EUROPEAN UNIVERSITY INSTITUTE
DEPARTMENT OF ECONOMICS

EUI Working Paper **ECO** No. 2003/21

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Published in Italy in October 2003
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*Revisions of this work can be found at: http://www.iue.it/ECO/JobMarket/CVs/renou.html

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October 13, 2003

*I owe Pierpaolo Battigalli and Larry Samuelson a special debt for their insightful comments and encouragements. I am also grateful to Bart Lipman, Jean-Marc Tallon, and Rabah Amir for helpful discussions. Special thanks to Guillaume Carlier for insightful discussions on Index Theory. I gratefully acknowledge the hospitality of UW-Madison where part of this research has been conducted.

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Abstract

The paper generalizes a costly state verification model along two dimensions: 1) diversity of opinions, and 2) endogenous formation of a financial intermediary, modeled as a multi-lender coalition. In contrast with previous contributions (e.g. [35] and [36]), our model can account for the coexistence of financial intermediation and direct lending (a non-trivial equilibrium). We prove the existence of such non-trivial equilibria and provide a complete characterization of them. Under some parameter conditions, the stronger the diversity of opinions, the smaller the expected coalition size.

Keywords : costly state verification, higher-order beliefs, multi-lender coalition, heterogenous prior, incomplete information.

JEL Classification Numbers : C7, D8, G2.

1 Introduction

The main purpose of this paper is to show that a costly state verification (henceforth, CSV) model with heterogenous and privately known beliefs can account for the coexistence of a financial intermediary and direct investors. This sharply contrasts with previous contributions (see [35] and [36]), where financial intermediation drives direct lending out of the system in equilibrium. In his seminal paper, Townsend [31] defines a CSV problem as a situation where some economic agents can freely observe the realization of a random variable while others have to pay a cost. Contracting problems between workers and firms ([12], [13]), insurers and insurees ([4], [25], [29]) or investors and corporations ([6], [7], [10], [14], [18], [35]) are a few examples of a CSV problem. Besides the CSV problem, a common assumption of all these models is the *common prior assumption*,[1] that is to say, all agents are assumed to share the same beliefs about the realizations of the random variable. Is this a palatable assumption? It is certainly reasonable to assume that an insuree and his insurer disagree on the likelihood of possible damages or that a firm disagrees with his outside financiers on the return of a new product. Moreover, "*some of the disagreement we see around us is neither due to dishonesty, nor error in reasoning, nor to friction in communication*" but rather to "*different personalistic views*" (Savage [30, p67]). In this paper, we fully subscribe to Savage's view in that differences in probabilities should *not only* reflect differences in information.

In the present paper, we focus on a CSV problem between investors and firms. Two important results are found in the literature. First, a CSV model provides a rationale for debt-like financial contracts with costly bankruptcy, like debentures[2] or corporate bonds, increasingly used by corporations (see, for instance, [14], [20], [35], [36]). Second, financial

[1] See [26] and references therein for an excellent survey on the common prior assumption. In fact, the CPA says that players' hierarchies of beliefs can be derived via conditioning from a common prior on the set of states of the world. This is more general than the assumption that players' beliefs about the external states are derived from a common prior.

[2] A debenture is a fixed-interest security issued by limited companies in return for a loan. Debenture interest must be paid whether the company makes a profit or not. In the event of non-payment, debentures holders can force liquidation. It is also worth noting that [8] shows how we can accommodate a CSV model to account for both financing by debt and equity.

intermediaries dominate markets in the allocation of saving to investment in CSV models (see, for instance, [11], [21], [35], [36]). Indeed, duplication of effort occurs in equilibrium with direct lending when intermediation is not permitted, in that each firm borrows from several lenders, and each of these lenders monitors in the case of default. A financial intermediary borrowing from a large number of lenders and lending to a large number of firms eliminates this duplication. Or in other words, since all agents agree on the investment decision to be taken, an intermediary playing the role of a *delegated monitor* behaves exactly as a single agent would behave, and in addition economizes on monitoring costs. Thus, direct lending does not coexist with financial intermediation in the equilibrium. But this theoretical result contradicts casual observations: Financial intermediation and direct lending do coexist in the real world. Now suppose that lenders have *heterogenous and privately known beliefs*. This implies that the choice of a delegated monitor does not necessarily agree with the choice that a direct investor would make, and thus there exists a trade-off between economizing on monitoring cost and disagreeing with the delegated monitor. A lender may then choose to invest directly instead of participating in a financial intermediary. The present paper focuses on this trade-off.

Formally, we extend a standard CSV model along two dimensions: 1) diversity of opinions and 2) endogenous formation of a financial intermediary, modeled here as a multi-lender coalition. To represent the diversity of opinions, we assume that no player (investor or borrower) knows the opponents' beliefs about the project return (first-order beliefs), and forms beliefs about the opponents' beliefs (second-order beliefs). This modeling of interactive beliefs is the simplest formulation that captures the natural idea that beliefs are *heterogenous and private knowledge*. The second extension, the endogenous formation of a financial intermediary, consists of adding a new stage to a standard CSV model: After observing the contract proposed by a unique entrepreneur, and before accepting or rejecting this contract, each investor can either participate in a unique multi-lender coalition (financial intermediation) or stand-alone (direct lending).

We show that this generalized model encompasses the CSV model analyzed in [35] as a limit case. Indeed, if all first-order beliefs are equal to the same belief, say p, and second-order beliefs are degenrate in $(p, ..., p)$, we are back to the CPA case where the results of

[35] hold. Besides this very special case, the model can account for the coexistence of a multi-lender coalition and stand-alone coalitions, a so-called non-trivial equilibrium. We prove the existence of such non-trivial equilibria and provide a complete characterization of them. In particular, any symmetric equilibrium is characterized by two thresholds $\underline{p}$ and $\overline{p}$, i.e., a most pessimistic belief and a most optimistic belief, such that any belief in-between participates in the coalition. Thus if $0 < \underline{p}$ and/or $\overline{p} < 1$, extreme beliefs stand alone. We also derive conditions under which the higher the heterogeneity of beliefs, the lower the expected coalition size i.e., more lenders are expected to invest directly. Finally, we show that the equilibrium contract offered by the borrower is such that pessimistic beliefs of any stand-alone lender do not finance the project. This result is essential since, otherwise, any belief of any lender finances the project, hence the potential disagreement among lenders is removed and only the grand coalition forms.

The closest contribution to ours is found in Allen and Gale [1] (henceforth, AG). In their model, lenders have different opinions about projects' returns and can either directly participate in financial markets, or alternatively, join a coalition (a bank) where the decision to invest in projects is delegated to a manager. By delegating to a manager, on the one hand an intermediary avoids the multiplication of information costs while, on the other, even if a manager does his best to choose projects he honestly believes profitable, some lenders might strongly disagree with his choice, and thus prefer to participate directly in financial markets. Clearly, this result would not hold if the CPA were imposed. However, our model departs from AG's model in two important ways. First, while AG assume passive borrowers, we consider an active borrower offering a debt contract to lenders. This seemingly innocuous modification introduces interesting interactions between the contracting and the coalition formation stage. Second, AG assume that forming a coalition requires the acceptance of all lenders. Hence, they cannot account for the coexistence of financial intermediation and direct lending, as our model does.

The remainder is organized as follows. Section 2 presents the CSV model. Sections 3 and 4 contain the main results: Section 3 analyses the endogenous formation of the multi-lender coalition, and section 4 solves the financial contracting problem. Finally, we briefly discuss possible extensions in section 5. The Appendix collects proofs.

2 A generalized CSV model

We consider a static, two-period economy with a single good used for both investment and consumption. There are $N+1$ agents in the economy: a unique borrower and N lenders. The borrower has no initial endowment, but has access to an investment project, described below. Each lender, indexed by $i \in I = \{1, \ldots, N\}$, is endowed with one unit of the investment good in the first period. All agents are assumed to be risk-neutral and to care only about second period consumption.

Investment can only occur in the first period using one or more of the following two technologies. First, there is a *commonly available, safe technology,* whereby one unit invested in the first period yields $r \geq 1$ unit(s) of output in the second period. Second, there is a *stochastic technology*, which converts current investment into future output. This risky project is of variable size, that is, there is no bound on the project feasibility. It produces ω units in period two per unit invested at the first date, where ω is a realization of the random variable $\widetilde{\omega}$. The probability law of $\widetilde{\omega}$ is the *unknown* cumulative distribution function G and we assume that G is parametrized by $\theta \in \{L, H\}$, which is *non verifiable ex-post.* We can interpret H as a state of boom and L as a state of bust. Lastly, we denote the cumulative distribution function parametrized by θ by $G(\cdot \mid \theta)$.

Assumption 0: For $\theta \in \{L, H\}$, the distribution $G(\cdot \mid \theta)$ has support $[a, b] \subset \mathbb{R}_+$ and admits the probability density $g(\cdot \mid \theta)$.

Assumption 1: $G(\cdot \mid H)$ dominates $G(\cdot \mid L)$ in the sense of *strong first order stochastic dominance* i.e., for all $\omega \in (a, b)$,

$$G(\omega \mid L) > G(\omega \mid H).$$

Additionally, there exists a technology which can be used by any lender to verify the realization ω of the project. This state verification technology is costly to use, requiring a utility cost of γ in the second period. It is common to interpret this monitoring cost as the cost to force a firm into bankruptcy such as lawyers' fees, cost to evaluate the value of the firm or resources needed to find a new firm manager (see [14] for a detailed discussion).

Assumption 2: For all $\omega \in [a, b]$ and $\theta \in \{L, H\}$,

$$1 - G(\omega \mid \theta) - \gamma g(\omega \mid \theta) \geq 0.$$

Assumption 2 is purely technical, ensuring that the expected payoff of a lender net of the monitoring cost is increasing in ω. Observe that it implies that the hazard rate of the distribution is bounded from above by $1/\gamma$. We also assume that the expected return, per unit invested, of the risky project, is strictly greater than the safe return, that is, $E(\widetilde{\omega} \mid \theta) := \int_{[a,b]} \omega g(\omega \mid \theta) \, d\omega > r$ for $\theta \in \{L, H\}$, and that $a < r$. Finally, *only* the borrower has the expertise to undertake the risky project. The next section constitutes the main point of departure from a standard CSV model.

2.1 Beliefs

Since the project return is random, lenders form beliefs on its possible realizations. In our model, this is equivalent to the formation of beliefs on the parameters of the cumulative distribution function G and the outcome ω. Following the literature on interactive epistemology (e.g. [2]), we define an *external state* as a pair (ω, θ), and call beliefs about external states *first-order beliefs.*

First, we assume that it is *common belief* among players that the probability law of the random variable $\widetilde{\omega}$ conditional on θ is given by the distribution $G(\cdot \mid \theta)$, $\theta \in \{L, H\}$. Defining $p_i(\theta)$ as the subjective probability of θ for lender i and denoting p_i for $p_i(H)$, it follows that for all $i \in I$, $p_i \in [0, 1]$, $[\omega', \omega''] \subseteq [a, b]$, lender i's first-order beliefs are

$$\begin{aligned} \pi_i^1 [p_i] \left([\omega', \omega''] \times \{H\} \right) &= p_i \int_{\omega'}^{\omega''} g(\omega \mid H) \, d\omega, \\ \pi_i^1 [p_i] \left([\omega', \omega''] \times \{L\} \right) &= (1 - p_i) \int_{\omega'}^{\omega''} g(\omega \mid L) \, d\omega. \end{aligned}$$

Hence lenders' first-order beliefs are parametrized by their subjective belief p_i. Second, we impose three further assumptions on beliefs:

B1 Every player regards the external state (ω, θ) and the subjective belief p_i of lenders (other than him, if he is a lender) as stochastically independent.

B2 All lenders assume that their opponents' subjective beliefs p_i are identically and independently distributed (*i.i.d.*), drawn from a probability measure μ on $[0,1]$, either absolutely continuous with respect to the Lebesgue measure or degenerate.

B3 The borrower assumes that the probability of event $[\theta = H]$ is $q(H)$ and that lenders' beliefs are *i.i.d.*, drawn from the measure μ.

It is common belief that B1-B3 hold. We can now define lenders' second-order beliefs, that is lenders' beliefs about external states and first-order beliefs of the other lenders. For all $i \in I$, $p_i \in [0,1]$, $[\omega', \omega''] \subseteq [a,b]$, $j \neq i$, $\emptyset \neq T_j \subseteq [0,1]$ (T_j measurable), lender i's second-order beliefs are

$$\begin{aligned} \pi_i^2 [p_i] \left([\omega', \omega''] \times \{H\} \times \prod_{j \neq i} T_j \right) &= p_i \int_{\omega'}^{\omega''} g(\omega \mid H) \, d\omega \times \prod_{j \in I \setminus \{i\}} \mu(T_j), \\ \pi_i^2 [p_i] \left([\omega', \omega''] \times \{L\} \times \prod_{j \neq i} T_j \right) &= (1 - p_i) \int_{\omega'}^{\omega''} g(\omega \mid L) \, d\omega \times \prod_{j \in I \setminus \{i\}} \mu(T_j). \end{aligned}$$

It is worth pointing out that beliefs are consistent in the sense that the marginal of π_i^2 over $[a,b] \times \{L, H\}$ is π_i^1. As we will see later, second-order beliefs will play an important role in the analysis of the game. Finally, since assuming B1-B3 and common beliefs in B1-B3 allow us to establish a one-to-one correspondence between p_i and i's hierarchy of beliefs, we will refer to p_i as the *epistemic type* (or type, for short) of lender i.

The above description of beliefs has adopted an extremely simple formulation to account for the natural idea that *first-order beliefs are heterogenous and private knowledge.* Let us comment on these assumptions. First, all lenders know that the random variable $\widetilde{\omega}$ is distributed according to either $G(\cdot \mid L)$ or $G(\cdot \mid H)$ but do not know the likelihood of θ being L or H. More precisely, they do not have common beliefs about such a likelihood. Differences in opinion are generated by assuming that lenders have different models for assessing the likelihood of θ. Moreover, we assume that each lender is *absolutely convinced* that his model is correct. Said differently, each lender believes that other lenders are basing their decisions on an incorrect model. As a familiar example (due to [19]), consider two well-trained economists, one from Chicago and the other from Cambridge, Massachusetts.

They have access to the same data, but if asked to comment on the consequences of current economic policies, they would certainly offer different predictions. Such an outcome is impossible if they use the same model to interpret the data. Furthermore, they would certainly not alter their predictions after observing the other's predictions. One might also think of differences in opinion resulting from differences in information and each lender being absolutely convinced that the others have received erroneous information. Along this line, Neeman [28] shows that agents can disagree when each assigns a small probability to the event that others are not rational. See also [15] and [33]. Formally, the stochastic independence of (ω, θ) and $p_{i'}$ in the eyes of lender i implies that he thinks of agent i''s beliefs as being *non-informative* about external states, and thus he believes that lender i' is using an incorrect model or is not rational or has received an incorrect signal.

Second, if μ is non-degenerate, then there is no *common prior*. In fact, it is impossible to reconcile the CPA with assumption B1, and the assumption that distinct epistemic types in the support of μ have different beliefs about the external state. Intuitively, the latter assumption and the CPA imply that each lender believes that other lenders' first-order beliefs are correlated with the external state, thus contradicting assumption B1. The CPA holds here when μ is degenerate in $q(H)$. Our representation of interactive beliefs is therefore tractable enough to allow for a model with heterogenous priors on external states as well as a model with a common prior.

2.2 The timing

The timing is the following (see Figure 1). The first period comprises three stages. In the first stage, the borrower publicly announces a debt contract, that is, a promise to repay a fixed interest per unit borrowed. Real-world counterparts of our contract are debentures, corporate bonds and, more broadly, fixed-interest securities.

In the second stage, all lenders *simultaneously* decide either to participate in a *unique* coalition or to stand-alone. If a lender decides to participate in the coalition, he delegates the investment decision to a manager, *chosen at random* among the coalition members, and the total payoff of the coalition is *equally shared* among coalition members. Notice that more sophisticated procedures to select the coalition manager such as a voting or bargaining

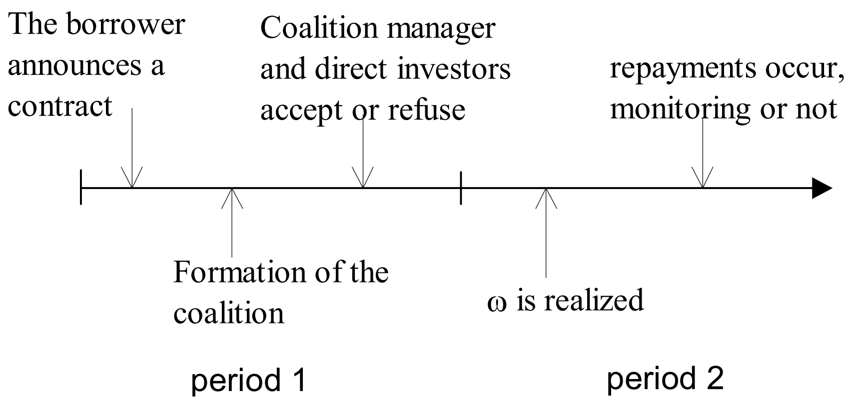

Figure 1: Timeline

procedure would not affect the main results of this paper. Indeed, no matter the procedure to select the manager, the crucial point is that his epistemic type is private knowledge. For simplicity, multiple coalitions and deviations from the unique coalition to subcoalitions (except stand-alone coalitions) are not considered. In the third stage, direct investors (i.e., lenders standing alone) and the coalition manager accept or not to fund the borrower. If a lender is indifferent between accepting or rejecting, he accepts. At this stage, it is worth noting that a lender might participate in a coalition, be chosen to be the manager and yet not finance the project, since he might have joined a coalition with too few lenders to make him financing the project. Indeed, since epistemic types are private knowledge, a lender is uncertain of the number of other lenders willing to participate in a coalition at an equilibrium.

In the second period, the investment return is realized. Monitoring takes place or not and repayments occur. When monitoring takes place, it is perfect in the sense that the true realization of the return is disclosed to the agent who requests (deterministic[3]) verification. Lenders bear the monitoring cost. Finally, it is assumed that when the coalition manager requests verification, the return ω is revealed to all members of the coalition. This simply

[3]Boyd and Smith [7] have shown that the welfare gain to use stochastic monitoring is rather weak, and thus we restrict ourselves to deterministic monitoring. (see also [25])

rules out the need to monitor the monitor (see [20] for more on this issue).

We can think of alternative timings. For instance, suppose that before forming a coalition, lenders accept or reject the contract. Then the potential disagreement among lenders is removed since a lender now has already financed the project or not before forming the coalition. Alternatively, lenders can decide whether to participate in a coalition or not before the contract is announced. I conjecture that this alternative timing would give similar results because a lender might also participate in a coalition, be chosen to be the manager and yet not finance the project (for instance, for low interest rate) as in the model we are going to analyze.

2.3 Debt contracts

Fix a finite subset of $[a, b]$ with $(k+1)$ evenly spaced elements $\Omega_k := \left\{a, a + \frac{b-a}{k}, a + 2\frac{b-a}{k}, \ldots, b\right\}$. For simplicity, we restrict ourselves to a particular type of contract called *simple debt contract.*

Definition 1 *A simple debt contract is a number $\overline{\omega} \in \Omega_k$ with the unit-repayment function $R(\omega) = \min\{\omega, \overline{\omega}\}$ for all $\omega \in [a, b]$ and the set of monitoring states $M := \{\omega : \omega < \overline{\omega}\}$.*

A simple debt contract comprises two distinct parts. The borrower promises to repay a fixed return $\overline{\omega}$, per unit borrowed, in non-monitoring states $[a, b] \diagdown M$ and, in monitoring states M, lenders *monitor the corporation and seize all the profit.* It is important to stress that by accepting a contract $\overline{\omega}$, a lender *commits* to monitoring if he is offered an actual return of $\omega < \overline{\omega}$. As apparent from Definition 1, monitoring exclusively occurs when the borrower defaults on the promised repayment, and, therefore, we can interpret monitoring states as bankruptcy states where investors force the liquidation of the corporation. Moreover, since the project is of variable size, we like to think of the contract as an announced interest rate at which lenders can lend any amount.

Several remarks are in order. First, it is easy to verify that the contract is individually rational for the borrower since $R(\omega) \leq \omega$ for all $\omega \in [a, b]$. A contract $\overline{\omega}$ is also truthtelling in that the borrower has no incentive to misreport the project return (see [20]). Second, the model does not intend to derive the optimal contract and its form is taken for granted.

Assuming a common prior on the distribution of the random variable $\widetilde{\omega}$, [14], [20] and [35], among others, have proven the optimality of simple debt contracts, in that simple debt contracts minimize the resources destroyed in the monitoring process. Boyd and Smith [6] have extended these debt-like financing contracts to the case of incomplete information (several types of borrowers) and have proven their optimality. However, to the best of my knowledge, there is no result related to our problem. We have chosen simple debt contracts for their similarities with real-world financial contracts like bonds and debentures. Third, the contract does not depend on the distribution parameter θ since it is unknown at the time of contracting and not verifiable ex-post, hence non-contractible. Fourth, for technical reasons, the borrower action space Ω_k is finite. This assumption is reasonable since prices of real-world financial contracts are generally expressed in multiples of a point (for example, a point of a five-year U.S. Treasury Notes Future is \$1000). Lastly, it is implicit in Definition 1 that the borrower does not offer a menu of contracts. We claim that, while this assumption is mainly made for simplicity, it has a grain of truth. First, issuing bonds or debentures is an extremely costly procedure (pricing, transaction costs, legal announcement, etc.), and thus we generally do not observe corporations offering bonds with multiple prices. Second, we can find a theoretical motivation in [9]. Allowing the borrower to offer a menu of contracts in a framework similar to ours, these authors provide an example where the optimal menu of contracts is the singleton menu.

3 Beliefs about beliefs and the multi-lender coalition

Throughout this section, we analyze the formation of the multi-lender coalition for a given contract proposal $\overline{\omega} \in \Omega_k$.

Notation: Hereafter, $]\underline{x}, \overline{x}[$ denotes the open set (interval) with endpoints $\underline{x}$ and $\overline{x}$ while $(\underline{x}, \overline{x})$ denotes the point in $\mathbb{R}^2$ with coordinates $\underline{x}$ and $\overline{x}$. Moreover, whenever an integral or the measure of a set appear, we assume that requirements of integrability and measurability are met. We say that a function $f : X \to \mathbb{R}$ is increasing if for $x > x'$, $f(x) > f(x')$ and nondecreasing if for $x > x'$, $f(x) \geq f(x')$.

3.1 Direct lending

The expected payoff of a lender i of epistemic type p_i investing directly is

$$\sum_{\theta\in\{L,H\}} p_i(\theta)\left(\int_a^{\overline{\omega}}(\omega-\gamma)\,dG(\omega\mid\theta)+\int_{\overline{\omega}}^b \overline{\omega}dG(\omega\mid\theta)\right). \tag{1}$$

The payoff consists of two parts. A fixed interest repayment $\overline{\omega}$ in solvency states $[\overline{\omega}, b]$, while in bankruptcy states, i.e. $\omega \in [a, \overline{\omega}]$, the lender seizes all the profit ω per unit invested and pays the monitoring cost γ. Abusing notations, we denote p_i the column vector $(1-p_i, p_i)$ and for $n \in \{1, \ldots, N\}$,

$$X(n) := \begin{pmatrix} X(n,L) \\ X(n,H) \end{pmatrix} := \begin{pmatrix} \int_a^{\overline{\omega}}\left(\omega-\frac{\gamma}{n}\right)dG(\omega\mid L)+\int_{\overline{\omega}}^b \overline{\omega}dG(\omega\mid L) \\ \int_a^{\overline{\omega}}\left(\omega-\frac{\gamma}{n}\right)dG(\omega\mid H)+\int_{\overline{\omega}}^b \overline{\omega}dG(\omega\mid H) \end{pmatrix}. \tag{2}$$

$X(n,\theta)$ is the lender payoff conditional on θ and n lenders sharing the monitoring cost. It is easy to verify that Assumption 1 on the return's distribution implies that $X(n,H) \geq X(n,L)$ [4] for all n with a strict inequality for $\overline{\omega} \neq a$, and therefore, *the more optimistic a lender (the higher p_i), the higher his expected payoff.* Equation (1) is rewritten as $p_i X(1)$.

3.2 Financial intermediation as a multi-lender coalition

Financial intermediaries provide both *brokerage*, such as financial advice, portfolio management and screening, and *qualitative assets transformation* services, such as liquidity provision and monitoring. Traditionally, depository intermediaries, like banks, provide all these services, whereas nondepository financial intermediaries, which do not raise funds through deposit, tend to be specialized in a few services.[5] In what follows, we only consider non-depository intermediaries, such as mutual funds, investment banks or venture capitalists, which are specialized in portfolio management and monitoring. A non-depository intermediary consists of one agent called the manager, who is liable for all debts and obligations

[4] Observe that a simple integration by parts yields:

$$X(n,H)-X(n,L) = -\int_a^{\overline{\omega}}[G(\omega\mid H)-G(\omega\mid L)]\,d\omega-\frac{\gamma}{n}[G(\overline{\omega}\mid H)-G(\overline{\omega}\mid L)]>0$$

[5] See [3] for an excellent survey on banking theory.

of the bank and who monitors investment projects, and one or more agents called coalition members, who contribute investment good as capital. The coalition members are not liable for the debts and obligations of the firm beyond the amount contributed. As in [5], our financial intermediary is best viewed as a *coalition* of lenders rather than a single agent, the intermediary, acting on the behalf of the other lenders, the depositors. The latter "single agent-intermediary" interpretation more closely resembles a commercial bank. However, the recent trend towards a higher specialization of financial intermediaries is largely consistent with our modeling. Specialized intermediaries are certainly more efficient than commercial banks at monitoring investments in increasingly complex technologies such as space technologies, IT technologies or biotechnologies where the lack of learning opportunities and the impressive complexity of such technologies strengthen our assumption of heterogenous beliefs.

Suppose that the lender i, member of a n-coalition, is chosen to be the manager. Remembering that the profit is equally shared, his expected payoff to financing the project is then

$$\sum_{\theta \in \{L,H\}} p_i(\theta) \left[\int_a^{\overline{\omega}} \left(\omega - \frac{\gamma}{n} \right) dG(\omega \mid \theta) + \int_{\overline{\omega}}^b \overline{\omega} dG(\omega \mid \theta) \right] = p_i X(n). \tag{3}$$

We can make two important observations. First, since i is risk-neutral, he either invests the n units of fund the intermediary has at its disposal or does not finance the project at all. Second, it is easy to see that we have an unambiguous ranking of the expected payoffs to form an intermediary i.e., $p_i X(n+1) \geq p_i X(n) \geq \ldots \geq p_i X(1)$ for a given type p_i (with strict inequalities if $\overline{\omega} \neq a$). Costly monitoring generates a role for an intermediary, and as it grows larger, the intermediary is more efficient at allocating saving to investment than direct investors are, because of economies of scale in monitoring.

3.3 The coalition formation game

3.3.1 The problem

For simplicity, we only consider symmetric Bayesian equilibria of the multi-lender coalition game. Let us illustrate, by means of an example, the problem a lender faces in making his decision to participate in a coalition or to stand-alone.

Example: Consider only two lenders i (he) and j (she), and let us put ourselves in the shoes of lender i. If lender i stands alone, he either finances the project (if his epistemic type p_i is such that $p_i X(1) \geq r$) or invests in the safe technology. Hence, his expected payoff to stand-alone is $\max(r, p_i X(1))$. Alternatively, if he decides to participate in a coalition and lender j does not participate (with probability φ), his expected payoff is again $\max(r, p_i X(1))$. Now if lender j also participates in the coalition (with probability $1-\varphi$), lender i forms an expectation on the decision of lender j if she were chosen to be the 2-coalition manager (with probability $1/2$ since it is at random). If lender j's belief is such that $p_j X(2) \geq r$, she does fund the project, and if her belief is such that $p_j X(2) < r$, she does not fund the project. However, i does not know the beliefs of j, and therefore he expects that j finances the project with probability β, that is, the probability that j finances the project conditional on j participating in the coalition and being the manager. With probability $1-\beta$, she invests in the safe technology. Moreover, lender i's expected payoff is $p_i X(2)$ if j finances the project, and r if she does not. Lastly, with probability $1/2$, i is chosen to be the manager, and his expected payoff is $\max(r, p_i X(2))$. Therefore, his expected payoff to participate in a coalition is

$$\varphi \max(r, p_i X(1)) + (1-\varphi)\frac{1}{2}\left[\max(r, p_i X(2)) + \left[\beta p_i X(2) + (1-\beta) r\right]\right]. \quad (4)$$

Hence, upon deciding to participate in a coalition or to stand-alone, lender i compares (4) with his stand-alone expected payoff $\max(r, p_i X(1))$. It is worth pointing out that φ and β have to be determined at an equilibrium.

Reasoning along the lines of the above example, suppose that lender i participates in a coalition of n lenders. With probability $(1/n)$, i is chosen to be the manager, and his expected payoff is $\max(r, p_i X(n))$. Hence, whether i does finance the project or not depends on his beliefs and the number of lenders participating in the coalition. Moreover, with probability $1-(1/n)$, i is not chosen to be the manager, and thus forms an expectation on the manager decision. Let $s : [0,1] \to \{0,1\}$, $p_i \mapsto s(p_i)$ be a symmetric equilibrium function, where "0" is interpreted as "stands alone" and "1" as "participates" and define

$$p^n := \begin{cases} 0 & \text{if} \quad X(n,L) \geq r, \\ \frac{r - X(n,L)}{X(n,H) - X(n,L)} & \text{if} \quad X(n,H) > r > X(n,L), \\ 1 & \text{if} \quad r \geq X(n,H). \end{cases} \tag{5}$$

For $X(n,H) > r > X(n,L)$, p^n is the subjective probability of the event $[\theta = H]$ that would make the manager of a n-coalition indifferent between funding the project and investing in the safe technology. Note that $p^{n+1} \leq p^n$, that p^n is decreasing in $\overline{\omega}$ and increasing in γ. If j also participates in the coalition and is chosen to be the manager of the n-coalition, she finances the project if and only if $p_j \geq p^n$, and invests in the safe technology, otherwise. Therefore, the probability $\beta(n,s)$ that she funds the project, conditional on participating in the coalition and being the manager of the n-coalition is[6]

$$\beta(n,s) := \Pr\left(p_j \geq p^n \mid p_j \in \{p \in [0,1] : s(p) = 1\}\right) \tag{6}$$

Furthermore, since epistemic types are private knowledge, lenders cannot infer the optimal action of others at the equilibrium path. *Consequently, a lender may participate in the coalition, be chosen as the manager, and yet not fund the project since he might have joined a coalition with too few lenders to make him financing the project.*

The probability that any lender $j \neq i$ participates in the coalition in a symmetric equilibrium is $\mu(\{p_j \in [0,1] : s(p_j) = 1\})$, the probability measure of the set of types participating in the coalition. And since epistemic types are $i.i.d$, the probability that exactly $(n-1)$ lenders other than i participate in the coalition is

$$\varphi(n-1,s) := [\mu(\{p_j \in [0,1] : s(p_j) = 1\})]^{n-1} [1 - \mu(\{p_j \in [0,1] : s(p_j) = 1\})]^{N-n} \begin{pmatrix} N-1 \\ n-1 \end{pmatrix}, \tag{7}$$

a binomial density with parameters $(\mu(\{p_j \in [0,1] : s(p_j) = 1\}), N-1)$. Hence, as the mathematical formulation of (6) and (7) makes clear, higher-order beliefs, that is to say, beliefs about beliefs, will play an important role in the analysis of this coalition formation game. It follows that the expected payoff of lender i of epistemic type p_i to participate in a coalition

[6] Obviously if $\mu\{p \in [0,1] : s(p) = 1\} = 0$, then $\beta(\cdot, s) \equiv 0$.

is given by

$$\begin{aligned}&\mathcal{E}^1 (p_i, s) := \\ &\sum_{n=1}^{N} \varphi (n-1, s) \left[\frac{1}{n} \max (r, p_i X (n)) \right. \\ &\left. + \frac{(n-1)}{n} [\beta (n, s) p_i X (n) + (1 - \beta (n, s)) r] \right]. \quad (8)\end{aligned}$$

Observe that $\mathcal{E}^1$ is a continuous, increasing function of p_i and linear by parts[7]. Alternatively, a lender can stand-alone, with stand-alone expected payoff

$$\mathcal{E}^0 (p_i) := \max (r, p_i X (1)). \quad (9)$$

Finally, we assume that if a lender is indifferent between joining a coalition or not, he does not join. Given that we abstract from considerations like monitoring the manager, electing the manager, etc., this break-even condition is natural.

3.3.2 Equilibrium analysis

Existence of an equilibrium

Proposition 1 *All symmetric equilibrium functions* $s : [0, 1] \to \{0, 1\}$ *are the indicator of some open interval* $]\underline{p}, \overline{p}[$.

Proposition 1 states that any equilibrium has a *double cutoff nature*: for all epistemic types $p_i \in [0, 1]$ such that $p_i \le \underline{p}$ and $p_i \ge \overline{p}$, a lender stands alone. As the epistemic type of a lender increases, his expected payoff to participate in a coalition increases as well as his expected payoff to stand-alone, and we can show that the difference of expected payoffs $\mathcal{E}^1(\cdot, s) - \mathcal{E}^0(\cdot)$ is increasing for $p_i < p^1$ and decreasing for $p_i \ge p^1$. Thus if we find a most pessimistic type $\underline{p}$ and a most optimistic type $\overline{p}$ such that these two types are indifferent between participating in the coalition and standing alone, then every type in-between participates.

[7] The inflection points being $p^1, ..., p^N$.

From Proposition 1, knowing the open interval $]\underline{p},\overline{p}[$ is isomorphic to knowing s, and thus we substitute s by $\underline{p},\overline{p}$ in Equations (6)-(8). Moreover, we have that the probability that any lender participates in the coalition is $\mu(]\underline{p},\overline{p}[)$ since $\{p\in[0,1]:s(p)=1\}=]\underline{p},\overline{p}[$ in a symmetric equilibrium. Hence the probability that exactly $(n-1)$ lenders other than i participate in the coalition follows a binomial density with parameters $(\mu(]\underline{p},\overline{p}[),N-1)$. Finally, Eq. (6) is rewritten as

$$\beta\left(n,\underline{p},\overline{p}\right)=\frac{\mu([p^{n},1]\cap]\underline{p},\overline{p}[)}{\mu(]\underline{p},\overline{p}[)}. \tag{10}$$

Remark 1 *A symmetric Bayesian equilibrium of the coalition formation game exists.*

Observe that the point $(\underline{p},\overline{p})=(1,1)$ is a trivial equilibrium, because $\varphi(0,1,1)=1$, hence, $\mathcal{E}^{0}(p_i)=\mathcal{E}^{1}(p_i,1,1)$ for all subjective beliefs p_i. Intuitively if each epistemic type of each lender conjectures that every type of the other lenders will not participate in the coalition, then each type is indifferent between standing alone and participating. Given our tie-breaking rule, it then follows that every type stands alone. Thus for each possible contract proposal in the first stage, there exists a trivial equilibrium in which any type of any lender stands alone.

Formally, define the map $\Gamma_{\overline{\omega}}:\Sigma:=\left\{(\underline{p},\overline{p})\in[0,1]\times[0,1]:\overline{p}\geq\underline{p}\right\}\to\mathbb{R}^{2}$, with

$$\Gamma_{\overline{\omega}}(\underline{p},\overline{p})=\left(\begin{array}{c}\Gamma_{\overline{\omega}}^{1}(\underline{p},\overline{p})\\ \Gamma_{\overline{\omega}}^{2}(\underline{p},\overline{p})\end{array}\right):=\left(\begin{array}{c}\mathcal{E}^{1}(\underline{p},\underline{p},\overline{p})-\mathcal{E}^{0}(\underline{p})\\ \mathcal{E}^{1}(\overline{p},\underline{p},\overline{p})-\mathcal{E}^{0}\left(\overline{p}\right)\end{array}\right). \tag{11}$$

Three remarks are worth making. First, we make explicit the dependence of Γ on $\overline{\omega}$. Second, observe that we cannot have $\mathcal{E}^{1}(p_i,\underline{p},\overline{p})-\mathcal{E}^{0}(p_i)<0$ for all $p_i\in[0,1]$, since then $s(p_i)=0$ for all p_i, hence $\underline{p}=\overline{p}$, implying that $\mathcal{E}^{1}(p_i,\underline{p},\overline{p})-\mathcal{E}^{0}(p_i)=0$ for all p_i. Third, $\Gamma_{\overline{\omega}}$ is a continuous function of $\underline{p}$ and $\overline{p}$.

It is then easy to see that an equilibrium $(\underline{p},\overline{p})$ is the solution of $(\underline{p},1-\overline{p})\cdot\Gamma_{\overline{\omega}}(\underline{p},\overline{p})\geq0$, with $\Gamma_{\overline{\omega}}(\underline{p},\overline{p})=0$ if $(\underline{p},\overline{p})\neq(0,1)$. As already mentioned, the set $\left\{(\underline{p},\overline{p}):\underline{p}=\overline{p}\right\}$ is contained in $\Gamma_{\overline{\omega}}^{-1}(0):=\left\{(\underline{p},\overline{p}):\Gamma_{\overline{\omega}}(\underline{p},\overline{p})=0\right\}$ for all $\overline{\omega}\in\Omega_k$. Moreover, it is easy to show that these points are *critical* points, that is to say, the Jacobian of $\Gamma_{\overline{\omega}}$ evaluated in $\left\{(\underline{p},\overline{p}):\underline{p}=\overline{p}\right\}$ does not have full rank.

Also observe that if $(\overline{\omega},\gamma)\in\Phi:=\left\{(\overline{\omega}',\gamma')\in\Omega_k\times(0,+\infty):p^N=1\right\}$, then

$$\left\{(\underline{p},\overline{p}):\underline{p}=\overline{p}\right\}=\Gamma_{\overline{\omega}}^{-1}(0).$$

Indeed, if $p^N=1$ (or equivalently, $X(N,H)\leq r$), not financing the project is a dominant strategy, and thus each epistemic type of each lender is indifferent between standing alone and participating in the coalition. Given our tie-breaking rule, it then follows that all equilibria are trivial equilibria. From the definition of $X(N,H)$ (see (2)), we have that $X(N,H)\leq r$ whenever the monitoring cost γ is relatively high[8] and/or $\overline{\omega}$ is relatively small (for instance, if $\overline{\omega}=a$, then $X(N,H)=X(N,L)=a<r$). However, estimations of the monitoring (bankruptcy) cost suggest that it is relatively small. For instance, based on a sample of firm failures in New York State, [34] estimated the monitoring cost to be about 3% of assets for firms that liquidated. Moreover, it is worth noting that whenever $\overline{\omega}\in\Phi$, the borrower is not financed, and therefore it is clearly suboptimal for him to propose such a contract. From now on, suppose that $(\overline{\omega},\gamma)\notin\Phi$.

What about the existence of non-trivial equilibria? A non-trivial equilibrium $(\underline{p},\overline{p})$ is a zero of $\Gamma_{\overline{\omega}}$ which does not belong to the set $\left\{(\underline{p},\overline{p}):\underline{p}=\overline{p}\right\}$. In a non-trivial equilibrium, the probability to participate in the coalition is strictly positive. Relying on arguments from Index Theory, we prove the existence of at least one non-trivial equilibrium. In what follows, we provide the reader with a rather informal discussion of the arguments involved.[9]

First, observe that a non-trivial equilibrium necessarily satisfies $(\underline{p},\overline{p})\in T\times T^c\subset\Sigma$ (T^c being the complement of T in $[0,1]$), with

$$T:=\left\{p_i\in[0,1]:p_i<p^1\right\},$$

the set of epistemic types which do not finance the project standing alone (i.e., for all $p_i\in T$, $\max\left(r,p_iX(1)\right)=r$). Suppose not. First, suppose that $(\underline{p},\overline{p})\in T\times T$, then we have

[8] A sufficient condition for $X(N,H)\leq r$ is given by $\gamma>N(b-r)$ independently of $\overline{\omega}$. To see this, observe that

$$X(N,H)=\overline{\omega}-\frac{\gamma}{N}G(\overline{\omega}\mid H)-\int_a^{\overline{\omega}}G(\omega\mid H)d\omega\leq b-\frac{\gamma}{N},$$

where the last inequality follows from the fact that $b\geq E(\omega\mid H)>\overline{\omega}-\int_a^{\overline{\omega}}G(\omega\mid H)d\omega$.

[9] A proof is available upon request.

$\mathcal{E}^1(\underline{p},\underline{p},\overline{p}) = r$ from the definition of T and an equilibrium. Since $\mathcal{E}^1$ is increasing in the epistemic type p_i (see (8)), we then have $\mathcal{E}^1(\overline{p},\underline{p},\overline{p}) > r$, a contradiction. Second, suppose that $(\underline{p},\overline{p}) \in T^c \times T^c$, then we have $\mathcal{E}^1(\overline{p},\underline{p},\overline{p}) - \overline{p}X\,(1) = 0$ from the definition of T^c and an equilibrium. However, we can show that $\mathcal{E}^1(\cdot,\underline{p},\overline{p}) - \mathcal{E}^0\,(\cdot)$ is decreasing in p_i, $p_i \in T^C$ (see Appendix A, Lemma 4), and thus $\mathcal{E}^1(\underline{p},\underline{p},\overline{p}) - \underline{p}X\,(1) > 0$, again a contradiction. Finally, if $(\underline{p},\overline{p}) = (0,1)$, it is trivially true. Therefore, at a non-trivial equilibrium, we have $\underline{p} < p^1 \leq \overline{p}$.

Second, we further characterize a non-trivial equilibrium. Observe that if $\underline{p} = p^N$, we have that $p^N X(1) \leq \ldots \leq p^N X(n) \leq \ldots \leq p^N X(N) = r$, hence $\max(r, p^N X(n)) = r$ for all n, and therefore $\mathcal{E}^1(p^N, p^N, \overline{p}) \leq r = \mathcal{E}^0(p^N)$, independently of $\overline{p} \in T^c$. The last equality comes from the fact that $\mathcal{E}^0(p^N) = \max(r, p^N X(1)) = r$. Similarly, as $\underline{p} \to p^1$, independently of $\overline{p} \in T^c \setminus \left\{p^1\right\}$ we have

$$\lim_{\underline{p}\to p^1} \mathcal{E}^1(\underline{p},\underline{p},\overline{p}) = K,$$

with

$$K = \sum_{n=1}^{N} \varphi(n-1, p^1, \overline{p}) \left[\frac{1}{n} + \frac{(n-1)}{n}\beta(n, p^1, \overline{p})\right] (p^1 X\,(n) - r) + r > r,$$

where the inequality follows from the definition (5) of p^1. We have excluded the point $\left\{\overline{p} = p^1\right\}$ since, otherwise, we converge to the trivial equilibrium (p^1, p^1). Therefore, independently of $\overline{p} \in T^c$, we have $\Gamma^1_{\overline{\omega}}(p^N, \overline{p}) \leq 0$ and $\Gamma^1_{\overline{\omega}}(p^1, \overline{p}) > 0$, hence $\underline{p} \in [p^N, p^1)$ (to see this, just apply the Intermediate Value Theorem).

Furthermore, if $\overline{p} = p^1$, it is easy to see that, independently of $\underline{p} \in T$, $\mathcal{E}^1(p^1, \underline{p}, p^1) > p^1 X(1) = r$, the last equality following from the definition of p^1, and thus $\Gamma^2_{\overline{\omega}}(\underline{p}, p^1) > 0$. Lastly, if $\overline{p} = 1$, we have either $\mathcal{E}^1(1, \underline{p}, 1) \geq X(1, H)$ or $\mathcal{E}^1(1, \underline{p}, 1) < X(1, H)$, independently of $\underline{p} \in T$. In the latter case, an equilibrium point $(\underline{p}, \overline{p})$ necessarily belongs to $[p^N, p^1[\times]p^1, 1[$, an open subset of Σ, while in the former case one might have $(\underline{p}, \overline{p}) \in [p^N, p^1[\times\{1\}$ if $\min_{\overline{p}\in[p^1,1]} \Gamma^2_{\overline{\omega}}(\underline{p}, \overline{p}) \geq 0$ or $(\underline{p}, \overline{p}) \in [p^N, p^1[\times]p^1, 1[$, otherwise.

The last step in proving the existence of a non-trivial equilibrium consists in proving the existence of a zero of $\Gamma_{\overline{\omega}}$. If $(\underline{p}, \overline{p}) \in [p^N, p^1[\times]p^1, 1[$, we show that $\Gamma_{\overline{\omega}}$ is homotopic to a mapping h admitting a unique zero in the interior of $\left\{(\underline{p}, \overline{p}) \in [0,1] \times [0,1] : \overline{p} \geq \underline{p}\right\}$. Since two homotopic mappings have the same degree and that the degree of h at 0 is non-nul, $\Gamma_{\overline{\omega}}$ admits a zero. If $(\underline{p}, \overline{p}) \in [p^N, p^1[\times \{1\}$, a straightforward application of the Intermediate Value Theorem proves the existence of at least one non-trivial equilibrium.

Further remarks

First, it is important to bear in mind that the two thresholds $\underline{p}$ and $\overline{p}$ characterizing the equilibrium depend on the contract proposal. For instance, suppose the contract proposal $\overline{\omega}$ is such that all epistemic types finance the project regardless of whether they are standing alone or participating in the coalition, that is, $p_i X(1) \geq r$ (i.e., $p^1 = 0$) for all p_i. It trivially follows that

$$\mathcal{E}^1(p_i, \underline{p}, \overline{p}) = \sum_{n=1}^{N} \varphi(n-1, \underline{p}, \overline{p}) p_i X(n),$$

a point in the convex hull of $\{p_i X(1), \ldots, p_i X(N)\}$ for all $p_i \in [0,1]$. Since $p_i X(n)$ is increasing in n, we then have that $p_i X(1)$ is the minimizer of $\mathcal{E}^1$ for all p_i. Therefore, the unique non-trivial equilibrium is the grand coalition since for $\underline{p} \neq \overline{p}$, $\mathcal{E}^1(p_i, \cdot) > p_i X(1)$ for all epistemic types p_i (observe that $\mathcal{E}^0(p_i) = p_i X(1)$ since all beliefs are in T^c). In words, there are only two equilibria, either all lenders stand alone independently of their types or all lenders participate in the coalition independently of their types. It is also worth noticing that the stand-alone equilibrium is in weakly dominated strategies.

Similarly, if the contract proposal is such that $p^2 = 0$, then *the grand coalition is the unique non-trivial equilibrium.* Intuitively, whenever $p^2 = 0$, any epistemic type of any manager of a $n-$coalition ($n \geq 2$) finances the project, and all types of each lender believe the project profitable ($p_i X(2) \geq r$, for all $p_i \in [0,1]$ and for all $i \in \{1, \ldots, N\}$) upon participating in a coalition with two or more lenders (a positive probability event in a non-trivial equilibrium). And since any lender's epistemic type is indifferent between participating in a singleton coalition or standing alone, it follows that the expected payoff to participate in a coalition is strictly higher than the expected payoff to stand-alone. Formally, it is easy to show that for all $p_i \in [0,1]$,

$$\mathcal{E}^1(p_i, \underline{p}, \overline{p}) - \mathcal{E}^0(p_i) = \sum_{n=2}^{N} \varphi(n-1, \underline{p}, \overline{p}) \left[p_i X(n) - \max(r, p_i X(1))\right] > 0,$$

implying that $(\underline{p}, \overline{p}) = (0,1)$. Participating in a coalition is a weakly dominant strategy.

Second, having the grand coalition with probability one is not an equilibrium whenever the contract proposal is such that $p^N > 0$.[10] In other words, a financial intermediary and direct investors do coexist. Intuitively, if $p^N > 0$, a lender with epistemic type $p_i < p^N$ strictly prefers to stand-alone since he expects the project return to be lower than the safe return regardless of whether he stands alone or participates in a coalition of any size, that is, $p_i X(n) < r$ for all $n \in \{1, \ldots, N\}$. These epistemic types of lender are so pessimistic about the project return that they prefer to not finance the project at all.

Lastly, we have the following Lemma.

Lemma 1 *If in two non-trivial equilibria* $(\underline{p}, \overline{p})$ *and* $(\underline{p}', \overline{p}')$*, the probability to participate in the coalition is the same, i.e.,* $\mu(]\underline{p}, \overline{p}[) = \mu(]\underline{p}', \overline{p}'[)$*, then they are identical i.e.,* $(\underline{p}, \overline{p}) = (\underline{p}', \overline{p}')$.

Equilibrium selection

In the previous discussion, we have shown that the coalition formation game possesses trivial equilibria and, at least, one non-trivial equilibrium.[11] This multiplicity of equilibria is highly problematic since, in a Perfect Bayesian equilibrium, the borrower has to make a correct conjecture in the first-stage game about the equilibrium played in the second stage game (the coalition formation game). To overcome this problem, we assume that it is *common knowledge* that lenders coordinate on a most comprehensive equilibrium, as defined below.

Definition 2 *An equilibrium* $(\underline{p}^*, \overline{p}^*)$ *is said to be a most comprehensive equilibrium if there does not exist another equilibrium* $(\underline{p}, \overline{p})$*, such that*

$$\mu(]\underline{p}, \overline{p}[) > \mu(]\underline{p}^*, \overline{p}^*[).$$

[10] Observe that $p_N \in]0, 1[$ if and only if the pair $(\overline{\omega}, \gamma)$ belongs to

$$\left\{ (\overline{\omega}', \gamma') : N \frac{\overline{\omega}' - \int_a^{\overline{\omega}'} G(\omega \mid L) d\omega - r}{G(\overline{\omega}' \mid L)} < \gamma' < N \frac{\overline{\omega}' - \int_a^{\overline{\omega}'} G(\omega \mid H) d\omega - r}{G(\overline{\omega}' \mid H)} \right\}.$$

[11] In fact, the argument used to prove the existence of at least one non-trivial equilibrium guarantees than there exists an odd number of trivial equilibria (see [22]). Moreover, they are locally unique.

Thus, in a most comprehensive equilibrium, the probability to participate in the coalition is maximal. Next, we show that a most comprehensive equilibrium has some appealing properties.

Lemma 2 *A most comprehensive equilibrium exists and is unique.*

The existence of a most comprehensive equilibrium is a direct application of the Maximum principle of Hausdorff and its uniqueness follows from Lemma 1 (see appendix D). A desirable, if not essential, property of a selected equilibrium is efficiency. For games of complete information, the concept of efficiency is clearly defined. However, for games of incomplete information, as ours, the concept of efficiency becomes more difficult to apprehend. In this paper, we use the concepts of *interim* efficiency (see Holmstrom and Myerson[12] [16]). For the sake of completeness, we recall its definition: If every lender prefers a given equilibrium over an alternative equilibrium when he knows his epistemic type, whatever his epistemic type might be, then the given equilibrium *interim* dominates the alternative one. And we say that an equilibrium is *interim* efficient if there exists no other equilibrium that *interim* dominates it. Thus *interim* efficiency is the appropriate concept of efficiency for games of incomplete information, in which the individuals already know their type when the play of the game begins.

Lemma 3 *The most comprehensive equilibrium is interim efficient.*

For any alternative equilibrium, there exists a set of types of positive measure participating in the coalition in the most comprehensive equilibrium and standing-alone in the alternative equilibrium and these epistemic types of any lender obtain a higher expected payoff in the most comprehensive equilibrium. Therefore no alternative equilibrium can interim dominate the most comprehensive equilibrium, hence the most comprehensive equilibrium is interim efficient. Formally, consider the most comprehensive equilibrium $(\underline{p}^*, \overline{p}^*)$ and an alternative equilibrium $(\underline{p}, \overline{p})$. By definition of an equilibrium, we have

$$\mathcal{E}^1(p_i, \underline{p}^*, \overline{p}^*) > \mathcal{E}^0(p_i) \geq \mathcal{E}^1(p_i, \underline{p}, \overline{p})$$

[12] Holmstrom and Myerson make the distinction between classical efficiency and incentive-compatible efficiency. In the paper, we refer to their concept of classical efiiciency.

for all epistemic type in $]\underline{p}^*, \overline{p}^*[\cap([0,1]\backslash]\underline{p}, \overline{p}[)$, a set of positive measure. [13] Hence the most comprehensive equilibrium cannot be interim dominated. Besides interim efficiency, the most comprehensive equilibrium has another interesting property.

Remark 2 *There exists an integer $\widehat{N}$ such that for $N > \widehat{N}$, the most comprehensive equilibrium minimizes the expected resources destroyed in the monitoring process.*

To fix idea, suppose (for the time being) that all lenders have financed the project and there are n lenders participating in the coalition. Since a lender commits to monitoring if he is offered an actual return $\omega < \overline{\omega}$, conditionally on being in a monitoring state (i.e., $\omega < \overline{\omega}$), the resources destroyed in the monitoring process are $(N-n)\gamma + \gamma$, a decreasing function of n. The more lenders in the coalition, the less resources are destroyed. However, matters are more complex since the numbers of lenders participating in the coalition is unknown and the probability to finance the project is not necessary the unity.

For any non-trivial equilibrium $(\underline{p}, \overline{p})$, conditionally on n lenders participating in a coalition and being in a monitoring state, the expected monitoring cost is

$$\gamma\left[(N-n)\mu[\overline{p},1] + \mu\left(]\max(\underline{p}, p^n), \overline{p}[\right)\right],$$

that is, the probability that $(N-n)$ lenders standing alone finance the project (remember that $\overline{p} \geq p^1 > \underline{p}$ in a non-trivial equilibrium) and the probability that the coalition finances the project. Moreover, the probability that exactly n lenders participate in the coalition is

$$\varphi\left(n, \underline{p}, \overline{p}\right) = \left[\mu(]\underline{p}, \overline{p}[)\right]^n \left[1 - \mu(]\underline{p}, \overline{p}[)\right]^{N-n} \begin{pmatrix} N \\ n \end{pmatrix};$$

hence, the total expected resources destroyed in the monitoring process are given by

$$\gamma\left[N(1-\mu(]\underline{p}, \overline{p}[))\mu\left([\overline{p},1]\right) + \sum_{n=0}^{N} \varphi(n, \underline{p}, \overline{p})\mu(]\max(\underline{p}, p^n), \overline{p}[)\right]. \tag{12}$$

Observe that for N sufficiently large (i.e., $N > \widehat{N}$), Eq. (12) is bounded from below by $\mu(]p^N, 1[)\gamma$ the expected monitoring cost in a grand coalition equilibrium and bounded from

[13] Observe that if $(\underline{p}, \overline{p})$ is a trivial equilibrium, then $\mu(]\underline{p}^*, \overline{p}^*[\cap([0,1]\backslash]\underline{p}, \overline{p}[)) = \mu(]\underline{p}^*, \overline{p}^*[)$, hence of positive measure. If $(\underline{p}, \overline{p})$ is a non-trivial equilibrium, then $]\underline{p}^*, \overline{p}^*[\cap]\underline{p}, \overline{p}[\neq \emptyset$ since $\underline{p}^* < p^1 < \overline{p}$ and equilibria are locally unique, hence $]\underline{p}^*, \overline{p}^*[\cap([0,1]\backslash]\underline{p}, \overline{p}[)$ is of positive measure.

above by $N\mu(]p^1, 1[)\gamma$ the expected monitoring cost in a trivial equilibria. Now consider two non-trivial equilibria $(\underline{p}^*, \overline{p}^*)$ and $(\underline{p}, \overline{p})$ such that $\mu(]\underline{p}^*, \overline{p}^*[) > \mu(]\underline{p}, \overline{p}[)$. We can easily show that the first term in the bracket is smaller for the equilibrium $(\underline{p}^*, \overline{p}^*)$ than $(\underline{p}, \overline{p})$. As for the second term, the complexity of the finite binomial sum of terms, which also depends on $\underline{p}$ and $\overline{p}$, does not make it possible to sign its variation. Nonetheless, it is clearly bounded.[14] As N gets larger, the variation in the first term dominates the variation in the second term, and thus we can conclude that for two equilibria $(\underline{p}^*, \overline{p}^*)$ and $(\underline{p}, \overline{p})$ such that $\mu(]\underline{p}^*, \overline{p}^*[) > \mu(]\underline{p}, \overline{p}[)$, fewer resources are destroyed in the monitoring process for an equilibrium with a higher participating probability.

A parametrized example. Suppose that $N = 2$ and μ is the Lebesgue measure on $[0, 1]$. It follows that for $n \in \{1, 2\}$

$$\beta\left(n, \underline{p}, \overline{p}\right) = \begin{cases} 0 & \text{if} \quad p^n \geq \overline{p} \\ \frac{\overline{p} - p^n}{\overline{p} - \underline{p}} & \text{if} \quad \overline{p} > p^n > \underline{p} \\ 1 & \text{if} \quad \underline{p} \geq p^n \end{cases}.$$

Moreover, the probability that a lender participates in the coalition is $(\overline{p} - \underline{p})$. The expected payoff to participate in a coalition is given by

$$\begin{aligned} \mathcal{E}^1\left(p_i, \underline{p}, \overline{p}\right) &= (1 - (\overline{p} - \underline{p}))\max(r, p_i X(1)) \\ &\quad + (\overline{p} - \underline{p})\frac{1}{2}\max(r, p_i X(2)) + \\ &\quad + (\overline{p} - \underline{p})\frac{1}{2}\left[\beta\left(2, \underline{p}, \overline{p}\right) p_i X(2) + \left(1 - \beta\left(2, \underline{p}, \overline{p}\right)\right) r\right], \end{aligned}$$

and the expected payoff to stand-alone by

$$\mathcal{E}^0\left(p_i\right) = \max(r, p_i X(1)).$$

Several cases are possible:

1. If $\overline{p} = \underline{p}$, then clearly $\mathcal{E}^1\left(p_i, \underline{p}, \overline{p}\right) = \mathcal{E}^0\left(p_i\right)$ for all $p_i \in [0, 1]$.

[14] Since $\left\{\mu(]\max(\underline{p}, p^n), \overline{p}[)\right\}_n$ is an increasing sequence in n, the sum is clearly bounded from below by $\mu(]p^1, \overline{p}[)$ and from above by $\mu(]\underline{p}, \overline{p}[)$. It follows that the maximal variation is $\left|\mu(]\underline{p}^*, \overline{p}^*[) - \mu(]p^1, \overline{p}[)\right|$.

2. If the contract proposal $\overline{\omega}$ is such that $p^2 = 1$, then we have $\max(r, p_i X(n)) = r$ for all $p_i \in [0,1]$, for all $n \in \{1,2\}$, and $\beta\left(2, \underline{p}, \overline{p}\right) = 0$, hence $\mathcal{E}^1\left(p_i, \underline{p}, \overline{p}\right) = \mathcal{E}^0\left(p_i\right)$ for all $p_i \in [0,1]$. Thus the equilibrium is trivial i.e., $\overline{p} = \underline{p}$.

3. If the contract proposal $\overline{\omega}$ is such that $p^1 = 0$, then for all $p_i \in [0,1]$, we have
$$\mathcal{E}^1\left(p_i, \underline{p}, \overline{p}\right) = \left(\overline{p} - \underline{p}\right)\left[p_i X(2) - p_i X(1)\right] + p_i X(1) > p_i X(1) = \mathcal{E}^0\left(p_i\right),$$
since any epistemic type finances the project whether standing alone or participating in a coalition. Hence the unique non-trivial equilibrium is the grand coalition.

4. If the contract proposal $\overline{\omega}$ is such that $1 \geq p^1 > 0$, then obviously $p^2 < 1$. An equilibrium $(\underline{p}, \overline{p})$ is the solution of
$$\left\{\begin{array}{c} \underline{p} \cdot \left[\left(\overline{p} - \underline{p}\right)\frac{1}{2}\left[\max\left(0, \underline{p}X\left(2\right) - r\right) + \beta\left(2, \underline{p}, \overline{p}\right)\left(\underline{p}X\left(2\right) - r\right)\right]\right] \geq 0 \\ \left(1 - \overline{p}\right) \cdot \left[\frac{1}{2} + \frac{1}{2}\beta\left(2, \underline{p}, \overline{p}\right) - \frac{\overline{p}[X(1,H) - X(1,L)] + X(1,L) - r}{\overline{p}[X(2,H) - X(2,L)] + X(2,L) - r}\right] \geq 0 \end{array}\right.,$$
with equality if $(\underline{p}, \overline{p}) \neq (0,1)$. Observe that if $p^2 > 0$, $\underline{p} = 0$ could not be part of an equilibrium since the first equation of the system is then negative. Moreover, a direct inspection of the first equation shows that $\underline{p} = p_2$ is part of an equilibrium. This implies that $\beta\left(2, \underline{p}, \overline{p}\right) = 1$, and thus $\overline{p} = 1$ is part of an equilibrium. Indeed, if $\overline{p} < 1$, the second equation of the system is strictly positive, contradicting the definition of an equilibrium for $\overline{p} \neq 1$. Therefore $(p_2, 1)$ is the unique non-trivial equilibrium.

3.3.3 Some comparative statics

Remark 3 *If μ is degenerate in $q(H)$, then the most comprehensive equilibrium is the grand coalition.*

Remark 3 stresses the crucial role of the common prior assumption in a CSV model. If μ is degenerate in $q(H)$, our coalition formation game is a standard game of coordination. In pure strategies, either the grand coalition or the stand-alone coalition form (of course, there exists a mixed equilibrium where lenders randomize between participating in the coalition or standing alone). Hence the most comprehensive equilibrium is the grand coalition. Consequently, when the CPA holds, financial intermediation drives direct lending out of the market in the most comprehensive equilibrium. This is literally the result contained in [35].

The next question concerns the change in the equilibrium points $(\underline{p}, \overline{p})$ as the contract proposal $\overline{\omega}$ varies. The usual method of comparative statics, namely applying the Implicit Function Theorem (IFT), has little power in our model. Indeed, although the continuity and smoothness conditions required by the IFT are met, the complexity of the system of equations (11) does not make it possible to sign the derivatives. Hence, we prefer to take advantage of a powerful tool for monotone comparative statics introduced by [23] and [24]: the lattice method. First, observe that

$$\Sigma := \left\{(\underline{p}, \overline{p}) \in [0,1] \times [0,1] : \overline{p} \geq \underline{p}\right\},$$

together with the coalition order $\geqslant_\mu$ i.e. for any pair $(\underline{p}, \overline{p})$, $(\underline{p}', \overline{p}') \in \Sigma$, $(\underline{p}, \overline{p}) \geqslant_\mu (\underline{p}', \overline{p}')$ if and only if $\mu(]\underline{p}, \overline{p}[) \geq \mu(]\underline{p}', \overline{p}'[)$, is a complete lattice. Indeed it is easy to see that every non-empty subset Σ_0 of Σ has a greatest lower bound and a least upper bound (with respect to the order $\geqslant_\mu$). Moreover, we define the highest zero of the map $\Gamma_{\overline{\omega}} : \Sigma \to \mathbb{R}^2$ as the point $(\underline{p}^*, \overline{p}^*)$ satisfying $\Gamma_{\overline{\omega}}(\underline{p}^*, \overline{p}^*) = 0$ and for all $(\underline{p}, \overline{p})$ such that $\Gamma_{\overline{\omega}}(\underline{p}, \overline{p}) = 0$, $(\underline{p}^*, \overline{p}^*) \geqslant_\mu (\underline{p}, \overline{p})$. Not surprisingly, the highest zero is the most comprehensive equilibrium, hence we can apply Theorem 3 of Milgrom and Roberts [23, p451] for the monotone comparative statics of extreme zeros. Second, Ω_k is obviously an ordered set. Third, $\Gamma_{\overline{\omega}}$ is a continuous map of $\underline{p}$ and $\overline{p}$.

Proposition 2 *There exists a $\widehat{\gamma} > 0$ such that for all $\gamma < \widehat{\gamma}$, an increase in the contract proposal increases the probability to participate in the coalition i.e., for $\overline{\omega} \geq \overline{\omega}'$, $(\underline{p}^*(\overline{\omega}), \overline{p}^*(\overline{\omega})) \geqslant_\mu (\underline{p}^*(\overline{\omega}'), \overline{p}^*(\overline{\omega}'))$.*

On the one hand, an increase in $\overline{\omega}$ increases the probability of monitoring the project (i.e., for $\omega < \overline{\omega}$), and thus gives an incentive to further share the monitoring cost. Ceteris paribus, it also increases the probability β that the coalition manager finances the project conditional on n lenders in the coalition (since p^n is decreasing in $\overline{\omega}$). On the other hand, it also increases the expected payoff to stand-alone since $\mathcal{E}^0(p_i, \overline{\omega})$ is increasing in $\overline{\omega}$. In the Appendix, we show that the former effect dominates the latter, hence Γ is monotone nondecreasing in $\overline{\omega}$.

Moreover, the condition stated in Proposition 2 ensures that Γ is monotone nondecreasing in the probability to participate in the coalition, i.e. for $(\underline{p}, \overline{p}) \geqslant_\mu (\underline{p}', \overline{p}')$, $\Gamma_{\overline{\omega}}(\underline{p}, \overline{p}) \geq$

$\Gamma_{\overline{\varpi}}(\underline{p}', \overline{p}')$. Observe that a change in the participating probability has an ambiguous effect on Γ. First, for a given β, it increases the likelihood that a coalition with many lenders will form, and thus increases the expected payoff to participate in the coalition through a reduction in the expected monitoring cost. Second, an increase in the participating probability has an ambiguous effect on β. As the participating probability increases, more pessimistic and more optimistic types might participate. If an increase in the participating probability implies that relatively more optimistic types participate, then β increases and the expected payoff to participate in a coalition unambiguously increases. However, if β decreases, the total variation in the expected payoff is ambiguous. A relatively small monitoring cost ensures that the first positive effect offsets the second negative effect. Formally, this insures that $p^N \approx p_2$ for all contract proposals $\overline{\varpi}$, hence $\beta(n, \underline{p}, \overline{p}) \approx 1$ for all n (since we know that $\underline{p} \geq p^N$ at any non-trivial equilibrium), and we are only left with the first positive effect. Lastly, since Γ is *monotone nondecreasing*, the result simply follows from Theorem 3 of Milgrom and Roberts [23, p451].

Remark 4 *For small monitoring costs ($\gamma < \widehat{\gamma}$), the most comprehensive equilibrium is the unique interim efficient equilibrium.*

For small monitoring costs, Γ is monotone nondecreasing. We first note that the set of non-trivial equilibria is ordered by the weak inclusion order (see Appendix E) when Γ is monotone nondecreasing. It follows that any type participating in the coalition in any alternative equilibrium also participates in the coalition in the most comprehensive equilibrium. These types are obviously strictly better-off in the most comprehensive equilibrium.[15] Any other type is obviously either better-off or strictly better-off in the most comprehensive equilibrium. Hence any alternative equilibrium is interim dominated by the most comprehensive equilibrium, and thus it is the unique interim efficient equilibrium.

A last question concerns the change in the multi-lender coalition size when the probability measure μ varies. More precisely, suppose that the measure μ is parametrized by $l \in L$, a partially ordered set; how is the most comprehensive equilibrium affected by an exogenous change in the parameter l? Once again, we take advantage of the lattice theory. We endow

[15] Since Γ is monotone nondecreasing in the participating probability, we have for $\mu(]\underline{p}, \overline{p}[) > \mu(]\underline{p}', \overline{p}'[)$, $\mathcal{E}^1(p_i, \underline{p}, \overline{p}) > \mathcal{E}^1(p_i, \underline{p}', \overline{p}')$ for all p_i.

Σ with the coalition order $\geqslant_{\mu_l}$, $l \in L$, i.e. for any pair $(\underline{p}, \overline{p})$, $(\underline{p}', \overline{p}') \in \Sigma$, $(\underline{p}, \overline{p}) \geqslant_{\mu_l} (\underline{p}', \overline{p}')$ if and only if $\mu_l(]\underline{p}, \overline{p}[) \geq \mu_l(]\underline{p}', \overline{p}'[)$. For any l, $(\Sigma, \geqslant_{\mu_l})$ is obviously a complete lattice. Notice the particularity of our problem, the order relation is also parametrized. We further need to define how the probability measures are ordered.

Definition 3 *Two probability measures μ_l and $\mu_{l'}$ $(l > l')$ are said to be ordered by the single crossing property in p if for any measurable subsets $[0, x[\subset \mathcal{B}([0,1])$, the Borel sigma-algebra on $[0,1]$,*

$$\begin{array}{ll} \mu_l([0,x[) \leq \mu_{l'}([0,x[) & \forall x < p \\ \mu_l([0,x[) > \mu_{l'}([0,x[) & \forall x \geq p \end{array}.$$

It is easy to see that if, in addition, μ_l and $\mu_{l'}$ have the same mean, then μ_l dominates $\mu_{l'}$ in the sense of the second order stochastic dominance. The lower l, the higher the heterogeneity of beliefs.

Proposition 3 *Suppose the set of probability measures $\{\mu_l\}_{l \in L}$ is ordered by the single crossing property in p^1 and the monitoring cost γ is smaller than $\widehat{\gamma}$. Then $(\underline{p}^*(t), \overline{p}^*(t))$ is monotone nondecreasing in the coalition order $\geqslant_{\mu_l}$, i.e. for $l > l'$, we have*

$$\mu_l(]\underline{p}^*(l), \overline{p}^*(l)[) \geq \mu_{l'}(]\underline{p}^*(l'), \overline{p}^*(l')[).$$

A sketch of the proof goes as follows. We already know that Γ is monotone nondecreasing under the condition stated in Proposition 3, i.e. for $(\underline{p}, \overline{p}) \geqslant_{\mu} (\underline{p}', \overline{p}')$, $\Gamma_{\overline{\omega}}(\underline{p}, \overline{p}, l) \geq \Gamma_{\overline{\omega}}(\underline{p}', \overline{p}', l)$ and for $l > l'$, $\Gamma_{\overline{\omega}}(\underline{p}, \overline{p}, l) \geq \Gamma_{\overline{\omega}}(\underline{p}, \overline{p}, l')$. The latter result comes from the fact that for $l > l'$, $\mu_l(]\underline{p}, \overline{p}[) \geq \mu_{l'}(]\underline{p}, \overline{p}[)$ by the single crossing property in p^1, hence the finite binomial sum $\mathcal{E}^1$ non decreases, and Γ is nondecreasing. Since Γ is monotone nondecreasing, applying the Theorem 3 of Milgrom and Roberts [23, p451] we have that $\mu_l(]\underline{p}^*(l), \overline{p}^*(l)[) \geq \mu_l(]\underline{p}^*(l'), \overline{p}^*(l')[)$. Furthermore, since $\underline{p}^*(l) < p^1 \leq \overline{p}^*(l)$, we have the desired result $\mu_l(]\underline{p}^*(l), \overline{p}^*(l)[) \geq \mu_{l'}(]\underline{p}^*(l'), \overline{p}^*(l')[)$ by the single-crossing property in p^1. If, in addition, the probability measures $\{\mu_l\}_{l \in L}$ have the same mean, then as we shift mass from the center to the tails, the risk that two lenders, randomly selected, strongly disagree is higher, and the most comprehensive equilibrium is characterized by a lower probability to participate in the coalition.

4 Debt contracts

4.1 The problem

In this section, we make explicit the dependence on the contract proposal $\overline{\omega}$. At the most comprehensive equilibrium $(\underline{p}^*(\overline{\omega}), \overline{p}^*(\overline{\omega}))$, a lender participates in the coalition when his epistemic type p_i is in the set $\left\{p_i \in [0,1] : \underline{p}^*(\overline{\omega}) < p_i < \overline{p}^*(\overline{\omega})\right\}$ and stands alone otherwise. If he stands alone, his expected payoff to financing the project is $p_i X(1)(\overline{\omega})$, and if he participates in an n-coalition, his expected payoff is $p_i X(n)(\overline{\omega})$. Observe that $p_i X(1)(\overline{\omega})$ and $p_i X(n)(\overline{\omega})$ are *increasing* functions of $\overline{\omega}$. Differentiating $p_i X(1)(\overline{\omega})$ with respect to $\overline{\omega}$, we have

$$\sum_{\theta \in \{L,H\}} p_i(\theta)\left[1 - G(\overline{\omega} \mid \theta) - \gamma g(\overline{\omega} \mid \theta)\right],$$

and this is positive by assumption 2. A similar argument holds for $p_i X(n)(\overline{\omega})$. Lenders can also abstain from financing the risky project, in which case they invest in the safe technology and obtain r.

Also, denote $V(\overline{\omega})$ the borrower's expected profit *per unit borrowed,* as defined below

$$V(\overline{\omega}) := \sum_{\theta \in \{L,H\}} q(\theta) \int_{\overline{\omega}}^{b} (\omega - \overline{\omega})\, g(\omega \mid \theta)\, d\omega. \tag{13}$$

We can readily check that V is *decreasing* in $\overline{\omega}$. The lower the promise to repay in solvency states, the higher the expected payoff per unit borrowed. Hence, the total expected payoff of the borrower is

$$V(\overline{\omega}) \sum_{m=0}^{N} m \Pr(m) = V(\overline{\omega}) \sum_{n=0}^{N} \Pr(n) \sum_{m=0}^{N} m \Pr(m \mid n),$$

with $\Pr(m \mid n)$ the probability that the borrower obtains exactly m units of funds conditional on n lenders participating in the coalition and $\Pr(n)$ the probability that exactly n lenders participate in the coalition. Let us compute $\Pr(m \mid n)$.

For $m < n$, the probability to receive exactly m units of fund is the probability that exactly m stand-alone lenders finance the project and the coalition does not finance the project. Notice that for $N - n < m < n$, $\Pr(m \mid n)$ is zero. For $m \geq n$, the probability to receive m units of funds is the probability that the coalition finances the project and exactly $m-n$ stand-alone lenders finance the project. For $N-n > m \geq n$, the probability to receive exactly m units is also the probability that exactly m stand-alone lenders finance the project and the coalition does not. Moreover, in a non-trivial equilibria, the probability that a lender standing alone finances the project is $\mu([\overline{p}^*(\overline{\omega}), 1])$, and since epistemic types are stochastically independent, the probability that exactly k stand-alone lenders will finance the project is

$$\mu([\overline{p}^*(\overline{\omega}), 1])^k (1-\mu([\overline{p}^*(\overline{\omega}), 1]))^{N-n-k} \begin{pmatrix} N-n \\ k \end{pmatrix}.$$

It then follows[16] that

$$\sum_{m=0}^{N} m \Pr(m \mid n) = (N-n)\mu([\overline{p}^*(\overline{\omega}), 1]) + n\mu(]\max(\underline{p}^*(\overline{\omega}), p^n), \overline{p}^*(\overline{\omega})[),$$

where $\mu(]\max(\underline{p}^*(\overline{\omega}), p^n), \overline{p}^*(\overline{\omega})[)$ is the probability that the manager of an n-coalition will finance the project. The former expression is fairly intuitive. Conditional on n lenders participating in the coalition, there are $N-n$ stand-alone lenders, each of whom finance the project with probability $\mu([\overline{p}^*(\overline{\omega}), 1])$ and the coalition has n units of fund at its disposal and finances the project with probability $\mu(]\max(\underline{p}^*(\overline{\omega}), p^n), \overline{p}^*(\overline{\omega})[)$. Lastly, the borrower expects that exactly n lenders participate in the coalition with probability

$$\Pr(n) = \varphi(n, \underline{p}^*(\overline{\omega}), \overline{p}^*(\overline{\omega})) = \left[\mu(]\underline{p}^*(\overline{\omega}), \overline{p}^*(\overline{\omega})[)\right]^n \left[1-\mu(]\underline{p}^*(\overline{\omega}), \overline{p}^*(\overline{\omega})[)\right]^{N-n} \begin{pmatrix} N \\ n \end{pmatrix};$$

[16] We let $p^0(\overline{\omega}) = p^1(\overline{\omega})$ for all $\overline{\omega}$. Denote $x_n(m)$ be the probability that exactly m lenders finance the project conditional on n lenders in the coalition. We have

$$\begin{aligned}\sum_{m=0}^{N} m \Pr(m \mid n) &= (1-\mu(]\max(\underline{p}^*(\overline{\omega}), p^n), \overline{p}^*(\overline{\omega})[)) \sum_{m=0}^{N-n} m x_n(m) \\ &\quad + \mu(]\max(\underline{p}^*(\overline{\omega}), p^n), \overline{p}^*(\overline{\omega})[) \sum_{m=0}^{N-n} (n+m) x_n(m).\end{aligned}$$

Manipulating this expression, we obtain the result in the text.

hence, we have

$$\sum_{n=0}^{N} \Pr(n) \sum_{m=0}^{N} m \Pr(m \mid n) =$$

$$N(1-\mu(]\underline{p}^*(\overline{\omega}), \overline{p}^*(\overline{\omega})[))\mu([\overline{p}^*(\overline{\omega}), 1])$$

$$+N \sum_{n=0}^{N} \varphi(n, \underline{p}^*(\overline{\omega}), \overline{p}^*(\overline{\omega})) \frac{n}{N} \mu(] \max(\underline{p}^*(\overline{\omega}), p^n), \overline{p}^*(\overline{\omega})[)$$

$$= NP(\overline{\omega}),$$

where $P(\overline{\omega})$ is the probability that the project is financed by an arbitrary lender (given that the second stage equilibrium is in symmetric strategies, it does not matter which lender it is). Note that $P(\overline{\omega})$ is bounded from above by $\mu([p^N(\overline{\omega}), 1[)$ for all contract proposal $\overline{\omega}$. The maximization problem of the borrower is then given by

$$\max_{\overline{\omega} \in \Omega_k} V(\overline{\omega}) NP(\overline{\omega}).$$

4.2 The equilibrium contract

First, the existence of an optimal contract $\overline{\omega}^*$ hence, of an equilibrium of the entire game is trivial. Indeed, there exists an equilibrium of the coalition game for all $\overline{\omega} \in \Omega_k$, and thus our equilibrium selection is well-defined. Given such selection, we simply have to note that the decision problem of the borrower is finite, and thus there exists an equilibrium of this two-stage game with incomplete information.

What about the variation of $P(\overline{\omega})$ with respect to $\overline{\omega}$? An increase of $\overline{\omega}$ has an ambiguous effect on $P(\overline{\omega})$. Under the condition stated in Proposition 2, for $\overline{\omega} \geq \overline{\omega}'$, we have $\overline{p}^*(\overline{\omega}) \geq \overline{p}^*(\overline{\omega}')$ and $\underline{p}^*(\overline{\omega}) \leq \underline{p}^*(\overline{\omega}')$. It follows that the first term of $P(\overline{\omega})$ is *decreasing* in $\overline{\omega}$. As for the second term, since $\left\{n\mu((\max(\underline{p}^*(\overline{\omega}), p^n), \overline{p}^*(\overline{\omega})))\right\}_{n=0}^{N}$ is an increasing sequence and

$$\mu(] \max(\underline{p}^*(\overline{\omega}), p^n), \overline{p}^*(\overline{\omega})[) > \mu(] \max(\underline{p}^*\left(\overline{\omega}'\right), p^n), \overline{p}^*\left(\overline{\omega}'\right)[)$$

for all n, then it is *increasing* in $\overline{\omega}$. Thus the total effect is ambiguous, and this is even more true if the condition stated in Proposition 2 does not hold.

Second, we show that the equilibrium contract $\overline{\omega}^*$ is such that $p^1(\overline{\omega}^*) > p^2(\overline{\omega}^*) > 0$ and $p^N(\overline{\omega}^*) < 1$. This result has interesting implications for the most comprehensive equilibrium. Previously we have shown that if the contract proposal $\overline{\omega}$ is such that $p^1(\overline{\omega}) = 0$ or $p^2(\overline{\omega}) = 0$, then the unique non-trivial equilibrium, hence the most comprehensive equilibrium, is the grand coalition. Thus our model would not have been able to account for the coexistence of direct investors and a multi-lender coalition at the most comprehensive equilibrium.

Proposition 4 *If $\overline{\omega}^*$ is an interior solution and the grid Ω_k is fine enough, the equilibrium contract $\overline{\omega}^*$ is such that $p^1(\overline{\omega}^*) > p^2(\overline{\omega}^*) > 0$ and $p^N(\overline{\omega}^*) < 1$.*

First of all, it is clearly optimal for the borrower to propose an equilibrium contract $\overline{\omega}^*$such that he is financed with positive probability, hence $p^N(\overline{\omega}^*) < 1$. Second, if the equilibrium contract is such that any epistemic type finances the project regardless of whether he stands alone or participates in the coalition (i.e., $p^1(\overline{\omega}) = 0$), the borrower obtains the N units of funds with probability one. Consider what happens to the borrower's expected profit if he decreases the repayment $\overline{\omega}$ by a small amount ε (assuming that the grid is fine enough to guarantee the existence of such ε deviation). The borrower will gain from every type who still accepts the contract, but will pay the penalty of causing some pessimistic types to not finance the contract anymore. Proposition 4 states that it is optimal to prevent some pessimistic types from directly funding the project since these types need to be extremely rewarded in order to finance the project. Note that the result depends crucially on the assumption of an absolutely continuous probability measure with respect to Lebesgue, and if the type space were discrete, the result would not necessarily hold. For instance, if all lenders were of the same type, then the borrower would make all lenders finance the project. Otherwise, he would not undertake the project and his payoff would be zero.

Lastly, if μ is degenerate in $q(H)$, the most comprehensive equilibrium is the grand coalition (see Proposition 3) and a borrower optimally sets $P(\overline{\omega}^*) = 1$. Otherwise, he would obtain no fund. This is obviously equivalent to maximizing $NV(\overline{\omega})$ with respect to $\overline{\omega}$ under the constraint $qX(N)(\overline{\omega}) \geq r$. This is exactly the problem solved in [35], and thus our model indeed encompasses the standard CSV model as a special case.

5 Further research

To conclude, I suggest a few extensions for future research. First, imagine that after observing the contract offered by the borrower, a selected lender (the banker) can propose deposit incentive-compatible contracts to the other lenders (the depositors). Does it exist an optimal menu of contracts that induces full participation? I conjecture that the answer is negative. Indeed a key feature of this problem is that the reservation utility of a lender is his expected payoff to stand-alone, hence type-dependent. Following [17], we have reasons to suspect that the optimal menu induces underparticipation, and thus some types of any lender would not participate in the bank. Second, at present the coalition formation game does not allow for multiple coalitions, i.e. either lenders stand alone or they form a unique coalition. The possibility of multiple coalitions, i.e. multiple financial intermediaries, would be an interesting extension. Third, we can also consider competition among several borrowers. A Bertrand-type competition would lead the profit of the firms to zero. Lastly, we can adopt a mechanism design approach for the first stage, i.e. the borrower selects among the set of equilibria of the second stage the one he prefers. Put differently, he would maximize his expected profit in $\overline{\omega}, \underline{p}$ and $\overline{p}$ subject to $(\underline{p}, \overline{p})$ being an equilibrium of the second stage. I conjecture that the borrower would select the most comprehensive equilibrium but cannot prove it.

A Proof of Proposition 1

Remember that

$$\mathcal{E}^1(p_i, \cdot) =$$
$$\sum_{n=1}^{N} \frac{1}{n} [\max(r, p_i X(n)) + (n-1)[\beta(n, \cdot) p_i X(n) + (1 - \beta(n, \cdot)) r]] \varphi(n-1, \cdot)$$

is increasing in p_i, and thus strictly quasi-concave. Also recall that

$$p_i X(n) = p_i [X(n, H) - X(n, L)] + X(n, L),$$

and

$$T := \{p_i \in [0, 1] : p_i < p^1\},$$

T being the set of epistemic types which does not finance the project standing-alone. Consider $p_i, p_i' \in [0,1] \times [0,1]$ such that $\mathcal{E}^1(p_i, \cdot) \geq \max(r, p_i X(1))$, $\mathcal{E}^1(p_i', \cdot) \geq \max(r, p_i' X(1))$, and any $\alpha \in [0,1]$. We shall show that

$$\mathcal{E}^1\left(\alpha p_i + (1-\alpha) p_i', \cdot\right) > \max\left(r, \left(\alpha p_i + (1-\alpha) p_i'\right) X(1)\right). \tag{14}$$

First, if $(p_i, p_i') \in T \times T$, (14) is trivially satisfied since $\mathcal{E}^1$ is strictly quasi-concave in p_i. Second, if $p_i \in T$, $p_i' \in T^c$, and $\alpha p_i + (1-\alpha) p_i' \in T$, we shall show that

$$\mathcal{E}^1\left(\alpha p_i + (1-\alpha) p_i', \cdot\right) > r.$$

One again, this is trivially true by the strict quasi-concavity of $\mathcal{E}^1$. Third, if $p_i \in T$, $p_i' \in T^c$, and $\alpha p_i + (1-\alpha) p_i' \in T^c$, we shall show that

$$\mathcal{E}^1\left(\alpha p_i + (1-\alpha) p_i', \cdot\right) > \left(\alpha p_i + (1-\alpha) p_i'\right) X(1). \tag{15}$$

To prove this last statement, we first need a Lemma.

Lemma 4 *For all $p_i \in T^c$, $\mathcal{E}^1(p_i, \cdot) - p_i X(1)$ is decreasing in p_i.*

Proof. First, observe that for all $p_i \in T^c$,

$$\mathcal{E}^1(p_i, \cdot) =$$
$$\sum_{n=1}^{N} \frac{1}{n} \left[p_i X(n) + (n-1)\left[\beta(n, \cdot) p_i X(n) + (1 - \beta(n, \cdot)) r\right]\right] \varphi(n-1, \cdot).$$

Its slope s is thus a point in the set S with

$$S := co\left\{(X(1,H) - X(1,L)), \ldots, \frac{1 + (N-1)\beta(N, \cdot)}{N}(X(N,H) - X(N,L))\right\},$$

the convex hull of $\left\{(X(1,H) - X(1,L)), \ldots, \frac{1+(N-1)\beta(N,\cdot)}{N}(X(N,H) - X(N,L))\right\}$. Since $\{(X(n,H) - X(n,L))\}_n$ is a decreasing sequence , we have

$$s^* := \underset{s \in S}{\arg\max}\, s = (X(1,H) - X(1,L)).$$

Finally, the slope of $p_i X(1)$ is $(X(1,H) - X(1,L))$, and thus $\mathcal{E}^1(p_i, \cdot) - p_i X(1)$ is decreasing in p_i since $(X(1,H) - X(1,L)) \geq s$, $\forall s \in S$. ■

By Lemma 4, it thus follows that (15) holds. Similarly,we can show that if $(p_i, p_i') \in T^c \times T^c$, and $\alpha p_i + (1-\alpha) p_i' \in T^c$, (15) holds. This completes the proof.

B Binomial formula

Let $a_1 \leq a_2 \leq ... \leq a_N$. Consider

$$f(p) = \sum_{n=0}^{N} a_n \binom{N}{n} p^n (1-p)^{N-n}.$$

Then

$$\begin{aligned}
f'(p) &= \sum_{n=0}^{N} a_n \binom{N}{n} \left[n p^{n-1} (1-p)^{N-n} - (N-n) p^n (1-p)^{N-n-1} \right] \\
&= \sum_{n=0}^{N} a_n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n - Np) \\
&= \sum_{n<Np} a_n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n-Np) + \sum_{n \geq Np} a_n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n-Np).
\end{aligned}$$

For $n < Np$ we have assumed $a_n \leq a_{[Np]}$, and since $n - Np < 0$ for such n, it follows that $a_n (n - Np) \geq a_{[Np]} (n - Np)$. Thus, the first summation satisfies

$$\sum_{n<Np} a_n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n-Np) \geq a_{[Np]} \sum_{n<Np} \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n-Np).$$

But also for the second summation it holds that

$$\sum_{n \geq Np} a_n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n-Np) \geq a_{[Np]} \sum_{n \geq Np} \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n-Np),$$

because $a_n \geq a_{[Np]}$ and $n - Np \geq 0$. Combining the two inequalities yields

$$\begin{aligned}
f'(p) &\geq a_{[Np]} \sum_{n=0}^{N} \binom{N}{n} p^{n-1} (1-p)^{N-n-1} (n-Np) \\
&= a_{[Np]} \sum_{n=0}^{N} n \binom{N}{n} p^{n-1} (1-p)^{N-n-1} - Np \sum_{n=1}^{N} \binom{N}{n} p^{n-1} (1-p)^{N-n-1} \\
&= a_{[Np]} (Np - Np) = 0.
\end{aligned}$$

This is the desired result, $f'(p) \geq 0$. Of course, if there is at least one strict inequality between the a_n's, a strict inequality for $f'(p)$ will follow. Moreover, if we consider a decreasing sequence i.e., $a_1 \geq a_2 \geq ... \geq a_N$, the reverse inequality trivially holds.

To see that a_n need not be increasing for f to be increasing in p, consider again the original expression for f'. Choose an arbitrary index $n_0 > Np$, fix a_n for $n \neq n_0$, and let $a_{n_0} \to \infty$. Then $f'(p) \to \infty$, and in particular, it is positive, independently of how you have fixed a_n for $n \neq n_0$.

C Proof of Lemma 2

Consider two non-trivial equilibria, $(\underline{p}, \overline{p})$ and $(\underline{p}', \overline{p}')$, such that $\mu(]\underline{p}, \overline{p}[) = q = \mu(]\underline{p}', \overline{p}'[)$. We have to show that $(\underline{p}', \overline{p}') = (\underline{p}, \overline{p})$. The proof proceeds by contradiction. Suppose $\overline{p}' > \overline{p}$. For all $p_i \in [0,1]$, a simple computation gives,

$$\mathcal{E}^1(p_i, \underline{p}, \overline{p}) - \mathcal{E}^1(p_i, \underline{p}', \overline{p}') =$$

$$q \sum_{m=0}^{M} q^m (1-q)^{M-m} \left[\mu(]\max(\underline{p}, p_{m+1}), \overline{p}[) - \mu(]\max(\underline{p}', p_{m+1}), \overline{p}'[)\right] \frac{m}{m+1} (p_i X(m+1) - r)$$

$$< 0,$$

since $\mu(]\max(\underline{p}, p_{m+1}), \overline{p}[) < \mu(]\max(\underline{p}', p_{m+1}), \overline{p}'[)$. It follows that $r = \mathcal{E}^1(\underline{p}, \underline{p}, \overline{p}) < \mathcal{E}^1(\underline{p}, \underline{p}', \overline{p}')$ implying that $\underline{p}' < \underline{p}$ for $(\underline{p}', \overline{p}')$ to be an equilibrium (i.e., $\mathcal{E}^1(\underline{p}', \underline{p}', \overline{p}') = r$), hence $(\underline{p}', \overline{p}') \supset (\underline{p}, \overline{p})$, contradicting $\mu(]\underline{p}, \overline{p}[) = \mu(]\underline{p}', \overline{p}'[)$. Therefore if two non-trivial equilibria have the same expected coalition size, they are identical.

D Existence of a most comprehensive equilibrium

Remember that $\Gamma_{\overline{\omega}} : \left\{(\underline{p}, \overline{p}) \in [0,1] \times [0,1] : \overline{p} \geq \underline{p}\right\} \to \mathbb{R}^2$, where

$$\Gamma_{\overline{\omega}}(\underline{p}, \overline{p}) := \left\{ \begin{array}{l} \mathcal{E}^1(\underline{p}, \underline{p}, \overline{p}) - \mathcal{E}^0(\underline{p}) \\ \mathcal{E}^1(\overline{p}, \underline{p}, \overline{p}) - \mathcal{E}^0(\overline{p}) \end{array} \right.$$

characterizes the equilibrium, and let $\Gamma_{\overline{\omega}}^{-1}(0) = \left\{(\underline{p}, \overline{p}) : \Gamma_{\overline{\omega}}(\underline{p}, \overline{p}) = 0\right\}$. We know that $\Gamma_{\overline{\omega}}^{-1}(0) \neq \emptyset$ and $\left\{(\underline{p}, \overline{p}) : \underline{p} = \overline{p}\right\} \subseteq \Gamma_{\overline{\omega}}^{-1}(0)$ for all $\overline{\omega}$. Also define $(\Gamma_{\overline{\omega}}^{-1}(0), \geqslant_\mu)$ as a partially ordered set with for all $(\underline{p}, \overline{p}), (\underline{p}', \overline{p}') \in \Gamma_{\overline{\omega}}^{-1}(0)$, $(\underline{p}, \overline{p}) \geqslant_\mu (\underline{p}', \overline{p}')$ if and only if

$$\mu(]\underline{p}, \overline{p}[) \geqslant_\mu \mu(]\underline{p}', \overline{p}'[).$$

Claim 1 *There exists an equilibrium point* $(\underline{p}^*, \overline{p}^*)$ *such that* $(\underline{p}^*, \overline{p}^*) \geqslant_\mu (\underline{p}, \overline{p})$ *for all* $(\underline{p}, \overline{p}) \in \Gamma_{\overline{\omega}}^{-1}(0)$.

Proof. We shall show that every simple ordered subset of $\Gamma_{\overline{\omega}}^{-1}(0)$ has an upper bound in $\Gamma_{\overline{\omega}}^{-1}(0)$, and thus a maximal element exists.
1) Suppose that $\left\{(\underline{p}, \overline{p}) : \underline{p} = \overline{p}\right\} = \Gamma_{\overline{\omega}}^{-1}(0)$. Then it is trivially true since every simple ordered subset of $\Gamma_{\overline{\omega}}^{-1}(0)$ is simply a singleton $\left\{\underline{p} = \overline{p}\right\}$ and an upper bound is trivially itself.
2) Suppose that $\left\{(\underline{p}, \overline{p}) : \underline{p} = \overline{p}\right\} \subset \Gamma_{\overline{\omega}}^{-1}(0)$. By the Maximum principle of Hausdorff (see p69, [27]), there exists a maximal simply ordered subset B of $\Gamma_{\overline{\omega}}^{-1}(0)$. Moreover, B has trivially an upper bound $(\underline{p}^*, \overline{p}^*)$ in $\Gamma_{\overline{\omega}}^{-1}(0)$. The element $(\underline{p}^*, \overline{p}^*)$ is then automatically a maximal element of B. For if there exists a $(\underline{p}^{**}, \overline{p}^{**}) \in \Gamma_{\overline{\omega}}^{-1}(0)$ such that $(\underline{p}^{**}, \overline{p}^{**}) \geqslant_\mu (\underline{p}^*, \overline{p}^*)$, then the set $B \cup \left\{(\underline{p}^{**}, \overline{p}^{**})\right\}$, which properly contains B, is simply ordered because $(\underline{p}^{**}, \overline{p}^{**}) \geqslant_\mu b$ for every $b \in B$. This fact contradicts the maximality of B. ■

E Monotone Comparative Statics

Part a: Theorem 3 of [23, p451].

We first need to slightly extend Theorem 3 of [23, p451] to take into account different orders on the range and domain of a function. Let $f : X \times T \to \mathbb{R}^n$, where X together with the order $\geq_X$ is a complete lattice, T is a partially ordered set and $\mathbb{R}^n$ is endowed with the component-wise order. We said that f is monotone nondecreasing if: for $t > t'$, $f(x,t) \geq f(x,t')$ and if for $x \geq_X x'$, $f(x,t) \geq f(x',t)$. One can easily show that the highest zero $x^*(t)$ of f is the $\sup \{x : f(x,t) \geq 0\}$ and is nondecreasing if f is nondecreasing and continuous. A sketch of the proof goes as follows: define $S(t) = \{x : f(x,t) \geq 0\}$, so that $x^*(t) = \sup S(t)$. By definition of $x^*(t)$, for all $x \geq_X x^*(t)$, then $f(x(t),t) \leq 0$, and thus $\lim_{x \downarrow x^*} \inf f(x(t),t) = f(x^*(t),t) \leq 0$. Next, by continuity, if $f(x^*(t),t) \leq 0$ and $f(x^*(t),t) \neq 0$, there exists a $\varepsilon > 0$ such that for all $x \in [x^* - \varepsilon, x^*]$, $f(x,t) < 0$, thus contradicting the definition of x^*, implying that $f(x^*(t),t) = 0$. Finally, as t increases, $S(t)$ becomes more exclusive since f is monotone nondecreasing in t. Hence, $x^*(t) = \sup S(t)$ is nondecreasing in t.

Part b: What do we need to apply the Theorem?

We shall show that for all $p_i \in [0,1]$, if $(\underline{p}, \overline{p}) \geqslant_\mu (\underline{p}', \overline{p}')$, then

$$\mathcal{E}^1\left(p_i, \underline{p}, \overline{p}, \overline{\omega}\right) - \mathcal{E}^0(p_i, \overline{\omega}) \geq \mathcal{E}^1\left(p_i, \underline{p}', \overline{p}', \overline{\omega}\right) - \mathcal{E}^0(p_i, \overline{\omega}), \qquad (i)$$

and for $\overline{\omega} > \overline{\omega}'$,

$$\mathcal{E}^1\left(p_i, \underline{p}, \overline{p}, \overline{\omega}\right) - \mathcal{E}^0(p_i, \overline{\omega}) \geq \mathcal{E}^1\left(p_i, \underline{p}, \overline{p}, \overline{\omega}'\right) - \mathcal{E}^0(p_i, \overline{\omega}'). \qquad (ii)$$

To see this, consider any pair, $(\underline{p}, \overline{p})$, $(\underline{p}', \overline{p}')$, of non-trivial equilibria such that $(\underline{p}, \overline{p}) \geqslant_\mu (\underline{p}', \overline{p}')$, then we have $\mathcal{E}^1\left(p_i, \underline{p}, \overline{p}, \overline{\omega}\right) \geq \mathcal{E}^1\left(p_i, \underline{p}', \overline{p}', \overline{\omega}\right)$ for all p_i by (i) In particular, it holds for $p_i = \overline{p}'$; hence,

$$\mathcal{E}^1\left(\overline{p}', \underline{p}, \overline{p}, \overline{\omega}\right) - \mathcal{E}^0(\overline{p}', \overline{\omega}) \geq \mathcal{E}^1\left(\overline{p}', \underline{p}', \overline{p}', \overline{\omega}\right) - \mathcal{E}^0(\overline{p}', \overline{\omega}) \geq 0.$$

Moreover, since $\mathcal{E}^1\left(\cdot, \underline{p}, \overline{p}, \overline{\omega}\right) - \mathcal{E}^0(\cdot)$ is decreasing for $p_i > p^1$, we have $\overline{p} \geq \overline{p}'$ for $(\underline{p}, \overline{p})$ to be an equilibrium. In addition, for $\overline{\omega} > \overline{\omega}'$, (i) and (ii) imply

$$\begin{gathered} \mathcal{E}^1\left(p_i, \underline{p}, \overline{p}, \overline{\omega}\right) - \mathcal{E}^0(p_i, \overline{\omega}) \geq \\ \mathcal{E}^1\left(p_i, \underline{p}', \overline{p}', \overline{\omega}\right) - \mathcal{E}^0(p_i, \overline{\omega}) \geq \\ \mathcal{E}^1\left(p_i, \underline{p}', \overline{p}', \overline{\omega}'\right) - \mathcal{E}^0(p_i, \overline{\omega}'); \end{gathered}$$

hence, in particular,

$$\mathcal{E}^1\left(\overline{p}', \underline{p}, \overline{p}, \overline{\omega}\right) - \mathcal{E}^0(\overline{p}', \overline{\omega}) \geq \mathcal{E}^1\left(\overline{p}', \underline{p}', \overline{p}', \overline{\omega}'\right) - \mathcal{E}^0(\overline{p}', \overline{\omega}') \geq 0.$$

Using once again the fact that $\mathcal{E}^1\left(\cdot, \underline{p}, \overline{p}, \overline{\omega}\right) - \mathcal{E}^0(\cdot)$ is decreasing implies that $\overline{p} \geq \overline{p}'$. A symmetric argument holds for $\underline{p} \leq \overline{p}'$. Thus, if $\mathcal{E}^1$ is monotone nondecreasing, then the set of non-trivial equilibria is ordered by the weak inclusion order and an increase in $\overline{\omega}$ increases the expected coalition size.

Part c: Conditions for (i) *and* (ii) *to hold.*

We first consider condition (ii). It is easy to see that $\mathcal{E}^1\left(p_i, \underline{p}, \overline{p}, \overline{\omega}\right)$ is a nondecreasing function of $\overline{\omega}$ since $p_i X(n)$ and $\beta(n, \underline{p}, \overline{p})$ are. Moreover, $\mathcal{E}^0(p_i, \overline{\omega})$ is also nondecreasing.

Since $\mathcal{E}^0(p_i) = r$ for all $p_i < p^1$, then (ii) trivially holds for $p_i < p^1$. For $p_i \geq p^1$,

$$\begin{aligned}
\mathcal{E}^1\left(p_i, \underline{p}, \overline{p}, \overline{\omega}\right) - \mathcal{E}^0(p_i, \overline{\omega}) &= \sum_{n=2}^{N} \varphi(n-1, \underline{p}, \overline{p}) \left[p_i X(n, \overline{\omega}) - p_i X(1, \overline{\omega})\right] \\
&\quad + \sum_{n=2}^{N} \varphi(n-1, \underline{p}, \overline{p}) \frac{n-1}{n} (\beta(n, \underline{p}, \overline{p}) - 1) p_i X(n, \overline{\omega}) \\
&= (p_i G(\overline{\omega} \mid H) + (1-p_i) G(\overline{\omega} \mid L)) \sum_{n=2}^{N} \varphi\left(n-1, \underline{p}, \overline{p}\right) \frac{\gamma(n-1)}{n} \\
&\quad + \sum_{n=2}^{N} \varphi(n-1, \underline{p}, \overline{p}) \frac{n-1}{n} (\beta(n, \underline{p}, \overline{p}) - 1) p_i X(n, \overline{\omega}),
\end{aligned}$$

is increasing in $\overline{\omega}$ and strictly increasing in the interior of Ω_k since (1) g has full support on (a, b), (2) p^n is decreasing in $\overline{\omega}$, hence $\beta(n, \underline{p}, \overline{p})$ increasing in $\overline{\omega}$ and (3) $p_i X(n, \overline{\omega})$ is increasing in $\overline{\omega}$.

Let us now consider condition (i). The idea of the proof is to show that condition (i) holds for p^2 sufficiently close to p^N i.e., for sufficiently small monitoring costs. And then to quantify the $\widehat{\gamma}$.

Since $\underline{p} \geq p^N$ in any non-trivial equilibrium, we have for $p^2 = p^N$, $\beta(n, \underline{p}, \overline{p}) = 1$ for $n \geq 2$, hence

$$\mathcal{E}^1\left(p_i, \underline{p}, \overline{p}, \overline{\omega}\right) - \mathcal{E}^0(p_i, \overline{\omega}) = \sum_{n=2}^{N} \varphi\left(n-1, \underline{p}, \overline{p}\right) \left[p_i X(n, \overline{\omega}) - \max(p_i X(1, \overline{\omega}), r)\right].$$

And since $\{p_i X(n, \overline{\omega}) - \max(p_i X(1, \overline{\omega}), r)\}_{n=2}^{N}$ is a strictly increasing sequence in n, the result follows with strict inequality. The condition $p^2 = p^N$ clearly holds if $N = 2$ or $\gamma = 0$. Let $\varepsilon > 0$. By continuity it also holds for $d(p^2, p^N) < \varepsilon$, an open ball around p^N. (d being the Euclidean metric). This is equivalent to

$$p^2 - p^N = \frac{r - X(2, L)}{X(2, H) - X(2, L)} - \frac{r - X(N, L)}{X(N, H) - X(N, L)} < \varepsilon.$$

Since the above expression is increasing in $\overline{\omega}$, a sufficient condition for all $\overline{\omega}$ is

$$p^2(\overline{\omega}) - p^N(\overline{\omega}) < p^2(b) - p^N(b) = \gamma \frac{N-2}{2N} \frac{1}{E(\omega \mid H) - E(\omega \mid L)} < \varepsilon$$

We also have that $\mathcal{E}^1\left(p_i, \underline{p}, \overline{p}, \overline{\omega} = b\right)$evaluated in $p^N = p^2$ is given by

$$
\begin{aligned}
& \sum_{n=2}^{N} \varphi(n-1, \underline{p}, \overline{p}) \left[p_i X\left(n, \overline{\omega} = b\right) - \max(p_i X(1), r)\right] \\
= & \sum_{n=2}^{N} \varphi(n-1, \underline{p}, \overline{p}) \left(p_i E(\omega \mid H) + (1-p_i) E(\omega \mid L) - \frac{\gamma}{n}\right) \\
& - \sum_{n=2}^{N} \varphi(n-1, \underline{p}, \overline{p}) \left(\max(p_i E(\omega \mid H) + (1-p_i) E(\omega \mid L) - \gamma, r)\right).
\end{aligned}
$$

And thus the limit of $\Delta(p^2) := (\mathcal{E}^1\left(p_i, \underline{p}, \overline{p}, \overline{\omega} = b\right) - \mathcal{E}^1\left(p_i, \underline{p}', \overline{p}', \overline{\omega} = b\right))(p^2)$ when p^2 converges to p^N is

$$
-\gamma \sum_{n=2}^{N} \left(\varphi(n-1, \underline{p}, \overline{p}) - \varphi(n-1, \underline{p}', \overline{p}')\right) \frac{1}{n} > 0.
$$

Finally define $p_*^2 = \inf\left\{p^2 : \Delta(p^2) = 0\right\}$ $(\inf\{\varnothing\} = +\infty)$. Hence $\varepsilon \leq p_*^2 - p^N(b)$. This implies that condition (i) holds for

$$
\gamma < \widehat{\gamma} := 2[(E(\omega \mid H) - E(\omega \mid L)) p_*^2 + E(\omega \mid L) - r].
$$

F Proof of Proposition 4

Proof. By contradiction. Suppose that the equilibrium contract $\overline{\omega}^* \in \Omega_k$ is such that $p^1(\overline{\omega}^*) = 0$ or $p^2(\overline{\omega}^*) = 0$. As shown in section 3, the most comprehensive equilibrium is then the grand coalition. Let $X^*(N)$ be the payoff associated with the optimal contract $\overline{\omega}^*$. For $\varepsilon > 0$, define

$$
A(\varepsilon) := \{p_i \in [0,1] : 0 \leq p_i X^*(N) - r \leq \delta_i(\varepsilon)\},
$$

with

$$-\delta_i(\varepsilon) :=$$

$$\sum_{\theta\in\{L,H\}} p_i(\theta)\left(\int_a^{\overline{\omega}^*-\varepsilon}\left(\omega-\frac{\gamma}{N}\right)g(\omega\mid\theta)\,d\omega+\int_{\overline{\omega}^*-\varepsilon}^{b}(\overline{\omega}^*-\varepsilon)\,g(\omega\mid\theta)\,d\omega\right)-$$

$$\sum_{\theta\in\{L,H\}} p_i(\theta)\left(\int_a^{\overline{\omega}^*}\left(\omega-\frac{\gamma}{N}\right)g(\omega\mid\theta)\,d\omega+\int_{\overline{\omega}^*}^{b}(\overline{\omega}^*-\varepsilon)\,g(\omega\mid\theta)\,d\omega\right)=$$

$$\sum_{\theta\in\{L,H\}} p_i(\theta)\left(-\int_{\overline{\omega}^*-\varepsilon}^{\overline{\omega}^*}\left(\omega-\frac{\gamma}{N}\right)g(\omega\mid\theta)\,d\omega+\overline{\omega}^*\left(G(\overline{\omega}^*\mid\theta)-G(\overline{\omega}^*-\varepsilon\mid\theta)\right)-\varepsilon\left(1-G(\overline{\omega}^*-\varepsilon\mid\theta)\right)\right),$$

the change in payoff when $\overline{\omega}^*$ decreases by ε. Since $X^*(N)$ is strictly increasing in $\overline{\omega}^*$, all $\delta_i(\varepsilon)$ are positive.

Let define $\Delta\pi$ as the change in profit when $\overline{\omega}^*$ decreases by ε,

$$\Delta\pi = N\int_{[0,1]\setminus A(\varepsilon)}\left(V(\overline{\omega}^*-\varepsilon)-V(\overline{\omega}^*)\right)d\mu(p_i)-N\int_{A(\varepsilon)}V(\overline{\omega}^*)\,d\mu(p_i).$$

Observe that if $\overline{\omega}^*$ is optimal, then either $A(0)$ contains a unique point or is empty. First, suppose that there exists a point in the grid Ω_k such that $\mathcal{U}_N(0)=r$. Now if $\mathcal{U}_N^*(0)>r$, the borrower can decrease the repayment $\overline{\omega}^*$ by a small fixed amount, leaving participation constraints non-binding and increasing his payoff. This would contradict the optimality of $\overline{\omega}^*$. Thus at the optimum, we should have $\mathcal{U}_N(0)=r$, and since $\mathcal{U}_N$ is increasing in p_i, this point is unique. Second, $A(0)$ is empty when there is no point in the finite grid Ω_k such that $\mathcal{U}_N(0)=r$. Moreover, the continuity of $\mathcal{U}^*$ and $\mathcal{U}_{\mathbf{n}}^*$ implies that $A(\varepsilon)$ is a continuous family of sets so that $A(\varepsilon)\to A(0)$ as $\varepsilon\to 0$.

The profit obtained from lenders in set $A(\varepsilon)$, denoted $\pi(\varepsilon)$, is

$$\pi(\varepsilon)=N\int_{A(\varepsilon)}V(\overline{\omega}^*)\,d\mu(p_i)=\mu(A(\varepsilon))\,NV(\overline{\omega}^*)$$

The last step consists in considering the change in profit caused by reducing the repayment $\overline{\omega}^*$ by ε. The entrepreneur repays ε less to all lenders who still do participate and his change in profit is

$$N\int_{[0,1]\setminus A(\varepsilon)}\left(V(\overline{\omega}^*-\varepsilon)-V(\overline{\omega}^*)\right)d\mu(p_i)=\theta(\varepsilon)\left(1-\mu(A(\varepsilon))\right),$$

with

$$\theta(\varepsilon) = V(\overline{\omega}^* - \varepsilon) - V(\overline{\omega}^*) =$$

$$\sum_{\theta=H,L} q(\theta) \left[\int_{\overline{\omega}^*-\varepsilon}^{\overline{\omega}^*} \omega g(\omega \mid \theta) \, d\omega - (\overline{\omega}^* - \varepsilon)(G(\overline{\omega}^* \mid \theta) - G(\overline{\omega}^* - \varepsilon \mid \theta)) + \varepsilon (1 - G(\overline{\omega}^* - \varepsilon \mid \theta)) \right]$$

$$\geq 0,$$

since V is decreasing in $\overline{\omega}$. Finally, we can write the total variation of profit as

$$\begin{aligned} \Delta\pi &= N[\theta(\varepsilon)(1 - \mu(A(\varepsilon))) - \mu(A(\varepsilon)) V(\overline{\omega}^*)] \\ &= N[\theta(\varepsilon)(1 - \mu(A(\varepsilon))) - \mu(A(\varepsilon))(\theta(\varepsilon) - V(\overline{\omega}^* - \varepsilon))]. \end{aligned}$$

$\Delta\pi$ therefore has the sign of

$$1 - \mu(A(\varepsilon)) - \frac{\mu(A(\varepsilon))(\theta(\varepsilon) - V(\overline{\omega}^* - \varepsilon))}{\theta(\varepsilon)} = 1 - 2\mu(A(\varepsilon)) + \frac{\mu(A(\varepsilon)) V(\overline{\omega}^* - \varepsilon)}{\theta(\varepsilon)}.$$

Since μ is absolutely continuous, $\mu(A(\varepsilon))$ converges uniformly to $\mu(A(0)) = 0$, and for any ε,

$$\frac{\mu(A(\varepsilon)) V(\overline{\omega}^* - \varepsilon)}{\theta(\varepsilon)} > 0,$$

therefore, $\Delta\pi$ has a positive sign for ε small enough, thus contradicting the optimality of $\overline{\omega}^*$.

Now since V, θ and μ are continuous, there exists a $\overline{\varepsilon}$ such that for $\varepsilon < \overline{\varepsilon}$, $\Delta\pi$ has a positive sign. Finally, we have to choose the grid $\Omega_k := \left\{a, a + \frac{b-a}{k}, a + 2\frac{b-a}{k}, \ldots, b\right\}$ such that $\frac{b-a}{\overline{\varepsilon}} < k$ in order to insure the existence of such profitable deviation. Thus $\left\{p_i \in [0,1] : p_i < p^N(\overline{\omega}^*)\right\}$ is non-empty at the optimum i.e., $p^N(\overline{\omega}^*) > 0$, and therefore

$$p^1(\overline{\omega}^*) > p^2(\overline{\omega}^*) > p^N(\overline{\omega}^*) > 0.$$

■

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