t (m_{t+1}) = E_t (m_{t+1}^*) \]

Then the unconditional mean of the gap (the log difference between SDFs) is equal to zero (see equation (3.6)). Therefore, the variance and the standard deviation of the gap can be used as measures of its volatility around zero, i.e. measures of the contingent risk-sharing inefficiency between the two countries over a certain period of time.

While the variance of the gap quantifies the risk-sharing inefficiency, this measure is silent about its causes. It does not indicate which are the sources of idiosyncratic risk against which countries are not fully insured. Moreover a high variance could be due either to the high frequency and strength of uninsurable shocks, or to a low degree of insurance against macro risk. In order to disentangle these effects and to study which sources of macroeconomic risk are left uninsured among two countries, we can regress the time-series for the gap on different sources of risk. Assuming there are only productivity disturbances, as in our DSGE model, we can run

\[ gap_t = \alpha + \beta \cdot \Delta \log Y_t + \gamma \cdot \Delta \log Y_t^* + \varepsilon_t \] (3.7)

where \( \Delta \) denotes first differences. The regressors capture the sources of macroeconomic risk that country H and F should ideally pool. The coefficient associated with each shock quantifies the percentual deviation from full risk-sharing that is created by a 1% variation in the corresponding macro aggregate, thus proxying for the degree of insurance against that particular source of risk. The lower the coefficient estimated, say, for country H GDP growth, the higher the degree of insurance against this source of risk. We can assess if some sources of risk are perfectly pooled by testing whether the corresponding coefficient equals zero. We can also estimate (3.7) over two distinct time periods and decompose changes in the variance of the gap over time into changes in the coefficients and changes in the variance of the regressors. This allows to attribute reductions in the risk-sharing inefficiency either to reductions in the extent of macro risk to be pooled (the variance of regressors) or to increases in the degree of insurance against different sources of risk (reductions in the coefficients).

---

3A rise in the degree of insurance against some disturbances could be due to three factors. First, an improvement in insurance possibilities to share risk across countries, e.g. an enlargement in the set of

European University Institute
DOI: 10.2870/19601
### 3.2. AN INSURANCE BENCHMARK FROM MACRO THEORY

**A note on the choice of deflators** Combining H and F budget constraints, it is easy to see that the *gap* can be expressed as

\[
\Delta \log Q_t - \rho (\Delta \log C_t - \Delta \log C_t^*) = -\mu (\Delta \log GDP_t - \Delta \log GDP_t^*) + \omega \Delta \log TOT_t + \Delta \log \nu_t + \phi \Delta \log NSAV_t - \kappa \Delta \log NFI_t + \Delta \log Q_t
\]

where TOT\(_t\) are country H terms of trade, \(\mu, \omega, \phi, \) and \(\kappa\) are positive constants, and \(\nu_t\) is the real relative value of the H and the F endowments. Thus, we have two options for estimating equation (3.7). We can either deflate output using the national CPIs or GDP deflators.\(^4\) If we choose the former, running (3.7) is equivalent to estimating

\[
gap_t = \alpha + \beta \cdot (\Delta \log Y_t + \Delta \log TOT_t) + \gamma \cdot (\Delta \log Y_t^* + \Delta \log TOT_t) + \varepsilon_t
\]

The regressors include the impact of terms of trade fluctuations on the relative value of endowments. Thus the coefficients proxy for the degree of insurance against GDP risk, abstracting from the insurance provided by relative price fluctuations that affect the relative value of endowments. Notice that this adjustment is not enough to clear completely the coefficients from the effects of price fluctuations, which enter (3.7) through real exchange rate movements.

Instead, if we deflate output using its own deflator, running (3.7) is equivalent to estimating

\[
gap_t = \alpha + \beta \cdot (\Delta \log Y_t) + \gamma \cdot (\Delta \log Y_t^*) + \varepsilon_t
\]

The coefficients proxy for the degree of insurance provided both by financial markets and relative price fluctuations. In this Chapter I choose this second solution – using national GDP deflators. The reason is the following. Recent contributions to the open-macro literature (see for instance Corsetti et al. (2008)) emphasize that the structure of financial markets and financial instruments traded internationally. Second, some other structural change in a shock process that makes the *available* set of assets more suitable to pool this specific source of risk across countries. For instance, two countries that can trade only in one uncontingent bond can self-insure against idiosyncratic risk quite efficiently if shocks are temporary. If disturbances are permanent, the same asset structure does not allow to insure against any share of risk (see Chapter 1). Finally, a rise in the degree of insurance could be due to an increase in the insurance properties of relative price fluctuations, either brought about by stronger financial deepness or not.

\(^4\)Notice that, from a purely theoretical point of view, deflating output with GDP deflators is not entirely correct in the other methods (for instance in the approach of Asdrubali, Sorensen and Yoshia (1996)). Since these empirical frameworks are based on an identity, using GDP deflators is equivalent to assuming no relative price fluctuations (see Chapter 2).
can influence the insurance properties of relative prices. In this sense, the impact on cross-border insurance of financial market integration may encompass also improvements in the insurance properties of relative prices, which should be included in our measure of the degree of insurance.\(^5\)

### 3.2.3 The new method in simulations

As in Chapter 2, I write a simple model in which both relative price fluctuations and financial flows can transfer purchasing power across countries, and simulate it under three different asset structures – Financial autarky, trade in uncontingent bonds, and trade in two equities. The model, the asset structures, and the parametrization are the same described in Chapter 2 and will not be repeated here. The risk-sharing properties of the three asset trade regimes are known and have been studied in Chapter 1. Financial autarky gives less or equal insurance against national supply shocks than the other asset structures. Trade in uncontingent bonds ranks second, while trade in two equities gives perfect risk-sharing against national disturbances.

I apply the new method to the simulated model to check if it identifies correctly full risk-sharing in the two-equities case, and if it captures the correct ranking of insurance among asset structures. Table 3.1 compares our measure of the bilateral risk-sharing inefficiency (the standard deviation of the \(\text{gap}\)), across different asset structures and values of the trade elasticity. For the Financial autarky case, our statistic signals correctly a high cross-border insurance from relative price fluctuations for \(\theta = 1.2\), and a low degree of insurance from terms of trade movements for \(\theta = 0.3\), as shown in Chapter 1. Thus, contrary to the coefficients estimated through some of the standard methods (the approach of Asdrubali, Sorensen and Yoshia (1996), and Obstfeld (1993), the standard deviation of the \(\text{gap}\) gives an indication of the welfare losses due to the absence of financial markets.\(^6\) Moreover, the standard deviation of the \(\text{gap}\) reflects the correct ranking of insurance among different asset trade regimes, and leads to a non-rejection of full insurance in the complete markets case.

Table 3.2 shows the OLS coefficients estimated by running (3.7). Since the model is fully symmetric \(\beta = -\gamma\). Standard errors are negligible and are not reported. The magnitude of the coefficients reflects the degree of insurance against national shocks (the lower the

---

\(^5\)Simulations carried out in our simple model (available upon request) show that the difference between the coefficients estimated using GDP and CPI deflators reveals whether relative price movements reduce or amplify the wealth divergence that would arise at constant prices. The implementation of this exercise is left for future research.

\(^6\)Under the maintained assumption that inefficient deviations from the law of one price are contained. See Chapter 2 for a comparison with other methodologies.
### 3.3 Empirical strategy

#### 3.3.1 Estimating Stochastic Discount Factors

The only assumptions we need to make in order to implement our method are the existence of a representative agent in each country, and the specific form of the SDFs. In this Chapter I adopt two strategies to estimate SDFs. First, I rely on the assumptions of standard open-macro models and postulate a constant risk-aversion; I estimate from the data the deep parameters characterizing the SDFs, and verify that the functional form assumed is not rejected by the data. Second, given the uncertainty on the specific form of the SDFs, I follow Campbell and Cochrane assuming a utility function with time-varying risk-aversion; I adopt the parametrization that was used by these authors to replicate the equity-premium puzzle and to capture the history of US stock prices.

---

#### Table 3.1: Gap standard deviation

<table>
<thead>
<tr>
<th>Gap standard deviation</th>
<th>$\theta = 0.3$</th>
<th>$\theta = 0.6$</th>
<th>$\theta = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>12.89</td>
<td>4.51</td>
<td>2.24</td>
</tr>
<tr>
<td>Bond</td>
<td>0.09</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Complete markets</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
3.3. EMPIRICAL STRATEGY

Table 3.2: Deviation from full risk-sharing due to a 1 percent increase in H GDP growth rate

<table>
<thead>
<tr>
<th></th>
<th>$\beta = -\gamma$</th>
<th>$\theta = 0.3$</th>
<th>$\theta = 0.6$</th>
<th>$\theta = 1.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial autarky</td>
<td>-6.61</td>
<td>2.28</td>
<td>-1.11</td>
<td></td>
</tr>
<tr>
<td>Bond</td>
<td>-0.03</td>
<td>0.011</td>
<td>-0.005</td>
<td></td>
</tr>
<tr>
<td>Complete markets</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

3.3.1.1 Stochastic Discount Factors from the traditional open-macro model

In accordance with what is assumed by most models of international interdependence, I assume that for all countries the period utility function is separable in consumption and depends on the current consumption level according to a standard CRRA function. Namely

$$U(C^i, \cdot) = \frac{(C^i)^{1-\rho}}{1-\rho} + \kappa$$  \hspace{1cm} (3.8)

where $\rho$ is the (common) coefficient of relative risk-aversion and the utility function is allowed to depend on other variables ($\kappa$) that do not enter the marginal utility of consumption. (3.8) implies that the gap between country $i$ and $j$ arising at time $t$ can be computed as

$$\text{gap}_{i,j}^t \equiv \log m_i^t - \log m_j^t = \Delta \log Q_{i,j}^t - \rho \left( \Delta \log C_i^t - \Delta \log C_j^t \right) + \log \beta^i - \log \beta^j$$  \hspace{1cm} (3.9)

where $\beta^i$ and $\beta^j$ denote the discount factors. In accordance with the literature, the deep parameters appearing in this formula can be estimated from the data by testing the equality of countries’ expected SDFs. The following equation can then be tested through non-linear GMM
EMPIRICAL STRATEGY

\[ E_t (m_{t+1}^i) = E_t (m_{t+1}^j) \Rightarrow \]
\[ \frac{\beta^i}{\beta^j} \cdot E_t \left\{ \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\rho} \frac{P_{t+1}^i}{P_t^i} \right\} = E_t \left\{ \left( \frac{C_{t+1}^j}{C_t^j} \right)^{-\rho} \frac{P_{t+1}^j}{P_t^j} \frac{S_{t+1}^{i,j}}{S_t^{i,j}} \right\} \]  
(3.10)

where \( S^{i,j} \) is the bilateral nominal exchange rate. This estimation allows to pin down \( \rho \) and the relative discount factor \( \frac{\beta^i}{\beta^j} \). Moreover, if (3.10) is not rejected, the mean of the \textit{gap} cannot be rejected to be zero. Therefore, the variance and the standard deviation of \( \textit{gap}^{i,j} \) can be conveniently used as measures of its volatility around zero, i.e. measures of the contingent risk-sharing inefficiency over a period of time.\(^7\) Notice also that failure to reject equation (3.10) implies that the data do not reject our assumption about the specific form of SDFs.

3.3.1.2 Stochastic Discount Factors with time-varying risk-aversion

The second strategy consists in assuming a functional form for SDFs following the literature on habit formation, in particular Campbell and Cochrane (1999). The period utility function depends on a stock of habit, and relative risk-aversion is time-varying and state-dependent. Campbell and Cochrane (1999) show that a standard model featuring this SDF solves the short and long-run equity premium puzzle, and captures much of the history of US stock prices. Rabitsch (2008) shows that the same preferences give a rationale for deviations from uncovered interest parity and match the volatility of exchange rates observed in the data. Following these contributions I assume that the utility function is

\[ \frac{(C_t - X_t)^{1-\rho} - 1}{1 - \rho} \]  
(3.11)

where \( X_t \) is the habit level. The relationship between consumption and habit is captured by the surplus consumption ratio

\[ S_t = \frac{C_t - X_t}{C_t} \]

Thus relative risk aversion (the local curvature of the utility function) is state-dependent and related to \( S_t \) by

\[ S^{i,j} \]
3.3. EMPIRICAL STRATEGY

\[ \eta_t = \frac{U_{CC}C}{U_C} = \frac{\rho}{S_t} \]

Assume further

\[ \log S_{t+1} = (1 - \phi) \log S + \phi \log S_t + \lambda (\log S_t) (\log C_{t+1} - \log C_t - \log G) \]

where \( G \) is the average growth rate of the economy and

\[ \lambda (\log S_t) = \frac{1}{S} \sqrt{1 - 2(\log S_t - \log \bar{S}) - 1}, \log S_t \leq \log S_{max} \]

\[ \lambda (\log S_t) = 0, \log S_t > \log S_{max} \]

and

\[ \bar{S} = \rho \sqrt{\frac{\rho}{1 - \phi - \frac{B}{\rho}}} \]

The Stochastic Discount Factors of H and F consumers read respectively

\[ m_t = \beta \left( \frac{S_{t+1}}{S_t} \right)^{-\rho} \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{P_t}{P_{t+1}} \]

\[ m_t^* = \beta^* \left( \frac{S_{t+1}^*}{S_t^*} \right)^{-\rho} \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\rho} \frac{P_t^*}{P_{t+1}^*} \frac{\varepsilon_t}{\varepsilon_{t+1}} \]

The gap is given by

\[ \text{gap}_t \equiv \log (m_t) - \log (m_t^*) = \Delta \log Q_t - \rho (\Delta \log C_t - \Delta \log C_t^* + \Delta \log S_t - \Delta \log S_t^*) \]  \hspace{1cm} (3.12)

For each country I compute \( G \) as the average growth rate of national consumption from the data. Following Campbell and Cochrane (1999) and Rabitsch (2008), I assume \( \rho = 2, \phi = 0.985, \) and \( B = -0.01. \)

3.3.2 Estimating the risk-sharing inefficiency

Having checked for both specifications of SDFs that the mean of the gap cannot be rejected to be zero in the data, I use its variance and standard deviation as measures of its
volatility around zero. The contingent ineficiency between country \(i\) and \(j\) over the period \(t = 1, ..., T\) will be measured by \(^8\)

\[
\hat{\sigma}^2 (gap^{i,j}) = \frac{1}{T} \sum_{t=1}^{T} (gap^{i,j} - E(gap^{i,j}))^2 \quad \text{or} \quad \hat{\sigma} (gap^{i,j}) = \left[ \hat{\sigma}^2 (gap^{i,j}) \right]^{1/2}
\]

### 3.3.3 Detecting significant changes in the inefficiency over time

Significant variations in the risk-sharing inefficiency between country \(i\) and \(j\) can be found by testing for structural changes in the variance of \(gap^{i,j}\). Since the dates of potential changes are unknown, I find it convenient to employ the Quasi-Maximum Likelihood methodology of Qu and Perron (2007), which allows to detect structural breaks at unknown dates, find their number, and test for their significance. More precisely, I estimate

\[
gap^{i,j}_t = \mu + \varepsilon_t \quad (3.13)
\]

where \(\mu\) is a constant and \(\varepsilon_t\) are \(i.i.d\). Normal disturbances. I assume no break occurred in \(\mu\), and test for structural breaks in the variance-covariance matrix of errors.\(^9\)

### 3.3.4 Macroeconomic risk versus degree of insurance

Following the same reasoning as for equation (3.7), if the relationship between country \(i\) and \(j\) was described by a model with exogenous GDP, government spending and monetary shocks, plus disturbances coming from currency speculation and from productivity in the rest of the world, it is easy to show that the risk-sharing inefficiency between country \(i\) and \(j\) would follow – in a first order approximation to the model – the following process

\[
gap^{i,j}_t = \alpha \cdot \Delta \log \text{gdp}^i_t + \beta \cdot \Delta \log \text{gdp}^j_t + \gamma \cdot \Delta \log \text{gov}^i_t + \delta \cdot \Delta \log \text{gov}^j_t + \eta \cdot \Delta \log s^{i,j}_t + \zeta \cdot \Delta \log \text{gdp}_t^{world} + \varepsilon_t \quad (3.14)
\]

where \(\Delta \log \text{gdp}^i_t\), \(\Delta \log \text{gdp}^j_t\), \(\Delta \log \text{gdp}_t^{world}\) denote respectively the growth rate of country \(i\), country \(j\) and “rest of the world” GDP, deflated through national GDP deflators and not including government expenditure.\(^{10}\) Rest of the world GDP affects the inefficiency between

---

\(^8\)Using the standard deviation allows to express the contingent inefficiency in terms of percentual deviations from perfect risk-sharing, while using the variance allows to run more easily formal tests on the inefficiency and to uncover its determinants through standard estimation procedures. I use the standard deviation to compare the extent of the inefficiency across countries, and the variance to detect changes in the inefficiency over time and to investigate their causes.

\(^9\)See Qu and Perron (2007) for additional details on the tests.

\(^{10}\)Equation (3.14) could also include lags of the independent variables as regressors.
3.3. EMPIRICAL STRATEGY

country i and j as long as it has an asymmetric impact on consumption in the two countries.\[^{11}\] \(\Delta \log \text{gov}_i^t, \Delta \log \text{gov}_j^t, \) and \(\Delta \log s_{i,j}^t\) are country i and country j final government expenditure expressed in local currency, and the nominal exchange rate between country i and country j currency. Nominal exchange rate growth is capturing both the effects of monetary shocks and the impact of possible disturbances coming from currency speculation. I use OECD GDP growth as a proxy for rest of the world GDP. I estimate equation (3.14) instrumenting nominal exchange rate growth with GDP and exchange rate growth lagged values.\[^{12}\]

Two kinds of analyses can be performed using (3.14). First we can establish if some sources of risk are perfectly pooled by testing whether the corresponding coefficients equal zero. Second, we can run (3.14) over two distinct time horizons in order to attribute changes in the variance of the gap over time to changes in the coefficients (the degree of insurance against shocks) and changes in the variance of regressors (the extent of macroeconomic risk to be pooled), according to the following procedure. First, any reduction in the variance of \(gap_{i,j}\) between the two subperiods pre and post (denoting the pre and post-break years) can be expressed in terms of changes in the variance of shocks and changes in the coefficients in (3.14) as

\[
\text{Var}(gap_{i,j})_{post} - \text{Var}(gap_{i,j})_{pre} = \left(\hat{\alpha}_{post}^2 - \hat{\alpha}_{pre}^2\right) \cdot \text{Var}\left(\Delta \log (gdp^i)\right)_{pre} + \\
\hat{\alpha}_{pre}^2 \cdot \left(\text{Var}\left(\Delta \log (gdp^j)\right)_{post} - \text{Var}\left(\Delta \log (gdp^j)\right)_{pre}\right) + \\
\left(\hat{\alpha}_{post}^2 - \hat{\alpha}_{pre}^2\right) \cdot \left(\text{Var}\left(\Delta \log (gdp^i)\right)_{post} - \text{Var}\left(\Delta \log (gdp^i)\right)_{pre}\right) + \ldots
\]

where the subscripts denote the pre and post-break years, and \(\hat{\alpha}_{pre}\) and \(\hat{\alpha}_{post}\) are the coefficient on country i GDP estimated on the two subsamples. Second, based on the decomposition in (3.15) we can compute the fraction of the reduction in \(\text{Var}(gap_{i,j})\) due to changes in each coefficient and in the variance of each shock. For instance, the share of the reduction in the variance of \(gap_{i,j}\) across the two periods that is due to changes in \(\alpha\) is given by

\[^{11}\]For instance, if a shock to Italian GDP had an asymmetric impact on the US and the UK, it would contribute to determine \(gap_{US,UK}\). Including rest of the world GDP as a regressor captures this effect, reducing the potential omitted-variable bias. Clearly \(\Delta \log gdp_{world}\) does not include country i and country j’s GDP.

\[^{12}\]Lags 1-3 or 3 only depending on the SDFs specification. I use Instrumental Variables for the following reasons. First, there could be other macro aggregates omitted in the regression that affect both the gap and the regressors. This problem is most likely to be relevant for a relative price, the nominal exchange rate, whose volatility is larger in the data and (partially) unexplained by fundamentals. Moreover, the estimation could be affected by a potential endogeneity bias arising from the nominal exchange rate: \(\Delta \log s_{US,j}^{i}\) is used to compute \(gap_{US,i}\), and might be correlated with the disturbance term \(\varepsilon_t\). Also a constant, not reported in the tables, was included in the regression.
3.4. RISK-SHARING AMONG THE US AND OECD COUNTRIES

\[
\text{share}(\alpha) = \frac{V(\alpha)}{\text{Var}(\text{gap}^{i,j})_{post} - \text{Var}(\text{gap}^{i,j})_{pre}}
\]  

(3.16)

where \( V(\alpha) \) is the reduction in \( \text{Var}(\text{gap}^{i,j}) \) explained by a fall in \( \alpha \) (see Appendix C for details on its computation). \( \text{share}(\alpha) \) can be interpreted as the share of the reduction in the contingent inefficiency due to better insurance against country \( i \) GDP risk.

Analogously we can compute the fraction of the change in \( \text{Var}(\text{gap}^{i,j}) \) due to a fall in the variance of each shock, that is in the extent of risk to be shared. For instance for country \( i \) GDP

\[
\text{share}(gdp^i) = \frac{V(gdp^i)}{\text{Var}(\text{gap}^{i,j})_{post} - \text{Var}(\text{gap}^{i,j})_{pre}}
\]

(3.17)

where \( V(gdp^i) \) is the reduction in the variance of the \( gap \) explained by a fall in the volatility of country \( i \) GDP across the two subsamples.

3.4 Risk-sharing among the US and OECD countries

I apply the methodology described above to estimate the bilateral risk-sharing inefficiency between the US and industrialized OECD countries, study its evolution over time, and investigate the causes of reductions.

Sample and data  The US is the reference country, the partner countries are 16 OECD industrialized economies: 4 extra-EU countries (Australia, Canada, Japan, and New Zealand), 8 Euro-Area countries (Belgium, Finland, France, Ireland, Italy, the Netherlands, Portugal, and Spain), and 4 European countries that did not join the Euro (Norway, Sweden, Switzerland, and the UK).\(^\text{13}\) The data are sampled at the quarterly frequency. Data on final consumption expenditure, consumption deflators, nominal exchange rates, government final consumption expenditure, and GDP, are from the OECD Economic Outlook. Data on population at the annual frequency were obtained from the OECD National Accounts. The period under consideration is 1970:2-2008:2.

3.4.1 SDFs from the traditional model: nominal exchange rate volatility as a source of risk

Deep parameters estimation  Assume CRRA utility function according to (3.8). Following Dedola and De Fiore (2005), I tested equation (3.10) simultaneously for all partner

\(^\text{13}\)Some OECD-members were not included in the sample because of data availability.
countries vis-a-vis the US through non-linear GMM. I used as instruments for each equation a constant, lags 1 and 2 of consumption factor growth of the US and partner country \(j\), US and country \(j\) inflation, and nominal depreciation of the US dollar.

Table 3.3 in Appendix D shows the results. Although in the specification of the SDFs we do not allow for taste shocks, the model seems to capture well the data, consistently with the results of previous GMM estimations of the same equation. The p-value of the \(J\) statistic indicates that the model cannot be rejected. The relative discount factors of the US and country \(j\), \(\frac{p^{r,s}}{p^r}\), cannot be rejected to be equal to 1 for all partner countries. The common coefficient of relative risk-aversion is estimated to be 1.32, well in the range of estimates derived in previous studies.

Although the risk-aversion coefficient is estimated very precisely, the estimation might not be immune from measurement errors in consumption and weak instrument issues. Since this could bias downwards the estimation, robustness checks on all results have been performed for higher values of this coefficient (up to \(\rho = 7\)) and are available upon request.

**Contingent risk-sharing inefficiency and its decrease over time** Given these results, for every partner country \(j\) in the sample I compute \(gap^{US,j}_t\) according to equation

---

14 I use continuously-updated GMM and employ the Newey-West adjustment for heteroskedasticity and autocorrelation. Given the interdependence between countries, it is more efficient to test the equations for all partners simultaneously. Testing each equation individually, though, did not lead to reject the model, consistently with the findings of Kollmann (1995).

The coefficient of risk-aversion is assumed to be the same for all countries. I found that, when assuming a different \(\rho\) for each partner, the model cannot be rejected, but some of the coefficients are estimated very imprecisely and are not significant. The coefficients that are significant are all between zero and 2. Therefore, since the main goals of the GMM estimation implemented here are (1) to establish whether the model can be rejected and (2) to estimate risk-aversion, this coefficient is assumed to be the same for all countries, which leads (again) not to reject the model and yields significant estimates for all the parameters. It should be noted that the common \(\rho\) estimated this way is in the range of the significant ones estimated assuming different risk-aversion coefficients for each country.

15 See, for instance, the results of Kollmann (1995) for several pairs of OECD economies, and Dedola and De Fiore (2005) for European countries vis-a-vis Germany. These results are robust to the inclusion of one additional instrument lag. I have chosen a reduced set of instruments in order to minimise the potential bias that could stem from the excess of overidentifying restrictions in small samples.

16 Stock and Wright (2000) show that when estimating risk-aversion inside a country (by using the traditional CAPM Euler Equation and national panel data), the estimated coefficient may be very low because of a weak instrument problem. It should be noticed, however, that cross-country estimations of the same equation tend to find low values for the the CRRA, but seem to be immune from the weak-instrument issue. Using non-linear GMM estimation and a cross-country version of the traditional CAPM Euler Equation, Dedola and De Fiore (2005) estimate a common CRRA of 0.39 for Euro countries. Selaive and Tuesta (2003) estimate the common CRRA of Australia and the US to be 1.58. By using the method of Stock and Wright (2000) to check for the presence of weak instruments, they establish that their result is not driven by this potential problem. Corsetti and Dedola (2006) also test the cross-country version of the traditional Euler equation through non-linear GMM and find it cannot be rejected for many countries vis-a-vis the US. They argue that testing the cross-country version of this equation may mitigate problems due to measurement errors and to the presence of potential peso problems common to the two countries. This might be the reason why the standard CAPM model is sometimes rejected in other studies that test its one-country version.

3.4. **RISK-SHARING AMONG THE US AND OECD COUNTRIES**

Exploiting the equality of the estimated discount factors for the US and each partner country \( \beta^{US} = \beta^j, \forall j \). Since the GMM estimation of equation (3.10) did not reject the equality of expected SDFs we can safely use the standard deviation (and the variance) of \( gap^{US,j} \) as a measure of the contingent risk-sharing inefficiency between the US and country \( j \).

Table 3.4 shows the standard deviation of \( gap^{US,j} \) for each partner country \( j \), using the \( \rho \) estimated from the data and other higher values. The estimated standard deviation of the gap between the US and Canada (in the first row) is statistically smaller than the standard deviation of the gap between the US and any other partner country.\(^{18}\) Results are robust to higher values of the risk-aversion coefficient. Over the period 1970-2008 the smallest contingent risk-sharing inefficiency has been the one between the US and Canada. Notice that this result could be due either to strong financial linkages between the two countries (a high degree of insurance via asset trade) or to a significant synchronization in their business cycles that could reduce the extent of purely idiosyncratic risk that the two countries should pool.

Figures 1 and 2 graph the gap between the US and, respectively, the UK and the Netherlands over the last forty years. The volatility of the gap between the US and these countries seems to decrease from the mid-Nineties. This intuition is confirmed by the results of the structural break analysis to detect significant changes in the variance of the gap, reported in Table 3.5. For every partner country, I show the number of breaks detected through the QML tests of Qu and Perron (2007), their dates, and the percentual reduction in the inefficiency across the pre- and post-break period. Asterisks next to the number of breaks indicate their significance according to the test of no change versus a number \( k \) of breaks.\(^{19}\)

\(^{18}\)I use a Wald test to establish whether this difference is statistically significant. The test statistic is

\[
W = \left[ r(b) - q \right] \left( R(b) \hat{V} R(b) \right)^{-1} \left[ r(b) - q \right] \sim \chi^2(1) \tag{3.18}
\]

where \( b \) is a vector of variances and covariances needed for the specific statistic, and \( r(b) \) is a function that maps these variances into the statistic of interest, in this case

\[
[\hat{\sigma} \left( gap^{US, CAN} \right) - \hat{\sigma} \left( gap^{US,j} \right)], j \neq CAN
\]

and \( R(b) = \frac{\partial r(b)}{\partial b} \). \( \hat{V} \) is the variance-covariance matrix of \( b \) and is estimated employing the Newey-West adjustment for autocorrelation and heteroskedasticity with a lag length of \( T^{1/4} \), where \( T \) is the sample size. Results are reported in Table 3.4 where asterisks next to the standard deviation of each \( gap^{US,j} \) indicate it is statistically different from \( \hat{\sigma} \left( gap^{US, CAN} \right) \). As usual, an asterisk indicates significance at the 10% level, two at the 5% level, and three at the 1% level.

\(^{19}\)See Qu and Perron (2007). Additional details on the estimation – including the estimated variance of errors before and after the breaks, and results for intermediate values of the risk-aversion coefficient – are available upon request. When computing the percentual reduction in the inefficiency, the pre-break period is assumed to start in 1978 in order to get subsamples of similar length.
The only breaks in the variance that are found to be statistically significant and robust to all the values $\rho$ are those estimated in 1992-1993 for several European countries, Italy, the Netherlands, Spain, Sweden, Switzerland, UK, Portugal, and Belgium.\footnote{A second break was detected for Italy, the Netherlands and Spain at the beginning of the Eighties, but it was found not to be robust to slightly higher values of the risk-aversion coefficient.} From the early Nineties the risk-sharing inefficiency between the US and these countries fell by 30 to 60%, depending on the partner. Figure 3 shows the rolling-window estimated standard deviation of the gap and its trend for some selected partners, illustrating this fall in the risk-sharing inefficiency.\footnote{The years indicated in the axes of the graphs correspond to the last year in each window. The series are smoothed by HP-filtering.}

**Macroeconomic risk vs degree of insurance** I estimate equation (3.14) for the pre and post-break periods (1978-1992 and 1993-2008) for the countries involved in the 1992-1993 break. Results are shown in Tables 3.6 and 3.7. Recall that each coefficient quantifies the percentual deviation from full risk-sharing associated with a 1% variation in the corresponding macro aggregate, and is therefore an inverse proxy for the degree of insurance against that particular source of risk. Asterisks next to the coefficients denote their significance computed using Newey-West adjusted standard errors with maximum lag length 10. (\textdagger) symbols next to 1993-2008 coefficients indicate that the coefficient changed significantly over time, and show the level of significance.\footnote{As in Dedola and De Fiore (2005), the null $\hat{a}_{\text{pre}} = \hat{a}_{\text{post}}$ was tested separately on the outcomes of the pre and post-break regressions. The post-break coefficient is statistically different from the pre-break one if and only if the null of equality is rejected by both tests.} The vast majority of coefficients are found to be significant, signaling that perfect insurance against the main sources of macro risk can be rejected in most of the cases. Only for some of the partner countries there has been a significant improvement in the degree of insurance against some sources of risk.

Tables 3.8 and 3.9 report the shares of the reduction in the variance of the gap between the pre and post-break period explained by falls in the variance-covariance of shocks and by changes in the coefficients. Shares were computed according to (3.16) and (3.17) using only statistically significant coefficients at the 10% level and variations in coefficients over time significant at the 10% level. Only for 3 countries out of 8 (the Netherlands, Spain, and the UK) more than 20% of the reduction in the welfare losses from imperfect risk-sharing can be attributed to improvements in the degree of insurance. For the other countries involved in the 1992-1993 break, most of the reduction in the risk-sharing inefficiency vis-a-vis the US was due to a decrease in nominal exchange rate volatility.\footnote{Also changes in the covariance of the regressors contribute to explain the reduction in the variance of the gap. I report them in the tables only when their effect is non-negligible.} Thus constant risk-aversion implies that nominal exchange rate volatility does not foster an efficient allocation of resources across
countries. It is rather an important source of macroeconomic risk whose decrease led to a significant reduction in insurance inefficiencies.

### 3.4.2 SDFs with time-varying risk-aversion: a higher degree of insurance and the effects of the Great Moderation

**Contingent risk-sharing inefficiency and its decrease over time** Assume the utility function with habits in (3.11). I adopt the parametrization of Campbell and Cochrane (1999). Table 3.10 shows the standard deviation of \( gap^{US,j} \) for each partner country \( j \). For each partner the mean of the \( gap \) could not be rejected to be zero in the data.

Table 3.12 shows the results of the structural break analysis on the variance of the \( gap \).

The risk-sharing inefficiency is found to decrease significantly over time for all partner countries but Ireland, Norway, and Portugal, for which significant increases were detected. The breaks seem to be clustered around two periods: last-Seventies/beginning-Eighties and end of the Nineties. Their magnitude varies between 20 and 60%. Figure 4 shows the rolling-window estimated standard deviation of the \( gap \) and its trend for some selected partners. The inefficiency between the US and, respectively, Switzerland and Belgium was subject to two reductions, at the beginning of the Eighties and at the end of the Nineties. The inefficiency relative to Italy and the Netherlands fell significantly only at the end of the Nineties.

**Macroeconomic risk vs degree of insurance** For the partner countries for which a reduction in the inefficiency was detected, I estimate (3.14) for the pre- and post-break period using as regressors also lagged values of the independent variables.\(^{24}\) Results are shown in Tables 3.13 – 3.16. Perfect insurance against the main sources of macro risk can be rejected in most of the cases. Half of the breaks are characterized by generalized falls in the coefficients, while the evidence is mixed for the remaining half.

Tables 3.17 and 3.18 report the shares of the reduction in the variance of the \( gap \) explained by reductions in the volatility of shocks and by changes in the coefficients. Half of the reductions in insurance inefficiencies (the UK from 1984, Switzerland from 1983 and 1999, the Netherlands from 1996, Finland from 1978, and Italy from 1998) were mostly due to a higher degree of insurance against macro shocks. Therefore time-varying risk-aversion

\(^{24}\)Using lagged values of the independent variables as regressors improves significantly the results. This is not surprising given that the risk-aversion coefficients used to compute SDFs and \( gap \) are history-dependent and thus embed the effects of past shocks. The inclusion of lagged regressors was not necessary in the case of constant risk-aversion.

In a few cases independent variables could explain only a small share of the variance of the \( gap \). These regressions are not reported.
suggests that financial markets integration was partially effective in raising the degree of insurance against macro risk.

The other half of the reductions were due mostly to a fall in the volatility of macro aggregates, mainly GDP of both the US and the partner countries, and its components. This is true for the breaks detected for Japan in 1978 and 1997, Canada in 1997, the UK in 1999, and Belgium in 1983. Notice that the beginning of a structural reduction in the volatility of US GDP (the beginning of the so-called Great Moderation) has been usually set at 1984 by the empirical literature on business-cycle fluctuations. This is consistent with the reduction in the risk-sharing inefficiency detected between the US and Belgium around 1983, which is found to be largely driven by falls in the volatility of US GDP and government spending. Summers (2005) finds that the Great Moderation began around the mid-Seventies in Japan, and documents another drop in the volatility of Japanese GDP around 1997-1998. Structural break analysis identifies a significant reduction in the variance of Canadian government spending in 1997 (results are available upon request). The timing of these episodes of reductions in macro volatility is consistent with the break-dates we estimate.

Thus time-varying risk-aversion implies that a reduction in the extent of risk to be shared internationally, requires a decrease in the volatility of macro fundamentals, as the one that occurred for several countries during the Great Moderation years.

3.5 Conclusions

This paper contributes to the literature on empirical risk-sharing measurement by proposing a theoretically-founded methodology to quantify international insurance, which is not invalidated by relative price movements, and by applying it to track the evolution of risk-sharing among industrialized countries. The main results have been outlined in the main text and will not be repeated here. This section will instead discuss some additional applications of this method and possible generalizations, left for future research.

The empirical analysis carried out assuming constant risk-aversion unveiled that the risk-sharing inefficiency between the US and some European economies fell from the early Nineties, mostly due to a decrease in nominal exchange rate volatility. The years 1992-1993, in which the break in the contingent inefficiency is detected, coincide with the signing of the Maastricht Treaty and with the end of the Exchange Rate Mechanism (ERM) crisis in Europe. Some of the partner countries for which the break was detected were directly involved in the ERM crisis (namely, Sweden, the UK, and Italy). All partners involved in the break (except Switzerland) signed the Maastricht Treaty, thus bounding themselves to take
part in the process of European integration and possibly to join the common currency. The end of currency turmoils combined with strong expectations of exchange rate stabilization might have been the factors driving the decrease in nominal exchange rate volatility between the US and European countries. Whether the same events reduced risk-sharing inefficiencies within European countries, is an interesting issue on which the method proposed in this Chapter could shed light.

Recent contributions to the open-macro literature (Kocherlakota and Pistaferri (2007) and (2008)) suggest that within-country lack of insurance may lead to empirical regularities that are customarily interpreted as cross-country risk-sharing inefficiencies (like the low correlation between relative cross-country consumption and real exchange rates). The method proposed in this Chapter could easily be extended to investigate this issue in the data. Relaxing the representative-agent assumption and generalizing the risk-sharing indicator adopted here would allow to investigate – using micro data from consumers’ surveys – both the within- and cross-country risk-sharing dimensions, focusing on their connections.
3.6. APPENDIX A: INSURANCE MEASURE AND CHANGES IN RELATIVE WEALTH

3.6 Appendix A: Insurance measure and changes in relative wealth

Using the model outlined in Chapter 1, I show that the gap reflects changes in countries’ relative wealth, for any asset structure we may assume in the model.

3.6.1 Changes in relative wealth under Financial autarky

First, let’s consider the Financial autarky case. Households have no means of smoothing consumption over time and in each period their consumption must equal the value of their income. H and F budget constraints read

\[ C_t P_t = P_{ht} Y_{ht} \quad \text{and} \quad C_t^* P_t^* = P_{ft} Y_{ft} \]

Agents’ intra-temporal decision can be solved through a standard expenditure minimization.\(^{25}\) The inter-temporal problem of H agents can be solved through a standard Lagrangian

\[ \max_{\{C_t, \lambda_t\}} : L = \frac{(C_t)^{1-\rho}}{1-\rho} + \lambda_t (P_{ht} Y_{ht} - P_t C_t), \forall t \]

where \(\lambda_t\) is the multiplier attached to H agents’ budget constraints, and represents the shadow value of current income, \(P_{ht} Y_t\). The first order condition with respect to consumption reads

\[ (C_t)^{-\rho} = \lambda_t P_t \]

(3.19)

Its analogue for the F agent is given by

\[ (C_t^*)^{-\rho} = \lambda_t^* P_t^* \]

(3.20)

where \(\lambda_t^*\) is the shadow value of F household’s current income.

Taking the ratio of (3.19) and (3.20), and dividing it by its analogue at time \((t-1)\) gives

\[ \left( \frac{C_t^*}{C_t} \right)^{-\rho} \left( \frac{C_{t-1}^*}{C_{t-1}} \right)^{\rho} = \frac{Q_t}{Q_{t-1}} \left( \frac{\lambda_t}{\lambda_t^*} \right) \left( \frac{\lambda_{t-1}^*}{\lambda_{t-1}} \right) \]

Taking logs

\[ \Delta \log (Q_t) - \rho (\Delta \log (C_t) - \Delta \log (C_t^*)) = gap_t = [\Delta \log (\lambda_t) - \Delta \log (\lambda_t^*)] \]

\(^{25}\)See for instance Obstfeld and Rogoff (1996).
3.6 APPENDIX A: INSURANCE MEASURE AND CHANGES IN RELATIVE WEALTH

where $\Delta$ denotes the first difference of a variable. The gap arising at time $t$ in response to country-specific shocks reflects asymmetric changes of the shadow value of income in the two countries.\textsuperscript{26}

3.6.2 Changes in relative wealth with trade in uncontingent bonds

Assume trade in uncontingent bonds. This amounts to modifying H household’s budget constraint to

$$P_t^B B_{t+1} = P_{ht} Y_{ht} - P_tC_t + P_t B_t$$

where $B_{t+1}$ denotes bonds purchased in period $t$, $P_t^B$ their unitary price in terms of the H consumption bundle. Bonds are assumed to be in zero net supply. H households’ inter-temporal problem is given by

$$\max_{\{C_t, B_{t+1}, \nu_t\}} : L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ (C_t)^{1-\rho} + \nu_t \left( P_{ht} Y_{ht} + P_t B_t - C_t P_t - P_t^B P_t B_{t+1} \right) \right] \right\}$$

$\nu_t$ represents the shadow value of income at time $t$. It can be shown that time $t$ income equals the present discount value of wealth

$$(P_{ht} Y_t + B_t P_t - P_tC_t) = \sum_{j=1}^{\infty} \prod_{i=0}^{j-1} P_{t+j}^B \left[ P_{t+j} C_{t+j} - P_{h,t+j} Y_{t+j} \right] \equiv PDV \text{ wealth}$$

Hence, the multiplier $\nu_t$ is the shadow value of wealth at time $t$.

H agent’s first order conditions are given by

$$(C_t)^{\rho} = \nu_t P_t$$

$$P_t^B = \beta E_t \left\{ \frac{\nu_{t+1} P_{t+1}}{\nu_t P_t} \right\}$$

From H agent’s first order condition with respect to consumption and from its analogue for the F consumer, we get

\textsuperscript{26}If H and F agents’ discount factors (the $\beta$’s) were different, the gap would reflect asymmetric wealth effects plus the (log) difference between the discount factors $\beta$ and $\beta^*$

$$\text{gap}_t = [\Delta \log (\chi_t) - \Delta \log (\chi^*_t)] + [\log (\beta) - \log (\beta^*)]$$

This shows that if agents are heterogeneous in the way they discount the future, for risk to be perfectly shared their relative wealth should change at the rate $[\log (\beta) - \log (\beta^*)]$. 

Viani, Francesca (2010), International Financial Markets, Cross-Border Transmission of Shocks and Risk-Sharing European University Institute

DOI: 10.2870/19601
The gap arising at time \( t \) corresponds to asymmetric changes in the shadow value of wealth in the two countries.\(^{27}\)

The proof above refers to a setup in which consumers can only smooth \textit{ex-post} their consumption. When agents can trade in contingent assets, gross portfolio holdings can be used to hedge \textit{ex-ante} against idiosyncratic risk: in response to any disturbance assets’ return differentials deliver automatic transfers of wealth into consumers’ budget constraints. As proved in Chapter 1, if households can form an optimal portfolio that hedges them \textit{ex-ante} – to some extent – against shocks, the gap still represents asymmetric changes in the shadow value of wealth in the two countries – the gap arising at time \( t \) is still described by equation (3.23). In this case, the asymmetric wealth effects mirrored in \([\Delta \log (\nu_t) - \Delta \log (\nu^*_t)]\) are residual after portfolio returns have delivered contingent transfers of income in households’ budget constraints.

3.7 Appendix B: Insurance measure and welfare

The first-best plan that delivers the maximum social welfare attainable in the workhorse model can be characterized as the outcome of a Social Planner maximization. The Social Planner must allocate consumption between the H and the F country, and that weights symmetrically the two economies. She solves the following problem

\[
\max : E_0 \left\{ \sum_{t=0}^{\infty} \Omega \beta^{t} \left( \frac{C_t}{1 - \rho} + (1 - \Omega) \beta^{t} \frac{C^*_t}{1 - \rho} \right) \right\} \tag{3.24}
\]

\[
s.t. : C_t = \left[ (\delta)^{1/\theta} (C_{ht})^{\frac{\theta - 1}{\theta}} + (1 - \delta)^{1/\theta} (C_{ft})^{\frac{\theta - 1}{\theta}} \right]^{\frac{\theta}{\theta - 1}} \tag{3.25}
\]

\[
C^*_t = \left[ (\delta)^{1/\theta} (C^*_{ht})^{\frac{\theta - 1}{\theta}} + (1 - \delta)^{1/\theta} (C^*_{ft})^{\frac{\theta - 1}{\theta}} \right]^{\frac{\theta}{\theta - 1}} \tag{3.26}
\]

\[
Y_t = C_{ht} + C^*_t \tag{3.27}
\]

\footnote{If \( \beta \neq \beta^* \), equation (3.23) would read
\[
gap_t = [\Delta \log (\nu_t) - \Delta \log (\nu^*_t)] + [\log (\beta) - \log (\beta^*)] \]
\[ Y_t^* = C_{ft} + C_{ft}^* \] (3.28)

In what follows \( \mu_t \) and \( \mu_t^* \) denote the multipliers attached to the CES bundle constraints, that is the shadow value of consumption, respectively, in the H and in the F country. In the decentralized problem whose solution corresponds to the Social Planner allocation, the ratio \( \left( \frac{\mu_t}{\mu_t^*} \right) \) corresponds to the real exchange rate \( Q_t \). The first order conditions with respect to \( C_t \) and \( C_t^* \) read

\[ \Omega \cdot (C_t)^{-\rho} = \mu_t \] (3.29)

\[ (1 - \Omega) \cdot (C_t^*)^{-\rho} = \mu_t^* \] (3.30)

The sequence of equations (3.29) and (3.30) evaluated at \( t = 0, \ldots, \infty \) characterize the allocation that constitutes the first-best plan that (by construction) delivers the maximum social welfare attainable in this economy, that is the sequence of allocations that maximize the Social Welfare function in (3.24). Equations (3.29) and (3.30) characterize the time \( t \) allocation that is part of this plan. The first-best allocations may coincide with the ones attained in a decentralized model with complete asset markets. In particular, in order for the Planner’s solution to correspond to a complete-markets decentralized allocation, the Planner’s weights \( \Omega \) and \( (1 - \Omega) \) should represent countries’ relative wealth determined by the initial conditions \( C_{-1}, C_{-1}^*, \mu_{-1}, \) and \( \mu_{-1}^* \), that is relative wealth at time \( t = -1 \).\(^{28}\) From (3.19), (3.20), and (3.21) we know that relative wealth at time \( t \) is pinned down by the following relationship

\[ \left( \frac{\omega_t}{\omega_t^*} \right) = \left( \frac{C_t}{C_t^*} \right)^{-\rho} \frac{P_t^*}{P_t} \] (3.31)

where \( \omega_t \) and \( \omega_t^* \) denote the shadow value of wealth of the H and the F consumer at time \( t \).\(^{29}\) Therefore, the ratio of the Social Planner’s weights must satisfy

\[ \frac{\Omega}{1 - \Omega} = \left( \frac{\omega_{-1}}{\omega_{-1}^*} \right) = \left( \frac{C_{-1}}{C_{-1}^*} \right)^{-\rho} \frac{P_{-1}^*}{P_{-1}} \]

Since only the ratio of the weights matters for the optimization, I assume without loss of generality

\(^{28}\)See Ljungqvist and Sargent (2004).

\(^{29}\)The shadow value of wealth in (3.31) may equal the shadow value of current income or the shadow value of the present discounted value of lifetime resources, depending on the asset structure assumed. Equation (3.31), however, holds for every asset structure - even in a setup with multiple assets.
Assume that countries’ relative wealth has always been constant from $t = -1$ until time $(t - 1)$ - i.e. assume the realized allocations have always coincided with the original first-best plan, that is conditions (3.29) and (3.30) have never been violated up to time $(t - 1)$. Then

$$
\Omega = \frac{(C_{-1})^{-\rho}}{P_{-1}} \quad \text{and} \quad (1 - \Omega) = \frac{(C_{*1})^{-\rho}}{P_{*1}}
$$

Substituting these expressions into the first order conditions (3.29) and (3.30) yields the benchmark allocation at time $t$ expressed as a function of the previous period allocation.

$$
\left( \frac{C_t}{C_{-1}} \right)^{-\rho} = \frac{\mu_t}{\mu_{-1}} \quad (3.32)
$$

$$
\left( \frac{C_t^*}{C_{*1}} \right)^{-\rho} = \frac{\mu_t^*}{\mu_{*1}} \quad (3.33)
$$

In economies in which asset markets are incomplete the Planner’s first order conditions (3.29) and (3.30) need not hold with equality. Assume that the incomplete market allocations have always coincided with the first-best plan from $t = 0$ up to time $(t - 1)$, but at time $t$ an idiosyncratic shock makes the incomplete market allocation deviate from the benchmark allocation. In this case

$$
\left( \frac{C_t^{IM}}{C_{t-1}^{IM}} \right)^{-\rho} \cdot \varphi_t = \frac{\mu_t^{IM}}{\mu_{t-1}^{IM}} \quad (3.34)
$$

$$
\left( \frac{C_t^{*IM}}{C_{t-1}^{*IM}} \right)^{-\rho} \cdot \varphi_t^* = \frac{\mu_t^{*IM}}{\mu_{t-1}^{*IM}} \quad (3.35)
$$

where $\varphi$ and $\varphi^*$ represent the wedge between marginal utility of consumption and CPI, and the IM superscript denotes the incomplete markets allocation. Taking logs of (3.34) and (3.35) gives

$$
\log (\varphi_t^*) = \log (\mu_t^{IM}) + \rho \log (C_t^{*IM}) - \log (\mu_{t-1}^{IM}) - \rho \log (C_{t-1}^{IM})
$$

$$
\log (\varphi_t) = \log (\mu_t^{IM}) + \rho \log (C_t^{IM}) - \log (\mu_{t-1}^{IM}) - \rho \log (C_{t-1}^{IM})
$$
Notice that if the shock is purely idiosyncratic, \( \log (\varphi_i) \) and \( \log (\varphi_i^*) \) must have opposite signs, that is if \( \log (\varphi_i) \) is positive, \( \log (\varphi_i^*) \) must be negative and vice versa. The incomplete markets gap coincides with the difference between \( \log (\varphi_i^*) \) and \( \log (\varphi_i) \)

\[
gap_t = \Delta \log (Q_i^{IM}) - \rho \left( \Delta \log (C_i^{IM}) - \Delta \log (C_i^{*IM}) \right) = \log (\varphi_i^*) - \log (\varphi_i)
\]

The opposite sign of \( \log (\varphi_i^*) \) and \( \log (\varphi_i) \) is sufficient to ensure that a higher gap (in absolute value) must be generated by an incomplete markets allocation that implies higher wedges between marginal utility of consumption and CPI, therefore a higher deviation from the Social Planner’s (logged) optimality conditions (3.29) and (3.30). Due to the concavity of the Social Welfare function in (3.24) as a sum of concave functions, a higher deviation from the Social Planner’s focs maps into a lower social welfare. Therefore, the higher the gap that arises in response to shocks in any incomplete markets setup, the higher the loss in social welfare caused by the disturbance with respect to the full risk-sharing Planner’s benchmark.

If countries’ relative wealth has changed between \( t = -1 \) and time \( (t - 1) \) the gap that opens up at time \( t \) reflects deviations from the second-best plan that would be implemented by a Social Planner who, given the previous deviation from the first-best plan, re-optimizes taking into account the new level of relative wealth.

Assume without loss of generality that only one deviation from the first-best plan has occurred, and that it took place at time \( (\tau - 1) < (t - 1) \). The second-best Planner’s problem reads

\[
\max : E_T \left\{ \sum_{j=0}^{\infty} \Omega_{\tau-1} \beta_j \left( \frac{C_{\tau+1}^*}{1 - \rho} \right)^{1 - \rho} + (1 - \Omega_{\tau-1}) \beta_j \left( \frac{C_{\tau+1}^*}{1 - \rho} \right)^{1 - \rho} \right\}
\]

subject to constraints (3.25)-(3.28) and to the initial conditions \( C_{\tau-1}, C_{\tau-1}^*, \mu_{\tau-1}, \mu_{\tau-1}^* \). The subscript \( (\tau - 1) \) attached to the weights signals that relative weights are chosen so as to reflect the new ranking of wealth determined by the time \( \tau \) shock. Following the same steps as above

\[
\Omega_{\tau-1} = \frac{(C_{\tau-1})^{-\rho}}{P_{\tau-1}} = \frac{(C_{\tau})^{-\rho}}{P_{\tau}} = \ldots = \frac{(C_{t-1})^{-\rho}}{P_{t-1}}
\]

\[
(1 - \Omega_{\tau-1}) = \frac{(C_{\tau-1}^*)^{-\rho}}{P_{\tau-1}^*} = \frac{(C_{\tau}^*)^{-\rho}}{P_{\tau}^*} = \ldots = \frac{(C_{t-1}^*)^{-\rho}}{P_{t-1}^*}
\]
These characterize the benchmark time $t$ allocation that delivers the maximum social welfare attainable in this economy given initial conditions $C_{t-1}$, $C^*_{t-1}$, $\mu_{t-1}$, $\mu^*_{t-1}$, i.e. the time $t$ component of the second-best sequence of allocations that maximizes the Social Welfare function in (3.36). Following the same steps as above it’s easy to show that the higher the gap arising in any incomplete markets setup at time $t$, the higher the deviation from $\text{focs}_{(3.32)}$ and (3.33), that is the higher the social welfare loss due to imperfect risk-sharing.

### 3.8 Appendix C: Decomposing changes in the variance of the gap

The reduction in the variance of $\text{gap}_{i;j}$ explained by a fall in the coefficient $\alpha$, associated with country $i$ GDP growth, is computed as

$$V(\alpha) = (\hat{\alpha}_{\text{post}} - \hat{\alpha}_{\text{pre}}) \text{Var} \left( \Delta \log gd{p}^i \right)_{\text{pre}} +$$

$$\frac{1}{2} (\hat{\alpha}_{\text{post}} - \hat{\alpha}_{\text{pre}}) \left( \text{Var} \left( \Delta \log gd{p}^i \right)_{\text{post}} - \text{Var} \left( \Delta \log gd{p}^i \right)_{\text{pre}} \right) +$$

$$\left( \hat{\alpha}_{\text{post}} - \hat{\alpha}_{\text{pre}} \right) \sum_{\lambda = \beta, \gamma, \delta, \eta, \zeta} 2 \hat{\lambda}^{\text{pre}} \text{Cov} \left( \Delta \log gd{p}^i, L^\lambda \right)_{\text{pre}} +$$

$$\left( \hat{\alpha}_{\text{post}} - \hat{\alpha}_{\text{pre}} \right) \sum_{\lambda = \beta, \gamma, \delta, \eta, \zeta} \hat{\lambda}^{\text{pre}} \left( \text{Cov} \left( \Delta \log gd{p}^i, L^\lambda \right)_{\text{post}} - \text{Cov} \left( \Delta \log gd{p}^i, L^\lambda \right)_{\text{pre}} \right) +$$

$$\left( \hat{\alpha}_{\text{post}} - \hat{\alpha}_{\text{pre}} \right) \sum_{\lambda = \beta, \gamma, \delta, \eta, \zeta} \left( \hat{\lambda}^{\text{post}} - \hat{\lambda}^{\text{pre}} \right) \text{Cov} \left( \Delta \log gd{p}^i, L^\lambda \right)_{\text{pre}} +$$

$$\left( \hat{\alpha}_{\text{post}} - \hat{\alpha}_{\text{pre}} \right) \sum_{\lambda = \beta, \gamma, \delta, \eta, \zeta} \left( \hat{\lambda}^{\text{post}} - \hat{\lambda}^{\text{pre}} \right) \frac{2}{3} \left( \text{Cov} \Delta \log \left( gd{p}^i \right), L^\lambda \text{post} - \text{Cov} \left( \Delta \log gd{p}^i, L^\lambda \right)_{\text{pre}} \right)$$

where $\lambda = \beta, \gamma, \delta, \eta, \zeta$ denote all other coefficients in equation (3.14) and $L^\lambda$ the source of shock with which each coefficient $\lambda$ is associated. The subscripts pre and post attached to second moments denote the subperiod (pre- or post-break) for which they are computed. The cross-product terms between the variation in the coefficient and the change in the variance are typically assumed to be negligible for small variations.
3.9 Appendix D: Figures and Tables

Figure 1 – Percentual deviations from full risk-sharing ($gap$) between the US and the UK.

Figure 2 – Percentual deviations from full risk-sharing ($gap$) between the US and the Netherlands.
Figure 3 – 10-years rolling-window estimates of the standard deviation of the gap for selected partner countries (CRRA preferences). The series are smoothed by HP-filtering.
3.9. APPENDIX D: FIGURES AND TABLES

Figure 4 – 10-years rolling-window estimates of the standard deviation of the gap for selected partner countries (CC preferences). The series are smoothed by HP-filtering.
Table 3.3: Results from GMM estimation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard error</th>
<th>P-value</th>
<th>J-stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-aversion coefficient</td>
<td>1.32</td>
<td>0.0676</td>
<td>6.25E-085</td>
<td>1.709</td>
</tr>
<tr>
<td>Rel discount factor AUS</td>
<td>1.0025</td>
<td>0</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor CAN</td>
<td>1.0004</td>
<td>0.0009</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor FRA</td>
<td>1.0043</td>
<td>0.0015</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor ITA</td>
<td>1.0047</td>
<td>0.0017</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor JAP</td>
<td>1.0064</td>
<td>0.0015</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor NDL</td>
<td>1.0068</td>
<td>0.0017</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor ESP</td>
<td>1.0053</td>
<td>0.0016</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor SWE</td>
<td>1.0048</td>
<td>0.0020</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor CH</td>
<td>1.0102</td>
<td>0.0015</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor UK</td>
<td>1.0026</td>
<td>0.0016</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor IRL</td>
<td>1.0024</td>
<td>0.0015</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor NOR</td>
<td>1.0021</td>
<td>0.0017</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor NZL</td>
<td>1.0058</td>
<td>0.0016</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor POR</td>
<td>1.0040</td>
<td>0.0017</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor BEL</td>
<td>1.0045</td>
<td>0.0016</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>Rel discount factor FIN</td>
<td>1.0032</td>
<td>0.0018</td>
<td>0.0000</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: Continuously-updated GMM. Newey-West adjustment for heteroskedasticity and autocorrelation.
### Table 3.4: Contingent risk-sharing inefficiency between the US and the partner countries (1970-2008) as measured by the std of the gap (CRRA preferences)

<table>
<thead>
<tr>
<th>Partner country</th>
<th>$\rho = 1.32$</th>
<th>$\rho = 4$</th>
<th>$\rho = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAN</td>
<td>2.5</td>
<td>4.39</td>
<td>5.21</td>
</tr>
<tr>
<td>AUS</td>
<td>4.18 ***</td>
<td>5.65 ***</td>
<td>6.39 **</td>
</tr>
<tr>
<td>FR</td>
<td>4.70 ***</td>
<td>5.84 ***</td>
<td>6.41 **</td>
</tr>
<tr>
<td>ITA</td>
<td>4.63 ***</td>
<td>5.99 ***</td>
<td>6.70 **</td>
</tr>
<tr>
<td>JAP</td>
<td>5.24 ***</td>
<td>7.05 ***</td>
<td>7.98 ***</td>
</tr>
<tr>
<td>NDL</td>
<td>5.05 ***</td>
<td>6.97 ***</td>
<td>7.89 ***</td>
</tr>
<tr>
<td>ES</td>
<td>4.89 ***</td>
<td>6.17 ***</td>
<td>6.80 ***</td>
</tr>
<tr>
<td>SWE</td>
<td>5.03 ***</td>
<td>7.81 ***</td>
<td>9.09 ***</td>
</tr>
<tr>
<td>CH</td>
<td>5.23 ***</td>
<td>6.05 ***</td>
<td>6.51 **</td>
</tr>
<tr>
<td>UK</td>
<td>4.77 ***</td>
<td>6.70 ***</td>
<td>7.64 ***</td>
</tr>
<tr>
<td>IRL</td>
<td>4.57 ***</td>
<td>6.07 ***</td>
<td>6.86 ***</td>
</tr>
<tr>
<td>NOR</td>
<td>4.53 ***</td>
<td>7.02 ***</td>
<td>8.22 ***</td>
</tr>
<tr>
<td>NZL</td>
<td>5.29 ***</td>
<td>9.05 ***</td>
<td>10.75 ***</td>
</tr>
<tr>
<td>POR</td>
<td>5.07 ***</td>
<td>7.65 ***</td>
<td>8.81 ***</td>
</tr>
<tr>
<td>BEL</td>
<td>4.87 ***</td>
<td>6.05 ***</td>
<td>6.66 ***</td>
</tr>
<tr>
<td>FIN</td>
<td>4.9 ***</td>
<td>7.30 ***</td>
<td>8.47 ***</td>
</tr>
</tbody>
</table>

**NOTE:** Asterisks next to each standard deviation denote the level of significance (* denotes the 10% level, ** the 5%, *** the 1%) of the Wald test for the null hypothesis $\sigma(\text{gap}^{US,CAN}) = \sigma(\text{gap}^{US,j})$. 

European University Institute
DOI: 10.2870/19601
Table 3.5: Reduction in the risk-sharing inefficiency between the US and the partner countries, detected as structural breaks in the variance of the gap (CRRA preferences)

<table>
<thead>
<tr>
<th>Partner country</th>
<th>No. breaks</th>
<th>Date</th>
<th>% reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITA</td>
<td>2 ***</td>
<td>1993</td>
<td>36.74%</td>
</tr>
<tr>
<td>NDL</td>
<td>2 ***</td>
<td>1993</td>
<td>35.15%</td>
</tr>
<tr>
<td>ES</td>
<td>2 ***</td>
<td>1993</td>
<td>35.15%</td>
</tr>
<tr>
<td>SWE</td>
<td>1 ***</td>
<td>1993</td>
<td>34.39%</td>
</tr>
<tr>
<td>CH</td>
<td>1 ***</td>
<td>1993</td>
<td>38.34%</td>
</tr>
<tr>
<td>UK</td>
<td>2 ***</td>
<td>1992</td>
<td>60.31%</td>
</tr>
<tr>
<td>POR</td>
<td>1 ***</td>
<td>1993</td>
<td>35.19%</td>
</tr>
<tr>
<td>BEL</td>
<td>1 *</td>
<td>1993</td>
<td>32.13%</td>
</tr>
<tr>
<td>AUS</td>
<td>0</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>CAN</td>
<td>0</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>FR</td>
<td>0</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>JAP</td>
<td>0</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>IRL</td>
<td>0</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>NOR</td>
<td>0</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>NZL</td>
<td>0</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>FIN</td>
<td>0</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

NOTE: Asterisks next to the number of breaks indicate their significance (* for 10%, ** for 5%, *** for 1%) according to the test of no change versus a number k of breaks. See Qu and Perron (2007) for details on this test.
### Table 3.6: IV regressions – dependent variable: gap(US,j), CRRA preferences

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UK</strong></td>
<td></td>
<td></td>
<td><strong>ITA</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US gdp</td>
<td>-0.97 ***</td>
<td>-0.63 *** (·)</td>
<td>US gdp</td>
<td>-1.06 ***</td>
<td>-0.52 *** (·)</td>
</tr>
<tr>
<td>UK gdp</td>
<td>1.02 ***</td>
<td>0.05 (· · ·)</td>
<td>ITA gdp</td>
<td>0.81 **</td>
<td>0.94 ***</td>
</tr>
<tr>
<td>US govt</td>
<td>-0.97 ***</td>
<td>-0.6 ***</td>
<td>US govt</td>
<td>-1.11 ***</td>
<td>-0.55 *** (· · ·)</td>
</tr>
<tr>
<td>UK govt</td>
<td>1.06 ***</td>
<td>-0.39 (· · ·)</td>
<td>ITA govt</td>
<td>0.61</td>
<td>0.33</td>
</tr>
<tr>
<td>NER</td>
<td>1.12 ***</td>
<td>0.99 *** (·)</td>
<td>NER</td>
<td>1.00 ***</td>
<td>1.04 ***</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>-0.17</td>
<td>-0.32</td>
<td>OECD gdp</td>
<td>0.11</td>
<td>-0.45 *** (· · ·)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>-0.75 ***</td>
<td>-1.19 *** (· · ·)</td>
<td>US gdp</td>
<td>-0.85 ***</td>
<td>-0.42 *** (· · ·)</td>
</tr>
<tr>
<td>NDL gdp</td>
<td>0.19 ***</td>
<td>0.49 *** (· · ·)</td>
<td>ESP gdp</td>
<td>0.36 ***</td>
<td>0.63 ***</td>
</tr>
<tr>
<td>US govt</td>
<td>-0.52 **</td>
<td>-0.92 *** (·)</td>
<td>US govt</td>
<td>-0.72 ***</td>
<td>-0.38 *** (· · ·)</td>
</tr>
<tr>
<td>NDL govt</td>
<td>-0.22</td>
<td>0.2</td>
<td>ESP govt</td>
<td>0.53 ***</td>
<td>0.73 ***</td>
</tr>
<tr>
<td>NER</td>
<td>1.11 ***</td>
<td>0.87 *** (· · ·)</td>
<td>NER</td>
<td>1.04 ***</td>
<td>0.98 *** (·)</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>0.03</td>
<td>-0.14</td>
<td>OECD gdp</td>
<td>0.03</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

**NOTE:** * next to the coefficients denotes their significance computed using Newey-West adjusted standard errors with maximum lag length 10. (· next to 1993-2008 coefficients indicates that the coefficient changed significantly with respect to its 1978-1992 value, and shows the level of significance.*
### Table 3.7: IV regressions – dependent variable: gap(US,j), CRRA preferences

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>-0.97 ***</td>
<td>-0.49 **</td>
<td>(·)</td>
<td>US gdp</td>
<td>-0.74 ***</td>
</tr>
<tr>
<td>SWE gdp</td>
<td>0.34 ***</td>
<td>0.34</td>
<td>CH gdp</td>
<td>0.29 *</td>
<td>0.3 ***</td>
</tr>
<tr>
<td>US govt</td>
<td>-0.69 **</td>
<td>-0.56 **</td>
<td>(·)</td>
<td>US govt</td>
<td>-0.78 ***</td>
</tr>
<tr>
<td>SWE govt</td>
<td>0.07</td>
<td>0.53 *</td>
<td>CH govt</td>
<td>0.65 ***</td>
<td>0.32 ***</td>
</tr>
<tr>
<td>NER</td>
<td>1.01 ***</td>
<td>0.99 ***</td>
<td>NER</td>
<td>1.01 ***</td>
<td>0.98 ***</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>0.64 **</td>
<td>-0.12 (·)</td>
<td>OECD gdp</td>
<td>-0.2 *</td>
<td>-0.31 ***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>-0.63 ***</td>
<td>-0.42 **</td>
<td>US gdp</td>
<td>-0.9 ***</td>
<td>-0.66 (·)</td>
</tr>
<tr>
<td>POR gdp</td>
<td>-0.2</td>
<td>0.23</td>
<td>BEL gdp</td>
<td>0.6 **</td>
<td>1.13 (·)</td>
</tr>
<tr>
<td>US govt</td>
<td>-0.66 ***</td>
<td>-0.7 ***</td>
<td>US govt</td>
<td>-0.91 ***</td>
<td>-0.79 ***</td>
</tr>
<tr>
<td>POR govt</td>
<td>0.16</td>
<td>0.32</td>
<td>BEL govt</td>
<td>0.97 **</td>
<td>1.1 **</td>
</tr>
<tr>
<td>NER</td>
<td>1.03 ***</td>
<td>0.94 ***</td>
<td>NER</td>
<td>0.95 ***</td>
<td>1.02 (·)</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>-0.02</td>
<td>-0.13</td>
<td>OECD gdp</td>
<td>0.02</td>
<td>-0.29 (·)</td>
</tr>
</tbody>
</table>

**NOTE:** See note to Table 3.6
Table 3.8: Share of the risk-sharing inefficiency between the US and the partner country explained by falls in the variance of shocks and improvements in the degree of insurance (CRRA preferences)

<table>
<thead>
<tr>
<th>UK</th>
<th>ITA</th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>US gdp</td>
</tr>
<tr>
<td>UK gdp</td>
<td>ITA gdp</td>
</tr>
<tr>
<td>US govt</td>
<td>US govt</td>
</tr>
<tr>
<td>UK govt</td>
<td>ITA govt</td>
</tr>
<tr>
<td>NER</td>
<td>NER</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>OECD gdp</td>
</tr>
<tr>
<td><strong>Insurance</strong></td>
<td><strong>Insurance</strong></td>
</tr>
<tr>
<td>Var(US gdp)</td>
<td>Var(US gdp)</td>
</tr>
<tr>
<td>Var(UK gdp)</td>
<td>Var(ITA gdp)</td>
</tr>
<tr>
<td>Var(UK gov)</td>
<td>Var(UK gov)</td>
</tr>
<tr>
<td>Var(NER)</td>
<td>Var(NER)</td>
</tr>
<tr>
<td>Var(OECD gdp)</td>
<td>Var(OECD gdp)</td>
</tr>
<tr>
<td>Cov(US gdp,NER)</td>
<td>Cov(UK gdp,NER)</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td><strong>Shocks</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NDL</th>
<th>ESP</th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>US gdp</td>
</tr>
<tr>
<td>NDL gdp</td>
<td>ESP gdp</td>
</tr>
<tr>
<td>US govt</td>
<td>US govt</td>
</tr>
<tr>
<td>NDL gov</td>
<td>ESP govt</td>
</tr>
<tr>
<td>NER</td>
<td>NER</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>OECD gdp</td>
</tr>
<tr>
<td><strong>Insurance</strong></td>
<td><strong>Insurance</strong></td>
</tr>
<tr>
<td>Var(US gdp)</td>
<td>Var(US gdp)</td>
</tr>
<tr>
<td>Var(NDL gdp)</td>
<td>Var(ESP gdp)</td>
</tr>
<tr>
<td>Var(UK gdp)</td>
<td>Var(UK gdp)</td>
</tr>
<tr>
<td>Var(NDL gov)</td>
<td>Var(ESP gov)</td>
</tr>
<tr>
<td>Var(NER)</td>
<td>Var(NER)</td>
</tr>
<tr>
<td>Var(OECD gdp)</td>
<td>Var(OECD gdp)</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td><strong>Shocks</strong></td>
</tr>
</tbody>
</table>

**NOTE:** Shares are computed using statistically significant coefficients at the 10% level.
Table 3.9: Share of the risk-sharing inefficiency between the US and the partner country explained by falls in the variance of shocks and improvements in the degree of insurance (CRRA preferences)

<table>
<thead>
<tr>
<th></th>
<th>SWE</th>
<th>CH</th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>3.54</td>
<td>US gdp</td>
</tr>
<tr>
<td>SWE gdp</td>
<td>0</td>
<td>CH gdp</td>
</tr>
<tr>
<td>US govt</td>
<td>0</td>
<td>US govt</td>
</tr>
<tr>
<td>SWE govt</td>
<td>0</td>
<td>CH govt</td>
</tr>
<tr>
<td>NER</td>
<td>0</td>
<td>NER</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>1,21</td>
<td>OECD gdp</td>
</tr>
<tr>
<td><strong>Insurance</strong></td>
<td><strong>4.75%</strong></td>
<td><strong>Insurance</strong></td>
</tr>
<tr>
<td>Var(US gdp)</td>
<td>5.48</td>
<td>Var(US gdp)</td>
</tr>
<tr>
<td>Var(SWE gdp)</td>
<td>0.88</td>
<td>Var(CH gdp)</td>
</tr>
<tr>
<td>Var(US govt)</td>
<td>0.36</td>
<td>Var(US govt)</td>
</tr>
<tr>
<td>Var(SWE govt)</td>
<td>0</td>
<td>Var(CH govt)</td>
</tr>
<tr>
<td>Var(S)</td>
<td>84.33</td>
<td>Var(S)</td>
</tr>
<tr>
<td>Var(OECD gdp)</td>
<td>0.34</td>
<td>Var(OECD gdp)</td>
</tr>
<tr>
<td>Cov(SWE gdp, NER)</td>
<td>7.64</td>
<td>Cov(SWE gdp, NER)</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td><strong>99.04%</strong></td>
<td><strong>Shocks</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>POR</th>
<th>BEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>0</td>
<td>US gdp</td>
</tr>
<tr>
<td>POR gdp</td>
<td>0</td>
<td>BEL gdp</td>
</tr>
<tr>
<td>US govt</td>
<td>0</td>
<td>US govt</td>
</tr>
<tr>
<td>POR govt</td>
<td>0</td>
<td>BEL govt</td>
</tr>
<tr>
<td>NER</td>
<td>0</td>
<td>NER</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>0</td>
<td>OECD gdp</td>
</tr>
<tr>
<td><strong>Insurance</strong></td>
<td><strong>0%</strong></td>
<td><strong>Insurance</strong></td>
</tr>
<tr>
<td>Var(US gdp)</td>
<td>2.64</td>
<td>Var(US gdp)</td>
</tr>
<tr>
<td>Var(POR gdp)</td>
<td>0</td>
<td>Var(BEL gdp)</td>
</tr>
<tr>
<td>Var(US govt)</td>
<td>0.38</td>
<td>Var(US govt)</td>
</tr>
<tr>
<td>Var(POR govt)</td>
<td>0</td>
<td>Var(BEL govt)</td>
</tr>
<tr>
<td>Var(NER)</td>
<td>95.17</td>
<td>Var(NER)</td>
</tr>
<tr>
<td>Var(OECD gdp)</td>
<td>0</td>
<td>Var(OECD gdp)</td>
</tr>
<tr>
<td>Cov(BEL gdp, NER)</td>
<td></td>
<td>Cov(BEL gdp, NER)</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td><strong>98.18%</strong></td>
<td><strong>Shocks</strong></td>
</tr>
</tbody>
</table>

NOTE: See note to Table 3.8
Table 3.10: Contingent risk-sharing inefficiency between the US and the partner countries (1970-2008) as measured by the std of the gap (CC preferences)

<table>
<thead>
<tr>
<th>Partner country</th>
<th>CC preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAN</td>
<td>23,2</td>
</tr>
<tr>
<td>AUS</td>
<td>25,32</td>
</tr>
<tr>
<td>FR</td>
<td>21,23</td>
</tr>
<tr>
<td>ITA</td>
<td>19,99</td>
</tr>
<tr>
<td>JAP</td>
<td>30,25</td>
</tr>
<tr>
<td>NDL</td>
<td>32,04</td>
</tr>
<tr>
<td>ES</td>
<td>18,26</td>
</tr>
<tr>
<td>SWE</td>
<td>41</td>
</tr>
<tr>
<td>CH</td>
<td>18,65</td>
</tr>
<tr>
<td>UK</td>
<td>27,61</td>
</tr>
<tr>
<td>IRL</td>
<td>23,61</td>
</tr>
<tr>
<td>NOR</td>
<td>37,04</td>
</tr>
<tr>
<td>NZL</td>
<td>41</td>
</tr>
<tr>
<td>POR</td>
<td>23,29</td>
</tr>
<tr>
<td>BEL</td>
<td>19,16</td>
</tr>
<tr>
<td>FIN</td>
<td>36,86</td>
</tr>
</tbody>
</table>
Table 3.11: Gap mean – Campbell and Cochrane preferences

<table>
<thead>
<tr>
<th>Partner country</th>
<th>Mean of $g^{(US,j)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>0.08</td>
</tr>
<tr>
<td>CAN</td>
<td>-0.09</td>
</tr>
<tr>
<td>FRA</td>
<td>0.55</td>
</tr>
<tr>
<td>ITA</td>
<td>0.48</td>
</tr>
<tr>
<td>JAP</td>
<td>0.12</td>
</tr>
<tr>
<td>NDL</td>
<td>-0.01</td>
</tr>
<tr>
<td>ES</td>
<td>0.36</td>
</tr>
<tr>
<td>SWE</td>
<td>-0.68</td>
</tr>
<tr>
<td>CH</td>
<td>0.71</td>
</tr>
<tr>
<td>UK</td>
<td>0.25</td>
</tr>
<tr>
<td>IRL</td>
<td>0.63</td>
</tr>
<tr>
<td>NOR</td>
<td>0.21</td>
</tr>
<tr>
<td>NZL</td>
<td>-0.99</td>
</tr>
<tr>
<td>POR</td>
<td>0.37</td>
</tr>
<tr>
<td>BEL</td>
<td>0.96</td>
</tr>
<tr>
<td>FIN</td>
<td>0.04</td>
</tr>
</tbody>
</table>

NOTE: For each partner and CRRA coefficient, the null hypothesis $E(g^{(US,j)})=0$ was tested using Newey-West adjusted standard errors with a maximum autocorrelation lag of 10. In no case could the null be rejected.
Table 3.12: Reductions in the risk-sharing inefficiency between the US and the partner countries, detected as structural breaks in the variance of the gap (CC preferences)

<table>
<thead>
<tr>
<th>Partner</th>
<th>No. breaks</th>
<th>Date and % reduction</th>
<th>Date and % reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>1 ***</td>
<td>1987 47%</td>
<td>/</td>
</tr>
<tr>
<td>CAN</td>
<td>2 ***</td>
<td>1977 43%</td>
<td>1997 44%</td>
</tr>
<tr>
<td>FRA</td>
<td>2 ***</td>
<td>1978 29%</td>
<td>1997 45%</td>
</tr>
<tr>
<td>ITA</td>
<td>1 ***</td>
<td>1998 51%</td>
<td>/</td>
</tr>
<tr>
<td>JAP</td>
<td>2 ***</td>
<td>1978 40%</td>
<td>1997 18%</td>
</tr>
<tr>
<td>NDL</td>
<td>1 ***</td>
<td>1996 30%</td>
<td>/</td>
</tr>
<tr>
<td>ESP</td>
<td>1 ***</td>
<td>1983 28%</td>
<td>/</td>
</tr>
<tr>
<td>SWE</td>
<td>2 ***</td>
<td>1978 55%</td>
<td>1998 44%</td>
</tr>
<tr>
<td>CH</td>
<td>2 ***</td>
<td>1983 29%</td>
<td>1999 33%</td>
</tr>
<tr>
<td>UK</td>
<td>2 ***</td>
<td>1984 39%</td>
<td>1999 43%</td>
</tr>
<tr>
<td>IRL</td>
<td>2 ***</td>
<td>1983 44%</td>
<td>2000 -105%</td>
</tr>
<tr>
<td>NOR</td>
<td>2 ***</td>
<td>1977 -111%</td>
<td>1999 47%</td>
</tr>
<tr>
<td>NZL</td>
<td>2 ***</td>
<td>1987 51%</td>
<td>1997 62%</td>
</tr>
<tr>
<td>POR</td>
<td>2 ***</td>
<td>1983 27%</td>
<td>1991 -96%</td>
</tr>
<tr>
<td>BEL</td>
<td>2 ***</td>
<td>1983 34%</td>
<td>1999 34%</td>
</tr>
<tr>
<td>FIN</td>
<td>1 ***</td>
<td>1978 61%</td>
<td>/</td>
</tr>
</tbody>
</table>

NOTE: See note to Table 3.5
Table 3.13: IV regressions – dependent variable: gap(US,j), CC preferences

<table>
<thead>
<tr>
<th></th>
<th>Break 1</th>
<th>Break 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAN</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US gdp</td>
<td>-13.6 ***</td>
<td>-5.81 ** (...</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-)</td>
</tr>
<tr>
<td>L1</td>
<td>21.4 ***</td>
<td>3.35 ** (...</td>
</tr>
<tr>
<td>L2</td>
<td>-8.14 **</td>
<td>-2.53 ** (...</td>
</tr>
<tr>
<td>CAN gdp</td>
<td>6.91 **</td>
<td>24.8 *** (...</td>
</tr>
<tr>
<td></td>
<td>(-)</td>
<td>(-)</td>
</tr>
<tr>
<td>L1</td>
<td>-13.03 **</td>
<td>-32.27 *** (...</td>
</tr>
<tr>
<td>L2</td>
<td>-6.84 ***</td>
<td>25.32 *** (...</td>
</tr>
<tr>
<td>US govt</td>
<td>-0.05</td>
<td>-2.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JAP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US gdp</td>
<td>-6.5 **</td>
<td>-5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-)</td>
</tr>
<tr>
<td>L1</td>
<td>6.5 **</td>
<td>12.6 ** (...</td>
</tr>
<tr>
<td>L2</td>
<td>-1.2</td>
<td>3.1 ** (...</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-)</td>
</tr>
<tr>
<td>JAP gdp</td>
<td>10.6 ***</td>
<td>25.6 *** (...</td>
</tr>
<tr>
<td></td>
<td>(-)</td>
<td>(-)</td>
</tr>
<tr>
<td>L1</td>
<td>-11.3 ***</td>
<td>-14 ***</td>
</tr>
<tr>
<td>L2</td>
<td>1.6</td>
<td>-4.2</td>
</tr>
<tr>
<td>US govt</td>
<td>-10.9 **</td>
<td>-0.5 (.)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-)</td>
</tr>
<tr>
<td>L1</td>
<td>10.6</td>
<td>17.9 *** (...</td>
</tr>
<tr>
<td>L2</td>
<td>-11.6 *</td>
<td>13.7 ** (...</td>
</tr>
<tr>
<td>JAP govt</td>
<td>17.1 ***</td>
<td>10 *** (...)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-)</td>
</tr>
<tr>
<td>L1</td>
<td>-18.3 ***</td>
<td>-21 ***</td>
</tr>
<tr>
<td>L2</td>
<td>3.8 ***</td>
<td>-7.6</td>
</tr>
<tr>
<td>NER</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>6.3</td>
<td>-9.9</td>
</tr>
</tbody>
</table>

NOTE: See note to Table 3.6. L1 and L2 denote first and second lags

European University Institute
DOI: 10.2870/19601
Table 3.14: IV regressions – dependent variable: gap(US,j), CC preferences

<table>
<thead>
<tr>
<th>CH</th>
<th>Break 1</th>
<th>Break 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>-4.6 *</td>
<td>-11.9 *** (⋯)</td>
</tr>
<tr>
<td>L1</td>
<td>13.5 ***</td>
<td>12 ***</td>
</tr>
<tr>
<td>L2</td>
<td>0.6</td>
<td>-3.5</td>
</tr>
<tr>
<td>CH gdp</td>
<td>-19.6 **</td>
<td>0.4 (⋯)</td>
</tr>
<tr>
<td>L1</td>
<td>25.6 ***</td>
<td>4.6 *** (⋯)</td>
</tr>
<tr>
<td>L2</td>
<td>-9.1</td>
<td>-2.6</td>
</tr>
<tr>
<td>US govt</td>
<td>-11.3 ***</td>
<td>10.7 *** (⋯)</td>
</tr>
<tr>
<td>L1</td>
<td>11.7 **</td>
<td>8 ***</td>
</tr>
<tr>
<td>L2</td>
<td>8.6</td>
<td>2</td>
</tr>
<tr>
<td>CH govt</td>
<td>-9.6</td>
<td>1.7</td>
</tr>
<tr>
<td>L1</td>
<td>0.1</td>
<td>1.5 * (⋯)</td>
</tr>
<tr>
<td>L2</td>
<td>9.4 *</td>
<td>-1.6 (⋯)</td>
</tr>
<tr>
<td>NER</td>
<td>-1.9</td>
<td>0.5 ** (⋯)</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>-1.6</td>
<td>1.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UK</th>
<th>Break 1</th>
<th>Break 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>-8.6 ***</td>
<td>-11 ***</td>
</tr>
<tr>
<td>L1</td>
<td>11.3 ***</td>
<td>17 *** (⋯)</td>
</tr>
<tr>
<td>L2</td>
<td>-3.5</td>
<td>-8.2 *** (⋯)</td>
</tr>
<tr>
<td>UK gdp</td>
<td>9.1 ***</td>
<td>1.9 (⋯)</td>
</tr>
<tr>
<td>L1</td>
<td>-11 ***</td>
<td>-0.3 (⋯)</td>
</tr>
<tr>
<td>L2</td>
<td>-0.1</td>
<td>-4.4</td>
</tr>
<tr>
<td>US govt</td>
<td>-13.5 *</td>
<td>-9.3 ***</td>
</tr>
<tr>
<td>L1</td>
<td>3.3</td>
<td>9.5 *** (⋯)</td>
</tr>
<tr>
<td>L2</td>
<td>-14.5 **</td>
<td>-2.1 (⋯)</td>
</tr>
<tr>
<td>UK govt</td>
<td>7.9 **</td>
<td>-0.03 (⋯)</td>
</tr>
<tr>
<td>L1</td>
<td>-11.2 **</td>
<td>-1.2 (⋯)</td>
</tr>
<tr>
<td>L2</td>
<td>0.3</td>
<td>-7.7</td>
</tr>
<tr>
<td>NER</td>
<td>1.8 ***</td>
<td>1</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>1.2</td>
<td>2.2</td>
</tr>
</tbody>
</table>

NOTE: See note to Table 3.13
Table 3.15: IV regressions – dependent variable: gap(US,j), CC preferences

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FRA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US gdp</td>
<td>-17,55 ***</td>
<td>-11,1 ***</td>
<td>1.3</td>
<td>-10.7 *** (\ldots)</td>
</tr>
<tr>
<td>L1</td>
<td>13,4 ***</td>
<td>9,9 *** (\ldots)</td>
<td>4.4</td>
<td>11 *** (\ldots)</td>
</tr>
<tr>
<td>L2</td>
<td>-1,9</td>
<td>1,4</td>
<td>-7,7 ***</td>
<td>6,7 **</td>
</tr>
<tr>
<td>FRA gdp</td>
<td>-15,5 **</td>
<td>23,5 ***</td>
<td>NDL gdp</td>
<td>7,1 *</td>
</tr>
<tr>
<td>L1</td>
<td>-2,1</td>
<td>14,7 *** (\ldots)</td>
<td>L1</td>
<td>-9,5 ***</td>
</tr>
<tr>
<td>L2</td>
<td>-5,3 ***</td>
<td>-9,8</td>
<td>L2</td>
<td>2,5</td>
</tr>
<tr>
<td>US govt</td>
<td>-23,5 ***</td>
<td>-14,3 ***</td>
<td>US govt</td>
<td>3,3</td>
</tr>
<tr>
<td>L1</td>
<td>15,6 ***</td>
<td>3,5 (\ldots)</td>
<td>L1</td>
<td>-5,6</td>
</tr>
<tr>
<td>L2</td>
<td>-6,4</td>
<td>10,1 ***</td>
<td>L2</td>
<td>0,2</td>
</tr>
<tr>
<td>FRA govt</td>
<td>-33,3 ***</td>
<td>17 (\ldots)</td>
<td>NDL govt</td>
<td>0,7</td>
</tr>
<tr>
<td>L1</td>
<td>-9,8 **</td>
<td>-13,8 (\ldots)</td>
<td>L1</td>
<td>-14,6 **</td>
</tr>
<tr>
<td>L2</td>
<td>17,2 ***</td>
<td>-15,4 *</td>
<td>L2</td>
<td>-8,9</td>
</tr>
<tr>
<td>NER</td>
<td>-1,05</td>
<td>2,3 ***</td>
<td>NER</td>
<td>4,2 **</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>13,9</td>
<td>-10 ***</td>
<td>OECD gdp</td>
<td>2,4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>-18,8 ***</td>
<td>-10,8 ***</td>
<td>US gdp</td>
<td>-17,8 **</td>
<td>-16,9 ***</td>
</tr>
<tr>
<td>L1</td>
<td>10,5 **</td>
<td>1,05 (\ldots)</td>
<td>L1</td>
<td>25,3 ***</td>
<td>9,9 *** (\ldots)</td>
</tr>
<tr>
<td>L2</td>
<td>8,1 *</td>
<td>4,8</td>
<td>L2</td>
<td>29,5 ***</td>
<td>1,2 (\ldots)</td>
</tr>
<tr>
<td>ITA gdp</td>
<td>17,4</td>
<td>9,5 ** (\ldots)</td>
<td>FIN gdp</td>
<td>10,8 ***</td>
<td>-12,4 ***</td>
</tr>
<tr>
<td>L1</td>
<td>-9,2</td>
<td>-1,8</td>
<td>L1</td>
<td>-26,9 ***</td>
<td>-10,7 *** (\ldots)</td>
</tr>
<tr>
<td>L2</td>
<td>3,4</td>
<td>-5,9 *** (\ldots)</td>
<td>L2</td>
<td>2,8</td>
<td>6,2 *** (\ldots)</td>
</tr>
<tr>
<td>US govt</td>
<td>-15,5 ***</td>
<td>-7,6 ***</td>
<td>US govt</td>
<td>124,5 ***</td>
<td>-10,5 ** (\ldots)</td>
</tr>
<tr>
<td>L1</td>
<td>-2,4</td>
<td>1,7</td>
<td>L1</td>
<td>50,8 ***</td>
<td>13,13 *** (\ldots)</td>
</tr>
<tr>
<td>L2</td>
<td>5,7</td>
<td>-2,4</td>
<td>L2</td>
<td>34,9 ***</td>
<td>13,92 ***</td>
</tr>
<tr>
<td>ITA govt</td>
<td>34,7 ***</td>
<td>1,7 (\ldots)</td>
<td>FIN govt</td>
<td>-22,2 ***</td>
<td>4,8 (\ldots)</td>
</tr>
<tr>
<td>L1</td>
<td>-2,6</td>
<td>7,2 ** (\ldots)</td>
<td>L1</td>
<td>-25,8 ***</td>
<td>-6 (\ldots)</td>
</tr>
<tr>
<td>L2</td>
<td>-12,5 *</td>
<td>-7,7 **</td>
<td>L2</td>
<td>71,4 ***</td>
<td>-28,6 *</td>
</tr>
<tr>
<td>NER</td>
<td>-2,4 ***</td>
<td>2,5 ***</td>
<td>NER</td>
<td>11,7</td>
<td>0,5</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>6,5</td>
<td>-2,5</td>
<td>OECD gdp</td>
<td>29,9 ***</td>
<td>41,1 ***</td>
</tr>
</tbody>
</table>

NOTE: See note to Table 3.13
Table 3.16: IV regressions – dependent variable: gap(US,j), CC preferences

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NZL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US gdp</td>
<td>-5,2</td>
<td>-8,5 ***</td>
</tr>
<tr>
<td>L1</td>
<td>27,2 ***</td>
<td>9,5 ***</td>
</tr>
<tr>
<td>L2</td>
<td>-17,7 **</td>
<td>-10 ***</td>
</tr>
<tr>
<td>NZL gdp</td>
<td>8,7 ***</td>
<td>3,4 (    )</td>
</tr>
<tr>
<td>L1</td>
<td>-10,8 ***</td>
<td>-5,7 ***</td>
</tr>
<tr>
<td>L2</td>
<td>1,4</td>
<td>0,3</td>
</tr>
<tr>
<td>US govt</td>
<td>6,9</td>
<td>-8,2 ***</td>
</tr>
<tr>
<td>L1</td>
<td>14,5 **</td>
<td>6,4 **</td>
</tr>
<tr>
<td>L2</td>
<td>-5</td>
<td>9,2 ***</td>
</tr>
<tr>
<td>NZL govt</td>
<td>4,1 **</td>
<td>3,3 (    )</td>
</tr>
<tr>
<td>L1</td>
<td>-12 ***</td>
<td>-7,7 ***</td>
</tr>
<tr>
<td>L2</td>
<td>4,7 **</td>
<td>1,3 (    )</td>
</tr>
<tr>
<td>NER</td>
<td>5,2 ***</td>
<td>0,9 *</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>-13,8 ***</td>
<td>1,3 (    )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BEL</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US gdp</td>
<td>-9,2 ***</td>
<td>-7,6 ***</td>
</tr>
<tr>
<td>L1</td>
<td>9,08</td>
<td>8,2 ***</td>
</tr>
<tr>
<td>L2</td>
<td>5,8</td>
<td>0,4</td>
</tr>
<tr>
<td>BEL gdp</td>
<td>0,4</td>
<td>9,7 *</td>
</tr>
<tr>
<td>L1</td>
<td>11 *</td>
<td>-2,7 (    )</td>
</tr>
<tr>
<td>L2</td>
<td>-13 ***</td>
<td>-0,8 (    )</td>
</tr>
<tr>
<td>US govt</td>
<td>-12,3 ***</td>
<td>-10,2 ***</td>
</tr>
<tr>
<td>L1</td>
<td>6,2</td>
<td>4,8 **</td>
</tr>
<tr>
<td>L2</td>
<td>9,7 **</td>
<td>3,3 **</td>
</tr>
<tr>
<td>BEL govt</td>
<td>0,1</td>
<td>5,7 ***</td>
</tr>
<tr>
<td>L1</td>
<td>30</td>
<td>0,2</td>
</tr>
<tr>
<td>L2</td>
<td>-27,3</td>
<td>-0,7</td>
</tr>
<tr>
<td>NER</td>
<td>0,5</td>
<td>1,04 ***</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>2,5</td>
<td>-8,8 ***</td>
</tr>
</tbody>
</table>

NOTE: See note to Table 3.13
Table 3.17: Share of the reduction in the risk-sharing inefficiency between the US and the partner country explained by falls in the variance of shocks and improvements in the degree of insurance (CC preferences)

<table>
<thead>
<tr>
<th>JAP</th>
<th>1978 break</th>
<th>1997 break</th>
<th>CAN</th>
<th>1997 break</th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>-54.32</td>
<td>7.55</td>
<td>US gdp</td>
<td>0</td>
</tr>
<tr>
<td>JAP gdp</td>
<td>-200.21</td>
<td>38.21</td>
<td>CAN gdp</td>
<td>0</td>
</tr>
<tr>
<td>US govt</td>
<td>-17.86</td>
<td>9.72</td>
<td>US govt</td>
<td>-200.42</td>
</tr>
<tr>
<td>JAP govt</td>
<td>44.59</td>
<td>15.68</td>
<td>CAN govt</td>
<td>-345.85</td>
</tr>
<tr>
<td>NER</td>
<td>0</td>
<td>-8.85</td>
<td>NER</td>
<td>0</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>0</td>
<td>-9.25</td>
<td>OECD gdp</td>
<td>0</td>
</tr>
<tr>
<td>Insurance</td>
<td>-227.7%</td>
<td>53%</td>
<td>Insurance</td>
<td>-546.2%</td>
</tr>
<tr>
<td>Var(US gdp)</td>
<td>-11.62</td>
<td>2.21</td>
<td>Var(US gdp)</td>
<td>11.33</td>
</tr>
<tr>
<td>Var(JAP gdp)</td>
<td>64.97</td>
<td>30.96</td>
<td>Var(CAN gdp)</td>
<td>311.6</td>
</tr>
<tr>
<td>Var(US gov)</td>
<td>-11.42</td>
<td>1.09</td>
<td>Var(US gov)</td>
<td>34.29</td>
</tr>
<tr>
<td>Var(JAP gov)</td>
<td>297.83</td>
<td>10.45</td>
<td>Var(CAN gov)</td>
<td>394.65</td>
</tr>
<tr>
<td>Var(NER)</td>
<td>0</td>
<td>0</td>
<td>Var(NER)</td>
<td>0</td>
</tr>
<tr>
<td>Var(OECD)</td>
<td>0</td>
<td>0</td>
<td>Var(OECD)</td>
<td>0</td>
</tr>
<tr>
<td>C(gdps)</td>
<td>-11.97</td>
<td>2.24</td>
<td>C(gdps)</td>
<td>-105.61</td>
</tr>
<tr>
<td>Shocks</td>
<td>327.7%</td>
<td>46.9%</td>
<td>Shocks</td>
<td>646.2%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>-189.77</td>
<td>12.58</td>
<td>US gdp</td>
<td>-58.93</td>
<td>30.14</td>
</tr>
<tr>
<td>UK gdp</td>
<td>191.43</td>
<td>47.3</td>
<td>CH gdp</td>
<td>223.4</td>
<td>-0.39</td>
</tr>
<tr>
<td>US govt</td>
<td>78.68</td>
<td>-4.06</td>
<td>US govt</td>
<td>0</td>
<td>77.15</td>
</tr>
<tr>
<td>UK govt</td>
<td>79.88</td>
<td>-36.9</td>
<td>CH govt</td>
<td>19.04</td>
<td>1.96</td>
</tr>
<tr>
<td>NER</td>
<td>0</td>
<td>-8.4</td>
<td>NER</td>
<td>-0.62</td>
<td>-18.24</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>0</td>
<td>0</td>
<td>OECD gdp</td>
<td>0</td>
<td>-1.02</td>
</tr>
<tr>
<td>Insurance</td>
<td>160.2%</td>
<td>10.5%</td>
<td>Insurance</td>
<td>182.8%</td>
<td>89.6%</td>
</tr>
<tr>
<td>Var(US gdp)</td>
<td>97.28</td>
<td>3.31</td>
<td>Var(US gdp)</td>
<td>61.89</td>
<td>5.35</td>
</tr>
<tr>
<td>Var(UK gdp)</td>
<td>29.34</td>
<td>84.34</td>
<td>Var(CH gdp)</td>
<td>-80.23</td>
<td>0</td>
</tr>
<tr>
<td>Var(US gov)</td>
<td>-111.33</td>
<td>1.84</td>
<td>Var(US gov)</td>
<td>-52.34</td>
<td>7</td>
</tr>
<tr>
<td>Var(UK gov)</td>
<td>2.32</td>
<td>0</td>
<td>Var(CH gov)</td>
<td>-15.32</td>
<td>-2.32</td>
</tr>
<tr>
<td>Var(NER)</td>
<td>-19.22</td>
<td>0</td>
<td>Var(NER)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Var(OECD)</td>
<td>0</td>
<td>0</td>
<td>Var(OECD)</td>
<td>0</td>
<td>0.37</td>
</tr>
<tr>
<td>C(gdps)</td>
<td>-31.62</td>
<td>0</td>
<td>C(gdps)</td>
<td>2.48</td>
<td></td>
</tr>
<tr>
<td>Shocks</td>
<td>-33.2%</td>
<td>89.4%</td>
<td>Shocks</td>
<td>-83.5%</td>
<td>10.4%</td>
</tr>
</tbody>
</table>

NOTE: See note to Table 3.8
Table 3.18: Share of the reduction in the risk-sharing inefficiency between the US and the partner country explained by falls in the variance of shocks and improvements in the degree of insurance (CC preferences)

<table>
<thead>
<tr>
<th>NDL</th>
<th>1996 break</th>
<th>BEL</th>
<th>1983 break</th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>-141,84</td>
<td>US gdp</td>
<td>-95,41</td>
</tr>
<tr>
<td>NLD gdp</td>
<td>0</td>
<td>BEL gdp</td>
<td>144,73</td>
</tr>
<tr>
<td>US govt</td>
<td>-190,91</td>
<td>US govt</td>
<td>99,48</td>
</tr>
<tr>
<td>NLD govt</td>
<td>0</td>
<td>BEL govt</td>
<td>-26,75</td>
</tr>
<tr>
<td>NER</td>
<td>591,52</td>
<td>NER</td>
<td>-64,5</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>0</td>
<td>OECD gdp</td>
<td>-58,54</td>
</tr>
<tr>
<td><strong>Insurance</strong></td>
<td><strong>258,77</strong></td>
<td><strong>Insurance</strong></td>
<td><strong>-0,99</strong></td>
</tr>
<tr>
<td>Var(US gdp)</td>
<td>9,3</td>
<td>Var(US gdp)</td>
<td>13,88</td>
</tr>
<tr>
<td>Var(NLD gdp)</td>
<td>-111,48</td>
<td>Var(BEL gdp)</td>
<td>54,77</td>
</tr>
<tr>
<td>Var(US govt)</td>
<td>0</td>
<td>Var(US govt)</td>
<td>32,34</td>
</tr>
<tr>
<td>Var(NLD govt)</td>
<td>-204,62</td>
<td>Var(BEL govt)</td>
<td>0</td>
</tr>
<tr>
<td>Var(NER)</td>
<td>166,6</td>
<td>Var(NER)</td>
<td>0</td>
</tr>
<tr>
<td>Var(OECD)</td>
<td>0</td>
<td>Var(OECD)</td>
<td>0</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td><strong>-140,2</strong></td>
<td><strong>Shocks</strong></td>
<td><strong>100,99</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FIN</th>
<th>1978 break</th>
<th>ITA</th>
<th>1999 break</th>
</tr>
</thead>
<tbody>
<tr>
<td>US gdp</td>
<td>-419,87</td>
<td>US gdp</td>
<td>35,42</td>
</tr>
<tr>
<td>FIN gdp</td>
<td>463,39</td>
<td>ITA gdp</td>
<td>-6,6</td>
</tr>
<tr>
<td>US govt</td>
<td>170,9</td>
<td>US govt</td>
<td>0</td>
</tr>
<tr>
<td>FIN govt</td>
<td>116,89</td>
<td>ITA govt</td>
<td>47,77</td>
</tr>
<tr>
<td>NER</td>
<td>0</td>
<td>NER</td>
<td>0</td>
</tr>
<tr>
<td>OECD gdp</td>
<td>0</td>
<td>OECD gdp</td>
<td>0</td>
</tr>
<tr>
<td><strong>Insurance</strong></td>
<td><strong>331,31</strong></td>
<td><strong>Insurance</strong></td>
<td><strong>76,59</strong></td>
</tr>
<tr>
<td>Var(US gdp)</td>
<td>-133,37</td>
<td>Var(US gdp)</td>
<td>-1,3</td>
</tr>
<tr>
<td>Var(FIN gdp)</td>
<td>182,24</td>
<td>Var(ITA gdp)</td>
<td>0</td>
</tr>
<tr>
<td>Var(US govt)</td>
<td>-461,97</td>
<td>Var(US govt)</td>
<td>1,98</td>
</tr>
<tr>
<td>Var(FIN govt)</td>
<td>91,13</td>
<td>Var(ITA govt)</td>
<td>29,46</td>
</tr>
<tr>
<td>Var(NER)</td>
<td>0</td>
<td>Var(NER)</td>
<td>6,12</td>
</tr>
<tr>
<td>Var(OECD)</td>
<td>93,47</td>
<td>Var(OECD)</td>
<td>0</td>
</tr>
<tr>
<td><strong>Shocks</strong></td>
<td><strong>-228,5</strong></td>
<td><strong>Shocks</strong></td>
<td><strong>36,26</strong></td>
</tr>
</tbody>
</table>

NOTE: See note to Table 3.8
BIBLIOGRAPHY


125
risk-sharing in a monetary union: updated evidence and policy implications for Europe”, CEPR discussion paper 4463


CHAPTER 4

THE MACROECONOMIC IMPLICATIONS OF SOVEREIGN WEALTH FUNDS

Joint work with Filipa Sá

4.1 Introduction

Sovereign Wealth Funds (SWFs) are government-owned investment funds, set up for a variety of purposes. The IMF (2008a) distinguishes five types of SWFs based on their main objective: (i) stabilization funds, set up to insulate the economy against swings in commodity prices; (ii) savings funds, which transform the income from non-renewable natural resources into a diversified portfolio of assets, accumulating savings for future generations; (iii) reserve investment corporations, established to increase the return on foreign exchange reserves; (iv) development funds, which help fund infra-structural projects to increase the country’s potential growth; and (v) contingent pension reserve funds, which complement resources from individual pension contributions to provide for pension liabilities on the government’s balance sheet.

SWFs are becoming increasingly important in the international monetary system. They are estimated to have between 2.1 and 3 trillion US dollars of assets under management. While this is relatively small in comparison with total global financial assets, estimated at 194 trillion dollars in 2006 (IMF (2008b)), it is a sizable amount and exceeds the size of hedge funds, estimated at 1.7 trillion dollars. Moreover, SWFs are projected to grow rapidly in the next decade and to have around 12 trillion dollars of assets under management in 2015 (Morgan Stanley (2008)).

Information about the portfolio structure of SWFs is relatively limited, since there is no uniform public disclosure of their assets and investment strategies. However, the information available suggests that the portfolios of SWFs are typically more diversified than traditional reserves held by central banks, with a larger share invested in equities and a wider geographical dispersion. Given these differences in investment strategies, the shift of reserve assets from central banks to SWFs could have implications for asset prices, the flow of funds between countries, exchange rates, and the evolution of global imbalances. In particular,
SWFs may increasingly diversify away from dollar assets. This might lead to a reduction in capital inflows into the US, a depreciation of the dollar and an increase in returns on dollar assets. SWFs may also diversify their portfolios away from low-risk, short term debt instruments, and into longer term equity assets, which might lead to changes in asset prices and rates of return.

It has been argued that the changes in asset returns generated by the growth in SWFs might induce a reduction in the so-called “exorbitant privilege” of the US. This term has been used by Gourinchas and Rey (2005) to denote the fact that the US receives higher returns on its foreign assets than it pays on its foreign liabilities. This excess return can be decomposed in two elements: a return effect - within each asset class, the return that the US pays to foreigners is smaller than the return that foreigners pay to the US; and a composition effect - the US tends to invest more in foreign equities, while foreigners tend to invest more in US bonds. The growth in SWFs may lead to a reduction in both components of the “exorbitant privilege”.

In this Chapter, we analyze the implications of the growth of SWFs using a dynamic general equilibrium model with two regions (the US and the rest of the world - ROW) and two goods (US and ROW produced goods). A distinctive feature of the model is the presence of two asset classes: equities and bonds. This allows us to study the implications of a diversification away from debt and into equity assets by SWFs. The real exchange rate determines the allocation of goods and assets across the two regions. Because each region issues both equities and bonds, there are four assets in total: US equities, US bonds, ROW equities, and ROW bonds. These assets are imperfect substitutes and their demand follows the specification in Blanchard, Giavazzi, and Sá (2005), where the share of wealth invested in each asset has an exogenous and an endogenous component. The exogenous component represents shocks to portfolio preferences. The endogenous component captures the reaction of asset demands to changes in the relative expected returns of different assets.\(^1\)

The general equilibrium nature of the model allows for endogenous adjustment in interest rates and asset prices. Therefore, we can see how they are affected by the growth in SWFs.

\(^1\)A growing body of literature derives portfolio holdings from optimization principles in stochastic general equilibrium settings. See, for instance, Devereux and Sutherland (2006) and Heathcote and Perri (2007). For our purposes, it proves more convenient not to derive portfolio shares from microfoundations. We rather choose to embed in a general equilibrium framework the specification of asset demands that characterize standard portfolio-balance models, in the spirit of Kouri (1976). The reasons are the following. First, this allows to match in the calibration international asset holdings with those observed in the data abstracting from considerations on the degree of international risk-sharing – a crucial issue for portfolio formation which is still largely debated in the literature. Second, in optimal portfolio-choice models, the returns on different assets coincide in a non-stochastic steady state. So, adopting that type of model and relying on standard approximation techniques, would prevent us from studying the impact of shifts in preferences of foreign investors on asset return differentials (for instance on the US exorbitant privilege).
and what the implications are for the “exorbitant privilege”. Following any shock to portfolio preferences, both equity and bond prices adjust to clear asset markets. Over time, investors rebalance their portfolios in response to changes in expected returns. This endogeneity of interest rates and asset prices is the key difference between our model and the one in Blanchard, Giavazzi, and Sá (2005), where interest rates are exogenous.

We calibrate the model to match asset holdings and returns observed in the data. We simulate a scenario where all “excess reserves” currently held by central banks in Emerging Market Economies (EMEs) are transferred to SWFs, where “excess reserves” are defined as being above the level that would be required for liquidity purposes. We consider two diversification paths: one in which SWFs keep the same asset allocation as central banks, i.e., the same investment shares in equities and bonds, but diversify away from dollar assets (path 1); and another in which they keep the same currency composition, but shift towards a riskier portfolio in the US market, with a larger share invested in US equities and a smaller share invested in US bonds (path 2). We focus on the implications for the dollar exchange rate, the US trade deficit and net debt, and the “exorbitant privilege”. The main purposes of our analysis are to provide a qualitative assessment of the impact of SWFs growth on asset returns, consumption, investment, the exchange rate, the trade deficit and net debt; to understand the channels through which changes in portfolio allocation impact on these variables; and to provide a rough quantification of the magnitude of the adjustments in these variables.

Our results show that in path 1 (currency diversification) the dollar depreciates in the period immediately after the shock, leading to a reduction in the US trade deficit and net debt. In subsequent periods, the return on US assets must increase to clear asset markets. This generates a rebalancing of the portfolios of foreign investors towards holding more dollar assets, which leads to an appreciation of the dollar. The “exorbitant privilege” of the US (the difference between the return it receives on its foreign assets and the return it pays on its foreign liabilities) decreases, and US net debt increases over time. In path 2 (asset diversification) the dollar depreciates and the US trade deficit decreases. However, US net debt increases over time due to a reduction in the “exorbitant privilege”.

Qualitatively, our results can be compared with the findings of an exercise conducted by the IMF (IMF (2008a)). It assumes that between 25 and 50 percent of new foreign currency inflows in countries that have recently established SWFs will be invested by those SWFs. The exercise is calibrated for two diversified portfolios: one which mimics the composition of Norway’s Government Pension Fund; and another which is based on information on asset allocation and currency composition provided in market analysis. These two stylized portfo-
4.2. **THE MODEL**

Lios are compared with a scenario where assets are kept as central bank reserves. The results, derived using the IMF’s Global Integrated Monetary Fiscal Model, suggest that the US real interest rate would increase by 10 to 20 bp, the dollar would depreciate by 2 to 5 percent, and the US current account deficit would improve by 0.25 to 0.5 pp of GDP. In the rest of the world, real interest rates would fall, currencies would appreciate, and domestic demand would increase.²

This Chapter is organized as follows. The structure of the model is explained in section 2. Section 3 presents the baseline parameters used in the calibration and section 4 explains the scenarios used in the simulations. The results of the baseline simulations are discussed in section 5, while section 6 presents a number of robustness checks. Section 7 concludes.

### 4.2 The Model

This section describes the general equilibrium model used for the simulations. The core structure of the model is based on Gosh (2007) and Meredith (2007). The model consists of two regions: the US and the rest of the world (ROW), each fully specialized in the production of one homogeneous good. In each region, there are two types of assets: equities and bonds. Equities are modelled as claims on the capital stock. Bonds are issued by the government, who must balance its budget each period.

Each country is populated by a representative firm and two types of representative households: entrepreneurs and portfolio investors. Firms in both regions produce output using capital and labour, and adjust their productive capacity by increasing or decreasing their stock of capital. We abstract from any nominal rigidity and from real economic growth. Entrepreneurs manage the firms and have all their wealth invested in home equities. Portfolio investors invest in bonds and equities at home and abroad. They supply labour inelastically and decide on the proportion of their income to be allocated to consumption and portfolio investment. They receive income in the form of wages and returns on their portfolio and pay taxes or receive transfers from the government. The presence of two types of households ensures internal consistency, while at the same time allowing to match steady state return differentials on different assets with those observed in the data. More specifically, for the model to be internally consistent, firms should discount future profits using the discount factor of the consumers who manage them. The steady-state discount factor of portfolio

---

²One of the key elements that differentiates our analysis from that exercise, is that the model used for simulations in IMF (2008a) does not allow explicitly for international trade in equities and ROW bonds. Therefore, contrary to what happens in our framework, investors' demand for different assets cannot react endogenously to changes in their prices. In this sense, the analysis in IMF (2008a) abstract from second round effects.
investors is a function of the returns on all assets in which they invest (home and foreign equities and bonds). On the other hand, US firms discount factor should equal in steady-state the user cost of capital, which coincides with the rate of return on US equities. If there were only portfolio investors in the economy, for the discount factor of US portfolio consumers to equal the discount factor of US firms, the steady-state returns on all four assets (home and foreign equities and bonds) would have to be the same. This would be an unattractive feature since we are interested in matching the return differential on assets observed in the data. We solve this problem by including two types of representative households: portfolio investors, who invest in all assets, and entrepreneurs, who only invest in home equities and manage the firms. This ensures that the model is internally consistent since the discount factor of entrepreneurs equals the discount factor of the firms.

The general equilibrium nature of the model lets any adjustment in interest rates and asset prices be determined endogenously, as asset demands react to changes in the relative expected returns of different assets. The supply of equities is determined by firms’ investment in physical capital. Because equity prices are determined endogenously, the model is able to account for capital gains on equity holdings, in addition to valuation effects caused by changes in the exchange rate.

4.2.1 Consumers

The size of the world population is normalized to 1, with a fraction $n$ in the US and $(1 - n)$ in the rest of the world. There are two types of representative households in each economy: entrepreneurs and portfolio investors. Entrepreneurs manage the firms and invest all their wealth in home equities. Portfolio investors supply labour inelastically, pay lump-sum taxes (or receive transfers), and invest their wealth in both equities and bonds, at home and abroad. We denote the fraction of entrepreneurs in the US population by $\alpha^E$ and the fraction of entrepreneurs in the ROW population by $\alpha^{E*}$.

Both entrepreneurs and portfolio investors decide on how much to consume given their wealth. US households derive utility from consuming the following CES bundle of US and ROW-produced goods:

$$C_t = \left\{ \left( \rho \right)^{1/\theta} (C_{US,t})^{\theta - 1} + (1 - \rho)^{1/\theta} (C_{ROW,t})^{\theta - 1} \right\}^{\theta/(\theta - 1)}$$ (4.1)

where $\rho > 0.5$ is a parameter capturing the degree of home bias in consumption and $\theta$ is the elasticity of substitution between goods produced in different regions. Consumption in the ROW is analogously defined, with starred variables denoting the corresponding quantities...
consumed by ROW households.

The consumer price indices can be derived from the households’ cost minimization problem. Taking the home good as the numéraire, the consumer price index for the US is given by:

\[ P_t = \{\rho + (1 - \rho) e_t^{\theta-1}\}^{\frac{1}{1-\theta}} \] (4.2)

where \( e_t \) is the real exchange rate between the US and ROW, defined as the relative price of the goods produced in the two regions, so that an increase in the exchange rate represents an appreciation of the dollar.

The demands of US consumers for domestically and foreign produced goods are obtained from standard cost minimization subject to 4.1:

\[ C_{US,t} = \rho (P_t) \theta C_t \] (4.3)

\[ C_{ROW,t} = (1 - \rho) (e_t P_t) \theta C_t \] (4.4)

An appreciation of the dollar makes ROW goods less expensive to US consumers and US goods more expensive to ROW households, shifting world demand from US to ROW-produced goods. In this sense, the real exchange rate determines the allocation of goods across markets.

Consumers optimally decide to allocate their income between consumption and savings. The utility maximization problem for entrepreneurs is given by:

\[
\max_{\{C_E^t\}_{s=t}^\infty} : E_t \left\{ \sum_{s=t}^\infty \beta^{s-t} \log \left( C_E^t \right) \right\} \\
\text{s.t. : } V_t^E = r_t^E V_{t-1}^E - P_t C_t^E
\]

where \( V_E^t \) represents the financial wealth of US entrepreneurs, and \( r_t^E \) is the rate of return on their portfolio defined in local currency. Because entrepreneurs invest all their wealth in home equities, the rate of return on their portfolio equals the return on home equities.

The utility maximization problem for portfolio holders is given by:

\[
\max_{\{C_P^t\}_{s=t}^\infty} : E_t \left\{ \sum_{s=t}^\infty \beta^{s-t} \log \left( C_P^t \right) \right\}
\]
4.2. THE MODEL

\[ s.t. : V^P_t = R^P_t V^P_{t-1} - P_t C^P_t + w_t - \tau_t \]

The key difference relative to the problem of entrepreneurs is in the budget constraint, which now includes wage earning \( w_t \) net of lump-sum taxes \( \tau_t \). \( R^P_t \) is the rate of return on the portfolio, which includes home and foreign equities and bonds.

The first order conditions for utility maximization deliver the standard Euler equations:

\[
1 = \beta E_t \left\{ R^E_{t+1} \frac{P^E_t C^E_t}{P^E_{t+1} C^E_{t+1}} \right\}
\]

where \( i = E, P \) for entrepreneurs and portfolio investors, respectively.

The assumption of logarithmic utility implies that consumption expenditure for entrepreneurs is optimally determined as:

\[
P^E_t C^E_t = (1 - \beta) R^E_t V^E_{t-1}
\]

Similarly, for portfolio investors:

\[
P^P_t C^P_t = (1 - \beta) \left\{ R^P_t V^P_{t-1} + H_t \right\}
\]

where \( H_t \) is the present discounted value of lifetime human wealth, in the form of labour income net of taxes.

4.2.2 Firms

Firms in both countries are fully specialized in the production of the regional good, which is available for consumption and investment in both countries. They produce using a constant returns to scale technology combining capital and labour. The US production function is given by:

\[ Y_t = A_t K^\eta_t (L_t)^{1-\eta} \]

\( Y_t \) is the output of the US-produced good, \( A_t \) is an exogenous productivity term, \( K_t \) is the capital input and \( L_t \) is the labour input. Since we assume that only portfolio investors supply labour and that labour supply is inelastic, for the labour market to clear in equilibrium the labour input must equal the fraction of these investors in the economy, i.e. \( L_t = (1 - \alpha^E) \). A share \( \eta \) of output is paid to capital and the remaining is paid to labour.
Firms adjust their productive capacity by deciding the optimal amount of physical investment, $I_t$, so as to maximize current and future cash flows. US firms solve the following problem:

$$\max_{(L_s,I_t,K_{t+1})} E_t \left\{ \sum_{s=t}^{\infty} \prod_{j=t+1}^{s} \Omega_{j,j+1} \left( A_s K_s^\eta (L_s)^{1-\eta} - w_s L_s - P_s I_s \left( 1 + \frac{I_s}{K_s} \right) - \delta P_s K_s \right) \right\}$$

(4.5)

subject to:

$$K_{s+1} - K_s = I_s$$

(4.6)

where $w_s$ denotes the wage, $\left( \frac{I_s}{K_s} \right)$ is the linear homogeneous installation cost of capital, $\delta$ is the depreciation rate, and $\Omega_{j,j+1}$ the discount factor of US entrepreneurs, used by firms to discount future cash flows. $P_s$ is the price index of the US investment bundle, which includes US and ROW-produced goods, built using the same CES aggregator used for the consumption good:

$$I_t = \left\{ \rho^{\frac{1}{\theta}} (I_{US,t})^{\frac{\theta-1}{\theta}} + (1 - \rho)^{\frac{1}{\theta}} (I_{ROW,t})^{\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}}$$

Standard cost minimization by firms delivers their demands for US and ROW-produced goods:

$$I_{US,t} = \rho P_t^\theta I_t$$

(4.7)

$$I_{ROW,t} = (1 - \rho) (e_t P_t)^\theta I_t$$

(4.8)

The price of the US investment bundle coincides with the price of the US consumption bundle, $P_t$, given by equation 4.2.

The firms’ problem in equations 4.5 and 4.6 can be stated recursively and its first order conditions written as:

$$w_t = A_t (1 - \eta) K_t^\eta (1 - \alpha^E)^{-\eta}$$

(4.9)

$$q_t = P_t \left( 1 + 2\phi \frac{I_t}{K_t} \right)$$

(4.10)

$$q_t = E_t \left\{ \Omega_{t,t+1} \left( q_{t+1} + \eta A_{t+1} K_{t+1}^{\eta-1} (1 - \alpha^E)^{1-\eta} - \delta P_{t+1} + \phi P_{t+1} \left( \frac{I_{t+1}}{K_{t+1}} \right)^2 \right) \right\}$$

(4.11)
4.2. **THE MODEL**

where $q_t$ is defined as the marginal value of capital and coincides with the price of US equities. Equation 4.9 determines the wage as the marginal product of labour and equation 4.10 determines the optimal amount of investment by US firms. Consistent with the standard $q$-theory, it implies that US firms increase their capital stock when the marginal value of capital $q_t$ exceeds the replacement cost of capital $P_t$. Equation 4.11 is an arbitrage condition stating that the marginal value of one unit of capital must be equal to the expected discounted value of returns one period ahead, which includes capital gains from equity ownership.

The realized gross return on US equities are given by:

$$r_t^E = \frac{q_t + \eta A_t K_t^{1-q} (1 - \alpha^E)^{1-q} - \delta P_t + \phi P_t \left( \frac{b_t}{K_t} \right)^2}{q_{t-1}}$$  \hspace{1cm} (4.12)

Each period US equities pay out the realized marginal product of capital plus capital gains or losses due to adjustments in the price of equities.

4.2.3 Government

In each period governments in both regions finance public expenditure and pay out the interest on the outstanding stock of public debt either by selling new bonds at the current market price or by levying taxes on home portfolio investors. We follow Meredith (2007) in assuming that taxation is lump-sum in order to abstract from distortionary effects on capital accumulation. The US government budget constraint is given by:

$$G_t + B_{t-1} = P_t^B B_t + (1 - \alpha^E) \tau_t$$  \hspace{1cm} (4.13)

Because taxes are paid only by portfolio investors, the amount of lump sum taxes raised by the government equals $(1 - \alpha^E) \tau_t$. For simplicity, we assume that $G_t$ and $B_t$ are exogenous and constant over time. Therefore, a reduction in the price of bonds, $P_t^B$, leads to an increase in lump sum taxes. In addition, we follow Meredith (2007) in assuming that the government consumes only the domestic good. This is a simplification, but is consistent with the evidence on home bias in government expenditure.

4.2.4 Portfolio Allocation

Entrepreneurs manage firms and allocate all their wealth to home equities. Portfolio investors make two types of decisions regarding their portfolios: they decide on the *geographical composition* (how much to invest in US and ROW assets) and on the *asset composition* (how
4.2. THE MODEL

much to invest in equities and bonds). In what follows, all asset returns and prices are measured in units of the domestic good.

Bonds are issued by the governments in the US and ROW and pay out one unit of the good produced in the country in which they are issued. Therefore, the rates of return on US and ROW bonds are:

\[ r^B_t = \frac{1}{P^B_t} \quad \text{and} \quad r^{B*}_t = \frac{1}{P^{B*}_{t-1}} \]

where \( P^B \) and \( P^{B*} \) are the prices of the bonds expressed in US and ROW currencies.

Equities represent the ownership of one unit of capital of US or ROW firms. The rate of return on US equities is given by equation 4.12. A similar expression gives the rate of return on ROW equities, \( r^{E*}_t \).

We denote by \( \alpha_t \) the share of US financial wealth invested in US assets. From this, a fraction \( \beta_t \) is allocated to equities and a fraction \( (1 - \beta_t) \) is allocated to bonds. Similarly, from the share of US wealth invested in ROW assets, a fraction \( \gamma_t \) is allocated to equities and a fraction \( (1 - \gamma_t) \) is allocated to bonds. The shares for ROW portfolio investors, denoted with a star, are defined analogously.

Asset demands are characterized by imperfect substitutability between different assets and follow a similar specification to the one used in Blanchard, Giavazzi, and Sá (2005). They have two components: an exogenous component, representing shocks to portfolio preferences, and an endogenous component, capturing the response of asset demands to changes in the relative returns of different assets. More specifically, \( \beta_t \), the fraction of wealth that US investors invest in equities in the US market, is given by:

\[ \beta_t = b^\beta \left\{ E_t \left( \frac{r^E_{t+1}}{r^B_{t+1}} \right) - \frac{r^E_t}{r^B_t} \right\} + s^\beta \tag{4.14} \]

where non time-indexed returns denote steady state values. The first term captures the reaction of \( \beta_t \) to changes in expected relative returns: if the return on US equities rises relative to the return on US bonds, investors allocate a relatively larger fraction of their wealth to equities. The parameter \( b^\beta \) captures the degree of substitutability between different assets (in this case between US equities and US bonds). A higher degree of substitutability makes portfolio shares more responsive to changes in expected relative returns. The second term, \( s^\beta_t \), is an exogenous shock to portfolio preferences.

---

3According to these definitions, at time \( t \) US portfolio investors invest \( \alpha_t \beta_t V^P_t \) dollars in US equities and \( \alpha_t (1 - \beta_t) V^P_t \) dollars in US bonds.
Similarly, \( \gamma_t \), the fraction of wealth that US households invest in equities in the foreign market, is given by:

\[
\gamma_t = b^\gamma \left\{ E_t \left( \frac{r_{E,t+1}}{r_{B,t+1}} - \frac{r_{E,*}}{r_{B,*}} \right) \right\} + s_t^\gamma \tag{4.15}
\]

An increase in the return on foreign equities relative to foreign bonds, increases the share of wealth that US investors invest in equities in the foreign market.\(^4\)

Turning to the currency composition of portfolios, the share of wealth that US portfolio investors invest in the domestic asset market, \( \alpha_t \), is given by:

\[
\alpha_t = b^\alpha \left\{ E_t \left( \frac{\beta \gamma_{E,t+1}}{\gamma_{E,t+1}} + (1 - \beta \gamma_t) \frac{r_{E,t+1}}{r_{B,t+1}} \right) - \frac{r_{E,*}}{r_{B,*}} \right\} + s_t^\alpha \tag{4.16}
\]

The share of financial wealth invested by US portfolio holders in the domestic market is increasing in the relative expected return on domestic assets, given the proportion of wealth allocated to bonds and equities in each market. An expected appreciation of the dollar increases the relative return on dollar assets. Therefore, in our model, the real exchange rate determines not only the allocation of consumption across US and ROW goods, but also the allocation of wealth across US and ROW assets.\(^5\)\(^6\)

Given these definitions of portfolio shares, we can express the total rate of return of US portfolio investors as:

\(^4\)The ROW analogues of equations 4.14 and 4.15 are:

\[
\beta_t^* = b^\beta^* \left\{ E_t \left( \frac{r_{E,t}}{r_{B,t}} - \frac{r_{E,*}}{r_{B,*}} \right) \right\} + s_t^\beta^*
\]

\[
\gamma_t^* = b^\gamma^* \left\{ E_t \left( \frac{r_{E,t}}{r_{B,t}} - \frac{r_{E,*}}{r_{B,*}} \right) \right\} + s_t^\gamma^*
\]

Expressing relative returns in deviations from steady state values simplifies the calibration of the model without affecting its dynamics.\(^5\)

\(^6\)An alternative way to model the portfolio allocation problem would be to choose one of the four assets (for example, US bonds) to be the reference asset and have investors decide how much to allocate to each asset depending on its return relative to the reference asset. However, this specification does not allow to differentiate between a shock to the currency composition (how much to invest in the US versus ROW) and a shock to the asset composition (how much to invest in equities versus bonds). The specification we adopt for portfolio shares allows us to differentiate between these two types of shocks: a shock to \( \beta_t \) changes the asset composition without affecting the currency composition, while a shock to \( \alpha_t \) changes the currency composition without affecting the asset composition.
\[ R_t^P = \alpha_{t-1} \beta_{t-1} r_t^E + \frac{(1 - \alpha_{t-1})}{e_t} \gamma_{t-1} r_t^{E*} e_{t-1} + \alpha_{t-1} (1 - \beta_{t-1}) r_t^B e_{t-1} + \frac{(1 - \alpha_{t-1})}{e_t} (1 - \gamma_{t-1}) r_t^{B*} e_{t-1} \]

Notice that valuation effects stemming from changes in the exchange rate affect the return US portfolio investors receive on their holdings of foreign assets.

### 4.2.5 Equilibrium and Balance of Payments Dynamics

Equilibrium requires goods and asset markets to clear. Combining equations 4.3, 4.4, 4.7, 4.8, and their foreign analogues, we can write market clearing conditions for the goods produced in the US and in ROW as:

\[ n_\lambda K_t^n (1 - \alpha^E)^{1-\eta} = \rho n (P_t)^\theta \left[ C_t + \delta K_t + I_t \left( 1 + \phi \frac{I_t}{K_t} \right) \right] + (1 - \rho^*) (1 - n) \left( \frac{P_t^*}{e_t} \right)^\theta \left[ C_t^* + \delta K_t^* + I_t^* \left( 1 + \phi \frac{I_t^*}{K_t^*} \right) \right] + n G_t \]

\[ (1 - n) A^K K_t^n (1 - \alpha^{E*})^{1-\eta} = (1 - \rho) n (P_t^*) \theta \left[ C_t^* + \delta K_t^* + I_t^* \left( 1 + \phi \frac{I_t^*}{K_t^*} \right) \right] + \rho^* (1 - n) \left( \frac{P_t^*}{e_t} \right)^\theta \left[ C_t^* + \delta K_t^* + I_t^* \left( 1 + \phi \frac{I_t^*}{K_t^*} \right) \right] + (1 - n) G_t^* \]

Since equity is a claim on the stock of capital, the supply of equities in each region equals the value of the capital stock in that region. Hence, the market clearing condition for the US equity market is given by:

\[ (1 - \alpha^E) n \alpha_t \beta_t V_t^P + \alpha^E n V_t^E + \frac{(1 - \alpha^{E*})(1 - n)(1 - \alpha_t^*)}{e_t} \gamma_t^* V_t^{P*} = n q_t K_t \]

This condition states that demand for US equities must equal supply. The first term on the left hand side gives the demand for US equities by US portfolio investors. There is a fraction \((1 - \alpha^E)n\) of these investors in the world population who invest a fraction \(\alpha_t \beta_t\) of their wealth in US equities. The second term gives the demand by US entrepreneurs, which equal a fraction \(\alpha^E n\) of the world population and invest all their wealth in home equities. Finally, the third term gives the demand by foreign portfolio investors, which equal a fraction \((1 - \alpha^{E*})(1 - n)\) of the world population and invest a share \((1 - \alpha)\gamma_t\) of their wealth in US equities.
4.2. THE MODEL

The market clearing condition for the ROW equity market is similarly given by:

\[(1 - \alpha^E)n(1 - \alpha_t)\gamma_t V_t^P e_t + \alpha^E(1 - n)V_t^{E*} + (1 - \alpha^E)(1 - n)\alpha^*_t \beta^*_t V_t^{P*} = (1 - n)q^*_t K^*_t\]

The market clearing conditions for the US and ROW bond markets are given by:

\[(1 - \alpha^E)n\alpha_t (1 - \beta_t) V_t^P + \frac{(1 - \alpha^E)(1 - n)(1 - \alpha^*_t)(1 - \gamma^*_t) V_t^{P*}}{e_t} = nB_t P_t^B\]

\[(1 - \alpha^E)n(1 - \alpha_t)(1 - \gamma_t) V_t^P e_t + (1 - \alpha^E)(1 - n)\alpha^*_t (1 - \beta^*_t) V_t^{P*} = (1 - n)B_t^* P_t^{B*}\]

The net debt position of the US is equal to the value of the stock of US assets, including equities and bonds, minus the value of financial wealth of US households:

\[F_t = q_t K_t + P_t^B B_t - (1 - \alpha^E)V_t^P - \alpha^E V_t^{E} \tag{4.18}\]

The US trade deficit equals the difference between total expenditure and total output:

\[TD_t = P_t \left[ C_t + \delta K_t + I_t \left( 1 + \phi \frac{I_t}{K_t} \right) + G_t \right] - A (K_t)^\eta (1 - \alpha^E)^{1-\eta} \tag{4.19}\]

Using the market clearing conditions for the equity and bond markets, we can rewrite equation 4.18 as:

\[F_t = \left[ r_t^E \gamma_{t-1} + r_t^B \left( 1 - \gamma_{t-1}^* \right) \right] F_{t-1} + T D_t - (1 - \alpha^E)n(1 - \alpha_{t-1}) V_{t-1}^P \cdot \left\{ \frac{r_t^E e_{t-1}}{\gamma_{t-1}} + \frac{r_t^B e_{t-1}}{e_t} \left( 1 - \gamma_{t-1}^* \right) - \gamma_t^E \gamma_{t-1}^* - r_t^B \left( 1 - \gamma_{t-1}^* \right) \right\} \tag{4.20}\]

This equation describes the dynamics of US net debt. The two terms on the right hand side are standard: net debt next period equals the return the US pays on its existing stock of external net debt plus the trade deficit. The last term captures the effect on US net debt of changes in returns on US assets and liabilities, and embeds valuation effects stemming from exchange rate adjustments. A higher positive spread between the return on US assets and liabilities implies a lower accumulation of net external debt. An appreciation of the dollar reduces the dollar value of the returns the US receives on its foreign assets, contributing to a rise in net debt.
The steady state of the model is characterized by zero physical capital investment \((I = I^* = 0)\) and constant portfolio shares. In steady state the current account of the two regions must be balanced, and equation 4.20 reads:

\[
[r^E \gamma^* + r^B (1 - \gamma^*) - 1] F - (1 - \alpha^E)n (1 - \alpha) V^P[r^E \gamma + r^B (1 - \gamma) - r^E \gamma^* - r^B (1 - \gamma^*)] + TD = 0
\]

We linearize the model to the first order around the steady state, and solve it using a numerical linear solver. The next section describes the calibration of the parameter values in steady state.

4.3 Calibration

We calibrate the model in the steady state to match asset returns and portfolio shares computed from the data, the ratio of US net external debt to GDP, and the ratio of US and ROW private consumption to GDP. Table 4.1 lists all parameter values used in the calibration.

The calculation of the portfolio shares is explained in detail in Appendix A. Consistent with the evidence in Gourinchas and Rey (2005), the share that US investors allocate to foreign equities (56\%) is substantially larger than the share that foreign investors allocate to US equities (31\%). In this sense, the US can be characterized as a “venture capitalist”. The steady state annual gross rates of returns on different assets are obtained from Forbes (2008), who presents rates of return disaggregated by three assets classes: FDI, portfolio equities and bonds. We treat FDI and portfolio equities as a single asset class and aggregate the returns on FDI and portfolio equities in Forbes (2008) by weighting them by the proportion of these types of assets on US external assets and liabilities using the data in Lane and Milesi-Ferretti (2007).

With this parameterization for asset shares and rates of returns, the model generates an exorbitant privilege equal to 3.85\% in steady state. This is close to the value 3.32\% computed by Gourinchas and Rey (2005) for the period 1973-2004. Gourinchas and Rey decompose the privilege into two components: a return effect, due to the fact that, within each asset class, the US receives higher returns on its foreign assets than it pays on its liabilities; and a composition effect, due to the fact that the composition of US portfolio is skewed towards high-yielding equity assets, while its liabilities are composed mostly of low-yielding debt. Table 4.2 presents the values for this decomposition generated by our model in steady state and compares them with the values in Gourinchas and Rey. We obtain
4.4 Shocks to Portfolio Preferences

that most of the exorbitant privilege (2.65%) is due to the return effect, consistent with the findings in Gourinchas and Rey.\(^7\)

We normalize the exchange rate and US total factor productivity to 1 for simplicity. In our benchmark calibration, we set the elasticity of substitution between US and ROW-produced goods, \(\theta\), to 0.97, which is the median value of long-run price elasticities of aggregate trade flows for the US and other G7 countries estimated by several studies and reported in Hooper and Marquez (1995). The parameters capturing the degree of substitutability between assets, \(b^\alpha, b^{\alpha*}, b^\beta, b^{\beta*}, b^\gamma\), and \(b^{\gamma*}\), are set to 1 following the central scenario in Blanchard, Giavazzi, and Sá (2005). As part of the sensitivity analysis, we check the robustness of our results to changes in \(\theta\) and the \(b^s\).

We set the shares of entrepreneurs in the US and ROW economies, \(\alpha^E\) and \(\alpha^{E*}\), to equal 20% and calibrate the relative wealth of entrepreneurs and portfolio holders to equal 20% as well.\(^8\) We also impose that the steady state values of the ratio of US net debt to GDP, US consumption to GDP and ROW consumption to US GDP match the values obtained from the data. Finally, we set \(n\) to equal the ratio of US population to world population, obtained from the US Census Bureau.

4.4 Shocks to Portfolio Preferences

To study the impact that growth in SWFs is likely to have on asset prices and returns, the level of US net debt and the dollar exchange rate, we need to make an assumption about the potential size of SWFs. A natural assumption is that the amount of “excess reserves” now held by central banks will be managed by SWFs in the future. “Excess reserves” are defined as being in excess of what would be justified for liquidity purposes. A rule of thumb frequently used to estimate the size of “excess reserves” is the Greenspan-Guidotti rule, according to which reserves should cover short term external debt. Using this rule, Bank of England estimates suggest that the amount of “excess reserves” held by central banks in emerging markets is around 3 trillion dollars, about the same as the current size of SWFs. We use our model to study what will happen if these 3 trillion dollars of “excess reserves” are managed by SWFs rather than central banks.

\(^7\)The exorbitant privilege is given by the difference between the return the US receives on its assets and the return it pays on its liabilities, i.e., \([\bar{r}^E\cdot \bar{y}^E + \bar{r}^{B*}\cdot (1 - \bar{y}^E)] - [\bar{r}^E\cdot \bar{y}^E + \bar{r}^{B}\cdot (1 - \bar{y}^E)]\). The return effect arises from the difference between the rates of return on assets and liabilities, evaluated at the average portfolio weights, i.e., \([\bar{r}^E\cdot \bar{y}^E + \bar{r}^{B}\cdot (1 - \bar{y}^E)]\). The composition effect arises from the difference between the weights on equities and bonds for assets and liabilities, evaluated at the average return, i.e., \([\bar{r}^E\cdot \bar{y}^E + \bar{r}^{B}\cdot (1 - \bar{y}^E)]\).

\(^8\)We tried alternative values for these shares and obtained very similar results.
4.4. SHOCKS TO PORTFOLIO PREFERENCES

There are two margins along which SWFs may diversify their portfolios relative to central banks: currency diversification (away from dollars towards other currencies) and cross-asset diversification (away from bonds towards equities). We consider two paths: one in which the currency composition changes and the asset composition remains constant (path 1), and one in which the asset composition changes and the currency composition remains constant (path 2).

To compute by how much the portfolio shares would change under each of these paths, we need information on the currency and asset composition of the portfolios of central banks and SWFs. For the currency composition, we use data from the IMF COFER dataset. For the asset composition, we use the data reported in IMF (2008a). This information is presented in Table 4.3. SWFs allocate a much smaller percentage of their wealth to dollar assets than central banks (38% compared to 60%) and allocate most of their wealth to equities.

4.4.1 Path 1. Shock to Currency Composition

Given the currency composition of the portfolios of SWFs and central banks reported in Table 4.3, if 3 trillion dollars of “excess reserves” held by central banks in emerging markets start being managed by SWFs, the amount of wealth that foreign investors invest in dollars will be reduced by \((0.6 - 0.38) \times 3 = 0.66\) trillion dollars. This corresponds to \(\frac{0.66}{142} \times 100 = 4.65\%\) of US GDP.

In terms of the parameters of our model, this shock can be seen as a reduction in the share of wealth that ROW portfolio investors invest in the US market, i.e., a reduction in \((1 - \alpha^*)\). The change in \(s^\alpha\) that generates a reallocation of wealth from dollars to other currencies equal to 4.65% of US GDP is given by:

\[
\Delta s^\alpha = \frac{4.65}{100} \cdot \frac{\text{US}_GDP}{(1 - n) \cdot (1 - \alpha^{E*}) \cdot V^{P*}} \cdot E = \frac{0.66}{(1 - n) \cdot (1 - \alpha^{E*}) \cdot V^{P*}} \cdot E \tag{4.21}
\]

The denominator in this expression is total wealth of ROW portfolio investors: there is a proportion \((1 - n) \cdot (1 - \alpha^{E*})\) of these investors in the world economy, each with wealth equal to \(V^{P*}\). This expression gives us the size of the shock for path 1.

4.4.2 Path 2. Shock to Asset Composition

A shift of 3 trillion dollars of “excess reserves” from central banks to SWFs reduces the amount that foreign investors invest in US bonds and increases the amount that they invest in US equities by \((1 - 0.29) \times 3 \times 0.6 = 1.278\) trillion dollars. This corresponds to \(\frac{1.278}{142} \times 100 = 9\%\) of US GDP.
4.5. BASELINE RESULTS

In terms of the parameters of our model, this corresponds to an increase in $\gamma^*$, the share of wealth that ROW portfolio investors invest in equities in the US market. The change in $s^{\gamma^*}$ that delivers an increase in investment in equities equal to 9% of US GDP is given by:

$$\Delta s^{\gamma^*} = \frac{9}{100} \cdot \frac{US_{GDP}}{(1-n) \cdot (1-\alpha^{E*}) \cdot V^{P*} \cdot E} = \frac{1.278}{(1-n) \cdot (1-\alpha^{E*}) \cdot V^{P*} \cdot E} \quad (4.22)$$

This gives us the size of the shock for path 2.

4.5 Baseline Results

For the baseline results, we calibrate the steady state using the numbers in Table 4.1. In this section, we show impulse responses for all the key variables in the model: US and ROW asset prices and returns, investment, capital stock, GDP, wages, consumption, the exchange rate, US trade deficit and net debt, and the exorbitant privilege. Looking at the full set of impulse responses allows us to understand the mechanisms through which the shocks operate. For the robustness checks we focus only on the responses of the exchange rate, US trade deficit and net debt and the exorbitant privilege.

4.5.1 Path 1. Shock to Currency Composition

Given the steady state parameters in Table 3.1 and our assumption about the size of the shock to portfolio preferences, equation 4.21 implies an increase in the share that foreign investors invest in ROW assets ($\alpha^*$) from 74% to 74.56%. This is a small increase and we should not expect it to have a large impact. We could assume a larger shock, for example, if we believe that currency diversification by SWFs will lead to herding behaviour, inducing other investors to also move away from dollar assets. However, our aim is not so much to analyze the quantitative impact of the shock, but to highlight the channels through which it impacts on the economy.

We expect that, as ROW investors shift demand from dollar assets to ROW assets, the price of US assets should fall and the price of ROW assets should rise. Chart 1 plots the evolution of the prices of US and ROW assets. There is a reduction in the price of US equities and bonds and an increase in the price of ROW equities and bonds. Since foreign investors are less willing to invest in dollar assets, we would expect the return on these assets to rise so that the US can continue attracting foreign investment and is able to maintain its current account balance. Chart 2 (a) and (b) shows the response of the return on US equities.

---

9 See Corsetti et al (2004) for a model in which a large trader may influence the actions of small traders.
and bonds. The return on equities includes capital gains or losses arising from movements in equity prices. Because the price of US equities falls in the first period after the shock, the return on US equities also falls, but it rises after that. The return on US bonds rises in response to the shock. The returns on ROW assets decrease (except in the first period after the shock), as illustrated in chart 2 (c) and (d). Because portfolio shares respond to movements in expected returns, changes in asset returns generate further changes in the portfolio shares over time. For example, as the expected return on US assets increases and the expected return on ROW assets falls, US investors invest a larger share of their wealth in US assets, i.e., $\alpha$ rises.

Chart 3 (a) to (c) illustrates the response of US investment, capital stock and GDP. Investment is driven by the marginal value of capital, $q_t$, which coincides with the price of US equities. Therefore, the evolution of investment in chart 9 mirrors the evolution of the price of US equities in chart 1. Because the price of US equities fall, investment also falls, leading to a reduction in the US capital stock and GDP. The opposite effects happen in ROW, as shown in chart 3 (d) to (f).

Chart 4 shows the evolution of wages and consumption in the US and ROW. The reduction in the capital stock in the US reduces the marginal product of labour, which is equal to the wage. The wealth of US portfolio investors falls, both because of the fall in the wage and because of the fall in the return on their portfolio, $R^P_t$, given by equation 4.17. Since there is home bias in portfolio investment ($\alpha > 0.5$) and the return on US assets falls, $R^P_t$ falls. The decrease in the wealth of portfolio investors leads to a reduction in their consumption. Turning to US entrepreneurs, they invest all their wealth in home equities. Therefore, the dynamics of their wealth is entirely determined by the return on US equities, given in chart 2 (a). This return falls in the period after the shock due to the capital losses generated by the fall in the price of US equities, but rises afterwards in order to attract foreign investment and maintain the current account balance. Therefore, consumption of US entrepreneurs falls in the period following the shock and rises afterwards. Chart 4 (d) shows the evolution of aggregate consumption by US households. This is dominated by consumption of portfolio holders, since in our calibration they represent 80% of the US population. For ROW, we obtain the opposite effects on wages and consumption.

The exchange rate is defined as the relative price of US and foreign-produced goods. Its evolution, shown in chart 5, is determined by the relative demand for US and foreign-produced goods, including both consumption and investment demands. Charts 3 and 4 show that investment and consumption fall in the US and increase in ROW as a result of the shock. Because there is home bias in consumption and investment ($\rho > 0.5$), this implies a
reduction in the world demand for US-produced goods and an increase in the world demand for foreign-produced goods. Therefore, the exchange rate depreciates following the shock. Given our calibration, we obtain an immediate depreciation of 0.58%. This is a small effect but it is not surprising given the small size of the shock that we are assuming. Following this initial depreciation, the exchange rate appreciates again, following the increase in investment in the US and the decrease in investment in ROW. These changes in investment are driven by the changes in asset prices. As the return in US equities increases to attract foreign investment back into the US and maintain the current account balance, the demand for US equities increases, leading to an increase in their price, a recovery in investment and an appreciation of the dollar.

Chart 6 shows the evolution of the trade deficit, which is obtained as the difference between total expenditure and total output (equation 4.19). The dynamics of the trade deficit is dominated by the evolution of consumption in the US and ROW (chart 4 (d) and (h)). The trade deficit falls significantly following the shock (by 0.24 percentage points of GDP) and continues falling afterwards, as US consumers reduce their consumption of both home and foreign-produced goods and ROW consumers increase their consumption of both varieties of goods.

The evolution of the US “exorbitant privilege” is shown in chart 7. The “exorbitant privilege” rises in the period following the shock because capital gains increase the return the US receives on its investment in ROW equities and capital losses reduce the return the US pays on US equities. After the first period the privilege is reduced, as the return the US receives on its foreign assets falls and the return it pays on its foreign liabilities increases. Table 4.4 shows the long run decomposition of the privilege. The spread between the return on US external assets and liabilities falls from 3.85% to 3.7%. This is fully explained by a reduction in the return effect.

The evolution of US net debt is given in chart 8 and can be explained by changes in the different terms in equation 4.20. The initial depreciation and the reduction in the trade deficit leads to a fall in net debt equal to 1.9 percentage points of GDP. The rapid increase in US net debt in subsequent periods can be explained by different factors. First, there is an increase in the return the US pays on its existing stock of debt, because the returns on US equities and bonds increase. Second, the reduction in the spread between the return on US assets and liabilities (the “exorbitant privilege”) implies a higher accumulation of US net debt over time. Finally, the expected appreciation of the dollar following the initial depreciation reduces the dollar value of the returns the US receives on its foreign assets, contributing to a rise in net debt.
4.5. BASELINE RESULTS

4.5.2 Path 2. Shock to Asset Composition

For path 2, we introduce a shock to the asset composition of the portfolios of foreign investors, assuming that they keep the same share of investment in dollar assets, but diversify away from bonds into equities. Given the parameter values we chose, equation 4.22 gives an increase in the share foreign investors allocate to equities in the US market, $\gamma^*$, from 31% to 32.11%.

Charts 9 and 10 plot the evolution of the prices and returns of US and ROW assets. Substitution away from US bonds into US equities by ROW investors leads to an increase in the price of US equities and a reduction in the price of US bonds. The return on US equities increases in the first period after the shock reflecting capital gains caused by the increase in the price of equities and falls in subsequent periods. The return on US bonds rises in order to attract investors and clear the market for US bonds. The changes in relative asset returns lead to changes in portfolio shares, generating small movements in the prices and returns of ROW assets. In particular, the decrease on the expected relative return on US equities versus US bonds reduces the return that US investors receive when they invest in the US (given that their portfolios at home consist mostly of equities), leading to a fall in the share of wealth that they invest in the US, $\alpha$. At the same time, foreign investors now receive a higher return on their investment in the US, because they moved away from bonds into higher yielding equities. This induces them to invest a higher fraction of their wealth in US assets, i.e., $\alpha^*$ decreases. In our calibration, the first effect dominates and demand for ROW assets increases, leading to an increase in their prices.

Chart 11 plots the response of investment, the capital stock and GDP. As before, the evolution of investment mirrors the evolution of the price of US equities, given in Chart 9 (a). Investment rises in the period after the shock, since the increase in demand for US equities drives up their price. This leads to an increase in the capital stock and GDP. In subsequent periods, investors rebalance their portfolios again towards bonds, as a response to the increase in the relative return on US bonds versus US equities. For this reason, the price of US equities falls and investment falls, leading to a reduction in the capital stock and GDP. The increase in investment in ROW can be attributed to the increase in the price of ROW equities documented in chart 9 (c).

The increase in the capital stock in the US raises the marginal product of labour, which is equal to the wage. In spite of the increase in wages, consumption of US portfolio holders falls because the reduction in the price of US bonds requires an increase in lump sum taxes in order for the government budget constraint (equation 4.13) to be satisfied. This reduces the wealth of US portfolio investors. Consumption of US entrepreneurs is driven by the return

European University Institute
DOI: 10.2870/19601
4.6. ROBUSTNESS CHECKS

on US equities, plotted in chart 10 (a). Consumption rises in the first period after the shock and falls in subsequent periods. Aggregate consumption in the US mirrors the evolution of consumption of US portfolio holders, since these represent 80% of the US population in our calibration.

The evolution of the dollar exchange rate, given in chart 13, is determined by the relative demands of US and ROW-produced goods. We have seen that the shock increases investment both in the US and ROW in the period after the shock and reduces it in subsequent periods. For aggregate consumption, we have seen that it decreases in both regions immediately after the shock. In subsequent periods, aggregate consumption falls in the US and rises in ROW. Because there is home bias in consumption and investment, this implies a depreciation of the dollar and a reduction in the US trade deficit (chart 14).

Chart 15 shows the evolution of the US “exorbitant privilege”. Because foreign investors moved away from US bonds into higher yielding US equities, the US must pay a higher return on its liabilities and its “exorbitant privilege” is reduced. This effect diminishes over time as investors rebalance their portfolios in response to endogenous changes in asset returns. Table 4.5 shows the quantification of the short run and long run effects on the US “exorbitant privilege”.

The evolution of US net debt, given in chart 16, can be interpreted by changes in the different elements of equation 4.20. The depreciation of the dollar reduces net debt through two channels: first, it reduces the trade deficit; second, it increases the dollar value of the return the US receives on its foreign assets. But there is a counterbalancing effect coming from the reduction in the “exorbitant privilege”. The reduction in the spread between the return the US receives on its foreign assets and the return it pays on its foreign liabilities leads to a higher accumulation of net external debt. In our calibration, this effect dominates and US net debt increases over time.

4.6 Robustness checks

4.6.1 Degree of substitutability between assets

To test the robustness of our results to different assumptions about the degree of substitutability between assets, we simulate path 1 under different values of the parameter $b$. In particular, we follow Blanchard, Giavazzi, and Sá (2005) and set $b = 1$ and $b = 0.1$. To show the effect of a very limited degree of substitutability we also use $b = 0.0001$.

With a lower degree of substitutability between assets, asset demands are less responsive to changes in relative returns. Therefore, asset prices and returns have to move more in
order for asset markets to clear. The price of US equities falls by more and the price of ROW equities rises by more when there is a lower degree of substitutability between assets. Because investment is driven by the price of equities, a low degree of substitutability increases the divergence in the response of US and ROW investment. Bond price movements and the consequent fiscal effects are amplified as well, which rises the gap between US and ROW consumption. Therefore, the lower the degree of substitutability between assets, the bigger the drop in the relative demand for the US and the ROW-produced goods, the bigger the fall in their relative price, and the larger the depreciation of the exchange rate, as chart 17 illustrates.

A larger depreciation of the dollar makes the US trade deficit fall by more, while more volatile asset prices amplify movements in the “exorbitant privilege”, as depicted in charts 18 and 19. The higher dollar depreciation, pronounced reduction in the US trade deficit, and the higher initial increase in the privilege associated with low asset substitutability, amplify the initial reduction in US net debt (chart 20). Over time, however, US net debt rises by more when the degree of substitutability between assets is low because the larger reduction in the “exorbitant privilege” facilitates a progressive transfer of financial wealth from the US to ROW.

4.6.2 Elasticity of substitution between goods

We have also looked at the sensitivity of our results to different values of the elasticity of substitution between US and ROW-produced goods ($\theta$). We compare the results obtained for path 1 with $\theta = 0.97$ (the benchmark) and $\theta = 0.6$ (following Kollman (2006)).

The lower the elasticity of substitution between US and ROW-produced goods, the larger the exchange rate depreciation required to absorb the excess demand for ROW-produced goods that opens up following a fall in ROW demand for US assets (chart 21). Since with a low elasticity of substitution the relative demand for US and ROW-produced goods is less reactive to changes in their relative price, the depreciation of the dollar generates a smaller reduction in the US trade deficit when $\theta = 0.6$ (chart 22). For this reason, we would expect a smaller reduction in US net debt when the elasticity of substitutions between goods is low. However, there is an additional effect which operates through exchange rate valuation effects. The depreciation of the exchange rate increases the dollar value of the return the US receives on its external assets. This effect is stronger when $\theta$ is low because in that case the depreciation is larger. This effect works towards reducing US net debt (chart 23).
4.7 Conclusions

Our analysis highlights the channels through which changes in the portfolio allocation of foreign investors (due, for example, to the expansion of SWFs) may impact on asset prices and returns, consumption, investment, the exchange rate and net debt. The framework we use allows for endogenous determination of asset prices and returns and portfolio rebalancing in response to changes in asset returns. In addition, the dynamics of net external debt incorporates valuation effects arising from movements in the exchange rate.

We look at two different scenarios: in one scenario, foreign investors move away from US assets but keep the same share of investment in equities and bonds; in another scenario, they do not change the currency composition of their portfolios, but move away from US bonds into US equities. In the first scenario, the dollar depreciates in the period immediately after the shock, leading to a reduction in the US trade deficit and net debt. In subsequent periods, the return on US assets must increase to clear asset markets. This generates a rebalancing of the portfolios of foreign investors towards holding more dollar assets, which leads to an appreciation of the dollar. The “exorbitant privilege” in the US, i.e., the difference between the return it receives on its foreign assets and the return it pays on its foreign liabilities, decreases, and US net debt increases over time. In the second scenario, the dollar depreciates and the US trade deficit decreases. However, US net debt increases over time due to a reduction in the “exorbitant privilege”.

European University Institute
DOI: 10.2870/19601
4.8 Appendix A: Construction of portfolio shares

Share of US wealth invested in the US market ($\alpha$)

In the first quarter of 2008, the value of US financial wealth was 44.1 US trillion, from Table L100 in Federal Reserve (2008) The value of US-owned assets abroad was 17.6 US trillion, according to BEA (2008). Combining this two numbers, the share of US wealth invested in the US market, $\alpha$, is given by:

$$\alpha = 1 - \frac{17.6}{44.1} = 0.60$$

Share of ROW wealth invested in the ROW market ($\alpha^*$)

From IMF (2008b), total world financial wealth in 2006 (equal to the sum of stock market capitalization and the value of debt securities) was equal to 120 US trillion. Subtracting the value of US financial wealth, we obtain a value of ROW financial wealth equal to 75.9 US trillion. According to BEA (2008), the value of foreign holdings of US assets in 2007 was equal to 20.1 US trillion. Therefore, the share of ROW wealth invested in ROW assets ($\alpha^*$) is equal to:

$$\alpha^* = 1 - \frac{20.1}{75.9} = 0.74$$

Share of US wealth in ROW market allocated to equities ($\gamma$)

Using the data from Lane and Milesi-Ferretti (2007), we compute the share of US foreign assets allocated to equities (the remainder is allocated to bonds). The data distinguishes between FDI and portfolio equities. Because the only difference between the two is the degree of ownership, we consider them as a single asset class. This gives a value $\gamma = 0.56$.

Share of ROW wealth in the US market allocated to equities ($\gamma^*$)

In a similar way, we can use the data from Lane and Milesi-Ferretti to compute the share of US liabilities allocated to equities, which corresponds to the share of ROW assets in the US allocated to equities. With this calculation we obtain $\gamma^* = 0.31$.

Share of US wealth in US market allocated to equities ($\beta$)

Using data from Table L100 in Federal Reserve (2008), we can construct the overall shares of wealth that US investors allocate to equities and bonds, considering both the domestic and the foreign markets. This gives us:

$$share^{US,E} = 0.73$$
$$share^{US,B} = 0.27$$

Combining these shares with $\alpha$ and $\gamma$, it is possible to compute the share of US wealth allocated to equities in the US market, $\beta$: 

\[ \beta = share^{US,E} \times \alpha + share^{US,B} \times \alpha^* + \gamma \]
Share of ROW wealth in ROW market allocated to equities ($\beta^*$)

Data for $\beta^*$ is calculated in a similar way to $\beta$. First, we need to obtain the overall shares of wealth that foreign investors allocate to equities and bonds, considering both the US and ROW markets. For the Euro Area, we can obtain these shares from Table 3.1 in ECB (2008):

\[ \text{share}^{\text{EA.E}} = 0.72 \]
\[ \text{share}^{\text{EA.B}} = 0.28 \]

For Japan, we can use data from Bank of Japan (2008):

\[ \text{share}^{\text{Japan.E}} = 0.71 \]
\[ \text{share}^{\text{Japan.B}} = 0.29 \]

The shares for the Euro Area and Japan are very similar. We take the Euro Area shares as representative of the ROW:

\[ \alpha^{\text{ROW.E}} = 0.72 \]
\[ \alpha^{\text{ROW.B}} = 0.28 \]

Combining these shares with $\alpha^*$ and $\gamma^*$, we compute the share of ROW wealth allocated to equities in the ROW market, $\beta^*$:

\[ \text{share}^{\text{ROW.E}} = \gamma^* (1 - \alpha^*) + \beta^* \alpha^* \]
\[ 0.72 = 0.31 \times 0.26 + \beta^* \times 0.74 \]
\[ \beta^* = 0.86 \]
4.9 Appendix B: Figures and Tables

4.9.1 Path 1: Shock to Currency Composition

Chart 1 (a) Price of US Equities

Chart 1 (b) Price of US Bonds

Chart 1 (c) Price of ROW Equities

Chart 1 (d) Price of ROW Bonds

Chart 2 (a) Return on US Equities

Chart 2 (b) Return on US Bonds

Chart 2 (c) Return on ROW Equities

Chart 2 (d) Return on ROW Bonds

European University Institute
DOI: 10.2870/19601
4.9. APPENDIX B: FIGURES AND TABLES

Chart 5 Dollar Exchange Rate

Chart 6 US Trade Deficit

Chart 7 US Exorbitant Privilege - % points

European University Institute
DOI: 10.2870/19601
4.9.2 Path 2: Shock to Asset Composition

European University Institute
DOI: 10.2870/19601
4.9. APPENDIX B: FIGURES AND TABLES

Chart 10 (a) Return on US Equities

Chart 10 (b) Return on US Bonds

Chart 10 (c) Return on ROW Equities

Chart 10 (d) Return on ROW Bonds

Chart 11 (a) US Capital Stock

Chart 11 (b) US Investment

Chart 11 (c) US GDP

Chart 11 (d) ROW Investment

Chart 11 (e) ROW Capital Stock

Chart 11 (f) ROW GDP
4.9. APPENDIX B: FIGURES AND TABLES

Chart 14 US Trade Deficit

Chart 15 US Exorbitant Privilege - % points

Chart 16 US net debt

European University Institute
DOI: 10.2870/19601
4.9.3 Robustness checks: Degree of substitutability between assets (b)
4.9.4 Robustness checks: Elasticity of substitution between goods ($\theta$)
4.9. APPENDIX B: FIGURES AND TABLES

Chart 22 US Trade Deficit

Chart 23 US Net Debt

European University Institute
DOI: 10.2870/19601
## Table 4.1: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Portfolio shares</strong></td>
<td></td>
</tr>
<tr>
<td>Share of US wealth invested in US market</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Share of ROW wealth invested in ROW market</td>
<td>$\alpha^*$</td>
</tr>
<tr>
<td>Share of US wealth in US market allocated to equities</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Share of ROW wealth in ROW market allocated to equities</td>
<td>$\beta^*$</td>
</tr>
<tr>
<td>Share of US wealth in ROW market allocated to equities</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Share of ROW wealth in the US market allocated to equities</td>
<td>$\gamma^*$</td>
</tr>
<tr>
<td><strong>Rates of return</strong></td>
<td></td>
</tr>
<tr>
<td>Rate of return on US equities</td>
<td>$r$</td>
</tr>
<tr>
<td>Rate of return on ROW equities</td>
<td>$r^*$</td>
</tr>
<tr>
<td>Rate of return on US bonds</td>
<td>$r$</td>
</tr>
<tr>
<td>Rate of return on ROW bonds</td>
<td>$r^*$</td>
</tr>
<tr>
<td><strong>Other parameters</strong></td>
<td></td>
</tr>
<tr>
<td>US home bias in goods</td>
<td>$\rho$</td>
</tr>
<tr>
<td>US capital share</td>
<td>$\eta$</td>
</tr>
<tr>
<td>ROW capital share</td>
<td>$\eta^*$</td>
</tr>
<tr>
<td>Elasticity of substitution between goods</td>
<td>$\theta$</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Installation cost of capital</td>
<td>$\phi$</td>
</tr>
<tr>
<td>Degree of substitutability between assets</td>
<td>$b$</td>
</tr>
<tr>
<td>US productivity</td>
<td>$A$</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>$E$</td>
</tr>
<tr>
<td>US net debt to GDP ratio</td>
<td>$F/Y$</td>
</tr>
<tr>
<td>US consumption to GDP ratio</td>
<td>$C/Y$</td>
</tr>
<tr>
<td>ROW consumption to US GDP ratio</td>
<td>$C^*/Y$</td>
</tr>
<tr>
<td>Share of entrepreneurs in US economy</td>
<td>$aE$</td>
</tr>
<tr>
<td>Share of entrepreneurs in ROW economy</td>
<td>$aE^*$</td>
</tr>
<tr>
<td>Relative wealth of entrepreneurs in US economy</td>
<td>$VE/VP$</td>
</tr>
<tr>
<td>Relative wealth of entrepreneurs in ROW economy</td>
<td>$VE^<em>/VP^</em>$</td>
</tr>
<tr>
<td>Relative size of US population</td>
<td>$n$</td>
</tr>
</tbody>
</table>
Table 4.2: Decomposition of the US exorbitant privilege

<table>
<thead>
<tr>
<th></th>
<th>Our calibration</th>
<th>Gourinchas and Rey (2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>3.85%</td>
<td>3.32%</td>
</tr>
<tr>
<td>Return effect</td>
<td>2.65%</td>
<td>2.45%</td>
</tr>
<tr>
<td>Composition effect</td>
<td>1.2%</td>
<td>0.86%</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations

Table 4.3: Currency and asset composition of the portfolios of central banks and SWFs

<table>
<thead>
<tr>
<th></th>
<th>Central Banks</th>
<th>SWFs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Currency composition</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USD</td>
<td>60%</td>
<td>38%</td>
</tr>
<tr>
<td>Other</td>
<td>40%</td>
<td>62%</td>
</tr>
<tr>
<td><strong>Asset composition</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equities</td>
<td>0%</td>
<td>71%</td>
</tr>
<tr>
<td>Bonds</td>
<td>100%</td>
<td>29%</td>
</tr>
</tbody>
</table>

Sources: Data on the currency composition of central banks reserves is from the IMF COFER dataset (numbers for developing countries). Data on the currency and asset composition of SWFs portfolios is from IMF (2008a)
Table 4.4: Path 1 (currency diversification) – Effect on the US exorbitant privilege

<table>
<thead>
<tr>
<th>Exorbitant privilege</th>
<th>Initial calibration</th>
<th>Initial impact (at T=1)</th>
<th>Long run impact (at T=100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>3.85%</td>
<td>4.05%</td>
<td>3.7%</td>
</tr>
<tr>
<td>Return effect</td>
<td>2.65%</td>
<td>2.82%</td>
<td>2.5%</td>
</tr>
<tr>
<td>Composition effect</td>
<td>1.2%</td>
<td>1.23%</td>
<td>1.2%</td>
</tr>
</tbody>
</table>

Table 4.5: Path 2 (asset diversification) – Effect on the US exorbitant privilege

<table>
<thead>
<tr>
<th>Exorbitant privilege</th>
<th>Initial calibration</th>
<th>Initial impact (at T=1)</th>
<th>Long run impact (at T=100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>3.85%</td>
<td>3.72%</td>
<td>3.8%</td>
</tr>
<tr>
<td>Return effect</td>
<td>2.65%</td>
<td>2.5%</td>
<td>2.7%</td>
</tr>
<tr>
<td>Composition effect</td>
<td>1.2%</td>
<td>1.22%</td>
<td>1.1%</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


166


