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Abstract
This paper reconsiders the division of the literature on electoral competition into models with forward-looking voters and those with backward-looking voters by combining ideas from both strands of the literature. As long as there is no uncertainty about voters' policy preferences and parties can commit in advance to a policy platform but not to a maximal level of rent extraction, voters can limit rents to the same extent as in a purely backward-looking model. At the same time, the policy preferred by the median voter is implemented as in a standard forward-looking model of political competition on an ideological policy dimension. Voters achieve this outcome by following a simple lexicographic voting strategy. They cast their vote in favor of their preferred policy position, but make their vote dependent on the incumbent parties' performance in office whenever they are indifferent. When uncertainty about the bliss point of the median voter is introduced into the model, voters have to accept higher rent payments, but they still retain some control over rent extraction.

Keywords
Elections, Accountability, Downsian Competition, Voting

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Jan Klingelhöfer
Max Weber Fellow, 2010-2011

1 Introduction

Do voters reward incumbents for past success and honesty or do they disregard the past and only consider the future when they vote? This is one of the most fundamental questions for a positive theory of electoral competition. However, theoretical models of elections usually assume either backward-looking or forward-looking voting. The voters' motivation at the ballot box is an assumption of the model rather than an outcome of the equilibrium analysis. In models of pre-election politics, candidates commit to their post-election actions before elections take place. In contrast, in models of postelection politics, politicians are free to decide about their policies when they are in office. However, in the successive elections, the voters can condition their vote on the performance of the incumbent party.[1]

In this paper, I combine a simple prospective model of Downsian spatial electoral competition on an ideological policy dimension and a simple retrospective model of electoral accountability with rent extraction. Specifically, parties can commit to a policy position before elections take place, as in Downs (1957), but decide on the level of rent extraction once they are in office, as in Barro (1973) and the simplified model of political accountability discussed in Persson and Tabellini (2000).

In the basic model in Section 2, I show that, as long as there is certainty about the position of the median voter, having voters with divergent policy preferences does not at all restrict the possibility of holding politicians accountable. The possible equilibrium rent levels are the same as they would be in a model without the additional ideological policy dimension. The voters achieve this by following a straightforward and intuitive lexicographic voting strategy. Specifically, if the parties commit to policy positions that differ in attractiveness to a voter, the voter casts her vote in favor of the party which minimizes her disutilty on the policy dimension. Only when a voter is indifferent with respect to the parties' policy platforms does she condition her vote on the rent extraction of the incumbent party. She supports the incumbent party only

[1] Models of preelection politics are especially popular for modeling spatial policy choices in the tradition of Downs (1957), where voters decide between announced policy positions, while models of postelection politics are often, but not exclusively, applied to accountability issues. Politicians are induced to put in more effort (Ferejohn 1986) or to limit rent extraction due to the possibility of losing the successive elections and office if they do not comply (Barro 1973). Essentially, these accountability models apply a principal-agent framework to elections with the politicians as agents and the voters as their principals. For an overview of both types of model, see Persson and Tabellini (2000). For an overview especially of models of accountability, see Besley (2006).

if the rents have not exceeded a maximum acceptable level. I call this voting strategy "lexicographic" because voters cast their votes as if they had lexicographic preferences over policy and rents. My model is the first one to show that lexicographic voting can achieve a reconciliation of backward-looking and forward-looking voting.[2] The lowest possible rent level that is sustainable in an equilibrium is positive but smaller than the maximum rent the incumbent party could take. Moreover, the existence of the ideological policy dimension does not influence the bound on rents.

The lexicographic voting strategy forces the parties to converge on the policy dimension, but also allows for control of the incumbent's party rent extraction. Moreover, it is intuitive that a voter who is indifferent will take past actions of the parties into account, whereas it is impossible for a rational forward-looking voter to consider the past when she is not indifferent with respect to the future.

Generally, the equilibria in backward looking models hinge on the fact that voters are indifferent between the incumbent party and the opposition and can therefore reward or punish past actions while playing undominated strategies. The fact that a simple strategy can solve the accountability problem in a model combining rent extraction with Downsian competition can be explained by the fact that competition on the ideological policy dimension forces both parties to choose the same platform so that voters are indeed indifferent between the parties in equilibrium. Full convergence of policy is a result of the lack of uncertainty over the preferences of the median voter in the basic model.

Section 3 shows that when uncertainty over voters' preferences is introduced into the model, the minimum equilibrium rent extraction by the incumbent party increases. Because the parties do not know the position of the median voter's bliss point with certainty, the opposition party now has a chance of winning office by offering a different policy position to that of the incumbent party. Nonetheless, the incumbent party has an incentive to accept somewhat reduced rent payments in return for being re-elected whenever the voters are indifferent because in this way, it can ensure that it will be re-elected with positive probability.

The paper proceeds as follows. Section 2 develops the main model with certainty about the position of the median voter and discusses its equilibrium. Section 3 shows

[2]The term lexicographic voting has been used before to describe similar voting strategies, for example in Dutter (1981) and Soberman and Sadoulet (2007). However, in these papers, lexicographic voting follows directly from lexicographic preferences. In my model, lexicographic voting is part of an equilibrium of the voting game, although the voters' preferences are not lexicographic.

that uncertainty over the positions of the median voter leads to less electoral control. An Appendix contains the proofs of the results in Section 3.

2 The model

I consider a polity with two parties interested in winning office only for rent-seeking purposes, and an odd number N of voters $i = 1, 2, ..., n$ interested in policy as well as rent reduction. The ideological policy space is the interval $[0, 1]$. Party $j \in \{x, y\}$ maximizes:

$$U_p^j = E_0 \sum_{t=0}^{\infty} \beta^t r_t^j, \tag{1}$$

where rents in future periods are discounted by the factor $\beta \in (0, 1)$. r_t^j is the rent extracted by party j in period t. The party in government (also called the incumbent party) in period t is denoted by $I_t \in \{x, y\}$. The opposition party in period t is denoted by $O_t \in \{x, y\}$, $O_t \neq I_t$. Parties decide how much rent $r_t \in [0, R]$ they extract in a period in which they are in office. R is the total amount of available public funds and constitutes the maximum per period rent. Parties out of office cannot acquire any rents. Hence $r_t^{O_t} = 0$ in all periods t.

Voters $i = 1, 2, ..., n$ maximize:

$$U_v^i = E_0 \sum_{t=0}^{\infty} \beta^t (-(p_t - b^i)^2 + (R - r_t)), \tag{2}$$

where b^i is the policy bliss point for voter i, r_t the rent extraction and p_t the policy implemented in period t. Because the policy platform announced by the incumbent is always implemented and $r_t^{O_t} = 0$ in all periods t we have $p_t = p_t^{I_t}$ and $r_t = r_t^{I_t}$. I abstract from any details on how rents are extracted and assume that rent payments reduce a given amount of public funds, which reduces every voter's utility in the same way. Hence, $R - r_t^{I_t}$ gives the amount of public funds that are used in the voters' interest. For simplicity, I assume that the utility from public good spending is uncorrelated with the ideological policy position. The variable p_t denotes the policy in period t and the vector $B = (b^1, b^2, ..., b^N)$ the policy bliss points of the voters. $b_m = median(B)$ is the bliss point of the median voter. For the moment, this is assumed to be constant over time. In Section 3, the more general case of uncertainty

about the median voter's position is discussed. Disutility in policy is quadratic in the distance from the bliss point. This standard functional form assumption is made for convenience of notation. All the following results only depend on increasing disutility in distance of policy from a voter's bliss point.

2.1 The order of moves

The order of moves is the following: In any period t, the policy position p_t^I of the incumbent party $I_t \in \{x, y\}$ is implemented, then rents $r_t^{I_t}$ and a new policy position p_{t+1}^I are chosen by the incumbent party. An alternative policy position $p_{t+1}^{O_t}$ is chosen by the opposition after observing the policy position of the incumbent party and the rent r_t. Then, elections take place and every voter i casts her vote $v_t^i \in \{x, y\}$. Abstentions are not possible.

Let $V_t = (v_t^1, v_t^2, .., v_t^N)$ be the vector containing the votes of all voters. After the elections have taken place, the new period $t + 1$ begins and the party with the majority of votes in period t becomes the incumbent party:

$$I_{t+1} = \text{mod}(V_t).$$

In Period 0, the identity and the policy positions of the incumbent party and the opposition are exogenously given.

The incumbent party is thus assumed to choose its position first, instead of the more standard assumption that policy positions are chosen simultaneously.[3] For the basic model, this is of no great importance (however, the best reply of the opposition is no longer unique), but it plays some role when I introduce uncertainty in Section 3, where it is essential for the existence of equilibria in pure strategies. The timing assumption is made to keep the analysis there as simple as possible.

2.2 Strategies

To denote the entire history of a variable z_t up to period t, I use a superscript t such that $z^t = \{z_0, z_1, z_2, ..., z_t\}$. Let $h_t = \{p^{y,t}, p^{x,t}, I^t, V^{t-1}, r^{t-1}\}$ be the history of the game up to the beginning of period t. A strategy for a party j is the decision

[3] This assumption is less common than simultaneous policy announcements, but has been made in many papers. For an early example see Wittman (1973).

about a policy platform $p^j_{t+1}(h_t) \in [0,1]$ for all possible histories with $j = I_t$ and $p^j_{t+1}(h_t, p^{I_t}_{t+1}, r_t) \in [0,1]$ for all possible histories with $j = O_t$. In addition, the strategy contains the rent payment $r^j_t(h_t)$ for all possible histories with $j = I_t$. Because the opposition can observe the policy position of the incumbent party, the party that is out of office can take both the policy position and the rent payment to the incumbent party into account when announcing its policy position, while the incumbent party cannot. A strategy for a voter i is a vote $v^i_t(h_t, p^y_{t+1}, p^x_{t+1}, r_t) \in \{y, x\}$ for every period t and every possible history up to the time of her voting decision.

Definition 1 *A strategy is history-independent if all decisions by a player in period t only depend on other variables that have been a) determined in the same period and b) before the decision is made.*[4]

Thus, a history-independent strategy for the incumbent party implies that its platform and rent extraction do not depend on moves in past periods at all, and thus they must be constant as long as the same party j is holding office: $p^{I_t}_{t+1} = p^I_j$ and $r_t = r^I_j$ whenever $I_t = j$ in period t. The reply of the opposition party only depends on the policy offer and rent extraction of the incumbent party and the votes only on the policy offers, the identity of the incumbent party and the rent extraction. If, in addition, both parties are assumed to play the same strategy, policy offers and rent extraction will be the same in all periods. Moreover, if the voters play pure strategies, the incumbent party is either always or never re-elected.

2.3 An equilibrium with lexicographic voting

The strategies formulated in Proposition 1 below constitute an interesting equilibrium which has all the essential features of a backward-looking model in the tradition of Barro (1973) and Ferejohn (1986) as well as those of a forward-looking model in the tradition of Downs (1957). Parties converge on the ideological dimension and the rents are at the lowest level sustainable in the purely backward-looking model without policy dimension. This is the result of an intuitive lexicographic voting strategy. A voter casts her ballot in favor of her preferred policy position. Only when she is indifferent in this respect does she decide according to past rent extraction by the

[4]This is often called a stationary strategy in political economics. However, it could be argued that the rent payment r_t should not play any role in a stationary strategy because it is a bygone by the time the voters cast their votes. I therefore avoid the term "stationary".

incumbent party. It is clear that with such a strategy, she encounters no credibility or time-inconsistency problem.

Proposition 1 *An equilibrium of the game is constituted by the following strategies:*

The parties play:

$$\begin{aligned} p_{t+1}^{j} &= b_m \text{ for } j = y, x \text{ in all } t, \\ r_t^{I_t} &= \bar{r} \text{ in all } t, \end{aligned} \tag{3}$$

where $\bar{r} = (1-\beta)R$.
The voters play:

$$v_t^i = \begin{cases} y \text{ if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 < 0 \\ x \text{ if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 > 0 \\ I_t \text{ if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 = 0 \text{ and } r_t \leq \bar{r} \\ O_t \text{ if } (p_{t+1}^y - b^i)^2 - (p_{t+1}^x - b^i)^2 = 0 \text{ and } r_t > \bar{r} \end{cases} \text{ in all } t. \tag{4}$$

Given the strategies, it follows that:

$$\begin{aligned} I_t &= I_0 \text{ in all } t, \\ p_t &= b_m \text{ in all } t \geq 1, \\ r_t &= \bar{r} \text{ in all } t. \end{aligned} \tag{5}$$

Proof. Given the voters' strategy, the median voter is decisive: If $v_t^m = j$, it follows that $(p_{t+1}^j - b^m)^2 - (p_{t+1}^{\sim j} - b^m)^2 \leq 0$. This implies that $(p_{t+1}^j - b^i)^2 - (p_{t+1}^{\sim j} - b^i)^2 \leq 0$ for all $b_i \leq b_m$ or all $b_i \geq b_m$ and therefore for a majority of voters. Thus, the majority of voters cast their votes for the same candidate as the median voter and the party with the support of the median voter wins. Given the equilibrium strategies of the parties, $(p_{t+1}^j - b^i)^2 = (p_{t+1}^{\sim j} - b^i)^2$ in all periods. Because $r_t = \bar{r}$ in all periods, all voters vote for the incumbent party, which remains in office and implements $p_{t+1}^I = b_m$.

Given the strategies of the parties, a voter in period t neither influences future rent payments nor future policy platforms (that is any p_s^j with $s > t+1$) with her vote. This is even true in the case with only one voter who is always pivotal. Therefore, a voter has no utility-increasing deviation from voting for the party that offers the policy closest to her bliss point in period $t+1$. In the case that a voter is indifferent

between the candidates' policy platforms in period $t+1$, there is no utility-increasing deviation from voting according to the past performance of the incumbent because, again, it does not influence future policy or rent payments.

The fact that the opposition party cannot be better off by deviating follows from the fact that given the position and rent extraction of the incumbent party and the strategy of the voters, it either wins with certainty or has no possibility of winning office and, moreover, it cannot influence any election results or rent payments in the future with its choice of policy position. For the incumbent party, any policy position different from $p^I_{t+1} = b_m$ leads to a loss of office (and therefore rent payments) forever because given the reply of the opposition, the latter is preferred by the median voter. The same is true for the combination of any policy position p^I_{t+1} with any rent $r_t > \bar{r}$. Therefore, re-election is only possible with $r \leq \bar{r}$. Hence, there is no possibility for the incumbent party of increasing its utility by deviating with a strategy that leads to its re-election. If it accepts defeat by deviating in an arbitrary period s, the incumbent party can, at most, achieve a rent of R in the period in which it deviates and then lose office and rents forever. This gives the same utility level that the incumbent party achieves by not deviating and receiving a rent of $r_t = \bar{r} = (1-\beta)R$ forever, because the present discounted value of future rent payments in period s is the same:

$$\sum_{t=0}^{\infty} \beta^t \bar{r} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \sum_{t=s}^{\infty} \beta^t \bar{r} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \sum_{t=s}^{\infty} \beta^s \frac{\bar{r}}{1-\beta} = \sum_{t=0}^{s-1} \beta^t \bar{r} + \beta^s R.$$

Therefore, no deviation from the given strategy increases the utility of the incumbent party. ■

Which party is the incumbent party in period 0 is exogenously given. This party remains in office forever, as in the standard case of backward-looking models without uncertainty. However, this will no longer be the case when I introduce some uncertainty in Section 3.

Corollary 1 *There is no equilibrium with a present discounted value of future rent payments in any period s of the game that is lower than the maximum per-period rent extraction R.*

Proof. Suppose that there is an equilibrium with $\sum_{t=s}^{\infty} \beta^{t+s} r_t < R$ in any period s. Then, the incumbent party in period s is better off by deviating and taking a rent of $r_s = R$. This is a contradiction. ■

Therefore, the equilibrium rent level in Proposition 1 gives a lower bound for rents in equilibrium.[5] The rent level is identical to the lower bound on rent extraction in a model without a policy dimension.[6]

Voters play as if they were always pivotal. This seems to be a reasonable assumption for a plausible equilibrium and helps to rule out equilibria which require a great deal of coordination of voters when they cast their votes. However, Corollary 1 is valid for all possible equilibria. Therefore, restricting strategies to being history-independent does not reduce electoral control at all.

The intuition is straightforward. Nothing can stop a party in power from taking the maximum rent R if it does not expect to get at least the same present discounted value in rents in later periods.

As is also common in models of political accountability, the given equilibrium is not unique and other equilibria with larger rent payments exist. However, the existence of this equilibrium is sufficient to establish that retrospective and prospective motives in voting are not inconsistent with each other. Voters have just one instrument, namely their single vote, but this is sufficient to control policy as well as to hold politicians accountable to a certain degree.

A voter is pivotal if her vote decides the winner of the elections because $\frac{N-1}{2}$ of the other voters vote for party x and $\frac{N-1}{2}$ vote for party y. Consequently, we can say that a voter votes as if she were pivotal if she votes for the party whose victory maximizes her utility.[7]

The Corollary 2 below shows that convergence on the policy dimension is the rule rather than the exception, but first I derive a useful Lemma:

Lemma 1 *If parties play symmetric history-independent strategies and voters vote as if they were pivotal even when they are not, then: a) A voter votes for a party that offers the bliss point minimizing her disutility from policy in the next period. b) A party's utility only depends on being the incumbent party in the next period and the*

[5] There are equilibria with a lower rent payment $r_t < \bar{r}$ in period t that are sustainable because the incumbent party expects higher rent payments in the future. However, from Corollary 1, we know that the present discounted value of rent extraction cannot be smaller than R. Equilibria with increasing rent payments over time seem rather implausible. The opposition party could in this case try to convince the voters that it would only demand a constant rent payment of $\bar{r}$ once in office.

[6] This can easily be established following the same line of reasoning as in the proof of corollary 1.

[7] This concept is only well-defined as long as a voter's vote itself does not influences future play by other players. However, as long as parties are restricted to history-independent strategies there is no problem.

rent extraction in the current period.

Proof. History independence together with symmetry of the parties strategies imply that, from period $t+1$ onwards, policy positions and rent extraction are decided independently of past periods. The only state variable is incumbency, but voters are indifferent to which party is in office and which party offers which policy position. From this, the lemma directly follows. ■

Corollary 2 *There is no equilibrium with symmetric history-independent strategies, voters who vote as if they were pivotal, rent payments $r_t < R$ and policy $p_{t+1} \neq b^m$ in any period t.*

Proof. From Lemma 1, it follows that in any equilibrium with history-independent symmetric strategies, a party's policy position influences its utility only in so far as it determines the winner of the elections and the rent extraction. Suppose that $r_t < R$. This can only be part of an equilibrium if the incumbent party is re-elected with positive probability; if not it would play $r_t = R$, because a lower rent r_t could not improve its situation once in opposition. If both parties play symmetric history-independent strategies, the incumbent party can only be re-elected with positive probability if it plays $p^I_{t+1} = b_m$, because all other positions would be beaten by $p^O_{t+1} = b_m$. To see this, consider the problem of a voter who votes as if she were pivotal: By definition of b_m, a majority of voters must prefer b_m to any $b \neq b_m$, and in equilibrium the opposition would have to choose a position that wins the election to maximize its utility. Therefore, if $r_t < R$ the incumbent party offers $p_{t+1} = b_m$ and, in equilibrium**,** a party offering b_m wins. ■

The existence of equilibria with $r_t = R$ and $p_{t+1} \neq b_m$ is due to the unusual timing assumption that the opposition party chooses its policy position after the incumbent party. There are, for example, history-independent equilibria where the incumbent party always takes R and is never re-elected. In such equilibria, the incumbent party has no incentive to take the median position. However, if the incumbent party does not take the median position, the opposition party does not have to take it either to win, because any policy position that is different from b_m can be beaten by another policy position that is different from b_m, but slightly closer to the bliss point of the median voter. With the standard timing assumption of simultaneous announcement of policy positions, this is not possible. However, a similar equilibrium in which policy

does not converge to the median position is possible in a purely Downsian framework with the incumbent party choosing its position first, and the result should therefore not be attributed to the combination of prospective and retrospective voting motives. On the contrary, only in combination with the outcome of $r_t = R$ in all periods can it be sustained in the combined model.

2.4 Discussion of the different treatment of rents and policy

A crucial assumption is that commitments to electoral platforms are credible in the policy dimension but lack credibility in the rent dimension. A first justification is that these are widely accepted standard assumptions for both types of models and that it is worth exploring whether combining them leads to results that cannot be found by looking at the models separately. Moreover, in the basic model as well as in the extension with uncertainty over the position of the median voter (Sections 2 and 3), parties have no reason to break their electoral promises with regard to policy because it does not enter their utility function. A further justification is that if parties announce policy motivated candidates who run for office, they can indeed credibly commit to policies, but not to limits of rent extraction. Osborne and Slivinski (1996) and Besley and Coate (1997) introduced citizen-candidates into the voting literature. In these models, not parties, but citizens with policy preferences run for election. Commitment to a policy position does not constitute a problem because voters vote for ideological candidates whom they know to implement their favorite policy. As long as there is a candidate with a certain ideology, voters can vote for that candidate. The principal-agent problem of the voters is solved by delegation to an agent with the right preferences. However, empirically, citizen-candidates who run independently of any party appear to be the exception rather than the rule. The basic idea that a certain type of candidate will implement a certain kind of policy can be incorporated into models with parties if the parties have the chance to decide before the elections who the candidate is and achieve office in case of victory, and if the choice of potential candidates is sufficiently large. I do not explicitly model such a candidate choice stage, but the fact that parties usually run with candidates who have their own ideology is a good justification for the assumption that parties can commit to a policy. However, as long as there are no candidates with purely altruistic motives without interest in rent payments available, parties cannot credibly commit to refrain from rent seeking.

3 Uncertainty about the median bliss point

So far, I have assumed that the identity of the median voter is known when parties decide on their policy platforms. How robust are the results to relaxing this assumption? This section shows that voters retain some control over rent extraction in a straightforward and plausible equilibrium where they follow the same lexicographic voting strategy as in Section 2.

The assumptions and the order of moves are the same as in Section 2. The only difference is that the favorite position of the median voter is now uncertain at the point when parties announce their policy positions. Voters keep some control over rent extraction, but the control is limited because sometimes the incumbent party loses office even when it does not deviate and therefore can demand higher rents in equilibrium.

For simplicity, I assume from now on that there is only one voter. She can be thought off as representing the decisive median voter.[8] Her expected utility is given by:

$$U_m = E_0 \sum_{t=0}^{\infty} \beta^t (-(p_t - b_t)^2 + R - r_t), \tag{2'}$$

where b_t is her bliss point in period t. This bliss point is now a random variable that is only determined after the parties have announced their policy positions for period t. The value of b_t is distributed identically and independently of past bliss points. The expected utility function of the parties $j = y, x$ is identical to the expected utility function in Section 2:

$$U_p^j = E_0 \sum_{t=0}^{\infty} \beta^t r_t^j. \tag{1}$$

Let there be K distinct possible policy bliss points b_k of the voter, all within the policy space $[0, 1]$. They are ordered such that $b_k < b_l$ if and only if $k < l$. Let q_k be the probability that the median voter of period t has the bliss point $b_t = b_k$. By assumption, this probability is the same in every period t. Then, $F(b_k) = \sum_{l=1}^{l=k} q_l$ is the cumulative distribution function of b_k. I define:

$$b_m = \min_{k \in K} F(b_k) \; s.t. \; F(b_k) \geq 0.5, \tag{6}$$

[8] This simplifies the notation because it rules out the possibility that the identity of the median voter changes between periods without changing the results significantly.

so that b_m is now the median of the possible bliss points of the voter.[9] Moreover, I define for the case $K \geq 2$:

$$b^*(b_k) = \begin{cases} b_2 & \text{for } k = 1 \\ b_{K-1} & \text{for } k = K \\ \left.\begin{array}{c} b_{k-1} \text{ if } F(b_{k-1}) \geq 1 - F(b_k) \\ b_{k+1} \text{ if } F(b_{k-1}) < 1 - F(b_k) \end{array}\right\} & \text{for } k \in \{2, 3, .., K-1\} \end{cases} \tag{7}$$

$$\pi^* = \begin{cases} F(b_m) \text{ if } b^*(b_m) > b_m \\ 1 - F(b^*(b_{m-1})) \text{ if } b^*(b_m) < b_m \end{cases} \tag{8}$$

$$r^* = \frac{((1-2\pi^*)\beta + 1)}{(1-\pi^*)\beta + 1} R \tag{9}$$

If $K = 1$, then $b^* = b_m = b_1$ and $\pi^* = 1$.

Proposition 2 *An equilibrium of the game entails the following strategies: The parties play:*

$$\begin{array}{c} p^I_{t+1} = b_m, \\ r_t = r^*, \\ p^O_{t+1} = \begin{cases} b^*(p^I_{t+1}) \text{ if } r_t \leq r^* \\ p^O_{t+1} = p^I_{t+1} \text{ if } r_t > r^* \end{cases} \end{array} \quad \text{in all } t. \tag{10}$$

The voter plays:

$$v_t = \begin{cases} y \text{ if } (p^y_{t+1} - b_{t+1})^2 - (p^x_{t+1} - b_{t+1})^2 < 0 \\ x \text{ if } (p^y_{t+1} - b_{t+1})^2 - (p^x_{t+1} - b_{t+1})^2 > 0 \\ I_t \text{ if } (p^I_{t+1} - b_{t+1})^2 - (p^O_{t+1} - b_{t+1})^2 = 0 \text{ and } r_t \leq r^* \\ O_t \text{ if } (p^I_{t+1} - b_{t+1})^2 - (p^O_{t+1} - b_{t+1})^2 = 0 \text{ and } r_t > r^* \end{cases} \quad \text{in all } t. \tag{11}$$

In every period, the probability that the incumbent party wins is π^*. If the incumbent party wins, b_m is implemented, if the incumbent party loses, $b^*(b_m)$ is implemented. If $K = 1$, the expected utility of the voter is $\frac{R-r^*}{1-\beta}$ because there is no uncertainty and her favorite policy is always implemented. In the case of $K \geq 2$, the expected utility

[9] Naturally, b_m was also the median of the possible median bliss points in Section 2, where the distribution of the median voter was degenerate. Therefore, there is no need to change the notation.

of the voter is:

$$u_{rv} = \begin{cases} \sum_{t=0}^{\infty}(\sum_{k=1}^{m-1} q_k\beta^t(-(b_{m-1}-b_k)^2+R-r^*) & \\ \quad +\sum_{k=m}^{K} q_k\beta^t(-(b_m-b_k)^2+R-r^*)) & \text{if } b^*=b_{m-1} \\ & \\ \sum_{t=0}^{\infty}(\sum_{k=1}^{m} q_k\beta^t(-(b_m-b_k)^2+R-r^*) & \\ +\sum_{k=m+1}^{K} q_k\beta^t(-(b_{m+1}-b_k)^2+R-r^*)) & \text{if } b^*=b_{m+1} \end{cases} \tag{12}$$

Proof. See the Appendix ■

The best position any incumbent party can choose is the median of the possible positions of the voter. The intuition is straightforward. The incumbent party must choose its position first. Because the incumbent party will not be re-elected if the voter prefers its opponent even if it constrains itself with respect to rent extraction, the best the incumbent party can do is to choose its position so that the opposition can only achieve less than 50% of the votes. The incumbent party can achieve this by announcing the median bliss point as policy position. The opposition party will then choose a position as close to the median position as possible to ensure the victory whenever the bliss point of the median voter is on the same side of the median position. It chooses the side of the median where this probability is the largest. Therefore, the most useful measure of uncertainty about the election outcome is given by:

$$\pi^* = min(F(b_m), 1-F(b_{m-1})).$$

It turns out that the larger π^*, the greater is the control of the voter over rent extraction by the parties. In the special case of no uncertainty about the bliss point of the voter, $\pi^*=1$, an incumbent party that does not extract too high rent payments is re-elected with certainty. The results of Section 2 are confirmed as a special case of the generalized model.

Restricting the strategies of parties to be history-independent and identical, and letting the strategy of the voter only depend on the current policy offers and the last rent payment seems intuitively plausible as the model is completely symmetric. Under these conditions, the equilibrium stated in Proposition 2 is the one with the lowest rent payment that the voter can achieve, as is shown by the following corollary:

Corollary 3 *There is no equilibrium with a rent $r_t < r^*$ if the voter's strategy only depends on rent extraction in the last period and the policy positions of the parties*

(that is $v_t(h_t, p^y_{t+1}, p^x_{t+1}, r_t) = v_t(r_t, I_t, p_{y,t+1}, p_{x,t+1})$*), when both parties play identical history-independent strategies (that is* $p^I_{t+1}(h_t) = p_I$*,* $r_t(h_t) = r$ *and* $p^O_{t+1}(h_t, r_t, p^I_{t+1}) = p^O_{t+1}(r_t, p^I_{t+1})$*.*

Proof. See the Appendix ■

From the voter's perspective, it would potentially enhance expected welfare if the candidates did not choose policy positions the way they actually do. Competition drives parties "almost" to convergence, but this is not necessarily in the voter's interest from an ex ante perspective. The reason is that if she has rather extreme preferences, both parties will offer a policy position that is rather centrist and she will suffer from lack of choice. The expected per-period utility of the voter before her preferences are revealed would increase if only one party chose a centrist position but the other an extreme one.

Bernhardt, Duggan, and Squintani (2009) show that such a lack of choice in policy provided by parties uncertain about the position of the median bliss point can make voters worse off. This may not be all that surprising in the light of the literature on spatial competition (Hotelling 1929).

Equilibrium rent extraction r^* is decreasing in π^*. The intuition is straightforward: The larger π^* is, the more likely it is that the incumbent party remains in office if it does not deviate. In addition, the incumbent party is also less likely to regain office once it loses it. Therefore, the rent that has to be paid to make the incumbent party willing to forgo the maximum rent R in favor of re-election decreases.

The voter is essentially playing the same lexicographic strategy as in the model without uncertainty in Section 2. However, she has to accept higher rent payments because there is no longer any guarantee that the incumbent party will be re-elected. Moreover, an incumbent party which loses office can regain office later, which also makes losing power less costly.

3.1 Two interesting cases

There are two interesting cases with intuitive results. First, there is the case of $\pi = 1$, which can only occur if $K = 1$; otherwise there would always be at least a small probability that the incumbent party loses. In this case, we are back to the set-up of Section 2 and it indeed turns out that $r^* = (1 - \beta)R = \bar{r}$. The incumbent

party once more faces the choice between either remaining in office forever or stealing R once.

The second case is $\pi = 0.5$, which happens if and only if $F(b_m) = 0.5$. Because the probability that $b_t \leq b_m$ is exactly equal to the probability that $b_t > b_m$, incumbents have no possibility of increasing their chances of re-election to more than 50% even when they accept limited rent extraction. This is also what would happen if there were a continuous function of possible positions of the median voter. In this case, $r^* = \frac{1}{0.5\beta+1}R$ or $(1 + 0.5\beta)r^* = R$. The reason is that when the incumbent party does extract the maximum amount of rent R, it loses $0.5\beta r^*$ in the next period, but from then onwards, it has the same chance of being the incumbent party (50%) that it would have without any deviation from its strategy.

3.2 Discussion of the timing assumption

Without the assumption of the incumbent party moving first, a lexicographic strategy by the voters can only be consistent with an equilibrium if the parties randomize over policy. The reason is that the incumbent party would always like to take the same position as the opposition and win with certainty, and therefore the opposition must randomize over its position. A somewhat similar model has been solved by Aragones and Palfrey (2002). In their set-up, voters are not indifferent because candidates differ in an exogenously given policy attribute, so that the candidate who is preferred in this dimension wins if he can take the same policy position as the other candidate. It should therefore be possible to solve an alternative model without the timing assumption and derive similar results with respect to accountability. However, finding optimal mixed strategies is not the focus of this paper.

4 Conclusion

It is surprising that until now, there seem to have been no attempts to combine models of retrospective voting with aspects of Downsian competition. My model shows that forward-looking and backward-looking motives can be reconciled in a single model. This should be considered in future empirical research because so far, the question seems to have been if voters vote retrospectively or prospectively. If there is not necessarily a contradiction, some empirical results might have to be re-evaluated.

As long as there is certainty about the position of the median voter, I find that on the policy dimension where commitment is possible, the usual median voter results apply, while rent extraction by politicians is limited to the same degree as in a standard model without a policy dimension. If there is uncertainty about the position of the median voter, voters cannot limit rent extraction to the same degree as in the certainty case, but accountability is not completely lost either. The reason is that even when the incumbent party complies with the voters demands for limited rent extraction, it will still lose office if the opposition party commits to a policy that is more attractive to the majority of voters. Models of political accountability can explain the often observed incumbency advantage, as is pointed out by Austen-Smith and Banks (1989). Models in the Downsian tradition, on the other hand, provide no explanation for an incumbency advantage. My basic model in Section 2 leads to the implausible result that in equilibrium, the incumbent party is always re-elected. In the extended model with uncertainty about the exact position of the median voter in Section 3, I find that the incumbent party always has a chance exceeding 50% of winning the elections and that its advantage depends on a measure of uncertainty about the preferences of the median voter. This result is consistent with election results in many countries. Incumbent parties win more often than not, but their victory is far from certain.

Appendix

Proofs Section 3

Proof of Proposition 2. The single deviation principle states that it is sufficient to show that no player can increase his expected utility by a single deviation to prove that the given strategies constitute a subgame perfect Nash Equilibrium. The single deviation principle applies to an infinite game when the overall payoffs are a discounted sum of the per-period payoffs that are uniformly bounded. This applies to the game in Section 3.[10]

First, I show that both the incumbent party and the opposition party maximize their chances of winning the election if they follow their given strategies. In the case of $r_t > r^*$, the opposition party wins with certainty by taking the same policy position as the incumbent party $p^O_{t+1} = p^I_{t+1}$ and wins office with certainty. In the case of $r_t \leq r^*$, if $p^O_{t+1} = p^I_{t+1}$ and therefore $(p^I_{t+1} - b_{t+1})^2 - (p^O_{t+1} - b_{t+1})^2 = 0$, the opposition loses with certainty. If $-(b_k - b_{t+1})^2 + (b_{k-1} - b_{t+1})^2 < 0$, then $(b_k - b_{t+1})^2 - (b_l - b_{t+1})^2 < 0$ for all $l \leq k-1$. Therefore, if $p^I_{t+1} = b_k$ and $r_t \leq r^*$, the opposition is at least as likely to win with $p^O_{t+1} = b_{k-1}$ as with any $p^O_{t+1} < b_{k-1}$. Similarly, if $-(b_k - b_{t+1})^2 + (b_{k+1} - b_{t+1})^2 < 0$, then $(b_k - b_{t+1})^2 - (b_l - b_{t+1})^2 < 0$ for all $l \geq k+1$ and therefore the opposition is at least as likely to win with $p^O_{t+1} = b_{k+1}$ as with any $p^O_{t+1} > b_{k+1}$. It follows that either $p^O_{t+1} = b_{k+1}$ or $p^O_{t+1} = b_{k-1}$ maximizes the probability of the opposition winning against $p^I_{t+1} = b_k$. Therefore, from the definition of $b^*(b_k)$, a policy that maximizes the probability of the opposition party winning is given by $p^O_{t+1} = b^*(p^I_{t+1})$. It remains to be shown that $p^I_{t+1} = b_m$ maximizes the prospects of the incumbent party given the reply $b^*(p^I_{t+1})$. From its definition and the voter's strategy, π^* gives the probability that the incumbent party wins when $r_t \leq r^*$, $p^I_{t+1} = b_m$ and $p^O_{t+1} = b^*(p^I_{t+1})$. From the definition of b_m, $F(b_{m-1}) < 0.5$ and $1 - F(b_m) \leq 0.5$. Therefore, $\pi^* \geq 0.5$. If $p^I_{t+1} \neq b_m$, the probability of winning for the opposition by choosing b_m itself is at least 0.5 and therefore, the probability that the opposition wins with $p^O_{t+1} = b^*(p^I_{t+1})$ for $p^I_{t+1} \neq b_m$ cannot be smaller than 0.5. Hence, $p^I_{t+1} = b_m$ maximizes the chances of the incumbent party remaining in power given the strategies of the other players, and π^* gives the probability of reelecting the incumbent party in the given equilibrium.

Given the strategies of the other players, the voter will encounter the two policy offers b_m and $b^*(b_m)$ and the rent extraction r^* in all future periods. Therefore,

[10] See Fudenberg and Tirole (1991) for a formal statement of the single deviation principle.

maximizing the current period utility, as she does by voting for the party she prefers if she is not indifferent, is maximizing her expected utility.

Let V denote the value of being in office and W denote the value of being out of office given the strategies. The present expected value of being out of office is determined by the value of being in office and the equilibrium probability of winning the next elections, $1 - \pi^*$:

$$W = (1 - \pi^*)\beta V + \pi^* \beta W \implies W = \frac{\beta(1 - \pi^*)V}{1 - \pi^* \beta}. \tag{13}$$

It follows that $W < V$ and being in office is better than being out of office. From this, it directly follows that deviating once from the strategy cannot make the opposition that maximizes its chances of becoming the next incumbent party better off, because a single deviation cannot change the future values of being in and out of office. Therefore, maximizing the probability of being incumbent and achieving V instead of W in the next period is optimal. The value of being the incumbent party depends on the equilibrium rent extraction r^* and the probability of being in and out off office in the next period is:

$$\begin{aligned} V &= r^* + \beta\pi^* V + \beta(1 - \pi^*)W = r^* + \beta\pi^* V + \beta(1 - \pi^*)\frac{\beta(1 - \pi^*)V}{1 - \pi^* \beta} \quad (14) \\ &= \frac{((1 - 2\pi^*)\beta + 1)}{(1 - \pi^*)\beta + 1} R + \beta\pi^* V + \beta(1 - \pi^*)\frac{\beta(1 - \pi^*)V}{1 - \pi^* \beta} \\ \implies V &= \frac{\pi^* \beta - 1}{\pi^* \beta + \beta^2 - \pi^* \beta^2 - 1} R. \end{aligned}$$

Given that the future values of being an incumbent party and in opposition cannot be changed by a one-time deviation, it is clear that the incumbent party should maximize the rent payment for a given probability of re-election. Therefore, any rent payment $r_t < r^*$ cannot make the incumbent party better off, because it decreases the rent without increasing the probability of re-election. From the fact that the incumbent party loses the election with certainty if $r_t > r^*$ independently of its chosen policy position, the only deviation that needs to be checked is $r_t = R$ in combination with any arbitrary policy position. The reason is that if the party were to be better off by extracting any rent r such that $r^* < r < R$, it must also be better off extracting R. The expected value of deviating in this way and then being in opposition in the next period is given by the sum of R and the present value in opposition in the next

period:

$$\begin{aligned} R+\beta W &= R+\beta\frac{\beta(1-\pi^*)V}{1-\pi^*\beta} \\ &= R+\beta\frac{\beta(1-\pi^*)}{1-\pi^*\beta}\frac{\pi^*\beta-1}{\pi^*\beta+\beta^2-\pi^*\beta^2-1}R \\ &= \frac{\pi^*\beta-1}{\pi^*\beta+\beta^2-\pi^*\beta^2-1}R = V. \end{aligned} \tag{15}$$

This gives the party the same utility V as following the strategy given in Proposition 2. Therefore, the incumbent party has no reason to deviate. None of the players is better off with a one time deviation and therefore, the strategies given in Proposition 2 constitute a subgame perfect Nash Equilibrium. ■

Proof of Corollary 3. Because $p^I_{t+1}(h_t) = p_I,\ r_t(h_t) = r$ and $p^O_{t+1}(h_t, r_t, p^I_{t+1})$ $= p^O_{t+1}(r_t, p^I_{t+1}) = p^O_{t+1}(r, p_I)$ for all t, the voter's decision can neither change her future policy choice nor future rent extraction. Therefore, in equilibrium, she votes for the party that offers the policy that is closest to her bliss point. Only if both parties offer the same policy position, is voting for either party consistent with an equilibrium. This gives the opposition party the possibility of being elected with a probability of at least $1-\pi^*$ for any rent payment r_t and the policy position of the incumbent party by offering $p_{O,t+1} = b^*(p_{I,t+1})$. The opposition party maximizes its utility by maximizing the probability of being voted into office since being in office must be better than being out of office. Only in office is any rent extraction possible and the history-independence of the strategies implies that future rents are given by some constant level r. Let $r^{\min}$ be the smallest rent payment that is consistent with an equilibrium. The value of being in office is given by $V(r^{\min}, \pi) = \frac{(1-\pi\beta)r^{\min}}{(1-\pi\beta)^2-\beta^2(1-\pi)^2}$, where π is the probability of re-election of the incumbent party. V is increasing in π, and the maximum π that is consistent with equilibrium is π^*. Therefore, the maximum V that is consistent with $r^{\min}$ and an equilibrium is given by $V(r_{\min}, \pi^*) = \frac{(1-\pi^*\beta)r^{\min}}{(1-\pi^*\beta)^2-\beta^2(1-\pi^*)^2}$. The second condition that must hold is $R \leq r^{\min} + \beta\pi V(r^{\min}, \pi^*) + \beta(1-\pi)\frac{\beta(1-\pi)r^{\min}V(,\pi^*)}{1-\pi\beta}$, because otherwise the incumbent party would be better off taking R and losing office. This condition can only hold if $r^{\min} \geq r^*$, hence it follows that $r^* = r^{\min}$. ■

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