



Department of Economics

Essays on Innovation, R&D Policy and Industrial Clusters

David Horan

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

Florence, April 2012

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Department of Economics

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Abstract

The first chapter studies how government should allocate R&D subsidies when firms interact through their R&D decisions. In practice, public agencies often distribute funding on a case-by-case (CbC) basis ignoring indirect effects that may arise due to firm interaction. The paper finds that when complementary knowledge spillovers occur among firms receiving support to research on independent products, knowledge spillovers induce complementary R&D interactions and CbC funding is socially excessive. By contrast, when no knowledge spillovers occur among firms receiving support to research on substitute products, product market rivalry induces substitutive R&D interactions and CbC funding is socially insufficient. An adjusted CbC rule is then proposed which corrects the inefficiency in CbC funding arising from a general pattern of bilateral influences.

The second chapter presents a model of the agglomeration of an oligopolistic industry to study the effect of cluster size on firm performance and firms' incentives to cluster. The model captures a distinctive feature of agglomeration among firms which produce close substitutes: the interaction of agglomeration externalities and negative pecuniary externalities. The paper finds that i) the performance of cluster firms exhibits a rise-and-fall pattern with respect to cluster size; ii) neither complete agglomeration nor complete dispersion of firms is socially desirable; and iii) firms' unilateral incentives drive them to agglomerate completely. This suggests that the private incentives to agglomerate of competing firms may be socially excessive. The paper also compares the performance of cluster and isolated firms, which is relevant for situations where geography or government constrains firms' location decisions.

The third chapter develops a model to study how government should subsidize investment, e.g. R&D, when firms are located in clusters and agglomeration externalities are present, e.g. local knowledge spillovers. The analysis focuses on industry spatial patterns characterized by a single core and peripheral cluster. The paper finds that asymmetries between core and peripheral cluster sizes create differential subsidy effects: i) the additionality effect of a subsidy on cluster firm investment is relatively stronger for a peripheral firm subsidy (expansion effect); and ii) the crowding-out effect of a subsidy on non-cluster firm investment is relatively stronger for a core firm subsidy (sitting-duck effect). We find the sitting-duck effect dominates the expansion effect. The main policy implication is that if government is justified in funding both core and peripheral firms then alongside core firm subsidies, government must provide adequate funding to peripheral firms to counteract the sitting-duck effect. The paper also finds that case-by-case subsidization is biased towards favouring firms in the core cluster.

ACKNOWLEDGMENTS

This thesis would not have been possible without the help and support of many people. Throughout my Ph.D studies I have had the opportunity to work under the supervision of Professor Pascal Courty. I would like to thank him for his guidance, patience, support and many helpful discussions and suggestions.

I am also grateful to my second reader Professor Andrea Mattozzi, as well as to Professor Alfonso Gambardella, Professor Luigi Guiso, Professor José Luis Moraga-González, Professor Omar Licandro, Professor Fernando Vega-Redondo and Professor Sean Dineen, for their help and comments on parts of the thesis.

During the last few years I have benefited from discussions with and comments by many of my colleagues at the EUI. These include Christoph Weiss, Oskar Nelvin, Sarah Stolting, Gizem Korkmaz, Charles Gottlieb, Anders Herlitz, Sami Stouli, Mark Jones, Oege Dijk, David Pothier and many others.

I would like to thank my family for their constant and unconditional support. I would also like to thank Shay Zabari and Micol Nacamulli for their patient support.

Finally, I would like to acknowledge the financial support I have received from the Irish Department of Education and Science and the European University Institute.

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Part I

Introduction

The main economic justification for R&D policy is R&D spillovers. Yet one of the most striking features of R&D spillovers is their diversity. Innovation tends to give rise to a rich array of externalities, both positive and negative, that can interact in non-trivial ways. Another striking feature of R&D spillovers is that proximity between firms, e.g. geographic or technological, tends to enhance inter-firm spillover exchanges. Therefore, it is difficult to formulate appropriate public policy towards R&D, e.g. in a specific industry, without a deep understanding of the nature of the R&D spillovers at play and how proximity (e.g. industrial clusters) shapes the intensity of these spillover effects.

On the other hand, a striking feature of R&D policy is that government agencies tend to use simple rules to allocate funding, e.g. funding decisions are often made on a case-by-case basis. Since the nature of R&D spillovers can differ greatly from one industry setting to the next, investigating the performance of government allocation rules for R&D subsidies, in the presence of different configurations of interacting R&D spillover effects, presents an avenue for improving R&D policy making.

The title of the thesis "*Essays on Innovation, R&D Policy and Industrial Clusters*" reflects its attempt to tackle some of these issues. The thesis studies R&D subsidy policy and the nature of R&D spillovers from an industrial organization perspective integrating tools from social network theory. All three chapters are theoretical. All three chapters deal in one way or another with the interplay of knowledge spillovers and business stealing effects stemming from innovation by product market rivals. The first chapter examines how the interplay between these two externalities, by shaping the nature of inter-firm R&D interaction, influences the performance of R&D subsidy programs in which funding decisions are made on a case-by-case basis. The second chapter highlights how a consideration of the interplay between local knowledge spillovers and business stealing effects can overturn conventional wisdom on why firms within a particular industry tend to cluster and the relationship between cluster size and firm innovative performance. Since industrial clusters are one of the most striking features of the economic landscape and venues of enhanced inter-firm knowledge exchanges, the last chapter examines how an industry's spatial pattern can influence the effect of R&D subsidies on firm R&D investment.

The first chapter studies how government should allocate R&D subsidies when firms interact through their R&D decisions. In practice, public agencies often distribute funding on a case-by-case (CbC) basis ignoring indirect effects that may arise due to firm interaction. The paper finds that when complementary knowledge spillovers occur among firms receiving

support to research on independent products, knowledge spillovers induce complementary R&D interactions and CbC funding is socially excessive. By contrast, when no knowledge spillovers occur among firms receiving support to research on substitute products, product market rivalry induces substitutive R&D interactions and CbC funding is socially insufficient. An adjusted CbC rule is then proposed which corrects the inefficiency in CbC funding arising from a general pattern of bilateral influences.

The second chapter presents a model of the agglomeration of an oligopolistic industry to study the effect of cluster size on firm performance and firms' incentives to cluster. The model captures a distinctive feature of agglomeration among firms which produce close substitutes: the interaction of agglomeration externalities and negative pecuniary externalities. The paper finds that i) the performance of cluster firms exhibits a rise-and-fall pattern with respect to cluster size; ii) neither complete agglomeration nor complete dispersion of firms is socially desirable; and iii) firms' unilateral incentives drive them to agglomerate completely. This suggests that the private incentives to agglomerate of competing firms may be socially excessive. The paper also compares the performance of cluster and isolated firms, which is relevant for situations where geography or government constrains firms' location decisions.

The third chapter develops a model to study how government should subsidize investment, e.g. R&D, when firms are located in clusters and agglomeration externalities are present, e.g. local knowledge spillovers. The analysis focuses on industry spatial patterns characterized by a single core and peripheral cluster. The paper finds that asymmetries between core and peripheral cluster sizes create differential subsidy effects: i) the additionality effect of a subsidy on cluster firm investment is relatively stronger for a peripheral firm subsidy (expansion effect); and ii) the crowding-out effect of a subsidy on non-cluster firm investment is relatively stronger for a core firm subsidy (sitting-duck effect). We find the sitting-duck effect dominates the expansion effect. The main policy implication is that if government is justified in funding both core and peripheral firms then alongside core firm subsidies, government must provide adequate funding to peripheral firms to counteract the sitting-duck effect. The paper also finds that case-by-case subsidization is biased towards favouring firms in the core cluster.

Part II

Chapters

CHAPTER 1

INTER-FIRM R&D INTERACTIONS AND THE CASE-BY-CASE SUBSIDY RULE

1.1 Introduction

Government agencies tend to apply simple rules to allocate R&D subsidies with funding often distributed on a case-by-case (CbC) basis even when firms interact through their R&D decisions.¹ For example, in Israel, OCS R&D programs funded all eligible projects at a predetermined percentage of the scheduled project cost. Most of this funding was allocated to large firms in the electronics and chemical industries and evidence suggests substantial substitutability between firms' R&D decisions may have occurred due to a serious shortage of skilled R&D workers (Lach, 2002).

This article identifies inter-firm interaction in R&D as a source of inefficiency in CbC funding. The paper finds that under CbC funding, government over-funds research in situations in which innovations tend to be complementary and under-funds research in situations in which innovations tend to be substitutive (provided sufficient external social benefits exist to justify funding in the first place). A modified case-by-case subsidy allocation rule is proposed which internalizes the externalities arising from a general pattern of interaction in R&D.

The majority of government programs which fund business R&D have a similar form: firms submit project applications to a public agency, the agency then decides which projects to fund and subsidies are determined.² The main justification behind these programs lies in the mass of evidence, both theoretical and empirical, that, in the absence of intervention, firms under-invest in R&D due to the existence of R&D spillovers (Arrow, 1962; Nelson, 1959; Griliches, 1992). However, the government's objective of funding R&D to a socially desirable level is complicated by the possibility that subsidies may displace, i.e. crowd-out

¹Governments spend a considerable amount of resources funding business R&D. For example, public support amounted to \$35 billion in OECD countries in 2005. This represents one-fifth of government R&D spending and 7% of business R&D spending (OECD, 2007).

²See, e.g., Tanayama (2006), Giebes et al. (2006), Duch-Brown et al. (2008) and Serrano-Velarde (2008) for descriptions of R&D subsidy programs.

or stimulate, privately-financed R&D (David et al., 2000; Klette et al., 2000).³

Apart from a few notable exceptions, there are few papers which study optimal allocation rules for R&D subsidies (Jaffe, 1996; Blum et al. 2003; Giebes et al., 2006). Previous research generally abstracts from the issue of funding in the presence of interaction in R&D decisions. Consequently, the displacement problem is typically seen in the context of the effect of a subsidy on the recipient firm's R&D. The selection rule usually recommended by the literature is that government select the best projects from a ranking of individual projects based on some measure of the spillover gap (Jaffe, 1996).

This spillover gap is the difference between a project's expected social and private returns. The argument is that by focusing funding on projects with large spillover gaps, the agency can "minimize displacement by maximizing the spillover gap" (Jaffe, 1996). A large spillover gap might signal the possibility that social returns are high at the same time that the risk of displacement is low (Jaffe, 1996). An important feature of the rule is it ignores possible indirect effects on R&D which may arise due to interaction between firms in R&D.

In general, the relationship between the R&D decisions of firms may be neutral, complementary or substitutive. Empirical evidence regarding the relationship between own and other's R&D suggests R&D complementarities are important in many cases (Jaffe, 1986; Levin, 1988; Bernstein, 1988; Cohen et al. 1993; Geroski et al., 1993; Branstetter et al., 1998). Bernstein (1988) finds a complementary relationship in R&D intensive industries and a substitute relationship in industries with moderate R&D spending. Popp (2009) finds a substitute relationship in the alternative energy industry and a neutral relationship in automotive manufacturing.⁴

This paper develops a two-stage game to investigate the performance of CbC funding in the presence of interaction in R&D. In the first stage, government announces an R&D subsidy for each firm. These subsidies are calculated on a CbC basis. In the second stage, firms decide on a level of investment in R&D. In this model, interaction in R&D is taken to be exogenous.

³If the agency subsidises a project with a high social return but in doing so stimulates (crowds-out) privately-financed R&D then the publicly funded level of investment will be socially excessive (insufficient) relative to the socially optimal level of investment. For example, suppose a project's *laissez faire* and socially optimal R&D spending are \$30m and \$50m respectively (m denotes million). Suppose the government subsidy is \$20m. If publicly funded R&D is \$60m (\$45m) then the subsidy stimulated (crowds-out) privately financed R&D by \$10m (\$5m) implying that publicly funded R&D is socially excessive (insufficient).

⁴According to Levin (1988), innovations in industries such as aircraft, computers, electronics components and communications equipment tend to be complementary and innovations in the chemical and drug industries tend to be substitutive.

The measure of the spillover gap used by government in this model is the investment gap, i.e. the difference between social optimum and laissez-faire R&D. It is assumed government knows the investment gap associated with each firm's R&D project and chooses a subsidy to "close the gap" ignoring possible indirect effects of subsidies on R&D.

The main results are that in the absence of interaction in R&D, CbC funding implements social optimum R&D. However, if R&D complementarity (substitutability) pervades among funded firms then the subsidy program indirectly stimulates (crowds-out) privately-financed R&D and CbC funding is socially excessive (insufficient).

The basic intuition behind the result is straight forward: when R&D complementarity (substitutability) pervades among firms, subsidizing a firm indirectly stimulates (crowds-out) investment by rival firms. When government moves in a case-by-case manner to fund one of these firms, it does not anticipate that the rival firm's investment gap is now smaller (larger) and consequently over-funds (under-funds) the firm.

The paper then investigates how the efficiency of the CbC rule depends on the relative intensity of knowledge spillovers and product market rivalry among funded firms. In particular, recent empirical evidence suggests that i) firm R&D performance is affected by at least two countervailing spillovers: positive effects from knowledge spillovers and negative business stealing effects from R&D by product market rivals;⁵ and ii) the relative intensity of these spillovers can differ greatly from one industry setting to the next (see, e.g., Branstetter et al. (2003), Bloom et al. (2007), and Gambardella et al. (2008)).⁶

A Cournot market competition stage is added to the model's set up which incorporates these two effects. The main result is that under CbC funding, government over-funds research in situations characterized by high knowledge spillovers and weak product market rivalry among funded firms. On the other hand, government under-funds research in situations characterized by low knowledge spillovers and strong product market rivalry among

⁵One of the primary motivations for firms to invest in R&D is to gain competitive advantage vis-a-vis their rivals (Branden and Spencer, 1983). Interaction in R&D can arise because of the interplay between innovation and market structure. On the one hand, firm incentives to innovate depend on the intensity of competition in the product market, e.g. number of rivals, industry concentration, degree of product substitutability, nature of competition (Arrow, 1962). On the other hand, innovation shapes market structure thereby influencing investment incentives, e.g. when R&D increases market share or keeps potential competition at bay (Gilbert et al., 1982).

⁶Bloom et al. (2007) develop a methodology to identify the separate effects of knowledge spillovers and product market rivalry on private inventive activity based on technological and product market closeness measures similar to Jaffe (1986). For evidence on differences across industries, see their findings on the pharmaceutical, computer hardware and telecommunications industries.

funded firms in which innovations create sufficiently large external social benefits to justify subsidization in the first place.⁷

The basic intuition behind the result is straight forward: if a rival firm innovates then i) receiving knowledge spillovers increases the profitability of a firm's R&D project thereby raising the firm's incentive to innovate; and ii) business stealing effects lower the profitability of the firm's R&D project thereby lowering the firm's incentive to innovate. Therefore, if the positive effect of knowledge spillovers offsets (is offset by) the negative effect of innovation by a product market rival then complementary (substitutive) interactions arise between firms in R&D.

The paper then proposes an adjusted CbC rule which is found to correct the inefficiency in CbC funding arising from a general pattern of bilateral influences. This rule involves two steps. First, government calculates the CbC subsidy program. Second, government adjusts each firm's CbC subsidy by subtracting the CbC subsidy of each rival weighted according to the (normalized) magnitude of bilateral influence exerted *by the rival on the firm*.

Under the adjusted CbC rule, if a firm's R&D project is complementary (substitutive) to projects with large investment gaps or intensely complementary (substitutive) to projects with moderate investment gaps then either of these relationships substantially lowers (raises) the firm's CbC subsidy (*cet. paribus*).

The article contributes to the literature on optimal allocation rules for R&D subsidies (Adam Jaffe, 1996, 2002; Blum et al. 2003; Giebes, 2009). The convention in this literature has been to abstract from interactions between firms in R&D decisions. This paper suggests ignoring such interactions results in over-funding of research in the presence of pervasive complementarity in R&D decisions and under-funding of research in the presence of pervasive substitutability in R&D decisions. The paper contributes to this literature by providing a simple allocation rule for correcting this inefficiency in CbC funding.

The econometric literature which studies the effect of R&D subsidies on private R&D spending offers a plethora of mixed evidence (see Hall et al. (2000) and Klette et al. (2000) for surveys of this literature. For more recent work, see Serrano (2009) and the references cited therein). A criticism of the empirical literature is the failure to distinguish key features among the various policy "experiments" being considered (Hall, 2000). This paper

⁷Goolsbee (1998) finds that firm R&D performance is negatively affected by an inelastic supply of R&D inputs. We incorporate this effect into our model and find that it is a source of substitutability in R&D and can remove (create) a CbC over-funding (under-funding) problem when knowledge spillovers dominate (are balanced by) product market rivalry.

contributes to this literature by providing a model which sheds new light on the conditions under which R&D subsidies stimulate or crowd-out private R&D. It is hoped the theoretical framework developed in this paper helps in interpreting and reconciling some of the literatures mixed empirical evidence.

The paper also contributes to the emerging literature on the different impacts of innovation across industries based on knowledge spillover-product market rivalry industry characteristics (see, e.g., McGahan and Silverman (2006), Bloom et al. (2007) and Gambardella et al. (2008)). The article contributes to this literature by providing theoretical evidence which suggests that, in addition to the different innovation impacts, the performance of R&D subsidies differs across industries according to these same industry characteristics.

1.2 Model and Analysis

Consider an industry consisting of n firms. Each firm $i = 1, \dots, n$ chooses R&D effort $x_i \geq 0$ given the subsidy s_i it receives from government. The interdependent, bilinear R&D payoffs of the firms are given by

$$\pi_i(x_1, \dots, x_n) = \alpha x_i - \frac{1}{2} \sigma_I x_i^2 + \sum_{j \neq i} \sigma_C x_i x_j + s_i x_i,$$

in which $\alpha > 0$, $\sigma_I > 0$ and σ_C are exogenously given constants.⁸ Interaction in R&D is captured by the cross derivative $\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} = \sigma_C$, for all $i \neq j$, which can be of either sign. This cross derivative measures the change in firm i 's incentive to innovate due to a marginal increase in firm j 's R&D effort. When $\sigma_C > 0$, the R&D efforts of firms are strategic complements and R&D complementarity is said to be pervasive in the industry. When $\sigma_C < 0$, the R&D efforts of firms are strategic substitutes and R&D substitutability is said to be pervasive in the industry. The restriction $\sigma_I > 2(n-1)\sigma_C$ ensures the existence of a unique interior Nash equilibrium denoted by $x^* = (x_1^*, \dots, x_n^*)$.

It is assumed that in the absence of government intervention firms under-invest in R&D. Formally, suppose social welfare is defined as the sum of industry profits and external social benefits arising from R&D spillovers to consumers and other firms outside the industry, minus the cost of the subsidy program:⁹

⁸This specification is a special case of the payoff functions presented in Ballester et al. (2006) in their research on peer effects.

⁹The model assumes R&D spillovers to firms outside the industry do not influence their R&D decisions. For example, innovations in computer software can significantly improve the productivity and hence profitability of other firms without influencing their R&D decisions.

$$W(x_1, \dots, x_n) = \sum_i \pi_i(x_1, \dots, x_n) + B(\sum_i x_i) - \sum_i s_i x_i,$$

in which $B \geq 0$ measures the value of these external social benefits. Denote by $x^{**} = (x_1^{**}, \dots, x_n^{**})$ the social optimum R&D profile associated with W and denote by $\hat{x} := x^*|_{(s_1, \dots, s_n)=0}$ the laissez-faire equilibrium R&D profile. Then, the investment gap of firm i , denoted Δ_i , is defined as the difference between the socially desirable and laissez faire R&D effort of the firm and must satisfy

$$\Delta_i := x_i^{**} - \hat{x}_i > 0, \text{ for all } i = 1, \dots, n.$$

Notice that in the presence of R&D substitutability, the investment gap is strictly positive provided external social benefits are sufficiently large.¹⁰ In the first stage, government calculates a subsidy for each firm under the CbC rule. Denote by s_i^{cbc} the CbC subsidy to firm i . It is assumed government knows the investment gap associated with each firm's R&D activity and correctly anticipates the direct effect of a subsidy on the recipient firm's R&D, denoted by $d_i^*(s_i)$. Given Δ_i and $d_i^*(s_i)$, government chooses s_i^{cbc} in order to close the firm's investment gap:

$$\Delta_i = d_i^*(s_i^{cbc}), \text{ for all } i = 1, \dots, n.$$

Notice that intervention takes the form of n independent subsidy decisions, one for each firm, and government ignores possible indirect subsidy effects that may arise from interaction in R&D.

1.2.1 Case-by-case Funding and R&D Interactions

The game is solved using backward induction. In the final stage, each firm chooses R&D effort to maximise its payoff given the R&D efforts of rival firms and the subsidy it receives from government. The first order condition is given by

$$\frac{\partial \pi_i}{\partial x_i} = \alpha + s_i - \sigma_I x_i + \sum_{j \neq i} \sigma_C x_j = 0.$$

Rearranging terms, the best response function of firm i is

$$b_i(s_i, (x_j)_{j \neq i}) = \frac{1}{\sigma_I} \alpha + \frac{1}{\sigma_I} s_i + \sum_{j \neq i} \frac{\sigma_C}{\sigma_I} x_j.$$

¹⁰When $\sigma_C < 0$ the restriction $B > -\frac{(n-1)\sigma_C}{\sigma_I - (n-1)\sigma_C}$ ensures $\Delta_i > 0$ for all $i = 1, \dots, n$.

Applying the implicit function theorem to this system of equations, there exists n differentiable implicit functions $\{x_i(s_1, \dots, s_n)\}_i$ such that equilibrium industry R&D can be written as

$$\sum_i x_i^* = \sum_i b_i(s_i, (x_j(s_1, \dots, s_n))_{j \neq i}).$$

Totally differentiating this expression with respect to s_i , around (x_1^*, \dots, x_n^*) , decomposes the impact of a subsidy on R&D into a direct and indirect effect:

$$\begin{aligned} \frac{d(\sum_j x_j^*)}{ds_i} = & \underbrace{\frac{\partial b_i}{\partial s_i}}_{\text{direct effect}(D_i)} + \underbrace{\left(\sum_{j \neq i} \frac{\partial b_i}{\partial x_j} \frac{\partial x_j}{\partial s_i} + \sum_{j \neq i} \left(\sum_{l \neq j} \frac{\partial b_j}{\partial x_l} \frac{\partial x_l}{\partial s_i} \right) \right)}_{\text{indirect effect}} \\ & \underbrace{\text{feedback effect}(F_i) + \text{cross effect}(C_{ji})}_{\text{indirect effect}} \end{aligned}$$

The direct effect of subsidy on R&D, D_i , is the recipient firm's response to the subsidy with the actions of rival firms held fixed. This direct effect can be computed from the firm's best response function and is given by $D_i = \frac{\partial b_i}{\partial s_i} = \frac{1}{\sigma_I}$. The indirect effect of a subsidy on R&D consists of feedback effect and $n-1$ cross effects. The cross effect, C_{ji} , is a rival firm's response to the subsidy-induced change in other firms' R&D. The feedback effect, F_i , is the recipient firm's response to the subsidy-induced change in rival firms' R&D.

Solving the system of equations and applying this decomposition, equilibrium R&D given the subsidy program can be written as

$$x_i^*(s_1, \dots, s_n) = \hat{x}_i + F_i s_i + \sum_{j \neq i} C_{ij} s_j + D_i s_i,$$

in which $F_i = \frac{(n-1)\sigma_C^2}{\sigma_I A}$, $C_{ij} = \frac{\sigma_C}{A}$, $D_i = \frac{1}{\sigma_I}$ are respectively the feedback, cross and direct effects and $\hat{x}_i = \frac{\sigma_I + \sigma_C}{A} \alpha$ with $A > 0$.¹¹ Notice that in the presence of R&D interactions, $\sigma_C \neq 0$, the subsidy program indirectly displaces firm i 's privately-financed R&D.¹² Firstly, since $F_i \geq 0$, the feedback effect is positive irrespective of the nature of interaction in R&D. Thus, subsidizing firm i indirectly stimulates the firm's privately-financed R&D. Secondly, notice that the sign of the cross effect depends on the nature of R&D interaction. On the one hand, if $\sigma_C > 0$ then $C_{ij} > 0$; hence in the presence of R&D complementarity, a subsidy to firm j indirectly stimulates firm i 's R&D. On the other hand, if $\sigma_C < 0$ then $C_{ij} < 0$;

¹¹See the Appendix for the derivation of this equilibrium R&D expression. Note that $\hat{x}_i = \frac{\sigma_I + \sigma_C}{A} \alpha$ is laissez-faire R&D and $A = (\sigma_I + \sigma_C)(\sigma_I - (n-1)\sigma_C) > 0$ since $\sigma_I > (n-1)\sigma_C$ by assumption.

¹²Note that individual R&D consists of a privately-financed component (laissez-faire plus indirect effects) and a publicly-financed component (direct effect).

hence in the presence of R&D substitutability, a subsidy to firm j indirectly crowds-out firm i 's R&D.

Invoking symmetry, we see that the net effect of the subsidy program, s , on a firm's equilibrium R&D is given by

$$x^*(s) = \hat{x} + Is + Ds,$$

in which $D = \frac{1}{\sigma_I}$ and $I = \frac{(n-1)\sigma_C}{\sigma_I(\sigma_I - (n-1)\sigma_C)}$ are respectively the direct and indirect effects of the subsidy program on individual R&D. This indirect effect consists of the feedback effect of the firm's subsidy and the cross effects of rival firms' subsidies on the firm's R&D. Notice that when complementary R&D interactions arise among firms, $\sigma_C > 0$, the subsidy program indirectly stimulates private R&D, $I > 0$. By contrast, when substitutive R&D interactions arise among firms, $\sigma_C < 0$, the subsidy program indirectly crowds-out private R&D, $I < 0$. These observations give the following result:

Proposition 1.1 *If R&D complementarity (substitutability) is pervasive among firms then the subsidy program stimulates (crowds-out) privately-financed R&D.*

In the first stage, subsidies are calculated under the CbC rule. Government sets the subsidy equal to the investment gap anticipating the direct effect of the subsidy on R&D, i.e. $Ds^{cbc} = \Delta$. Substituting in for the direct effect reveals that the CbC subsidy is given by

$$s^{cbc} = \sigma_I \Delta.$$

Notice that $s^{cbc} > 0$.¹³ The optimal subsidy, denoted by s^* , equates social optimum R&D and publicly funded R&D, i.e. $x^{**} = x^*(s^*)$. Simple calculations reveal the optimal subsidy is given by $s^* = \sigma_I \Delta - (n-1)\sigma_C \Delta$.¹⁴ Notice that $s^* > 0$.¹⁵ The CbC funding problem then is the difference between the CbC subsidy and the optimal subsidy and is given by

$$s^{cbc} - s^* = (n-1)\sigma_C \Delta.$$

Therefore, the CbC funding problem is proportional to the investment gap, weighted by the intensity of R&D interaction across the industry. In the benchmark case, the situation in which there is no interaction in R&D, $\sigma_C = 0$, CbC funding is socially optimal $s^{cbc} = s^*$.

¹³The concavity restriction $\sigma_I > 0$ and the under-investment assumption $\Delta > 0$ ensure $s^{cbc} > 0$.

¹⁴See the Appendix for the derivation of the optimal subsidy.

¹⁵The equilibrium restriction $\sigma_I > (n-1)\sigma_C$ and the under-investment assumption $\Delta > 0$ ensure $s^* > 0$.

Proposition 1.2 *In the absence of interaction in R&D, CbC funding is socially optimal.*

Since the investment gap is positive $\Delta > 0$, the direction of the CbC funding problem depends on the nature of the R&D interactions among the firms receiving support. If $\sigma_C > 0$ then $s^{cbc} > s^*$; hence when R&D complementarity pervades the industry, government over-subsidizes firms under the CbC rule. On the other hand, if $\sigma_C < 0$ then $s^{cbc} < s^*$; hence when R&D substitutability pervades the industry, government under-subsidizes firms under the CbC rule (assuming external social benefits are sufficiently large).¹⁶

Proposition 1.3 *If R&D complementarity (substitutability) pervades among funded firms then CbC funding is socially excessive (insufficient).*

The intuition behind the result is as follows: if R&D complementarity (substitutability) pervades among the firms receiving support then, under the CbC rule, government does not internalize that subsidizing a firm indirectly stimulates (crowds-out) investment by rival firms thereby reducing (widening) the investment gap associated with their R&D activities. When government moves to decide funding for one of these rival firms, it does not take into account that the rival's investment gap is now smaller (larger). As a result, government over-subsidizes (under-subsidizes) the rival firm. Since, it does this for all firms in the industry, government over-funds (under-funds) research under the rule.

Consider now the effect of the intensity of interaction in R&D on the CbC funding problem. Recall the social welfare function $W = \sum_i \pi_i(x_1, \dots, x_n) + B(\sum_i x_i)$ presented earlier. The investment gap associated with W is given by

$$\Delta = \frac{B}{(\sigma_I - 2(n-1)\sigma_C)} + \frac{(n-1)\sigma_C\alpha}{(\sigma_I - (n-1)\sigma_C)(\sigma_I - 2(n-1)\sigma_C)}.$$
¹⁷

First, notice that $\frac{\partial \Delta}{\partial B} > 0$ implies $\frac{\partial(s^{cbc} - s^*)}{\partial B} > 0$; hence in the presence of interaction in R&D, the magnitude of the funding problem is increasing in the intensity of external social benefits.

Second, if $\sigma_C > 0$ then $\frac{\partial(s^{cbc} - s^*)}{\partial \sigma_C} > 0$; hence the magnitude of CbC over-funding is increasing in the intensity of R&D complementarity. On the other hand, if $\sigma_C < 0$ then $\frac{\partial(s^{cbc} - s^*)}{\partial \sigma_C} > 0$ provided B is sufficiently large; hence the magnitude of CbC under-funding

¹⁶Notice that if $\Delta < 0$ then government over-taxes when complementarity in effort is pervasive and under-taxes when substitutability in effort is pervasive.

¹⁷See the Appendix for the derivation of the investment gap and results concerning the comparative static properties of the CbC funding problem.

is increasing in the intensity of R&D substitutability provided external social benefits are sufficiently large. Otherwise, the magnitude of CbC under-funding is decreasing in the intensity of R&D substitutability. These observations give the following result:

Proposition 1.4 *The CbC funding problem is i) increasing in the intensity of pervasive R&D complementarities; and ii) increasing in the intensity of pervasive R&D substitutability provided external social benefits are sufficiently large.*

The intuition behind these results is as follows: more intense complementary R&D interactions among funded firms has two main effects on the optimal subsidy: a negative effect due to the increase in the magnitude of bilateral influence and a positive effect due to the widening of the investment gap. The CbC rule ignores the first effect and consequently the CbC subsidy rises at a faster rate than the optimal subsidy resulting in greater over-funding under the rule.

On the other hand, more intense substitutive R&D interactions among funded firms has two main effects on the optimal subsidy: a positive effect due to the increase in the magnitude of bilateral influence and a negative effect due to the reduction in the investment gap. The CbC rule ignores the first effect and consequently the CbC subsidy falls at a faster rate than the optimal subsidy resulting in more extensive under-funding of R&D under the rule.

1.2.2 Spillover-induced R&D Interactions

The purpose of this section is to examine how the efficiency of the CbC rule depends on the relative intensity of knowledge spillovers and product market rivalry among funded firms. A third stage in which firms compete in the product market is added to the model of the previous section and reduced-form R&D payoff functions are derived.

Firms compete in a symmetric differentiated-good Cournot oligopoly (the case of no product differentiation is a special case). Each firm sets output q_i and faces a linear inverse demand function $p_i(q_1, \dots, q_n)$ given by

$$p_i(q_1, \dots, q_n) = a - q_i - \delta(\sum_{j \neq i} q_j),$$

in which $a > 0$ and $0 \leq \delta \leq 1$. The parameter δ is the degree of symmetric product differentiation. If $\delta = 0$ then the products are independent products and the firms behave as independent monopolists. If $\delta = 1$ then the products are perfect substitutes and the

firms compete in a homogenous-product oligopoly.¹⁸ Holding industry size n constant, the parameter δ measures the intensity of product market rivalry among the firms.

R&D, x_i , delivers a process innovation which reduces marginal production costs. These costs are linear and given by

$$c_i(x_1, \dots, x_n) = c - x_i - \beta(\sum_{j \neq i} x_j),$$

with $a > c \geq x_i + \beta(\sum_{j \neq i} x_j) \geq 0$ and $0 \leq \beta \leq 1$. The parameter β is the degree of symmetric knowledge spillover among the firms. This spillover implies some or all of the benefits of each firm's R&D flow without payment to rival firms. If $\beta = 1$ then the spillover is perfect. If $\beta = 0$ then there is no knowledge leakage.

R&D subsidies, s_i , lower the effective cost of R&D. This effective cost is given by

$$y_i(x_i, s_i) = -s_i x_i + \frac{1}{2} \psi x_i^2,$$

with $\psi \geq 1$. Notice that the subsidy reduces the intercept term on the firm's marginal cost of R&D.

Bringing together expressions, firm profits are $\pi_i = p_i(q_1, \dots, q_n)q_i - c_i(x_1, \dots, x_n)q_i - y_i(s_i, x_i)$. Social welfare is the sum of consumer surplus, industry profits and pure external benefits, deriving from R&D spillovers to firms and consumers in other industries, minus the cost of the subsidy program: $W = (\sum_i \frac{q_i^2}{2} + \frac{\delta}{2} \sum_{i,j,i \neq j} q_i q_j) + \sum \pi_i + B(\sum_i x_i) - \sum_i s_i x_i$, in which $B \geq 0$.¹⁹ As in the previous section, it is assumed that in the absence of intervention each firm under-invests in R&D, i.e. $\Delta_i > 0$ for all $i = 1, \dots, n$.

Computing the Nash equilibrium outputs of the production stage, deriving the reduced-form R&D payoff functions and taking the first order condition yields:²⁰

$$\frac{\partial \pi_i}{\partial x_i} = \alpha + s_i - \sigma_I x_i + \sum_{j \neq i} \sigma_C x_j = 0,$$

in which

$$\alpha = \frac{2(2-\delta)(2-\delta+\delta(1-\beta)(n-1))(a-c)}{((2-\delta)(2+\delta(n-1)))^2}, \text{ and } \sigma_I = \psi - 2\left(\frac{(2-\delta+\delta(1-\beta)(n-1))}{(2-\delta)(2+\delta(n-1))}\right)^2$$

and

¹⁸This formulation of the inverse demand functions is taken from Yi (1997).

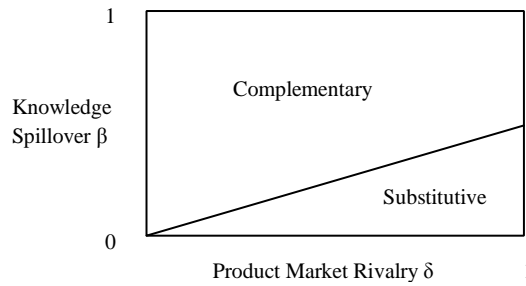
¹⁹The social optimum is in the spirit of the second best since production is chosen non-cooperatively.

²⁰See the Appendix for these derivations.

$$\sigma_C = \frac{2(2-\delta+\delta(1-\beta)(n-1))}{((2-\delta)(2+\delta(n-1)))^2} (2\beta - \delta).$$

Notice that the nature of interaction in R&D is determined by the relative intensity of knowledge spillovers and product market rivalry among firms: if $2\beta > \delta$ (resp. $2\beta < \delta$) then $\sigma_C > 0$ (resp. $\sigma_C < 0$); hence if knowledge spillovers dominate (are dominated by) product market rivalry then R&D complementarity (substitutability) is pervasive in the industry. Finally, if $2\beta = \delta$ then $\sigma_C = 0$; hence when knowledge spillovers and product market rivalry are balanced, no interaction occurs between firms in R&D.

Figure 1: Spillover-induced R&D Interactions



In particular, note that if the firms seeking support undertake research on independent products, $\delta = 0$, and knowledge spillovers occur among firms, $\beta > 0$, then knowledge spillovers induce complementary R&D interactions among the firms. By contrast, if the firms seeking support undertake research on substitute products, $\delta > 0$, and there are no knowledge spillovers between firms, $\beta = 0$, then product market rivalry induces substitutive R&D interactions among the firms.

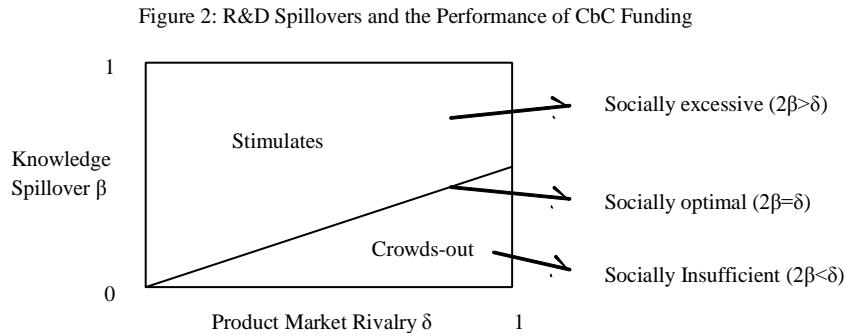
The intuition behind spillover-induced R&D interactions is as follows: if a firm innovates then on the one hand receiving knowledge spillovers from the innovation technologically advances the firm's R&D project increasing its profitability thereby stimulating investment. On the other hand, receiving business stealing effects from the commercialization of an innovation by a product market rival reduces the profitability of the firm's R&D project thereby dampening investment. If the positive effect of knowledge spillovers offsets (is offset by) the negative business stealing effect of innovation by a product market rival then, by symmetry, complementary (substitutive) R&D interactions arise among firms. Applying the results of the previous section, the CbC funding problem is

$$s^{cbc} - s^* = (n-1)\sigma_C \Delta$$

Notice that the direction of the CbC funding problem depends on the relative intensity of knowledge spillovers and product market rivalry among firms. If knowledge spillovers dominate product market rivalry, $2\beta > \delta$, then the subsidy program indirectly stimulates privately-financed R&D thereby lowering the required level of subsidization and CbC funding is socially excessive, $s^{cbc} > s^*$. However, if product market rivalry dominates knowledge spillovers, $\delta > 2\beta$, then the subsidy program indirectly crowds-out privately-financed R&D thereby raising the required level of subsidization and CbC funding is socially insufficient, $s^{cbc} < s^*$. Finally, if knowledge spillovers and product market rivalry are balanced, $2\beta = \delta$, then CbC funding is socially optimal, $s^{cbc} = s^*$. These observations give the following result:

Proposition 1.5 *In the symmetric differentiated-good Cournot model of R&D, CbC over-funding (under-funding) occurs in situations characterized by high (low) knowledge spillovers and relatively weaker (stronger) product market rivalry among funded firms.*

Figure 2 summarizes our findings on the relationship between the direction of the CbC funding problem and an industry's knowledge spillover-product market rivalry characteristics.



In particular, note that if the firms seeking support undertake research on independent products, $\delta = 0$, and knowledge spillovers occur among firms, $\beta > 0$, then government over-funds research under the case-by-case rule. However, if the firms seeking support undertake research on substitute products, $\delta > 0$, and there are no knowledge spillovers between firms, $\beta = 0$, then government under-funds research under the case-by-case rule. Thus, whereas knowledge spillovers are a source of additionality and CbC over-funding of R&D, product market rivalry among firms is a source of crowding-out and CbC under-funding of R&D.

By contrast, there are a number of distinct situations in which case-by-case funding is efficient. For instance, if the firms research on independent products, $\delta = 0$, and there are

no knowledge spillovers among firms, $\beta = 0$, then subsidization under the case-by-case rule implements the socially optimal amount of R&D. Moreover, if the firms develop substitute products, $\delta > 0$, and knowledge spillovers among firms are sufficiently large to offset the product market rivalry effects of innovation, $2\beta = \delta$, then use of the case-by-case subsidy rule is socially optimal.

What is the effect of an inelastic supply of R&D inputs on the direction of the CbC funding problem? Suppose that the effective R&D cost is given by

$$y_i(x_1, \dots, x_n, s_i) = -s_i x_i + \phi(\sum_{j \neq i} x_j) x_i + \frac{1}{2} \psi x_i^2.$$

Interaction in R&D costs is captured by the parameter $\phi > 0$. This parameter measures the extent to which R&D undertaken by rival firms increases the R&D cost of firm i . Solving the model as before, the first order condition of the firm's R&D decision is given by

$$\frac{\partial \pi_i}{\partial x_i} = \alpha + s_i - \sigma_I x_i + \sum_{j \neq i} (\sigma_C - \phi) x_j = 0.$$

Notice that an inelastic supply of R&D inputs is a source of substitubility between firms in R&D. R&D undertaken by a rival increases the firm's R&D cost reducing the profitability of its R&D project and thereby lowering investment. Applying the results of the previous section, the CbC funding problem is

$$s^{cbc} - s^* = (n - 1)(\sigma_C - \phi)\Delta.$$

Observe that if knowledge spillovers dominate business stealing effects then limited R&D resources can remove a CbC over-funding problem. On the other hand, if business stealing effects and knowledge spillovers are balanced then limited R&D inputs creates a CbC under-funding problem. Therefore, we see that under certain circumstances an inelastic supply of R&D inputs can improve the efficiency of CbC funding.

Finally, it should be noted the CbC rule performs well in many distinct settings. Consider the following examples in which CbC funding is socially optimal: i) if $2\beta = \delta$ and $\phi = 0$ then $s^{cbc} = s^*$; hence when knowledge spillovers and product market rivalry are balanced and there is no interaction in R&D costs; ii) if $2\beta > \delta$ and $\phi > 0$ with $\sigma_C = \phi$ then $s^{cbc} = s^*$; hence when knowledge spillovers dominate product market rivalry and the supply of R&D inputs is sufficiently inelastic; and iii) if $\delta > 2\beta$ and $\phi < 0$ with $\sigma_C = \phi$ then $s^{cbc} = s^*$; hence when product market rivalry dominates knowledge spillovers and positive synergies exist between firms' R&D costs.

1.2.3 The Adjusted Case-by-case Rule

This section proposes a simple allocation rule for correcting inefficiencies in CbC funding arising from a general pattern of R&D interactions between firms. Presenting this rule in a more general framework has the obvious advantages of i) providing greater insight into the details of the functioning of the rule; and ii) broadening the set of possible applications of the rule for government bureaucrats.

Consider the same model presented in Section 2 but in the following more general setting: suppose the R&D payoff of firm i given the subsidy s_i it receives from government is

$$\pi_i(x_1, \dots, x_n, s_i) = \alpha_i x_i - \frac{1}{2} \sigma_{ii} x_i^2 + \sum_{j \neq i}^n \sigma_{ij} x_i x_j + s_i x_i,$$

in which $\alpha_i, \sigma_{ii} > 0$, for all $i = 1, \dots, n$. Bilateral influences are captured by the cross-derivatives $\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} = \sigma_{ij}$, $i \neq j$, which are linear, pair-dependent and can be of either sign. This cross derivative measures the change in firm i 's incentive to innovate due to marginal increase in firm j 's R&D effort. When $\sigma_{ij} > 0$ ($\sigma_{ij} < 0$), the R&D efforts of firm i and j are strategic complements (substitutes) from i 's perspective. The square matrix of cross effects is denoted by $H := (\hat{\sigma}_{ij})_{1 \leq i, j \leq n}$, in which $\hat{\sigma}_{ij} = \sigma_{ij}$ for all $i \neq j$, and $\hat{\sigma}_{ii} = -\sigma_{ii}$ for all $i = 1, \dots, n$. It is assumed that H is negative definite. Suppose $W(x_1, \dots, x_n)$ is a strictly concave social welfare function such that the vector of investment gaps $\Delta = (\Delta_i)_{i=1}^n$ associated with W satisfies $\Delta > 0$. We use (H, Δ) as short-hand notation for the game.

In the Appendix we show that in the game (H, Δ) , the optimal subsidy to firm i is $s_i^* = \sigma_{ii} \Delta_i - \sum_{j \neq i}^n \sigma_{ij} \Delta_j$, and the CbC subsidy to firm i is $s_i^{cbc} = \sigma_{ii} \Delta_i$. Therefore, taking the difference, the CbC funding problem is $s_i^{cbc} - s_i^* = \sum_{j \neq i}^n \sigma_{ij} \Delta_j$. Notice that under the CbC rule, if $\sum_{j \neq i}^n \sigma_{ij} \Delta_j > 0$ then government over-funds firm i and if $\sum_{j \neq i}^n \sigma_{ij} \Delta_j < 0$ then government under-funds firm i .

Consider now the following adjusted CbC rule which involves two steps. It is assumed government knows the vector of investment gaps Δ and the pattern of cross effects H . First, government calculates the CbC subsidy program. Second, government adjusts each firm's CbC subsidy by subtracting the CbC subsidy of each rival firm weighted according to the magnitude of bilateral influence exerted by the rival on the firm (normalized for the idiosyncratic concavity of the rival's R&D). Formally, the adjusted CbC subsidy to firm i , denoted by s_i^{adj} , is given by

$$s_i^{adj} = s_i^{cbc} - \sum_{j \neq i}^n \frac{\sigma_{ij}}{\sigma_{jj}} s_j^{cbc}, \text{ for all } i = 1, \dots, n.$$

Substituting in for the rival firms' CbC subsidy, we see that the adjusted CbC subsidy to firm i can be written as

$$s_i^{adj} = s_i^{cbc} - \sum_{j \neq i}^n \sigma_{ij} \Delta_j, \text{ for all } i = 1, \dots, n.$$

Notice that under the adjusted CbC rule: i) if a firm's R&D project is complementary (substitute) to projects with large investment gaps then this relationship substantially lowers (raises) the firm's CbC subsidy (*cet. paribus*); and ii) if a firm's R&D project is intensely complementary (substitutive) to projects with moderate investment gaps then this relationship substantially lowers (raises) the firm's CbC subsidy (*cet. paribus*).

Establishing a comparison between the adjusted CbC subsidy and the optimal subsidy yields $s_i^{adj} = s_i^*$ for all $i = 1, \dots, n$. Therefore, under the adjusted CbC rule, government subsidy policy implements socially optimal R&D. This observation gives the following result:

Proposition 1.6 *In the game (H, Δ) , government subsidy policy under the adjusted CbC rule is socially optimal.*

Therefore, given information on the investment gaps and bilateral influences, the agency can apply this simple allocation rule and correct for the inefficiency in CbC funding arising from inter-firm R&D interactions. In particular, it should be noted that firms which research on independent products that benefit from complementary knowledge spillovers receive a smaller subsidy than under the CbC rule. By contrast, firms which research on substitute products for which there are no knowledge spillovers among firms receive a larger subsidy than under the CbC rule.

1.3 Policy Implications

In view of the underlying scarcity of society's R&D resources, scientists and engineers, and the variety of research paths available for technological advance, an important question is whether governments over-fund or under-fund research in specific industries/technology areas. For instance, if subsidy programs channel too much of society's R&D inputs into a few narrow research trajectories then other research paths may suffer from less effective R&D investment due to higher R&D input costs and/or inadequate government support (Goolsbee, 1998). The model of this paper suggests that governments may be i) over-funding research in sectors characterized by high knowledge spillovers and relatively weaker product market rivalry among firms; and ii) under-funding research in sectors characterized

by low knowledge spillovers and relatively stronger product market rivalry among firms in which innovations create significant external social benefits. Since the situation can differ greatly across industry settings, reallocating some funding from type (i) to type (ii) situations could significantly improve the performance of government support.²¹

For example, many research trajectories in high-tech industries tend to be characterized by i) high knowledge spillovers as suggested by the large levels of inter-firm R&D partnering (Hagedoorn, 2002); and ii) product market relationships between firms which are often complementary rather than rivalrous. Consider the case of Japanese government support to domestic research consortia. The government heavily subsidized participating firms, covering on average two-thirds of a project's cost. Funding was mainly concentrated in the computers/semiconductors and telecommunications sectors even though evidence suggests strong complementarities existed between firms in R&D activities (see Branstetter et al., 1998, 2002).

On the other hand, the power generation sector tends to be characterized by i) low inter-technology knowledge spillovers, e.g. developments in nuclear technology are not easily translated into advances in wind technology and vice versa; ii) business stealing effects because of the homogenous nature of the end-product (electricity) and the very limited number of niche markets; and iii) high external social benefits, e.g. energy security and environmental protection. Therefore, our model suggests that allocating the public energy R&D budget in a case-by-case manner is likely to contribute to the under-funding of energy technologies.

The model also suggests complementarity in R&D presents fewer difficulties than substitutability in R&D for government subsidy policy. On the one hand, complementary relationships among firms' R&D decisions tend to facilitate government efforts by stimulating private R&D. If these complementarities are intense then a "gentle push" from government in the form of a small subsidy can stimulate a disproportionately larger amount of innovative activity. Biased government preferences, e.g. picking winners, are unlikely to cause government failure because subsidizing in the presence of complementarities has an effect similar to "a rising tide raises all boats".²² The main concern is the agency over-funds R&D.

²¹Note that the distribution of government R&D budget outlays across industries is often highly skewed (OECD, 2007).

²²In practice, agencies may use R&D subsidies as a tool to achieve additional objectives other than correcting market failure. Examples include i) attracting firms to a location; ii) fostering national champions; iii) encouraging technological upgrading in declining industries (Blanes et al., 2004).

On the other hand, substitutive relationships among firms' R&D decisions can severely complicate government efforts to stimulate private R&D. In these circumstances, the agency is similar to a "tight-rope walker" in that successful intervention requires a "careful balancing act": if government pushes a single firm too much with overly generous funding then it may fully or partially crowd-out the innovative efforts of other firms. In particular, government should avoid targeting winners. Even if government offers support to a broad number of firms, biased government preferences may result in an unbalanced subsidy program leading to substantial crowding-out and under-funding of R&D.

For example, in response to the oil crises of the 1970s, most of the increase in government energy R&D was focused on developing nuclear technology at a time when high oil prices were inducing innovation in energy-saving technologies. Due to this targeting of funding on a single technology, governments may have partially or even fully crowded-out advances in other energy technologies.

1.4 Summary and Concluding Remarks

Apart from a few notable exceptions, little systematic attention has been paid to the issue of optimal allocation rules for R&D subsidies. In practice, government agencies tend to distribute funding on a case-by-case manner ignoring interactions between firm R&D decisions. This paper develops a model which incorporates these two features. The article identifies some conditions under which a CbC funding problem can occur and provides a simple allocation rule for internalizing externalities arising from a general pattern of interaction in R&D.

The paper finds that under CbC funding, government over-funds research in industries in which innovations tend to be complementary and under-funds research in industries in which innovations tend to be substitutive. The article then reveals a relationship between the inefficiency in CbC funding and an industry's knowledge spillover-product market rivalry characteristics.

We note that the analysis makes strong assumptions about the information available to government. It is clear government agencies face information problems in making funding decisions. The open question here is whether these information problems are so acute that addressing inefficiencies cannot be suitably achieved in practice.

There is nothing in the main model which formally restricts its interpretation to R&D. Applications range from R&D subsidies, structural funds to funding of charitable organiza-

tions and university research grants. The paper makes some restrictive assumptions which need to be relaxed when evaluating CbC funding in more complex situations, e.g. biased government preferences, multiple intervening agencies and uncoordinated government policy tools.

For instance, several markets have become global in recent decades with governments eager to support the research of their national champions, e.g Airbus and Boeing. The interplay of national interests and international interaction in R&D may have positive implications for the performance of publicly funded R&D. For example, under international cooperation, CbC funding would be socially excessive in the presence of trans-national R&D complementarities. The uncoordinated pursuit of national interest may restrict the extent of this over-funding.

Patent systems and competition policies are currently in use in many countries. These policy instruments, by altering industry knowledge spillover-product market rivalry characteristics among firms, may have a positive or negative effect on the efficiency of CbC funding. For example, if competition policy succeeds in increasing the intensity of product market rivalry in an industry then it may remove a CbC over-funding problem if knowledge spillovers are high or create a CbC under-funding problem if knowledge spillovers are weak.

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1.5 Appendix

Nash equilibrium R&D given subsidy policy: The best response function of each firm is $b_i((x_j)_{j \neq i}, s_i) = \frac{1}{\sigma_I} \alpha + \frac{1}{\sigma_I} s_i + \sum_{j \neq i} \frac{\sigma_C}{\sigma_I} x_j$. Solving this linear system of equations yields Nash equilibrium R&D as a function of the government’s subsidy program: $x_i^*(s_1, \dots, s_n) = \hat{x}_i +$

$\frac{\sigma_I(\sigma_I-(n-2)\sigma_C)}{A}s_i + \sum_{j \neq i} \frac{\sigma_C}{A}s_j$, in which $A = (\sigma_I + \sigma_C)(\sigma_I - (n-1)\sigma_C) > 0$. The term $T_i := \frac{\sigma_I(\sigma_I-(n-2)\sigma_C)}{A}$ denotes the total effect of a subsidy on the recipient firm's R&D.

From the decomposition of the effect of a subsidy, this total effect consists of a direct effect D_i and a feedback effect F_i , that is $T_i = D_i + F_i$. From the firm's best response function $D_i = \frac{1}{\sigma_I}$. Substituting in the direct effect and the total effect and solving reveals that the feedback effect is given by $F_i = T_i - D_i = \sum_{j \neq i} \frac{(n-1)\sigma_C^2}{\sigma_I A}$. Therefore, equilibrium R&D given subsidies can be rewritten as

$$x_i^*(s_1, \dots, s_n) = \hat{x}_i + F_i s_i + \sum_{j \neq i} C_{ij} s_j + D_i s_i,$$

in which $F_i = \frac{(n-1)\sigma_C^2}{\sigma_I A}$, $C_{ij} = \frac{\sigma_C}{A}$ and $D_i = \frac{1}{\sigma_I}$ are respectively the feedback, direct and cross effects of the subsidy on R&D.

Optimal subsidy: Optimal policy requires government equates $x^{**} = x^*(s)$. The policy equilibrium is given by $x^*(s) = \hat{x} + Is + Ds$. Therefore, the optimal subsidy must satisfy $x^{**} = \hat{x} + Is + Ds$. Rearranging and using the fact that the investment gap is given by $\Delta = x^{**} - \hat{x}$ leads to the equation $\Delta = (D + I)s$. Substituting in $D = \frac{1}{\sigma_I}$, and $I = \frac{(n-1)\sigma_C}{\sigma_I(\sigma_I-(n-1)\sigma_C)}$ gives $\Delta = \frac{1}{(\sigma_I-(n-1)\sigma_C)\Delta}s$. Therefore, the optimal subsidy is $s^* = \sigma_I \Delta - (n-1)\sigma_C \Delta$.

Investment gap: Social welfare is $W = \sum \pi_i(x_1, \dots, x_n) + B(\sum x_i)$. The first order conditions on the planner's problem are given by $\alpha - \sigma_I x_i + 2 \sum_{j \neq i} \sigma_C x_j + B = 0$, for all $i = 1, \dots, n$. The restriction $\sigma_I > 2(n-1)\sigma_C$ ensures the existence of a unique interior social optimum. Invoking symmetry and rearranging gives socially optimal R&D: $x^{**} = \frac{B+\alpha}{\sigma_I-2(n-1)\sigma_C}$. In the laissez-faire scenario, each firm equates $\alpha - \sigma_I x_i + \sum_{j \neq i} \sigma_C x_j = 0$. Invoking symmetry and solving yields laissez faire R&D $\hat{x} = \frac{\alpha}{\sigma_I-(n-1)\sigma_C}$. Taking the difference and rearranging, the investment gap is $\Delta = \frac{B}{(\sigma_I-2(n-1)\sigma_C)} + \frac{(n-1)\sigma_C \alpha}{(\sigma_I-(n-1)\sigma_C)(\sigma_I-2(n-1)\sigma_C)}$. Notice that if $\sigma_C < 0$ then $\Delta > 0$ provided $B > -\frac{(n-1)\sigma_C \alpha}{(\sigma_I-(n-1)\sigma_C)}$.

Comparative Static properties of the CbC funding problem:

External social benefits: Notice that $\frac{\partial(s^{CbC}-s^*)}{\partial B} = (n-1)\sigma_C \frac{\partial \Delta}{\partial B}$ and $\frac{\partial \Delta}{\partial B} = \frac{B}{(\sigma_I-2(n-1)\sigma_C)} > 0$. It follows that $\frac{\partial(s^{CbC}-s^*)}{\partial B} > 0$.

Effect of R&D interaction on the investment gap: Notice that

$$\frac{\partial \Delta}{\partial \sigma_C} = \frac{2(n-1)B}{(\sigma_I-2(n-1)\sigma_C)^2} + \frac{(n-1)(\sigma_I^2-2(n-1)^2\sigma_C^2)\alpha}{[(\sigma_I-(n-1)\sigma_C)(\sigma_I-2(n-1)\sigma_C)]^2} > 0.$$

Effect of intensity of R&D interaction on the magnitude of the CbC funding problem:

From the product rule of differentiation, $\frac{\partial(s^{CbC}-s^*)}{\partial \sigma_C} = (n-1)(\sigma_C \frac{\partial \Delta}{\partial \sigma_C} + \Delta)$. Recall that

$\frac{\partial \Delta}{\partial \sigma_C} > 0$ and by assumption $\Delta > 0$. Therefore, if $\sigma_C > 0$ then $\frac{\partial(s^{CbC}-s^*)}{\partial \sigma_C} > 0$. On the other hand, if $\sigma_C < 0$ then $\frac{\partial(s^{CbC}-s^*)}{\partial \sigma_C} > 0$ if and only if $\Delta > -\sigma_C \frac{\partial \Delta}{\partial \sigma_C}$. Notice that $\frac{\partial(s^{CbC}-s^*)}{\partial \sigma_C} = \frac{(n-1)B\sigma_I}{(\sigma_I-2(n-1)\sigma_C)^2} + \frac{(n-1)\sigma_C\sigma_I(2\sigma_I-3(n-1)\sigma_C)\alpha}{[(\sigma_I-(n-1)\sigma_C)(\sigma_I-2(n-1)\sigma_C)]^2}$ implies that $\frac{\partial(s^{CbC}-s^*)}{\partial \sigma_C} > 0$ provided that $B > -\frac{(n-1)\sigma_C\sigma_I(2-3(n-1)\sigma_C)}{(\sigma_I-(n-1)\sigma_C)^2}$. A negative relationship, $\frac{\partial(s^{CbC}-s^*)}{\partial \sigma_C} < 0$, occurs when B satisfies $-\frac{(n-1)\sigma_C}{(\sigma_I-(n-1)\sigma_C)} < B < -\frac{(n-1)\sigma_C\sigma_I(2-3(n-1)\sigma_C)}{(\sigma_I-(n-1)\sigma_C)^2}$.

Derivation of the reduced-form R&D payoff functions: Firm profits are $\pi_i = p_i(q_1, \dots, q_n)q_i - c_i(x_1, \dots, x_n)q_i - y_i(s_i, x_i)$. Standard derivations reveal the Nash Cournot outputs are given by $q_i(c_1, \dots, c_n) = \frac{(2-\delta)a-(2+\delta(n-2))c_i+\sum_{j \neq i} \delta c_j}{(2-\delta)(2+\delta(n-1))}$. If $\delta = 0$ then firms produce the monopoly level of output $q_i = \frac{a-c_i}{2}$. If $\delta = 1$ then firms produce the well known the homogenous product Cournot level of output $q_i = \frac{a-c_i+\sum_{j \neq i} c_j}{n-1}$. Substituting in the cost formulation and rearranging terms gives $q_i(x_1, \dots, x_n) = \frac{(2-\delta)(a-c)+(2-\delta+\delta(1-\beta)(n-1))x_i+\sum_{j \neq i} (2\beta-\delta)x_j}{(2-\delta)(2+\delta(n-1))}$. Substituting these outputs into the profit function yields the R&D payoff function of the firm:

$$\pi_i(x_i, x_j, s_i) = \left[\frac{(2-\delta)(a-c)+(2-\delta+\delta(1-\beta)(n-1))x_i+\sum_{j \neq i} (2\beta-\delta)x_j}{(2-\delta)(2+\delta(n-1))} \right]^2 + s_i x_i - \frac{1}{2} \psi x_i^2.$$

Taking the first order condition yields

$$\frac{\partial \pi_i}{\partial x_i} = \alpha + s_i - \sigma_I x_i + \sum_{j \neq i} \sigma_C x_j = 0,$$

in which

$$\alpha = \frac{2(2-\delta)(2-\delta+\delta(1-\beta)(n-1))(a-c)}{((2-\delta)(2+\delta(n-1)))^2}, \text{ and } \sigma_I = \psi - 2\left(\frac{(2-\delta+\delta(1-\beta)(n-1))}{(2-\delta)(2+\delta(n-1))}\right)^2$$

and

$$\sigma_C = \frac{2(2-\delta+\delta(1-\beta)(n-1))}{((2-\delta)(2+\delta(n-1)))^2} (2\beta - \delta).$$

Notice that the interplay of knowledge spillovers and product market rivalry among firms exacerbates the appropriation problem by undermining the idiosyncratic incentive to innovate. Interestingly, knowledge spillovers only discourage R&D investment provided the intensity of market rivalry is not too large: if $\delta = 0$ then $\frac{\partial}{\partial \beta} \left(\frac{\partial \pi_i}{\partial x_i} \right) > 0$ and if $\delta = 1$ then $\frac{\partial}{\partial \beta} \left(\frac{\partial \pi_i}{\partial x_i} \right) < 0$.

The Adjusted CbC rule: i) Proof that $s_i^* = \sigma_{ii}\Delta_i - \sum_{j \neq i}^n \sigma_{ij}\Delta_j$. In matrix notation, the policy equilibrium $x^*(s)$ must satisfy $xH^T = -\alpha - s$, in which $\alpha = (\alpha_i)_{i=1}^n$ and H^T is the transpose of H . Since H is invertible, the matrix equation has a unique solution. By inverting the matrix, we get that Nash equilibrium of the final stage is given by $x^*(s) = \hat{x} - s(H^T)^{-1}$, in which $\hat{x} = -\alpha(H^T)^{-1}$ is the vector of laissez-faire efforts. Optimal policy equates social optimum

effort and publicly funded effort, i.e. s^* is an optimal subsidy program if and only if s^* satisfies $x^{**} = \hat{x} - s^*(H^T)^{-1}$. Since $\Delta = x^{**} - \hat{x}$, optimal policy must satisfy $s^*(H^T)^{-1} = -\Delta$. Therefore, optimal subsidy policy is given by $s^* = -\Delta((H^T)^{-1})^{-1} = -\Delta H^T$.

ii) Proof that $s_i^{cbc} = \sigma_{ii}\Delta_i$ and $s_i^{cbc} - s_i^* = + \sum_{j \neq i}^n \sigma_{ij}\Delta_j$: From the first order condition, the best response function is given by $b_i = \frac{1}{\sigma_{ii}}\alpha_i + \frac{1}{\sigma_{ii}}s_i + \sum_{j \neq i} \frac{\sigma_{ij}}{\sigma_{ii}}x_j$. Applying the implicit function theorem to this system of equations, there exists n differentiable implicit functions $\{x_i(s_1, \dots, s_n)\}_i$ such that equilibrium aggregate effort can be written as $\sum_i x_i^* = \sum_i b_i(s_i, (x_j(s_1, \dots, s_n)_{j \neq i})^n)$. Totally differentiating this expression with respect to s_i around the Nash equilibrium decomposes the impact of a subsidy on R&D into a direct and indirect effect:

$$\frac{d(\sum_i x_i)}{ds_i} = \underbrace{\frac{\partial b_i}{\partial s_i}}_{\text{direct effect}} + \underbrace{\left(\sum_{j \neq i} \frac{\partial b_i}{\partial x_j} \frac{\partial x_j}{\partial s_i} \right) + \sum_{j \neq i} \left(\sum_{l \neq j} \frac{\partial b_j}{\partial x_l} \frac{\partial x_l}{\partial s_i} \right)}_{\text{indirect effect}}.$$

From the decomposition, we see that the direct effect can be computed from the firm's best response function and is given by $D_i = \frac{1}{\sigma_{ii}}$. Under the CbC rule, government ignores the indirect effect of subsidies and sets $\Delta_i = D_i s_i^{cbc}$. Therefore, it follows that $s_i^{cbc} = \sigma_{ii}\Delta_i$. Comparing the CbC subsidy and optimal subsidy of firm i yields the CbC funding problem of firm i : $s_i^{cbc} - s_i^* = + \sum_{j \neq i}^n \sigma_{ij}\Delta_j$.

CHAPTER 2

INDUSTRIAL CLUSTERS, STRENGTH EFFECTS AND FIRM PERFORMANCE

2.1 Introduction

Many industries are characterized by a high level of geographic clustering and a significant fraction of this clustering occurs among horizontally related firms, i.e. where firms exhibit some degree of product market rivalry. Popular examples of this phenomena include carpet manufacture in Dalton, Georgia (Krugman, 1991), auto manufacturers around Detroit, Michigan (Porter, 1998), software developers in Silicon Valley (Rosenthal and Strange, 2004) and the laser and electric optics industry in Orlando, Florida (Pouder and St. John, 1996). Ever since the pioneering work of Marshall (1920), the literature on agglomeration typically identifies three main benefits driving industrial clustering among competing firms: access to a pool of specialized labour; access to a pool of specialized input suppliers; and access to local knowledge spillovers among competitors.¹ It is thought that these benefits tend to increase with cluster size, "the net benefits of being in a location increase with the number of firms in the location" (Arthur, 1990), generating economies of agglomeration, i.e. production costs fall as cluster size increases, thereby improving the performance of cluster firms and enhancing the attractiveness of the location to other firms, an example of circular causation.

This paper develops a model of the agglomeration of an oligopolistic industry which sheds light on the nature of the externalities driving industrial clustering among competing firms. An often overlooked aspect of agglomeration among firms which produce close substitutes is that there are at least two externalities at play, agglomeration externalities and negative pecuniary externalities, and these externalities tend to interact in non-trivial ways.

Agglomeration externalities are cluster bounded positive externalities that improve the competitiveness of geographically proximate firms. Examples of activities which generate

¹There tends to be two broad explanations for agglomeration of economic activity within a particular industry, exogenous natural advantages and agglomeration economies (see Ellison and Glaeser; 1997). Ellison and Glaeser (1999) found that (i) only 20% of the clusters they studied can be explained by exogenous natural advantages; and (ii) there remain a large number of highly geographically concentrated industries in which inter-firm spillovers seem to be important.

agglomeration spillovers include R&D that creates local knowledge spillovers, investment in hiring and worker training programs that develop thicker local labour markets and investment in capital stock that attracts mobile input suppliers and/or enables local suppliers to exploit economies of scale. As pointed out by Shaver et al. (2000), "firms contribute to the externality in addition to benefiting from the externality". Contributing to the externality creates benefits which the firm cannot capture resulting in an *intra-cluster appropriation problem*. On the other hand, clustering enhances a firm's *absorptive capacity*, i.e. the ability to receive agglomeration spillovers, and receiving the externality leads to the possibility of *localized complementarities* in the externality generating investment activity.

On the other hand, negative pecuniary externalities refer to the business stealing effects that stem from product market rivalry between firms.² In contrast to agglomeration spillovers, these negative externalities are "global" and hence we may distinguish between two types of business stealing effects, intra-cluster and inter-cluster competitive effects.

Firstly, the interplay of intra-cluster competitive effects and agglomeration externalities tends to exacerbate the intra-cluster appropriation problem thereby undermining spillover generating investment and weakening the intensity of localized synergies. Not only are there benefits which the spillover generating firm cannot capture, but the improvement in the competitive position of rival cluster firms competes away the firm's own profits lowering the private return on spillover generating investment activities.³

Secondly, inter-cluster competitive effects imply that investment by a cluster firm crowds-out the investment activities of non-cluster firms. In particular, agglomeration externalities arising from the investment, by improving the competitiveness of cluster firms, exacerbate the negative pecuniary externalities experienced by non-cluster firms. Under these conditions, clusters tend to exhibit "*strength in numbers*", i.e. the intensity of the crowding effect of a cluster firm investment on the investment activities of non-cluster firms increases with cluster size.⁴

On the other hand, agglomeration externalities help cluster firms withstand the negative pecuniary externalities stemming from investment by a non-cluster firm. To recover compet-

²Business stealing effects are a pecuniary externality, i.e. an externality which works through the price system. In the absence of market imperfections, pecuniary externalities do not affect the pareto-optimality of competitive general equilibrium. However, in the presence of imperfect competition and agglomeration externalities, pecuniary externalities have welfare significance and ought to be treated like any other externality. See Krugman (1991) for discussion on this topic.

³Jaffe (1996) clarified this point in the context of R&D. This paper extends this logic to agglomeration.

⁴For example, if innovation diffuses faster within a cluster than after an innovation occurs, the number of tougher rivals faced by a non-cluster firm equals the size of the cluster from which the innovation emerged.

itiveness, cost reductions can be achieved more efficiently in the presence of agglomeration externalities. Under these conditions, clusters tend to exhibit "*safety in numbers*", i.e. the resilience of the investments of cluster firms to the crowding-out effects of investment by a non-cluster firms increases with cluster size.⁵ We refer to both strength in numbers and safety in numbers as *cluster strength effects*.

The model of this paper captures clusters and agglomeration externalities using the tools of social network theory. Nodes represent firms and links represent geographic proximity between firms.⁶ It is assumed each firm has only one geographic location with the implication that links are transitive and network components represent clusters. The network then defines the industry's spatial pattern and is characterized by a set of geographic clusters. Agglomeration externalities are modelled as local cost spillovers, i.e. component restricted spillovers, arising from the cost-reducing investments of firms.

An interesting feature of agglomeration is that the formation of a geographic connection between firms is the outcome a unilateral decision i.e. link formation is one-sided. In addition, switching location can involve the severing of more than one link. Therefore, the appropriate equilibrium concept for the analysis of geographic location decisions is that of Nash equilibrium in contrast to the stability concept frequently employed in the literature on social networks.

The paper adapts a model developed by Goyal and Moraga-Gonzalez (2001) in order to study the effect of agglomeration on firm performance and firms' incentives to agglomerate. In the first stage, each firm chooses its geographic location. In the second stage, given the industry's geography, firms invest in cost reduction. Cost reductions spillover to geographically proximate firms reducing their marginal production costs. In the final stage, given the industry's cost configuration, firms compete in a homogenous-product Cournot oligopoly. We adopt the interpretation that firms invest in cost-reducing R&D and agglomeration externalities arise from local knowledge spillovers. However, the model is sufficiently general as to admit several other relevant interpretations.⁷

⁵Note that in addition, since cluster size enhances a firm's absorptive capacity, firms in larger clusters are less likely to be exposed to a spillover generating investment that undermines the competitive position of the firm.

⁶A social network $g = (N, G)$ is defined by a set of agents N and a set of links $G = \{g_{ij} : g_{ij} \in \{0, 1\}, i \neq j \in N\}$. A link represents a pairwise relationship between agents. If $g_{ij} = 1$ then a link is said to exist between agents i and j . Otherwise, no link exists and $g_{ij} = 0$. Links are said to be transitive if and only if for all $i, j, k \in N$, $g_{ij} = 1$ and $g_{jk} = 1$ implies $g_{ik} = 1$.

⁷Some examples include: (i) investment in capital stock that creates a local pool of specialized inputs (ii) investment in worker training or university programs for the accumulation of industry specific human capital

The model identifies the following trade-off: on the one hand, firms in relatively larger clusters experience a weaker idiosyncratic incentive to innovate due to a more severe local appropriation problem but on the other hand, they benefit more from cluster strength effects, i.e. the crowding-out effect of an innovation by a cluster firm on non-cluster firms' R&D is relatively stronger in larger clusters. Local complementarities in R&D tend to be intense for cluster sizes which absorb a small fraction of the industry's firms but weaken considerably for larger cluster sizes as the appropriation problem takes over.

The analysis then focuses on dominant cluster spatial patterns, which are a particular type of core-periphery pattern. This spatial pattern is characterized by a single core cluster and a periphery of isolated firms. Firms choose between locating in the cluster and locating at an isolated location.⁸

In the class of dominant cluster spatial patterns, the main results concerning cluster firms are (i) innovation is lowest under geographic concentration; (ii) innovation is highest at an intermediate cluster size provided R&D is not too costly; (iii) unit production costs are lowest at an intermediate cluster size; and (iv) output and profits are highest at an intermediate cluster size. We then use a series of plots to provide intuition on the behaviour of innovation, production costs and firm performance.

We find that innovation by cluster firms exhibits a rise-and-fall pattern with respect to cluster size. For relatively small cluster sizes, local complementarities and strength effects dominate the appropriation problem stimulating innovation. For larger cluster sizes, innovation falls as the appropriation problem takes over and local complementarities weaken considerably. Strength effects become less important as innovation by isolated firms drops significantly. Under geographic concentration, a severe appropriation problem strongly discourages innovative effort.

Turning to the production costs of cluster firms, we find that economies of agglomeration dominate below a threshold cluster size and are driven by a combination of local complementarities, strength effects and improved absorptive capacity. However, diseconomies of

which increase the local pool of specialized labour; (iii) investment in hiring programs that reduce search costs resulting in thicker local labour market (iii) investment in lobbying aimed at increasing the supply of local public goods which enhance the attractiveness of the location for mobile workers and suppliers. Examples of such local public goods include public amenities, childcare facilities, cultural or entertainment goods, transport infrastructure.

⁸The U.S. automobile industry with Detroit as its core cluster is an example of a dominant cluster industry spatial pattern.

agglomeration eventually dominate after a threshold as the intra-cluster appropriation problem becomes severe. Moving to output and profits, we find the performance of cluster firms exhibits a rise-and-fall pattern with respect to cluster size. Agglomeration economies drive improvements in firm performance. However, firm performance falls steadily in the presence of diseconomies of agglomeration.

The paper then investigates the efficient and equilibrium cluster sizes. First, complete dispersion of firms is found to be inefficient. Welfare is shown to be higher under complete agglomeration. Although completely dispersed firms have stronger incentives to invest in R&D, there is *underutilization* of created knowledge. In particular, the industry achieves the same level of cost reduction under complete agglomeration and at a lower cost to society. Thus, the presence of at least some cluster firms reduces the level of R&D required to achieve a given level of cost reduction.

Second, complete agglomeration of firms is not efficient. Welfare is higher at an intermediate cluster size. In a geographically concentrated industry, an overly severe appropriation problem leads to too little innovative effort resulting in *underproduction* of knowledge. In particular, the industry-wide level of innovative activity is found to be higher at the intermediate cluster size. Thus, the presence of at least some isolated firms helps alleviate the cluster's appropriation problem thereby stimulating R&D to a more socially desirable level. In this sense, non-cluster firms keep cluster firms "on their toes" to the benefit of society at large.

We find that the unique equilibrium spatial pattern is a geographically concentrated industry. Thus, the private incentives to agglomerate of competing firms are socially excessive. Cluster strength effects ensure the intra-cluster appropriation problem does not undermine the incentive to cluster. Switching to an isolated location is a profitable deviation providing the deviating firm can innovate sufficiently. However, the deviating firm faces an "uphill" battle due to asymmetries in the competitive effects of innovation by cluster and isolated firms. It is the inability of the isolated firm to overcome the cluster's strength effects that drives firms to agglomerate completely, i.e. by joining the cluster, a firm can escape the cluster's strength effects.

This article contributes to the literature on agglomeration pioneered by Marshall (1920). Firstly, formal models of agglomeration economies typically assume away the competitive interaction which is central to this paper's arguments. For instance, Krugman's (1991) model

is formulated as a model of monopolistic competition. Incorporating competitive interaction, this paper sheds light on the role of cluster strength effects in industry agglomeration.

Secondly, whereas most theoretical work on agglomeration focuses on input-output linkages, following Abdel-Rahman and Fujita (1990) and Krugman (1991), or labour market pooling, following Hesley and Strange (1990) and Combes and Duranton (2006), knowledge spillover driven clustering has only rarely been studied (Spence, 2009). Moreover, work by Fosfuri and Ronde (2004) and Alseben (2005) among others caution that knowledge spillovers may provide a rationale against industrial clustering since opposing the benefit of absorbing external spillovers is the cost of sharing private knowledge with competitors. This paper contributes to this literature by deriving a model of agglomeration based on local knowledge spillovers. Moreover, the paper suggests that the firms may engage in a socially excessive level of agglomeration in the presence of local knowledge spillovers.

The paper also contributes to the literature on R&D (Arrow, 1962; Nordhaus, 1969; Spence, 1984; Katz, 1986). Spence (1984) clearly established that imperfect R&D appropriability results in a trade-off between incentives for the socially efficient production of knowledge and incentives for its socially efficient diffusion. This literature proposes several policy approaches aimed at striking a balance between the conflicting problems of underproduction and underutilization of knowledge, e.g. cooperative R&D, patents and R&D subsidies. Our main contribution to this literature is that if knowledge spillovers are local then geography can help resolve this trade-off, but that firms' private incentives to agglomerate may be socially excessive.

The article is also related to the literature on R&D cooperation in oligopoly (see, e.g., Katz (1986), d'Aspremont and Jacquemin (1988), Kamien, Muller and Zang (1992), Suzumura (1992) and Leahy and Neary (1997)). In the terminology of Kamien, Muller and Zang (1992) our model is a research joint venture competition type of cooperative agreement, where firms commit to completely share R&D results undertaken unilaterally. This literature typically compares the welfare effects of an industry-wide RJV with the R&D rivalry situation. This paper endogenizes the size of this RJV. The main contribution of our article to this literature is the finding that intermediate levels of cooperation better resolve the trade-off identified by Spence (1984), but that firms' private incentives to participate in the RJV may be socially excessive.

2.2 Model and Analysis

Consider an industry consisting of $n \geq 3$ ex-ante symmetric firms. Each firm locates in one of m , ex-ante identical, geographic locations. A pair of firms is said to be linked and referred to as neighbours if they locate at the same location. Let C_r denote the cluster of firms at location r and C_r^i denote the set of cluster neighbours of firm $i \in C_r$. The size of a cluster is taken to be the number of co-located firms, $|C_r| = n_r$. The set of possible industry spatial patterns is given by

$$G = \{(n_r)_{r=1}^m : \sum_{r=1}^m n_r = n, n_r \geq 0\}.$$

Given the industry's geography $g \in G$, each firm invests in R&D, denoted x_i , which reduces the firm's marginal cost of production. Innovations spillover to neighbouring firms lowering their marginal costs of production. The marginal cost of production of firm i located in cluster r is given by

$$c_i(x_i, (x_j)_{j \in C_r^i(g)}) = c - x_i - \sum_{j \in C_r^i(g)} x_j,$$

in which the initial constant marginal cost satisfies $c \geq x_i + \sum_{j \in C_r^i(g)} x_j > 0$ for all $g \in G$. Notice that the total cost reduction depends upon the aggregate level of innovation achieved by the cluster's firms. Knowledge spillovers are local and do not spread to non-neighbouring firms. The cost of R&D is quadratic reflecting the existence of diminishing returns to R&D expenditures and is given by $y(x_i) = \psi x_i^2$ with $\psi \geq 1$.

The industry's spatial pattern g leads to a vector of R&D efforts $\{x_i(g)\}_{i=1}^n$, which in turn determines the industry's cost configuration $\{c_i(g)\}_{i=1}^n$. In the final stage, given these marginal costs, firms compete in a homogenous-product oligopoly. Each firm sets output, q_i , and faces a linear inverse demand function given by $p = a - \sum_{i=1}^n q_i$, in which $a > c$. Bringing together expressions, the profits of firm i are

$$\pi_i(g) = [a - Q(g) - c_i(g)]q_i(g) - \psi x_i^2(g),$$

in which $Q(g) = \sum_{i=1}^n q_i(g)$. In the first stage, each firm chooses a location $r \in \{1, \dots, m\}$ in order to maximize profits taking as given the location decisions of rival firms. An industry spatial pattern $g = (n_r)_{r=1}^m \in G$ constitutes a Nash equilibrium if and only if for each firm i , $i \in C_r$ and $r \in \{1, \dots, m\}$, there is no incentive to switch location, i.e.

$$\pi_i(n_1, \dots, n_m) \geq \pi_i(n_1, \dots, n_{r-1}, n_r - 1, n_{r+1}, \dots, n_{v-1}, n_v + 1, n_{v+1}, \dots, n_m),$$

for all $v \neq r \in \{1, \dots, m\}$. Social welfare is defined as the sum of consumer surplus and industry profits and is given by

$$W(g) = \frac{1}{2}Q^2(g) + \sum_{i=1}^n \pi_i(g).$$

An industry spatial pattern $g \in G$ is said to be efficient if and only if $W(g) \geq W(\hat{g})$, for all $\hat{g} \in G$. The concept of efficiency employed here is in the spirit of the second best, since innovation and production are chosen non-cooperatively.

2.2.1 Performance Implications of Agglomeration

The game is solved using backward induction. In the market competition stage, given an industry cost configuration $\{c_i(g)\}_i$, each firm behaves as a Cournot monopolist and chooses output to maximise profits. The Cournot equilibrium output of firm i is given by

$$q_i(g) = \frac{a - nc_i(g) + \sum_{l \neq i}^n c_l(g)}{n+1},$$

and the profits of the Cournot competitors are

$$\pi_i(g) = \left[\frac{a - nc_i(g) + \sum_{l \neq i}^n c_l(g)}{n+1} \right]^2 - \psi x_i^2.$$

Without loss of generality, suppose firm i locates in cluster r . Note that there are three types of firms; the representative firm i , the $n_r - 1$ cluster neighbours of firm i and the $n - n_r$ non-neighbours of firm i (i.e. those firms which do not locate in firm i 's cluster).

Suppose the neighbours and non-neighbours of firm i are represented by the subscripts j and k respectively. Let x_j denote the R&D investment undertaken by a neighbour of firm i and let x_k denote the corresponding R&D investment of a non-neighbour of firm i . Plugging in the cost reduction formulation and rearranging yields the reduced-form profit function of firm i

$$\pi_i(g) = \left[\frac{(a-c) + (n+1-n_r)x_i + \sum_{j \in C_r^i} (n+1-n_r)x_j - \sum_{t \neq r}^m (\sum_{k \in C_t} n_t x_k)}{n+1} \right]^2 - \psi x_i^2(g).$$

Prior to market competition, each firm invests in R&D in order to maximise reduced-form profits taking as given the R&D choices of rival firms and the industry's geography $g = (n_r)_{r=1}^m$. Taking the first order condition, the incentive to innovate can be decomposed into an idiosyncratic component, a local complementarity component and $m - 1$ inter-cluster substitutability components,⁹

⁹The restriction $\psi \geq 1$ ensures that the second order condition is satisfied.

$$\frac{\partial \pi_i}{\partial x_i} = \underbrace{\alpha_r - \sigma_r x_i}_{\text{idiosyncratic}} + \underbrace{\sum_{j \in C_r^i} \sigma_{rr} x_j}_{\text{local}} - \sum_{t \neq r}^m \underbrace{\left(\sum_{k \in C_t} \sigma_{rt} x_k \right)}_{\text{inter-cluster}} = 0$$

in which

$$\alpha_r = \left(1 - \frac{n_r}{n+1}\right) \frac{(a-c)}{n+1}, \quad \sigma_r = \psi - \left(1 - \frac{n_r}{n+1}\right)^2,$$

and

$$\sigma_{rr} = \left(1 - \frac{n_r}{n+1}\right)^2, \quad \sigma_{rt} = \left(1 - \frac{n_r}{n+1}\right) \frac{n_t}{n+1}.$$

Local bilateral influence is captured by the cross-derivatives $\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} = \sigma_{rr}$ for all $j \in C_r^i$.¹⁰ Notice that $\sigma_{rr} > 0$ implies that the R&D investments of neighbouring firms are strategic complements. The term σ_{rr} measures the increase in firm i 's incentive to innovate due to a marginal increase in a neighbouring firm's R&D effort and reflects the *intensity of local complementarity* in cluster r .

Inter-cluster bilateral influences are captured by the cross-derivatives $\frac{\partial^2 \pi_i}{\partial x_i \partial x_k} = -\sigma_{rt}$ for all $k \in C_t, t \neq r$.¹¹ Notice that $\sigma_{rt} > 0$ for all $t \neq r$, implies that the R&D investments of non-neighbouring firms are strategic substitutes. In addition, observe that $\sigma_{rt} \neq \sigma_{tr}$ for all $t \neq r$; hence inter-cluster bilateral influences are not symmetric. The term σ_{rt} measures the reduction in firm i 's incentive to innovate due to a marginal increase in a non-neighbouring firm's R&D effort and reflects the *degree of vulnerability* of firms in cluster r to innovations made by firms in cluster $t \neq r$.

We investigate the effect of cluster size on the incentive to innovate of cluster firms holding constant the number of firms in the industry.¹² First, notice that if $\tilde{n}_r < n_r$ then $\alpha_r < \tilde{\alpha}_r$ and $\tilde{\sigma}_r < \sigma_r$; hence an increase in cluster size, by exacerbating the intra-cluster appropriation problem, reduces the idiosyncratic incentive to innovate.¹³

Second, clusters exhibit "safety in numbers". Observe that $\tilde{n}_r < n_r$ implies $\sigma_{rt} < \tilde{\sigma}_{rt}$ for all $t \neq r$; hence firms in larger clusters are more resilient to innovations emanating

¹⁰Note that these local bilateral influences are uniform across all pairs of neighbouring firms.

¹¹Note that for any two clusters $t, r \in \{1, \dots, m\}, t \neq r$, intercluster bilateral influences are uniform across all pairs of non-neighbouring firms.

¹²An alternative comparative static exercise is to examine the effect of a new entrant on the incentive to innovate of its neighbouring firms. This exercise does not change the qualitative nature of our comparative static results, since the cluster quotient, i.e. the fraction of firms in the industry locating in the cluster, increases when the entrant firm joins the cluster.

¹³In the absence of competitive effects, an increase in cluster size does not influence the idiosyncratic incentive to innovate, see Section 2.3.

from non-cluster firms. Furthermore, clusters exhibit "strength in numbers". Notice that $\tilde{n}_r < n_r$ implies $\tilde{\sigma}_{tr} < \sigma_{tr}$ for all $t \neq r$; hence non-cluster firms are more vulnerable to innovations made by firms in larger clusters.¹⁴

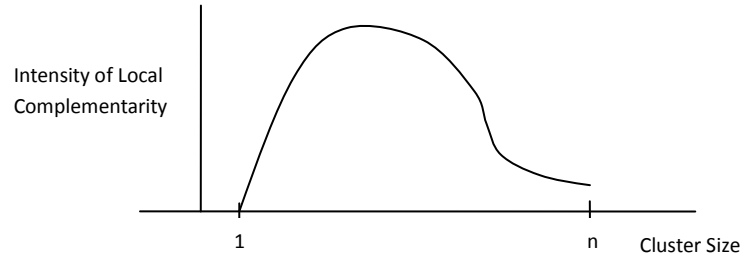
Third, the effect of cluster size on the intensity of local complementarity involves two opposing effects. On the one hand, if a neighbouring firm innovates then the extent to which this innovation stimulates firm investment is affected by the severity of the intra-cluster appropriation problem (appropriation effect). Notice that $\tilde{n}_r < n_r$ implies $\sigma_{rr} < \tilde{\sigma}_{rr}$; hence cluster size, by exacerbating the intra-cluster appropriation problem, weakens the intensity of local complementarity. On the other hand, the potential spillover pool of a firm increases with cluster size enhancing the firm's absorptive capacity (absorption effect).

In order to investigate which of these effects dominate, symmetry is imposed on neighbouring firms. Let $x_j = \bar{x}_j$ and $x_k = x_t$ for all $j \in C_r^i$, $k \in C_t$, $t \neq r$. The first order condition then becomes

$$\frac{\partial \pi_i}{\partial x_i} = \alpha_r - \sigma_r x_i + (n_r - 1)\sigma_{rr}\bar{x}_j - \sum_{t \neq r}^m n_t \sigma_{rt} x_t = 0.$$

The term $(n_r - 1)\sigma_{rr}$ captures the intensity of local complementarity in cluster r . Suppose n_r is a continuous variable, the graph of this function is plotted in Figure 1.

Figure 1: Intensity of Local Complementarity and Cluster Size



Observe that as cluster size increases, the intensity of local complementarity (i) rises rapidly initially, (ii) remains relatively stable around its maximum $n_r^{\max} = \frac{1}{3}n + 1$, i.e. when the cluster absorbs approximately one-third of the industry's firms, (iii) declines rapidly around the point of inflection $n_r^{in} = \frac{2}{3}n + 1$, i.e. when the cluster absorbs roughly two-thirds of the industry's firms, and (iv) bottoms-out as the cluster absorbs all firms in the industry.¹⁵ Notice that the absorption effect dominates the appropriation effect for relatively small cluster sizes and vice versa.

¹⁴In the absence of competitive effects, these cluster strength effects do not occur, see Section 2.3.

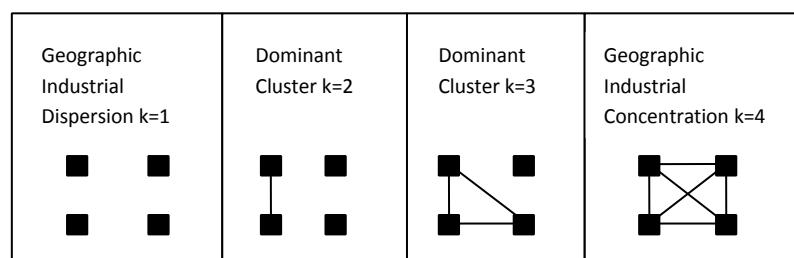
¹⁵In the language of social network theory, network effects are not monotonically increasing in network (cluster) size.

To summarize, there is a trade-off between innovation and cluster size: firms in relatively larger (smaller) clusters experience a weaker (stronger) idiosyncratic incentive to innovate, since they appropriate less (more) of the return on their investment, but they tend to be more resilient (vulnerable) to innovations achieved by non-cluster firms. Local complementarities tend to be intense for cluster sizes which absorb a small fraction of the industry's firms but weaken considerably for larger cluster sizes.

Dominant Cluster Industry Spatial Patterns

The paper focuses on dominant cluster industry spatial patterns. This spatial pattern is characterized by one cluster with k firms and $n - k$ isolated firms. In this simplified setting, firms choose to join the cluster or to be isolated. Figure 2 illustrates the set of possible dominant cluster spatial patterns in an industry consisting of four firms.

Figure 2: Dominant Cluster Industry Spatial Patterns $n=4$



If the industry is geographically dispersed then each firm has a strong idiosyncratic incentive to innovate. However, since firms are isolated, they do not benefit from localized complementarities and each firm is vulnerable to innovations achieved by rival firms. If two firms colocate then the intra-cluster appropriation problem weakens their idiosyncratic incentives to innovate but the firms benefit from intense localized complementarities and increased resilience to innovations from isolated firms. On the other hand, isolated firms are now more vulnerable to innovations from their clustered rivals.

In comparison to a cluster size of two, if three firms colocate then the appropriation problem further weakens the idiosyncratic incentive to innovate of cluster firms. Local complementarities become less intense but cluster firms are now more resilient to innovations from the remaining isolated firm. This isolated firm is now much more vulnerable to the innovations of rival firms. If the industry is geographically concentrated then the appropriation problem is severe. Both the idiosyncratic incentives to innovate and local

complementarities are at their lowest level. However, firms are not adversely affected by the innovations of rival firms. Notice that regardless of cluster size, isolated firms have the same strong idiosyncratic incentive to innovate.

Suppose cluster firms and isolated firms are represented with the subscripts D and I respectively. Substituting the dominant cluster spatial pattern into the first order condition, invoking intra-cluster symmetry and solving gives the Nash equilibrium R&D investments.¹⁶ The equilibrium R&D investments of cluster and isolated firms as a function of cluster size k are respectively given by

$$x_D(k) = \frac{\frac{1}{n+1}(\psi - \frac{n}{n+1})(1 - \frac{k}{n+1})(a-c)}{\psi^2 - (k(1 - \frac{k}{n+1})^2 + \frac{n(k+1)}{(n+1)^2})\psi + \frac{nk}{(n+1)^2}(1 - \frac{k}{n+1})}$$

for $k \in \{1, \dots, n\}$ and

$$x_I(k) = \frac{\frac{n}{(n+1)^2}(\psi - k(1 - \frac{k}{n+1}))(a-c)}{\psi^2 - (k(1 - \frac{k}{n+1})^2 + \frac{n(k+1)}{(n+1)^2})\psi + \frac{nk}{(n+1)^2}(1 - \frac{k}{n+1})}$$

for $k \in \{1, \dots, n-1\}$. We would like to derive and classify the critical and inflection points for each of these functions. Unfortunately, the expressions become quite complicated.¹⁷ In order to proceed, we follow a method of analysis similar to that of Goyal and Moraga-Gonzalez (2001).

We first show that if R&D is not too expensive then there exists an intermediate cluster size \bar{k}_D , satisfying $1 < \bar{k}_D < n$, for which innovation by cluster firms is maximized. To establish this result, first note that innovation achieves a minimum under geographic concentration. The denominator and numerator of $x_D(k)$, denoted $d(k)$ and $u_D(k)$ respectively, satisfy $d(k) < d(n)$ and $u_D(k) > u_D(n)$ for $k \in \{1, \dots, n-1\}$.¹⁸ Therefore, it follows that $x_D(k) > x_D(n)$ for all $k \neq n$.

To complete the argument, it is now sufficient to show that provided R&D is not too expensive, innovation rises initially with cluster size. Otherwise, innovation achieves a maximum under geographic dispersion. In the Appendix we show there exists a threshold

¹⁶The restriction $\psi > \psi_{NE} = \max\{\frac{1}{4}(n+1), \frac{n^2}{(n+1)^2} \frac{a}{c}\}$ ensures that the firms' effective costs are positive, second-order conditions are satisfied and that a unique interior Nash equilibrium exists. See the Appendix for the derivations.

¹⁷Innovation by cluster firms is a linear/cubic rational function in cluster size. The linear numerator is monotonically decreasing in cluster size. On the other hand, innovation by isolated firms is a quadratic/cubic function in cluster size. The quadratic numerator exhibits a U shape. The common cubic denominator follows a U shape that is skewed to the left. The coefficients alternate in sign and the leading coefficient is negative.

¹⁸See the Appendix for the proof.

value of ψ , denoted ψ_t , with $\psi_t > \psi_{NE}$, such that if $\psi < \psi_t$ then $x_D(1) < x_D(2)$ and if $\psi > \psi_t$ then $x_D(1) > x_D(k)$ for all $k \in \{2, \dots, n\}$. These observations give the following result:

Proposition 2.1 *In the class of dominant cluster spatial patterns, innovation by cluster firms is minimized under geographic concentration. If R&D is not too costly then innovation by cluster firms is maximized at an intermediate cluster size. Otherwise, it is maximized under geographic dispersion.*

We next show that provided the industry is not too small i.e. $n \geq 4$, there exists an intermediate cluster size \bar{k}_I , satisfying $1 < \bar{k}_I < n - 1$, for which innovation by isolated firms is minimized. To establish this result, first notice that innovation by isolated firms attains a maximum under geographic dispersion,

$$x_I(1) - x_I(k) = \frac{nk(n-k)(k-1)(a-c)}{d(1)d(k)(n+1)^4} \left(\psi - \frac{n}{n+1} \right) > 0$$

for all $k = 2, \dots, n - 1$.¹⁹ To complete the argument, it is now sufficient to show that innovation eventually rises with cluster size. In the Appendix we prove that $x_I(n - 1) - x_I(n - 2) > 0$ for all $n \geq 4$.

We next show that if the R&D cost parameter is not too low then isolated firms are more innovative than cluster firms after a threshold cluster size \bar{k} , satisfying $1 \leq \bar{k} \leq n - 1$. Comparing innovation by cluster and isolated firms gives

$$x_D(k) - x_I(k) = \frac{(a-c)}{(n+1)d(k)} (k - 1) \left(k - \left(n + 1 - \frac{n+1}{n} \psi \right) \right).$$

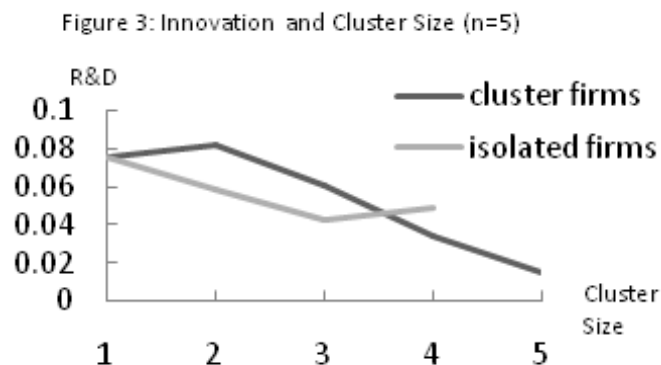
Notice that there exists a threshold cluster size $\bar{k} = n + 1 - \frac{n+1}{n} \psi$, with $\bar{k} \leq n - 1$ provided $\psi \geq \frac{2n}{n+1}$, such that $x_D(k) > x_I(k)$ for $k < \bar{k}$ and $x_D(k) \leq x_I(k)$ for $k \geq \bar{k}$.²⁰ These observations give the following proposition:

Proposition 2.2 *In the class of dominant cluster spatial patterns, innovation by isolated firms is maximized under geographic dispersion and minimized at an intermediate cluster size. If the R&D cost parameter is not too low then after a threshold cluster size, isolated firms are more innovative than cluster firms.*

¹⁹See the Appendix for details of the derivation.

²⁰If R&D is sufficiently expensive, namely $\psi > \frac{n(n-1)}{n+1}$, then $\bar{k} < 2$ and hence non-cluster firms innovate more than cluster firms for all possible cluster sizes k .

The intuition behind these results may be seen with the help of an example. Figure 3 depicts innovation by cluster and isolated firms in an industry consisting of 5 firms. The parameters values are $\psi = 2$ and $a - c = 1$, which can be interpreted as the initial market size normalized to one. The horizontal axis measures cluster size. At the left end of this axis is geographic industrial dispersion. At the right end of the axis is geographic industrial concentration.



Innovation by cluster firms exhibits a rise-and-fall pattern. As cluster size rises, increased resilience to reduced innovation by isolated firms and local complementarities dominate the appropriation problem stimulating innovation which achieves its maximum when the cluster is roughly one third of the industry's size. After the maximum, the appropriation problem dominates and innovation decreases monotonically. Innovation declines rapidly when cluster size is roughly two thirds of the industry's size as local complementarities weaken considerably. Under geographic concentration, a severe local appropriation problem and negligible local complementarities strongly discourage innovative effort.

Innovation by isolated firms exhibits a fall-and-rise pattern. Innovation is highest under geographic dispersion. As cluster size increases, innovation falls because of higher innovation by cluster firms and the increased vulnerability of isolated firms. As cluster size increases further, innovation continues to decline as the strength in numbers effect offsets reductions in cluster firm innovation. Notice that innovation is lowest at an intermediate cluster size. For larger cluster sizes, innovation rises as the intra-cluster appropriation problem offsets the increased vulnerability of isolated firms. However, the rise in innovation is weak because the strength in numbers effect continues to hold back innovation by isolated firms.

Substituting R&D by cluster firms into the marginal cost specification gives the unit production costs of cluster firms as a function of cluster size

$$c_D(k) = c - kx_D(k).$$

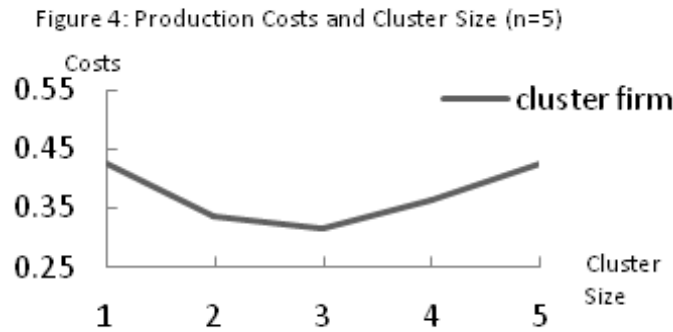
We now show that there exists an intermediate cluster size \bar{k}_c , satisfying $1 < \bar{k}_c < n$, at which the unit production costs of cluster firms are minimized. It is easily verified that unit production costs satisfy $c_D(1) = c_D(n)$. To establish the result, it is now sufficient to show that the unit production costs achieve a maximum under geographic dispersion and geographic concentration,

$$c_D(n) - c_D(k) = \frac{(n-k)(k-1)(a-c)}{(n+1)^2 d(n)d(k)} \left(\psi - \frac{n}{(n+1)} \frac{(k+1)}{(n+1)} \right) > 0$$

for $k \in \{2, \dots, n-1\}$.²¹ In particular, notice that $c_D(1) > c_D(2)$ and $c_D(n-1) < c_D(n)$; agglomeration economies dominate initially but eventually give way to diseconomies of agglomeration. The effect of cluster size on the unit production costs of cluster firms is non-linear. These observations give the following result:

Proposition 2.3 *In the class of dominant cluster spatial patterns, the unit production costs of cluster firms are minimized at an intermediate cluster size.*

Continuing our example (i.e. $n = 5$, $\psi = 2$, $a - c = 1$) and taking $c = 0.5$, Figure 4 depicts unit production costs of cluster firms as a function of cluster size.



The relationship between the unit production costs of cluster firms and cluster size is U-shaped. As cluster size increases, economies of agglomeration dominate because of higher innovation and R&D sharing. Agglomeration economies then weaken but continue to dominate as the effects of lower innovative effort are offset by R&D sharing. It can be seen that production costs are lowest at an intermediate cluster size. Diseconomies of agglomeration then dominate as the effects of lower innovative efforts take over.

²¹See the Appendix for details of the derivation.

Notice that economies of agglomeration dominate below a threshold cluster size and are driven by a combination of local complementarities, cluster size strength effects and improved absorptive capacity. Diseconomies of agglomeration play an increasing role as cluster size increases and eventually dominate after a threshold as the intra-cluster appropriation problem takes over and becomes severe, i.e. when innovation reductions dominate R&D sharing.

We next examine the effect of agglomeration on firm profits. Substituting the production costs into the profit functions and rearranging reveals that the profits of cluster and isolated firms as a function of cluster size k are respectively given by

$$\pi_D(k) = \frac{\psi(\psi - (1 - \frac{k}{n+1})^2)(\psi - \frac{n}{n+1})^2(a-c)^2}{(n+1)^2 d^2(k)},$$

and

$$\pi_I(k) = \frac{\psi(\psi - (\frac{n}{n+1})^2)(\psi - k(1 - \frac{k}{n+1}))^2(a-c)^2}{(n+1)^2 d^2(k)}.$$

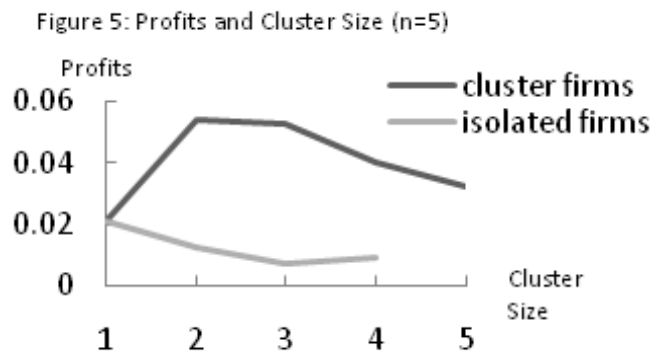
We first show there exists an intermediate cluster size \bar{k}_D^π , satisfying $1 < \bar{k}_D^\pi < n$, for which the profits of cluster firms are maximized. To establish this result, first note that profits attain a minimum under geographic dispersion. In the Appendix we prove that $\pi_D(k) - \pi_D(1) > 0$ for all $k \in \{2, \dots, n\}$. To complete the argument, it is sufficient to show that profits eventually decrease with cluster size. In the Appendix we prove that $\pi_D(n-1) - \pi_D(n) > 0$.

We next show that, provided the industry is not too small i.e. $n \geq 4$, there exists an intermediate cluster size $k_{\pi_I}^{\min}$, with $1 < k_{\pi_I}^{\min} < n-1$ at which the profits of isolated firms are minimized. First note that profits attain a maximum under geographic dispersion. In the Appendix we prove that $\pi_I(1) > \pi_I(k)$ for all $k \in \{2, \dots, n-1\}$. To establish the result, it is now sufficient to show that profits eventually increase with cluster size. In the Appendix we show that $\pi_I(n-2) < \pi_I(n-1)$ for all $n \geq 4$.

Finally we show that cluster firms earn higher profits than isolated firms. The profits of cluster firms are bounded below by $\pi_D(1)$ and this lower bound is an upper bound for the profits of isolated firms. It follows that $\pi_D(k) > \pi_D(1) > \pi_I(k)$ for all $k \in \{2, \dots, n-1\}$.

Proposition 2.4 *In the class of dominant cluster spatial patterns, (i) the profits of cluster firms are maximized at an intermediate cluster size; (ii) the profits of isolated firms are minimized at an intermediate cluster size; and (iii) cluster firms earn higher profits than isolated firms.*

Analogous results hold for firm output (see the Appendix). Continuing our example (i.e. $n = 5$, $\psi = 2$, $a - c = 1$), Figure 5 depicts the profits of cluster and isolated firms as a function of cluster size.



The profits of cluster firms exhibit a rise-and-fall pattern. Profits are lowest under geographic dispersion. As cluster size increases, profits rise because agglomeration economies raise the relative efficiency of cluster firms. Notice that the profits of cluster firms are highest at an intermediate cluster size. Profits then decline slightly because the effects of more intense competition offset cost reductions stemming from agglomeration economies. For large cluster sizes, firm performance decreases steadily in the presence of diseconomies of agglomeration.

The profits of isolated firms exhibit a fall-and-rise pattern. Profits are highest under geographic dispersion. As cluster size increases, firm performance falls as cluster firms become relatively more efficient than isolated firms because of economies of agglomeration. It can be seen that profits are lowest at an intermediate cluster size. Profits then recover as isolated firms become relatively more efficient in the presence of diseconomies of agglomeration.

2.2.2 Incentives to Agglomerate

In the first stage, firms decide whether to locate in the cluster or at an isolated location and face the following trade-off: firms which locate in the cluster suffer from a local appropriation problem but benefit from cluster strength effects and improved absorptive capacity. On the other hand, firms which locate at an isolated location escape the cluster's appropriation problem but they do not access knowledge spillovers from competitors and they are exposed to the cluster's strength effects.

A spatial pattern g constitutes a Nash equilibrium provided no firm has an incentive to switch location. In the context of dominant cluster spatial patterns, this equilibrium

condition requires that cluster firms do not have an incentive to switch to an isolated location and isolated firms do not have an incentive to move to the cluster.

Recall that the profits of cluster firms are bounded below, i.e. $\pi_D(k) > \pi_D(1)$ for all $k \in \{2, \dots, n\}$, and this lower bound is an upper bound for the profits of isolated firms, i.e. $\pi_D(1) > \pi_I(k)$ for all $k \in \{2, \dots, n-1\}$. Combining expressions we see that $\pi_D(k+1) > \pi_I(k)$ for all $k \in \{1, \dots, n-1\}$. Hence, given the location decisions of rival firms, cluster firms never have an incentive to switch to an isolated location. On the other hand, isolated firms always have an incentive to move to the location of the cluster. Therefore, the intra-cluster appropriation problem does not undermine the incentive to cluster. These observations give the following result:

Proposition 2.5 *In the class of dominant cluster spatial patterns, the unique Nash equilibrium spatial pattern is a geographically concentrated industry, i.e. $k^* = n$.*

We see that firms have strong incentives to cluster together. Even in the presence of diseconomies of agglomeration, cluster firms do not have an incentive to switch to an isolated location. The intuition behind the result is that cluster strength effects ensure the intra-cluster appropriation problem does not undermine the incentive to cluster.

Notice that switching to an isolated location is a profitable deviation provided the deviating firm can innovate sufficiently. However, the deviating firm "faces an uphill battle" because of asymmetries in the inter-cluster competitive effects of innovation.²² On the one hand, due to strength in numbers, the incentive to innovate of the deviating firm is highly vulnerable to innovation by the remaining cluster firms. On the other hand, due to safety in numbers, the incentive to innovate of the remaining cluster firms is highly resilient to innovation by the deviating firm. These asymmetries restrict innovation by isolated firms and it is the inability of isolated firms to overcome these asymmetries which motivates firms to cluster together. By clustering, firms can avoid the cluster's strength effects.

Consider now the social welfare aspects of geographic clustering. First, geographic industrial dispersion is not efficient. Notice that social welfare is higher under geographic industrial concentration than under geographic industrial dispersion:

$$W(n) - W(1) = \frac{(n-1)n\psi(a-c)^2}{(n+1)(\psi - \frac{n}{n+1})^2} > 0.$$

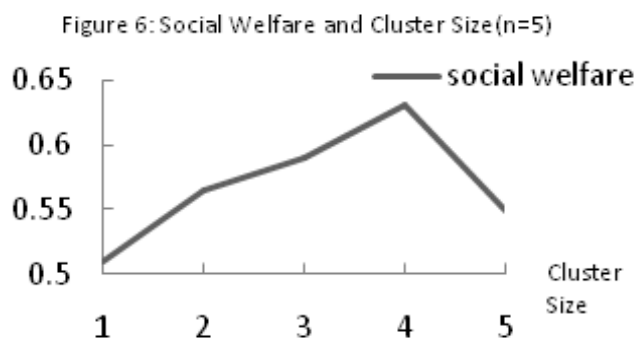
²²Notice that the asymmetry in the inter-cluster bilateral effects of innovation for cluster size k is given by $\sigma_{DI} = \frac{n+1-k}{(n+1)^2} < \frac{nk}{(n+1)^2} = \sigma_{ID}$ for $k > 1$, which is particularly severe when $k = n-1$.

This observation reveals that at least some level of agglomeration is socially desirable. Under geographic industrial dispersion, firms have strong incentives to invest in R&D but there is *underutilization* of created knowledge. Notice that whereas the industry-wide level of innovative activity is higher under complete dispersion than under complete agglomeration of firms, i.e. $nx_I(1) > nx_D(n)$, the unit productions costs of firms are the same, i.e. $c_I(1) = c_D(n)$. Consequently, the level of cost reduction obtained under geographic dispersion is achieved at an excessively high cost to society. Thus, the presence of some cluster firms reduces the level of R&D required to achieve a given level of cost reduction.

Second, social welfare is highest at an intermediate cluster size. To establish this result, it is sufficient to prove that geographic industrial concentration is not efficient. In the Appendix we show that $W(n-1) > W(n)$. This observation reveals that the presence of at least some isolated firms is socially desirable. Under geographic industrial concentration, firms experience an overly severe appropriation problem resulting in *underproduction* of knowledge. In the Appendix it is shown that the industry-wide level of innovative activity is higher for cluster size $k = n-1$ than under complete agglomeration of firms. Thus, the presence of some isolated firms helps alleviate the cluster's appropriation problem thereby stimulating R&D to a more socially desirable level. Taken together these observations give the following result:

Proposition 2.6 *Neither complete agglomeration nor complete dispersion of firms is socially desirable, i.e. $1 < k^{**} < n$. In the class of dominant cluster spatial patterns, firms engage in a socially excessive level of clustering, i.e. $k^{**} < k^*$.*

We see that the equilibrium level of industry clustering is socially excessive. Under geographic concentration, a severe intra-cluster appropriation problem leads to too little innovative effort. Continuing our example, Figure 6 depicts social welfare as a function of cluster size in an industry consisting of five firms.



Observe that social welfare exhibits a rise-and-fall pattern with respect to cluster size. Notice that welfare is lowest under geographic concentration. It can be seen that intermediate cluster sizes dominate the equilibrium spatial pattern from a welfare perspective. At intermediate cluster size, the presence of non-cluster firms helps alleviate the intra-cluster appropriation problem thereby stimulating innovation to a more socially desirable level. In this sense, non-cluster firms keep cluster firms "on their toes" to the benefit of society at large.

2.2.3 The Role of Competitive Effects

To clarify the role of negative pecuniary externalities in our model of agglomeration, consider the same model as before except now suppose firms behave as independent monopolists i.e. the inverse demand function is given by $p_i = a - q_i$.

Firm output and profits are then respectively given by $q_i(g) = \frac{a-c_i(g)}{2}$ and $\pi_i(g) = (\frac{a-c_i(g)}{2})^2 - \psi x_i^2(g)$. Plugging in the cost formulation, the reduced form profit function is $\pi_i(g) = (\frac{a-c+x_i+\sum_{j \in C_r^i(g)} x_j}{2})^2 - \psi x_i^2(g)$. Taking the first order condition, the incentive to innovate decomposes into an idiosyncratic component and a local complementarity component:

$$\frac{\partial \pi_i}{\partial x_i} = \underbrace{\frac{a-c}{4} - (\psi - \frac{1}{4})x_i}_{\text{idiosyncratic}} + \underbrace{\frac{1}{4} \sum_{j \in C_r^i(g)} x_j}_{\text{local}} = 0.$$

First note the absence of cluster strength effects. Second, cluster size does not influence the idiosyncratic incentive to innovate. Therefore, the appropriation problem does not undermine private innovation incentives. Invoking symmetry on cluster neighbours, the first order condition becomes

$$\frac{\partial \pi_i}{\partial x_i} = \frac{a-c}{4} - (\psi - \frac{1}{4})x_i + \frac{1}{4}(n_r - 1)x_j = 0.$$

Notice that the intensity of local complementarity increases with cluster size. Due to the absence of the appropriation effect, the absorption effect dominates for all cluster sizes. In the class of dominant cluster spatial patterns, equilibrium R&D of cluster and isolated firms are respectively given by

$$x_D(k) = \frac{a-c}{4(\psi - \frac{k}{4})}; \text{ and } x_I(k) = \frac{a-c}{4(\psi - \frac{1}{4})}.$$

Observe that innovation by cluster firms is monotonically increasing in cluster size because of local complementarities.²³ The unit production costs of cluster firms are given by $c_D(k) = c - \frac{k}{4(\psi - \frac{k}{4})}(a - c)$. Notice that economies of agglomeration dominate for all cluster sizes. The profits of cluster and isolated firms are respectively given by

$$\pi_D(k) = \frac{\psi(\psi - \frac{1}{64})(a - c)^2}{4(\psi - \frac{k}{4})^2}; \text{ and } \pi_I(k) = \frac{\psi(\psi - \frac{1}{64})(a - c)^2}{4(\psi - \frac{1}{4})^2}.$$

The performance of cluster firms monotonically increases with cluster size. Notice that $\pi_D(k) > \pi_D(1)$ for all $k > 1$ and $\pi_D(1) = \pi_I(k)$ for all $k \geq 1$. It follows that $\pi_D(k+1) > \pi_I(k)$. Therefore, the unique equilibrium spatial pattern is geographic industrial concentration.

Simple manipulations reveal that in the class of dominant cluster spatial patterns, social welfare can be written as $W(k) = n(q_D^2(1) + \pi_D(1)) + k(q_D^2(k) - q_D^2(1)) + k(\pi_D(k) - \pi_D(1))$.²⁴ Since both the output and profits of cluster firms increase with cluster size, it follows that social welfare is increasing in cluster size. Therefore, the equilibrium spatial pattern is efficient.

We see that i) firms cluster because agglomeration externalities generate agglomeration economies; ii) firm performance increases with cluster size; and iii) the equilibrium level of industry clustering is efficient. Therefore, the agglomeration economies story adequately explains industry clustering in this setting in which competitive effects are absent.

This variation of the model clarifies that it is the interplay of agglomeration externalities and negative pecuniary externalities which leads to i) an intra-cluster appropriation problem that discourages spillover generating effort and weakens the intensity of local complementarities; and ii) cluster strength effects. The example highlights that the interaction of these two externalities is behind the rise-and-fall relationship between firm performance and cluster size and the socially excessive level of geographic clustering among competing firms.

²³The restriction $\psi > \max\{\frac{n}{4}, \frac{n}{4} \frac{a}{c}\}$ ensures the interiority of the solutions for all the optimization problems and the existence of a unique interior Nash equilibrium.

²⁴The output of cluster and isolated firms is respectively given by $q_D = \frac{\psi(a-c)}{2(\psi - \frac{k}{4})}$ and $q_I = \frac{\psi(a-c)}{2(\psi - \frac{1}{4})}$. Social welfare is the sum of consumer surplus and industry profits: $W(g) = \sum_i \frac{q_i^2(g)}{2} + \sum_i \pi_i(g)$. In the class of dominant group spatial patterns, social welfare is given by $W(k) = kq_D^2(k) + (n - k)q_I^2(k) + k\pi_D(k) + (n - k)\pi_I(k)$.

2.3 Conclusion

There is a large body of research which studies the geographic concentration of economic activity within specific industries. This research tends to explain industrial agglomeration by emphasizing the role of agglomeration economies. This article develops a model of agglomeration which captures an often overlooked aspect of agglomeration among firms which produce close substitutes: the interaction of agglomeration externalities and negative pecuniary externalities. The analysis sheds light on the nature of the externalities that lead to the localization of specific industries, firms' incentives to cluster and the relationship between firm performance and geographic cluster size.

We find that the interaction of these two externalities is behind the rise-and-fall pattern in the performance of dominant cluster firms and the socially excessive level of geographic clustering among competing firms. Firms still have strong incentives to locate in the dominant cluster, even in the presence of diseconomies of agglomeration, because of cluster strength effects which push isolated firms to the fringes of the market. The model makes a number of restrictive assumptions which imply that these results may not hold in more general settings. In particular, the analysis focuses on the class of dominant cluster spatial patterns, ex-ante symmetric firms, with specific functional forms for demand and cost conditions.

It is hoped that future research will explore the relationship between cluster size, firms' incentives to cluster and firm performance in more general industry settings. For instance, our model does not succeed in explaining why in many industries a significant fraction of firms locate outside the major industry cluster (see Prevezer, 1997; and Folta et al., 2006). Allowing for heterogeneity in firm R&D productivity (see Shaver et al. 2000) or relaxing the intensity of the competitive effects in our model might help explain this empirical regularity. In addition, the insights of this paper concerning the payoff structures for general industry spatial patterns may provide a useful building block for dynamic analyses of industry clusters, e.g agent-based modelling approaches.

The ideas presented in this paper are likely to be much more general than suggested by the industrial agglomeration setting. These ideas apply to situations in which the incentives of group members exhibit the pattern of interaction displayed in our first order condition: an intra-group appropriation problem, "local" interactions and group strength effects. It is hoped the notion of group strength effects might prove useful for the study of R&D, academic research, terrorism, crime, herding behaviour and social movements.

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2.4 Appendix

Equilibrium R&D: The R&D choice sets, $[0, c]$, are non-empty, convex and compact and the R&D payoff functions are continuous and strictly concave in own strategy (recall the assumption $\psi \geq 1$). Therefore, a Nash equilibrium exists in pure strategies for general industry spatial patterns. Substituting the dominant cluster spatial pattern into the first order condition and invoking intra-cluster symmetry, the first order conditions in matrix notation are:

$$\begin{pmatrix} \psi - k(1 - \frac{k}{n+1})^2 & (1 - \frac{k}{n+1})(\frac{n-k}{n+1}) \\ \frac{nk^2}{(n+1)^2} & \psi - \frac{n(k+1)}{(n+1)^2} \end{pmatrix} \begin{pmatrix} x_D \\ x_I \end{pmatrix} = \begin{pmatrix} (1 - \frac{k}{n+1})(\frac{a-c}{n+1}) \\ \frac{n}{n+1}(\frac{a-c}{n+1}) \end{pmatrix}.$$

The determinant of the matrix of this system is given by $d(k) = \psi^2 - (k(1 - \frac{k}{n+1})^2 + \frac{n(k+1)}{(n+1)^2})\psi + \frac{nk}{(n+1)^2}(1 - \frac{k}{n+1})$. The restriction $\psi > \frac{n+1}{4}$ ensures that $d(k) > 0$, for all $k \in \{1, \dots, n\}$. Solving the matrix equation, simple algebra yields the unique solution for equilibrium R&D of cluster and isolated firms:

$$\begin{pmatrix} x_D \\ x_I \end{pmatrix} = \frac{1}{d(k)} \begin{pmatrix} (\psi - \frac{n}{n+1})(1 - \frac{k}{n+1})(\frac{a-c}{n+1}) \\ ((\psi - k(1 - \frac{k}{n+1}))\frac{n}{n+1})(\frac{a-c}{n+1}) \end{pmatrix}.$$

The restriction $\psi > \frac{n+1}{4}$ ensures $\psi - k(1 - \frac{k}{n+1}) > 0$ for all $k \in \{1, \dots, n-1\}$. Therefore, this restriction implies that both $x_D(k) > 0$ for all $k \in \{1, \dots, n\}$, and $x_I(k) > 0$ for all $k \in \{1, \dots, n-1\}$.

The firms' effective costs are positive provided that both $c - x_I(k) > 0$ for all $k \in \{1, \dots, n-1\}$ and $c - kx_D(k) > 0$ for all $k \in \{1, \dots, n\}$. It can be verified that $nx_D(1) > kx_D(k)$ for all $k \in \{1, \dots, n\}$. The restriction $\psi > \frac{n^2}{(n+1)^2} \frac{a}{c}$ ensures that $c > nx_D(1)$. It follows that under this assumption, the firms' effective costs are positive and $x_D(k) < c$ for all $k \in \{1, \dots, n\}$, and $x_I(k) < c$ for all $k \in \{1, \dots, n-1\}$. Therefore, taking $\psi > \max\{\frac{n+1}{4}, \frac{n^2}{(n+1)^2} \frac{a}{c}\}$ the Nash equilibrium is unique and interior.

Proof of proposition 2.1: (i) We prove that $x_D(k) > x_D(n)$ for $k \in \{1, \dots, n-1\}$. Establishing a comparison between $x_D(k)$ and $x_D(n)$ yields $x_D(k) - x_D(n) = \frac{A(\psi - \frac{n}{n+1})(a-c)}{(n+1)d(k)d(n)}$, in which $A = (1 - \frac{k}{n+1})d(n) - \frac{1}{n+1}d(k)$. Notice that $x_D(k) > x_D(n)$ if and only if $A > 0$. First, it is easily seen that $(1 - \frac{k}{n+1}) > \frac{1}{n+1}$ for $k \in \{1, \dots, n-1\}$. Second, it is easily verified that $d(n) \geq d(k)$ provided that $(k-1)(\psi - \frac{1}{(n+1)(n-k)}) \geq 0$. It can be seen that this inequality holds for all $k \geq 1$ and $\psi \geq 1$. Therefore, the result follows.

(ii) We prove that if $R\mathcal{E}D$ is not too expensive then $x_D(2) > x_D(1)$.

Establishing a comparison between $x_D(1)$ and $x_D(k)$ yields $x_D(1) - x_D(k) = \frac{A(\psi - \frac{n}{n+1})(a-c)}{(n+1)d(1)d(k)}$, in which $A = \frac{n}{n+1}d(k) - (1 - \frac{k}{n+1})d(n)$. Notice that $x_D(1) > x_D(k)$ if and only if $A > 0$.

Substituting in the expressions for the denominator and arranging as a polynomial in k , we see that $A > 0$ provided that $(k-1)(\frac{n\psi}{(n+1)^2}k^2 - \frac{(n(2n+1)(n+1)\psi - n^2)}{(n+1)^3}k + (\frac{-(n+1)^2\psi^2 + (n^2 + 2n + 2)n\psi - n^2}{(n+1)^2})) \leq 0$. Since $k \geq 1$, this inequality is satisfied provided the quadratic polynomial is negative. The roots of this polynomial are given by

$$k = \frac{(2n+1)(n+1)n\psi - n^2}{2(n+1)\psi n} \pm \frac{\sqrt{4(n+1)^4n\psi^3 - (4n+7)(n+1)^2n^2\psi^2 + 2(n+1)n^3\psi + n^4}}{2(n+1)\psi n}.$$

It can be seen that the discriminant of the quadratic polynomial is increasing in ψ and positive for $\psi = 1$. Hence, the polynomial has two real and distinct roots, denoted by r^- and r^+ . Since the leading coefficient is positive, $A > 0$ provided the roots satisfy $r^- \leq 2$ and $r^+ \geq n$. First, we prove that $r^+ > n$ and $r^- < n$. It is easily verified that $\frac{(2n+1)(n+1)n\psi - n^2}{2(n+1)\psi n} > n$ for all $\psi > \frac{n}{n+1}$. Therefore, it follows that $r^+ > n$. Notice that if $r^- > n$ then $x_D(k) > x(1)$, for all $k \in \{2, \dots, n\}$. This contradicts the fact that $x_D(n) < x(1)$. Therefore, it follows that $r^- < n$.

Notice that $x_D(2) > x_D(1)$ if and only if $r^- > 2$. Substituting in for r^- , gathering terms and squaring both sides of the inequality, we see that $r^- > 2$ if and only if $[(n+1)n\psi(2n-3) - n^2]^2 > 4(n+1)^4n\psi^3 - (4n+7)(n+1)^2n^2\psi^2 + 2(n+1)n^3\psi + n^4$. Arranging terms as a polynomial in ψ and multiplying across by $\frac{-1}{4n(n+1)\psi}$, we find that $r^- > 2$ if and only if the following quadratic polynomial is negative: $(n+1)^3\psi^2 - (n+1)n(n^2 - 2n + 4)\psi + n^2(n-1) \leq 0$. The roots of this polynomial are $\psi = \frac{n(n^2 - 2n + 4)}{2(n+1)^2} \pm \frac{n\sqrt{(n^2+4)(n^2-4n+4)+4}}{2(n+1)^2}$. It easily seen that the discriminant is strictly positive. Hence, there exists two real and distinct roots, denoted z^- and z^+ . Since the polynomial's leading coefficient is positive, it follows that $r^- > 2$ if and only if ψ satisfies $z^- \leq \psi \leq z^+$.

Recall the restriction $\psi \geq \frac{n+1}{4}$ which ensures the existence of an interior Nash equilibrium. First, we show that $z^+ > \frac{n+1}{4}$. It is easily seen that the discriminant is bounded below: $\sqrt{(n^2+4)(n^2-4n+4)+4} > 4$ for all $n \geq 3$. Hence, the inequality is satisfied provided that $\frac{n(n^2-2n+4)+4n}{2(n+1)^2} > \frac{n+1}{4}$. Rearranging as a polynomial in n , we see that this inequality holds provided $f(n) := n^3 - 7n^2 + 13n - 1 > 0$. Simple calculations reveal that $f(3) > 0$, $f(4) > 0$ and $\frac{df}{dn} > 0$ for all $n \geq 4$. Hence $\frac{n+1}{4} < z^+$.

Second, we show that $z^- < 1$. Notice that $z^- < 1$ if and only if $\frac{n(n^2-2n+4)}{2(n+1)^2} - \frac{n\sqrt{(n^2+4)(n^2-4n+4)+4}}{2(n+1)^2} < 1$. Rearranging, squaring both sides and writing the resulting expression as a polynomial in n , we see that $z^- < 1$ if and only if $h(n) = 4n^5 - 8n^4 - 12n^3 + 4n^2 - 4 > 0$. Simple computations reveal that $h(3) > 0$. It can be seen $h(n)$ is increasing in n . Hence, $h(n) > 0$ for all $n \geq 3$.

We have now proved that $z^- < 1$ and $\frac{n+1}{4} < z^+$. Therefore, there exists $\psi_t = z^+ > \frac{n+1}{4}$ such that if $\psi < \psi_t$ then $x_D(2) > x_D(1)$ and if $\psi > \psi_t$ then $x_D(1) > x_D(k)$ for all $k \in \{2, \dots, n\}$.

Proof of proposition 2.2: (i) We prove $x_I(1) > x_I(k)$ for $k \in \{2, \dots, n-1\}$. Establishing a comparison between $x_I(1)$ and $x_I(k)$ yields $x_I(1) - x_I(k) = \frac{nA(a-c)}{(n+1)^2 d(1)d(k)}$, in which $A = (\psi - \frac{n}{n+1})d(k) - (\psi - k(1 - \frac{k}{n+1}))d(1)$. Observe that $x_I(1) > x_I(k)$ if and only if $(\psi - \frac{n}{n+1})d(k) > (\psi - k(1 - \frac{k}{n+1}))d(1)$.

Substituting in the expressions for the denominator, arranging terms as a polynomial in ψ and simplifying, we find that $x_I(1) > x_I(k)$ if and only if $\frac{k(n-k)(k-1)}{(n+1)^2}(\psi - \frac{n}{n+1}) > 0$. It is easily seen that this inequality is satisfied for all $k \in \{2, \dots, n-1\}$ and $\psi \geq 1$.

(ii) We prove that $x_I(n-1) - x_I(n-2)$ for $n \geq 4$. Establishing a comparison between $x_I(n-1)$ and $x_I(n-2)$ yields $x_I(n-1) - x_I(n-2) = \frac{nB(a-c)}{(n+1)d(n-1)d(n-2)}$, in which $B = (\psi - \frac{2(n-1)}{n+1})(\psi^2 - (\frac{n^2+2n-6}{(n+1)^2})\psi + \frac{3n(n-2)}{(n+1)^3}) - (\psi - \frac{3(n-2)}{n+1})(\psi^2 - (\frac{n^2+4n-4}{(n+1)^2})\psi + \frac{2n(n-1)}{(n+1)^3})$. Notice that $x_I(n-1) > x_I(n-2)$ provided that $B > 0$.

Rearranging terms as a polynomial in ψ and simplifying terms, we see that $B > 0$ if and only if $\psi - \frac{(n-1)(n+6)}{(n+1)^2} > 0$. Recall that the condition for an interior equilibrium requires $\psi > \frac{n+1}{4}$. It is easily verified that $\frac{n+1}{4} > \frac{(n-1)(n+6)}{(n+1)^2}$ if and only if $h(n) = n^3 - n^2 - 17n + 25 > 0$. It can be seen that $h(3) < 0$ and $h(4) > 0$. Taking the derivative, we see that $\frac{dh}{dn} = 3n^2 - 2n - 17 > 0$ for all $n \geq 3$. Hence $x_I(n-1) > x_I(n-2)$ for all $n \geq 4$.

(iii) If the R&D cost parameter is not too low, isolated firms innovate more than cluster firms after a threshold cluster size. Establishing a comparison between $x_D(k)$ and $x_I(k)$ yields $x_D(k) - x_I(k) = \frac{C(a-c)}{(n+1)d(k)}$, in which $C = (\psi - \frac{n}{n+1})(1 - \frac{k}{n+1}) - \frac{n}{n+1}(\psi - k(1 - \frac{k}{n+1}))$. We find that $C > 0$ if and only if the following quadratic polynomial in k satisfies $-nk^2 - (n+1)\psi - ((n+1)\psi - (n^2 + 2n))k + (n+1)\psi - n(n+1) > 0$.

Notice that the leading coefficient is negative. The discriminant of this polynomial is positive. Hence, there exists two real and distinct roots given by $r^- = 1$ and $r^+ = (n+1) - \frac{n+1}{n}\psi$. Notice that if $\psi > \frac{2n}{n+1}$ then $r^+ < n-1$ and if R&D is not too expensive, that is $\psi \leq \frac{n(n-1)}{n+1}$, then $r^+ \geq 2$. Therefore, setting $\bar{k} = r^+$, it follows that if $k < \bar{k}$ then $x_D(k) > x_I(k)$ and if $k > \bar{k}$ then $x_I(k) > x_D(k)$.

Proof of proposition 2.3: We prove that the unit production costs of cluster firms are lowest at an intermediate cluster size. Note that $nx_D(n) = x_D(1)$ implies $c_D(n) = c_D(1)$. It is now sufficient to prove that $c_D(n) > c_D(k)$ for all $k \in \{2, \dots, n-1\}$. Observe that $c_D(n) > c_D(k)$ if and only if $kx_D(k) > nx_D(n)$. Establishing a comparison between $kx_D(k)$ and $nx_D(n)$ yields $kx_D(k) - nx_D(n) = \frac{A(\psi - \frac{n}{n+1})(a-c)}{(n+1)d(n)d(k)}$, in which $A = k(1 - \frac{k}{n+1})d(n) - \frac{n}{n+1}d(k)$.

Observe that $c_D(n) > c_D(k)$ if and only if $k(1 - \frac{k}{n+1})d(n) > \frac{n}{n+1}d(k)$. Substituting in terms, multiplying out as a polynomial in ψ and simplifying, we find that $c_D(n) > c_D(k)$ if and only if $\frac{(n-k)(k-1)}{n+1}(\psi - \frac{n(k+1)}{(n+1)^2}) > 0$. It is easily seen that this inequality holds for $k \in \{2, \dots, n-1\}$ and $\psi \geq 1$.

Proof of proposition 2.4: (i) *We prove that $\pi_D(k) > \pi_D(1)$ for $k \in \{2, \dots, n\}$.* Setting $k = 1$ yields $\pi_D(1) = \frac{\psi(\psi - (\frac{n}{n+1})^2)(\psi - \frac{n}{n+1})^2(a-c)^2}{(n+1)^2(d(1))^2}$. Establishing a comparison between $\pi_D(k)$ and $\pi_D(1)$ yields $\pi_D(k) - \pi_D(1) = \frac{\psi A(\psi - \frac{n}{n+1})^2(a-c)^2}{(n+1)^2(d(k))^2(d(1))^2}$, in which $A = (\psi - (1 - \frac{k}{n+1})^2)(d(1))^2 - (\psi - (\frac{n}{n+1})^2)(d(k))^2$. The sign of $\pi_D(k) - \pi_D(1)$ is positive if and only if $(\psi - (1 - \frac{k}{n+1})^2)(d(1))^2 > (\psi - (\frac{n}{n+1})^2)(d(k))^2$. Recall that $d(1) \geq d(k)$ for all $k > 1$. Observe that $\psi - (1 - \frac{k}{n+1})^2 > \psi - (\frac{n}{n+1})^2$ provided that $k > 1$. Hence the result follows.

(ii) *We prove that $\pi_D(n-1) > \pi_D(n)$.* Setting $k = n-1$ and $k = n$ respectively, yields $\pi_D(n-1) = \frac{\psi(\psi - \frac{4}{(n+1)^2})(\psi - \frac{n}{n+1})^2(a-c)^2}{(n+1)^2(d(n))^2}$ and $\pi_D(n) = \frac{\psi(\psi - \frac{1}{(n+1)^2})(\psi - \frac{n}{n+1})^2(a-c)^2}{(n+1)^2(d(n))^2}$. Establishing a comparison between $\pi_D(n-1)$ and $\pi_D(n)$ yields $\pi_D(n-1) - \pi_D(n) = \frac{\psi B(\psi - \frac{n}{n+1})^2(a-c)^2}{(n+1)^2(d(n-1))^2(d(n))^2}$, in which $B = (\psi - \frac{4}{(n+1)^2})(d(n))^2 - (\psi - \frac{1}{(n+1)^2})(d(n-1))^2$. Notice that the sign of $\pi_D(n-1) - \pi_D(n)$ is positive if and only if $[(\psi - \frac{4}{(n+1)^2})d(n)]d(n) > [(\psi - \frac{1}{(n+1)^2})d(n-1)]d(n-1)$.

First, it is easily verified that $\psi \geq 1$ ensures $d(n) > d(n-1)$. Second, substituting in for the denominator and arranging as a polynomial in ψ , we find that $(\psi - \frac{4}{(n+1)^2})d(n) > (\psi - \frac{1}{(n+1)^2})d(n-1)$ provided the following inequality holds: $f(\psi) = (2n^3 - 3n^2 - 12n - 7)\psi^2 - (n^3 - 4n^2 - 6n - 4)\psi - 2n > 0$. Setting $\psi = 1$ in the inequality, we obtain $n^3 + n^2 - 8n - 3 > 0$ for all $n \geq 3$. Taking the derivative, we see that $\frac{df}{d\psi} = (4n^3 - 6n^2 - 24n - 14)\psi - (n^3 - 4n^2 - 6n - 4)$ which is increasing in ψ for all $n \geq 4$. If $n = 3$ then $f(\psi) = -30\psi^2 + 31\psi + 6 > 0$ if and only if ψ is not too large. Therefore, we have proven that $\pi_D(n-1) > \pi_D(n)$ for all $n \geq 4$ and $\pi_D(n-1) > \pi_D(n)$ for $n = 3$ provided that ψ is not too large.

(iii) *We prove that $\pi_I(1) > \pi_I(k)$ for $k \in \{2, \dots, n-1\}$.* Establishing a comparison between $\pi_I(1)$ and $\pi_I(k)$ yields $\pi_I(1) - \pi_I(k) = \frac{\psi C(\psi - (\frac{n}{n+1})^2)(a-c)^2}{(n+1)^2(d(1))^2(d(k))^2}$, in which $C = (\psi - \frac{n}{n+1})^2(d(k))^2 - (\psi - k(1 - \frac{k}{n+1}))^2(d(1))^2$. Notice that $\pi_I(1) > \pi_I(k)$ if and only if $(\psi - \frac{n}{n+1})(d(k)) > (\psi - k(1 - \frac{k}{n+1}))(d(1))$. Substituting in for the denominators, arranging as a polynomial in ψ and simplifying, we find that $\pi_I(1) > \pi_I(k)$ if and only if $\frac{k(n-k)(k-1)}{(n+1)^2}(\psi - \frac{n}{n+1}) > 0$. It is easy to see this inequality holds for all $k \in \{2, \dots, n-1\}$ and $\psi \geq 1$.

(iv) *We prove that $\pi_I(n-1) > \pi_I(n-2)$ for all $n \geq 4$.* Setting $k = n-1$ and $k = n-2$ respectively, yields $\pi_I(n-1) = \frac{\psi(\psi - (\frac{n}{n+1})^2)(\psi - \frac{2(n-1)}{n+1})^2(a-c)^2}{(n+1)^2(d(n-1))^2}$ and $\pi_I(n-2) = \frac{\psi(\psi - (\frac{n}{n+1})^2)(\psi - \frac{3(n-2)}{n+1})^2(a-c)^2}{(n+1)^2(d(n-2))^2}$. Establishing a comparison between $\pi_I(n-1)$ and $\pi_I(n-2)$ yields $\pi_I(n-1) - \pi_I(n-2) =$

$\frac{\psi D(\psi - (\frac{n}{n+1})^2)(a-c)^2}{(n+1)^2(d(n-1))^2(d(n-2))^2}$, in which $D = (\psi - \frac{2(n-1)}{n+1})^2(d(n-2))^2 - (\psi - \frac{3(n-2)}{n+1})^2(d(n-1))^2$. Notice that $\pi_I(n-1) > \pi_I(n-2)$ if and only if $(\psi - \frac{2(n-1)}{n+1})d(n-2) > (\psi - \frac{3(n-2)}{n+1})d(n-1)$. We proved in proposition 2 (ii) that this inequality is satisfied for all $n \geq 4$. Therefore, the result follows.

Results concerning firm output: Substituting the production costs into the output functions and rearranging yields the equilibrium output of cluster and isolated firms:

$$q_D(k) = \frac{\psi(\psi - \frac{n}{n+1})(a-c)}{(n+1)d(k)}; \text{ and } q_I(k) = \frac{\psi(\psi - k(1 - \frac{k}{n+1}))(a-c)}{(n+1)d(k)}.$$

We derive the following results (i) the output of cluster firms is maximized at an intermediate cluster size; (ii) cluster firms produce more than isolated firms; and (iii) if the industry is not too small, i.e. $n \geq 4$, the output of isolated firms is minimized at an intermediate cluster size.

Proof of (i): Since $d(1) = d(n)$, it is easily seen that $q_D(1) = q_D(n)$. It now sufficient to show that $q_D(k) > q_D(1)$ for all $k \in \{2, \dots, n-1\}$. It is easily verified that $d(1) > d(k)$ for all $k \in \{2, \dots, n-1\}$. Therefore, it follows that $q_D(k) > q_D(1)$ for all $k \in \{2, \dots, n-1\}$.

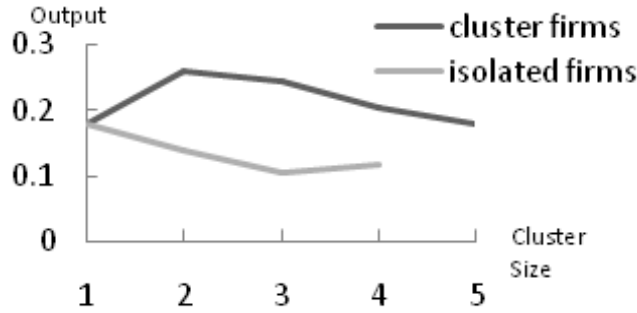
Proof of (ii): Establishing a comparison between $q_D(k)$ and $q_I(k)$ yields $q_D(k) - q_I(k) = \frac{\psi A(a-c)}{(n+1)d(k)}$, in which $A = (\psi - \frac{n}{n+1}) - (\psi - k(1 - \frac{k}{n+1}))$. Observe that $A > 0$ if and only if $\frac{(n-k)(k-1)}{n+1} > 0$, which is true for all $k \in \{2, \dots, n-1\}$. Therefore, it follows that $q_D(k) > q_I(k)$ for all $k \in \{2, \dots, n-1\}$.

Proof of (iii) We first prove that $q_I(1) > q_I(k)$ for all $k \in \{2, \dots, n-1\}$. Establishing a comparison between $q_I(1)$ and $q_I(k)$ yields $q_I(1) - q_I(k) = \frac{\psi B(a-c)}{(n+1)d(1)d(k)}$, in which $B = (\psi - \frac{n}{n+1})d(k) - (\psi - k(1 - \frac{k}{n+1}))d(n)$. The sign of $q_I(1) - q_I(k)$ is positive if and only if $B > 0$. Substituting in the expressions for the denominator, multiplying out as a polynomial in ψ and simplifying, we find that $B > 0$ if and only if $\frac{(n-k)(k-1)}{n+1}(\psi - \frac{n}{n+1}) > 0$. It is easily seen this inequality is satisfied for all $k \in \{2, \dots, n-1\}$ and $\psi \geq 1$.

It now sufficient to show that $q_I(n-2) < q_I(n-1)$. Establishing a comparison between $q_I(n-1)$ and $q_I(n-2)$ yields $\pi_I(n-1) - \pi_I(n-2) = \frac{\psi D(a-c)}{(n+1)d(n-1)d(n-2)}$, in which $D = (\psi - \frac{2(n-1)}{n+1})(d(n-2)) - (\psi - \frac{3(n-2)}{n+1})(d(n-1))$. Notice that $q_I(n-1) > q_I(n-2)$ if and only if $D > 0$. We proved in proposition 2 (ii) that this inequality is satisfied for all $n \geq 4$. Therefore, the output of isolated firms is lowest at an intermediate cluster size \bar{k}_q , satisfying $1 < \bar{k}_q < n-1$, for all $n \geq 4$.

Figure 7 depicts the outputs of cluster and isolated firms as a function of cluster size.

Figure 7: Production and Cluster Size (n=5)



Proof of Proposition 2.6: (i) We first prove that $W(n) > W(1)$. Simple derivations show that $W(1) = \frac{n\psi((n+2)\psi - \frac{2n^2}{(n+1)^2})(a-c)^2}{2(n+1)^2(\psi - \frac{n}{(n+1)^2})^2}$ and $W(n) = \frac{n\psi((n+2)\psi - \frac{2}{(n+1)^2})(a-c)^2}{2(n+1)^2(\psi - \frac{n}{(n+1)^2})^2}$. Establishing a comparison between these social welfare levels yields

$$W(n) - W(1) = \frac{(n-1)n\psi(a-c)^2}{(n+1)(\psi - \frac{n}{(n+1)^2})^2}.$$

It is easily seen that the sign of $W(n) - W(1)$ is positive.

(ii) We next prove that $W(n-1) > W(n)$. It suffices to show that $Q(n-1) > Q(n)$ and $\Pi(n-1) > \Pi(n)$, where $\Pi(k) := k\pi_D(k) + (n-k)\pi_I(k)$ denotes industry profits. We first prove that $Q(n-1) > Q(n)$. Simple derivations show that $Q(n-1) = \frac{n\psi(\psi - \frac{(n-1)(n+2)}{n(n+1)})(a-c)}{(n+1)(\psi^2 - \frac{(n^2+4n-4)}{(n+1)^2}\psi + \frac{2n(n-1)}{(n+1)^3})}$ and $Q(n) = \frac{n\psi(a-c)}{(n+1)(\psi - \frac{n}{(n+1)^2})}$. Establishing a comparison between these aggregate output levels yields

$$Q(n-1) - Q(n) = \frac{A(n-1)(n-2)n\psi(a-c)}{(n+1)(\psi^2 - \frac{(n^2+4n-4)}{(n+1)^2}\psi + \frac{2n(n-1)}{(n+1)^3})(\psi - \frac{n}{(n+1)^2})}$$

in which $A = \frac{1}{n(n+1)^2}\psi - \frac{1}{(n+1)^3}$. The sign of $Q(n-1) - Q(n)$ is positive if and only if $\psi > \frac{n}{(n+1)}$. Recall that $\psi \geq 1$. Therefore, the result follows.

We next show that $\Pi(n-1) > \Pi(n)$. Simple derivations show that $\Pi(n-1) = \frac{n\psi B(a-c)^2}{(n+1)^2(\psi^2 - \frac{(n^2+4n-4)}{(n+1)^2}\psi + \frac{2n(n-1)}{(n+1)^3})}$ and $\Pi(n) = \frac{n\psi(\psi - \frac{1}{(n+1)^2})(a-c)^2}{(n+1)^2(\psi - \frac{n}{(n+1)^2})^2}$

in which $B = \psi^3 - \frac{2n^3+5n^2+2n-8}{n(n+1)^2}\psi^2 + \frac{(n-1)(n^3+9n^2+8n-4)}{n(n+1)^3}\psi - \frac{4n^2(n-1)}{(n+1)^4}$. Establishing a comparison between the aggregate profits levels yields

$$\Pi(n-1) - \Pi(n) = \frac{n\psi C(a-c)^2}{(n+1)^2(\psi^2 - \frac{(n^2+4n-4)}{(n+1)^2}\psi + \frac{2n(n-1)}{(n+1)^3})^2(\psi - \frac{n}{(n+1)^2})^2}$$

$$\text{with } C = \frac{3n^8+13n^7+7n^6-63n^5-175n^4-217n^3-147n^2-53n-8}{n(n+1)^8}\psi^4 + \frac{n^8+12n^7+53n^6+80n^5-n^4-114n^3-89n^2-16n+4}{n(n+1)^8}\psi^3 \\ - \frac{2n^8+11n^7+39n^6+33n^5-21n^4-36n^3-20n^2-8n}{n(n+1)^8}\psi^2 + \frac{n^7-3n^6-9n^5-n^4+8n^3+4n^2}{n(n+1)^8}\psi - \frac{4n^6-8n^5+4n^4}{n(n+1)^8}.$$

The sign of $\Pi(n-1) - \Pi(n)$ is positive if and only if $\frac{3n^8+13n^7+7n^6-63n^5-175n^4-217n^3-147n^2-53n-8}{n(n+1)^8}\psi^4 + \frac{n^8+12n^7+53n^6+80n^5-n^4-114n^3-89n^2-16n+4}{n(n+1)^8}\psi^3 + \frac{n^7-3n^6-9n^5-n^4+8n^3+4n^2}{n(n+1)^8}\psi > \frac{2n^8+11n^7+39n^6+33n^5-21n^4-36n^3-20n^2-8n}{n(n+1)^8}\psi^2 + \frac{3n^6+9n^5+n^4-8n^3-4n^2}{n(n+1)^8}\psi + \frac{4n^6-8n^5+4n^4}{n(n+1)^8}$. The coefficients of the polynomial on the left hand side and on the right hand side are positive for all $n \geq 3$. Observe that both sides of the inequality are increasing in ψ . The left hand side rises at a faster rate than the right hand side for all $n \geq 3$ because of the larger coefficients on the higher positive power terms. Hence, if the inequality holds for the minimum value of ψ , then the result follows. Setting $\psi = 1$ in the inequality, we get that $2n^8 + 15n^7 + 14n^6 - 17n^5 - 160n^4 - 287n^3 - 212n^2 - 61n - 4 > 0$ for all $n \geq 3$.

(iii) We prove that $X(1) > X(n)$ and $X(n-1) > X(n)$ where $X(k) = kx_D(k) + (n-k)x_I(k)$ denotes the industry-wide level of innovative activity: First, it was shown in the proof of proposition 2.1 that $x_I(1) = x_D(1) > x_D(n)$. Therefore, it follows that $nx_I(1) = X(1) > X(n) = nx_D(n)$.

Second, notice that $X(n-1) > X(n)$ if and only if the following inequality holds: $d(n)(\frac{2(n-1)}{(n+1)^2}(\psi - \frac{n}{n+1}) + \frac{n}{(n+1)^2}(\psi - \frac{2(n-1)}{n+1})) > d(n-1)(\frac{n}{(n+1)^2}(\psi - \frac{n}{n+1}))$. Recall that $d(n) > d(n-1)$. Therefore, the inequality is satisfied provided $\frac{2(n-1)}{(n+1)^2}(\psi - \frac{n}{n+1}) + \frac{n}{(n+1)^2}(\psi - \frac{2(n-1)}{n+1}) > \frac{n}{(n+1)^2}(\psi - \frac{n}{n+1})$. Mutiplying out this expression and rearranging, we see that $X(n-1) > X(n)$ if $2(n-1)\psi - \frac{n}{(n+1)}(3n-4) > 0$, which is true provided $\psi > \frac{n(3n-4)}{2(n^2-1)}$. Recall that a unique equilibrium requires $\psi > \frac{n+1}{4}$. Therefore, the inequality holds if $\frac{n+1}{4} > \frac{n(3n-4)}{2(n^2-1)}$. Multiplying out, we see that $X(n-1) > X(n)$ if $p(n) = n^3 - 5n^2 + 7n - 1 > 0$. First, it is easily shown that $p(n)$ satisfies $\frac{dp}{dn} > 0$, for all $n \geq 3$. Thus, the inequality holds provided $p(3) > 0$. Substituting $n = 3$ into $p(n)$, we see that $p(3) = 2 > 0$. Therefore, the result follows.

CHAPTER 3

INDUSTRIAL CLUSTERS, SUBSIDIES AND THE SITTING-DUCK EFFECT

3.1 Introduction

Policies to promote and develop clusters are widely popular in political circles and considerable amounts of money are often spent on cluster initiatives, e.g. regional development grants, cluster-oriented R&D subsidy programs. However, there is little empirical or theoretical evidence on the effect of cluster policies on firm performance. The present paper seeks to redress this imbalance. To our knowledge, it is the first one to undertake a micro theoretical analysis of how an industry's spatial pattern influences the effect of government subsidies on firm investment outcomes.

This paper suggests that differences between the sizes of core and peripheral clusters create differential subsidy effects on investment. In particular, the intensity of the crowding-out effect of a subsidy on non-cluster firm investment tends to be relatively stronger for core firm subsidies (sitting-duck effect). The main policy implication of this finding is that if government is justified in funding both core and peripheral firms, then alongside subsidies to core firms, government ought to provide adequate funding to peripheral firms in order to counteract the sitting-duck effect.

The type of cluster studied in this paper is that of a group of colocated firms that compete in a particular industry in which agglomeration externalities are present among proximately located firms. The cluster may be technological, e.g. firms which research on the same technology, or geographic, e.g. firms located in the same geographic area. Marshall (1920) identified local knowledge spillovers, labour market pooling and vertical linkages as the main sources of (localized) agglomeration externalities and there exists a substantial amount of empirical evidence supporting the existence of each of these channels of agglomeration spillover (see, e.g., Jaffe (1986), Jaffe et al. (1993), Audretsch et al. (1996), Branstetter et al. (2002), Bloom et al. (2007), Lychagin et al. (2010)).

Clusters and agglomeration externalities are modelled using the tools of social network theory. Nodes represent firms and links between firms indicate proximity e.g. researching

on the same technology or locating in the same geographic area. It is assumed each firm has only one location with the implication that links are transitive and network components represent clusters. The network then defines the industry's spatial pattern. Agglomeration externalities are modelled as local cost externalities, i.e. component restricted spillovers, arising from the cost-reducing investment activities of firms, e.g. local knowledge spillovers from R&D, worker training and hiring programs that create a local pool of specialized labour, investment in capital stock that attracts mobile input suppliers.

There are two main types of cluster policies typically used by governments (Martin et al., 2011). First, cluster policies could seek to increase the size of a given cluster by attracting firms to the location. Second, given the distribution of cluster sizes, cluster policies could seek to improve the workings of the externalities via subsidies. This paper focuses on a latter type of cluster policy namely investment subsidies.

In many industries a significant fraction of firms choose to locate outside the industry's core cluster, (see, e.g. Prevezer (1997), Folta (2006)). Therefore, this paper focuses on government subsidy policy in the presence of core-periphery industry spatial patterns. For simplicity, we focus on the special case of a single core and single peripheral cluster in which core firms are firms which reside in the relatively larger cluster. However, we believe the papers main insights hold in more general cases.

The paper develops a three-stage game in order to study how government should subsidize investment when firms are located in clusters and local cost externalities are present. The model is similar to Goyal and Moraga-Gonzalez (2001) except the industry's spatial pattern (network) is exogenous and we introduce subsidies. In the first stage, government announces a subsidy for each firm taking as given the industry's spatial pattern. Given these subsidies, each firm invests in cost reduction which reduces the firm's marginal production costs. Cost reductions spillover to neighbouring firms. In the final stage, given the industry's cost configuration, firms compete in a homogenous-product Cournot oligopoly. The game is solved using backward induction.

The model is an appropriate description of a situation in which the industry's spatial pattern is stable for a relatively long period while subsidies are decided for a relatively shorter period. Although moving location is observed in practice, the costs of switching location can be quite high. For example, changing a firm's R&D focus can involve substantial "catch-up" costs as the firm seeks to build up innovative capacity in the new research area. On the other hand, switching geographic location can involve significant set-up costs. It is impicity

assumed in our model that the cost of switching from one location to another is sufficiently high to deter such behaviour.

The analysis reveals that asymmetries between the sizes of core and peripheral clusters create differential investment incentives for core and peripheral firms. In the core cluster, cost reductions spillover to a relatively larger fraction of the industry's firms. Therefore, on the one hand, core firms suffer from a relatively more severe local appropriation problem resulting in a relatively lower idiosyncratic incentive to invest and a relatively weaker intensity of local complementarity in the core cluster.

On the other hand, the business stealing effects of core firm investment experienced by peripheral firms is more severe than the business stealing effects of peripheral firm investment experienced by core firms. Consequently, core firms benefit from more intense cluster strength effects, i.e. the investments of peripheral firms tend to be relatively more vulnerable to cost reductions achieved by core firms.

These differential investment incentives create differential effects of subsidies on the investments of core and peripheral firms. On the one hand, the intensity of the additionality effect of a subsidy on the recipient firm's investment is relatively stronger for a peripheral firm subsidy because of the periphery cluster's relatively weaker local appropriation problem (expansion effect). On the other hand, the crowding-out effect of subsidy on non-cluster firm investment is relatively stronger for a core firm subsidy because core firms benefit from more intense cluster strength effects (sitting-duck effect).

We find that the sitting-duck effect dominates the expansion effect. The main policy implication is that if government is justified in subsidizing both core and peripheral firms then alongside subsidies to core firms, government must provide adequate funding to peripheral firms in order to counteract the sitting duck effect.

The paper contributes to the literature on the effect of cluster policies on firm performance (see, e.g. Criscuolo et al. (2007), Martin et al. (2011)). This literature finds evidence that subsidies to peripheral firms tend to have no significant effect on firm performance. One interpretation of the apparent poor performance of government subsidies to peripheral firms is that these subsidies largely protected peripheral firms from the crowding-out effects of core firm subsidies.

The paper also contributes to the literature on the effect of R&D subsidies on private R&D spending (see, e.g. Klette et al (2000) and Hall et al. (2000) for a review). This

literature generally abstracts from the effect of an industry's spatial pattern on the performance of R&D subsidies. This paper contributes to this literature by providing a model which sheds light on the differential effects of subsidies on firm R&D investment that can arise from asymmetries in an industry's spatial pattern.

3.2 The Model

Consider an industry consisting of n firms and $m = 2$ clusters. It is assumed each firm has just one location. A pair of firms is said to be linked and referred to as neighbours if they are located in the same cluster. Let C_r denote the set of firms in cluster r and C_r^i denote the set of cluster neighbours of firm $i \in C_r$. The size of a cluster is taken to be the number of co-located firms $|C_r| = n_r$. If $n_r > n_t$ then cluster r is said to be core with respect to cluster t and cluster t is said to be peripheral with respect to cluster r . The industry's spatial pattern $g = (n_r, n_t)$, in which $n_r + n_t = n$, is taken to be exogenous.

Given the industry's spatial pattern g , each firm invests in a level of cost reduction $x_i \geq 0$ which reduces the firm's marginal cost of production. Cost reductions spillover to neighbouring firms. The marginal cost of production of firm i located in cluster r is given by

$$c_i(g) = c - x_i - \sum_{j \in C_r^i(g)} x_j,$$

in which the initial constant marginal cost satisfies $c \geq x_i + \sum_{j \in C_r^i(g)} x_j > 0$ for all g . Notice that the total cost reduction depends upon the aggregate level of investment undertaken by the cluster's firms. Cost externalities are local and do not spread to non-neighbouring firms. Government subsidies, s_i , lower the effective cost of this investment. This effective cost is given by

$$y_i(x_i, s_i) = -2s_i x_i + \psi x_i^2,$$

in which $\psi \geq 1$. Notice that the investment subsidy reduces the intercept term on the firm's marginal cost of capital. In the final stage, given the industry's marginal cost configuration $\{c_i(g)\}_{i=1}^n$, firms compete in a homogenous-product oligopoly. Each firm sets output, q_i , and faces a linear inverse demand function given by $p = a - \sum_{i=1}^n q_i$, in which $a > c$. Bringing together expressions, the profits of firm i are

$$\pi_i(g) = [a - Q(g) - c_i(g)]q_i(g) + 2s_i x_i(g) - \psi x_i^2(g),$$

in which $Q(g) = \sum_{i=1}^n q_i(g)$. In the first stage, given the industry's spatial pattern g , government chooses a subsidy program $\{s_i\}_i$ in order to maximise social welfare correctly anticipating the behaviour of the firms. Social welfare is defined as the sum of consumer surplus, industry profits and pure external benefits less the cost of the subsidy program:

$$W(g) = \frac{1}{2}Q^2(g) + \sum_{i=1}^n \pi_i(g) + \sum_r (\sum_{i \in C_r} \varepsilon_r q_i(g)) - \sum_i 2s_i x_i,$$

in which $\varepsilon_r \geq 0$ measures the intensity of pure external benefits arising from consumption of firm i 's product. Notice that pure external benefits increase the social return of a firm's investment but do not effect its private return.

3.2.1 Analysis of Subsidy Effects

The game is solved using backward induction. In the market competition stage, given an industry cost configuration $\{c_i(g)\}_i$, each firm behaves as a Cournot monopolist and chooses output to maximise profits. The Cournot equilibrium output of firm i is given by

$$q_i(g) = \frac{a - nc_i(g) + \sum_{l \neq i}^n c_l(g)}{n+1},$$

and the profits of the Cournot competitors are

$$\pi_i(g) = \left[\frac{a - nc_i(g) + \sum_{l \neq i}^n c_l(g)}{n+1} \right]^2 + 2s_i x_i(g) - \psi x_i^2(g).$$

Without loss of generality, suppose firm i is located in cluster r . Note that there are three types of firms; the representative firm i , the $n_r - 1$ cluster neighbours of firm i and the n_t non-neighbours of firm i (i.e. those firms which are not located in firm i 's cluster).

Suppose the neighbours and non-neighbours of firm i are represented by the subscripts j and k respectively. Let x_j denote the level of cost reduction undertaken by a neighbour of firm i and let x_k denote the corresponding level of cost reduction undertaken by a non-neighbour of firm i . Plugging in the cost reduction formulation and rearranging yields the reduced-form profit function of firm i

$$\pi_i(g) = \left[\frac{(a-c) + (n+1-n_r)x_i + \sum_{j \in C_r^i} (n+1-n_r)x_j - (\sum_{k \in C_t} n_t x_k)}{n+1} \right]^2 + 2s_i x_i - \psi x_i^2.$$

Prior to market competition, each firm invests in a level of cost reduction in order to maximize reduced-form profits taking as given the investment choices of rival firms, the subsidy it receives from government and the industry's spatial pattern $g = (n_r, n_t)$. Taking the first order condition, the incentive to invest can be decomposed into an idiosyncratic component, a local complementarity component and an inter-cluster substitutability component,¹

¹The restriction $\psi \geq 1$ ensures that the second order condition is satisfied.

$$\frac{\partial \pi_i}{\partial x_i} = s_i + \underbrace{\alpha_r - \sigma_r x_i}_{\text{idiosyncratic}} + \underbrace{\sum_{j \in C_r^i} \sigma_{rr} x_j}_{\text{local}} - \underbrace{\left(\sum_{k \in C_t} \sigma_{rt} x_k \right)}_{\text{inter-cluster}} = 0$$

in which

$$\alpha_r = \left(1 - \frac{n_r}{n+1}\right) \frac{(a-c)}{n+1}, \quad \sigma_r = \psi - \sigma_{rr},$$

and

$$\sigma_{rr} = \left(1 - \frac{n_r}{n+1}\right)^2, \quad \sigma_{rt} = \left(1 - \frac{n_r}{n+1}\right) \frac{n_t}{n+1}.$$

Local bilateral influence is captured by the cross-derivatives $\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} = \sigma_{rr}$ for all $j \in C_r^i$.² Notice that $\sigma_{rr} > 0$ implies that the investments of neighbouring firms are strategic complements. The term σ_{rr} measures the increase in firm i 's incentive to invest due to a marginal increase in a neighbouring firm's investment level and reflects the *intensity of local complementarity* in cluster r .

Inter-cluster bilateral influences are captured by the cross-derivatives $\frac{\partial^2 \pi_i}{\partial x_i \partial x_k} = -\sigma_{rt}$ for all $k \in C_t$, $t \neq r$.³ Notice that $\sigma_{rt} > 0$ for all $t \neq r$, implies that the investments of non-neighbouring firms are strategic substitutes. In addition, observe that $\sigma_{rt} \neq \sigma_{tr}$ for all $t \neq r$; hence inter-cluster externalities are not symmetric. The term σ_{rt} measures the reduction in firm i 's incentive to invest due to a marginal increase in a non-neighbouring firm's investment level and reflects the *degree of vulnerability* of firms in cluster r to investment undertaken by firms in cluster $t \neq r$.

We now investigate how asymmetries between core and periphery cluster sizes influence firm investment incentives. On the one hand, core firms suffer from a more severe *local appropriation problem* than peripheral firms. In the core cluster, cost reductions spillover to a relatively larger fraction of the industry's firms which more intensely competes away the private return on a core firm's investment effort. Notice that if $n_r > n_t$ then $\alpha_r < \alpha_t$, $\sigma_r > \sigma_t$ and $\sigma_{rr} < \sigma_{tt}$; hence the relatively more severe local appropriation problem implies i) core firms experience a lower idiosyncratic incentive to invest than peripheral firms; and ii) the intensity of local complementarity is weaker among core firms than among peripheral firms even though the potential spillover pool of a core firm is relatively larger.

²Note that these local bilateral influences are uniform across all pairs of neighbouring firms.

³Note that the intercluster bilateral influences are uniform across all pairs of non-neighbouring firms.

On the other hand, the crowding-out effect of a core firm investment on peripheral firms' investment is more severe than the crowding-out effect of a peripheral firm investment on core firms' investment. Since investment by a core firm lowers the marginal production costs of a relatively larger fraction of the industry's firms, the business stealing effects from a core firm investment experienced by peripheral firms are more severe than the business stealing effects from a peripheral firm investment experienced by core firms. Observe that if $n_r > n_t$ then $\sigma_{rt} < \sigma_{tr}$; hence whereas investment by core firms tends to be more resilient to cost reductions achieved by peripheral firms, investment by peripheral firms tends to be more vulnerable to cost reductions achieved by core firms. We say that core firms benefit more from *cluster strength effects* than peripheral firms.⁴

Therefore, on the one hand, peripheral firms experience a relatively less severe local appropriation problem but on the other hand, their investments tend to be relatively more vulnerable to cost reductions achieved by core firms.

Rearranging the first order condition yields the best response function of firm i which is given by

$$b_i(s_i, (x_j)_{j \in C_r^i}, (x_k)_{k \in C_t}) = \frac{1}{\psi - \sigma_{rr}} \alpha_r + \frac{1}{\psi - \sigma_{rr}} s_i + \sum_{j \in C_r^i} \frac{\sigma_{rr}}{\psi - \sigma_{rr}} x_j - \sum_{k \in C_t} \frac{\sigma_{rt}}{\psi - \sigma_{rr}} x_k.$$

Applying the implicit function theorem to this system of equations, there exists n differentiable implicit functions $\{x_i(s_1, \dots, s_n)\}_i$ such that equilibrium industry investment can be written as

$$\sum_i x_i^* = \sum_i b_i(s_i, (x_j(s_1, \dots, s_n))_{j \in C_r^i}, (x_k(s_1, \dots, s_n))_{k \in C_t}).$$

Totally differentiating this expression with respect to s_i , around (x_1^*, \dots, x_n^*) , decomposes the impact of a subsidy on investment into a direct and indirect effect. The indirect effect of a subsidy consists of a feedback effect, $n_r - 1$ local cross effects and n_t inter-cluster cross effects:

$$\frac{d(\sum_j x_j^*)}{ds_i} = \underbrace{\frac{\partial b_i}{\partial s_i}}_{\text{direct effect}(D_r)} + \underbrace{\sum_{j \in C_r^i} \frac{\partial b_i}{\partial x_j} \frac{\partial x_j}{\partial s_i} + \sum_{k \in C_t} \frac{\partial b_i}{\partial x_k} \frac{\partial x_k}{\partial s_i}}_{\text{feedback effect}(F_r)} + \underbrace{\sum_{j \in C_r^i} \left(\sum_{l \neq j} \frac{\partial b_j}{\partial x_l} \frac{\partial x_l}{\partial s_i} \right) + \sum_{k \in C_t} \left(\sum_{l \neq j} \frac{\partial b_k}{\partial x_l} \frac{\partial x_l}{\partial s_i} \right)}_{\text{cross effects}(C_{rr}, C_{tr})}.$$

The local (resp. inter-cluster) cross effect, denoted C_{rr} (resp. C_{tr}), is the response of a neighbouring (resp. non-neighbouring) firm to the subsidy-induced change in rival firms'

⁴See Horan (2011) for further discussion of cluster strength effects.

investment. The feedback effect, denoted F_r , is the response of the recipient firm to subsidy-induced change in rival firms' investment behaviour. This feedback effect consists of $n_r - 1$ local feedback effects, denoted F_{rr} , and n_t inter-cluster feedback effects, denoted F_{tt} . Finally, the direct effect of subsidy on investment, denoted D_r , is the response of the recipient firm to the subsidy abstracting from investment interactions between recipient and rival firms. This direct effect can be computed from the firm's best response function and is given by $D_r = \frac{\partial b_i}{\partial s_i} = \frac{1}{\psi - \sigma_{rr}}$.

Solving the system of equations and applying this decomposition,⁵ equilibrium firm investment given the subsidy program and the industry's spatial pattern can be written as

$$x_i^*(s_1, \dots, s_n, g) = \hat{x}_i + D_r s_i + ((n_r - 1)F_{rr} + n_t F_{rt})s_i + \sum_{j \in C_r^i} C_{rr} s_j - \sum_{k \in C_t} C_{rt} s_k,$$

in which

$$F_{rr} = \frac{\sigma_{rr}^2 (\psi - n_t \sigma_{tt})}{\psi (\psi - \sigma_{rr}) A}, \quad F_{rt} = \frac{n_r \sigma_{rt} \sigma_{tr}}{(\psi - \sigma_{rr}) A},$$

and

$$C_{rr} = \frac{(\psi - n_t \sigma_{tt}) \sigma_{rr}}{\psi A}, \quad C_{rt} = \frac{\sigma_{rt}}{A},$$

are respectively the local and inter-cluster feedback and cross effects in which $A > 0$ and $\hat{x}_i = \frac{(\psi - n_t \sigma_{tt}) \alpha_r - n_t \sigma_{rt} \alpha_t}{A}$ denotes the firm's laissez-faire investment level.⁶ We now examine how the industry's spatial pattern influences the effect of the subsidy program on firm investment outcomes. Since $F_{rr}, F_{rt} > 0$ the feedback effect is positive and therefore subsidizing a firm indirectly stimulates the recipient firm's privately-financed investment.

Notice that $\sigma_{rr} > 0$ implies $C_{rr} > 0$; hence local complementarity in investment decisions implies subsidizing a firm indirectly stimulates investment by neighbouring firms. In the Appendix we prove that if $n_r > n_t$ and $n_t > 1$ then $F_{rr} < F_{tt}$ and $C_{rr} < C_{tt}$; hence the magnitudes of the local feedback and local additionality effects of a subsidy are weaker for a core firm subsidy than for a periphery firm subsidy. This observation gives the following result:

Proposition 3.1 *The intensity of the additionality effect of a subsidy on a neighbouring firm's investment is relatively higher for a periphery firm subsidy.*

⁵See the Appendix for the derivation of this equilibrium R&D expression.

⁶The assumption $\psi > \max\{n_r \sigma_{rr} + n_r \sigma_{tr}, \frac{n+1}{4}, \frac{n^2}{(n+1)^2} \frac{a}{c}\}$ ensures i) a unique interior laissez-faire Nash equilibrium exists; and ii) $A = (\psi - n_r \sigma_{rr})(\psi - n_t \sigma_{tt}) - n_r n_t \sigma_{rt} \sigma_{tr} > 0$.

The intuition behind this result is that the relatively lower intensity of local complementarity in the core cluster, due to the relatively more severe local appropriation problem, creates differential local subsidy effects.

Notice that $\sigma_{rt} > 0$ implies $C_{rt} > 0$; hence inter-cluster substitutability in investment decisions implies subsidizing a firm crowds-out investment by non-neighbouring firms. Simple comparisons reveal if $n_r > n_t$ then $F_{rt} > F_{tr}$ and $C_{tr} > C_{rt}$; hence the inter-cluster feedback and inter-cluster crowding-out effects of a subsidy are stronger for a core firm subsidy than for a peripheral firm subsidy. This observation gives the following result:

Proposition 3.2 *The intensity of the crowding-out effect of a subsidy on a non-neighbouring firm's investment is relatively stronger for a core firm subsidy.*

The intuition behind this result is that since core firms benefit more from cluster strength effects than peripheral firms, i) the investments of peripheral firms are relatively more vulnerable to cost reductions induced by a core firm subsidy; and ii) the investments of core firms are relatively more resilient to cost reductions induced by a periphery firm subsidy. Thus, asymmetry in the resilience of the investments of core and peripheral firms creates differential inter-cluster subsidy effects.

We now investigate how the industry's spatial pattern influences the effect of a cluster subsidy on firm investment. Suppose $s_r = s_i = s_j$, for all $i, j \in C_r$, denotes the subsidy to a firm in cluster r and suppose $s_t = s_k$, for all $k \in C_t$, denotes the subsidy to a firm in cluster t . Invoking intra-cluster symmetry, we obtain equilibrium investment as a function of government cluster policy and the industry's spatial pattern:

$$x_r^*(s_r, s_t, g) = \hat{x}_r + \underbrace{(D_r + F_r + (n_r - 1)C_{rr})}_{\text{additionality}} s_r - \underbrace{n_t C_{rt}}_{\text{crowding-out}} s_t,$$

in which simple derivations reveals that $D_r + F_r + (n_r - 1)C_{rr} = \frac{(\psi - n_t \sigma_{tt})}{A} > 0$. In the Appendix we show that if $n_r > n_t$ then $\frac{(\psi - n_r \sigma_{rr})}{A} > \frac{(\psi - n_t \sigma_{tt})}{A}$; hence the relatively less severe local appropriation problem in the periphery cluster implies that the extent to which a periphery cluster subsidy stimulates peripheral firm investment is greater than the extent to which a core cluster subsidy stimulates core firm investment. This observation gives the following result:

Proposition 3.3 *The intensity of the additionality effect of a cluster subsidy on cluster firm investment is relatively higher for a periphery cluster subsidy.*

We refer to the difference between the intensity of the additional effect of a periphery and core cluster subsidy as the *expansion effect*. Formally, the expansion effect, denoted e , is given by

$$e := \frac{(\psi - n_r \sigma_{rr})}{A} - \frac{(\psi - n_t \sigma_{tt})}{A} > 0.$$

On the other hand, if $n_r > n_t$ then $n_r C_{tr} > n_t C_{rt}$; hence the more intense strength effects of the core cluster imply the degree to which a periphery cluster subsidy crowds-out core firm investment is weaker than the degree to which a core cluster subsidy crowds-out peripheral firm investment. This observation gives the following result:

Proposition 3.4 *The intensity of the crowding-out effect of a cluster subsidy on non-cluster firm investment is relatively stronger for a core cluster subsidy.*

We refer to this difference between the intensity of the crowding-out effect of a core and periphery cluster subsidy as the *sitting-duck effect*. Formally, the sitting-duck effect, denoted δ , is given by

$$\delta := \frac{n_r \sigma_{tr}}{A} - \frac{n_t \sigma_{rt}}{A} > 0.$$

These observations suggest that if government seeks to stimulate the investments of core and peripheral firms by an equal amount then on the one hand, peripheral firms require a smaller subsidy than core firms because of the expansion effect but on the other hand, peripheral firms require a larger subsidy because of the sitting-duck effect. In the next section, we investigate the implications of the interplay between the expansion effect and the sitting-duck effect for optimal subsidy policy.

3.2.2 Optimal Subsidy Policy and the Sitting-duck Effect

It is assumed that in the absence of government intervention both core and peripheral firms under-invest in cost reduction. Recalling the social welfare function presented earlier, suppose x_r^{**} denotes the socially optimal investment of a firm in cluster r . We define the investment gap of a firm in cluster r , denoted Δ_r , as the difference between its socially optimal and laissez-faire investment which must satisfy

$$\Delta_r := x_r^{**} - \hat{x}_r > 0.$$

Under this assumption, government is justified in subsidizing both core and peripheral firms. An optimal policy equates social optimum investment and publicly-funded investment, i.e. the optimal subsidy program must satisfy $x_r^{**} = x_r^*(s_r^*, s_t^*)$. Substituting in the policy equilibrium yields $\Delta_r = \frac{(\psi - n_t \sigma_{tt})}{A} s_r^* - \frac{n_t \sigma_{rt}}{A} s_t^*$. Solving this system of linear equations gives the optimal subsidy program as a function of the industry's spatial pattern

$$s_r^*(g) = (\psi - n_r \sigma_{rr}) \Delta_r + n_t \sigma_{rt} \Delta_t.$$

We now investigate under what conditions peripheral firms receive a larger subsidy. Suppose throughout $n_r > n_t$ and let the parameter ρ denote the difference between the investment gaps of core and peripheral firms, i.e. $\Delta_r = (1 + \rho) \Delta_t$. Establishing a comparison between the optimal core and periphery cluster subsidy, we see that core firms receive a larger subsidy, i.e. $s_r^*(\rho) > s_t^*(\rho)$, if and only if the investment gap differential satisfies the following inequality

$$\rho > \rho^* = \frac{\delta - e}{\psi - n_r \sigma_{rr} - n_r \sigma_{tr}} A,$$

in which δ denotes the sitting-duck effect and e denotes the expansion effect.⁷ The equilibrium restrictions imply $A > 0$ and $\psi > n_r \sigma_{rr} + n_r \sigma_{tr}$. Therefore, if the sitting-duck effect dominates (resp. is dominated by) the expansion effect then $\rho^* > 0$ (resp. $\rho^* < 0$). Substituting in the expressions for the bilateral influences and establishing a comparison between the expansion effect and the sitting-duck effect, it is easily verified that

$$\delta - e = (n_r - n_t) \left(1 - \frac{n_r + n_t}{n+1} + \frac{n_r^2 + n_t^2}{(n+1)^2} \right) > 0.$$

Thus, the sitting-duck effect dominates the expansion effect and $\rho^* > 0$.⁸ Notice that if the investment gap differential satisfies $\rho < \rho^*$ then peripheral firms receive a larger subsidy. In particular, if $0 < \rho < \rho^*$ then $\Delta_r > \Delta_t$ and $s_t^* > s_r^*$. Therefore, even if the investment projects of core firms are more attractive from a social welfare perspective, government may still need to provide peripheral firms with a larger subsidy in order to counteract the sitting-duck effect. These observations give the following result:

Proposition 3.5 *The sitting-duck effect dominates the expansion effect and if government is justified in funding both core and peripheral firms then peripheral firms require additional funding to counteract the sitting-duck effect.*

⁷See the Appendix for the derivation of this inequality.

⁸See the Appendix for the derivation of this inequality.

To clarify the point, suppose government seeks to stimulate the investments of core and peripheral firms by an equal amount, i.e. $\Delta_r = \Delta_t$. Since the sitting-duck effect dominates the expansion effect, peripheral firms require a larger subsidy in order to offset the more intense crowding-out effects of the core cluster subsidy, i.e. $\delta > e$ implies $\rho^* > 0$ and hence $s_t^* > s_r^*$. Therefore, in the presence of government subsidy policy, peripheral firms are said to be sitting-ducks with respect to core firms.

3.2.3 Case-by-case Funding and Core Bias

This short section compares the optimal subsidy program and the subsidy program that emerges when government allocates funding under the case-by-case (CbC) rule. Denote by s_r^{cbc} the CbC subsidy to a firm in cluster r . It is assumed government knows the investment gap associated with each firm's investment activity and correctly anticipates the direct effect of a subsidy on investment. Given Δ_r and D_r , government chooses s_r^{cbc} in order to close the firm's investment gap ignoring the indirect effect of the subsidy on investment: $D_r s_r^{cbc} = \Delta_r$. Substituting in for the direct effect and rearranging yields the CbC subsidy to a firm in cluster r :

$$s_r^{cbc} = (\psi - \sigma_{rr})\Delta_r.$$

We now investigate under what conditions core firms receive a larger subsidy under the CbC rule. Suppose throughout $n_r > n_t$ and as before let the parameter ρ denote the difference between the investment gaps of core and peripheral firms, i.e. $\Delta_r = (1 + \rho)\Delta_t$. Establishing a comparison between the CbC subsidies of core and peripheral firms, we see that core firms receive a larger subsidy, i.e. $s_r^{cbc}(\rho) > s_t^{cbc}(\rho)$, if and only if the investment gap differential satisfies

$$\rho > \rho^{cbc} = -\frac{(\sigma_{tt} - \sigma_{rr})}{(\psi - \sigma_{rr})}.$$

Notice that since core firms suffer from a more severe local appropriation problem than peripheral firms, it follows that $\sigma_{tt} > \sigma_{rr}$ and hence $\rho^{cbc} < 0$.⁹ Comparing the investment gap differential cut-offs of the optimal subsidy program and the CbC subsidy program we see that the indifference point of the CbC rule is lower than the optimal indifference point:

$$\rho^* > 0 > \rho^{cbc}.$$

⁹See the Appendix for the derivation of the CbC indifference point.

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Therefore, if the investment gap differential, ρ , satisfies $\rho^* > \rho > \rho^{cbc}$ then whereas in the optimal subsidy program peripheral firms receive a larger subsidy, $s_t^* > s_r^*$, in the CbC subsidy program core firms receive a larger subsidy, $s_r^{cbc} > s_t^{cbc}$. This finding highlights that subsidization under the CbC rule is biased towards favouring firms in larger clusters. This core bias stems from the fact that when allocating subsidies, the CbC rule is guided by the extent of the local appropriation problem and does not take into account the sitting-duck effect.

Notice that since the local appropriation problem is relatively more severe in the core cluster, the direct effect of a subsidy on investment is relatively weaker for a core firm subsidy, i.e. $D_r < D_t$. Thus, under the rule government feels compelled to provide core firms with additional funding to counteract its relatively more severe local appropriation problem. In particular, notice that if $0 > \rho > \rho^{cbc}$ then the investment gap of peripheral firms is larger than core firms, $\Delta_t > \Delta_r$, but under the CbC rule core firms receive a suboptimally larger subsidy than peripheral firms, $s_r^{cbc} > s_t^{cbc}$.

3.3 Advanced Technology Vehicle Manufacturing and Loan Program

First we describe the ATVMLP program and highlight the connection with our model. Then we discuss the relevance of our policy findings.

The U.S. Department of Energy ATVMLP is a \$25 billion direct loan program which provides conditional low interest loans, "soft loans", to automobile manufacturers for the purpose of funding projects that help vehicles manufactured in the U.S. meet higher millage requirements and lessen U.S. dependence on foreign oil. To qualify, automakers must promise to increase the fuel economy of their products by 25% and apply the loans to future investments "reasonably related to the reequipping, expanding, or establishing of a manufacturing facility in the U.S."

The first tranche of soft loans was announced in 2009. The loan commitments to U.S. automakers include a \$5.9 billion loan to Ford for upgrading factories to produce 13 more fuel-efficient gasoline vehicle models equipped with EcoBoost technology for the internal combustion engine, a \$465 million loan to Tesla Motors to manufacture electric drive-train technology and its new electric vehicle the Model S, and a \$528.7 million loan to Fisker Automotive for the development of plug-in hybrid electric vehicles.

It is widely accepted that the core cluster of the U.S. automobile industry is centred around Detroit and the "Big Three" motor vehicle corporations (General Motors, Ford and

Chrysler). These firms are linked not only geographically but also technologically as their research efforts are overwhelmingly focused on advancing gasoline vehicle technology. On the other hand, the electric car start-ups Tesla Motors and Fisker Automotive are linked technologically through their investments in electric vehicle technology and are part of a peripheral (electric vehicle) technology cluster of the U.S. automobile industry.

Cost spillovers between linked firms are likely to be quite high because firms which work on the same technology tend to draw on a common pool of specialized inputs. The supply of these often complementary intermediate goods depends on many intermediate firms exploiting economies of scale. These firms will try to bet on the winning technologies which is likely to be a function of the sum of firm investments and subsidies. Consequently, their location and investment decisions are likely to be the main driving force behind cost reductions between linked firms.

The analysis of this paper's model suggests the peripheral electric cluster may be a sitting duck with respect to the core gasoline cluster. Subsidies to the Detroit core are likely to exert relatively stronger crowding effects on the investments of the electric start-ups. On the one hand, common sense suggests the investment behaviour of firms in the Detroit core is likely to prove quite resilient to the modest subsidies allocated to the electric start-ups. On the other hand, government fuelled developments in advanced gasoline technology will probably steal business in green niche markets lowering the innovation and investment incentives of electric car makers.

3.4 Conclusion

Little systematic attention has been paid to understanding how an industry's spatial pattern influences the effect of government subsidies, e.g. R&D subsidies, on firm investment outcomes. This paper develops a model in order to study how government should subsidize investment when firms are located in clusters and local cost externalities are present. The analysis sheds light on the differential investment incentives and the differential subsidy effects experienced by core and peripheral firms.

The paper finds that although core firms suffers from a more severe local appropriation problem, they benefit from more intense cluster strength effects. Consequently, the additionality effect of subsidy on investment tends to be stronger for peripheral firm subsidies (expansion effect). On the other hand, the crowding-out effect of subsidy on non-cluster firm investment tends to be stronger for core firm subsidies (sitting-duck effect). We find

that the sitting duck effect dominates and if government subsidizes core firms then peripheral firms require additional funding to counteract the sitting-duck effect.

We note that the analysis was conducted in a simple setting of intense product market competition, perfect cost externalities and for the class of single core-periphery industry spatial patterns. We hope in future research to examine the relationship between the differential investment incentives of cluster firms and differential effects of cluster policies in more general settings.

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3.5 Appendix

Nash equilibrium R&D given subsidy policy: The best response function of the representative firm i and neighbouring firm j are respectively given by

$$x_i = \frac{\alpha_r + s_i + \sum_{j \in C_r^i} \sigma_{rr} x_j - \sum_{k \in C_t} \sigma_{rt} x_k}{\psi - \sigma_{rr}}; \quad x_j = \frac{\alpha_r + s_j + \sum_{\tilde{j} \neq j \in C_r^i} \sigma_{rr} x_{\tilde{j}} + \sigma_{rr} x_i - \sum_{k \in C_t} \sigma_{rt} x_k}{\psi - \sigma_{rr}}.$$

Invoking symmetry on the neighbouring firm's cluster neighbours yields

$$x_j = \frac{\alpha_r + s_j + \sigma_{rr} x_i - \sum_{k \in C_t} \sigma_{rt} x_k}{\psi - (n_r - 1)\sigma_{rr}}.$$

Substituting this expression into the best response function of firm i yields

$$x_i = \frac{\psi \alpha_r + (\psi - (n_r - 1)\sigma_{rr}) s_i + \sum_{j \in C_r^i} \sigma_{rr} s_j - \sum_{k \in C_t} \psi \sigma_{rt} x_k}{\psi(\psi - n_r \sigma_{rr})}.$$

By symmetry, the best response function of the representative non-neighbouring firm is given by

$$x_k = \frac{\psi \alpha_t + (\psi - (n_t - 1)\sigma_{tt}) s_k + \sum_{j \in C_r^i} \sigma_{tt} s_{\tilde{k}} - \sum_{i \in C_r} \psi \sigma_{tr} x_i}{\psi(\psi - n_t \sigma_{tt})}.$$

Substituting this expression into x_i and rearranging terms yields

$$x_i = \widehat{x}_i + \frac{(\psi - (n_r - 1)\sigma_{rr})(\psi - n_t\sigma_{tt})}{\psi A} s_i + \sum_{j \in C_r^i} \frac{(\psi - n_t\sigma_{tt})\sigma_{rr}}{\psi A} s_j - \sum_{k \in C_t} \frac{\sigma_{rt}}{A} s_k,$$

in which $A = (\psi - n_r\sigma_{rr})(\psi - n_t\sigma_{tt}) - n_r n_t \sigma_{rt} \sigma_{tr} > 0$ and $\widehat{x}_i = \frac{(\psi - n_t\sigma_{tt})\alpha_r - n_t\sigma_{rt}\alpha_t}{A}$. The term $T_r := \frac{(\psi - (n_r - 1)\sigma_{rr})(\psi - n_t\sigma_{tt})}{\psi A}$ is the total effect of a subsidy on the recipient firm's investment. From the decomposition of the effect of a subsidy, this total effect consists of a direct effect and a feedback effect, denoted F_r , i.e. $T_r = D_r + F_r$. Substituting in for the direct effect and the total effect and solving reveals that the feedback effect is given by $F_r = (n_r - 1) \frac{\sigma_{rr}^2(\psi - n_t\sigma_{tt})}{\psi(\psi - \sigma_{ee})A} + n_t \frac{n_r\sigma_{rt}\sigma_{tr}}{(\psi - \sigma_{rr})A}$. From the decomposition, this feedback effect consists of $n_r - 1$ intra-cluster feedback effects denoted F_{rr} , and n_t inter-cluster feedback effects denoted F_{rt} , i.e. $F_r = \sum F_{rr} + \sum F_{rt}$. It is easily seen that $F_{rr} = \frac{\sigma_{rr}^2(\psi - n_t\sigma_{tt})}{\psi(\psi - \sigma_{ee})A}$ and $F_{rt} = \frac{n_r\sigma_{rt}\sigma_{tr}}{(\psi - \sigma_{rr})A}$. Therefore, equilibrium R&D given the subsidy program can be rewritten as

$$x_i^*(s_1, \dots, s_n) = \widehat{x}_i + D_r s_i + \left(\sum_{j \in C_r^i} F_{rr} + \sum_{k \in C_t} F_{rt} \right) s_i - \sum_{j \in C_r^i} C_{rr} s_j + \sum_{k \in C_t} C_{rt} s_k,$$

in which $D_r = \frac{1}{\psi - \sigma_{rr}}$, $F_{rr} = \frac{\sigma_{rr}^2(\psi - n_t\sigma_{tt})}{\psi(\psi - \sigma_{rr})A}$, $F_{rt} = \frac{n_r\sigma_{rt}\sigma_{tr}}{(\psi - \sigma_{rr})A}$, $C_{rr} = \frac{(\psi - n_t\sigma_{tt})\sigma_{rr}}{\psi A}$ and $C_{rt} = \frac{\sigma_{rt}}{A}$ are respectively the direct, feedback and cross effects of the subsidy program on the representative firm's R&D.

Analysis of Subsidy Effects: i) We prove that if $n_r > n_t$ then $C_{tt} > C_{rr}$. Establishing a comparison between C_{tt} and C_{rr} yields

$$C_{tt} - C_{rr} = \frac{(\psi - n_r\sigma_{rr})\sigma_{tt} - (\psi - n_t\sigma_{tt})\sigma_{rr}}{\psi A}.$$

The sign of $C_{tt} - C_{rr}$ is positive if and only if $(\psi - n_r\sigma_{rr})\sigma_{tt} > (\psi - n_t\sigma_{tt})\sigma_{rr}$. First, notice that $n_r > n_t$ ensures $\sigma_{tt} > \sigma_{rr}$. Second, the inequality $\psi - n_r\sigma_{rr} > \psi - n_t\sigma_{tt}$ is satisfied if and only if $n_t\sigma_{tt} > n_r\sigma_{rr}$. Substituting in the expressions for the intensity of local complementarity, simple derivations reveal $n_t(1 - \frac{n_t}{n+1})^2 - n_r(1 - \frac{n_r}{n+1})^2 = -(n_r - n_t) \frac{(1 - n_r n_t)}{(n+1)^2} > 0$ for all $n_r > n_t$ and $n_t \geq 1$. Therefore, the result follows.

ii) We prove that if $n_r > n_t$ then $F_{tt} > F_{rr}$. Establishing a comparison between F_{tt} and F_{rr} yields

$$F_{tt} - F_{rr} = \frac{(\psi - \sigma_{rr})(\psi - n_r\sigma_{rr})\sigma_{tt}^2 - (\psi - \sigma_{tt})(\psi - n_t\sigma_{tt})\sigma_{rr}^2}{\psi A}.$$

The sign of $F_{tt} - F_{rr}$ is positive if and only if $(\psi - \sigma_{rr})(\psi - n_r\sigma_{rr})\sigma_{tt}^2 > (\psi - \sigma_{tt})(\psi - n_t\sigma_{tt})\sigma_{rr}^2$. First, notice that $n_r > n_t$ ensures $\psi - \sigma_{rr} > \psi - \sigma_{tt}$ and $\sigma_{tt}^2 > \sigma_{rr}^2$. Second, in (i) it was proved that if $n_r > n_t$ then $\psi - n_r\sigma_{rr} > \psi - n_t\sigma_{tt}$. Therefore, the result follows.

iii) I prove that if $n_r > n_t$ then $\frac{(\psi - n_r \sigma_{rr})}{A} > \frac{(\psi - n_t \sigma_{tt})}{A}$. It was proved in (i) that if $n_r > n_t$ then $(\psi - n_r \sigma_{rr}) > (\psi - n_t \sigma_{tt})$. Therefore, the result follows.

Optimal Subsidy Policy: i) We prove that if $\rho^* = \frac{\delta - e}{\psi - n_r \sigma_{rr} - n_r \sigma_{tr}} A$. Establishing a comparison between s_r^* and s_t^* yields

$$s_r^* - s_t^* = (\psi - n_r \sigma_{rr}) \Delta_r + n_t \sigma_{rt} \Delta_t - ((\psi - n_t \sigma_{tt}) \Delta_t + n_r \sigma_{tr} \Delta_r)$$

Substituting in for $\Delta_r = (1 + \rho) \Delta_t$ and rearranging yields

$$(((\psi - n_r \sigma_{rr}) - (\psi - n_t \sigma_{tt})) - (n_r \sigma_{tr} - n_t \sigma_{rt})) \Delta_t + \rho((\psi - n_r \sigma_{rr} - n_r \sigma_{tr})) \Delta_t$$

The sign of $s_r^* - s_t^*$ is positive if and only if $\rho > \rho^* = \frac{-((\psi - n_r \sigma_{rr}) - (\psi - n_t \sigma_{tt}) - (n_r \sigma_{tr} - n_t \sigma_{rt}))}{(\psi - n_r \sigma_{rr} - n_r \sigma_{tr})}$. Multiplying above and below by A and substituting in for the expansion effect and the sitting-duck effect gives $\rho^* = \frac{\delta - e}{\psi - n_r \sigma_{rr} - n_r \sigma_{tr}} A$.

ii) We prove that $\delta - e = (n_r - n_t)(1 - \frac{n_r + n_t}{n+1} + \frac{n_r^2 + n_t^2}{(n+1)^2}) > 0$. Establishing a comparison between δ and e yields

$$\delta - e = (n_r \sigma_{tr} - n_t \sigma_{rt}) - ((\psi - n_r \sigma_{rr}) - (\psi - n_t \sigma_{tt}))$$

The sign of $\delta - e$ is positive if and only if $n_t(\sigma_{tt} + \sigma_{tr}) - n_r(\sigma_{rr} + \sigma_{rt}) > 0$. Substituting in the expressions for the bilateral influences, simple derivations reveal that $n_r > n_t$ implies

$$\delta - e = (n_r - n_t)(1 - \frac{n_r + n_t}{n+1} + \frac{n_r^2 + n_t^2}{(n+1)^2}) > 0,$$

and the result follows.

Case-by-Case Subsidy Policy: We prove that if $\rho^{cbc} = -(\frac{\sigma_{tt} - \sigma_{rr}}{\psi - \sigma_{rr}})$. Establishing a comparison between s_r^{cbc} and s_t^{cbc} yields

$$s_r^{cbc} - s_t^{cbc} = (\psi - \sigma_{rr}) \Delta_r - (\psi - \sigma_{tt}) \Delta_t$$

Substituting in for $\Delta_r = (1 + \rho) \Delta_t$ and rearranging yields

$$s_r^{cbc} - s_t^{cbc} = ((\psi - \sigma_{rr}) - (\psi - \sigma_{tt})) \Delta_t + \rho((\psi - \sigma_{rr})) \Delta_t$$

The sign of $s_r^{cbc} - s_t^{cbc}$ is positive if and only if $\rho > \rho^{cbc} = \frac{-((\psi - \sigma_{rr}) - (\psi - \sigma_{tt}))}{(\psi - \sigma_{rr})}$. and therefore it follows that $\rho^{cbc} = -(\frac{\sigma_{tt} - \sigma_{rr}}{\psi - \sigma_{rr}})$.

