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On the Existence of Equilibrium Configurations in a Class of Asymmetric Market Entry Games

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ON THE EXISTENCE OF EQUILIBRIUM CONFIGURATIONS IN A CLASS OF ASYMMETRIC MARKET ENTRY GAMES

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Abstract

Sufficient conditions for the existence of subgame-perfect equilibria in a class of two-stage "entry games" are provided. In this class of games, players first decide to enter (participate) or not, and then play a non-cooperative game with a unique equilibrium in the second stage. The assumptions used do not exclude asymmetry of the players. The result is applicable to asymmetric oligopoly models with technological non-convexities such as fixed costs.

0. Introduction

We consider here a class of non-cooperative two-stage games, with a finite set of players. In the first stage, players decide to "enter" or to "remain outside". In the second stage, those players who decided to enter play a game whose non-cooperative equilibrium is assumed to be unique. The payoff of the players who decided not to participate in the second stage game is equal to zero. An *equilibrium* or *"stable" configuration* is defined as a subset of the initial set of players such that: (i) each player in the subset enters, chooses a non-cooperative equilibrium strategy, and receives a non-negative payoff, (ii) all players who remain outside would receive a negative payoff if they decided to enter and join the subset of "active" players.

Sufficient conditions for the existence of "stable" configurations are provided. These conditions restrict the structure of equilibrium payoffs in an interesting an intuitive manner, but they do not relie on symmetry assumptions.

The result is clearly applicable to asymmetric oligopolistic market models in which participating firms must take a preliminary entry decision and then compete in a second stage. It also sheds some light on the more general problem of market equilibrium under (technological) non-convexities, such as the presence of (large and asymmetric) fixed costs.

Finally, this existence result is a generalization of a theorem proved by Selten and Güth (1982).

1. A class of games

1.1. Description of the two-stage game

Let J(n)={1,...n} denote the set of players or *firms*, indexed by j. Let P(n) denote the set of all subsets of J(n).

In the first stage, each firm j decides to enter or not. Firm j's equilibrium payoff is denoted π_j. The second stage equilibrium strategies are assumed to be unique.

1.2. Stable configurations

For all j, firm j's equilibrium payoff is defined as a function $\pi_j : P(n) \rightarrow \mathbf{R}$. This must be interpreted as follows. For every subset $A \in P(n)$, $\pi_j(A)$ is firm j's equilibrium payoff when the firms $k \in A$ have decided to enter. If $j \in J(n) \backslash A$, then $\pi_j(A) = 0$.

Definition: A subset A* included in P(n) is an *equilibrium configuration* (or is a *"stable" configuration*) if and only if:

(1) $A^* = J(n)$ and $\pi_j(J(n)) \geq 0$ for all $j \in J(n)$.

or (2) $A^* = \emptyset$ and $\pi_j(\{j\}) < 0$ for all $j \in J(n)$.

or (3) $J(n) \supset A^*$, $\pi_j(A^*) \geq 0$ for all $j \in A^*$, and $\pi_k(A^* \cup \{k\}) < 0$ for all $k \in J(n) \setminus A^*$.

The problem can now be formulated: under which conditions does a *"stable" configuration* A* exist?

1.3. Monotonicity of payoff functions

1.3.1. Counterexample with n=2

In the case of 2 firms, let equilibrium payoffs be defined as follows:

$$\pi_1(\{1\}) \geq 0; \quad \pi_1(\{1,2\}) < 0;$$
$$\pi_2(\{2\}) < 0; \quad \pi_2(\{1,2\}) \geq 0.$$

It is clear that for all $A \in P(2)$, A is *unstable*. Firm 2 wishes to enter when firm 1 is already operating on the market, but when firm 2 enters, firm 1 makes losses, and firm 2 does not want to stay alone on the market. This type of situation is excluded under the following assumption.

1.3.2. Monotonicity

Hypothesis H(1):
For all $A \in P(n)$, all $j \in A$, $\pi_j(A) \geq \pi_j(A \cup \{k\})$ for all $k \in J(n) \setminus A$.

In example 1.3.1, one has $\pi_2(\{2\}) < 0$ and $\pi_2(\{1,2\}) \geq 0$, so that assumption H(1) is violated.

1.4. Acyclicity of dominance relations

Assumption H(1) is not sufficient to ensure the existence of a stable configuration, as shown by the following example with three firms.

1.4.1. Counterexample with n=3

$$
\begin{array}{lll}
 & \pi_1(\{1,2\}) < 0; & \\
\pi_1(\{1\}) \geq 0; & \pi_2(\{1,2\}) \geq 0; & \pi_1(\{1,2,3\}) < 0; \\
\pi_2(\{2\}) \geq 0; & \pi_2(\{2,3\}) < 0; & \pi_2(\{1,2,3\}) < 0; \\
\pi_3(\{3\}) \geq 0; & \pi_3(\{2,3\}) \geq 0; & \pi_3(\{1,2,3\}) < 0; \\
 & \pi_1(\{1,3\}) \geq 0; & \\
 & \pi_3(\{1,3\}) < 0. &
\end{array}
$$

In this example, there is clearly no stable configuration with either 0 firms, or with 3 firms. In addition, if firm i is on the market, firm j wishes to enter and i makes losses. Therefore, there is no stable configuration either with a single firm or with two firms operating on the market.

1.4.2. 'Dominance' relation among firms

Definition : Define the binary relation $>^\circ$ as follows. Let i, j be two different firms in J(n). By definition, $i >^\circ j$ if and only if there exists a subset $S \supset \{i, j\}$ such that $\pi_i(S) \geq 0$ and $\pi_j(S) < 0$. In the above example 1.4.1, there is a cycle

$$1 >^\circ 3 >^\circ 2 >^\circ 1,$$

and the example satisfies the monotonicity requirement H(1). Considering all pairs {i, j} in 1.4.1, it is sufficient to reverse just one among the six inequalities of the type $\pi_i(\{i, j\}) < 0$ (or ≥ 0) to suppress the cycle and a stable set A with either one or two elements appears. This type of situation is excluded under the following assumption.

1.4.3. Acyclicity

Hypothesis H(2):
$>^\circ$ is acyclic.

2. Existence of stable configurations

Lemma: Under assumptions H(1) and H(2), there exists a stable configuration for all n.

2.1. Proof of the Lemma

Clearly, a stable set exists for n=1 (and n=2) under H(1) and H(2). The result is proved by induction over n.

Step 1. Suppose that the Lemma holds true for n and suppose that a stable set does not exist under H(1) and H(2) in a game with n+1 firms. Define $J(-k) = J(n+1)\setminus\{k\}$ for all $k \in \{1,...,n+1\}$. The restrictions of the functions (π_j) to J(-k) and its subsets constitute a well-defined sub-problem (a game with only n players). If H(1) and H(2) hold in the game with n+1 players, they are clearly also satisfied in the restricted game with J(-k) as a set of players. Hence, by the induction hypothesis, for all $k \in J(n+1)$, there exists a "stable" configuration, denoted A(-k), in the sub-problem defined by J(-k) and the corresponding restrictions of the functions (π_j).

Since there is no stable set in the complete game with n+1 players, firm k necessarily wishes to enter and "join" A(-k) for all $k \in J(n+1)$. Formally, for all $k \in J(n+1)$, one must have $\pi_k(A(-k) \cup \{k\}) \geq 0$, for otherwise, A(-k) would be a stable set in J(n+1), since $\pi_j(A(-k)) \geq 0$ for all $j \in A(-k)$ and $\pi_i(A(-k) \cup \{i\}) < 0$ for all $i \in J(-k) \setminus A(-k)$.

Step 2. Furthermore, each "outsider" k dominates an element of A(-k). To show this, suppose that it is not true. Then, for all $j \in A(-k)$, $\pi_j(A(-k) \cup \{k\}) \geq 0$ and for all $i \in J(-k) \setminus A(-k)$,

$\pi_i(A(-k) \cup \{k\} \cup \{i\}) < 0,$

since by H(1), one necessarily has

$\pi_i(A(-k) \cup \{k\} \cup \{i\}) \leq \pi_i(A(-k) \cup \{i\}) < 0$

for all $i \in J(-k) \setminus A(-k)$. But if these inequalities were true, they would implie that $A(-k) \cup \{k\}$ is a stable configuration in the complete problem with n+1 players, which contradicts the assumption that a stable set doesn't exist.

Step 3. Thus, if a stable configuration does not exist in the complete game (in J(n+1)), for all $k \in J(n+1)$, there exists $j_k \neq k$ such that $k >^\circ j_k$, with $j_k \in J(-k)$.

The elements of J(n+1) can thus be relabeled to provide

$j_1 >^\circ j_2 >^\circ ... >^\circ j_k >^\circ ... >^\circ j_{n+1}$.

But it has been shown that there exists $j_{n+2} \in J(n+1)$ such that

$j_{n+1} >^\circ j_{n+2}$.

Therefore, there must be a cycle, and H(2) is violated. This contradicts the assumption that a stable set does not exist in the complete game. Q.E.D.

2.2 H(1) and H(2) are not necessary for the existence of an equilibrium configuration.

By means of a couple of examples, it is easy to show that the sufficient conditions H(1) and H(2) are not necessary conditions.

2.2.1. Monotonicity is not necessary

Let n = 3, and consider the equilibrium payoff functions defined as follows.

$$\begin{array}{lll} & \pi_1(\{1,2\}) \geq 0; & \\ \pi_1(\{1\}) < 0; & \pi_2(\{1,2\}) \geq 0; & \pi_1(\{1,2,3\}) \geq 0; \\ \pi_2(\{2\}) < 0; & \pi_2(\{2,3\}) \geq 0; & \pi_2(\{1,2,3\}) \geq 0; \\ \pi_3(\{3\}) < 0; & \pi_3(\{2,3\}) \geq 0; & \pi_3(\{1,2,3\}) \geq 0; \\ & \pi_1(\{1,3\}) \geq 0; & \\ & \pi_3(\{1,3\}) \geq 0. & \end{array}$$

It is easy to check that $A^*=\{1,2,3\}$ is stable.

2.2.2. Acyclicity is not necessary

Let n = 4, and consider the payoff functions defined as follows.

$$
\begin{array}{llll}
 & \pi_1(\{1,2\}) \geq 0; & & \\
\pi_1(\{1\}) \geq 0; & \pi_2(\{1,2\}) < 0; & \pi_i(\{i,j,k\}) < 0; & \text{and} \\
\pi_2(\{2\}) \geq 0; & \pi_2(\{2,3\}) \geq 0; & \pi_j(\{i,j,k\}) < 0; & \pi_i(J(4)) < 0; \\
\pi_3(\{3\}) \geq 0; & \pi_3(\{2,3\}) < 0; & \pi_k(\{i,j,k\}) < 0; & \text{for all } i \in J(4). \\
\pi_4(\{4\}) \geq 0; & \pi_3(\{3,4\}) \geq 0; & \text{for all} & \\
 & \pi_4(\{3,4\}) < 0; & \{i,j,k\} \in P(4); & \\
 & \pi_1(\{1,3\}) \geq 0; & & \\
 & \pi_3(\{1,3\}) < 0; & & \\
 & \pi_1(\{1,4\}) < 0; & & \\
 & \pi_4(\{1,4\}) < 0; & & \\
 & \pi_2(\{2,4\}) < 0; & & \\
 & \pi_4(\{2,4\}) \geq 0; & &
\end{array}
$$

In this last example, acyclicity is violated: one finds $2 >^\circ 3 >^\circ 4 >^\circ 2$, but $A^*=\{1\}$ is a stable set. Note that monotonicity is satisfied.

3. Applications

The above result is clearly applicable to some asymmetric oligopolistic market models with a preliminary entry decision of firms.

For instance, consider a Cournot oligopoly model such that firms face the same demand function and have the same variable costs, but such that fixed costs are different across firms. One then easily finds that the dominance relation is complete and transitive. Firm i dominates firm j if and only if firm i's fixed cost is strictly lower than firm j's fixed cost. In this type of model, the monotonicity assumption introduced above is also satisfied under standard assumptions (downward sloping demand), and the existence of an equilibrium configuration of the market is guaranteed.

The loose structure of the second stage game studied above permits one to consider a much wider class of situations. It is sufficient to have a set of well-defined equilibrium payoff functions for each possible subgame (or each possible subset of participating players). For instance, the result could be useful to study the existence of equilibria in general equilibrium models with imperfect competition and production sets of the type $\{0\} \cup K$, $0 \notin K$, K convex and compact (see Novshek and Sonnenschein (1986)). Assumptions H(1) and H(2), although intuitively reasonable, might induce overly strong restrictions on the underlying economic data of the model considered. It remains to be shown that sufficiently general and interesting economic situations lead to a payoff structure satisfying the above introduced monotonicity and acyclicity requirements.

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