Department of Economics

Moment Condition Models in Empirical Economics

Sara Maria de Almeida Duarte Lopes Riscado

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

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DEDICATION

To Ana and Ema
To my parents, Ana and Máximo
To my grandparents, Céu and João, and my aunt and uncle, Lurdes and Lito

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And of course, the nicest outcome of the process of writing this dissertation was that I made very special friends: Dimitri, Régis, Sophie and Hugo, to whom I am grateful for being always next to me in the worst times. Without them I would not have arrived to this stage.

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Part I

Introduction

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0.1 Abstract

In the first chapter of this dissertation, we approach the estimation of dynamic stochastic general equilibrium models through a moments-based estimator, the empirical likelihood. We try to show that this inference process can be a valid alternative to maximum likelihood. The empirical likelihood estimator only requires knowledge about the moments of the data generating process of the model. In this context, we exploit the fact that these economies can be formulated as a set of moment conditions to infer on their parameters through this technique. For illustrational purposes, we consider the standard real business cycle model with a constant relative risk adverse utility function and indivisible labour, driven by a normal technology shock.

In the second chapter, we explore further aspects of the estimation of dynamic stochastic general equilibrium models using the empirical likelihood family of estimators. In particular, we propose possible ways of tackling the main problems identified in the first chapter. These problems resume to: (i) the possible existence of dependence between the random variables; (ii) the definition of moment conditions in the dynamic stochastic general equilibrium models setup; (iii) the alternatives to the data generation process used in the first chapter.

In the third chapter, we investigate the short run effects of macroeconomic and fiscal volatility on the decision of the policy maker on how much to consume and how much to invest. To that end, we analyse a panel of 10 EU countries during 1991—2007. Our results suggest that increases in the volatility of regularly collected and cyclical revenues such as the VAT and income taxes tend to tilt the expenditure composition in favour of public investment. In contrast, increases in the volatility of ad hoc –type of taxes such as capital taxes tend to favour public consumption spending, albeit only a little.

0.2 Some Remarks

The third chapter of this dissertation is a joint research work developed during my internship in the European Investment Bank. It is a co-authored article with Juraj Stancik, from CERGE-EI, Charles University Prague, Academy of Sciences of the Czech Republic, and Timo Valila, from the European Investment Bank. Juraj helped me to assemble the dataset and Timo redacted the text. My contribution consisted in reviewing literature and performing all the econometric analysis.

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Part II

Two Essays on Empirical Likelihood and an Application of the Arellano and Bond Estimator to Public Policy

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CHAPTER 1

ON THE ESTIMATION OF DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM MODELS: AN EMPIRICAL LIKELIHOOD APPROACH

1.1 Introduction

Dynamic stochastic general equilibrium (DSGE) models are widely used as the standard tool of macroeconomic analysis. In a nutshell, a DSGE model represents the economy as constrained optimization problem(s), subject to the definition of preferences and technology, the laws of motion of the economic variables, and the resource constraints of both the whole economy and the agents. Furthermore, it is possible to introduce a variety of sources of stochasticity, such as productivity, preferences and/or policy shocks. In this way, it is possible to simulate the economy's equilibrium path under uncertain conditions and compare it with what is observed in the real world. It is also possible to shock the economy, and observe its behaviour as it converges to its steady state. This is also useful from the policy point-of-view. As Ruge-Murcia (2003) well explains, "DSGE models are attractive because they specify explicitly the objectives and constraints faced by households and firms, and then determine the prices and allocations that result from their market interaction in an uncertain environment".

From an econometric pespective these models are interesting in two main ways.

First, as Sargent (1989) showed, it is possible to derive a likelihood function from a DSGE model and, in this way, it is possible to estimate its structural parameters. Sargent (1989) proposed to linearize the equilibrium conditions of the economy around the steady state or, alternatively to generate a quadratic approximation to the utility function of the economic agents. From here, one would be able to obtain approximated policy functions of the model (that is, the optimal choices of the economic agents, at each period of time), upon which the likelihood function would be written. Sargent's seminal work originated a very extensive literature on the maximum likelihood estimation of DSGE models, from which we outline the recent studies of Fernandez-Villaverde and Rubio-Ramirez (2004, 2005 and 2007). However, the implementation of maximum likelihood is analytically and computationally demanding, due to the two issues that have to be addressed in order to reach the estimation stage: (i)

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to write down the likelihood function; (ii) to evaluate the likelihood function.

To write down the likelihood function requires to solve the DSGE model and to obtain its policy functions. The specific analytical structure of the system of equilibrium conditions of these models, which is generally a nonlinear system of difference equations, yields closed form solutions only in very few settings. Therefore, for the vast majority of DSGE models it is only possible to obtain approximated policy functions. The standard procedure in the literature to obtain these approximated policy functions has been to (log)linearize the system of equilibrium conditions around the steady state, and then solving the resulting system through one of the available linear methods. The resulting policy funcions will be linear on the states of the economy. As an alternative to the linear option, there is also a set of numerical nonlinear methods to compute approximations to the policy functions (see, for instance, Aruoba *et al*, 2003, for a survey, classification and comparison of numerical nonlinear methods).

The next step is to evaluate the likelihood and this depends crucially on the solution method chosen. If one choses the linear path was, one has to apply the Kalman filter to evaluate the likelihood function, and assume that the stochastic component of the model follows a normal distribution. If one choses the nonlinear paths, one has to apply nonlinear filters, such as the Sequential Monte Carlo filter proposed by Fernandez-Villaverde and Rubio-Ramirez (2004).

Then, the maximization of the likelihood function follows. At this stage of the likelihood estimation process, we can choose between the classical inference and the Bayesian estimation. The classical inference consists of maximizing the likelihood function with respect to the parameters and computing their asymptotic variance-covariance matrix. The Bayesian inference, instead, implies obtaining a posterior distribution for the parameters proportional to the observables likelihood function times the priors on the parameters.

The choice of the solution method, and consequently of the filter applied on the estimation, has generated discussion on the accuracy, implementation and computation costs of each of the methods. Fernandez-Villaverde and Rubio-Ramirez (2005) address this question, by comparing linear with nonlinear likelihood based inference in a DSGE framework. They argue that eliminating the nonlinearities of the economy may not deliver sufficiently accurate policy functions, specially if we depart from the steady state of the model and if the nonlinearities are important features of the economy. Their results prove that the nonlinear choice is more suitable to take the model to the data, even when considering a relatively linear model. This discussion about the accuracy of the solution and evaluation methods has lead to a more subtle thread, which is the problem of performing inference with an ap-

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proximated likelihood function instead of the exact one. Fernandez-Villaverde *et al* (2006) are concerned with the possibility of having nonnegligible biases on estimates, if they use an approximated likelihood function instead of the true one. Ackerberg *et al* (2009) investigate this same question and prove that "*second order approximation errors in the policy function* have at most *second order effects on parameter inference*"[1] meaning that the impact of the approximation error may not cause important biases in the estimates. Nonetheless, these authors note that these are asymptotic results, so the level of accuracy of estimates is the same as the level of accuracy of the approximated likelihood function (i.e. how close the approximated likelihood function is to the true one). In this way, the researcher should carefully choose the method that can deliver the most accurate policy functionst.

Second, a DSGE model can also be represented by a set of moment conditions. This set of moments can be derived from the optimality conditions of the economic agents, and from the assumptions on the economy itself, such as the laws of motion of the economic variables and the moments of the stochastic shocks considered. This fact implies that it is also possible to estimate the structural parameters of DSGE models through the general method of moments and other moments-based family of estimators. The general method of moments is a less demanding estimation method in terms of distributional assumptions than the maximum likelihood estimation. In the context of the estimation of DSGE models this proves to be of great benefit. In the DSGE literature is commonly assumed that shocks are normally distributed, which is very convenient when coming to write their likelihood function. However, authors like Geweke (1993) and, more recently, Justiniano and Primiceri (2005) and Fernandez-Villaverde and Rubio-Ramirez (2006) argue that this is an extremely strong assumption and that, very often, the observed data show "fat tails" and could be better described by other distributions. These authors show that the fit of an ARMA process to US output data improves substancially when the errors are distributed as a student-t distribution instead of a normal one. Kim and Nelson (1999), McConnel and Perez-Quirós (2000) and Stock and Watson (2002) seem to confirm this argument, defending that shocks in DSGE models may have a richer structure than normal innovations.

In this paper, we also exploit the moment conditions of a DSGE model to estimate the structural parameters of the economy, but in a different framework. We approach the estimation of DSGE models with the empirical likelihood estimator. The empirical likelihood is also a moments-based estimador, which was first introduced by Owen (1988) as a likelihood ratio test, but it was soon found that it could be used to perform inference in a great variety of contexts, as explained by Qin and Lawless (1994). Briefly, the empirical likelihood esti-

[1] Authors' emphasis in their original text.

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mator consists in a nested optimization of an empirical likelihood function, subject to a set of moment conditions. This optimization problem is solved by first optimizing with respect to the weights assigned to each element of the sample, provided that the moment conditions are satisfied, and then to the parameters embodied in the moments. As in the case of the general method of moments, the empirical likelihood estimator only requires knowledge on the moments of the data generating process of the model and not on the distribution function itself. In addition, it accomodates all types of stochasticity and respects the nonlinear struture of the economy considered. Furthermore, it is a maximum-likelihood type of optimization problem, which implies good high order asymptotic properties. Moreover, we can skip the cumbersome and discretionary steps of finding the most accurate policy functions and of evaluating the likelihood, which we have described above.

Hence, we shall attempt to prove that the empirical likelihood estimator can be a valid way to perform inference with DSGE models. For that purpose, we will try to estimate the parameters of the standard business cycle model with a constant risk aversion utility function and indivisible labour, driven by a normal technology shock, using the empirical likelihood estimator.

In this exercise, we face two main problems. One is to find the adequate set of moment conditions, since our model will not deliver enough original moment conditions to estimate all the parameters. The other is to tackle the singularity problem arising from having less sources of stochasticity than control variables.

The rest of this paper is organized as follows. In section 2 we describe the empirical likelihood estimator and discuss the definition of moments in the DSGE context. In section 3 we present the economy which will illustrate the inference. In section 4 we describe the estimation strategy followed and present some preliminary results. Section 5 concludes.

1.2 The empirical likelihood framework

We present the empirical likelihood estimator, explain how it adapts to our estimation problem and discuss the definition and selection of the moment conditions.

1.2.1 The empirical likelihood estimator

Setup. Let $x_1, ..., x_m$ be identically independently distributed random variables drawn from an unknown distribution F, and $\omega(F)$ a $q-$dimensional vector of parameters, associated with the distribution F. Assume also that there is available information about ω and F in the form of r independent functions $g_j(x_i, \omega), j = 1, ..., r$, with $r \geq q$, such that $E\{g_i(x_i, \omega)\} = 0, \forall j$,

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under F. We can summarize the required setting in the following vector form:

$$g(x_i, \omega) = (g_1(x_i, \omega), ..., g_r(x_i, \omega))' \tag{1.1}$$

where

$$E_F\{g(x_i, \omega)\} = 0 \tag{1.2}$$

Condition (1.2) delivers a set of general estimating equations, which will be imposed in the sample considered for estimation purposes. In this setting, we define the empirical likelihood function as follows:

$$L_{EL}(\omega, p) = \left\{ \prod_{i=1}^{m} p_i | p_i > 0, \sum_{i=1}^{m} p_i = 1, \sum_{i=1}^{m} [p_i \cdot g(x_i, \omega)] = 0 \right\} \tag{1.3}$$

where p is the vector of the weights assigned to the sample elements.

The empirical likelihood function described in (1.3) is simply the product of the probabilities assigned to the sample elements, which satisfies three restrictions. The weights, or probabilities, p_i assigned to the sample elements have to be strictly positive and have to sum to one. Also, the sample counterparts of the distribution moments defined by (1.2), computed with the probabilities p_i, have to be satisfied.

Optimization problem. The maximum empirical likelihood estimator of $\{\omega, p\}$ is given by:

$$\{\hat{\omega}, \hat{p}\} = \arg\max_{\omega, p} L_{EL}(\omega, p) \tag{1.4}$$

The optimization problem defined by (1.4) consists of a reweighting problem since we optimize with respect to the vector of weights p. Note that this is a nested optimization problem, and not a two step estimator. Hence, once we obtain an estimate of p we use it to estimate ω, which is then substituted again into the problem and we reoptimize with respect to p. The optimization proceeds in this way until we find the pair $(\hat{\omega}, \hat{p})$ that maximizes our objective function, given by (1.3). The main idea underlying the empirical likelihood inference is to use optimally all the information embodied in the moments and in the sample in order to estimate the parameters in vector ω and the distribution F (see section A.4 of the Appendix for details on the solution of this problem). As pointed out by Schennach (2010), the empirical likelihood estimation "seeks to reweight the sample so that it satisfies the moment conditions exactly, while maximizing the likelihood function of a multinomial supported on the sample". In other words, unlike what happens with the general method of moments, we impose that the sample moments coincide with the theorectical ones, and we choose the proper weights of the sample elements that deliver exactly that result.

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It is also possible to make a parallel between the empirical likelihood function and the bootstrap technique. As Kitamura (1997) explains "both are based in nonparametric likelihood", but "while the bootstrap assigns $1/N$ probability mass to each observation, the empirical likelihood "chooses" probability mass under linear constraints".

1.2.2 Exponential Tilting Empirical Likelihood

This is an important extension of the empirical likelihood estimator and can be especially useful in our DSGE setup, because of possible misspecification problems. Schennach (2007) shows that, in the presence of model misspecification, the good high-order properties exhibited by the empirical likelihood estimator, are no longer verified. This author proposes to tilt the empirical likelihood with an exponential functional form, to tackle the problem. In comparison with the simple empirical likelihood setting described by problem (1.3), the setting of the exponential tilted empirical likelihood is modified as follows:

$$\hat{\theta} = \arg\min_{\theta} \left(\frac{1}{n} \sum_{i=1}^{n} \tilde{h} \left(\hat{p}_i \left(\theta \right) \right) \right) \tag{1.5}$$

where $\hat{p}_i(\theta)$ is the solution to

$$\min_{p_i} \left\{ \frac{1}{n} \sum_{i=1}^{n} h \left(p_i \right) | \sum_{i=1}^{n} p_i = 1; \sum_{i=1}^{n} \left[p_i g \left(x_i, \theta \right) \right] = 0 \right\} \tag{1.6}$$

with $\tilde{h}(p_i) = -\log(np_i)$ and $h(p_i) = np_i \log(np_i)$. The idea underlying this problem is to use the exponential tilted likelihood to find $\hat{p}_i(\theta)$ and use the empirical likelihood function to obtain the estimate of θ. In this way, the empirical likelihood function is combined with the entropy of the exponential tilted to estimate θ. A possible extension would be to use the exponential tilted empirical likelihood estimator, besides the simple empirical likelihood. It is better behaved than the general method of moments and empirical likelihood in the presence of model misspecification, $\inf_{\theta \in \Theta} \| E[g(x_i, \theta)] \| \neq 0$.

1.2.3 General estimation equations in the DSGE models context

The definition of a set of "good" moment conditions is crucial in the empirical likelihood framework. DSGE models can be formulated as a set of moments. However, the definition of the set of moments can be very involving, in this type of models.

The stylized nature of DSGE models results in the impossibility of having a sufficient number of moment conditions to estimate all the parameters of interest. Therefore, in most

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settings, we have to build more independent estimating equations. This can be, to a considerable extent, an arbitrary, task. Hence, we should be careful when choosing the criteria to follow when constructing these "artificial" moments. Furthermore, even the selection of original moments, which derive directly from the equilibrium conditions of the economy, is not straighforward. We stress the fact that the moments' function is actually a hybrid object, since it accumulates economic and statistical characteristics. The moment conditions should represent the economy under study and, consequently, we should be able to interpret them at the light of the theory underlying our model. At the same time, they should comply with some statistical requirements such as low cross correlation and high sensibility to "false" values of the parameters. Therefore, each moment condition has to intertwin the economic interpretation with the desirable statistical features. Hence, the main difficulty arising when defining the set of moment conditions, whether they are original or not, is to combine the statistic requirements with the economic interpretation.

In this preliminary analysis we will consider two sets of moment conditions: one constructed with instruments (Canova (2007)); and the other chosen according to the correlation criteria.

1.3 An illustration

We illustrate our analysis with the standard stochastic neoclassical model with indivisible labour, assuming a constant relative risk aversion utility function, a constant returns to scale Cobb-Douglas production function, and a technology shock. In this section, we describe the economy in more detail, define and solve the representative household problem and write down the competitive equilibrium of the economy. We choose this economy because it is simple and general enough, so that the estimation process can be adapted to other models with a similar structure, but different specifications of utillity and production functions, laws of motion and more types of shocks.

1.3.1 Description of the economy

Preferences. The instantaneous utility function describing households' preferences is defined as:

$$U(c_t, n_t) = \frac{[c_t^{\theta}\,(1-n_t)^{1-\theta}]^{1-\tau}}{1-\tau} \tag{1.7}$$

Equation (1.7) is a constant relative risk aversion (or constant elasticity of intertemporal substitution) utility function with two arguments: consumption, c_t, and leisure time, l_t. We have normalized time endowment to 1, so $l_t = 1 - n_t$, where n_t is labour hours supplied, and therefore household's total time endowment is allocated between leisure and hours worked. The parameter $\theta \in [0,1]$ weights the relative importance of leisure and consumption in the bundle of the representative household. The curvature parameter or coefficient of relative risk aversion, $\tau \geq 0$, measures how averse to risk the representative household is. The higher τ is, the more risk averse and the less willing to change her pattern of consumption the representative household is.

Production. We assume that output, y_t, is produced combining capital, k_t, and labour, n_t, and for an exogenous level of productivity z_t (which will be described later in more detail), through the following Cobb-Douglas production function:

$$y_t = f(k_t, n_t) = e^{z_t} k_t^{\alpha} n_t^{1-\alpha} \tag{1.8}$$

Equation (1.8) satisfies the usual neoclassical properties, namely decreasing marginal factor productivities and the Inada conditions. The parameter $\alpha \in (0,1)$ measures the elasticity of output with respect to capital.

Laws of motion. We assume in our economy that the capital stock and the technology evolve according to the following laws of motion.

$$k_{t+1} = i_t + (1-\delta) k_t \tag{1.9}$$

$$z_t = \rho z_{t-1} + \epsilon_t, \epsilon_t \sim N\left(0, \sigma_\epsilon^2\right) \tag{1.10}$$

Equation (1.9) describes how capital evolves over time when the economy invests i_t and capital depreciates by a fraction $\delta \in [0,1]$. Equation (1.10) describes the technology or productivity process of the economy. In each period t, technology depends on two components: the persistence of past shocks, measured by ρ, and a normally distributed technology shock, ϵ_t. Note that we assume $\rho \in (-1,1)$, which implies a stationary technology process.

Feasibility. The feasibility condition or resource constraint of this economy is simply:

$$y_t = c_t + i_t \tag{1.11}$$

1.3.2 Representative household's problem

The representative household solves the following maximization problem:

$$\max_{\{c_t, n_t, k_{t+1}\}_{t=0}^{+\infty}} E_t \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^{\theta} (1-n_t)_t^{1-\theta}\right]^{1-\tau}}{1-\tau}$$

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subject to

$$\begin{aligned} y_t &= e^{z_t} k_t^{\alpha} n_t^{1-\alpha} \\ k_{t+1} &= i_t + (1-\delta) k_t \\ z_t &= \rho z_{t-1} + \epsilon_t \\ y_t &= c_t + i_t \end{aligned}$$

and k_0, z_0 are known, and where β is the household's subjective discount factor, which reflects the households' rate of time preference, i.e. households' valuation of future consumption and leisure, relatively to today's consumption and leisure. It is assumed that households are "impatient", and therefore the utility derived from future consumption and leisure is valued less the later it is obtained, so $\beta \in (0,1)$.

Optimality conditions. From the optimization problem, the representative household derives the following first order conditions:

$$\frac{\left[c_t^{\theta}(1-n_t)^{1-\theta}\right]^{1-\tau}}{c_t} = \beta E_t \left[\begin{array}{c} \frac{\left[c_{t+1}^{\theta}(1-n_{t+1})^{1-\theta}\right]^{1-\tau}}{c_{t+1}} \cdot \\ \cdot \left(1-\delta+\alpha \frac{y_{t+1}}{k_{t+1}}\right) \end{array} \right] \quad (1.12)$$

$$\frac{1-\theta}{\theta} \cdot \frac{c_t}{1-n_t} = (1-\alpha) \frac{y_t}{n_t} \quad (1.13)$$

The optimality conditions (1.12) and (1.13) embody the intertemporal and intratemporal choices of the representative consumer. The Euler equation, (1.12), represents the trade-off between consuming today and postponing consumption into the future. This depends on the marginal rate of substitution between consumption today and tomorrow and on the marginal productivity of capital, since postponing consumption means investing more today and consuming more tomorrow. The intratemporal condition, (1.13), equates the marginal rate of substitution between consumption and leisure to the marginal productivity of labour. Note that the stochastic shock that drives this economy is incorporated into both the optimality conditions of the agents through the production function, y_t.

1.3.3 Competitive equilibrium

The competitive equilibrium of the economy consists of a sequence of real allocations $\{c_t, n_t, i_t, k_{t+1}, y_t\}_{t=0}^{\infty}$ that maximize consumer's utility and firm's profit, and clear the markets. Hence, the competitive equilibrium of the economy is described by equations (1.8) to (1.13).

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1.4 Estimation strategy

We define the set of moment conditions that will be considered in the estimation procedure. We also describe the inference steps followed as well as the problems faced in the procedure, and show some preliminary results.

1.4.1 The moment conditions

Consider the competitive equilibrium defined by equations (1.8) to (1.13), and notice that our economy is parameterized by the vector $\omega = \{\beta, \theta, \tau, \delta, \alpha, \rho, \sigma_\epsilon\}$. According to the framework described by (1.1) and (1.2), it is required that we formulate our economy as a set of moment conditions, that is, we need to define our general estimating equations $g(x_i, \omega) = (g_1(x_i, \omega), ..., g_r(x_i, \omega))'$ such that $E\{g_j(x_i, \omega)\} = 0, \forall\ j = 1, ..., r \geq 7$. Note that we require at least seven equations to solve the system for the seven unknows, which are the parameters of our economy.

In fact, our economy will not generate enough moments so we will not have sufficient general estimating equations to retrieve the estimates of our seven parameters.

We can write at least five initial (we call them "original") moment conditions picking equations (1.9), (1.10), (1.12), and (1.13) from the competitive equilibrium describing our economy. These "original" moment conditions are:

$$E\left[\beta\left(\frac{c_{t+1}^{\theta}(1-n_{t+1})^{1-\theta}}{c_t^{\theta}(1-n_t)^{1-\theta}}\right)^{1-\tau}\left(\frac{c_t}{c_{t+1}}\right)\left(1-\delta+\alpha\frac{y_{t+1}}{k_{t+1}}\right)-1\right] = 0 \quad (1.14)$$

$$E\left[\frac{1-\theta}{\theta}\frac{c_t}{1-n_t}-(1-\alpha)\frac{y_t}{n_t}\right] = 0 \quad (1.15)$$

$$E\left[\delta-1+\frac{k_{t+1}}{k_t}-\frac{i_t}{k_t}\right] = 0 \quad (1.16)$$

$$E[z_t-\rho z_{t-1}] = 0 \quad (1.17)$$

$$E[(z_t-\rho z_{t-1})^2-\sigma_\epsilon^2] = 0 \quad (1.18)$$

The moment condition given by (1.14) results from the only expectational equation that arises in our economy, the Euler equation, (1.12). Condition (1.15) is obtained by applying the expectation operator to the intratemporal condition, equation (1.13). In a similar way, condition (1.16) is obtained by applying the expectation operator to the law of motion of capital, equation (1.9). Moment conditions (1.17) and (1.18) are obtained from equation (1.10), by exploiting the assumption $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. Then we can write:

$$E_t[\epsilon_t] = 0 \Leftrightarrow E(z_t - \rho z_{t-1}) = 0$$

and also

$$\begin{aligned} E\left[\left(\epsilon_t^2 - E\left(\epsilon_t\right)^2\right)\right] &= \sigma_\epsilon^2 \Leftrightarrow \\ &\Leftrightarrow E\left(\epsilon_t^2\right) = \sigma_\epsilon^2 \Leftrightarrow E\left(z_t - \rho z_{t-1}\right)^2 = \sigma_\epsilon^2 \Leftrightarrow \\ &\Leftrightarrow E[(z_t - \rho z_{t-1})^2 - \sigma_\epsilon^2] = 0 \end{aligned}$$

Since the technology process is not observable in the real data we can use the production function to write moments (1.17) and (1.18) in terms of observables. From the production function, equation (1.8), we can write the technology level, z_t, as follows:

$$z_t = \ln y_t - \alpha \ln k_t - (1-\alpha) \ln n_t$$

and the moments (1.17) and (1.18) can be redefined as:

$$E\left[\begin{array}{c} (\ln y_t - \rho \ln y_{t-1}) - \alpha(\ln k_t - \rho \ln k_{t-1}) \\ -(1-\alpha)\left(\ln n_t - \rho \ln n_{t-1}\right) \end{array}\right] = 0 \tag{1.19}$$

$$E\left[\left(\begin{array}{c} (\ln y_t - \rho \ln y_{t-1}) - \alpha(\ln k_t - \rho \ln k_{t-1}) \\ -(1-\alpha)\left(\ln n_t - \rho \ln n_{t-1}\right) \end{array}\right)^2 - \sigma_\epsilon^2\right] = 0 \tag{1.20}$$

Hence, we can define a starting $g_{original}(x_i, \omega)$ vector as follows:

$$\begin{aligned} g_{original}\left(x_i, \omega\right) &= \left(g_1\left(x_i, \omega\right), ..., g_5\left(x_i, \omega\right)\right)' = \\ &= \left[\begin{array}{c} \beta\left(\frac{c_{t+1}^{\theta}(1-n_{t+1})^{1-\theta}}{c_t^{\theta}(1-n_t)^{1-\theta}}\right)^{1-\tau}\left(\frac{c_t}{c_{t+1}}\right)\left(1-\delta+\alpha\frac{y_{t+1}}{k_{t+1}}\right) - 1 \\ \frac{1-\theta}{\theta}\frac{c_t}{1-n_t} - (1-\alpha)\frac{y_t}{n_t} \\ \delta - 1 + \frac{k_{t+1}}{k_t} - \frac{i_t}{k_t} \\ \left[\begin{array}{c} (\ln y_t - \rho \ln y_{t-1}) - \alpha(\ln k_t - \rho \ln k_{t-1}) + \\ -(1-\alpha)\left(\ln n_t - \rho \ln n_{t-1}\right) \end{array}\right] \\ \left[\left(\begin{array}{c} (\ln y_t - \rho \ln y_{t-1}) - \alpha(\ln k_t - \rho \ln k_{t-1}) + \\ -(1-\alpha)\left(\ln n_t - \rho \ln n_{t-1}\right) \end{array}\right)^2 - \sigma_\epsilon^2\right] \end{array}\right] \end{aligned}$$

where the random variable $x_i's$ is a multivariate random vector including the economic variables of our economy, as well as their lags and leads. Therefore, the function $g_{original}\left(x_i, \omega\right)$ gathers all the essential information of the economy: the optimal choices of the representative household, the endogenous evolution of the capital stock, the autoregressive technology process, and the statistics of the exogenous technology shock hitting the economy every period.

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In this setting, we have to identify seven parameters. From condition (1.16) it is possible to identify δ. Moments (1.19) and (1.20) allow the identification of the persistence parameter, ρ, and of the standard deviation of the technology shock, σ_ϵ. From conditions (1.14) and (1.15) we need to identify four parameters: $\beta, \theta, \tau, \alpha$. This can be done by constructing a least two additional moment conditions from the original moments (1.14) and (1.15). Hence, in order to have a just identified system of moment conditions, we have to write down two more moment conditions from conditions (1.14) to (1.16), (1.19), and (1.20).

We are forced to build at least two more moment conditions, in order to have a just identified system of moments. These two "artificial" moments should be preferably theory driven and satisfying the statistical property of low correlation. According to the related literature on the definition of moment conditions (check, for instance, Canova (2007)), one can use instruments to define more moment conditions, i.e. one can use lagged values of the variables of interest. Thus, let us define the following moments, based on lagged values of the series of consumption, hours, and of the neutral technology process (written from the production function):

$$\begin{aligned} g^1_{artificial}(x_i,\omega) &= \left(g_6(x_i,\omega), g_7(x_i,\omega)\right)' = \\ &= \begin{bmatrix} g_1(x_i,\omega)\cdot\left(\frac{c_{t-2}}{n_{t-2}}\right) \\ g_4(x_i,\omega)\cdot\left(\ln y_{t-2}-\alpha\ln k_{t-2}-(1-\alpha)\ln n_{t-2}\right)\end{bmatrix}\end{aligned}$$

However, when we compute the matrix of correlations of the set of moment conditions $\left(g_{original}(x_i,\omega), g^1_{artificial}(x_i,\omega)\right)$ to control for the "low correlation" criterium we obtain the following results:

Table 1. Moments correlation matrix

	g_1	g_2	g_3	g_4	g_5	g_6	g_7
g_1	1.000						
g_2	-0.001	1.000					
g_3	-0.006	-0.035	1.000				
g_4	0.032	-0.045	0.025	1.0000			
g_5	-0.055	-0.110	-0.078	-0.033	1.000		
g_6	-0.999	-0.001	-0.005	-0.031	-0.056	1.000	
g_7	-0.031	0.102	-0.419	-0.176	-0.053	-0.029	1.000

We observe that the first and sixth moments are almost perfectly correlated. This is expected since the sixth moment is constructed by multiplying the Euler equation by the lagged ratio of consumption and hours. This may be a problem for our inference process, since the

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sixth moment may not deliver the information needed so that our system of moments is just identified. In this way we will also define another set of "artificial" moment conditions $E\left[g^2_{artificial}\left(x_i,\omega\right)\right]=0$, in order to obtain an overidentified system.

Hence, our second approach to the definition of moment conditions was based solely on the "low correlation" criterion. We have considered different nonlinear combinations between the moments and computed their correlation matrix (including also the correlation with the original moments). Then, we suggest the following "artificial" functions:

$$\begin{aligned} g^2_{artificial}\left(x_i,\omega\right) &= \left(g_6\left(x_i,\omega\right),...,g_{10}\left(x_i,\omega\right)\right)' = \\ &= \begin{bmatrix} \left(g_1\left(x_i,\omega\right)\right)^2\cdot g_2\left(x_i,\omega\right) \\ \left(g_2\left(x_i,\omega\right)\right)^2\cdot g_1\left(x_i,\omega\right) \\ \left(g_1\left(x_i,\omega\right)\right)^2\cdot g_3\left(x_i,\omega\right) \\ \left(g_3\left(x_i,\omega\right)\right)^2\cdot g_1\left(x_i,\omega\right) \\ g_4\left(x_i,\omega\right)\cdot\left(\ln y_{t-1}-\alpha\ln k_{t-1}-\left(1-\alpha\right)\ln n_{t-1}\right) \end{bmatrix} \end{aligned}$$

to which corresponds the following correlation matrix:

Table 2. Moments correlation matrix

	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9
g_1	1.000								
g_2	-0.001	1.000							
g_3	-0.006	-0.035	1.000						
g_4	0.032	-0.045	0.025	1.0000					
g_5	-0.055	-0.110	-0.078	-0.033	1.000				
g_6	-0.006	0.618	-0.007	-0.001	0.032	1.000			
g_7	0.400	-0.041	-0.004	0.017	-0.070	-0.162	1.000		
g_8	-0.067	-0.055	0.534	0.072	-0.115	-0.142	-0.043	1.000	
g_9	0.390	-0.009	0.082	0.028	-0.051	-0.003	0.167	0.097	1.000
g_{10}	-0.031	0.102	-0.419	-0.176	-0.053	0.022	0.003	-0.283	-0.162

We still find some relatively high correlation values between the "artificial" and the "original" moments, and between the "artificial" moments themselves. The "original" moments are not very much correlated. We note that function $g_{10}\left(x_i,\omega\right)$ actually derives from the assumption of no autocorrelation in the technology shock of the economy, that is $E\left(\epsilon_t\epsilon_{t-1}\right)=0,\forall t$. However, we acknowledge that it will also be necessary to explain the economic reasoning behind all the other functions.

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The general estimating equations that will be considered in our estimation problem are given by:

$$g^1(x_i,\omega) = \left(g'_{original}, g^{1\prime}_{artificial}\right)' \quad (1.21)$$
$$g^2(x_i,\omega) = \left(g'_{original}, g^{2\prime}_{artificial}\right)' \quad (1.22)$$

such that

$$E\{g^s(x_i,\omega)\} = 0 \Leftrightarrow E\left(g'_{original}, g'_{artificial}\right)' = 0, s = 1,2 \quad (1.23)$$

Given the two sets of moment conditions defined by (1.23), we can proceed to the estimation process itself.

1.4.2 Inference procedure

Data generating process. We choose to perform our estimation experiment with quarterly "artificial data", generated from our economy. The final objective of this exercise is to obtain estimates on the parameters of our model and compare them with their "true" values. In this way, we can control the accuracy of the estimates obtained. In order to compute the policy functions and simulate the data, we assign values to the set $\{\beta,\theta,\tau,\delta,\alpha,\rho,\sigma\}$, which we will assume as "true" values. Since we wish to assume reasonable "true" values, we follow the suggestions of Aruoba *et al* (2003), since they use a similar economy. The parameter values proposed by Aruoba *et al* (2003) are summarized on Table 1 (more details about these values can be found in section A.3 of the Appendix).

Table 3. "True" parameter values

Parameter	β	θ	τ	δ	α	ρ	σ_ϵ
Value	0.9896	0.375	2	0.0196	0.4	0.95	0.007

Hence, given the above values, we obtain the decision rules of our economy by loglinearizing the competitive equilibrium conditions, equations (1.8) to (1.13), and then applying the Blanchard and Kahn linear algorithm (see sections A.1 and A.2 of the Appendix for details about these two steps).

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The approximated policy functions computed are of the form:

$$\begin{bmatrix} \hat{k}_{t+1} \\ z_t \end{bmatrix} = S(\omega) \begin{bmatrix} \hat{k}_t \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \epsilon_t \tag{1.24}$$

$$\begin{bmatrix} \hat{y}_t \\ \hat{\imath}_t \\ \hat{n}_t \\ \hat{c}_t \end{bmatrix} = C(\omega) \begin{bmatrix} \hat{k}_t \\ \hat{z}_t \end{bmatrix} \tag{1.25}$$

where $S(\omega)$ and $C(\omega)$ are the solution matrices resulting from the Blanchard and Khan algorithm, whose elements depend on the parameters of our economy. Notice that at this point the variables are still measured in terms of log deviations from the steady state, as a result of the loglinearization procedure (we will take their exponential in order to evaluate $g(x_i, \omega)$, where they are considered in levels).

Notice that the decision rules (1.24) and (1.25) are linear on the states of our economy and that all the stochasticity is induced by just one source, the technology shock, ϵ_t. Since the number of sources of stochasticty is less than the number of endogenous variables (output, investment, hours and consumption), the variables y_t, n_t, i_t and c_t are, in fact, linear combinations of the states, k_t and z_t. This is not enough to avoid deterministic relations between the endogenous variables of our economy, i.e. "there are linear combinations of these variables that are predicted without noise", as Ruge-Murcia (2003) explains. Therefore, we face a singularity problem, which is usually tackled in the literature by introducing measurement errors in the policy functions. In this way, we define the following measurement errors $\nu_1 \sim N(0, \sigma_1^2), \nu_2 \sim N(0, \sigma_2^2), \nu_3 \sim N(0, \sigma_3^2)$ and introduce them linearly in the policy functions of output, investment and hours worked:

$$\begin{bmatrix} \hat{y}_t \\ \hat{\imath}_t \\ \hat{n}_t \\ \hat{c}_t \end{bmatrix} = C(\omega) \begin{bmatrix} \hat{k}_t \\ \hat{z}_t \end{bmatrix} + \begin{bmatrix} \nu_{1,t} \\ \nu_{2,t} \\ \nu_{3,t} \\ 0 \end{bmatrix} \tag{1.26}$$

Note that we do not introduce a measurement error in the policy function of the consumption since the feasibility condition of the economy has to be satisfied. Now, we have to assign values to the standard deviations of the measurement errors. Again, we wish to assign reasonable values to the measurement errors standard deviations. Here we follow the suggestions of Fernandez-Villaverde and Rubio-Ramirez (2005) summarized in the following table (the standard deviations are a proportion of the steady state values of output, investment and hours worked, based on the relative importance of these measurement errors in

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the National Income and Product Accounts (NIPA) of the US):

Table 4. Measurement errors standard deviations

Parameter	σ_1	σ_2	σ_3
Value	$y \times 0.0001$	$i \times 0.002$	$n \times 0.0035$

The introduction of measurement errors will not imply any further assumption or restriction on our model. However, we acknowledge that it lacks a theorectical explanation and may capture instead misspecification errors.

The simulated series for our economic variables, y_t, n_t, c_t, k_t and i_t will be generated by equations (1.24) and (1.26), for 500 periods.

Optimization At this stage we should put together the two essential elements of this estimation procedure: the "artificial" data we generated and the moment conditions we defined. Therefore, we should evaluate function $g(x_i, \omega)$, given by (1.21),with the simulated series of our economic variables for starting values of the parameters in ω. Then the optimization algorithm associated with the empirical likelihood problem stated in (1.4) would deliver the estimates of the parameters of our economy. Note that this algorithm requires that we set starting values for our parameters. The inference would proceed then with the repetition of the optimization algorithm in a Monte Carlo way, and with the computation of means and relevant statistics of the estimates.

1.4.3 Findings

We show some very preliminary results associated with the estimator proposed.

We start by naively estimating our model, i.e. we abstract from the fact that we had to add measurement errors to our policy functions, in order to tackle the singularity problem. Moreover, we are aware they maybe our two sets of moment conditions may not be exactly satisfied. This may be due to the fact that we are generating our data from approximated policy functions, but it may also be that the moment conditions, in particular, the "artificial" ones are not well specified. To minimize the misspecification problem, we will estimate our model using the exponential tilting empirical likelihood setup, proposed by Schennach (2007) and that was described above. As we have mentioned the objective function of this estimator is the entropy function. We will then compare the results of the exponential tilting empirical likelihood estimation with the ones obtained with the simple general method of moments estimator (see Appendix, section A.5, for a brief description of the general method of moments setting). We believe that estimating our problem through the standard method

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of moments can give us some insights on what we can expect from the empirical likelihood case. We assign starting values to ω which are close to their "true" values. Then, we replicate the inference procedure 100 times[2]. The results from the general method of moments (GMM) obtained are summarized on Tables 5 and 6 showed in section A.6 of the Appendix. The exponential tilting empirical likelihood (ETEL) estimates are summarized in Tables 7 and 8 showed in section A.6 of the Appendix.

Comparing the "true" values of the parameters and their estimated means, we conclude that, for both methods, we obtain relatively precise estimates of β, δ, α and σ_ϵ. However, we were not able to identify the parameters θ and τ. We observe that the optimization process didn't move far from the starting values of the parameters, even if these where set very close to the "true" values. We believe that this result is probably linked to the "quality" of the moments that allow us to identify θ and τ. Recall that we had to build additional moments because it was not possible to identify these two parameters from the original moment conditions. Also, note that the relatively high correlation between the "artificial" moments and the "original" ones may indicate that the explanatory power of the "artificial" moments is low. This is a worrying result since the estimation of curvature parameters, θ and τ, is extremely important in the context of the estimation of DSGE models. These two parameters characterise the utility function of the representative household and are responsible for most of the nonlinear features of our economy.

Comparing now the ETEL estimates with the GMM ones, we observe that one obtains more precise estimates with the general method of moments: the bias is, in general, smaller for all the parameters, and the same happens for standard deviations[3]. However, we know that the ETEL estimates take into account possible missepecification problems of our model, which is an advantage over the general method of moments.

1.5 Concluding remarks: Limitations

This paper is a preliminary attempt to present empirical likelihood as a valid inference method to estimate DSGE models. We have concluded that, compared with the general method of moments, empirical likelihood (or, more precisely, the exponential tilting empirical likelihood) estimates are not so close to the true values of the parameters and have higher standard deviations. However, both estimates share the difficulty in identifying the curvature

[2]We are aware that 100 replications are too few in the Monte Carlo simulation context, however, we just wanted to have some preliminary estimates.

[3]Since the GMM alternative seemed to be better than the ETEL one, we also show in section A.6 of the Appendix, the estimates resulting from 1000 replications for GMM, considering an overindentified model in Table 9.

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parameters, that is, the coefficient of relative risk aversion and the relative weight of leisure in the utility function.

However, we stress the fact that the empirical likelihood setup, and in particular the exponential tilting empirical likelihood, presents some important advantages, so it is worth to invest in further research on the topic. As we have pointed out he empirical likelihood framework does not require that we compute the policy functions of the economy. Instead, we can work directly with the equilibrium conditions that describe our model. Moreover, we do not need to assume that the stochastic processes of our economy are normally distributed. Finally, we should note that the empirical likelihood estimation setup preserves the nonlinear structure of the equilibrium conditions of the economy.

For further research, we have to address the question of the definition and selection of the moment conditions. This is a common difficulty shared with the general moments estimator: the definition of "artificial" moments is, to a great extent, an arbitrary task. The difficulty of selecting "good" moments is reflected on the results obtained from the estimation with either the general method of moments and the exponential tilting empirical likelihood estimator. In particular, we experience difficulties when it comes to identify the relative weight of consumption and leisure in the household's bundle and of the coefficient of risk aversion in empirical likelihood framework. Hence, we conclude that more work is needed in order to select "good artificial moments". In fact, we have presented a naive estimation of our example economy, since our analysis lacks still a complete time series analysis of the set of moment conditions (for instance, we haven't made any remark about the existence of unit roots or cointegrated relations of our series). So far we have studied the moments in terms of their correlation, in order to define the additional moments required for identifying all the parameters of our economy. Another problem that might be related with the difficulty of obtaining the plain empirical likelihood estimates of the parameters of interest is the data generated from a loglinearized economy, i.e. from approximated policy functions (and we share this problem with the maximum likelihood estimation). The fact that the generated series are not quite the ones that solve the original nonlinear equilibrium system might also create difficulties to the empirical likelihood estimators. Moreover, it may be the case the that the artificial moment conditions are not well specified. In order to tackle these problems, we follow Schennach (2007) and use as an objective function of the empirical likelihood setup the entropy function, i.e. we use the exponential tilting empirical likelihood estimator. This allowed us to obtain the preliminary estimates we have presented above.

To summarize, the main limitations of this analysis are (i) the issue the definition of "artificial" moment condition for identification purposes; (ii) the data generation process

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based upon approximated policy functions.

Finally, together with the two issues that should be addressed in further research, it would be interesting to find estimates using real economic data.

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1.6 Appendix

A.1 Loglinearized competitive equilibrium and steady state values. In this section we present the loglinearized competitive equilibrium of the economy, obtained from equations (1.8) to (1.13), and the nonstochastic steady state values of our variables ($\epsilon_t = 0$).

The loglinearization process consists of taking the logs of each equation, and then writing a first order Taylor approximation around the steady state of each variable for each equation of the competitive equilibrium.

$$\frac{1}{\beta}\eta E_t\hat{c}_{t+1} + \frac{1}{\beta}\varphi E_t\hat{n}_{t+1} = \frac{1}{\beta}\eta\hat{c}_t + \frac{1}{\beta}\varphi\hat{n}_t + \gamma E_t\hat{y}_{t+1} - \gamma\hat{k}_{t+1} \quad (1.27)$$

$$\hat{y}_t = \hat{c}_t + \phi\hat{n}_t \quad (1.28)$$

$$\hat{y}_t = \alpha\hat{k}_t + (1-\alpha)\,\hat{n}_t + z_t \quad (1.29)$$

$$\hat{k}_{t+1} = (1-\delta)\,\hat{k}_t + \delta\hat{\imath}_t \quad (1.30)$$

$$z_t = \rho z_{t-1} + \epsilon_t \quad (1.31)$$

$$\gamma\hat{y}_t = (\gamma - \alpha\delta)\,\hat{c}_t + \alpha\delta\hat{\imath}_t \quad (1.32)$$

where $\gamma = \frac{1}{\beta} - 1 + \delta$, $\eta = 1 - (1-\tau)\,\theta$, $\varphi = \frac{\theta\gamma(1-\tau)(1-\alpha)}{\gamma-\alpha\delta}$ and $\phi = 1 + \frac{\varphi}{(1-\theta)(1-\tau)}$. The circumflex represents percentage deviations from the steady state value of each variable. Equations (1.27), (1.28), (1.29), (1.30), and (1.32) are the loglinearized version of equations (1.12), (1.13), (1.8), (1.9) and (1.11), respectively. Note that equation (1.31) is exactly the same as equation (1.10). In the loglinerization process we made also use of the steady state values of the model variables: $z = 0$, $k = \frac{\alpha}{\gamma}y$, $i = \frac{\alpha\delta}{\gamma}y$, $c = \left(1 - \frac{\alpha\delta}{\gamma}\right)y$, $n = \left[\frac{1-\theta}{\theta(1-\alpha)}\left(1 - \frac{\alpha\delta}{\gamma}\right) + 1\right]^{-1}$, $y = \left(\frac{\alpha}{\gamma}\right)^{\frac{\alpha}{1-\alpha}}\left[\frac{1-\theta}{\theta(1-\alpha)}\left(1 - \frac{\alpha\delta}{\gamma}\right) + 1\right]^{-1}$.

A.2 Application of Blanchard and Kahn algorithm. Given the loglinearized competitive equilibrium defined by equations (1.27) to (1.32), we can find the policy functions of this economy by applying a linear algorithm, like the Blanchard and Kahn one (actually, it is indifferent which linear algorithm we use since all deliver the same solution). In order to apply the Blanchard and Kahn method let us define the following vectors: $u_t^0 = \left[\begin{array}{cc} \hat{k}_t & \hat{c}_t \end{array}\right]'$ and $v_t^0 = \left[\begin{array}{ccc} \hat{y}_t & \hat{\imath}_t & \hat{n}_t \end{array}\right]'$. The vector u_t^0 contains the model's dynamic predetermined and nonpredetermined endogenous variables and the vector v_t^0 contains the model's static nonpredetermined variables.

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We can write equations (1.27) to (1.32) into the following matrix form:

$$
\begin{aligned}
DE_t u^0_{t+1} + FE_t v^0_{t+1} &= Gu^0_t + Hv^0_t && (1.33)\\
Iv^0_t &= Ju^0_t + Kz_t && (1.34)\\
z_{t+1} &= \rho z_t + \epsilon_{t+1} && (1.35)
\end{aligned}
$$

where

$$
D = \begin{bmatrix} 1 & 0 \\ \theta & \frac{\eta}{\beta} \end{bmatrix}; F = \begin{bmatrix} 0 & 0 & 0 \\ -\gamma & 0 & \frac{\varphi}{\beta} \end{bmatrix}; G = \begin{bmatrix} 1-\delta & 0 \\ 0 & \frac{\eta}{\beta} \end{bmatrix}; H = \begin{bmatrix} 0 & \delta & 0 \\ 0 & 0 & \frac{\varphi}{\beta} \end{bmatrix}
$$

and

$$
I = \begin{bmatrix} 1 & 0 & \alpha-1 \\ \gamma & -\delta\alpha & 0 \\ 1 & 0 & -\phi \end{bmatrix}; J = \begin{bmatrix} \alpha & 0 \\ 0 & \gamma-\delta\alpha \\ 0 & 1 \end{bmatrix}; K = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
$$

We focus on the autoregressive process for the technology shock. Equation (1.31) can be solved "forward", in the following way. Take expectations on both sides of (1.31) to have:

$$z_t = \frac{1}{\rho}E_t z_{t+1} - \frac{1}{\rho}E_t \epsilon_{t+1} \tag{1.36}$$

Then we iterate equation (1.36) one period into the future:

$$z_{t+1} = \frac{1}{\rho}E_{t+1} z_{t+2} - \frac{1}{\rho}E_{t+1} \epsilon_{t+2} \tag{1.37}$$

We substitute (1.37) into (1.36). We also use the law of iterated expectations, and the fact that $\epsilon_t \sim N(0, \sigma^2)$ to write z_t as follows:

$$z_t = \frac{1}{\rho^2}E_t z_{t+2} \tag{1.38}$$

If we keep iterating and repeatedly substituting on (1.38) we end up with:

$$E_t z_{t+j} = \rho^j z_t \tag{1.39}$$

Then we can substitute the autoregressive process of the technology shock in (1.35) by (1.39).

Assuming that $\exists I^{-1}$, we obtain from (1.34)

$$v^0_t = I^{-1}Ju^0_t + I^{-1}Kz_t$$

and recalling that $E_t z_{t+j} = \rho^j z_t$, we can write (1.33) as follows:

$$\begin{aligned} DE_t u_{t+1}^0 + FE_t \left(I^{-1} J u_{t+1}^0 + I^{-1} K z_{t+1}\right) &= G u_t^0 + H \left(I^{-1} J u_t^0 + I^{-1} K \hat{z}_t\right) \\ \left(D + FI^{-1} J\right) E_t u_{t+1}^0 &= \left(G + HI^{-1} J\right) u_t^0 + \left(HI^{-1} K - FI^{-1} K \rho\right) z_t \end{aligned}$$

Assuming that $\exists \left(D + FI^{-1} J\right)^{-1}$ and defining:

$$\begin{aligned} A &= \left(D + FI^{-1} J\right)^{-1} \left(G + HI^{-1} J\right) \\ B &= \left(D + FI^{-1} J\right)^{-1} \left(HI^{-1} K - FI^{-1} K \rho\right) \end{aligned}$$

we obtain the required Blanchard and Kahn setup:

$$\begin{aligned} E_t u_{t+1}^0 &= A u_t^0 + B \hat{z}_t \Leftrightarrow \\ &\Leftrightarrow \begin{bmatrix} \hat{k}_{t+1} \\ E_t \hat{c}_{t+1} \end{bmatrix} = A \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} + B z_t \end{aligned} \quad (1.40)$$

The solution of (1.40) is derived through the Jordan canonical form of matrix $A = M^{-1} \Lambda M$, where Λ is a diagonal matrix whose non zero elements are the eigenvalues of A, λ_1 and λ_2, ordered by increasing absolute value, and M^{-1} is a (2×2) matrix whose colunms contain the associated eigenvectors of A. That is:

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}; M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

Also, we let

$$B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Hence, we can rewrite the Blanchard and Kahn system of difference equations as:

$$\begin{aligned} \begin{bmatrix} \hat{k}_{t+1} \\ E_t \hat{c}_{t+1} \end{bmatrix} &= M^{-1} \Lambda M \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} + B z_t \\ M \begin{bmatrix} \hat{k}_{t+1} \\ E_t \hat{c}_{t+1} \end{bmatrix} &= \Lambda M \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} + M B z_t \\ \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \hat{k}_{t+1} \\ E_t \hat{c}_{t+1} \end{bmatrix} &= \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{c}_t \end{bmatrix} \\ &+ \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} z_t \\ &\Leftrightarrow \begin{cases} E_t u_{1t+1}^1 = \lambda_1 u_{1t}^1 + q_1 z_t \\ E_t u_{2t+1}^1 = \lambda_2 u_{2t}^1 + q_2 z_t \end{cases} \end{aligned} \quad (1.41)$$

with

$$\begin{aligned}
u_{1t}^1 &= m_{11}\hat{k}_t + m_{12}\hat{c}_t \\
q_1 &= m_{11}b_1 + m_{12}b_2 \\
u_{2t}^1 &= m_{21}\hat{k}_t + m_{22}\hat{c}_t \\
q_2 &= m_{21}b_1 + m_{22}b_2
\end{aligned}$$

Assuming that λ_2 lies outside the unit circle, $E_t u_{2t+1}^1 = \lambda_2 u_{2t}^1 + q_2\hat{z}_t$ can be solved "forward". Solving the equation for u_{2t}^1 yields:

$$u_{2t}^1 = \frac{1}{\lambda_2}E_t u_{2t+1}^1 - \frac{q_2}{\lambda_2}z_t \tag{1.42}$$

Iterating on (1.42) one period forward gives:

$$u_{2t+1}^1 = \frac{1}{\lambda_2}E_{t+1}u_{2t+2}^1 - \frac{q_2}{\lambda_2}z_{t+1} \tag{1.43}$$

We then substitute (1.43) into (1.42) and obtain:

$$u_{2t}^1 = \frac{1}{\lambda_2^2}E_t u_{2t+2}^1 - \frac{q_2}{\lambda_2}(\frac{1}{\lambda_2}E_t z_{t+1} + z_t)$$

Successive forward substitution yields the following general expression:

$$\begin{aligned}
u_{2t}^1 &= -\frac{q_2}{\lambda_2}\sum_{j=0}^{\infty}\left(\frac{1}{\lambda_2}\right)^j E_{tt+j} \Leftrightarrow \\
&\Leftrightarrow \; u_{2t}^1 = \frac{q_2}{\rho - \lambda_2}z_t
\end{aligned} \tag{1.44}$$

At this point we are very close to the final solution, since:

$$\begin{aligned}
u_{2t}^1 &= m_{21}\hat{k}_t + m_{22}\hat{c}_t \Leftrightarrow \\
&\Leftrightarrow \; \hat{c}_t = -\frac{m_{21}}{m_{22}}\hat{k}_t + \frac{1}{m_{22}}\frac{q_2}{\rho - \lambda_2}z_t
\end{aligned}$$

or

$$\hat{c}_t = \phi_{ck}\hat{k}_t + \phi_{cz}z_t \tag{1.45}$$

where $\phi_{ck} = -\frac{m_{21}}{m_{22}}$ and $\phi_{cz} = \frac{1}{m_{22}}\frac{q_2}{\rho - \lambda_2}$.

Notice that ϕ_{ck} and ϕ_{cz} are nonlinear combinations of the eigenvectors and eigenvalues of A and also of the model parameters.

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It is easy to obtain the solution for $\hat{k}_{t+1}$ given that:

$$\begin{aligned} u_{1t}^1 &= m_{11}\hat{k}_t + m_{12}\hat{c}_t \Leftrightarrow \\ &\Leftrightarrow u_{1t}^1 = m_{11}\hat{k}_t + m_{12}\left(\phi_{ck}\hat{k}_t + \phi_{cz}\hat{z}_t\right) \end{aligned} \tag{1.46}$$

Thus:

$$\begin{aligned} E_t u_{1t+1}^1 &= \lambda_1 u_{1t}^1 + q_1 \hat{z}_t \\ E_t\left[m_{11}\hat{k}_{t+1} + m_{12}\left(\phi_{ck}\hat{k}_{t+1} + \phi_{cz}\hat{z}_{t+1}\right)\right] &= \lambda_1\left[m_{11}\hat{k}_t + m_{12}\left(\phi_{ck}\hat{k}_t + \phi_{cz}\hat{z}_t\right)\right] + q_1\hat{z}_t \\ \hat{k}_{t+1} &= \lambda_1\hat{k}_t + \left(m_{11} + m_{12}\phi_{ck}\right)^{-1} \cdot \\ &\quad \cdot \left(q_1 + \lambda_1 m_{12}\phi_{ck} - m_{12}\phi_{cz}\rho\right)\hat{z}_t \end{aligned}$$

or

$$\hat{k}_{t+1} = \phi_{kk}\hat{k}_t + \phi_{kz}\hat{z}_t \tag{1.47}$$

where $\phi_{kk} = \lambda_1$ and $\phi_{kz} = \left(m_{11} + m_{12}\phi_{ck}\right)^{-1}\left(q_1 + \lambda_1 m_{12}\phi_{cz} - m_{12}\phi_{cz}\rho\right).$

Therefore, the linear decision rules derived for this economy are given by equations (1.45) and (1.47).

Furthermore, it is also possible to write a system where the endogenous variables $\hat{y}_t$, $\hat{\imath}_t$ and $\hat{n}_t$ are predicted by the predetermined level of capital and the technology shock. That is, we can also obtain the decision rules for $\{\hat{y}_t, \hat{\imath}_t, \hat{n}_t\}$. Recall that the vector of endogenous static non-predetermined variables could be written as:

$$\begin{aligned} v_t^0 &= I^{-1}Ju_t^0 + I^{-1}K\hat{z}_t = \\ &= I^{-1}J\begin{bmatrix}\hat{k}_t \\ \hat{c}_t\end{bmatrix} + I^{-1}K\hat{z}_t = \\ &= I^{-1}J\begin{bmatrix}\hat{k}_t \\ \phi_{ck}\hat{k}_t + \phi_{cz}\hat{z}_t\end{bmatrix} + I^{-1}K\hat{z}_t = \\ &= I^{-1}J\begin{bmatrix}1 \\ \phi_{ck}\end{bmatrix}\hat{k}_t + \left(I^{-1}K + I^{-1}J\begin{bmatrix}0 \\ \phi_{cz}\end{bmatrix}\right)\hat{z}_t \end{aligned}$$

or

$$v_t^0 = \Phi_1\hat{k}_t + \Phi_2\hat{z}_t \tag{1.48}$$

with

$$\Phi_1 = I^{-1}J\begin{bmatrix}1 \\ \phi_{ck}\end{bmatrix}; \Phi_2 = \left(I^{-1}K + I^{-1}J\begin{bmatrix}0 \\ \phi_{cz}\end{bmatrix}\right)$$

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Finally, we can write the decision rules of all the non-predetermined variables as a function of the predetermined and the exogenous variables as

$$\begin{aligned}
\begin{bmatrix} v_t^0 \\ \hat{c}_t \end{bmatrix} &= \begin{bmatrix} \Phi_1 & \Phi_2 \\ \phi_{ck} & \phi_{cz} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{z}_t \end{bmatrix} \Leftrightarrow \\
\Leftrightarrow \begin{bmatrix} \hat{y}_t \\ \hat{i}_t \\ \hat{n}_t \\ \hat{c}_t \end{bmatrix} &= \begin{bmatrix} \phi_{yc} & \phi_{yz} \\ \phi_{ik} & \phi_{iz} \\ \phi_{nk} & \phi_{nz} \\ \phi_{ck} & \phi_{cz} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{z}_t \end{bmatrix} \Leftrightarrow \\
\Leftrightarrow \begin{bmatrix} \hat{y}_t \\ \hat{i}_t \\ \hat{n}_t \\ \hat{c}_t \end{bmatrix} &= C \begin{bmatrix} \hat{k}_t \\ \hat{z}_t \end{bmatrix} \qquad (1.49)
\end{aligned}$$

and for the endogenous states:

$$\begin{aligned}
\begin{bmatrix} \hat{k}_{t+1} \\ z_t \end{bmatrix} &= \begin{bmatrix} \phi_{kk} & \phi_{kz} \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{k}_{t+1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \epsilon_t \\
\Leftrightarrow \begin{bmatrix} \hat{k}_{t+1} \\ z_t \end{bmatrix} &= S \begin{bmatrix} \hat{k}_{t+1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \epsilon_t \qquad (1.50)
\end{aligned}$$

where

$$C = \begin{bmatrix} \phi_{yc} & \phi_{yz} \\ \phi_{ik} & \phi_{iz} \\ \phi_{nk} & \phi_{nz} \\ \phi_{ck} & \phi_{cz} \end{bmatrix}; S = \begin{bmatrix} \phi_{kk} & \phi_{kz} \\ 0 & \rho \end{bmatrix}$$

Matrices C and S are solution matrices. From them we obtain the approximated policy functions of the economy and we generate simulated data for our variables. We then use the simulated data to estimate our model.

A.3 Some notes on the calibration proposed by Aruoba *et al* (2003). The calibration chosen by Aruoba et al (2003) is justified by the authors in the following way. The subjective discount rate of the representative household, β, matches an annual interest rate of 4%. The relative weight of consumption and leisure in the household bundle, θ, is set so that labour supply amounts to 31% of the available time in the steady state. The value of the coefficient of risk aversion, τ, is a common choice in the literature. The depreciation rate, δ, sets the investment/capital ratio. The elasticity of output to capital, α, matches the labour share

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of US national income. The values of the persistence parameter, ρ, and of the standard deviation of the technology shock, σ_ϵ, were chosen so that they match the properties of the Solow residual for the US economy.

A.4 Solutions of the empirical likelihood problem. In this section we explain analytically how to solve the empirical likelihood problem stated in (1.4).

First we apply logs to the empirical likelihood function given by (1.3) and rewrite the optimization problem as follows:

$$\max_{p_i} \left\{ \sum_{i=1}^{m} \log p_i | \sum_{i=1}^{m} p_i = 1; \sum_{i=1}^{m} [p_i \cdot g(x_i, \omega)] = 0 \right\} \tag{1.51}$$

The Lagrangean function associated with the optimization problem in (1.51) can be written as follows:

$$\mathcal{L} = \sum_{i=1}^{m} \log p_i - \xi \left(\sum_{i=1}^{m} p_i - 1 \right) - \mu' \left(\sum_{i=1}^{m} [p_i \cdot g(x_i, \omega)] \right) \tag{1.52}$$

where ξ and $\mu = (\mu_1, ..., \mu_r)'$ are the lagrangean multipliers associated with the constraints of the problem.

The first order condition derived from (1.52) with respect to p_i is:

$$\frac{1}{p_i} - \xi - \mu' g(x_i, \omega) = 0 \forall i \tag{1.53}$$

Summing condition (1.53) over all $i's$ we obtain that $\xi = m$. We plug it back on (1.53) and find the expression for the optimal weight:

$$p_i^* = \frac{1}{m} \frac{1}{1 + \tilde{\mu}' g(x_i, \omega)} \tag{1.54}$$

where $\tilde{\mu} = -\frac{1}{m}\mu$.

The empirical likelihood evaluated at the optimal weight is then:

$$L_{EL}(\omega, p) = \prod_{i=1}^{m} \frac{1}{m} \frac{1}{1 + \tilde{\mu}' g(x_i, \omega)}$$

and its log counterpart:

$$\begin{aligned} \log L_{EL}(\omega, p) &= \sum_{i=1}^{m} \log \left(\frac{1}{m} \frac{1}{1 + \tilde{\mu}' g(x_i, \omega)} \right) = \\ &= -\sum_{i=1}^{m} \log m - \sum_{i=1}^{m} (1 + \tilde{\mu}' g(x_i, \omega)) \end{aligned} \tag{1.55}$$

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The next step consists on maximizing (1.55) with respect to ω and also to $\tilde{\mu}$. This is equivalent to:

$$\min_{\omega,\tilde{\mu}} \left\{ \sum_{i=1}^{m} \left(1 + \tilde{\mu}' g\left(x_i, \omega\right)\right) \right\} \tag{1.56}$$

The first order conditions of the problem in (1.56) are:

$$\omega \quad : \quad \sum_{i=1}^{m} \frac{1}{1 + \tilde{\mu}' g\left(x_i, \omega\right)} \left(\frac{\partial g\left(x_i, \omega\right)}{\partial \omega} \right)' \tilde{\mu} = 0 \tag{1.57}$$

$$\tilde{\mu}' \quad : \quad \sum_{i=1}^{m} \frac{1}{1 + \tilde{\mu}' g\left(x_i, \omega\right)} g\left(x_i, \omega\right) = 0 \tag{1.58}$$

Equations (1.57) and (1.58) form a system of two equations of two unknowns, and by solving this system we can obtain the expressions for the optimal ω and $\tilde{\mu}$.

A.5 General method of moments formulation of the problem. We consider the same setup as the one assumed in the empirical likelihood context, that is, we consider the framework given by (1.1) and (1.2). However, we solve now the following optimization problem:

$$\min_{\omega} \frac{1}{m} \left[\sum_{i=1}^{m} g\left(x_i, \omega\right) \right]' W \left[\sum_{i=1}^{m} g\left(x_i, \omega\right) \right] \tag{1.59}$$

where W is a positive definite weighting matrix.

A.6 Estimation Results

Table 5. GMM estimates for the first set of moments $g^1\left(x_i, \omega\right)$

(500 periods; 100 Monte Carlo replications)

Parameter	True	Start	Mean	Bias	StDeviation
β	0.9896	0.9600	0.9892	0.000418	1.882×10^{-3}
θ	0.3750	0.4000	0.40198	-0.044984	1.654×10^{-4}
τ	2.0000	2.2000	2.20005	-0.020005	8.792×10^{-6}
δ	0.0196	0.0200	0.0197	-0.00005	1.245×10^{-5}
α	0.4000	0.5000	0.4061	-0.00614	0.0250
ρ	0.9500	0.9000	0.9912	-0.04118	2.746×10^{-3}
σ_ϵ	0.0070	0.0100	-0.0016	0.00859	6.993×10^{-3}

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Table 6. GMM estimates for the second set of moments $g^2(x_i, \omega)$ (500 periods; 100 Monte Carlo replications)

Parameter	True	Start	Mean	Bias	StDeviation
β	0.9896	0.9600	0.9890	0.00057	1.803×10^{-3}
θ	0.3750	0.4000	0.4024	-0.04536	1.507×10^{-3}
τ	2.0000	2.2000	2.20008	-0.20008	1.289×10^{-5}
δ	0.0196	0.0200	0.01965	-0.00005	1.243×10^{-5}
α	0.4000	0.5000	0.4082	-0.00082	0.02419
ρ	0.9500	0.9000	0.9914	-0.04143	3.094×10^{-3}
σ_ϵ	0.0070	0.0100	0.00643	0.00057	3.143×10^{-3}

Table 7. ETEL estimates for the first set of moments $g^1(x_i, \omega)$ (500 periods; 100 Monte Carlo replications)

Parameter	True	Start	Mean	Bias	StDeviation
β	0.9896	0.9600	0.9761	0.0135	0.0120
θ	0.3750	0.4000	0.4104	-0.0534	0.0853
τ	2.0000	2.2000	2.1816	-0.1816	0.8513
δ	0.0196	0.0200	0.0196	0.00003	0.00054
α	0.4000	0.5000	0.4712	-0.0712	0.0352
ρ	0.9500	0.9000	0.8856	0.0644	0.2771
σ_ϵ	0.0070	0.0100	0.020	-0.0139	0.0463

Table 8. ETEL estimates for the second set of moments $g^2(x_i, \omega)$ (500 periods; 100 Monte Carlo replications)

Parameter	True	Start	Mean	Bias	StDeviation
β	0.9896	0.9600	0.9798	0.0098	0.0117
θ	0.3750	0.4000	0.4459	-0.0889	0.1084
τ	2.0000	2.2000	2.2001	-0.200	0.0003
δ	0.0196	0.0200	0.1956	0.00004	0.0003
α	0.4000	0.5000	0.4588	-0.0588	0.0458
ρ	0.9500	0.9000	0.9166	0.0334	0.0201
σ_ϵ	0.0070	0.0100	0.0245	-0.0175	0.0229

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Table 9. GMM estimates for the second set of moments $g^2(x_i, \omega)$ (500 periods; 1000 Monte Carlo replications)

Parameter	True	Start	Mean	Bias	StDeviation
β	0.9896	0.9600	0.98890	0.00070	1.077×10^{-3}
θ	0.3750	0.4000	0.40235	-0.04535	1.626×10^{-3}
τ	2.0000	2.2000	2.20008	-0.20008	1.093×10^{-5}
δ	0.0196	0.0200	0.01965	-0.00005	1.220×10^{-5}
α	0.4000	0.5000	0.40994	-0.00994	1.460×10^{-2}
ρ	0.9500	0.9000	0.99129	-0.04129	3.114×10^{-3}
σ_ϵ	0.0070	0.0100	0.00670	0.00030	2.527×10^{-3}

CHAPTER 2

EXPLORING FURTHER THE EMPIRICAL LIKELIHOOD FAMILY OF ESTIMATORS IN THE DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM FRAMEWORK

2.1 Introduction

In this paper, we try to investigate further the estimation of dynamic stochastic general equilibrium (DSGE) models with the empirical likelihood family of estimators. Basically we try to address the main problems and limitations of the estimation of DSGE models with an empirical likelihood estimator. More specifically, we explore further the problems of defining more moment conditions and of generating simulated series that can satisfy exactly the moment conditions. Moreover, we will focus on an issue that assumes particular importance when working with DSGE models, i.e. the hypothesis of independence of our economic variables. In order to be able to estimate our model, we have assumed that all our variables - production, consumption, investment, capital and hours - were all identically, independent random variables. This may not be so in the context of a DSGE model, since these variables are linked between each other through relations such as the optimality conditions of the agents, the production function, the budget constraint, and laws of motion. We have tried to tackle this dependence problem in practical terms, by adding measurement errors to the approximated policy functions, and by considering uncorrelated moment conditions. However, this problem has also to be dealt with from a theorectical point of view. In this way, we present a possible method of working with dependent variables, in the empirical likelihood framework.

As we have explained in the previous chapter, and according to Schennach (2007), a statistical model consisting in moment conditions of the form $E\left[g\left(x,\theta\right)\right]=0$, where $g\left(x,\theta\right)$ is a nonlinear function of the variables in vector x and of the vector of parameters θ, do not require any distributional assumption on the data generating process. In this way, the probability of obtaining inconsistent estimates as a consequence of a wrong distribution choice, is dramatically reduced. The empirical likelihood estimator fall in this category of estimators and, at the same time, exhibits the same properties enjoyed by maximum

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likelihood parametric estimators, which is of great advantage.

Although at the theoretical level the estimation of DSGE models through empirical likelihood seem to be a quite attractive alternative, we have identified in the previous chapter several problems when we took the estimation into practice. More specifically, these problems came out when we tried to apply the "R empirical likelihood routine" to our set of moment conditions, and to the estimation strategy chosen. These problems may be divided in two types: (i) the choice of the "artificial" moment conditions, and (ii) the use of approximated policy functions to generate artificial observations of the economic variables series.

In what concerns the choice of the "artificial" moment conditions, recall that the criterium defined to write "new" moment conditions was the existence of low correlations among the "artificial" moments themselves, and between these and the "original" ones. As we will show later on, this criterium is not sufficient to define more moment conditions related with our economy. In fact, it does not guarantee that the moments are satisfied, that is that $E_F\left\{g\left(x_i,\omega\right)\right\} = 0$. In what concerns the use of approximated policy functions, we note that the use of "approximated" policy functions to generate simulated data implies that the moment conditions defined are also met approximately. This fact may induce nonnegligible errors on the estimation process proposed. In order to tackle this last issue, we propose a new illustrational example: we will assume an economy for which it is possible to obtain closed form solutions (exact policy functions) and that is not so richly parameterized (in this setup, we will see that the equilibrium conditions of our DSGE model are enough to have a just identified estimation problem.

The remainder of the paper is organized as follows. Section 2 adds some more details to the empirical likelihood estimation methods. In section 3 we present a possible way of working with dependent random variables in the empirical likelihood context. In section 4, we show that the the artificial moment conditions defined in chapter 1 may not be exactly met, and justify in this way the need to use the exponential tilting empirical likelihood estimator, instead of the empirical likelihood one. In section 5 we describe the "closed form" economy, define the moment conditions and explain the estimation procedure to follow. Section 6 presents a research agenda on future investigation in this field.

2.2 The empirical likelihood family of estimators

In this section we present a short review of literature on the empirical likelihood family of estimators, in order to understand this estimator in a broader sense.

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2.2.1 Origins

In his seminal paper on the empirical likelihood topic, Owen (1988) shows that the sample-based empirical distribution function is the maximum likelihood estimate of the distribution from which the sample was taken. This author uses the empirical distribution function to define a likelihood ratio test for distributions and also to construct confidence intervals for the sample mean for a class of M-estimates, and for differentiable statistical functionals.

According to Owen (1988), suppose that $X_1, ..., X_n$ are independent observations from a distribution function F_0. Then, the empirical distribution function $F_n(x) = \frac{1}{n}\sum_{i=1}^{n} 1_{x_i \leq x}$ is the nonparametric maximum likelihood estimate of F_0, because it maximizes the objective function $L(X) = \prod_{i=1}^{n} \{F(X)_i - F(X_i-)\}$, over all distributions F.

In this way, it is possible to define the empirical likelihood ratio as follows:

$$R(F) = \frac{L(F)}{L(F_n)}$$

and Owen (1988) shows that, under some weak assumptions, sets like

$$\{T(F) = R(F) \geq c\}$$

may be used as a confidence region for $T(F_0)$, with $T(.)$ being a statistical functional.

Moreover, Owen (1988) proves that empirical likelihood ratio statistics for various parameters $\theta(F)$ of an unknown distribution F have a limiting chi-square distribution. This is an important result because it may be used to compute tests and construct confidence intervals in a similar way to that of the parametric likelihoods.

Qin and Lawless (1994) build on Owen (1988) and establish a link between general estimating functions (or equations) and the empirical likelihood. More specifically, these authors developed methods of extracting more information about parameters, by assuming that information about F and θ is available in the form of unbiased estimation functions. Moreover, these authors show that the empirical likelihood estimator has analogous properties to those of parametric likelihood, namely in terms of efficiency.

According to Qin and Lawless (1994), the empirical likelihood estimator is defined as follows. Let Let $x_1, ..., x_n$ be identically independently distributed random variables drawn from an unknown distribution F, and $\theta(F)$ a $q-$dimensional vector of parameters, associated with the distribution F. Suppose that there is available information about θ and F in the form of r independent functions $g_j(x_i, \theta), j = 1, ..., r$, with $r \geq q$, such that $E\{g_i(x_i, \theta)\} = 0, \forall j$,

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under F. Then the empirical likelihood function is defined as follows:

$$L_{EL}(\theta, p) = \left\{ \prod_{i=1}^{m} p_i | p_i > 0, \sum_{i=1}^{m} p_i = 1, \sum_{i=1}^{m} \left[p_i \cdot g(x_i, \theta) \right] = 0 \right\} \tag{2.1}$$

where p is the vector of the weights/probabilities assigned to each sample observation.

Hence, the maximum likelihood estimator[1] of $\{\theta, p\}$ is

$$\left\{ \hat{\theta}, \hat{p} \right\} = \arg\max_{\theta, p} L_{EL}(\theta, p) \tag{2.2}$$

Qin and Lawless (1994) have also derived the asymptotic distribution of the empirical likelihood estimator. Suppose $D = E\left[\nabla_\theta g(x, \theta_0)\right]$ and $S = \left[g(x, \theta_0)\, g(x, \theta_0)'\right]$. Then:

$$\sqrt{n}\left(\hat{\theta}_{EL} - \theta_0\right) \xrightarrow{d} N\left(0, (D'SD)^{-1}\right)$$

where the asymptotic variance given by matrix $(D'SD)^{-1}$ achieves the semiparametric efficiency bound of Chamberlain (1987).

Owen (2001) argues that the empirical likelihood estimator, as a nonparametric inference method, allows using likelihood-based methods without having to assume that the data come from a particular family of distributions. In fact, as this author observes, there is no reason to assume that a newly set of data belongs to any of the known parametric families of distributions. Considering such a strong assumption may introduce misspecification problems in our econometric analysis, which leads to inefficient likelihood estimates and confidence intervals and tests may not be valid anymore. In this way, Owen (2001) points out that the main advantage of the empirical likelihood estimator consists in the fact that it combines the reliability o the nonparametric methods with the "good" properties of the likelihood based inference methods.

Intuitively, in the nonparametric setting we are interested in estimating the distribution F through $T(F)$, and we prefer to obtain a correct approximation of F than to assume the wrong distribution with certainty.

2.2.2 Recent developments

Kitamura (2006) makes a review of the literature on the most recent developments in the empirical likelihood field of research. According to this author, the empirical likelihood inference method may be interpreted in two ways.

[1]For more details on the empirical likelihood problem formulation and computation of the respective solutions please check the appendix of the first chapter.

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First, the empirical likelihood estimator can be considered as a nonparametric maximum likelihood estimatior (NPMLE) procedure, and, as a likelihood based inference method it is asymptotically efficient, under mild regularity conditions. The approach of NPMLE follows the line of research of Owen (1988 and 2001) and Qin and Lawless (1994) discussed above.

Second, the empirical likelihood estimator can also be regarded as special case of the generalized contrast estimator (GMC), which is a robust estimator in terms of distribution assumptions, though losing some efficiency. Suppose a function D that measures the divergence between two probability measures P and Q that is defined as:

$$D(P,Q) = \int \phi\left(\frac{dP}{dQ}\right) dQ$$

satisfying the following regularity conditions: (i) ϕ is convex; (ii) $D(.,P)$ is minimized at P. Assume also that M is the set of all probability measures in $\mathbb{R}^p$ and

$$\tilde{P}(\theta) = \left\{P \in M : \int g(x,\theta)\, dP = 0\right\}$$

which is defined as the set of all probability measures that are compatible with the moment condition $\int g(x,\theta)\, dP = 0$. Suppose, finally, that μ is the true measure and that it is included in $\tilde{P}$. Then the GMC optimization problem can be written as follows, at the population level:

$$\inf_{\theta\in\Theta} \rho(\theta,\mu), \text{ where } \rho(\theta,\mu) = \inf_{P\in\tilde{P}(\theta)} D(P,\mu) \tag{2.3}$$

If there is no misspecification problem with the model, then $\theta = \theta_0$ achieves the minimum of the contrast function $\rho(.,.)$. We may write (2.3) as:

$$v(\theta) = \inf_p \int \phi(p)\, d\mu \text{ s.t. } \int g(x,\theta)\, p d\mu = 0, \int p d\mu = 1 \tag{2.4}$$

where $p = \frac{dP}{d\mu}$, $D(P,\mu) = \int \phi(p)\, d\mu$, and $v(\theta)$ is a value function for a particular choice of θ. The problem represented by (2.4) already resembles the one of empirical likelihood in (2.1). And the sample counterpart of (2.4) can be defined as:

$$\min \frac{1}{n}\sum_{i=1}^{n} \phi(np_i) \text{ s.t. } \sum_{i=1}^{n} p_i g(x_i,\theta) = 0, \sum_{i=0}^{n} p_i = 1, \theta \in \Theta \tag{2.5}$$

If now we chose function $\phi(.)$ to be $\phi(.) = -\log(.)$, we obtain the empirical likelihood setting.

Problems of model misspecification in the context of the empirical likelihood estimation have also been studied recently. This is specially important in the context of our

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research, since DSGE models suffer from this kind of problem. Schennach (2007) proves that, although the empirical likelihood estimator exhibits desirable high order asymptotical properties, namely high order efficiency, in the presence of model misspecification empirical likelihood inference is not so well behaved. To tackled this problem, Schennach (2007) proposes to using the exponential tilted empirical likelihood estimator, that we have also considered and described in chapter 1. She shows that this alternative estimator is well-behaved in the presence of misspecification and still enjoy the high-order asymptotic properties of the empirical likelihood.

Schennach (2005) shows that it is also possible to make a connection between empirical likelihood and bayesian estimation. She shows that "a likelihood function closely related to empirical likelihood naturally arises as the nonparametric limit of a Bayesian procedure which places a type of noninformative prior on the space of distributions". Schennach (2005) notes that this is an important development in the study of the empirical likelihood estimation, because so far the literature had no formal probabilistic interpretation for this estimator.

2.3 The independence assumption

When we presented the empirical likelihood estimation method in chapter 1, we have assumed a set of $x_1, ..., x_m$ identically, independently distributed random variables drawn from an unknown distribution F. In the context of the real business cycle model considered in the same chapter, this set of variables consists in the series of production, consumption, investment, capital and labour hours. In the DSGE framework, we may doubt about the independence of the above variables, since they are linked through several economic relations, such as the optimality conditions of the economic agents, the production function and the laws of motion. Note that these relations are used as general estimating equations that will be used in the empirical likelihood estimating process. However, if the independence assumption is, in fact, violated, the empirical likelihood estimation fails (see explanation in section A.1 of the appendix of this chapter).

Kitamura (1997) poposes a "blockwise empirical likelihood" to solve the variable dependence problem. According to the author, this method "preserves the dependence of data, and the resulting likelihood ratios can be used to construct asymptotically valid condidence intervals". For the framework of general estimating equations Kitamura (1997) defines an efficient estimator, which is derived by maximizing a blockwise empirical likelihood.

Consider the general estimating equations setting defined by Qin and Lawless (1994) described above, assuming a weakly dependent data generating process. Under regularity conditions, the estimates obtained through the maximum empirical likelihood method are

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consistent, but then are not efficient. Moreover, the likelihood ratio statistics is not anymore asymptotically converging to a chi-square distribution. In this situation, Kitamura (1997) suggests using the following blockwise empirical likelihood estimator for general estimating equations. Assume an estimating function $f(X_t, \theta_0)$ that satisfies the moment condition

$$E\left[f(X_t, \theta_0)\right]$$

where θ_0 is a vector of parameters with dimension p, and $r \geq p$ (r is the number of estimating equations).

Now considere the function

$$T_i(\theta) = \phi_M(B_i, \theta) = \sum_{n=1}^{M} f\left(X_{(i-1)L+n}, \theta\right)/M$$

where B_i is the i^{th} block of M consecutive observations $\left(X_{(i-1)L+1}, ..., X_{(i-1)L+M}\right)$[2]. From here, it is possible to define the following blockwise empirical likelihood maximization problem:

$$L_{EL}(\theta, p) = \max_{\theta, p}\left\{\prod_{i=1}^{Q} p_i | p_i > 0, \sum_{i=1}^{Q} p_i = 1, \sum_{i=1}^{Q} p_i T_i(\theta) = 0\right\}$$

where $Q = [(N-M)/L]$, and which can be solved in the standard way (see Qin and Lawless (1994)). Thus, from the lagrangean function

$$\mathcal{L}(\theta, p, \lambda, \gamma) = \sum_{i=1}^{Q} \log p_i + \lambda\left(1 - \sum_{i=1}^{Q} p_i\right) - Q\gamma' \sum_{i=1}^{Q} p_i T_i(\theta)$$

one can obtain the first order conditions with respect to p and θ. Hence, for the optimum p:

$$L(\theta) = \Pi_{i=1}^{Q}\left\{\left(\frac{1}{Q}\right)\frac{1}{\left[1 + \gamma_N(\theta)' T_i(\theta)\right]}\right\} \tag{2.6}$$

and the blockwise empirical likelihood estimate is $\hat{\theta}$ such as (2.6) is maximum.

2.4 The problem of defining more moment conditions

Recall that, in the first chapter, we alert to the fact that defining new moment conditions considering just their correlation may not be such a good option since these moments had no economic meaning whatsoever. This may trouble us as economists, however, this is not the only problem with the choice of moments made. A more serious problem has to do

[2] M is the "window width" and L is the separation between the block starting points.

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with the fact that the "artificial moments" considered in chapter 1 are not met, that is $E\left[g_{artificial}\left(x_i,\omega\right)\right] = E\left[\left(g_6\left(x_i,\omega\right),...,g_{10}\left(x_i,\omega\right)\right)'\right] \neq 0$, under the unknown theorethical distribution F.

Consider, for instance, the "artificial" moment condition defined as follows:

$$E\left[g_6\left(x_i,\omega\right)\right] = E\left[\left(g_1\left(x_i,\omega\right)\right)^2 \cdot g_2\left(x_i,\omega\right)\right] \tag{2.7}$$

It is assumed that:

$$E\left[g_6\left(x_i,\omega\right)\right] = E\left[\left(g_1\left(x_i,\omega\right)\right)^2 \cdot g_2\left(x_i,\omega\right)\right] = 0 \tag{2.8}$$

however, we cannot assure that (2.8) is met because:

$$E\left[\left(g_1\left(x_i,\omega\right)\right)^2 \cdot g_2\left(x_i,\omega\right)\right] = E\left[\left(g_1\left(x_i,\omega\right)\right)^2\right] \cdot E\left[g_2\left(x_i,\omega\right)\right] + cov\left[\left(g_1\left(x_i,\omega\right)\right)^2, g_2\left(x_i,\omega\right)\right]$$

While

$$E\left[\left(g_1\left(x_i,\omega\right)\right)^2\right] \cdot E\left[g_2\left(x_i,\omega\right)\right] = 0$$

because

$$E\left[g_2\left(x_i,\omega\right)\right] = E\left[\frac{1-\theta}{\theta}\frac{c_t}{1-n_t} - \left(1-\alpha\right)\frac{y_t}{n_t}\right] = 0$$

we do not know if:

$$cov\left[\left(g_1\left(x_i,\omega\right)\right)^2, g_2\left(x_i,\omega\right)\right] = 0$$

The fact that we have defined moments that are not met poses serious problems to our empirical likelihood estimation procedure. This may require the use of the exponential tilting empirical likelihood estimator instead of the empirical likelihood one, when we do not have enough moment conditions to estimate the parameters of our economy. As Schennach (2007) shows, the exponential tilting empirical likelihood estimator can deal better with problems of misspecification than the empirical likelihood estimator.

2.5 The (simplest) economy

In this paper, we will consider an economy that admits close form solutions, that is, that allows the analytical computation of its correspondent policy functions. This requires some simplifications relatively to the economy considered in the previous one. We will assumed a less general utility function and also that the depreciation parameter is equal to one (capital fully depreciates). On the one hand, these assumptions may restrict the generalization of our conclusions. But, on the other hand, these assumptions may help to shed some light in the inference procedure, since it eliminates possible errors in the data generating process, induced by the approximated policy functions. In this way, the moment conditions derived from this economy will be exactly met by the simulated data.

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Preferences. The instantaneous utility function describing households' preferences is defined as:

$$U(c_t, n_t) = \log(c_t) + \psi \log(1 - n_t) \tag{2.9}$$

Equation (2.9) is a log-additive utility function[3] with two arguments: consumption, c_t, and leisure time, l_t. Household's total time endowment is allocated between leisure and hours worked. We have normalized time endowment to 1, so $l_t = 1 - n_t$, where n_t is labour hours supplied. The parameter $\psi > 0$ controls the supply of labour in the bundle of the representative household.

Production. We assume that output, y_t, is produced combining capital, k_t, and labour, n_t, given an exogenous level of productivity z_t, in a Cobb-Douglas way:

$$y_t = f(k_t, n_t) = e^{z_t} k_t^{\alpha} n_t^{1-\alpha} \tag{2.10}$$

Equation (2.10) satisfies the usual neoclassical properties: decreasing marginal factor productivities and the Inada conditions. The parameter $\alpha \in (0, 1)$ measures the elasticity of output with respect to capital.

Laws of motion. We assume that the capital stock and the technology evolve according to the following laws of motion.

$$k_{t+1} = i_t \tag{2.11}$$

$$z_t = \rho z_{t-1} + \epsilon_t, \epsilon_t \sim N\left(0, \sigma_\epsilon^2\right) \tag{2.12}$$

Equation (2.11) describes how capital evolves over time when the economy invests i_t and capital fully depreciates ($\delta = 1$).

Equation (2.12) describes the general purpose technology process of the economy. In each period t, technology depends on two components: the persistence of past shocks, measured by ρ, and a normally distributed technology shock, ϵ_t. Note that we technology shock follows a stationary process since we assume that $\rho \in (-1, 1)$.

Feasibility. The feasibility condition or resource constraint of this economy is:

$$y_t = c_t + i_t \tag{2.13}$$

[3]This functional form is a particular case of the more general constant intertemporal elasticity of substitution utility function $\frac{\left(c_t^{\theta}(1-n_t)^{1-\theta}\right)^{1-\tau}}{1-\tau}$, when the coefficent of relative risk aversion τ is set to 1 (then ψ would stand for the ratio $\frac{1-\theta}{\theta}$).

Maximization problem. The problem facing any household can be centralized by the social planner[4], who solves the following maximization problem:

$$\max_{\{c_t, n_t, k_{t+1}\}_{t=0}^{+\infty}} E_t \sum_{t=0}^{\infty} \beta^t \{\log(c_t) + \psi \log(1 - n_t)\}$$
$$s.t.\ c_t + k_{t+1} = e^{z_t} k_t^{\alpha} n_t^{1-\alpha}$$
$$k_0, z_0 \text{ known} \tag{2.14}$$

where β is the household's subjective discount factor, reflecting the households' rate of time preference, i.e. households' valuation of future consumption and leisure, relatively to today's consumption and leisure. We assume that $\beta \in (0,1)$, which means consumers are "impatient".

Optimality conditions. From the optimization problem, the social planner derives the following first order conditions:

$$\frac{1}{c_t} = \alpha \beta E_t \frac{1}{c_{t+1}} e^{z_{t+1}} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} \tag{2.15}$$
$$\psi \frac{c_t}{1 - n_t} = (1 - \alpha) e^{z_t} k_t^{\alpha} n_t^{-\alpha} \tag{2.16}$$

The optimality conditions (2.15) and (2.16) embody the intertemporal and intratemporal choices of the representative consumer. The Euler equation, (2.15), represents the trade-off between consuming today and postponing consumption into the future. This depends on the marginal rate of substitution between consuming today and consuming tomorrow and on the marginal productivity of capital (postponing consumption means investing more today and consuming more tomorrow). The intratemporal condition, (2.16), equates the marginal rate of substitution between consumption and leisure to the marginal productivity of labour. Note that the stochastic shock that drives this economy is incorporated into both the optimality conditions of the agents through the production function.

2.5.1 Estimation strategy

We define the set of moment conditions that will be considered in the estimation procedure. We also describe the inference steps to be followed.

[4]The Second Welfare Theorem applies here.

2.5.1.1 The moment conditions

Consider the chosen real business cycle model, and notice that our economy is parameterized by the vector $\omega = \{\beta, \psi, \alpha, \rho, \sigma_\epsilon^2\}$. According to the empirical likelihood framework described before, it is required that we formulate our economy as a set of moment conditions, that is, we need to define our general estimating equations $g(x_i, \omega) = (g_1(x_i, \omega), ..., g_r(x_i, \omega))'$ such that $E\{g_j(x_i, \omega)\} = 0, \forall\ j = 1, ..., r \geq 5$. Note that we require at least five equations to solve the system for the five unknows, which are the parameters of our economy.

We define the following moment conditions:

$$E\left[\alpha\beta\frac{c_t}{c_{t+1}}e^{z_{t+1}}k_{t+1}^{\alpha-1}n_{t+1}^{1-\alpha} - 1\right] = 0 \quad (2.17)$$

$$E\left[\psi\frac{c_t}{1-n_t} - (1-\alpha)e^{z_t}k_t^{\alpha}n_t^{-\alpha}\right] = 0 \quad (2.18)$$

$$E\left[c_t + k_{t+1} - e^{z_t}k_t^{\alpha}n_t^{1-\alpha}\right] = 0 \quad (2.19)$$

$$E\left[z_t - \rho z_{t-1}\right] = 0 \quad (2.20)$$

$$E\left[(z_t - \rho z_{t-1})^2 - \sigma_\epsilon^2\right] = 0 \quad (2.21)$$

The moment condition given by (2.17) results from the only expectational equation that arises in our economy, the Euler equation, (2.15). From this equation we can identify the discount factor β. Condition (2.18) is obtained by applying the expectation operator to the intratemporal condition, equation (2.16), and it allows us to identify the parameter controlling the weight of leisure in the consumer bundle, ψ. In a similar way, condition (2.19) is obtained by applying the expectation operator to the feasibilty condition, equation (2.13), and from this we can estimate the parameter α. Moment conditions (2.20) and (2.21) are obtained from equation (2.12), by exploiting the assumption $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. These two moments allow the identification of the parameters characterizing technology in this economy: persistence, ρ, and volatility, σ_ϵ^2.

Hence, we can define a $g(x_i, \omega)$ vector as follows:

$$g(x_i, \omega) = (g_1(x_i, \omega), ..., g_5(x_i, \omega))' = \begin{bmatrix} \alpha\beta\frac{c_t}{c_{t+1}}e^{z_{t+1}}k_{t+1}^{\alpha-1}n_{t+1}^{1-\alpha} - 1 \\ \psi\frac{c_t}{1-n_t} - (1-\alpha)e^{z_t}k_t^{\alpha}n_t^{-\alpha} \\ c_t + k_{t+1} - e^{z_t}k_t^{\alpha}n_t^{1-\alpha} \\ z_t - \rho z_{t-1} \\ (z_t - \rho z_{t-1})^2 - \sigma_\epsilon^2 \end{bmatrix} \quad (2.22)$$

where the random variable $x_i's$ is a multivariate random vector including the economic variables of our economy, as well as their lags and leads. Therefore, the function $g(x_i, \omega)$ gathers

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all the essential information of the economy: the optimal choices of the representative household, the endogenous evolution of the capital stock, the autoregressive technology process, and the statistics of the exogenous technology shock hitting the economy every period. And:

$$E\left[g\left(x_i,\omega\right)\right]=\left[\left(g_1\left(x_i,\omega\right),...,g_5\left(x_i,\omega\right)\right)'\right]=\left[\overrightarrow{0}\right] \tag{2.23}$$

As we may observed, the choice of this simpler economy has also solved the problem of defining more moment conditions, since we have an exactly identified system[5].

2.5.1.2 Inference procedure

We will follow the same inference procedure as in chapter 1. Hence, we will briefly outline the main steps and some minor changes, required by the economy considered in this case.

Data generating Process Again we will assign "true" values to the parameters of our economy and then generate "artificial data" for each of our economic variables. In this way, we will be able to control the accuracy of the estimation results by comparing them with the "true" values assigned to each parameter. However, this time we are able to generate this data from the exact policy functions that solve the representative household's problem. Moreover, in this case, we are also sure that these data meet exactly the moment conditions we have defined.

The series for capital and consumption are then generated from the following equations (please check section A.1. of the Appendix for details on the computation of the policy functions):

$$\ln k_{t+1} = \Gamma_k+\alpha\ln k_t+\Sigma_{j=0}^{t}\rho^j\epsilon_{t-j} \tag{2.24}$$

$$\ln c_t = \Gamma_c+\alpha\ln k_t+\Sigma_{j=0}^{t}\rho^j\epsilon_{t-j} \tag{2.25}$$

where $\Gamma_k=\ln\alpha\beta n^{1-\alpha}$ and $\Gamma_c=\ln\left(1-\alpha\beta\right)n^{1-\alpha}$.

[5]In the case, we want to estimate an overidentified model, we can construct more moments by exploiting, for example, the Leibniz Integral Rule. According to this rule:

$$\frac{d}{d\omega}\int_b^a g\left(x,\omega\right)dx=\int_b^a\frac{\partial}{\partial\omega}g\left(x,\omega\right)dx$$

which implies

$$\frac{d}{d\omega}E\left[g\left(x,\omega\right)\right]=E\left[\frac{\partial}{\partial\omega}g\left(x,\omega\right)\right]=0$$

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Recall that our set of parameter is $\omega = \{\beta, \psi, \alpha, \rho, \sigma_\epsilon^2\}$, in this setting. Following once again the calibration of Aruoba *et al* (2003)[6], we will have:

Table 1. "True" parameter values

Parameter	β	ψ	α	ρ	σ_ϵ
Value	0.9896	1.67	0.4	0.95	0.007

Finally note that in this economy the series of capital coincides with the one of investment and we assumed that the number of hours worked was constant throught the household's lifetime. So in this case, we will only have to include in the data generating process one more source of stochasticity, that is, we will introduce a measurement error to generate the series of the product and tackle the singularity problem. Let us then define the measurement error as $\nu \sim N(0, \sigma_\nu^2)$ with $\sigma_\nu = y \times 0.0001$ (Fernandez-Villaverde and Rubio-Ramirez, 2005).

Optimization The inputs for our optimization algorithm are the simulated series for each of the economic variables and the moment conditions defined in (2.22).

We start the optimization algorithm with an "educated" guess on the parameter vector ω, that is, we will assign starting values to the parameters close to the true ones. We repeat the optimization in a Monte Carlo style and compute means and standard deviations of our estimates.

2.6 Further research

We have explored further the main problems of estimating DSGE models with and empirical likelihood estimator identified in chapter 1. We have defined a simpler economy in order to better understand the empirical likelihood inference method when applied to DSGE models[7]. We have also presented a possible way of working with dependent random variables in the context of the empirical likelihood estimation.

The next step of this research is to improve on the computing side of this project. Also, as we have mention in chapter 1, the next exercise would be to estimate with the empirical likelihood estimator a DSGE model with real economic data.

In what follows we present a research agenda on possible topics to be investigated further in the context of the empirical likelihood estimation of DSGE models.

A possible extension would be to build upon the work of Moon and Schorfheide (2006) and use moment base estimators in the context of macroeconomic models with inequality

[6]Note that $\psi = \frac{1-\theta}{\theta}$ and τ and δ are set to one.

[7]Of course, we may use the Leibniz Integral Rule in case we wish to define more moment conditions in order to overidentify the parameters of the economy defined in this chapter.

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moment conditions, that is, $E\left[g\left(x_i,\theta\right)\right] \geq 0$. Inequality moment conditions arise, for instance, when the Euler equations are violated in the presence of borrowing constraints, or in the presence of endogeneity. These authors derive limit distributions of empirical likelihood estimators for models in which inequality moment conditions provide overidentifying information and show that this information reduce the asymptotic mean-square estimation error (see section A.2 of the Appendix for details on the Moon and Schorfheide empirical likelihood setting).

Finally, another interesting extension is to study the identification problems in the DSGE models context and try to understand if the empirical likelihood estimation method can help to solve these problems. Canova and Sala (2006) describe the main identification problems that can be found when estimating DSGE models. These authors define several types of identification issues and their consequences for parameter estimation and model evaluation. In general, identifiability is related with the objective function having a unique zero and displaying enough curvature in all relevant dimensions. The authors argue that, since impulse responses depend nonlinearly on the structural parameters, it is unknown if the identifiability conditions are met. This problem is hard to solve because: stationary solutions are found with numerical methods so the mapping from structural parameters to impulse responses is not analytical available; since the objective function can be evaluated only at a finite number of points, it is difficult to infer its properties in high dimensional parameters spaces. They define identification as the ability to draw inference about parameters of a theoretical model from an observed sample, and formalize this concept as follows:

$$\min g\left(y,T,m,\theta\right) = \left(ir^d(y,T) - ir^m\left(m,\theta\right)\right) W\left(T\right) \left(ir^d(y,T) - ir^m\left(m,\theta\right)\right)'$$

where y, vector of data, T, sample size, m, DSGE model, and θ, vector of parameters, $ir^d\left(y,T\right)$, vector of data-based structural responses, $id^m\left(m,\theta\right)$, vector of model-based responses, $W\left(T\right)$, weighting matrix (see details on the different types of identification problems in section A.3 of the Appendix).

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2.7 Appendix

A.1 The empirical likelihood estimator under variable dependence Following Kitamura (1997), assume that one is interested in estimating, using the empirical likelihood, the mean θ_0 of a set of identically distributed random vectors $X_t, t = 1, ..., N$.Suppose we start by treating these vectors X_t as if they were independent variables. The empirical likelihood function in this setup is defined by

$$\Pi_{t=1}^{N} p_t$$

and the maximization problem is

$$\max \left\{ \Pi_{t=1}^{N} p_t, s.t. \sum_t p_t = 1; \sum_t p_t X_t = \theta \right\}$$

If one considers only the first of the two constraints, then the empirical likelihood function achieves its maximum value when the empirical distribution is $p_t = 1/N, \forall t$. Consequently, the estimate of θ_0 is $\hat{\theta} = \bar{X} = N^{-1} \sum_t X_t$.

The empirical likelihood ratio (ratio between the theorectical maximum value of the likelihood and the value of the likelihood function evaluated at the estimate) is

$$R(\theta) = \frac{L(\theta)}{L(\bar{X})}$$

which, according to Owen (1988) follows a limiting chi-square distribution and, under some mild regularity conditions, the following approximation holds

$$-2 \log R(\theta_0) = N \left(\bar{X} - \theta_0\right)' \bar{\Sigma}^{-1} \left(\bar{X} - \theta_0\right) + o_p(1)$$

where $\bar{\Sigma} = N^{-1} \sum_t \left(\bar{X} - \theta_0\right) \left(\bar{X} - \theta_0\right)'$.

However, if X_t is a dependent variable, $\bar{\Sigma} = N^{-1} \sum_t \left(\bar{X} - \theta_0\right) \left(\bar{X} - \theta_0\right)' \xrightarrow{p} Var(X_t)$ instead of converging in probability to $\sum_{-\infty}^{+\infty} Cov\left(X_t, X_{t-j}\right)$. As Kitamura (1997) concludes, the empirical likelihood fails, because it ignores the dependence structure of the data.

A.2 Deriving the policy functions. Consider the system of equations which characterize the competitive equilibrium of the economy:

$$\begin{cases} \frac{1}{c_t} = \alpha\beta E_t \frac{1}{c_{t+1}} e^{z_{t+1}} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} \\ \psi \frac{c_t}{1-n_t} = (1-\alpha) e^{z_t} k_t^{\alpha} n_t^{-\alpha} \\ c_t + k_{t+1} = e^{z_t} k_t^{\alpha} n_t^{1-\alpha} \end{cases} \tag{2.26}$$

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In order to solve the system of equations in (2.26) we will follow the "guess and verify" method.

We will: (i) assume that the amount of working hours supplied by the households is constant over their lifetime and do not vary with the stochastic state of the economy; and (ii) guess that the optimal stochastic sequence of capital of this economy is:

$$k_{t+1} = \alpha\beta e^{z_t} k_t^{\alpha} n^{1-\alpha} \tag{2.27}$$

with

$$n_t = n, \forall t \tag{2.28}$$

Then, using the resource constraint of the economy, consumption is equal to:

$$c_t = (1-\alpha\beta)\, e^{z_t} k_t^{\alpha} n^{1-\alpha} \tag{2.29}$$

The next step consists in checking whether the policy functions (2.27) and (2.29) are in fact the solutions of (2.26).

For that purpose, we have to substitute (2.28) and (2.29) into the Euler equation, and check if this condition is satisfied.

$$\begin{aligned}
\frac{1}{c_t} &= \alpha\beta E_t \frac{1}{c_{t+1}} e^{z_{t+1}} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} \Leftrightarrow \\
&\Leftrightarrow \frac{1}{(1-\alpha\beta)\, e^{z_t} k_t^{\alpha} n^{1-\alpha}} = \alpha\beta E_t \frac{e^{z_{t+1}} k_{t+1}^{\alpha-1} n^{1-\alpha}}{(1-\alpha\beta)\, e^{z_t+1} k_{t+1}^{\alpha} n^{1-\alpha}} \Leftrightarrow \\
&\Leftrightarrow \frac{1}{(1-\alpha\beta)\, e^{z_t} k_t^{\alpha} n^{1-\alpha}} = \alpha\beta E_t \frac{1}{(1-\alpha\beta)\, \alpha\beta e^{z_t} k_t^{\alpha} n^{1-\alpha}} \Leftrightarrow \\
&\Leftrightarrow \frac{1}{(1-\alpha\beta)\, e^{z_t} k_t^{\alpha} n^{1-\alpha}} = \frac{1}{(1-\alpha\beta)\, e^{z_t} k_t^{\alpha} n^{1-\alpha}}
\end{aligned}$$

Then, from the intratemporal optimality condition, we obtain the expression for labour supply:

$$\begin{aligned}
\psi \frac{c_t}{1-n_t} &= (1-\alpha)\, e^{z_t} k_t^{\alpha} n_t^{-\alpha} \Leftrightarrow \\
&\Leftrightarrow \psi \frac{(1-\alpha\beta)\, e^{z_t} k_t^{\alpha} n^{1-\alpha}}{1-n} = (1-\alpha)\, e^{z_t} k_t^{\alpha} n_t^{-\alpha} \Leftrightarrow \\
&\Leftrightarrow \frac{n}{1-n} = \frac{1-\alpha}{\psi (1-\alpha\beta)} \Leftrightarrow \\
&\Leftrightarrow n = \frac{1-\alpha}{1-\alpha+\psi(1-\alpha\beta)}
\end{aligned}$$

which is constant (as assumed) and depends only on the economy's parameters.

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The exact policy functions will be necessary in the estimation procedure, namely in what concerns the data generating process. Since this is the case, writing the policy functions as backward representations seems more convenient. First, we take the logs of (2.27) and (2.29):

$$\begin{aligned} \ln k_{t+1} &= \Gamma_k + z_t + \alpha \ln k_t \\ \ln c_t &= \Gamma_c + z_t + \alpha \ln k_t \end{aligned}$$

where $\Gamma_k = \ln \alpha\beta n^{1-\alpha}$ and $\Gamma_c = \ln (1-\alpha\beta) n^{1-\alpha}$.

Next, we recursively substitute z_{t-1} by its past values, assuming for simplicity that $z_0 = 0$. In this way we obtain a representation of the behaviour of the economy as a function of today's capital and past shocks:

$$\begin{aligned} \ln k_{t+1} &= \Gamma_k + \alpha \ln k_t + \Sigma_{j=0}^{t} \rho^j \epsilon_{t-j} \\ \ln c_t &= \Gamma_c + \alpha \ln k_t + \Sigma_{j=0}^{t} \rho^j \epsilon_{t-j} \end{aligned}$$

A.3 Moon and Schorfheide (2006) empirical likelihood problem. Moon and Schorfheide (2006) model the problem in the empirical likelihood setting as follows:

$$\max L_{EL}(\theta, p) = \prod_{i=1}^{m} p_i$$

subject to

$$p_i > 0; \sum_{i=1}^{m} p_i = 1; \sum_{i=1}^{m} [p_i g_1(x_i, \theta)] = 0; \sum_{i=1}^{m} [p_i g_2(x_i, \theta)] \geq 0$$

The Lagrangean of the problem would then be:

$$\mathcal{L} = \sum_{i=1}^{m} \log p_i - \xi \left(\sum_{i=1}^{m} p_i - 1 \right) - \mu_1' \left(\sum_{i=1}^{m} [p_i g(x_i, \theta)] \right) - \mu_2' \left(\sum_{i=1}^{m} [p_i g_2(x_i, \theta)] \right)$$

from which the first order condition for each p_i is obtained (the derivation process is similar to the one in which only equality moment conditions are considered, but now we have to write also Khun-Tucker conditions):

$$p_i = \frac{1}{m} \frac{1}{1 + \lambda_1 g_1(x_i, \theta) + \lambda_2 g_2(x_i, \theta)}$$

with $\lambda_1 = -\frac{\mu_1}{m}$ and $\lambda_2 = -\frac{\mu_2}{m}$. Then, similarly, the saddlepoint formulation of the problem would be:

$$G(\theta, \lambda_1, \lambda_2) = \frac{1}{m} \sum_{i=1}^{m} \log (1 + \lambda_1 g_1(x_i, \theta) + \lambda_2 g_2(x_i, \theta))$$

and the empirical likelihood estimator can be expressed as:

$$\hat{\theta} = \arg \min_{\theta \in \Theta} \max_{\lambda_1, \lambda_2 \leq 0} G_m(\theta, \lambda_1, \lambda_2)$$

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A.4 Identification problems in the DSGE models context. Canova and Sala (2006) specify the following type of identification problems:

1. Observational equivalence: the objective function does not have a unique maximum, the mapping between structural parameters and reduced form statistics is not unique, and different structural models having different economic interpretations may be indistinguishable. Formally, two models, with parameter vectors θ and ξ, are observationally equivalent if $g(y,T,m_1,\theta^*) = g(y,T,m_2,\xi^*) = 0$. In other words, $\frac{\partial g(y,T,m_1,\theta)}{\partial \theta}|_{\theta^*} = \frac{\partial g(y,T,m_2,\xi)}{\partial \xi}|_{\xi^*} = 0$ and both $\frac{\partial^2 g(y,T,m_1,\theta)}{\partial \theta \partial \theta'}|_{\theta^*}$ and $\frac{\partial^2 g(y,T,m_2,\xi)}{\partial \xi \partial \xi'}|_{\xi^*}$ are positive definite.

2. Underidentification or partial identification: the objective function may be independent of certain structural parameters - this happens often after linearization -, being constant for all values of the parameter. Let $\theta = [\theta_1, \theta_2]$ and partition $\Theta = [\Theta_1, \Theta_2]$, then θ is locally underidentified if $g(y,T,m,\theta_1,\theta_2) = g(y,T,m,\theta_2)\,\forall \theta_1 \in \Theta_1 \subset \Theta$. On the other hand, $g(y,T,m,\theta) = g(y,T,m,\theta_1 f(\theta_2))\,, \forall \theta_1 \in \Theta_1 \subset \Theta^1$, and $\theta_2 \in \Theta_2 \subset \Theta^2$ for some continuous and differentiable f, then θ is locally partially identified. Global under and partial identification occur when $\Theta_1 = \Theta^1$.

3. Weak identification: though the objective function is globally concave, its curvature may be "insufficient"; This means that a parameter θ is locally weak identified if there exist a unique θ^* such that $g(y,T,m_1,\theta^*) = 0$ but $g(y,T,m_1,\theta) < \varepsilon \forall \theta \in \Theta^{\dagger} \subset \Theta$ and globally weakly identified if this occur for all $\theta \in \Theta$. Furthermore, it implies that $\frac{\partial g(y,T,m_1,\theta)}{\partial \theta}|_{\theta^*} = 0$, $\frac{\partial^2 g(y,T,m_1,\theta)}{\partial \theta \partial \theta'}|_{\theta^*}$ is positive definite but its eigenvalues are small. (The authors also formally define the case of asymmetric weak identification: right and left Hessians differ and at least one eigenvalue of one of the two is zero).

4. Limited information: if the objective function considers only a subset of the implications (limited number of shocks or a subset of responses to all shocks), different responses may carry different information about the parameters and, in this way, a parameter can remain under-identified if one shock or one particular set of responses is used (weighting matrix $W(T)$ factorized as $W(T) = \mathcal{S}\mathcal{W}(T)$, where $\mathcal{S}$ is a selection matrix of zeros and ones).

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CHAPTER 3

MACROECONOMIC AND FISCAL VOLATILITY AND THE COMPOSITION OF PUBLIC SPENDING

3.1 Introduction

The volatility of macro-economic and fiscal variables has become an increasingly fashionable topic. A number of recent empirical papers, reviewed below, have sought to assess the link between macro-fiscal volatility and the volatility or cyclicality of public spending. Others have examined the link between macro-fiscal volatility and countries' growth performance.

Earlier literature has thus addressed some issues related to the longer-term (growth) and medium-term (cyclicality) aspects of macro-fiscal volatility but, to the best of our knowledge, its shorter-term effects have not been examined at all. Specifically, volatility and changes in volatility tell us something about "news" to the policy maker, and it seems that our knowledge of how fiscal policy responds to such news is almost non-existent.

To start filling this gap, we consider the impact of short-term macro-fiscal volatility on the composition of government spending. That is, we study how the volatility in macroeconomic and fiscal (revenue-side) variables affect the relative weights of government investment and government consumption spending. As a result, we seek to gain some first insights into fiscal policy responses to short-term macro-fiscal volatility.

Before proceeding, it is important to clarify two issues related to the terminology used in the remainder of the paper.

First, the terms "public investment (spending)" and "government investment (spending)" will be used interchangeably, as is customary in related literature. However, as pointed out by Gonzalez Alegre et al. (2008), government investment comprises gross fixed asset formation by the general government, while public investment also includes investment in government-owned corporations, such as many utilities. Thus, although we succumb to custom and frequently refer to public investment (spending) below, our sole focus is on government investment (spending).

Second, as will become clear in next section, earlier studies have examined public expenditure composition in number of different ways, relating the evolution of public investment to different spending categories such as public consumption expenditure, primary spending,

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current expenditure, or even total outlays. Our focus will be on the relationship between investment and consumption spending, both because that is arguably the economically most relevant comparison, and because consumption is most similar to investment as a policy maker's decision variable; thus, by focusing on consumption rather than, e.g., current spending we reduce unwelcome noise due to inherent differences between the components constituting our dependent variable.

These caveats duly noted we proceed to a review of earlier related literature (section 2). Section 3 presents the empirical analysis, section 4 interprets the results and section 5 concludes.

3.2 Related Literature

3.2.1 Empirical Studies

A few recent empirical papers have considered indicators of macro-fiscal volatility either as right-hand side or as left-hand side variables. First, the impact of macro-fiscal volatility on economic growth has been assessed (Afonso and Furceri, 2008; see also Ramey and Ramey, 1995). Second, determinants of government spending volatility have been assessed in general (Furceri and Ribeiro, 2008), and the cyclicality of different categories of government spending has been studied in particular (Lane, 2003), with a special focus on the impact of output volatility.

Starting with macro-fiscal volatility as a determinant of economic growth, Afonso and Furceri (2008) estimate the effects of the size and volatility of government spending and revenues on output growth. They observe 28 EU and OECD countries over seven five-year periods between 1970 and 2004 and specify separate panel growth models for government revenues, including their volatility, and government expenditure, again including their volatility. They find that both the size and the volatility of government spending and revenues have a negative impact on growth. Specifically, indirect taxes (size and volatility); social contributions (size and volatility); government consumption (size and volatility); subsidies (size); and government investment (volatility) have a sizeable, negative and statistically significant effect on growth.

Ramey and Ramey (1995) investigate the relation between macro-fiscal volatility and growth in a panel of 92 countries in the period 1960-85, and in a subset of 24 OECD countries in the period 1950-88. They first regress the mean of GDP growth on its volatility, finding a significant and negative relationship. They then regress per capita GDP growth on a set of control variables—including government spending volatility—and on the volatility

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of the regression residuals, finding a strongly significant and negative relationship between government spending volatility and growth.

Turning then to the determinants of government spending volatility, Furceri and Ribeiro (2008) examine the link between country size and government spending volatility. The sample includes observations for 160 countries from 1960 to 2000. The authors regress the standard deviation of annual growth in government consumption spending on the (log of) population and controls for demographic, geographical and macroeconomic factors (GDP per capita, openness, CPI inflation, and government size). They conclude that smaller countries tend to have more volatile government (consumption) spending.

Finally, considering the impact of output volatility on the composition of government spending, Lane (2003) seeks to explain the cyclical behaviour of fiscal policy by analysing the effects of output volatility (and also power dispersion) on various categories of government spending. The sample comprises 22 OECD countries observed over the period 1960-98 (annual data). The categories of government spending considered are total government spending, government consumption and its breakdown between wage and non-wage components, government investment, and non-interest total government spending. Lane constructs measures of cyclicality for each of these spending categories and regresses them on output volatility, political power dispersion, output per capita, openness and the share of public sector employment. He concludes that investment is the most pro-cyclical component of government spending, while current spending is mildly counter-cyclical. Further, countries with volatile output and dispersed political power are most likely to run pro-cyclical fiscal policies, with government wage expenditure as the most important channel through which this effect operates.

To sum up, recent work has cast some light on how fiscal volatility affects economic growth and on what determines the volatility (or cyclicality) of different types of government spending. In general, it has been found that macro-fiscal volatility is detrimental for growth and that country size, output volatility and political dispersion all affect government spending volatility or cyclicality.

3.2.2 Some Theoretical Considerations

There is, to the best of our knowledge, no body of theoretical literature explicitly examining the impact of short-term revenue volatility on the composition of public spending. We can, however, consider how cyclical revenue volatility might affect public consumption and investment in both Keynesian and neoclassical frameworks. To that end, we first review briefly the arguments pertaining to the cyclicality of fiscal policy in Lane (2003) as well as

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Talvi and Vegh (2000). We then discuss the applicability of such cyclical considerations in a shorter-term setting, using arguments from the recent literature on "real time" fiscal policy (Cimadomo, 2008). To link the discussion to real-world fiscal policymakin, we refer throughout to the recent recession, which started in late 2007 in the US and in the course of 2008 in most European countries, while obviously acknwledging that the recession was more severe than a typical cyclical downturn. These references are meant to be merely illustrative; the subsequent empirical work will focus exclusively on the pre-crisis period.

Starting with the Keynesian view, both public consumption and investment spending are considered the policymaker's decision variables, to be used in a countercyclical manner. The higher the multiplier value of a specific spending type is, the more desirable it is as a cyclical stabilisation instrument. To the extent that public investment multipliers are greater than those of public consumption, an active Keynesian fiscal stabilisation policy would imply an increase in the relative share of public investment in cyclical downturns.

In contrast, the neoclassical tax smoothing argument (Barro, 1979) has it that the path of public spending be set *ex ante*, and any cyclical variability of government revenues be allowed to move the fiscal balance in a procyclical manner. If, however, public spending is considered as a decision variable, the optimal relationship between public and private consumption depends on their substitutability in utility. With substitutability government consumption would move countercyclically; with complementarity it would move procyclically.

As regards public investment, the neoclassical view suggests that it be analyzed in a long-run growth framework rather than in a cyclical stabilisation context (Barro and Sala-i-Martin, 1995). However, Lane (2003) suggests that even if public investment planning is undertaken with a long-run focus, its optimal execution may be countercyclical, if the relative cost of public investment declines in cyclical downturns.

In sum, the Keynesian view suggests that the relative share of public investment be increased in downturns, provided that its multiplier is greater than that of public consumption. The neoclassical view is less straight-forward but can be interpreted as suggesting that the relative share of public consumption be increased (reduced) in downturns with substitutability (complementarity) of public and private consumption.

Against this background, how could one charaterise the fiscal policy response to the recent recession? Many governments did indeed appear Keynesian in their initial response. In the UK, France and Germany, to name a few examples, the discretionary fiscal expenditure measures announced early on in the recession comprised an increase in public investment, while no additional public consumption (excluding transfers) was announced. In contrast, the US (federal) fiscal stimulus package boosted public consumption; that was also the case in

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Italy (OECD, 2009). One could interpret these countries' fiscal stimulus as more neoclassical in character; alternatively, it could also be interpreted as Keynesian in the presence of relatively high public consumption multipliers. The public consumption multipliers in the US and Italy do indeed appear significantly higher than in Germany (OECD, 2009); at the same time even in the US and Italy, the multiplier for public infrastruture investment exceeds that for public consumption. All in all, these countries' initial stimuli do not lend itself to a clear-cut classification along the theoretical lines depicted earlier - which is possible related to the unique character of te recession and the perceived need to try different fiscal responses to it.

Consider now the applicability of such cyclical considerations in a shorter-term setting. To that end, it is useful to sketch the basic characteristics of the budgetary planning process. Typically, the budget for the next fiscal year (call it t) is prepared and approved before the end of *t-1*. That budget, including notably the composition of spending, is based on information available at the end of *t-1*. As argued by Cimadomo (2008), the information available at the time of budget preparation and approval tends to be incomplete as regards the prevailing cyclical conditions. Moreover, the cyclical outcome in t can deviate from the expected one. This incompleteness of information about both prevailing and future cyclical conditions can render the budget sub-optimal.

The fiscal policymaker can try to alleviate such sub-optimality in two ways. First, he can use indicators other than output to derive information about (changes in) the cyclical conditions. Output data tends to be available only with a delay and subject to significant revisions. Thus, the use of more timely and accurate indicators that are closely correlated with output movements is warranted, both at the time of budget preparation and approval in *t-1* and in the course of t as the cycle unfolds.

Second, the fiscal policymaker can address deviations from expected cyclical conditions during t by using his discretion in implementing the budget or, should the deviations be major, by preparing a supplementary budget. The degree of discretion in implementing an approved budget is generally limited, excluding for example major overspending. However, such discretion may allow limited reallocation of spending between categories, thus possibly affecting its composition.

Combining now the discussion above of cyclical and "real time" aspects of fiscal policy, consider a Keynesian policymaker. He receives "news" about the cyclical situation at the time of budgeting in *t-1* and in the course of *t*. Such news can include output data, but they can also include other, higher-frequency indicators. Assume that, based on such news, he concludes that a negative shock has hit the economic environment. He would then fine-tune fiscal policy by making an appropriate adjustment to the budget by increasing the budgeted share of government investment relative to consumption spending. If the shock is observed during *t*, he would use his discretion or, if warranted, prepare a supplementary budget for fine-tuning purposes, in both cases again increasing the share of government investment relative to consumption spending.

In contrast, the reaction of a neoclassical policymaker to such news would depend on the substitutability of public and private consumption such that he would increase (reduce) the relative share of public consumption in downturns with substitutability (complementarity).

The fiscal reaction to the recent recession involved the introduction of significant discretionary stimulus packages in late 2008, in many cases on top if the draft budgets that had already be prepared for 2009. These stimulus packages were a reaction to the rapid and significant deterioration of the cyclical situation and outloook in the autumn of 2008 (especially following the collapse of Lehman Brothers in September); hence, they are readily interpretable in the framework of real-time fiscal policy-making. As pointed out before, the unique magnitude and character of the negative shock caused some governments to implement a fiscal package that comprised both Keynesian and neoclassical characteristics.

While they lend themselves to an illustration of the fiscal response to the recent recession, these insights from theory do not provide us with unambiguous and empirically testable hypotheses. They do, however, suggest that a link between short-term revenue volatility and the composition of public spending exists. As reviewed above, that link has been empirically examined in a cyclical context but not in a short-term context.

3.3 Empirical Analysis

3.3.1 Model and Estimation Methodology

Our goal is to model the determinants of the composition of public expenditure, with a special focus on variables that provide the fiscal policymaker with "news" about the prevailing and expected cyclical conditions. In addition to the most widely-used variables (GDP and inflation), we also consider higher-frequency indicators that are highly correlated with cyclical conditions and that are directly observable to the policymaker. Most notably, we consider different types of tax revenues to that end, as they are good coincident indicators of economic activity (we will confirm this in our sample below), as they are observable in a timely manner, and as they tend to be accurate.

We consider the ratio of public investment to public consumption spending as our dependent variable. We are interested in estimating short-term impacts on the composition of

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public spending, so we make use of the observation that the ratio of public investment to consumption expenditure has shown persistence over time[1] and specify a dynamic model in reduced form as follows:

$$\left(\frac{I}{C}\right)_{it} = \alpha \left(\frac{I}{C}\right)_{it-1} + \sum_{j} \beta_j \sigma_{jt} + \sum_{k} \delta_k X_{kt} + \gamma_i + u_{it} \tag{3.1}$$

where $u_{it}\tilde{}\,iid\,(0, \sigma_u^2)$, with subscript i referring to observations in the cross-section dimension (individual countries) and t to observations in the time dimension.

The dependent variable is the ratio of public investment to public consumption spending (I/C).

Our macro-fiscal "news" variables of interest are collected in the second term of equation (3.1). To measure "news", we consider the volatility rather than level of the variables. This dynamic specification of our model (3.1) allows us to interpret the impact of the volatility measures as "news" about the underlying economic conditions to which the composition of public spending reacts[2]. The volatility variables measure only the impact of any contemporaneous change in the standard deviation, as the volatility history is entirely captured by the lagged dependent variable. Note, however, that we construct our volatility measures in a backward-looking way, as detailed below, which allows us to capture volatility innovations both at the time of budget preparation in *t-1* and during its execution in *t*.

In what is to come, such contemporaneous volatility innovations are interpreted as news to the policy maker to which he reacts by changing the composition of public spending.

The macro-volatility variables include the volatility (standard deviations, denoted by σ) of real GDP, CPI inflation and also total tax revenues. The more specific fiscal volatility variables include the standard deviation of four types of tax revenues (taxes on capital, current taxes on income and wealth, taxes on production and imports, the Value Added Tax (VAT)). These taxes are more closely described in the next section; suffice it to mention here that we consider the volatility of these taxes both in levels and in relation to GDP.

The third term on the right-hand side of equation (3.1) contains a number of control variables X. Their role is simply to render the model empirically well-specified, and we do not seek to give them any economic interpretation. The selection of controls is based on earlier empirical literature summarised in section 2, with a special focus on controlling for any cyclical influences.

The estimation of equation (3.1) will have to account for the correlation between the

[1] The first-order autocorrelation of that ratio in our sample, described in detail in the next section, is as high as 0.862.

[2] We adopt here the term "news", based on Greene (2003, p. 307).

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regressors (lagged dependent) and the composite term $(\gamma_i + u_{it})$ where γ_i denotes country-specific random effects, which renders least squares estimators inconsistent even asymptotically. To circumvent this problem we employ General Method of Moments (GMM) estimation (Arellano and Bond, 1991). The Arellano and Bond estimator adapts perfectly to the estimation set-up described above since it explores linear moment restictions in the context of an estimating equation that includes a lagged dependent variable, individual fixed effects and possible endogenous control variables. More specifically, the assumptions and process underlying GMM estimation are as follows. The levels equation, as in equation (3.1), is first-differenced, which eliminates any constant or fixed effects as well as any deterministic trend. A set of (internal) instrumental variables that are orthogonal to the error term, Δu_i, is then specified. Assuming that the error term is not serially correlated and that the explanatory variables are weakly exogenous, higher-order lags of the dependent variable constitute valid instruments[3]. This results in the following set of instruments, denoted by I_c, and stacked in the following matrix:

$$Z_{c,i} = \begin{bmatrix} I_{c,it-2} & 0 & 0 & ... & 0 & ... & 0 \\ 0 & I_{c,it-2} & I_{c,it-3} & ... & 0 & ... & 0 \\ . & . & . & ... & . & ... & . \\ 0 & . & 0 & ... & I_{c,it-2} & ... & I_{c,i1} \end{bmatrix}$$

While identification requires the number of instruments to equal the number of explanatory variables, overidentification is in practice necessary, as it both allows the testing of the moment conditions, as explained below, and improves efficiency[4],[5].

The orthogonality requirement is equivalent to conditions on the first moment of the sample data.

$$E\left[(Z_{c,i})'\,\Delta u_i\right] = 0, i = 1, 2, ..., N$$

where $\Delta u_i = (\Delta u_{i3}, \Delta u_{i4}, ..., \Delta u_{iT})$. That is, the expected value of the product between the instruments matrix and the first difference of the error term is zero. The fulfilment of these moment conditions is a sufficient condition for the asymptotic consistency of GMM estimators.

[3]Higher-order lags of other explanatory variables can also be used as instruments under the same assumptions.

[4]Note that GMM estimation dominates least squares estimation with instrumental variables, as GMM can be shown to be asymptotically efficient, which is not the case for least squares estimation with instrumental variables.

[5]There is, however, a possible trade-off between bias and efficiency when the number of instruments (moment conditions) is increased with small samples like ours (see, e.g. Roodman, 2007). We employ the Sargan overidentification test, together with a consideration of the robustness of coefficient estimates to different instrument sets, as a criterion to manage this trade-off.

Then, the GMM estimator of our vector of parameters, $\hat{\theta} = \begin{bmatrix} \hat{\alpha} & \hat{\beta} & \hat{\delta} \end{bmatrix}$ is given by:

$$\hat{\theta} = \arg\min_{\theta} \left[\left(\Delta u' Z \right) W_N \left(Z' \Delta u \right) \right]$$

where $W_N = \left[\frac{1}{N} \sum_{i=1}^{N} \left(Z_{c,i} \right)' H Z_{c,i} \right]^{-1}$ is the one-step GMM-estimator weighting matriz. H is a $(T-2)$ square matrix with 2s on the main diagonal, -1s on the first off-diagonals, and zero elsewhere, as an initial "guess". According to Hansen (1982), the optimal choice for the weighting matrix is given by the estimated covariance matrix of $Z'\Delta u$, i.e.

$$\hat{W}_N = \left[\frac{1}{N} \sum_{i=1}^{N} \left(Z_i' \Delta \hat{u}_i \right)' \left(\Delta \hat{u}_i' Z_i \right) \right]^{-1}$$

While we will test for the validity of the underlying assumptions concerning the orthogonality of instruments and the serial correlation properties of the error term, a concluding note on the validity of using the Arellano-Bond estimator in view of our sample size is warranted. Initially, the estimator was developed for samples with a large cross-sectional (N) and a small time (T) dimension. In our case, as is typical for studies using macro-economic data, neither of the dimensions is particularly large, and the T dimension exceeds the N dimension. However, simulation exercises have shown that the GMM estimator performs relatively well even when both N and T dimensions are small. Specifically, while the bias in the comparable least squares dummy variable (LSDV) estimator can be sizeable regardless of sample size, the small-sample bias in the GMM estimator has proved sufficiently small to make it the estimator of choice for dynamic macro-economic panel data models (Judson and Owen, 1999).

3.3.2 Data

The dataset consists of a panel of 10 EU member states[6], with annual data for the period 1991-2007. Due to the unbalancedness of the panel the total number of observations in the estimations is 121—138.

The ratio of public investment to public consumption expenditure is depicted in Figure 1, in the Appendix. That ratio is, on average, about 0.1, which seems high at the outset, given that total government expenditure in our sample is roughly 50 percent of GDP while investment only amounts to some 2.5 percent of GDP. While we consider government investment as is customary (gross fixed capital formation of the general government), our focus on

[6] Austria, Belgium, Denmark, Finland, France, Germany, Italy, the Netherlands, Sweden, and the UK.

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government consumption means that some categories of current spending, such as interest payments and some subsidies and transfers, are excluded from our denominator.

More specifically, and following Straub and Tchakarov (2007), we employ the variable "final consumption expenditure of the general government", as defined in the UN System of National Accounts, to account for governments' true consumption spending. It comprises non-market output and social transfers in kind related to expenditure on products supplied to households via market producers. Final consumption expenditure thus defined amounts on average to 25 percent of GDP in our sample. All data on government investment are obtained from Eurostat, while the data on government consumption originate from the OECD.

Turning then to our volatility variables of interest, we construct a time series of annual observations on the standard deviation of each. We assume that public expenditure composition in year t can be affected by the volatility of revenues in year t and t-1 but that further lags do not add any explanatory power. The annual observations on the volatility of revenues are computed on the basis of a rolling window covering eight quarters - the year of the observation on the dependent variable and the preceding year. In other words, the volatility variables explaining public expenditure composition in year t are calculated on the basis of quarterly observations in year t and t-1, allowing us to capture volatility innovations ("news") both at the time of budget preparation and during its execution. The standard deviations are calculated using seasonally adjusted quarterly data.

The macro-volatility variables comprise real GDP (labelled volrealgdp henceforth; source Eurostat) and CPI inflation ("volinflation"; source OECD). We also consider the volatility of total tax revenues, both in levels and as a share of GDP ("voltaxtot" and "voltaxtot_gdp", respectively; source Eurostat), among the macro-volatility indicators.

Volatility indicators for individual sub-groups of taxes are based on the breakdown of taxes according to European System of Accounts (ESA), version 1995, and include current taxes on income and wealth (abbreviated "taxiw" in subsequent tables); taxes on capital ("taxc"); taxes on production and imports ("taxpm"); and the VAT ("vat").

- Current taxes on income and wealth comprise all taxes levied regularly on personal and corporate income, as well as taxes on capital gains.

- Taxes on capital include ad hoc taxes such as inheritance taxes, death duties, taxes on gifts and so-called betterment levies (e.g., taxes on the increase in land value due to planning permissions).

- Taxes on production and imports include taxes on products except the VAT (general sales or turnover taxes, excise duties, stamp taxes, taxes on financial and capital transactions, car registration taxes, export duties, etc.); taxes on imports (import duties and all

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other taxes on imports, excluding the VAT); and other taxes on production (e.g., payroll taxes, property taxes on enterprises, licence fees, pollution taxes).

All these taxes, including also the VAT, are reported at the level of the general government. The volatility measures are constructed as explained above, based on a backward-looking eight-quarter rolling window. The volatilities are calculated on each tax type in level terms and in relation to GDP.

Finally, the set of significant controls include log real GDP per capita (log_gdp_pc; source OECD); public debt relative to GDP (debt_gdp; source Ameco); and external trade balance relative to GDP (extbal_gdp; source OECD). We also report the (insignificant) coefficient estimates for a dummy variable indicating EMU participation (emu). We also consider the interaction term of each tax volatility variable (in level terms) and the real GDP variable, so as to discern the direct effect of the tax volatility on the composition of public spending and its indirect effect through GDP (business cycle).

Another possible control variable would be the level of government spending, which has been empirically associated with its volatility (e.g., van Riet, 2010; Albuquerque, 2010). That relationship arises primarily as a result of stronger automatic stabilizers at higher levels of public spending (van Riet, 2010); indeed, Albuquerque (2010) finds that higher levels of public spending are associated with less volatility in discretionary public spending. As we control directly for the impact of the economic cycle, we control indirectly for the impact of automatic stabilizers. For this reason, we choose not to include the level of government spending as yet another control variable.

Table 1, in the Appendix, reports descriptive statistics of the variables employed in the estimation.

Panel unit root test results are reported in Table 2 of the Appendix, including both the Levin, Lin and Chu test assuming homogeneity in the individual unit root processes, and the Im, Pesaran and Shin test allowing for individual heterogeneity in the unit root processes. Both tests confirm that all variables are difference stationary.

Table 3 in the Appendix reports correlation coefficients between the volatility variables. The macro-economic volatility variables are highly correlated with the tax volatility variables, especially in level terms, so we perform separate analyses of how the former affect the composition of public spending and how the latter affect it.

Similarly, the correlation coefficients among the tax volatility variables, when based on real levels data, are high, in some cases 0.8-0.9. The only exception is the capital tax volatility variable, whose correlation coefficient with the other tax volatilities never exceeds 0.5. We therefore group the tax volatilities into three groups within which correlations are low—

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combining the capital tax volatility with each of the others—and run separate regressions for each group. In contrast, when measured relative to GDP the tax volatilities are much less correlated with one another, with the correlation coefficient always below 0.31, so we can include all of them in one regression.

3.3.3 Results

In this section we report the estimation results for the preferred specifications of model (3.1). The results with macro-economic volatility variables as regressors are reported first, including the volatility of aggregate tax revenues, followed by the results with the volatility of sub-groups of taxes as regressors. The tables in the Appendix show the preferred model specifications in terms of the variables treated as endogenous; the number of lags included as instruments; and the set of control variables employed. The robustness of the estimation results to changes in the specification is discussed as appropriate. The interpretation of the results from an economic perspective is done in section 4.

Starting with the impact of macro-economic volatility on the composition of public spending, Table 4 shows the results with real GDP volatility, CPI inflation volatility, and total tax revenue volatility as regressors. Table 4 shows seven different model specifications (A-G) which differ mainly in terms of the controls and variables treated as endogenous. In all cases the number of lags of the dependent and the endogenous variables used as instruments is three. This choice is based on the Sargan test for overidentifying restrictions (shown at the bottom of the table), as well as on the observation that coefficient estimates change materially as the number of lags is increased. This suggests a possible bias from employing too many instruments; hence, we opt for a small number of lags, possibly losing some efficiency in the estimation. Tests for the first and second order residual autocorrelation are also shown at the bottom of the table.

We note based on the diagnostic test results that all seven models are well specified; however, the test statistic for the Sargan test suggests a possible problem with the set of overidentifying restrictions for specification C, where GDP alone is included and is considered endogenous. That is also the only specification where the coefficient for the lagged dependent is insignificant and much smaller in magnitude than otherwise.

In specifications A, D and E both GDP and public debt are endogenous. GDP is insignificant throughout, but lagged public debt (to GDP) is significant. Note that we can interpret the GDP variable as controlling for the impact of business cycles, given that the estimation is done in first differences (which eliminates any trend effects) and given that the data frequency is annual (which eliminates any seasonal effects).

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Both real GDP volatility and CPI inflation volatility are strongly insignificant throughout, as is the control variable trade openness. (In A the EMU dummy was also included but is not shown due to insignificance.) The macro-economic volatility variables are also insignificant in the most parsimonious specification B.

Consequently, innovations to neither GDP volatility nor inflation volatility affect the composition of public spending in our sample. Note that the dynamic specification of the model implies that the estimated impact of the volatility variables measures the impact of contemporaneous innovations to them, over and above the impact of the lagged dependent.

Columns F and G show the preferred specification with the volatility of total tax revenues (in levels) and the volatility of total tax revenues as a share of GDP, respectively, as a regressor. The volatility of total tax revenues is insignificant when measured in levels, but weakly significant when measured in relation to GDP. In the latter case it has a negative sign, implying that a contemporaneous increase in volatility tends to reduce the dependent variable, that is, increase the relative share of public consumption spending at the cost of public investment. The magnitude of that effect is such that a one standard deviation increase in the volatility of total tax revenues, as a share of GDP, reduces the ratio of public investment to consumption from 10.4% to about 10.1% at sample mean.

As the evidence concerning the impact of the volatility of total tax revenues is weak and inconclusive, we consider next the volatility of individual sub-groups of taxes on the composition of public spending. The estimation results are shown in Tables 5-7 in the Appendix. Each table shows a different combination of tax volatilities in levels, based on the sample correlation properties as explained above. Both (the log of per capita) GDP and public debt relative to GDP are considered endogenous to ensure robustly satisfactory diagnostic test results. To test the robustness of the estimation results with respect to changes in controls (including interaction terms), each table shows eight different model specifications (A-H), together with the corresponding diagnostic test results. The coefficient estimates of interest are indicated in bold. When discussing the results we bear in mind the interpretation of the coefficient estimates based on the dynamic model specification, although the discussion is given a more straightforward spin for ease of comprehension.

Table 5 considers the volatility of taxes on income and wealth as well as on capital (in levels, specifications A-D), as well as all tax volatilities in relation to GDP (specifications E-H). The difference between Tables 5-7 in terms of specifications E-H concerns the interaction terms; Table 5 focuses on the interaction terms between GDP and the volatilities of taxes on income and wealth as well as capital.

Starting with the tax volatilities in levels, we note that the volatility of taxes on income

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and wealth is significant and positive throughout, while the volatility of capital taxes is significant and negative throughout. The interaction term between GDP and the volatility of taxes on income and wealth is significant and negative, while the interaction term between GDP and the volatility of capital taxes is insignificant. Of the controls, GDP and public debt (to GDP) are both significant, especially their lags, while neither trade openness nor the EMU dummy is significant.

In sum, innovations to the volatility of taxes on income and wealth tend to increase public investment relative to consumption spending, but their indirect impact through GDP dampens that increase. Innovations to the volatility of capital taxes tend to reduce public investment relative to consumption spending.

Consider then specifications E-H, focusing on the volatilities of these taxes relative to GDP. All controls behave as above, with the estimated coefficients remarkably stable. The volatility of taxes on income and wealth relative to GDP is predominantly significant and positive, and the volatility of capital taxes relative to GDP is predominantly significant and negative. The interaction terms behave as above.

As regards the estimated magnitudes, Table 5 suggests that a one percentage point increase in the volatility of taxes on income and wealth (capital) relative to GDP will increase (reduce) the ratio of public investment to consumption by about 0.08 percentage points (0.3 percentage points). To be more concrete, a one standard deviation increase in the volatility of taxes on income and wealth relative to GDP will increase the ratio of public investment to consumption by about 0.1 percentage point (from 10.4 to 10.3 percent at sample mean).

All in all, the volatility of taxes on income and wealth has a robustly positive direct impact, increasing public investment relative to consumption. The indirect cyclical effect dampens that relative gain of public investment. The volatility of capital taxes has a robust negative direct impact, decreasing public investment relative to consumption.

Next, consider Table 6 in the Appendix showing the estimation results with a focus on taxes on production and imports. The volatility of capital taxes is considered alongside as above as a robustness check, and we note that the results with respect to it are remarkably similar to those reported in Table 5.

Measured in level terms the volatility of taxes on production and imports has a significant and positive impact, with the GDP interaction dampening it. However, measured relative to GDP, that volatility is no longer significant either directly or through its interaction with GDP. In sum, the volatility of taxes on production and imports does not have an unambiguously significant effect on the composition of public spending.

Finally, Table 7 in the Appendix reports the estimation results with a special focus on the

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VAT. Again, we confirm the robustness of the results pertaining to the volatility of capital taxes.

Measured in level terms, the volatility of VAT revenues has a significant and positive direct effect, dampened by its interaction with GDP. These results are confirmed when the volatility of VAT receipts is measured relative to GDP.

Note that in terms of the estimated magnitude of the impact, volatility of VAT and volatility of capital taxes are relatively similar. Their volatility is orders of magnitude bigger than the volatility of taxes on income and wealth. Thus, a one standard deviation increase in the volatility of taxes on VAT relative to GDP will increase the ratio of public investment to consumption by about 1.4 percentage points (from 10.4 to 11.8 percent at sample mean).

3.4 Economic Interpretation of Results

Our key results can be summarised as follows:

- The volatility of GDP or CPI inflation, and innovations to it, do not directly affect the composition of public spending. There is some evidence that the volatility of total tax revenues increases public consumption spending at the cost of investment; however, that evidence is weak and inconclusive;
- The volatility of taxes on income and wealth as well as of VAT tend to increase public investment relative to consumption spending, but their indirect impact through GDP dampens that increase;
- The volatility of capital taxes tend to reduce public investment relative to consumption spending;
- The volatility of taxes on production and imports does not have an unambiguously significant effect on the composition of public spending;
- In terms of the estimated magnitudes, the volatility of VAT and capital taxes has the biggest impact on the composition of public spending.

In other words, the composition of public spending is not affected directly by macroeconomic "news". This finding is in line with earlier literature (Afonso, Agnello and Furceri, 2008), which considers not only fiscal "responsiveness" to output fluctuations but also fiscal "persistence" (inertia) and fiscal "discretion". Considering all three aspects of fiscal policy at the same time leads to the conclusion that fiscal persistence is the dominant aspect and, when accounting for it, fiscal responsiveness becomes empirically weak or insignificant. As we account for fiscal persistence in our dynamic set-up of the empirical model, the finding that cyclical responsiveness appears insignificant is, indeed, to be expected.

However, we find that the macro-economic conditions do affect public expenditure com-

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position indirectly through their revenue impact, which is clearly visible at the level of individual tax groups, less so at the level of total tax revenues. Note that the volatility of GDP is very highly correlated with the volatility of tax revenues; given that tax revenues can be observed more directly and more frequently by the policy maker than GDP, it is reasonable to assume that he uses tax revenues as a primary source of information about the underlying economic conditions.

The relative share of public investment increases following increases in the volatility of income taxes and the VAT, despite the fact that the indirect impact of these volatilities through the business cycle (GDP) works in the opposite direction. The relative share of public consumption spending, in turn, increases with increases in the volatility of capital taxes.

So we have found that even controlling for the effect of business cycles (as well as for fiscal persistence), volatility innovations to tax receipts have a significant impact on the composition of public spending. Changes in the volatility of VAT and income tax receipts tilt the composition of public spending in favour of public investment, while changes in the volatility of capital taxes tilt the composition against it.

These findings raise two broad questions. First, why is the impact of tax volatility visible at the level of individual tax groups but not at the level of total tax revenues? And second, why do innovations to the volatility of individual tax groups affect the composition of public spending the way they do? We will address both these broad questions in turn.

The first question is empirical in character. The near-insignificance of the volatility of total tax revenues is obviously a sum of effects at the level of individual tax groups that offset one another to a great extent. The individual tax groups respond to different underlying economic factors, so such individual effects can provide valuable and timely news to the policy maker about underlying economic conditions. After all, many taxes are collected on a monthly basis, so news embedded in their collection can convey information about different kinds of incipient changes in the economic environment.

But what exactly can such news tell the policy maker? This leads us to the second question concerning possible reasons for the observed effects at the level of individual tax groups. This question, in turn, breaks down into two sub-questions: Why do tax volatilities change (which is an empirical question) and why do those changes have the observed effect on public expenditure composition (which is a theoretical question in view of Section 2.2).

There are, in principle, two possible reasons for changes in tax revenues and, hence, their volatility: changes in tax rates or tax bases.

First, changes in contemporaneous tax volatility can reflect changes in tax rates. Such rate changes are known to the policymaker in advance, and he can change the expenditure composition based on that advance knowledge. Thus, changes in the VAT and income tax rates could, ceteris paribus, raise the contemporaneous volatilities of VAT and income tax revenues temporarily, and that contemporaneous and temporary increase in volatility could be accompanied with a shift in the expenditure composition. However, tax rates do not change very often, so changes in tax rates are unlikely to be the dominant driver of tax volatilities.

Second, changes in contemporaneous tax volatility can reflect changes in tax bases. The dominant driver of changes in tax bases—especially those for the VAT and income taxes—is the business cycle. However, to the extent that the cyclical situation differs from what was known or expected at the time of budgeting revenues, the tax base; tax revenues; and their volatility are also different from what was expected. Such unexpected changes could conceivably translate into the kinds of effects observed in this study: a sudden change in the cyclical outlook could prompt the policy maker to change the composition of public spending.

Specifically, the finding that volatility innovations to these types of taxes tend to increase the relative share of public investment could be interpreted in light of the theoretical considerations presented in Section 2.2. as follows.

Consider first an increase in revenue volatility due to a negative event (e.g., unusually low VAT collection). This could signal to the fiscal policymaker that the cyclical situation is worsening. An active Keynesian policymaker would, ceteris paribus, be inclined to boost the relative share of public investment, assuming that its multiplier is higher than that of public consumption. A neoclassical policymaker would do the same, assuming that public and private consumption are complements in utility.

Consider next an increase in revenue volatility due to a positive event (e.g., unusually high VAT collection). An increase in the relative share of public investment would now only take place under a neoclassical policymaker regarding public and private consumption as substitutes.

Capital taxes, as explained in section 3.2, are ad hoc in character, so their base has a significant random element to it—as also suggested by the volatile capital tax revenues in our sample (see Table 1). Thus, innovations to the volatility of capital tax base and revenues are by nature unexpected and temporary. They do, therefore, not necessarily convey useful information to the fiscal policymaker about the cyclical situation.

That such one-off volatility shocks tend to increase the relative share of public consumption may be related to the long-term character of public investment projects, mentioned in

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Section 2.2. One-off positive revenue shocks do not provide a basis for financing long-term investment projects; hence, they will be spent on consumption (they could also be saved, of course). The link between one-off negative revenue shocks and an increase in the relative share of public consumption is more difficult to interpret. That a one-off negative revenue shock would cause an increase in public consumption does not seem plausible. It may, however, cause a temporary decline in public investment, if a temporary halt to the financing of long-term investment causes less economic or political damage than disruptions to government consumption.

In any event, such one-off shocks tend to have a quantitatively small impact on public spending composition, so we should not over-interpret their repercussions.

To sum up, the observed effect of tax volatilities on the composition of public spending may be related to the way policy makers react to news embedded in tax revenues, among other similar sources of news. Revenue surprises linked to the cyclical situation could conceivably boost the relative share of public investment. Revenue surprises linked to temporary factors, in turn, are of smaller importance and more difficult to interpret.

These explanations for the results obtained are, of course, speculative. The theoretical discussion did not provide us with unambiguous and empirically testable hypotheses, so the link between theory and the empirical results should not be overemphasised. The explanations suggested above are, however, economically plausible and something for future research to validate or challenge.

3.5 Conclusion

While earlier literature has considered long-term (growth) and medium-term (cyclical) aspects of macro-fiscal volatility, our study has focussed on its short-term impact. Specifically, we consider contemporaneous changes in the volatility of macro-economic and fiscal (revenue-side) variables as news to the policy maker and seek to examine the impact of such news on the composition of public spending.

We find that news about growth or inflation are immaterial for the composition of public spending as such; however, they do have a significant impact through news about revenue collections, visible at the level of individual tax groups and less so at the level of total tax revenues. Contemporaneous increases in the volatility of taxes such as the VAT or income taxes are that are frequently collected tend to increase the share of public investment relative to consumption spending, possibly because they convey news about cyclical changes in the underlying economic conditions. In contrast, contemporaneous increases in the volatility of ad hoc of taxes such as capital taxes tend to increase the relative share of public consumption

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spending, but that impact is relatively weak and more difficult to interpret.

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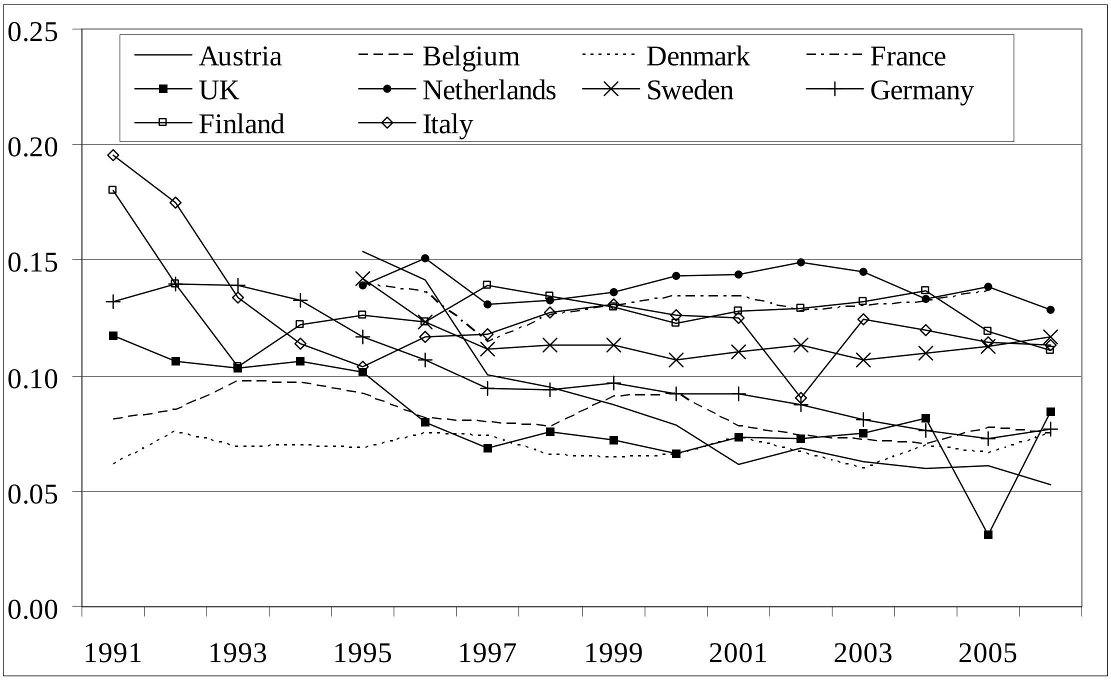
3.6 Appendix A: Figures

Figure 1: Dependent Variable: ratio of public investment to public consumption expenditure. (Source: Eurostat, OECD and authors' calculations)



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3.7 Appendix B: Tables

Table 1: Descriptive Statistics of the Data

	N	mean	max	min	sd
IC	160	0.104	0.177	0.025	0.031
volrealgdp	158	2549.882	10433.5	247.124	2504.076
volinfl	142	0.330	1.156	0.069	0.181
voltaxtot	158	1751.951	7529.375	86.280	1652.336
voltaxtot_gdp	158	0.223	1.419	0.009	0.210
voltaxiw	158	700.413	3589.953	7.009	755.090
voltaxiw_gdp	158	0.148	0.520	0.006	0.123
voltaxc	158	29.264	316.740	0.132	48.525
voltaxc_gdp	158	0.009	0.093	0.000	0.015
voltaxpm	158	703.948	2653.672	28.039	645.411
voltaxpm_gdp	158	0.103	1.044	0.003	0.146
voltaxvat	158	350.745	1287.957	18.817	320.985
voltaxvat_gdp	158	0.052	0.221	0.002	0.041
log_GDP_pc	160	8.790	12.587	4.259	3.050
debt_gdp	160	0.671	1.340	0.221	0.255
extbal_gdp	160	0.029	0.115	-0.032	0.033
emu	170	0.618	1.000	0.000	0.487

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Table 2: Unit Root Tests

			Levin, Lin, Chu		Im, Pesaran & Shin	
	stationarity	N	statistics	p-value	statistics	p-value
IC	level	147	-2.219	0.013	-2.654	0.004
	difference	139	-9.151	0.000	-7.323	0.000
volrealgdp	level	145	-5.360	0.000	-4.548	0.000
	difference	135	-10.748	0.001	-9.395	0.000
volinfl	level	123	-5.384	0.000	-4.353	0.000
	difference	118	-7.457	0.000	-4.811	0.000
voltaxtot	level	130	-1.553	0.060	-2.652	0.004
	difference	123	-7.268	0.000	-5.525	0.000
voltaxtot_gdp	level	136	-3.459	0.000	-2.526	0.006
	difference	129	-4.580	0.000	-3.788	0.000
voltaxiw	level	131	2.483	0.994	-1.003	0.158
	difference	125	-4.530	0.000	-3.652	0.000
voltaxiw_gdp	level	134	-4.879	0.001	-3.133	0.001
	difference	131	-6.587	0.002	-5.034	0.000
voltaxc	level	134	-1.240	0.108	-1.150	0.125
	difference	127	-6.064	0.000	-4.381	0.000
voltaxc_gdp	level	135	-1.621	0.053	-2.105	0.018
	difference	127	-5.742	0.000	-4.628	0.000
voltaxpm	level	131	-7.278	0.001	-8.748	0.000
	difference	122	-7.570	0.002	-6.886	0.001
voltaxpm_gdp	level	136	-1.948	0.026	-1.891	0.029
	difference	131	-7.432	0.000	-4.829	0.000
voltaxvat	level	133	-5.905	0.000	-3.171	0.001
	difference	121	-9.755	0.001	-7.590	0.000
voltaxvat_gdp	level	137	-8.378	0.002	-5.530	0.000
	difference	128	-5.653	0.000	-4.439	0.000
log_gdp_pc	level	146	-1.256	0.105	3.624	1.000
	difference	140	-6.265	0.000	-4.779	0.000
debt_gdp	level	143	-1.348	0.089	-0.955	0.170
	difference	138	-4.696	0.000	-2.887	0.002
extbal_gdp	level	148	-0.068	0.473	1.953	0.975
	difference	140	-7.668	0.000	-6.105	0.000

Note: Automatic selection of lags by SIC

Table 3: Correlation Matrix

	voltaxtot	volrealgdp	volinflation	voltaxiw	voltaxpm	volvat	voltaxc	log_gdp_pc	debt_gdp	extbal_gdp
voltaxtot	1.000									
volrealgdp	0.801	1.000								
volinflation	-0.385	-0.407	1.000							
voltaxiw	0.947	0.757	-0.348	1.000						
voltaxpm	0.957	0.732	-0.384	0.826	1.000					
volvat	0.959	0.794	-0.423	0.847	0.954	1.000				
voltaxc	0.439	0.299	-0.362	0.440	0.452	0.396	1.000			
log_gdp_pc	-0.161	-0.146	0.201	-0.013	-0.181	-0.152	-0.411	1.000		
debt_gdp	-0.134	-0.145	-0.215	-0.132	0.005	-0.120	0.401	-0.227	1.000	
extbal_gdp	-0.530	-0.503	0.535	-0.516	-0.514	-0.530	-0.394	0.043	-0.177	1.000

	voltaxtot_gdp	volrealgdp	volinflation	voltaxiw_gdp	voltaxpm_gdp	volvat_gdp	votaxc_gdp	log_gdp_pc	debt_gdp	extbal_gdp
voltaxtot_gdp	1.000									
volrealgdp	-0.097	1.000								
volinflation	0.086	-0.407	1.000							
voltaxiw_gdp	0.623	-0.045	0.069	1.000						
voltaxpm_gdp	0.805	-0.069	0.267	0.307	1.000					
volvat_gdp	0.252	-0.133	0.262	0.086	0.266	1.000				
voltaxc_gdp	0.068	-0.009	-0.171	0.022	0.042	-0.061	1.000			
log_gdp_pc	0.187	-0.146	0.201	0.006	0.141	0.146	-0.417	1.000		
debt_gdp	0.162	-0.145	-0.215	0.012	0.129	-0.024	0.608	-0.227	1.000	
extbal_gdp	0.128	-0.503	0.535	0.171	0.117	0.041	-0.113	0.043	-0.177	1.000

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Table 4: Estimation results: Macro-volatility (dependent variable: ratio of public investment to public consumption spending)

	A coef/p-value	B coef/p-value	C coef/p-value	D coef/p-value	E coef/p-value	F coef/p-value	G coef/p-value
LD.i_c	0.335***	0.467***	0.119	0.346***	0.335***	0.673***	0.256**
	(0.000)	*(0.000)*	*(0.320)*	*(0.000)*	*(0.000)*	*(0.000)*	*(0.006)*
D.log_gdp_pc	0.077		-0.201	0.097	0.077		0.040
	(0.599)		*(0.106)*	*(0.502)*	*(0.597)*		*(0.769)*
LD.log_gdp_pc	-0.136		0.160	-0.162	-0.136		-0.116
	(0.345)		*(0.186)*	*(0.246)*	*(0.343)*		*(0.375)*
D.debt_gdp	0.038			0.041	0.038		0.022
	(0.423)			*(0.393)*	*(0.421)*		*(0.618)*
LD.debt_gdp	-0.1090**			-0.114**	-0.109**		-0.079*
	(0.021)			*(0.015)*	*(0.021)*		*(0.056)*
D.volrealgdp	0.000	0.000	0.000	0.000	0.000		0.000
	(0.221)	*(0.304)*	*(0.102)*	*(0.217)*	*(0.219)*		*(0.243)*
D.volinflation	0.002	0.003	-0.001	0.003	0.002		-0.004
	(0.831)	*(0.779)*	*(0.947)*	*(0.772)*	*(0.830)*		*(0.659)*
D.voltaxtot						0.000	
						(0.794)	
D.voltaxtot_gdp							-0.012*
							(0.072)
D.extbal_gdp	-0.069				-0.069		
	(0.424)				*(0.422)*		
Number of observations	121	121	121	121	121	121	121
lags	*3*	*3*	*3*	*3*	*3*	*3*	*3*
p_sargan	*0.289*	*0.279*	*0.075*	*0.260*	*0.269*	*0.238*	*0.130*
p_ar1	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*	*0.000*
p_ar2	*0.623*	*0.695*	*0.614*	*0.605*	*0.622*	*0.832*	*0.617*

note: *** p<0.01, ** p<0.05, * p<0.1

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Table 5. Estimation results: Focus on taxes on income and wealth as well as on capital (dependent variable: ratio of public investment to public consumption spending)

	A	B	C	D	E	F	G	H
	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value
LD.i_c	0.228793***	0.251217***	0.207281**	0.230412***	0.300198***	0.240334***	0.232415***	0.270123***
	(0.007545)	*(0.002693)*	*(0.020572)*	*(0.005855)*	*(0.000435)*	*(0.005975)*	*(0.006836)*	*(0.002012)*
D.log_gdp_pc	0.168883	0.189519*	0.194885*	0.180651*	0.174148	0.152396	0.165895	0.137008
	(0.110378)	*(0.075607)*	*(0.063241)*	*(0.088136)*	*(0.112543)*	*(0.162703)*	*(0.121339)*	*(0.216727)*
LD.log_gdp_pc	-0.224076**	-0.249594**	-0.238574**	-0.221615**	-0.238261**	-0.258703**	-0.271241**	-0.222620**
	(0.031779)	*(0.015652)*	*(0.019928)*	*(0.032446)*	*(0.030571)*	*(0.019607)*	*(0.012313)*	*(0.045885)*
D.debt_gdp	0.075490*	0.071932	0.087893**	0.077915*	0.034199	0.026144	0.024537	0.020779
	(0.076538)	*(0.100871)*	*(0.034397)*	*(0.072627)*	*(0.414874)*	*(0.507364)*	*(0.532743)*	*(0.605960)*
LD.debt_gdp	-0.150808***	-0.148278***	-0.153946***	-0.137844***	-0.102235**	-0.106416***	-0.108052***	-0.093779**
	(0.000479)	*(0.000502)*	*(0.000367)*	*(0.001073)*	*(0.016209)*	*(0.008714)*	*(0.007631)*	*(0.023620)*
D.voltaxiw	**0.000004***	**0.000004***	**0.000018****	**0.000018****				
	(0.065290)	*(0.087637)*	*(0.025375)*	*(0.012871)*				
D.voltaxc	**-0.000139*****	**-0.000134*****	**-0.000126**	**-0.000145*****				
	(0.000101)	*(0.000105)*	*(0.503659)*	*(0.000041)*				
D.extbal_gdp	-0.102777		-0.121021	-0.102942	-0.058021	-0.036545	-0.033876	-0.016660
	(0.159396)		*(0.103982)*	*(0.147609)*	*(0.436528)*	*(0.620785)*	*(0.644298)*	*(0.823450)*
D.emu	0.004166		0.003843		0.001825	0.003969	0.003716	0.002531
	(0.344540)		*(0.385678)*		*(0.689219)*	*(0.381110)*	*(0.408369)*	*(0.575448)*
D.voltaxiw_loggdppc			**-0.000002**	**-0.000002****				
			(0.105388)	*(0.030561)*				
D.voltaxc_loggdppc			**-0.000006**					
			(0.893333)					
D.voltaxiw_gdp					**-0.008966**	**0.078863****	**0.081129****	**0.083157****
					(0.415097)	*(0.028891)*	*(0.021125)*	*(0.021201)*
D.voltaxpm_gdp					-0.009969	-0.030447	-0.014925	-0.001996
					(0.249997)	*(0.505403)*	*(0.128826)*	*(0.826803)*
D.volvat_gdp					0.050400*	0.355505***	0.335552***	0.062590**
					(0.090650)	*(0.001426)*	*(0.000147)*	*(0.038484)*
D.voltaxc_gdp					**-0.305184*****	**-0.450706**	**-0.299578*****	**-0.432723**
					(0.006127)	*(0.100714)*	*(0.005836)*	*(0.121187)*
D.voltaxiwgdp_ loggdppc						**-0.009731****	**-0.009819****	**-0.010373*****
						(0.012176)	*(0.010272)*	*(0.008174)*
D.voltaxpmgdp_ loggdppc						0.001395		
						(0.706853)		
D.volvatgdp_ loggdppc						-0.031659***	-0.029762***	
						(0.003603)	*(0.001329)*	
D.voltaxcgdp_ loggdppc						**0.027787**		**0.016449**
						(0.571248)		*(0.743656)*
Number obs	138	138	138	138	138	138	138	138
lags	3	3	3	3	3	3	3	3
p_sargan	*0.280650*	*0.239957*	*0.307360*	*0.275901*	*0.163144*	*0.190828*	*0.161312*	*0.198632*
p_ar1	*0.000001*	*0.000001*	*0.000001*	*0.000001*	*0.000000*	*0.000000*	*0.000000*	*0.000000*
p_ar2	*0.433567*	*0.449710*	*0.377300*	*0.402811*	*0.405322*	*0.476528*	*0.501198*	*0.559585*

note: *** p<0.01, ** p<0.05, * p<0.1

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Table 6. Estimation results: Focus on taxes on production and imports (dependent variable: ratio of public investment to public consumption spending)

	A	B	C	D	E	F	G	H
	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value
LD.i_c	0.212915**	0.226054***	0.202533**	0.223137***	0.300198***	0.240334***	0.232415***	0.306414***
	(0.012424)	*(0.007100)*	*(0.021245)*	*(0.008055)*	*(0.000435)*	*(0.005975)*	*(0.006836)*	*(0.000401)*
D.log_gdp_pc	0.206197**	0.205139*	0.242151**	0.215161**	0.174148	0.152396	0.165895	0.177941
	(0.047654)	*(0.051874)*	*(0.021800)*	*(0.041241)*	*(0.112543)*	*(0.162703)*	*(0.121339)*	*(0.113302)*
LD.log_gdp_pc	-0.259136**	-0.253042**	-0.282501***	-0.254565**	-0.238261**	-0.258703**	-0.271241**	-0.243381**
	(0.011664)	*(0.014145)*	*(0.006133)*	*(0.013450)*	*(0.030571)*	*(0.019607)*	*(0.012313)*	*(0.031816)*
D.debt_gdp	0.086420**	0.083033*	0.115886***	0.097937**	0.034199	0.026144	0.024537	0.033963
	(0.043797)	*(0.058285)*	*(0.006644)*	*(0.026840)*	*(0.414874)*	*(0.507364)*	*(0.532743)*	*(0.425348)*
LD.debt_gdp	-0.159971***	-0.150626***	-0.178958***	-0.158141***	-0.102235**	-0.106416***	-0.108052***	-0.104348**
	(0.000207)	*(0.000388)*	*(0.000028)*	*(0.000196)*	*(0.016209)*	*(0.008714)*	*(0.007631)*	*(0.015112)*
D.voltaxpm	**0.000004***	**0.000004**	**0.000021***	**0.000022***				
	(0.096032)	*(0.116488)*	*(0.003715)*	*(0.002905)*				
D.voltaxc	**-0.000124***	**-0.000118***	**-0.000114**	**-0.000104***				
	(0.000164)	*(0.000248)*	*(0.374081)*	*(0.001664)*				
D.extbal_gdp	-0.125453*	-0.110758	-0.152973**	-0.139947*	-0.058021	-0.036545	-0.033876	-0.058357
	(0.085165)	*(0.121475)*	*(0.042262)*	*(0.054425)*	*(0.436528)*	*(0.620785)*	*(0.644298)*	*(0.436119)*
D.emu	0.004191		0.002858		0.001825	0.003969	0.003716	0.002408
	(0.332338)		*(0.514401)*		*(0.689219)*	*(0.381110)*	*(0.408369)*	*(0.602559)*
D.voltaxpm_loggdppc			**-0.000002****	**-0.000002*****				
			(0.011062)	*(0.008455)*				
D.voltaxc_loggdppc			**0.000002**					
			(0.928035)					
D.voltaxiw_gdp					-0.008966	0.078863**	0.081129**	-0.006239
					(0.415097)	*(0.028891)*	*(0.021125)*	*(0.598262)*
D.voltaxpm_gdp					**-0.009969**	**-0.030447**	**-0.014925**	**0.021112**
					(0.249997)	*(0.505403)*	*(0.128826)*	*(0.577333)*
D.volvat_gdp					0.050400*	0.355505***	0.335552***	0.037549
					(0.090650)	*(0.001426)*	*(0.000147)*	*(0.265163)*
D.voltaxc_gdp					**-0.305184****	**-0.450706+**	**-0.299578****	**-0.372931**
					(0.006127)	*(0.100714)*	*(0.005836)*	*(0.185777)*
D.voltaxiwgdp_ loggdppc						-0.009731**	-0.009819**	
						(0.012176)	*(0.010272)*	
D.voltaxpmgdp_ loggdppc						**0.001395**		**-0.002708**
						(0.706853)		*(0.408084)*
D.volvatgdp_ loggdppc						-0.031659***	-0.029762***	
						(0.003603)	*(0.001329)*	
D.voltaxcgdp_ loggdppc						**0.027787**		**0.015165**
						(0.571248)		*(0.765019)*
Number obs	138	138	138	138	138	138	138	138
lags	3	3	3	3	3	3	3	3
p_sargan	*0.382172*	*0.341106*	*0.450411*	*0.489931*	*0.163144*	*0.190828*	*0.161312*	*0.211623*
p_ar1	*0.000002*	*0.000001*	*0.000002*	*0.000001*	*0.000000*	*0.000000*	*0.000000*	*0.000000*
p_ar2	*0.429669*	*0.470479*	*0.516694*	*0.507533*	*0.405322*	*0.476528*	*0.501198*	*0.405528*

note: *** p<0.01, ** p<0.05, * p<0.1

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Table 7. Estimation results: Focus on VAT (dependent variable: ratio of public investment to public consumption spending)

	A	B	C	D	E	F	G	H
	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value	coef/p-value
LD.i_c	0.204621**	0.218443***	0.195559**	0.214629**	0.300198***	0.240334***	0.232415***	0.265101***
	(0.015793)	*(0.008936)*	*(0.025795)*	*(0.010452)*	*(0.000435)*	*(0.005975)*	*(0.006836)*	*(0.001951)*
D.log_gdp_pc	0.199339*	0.197849*	0.230149**	0.205006*	0.174148	0.152396	0.165895	0.186252*
	(0.057263)	*(0.062831)*	*(0.030042)*	*(0.052990)*	*(0.112543)*	*(0.162703)*	*(0.121339)*	*(0.090976)*
LD.log_gdp_pc	-0.257401**	-0.250462**	-0.269611***	-0.242658**	-0.238261**	-0.258703**	-0.271241**	-0.272604**
	(0.012961)	*(0.016046)*	*(0.009848)*	*(0.019667)*	*(0.030571)*	*(0.019607)*	*(0.012313)*	*(0.014689)*
D.debt_gdp	0.084559**	0.081141*	0.111675***	0.094209**	0.034199	0.026144	0.024537	0.043647
	(0.045462)	*(0.060811)*	*(0.008065)*	*(0.031004)*	*(0.414874)*	*(0.507364)*	*(0.532743)*	*(0.292637)*
LD.debt_gdp	-0.157010***	-0.147456***	-0.174530***	-0.152954***	-0.102235**	-0.106416***	-0.108052***	-0.117874***
	(0.000211)	*(0.000407)*	*(0.000032)*	*(0.000239)*	*(0.016209)*	*(0.008714)*	*(0.007631)*	*(0.005163)*
D.volvat	**0.000011***	**0.000010***	**0.000036****	**0.000036****				
	(0.053394)	*(0.063173)*	*(0.020011)*	*(0.017205)*				
D.voltaxc	**-0.000122*****	**-0.000116*****	**-0.000120**	**-0.000110*****				
	(0.000187)	*(0.000289)*	*(0.352526)*	*(0.000661)*				
D.extbal_gdp	-0.124410*	-0.109617	-0.146973*	-0.134776*	-0.058021	-0.036545	-0.033876	-0.077023
	(0.085970)	*(0.123296)*	*(0.050751)*	*(0.063734)*	*(0.436528)*	*(0.620785)*	*(0.644298)*	*(0.297038)*
D.emu	0.004320		0.003668		0.001825	0.003969	0.003716	0.003440
	(0.317459)		*(0.401107)*		*(0.689219)*	*(0.381110)*	*(0.408369)*	*(0.444372)*
D.volvat_loggdppc			**-0.000004***	**-0.000004***				
			(0.061881)	*(0.060314)*				
D.voltaxc_loggdppc			**0.000002**					
			(0.941658)					
D.voltaxiw_gdp					-0.008966	0.078863**	0.081129**	-0.006914
					(0.415097)	*(0.028891)*	*(0.021125)*	*(0.530535)*
D.voltaxpm_gdp					-0.009969	-0.030447	-0.014925	-0.022466**
					(0.249997)	*(0.505403)*	*(0.128826)*	*(0.021331)*
D.volvat_gdp					**0.050400***	**0.355505*****	**0.335552*****	**0.347602*****
					(0.090650)	*(0.001426)*	*(0.000147)*	*(0.000124)*
D.voltaxc_gdp					**-0.305184*****	**-0.450706**	**-0.299578*****	**-0.428606**
					(0.006127)	*(0.100714)*	*(0.005836)*	*(0.119209)*
D.voltaxiwgdp_ loggdppc						-0.009731**	-0.009819**	
						(0.012176)	*(0.010272)*	
D.voltaxpmgdp_ loggdppc						0.001395		
						(0.706853)		
D.volvatgdp_ loggdppc						**-0.031659*****	**-0.029762*****	**-0.032515*****
						(0.003603)	*(0.001329)*	*(0.000605)*
D.voltaxcgdp_ loggdppc						**0.027787**		**0.033070**
						(0.571248)		*(0.503771)*
Number obs	138	138	138	138	138	138	138	138
lags	3	3	3	3	3	3	3	3
p_sargan	*0.377504*	*0.335572*	*0.402109*	*0.426898*	*0.163144*	*0.190828*	*0.161312*	*0.185912*
p_ar1	*0.000002*	*0.000001*	*0.000003*	*0.000001*	*0.000000*	*0.000000*	*0.000000*	*0.000000*
p_ar2	*0.405149*	*0.448490*	*0.460832*	*0.467493*	*0.405322*	*0.476528*	*0.501198*	*0.315477*

note: *** p<0.01, ** p<0.05, * p<0.1