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GIANCARLO CORSETTI, LUCA LEDOLA

and

SYLVAIN LEDUC

BADIA FIESOLANA, SAN DOMENICO (FI)
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Giancarlo Corsetti\textsuperscript{a} Luca Dedola\textsuperscript{b} Sylvain Leduc\textsuperscript{c}

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\textsuperscript{a}European University Institute and CEPR; email: Giancarlo.Corsetti@iue.it.

\textsuperscript{b}Bank of Italy; email: dedola.luca@insedia.interbusiness.it.

\textsuperscript{c}Federal Reserve Bank of Philadelphia; email: Sylvain.Leduc@phil.frb.org.
Abstract

A central puzzle in international finance is that real exchange rates are volatile and, in stark contradiction to efficient risk-sharing, negatively correlated with relative consumptions across countries. This paper shows that a model with incomplete markets and a low price elasticity of tradables can account for these properties of real exchange rates. The low price elasticity stems from introducing distribution services, intensive in local inputs, which drive a wedge between producer and consumer prices and lower the impact of terms-of-trade changes on optimal agents' decisions.

In our model, two very different patterns of the international transmission of productivity shocks generate the observed degree of risk-sharing: one associated with an improvement, the other with a worsening of the country’s terms of trade and real exchange rate. We provide VAR evidence on the effect of technology shocks to U.S. manufacturing, identified through long-run restrictions, in support of the first transmission pattern. These findings are at odds with the presumption that terms-of-trade movements foster international risk-pooling.

JEL classification: F32, F33, F41

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1 Introduction

Why isn’t domestic consumption higher relative to foreign consumption when its relative price is lower? With the development of international financial markets, domestic households should be able to insure their consumption streams against country-specific shocks, as to benefit from income transfers when consumption is relatively cheap. However, as first shown by Backus and Smith [1993], this prediction is clearly at odds with the data. For the OECD countries, the correlation between relative consumption and the real exchange rate (i.e., the relative price of consumption across countries) is generally low, and even negative. The Backus-Smith evidence is obviously hard to replicate with models assuming complete international asset markets. But, as emphasized by Chari, Kehoe and McGrattan [2002], it remains an outstanding challenge to models restricting international trade in assets and allowing for different market frictions and imperfections — including nominal price rigidities — to address the main puzzles in international finance.

The standard Mundell-Fleming-Dornbusch model suggests a way to rationalize the Backus-Smith observation, considering shocks to real demand that drive up domestic expenditure and consumption, while at the same time appreciating the currency in real terms. This is because some external demand needs to be “crowded out” in order to make “more room” for domestic demand. Thus this model seems consistent with the above evidence, but only to the extent that international business cycles and real exchange rate fluctuations can be described as driven by demand shocks.

In a general-equilibrium framework, however, very different shocks can have demand effects. Specifically, technology improvements raise not only domestic supply but also affect demand by impinging on wealth. Country-specific shocks that move the terms of trade and the real exchange rate change the equilibrium valuation of domestic output relative to the rest of the world. If risk-pooling is only partial, large swings in international prices may have large, uninsurable effects on relative wealth and demand.

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1 Rather than a high cross-country correlation of consumption, this is the main implication of efficient risk-sharing in the presence of real exchange rate fluctuations, as discussed in Section 2.

2 Chari, Kehoe and McGrattan [2002] show that in a model in which prices are sticky in the importer currency the correlation between relative consumption and the real exchange rate is close to 1 even when the only internationally traded asset is a nominal bond.

In this paper, we study the link between the high exchange rate volatility characteristic of the international economy (the exchange rate volatility puzzle) and the observed low degree of international consumption risk-sharing (the Backus-Smith puzzle), deriving its implications for the connection of business cycles across countries. We proceed in two steps. First, we build a two-country model where asset markets are incomplete, and because of a low price elasticity of tradables, the terms of trade and the real exchange rate are highly volatile in response to productivity shocks. When we calibrate our model to match the U.S. real exchange rate volatility, we find that it generates a low degree of risk-sharing. The predicted correlation between the real exchange rate and relative consumption is negative, and the comovements in aggregates across countries are broadly in line with those in the data. An important feature of our model is the presence of distribution services, produced with the intensive use of local inputs. As in Corsetti and Dedola [2002], distribution contributes to generate a low price elasticity of tradables. However, nominal rigidities play no role in our results — in our specification all prices and wages are flexible. The main predictions of the model are reasonably robust to extensive sensitivity analysis.

Using our model, we show that a low degree of risk-sharing can be generated by two very different patterns of the international transmission of productivity shocks, each corresponding to a plausible set of parameter values. In our benchmark calibration, for a price elasticity slightly above 1/2 international spillovers in equilibrium are large and positive. A positive transmission is a standard prediction of the international business cycle literature: a productivity increase in the domestic tradable sector leads to a deterioration of the terms of trade and a depreciation of the real exchange rate. However, in our baseline economy the deterioration is so large that relative domestic wealth decreases, driving foreign consumption above domestic consumption. For a price elasticity slightly below 1/2, international spillovers are still large but — strikingly — negative. With a negative transmission, following a productivity increase, the home terms of trade and the real exchange rate appreciate, reducing relative wealth and consumption abroad.

The latter pattern of international transmission is due to a combination of an unconventionally sloped demand curve, and nontrivial general equilibrium effects. Because of home bias in consumption, domestic tradables are mainly demanded by domestic households. With a low price elasticity, a terms-of-trade depreciation that reduces domestic wealth relative to the rest of the world would actually result in a drop of the world demand for do-
mestic goods — the negative wealth effect in the home country would more than offset any global positive substitution and wealth effect. Therefore, for the world markets to clear, a larger supply of domestic tradables must be matched by an increase in their relative price, that is, an appreciation of the terms of trade — driving up domestic demand.

Second, we investigate empirically whether the international transmission of productivity shocks to tradables in the U.S. data bear any resemblance to the above patterns. Using structural VARs, we identify technology shocks to manufacturing (our measure of tradables) by means of long-run restrictions — in doing so, we extend the work by Galí [1999] and Christiano, Eichenbaum and Vigfusson [2003], to an open-economy framework. Our VAR analysis yields two important results. First, we provide novel evidence in support of the prediction of a negative conditional correlation between relative consumption and the real exchange rate. Following a permanent positive shock to U.S. labor productivity in manufacturing, U.S. output and consumption increase relative to the rest of the world, while the real exchange rate appreciates. Second, the same productivity shock improves the terms of trade, as suggested by our model under the negative transmission.

In light of these results, the Backus-Smith evidence appears less puzzling yet more consequential for the construction of open-economy general-equilibrium models. Our VAR evidence questions the international transmission mechanism in a wide class of general equilibrium models, with potentially strong implications for welfare and policy analysis. In fact, if a positive shock to productivity translates into a higher, rather than lower, international price of exports, foreign consumers will be negatively affected. Terms-of-trade movements do not contribute at all to consumption risk-sharing. Gains from international portfolio diversification may thus well be large, relative to the predictions of standard open-economy models.

The text is organized as follows. The following section presents the key implications of standard two-goods open-economy models for the link between relative consumption and the real exchange rate, and briefly summarizes some evidence on their correlations for industrialized countries. In

\footnote{Conditional on a productivity increase in tradables, an appreciation of the real exchange rate and an increase in domestic consumption are also predicted by the Balassa-Samuelson model with no terms-of-trade effect (because of perfect substitutability of domestic and foreign tradables). Yet, as shown by our numerical experiments, a model with high elasticity of substitution between tradables cannot generate either enough volatility of the real exchange rate and terms of trade or replicate the negative Backus-Smith unconditional correlation.}
Section 3, we introduce the model, whose calibration is presented in Section 4. Section 5 explores the quantitative predictions of the model in numerical experiments. Section 6 presents the VAR evidence on the effects of productivity shocks in the open-economy. Finally, Section 7 summarizes and qualifies the paper’s results, suggesting directions for further research.

2 International consumption risk-sharing: reconsidering the Backus-Smith puzzle

In this section, we first restate the Backus and Smith [1993] puzzle, looking at the data for most OECD countries. Second, we reconsider the general equilibrium link between relative consumption and the real exchange rate. Focusing on a simple model with tradable goods only we show that the link between these variables can have either sign depending on the price elasticity of tradables: a low elasticity can generate the negative pattern observed in the data. But since a low price elasticity also means that quantities are not very sensitive to price movements, a negative correlation between the real exchange rate and relative consumption will be associated with a high volatility of the real exchange rate and the terms of trade relative to fundamentals and other endogenous macroeconomic variables — in accord with an important set of stylized facts of the international economy.

2.1 Stating the puzzle

As pointed out by Backus and Smith [1993], an internationally efficient allocation implies that the marginal utility of consumption, weighted by the real exchange rate, should be equalized across countries:

\[
\frac{P_t^*}{P_t} U_{c,t} = U_{c^*,t}^*,
\]  

(1)

where the real exchange rate (RER) is customarily defined as the ratio of foreign \((P_t^*)\) to domestic \((P_t)\) price level, expressed in the same currency units (via the nominal exchange rate), \(U_{c,t} (U_{c^*,t}^*)\) denotes the marginal utility of consumption, and \(C_t\) and \(C_t^*\) denote domestic and foreign consumption, respectively. Intuitively, a benevolent social planner would allocate consumption across countries such that the marginal benefits from an extra unit of foreign consumption equal its marginal costs, given by the domestic
marginal utility of consumption times the real exchange rate \( \frac{P_t^*}{P_t} \), i.e., the relative price of \( C_t^* \) in terms of \( C_t \).

If a complete set of state-contingent securities is available, the above condition holds in a decentralized equilibrium independently of trade frictions and goods market imperfections (including shipping and trade costs, as well as sticky prices or wages) that can cause large deviations from the law of one price and purchasing power parity (PPP). It is only when PPP holds (i.e., \( RER = 1 \)) that efficient risk-sharing implies equalization of the \( ex-post \) marginal utility of consumption — corresponding to the simple notion that complete markets imply a high cross-country correlation of consumption.

Under the additional assumption that agents have preferences represented by a time-separable, constant-relative-risk-aversion utility function of the form

\[
\frac{C^{1-\sigma} - 1}{1-\sigma}, \quad \text{with } \sigma > 0,
\]

(1) translates into a condition on the correlation between the (logarithm of the) ratio of domestic to foreign consumption and the (logarithm of the) real exchange rate.\(^5\) Against the hypothesis of perfect risk-sharing, many studies have found this correlation to be significantly below one, or even negative, in the data (in addition to Backus and Smith [1993], see for instance Kollman [1995] and Ravn [2001]).

Table 1 reports the correlation between real exchange rates and relative consumption for OECD countries relative to the U.S. and to an aggregate of the OECD countries, respectively. Since we use annual data, we report the correlations for both the HP-filtered and first-differenced series. As shown in the table, real exchange rates and relative consumption are negatively correlated for most OECD countries. The highest correlation is as low as 0.53 (Switzerland vis-à-vis the rest of the OECD countries), and most correlations are in fact negative — the median of the table entries in the first two columns are \(-0.30\) and \(-0.27\), respectively.

Consistent with other studies, Table 1 presents strong \emph{prima facie} evidence against open-economy models with a complete set of state-contingent securities. Given that debt and equity trade, the most transparent means of consumption-smoothing, are far less operative across borders than within.

\(^5\)Clearly, one can envision shocks, e.g., taste shocks, that move the level of consumption and the marginal utility of consumption in opposite directions. These shocks may help in attenuating the link between the real exchange rate and relative consumption. However, it would be quantitatively quite challenging to identify shocks with this property, which can account for the low or negative correlations reported in Table 1 below.

Likewise, Lewis [1996] rejects nonseparability of preferences between consumption and leisure as an empirical explanation of the low correlation of consumption across countries.
them, a natural first step to account for the apparent lack of risk-sharing is to assume that financial assets exist only on a limited number of securities. Restricting the set of assets that agents can use to hedge country-specific risk breaks the tight link between real exchange rates and the marginal utility of consumption implied by (1). It should therefore be an essential feature of models trying to account for the stylized facts summarized in Table 1.

Unfortunately, it is now well understood that allowing for incomplete markets may not be enough to bring models in line with these facts. To start with, in the face of transitory shocks, trade in an international, uncontingent bond may be enough to bring the equilibrium allocation quite close to the efficient one (see e.g., Baxter and Crucini [1993]). Intuitively, if agents in one country get a positive output shock, they will want to lend to the rest of the world, so that consumption increases both at home and abroad. This result has generally been derived in one-good models, abstracting from movements in relative prices. However, terms-of-trade movements can also impinge on the international transmission of shocks and even ensure perfect risk-sharing independently of trade in financial assets — a point underscored by Cole and Obstfeld [1991] and Corsetti and Pesenti [2001a, b]. Positive productivity shocks in one country that moderately depreciate the domestic terms of trade and the real exchange rate will allow consumption abroad to increase to some extent, though less than domestic consumption, thus resulting in a tight positive link between international relative prices and cross-country consumption.

In light of these considerations, the Backus-Smith anomaly provides an important test of open economy models with frictions — more specifically, of the international transmission mechanism envisioned in the theory. To account for the anomaly, it seems that terms-of-trade movements need to hinder risk-sharing and reduce the scope for risk-pooling in response to country-specific shocks provided by the assets available to agents. In what follows, we will build on a simple setting due to Cole and Obstfeld [1991], to provide an intuitive account of the determinants of the comovements between the real exchange rate and relative consumption with incomplete financial markets.
2.2 Into the puzzle

2.2.1 Volatility and international transmission

This section develops a simple model — a special case of the model presented in section 3 — with the aim of providing an intuitive yet analytical account of the main mechanisms driving our quantitative results below. We will first relate the sign and magnitude of the transmission of shocks across borders to the price elasticity of tradables. We will then relate the pattern of international transmission to risk-sharing.

Consider a two-country, two-good endowment economy under the extreme case of financial autarky. We will refer to the two countries as ‘Home’ and ‘Foreign’. For the Home representative consumer, consumption is given by the following CES aggregator

\[ C = C_T = \left[ a_H^{1-\rho} C_H^\rho + a_F^{1-\rho} C_F^\rho \right]^\frac{1}{\rho}, \]  

(2)

where \( C_{H,t} \) (\( C_{F,t} \)) is the domestic consumption of Home (Foreign) produced good, \( a_H \) is the share of the domestically produced good in the Home consumption expenditure, \( a_F \) is the corresponding share of imported goods, with \( a_F = 1 - a_H \). Let \( P_{H,t} \) (\( P_{F,t} \)) denote the price of the Home (Foreign) good, and \( \tau = \frac{P_F}{P_H} \) the terms of trade, the relative price of Foreign goods in terms of Home goods. The consumption-based price index \( P \) is

\[ P = P_T = \left[ a_H P_H^{\rho-1} + (1 - a_H) P_F^{\rho-1} \right]^\frac{\rho-1}{\rho}. \]  

(3)

Let \( Y_H \) denote Home (tradable) output. In financial autarky, consumption expenditure has to equal current income, i.e., \( \frac{PC}{P_H} = Y_H \). Domestic demand for Home goods can then be written:

\[ C_H = a_H \left( \frac{P_H}{P} \right)^{-\omega} C = \frac{a_H}{a_H + (1 - a_H) \tau^{1-\omega}} Y_H \]

where the demand’s price elasticity coincides with the elasticity of substitution across the two goods, \( \omega = (1 - \rho)^{-1} \). Analogous expressions hold for the Foreign country. Using an asterisk to denote foreign variables, the foreign demand for the Home goods is

\[ C_H^* = \frac{a_H^*}{a_H^* + (1 - a_H^*) \tau^{1-\omega}} Y_F^*, \]
where $a^*_H$ is the share of Home goods in the foreign consumption basket. As above, we used the fact that, from the trade balance condition, \[
\frac{P_F Y^*_F}{P_H} = \frac{P_F}{P_H} Y^*_F \quad \text{where } Y^*_F \text{ is foreign (tradable) output.}
\]

Now, taking the derivative of $C_H$ with respect to $\tau$:

\[
\frac{\partial C_H}{\partial \tau} = \left( \frac{\omega}{SE} - \frac{1}{IE} \right) \frac{a_H (1 - a_H) \tau^{-\omega}}{[a_H + (1 - a_H) \tau^{1-\omega}]^2} Y_H > 0 \iff \omega > 1,
\]

(4)

makes it clear that the Home demand for the Home good $C_H$ can be either increasing or decreasing in the terms of trade $\tau$, depending on $\omega$. When $\omega > 1$, a fall in the relative price of the domestic tradable — an increase in $\tau$ — will raise its domestic demand. This is the case when the positive substitution effect (SE) from lower prices is larger in absolute value than the negative income effect (IE) from a lower valuation of $Y_H$.

Conversely, when $\omega < 1$ the negative income effect will more than offset the substitution effect. Thus, a terms-of-trade depreciation will reduce the domestic demand for the Home tradable. The foreign demand for Home tradables $C^*_H$ will instead always be increasing in $\tau$, independently of $\omega$:

\[
\frac{\partial C^*_H}{\partial \tau} = \left[ \frac{\omega (1 - a^*_H) \tau^{1-\omega}}{SE} + \frac{a^*_H}{IE} \right] \frac{a^*_H}{[(1 - a^*_H) \tau^{1-\omega} + a^*_H]^2} Y^*_F > 0;
\]

the substitution and income effects are both positive.

Putting these very basic relations together, it is apparent that a positive shock to Home output $Y_H$ will cause the Home terms of trade to depreciate only if $\omega$ is large enough that the world demand $C_H + C^*_H$ is increasing in $\tau$.

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6 Formally, by a straightforward derivation of the Slutsky equation, the substitution effect is obtained from the compensated demand function $x_H$:

\[
\frac{\partial x_H}{\partial \tau} = \omega \frac{a_H (1 - a_H) \tau^{-\omega}}{[a_H + (1 - a_H) \tau^{1-\omega}]^2} Y_H.
\]

7 In the presence of negative net foreign asset positions, the income effect can be lower than -1, as argued by Benigno and Thonissen [2002]. However, this effect is unlikely to be quantitatively important for the OECD countries: for instance, Kraay and Ventura [2000] estimate that the largest foreign debt as a percentage of total assets on average over the 1973-1995 period amounts to less than 0.1 for Finland. The U.S. position is positive and equal to 5% of total assets.
(i.e., decreasing in the relative price of Home goods). Note that in this case foreign consumption of Home tradables will rise, responding to the fall in the relative price of imports. If \( \omega \) is sufficiently below 1 and \( a^*_H \) is large relative to \( a^*_F \), however, the world demand for the Home good will be dominated by its domestic component, and will be falling in \( \tau \). The negative income effect of a depreciation of the domestic terms of trade on Home demand will be so strong as to more than offset any positive substitution and income effect abroad. For a positive supply shock to \( Y_H \) to be matched by an increase in world demand, the terms of trade need to appreciate — with a negative impact on demand abroad. Moreover, for values of \( \omega \) in the region where the slope of world demand changes sign (and is rather flat), small changes in \( Y_H \) will bring about large movements in the terms of trade and the real exchange rate.

To make these points formally, we take a log-linear approximation of the market clearing condition for Home tradables \( Y_H = C_H + C^*_H \) around a symmetric equilibrium (with \( a^*_H = 1 - a^*_H \) and \( Y_H = Y^*_H \)). The equilibrium link between relative output (endowment) changes, and the terms of trade/real exchange rate can be expressed as

\[
\bar{\tau} = \frac{\bar{Y}_H - \bar{Y}_F^*}{1 - 2a_H (1 - \omega)}, \tag{5}
\]

\[
\text{REER} = \frac{2a_H - 1}{1 - 2a_H (1 - \omega)} \left( \bar{Y}_H - \bar{Y}_F^* \right), \tag{6}
\]

where a “~” represents a variable’s percentage deviation from the symmetric values. For given movements in relative output, the sign of the coefficients in the above expressions depends on \( \omega \), while the volatility of the terms of trade and the real exchange rate follows a hump-shaped pattern as \( \omega \) increases. These features are crucial determinants of our theoretical and empirical results in the following sections. We discuss them in turn.

First, with home bias in consumption \( (a_H > 1/2) \) and a sufficiently low price elasticity of imports, that is, \( 0 < \omega < \frac{2a_H - 1}{2a_H} < 1/2 \), the ratio on the right-hand side of (6) is negative and increasing in \( \omega \). The domestic and world demand schedules for Home tradables will be negatively sloped, so that relative output will move in opposite directions relative to the real exchange rate and the terms of trade — which will both appreciate in response to a

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8We are grateful to Fabrizio Perri for suggesting this line of exposition.
positive Home supply shock. As shown above, what is key to this result is a weak substitution effect relative to the income effect following changes in relative prices.

Second, since the substitution effect is increasing in \( \omega \) the demand schedule becomes flatter, the closer \( \omega \) is to \( \frac{2a_H - 1}{2a_H} \), the upper bound of the region with a downward-sloping world demand. The coefficient relating \( \hat{Y}_H - \hat{Y}_F^* \) to \( \hat{RE}R \) and \( \hat{\tau} \) in the above expressions becomes quite high in absolute value, driving up the volatility of the real exchange rate and the terms of trade in terms of changes in relative output.

For higher values of the price elasticity, namely \( \omega > \frac{2a_H - 1}{2a_H} \), the ratio on the right-hand side of (6) becomes positive and decreasing in \( \omega \). The slope of world demand is now positive and increasing in \( \omega \). As a result, higher values of \( \omega \) reduce the coefficient relating \( \hat{Y}_H - \hat{Y}_F^* \) to \( \hat{RE}R \) and \( \hat{\tau} \): in this region, the larger the price elasticity, the lower the volatility of the real exchange rate and the terms of trade in terms of changes in relative output. Therefore, there will be in general two values of \( \omega \) below and above \( \frac{2a_H - 1}{2a_H} \) that yield the same volatility of the terms of trade and real exchange rate, each associated to a different sign of the response of relative prices to country-specific shocks.

### 2.2.2 Risk-sharing

So far, we have shown that there can be different patterns of relative price movements, shaping the international transmission of supply shocks in terms of both its magnitude and sign. We can now derive the implications of our results for risk-sharing, looking at the equilibrium comovements between the real exchange rate and relative consumption. With incomplete markets (under financial autarky) the scope for insurance against country-specific shocks is limited, and equilibrium movements in international relative prices will expose consumers to potentially strong relative wealth shocks.

In our simple model, using the balanced-trade condition, it is easy to write relative consumption as a function of the terms of trade:

\[
\tau C_F = C_H^* \iff \frac{C}{C^*} = \frac{C}{C^*} = \left[ \frac{(1 - a_H^*) \tau^{1 - \omega} + a_H^*}{a_H \tau^\omega + (1 - a_H) \tau} \right] ^{\frac{1}{\omega}} ;
\]

(7)

from this, we can then derive the following log-linearized relationship be-
tween the real exchange rate and relative consumption:

$$\bar{RE}R = \frac{2a_H - 1}{2a_H \omega - 1} \left( \bar{C} - \bar{C}^* \right).$$

(8)

The relation between real exchange rates and relative consumption can have either sign, depending again on the values of $a_H$ and $\omega$. Specifically, assume again that countries’ preferences are characterized by home bias in consumption. Then the ratio on the right-hand side of (8) will be negative when $\omega < \frac{1}{2a_H} < 1$.

We have seen above that, for a given change in the terms of trade and the real exchange rate, the international transmission of shocks can be positive or negative, depending on whether $\omega$ is above or below $\frac{2a_H - 1}{2a_H}$. But this cutoff point is smaller than $\frac{1}{2a_H}$. Hence, a negative correlation between the real exchange rate and relative consumption can correspond to different patterns of the international transmission. Consider the equilibrium response to a Home supply shock. For $\omega < \frac{2a_H - 1}{2a_H}$, the Home terms of trade improve and the real exchange rate appreciates, while Home consumption rises relative to Foreign consumption. For $\frac{2a_H - 1}{2a_H} < \omega < \frac{1}{2a_H}$, a Home supply shock reduces the relative price of Home exports, worsening the Home terms of trade and depreciating the Home real exchange rate. Because of the size of the price movements, consumption abroad increases relative to consumption at Home (which may or may not fall). With $\omega > \frac{1}{2a_H}$, there is again a depreciation, but consumption abroad increases by less than consumption at Home.

Contrast these results with the benchmark economy constructed by Cole and Obstfeld [1991], which is a special case of our economy with $\omega = 1$ and $a_H = a_H^* = 1/2$. This contribution — as well as Corsetti and Pesenti [2001a] — builds examples where productivity shocks to tradables bring about relative price movements that exactly offset changes in output, leaving cross-country relative wealth unchanged. Even under financial autarky, agents can achieve the optimal degree of international risk-sharing.

But optimal risk-sharing via terms-of-trade movements is likely to be an extreme case, since according to the evidence, both the sign of the transmission and the magnitude of relative price movements appear to be different from what is required to support an efficient allocation. Even when the in-
ternational transmission is positive — as should be in the examples by Cole and Obstfeld and Corsetti and Pesenti — equilibrium fluctuations in real exchange rates and the terms of trade of the magnitude of those observed in the data may be excessive relative to the benchmark case of optimal transmission, as is the case when \( \frac{2a_H - 1}{2a_H} < \omega < \frac{1}{2a_H} \).

Our analysis above unveils that an “excessively positive” international transmission of productivity shock generates an empirical pattern of low risk-sharing and can therefore rationalize the Backus-Smith anomaly: a terms-of-trade and real exchange rate depreciation will be reflected in a reduction in relative consumptions. Risk-sharing is of course hindered by a negative transmission, which prevails when \( \omega < \frac{2a_H - 1}{2a_H} \). A terms of trade appreciation in response to a productivity shock raises domestic real import and consumption, while reducing wealth abroad — again in line with the Backus-Smith evidence, but at odds with risk-sharing via relative price movements.

2.3 The way ahead

Using a stylized two-country, two-good model with financial autarky and endowment (productivity) shocks, we have shown that, depending on the price elasticity of tradables, the correlation between relative consumption and the real exchange rate can have either sign. By emphasizing a low price elasticity, this analysis suggests what we see as a promising modelling strategy to address the Backus-Smith anomaly. As shown below, our strategy consists of building a model in which a low price elasticity of tradables is not exclusively related to a low elasticity of substitution \( \omega \) but is an implication of assuming a realistic structure of the goods market with distributive trade. In the next sections we will study the quantitative implications of our dynamic model with capital accumulation, assuming that only uncontingent bonds are traded internationally. In particular, we want to check whether versions of the model, with and without a retailing sector, can give rise to international spillovers of productivity shocks consistent with the low degree of risk-sharing implied by the Backus-Smith anomaly, when \( \omega \) is set to match the observed volatility of the real exchange rate relative to that of output. This framework leads to empirically plausible predictions that find striking support in the data.

Before proceeding, it is worth noting that nominal rigidities do not seem to play a crucial role in explaining the Backus-Smith puzzle — as pointed
out by Chari, Kehoe and McGrattan [2002] in a model in which exporters fix their price in the currency of the market of destination. To see why, consider a version of our simple economy with production and prices fixed in local currencies. It is easy to see that the correlation between the real exchange rate and relative consumption will remain strongly positive, irrespective of the value of $\omega$. Under financial autarky, the counterpart of the balanced trade condition (7) implies that relative consumption is proportional to the inverse of the terms of trade. A shock that increases Home consumption relative to Foreign consumption must thus appreciate the terms of trade to ensure zero net exports; but since prices are fixed in local currencies, a terms of trade appreciation can only occur because of a nominal currency depreciation that, again owing to local-currency price-stickiness, will coincide with a real depreciation. In what follows, we will abstract from nominal rigidities.

3 The model

In this and the next section, we develop our model. In section 5 we will employ standard numerical techniques to solve it, with the specific goal of quantifying the link between the real exchange rate and the level of consumption across countries when the economy is hit by productivity shocks.

Our world economy consists of two countries of equal size, denoted H and F, each specialized in the production of an intermediate, perfectly tradable good. In addition, each country produces a nontradable good. This good is either consumed or used to make intermediate tradable goods H and F available to domestic consumers. In what follows, we describe our setup focusing on the Home country, with the understanding that similar expressions also characterize the Foreign economy — whereas starred variables refer to Foreign firms and households.

3.1 The Firms’ Problem

Firms producing Home tradables (H) and Home nontradables (N) are perfectly competitive and employ a technology that combines domestic labor and capital inputs, according to the following Cobb-Douglas functions:

\[
Y_H = Z_H K_H^{1-\xi} L_H^{\xi}, \\
Y_N = Z_N K_N^{1-\xi} L_N^{\xi},
\]

where $Z_H$ and $Z_N$ are exogenous random disturbance following a statistical process to be determined below. We assume that capital and labor are
freely mobile across sectors. The problem of these firms is standard: they hire labor and capital from households to maximize their profits:

\[ \pi_H = P_{H,t} Y_{H,t} - W_t L_{H,t} - R_t K_{H,t}, \]

\[ \pi_N = P_{N,t} Y_{N,t} - W_t L_{N,t} - R_t K_{N,t}, \]

where \( P_{H,t} \) is the wholesale price of the Home traded good and \( P_{N,t} \) is the price of the nontraded good. \( W_t \) denote the wage rate, while \( R_t \) represents the capital rental rate.

Firms in the distribution sector are also perfectly competitive. They buy tradable goods and distribute them to consumers using nontraded goods as the only input in production. In the spirit of Erceg and Levin [1996] and Burstein, Neves and Rebelo [2001], we assume that bringing one unit of traded goods to Home (Foreign) consumers requires \( \eta \) units of the Home (Foreign) nontraded goods.

### 3.2 The Household’s Problem

#### 3.2.1 Preferences

The representative Home agent in the model maximizes the expected value of her lifetime utility, given by:

\[ E \left\{ \sum_{t=0}^{\infty} U [C_t, \ell_t] \exp \left[ \sum_{\tau=0}^{t-1} -\nu (U [C_\tau, \ell_\tau]) \right] \right\} \]

where instantaneous utility \( U \) is a function of a consumption index, \( C \), and leisure, \((1-\ell)\). Foreign agents’ preferences are symmetrically defined. These preferences guarantee the presence of a locally unique steady state, independent of initial conditions.\(^9\)

The full consumption basket, \( C_t \), in each country is defined by the following CES aggregator

\[ C_t \equiv \left[ a_T^{1-\phi} C_{T,t}^\phi + a_N^{1-\phi} C_{N,t}^\phi \right]^{\frac{1}{\phi}}, \quad \phi < 1, \]

where \( a_T \) and \( a_N \) are the weights on the consumption of traded and nontraded goods, respectively and \( \phi \) is the constant elasticity of substitution.

\(^9\)A unique invariant distribution of wealth under these preferences will allow us to use standard numerical techniques to solve the model when only a non-contingent bond is traded internationally (see Obstfeld [1990], Mendoza [1991], and Schmitt-Grohe and Uribe [2001]).
between $C_{N,t}$ and $C_{T,t}$. As in Section 2, the consumption index of traded goods $C_{T,t}$ is given by (2).

### 3.2.2 Price indexes

A notable feature of our specification is that, because of distribution costs, there is a wedge between the producer price and the consumer price of each good. Let $\overline{P}_{H,t}$ and $P_{H,t}$ denote the price of the Home traded good at the producer and consumer level, respectively. Let $P_{N,t}$ denote the price of the nontraded good that is necessary to distribute the tradable one. With competitive firms in the distribution sector, the consumer price of the traded good is simply

$$P_{H,t} = \overline{P}_{H,t} + \eta P_{N,t}. \quad (11)$$

We hereafter write the utility-based CPIs, whereas the price index of tradables is given by (3):

$$P_t = \left[ a_T P_{T,t}^{\frac{\phi}{\phi+1}} + a_N P_{N,t}^{\frac{\phi}{\phi+1}} \right]^{\frac{\phi+1}{\phi}}. \quad (12)$$

Foreign prices, denoted with an asterisk and expressed in the same currency as Home prices, are similarly defined. Observe that the law of one price holds at the wholesale level but not at the consumer level, so that $\overline{P}_{H,t} = \overline{P}_{H,t}^*$ but $P_{H,t} \neq P_{H,t}^*$. In the remainder of the paper, the price of Home aggregate consumption $P_t$ will be taken as the numeraire. Hence, the real exchange rate will be given by the price of Foreign aggregate consumption $P_t^*$ in terms of $P_t$.

### 3.2.3 Budget constraints and asset markets

Home and Foreign agents hold an international bond, $B_H$, which pays in units of Home aggregate consumption and is zero in net supply. They derive income from working, $W_t l_t$, from renting capital to firms, $R_t K_t$, and from the proceeds from holding the international bond, $(1 + r_t)B_{H,t}$, where $r_t$ is the real bond’s yield, paid at the beginning of period $t$ but known at time $t - 1$. The individual flow budget constraint for the representative agent in
the Home country is therefore:

\[ P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P_{N,t}C_{N,t} + B_{H,t+1} + \mathcal{P}_{H,t}I_{H,t} \leq W_{t}\ell_{t} + R_{t}K_{t} + (1 + r_{t})B_{H,t}. \]  

We assume that investment is carried out in Home tradable goods and that the capital stock, \( K \), can be freely reallocated between the traded (\( K_{H} \)) and nontraded (\( K_{N} \)) sectors:

\[ K = K_{H} + K_{N}. \]

Moreover, contrary to the consumption of tradables, we assume that investment is not subject to distribution services. The price of investment is therefore the wholesale price of the domestic traded good, \( \mathcal{P}_{H,t} \). The law of motion for the aggregate capital stock is given by:

\[ K_{t+1} = I_{H,t} + (1 - \delta)K_{t} \]  

The household’s problem then consists of maximizing lifetime utility, defined by (9), subject to the constraints (13) and (14).

3.3 Competitive Equilibrium

Let \( s_{t} = \{ B_{H}; Z \} \) denote the state of the world at time \( t \), where \( Z = \{ Z_{H}, Z_{F}, Z_{N}, Z_{N}^{*} \} \). A competitive equilibrium is a set of Home agent’s decision rules \( C_{H}(s), C_{F}(s), C_{N}(s), I_{H}(s), l(s), B_{H}(s) \); a set of Foreign agent’s decision rules \( C_{H}^{*}(s), C_{F}^{*}(s), C_{N}^{*}(s), l_{*}^{*}(s), B_{H}^{*}(s) \); a set of Home firms’ decision rules \( K_{H}(s), K_{N}(s), L_{H}(s), L_{N}(s) \); a set of Foreign firms’ decision rules \( K_{H}^{*}(s), K_{N}^{*}(s), L_{H}^{*}(s), L_{N}^{*}(s) \); a set of pricing functions \( P_{H}(s), P_{F}(s), \mathcal{P}_{H}(s), \mathcal{P}_{F}(s), P_{N}(s), P_{N}^{*}(s), W(s), W^{*}(s), R(s), R^{*}(s), r(s) \) such that (i) the agents’ decision rules solve the households’ problems; (ii) the firms’ decision rules solve the firms’ problems; and (iii) the appropriate market-clearing conditions (for the labor market, the capital market and the bond market) hold.

\(^{10}B_{H,t}\) denotes the Home agent’s bonds accumulated during period \( t - 1 \) and carried over into period \( t \).

\(^{11}\)We also conduct sensitivity analysis on our specification of the investment process, below.
3.4 A remark on distribution and the price elasticity of tradables

The introduction of a distribution sector in our model is a novel feature relative to standard business cycle models in the literature. Before delving into numerical analysis, it is appropriate to discuss an important implication of this feature regarding the volatility of the terms of trade. From the representative consumer’s first-order conditions (regardless of frictions in the asset and goods markets), optimality requires that the relative price of the imported good in terms of the domestic tradable at consumer level be equal to the ratio of marginal utilities:

\[
\frac{P_{F,t}}{P_{H,t}} = \frac{\eta P_{N,t}}{\eta P_{N,t} + \frac{1}{a_H} C_{H,t}} \left( \frac{C_{H,t}}{C_{F,t}} \right)^{\frac{1}{\omega}},
\]

where \( \omega = (1 - \rho)^{-1} \) is equal to the elasticity of substitution between Home and Foreign tradables in the consumption aggregator \( C_{T,t} \), and thus to the consumer price elasticity of these goods. Note that \( \frac{C_{H,t}}{C_{F,t}} \) is the inverse of the ratio of real imports to nonexported tradable output net of investment. In analogy to the literature, we can refer to this ratio as the (tradable) import ratio.

Because of distribution costs, the relative price of imports in terms of Home exports at the consumer level does not coincide with the terms of trade \( \frac{P_{F,t}}{P_{H,t}} \) — as in most standard models (e.g. Lucas [1982]). Let \( \mu \) denote the size of the distribution margin in steady state, i.e., \( \mu = \eta P_{N} / P_{H} \). By log-linearizing (15), we get:

\[
\hat{\tau} = \frac{1}{\omega (1 - \mu)} \left( \hat{C}_{H,t} - \hat{C}_{F,t} \right).
\]

where the terms of trade \( \tau \) is measured at the producer-price level so that \( \omega (1 - \mu) \) can be thought of as the producer price elasticity of tradables. Clearly, both \( \omega \) and \( \mu \) impinge on the magnitude of the international transmission of country-specific shocks through the equilibrium changes in the terms of trade. It is well known that for any given change in \( \hat{C}_{H,t} - \hat{C}_{F,t} \), a lower \( \omega \) transpires into larger changes in the terms of trade. In our model, a larger distribution margin \( \mu \) (i.e., a larger \( \eta \)) has a similar effect. Accounting for distributive trade introduces a novel amplification channel of fluctuations in international relative prices for any given variability in real
quantities. So, for given $\omega$ and $\mu$, large movements in the difference between the real consumption of domestic and imported tradables $\frac{C_{H,t}}{C_{F,t}}$ (the inverse of the import ratio) will be reflected in highly volatile terms of trade and deviations from the law of one price.\(^{12}\) Remarkably, it will be shown below that in the U.S. data the absolute standard deviation of this ratio is very close to that of the terms of trade (4.13 and 3.68 per cent, respectively).

Note that under financial autarky the counterpart of condition (4) in our fully-specified model with distribution services is:

$$\frac{\partial C_H}{\partial \tau} > 0 \iff \left( \frac{\omega(1-\mu)}{SE} - \frac{1}{IE} \right) \left( 1 - a_H \right) \left( \frac{P_F}{P_H} \right)^{1-\omega} - \frac{a_H \mu}{IE} > 0.$$  

Not only does a positive distribution margin $\mu$ reduce below $\omega$ the substitution effect ($SE$) from a deterioration in the terms of trade. It also makes the income effect ($IE$) more negative, as the presence of distributive trade causes the consumer price to fall less than one-to-one relative to the relative price of domestic tradables.

\section{Model calibration}

Table 2 reports our benchmark calibration, which we assume symmetric across countries. Several parameter values are similar to those adopted by Stockman and Tesar [1995] and Chari, Kehoe, and McGrattan [2002], who calibrate their models to the United States relative to a set of OECD countries. Throughout the exercise, we will carry out sensitivity analysis and assess the robustness of our results under the benchmark calibration. In particular, we are interested in the sensitivity of our results to changes in the elasticity of substitution for tradables $\omega$.

\textbf{Productivity shocks} We previously defined the exogenous state vector as $Z = \{Z_H, Z_F, Z_N, Z_N^*\}$. We assume that disturbances to technology follow a trend-stationary AR(1) process

$$Z' = \lambda Z + u,$$  

whereas $u = (u_H, u_F, u_N, u_N^*)$ has variance-covariance matrix $V(u)$, and $\lambda$ is a 4x4 matrix of coefficients describing the autocorrelation properties of the

\(^{12}\)In particular, the tradable import ratio will display more variability, \textit{ceteris paribus}, when changes in absorption of domestic and imported tradables have opposite signs.
shocks. Since we assume a symmetric economic structure across countries, we also impose symmetry on the autocorrelation and variance-covariance matrices of the above process.

Consistent with our model and other open-economy studies (e.g., Backus, Kehoe and Kydland [1995]), we identify technology shocks with Solow residuals in each sector, using annual data in manufacturing and services from the OECD STAN database. Since hours are not available for most other OECD countries, we use sectoral data on employment. An appendix describes our data in more detail.

The bottom panel of Table 2 reports our estimates of the parameters describing the process driving productivity. As found by previous studies, our estimated technology shocks are fairly persistent. On the other hand, in line with empirical studies, we find that spillovers across countries and sectors are not negligible.\(^{13}\)

**Preferences and production** Consider first the preference parameters. Assuming a utility function of the form:

\[
U [C_t (j), \ell_t(j)] = \frac{\left[ C_t^\alpha (j) (1 - \ell_t(j))^{1-\alpha} \right]^{1-\sigma}}{1 - \sigma} - 1, \quad 0 < \alpha < 1, \quad \sigma > 0,
\]

we set \(\alpha\) so that in steady state, one third of the time endowment is spent working; \(\sigma\) (risk aversion) is set equal to 2. Following Schmitt-Grohe and Uribe [2001], we assume that the endogenous discount factor depends on the average per capita level of consumption, \(C_t\), and hours worked, \(\ell_t\), and has the following form:

\[
\nu (U [C_t, \ell_t]) = \begin{cases} 
\ln (1 + \psi [C_t^\alpha (1 - \ell_t(j))^{1-\alpha}]) & \sigma \neq 1 \\
\ln (1 + \psi [\alpha \ln C_t + (1 - \alpha) \ln(1 - \ell_t)]) & \sigma = 1 
\end{cases}
\]

whereas \(\psi\) is chosen such that the steady-state real interest rate is 4 percent per annum, equal to 0.08.

The value of \(\phi\) is selected based on the available estimates for the elasticity of substitution between traded and nontraded goods. We use the estimate

\(^{13}\text{See Costello [1993]. The persistence of the estimated shocks, though in line with estimates both in the closed (e.g., Cooley and Prescott [1995]) and open-economy (Heathcote and Perri [2002]) literature, is higher than that reported by Stockman and Tesar [1995]. The difference can be attributed to the fact that they compute their Solow residuals from HP-filtered data - while we and most of the literature compute them using data in (log) levels.}\)
by Mendoza [1991] referred to a sample of industrialized countries and set that elasticity equal to 0.74. Stockman and Tesar [1995] estimate a lower elasticity (0.44), but their sample includes both developed and developing countries.

According to the evidence for the U.S. economy in Burstein, Neves and Rebelo [2001], the share of the retail price of traded goods accounted for by local distribution services ranges between 40 percent and 50 percent, depending on the industrial sector. We follow their calibration and set it equal to 50 percent.

As regards the weights of domestic and foreign tradables in the tradables consumption basket ($C_T$), $a_H$ and $a_F$ (normalized $a_H + a_F = 1$) are chosen such that imports are 5 percent of aggregate output in steady state. This corresponds to the average ratio of U.S. imports from Europe, Canada and Japan to U.S. GDP between 1960 and 2002. The weights of traded and non-traded goods, $a_T$ and $a_N$, are chosen as to match the share of nontradables in the U.S. consumption basket. Over the period 1967-2002, this share is equal to 53 percent on average. Consistently, Stockman and Tesar [1995] suggest that the share of nontradables in the consumption basket of the seven largest OECD countries is roughly 50 percent.

We calibrate $\xi$ and $\zeta$, the labor shares in the production of tradables and nontradables, based on the work of Stockman and Tesar [1995]. They calculate these shares to be equal to 61 percent and 56 percent, respectively.

The elasticity of substitution between Home and Foreign tradables

The quantitative literature has proposed a variety of values for the elasticity of substitution between traded goods. For instance, Backus, Kydland, and Kehoe [1995] set it equal to 1.5, whereas Heathcote and Perri [2002] estimate it to be 0.9. Here, we set the elasticity of substitution $\omega$ to match the volatility of the U.S. real exchange rate relative to that of U.S. output, equal to 3.28 (see Table 3). In Section 2.2, we have used our simple model to show that the volatility of international prices is hump-shaped in $\omega$, and discussed at length the mechanism underlying this pattern. Consistently, in our model we find two values for the elasticity $\omega$ such that the model matches the volatility of the U.S. real exchange rate, namely, $\omega = 0.99$ and $\omega = 1.11$. While apparently close to each other, these values imply quite different

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14 There is considerable uncertainty regarding the true value of trade elasticities, directly related to this parameter. For instance, Taylor [1993] estimates the value for the U.S. to be 0.39, while Whalley [1985], in the study quoted by Backus et al. [1995], reports a value of 1.5. For European countries most empirical studies suggest a value below 1.
dynamics and international transmission patterns for shocks to tradables productivity. These differences will become central to our discussion of the evidence in Section 6.

5 Real exchange rate volatility and the international transmission of productivity shocks

Our goal in this section is to verify whether our model can match the empirical evidence on the unconditional correlation between international prices and quantities, as well as the their relative volatilities. The evidence is summarized by the statistics reported in the first column of Tables 3 and 4. The statistics for the data — all filtered using the Hodrick and Prescott filter — are computed with the United States as the home country and an aggregate of the OECD comprising the European Union, Japan and Canada as the foreign country.\footnote{Here we follow Heathcote and Perri [2002]. See the Data Appendix for details.} Notably, the Backus Smith correlation between relative consumption and the real exchange rate is equal to -0.45.

In what follows, we will show that, different from standard open-economy models, our artificial economy performs quite well in this dimension. Throughout our exercises, we will compute statistics by logging and filtering the model’s artificial time series using the Hodrick and Prescott filter and averaging moments across 100 simulations. The results for our baseline model and some variations on it are also shown in Tables 3 and 4.

5.1 Volatilities and correlation properties

The real exchange rate and the terms of trade Using our framework, we can write the real exchange rate ($RER$) in the following log-linear form, reflecting movements in the terms of trade as well as in the relative price of non-traded goods:

$$
\hat{RER}_t = (1 - \mu) (2a_H - 1) \hat{\pi}_t + \mu \left( \hat{P}_{N,t}^* - \hat{P}_{N,t} \right) + \Omega \left( \hat{q}_t - \hat{q} \right),
$$

(19)

where $0 < \Omega < 1$ and $\hat{q}_t$ represents the relative price of nontradables.\footnote{Namely, $\Omega = a_N \bar{q}^{\frac{\sigma - 1}{\sigma}} / (a_T + a_N \bar{q}^{\frac{\sigma - 1}{\sigma}}) > 0$, where $\bar{q}$ denotes a steady-state value and $\frac{1}{1 - \phi}$ is the elasticity of substitution between tradables and nontradables.} In our numerical results, it is the first two components, arising from deviations
of the law of one price for the CPI of tradables, which turn out to dominate real exchange-rate movements.

In our baseline economy the real exchange rate and the terms of trade are tightly related. Their correlation is positive (and equal to 0.97 for both values of \( \omega \)), though higher than in the data (0.6). A positive sign for this correlation is an important result relative to alternative models that — like ours — allow for deviations from the law of one price but do so by assuming sticky prices in the buyer’s currency. As argued by Obstfeld and Rogoff [2001], these models can generate high exchange rate volatility as well, but at the cost of inducing a counterfactual negative correlation between the real exchange rate and the terms of trade.

The terms of trade is very volatile, even more than in the data. The volatility of the terms of trade relative to output is 3.04 with \( \omega = 0.99 \), and 4.34 with \( \omega = 1.11 \), compared to 1.79 in the data. In this sense, our model suggests that high volatility of the international prices per se is not a measure of their ‘disconnect’ from fundamentals. To highlight this point, consider the volatility of the import ratio (IR), defined as the ratio of real imports to nonexported tradable output net of investment (empirically, we compute this ratio using manufacturing output). As shown in Table 4, the standard deviation of the import ratio is 4.13 percent in the data. In our benchmark parametrization, it is equal to 2.78 for the smaller \( \omega \), but increases to 4.44 percent for the larger \( \omega \). Hence, as in Backus et al. [1995] and Heathcote and Perri [2002], the variability of international prices is positively related to the variability of the IR, which, in turn, is increasing in \( \omega \).17

Moreover, with \( \omega = 0.99 \) the model is consistent with the ranking of variability in international prices observed in the data: the real exchange rate is more volatile than the terms of trade. The difference in volatility may be due either to the volatility of deviations from the law of one price (which drives a wedge between the terms of trade and relative prices at consumer levels) or to the volatility of nontradable prices, or a combination of the two. For this reason, the correct ranking of volatility is very hard to replicate using models that abstract from the features above (see Heathcoate and Perri [2002]).

We find that the relative price of nontradables across countries is not the main force driving the high volatility of the model’s real exchange rate. Table

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17Remarkably, the data supports the tight and negative link between the terms of trade and the real exchange rate, on the one hand, and the import ratio, on the other hand, predicted by the theory. In the data these correlations stand at -0.68 and -0.41, respectively, against -1 and -0.97 predicted by the model with for either value of \( \omega \).
3 shows that the volatility of the relative price of nontradables predicted by our model is quite in line with that in the data: depending on $\omega$, this volatility is 1.72 and 1.43, against an empirical estimate of 1.73. When we compute the ratio between the standard deviation of the relative price of nontradables across countries, and the standard deviation of the real exchange rate, this ratio is roughly 20 percent. This figure is slightly lower than that estimated by Betts and Kehoe [2001], who find this ratio to be between 35 and 44 percent using a weighted average of U.S. bilateral real exchange rates.\footnote{Following a different procedure, Engel [1999] finds that deviations from the law of one price in traded goods virtually account for all of the volatility of the U.S. real exchange rate.}

**The Backus-Smith correlation** The main result of our baseline model shown in Table 3 is that the correlation between relative consumption and the real exchange rate is not only negative, but also quite close to its empirical counterpart. With both $\omega = 0.99$ and $\omega = 1.11$, the correlation generated by the model is equal to -0.55, against our empirical estimate of -0.45. A similar pattern emerges for the terms of trade: its correlation with relative consumption is -0.72 and -0.73 in the model, against an empirical estimate of -0.53.

Since our two values of $\omega$ are set so that the model fits the empirical volatility of the real exchange rate, our results show that the price elasticity that is consistent with a realistic volatility in international prices also implies a realistic pattern of risk-sharing, quite in line with the data. What generates a negative Backus-Smith correlation is the mechanism linking volatility and risk-sharing derived and discussed in Section 2 using a very simple setting under financial autarky. The same mechanism holds quantitatively in our baseline economy with traded and nontraded goods, capital accumulation and international borrowing and lending.

When international asset trade is limited to uncontingent bonds, the relation between the real exchange rate and marginal utilities of consumption only holds in expected first-differences — log-linearizing the Euler equations for the bond yields (abstracting from the time-varying discount factor):

$$E_t \left( \tilde{R}ER_{t+1} - \tilde{R}ER_t \right) \approx E_t \left[ \left( \tilde{U}_{c,t+1}^* - \tilde{U}_{c,t}^* \right) - \left( \tilde{U}_{c,t+1} - \tilde{U}_{c,t} \right) \right].$$

Without uncertainty, the correlation between the rate of real depreciation and the cross-country differential in the growth rate of consumption will be...
very high and positive, and this tight link will be inherited by the levels of these variables — running against the Backus-Smith evidence. In a stochastic environment, however, the bond is traded only after the resolution of uncertainty and does not provide households with ex-ante insurance against country-specific income shocks — it only makes it possible to reallocate wealth and smooth consumption over time. The impact effect of a shock to tradables in a bond economy will thus be roughly the same as under financial autarky, moving relative consumption and the real exchange rate in opposite directions (for an appropriately low value of the price elasticity).19

The Backus-Smith correlation will therefore be negative on impact, but positive in the aftermath of a shock, when the dynamics of relative consumption and the real exchange rate is determined by the above equation. For this reason, the Backus-Smith correlation in a bond economy will be less negative than under financial autarky. It will also become higher and closer to that implied by complete markets, the weaker the impact response (in absolute value) of the real exchange rate — i.e., the less volatile the real exchange rate and the terms of trade on impact. Obviously, when shocks are permanent, bonds do not provide any means to smooth consumption, so that the bond economy will behave like an economy under financial autarky.20

**International relative prices and business cycles** Consider now the rest of the statistics for the baseline economy in Tables 3 and 4. As is well known, most open-economy models — including those allowing for nominal rigidities and monetary shocks — predict a strong and positive link between relative output and real exchange rates. As Stockman [1998] points out, this prediction is at variance with the data: the empirical correlation shown in Table 3 is -0.23. A similar shortcoming concerns the correlation between relative output and the terms of trade, which is negative in the data (and equal to -0.20), while it tends to be positive in quantitative models.

Our baseline economy yields contrasting results on this issue. The correlation between relative output and the real exchange rate (the terms of

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19The model can also get close to the Backus-Smith statistics even when we look at first-differenced data. As Ravn [2001] argues, the availability of an international bond should imply that the (expected) relative growth rate of consumption across countries be positively and strongly correlated with the (expected) real rate of currency depreciation. In our economy this correlation ex-post is -0.46 (-0.61) when ω equals 0.97 (1.13).

20Since the shock to nontradables is very close to a random walk (its estimated serial correlation with annual data is 0.99), the correlation between relative consumption and the real exchange rate conditional on shocks to the nontraded goods sector only is very close to -1, lower than this same correlation conditional on shocks to tradables.
trade) is high and positive — equal to 0.78 and 0.90 respectively — with \( \omega = 1.11 \), but becomes strongly negative with \( \omega = 0.99 \). This is because, with the lower \( \omega \), positive productivity shocks in the tradable sector appreciate the terms of trade and the real exchange rate — a result that we will discuss in greater detail below. We observe here that this very mechanism also accounts for the ability of the model to match the observed positive correlation between international relative prices and net exports, also shown at the bottom of the table.

In Table 4, we see that the cross-country correlation of output in the model (0.45 and 0.43 depending on \( \omega \)) is very close to that in the data (0.49), and higher than that of consumption. The cross-correlation of consumption is lower than in the data (0.14 and 0.11, against 0.32), while the cross-correlations of investment and employment are higher. However, the model does relatively better in this dimension than the standard real business cycle model — Backus, Kehoe and Kydland [1995] dub this empirical incongruity the ‘quantity anomaly’. It is well known that this class of models — even assuming incomplete markets, with a real bond as the only internationally traded asset (see Heathcote and Perri [2002]) — predicts that consumption should be more correlated across countries than output, and that the correlation across countries of investment and employment is negative.

Finally, a minor discrepancy between the benchmark model and the data is that — relative to output — consumption, investment, and employment are slightly less volatile than in data; net exports are about half as volatile in the model as in the data (0.29/0.40 against 0.63). However, note that our results with \( \omega = 0.99 \) account for countercyclical net exports. Their correlation with GDP is -0.53 in the model, and -0.51 in the data.

**The Arrow-Debreu Economy**  The fourth column of Tables 3 and 4 reports results for an economy with a complete set of Arrow-Debreu securities. Since in such an economy the volatility of the real exchange rate is to a large extent independent of the price elasticity of imports, we only show numerical results for the lower value of \( \omega \) — basically replicating the parameterization in Stockman and Tesar [1995]. As expected, including distribution services in such an environment is not enough to account for the Backus-Smith anomaly. The correlation between the real exchange rate and relative consumption is approximately equal to one. Moreover, the volatility of the real exchange rate, the terms of trade, the import ratio and net
exports is several times lower than that in the data.

Nevertheless, this model generates a negative correlation between the real exchange rate and relative output, in line with the observed one. This is because a productivity gain in the Home tradable sector raises relative output, worsens the Home terms of trade, but appreciates the real exchange rate — the real appreciation reflecting a higher relative price of nontradables and a fall in relative consumption in the period following the shock, driven by a drop in the consumption of nontradables. On the other hand, contrary to the data, the correlation between the terms of trade and relative output is positive, while that between the real exchange rate and the terms of trade is negative.

5.2 Sensitivity analysis

We now assess the sensitivity of our results to (a) removing the distribution sector from our baseline economy, (b) removing cross-country spillovers from the process driving productivity shocks, and (c) using different specifications of investment. We also check whether the Backus-Smith correlation could be explained by a Balassa-Samuelson effect of productivity shocks on consumption and the real exchange rate. Results from these exercises are shown in Tables 3 and 4.

Changing the distribution margin and the elasticity of substitution

When we abstract from distributive trade and set $\eta = 0$, the two values of $\omega$ for which the relative volatility of the real exchange rate in the model is the same as in the data are 0.33 and 0.41, a good deal lower than in our benchmark economy. As discussed in section 3.4, the need to combine tradables with retailing in our baseline economy lowers the price elasticity of imports. Without retailing, for the model to fit the volatility of the exchange rate, we need to assume a relatively lower elasticity of substitution between Home and Foreign goods.

With a lower elasticity of substitution but no retailing, the model still performs remarkably well with respect to the Backus-Smith anomaly: the correlation between the real exchange rate and relative consumption is negative and equal to -0.37 (-0.77) for $\omega = 0.33$ (0.41). The underlying mechanism has already been thoroughly discussed in sections 2.2 and 2.3.

With $\eta = 0$, however, there are no deviations from the law of one price, contradicting an important stylized fact of the international economy (e.g., see Engel [1999]). As a consequence, movements in the relative price of
nontradables across countries contribute to real exchange-rate fluctuations much more than in our benchmark economy. The standard deviation of the relative price of nontradables across countries is now 78 percent of that of the real exchange rate, a fraction which much higher than in the data. Moreover, the relative price of nontradables is more than twice as volatile as in our baseline model with distribution (3.67 and 2.28 against 1.72 and 1.43 depending on $\omega$), as well as in the data (1.73).

**Balassa-Samuelson effects** An interesting issue is whether the Backus-Smith anomaly could be accounted for by a Balassa-Samuelson effect, linking exchange-rate fluctuations to movements in the relative price of nontradables. The idea is as follows. Consider a model in which domestic and foreign tradables are highly substitutable. A positive productivity shock to the tradable sector should appreciate the real exchange rate (terms of trade movements are tiny), and drive up domestic relative to foreign consumption. Is the Backus-Smith correlation driven mainly by this effect?

To address this issue, we abstract from distributive trade $\eta = 0$ and assume a rather high value of $\omega$, equal to 10 — so as to make tradables more homogeneous across countries and reduce the role of the terms of trade in exchange-rate fluctuations (results are the same for higher $\omega$). With such a high elasticity of substitution, the correlation between the real exchange rate and relative output becomes very negative (-0.72), but the corresponding correlation with relative consumption remains close to one, i.e. as high as 0.92. In addition, both the real exchange rate and the terms of trade are a great deal less volatile than output (0.95 and 0.20), while their cross-correlation is substantially lower than in the data (0.13).

**Absence of Spillovers** As shown in Table 2, the process driving productivity that we estimate and use in our model displays substantial cross-country spillovers. How much of our results can be attributed to the magnitude of such spillovers? It turns out that removing them altogether in our numerical exercises does not substantially affect our main conclusions. Adopting the productivity process without spillovers, we again calibrate our economy such that the real exchange rate is as volatile as in the data, obtaining $\omega = 0.93$ and $\omega = 1.16$. The Backus-Smith correlation remains close to the one in our baseline economy: -0.64 and -0.39. However, one significant implication of removing spillovers is that consumption becomes negatively correlated across countries for $\omega = 0.93$. 

27
Changing the investment specification  In our baseline economy investment is carried out solely in domestically produced tradable goods. In our last exercise, we allow for a more general specification in which investment is a composite good comprising both Home and Foreign tradables. We assume that investment goods are given by the following CES aggregator

$$I_{T,t}(j) \equiv \left[a_H^{1-\rho} I_{H,t}(j)^\rho + a_F^{1-\rho} I_{F,t}(j)^\rho\right]^{\frac{1}{\rho}}$$

where $I_{H,t}$ ($I_{F,t}$) is the level of investment in terms of the domestic (imported) traded good. As in our baseline calibration, we set $a_H$ and $a_F$ such that imports (which now also include investment) are 5 percent of aggregate output in steady state. We continue to assume that distribution services are required only to bring tradables to consumers. In Tables 3 and 4 results are shown under the heading “CES Investment.”

With the more general CES specification for investment, the values of $\omega$ needed to reproduce the volatility of the real exchange rate relative to that of output are smaller than under our benchmark calibration. This is because investment goods can now be imported from abroad, and investment does not use distribution services. Thus, any given price elasticity of imports corresponds to a lower elasticity of substitution relative to our baseline specification. Nonetheless, the model still succeeds in generating a significant departure from the complete markets outcome. Although the real exchange rate and relative consumption are not as negatively correlated as in our previous experiments, their correlation remains well below unity. When $\omega = 0.57$, the model predicts a slightly negative correlation of -0.08.

Finally, we report results for an economy without capital accumulation (shown under the heading “No Capital” in the tables). Excluding capital does not substantially change the match of the model with the data along most dimensions. However, for $\omega = 0.97$, consumption becomes more volatile than output (1.09), while the volatility and cross-country correlation of employment are very low (0.12 and -0.52).

5.3 The international transmission of productivity shocks to tradables

In our model, given a value for the distribution margin $\mu$, there are two values of price elasticity and thus of $\omega$ that generate a real exchange-rate volatility matching the evidence. In this subsection, we analyze the difference between these two parameterizations by looking at theoretical impulse responses to
a shock to the traded goods sector. In the next section, we will compare these responses to the estimated ones from an identified VAR.

Our experiments consist of shocking the exogenous process for sectoral productivity by once 1 percent at date 0, when both countries are at their symmetric, deterministic steady state. Figure 1 draws the responses of the following economic variables: (a) the real exchange rate; (b) the terms of trade; (c) relative consumption; (d) relative aggregate output; (e) the ratio of net exports to output. The two columns in Figure 1 report impulse responses for $\omega = 0.99$ and $\omega = 1.11$, respectively.

Consider first the impulse responses under the higher $\omega$ (first column in the figure). Since for this value of the price elasticity world demand for Home tradables is increasing in its relative price, the increase in the supply of Home traded goods relative to the Foreign goods worsens the Home country’s terms of trade. Note that an adverse effect of productivity shocks on the real exchange rate and the terms of trade is predicted by all standard models with product specialization and homothetic preferences (e.g., Lucas [1982] and Backus et al. [1995]).

The notable feature of our specification with incomplete markets is that a relatively low price elasticity of imports (also owing to the presence of retailing) magnifies the deterioration of the Home terms of trade and real exchange rate, increasing the ensuing negative wealth effect for the domestic household. As a result, consumption abroad rises by more than domestic consumption, while domestic output rises relative to the foreign one. Thus, the real exchange rate, the terms of trade and relative output on the one hand, and relative consumption on the other move in the opposite direction, as the large terms of trade worsening entails an excessively positive transmission of the productivity shock in favor of the Foreign country.

The response of the economy to an innovation in the productivity of the domestic traded sector is widely different when $\omega = 0.99$. In this case, relative output still rises, but the real exchange rate and the terms of trade now appreciate. Remember from Section 2 that for a low enough price elasticity

\[ \omega \rightarrow \infty \]
(low enough \( \omega \)), world demand for Home tradables will be negatively sloped in the terms of trade, owing to a prevailing negative income effect for the domestic household. An increase in the relative supply of Home tradables will thus require a terms-of-trade appreciation in equilibrium to bring about market clearing. And as the terms of trade improve, Home consumption rises by more than Foreign consumption. As a result, the real exchange rate, the terms of trade and relative consumption are again negatively correlated, but now relative output will move in the same direction as relative consumption, though by a lesser amount.

To summarize, a productivity shock to the export sector always induces an increase in relative output and (conditional) negative comovements between the real exchange rate, the terms of trade and relative consumption. Depending on the strength of the price-elasticity of imports and thus on the slope of world demand, however, relative consumption can increase or fall in response to a positive shock.

6 Productivity shocks, the real exchange rate and the terms of trade: VAR evidence for the U.S.

In this section we study empirically the comovements between the real exchange rate, the terms of trade, and relative consumption in response to productivity shocks. We adopt a structural VAR approach, extending the work by Galí [1999] — where technology shocks are identified via long-run restrictions — to an open-economy context. We focus our study on the U.S. economy vis-à-vis an aggregate of other OECD countries.

A number of recent papers have investigated the effects on closed-economy macroeconomic variables of technology shocks identified using long-run restrictions. Galí [1999] uses the insight from the standard stochastic growth model that only technology shocks should have a permanent effect on labor productivity to identify economy-wide technology shocks in the data, while there are no analogous long-run restrictions with respect to other macroeconomic variables. In particular, other kinds of shocks can have permanent effects on output, consumption, and investment and external variables like the real exchange rate, the terms of trade, and the trade balance.\(^\text{22}\)

\(^{22}\)See Shapiro and Watson [1988], Blanchard and Quah [1989], Christiano et al. [2003], and Francis and Ramey [2001], among others. Following Blanchard and Quah [1989], some open-economy papers use long-run restrictions derived from the traditional framework of aggregate-demand and aggregate-supply analysis. For instance, Clarida and Galí [1994]
Following these insights, we examine the effects of technology shocks to the U.S. manufacturing sector (a proxy for traded goods) on the real exchange rate, the terms of trade, and relative consumption, by augmenting with these variables the specifications used by the above authors. Moreover, since Chang and Hong [2002] show that using total factor productivity (TFP) instead of labor productivity may affect results for the manufacturing sector, we also assess the robustness of our results to the use of (annual) TFP data. The use of TFP should also control for long-run effects on labor productivity brought about by changes in the long-run capital labor ratio. Leaving to the data appendix a more detailed description of data sources, hereafter we briefly describe our approach and discuss the main results.

Over the period 1970 to 2001, we estimate two specifications of the following structural VAR model

\[
\begin{pmatrix}
\Delta x_t \\
\Delta y_t 
\end{pmatrix} =
\begin{pmatrix}
C^{xz} (L) & C^{xm} (L) \\
C^{yz} (L) & C^{ym} (L)
\end{pmatrix}
\begin{pmatrix}
\Delta \varepsilon^z_t \\
\Delta \varepsilon^m_t
\end{pmatrix}.
\]

Here \(x_t\) denotes the variable that is assumed to be affected in the long run only by permanent technology shocks: in our two different specifications, this variable is equal to (the log of) U.S. quarterly manufacturing labor productivity and (the log of) annual manufacturing TFP, respectively, both measured in deviation from labor productivity in an aggregate of other OECD countries. \(y_t\) is a 3x1 vector of variables, including (the log of) U.S. consumption relative to that of an aggregate of other OECD countries, (the log of) the U.S. real effective (trade-weighted) exchange rate, and the terms of trade (computed as the non-energy imports deflator over the exports deflator).

\(C (L)\) is a polynomial in the lag operator; \(\varepsilon^z_t\) denotes the technology shock to manufacturing, and \(\varepsilon^m_t\) the other structural, non-technology shocks.\(^{23}\) In addition to the usual assumption that the structural shocks are uncorrelated, positing that \(C^{xm} (1) = 0\) is enough to identify \(\varepsilon^z_t\). This restricts the unit root in the variable \(x_t\) to originate solely in the technology shock. Although not necessary for identification, implicit in this benchmark specification is the assumption that all the other variables also have a unit root;

\(^{23}\)We include up to four lags for quarterly data and one for annual data, based on a BIC criterion and tests of residual serial correlation.
this assumption is not rejected by the data over our sample.24

Figure 2 shows the effects of the identified technology shocks on the levels of productivity, relative consumption, the real exchange rate, and the terms of trade.25 The first column is obtained from quarterly data, the second one from annual data. We report standard error bands for the significance levels of 68 percent and 90 percent (corresponding to the darker and lighter shaded areas, respectively).26

The first column in Figure 2 shows the impulse responses using Gali’s identification scheme, with \( x_t \) equal to (relative) U.S. manufacturing labor productivity.27 Following a positive technology shock to manufacturing, U.S. total consumption increases gradually but permanently relative to the rest of the world. Moreover, the real exchange rate and the terms of trade strongly appreciate on impact and remain permanently stronger, by an amount that is larger in the case of the real exchange rate, but that for both variables outsizes the increase in productivity. However, the real exchange rate response is somehow less significant in the long run.

The second column in Figure 2 reports the effects of a technology shock identified as the only shock that permanently affects TFP in U.S. manufacturing. Our results are broadly robust across different long-run identification schemes. In the annual VAR also a positive technology shock to the U.S. production of tradables appears to lead to an increase in domestic consumption relative to the rest of the world, while improving the terms of trade and appreciating the real exchange rate for at least a year.28

---

24 Following the suggestions in Christiano et al. [2003], we also estimated specifications of our VAR with those variables, for which the unit root null is not rejected only marginally, in levels. Our main findings that a technology improvement leads to a persistent terms-of-trade deterioration and real exchange rate depreciation are basically unaltered. These results are not included to save on space, but they are available upon request.

25 We also estimated specifications of the model, adding more U.S. and international variables, like GDP, investment, aggregate hours, and net exports. In all cases we obtain very similar results to those discussed in the text.

26 The standard error bands were computed using a bootstrap Monte Carlo procedure with 500 replications. We thank Yongsung Chang for graciously providing us with his bootstrapping Matlab codes.

27 Despite the changes in variables and the shorter sample period, the results on productivity and hours are very similar to Gali’s results. An identified technology shock to manufacturing leads to an immediate and permanent rise in productivity, while hours worked somehow decline and do not return to near normal for about six quarters.

28 Using cointegrating techniques, Alquist and Chinn (2002) find that each percentage point increase in the U.S.-Euro area economy-wide labor productivity differential results in a 5-percentage-point real appreciation of the dollar in the long run.
To summarize, U.S. consumption relative to the rest of the world and the real exchange rate move in opposite directions, in sharp contrast with the predictions of the perfect risk-sharing hypothesis. Consistent with the Backus-Smith anomaly, the results in this section indicate that following a technology shock to the traded goods sector, real exchange rates and relative consumption can indeed be negatively correlated. Most interestingly, the appreciation of the real exchange rate, and especially the terms of trade, in response to a positive technology shock to domestic tradables, is qualitatively consistent with the transmission mechanism at work in our setup under the lower value of $\omega$. Conversely, it is at odds with the predictions of a vast class of models of international fluctuations, which link increasing world supply of a good to a fall in its relative price.

7 Concluding remarks

In this paper, we develop a model with incomplete asset markets and a low price elasticity of tradables arising from the need to employ distribution services in order to reach final consumers. In numerical exercises with a plausible parameterization of our world economy, we study the international transmission of productivity shocks and account for the high volatility of international prices and the (unconditional) negative link between the real exchange rate and relative consumption observed in the data.

Many contributions to the literature have stressed that movements in the terms of trade in response to country-specific shocks may provide risk insurance to countries specialized in different types of goods. In our model, however, because of deviations from the law of one price and low price-elasticities, these large terms of trade movements are much less effective in providing insurance against production risk and are even counterproductive, in the sense of amplifying the wedge in wealth across countries stemming from asymmetric productivity shocks.

Using structural VAR techniques, we apply long-run restrictions to identify productivity shocks to manufacturing (our measure of tradable goods). We find evidence supporting our prediction of a negative conditional correlation between relative consumptions and international relative prices. Following a permanent positive shock to U.S. labor productivity in manufacturing, domestic output and consumption increase relative to the rest of the world, while both the terms of trade and the real exchange rate appreciate, consistent with the predictions of our model. This result is reasonably robust
to the definition of the terms of trade and the use of TFP instead of labor productivity.

By showing that the terms of trade appreciate in response to a positive productivity shock to tradables, however, our VAR evidence questions the model of international transmission of productivity shocks in most theoretical and empirical contributions to open macro. This result is a challenge to standard open macro models that predict a drop in the international relative price of domestic tradables, generating some degree of risk-sharing even with severe goods and financial markets segmentation. Moreover, several VAR studies have found that the U.S. real exchange rate and terms of trade depreciate following an expansionary monetary policy shock.\textsuperscript{29} Given the relevance of this issue to our understanding of the international transmission of supply shocks and the mechanism of international risk-sharing, further empirical and theoretical work trying to reconcile these apparently conflicting results would prove extremely helpful.

\textsuperscript{29}Clarida and Gali [1994], using long-run restrictions, found that a permanent increase in U.S. relative output appreciates the real exchange rate vis-à-vis Japan and Germany, while an expansionary monetary policy triggers a currency depreciation.
References


A Data Sources

This appendix describes the data used in this paper. The complete dataset is available from the authors upon request.

Calibration dataset To calibrate the process of the shocks for the Home country labor productivity in tradables and nontradables we use the annual BLS series “Index of output per hour in manufacturing” and “Index of output per hour in private services,” respectively. For the Foreign country we use an aggregation of the index of manufacturing output and output in services divided by sectoral total employment for OECD countries obtained from the OECD STAN sectoral database.

U.S. GDP and consumption are chain-weighted 1996 dollar NIPA series from the BEA. World GDP and consumption are constant 1995 PPP dollar series for the total of the OECD countries from the OECD Quarterly National Accounts.

The series for U.S. imports and exports at current and constant prices are NIPA series from the BEA. The series for the U.S. real exchange rate is a trade-weighted measure of the real value of the dollar computed by J.P. Morgan; the series for the U.S. (ex-oil) terms of trade is the ratio of the NIPA (non-oil) import price deflator over the export price deflator from the BEA.

VAR dataset In the estimation of the VAR models we use quarterly data from 1970:1 to 2001:4 and annual data from 1970 to 2001. For the series on U.S. labor productivity (quarterly), total factor productivity (annual), and labor input (quarterly and annual) we use the BLS series “Index of output per hour in manufacturing,” “Index of total factor productivity in manufacturing,” and “Index of hours in manufacturing,” respectively. Hours are put on a per capita basis by dividing by the population of age 16 and above. The quarterly real wage measure is the BLS measure of nominal hourly compensation in manufacturing divided by the BLS producer price index.
Table 1: Correlations between real exchange rates and relative consumptions\(^a\)

<table>
<thead>
<tr>
<th>Country</th>
<th>HP-Filtered</th>
<th>First-Difference</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S.</td>
<td>OECD</td>
<td>U.S.</td>
</tr>
<tr>
<td>Australia</td>
<td>-0.01</td>
<td>0.05</td>
<td>-0.09</td>
</tr>
<tr>
<td>Austria</td>
<td>-0.35</td>
<td>-0.54</td>
<td>-0.20</td>
</tr>
<tr>
<td>Belgium</td>
<td>-0.12</td>
<td>0.15</td>
<td>-0.11</td>
</tr>
<tr>
<td>Canada</td>
<td>-0.41</td>
<td>-0.10</td>
<td>-0.20</td>
</tr>
<tr>
<td>Denmark</td>
<td>-0.16</td>
<td>-0.27</td>
<td>-0.20</td>
</tr>
<tr>
<td>E.U.</td>
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<td>-0.10</td>
<td>-0.23</td>
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<tr>
<td>Finland</td>
<td>-0.27</td>
<td>-0.64</td>
<td>-0.40</td>
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<tr>
<td>France</td>
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<td>-0.21</td>
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<tr>
<td>Germany</td>
<td>-0.27</td>
<td>-0.17</td>
<td>-0.13</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.26</td>
<td>-0.51</td>
<td>-0.27</td>
</tr>
<tr>
<td>Japan</td>
<td>0.09</td>
<td>0.27</td>
<td>0.04</td>
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<tr>
<td>South Korea</td>
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<td>Mexico</td>
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<td>-0.77</td>
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<td>Netherlands</td>
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<td>-0.37</td>
<td>-0.27</td>
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<tr>
<td>Portugal</td>
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<td>-0.73</td>
<td>-0.48</td>
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<td>-0.39</td>
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<td>Switzerland</td>
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<td>0.09</td>
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<td>-0.34</td>
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<td>U.K.</td>
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<td>N/A</td>
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<td>Median(^b)</td>
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<td>-0.27</td>
<td>-0.27</td>
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\(^a\)Data are from the OECD Main Economic Indicators dataset.

\(^b\)In parenthesis the cross-sectional 68 percent confidence bands.

40
Table 2. Parameter values

Benchmark Model

Preferences and Technology

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<th>Value</th>
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<td>Risk aversion</td>
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<tr>
<td>Consumption share</td>
<td>$\alpha = 0.34$</td>
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Elasticity of substitution between:

<table>
<thead>
<tr>
<th>Goods</th>
<th>Value</th>
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<tbody>
<tr>
<td>Home and Foreign traded goods</td>
<td>$\frac{1}{1-\rho} = {0.99, 1.11}$</td>
</tr>
<tr>
<td>Home and Foreign traded goods</td>
<td>$\frac{1}{1-\varrho} = 0.74$</td>
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</tbody>
</table>

<table>
<thead>
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<th>Goods</th>
<th>Value</th>
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<tbody>
<tr>
<td>Share of Home Traded goods</td>
<td>$a_H = 0.72$</td>
</tr>
<tr>
<td>Share of non-traded goods</td>
<td>$a_N = 0.45$</td>
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Elasticity of the discount factor with respect to $C$ and $L$

<table>
<thead>
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<tbody>
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<td>$\psi$</td>
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Distribution Margin

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<td>$\eta$</td>
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Labor Share in Tradables

<table>
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<tr>
<td>$\xi$</td>
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Labor Share in Nontradables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta$</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Productivity Shocks

$$\lambda = \begin{bmatrix}
0.78 & 0.11 & 0.19 & 0.31 \\
0.11 & 0.78 & 0.31 & 0.19 \\
-0.04 & 0.01 & 0.99 & 0.05 \\
0.01 & 0.04 & 0.05 & 0.99
\end{bmatrix}$$

Variance-Covariance Matrix (in percent)

$$\lambda = \begin{bmatrix}
0.054 & 0.026 & 0.003 & 0.015 \\
0.026 & 0.054 & 0.015 & 0.003 \\
0.003 & -0.001 & 0.008 & 0 \\
-0.001 & 0.003 & 0 & 0.008
\end{bmatrix}$$
### Variations on the benchmark economy

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<thead>
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<th>Statistics</th>
<th>Data</th>
<th>Benchmark Economy</th>
<th>Arrow-Debreu Economy</th>
<th>No Spillover</th>
<th>No Investment</th>
<th>No Capital</th>
<th>No Distribution</th>
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<td></td>
<td></td>
<td>$\omega = 0.99$</td>
<td>$\omega = 1.11$</td>
<td>$\omega = 0.99$</td>
<td>$\omega = 1.16$</td>
<td>$\omega = 0.40$</td>
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<td>Real exchange rate</td>
<td>3.28</td>
<td>3.28</td>
<td>3.28</td>
<td>0.79</td>
<td>3.28</td>
<td>3.28</td>
<td>3.28</td>
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<td>Terms of trade</td>
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<td>0.61</td>
<td>3.23</td>
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<td>2.49</td>
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<td>Relative price of nontradables</td>
<td>1.73</td>
<td>1.72</td>
<td>1.43</td>
<td>1.24</td>
<td>1.64</td>
<td>1.51</td>
<td>1.47</td>
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<td>Cross-correlations</td>
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<td></td>
<td></td>
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<td>Between real exchange rate and Relative GDPs</td>
<td>-0.23</td>
<td>-0.97</td>
<td>0.78</td>
<td>-0.48</td>
<td>-0.98</td>
<td>0.70</td>
<td>-0.88</td>
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<tr>
<td>Relative consumptions</td>
<td>-0.45</td>
<td>-0.55</td>
<td>-0.55</td>
<td>0.98</td>
<td>-0.61</td>
<td>-0.39</td>
<td>0.31</td>
</tr>
<tr>
<td>Net exports</td>
<td>0.39</td>
<td>0.95</td>
<td>0.95</td>
<td>-0.73</td>
<td>0.94</td>
<td>0.94</td>
<td>0.88</td>
</tr>
<tr>
<td>Terms of trade</td>
<td>0.60</td>
<td>0.97</td>
<td>0.97</td>
<td>-0.12</td>
<td>0.96</td>
<td>0.96</td>
<td>0.99</td>
</tr>
<tr>
<td>Between terms of trade and Relative GDPs</td>
<td>-0.20</td>
<td>-0.91</td>
<td>0.90</td>
<td>0.82</td>
<td>-0.82</td>
<td>0.87</td>
<td>-0.89</td>
</tr>
<tr>
<td>Relative consumptions</td>
<td>-0.53</td>
<td>-0.72</td>
<td>-0.73</td>
<td>0.03</td>
<td>-0.81</td>
<td>-0.61</td>
<td>0.21</td>
</tr>
<tr>
<td>Net exports</td>
<td>0.43</td>
<td>0.99</td>
<td>0.99</td>
<td>0.74</td>
<td>0.99</td>
<td>0.99</td>
<td>0.92</td>
</tr>
</tbody>
</table>

*HER is the real exchange rate, TOT is the terms of trade.
Table 4. Business cycle statistics in the theoretical economies

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Benchmark Economy</th>
<th>Arrow-Debreu Economy</th>
<th>No Spillover</th>
<th>CES Investment</th>
<th>No Capital</th>
<th>No Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(\omega = 0.99)</td>
<td>(\omega = 1.11)</td>
<td>(\omega = 0.93)</td>
<td>(\omega = 1.16)</td>
<td>(\omega = 0.40)</td>
<td>(\omega = 0.57)</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.92</td>
<td>0.64</td>
<td>0.56</td>
<td>0.55</td>
<td>0.49</td>
<td>0.56</td>
<td>0.55</td>
</tr>
<tr>
<td>Investment</td>
<td>4.25</td>
<td>3.88</td>
<td>3.87</td>
<td>3.88</td>
<td>3.74</td>
<td>3.73</td>
<td>4.38</td>
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<tr>
<td>Employment</td>
<td>1.09</td>
<td>0.67</td>
<td>0.68</td>
<td>0.67</td>
<td>0.63</td>
<td>0.64</td>
<td>0.69</td>
</tr>
<tr>
<td>Import ratio</td>
<td>4.13</td>
<td>2.78</td>
<td>4.44</td>
<td>0.54</td>
<td>2.74</td>
<td>4.45</td>
<td>0.92</td>
</tr>
<tr>
<td>Net exports over GDP</td>
<td>0.63</td>
<td>0.29</td>
<td>0.40</td>
<td>0.04</td>
<td>0.30</td>
<td>0.38</td>
<td>0.04</td>
</tr>
</tbody>
</table>

**Cross-correlations**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Benchmark Economy</th>
<th>Arrow-Debreu Economy</th>
<th>No Spillover</th>
<th>CES Investment</th>
<th>No Capital</th>
<th>No Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between foreign and domestic GDP</td>
<td>0.49</td>
<td>0.45</td>
<td>0.43</td>
<td>0.45</td>
<td>0.44</td>
<td>0.42</td>
<td>0.43</td>
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<tr>
<td></td>
<td>0.32</td>
<td>0.14</td>
<td>0.11</td>
<td>0.48</td>
<td>-0.17</td>
<td>0.13</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
<td>0.48</td>
<td>0.45</td>
<td>0.47</td>
<td>0.48</td>
<td>0.46</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>0.32</td>
<td>0.47</td>
<td>0.41</td>
<td>0.46</td>
<td>0.49</td>
<td>0.43</td>
<td>0.37</td>
</tr>
<tr>
<td>Between net exports and GDP</td>
<td>-0.51</td>
<td>-0.53</td>
<td>0.58</td>
<td>0.58</td>
<td>-0.51</td>
<td>0.58</td>
<td>-0.62</td>
</tr>
</tbody>
</table>

\(\omega = \frac{1}{1 - \rho}\) denotes the elasticity of substitution between Home and Foreign traded goods.
Figure 1
Theoretical Responses to a Technology Shock in the Traded-Goods Sector

<table>
<thead>
<tr>
<th>Low Elasticity of Substitution</th>
<th>High Elasticity of Substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RER</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td><strong>TOT</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td><strong>C - C*</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
<tr>
<td><strong>Y - Y*</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
</tr>
<tr>
<td><strong>NX/Y</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image9" alt="Graph" /></td>
<td><img src="image10" alt="Graph" /></td>
</tr>
</tbody>
</table>

All series are in percent.
Figure 2
Impulse Responses to a Technology Shock in the Traded-Goods Sector

Quarterly Data

Annual Data

Productivity

RER

TOT

C - C*

Y - Y*

NX/Y

All series are in percent.