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**COMPETITIVE EQUILIBRIA
WITH INCREASING RETURNS**

by

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Abstract

This paper proposes a concept of competitive equilibrium at which firms maximise profits given the prices and the demand for their outputs. The equilibrium is called "competitive" because it combines voluntary trading with a minimality condition on output prices. When production sets are convex, the set of equilibria as defined here coincides with the usual set of competitive equilibria. Existence is proved without imposing convexity assumption on either individual or aggregate technologies.

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I. INTRODUCTION

1. One of the great success stories in economics is that of the competitive equilibrium, defined by a feasible allocation and a price system such that, for each agent, the allocation is self-financing, and, at those prices, no further self-financing transaction is positively desired. Competitive equilibria possess several attractive properties, in particular: (i) a *decentralisation* property: all agents act independently, pursuing their interests, and their actions are coordinated by a set of signals (the prices) of minimal dimension (see e.g., Mount and Reiter, 1974), signals which are easily transmitted and interpreted; and (ii) an *efficiency* property: under the minimal requirement of local non-satiation, competitive equilibria are Pareto efficient (see e.g., Debreu, 1959)¹.

These attractive features must be weighed against two major drawbacks. In the first place, only under extreme, unrealistic assumptions about the production technology do competitive equilibria exist, and convincing examples with non-existence are common place. The critical assumption is that production sets should be convex, ruling out fixed costs and increasing returns to scale for new investments, or for existing plants as well under the additional standard assumption that production sets allow for inactivity. In reality, non convexities arise from plant size, set-up costs and production runs, ordering costs and lot-sizes, product development and marketing, transportation, specialised management, etc... . Even in many activities typically carried out by small firms (in handicraft, farming or services), non-convex production sets are the rule rather than the exception. Recently, some of these features have received renewed attention under the label of "economies of scope" (see e.g., Panzar and Willig, 1981).

A second drawback is that, even when competitive equilibria are known to exist, only under additional extreme assumptions will they emerge naturally as stable solutions of the economic game. The only assumption yielding a fully convincing argument to that effect is that of a constant returns to scale technology freely available to all firms (see e.g., Debreu and Scarf, 1963). But that assumption rules out non-reproducible assets, of which natural sites and resources are the most obvious example in the long run. (In the short run, existing plants, equipments and inventories provide another omnipresent example.)

Outside of such a technological assumption, extreme behavioural or institutional assumptions would be required to validate the natural emergence of competitive equilibria. For instance, firms could be assumed to anticipate infinite price elasticities of demand; or a mixture of regulation and public controls could be deemed effective in bringing about price taking profit maximisation.

Why then so much fascination with a concept so obviously inadequate? Because

(we surmise) it still provides a very useful benchmark for positive and normative analysis alike. Even in the absence of convexity, prices remain a standard device for guiding and coordinating individual activities. Under price decentralisation, "competitive forces" are at play, as agents seek to identify mutually advantageous transactions, and are prepared to outbid each other in carrying these out. Economists studying the likely outcomes of such processes find it convenient to use competitive equilibria as a benchmark, sometimes to conclude that they provide a good approximation to the likely outcomes; at other times to bring out significant departures from it. Also, given the normative appeal of competitive equilibria (efficiency and fairness - two incomplete but still useful yardsticks) economists have studied policies (like antitrust policies) aimed at bringing the outcomes closer to the competitive benchmark.

Until recently, the emergence of competitive, or nearly competitive, equilibria in economies with non-convex technologies had been obtained through large numbers; that "classical" viewpoint underlies the work of Arrow and Hahn (1971, Chap. 8), or Novshek and Sonnenschein (1980). A new approach has been introduced with the theory of "sustainable equilibria" in "contestable markets" (see e.g., Baumol, Panzar and Willig, 1982; and Sharkey, 1982) which avoids the unnatural, or restrictive, assumption of large numbers, but remains confined to a partial equilibrium analysis.

2. The purpose of the present paper is to suggest an equilibrium concept for economies with general, non-convex technologies, that embodies some important features of the competitive equilibria. That concept could be labeled "competitive equilibrium with price-and-quantity taking firms" - but we shall refer to it simply as a competitive equilibrium.

A first and absolute requirement is that our concept should coincide with the usual competitive equilibrium when production sets are convex.

A second, and in our minds, foremost requirement is that our equilibrium should be decentralised through simple signals and natural incentives. Agents will indeed be assumed to optimise at given signals. The framework is the standard private ownership economy and the consumers behave in the usual way namely, they act at given prices and income, the latter incorporating the profits. The producers on the other hand are assumed to act at given prices *and demand levels*; their willingness to meet demand in full at those prices, rather than selling less endows their supply behaviour with the property of "voluntary trading" - a property which is always satisfied at a competitive equilibrium. This has already far reaching implications and in itself excludes approaches inspired by efficiency considerations like marginal cost pricing, which has received much (well deserved) attention lately.

The decentralisation requirement is however not sharp enough to characterise

fully competitive equilibria in the convex case. Accordingly we introduce as a further requirement that output prices should not embody elements of monopolistic exploitation. In the convex case, this requirement is sufficient, in conjunction with voluntary trading, to characterise fully competitive equilibria. There is however some latitude in defining that requirement precisely. Alternative definitions, which are equivalent in the convex case, do have slightly different contents in the general case. We have adopted a broad definition, namely that *output prices should be minimal* (in a suitable technical sense and at given input prices) *over the set of prices consistent with voluntary trading*. Our definition is analogous to that of "supply prices" used by Marshall: "... the price the expectation of which will just suffice to maintain the existing aggregate amount of production..." (1920, p. 343; see also Keynes, 1936, p. 24).

This third requirement has also important implications. In particular, it excludes approaches inspired by the consideration that increasing returns may introduce monopolistic features in the analysis. Our intention is precisely to define an equilibrium concept which is free of such features². From a normative viewpoint, our equilibrium concept is directly relevant to the discussion of price regulation in a general equilibrium framework. More specifically, our existence result shows that an approach whereby regulation imposes minimal output prices subject to voluntary trading is consistent, at a general equilibrium level.

3. In this first section, we shall introduce intuitively the definition of our equilibrium concept. The economy we consider is the standard private ownership economy. An equilibrium is defined by a *feasible* allocation and a price system satisfying the following conditions:

(i) given the prices and the profits, the allocation corresponds to a best choice for the consumers in their budget set;

(ii) at this allocation, each producer minimises its costs and engages in voluntary sales;

(iii) for each producer, output prices could not be lowered without violating the voluntary trading condition (ii).

To formalise the content of the last two conditions, we shall consider a single producer. The exposition will be made easier by considering a technology which distinguishes unambiguously inputs from outputs. Also, because our main interest lies with increasing returns to scale of outputs, we shall assume that input requirement sets (isoquants) are convex, and we shall work with cost functions. The more general case of a technology for which no a priori distinction between outputs and inputs is made and isoquants are not necessarily convex will be considered in Section III.

Thus, we shall consider a production set $Y \subset \mathbb{R}^L$, with elements $y = (a, b)$, $a \in \mathbb{R}_+^m$ and $b \in \mathbb{R}_+^n$. The only restriction placed on Y , beyond those already mentioned, are that it be a closed subset of \mathbb{R}^L containing the origin and satisfying free disposal and absence of free production, i.e., $Y + \mathbb{R}_+^L \subset Y$ and $Y \cap \mathbb{R}_+^L = \{0\}$. Price systems are denoted by $p = (p_a, p_b)$, $p \in \mathbb{R}_+^L$ and the associated cost function is denoted by $c(b, p_a)$, i.e.

$$c(b, p_a) = - \max_{(a, b) \in Y} p_a a.$$

4. The decentralisation property of competitive equilibria is captured by the simple condition that the chosen production plan \bar{y} yields, at the prevailing price system p , profits $p \cdot \bar{y}$ at least as high as those of any other feasible alternative, i.e., $p \cdot \bar{y} \geq p \cdot y$ for all $y \in Y$. It is readily verified that such a condition cannot be satisfied in general with non-convex production sets, as shown in Figure 1 where the only prices consistent with profit maximisation are those such that $p \cdot y \leq 0$ for all $y \in Y$, i.e., $p_b = 0$ and $p_a \geq 0$.

A less stringent and more reasonable condition simply requires that no smaller output yields higher profits; in other words, that the firm should be prepared to meet in full the demand that materialises at the prevailing prices. This defines the equilibrium condition (ii), labeled "voluntary trading", which may be written as³:

$$(*) \quad p_a \bar{a} + p_b \bar{b} \geq p_a a + p_b b \quad \text{for all } (a, b) \in Y, b \leq \bar{b}$$

It characterises what we call "decentralisation prices" and says that profits are maximised *subject to a sales constraint*, namely $b \leq \bar{b}$. This describes well the situation of a producer who operates by meeting the demand which materialises at the going prices⁴. Under diminishing marginal costs, these producers operate with excess supply, meaning that they would prefer to sell more at the going prices but accept the quantity constraint implied by the demand signal⁵. And because inactivity is feasible, decentralisation prices always yield non-negative profits.

Condition (*) can be seen as defining implicitly the set of output prices p_b compatible with trading voluntarily the output vector \bar{b} given the input prices p_a . Indeed, *voluntary trading implies cost minimisation* and condition (*) can equivalently be written as $p_b \in \phi(\bar{b}, p_a)$, where ϕ is the correspondence defined by:

$$\phi(b, p_a) = \{p_b \in \mathbb{R}_+^n \mid p_b b - c(b, p_a) \geq p_b b' - c(b', p_a), \forall b' \leq b\}.$$

Alternatively, $\phi(\bar{b}, p_a)$ can be defined as the subdifferential of the convexified cost function restricted to $b \leq \bar{b}$. In Figure 2, $\phi(\bar{b}, p_a)$ is the set of all prices p_b

greater than or equal to average cost at \bar{b} , given p_a . More generally, it may happen that *all* decentralisation prices exceed average cost and therefore entail positive profits. Figure 3 depicts the case of a technology with facilities (machines) acquired at a fixed cost and capable of producing each an output not exceeding β at a constant marginal cost. For outputs between 0 and β , the situation is the same as in Figure 2. But for outputs *inside the range* $[\beta, 2\beta]$, decentralisation prices yield positive profits⁶.

5. It seems clear that voluntary trading belongs - explicitly or implicitly - to any reasonable definition of a *competitive equilibrium*. It is equally clear that many price systems compatible with voluntary trading are not "competitive" because output prices are too high, amounting to "exploitation" of buyers by sellers and entailing monopolistic profit margins. Going back to Figure 2, it seems natural to suggest that the *only* competitive output prices are those corresponding to average cost, implying zero profits.

To formalise the content of equilibrium condition (iii), it is instructive to study the convex case first. Clearly, voluntary trading allows in that case output prices which exceed competitive levels by an arbitrary margin. Indeed, if marginal costs are well defined at \bar{b} , hence equal to the gradient vector $\nabla_b c(b, p_a)$, the set of decentralisation prices *in the convex case* is given by

$$\phi(b, p_a) = \{p_b \in \mathbb{R}_+^n \mid p_b \geq \nabla_b c(b, p_a)\}.$$

This is illustrated in Figure 4 for the case of a differentiable cost function. Competitive prices are then extracted from the set of decentralisation prices by the straightforward condition that output prices should be minimal in that set. This additional condition can in general be written as $p_b \in \phi^*(\bar{b}, p_a)$ where ϕ^* is the correspondence defined by

$$\phi^*(b, p_a) = \{p_b \in \phi(b, p_a) \mid \nexists p'_b \in \phi(b, p_a), p'_b < p_b\}.$$

In our differentiable convex case, $\phi^*(\bar{b}, p_a) \equiv \nabla_b c(\bar{b}, p_a)$ i.e., minimal decentralisation output prices are marginal costs. And competitive output is characterised indifferently by the property that profit is maximised at given prices or by the property that output prices are minimal decentralisation prices.

When the cost function is convex but not differentiable, this equivalence no longer holds and the minimality condition must be relaxed. In the convex case, the set of output prices p_b at which an output \bar{b} can be sustained as a competitive output given input prices p_a is defined by the property that p_b is a subgradient⁷ of c at (\bar{b}, p_a) , i.e., $p_b \in \partial_b c(\bar{b}, p_a)$. Furthermore, the correspondence ϕ can alternatively

be defined as⁸

$$\phi(b, p_a) = \partial_b c(b, p_a) + \mathbb{R}_+^n$$

and therefore, at a point \bar{b} where the cost function is not differentiable, $\phi^*(\bar{b}, p_a)$ is typically a proper subset of $\partial_b c(\bar{b}, p_a)$. This is illustrated by Figure 5 in which $\phi^*(\bar{b}, p_a) = \underline{p}_b$ where \underline{p}_b is the average cost and corresponds to the lower element of the subdifferential $\partial_b c(\bar{b}, p_a)$; all prices p_b , $\underline{p}_b \leq p_b \leq \bar{p}_b$, sustain \bar{b} as a competitive output and are thus accepted by the concept of competitive equilibrium.

A convex function is almost everywhere differentiable and its subdifferential can be expressed in terms of limits of gradients⁹. More precisely, for a convex cost function,

$$\partial_b c(\bar{b}, p_a) = \text{Co Lim Sup}_{b^v \rightarrow \bar{b}} \nabla_b c(b^v, p_a),$$

where c is differentiable along all the sequences (b^v) . Therefore, when *all* possible converging sequences are considered, the following inclusion holds:

$$\partial_b c(\bar{b}, p_a) \subseteq \text{Co Lim Sup}_{b^v \rightarrow \bar{b}} \phi^*(b^v, p_a).$$

But $\phi^*(b, p_a) \subset \partial_b c(b, p_a)$ and $\partial_b c(b, p_a)$ is a convex set which defines a closed correspondence on \mathbb{R}_+^n . Consequently, the converse inclusion holds, offering on *equivalent* characterisation of the subdifferential of a convex function.

This analysis suggests that the equilibrium condition (iii) could in general be written as $p_b \in \psi(\bar{b}, p_a)$ where the correspondence ψ is defined by

$$\psi(b, p_a) = \text{Co Lim Sup}_{b^v \rightarrow b} \phi^*(b^v, p_a).$$

Because ϕ is a closed correspondence with convex values, $\psi(b, p_a) \subset \phi(b, p_a)$. As a consequence, the equilibrium conditions (ii) and (iii) are covered simultaneously by the condition $p_b \in \psi(\bar{b}, p_a)$ which fully characterises competitive equilibria in the convex case. In other words, a feasible allocation and a price system, at which consumers optimise in their budget sets, producers minimise costs, and output prices are minimal decentralisation prices, or limits of minimal decentralisation prices, or convex combinations thereof, is an equilibrium. The logic of the limiting process is the same as that underlying the definition of generalised gradients by Clarke (1975), a logic also adopted in the recent literature on "marginal cost pricing equilibria" - see e.g., Bonnisseau and Cornet (1986). Intuitively, generalised gradients collect derivatives in all possible directions, and we collect "supply prices" for all possible perturbations of output. Furthermore, at an equilibrium, "convex producers"

maximise profits at given prices; and in a convex economy, such an equilibrium is a competitive equilibrium in the usual sense.

6. Before we give further justifications to our condition of minimal output prices, two shortcomings must be recognised. First, our criterion remains incomplete, in the same sense that marginal cost pricing is incomplete¹⁰, because with increasing returns it still admits some allocations which one would prefer to exclude, in order to retain only a subset of equilibria. Second, the criterion of "minimal output prices" is not the only complement to "voluntary trading" which leads to competitive equilibria in the convex case. One alternative would be "minimal profits" or possibly "competitive rates of returns". But in the general, non-convex case, these alternatives no longer have the same implications as "minimal output prices"; they are typically more selective.

Some of the ambiguities that we face here have their counterpart in the convex case. In the economy of Figure 6, the (unique) competitive equilibrium y^* is also the single Pareto optimum; but it is perhaps not a realistic *forecast* in the absence of anti-trust policy. What convexity contributes is an unequivocal guideline for anti-trust policy. By contrast, in the economy of Figure 7, if one insists on a natural decentralisation property, namely that consumption should be decentralised through prices at which production satisfies voluntary trading, there remain three non-trivial equilibria, labeled A, B and C respectively. In this example, C Pareto dominates B which Pareto dominates A. But A is locally undominated: information of a global nature is required to conclude that it is actually dominated by B which is locally dominated, and by C which is the global optimum under the decentralisation constraint.

In the example of Figure 7, our definition admits A, B and C as equilibria, and we are unwilling to introduce further conditions that would eliminate A, because such conditions would have to rest upon information of a global nature. Although B could be eliminated through conditions involving only local information, we refrain from doing so here because the *nature* of that information, namely elasticities of demand, is more sophisticated than the simple signals on which we rely.

Accepting the logic of "simple signals" used in the previous paragraph, one may wish to go back to the criterion of "voluntary trading" and object that it embodies global information about production sets. But the intuitive appeal of voluntary trading seems sufficient to overcome that objection. And it is important to notice that each firm acting in isolation can discover that an alternative production plan with smaller outputs leads to higher profits. At a long run or "final" equilibrium, voluntary trading seems compelling.

7. Let us review the economic arguments which suggest that "minimal output prices under voluntary trading" is a competitive-like property, in the light of the arguments put forward to justify the concept of competitive equilibrium in the convex case. We shall treat positive and normative aspects separately.

From a positive viewpoint, competitive equilibria have predictive appeal in a convex economy at two distinct logical levels. A deep argument applicable most clearly to the case of a constant returns technology freely available to all, says that positive profits will be eliminated by entry of new firms or by spontaneous coalition formation. That argument is clearly not applicable under increasing returns¹¹.

That argument does not apply either when some (convex) firms use non-reproducible resources, to which potential contestants do not have access, and to which a rent could accrue in the form of profits. In such situations, corresponding to strictly diminishing returns, a competitive equilibrium need not emerge naturally from the decentralised decisions of self-interested agents or coalitions. It will only emerge if the firms follow certain rules of behaviour, like putting on the market quantities of output which maximise profits at given prices; or if the economic organisation is such as to privilege competitive outcomes, for instance through auction markets.

The positive appeal of our equilibrium concept is of this second kind - and can also be spelled out alternatively in terms of rules of behaviour of the firms, or in terms of market organisation.

If firms quote prices; meet whatever demand materialises at these prices; revise prices upward when demand exceeds supply; but revise prices downward when demand falls short of supply, only to the extent compatible with voluntary trading, then an equilibrium will be characterised by "minimum output prices under voluntary trading".

Alternatively, if markets are organised with auctioneers who adjust prices in the direction of excess supply, subject to downward rigidities reflecting the voluntary trading condition - then an equilibrium will again be characterised by "minimum output prices under voluntary trading".

This indicates that an equilibrium concept could also be justified, from an altogether different viewpoint, as an *equilibrium with quantity constraints* imposed on output supplies, when and only when supply conditions introduce downward rigidity on prices, where the rigidities reflect the voluntary trading condition. This line of justification is bound to be less appealing to those who regard quantity constraints as unnatural. Yet, the example of Figure 1 should convince anyone that excess supply

at given prices is a normal byproduct of increasing returns. Once that conclusion is accepted, it seems natural to require that sales constraints only set in when prices become downward rigid; and prices have no reason to be downward rigid if they are not minimal, barring monopolistic elements.

An additional step would consist in studying a tâtonnement process on prices and quantity constraints (on output supplies) whereby prices are lowered whenever there is excess supply, whereas under excess demand quantity constraints are relaxed until they cease to be binding and prices are raised thereafter¹². A limit point of such a process would be an appealing equilibrium but the assumptions ensuring its quasi-stability would definitely be more restrictive than the ones used here¹³.

Short of modelling such a process, we prove existence of an equilibrium by a fixed point argument which involves a correspondence under which:

- market prices respond to excess demands;
- consumers announce their demands at market prices and at incomes incorporating profits computed at market prices;
- producers announce their production plans and prices such that
 - production costs of announced outputs are minimal at the announced input prices,
 - output prices are minimal subject to voluntary trading at the announced input prices:
- producers revise their production plans in the direction of discrepancies between market prices and the prices which they announce.

We show that the fixed points of that correspondence define competitive equilibria in our sense (we do *not* show that that correspondence defines a quasi-stable adjustment process...).

8. From a normative viewpoint, competitive equilibria in a convex economy have the compelling appeal of Pareto optimality. When applied to a specific market or product, the normative argument for competitive pricing relies on the strong assumption that the rest of the economy is competitive.

Our equilibria formalise the natural tendency of regulators to impose minimal output prices¹⁴, a tendency presumably based on the notion that lower prices compatible with voluntary trading (covering marginal costs) are better from a welfare viewpoint.

The limitations of that argument have been brought out by the second-best literature. In particular, Ramsey-Boiteux prices (see Boiteux, 1956) take demand elasticities into account - a level of sophistication typically absent from regulation, and unnecessary in the single output case. Our equilibria may thus be viewed

as third-best, where the additional constraint (beyond voluntary trading) is that only minimal information be used (namely, prices and quantities, *not* elasticities); a constraint that is innocuous at the level of partial, market by market analysis.

Simple examples reveal that second-best Pareto optimality may require violation of the voluntary trading condition; see Figure 8. Examples with several outputs reveal that, when the voluntary trading condition is imposed, the requirement of minimal output prices may again conflict with (constrained) optimality. Still, we feel that decentralisation based on simple signals defines an interesting equilibrium concept, extending naturally the competitive ideas to non-convex economies.

II. DESCRIPTION OF THE ECONOMY

The model we shall consider is the private ownership economy as described for instance in Debreu's "*Theory of Value*" (1959).

There are ℓ commodities, n producers and m consumers. Producer j is characterised by a production set Y_j . Consumer i is characterised by a consumption set X_i , a preference relation \succsim_i , an initial endowment ω_i , and shares in profits $(\theta_{i1}, \dots, \theta_{in})$. By construction, the latter satisfy $0 \leq \theta_{ij} \leq 1$ for all i and j , and $\sum_j \theta_{ij} = 1$ for all i .

We make the following assumptions on the consumers' characteristics:

- C.1 for all i , X_i is a closed subset of \mathbb{R}^ℓ , convex and bounded below;
- C.2 for all i , \succsim_i is a complete, continuous, convex¹⁵ and non-satiated preordering of X_i ;
- C.3 for all i , there exists $\hat{x}_i \in X_i$ such that $\hat{x}_i \ll \omega_i$.

These are usual assumptions, exactly as they appear in "*Theory of Value*". Although they could be weakened through more specific assumptions on the preference relations, *our focus is here on the production side*. We make the following assumptions on the producers:

- P.1 for all j , Y_j is a closed subset of \mathbb{R}^ℓ ;
- P.2 for all j , $Y_j + \mathbb{R}_-^\ell \subset Y_j$;
- P.3 for all j , $Y_j \cap \mathbb{R}_+^\ell = \{0\}$.

These are again usual assumptions, except that the aggregate production set ΣY_j is *not* assumed to be convex. Furthermore, free disposal (P.2) and absence of free production (P.3) are assumed to hold at the *individual* level. Note that P.3 implies

that *inactivity is feasible*, i.e., $0 \in Y_j$ for all j .

The set of *feasible allocation* is given by

$$\mathcal{A} = \{(y_1, \dots, y_n, x_1, \dots, x_m) \in \Pi Y_j \times \Pi X_i \mid \sum x_i \leq \sum \omega_i + \sum y_j\}$$

It is a subset of $\mathbb{R}^{\ell(m+n)}$ which is non-empty as a consequence of the assumptions C.3 and P.3. These indeed imply that $(0, \hat{x}_1, \dots, \hat{x}_m) \in \mathcal{A}$. The following assumption:

B. for all $z \in \mathbb{R}^{\ell}$, the set $\{(y_1, \dots, y_n) \in \Pi Y_j \mid \sum y_j \geq z\}$ is bounded in $\mathbb{R}^{\ell n}$,

is introduced to ensure that the set \mathcal{A} is itself bounded. Following Bonnisseau and Cornet (1986), this direct assumption has been preferred to the usual set of assumptions on the asymptotic cone of the aggregate production set because it is **actually less restrictive, especially in a context where increasing returns prevail**¹⁶.

Altogether the assumptions which have been introduced would ensure the existence of a (standard) competitive equilibrium, had we assumed the convexity of the aggregate production set; cfr. Debreu (1959).

The behaviour of the consumers is the usual one: they take the prices and profits as given when choosing the consumption plans which are best, with respect to their preferences, in their budget sets. The behaviour of the producers differs from the usual one and is the subject of the next section.

III. BEHAVIOUR OF THE PRODUCERS

In this section, we shall be concerned with a given producer characterised by some production set Y which will be assumed to satisfy the assumptions P.1, P.2 and P.3: Y is therefore a closed and comprehensive¹⁷ subset of \mathbb{R}^{ℓ} , whose intersection with the positive orthant coincides with the origin.

In the standard competitive model, the production sets are convex and the behaviour of the producers is summarized by their supply correspondences which define profit maximising production plans corresponding to given prices. Here instead, we proceed along the lines initiated by Dierker, Guesnerie and Neufeind (1985). They use the concept of "*pricing schemes*" which define "*acceptable prices*" associated with given production plans¹⁸. More precisely, a pricing scheme is a correspondence $\psi: Y \rightarrow \mathbb{R}_+^{\ell}$. A price system $p \in \mathbb{R}_+^{\ell}$ associated with a production plan $y \in Y$ is then said to be "*in equilibrium*" if and only if $p \in \psi(y)$. When Y is a convex set, profit maximisation on Y at given prices is obtained by defining the pricing scheme as the *normal cone*, i.e., $\psi(y) = \mathbb{N}_Y(y)$. Indeed, in that case, the condition

$p \in \psi(y)$ means $py \geq py'$ for all $y' \in Y$, see Rockafellar (1970) for a definition¹⁹.

To prove existence of an equilibrium for given pricing schemes, one shows that the latter are closed correspondences whose values are non-degenerated convex cones with vertex zero. In particular, these properties ensure that, when intersected with the unit simplex, the pricing schemes yield well defined uhc (upper hemi-continuous) correspondences. The subject of the present section is precisely to construct a pricing scheme ψ which embodies the ideas of voluntary trading and minimality of output prices, while satisfying these existence requirements. That pricing scheme ψ is obtained after a sequence of intermediate definitions - ϕ , ϕ^* , ψ^* - which retrace more formally and more generally the reasoning in the introduction. The impatient reader may thus look at these formal definitions alone and skip the commentaries. As we proceed, we use systematically the convex case as a benchmark, and we provide in Lemmata 1-3 characterisations of our concepts when Y is convex. After defining our pricing scheme ψ formally, we establish (Lemma 4) the important property that it is a closed correspondence. And we prove (Proposition 1) that it corresponds to profit maximisation when the production set Y is convex.

Let $\partial Y = \{y \in Y \mid \nexists y' \in Y, y' \gg y\}$. For $y \in \partial Y$, the set

$$\phi(y) = \{p \in \mathbb{R}_+^l \mid py \geq py' \quad \forall y' \in Y, y' \leq y^+\},$$

defines the price systems which are compatible with voluntary trading at y : given $p \in \phi(y)$, it is profitable for the firm to meet fully the demand as given by y^+ , instead of producing less²⁰.

The restriction to prices in \mathbb{R}_+^l is a consequence of the assumption of free disposal (comprehensiveness of Y). The restriction to (weakly) efficient production plans ($y \in \partial Y$) comes from the fact that $\phi(y) = \{0\}$ whenever $y \in \text{Int } Y$. Clearly, for all $y \in \partial Y$, $\phi(y)$ is a non-degenerated, closed and convex cone with vertex zero. It can equivalently be defined as the normal cone to the convex set²¹
Co $\{y' \in Y \mid y' \leq y^+\}$ at y . Furthermore, it defines a correspondence $\phi: \partial Y \rightarrow \mathbb{R}_+^l$ whose graph is closed, as a consequence of the closedness of Y . Finally, if $y \in \partial Y \cap \mathbb{R}_-^l$, $\phi(y)$ coincides with the normal cone of \mathbb{R}_-^l at y which is given by $\{p \in \mathbb{R}_+^l \mid py = 0\}$ and is uniquely defined (up to a multiplicative constant) whenever $y \neq 0$. In particular, $\phi(0) = \mathbb{R}_+^l$.

As we do not wish to make an a priori distinction between inputs and outputs, we must define the set of inputs and the set of outputs for every production plan. The set of inputs at $y \in Y$ is a subset of $\{1, \dots, l\}$ defined by

$$I(y) = \{h \mid y_h < 0 \text{ or } y_h' \leq 0 \text{ for all } y' \in Y\}.$$

It is the index set of the commodities which are either effectively used as input at y or never appear as output. It therefore includes the commodities which are never involved in the production process. Its complement defines the set of outputs at y : commodity h is an output at y if $y_h \geq 0$ and $y'_h > 0$ for some $y' \in Y$. The absence of free production (P.3) ensures that $I(y)$ is a non-empty set for all $y \in Y$, $y \neq 0$. Hence, $I(y) = \emptyset$ for some $y \in Y$ implies $y = 0$.

The following lemma provides a simple characterisation of the set $\phi(y)$ in the convex case:

LEMMA 1: If Y is a convex set, then for all $y \in \partial Y$, $\phi(y) = N_Y(y) + C(y)$ where $C(y) = \{p \in \mathbb{R}_+^L \mid p_h = 0 \ \forall h \in I(y)\}$.

(The proof of the lemmata are given in Appendix.)

As an immediate consequence of this lemma, the normal cone is seen to be a subset of $\phi(y)$ in the convex case. The usefulness of Lemma 1 will become clear after the following definition. The set of price systems for which output prices are minimal subject to the condition of voluntary trading is given by

$$\phi^*(y) = \{p \in \phi(y) \mid \nexists p' \in \phi(y), p' < p, p'_h = p_h \ \forall h \in I(y)\}.$$

It is a cone with vertex zero which is generally non-convex and may be degenerated. If $y \in \partial Y \cap \mathbb{R}_-^L$, then $\phi^*(y) = \phi(y) = \{p \in \mathbb{R}_+^L \mid p_h = 0 \ \forall h \in I(y)\}$ and $\phi^*(y) \neq \{0\}$ when $y \neq 0$. On the other hand, $\phi^*(y)$ is equal to the origin if and only if either (i) $y = 0$ or (ii) $y \in \partial Y \setminus \mathbb{R}_-^L$ but $p \in \phi(y)$ implies $p_h = 0$ for all $h \in I(y)$; in this second case, $y_h < 0$ for all $h \in I(y)$. Hence, if $y \notin \partial Y \cap \mathbb{R}_-^L$, $\phi^*(y) = \{0\}$ if and only if *voluntary trading at y imposes zero input prices*, implying that all potential inputs are in effective use. Such a situation may occur at points where isoquants are not convex or at inefficient boundary points. This is a consequence of the fact that *costs are minimised* at (p, y) whenever $p \in \phi(y)$. These cases are illustrated in Figures 9 and 10. Such a situation may also occur at efficient points, at which some non-zero elements of the normal cone - *generalised*²² in the sense of Clarke - have zero input coordinates. Illustration of this case is also given in Figure 10. It should be noticed that $\phi^*(y) = \{0\}$ may occur in the convex case as well as in the non-convex case.

From Lemma 1, we can immediately conclude that, in the convex case, the following sequence of inclusions holds for all $y \in \partial Y$:

$$\phi^*(y) \subset N_Y(y) \subset \phi(y).$$

Furthermore, when the production set is both *convex and smooth*²³, $\phi^*(y)$ actually coincides with the normal cone whenever $\phi^*(y) \neq \{0\}$:

LEMMA 2: Assume that Y is convex and consider a point $y \in \partial Y$ at which $\phi^*(y) \neq \{0\}$. Then $\phi^*(y) = N_Y(y)$ whenever $N_Y(y)$ consists of a half-line.

Hence, in the convex and smooth case, voluntary trading and minimality of output prices is equivalent to profit maximisation, except in the extreme situation where all inputs are in use and voluntary trading imposes zero input prices.

At points where the boundary of the production set is not smooth, $N_Y(y)$ is typically larger than $\phi^*(y)$, as shown in Figure 11. Actually, $\phi^*(y)$ is generally not convex whenever more than one output is involved. Furthermore, as a correspondence, ϕ^* is not necessarily closed. However, the following result holds in the convex case:

LEMMA 3: Assume that Y is a convex set and consider a point $y \in \partial Y$ at which $\phi^*(y) \neq \{0\}$. Then,

$$\text{Co Lim Sup } \phi^*(y^v) = N_Y(y)$$

where the sequences (y^v) are taken on the boundary ∂Y and converge to y .²⁴

Hence, in the convex case, the normal cone coincides with the smallest closed correspondence with convex values which contains ϕ^* , whenever $\phi^*(y) \neq \{0\}$. Actually this result remains true when $\phi^*(y) = \{0\}$ but $\phi^*(y^v) \neq \{0\}$ along (at least) one converging sequence in ∂Y .

From Lemma 3, we conclude that $\text{Co Lim Sup } \phi^*$ is a natural candidate for an pricing rule in the case where $\phi^*(y) \neq \{0\}$. For any given $y \in \partial Y$, we shall actually distinguish the following three cases:

Case 1: $\phi^*(y) \neq \{0\}$

Case 2: $\phi^*(y) = \{0\}$ and $\phi(y) \cap N_Y(y) \neq \{0\}$

Case 3: $\phi^*(y) = \{0\}$ and $\phi(y) \cap N_Y(y) = \{0\}$

The second case can arise with convex as well as non-convex production sets. The third case arises only when Y is not convex. The second case is illustrated in Figure 12.a which involves one input and two outputs. In that situation, $\phi(y) = \{p \in \mathbb{R}_+^3 | p_1 = 0\}$ and therefore $\phi^*(y) = \{0\}$. In this case, $N_Y(y) = \{p \in \mathbb{R}_+^3 | p_1 = 0, p_2 = p_3\}$ and consequently $\phi(y) \cap N_Y(y) = N_Y(y)$. Such a situation can arise in the convex case as well. The third case is illustrated in Figure 12.b which involves two inputs and one output, with a concave isoquant. In that situation, $\phi(y) = \{p \in \mathbb{R}_+^3 | p_1 = p_2 = 0\}$ and $\phi^*(y) = \{0\}$. In this case, $N_Y(y) = \{p \in \mathbb{R}_+^3 | p_1 = p_2 = p_3\}$ and, consequently $\phi(y) \cap N_Y(y) = \{0\}$.

When Y is convex, the third case does not arise. Indeed, Lemma 1 implies that $\phi(y) \cap \mathbb{N}_Y(y) = \mathbb{N}_Y(y)$, where $\mathbb{N}_Y(y) \neq \{0\}$ for all $y \in \partial Y$. We conjecture that the third case does not arise *when the isoquants are convex*:

CONJECTURE: Assume that the set $\{y' \in Y | y'^+ = y^+\}$ is convex for all $y \in \partial Y$. Then if $\phi^*(y) = \{0\}$, there exists $p \in \mathbb{N}_Y(y)$, $p \neq 0$, such that $p_h = 0$ for all $h \in I(y)$.

Let us consider the correspondence $\psi^*: \partial Y \rightarrow \mathbb{R}_+^{\ell}$ defined by

$$\psi^*(y) = \begin{cases} \phi^*(y) & \text{in case 1} \\ \phi(y) \cap \mathbb{N}_Y(y) & \text{in case 2} \\ \phi(y) & \text{in case 3.} \end{cases}$$

The pricing scheme $\psi: \partial Y \rightarrow \mathbb{R}_+^{\ell}$ we shall work with is then defined by:

$$\psi(y) = \text{Co Lim Sup } \psi^*(y^V)$$

where the sequences (y^V) are taken on ∂Y and converge to y . By construction, $\psi(y)$ is a non-degenerated *convex* cone with vertex zero. However, because the convex hull of a closed correspondence is not necessarily closed, we must prove the following:

LEMMA 4: The correspondence ψ is *closed*.

As a consequence of the fact that ϕ is a closed correspondence with convex values, the following result holds:

LEMMA 5: $\psi(y) \subset \phi(y)$ for all $y \in \partial Y$.

Hence, price systems in $\psi(y)$ are compatible with voluntary trading. Therefore, inactivity being feasible, $p y \geq 0$ whenever $p \in \psi(y)$ and $y \in \partial Y$.

In the convex case, ψ coincides with the normal cone:

PROPOSITION 1: If Y is a convex set, then $\psi(y) = \mathbb{N}_Y(y)$ for all $y \in \partial Y$.

PROOF

Once in the convex case, the third case does not arise and the proof of Lemma 3 can be transposed, replacing ϕ^* by ψ^* , to obtain the result.

To conclude this section, we remark that, under our definition of inputs and outputs, imposing minimal profits subject to voluntary trading does *not* select a subset of $\phi^*(y)$, i.e., $\hat{\phi}(y) \not\subset \phi(y)$ for all $y \in \partial Y$, where

$$\hat{\phi}(y) = \operatorname{argmin}_{p \in \phi(y)} py.$$

This inclusion would hold if an alternative definition of outputs had been adopted, restricting attention to effectively produced commodities. Nevertheless, replacing ϕ^* by $\hat{\phi}$ leads to a pricing scheme $\hat{\psi}$ which will again reproduce the normal cone in the convex case but will be typically "smaller" than ψ .

IV. THE EQUILIBRIUM CONCEPT

In this section, we shall formally define what we mean by a "competitive equilibrium with price-and-quantity taking firms", abbreviated to a "competitive equilibrium", when there are non-convexities in production and we shall prove its existence under the assumptions introduced in Section II.

A *competitive equilibrium* is defined by a price system $\bar{p} \neq 0$, a set of production plans $(\bar{y}_1, \dots, \bar{y}_n)$ and a set of consumption plans $(\bar{x}_1, \dots, \bar{x}_m)$ satisfying the following conditions:

E.1 It is a feasible allocation, up to free disposal, i.e.,

$$\sum \bar{x}_i \leq \sum \omega_i + \sum \bar{y}_j$$

with equality for the commodities whose price is positive.

E.2 It is a best choice for the consumers, given the prices and profits, i.e., for all i , \bar{x}_i is \succeq_i -maximal in the budget set

$$\{x_i \in X_i \mid \bar{p}x_i \leq \bar{p}\omega_i + \sum_{ij} \theta_{ij} \bar{p}y_j\}.$$

E.3 It is a best choice for the producers, given the prices and demand levels, i.e., for all j , \bar{y}_j maximises $\bar{p}y_j$ on the set

$$\{y_j \in Y_j \mid y_j \leq \bar{y}_j^+\}.$$

E.4 For every producer, the output prices are minimal with respect to condition E.3, in the sense that

$$\bar{p} \in \psi_j(\bar{y}_j) \text{ for all } j.$$

The first two equilibrium conditions are standard²⁵. The third condition imposes *voluntary trading*: at the going prices, each producer chooses to satisfy demand fully, and minimises the associated production costs. Furthermore, $\bar{y}_j \in \partial Y_j$ as a consequence of E.3. In terms of the correspondences which were introduced in Section 3, the condition E.3 simply reads:

$$\bar{p} \in \phi_j(\bar{y}_j) \text{ for all } j.$$

However, because $\psi_j(\bar{y}_j) \subset \phi_j(\bar{y}_j)$ by Lemma 5, condition E.4 actually embodies condition E.3 when $y_j \in \partial Y_j$.

Following Proposition 1, at an equilibrium, the convex producers actually maximise their profits at given prices. Indeed, in that case, condition E.4 reads:

$$\text{for all } j, \bar{p} y_j \geq p y_j \quad \forall y_j \in Y_j,$$

i.e. $\bar{p} \in \Pi_Y(\bar{y}_j)$ for all j . As a consequence, in a convex economy, the equilibrium concept we have introduced coincides with the standard concept of competitive equilibrium as suggested by our title.

It should be noticed that our equilibrium is in general *not production efficient in the aggregate*, i.e., $\Sigma \bar{y}_j \notin \partial \Sigma Y_j$. As indicated by Beato and Mas Colell (1985), this appears to be a natural consequence of the aggregation of production sets when some of them are non-convex.

V. EXISTENCE OF EQUILIBRIUM

PROPOSITION 2: Under the assumptions C.1 to C.3, P.1 to P.3 and B, there exists a competitive equilibrium.

PROOF:²⁶

The set of attainable states is non-empty and bounded in $\mathbb{R}^{\ell(m+n)}$, and so are the individual attainable consumption and production sets. Therefore, there exists a closed cube K in \mathbb{R}^{ℓ} with length k , centered at the origin and containing in its interior all the (individual) attainable consumption and production sets; cfr. Debreu, (1959, p. 85). We then define $\bar{X}_i = X_i \cap K$ and

$$\bar{Y}_j = \{y_j \in Y_j' \mid \nexists y_j' \in Y_j', y_j' \leq y_{jh} \text{ for all } h, \text{ with strict inequality if } y_{jh} > 0\}$$

where $Y_j' = \partial(Y_j + \{ke\}) \cap \mathbb{R}_+^{\ell}$ and $e = (1, \dots, 1)$.

Let f_j denote the projection of points in $\mathbb{R}_+^{\ell}/\{0\}$ on S , the unit simplex of \mathbb{R}_+^{ℓ} . It is well known that, under P.1 to P.3 and B, as a function from \bar{Y}_j into S , f_j defines a *homeomorphism* which satisfies:

$$(1) \quad f_j(y_j) \gg 0 \text{ if and only if } y_j \gg 0.$$

See for instance Brown, Heal, Khan and Vohra (1984). Defining $g_j(s) = f_j^{-1}(s) - ke$, P.2 ensures that $g_j(s) \in \partial Y_j$ for all $s \in S$.

We shall now construct a correspondence Φ whose fixed points are equilibria.

For each i , the (quasi) demand correspondence $\xi_i: S^{n+1} \rightarrow \bar{X}_i$ is defined by $\xi_i(p, s_1, \dots, s_n)$, the set of \sum_i -maximisers on the set

$$\{x_i \in \bar{X}_i \mid p x_i \leq p \omega_i + \sum_j \theta_{ij} p g_j(s_j)\}$$

if $p \omega_i + \sum_j \theta_{ij} p g_j(s_j) > \text{Min } p \bar{X}_i$; and by $\text{argmin } p \bar{X}_i$ if not. This correspondence is known to be *uhc*, and it has convex and compact values; see Debreu (1962, p. 261). Note that free disposal ensures that $\text{Min } p \bar{X}_i = \text{Min } p X_i$.

For each j , the supply correspondence $\beta_j: S^3 \rightarrow S$ is defined by the function

$$\beta_j(p, q_j, s_j) = \frac{1}{\lambda_j(\cdot)} F_j(\cdot)$$

where $F_{jh}(\cdot) = \text{Max}(0, s_{jh} + p_h - q_{jh})$ and $\lambda_j(\cdot) = \sum_h F_{jh}(\cdot)$. Clearly, $\lambda_j(\cdot) \geq 1$ on S^3 , ensuring the continuity of β_j . Here p denotes "market prices" as opposed to the q_j 's which denote "producer's prices".

Market prices are determined through the usual correspondence $\mu: \prod \bar{X}_i \times S^n \rightarrow S$ defined by

$$\mu(x_1, \dots, x_m, s_1, \dots, s_n) = \underset{p \in S}{\text{argmax}} p(\sum x_i - \sum \omega_i - \sum g_j(s_j)).$$

The prices of producer j are determined through the correspondences $\bar{\psi}_j: S \rightarrow S$ defined by

$$\bar{\psi}_j(s) = \psi_j(g_j(s)) \cap S.$$

Because the ψ_j 's are closed correspondences whose values are non-degenerate convex cones, the correspondences $\bar{\psi}_j$ are *uhc* and have convex and compact values; see Hildenbrand (1974, p. 23).

The correspondence Φ from $S^{2n+1} \times \prod \bar{X}_i$ into itself is then defined as follows:

$$\Phi(p_1, q_1, \dots, q_n, s_1, \dots, s_n, x_1, \dots, x_m) = \mu(\cdot) \times \prod \beta_j(\cdot) \times \prod \bar{\psi}_j(\cdot) \times \prod \xi_i(\cdot).$$

It satisfies the conditions of Kakutani's theorem according to which Φ has a fixed point. We denote that fixed point by $(\bar{p}, \bar{q}_1, \dots, \bar{q}_n, \bar{s}_1, \dots, \bar{s}_n, \bar{x}_1, \dots, \bar{x}_m)$ and we define $\bar{y}_j = g_j(\bar{s}_j)$ and $\bar{z} = \sum \bar{x}_i - \sum \bar{\omega}_i - \sum \bar{y}_j$. Then, $\bar{y}_j \in \partial Y_j$ for all j and the following conditions are satisfied:

$$(2) \quad \bar{s}_j = \beta_j(\bar{p}, \bar{q}_j, \bar{s}_j) \text{ for all } j;$$

$$(3) \quad \bar{x}_i \in \xi_i(\bar{p}, \bar{s}_1, \dots, \bar{s}_n) \text{ for all } i;$$

$$(4) \quad \bar{p}z \leq \bar{p}z \text{ for all } p \in S;$$

$$(5) \quad \bar{q}_j \in \psi_j(\bar{y}_j) \cap S \text{ for all } j.$$

Let $\bar{\lambda}_j = \lambda_j(\bar{p}, \bar{q}_j, \bar{s}_j)$. Then (2) implies

$$(6) \quad \bar{\lambda}_j \bar{s}_{jh} \geq \bar{s}_{jh} + \bar{p}_h - \bar{q}_{jh}$$

with equality whenever $\bar{s}_{jh} > 0$. Multiplying both sides of (6) by \bar{s}_{jh} and summing over all h , we then get

$$(\bar{\lambda}_j - 1) \bar{s}_j \bar{s}_j = (\bar{p} - \bar{q}_j) \bar{s}_j$$

where $\bar{\lambda}_j \geq 1$ and $\bar{s}_j \bar{s}_j \geq 1/\ell$. We therefore have the following set of inequalities

$$(7) \quad (\bar{p} - \bar{q}_j) \bar{s}_j \geq 0 \text{ for all } j.$$

By definition of f_j , there exists $\bar{\mu}_j > 0$ such that $\bar{s}_j = \bar{\mu}_j(\bar{y}_j + ke)$. Using the fact that $(\bar{p} - \bar{q}_j)e = 0$, we then have

$$(\bar{p} - \bar{q}_j) \bar{s}_j = \bar{\mu}_j(\bar{p} - \bar{q}_j) \bar{y}_j$$

which, combined with (7) gives

$$(8) \quad \bar{p} \bar{y}_j \geq \bar{q}_j \bar{y}_j \text{ for all } j.$$

By (5), $\bar{q}_j \bar{y}_j \geq 0$. Therefore, $\bar{p} \bar{y}_j \geq 0$ for all j and C.3 ensures that $\bar{p} \omega_i + \sum_{ij} \theta_{ij} \bar{p} \bar{y}_j > \min_i \bar{p} \bar{x}_i$ for all i . Using (3) and summing over all budget constraints, we get $\bar{p}z \leq 0$. Combining this with (4) gives $z \leq 0$. The fixed point therefore defines an attainable state and consequently $f_j^{-1}(\bar{s}_j) \gg 0$ for all j . Hence, by (1), $\bar{s}_j > 0$ for all j ; and by (6), $\bar{\lambda}_j = 1$ and $\bar{q}_j = \bar{p}$ for all j . Conditions E.3 and E.4 are therefore satisfied. Condition E.2 follows from (3) by a standard argument; see Debreu (1959, p, 87). On the other hand, C.2 implies local non-satiation. As a consequence, the budget constraints hold with equality; $\bar{p}z = 0$ and condition E.1 follows. QED

To conclude this section, let us mention that our definition of equilibrium does not allow for a trivial existence proof whenever $\psi_j(0) \neq R_{+L}$ for some non-convex j . When $\psi_j(0) = R_{+L}$ for all non-convex j , a proof would consist in constraining the non-convex producers to inactivity and in looking at the competitive

equilibrium in the resulting convex economy. If $\psi(0) = R_{+L}$, then $N_Y(0) = R_{+L}$ and this may be used as a definition of a technology with *set-up costs*, a case where "marginal costs" are infinite at the origin.

APPENDIX

PROOF OF LEMMA 1

When Y is convex, $\phi(y)$ is the normal cone at y of the intersection of Y with $\{y' \in \mathbb{R}^L | y' \leq y^+\}$. Therefore, following Rockafellar (1970, p; 233), $\phi(y)$ can be written as $\phi(y) = N_Y(y) + C'(y)$, where $C'(y) = \{p \in \mathbb{R}_+^L | p_h = 0 \text{ for all } h \text{ such that } y_h < 0\}$ is the normal cone to $\{y \in \mathbb{R}^L | y' \leq y^+\}$ at y .

Let us fix $p \in N_Y(y)$ and assume that, for some h , $y_h = 0$ and $y'_h \leq 0$ for all $y' \in Y$. Then, any vector p' obtained from p by adding a positive quantity δ to its h^{th} coordinate is again an element of $N_Y(y)$. Indeed, we then have $p'y = py \geq p'y' \geq py' + \delta y'_h = p'y'$ for all $y' \in Y$.

We can therefore conclude that $\phi(y) = N_Y(y) + C(y)$. □

PROOF OF LEMMA 2

From Lemma 1, $\phi^*(y) \subset N_Y(y)$. Hence, if $\phi^*(y) \neq \{0\}$, $\phi^*(y) = N_Y(y)$ when $N_Y(y)$ is a half-line. □

PROOF OF LEMMA 3

On the one hand, $\phi^*(y) \subset N_Y(y)$ implies

$$\text{Co Lim Sup } \phi^*(y^\vee) \subset N_Y(y).$$

This follows immediately from the fact that $N_Y(\cdot)$ defines a correspondence with closed graph and convex values; cfr. Rockafellar (1970). The convex inclusion follows from a result which has been established by Cornet (1986) according to which "the boundary of a convex set is almost everywhere smooth":

Let X be a closed, convex and comprehensive subset of \mathbb{R}^L . Then, there exist a subset $E \subset \partial X$ of measure zero such that $\text{cl}(\partial X/E) = \partial X$, and a continuous function $q: \partial X/E \rightarrow \mathbb{R}^L$ such that $N_X(x) = \{p \in \mathbb{R}^L | p = \lambda q(x), \lambda \geq 0\}$ for all $x \in \partial X/E$. Moreover, for all $x \in X$, the normal cone can be defined as $N_X(x) = \text{Co Lim Sup } N_X(x^\vee)$ where the sequence (x^\vee) are taken in $\partial X/E$ and converge to x .

From Lemma 2, we can then immediately conclude that $N_Y(y) = \text{Co Lim Sup } \phi^*(y^\vee)$ where the sequences are taken on a subset of ∂Y . Therefore $N_Y(y) \subset \text{Co Lim Sup } \phi^*(y^\vee)$ where the sequences are taken on the entire boundary of Y . □

PROOF OF LEMMA 4

The correspondence $\gamma: \partial Y \rightarrow \mathbb{R}_+^{\ell}$ defined by

$$\gamma(y) = \text{Co} (S \cap \text{Lim Sup } \psi^*(y^v))$$

where S is the unit simplex of \mathbb{R}_+^{ℓ} is uhc; see Hildenbrand (1974, pp. 23 and 26).

Let C be a non-degenerate cone of \mathbb{R}^{ℓ} with vertex zero. Then $\text{Co } C = \text{cone} (\text{Co} (C \cap S))$, where the right-hand side denotes the cone with vertex zero generated by $\text{Co} (C \cap S)$. Because the latter is a convex set containing C , a first inclusion immediately follows. To establish the converse inclusion, let $p \in \text{cone} (\text{Co} (C \cap S))$, $p \neq 0$. Then, by Caratheodory's theorem, there exist $\lambda > 0$ and $(\lambda^i, p^i)_{i=0}^{\ell}$ such that $\sum \lambda^i = 1$, $p = \sum \lambda^i p^i$, $\lambda^i \geq 0$ and $p^i \in C \cap S$ for all i . Because C is a cone, $\lambda p^i \in C$ and therefore $p = \sum \lambda^i (\lambda p^i) \in \text{Co } C$.

Hence, $\psi(y) = \text{cone } \gamma(y)$. Consider now sequences (y^v) and (p^v) such that $y^v \rightarrow y$, $p^v \rightarrow p$ and $p^v \in \psi(y^v)$ for all v . Then, there exist sequences (λ^v) and (q^v) such that $p^v = \lambda^v q^v$ and $q^v \in \gamma(y^v)$ for all v . Because $\gamma(y) \subset S$ for all y , $\lambda^v = \sum p_h^v + \lambda = \sum p_h^v$, and there exists a converging subsequence (q^{v_q}) , $q^{v_q} \rightarrow q$. Furthermore, $q \in \gamma(y)$ as a consequence of the fact that γ is uhc, and $\lambda q = p$. Hence, $p \in \psi(y)$ establishing that ψ is a closed correspondence. \square

PROOF OF LEMMA 5

Let $p \in \psi^*(y)$ for some $y \in \partial Y$. Then $p = \text{Lim } p^v$ where $p^v \in \psi^*(y^v)$ for some sequence (y^v) converging to y along ∂Y . Because $\psi^*(y) \subset \phi(y)$, $p^v \in \phi(y^v)$ for all v . Therefore, ϕ being a closed correspondence, $p \in \phi(y)$. Hence $\text{Lim Sup } \psi^*(y^v) \subset \phi(y)$ and $\phi(y)$ being convex, $\psi(y) \subset \phi(y)$. \square

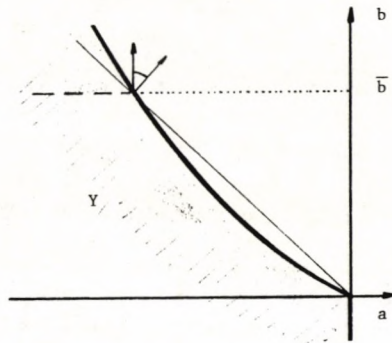


Fig. 1

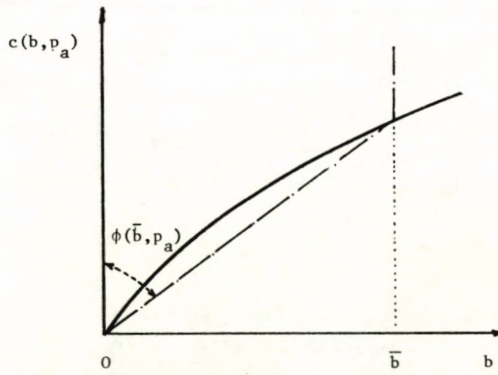


Fig. 2

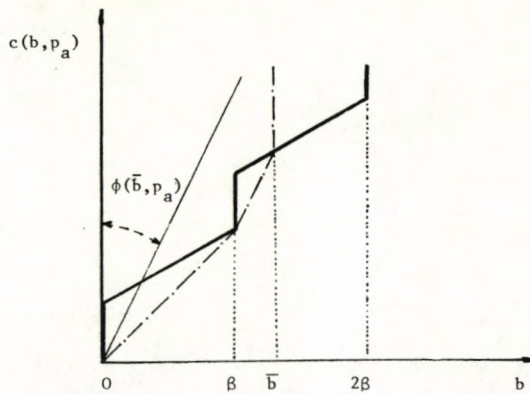


Fig. 3

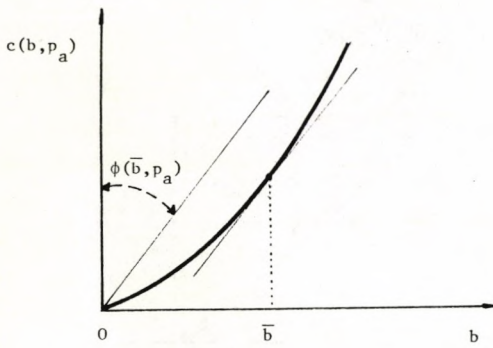


Fig. 4

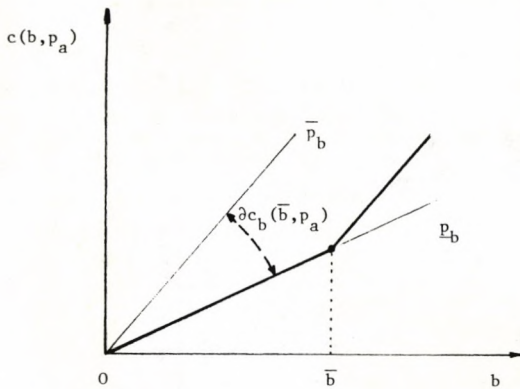


Fig. 5

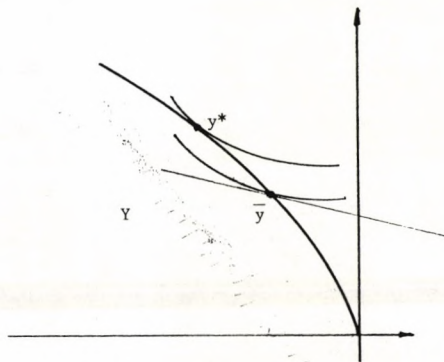


Fig. 6

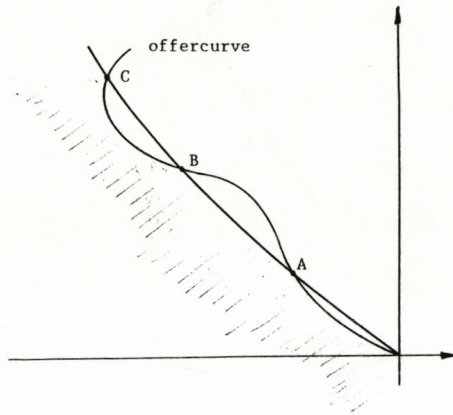


Fig. 7

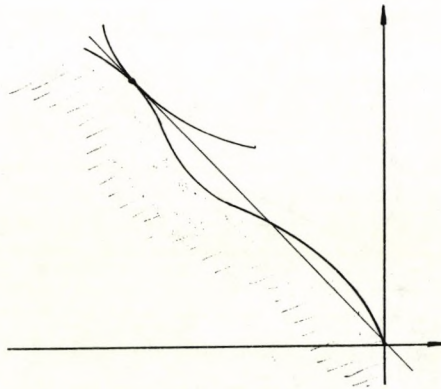


Fig. 8

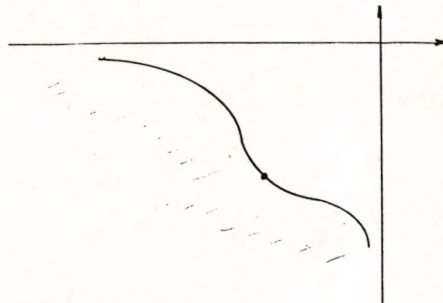


Fig. 9

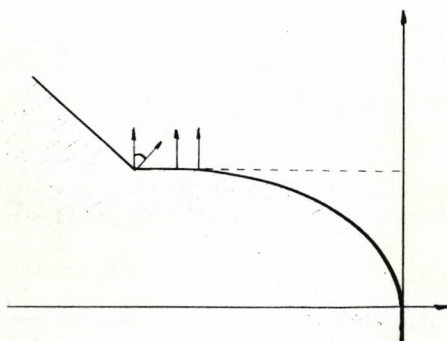


Fig. 10

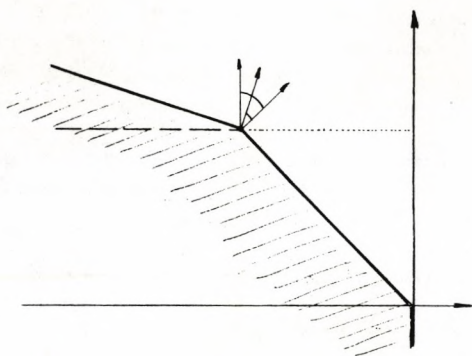


Fig. 11

Fig. 12.a

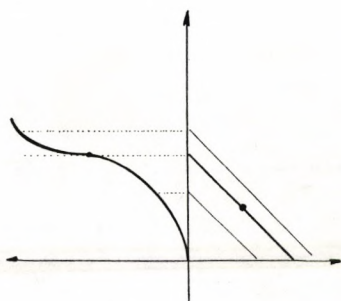
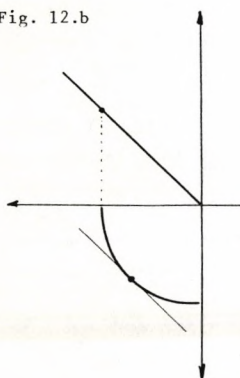


Fig. 12.b



FOOTNOTES

- ¹ One should perhaps add to this list a fourth property of simplicity and elegance: the concept is easily stated and understood; given the taste of some of our esteemed contemporaries for technical intricacies, we refrain from treating such controversial grounds.
- ² This does not exclude that our approach may serve as a stepping stone to study monopolistic equilibria in the tradition of Negishi (1961).
- ³ For vector inequalities, we adopt the following sequence of symbols: $\geq, >, \gg$.
- ⁴ Under convexity, when supply is defined by a correspondence rather than then by a function, meeting demand naturally determines output levels.
- ⁵ This is to be contrasted with the work of Scarf (1963). He considers an economy whose production sector is described by an aggregate production set which displays a form of increasing returns to scale. At an equilibrium, the production sector maximises profits *subject to an input constraint*, at given prices
- ⁶ In the break even case, the cost function is supportable (see Sharkey and Telser, 1973) and the output price vector is unanimously equitable (see Faulhaber and Levinson, 1981).
- ⁷ See Rockafellar (1970) for definitions of the concepts of convex analysis.
- ⁸ We use here (and latter in Section I) the Theorem 25.6 of Rockafellar (1970, p. 246).
- ⁹ This is again a consequence of Theorem 25.6 of Rockafellar. In Section III, we shall use a generalisation of this result to the normal cone, which is due to Cornet (1986).
- ¹⁰ In the absence of convexity, marginal cost pricing is necessary, but not sufficient, for Pareto efficiency; see Guesnerie (1975).
- ¹¹ It is interesting to note that, when production set reflects technological knowledge alone ($Y \supset Y + Y$), the allocation of output among several firms may result in higher prices than concentration in a single firm, as in the case of Figure 2, but may also result in higher prices, as in the case of Figure 3 for an aggregate output \bar{b} .
- ¹² See Drèze (1986) for an example of such a process which leads to a supply-constrained equilibrium as defined in Dehez and Drèze (1984).
- ¹³ Drèze (1986) assumes absence of inferior goods and notes that this assumption plays for quantity adjustment a role similar to that played by gross substitutability for price adjustments; combining both assumptions is definitely unappealing.

- ¹⁴ Regulation often takes the related, yet distinct, form of imposing "competitive" (i.e., minimal subject to voluntary trading) rates of return on investment.
- ¹⁵ By convex, we mean: $x \succ x'$ implies $\lambda x + (1 - \lambda)x' \succ x'$ for all λ , $0 < \lambda \leq 1$.
- ¹⁶ See for instance Dierker, Guesnerie and Neufeind (1985) for a set of assumptions ensuring the boundedness of \mathbb{A} . These assumptions on the aggregate production set and its asymptotic cone exclude in particular the case of a production set defined by the function $y_2 = (y_1)^2$.
- ¹⁷ A set Y is comprehensive if it satisfies free disposal, i.e., if $Y + \mathbb{R}_-^{\ell} \subset Y$.
- ¹⁸ These authors actually use the term *pricing rule*. We have preferred to use the term *pricing scheme* because our focus is not on regulation. They use a *joint* pricing rule which depends in addition on prices. More generally, a pricing scheme may depend also on production and
- ¹⁹ Formally, the *normal cone* to a convex set X at some point $x \in X$ is defined by $N_X(x) = \{p \in \mathbb{R}^{\ell} \mid p x \geq p x' \text{ for all } x' \in X\}$.
- ²⁰ For any vector $x \in \mathbb{R}^{\ell}$, x^+ denotes the vector whose coordinates are $\text{Max}(0, x_h)$.
- ²¹ Here, $\text{Co } X$ denotes the *convex hull* of the set X .
- ²² Clarke (1975) has proposed a definition of a generalised normal cone which coincides with the normal cone in the convex case and is always non-degenerate; see also Clarke (1983).
- ²³ A set X is "*smooth*" at a point $x \in \partial Y$ if and only if $N_X(x)$ is a half-line, i.e., there exists $q(x) \in \mathbb{R}^{\ell}$, $q(x) \neq 0$, such that $p \in N_X(x)$ implies $p = \lambda q(x)$ for some $\lambda \geq 0$.
- ²⁴ Here Lim Sup is defined as $\{p \in \mathbb{R}_+^{\ell} \mid \exists (p^{\vee}, y^{\vee}), y^{\vee} \in \partial Y, p^{\vee} \in \phi^*(y^{\vee}) \forall y^{\vee}, (p^{\vee}, y^{\vee}) \rightarrow (p, y)\}$.
- ²⁵ If there was a producer with a *convex* production set satisfying free disposal, condition E.1 could be written with equality.
- ²⁶ The proof given here is inspired by an existence proof proposed by Vohra (1986) It can be viewed as the proof of existence of an equilibrium for pricing schemes entailing no loss (like for instance average cost pricing). It could be generalised to pricing schemes depending on all production and consumption plans and on prices. Existence proofs for pricing schemes entailing losses (like for instance marginal cost pricing) are available. The most general one is due to Bonnisseau and Cornet (1986) and applies to the case where losses are uniformly bounded below.

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