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Imperfect Information and Financial Markets
A General Equilibrium Model*

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1. Introduction

The informational approach to economic theory has challenged several traditional views on the working of financial markets. It is by now commonly held that there are typical informational asymmetries which are likely to affect the market-determined allocation of capital across firms: these account for a number of features - both behavioural and institutional - which had remained hitherto unexplained.

As is well known, two information related mechanisms stand out as most significant in explaining widespread market failures: moral hazard and adverse selection. As put forth by Laffont and Maskin (1980), the former is generated by an agent's behaviour being unobservable by others; the latter is rather due to the information of one side of the market being imperfectly known to the other. This distinction is relevant, among other things, because of the competitive assumptions nested in each framework. Moral hazard is mainly a noncompetitive phenomenon: it arises when some agent(s) control(s) some variable(s) which cannot be freely observed by other(s), and can therefore be built in - indeed, it is fairly typical of - models of two-party contractual arrangements. However, one need have a "proper" market (with "many" agents) for adverse selection to work: there is a competitive bent in agents taking both their individual quality and the market-determined average quality as given[1].

In this paper we focus on the simplest example of informational asymmetries and their effect on the equilibrium capital allocation of a competitive economy. More precisely, we consider a (nonstochastic) two-period general equilibrium setup: on the one hand, competitive

firms facing a nonlinear (convex) technology choose their capital requirements, to be financed by share issuing; on the other hand, consumers allocate their endowments between current and future consumption, using the firms' shares as the only store of value. The existence of a perfectly competitive share market allows the resulting equilibrium capital allocation to be jointly determined together with an equilibrium share allocation. It is in this framework that asymmetric information is introduced, and adverse selection effects examined.

Asymmetric information is brought in under the simple assumption that no shareholder knows the technology each firm is using, neither can he improve upon his knowledge. Adverse selection follows from this assumption, which appears to suit the competitive bent of the model. Now, that adverse selection itself may lead to inefficient allocations is actually an intuitive and well known result. However, the purpose of the model is to give a general equilibrium outlook to such result. On the one hand, this allows to bring out explicitly the overall effects of the agents' imperfect information on the firms' choices; on the other hand, it points to the relationship between the adverse selection mechanism and the intertemporal pattern of resource allocation enforced by a competitive equilibrium on the share market.

The above implications are studied within the stockholders' equilibrium notion, which Drèze (1972, 1974) developed in a different framework. This notion is required to overcome a potential inconsistency between the firms' and the investors' behaviours, arising when a share market is explicitly modeled in which both act as competitive price takers: while any firm's optimal capital requirement is based on marginal profit evaluation, any shareholder's investment decision is determined by the unit average return on such investment.

The main result of the paper is that the agents'

intertemporal preferences do interact with adverse selection, so that there may be positive trade equilibria even when agents are very pessimistic about the firms' (unknown) quality. Moreover, the agents' attitude towards risk affects the equilibrium outcome in such a way that different (subjectively held) quality evaluations are consistent with equilibrium only when traders have different, risk averse preferences.

The paper is organized as follows. In the next section the basic model is presented. In a nonstochastic competitive economy, a set of agents owning "technologies" try to sell them to a set of consumers. The latter exploit these technologies by setting up equity financed firms. Section 3 examines the natural result that, under complete information, bad quality technologies may not be used. Section 4 takes up the asymmetric information case: if consumers cannot distinguish good from bad quality, the equilibrium capital allocation is independent of firms' (and technologies') types but depends on the agents' quality evaluations; some efficiency aspects are also discussed. Finally, section 5 provides some concluding remarks. All proofs are gathered in the Appendix.

2. The basic model

We consider a one commodity, two-period economy, where production has to be implemented in the first period to yield output in the second[2]. Since consumption takes place in both periods, an intertemporal allocation problem arises: consumers exchange part of their wealth in period t=1 for output to be delivered at t=2, so that the transfer of resources over time involves trading in assets. Here follows a more detailed description of the economy.

There are two classes of agents: technology sellers (TS's) and consumers. Both own commodity endowments, but the

former are also endowed with "technologies", i.e. blueprints which specify how output can be produced. As was pointed out, only one perishable good ("corn") is traded in this economy: as input in the first period and output in the second, as consumption good in both periods.

2.1. Preferences, technologies, resources

In the economy there are m TS's, belonging to the set $M:=\{1,\ldots,m\}$. Each of them, $j\in M$, lives two periods, $t=1,2$, and is identified by his preferences and by the technology he owns prior to trade. For each $j\in M$, a consumption plan is a nonnegative vector $c_j:=(c_{j1},c_{j2})\in\mathbb{R}^2_+$, his consumption set. His intertemporal preferences are then described by a utility function $u_j:\mathbb{R}^2_+\to\mathbb{R}_+$, which is monotonically increasing, twice continuously differentiable and strictly quasi-concave.

Each agent $j\in M$ also owns a technology, which is decribed by a production set, Y_j. Production takes one period to be carried out. Each point in Y_j is a pair of input (x_j) employed at t=1 and output (y_j) delivered at t=2. For each TS $j\in M$, therefore, we define

$$Y_j:=\{(x_j,y_j)\,|\,(x_j,y_j)\in\mathbb{R}^2_+;\ y_j\leq f_j(x_j)\} \qquad (1)$$

where inputs are measured by nonnegative real numbers. The production function $f_j(x_j):\mathbb{R}_+\to\mathbb{R}_+$ is used as a shorthand to describe technology j. We assume that any production function exhibits decreasing returns to scale. We also assume that there is no positive input value at which two different technologies yield the same (absolute as well as marginal) level of output: as will be clear, this simplifies the criterion by which technologies can be ranked. Our assumptions on production functions are summarized as follows.

A.1. For all $j \in M$, $f_j(x_j)$ is at least twice continuously differentiable and such that:

(a) $f_j(0)=0$;

(b) $0 < f'_j(x_j) < \infty$, for all $x_j \geq 0$;

(c) $f''_j(x_j) < 0$, for all $x_j > 0$;

(d) for any $j,k \in M$ ($j \neq k$), there is no $x>0$ such that $f_j(x) = f_k(x)$ or $f'_j(x) = f'_k(x)$.

Like TS's, consumers - belonging to the set $N:=\{m+1,\ldots,n\}$ - live two periods. Each of them is accordingly endowed with a monotonically increasing, twice continuously differentiable and strictly quasi-concave utility function, $u_i: R^2_+ \to R_+$, defined over consumption plans $c_i \in R^2_+$, $i \in N$. It is assumed that, for at least one consumer, the marginal rate of substitution between current and future consumption vanishes as the latter goes to zero. Our assumptions on preferences are summarized as follows[3].

A.2. For any $a \in A := M \cup N$, $u_a(c_{a1},c_{a2})$ is twice continuously differentiable and such that:

(a) $(\delta u_a/\delta c_{at})>0$, for all $c_{at} \geq 0$, $t=1,2$;

(b) for any c_a and c'_a such that $u_a(c_a) \geq u_a(c'_a)$, $u_a[\mu c_a+(1-\mu)c'_a]>u_a(c'_a)$, for all μ, $0<\mu<1$;

(c) for at least one $i \in N$ $\lim_{c_{i1} \to 0}(\delta u_i/\delta c_{i1})/(\delta u_i/\delta c_{i2})=0$.

We finally consider the resource endowment of our economy. It is assumed that such endowment is made up by an amount W_t of resources available in each period ($t=1,2$) such that $W_1 = W>0$ and $W_2 = 0$.

Some comments are now in order about these assumptions. As far as A.1 is concerned, points (b) and (d) are not standard. The former violates the so-called Inada conditions. It is made here because it allows to use the marginal product at zero as a quality index, as will be clear in the sequel. Point (d) is then required to prevent

this quality ordering from depending on the activity level. An example of a production function satisfying A.2 is the logarithmic form

$$f_j(x_j) = a_j \log(x_j + 1), \quad a_j > 0$$

Assumption A.2 is standard: differentiability and strict quasi-concavity ensure the existence of a unique solution[4] for the agent's intertemporal allocation problem. Point (c) rules out corner solutions, at which no consumer is willing to consume in the second period. An obvious example of a utility function satisfying A.2 is the standard Cobb-Douglas, of the type

$$u_a(c_{a1}, c_{a2}) = c_{a1}^{\beta} \, c_{a2}^{1-\beta}, \quad \beta \in (0,1)$$

Finally, the way resources are modeled also deserves some comments. First, we need not specify how the economy's endowment is distributed across agents, since optimality is our only concern. Second, shares in (future) production are assumed to be the only asset - as will be explained later in more detail. This implies that there are no credit markets open, i.e. no exchange of future resources is allowed. Hence, we do not loose in generality by assuming that $W_2=0$.

2.2. Further technological and institutional constraints

Assuming $W_2=0$ entails that second period consumption must be wholly financed by implementing the available technologies. Under this respect, we characterize more sharply our economy by assuming that TS's are not allowed to invest their first period wealth: in the second period, each TS only consumes out of the proceeds from the sale of his technology[5]. The latter is modelled as follows: each TS

imposes a fee in exchange for the technology he is selling. This fee is proportional to the quantity of output actually produced, and is exogenously given in the model. Let the quantity of capital for technology $j \in M$ be x_j; then agent j will get a fee $\delta_j f_j(x_j)$, to be paid at $t=2$ out of the actual production delivered in the second period. Since δ_j is taken as given outside the model, there is no loss of generality in assuming that it is the same for all TS's: for all $j \in M$, $\delta_j = \delta > 0$[6]. Each TS can sell his technology only once, so that no more than one firm can use j's technology. Thus, the TS's behaviour can be described as follows: in the first period, they consume their wealth and try to sell their technology. Those who succeed get a fee in the second period, which is then (immediately) consumed. This constraint on the TS's behaviour can be formalized as

$$c_{j2} \leq \delta_j f_j(x_j), \quad \delta_j = \delta > 0,$$

holding for all $j \in M$.

One basic picture emerges from the above description. In order to achieve a satisfactory intertemporal pattern of consumption, the TS's have to sell their technologies. Consumers, on the other hand, have to invest their wealth in the available technologies. However, the set of production possibilities made available by the TS's does not exhaust the range of investment opportunities for the consumers. It is also assumed that there is a technology $j=0$ which provides constant returns to scale (CRS). The set of available technologies is therefore $T := \{0, 1, \ldots, m\}$. The CRS technology, unlike any other, is freely available. No fee being imposed (i.e., $\delta_0 = 0$), it grants a given return at $t=2$ for each unit invested at $t=1$. Thus the following definition can be given.

$$Y_0 := \{(x_0, y_0) \mid (x_0, y_0) \in R^2_+;\ y_0 \leq r x_0,\ r > 1\} \qquad (2)$$

The CRS technology provides a simple benchmark within the quality ranking of the available technologies that we are going to impose. The real number r is assumed to be greater than unity. Although not strictly necessary, this is however convenient: it captures the idea that agents can always invest their wealth at a given positive interest rate,(r-1). For notational convenience one can define $f_0(x_0) := rx_0$, and summarize all this in the following assumption:

A.3. There exists a CRS technology Y_0 such that $f_0(x_0) := rx_0$, $r>1$ and $\delta_0 = 0$.

The CRS technology being always freely available has clearly a bearing upon the investment cost. As will turn out, the existence of this fee system amounts to saying that the unit capital cost to investors for $j \in M$ is $R_j = r/(1-\delta_j)$. It is made up by the opportunity cost r of foregoing investment in the CRS technology, and the actual cost δ_j of having access to any technology $j \in M$. The exogenous fee being the same for all TS's (i.e., $\delta_j = \delta$ for all $j \in M$) and nil for the CRS technology (i.e., $\delta_0 = 0$), the unit capital cost will be

$$R_j = r/(1-\delta) := R, \quad \text{for all } j \in M;$$

$$R_0 = r$$

Each technology is identified by a production function: we can formalize the idea of quality differences across technologies by giving a simple ranking criterion for such functions. In order to do so, let us consider the following definition:

$$q_j(x_j) := f'_j(x_j)/R_j, \quad j \in T \tag{3}$$

$q_j(x_j)$ is just the ratio of the marginal product technology

j yields at the input level x_j, to the unit cost of capital[7]. Using the above definition of capital cost, we have that

$$q_j(x_j) = f'_j(x_j)/R, \qquad j\in M \tag{3'}$$

while $q_0(x_0)=1$ for any activity level.

We can use $q_j(0)$ to rank different technologies[8]. With no loss of generality, one can order them according as the $q_j(0)$'s are greater (or smaller) than $q_0=1$, and therefore assume that

$$q_1(0) > \ldots > q_k(0) > 1 > q_{k+1}(0) > \ldots > q_m(0)$$

We now define the sets $G:=\{j\in M \mid q_j(0)>1\}$, $B:=\{j\in M \mid q_j(0)<1\}$, $\tilde{B}:=\{j\in M \mid q_j(0)=1\}$. We assume that $\tilde{B}$ is empty, so that G and B make up a partition of M such that $G:=\{1,\ldots,k\}$ and $B:=\{k+1,\ldots,m\}$. By (d) of assumption A.2 it follows trivially that, if $q_j(0)>q_{j+1}(0)$, then $f_j(x)/x > f_{j+1}(x)/x$, for any $x>0$ and any $j\in M$. Thus we can unambiguously say that technology j is "better" than technology j+1. $q_j(0)$ is therefore an unambiguous "quality index" in this framework. Clearly, all this also applies to any pair $(j,j+1)$ such that $j\in G$ and $(j+1)\in B$.

Remark 1: The last assumptions bring in the model the main structural departure from Drèze's original work. This is particularly true as far as the existence of an exogenous fee system is concerned. Its use to evaluate technologies is a rough shortcut to modeling a market for technologies. In the present framework this is used only to introduce asymmetric information (section 4). Under this respect, TS's not investing in the first period is an innocuous assumption, which simplifies the algebra. However, modeling explicitly a market for technologies (i.e. endogenizing δ)

is, in principle, an independent issue.

As far as the quality ranking is concerned, the criterion we adopted is consistent with any model with strictly convex production sets: it is introduced here so as to have a ready benchmark in the analysis of the asymmetric information case. In the previous example where $f_j(x_j) = a_j \log(x_j + 1)$, $q_j(0) = a_j/R$, so that $a_j > R \Rightarrow j \in G$. Under our assumptions the standard marginal productivity (optimality) condition for any firm $j \in M$ is obviously given by $q_j(\bar{x}_j) = 1$, which in the same example gives $\bar{x}_j = (a_j - R)/R = q_j(0) - 1$.

3. Full information equilibria

In this section the polar case of complete information will be examined. At the beginning of the first period the markets open. Each TS exhibits his technology; consumers observe the available technologies and decide which firms should be set up and what is the amount of capital required. In doing so, they take into account that any firm should pay a fee on output when production is over[9] and that investment in the CRS technology is always possible. Each firm acquires its capital by issuing claims on output to be delivered in period t=2, defined as shares of the actual production; consumers grant that capital by purchasing those assets (choosing a portfolio which may include investment in the CRS technology) trading off current and future consumption, while TS's consume their endowment. When the share market has cleared, firms begin their production. This is completed at t=2, when output is distributed to both asset owners and successful TS's, and second period consumption takes place; unsuccessful TS's - who could not sell their technologies in the first period - are allocated a zero ("subsistence") level of consumption.

We model this story by looking at capital allocations across firms: any of them corresponds to a share allocation

across consumers, which can be supported by a share price vector. Consider a given set of existing firms (active technologies), the share allocation which is associated to it and the share price system which supports that allocation. This is an equilibrium situation if the following holds: (a) no shareholder is willing to invest more wealth in any existing firm, (b) there are no inactive technologies which consumers might want to fund with positive capital amounts and (c) no consumer is willing to trade shares at the going price.

This equilibrium is decribed by an array of vectors in R^{2}_{+}, corresponding to consumption and production of the n agents in the two periods, to which a set of existing firms is associated. Such an equilibrium has to satisfy certain conditions. In order that these be derived, we take advantage of a work by Drèze (1974), where the following strategy is suggested. First, taking as given the allocation of shares among agents and the production plans of all existing firms but one, the optimal choice of the remaining firm is considered and a pseudoequilibrium for that firm is defined. Second, taking as given the existing firms' decisions about production plans, the optimal allocation of shares among agents is examined, and a price equilibrium for this problem is defined. Third, these two equilibrium concepts are put together to yield a stockholders' equilibrium. The economy considered in this paper is actually different from Drèze's. In particular, the world described here is nonstochastic, which simplifies somewhat the analysis. Once a stockholders' equilibrium has been defined, its existence can quite simply be established for this economy. Asymmetric information will be introduced in the next section.

3.1. Feasibility

Any equilibrium must obviously be feasible. The feasible set for this economy is decribed by the set of production (x_j) and consumption decisions (c_{at}) which satisfy:

$$\Sigma_j x_j + \Sigma_a c_{a1} \leq W \tag{4a}$$

$$\Sigma_a c_{a2} \leq \Sigma_j f_j(x_j) \tag{4b}$$

$$x_j \geq 0,\ c_{at} \geq 0,\ j \in T,\ a \in A,\ t=1,2 \tag{4c}$$

As is clear, the feasible set exhibits some standard properties. In particular, it is compact and convex. However, this set does not exhaustively describe the relevant constraints under which the economy is operating. Actually, one has to take into account that any consumption and production plan must be implemented within the existing fee system and through joint stock ownership of the firms. In other words, we have to limit ourselves to considering the stock ownership feasible set of this economy, which is a (proper) subset of the feasible set described by (4). This stock ownership feasible set is the set of production and consumption decisions which satisfy:

$$\Sigma_{j \in T} x_j + \Sigma_{a \in A} c_{a1} \leq W \tag{5a}$$

$$c_{j2} \leq \delta f_j(x_j), \quad \text{all } j \in M \tag{5b}$$

$$c_{i2} \leq \Sigma_{j \in T} \theta_{ij} \bar{Q}_j(x_j), \quad \text{all } i \in N \tag{5c}$$

$$\Sigma_{i \in N} \theta_{ij} \leq 1, \quad \text{all } j \in T \tag{5d}$$

$$c_{at} \geq 0,\ x_j \geq 0,\ \theta_{ij} \geq 0, \text{ all } i \in N, \text{ all } j \in T, \ t=1,2 \tag{5e}$$

where $\bar{Q}_j(x_j) := (1-\delta) f_j(x_j)$, $j \in M$, and $\bar{Q}_0(x_0) := f_0(x_0) := r x_0$.

Clearly, $\bar{Q}_j(.)$ is the aggregate amount to be paid out as (net) dividends; θ_{ij} is an element of the $(m+1)\times(n-m)$ matrix $\boldsymbol{\theta}:=[\theta_{ij}]_{j\in M}^{i\in N}$, which describes an allocation of shares[10] among consumers; finally, the parameter δ is constrained to lie in the interval $[0,1]$ if (5e) is to be satisfied. Using the vector notation $c_a:=(c_{a1},c_{a2})$, we can define the set of stock ownership feasible programmes as

$$Z:=\{z:=(x_j,c_a,\boldsymbol{\theta}) \mid j\in T,\ a\in A;\ (5)\ \text{is satisfied}\} \qquad (6)$$

This set is clearly compact. It is however not generally convex, due to the individual constraint (5c), imposing that the economy's real allocations must be attained through share allocations. This nonconvexity may be illustrated in the following way[11].

Consider two points $z,z^{\circ}\in Z$, such that $x_o>x_o^{\circ}>0$ and $\theta_{ij}>\theta_{ij}^{\circ}>0$. Assume further that, for all $j\in M$, $x_j=x_j^{\circ}=0$, and

$$[c_{i2}-\theta_{io}f_o(x_o)]=[c_{i2}^{\circ}-\theta_{io}^{\circ}f_o(x_o^{\circ})]=0.$$

Then, for any $\mu\in(0,1)$

$$\mu c_{i2}+(1-\mu)c_{i2}^{\circ}-[\mu\theta_{io}+(1-\mu)\theta_{io}^{\circ}][\mu x_o+(1-\mu)x_o^{\circ}]r$$

$$=\mu(1-\mu)[\theta_{io}(x_o-x_{io}^{\circ})-\theta_{io}^{\circ}(x_{io}-x_{io}^{\circ})]r$$

$$=\mu(1-\mu)(x_o-x_o^{\circ})(\theta_{io}-\theta_{io}^{\circ})r>0$$

Hence, the (strictly) convex combination of z and z° does not belong to Z.

Remark 2: The nonconvexity just considered gives a rationale for the two-stage procedure suggested by Drèze: when the share allocation problem is not explicitly and separately taken care of, the firms' choices may not be consistent with

the shareholders'. That is, an equilibrium where the share and capital allocations are simultaneously Pareto optimal cannot be achieved[12].

Let each shareholder behave competitively and take as given the unit return on capital paid out by any share. Then, investment in a given firm is optimal to him when the average return and the unit cost of capital are equal ("arbitrage" solution), which is inconsistent with profit maximizing (Pareto optimum). This inconsistency is formalized through a simple game in appendix A. Each shareholder takes into account the depressing effect that his own investment has on the average rate of return, but takes as given the other investors' behaviour. Indeed, the (Nash) solution to this game tends to the arbitrage solution as the number of shareholders goes to infinity, while the Pareto optimum follows if such number is set equal to one (cooperative solution). The nonconvexity of the (stockholders') feasible set entails that any equilibrium allocation will be, at best, constrained Pareto efficient. As the game theoretic example shows, this is a consequence of the model's competitive setup. Further comments on this point will be offered in remark 3.

3.2. Pseudoequilibrium

Let us consider a given point $\bar{z}=([\bar{x}_j]_{j\in T},[\bar{c}_a]_{a\in A},\bar{\theta})\in Z$. To this point there corresponds a given set of firms, $L(\bar{z})\subseteq M$, defined as the set of firms "existing" at $\bar{z}$: a firm using technology $j\in M$ (firm $j\in M$ for short) "exists" at $\bar{z}$ if at least one consumer holds positive shares in it, that is $L(\bar{z}):=\{j\in M \mid I i\in N \text{ s.t. } \bar{\theta}_{ij}>0\}$[13].

For any $j\in L(\bar{z})$, consider the consumption and production plans which may be attained by redistributing the invested capital between technology j and the shareholders' current consumption. The set $F_j(\bar{z})$ includes all such plans, which

can be carried out while leaving unchanged both the share allocation and the amount of capital invested in technologies other than j. Given $\bar{z}$, any other point in Z generated by reallocating resources to (or from) first period consumption must then lie in $F_j(\bar{z})$, to which $\bar{z}$ itself obviously belongs. We therefore define

$$F_j(\bar{z}) := \{z \in Z \mid x_h = \bar{x}_h, \text{ all } h \in T, h \neq j; \theta = \bar{\theta}\} \qquad (7)$$

The set $F_j(\bar{z})$ is compact and convex[14]. Within such a set, we are now looking for firm j's production plans that would be accepted as optimal by its shareholders. To this end, given monotonicity of preferences, we can define an <u>induced</u> utility function, representing the consumers' preferences over alternative points in $F_j(\bar{z})$, that is

$$V^i(c_{i1}, x_j) := u_i[c_{i1}, \bar{\theta}_{ij}\bar{Q}_j(x_j) + \Sigma_{h \in T}\bar{\theta}_{ih}\bar{Q}_h(\bar{x}_h)] \qquad (8)$$

which, under our assumptions, is a continuous, strictly quasi-concave monotonic (increasing) function[15]. The Pareto optimal allocation of capital within $F_j(\bar{z})$ can now be characterized by noting that, given the share allocation θ, any variation in x_j affects second period consumption of <u>all</u> shareholders. Thus, x_j can be treated as a public good for the solution of our problem. Finding a Pareto optimal allocation within $F_j(.)$ then amounts to considering a public good economy, where (a) $V^i(.,.)$ describes the consumers' preferences over one private good (c_1) and one public good (x_j); (b) the transformation set between the private and the public good is described by the linear constraint

$$\Sigma_{a \in A} c_{a1} + x_j \leq W - \Sigma_{h \in T}\bar{x}_h, \quad h \neq j \qquad (9)$$

A Pareto optimal allocation can therefore be characterized by solving the following programme w.r.t. (c_{i1}, x_j), with μ^i ($i \in N$) as arbitrary positive weights.

$$\text{Max} \quad \Sigma_{i\in N}\mu^i V^i(c_{i1}, x_j)$$

$$\text{s.t.} \quad \Sigma_{a\in A} c_{a1} + x_j \leq W - \Sigma_{h\in T}\bar{x}_h, \quad h\neq j \tag{10}$$

It is well known (e.g., Malinvaud, 1972, p.212) that any Pareto allocation for a public good economy must satisfy the equality between the sum of the consumers' marginal rates of substitution among private and public goods and the marginal rate of transformation of private into public goods. In this particular instance, this condition becomes:

$$\bar{Q}'(\hat{x}_j)\Sigma_{i\in N}\hat{\pi}_{ij}\bar{\theta}_{ij} = 1 \tag{11}$$

where $\hat{\pi}_{ij} := \pi_i(\hat{c}_{i1}, \hat{x}_j) = (\delta u_i/\delta c_{i2})/(\delta u_i/\delta c_{i1})$ represents consumer (shareholder) i's marginal substitution rate over time, evaluated at $(\hat{c}_{i1}, \hat{x}_j)$, given $\bar{x}_h$ $(h\neq j)$ and $\bar{\theta}$. Equation (11) is an optimality condition on the amount of capital $\hat{x}_j$ required by any firm $j\in L(\bar{z})$ to maximize its shareholders' welfare, given the share allocation $\bar{\theta}$: it is taken as the pseudoequilibrium condition for firm j. This equation can be set in terms of $q_j(.)$: by straightforward manipulations one obtains:

$$q_j(\hat{x}_j) = (r\Sigma_{i\in N}\hat{\pi}_{ij}\bar{\theta}_{ij})^{-1} \tag{12}$$

which allows an easier comparison with the standard optimality condition, $q_j(.)=1$.

Only firms $j\in L(\bar{z})$ have been considered till now. We now briefly turn to the remaining cases. A firm not existing at $\bar{z}$, $k\in M\setminus L(\bar{z})$, is formally identified with $\bar{\theta}_{ik}=0$ for all $i\in N$. When this is so, x_k disappears from all agents' utility functions: it is not traded in the fictitious public good economy. Hence, $\bar{\theta}_{ik}=0$, for all $i\in N$, implies $\hat{x}_k=0$. Finally, a pseudoequilibrium can be defined also for the CRS technology: this can be seen as a firm for which no TS

charges a fee. The pseudoequilibrium condition for j=0 is:

$$r\Sigma_{i\in N}\hat{\pi}_{io}\bar{\theta}_{io} = 1 \qquad (13)$$

so that $q_o(x_o)=1$. We can at this point give an explicit definition of a pseudoequilibrium (Drèze, 1987, p.272).

> Given any point $\bar{z}\in Z$, a **pseudoequilibrium** for $j\in T$ is a point $\hat{z}\in F_j(\bar{z})\subset Z$, such that
> (a) $V^i(c_{i1},x_j) > V^i(\hat{c}_{i1},\hat{x}_j) \Rightarrow c_{i1}+\hat{\pi}_{ij}\bar{\theta}_{ij}\bar{Q}_j(x_j) > \hat{c}_{i1}+\hat{\pi}_{ij}\bar{\theta}_{ij}\bar{Q}_j(\hat{x}_j)$, for all $i\in N$;
> (b) $q_j(\hat{x}_j)=(r\Sigma_{i\in N}\pi_{ij}\bar{\theta}_{ij})^{-1}$ for $j\in L(\bar{z})$;
> (c) $\hat{x}_j=0$ for $j\in M\setminus L(\bar{z})$, and $q_o(x_o)=1$.

A pseudoequilibrium is therefore such that the shareholders' welfare is maximized (a) by choosing an appropriate activity level (b,c). Formally, this is really the standard definition of a "Lindahl equilibrium" for a public good economy, where $\hat{\pi}_{ij}$ are "Lindahl prices"(e.g., Milleron, 1972). One has now to show that, given $\bar{z}\in Z$ (and the related share allocation $\bar{\theta}$), there indeed exists a set $\hat{\Pi}^j:=\{\hat{\pi}_{ij}|i\in N\}$ of weights which warrant existence of a pseudoequilibrium for any $j\in T$. This is established in the following proposition.

> **Proposition 1.** Under assumptions A.1 to A.3, and given any point $\bar{z}\in Z$ such that $\Sigma_{i\in N}\bar{c}_{i1}>0$, for any $j\in T$:
> (a) there exists a point $\hat{z}\in F_j(\bar{z})$ which is a pseudoequilibrium;
> (b) $\hat{z}$ is a Pareto optimum within $F_j(\bar{z})$;
> (c) to any such optimum one can associate a set of weights $\hat{\Pi}^j$, such that $(\hat{z},\hat{\Pi}^j)$ is a pseudoequilibrium.

Given convexity and compactness of $F_j(.)$, this result

follows by standard arguments[16]. It is simply a version of Drèze's Theorem 3.1 (Drèze, 1987, p.273).

3.3. Price equilibrium

We now turn to the investors' optimal choices given the firms' production plans. Starting from any point $\bar{z}\in Z$, alternative points can be generated via alternative allocations of shares, i.e. through changes in $\bar{\theta}$ matched by variations in the consumers' first period consumption. Following again Drèze (1987, p.275) we denote by $E(\bar{z})$ the set of stock ownership feasible programmes which can be obtained by reallocating first period consumption and shares:

$$E(\bar{z}) := \{z\in Z \mid x_j = \bar{x}_j, \text{ all } j\in T\} \qquad (16)$$

We can treat $E(\bar{z})$ as the set of feasible allocations for an exchange economy, where first period commodity and shares are the commodities being traded. This set is compact and convex, $\bar{x}_j$ ($j\in T$) being given[17]. We now look for the consumption - share allocations which are Pareto optimal within $E(\bar{z})$. Like in the former case, we can identify each consumer's preferences over allocations in $E(\bar{z})$ by the induced utility function

$$W^i(c_{i1},\theta_i) := u_i[c_{i1},\ \Sigma_{j\in T}\theta_{ij}\bar{Q}_j(\bar{x}_j)] \qquad (17)$$

where $\theta_i := [\theta_{ij}]_{j\in T}$ is the $(m+1)$ vector of shares traded by consumer i. This function is continuous, strictly quasi-concave and (monotonically) increasing. The set of Pareto optimal share allocations, given a point $\bar{z}\in Z$ at which $x_j=\bar{x}_j$ for all $j\in T$, coincides with the set of Pareto optimal allocations of a fictitious economy such that: (a) $E(\bar{z})$ describes its feasible set; (b) W^i, $i\in N$, describes its

traders' preferences. Given the outlined features of W^i and $E(\bar{z})$, such an allocation exists, and can be decentralized by an appropriate price system. It is therefore termed <u>price equilibrium</u>. We now, first, define a price equilibrium, then establish its existence, and finally look more precisely into its features.

To define a price equilibrium, consider the fictitious economy referred to above, where (m+2) commodities are traded. We define a price vector $p \in \mathbb{R}_{++}^{m+2}$ for this economy, and normalize the price of the consumption good $p_c=1$[18]. The market clearing conditions are given by

$$\Sigma_{i \in N} \theta_{ij} \leq 1 \qquad (18a)$$

$$\Sigma_{a \in A} c_{a1} \leq W - \Sigma_{j \in T} \bar{x}_j \qquad (18b)$$

The following definition can now be given:

A **price equilibrium** is a pair $(z^*, p^*) \in E(\bar{z}) \times \mathbb{R}_{++}^{m+2}$, $p_c^*=1$, $\bar{z} \in Z$, such that:
(a) for all $i \in N$, $W^i(c_{i1}, \theta_i) > W^i(c_{i1}^*, \theta_i^*) \Rightarrow c_{i1} + \Sigma_{j \in T} \theta_{ij} p_j^* > c_{i1}^* + \Sigma_{j \in T} \theta_{ij}^* p_j^*$;
(b) $\Sigma_{i \in N} c_{i1}^* = W - \Sigma_{j \in T} \bar{x}_j$;
(c) for all $j \in T$ such that $\bar{x}_j > 0$, $\Sigma_{i \in N} \theta_{ij}^* = 1$.

This is the definition of a competitive equilibrium for the exchange economy described by (16) and (17). We now establish its existence, i.e. that of a Pareto optimal share allocation (and of a price vector which supports it), given the firms' production plans. This is done in the following proposition:

Proposition 2. Under assumptions A.1 to A.3, given any $\bar{z} \in Z$ such that $\Sigma_{i \in N} \bar{c}_{i1} > 0$:
(a) there exists a price equilibrium;
(b) any price equilibrium is a Pareto optimum;

(c) to any Pareto optimum $z^* \in E(\bar{z})$ one can associate a price vector $p^* \in \mathbb{R}^{m+2}_{++}$ such that (z^*, p^*) is a price equilibrium.

Given convexity and compactness of $E(\bar{z})$, this result follows by standard competitive arguments.

We now characterize a price equilibrium. To do so, notice that, at such an equilibrium, the optimal share portfolio for each shareholder must satisfy (Drèze, 1987, p.275):

$$\frac{\delta W^i / \delta \theta^*_{ij}}{\delta W^i / \delta c^*_{i1}} \leq p^*_j$$

with equality if $\theta^*_{ij} > 0$, as the standard marginal utility - price ratio optimality rule. In the present framework this becomes

$$\pi^*_i \bar{Q}_j(\bar{x}_j) \leq p^*_j \qquad (19)$$

where $\pi^*_i := \pi_i(c^*_{i1}, \theta^*_i)$ is the usual marginal rate of substitution over time, evaluated at (c^*_{i1}, θ^*_i), given $\bar{x} := [\bar{x}_j]_{j \in T}$. The equilibrium condition (19) entails that positive trades of shares, $\theta^*_{ij} > 0$, take place at a price

$$p^*_j = \pi^*_i \bar{Q}_j(\bar{x}_j), \quad \text{all } j \in T, \text{ all } i \in N.$$

Hence, p^*_j and $\bar{x}_j$ being given to any agent $i \in N$, π^*_i is equal for all consumers at a price equilibrium, $\pi^*_i = \pi^*$, all $i \in N$. This is not surprising: in equilibrium, the marginal rate of substitution over time must be the same to all consumers. Since $\bar{Q}_j(0) = 0$, the shares $j \in T$ such that $\bar{x}_j = 0$ disappear from the agents' utility function and are not traded: $\bar{x}_j = 0$ implies $\theta^*_{ij} = 0$ for all $i \in N$.

3.4. Full information equilibrium

We now specify a full information stockholders' equilibrium (FIE) for our economy. A FIE is both a pseudoequilibrium for each firm and a price equilibrium for the consumers. It can therefore be defined as follows:

> A (full information) **stockholders' equilibrium** is a point $z^* \in Z$ such that:
> (a) for every $j \in T$, there exists a set of weights π^{j*} such that (z^*, π^{j*}) is a pseudoequilibrium for j;
> (b) there exists a price system $p^* \in \mathbb{R}^{m+2}_{++}$, such that (z^*, p^*) is a price equilibrium.

We have to establish the existence of a FIE, and then characterize it. Before doing so, however, let us recap the general argument followed so far. Consider any point $\bar{z} \in Z$ such that $\Sigma_{i \in N} \bar{c}_{i1} > 0$ and any $j \in T$. Under proposition 1, one can associate at $\bar{z}$ a set of weights $\hat{\pi}^j := \{\hat{\pi}_{ij} \mid i \in N\}$ such that $(\hat{z}, \hat{\Pi}^j)$, $\hat{z} \in F_j(\bar{z})$, is a pseudoequilibrium. On the other hand, by proposition 2, to $\bar{z}$ there correspond also a point $z^* \in E(\bar{z})$ and a price vector $p^* \in \mathbb{R}^{m+2}_{++}$ such that (z^*, p^*) is a price equilibrium. The latter also identifies an equilibrium set of weights $\Pi^* := \{\pi^*_i \mid i \in N\}$, having the property that $\pi^*_i = \pi^*$ for all $i \in N$. As a natural consistency requirement, we therefore expect that $\hat{\Pi}^j = \Pi^*$ for all $j \in T$ at a FIE.

The following proposition establishes existence of a FIE:

> **Proposition** 3. Under assumptions A.1 to A.3, there exists a full information (stockholders') equilibrium.

See Drèze (1987), Theorem 3.3, for a proof of this proposition.

Consider now a FIE, $z^* \in Z$. It can be characterized by using the FIE definition and propositions 1 and 2. The

following conditions must hold at a FIE:

(a) $\pi^*_{ij} = \pi^*$, all $i \in N$, all $j \in T$, as a purely FIE condition: each agent's marginal rate of substitution, $\pi_i(c_{i1}, \theta, x)$ must satisfy both price and pseudoequilibrium conditions;

(b) $\bar{Q}'_j(x^*_j)\pi^* \Sigma_{i \in N}\theta^*_{ij} = 1$, as a pseudoequilibrium condition holding for all $j \in T$ such that $\theta^*_{ij} > 0$ for some $i \in N$;

(c) $\pi^* \bar{Q}_j(x^*_j) = p^*_j$, as a price equilibrium condition, holding for all $j \in T$ such that $x^*_j > 0$ and $\theta^*_{ij} > 0$ for some $i \in N$;

(d) $\Sigma_{i \in N}\theta^*_{ij} = 1$, as a price equilibrium condition, holding for all $j \in T$ such that $x^*_j > 0$.

By combining these features, one obtains the following noteworthy results, summarized in proposition P.4.

Proposition 4. Under assumptions A.1 to A.3, the following holds at a FIE:

(i) $\pi^* = 1/r$ for $x^*_0 > 0$; for $x^*_0 = 0$, $\pi^* \leq 1/r$.

(ii) Define $\tilde{x}_j$ by $q_j(\tilde{x}) = 1$: then, $\tilde{x}_j > 0$, all $j \in G$, and $\tilde{x}_j = 0$, all $j \in B$. If $\pi^* = 1/r$, then $x^*_j = \tilde{x}_j$, all $j \in M$, and $L(z^*) = G$. If $\pi^* < 1/r$, then: $0 \leq x^*_j < \tilde{x}_j$, all $j \in G$; $x^*_j = \tilde{x}_j$, all $j \in B$; $L(z^*) \subseteq G$.

(iii) For all $j \in L(z^*)$, $\Sigma_{i \in N}\theta^*_{ij} = 1$, and $p^*_j = \bar{Q}_j(x^*_j).\pi^*$. For all $j \notin L(z^*)$, $x^*_j = \theta^*_{ij} = 0$, all $i \in N$.

(iv) z^* is <u>constrained</u> (and <u>not</u> fully) Pareto efficient.

The proof is given in Appendix B. We briefly comment on each point separately. Point (i) is implied by the certain environment agents face: it is the usual Fisherian condition for intertemporal maximization, including the possibility of corner solutions. Point (ii) is a statement of full efficiency <u>in production</u>. Under full information, only

"good" firms are served. The pseudoequilibrium condition, together with share market clearing, leads to a Pareto optimal production level. Point (iii) is an implication of the price equilibrium condition. The market clearing share price is, not surprisingly, the firm's value. Finally, Pareto optimality (point (iv)) deserves some more detailed remarks.

Remark 3: By point (ii), production is clearly efficient at a FIE. However, consumption is not fully, but only constrained efficient: each agent $i \in N$ chooses a (first period) consumption level $c^*_{i1} < \bar{c}_{i1}$, the Pareto optimal level. This can be seen in the following figure 1, where for simplicity's sake we consider a firm $j \in M$ at a FIE where $x^*_o = 0$, $x^*_j > 0$ and $\theta^*_{ij} = 1$.

On the top right hand side we represent net dividends for firm j, $\bar{Q}_j(x_j) := (1-\delta) f_j(x_j)$. On the bottom right hand side the indifference curves represents induced utility in the (c_1, x_j) space, while on the bottom left hand side the indifference curves represent "actual" utility in the (c_1, c_2) space. In the graph, we can identify the FIE by a consumption vector (c^*_{i1}, c^*_{i2}) and an activity level x^*_j. The following should be noticed.

(a) x^*_j is Pareto optimal: actually, $\bar{Q}'_j(x^*_j) = r$, i.e. $q_j(x^*_j) = 1$;

(b) by proposition 4, the equilibrium share price is equal to the firm's value: $p^*_j = \bar{Q}_j(x^*_j)/r$. This implies that the left hand budget line has a slope $(dc_1/dc_2) = -(1/r)$ (solid line);

(c) if the share allocation was exogenously given, the left hand budget line would have a slope $(dc_1/dc_2) = -[x^*_j/\bar{Q}_j(x^*_j)]$ (dotted line). In that case, a first period consumption of $\bar{c}_{i1} > c^*_{i1}$ is fully consistent with x^*_j and c^*_{i2}.

One might ask why (c) does not hold when a share market is explicitly allowed for. The answer is that, when both choose simoultaneously, the firm's marginal evaluation and

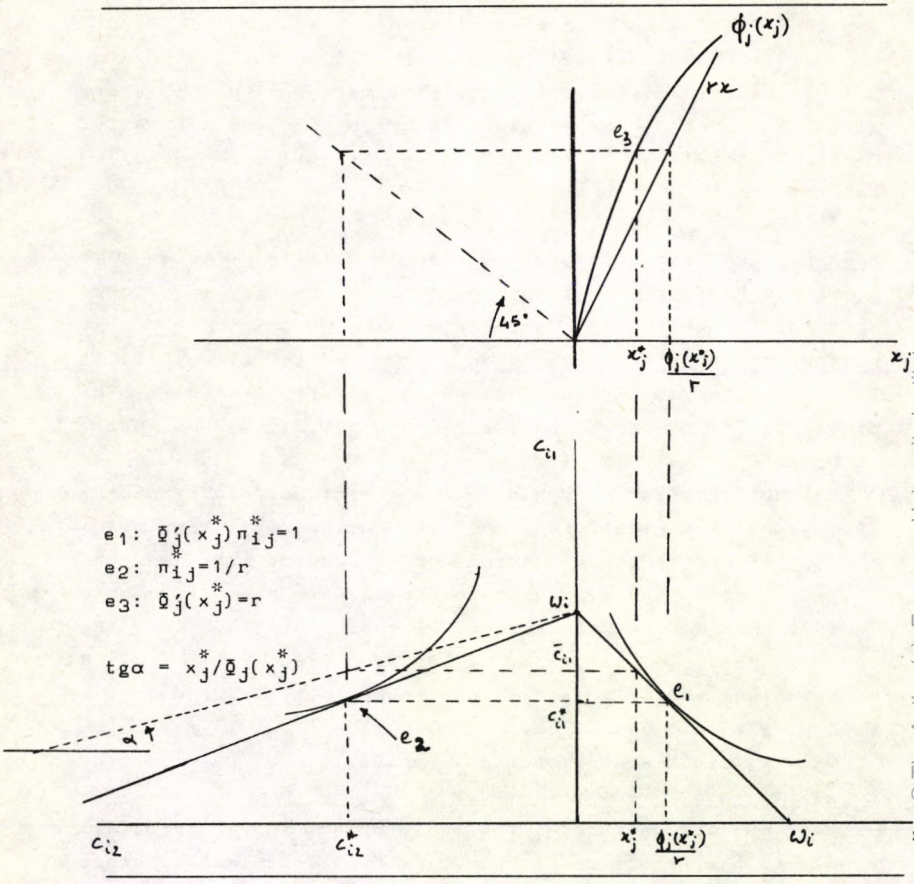

Figure 1: A Full Information Equilibrium

the shareholders' <u>average</u> evaluation are inconsistent with each other. Actually, consider any strictly concave continuously differentiable function $f(x)$, such that there exists a value $x^{\circ}>0$ for which $f'(x^{\circ})=a>0$; surely, if x^{+} satisifies $f(x^{+})/x^{+}=a$, then $x^{+}>x^{\circ}$. The nonconvexity of the

(stock ownership) feasible set Z reflects the fact that, while firms solve the former kind of problem, shareholders solve the latter: when there is competitive share market, shareholders face a share allocation problem of their own[19]. Drèze's construction overcomes this inconsistency by requiring that each side, given the other's plans, be maximizing. Thus we have tangency equilibria for both consumers (e_1, e_2) and for firms (e_3)[20]. However, this implies that, given x_j^*, the share value in each consumer's budget (evaluated at the firm's value: $\theta_{ij}^* \bar{Q}_j(x_j^*)/r$) is larger than the capital amount owned by the same consumer $(\theta_{ij}^* x_j^*)$. Hence, consumption is not Pareto optimal.

The TS's consumption

As a final comment to this section, it is clear that in a FIE the TS's are paid according to the activity level of the technologies they sell. Given monotonicity of their preferences, the equilibrium consumption of each TS $j \in M$ will satisfy

$$\Sigma_{j \in M} c_{j1}^* = W - \{\Sigma_{i \in N} c_{i1}^* + \Sigma_{j \in T} x_j^*\} \qquad (20a)$$

$$c_{j2}^* = \delta f_j(x_j^*) \qquad (20b)$$

The technology sellers are a passive component of the economy. They do not make any relevant decision, but have an incentive to sell bad technologies in the asymmetric information case. This is what we now turn to.

4. Equilibria with asymmetric information

We begin this section by assuming that, in addition to the CRS technology, there are only two technologies, and therefore two types of firms which can possibly be established. This is formalized in the following assumption:

A.4. Each agent $j \in M$ is endowed with a production possibility set Y_j, such that:
$Y_j = Y_g$, for all $j \in G$;
$Y_j = Y_b$, for all $j \in B$.

This assumption allows a simpler analytical treatment of the asymmetric information case.

The question we pose is the following: what happens if the market cannot perceive the difference between technologies. In general, the reason for agents being unable to distinguish good from bad quality is that information is costly. Thus, we have an implicit assumption: it is prohibitively costly to disseminate information. Each TS knows the technology he is about to sell, but neither can he profitably signal, nor can the market screen, this technology. Outsiders only know that there are "good" and "bad" technologies on sale. We follow a "bounded rationality" approach in assuming that they do not know the "true" quality distribution. The latter is described by the parameter $\bar{\alpha} := (k/m) > 0$, the a priori probability that a technology (independently) drawn at random be "good". Each consumer $i \in N$ does not know such parameter and has a given point expectation on its value, α_i. The vector of the consumers' subjective quality evaluations is $\boldsymbol{\alpha} := [\alpha_i]_{i \in N}$. Consumers, when faced with uncertain outcomes, are assumed to be expected utility maximizers: the informational asymmetry implies that the agents' choices are restricted between the safe technology $j=0$ and a technology (called u, $u \in M$), whose quality is uncertain. The actual quality of a technology is only revealed when production is completed and output is delivered. The following assumption describes the economy's informational structure.

A.5. No agent $i \in N$ can observe the production set Y_j. For each $z \in Z$ and $j \in L(z)$, y_j is freely observable by

all agents at t=2. For each $i \in N$, the subjective quality distribution is $[\alpha_i, (1-\alpha_i)] \in (0,1)^2$, with respect to which he acts as an expected utility maximizer. The objective distribution is $[\bar{\alpha}, (1-\bar{\alpha})]$, $\bar{\alpha} := (k/m)$.

We now apply the former concepts of pseudo- and price equilibria to this new situation, and finally define an asymmetric information (stockholders') equilibrium (AIE).

4.1. Pseudoequilibrium

Consider any given point $\bar{z} \in Z$. A set of existing firms, $L(\bar{z}) \subseteq M$ and an allocation $\bar{\theta}$ of shares across agents correspond to such point. Since quality is revealed only at t=2, all existing firms look alike from outside. Consider any of them, $u \in L(\bar{z})$: its optimal production plan must be chosen (and carried out) while ignoring the quality of the technology. Its probability of being "good" is differently evaluated by the shareholders, each of whom has his own subjective evaluation $\alpha_i \in (0,1)$. Hence, firm u's optimal production plan (given the share allocation $\bar{\theta}$ and the other firms' production plans) must lie in the set

$$F_u(\bar{z}) := \{z \in Z \mid x_h = \bar{x}_h \text{ all } h \in M,\ h \neq u;\ \theta = \bar{\theta}\} \qquad (21)$$

As in the full information case, $F_u(\bar{z})$ is a convex and compact set. Recalling Drèze's procedure, we now look for the firm's production plans that yield the highest (expected) utility to the shareholders. The agents' induced utility is as follows:

$$\bar{V}^i(c_{i1}, x_u) := \alpha_i . u_i[c_{i1}, \bar{\theta}_{iu} \bar{Q}_g(x_u) + K] + \\ + (1-\alpha_i) . u_i[c_{i1}, \bar{\theta}_{iu} \bar{Q}_b(x_u) + K] \qquad (22)$$

where $K:=\Sigma_{h\in T}\bar{\theta}_{iu}\bar{Q}_h(\bar{x}_h)$, $h\neq u$. It is continuous, strictly quasi-concave and (monotonically) increasing. We consider the fictitious public good economy where c_1 (private good) and x_u (public good) are traded; $F_u(\bar{z})$ represents the feasible set and $\bar{V}^i(.,.)$ agent i's ($i\in N$) utility. Optimal allocations for such an economy are the solutions of the following programme:

$$\text{Max } \Sigma_{i\in N}\mu^i\bar{V}^i(c_{i1},x_u)$$

$$\text{s.t. } \Sigma_{a\in A}c_{a1}+x_u\leq W-\Sigma_{h\in T}\bar{x}_h,\ h\neq u \tag{23}$$

Any solution $(\hat{c}_{i1},\hat{x}_u)$ satisfies the first order condition:

$$\bar{Q}_g'(\hat{x}_u).\Sigma_{i\in N}\alpha_i\hat{\pi}^u_{ig}\bar{\theta}_{iu}+\bar{Q}_b'(x_u).\Sigma_{i\in N}(1-\alpha_i)\hat{\pi}^u_{ib}\bar{\theta}_{iu}=1 \tag{24}$$

where $\hat{\pi}^u_{ij}:=\pi_i[\hat{c}_{i1},\bar{Q}_j(\hat{x}_u)]$ is the consumer's marginal rate of substitution over time, $(\delta u_i/\delta c_{i2})/(\delta u_i/\delta c_{i1})$, evaluated at $[\hat{c}_{i1},\bar{Q}_j(\hat{x}_u)]$, $j=g,b$, given $\bar{x}_h$ ($h\neq u$) and $\bar{\theta}$. More precisely, if we define $u_{ij}:=u_i[c_{i1},\bar{\theta}_{ij}\bar{Q}_j(x_u)+K]$, we have

$$\pi^u_{ij}:=(\delta u_{ij}/\delta c_{i2})/[\alpha_i(\delta u_{ig}/\delta c_{i1})+(1-\alpha_i)(\delta u_{ib}/\delta c_{i1})] \tag{25}$$

Hence, π^u_{ij} is the rate at which consumer $i\in N$ would trade off current vs future consumption if the actual quality turned out to be $j=g,b$ in the second period, given his ignoring such quality when making his decision in the first period.

Like in the full information case, $\hat{x}_u$ is the equilibrium amount of capital which maximizes the shareholders' welfare given the share allocation and the quality evaluation vector $\boldsymbol{\alpha}$. Therefore, $\hat{x}_u$ is the production plan that the shareholders of an existing firm u (given their ignorance of the firm's actual quality) would accept as optimal. It defines an asymmetric information (AI) pseudoequilibrium for the firm. We can re-write (24) in a more compact form as

$$\Sigma_{i\in N}\bar{q}_i(\hat{x}_u)\bar{\theta}_{iu}= 1 \qquad (26)$$

where $\bar{q}_i(.):=\alpha_i\bar{q}_g(.)\pi_{ig}+(1-\alpha_i)\bar{q}_b(.)\pi_{ib}$ is each shareholder's expected marginal evaluation of $\bar{q}_u(.)$. If we similarly define

$$q_i(.):=\alpha_i q_g(.)\pi_{ig}+(1-\alpha_i)q_b(.)\pi_{ib}$$

we can also write the pseudoequilibrium condition as

$$\Sigma_{i\in N}q_i(\hat{x}_u)\bar{\theta}_{iu}= 1/r \qquad (27)$$

Only firms $j\in L(\bar{z})\subseteq M$ have been considered so far. For the technologies not belonging to $L(\bar{z})$, what has been said in the full information case applies here: $\bar{\theta}_{iu}=0$ for all $i\in N$ implies $\hat{x}_u=0$. As to the CRS technology, it is fully known by agents. Hence, the pseudoequilibrium condition is the same as (13), i.e.,

$$r\ \Sigma_{i\in N}\hat{\pi}_{io}\bar{\theta}_{io} = 1 \qquad (28)$$

Here is the explicit definition of an AI pseudoequilibrium:

> An (asymmetric information) **pseudoequilibrium** for an unrecognizable firm $u\in M$, given an evaluation vector α, is a point $\hat{z}\in F_u(\bar{z})$, $\bar{z}\in Z$, such that
>
> (a) $\tilde{V}^i(c_{i1},x_u) > \tilde{V}^i(\hat{c}_{i1},\hat{x}_u)$
> $\Rightarrow\ c_{i1} + [\alpha_i\hat{\pi}^u_{ig}\bar{q}_g(x_u)+(1-\alpha_i)\hat{\pi}^u_{ib}\bar{q}_b(x_u)]\bar{\theta}_{iu} > \hat{c}_{i1}+[\alpha_i\hat{\pi}^u_{ig}\bar{q}_g(x_u)+(1-\alpha_i)\hat{\pi}^u_{ib}\bar{q}_b(\hat{x}_u)]\bar{\theta}_{iu}$,
> for all $i\in N$;
>
> (b) $\Sigma_{i\in N}q_i(\hat{x}_u)\bar{\theta}_{iu}= 1/r$, for all $u\in L(\bar{z})$;
>
> (c) $\hat{x}_u= 0$ for $u\in M\setminus L(\bar{z})$ and $r\ \Sigma_{i\in N}\hat{\pi}_{io}\bar{\theta}_{io}= 1$.

which has the same Lindahl-equilibrium interpretation as in the full information case. The existence of an AI

pseudoequilibrium can now be established:

Proposition 5. Under assumptions A.1 to A.5, and given any point $\bar{z}\in Z$ such that $\Sigma_{i\in N}\bar{c}_{i1}>0$, for any $u\in M$:

(a) there exists a point $\hat{z}\in F_u(\bar{z})$ which is a pseudoequilibrium;

(b) $\hat{z}$ is a Pareto optimum within $F_u(\bar{z})$;

(c) to any such optimum one can associate a set of weights $\hat{\Pi}^u:=\{\hat{\pi}^u_{ij}\,|\,i\in N,\ j=g,b\}$ such that $(\hat{z},\hat{\Pi}^u)$ is an AI pseudoequilibrium for u.

The proof of this proposition is given in Appendix C.

4.2. Price equilibrium

We turn to the investors' optimal choices given the firms' production plans. In this AI case, we consider a point $\bar{z}\in Z$, and define the set of feasible share allocations given $\bar{z}$. This is

$$E(\bar{z}):=\{z\in Z\,|\,x_h=\bar{x}_h,\ \text{all } h\in T\} \qquad (29)$$

Like before, we can treat $E(\bar{z})$ as the set of feasible allocations for an economy where first period consumption and the (m+1) shares are traded. Such set is compact and convex, $\bar{x}_h$ (all $h\in T$) being given. In this AI case, however, any firm $u\in M$ is indistinguishable for any other: shares in different firms are really the same commodity for each consumer's standpoint.

Now, consumer i's induced utility is

$$\tilde{W}^i(c_{i1},\theta_i):=\alpha_i.u_i[c_{i1},\ \Sigma_{u\in M}\theta_{iu}\bar{g}_g(\bar{x}_u)+\theta_{io}r\bar{x}_o]+$$
$$+(1-\alpha_i).u_i[c_{i1},\Sigma_{u\in M}\theta_{iu}\bar{g}_b(\bar{x}_u)+\theta_{io}r\bar{x}_o] \qquad (30)$$

where, for each $u \in M$, $\theta_{iu}\bar{Q}_j(\bar{x}_u)$ is the return of firm j, $j=g,b$. This function is continuous, strictly quasi-concave and (monotonically) increasing. The set of Pareto optimal share allocations under AI, given the capital allocation - the AI price equilibrium set - corresponds therefore to the set of Pareto allocations of the competitive economy described by (29) and (30). We have the usual market clearing conditions

$$\Sigma_{i \in N}\theta_{iu} \leq 1, \quad \text{all } u \in M \tag{31a}$$

$$\Sigma_{i \in N}\theta_{io} \leq 1 \tag{31b}$$

$$\Sigma_{a \in A}c_{a1} \leq W - \Sigma_{h \in T}\bar{x}_h \tag{31c}$$

and again normalize for $p_c=1$, while working with price vectors $p \in \mathbb{R}_{++}^{m+2}$. The following definition can now be given:

An (asymmetric information) **price equilibrium** is a pair $(z^*,p^*) \in E(\bar{z}) \times \mathbb{R}_{++}^{m+2}$, $p_c=1$, $\bar{z} \in Z$, such that:

(a) for all $i \in N$, $\tilde{W}^i(c_{i1},\theta_i) > \tilde{W}^i(c^*_{i1},\theta^*_i) \Rightarrow c_{i1}+\Sigma_{u \in M}\theta_{iu}p_u+\theta_{io}p_o > c^*_{i1}+\Sigma_{u \in M}\theta^*_{iu}p^*_u+\theta^*_{io}p^*_o$;

(b) $\Sigma_{a \in A}c^*_{a1} = W - \Sigma_{h \in T}\bar{x}_h$;

(c) for all $h \in T$ such that $\bar{x}_h > 0$, $\Sigma_{i \in N}\theta^*_{ih} = 0$.

The existence of a price equilibrium is now established in the following proposition.

Proposition 6. Under assumptions A.1 to A.5, given any point $\bar{z} \in Z$ such that $\Sigma_{i \in N}\bar{c}_{i1} > 0$,

(a) there exists a price equilibrium;

(b) any price equilibrium is a Pareto optimum within $E(\bar{z})$;

(c) to any such optimum $z^* \in E(\bar{z})$ one can associate a price vector $p^* \in \mathbb{R}_{++}^{m+2}$, such that (z^*,p^*) is a price equilibrium.

The proof of this proposition is given in Appendix C. Although proposition 6 has quite the same wording as proposition 2, its meaning is actually different: like in the AI pseudoequilibrium, Pareto optimality is in terms of expected utility, and is therefore to be meant as an ex-ante concept.

It is straightforward to check that the price equilibrium condition at which positive share trades must take place is:

$$p_u^* = \alpha_i \cdot \pi_{ig}^{u*} \bar{Q}_g(\bar{x}_u) + (1-\alpha_i) \cdot \pi_{ib}^{u*} \bar{Q}_b(\bar{x}_u) \qquad (32a)$$

$$p_o^* = \alpha_i \cdot \pi_{ig}^{u*} R\bar{x}_o + (1-\alpha_i) \cdot \pi_{ib}^{u*} r\bar{x}_o \qquad (32b)$$

for all $i \in N$. Equation (32) is the standard price-marginal utility optimality rule[21]. Since $\bar{Q}_h(0)=0$, all $h \in T$, $\bar{x}_h=0$ means that θ_{ih} appears in no utility functions: $\bar{x}_h=0$ implies $\theta_{ih}=0$, all $i \in N$.

4.3. Asymmetric information equilibrium

At this point, we can put together the previous notions, define an asymmetric information (stockholders') equilibrium, and establish its existence.

> An (asymmetric information) **stockholders' equilibrium** is a point $z^* \in Z$, which is both a pseudo- and a price equilibrium, i.e. such that:
> (a) for any firm $u \in M$, there exists a set of weights Π^{u*} such that (z^*, Π^{u*}) is a pseudoequilibrium for u;
> (b) there exists a price system $p^* \in \mathbf{R}_{++}^{m+2}$ such that (z^*, p^*) is a price equilibrium.

We now establish the existence of an AIE, and then characterize it. Before doing so, however, let us recap the

main line of argument. Consider a point $\bar{z}\in Z$ such that $\Sigma_{i\in N}\bar{c}_{i1}>0$. By proposition 5, one can associate to $\bar{z}$ a set of weights $\hat{\pi}^u$ such that $(\hat{z},\hat{\pi}^u)$, $\hat{z}\in F_u(\bar{z})$, is a pseudoequilibrium for u. Since no technology can be recognized ex-ante, the pseudoequilibrium amount of capital can differ across firms only because of the (given) share allocation. On the other hand, by proposition 6, to $\bar{z}$ there corresponds also a point $z^*\in E(\bar{z})$, and a price vector $p^*\in R_{++}^{m+2}$, such that (z^*,p^*) is a price equilibrium. At a price equilibrium, according to (32), p^* depends only on the given capital allocation $\bar{x}:=[\bar{x}_h]_{h\in T}$. In the full information case, the price equilibrium condition identified a set of weights whose equilibrium values were the same for all consumers. In this AI case, expected marginal rates of substitution will clearly be considered. We can now establish the following proposition.

Proposition 7. Under assumptions A.1 to A.5 there exists an asymmetric information (stockholders') equilibrium.

The proof is given in Appendix C.

Consider now an AIE, $z^*\in Z$: being both a pseudo- and a price equilibrium, it must satisfy the following.

(a) $\Sigma_{i\in N}\bar{\theta}_i(x_u^*)\theta_{iu}^* = 1$, as a pseudoequilibrium condition, holding for any $u\in M$ such that $\theta_{iu}^*>0$ for some $i\in N$;

(b) $r\,\Sigma_{i\in N}\pi_{io}^*\theta_{io}^* = 1$, as a pseudoequilibrium condition for the CRS technology, which applies when $\theta_{io}^*>0$ for some $i\in N$;

(c) $p_u^* = \alpha_i.\pi_{ig}^{u*}\bar{\theta}_g(x_u^*)+(1-\alpha_i).\pi_{ib}^{u*}\bar{\theta}_b(x_u^*)$, as a price equilibrium condition, holding for all $i\in N$ and any $u\in M$ such that $x_u^*>0$ and θ_{iu}^* for some $i\in N$;

(d) $p_o^* = rx_o^*.[\alpha_i\pi_{ig}^{u*}+(1-\alpha_i\pi_{ib}^{u*}]$, as a price equilibrium

condition for the CRS technology, holding for all $i \in N$ when $x_o^* > 0$;

(e) $\Sigma_{i \in N} \theta_{ih}^* = 1$, as a price equilibrium condition, holding for all $h \in T$ such that $x_h^* > 0$.

We combine these features to obtain the following results, which characterize an equilibrium with asymmetric information on technological quality.

Proposition 8. An (asymmetric information) stockholders' equilibrium $z^* \in Z$ has the following properties:

(i) $\alpha_i \pi_{ig}^{u*} + (1-\alpha_i)\pi_{ib}^{u*} = 1/r$, all $i \in N$, for $x_o^* > 0$; for $x_o^* = 0$, $\alpha_i \pi_{ii}^{u*} + (1-\alpha_i)\pi_{ib}^{u*} \leq 1/r$;

(ii) If z^* is such that $x_o^* > 0$, then $x_u^* = x^* > 0$, all $u \in M$, i.e. $L(z^*) = M$;

(iii) In the case sub (ii), $p_u^* = p^*$, all $u \in M$.

The proof is given in Appendix D.

We briefly comment on each point separately, and devote the next subsection to the efficiency aspects. Point (i) is simply the extension of (i) of proposition 4 to the case of quality uncertainty: the marginal rate of substitution over time is replaced here by the expected marginal rate.

Point (ii) deserves a somewhat detailed comment. First, $x_u^* = x^*$ is a natural implication of technologies being unrecognizable a priori: it is not surprising that a quality independent amount of capital be allocated to all firms alike. However, and more importantly, $x^* > 0$ always, i.e. whatever the distribution of α_i. In other words, in spite of the simple binomial quality distribution, there is no zero trade result of the kind discussed by Akerlof (1970). This is due to the model's intertemporal setting. Provided the marginal rate of substitution over time goes to infinity as second period consumption goes to zero, the consumers'

"impatience" can always counterbalance whatever pessimistic opinion about quality they may have. This holds for any AIE such that $x_o^*>0$. This is actually the really interesting case: given A.2, agents <u>always</u> wish to consume in the second period so that, if $L(z^*)$ was empty, x_o^* would necessarily be positive. However, $x_o^*>0$ is associated with $L(z^*)=M$. The interrelation between intertemporal allocation patterns and asymmetric information has some natural efficiency implications that will be taken up in the next subsection.

Point (iii) is a consequence of point (ii), in the sense that $p_u^*=p_u(x_u^*)$. Actually, the equilibrium share price does not depend directly on the agents' quality evaluations, but only on the equilibrium capital allocation (which, of course, does depend on α). Hence, the share market in this model works exactly as in the full information case, with no direct influence of the agents' subjective evaluations on share trading as such.

4.4. Efficiency aspects and subjective evaluations

We concentrate here on the case where $x_o^*>0$ at an AIE, in fact the most relevant case. We refer to Appendix D for a proof that the following equation characterizes an AIE in such a case:

$$S(c_1^*, x^*) = \mu(x^*)/r \qquad (33)$$

where we use the vector notation $c_1:=[c_{i1}]_{i\in N}$. The following definition holds:

$$S(c_1, x) := \Sigma_{i\in N}\alpha_i \pi_{ig}^u \theta_{iu}$$

For each $i\in N$ and $u\in M$, α_i is the subjective probability that u be "good", while π_{ig}^u is the expected marginal rate of substitution between current and future consumption,

evaluated when the latter is financed by investment in a "good" firm. Thus, $S(c_1, x)$ is the weighted sum of expected marginal rates of substitution over time relative to the shareholders of firm u, under the assumption that the latter is using a "good" technology[22].

On the other hand, the ratio $\mu(x)/r$ is the marginal rate of transformation between first and second period consumption. The latter is generated by trading off investment in the CRS technology and an ex-ante uncertain technology (yielding $\bar{Q}_g(.)$ and $\bar{Q}_b(.)$ with different probabilities) under the constraint that the ex-post outcome is "good"[23].

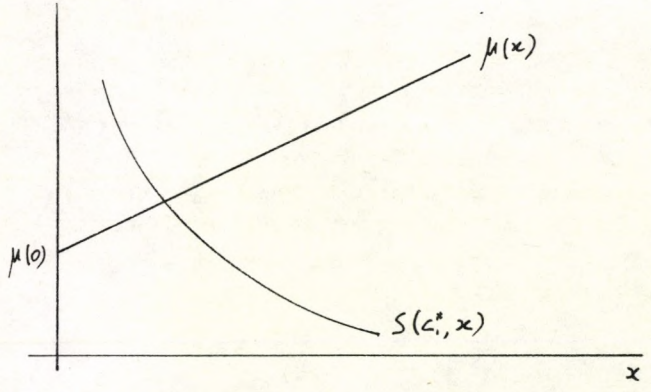

Figure 2: An equilibrium with positive trades

Equation (33) can be read as an equilibrium condition. On the left hand side we have a weighted sum of the marginal rates at which consumers exchange current against future consumption; on the right hand side we have the marginal rate of technical transformation between current consumption

and future production. Both rates are evaluated under the constraint that the ex-ante uncertain technology takes an ex-post value "good". Figure 2 describes the equilibrium condition identified by equation (33). The function $\mu(x)$ is increasing in x and such that $\mu(0)>0$, while $S(c_1,x)$ is smoothly decreasing in x, never reaching the axis[24]. We refer to appendix D for details.

In the remaining part of this section we shall comment on the following points: (i) the relationship between Pareto optimality and the agents' subjective evaluations; (ii) the relationship between the agents' risk attitudes, their subjective evaluations and the AIE intertemporal allocation; (iii) the role played by the assumption that agents differ in their preferences.

(i) In principle, given the agents' quality evaluations, an AIE has - in ex ante terms, i.e. with respect to those evaluations - the same efficiency properties as the FIE: it is constrained Pareto efficient, given the nonconvexity of the (stockownership) feasible set Z.

Now, consider any agent i's first period consumption at the AIE z^*:

$$c_{i1}^* = w_i - [p^* \Sigma_{u\in M} \theta_{iu}^* + p_o^* \theta_{io}^*] \qquad (34)$$

where $w_i>0$ is his first period commodity endowment. Given the AIE capital allocation x^*, the maximum feasible (ex-ante Pareto optimal) first period consumption for i is clearly

$$\bar{c}_{i1} = w_i - [x^* \Sigma_{u\in M} \theta_{iu}^* + x_o^* \theta_{io}^*].$$

In the case of a FIE ($\bar{z}$, say), the consumption inefficiency arose because agents placed an evaluation $\bar{p}_j = \bar{Q}_j(\bar{x}_j)/r > \bar{x}_j$ on each firm $j\in L(\bar{z})$. At an AIE, however, p^* is given by (see Appendix D)

$$p^*(x^*) = \{\mu(x^*).[\bar{\varrho}_g(x^*) - \bar{\varrho}_b(x^*)] + \bar{\varrho}_b(x^*)\}/r \qquad (35)$$

which is not necessarily greater than x^*. The (unrecognizable) firm's equilibrium value might indeed be $p^* = x^*$. This is so whenever α is such that equation (33), (34) and (35) are mutually consistent at a value $p^* = x^*$. In such a case, the distribution of the agents' quality evaluations leads each consumer to choose the ex-ante Pareto optimal consumption. This happens only when α is different from $\bar{\alpha}$. The agents' wrong evaluations, <u>because of their being wrong</u>, may lead them to pick up the first period consumption allocation that a planner would choose ex-ante[25].

(ii) As already noticed, there is an interrelation between the adverse selection problem and the intertemporal allocation mechanism. Using the pseudoequilibrium notion brings the agents' attitudes with respect to time and risk in the firm optimization, so that a link is established between the agents' risk aversion and the role played by their quality evaluations.

A starting point for discussion can be provided by the well known no-trade Akerlof result and its relevance in this model. Assumption A.2 on utility makes the agents' intertemporal substitution rates depend smoothly on x, and allows therefore to find always an equilibrium with positive trades. Asymmetric information leads to "wrong" capital allocations, but does not drive good quality out of the market.

Actually, the zero trade result requires as a necessary (but not sufficient) condition that the function $S(.,x)$ takes a finite value, $\bar{S}$ say, when x goes to zero. That is, the smoothness assumption is violated: there must be a finite value $\bar{\pi}^u_{ig}$ such that $\pi^u_{ig} = \bar{\pi}^u_{ig} \neq 0$ for $c_{i2}=0$, <u>all</u> $i \in N$. This no trade equilibrium also requires that $\mu(0)/r \geq \bar{S}$ (figure 3).

Provided A.2 holds, then, a positive trade equilibrium

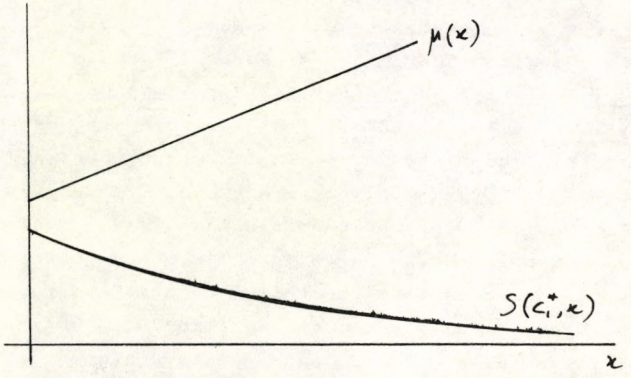

Figure 3: An equilibrium with zero trade

always exists. This assumption, however, implies risk aversion which - not surprisingly - does affect the pattern of intertemporal allocation. At a FIE, we had the obvious feature that

$$c^*_{i1}+(c^*_{i2}/r)= w_i$$

for all $i\in N$. At an AIE with risk averse traders, this intertemporal consumption pattern cannot be achieved by agents correctly perceiving the average quality, i.e. when $\alpha_i= \bar{\alpha}$, all $i\in N$. One can show that this pattern can indeed be achieved if α is such that the equations

$$rS(\mathbf{c}^*_1, x) = \bar{\alpha}$$
$$\mu(x) = \bar{\alpha}$$

are consistent for a positive value of x. A vector $\alpha\neq\bar{\alpha}$ may exist, such that an AIE enforces the same intertemporal

consumption pattern as the FIE: some incorrect subjective evaluation vector might counterbalance the distortion induced by risk aversion. We now want to rule out such distortion, and consider the risk neutrality case.

If all $i\in N$ are risk neutral, π^u_{ig} is independent of the quantity of capital x for all i: the smoothness assumption (A.2) is violated ipso facto. In this case, $S(c_1,x)$ is a constant, $\tilde{S}$ say, and the no trade result follows when $\mu(0)/R \geq \tilde{S}$. We can further characterize this point: risk neutrality implies $\pi^u_{ig} = \pi^u_{ib} = \bar{\pi}_i$, say, so that (i) of proposition 8 reduces to a constraint on utility functions, $\bar{\pi}_i = 1/r$. Hence, existence of equilibrium under risk neutrality requires that all agents have the same intertemporal preferences obeying that constraint, although their subjective evaluations may differ. If this is indeed the case, then (33) becomes

$$\tilde{S} := (1/r)\Sigma_{i\in N}\alpha_i\theta^*_{iu} = \mu(x) \qquad (33')$$

and equilibrium (be it positive or zero trade) depends only on the agents' evaluations. A zero trade equilibrium occurs when these are sufficiently pessimistic, since their (constant) rate of intertemporal substitution cannot adjust to compensate for them.

In order to separate out the role of the assumptions on risk attitudes on the one hand, and on the distribution of α_i on the other, we now compare a risk neutral AIE with a FIE. Consider therefore an existing AIE at which $\pi^u_{ig} = \pi^u_{ib} = 1/r$, all $i\in N$. In general, second period output at an AIE is given by

$$\Sigma_{h\in T} f_h(x^*) = \delta\Sigma_{h\in M} f_h(x^*_h) + m.\bar{\varrho}(x^*) + rx^*_o \qquad (36)$$

where $\bar{\varrho}(.) = \bar{\alpha}\varrho_g(.) + (1-\bar{\alpha})\varrho_b$ is the average dividend. Each consumer $i\in N$ and each TS $j\in M$ will obtain a second period consumption of

$$c^*_{i2} = \bar{Q}(x^*).\Sigma_{u\in M}\theta^*_{iu} + \theta^*_{io}rx^*_o \qquad (37a)$$

$$c^*_{j2} = \delta f_j(x^*) \qquad (37b)$$

We already saw the full information result, $c^*_{i1} + (c^*_{i2}/r) = w_i$. In order to see whether this holds at a risk neutral AIE, we recall from (34) agent i's first period consumption under AI:

$$c^*_{i1} = w_i - [p^*\Sigma_{u\in M}\theta^*_{iu} + p^*_o\theta^*_{io}]$$

Since we are assuming $\pi^u_{ig} = \pi^u_{ib} = 1/r$, by equation (32b) we have $p^*_o = x^*_o$. After some manipulations, we obtain the following relationship, holding at an AIE:

$$c^*_{i1} + (c^*_{i2}/r) = w_i - \{p^* - [\bar{Q}(x^*)/r]\}\Sigma_{u\in M}\theta^*_{iu} \qquad (38)$$

The expression in curly brackets is the difference between the evaluation placed on any unrecognizable firm (p^*) and the present value of the average dividend, estimated using the objective distribution $[\bar{Q}(x^*)/r]$. It is straightforward to check that this difference is zero (in this risk neutral case) when $\alpha_i = \bar{\alpha}$ for all $i\in N$, i.e. when agents perceive correctly the quality distribution. Risk neutrality and correct perception of the distribution yield the same intertemporal consumption pattern as full information.

(iii) In general, different agents' characteristics play the role of counterbalancing each other to "average out" the market outcome. Not surprisingly, this is so also in the present case. With identical preferences, equilibrium requires that the agents' subjective quality evaluations be consistent with each other in the sense of being equal, although they need not coincide with the objective distribution. This can be seen directly from (i) of proposition 8. When $u_i(.,.) = u_h(.,.)$, all $i,h\in N$, the AIE

condition

$$\alpha_i \pi^u_{ig} + (1-\alpha_i)\pi^u_{ib} = 1/r$$

solves for a unique value of α_i, $\bar{\alpha}$ say, for all $i \in N$[26]. Thus, when all agents have the same preferences the existence of an AIE imposes a consistency requirement on the agents' subjective evaluations, though, of course, it does not impose that $\bar{\alpha}=\bar{\alpha}$. If however this is indeed the case (agents are identical and perceive correctly the objective quality distribution), an AIE exists only when $\pi_g = \pi_b = 1/r$: i.e., we are back to the risk neutral case. Thus, the spreading of agents' characteristics plays a key role. Nondegenerate distributions of $\alpha=[\alpha_i]_{i \in N}$ can sustain an AIE only because of the traders' differences. Whenever this is not so, for equilibrium to exist the distribution $[\alpha_i]_{i \in N}$ must be collapsed to a single value. Thus, it is really the agents' different utilities which allow different subjective evaluations to be consistently averaged out at an AIE.

5. Concluding remarks

In this paper a standard two-quality asymmetric information problem has been analyzed with reference to a competitive capital market, modeled within a stockholders' equilibrium framework. Drèze's original notion allows to determine jointly the equilibrium allocations of consumption, capital and shares. In this paper, his framework has been modified to include asymmetric information on technological quality. The main results are that intertemporal preferences and asymmetric information may interact to yield positive equilibrium trades, irrespective of the agents' subjective evaluations. Moreover, the agents' different attitudes toward time and risk play a key role in determining the equilibrium

allocations of capital and shares.

The key assumptions of the paper are clearly the simple binomial quality distribution[27], the subjective quality expectations being exogenous and held with certainty[28] and, last but not least, the TS's fee being exogenously given. As a first step towards a more general model, the quality evaluations should be endogenized. This might involve making them depend on the actual quality distribution, so that learning behaviour can be modeled and the convergence of the subjective to the objective quality distribution studied. A complete account of a general equilibrium economy with quality uncertainty, however, should certainly include - as a further step - an endogenous objective quality distribution. This arises form the interplay between the behaviour over time of the set of existing firms, the agents' subjective evaluations and their learning behaviour. All this clearly involves a more sophisticated modeling of the quality distributions, possibly within a full fledged temporary equilibrium framework.

As to the TS's fees, these should be endogenized at the pseudoequilibrium level to model the "dividend policy" of the firm. Irrespective of asymmetric information, it could be a tentative solution to the problem of modeling an exchange between agents owning a technology and agents who are simply outside investors.

These (many) possible amendments might perhaps confirm the basic result of the paper: in the presence of asymmetric information, the presence of many price-taking shareholders allows the traders' different intertemporal preferences to compensate for their different subjective quality evaluations. As a consequence, inefficient outcomes can be sustained, irrespective of prevailing pessimistic opinions about the quality present in the market.

FOOTNOTES

[1] A general survey on information dependent market mechanisms involving price-quality links is provided by Stiglitz (1987), with an extended bibliography; see also Laffont and Maskin (1982). Green (1985) provides a reference framework for analyzing the role of private information.

[2] The use of two-period models for the analysis of financial markets has a long standing in the literature: e.g., Diamond (1967), Mossin (1971), Drèze (1974). Our model differs from this literature mainly in its ruling out environmental uncertainty.

[3] Drèze points out that strict quasi concavity is really not necessary; it is nevertheless assumed here for the sake of simplicity. It implies "diminishing marginal rate of time preference": see Drèze (1987, p.265). All quotations of Drèze's work referred to as Drèze (1987) are from his 1987 version of Drèze (1974).

[4] Strictly speaking, an interior solution for $i \in N$ is granted when u_i is strongly quasi-concave, i.e. such that its Hessian matrix is negative definite (e.g., Barten and Böhm, 1982).

[5] This assumption does not alter our results, so long as each TS is forbidden to invest in his own technology.

[6] As will be argued in the concluding remarks (section 5), endogenizing this parameter should allow to model explicitly a sort of "dividend policy" for the firm. This deserves treatment in its own right, independently of informational asymmetries. Further research on this point is being currently undertaken.

[7] In this sense, $q_j(.)$ is actually similar, but not identical, to Tobin's well known "marginal q": see, e.g., Yoshikawa (1980).

[8] We use $q_j(.)$ for the sake of generality. Since $\delta_j = \delta$ for all $j \in M$ implies $R_j = R$ (a constant), we might as well have used $f'_j(.)$.

[9] We assume honest behaviour: the fee is paid anyway.

[10] Notice that constraint (5e) rules out short-holdings of securities. Allowing them should not alter our results, so long they are bounded (Drèze, 1987, p.268, fn.10).

[11] This is similar to Drèze's account (1987, pp.268-269).

[12] This should be contrasted, e.g., with Debreu's (1959) treatment, where the share allocation is exogenously given: convexity of the production sets is enough to ensure Pareto optimality of the equilibrium.

[13] We reserve the word "firm" for active technologies $j \in M$.

[14] Notice that

$$F_j(\bar{z}) := \{z \in Z \mid \Sigma_{a \in A} c_{a1} + x_j \leq W - \Sigma_{h \in T} \bar{x}_h,\ h \neq j;\ c_{i2} \leq \bar{\theta}_{ij} \bar{Q}_j(x_j) + \Sigma_{h \in T} \bar{\theta}_{ih} \bar{Q}_h(\bar{x}_h),\ h \neq j,\ \text{all } i \in N\}$$

which is clearly compact and convex, $[\bar{x}_h]_{h \in T}$ $(h \neq j)$ and $\bar{\theta}$ being given.

[15] This transformation preserves strict quasi-concavity.

[16] Here, like in all subsequent propositions, the proviso $\Sigma_{i \in N} \bar{c}_{i1} > 0$ is not necessary if $\lim (\delta u_i / \delta c_{i1}) / (\delta u_i / \delta c_{i2}) = \infty$.

17 Notice that

$$E(\bar{z}) := \{z \in Z \mid \Sigma_{a \in A} c_{a1} \leq W - \Sigma_{j \in T} \bar{x}_j;\ c_{i2} \leq \Sigma_{j \in T} \theta_{ij} \bar{\varphi}_j(\bar{x}_j),\ \text{all } i \in N;\ \Sigma_{i \in N} \theta_{ij} \leq 1,\ \text{all } j \in T\}$$

which is clearly compact and convex, $[\bar{x}_j]_{j \in T}$ being given.

18 If $p := [p_j]_{j \in T}$ is the share price vector faced by each consumer and $w_i > 0$ his wealth, i's budget constraint will read $B_i(p) := \{(c_{i1}, \theta_i) \mid c_{i1} + \Sigma_{j \in T} p_j \theta_{ij} \leq w_i\}$. Not surprisingly, p_j will turn out to be the firm j's value.

19 The consistency between the firm's and its shareholders' maximum problems may be analyzed within a simple game, as shown in Appendix A.

20 We refer to Drèze (1972, 1974, 1987) for a complete analysis of the optimality properties of a stockholders' equilibrium.

21 In other words, equations (32) are simply the explicit formulation of the implicit optimality condition

$$(\delta \tilde{W}^i / \delta \theta^*_{ij}) / (\delta \tilde{W}^i / \delta c^*_{i1}) \leq p^*_j, \quad j = u, 0$$

holding with equality for positive trades.

22 In the following discussion we might use indifferently an equilibrium condition derived using "bad" technologies, of the kind

$$\Sigma_{i \in N} (1 - \alpha_i) \pi^{u*}_{ib} \theta^*_{iu} = v(x^*)/r$$

where both the marginal rates of substitution (π^u_{ib}) and the marginal rate of transformation ($v(x)$) refer to a bad technology. Notice that $v(x) + \mu(x) = 1$. Condition (33) involves

a summation over agents because of the externality implied by a pseudoequilibrium.

23 $\mu(x)$ is independent of any such probability: see Appendix D.

24 Using our example, $f_j(x_j)=a_j\log(x_j+1)$, $j=g,b$, $\mu(x)$ is linear, i.e., $\mu(x) = [R(x+1)-a_b]/(a_g-a_b)$.

25 Clearly, a complete analysis of this point should require modeling an endogenous distribution of the agents' quality evaluations. Some brief remarks are offered in section 5.

26 This is so, as long as α_i is strictly less than unity and u_i is (monotonic and) strictly increasing.

27 Actually, this is a rather standard assumption in the literature (e.g., Campbell and Kracaw, 1980), although not completely innocuous.

28 There is another feature of the model which may deserve some comment: firms are imperfectly informed on the quality of the technologies they are using. However, it is the consumers who are actually not informed: they evaluate unknown production processes, under a veil of ignorance which affects their decision to accept as optimal (or not) any given production plan.

APPENDIX A: Optimal Investment with Many Shareholders

In this appendix we consider the problem of optimal investment in a technology with strictly decreasing returns to scale (DRS), in the case where there are n "investors", indexed by $i \in N := \{1,\ldots,n\}$. We model this problem as a simple noncooperative game, as follows. The investors are assumed to allocate their wealth between a constant return to scale (CRS) and the DRS technology. For simplicity's sake, we characterized these investors by their wealth $(W_1,\ldots,W_n)$, and assume that each of them chooses his investment plan so as to maximize his overall return. An investment plan for $i \in N$ is a pair $(a_i, b_i) \geq 0$ such that $a_i + b_i \leq W_i$: a_i denotes the amount invested in the CRS technology, yielding Ra_i, and b_i denotes the amount invested in the DRS technology. We describe the latter with a strictly concave, continuously differentiable production function $f(.)$, such that the investment b_i yields a return $(b_i/\Sigma_{k \in N} b_k) f(\Sigma_{k \in N} b_k)$ to agent i. This return then depends on the other agents' investment plans. A Nash equilibrium is defined by $(a_i^*, b_i^*)_{i \in N}$, such that

(a_i^*, b_i^*) maximizes

$$Ra_i + \frac{b_i}{b_i + \Sigma_{k \neq i} b_k} f(b_i + \Sigma_{k \neq i} b_k)$$

for all $i \in N$. Let us consider investor i. Denoting by B the amount invested by the other investors, i's problem is to maximize

$Ra_i+[b_i/(b_i+B)]\ f(b_i+B)$, subject to

$(a_i,b_i)\geq 0$

$a_i+ b_i\leq W_i$

At an interior solution, we have

$$a_i^*+ b_i^*= W_i \qquad (A.1)$$

$$\frac{1}{b_i^*+ B}\, f(b_i^*+ B)\left(1 - \frac{b_i^*}{b_i^*+ B}\right)+ \frac{b_i^*}{b_i^*+ B}\, f'(b_i^*+ B)=R \qquad (A.2)$$

Equation (A.2) implies that a Nash equilibrium is necessarily symmetrical in terms of b_i^*, i.e.

$b_i^*= b^*$, all $i\in N$,

where b^* satisifies

$$[1-(1/n)]\ \frac{f(nb^*)}{nb^*} + (1/n)\ f'(nb^*)=R \qquad (A.2')$$

i.e., b^* is such that R equals the convex combination of the marginal and average productivities. Because of the DRS assumption on $f(.)$, we have

$$f'(nb^*)< R < [f(nb^*)/nb^*)$$

When n=1, $f'(b^*)=R$, which characterizes the Pareto optimal (cooperative) solution. By contrast, as n goes to infinity the aggregate investment B^* tends to the solution of $f(B^*)/B^*=R$. The latter is referred to as "arbitrage" solution in the text, and can be seen as the competitive solution holding when each investor's individual weight is negligible.

APPENDIX B: Proof of Proposition 4

We derive in detail the four points discussed in the text. The FIE conditions are:

(a) $\pi^*_{ij}=\pi^*$, all $j\in T$, all $i\in N$;

(b) $\bar{Q}'_j(x^*_j).\pi^*\Sigma_{i\in N}\theta^*_{ij}=1$, all $j\in T$ s.t. $\theta^*_{ij}>0$ for some $i\in N$;

(c) $\pi^*\bar{Q}_j(x^*_j)=p^*_j$, all $j\in T$, s.t. $x^*_j>0$ and $\theta^*_{ij}>0$ for some $i\in N$;

(d) $\Sigma_{i\in N}\theta^*_{ij}=1$, all $j\in T$ s.t. $x^*_j>0$.

Point (i)

By (a), $\pi^*_{ij}=\pi^*$. Then (d) becomes

$$\bar{Q}'_j(x^*_j).\Sigma_{i\in N}\theta^*_{ij} = 1/\pi^*$$

Take $j=0$: either $x^*_0=0$, or $x^*_0>0$. If the latter, by (d) $\bar{Q}'_0(x^*_0)=1/\pi^*$, which implies $\pi^*=1/r$. If, on the other hand, $x^*_0=0$, take any $j\in L(z^*)$; then $x^*_j>0$ and, by (a), (b) and (c), $\bar{Q}'_j(x^*_j)=1/\pi^*$. Suppose now $1/\pi^*<r$: then $\bar{Q}'_j(x^*_j)<r$ and, by (12), $q_j(x^*_j)<1$, which cannot be a pseudoequilibrium for firm j, since $V^i(c^*_{i1},x^*_j)<V^i(c^*_{i1},\tilde{x}_j)$, $q_j(\tilde{x}_j)=1$, $x^*_j>\tilde{x}_j$, all $i\in N$ s.t. $\theta^*_{ij}>0$. Hence $1/\pi^*\geq r$.

Point (ii)

First, consider $\pi^*=1/r$ and take any $j\in M$ s.t. $x^*_j>0$. Then, by (d) and (b), $\bar{Q}'_j(x^*_j)=r$, i.e. $q_j(x^*_j)=1$. Given $x^*_j>0$, $q_j(0)>1$ since $f''_j(.)<0$ for all nonnegative arguments: hence, $j\in G$. As a consequence, $\{j\in M \mid x^*_j=0\}=B$.

Consider now $\pi^*<1/r$, and take any $j\in M$ s.t. $x^*_j>0$. By (d) and (b), $\bar{Q}'_j(x^*_j)=1/\pi^*>r$, i.e. $q_j(x^*_j)>1$; hence, $q_j(0)>1$ and $j\in G$, although $x^*_j<\tilde{x}_j$. Thus any active firm belongs to G. The converse, however, is not true: take a firm $j\in G$ s.t. $(1/\pi^*)\geq q_j(0)>1$; clearly, $x^*_j=0$ in this case. Define therefore

$\bar{L}(\pi^*) := \{j \in G \mid q_j(0) \leq 1/\pi^*\}$; then $M \setminus L(z^*) := \{j \in M \mid q_j(0) \leq 1/\pi^*\} = B + \bar{L}(\pi^*)$, so that indeed $L(z^*) \subseteq G$, with strict inclusion holding when $\bar{L}(\pi^*)$ is nonempty.

Point (iii)

Take any $j \in L(z^*)$. Then both (c) and (d) apply. If $j \notin L(z^*)$, then $\theta^*_{ij} = 0$ all $i \in N$ by definition and hence, by the price equilibrium condition, $x^*_j = 0$.

Point (iv)

We take the case $\pi^* = 1/r$. Then $L(z^*) = G$ and $q_j(x^*_j) = 1$, all $j \in L(z^*)$: hence production is fully efficient. However, consider any shareholder $i \in N$, whose first period endowment is $w_i > 0$, s.t. $\theta^*_{ij} > 0$ for some $j \in L(z^*)$; his first period consumption at a FIE is $c^*_{i1} = w_i - \Sigma_{j \in T} p^*_j \theta^*_{ij}$. By (c), $p^*_j = \bar{Q}_j(x^*_j)/r$, so that $c^*_{i1} < \bar{c}_{i1} := w_i - \Sigma_{j \in T} x^*_j \theta^*_{ij}$, the Pareto optimal level; i.e., $u_i(c^*_{i1}, c^*_{i2}) < u_i(\bar{c}_{i1}, c^*_{i2})$.

APPENDIX C: Proofs of Propositions 5, 6 and 7

(A) Proposition 5

Consider any point $\bar{z}$ s.t. $\Sigma_{i\in N}\bar{c}_{i1}>0$, and take any $u\in M$. Then $F_u(\bar{z})$, defined in (21), and $\tilde{V}^i(c_{i1},x_u)$, defined in (22), describe the fictitious public good economy we are interested in.

(a) To establish the existence of a pseudoequilibrium, notice that: (i) $F_u(\bar{z})$ is compact, (ii) $F_u(\bar{z})$ is convex, (iii) $\tilde{V}^i(.,.)$ is a stricitly quasi-concave, monotonically increasing, continuously differentiable function.

(i) By definition,

$$F_u(\bar{z}):=\{z\in Z \mid \Sigma_{a\in A}c_{a1}+x_u \leq W-\Sigma_{h\in T}\bar{x}_h,\ h\neq u;\quad c_{i2}\leq \leq\bar{\theta}_{iu}\bar{g}_u(x_u)+\Sigma_{h\in T}\bar{\theta}_{ih}\bar{g}_h(\bar{x}_h),\ h\neq u,\ \text{all } i\in N\}$$

so that compactness follows from $\Sigma_{a\in A}c_{a1}+x_u\leq W-\Sigma_{h\in T}\bar{x}_h$, $h\neq u$, from which c_{i1} and x_u are bounded above. By the definition of Z, $c_{i1}\geq 0$ and $x_u\geq 0$ bound c_{i1} and x_u from below. All constraints are closed.

(ii) follows immediately by noting that the transformation surface described by (23b) is linear.

(iii) follows from A.2, since $\tilde{V}^i(.,.)$ is a strictly increasing and continuous transformation of u_i, α_i being strictly in (0,1).

Under (i), (ii) and (iii) there exits a pseudoequilibrium (see also proposition 1), $\hat{z}\in F_u(\bar{z})$, at which the following must hold:

$$\Sigma_{i\in N}\{(\delta\tilde{V}^i/\delta\hat{x}_u)/(\delta\tilde{V}^i/\delta\hat{c}_{i1})\}=1 \qquad (C.1)$$

(b) To establish (ex ante) Pareto optimality, notice simply that (C.1) is a necessary and sufficient condition for Pareto optimality in a convex public good economy (e.g., Malinvaud, 1972, p. 212).

(c) To establish the existence of $\hat{\Pi}^u$, note that at a pseudoequilibrium, (C.1) must hold. For any $i\varepsilon N$, we have

$$[\delta\bar{V}^i/\delta x_u]/[\delta\bar{V}^i/c_{i1}] = \alpha_i.\pi^u_{ig}\theta_{iu}\bar{Q}'_g + (1-\alpha_i).\pi^u_{ib}\theta_{iu}\bar{Q}'_b \qquad (C.2)$$

which takes finite values at $\bar{z}$ for all $i\varepsilon N$ such that $\bar{c}_{i1}>0$. One requires that there is some $x_u=\hat{x}_u>0$ such that $\hat{\pi}^u_{ij}:=\pi_i[c_{i1},\bar{Q}_j(\hat{x}_u)]$, $j=g,b$, satisfies (C.1). Following definition (25) π^u_{ij} is, by A.2, a continuously differentiable function of c_{i2} and hence of x_u, defined over all x_u. Hence, at pseudoequilibrium where (C.1) holds, the pair $(\hat{\pi}^u_{ig}, \hat{\pi}^u_{ib})$ exists.

(B) Proposition 6

Take any $\bar{z}\varepsilon Z$ such that $\Sigma_{i\varepsilon N}\bar{c}_{i1}>0$. Then $E(\bar{z})$, defined in (29), and $\tilde{W}^i(c_{i1},\theta_i)$, defined in (30), describe the fictitious economy, a competitive equilibrium of which is a price equilibrium for our economy. $E(\bar{z})$ is the feasible set, while $\tilde{W}^i(.,.)$ represents consumer i's preferences over shares, $\theta_i:=[\theta_{iu}]_{u\varepsilon M}$, and first period consumption, c_{i1}. All three points, (a), (b) and (c) of proposition P.6 follow from the fact that:

(i) $E(\bar{z})$ is compact;

(ii) $E(\bar{z})$ is convex;

(iii) $\tilde{W}^i(.,.)$ is strictly quasi-concave.

(i) By definition,

$$E(\bar{z}) := \{z \in Z \mid \Sigma_{a \in A} c_{a1} \leq W - \Sigma_{h \in T} \bar{x}_h;\ c_{i2} - \Sigma_{h \in T} \theta_{ih} \bar{g}_h(\bar{x}_h) \leq 0,$$
$$\text{all } i \in N;\ \Sigma_{i \in N} \theta_{ih} \leq 1, \text{ all } h \in T\}$$

so that c_{i1} and θ_i are both bounded from above and below, all such bounds being closed;

(ii) follows, since all constraints are linear;

(iii) follows from $\bar{W}^i(.,.)$ being strictly quasi-concave, as a strictly increasing and continuous transformation of u_i, α_i being strictly in $(0,1)$.

(C) Proposition 7

Consider a point $z \in Z$, such that $\Sigma_{i \in N} c_{i1} > 0$. By construction, $z \in F_u(z)$ (all $u \in T$) and $z \in E(z)$. Under our assumptions on preferences and technologies, there is a point $z^u := g_u(z) \in F_u(z)$ which is a pseudoequilibrium for u at z. Also, there is a point $\bar{z} := h(z) \in E(z)$ which is a pseudoequilibrium for the economy at z. These results follow from propositions 5 and 6. To prove existence of an AIE, one has to prove that there is a point z^* such that $g_u(z^*) = z^*$ for all $u \in T$, and $h(z^*) = z^*$.

We first consider pseudoequilibria and prove that, if $\hat{z}$ is a pseudoequilibrium for u, then $g_u(\hat{z}) := z^u = \hat{z}$. Consider a point $\hat{z} \in Z$ and the related pseudoequilibrium for u, $z^u := ([c^u_a]_{a \in A}, x^u, \theta^u) = g(\hat{z})$, where $c_a := (c_{a1}, c_{a2})$ and $x := [x_h]_{h \in T}$. By definition of a pseudoequilibrium, given any $i \in N$, $\bar{V}^i(c^u_{i1}, x^u) \geq \bar{V}^i(\hat{c}_{i1}, \hat{x})$. Then, either $z^u = \hat{z}$, or the inequality holds strictly. If the latter, $F_u(\hat{z})$ - a convex and compact set containing both z^u and $\hat{z}$ - contains $(z^u + \hat{z})/2 := \tilde{z}$, say, which is such that $\bar{V}^i(c_{i1}, \tilde{x}) \geq \bar{V}^i(c^u_{i1}, x^u)$; this contradicts the Pareto optimality of z^u. Hence, indeed $z^u = \hat{z}$. Now, by proposition 5 there exist weights $\hat{\Pi}^u$ such that a point $\hat{z}$ is a pseudoequilibrium for all $u \in M$. Hence, at $\hat{z}$,

$z^u=\hat{z}$ for all $u\in T$.

We now take up price equilibria. Consider the pseudoequilibrium $\hat{z}\in Z$. Associated to $\hat{z}$ there is a price equilibrium $\bar{z}:=h(\hat{z})\in E(\hat{z})$. Since $\bar{z}$ is Pareto optimal within $E(\hat{z})$, it is such that $\tilde{W}^i(\bar{c}_{i1},\bar{\theta}_i)\geq\tilde{W}^i(\hat{c}_{i1},\hat{\theta}_i)$, for all $i\in N$. Again, either $\bar{z}=\hat{z}$ or the strict inequality holds. If the latter, any convex combination of $\bar{z}$ and $\hat{z}$, z° say, is contained in $E(\hat{z})$, a convex set containing both $\bar{z}$ and $\hat{z}$. Then, for all $i\in N$ it must be that $\tilde{W}^i(c^\circ_{i1},\theta^\circ_i)\geq\tilde{W}^i(\bar{c}_{i1},\bar{\theta}_i)$, contradicting the Pareto optimality of $\bar{z}$. Hence, there is a point $z^*=\hat{z}=\bar{z}$ that is both a price and a pseudoequilibrium: it is an AIE.

APPENDIX D: Proof of Proposition 8

We derive in detail the three points discussed in the text. The AIE conditions are:

(a) $\Sigma_{i\in N}\bar{Q}_i(x^*_u)\theta^*_{iu} = 1$, all $u\in M$ s.t. $\theta^*_{iu}>0$ for some $i\in N$,

(b) $r\ \Sigma_{i\in N}\pi^*_{io}\theta^*_{io} = 1$, when $\theta^*_{io}>0$ for some $i\in N$;

(c) $p^*_u = \alpha_i.\pi^{u*}_{ig}\bar{Q}_g(x^*_u)+(1-\alpha_i).\pi^{u*}_{ib}\bar{Q}_b(x^*_u)$, all $i\in N$, all $u\in M$ s.t. $\theta^*_{iu}>0$ for some $i\in N$;

(d) $p^*_o=[\alpha_i.\pi^{u*}_{ig}+(1-\alpha_i).\pi^{u*}_{ib}]rx^*_o$, for $x^*_o>0$, all $i\in N$,

(e) $\Sigma_{i\in N}\theta^*_{ih}= 1$, all $h\in T$ s.t. $x^*_h>0$.

Point (i)

Take the case $x^*_o>0$. Then, by (d), $\alpha_i\pi^{u*}_{ig}+(1-\alpha_i)\pi^{u*}_{ib}$ is equal across i in equilibrium. By (e), $\theta^*_{io}>0$ for some $i\in N$: hence (b) applies. The CRS technology being known with certainty, $\pi^*_{io}=\alpha_i\pi^{u*}_{ig}+(1-\alpha_i)\pi^{u*}_{ib}$, from which the result follows.

Take now the case $x_o^*=0$. Take any $u\in L(z^*)$, and consider $\alpha_i\pi_{ig}^{u*}+(1-\alpha_i)\pi_{ib}^{u*}=\pi_i^*$, say; by (27), (a) can be written as $\Sigma_{i\in N}q_i(x_u^*)\theta_{iu}^*=1/r$. Suppose $\pi_i^*>1/r$; then $\Sigma_{i\in N}q_i(x_u^*)\theta_{iu}^*>1/r$. But this cannot be an equilibrium, since it violates the pseudoequilibrium condition (27): actually, define $\bar{x}_u$ by $\Sigma_{i\in N}q(\bar{x}_u)\theta_{iu}^*=1/r$; then $\bar{x}_u<x_u^*$, and therefore $\tilde{V}^i(c_{i1}^*,\bar{x}_u)>\tilde{V}^i(c_{i1}^*,x_u^*)$, all $i\in N$. Hence $\pi_i^*\leq 1/r$.

Point (ii)

We concentrate on the case $x_o^*>0$. By (i) of proposition 8, $(1-\alpha_i)\pi_{ib}^{u*}=(1/r)\alpha_i\pi_{ig}^{u*}$. Assume $\theta_{iu}^*>0$ for some $i\in N$. Then (a) can be written as

$$\Sigma_{i\in N}\alpha_i\pi_{ig}^{u*}\theta_{iu}^*=(1/r)\{\mu(x_u^*)+q_b(x_u^*)[q_g(x_u^*)+-q_b(x_u^*)]^{-1}(1-\Sigma_{i\in N}\theta_{iu}^*)\} \qquad (D.1)$$

where $\mu(x):=[1-q_b(x)]/[q_g(x)-q_b(x)]$ is a continuous function from $\mathbb{R}_+$ into itself, such that:

$$\mu(0)>0;$$
$$\mu'(x)=[q_g-q_b]^{-2}.[q_g'(q_b-1)+q_b'(1-q_g)]>0 \text{ all } x\geq 0;$$

Indeed, $\mu(x)$ is the marginal rate of transformation between first and second period consumption, the latter being financed by a technology $f_g(x)$. Define

$$\tilde{c}_2:=\alpha_i c_{2g}+(1-\alpha_i)c_{2b}, \text{ any } i\in N$$
$$\tilde{Q}(x):=\alpha_i Q_g(x)+(1-\alpha_i)Q_b(x)$$

such that the following constraints hold:

$$c_1\leq W-x-x_o$$
$$\tilde{c}_2\leq \tilde{Q}(x)+rx_o$$
$$c_{2j}\leq Q_j(x)+rx_o, \quad j=g,b$$

By solving for c_1, $\tilde{c}_2$ and c_{2g} and using a linear

differentiation,

$$rdc_1 + dc_2 = [\bar{Q}'(x)-r]dx$$

$$rdc_1 + dc_{2g} = [\bar{Q}_g'(x)-r]dx$$

This solves for $(dc_1/dc_{2g})|_{c2b=0}$ to get $(dc_1/dc_{2g}) = (1/r)\mu(x)$, which is independent of α_i.

We now prove that $x_u^*>0$, all $u\in M$. Consider $\pi_{ig}^u = \pi_i[c_{i1}, \bar{Q}_g(x_u)]$. By assumption A.2, $\pi_{ig}^u>0$, $\delta\pi_{ig}^u/\delta x_u \leq 0$ for all $x_u \geq 0$ and, for at least one $i\in N$, $\lim_{x\to 0}\pi_{ig}^u=\infty$. Since $\alpha_i>0$ all $i\in N$ and $\theta_{iu}^*>0$ for some $i\in N$, this is so also for the continuous function $S(\mathbf{c}_1, x_u) := \Sigma_{i\in N}\alpha_i\pi_{ig}^u\theta_{iu}$, where $\mathbf{c}_1 := [c_{i1}]_{i\in N}$. Given that $\mu(0)>0$ and $\mu'(x_u)>0$, (D.1) implies $x_u^*>0$.

We now prove that $x_u^*=x^*$, all $u\in M$. From AIE condition (c) and (i) of proposition 8, we can write

$$\alpha_i\pi_{ig}^{u*} = \{[Ap_u^* - \bar{Q}_b(x_u^*)]/[\bar{Q}_g(x_u^*) - \bar{Q}_b(x_u^*)]\}^{1/r} \qquad (D.2)$$

such that $\alpha_i\pi_{ig}^{u*}$ is equal for all i in equilibrium. By (D.1), this implies that $x_u^*=x^*$ all $u\in M$, provided that $\Sigma_{i\in N}\theta_{iu}^*=1$, i.e., by (e), provided $x_u^*>0$, as is indeed the case. At an AIE (D.1) becomes

$$S(\mathbf{c}_1^*, x^*) = \mu(x^*)/r \qquad (D.1')$$

Point (iii)

We concentrate on the case where $x_o^*>0$. By (ii) of proposition 8, (D.1) solves for $x^*>0$. Also, (D.2) holds for $x^*>0$ and $\Sigma_{i\in N}\theta_{iu}^*=1$, all $u\in M$. By substituting for $\alpha_i\pi_{ig}^{u*}$ we obtain

$$p_u^*(x^*) = (1/r)\{\mu(x^*)[\bar{Q}_g(x^*) - \bar{Q}_b(x^*)] + \bar{Q}_b(x^*)\} \qquad (D.3)$$

so that p_u^* does not depend directly on α or θ. Hence, $p_u^*(x^*) = p^*$, all $u \in M$.

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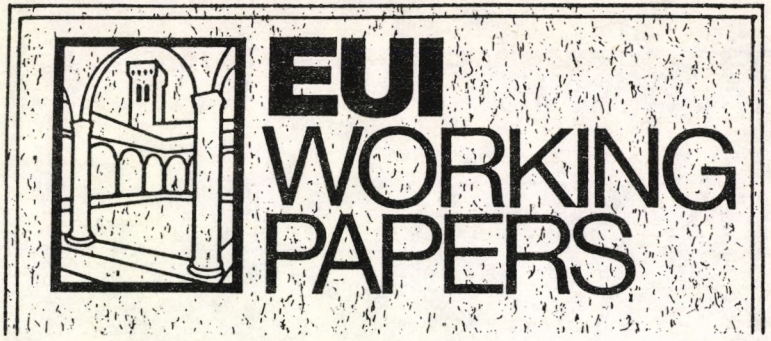

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