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LEARNING ABOUT THE MATCH QUALITY: INFORMATION  
FLOWS AND LABOR MARKET OUTCOMES OF SKILL GROUPS

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*Learning about the Match Quality:  
Information Flows and Labor Market Outcomes of Skill Groups*

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**Abstract**

Workers with higher skills (more education) have lower unemployment rates, lower probabilities of separating from their jobs, and lower probabilities of losing their jobs conditional on tenure. This paper analyzes to what extent these differences can come from variations in the speed of learning about the suitability of an employee for the job (match quality) by skill. The speed of learning would affect labor market outcomes not only directly, but also through affecting firms' selectiveness in choosing whom to hire.

**Keywords**

Unemployment, Skill, Learning, Hiring Strategies, Job Search

**JEL Code:** E24, J64, J63.



Firms want to hire the worker most compatible for their vacant position, as not all workers are equally productive in the same job. However, whether a worker is compatible is unknown when firms decide who to hire. Firms invest in anticipating the most compatible worker among applicants (i.e., they decide how much effort to put into the selection of workers). They select a worker if the expectation that the worker is a good fit for the job is high enough. After the start of the employment relationship, they begin to learn about the true fit (the quality of this match) between the employee and the job, and decide whether to keep the worker.

This paper has two objectives. First, it aims to explore the effect of the speed of learning about the match quality on the choice of hiring strategies, thus contributing to our comprehension of the determinants of firms' choices of hiring strategies. Clearly, how much firms invest in hiring in order to increase their chances of finding a worker that is compatible depends on how soon after hiring they expect to learn about the true quality of the employment relationship. One expects the speed of learning about the match quality to have an adverse effect on hiring strategies. The longer it takes for a firm to be sure about the compatibility of a worker, the harder it becomes to detect the mistake of having hired the "wrong" worker. Thus, the slower the "learning" the more likely it is that the firm will want to reduce the risk of mistake by investing more in hiring.

The second goal of this paper is to further our understanding of differences in labor market outcomes by skill, which are discussed below. We expect the speed of learning about the quality of a match to depend on the skill requirements of the job. Although there is no direct evidence, due to the complexity of tasks at highly-skilled jobs, it would be harder for firms to recognize whether the employee is a good fit. As a result, we expect the speed of learning to be slower for highly-skilled jobs. To what extent do differences between the speed of learning among different skill groups account for the observed disparities between their labor market outcomes? This paper aims to shed light on this question.

As stated earlier, labor market outcomes differ by skill. Highly-skilled (highly educated) workers have lower unemployment rates and lower job separation probabilities in comparison to their low-skilled counterparts (for recent data, see Sengul (2009)). The difference between unemployment rates is mostly driven by the differences in job separation probabilities. Moreover, vacancy filling probabilities are

lower for jobs with higher skill requirements (Van Ours and Ridder 1993). Not only vacancy duration, but also how these vacancies are filled differ across skill groups. Barron, Berger, and Black (1997) show that employers “search more” when they hire workers with more education and prior experience, and for jobs with higher training requirements. As the education requirement of a vacancy increases, the number of interviews and the number of applicants per offer goes up.

Table 1: Labor Market Outcomes of Skill Groups;  
Tenure

	<b>Unemployment Rate</b>	<b>Job Finding Probability</b>	<b>Separation Probability</b>
Low-skill*	.058 (.02)	.36 (.06)	.026 (.004)
High-Skill	.023 (.006)	.32 (.07)	.009 (.002)

Flow Averages over Monthly CPS Data ( prime age [25–50] males), February 1976 - Dec 2008. \*: Workers with no college degree.

There are also differences in tenure profiles across skill. Abraham and Farber (1987) report that highly-skilled jobs have longer durations than low-skilled jobs. They also find that the fraction of workers with less than a year of completed tenure is lower in professional and managerial occupations than in blue collar occupations in the US. Although it is well documented that job separation probability decreases with tenure (see, for example, Farber (1999)), there is not much evidence on how this varies by skill. Bagger and Henningsen (2008) use Danish and Norwegian data and look at the job ending hazard rates by skill. They find that for all skill levels the likelihood of a job ending decreases with tenure. Furthermore, at low levels of tenure, low-skill workers are more likely to separate from their jobs, and the difference diminishes as the tenure increases.

There are few papers that explore separation rate differences across skill. Sengul (2009) shows that differences in hiring strategies are the key to explaining the observed differences in job separation probabilities across skill, while Nagypal (2007) argues that the differences in learning about match quality can also help theoretically explain the disparity in job separation probabilities.

I use a labor search-matching model, which builds on the model used by Pries and



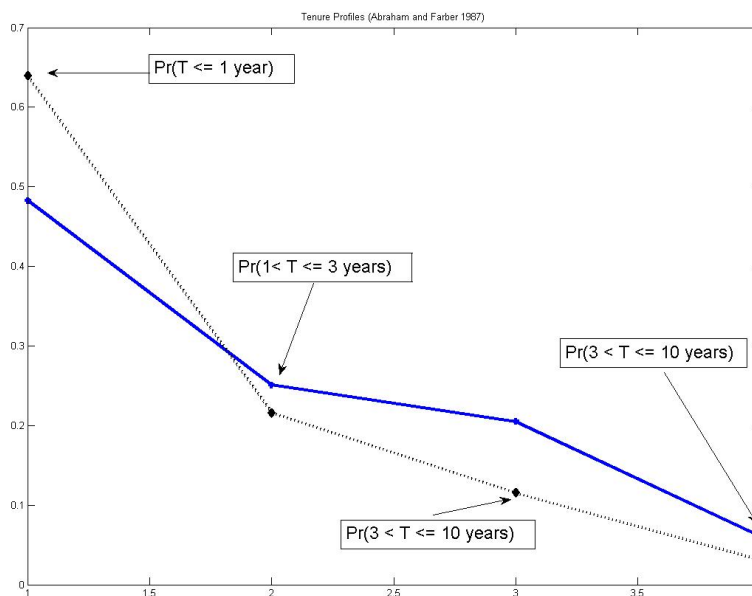


Figure 1: *Completed tenure (in years) distribution of employees. From Abraham and Farber (AER, 1987), PSID data.*

Rogerson (2005) to study the question posed above. There are two types of workers in the model: highly-skilled and low-skilled workers. The skill type is exogenous and workers are homogenous within a skill group. Highly-skilled workers are more productive than low-skilled workers. There are highly-skilled and low-skilled jobs which can employ only the workers from their own skill type. Firms are free to choose the type of jobs they want to create.

For each skill type, an employment relationship between a firm and a worker can be of either good or bad quality. Good-quality matches are expected to produce higher output than bad-quality matches within a skill group. The bad-quality matches are not profitable, and thus firms and workers do not want to be in a bad-quality match, regardless of the skill type.

When a firm with a vacancy meets an unemployed worker, the parties receive information regarding the probability of this match being of good quality. Based on this information (probability), they decide whether to form the employment relationship. If the information is good enough (if it meets the firm's hiring requirements/screening), then the worker is hired. Hence, there is a threshold probability value in equilibrium, above which all meetings result in an employment relationship.

The information parties receive is the outcome of the following process. The probability of a good-quality match is drawn from a "basic" distribution with probability  $1 - \lambda$  and from a "better" distribution with probability  $\lambda$ . The better distribution is more likely to deliver good-quality matches. Firms can choose the  $\lambda$  probability, incurring some cost. The higher the chosen value of  $\lambda$  is, the more likely it is that firms get a good-quality match. When deciding how much effort to put into hiring (what  $\lambda$  to choose), firms compare the expected value of returns to using the "better" distribution to the expected returns from using the "basic" distribution.

The output a match produces is a signal of the match quality. After the vacant firm and the unemployed worker decide to form the match, they observe the output every period and they either ascertain the true quality of the match or continue with the same beliefs about the match quality. How soon the parties can learn, i.e. the probability of learning the match quality, is exogenous. The speed of learning about the match quality is slower for highly-skilled jobs than it is for low-skilled jobs. This is the second difference between high- and low-skilled jobs. If the worker-firm pair learns that the match is of good quality, they stay attached until hit by an exogenous separation shock. If the match is revealed to be of bad quality, the parties unanimously decide to separate as bad-quality matches are not profitable. The separation decision is unanimous as parties share the net surplus from the match and there is no net surplus to share, hence no wage to agree on, for bad quality matches.

Preliminary results suggest that indeed firms invest more in hiring if they have a slower speed of learning. More investment in hiring makes employment relationships more stable on average, yielding lower endogenous separations. Hence, in this model, in which highly-skilled firms are more productive and have a slower speed of learning, labor market outcomes of highly-skilled workers are consistent with what we observe in the data.

The rest of the paper is organized as follows. The next section lays out the model. The equilibrium of the model is defined and analyzed in section 2. Section 3 presents the quantitative results and is followed by concluding remarks.

## 1 Model

To analyze the question posed above, I use a labor search-matching model that builds on the model used by Pries and Rogerson (2005). I assume that there are two types

of workers: highly-skilled and low-skilled. The skill type is exogenous and each type has a total mass equal to one.<sup>1</sup> Moreover, there are highly-skilled and low-skilled jobs which can employ only the workers of the same skill type. Firms are free to choose the type of jobs they want to create. I assume that there is no other interaction between skill types.

Within each skill group, firms and workers are ex-ante identical. Workers can be unemployed or employed, and there are filled and vacant jobs. A firm can employ at most one worker. All agents are risk neutral and they discount the future at a rate  $\beta$ . A production unit in the model is a firm-worker pair (match hereafter). A match between a firm and a worker can be of either good or bad quality. The output a match produces depends on the skill type of the firm-worker pair as well as the quality of the match. Good-quality employment relationships are expected to produce higher output than bad-quality matches within a skill group. Let  $y_j^i = y_j^i + \epsilon_j$  be the output produced by a match of skill type  $j$  and of quality  $i$ , where  $i$  can be good ( $g$ ) or bad ( $b$ ) and  $j$  can be highly-skilled ( $hs$ ) or low-skilled ( $ls$ ).  $y_j^i$  is the deterministic part of the output whereas  $\epsilon_j$  is the stochastic part. Observe that the noise term only depends on the skill type of the match. The assumption that good-quality matches produce higher output than bad-quality matches within a skill group implies  $y_j^g > y_j^b$  for all  $j$ . Moreover, I assume that the productivity gap between good and bad quality matches is larger for highly-skilled workers than it is for low-skilled workers, i.e.,  $y_{hs}^g - y_{hs}^b > y_{ls}^g - y_{ls}^b$ .

The stochastic term  $\epsilon_j$  is uniformly distributed over the interval  $[-\tilde{\epsilon}_j, \tilde{\epsilon}_j]$ , where  $\tilde{\epsilon}_j > \frac{y_j^g - y_j^b}{2}$ . Under this assumption, parties either ascertain the true quality of the match with probability  $\pi_j = \frac{y_j^g - y_j^b}{2\tilde{\epsilon}_j}$ , or continue with the same beliefs (i.e., all-or-nothing learning).<sup>2</sup> I assume that  $\tilde{\epsilon}_j$ , and hence  $\pi_j$ , is exogenous. Observe that  $\pi_j$  determines the speed of ex-post learning. If, for instance,  $\pi_j = 1$ , then parties learn the match quality after observing the first period of output as there would not be any output value which both a good and a bad quality match could have produced.

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<sup>1</sup>Since I assume that there is no interaction across skill groups, the size of these groups does not matter per se.

<sup>2</sup>Note that if  $y_j < y_j^g - \tilde{\epsilon}_j$ , parties know that the match must be of bad quality, while  $y_j > y_j^b + \tilde{\epsilon}_j$  indicates a good quality match. Hence, the probability that the match quality reveals,  $\pi_j$ , is

$$\pi_j = 1 - P(y_j^g - \tilde{\epsilon}_j < y_j^i + \epsilon_j < y_j^b + \tilde{\epsilon}_j) = 1 - \frac{y_j^b + \tilde{\epsilon}_j - y_j^g + \tilde{\epsilon}_j}{2\tilde{\epsilon}_j} = \frac{y_j^g - y_j^b}{2\tilde{\epsilon}_j}.$$

I assume that  $\frac{\tilde{\epsilon}_{hs}}{\tilde{\epsilon}_{ls}} > \frac{y_{hs}^g - y_{hs}^b}{y_{ls}^g - y_{ls}^b}$ , which implies that  $\pi_{hs} < \pi_{ls}$ .

In the model, unemployed workers and vacant jobs find each other via a function that endogenously determines workers' and firms' meeting probabilities. The  $M(u, v)$  function, generically named as the matching function, takes the numbers of unemployed workers ( $u$ ) and vacant jobs ( $v$ ) and determines the number of meetings between these parties ( $M$ ). The number of meetings determines the probability that an unemployed worker meets a vacancy ( $h_w = M(u, v)/u$ ) and the probability that a vacant job meets a worker ( $h_f = M(u, v)/v$ ). The matching function is such that  $h_w$  and  $h_f$  fall into the unit interval.

When a firm with a vacancy meets an unemployed worker, the parties learn the probability of this match being of good quality,  $\gamma$ , and decide whether to form the employment relationship. The probability of a good-quality match,  $\gamma$ , is drawn from either a "basic" distribution  $\Gamma$  or from a "better" distribution  $\Omega$ . These distributions are defined over the unit interval and have the standard properties of a cdf. Moreover,  $\Omega$  first order stochastically dominates  $\Gamma$ , i.e., in expectation, the better distribution is more likely to yield a good-quality match than the basic distribution. Let  $\lambda$  be the probability that the good quality match likelihood is drawn from the better distribution,  $\Omega$ . Firms choose  $\lambda$ , with a cost of  $C(\lambda)$ .<sup>3</sup>

Once the match is formed, the firm and the worker observe the level of output and learn the match quality with probability  $\pi$ . If the worker-firm pair learns the match is good quality, they stay attached until hit by an exogenous separation shock. If the match is revealed to be of a bad quality, the parties unanimously decide to separate. I assume that  $y^b \leq b$  and  $y^g > b$ . Under this assumption, bad-quality matches are not profitable in equilibrium.

## 1.1 Firms' Bellman Equations

I start by formalizing the firms' decision problem. A firm needs to form beliefs about the search strategies of other firms in its optimization problem as others' actions will affect wages. Let  $\bar{\lambda}$  be the belief about the other firms' search effort. Let me start with the value of a firm with a posted vacancy,  $V(\lambda, \bar{\lambda})$ . The value of a vacancy is

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<sup>3</sup>Alternatively, one could have modeled the selection as firms choosing a probability of having a good quality math. However this would not give results such that high- and low-skill workers have similar job finding probabilities and substantially different vacancy filling probabilities.

the discounted value of expected profits, net of the cost of the vacancy.

$$V(\lambda, \bar{\lambda}) = -c + \max_{\lambda \in [0,1]} \left\{ -C(\lambda) + \beta(1 - h_f)V(\lambda, \bar{\lambda}) + \beta h_f \left[ \lambda \int J(\gamma, \lambda, \bar{\lambda}) d\Omega + (1 - \lambda) \int J(\gamma, \lambda, \bar{\lambda}) d\Gamma \right] \right\}, \quad (1)$$

where  $h_f$  is the likelihood that the firm meets a worker, which is a function of the number of vacancies and unemployment, and  $J(\gamma, \lambda, \bar{\lambda})$  is the value of the firm when in a match which is of good quality with  $\gamma$  probability. Note that the firm pays a vacancy cost regardless of its search strategy. The firm's search choice depends on the cost of the search and how this choice affects  $J(\gamma, \lambda, \bar{\lambda})$ . Equation (2) formalizes the problem of a firm that is in a match with a worker.

$$J(\gamma, \lambda, \bar{\lambda}) = \max \left\{ V(\lambda, \bar{\lambda}), E(y|\gamma) - w(\gamma, \lambda, \bar{\lambda}) + \beta \delta V(\lambda, \bar{\lambda}) + \beta(1 - \delta) \left[ \pi(\gamma J(1, \lambda, \bar{\lambda}) + (1 - \gamma)J(0, \lambda, \bar{\lambda})) + (1 - \pi)J(\gamma, \lambda, \bar{\lambda}) \right] \right\} \quad (2)$$

where  $\delta$  is the exogenous probability of job destruction,  $w(\gamma, \lambda, \bar{\lambda})$  is the wage the firm pays, and  $E(y|\gamma)$  is the expected value of output, which is defined as  $E(y|\gamma) = y^h \gamma + y^l (1 - \gamma)$ .

The firm compares the expected discounted value of profits from producing output with the current worker to the discounted present value of separating from the worker (being vacant). The value the firm gets from producing is the sum of the current period profit, which is the expected value of output produced net of the wage paid to the worker, and the discounted value of being in a match with the same worker in the subsequent period, if the match survives.

## 1.2 Workers' Bellman Equations

Workers face a choice problem that is similar to that of the firms, except for the choice of search strategy. Let  $U(\bar{\lambda})$  be the value of unemployment to a worker, and  $W(\gamma, \bar{\lambda}, \bar{\lambda})$  be the value of being in a match with a firm where  $\gamma$  is the probability that the match is good quality and  $\bar{\lambda}$  is the workers' belief about the firms' choice. If a worker is unemployed, she gets the unemployment income,  $b$ , for the current period. With probability  $1 - h_w$  the worker does not meet any firms, and thus continues to be unemployed in the subsequent period. The worker meets a firm with probability

$h_w$  and gets an expected value from being in a match with a firm. The equation below states this sequence of events:

$$U(\bar{\lambda}) = b + \beta(1 - h_w)U(\bar{\lambda}) + \beta h_w \left[ \bar{\lambda} \int W(\gamma, \bar{\lambda}, \bar{\lambda}) d\Omega + (1 - \bar{\lambda}) \int W(\gamma, \bar{\lambda}, \bar{\lambda}) d\Gamma \right]. \quad (3)$$

Note that the expected value the worker gets from being in a match with the firm depends on the selection strategy the firm has chosen. The formal statement of a worker's decision problem when she is in a match is as follows:

$$W(\gamma, \bar{\lambda}, \bar{\lambda}) = \max \left\{ U(\bar{\lambda}), w(\gamma, \bar{\lambda}, \bar{\lambda}) + \beta\delta U(\bar{\lambda}) + \beta(1 - \delta) \left[ \pi(\gamma W(1, \bar{\lambda}, \bar{\lambda}) + (1 - \gamma)W(0, \bar{\lambda}, \bar{\lambda})) + (1 - \pi)W(\gamma, \bar{\lambda}, \bar{\lambda}) \right] \right\}. \quad (4)$$

The worker needs to decide between being unemployed and being in a match which is of a good quality with  $\gamma$  probability. If the worker chooses to be in the match, she receives a wage and continues to get a value which depends on the quality of the match in the subsequent period and whether the match survives the risk of destruction.

### 1.3 Wage Determination and Flows Across Employment States

Wages are determined according to the Nash bargaining rule, where workers' bargaining power is  $\mu$ . The wage rate that solves the bargaining problem is such that a worker gets a constant ( $\mu$ ) fraction of the net value generated by the worker-firm union. The wage is a weighted average of the expected output the match will produce and the worker's outside option, which is the value of unemployment.<sup>4</sup> After

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<sup>4</sup>Let the second argument of the maximization operation in equation (4) and equation (2) be  $\tilde{W}(\gamma)$  and  $\tilde{J}(\gamma)$ , respectively.

$$\tilde{W}(\gamma) = w(\gamma, \lambda, \bar{\lambda}) + \beta\delta U(\bar{\lambda}) + \beta(1 - \delta) \left[ \pi(\gamma W(1, \lambda, \bar{\lambda}) + (1 - \gamma)W(0, \lambda, \bar{\lambda})) + (1 - \pi)W(\gamma, \lambda, \bar{\lambda}) \right]$$

$$\tilde{J}(\gamma) = E(y|\gamma) - w(\gamma, \lambda, \bar{\lambda}) + \beta\delta V(\lambda, \bar{\lambda}) + \beta(1 - \delta) \left[ \pi(\gamma J(1, \lambda, \bar{\lambda}) + (1 - \gamma)J(0, \lambda, \bar{\lambda})) + (1 - \pi)J(\gamma, \lambda, \bar{\lambda}) \right]$$

Then, the wage is such that

$$\tilde{W}(\gamma) - U(\bar{\lambda}) = \mu \left\{ \tilde{W}(\gamma) - U(\bar{\lambda}) + \tilde{J}(\gamma) - V \right\}$$

some algebra, one can show that

$$w(\gamma, \lambda, \bar{\lambda}) = \mu[y^h\gamma + y^l(1 - \gamma)] + (1 - \mu)(1 - \beta)U(\bar{\lambda}). \quad (5)$$

The Nash bargaining assumption guarantees the unanimity of the separation or match formation decision. That is because parties bargain over the net surplus of the match, and if the surplus is positive (negative) they decide to form the match (separate). Hence there is no inconsistency across parties in decision making.

Note that at any period there will be two types of employment relationships; those that are known to be good quality matches and those with unknown quality. Let  $e_t^g$  and  $e_t^n$  denote the number of workers with good and unknown quality matches, respectively. Moreover, let

$$E(\gamma|\gamma^*) = \bar{\lambda} \frac{\int_{\gamma^*} \gamma d\Omega}{\int_{\gamma^*} d\Omega} + (1 - \bar{\lambda}) \frac{\int_{\gamma^*} \gamma d\Gamma}{\int_{\gamma^*} d\Gamma} \quad (6)$$

be the conditional expected value of  $\gamma$ .

The number of workers who have known (good) quality matches is the sum of the current period good quality matches that survive and the current period unknown quality matches that survive and turn out to be good quality.

$$e_{t+1}^g = (1 - \delta)e_t^g + e_t^n(1 - \delta)\pi E(\gamma|\gamma^*) \quad (7)$$

The number of unknown quality matches in the subsequent period are the current period unknown quality matches that survive and stay unknown and the newly-formed matches.

$$e_{t+1}^n = (1 - \delta)e_t^n(1 - \pi) + f(\theta)u_t \quad (8)$$

Unemployment evolves according to the following equation:

$$u_{t+1} = (1 - f(\theta))u_t + e_t^n(\delta + (1 - \delta)\pi(1 - E(\gamma|\gamma^*))) + e_t^g\delta \quad (9)$$

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Substituting  $\tilde{W}(\gamma)$  and  $\tilde{J}(\gamma)$  into one of these last two equations and solving for  $w(\gamma, \lambda, \bar{\lambda})$  gives the wage equation.

## 2 Equilibrium

It is more convenient to work with surplus equations, the total net value of a match, rather than the value of a match to the firm and the worker. Let  $S(\gamma, \lambda, \bar{\lambda})$  be the net value a match generates.

$$S(\gamma, \lambda, \bar{\lambda}) = W(\gamma, \lambda, \bar{\lambda}) - U(\bar{\lambda}) + J(\gamma, \lambda, \bar{\lambda}) - V(\lambda, \bar{\lambda}) \quad (10)$$

Substituting equations (4) and (2) into equation (10) for  $W(\gamma, \lambda, \bar{\lambda})$  and  $J(\gamma, \lambda, \bar{\lambda})$  respectively yields

$$S(\gamma, \lambda, \bar{\lambda}) = \max \left\{ 0, \frac{E(y|\gamma) + \beta(1-\delta)\pi\gamma S(1, \lambda, \bar{\lambda}) - (1-\beta)U(\bar{\lambda}) - (1-\beta)V(\lambda, \bar{\lambda})}{1 - \beta(1-\delta)(1-\pi)} \right\}. \quad (11)$$

**Definition:** The steady state equilibrium, for each sector, is a list  $\{v, u, \lambda^*, \bar{\lambda}, e^g, e^n, \gamma^*, w(\gamma), J(\gamma), V, W(\gamma), U, h_w, h_f\}$  such that

- $\{J(\gamma), V, W(\gamma), U\}$  satisfy equations (3), (4), (1), and (2).
- $V = 0$ .
- $w(\gamma, \bar{\lambda})$  is the solution to the Nash bargaining.
- $\gamma^*$  solves

$$0 = \frac{E(y|\gamma^*) + \beta(1-\delta)\pi\gamma^* S(1, \bar{\lambda}, \bar{\lambda}) - (1-\beta)U(\bar{\lambda}) - (1-\beta)V(\bar{\lambda}, \bar{\lambda})}{1 - \beta(1-\delta)(1-\pi)}$$

- employment stocks are such that

$$\begin{aligned} \delta e^g &= e^n(1-\delta)\pi E(\gamma|\gamma^*) \\ e^n &= (1-\delta)e^n(1-\pi) + f(\theta)u \\ u &= (1-f(\theta))u + e^n(\delta + (1-\delta)\pi(1-E(\gamma|\gamma^*))) + e^g\delta \end{aligned}$$

- $\lambda^*$  is selected optimally, and firms do not want to deviate from it (i.e.  $\bar{\lambda} = \lambda^*$ ).



## 2.1 Optimal Threshold Value

Firms and workers decide on a threshold probability of a good-quality match,  $\gamma^*$ , at which firms and workers are indifferent between the match and their outside options, i.e.  $S(\gamma^*, \bar{\lambda}, \bar{\lambda}) = 0$ . One can think of  $\gamma^*$  as the hiring standards firms impose. If a worker cannot meet these standards, i.e. the probability that this will be a good quality match is low, she will not be hired. It is formally defined as

$$\gamma^* = \frac{\left[ (1 - \beta)U(\bar{\lambda}) - y^l \right] (1 - \beta(1 - \delta))}{(y^h - y^l)(1 - \beta(1 - \delta)) + \beta(1 - \delta)\pi(y^h - (1 - \beta)U(\bar{\lambda}))}. \quad (12)$$

Assumptions  $y^b = b$  and  $y^g > b$  suffice for the existence of a threshold value.<sup>5</sup>

The equilibrium value of firms' effort choice in hiring affects the requirements they will pose. A higher value of  $\bar{\lambda}$  increases the  $\gamma^*$ . This is because the workers' outside options depend on the firms' search behavior. As firms increase their effort in hiring, they have better prospects of matches, and thus they offer higher wages, which increases workers' outside options. When the workers' outside options increase, the marginal worker and firm are not indifferent since the net surplus for this match is now lower. Hence, the threshold value goes up. Moreover, the higher the market tightness, the higher the threshold. A higher market tightness implies a better chance of workers meeting a vacant firm, and thus on average a better chance of having a good-quality match, compared to the case with lower market tightness. Hence, workers' outside options, and thus the threshold value, would rise.

The threshold likelihood of a good-quality match depends negatively on the speed of learning about the match quality. As firms and workers learn about the match quality faster, the expected profit from a prospective match increases, making the opportunity cost of forgoing the match increase. Hence firms impose lower hiring

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<sup>5</sup>Note that

$$S(1, \lambda, \bar{\lambda}) = \max \left\{ 0, \frac{y^g - (1 - \beta)U(\bar{\lambda}) - (1 - \beta)V(\lambda, \bar{\lambda})}{1 - \beta(1 - \delta)} \right\},$$

$$S(0, \lambda, \bar{\lambda}) = \max \left\{ 0, \frac{y^b - (1 - \beta)U(\bar{\lambda}) - (1 - \beta)V(\lambda, \bar{\lambda})}{1 - \beta(1 - \delta)} \right\}$$

As long as  $y^b < (1 - \beta)U(\bar{\lambda})$  and  $y^g > (1 - \beta)U(\bar{\lambda})$ , good quality matches are desirable while bad quality ones are undesirable (as  $V(\lambda, \bar{\lambda}) = 0$  in equilibrium). Thus, there will be a positive value for  $\gamma^*(\lambda, \bar{\lambda})$ , such that firms and workers are indifferent between forming a match vs. not forming one. Moreover, since  $(1 - \beta)U(\bar{\lambda}) = b + \frac{\theta\mu(c+C(\bar{\lambda}))}{(1-\mu)}$ , assumptions  $y^b = b$  and  $y^g > b$  guarantee the existence of an equilibrium (for sufficiently large  $y^g - b$ ).

standards.

## 2.2 Optimal Selection Effort

Notice that the surplus function is linearly increasing in  $\gamma$ . Thus, we can write  $S(\gamma) = S'(\gamma - \gamma^*)$ ,  $\forall \gamma \geq \gamma^*$  where

$$S' = \frac{y^h - y^l}{1 - \beta(1 - \delta)(1 - \pi\gamma^*)}.$$

We can rewrite a firm's effort choice in hiring using surplus functions as follows:

$$\max_{\lambda \in [0,1]} \left\{ -C(\lambda) + \beta h_f(1 - \mu)S'(\gamma^*) \left[ \lambda \int (\gamma - \gamma^*)d\Omega + (1 - \lambda) \int (\gamma - \gamma^*)d\Gamma \right] \right\}.$$

The first order condition for the selection effort, in the interior, is:

$$C'(\lambda) = \beta h_f(1 - \mu)S' \left[ \int_{\gamma^*} (\gamma - \gamma^*)d\Omega - \int_{\gamma^*} (\gamma - \gamma^*)d\Gamma \right]. \quad (13)$$

Higher productivity (a higher  $y^h$ ) increases the value of a good quality match, thus making net returns from putting in more effort go up. As a result, a firm would want to put in more effort, for a given value of equilibrium variables.

How does the choice of selection effort depend on the speed of learning? As the speed of learning increases, firms' losses from a possible bad quality match go down (as firms would realize the loss and terminate the relationship faster with a higher speed of learning), while returns to a good quality match do not change. Hence expected returns from a match, conditional on match formation, increase. Match formation probability is also affected by the effort level firms choose. Hence, whether the firms want to put more effort into selection depends on the threshold value. For low threshold values the only difference between the basic and the better distributions is in their likelihood of giving a good quality match. Hence a faster speed of learning would reduce a firm's incentive to invest in effort. If the threshold value is high, then there is a significant probability that the firm will not have an acceptable match. In this case, a faster speed of learning makes a match more valuable, hence firm is more willing to get an acceptable match. However, for higher threshold values, the better distribution is more likely to yield an acceptable match, hence the firm is willing to put more effort in.

Whether a higher speed of learning delivers a higher vetting effort in equilibrium depends on how the value of a vacancy ( $V$ ) changes with the change in optimal effort. Recall that

$$V(\lambda) = -c - C(\lambda) + \beta h_f(1 - \mu)S'(\gamma^*) \left[ \lambda \int (\gamma - \gamma^*) d\Omega + (1 - \lambda) \int (\gamma - \gamma^*) d\Gamma \right].$$

Using the first order condition, we can rewrite this value as:

$$V(\lambda) = -c - C(\lambda) + \lambda C'(\lambda) + \beta h_f(1 - \mu)S'(\gamma^*) \int (\gamma - \gamma^*) d\Gamma.$$

Note that  $\beta h_f(1 - \mu)S'(\gamma^*) \int (\gamma - \gamma^*) d\Gamma$  is the deterministic part of the surplus from a match while  $\lambda' C'(\lambda') - C(\lambda')$  is the net value, the expected value net of the cost, of using the better distribution. When the speed of learning increases firms choose a new optimal vetting effort, say  $\lambda'$ . The new optimal effort would also satisfy the first order condition. Note that a change in the optimal effort level changes the net value-added of the better distribution as well as the deterministic surplus via changing workers' outside options. The net value-added moves in the same direction as the effort, while the change in deterministic surplus value moves in the opposite direction. Whether in equilibrium effort would increase or not depends on which of these effects dominate <sup>6</sup>

### 3 Implications for Labor Market Outcomes

In this model, the job finding probability and vacancy filling probability ( $f$  and  $q$ ) are

$$\begin{aligned} f &= h_w(\theta) \left( \bar{\lambda} \int_{\gamma^*(\bar{\lambda})} d\Omega + (1 - \bar{\lambda}) \int_{\gamma^*(\bar{\lambda})} d\Psi \right) \\ q &= h_f(\theta) \left( \bar{\lambda} \int_{\gamma^*(\bar{\lambda})} d\Omega + (1 - \bar{\lambda}) \int_{\gamma^*(\bar{\lambda})} d\Psi \right). \end{aligned}$$

Note that how match formation probability, which is  $\left( \bar{\lambda} \int_{\gamma^*(\bar{\lambda})} \gamma d\Omega + (1 - \bar{\lambda}) \int_{\gamma^*(\bar{\lambda})} d\Psi \right)$ , changes with  $\bar{\lambda}$  depends on the distribution and the threshold value. This is because, although a higher  $\bar{\lambda}$  puts more weight on the better distribution, it also increases

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<sup>6</sup>For instance, if the cost of vetting is convex enough, then a higher speed of learning results in a smaller effort of vetting in equilibrium.

the threshold value, which reduces the acceptance probability. Thus, a higher level of vetting may reduce the match formation probability. Moreover, a higher value of market tightness would increase the vacancy filling rate while decreasing the job finding probability. Since the effect of a higher level of vetting on match formation probability is ambiguous, so is the effect on job finding and separation probabilities.

The total separation rate  $s$  is

$$s = \delta \frac{\delta + (1 - \delta)\pi}{\delta + (1 - \delta)\pi E(\gamma|\gamma^*)}.$$

One can show that increase in the speed of learning increases the job separation probability, while an increase in effort decreases it.

The unemployment rate  $u$  will be

$$u = \frac{\delta \frac{\delta + (1 - \delta)\pi}{\delta + (1 - \delta)\pi E(\gamma|\gamma^*)}}{f + \delta \frac{\delta + (1 - \delta)\pi}{\delta + (1 - \delta)\pi E(\gamma|\gamma^*)}} = \frac{s}{f + s}.$$

Since the change in  $f$  is ambiguous, how unemployment responds to a change in  $\pi$  is also ambiguous.

A higher speed of learning will affect tenure directly, as most of the bad-quality matches are sorted out in the earlier periods of employment. Thus one would expect the empirical hazard rate to be higher at the beginning, and then decline fast. It would also affect tenure by changing the  $\bar{\lambda}$  and the threshold value. A lower threshold value means an expectation of a higher number of bad quality matches, and thus increases separations at all tenure levels for a given  $\pi$ . A higher investment in selection reduces the number of bad quality matches, resulting in a longer expected tenure.

Workers will start to work for a wage that depends on the likelihood of the match being of good quality. As production takes place, if the parties learn the true match quality the worker either loses her job or gets a wage raise. Note that workers will not get any further raise once they learn that they are in a good quality match. Recall the wage equation (equation (5)):

$$w(\gamma, \bar{\lambda}) = \mu(\gamma y^h + (1 - \gamma)y^l) + (1 - \mu)(1 - \beta)U(\bar{\lambda}).$$

Note that there is no direct effect of ex-post learning speed on a wage in a particular period. However, it will affect the distribution of wages. This is because

as ex-post learning increases, so does the fraction of matches that are good quality. Selection, i.e.  $\bar{\lambda}$ , also affects the distribution of wages, as well as the level.

An increase in  $\bar{\lambda}$  will increase the average wage rate as both workers' outside options and the conditional expected likelihood of good quality matches increase with  $\bar{\lambda}$ .<sup>7</sup> The effect on variance of wages is ambiguous because values of both  $\gamma$  and the conditional mean go up. Whether the variation will be larger or not depends mainly on the shape of the distributions.

## 4 A Quantitative Analysis

As the discussion above demonstrates, although we can explore the direct effects of a change in the speed of learning and the vetting effort on labor market outcomes, the total effects are mostly ambiguous. To explore the general equilibrium effects and equilibrium outcomes, I carry out the following numerical exercises.

Let me start by assigning values to parameters that are common across skills. The model period is one month. I choose the exogenous destruction rate  $\delta = 0.006$ , discount factor  $\beta = 0.9967$ , and bargaining parameter  $\mu = 0.5$ . I use the following functional form for the matching function:  $M = \frac{uv}{(u^\alpha + v^\alpha)^{\frac{1}{\alpha}}}$  where  $\alpha = 1$ . I normalize  $b_{ls}$  for low-skilled workers to 1. Moreover I assume  $C(\lambda) = \kappa\lambda^{1.5}$ .<sup>8</sup> I assume

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<sup>7</sup>Let  $w_s$  denote the starting wage of a new hire. Then

$$\begin{aligned}
 Ew_s &= \mu(y^h - y^l)E(\gamma|\gamma^*) + \mu y^l + (1 - \mu)(1 - \beta)U(\bar{\lambda}) \\
 \sigma_s^2 &= \frac{\bar{\lambda} \int_{\gamma^*(\bar{\lambda})} (w(\gamma, \bar{\lambda}) - Ew)^2 d\Omega}{\int_{\gamma^*(\bar{\lambda})} d\Omega} + \frac{(1 - \bar{\lambda}) \int_{\gamma^*(\bar{\lambda})} (w(\gamma, \bar{\lambda}) - Ew)^2 d\Gamma}{\int_{\gamma^*(\bar{\lambda})} d\Gamma} \\
 \sigma_s^2 &= (\mu(y^h - y^l))^2 \left[ E(\gamma^2|\gamma^*) - E(\gamma|\gamma^*)^2 \right]
 \end{aligned}$$

The average and the standard deviation of overall wages are:

$$\begin{aligned}
 Ew &= \frac{\delta \mu \left( (y^h - y^l)E(\gamma|\gamma^*) + y^l \right)}{\delta + (1 - \delta)\pi E(\gamma|\gamma^*)} + \frac{(1 - \delta)\pi E(\gamma|\gamma^*) \mu y^h}{\delta + (1 - \delta)\pi E(\gamma|\gamma^*)} + (1 - \beta)U(\bar{\lambda}) \\
 \sigma^2 &= \mu^2 (y^h - y^l)^2 \left[ E(\gamma^2|\gamma^*) - 2 \frac{(\delta + (1 - \delta)\pi)(E(\gamma|\gamma^*))^2}{\delta + (1 - \delta)\pi E(\gamma|\gamma^*)} + \frac{(\delta + (1 - \delta)\pi)^2 (E(\gamma|\gamma^*))^2}{(\delta + (1 - \delta)\pi E(\gamma|\gamma^*))^2} \right].
 \end{aligned}$$

<sup>8</sup>A steeper marginal cost for lower values of vetting effort makes a gap in chosen effort between high- and low-skilled firms.

Table 2: Parameter Values

Parameter	Value	Definition
$\beta$	0.996	4% discount factor
$\mu$	0.5	workers' bargaining power
$\alpha$	0.85	Matching function parameter
$\delta$	0.006	Exogenous fraction of job destructions
$c$	1.2	Vacancy creation cost
$\kappa$	3	Search cost parameter
$b_{ls}$	1	unemployment income, low-skilled
$y_{ls}^l$	$b_{ls}$	bad-quality output, low-skilled
$y_{ls}^h$	2	good-quality output, low-skilled
$\pi_{ls}$	1/6	low-skilled learning speed
$\pi_{hs}$	1/12	high-skilled learning speed
$b_{hs}$	1	unemployment income, high-skilled
$y_{hs}^l$	$b_{hs}$	bad-quality output, high-skilled
$y_{hs}^h$	$2y_{ls}^h$	good-quality output, high-skilled

that the speeds of learning for low-skilled and highly-skilled jobs are such that, on average, the match quality is revealed within 6 and 12 periods of employment for low-skilled and highly-skilled respectively. The values of all parameters are reported in Table 2.

I assume the distributions from which the good-quality match likelihood comes are power distributions where the basic distribution  $\Psi(\gamma) = \gamma^\alpha$ , where  $\alpha$  is 0.5 for both skill groups. Moreover,  $\Omega(\gamma) = n\Psi(\gamma)^{(n-1)}\psi(\gamma) = n\alpha\gamma^{n\alpha-1}$ , where  $n = 3$ .

Given the parameter values above; I run the model for both high-skill and low-skill sectors, for different values of ex-post learning. First, notice that, for each skill group, as the speed of learning increases the vetting ( $\lambda$ ) decreases (as well as the threshold probability for an acceptable match). At the same time, market tightness goes up. As a result, a worker's probability of meeting a firm and the match formation probability go up, resulting in a higher probability of finding a job. Moreover, average output increases because, despite less vetting, the fraction of matches that are good quality increases (since bad quality ones are detected faster). Also notice that as the speed of learning increases the probability of losing one's job in a year increases (see the appendix for the calculations of these probabilities). These changes, although different quantitatively, are the same qualitatively across

skill groups.

One can see the effects of a change in productivity on labor market outcomes by comparing the outcomes for different skill groups for a given speed of learning about the match quality. When the productivity is higher, the unemployment and job separation probabilities are lower, while the job finding probability is higher. Moreover, the vetting and the threshold probability values are also higher. The table also reveals that productivity does not have large effects on the probability of keeping one's job for more than a year.

## 5 Concluding Remarks

Could differences in the speed of ex-post learning across skills explain the differences in employers' search strategies? We should expect that the speed of learning depends on the skill requirements of a job. Due to the complexity of tasks in more highly-skilled jobs, it would be harder for firms to recognize whether the employee is a good fit. As a result, we expect the speed of ex-post learning to be slower for highly-skilled jobs. In return, one anticipates that firms would invest more in searching for a worker for jobs with higher skill requirements.

This paper shows that slower ex-post learning increases firms' ex-ante investment decision in selection while reducing the average productivity. I use a labor search-matching model, which builds on the model used by Pries and Rogerson (2005), with two types of workers: highly-skilled and low-skilled workers. For each skill type, an employment relationship between a firm and a worker can be of either good or bad quality. Good-quality matches are expected to produce higher output than bad-quality matches within a skill group. I assume that the bad-quality matches are not profitable, and thus firms and workers do not want to be in a bad-quality match, regardless of the skill type.

In this environment, high-skilled firms, the ones that have higher productivity and a slower speed of learning, are more likely to invest more in vetting. The numerical exercises show that the differences in ex-post learning is important not only explain the differences in employers' selection strategies across skill, but also inequalities in these skill groups' labor market outcomes.

Table 3: Some Numerical Exercises ( $C(\lambda) = \kappa(1 + \lambda)^\phi = 0.65(1 + \lambda)^{1.7}$ )

	( $\pi = 1/12$ )	( $\pi = 1/6$ )	( $\pi = 1/3$ )
<b>High-skill</b>			
$\lambda$	0.946	0.713	0.372
unemployment rate	0.053	0.051	0.052
job finding pr.	0.288	0.346	0.407
job separation pr.	0.016	0.019	0.022
vacancy filling pr.	0.396	0.383	0.346
match formation pr.	0.771	0.825	0.850
$h_{worker}$	0.373	0.420	0.478
$h_{firm}$	0.513	0.465	0.407
$\theta$	0.727	0.903	1.174
$\gamma^*$	0.201	0.149	0.097
$\frac{e^n}{1-u}$	0.180	0.109	0.066
average output	3.555	3.735	3.836
$Pr(\text{Tenure} \leq 1\text{year})$	0.451	0.634	0.752
$Pr(1 < \text{Tenure} \leq 3\text{years})$	0.257	0.125	0.040
<b>Low-skill</b>			
$\lambda$	0.6	0.314	0.042
unemployment rate	0.106	0.102	0.099
job finding pr.	0.152	0.193	0.242
job separation pr.	0.018	0.022	0.027
vacancy filling pr.	0.606	0.582	0.575
match formation pr.	0.833	0.858	0.911
$h_{worker}$	0.183	0.224	0.265
$h_{firm}$	0.728	0.679	0.631
$\theta$	0.251	0.331	0.421
$\gamma^*$	0.130	0.089	0.056
$\frac{e^n}{1-u}$	0.204	0.127	0.079
average output	1.811	1.882	1.929
$Pr(\text{Tenure} \leq 1\text{year})$	0.477	0.673	0.793
$Pr(1 < \text{Tenure} \leq 3\text{years})$	0.266	0.125	0.035



Table 4: Some Numerical Exercises - 2 ( $C(\lambda) = ??$ )

	( $\pi = 1/18$ )	( $\pi = 1/12$ )	( $\pi = 1/6$ )
<b>High-skill</b>			
$\lambda$	0.815	0.732	0.517
unemployment rate	0.058	0.056	0.054
job finding pr.	0.240	0.278	0.340
job separation pr.	0.015	0.016	0.019
vacancy filling pr.	0.342	0.352	0.344
match formation pr.	0.656	0.711	0.774
$h_{worker}$	0.366	0.391	0.44
$h_{firm}$	0.521	0.495	0.445
$\theta$	0.702	0.79	0.988
$\gamma^*$	0.221	0.194	0.143
$\frac{e^n}{1-u}$	0.243	0.185	0.113
average output	3.348	3.52	3.713
$Pr(\text{Tenure} \leq 1\text{year})$	0.353	0.456	0.643
$Pr(1 < \text{Tenure} \leq 3\text{years})$	0.300	0.258	0.125
<b>Low-skill</b>			
$\lambda$	0.547	0.426	0.183
unemployment rate	0.112	0.108	0.102
job finding pr.	0.133	0.154	0.198
job separation pr.	0.017	0.019	0.023
vacancy filling pr.	0.569	0.564	0.558
match formation pr.	0.768	0.790	0.84
$h_{worker}$	0.173	0.194	0.236
$h_{firm}$	0.741	0.714	0.665
$\theta$	0.233	0.272	0.355
$\gamma^*$	0.149	0.126	0.087
$\frac{e^n}{1-u}$	0.273	0.21	0.132
average output	1.725	1.792	1.874
$Pr(\text{Tenure} \leq 1\text{year})$	0.374	0.483	0.681
$Pr(1 < \text{Tenure} \leq 3\text{years})$	0.313	0.268	0.125

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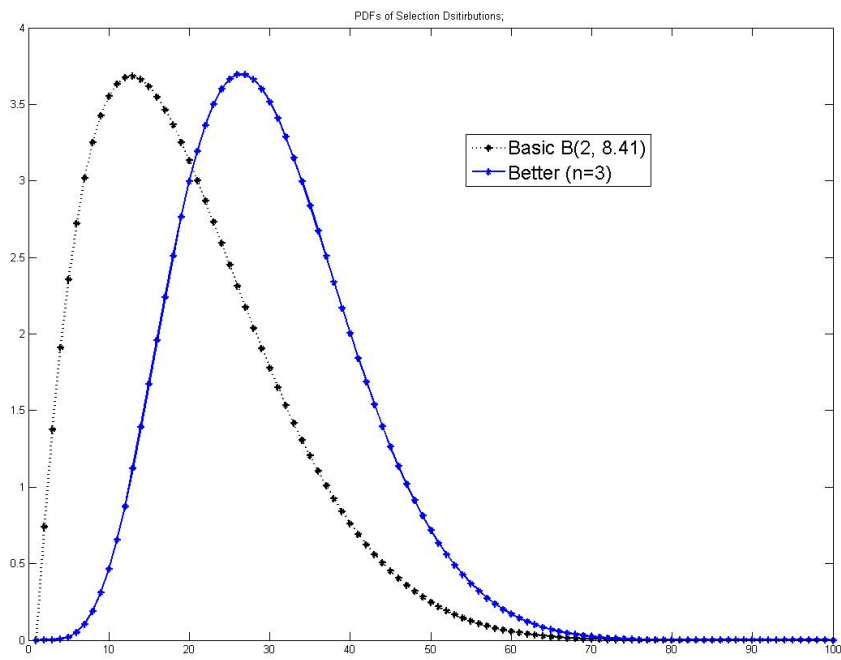
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## A Appendix

### A.1 Tenure

An employed worker is identified by her tenure and the belief about the match quality. Let  $n^u(\tau, \gamma)$  be the number of workers with  $\tau$  periods of tenure and with  $\gamma$  probability of good quality match in period t where  $\gamma < 1$ . Let  $n^k(\tau)$  be the number

Figure A.1: *PDFs of Vetting Distributions*



of known (good) quality matches with tenure  $\tau$ . Moreover, let  $X(\gamma) \in \{0, 1\}$  reflect the decision whether to form the match.

$$n_t^u(\tau, \gamma) = \begin{cases} u_{t-1} h_w(\theta_t) \left( \bar{\lambda} X(\gamma) d\Omega + (1 - \bar{\lambda}) X(\gamma) d\Gamma \right) & \tau = 1 \\ (1 - \delta)(1 - \pi) n_{t-1}^u(\tau - 1, \gamma) & \tau > 1 \end{cases} \quad (\text{A.1})$$

$$n_t^k(\tau, \gamma) = (1 - \delta) n_{t-1}^k(\tau - 1) + (1 - \delta) \pi \int_{\gamma} \gamma n_{t-1}^u(\tau - 1, \gamma) d\gamma. \quad (\text{A.2})$$

To see the effects on tenure, let us look at evolution of employment. Let  $\Delta(t, \gamma)$  be the fraction of  $\gamma$  quality matches among all the matches with  $t$  periods of tenure. After some algebra, one can show that,  $\forall \tau \geq 1$

$$n(\tau + 1, \gamma) = (1 - \delta)(1 - \pi) \Delta(\tau, \gamma) n(\tau),$$

$$n(\tau + 1, 1) = (1 - \delta) n(\tau) \left( 1 - \int_{\gamma} \Delta(\tau, \gamma) d\gamma + \pi \int_{\gamma} \gamma \Delta(\tau, \gamma) d\gamma \right).$$

$$n(\tau + 1) = (1 - \delta) n(\tau) \left( 1 - \int_{\gamma} \Delta(\tau, \gamma) d\gamma + \pi \int_{\gamma} \gamma \Delta(\tau, \gamma) d\gamma + (1 - \pi) \int_{\gamma} \Delta(\tau, \gamma) d\gamma \right).$$

Thus, the expression for  $\Delta(\tau, \gamma)$  becomes

$$\Delta(\tau + 1, \gamma) = \frac{(1 - \pi) \Delta(\tau, \gamma)}{1 - \pi \int \Delta(\tau, \gamma) d\gamma + \pi \int \gamma \Delta(\tau, \gamma) d\gamma}.$$

By iterations, one can show that

$$\Delta(\tau + 1, \gamma) = \frac{(1 - \pi)^\tau \left( \bar{\lambda} d\Omega(\gamma) + (1 - \bar{\lambda}) d\Gamma(\gamma) \right)}{(1 - \pi)^\tau \left( \bar{\lambda} \int_{\gamma^*} d\Omega + (1 - \bar{\lambda}) \int_{\gamma^*} d\Gamma \right) + \pi \sum_{j=0}^{\tau-1} (1 - \pi)^j \left( \bar{\lambda} \int_{\gamma^*} \gamma d\Omega + (1 - \bar{\lambda}) \int_{\gamma^*} \gamma d\Gamma \right)}.$$

Observe that one can write the probability of separating from a job in period

$\tau + 1$ , given that the employment has survived  $\tau$  periods, as follows:<sup>9</sup>

$$n(\tau + 1) = (1 - \delta)n(\tau) \left( 1 - \pi \int_{\gamma} \Delta(\tau, \gamma) d\gamma + \pi \int_{\gamma} \gamma \Delta(\tau, \gamma) d\gamma \right).$$

$$h(\tau + 1|\tau) = 1 - \frac{n(\tau + 1)}{n(\tau)} = 1 - (1 - \delta) \left( 1 - \pi \int_{\gamma} \Delta(\tau, \gamma) d\gamma + \pi \int_{\gamma} \gamma \Delta(\tau, \gamma) d\gamma \right),$$

Moreover, the likelihood of a job lasting at most  $T$  periods is then

$$Pr(\tau \leq T) = 1 - Pr(\tau > T) = 1 - \prod_{k=1}^{T-1} (1 - h(k + 1|k)).$$

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9

$$h(\tau + 1|\tau) = 1 - \frac{n(\tau + 1)}{n(\tau)} = 1 - (1 - \delta) \frac{(1 - \pi)^\tau \left( \bar{\lambda} \int_{\gamma^*} d\Omega + (1 - \bar{\lambda}) \int_{\gamma^*} d\Gamma \right) + \pi \sum_{j=0}^{\tau-1} (1 - \pi)^j \left( \bar{\lambda} \int_{\gamma^*} \gamma d\Omega + (1 - \bar{\lambda}) \int_{\gamma^*} \gamma d\Gamma \right)}{(1 - \pi)^{\tau-1} \left( \bar{\lambda} \int_{\gamma^*} d\Omega + (1 - \bar{\lambda}) \int_{\gamma^*} d\Gamma \right) + \pi \sum_{j=0}^{\tau-2} (1 - \pi)^j \left( \bar{\lambda} \int_{\gamma^*} \gamma d\Omega + (1 - \bar{\lambda}) \int_{\gamma^*} \gamma d\Gamma \right)}$$

