Department of Economics

Essays on Firm Dynamics, Endogenous Growth and International Trade

Cristiana Benedetti Fasil

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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Examining Board:

Professor Omar Licandro, Supervisor, Instituto de Análisis Económico (CSIC)
Professor Fernando Vega Redondo, European University Institute
Professor Boyan Jovanovic, New York University
Professor Hugo Hopenhayn, UCLA

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Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
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Abstract

Recent empirical firm level studies reveal the structural heterogeneity of firms in process and product innovation, as well as the central role of product quality in determining world trade patterns and intensities. This calls for a better understanding of the link between firm heterogeneity and the innovation and export decisions of firms which are at the base of productivity growth and, hence, economic growth and development.

My dissertation contributes to this debate focusing on the supply side. I propose a novel way to model the production technology of firms by introducing two attributes of firm heterogeneity: cost efficiency and product quality. The goal of the first thesis chapter is to study the effects of process and product innovation on firm dynamics, productivity and endogenous long run growth. In the second chapter an open economy framework with trade between symmetric countries is analyzed. Here the focus is on quantifying the impact of trade as well as trade liberalization on firm innovation dynamics and productivity- and aggregate growth. The third chapter abstracts from endogenous growth and examines the role of the two attributes of firm heterogeneity in shaping the trade patterns and intensities within and across developed and developing countries.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
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Contents

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

List of Figures

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

List of Tables

Dedicato a mia mamma Antonella per una promessa mantenuta

e a mio papà Silvio per la sua continua presenza

 DOI: 10.2870/28221

Part I

Introduction

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

In the last decades a growing availability of data at the firm level covering production, innovation investments, financial systems, and exports has challenged both empirical and theoretical researchers in answering new questions. A key and common issue has become the understanding of the effects of firm decisions on the mechanism of resource reallocation from exiting and contracting firms to new and expanding ones and how this translates into persistent firm level heterogeneity and growth. This thesis collocates within this research area emphasizing the different role played by heterogeneity in firm efficiency and product quality in shaping firms' innovation and export decisions and their impact on firm size, pricing, productivity- and aggregate growth, and direction of trade. I believe that this is an important research area as innovation and international trade are among the main factors leading a country growth. Hence, contributing in understanding their causes and consequences could help to explain differences in the growth rate of industries and hence countries and to design policies aimed at promoting growth and development.

The first chapter is directly motivated by this recent empirical evidence concerning firms innovation investments. In particular, it is shown that firms are heterogeneous also in their innovation activities and that process and product innovations have different effects on firms productivity levels, productivity- and aggregate growth. To explain this evidence, this chapter develops an endogenous growth model with two sources of firm heterogeneity: production efficiency and product quality. Both attributes evolve endogenously through firms' innovation choices and permanent idiosyncratic shocks. Growth is driven by innovation and self-selection of unsuccessful firms and sustained by entrants who imitate successful incumbent firms. Calibrating the economy to match the Spanish manufacturing sector, the model enables to quantify the different effects of selection, innovation, and imitation as well as product and process innovation on growth. Moreover, it provides a complete characterization of firms' innovation choices explaining the partition of firms along different innovation strategies and generating consistent firm size distributions.

In the second chapter this model is applied to study how symmetric trade affects the decisions of firms to invest in process and product innovation and how this generates firm level- and aggregate growth. In particular, costly trade impacts on the growth mechanism through a tougher selection of unsuccessful firms, a selection of the marginal innovators, and a higher innovation intensity. The quantitative analysis shows that the combination of these factors has a positive effect on the growth rate. Hence, exposure to trade increases unambiguously growth. This comes together with a more concentrated industry and a higher share of product innovators than in the closed economy. Concerning the debate on trade liberalization, the model yields interesting predictions. A reduction of the variable cost of trade unambiguously promotes growth and fosters the

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

diffusion of higher product quality. Instead a too strong reduction of the fixed export cost is detrimental for growth and it is accompanied by a reduction of product quality in favor of cheaper varieties.

The third chapter is a joint work with Teodora Borota. We abandon endogenous growth and analyze the role of production efficiency and product quality in shaping the trade patterns and trade intensities within and across two groups of countries, the developed and richer North and the developing South. Taking prices as a proxy for quality, recent empirical literature identifies a positive relation between income per capita and both export and import prices, suggesting that rich countries trade goods of relatively higher quality. The novelty of this model is that instead of relying on specific demand side mechanisms such as non-homothetic preferences for explaining these findings, it focuses on the supply side and North-South differences in technology as the key determinants of trade specialization over quality. We employ a four country North-South trade model with two dimensions of firm heterogeneity. Differences in firms product qualities and cost efficiencies result in a price distribution which, when the fixed cost of trade is applied, generate different consumption bundles and the predicted export and import prices across income levels. Furthermore, the resulting total expenditure allocation across quality shows that the North (South) spends a larger share of its income on high (low) quality even with the same homothetic preferences across regions.

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Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

Part II

Chapters

Chapter 1

Product and Process Innovation in a Growth Model of Firm Selection

1.1 Introduction

Globalization and the rise of new technologies have challenged firms' abilities in developing innovation strategies to face increasing market competition. Innovation has become a fundamental source of firm survival and growth.[1] The literature has widely analyzed the relationship between innovation and economic growth.[2] However, little attention has been paid to the relationship between firm heterogeneity and innovation activities and even less to the relationship between firm heterogeneity and different innovation strategies as well as to their impact on firms' competitiveness and productivity growth. The channel between firm growth and aggregate growth is still comparatively unexplored. Understanding the determinants of firms' innovation strategies and the mechanism of resource reallocation through which they impact on aggregate growth is therefore crucial and can also contribute to enhance the effectiveness of policies aimed at fostering economic growth and welfare.

[1] For instance, on a panel of Dutch firms Cefis and Marsili (2005) find that the expected longevity of innovative firms is 11% higher than non-innovative firms while Doraszelski and Jaumandreu (2008) using a Spanish panel estimate that the sole contribution of firms that perform R&D explains between 45% and 85% of productivity growth in the industry with intermediate or high innovation activity. Moreover, Bartelsman and Doms (2000) report evidence of a self-reinforcing mechanism between productivity and innovation. Profitable firms have a higher propensity to innovate and innovation is positively related with productivity and productivity growth.

[2] Few examples are Aghion and Howitt (1992), Grossman and Helpman (1991) and Romer (1990).

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
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This need comes together with an increasing availability of data at the firm-level which distinguish between process and product innovation.[3] These data have stimulated a series of empirical studies which highlight three main pieces of evidence: innovations are *heterogeneous*, *asymmetric*, and *complementary*.

Firstly, innovation are *heterogeneous* in the sense that some firms do not innovate, some firms specialize in process innovation, others in product innovation and some in both types of innovations. Thus, firms have different incentives to invest either in product or process innovation. Table 1 shows the share of firms across the different innovation strategies for four European countries.[4] Huergo and Jaumandreu (2004) finds in a sample of Spanish firms in the manufacturing sector that half of the firms never innovate, 30% undertake either process or product innovation and 20% of the firms undergo both types of innovations. Similar statistics are also available for Germany and Great Britain (Harrison et. al. (2008)) and the Netherlands (Cefis and Marsili (2005)).

Table 1.1: Heterogeneity in Innovation Strategies

Country	**Share of Innovative Firms**			
	No Innovation	**Process**	**Product**	**Process and Product**
Spain	55.4%	12.2%	12.4%	20%
Germany	41%	10.2%	21%	27.4%
Great Britain	60.5%	11%	14.2%	14.3%
Netherlands	36.6%	5.8%	18.8%	42.7%

Secondly, the innovation strategies are *asymmetric*. Parisi et. al. (2006) estimate on an Italian panel that process innovation increases productivity by 14% and product innovation by 4% over a three year period. As expected, innovating firms are characterized by a productivity distribution that stochastically dominates the productivity distribution of non-innovators. But in the case of product innovation the distribution becomes more skewed to the right. Huergo and Jaumandreu (2004) show similar results for Spain and highlight a relation betwen firm size and type of innovation. Small firms are more likely

[3]The European Commission has developed a program aimed at studying the innovation systems of the States member of the European Union with the scope of promoting innovation and growth. The core of the program is based on firm-level surveys (Community Innovation Surveys) which ask detailed questions about the innovation investments of firms distinguishing between process and product innovations. In particular, process innovation occurs when firms introduce some significant modification of the productive process as the introduction of new machines or the introduction of new methods of organization, while product innovation occurs when firms report a new or improved good. This information is then merged with structural and macroeconomic data drawn from OECD surveys. Additionally, some European Countries carry out nation-specific surveys. For instance, in Spain there is the *Encuestas Sobre Estrategias Empresariales* that is issued every three years. The same analysis becomes more difficult with American data where innovation is measured as patents and therefore the two innovations cannot be distinguished. However, for a concise summary Klette and Kortum (2004) report a list of stylized facts concerning firm R&D, innovation, and productivity.

[4]It should be noticed that the data sets are not homogeneous. Hence table 1 does not allow comparisons across countries but only the ability to observe the stated heterogeneity in the innovation choices.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

to undertake product innovation while large firms are more likely to undertake process innovation.

Thirdly, innovations are *complements*. Process innovation is more frequent than product innovation, while the probability of introducing a product innovation is higher for firms that also introduce a process innovation in the same period. However process innovation does not necessarily imply product innovation.[5] Firms innovate on their existing products, aiming at increasing product differentiation and hence prices, in the hope of exploiting consumers' willingness to pay for a higher quality good. Instead process innovation increases the firms' production efficiency. This leads to higher firm productivity, lower prices and a larger scale of production.[6] Complementarity between process and product innovation then arises: product innovation allows new product designs but these new designs become profitable only when they are affordable for the consumers.

Entry and exit play an important role in explaining the reallocation of resources from less productive firms to more productive firms and therefore growth.[7] In addition, Huergo and Jaumadreu (2004) show that exit is associated with a lower level of pre-exit innovations, while entrants present a high probability of innovation.

Existing growth literature cannot explain all these pieces of evidence as it treats quality upgradings and cost reduction innovations as interchangeable. Moreover, the literature on heterogeneous firms is usually based only on one factor of heterogeneity, either cost efficiency or the ability of producing quality. In these models a single attribute monotonically predicts firms' revenue, competitiveness, and innovation. This characteristic then implies a threshold firm size above which all firms innovate and below none do and hence predictions not in line with the empirical results.

Hence, motivated by the discrepancy between the existing theoretical literature and the empirical evidence, this paper proposes a new framework able to explain and quantitatively replicate the empirical regularities discussed. It analyzes the effects of cost reduction (process) and quality improving (product) innovations on firm dynamics, productivity- and aggregate growth, highlighting the importance of product quality in the growth process. For this purpose, I develop a general equilibrium model with endogenous process and product innovation. The industry dynamics are taken from Hopenhayn (1992) using monopolistic competition as in Melitz (2003). Firms produce differentiated

[5]See Miravate and Pernias (2004) on data for the ceramic tile industry in Spain, Martinez-Ros (1999) for Spanish manufacturing firms and Parisi et. al. (2006) for Italy.

[6]See Smolny (1998) for an empirical study on the effects of process and product innovation on the prices charged by German firms.

[7]Foster et. al. (2001) on data from the US manufacturing sector find that more than 25% of the growth between 1997 and 1998 was due to net entry. However, Bartelsman et. al. (2004) find that in Europe the contribution of net entry is comparatively low than in US.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

goods and are heterogeneous in their production efficiency and in their product quality. The evolution of both efficiency and quality is given by an idiosyncratic permanent component and by an endogenous component proportional to the optimal investment decision taken by the firm. Product innovation increases firms product quality while process innovation increases firm production efficiency. In each period non profitable incumbents exit the industry, and are replaced by new firms. Entrants imitate the average incumbent as in Gabler and Licandro (2005) and Luttmer (2007) and on average they are more productive than exiting firms increasing the average productivity of the industry. Hence, growth arises due to firms' innovation and firms' self-selection and is sustained endogenously by entrants' imitation.

The model is calibrated to match the Spanish manufacturing sector for which there is a large availability of firm-level data and related empirical studies on both firm dynamics and innovation dimensions. Besides matching closely the data, the model generates moments and a firm size distribution consistent with the empirical evidence. The interplay between the two sources of firm heterogeneity and costly innovation results in a non-monotonic relation between firm size and innovation strategies. Small firms undertake product innovation, medium firms both process and product innovation while large firms specialize mainly in process innovation. Moreover, it emphasizes the importance of the reallocation of resources among incumbents and innovators as the main source of growth. In fact, firms' turnover explains only 8.13% of aggregate growth and when innovation is banned output growth declines by 3.1 percentage points. Another interesting prediction that can be empirically tested is the contribution of the growth in production efficiency and product quality in explaining productivity growth. The model predicts that efficiency growth plays the major role explaining 69.8% of output growth. Additionally, this model contributes to the literature that tries to understand why firm heterogenity is persistent endogenizing the evolution of firm technology.

In this model the relationship between firm size and innovative strategies is more articulate in explaining why different firms choose optimally different innovation strategies. Additionally, comparing industries that differ for innovation costs or for entry barriers allows for a better understanding of the growth rate composition and how it is affected by changes in the industry structure. Hence this model provide a suitable framework for the analysis of policy implications aimed at fostering growth.

1.1.1 Related Literature

This paper attempts to link the literature on firm dynamics and endogenous growth theory by explicitly modeling different types of firm-level innovations. As in the seminal

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

models of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), innovation is firm-specific and it is motivated by the appropriation of revenues associated with a successful R&D investment. In Romer (1990) growth is driven by two elements. The first one is the invention of new inputs which make the production of the final good sector more efficient. In this sense and from the point of view of the final good firm it can be seen as process innovation. The second one is knowledge spillovers from past R&D: the higher the stock of knowledge, the easier the invention of new varieties. In this paper there is a similar spillover, which is the imperfect imitation of incumbent firms by entrants. Grossman and Helpman (1991) introduce growth through quality improving innovation of existing products. However, in their model, different qualities are perceived as perfect substitutes and hence the representative consumer buys only the cheapest variety (adjusted by quality). Instead, in my model each variety is perceived as different by the consumer and higher quality varieties give higher utility. In Aghion and Howitt (1992) growth is based on the idea of Schumpeterian creative destruction in which new innovations replace the previous ones driving the incumbent monopolist out of the industry. The creative destruction mechanism is not far from the idea of firm selection. Successful firms grow and drive out of the market unsuccessful ones. Based on these general features my work adds firm heterogeneity, permanent idiosyncratic shocks that hit both production efficiency and product quality, and endogenous investment choices made by incumbent firms. These new elements endogenously link aggregate growth with firm-specific growth and hence with the mechanism of resource reallocation from non-innovators to innovators and from exiting to active firms. The resulting distribution of firm size is consistent with the data.

The idea of firm selection was already present in Jovanovic (1982). He introduces the first model with firm-specific stochastic productivities with unknown mean but known variance. As time goes by firms learn their productivity and the inefficient firms exit. As firms learn their productivity the effects of selection on firms evolution dies out and eventually the industry converges to a stationary equilibrium without entry and exit. For this reason, this paper takes the industry structure from Hopenhayn (1992), who develops a partial dynamics stochastic heterogeneous firms' model which generates a stationary equilibrium with entry and exit that is capable of studying the effects of structural changes in the industry on the distribution of firm size and age. Hopenhayn and Rogerson (1993) analyze the general equilibrium of the Hopenhayn model focusing on the process of labor reallocation. Both papers study the stationary equilibrium in which each firm is hit by shocks characterized by a stationary AR(1) process. However, both papers focus only on firm productivity growth between cohorts and disregard the effects on aggregate growth.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

The link between the process of resource reallocation due to selection at the firm level and economic growth is studied in Gabler and Licandro (2005) and in Luttmer (2007). In both papers firm technology is hit by permanent shocks which together with firm selection and entrant imitation generates endogenous growth. The resulting stationary distribution is a consequence of the knowledge spillover that links the distribution of entrants productivities to the distribution of incumbents productivities. This assumption is necessary to generate endogenous growth. In fact without imitation, as incumbent firms become more productive through selection, the incentives to enter the industry diminish and eventually vanish. In the end no new firms enter into the industry and the equilibrium is characterized by the absence of entry and exit similarly as Jovanovic (1982). Gabler and Licandro (2005) model a competitive equilibrium with heterogeneous firms using both labor and capital as inputs. When calibrating their model on US data they show that selection and imitation account for a fifth of productivity growth. This represents a lower bound. Luttmer (2007) instead considers a monopolistic competition market in which each firm produces a different variety and it is subjected to shocks to both productivity and demand. Calibrating his model to US data he finds that half of output growth can be attributed to selection and imitation. This can be seen as an upper bound.

This paper attempts to extend Gabler and Licandro (2005) and Luttmer (2007) by considering alongside their models the role of innovation in linking firm level growth to aggregate growth. Modeling endogenously firm innovation investments in both firm efficiency and product quality can help to distinguish the differing contributions of selection and imitation versus innovation in process and product when explaining economic growth.

The other papers that shed light on the relationship between innovation, firm heterogeneity and the role of resource reallocation of the growth process are Klette and Kortum (2004) and Lenz and Mortensen (2008). The former, building on Grossman and Helpman (1991), introduces firms that exogenously differ in the profits earned by selling their own products. Endogenous growth is then generated through innovation investments aimed at increasing the number of goods produced by each firm and firms adjust the production lines in response to their own and competitors' investment in R&D. However they posit permanent exogenous differences across firm profitability and hence across the size of the innovative step. This simplification results in a distribution of innovative firms that have the same volatility as the distribution of the firms that do not innovate. This model, defining innovation as an endogenous drift into the stochastic evolution of firm productivity and quality, can account for the differing variances of the distribution of innovators and non-innovators. Lenz and Mortensen (2008) relate to Klette and Kortum (2004) introducing heterogeneity in the expected productivity of the new variety

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

produced. But as in both models the engine of growth is a mechanism of creative destruction on the numbers of goods existing in the economy at a given point in time, they can analyze only one channel of innovation.

More recently, Atkeson and Burstein (2007) address the relation between the decision of heterogeneous firms to innovate and engage in international trade by introducing two types of stochastic innovation activities. Though their model abstracts from endogenous growth, they define as process innovation the decision to increase the stock of firm-specific factors that then translates in higher profits opportunities. This is analogous to process innovation defined in this model. They define as product innovation the creation of a new firm and hence a new product. This is the analogous to firm entry discussed in this model. In fact, this model defines differently from them as product innovation the decision of firms to improve the quality of an exiting variety. Moreover, the jump in the efficiency and/or quality scale are, in this paper, proportional to the research intensity.

Finally two other papers of note, Melitz (2003) and Hallak and Sivadasan (2008). Melitz (2003) proposes a static model with heterogeneous firms in which the exposure to international trade increases firm selection and generates a partition among firms such that the more productive firms are the ones who gain access to foreign markets. Hallak and Sivadasan (2008), building on Melitz (2003), introduce a partial and static equilibrium model in which firms differ in two attributes: labor efficiency and ability to produce high quality varieties. Under the assumption of minimum quality requirements they study how openness affects firm distribution. In their model as in Melitz (2003) the partition of firms between domestic producers and exporters is generated by the presence of a fixed cost to enter the foreign market. Here the same mechanism is used to generate the partition of firms among the different innovation strategies. However, the firm partition and the effects on the size distribution of firms is not the result of a one-shot change but it is the result of the combination of permanent shocks on both states and inter-temporal innovation decisions.

1.2 The Model

This section develops a general equilibrium model in discrete time with infinite horizon.

1.2.1 Consumer Problem

The representative consumer maximizes his utility choosing consumption and supplying labor inelastically at the wage rate w. Its lifetime utility is assumed to take the following

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

form:

$$U = \sum_{t=0}^{\infty} \beta^t \ln(U_t) \tag{1.1}$$

where $\beta < 1$ is the discount factor and t is the time index. In every period the consumer faces the problem of maximizing his current consumption across a continuum of differentiated products indexed by $i \in I$ where I is a measure of the available varieties in the economy. Specifically, the preferences are represented by an augmented Dixit-Stiglitz utility function with constant elasticity of substitution between any two goods $\sigma = 1/(1-\alpha) > 1$ with $\alpha \in (0,1)$. Hence, the utility function at time t is:

$$U_t = \left(\int_{i \in I} (q_t(i) x_t(i))^{\alpha} \mathrm{d}i \right)^{\frac{1}{\alpha}}. \tag{1.2}$$

where $x(i)$ is the quantity of variety $i \in I$ and $q(i)$ is the corresponding quality. This utility function is augmented to account for quality variation across products and quality acts as an utility shifter: for a given price the consumer prefers products with high quality rather than products with low quality.

The per period budget constraint is $E_t = \int_{i \in I} p_t(i) x_t(i) di$ where E_t is total expenditure at time t and $p_t(i)$ is the price of variety $i \in I$ at time t. Solving the intra-temporal consumer problem yields the demand for each variety $i \in I$:

$$x_t(i) = \left(\frac{P_t q_t^{\alpha}(i)}{p_t(i)} \right)^{\frac{1}{1-\alpha}} X_t = \left(\frac{P_t^{\alpha} q_t^{\alpha}(i)}{p_t(i)} \right)^{\frac{1}{1-\alpha}} E_t \tag{1.3}$$

with:

$$P_t = \left(\int_{i \in I} \left(\frac{p_t(i)}{q_t(i)} \right)^{\frac{\alpha}{\alpha-1}} \mathrm{d}i \right)^{\frac{\alpha-1}{\alpha}} \quad \text{and} \quad X_t = U_t. \tag{1.4}$$

P_t is the price quality index at time t of all the bundle of varieties consumed and X_t is the aggregate set of varieties consumed.

Finally, the optimal inter-temporal allocation of consumption yields the standard Euler equation:

$$\frac{X_{t+1}}{X_t} = \beta(1 + r_t). \tag{1.5}$$

where r_t is the return on asset holding.

1.2.2 Firms

This section outlines a dynamic two factors heterogeneous firm model. The first source of heterogeneity is production *efficiency*, $a(i) \in \mathbb{R}_{++}$, which increases the marginal productivity of labor, as in the seminal paper of Hopenhayn (1992), and the second

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

source is *quality* of the firm's variety, $q(i) \in \mathbb{R}_{++} \setminus (0,1)$, which decreases the marginal productivity of labor. In this respect, a higher quality variety has a higher variable cost. Firms are distributed over productivity and quality. $\widetilde{\mu}(a,q) = \mu(a,q)I$ is the measure of firm with state (a,q) at time t, where I is the number of firms in the industry and $\mu(a,q)$ is a density function. It is assumed that each firm produces only one variety so that the index i identifies both the firm and the corresponding variety produced by that firm and I represents both the set of varieties and the mass of incumbent firms active in the industry. The following definition are used, A is the set of all production efficiencies, Q is the set of all product qualities, and $\Omega \equiv A \times Q$ is the state space.

1.2.2.1 Production Decision

After paying a fixed operational cost, c_f, expressed in terms of labor, active firms receive their new technology level, (a,q). Firms produce and price their own products under the assumption of monopolistic competition. As in Hallak and Sivadasan (2008), the production function is assumed to be linear in labor, n, which is the unique input, increasing in firm efficiency, a, and decreasing in firm product quality, q. That is, $x_t(i) = a_t(i)q_t(i)^{-\eta}n_t(i)$ with $\eta \in (0,1)$. The parameter η introduces asymmetry between firm efficiency and product quality and measures the difficulties in producing a higher quality variety: the higher η, the more difficult and costly it becomes to produce a high quality product. This particular functional form is justified by empirical evidence: it generates a price distribution consistent with the estimates of Smolny (1998) and moreover complementarity between process and product innovation is obtained.

The profit maximization problem, faced by each firm, is:

$$\pi_t(a(i), q(i)) = \max_{p(i)} p_t(i)x_t(i) - w_t n_t(i) - w_t c_f \tag{1.6}$$

where w_t is the wage rate at time t common to all firms. The first order condition with respect to price yields the optimal pricing rule:

$$p_t(a(i), q(i)) = \frac{w_t q_t^{\eta}(i)}{\alpha a_t(i)}. \tag{1.7}$$

$1/\alpha$ is the constant mark-up associated with the CES demand function. In contrast to the standard models with a single factor of firm heterogeneity, firms' prices depend on both firms' efficiency and quality. Consistent with both the theoretical predictions and the empirical estimates, the price schedule is increasing in product quality and

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

decreasing in efficiency.[8] As in Melitz (2003) the nominal wage is normalized to one. Using the monopolistic price to solve for the optimal demand for each variety yields:

$$x_t(a(i), q(i)) = \left(\frac{\alpha a_t(i) P_t^\alpha}{q_t(i)^{\eta-\alpha}}\right)^{\frac{1}{1-\alpha}} E_t. \tag{1.8}$$

Firm output is an increasing function of both the aggregates and of the efficiency level of firms. The relationship between product quality and output is ambiguous and depends on the comparison between α, related to consumer preferences, and η, coming from firm production function. If $\eta > \alpha$ then firm output is decreasing in the product quality: high quality varieties are characterized by a relatively lower market share. In this case, the positive effect of quality on consumer utility is completely offset by the related high market price. The opposite is true when $\alpha > \eta$.

The optimal labor demand is given by:

$$n_t(a(i), q(i)) = \left(a_t(i) q_t(i)^{1-\eta}\right)^{\frac{\alpha}{1-\alpha}} \left(\alpha P_t^\alpha\right)^{\frac{1}{1-\alpha}} E_t. \tag{1.9}$$

Labor input is an increasing function of both firms' state variables. Consequently, firms with more advanced technology demand more labor input. Finally, the net per period profit of firm i is given by:

$$\pi_t(a(i), q(i)) = \left(a_t(i) q_t(i)^{1-\eta} \alpha\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) P_t^{\frac{\alpha}{1-\alpha}} E_t - c_f. \tag{1.10}$$

Although product quality has an ambiguous effect on the optimal output of firms, profits are increasing in both labor efficiency and product quality. This provides incentives for firms to improve endogenously their position in the technology distribution via firms' innovation policies. In this respect, the model predicts that a change in efficiency impacts more a firm's profit than a change in quality.

The different effects of firm efficiency and quality on the monopolistic price, on the output, and on the profits provide a suitable framework in which to study the interplay among different innovation choices taken by a firm and their effects on a firm's competitiveness.[9]

[8]Smolny (1998), studying a panel of West German firms in the manufacturing sector in the period 1980-1992, estimates that product innovation increases the probability and the frequency of positive net prices increases by more than 18% while process innovation does not reveal a conclusive effect on firm pricing strategies. However, he clearly estimates that process innovations increases the probability of employment and especially output increases. Making increases in output and employment without a lower price is difficult. Hence the effects on output and employment support the relevance of price effects and of the complementarity between the two forms of innovation.

[9]An innovation in product, aimed at increasing product quality, results in a higher market price for the given variety and, for appropriate parameters, in a contraction of the market quota. This then determines an incentive to invest also in process innovation and hence to increase firm efficiency. That in turn leads to a lower market price and to an unambiguous larger market share.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

1.2.2.2 Innovation Decision

Firms receive idiosyncratic permanent shocks on both states. That is, firms' log efficiency and log quality follow a random walk. This is a way of capturing the role of firm-specific characteristics and the persistence of firm productivity which is established in the empirical literature.[10] Besides the exogenous random walks, firms can endogenously affect the evolution of their states through private innovation activities. In line with the terminology used in the surveys at the firm-level, this paper identify two different types of innovation: *process innovation* and *product innovation.* Process innovation refers to the decision of firms to invest labor, with the aim of lowering firm production costs, while product innovation refers to the decision of firms to direct labor investment at increasing the quality of the varieties produced.

According to the theoretical growth literature, the benefits derived by firms' innovation investments are proportional to the amount of resources spent. In particular, innovation introduces an endogenous drift in the random walk processes which reflects the amount of variable labor that firms optimally invest in R&D. The innovation choice is history dependent as today investment in process or product innovation results in tomorrow higher firm production efficiency and/or product quality. In addition, firms have to pay also a fixed cost of innovation, c_a and c_q, for process and product innovation, respectively. This is a way of capturing the costs necessary to set up an R&D department, to conduct market analysis and technically it determines the partition of firms among different innovation strategies. Depending on the firms' technology state, some firms decide to innovate either in process or in product or in both types of innovation. In whichever form innovation comes, it represents a first source of endogenous growth since it shifts the bivariate firms' distribution to the right.

Specifically, log efficiency is assumed to evolve according to:

$$\log a_{t+1} = \begin{cases} \log a_t + \varepsilon^a_{t+1} & \text{when } \; z_t = 0 \\ \log a_t + \lambda^a \log z_t(a,q) + \varepsilon^{az}_{t+1} & \text{otherwise} \; . \end{cases} \tag{1.11}$$

Shocks are firm-specific and distributed as $\varepsilon^a_{t+1} \sim N(0, \sigma^2_a)$, $\varepsilon^{az}_{t+1} \sim N(0, \sigma^2_{az})$ where σ^2_a is the variance of the random walk when innovation does not occur and σ^2_{az} is the variance of the process when innovation takes place. $z_t(a, q) > 0$ is the labor that a firm with states (a, q) decide optimally to invest in process innovation. $\lambda^a > 0$ is a parameter that, together with the log form of the innovation drift, scales the effects of innovation. The log functional form chosen for the innovation drift is important as together with firm

[10]For instance, the idiosyncratic shocks can capture factors as absorption techniques, managerial ability, gain and losses due to the change in the labor composition and so on.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

selection assure a bounded growth and hence the existence of a stationary distribution. Similarly log quality evolves as:

$$\log q_{t+1} = \begin{cases} \log q_t + \varepsilon^q_{t+1} & \text{when } \; l_t = 0 \\ \log q_t + \lambda^q \log l_t(a,q) + \varepsilon^{ql}_{t+1} & \text{otherwise } \; . \end{cases} \tag{1.12}$$

Again $\varepsilon^q_{t+1} \sim N(0, \sigma^2_q)$, $\varepsilon^{ql}_{t+1} \sim N(0, \sigma^2_{ql})$ where σ^2_q and σ^2_{ql} are the two variances without and with innovation. $l_t(a, q)$ is the variable labor devoted to product innovation and $\lambda^q > 0$ is the related scale parameter. The means of the efficiency and quality shocks are normalized to zero eliminating exogenous sources of growth. In fact, abstracting from innovation and firm selection, in expectation firms do not grow.

The random component ε is independent both across firms and over time. Moreover, the two processes, efficiency and quality, are independent.[11] Define the density function of a_{t+1} conditional on a_t as $f(a_{t+1}|a_t)$, and the density functions of q_{t+1} conditional on q_t as $p(q_{t+1}|q_t)$. The transition of the two state variables depends on the firms' innovation decisions and the idiosyncratic shocks. Considering jointly the two transition functions, $\Phi : \Omega \to \Omega$ can be defined as the joint transition function, which moves firms' quality and efficiency states. The corresponding transition probability function is defined as $\phi : \Omega \times \Omega \to [0, 1]$, which gives the probability of going from state (a, q) to state (a', q'). The transition probability takes different forms depending on the innovation decisions and on the exit decision defined below. If the two processes are independent then $\phi(\cdot) = f(\cdot)p(\cdot)$.

1.2.2.3 Firm Value Function

Incumbent firms face a dynamic optimization problem of maximizing their expected value. Once abstracted from the innovation decision this is a particularly simple problem since it is a sequence of static optimizations. With the innovation scheme, current investments in innovation affect the transition probabilities and thus the value of future technology. This generates a dynamic interplay between firm technology and the innovative position taken by the firm. This is summarized by the following value function:

$$v(a, q) = \max\{v^P(a, q), v^A(a, q), v^{AQ}(a, q), v^Q(a, q)\}. \tag{1.13}$$

The max operator indicates that in each period firms face different discrete choices which depend on the current level of production efficiency and product quality. $v^P(a, q)$

[11]This simplification does not affect qualitatively the model predictions, but it has the advantage to narrow the set of parameters to calibrate since it is possible to ignore the covariances of the two processes.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

is the value when no innovation investments occurred, $v^A(a,q)$ when a firm produces and innovates in process, $v^{AQ}(a,q)$ when both process and product innovation are undertaken and $v^Q(a,q)$ when a firm specializes only in product innovation.

Using $J = \{P, A, Q, AQ\}$ and defining with prime the next period variables, the Belman equation for each choice is given by:

$$v^J(a,q) = \max_p \left\{ \pi^J(a,q) + \frac{1}{1+r} \max\left\{ \int_\Omega v(a',q')\phi(a',q'|a,q)\mathrm{d}a'\mathrm{d}q', 0 \right\} \right\}. \quad (1.14)$$

where $\pi^P(a,q)$ is given by equation (11), $\pi^A(a,q) = \pi(a,q) - z(a,q) - c_a$, $\pi^{AQ}(a,q) = \pi(a,q) - (z(a,q) + l(a,q)) - c_a - c_q$, and $\pi^Q(a,q) = \pi(a,q) - l(a,q) - c_q$.

These value functions characterize a partition of firms among the different decisions (only produce or produce and innovate, and in the latter case if process, or product or both at the same time) which depends on the relation between the technological state of each firm and the fixed costs. In fact, given the specific position of a firm inside the bivariate distribution of technology, the fixed costs of innovation generate different firms decisions consistently with equation (14). Two sources of firm heterogeneity implies that the thresholds, characterizing the border among the different innovation strategies, are given by infinite combinations of (a,q) couples. For this reason, it becomes convenient to express the reservation values in terms of efficiency as a function of quality, $a(q)$ and to obtain *cutoff functions* rather than cutoff values as in one factor heterogeneous firm models. For given $q \in Q$ it is possible to define the following cutoff functions: $a_A(q)$ delimits the area in which process innovation is optimal, $a_Q(q)$ delimits the area in which product innovation is optimal, and $a_{AQ}(q)$ delimits the area in which both innovations are chosen by the firms.[12] Appendix A provides a formal definition of these cutoff functions.

The cutoff functions are decreasing in q and hence also less efficient firms but characterized by a product with high quality may innovate. Notice that firm profits, $\pi(a,q)$, are increasing in both efficiency and quality generating the incentives to innovate which are slowed down by the log form in which the innovation drift is modeled. Abstracting from the discontinuity in the value function due to the fixed costs of innovation, the more advanced the firm technology, the higher the innovation investment but the lower the benefit due to the diminishing returns of innovation.

[12]It is equivalent to express product quality as a function of efficiency, $q(a)$. Using a specific formulation for the cutoff function does not affect the implications of the model.

 DOI: 10.2870/28221

1.2.2.4 The Exit Decision

Firms exit the industry after a bad technological draw such that the expected value of continuing is lower than the exit value which has been normalized to zero.[13] Since firm value is increasing in both states the exit reservation value is decreasing in both of them. Again a cutoff function $a_x(q)$ can be defined such that:

$$E[v(a'(q), q')|(a_x(q), q)] = 0. \tag{1.15}$$

For each quality level, there is a maximum efficiency level such that below this maximum firm value is negative and therefore firms find optimally to exit the industry. Interestingly, the cutoff function $a_x(q)$ is decreasing in quality: for given efficiency firms with a high quality product can survive longer in the market when hit by a bad efficiency shock.

Firms innovation decisions, exit and the law of motion of (a, q) define the transition function $\Phi_{xI} : A \setminus A_x \times Q \to (A_p \cup A_A \cup A_Q \cup A_{AQ} \cup A_x) \times Q$ where the support of efficiency is partitioned into the exit support, A_x, the production support, A_P, the process innovation support, A_A, the product innovation support, A_Q, and the process and product innovation support, A_{AQ}. These partitions differ across different elements of Q.[14] The corresponding transition probability of going from state $(a, q) \in (A_p \cup A_A \cup A_Q \cup A_{AQ}) \times Q$ to $(a', q') \in (A_p \cup A_A \cup A_Q \cup A_{AQ} \cup A_x) \times Q$ is given by a function $\phi_{xI}(\cdot)$.

1.2.2.5 Firms Entry

Every period there is a mass of potential entrants in the industry which are *a priori* identical. To enter firms have to pay a sunk entry cost, c_e, expressed in terms of labor. This cost can be interpreted as an irreversible investment into setting up the production facilities. After paying the initial cost, firms draw their initial a and q from a common bivariate density function, $\gamma(a, q)$. The associated distribution is denoted by $\Gamma(a, q)$ and has support in $\mathbb{R}_+ \times \mathbb{R}_+$. Define $\overline{\gamma}_e$ the mean of the joint distribution and σ^2_{ea} and σ^2_{eq} the variances of the entrants efficiency and quality processes.[15] Moreover, as in Gabler and Licandro (2005) and Luttmer (2007) I assume that entrants are on average less productive than successful incumbent and that they imitate them. In particular, the

[13]Notice that exit is triggered by the assumption of fixed operational costs, c_f, paid by active firms in each period. Without fixed operational costs, firms hit by bad shocks instead of exiting the market could temporary shut down their production and just wait for better periods when positive shocks hit their technology and then start again producing.

[14]Appendix A defines mathematically these supports.

[15]The covariance is zero given the current assumption of independence between the evolution of the two states.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

mean of the entrant distribution is a constant fraction $\psi_e \in (0,1)$ of the mean of the joint distribution of incumbents defined as $\overline{\mu}$. That is, $\overline{\gamma}_e = \psi_e \overline{\mu}$. This knowledge spillover, that goes from incumbent firms to entrants, is the only externality of the model and combined with firm selection and innovation generates endogenous growth.[16]

In equilibrium the free entry condition holds: potential entrants enter until the expected value of entry is equal to the entry cost:

$$v^e(a,q) = \int_{\Omega_e} v(a,q) d\Gamma(a,q) = c_e, \tag{1.16}$$

M_t is the mass of firms that enter in the industry at time t. At the stationary equilibrium also a stability condition holds: the mass of new entrants exactly replaces the mass of unsuccessful incumbents who are hit by a bad shock and exit the market. That is, $M' = \int_0^{a_x(q)} \int_Q I\mu(a,q)$.

1.2.3 Cross Sectional Distribution and Aggregates

All firms' choices and the processes for the idiosyncratic shocks yield the low of motion of firms distribution across efficiencies and qualities, $\mu(a,q)$. That is:

$$\begin{aligned} I'\mu'(a',q') = I\Bigg(& \int_{A_P}\int_Q \mu(a,q)\phi(a',q'|a,q)\mathrm{d}q\mathrm{d}a + \\ & \int_{A_A}\int_Q \mu(a,q)\phi(a',q'|a,q,z)\mathrm{d}q\mathrm{d}a + \int_{A_{AQ}}\int_Q \mu(a,q)\phi(a',q'|a,q,z,l)\mathrm{d}q\mathrm{d}a + \\ & \int_{A_Q}\int_Q \mu(a,q)\phi(a',q'|a,q,l)\mathrm{d}q\mathrm{d}a\Bigg) + M'\gamma(a',q') \end{aligned} \tag{1.17}$$

Tomorrow density is given by the contribution of all surviving firms (the domain of the integrals is restricted to surviving firms only) and of entrants. The contribution of new firms is represented by the last term of (17). The first integral represents the share of surviving firms that only produce and do not innovate, the second integral shows the contribution of the firms that successfully produce and invest in process innovation. The third one instead represents the firms that produce and undertake both types of innovation and finally the forth one highlights the share of producers that specialize in product innovation only.[17]

[16] Eeckhout and Jovanovic (2002) used a wider mechanisms of knowledge spillover in which all firms and not only entering firms, can imperfectly imitate the whole population of firms.

[17] Since the industry is populated by a continuum of firms and only independent idiosyncratic shocks occur the aggregate distribution evolves deterministically. As a consequence, though the identity of any firms i associated with a couple (a,q) is not determined, their aggregate measure is deterministic. For the same reason the other aggregate variables evolve deterministically.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

To summarize the information about the average firm efficiency and product quality, a weighted mean of firm technology can be introduced. That is:

$$\overline{\mu} = \left(\int_{a_x(q)} \int_Q \left(aq^{1-\eta}\right)^{\frac{\alpha}{1-\alpha}} \mu(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{1-\alpha}{\alpha}}. \tag{1.18}$$

Notice that $aq^{1-\eta}$ is an index of firm level technology that maps one to one to firms' profits and size. Differing from Melitz (2003), this weighted mean not only depends on two states, efficiency and quality, but also the weights reflect the relative quality adjusted output shares of firms with different technology levels rather than the simple output shares. Moreover, the weighted mean can be also seen as the aggregate technology incorporating all the information contained in $\mu(a,q)$. In fact, it has the property that the aggregate variables can be expressed as a function of only $\overline{\mu}$ disregarding the technology distribution, $\mu(a,q)$.[18]

1.2.4 Equilibrium Definition

In equilibrium the representative consumer maximizes its utility, firms maximize their discounted expected profit and markets clear. The stationary equilibrium of this economy is a sequences of prices $\{p_t\}_{t=0}^{\infty}$, $\{P_t\}_{t=0}^{\infty}$, real numbers $\{I_t\}_{t=0}^{\infty}$, $\{M_t\}_{t=0}^{\infty}$, $\{X_t\}_{t=0}^{\infty}$ functions $n(a,q;\mu)$, $z(a,q;\mu)$, $l(a,q;\mu)$, $v(a,q;\mu)$, cutoff functions $a_x(q)$, $a_A(q)$, $a_{AQ}(q)$, and $a_Q(q)$ and a sequence of probability density function $\{\mu_t\}_{t=0}^{\infty}$ such that:

- the representative consumer chooses asset holding and consumption optimally so that to satisfy the Euler Equation (5),
- all active firms maximize their profits choosing a price that satisfies (7) and employment and innovation policies that satisfy $n(a,q;\mu)$, $z(a,q;\mu)$, and $l(a,q;\mu)$ yielding the value function $v(a,q)$ as specified by equation (13) and its components,
- innovation is optimal such that the cutoff functions $a_A(q)$, $a_{AQ}(q)$, and $a_Q(q)$ satisfy the previous conditions,
- exit is optimal such that $a_x(q)$ is given by equation (15) and firms exit if $a(q) < a_x(q)$,
- entry is optimal: firms enter until equation (16) and the aggregate stability condition are satisfied,

[18] See Appendix B for more details.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

- the number of active firms I adjusts till the labor market clears: $L^P + L^I + Ic_f + M'c_e$.[19]
- the stationary distribution of firms evolves accordingly to (17) given μ_0, I, M and the cutoff values,
- the stability condition, $M' = \int_0^{a_x(q)} \int_Q I\mu(a, q)$, holds.

In equilibrium a_x, a_A, a_{AQ}, a_Q, I and M are such that the sequence of firms distribution is consistent with the law of motion generated by the entry and exit rules.[20]

1.3 Endogenous Growth

1.3.1 Balanced Growth Path

In general, on the Balanced Growth Path output, consumption, real wage, prices and the aggregate technology grow at a constant rate, the bivariate distribution of efficiency and quality shifts to the right by constant steps, its shape is time invariant, and the interest rate, the aggregate expenditure, the aggregate profit, the profit and the labor demand distributions, the number of firms, the firm turnover rate, and the other characteristics of the firms' distribution are constant.

Define g as the average growth rate of firm productivity, $\overline{\mu}$. It is given by a combination of the growth rate of the efficiency state, denoted by g_a, and of the growth rate of the product quality state, indicated by g_q. Intuitively, growth arises because in every period the log of the joint aggregate technology shifts to the right by a factor g, meaning that the average efficiency and the average product quality of the industry grow. Defining the growth factors of firm efficiency and product quality by $G_A = \frac{a_{t+1}}{a_t} = 1 + g_a$ and $G_Q = \frac{q_{t+1}}{q_t} = 1 + g_q$, the Balanced Growth Path can be found as follows. From the labor market clearing condition, given the assumption of a constant labor supply, N_s, also the number of incumbent firms, I, and the number of entrants, M, have to be constant as well as the share of labor allocated to production and innovation.[21] Aggregate expenditure, E, has to be equal to the aggregate labor income, N_s, given the wage normalization. This in turn implies that E is constant and hence also Π has to be constant. The profit

[19] Where $L^P = \int_A \int_Q n(a,q) I\mu(a,q) \mathrm{d}q\mathrm{d}a$ is the production labor and $L^I = \int_A \int_Q (l(a,q) + z(a,q)) I\mu(a,q)\mathrm{d}q\mathrm{d}a + I\int_{A_A}\int_Q \mu(a,q)c_a \mathrm{d}q\mathrm{d}a + I\int_{A_Q}\int_Q \mu(a,q)c_r \mathrm{d}q\mathrm{d}a + I\int_{A_{AQ}}\int_Q \mu(a,q)(c_a + c_r)\mathrm{d}q\mathrm{d}a$ is the innovation labor considering both the variable and fixed costs.

[20] Hopenhayn (1992)'s paper proves the existence of equilibrium for similar economies.

[21] If there was population growth then the number of varieties, and the number of entrant firms would grow at the same rate as population grows.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

distribution, equation (10), shows that $\pi(a, q)$ has to be constant because of constant fixed operational costs. Given a constant expenditure, profits are constant only if $aq^{1-\eta}P$ is constant. For positive growth rate of the technology, the previous condition holds if the price index growth factor is inversely related to the average technology growth factor, $G_P = (G_A G_Q^{1-\eta})^{-1}$. In other words, as the industry grows and the average technology advances, the price index diminishes. With the same reasoning also the distribution of manufacturing labor, equation (9), is time invariant, which together with the labor market clearing condition implies that also the distributions of the labor hired for the innovation activities, $z(a, q)$ and $l(a, q)$, are constant. From the consumer problem $E = PX$, which holds only if the aggregate consumption X grows at a constant factor $(G_A G_Q^{1-\eta})$. This results in a constant interest rate as shown by the Euler equation, $r = (1+g)\beta - 1$. The price distribution, $p(a, q)$, decreases at a factor equal to $\frac{G_Q^{\eta}}{G_a}$ which is lower than the growth rate of the price index. This is a consequence of the fact that the price index is adjusted to consider the growth in the product quality. Finally, $x(a, q)$ grows at a factor of $\frac{G_A}{G_Q^{\eta}}$.

A Balanced Growth Path equilibrium exists if there is a g_a and a g_q consistent with the stationary equilibrium. To find these growth rates and to characterize the equilibrium itself and the stationary firms' distribution it is necessary to transform the model such that all the variables are constant along the Balanced Growth Path. Hence, all growing variables need to be divided by the corresponding growth factor, $\widetilde{s} = s/G_s^t$ and the stochastic processes in efficiency and quality need to be de-trended by the respective growth rates, $\log \widetilde{a}_t = \log a_t - g_a t$ and $\log \widetilde{q}_t = \log q_t - g_q t$, where "$\sim$" denotes the stationarized variables. In expected terms both average firm efficiency and average quality increase and thus in expectation in every period each firm falls back relative to the distribution. This transformation affects also the transition functions and hence log efficiency and log quality, in the stationarized economy, which evolve according to:

$$\log \widetilde{a}_{t+1} = \begin{cases} \log \widetilde{a}_t - g_a + \varepsilon^a_{t+1} \\ \log \widetilde{a}_t - g_a + \lambda^a \log \widetilde{z}_t + \varepsilon^{az}_{t+1} \end{cases} \tag{1.19}$$

$$\log \widetilde{q}_{t+1} = \begin{cases} \log \widetilde{q}_t - g_q + \varepsilon^q_{t+1} \\ \log \widetilde{q}_t - g_q + \lambda^q \log \widetilde{l}_t + \varepsilon^{ql}_{t+1}. \end{cases} \tag{1.20}$$

These negative trends together with decreasing return in innovation determine a finite expected lifetime for any level of technology (a, q). Any successful firm which performs innovation will not be an innovator forever but eventually it will exit the market, leading to a finite expectation and to a finite variance of the incumbent firm distribution and hence assuring the existence of a stationary distribution in the de-trended economy.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

The previous discussion leads to the following proposition:
Proposition 1: *Given G_a and G_q growth factors of firms efficiency and quality the economy admits a Balanced Growth Path along which the mean of the joint distribution of incumbent firms and of entrant firms and the aggregate consumption grow at a rate $G_a G_q^{1-\eta}$, the price index decreases at a rate $G_a G_q^{1-\eta}$, the output distribution grows at a rate G_a / G_q^{η}, the price distribution grows at a rate G_q / G_a^{η} and the number of firms, the number of entrants, the aggregate expenditure, the aggregate profits, the profit distribution, and the labor distributions are constant.*

1.3.2 Growth Rate Determinants

Firms' *Selection* and *Innovation* drive endogenous growth which is then sustained by entrants' *Imitation*. Firm selection results from the assumption of a random walk process for both the evolution of labor efficiency and product quality together with firm exit. Considering only a cohort of firms and abstracting from the endogenous drift introduced by innovations, in the growing economy the random walk processes are characterized by constant expectations and by variances of the distribution of those firms that increase over time. However, among the given firms the ones with low efficiency and low quality exit the industry truncating the joint distribution from below. This implies that the distribution can spread only towards higher level of efficiency and quality resulting in a higher average productivity of the remaining firms in the cohort.

Firms' innovation reinforces growth. For a given set of innovative firms also the productivity and quality expectations increase over time and they depend on the initial states and on the sequences of innovation investments. In fact, after every successful innovation the average technology shifts upwards due to the endogenous drifts generating growth. However, innovation has decreasing returns through the log form in which the innovation drift is modeled. For this reason the resource reallocation effect from non-innovators to innovators is controlled by the selection effect and the result is that growth is reinforced but still bounded. As a result the average productivity of innovators grows slower than the exit cutoff. Consequently, as time goes by firms keep exiting the industry and the distribution shrinks.

Hence, entrants' imitation is needed to sustain growth and assure the existence of a stationary distribution with entry and exit. In equilibrium the mass of entrants has to be equal to the mass of firms exiting the market. However entrants are on average more productive than exiting firms otherwise they would not find optimal to enter the market. Since exiting firms are replaced by entrants with on average better efficiency and quality levels, the resulting firm distribution moves every period upwards towards

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

higher technological levels.[22] Notice that innovation affects growth also allowing for better imitation.

When innovation occurs the efficiency and quality processes have also higher variances of the stochastic component. This increases the probability of a bad shock hitting the innovative firms and the dispersion of the innovator distribution against the distribution of non-innovators and exiting firms. On the one hand, selection results in a higher average technology for innovators because relatively bad firms fall among the pool of non-innovators resulting in a scenario where only relatively low cost and high quality firms keep innovating. On the other hand, the pool of non-innovators becomes larger, implying a higher weight to the distribution of non-innovators which has a lower average technology. The final effect of higher variances of the innovation random walks on the mean of the joint distribution is ambiguous. However, calibrating the model to match the Spanish data shows that the positive effect of innovation always outweighs the negative effect.

1.3.3 Growth Rate Decomposition

On the Balanced Growth Path the growth rate of aggregate and average consumption is the same and can be rewritten and approximated (the derivations are in the Appendix) as:

$$g \approx \frac{1}{\alpha \bar{X}^{\alpha}} \left\{ \int_A \int_Q \hat{x}(a,q)^{\alpha} \left[\Phi_{xI}\mu(a,q) - \left(1 - \frac{M}{I}\right)\mu(a,q) + \frac{M}{I}\big(\gamma(a,q) - \mu(a,q)\big) \right] \mathrm{d}q\mathrm{d}a \right\}, \tag{1.21}$$

where $\bar{X}$ is the average consumption, $\hat{x}(a,q) = qx(a,q)$ is the firm's quality weighted output, Φ_{xI} is the transition function with the exit and innovation rules and M/I is the entry/exit equilibrium rate. The first difference into the squared bracket represents the growth contribution of selection and innovation. That is, the difference between the quality-output weighted average productivity of surviving firms (both innovators and non innovators) and the one of the previous period incumbents. The more significant the innovation investment is, the larger $\Phi_{xI}\mu$ and the tougher selection is, the smaller $(1 - M/I)\mu$. Hence, both more innovation and tougher selection promotes growth. The second difference instead represents the contribution of entrants' imitation. The easier or cheaper the imitation mechanism (the smaller the distance between the entrants' and incumbents' distributions) the larger the contribution of entrants to the aggregate growth. Adopting the terminology introduced by Poschke (2008), μ can be divided into

[22]Randomness and innovation are important to emphasize the fundamental role of reallocation of resources in the growth process. Growth could still be generated without selection and innovation assuming that the joint mean of the entrants distribution shifts every period exogenously by g. However in this way growth would just result from entry and exit.

μ_{con}, continuing firms, and μ_{exit}, exiting firms. This allows for a further disaggregation of the aggregate growth rate:

$$\begin{aligned} g \quad \approx \quad & \frac{1}{\alpha \bar{X}^{\alpha}} \left\{ \int_A \int_Q \hat{x}(a,q)^{\alpha} \left[\Phi \mu_{con}(a,q) - \mu_{con}(a,q) \right] \mathrm{d}q \mathrm{d}a + \right. \\ + \quad & \left. \int_A \int_Q \hat{x}(a,q)^{\alpha} \left[\frac{M}{I} \gamma(a,q) - \mu_{exit}(a,q) \right] \right\}. \end{aligned} \quad (1.22)$$

The first integral catches the share of growth due to firms' innovation activities and due to the idiosyncratic shocks hitting surviving firms' level technology.[23] The second integral instead represents the share of growth due to net entry. It is clear that the selection of inefficient firms exiting the market and the imitation of new entrants generate positive growth only if entrants are on average more productive than exiting firms. This condition holds in the stationary equilibrium with positive entry. Furthermore, splitting the density of continuing firms between the densities of firms that only produce, μ_p, and of firms that innovate and produce, μ_i, the first integral in equation (22) can be further disaggregated in:

$$\begin{aligned} & \int_A \int_Q \hat{x}(a,q)^{\alpha} \left[\Phi \mu_{con}(a,q) - \mu_{con}(a,q) \right] \mathrm{d}q \mathrm{d}a = \\ & \int_A \int_Q \hat{x}(a,q)^{\alpha} \left[(\Phi \mu_p(a,q) - \mu_p(a,q)) + (\Phi \mu_i(a,q) - \mu_i(a,q)) \right] \mathrm{d}q \mathrm{d}a. \end{aligned} \quad (1.23)$$

Among surviving firms it is now possible to calculate the share of growth that is due to only firms' experimentation based on the random walk processes without drift and the share of growth due to both experimentation and firms' innovation. The numerical analysis of the model will then quantify the share of growth due to net entry, innovation together with experimentation, and firms' experimentation.

The innovation investments of firms affect aggregate growth both directly and indirectly through a better imitation. In fact, innovation results in a higher joint mean of the incumbents' distribution and hence on entrants that can draw their initial technology from a distribution that stochastically dominates the distribution of entrants in an economy without innovation. Given that $\bar{\mu}$ is the key variable in the imitation process, the contribution of innovation on a better imitation can be assessed rewriting $\bar{\mu}$ as:

$$\bar{\mu} = \left(\int_{A_P} \int_Q (aq^{1-\eta})^{\frac{\alpha}{1-\alpha}} \mu_p(a,q) \mathrm{d}a \mathrm{d}a + \int_{A_I} \int_Q (aq^{1-\eta})^{\frac{\alpha}{1-\alpha}} \mu_i(a,q) \mathrm{d}q \mathrm{d}a \right)^{\frac{1-\alpha}{\alpha}} \quad (1.24)$$

[23]Without weighting the firm distribution by the share of quality weighted output the resulting expected growth rate of the average technology of continuing firms would be zero. However, given that the optimal consumption is a convex function of the technology index $aq^{1-\eta}$, by Jensen inequality, the average growth rate of the output weighted technology is positive.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

and using the following equation:

$$1 = \frac{1}{\bar{\mu}^{\frac{\alpha}{1-\alpha}}} \left(\int_{A_P} \int_Q (aq^{1-\eta})^{\frac{\alpha}{1-\alpha}} u_p(a,q) \mathrm{d}q\mathrm{d}a + \int_{A_I} \int_Q (aq^{1-\eta})^{\frac{\alpha}{1-\alpha}} u_i(a,q) \mathrm{d}q\mathrm{d}a \right), \quad (1.25)$$

where A_P is the support of surviving firms that produce but do not innovate while $A_I = A_A \cup A_Q \cup A_{AQ}$ is the support of firms that produce and innovate. The second integral captures the contribution of innovation in determining the joint mean of the incumbent firms. It is clear that the larger this term is, the higher the indirect growth contribution of innovation via a better imitation.

1.4 Numerical Analysis

The algorithm, used to solve the model in the stationary equilibrium, is explained in Appendix D.

1.4.1 Calibration

Sixteen parameters, linked to firm dynamics characteristics, firms specific innovation behavior and the general economic environment, need to be chosen. Since all of them interact with each other to determine the stationary equilibrium only the discount factor, β, the preference parameter, α, and the imitation parameter, ψ_e are chosen *a priori*. The others are jointly calibrated to match the Spanish manufacturing sector.[24] In detail, β is set equal to 0.95 to analyze a yearly time span. Accordingly to Ghironi and Melitz (2003), α is set equal to 0.73, so that the price mark-up charged by the monopolistic firm is of 36% over the marginal cost.[25] ψ_e, relating the mean of the entrants distribution with the mean of the incumbents, is a key parameter in determining growth. For this reason it is set individually to match its empirical counterpart. That is, ψ_e is chosen such that the average size of entrants is 38% of the size of incumbent firms as estimated by Gracia and Puente (2006).

[24]The Spanish economy has been empirically widely studied in both the dimensions object of this paper: the new dimension related to firm innovation behavior and the traditional dimension related to firm dynamics. Hence, from the Spanish data it is possible to obtain enough information to calibrate successfully the model. Similar studies are available also for other European countries (Bartelsman et al. (2004), Bartelsman et al. (2003) for OECD countries; Cefis and Marsili (2005) for the Netherlands, Smolny (2003) and Fritsch and Meschede (2001) for Germany).

[25]This high mark-up could be seen at odds with the macro literature that delivers a standard mark-up of around 20% over the marginal/average cost. In this model, a higher mark-up is justified by the presence of the fixed costs. In fact, given the free entry condition, firms on average break even. Hence on average, firms price at the average cost leading to reasonaby high mark-ups over the average cost.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

Twelve parameters are calibrated using a genetic algorithm as described by Dorsey and Mayer (1995).[26] These are: the ratio among the fixed costs, c_e/c_f, c_a/c_f, and c_q/c_f, the quality parameter η, the four variances of the incumbent random walks σ_a, σ_{az}, σ_q, and σ_{ql}, the two variances of the entrant random walks, σ_{ea} and σ_{eq}, and finally the two parameters that scale the innovation drifts into the stochastic processes, λ_a and λ_q. These parameters jointly determine the shape, the truncation functions of the stationary distribution of firms, and the partition of firms among the different innovation strategies. They are calibrated, using as targets, static and dynamic empirical moments that are informative and related to the main objective of the paper. It is possible to distinguish between two sets of targets.

Firstly, I use moments related to the literature on firm dynamics. These are firms' survival rates after two and five years upon entry, firms' yearly turnover rate, the job creation rate due to entry, the fraction of firms below average productivity, and the productivity spread, which calibrate the six variances of the model and the size of entrants with respect to exiting firms which gives information about the entry cost. Accordingly to Garcia and Puente (2006), the two and five year survival rates for Spanish manufacturing firms are estimated to be 82% and 58%, respectively.[27] They report also a yearly firm turnover rate of 9% and a job creation rate due to entry equal to 3%.[28] Garcia and Puente (2006), estimate that entrants firms are 23% bigger than exiting firms in terms of employment. Bartelsman et al. (2004) estimate that the fraction of Spanish firms below average productivity is equal to 83%, highlighting a right skewed firm size distribution. The last moment is the productivity spread between the 85^{th} and 15^{th} percentile which is estimated to be between 3 and 4.

A second set of moments are instead taken from the empirical literature on firm innovation. The targets used are the share of Spanish manufacturing firms performing process innovation, product innovation and the share of firms that do not innovate and the intensity of the innovation investments in process and product, respectively. In the scope of this paper these are relevant moments that help to calibrate the fixed cost of process and product innovation, η, λ_a, and λ_q. Harrison et al. (2008) working on data derived from the CIS report that 12.2% of Spanish firms in the manufacturing sector declared process innovation between 1998 and 2000, while 12.4% declare product innovation and

[26] The object of the algorithm is to jointly calibrate the parameters in order to minimize the mean relative squared deviation of twelve model moments with respect to the corresponding moments in the data. Since the problem is highly non-linear, the minimization can be characterized by many local minima and the genetic algorithm used has the nice feature to increase the probability of choosing the global minimum.

[27] Those numbers are aligned to the one reported by other developed countries as UK, Germany and Nederland (Bartelsman et al. (2003)).

[28] Firms' turnover is computed as the sum of the number of entering and exiting firms over the total number of firms while job creation rate is computed as the total amount of labor employed by entering firms in a year divided by the total employment in the same year.

more than half of the firms do not innovate in the time span considered. This numbers are very close to the one published by the National Statistics Institute (www.ine.es) using the ESEE. The innovation intensity, computed as the ratio between the aggregate investment in innovation and the aggregate sales, in the 1998 is of 1.71%, process innovation intensity accounts for 1.26% while product innovation intensity accounts for the remaining 0.44%.[29]

Finally, the last parameter to calibrate is the growth rate of the economy, g. In fact, the aim of this paper is to provide a model able to disentangle the contribution of efficiency and quality improvements in explaining the economy growth rate and not to test the ability of the model in matching the aggregate growth rate. For this reason g is set equal to 0.042 accordingly to the European Innovation Scoreboard (2001) and represents the labor productivity growth measured in terms of value added per worker as average over the nineties.

Table 1.2: Calibration

Parameter	**Value**	**Description**
Calibrated Parameters		
c_e	142.28%	Entry cost, % of average firm size
c_f	3.85%	Fixed cost, % of average firm size
c_a	31.96%	Process innovation cost, % of average firm size
c_q	16.29%	Product innovation cost, % of average firm size
η	0.74	Quality parameter
σ_a	0.15	Variance of efficiency shock
σ_{az}	0.9	Variance of efficiency shock with innovation
σ_q	0.32	Variance of quality shock
σ_{ql}	1.2	Variance of quality shock with innovation
σ_{ea}	0.40	Variance of efficiency distribution of entrants
σ_{eq}	0.48	Variance of quality distribution of entrants
λ_a	0.083	Scale coefficient for process innovation
λ_q	0.025	Scale coefficient for product innovation
Parametrization		
β	0.95	Discount factor
α	0.73	Preference parameter
θ	0.38	Relative entrant mean

Table 2 shows the values assigned to the parameters characterizing the economy. The fixed costs are expressed in relation to the average employment devoted to production.

[29]The European Innovation Scoreboard 2001 reports an innovation intensity for the Spanish manufacturing sector in the 1998 of 2.4% of aggregate sales. This number has been computed on the basis of the CIS which includes also external $R\&D$ investments. This can explain the different numbers between the Euroean Commission survey and the INE statistics.

Table 1.3: Empirical Targets and Model Statistics

Targets	**Data**	**Model**
Targets for Calibration		
Share process innovation	12.2%	13.4%
Share no innovation	55.4%	60.92%
Share product innovation	12.4%	11.1%
Product innovation intensity	0.44%	0.5%
Process innovation intensity	1.26%	1.29%
2 year survival rate	0.8	0.74
5 year survival rate	0.58	0.6
Firm turnover rate	0.09	0.086
Firm below average productivity	0.83	0.78
Job creation due to entry	0.03	0.02
Size entrants wrt exiting firms	1.23	1.31
Productivity spread	[2, 3]	2.48
Targets for Parametrization		
Entrant size/incumbent size	0.38	0.38
Mark-up over marginal cost	0.37	0.37
Growth rate of labor productivity	0.042	0.042

As expected the entry cost, which represents a sunk entry investment, is the highest. Reasonable values are attributed to the fixed cost of both process and product innovation. The parameter associated with the difficulty to produce high quality, η, is just above α.[30] When new firms enter the market there is high uncertainty on their profitability, and the probability of surviving the market competition is low. However, the growth rate of surviving young firms is on average higher than the growth rate of incumbents. This fragility is represented by a variance of the entrants distribution that is higher than the variance of the random walk process associated with a and q when firms only produce.[31] Innovation also increases uncertainty. This is reflected by higher variances of the corresponding random walk processes. In particular, a very high variance is associated with product innovation.[32]

Table 3 reports the empirical targets used and the corresponding model moments. Despite the large number of parameters to calibrate, the model statistics match closely

[30]Bils and Klenov (2001) estimate quality Engel curves for 66 durable goods in US using data on consumers expenditures. They find that the weighted average slope of the quality Engel curve is of 0.76. This number is very closed to the calibrated η of this model.

[31]For OECD countries the higher uncertainty faced by entering firms is documented by Bartelsman et. al. (2004).

[32]The higher uncertainty of product innovation is, for instance, documented by Parisi et. al. (2006).

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

the data in both sets of targets. Hence, the innovation choices of firms, the shape of the distribution, its dynamic characteristics, and entrants' behavior seem to reproduce accurately the Spanish manufacturing sector.

1.4.2 The Role of Innovation

After setting g equal to 4.2%, the model predicts an annual growth rate of firms' production efficiency, g_a, of 2.93% and of product quality, g_q, of 4.64%. Using that $g \approx g_a + (1-\eta)g_q$, 69.8% of the aggregate growth is due to the growth in firms' level efficiency and that only 29.81% is due to the growth in product quality.[33] Though these figures represents the growth in efficiency and quality due to both innovation and randomness, they confirm a higher impact of efficeny in explaing growth accordingly the estimates reported by Huergo and Jamandreu (2004).

Equations (22) and (23) are used to distinguish the effect of innovation and firm experimentation, selection, and imitation in determining the aggregate growth rate. The model predicts that 8.63% of the growth is due to entry (10.61%) and exit (−1.98%) and the remaining 91.37% is due to both experimentation and innovation of the firms that remain active in the industry. Hence, incumbent firms represent the main source of growth in the Spanish manufacturing sector.[34] Decomposing further the growth contribution of incumbents in the contribution of non-innovators and innovators helps to asses the important role played by innovative firms in determining the aggregate growth rate. In fact, the growth contribution of non-innovators is negative (−8.34% of the 91.37%). These firms are characterized by a low level of technology and are destined to exit the market after a series of bad shocks. The high likelihood of receiveing a bad shock and the firm's powerlesseness to escape exit explains their negative contribution to growth. This negative effect is more than compensated by the growth contribution of innovative firms that develops to be the leading force of aggregate growth. However, it should be noticed that the growth derived by innovators is a combined effect of the within firm growth, of the reallocation of resources between incumbents and of tougher selection.

[33] In equilibrium $(1+g) = (1+g_a)(1+g_q)^{(1-\eta)}$ holds. Approximating it using a logarithmic transformation yields $g \approx g_a + (1-\eta)g_q$.

[34] Farina and Ruano (2004) estimate that the within firm growth accounts for 58% of the aggregate Spanish productivity growth while net entry accounts between 5% and 10% and the remaining part is due to reallocation of resources between contractiong and expanding incumbents. This numbers are in line with Bartelsman et. al. (2004). Their general finding is that the role of entry and exit in explaining productivity growth is marginal compared with US. Foster et. al. (2001) find that in the U.S. Census Manufactures, more than a quarter of the increase in aggregate productivity between 1978 and 1997 was due to entry and exit. Moreover, Lenz and Mortensen (2008) estimating their model on a panel of Danish firms find that entry and exit of firms can account for 20% of the aggregate growth while within firm growth account for 55%.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

More insights on the importance of innovation can be obtained simulating an economy with the same parameters values in which innovation is shut down and growth is generated by only selection and imitation. In this example the share of aggregate growth due to g_a is fixed to 69.8% given the previous results and the aggregate growth rate, g is now determined endogenously. In the absence of innovation the growth rate is 1.1% falling of 3.1 percentage points. This confirm the fundamental role of innovation in explaing productivity growth in the Spanish manufacturing sector.[35]

Additionally, innovative firms have a higher weighted mean of their technology index than non-innovators. This implies that innovation increases the weighted mean of the technology distribution of active firms, that is used as reference by the entering firms. Hence innovation also means better imitation and therefore higher growth. Applying equation (25), it is possible to conclude that 84.31% of the joint mean is due to the average technology level reached by the innovative firms.

1.4.3 Firms Partition and Cutoff Functions

Figure 1 displays how the two attributes of firm heterogeneity together with the fixed operational and innovation costs determine the partition of firms between those exiting and remaining, and among process innovators, product innovators, and both types of innovators or non-innovators. Hence, it illustrates the equilibrium cutoff functions and the combinations of efficiency (x-axis) and quality (y-axis) for which the different choices faced by firms are optimal. The firm distribution over the two dimensions of technology (Figure 2, left) is right skewed in both states as the largest mass of firms is concentrated in the bottom-left corner. This information complements the partition of firms and strengthens the subsequent interpretation.

The first area on the left represents the firms with production efficiency and product quality lower than $a_x(q)$ which optimally exit the market. These area represent about 9% of the total mass of firms given by the sum of incumbents and of entrants. The exit cutoff function is the border between the exit region and the region where firms remain active and only produce. Due to the trade-off between quality and efficiency this cutoff function is decreasing in quality: relatively high cost firms can survive longer in the market when the quality of their variety is high. In the second region, for slightly higher level of efficiency and quality, firms are sufficiently profitable to stay in the market but not enough to innovate, $v(a, q) = v^P(a, q)$. These are firms with relatively high level

[35]The growth reduction is accompanied by a lower turnover rate equal to only 1.57% showing how innovation increases also market selection. Using equation (22) the growth contribution of net entry reaches 12.1% confirming the importance of within firm growth.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

of cost but with all the possible levels of quality. In fact, product quality has a lower impact on firm profitability than production efficiency.

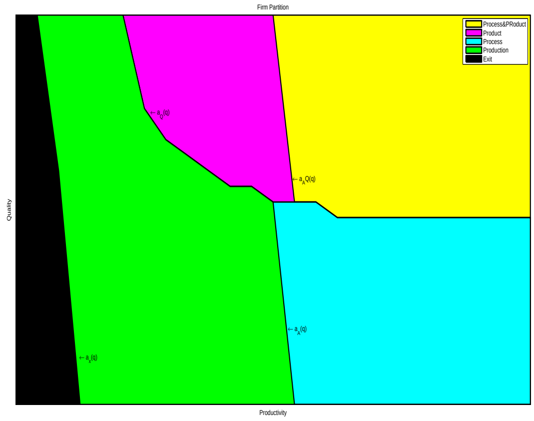

FIGURE 1.1: Firms Partition

Moving along the efficiency dimension, for relatively small level of quality, it is optimal for firms to pay c_a and undertake process innovation while for relatively high level of quality it is optimal to pay c_q and undertake product innovation. This is the result of the interplay between the fixed costs of innovation and the convexity of the profit function in a. The higher the efficiency level reached by the firm the higher the gain in terms of profitability resulting from a marginal reduction of the production cost. This explains why it is optimal for firms to innovate in process when their efficiency has already reached a minimum level. The same is true for the quality dimension, though the profit function is concave in q. However this disadvantage is compensated by the lower fixed cost of product innovation. The last region is represented by firms with high efficiency and high quality that optimally innovate in both process and product.

TABLE 1.4: Conditional Probabilities

	Exit	**No Innovation**	**Process**	**Product**	**Both**
No Innovation	5.1%	87.84%	0.84%	5.6%	0.21%
Process	0	4.5%	75.9%	0.95%	18.65%
Product	0	34.65%	1.22%	51.84%	12.3%
Both	0	1.83%	33.26%	3.3%	61.61%

Table 4 shows the equilibrium conditional probabilities of switching actions after a one-year period given the current decision of incumbent firms.[36] The first column lists the current action of the firms and the rows give the transition probabilities of each

[36]This information is contained in the optimal transition function T_{XI} and the derivations are in the Appendix.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

future decision. Due to the persistence of the random walk process a high probability is attached to the repetition of the current action.[37] Interestingly, consistent with the Spanish empirical evidence shown by Huergo and Jaumandreu (2004), this persistence appears less strong in the case of product innovators: 34% of product innovators today will not innovate tomorrow while 15% will switch to process innovation, both alone and with product innovation, and only 51% will repeat an innovation in product quality. The relative low persistence in quality enhancing innovation is due to the high variance associated with this decision. A high variance implies that the probability of receiving a bad shock is high as well as the probability of switching to a differnt strategy. Empirical evidence emphasises that exit is associated with a low level of pre-exit innovation (Huergo and Jamandreu (2004) for evidence on Spanish firms). This model predicts that an incumbent firm exits the market with 5% of probabilty only if in the current year no innovation has been introduced. This also implies that an innovative firm, before exiting the market, has to receive a bad shock and become a non-innovator.

1.4.4 Firms Distribution

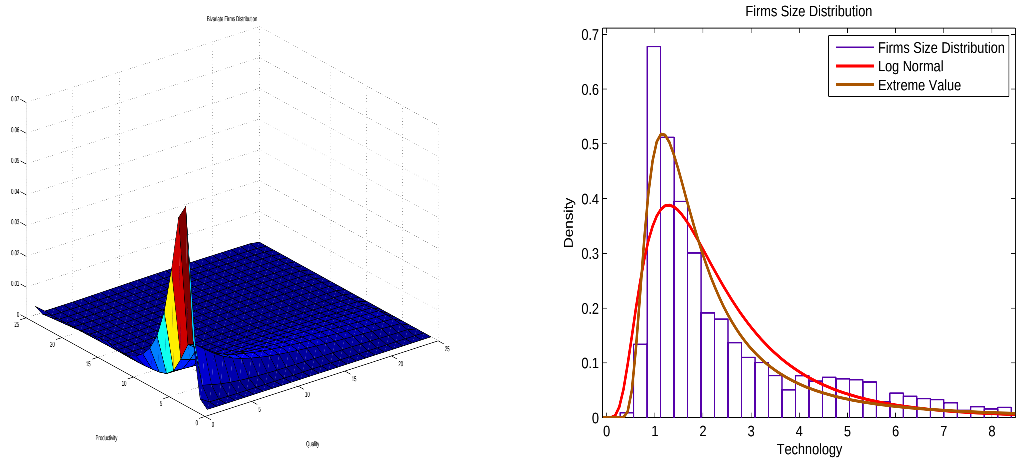

FIGURE 1.2: Bivariate and Univariate Firms Distribution

The equilibrium distribution of firms is determined endogenously and it is shaped by the static and dynamic decisions of incumbent firms together with entrants imitation. Figure 2, left panel, shows the bivariate firms distribution over the two attributes of firm heterogeneity. However, empirical studies are not able to distinguish these two dimensions and hence Figure 2, right panel, displays the corresponding univariate firm size distribution over a technological index that summarizes the information contained

[37]This can be read as persistent firms productivity which is documented by the empirical literature in the case of Spain by Garcia et. al. (2008).

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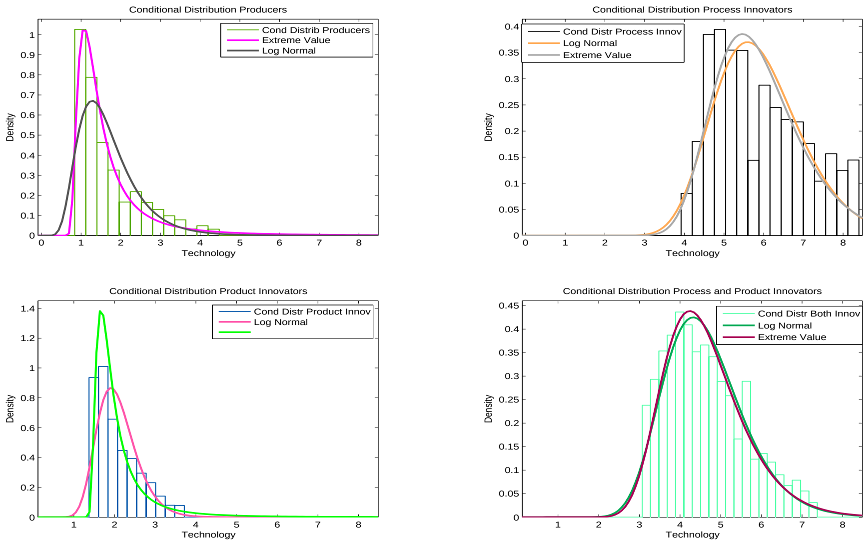

FIGURE 1.3: Conditional Firms Size Distributions

in a and q. That is, $aq^{1-\eta}$. Notice that this is the equivalent of the employment distribution of firms which is observed in the data. The univariate firm distribution looks right skewed and hence with a right thick tail (the moments of the distribution are reported in Table 5).[38] In fact, a log-normal distribution fits the date well. However, empirically there is not much information about the moments of the size distribution of the manufacturing firms in the Spanish economy but in general it is possible to conclude that it is right skewed.[39]

The conditional distribution of firms that only produce and do not innovate is concentrated at lower levels of the technological index $aq^{1-\eta}$ than the conditional distributions of innovators (Figure 3 and Table 5). Consistently with the empirical evidence (see Doraszelski and Jaumandreu (2007)) innovative firms have a higher labor productivity and are bigger than firms that do not innovate. The comparison among innovators is more interestingly: on average small firms do product innovation, medium and large firms do both product and process innovation and large firms do process innovation.[40] Finally, the conditional distribution of product innovators is more right skewed than the distribution of firms that do process innovation or do not innovate. Also this last feature is confirmed by empirical estimations of the firm size distribution in the Spanish manufacturing sector.

[38] The underlying distribution used to compute the skewness in Table 5 is a log-normal distribution.

[39] See Doraszelski and Jaumandreu (2007) and Garcia and Puente (2006) for Spanish firms. Cabral and Mata (2003) estimate that the distribution of Portuguese firms converge to a log-normal distribution.

[40] Huergo and Jaumandreu (2004) find that innovation is systematically related to size: large firms have a higher probability of innovating but this size advantage reduces in the case of product innovation.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

Table 1.5: Descriptive Statistics of Firms Distributions

	Mean	**Variance**	**Coef. of Variation**	**Skewness**
Size Distribution	2.41	3.05	0.72	0.95
Cond. on Process Innov.	5.9	1.26	0.19	0.89
Cond. on Product Innov.	2.08	0.24	0.23	2.32
Cond. on Both Innov.	4.63	0.98	0.21	1.1
Cond. on No innovation	1.67	3.05	0.44	0.95

1.5 Comparative Statics

This section analyzes how changes in the key parameters of the model, which characterize the industry structure, affect the process of labor reallocation among firms and hence the equilibrium growth rates of the economy. In particular, changes in the innovation costs, c_a and c_q, as well as changes in the entry cost, c_e, are analyzed. Both types of costs are directly linked to growth: changes in c_a and c_q bring changes in the composition of the pool of innovative firms and changes in c_e affect the imitation process of entrants firms. High entry cost are seen as barrier to enter the industry and they are often regarded as a protection of incumbent firms and hence as a stimulus to innovation. On the other hand, high innovation costs are seen as detrimental of innovation. Hence, it becomes important to understand how the economy responds to changes in these key parameters in order to design policy recommendations aimed at fostering growth.

Using the quantitative results of Section 4.3 let fix the fraction of growth explained by the growth in efficiency to 69.8% and determine edogenously the aggregate growth rate.

Figure 5, left panel, plots the equilibrium growth rate for different values of the fixed costs of innovation: on the x-axis the cost of doing product innovation, c_q, while on the y-axis the cost of doing process innovation, c_a. As both the innovation fixed costs decline two opposite effects arise. On the one hand, innovation becomes cheaper and more firms find it profitable. Hence the pool of innovative firms increases and this affects positively and directly the growth rate of the economy (Figure 4). This positive effect is then reinforced by an indirect effect. If the mass of innovators is larger, more firms will pay the fixed costs. This sustains the demand of labor and hence the wage rate, thus assuring a strong selection. On the other hand, if the innovation costs are reduced, less labor is demanded by the individual innovative firm. Consequently, the demand of labor by an innovative firm declines and hence the real wage declines to satisfy the labor market clearing condition. A lower wage translates into a weaker selection and hence in a lower effect on the economy growth rate. The final response of the growth rate to the changes in the innovation costs results from the combination of these two effects. Generally,

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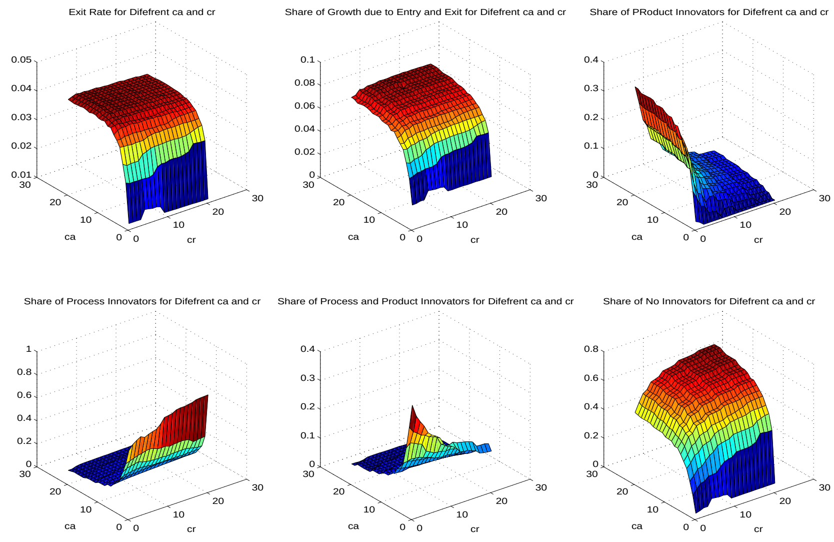

FIGURE 1.4: Comparative statics for different c_a and c_q

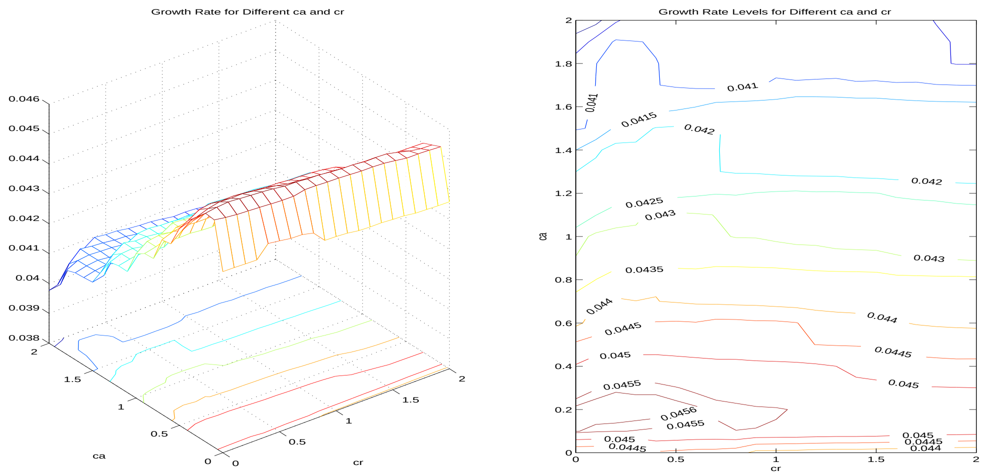

FIGURE 1.5: g for different c_a and c_q

the positive effect prevails. The lower the innovation costs, the higher the growth rate. This holds true for all the values of the fixed cost of undertaking product innovation but only for high and intermediate value of the fixed cost of doing process innovation. The maximum growth rate is obtained for $c_q = 0$ but small and positive c_a, showing that for very low levels of c_a the negative effect offsets the positive one. Additionally, the economy growth rate is more sensitive to changes in c_a than to changes in c_q. Hence, a policy aimed at promoting only growth would be more successful when used to address an increase in process innovation.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

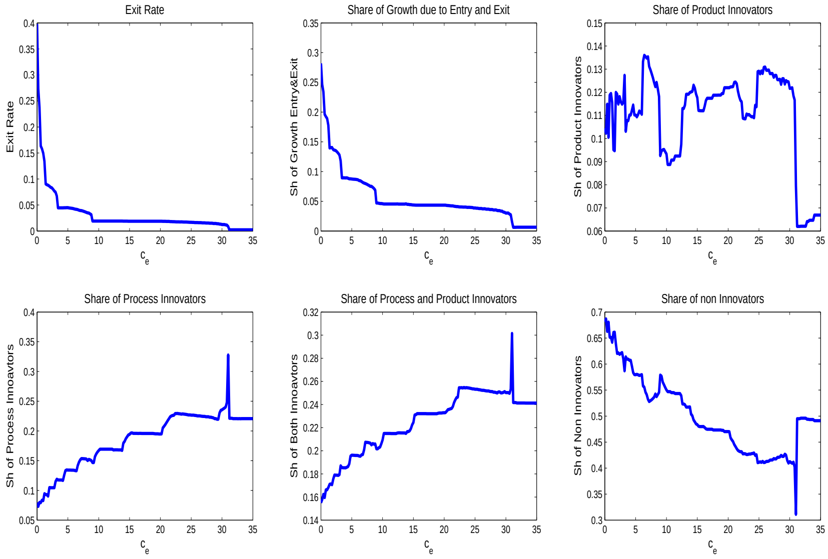

FIGURE 1.6: Comparative Statics for different c_e

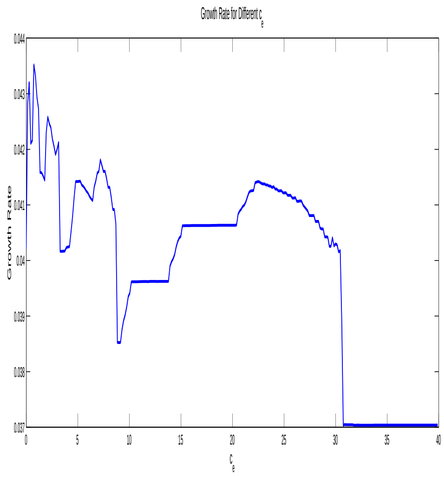

FIGURE 1.7: g for different c_e

When entry cost are low, imitation is cheap (Figure 6), and many firms enter and exit the market, which results in a high growth rate (Figure 7). As the entry cost increases firm selection and imitation become weaker and the growth rate declines. However higher c_e leads to a higher expected value of entrants which in turn imply that the discounted expected profits of incumbents need to be higher. Hence, progressively the mass of innovative firms increases and this generates an inversion in the direction of the growth rate. However, as the entry barrier increases further the industry becomes more and more concentrated and the number of innovators slightly declines. Thought few firms enter the industry they drain a lot of labor increasing the wage rate and hence

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

innovation becomes more costly.[41]

1.6 Final Remarks

This paper proposes an endogenous growth model with heterogeneous firms where firms differ in two dimensions: production efficiency and product quality. Both dimensions are subject to idiosyncratic permanent shocks but firms can affect endogenously their evolution through process, product or both types of innovations. Growth arises due to incumbent firms' innovation and selection and is sustained by entrants' imitation. Selection eliminates the inefficient firms from the market, thereby increasing the average productivity of incumbents. Innovation amplifies this not only increasing directly the average technology of firms but also increasing selection. Entrants imitate the average incumbent and are, on average, more productive than exiting firms. The result is that the firm distribution shifts upwards, generating growth.

The economy is calibrated to the Spanish manufacturing sector and closely matches static and dynamic moments related to the firm distribution and new moments related to the innovation behavior of firms. Hence, the model provides an accurate representation of the Spanish economy and an explanation of the heterogeneity in the innovation activities among firms. Improvements in production efficiency explain 69.8% of the output growth while quality upgrading contributes only for the remaining 30.2%. Moreover, decomposing the aggregate growth in the contribution of firm turnover and innovation and experimantation by incumbents shows that net entry contributes only marginally. In fact, more than 90% of growth is due to within and between firms growth and when innovation is banned output growth declines of almost 74%. Innovation is also necessary to survive market competion: only non-innovative firms exit the industry. An unanswered question is to identify which type of innovation, between process and product innovation, allows for a greater period of firms' longevity.

The endogenous firm size distribution is right skewed and approximated well by a log-normal distribution. The conditional distributions of innovators are consistent with the data: innovators are larger than non-innovators and in the case of product innovators also more right skewed. Additionally, small firms do product innovation, intermediate firms do both product and process innovation and large firms do process innovation. Hence, there is a non-monotonic relation between firm size and innovation though firm size is still an indicator of the type of innovation undertaken by firms. The industry growth

[41] Notice that when the entry cost is very high the industry is characterized by the absence of entering and exiting firms. This generates the irregularities in the pictures. However, the discussion of the properties of this scenario are not in the object of this paper.

rate reacts positively to reductions in the innovation costs, however the model predicts that its maximum is reached for a positive but small cost of process innovation. Though entry barriers protect and stimulates innovation, growth is maximized for relatively low entry costs which are accompanied by a more dynamic industry with a high turnover. As the industry becomes more concentrated, the aggregate share of innovators increases however growth is impacted less strongly.

These considerations leads to attractive policy recommendations aimed at fostering growth and welfare. The next step is therefore to compute the optimal allocation and design innovation policies that can implement the first best in the decentralized economy.

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Appendix

A Partitions and Innovation Cutoff Functions

Define $A_x = \{(a,q) : a \in A, q \in Q : a(q) < a_x(q)\}$ the exit support, $A_P = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^P(a,q)\}$ the production support, $A_A = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^A(a,q)\}$ the process innovation support, $A_Q = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^Q(a,q)\}$ the product innovation support and $A_{AQ} = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^{AQ}(a,q)\}$ the process and product innovation support.

Let $B = \{(a+\epsilon, q+\epsilon)\}$ for $|\epsilon| > 0$ arbitrarily small. The innovation cutoff function are defined as $a_A = \{(a,q) : (a,q) \in A_A \wedge (A_P \cup A_Q \cup A_{AQ}) \setminus A_A \neq \emptyset\}$, $a_Q = \{(a,q) : (a,q) \in A_Q \wedge (A_P \cup A_A \cup A_{AQ}) \setminus A_Q \neq \emptyset\}$ and $a_{AQ} = \{(a,q) : (a,q) \in A_{AQ} \wedge (A_P \cup A_A \cup A_Q) \setminus A_{AQ} \neq \emptyset\}$.

B Aggregate Variables

Using the information contained in equation (19), the price index, the aggregate consumption, and the aggregate profits can be rewritten as:

$$P = \left(\int_{a_x(q)} \int_Q \left(\frac{p(a,q)}{q(a,q)} \right)^{\frac{\alpha}{\alpha-1}} I\mu(a,q) \mathrm{d}q \mathrm{d}a \right)^{\frac{\alpha-1}{\alpha}} = I^{\frac{\alpha-1}{\alpha}} p(\overline{\mu}), \tag{1.26}$$

$$X = \left(\int_{a_x(q)} \int_Q \big(qx(a,q)\big)^{\alpha} I\mu(a,q) \mathrm{d}q \mathrm{d}a \right)^{\frac{1}{\alpha}} = I^{\frac{1}{\alpha}} x(\overline{\mu}). \tag{1.27}$$

$$\Pi = \left(\int_{a_x(q)} \int_Q \pi(a,q) I\mu(a,q) \mathrm{d}q \mathrm{d}a \right) = I\pi(\overline{\mu}). \tag{1.28}$$

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

C Growth Rate Disaggregation

On the Balanced Growth Path, given that the number of firms is constant, the growth factor of aggregate (X) and average ($\bar{X}$) consumption coincides:

$$G = \frac{X'}{X} = \frac{\bar{X}'}{\bar{X}}. \tag{1.29}$$

Defining the firm's quality weighted output with $\hat{x}(a,q)$, the growth factor can be rewritten as:

$$G = \frac{\left(\int_{a_x(q)} \int_Q \hat{x}(a,q)^\alpha \mu'(a,q) \mathrm{d}q \mathrm{d}a\right)^{\frac{1}{\alpha}}}{\bar{X}}. \tag{1.30}$$

Rewrite μ' using its law of motion yields:

$$G = \left(\frac{\int_A \int_Q \hat{x}(a,q)^\alpha \left(\Phi_{xI}\mu(a,q) + \frac{M}{I}\gamma(a,q)\right) \mathrm{d}q \mathrm{d}a}{\bar{X}^\alpha}\right)^{\frac{1}{\alpha}}, \tag{1.31}$$

where Φ_{xI} is the optimal transition function with the exit and innovation rules. Adding and subtracting $\bar{X}^\alpha = \int_{a_x(q)} \int_Q \hat{x}(a,q)^\alpha((1-M/I)\mu(a,q)+M/I\mu(a,q))$ to the numerator and rearranging the equation gives:

$$G = \left(\frac{\int_A \int_Q \hat{x}(a,q)^\alpha \left(\Phi_{xI}\mu(a,q) - \left(1-\frac{M}{I}\right)\mu(a,q) + \frac{M}{I}\left(\gamma(a,q) - \mu(a,q)\right)\right) \mathrm{d}q \mathrm{d}a}{\bar{X}^\alpha} + 1\right)^{\frac{1}{\alpha}}. \tag{1.32}$$

The last step to obtain the growth rate decomposition consists in taking the logarithm of both terms of the equation and approximating them using the rule $\ln(G) \approx g$, given that g is a small number. This results in:

$$g \approx \frac{1}{\alpha \bar{X}^\alpha} \left\{ \int_A \int_Q \hat{x}(a,q)^\alpha \left[\Phi_{xI}\mu(a,q) - \left(1 - \frac{M}{I}\right)\mu(a,q) + \frac{M}{I}\left(\gamma(a,q) - \mu(a,q)\right)\right]\right\}, \tag{1.33}$$

which is equation (29) in the main body of the paper.

D Algorithm

The state space $A \times Q$ is discretized. The grid chosen is of 30 points for each state yielding 900 technology combinations, (a,q).[42] Firms' value function is computed through

[42] The choice of 30 grid points for each state is due to the fact that the algorithm is computationally heavy given the presence of two states and the endogenization of the dynamic choice of the innovation investment. Increasing the grid size would improve the precision of the calibration but would not affect qualitatively the results. On the other hand, the technology combination (a,q) available to firms would increase quadratically in the grid size and the code would eventually become unfeasible. Hence, given

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

value function iteration. The unknown variables are the growth rates g_a and g_q, which combines in the growth rate of the aggregate technology g, and the aggregate expenditure and price index summarized by $k = P^{\frac{\alpha}{1-\alpha}}E$. The growth rate of labor productivity, g, is fixed exogenously. For given g_a, $g_q = (G/G_a)^{\frac{1}{1-\eta}} - 1$, and k compute the stationary profit $\widetilde{\pi}(a, q; g_a, k)$ and then the firm value function $\widetilde{v}(a, q; g_a, k)$.[43] While iterating the value function, the optimal policies for the investment in process and product innovation, $\widetilde{z}(a, q; g_a, k)$ and $\widetilde{l}(a, q; g_a, k)$, are computed and the random walk processes, that govern the transition of firm productivity and product quality, are approximated using the method explained by Tauchen (1987). This step is time consuming since each firm's problem has to be solved via first order conditions for each single couple of states, (a, q), till convergence is reached. Once the value function is approximated the algorithm computes the cutoff functions $a_x(q; g_a, k)$, $a_A(q; g_a, k)$, $a_Q(a; g_a, k)$, and $a_{AQ}(q; g_a, k)$. Then the transition matrix Φ_{xI} is computed. This is the final transition matrix which takes into account the exit and the innovation decisions. After guessing an initial distribution for entrant firms and normalizing its initial joint mean to zero, the expected value of entry is computed. The free entry condition is used to pin down the equilibrium value of k resulting from the first iteration of the algorithm. Using the equilibrium k, the firm value, the cutoff functions, and the transition matrix can be found for given initial g_a. The bivariate firm distribution is then determined using the formula for the ergodic distribution $\widetilde{\mu} = (I - \Phi_{xI})^{-1}\Gamma$ as proved by Hopenhayn (1992). The algorithm is closed using the condition on the mean of the entrant distribution, $\overline{\gamma}_e = \psi_e\overline{\mu}$, and pinning down the equilibrium growth rate, g_a, that satisfies this equation. Once g_a is determined, g_q is determined as well. All these steps are repeated until all conditions are jointly satisfied and convergence is reached.

E Conditional Probabilities

The final transition function $T_{XI}(a', q'|a, q)$ contains all the information to compute the probability that tomorrow a firm will optimally decide to do action $Y \in A'$ given that today it choses action $X \in A$ where A' ={Exit, Not to Innovate, Do Process Innovation, Do Product Innovation, Do Both Innovations} and A ={Not to Innovate, Do Process Innovation, Do Product Innovation, Do Both Innovations}. Weighting these probabilities by the firm density in each state allows to calculate the fraction of firms that today chose action X and tomorrow will switch to action Y. Simplify the notation and define a vector of states, s, of all the possible combinations of a and q couples.

that the results are not qualitatively affected by the grid size, a quality and productivity grid of 30 points is a reasonable restriction.

[43]Notice that all the variables depend on both g_a and g_q. However for notational convenience g_q is omitted since it is a function of g_a.

 DOI: 10.2870/28221

Indicating with "′" the next period variables the conditional probabilities are computed as follows

$$P(Y|X) = \frac{1}{\int_{s:A=X} \mu(s)\mathrm{d}s} \int_{s':A'=Y} \int_{s:A=X} \phi(s'|s)\mu(s)\mathrm{d}s\mathrm{d}s'. \tag{1.34}$$

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

Chapter 2

Trade and Growth: Selection versus Process and Product Innovation

2.1 Introduction

A growing empirical literature based on firm level data has documented the impact of trade in affecting industry dynamics and firm level productivity.[1] A robust prediction is that trade increases on average firm level productivity through a mechanism of self-selection of both unprofitable firms exiting the industry and exporters that on average are more productive than domestic firms. A less clear answer is given when analyzing the dynamic effects of trade on firm productivity growth and industry growth. In this respect a key point to investigate is the effect of trade on firms' innovation investments and hence on productivity growth. A series of empirical works find that exporting and innovation are complements. That is, firms are more likely to be exporters if they innovate and are more likely to innovate when they can increase their market quota through trade.[2]

Though innovation is a fundamental force through which trade policies can affect growth one element that is disregarded by this literature is the possibility of firms to undertake different types of innovation. Recent evidence coming from the availability of micro data emphasizes that firms perceive differently innovations aimed at reducing the production

[1] See Bernard et al. (2007) for a survey on this literature.

[2] Complementarity is documented by Aw et al. (2009), Lileeva and Trefler (2007), and Bustos (2007).

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

costs or increasing the product quality.[3] Not only firms have different incentives to undertake one or the other innovation investment, but also their impact on firms' pricing strategies, productivity, and TFP growth is different.

Motivated by this empirical evidence this paper presents a theoretical model that attempts to examine the impact of openness and trade liberalization on the decisions of heterogeneous firms to invest in cost reducing innovations, *process innovation*, or in product quality enhancing innovations, *product innovation*, and how this generates firm level- and aggregate growth. At this scope a general equilibrium dynamic model in which firms are heterogeneous in their production efficiency and in their product quality is developed. The competitive structure is taken from Melitz (2003) but introducing industry dynamics as in Hopenhayn (1992). The evolution of both efficiency and quality is given by a stochastic permanent component and by an endogenous component proportional to firms' innovation investments. In each period non profitable incumbents exit the industry and new firms enter the market imitating the average incumbent as in Gabler and Licandro (2005) and Luttmer (2007). In the closed economy endogenous growth arises due to firms innovation and selection and is sustained by entrants imitation. Opening to costly trade generates three main mechanisms that affect growth: (*i*) the selection of inefficient firms becomes tougher; (*ii*) the mass of innovative firms decreases eliminating the marginal innovators; but (*ii*) the average innovation intensity increases as the share of innovators that is also exporting can enjoy a higher market share. The selection of innovators is a general equilibrium result. When an economy is exposed to costly trade part of its resources are used to pay the export costs increasing the labor demand and as a consequence the wage rate. Innovation becomes more expensive and thus the marginal innovators are forced to become non-innovators. Hence, the economy resources are reallocated not only from less efficient firms to more successful firms but also from less efficient innovators to more efficient innovators and to exporters.

Calibrating the model parameters to match empirical moments related to the Spanish manufacturing sector shows that the positive effects of trade completely off-set the negative one leading to a higher growth rate in the open economy. Moreover, the model yields several interesting predictions that could be further empirically tested. In particular, exposure to trade results in a more concentrated industry and in a larger share of non innovators. In addition, in this model firm efficiency is not the only factor that determine the export decisions of firms. In fact, also relatively less efficient firms can access the foreign market when their product is of high quality. This is a result that derives from the assumption of two attributes of firms heterogeneity.

[3] Harrison et al. (2008), Huergo and Jamandreu (2004), Fritsch and Meschede (2001), and Smolny (2003) are some references studying the effects of cost reduction and quality improving innovations on firm dynamics in different European countries.

Another important result of this model concern the effects of trade liberalization on economic growth. A reduction of the variable trade cost unambiguously promotes growth and fosters the diffusion of higher quality variety as the share of firms undertaking product innovation increases. Instead, changes in the fixed cost of trade promote growth only when the fixed cost is not too low ensuring a sustained self-selection of exporters. When the fixed export cost is low all the firms gain access to the foreign market and hence the competition in both the domestic and foreign market increases. More firms start innovating mainly in process challenged by the tougher competition. However the intensity of the innovation investment is low given the reduced market quota. This could also bring to a growth rate in the open economy that is lower than the growth rate in the closed economy.

This model is related to several models in the literature that try to understand how trade impacts on the innovation investments of heterogeneous firms. Bustos (2007), Yeaple (2005), and Navas and Sala (2007) study the static gain of technology adoption in response to changes in the trade costs. Costantini and Melitz (2007) introduce a one-off innovation that results in a one-time stochastic jump in productivity and then they analyze the transitional dynamics induced by trade reforms. Van Long et al. (2008) introducing oligopolistic competition studies how openness affects the process innovation incentives in a static framework. The main result of the literature is that, trade liberalization leads to two effects: a direct effect through which cost reducing innovations affect firm level productivity and a selection effect due to inefficient firms are forced to leave the market.[4] While the latter effect always increases firms productivity the former can either rise or reduce productivity depending if the trade cost are high or low. Generally, the overall effect of trade liberalization is positive.

More closely to my work, Atkenson and Burstein (2007) and Impullitti and Licandro (2009) focus on the joint continuous decision of exporting and innovating in a dynamic set-up.[5] Atkenson and Burstein (2007)'s paper presents a general equilibrium dynamic model of firms process and product innovation but without endogenous growth. While process innovation is stochastic and if successful upgrades firms' productivity, product innovation is seen as the creation of a new product and hence it is equivalent to firm entry. Their main finding is that changes in the marginal trade costs do not impact on aggregate productivity though they generate a substantial impact at the firm level.

[4]Since Melitz (2003) the selection effect is a feature of models with heterogeneous firm. Stoelting (2009) extends the Melitz (2003)'s model introducing endogenous growth due to persistent productivity shocks and firm selection. She finds that moving from a close economy to an open economy increases permanently the growth rate. Bernard et al. (2009) introducing multi-product firms find that the selection channel works not only eliminating the least efficient firms but also eliminating the marginal products in the firm's portfolio.

[5]My model is also related to Klette and Kortum (2004) and Lenz and Mortensen (2008) that shed light on the link between innovation, firm heterogeneity and the role of resource reallocation in the growth process of a closed economy.

 DOI: 10.2870/28221

Impullitti and Licandro (2009) studying process innovation in a oligopolistic framework find that trade liberalization leads to a higher number of firms and lower markups. This in turns generates a dynamic selection effect which affects positively aggregate growth. An ambiguous effect of trade liberalization on growth is instead found by Boldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2006). However, it relies on the nature of the knowledge spillowers.

This model complements the work of Irarrazabal and Opromolla (2009) that extends Luttmer (2007)'s model to an open economy set up. They show that the export decision of firms becomes history dependent and that also small firms can be exporters when the export costs are sunk.[6] This result is consistent to the empirical findings of Eaton et al. (2008) for Columbian plants and by Hallak and Sivadasan (2008) for exporters in India, U.S, Chile and Columbia.[7]

In the following I present the open economy version of the model developed in Chapter 1.

2.2 Open Economy

The world economy is characterized by two symmetric countries with the same preferences, technologies, wage rate, and aggregate variables. The access to the foreign market is costly: firms willing to export have to pay fixed export costs, c_{ex} expressed in terms of labor, and variable costs of the iceberg type, $\tau > 1$. The export fixed cost is necessary to generate a partition of firms between domestic firms and exporters.

Households face the same problem as in the closed economy implying that the demand for each variety i stays the same (equation (1.3)). The only difference arises in the composition of the consumption basket which is now given by the varieties produced domestically plus the varieties imported, $I = I^D + I^{EX*}$. From now on, the superscripts D, EX and $EX*$ indicate the domestic variables, the export of the domestic country, and the imports of the domestic country, respectively. Firms face a more complicated problem: after drawing their technology level they have to evaluate the choice of entering or not the export market. This is a per-period choice that impacts on the dynamic choices of innovation.

[6] A similar result is shown also in Arkolakis (2008) in which the rational for the existence of small firms is given by per-consumer access costs. Firms can decide the fraction of the market they want to serve and the fixed entry costs increases with the number of consumers reached.

[7] This paper is also related to the trade literature that focuses on vertical differentiation and hence on the prominent role of quality in shaping the intra-industry trade patterns. Few examples are Schott (2004), Hallak (2006) and Hallak and Sivadasan (2008).

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

2.2.1 Production and Innovation

Firms that serve the domestic market face the same maximization problem as in the closed economy. Thus, the optimal monopolistic price for the domestic products, $p^D(a,q)$, is given by equation (1.7) and the optimal domestic profits, $\pi^D(a,q)$, by equation (1.10) . On the other hand, when firms access the foreign market they have to pay fixed and variable trade costs that affect the export profits and prices. It follows that:

$$\pi^{EX}(a,q) = \left(\frac{a_t q_t^{1-\eta}\alpha}{\tau}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) P_t^{\frac{\alpha}{1-\alpha}} E_t - c_{ex} \tag{2.1}$$

and:

$$p_t^{EX}(a,q) = \frac{\tau q_t^{\eta}}{\alpha a_t}. \tag{2.2}$$

The total profits that a firm with technology level (a,q) receives in period t are given by the profits obtained selling in the domestic market and the profits obtained by serving the foreign market but only if it is profitable. That is:

$$\pi_t(a,q) = \pi^D(a,q) + \max\{\pi^{EX}(a,q), 0\}. \tag{2.3}$$

The export decision does not affect the modeling strategies of innovation and of the evolution of firms production efficiency and product quality. Hence, the evolution of $\log a_t$ and $\log q_t$ are the same in both the closed and the open economy.

2.2.1.1 Firm Dynamic Optimization

In the open economy, the maximization of the expected discounted value of firms is slightly more complicated as also the export choice needs to be considered. A firm with technology (a,q) will export only if the value of exporting is higher than the value of non exporting:

$$v(a,q) = \max\{v^D(a,q), v^{EX}(a,q)\}. \tag{2.4}$$

Notice that the return functions of $v^D(a,q)$ and $v^{EX}(a,q)$ are different: in $v^D(a,q)$ the profits come only from the domestic market while in $v^{EX}(a,q)$ the profits come from both the domestic and the foreign market. Different return functions imply different dynamic paths for the innovation decisions. After drawing a technology (a,q) a firm decide whether to produce only for the domestic market or also for the foreign market. Then within this decision a firm optimally innovate. Hence, nested within the export

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

decision there are the different innovation strategies:

$$v^D(a,q) = \max\{v^{DP}(a,q), v^{DA}(a,q), v^{DAQ}(a,q), v^{DQ}(a,q)\}, \tag{2.5}$$

for the producers that supply only the domestic market and:

$$v^{EX}(a,q) = \max\{v^{EXP}(a,q), v^{EXA}(a,q), v^{EXAQ}(a,q), v^{EXQ}(a,q)\} \tag{2.6}$$

for the producers that supply both the domestic and the foreign market. Again $v^{DP}(a,q)$, $v^{DA}(a,q)$, $v^{DAQ}(a,q)$, and $v^{DQ}(a,q)$, ($v^{EXP}(a,q)$, $v^{EXA}(a,q)$, $v^{EXAQ}(a,q)$, and $v^{EXQ}(a,q)$) are the value when a firm do not innovate, innovate in process, in both process and product, or only in product, and serves only the domestic market (and serves both the domestic and the foreign market). Trade affects the innovation choices of the firms since firms face different profits and hence different incentives to innovation. In the Appendix the several components of the value function are shown.

2.2.2 Exit, Entry, and the Cutoff Functions

The entry and exit conditions are the same as in the closed economy: firms exit when their continuation value is negative and firms enter until the free entry condition is satisfied. The innovation and the exit cutoff functions are defined as before. Upon these cutoffs another cutoff function related to the export decisions can be introduced. That is, $a_{ex}(q)$ such that $a_{ex}(q) > a_x(q)$ and $v^D(a_{ex}(q), q) = v^{EX}(a_{ex}(q), q)$. Hence, the export cutoff function is given by all the technology levels such that firms stay in the industry and are indifferent between exporting or not exporting. Every firm with $a(q) \geq a_{ex}(q)$ choses to produce also for the foreign market. Also the export cutoff is decreasing in the quality dimension. For given productivity, a firm producing a high quality variety has a easier access to the export market.

Opening the model to trade slightly modify the transition function that summarizes all firms' decisions and the corresponding supports. Define $\Phi_{xI}^{Open} : A \setminus A_x^{Open} \times Q \rightarrow (A_P^{Open} \cup A_A^{Open} \cup A_Q^{Open} \cup A_{AQ}^{Open} \cup A_x^{Open}) \times Q$ where the support of efficiency is partitioned into $A_x^{Open} = \{a \in A \wedge q \in Q : a(q) < a_x(q)\}$ (exit support), $A_P^{Open} = \{a \in A \wedge q \in Q : v(a,q) = v^{DP}(a,q) \vee v(a,q) = v^{EXP}(a,q)\}$ (production support), $A_A^{Open} = \{a \in A \wedge q \in Q : v(a,q) = v^{DA}(a,q) \vee v(a,q) = v^{EXA}(a,q)\}$ (process innovation support), $A_Q^{Open} = \{a \in A \wedge q \in Q : v(a,q) = v^{DQ}(a,q) \vee v(a,q) = v^{EXQ}(a,q)\}$ (product innovation support), and $A_{AQ}^{Open} = \{a \in A \wedge q \in Q : v(a,q) = v^{DAQ}(a,q) \vee v(a,q) = v^{EXAQ}(a,q)\}$ (process and product innovation support).

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

2.2.3 Firm Distribution

Firm density function is still shaped by the entry, exit and innovation decisions of firms and similarly as the closed economy is given by:

$$I^{D\prime}\mu'(a',q') = I^D\Bigg(\int_{A_P^{Open}}\int_Q \mu(a,q)\phi(a',q'|a,q)\mathrm{d}q\mathrm{d}a + \tag{2.7}$$
$$\int_{A_A^{Open}}\int_Q \mu(a,q)\phi(a',q'|a,q,z)\mathrm{d}q\mathrm{d}a + \int_{A_{AQ}^{Open}}\int_Q \mu(a,q)\phi(a',q'|a,q,z,l)\mathrm{d}q\mathrm{d}a$$
$$+\int_{A_Q^{Open}}\int_Q \mu(a,q)\phi(a',q'|a,q,l)\mathrm{d}q\mathrm{d}a\Bigg) + M'\gamma(a',q').$$

The support of each integral is corrected to take into account that the innovation decisions are now taken by both exporters and non-exporters. In the open economy the mass of domestic firms is denoted by I^D. This measure of firms includes also the fraction of domestic firms that export.

Finally, the output weighted mean adjusted by quality is a weighted average between the output weighted average of the domestic firms and the output weighted average of the exporters. This last term includes only the exporting market shares and reflects the technology gains obtained by the additional market share enjoyed by exporters corrected by the output loss due to the iceberg cost, τ. That is:

$$\overline{\mu} = \left(\frac{I_t^D}{I_t}\overline{\mu}_t^{D\frac{\alpha}{1-\alpha}} + \frac{I_t^{EX}}{I_t}\left(\frac{\overline{\mu}_t^{EX}}{\tau}\right)^{\frac{\alpha}{1-\alpha}}\right)^{\frac{1-\alpha}{\alpha}} \tag{2.8}$$

where $I^{EX} = I^D\int_{a_{ex}(q)}\int_Q \mu(a,q)$ is the mass of domestic firms that export and I is the total mass of firms selling in the domestic market, and in both the domestic and the foreign market.[8] Hence I represents the mass of available varieties in each country. The weighted mean that considers only the domestic production is given by:

$$\overline{\mu}_t^D = \left(\int_{a_x(q)}\int_Q \left(aq^{1-\eta}\right)^{\frac{\alpha}{1-\alpha}}\mu_t(a,q)\mathrm{d}q\mathrm{d}a\right)^{\frac{1-\alpha}{\alpha}}, \tag{2.9}$$

while the weighted mean related to the exporters production is given by:

$$\overline{\mu}_t^{EX} = \left(\frac{1}{\int_{a_{ex}(q)}\int_Q \mu_t(a,q)\mathrm{d}q\mathrm{d}a}\int_{a_{ex}(q)}\int_Q \left(aq^{1-\eta}\right)^{\frac{\alpha}{1-\alpha}}\mu_t(a,q)\mathrm{d}q\mathrm{d}a\right)^{\frac{1-\alpha}{\alpha}}. \tag{2.10}$$

[8] Given the symmetry between the two countries, I^{EX} is also the mass of foreign firms that import.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

2.2.4 Equilibrium and Balanced Growth Path

A stationary equilibrium in this economy is a collection of sequences of prices $\{p_t^D\}_{t=0}^{\infty}$, $\{p_t^{EX}\}_{t=0}^{\infty}$, $\{P_t\}_{t=0}^{\infty}$, real numbers $\{I_t^D\}_{t=0}^{\infty}$, $\{I_t^{EX}\}_{t=0}^{\infty}$, $\{M_t\}_{t=0}^{\infty}$, $\{X_t\}_{t=0}^{\infty}$, functions $n(a,q;\mu)$, $z(a,q;\mu)$, $l(a,q;\mu)$, $v(a,q;\mu)$, cutoff functions $a_x(q)$, $a_A(q)$, $a_{AQ}(q)$, $a_Q(q)$, and $a_{ex}(q)$, and probability density function $\{\mu_t\}_{t=0}^{\infty}$ such that consumers maximize their utility given their budget constraints, active firms maximize their expected discounted value, the free entry condition holds, the exit and the export decisions are optimal, the good and the labor markets clear, the firm distribution evolves as described before, and the stability condition is satisfied.

The BGP is found similarly as in the closed economy. The economy admits a BGP along which the shape of the firms' distribution is invariant but its mean, the mean of the entering firms, and the aggregate consumption grow at a rate $G = G_a G_q^{1-\eta}$, the price index decreases at the same rate $G = G_a G_q^{1-\eta}$, the domestic and exported output distributions grow at a rate G_a/G_q^{η}, the domestic and export price distributions grow at a rate G_q/G_a^{η} and the number of firms, the number of exporters, the number of entrants, the aggregate expenditure, the aggregate profits, the profit distribution, the labor distributions are constant.

The model along the BGP can be stationarized as in the closed economy and then it can be solved numerically.

2.3 Quantitative Analysis

2.3.1 Calibration

Table 1 lists all the parameters used in the open economy. Fourteen parameters are calibrated to match empirical targets related to the Spanish manufacturing sector and five are fixed accordingly to the literature or to directly match their empirical counterpart. These last parameters are the discount factor, the preference parameter, the imitation parameter, the growth rate of labor productivity, and the iceberg cost of trade. β is set equal to 0.95 to analyze a yearly time period. Accordingly to Ghironi and Melitz (2003), α is set equal to 0.73, so that the price mark-up charged by the monopolistic firm is of 36% over the marginal cost.[9] The imitation parameter ψ_e is chosen such that

[9] A mark-up of 36% over the marginal cost could be seen high and at odds with the macro literature that delivers a standard mark-up of around 20% over the marginal/average cost. In this model, a higher mark-up is justified by the presence of the fixed costs. In fact, given the free entry condition firms on average break even: on average firms price at the average cost leading to reasonable high mark-ups over the average cost.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

the average size of entrants is 38% of the size of incumbent firms as estimated by Gracia and Puente (2006). From the European Innovation Scoreboard (2001) the growth rate of labor productivity is fixed to 0.042, measured in terms of value added per-worker as average over the nineties. Fixing g enables to distinguish endogenously the growth contributions of efficiency, g_a, and quality, q_q. Once these growth contributions are assesed it is possible to fixed them and to evaluate the impact of trade on the aggregate growth rate. Finally, τ is set equal to 1.099 accordingly to Dovis and Milgram-Baleix (2009), who find that the average trade tarif of the Spanish manufacturing sector is 9.9%.

Table 2.1: Calibration

Parameter	**Value**	**Description**
Calibrated Parameters		
c_e	36.72%	Entry cost, % of average firm size
c_f	1.61%	Fixed cost, % of average firm size
c_a	8.21%	Process innovation cost, % of average firm size
c_q	2.42%	Product innovation cost, % of average firm size
c_{ex}	11.27%	Export cost, % of average exporting firm size
η	0.75	Quality parameter
σ_a	0.15	Variance of productivity shock
σ_{az}	0.9	Variance of productivity shock with innovation
σ_q	0.32	Variance of quality shock
σ_{ql}	1.2	Variance of quality shock with innovation
σ_{ea}	0.40	Variance of log productivity distribution of entrants
σ_{eq}	0.32	Variance of log quality distribution of entrants
λ_a	0.083	Scale coefficient for process innovation
λ_q	0.025	Scale coefficient for product innovation
Parametrization		
β	0.95	Discount factor
α	0.73	Preference Parameter
θ	0.38	Relative entrant mean
τ	1.099	Iceberg cost of exporting
g	0.042	Growth rate of labor productivity

The remaining parameters are calibrated using a genetic algorithm as described by Dorsey and Mayer (1995).[10] These parameters are: the ratio among the fixed costs, c_e/c_f, c_a/c_f, c_q/c_f, and c_{ex}/c_f, the quality parameter η, the four variances of the incumbent random walks, σ_a, σ_{az}, σ_q, and σ_{ql}, the two variances of the entrant random

[10]The aim of the algorithm is to jointly calibrate the parameters such that the mean relative squared deviation of thirteen model moments with respect to the corresponding moments in the data is minimized. Since the problem is highly non-linear, this optimization can be characterized by many local minima and the genetic algorithm used has the nice feature to increase the probability of choosing the global minimum.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

Table 2.2: Empirical Targets and Model Statistics

Targets	**Data**	**Model**
Targets for Calibration		
Share process innovation	12.2%	10.38%
Share product innovation	12.4%	13.17%
Share process and product innovation	20%	16.32%
Product innovation intensity	0.44%	0.47%
Process innovation intensity	1.26%	1.41%
2 year survival rate	0.8	0.76
5 year survival rate	0.58	0.57
Firm turnover rate	0.09	0.10
Firm below average productivity	0.83	0.75
Job creation due to entry	0.03	0.03
Size entrants wrt exiting firms	1.23	1.37
Productivity spread	[2, 3]	2.21
Share of exporters	33%	28.38%
Targets for Parametrization		
Entrant size/incumbent size	0.38	0.38
Mark-up over marginal cost	0.37	0.37
Average tariff	0.09	0.09

walks, σ_{ea}, and σ_{eq} and the two parameters that scale the innovation drifts into the stochastic processes, λ_a and λ_q. These parameters jointly determine the shape, the truncation functions of firm stationary distribution, and the partition of firms among the different innovation strategies and among exporters and non exporters. They are calibrated using as targets both static and dynamic empirical moments that are informative about the industry characteristics, the innovation decisions, and firms' export status.

A first group of moments refers to a set of targets traditionally used in the firm dynamic literature. These are firms' survival rates after two and five years upon entry, firms' yearly turnover rate, the job creation rate due to entry, the fraction of firms below average productivity, the productivity spread, and the size of entrants with respect to exiting firms which calibrate the six variances of the model and the entry cost. Accordingly to Garcia and Puente (2006), the two and five year survival rates for Spanish manufacturing firms are estimated equal to 82% and 58%, respectively.[11] They report also a yearly firm

[11]Those numbers are aligned to the one reported by other developed countries as UK, Germany and Nederland (Bartelsman et al. (2003)).

turnover rate of 9% and a job creation rate due to entry equal to 3%.[12] Moreover they show that entrants firms are 23% bigger than exiting firms in terms of employment. Bartelsman et al. (2004) estimate that the fraction of Spanish firms below average productivity is equal to 83%, highlighting a right skewed firm size distribution. The last moment is the productivity spread between the 85^{th} and 15^{th} percentile which is widely accepted to be between 3 and 4.

A second set of empirical moments gives information related to the innovation behavior of firms: the share of firms performing process innovation, product innovation, both process and product innovation, and the intensity of the innovation investments in both process and product. These are new statistics coming from European and national surveys at the firm level. In the scope of this paper these are relevant moments that help to calibrate the fixed cost of process and product innovation, the quality parameter η, λ_a, and λ_q. Harrison et al. (2008), working on data derived from the CIS, report that 12.2% of Spanish firms in the manufacturing sector declared a process innovation between 1998 and 2000, while 12.4% declare a product innovation and 20% decleare both process and product innovation. These numbers are close to the one published by the National Statistics Institute (www.ine.es) using the ESEE. The innovation intensity of the Spanish manufacturing sector, computed as the aggregate innovation expenditure over the aggregate sales, in the 1998, is of 1.71%. Process innovation intensity accounts for 1.26% while product innovation intensity accounts for the remaining 0.44%.

Finally, the last parameter to calibrate is the fixed cost of export. The empirical moment used as target is the share of exporting firms set equal to 33% as Dovis and Milgram-Baleix (2009) reported. This moment represent the natural candidate given the fundamental role played by the fixed cost of export in determining the partition of firms between exporters and non exporters.

Table 2 shows the value assigned to the targets and the corresponding model moments. Despite the large number of parameters to calibrate, the model statistics match closely the data. Hence, this model seems to reproduce accurately the Spanish manufacturing sector.

For a given growth rate of labor productivity the model generates an average production efficiency growth rate, g_a, equal to 3.27% and an average product quality growth rate, g_q, equal to 3.64%, (Table 3). That is, 77.90% of aggregate growth is due to firms level efficiency growth.[13] This figure is very close to the estimates reported by Huergo and

[12]Firm turnover is computed as the sum of the number of entering and exiting firms over the total number of firms while job creation rate is computed as the total amount of labor employed by entering firms in a year divided by the total employment in the same year.

[13]In equilibrium the growth rate can be approximated using a logarithmic transformation which yields $g \approx g_a + (1-\eta)g_q$. From this equation is then possible to distinguish the growth contribution of efficiency from the growth contribution of quality.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

Jamandreu (2004) confirming the validity of the model in explaining the dynamics of the Spanish manufacturing sector.[14]

Table 2.3: Growth Rates in the Open Economy

g	g_a	g_q
4.2%	3.27%	3.64%

2.3.2 Closed and Open Economy

The object of this section is to study the effects of openness on firms' innovation decisions and hence on aggregate growth. To achieve this aim the closed economy is simulated using the parameters listed in Table 1 and fixing the share of growth due to efficiency and quality accordingly to what discussed in the previous section. Appendix C explains the algorithm used to solve for the stationary solution in both the closed and the open economy.

Opening up to trade increases unambiguously the aggregate growth rate. The growth rate in the closed economy is equal to 3.81% while the growth rate in the open economy is equal to 4.2% (Table 4).

Table 2.4: Growth Rates in the Closed and Open Economy

	Closed Economy	**Open Economy**
Growth Rate	3.81%	4.2%

This positive growth differential induced by costly trade results from the combination of tougher selection of unprofitable firms, tougher selection of marginal innovators, and higher innovation intensity as Appendix E shows. On the one hand, trade induces a higher turnover rate which affects positively and permanently growth shifting to the right the exit cutoff function, $a_x(q)$. On the other hand, in the open economy innovation is more costly and hence less firms find optimal to innovate. However, mostly exporting firms are also innovators. Since they enjoy a larger market share and larger profits due to both domestic and foreign sales, their incentives to invest in both product and process R&D increase. The two positive growth effects completely offset the negative effect of the selection of innovators.[15]

[14]The model can be equivalently solved fixing the growth contribution of productivity equal to 77.9% and obtaining endogenously an aggregate growth rate, g, equal to 4.2%.

[15]It should be noticed that tougher selection affect growth also indirectly leading to better entrants imitation.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

2.3.2.1 Firms Partition and Distributions

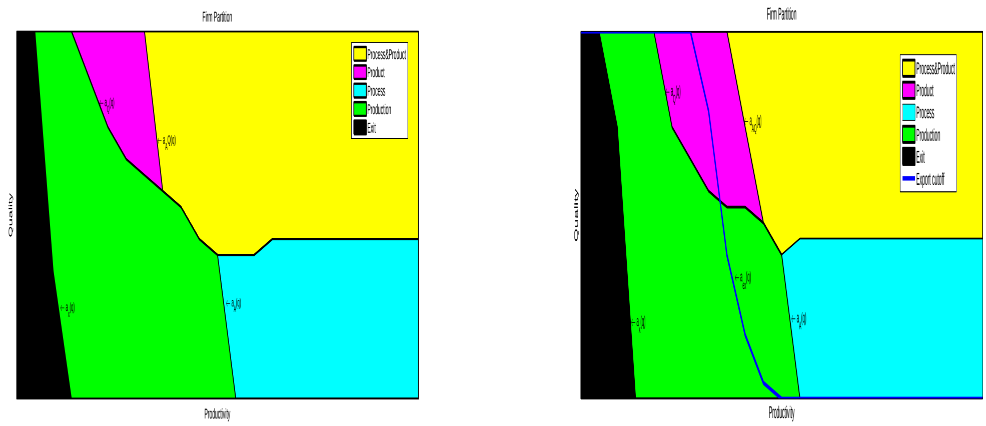

FIGURE 2.1: Firms Partition, Closed (Left) vs. Open (Right)

Figure 1 shows the partition of firms among exiting firms (in black), non-innovators (in green), product innovators (in pink), process innovators (in blue), and both process and product innovators (in yellow), and the corresponding cutoff functions in the closed (left) and open economy (right) for different combination of efficiency (x-axis) and quality (y-axis).[16] Moreover, in the left panel the export cutoff function can be identified. In the open economy there are less innovators. These are taken from process and from both process and product innovation but not from product innovators. In fact, trade advantages product innovators which represent only 27.01% of the innovators in autarchy and 33.03% of the innovators in the open economy. Due to lower product innovation fixed cost, the benefit-cost ratio of the R&D investments is such that improvements in quality are prefered to improvements in efficiency leading to a higher product quality. Less innovators but relatively more product innovators generate a firm distribution over a and q which is more concentrate towards the quality dimesion (Figure 2). This higher weight on quality shapes a univariate firm distribution over a technology index $aq^{1-\eta}$ that is more concentrated in the open economy (Figure 3, left).[17]

Product quality and the different innovation investments generates a non-monotonic relation between quality weighted labor productivity and export status. Figure 3 (right panel) shows that also firms with low labor productivity can become exporters. These

[16] See Benedetti Fasil (2009) for more details on the composition of the partition among the different choices faced by firms.

[17] While in the closed economy this distribution maps one-to-one to the firm size distribution, in the open economy it needs to be corrected by the additional labor used by exporting firms to serve the foreign market. Hence, no direct conclusion on the firm size distribution in the open economy can be driven.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

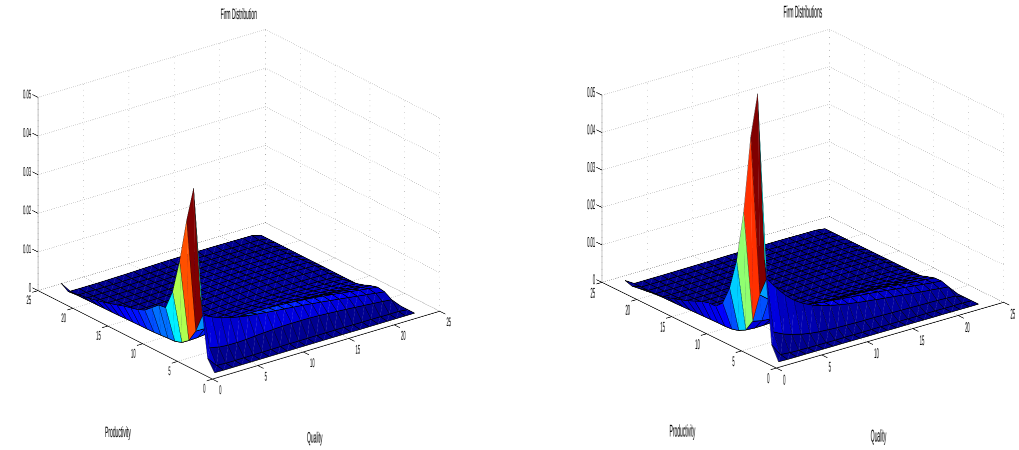

FIGURE 2.2: Bivariate Firms Distribution, Closed (Left) vs. Open (Right)

are firms characterized by a relatively low a (but still higher than the export cutoff) but high product quality.[18]

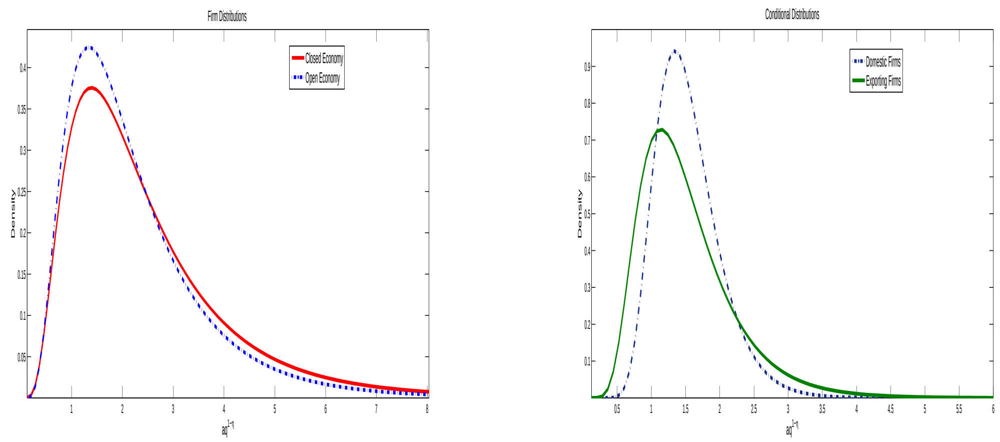

FIGURE 2.3: Firms Distribution, Closed vs. Open (Left), Non Exporters vs. Exporters (Right)

2.3.3 Trade Liberalization

This section studies the impact of trade liberalization on economic growth. Trade liberalization calls for a reduction of the trade costs. Firstly, the attention is focused on the effects of changes in the iceberg cost of trade, τ. Figure 4 plots the growth differential

[18]Through the innovation investments the export decision becomes history dependent. Firms with relatively low labor productivity, caused mainly by high quality, choose to become exporters as they can benefit from higher profits opportunities. These better opportunities are also generated by the expectation on future R&D investments.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

between the open and the closed economy. In general, as trade liberalizes the growth differential rises. However, for high τ it is negative while for intermediate and low level of τ it is positive. Hence, when trade liberalization is at an early stage it leads to a growth rate in the open economy that is lower than the growth rate in autarchy.

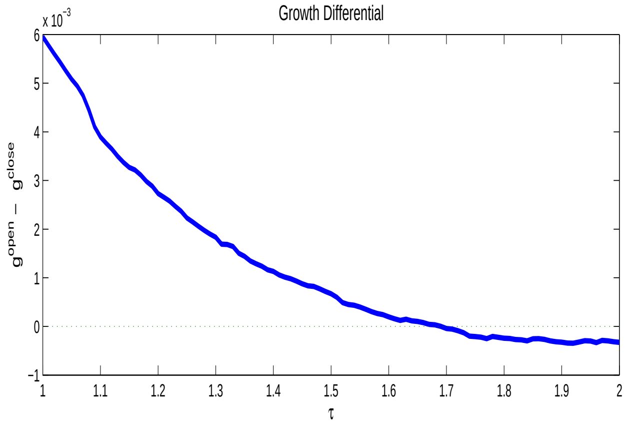

FIGURE 2.4: Growth Differential for different τ

When τ is high the exported varieties are very expensive and their demand is low. Hence, only few firms serve the foreign market profitably shifting to the right the export cutoff, $a_{ex}(q)$. This implies a low labor demand and a low wage rate which relaxes the exit cutoff, $a_x(q)$. Selection is weak and many marginal firms survive in the market. A lower wage rate eases R&D increasing the share of innovative firms which is higher than in the closed economy. However, the majority of innovators serves only the domestic market and this reduces the innovation intensity. The consequence is a negative growth differential. The result reverses when the iceberg cost of trade decreases. The export cutoff shifts to the left and more firms enter successfully into the export market demanding more labor. As a result the wage rate increases and the selection of unprofitable firms becomes tougher. Innovation is more expensive and attracts less firms. However the innovation intensity of these firms is higher given that many of them export. Interestingly, a reduction in the iceberg cost of trade together with asymmetries in the innovation costs favor product innovation. This results in a range of exported varieties characterized by higher quality and by a better quality-price ratio. Figure 6 in Appendix D plots the comparative statics discussed.

The scenario changes when trade liberalization is implemented through a reduction of the fixed export cost, c_{ex}. As can be seen from Figure 5 the growth differential between the open and the closed economy is not monotonically related to c_{ex} and it sharply declines up till it becomes negative for low fixed costs.[19]

[19]Notice that the export cost is expressed as a percentage of the average firm size and that the x-axis is cut for $c_{ex} = 100$. After this point the growth differential does not change substantially. The same is

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

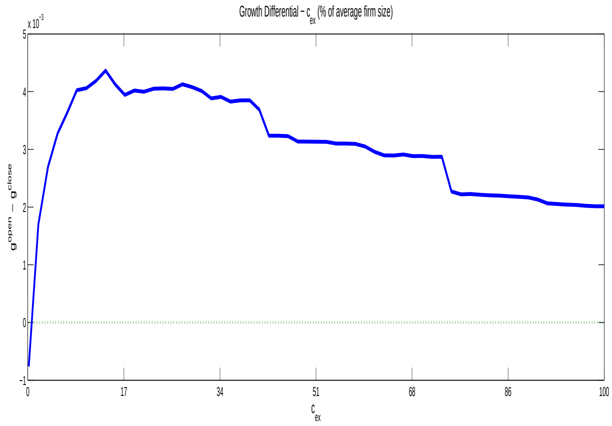

FIGURE 2.5: Growth Differential for different c_{ex}

When trade liberalization starts (high value of c_{ex}) the mechanism of resources reallocation from exiting and domestic firms to innovators and exporting firms works as discusses in the previous paragraph. Hence higher selection, higher innovators selection, higher investment intensity, and higher growth are the result. However, as c_{ex} declines a sustained selection is fostered mainly by an increasing number of innovators than by an increasing number of exporters. In fact, lower export cost, though accompanied by more exporters, reduces the demand of labor and hence the wage rate as c_{ex} declines more rapidly than the rate at which the share of exporters increases. Innovation becomes cheaper and the composition of the pool of innovative firms changes. The share of product innovators progressively diminishes in favor of the share of process and both process and product innovators. Since process innovation is more expensive than product innovation the wage rate is sustained and firm selection is tough. This together with more innovators and higher intensity, particularly by process innovators, results in a high growth differential. However, if c_{ex} declines further many inefficient firms are able to enter the foreign market. This together with a higher competition in the domestic market, due to the introduction of many imported products, reduce the market share of each domestic and exporting firm. Challenged by this increasing competition, more firms undertake process innovation. However, though the number of innovator increases their investments reduces and also the demand of labor declines weakening selection. The result is a decline in the growth differential until it becomes negative. A too strong trade liberalization, when implemented through changes in the fixed cost of trade, leads to a growth rate in the open economy that is lower than the growth rate in autarchy.

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not true for the comparative statics displayed in Figure 7 in the Appendix. In this case the maximum c_{ex} is set equal to 240% of the average firm size.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

2.4 Final Remarks

This paper studies the effects of intra-industry trade on firms' exit, process and product innovation decisions and how these firm level dynamics impact on aggregate growth. At this scope a general equilibrium model with endogenous growth is developed. Firms differ in their production efficiency and product quality. Both factors evolve through permanent shocks but incumbent firms endogenously affect their evolution through process and product innovations.

Calibrating the model parameters to match the Spanish manufacturing sector allows several interesting implications. Costly trade unambiguously increases growth not only through the standard tougher selection of inefficient firms but also through the selection of inefficient innovators. In fact, when an economy is exposed to trade some labor is reallocated from exiting and innovative firms to the payment of the export costs. Hence, the share of innovators decreases. However, the remaining innovators are often also exporters and given the higher market quota, domestic and foreign, the intensity of their investments increases. Moreover, the resulting more concentrated industry favors product innovation and the average quality of the varieties produced increases. The inter-temporal link between export and product innovation determines that small firms with a product of high quality have an easier access to the export market than large firms with a low product quality.

Concerning the debate on the effects of free-trade agreements on growth this model provides the following contribution. As long as trade liberalization is implemented through a reduction of the variable cost of trade it is beneficial for growth and for the production and diffusion of high quality products. More attention has to be paid when freer trade is obtained reducing the fixed cost of export. In this case, a too sharp liberalization could cause a decline of the growth rate that could become even lower than the growth rate obtained in autarchy. This decline would be accompanied by a reduction of product quality in favor of cheaper varieties.

These long run predictions are obtained by analyzing the economy at its steady state. A complete understanding of their implications on growth and also on consumers' welfare requires the study of the transitional dynamic of the model. The research agenda is therefore concentrated on this point.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
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Appendix

A Innovation Cutoff Functions

Define $A_P = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^P(a,q)\}$ the production support, $A_A = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^A(a,q)\}$ the process innovation support, $A_Q = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^Q(a,q)\}$ the product innovation support and $A_{AQ} = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^{AQ}(a,q)\}$ the process and product innovation support $a_A(q)$. Moreover, let $B = \{(a+\epsilon, q+\epsilon)\}$ for $|\epsilon| > 0$ arbitrarily small. The innovation cutoff function are defined as $a_A = \{(a,q) : (a,q) \in A_A \wedge (A_P \cup A_Q \cup A_{AQ}) \setminus A_A \neq \emptyset\}$, $a_Q = \{(a,q) : (a,q) \in A_Q \wedge (A_P \cup A_A \cup A_{AQ}) \setminus A_Q \neq \emptyset\}$ and $a_{AQ} = \{(a,q) : (a,q) \in A_{Aq} \wedge (A_P \cup A_A \cup A_Q) \setminus A_{AQ} \neq \emptyset\}$. The innovation cutoffs in the open economy case are defined in a similar way though the required notation becomes heavier.

B Value Function in the Closed Economy

A firm with technology (a,q) has the following value function:

$$v(a,q) = \max\{v^P(a,q), v^A(a,q), v^{AQ}(a,q), v^Q(a,q)\}. \tag{2.11}$$

Defining with prime the next period variables:

$$v^P(a,q) = \max_p \left\{ \pi(a,q) + \frac{1}{1+r} \max\left\{ \int_\Omega v(a',q')\phi(a',q'|a,q)\mathrm{d}a'\mathrm{d}q', 0 \right\} \right\} \tag{2.12}$$

is the Belman equation when no innovation investments occurred and a firm takes only the static decision about pricing and production. The profit function includes the fixed operational cost and the inner max operator indicates the option to exit the market. Next:

$$v^A(a,q) = \max_{p,z} \left\{ \pi(a,q) - z(a,q) - c_a + \frac{1}{1+r} \max\left\{ \int_\Omega v(a',q')\phi(a',q'|a,q,z)\mathrm{d}a'\mathrm{d}q', 0 \right\} \right\}, \tag{2.13}$$

is the firm value when a firm produces and innovates in process aiming at increasing next period productivity. The fixed cost and the variable cost related to process innovation are

 DOI: 10.2870/28221

c_a and $z(a,q)$, respectively. Analogously, the value function when, besides production, both process and product innovation occur reads:

$$\begin{aligned} v^{AQ}(a,q) &= \max_{p,z,l}\Bigg\{\pi(a,q) - (z(a,q) + l(a,q)) - c_a - c_q + \\ &+ \frac{1}{1+r}\max\Big\{\int_\Omega v(a',q')\phi(a',q'|a,q,z,l)\mathrm{d}a'\mathrm{d}q', 0\Big\}\Bigg\}. \end{aligned} \tag{2.14}$$

This time the fixed cost are given by the sum of $c_a + c_q$ and the variable costs by $z(a,q) + l(a,q)$. Finally,

$$v^{Q}(a,q) = \max_{p,l}\Bigg\{\pi(a,q) - l(a,q) - c_q + \frac{1}{1+r}\max\Big\{\int_\Omega v(a',q')\phi(a',q'|a,q,l)\mathrm{d}a'\mathrm{d}q', 0\Big\}\Bigg\}. \tag{2.15}$$

is the value function when a firm optimally specializes only in product innovation.

C Value Function in the Open Economy

A firm with technology (a,q) has the following value function:

$$v(a,q) = \max\{v^D(a,q), v^{EX}(a,q)\}, \tag{2.16}$$

where:

$$v^D(a,q) = \max\{v^{DP}(a,q), v^{DA}(a,q), v^{DAQ}(a,q), v^{DQ}(a,q)\}, \tag{2.17}$$

and:

$$v^{EX}(a,q) = \max\{v^{EXP}(a,q), v^{EXA}(a,q), v^{EXAQ}(a,q), v^{EXQ}(a,q)\}. \tag{2.18}$$

$v^D(a,q)$ is the value if a firm produce only for the domestic market. Hence the profit function is given by only the first part of equation (3), $\pi(a,q) = \pi^D(a,q)$. Using these profits in the value functions listed above and consistently changing the superscripts gives the values for the domestic firms in the open economy. Similarly, $v^{EX}(a,q)$ is the value of a firm that operates both domestically and abroad. Its profits are given by both components of equation (3), $\pi(a,q) = \pi^D(a,q) + \pi^{EX}(a,q)$. Substituting these profits in the previous value functions and accordingly changing the superscripts yield the values for the exporting firms.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

D Algorithm

The state space $A \times Q$ is discretized. The grid chosen is of 25 points for each state yielding 625 technology combinations, (a, q).[20]

In the closed economy the unknown variables are the growth rates, g_a and g_q, the aggregate expenditure, and price index summarized by $k = P^{\frac{\alpha}{1-\alpha}} E$. The growth rate of labor productivity, g, is fixed exogenously. For given g_a, $g_q = (G/G_a)^{\frac{1}{1-\eta}} - 1$, and k compute the stationary profit $\widetilde{\pi}(a, q; g_a, k)$ and then the firm value function $\widetilde{v}(a, q; g_a, k)$. Firms' value function is computed through value function iteration. While iterating the value function, the optimal policies for the investment in process and product innovation, $\widetilde{z}(a, q; g_a, k)$ and $\widetilde{l}(a, q; g_a, k)$, are computed. The random walk processes, that govern the transition of a and q, are approximated using the method explained by Tauchen (1987). This step is time consuming since a firm's problem has to be solved via first order conditions for each single couple (a, q), till convergence is reached. Once the value function is approximated the algorithm computes the cutoff functions $a_x(q; g_a, k)$,$a_A(q; g_a, k)$, $a_Q(a; g_a, k)$, and $a_{AQ}(q; g_a, k)$. Then the transition matrix Φ_{xI} is computed. After guessing an initial γ and normalizing its initial joint mean to zero, compute the expected value of entry. The free entry condition pins down the equilibrium value of k. Using the equilibrium k then compute the firm value, the cutoff function and the transition matrices for given initial g_a. The binomial firm distribution is then determined using $\widetilde{\mu} = (I - T_{xI})^{-1} G$ as proved by Hopenhayn (1992). The algorithm is closed using the condition on the mean of the entrant distribution, $\overline{\gamma}_e = \psi_e \overline{\mu}$, and pinning down the equilibrium growth rate, g_a, that satisfies this equation. Once g_a is determined, g_q can be computed. All these steps are repeated until all conditions are jointly satisfied and convergence is reached.

In the open economy case, the algorithm needs to consider also the export decisions and hence in the value function iteration the export and domestic profits are evaluated nesting in each of them the innovation decisions. Again this step yields the innovation policy functions and all the cutoff functions that are then used to compute the final transition matrix Φ_{xI}. The remaining part of the algorithm is the same as the one used for the closed economy.

[20]This discretization is due to the fact that the algorithm is computationally heavy given the presence of two states and the endogenization of the innovation choice. On the one hand, increasing the grid size would improve the precision of the calibration but would not affect qualitatively the results. On the other hand, the (a, q) combinations available would increase quadratically in the grid size and the code would eventually become unfeasible. Hence, given that the results are not qualitatively affected by the grid size, a quality and productivity grid of 25 points is a reasonable restriction.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

E Closed vs. Open Economy and Trade Liberalization

Table 2.5: Model Statistics in the Closed and Open Economy

	Closed Economy	**Open Economy**
Turnover Rate	7.95%	10.26%
Process Innovation	13.47%	10.38%
Product Innovation	11.40%	13.17%
Process and Product Innovation	18.50%	16.32%
Non Innovators	57.08%	60.13%
Process Innovation Intensity	1.21%	1.41%
Product Innovation Intensity	0.42%	0.47%

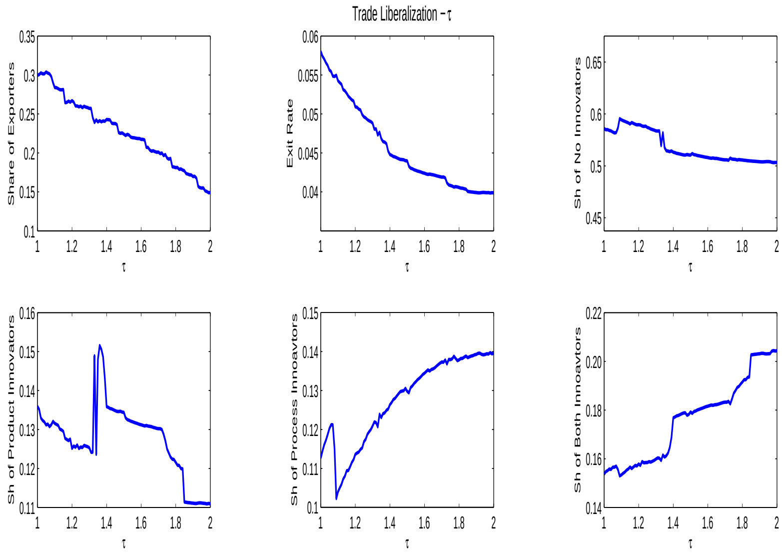

Figure 2.6: Trade Liberalization - τ

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

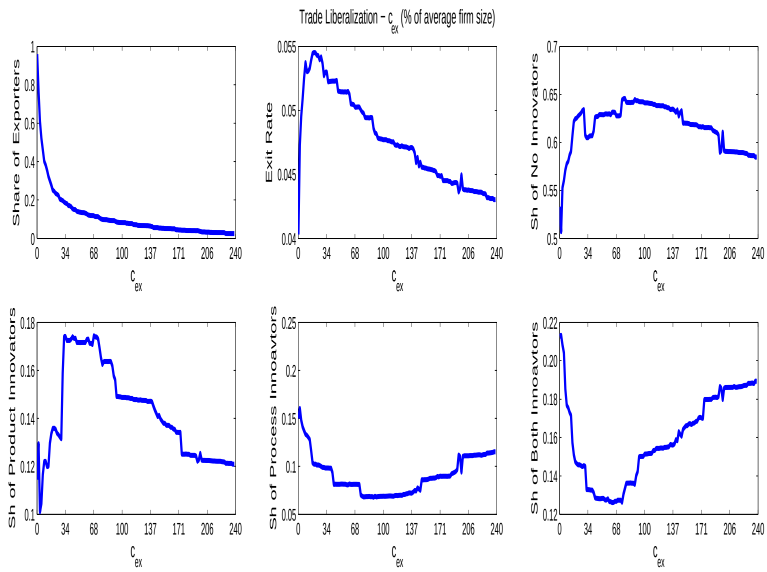

FIGURE 2.7: Trade Liberalization - c_{ex}

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

Chapter 3

World Trade Patterns and Prices: The Role of Cost and Quality Heterogeneity

joint work with Teodora Borota

3.1 Introduction

World trade patterns and their relation to the technological development and income per capita levels of the trading partners have been studied extensively in the theoretical and empirical literature. In several recent studies, data on export and import prices has been exploited as evidence of countries' technological development (particularly as the ability to produce higher quality), trade specialization and demand schedules.[1] On the export side, Schott (2004) presents evidence on positive variation of US import prices depending on the exporter's income per capita, suggesting positive relation between prices and exporters income per capita within the same product category. Fieler (2007) finds that export prices increase with income per capita of the origin country. On the import side, the same paper reports that import prices are positively related to income per capita, as well as that countries of different income per capita import goods of different prices from the *same* exporter. To the extent that prices may be used as a proxy for quality, this evidence suggests that rich countries not only specialize in the production and export of relatively higher quality goods, but that they devote larger share of income on higher

[1] We focus on empirical evidence that refers to product-level trade prices, and also the aggregate prices. Manova and Zhang (2009) analyze the firm-level prices and relate the quality dimension of firm's productivity to it's export status, import and export prices, trade values and the choice of trading partners, which also relates to the present study.

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quality imports and possibly high quality total consumption.[2] Most of the literature that proposes a theoretical basis for this analysis starts from either non-homothetic preferences, where different income levels generate different demand structures, or standard preferences with arbitrarily imposed different "love for quality" parameters in the North and the South. The supply side mechanisms result in a comparative advantage in the production of goods that are in high domestic demand.[3] Non-homothetic preferences might be the immediate natural assumption for explaining reported increase in traded goods' prices with income per capita, but are certainly not the only factor. Although the arbitrary parametrization of preferences might be regarded as a way around modeling the black box of demand heterogeneity across countries, non-homothetic preferences do have some empirical support in the micro-level data. The purpose of this paper is not to contradict these findings, but to show that when the attention is shifted from modeling preferences to modeling technology more closely, standard preferences model with fixed operational and trade cost can yield the stated predictions as well.

We wish to give more weight to the supply side mechanisms and their role in shaping the demand structure and therefore, we use homothetic preference structure. Specifically, the focus is on the technology endowments of the North and the South which are the main determinants of the production and export specialization, and the relative income per capita of the two regions. The North has a higher level of technological development, while the South lags behind the North and uses a lower level of technology. Firms in each region are heterogeneous in two technology (productivity) dimensions: product quality and labor efficiency which together determine the firms' domestic and foreign market profitability. Existing models of trade and heterogeneous firms that introduce only one productivity dimension, such as Melitz (2003), predict a negative relation between export prices and income per capita since higher technological development implies higher income but also higher cost efficiency and thus lower prices. Empirical evidence on export prices calls for the introduction of a different productivity dimension in a way that it generates positive relation between productivity and price. Several papers introduce the quality dimension of firm heterogeneity. In this sense, Northern technology allows this region to produce relatively higher productivity-higher price varieties, while the South specializes in the production of lower quality-lower price varieties.

Baldwin and Harrigan (2007) develops a model of trade and heterogeneous firms in the quality dimension. They assume that quality rises faster than marginal cost and thus high quality-high cost varieties are the most profitable ones. Therefore, export profitability is increasing in quality (and price) monotonically. Johnson (2010) introduces

[2]These findings, however, should not be taken as a straightforward support for the differences in expenditure distribution over quality in the North and the South, as traded goods might present only a minor share of total consumption.

[3]See Fajgelbaum, Grossman and Helpman (2009) for a recent discussion.

two dimensions of firm heterogeneity, but for the purpose of empirical analysis, two dimensions again collapse to a single by assuming that quality is mechanically related to capability (quality-cost ratio). Using this set-up for the analysis of the North-South trade, counterfactual predictions are derived. Lower aggregate expenditure of the South implies that only the most profitable, so highest price firms can cover the fixed cost of trade and export to the South, while the pool of exporters to the North is larger. This prediction does not match the empirical evidence, as it results in the negative relationship between import prices and income per capita *conditional* on exporter.

We wish to separate the quality and efficiency dimensions and introduce a measure of cost efficiency which affects the marginal cost independently of the quality. Each firm (variety) is characterized by a quality level which affects positively both utility and the cost of production, and by a labor efficiency level which decreases the marginal cost. These two dimensions together determine the productivity level of the firms, which are distributed across quality-efficiency pairs, with the Southern joint distribution having a lower mean due to its technological lag behind the North. In this framework, the export decision of any firm depends on its productivity pair which determines the profitability and thus the ability to cover the fixed cost of exporting. Less profitable firms that export only to the North, also include those with highest quality but lower efficiency, and therefore a higher price. This contributes to a rise in the average import price with income per capita conditional on exporter. In this sense, Northern average import price is higher not because it consumes higher quality than the South, but due to the fact that it consumes also the high priced - high quality varieties. Given the right-skewed distribution of firms in equilibrium, varieties of this type are relatively numerous and this amplifies the effect on the average import price and insures that North imports higher price varieties on average.

Two dimensions of firm productivity have been identified also in the industry surveys. Several empirical studies document that firms distinguish between two different types of investment in R&D - process or product innovation, which raise the firms' efficiency or product quality, respectively. Huergo and Jaumandreu (2004a) report a survey of Spanish firms while Parisi et al. (2006) present a classification of Italian firms based on their R&D strategy (process, product, both or none). Similar data are also available for Germany, Great Britain and Nederlands. Moreover, Huergo and Jamandreu (2004b) estimate that process and product innovation have different contributions to firms' growth.

An important justification for the introduction of two productivity dimensions is found in the recent debates in the literature on how valid unit values actually are as a proxy for the product quality. Hallak and Schott (2010) oppose the large literature that associates cross-country variation in export unit-values with variation in product quality, implicitly assuming away cross-country variation in quality-adjusted prices. They allow for price variation induced by factors other than quality, e.g., comparative advantage or currency misalignment, and find that observed unit value ratios can be a poor approximation for relative quality differences, that quality is converging more rapidly than income levels across countries, and that countries differ in growth strategies - high-quality versus low-price. These findings directly provide support for our modeling of firms' productivity.[4]

In aggregate terms, the greater income of the North compared to the South implies not only a greater expenditure on any good that is available in both regions, but higher levels in equal proportion across goods, due to homothetic preferences. However, with fixed cost of export only a subset of varieties is exported to foreign markets, and the resulting expenditure shares on certain quality are not equal across regions. The North spends a lower share of income on low quality varieties originated from the South, while the South spends a lower share on high quality produced in the North, both relative to the other region's share of expenditure on those varieties. If the income difference between the regions is sufficiently large, the statement above holds also in absolute terms.

The analysis of trade intensities within and across regions refers to the Linder hypothesis. Linder (1961) argues that on the demand side, countries with high (low) income per capita spend a larger fraction of their income on high (low) quality goods. On the supply side, countries develop a comparative advantage in the goods that are in high domestic demand, so high (low) income countries produce high (low) quality goods. Both these premises are predicted by our model, but Linder's hypothesis goes further. The demand and supply premises are combined in order to argue that the overlap of production and consumption patterns between countries of similar income per capita should induce them to trade more intensively with one another. Rich trade more with rich, while poor trade with poor. Our model predicts the highest intensity and value of the North-North trade. The ordering of the South-South and the North-South trade depends on the fixed and/or variable costs of trade, in particular on their asymmetries that are conditional on the origin and destination country. With symmetric costs, North-South trade is of higher value, but the result is reversed when stronger restrictions on Southern exports to the North are imposed. However, there is no robust empirical support of the Linder hypothesis. Namely, it is important to ascertain the level of aggregation at which the "Linder" mechanism might operate. Hallak (2008) shows that the trade intensities prediction is valid on both sides of income per capita distribution at the sectoral level (for some sectors), but is strongly rejected when data is aggregated over sectors.

[4] See also Khandelwal (2010) who estimates the quality of U.S. imports using a procedure that relaxes the strong quality- equals-price assumption.

The rest of the paper is organized as follows, Section 2 presents the closed economy model set-up, Section 3 present the open versions of the model with symmetric and asymmetric countries, Section 4 discusses the results of the numerical exercise with a 4-country North-South scenario, while Section 5 concludes.

3.2 The Model Set-up

3.2.1 Consumers

Consumers have homothetic preferences and every period they choose consumption and supply labor inelastically at the wage rate w. The aggregate measure of population (labor) is L. Consumers allocate optimally the aggregate consumption X across differentiated varieties produced by operating firms. The utility function is given by a quality augmented Dixit-Stiglitz utility function,

$$U(t) = \left(\int_{i \in I} (q(i)x(i,t))^{\alpha} di \right)^{\frac{1}{\alpha}}, \tag{3.1}$$

where $x(i,t)$ is the quantity and $q(i)$ is the quality of a variety $i \in I$ consumed at time t. The standard CES utility index is augmented to account for the quality variation across products where quality acts as a utility shifter: a consumer prefers high quality over low quality products. The elasticity of substitution between any two goods is constant and equal to $\sigma = 1/(1-\alpha) > 1$, with $\alpha \in (0,1)$.

Consumers derive the optimal demand for each good, maximizing their utility subject to the individual budget constraint $E(t) = \int_{i \in I} p(i,t)x(i,t)di$, where $E(t)$ presents total expenditure in the country and $p(i,t)$ is the price of variety $i \in I$ at time t. The demand for product $x(i,t)$ is given by

$$x(i,t) = \left(\frac{P(t)q(i)^{\alpha}}{p(i,t)} \right)^{\frac{1}{1-\alpha}} X(t) = \left(\frac{q(i)^{\alpha}}{p(i,t)} \right)^{\frac{1}{1-\alpha}} P(t)^{\frac{\alpha}{1-\alpha}} E(t) \tag{3.2}$$

with $P(t)$ as the price-quality index defined by

$$P(t) = \left(\int_{i \in I} \left(\frac{p(i,t)}{q(i,t)} \right)^{\frac{\alpha}{\alpha-1}} \mathrm{d}i \right)^{\frac{\alpha-1}{\alpha}} \quad \text{and} \quad X_t = U_t. \tag{3.3}$$

Given the aggregates, the optimal expenditure $(r(i,t))$ decision across varieties is

$$r(i,t) = \left(\frac{P(t)q(i)}{p(i,t)} \right)^{\frac{\alpha}{1-\alpha}} E(t). \tag{3.4}$$

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

This paper focuses on the analysis of the steady-state equilibrium in which all variables are constant and we omit the time subscripts in the further text.

3.2.2 Firms

Firms differ in two dimensions of heterogeneity. The first source of heterogeneity is *labor efficiency* (in further text, efficiency), $a(i) \in \mathbb{R}_{++}$, which increases the marginal productivity of labor, as in the seminal paper of Hopenhayn (1992). The second source is *quality* of a firm's variety, $q(i) \in \mathbb{R}_{++} \setminus (0,1)$, which decreases the marginal productivity of labor. In this respect, a higher quality variety implies a higher variable cost as in Verhoogen (2008), but contributes positively to consumers' utility. The production technology has the following form

$$x(i) = \frac{a(i)^{\chi}}{q(i)^{\eta}} n(i), \tag{3.5}$$

where $n(i)$ is the production labor employed by firm i and $\chi, \eta \in (0,1)$. Firms distribute over quality and efficiency, and we assume that each firm produces only one variety so that the index i identifies both the firm and the corresponding variety it produces. Firms enter and exit the market and the industry is characterized at the steady-state equilibrium.

3.2.2.1 Production Decision

Each firm is the monopolistic producer of its own variety. Firms pay a fixed operational cost, c_f, expressed in terms of labor in order to produce and this cost is responsible for firms' exit from the market. Solving the standard monopolistic problem, firms charge a price $p(i)$, that is

$$p(i) = \frac{wq(i)^{\eta}}{\alpha a(i)^{\chi}}, \tag{3.6}$$

where common wage rate, w, is hereafter normalized to one. Substituting the expression for prices in the demand function,

$$x(i) = (a(i)^{\chi} q(i)^{\alpha-\eta} \alpha)^{\frac{1}{1-\alpha}} P^{\frac{\alpha}{1-\alpha}} E, \tag{3.7}$$

it follows that $x(i)$ is increasing in a and it is decreasing in q *iff* $\eta > \alpha$. We restrict our attention to the specification when this condition holds.

Firms revenues and profits are then given by

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

$$\begin{aligned} r(a,q) &= (a^{\chi}q^{1-\eta})^{\frac{\alpha}{1-\alpha}}(\alpha P)^{\frac{\alpha}{1-\alpha}}E \\ \pi(a,q) &= (1-\alpha)(a^{\chi}q^{1-\eta})^{\frac{\alpha}{1-\alpha}}(\alpha P)^{\frac{\alpha}{1-\alpha}}E - c_f, \end{aligned} \tag{3.8}$$

where the ratio of the revenues of any two firms is a function of the ratio of their productivities,

$$\frac{r(a_i,q_i)}{r(a_j,q_j)} = \left(\frac{a_i^{\chi}q_i^{1-\eta}}{a_j^{\chi}q_j^{1-\eta}}\right)^{\frac{\alpha}{1-\alpha}}. \tag{3.9}$$

It is important to note here that profitability of a firm is increasing with its productivity (in either dimension), but it is not a monotonous function of the price. Price is increasing in quality but decreasing in efficiency, while profits increase in both productivity dimensions. In this sense the patterns present in previous literature, monotonously negative (Melitz 2003) or positive (Baldwin and Harrigan 2007) relation between price and profitability, is broken in this paper. This relationship will become crucial for shaping the average price pattern across the firm partitioning space, particularly concerning the exporter/non-exporter partitioning in the open economy scenario. The most profitable firms are the most productive in both dimensions, so their varieties have neither the highest nor the lowest price. Less productive firms have lower efficiency and/or quality, and they include both the firms that charge lower price compared to the most productive, but also those with the highest prices (high quality-low efficiency firms). Therefore, in the context of the closed economy, the average price of the exiting firms may as well be higher than the average price of the surviving varieties.

On the other hand, the specification of χ and η affects the concavity of profits and the price function in the two productivity dimensions, but also the ratio of the elasticities with respect to each dimension. With χ bigger (smaller) than $1-\eta$ the profits increase faster along the efficiency (quality) dimension, which shapes the isoprofit curves in the (a,q) space and thus the exit productivity threshold functions.

3.2.2.2 The Exit Decision

Every firm faces an exogenous probability of a bad shock δ which forces the firm to exit the market. Besides this exogenous exit, firms exit the market when their profits are not enough to cover the fixed operational cost, c_f. The two sources of firm heterogeneity imply that the thresholds that characterize the border between exit and survival in the market are given by the infinite combinations of the (a,q) couples. For this reason, it becomes convenient to express the reservation values in terms of efficiency as a function of quality[5], $a(q)$, and to obtain a *cutoff function* rather than cutoff values as in one factor heterogeneous firm models. For a given $q \in Q$ it is possible to define the following exit cutoff functions

$$a_x(q) = \left[\left(\frac{c_f}{(1-\alpha)P^{\frac{\alpha}{1-\alpha}}E} \right)^{\frac{1-\alpha}{\alpha}} \frac{1}{\alpha} \frac{1}{q^{1-\eta}} \right]^{\frac{1}{\chi}}. \tag{3.10}$$

The exit cutoff functions are decreasing in quality produced: high quality allows for an easier survival. A firm characterized by a low level of efficiency but a high quality may still find it optimal to produce. However, with $\chi > 1 - \eta$, the cutoff efficiency is decreasing in quality at a decreasing rate. We assume this condition holds, as it captures the idea of increasing difficulty in keeping the market shares for the firms that produce high quality varieties with low efficiency which results in a high price. In other words, this assumption represents minimum (cost) efficiency requirements for survival. This also relates to the literature on the types of R&D investment and their contributions to firms' profitability and growth. Huergo and Jamandreu (2004b) estimate that process innovation contributes for about 77% of the yearly growth rate of aggregate productivity, while product innovation can account for about 23%. The estimates do not apply directly to our specification, but may point to higher returns to firm's efficiency then product quality.

3.2.2.3 Firms Entry

Each period, M firms enter the industry and pay a sunk entry cost, c_e, expressed in terms of labor. After paying the entry cost they draw the product quality and efficiency level (productivity vector (a,q)) from a bivariate distribution $G(a, q)$, with corresponding density $g(a, q)$.

We assume that the free entry condition holds in equilibrium. Firms enter the industry until the expected value of the firm, $\overline{v}$, is equal to the entry costs. With the value of the firm given as the discounted future flow of profits, and with no time discounting as in Melitz (2003), the free entry condition reads

$$\overline{v} = \int_{a_x(q)} \int_Q \frac{\pi(a,q)}{\delta} g(a,q) \mathrm{d}q \mathrm{d}a = c_e. \tag{3.11}$$

[5]It is equivalent to express product quality as a function of efficiency, $q(a)$. Using a specific formulation for the cut-off function does not affect the implications of the model.

 DOI: 10.2870/28221

3.2.3 Cross Sectional Distribution and Aggregates

The density of firms conditional on successful entry is computed as

$$\mu(a,q) = \begin{cases} \frac{g(a,q)}{P_{in}} & \text{if } a \geq a_x(q) \\ 0 & \text{otherwise} \end{cases} \tag{3.12}$$

where $P_{in} = \int_{a_x(q)} \int_Q g(a,q)\mathrm{d}q\mathrm{d}a$ is the ex-ante probability of firm survival.

The average productivity measure as a function of the exit cutoff is computed as

$$\tilde{\mu} = \left(\int_{a_x(q)} \int_Q (a^\chi q^{1-\eta})^{\frac{\alpha}{1-\alpha}} \mu(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{1-\alpha}{\alpha}}. \tag{3.13}$$

The average productivity level is determined by the cutoff function, $a_x(q)$, and thus the average revenue and profit, as the functions of the average productivity, also depend on the cutoff function. Using (3.9), for any given q, we obtain

$$\begin{aligned} \overline{r} &= r(\tilde{\mu}) = \left(\frac{\tilde{\mu}}{a_x(q)^\chi q^{1-\eta}} \right)^{\frac{\alpha}{1-\alpha}} r(a_x(q), q) \\ \overline{\pi} &= \pi(\tilde{\mu}) = \left(\frac{\tilde{\mu}}{a_x(q)^\chi q^{1-\eta}} \right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) r(a_x(q), q) - c_f. \end{aligned} \tag{3.14}$$

As the profit of a cutoff firm equals zero and it's revenue is equal to $\frac{c_f}{1-\alpha}$, it follows that the relationship between the average profits and the exit cutoff function can be expressed as

$$\overline{\pi} = \left[\left(\frac{\tilde{\mu}}{a_x(q)^\chi q^{1-\eta}} \right)^{\frac{\alpha}{1-\alpha}} - 1 \right] c_f.$$

3.2.4 Steady-State Equilibrium

The free entry condition also represents a relation between the average profits and the cutoff productivity, i.e. cutoff efficiency for any given level of quality. Therefore, the two equilibrium conditions,

$$\begin{aligned} \overline{\pi} &= \left[\left(\frac{\tilde{\mu}}{a_x(q)^\chi q^{1-\eta}} \right)^{\frac{\alpha}{1-\alpha}} - 1 \right] c_f && \textit{Zero Cutoff profit} \\ \overline{\pi} &= \frac{\delta c_e}{1 - G(a_x(q,q))} && \textit{Free Entry}, \end{aligned} \tag{3.15}$$

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

define the equilibrium average profits and the cutoff productivity. The aggregate stability condition requires that the mass of successful entrants in the market equals the mass of exiting firms, i.e. $P_{in}M = \delta I$. The labor market clearing condition assumes that the total labor is used either in production, where aggregate income equals the difference between aggregate revenue and aggregate profits, or to pay the fixed cost of entry, Mc_e. Therefore, using the stability and free entry conditions,

$$L = (R - \Pi) + Mc_e = (R - \Pi) + \frac{\delta I}{P_{in}} c_e = (R - \Pi) + I\overline{\pi} = (R - \Pi) + \Pi = R.$$

The mass of operating firms is then derived as

$$I = \frac{R}{\overline{r}} = \frac{L(1-\alpha)}{\overline{\pi} + c_f}$$

which in turn determines the equilibrium price-quality index as $P = \frac{1}{\alpha}\frac{1}{\tilde{\mu}} I^{\frac{\alpha-1}{\alpha}}$. This closes the characterization of the steady-state equilibrium.

3.3 Equilibrium in the Open Economy

3.3.1 Symmetric Countries

We now assume that there are two regions open to trade, home and foreign (denoted by $*$), which are symmetric in all preference and technology dimensions except that they produce different varieties. Consumers have the same homothetic preferences and they supply labor inelastically at the wage rate w, with $w = w^*$. Labor is not mobile across regions and the aggregate measure of population in a region is L, $L = L^*$. Consumers now allocate consumption X across differentiated varieties produced by domestic firms and those imported from abroad. The measure of available goods is hence given by domestic goods of measure I^D and imports from abroad I^{*X}, and similarly for the foreign region, $I^* = I^{*D} + I^X$. Although consumer preferences are the same in both regions, the bundles of varieties consumed are different. Due to firm selection into exporters and non-exporters firms, a subset of varieties in each country is not exported, resulting in a different consumption composition. However, due to symmetry in technology, productivity levels and prices of non-exported and exported goods will be the same across countries, and thus the price-quality indices will be the same, although relating to different bundles. This also assumes that we abstract from the variable trade costs which may differ across origin and destination market and thus distort the relative prices of tradables, and compared to non-tradables. Namely, we are interested in trade patterns and prices that are a result of regions' technologies and firm partitioning, and thus we assume no trade cost except for the fixed cost of becoming an exporting firm. Therefore, conditional on being exporter, a firm charges the same price in domestic and foreign market.

Firms still pay a fixed operational cost, c_f, expressed in terms of labor in order to produce, but now also incur a fixed export cost c_{ex}, expressed in terms of labor, in order to export. The fixed export cost generates the partition between exporter and non exporter firms and it is assumed to be the same across regions.

Firms total profits are the sum of the profits obtained in the domestic market and the profits from the foreign markets when it is profitable to export. The optimal profits for home region are given by

$$\begin{aligned} \pi(a,q) &= \pi^D(a,q) + \max\{0, \pi^X(a,q)\} \\ \pi^D(a,q) &= \left(\frac{a^\chi q^{1-\eta}\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) P^{\frac{\alpha}{1-\alpha}} E - wc_f \\ \pi^X(a,q) &= \left(\frac{a^\chi q^{1-\eta}\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) P^{*\frac{\alpha}{1-\alpha}} E^* - wc_{ex} \end{aligned} \quad (3.16)$$

The max operator in π indicates the choice of each firm to specialize only in the domestic market, or to open to foreign markets when the profits derived from exporting exceed the fixed cost of export, c_{ex}. As the specification of χ and η shapes the isoprofit curves in the (a,q) space, this also has implications for the export productivity threshold functions.

Similarly to the closed economy cutoff functions, it is convenient to express the export reservation value in terms of efficiency as a function of quality, $a(q)$. For a given $q \in Q$ it is possible to define the following export cutoff function for the home region,

$$a_{ex}(q) = \left[\left(\frac{wc_{ex}}{(1-\alpha)P^{*\frac{\alpha}{1-\alpha}}E^*}\right)^{\frac{1-\alpha}{\alpha}} \frac{1}{\alpha}\frac{w}{q^{1-\eta}}\right]^{\frac{1}{\chi}} \quad (3.17)$$

As in the case of exit cutoff, the export cutoff function is decreasing in quality which implies that a firm characterized by a low level of efficiency but a high quality may still find it optimal to export. With $\chi > 1-\eta$, the cutoff efficiency is decreasing in quality at a decreasing rate which represents the minimum (cost) efficiency requirements for exporting.

The cutoff functions are increasing in the wage as higher wage implies higher fixed cost of export and higher export price, while they decrease in the total expenditure and the price index. Higher expenditure (income) of the destination market implies higher

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

purchasing power of the market, while higher price index represents lower competition pressures on the exporting firm. As the total expenditure depends on the size of the population in the destination country, it follows that a larger export market implies higher profitability and lower cutoff productivity levels.

With symmetric wages and technology level of exporters and non-exporters across regions, and thus price-quality indeces and expenditures, the optimal profits and cutoff functions are symmetric and the $*$ superscript can be dropped. The export cutoff function differs from the exit cutoff function only in the fixed cost term, c_{ex} and c_f. With $c_{ex} > c_f$, the exit cutoffs are associated with lower productivity levels than the export cutoffs.

3.3.1.1 Cross Sectional Distribution and Aggregates

The density of firms conditional on successful entry is computed as in the closed economy scenario, equation (3.12). The ex-ante probability of firm survival is still given by $P_{in} = \int_{a_x(q)} \int_Q g(a,q)\mathrm{d}q\mathrm{d}a$, and we define the ex-ante probability that a successful firm exports as $P_{ex} = \frac{1-G(a_{ex}(q),q)}{P_{in}}$. To compute the weighted mean of productivity, we define the mass of incumbents in each country. Hence, I^D also represents the measure of varieties produced in each country, so $I_{ex} = P_{ex}I^D$ is the mass of exporting firms and exported varieties. This means that the mass of available varieties in each region is given by the mass of varieties produced domestically plus the mass of varieties imported: $I = I^D + I^*_{ex}$. With symmetry, $I_{ex} = I^*_{ex}$.

The average weighted productivity is computed taking into account not only the output share of the domestic firms, but the additional export share of the more productive firms:

$$\tilde{\mu} = \left(\frac{I^D}{(I^D + I_{ex})} \tilde{\mu}_x^{\frac{\alpha}{1-\alpha}} + \frac{I_{ex}}{(I^D + I_{ex})} \tilde{\mu}_{ex}^{\frac{\alpha}{1-\alpha}} \right)^{\frac{1-\alpha}{\alpha}} \tag{3.18}$$

where

$$\tilde{\mu}_x = \left(\int_{a_x(q)} \int_Q (a^\chi q^{1-\eta})^{\frac{\alpha}{1-\alpha}} \mu(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{1-\alpha}{\alpha}} \tag{3.19}$$

$$\tilde{\mu}_{ex} = \left(\int_{a_{ex}(q)} \int_Q (a^\chi q^{1-\eta})^{\frac{\alpha}{1-\alpha}} \mu_{ex}(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{1-\alpha}{\alpha}},$$

with $\mu_{ex}(a,q)$ as the conditional distribution of exporting firms, given that the firm survives in the market. Zero cutoff profit and free entry conditions determine the steady state equilibrium in open economy, but also taking into account the partitioning of firms into exporters and non-exporters and the associated export cutoff function. The model is solved in the same manner as described in the closed economy section.

3.3.2 Asymmetric Countries

We now assume two asymmetric regions, home and foreign, which have the same preference structure but differ in two technology dimensions and produce different varieties. The consumers allocate their expenditure on domestic and foreign varieties, but due to asymmetry in productivity levels and thus the wages and prices of goods, the resulting consumption composition and price schedules will be different across regions. This yields different price indices as averages of the quality weighted prices of all varieties consumed by a region, domestically produced and imported.

Firms in both regions distribute over quality and efficiency, and since the regions' asymmetry takes the form of different level of productivity, we refer to the regions as North (N) and South (S), the technologically developed and the developing region, respectively. Firms in the North lead in both productivity dimensions while firms in the South lag behind the more advanced Northern technology.

The wage rate is w^N in the North and w^S in the South, with $w^N > w^S$. Labor is not mobile across regions and the aggregate measure of population in each country in the North and the South regions is L^N and L^S, respectively. The fixed operational cost incurred by firms triggers firm exit while the fixed export cost generates the partition between exporter and non exporter firms. Given the same labor requirement for the fixed cost of operation and export in the North and the South, it follows that both costs are higher in the North due to its higher wage.

3.3.2.1 Firms Entry

After paying the entry cost, firms in both regions draw the product quality and efficiency level (productivity vector (a,q)) from a bivariate distribution $G^J(a,q)$, $J = \{N, S\}$, with corresponding density $g^J(a,q)$. The density function in the North, $g^N(a,q)$, is assumed to be log-normal and exogenous while $g^S(a,q|\overline{\mu}^N)$ is log-normal but its mean, $\overline{g}^S$, is determined as a θ fraction of the incumbents joint mean in the North, $\overline{\mu}^N$.[6] The assumption attempts to capture the idea of imitative R&D in the South which copies the technology of the North at a certain lag due to high difficulty of copying the advanced goods. As we don't model the R&D process endogenously, we might justify

[6]This specification is similar to the one used in Gabler and Licandro (2005).

this assumption by the evidence on differences in North-South TFP levels documented in the literature.[7]

When solving the model, we define another equilibrium condition besides the zero cutoff profit and free entry conditions. This is the trade balance requirement which equates the values of Northern and Southern exports. At the same time, it is the third equation linking the relative South-North wage (Southern wage when Northern is taken as numeraire and normalized to one) and the parameter measuring the technological lag of the South, θ. This allows for solving the model for the South-North relative wage.

3.3.3 Four Countries, Open Economy Model

We wish to analyze the trade patterns and prices of tradables at the regions' aggregate level but also conditional on importer/exporter GDP per capita, and thus we construct a four countries scenario. We propose a two region North-South trade model where each region, the North and the South, consists of two symmetric countries (two symmetric North and two symmetric South).[8] The measure of available goods in each country is hence given by domestic goods of measure I^{JD}, imports from the other country of the same region, I^{JJ}, and from the two countries of the other region, I^{JK}, with $J, K = \{N, S\}, J \neq K$. Thus, $I^N = I^{ND} + I^{NN} + 2I^{SN}$ for the North and similarly for the South, $I^S = I^{SD} + I^{SS} + 2I^{NS}$. We use the same index to represent both the region and the country of a particular region, as we assume symmetry in all environment dimensions of the countries within a region. However, the varieties they produce are perceived as different by the consumers and thus are all in demand, i.e. each country's consumers demand varieties from the other country of the same region as well as the goods of both countries of the other region.

3.3.3.1 Production and Export

Firms total profits are the sum of the profits obtained in the domestic market and the profits from the foreign markets when it is profitable to export. Hence the optimal

[7] See for example, Cordoba and Ripoll (2008), Jerzmanowski (2007), Hall and Jones (1999).

[8] With four countries, we can analyze the difference in variables concerning e.g. Northern exports to both Southern and other Northern country, as well as its imports from countries at different income level. In other words, this model specification at the same time represents both a North-North and a North-South trade model.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

profits with $J, K = \{N, S\}, J \neq K$ are given by

$$\begin{aligned}\pi^{J}(a,q) &= \pi^{JD}(a,q) + \max\{0, \pi^{JJ}(a,q)\} + 2\max\{0, \pi^{JK}(a,q)\} \qquad (3.20)\\ \pi^{JD}(a,q) &= \left(\frac{a^{\chi}q^{1-\eta}\alpha}{w^{J}}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)P^{J\frac{\alpha}{1-\alpha}}E^{J} - w^{J}c_{f}\\ \pi^{JJ}(a,q) &= \tau^{\frac{\alpha}{\alpha-1}}\left(\frac{a^{\chi}q^{1-\eta}\alpha}{w^{J}}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)P^{J\frac{\alpha}{1-\alpha}}E^{J} - w^{J}c_{ex}\\ \pi^{JK}(a,q) &= \tau^{\frac{\alpha}{\alpha-1}}\left(\frac{a^{\chi}q^{1-\eta}\alpha}{w^{J}}\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)P^{K\frac{\alpha}{1-\alpha}}E^{K} - w^{J}c_{ex}\end{aligned}$$

where superscript JJ denotes exports to the symmetric country of the same region, while JK stands for export to a country of the other region.

Since export profits depend on the aggregate variables of the foreign region, this is the channel through which the aggregate economy of the foreign region affects the profitability of the domestic firms.

For a given $q \in Q$ we define the following export cutoff functions for the North and the South,

$$\begin{aligned}a_{ex}^{JJ}(q) &= \left[\left(\frac{w^{J}c_{ex}}{(1-\alpha)P^{J\frac{\alpha}{1-\alpha}}E^{J}}\right)^{\frac{1-\alpha}{\alpha}}\frac{1}{\alpha}\frac{w^{J}\tau}{q^{1-\eta}}\right]^{\frac{1}{\chi}} \qquad (3.21)\\ a_{ex}^{JK}(q) &= \left[\left(\frac{w^{J}c_{ex}}{(1-\alpha)P^{K\frac{\alpha}{1-\alpha}}E^{K}}\right)^{\frac{1-\alpha}{\alpha}}\frac{1}{\alpha}\frac{w^{J}\tau}{q^{1-\eta}}\right]^{\frac{1}{\chi}}.\end{aligned}$$

The order of the cutoffs for export to different regions is determined by the ratio of the aggregates of the two regions, $P^{\frac{\alpha}{1-\alpha}}E$. However, the exit cutoffs depend only on the domestic aggregates. For a given quality firm partition in both the North and the South is such that firms with low level of efficiency (a) exit the industry, firms with intermediate levels produce only for the domestic market, while the most efficient firms produce for both the domestic and the foreign markets, first for the market in the North and then for the foreign markets in both regions. The stated order of the firm partition is assured by the conditions on the fixed costs of operation and export.[9]

[9]See Appendix A. for the discussion on exit and export cutoffs.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

3.3.3.2 Cross Sectional Distribution and Aggregates

The density of firms conditional on successful entry is computed as

$$\mu^N(a,q) = \begin{cases} \frac{g^N(a,q)}{P_{in}^N} & \text{if } a \geq a_x^N(q) \\ 0 & \text{otherwise} \end{cases} \tag{3.22}$$

for the North firms and similarly for the South firms,

$$\mu^S(a,q) = \begin{cases} \frac{g^S(a,q)}{P_{in}^S} & \text{if } a \geq a_x^S(q) \\ 0 & \text{otherwise,} \end{cases} \tag{3.23}$$

where $P_{in}^N = \int_{a_x^N(q)} \int_Q g^N(a,q) \mathrm{d}q \mathrm{d}a$ and $P_{in}^S = \int_{a_x^S(q)} \int_Q g^S(a,q|\overline{\mu}^N) \mathrm{d}q \mathrm{d}a$ are the ex-ante probabilities of surviving for the firms in the North and the South, respectively. In a similar way we can define the ex-ante probability that a successful firm exports. That is, $P_{ex}^{NN} = \frac{1-G(a_{ex}^{NN}(q),q)}{P_{in}^N}$, $P_{ex}^{NS} = \frac{1-G(a_{ex}^{NS}(q),q)}{P_{in}^N}$, $P_{ex}^{SN} = \frac{1-G(a_{ex}^{SN}(q),q)}{P_{in}^S}$ and $P_{ex}^{SS} = \frac{1-G(a_{ex}^{SS}(q),q)}{P_{in}^S}$ for North and South.

I^{ND} and I^{SD} represent the measure of varieties produced in each country of the North and the South, so $I_{ex}^{NN} = P_{ex}^{NN} I^{ND}$, $I_{ex}^{NS} = P_{ex}^{NS} I^{ND}$, $I_{ex}^{SN} = P_{ex}^{SN} I^{SD}$ and $I_{ex}^{SS} = P_{ex}^{SS} I^{SD}$ are the masses of exporting firms and exported varieties in the North and the South, respectively. This means that the mass of available varieties in each country is given by the mass of varieties produced domestically plus the mass of varieties imported: $I^N = I^{ND} + I_{ex}^{NN} + 2I_{ex}^{SN}$ for the North, and $I^S = I^{SD} + I_{ex}^{SS} + 2I_{ex}^{NS}$ for the South.

The average weighted productivity for the North is given by

$$\begin{aligned} \tilde{\mu}^J &= \left(\frac{I^{JD}}{(I^{JD} + I_{ex}^{JJ} + 2I_{ex}^{JK})} \tilde{\mu}_x^{JD\frac{\alpha}{1-\alpha}} + \frac{I_{ex}^{JJ}}{(I^{JD} + I_{ex}^{JJ} + 2I_{ex}^{JK})} \tilde{\mu}_{ex}^{JJ\frac{\alpha}{1-\alpha}} \right. \\ &+ \left. \frac{2I_{ex}^{JK}}{(I^{JD} + I_{ex}^{JJ} + 2I_{ex}^{JK})} \tilde{\mu}_{ex}^{JK\frac{\alpha}{1-\alpha}} \right)^{\frac{1-\alpha}{\alpha}} \end{aligned} \tag{3.24}$$

where $J, K = \{N, S\}$, J≠K and

$$\begin{aligned} \tilde{\mu}_x^{JD} &= \left(\int_{a_x^J(q)} \int_Q (a^\chi q^{1-\eta})^{\frac{\alpha}{1-\alpha}} \mu^J(a,q) \mathrm{d}q \mathrm{d}a \right)^{\frac{1-\alpha}{\alpha}} \\ \tilde{\mu}_{ex}^{JJ} &= \left(\int_{a_{ex}^{JJ}(q)} \int_Q (a^\chi q^{1-\eta})^{\frac{\alpha}{1-\alpha}} \mu_{ex}^{JJ}(a,q) \mathrm{d}q \mathrm{d}a \right)^{\frac{1-\alpha}{\alpha}} \\ \tilde{\mu}_{ex}^{JK} &= \left(\int_{a_{ex}^{JK}(q)} \int_Q (a^\chi q^{1-\eta})^{\frac{\alpha}{1-\alpha}} \mu_{ex}^{JK}(a,q) \mathrm{d}q \mathrm{d}a \right)^{\frac{1-\alpha}{\alpha}}. \end{aligned} \tag{3.25}$$

Variables $\mu_{ex}^{JJ}(a,q)$ and $\mu_{ex}^{JK}(a,q)$ are the conditional distributions of firms exporting to the North and of firms exporting to both regions, respectively, given that the firm survives in the market.

3.3.3.3 Steady-State Equilibrium

The steady state equilibrium is characterized by prices (p^{JD}, p^{JX}), wages (w^J), exit and export cutoff functions $(a_x^J(q), a_{ex}^{JJ}(q), a_{ex}^{JK}(q))$, firm distributions ($\mu^J$, μ_{ex}^{JJ} and μ_{ex}^{JK}), number of firms in each region (I^{JD}) and the aggregate expenditure and price indices (E^J, P^J) such that

- consumers choose consumption optimally and firms choose prices to maximize their profits
- exit and export cutoff functions satisfy the conditions given by (3.10) and (3.21)
- entry and exit are such that the stability condition $\delta I^{JD} = P_{in}^J M^J$ and the free entry condition are satisfied
- distribution of firms in the North and the South are given by (3.25)
- number of operating firms is such that the labor markets clear, i.e. total labor is used for domestic and export production and also for the fixed cost of entry, operation and export

$$\begin{aligned} L^J &= \int_{a_x^J(q)}\int_Q n(a,q)\mu^J(a,q)I^{JD}\mathrm{d}q\mathrm{d}a + \int_{a_{ex}^{JJ}(q)}\int_Q n(a,q)\mu^J(a,q)I^{JD}\mathrm{d}q\mathrm{d}a \\ &+ \int_{a_{ex}^{JK}(q)}\int_Q n(a,q)\mu^J(a,q)I^{JD}\mathrm{d}q\mathrm{d}a + c_e M^J + c_{ex}(P_{ex}^{JJ} + P_{ex}^{JK})I^{JD} + c_f I^{JD} \end{aligned} \tag{3.26}$$

- the trade balance condition is satisfied, implying that the bilateral North-North, South-South, North-South and South-North trade is balanced.[10]

We solve the model numerically using the value of parameters which are calibrated to match the recent data on the aggregate trade values (shares of North-North, North-South and South-South exports in the total world exports, relative wage of the South compared to the North) and the firm-level variables.

[10]Due to symmetry between the countries of the same region, trade balance depends only on the values of export flows between countries of different regions in equilibrium.

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3.3.4 Calibration

In our quantitative exercise we choose the preference parameter, α, exponents on productivity and quality in the production function, χ and η, exogenous exit probability, δ, the size of the countries, L^N and L^S, and the mean of the entrants in the North, $\overline{g}^N$. α is set equal to 0.73 to match a mark-up over the marginal cost of 36%.[11] χ and η are equal to 0.5 and 0.86, respectively. The results do not change qualitatively if χ and η change as long as the conditions on these two exponent are satisfied.[12] The exogenous death probability is fixed equal to 0.5% and hence firms's life expectancy is *a priori* of 200 years.[13] Finally, L^N, L^S, and $\overline{g}^N$ scale and locate the economy in the space (a, q). The population is assumed to be the same in both the North and the South and normlized to one while $\overline{g}^N$ is set equal to 4.

The remaining parameters are the technological gap between the North and the South, θ, the fixed cost of entry, c_e, the fixed operational cost, c_f, the fixed cost of export, c_{ex}, and the entrants distribution variance for the North and the South (assuming equal variance over productivity and quality and across countries). These parameters are calibrated to match a number of salient features related to the 2006 data on the within and across region export shares in the total world exports, exit and entry rates in the manufacturing industry and the South-North relative wage. The data on export shares are taken from The OECD Policy Brief "South-South Trade:Vital for Development", August 2006, available at: www.oecd.org/publications/Policybriefs and Goksel (2008). The reported export shares are 52.69% for the North-North trade, 40.86% for the North-South and 6.45% for the South-South exports. Bartelsman et al. (2004) compute that the average firms exit rate in the data for the North is around 10%, while it is slightly higher in the South, 20%. Accordingly to the World Bank, International Comparison Program database, online edition, 2009 the relative South-North wage in the manufacturing sector is on average 0.4.

Table 2 in Appendix B summarizes the parameters values both exogenously set and calibrated, the empirical targets used for the calibration and the corresponding model moments.

[11] For more details on mark-ups in models with heterogenous firms and fixed costs see Ghironi and Melitz 2005.

[12] This also includes the specification with $\chi = \eta > 0.5$

[13] Atkeson and Burstein (2007) and Luttmer (2007) find the same value calibrating δ.

 DOI: 10.2870/28221

3.4 Four-Country Scenario Results

This section presents the numerical results of the North-South trade model with four countries, two symmetric Norths and two symmetric Souths. Given the productivity lag of the entrants in the South behind the incumbents in the North, the selection of the firms in the equilibrium results in the distribution of operating firms over productivity vectors in the North and the South as presented in Figure 1. The equilibrium productivity lag of the South results in the positive North-South wage differential in equilibrium.

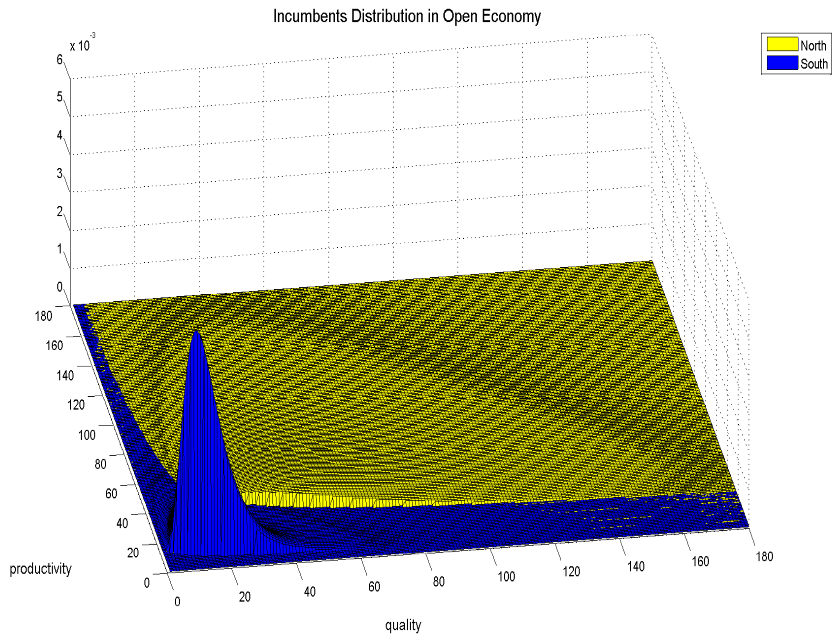

FIGURE 3.1: Incumbents Distribution over Productivity and Quality

When the North and the South are open to trade, the South produces the low productivity varieties that are demanded domestically but also by the North whose international competitiveness in this portion of the distribution is weakened due to lower production cost in the South. On the other hand, Northern firms are more spread out on the whole remaining area of the productivity space, higher efficiency and higher quality. Few firms in the South reach these productivity levels and thus the North specializes in the production and export of higher (a, q) varieties.

Figure 2. presents the partitioning of the firms across the (a, q) space into exiting firms, domestic producers and exporters of two types, those that export only to the North and those that export both to the North and the South. Analyzing the partition over the efficiency dimension, the lowest a firms exit the industry in both regions, but the exit cutoff in the North is higher than in the South due to higher absolute value of the fixed operational cost. Therefore, it can be observed that the low efficiency

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

varieties are consumed exclusively by the South as the North exits this market, and as the South does not export due to low profitability. The North-South head-on competition occurs in the intermediate efficiency range of varieties. Southern varieties are more competitive and are exported to the North, while the North produces them only for the domestic consumption at a reduced scale. At even higher levels of efficiency, the number of Southern firms (varieties) decreases. This is principally the market for Northern exporters who employ a large share of the total labor force in the North. Details on labor (size) distribution of firms and the values of average productivities across different areas of the (a, q) space in the North and the South are presented in Appendix C.

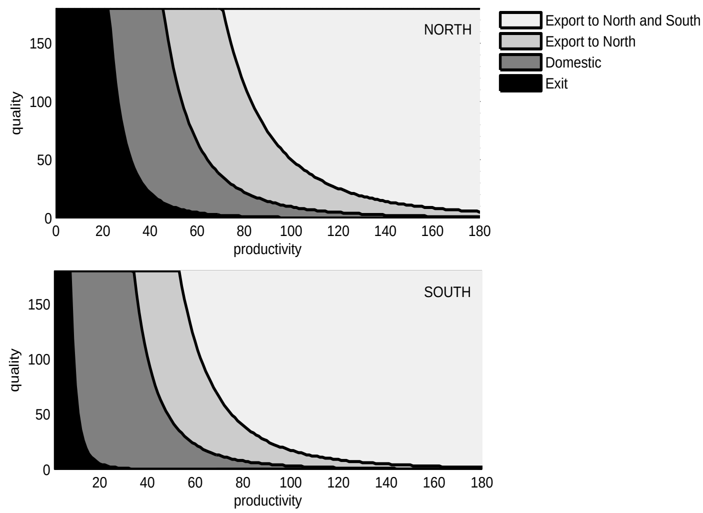

Figure 3.2: Firms Partition

Bearing in mind the price schedule over the (a, q) space, the partitioning graph provides a graphical explanation for positive relationship between the average export and import prices on one side and income per capita on the other. With $\chi > 1 - \eta$ the profits increase faster along the efficiency dimension, which shapes the isoprofit curves (cutoff functions) in the (a, q) space as presented in Figure 3.

The shape of the cutoff functions determines the quality and price composition of the domestic and import bundles of the two regions. The most profitable firms export both to the North and the South, while less profitable export only to the North. With $\chi > 1 - \eta$, the bigger share of the relatively higher priced varieties (high q and low a) are not exported to the South and are shipped only to the North.[14]

Thus, the resulting average import price is higher for the North. This result holds for all exporter, and also conditional on a particular exporting country. Northern imports

[14] As opposed to the case with $\chi < 1 - \eta$ when relatively low priced varieties are excluded from exports to the South in a larger share than the high priced varieties.

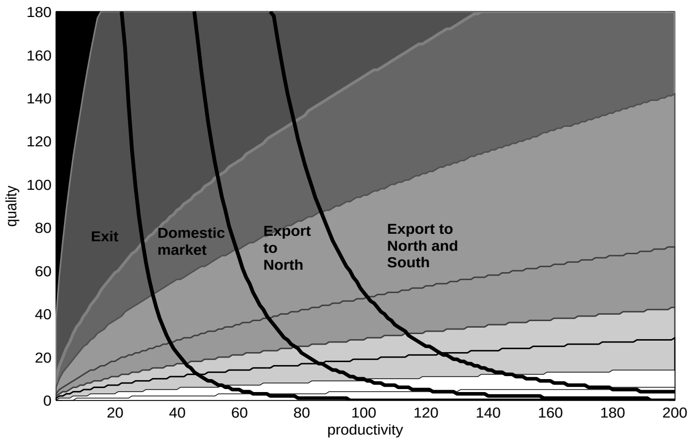

Figure 3.3: Distribution of Prices

are of higher average price relative to the imports of the South as more high quality-low efficiency varieties are included in its import bundle. In other words, it imports goods of higher average price not as it consumes higher quality than the South but due to the fact that it additionally consumes the high priced high quality varieties. The analogue reasoning applies to the imports from the South.This effect is not present with only one dimension of firms heterogeneity as the profits are just a monotonic transformation of the price and the unique productivity measure.

On the export side, the North abandons the export of low price varieties due to competition from the South, which results in higher export prices of the North. Average prices of export and import are presented in Table 1.

Average Price	**North**	**South**
Exports	4.0739	0.9495
Imports	1.0072	0.9101
Imports from North	4.2514	3.9861
Imports from South	1.0008	0.9054

Table 3.1: Average Import Prices

The following graph (Figure 4.) presents the expenditure shares distribution of the two regions across different levels of quality for a given efficiency of the firm. Northern demand is relatively higher for the varieties produced by the high quality firms, and the South is demanding relatively more of the goods in the lower quality portion of the distribution, which is the effect of the fixed cost of trade. With no fixed cost, the homothetic preferences would result in a lower demand from the South but still in levels exactly proportional to those of the North. Once the fixed cost of export is introduced in both the North and the South, this results in subsets of firms with only domestic sales, which subsequently distorts the proportionality of the consumption shares of the two regions across varieties.

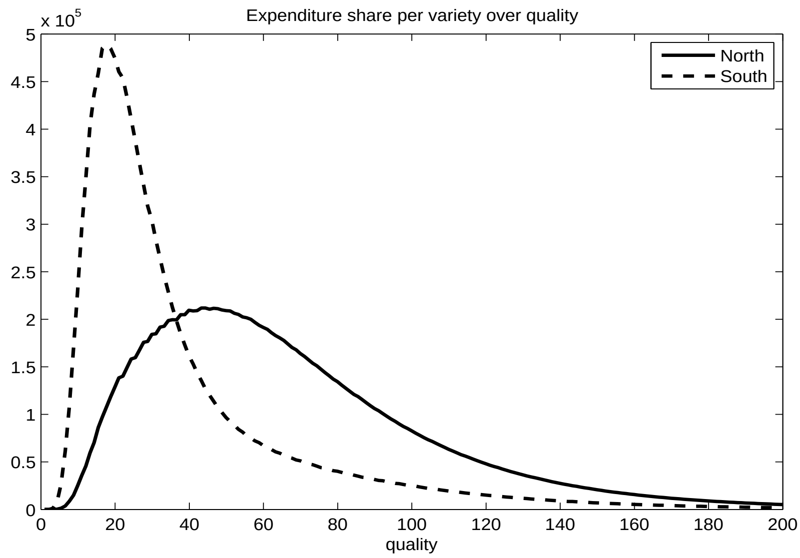

FIGURE 3.4: Expenditure Shares Distribution over Quality

Figure 5. shows the total trade values within and across two groups of countries with no asymmetries in the variable costs of trade. The model implies that larger shares of Northern export revenue is coming from the North due to higher profitability requirements for the export to the South and low absolute expenditure of the South. This implies higher trade intensity between countries of the North. As a result, the North-North trade is the largest compared to the other trade flows, North-South and South-South. In this set-up North-South trade is of higher value than the South-South trade, but the ranking reverses when the asymmetric variable costs of trade are introduced, with the highest cost imposed on Southern exports to the North. Some empirical evidence points to these asymmetries in the form of higher export barriers imposed on the exporters from the South (such as iceberg trade cost, quality requirements, tariffs). In sectors with these asymmetries, our model's results might support the final conjecture of the Linder hypothesis, besides predicting the demand and supply premises.

3.5 Conclusions

This paper analyzes the role of efficiency and quality in shaping the trade patterns and trade intensities within and across two groups of countries, the developed and richer North and the developing South. We employ a four country North-South trade model

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

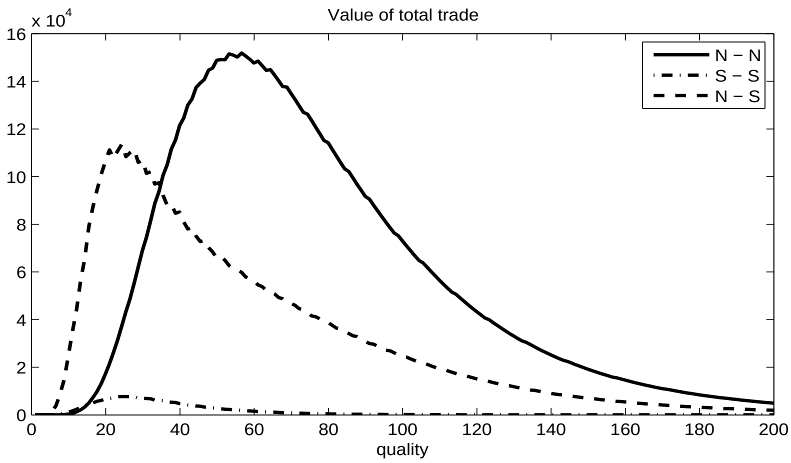

FIGURE 3.5: Total Trade Values Within and Across Regions

with two dimensions of firm heterogeneity. Matching the empirical values of within and across region export shares in the total world exports, we show that the equilibrium results support the ranking of the average prices of tradables within and across regions as found in the data. This result is not previously found in the literature since using only one technology dimension does not simultaneously allow for increasing relation between export prices, import prices and import prices conditional on exporter on one side and income per capita on the other.

Furthermore, we find differences in the consumption bundles across regions even though the preferences are of standard, homothetic form. Namely, the resulting total expenditure allocation across quality shows that the North spends a larger share of its income on high quality while the South allocates more of its expenditure on low quality varieties. Therefore, we wish to stress that the trade patterns in this model are not determined by the non-homotheticity of preferences and therefore do not originate exclusively from the demand structures. The results mainly come from the supply side through the productivity distribution of incumbents and its effect on prices. This in turn allows the fixed cost of exporting to act in a way that the empirically observed trading pattern is replicated. In other words, it is not that the consumers alone have different preferences over qualities based on their income but differences in productivity and income (coming endogenously from the productivity level) are the principal deciding factors.

The future research agenda calls for the development of an endogenous R&D mechanism which will determine technology level of the North and the South in equilibrium. In this hypothetical set-up, firm would choose the level of their investment in technology, which would affect the initial productivity draw through the innovation in the North and technology adoption in the South. R&D incentives would come partly from the domestic demand structure but also as a response to foreign demand, which would together shape the comparative advantage of each region over quality segments. This allows for the analysis of several issues such as trade liberalization, income inequality and R&D subsidies to promote welfare. Furthermore, it should be noted that the set-up is easily extendable to include n countries which allows for more empirically testable predictions.

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Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

Appendix

A Conditions on Fixed Costs and Technological Lag

The setup of the model requires that the exit cutoff in any region, $a_x^J(q)$, is lower than the export cutoff, $a_{ex}^{JK}(q)$, in order to rule out the possibility of firms not operating domestically, and producing only for the export market. To insure this we impose conditions on the fixed costs of production and export, and on the level of the technological lag of the South behind the North. With fixed export cost c_{ex} higher than the fixed operational cost c_f, the cutoff for exporting to the other country of the same region (North-North and South-South trade) will be higher than the exit cutoff. However, to insure higher cutoff for exporting to the other region (North-South trade) than the exit cutoff, the following condition is required

$$\frac{c_f}{c_{ex}} < \frac{P^{N\frac{\alpha}{1-\alpha}} L^N w^N}{P^{S\frac{\alpha}{1-\alpha}} L^S w^S} < \frac{c_{ex}}{c_f} \tag{3.27}$$

As the equlibrium wage and price indices are functions of the technological lag θ, it follows that the three parameters together determine whether the condition above holds. The relative size of the population in the two regions affects the relative size of the aggregates and therefore the ratio of exit cutoffs in the North and the South, and the ordering of export cutoffs conditional on the destination country. In general, if the South is sufficiently larger than the North, the aggregates of the South might be larger than those of the North even with the relative wage smaller than one. However, the calibration exercise shows that such a large South would neither match the data on the actual size of trading partners in the North and the South nor the model could be considered as the model of North-South trade as the share of the Southern firms exporting to the North would be approaching zero. Therefore, without the loss of generality, we assume equal sizes of the regions. We find that under the wide range of c_f, c_{ex} and θ that satisfy the stated condition, the resulting ordering of the cutoffs is such that the exit cutoff is higher in the North than in the South. Moreover, the exporters of relatively lower productivity export only to the North, while the highest productivity firms export also to the South.

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

B Calibration

Table 3.2: Targets and Parameters

Targets	**Data**	**Model**
North-North Export Share	52.69%	54.95%
North-South Export Share	40.86%	42.49%
North Exit Rate	10%	10.43%
South Exit Rate	20%	23.43%
Wage Ratio w^s/w^N	0.4	0.41
Calibrated Parameters		
θ	0.18	
σ	0.5	
c_f	11.42%	of avg North domestic employment
c_{ex}	29.51%	of avg North domestic employment
c_e	38%	of avg North domestic employment
Other Parameters		
α	0.73	
ξ	0.5	
η	0.86	
δ	0.5%	
τ	1	
$\overline{g}^N$	4.1	
$L^N = L^S$	1	

C Size Distribution and Average Productivities

Table 3.3: Weighted Average Technology Across Firm Partition

Weighted Average Technology	**North**	**South**
Total	16.76	8.38
Domestic	15.01	8.05
Export to North	17.23	13.29
Export to N and S	19.79	16.18

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
European University Institute DOI: 10.2870/28221

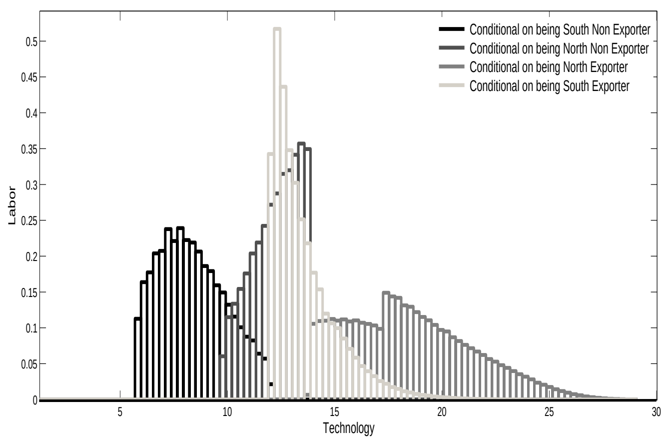

FIGURE 3.6: Conditional Labor Distribution over Technology

Benedetti Fasil, Cristiana (2011), Essays on Firm Dynamics, Endogenous Growth and International Trade
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