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STABILIZING COMPETITIVE BUSINESS CYCLES

by

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We study how stylized monetary (proportional money transfers), fiscal (lump sum money transfers) and public expenditure policies may eliminate endogenous deterministic cycles and stationary Markov sunspot equilibria in a simple version of the overlapping generations model. We present also new, constructive methods for analysing stationary Markov sunspot equilibria in such a framework. J. Econ. Theory,

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STABILIZING COMPETITIVE BUSINESS CYCLES

Jean-Michel GRANDMONT*

Many studies have shown that a competitive economy of which the characteristics are deterministic and stationary may display highly complex endogenous fluctuations under laisser faire, be they deterministic cycles (Benhabib and Day [3], Benhabib and Nishimura [4], Grandmont [9], Woodford [15]) or stochastic sunspot equilibria (Shell [14], Azariadis [1], Cass and Shell [6], Azariadis and Guesnerie [2], Woodford [15], Grandmont [12]). In the present paper, we extend the results of Grandmont [9], and study how stylized monetary (proportional money transfers), fiscal (lump sum money transfers) and budgetary (public expenditures) Government policies may help reducing the cyclical properties of a competitive economy.

We focus attention on a simple version of the overlapping generations model that involves only one good and one asset, in which agents live two periods, as in [9] (Section 1). We analyse in Sections 2 and 3 how variations of predetermined rates of growth of the money supply through the three above mentioned channels affect the "cyclical" properties of the economy, i.e. how they influence the set of deterministic cycles with perfect foresight (Section 2) and the set of stationary Markov sunspot equilibria (Section 3). Predetermined proportional money transfers are found as in [9] to be superneutral, i.e. they do not alter the set of these equilibria. By contrast, increases of the predetermined (and constant over time) money growth rate through lump sum transfers and/or through public expenditures are found to have a stabilizing impact. They eliminate in particular, if large enough, all cycles with perfect foresight that have a period $k \geq 2$ (section 2).

We establish in Section 3 a new, constructive characterization of stationary Markov sunspot equilibria involving an arbitrary number of states in that sort of model, in which the underlying deterministic perfect foresight dynamics in onedimensional. We get then as a corollary that such sunspot equilibria exist if and only if there is a "noncritical" period 2 cycle, a result obtained in the case of laisser faire by Azariadis and Guesnerie [2] by different techniques. We get also as a corollary that stationary sunspot equilibria exist arbitrarily close to the deterministic monetary stationary state if and only if it is asymptotically stable in the (forward) perfect foresight dynamics that is defined locally in its neighbourhood (for similar results, see Gourieroux, Laffont, Monfort [7], Woodford [15]). The results of Sections 2 and 3 imply in particular that predetermined changes of the money growth rate through lump sum transfers and/or public expenditures that eliminate nontrivial cycles with perfect foresight do eliminate stationary Markov sunspot equilibria as well.

Such predetermined changes of the growth rate of the money supply do not guarantee, however, the disappearance of the forecasting mistakes that the traders may make. There are still many possible nonstationary intertemporal equilibria with self-fulfilling expectations and the inference problem to be solved by the agents is a very difficult one. We show in Section 4 how deterministic <u>policy rules</u> can, if believed and understood by private units, solve this expectations coordination problem and guarantee perfect foresight. It was proved in [9] that monetary policy alone, through nominal interest payments, could achieve this goal and moreover, force the economy to converge to the Golden rule stationary state, provided that it was asymptotically stable in the forward perfect foresight dynamics that is defined in its neighbourhood. We show in Section 4 that with lump sum transfers and public expenditures, the stability requirement can be dropped : there are policy rules through these channels that ensure perfect foresight stability of the target equilibrium. The design of such policies necessitates only the knowledge of the local behaviour of the economy around the Golden Rule.

1. SHORT RUN EQUILIBRIUM

This section is devoted to the presentation of the assumptions of the model and of the relations determining a competitive equilibrium at any date.

We consider the simple specification of the overlapping generations model, without bequests, in which production is taken for simplicity as exogenous, that was used in Grandmont [9]. The model involves one nonstorable good and a single asset, "money", that is employed for transferring wealth from one period to the next. Agents live two periods and are identical (equivalently there is a single agent) in each generation. The agents' endowments of the good at each age $\tau = 1,2$ are

$$\ell_1^* > 0, \ \ell_2^* > 0 \tag{1.a}$$

The agents' tastes among consumption streams $a_1 \geqq 0$, $a_2 \geqq 0$ are described by the separable utility function $V_1(a_1) + V_2(a_2)$ with

(1.b) <u>For each</u> $\tau = 1,2$, $V_\tau(a_\tau)$ <u>is continuous on</u> $[0,+\infty)$ <u>and twice continuously differentiable on</u> $(0,+\infty)$. <u>Moreover</u>, $V'_\tau(a_\tau) > 0$, $V''_\tau(a_\tau) < 0$ <u>for</u> $a_\tau > 0$, <u>and</u> $\lim_{a_\tau \to 0} V'_\tau(a_\tau) = +\infty$, $\lim_{a_\tau \to +\infty} V'_\tau(a_\tau) = 0$.

We focus on the <u>Samuelson case</u>, in which autarchy is inefficient, that is

$$\bar{\theta} = V'_1(\ell_1^*)/V'_2(\ell_2^*) < 1 \tag{1.c}$$

We assume that there is in addition a "Government" that has three available instruments. At date t, it makes a money transfer that is proportional to the money holding of the old agent, at the rate $x_t - 1$ (<u>nominal interest payments</u>). It gives also a <u>lump sum</u> money subsidy S_t (to be interpreted as a lump sum tax if negative) to the old agent and issues the amount of money G_t when purchasing (or selling) some quantity of the good[1]. In the sequel it will be often convenient to consider as the Government's instrumental variables the rates of growth of the money supply that are attributable to lump sum transfers and to the deficit $D_t = S_t + G_t$, excluding interest payments, i.e.

$$s_t = (M_{t-1} x_t + S_t) / M_{t-1} x_t \tag{1.1}$$

$$d_t = (M_{t-1} x_t + S_t + G_t) / M_{t-1} x_t$$

where $M_{t-1} > 0$ designates the money stock at the outset of period t. The evolution of the money supply is then ruled by

$$M_t = M_{t-1} x_t d_t > 0 \ , \quad M_o > 0 \quad \underline{\text{given}} \tag{1.2}$$

It will be assumed for all t

$$x_t > 0 \ , \ d_t > 0 \ , \ s_t > 0 \tag{1.d}$$

The assumption $s_t > 0$ is made for analytical convenience and means that lump sum taxes do not wipe out the old agent's money balance. Conditions (1.a) through (1.d) will be assumed to hold without any further reference.

A newborn trader has to solve the following decision problem. Let $p > 0$ be the current money price of the good, and let $p^e > 0$, $x^e > 0$ and Δm^e stand for the price of the good, the proportional money transfer and the lump sum subsidy that he expects (with subjective certainty) for the next date, respectively. The consumer must choose then his current and future consumptions $a_1 \geqq 0$, $a_2 \geqq 0$, and his money demand $m \geqq 0$ so as to maximize his utility function subject to

$$pa_1 + m = p\ell_1^* \quad \text{and} \quad p^e a_2 = p^e \ell_2^* + mx^e + \Delta m^e \tag{1.3}$$

The solution of this problem depends only on $\theta = px^e/p^e$ and on $\sigma = \Delta m^e/p^e$. The problem has a solution, which is unique, if and only if $\theta\ell_1^* + \ell_2^* + \sigma \geqq 0$. In that case, we denote the optimum excess demands for the good $a_\tau - \ell_\tau^*$ as $z_\tau(\theta,\sigma)$. They satisfy

$$\theta z_1(\theta,\sigma) + z_2(\theta,\sigma) - \sigma \equiv 0 \tag{1.4}$$

On the other hand, money demand is given by

$$m^d(p,p^e/x^e,\sigma) \equiv -p\, z_1(\theta,\sigma) \equiv (p^e/x^e)[z_2(\theta,\sigma) - \sigma] \tag{1.5}$$

An important point to note is that when $\theta\ell_1^* + \ell_2^* + \sigma > 0$, and when money demand is positive, it is the unique value of m that satisfies the first order condition

$$V_1'(\ell_1^* - \frac{m}{p}) = \theta\, V_2'(\ell_2^* + \frac{mx^e}{p^e} + \sigma) \tag{1.6}$$

In the limiting case $\theta\ell_1^* + \ell_2^* + \sigma = 0$, money demand $m = p\ell_1^*$ may be viewed also as the unique solution of (1.6) although the latter reads then $+\infty = +\infty$.

The equilibrium condition at date t is now easy to state. The excess demand for the good of the old trader and of the Government is equal to the real value M_t/p_t of the money supply, in which $p_t > 0$ is the current money price of the good. Competitive equilibrium of the good market at date t reads then, with obvious notations

$$z_1(p_t\, x_{t+1}^e/p_{t+1}^e\,,\, S_{t+1}^e/p_{t+1}^e) + (M_t/p_t) = 0 \tag{1.7}$$

in which the superscript "e" denotes values that are anticipated by the young trader at t for the next date. This formulation takes implicitly into account the restriction

$$p_t x^e_{t+1} \ell^*_1 + p^e_{t+1} \ell^*_2 + S^e_{t+1} \geq 0 \tag{1.8}$$

which is necessary for the function z_1 to be well defined. By Walras' law, the money equilibrium equation may be obtained by multiplying both members of (1.7) by p_t — see (1.5).

The equilibrium condition at t can be given another equivalent form, which is often more convenient, by setting m equal to M_t in the first order condition (1.6). This yields

$$V'_1(\ell^*_1 - \frac{M_t}{p_t}) = (p_t x^e_{t+1} / p^e_{t+1}) V'_2(\ell^*_2 + (M_t x^e_{t+1} + S^e_{t+1}) / p^e_{t+1}) \tag{1.9}$$

Here again, the restriction (1.8) is implicitly taken into account since the arguments of V'_1 and V'_2 must be nonnegative.

2. *CYCLES WITH PERFECT FORESIGHT*

We assume in this section that the (doubly infinite) sequences of policy parameters (x_t), (s_t), (d_t) are <u>predetermined</u>, with $s_t = s$ and $d_t = d$ for all t . Our goal will be to analyse the influence of these policy parameters on the set of periodic equilibria of the economy, under the assumption of perfect foresight. It will be shown in particular that less and less cycles with perfect foresight do exist when s and/or d gets larger. In this sense, an increase of the size of the Government's intervention through these channels has a strong "stabilizing" impact.

Trajectories with perfect foresight

Given (x_t) , s , d and M_o , (1.2) determines the money supplies M_t while S_t and G_t are given by (1.1). An intertemporal competitive equilibrium with perfect foresight corresponding to M_o and the predetermined policy parameters is then a (again doubly infinite) sequence of prices (p_t) that verifies (1.7), or (1.9), for all t , where the superscripts "e" have been suppressed.

Such an intertemporal equilibrium is most conveniently described in terms of the variable $\theta_t = p_t x_{t+1}/p_{t+1}$, which is equal to one plus the real interest rate between dates t and t+1 , and the real balance $\mu_t = M_t/p_t$. Indeed suppressing the superscript "e" and multiplying both members of (1.9) by M_t/p_t yields

$$\mu_t V_1'(\ell_1^* - \mu_t) = d^{-1} \mu_{t+1} V_2'(\ell_2^* + s d^{-1} \mu_{t+1}) \tag{2.1}$$

If one defines next

$$v_1(\mu) = \mu V_1'(\ell_1^* - \mu) \quad \text{for} \quad \mu \text{ in } [0,\ell_1^*) \tag{2.2}$$

$$v_2(\mu) = \mu V_2'(\ell_2^* + \mu) \quad \text{for} \quad \mu \text{ in } [0,+\infty)$$

the function v_1 is increasing and maps $[0,\ell_1^*)$ onto $[0,+\infty)$, while v_2 maps $[0,+\infty)$ into itself. Thus v_1 has an inverse and (2.1) leads to

$$\mu_t = v_1^{-1}[s^{-1} v_2(sd^{-1} \mu_{t+1})] \equiv \chi_{s,d}(\mu_{t+1}) \tag{2.3}$$

An intertemporal equilibrium with perfect foresight may then be characterized by a sequence (μ_t) that satisfies (2.3) for all t . The corresponding sequence of real interest rates is given in turn by (1.2), which reads here

$$\mu_{t+1} = \mu_t \theta_t d \tag{2.4}$$

These equations show that the <u>set</u> of intertemporal equilibria with perfect foresight is independent, in real terms, of the initial money stock M_o (money is <u>neutral</u>), and of the predetermined sequence (x_t) (proportional money transfers are <u>superneutral</u>). By contrast, the parameters s , d (fiscal and budgetary policies) <u>do influence</u> the set of real perfect foresight equilibrium magnitudes, although they are predetermined.

The following result gives useful information on the <u>backward perfect foresight (b.p.f.) map</u> on real balances $\chi_{s,d}$ that is defined in (2.3). Its proof is omitted since it follows readily from the definitions.

PROPOSITION 2.1. <u>The map</u> $\chi_{s,d}$ <u>from</u> $[0,+\infty)$ <u>into</u> $[0,\ell_1^*)$ <u>is continuously differentiable</u>, <u>with</u> $\chi_{s,d}(0) = 0$ <u>and</u> $\chi'_{s,d}(0) = (d\bar{\theta})^{-1}$. <u>Moreover</u>,

1) <u>The intersection of the graph of</u> $\chi_{s,d}$ <u>with the line</u> $\mu_t = (d\theta)^{-1} \mu_{t+1}$ <u>is composed of the origin</u> $\mu_{t+1} = \mu_t = 0$ <u>and of at most another point</u>. <u>The two points of intersection coincide if only if</u> $\theta \leqq \bar{\theta}$.

2) <u>If</u> $d\bar{\theta} \geqq 1$, $\chi_{s,d}$ <u>has a unique fixed point at the origin (a nonmonetary stationary state). If</u> $d\bar{\theta} < 1$, $\chi_{s,d}$ <u>has another fixed point</u> $\bar{\mu}_{s,d} > 0$ (<u>the unique monetary stationary state</u>).

Fig. 1 shows how the b.p.f. dynamics on real balances implied by (2.3) may be represented by using the graph of $\chi_{s,d}$. It describes also the corresponding b.p.f. dynamics on real interest rates θ_t that is implied by (2.4).

Figure 1

Existence of cycles

The graph of the map $\chi_{s,d}$ is a deformation of the graph of the b.p.f. map $\chi = v_1^{-1} \circ v_2$ on real balances that is obtained under laisser faire, i.e. when $s = d = 1$. It is known that the graph of χ represents the traders' offer curve under laisser faire and that there may exist many cycles with perfect foresight if this offer curve displays a significant hump (see [3], [9]). We wish to see here how changes of the policy parameters s and d affect the shape of the offer curve and may reduce the size of the set of cycles with perfect foresight, i.e. of the periodic points of $\chi_{s,d}$.

To fix ideas, assume that the laisser faire map χ has a cycle of period $k \geqq 2$. This implies that χ , or equivalently v_2 , has at least one local maximum and that the least value for which a local maximum occurs, say $\mu' > 0$, satisfies $\chi(\mu') > \mu'$. From (2.3) the least value of μ for which $\chi_{s,d}$ has a local maximum is $s^{-1} d\,\mu'$. It is then clearly possible to choose s and/or d so that $\chi_{s,d}$ has no cycle other than the stationary states : it suffices to bring the point of the graph of χ corresponding to $\mu = s^{-1} d\,\mu'$ on or below the diagonal. Formally

PROPOSITION 2.2. <u>Let</u> $\mu' > 0$ <u>be the least value for which</u> v_2 <u>has a local maximum. Then</u> $\chi_{s,d}$ <u>has no cycle with a period</u> $k \geq 2$ <u>if</u>

$$\chi_{s,d}(s^{-1} d\,\mu') = v_1^{-1}[s^{-1} v_2(\mu')] \leq s^{-1} d\,\mu'$$

The foregoing result encompasses three interesting particular cases.

1. Consider an increase of d from 1 to $d > 1$, with s being fixed and equal to 1 (change of the money growth rate through the Government's purchases of the good, no lump sum transfers). Then the graph of $\chi_{1,d}$ is the <u>affine transformation</u> of the offer curve under laisser faire χ, parallel $\overrightarrow{0\mu}_{t+1}$ and of ratio d . If d is large enough, $\chi_{1,d}$ has no nontrivial cycle. One must keep $d\bar{\theta} < 1$ to ensure the existence of a monetary stationary state.

2. Consider an increase of the rate of growth of the money supply through lump sum transfers from 1 to $s > 1$, the Government being inactive on the good market, i.e. d is constantly equal to s . The graph of the laisser faire map χ is <u>pushed down</u>, and the critical points of $\chi_{s,s}$ are the same as those of χ . If $s = d$ is large enough, $\chi_{s,s}$ has no cycle of a period $k \geq 2$. Here again, one must have $d\bar{\theta} < 1$ for a monetary stationary state to exist.

3. Let the Government increase its expenditures but balance its budget, i.e. d is fixed and equal to 1 , while s decreases from 1 to $s < 1$. The associated map $\chi_{s,1}$ is given by

$$\chi_{s,1}(\mu) = v_1^{-1}[\mu\, V_2'(\ell_2^* + s\mu)]$$

Thus its graph <u>goes up</u> as s decreases, the slope of the tangent at the origin being constant and equal to $\bar{\theta}^{-1}$. The least value for which $\chi_{s,1}$ has a local maximum is $s^{-1}\mu'$, which increases and gets eventually larger than ℓ_1^*

when s gets lower than μ'/ℓ_1^* . Since $\chi_{s,1}(s^{-1}\mu')$ is bounded above by ℓ_1^* , the corresponding critical point of the graph $\chi_{s,1}$ is bound to cross the diagonal when s is low enough.

Remark 2.1 : Bifurcations. The above arguments can be used to describe how the set of cycles with perfect foresight is progressively reduced when one changes continuously the policy parameters s and/or d , by applying the theory of the bifurcations of onedimensional discrete dynamical systems, as in [9,10]. If the laisser faire offer curve χ is unimodal with the unique maximum occurring at μ^* , the graph of $\chi_{s,d}$ is also unimodal from (2.3), the critical point being $s^{-1}d\mu^*$. Assume now that under laisser faire, one has $\chi^3(\mu^*) < \mu^*$, which implies that χ has a cycle of period 3 (see [9]). Then if one follows a path in the plane (s,d) that starts from laisser faire (s = d = 1) and ends at a configuration verifying the conditions of Proposition 2.2, the resulting family of maps $\chi_{s,d}$ will typically undergo eventually a succession of "period halving" bifurcations : the bifurcation diagram will be qualitatively similar to e.g. [9, Fig. 4], with a reversed orientation [(2)].

3. STATIONARY MARKOV SUNSPOT EQUILIBRIA

Deterministic cycles with perfect foresight are but degenerate cases of stationary Markov "sunspot" equilibria, in which agents predict that prices are affected by random factors, although they do not influence the real characteristics of the economy, and in which this prophecy is self-fulfilling. We establish in this section, under the assumption that (x_t) , $s_t = s$, $d_t = d$ are predetermined, a new, constructive characterization of stationary Markov sunspot equilibria involving an arbitrary number of states in that sort of model, in which the underlying perfect foresight dynamics is one-dimensional. We get as a corollary that such sunspot equilibria exist if and only if there is a "noncritical" period 2 cycle, a result obtained in the case of laisser faire by Azariadis and Guesnerie [2] by different techniques. Another corollary of the analysis is that stationary sunspot equilibria exist arbitrarily close to the deterministic monetary stationary state if and only if it is asymptotically stable in the forward perfect foresight dynamics that is defined locally in its neighbourhood (for similar results see, Gourieroux, Laffont, Monfort [7], Woodford [15]).

Consider a stationary Markov chain of random variables (sunspots) taking values in some space Ω. In order to simplify the exposition, we assume that Ω has r elements, $\Omega = \{\omega_1, \dots, \omega_r\}$, $r \geqq 2$ and that the transition probabilities from ω_i to ω_j, say q_{ij}, satisfy $q_{ij} > 0$ for all i,j . The agents are supposed to observe the value of the sunspot at each date, and to know the transition probabilities.

Let (x_t), $s_t = s$, $d_t = d$ be fixed and known by the agents. Given $M_o > 0$, the evolution of the money supply is then ruled by $M_t = M_{t-1} x_t d$, while S_t, G_t are given by (1.1) with $s_t = s$, $d_t = d$. The agents are supposed to know or to forecast correctly these quantities. Assume now that although sunspots do not alter the utility functions or the endowments, the agents believe nevertheless that they influence the equilibrium price at each date through the relation $p(\omega,M)$, in which ω is the sunspot observed at the date under consideration and M is the end-of-the-period money stock, with

$$p(\omega_i,M) = M/\mu_i \quad , \text{all } i \ , \ 0 < \mu_1 \leqq \dots \leqq \mu_r \ , \ \mu_1 \neq \mu_r \ . \tag{3.1}$$

The above ordering can be obtained through a relabeling of the sunspots. The condition $\mu_1 \neq \mu_r$ means that there is genuine uncertainty about prices.

Since s and d are fixed, this will imply (see (3.6)) that agents believe that equilibrium prices are proportional to M_o (they believe that money is neutral) and to the x_t (agents believe that proportional money transfers are superneutral). The prophecy described in (3.1) will be self-fulfilling, and will lead to a stationary Markov sunspot equilibrium if, when agents act in accordance to the belief (3.1) , $p(\omega_i, M)$ is indeed an equilibrium price when ω_i is observed, and when the money supply is M .

In order to translate this notion into a formal definition, consider a young agent at an arbitrary date t who observes ω_i and $p_t > 0$ in the current period. If he believes that prices obey (3.1) , he expects the price $p_j = p(\omega_j, M_{t+1})$ to occur with probability q_{ij} at the next date. His problem is then to choose his current consumption $a_1 \geqq 0$ and his money demand $m \geqq 0$, as well as his future consumption $a_{2j} \geqq 0$ in the event where ω_j will be observed at the next date, so as to maximize the mathematical expectation of his utility

$$V_1(a_1) + \Sigma_j q_{ij} V_2(a_{2j}) \tag{3.2}$$

subject to the current and expected budget constraints

$$p_t a_1 + m = p_t \ell_1^* \tag{3.3}$$

$$p_j a_{2j} = p_j \ell_2^* + m x_{t+1} + S_{t+1} \quad , \quad j = 1,\dots r$$

where $S_{t+1} = (s-1) M_t x_{t+1}$. This decision problem has a solution, which is then unique, if and only if $x_{t+1} p_t \ell_1^* + p_j \ell_2^* + S_{t+1} \geqq 0$ for all j .

The most convenient way to characterize the solution is to look at the first order condition. When money demand is positive, the optimum solution satisfies

$$V_1'(a_1)/p_t = \Sigma_j\, q_{ij}\, x_{t+1}\, V_2'(a_{2j})/p_j$$

Therefore, money demand, when it is positive, is the only positive value of m that verifies

$$\frac{m}{p_t} V_1'(\ell_1^* - \frac{m}{p_t}) = \Sigma_j\, q_{ij} \frac{mx_{t+1}}{p_j} V_2'(\ell_2^* + \frac{mx_{t+1} + S_{t+1}}{p_j}) \qquad (3.4)$$

Equilibrium at t is obtained by setting the young trader's demand m equal to M_t in (3.4) . By using $M_t\, x_{t+1} = d^{-1}\, M_{t+1}$ and $M_t\, x_{t+1} + S_{t+1} = sd^{-1}\, M_{t+1}$ to rearrange the right hand member, and by employing the functions v_1 , v_2 introduced in (2.2), one gets

$$v_1(\frac{M_t}{p_t}) = \Sigma_j\, q_{ij}\, s^{-1}\, v_2(sd^{-1}\, \mu_j) \qquad (3.5)$$

Since v_1 is an increasing function from $[0,\ell_1^*)$ onto $[0,+\infty)$, the condition (3.5) determines <u>uniquely</u> the equilibrium price p_t at an arbitrary date t as a function of ω_i and of M_t . This equation leads therefore to a well defined causal (stochastic) dynamics of temporary equilibria, in which the past determines the present [(3)] .

<u>Given</u> M_o , (x_t) , $s_t = s$ <u>and</u> $d_t = d$ <u>for all</u> t , <u>a</u> <u>stationary Markov sunspot equilibrium (with</u> r <u>states) is a price function</u> $p(\omega_i, M)$ <u>satisfying</u> (3.1) <u>such that the solution in</u> p_t <u>of</u> (3.5) <u>given</u> ω_i <u>and</u> M_t , <u>is indeed</u> M_t/μ_i , <u>or equivalently such that for all</u> i

$$v_1(\mu_i) = \Sigma_j\, q_{ij}\, s^{-1}\, v_2(sd^{-1}\, \mu_j) \qquad (3.6)$$

The agents' belief is then self-fulfilling, and $M/p(\omega,M)$ follows a Markov chain on the set $\{\mu_1,\ldots,\mu_r\}$ with transition probabilities q_{ij}. In the limit case $r = 2$, $q_{12} = q_{21} = 1$, one gets of course a cycle with perfect foresight of period 2, see (2.3).

We characterize now the real balances appearing in (3.6) by using the map $\chi_{s,d}$. To simplify notations, since s,d are given, we shall pose until the end of this section

$$\bar{\chi} = \chi_{s,d}\ ,\ \bar{\mu} = \bar{\mu}_{s,d}\ \text{ and }\ \bar{v}_2(\mu) = s^{-1}\, v_2(sd^{-1}\,\mu) \tag{3.7}$$

Consider a sunspot equilibrium $(\mu_1,\ldots,\mu_r)$ as in (3.6). Let now m and n be integers such that $\bar{v}_2(\mu_n) \leq \bar{v}_2(\mu_i) \leq \bar{v}_2(\mu_m)$ for all i. It follows from (3.6) that one has for all i, $\bar{v}_2(\mu_n) \leq v_1(\mu_i) \leq \bar{v}_2(\mu_m)$. Since $\mu_1 < \mu_r$, the expressions $\bar{v}_2(\mu_n)$ and $\bar{v}_2(\mu_m)$, and thus μ_m and μ_n, must differ, and one has actually $\bar{v}_2(\mu_n) < v_1(\mu_i) < \bar{v}_2(\mu_m)$ for all i, since the probabilities q_{im} and q_{in} are both positive. Applying this fact to m and n, one gets $\bar{v}_2(\mu_n) < v_1(\mu_n)$ and $v_1(\mu_m) < \bar{v}_2(\mu_m)$, or equivalently $\bar{\chi}(\mu_n) < \mu_n$ and $\mu_m < \bar{\chi}(\mu_m)$. The final step is to remark that the configuration $\mu_n < \mu_m$ is impossible, since it would imply that $\bar{\chi}$ has two distinct monetary stationary states. Hence we obtain that there exist two integers m,n such that $\mu_m < \mu_n$ and

$$\bar{v}_2(\mu_n) < v_1(\mu_1) < v_1(\mu_r) < \bar{v}_2(\mu_m) \tag{3.8}$$

or equivalently

$$\bar{\chi}(\mu_n) < \mu_1 < \mu_r < \bar{\chi}(\mu_m) \qquad (3.9)$$

Consider conversely a sequence $0 < \mu_1 \leqq \ldots \leqq \mu_r$ and assume that there exist $\mu_m < \mu_n$ such that (3.9) or (3.8) holds. Is there a Markov chain on Ω such that $(\mu_1,\ldots,\mu_r)$ defines a stationary Markov sunspot equilibrium, i.e. satisfies (3.6) ? Here the unknowns are the transition probabilities q_{ij} and it is clear that given (3.8) , one can find probabilities $q_{ij} > 0$ acting on Ω so as to satisfy (3.6). The simplest way to generate a solution is to solve (3.6) by setting first $q_{ij} = 0$ when $j \neq m,n$ (this determines uniquely $q_{im} > 0$ and $q_{in} > 0$ for all i) and then to perturbate slightly the probabilities so as to make all of them positive. Obviously, the set of solutions is nonempty, and open. We have thus obtained

PROPOSITION 3.1. Given $\Omega = \{\omega_1,\ldots,\omega_r\}$, the real balances $0 < \mu_1 \leqq \ldots \leqq \mu_r$ define a stationary Markov sunspot equilibrium if and only if there exist $\mu_m < \mu_n$ that satisfy (3.9).

This characterization yields the most general way to construct stationary Markov sunspot equilibria involving r states : it suffices to find two values μ' , μ'' satisfying $\bar{\chi}(\mu'') < \mu' < \mu'' < \bar{\chi}(\mu')$, and to distribute arbitrarily $\mu_1,\ldots,\mu_r$ in the interval $(\bar{\chi}(\mu'') , \bar{\chi}(\mu'))$, two of these values being equal to μ' and μ'' . The analysis yields also as an immediate corollary that such a sunspot equilibrium exists in the present framework if and only if the map $\bar{\chi}$ has a cycle of period 2 . In fact, since we have only considered "nondegenerate" sunspot equilibria ($q_{ij} > 0$ for all i,j and $\mu_1 \neq \mu_r$) , we wish to show this equivalence with cycles that are also "nondegenerate". The situation to be avoided is the limit case where $\bar{\chi}$ has a cycle of period 2 — in which case $\bar{\chi}'(0) = (d\bar{\theta})^{-1} > 1$ and the second iterate of $\bar{\chi}$, i.e. $\bar{\chi}^2$, has a fixed point that differs from 0 and from the monetary stationary state $\bar{\mu} = \bar{\mu}_{s,d}$ — but where the graph of $\bar{\chi}^2$ has no point below the diagonal when $0 < \mu < \bar{\mu}$, nor any point above it when $\mu > \bar{\mu}$. Formally, we say that $\bar{\chi}$ <u>has a noncritical cycle of period</u> 2 if there exists a period 2 cycle and if there exists either μ' in $(0,\bar{\mu})$ such that $\bar{\chi}^2(\mu') < \mu'$, or $\mu'' > \bar{\mu}$ such that $\bar{\chi}^2(\mu'') > \mu''$.

It was shown in Grandmont [12, (3.12)] that the laisser faire map $\chi = \chi_{1,1}$ had a noncritical cycle of period 2 if and only if there existed μ' and μ'' verifying $\chi(\mu'') < \mu' < \mu'' < \chi(\mu')$. The proof — which is straightforward — applies without any change to $\bar{\chi}$. Thus

PROPOSITION 3.2. <u>The map</u> $\bar{\chi} = \chi_{s,d}$ <u>has a noncritical cycle of period</u> 2 <u>if and only if there exist</u> μ' and μ'' <u>such that</u> $\bar{\chi}(\mu'') < \mu' < \mu'' < \bar{\chi}(\mu')$.

Propositions 3.1 and 3.2 establish the equivalence we were looking for.

Proposition 3.1 allows also characterizing the conditions under which stationary sunspots equilibria exist near the stationary state $\bar{\mu}$. Let $\bar{\chi}'(0) > 1$ and assume away the "nongeneric" case $\bar{\chi}'(\bar{\mu}) = 1$. If stationary Markov sunspot equilibria exist arbitrarily close to $\bar{\mu}$, there are pairs (μ_m, μ_n) in every neighbourhood of $\bar{\mu}$ such that

$$\bar{\chi}(\mu_n) < \mu_m < \bar{\mu} < \mu_n < \bar{\chi}(\mu_m)$$

For each such pair, by continuity, there exists μ in $(\mu_m, \bar{\mu})$ such that $\mu_n = \bar{\chi}(\mu)$ and thus $\bar{\chi}^{2}(\mu) < \mu$. Since μ is arbitrarily close to $\bar{\mu}$, this implies that $D\bar{\chi}^{2}(\bar{\mu}) = [\bar{\chi}'(\bar{\mu})]^2 > 1$ in which case $\bar{\chi}'(\bar{\mu}) < -1$.

Assume conversely that $\bar{\chi}'(\bar{\mu}) < -1$. The local picture around $\bar{\mu}$ is represented in Fig. 2. <u>If</u> μ_1 <u>is less than but close enough to</u> $\bar{\mu}$, <u>then</u> $\mu_1 \leq \ldots \leq \mu_r$ <u>satisfies the conditions of Proposition</u> 3.1 <u>and thus defines consistently a stationary Markov sunspot equilibrium if and only if</u> μ_r <u>belongs to the interval</u> $(\mu', \bar{\chi}(\mu_1))$ (one takes $m = 1$ and $n = r$ in this case). Thus if $\bar{\chi}'(\bar{\mu}) < -1$, there are stationary Markov sunspot equilibria arbitrarily near $\bar{\mu}$.

<u>Figure 2</u>

To sum up, if we assume away the specific case $\bar{\chi}'(\bar{\mu}) = 1$, a necessary and sufficient condition for the existence of stationary Markov sunspot equilibria with r states in every neighbourhood of $\bar{\mu}$ is $\bar{\chi}'(\bar{\mu}) < -1$. This means that the monetary stationary state is unstable in the <u>backward</u> perfect foresight dynamics that is implied by the map $\bar{\chi}$. Or equivalently, that $\bar{\mu}$ is asymptotically stable in the <u>forward</u> perfect foresight dynamics that is defined near $\bar{\mu}$ by inverting locally the map $\bar{\chi}$. Formally,

PROPOSITION 3.3. <u>Assume</u> $\bar{\chi}'(0) > 1$ <u>and</u> $\bar{\chi}'(\bar{\mu}) \neq 1$. <u>There exists a stationary markov sunspot equilibrium with</u> r <u>states in every neighbourhood of</u> $\bar{\mu}$ <u>if and only if</u> $\bar{\chi}'(\bar{\mu}) < -1$.

The above arguments show that given r , the set of stationary Markov sunspot equilibria with r states (characterized by $(\mu_1,...,\mu_r)$ and the probabilities $q_{ij} > 0$ for all i,j) is finite dimensional and open. The set of all such Markov sunspot equilibria, when the number of states r is variable, is by contrast <u>infinite dimensional</u> whenever it is nonempty.

The results of this section extend easily to the case where Ω is a <u>compact metric space</u>, endowed with its Borel σ-field. Consider a Markov process on Ω given by the stationary transition probabilities $q(\omega,B)$ where the map that associates to every ω the probability $q(\omega,.)$ is <u>continuous</u> when the space of probabilities on Ω is endowed with the topology of weak convergence (Billingsley [5] or Parthasarathy [13]). We assume that $q(\omega,B) > 0$ for every ω and every nonempty <u>open</u> set B . A price function is defined, as in (3.1), as $p(\omega,M) = M/\mu(\omega)$ where $\mu(\omega)$ is <u>continuous and bounded away from 0</u> , i.e. $\mu(\omega) \geq a$ for all ω and some $a > 0$, and $\mu(\omega') \neq \mu(\omega'')$ for some ω' , ω'' . The counterpart of Proposition 3.1 is that $\mu(\omega)$ determines a stationary Markov sunspot equilibrium if and only if there are ω' and ω'' with $\mu(\omega') < \mu(\omega'')$ and $\bar{\chi}(\mu(\omega'')) < \mu(\omega) < \bar{\chi}(\mu(\omega'))$ for all ω .

Such a Markov sunspot equilibrium is described by the continuous map $\mu(\omega)$ and the continuous map that associates to every ω the probability $q(\omega,.)$. The set of stationary Markov sunspot equilibria is nonempty and <u>open</u> (in the topology of uniform convergence of maps) if and only if there is a noncritical period 2 cycle. If Ω is infinite dimensional, so is the set of stationary Markov sunspot equilibria whenever it is nonempty.

<u>Remark 3.1</u>. The methods developed in the text for analysing stationary Markov sunspot equilibria are rather general. To illustrate their power, consider the case where (x_t) is predetermined, but where $S_t = 0$, $G_t = p_t \gamma$ for all t , $\gamma > 0$ (no lump sum transfers, the Government's purchases of the good are pegged at γ) . In view of (1.9), the corresponding b.p.f. map on real balances is easily seen to be

$$\mu_t = v_1^{-1}[v_2(\mu_{t+1} - \gamma)] \equiv \bar{\chi}(\mu_{t+1}) \qquad (3.10)$$

(we keep the same notation $\bar{\chi}$ as in the text to facilitate the comparison, but no confusion should arise). If we think of Fig. 1 as representing the laisser faire offer curve, i.e. the graph of $\chi = v_1^{-1} \circ v_2$, then the graph of the map $\bar{\chi}$ defined in (3.10) is obtained by translating this offer curve to the right along the axis $\overrightarrow{0\mu_{t+1}}$ by the quantity γ . As a result, there may exist now <u>several</u> monetary stationary states, i.e. several $\bar{\mu} > 0$ such that $\bar{\mu} = \bar{\chi}(\bar{\mu})$.

The analogue of (3.6) becomes in this context

$$v_1(\mu_i) = \Sigma_j \, q_{ij} \, v_2(\mu_j - \gamma) \qquad (3.11)$$

Then replicating the argument leading to Proposition 3.1 yields immediately that $\mu_1 \leq \ldots \leq \mu_r$ defines a stationary Markov sunspot equilibrium if and only if there exist m,n with $\mu_m < \mu_n$ that verify either (3.9) or

$$\bar{\chi}(\mu_m) < \mu_1 < \mu_r < \bar{\chi}(\mu_n) \tag{3.12}$$

The configuration (3.12) could not occur in the case studied in the text, as remarked in the proof of Proposition 3.1. It may of course here : assume that the least fixed point $\bar{\mu}$ of $\bar{\chi}$ satisfies $\bar{\chi}'(\bar{\mu}) \neq 1$, and think of $\mu_1, \ldots, \mu_r$ as being clustered near it.

It follows that in this case, the existence of a noncritical period 2 cycle is no longer necessary for the occurrence of such stationary sunspot equilibria (think of the case where $\bar{\chi}$ is increasing everywhere). What can be shown nevertheless is that stationary Markov sunspot equilibria corresponding to the configuration (3.9) exist if and only if there is a "noncritical" period 2 cycle.

A trivial adaptation of the proof of Proposition 3.3 shows that there exists a stationary Markov sunspot equilibrium in every neighbourhood of a given fixed point $\bar{\mu} > 0$ of $\bar{\chi}$, with $\bar{\chi}'(\bar{\mu}) \neq 1$, if and only if either $\bar{\chi}'(\bar{\mu}) < -1$ or $\bar{\chi}'(\bar{\mu}) > 1$. In the first case, all such sunspot equilibria near $\bar{\mu}$ correspond to the scheme (3.9) (and Fig. 2 applies). In the second case, they all verify (3.12). This implies in particular that if $\bar{\mu}$ is the least fixed point of the map $\bar{\chi}$ defined in (3.10), with $\bar{\chi}'(\bar{\mu}) \neq 1$, there are stationary sunspot equilibria involving an arbitrary number of states near it, a result obtained by a different route in a similar model by Woodford [15].

4. POLICY RULES

The results of the previous two sections imply that a manipulation of the predetermined money growth rates $s_t = s$, $d_t = d$ may succeed in eliminating all deterministic cycles with perfect foresight with a period $k \geq 2$, and thus all stationary Markov sunspot equilibria as well. Such a reduction of the "cyclical" properties of the economy obtains, however, only under the assumption that the traders' forecasts are correct. This assumption supposes a coordination across time of expectations : forecasts made at t must be validated by the behaviour of the next generation, i.e. their expectations, and so on ad infinitum. Such an intertemporal coordination of expectations, however, is quite unlikely to occur in a decentralized economy out of long run "stationary" equilibria. On the transition path, one should expect the agents to make significant forecasting errors while they attempt to learn the dynamic laws of the signals they receive. The corresponding efficiency losses will disappear only in the long run, provided that the traders' environment becomes eventually repetitive and simple enough to enable them to forecast correctly the future — if the induced dynamic process converges to anything at all in calendar time.

We show in this section how deterministic policy rules can, if believed and understood by private units, solve this expectations coordination problem and guarantee perfect foresight. It was proved in [9] that monetary policy alone, through nominal interest payments, could achieve this goal and moreover, force the economy to converge to the laisser faire Golden Rule stationary state, provided that it was asymptotically stable in the forward perfect foresight dynamics that is defined in its neighbourhood. We show here that with lump sum transfers and public expenditures, the stability requirement can be dropped : there are policy rules using these channels that ensure perfect foresight stability of the target equilibrium.

Perfect foresight

It is convenient to assume here that the Government, instead of controlling the rates of growth of the money supply through lump sum transfers and public expenditures, ties the real values of the subsidies S_t and of the deficits $D_t = S_t + G_t$ to real interest rates. As a preliminary step, we characterize intertemporal equilibria with perfect foresight when the sequence of proportional money transfers (x_t) is predetermined and when S_t, D_t evolve, for all t, according to the <u>policy rules</u>

$S_t = p_t\, \sigma(p_{t-1} x_t / p_t)$ <u>and</u> $D_t = p_t\, \delta(p_{t-1} x_t / p_t)$. <u>The functions</u> (4.a)

σ , δ <u>are defined on</u> $(0,+\infty)$ <u>and satisfy</u> $\sigma(1) = \delta(1) = 0$,

$\theta \ell_1^* + \ell_2^* + \sigma(\theta) \geqq 0$.

The last inequality appearing in (4.a) is the counterpart in the present context of the feasibility condition (1.8). It implies in particular that the functions

$$Z_1(\theta) = z_1(\theta,\sigma(\theta)) \quad \text{and} \quad Z_2(\theta) = z_2(\theta,\sigma(\theta)) + \delta(\theta) - \sigma(\theta) \tag{4.1}$$

as well defined for all positive θ .

An intertemporal (monetary) equilibrium with perfect foresight corresponding to the predetermined sequence (x_t) and to the policy (4.a), is a sequence of prices (p_t) that satisfies, for all t , the equation (1.7) after the suppression of the superscript "e", the evolution of the money supply being governed by

$$M_t = M_{t-1} x_t + D_t > 0 \text{ , } M_o > 0 \text{ \underline{given}}$$

Here again, such an equilibrium is most conveniently described with the variable $\theta_t = p_{t-1} x_t / p_t$ and the real balance $\mu_t = M_t / p_t$. By employing the notation introduced in (4.1), the equilibrium equation (1.7) reads under perfect foresight

$$Z_1(\theta_t) + \mu_t = 0 \tag{4.2}$$

One may remark that μ_t is the excess demand for the good of the old trader and of the Government at date t . Under perfect foresight, it is given by

$$\mu_t = Z_2(\theta_{t-1}) \tag{4.3}$$

Combining (4.2) and (4.3), we get that an intertemporal equilibrium with perfect foresight is characterized by a sequence (θ_t) that verifies

$$Z_1(\theta_t) + Z_2(\theta_{t-1}) = 0 \tag{4.4}$$

such that the real balances that are implied by (4.2), or (4.3), are all positive. Here again, the set of such equilibria with perfect foresight is, in real terms, independent of M_o and of the predetermined sequence (x_t) .

In view of the assumption $\sigma(1) = \delta(1) = 0$, a particular equilibrium is obtained by setting $\theta_t = 1$ for all t : it is in fact identical to the laisser faire Golden Rule stationary state. Then if $Z_1'(1) \neq 0$, one can solve (4.4) in θ_t in a neighbourhood of the stationary state $\theta = 1$. An important question is whether one can choose appropriately the policy rules σ , δ so as to make $\theta = 1$ asymptotically stable in this (local) forward perfect foresight dynamics. Or equivalently, from the implicit function theorem, whether one can choose $\sigma'(1)$, $\delta'(1)$ to guarantee $Z_1'(1) > |Z_2'(1)|$. An answer is provided by the following result.

PROPOSITION 4.1. _Assume_ (4.a) _and that the policy functions_ σ , δ _are continuously differentiable. Then if_ $\delta'(1) \geq 0$, _one has_ $Z_1'(1) + Z_2'(1) > 0$. _Given such a choice of_ $\delta'(1)$, _one has_ $Z_1'(1) - Z_2'(1) > 0$ _and thus_ $Z_1'(1) > |Z_2'(1)|$ _if_ $\sigma'(1)$ _is chosen large enough_.

The proof is straightforward. By differentiating (4.1) and the budget equation (1.4), one gets

$$Z_1'(1) + Z_2'(1) = -z_1(1,0) + \delta'(1)$$

$$Z_1'(1) - Z_2'(1) = z_{1\theta}'(1,0) - z_{2\theta}'(1,0) + 2z_{1\sigma}'(1,0)\,\sigma'(1) - \delta'(1)$$

Since the young trader's excess demand for the good $z_1(1,0)$ at the Golden rule is negative, $\delta'(1) \geqq 0$ implies $Z_1'(1) + Z_2'(1) > 0$. On the other hand, goods are normal since the traders' utility function is separable, so that $0 < z_{1\sigma}'(1,0) < 1$. Thus, given $\delta'(1) \geqq 0$, one gets $Z_1'(1) - Z_2'(1) > 0$ if $\sigma'(1)$ is large enough.

A simple application of the implicit function theorem yields then

COROLLARY 4.2. <u>Let the conditions of Proposition</u> 4.1 <u>be satisfied, so that</u> $Z_1'(1) > |Z_2'(1)|$, <u>and consider an interval of the form</u> $I = (1-\varepsilon, 1+\varepsilon)$. <u>Then if</u> ε <u>is small enough, for any</u> θ_{t-1} <u>in</u> I , <u>there is a unique</u> $\theta_t = f(\theta_{t-1})$ <u>that lies in</u> I <u>and verifies</u> (4.4), <u>and the resulting map</u> f <u>is continuously differentiable with</u> $|f'(\theta)| \leqq k$ <u>for some</u> $k < 1$ <u>and all</u> θ in I .

Of course, if ε is small enough, for any θ_o in I , the sequence generated by $\theta_t = f(\theta_{t-1})$ will converge to 1 , while the associated real balances $\mu_t = Z_2(\theta_{t-1})$ will be positive and converge to the laisser faire Golden rule real balance $\bar{\mu} = z_2(1,0)$.

<u>Stabilization Policies</u>

The preceding results concerned the possibility of designing policy rules through lump sum transfers and public expenditures that make the Golden rule stationary state asymptotically stable under the assumption of perfect foresight. As explained at the outset of this section, perfect foresight supposes an intertemporal coordination of expectations that is quite unlikely to obtain in a decentralized economy [4]. We show now how a combination of a <u>policy rule</u> using proportional money transfers and of policy rules like those contemplated in Proposition 4.1, can solve this expectations coordination problem and make the economy to converge to the Golden rule. The argument will be in fact the same as in [9, Section 6].

Assume that the economy evolves under laisser faire prior to date $t = 0$. At the outset of period 0, the Government announces that it will implement a policy — starting at $t = 1$ — that ties the real interest rate, i.e. $p_{t-1}x_t/p_t$, the real values of the subsidy S_t/p_t and of the deficit D_t/p_t, to the previous rate of inflation, according to the rule

$$x_t = 1 \text{ , } S_t = D_t = 0 \text{ \underline{for} } t \leqq 0 \text{ , \underline{and} } x_t = (p_t/p_{t-1})\, \xi(p_{t-1}/p_{t-2}) \text{ ,} \tag{4.b}$$

$S_t = p_t\, \sigma(p_{t-1}\, x_t/p_t)$, $D_t = p_t\, \delta(p_{t-1}\, x_t/p_t)$ <u>for all</u> $t \geqq 1$, <u>where</u> ξ <u>maps</u> $(0,+\infty)$ <u>into itself and</u> σ , δ <u>satisfy</u> (4.a)

The public adoption of the rule (4.b) modifies the informational content of prices. If the traders understand and believe in the rule, their expectations will obey for all $t \geqq 0$

$$p_t\, x^e_{t+1}/p^e_{t+1} = \xi(p_t/p_{t-1}) \tag{4.5}$$

$$S^e_{t+1}/p^e_{t+1} = \sigma(\xi(p_t/p_{t-1}))$$

The traders may be wrong about x^e_{t+1} , p^e_{t+1} , S^e_{t+1} , but they will predict correctly $\theta_t = p_t\, x_{t+1}/p_{t+1}$ and S_{t+1}/p_{t+1} . The economy will accordingly follow from $t = 0$ on a forward sequence of temporary equilibria obtained by plugging (4.5) into the temporary equilibrium equation (1.7), along which traders have perfect foresight about real interest rates and the real values of lump sum transfers. All sunspots phenomena will thus be ruled out.

By using the notation introduced in (4.1), one can formulate the temporary equilibrium condition for $t = 0$ as

$$Z_1(\xi(p_o/p_{-1})) + (M_o/p_o) = 0 \tag{4.6}$$

in which $M_o > 0$ are p_{-1} are given by past history. With the same notation, since there are no forecasting errors along the sequence, the temporary equilibrium condition for $t \geqq 1$ will read

$$Z_1(\xi(p_t/p_{t-1})) + Z_2(\xi(p_{t-1}/p_{t-2})) = 0 \tag{4.7}$$

together with the condition that money stocks must be positive

$$M_t/p_t = - Z_1(\xi(p_t/p_{t-1})) > 0 \tag{4.8}$$

The variables $\theta_t = \xi(p_t/p_{t-1})$ will therefore satisfy the perfect foresight recurrence equation (4.4), with the additional restriction that they must lie in the range of ξ. The strategy to follow now is clear. We choose $\delta'(1) \geqq 0$ and $\sigma'(1)$ large enough as in Proposition 4.1 so that $Z_1'(1) > |Z_2'(1)|$. As for monetary policy, we choose the same as in [9, Section 6], that is

(4.c) ξ <u>is continuously differentiable, with</u> $\xi'(x) < 0$ <u>for all</u> x, <u>and range</u> $I = (1-\varepsilon, 1+\varepsilon)$. <u>Moreover</u> $\xi(x^*) = 1$.

Monetary policy slows down the real rate growth of the money supply attributable to nominal interest payments, i.e. $\theta_t = p_t x_{t+1}/p_{t+1}$, when inflation has been high in the immediate past, and constrains the variables θ_t to lie in the interval I . Finally, $x^* - 1$ is the rate of inflation to be achieved in the long run together with the goal $\theta = 1$.

If ε is small enough, (4.6) has a unique solution p_o given M_o and p_{-1} and it is stable in any Walrasian tatonnement at date 0 . By applying Corollary 4.2 to (4.7) recursively, one gets finally

PROPOSITION 4.3. Assume that the policy rules ξ , σ , δ satisfy (4.b), (4.c). Assume that σ , δ are continuously differentiable and choose $\delta'(1) \geqq 0$ and $\sigma'(1)$ large enough to ensure $Z_1'(1) > |Z_2'(1)|$. Then if ε is small enough,

1) there is a unique sequence of temporary equilibrium prices $(p_t), t \geqq 0$ that satisfies (4.6), and (4.7), (4.8) for all $t \geqq 1$, given M_o and p_{-1} . Each temporary equilibrium price p_t is stable in any Walrasian tatonnement at date $t \geqq 0$.

2) there are no forecasting errors about real interest rates and the real value of lump sum transfers along the sequence. Moreover $\theta_t = \xi(p_t/p_{t-1})$ tends to 1 and p_t/p_{t-1} tends to x^* as t tends to $+\infty$.

FOOTNOTES

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1. We admit in the sequel that the Government may sell some amount of the good as well as purchase it. We maintain this fiction for analytical simplicity as a proxy for the more realistic case where public consumption would enter the trader's utility functions (production of a public good) and where the Golden Rule equilibrium would entail accordingly a <u>positive</u> consumption by the Government.

2. Specifically, under the assumptions of [9, Lemma 4.6] on the functions v_1, v_2, each member of the family has a negative Schwarzian derivative and one can apply [10, Theorems 7 and 9]. The details are to the reader.

3. This result is due to the fact that the specification of the traders' beliefs through (3.1) generates an expectation function linking a trader's forecast at date t, i.e. the probability distribution $\psi(\omega_i, M_t)$ which assigns probability q_{ij} to the price $p(\omega_j, M_{t+1})$, to his information (p_t, ω_i, M_t) at that date. Note that the expectation function is independent of the currently observed price, a condition that is familiar in temporary equilibrium analysis in order to get existence, see Grandmont [8, 11].

4. A further problem is that, in view of the results of Section 3, many stationary sunspot equilibria are likely to exist near the Golden rule, if it is made stable as in Proposition 4.1, which would make the inference problem to be solved by the agent quite difficult.

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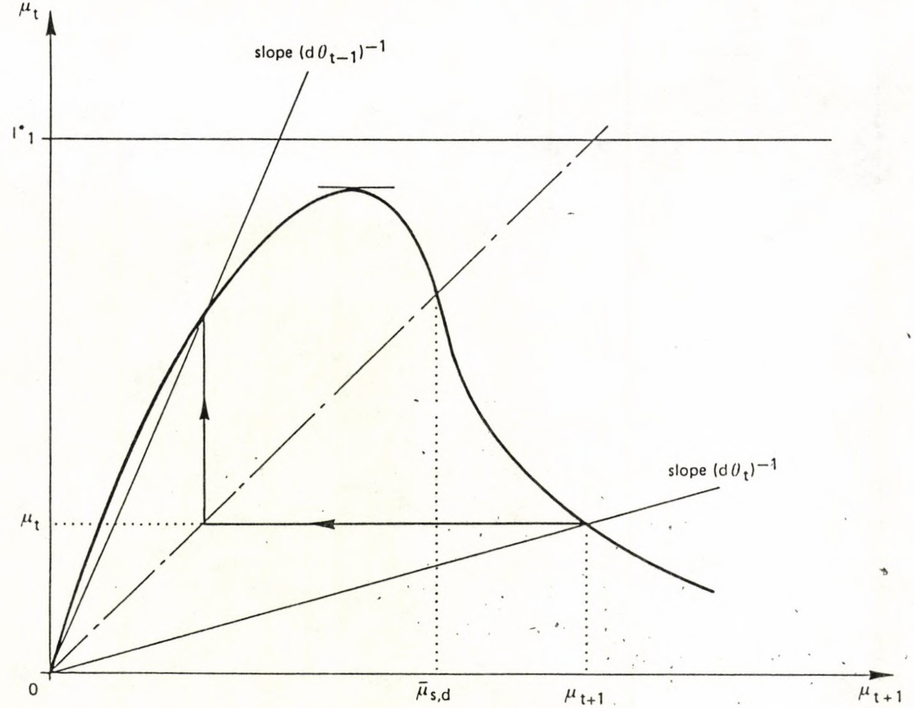
μ_t
slope $(d\theta_{t-1})^{-1}$
l^*_1
slope $(d\theta_t)^{-1}$
μ_t
0
$\bar{\mu}_{s,d}$
μ_{t+1}
μ_{t+1}

Figure 1

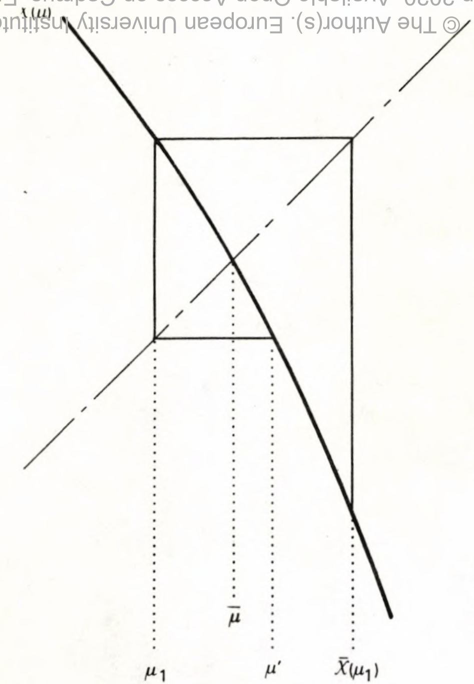

Figure 2

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