EUROPEAN UNIVERSITY INSTITUTE, FLORENCE

DEPARTMENT OF ECONOMICS

EUI WORKING PAPER No. 89/387

Modeling Oligopolistic Interaction

STEPHEN MARTIN

BADIA FIESOLANA, SAN DOMENICO (FI)

Printed in Italy in May 1989
European University Institute
Badia Fiesolana
– 50016 San Domenico (FI) –
Italy

MODELING OLIGOPOLISTIC INTERACTION

Stephen Martin

Department of Economics

European University Institute

May 1989

JEL Classification Number: 610

ABSTRACT: It is shown that standard versions of overt and tacit collusion under quantity-setting oligopoly are formally equivalent. The two approaches are combined to model collusive behavior when firms expect rivals to react to output changes.

I. Introduction

Three distinct themes appeared in the earliest scholarly reactions to Cournot's [1838] analysis of duopoly. Perhaps the most prominent was that error lay in specifying quantity rather than price as the decision variable (Bertrand [1883, p. 503]; Fisher [1898, p. 126]; Edgeworth [1922]). The current consensus is that it is useful to model both price-setting and quantity-setting markets.

The two remaining themes, also introduced by Bertrand [1883, p. 503], are stated concisely by Fisher [1898, p. 126]:

> The fault to be found in [Cournot's] reasoning is in his premise that each individual will act on the assumption that his rival's output is constant, and will strive only to so regulate his own output as to secure the largest profit.

The first criticism is that Cournot analyzed a duopoly in which each duopolist acted in the belief that the other's control variable was fixed. The second criticism is that Cournot had each duopolist maximize its own profit, and only its own profit.

Although economists had been tardy in recognizing the importance of Cournot's work, they were not slow explore alternative specifications, in attempts to relax one or the other of these assumptions. Edgeworth [1881, p. 53] introduced what has come to be the "coefficient of cooperation" approach, and formally explored the implications of partial joint profit maximization.[1] Bowley [1924, p. 38]

---

1. Edgeworth used the kinder, gentler term "coefficient of effective sympathy." "Coefficient of cooperation," due to Cyert and DeGroot [1975] seems by now well established in the literature.

wrote out first-order conditions for conjectural derivatives in duopoly, specifying a model in which expectations of rivals' reactions are not set to zero by assumption.[2]

Three different ways of modeling conjectural variations have appeared in the literature, and two of these have been widely used. Although equivalent in equilibrium, the alternative approaches to modeling conjectures have different implications for firms' behavior out of equilibrium, and for the specification of tests of market power. The relationship among the three specifications is discussed below.

Only recently have economists begun to compare and contrast the conjectural variation and coefficient of cooperation approaches to modeling oligopolistic interactions (Mueller [1987, pp. 56-61]).[3] In what follows, I show that the coefficient of cooperation approach is formally equivalent to one of the common conjectural variation specifications. I also show that the two approaches can be combined, to obtain a model of cooperation with conjectures as to rivals' behavior.

---

2. Bowley's discussion, "außerordentlich kurze," received prompt recognition; see Schneider [1932, pp. 167-169].

3. It is interesting to note that a textbook treatment of conjectural variations (Cohen and Cyert [1965, pp. 230-241]) included a discussion of collusion/joint profit maximization [pp. 235-236].

II. Conjectural Variations Models

A. Alternative Specifications of Conjectures

Consider an oligopoly of n firms which supply a standardized product to a market with inverse demand curve

(1) $$p = a - bQ = a - b(q_1 + q_2 + \dots + q_n) .$$

Let c_i be the constant marginal and average cost of firm i,[4] and for notational convenience, define

(2) $$Q_{-i} = Q - q_i$$

as the output of all firms <u>except</u> firm i. Three specifications of firm conjectures have appeared in the literature.

Frisch [1933, p. 252] discusses conjectural elasticities, which in the current accepted notation can be defined as

(3) $$\alpha_i = \frac{\partial \log Q_{-i}}{\partial \log q_i} = \frac{q_i}{Q_{-i}} \frac{dQ_{-i}}{dq_i} .$$

α_i is the percentage change in rivals' output which firm i expects in response to a unit percentage change in its own output. Recent use of the conjectural elasticity approach can be traced to Dickson [1982] and Clarke and Davies [1982].[5]

In contrast, Hicks [1935] defines conjectural derivatives, of the form

(4) $$\lambda_i = \frac{dQ_{-i}}{dq_i} .$$

---

4. For ease of exposition, I suppose fixed costs are zero. Positive fixed costs would introduce discontinuities in reaction curves and the potential for non-existence of equilibrium.

5. Kwoka and Ravenscraft [1985] refer to (3) as the Clarke-Davies model. Bresnahan [1983] has noted the difficulty of establishing priority of authorship in this ancient and cyclic literature. The labels used here will be descriptive rather than attributive.

λ_i is the absolute change in rivals' output expected by firm i in response to a unit absolute change in its own output. This formulation of conjectural beliefs is probably dominant in theoretical discussions. Despite some earlier appearances,[6] the "growth industry" phase of the literature using conjectural derivatives can be traced to Cowling [1976] and Bresnahan [1981].

Gallop and Roberts [1979] introduce a relative specification of conjectural beliefs:

$$\beta_i = \frac{\partial \log Q_{-i}}{\partial q_i} = \frac{1}{Q_{-i}} \frac{dQ_{-i}}{dq_i} . \tag{5}$$

β_i is the percentage change in rivals' output which firm i expects in response to a unit absolute change in its own output. Use of the relative conjecture specification in standard oligopoly models frequently yields results which are of limited tractability. In one respect, however, the relative conjecture approach has implications which are more palatable than those of either the conjectural derivative or the conjectural elasticity approach.

---

6. Fellner [1949, footnote 1, pp. 55-56] gives references to much of the early literature. His own discussion [1949, pp. 71-77] is in terms of conjectural derivatives. See also Fama and Laffer [1972] and Kamien [1975].

B. Reaction Curves in Conjectural Oligopoly

If firm i selects output to maximize its own profit,

(6) $$\pi_i = (p - c_i)q_i \ ,$$

the resulting first-order condition is

(7) $$\left(2 + \frac{\partial Q_{-i}}{\partial q_i}\right)q_i + Q_{-i} = S_i$$

where

(8) $$S_i = \frac{a - c_i}{b}$$

is a natural measure of the size of the market from the point of view of firm i: the quantity which would be demanded if price were equal to firm i's marginal cost.

Substituting (3), (4), and (5), respectively, into (7) yields alternative expressions for firm i's reaction curve:

(9) $$q_i = \frac{S_i - Q_{-i}}{2 + \lambda_i} \ ,$$

(10) $$q_i = \tfrac{1}{2}[S_i - (1 + \alpha_i)Q_{-i}]$$

and

(11) $$q_i = \frac{S_i - Q_{-i}}{2 + \beta_i Q_{-i}} \ .$$

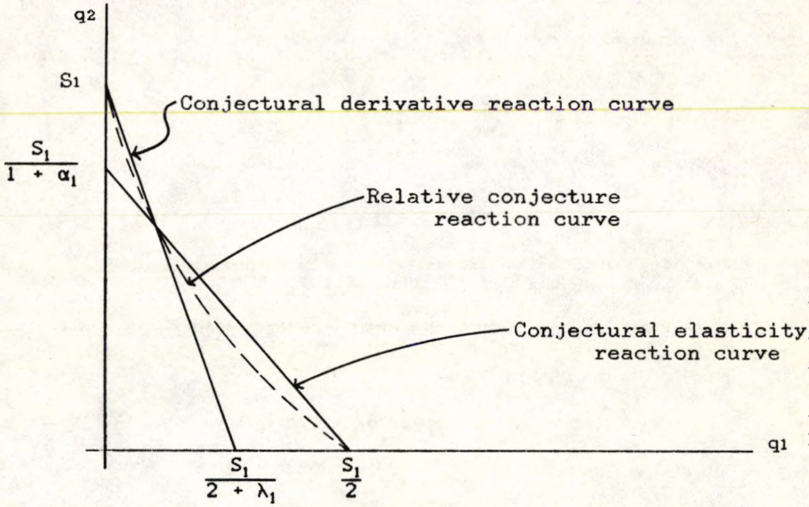
Note that in each case the parameters which describe firm cost and market demand affect reaction curves (and therefore equilibrium) only insofar as they affect the market size index S_i.

There are obvious algebraic relationships among α_i, β_i, and λ_i:

(12) $$\alpha_i = \frac{q_i}{Q_{-i}} \lambda_i \qquad \beta_i = \frac{1}{Q_{-i}} \lambda_i \ .$$

It follows that if (for example) we take α_i to be a constant and require that β_i and λ_i be everywhere equivalent to α_i, then β_i and λ_i will be variable. If, less restrictively, we require only that

Figure 1: Conjectural Derivative/Conjectural Elasticity Reaction Curves, Firm 1



Notes: drawn for duopoly, $p = 10 - Q$, $c_1 = 1$, $\lambda_1 = \frac{1}{2}$

α_1, β_1 and λ_1 be equivalent in equilibrium, we need only substitute the equilibrium values of q_i and q_{-i} in (12).

Imposing equivalence in equilibrium, one obtains three distinct reaction curves, as shown in Figure 1.

With conjectural derivatives, equation (9) indicates that rivals must supply S_1, the quantity demanded at a price equal to firm i's marginal cost, to drive firm i from the market. This accords with economic intuition: for rival output less than S_1, firm i can always make some profit by acting as a monopolist along the residual demand curve.

In contrast, and counterintuitively, with conjectural elasticities equation (10) shows that rivals can drive firm i from the market while producing somewhat less than S_i. Although price at output $\frac{S_i}{1 + \alpha_i}$ is above firm i's marginal cost, with a constant conjectural elasticity firm i believes that if it were to operate as a monopolist on the residual demand curve, rivals would expand output sufficiently to push price below marginal cost.

If rivals drop out of the market, firm i is a monopolist. With conjectural elasticities, firm i's reaction curve gives the intuitive result: for $Q_{-i} = 0$, firm i produces the monopoly output. But with conjectural derivatives, if rivals drop out of the market, firm i chooses to produce something less than the monopoly output. Although rivals produce nothing, firm i believes that if it expanded output to the monopoly level, rivals would expand output sufficiently to result in a net loss of profit to firm i.

Thus the conjectural derivative reaction curve gives the intuitive result when firm i drops out of the market, and the conjectural elasticity reaction curve gives the intuitive result when rivals drop out of the market. As shown in Figure 1, the relative conjecture reaction curve, equation (11), (which is a rotated hyperbola) gives the intuitive result at both extremes. With relative conjectures, a firm will produce the monopoly output if rivals drop out of the market, and rivals will have to supply the entire market at a price equal to a firm's marginal cost before the firm will withdraw from the market. In this sense, relative conjectures yield a description of oligopoly behavior which is more appealing than that provided by either the conjectural derivative or the conjectural elasticity approach.

Table 1: Test of Conjectural Derivative vs. Conjectural Elasticity Specifications

Company	Intercept	Share	Capital-Sales Ratio	R^2
Anderson, Clayton & Co.	0.0328 (1.2567)	0.3946 (1.8904)	-0.0252 (0.5318)	0.0999
Beatrice	0.0473 (3.7775)	0.1303 (2.3711)	0.0186 (1.6096)	0.2368

C. A Test of Specification (I)[7]

The conjectural derivative and conjectural elasticity specifications have been widely used in theoretical and empirical work in industrial economics. It is possible to test which specification better describes data for an individual firm.

The conjectural derivative specification implies that firm i's Lerner index of market power is[8]

$$\frac{p - c_i}{p} = \frac{s_i}{\epsilon_{Qp}/(1 + \lambda_i)} . \tag{13}$$

7. For comparison with (13) and (14), note that the Lerner index for the relative conjecture specification is

$$\frac{p - c_i}{p} = \frac{1 + \beta_i Q_{-i}}{\epsilon_{Qp}} s_i .$$

8. Cowling [1976] (for the case of equal-sized firms).

In this model, an increase in the conjectural derivative λ_i has the same effect as a decrease in the price elasticity of demand: to increase the Lerner index.

The Lerner index for the conjectural elasticity specification is (Clarke and Davies [1982])

(14)
$$\frac{p - c_i}{p} = \frac{\alpha_i + (1 - \alpha_i)s_i}{\varepsilon_{Qp}}$$

where s_i is firm i's market share.

It is useful to think of the numerator on the right in (14) as a weighted average of 1 (the monopoly market share) and s_i, the firm's market share. An increase in the conjectural elasticity α_i increases the weight given to 1, reduces the weight given to s_i, and increases the Lerner index.

An empirical specification which includes (13) and (14) as special cases is[9]

(15)
$$\frac{pq_i - VC_i}{pq_i} = a_0 + a_1 s_i + \rho_i \frac{p^k K_i}{pq_i}$$

where VC_i is variable costs and the final term on the right, the capital-sales ratio, controls for the normal rate of return on capital.

---

9. Martin [1984]. I assume here constant returns to scale. For an extension to the case of non-constant returns to scale in a related model, see Martin [1988].

Comparing (13) and (15), the conjectural derivative model implies that the intercept term on the right in (15), a_0, is zero. An intercept term in (15) which is significantly different from zero is consistent with the conjectural elasticity model but not the conjectural derivative model.

Table 1 reports estimates of equation (15) for two firms in the U.S. food processing industry.[10] The estimate of the intercept term for Anderson, Clayton & Co. is insignificantly different from zero, which (in the context of the models discussed in this section) suggests that the conjectural derivative specification is appropriate for Anderson, Clayton & Co.

In contrast, the intercept term in the estimate of (15) for the Beatrice Company is significantly different from zero. This is consistent with the implications of the conjectural elasticity specification.

---

10. Sales and margin data, taken from Compustat data tapes, are quarterly, from 1973.2 through 1982.4 for Anderson, Clayton and Company and from 1973.1 through 1982.4 for Beatrice. Estimates of the economic values of capital stock were computed as in Martin [1988].

III. The Coefficient of Cooperation Model

A. Formal Equivalence to the Conjectural Elasticity Model

Suppose now that firm i maximizes a weighted average of its own profit and joint profit:

$$G_i = \Pi_i + \theta_i \sum_{j \neq i} \Pi_j = (1 - \theta_i)\Pi_i + \theta_i \sum_{j=1}^{n} \Pi_j. \tag{16}$$

θ_i is firm i's coefficient of cooperation, which indexes the weight firm i gives to other firms' profit when it takes its own decisions. Such an objective function might be thought of as describing collusive behavior. Alternatively, it could be thought of as describing the behavior of a firm which owns a fraction θ_i of its rivals, so sharing in their profits (Bresnahan and Salop [1986], Reynolds and Snapp [1986]).

It is straightforward to show that, on the Cournot conjectural assumption, the first-order condition for (16) implies a reaction curve

$$q_i = \frac{1}{2}[S_i - (1 + \theta_i)Q_{-i}] \tag{17}$$

and a Lerner index

$$\frac{p - c_i}{p} = \frac{\theta_i + (1 - \theta_i)s_i}{\epsilon_{Qp}} . \tag{18}$$

Comparing (17) and (18) with (10) and (14), respectively, it is apparent that the conjectural derivative model and the coefficient of cooperation model are formally equivalent.

It follows that the empirical literature motivated by models of conjectural elasticities may be reinterpreted as providing tests of the coefficient of cooperation model. Without additional theoretical development (see below) the two models have identical implications.

The formal identity of the conjectural elasticity and the conjectural elasticity approach has certain implications for the consistent conjectures literature, although that literature has developed in the context of the conjectural derivative model. In the context of the conjectural elasticities model, the consistent conjectures literature would suggest exploring the implications of the requirement that the conjectural elasticity of rivals' output equal the actual elasticity of rivals' output. In a study of conjectural interactions, this is a natural line of investigation. Problems with the existence of consistent conjectures have led some to suggest that game theoretic techniques, and those alone, are the appropriate techniques for the analysis of oligopolistic interactions (Makowski [1987]; see also Bhaskar [1989]).

The formal equivalence of the coefficient of cooperation approach and the conjectural elasticity approach suggests that the obituary of Cournot, conjectural, coefficient of cooperation, reaction function approach is premature. In a coefficient of cooperation model, there is no equivalent of the "rational conjecture." Firms make their own decisions without knowing or having beliefs about the coefficients of cooperation of their rivals. The existence or nonexistence of consistent beliefs about rivals' coefficients of cooperation is irrelevant to the insights yielded by the coefficient of cooperation model. Yet the coefficient of cooperation model is formally equivalent to one of the two widely-used conjectures models. If - as seems likely - the coefficient of cooperation approach has a place in the industrial economist's toolbox, then so does the conjectural variation approach.

B. Cooperation With Conjectures

The Cournot behavioral assumption is no more appealing in a coefficient of cooperation model than elsewhere in the analysis of oligopoly. If the Cournot behavioral assumption is abandoned in the coefficient of cooperation model, the result is a model of partial cooperation with conjectures concerning rivals' behavior.

1. Reaction curves

Conjectural derivatives

Suppose all firms have the same marginal and average cost ($c_i = c$ for all i). It is then possible to derive equations for the reaction curves of one firm and of all other firms, on the assumption that all other firms produce the same output. This assumption is met in equilibrium. Its convenience is that it allows us to condense the equations of n reaction curves to two, and permits graphical depiction of the reaction curves.

If we further assume that all firms have the same coefficients of cooperation and conjectural variations, equilibrium will be symmetric, in the sense that all firms produce the same output in equilibrium. These assumptions simplify the algebra without altering the qualitative nature of the results.

If conjectural derivatives are grafted onto the coefficient of cooperation model, the reaction curves of firm 1 and of all other firms are

(19a) $$q_1 = \frac{S - [1 + \theta(1 + \lambda)](n - 1)q_{-1}}{2 + \lambda}$$

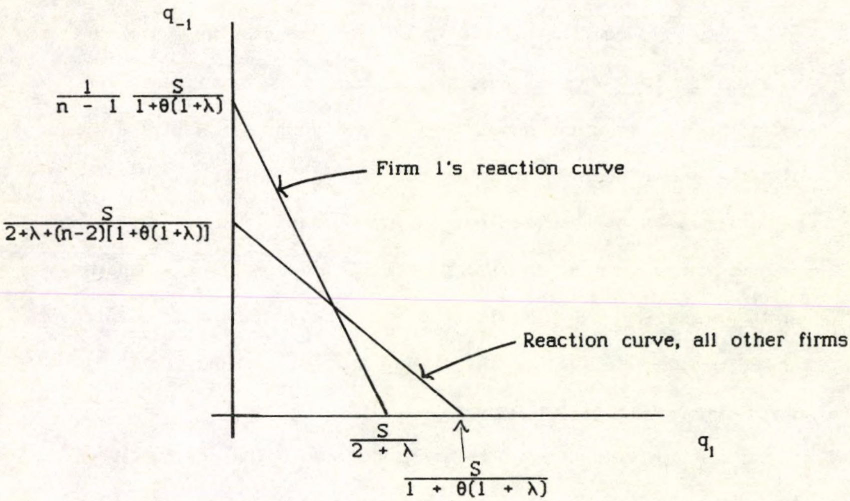
and

(19b) $$q_{-1} = \frac{S - [1 + \theta(1 + \lambda)]q_1}{n + \lambda + (n - 2)\theta(1 + \lambda)}$$

respectively, where q_{-1} is output per firm of all firms other than firm 1.

The reaction curves have the advantage of being linear. It is straightforward to solve for equilibrium values. Equilibrium, as shown in Figure 2, is symmetric and stable.

Figure 2: Reaction Curves, Coefficient of Cooperation With Conjectural Derivatives



Conjectural elasticities[11]

If conjectural elasticities are combined with the coefficient of cooperation model, the reaction curves of firm 1 and all other firms are

$$(20a)\quad 2q_1^2 + (n - 1)(1 + \alpha)(1 + \theta)q_1q_{-1} + 2(n - 1)^2\alpha\theta q_{-1}^2 - Sq_1 - (n - 1)\alpha\theta Sq_{-1} = 0$$

and

$$2\alpha\theta q_1^2 + [(1 + \alpha)(1 + \theta) + 4(n - 2)\alpha\theta]q_1q_{-1}$$

$$(20b)\quad + \{2 + (n - 2)[(1 + \alpha)(1 + \theta) + 2(n - 2)\alpha\theta]\}q_{-1}^2$$

$$- \alpha\theta Sq_1 - [1 + (n - 2)\alpha\theta]Sq_{-1} = 0 .$$

respectively.

In symmetric equilibrium (letting $q_1 = q_{-1} = q$), price and output per firm are

$$(21)\quad p = c + \frac{(n - 1)[(1 + \alpha)(1 + \theta) - (n - 2)(1 - \alpha\theta)]}{2 + (n - 1)(1 + \alpha)(1 + \theta) + 2(n - 1)^2\alpha\theta}bS$$

and

$$(22)\quad q = \frac{1 + (n - 1)\alpha\theta}{2 + (n - 1)(1 + \alpha)(1 + \theta) + 2(n - 1)^2\alpha\theta}bS$$

respectively.

The reaction curves (20) are rotated hyperbolae.[12] The number of terms in (20b) which vanish if $n = 2$ suggests that duopoly

11. Reaction curves for the conjectural elasticity-coefficient of cooperation case have been analyzed using BASIC programs vetted by comparison with Borland's Eureka. These programs are available on request.

12. If α and θ are both negative, the reaction curves are ellipses.

is a special case, and this proves to be correct. Taking $\theta > 0$, duopoly reaction curves typically exhibit a single equilibrium, which is symmetric and stable.

Moving beyond duopoly, it is useful to distinguish 2 cases. $\theta > 0$ and $\alpha < 0$ may be thought of as collusion with distrust. In this case firms partially maximize joint profit, but each firm lacks confidence in the behavior of its fellows. Each firm expects that as it restricts output, rivals will expand output. Unless α is highly negative, there is typically a single equilibrium in which all firms have positive output. This is the symmetric equilibrium, and it is stable. Distrust may be bad for the soul, but it is apparently good for oligopolistic equilibrium.

Things are otherwise if $\theta > 0$ and $\alpha > 0$. A symmetric equilibrium in which all firms have positive output typically exists. Nonsymmetric equilibria in which all firms have positive output may also exist. The symmetric equilibrium may be stable or unstable. In some cases, there are stable equilibria in which one firm has radically larger output than other firms.[13] Trust may be good for the soul, but it is not necessarily good for market position.

---

13. If $n = 4$, $\alpha = .4$, $\theta = .6$, and $S = 90$, there is a unstable equilibrium in which each firm produces 11.87 units of output. There is a stable equilibrium in which one firm produces 29.57 units of output and each of the remaining three firms produces 5.75 units.

2. The Lerner Index

The Lerner index for the coefficient of cooperation-conjectural derivative model is

$$\text{(23)} \qquad \frac{p - c_i}{p} = \frac{\theta_i + (1 - \theta_i)s_i}{\epsilon_{Qp}/(1 + \lambda_i)} .$$

The right-hand side of equation (23), like that of (18), has a constant term and is linear in market share. (23), like (14) or (18), could be used to motivate the estimating equation (15).

The Lerner index for the coefficient of cooperation-conjectural elasticity model is

$$\text{(24)} \qquad \begin{aligned} \frac{p - c_i}{p} &= \frac{\theta_i + (1 - \theta_i)s_i}{\epsilon_{Qp}} \, \frac{\alpha_i + (1 - \alpha_i)s_i}{s_i} \\ &= \frac{(\alpha_i + \theta_i - 2\alpha_i\theta_i)}{\epsilon_{Qp}} + \frac{(1 - \alpha_i)(1 - \theta_i)}{\epsilon_{Qp}}s_i + \frac{\alpha_i\theta_i}{\epsilon_{Qp}} \frac{1}{s_i} . \end{aligned}$$

3. A Test of Specification (II)

An empirical specification which includes (23) and (24) as special cases is

$$\text{(25)} \qquad \frac{pq_i - VC_i}{pq_i} = b_0 + b_1 s_i + b_2\left(\frac{1}{s_i}\right) + \rho_i \frac{p^k K_i}{pq_i}$$

Within the context of the coefficient of cooperation model, it is possible to distinguish between the conjectural derivative and the conjectural elasticity specifications by examining the significance of the estimated coefficient of the inverse market share term on the right in (25).

If this coefficient is significantly different from zero, as for Anderson, Clayton & Co. in Table 2, the results are consistent with the coefficient of cooperation, conjectural elasticity model. Comparing Tables 1 and 2, it is evident that use of the inverse market share term substantially increases the statistical significance of coefficient estimates and the explanatory power of the regression for Anderson, Clayton & Co.

Table 2: Test of Conjectural Derivative vs. Conjectural Elasticity Specifications

Company	Intercept	Share	1/Share	Capital-Sales Ratio	R^2
Anderson, Clayton & Co.	-0.6104 (2.4215)	4.3162 (2.7991)	0.0258 (2.5636)	-0.0379 (0.8541)	0.2422
Beatrice	0.0969 (0.2614)	0.0251 (0.0319)	-0.0057 (0.1339)	0.0176 (1.2494)	0.2372

No such effect is apparent for Beatrice. An insignificant coefficient of the inverse market share term suggests rejection of the coefficient of cooperation-conjectural elasticity approach in favor of the coefficient of cooperation-conjectural derivative model.

IV. Final Remarks

In terms of their implications for firm behavior, market equilibrium, and market performance, the coefficient of cooperation and model and the conjectural elasticity models are equivalent.

It is possible to test which conjectural specification provides the better explanation of firm performance. This is so whether or not the conjectural model is combined with the coefficient-of-cooperation approach.

REFERENCES

Bertrand, Joseph "Review," Journal des Savants Volume 68, 1883, pp. 499-508; reprinted in English translation in Andrew F. Daughety, editor, Cournot oligopoly. Cambridge: Cambridge University Press, 1988, pp. 73-81.

Bhaskar, V. "Quick Responses in Duopoly Ensure Monopoly Pricing," Economics Letters Volume 29, Number 2, 1989, pp. 103-107.

Bowley, A. L. The Mathematical Groundwork of Economics. Oxford: Oxford University Press, 1924.

Bresnahan, Timothy F. "Duopoly Models with Consistent Conjectures," American Economic Review Volume 71, Number 5, December 1981, pp. 934-945.

----- "Duopoly Models with Consistent Conjectures: Reply," American Economic Review Volume 73, Number 1, March 1983, pp. 240-241.

Bresnahan, Timothy F. and Snapp, Bruce R. "Quantifying the competitive effects of production joint ventures," International Journal of Industrial Organization Volume 4, Number 2, June 1986, pp. 155-175.

Clarke, Roger and Davies, Stephen W. "Market Structure and Price-Cost Margins," Econometrica, Volume 49, August 1982, pp. 277-287.

Cohen, Kalman J. and Cyert, Richard M. Theory of the Firm: Resource Allocation in a Market Economy. Englewood Cliffs, New Jersey: Prentice-Hall, 1965.

Cournot, Augustin Researches into the Mathematical Principles of the Theory of Wealth. original Paris: L. Hachette, 1838; English translation by Nathaniel T. Bacon New York: The Macmillan Company, 1897.

Cowling, Keith "On the Theoretical Specification of Industrial Structure-Performance Relationships," European Economic Review Volume 8, 1976, pp. 1-14.

Cyert, Richard M. and DeGroot, Morris H. "An Analysis of Cooperation and Learning in a Duopoly Context," American Economic Review, Volume 63, Number 1, March 1973, pp. 24-37.

Edgeworth, F. Y. Mathematical Psychics. London: Kegan Paul, 1881.

----- "The Mathematical Economics of Professor Amoroso," Economic Journal Volume 32, Number 127, September 1922, pp. 400-407.

Fellner, William Competition Among the Few. New York: Alfred A. Knopf, 1949.

Fisher, Irving "Cournot and Mathematical Economics," Quarterly Journal of Economics Volume 12, Number 2, January 1898, pp. 119-138

Frisch, R. "Monopole--Polypole--La Notion de Force dans l'Economie," Nationalokonomisk Tidsskrift, Volume 71, 1933, pp. 241-259.

Gallop, Frank M. and Roberts, Mark J. "Firm Interdependence in Oligopolistic Markets," Journal of Econometrics Volume 10, Number 3, August 1979, pp. 313-331.

Hicks, J. R. "Annual Survey of Economic Theory: The Theory of Monopoly," Econometrica Volume 3, Number 1, January 1935 pp. 1-20

Kwoka, John E. and Ravenscraft, David J. "Cooperation v. Rivalry: Price-Cost Margins by Line of Business," Economica Volume 53, August 1986, pp. 351-363.

Martin, Stephen "The misuse of accounting rates of return: Comment," American Economic Review Volume 74, June 1984, pp. 501-506.

----- "The measurement of profitability and the diagnosis of market power," International Journal of Industrial Organization Volume 6, Number 3, September 1988, pp. 301-321.

Makowski, Louis "Are 'Rational Conjectures' Rational," Journal of Industrial Economics Volume 36, Number 1, September 1987, pp. 35-47.

Mueller, Dennis C. Profits in the Long Run. Cambridge, England: Cambridge University Press, 1986.

Reynolds, Robert J. and Snapp, Bruce R. "The competitive effects of partial equity interests and joint ventures," International Journal of Industrial Organization Volume 4, Number 2, June 1986, pp. 141-153.

Schneider, Erich Reine Theorie monopolistischer Wirtschaftsformen. Tübingen: J. C. B. Mohr (Paul Siebeck), 1932.

WORKING PAPERS ECONOMICS DEPARTMENT

86/206:	Volker DEVILLE	Bibliography on The European Monetary System and the European Currency Unit.
86/212:	Emil CLAASSEN Melvyn KRAUSS	Budget Deficits and the Exchange Rate
86/214:	Alberto CHILOSI	The Right to Employment Principle and Self-Managed Market Socialism: A Historical Account and an Analytical Appraisal of some Old Ideas
86/218:	Emil CLAASSEN	The Optimum Monetary Constitution: Monetary Integration and Monetary Stability
86/222:	Edmund S. PHELPS	Economic Equilibrium and Other Economic Concepts: A "New Palgrave" Quartet
86/223:	Giuliano FERRARI BRAVO	Economic Diplomacy. The Keynes-Cuno Affair
86/224:	Jean-Michel GRANDMONT	Stabilizing Competitive Business Cycles
86/225:	Donald A.R. GEORGE	Wage-earners' Investment Funds: theory, simulation and policy
86/227:	Domenico Mario NUTI	Michal Kalecki's Contributions to the Theory and Practice of Socialist Planning
86/228:	Domenico Mario NUTI	Codetermination, Profit-Sharing and Full Employment
86/229:	Marcello DE CECCO	Currency, Coinage and the Gold Standard
86/230:	Rosemarie FEITHEN	Determinants of Labour Migration in an Enlarged European Community
86/232:	Saul ESTRIN Derek C. JONES	Are There Life Cycles in Labor-Managed Firms? Evidence for France
86/236:	Will BARTLETT Milica UVALIC	Labour Managed Firms, Employee Participation and Profit Sharing - Theoretical Perspectives and European Experience.
86/240:	Domenico Mario NUTI	Information, Expectations and Economic Planning
86/241:	Donald D. HESTER	Time, Jurisdiction and Sovereign Risk

87/283:	Leslie T. OXLEY Donald A.R. GEORGE	Perfect Foresight, Non-Linearity and Hyperinflation
87/284:	Saul ESTRIN Derek C. JONES	The Determinants of Workers' Participation and Productivity in Producer Cooperatives
87/285:	Domenico Mario NUTI	Financial Innovation under Market Socialism
87/286:	Felix FITZROY	Unemployment and the Share Economy: A Sceptical Note
87/287:	Paul HARE	Supply Multipliers in a Centrally Planned Economy with a Private Sector
87/288:	Roberto TAMBORINI	The Stock Approach to the Exchange Rate: An Exposition and a Critical Appraisal
87/289:	Corrado BENASSI	Asymmetric Information and Financial Markets: from Financial Intermediation to Credit Rationing
87/296:	Gianna GIANNELLI	On Labour Market Theories
87/297:	Domenica TROPEANO	The Riddle of Foreign Exchanges: A Swedish-German Debate (1917-1919)
87/305:	G. VAN DER LAAN A.J.J. TALMAN	Computing Economic Equilibria by Variable Dimension Algorithms: State of the Art
87/306:	Paolo GARELLA	Adverse Selection and Intermediation
87/307:	Jean-Michel GRANDMONT	Local Bifurcations and Stationary Sunspots
87/308:	Birgit GRODAL Werner HILDENBRAND	Income Distributions and the Axiom of Revealed Preference
87/309:	Eric PEREE Alfred STEINHERR	Exchange Rate Uncertainty and Foreign Trade
87/312:	Pietro REICHLIN	Output-Inflation Cycles in an Economy with Staggered Wage Setting
87/319:	Peter RAPPOPORT Lucrezia REICHLIN	Segmented Trends and Nonstationary Time Series
87/320:	Douglas GALE	A Strategic Model of Labor Markets with Incomplete Information
87/321:	Gianna GIANNELLI	A Monopoly Union Model of the Italian Labour Market: 1970-1984

87/322:	Keith PILBEAM	Sterilization and the Profitability of UK Intervention 1973-86
87/323:	Alan KIRMAN	The Intrinsic Limits of Modern Economic Theory
87/324:	Andreu MAS-COLELL	An Equivalence Theorem for a Bargaining Set
88/329:	Dalia MARIN	Assessing Structural Change: the Case of Austria
88/330:	Milica UVALIC	"Shareholding" in Yugoslav Theory and Practice
88/331:	David CANNING	Convergence to Equilibrium in a Sequence of Games with Learning
88/332:	Dalia MARIN	Trade and Scale Economies. A causality test for the US, Japan, Germany and the UK.
88/333:	Keith PILBEAM	Fixed versus Floating Exchange Rates Revisited.
88/335:	Felix FITZROY Kornelius KRAFT	Piece Rates with Endogenous Monitoring: Some theory and evidence
88/337:	Domenico Mario NUTI	On Traditional Cooperatives and James Meade's Labour-Capital Discriminating Partnerships
88/338:	Pietro REICHLIN Paolo SICONOLFI	Government Debt and Equity Capital in an Economy with Credit Rationing
88/339:	Alfred STEINHERR	The EMS with the ECU at Centerstage: a proposal for reform of the European rate system
88/340:	Frederick VAN DER PLOEG	Monetary and Fiscal Policy in Inter-dependent Economies with Capital Accumulation, Death and Population Growth
88/341:	David CANNING	Optimal Monetary Policy in an Economy without a Forward Market for Labour
88/344:	Joerg MAYER	Intervention Mechanisms and Symmetry in the EMS
88/345:	Keith PILBEAM	Exchange Rate Management and the Risk Premium

Spare copies of these Working Papers can be obtained from the Econaomics Department secretariat.

EUI WORKING PAPERS

EUI Working Papers are published and distributed by the European University Institute, Florence.

Copies can be obtained free of charge - depending on the availability of stocks - from:

The Publications Officer
European University Institute
Badia Fiesolana
I - 50016 San Domenico di Fiesole (FI)
Italy

Please use order form overleaf

PUBLICATIONS OF THE EUROPEAN UNIVERSITY INSTITUTE

To The Publications Officer
European University Institute
Badia Fiesolana
I - 50016 San Domenico di Fiesole (FI)
Italy

From Name . . . .
Address . . . .
. . . .
. . . .
. . . .
. . . .

Please send me the following EUI Working Paper(s):

No. . . . .
Author, title: . . . .
. . . .
. . . .
. . . .
. . . .

Date . . . .

Signature
. . . .

88/329:	Dalia MARIN	Assessing Structural Change: The Case of Austria *
88/330:	Milica UVALIC	"Shareholding" in Yugoslav Theory and Practice
88/331:	David CANNING	Convergence to Equilibrium in a Sequence of Games with Learning
88/332:	Dalia MARIN	Trade and Scale Economies. A causality test for the U.S., Japan, Germany and the UK
88/333:	Keith PILBEAM	Fixed versus Floating Exchange Rates Revisited
88/334:	Hans Ulrich Jessurun d'OLIVEIRA	Die EWG und die Versalzung des Rheins
88/335:	Felix Fitzroy and Kornelius Kraft	Piece Rates with Endogenous Monitoring Some Theory and Evidence
88/336:	Norbert LORENZ	Die Übertragung von Hoheitsrechten auf die Europäischen Gemeinschaften - verfassungsrechtliche Chancen und Grenzen einer europäischen Integration erläutert am Beispiel der Bundesrepublik Deutschland, Frankreichs und Italiens -
88/337:	Domenico Mario NUTI	On Traditional Cooperatives and James Meade's Labour-Capital Discriminating Partnerships
88/338:	Pietro REICHLIN and Paolo SICONOLFI	Government Debt and Equity Capital in an Economy with Credit Rationing
88/339:	Alfred STEINHERR	The EMS with the ECU at Centerstage: A proposal for reform of the European Exchange rate system
88/340:	Frederick VAN DER PLOEG	Monetary and Fiscal Policy in Interdependent Economies with Capital Accumulation, Death and Population Growth
88/341:	David CANNING	Optimal Monetary Policy in an Economy without a Forward Market for Labour
88/342:	Gunther TEUBNER	"And God Laughed..." Indeterminacy, Self-Reference and Paradox in Law
88/343:	Jean BLONDEL	Ministerial Careers in Western European Governments

* :Working Paper out of print

88/344:	Joerg MAYER	Intervention Mechanisms and Symmetry in the EMS
88/345:	Keith PILBEAM	Exchange Rate Management and the Risk Premium
88/346:	Efisio ESPA	The Structure and Methodology of International Debt Statistics
88/347:	Francesc MORATA and and Jaume VERNET	Las Asambleas Regionales en Italia y España: Organizacion Institucional y Reglas de Funcionamiento
88/348:	Milica UVALIC	The Investment Behaviour of the Labour-Managed Firm: An Econometric Analysis
88/349:	Massimo PANEBIANCO	Inter-Regional Co-Operation in the North-South Dialogue Latin America and the European Community
88/350:	Gregorio ROBLES	La Cour de Justice des CE et les Principes Généraux du droit
88/351:	Alan KIRMAN	On Ants and Markets
88/352:	Gianna GIANNELLI	Labour Demand, Pricing and Investment Decisions in Italy: An Econometric Analysis
88/353:	Niall O'HIGGINS	The Progressivity of Government Taxes and Benefits in Ireland: A Comparison of Two Measures of Redistributive Impact
88/354:	Christian JOERGES	Amerikanische und deutsche Traditionen der soziologischen Jurisprudenz und der Rechtskritik
88/355:	Summary of Conference, debates and abstracts of selected interventions	The Future Financing of the EC Budget: EPU Conference 16-17 October 1987
88/356:	Mary MCCARTHY and Lucrezia REICHLIN	Do Women Cause Unemployment? Evidence From Eight O.E.C.D. Countries
88/357:	Richard M. GOODWIN	Chaotic Economic Dynamics
88/358:	Fernando PACHECO Eric PEERE and Francisco S. TORRES	Duopoly Under Demand Uncertainty
88/359:	Jaakko NOUSIAINEN	Substance and Style of Cabinet Decision-Making

* :Working Paper out of print

PUBLICATIONS OF THE EUROPEAN UNIVERSITY INSTITUTE APRIL 1989

88/360:	Domenico Mario NUTI	Economic Relations between the European Community and CMEA
88/361:	Domenico Mario NUTI	Remonetisation and Capital Markets in the Reform of Centrally Planned Economies
88/362:	Domenico Mario NUTI	The New Soviet Cooperatives: Advances and Limitations
88/363:	Reiner GRUNDMANN	Marx and the Domination of Nature Alienation, Technology and Communism
88/364:	Tony PROSSER	The Privatisation of Public Enterprises in France and Great Britain The State, Constitutions and Public Policy
88/365:	Silke BRAMMER	Die Kompetenzen der EG im Bereich Binnenmarkt nach der Einheitlichen Europäischen Akte
88/366:	Goesta ESPING-ANDERSEN	The Three Political Economies of the Welfare State
88/367:	Goesta ESPING-ANDERSEN Paul FARSUND and Jon Eivind KOLBERG	Decommodification and Work Absence in the Welfare State
88/368:	Stephen MARTIN	Joint Ventures and Market Performance in Oligopoly
88/369:	Giuseppe RAO	The Italian Broadcasting System: Legal and Political Aspects
89/370:	B. BENSAID/ S. FEDERBUSCH/ R.J. GARY BOBO	The Strategic Aspects of Profit Sharing in the Industry
89/371:	Klaus-Dieter STADLER	Die Europäische Zusammenarbeit in der Generalversammlung der Vereinten Nationen zu Beginn der Achtziger Jahre
89/372:	Jean Philiippe Robé	Countervailing Duties, State Protectionism and the Challenge of the Uruguay Round
89/373:	Giovanni FEDERICO/ Antonio TENA	On the Accuracy of Historical International Foreign Trade Statistics Morgenstern Revisited
89/374:	Francisco TORRES	Small Countries and Exogenous Policy Shocks

* :Working Paper out of print

* :Working Paper out of print