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A Re-Interpretation of the Linear-Quadratic Model when Inventories and Sales are Polynomially Cointegrated

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A Re-interpretation of the Linear-Quadratic Model When Inventories and Sales are Polynomially Cointegrated[1]

by

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Abstract

Estimation of the linear quadratic model, the workhorse of the inventory literature, traditionally takes inventories and sales to be first-difference stationary series, and the ratio of the two to be stationary. However, these assumptions do not match the properties of the data for the last two decades in the US and the UK. We offer a model that allows for the non-stationary characteristics of the data, using polynomial cointegration. We show that the closed-form solution has other recent models as special cases. The resulting model performs well and shows good forecasting properties.

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1. *Introduction*

The linear-quadratic (L-Q) model, developed by Holt *et al*. (1960) and Lovell (1961), has been the workhorse of the literature on inventories and stock adjustment (see Blinder and Maccini, 1991, West, 1995, and Ramey and West, 1999). The approach has given rise to the 'production smoothing' class of inventory models, in which inventories act as a buffer between demand and supply. From this approach it is possible to derive a relationship between a firm's stocks and sales, and the relationship can be estimated using time series methods (see *inter alia* Kashyap and Wilcox, 1993, West, 1995, and Ramey and West, 1999). A common feature in models of the sales-inventories relationship, characterized by Kashyap and Wilcox (1993), Rosanna (1995, 1998) and Hamilton (2002), is the requirement that inventories and sales be stationary in first differences. This implies that the series for inventories and sales should each have a single unit root, and a linear combination (i.e. a cointegrating relationship), for which the the ratio of inventories to sales is a special case, should be a stationary series. The dynamic adjustment process could then be written as a straightforward error-correction process in first differenced variables and the cointegrating linear combination. Rosanna (1995, 1998) verifies that a sufficient condition for cointegration between sales and inventories is the assumption that cost shocks are stationary.

A recent paper by Hamilton (2002) notes that the traditional formulation of the L-Q model implies cointegration between inventories and sales. But the traditional interpretation also has some unappealing characteristics, since marginal production and inventory management costs tend to infinity while profits tend to minus infinity under standard assumptions about the driving variables. Hamilton's solution is to consider productivity shocks as an I(1) process and re-configure the system to allow cointegration between sales and productivity shocks, which preserves an identical dynamic structure to the original model. This solution is ingenious since maximum likelihood estimation would result in a value of the likelihood function numerically identical to the original model, the only difference being the interpretation given to the coefficients of the cointegrating relationship i.e. the mapping to the structural parameters of the model.

Both the traditional model and Hamilton's reconfiguration require that a linear combination of inventories-to-sales be stationary. However, examination of the data in Figures 1 and 2 illustrates that during the last two decades there has been a pronounced downward trend in the ratio of inventories to sales in both the US and the UK[4]. This gives rise to the possibility that the linear combination based on the ratio is not stationary, as the majority of empirical papers assume, but is an integrated series. It is well known that allowance for this form of 'non-stationarity' cannot be made by the straightforward incorporation of deterministic time polynomials (see, *e.g.* Phillips 1986, 1987a, 1987b). Nor is there evidence of a cointegrating relationship for other linear combinations of inventories and sales, over the sample period from the early 1980s to the present[5]. Rather the combination of inventories and sales appears to be an I(1) series for any linear combination.

This has two important consequences. First, it explains why forecasting of the inventories component of GDP based on detrended or first differenced data has been poor. Albertson and Aylen (2003 forthcoming) document the relatively poor performance of simple error correction models offered by Cuthbertson and Gasparo (1993) and Bank of England (2000) versus alternatives on UK data. Historically, stockbuilding has been a good leading indicator of GDP growth on an annual basis, but allowing for the decline in the ratio over the 1980s and 1990s has proven to be one of the most difficult components in the forecast. The recent performance of the detrended stocks model shows particularly poor performance against simple alternatives.

Second, the downward trend in the inventories-to-sales ratio undermines the case for a simple reduction in the order of integration through an CI(1,0) error correction system. Inspection of the properties of the data suggest that the series for sales and inventories are in fact I(2) series, and the linear combination is nonstationary since it is I(1)[6]. This implies that a more generalized approach to the dynamic model is required in which we allow for cointegrating systems of the CI(2,1) variety. In this respect, this paper draws on earlier work by Dolado *et al.* (1991) where

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[4] We can offer two potential explanations for this phenomenon, since improvements to stock management processes such as computerized stock control and just-in-time delivery, and the relative decline in the manufacturing sector as a share of GDP (a major contributor to total stockholding), may have reduced inventories (See Cuthbertson and Gasparo (1993), Sensier, (1997), and Mizen (2003), for further analysis).

[5] This is referred to as $(H - qS)$ in the terminology of Ramey and West (1999), where H denotes inventories, S denotes sales and q is a parameter to be estimated.

[6] Earlier investigations of cumulated series by Engle and Yoo (1991), Granger and Lee (1989) and the recent interest in multicointegration (Engsted and Haldrup (1999a), Engsted and

integrated variables are introduced to L-Q models in general form. There are similarities with the applications of the generalized approach of Dolado *et al*. (1991) to labor and money demand equations arising from a L-Q model, reported in Engsted and Haldrup (1994, 1997, 1999b). As far as we are aware, no application has been made to inventories, where the issue is pertinent to the current debate.

In this paper we consider a simple generalization of the cointegrating system offered by Hamilton (2002) – which is itself a representation of the model of Ramey and West (1999) – that can allow for more sophisticated dynamics involving polynomial cointegration (Dolado *et al*., (1991), Engsted and Haldrup (1999a), and Engsted and Johansen, (1999))[7]. In this case we do not require the variables to be I(1), nor the ratio to be stationary. In fact, the variables should have I(2) properties, and the error-correction system will exhibit cointegration between the levels and the differences of the series. We provide a closed-form solution for the dynamic vector error correction model under polynomial cointegration, with Hamilton (2002) as a special case. We illustrate our model using data for the US and the UK, and have some success in modeling the decline in the inventories-to-sales ratio in recent years. Finally we show that such a model is successful in forecasting inventory growth.

We start in the next section by generalizing the interpretation of cointegration in the L-Q model to allow for polynomial cointegration. Section 3 explains how the polynomial approach is implemented. Section 4 reports two illustrative examples using US and UK data and offers some forecasts. Section 5 concludes the paper.

2. A Generalisation of the Cointegration Interpretation

A. Hamilton's I(1) Model

Following Ramey and West (1999), the model used by Hamilton (2002) to illustrate the interpretation of cointegration in the L-Q setting is the decision problem for the representative firm:

(1) $$\max_{\{Q_t, H_t\}_{t=0}^{\infty}} E_0 \left\{ \sum_{i=0}^{\infty} r^i (P_t S_t - C_t) \right\}$$

Johansen (1999)) show that I(2) properties are common in stock-flow models such as inventory-sales relationships.

[7] Since we use a generalized model the I(1) case in Hamilton (2002) is a special case of our approach.

s.t. $$\begin{aligned} C_t &= (\tfrac{1}{2})\left\{a_0(\Delta Q_t)^2 + a_1[(Q_t - U_{ct})^2 + a_2(H_{t-1} - a_4 - a_3 S_{t-1})^2]\right\} \\ Q_t &= S_t + \Delta H_t \end{aligned}$$

where P_t is the price of the good, S_t is unit sales, C_t is the cost of production and Q_t the level of production. H_t is the level of inventories, and the change in inventories is equal to the difference between production and sales by an identity. U_{ct} is a shock to marginal production, and r is the discount rate[8]. Sales follow a random walk with drift, $S_t = S_{t-1} + a_5 + v_{st}$, and the first order condition in this case (setting $a_0 = 0$ for simplicity) is

(2) $$E_t\left[\Delta H_t - S_t - v_{ct} - r(\Delta H_{t+1} + S_t + a_5) + r\, a_2(H_t - a_4 - a_3 S_t - a_3 a_5)\right] = 0$$

And the resulting error correction system when productivity shocks are stationary (i.e. $U_{ct} = v_{ct}$) is

(3) $$\Delta H_t = (l_1 - 1)(H_{t-1} + g_0 - g_1 S_{t-1}) + \frac{g_1 a_5(l_1 - 1)}{1 - l_1 r} + l_1 v_{ct} + (l_1 - 1) g_1 v_{st}$$

where l_1 is the real root of the difference equation and

$$\begin{aligned} g_0 &= -(a_5 / a_2) - a_4 - a_3 a_5 \\ g_1 &= \tfrac{1-r}{r a_2} - a_3 \end{aligned}$$

When productivity shocks are non-stationary (i.e. $U_{ct} = S_t + v_{ct}$) then the system can be written in an almost identical format as

(4) $$\Delta H_t = (l_1 - 1)(H_{t-1} + \tilde{g}_0 - \tilde{g}_1 S_{t-1}) + \frac{\tilde{g}_1 a_5(l_1 - 1)}{1 - l_1 r} + l_1 v_{ct} + (l_1 - 1)\tilde{g}_1 v_{st}$$

but the coefficients are now defined as:

$$\begin{aligned} \tilde{g}_0 &= -a_4 - a_3 a_5 \\ \tilde{g}_1 &= -a_3 \end{aligned}$$

Hamilton's recommendation is that the structural interpretation of the coefficients in the cointegrating vector should follow the second model to avoid the implausible scenario of profits falling to minus infinity.

The model that is explored by Hamilton is ingenious, and proposes a useful re-interpretation of cointegration in the standard L-Q model. However, it makes some simplifications in order to allow the key point of the paper to be illustrated more

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[8] This is representative of the L-Q model reported in Ramey and West (1999).

clearly. For example, the seemingly innocuous simplification of setting the coefficient a_0 to zero, ensures that the dynamic model is simplified, but it also has important drawbacks. First, if adjustment of production were costless (as it would be if a_0 = 0) then the reason for a firm to hold inventories is considerably undermined. An unexpected shock to demand could be met by costlessly adjusting production, and in this case the cost of producing the extra output would simply be the marginal cost of the producing the extra output (which is not the same as the cost of a *change* in output), defined by the second term in the cost function (1) as a_1. If the marginal cost of the extra production amounted to less than the cost of holding inventories (which is positive in the third term of the cost function) then inventories would be eliminated in favor of a direct response to demand shocks from production. Second, the restriction would also imply, if inventories were eliminated, that the firm set marginal revenue (prices) equal to marginal costs. Third, Ramey and West (1999) note that, although a model in which a_0 = 0 is useful for illustrative purposes (Hamilton's main reason for using the restriction), it is 'not a good [assumption] empirically' p889. Recent data suggests that manufacturing firms hold between 20 and 30 weeks of output in the form of inventories, hence we suspect that restricting a_0 = 0 is an inappropriate restriction. We therefore investigate the implications of allowing a_0 ? 0 within this framework.

B. Generalizing Hamilton's Approach to I(2)

Suppose that a_0 ? 0. The cost function (1) can be solved to generate an Euler equation and a dynamic vector error correction representation as before. The Euler equation now becomes

$$(5)\quad E_t\begin{bmatrix} a_0\left\{(\Delta S_t + \Delta^2 H_t) - 2r(\Delta S_{t+1} + \Delta^2 H_{t+1}) + r^2(\Delta S_{t+2} + \Delta^2 H_{t+2})\right\} \\ - ra_1(\Delta H_{t+1} + S_t + a_5 + \Delta S_t) + ra_1 a_2(H_t - a_4 - a_3 S_t - a_3 a_5 - a_3 \Delta S_t) \\ + a_1(\Delta H_t - S_t) - v_{ct}) \end{bmatrix} = 0$$

A feature that differs between our representation and that of Hamilton is that a term in ΔS_t appears in the second and third terms of (5). This is because we specify the driving process for S_t as a driftlesss I(2) process, $\Delta S_t = \Delta S_{t-1} + v_{st}$, which can also be written as $(1-q_1 L)(1-L)S_t = a_5 + v_{st}$ where a_5 = 0 and q_1 = 1. This assumption is an integral part of the polynomial cointegration approach that the generalization of the cost function allows, but recovery of Hamilton's original specification can be achieved if we set $a_0 = 0$, a_5 ? 0 and $q_1 = 0$; therefore Hamilton's version is a special case of our model.

We can then define

$$g_0 = -(a_5 / a_2) - a_4 - a_3 a_5$$
$$g_1 = \tfrac{1-r}{ra_2} - a_3$$
$$g_2 = -a_3 + \tfrac{a_0 - ra_1}{ra_1 a_2}$$

and the polynomial cointegrating relationship is

(6) $$w_t \equiv H_t + g_0 + g_1 S_t + g_2 \Delta S_t$$

We can then make use of this equation to define

$$\Delta^j H_{t+i} = \Delta^j w_{t+i} - g_1 \Delta^j S_{t+i} - g_2 \Delta^{j+1} S_{t+i} \quad \text{for } i, j = 0, 1, 2, ...$$

Substituting out terms in $\Delta^j H_{t+i}$, and making use of the property that terms in $E_t \Delta^j S_{t+i} = E(v_{st+i}) = 0$ for i, j =1, 2,... we can then write the Euler equation as:

(7) $$E_t \left[a_0 \left\{ \Delta^2 w_t - 2r\Delta^2 w_{t+1} + r^2 \Delta^2 w_{t+2} \right\} - ra_1 \Delta w_{t+1} + ra_1 a_2 w_t + a_1 \Delta w_t \right] \\ = a_1 v_{ct} + g_1 (a_0 + a_1) v_{st} - g_2 a_0 (v_{st} - v_{st-1})$$

This can be solved in the same way as Hamilton's problem using the methods for second-order difference equations in Sargent (1987, p 201) since the equation is identical to Hamilton's except that it includes the terms in the first bracket. Thus:

(8) $$E_t \left[a_0 \left\{ (1 - rL^{-1})(1 - rL^{-1})(1-L)(1-L) \right\} w_t + ra_1 \left\{ 1 - \tfrac{1+r+ra_2}{r} L + r^{-1} L^2 \right\} L^{-1} w_t \right] \\ = a_1 v_{ct} + [\, g_1 (a_0 + a_1) - g_2 a_0 (1-L)] v_{st} = y_t$$

Since $ra_1 \left\{ 1 - \tfrac{1+r+ra_2}{r} L + r^{-1} L^2 \right\} = -a_1 (ra_2 + (1 - rL^{-1})(1-L))$ we can factorize the above expression to give:

(9) $$E_t \left[(z^2 - \tfrac{a_1}{a_0} z - \tfrac{ra_1 a_2}{a_0}) w_t \right] = y_t$$

where $z = (1-r^{-1}L)(1-L)$.

If we take $(z^2 - \tfrac{a_1}{a_0} z - \tfrac{ra_1 a_2}{a_0}) = (1 - l_1 z)(1 - l_2 z)$ where 0< l_1 <1 and l_2 >1, for ($l_1 + l_2$) = - a_1/a_0 and $l_1\ l_2 = -\ ra_1a_2/a_0$, the solution to equation (9) is:

(10) $$w_t = l_1 \Delta w_t - r^{-1} l_1 (1-L) w_{t-1} + h_t$$

where $h_t = -l_1 \sum_{i=0}^{\infty} (\frac{1}{l_2})^i y_{t+i}$. This has a natural polynomial cointegration interpretation since (10) can be rewritten as:

$$(11) \qquad \Delta w_t = (1 - l_1)^{-1} [r^{-1} l_1 (1 - L) - 1] w_{t-1} + h_t$$

and replacing the identities in w_t derived from (6) we obtain

$$(12) \qquad \Delta H_t = q_0(L)(H_{t-1} + g_0 + g_1 S_{t-1} + g_2 \Delta S_{t-1}) + q_1(L) \Delta S_t + h_t$$

where $q_0(L) = (1 - l_1)^{-1}[r^{-1} l_1 (1 - L) - 1]$ and $q_1(L) = -[g_1 + g_2(1 - L)]$. Here the first term on the RHS is the polynominal cointegrating relationship between levels and differences of the stocks and sales variables. The requirement for stability of the dynamic system is that $q_0(L) < 0$, which is also a condition that ensures that a polynomial cointegrating relationship exists. In the next section we demonstrate how a polynomial cointegrating relationship can be implemented in practice. The following section will then use inventories and sales data for the US and UK to illustrate the point.

3. Implementing a polynomial cointegration approach

In our two empirical examples there is the possibility that the I(2) variables to cointegrate directly to a stationary variable, however, it is more likely that polynomial cointegrating occurs[9]. That is, the variables may form a linear combination that is integrated of order one, and this may then form a further cointegrating relationship with the first differences, which are also I(1), since the original variables are I(2) in log-levels. This linear combination would then be an I(0) variable and therefore the I(2) variables would be polynomially cointegrated.

Consider a k^{th}-order vector autoregression of the core variables, x_t, of dimension $n \times 1$:

$$(13) \qquad \Delta x_t = \Pi x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Phi D_t + m + e_t$$

where $\Pi = ab'$, m is a constant term that may be unrestricted and D_t is a vector of trends and dummy variables[10]. Equation (13) may be re-written:

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[9] This terminology was established by Yoo (1986), Johansen (1992, 1995b), Gregoir and Laroque (1993, 1994), and Juselius (1998).

[10] The trend and dummy variables may or may not enter the cointegrating space depending on the restrictions imposed during estimation of the system.

(14) $\Delta^2 x_t = \sum_{i=1}^{k-2} \Psi_i \Delta^2 x_{t-i} + \Pi x_{t-1} - \Gamma \Delta x_{t-1} + \Phi D_t + m + e_t$

where $\Psi_i = -\sum_{j=i+1}^{k-1} \Gamma_j$, $i = 1,...k-2$, $\Pi = ab'$ and $\Gamma = I - \sum_{i=1}^{k-1} \Gamma_i$. The variable e_t is a *n*-dimensional vector of errors assumed to be Gaussian with mean vector *0* and variance matrix Σ. The parameters $(\Psi_i, \Pi, \Gamma, \Phi, m, \Sigma)$ are assumed to be variation free.

For a system to be I(2) requires not only that the long-run matrix $\Pi = ab'$ is of reduced rank but that $a'_{\perp} \Gamma\ b_{\perp}$, is also of reduced rank *s* (where the $_{\perp}$ notation refers to the orthogonal complement). $a'_{\perp} \Gamma\ b_{\perp}$ is therefore expressible as $a'_{\perp} \Gamma\ b_{\perp} = xh'$ where x and h are matrices of order $(n-r) \times s$ with $s < n-r$. The I(2) system then can be decomposed into I(0), I(1) and I(2) directions with dimensions r, s and $n-r-s$ respectively. Moreover, the r cointegrating relationships are further decomposable into r_0 directly cointegrating relationships where the levels of the I(2) variables cointegrate directly to an I(0) variable and r_1 polynomially cointegrating relationships where the levels cointegrate with the differences of the levels to give an I(0) variable. Thus:

$b_0' x_t \sim I(0)$ where b_0 is $n \times r_0$ with rank r_0;

$b_1' x_t + k' \Delta x_t \sim I(0)$ where b_1 and k are $n \times r_1$;

$r_0 + r_1 = r$.

It is possible of course for either r_0, r_1 or both to be zero. In general the number of polynomially cointegrating relationships equals the number of I(2) common trends in the system such that $r = r_0 + r_1$ and $r_1 = n - r - s \equiv s_2$. If s_2 equals zero, or equivalently $n - r = s$, the I(2) system collapses to the I(1) case.

4. Two Illustrative Examples

A. Data[11]

To illustrate our point we use quarterly inventory and sales data for the United States and the United Kingdom. For the United States, the measure of inventories is the

[11] The data used in this paper are freely available from us on request.

private inventories (billions of dollars, seasonally adjusted quarterly totals) taken from the BEA database. The series is in logarithms and is deflated to real terms using the inventories deflator (1996=100). We denote the logarithm of real inventories as h_t^{US}. Private Domestic Final Sales (billions of dollars, seasonally adjusted quarterly totals) are obtained from the same source. Final sales of domestic business equals final sales of domestic product less gross product of households and institutions and of general government, and it includes a small amount of final sales by farm and by government enterprises. These data are deflated using the CPI-U for all urban consumers and this variable in logarithms is denoted s_t^{US}. The time span for all the series for the US is 1982:q1 – 2002:q2.

The United Kingdom data, for the time span 1981:q2 to 2001:q2 are constructed by cumulating the flows (seasonally adjusted at constant 1995 prices) on the 1995 real stock figure. The data are provided by the ONS and Bank of England. The proxy for private sales is based on whole economy gross domestic product (seasonally adjusted at constant 1995 prices) less the change in real inventories (corresponding to the identity in equation 1). Again we take logarithms and refer to inventories and sales as h_t^{UK} and s_t^{UK} respectively.

These definitions were the basis for a comparison of inventories-to-sales ratios for the US and the UK published in the May 2002 *Inflation Report* (Bank of England, 2002), and they were reproduced in the introduction to this paper as Figures 1 and 2 for the sample 1982 q1– 2001q2. We refer to the inventories-to-sales ratio for the US and the UK as R_t^{US} and R_t^{UK} , respectively. The pronounced downward trend in both R_t^{US} and R_t^{UK} after 1980 suggests that the ratio is not a stationary variable, but rather it appears to be integrated of order one. When we investigate the properties of the data for both US and UK we can confirm that the series for sales and inventories are stationary only after differencing *twice* (i.e. they are I(2)), and the ratio is stationary in *first* differences, hence it is I(1)[12].

B. Estimating a Polynomially Cointegrating System

The results, taken from the estimates of an I(2) system, are described for the US and UK data. There are several ways of specifying the system to allow for a constant, dummies and a trend. The system is estimated without any restrictions

[12] The unit root tests are available from the authors on request.

except for the exclusion of quadratic trends, and is reported for both the US and the UK with and without a trend in the cointegrating space. Estimation is undertaken with four lags for the US and five lags for the UK. A single significant impulse dummy is inserted in the UK systems in 1998q4 (to obtain better diagnostics) while none is needed in the US systems.

Table 1 shows that r = 1, s = 0, and $n-r-s=1$ is the chosen cell for both the US and UK cases when we compare the joint trace test statistics against the computed critical values for the 95% quantile taken from Paruolo (1996) and Rahbek *et al.* (1999). Since the number of I(2) trends in the model equals the number of polynomially cointegrating relationships, the arithmetic implies that the only cointegrating relationship detected above must be of the polynomially cointegrating variety. There is no I(1) common trend $(s=0)$ and one I(2) common trend $(n-r-s=1)$, suggesting that there ought to be two unit roots in the companion matrix. The five largest roots in modulus of the characteristic polynomial are 1.0000, 0.9066, 0.7927, and 0.7927 for the US (with the last two being complex conjugates) and 1.0000, 0.9196, 0.9196, 0.7703, 0.7546 for the UK. While the third root for the UK is also close to unity, we proceed under the maintained assumption of one cointegrating vector and one I(2) trend.

An important difference between the two models is that the US system marginally supports a trend in the cointegrating space, although when we report the results for the US systems with and without a trend in the cointegrating space in Table 2 we do not find that the properties of the system are dramatically altered. The test statistic is $c^{2}(1)$ = 3.86, p-value = 0.05. This observation suggests that dealing with the downward trend in the inventories-to-sales ratio by including a deterministic trend process in the model will not restore stationarity.[13]

The results of the I(2) system estimation deliver a normalized cointegrating vector b_1' and the polynomial component based on the parameter estimates, k', with the following representations:

US: $h_{\mathrm{t}}^{\mathrm{US}} = 1.333 s_t^{US} - 2.967 \Delta h_t^{US} - 3.337 \Delta s_t^{US}$

UK: $h_{\mathrm{t}}^{\mathrm{UK}} = 0.471 s_t^{UK} + 0.924 \Delta h_t^{UK} + 1.963 \Delta s_t^{UK}$

where the superscript distinguishes the US from the UK results.

Estimation of the I(2) system shows that the I(1) direction of the data is given by the vectors $b_1' \equiv (1,-a)$, thus the vector $b_3' \equiv (1,1/a)$ provides the I(2) direction. These vectors are orthogonal to each other and, the first lies in the space orthogonal to b_3'. A basis for this space is given by the matrix $H = \begin{pmatrix} 1 \\ -a \end{pmatrix}$. Thus $\begin{pmatrix} H'x_t \\ b'\Delta x_t \end{pmatrix}$, where a is any 2 × 1 vector that satisfies the restriction that $b'b_3 \neq 0$, provides the transformation to I(1) which keeps all the cointegrating and polynomially cointegrating information. Hence if we take b to be $(0,1)'$, then the bivariate system given by the I(1) representation $\begin{pmatrix} h_t - a\,s_t \\ \Delta s_t \end{pmatrix}$ is a valid full reduction. We define $x_t = (h_t, s_t)'$ in this notation.

To investigate the properties of the system in an I(1) format we re-estimate the model and report the findings in Table 3. There is a single cointegrating relationship reported for the ratio of inventories-to-sales using both US and UK data, which is confirmed in the first four roots in modulus of the companion matrix being given by 1.0000, 0.6931, 0.1975, 0.1975 (US) and 1.0000, 0.9114, 0.9114, 0.8193 (UK). The error correction terms are calculated from the cointegrating matrix b' in this system and are given by

$$ECM_t^{US} \equiv (h_t^{US} - 1.333 s_t^{US} + 0.003t) + 4.486 \Delta s_t^{US}$$

and

$$ECM_t^{UK} \equiv (h_t^{UK} - 0.471 s_t^{UK}) + 4.318 \Delta s_t^{UK} .$$

We can reject the null that the coefficient on sales should be restricted to one, but the linear combination of the I(2) variables is polynomially cointegrated with the change in sales. The coefficient on the change in sales is a similar magnitude in both the UK and the US cases. Economically, this relationship represents a stable long-run relation between the log-levels of stocks and sales that has a positive response to sales growth.

Table 4 reports the results from estimating the inventories and sales equations in a system, using the dynamic error correction mechanisms given above. The system can be specified rather easily for the US, although with long lags of the second-

[13] For the UK the trend can be tested out of the cointegrating space. The results for the UK are

difference variables in the equation for inventories. For the UK, the autocorrelation structure of the data requires the incorporation of either (I) the inclusion of spike dummies at the quarters indicated in the table or (II) long lags of the second-differenced variables. The results of estimating the models (under the two alternative strategies) are reported as Model (I) and Model (II) respectively in Table 4. The choice between the two could be based either on forecasting performance or a preference for models without structural shift dummies. The forecasting performance of all three models are discussed below.

Excluding the insignificant variables in Table 4 on a 5 per cent *t*-criterion the final form of the inventories and sales equations in the system can be represented (with the length of the maximal lag for each variable (for either country) indicated in each case) as:

$$(15)\quad \Delta^2 h_t = m_1 - a_1\left(h+q_1 s+q_2\,\Delta s\right)_{t-1} + \sum\nolimits_{i=1}^{7} t_{1i}\Delta^2 s_{t-i} + \sum\nolimits_{i=1}^{8} t_{2i}\Delta^2 h_{t-i} + f_1' D_t + e_{1t}$$

$$(16)\quad \Delta^2 s_t = m_2 - a_2\left(h+q_1 s+q_2\,\Delta s\right)_{t-1} + \sum\nolimits_{i=1}^{7} t_{3i}\Delta^2 s_{t-i} + \sum\nolimits_{i=1}^{4} t_{4i}\Delta^2 h_{t-i} + f_2' D_t + e_{2t}$$

The estimated system as represented by (15) and (16) describes an economy where disequilibrium from the long-run relationship is corrected by changes in sales over a horizon not longer than two years (eight quarters). The system is specified as a parsimonious vector error correction model, with the polynomial error correction term playing a significant role in both equations. The equations are well specified and have satisfactory diagnostics for both US and UK data.[14]

C. Forecasting

We have noted above the fact that simple error correction models of the kind offered by Cuthbertson and Gasparo (1993) and the Bank of England (2000), which do not allow for dynamic or polynomial cointegration, do not perform well for the UK in many forecasting exercises. We hypothesize that this is caused by an inadequate

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therefore presented without a trend included.

[14] For the sake of concision Table 4 reports only the systems diagnostics. Individual-equation diagnostics which are also available from the authors on request indicate proper specification in all cases.

parameterization of the equilibrium relation or, in other words, by a mis-specification of the adjustment process.[15] In this section we verify the gains in forecasting obtainable from our more general form of modeling, and reject the possibility that restrictions that would result in a simple error correction model are valid on forecasting grounds.

In order to conduct the forecasting exercise and comparison, our model for the UK is estimated recursively, starting with a base sample of 1982:1 – 1995:4. At each recursion, the estimated coefficients are used to compute the forecast from our model for time l-periods ahead, where l is four or eight (i.e. up to 1996:4 in this particular example for l=4). This is repeated (augmenting the sample by one sample point at each stage and re-estimating the model, to derive a *sequence* of 4-step or 8-step ahead forecasts) from which the root mean squared forecast error (RMSE) is calculated by comparison of the forecast with the actual numbers for these series. The exercise is repeated for the US.

Table 5 provides the results for the US, while Table 6 gives the corresponding results for the UK in the two versions of the models. Figures 3 and 4 provide the same information graphically. The root mean-squared error of the forecasts for l = 4 for both the US and both UK models is within one standard deviation of the change in inventories while it is about one and a half standard deviations for l=8 for the US and UK Model II. The root mean squared error for UK Model I for l=8 is again within one standard deviation of the change in inventories.

Turning points are successfully picked up, emphasizing the usefulness of this methodology in modeling the dynamics, and the method appears to be relatively robust to the presence of structural breaks which are likely to be very important within this time period. There is some evidence of the need for an intercept correction in the US-forecast series which would bring the forecasts back on track and reduce the root mean squared error even further. The success of our forecasting model offers further evidence in favor of a polynomially cointegrated approach to modeling inventories and sales.

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[15] We were unable to replicate the dynamic features of the equation reported for the inventories-sales relationship in Bank of England (2000). Although the long-run relationship (despite our view that this is misspecified) could be recovered by means of a Engle-Granger static regression, we were not able to estimate the short run relationship. The principal reason for this is the fact that the real time data, which we have tried our best to acquire, has been retrospectively revised and combines vintages of data which are unknown and unavailable to us. Therefore we cannot expect to replicate the results

5. *Conclusions*

We have argued the case in this paper for a much richer modelling framework for stocks and sales. We offer a theoretical model that allows for the non-stationary characteristics of the data, using polynomial cointegration., and we show that the closed-form solution has other recent models as special cases. The resulting I(2) model performs well when put to the test on UK and US data relating to inventories and sales and forecasts better than the existing models.

Our empirical modelling strategy is derived directly from theoretical considerations. Indeed the possibility of casting the problem within an I(2) framework arises directly from the solution to the optimisation problem and estimating the polynomially cointegrating relationship may be seen as the empirical analogue of the theoretical solution. Viewed in this light, our work offers well-specified dynamic models in accord with the time series properties of the data, and better forecasts result. Our approach fits in with existing theoretical interpretations presented not only here but also in the general formulation reported by Dolado *et al*. (1991). It also marks the way forward for modelling inventory-sales relations extending Ramey and West's and Hamilton's excellent contributions in this area.

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on the same data span. Taking the short run equation as given, however, we find that the forecasting performance of the Bank of England equation is very weak in comparison with our own equations.

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Figures 1 and 2

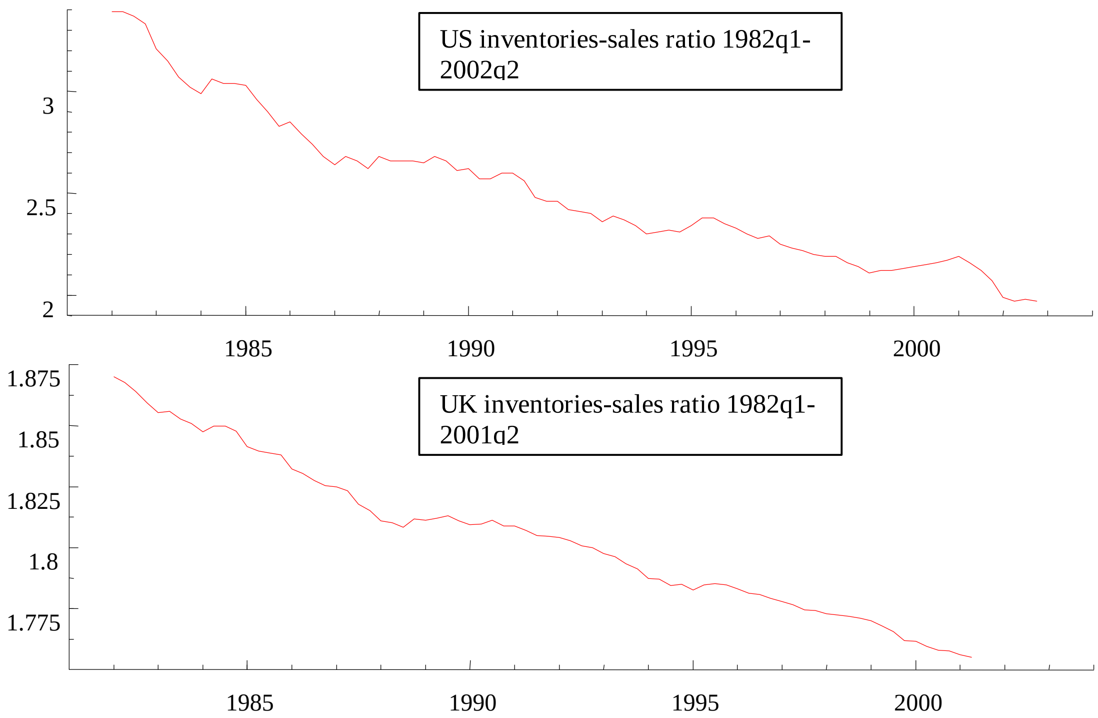

Figure 3: Forecasting the change in inventories for the US

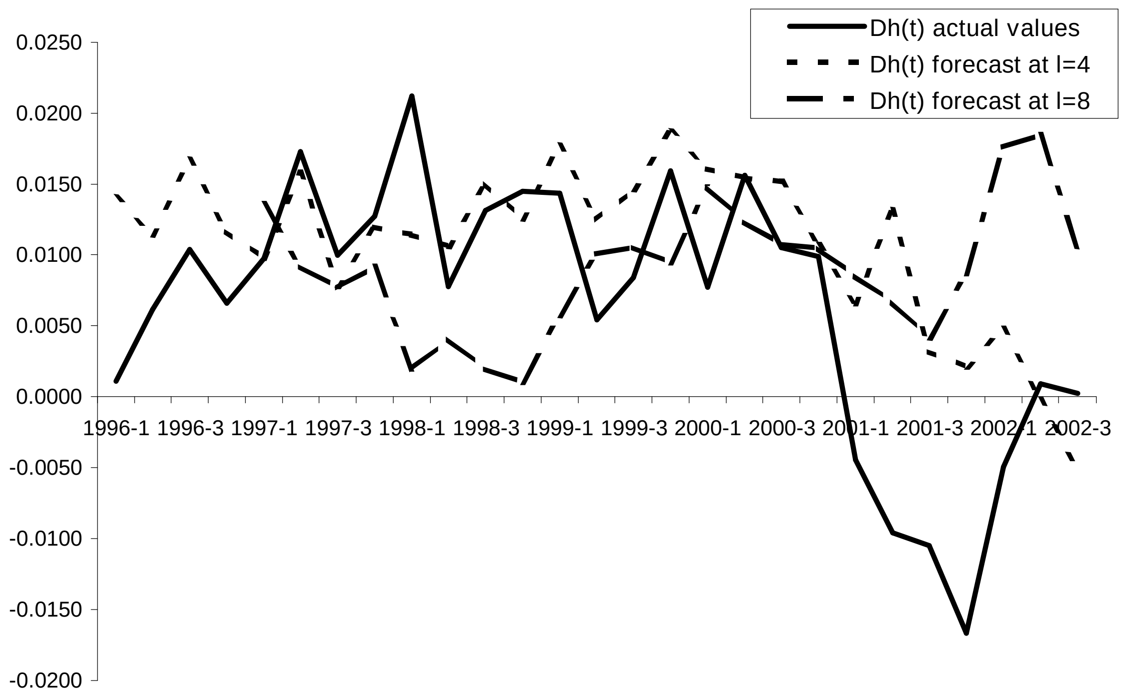

Figure 4a: Forecasting change in inventories for the UK -Model I

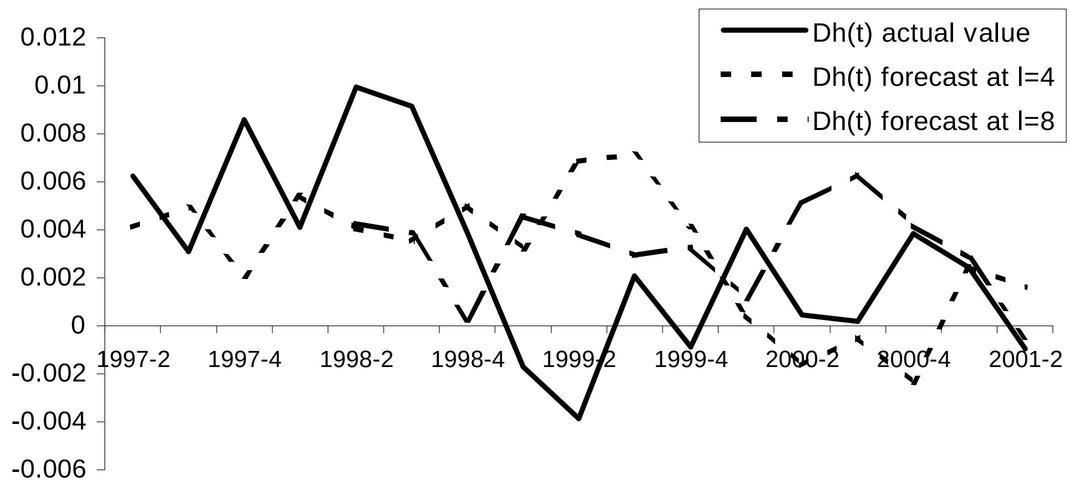

Figure 4b: Forecasting change in inventories for the UK -Model II

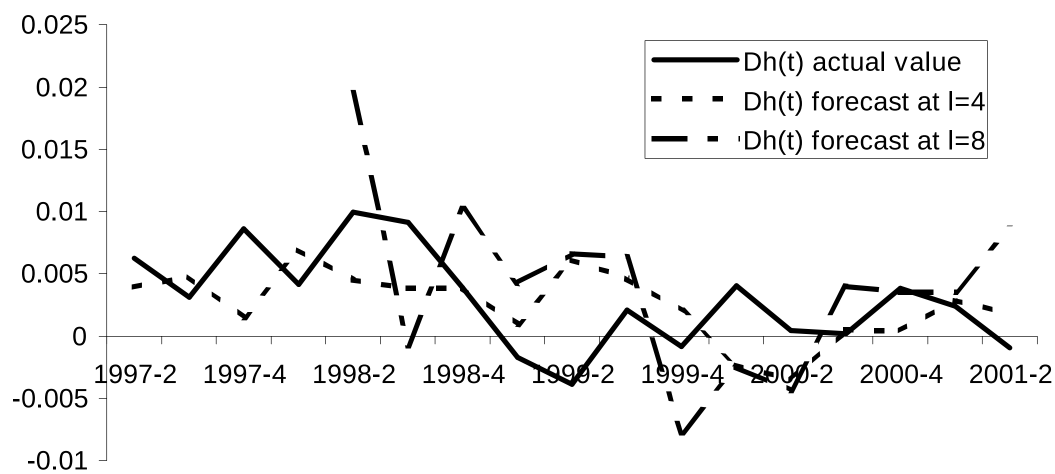

Table 1: Polynomial Cointegration in an I(2) System

(a) US results

(i) with trend

$n-r$	r			Q(r)
2	1	64.15	35.40	27.46
1	2		15.02	8.30
$n-r-s$		2	1	0

(ii) without trend

$n-r$	r			Q(r)
2	1	53.15	24.40	16.47
1	2		5.99	1.16
$n-r-s$		2	1	0

(b) UK results

(ii) without trend

$n-r$	r			Q(r)
2	1	48.72	25.04	18.55
1	2		12.22	1.80
$n-r-s$		2	1	0

Notes: Statistics are computed with 4 lags (US) and 5 lags (UK). The estimation sample is 1983q1 – 2002q2 (78 observations, 69 deegrees of freedom) for the US and 1982q3 to 2001q2 (76 observations, 64 degrees of freedom) for the UK. Critical values (95%) for the Joint Trace Test *S(s, r)* are (with trend) from Rahbek *et al.* (1999), reported in roman, and (without trend) from Paruolo (1996), reported in italics.

$n-r$	r			
2	1	47.60 *36.12*	34.40 *22.60*	25.40 *15.34*
1	2		19.90 *12.93*	12.50 *3.84*
$n-r-s$		2	1	0

Table 2: System Diagnostics for the Polynomial Cointegration Models

(a) United States

Tests for Serial Correlation	
Ljung-Box (19)	c^2 (62) = 47.985, p-value = 0.90
LM(1)	c^2 (4) = 5.411, p-value = 0.25
LM(4)	c^2 (4) = 2.729, p-value = 0.60
Test for Normality:	
Doornik-Hansen Test	c^2 (4) = 6.129, p-value = 0.19

(b) United Kingdom

Tests for Serial Correlation	
Ljung-Box (23)	c^2 (195) = 78.315, p-value = 0.04
LM(1)	c^2 (4) = 1.403, p-value = 0.84
LM(4)	c^2 (4) = 3.695, p-value = 0.45
Test for Normality:	
Doornik-Hansen Test	c^2 (4) = 7.638, p-value = 0.11

Table 3: Cointegration in an I(1) System

US Results

Null $H_O : r$	Eigenvalues	Estimated L-Max Statistic	Estimated Trace Statistic Q(r)	$p - r$
0	0.3287	31.88	33.86	2
1	0.0244	1.98	1.98	1

b	h_t^{US}-as_t^{US}	Δs_t^{US}
(normalised coefficients)	1.000	-4.486

a	$\Delta(h_t^{US}$-$as_t^{US})$	$\Delta^2 s_t^{US}$
Coefficients	-0.184	0.104
t-values	-6.191	5.877

UK Results

Null $H_O : r$	Eigenvalues	Estimated L-Max Statistic	Estimated Trace Statistic Q(r)	$p - r$
0	0.2845	25.88	29.38	2
1	0.0488	3.49	3.49	1

b	h_t^{UK}-as_t^{UK}	Δs_t^{UK}
(normalised coefficients)	1.000	-4.318

a	$\Delta (h_t^{UK}$-$as_t^{UK})$	$\Delta^2 s_t^{UK}$
Coefficients	-0.105	-0.014
t-values	-4.649	-0.728

Notes: Statistics are computed with 4 lags (US) and 5 lags (UK). The estimation sample is 1983q1 – 2002q2 (78 observations, 69 deegrees of freedom) for the US and 1982q3 to 2001q2 (76 observations, 64 degrees of freedom) for the UK.

Table 4: The Dynamic System

	US results		UK results (Model I)		UK Results (Model II)	
	$\Delta^2 h_t^{US}$	$\Delta^2 s_t^{US}$	$\Delta^2 h_t^{UK}$	$\Delta^2 s_t^{UK}$	$\Delta^2 h_t^{UK}$	$\Delta^2 s_t^{UK}$
$\Delta^2 h_{t-1}$	-0.288 (0.103)	0.195 (0.111)	-0.557 (0.081)	-0.156 (0.058)	-0.572 (0.112)	-0.161 (0.081)
$\Delta^2 h_{t-2}$	-0.210 (0.106)		-0.358 (0.090)		-0.378 (0.127)	-0.091 (0.082)
$\Delta^2 h_{t-3}$	-0.252 (0.109)		-0.430 (0.086)	-	-0.346 (0.115)	
$\Delta^2 h_{t-4}$	-0.277 (0.101)			-		
$\Delta^2 h_{t-5}$	-0.339 (0.097)		-0.177 (0.068)		-0.203 (0.098)	
$\Delta^2 h_{t-8}$	-0.173 (0.085)					
$\Delta^2 s_{t-1}$		-0.472 (0.118)	0.503 (0.111)	-0.412 (0.091)	0.588 (0.161)	-0.349 (0.122)
$\Delta^2 s_{t-2}$	0.416 (0.096)	-0.286 (0.092)		-0.371 (0.097)	0.282 (0.177)	-0.352 (0.117)
$\Delta^2 s_{t-3}$	0.303 (0.124)		0.445 (0.100)	-0.264 (0.087)	0.740 (0.184)	-0.158 (0.107)
$\Delta^2 s_{t-4}$	0.365 (0.129)		-	-0.191 (0.082)	0.373 (0.179)	
$\Delta^2 s_{t-5}$	0.232 (0.134)				0.279 (0.160)	
$\Delta^2 s_{t-6}$	0.341 (0.129)				0.207 (0.133)	0.163 (0.091)
$\Delta^2 s_{t-7}$	0.265 (0.102)					0.074 (0.089)
ECM_{t-1}	-0.064 (0.016)	0.112 (0.021)	-0.049 (0.022)	-0.071 (0.017)	-0.056 (0.029)	-0.066 (0.022)
D831				0.015 (0.003)		
D842				-0.015 (0.004)		
D851				0.011 (0.004)		
D873				0.012 (0.004)		
D884			0.017 (0.004)			
D894			-0.017 (0.005)			
D903			-0.011 (0.005)			
D952				-0.011 (0.004)		
D962			-0.015 (0.005)	-0.003 (0.004)		
Constant	0.053 (0.013)	-0.093 (0.017)	0.088 (0.041)	0.125 (0.030)	0.100 (0.051)	0.117 (0.040)

Standard errors in parentheses

System Diagnostics

US	$\Delta^2 h_t^{US}$	$\Delta^2 s_t^{US}$
AR(1-5)	F(5,56) = 2.228 [0.06]	F(5,56) = 1.443 [0.22]
ARCH(1-4)	F(4, 62) = 0.817 [0.52]	F(4, 62) = 0.539 [0.71]
Jarque-Bera Normality (2)	$c^2(2)$ = 0.047 [0.98]	$c^2(2)$ = 2.537 [0.28]
Heteroskedasticity test	F(34, 35)= 0.807 [0.73]	F(34, 35)= 0.770 [0.77]
UK (MODEL I)	$\Delta^2 h_t^{UK}$	$\Delta^2 s_t^{UK}$
AR(1-5)	F(5,50) = 1.903 [0.11]	F(5,50) = 2.369 [0.05]
ARCH(1-4)	F(4, 55) = 0.501 [0.73]	F(4, 55) = 0.190 [0.94]
Jarque-Bera Normality (2)	$c^2(2)$ = 1.581 [0.45]	$c^2(2)$ = 0.707 [0.70]
Heteroskedasticity test	F(31, 31)=1.14 [0.36]	F(31, 31)= 0.768 [0.77]
UK (MODEL II)	$\Delta^2 h_t^{UK}$	$\Delta^2 s_t^{UK}$
AR(1-5)	F(5,58) = 2.331 [0.05]	F(5,50) = 2.364 [0.05]
ARCH(1-4)	F(4, 60) = 1.254 [0.298]	F(4, 60) = 2.608 [0.04]
Jarque-Bera Normality (2)	$c^2(2)$ = 2.818 [0.24]	$c^2(2)$ = 0.329 [0.85]
Heteroskedasticity test	F(31, 31)=0.686 [0.85]	F(31, 31)= 1.459 [0.136]

p-values are in square parentheses

Table 5: Forecast performance of Banerjee and Mizen model (as reported in Table 4) for US

		l=4		*l*=8	
	Δh_t^{US}	$\Delta \hat{h}_t^{US}$	error	$\Delta \hat{h}_t^{US}$	error
1996-1	0.0011	0.0141	-0.0130		
1996-2	0.0062	0.0114	-0.0052		
1996-3	0.0104	0.0167	-0.0063		
1996-4	0.0066	0.0117	-0.0051		
1997-1	0.0098	0.0097	0.0000	0.0136	-0.0038
1997-2	0.0173	0.0158	0.0015	0.0092	0.0081
1997-3	0.0100	0.0078	0.0022	0.0077	0.0023
1997-4	0.0127	0.0120	0.0008	0.0092	0.0035
1998-1	0.0212	0.0114	0.0098	0.0019	0.0193
1998-2	0.0078	0.0106	-0.0028	0.0040	0.0037
1998-3	0.0131	0.0150	-0.0019	0.0020	0.0112
1998-4	0.0145	0.0125	0.0020	0.0010	0.0135
1999-1	0.0144	0.0177	-0.0034	0.0057	0.0086
1999-2	0.0054	0.0125	-0.0071	0.0101	-0.0046
1999-3	0.0084	0.0146	-0.0062	0.0105	-0.0021
1999-4	0.0159	0.0187	-0.0028	0.0094	0.0065
2000-1	0.0077	0.0161	-0.0084	0.0148	-0.0071
2000-2	0.0156	0.0154	0.0002	0.0123	0.0033
2000-3	0.0105	0.0152	-0.0047	0.0107	-0.0002
2000-4	0.0099	0.0107	-0.0008	0.0105	-0.0006
2001-1	-0.0045	0.0065	-0.0110	0.0085	-0.0130
2001-2	-0.0096	0.0132	-0.0228	0.0067	-0.0163
2001-3	-0.0105	0.0032	-0.0137	0.0041	-0.0146
2001-4	-0.0167	0.0021	-0.0188	0.0087	-0.0253
2002-1	-0.0050	0.0048	-0.0098	0.0176	-0.0226
2002-2	0.0009	-0.0002	0.0011	0.0185	-0.0176
2002-3	0.0002	-0.0051	0.0053	0.0105	-0.0103
RMSE		0.0084		0.0118	

Table 6a: Forecast performance of Banerjee and Mizen Model I (as reported in Table 4) for UK

		l=4		l=8	
	Δh_t^{UK}	$\Delta\hat{h}^{UK}$	Error	$\Delta\hat{h}^{UK}$	error
1997-2	0.006255	0.00411	0.005274		
1997-3	0.003109	0.004935	0.002394		
1997-4	0.008599	0.002075	-0.00195		
1998-1	0.004125	0.005425	-0.00163		
1998-2	0.009964	0.004056	-0.00099	0.004268	-0.0012
1998-3	0.009151	0.003528	0.007298	0.003873	0.006953
1998-4	0.003869	0.005027	0.001228	0.00026	0.005995
1999-1	-0.00171	0.003212	-0.0001	0.004562	-0.00145
1999-2	-0.00385	0.006871	0.001728	0.003828	0.004771
1999-3	0.002094	0.007145	-0.00302	0.002943	0.001182
1999-4	-0.00086	0.004137	0.005827	0.003289	0.006675
2000-1	0.00404	0.000447	0.008704	0.001151	0.008
2000-2	0.000448	-0.00163	0.005504	0.0051	-0.00123
2000-3	0.000183	-0.00048	-0.00123	0.006324	-0.00804
2000-4	0.003846	-0.00237	-0.00149	0.00419	-0.00804
2001-1	0.002424	0.002473	-0.00038	0.002818	-0.00072
2001-2	-0.00095	0.0016	-0.00246	-0.00046	-0.0004
RMSE		0.0039		0.0052	

Table 6b: Forecast performance of Banerjee and Mizen Model II (as reported in Table 4) for UK

		l=4		l=8	
	Δh_t^{UK}	$\Delta\hat{h}^{UK}$	Error	$\Delta\hat{h}^{UK}$	error
1997-2	0.006255	0.003916	0.005468		
1997-3	0.003109	0.004779	0.00255		
1997-4	0.008599	0.001499	-0.00137		
1998-1	0.004125	0.006939	-0.00315		
1998-2	0.009964	0.004532	-0.00147	0.019551	-0.01649
1998-3	0.009151	0.003822	0.007004	-0.0008	0.011624
1998-4	0.003869	0.003872	0.002383	0.010234	-0.00398
1999-1	-0.00171	0.000958	0.002151	0.004234	-0.00112
1999-2	-0.00385	0.006105	0.002493	0.006624	0.001974
1999-3	0.002094	0.004655	-0.00053	0.006357	-0.00223
1999-4	-0.00086	0.002047	0.007917	-0.00776	0.017729
2000-1	0.00404	-0.00235	0.011503	-0.00249	0.011645
2000-2	0.000448	-0.00351	0.007376	-0.00437	0.008237
2000-3	0.000183	0.00048	-0.00219	0.003975	-0.00569
2000-4	0.003846	0.000465	-0.00432	0.003545	-0.0074
2001-1	0.002424	0.002878	-0.00078	0.003553	-0.00146
2001-2	-0.00095	0.001796	-0.00266	0.008641	-0.0095
RMSE		0.0048		0.0093	