DEPOSIT INSURANCE WITHOUT COMMITMENT:
WALL ST. VERSUS MAIN ST.

Russell Cooper and Hubert Kempf
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and

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Russell Cooper† and Hubert Kempf‡

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Abstract

This paper studies the provision of deposit insurance without commitment in an economy with heterogeneous households. When households are identical, deposit insurance will be provided ex post to reap insurance gains. But the ex post provision of deposit insurance redistributes consumption when households differ in their claims on the banking system as well as in their tax obligations to finance the deposit insurance. Deposit insurance will not be provided ex post if it requires a (socially) undesirable redistribution of consumption which outweighs insurance gains.

1 Introduction

Within the framework of Diamond and Dybvig (1983), the implications of deposit insurance are well understood. If agents believe that deposit insurance will be provided, then bank runs, driven by beliefs, will not occur. In equilibrium, the government need not act: deposit insurance is never provided and costly liquidations need not occur. Instead, deposit insurance works through its effects on beliefs, supported by the commitment of a government to its provision.

Yet, recent events during the financial crisis leads one to question this commitment of the government. In many countries, such as the US, the parameters of deposit insurance were adjusted during the crisis period. In other countries, such as UK, ambiguities about the deposit insurance program contributed to banking instability. In yet other countries, such as China, the exact nature of deposit insurance is not explicit. And, in Europe, the combination of a common currency, the commitment of the ECB not to bailout member governments and fiscal restrictions, casts some doubt upon the ability of individual countries to finance deposit insurance as needed.

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†Economics Department, European University Institute, Florence, Italy and Department of Economics, University of Texas, Austin, russelcoop@gmail.com

‡Banque de France and Paris School of Economics, kempf@univ-paris1.fr
Finally, in all of these instances, there is also the question of how broadly to define a bank and thus the types of financial arrangements deposit insurance (in some cases interpretable as an \textit{ex post} bailout) might cover.\footnote{This was brought out clearly in a presentation, \url{http://www.federalreserve.gov/newsevents/speech/bernanke20100924a.htm}, by Ben Bernanke at Princeton University in September 2010.} The bailout of AIG, for example, along with the choice not to bail-out Lehman Brothers, makes clear that some form of deposit insurance is possible \textit{ex post} for some, but not all, financial intermediaries.

These events highlight ambiguities about the provision and extent of deposit insurance. This motivates our study of deposit insurance without commitment to identify conditions under which this insurance will be provided. A finding that deposit insurance will be provided \textit{ex post} establishes a firmer basis for the benefits of this insurance. A finding that deposit insurance will not be provided \textit{ex post} suggests guidelines for policy design \textit{ex ante} to change these \textit{ex post} incentives.

There are two central building blocks for our analysis. First there is the standard argument about gains to deposit insurance, as in Diamond and Dybvig (1983). These are present in the \textit{ex post} choice of providing deposit insurance since agents face the risk of obtaining a zero return on deposits in the event of a run.

Second, there are potential costs of redistribution across heterogeneous households that may not be desired. This depends on the social objective function. These costs of redistribution play a key role in the Cooper, Kempf, and Peled (2008) study of bailout of one region by others in a fiscal federation. That analysis highlights two motives for a bailout, the smoothing of consumption and the smoothing of distortionary taxes across regions.

Here, instead of regions, we consider an economy with heterogeneous households. The central trade-off we study is between the insurance gains of deposit insurance and the costs of the redistribution. The redistribution arises both from the distribution of deposits across heterogeneous households and the tax obligations needed to finance deposit insurance. As long as the insurance gains dominate, deposit insurance will be provided \textit{ex post} and there is no commitment problem. But, if the deposit insurance entails a redistribution from relatively poor households to richer households and the social welfare function places sufficient weight on poor households, then deposit insurance will not be provided.

In the bank runs literature following Diamond and Dybvig (1983), Ennis and Keister (2009) focus on \textit{ex post} interventions in the form of a “deposit freeze” and payment rescheduling. An important feature of that analysis is the lack of commitment: the decision on the policy intervention arises during the run. Keister (2010) studies the trade-off between the \textit{ex ante} incentive effects and \textit{ex post} gains to a bailout. Here the attention is on the design of \textit{ex ante} measures given the prospect of a bailout \textit{ex post}.

Neither of those papers focus on the heterogeneity across households and thus the redistributive aspects of deposit insurance. The redistributive effects of different forms of bailouts are surely present in the ongoing political debate, summarized as “Wall St. vs. Main St.”. These effects are central to the contribution of this paper.

Our presentation explores the trade-off between deposit insurance and redistribution. We begin with a planner’s problem in which the central authority has a sufficiently rich set of tools to redistribute across
agents both \textit{ex ante} and, in the event of a bank run, \textit{ex post} as well. If a bank run arises, the planner retakes control of the allocation process, choosing the allocation of consumption as well as the liquidation of any long term investments. We find that the planner will have an incentive to undertake this reallocation in the event of a bank run and that, given the prospect of this reallocation, a bank run will never arise.

We then turn to decentralized environments where the capacity for redistribution is progressively limited so that a trade-off emerges between redistribution and insurance. Richer households will gain from deposit insurance simply because of their larger claims on the banking system. Unless these claims are offset by progressive taxes to fund deposit insurance, the provision of deposit insurance is regressive: transferring resources from the poor to the rich. If this redistribution runs counter to social welfare, then deposit insurance may not be provided \textit{ex post} despite insurance gains.

\section{Planner’s Problem}

We begin with the planner’s problem to introduce the environment and to make clear conditions under which there is no tension between insurance and redistribution. The model is a version of Diamond and Dybvig (1983) with heterogeneity across agents. The model is structured to highlight a tension across agents based upon their claims on the financial system. Other differences across agents, perhaps in terms of the types of financial institutions they have access to, are not part of the focus of the model.

\subsection{Environment}

There are three periods, with \( t = 0, 1, 2 \). In periods 0 and 1, each household receives an endowment of the single good denoted \( \alpha = (\alpha^0, \bar{\alpha}) \). We index households by their period 0 endowment and refer to them as type \( \alpha^0 \). Let \( f(\alpha^0) \) be the pdf and \( F(\alpha^0) \) the cdf of the period 0 endowment distribution.

Households consume in either period 1, or in period 2. In the former case, households are called “early” consumers, in the latter case, “late” consumers. The fraction of early consumers for each type of household is \( \pi \). The preferences of households are determined at the start of period 1, after any saving decision. The utility from period 0 consumption is represented by \( u(c^0) \). Utility in periods 1 and 2 is given by \( v(c^E) \) if the household is an early consumer and by \( v(c^L) \) if the household is a late consumer. Both \( u(\cdot) \) and \( v(\cdot) \) are assumed to be strictly increasing and strictly concave.

There are two storage technologies available in the economy. There is a one period technology which generates a unit of the good in period \( t + 1 \) from each unit stored in period \( t \). Late households can store their period 1 endowment using this technology.

There is also a two period technology which yields a return of \( R > 1 \) in period 2 for each unit stored in period 0. This technology is illiquid though: it has a return of \( \epsilon < 1 \) if it is interrupted in period 1. The assumption of \( \epsilon < 1 \) implies that there is a non-trivial choice between investing in the two technologies.

\footnote{Here there are two important assumptions. First, \( \pi \) is independent of \( \alpha^0 \) and second there is no aggregate uncertainty in \( \pi \).}
2.2 Optimal Allocation

For the planner’s problem, we assume that the household type is observable so that the contract is contingent on the household’s endowment \( \alpha^0 \). In contrast, the household’s preferences are not assumed to be observed by the planner. So, though the contract is dependent upon realized household preferences, the allocation must be incentive compatible.

The planner chooses the type dependent functions \( (d(\alpha^0), x^E(\alpha^0), x^L(\alpha^0)) \) and the fraction of deposits to invest in the one period technology, \( \phi \), to maximize:

\[
\int \omega(\alpha^0)[u(\alpha^0 - d(\alpha^0)) + \pi v(\alpha^0 + x^E(\alpha^0)) + (1 - \pi) v(\alpha^0 + x^L(\alpha^0))] f(\alpha^0) d\alpha^0. \tag{1}
\]

Here the period 0 consumption of the household is its endowment less a deposit, \( \alpha^0 - d(\alpha^0) \). The period 1 consumption for an early consumer is the household’s endowment plus its transfer under the contract, \( \alpha^0 + x^E(\alpha^0) \). Likewise the period 2 consumption if the household is a late consumer is \( \alpha^0 + x^L(\alpha^0) \). For late consumers, the endowment in period 1 is saved to period 2 using the one-period technology.

The resource constraints for the planner are:

\[
\phi D = \pi \int x^E(\alpha^0) f(\alpha^0) d\alpha^0 \tag{2}
\]
and

\[
(1 - \phi) DR = (1 - \pi) \int x^L(\alpha^0) f(\alpha^0) d\alpha^0. \tag{3}
\]

Here \( \phi \) is the fraction of the overall deposits put into the one-period technology and \( d(\alpha^0) \) is the “deposit” of agent of type \( \alpha^0 \). Total deposits are denoted \( D = \int d(\alpha^0) f(\alpha^0) d\alpha^0 \). In (1) the welfare weight of a type \( \alpha^0 \) agent is \( \omega(\alpha^0) \). The weights sum to one without loss of generality.

The first-order condition with respect to \( d(\alpha^0) \) for this problem is:

\[
\omega(\alpha^0) u'(\alpha^0 - d(\alpha^0)) = \lambda \tag{4}
\]
for all \( \alpha^0 \) where \( \lambda \) is the multiplier on (2). This condition implies that the marginal utility of period 0 consumption, weighted by \( \omega(\alpha^0) \), is equal across households. Difference between the consumption allocation and endowment distribution in period 0 reflects redistribution through the tax system across heterogeneous agents.

The other first-order conditions are:

\[
v'(\alpha^0 + x^E(\alpha^0)) = Rv'(\alpha^0 + x^L(\alpha^0)) \tag{5}
\]
and

\[
v'(\alpha^0 + x^E(\alpha^0)) = u'(\alpha^0 - d(\alpha^0)). \tag{6}
\]
Condition (5) stipulates optimal insurance across being an early and a late consumer. The final condition ties down the intertemporal dimension of the consumption profile. Further, from (5), \( x^E(\alpha^0) < x^L(\alpha^0) \) and thus \( c^E(\alpha^0) < c^L(\alpha^0) \) as \( R > 1 \).

As a special case, suppose the weights are independent of the household endowment, i.e. \( \omega(\alpha^0) = \bar{\omega} \). Then these conditions imply that the consumption levels of all agents were independent of \( \alpha^0 \): there would be complete redistribution along with optimal risk sharing.

### 2.3 Runs and Deposit Insurance

Despite the presence of a planner, there is still the possibility of “runs”. Since we do not assume that planner observes the tastes of each household, we implement this allocation through a direct mechanism in which households announce their taste types to the planner.

In particular, at the start of period 1, after tastes are realized, households announce if they are early or late consumers. If there are not enough resources available from the liquid technology to meet the demands of the early consumers, then the planner retakes control of the allocation mechanism deciding whether to liquidate the long-term investment and how to distribute the existing goods among the claimants. As discussed in Diamond and Dybvig (1983), a key part of deposit insurance is avoiding inefficient liquidation of long-term illiquid investments. As they put it, “What is crucial is that deposit insurance frees the asset liquidation policy from strict dependence on the volume of withdrawals.” Thus it is important that not only deposit insurance be provided in some form but that liquidation be prevented as well.

In the spirit of sequential service, instead of making announcements about their tastes, households would instead line up to obtain their promised allocation of \( x^E(\alpha^0) \). Those near the front of the line would be served, others would not.

One equilibrium is truth-telling which implements the allocation solving (1). Since \( c^L(\alpha^0) > c^E(\alpha^0) \), late households have no incentive to claim to be early households as long as all others tell the truth.

But there is the possibility that each household would announce their taste to be “early” consumer, given that others are doing the same. If so, this is akin to a bank run. Only a fraction of the households who announced they were early consumers would be served.

If \( \epsilon \) is sufficiently close to zero, \( \pi < 1 \) implies that there is always a bank run equilibrium. To see this, note that (2) implies \( \phi D < \int x^E(\alpha^0)f(\alpha^0)d\alpha^0 \). The left side is the total amount of resources available to the economy while the right side, which is larger, is the total demands for consumption in period 1 if all agents announce they are early consumers. Since there are not enough resources to meet the demands of the households, each would strictly prefer to announce he is an early rather than a late consumer in order to have a positive probability of obtaining positive consumption.

More generally, regardless of the liquidation value, runs may still occur. Sufficient conditions for runs are discussed in Cooper and Ross (1998). The condition for runs depends jointly on the liquidation value, \( \epsilon \), and the curvature of \( v(\cdot) \). From (5), preferences will determine the magnitude of \( x^E(\alpha^0) \) relative to \( x^L(\alpha^0) \) and
thus the optimal choice of $\phi$. For a given $\epsilon$, a small value of $\phi$, reflecting $x^L(\alpha^0)$ large relative to $x^E(\alpha^0)$, is sufficient for a bank run.

In the event of a run, let $\zeta$ be the probability that a household is able to withdraw $x^E(\alpha^0)$ from the intermediation process. We assume $\zeta$ is not dependent on the household type. Since the total resources in the event of a run are $\phi D$, then $\zeta = \frac{\phi D}{\int x^E(\alpha^0)f(\alpha^0)d\alpha^0}$. The expected utility of a type $\alpha^0$ household during a bank run is

$$\zeta v(c^E(\alpha^0)) + (1-\zeta)v(\bar{\alpha}).$$

(7)

Thus sequential service implies that agents face consumption risk in a bank run with promised consumption of $c^E(\alpha^0)$ going to those served while the others consume their period 1 endowment, $\bar{\alpha}$.

When a run is under way, the planner reallocates the resources given the announced taste types of the households. In this new allocation, the planner will determine the consumption of early consumers and late consumers based upon their announced types. In addition, the planner can choose to liquidate some of the two-period investment or save some of the proceeds from the one-period technology to period 2. So this policy is comprehensive: it gives the planner complete control of the intermediation process.

**Proposition 1**

*Given a bank run, the planner has an incentive to reallocate consumption relative to the outcome under sequential service.*

**Proof.** The planner chooses the consumption allocations for the early and late households ($\tilde{x}^E(\alpha^0), \tilde{x}^L(\alpha^0)$) along with the storage from period 1 to 2, $S$, and the liquidation of the two-period technology, $Z$, to maximize:

$$\int \omega(\alpha^0)[\pi + \nu(\alpha^0)(1-\pi)][v(\bar{\alpha} + \tilde{x}^E(\alpha^0))]f(\alpha^0)d\alpha^0 + \int \omega(\alpha^0)[(1-\nu(\alpha^0))(1-\pi)][v(\bar{\alpha} + \tilde{x}^L(\alpha^0))]f(\alpha^0)d\alpha^0$$

(8)

where $\nu(\alpha^0)$ is the fraction of type $\alpha^0$ late consumers who announced they were early consumers. The resource constraint in period 1 is:

$$\int [\pi + \nu(\alpha^0)(1-\pi)]\tilde{x}^E(\alpha^0)f(\alpha^0)d\alpha^0 = \phi D - S + \epsilon Z.$$  

(9)

The resource constraint in period 2 is

$$(1-\pi)\int (1-\nu(\alpha^0))\tilde{x}^L(\alpha^0)f(\alpha^0)d\alpha^0 = (\phi D - Z)R + S.$$  

(10)

In these constraints, $S$ is additional storage from period 1 to period 2 and $Z$ is the amount of the two-period investment that is liquidated. Each unit liquidated yields $\epsilon$ units in period 1.

It is sub-optimal to get both $S > 0$ and $Z > 0$. Further, as there is a bank run, $S > 0$ is transferring resources into period 2 when they are needed in period 1. Thus we focus on the case of $S = 0$ and $Z \geq 0$.

If there is any liquidation, the first-order conditions imply
\[ v'(\bar{\alpha} + \tilde{x}_E(\alpha_0)) = \frac{R}{\epsilon} v'(\bar{\alpha} + \tilde{x}_L(\alpha_0)) \]  

(11)

and

\[ \omega(\alpha_0) v'(\bar{\alpha} + \tilde{x}_E(\alpha_0)) = \lambda^E \]

(12)

for all \( \alpha_0 \) where \( \lambda^E \) is the multiplier on (9).

Under the alternative allocation of sequential service, agents would face a probability of not being served. Some agents would receive their promised \( x^E(\alpha_0) \) while others would receive nothing.

Clearly this allocation, summarized by (7), is feasible for the planner \textit{ex post} but does not solve (8).

**Corollary 1** In the allocation characterized in Proposition 1, there is no bank run.

**Proof.** From (11), \( \tilde{x}_L(\alpha_0) > \tilde{x}_E(\alpha_0) \) since \( \frac{R}{\epsilon} > 1 \). That is, the consumption of late consumers exceeds that of the early consumers. Anticipating this reallocation in the event of a run, a late consumer would not have any incentive to run, regardless of the choices of the other late consumers. Hence there is no bank run: truth-telling is a dominant strategy.

Together, these results suggest that deposit insurance, in the form of this reallocation, will be provided without any need of commitment. The planner has both the incentive and the power to reallocate resources in a response to a run in an optimal fashion. The resulting allocation provides higher consumption for late consumers and an incentive for them not to run.

### 3 Decentralization

Instead of the optimal allocation from the planner’s perspective, we can also study the decentralized allocation through bank contracts. This approach has a couple of advantages. First, it allows us to focus on government provision of deposit insurance and the related taxation of period 1 endowments independent of period 0 redistribution. This provides some insights into the trade-off between redistribution and insurance. Second, we are able to use this structure to look at runs at a subset of banks.

Competitive banks offer contracts to households. Through this competition, the equilibrium outcome will maximize household utility subject to a zero expected profit constraint. Since household types are observable, the contracts will be dependent on \( \alpha_0 \).

For now, as in Diamond and Dybvig (1983), assume that neither the bank nor its customers place positive probability on a bank run. We study the possibility of runs given this optimal contract.

#### 3.1 Household Optimization

Given a contract stipulating a return on deposits in the two periods, \((r_1(\alpha_0), r_2(\alpha_0))\), the type \( \alpha_0 \) household chooses its deposit level to solve:

\[ ^3\text{Later we explore a case with restricted contracts in which private information makes this dependence infeasible.} \]
\[
\max_d u(\alpha^0 - d) + \pi v(\bar{\alpha} + r^1(\alpha^0)d) + (1 - \pi)v(\bar{\alpha} + r^2(\alpha^0)d).
\]

The first-order condition for the household is
\[
u'(\alpha^0 - d) = \pi r^1(\alpha^0)v'(\bar{\alpha} + r^1(\alpha^0)d) + (1 - \pi)r^2(\alpha^0)v'(\bar{\alpha} + r^2(\alpha^0)d)
\]
Denote the optimal deposit level as \(d(\alpha^0)\) and the value of this problem as \(U_{\alpha^0}(r^1(\alpha^0), r^2(\alpha^0))\).

### 3.2 Banks

The bank will choose a contract and an investment plan, \((r^1(\alpha^0), r^2(\alpha^0), \phi(\alpha^0))\) to maximize household utility, \(U_{\alpha^0}(\cdot)\), subject to a zero expected profit constraint for each type \(\alpha^0\). The bank will place a fraction of deposits, \(\phi(\alpha^0)\) into the liquid storage technology which yields a unit in either period 1 per unit deposited in period 0. The remainder is deposited into the illiquid technology.

The zero expected profit condition for a type \(\alpha^0\) contract is:
\[
r^1(\alpha^0)\pi d(\alpha^0) + r^2(\alpha^0)(1 - \pi)d(\alpha^0) = \phi(\alpha^0)d(\alpha^0) + (1 - \phi(\alpha^0))d(\alpha^0)R.
\]
To be sure the bank can meet the needs of customers, the following constraints must hold as well:
\[\phi(\alpha^0)d(\alpha^0) \geq r^1(\alpha^0)d(\alpha^0)\pi \quad \text{and} \quad (1 - \phi(\alpha^0))d(\alpha^0)R \geq r^2(\alpha^0)(1 - \pi)d(\alpha^0).\]

Clearly if the two constraints in (16) hold with equality, then the zero expected profit condition is met. Note that these conditions hold for any level of deposits.

### 3.3 Decentralized Allocation

The decentralized allocation maximizes \(U_{\alpha^0}(r^1(\alpha^0), r^2(\alpha^0))\) subject to (15) and (16) for each \(\alpha^0\). The first-order condition implies
\[
v'(\bar{\alpha} + r^1(\alpha^0)d(\alpha^0)) = Rv'(\bar{\alpha} + r^2(\alpha^0)d(\alpha^0)).
\]
In addition, the constraints in (16) are binding so that (15) holds.

Condition (17) is similar to condition (5) from the planner’s problem. Both conditions characterize optimal insurance across the two preference states for a household of type \(\alpha^0\). Of course, the levels of consumption need not be the same in the two solutions since the planner’s allocation allowed for redistribution through the choice of \(d(\alpha^0)\). Importantly, the welfare weights, \(\omega(\alpha^0)\) are not present in the decentralized allocation.
4 Systemic Runs and Deposit Insurance

Given that the optimal contract written is without any consideration of bank runs, we ask two questions. First, can there be a run without Deposit Insurance (DI)? Second, if so, will the government have an incentive to provide DI \textit{ex post}?

In this section, we assume there are runs by all agents on all banks in the system. We refer to this situation as “systemic runs”. Later, we study the case where there are runs on only a subset of the banks.

4.1 Are there runs?

For this decentralized allocation, if the two period technology is assumed to have essentially no value ($\epsilon$ near zero), a sufficient condition for runs is that the amount owed to all agents claiming to be early consumers is less than the resources available to meet these demands for each $\alpha^0$. This is equivalent to $\phi(\alpha^0) < r^1(\alpha^0)$ for each $\alpha^0$. Since (16) is binding, $\pi < 1$ implies $\phi(\alpha^0) < r^1(\alpha^0)$. In contrast to Diamond and Dybvig (1983) and Cooper and Ross (1998), the condition for runs is simple due to our assumption that the two period technology has essentially no liquidation value. For $\epsilon < 1$ but non-negligible, the conditions for runs are discussed in Cooper and Ross (1998). Since our focus is on the consequence rather than the conditions for runs, we focus here on the case of $\epsilon$ near zero. We indicate where this assumption is important and then discuss relaxing it in section 6.

4.2 Deposit Insurance

To study the provision of deposit insurance in the decentralized model, we adopt the same timing in period 1 as described for the implementation of the planner’s solution. The households announce their taste types, either through direct revelation or by being in line to withdraw. If a bank is unable to meet these claims, then government assistance through deposit insurance is requested. We study the conditions under which deposit insurance will be provided.

For the analysis of the government decision to provide deposit insurance, we assume that $\epsilon$ is close to zero. This allows us to focus on the provision of deposit insurance separately from the decision on the liquidation of the two period technology. With $\epsilon$ near zero, the liquidation decision is trivial. We return to this below when we consider the issue of whether the provision of deposits insurance prevents runs.

Deposit insurance provides to each household its promised return of $r^1(\alpha^0)d(\alpha^0)$ under its deposit contract. This insurance is funded by the levy of a tax, $T(\alpha^0)$, on households.

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4As in Cooper and Ross (1998), we could also study the choice of a deposit contract given that runs are possible. This is of interest if the government does not have an incentive to provide deposit insurance.
4.2.1 Household Period 1 Utility under Deposit Insurance

If, *ex post* the government provides deposit insurance in the event of a run by all households, then welfare is:

\[ W^{DI} = \int \omega(\alpha^0) v(\bar{\alpha} + \chi(\alpha^0) - T(\alpha^0)) f(\alpha^0) d\alpha^0 \]  

(18)

where \( \chi(\alpha^0) \equiv r^1(\alpha^0)d(\alpha^0) \) is the total promised by the bank to an early household of type \( \alpha^0 \). If \( \omega(\alpha^0) \) is a constant, then the objective of the government is just a population weighted average of household expected utility. In general, the structure of \( \omega(\alpha^0) \) will be relevant for gauging the costs and benefits of the redistribution associated with DI.

Another key element in the redistribution is the tax system used to pay for DI. In (18), \( T(\alpha^0) \) is the tax paid by an agent of type \( \alpha^0 \). Government budget balance requires

\[ \int [T(\alpha^0) + \phi(\alpha^0)d(\alpha^0)] f(\alpha^0) d\alpha^0 = \int \chi(\alpha^0) f(\alpha^0) d\alpha^0. \]  

(19)

The left-hand side of this expression is the total tax revenues collected by the government plus the liquid investment and the right-hand side is the total paid to depositors. If, *ex post*, there is no deposit insurance, then welfare is given by:

\[ W^{NI} = \int \omega(\alpha^0)[\zeta v(\bar{\alpha} + \chi(\alpha^0)) + (1 - \zeta)v(\bar{\alpha})] f(\alpha^0) d\alpha^0. \]  

(20)

Here \( \zeta \) is again the probability a household obtains the full return on its deposit.

The welfare values with and without DI are both calculated at the start of period 1. This is because the government lacks the ability to commit to DI before agents make their deposit decisions. The government can only react to an actual bank run in period 1.

4.2.2 Welfare Effects of DI

The government has an incentive to provide deposit insurance iff \( \Delta = W^{DI} - W^{NI} \geq 0 \). We can write this difference as

\[ \Delta = \int \omega(\alpha^0)[v(\chi(\alpha^0) + \bar{\alpha} - T(\alpha^0)) - v(\chi(\alpha^0) + \bar{\alpha} - \bar{T})] f(\alpha^0) d\alpha^0 + \int \omega(\alpha^0)[v(\chi(\alpha^0) + \bar{\alpha}) - v(\zeta \chi(\alpha^0) + \bar{\alpha})] f(\alpha^0) d\alpha^0 + \int \omega(\alpha^0)[v(\zeta \chi(\alpha^0) + \bar{\alpha}) - \zeta v(\chi(\alpha^0) + \bar{\alpha}) - (1 - \zeta)v(\bar{\alpha})] f(\alpha^0) d\alpha^0 \]  

(21)

where \( \bar{T} = \int T(\alpha^0) f(\alpha^0) d\alpha^0 \).

\(^5\)If the liquidation value of the illiquid investment was non-trivial, then any liquidated long-term investment would appear as additional resources on the left-side of the government budget constraint.
Here there are three terms. The first two terms capture the two types of redistribution through deposit insurance. One effect is through differences in tax obligations and the other effect comes from differences in deposit levels across types. The third term is the insurance effect of deposit insurance.

Specifically, the first term captures the redistribution from taxes. It is the utility difference between consumption with deposit insurance and type dependent taxes and consumption with deposit insurance and type independent taxes, $\bar{T}$.

The second term captures the effects of redistribution through deposit insurance. The term $v(\chi(\alpha^0) + \bar{\alpha} - \bar{T}) - v(\zeta\chi(\alpha^0) + \bar{\alpha})$ is the difference in utility between the consumption allocation if a type $\alpha^0$ household gets his promised allocation and bears a tax of $\bar{T}$ and the allocation obtained if all households received a fraction $\zeta$ of their promised allocation. This second part is the utility of the expected consumption if there are runs without deposit insurance.

The third term captures the insurance gains from DI. It is clearly positive if $v(\cdot)$ is strictly concave. These gains are independent of the shape of $\omega(\alpha^0)$.

Thus the key trade-off to the provision of DI ex post is whether the insurance gains dominate the redistribution effects. Importantly, this trade-off was not present in the discussion of the planner’s solution. In that case, the ability of the planner to redistribute across the heterogenous households implied that the insurance gains from DI were independent of the redistribution. But in this decentralized economy they are coupled.

Assume there is no heterogeneity across households, so $F(\alpha^0)$ is degenerate. In this case, deposit insurance is valued as it provides risk sharing across households of the uncertainty coming from sequential service.

**Proposition 2** If $F(\alpha^0)$ is degenerate, $v(c)$ is strictly concave, then the government will have an incentive to provide deposit insurance.

**Proof.** In this case, the first two terms of (21) are zero. The third term is strictly positive since $v(\cdot)$ is strictly concave. Hence $\Delta > 0$. ■

This is parallel to the standard result in the Diamond and Dybvig (1983) model although it obtained here without commitment. It highlights the insurance gain from DI when there are no costs of redistribution. Here we see that the insurance benefit is enough to motivate the provision of deposit insurance without commitment.

In our environment of heterogeneous households, this results may also reflect the consequence of redistribution across households in period 0. In particular, consider (1) with the restriction of equal welfare weights across households. When welfare weights are equal across households, the first-order conditions for the planner imply that consumption allocations are equal as well: $(\rho(\alpha^0), \rho(\alpha^0), \rho(\alpha^0)) = (\bar{\rho}, \bar{\rho}, \bar{\rho})$ for all $\alpha^0$. Given this equality of consumption allocations, Proposition 2 applies.

Note though that these insurance gains arise without any apparent costs because redistribution occurred in the initial period. Any heterogeneity across households was offset by taxes and transfers so that, as noted
earlier, consumption allocations were independent of $\alpha^0$ in the optimal allocation. Hence DI was provided for insurance reasons alone, as in Proposition 2.

When there is heterogeneity across households, these insurance gains may be offset by redistribution costs. The next two subsections study these redistribution effects with type dependent taxes. In doing so, we consider two situations. In the first one, the tax system to fund DI is set at the same time the decision is made to provide DI or not. In this case, there is enough flexibility in the tax system to offset any redistribution effects of DI. In the second scenario, we take the tax system as given and explore the incentives to provide DI.

4.3 Taxation \textit{ex post}: DI Will Be Provided

Consider a government which can choose the tax system used to finance DI at the same time it is choosing to provide insurance or not. This can be viewed as the choice of a supplemental tax to fund DI.

In this setting, $W^{DI}$ is the solution to an optimal tax problem:

$$W^{DI} = \max_{T(a^0)} \int \omega(\alpha^0)v(\chi(\alpha^0) + \bar{\alpha} - T(\alpha^0))f(\alpha^0)d\alpha^0$$

subject to a government budget constraint (19). The first-order condition implies that $\omega(\alpha^0)\frac{v'(\chi(\alpha^0) + \bar{\alpha} - T(\alpha^0))}{v'(\chi(\alpha^0) + \bar{\alpha} - T(\alpha^0))}$ independent of $\alpha^0$. This creates a connection between $\omega(\alpha^0)$ and $T(\alpha^0)$ which can be used to evaluate the gains to DI.

\textbf{Proposition 3} If $T(\alpha^0)$ solves the optimization problem (22), then deposit insurance is always provided.

\textbf{Proof.}

Using the first-order condition from (22), we rewrite (21) as:

$$\Delta = \int \left[ \frac{v(\chi(\alpha^0) + \bar{\alpha} - T(\alpha^0)) - v(\chi(\alpha^0))}{v'(\chi(\alpha^0) + \bar{\alpha} - T(\alpha^0))} f(\alpha^0) d\alpha^0 \right] +$$

$$\int \omega(\alpha^0)[v(\chi(\alpha^0) + \bar{\alpha} - T(\alpha^0)) - \zeta v(\chi(\alpha^0)) + (1 - \zeta) v(\bar{\alpha})] f(\alpha^0) d\alpha^0$$

The second term is positive as argued previously. The first term can be shown to be positive as well.

To see this, do a second-order approximation of the second part of the first term, $v(\chi(\alpha^0) + \bar{\alpha})$, around the first part, $v(\chi(\alpha^0) + \bar{\alpha} - T(\alpha^0))$. Using the fact that $\int T(\alpha^0) f(\alpha^0) d\alpha^0 = (1 - \zeta) \int \chi(\alpha^0) f(\alpha^0) d\alpha^0$, the first term reduces to

$$\int \frac{-((1 - \zeta)\chi(\alpha^0) - T(\alpha^0))^2 v''(\chi(\alpha^0) + \bar{\alpha} - T(\alpha^0))}{v'(\chi(\alpha^0) + \bar{\alpha} - T(\alpha^0))} f(\alpha^0) d\alpha^0$$

which is positive as $v(\cdot)$ is strictly concave. Thus $\Delta > 0$. ■

Why is there always a gain to deposit insurance here? Because with this \textit{ex post} tax scheme, the current government can undo any undesirable redistribution coming from DI. Thus the redistribution costs are not present.
This result is important for the design of policy. As governments strive to make clear the conditions under which deposit insurance and other financial bailouts will be provided \textit{ex post}, they ought to articulate how revenues will be raised to finance those transfers. If a government says it will not rely on existing tax structures but instead will, in effect, solve (22), then private agents will know that the government will have enough flexibility in taxation to overcome any redistributive costs of deposit insurance. This will enhance the credibility of a promise to provide deposit insurance \textit{ex post}.

4.4 Taxation \textit{ex ante}: Will DI Be Provided with Type Independent Taxes?

If the tax system to fund DI is not set \textit{ex post}, costly redistribution may arise. Then the trade-off between insurance gains and redistribution costs emerges. As we shall see, these redistribution effects can be large enough to offset insurance gains.

To study these issues, we return to (21) which cleanly distinguishes the redistribution and insurance effects. We start with a case in which taxes are independent of type to gain some understanding of the trade-off and then return to the more general case where taxes depend on agent types.

To focus on one dimension of the redistributive nature of deposit insurance, assume that taxes are type independent: $T(\alpha) = \bar{T}$ for all $\alpha$. Under this tax system, (21) simplifies to:

$$
\Delta = \int \omega(\alpha^0)[v(\chi(\alpha^0) - \bar{T} + \bar{\alpha}) - v(\zeta \chi(\alpha^0) + \bar{\alpha})]f(\alpha^0)d\alpha^0 + \int \omega(\alpha^0)[v(\zeta \chi(\alpha^0) + \bar{\alpha}) - v(\chi(\alpha^0) + \bar{\alpha}) - (1 - \zeta)v(\bar{\alpha})]f(\alpha^0)d\alpha^0. \tag{24}
$$

If taxes are independent of type, then the government budget constraint implies

$$
\bar{T} = \int [\chi(\alpha^0) - \phi(\alpha^0)d(\alpha^0)]f(\alpha^0)d\alpha^0. \tag{25}
$$

With type independent taxes, redistribution arises solely from differences in deposits across types. In some cases, this redistribution can be costly to society.

**Proposition 4** If $\omega(\alpha^0)$ is weakly decreasing in $\alpha^0$, then the redistribution effect of deposit insurance reduces social welfare.

**Proof.** The effects of redistribution are captured by the first term in (24). Using $\zeta = \frac{\int \phi(\alpha^0)d(\alpha^0)d\alpha^0}{\int \chi(\alpha^0)f(\alpha^0)d\alpha^0}$, $\bar{T} = (1 - \zeta) \int \chi(\alpha)f(\alpha)d\alpha$. Letting $\hat{\bar{c}}(\alpha^0) = \zeta \chi(\alpha^0) + \bar{\alpha}$ and $\hat{\bar{c}} \equiv \int \hat{\bar{c}}(\alpha^0)f(\alpha^0)d\alpha^0$, the first term in (24) becomes

$$
\int \omega(\alpha^0)[v(1 - \zeta)(\hat{\bar{c}}(\alpha^0) - \hat{\bar{c}}) + \hat{\bar{c}} - v(\hat{\bar{c}}(\alpha^0))]f(\alpha^0)d\alpha^0. \tag{26}
$$

The first consumption allocation, $\frac{1}{\zeta}(\hat{\bar{c}}(\alpha^0) - \bar{\alpha}) + \hat{\bar{c}}$, is a mean-preserving spread of the second, $\hat{\bar{c}}(\alpha^0)$. Both have the same mean of $\bar{\alpha}$ and since $1 > \zeta$ the variance of the first consumption allocation is larger. From the results on mean-preserving spreads, if $v(c)$ is strictly concave
\[
\int [v(\chi(\alpha^0) - \bar{T} + \bar{\alpha}) - v(\zeta\chi(\alpha^0) + \bar{\alpha})]f(\alpha^0)d(\alpha^0) < 0.
\] (27)

Using the fact that the welfare weights integrate to one, we can write the first term in (24) as

\[
\int [v(\chi(\alpha^0) - \bar{T} + \bar{\alpha}) - v(\zeta\chi(\alpha^0) + \bar{\alpha})]f(\alpha^0)d(\alpha^0) + \text{cov}(\omega(\alpha_0), v(\chi(\alpha^0) - \bar{T} + \bar{\alpha}) - v(\zeta\chi(\alpha^0) + \bar{\alpha})).
\] (28)

From the discussion above, the first term is negative. If \(\omega(\alpha^0)\) is independent of \(\alpha^0\), then the covariance term in (28) is zero and so (28) is negative. This corresponds to costly redistribution.

If \(\omega(\alpha^0)\) is decreasing in \(\alpha^0\), then social welfare puts less than the population weight on high \(\alpha^0\) agents. The difference, \(v(\chi(\alpha^0) - \bar{T} + \bar{\alpha}) - v(\zeta\chi(\alpha^0) + \bar{\alpha})\) in (28) is increasing in \(\alpha^0\) if \(\chi(\alpha^0)\) is monotonically increasing.

To see that \(\chi(\alpha^0)\) is increasing in \(\alpha^0\), the first-order condition for the household, (14), can be written as

\[
u'(\alpha^0 - d(\alpha^0)) = \pi r^1 v'(\chi(\alpha^0) + \bar{\alpha}) + (1 - \pi) r^2 v'(\alpha + r^2(\alpha^0)d(\alpha^0)).
\] (29)

The feasibility constraint for the bank, (16), along with the first-order condition for the optimal deposit contract, (17), implies

\[
u'(\alpha^0 - d(\alpha^0)) = v'(\chi(\alpha^0) + \bar{\alpha}).
\] (30)

From this expression, an increase in \(\alpha^0\) will lead to an increase in consumption in both period 0 and in period 1, for early consumers. For this to be the case, \(\chi(\alpha^0)\) must increase with \(\alpha^0\).

As a consequence, the covariance term in (28) is negative. Thus the redistribution effects reduce welfare if either \(\omega(\alpha^0)\) is either independent of, or decreasing in, \(\alpha^0\).

Proposition 4 makes clear that the provision of DI may entail distribution effects that are socially undesirable. There are two key pieces of the argument. First, if welfare weights are type independent, then the provision of deposit insurance financed by a lump-sum tax creates a mean preserving spread in consumption. This is welfare reducing. Second, if welfare weights are decreasing so that the rich are valued less than the poor in the social welfare function, then the redistribution from poor to rich from the provision of deposit insurance reduces social welfare further. This second influence is captured by the covariance term in (28).

This result contrasts with Proposition 2 which eliminates by assumption the redistribution issue and thus highlights the gains from the provision of deposit insurance. One important factor in the trade-off between insurance and redistribution is the underlying distribution of income and thus of deposits. In the following proposition we look at changes in the distribution of bank deposits, denoted \(H(\chi)\).

**Proposition 5** If \(v''(\alpha) < 0\) and \(\omega(\alpha_0)\) is constant, then \(\Delta\) is lower when \(H(\cdot)\) is replaced by a mean-preserving spread.
Proof.

We rewrite (24) to express the gains from deposit insurance using the distribution over claims on the bank, $\chi$ rather than endowments:

$$\tilde{\Delta} = \int [v(\chi - \bar{T} + \bar{\alpha}) - \zeta v(\chi + \bar{\alpha}) - (1 - \zeta)v(\bar{\alpha})]h(\chi)d\chi \quad (31)$$

where $h(\cdot)$ is the pdf over bank claims.

If $v'''(\cdot) < 0$, then by differentiation, $v(\chi - \bar{T} + \bar{\alpha}) - \zeta v(\chi + \bar{\alpha})$ is strictly concave in $\chi$. Thus if we replace $H(\cdot)$ with a mean-preserving spread, $\tilde{\Delta}$ will be lower.

These propositions highlight the redistributive effects of deposit insurance. Proposition 4 provides sufficient conditions for redistribution to be costly. Proposition 5 makes clear that these losses from redistribution depend on the distribution of deposits.

Of course, deposit insurance also has an insurance gain, as captured by the second term of (24). These gains can outweigh the redistribution costs and thus rationalize the provision of deposit insurance ex post.

To gauge the magnitude of this trade-off, we turn to an example.

4.4.1 Example

Here we consider a specific example to illustrate conditions for the provision of deposit insurance. Assume there are two types of households, rich and poor. The rich households have an endowment in youth of $\alpha^0 = \alpha^r$ and the poor have an endowment in youth of $\alpha^0 = \alpha^p$. Let the fraction of rich households be given by $f$. Assume $u(c) = \frac{c^{1-\gamma_0}}{1-\gamma_0}$ and that $v(c) = \beta \frac{c^{1-\gamma_1}}{1-\gamma_1}$. Thus there are two curvature parameters, $\gamma_0$ and $\gamma_1$.

Throughout this example, we set $\gamma_0 = 2$.

To compute an equilibrium, we solve for the optimal contract offered by a bank to a type $\alpha^0$ household. This involves finding a level of deposits and interest rates for early and late consumers that satisfy (14), (16) and (17). We also check that bank investment in each of the two technologies is non-negative and that interest rates are non-negative as well.

Given the contract, we can evaluate the social gains from deposit insurance by calculating $\Delta \equiv W^{DI} - W^{NI}$ using some welfare weights. Taxes are type independent.

Figures 1 and 2 provide some results. For these figures, $\bar{\alpha} = 1$, $\beta = 0.9$, $R = 1.10$ and the fraction of rich households was set at 50%. The utility difference from the provision of deposit insurance, $\Delta$, is shown on the vertical axis.

The effects of variation in risk version, $\gamma_1$, in $v(\cdot)$ are shown in Figure 1. Here the initial endowments were fixed at $\alpha^p = 3$ and $\alpha^r = 5$. The welfare weight for the poor is 0.80, larger than their population share.

For low values of risk aversion, the net gains to the provision of deposit insurance are negative. That is, the costs of redistribution offset the insurance gains. The utility difference increases as $\gamma_1$ increases and eventually becomes positive. Interestingly, for high enough values of $\gamma_1$, the utility gains from deposit insurance again start to fall off as the costs of redistribution become stronger.
Figure 2 studies the effects of income distribution on the gains from deposit insurance. On the horizontal axis is the welfare weight placed on poor households. One curve is the base case and the second, steeper, curve comes from a mean-preserving spread of endowments such that $\alpha^p = 2.8$ and $\alpha^r = 5.2$. This is the case labelled “MPS” in the figure. The curvature of $v(\cdot)$ was set at $\gamma_1 = 2$.

As is clear from this figure, the MPS of endowments reduces the gains to DI for all levels of the welfare weight for the poor below around 20%. The reason is that the spread of the endowments exacerbates the redistribution costs of the provision of deposit insurance. Only when the poor have a low welfare weight, does the redistribution in favor of the rich, combined with the provision of insurance, increase social welfare. After the MPS in endowments, if the weight on the poor exceeds 0.7, then deposit insurance will not be provided. This compares to a critical weight of about 0.8 in the baseline case.

### 4.4.2 Restricted Contract

A second way of highlighting the trade-off between redistribution and insurance is through the outcome of the model with a restricted contract. In particular, assume that the intermediary is restricted to offer the same contract to all agents: type dependent returns are not feasible. Further, suppose that a deposit contract is summarized by a single interest rate, denoted $r$, which is the annual gross return. So deposits for one period earn $r$ and deposits for two periods earn $r^2$.

With this simplified contract we continue to explore the trade-off between redistribution and insurance. The analysis of the household and banking problems with this restricted contract are similar to the more
The deposit of a type $\alpha_0$ household is given by $d(r, \alpha_0)$. The deposit is increasing in the endowment $\alpha_0$ and increasing in the deposit return $r$. Importantly, even if $v''(c) = 0$, the household will have a well defined deposit level as long as $u''(c) < 0$. Thus we can study the special case of risk neutrality in periods 1 and 2 in this model.\footnote{In the previous specification where the returns could differ for early and late consumers, at $v''(\cdot) = 0$, there consumption for early households went to zero. The restricted contract has the benefit of being better behaved when $v(\cdot)$ is linear.}

Given the deposit demand functions, a bank will choose a return $r$ and a portfolio to maximize expected utility of the households subject to zero profit and feasibility constraints. This is analogous to the problem specified above, though with a much simpler contract.

Using this model, we return to our discussion of costly redistribution and the risk sharing benefits of deposit insurance. For the restricted contract, if households are almost risk neutral with respect to variations in early and late consumption, DI will not be provided ex post if redistribution is costly enough.

**Proposition 6** If households are not too risk averse and $\omega(\alpha^0)$ is strictly decreasing in $\alpha^0$, then a government will not have an incentive to provide deposit insurance.

**Proof.**

With the restricted contract, (24) becomes

\[
\text{Figure 2: MPS on Endowment Distribution}
\]
Δ = \int \omega(\alpha^0)[v(rd(r, \alpha^0) - \bar{T} + \bar{\alpha}) - v(\zeta rd(r, \alpha^0) + \bar{\alpha})]f(\alpha^0)d\alpha^0 + \\
\int \omega(\alpha^0)[v(\zeta rd(r, \alpha^0) + \bar{\alpha}) - \zeta v(rd(r, \alpha^0) + \bar{\alpha}) - (1 - \zeta)v(\bar{\alpha})]f(\alpha^0)d\alpha^0.

(32)

Suppose \(v(\cdot)\) is linear. Then there are no insurance gains and the second term in (32) is zero, and therefore

\[\Delta = \int \omega(\alpha^0)[(1 - \zeta)rd(r, \alpha^0) - \bar{T}]f(\alpha^0)d\alpha^0.\]

(33)

Using \(\zeta = \frac{\int \phi(\alpha^0)d(r, \alpha^0)f(\alpha^0)d\alpha^0}{\int rd(r, \alpha^0)f(\alpha^0)d\alpha^0}\), \(\bar{T} = (1 - \zeta) \int d(r, \alpha)f(\alpha)d\alpha\). Let \(\bar{d}(r) \equiv \int d(r, \alpha)f(\alpha)d\alpha\), (34) becomes

\[(1 - \zeta) \int \omega(\alpha^0)[(d(r, \alpha^0) - \bar{d}(r))]f(\alpha^0)d\alpha^0 = \]

\[(1 - \zeta) r \times \text{cov}(\omega(\alpha^0), d(r, \alpha^0) - \bar{d}(r))\]

(34)

Since \(d(r, \alpha^0)\) is increasing in \(\alpha^0\), the provision of deposit insurance redistributes from low to high \(\alpha^0\) households. This redistribution reduces social welfare if \(\omega(\alpha^0)\) is strictly decreasing.

If \(v(\cdot)\) is close enough to linearity, then the insurance gain from deposit insurance, the second term in (32) can be made arbitrarily small. Thus the insurance gains are dominated by the costs of redistribution when \(\omega(\alpha^0)\) is strictly decreasing.

This proposition highlights the redistributive aspect of DI. Since the total resources in the economy are predetermined and agents are nearly risk neutral, the only role of DI is to redistribute consumption. The nature of that redistribution depends on the deposits of each type, \(d(r, \alpha^0)\) and the tax system. The social value of the redistribution is determined by \(\omega(\alpha^0)\). When this is decreasing, so that the rich households have a lower weight and households are not very risk averse, then DI will not be provided \textit{ex post}.

4.5 Taxation \textit{ex ante}: Will DI Be Provided with Type Dependent Taxes?

The discussion of \textit{ex ante} taxation has thus far assumed type independent taxes. This allows us to highlight the effects of the distribution of deposit claims across households which, given type independent taxes, translates into the distribution of consumption across households.

However, when \textit{ex ante} taxes are type dependent, then the tax system itself influences the distribution of consumption. All else the same, a tax system that redistributes from the poor to the rich will reduce the desirability of deposit insurance. For this discussion, we assume the welfare weights, \(\omega(\alpha^0)\), are constant to highlight the effects of \(T(\alpha^0)\).

**Proposition 7** Compare two tax schedules, \(T(\cdot)\) and \(\tilde{T}(\cdot)\). If \(\tilde{T}(\cdot)\) induces a MPS on disposable income relative to \(T(\cdot)\) then \(\Delta \) falls when we replace \(T(\cdot)\) with \(\tilde{T}(\cdot)\).
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**Proof.** Since social welfare is the integral of strictly concave functions, a mean-preserving spread on disposal income, which is the same as consumption, will reduce social welfare. □

To further investigate this issue, following Benabou (2002), consider a tax scheme which maps from income $\varphi$ to consumption, $c(\varphi)$ according to

$$c(\varphi) = \varphi^{1-\tau} \bar{T}^\tau.$$  \hfill (35)

Here $\tau$ is a tax rate and $\bar{T}^\tau$ is a common tax to guarantee feasibility: $\int c(\varphi) dF(\varphi) = \int \varphi dF(\varphi)$, where $F(\cdot)$ is the cdf of gross income. The constant elasticity of after-tax income (equivalently consumption here) is simply $1 - \tau$. The average tax rate, $\frac{\varphi - c(\varphi)}{\varphi} = 1 - \bar{T}^\tau \varphi^{-\tau}$, is increasing in $\varphi$ if $\tau > 0$.

Applying this to our model, the consumption allocation of an agent claiming to be an early consumer is

$$c(\alpha^0) = (\bar{\alpha} + \chi(\alpha^0)) (1-\tau) \bar{T}^\tau.$$ Here $\bar{T}^\tau$ guarantees that the allocation is feasible.

**Proposition 8** Assume consumption allocations are given by (35). Compare two tax rates, $\tau^L$ and $\tau^H$ with $\tau^H > \tau^L > 0$, then $\Delta$ is higher under the tax rate $\tau^H$ compared to $\tau^L$.

**Proof.**

From Atkinson (1970), social welfare, defined as the integral of an increasing strictly concave function of individual consumption, is higher for a consumption distribution with a Lorenz curve that lies entirely above the Lorenz curve for another distribution of consumption. The social welfare associated with the provision of deposit insurance, (18), satisfies these requirements. Hence a distribution of consumption which Lorenz-dominates another generates higher social welfare.

Kakwani (1977) provides the link between income taxes and the ordering of Lorenz curves. Note that the elasticity of consumption, $c(\alpha^0)$ with respect to $\bar{\alpha} + \chi(\alpha^0)$ is $1 - \tau$. Theorem 1 of Kakwani (1977) is interpreted in terms of the elasticity of after-tax income with respect to pre-tax income, denoted $g(x)$ in that theorem. In our application that elasticity is $1 - \tau$. Hence the elasticity is lower for $\tau^H$ compared to $\tau^L$.

From Theorem 1 of Kakwani (1977), this implies that the distribution of consumption under the tax rate $\tau^H$ Lorenz-dominates the distribution of consumption under the tax rate $\tau^L$. Putting these results together, social welfare is higher under the tax rate $\tau^H$ than under the tax rate $\tau^L$. □

These results highlight the implications of the tax schedule for the costs of redistribution associated with the provision of deposit insurance. Essentially, the more progressive is a tax system, the lower are these redistribution costs and thus the higher is the welfare gain (the lower is the welfare cost) from the provision of deposit insurance.

5 Partial Bank Runs and Deposit Insurance

Bank runs are not always systemic but instead may impact only a subset of banks. We refer to this situation as a “partial bank run”. In this section we explore the issue of whether DI will be provided in the event of

\[\text{We are grateful to Roland Benabou for discussions on this application and outlining the proof.}\]
partial bank runs. The fact that runs occur in a subset of banks implies that there is a second dimension for redistribution: across groups of agents depending on the state of their bank as well as across types of agents based on their endowments.

Suppose there is a run at a set of banks covering a fraction \( n \) households.\(^8\) This creates two groups of agents, one group experiencing a bank run and the other with no run. As before, assume lump-sum taxation levied on all agents. Then we can write the payoff from DI as:

\[
W^{DI} = n \int \omega(\alpha_0)v(\bar{\alpha} + \chi(\alpha^0) - \bar{T})f(\alpha^0)d\alpha^0 + (1 - n) \int \omega(\alpha_0)v(\bar{\alpha} + \chi(\alpha^0) - \bar{T})f(\alpha^0)d\alpha^0. \tag{36}
\]

The two terms here highlight the two groups of households even though with DI the consumption levels are the same for each type. If a fraction \( n \) of households are involved in a bank run, the lump-sum tax per household would be given by \( \bar{T} = n \int [\chi(\alpha^0) - \phi(\alpha^0)d(\alpha^0)]f(\alpha^0)d\alpha^0 \).

If, \textit{ex post}, there is no deposit insurance, then social welfare is given by:

\[
W^{NI} = n \int \omega(\alpha_0)[\zeta v(\bar{\alpha} + \chi(\alpha^0)) + (1 - \zeta)v(\bar{\alpha})]f(\alpha^0)d\alpha^0 + (1 - n) \int \omega(\alpha_0)v(\bar{\alpha} + \chi(\alpha^0))f(\alpha^0)d\alpha^0. \tag{37}
\]

The two terms indicate the differential treatment across groups: in one group, a run creates the uncertainty from sequential service while in the other there is financial stability.

The key point of the scenario with multiple groups is that the tax paid by depositors in the failed bank is smaller due to the presence of the other banks because depositors in the other banks pay a share of the deposit insurance. Whether deposit insurance is then paid \textit{ex post} depends, in part, on the relative size of these gains and costs.

In the case of a partial bank run, the net gain to deposit insurance is

\[
\Delta = n \int \omega(\alpha_0)[v(cE(\alpha^0) - \bar{T}) - \zeta v(\bar{\alpha} + \chi(\alpha^0)) - (1 - \zeta)v(\bar{\alpha})]f(\alpha^0)d\alpha^0 + (1 - n) \int \omega(\alpha_0)v(cE(\alpha^0) - \bar{T}) - v(cE(\alpha^0))]f(\alpha^0)d\alpha^0. \tag{38}
\]

The following results use this definition of the welfare differential.

Drawing upon the arguments in Cooper, Kempf, and Peled (2008) that consumption smoothing across regions will lead to bailouts, DI will in fact be provided if the only difference across households is due to the status of their bank, that is, whether it is subject to a run or not.

**Proposition 9** If \( F(\alpha^0) \) is degenerate, then the gains from deposit insurance are positive for any \( n \).

**Proof.** When all households are identical, from (38), the expected utility difference across regions is given by:

\[
\Delta = [v(cE - \bar{T}) - n[\zeta v(\bar{\alpha} + \chi^E) + (1 - \zeta)v(\bar{\alpha})] - (1 - n)v(cE)]. \tag{39}
\]

\(^8\)We do not consider interbank loans since the runs are totally unanticipated and banks are symmetric \textit{ex ante} and, aside from the partial run, \textit{ex post} as well. So if deposit insurance is provided, there is nothing for the banks to trade.

20
To see that \( \Delta > 0 \), combine the second group of terms, subtracted from the first term. Since \( v(\cdot) \) is strictly concave, this combining of terms decreases \( \Delta \). Hence we have

\[
\Delta > [v(c^E - \bar{T}) - v(\chi^E(n\zeta + (1 - n)) + \bar{\alpha})].
\]  
(40)

Using \( \bar{T} = \chi^E(1 - \zeta)n \) and arranging terms,

\[
\Delta > [v(c^E - \chi^E(1 - \zeta)n) - v(\chi^E(n\zeta + (1 - n)) + \bar{\alpha})].
\]  
(41)

Since \( c^E = \chi^E + \bar{\alpha} \), the term on the right of (41) is zero implying \( \Delta > 0 \). This argument holds for any \( n \).

If the distribution of \( \alpha^0 \) is not degenerate, then the provision of DI entails redistribution in two dimensions: across groups and across household types. Proposition 9 makes clear that if there is only redistribution across groups, then DI will be provided. But we know from Proposition 4 that in some cases, the redistribution across types created by DI may be welfare reducing so that this insurance is not provided.

To better appreciate this trade-off, we extended the example introduced in section 4.4.1 to allow for runs at a subset of banks. Recall that in the example taxes were type independent. We used the same parameter values as earlier with \( \gamma_0 = \gamma_1 = 2 \) and the welfare weight of the poor equal to 0.90. Hence, from Figure 1, if there is a run in all banks, we know that deposit insurance is not welfare improving.

Figure 3 shows the results of our experiment. Along the horizontal axis is the fraction of households involved in a bank run. If all households are in a run, then there is a social utility loss from deposit insurance. This utility loss falls as the fraction of household involved in a bank run falls. The utility loss is zero when the fraction is about 80%. Below that critical value, the utility difference is positive and hence deposit insurance will be provided.

Thus this figure illustrates the trade-offs involved when there are two dimensions of heterogeneity. If there are runs at all banks, the costly redistribution across income classes outweighs the insurance gains from deposit insurance. But, if the fraction of banks is sufficiently small, then the costs of redistribution across income classes fall relative to the insurance gains across households experiencing runs and those not experiencing runs.

6 Does Deposit Insurance Eliminate Bank Runs?

The final step of the analysis connects the provision of deposit insurance with the elimination of bank runs. It might seem immediate that once a government has an incentive to provide deposit insurance \( \text{ex post} \), bank runs will be eliminated. But, in fact, this is not the case. Whether the provision of deposit insurance eliminates bank runs depends upon the liquidation decision of the illiquid investment.

It is possible that the government has an incentive to provide deposit insurance, agents understand this and yet a bank run occurs. In particular, if the two period technology is completely liquidated in the run,
then the provision of deposit insurance may be an optimal way to redistribute the given resources. Yet, from the standpoint of a late household, the incentive to run remains given the liquidation of the two period technology.

As long as \( \epsilon \) exceeds zero, there is a non-trivial liquidation decision to be made. The issue is who makes that decision: the bank or the government?

### 6.1 Runs with Deposit Insurance: Costly Bank Liquidations

To start, suppose that a bank is obligated to meet the sequential demands of depositors. In the face of a run, it will be induced to liquidate the long-term investment. Only when its resources are totally exhausted, will the provision of deposit insurance be an issue.

Our results indicate that as long as the insurance gains dominate the costly redistribution, deposit insurance will be provided. Those results were obtained under an assumption of a negligible liquidation value of long-term investment. As noted above, if \( \epsilon \) is not negligible, the resources available to redistribute in the event of a run will be larger. Yet the trade-off between insurance and redistribution will remain.

More importantly, when the bank is forced to liquidate the long-term investment in the face of a run, the provision of deposit insurance does not prevent costly liquidation. This has two consequences. First, there is an inefficiency because of the liquidation. Second, the provision of deposit insurance does not prevent runs. If we think of deposit insurance as a redistribution among agents claiming to be early consumers, then those claiming to be late consumers will have nothing to consume given the liquidation. Thus it is in their interest
to misrepresent and claim to be early consumers as well. The bank run is not eliminated.

6.2 Preventing Runs: Comprehensive Deposit Insurance

Alternatively, suppose that it is the authority providing deposit insurance which makes the liquidation decision. As soon as a bank realizes that a run is underway and prior to liquidation, the authority is given control over the liquidation decision and the consumption allocations of early and late consumers. This returns the discussion to Proposition 1. There we characterized the optimal choice of the planner with regards to both liquidation and redistribution given the announcements of agents. We argued that some liquidation might occur and that resources would be optimally allocated across early and late consumers. In this way, deposit insurance is provided to agents claiming to be early and late consumers. Importantly, bank runs were eliminated as truthtelling is a dominant strategy.

Once we grant the power to decide upon the liquidation of the long-term investment along with the taxation to scheme to provide deposit insurance, then the regulator and planner’s problems are the same. To see how this logic applies in the decentralized model, consider again the discussion of *ex post* optimal taxation in Section 4.3. There we argued that there exists a tax system which would finance deposit insurance and lead to an allocation that improved upon that generated through sequential service when all agents participated in a bank run.⁹ If there was some liquidation value to the long-term investment, then the regulator would jointly determine the amount to be liquidated along with designing the tax system. Since households value their consumption allocation, a regulator able to design an optimal tax system with full deposit insurance is able to choose the same allocation as the planner in Proposition 1. And the regulator can make the same liquidation decision as the planner. Thus in the case of *ex post* optimal taxation, a promise to provide deposit insurance is credible, the liquidation decision is optimal and the bank run is eliminated.

In the other cases of *ex ante* taxation, we argued that deposit insurance would be provided as long as the redistribution costs were not large enough. If there is some liquidation value to the long-term investment, the incentive to provide deposit insurance to those claiming to be early consumers will remain. The long-term investment could be left intact, providing for the consumption of household claiming to be late consumers. This allocation dominates that provided under sequential service and eliminates the bank run. Of course, the regulator might improve upon this allocation by solving an optimization problem allowing optimal liquidation but constrained by the existing tax system.

The regulator’s power to optimally choose liquidation of long-term investment can eliminate bank runs. However, this requires governments to assume control of troubled banks and thus prevent costly liquidation. This analysis makes clear that this is an important tool of regulation. Without it, banks may be induced to undertake costly liquidations despite the provision of deposit insurance.

⁹If only a fraction of agents participate in the run, the argument again follows that in the proof of Proposition 1.
7 Conclusion

This paper studies the provision of deposit insurance in the absence of commitment. We interpret deposit insurance broadly to encompass a variety of forms of \textit{ex post} bailout of financial intermediaries. While steps taken recently to support the financial system in a number of countries may have been warranted, these \textit{ex post} interventions have a consequence: agents will now realize that governments will make \textit{ex post} decisions on deposit insurance.

If so, it is natural to understand the conditions under which deposit insurance will be supplied \textit{ex post}. In our environment, the planner’s allocation involves both redistribution and the provision of deposit insurance. But, in decentralized settings in which household differences appear as differences in deposit levels, a trade-off emerges between risk sharing and the redistribution created by the funding of the transfers inherent in a deposit insurance system. In some cases, these redistribution costs may be large enough to offset insurance gains. These costs are reflected in the ongoing discussion of bailouts in the U.S. and other countries insofar as those policies entail a regressive redistribution from Main St. to Wall St.

In the absence of commitment to deposit insurance, the concerns for financial stability first illustrated by Diamond and Dybvig (1983) resurface. From our analysis, the tax system used to finance payments to depositors plays a crucial role in determining whether deposit insurance will be provided. As we have seen, the claims on the banking system reflect the underlying heterogeneity in wealth. Thus the provision of deposit insurance transfers more to rich than to poor households. If the tax system used to finance these transfers is sufficiently redistributive, then it will reduce the redistribution costs of deposit insurance and is conducive to the \textit{ex post} provision of deposit insurance. As we argued in this paper, if the tax system is set \textit{ex post} along with deposit insurance, then the government can optimally choose the net transfer and avoid the conflict between insurance and redistribution. But if the deposit insurance must be financed by an \textit{ex ante} tax system that allows for redistributions from the poor to the rich through the provision of deposit insurance, then the credibility of deposit insurance is weakened. This was illustrated through our discussion of lump-sum taxes.

In addition, \textit{ex post} intervention raises the case of the extent of a bank run: does it cover all banks or just a subset of them. The latter case opens the possibility of redistribution across groups of depositors, and thus alters the trade-off between insurance gains and redistribution costs.

Whether the provision of deposit insurance is enough to prevent runs depends on how comprehensive is the control of the banks by the regulatory authority. If the banks are prevented from costly liquidation at the same time the deposit insurance is provided, then the bank runs equilibrium is avoided.

There is another intriguing situation to study the provision of deposit insurance: too big to fail. In that setting, there is a fundamental heterogeneity across banks. Some are more essential to the financial system than others. It will be of interest to extend this study to allow those asymmetries across financial institutions and understand conditions for deposit insurance in that environment.
References


