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NONLINEAR CAUSALITY TESTING WITH STEPWISE MULTIVARIATE FILTERING

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ABSTRACT

This study explores the direction and nature of causal linkages among six currencies denoted relative to United States dollar (USD), namely Euro (EUR), Great Britain Pound (GBP), Japanese Yen (JPY), Swiss Frank (CHF), Australian Dollar (AUD) and Canadian Dollar (CAD). These are the most liquid and widely traded currency pairs in the world and make up about 90% of total Forex trading worldwide. The data covers the period 3/20/1987-11/14/2007, including the Asian crisis, the dot-com bubble and the period just before the outbreak of the US subprime crisis. The objective of the paper is to test for the existence of both linear and nonlinear causal relationships among these currency markets. The modified Baek-Brock test for nonlinear non-causality is applied on the currency return time series as well as the linear Granger test. Further to the classical pairwise analysis causality testing is conducted in a multivariate formulation, to correct for the effects of the other variables. A new stepwise multivariate filtering approach is implemented. To check if any of the observed causality is strictly nonlinear, the nonlinear causal relationships of VAR/VECM filtered residuals are also examined. Finally, the hypothesis of nonlinear non-causality is investigated after controlling for conditional heteroskedasticity in the data using GARCH-BEKK, CCC-GARCH and DCC-GARCH models. Significant nonlinear causal linkages persisted even after multivariate GARCH filtering. This indicates that if nonlinear effects are accounted for, neither FX market leads or lags the other consistently and currency returns may exhibit statistically significant higher-order moments and asymmetries.

Keywords: nonparametric Granger causality; filtering; multivariate GARCH models; spillovers

JEL classification: C14; C51; F31

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1. INTRODUCTION

During the Great Moderation period and in particular during the 1990s, currency markets have grown more similar. This led to lower exchange rate volatility and caused asymmetry in reactions toward macroeconomic developments to significantly decrease (Laopodis, 1998)¹. The result of the gradual abolition of capital controls and trade barriers provided the foundation for liberalized and deregulated currency markets. A rich empirical literature exists on the spillovers of US currency volatility across other foreign exchange markets and on “stylized facts” like leptokurtosis and volatility clustering. These studies focus on the investigation of the stochastic behavior of the US dollar, mostly employing the autoregressive conditional heteroskedastic (ARCH) methodology of Engle (1982) (Engle and Bollerslev, 1986; Boothe and Glassman, 1987; Hsieh, 1989; Baillie and Bollerslev, 1989, 1990; Engle *et al.*, 1990). The nature of the transmission mechanism as well as the degree of price information efficiency was already investigated during the 1980s in the beginning of the higher integration of FX rates vis-à-vis the USD (Hogan and Sharpe, 1984; Ito and Roley, 1987). Empirical evidence by Koutmos and Booth (1995) and Laopodis (1997) suggests that the size and sign of an innovation in US exchange rates may seriously affect the extent of dependence and spillovers across markets. Given the status of the USD as the anchor currency, it should be interesting to examine its volatility transfers and more general the nature of causal linkages with other major currencies.

The nature of causality in currency markets, i.e. linear or nonlinear is also a matter for investigation. Ever since the influential work of Meese and Rogoff (1983) in which they examined the failure of some linear exchange rate models, several more recent studies have provided further evidence of the empirical failure of the linear models. The theoretical extension of the linear exchange rate framework to nonlinear models has been growing in the literature. According to Ma and Kanas (2000) these nonlinear extensions include the concept of bubbles with self-fulfilling expectations (Blanchard and Watson, 1982), target zone models (Krugman, 1991), models of micro-foundation of trading behaviour (Krugman and Miller,

¹ According to Stock and Watson (2003) the Great Moderation period initiated around the mid-1980s and lasted until the beginning of the 2000s. During that period, the growth variance of the G7 countries was considerably lower, from 50% to 80% in comparison to the pre- and the post –Great Moderation period.

1993), nonlinear monetary policies (Flood and Isard, 1989) and noise trading (Black, 1986; De Long *et al.*, 1990). Empirical studies have mainly tested for nonlinearities due to target zones, and have failed to support such nonlinearities (Lindberg and Soderlind, 1994). Yet, it still remains an open question whether other types of nonlinear interdependencies across FX markets exist.

The empirical evidence on market co-movement is primarily based on the linear Granger causality test (Granger, 1969). This test assumes a parametric, linear time series model for the conditional mean. The approach is appealing, as the test reduces to determining whether the lags of one variable enter into the equation for another variable, although it requires the linearity assumption. Moreover, this test is sensitive only to causality in the conditional mean while covariables may influence the conditional distribution of the time series in nonlinear ways. Baek and Brock (1992) noted that parametric linear Granger causality tests have low power against certain nonlinear alternatives. Various nonparametric causality tests have been proposed in the literature. The test by Hiemstra and Jones (1994) which is a modified version of the Baek and Brock (1992) test is regarded as a test for a nonlinear dynamic relationship. This test can detect the nonlinear Granger-causal relationship between variables by testing whether the past values influence present and future values.

The objective of the current paper is to test for the existence of both linear and nonlinear causal relationships among six currencies denoted relative to United States dollar (USD), namely Euro (EUR), Great Britain Pound (GBP), Japanese Yen (JPY), Swiss Frank (CHF), Australian Dollar (AUD) and Canadian Dollar (CAD). The prime motivation for choosing these exchange rates (also known as “FX majors”) comes from the fact that they are the most liquid and widely traded exchange rates in the world². The data cover a pre- and a post-Asian crisis period and the period before the outbreak of the US subprime financial crisis³. Thus, it is worth investigating whether the time period after the 1997 Asian financial crisis and before the global

² Trades involving “majors” make up about 90% of total Forex trading worldwide.

³ The Asian crisis started with a 15 - 20% devaluation of Thailand’s Bath which took place on July 2, 1997. It was followed by devaluations of the Philippine Peso, the Malaysian Ringgit, the Indonesian Rupiah and the Singaporean Dollar. In addition, the currencies of South Korea and Taiwan suffered. Further in October, 1997 the Hong Kong stock market collapsed with a 40% loss. In January 1998, the currencies of most South-East Asian countries regained parts of the earlier losses.

financial crisis may have changed the direction and strength of the causal relationships among the currencies under study.

In this study a three-step framework for examining dynamic relationships among foreign exchange markets is introduced. First, linear and nonlinear dynamic linkages are explored between exchange rates, applying both a parametric Granger causality test and the nonparametric modified Baek-Brock causality test (Hiemstra and Jones, 1994). Then, after filtering return series pairwise, as well as in a six-variate formulation (full model) for linear Vector AutoRegressive (VAR) structure, the series of residuals are examined pairwise by the nonparametric modified Baek-Brock causality test. This step ensures that any remaining causality is strictly nonlinear in nature, as the VAR model has already purged the residuals off linear causality. Finally, in the last step, the hypothesis of nonlinear non-causality after controlling for conditional heteroskedasticity is investigated in the data, using multivariate GARCH-BEKK models, both pairwise and in a full model representation. This approach allows the entire variance-covariance structure of the currency interrelationship to be incorporated. The methodology employed via the multivariate heteroscedastic models can help not only to understand the short-run movements but also explicitly capture the volatility spillover mechanism. The method's advantage rests with its ability to examine all markets concurrently and paired, assuming that spillovers are realizations of a process of international news affecting the examined markets. Improved knowledge of the direction and nature of causality and interdependency between the currency markets and consequently the degree of their integration will expand the information set available to international portfolio managers, multinational corporations, and policymakers for decision-making.

The remainder of the paper is organized as follows. Section 2 provides a description of the nonlinear nonparametric test for nonlinear Granger causality. Section 3 describes the data used and Section 4 presents the results. Section 5 concludes with suggestions for future research.

2. NONPARAMETRIC CAUSALITY TESTING

Let $F(x_t | \Theta_{t-1})$ denote the conditional probability distribution of x_t given the information set Θ_{t-1} , which consists of an L_x -length lagged vector of x_t ,

$\mathbf{x}_{t-L_x}^{L_x} \equiv (x_{t-L_x}, x_{t-L_x+1}, \dots, x_{t-1})$ and an L_y -length lagged vector of y_t ,

$\mathbf{y}_{t-L_y}^{L_y} \equiv (y_{t-L_y}, y_{t-L_y+1}, \dots, y_{t-1})$. The null hypothesis tested for a given pair of lags L_x and L_y is

$$H_0 : F(x_t | \Theta_{t-1}) = F(x_t | \Theta_{t-1} - \mathbf{y}_{t-L_y}^{L_y}) \quad (1)$$

which states that taking the vector of past y -values out of the information set does not affect

the distribution of current x -values. Denoting the m -length lead vector of x_t by x_t^m , the

vectors defined so far can be summarized as $\mathbf{x}_t^m \equiv (x_t, x_{t+1}, \dots, x_{t+m-1})$,

$\mathbf{x}_{t-L_x}^{L_x} \equiv (x_{t-L_x}, x_{t-L_x+1}, \dots, x_{t-1})$ and $\mathbf{y}_{t-L_y}^{L_y} \equiv (y_{t-L_y}, y_{t-L_y+1}, \dots, y_{t-1})$, for $t \in \mathbf{Z}$. The claim made by

Hiemstra and Jones (1994) is that the null hypothesis given in Eq. (1) implies for all $\varepsilon > 0$

$$\begin{aligned} P \left(\left\| \mathbf{x}_t^m - \mathbf{x}_s^m \right\| < \varepsilon \left\| \mathbf{x}_{t-L_x}^{L_x} - \mathbf{x}_{s-L_x}^{L_x} \right\| < \varepsilon, \left\| \mathbf{y}_{t-L_y}^{L_y} - \mathbf{y}_{s-L_y}^{L_y} \right\| < \varepsilon \right) \\ = P \left(\left\| \mathbf{x}_t^m - \mathbf{x}_s^m \right\| < \varepsilon \left\| \mathbf{x}_{t-L_x}^{L_x} - \mathbf{x}_{s-L_x}^{L_x} \right\| < \varepsilon \right) \end{aligned} \quad (2)$$

where $\| \cdot \|$ is the supremum norm, which for a d -dimensional vector $\mathbf{x} = (x_1, \dots, x_d)^T$ is given

by $\| \mathbf{x} \| = \sup_{i=1}^d |x_i|$. Equation (2) states that the conditional probability that two arbitrary m -

length lead vectors of $\{x_t\}$ are within distance ε , given that the corresponding lagged L_x -

length lag vectors of $\{x_t\}$ are ε -close, is the same as when in addition one also conditions on

the L_y -length lag vectors of $\{y_t\}$ being ε -close.

For the time series of realizations $\{x_t\}$ and $\{y_t\}$, $t = 1, \dots, T$, the nonparametric test consists of choosing a value for ε typically in $[0.5, 1.5]$ after unit variance normalization, and

testing Eq. (2) by expressing the conditional probabilities in terms of the corresponding ratios of joint probabilities

$$\begin{aligned}
C_1(m + L_x, L_y, \varepsilon) &\equiv P\left(\left\|\mathbf{x}_{t-L_x}^{m+L_x} - \mathbf{x}_{s-L_x}^{m+L_x}\right\| < \varepsilon, \left\|\mathbf{y}_{t-L_y}^{L_y} - \mathbf{y}_{s-L_y}^{L_y}\right\| < \varepsilon\right) \\
C_2(L_x, L_y, \varepsilon) &\equiv P\left(\left\|\mathbf{x}_{t-L_x}^{L_x} - \mathbf{x}_{s-L_x}^{L_x}\right\| < \varepsilon, \left\|\mathbf{y}_{t-L_y}^{L_y} - \mathbf{y}_{s-L_y}^{L_y}\right\| < \varepsilon\right) \\
C_3(m + L_x, \varepsilon) &\equiv P\left(\left\|\mathbf{x}_{t-L_x}^{m+L_x} - \mathbf{x}_{s-L_x}^{m+L_x}\right\| < \varepsilon\right) \\
C_4(L_x, \varepsilon) &\equiv P\left(\left\|\mathbf{x}_{t-L_x}^{L_x} - \mathbf{x}_{s-L_x}^{L_x}\right\| < \varepsilon\right)
\end{aligned} \tag{3}$$

Thus, Eq. (2) can be formulated as

$$\frac{C_1(m + L_x, L_y, \varepsilon)}{C_2(L_x, L_y, \varepsilon)} = \frac{C_3(m + L_x, \varepsilon)}{C_4(L_x, \varepsilon)} \tag{4}$$

In order to test the condition in Eq. (4), correlation-integral estimators of the joint probabilities in Eq. (3) are used. Let $n = T + 1 - m - \max(L_x, L_y)$ and $t, s = \max(L_x, L_y) + 1, \dots, T - m + 1$, while $I(\mathbf{z}_1, \mathbf{z}_2, \varepsilon)$ denote a kernel that equals 1 when two vectors $\mathbf{z}_1, \mathbf{z}_2$ are within the maximum norm distance ε of each other and zero otherwise. The correlation-integral estimators of the joint probabilities can be written

$$\begin{aligned}
C_1(m + L_x, L_y, \varepsilon, n) &\equiv 2/[n \cdot (n - 1)] \cdot \sum_{t < s} \sum I(\mathbf{x}_{t-L_x}^{m+L_x}, \mathbf{x}_{s-L_x}^{m+L_x}, \varepsilon) \cdot I(\mathbf{y}_{t-L_y}^{L_y}, \mathbf{y}_{s-L_y}^{L_y}, \varepsilon) \\
C_2(L_x, L_y, \varepsilon, n) &\equiv 2/[n \cdot (n - 1)] \cdot \sum_{t < s} \sum I(\mathbf{x}_{t-L_x}^{L_x}, \mathbf{x}_{s-L_x}^{L_x}, \varepsilon) \cdot I(\mathbf{y}_{t-L_y}^{L_y}, \mathbf{y}_{s-L_y}^{L_y}, \varepsilon) \\
C_3(m + L_x, \varepsilon, n) &\equiv 2/[n \cdot (n - 1)] \cdot \sum_{t < s} \sum I(\mathbf{x}_{t-L_x}^{m+L_x}, \mathbf{x}_{s-L_x}^{m+L_x}, \varepsilon) \\
C_4(L_x, \varepsilon, n) &\equiv 2/[n \cdot (n - 1)] \cdot \sum_{t < s} \sum I(\mathbf{x}_{t-L_x}^{L_x}, \mathbf{x}_{s-L_x}^{L_x}, \varepsilon)
\end{aligned} \tag{5}$$

Using these estimators and under the assumptions that $\{x_t\}$ and $\{y_t\}$ are strictly stationary, weakly dependent and satisfy the mixing conditions of Denker and Keller (1983), Hiemstra and Jones (1994) show that

$$\sqrt{n} \left(\frac{C_1(m + L_x, L_y, \varepsilon, n)}{C_2(L_x, L_y, \varepsilon, n)} - \frac{C_3(m + L_x, \varepsilon, n)}{C_4(L_x, \varepsilon, n)} \right) \sim N\left(0, \sigma^2(m, L_x, L_y, \varepsilon)\right) \tag{6}$$

with $\sigma^2(m, L_x, L_y, \varepsilon)$ as given in their appendix. One-sided critical values are used based on this asymptotic result, rejecting when the observed value of the test statistic in Eq. (6) is too large. This “causality-in-probability” test is regarded as a test for a nonlinear dynamic relationship (Baek and Brock, 1992). It detects the nonlinear causal relationship between variables, including second- or higher-order moment effects, by testing whether the past values influence present and future values (Hiemstra and Jones, 1994).

3. DATA

The data comprise six time series of daily closing foreign exchange rates denoted relative to United States dollar (USD). In particular Euro (EUR), Great Britain Pound (GBP), Japanese Yen (JPY), Swiss Frank (CHF), Australian Dollar (AUD) and Canadian Dollar (CAD) are investigated. The exact ratios represent EUR/USD, GBP/USD, USD/JPY, USD/CHF, AUD/USD and USD/CAD respectively. These are the most liquid and widely traded currency pairs in the world and make up about 90% of total Forex trading worldwide. The data covers the period 3/20/1987-11/14/2007. It includes the on-set of the Asian financial crisis which took place on July 2, 1997 with the devaluation of Thailand's Bath and was followed by the devaluations of the Philippine Peso, the Malaysian Ringgit, the Indonesian Rupiah and the Singaporean Dollar. In October, 1997 the Hong Kong stock market collapsed with a 40% loss. In January 1998, the currencies of South-East Asian countries began to regain part of the earlier losses. Additionally, the sample includes the dot-com bubble “burst” (Greenspan, 2007). On March 10, 2000 the technology NASDAQ Composite index peaked at 5,048.62 (intra-day peak 5,132.52), more than double its value just a year before. Moreover, the period just before the outbreak of the US subprime crisis is also included. The US financial crisis was triggered by a liquidity shortfall in the US banking system, which resulted in the collapse of large financial institutions, the "bail out" of banks by national governments and turbulence in stock markets around the world (Krugman, 2009).

4. EMPIRICAL RESULTS

The results are reported in Tables 1 and 2. The notation “**” is used to indicate that the corresponding p -value of a particular causality test is smaller than 1% and “*” that the corresponding p -value of a test is in the range 1-5%; Directional causalities are denoted by the functional representation \rightarrow . The stepwise filtering methodology for examining dynamic relationships among foreign exchange markets is implemented thereafter.

4.1 Step1: Testing causality on raw returns

Considering that \mathbf{Y}_t is the vector of endogenous variables and ℓ number of lags, then the VAR(ℓ) model is given as follows

$$\mathbf{Y}_t = \sum_{s=1}^{\ell} \mathbf{A}_s \mathbf{Y}_{t-s} + \boldsymbol{\varepsilon}_t \quad (6)$$

where $\mathbf{Y}_t = [Y_{1t}, \dots, Y_{\ell t}]$ the $\ell \times 1$ vector of endogenous variables, \mathbf{A}_s the $\ell \times \ell$ parameter matrices and $\boldsymbol{\varepsilon}_t$ the residual vector, for which $E(\boldsymbol{\varepsilon}_t) = \mathbf{0}$, $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_s') = \begin{cases} \boldsymbol{\Sigma}_\varepsilon & t = s \\ \mathbf{0} & t \neq s \end{cases}$.

Specifically, in case of two stationary time series $\{X_t\}$ and $\{Y_t\}$ the bivariate VAR model is given by

$$\begin{aligned} X_t &= A(\ell)X_t + B(\ell)Y_t + \varepsilon_{X,t} \\ Y_t &= C(\ell)X_t + D(\ell)Y_t + \varepsilon_{Y,t} \end{aligned} \quad t = 1, 2, \dots, N \quad (7)$$

where $A(\ell), B(\ell), C(\ell)$ and $D(\ell)$ are all polynomials in the lag operator with all roots outside the unit circle. The error terms are separate *i.i.d.* processes with zero mean and constant variance.

The test whether Y strictly Granger causes X is simply a test of the joint restriction that all the coefficients of the lag polynomial $B(\ell)$ are zero, whilst similarly, a test of whether X strictly Granger causes Y is a test regarding $C(\ell)$. In each case, the null hypothesis of no Granger causality is rejected if the exclusion restriction is rejected. If both $B(\ell)$ and $C(\ell)$ joint tests for significance show that they are different from zero, the series are bi-causally related. For each of the six raw return series linear causality testing was carried out using the Granger's test. The lag lengths of the VAR specification were selected using the Schwartz Information

Criterion (SIC). However, if cointegration is detected, a vector autoregression model in error correction form (Vector Error Correction Model-VECM) should be used instead of a VAR to estimate Granger causality (Engle and Granger, 1987; Johansen, 1988; Johansen and Juselius, 1990). Hence, the trace and maximum eigenvalue statistics were applied on the FX log-prices series. As they did not identify any cointegrating vectors, the null of no cointegration was not rejected and the VAR specification was properly used. Finally, for the modified Baek-Brock causality test the lags $\ell_X = \ell_Y = 1$ are used.

The results presented in Tables 1 and 2 allow for the following observations: GBP, JPY and CHF linearly cause EUR. Further, EUR presents a significant unidirectional linear relationship EUR→AUD as well as JPY weakly Granger causes CHF. All other currencies appear to lack any causal relationship. Next, the results of the modified Baek-Brock causality test are discussed. Interestingly, there is now strong evidence of the bi-directional nonlinear relationships EUR↔GBP, AUD↔CAD, EUR↔AUD, GBP↔CHF and JPY↔AUD with small differences in statistical significance. Finally, a strong unidirectional nonlinear causality appears from CHF to EUR and CHF to JPY. Next, in considering the full model (six-variate) some differences are observed, such as the JPY→EUR and EUR→AUD have disappeared and the GBP→JPY linkage has emerged.

[Insert Table 1 here]

[Insert Table 2 here]

4.2 Step2: Testing causality on VAR residuals

Previous results suggest that there are some significant and persistent linear and nonlinear causal linkages among the FX rates. However, even though nonlinear causality was detected, the modified Baek-Brock test should be re-applied on the filtered VAR-residuals to ensure that any causality found is strictly nonlinear in nature. The lag lengths of the VAR specification were selected by the Schwartz Information Criterion (SIC). The application of the test on the VAR residuals points towards the preservation of the results reported for the raw returns in the pairwise and six-variate implementation. Comparing the summary results in Table 1, it is interesting to see that they show identical significant causal nonlinear relationships both for the bivariate and full model specifications, except for the absence now of the

EUR→AUD causal relationship. The nature and source of the detected nonlinearities are different from that of the linear Granger causality test and may also imply a temporary, or long-term, causal relationship between the currencies. For example, exchange rate volatility might induce nonlinear causality. Given the status of the USD as the anchor currency, an innovation in the US stock market or an increase in the interest rates from the Federal Reserve System, may seriously affect the extent of dependency and volatility spillovers across currencies. The nature of the volatility transmission mechanism can be investigated after controlling for conditional heteroskedasticity using multivariate GARCH models. This is conducted both pairwise and in a full model representation, thus allowing the entire variance-covariance structure of the currency interrelationship to be incorporated.

4.3 Step3: Causality testing on Multivariate GARCH-filtered residuals

If the statistical evidence of nonlinear Granger causality lies in the conditional variances and covariances then it would be strongly reduced when the appropriate multivariate GARCH model is fitted to the raw or linearly filtered data. The use of the nonlinear test on filtered data with a multivariate GARCH model enables to determine whether the posited model is sufficient to describe the relationship among the series. However, failure to accept the no-causality null hypothesis may also constitute evidence that the selected multivariate GARCH model was incorrectly specified⁴. Many GARCH models can be used for this purpose.

In the present study the GARCH-BEKK, the CCC-GARCH and the DCC-GARCH multivariate models are used. Let $\{y_t\}$ be a vector stochastic return process of dimension $N \times 1$ and ω a finite vector of parameters. Then $y_t = \mu_t(\omega) + \varepsilon_t$ where $\mu_t(\theta)$ is the conditional mean vector and $\varepsilon_t = H_t^{1/2}(\omega)z_t$ where $H_t^{1/2}(\omega)$ is a $N \times N$ positive definite matrix. Furthermore, the $N \times 1$ random vector z_t have $E(z_t) = 0$ and $Var(z_t) = I_N$ as the first two moments where I_N is the identity matrix. Hence H_t is the conditional variance matrix of y_t .

⁴ This line of analysis is similar to the use of the univariate BDS test on raw data and on GARCH models (Brock *et al.*, 1996; Brooks, 1996; Hsieh, 1989)

4.3.1 GARCH-BEKK model

As it is difficult to guarantee the positivity of H_t in the VEC-GARCH representation of Bollerslev *et al.* (1988) without imposing strong restrictions on the parameters, Engle and Kroner (1995) proposed a new parametrization of H_t that imposes its positivity, namely the Baba-Engle-Kraft-Kroner (BEKK) model. The full $BEKK(1, 1, K)$ model is defined as:

$$H_t = C^{*'}C^* + \sum_{k=1}^K \Lambda_k^{*'} \varepsilon_{t-1} \varepsilon_{t-1}' \Lambda_k^* + \sum_{k=1}^K M_k^{*'} H_{t-1} M_k^* \quad (8)$$

where C^* , Λ_k^* and M_k^* are $N \times N$ matrices but C^* is upper triangular. The summation limit K determines the generality of the process and the sufficient conditions to identify BEKK models are that $\Lambda_{k,11}^*$, $M_{k,11}^*$ and the diagonal elements of C^* are restricted to be positive. To reduce the $N(5N + 1)/2$ number of parameters in the $BEKK(1,1,1)$ model and consequently to reduce the generality, a diagonal BEKK model can be imposed, i.e. Λ_k^* and M_k^* in (8) are diagonal matrices. Maximum likelihood estimation is used for the BEKK model.

4.3.2 Conditional correlation models

Bollerslev (1990) proposes a class of MGARCH models in which the conditional correlations are constant and conditional covariances are proportional to the product of the corresponding conditional standard deviations. This restriction greatly reduces the number of unknown parameters and simplifies the estimation. The constant conditional correlation model (CCC) is defined as

$$H_t = G_t R G_t = \left(\rho_{ij} \sqrt{h_{iit} h_{jji}} \right) \quad (9)$$

where $G_t = \text{diag} \left(h_{11t}^{1/2} \dots h_{NNt}^{1/2} \right)$. It should be noted that h_{iit} can be defined as any univariate GARCH model, and $R = \left(\rho_{ij} \right)$ is a symmetric positive definite matrix containing the constant conditional correlations with $\rho_{ii} = 1, \forall i$. The classical CCC model has a GARCH(1,1) specification for each conditional variance in G_t . The unconditional variances are easily obtained, as in the univariate case, but the unconditional covariances are difficult to calculate because of the nonlinearity in Eq. (9).

The assumption of a constant conditional correlation often seems unrealistic in many empirical applications. Christodoulakis and Satchell (2002), Engle (2002) and Tse and Tsui (2002) propose a generalization of the CCC model by making the conditional correlation matrix time-dependent. They propose a dynamic conditional correlation (DCC) model, with the additional assumption that the time-dependent conditional correlation matrix has to be positive definite $\forall t$. The DCC model of Engle (2002) is genuinely multivariate and particularly useful when modelling high-dimensional data samples. The DCC model of Engle (2002) is defined as

$$H_t = G_t R_t G_t \quad (10)$$

with $R_t = \text{diag}\left(w_{11,t}^{-1/2} \dots w_{NN,t}^{-1/2}\right) W_t \text{diag}\left(w_{11,t}^{-1/2} \dots w_{NN,t}^{-1/2}\right)$. The $N \times N$ symmetric positive definite matrix $W_t = (w_{ij,t})$ is given by $W_t = (1 - \alpha - \beta)\bar{W} + \alpha u_{t-1} u_{t-1}' + \beta W_{t-1}$, with $u_{i,t} = (\varepsilon_{i,t} / \sqrt{h_{ii,t}})$. \bar{W} is the $N \times N$ unconditional variance matrix of u_t , and α and β are non-negative scalar parameters satisfying $\alpha + \beta < 1$. The correlation coefficient in the bivariate case is

$$\rho_{12t} = \frac{(1 - \alpha - \beta)\bar{w}_{12} + \alpha u_{1,t-1} u_{2,t-1} + \beta w_{12,t-1}}{\sqrt{\left((1 - \alpha - \beta)\bar{w}_{11} + \alpha u_{1,t-1}^2 + \beta w_{11,t-1}\right)\left((1 - \alpha - \beta)\bar{w}_{22} + \alpha u_{2,t-1}^2 + \beta w_{22,t-1}\right)}} \quad (11)$$

The DCC model can be estimated consistently using the two-step quasi-loglikelihood approach of Engle and Sheppard (2001).

In Tables 1 and 2 the results before and after multivariate GARCH filtering are shown. The order parameters were determined for the time series in terms of the minimal SIC. One conclusion after the nonlinear causality testing is that in some cases the statistical significance is weaker after filtering than before. These differences in statistical significance indicate that the nonlinear causality is largely due to simple volatility effects. However, this is not indicative of a general conclusion. Instead, significant nonlinear interdependencies remain after the pairwise and full model GARCH filtering revealing that volatility effects and spillovers are probably not the only ones inducing nonlinear causality. In particular, the pairwise nonlinear causality reveals that the linkages EUR \leftrightarrow GBP, EUR \leftrightarrow JPY, CHF \rightarrow EUR, CHF \rightarrow JPY and AUD \leftrightarrow CAD remain in all MGARCH specifications. Thus, there is strong evidence of the influence of the aforementioned currencies on EUR. This is perhaps an after-effect of the independent and robust Euro zone behavior against the USD (Bénassy-Quéré *et al.*, 2000; Wang *et al.*, 2007;

Bekiros and Diks, 2008) that was observed right before the outbreak of the US financial crisis. A potential increase/decrease in the US dollar volatility affected the Euro zone currencies less than (and with a significant delay) the USD closest dependent economies of Canada and Australia. Additionally, some differences are observed for the other relationships detected for VAR-residuals depending on the GARCH specification employed.

Next, incorporating the effect of all the currencies in a full model (six-variate) GARCH framework, the “whitened” residuals present different causal relationships than before. Specifically, only two linkages remain as tested in VAR-residuals, namely CHF→EUR and AUD↔CAD. The other links are seriously weakened or even purged in most of the GARCH specifications. Thus, the second-moment filtering via a six-variate representation is proven to be significantly more effective in detecting spillover effects than the pairwise filtering. However, in all results, third moment causality may be also a significant factor of the remaining interdependence. Finally, mostly in pairwise modelling but also in six-variate, the conditional correlation models appear to be much better in capturing spillover effects than the BEKK model.

5. CONCLUSIONS

In this study we investigated the existence of linear and nonlinear causal relationships among the six most liquid and widely traded currencies in the world denoted relative to the USD. It was shown that almost all FX markets considered here have become more internationally integrated after the Asian financial crisis and before the financial crisis of 2007-2010. Interestingly, whilst the linear causal relationships detected on the returns have disappeared after proper filtering, nonlinear causal linkages in some cases emerged and more importantly persisted even after multivariate GARCH filtering. These results, apart from offering a much better understanding of the dynamic linear and nonlinear relationships underlying the major currency markets, may have important implications for market efficiency. They may be useful in future research to quantify the process of financial integration or may influence the greater predictability of these markets.

An important subject is the nature and source of the nonlinear causal linkages. It was shown that volatility effects might partly induce nonlinear causality. The fitted GARCH-BEKK,

CCC-GARCH and DCC-GARCH models accounted for a large part of the nonlinearity in daily exchange rates, but only in some cases. Perhaps other short-term asset-pricing models should be developed to explain this stylized fact. Among the multivariate heteroscedastic models utilized in this paper, the conditional correlation models captured the transmission mechanism of the volatility shocks more efficiently than the GARCH-BEKK, especially in the full-variate representation. Moreover, currency returns may exhibit statistically significant higher-order moments (Scheinkman and LeBaron, 1989). This may explain why multivariate GARCH filtering does not capture all the nonlinearity in currency returns.

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TABLE 1: CAUSALITY RESULTS (PAIRWISE)

Pair		Linear Granger Causality		Nonlinear Causality									
X	Y	Raw data		Raw data		VAR Residuals		Multivariate GARCH Residuals					
		X→Y	Y→X	X→Y	Y→X	X→Y	Y→X	BEKK	X→Y CCC	DCC	BEKK	Y→X CCC	DCC
EUR	GBP		**	**	**	*	**	*	*	*	**	**	**
EUR	JPY		**	*	**	*	**	*	*	*	**	**	**
EUR	CHF		**		**		**	*			**	**	**
EUR	AUD	*		*	*		*				*		
EUR	CAD												
GBP	JPY												
GBP	CHF			*	**	*	*	*			**	*	*
GBP	AUD												
GBP	CAD												
JPY	CHF	*			*		*				*	*	*
JPY	AUD			*	**	*	**	*			**		
JPY	CAD												
CHF	AUD												
CHF	CAD												
AUD	CAD			**	**	**	**	**	**	**	**	*	*

X→Y: r_X does not Granger Cause r_Y . Statistical significance 5% (*), 1% (**). The foreign exchange rates Euro (EUR), Great Britain Pound (GBP), Japanese Yen (JPY), Swiss Frank (CHF), Australian Dollar (AUD) and Canadian Dollar (CAD) are denoted relative to United States dollar (USD). The exact ratios represent EUR/USD, GBP/USD, USD/JPY, USD/CHF, AUD/USD and USD/CAD respectively. Total period: 3/20/1987 - 11/14/2007.

Panel A: Linear Granger Causality

All data (log-levels) were investigated with a VECM specification and the null of no cointegration was not rejected. The VAR lags were determined using the Schwartz Information Criterion (SIC). The number of lags identified for all variables in all periods is one (1) except for EUR-CHF which is two (2). For testing reasons linear Granger causality was further investigated in the VAR or GARCH residuals, but was not detected.

Panel B: Non-Linear Causality

The number of lags used for the nonlinear causality test are $\ell_X = \ell_Y = 1$. The data used are log-returns. As the log-levels were not found to be cointegrated (Panel A) the nonlinear causality was investigated on the VAR residuals. The number of lags identified for the VAR specification are reported in Panel A. The second moment filtering was performed with a GARCH-BEKK (1,1), GARCH-CCC (1,1) and GARCH-DCC (1,1,1,1) model.

TABLE 2: CAUSALITY RESULTS (FULL MODEL)

Pair		Linear Granger Causality		Nonlinear Causality									
X	Y	Raw data		Raw data		VAR Residuals		Multivariate GARCH Residuals					
		X→Y	Y→X	X→Y	Y→X	X→Y	Y→X	BEKK	X→Y CCC	DCC	BEKK	Y→X CCC	DCC
EUR	GBP		**	**	**	*	**		*		**	**	**
EUR	JPY			*	**	*				*			
EUR	CHF		**		**		**				**	**	**
EUR	AUD			*	*	*		*					
EUR	CAD												
GBP	JPY	**											
GBP	CHF			*	**	*	*		*	*		*	*
GBP	AUD											*	
GBP	CAD												
JPY	CHF	*			*		*				*		
JPY	AUD			*	**	*	**	*					
JPY	CAD												
CHF	AUD												
CHF	CAD												
AUD	CAD			**	**	**	**		**	**	**	*	*

X→Y: rx does not Granger Cause ry. Statistical significance 5% (*), 1% (**). The foreign exchange rates Euro (EUR), Great Britain Pound (GBP), Japanese Yen (JPY), Swiss Frank (CHF), Australian Dollar (AUD) and Canadian Dollar (CAD) are denoted relative to United States dollar (USD). The exact ratios represent EUR/USD, GBP/USD, USD/JPY, USD/CHF, AUD/USD and USD/CAD respectively. Total period: 3/20/1987 - 11/14/2007.

Panel A: Linear Granger Causality

The 6x6 system of the data (log-levels) was investigated with a VECM specification and the null of no cointegration was not rejected. The VAR lags of the system were determined using the Schwartz Information Criterion (SIC) and found in all periods one (1). For testing reasons linear Granger causality was further investigated in the VAR or GARCH residuals, but was not detected.

Panel B: Non-Linear Causality

The number of lags used for the nonlinear causality test are $\ell_x = \ell_y = 1$. The data used are log-returns. As the 6-variate system of log-levels was not found to be cointegrated (Panel A) the nonlinear causality was investigated on the VAR residuals. The number of lags identified for the VAR specification are reported in Panel A. The second moment filtering was performed with a GARCH-BEKK (1,1), GARCH-CCC (1,1) and GARCH-DCC (1,1,1,1) model.

