Econometric Models for Mixed-Frequency Data

Claudia Foroni

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence
September 2012
Econometric Models for Mixed-Frequency Data

Claudia Foroni

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

Jury Members:
Prof. Massimiliano Marcellino, EUI, Supervisor
Prof. Tommaso di Fonzo, Università di Padova
Prof. Eric Ghysels, University of North Carolina
Prof. Helmut Lütkepohl, Humboldt University Berlin

© 2012, Claudia Foroni
No part of this thesis may be copied, reproduced or transmitted without prior permission of the author
To my parents, for what they have taught and keep teaching me.
ACKNOWLEDGMENTS

Writing this PhD thesis has required the contribution of many, to whom I want to express my gratitude.

First and foremost, I am deeply indebted to my supervisor, Massimiliano Marcellino, who helped me much more than what it is required to a supervisor. He provided invaluable guidance and support throughout the years, teaching me what doing research means. And I need to thank him even more for being constantly present and available, for the time he spent whenever I knocked at his door, for the encouragement he provided especially when I was losing confidence in what I was doing.

I am also very grateful to my second advisor, Helmut Luetkepohl, who patiently read and commented the draft of every chapter at different stages.

The thesis benefited also from the comments of numerous seminar and conference participants, and in particular I would like to thank Eric Ghysels, Mike McCracken and Christian Schumacher for their helpful suggestions.

During the PhD I had the privilege of spending some of my time in the Research Department of the Deutsche Bundesbank. In particular, I would like to thank Heinz Herrmann and Christian Schumacher, for the possibility they gave me to have an internship there and to write one of the chapters of the thesis in that stimulating environment.

I would like to extend my gratitude to the staff at the Economics Department of the EUI, and especially to Jessica and Lucia: with their efficiency and extreme friendliness, they have been a reference point in my VSP-life.

Many thanks go to all my classmates and friends who contributed to make my last four years more pleasant. In particular, a special mention goes to Agnese, Alessia, Lena and Mario, for all the advice, encouragement and support that only those who pass through the PhD experience can provide. And to Chiara, Eleonora and Eliana, for reminding me that there is an entire life outside the PhD.

My gratitude goes to Stefan, for making me smile even in the most difficult times. His constant encouragement and his confidence in me have been invaluable.

Finally, I thank my family, my parents, my aunt and my grandmother, who have always supported me unconditionally, encouraging me to do what I really like. Knowing that I have such a safe place where to go back makes my life much easier.
This thesis addresses different issues related to the use of mixed-frequency data. In the first chapter, I review, discuss and compare the main approaches proposed so far in the literature to deal with mixed-frequency data, with ragged edges due to publication delays: aggregation, bridge-equations, mixed-data sampling (MIDAS) approach, mixed-frequency VAR and factor models. The second chapter, a joint work with Massimiliano Marcellino, compares the different approaches analyzed in the first chapter, in a detailed empirical application. We focus on now- and forecasting the quarterly growth rate of Euro Area GDP and its components, using a very large set of monthly indicators, with a wide number of forecasting methods, in a pseudo real-time framework. The results highlight the importance of monthly information, especially during the crisis periods. The third chapter, a joint work with Massimiliano Marcellino and Christian Schumacher, studies the performance of a variant of the MIDAS model, which does not resort to functional distributed lag polynomials. We call this approach unrestricted MIDAS (U-MIDAS). We discuss the pros and cons of unrestricted lag polynomials in MIDAS regressions. In Monte Carlo experiments and empirical applications, we compare U-MIDAS to MIDAS and show that U-MIDAS performs better than MIDAS for small differences in sampling frequencies. The fourth chapter, a joint work with Massimiliano Marcellino, focuses on the issues related to mixed-frequency data in structural models. We show analytically, with simulation experiments and with actual data that a mismatch between the time scale of a DSGE or structural VAR model and that of the time series data used for its estimation generally creates identification problems, introduces estimation bias and distorts the results of policy analysis. On the constructive side, we prove that the use of mixed-frequency data can alleviate the temporal aggregation bias, mitigate the identification issues, and yield more reliable policy conclusions.
CONTENTS

I Introduction vii

II Chapters 1

1 A SURVEY OF ECONOMETRIC METHODS FOR MIXED-FREQUENCY DATA 3
  1.1 Introduction ......................................................... 3
  1.2 Models for mixed-frequency data ..................................... 4
    1.2.1 Aggregation and interpolation ............................... 6
    1.2.2 Bridge equations .............................................. 6
    1.2.3 Mixed-Data Sampling ......................................... 7
    1.2.4 Mixed-frequency VAR ......................................... 15
    1.2.5 Mixed-frequency factor models ............................. 18
  1.3 Ragged-edge data ................................................... 25
    1.3.1 Estimating the MF-VAR with missing observations ......... 25
    1.3.2 Estimating the factors with ragged-edge data ................ 26
  1.4 A comparison of the different methods .......................... 30
  1.5 An overview of empirical studies ................................ 31
    1.5.1 Bridge equations .............................................. 31
    1.5.2 MIDAS models .................................................. 33
    1.5.3 Mixed-frequency VAR models ................................ 35
    1.5.4 Factor models .................................................. 37
  1.6 Conclusions ....................................................... 42

2 A COMPARISON OF MIXED FREQUENCY APPROACHES FOR MODELLING EUROC Area MACROECONOMIC VARIABLES 51
  2.1 Introduction ....................................................... 51
  2.2 Data ................................................................. 54
  2.3 Model Specification ............................................... 55
    2.3.1 The Bridge Model approach ................................. 56
    2.3.2 The MF-VAR approach ....................................... 57
    2.3.3 The MIDAS approach ......................................... 60
  2.4 Results for Euro area GDP ....................................... 61
  2.5 Forecast pooling .................................................. 63
CONTENTS

2.6 Robustness analysis ......................................................... 65
2.7 Large scale models ....................................................... 67
  2.7.1 Quarterly factor model .............................................. 68
  2.7.2 Factor-MIDAS models .............................................. 68
  2.7.3 Results ............................................................... 69
  2.7.4 Alternative mixed frequency factor models ......................... 70
2.8 Results for euro area GDP components ................................ 71
2.9 Conclusions .............................................................. 72
2.10 Tables ................................................................. 74
2.11 Appendix A: Robustness analysis ...................................... 84
  2.11.1 Extending the forecast horizon .................................. 84
  2.11.2 Temporal stability ................................................ 85
  2.11.3 Subsample analysis .............................................. 86
  2.11.4 Several frequencies ................................................ 88
  2.11.5 The role of real-time data ...................................... 89
2.12 Appendix B: Results for euro area GDP components ................. 102
2.13 Appendix C: Dataset description ..................................... 110

3 U-MIDAS: MIDAS REGRESSIONS WITH UNRESTRICTED LAG POLYNOMIALS .......................................................... 115
  3.1 Introduction ............................................................ 115
  3.2 The rationale behind U-MIDAS and its use in forecasting ............ 118
    3.2.1 U-MIDAS regressions in dynamic linear models ................. 118
    3.2.2 Forecasting with U-MIDAS ...................................... 121
    3.2.3 U-MIDAS and MIDAS with exponential Almon lags ............... 122
  3.3 Monte Carlo experiments .............................................. 123
    3.3.1 The simulation design .......................................... 124
    3.3.2 The models under comparison .................................... 125
    3.3.3 Monte Carlo in-sample comparison results ...................... 126
    3.3.4 Monte Carlo forecast comparison results ....................... 128
    3.3.5 The role of BIC for lag length selection ....................... 128
    3.3.6 An alternative DGP with non-recursive VAR structure ........... 131
    3.3.7 Using MIDAS as DGP ............................................ 133
  3.4 Empirical examples .................................................. 133
    3.4.1 An application to Euro area GDP growth ....................... 135
<table>
<thead>
<tr>
<th>CONTENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4.2 An application to US GDP growth ........................................... 139</td>
</tr>
<tr>
<td>3.5 Conclusions .................................................................................. 143</td>
</tr>
<tr>
<td>3.6 Appendix A: Identification of the disaggregate process .................. 149</td>
</tr>
<tr>
<td>3.7 Appendix B: Monthly indicators .................................................... 150</td>
</tr>
<tr>
<td>3.7.1 Monthly Euro area indicators .................................................... 150</td>
</tr>
<tr>
<td>3.7.2 Monthly US data ................................................................. 150</td>
</tr>
<tr>
<td>4 MIXED-FREQUENCY STRUCTURAL MODELS: IDENTIFICATION, ESTIMATION, AND POLICY ANALYSIS 151</td>
</tr>
<tr>
<td>4.1 Introduction .................................................................................. 151</td>
</tr>
<tr>
<td>4.2 Time aggregation ......................................................................... 153</td>
</tr>
<tr>
<td>4.3 The time scale problem in a SVAR context .................................. 155</td>
</tr>
<tr>
<td>4.3.1 The common approach: aggregation at quarterly frequency ........ 156</td>
</tr>
<tr>
<td>4.3.2 Exploiting data at different frequencies ................................... 157</td>
</tr>
<tr>
<td>4.4 The time scale problem in a DSGE model .................................. 160</td>
</tr>
<tr>
<td>4.4.1 A basic New Keynesian model: mapping from monthly to quarterly specification .................................................... 161</td>
</tr>
<tr>
<td>4.4.2 A second New Keynesian model: aggregation and loss of identification 163</td>
</tr>
<tr>
<td>4.4.3 Exploiting mixed-frequency data to deal with identification issues .................................................... 165</td>
</tr>
<tr>
<td>4.5 Estimation methods ...................................................................... 166</td>
</tr>
<tr>
<td>4.5.1 Estimation of a mixed frequency SVAR model .......................... 166</td>
</tr>
<tr>
<td>4.5.2 Estimation of a mixed frequency DSGE model ......................... 168</td>
</tr>
<tr>
<td>4.6 Monte Carlo experiments ............................................................. 169</td>
</tr>
<tr>
<td>4.6.1 A Monte Carlo exercise within a SVAR framework .................. 170</td>
</tr>
<tr>
<td>4.6.2 A Monte Carlo exercise within a DSGE framework ................ 174</td>
</tr>
<tr>
<td>4.7 Empirical applications ................................................................ 175</td>
</tr>
<tr>
<td>4.7.1 SVAR model with US data ....................................................... 175</td>
</tr>
<tr>
<td>4.7.2 Small-scale New Keynesian model with US data ..................... 177</td>
</tr>
<tr>
<td>4.8 Conclusions ................................................................................ 178</td>
</tr>
<tr>
<td>4.9 Figures and tables ....................................................................... 180</td>
</tr>
<tr>
<td>4.10 Appendix: .................................................................................. 191</td>
</tr>
<tr>
<td>4.10.1 Quarterly SVAR: identification issues ..................................... 191</td>
</tr>
<tr>
<td>4.10.2 SVAR with mixed frequency data: identification issues .......... 192</td>
</tr>
<tr>
<td>4.10.3 High-frequency dependent variables ..................................... 194</td>
</tr>
<tr>
<td>4.10.4 A basic New Keynesian model: mapping and identification issues .................................................... 195</td>
</tr>
</tbody>
</table>
4.10.5 Introducing more dynamic in the Euler equation .................. 197
4.10.6 Obtaining identification by exploiting mixed-frequency data .... 199
Part I

Introduction
Economic time series differ substantially with respect to their sampling frequency. As an example, the Gross Domestic Product (GDP) is available at quarterly frequency and with a considerable publication lag, while a range of leading and coincident indicators is available at a monthly or even higher frequency. This raises the problem of how to conduct empirical analyses on the relationships between variables sampled at different frequencies. Policy-makers, in particular, need to assess the current state of the economy in real-time, when only incomplete information is available. The unbalancedness of datasets arises due to the different sampling frequency with which the indicators are available, and also to the so-called "ragged-edge" problem, namely publication delays of indicators cause missing values of some of the variables at the end of the sample.

The simplest solution is to work at the lowest frequency in the data, but this requires time aggregation of high-frequency variables with a loss of potentially relevant high-frequency information, and a convolution of the dynamic relationships among the variables. In recent times, the interest in direct modelling of mixed-frequency data has increased substantially. Nowadays, there is a growing strand of the literature which takes into account the information in the unbalanced datasets.

In the first chapter of this thesis, I review the main approaches proposed so far in the literature to deal with mixed-frequency data, possibly with ragged edges due to publication delays: aggregation, bridge-equations, mixed-frequency VAR (MF-VAR) and factor models, and the more recent mixed-data sampling (MIDAS) approach. After discussing and comparing the theoretical properties of the different models, I survey the main empirical applications, and identify the main achievements and still open questions in this literature. This first chapter paves the way to the next chapters, which all address different issues related to the use of mixed-frequency data.

The second chapter, a joint work with Massimiliano Marcellino\(^1\), compares the different approaches analyzed in the first chapter, in an extensive and detailed empirical application. Specifically, we focus on now- and forecasting the quarterly growth rate of Euro Area GDP and its components, using a very large set of monthly indicators, with a wide number of forecasting methods. We both investigate the behavior of single indicator models and combine first the forecasts within each class of models and then the information in the dataset by means of factor models, in a pseudo real-time framework. Moreover, in order to replicate the real-time situation in which policy makers and institutions need to assess the state of the economy, we investigate the behavior of a small group of indicators in a genuine real-time context, using data as they were at that moment, which allows taking into account

\(^1\)Affiliation: European University Institute, Bocconi University and CEPR.
data revisions. We find that MIDAS without an AR component performs worse than the corresponding approach which incorporates it, and MF-VAR seems to outperform the MIDAS approach only at longer horizons. Bridge equations have overall a good performance. Pooling many indicators within each class of models is overall superior to most of the single indicator models. Pooling information with the use of factor models gives even better results, at least at short horizons. A battery of robustness checks highlights the importance of monthly information during the crisis more than in stable periods. Extending the analysis to a real-time context highlights that revisions do not influence substantially the results.

The third chapter, a joint work with Massimiliano Marcellino and Christian Schumacher\(^2\), focuses on one of the mixed-frequency approaches presented in the first two chapters, the MIDAS model, and studies the performance of a variant of it, which does not resort to functional distributed lag polynomials. We call this approach unrestricted MIDAS (U-MIDAS). One reason that motivates the use of U-MIDAS in macroeconomic applications is that the difference between sampling frequencies is in many applications not so high. In such a case, it might not be necessary to employ distributed lag functions. We discuss the pros and cons of unrestricted lag polynomials in MIDAS regressions. We derive U-MIDAS regressions from linear high-frequency models, discuss identification issues, and show that their parameters can be estimated by OLS. In Monte Carlo experiments, we compare U-MIDAS to MIDAS with functional distributed lags estimated by NLS. We show that U-MIDAS performs better than MIDAS for small differences in sampling frequencies. On the other hand, with large differing sampling frequencies, distributed lag-functions outperform unrestricted polynomials. The good performance of U-MIDAS for small differences in frequency is confirmed in an empirical application on nowcasting Euro area and US GDP using monthly indicators.

The fourth chapter deviates from the statistical models considered so far, and concentrates on the issue of unbalanced datasets in the context of structural models. In the paper, which is a joint work with Massimiliano Marcellino, we show analytically, with simulation experiments and with actual data that a mismatch between the time scale of a DSGE or structural VAR (SVAR) model and that of the time series data used for its estimation generally creates identification problems, introduces estimation bias and distorts the results of policy analysis. On the constructive side, we prove that the use of mixed-frequency data, combined with a proper estimation approach, can alleviate the temporal aggregation bias, mitigate the identification issues, and yield more reliable policy conclusions. The problems and possible remedy are illustrated in the context of standard structural monetary policy models.

\(^2\)Affiliation: Deutsche Bundesbank.
Part II

Chapters
Chapter 1

A Survey of Econometric Methods for Mixed-Frequency Data

1.1 Introduction

In recent times, econometric models that take into account the information in unbalanced datasets have attracted substantial attention. Policy-makers, in particular, need to assess in real-time the current state of the economy and its expected developments, when only incomplete information is available.

In real-time, the unbalancedness of datasets arises mainly due to two features: the different sampling frequency with which the indicators are available and the so-called "ragged-edge" problem, namely, publication delays cause missing values for some of the variables at the end of the sample, see Wallis (1986). As an example, one of the key indicators of macroeconomic activity, the Gross Domestic Product (GDP), is released quarterly and with a considerable publication lag, while a range of leading and coincident indicators is available more timely and at a monthly or even higher frequency.

In this paper we review the methods proposed so far in the literature to deal with mixed-frequency data and missing values due to publication lags. The simplest technique is to aggregate the data to obtain a balanced dataset at the same frequency and to work with a "frozen" final vintage dataset, which eliminates the ragged edge problem. However, in the literature there are also a few methods to avoid pre-filtering associated with temporally aggregated and disaggregated data, and to exploit the information contained in the large number of series available in real-time at different frequencies. In what follows, we depict the main features of the temporal aggregation methods, followed by an overview of
the bridge equations models, often employed in central banks and other policy making institutions, especially for nowcasting and short-term forecasting, see e.g. Baffigi, Golinelli and Parigi (2004) and Diron (2008). We then move to one of the main strands of the literature, mixed-data sampling (MIDAS) models, parsimonious specifications based on distributed lag polynomials, which flexibly deal with data sampled at different frequencies and provide a direct forecast of the low-frequency variable (see e.g. Ghysels et al. (2004), Clements and Galvao (2008)). Finally, we consider the state-space approaches, presenting mixed-frequency VAR (MF-VAR) and factor models. Both are system approaches that jointly describe the dynamics of the variable to be explained and of the indicators, where the use of the Kalman filter provides not only predictions of the future observations but also estimates of the current latent state (see Mariano and Murasawa (2003, 2010)). A natural extension in the literature is the combination of the factors with the MIDAS models, and it is based on the use of factors as explanatory variables to exploit the information in large mixed-frequency datasets. The resulting model is labelled Factor-MIDAS by Marcellino and Schumacher (2010).

The paper is organized as follows. In Section 1.2, we survey the different approaches to model mixed frequency variables. In Section 1.3, we discuss the additional estimation issues arising with a ragged-edge structure of the dataset. In Section 1.4 we compare the main features of the different approaches. In Section 1.5 we present a summary of the most significant empirical applications appeared so far in this literature. Finally, in Section 1.6 we summarize and conclude.

1.2 Models for mixed-frequency data

Typical regression models relate variables sampled at the same frequency. To ensure the same frequency, researchers working with time series data either aggregate the higher-frequency observations to the lowest available frequency or interpolate the lower-frequency data to the highest available frequency (see Section 1.2.1). The most common solution in empirical applications is the former, temporal aggregation. The higher-frequency data are aggregated to the lowest-frequency by averaging or by taking a representative value (for example, the last month of a quarter). In pre-filtering the data so that left- and right-hand variables are available at the same frequency, a lot of potentially useful information might be destroyed, and mis-specification inserted in the model. Hence, direct modelling of mixed frequency data can be useful.¹

¹A review of the models to deal with mixed-frequency data has been proposed by Wolhrabe (2009). However, his review focuses more on the earliest attempts to tackle the mixed-frequency issues. In particular he reviews in detail the aggregation and interpolation of data, and the bridge and linkage models. In our
One of the early approaches to deal with mixed-frequency data focuses on forecasting and relies on bridge equations, see e.g. Baffigi, Golinelli, Parigi (2004), i.e. equations that link the low-frequency variables and time-aggregated indicators. Forecasts of the high-frequency indicators are provided by specific high-frequency time series models, then the forecast values are aggregated and plugged into the bridge equations to obtain the forecast of the low-frequency variable. Details are provided in Section 1.2.2.

In Section 1.2.3 we provide a more detailed overview of one of the most recent and competitive univariate approaches, the mixed-data sampling method originally proposed by Ghysels, Santa-Clara and Valkanov (2004). Mixed-data sampling (MIDAS) models handle series sampled at different frequencies, where distributed lag polynomials are used to ensure parsimonious specifications. Whereas early MIDAS studies focused on financial applications, see e.g. Ghysels, Santa-Clara and Valkanov (2006), recently this method has been employed to forecast macroeconomic time series, where typically quarterly GDP growth is forecasted by monthly macroeconomic and financial indicators, see e.g. Clements and Galvao (2008, 2009).

Another common approach in the literature is the state-space representation of the model, where to handle data with different frequencies, the low-frequency variable is considered as a high-frequency one with missing observations. The Kalman filter and smoother is then applied to estimate the missing observations and to generate forecasts. Moreover, the dynamics of the low and high-frequency series are jointly analyzed. One of the most compelling approaches at the moment is the one proposed by Zadrozny (1988) for directly estimating a VARMA model with series sampled at different frequencies. In the same fashion, Mariano and Murasawa (2010) set what they call mixed-frequency VAR (MF-VAR from now on), i.e. they introduce a VAR model for partially latent time series and cast it in state-space form, see Section 1.2.4 for more details. Among the state-space approaches we can also list mixed-frequency factor models employed, for example, to extract an unobserved state of the economy and create a new coincident indicator or forecast and nowcast GDP, see e.g. Mariano and Murasawa (2003, 2010) in Section 1.2.5.1 for small scale applications and Giannone, Reichlin and Small (2008) and Banbura and Rünstler (2011) for large scale models in Section 1.2.5.2 and 1.2.5.3. A similar approach is also followed by Frale et al. (2010, 2011): differently from the other studies, dynamic factor models are applied to a set of small datasets where variables are grouped according to economic theory and institutional considerations, rather than to the entire information set. The separate small factor models are then linked...
together within a state-space framework. Finally, in Section 1.2.5.4 we review the literature that proposes to merge the two recent strands in the mixed sampling econometrics: factor models and MIDAS approach. Marcellino and Schumacher (2010) introduce Factor-MIDAS, an approach for now- and forecasting low-frequency variables exploiting information in large sets of higher-frequency indicators.

1.2.1 Aggregation and interpolation

In most of the empirical applications, the common solution in the presence of a mixed sample frequency is to pre-filter the data so that the left- and right-hand side variables are sampled at the same frequency. In the process, a lot of potentially useful information can be destroyed and mis-specification included in the model.

The standard aggregation methods depend on the stock/flow nature of the variables and, typically, it is the average of the high-frequency variables over one low-frequency period for stocks, and the sum for flows.

Taking the latest available value of the higher frequency variable is another option for both stock and flow variables. The underlying assumption is that the information of the previous high-frequency periods is reflected in the latest value, representative of the whole low-frequency period.

The second option to match frequencies is the interpolation of the low frequency variables, which is rarely used. There are several different interpolating methods, see e.g. Lanning (1986), Marcellino (1998) and Angelini et al. (2006). A common approach is a two-step procedure: first missing data are interpolated, then model parameters are estimated using the new augmented series, possibly taking into account the measurement error induced by disaggregation. Both steps can be conveniently and jointly run in a Kalman filter set-up, starting with a state-space representation of the model, see e.g. Harvey (1989) and Sections 1.2.4 and 1.2.5 below.

1.2.2 Bridge equations

One of the early econometric approaches in the presence of mixed-frequency data relies on the use of bridge equations, see e.g. Baffigi, Golinelli, Parigi (2004) and Diron (2008). Bridge equations are linear regressions that link ("bridge") high frequency variables, such as industrial production or retail sales, to low frequency ones, e.g. the quarterly real GDP growth, providing some estimates of current and short-term developments in advance of the release. The "Bridge model" technique allows computing early estimates of the low-frequency
variables by using high frequency indicators. They are not standard macroeconometric models, since the inclusion of specific indicators is not based on causal relations, but on the statistical fact that they contain timely updated information. In principle, bridge models require that the whole set of regressors should be known over the projection period, allowing for an estimate only of the current period. In practice, anyway, this is not the case, even though the forecasting horizon of the bridge models is quite short, one or two quarters ahead at most.

Taking forecasting GDP as an example, since the monthly indicators are usually only partially available over the projection period, the predictions of quarterly GDP growth are obtained in two steps. First, monthly indicators are forecasted over the remainder of the quarter, usually on the basis of univariate time series models (in some cases VAR have been implemented in order to obtain better forecasts of the monthly indicators), and then aggregated to obtain their quarterly correspondent values. Second, the aggregated values are used as regressors in the bridge equation which allows to obtain forecasts of GDP growth.

Therefore, the bridge model to be estimated is:

$$y_{tq} = \alpha + \sum_{i=1}^{j} \beta_i (L) x_{itq} + u_{tq}$$

where $\beta_i (L)$ is a lag polynomial of length $k$, and $x_{itq}$ are the selected monthly indicators aggregated at quarterly frequency.

The selection of the monthly indicators included in the bridge model is usually based on a general-to-specific methodology and relies on different in-sample or out-of-sample criteria, like information criteria or RMSE performance.

In order to forecast the missing observations of the monthly indicators which are then aggregated to obtain a quarterly value of $x_{itq}$, it is common practice to use autoregressive models, where the lag length is based on information criteria.

### 1.2.3 Mixed-Data Sampling

Distributed lag (DL) models have been typically employed in the literature to describe the distribution over time of the lagged effects of a change in the explanatory variable. In general, a stylized distributed lag model is given by

$$y_{tq} = \alpha + B (L) x_{tq} + \varepsilon_{tq}$$

where $B (L)$ is some finite or infinite lag polynomial operator.
This kind of models underlies the construction of the bridge equations, once all the high
frequency values are aggregated to the corresponding low-frequency values.

In order to take into account mixed-frequency data, Ghysels et al. (2004) introduce the
Mixed-Data Sampling (MIDAS) approach, which is closely related to the distributed lag
model, but in this case the dependent variable $y_{t_q}$, sampled at a lower-frequency, is regressed
on a distributed lag of $x_{t_m}$, which is sampled at a higher-frequency.

In what follows, we first present the basic features of the model as presented by Ghysels et
al. (2004), the corresponding unrestricted version as in Foroni, Marcellino and Schumacher
(2012), and then the extensions that have been introduced in the literature.

In terms of notation, $t_q = 1, \ldots T_q$ indexes the basic time unit (e.g. quarters), and $m$
is the number of times the higher sampling frequency appears in the same basic time unit.
For example, for quarterly GDP growth and monthly indicators as explanatory variables,
$m = 3$. $w$ is the number of monthly values of the indicators that are earlier available than
the lower-frequency variable to be estimated. The lower-frequency variable can be expressed
at the high frequency by setting $y_{t_m} = y_{t_q}$, $\forall t_m = mt_q$, where $t_m$ is the time index at the high
frequency.

1.2.3.1 The basic MIDAS model

MIDAS regressions are essentially tightly parameterized, reduced form regressions that in-
volve processes sampled at different frequencies. The response to the higher-frequency ex-
planatory variable is modelled using highly parsimonious distributed lag polynomials, to
prevent the proliferation of parameters that might otherwise result, as well as the issues
related to lag-order selection.

The basic MIDAS model for a single explanatory variable, and $h_q$-step-ahead forecasting,
with $h_q = h_m/m$, is given by:

$$y_{t_q + mh_q} = y_{t_m + h_m} = \beta_0 + \beta_1 b \left( L_{m} ; \theta \right) x_{t_m}^{(m)} + \varepsilon_{t_m + h_m}$$  \hspace{1cm} (1.3)

where $b \left( L^{1/m} ; \theta \right) = \sum_{k=0}^{K} c \left( k ; \theta \right) L_{m}^{k}$, and $L_{m} x_{t_m}^{(m)} = x_{t_m}^{(m)} - x_{t_m}^{(m-1)}$. $x_{t_m}^{(m)}$ is skip-sampled from the high
frequency indicator $x_{t_m}$.

The parameterization of the lagged coefficients of $c \left( k ; \theta \right)$ in a parsimonious way is one
of the key MIDAS features. One of the most used parameterizations is the one known as
“Exponential Almon Lag”, since it is closely related to the smooth polynomial Almon lag
functions that are used to reduce multicollinearity in the Distributed Lag literature. It is
often expressed as
\[
c(k; \theta) = \exp \left( \theta_1 k + \ldots + \theta_Q k^Q \right) \sum_{k=1}^K \exp \left( \theta_1 k + \ldots + \theta_Q k^Q \right)
\] (1.4)

This function is known to be quite flexible and can take various shapes with only a few parameters. These include decreasing, increasing or hump-shaped patterns. Ghysels, Santa-Clara and Valkanov (2005) use the functional form with two parameters, which allows a great flexibility and determines how many lags are included in the regression.

Notice that the standard practice in bridge equations of calculating a quarterly series from the monthly indicators corresponds to imposing restrictions on this parameterization function. To be concrete, in the case of the quarterly-monthly example, taking the last month in the quarter to produce a quarterly series amounts to setting \( c(2; \theta) = c(3; \theta) = c(5; \theta) = c(6; \theta) = \ldots = c(11; \theta) = c(12; \theta) = 0 \).

Another possible parameterization, also with only two parameters, is the so-called “Beta Lag”, because it is based on the Beta function:
\[
c(k; \theta_1, \theta_2) = \frac{\int \left( \frac{k}{\theta_1}, \theta_1; \theta_2 \right) \Gamma(\theta_1)}{\Gamma(\theta_1, \theta_2)}
\] (1.5)

where \( c(x, a, b) = \frac{x^{a-1}(1-x)^{b-1} \Gamma(a+b)}{\Gamma(a) \Gamma(b)} \) and \( \Gamma(a) = \int_0^\infty e^{-x} x^{a-1} dx \).

Ghysels, Rubia and Valkanov (2009) propose also three other different parameterizations of the lag coefficients: a linear scheme, with \( c(k; \theta) = \frac{1}{K} \), where there are no parameters to estimate in the lagged weight function; an hyperbolic scheme, with \( c(k; \theta) = \frac{g(k; \theta)}{\sum_{k=1}^K g(k; \theta)} \), where the gamma function has only one parameter to estimate, but it’s not as flexible as the Beta specification; a geometric scheme, with \( c(k; \theta) = \frac{\theta^k}{\sum_{k=1}^\infty \theta^k} \), |\( \theta \)| ≤ 1 and \( c(k; \theta) \) are normalized so that they sum up to one.

The parameterizations described above are all quite flexible. For different value of the parameters, they can take various shapes: weights attached to the different lags can decline slowly or fast, or even have a hump shape. Therefore, estimating the parameters from the data automatically determines the shape of the weights and, accordingly, the number of lags to be included in the regression.

The MIDAS model can be estimated using nonlinear least squares (NLS) in a regression of \( y_t \) onto \( x_{t-h}^{(m)} \). Ghysels, Santa-Clara and Valkanov (2004) show that MIDAS regressions...
always lead to more efficient estimation than the typical approach of aggregating all series to the least frequent sampling. Moreover, they also show that discretization biases are the same for MIDAS and distributed lag models and vanish when regressors are sampled more frequently.

The forecast is given by

$$\hat{y}_{T+h_m} = \hat{\beta}_0 + \hat{\beta}_1 b \left( L_m; \hat{\theta} \right) x_{T_m}^{(m)}.$$  \hfill (1.6)

Note that MIDAS is $h$-dependent, and thus needs to be re-estimated for each forecast horizon.

### 1.2.3.2 The AR-MIDAS model

Since autoregressive models often provide competitive forecasts to those obtained with models that include explanatory variables, the introduction of an autoregressive term in the MIDAS model is a desirable extension, although not straightforward. Ghysels, Santa-Clara and Valkanov (2004) show that the introduction of lagged dependent variables creates efficiency losses. Moreover, it would result in the creation of seasonal patterns in the explanatory variables.

Consider adding a lower-frequency lag of $y_{t_m}$, $y_{t_m-3}$, to the basic model with $m = 3$ ($x$ is monthly and $y$ is quarterly):

$$y_{t_m} = \beta_0 + \lambda y_{t_m-3} + \beta_1 b \left( L_m; \theta \right) x_{t_m+w-3}^{(3)} + \varepsilon_{t_m}.$$  \hfill (1.7)

As highlighted in Clements and Galvao (2009), this strategy is in general not appropriate. The reason becomes clear when we write the model as:

$$y_{t_m} = \beta_0 \left( 1 - \lambda \right)^{-1} + \beta_1 \left( 1 - \lambda L_m^3 \right)^{-1} B \left( L_m; \theta \right) x_{t_m+w-3}^{(3)} + \varepsilon_{t_m}.$$  \hfill (1.8)

The polynomial on $x_{t-1}^{(3)}$ is a product of a polynomial in $L^{1/3}$ and a polynomial in $L$. This product generates a seasonal response of $y$ to $x^{(3)}$, irrespective of whether $x^{(3)}$ displays a seasonal pattern.

To avoid this inconvenience, the authors suggest the introduction of the AR dynamics as a common factor:

$$y_{t_m} = \beta_0 + \lambda y_{t_m-3} + \beta_1 b \left( L_m; \theta \right) \left( 1 - \lambda L_m^3 \right) x_{t_m+w-3}^{(3)} + \varepsilon_{t_m}.$$  \hfill (1.9)

so that the response of $y$ to $x^{(3)}$ remains non-seasonal.
The analogous multi-step model is written as:

\[ y_{tm} = \beta_0 + \lambda y_{tm-h_m} + \beta_1 b(L_{m}; \theta) (1 - \lambda L_{m}^{h_m}) x_{t+w-h_m} + \varepsilon_{tm}. \]  

(1.10)

To estimate the MIDAS-AR model, the common procedure is to estimate the standard MIDAS (the basic model), take the residuals \( \hat{\varepsilon}_{tm} \) and estimate an initial value for \( \lambda \), say \( \hat{\lambda}_0 \), where

\[ \hat{\lambda}_0 = \left( \sum \hat{\varepsilon}_{tm}^2 \right)^{-1} \sum \hat{\varepsilon}_{tm} \hat{\varepsilon}_{tm+w-h_m}. \]

Then construct \( y^{*}_{tm} = y_{tm} - \hat{\lambda}_0 y_{tm-h_m} \) and \( x_{tm+w-h_m} = x_{tm+w-h_m} - \hat{\lambda}_0 x_{tm-(h_m-w)-h_m} \). The estimator \( \hat{\theta}_1 \) is obtained by applying nonlinear least squares to:

\[ y^{*}_{tm} = \beta_0 + \beta_1 b(L_{m}; \theta) x^{*}_{tm+w-h_m} + \varepsilon_{tm}. \]  

(1.11)

A new value of \( \lambda, \hat{\lambda}_1 \), is obtained from the residuals of this regression. Then a new step is run, using \( \hat{\lambda}_1 \) and \( \hat{\theta}_1 \) as the initial values. In this way, the procedure gets the estimates and \( \hat{\lambda} \) and \( \hat{\theta} \) that minimize the sum of squared residuals.

### 1.2.3.3 The Unrestricted MIDAS model

Foroni, Marcellino and Schumacher (2012) study the performance of a variant of MIDAS which does not resort to functional distributed lag polynomials. In the paper, the authors discuss how unrestricted MIDAS (U-MIDAS) regressions can be derived in a general linear dynamic framework, and under which conditions the parameters of the underlying high-frequency model can be identified\(^2\).

The U-MIDAS model based on a linear lag polynomial such as

\[ c(L^m)w(L)y_{tm} = \delta_1(L)x_{1tm-1} + ... + \delta_N(L)x_{Ntm-1} + \varepsilon_{tm}, \]  

(1.12)

where \( c(L^m) = (1-c_1L^m-...-c_cL^{mc}) \), \( \delta_j(L) = \delta_{j,0} + \delta_{j,1}L + ... + \delta_{j,v}L^v \), \( j = 1, ..., N \).

Note that if we assume that the lag orders \( c \) and \( v \) are large enough to make the error term \( \varepsilon_{tm} \) uncorrelated, then, all the parameters in the U-MIDAS model (1.12) can be estimated by simple OLS (while the aggregation scheme \( \omega(L) \) is supposed known). From a practical point of view, the lag order \( v \) could differ across variables, and \( v_i \) and \( c \) could be selected by an information criterion such as BIC.

A simple approach to forecasting is to use a form of direct estimation and construct the forecast as

\[ \tilde{y}_{T_m^{n+m}|T_m} = \tilde{c}(L^k)y_{T_m}^{n} + \tilde{\delta}_1(L)x_{1T_m^{n}} + ... + \tilde{\delta}_N(L)x_{NT_m^{n}}, \]  

(1.13)

---

\(^2\)Koenig, Dolmas, and Piger (2003) already proposed U-MIDAS in the context of real-time estimation. However, they did not systematically study the role of the functional form of the lag polynomial.
where the polynomials $\tilde{c}(Z) = \tilde{c}_1L^m + ... + \tilde{c}_cL^{mc}$ and $\tilde{\delta}_i(L)$ are obtained by projecting $y_t$ on information dated $m_t - m$ or earlier, for $t = 1, 2, ..., T_m$. We will use this approach in the Monte Carlo simulations and empirical applications. In general, the direct approach of (1.13) can also be extended to construct $h_m$-step ahead forecasts given information in $T_m^x$:

$$\tilde{y}_{T_m^x+h_m|T_m^x} = \tilde{c}(L^k)y_{T_m^x} + \tilde{\delta}_1(L)x_{1T_m^x} + ... + \tilde{\delta}_N(L)x_{NT_m^x},$$

(1.14)

where the polynomials $\tilde{c}(Z)$ and $\tilde{\delta}_i(L)$ are obtained by projecting $y_t$ on information dated $m_t - h_m$ or earlier, for $t = 1, 2, ..., T_m^x$.

As a final comment, we can notice that in the case of U-MIDAS, the autoregressive term can be included easily without any common factor restriction as in Clements and Galvao (2009).

1.2.3.4 Extensions of the MIDAS model

Different extensions of the MIDAS models have been analyzed in the literature, to introduce the use of mixed-frequency data in specific applications or studies, in which there is a need to capture particular features. We can see for example the studies which incorporate regime changes in the parameters or those which incorporate asymmetric reactions to negative or positive values of the explanatory variables.

In what follows, we provide a brief overview of the extensions of the MIDAS models discussed so far in the literature.

Multiple explanatory variables

To allow for the inclusion of several additional explanatory variables into the MIDAS framework, it is necessary to extend the basic model above as follows:

$$y_t = \beta_0 + \beta_1 b(L_m; \theta_1) x_{1t}^{(m)} + \beta_2 b(L_m; \theta_2) x_{2t}^{(m)} + \varepsilon_t.$$ (1.15)

In this case, we consider $x_1$ and $x_2$ as two different explanatory variables. The values of the theta parameters are assumed to take on independent values and are thus represented by two independent vectors for the parameters, which may have different lag lengths.

Obviously, the above specification may be extended to allow for the inclusion of more than two explanatory variables (or more than two lags), and for the presence of an autoregressive structure. The most general MIDAS linear regression model can then be written as

$$y_t = \beta_0 + \sum_{i=1}^{K} \sum_{j=1}^{L} b_{ij} (L_{m_i}) x_{t}^{(m_i)} + \varepsilon_t.$$ (1.16)
1.2. MODELS FOR MIXED-FREQUENCY DATA

Within the more general framework, it is also possible to include explanatory variables at different frequencies, since each indicator is modelled with its own polynomial parameterization. As an example, quarterly GDP growth can be explained not only by monthly indicators but also by weekly financial variables, with the explanatory variables, therefore, sampled at two different frequencies.

**Nonlinear MIDAS models**

Ghysels, Sinko and Valkanov (2007) further generalize (1.16) to:

\[ y_{t_m} = \beta_0 + f \left( \sum_{i=1}^{K} \sum_{j=1}^{L} b_{ij} (L_{m_i}; \theta) g \left( x_{t_m+w-h_m}^{(m)} \right) \right) + \varepsilon_{t_m}, \quad (1.17) \]

where the functions \( f \) and \( g \) can be either fully known or parameter dependent. This model is inspired by the EGARCH model, and can be useful especially in volatility applications and risk-return trade-off studies.

**Asymmetric MIDAS models**

Ghysels, Santa-Clara and Valkanov (2005) introduce the asymmetric MIDAS model given by:

\[ y_{t_m} = \beta_0 + \beta_1 \left( \phi b \left( L_m; \theta^- \right) 1_{t_m-h_m}^{(m)} x_{t_m+w-h_m}^{(m)} + (2 - \phi) b \left( L_m; \theta^+ \right) 1_{t_m-h_m}^{+} x_{t_m+w-h_m}^{(m)} \right) + \varepsilon_{t_m} \quad (1.18) \]

where \( 1_{t_m-h_m}^{+} \) denotes the indicator function for \( x_{t_m+w-h_m}^{(m)} \geq 0 \) and \( 1_{t_m-h_m}^{-} \) for \( x_{t_m+w-h_m}^{(m)} < 0 \), and \( \phi \in (0, 2) \) in order to ensure that the total weights sum up to one. This formulation allows for a different impact of negative and positive values of the regressor \( x \). The value of \( \phi \) controls the different weight put on positive and negative impacts. Allowing for an asymmetric impact of the indicator is important in financial applications, especially in examining the asymmetric reaction of volatility in positive and negative return shocks.

**Smooth Transition MIDAS models**

Galvao (2007) proposes a new regression model which combines a smooth transition regression with a mixed data sampling approach:

\[ y_{t_m} = \beta_{0,h_m}^{(m)} + \beta_{1,h_m}^{(m)} x_{t_m+w-h_m}^{(m)} \left[ 1 - G_{t_m+w-h_m} \left( x_{t_m+w-h_m}^{(m)}; \gamma, c \right) \right] + \]

\[ + \beta_{2,h_m}^{(m)} x_{t_m+w-h_m}^{(m)} \left[ G_{t_m+w-h_m} \left( x_{t_m+w-h_m}^{(m)}; \gamma, c \right) \right] + \varepsilon_{t_m} \quad (1.19) \]

where

\[ G_{t_m+w-h_m} \left( x_{t_m+w-h_m}^{(m)}; \gamma, c \right) = \frac{1}{1 + \exp \left( -\gamma \sqrt{\sigma_x} \left( x_{t_m+w-h_m}^{(m)}; \gamma, c \right) \right)} \quad (1.20) \]
The transition function is a logistic function that depends on the weighted sum of the explanatory variable in the current quarter.

The time-varying structure allows for changes in the predictive power of the indicators. This can be particularly relevant when one wants to use asset returns for forecasting macroeconomic variables, since changes in the predictive power of asset returns on economic activity may be related to business cycle regimes.

**Markov-Switching MIDAS models**

Guerin and Marcellino (2011) incorporate regime changes in the parameters of the MIDAS models. The basic version of the Markov-Switching MIDAS (MS-MIDAS) regression model they propose is:

$$y_{tm} = \beta_0 (S_{tm}) + \beta_1 (S_{tm}) B (L_m; \theta) x_{tm+w-h_m}^{(m)} + \varepsilon_{tm} (S_{tm})$$ (1.21)

where $\varepsilon_{tm} | S_{tm} \sim NID (0, \sigma^2 (S_{tm}))$. The regime generating process is an ergodic Markov-chain with a finite number of states $S_{tm}$.

These models allow also mixed-sample estimation of the probabilities of being in a given regime, which are relevant, for example, when one wants to predict business cycle regimes.

**MIDAS with step functions**

Forsberg and Ghysels (2007) introduce a MIDAS regression with step functions, where the distributed lag pattern is approximated by a number of discrete steps. To define this MIDAS regression, we consider the regressors $X (t_m, K_i) = \sum_{j=1}^{K_i} x_{tm-j}^{(m)}$, which are partial sums of the high frequency variables. Then the MIDAS regression with $M$ steps is:

$$y_{tm} = \beta_0 + \sum_{i=1}^{M} \beta_i X (t_m, K_i) + \varepsilon_{tm}.$$ (1.22)

This special case of MIDAS models can be reconnected to the U-MIDAS case we have analyzed in Section 1.2.3.3, in which the steps are the single individual observations.

**Multivariate MIDAS models**

Regression (1.16) can be generalized to multivariate specifications:

$$Y_{tm} = B_0 + \sum_{i=1}^{K} \sum_{j=1}^{L} B_{ij} (L_{m_i}; \theta) X_{tm+w-h_m}^{(m_i)} + \varepsilon_{tm},$$ (1.23)
where $Y$, $\varepsilon$ and $X$ are $n$-dimensional vector processes $B_0$ in an $n$-dimensional vector and $B_{ij}$ are $n \times n$ matrices of polynomials. The main issue is how to handle parameter proliferation in a multivariate context. One approach is to consider all the off-diagonal elements controlled by one polynomial, while the diagonal elements by a second one. Of course, the restrictions may not be valid, and will be chosen depending on the application.

Considering multivariate MIDAS regressions allows to address Granger causality issues, avoiding temporal aggregation error that can disguise or create spurious causality.

### 1.2.4 Mixed-frequency VAR

While so far, we have seen models which take into account mixed-frequency data in a univariate approach, we now focus on multivariate methods which jointly specify the dynamics of the indicators and of the variable to be explained. To exploit the information available in series released at different frequencies and jointly analyze them, there is a growing literature which looks at mixed-frequency VARs, which aim to characterize the co-movements in the series and summarize the information contained in the mixed-frequency data.

Nowadays, in the literature, we are seeing two kinds of approaches to estimate MF-VAR models, the classical and the Bayesian. In what follows, we describe the main features of these two classes of estimation, following two of the most representative studies in the literature, Mariano and Murasawa (2010) for the classical approach and Schorfheide and Song (2011) for the Bayesian approach.

#### Classical framework

One of the most compelling approaches in the literature to deal with mixed-frequency time series at the moment is the one proposed by Zadrozny (1988) for directly estimating a VARMA model sampled at different frequencies, see also Harvey (1989). The approach treats all the series as generated at the highest frequency, but some of them are not observed. Those variables that are observed only at the low frequency are therefore considered as periodically missing.

Following the notation of Mariano and Murasawa (2010), we consider the state-space representation of a VAR model in a classical framework, treating quarterly series as monthly series with missing observations and taking GDP growth as an example. The disaggregation of the quarterly GDP growth, $y_{tq}$, observed every $t_q = 3, 6, 9, ..., T_q$, into the month-on-month GDP growth, $y_{tm}^*$, never observed, is based on the following aggregation equation:
This aggregation equation comes from the assumption that the quarterly GDP series (in log levels), $Y_{t_m}$, is the geometric mean of the latent monthly random sequence $Y_{t_m}^*, Y_{t_m-1}^*, Y_{t_m-2}^*$. Taking the three-period differences and defining $y_{t_m} = \Delta_3 Y_{t_m}$ and $y_{t_m}^* = \Delta Y_{t_m}^*$, we obtain eq. (1.24).

Let for all $t_m$ the latent month-on-month GDP growth $y_{t_m}^*$ and the corresponding monthly indicator $x_{t_m}$ follow a bivariate VAR($p$) process

$$
\phi(L_m)
\begin{pmatrix}
  y_{t_m}^* - \mu_y^* \\
  x_{t_m} - \mu_x
\end{pmatrix} = u_{t_m},
$$

where $u_{t_m} \sim N(0, \Sigma)$. The VAR($p$) process in eq. (1.25) together with the aggregation equation (1.24) is then cast in a state-space representation.

Assuming $p \leq 4^3$ and defining

$$s_{t_m} = \begin{pmatrix} z_{t_m} \\ \vdots \\ z_{t_m-4} \end{pmatrix}, \quad z_{t_m} = \begin{pmatrix} y_{t_m}^* - \mu_y^* \\ x_{t_m} - \mu_x \end{pmatrix},$$

a state-space representation of the MF-VAR is

$$
\begin{align*}
    s_{t_m} &= Fs_{t_m-1} + Gv_{t_m}, \quad (1.26) \\
    \begin{pmatrix} y_{t_m} - \mu_y \\ x_{t_m} - \mu_x \end{pmatrix} &= Hs_{t_m}, \quad (1.27)
\end{align*}
$$

with $\mu_y = 3\mu_y^*$ that holds, and $v_{t_m} \sim N(0, I_2)$.

\footnote{For the sake of conciseness, we do not report the state-space representation for $p > 4$. Details for this case can be found in Mariano and Murasawa (2010).}
In the notation,

\[
F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}; \quad F_1 = \begin{bmatrix} \phi_1 & \ldots & \phi_p & 0_{2 \times 2(5-p)} \end{bmatrix} ; \quad F_2 = \begin{bmatrix} I_8 & 0_{8 \times 2} \end{bmatrix}, \quad (1.28)
\]

\[
G = \begin{bmatrix} \Sigma^{1/2} \\ 0_{8\times2} \end{bmatrix}; \quad H = \begin{bmatrix} H_0 & \ldots & H_4 \end{bmatrix} \quad (1.29)
\]

where \( H \) contains the lag polynomial

\[
H(L_m) = \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2/3 & 0 \\ 0 & 0 \end{bmatrix} L_m + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} L_m^2 + \begin{bmatrix} 2/3 & 0 \\ 0 & 0 \end{bmatrix} L_m^3 + \begin{bmatrix} 1/3 & 0 \\ 0 & 0 \end{bmatrix} L_m^4 \quad (1.30)
\]

The state-space model consisting of equations (1.26) and (1.27) can be estimated with maximum-likelihood techniques or the expectation-maximization algorithm, where we have to take into account missing values due to publication lags and the low-frequency nature of the GDP. We illustrate the estimation and forecasting issues later on, in Section 1.3.1, where we review the problems related to ragged-edge data.

**Bayesian framework**

The estimation of MF-VAR model with Bayesian techniques has been recently been considered as an alternative framework in the literature. One of the earliest studies on this is the paper by Chiu et al. (2011). In this paper, the authors develop a Gibbs sampling approach to estimate a VAR with mixed and irregularly sampled data. The algorithm they develop is a Gibbs sampler which iterates over the draws from the missing data and from the unknown parameters in the model. Under the assumption of a normally distributed error term, the algorithm allows for draws from Gaussian conditional distributions for estimating the missing data, and for draws from Gaussian and inverse Wishart conditional posterior distributions for the parameters in the model.

As an example for the Bayesian estimation of a MF-VAR, we present the algorithm developed by Schorfheide and Song (2011). The authors represent the MF-VAR as a state-space model, and use MCMC methods to conduct Bayesian inference for model parameters and unobserved monthly variables.

The state equation of the model is represented by the VAR(\( p \)) model written in the companion form:

\[
z_{tm} = F_1(\Phi) z_{tm-1} + F_c(\Phi) + v_{tm}, \quad v_{tm} \sim iidN(0, \Omega(\Sigma)) \quad (1.31)
\]

To write the measurement equation, the authors need to write the aggregation equation, which is in this case different from the one considered by Mariano and Murasawa (2010). In
this case, the quarterly variable is seen as the three-month average of the monthly process, which in the previous notation is:

\[ y_{tm} = \frac{1}{3} \left( y_{tm}^* + y_{tm-1}^* + y_{tm-2}^* \right) = \Lambda_{mz} z_{tm}. \]  

(1.32)

However, since \( y_{tm} \) is observed only every third month, there is a need of a selection matrix that equals the identity matrix if \( t_m \) corresponds to the last month of the quarter and is empty otherwise. Therefore, the measurement equation can be written as

\[
\begin{pmatrix} y_{tm} \\ x_{tm} \end{pmatrix} = M_{tm} \Lambda_z z_{tm},
\]

(1.33)

where \( M_{tm} \) is the selection matrix. A Minnesota prior that shrinks the VAR coefficients toward univariate random walk representations is introduced to cope with the issue of dimensionality.

### 1.2.5 Mixed-frequency factor models

Closely related to the MF-VAR for their state-space representation, factor models have also been employed in the literature to handle data with different frequencies. These models have been utilized to extract an unobserved state of the economy and create a new coincident indicator, but also to exploit more information and obtain more precise forecasts. In what follows, we discuss the Mariano and Murasawa (2003) small scale mixed-frequency factor model, developed to extend the Stock–Watson coincident index for the US economy by combining quarterly real GDP and monthly coincident business cycle indicators. Interesting applications of a similar approach can be found in Frale et al. (2010, 2011). Then, we present an example of large scale mixed-frequency factor model, as proposed by Giannone, Reichlin and Small (2008), which aim is to bridge the information in the large monthly dataset with the forecast of the quarterly variable. As an extension to it, we present the mixed-frequency state-space framework as proposed by Banbura and Rünstler (2011). Finally, looking at Marcellino and Schumacher (2010), we analyze the new approach which merges the factor models with the MIDAS framework as presented before.

#### 1.2.5.1 Mixed-frequency small scale factor models

Factor models have a long tradition in econometrics and they are also appealing from an economic point of view. In fact, they decompose each time series under analysis into a
common component, driven by few factors that represent the key economic driving factor, and an idiosyncratic component.

Mariano and Murasawa (2003) set up a static one-factor model for a small set of monthly series, including latent series underlying quarterly series, and derive a state-space model for observable monthly and quarterly series.

Following their notation, consider a one-factor model for $y_t^*$, such that for all $t_m$,

$$
y_t^* = \mu^* + \Lambda f_{t_m} + u_{t_m} \tag{1.34}
$$

$$
\Phi_f (L) f_{t_m} = v_{t_m} \tag{1.35}
$$

$$
\Phi_u (L) u_{t_m} = w_{t_m} \tag{1.36}
$$

where $\Phi_f (.)$ is a $p$th-order polynomial on $\mathbb{R}$ and $\Phi_u (.)$ is a $q$th-order polynomial on $\mathbb{R}^{N \times N}$. In order to have identification, we assume $\Lambda := [I, \Lambda_2']$ and $\Phi_u (.)$ and $\Sigma_{ww}$ diagonal.

**State-space representation**

Assuming $p,q \leq 4$, for all $t_m$, and defining

$$
s_t = \begin{pmatrix}
    f_{t_m} \\
    \vdots \\
    f_{t_m-4} \\
    u_{t_m} \\
    \vdots \\
    u_{t_m-4}
\end{pmatrix},
$$

the state-space representation of the factor model is

$$
s_{t_m+1} = Fs_{t_m} + Gv_{t_m} \tag{1.38}
$$

$$
y_{t_m} = \mu + Hs_{t_m} \tag{1.39}
$$
with \( v_{tm} \sim N(0, I_3) \), where

\[
F = \begin{bmatrix} F_1 & F_2 \\ F_3 & F_4 \end{bmatrix}; \quad F_1 = \begin{bmatrix} \Phi_{f,1} \cdots \Phi_{f,p} & 0_{1 \times (5-p)} \\ I_4 & 0_{4 \times 1} \end{bmatrix}; \quad F_2 = 0_{5 \times 10}; \quad (1.40)
\]

\[
F_3 = 0_{10 \times 5}; \quad F_4 = \begin{bmatrix} \Phi_{u,1} \cdots \Phi_{u,q} & 0_{1 \times (5-q)} \\ I_8 & 0_{8 \times 2} \end{bmatrix}
\]

\[
G = \begin{bmatrix} \Sigma_{u/w}^{1/2} & 0_{1 \times 2} \\ 0_{4 \times 1} & \Sigma_{u/w}^{1/2} \\ 0_{2 \times 1} & 0_{2 \times 2} \\ 0_{8 \times 1} & 0_{8 \times 2} \end{bmatrix}; \quad H = \begin{bmatrix} H_0 \Lambda & \cdots & H_{4} \Lambda & H_0 & \cdots & H_4 \end{bmatrix} \quad (1.41)
\]

where \( H(L_m) \) is defined as in equation (1.30).

In the estimation, Mariano and Murasawa (2003) cannot use the standard EM algorithm, since the measurement equation has unknown parameters. The procedure they followed is similar to the one described in Section 1.3.1.

The dynamic factor model as extended by Mariano and Murasawa (2003) is also used in Frale et al. (2011) to handle mixed frequency data, in order to obtain estimates of the monthly Euro area GDP components from the output and expenditure sides, to be later aggregated into a single indicator, called EUROMIND. Broadly speaking, GDP is disaggregated by supply sectors and demand components. For each of these sectors and components, timely and economically sensible observable monthly indicators are then selected and represented with a dynamic factor model, as described above. The single models are then linked together based on the composition of GDP.

### 1.2.5.2 Bridging with factors

We now discuss a large mixed frequency factor model as proposed by Giannone, Reichlin and Small (2008), which exploits a large number of series that are released at different times and with different lags. The methodology the authors propose relies on the two-step estimator by Doz et al. (2011). This framework combines principal components with the Kalman filter. First, the parameters of the model are estimated by OLS regression on the estimated factors, where the latter are obtained through principal components calculated on a balanced version of the dataset. Then, the Kalman smoother is used to update the estimate of the signal variable on the basis of the entire unbalanced panel.
The model

The dynamic factor model of Doz et al. (2011) is given by

\[ x_{tm} = \Lambda f_{tm} + \xi_{tm}, \quad \xi_{tm} \sim N(0, \Sigma \xi) \]  
\[ f_{tm} = \sum_{i=1}^{p} A_i f_{tm-i} + B \eta_{tm}, \quad \eta_{tm} \sim N(0, I_q) \]  

Equation (1.42) relates the \( n \) monthly series \( x_{tm} \) to a \( r \times 1 \) vector of latent factors \( f_{tm} \), through a matrix of factor loadings \( \Lambda \), plus an idiosyncratic component \( \xi_{tm} \), assumed to be a multivariate white noise with diagonal covariance matrix \( \Sigma \). Equation (1.43) describes the law of motion of the latent factors, which are driven by a \( q \)-dimensional standardized white noise \( \eta_{tm} \), where \( B \) is a \( r \times q \) matrix \(( r \leq q \)). Hence, \( \xi_{tm} \sim N(0, BB') \).

To deal with missing observations at the end of the sample, the authors adapt the large FM typically used in the literature, using a two-step estimator. In the first step, the parameters of the model are estimated consistently through principal components on a balanced panel, created by truncating the data set at the date of the least timely release. In the second step, the Kalman smoother is applied to update the estimates of the factor and the forecast on the basis of the entire unbalanced data set (see Section 1.3.2.3 for more details on the estimation method).

The model is then complemented by a forecast equation for mean-adjusted quarterly GDP. The forecast is defined as the projection of the quarterly GDP growth on the quarterly aggregated estimated common factors:

\[ \hat{y}_{tq} = \alpha + \beta \hat{f}_{tq}, \]  

where \( \hat{f}_{tq} \) is the quarterly aggregated correspondent of \( \hat{f}_{tm} \).

If we look at eq. (1.44), we see that this is exactly what we analyzed in Section (1.2.2) for the bridge equations. The framework has to be understood as a large bridge model which uses a large number of variables and bridges monthly data releases with the forecast of the quarterly variable.

1.2.5.3 Factor models in a mixed-frequency state-space representation

Banbura and Rünstler (2011) extend the model of Giannone et al. (2008), integrating into the model a forecast equation for quarterly GDP. More specifically, they introduce the forecast of monthly GDP growth \( y_{tm} \) as a latent variable, related to the common factors by the static equation

\[ y_{tm} = \beta' f_{tm} + \varepsilon_{tm}, \quad \varepsilon_{tm} \sim N(0, \sigma_{\varepsilon}^2). \]  

Foroni, Claudia (2012), Econometric Models for Mixed-Frequency Data
European University Institute
DOI: 10.2870/45897
The quarterly GDP growth, $y_{tm}$, is assumed to be the quarterly average of the monthly series:

$$y_{tm} = \frac{1}{3} (y_{tm}^* + y_{tm-1}^* + y_{tm-2}^*) .$$  \hspace{1cm} (1.46)

The innovations $\varepsilon_{tm}, \eta_{tm}, \xi_{tm}$ are assumed to be mutually independent at all leads and lags.

Equations (1.42) to (1.46) can be cast in state-space form. $y_{tm}$ is constructed in such a way that it contains the quarterly GDP growth in the third month of each quarter, while the other observations are treated as missing.

**State-space representation**

The state-space representation, when $p = 1$, is:

$$
\begin{bmatrix}
  x_{tm} \\
  y_{tm}
\end{bmatrix} =
\begin{bmatrix}
  \Lambda & 0 & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  f_{tm} \\
  y_{tm}^C \\
  y_{tm}^C \Xi_{tm}
\end{bmatrix} +
\begin{bmatrix}
  \varepsilon_{tm} \\
  \varepsilon_{tm}
\end{bmatrix} 
= I_r
\begin{bmatrix}
  f_{tm+1} \\
  y_{tm+1}^C \\
  y_{tm+1}^C \Xi_{tm+1}
\end{bmatrix} +
\begin{bmatrix}
  A_1 & 0 & 0 \\
  0 & 0 & 0 \\
  0 & 0 & \Xi_{tm+1}
\end{bmatrix}
\begin{bmatrix}
  f_{tm} \\
  y_{tm}^C \\
  y_{tm}^C \Xi_{tm+1}
\end{bmatrix} +
\begin{bmatrix}
  B\eta_{tm+1} \\
  0 \\
  0
\end{bmatrix} \hspace{1cm} (1.47)
$$

The aggregation rule (1.46) is implemented in a recursive way, by introducing a latent cumulator variable $y_{tm}^C = \Xi_{tm}y_{tm}^C + \frac{1}{3} y_{tm}^*$, where $\Xi_{tm} = 0$ for $t_m$ corresponding to the first month of the quarter and $\Xi_{tm} = 1$ otherwise. The estimation of the model parameters follows Giannone, Reichlin and Small (2008).

### 1.2.5.4 Factor-MIDAS

It is possible to augment the MIDAS regressions with the factors extracted from a large dataset to obtain a richer family of models that exploit a large high-frequency dataset to predict a low-frequency variable.

While the basic MIDAS framework consists of a regression of a low-frequency variable on a set of high-frequency indicators, the Factor-MIDAS approach exploits estimated factors rather than single or small groups of economic indicators as regressors.

Marcellino and Schumacher (2010) propose alternative MIDAS regressions. In the standard MIDAS case, they follow Clements and Galvao (2008), while as a modification they evaluate a more general regression approach, labeled unrestricted Factor-MIDAS, where the dynamic relationship between the low-frequency variables and the high-frequency indicators is unrestricted, in contrast to the distributed lag functions as proposed by Ghysels et al. (2007). As a third alternative, they consider a regression scheme proposed by Altissimo et
al. (2010), which considers only correlation at certain frequencies between variables sampled at high- and low- frequencies. This approach is called smoothed MIDAS, since the regression essentially eliminates high-frequency correlations.

The information set consists of a large set of stationary monthly indicators, $X_{tm}$. The last observation is at time $T_m + w$, $w > 0$, that is we allow for at most $w > 0$ monthly values of the indicators that are earlier available than the lower-frequency variable to be estimated. $X_{tm}$ is modeled using a factor representation, where $r$ factors $F_{tm}$ are estimated in order to summarize the information in $X_{tm}$. After having estimated the factors, $\hat{F}_{tm}$, they will be used in the projection for the quarterly-frequency variable.

We now describe in details the three alternative Factor-MIDAS approaches proposed by Marcellino and Schumacher (2010), assuming again that the target variable is GDP. These approaches are tools for direct multi-step now- and forecasting, thus each model is for a specific forecast horizon.

The Basic Factor-MIDAS approach

In the basic Factor-MIDAS approach the explanatory variables used as regressors are estimated factors. Assume for simplicity $r = 1$, so that there is only one factor $\hat{f}_{tm}$. The Factor-MIDAS model for forecast horizon $h_q$ quarters with $h_q = h_m/3$ is

$$ y_{t+h_q} = y_{tm+h_m} = \beta_0 + \beta_1 b(L_m; \theta) \hat{f}_{tm}^{(3)} + \epsilon_{tm+h_m}, \quad (1.49) $$

where $\beta(L_m; \theta) = \sum_{k=0}^{K} c(k; \theta) L_m^k$ and $c(k; \theta) = \frac{\exp(\theta_1 k + \theta_2 k^2)}{\sum_{k=0}^{K} \exp(\theta_1 k + \theta_2 k^2)}$.

$\hat{f}_{tm}^{(3)}$ is skip-sampled from the monthly factor $\hat{f}_{tm}$. Every third observation starting from the final, one is included in the regressor $\hat{f}_{tm}^{(3)}$, i.e. $\hat{f}_{tm}^{(3)} = \hat{f}_{tm+w}, \forall t_m + w = ..., T_m + w - 6, T_m + w - 3, T_m + w$. Note that we take into account the fact that a monthly indicator is typically available also within the quarter for which no GDP figure is available. As described above in the MIDAS models, the exponential lag function provides a parsimonious way to consider monthly lags of the factors.

The model can be estimated using nonlinear least squares in a regression of $y_{tm}$ onto the factors $\hat{f}_{tm+w-h}$. The forecast is given by

$$ y_{T_m+h_m|T_m+w} = \hat{\beta}_0 + \hat{\beta}_1 b(L_m; \hat{\theta}) \hat{f}_{T_m+w}^{(3)}. \quad (1.50) $$

The projection is based on the final values of estimated factors.

MIDAS regression can be generalized to more than one factor and extended with the addition of autoregressive dynamics. Details on factor estimation are provided in Section 1.3.2.
Smoothed MIDAS

A different way to formulate a mixed-frequency projection is proposed by Altissimo et al. (2010). The projection can be written as:

\[ y_{T_m+h_m|T_m+w} = \hat{\mu} + G\hat{F}_{T_m+w} \]
\[ G = \hat{\Sigma}_{yF}(h_m - w) \times \hat{\Sigma}_F^{-1}, \]

where \( \hat{\mu} \) is the sample mean of GDP, \( G \) is the projection coefficient matrix, \( \hat{\Sigma}_F \) is the estimated sample covariance of the factors, and \( \hat{\Sigma}_{yF}(k) \) is a particular cross-covariance with \( k \) monthly lags between GDP and the factors. \( \hat{\Sigma}_{yF}(k) \) is not an estimate of the sample cross-covariance between factors and GDP, but a cross-covariance between smoothed GDP and factors. The smoothing aspect is introduced in \( \hat{\Sigma}_{yF}(k) \) as we describe now. The covariance between \( \hat{F}_{t_m-k} \) and \( y_{t_m} \) (both demeaned) is estimated by

\[ \hat{\Sigma}_{yF}(k) = \frac{1}{T^* - 1} \sum_{t_m=M+1}^{T_m} y_{t_m} \hat{F}_{t_m-k}^{(3)} \]

where \( T^* \) is the number of observations available to compute the cross-covariances. Note the use of skip-sampled factors, since GDP is available only quarterly. Given \( \hat{\Sigma}_{yF}(k) \), we estimate the cross-spectral matrix

\[ \hat{S}_{yF}(\omega_j) = \sum_{k=-M}^{M} \left( 1 - \frac{|k|}{M + 1} \right) \hat{\Sigma}_{yF}(k) e^{-i\omega_j k} \]

at frequencies \( \omega_j = \frac{2\pi i}{2H} \) for \( i = -H, ..., H \) using a Bartlett lead-lag window. The low-frequency relationship between \( \hat{F}_{t_m-k} \) and \( y_{t_m} \) is obtained by filtering out cross fluctuations at frequency higher than a certain threshold \( \pi/q \), using the frequency-response function \( \alpha(\omega_j) \), defined as \( \alpha(\omega_j) = 1, \forall |\omega_j| < \pi/q \) and zero otherwise. The autocovariance matrix \( \hat{\Sigma}_{yF}(k) \) reflecting low-frequency co-movements between \( \hat{F}_{t_m-k} \) and \( y_{t_m} \) is obtained by inverse Fourier transform:

\[ \hat{\Sigma}_{yF}(k) = \frac{1}{2H + 1} \sum_{j=\pm H}^{H} \alpha(\omega_j) \hat{S}_{yF}(\omega_j) e^{iwj k}. \]

Note that \( \hat{\Sigma}_{yF}(k) \) is a consistent estimator of the true cross-covariance, if the sample size is sufficiently large.
1.3. RAGGED-EDGE DATA

Unrestricted MIDAS

As an alternative to the two previous models, we use the unrestricted lag order model, as described in the above Section 1.2.3.3:

\[ y_{T_m+h_m} = \beta_0 + D(L_m) \hat{F}_{t_m+w}^{(3)} + \epsilon_{t_m+h_m}, \]  

(1.56)

where \( D(L_m) = \sum_{k=0}^{K} D_k L^k_m \) is an unrestricted lag polynomial of order \( K \).

\( D(L_m) \) and \( \beta_0 \) are estimated by OLS. To specify the lag order in the empirical application, Marcellino and Schumacher consider a fixed scheme with \( k = 0 \) and an automatic lag length selection using Bayesian information criterion.

1.3 Ragged-edge data

After having analyzed the various techniques to deal with mixed-frequency data, in this section we review the estimation methods that can handle ragged-edge data, that is datasets which are not balanced, due to the presence of missing values at the end of the sample for some indicators.

First, we discuss the issues of estimation and forecasting MF-VAR in the presence of missing observations due to publication lags and to the low-frequency nature of one variable. We follow Mariano and Murasawa (2010) in the discussion of estimation of the state-space form and forecasting with the use of Kalman filter and/or smoother.

Going further, we analyze issues related to factor estimation in the presence of ragged-edge data. Marcellino and Schumacher (2010) review three different methods to tackle it. First, the method proposed by Altissimo et al. (2010), who realign each time series in the sample in order to obtain a balanced dataset, and then estimate the factors with dynamic PCA. As an alternative, to consider missing values in the data for estimating factors Stock and Watson (2002) propose an EM algorithm together with the standard principal component analysis (DPCA). As a third method, Doz et al. (2011) propose a factor estimation approach based on a complete parametric representation of the large factor model in state-space form.

1.3.1 Estimating the MF-VAR with missing observations

As already anticipated in Section 1.2, Kalman filtering techniques can handle ragged-edge data and missing values due to publication lags and the low-frequency nature of a time series.
Estimation

The state-space representation of the mixed-frequency VAR model is described by equations (1.26) and (1.27), reported also here:

\[
\begin{align*}
    s_{t_m} &= F s_{t_m-1} + G v_{t_m} \\
    \begin{pmatrix}
        y_{t_m} - \mu_y \\
        x_{t_m} - \mu_x
    \end{pmatrix}
    &= H s_{t_m}.
\end{align*}
\]

It can be estimated by maximum-likelihood even in the presence of missing observations due to publication lags and the low-frequency nature of GDP. However, as Mariano and Murasawa (2010) mention in their paper, when the number of parameters is large, the ML method can fail to converge.

In these cases, it is useful to implement the EM algorithm modified to allow for missing observations. Mariano and Murasawa (2010) consider the missing values as realizations of some iid standard normal random variables, i.e.

\[
    y_{t_m}^+ = \begin{cases} 
        y_{t_m} & \text{if } y_{t_m} \text{ is observable} \\
        \zeta_{t_m} & \text{otherwise}
    \end{cases}
\]

where \( \zeta_{t_m} \) is a draw from a standard normal distribution independent of the model parameters.

The measurement equation is modified accordingly in the first two months of each quarter, where the upper row of \( H \) is set to zero and a standard normal error term is added, so that the Kalman filter skips the random numbers. Since the realizations of the random numbers do not matter in practice, the authors suggest to replace the missing values with zeros. Then, the EM algorithm is employed to obtain estimates of the parameters.

Estimation of latent monthly real GDP

Mariano and Murasawa (2010) use the Kalman smoother instead of the Kalman filter, because it uses more information and also simplifies the formulation of the state-space model. Although GDP growth for a particular month is not available, the smoother considers the monthly indicators available for the same quarter, so that nowcasting is also possible. For the months in which no observations are available also for the monthly indicators, the Kalman smoother acts exactly as the Kalman filter.

1.3.2 Estimating the factors with ragged-edge data

Factor forecasting with large, single-frequency data is often carried out using a two-step procedure. First, the factors are estimated and, second, a dynamic model for the variable to
be predicted is augmented with the estimated factors. The same procedure can be used also in case of mixed-frequency data. As an alternative, factor estimation and forecasting can be conducted in a single step, in the contest of a parametric model.

The literature provides various ways to estimate the factors, in case of balanced datasets. However, in the following subsections we describe the methods that can handle ragged-edge data, that is datasets which are not balanced due to the presence of missing values at the end of the sample for some indicators.

### 1.3.2.1 Vertical realignment of data and DPCA

Altissimo et al. (2010) provide a convenient way to solve the ragged-edge problem. They propose to realign each time series in the sample in order to obtain a balanced dataset. Assume that variable $i$ is realized with $k_i$ months of publication lags. Thus, given a dataset in period $T_m + w$, the final observation of time series $i$ is for period $T_m + w - k_i$. Altissimo et al. (2010) propose to realign the series in this way:

$$ \tilde{x}_{i,T_m+w} = x_{i,T_m+w-k_i}, $$

for $t_m = k_1 + 1, ..., T_m + w$. Once applied to each time series, the result is a balanced dataset $\tilde{X}_{t_m}$, for $t_m = \max \{k_i\}_{i=1}^N + 1, ..., T_m + w$.

Given this balanced dataset, Altissimo et al. (2010) propose dynamic PCA to estimate the factors. The two-step estimation procedure introduced by Forni et al. (2005) directly applies, since the dataset is balanced.

One of the main advantages of this method is the simplicity. A drawback is that the availability of data determines cross-correlations between variables. Moreover, data releases are not the same over time, so that dynamic correlations within the data change and, as a consequence, factors change over time. The same happens if factors are reestimated at a higher frequency than the one of the factor model, for example in the case of a monthly factor model reestimated several times within a month, in correspondence of new releases of the data.

### 1.3.2.2 EM algorithm and PCA

As an alternative, to consider missing values in the data for estimating factors, Stock and Watson (2002) propose an EM algorithm combined with the standard PCA. Call $X_i$ the column $i$ of the dataset $X_{t_m}$: not all observations are available, due to publication lags. The vector $X_i^{obs}$ contains the observations available for variable $i$, a subset of $X_i$. More precisely,
the relation between observed and not fully observed variables is

\[ X_{i}^{\text{obs}} = A_{i}X_{i}, \quad (1.59) \]

where \( A_{i} \) is the matrix that tackles missing values. \( A_{i} \) is equal to the identity matrix, in case there are no missing values in the series. When an observation is missing at the end of the sample, the corresponding final row of the identity matrix is removed.

The EM algorithm consists in the following steps:

1. Provide an initial guess \( \hat{X}_{i}^{(0)}, \forall i \). These guesses together with the fully available series provide a balanced dataset \( \hat{X}^{(0)} \). With a balanced dataset, standard PCA gives initial monthly factors \( \hat{F}^{(0)} \) and loadings \( \hat{\Lambda}^{(0)} \).

2. E-step: an updated estimate of the missing observations for variable \( i \) is provided by the expectation of \( X_{i} \) conditional on \( X_{i}^{\text{obs}} \), factors \( \hat{F}^{(j-1)} \) and loadings \( \hat{\Lambda}^{(j-1)} \) from the previous iteration

\[
\hat{X}_{i}^{(j)} = \hat{F}^{(j-1)}\hat{\Lambda}^{(j-1)} + A_{i}'(A_{i}'A_{i})^{-1}\left(X_{i}^{\text{obs}} - A_{i}\hat{F}^{(j-1)}\hat{\Lambda}^{(j-1)}\right). \quad (1.60)
\]

We can recognize two components in the update: the common component from the previous iteration \( \hat{F}^{(j-1)}\hat{\Lambda}^{(j-1)} \), and a low-frequency idiosyncratic component \( X_{i}^{\text{obs}} - A_{i}\hat{F}^{(j-1)}\hat{\Lambda}^{(j-1)} \), distributed by the projection coefficient \( A_{i}'(A_{i}'A_{i})^{-1} \) on the high-frequency periods, see Breitung and Schumacher (2008).

3. M-step: repeat the E-step for each variable \( i \), in order to obtain a balanced dataset. Reestimate all the factors \( \hat{F}^{(j)} \) and loadings \( \hat{\Lambda}^{(j)} \) by PCA. Go back to step 2 until convergence.

After convergence, the EM algorithm provides both the monthly factor estimates \( \hat{F}_{m} \) and the estimates of the missing values of the time series, see also Angelini et al. (2006).

1.3.2.3 Large parametric factor model in state-space form

Doz et al. (2011) propose a factor estimation approach based on a complete representation of the large-factor model in state-space form. The full state-space model has the form

\[
\begin{align*}
X_{tm} &= \Lambda F_{tm} + \xi_{tm}, \\
\Psi (L_{m}) F_{tm} &= B\eta_{tm}.
\end{align*} \quad (1.61, 1.62)
\]
Equation (1.61) is the static factor representation of $X_{tm}$. Equation (1.62) specifies a VAR structure of the factors, with lag polynomial $\Psi (L_m) = \sum_{i=1}^{p} \Psi_i L_m^i$. $\eta_{tm}$ is a $q$-dimensional vector that contains the orthogonal dynamic shocks that drive the $r$ factors. The factors $F_{tm}$ represent the states, while $\xi_{tm}$ is the stationary idiosyncratic component which admits a Wold representation. The shocks driving the factors and the idiosyncratic components are assumed to be independent. If the $X_{tm}$ is of a small dimension, the model can be estimated by iterative maximum likelihood. If the dimension is large, iterative ML is infeasible, so Doz et al. (2011) propose a quasi-ML to estimate the factors. For a given number of factors, $r$, and dynamic shocks, $q$, the estimation follows the steps illustrated below:

1. Estimate $\hat{F}_{tm}$ using PCA as an initial estimate. The estimation is based on the balanced part of the data, obtained by removing the values at the end of the sample that create the unbalancedness.

2. Estimate the loadings $\hat{\Lambda}$ by regressing $X_{tm}$ on the factors estimated in the previous step, $\hat{F}_{tm}$. Estimate also the covariance of the idiosyncratic components $\hat{\xi_{tm}}$, denoted as $\hat{\Sigma}_\xi$.

3. Estimate a VAR($p$) on the factors $\hat{F}_{tm}$, obtaining $\hat{\Psi} (L_m)$, and the residual covariance of $\hat{\xi_{tm}} = \hat{\Psi} (L_m) \hat{F}_{tm}$, denoted as $\hat{\Sigma}_\xi$.

4. To obtain an estimate of $B$, given the number of dynamic shocks $q$, apply an eigenvalue decomposition of $\hat{\Sigma}_\xi$. Call $M$ the $(r \times q)$-dimensional matrix of the eigenvectors corresponding to the $q$ largest eigenvalues, and call $P$ the $(q \times q)$-dimensional matrix with the largest eigenvalues on the main diagonal and zero otherwise. The estimate of $B$ is $\hat{B} = M \times P^{-1/2}$. All the parameters and coefficients in the system of equations (1.61) and (1.62) are then fully specified. The model is cast into state-space form.

5. The Kalman filter or smoother yields new estimates of the monthly factors. The dataset used now is the unbalanced one, where $T_m$ is the last observation available in the whole set on monthly series. The Kalman filter also provides estimates and forecasts for the missing values conditional on the model structure and properties of the shocks.

Note that the coefficients in the system have to be estimated from a balanced sub-sample of data, as in step 1 there is the need of a fully balanced dataset for PCA initialization. Nevertheless, in step 5 the factor estimation based on the Kalman filter applies to the unbalanced dataset. The solution is to estimate the coefficients outside the state-space model and avoid to estimate a large number of coefficients by iterative ML.
1.4 A comparison of the different methods

So far, we have seen that several methods have been proposed in the literature to deal with mixed-frequency data, possibly with a ragged edge structure. In general, there is an agreement on the fact that exploiting data at different frequencies matters. We now try to summarize the advantages and disadvantages of the different methods, comparing their most important features.

Bridge equations are still one of the most used techniques, especially in short run forecasting, because they are pretty easy to estimate and allow computing early estimates of the low-frequency variable. The drawback is that they are purely statistical models, where the regressors are included only because they contain timely updated information. Therefore, if the model that exploits the high-frequency information is mis-specified, the error transmits to the bridge equation and to the forecasts that are obtained recursively.

A more sophisticated way to deal with data sampled at different frequencies is the state-space approach. Casting the model in state-space form has the advantage of jointly specifying the dynamics of the indicators and of the variable to be explained without imposing any a-priori restriction. Moreover, since the low-frequency series is seen as a high-frequency series with missing values, the use of the Kalman filter permits the estimation of these missing data. As shown in Bai, Ghysels and Wright (2011), the Kalman filter results to be the optimal filter in population, when ignoring parameter estimation errors and assuming that the model is correctly specified. Therefore, under these ideal circumstances, the state-space approach cannot be beaten by any other method. On the other side, there are also some drawbacks from the use of this approach: first of all, in most of the cases it is computationally complex, and the complexity increases with the number of variables involved, so that most of the time only small-scale models can be estimated. Moreover, the state space approach requires the correct specification of the model in high frequency, which is even more complex than usual given the missing observations in the dependent variable.

An alternative way to deal with mixed-frequency data is the MIDAS approach. Even though in population, when the process is correctly specified, MIDAS is coarse than the optimal Kalman filter, it can be more robust in the presence of mis-specification. Moreover, the lag polynomials are based on a very small number of parameters, allowing the MIDAS models to be parsimonious, even though it is still not clear which is the best polynomial specification. Contrary to what stated for the state-space models, MIDAS models can be easily estimated by NLS. However, it is only possible to obtain a high frequency update of the expected low frequency realization, not an estimate of the missing values in the low
frequency variable.

Both the state-space and the MIDAS approaches can be combined with a factor specification, in order to use the information in a large dataset, possibly with a ragged edge. Whether factor methods provide more precise estimates and forecasts than VARs or single equation methods is a matter for empirical investigation, since there is a trade-off between model complexity and extended information set.

1.5 An overview of empirical studies

In this section we review the empirical literature on forecasting with mixed-frequency and ragged-edge data, providing some examples of all the models and estimation methods outlined in the previous sections.

1.5.1 Bridge equations

Bridge equations have been one of the first methods employed in nowcasting the current state of the economy, making use of the monthly information available. Studies of this kind have been conducted for the nowcasts of different economies. A common finding of these studies is that the exploitation of intra-period information reduces the forecasting error in the majority of the cases. The applications concern both "supply-side" and "demand-side" models.

Looking at US data, Trehahn and Ingenito (1996) construct a model that predicts current quarter real GDP based on knowledge of nonfarm payrolls, industrial production and real retail sales, which have the advantage of being released at a monthly frequency by the middle of the following month. In order to produce a model that predicts real GDP, the authors rely on auxiliary models that generate forecasts of the indicator variables themselves. Their evidence shows that consumption data provide key information about current output, and that retail sales release allows to have a good forecast of contemporaneous consumption.

Stark (2000) presents evidence on the usefulness of conditioning quarterly model forecasts on monthly current-quarter data, in the case of the US economy. Starting by generating a one-step-ahead forecast from a quarterly Bayesian vector error correction model, the author then specifies a monthly statistical model for variables that are thought to carry information about each of the variables in the quarterly model and uses it to generate sequences of current-quarter quarterly-average forecasts from the monthly indicators. Once he has these quarterly-average monthly indicator forecasts, he forms updated estimates of
the quarterly model’s current-quarter forecast. The findings show that exploiting monthly information produces economically and statistically significant improvements, particularly large especially during periods of recession.

A study by Barhoumi et al. (2011) presents a model to predict French gross domestic product (GDP) quarterly growth rate. The authors employ the bridge equations to forecast each component of the GDP, and select the monthly explanatory variables among a large set of hard and soft data. They find that changing the set of equations over the quarter is superior to keeping the same equations over time. These models turn out to beat the benchmark in terms of forecasting performance.

Studies involving bridge equations can be found for many other countries. In particular, bridge models have been employed also for nowcasting Euro Area GDP growth. As an example, we see Baﬁgì et al. (2004). In this paper, bridge models are estimated for aggregate GDP and components, both area-wide and for the main countries of the Euro Area. Their short-term performance is then assessed with respect to benchmark univariate and multivariate standard models, and a small structural model. The results shown in the paper are clear-cut: bridge models performance is always better than benchmark models, provided that at least some indicators are available over the forecasting horizon. As far as the type of aggregation is concerned, the supply-side approach (modelling aggregate GDP) performs better than the demand-side approach (aggregation of forecasts by national account components). The supply-side models highlight the role of industrial production and manufacturing surveys as the best monthly indicators. Looking at the demand-side models, from the different equations estimated in this paper, private consumption results well tracked by retail sales index, while the consumer confidence index plays a minor role; in the case of investment a major role seems to be played by survey variables.

Diron (2008) makes use of bridge equations with Euro Area data to provide an assessment of forecast errors, which takes into account data-revisions. Using four years of data vintages, the paper provides estimates of forecast errors for Euro Area real GDP growth in genuine real-time conditions and assesses the impact of data revisions on short-term forecasts of real GDP growth. Given the size of revision to indicators of activity, the assessment of reliability of short-term forecasts based on revised series could potentially give a misleading picture. Nevertheless, averaging across all bridge equations, forecasts of individual quarters tend to be similar whether they are based on preliminary or revised data. More specifically, the RMSEs based on real-time and pseudo real-time exercises are quite similar and both smaller compared with AR forecasts of GDP, considered as the benchmark. The difference in forecast accuracy is significant according to Diebold and Mariano tests, highlighting that
short-term forecasts based on bridge equations are informative. Moreover, the paper investigates the contributions of the various sources to the overall forecasting errors. Revisions to the monthly variables and to GDP growth account only for a small share of the overall forecast error, while the main sources are from extrapolation of the monthly indicators. The relative contributions of extrapolation and of revision of monthly indicators vary depending on whether the equations include hard data, in which case both sources are significant, or survey and financial variables, in which case these two sources tend to have a smaller weight.

In the context of nowcasting, it has become more common to exploit now the information coming from a large set of variables. Therefore, recent studies combine the bridge models with factors, in what in is called "bridging with factors". This new kind of model is related to the one described in Section 1.2.5.2 by Giannone et al. (2008). In Section 1.5.4.2, we will review these studies and compare the performance of this new kind of bridge with factors to the standard bridge models and other benchmarks.

1.5.2 MIDAS models

In the first applications, MIDAS models have been applied to financial data, investigating the relation between the conditional mean and the conditional variance of the stock market returns or future volatility, see Ghysels et al. (2005) as an example. Clements and Galvao (2008) are the first to apply MIDAS regressions to macroeconomic data. In the next paragraphs we will overview applications both to financial and macroeconomic data. Recently, this framework has been exploited to include financial series in the forecasting of macroeconomic variables.

Ghysels, Santa-Clara and Valkanov (2005) study the intertemporal relation between the conditional mean and the conditional variance of the aggregate market return. In support of Merton’s ICAPM, the authors find a positive significant and robust relation between risk and return. They also find that the MIDAS estimator is a better forecaster of the stock market variance than two other benchmark models: rolling window and GARCH estimators. The authors also focus on the asymmetric reaction of volatility to positive and negative shocks. They find that positive shocks have a bigger impact overall on the conditional mean of returns, are slower to be incorporated in the conditional variance, and are much more persistent, while negative shocks have a large initial, but temporary, effect on the variance of returns. Ghysels, Santa-Clara and Valkanov (2006) consider various MIDAS regressions to predict volatility in a parsimonious way with data at different frequency. They find that daily realized power is the best predictor of future increments in quadratic variation. Surprisingly, the direct use of high-frequency (5 minutes) data does not improve volatility prediction.
Ghysels, Rubia and Valkanov (2009) compare three different approaches of producing multi-period-ahead forecasts of volatility: iterated, direct and MIDAS. The comparison is conducted out-of-sample using returns data of the US stock market portfolio and a cross section of size, book-to-market and industry portfolios, in terms of the average forecasting accuracy, using the MSFE. The direct approach provides the worst forecasts. Iterated forecasts are suitable for shorter horizons, while MIDAS forecasts perform well at long horizons.

Clements and Galvao (2008) introduce the use of MIDAS regressions in forecasting macroeconomic data. They also look at whether a mixed-data sampling approach including an autoregressive term can improve forecasts of US real output growth. They conduct a real-time forecasting exercise that exploits monthly vintages of the indicators and the quarterly vintages of the output growth, consistent with the timing of the releases of the different data vintages. The authors find that the use of within-quarter information on monthly indicators can result in a marked reduction in RMSE compared with the more traditional quarterly-frequency AR or AR distributed-lag models. Moreover, Clements and Galvao (2009) evaluate the predictive power of leading indicators for output growth up to one year, using MIDAS approach to combine multiple leading indicators in a parsimonious way. The results confirm that MIDAS is a useful instrument to improve forecasts. Moreover, they show that the use of real-time vintage data improves forecast performance and that the predictive power of the indicators is stronger when the aim is to forecast final data rather than first-released data, although first releases can generally be forecasted more accurately.

Foroni, Marcellino and Schumacher (2012) compare the performance of the MIDAS with functional distributed lags estimated with NLS to the one of the U-MIDAS, the unrestricted version analyzed in Section 1.2.3.3. In Monte Carlo experiments, they show that U-MIDAS generally performs better than MIDAS when mixing quarterly and monthly data. On the other hand, with larger differences in sampling frequencies, distributed lag-functions outperform unrestricted polynomials. In an empirical application on out-of-sample nowcasting GDP in the Euro area and the US using monthly predictors, they find a good performance of U-MIDAS for a number of indicators, albeit the results depend on the evaluation sample.

In the recently increasing literature, which is exploiting the availability of a huge number of financial time series on a daily basis to forecast macroeconomic time series, the empirical evidence in support of the use of high-frequency financial series is rather mixed. On the one side, it is useful to use this great amount of timely information, but on the other side there is a question on how to weight the daily observations and to filter these data, to get rid of the possible noise. Results from recent studies suggest that daily variables seem to have useful information for forecasting inflation and economic activity.
Among these studies, Ghysels and Wright (2009) propose methods for using asset price data to construct daily forecasts of upcoming survey releases, employing MIDAS regression models and a more structural approach based on the Kalman filter to estimate what forecasters would predict if they were asked to make a forecast each day, treating their forecasts as missing data to be interpolated. Their aim is to obtain high-frequency measures of forecasters’ expectations. The authors consider two surveys in their empirical work: the Survey of Professional Forecasters and the Consensus Forecasts, and use the daily asset prices to predict the upcoming releases of either of the two surveys. In an in- and out-of-sample forecasting exercise, both approaches (MIDAS and Kalman filter) perform better than the simple random walk benchmark forecasts.

Andreou, Ghysels and Kourtellos (2010) assess whether daily financial data can improve macroeconomic forecasting, employing MIDAS regression models. They forecast US quarterly inflation rate and economic growth using a dataset including daily, monthly and quarterly indicators. An important advantage of the MIDAS model is that it can provide new forecasts as daily data become available. The authors find that on average daily financial predictors improve the forecasts of quarterly inflation and GDP relative to the AR benchmark model.

Monteforte and Moretti (2010) present a mixed frequency model for daily forecasts of Euro area inflation in real-time. The model they use allows to combine a monthly core inflation estimated from a dynamic factor model with daily financial market variables, which provide timely information on the recent shocks. They compare the results of this mixed-frequency model with standard univariate and multivariate models with monthly data, and also with the forecasts implied by financial market expectations extracted from future contracts. In both cases, the mixed frequency approach shows a superior predictive power.

1.5.3 Mixed-frequency VAR models

As we have seen in Section 1.2.4, studies on MF-VAR models have been conducted both in a classical and in a Bayesian context. In this review, we first outline the most important examples of empirical studies conducted in both frameworks.

Mittnik and Zadrozny (2005) evaluate a Kalman-filtering-based maximum-likelihood estimation method for forecasting German real GDP at monthly intervals. They estimate a VAR(2) model of quarterly GDP and up to three monthly indicator variables (industrial production, current and expected business conditions). They find that in general monthly models produce better short-term GDP forecasts, while quarterly models produce better long-term GDP forecasts.
Mariano and Murasawa (2010) apply the MF-VAR method to construct a new coincident indicator, i.e. an estimate of monthly real GDP. What they find is that the coincident index based on the VAR model is close to the one obtain by a factor model, and they both track well quarterly real GDP, although they are quite volatile.

Kuzin, Marcellino and Schumacher (2011) compare the MF-VAR, as presented in Mariano and Murasawa (2010), with the MIDAS approach to model specification in the presence of monthly and quarterly series. MIDAS leads to parsimonious models, while MF-VAR does not restrict the dynamics but suffers from the curse of dimensionality. The authors argue that it is difficult to rank the different approaches a priori, so they compare their performance empirically, considering an AR process as a benchmark. The two approaches tend to be more complementary than substitutes, since the MF-VAR performs better for longer horizons, whereas MIDAS for shorter horizons. Looking at the relative MSE of the different models with respect to the benchmark, the mixed-frequency models perform relatively well, especially when forecast combinations are adopted.

Similar evidence is found by Foroni and Marcellino (2012) in their paper which focuses on different methods proposed in the literature to deal with mixed-frequency and ragged-edge datasets. The authors discuss the performance of the different methods on now- and forecasting the quarterly growth rate of the Euro Area GDP and its components, using a very large set of monthly indicators. They also find that MF-VAR outperforms the MIDAS approach only at longer horizons.

Ghysels (2011) introduces a different MF-VAR representation, in which he constructs the MF-VAR process as stacked skip-sampled processes. In this paper, the author characterizes explicitly the mis-specification of a traditional low frequency VAR and the consequent mis-specifications in the impulse response functions. Moreover, since the MF-VAR specified in this way can also characterize the timing of information releases, he shows how Choleski factorizations are a more natural tool for impulse response analysis because the elements in the vector represent a sequence of time events. As another contribution, he studies a Bayesian approach which accommodates the potentially large set of parameters to be estimated.

One of the earliest studies on Bayesian estimation of MF-VAR is the paper by Chiu et al. (2011). In this paper, the authors develop a Gibbs sampling approach to estimate a VAR with mixed and irregularly sampled data. The focus of the paper is on the parameter estimation. In an exercise with simulated data, the authors show that taking into account mixed-frequency data allows to obtain smaller root mean squared errors for all the parameter estimates regardless of the sample size and of the correlation between the variables of the system. These results find confirmation also in the two empirical examples, conducted with
Another study by Viefers (2011) reconsiders the estimation of a MF-VAR as in Mariano and Murasawa (2010). First, the author makes use of the Bayesian MCMC algorithm to simulate and estimate the model, and second he extends the MF-VAR to allow for regime switching. In his model, the inference is based on the joint posterior density of all the unknowns. The findings of the simulation study suggest that the inference of the latent series and the regime processes is fairly precise, although there is a more pronounced imprecision in the estimation of the VAR parameters. In the empirical exercise, the author considers monthly and quarterly data for the US economy. The results on the regime probabilities show a relative high ability to identify the same recession dates provided by the NBER, although the probabilities tend to be more erratic and much worse in the most recent years.

Schorfheide and Song (2011) conduct a forecasting exercise on US data exploiting MF-VAR models. The goal of their paper is to study the extent to which the incorporation of monthly information improves the forecasts compared to models based on quarterly aggregated data. The analysis is conducted for 11 US variables, of which 3 observed at quarterly level, in a real-time context. The authors find that the monthly series provide important information in the short run, with significant RMSE reductions obtained with the mixed-frequency model. Moreover, the more intra-quarter information is available, the increasing the improvements.

### 1.5.4 Factor models

#### 1.5.4.1 Applications of small-scale factor models

Small-scale factor models have been frequently employed in the literature to construct a coincident indicator, which is able to track the development of the economy in real-time. In what follows, we describe the main studies which employ small-scale factor models which extract an index and provide, in some of the cases presented, short-term forecasts of the real GDP growth.

As described also in Section 1.2.5.1, Mariano and Murasawa (2003) propose a new coincident index of business cycles that relies on both monthly and quarterly indicators. Stock and Watson (1989) construct a coincident index by applying maximum likelihood (ML) factor analysis to the four monthly coincident indicators. Mariano and Murasawa extend the Stock–Watson coincident index by including quarterly real GDP and compare the turning
point detection performance of the two indexes. What they find is that the behavior of the common component is quite different from monthly real GDP, and more generally that the behavior of the common factor depends on the choice of the component indicators and therefore the monthly real GDP and the common factor component can have different turning points.

A different application of the Mariano and Murasawa model can be found in Frale et al. (2011). This paper proposes a new monthly indicator of the euro area economic conditions, EUROMIND, based on tracking real GDP on a monthly basis. The construction of this new monthly indicator of GDP is carried out indirectly through the temporal disaggregation of the value added by supply sectors from the output side and at the same time through the temporal disaggregation of the main components of the demand from the expenditure side. The two estimates are combined with optimal weights reflecting their precision. Therefore, the indicator is based on information at both monthly and quarterly level, modelled with a dynamic factor specification cast in state-space form, where computational efficiency is achieved by implementing univariate filtering and smoothing procedures, which also allows to handle the ragged-edge problem and other data irregularities in a unified framework. The authors find satisfactory results in the application of the model to the sectorial data, while the results are less convincing on the expenditure side. In a second paper, Frale et al. (2010) introduce a modification in the model which consists of the introduction of a second common factor, capturing the contribution of the survey variables as coincident indicators. What they find is that the second factor loads significantly on the survey variables for the industry sector and for exports. Moreover, they also attempt to isolate the news content of each block of series by using a real-time database: the analysis of the revisions in the data indicates that the contribution of surveys is not negligible.

Camacho and Perez-Quiros (2010) construct a different coincident indicator of the Euro area economy, the so-called Euro-STING indicator, which evolves accordingly to the Euro area dynamics and it is also based on an extension of the dynamic factor model described in Mariano and Murasawa (2003). The authors accommodate the GDP releases (flash, first and second estimate) in a statistical model to examine the impact of preliminary announcements and data revisions in the accuracy of real time forecasting. They assume that monthly growth rates of quarterly series and monthly growth rates of hard indicators have a direct relation with the common factor, which represents the common component that drives the series dynamics. However, they model the relation of the common factor with the soft indicators in a different way, and precisely they relate the level of the soft indicators considered with the year-on-year common growth rate, written as the sum of current values of the common factors.
and the last eleven lagged values. In their empirical application, they deal with a relatively small number of indicators. What they find is that exploiting information within each quarter their model improves upon the accuracy of preliminary announcements in forecasting GDP and forecasting uncertainty decreases during the forecasting period. Moreover, not only hard indicators are useful in forecasting GDP but also business surveys are relevant, especially in the months when real activity data are not available yet due to publication lags.

What Camacho and Perez-Quiros (2010) do for the Euro area is closely related to the empirical work done by Evans (2005) for the US, who applies a model that allows for variable reporting lags and temporal aggregation to a wide range of US macroeconomic data releases. The author models the growth in GDP as the quarterly aggregate of an unobserved daily process and specifies the relationship between GDP, data releases on GDP growth and on other macroeconomic variables in such a way to accommodate the different timing of data releases. By writing the model in state-space form (similar to Mariano and Murasawa (2003) but accommodating for a more complex timing of the releases), Evans (2005) obtains a real-time estimate of GDP on a daily basis as a product of the Kalman filter applied to estimate the model. The results seem promising, showing that within quarter data releases contain useful information for real-time estimation and forecasting of GDP. However, gaps between the real-time estimates and ex-post GDP data remain both persistent and significant.

Another extension to the small-scale factor model of Mariano and Murasawa (2003) has been analyzed by Marcellino, Porqueddu and Venditti (2012), to investigate business cycle dynamics and for forecasting GDP growth at short-term horizons in the euro area. While so far the parameters of the model have been considered as constant, the authors consider time variation in the variance of the shocks, and they generalize the setup of Mariano and Murasawa (2003) to allow for continuous shifts in the volatility of the shocks both to the common components and to the single indicators. To do so, they model volatility shifts as independent random walks. Moreover, differently from the other studies, the model is estimated with a Bayesian technique, using a Gibbs sampling procedure. The authors use the model to evaluate the impact of macroeconomic releases on point and density forecast accuracy and on the width of forecast intervals, and they show how their setup allows to make a probabilistic assessment of the contribution of releases to forecast revisions. From a pseudo out of sample forecasting exercise, they find that stochastic volatility contributes to an improvement in density forecast accuracy.
1.5.4.2 Bridging with factors

As we have seen in Section 1.2.5.2, Giannone et al. (2008) provide a framework that formalizes the updating of the nowcast and forecast of output and inflation as data are released throughout the month and that can be used to evaluate the marginal impact of new data releases on the precision of the now- and forecasts as well as the marginal contribution of different groups of variables. The framework they propose is adapted from the parametric dynamic factor model in state-space form proposed by Doz, Giannone and Reichlin (2011) that helps handling ragged-edge data. They extract monthly factors and use them in a state-space framework to forecast monthly GDP. The authors construct pseudo intra-month vintages according to a stylized data release calendar. As a new block of information is released, the factors are reestimated and the nowcast updated. The main finding is that information matters: the precision of the signal increases monotonically within the month, with the release of new data. Timeliness of the release and quality matters for decreasing uncertainty.

Barhoumi et al. (2008) compare small bridge equations and forecast equations in which the bridging between monthly and quarterly data is achieved through a regression on factors extracted from large monthly datasets. The authors consider the framework proposed by Giannone et al. (2008), but they also extract the factors following Forni et al. (2005), using generalized principal components which allow to take into account the ragged-edge structure of the dataset. In their paper, they focus on the Euro Area as a whole as well as on single Euro Area countries. The results obtained for the Euro Area countries show that models that exploit timely monthly releases fare better than quarterly models, and among those, factor models do generally better than averages of bridge equations.

1.5.4.3 Factor models in a mixed-frequency state space representation

Enriching the model proposed by Giannone et al. (2008), Banbura and Rünstler (2011) and Banbura and Modugno (2010) make use of a large state-space model allows for joint estimation of GDP and the factors in a single framework.

Banbura and Rünstler (2011) develop measures for understanding the importance of an individual series in the forecasts: they derive the weights of the series in the forecast and use them to calculate their contributions to the forecasts. Moreover, they assess the gains in forecast precision due to certain series by measuring the increase in uncertainty once the series have been removed from the explanatory variables. Banbura and Rünstler (2011) use a factor model which implements the common factors as unobserved components in a state-
1.5. AN OVERVIEW OF EMPIRICAL STUDIES

space form, and integrate the monthly factor model and a forecast equation for quarterly GDP in a single state-space representation, using a mixed-frequency setup. The authors confirm the finding of the importance of intra-quarter information, showing that a quarterly AR model is clearly outperformed by the factor model. Moreover they find evidence that differences in the timeliness of data releases can have strong effects on the optimal weights of individual series in the forecast and on their contribution to forecast precision.

Banbura and Modugno (2010) extend the analysis in Banbura and Rünstler (2011) by augmenting the dataset by short history indicators and quarterly series. Moreover, they allow the model to have an AR(1) idiosyncratic component. In their pseudo real-time exercise, they recursively forecast the Euro area GDP on the basis of a large monthly dataset. Compared to the previous data employed in Banbura and Rünstler (2011), they introduce short-history indicators, the Purchasing Managers’ Surveys, available only from mid 1997. The results obtained including these new short history monthly indicators are similar to the results obtained without including them, therefore it seems that these additional series do not improve the precision of the projections. Also with the explicit modelling of the serial autocorrelation of the idiosyncratic component the results do not improve significantly.

Moreover, Angelini et al. (2008) provide an out-of-sample evaluation of the method presented by Banbura and Rünstler (2011), and compare the forecasting performance of this approach to the one obtained by pooling the forecasts from different selected bridge equations. In order to evaluate the impact of new data releases on current GDP nowcast throughout the quarter, they update the model two times per month, measuring the accuracy of the forecasts computed using the information available at each date. The results they find show that the factor model forecast tracks GDP more accurately, most likely due to the fact that the factors take into account the information content of cross correlations across series.

1.5.4.4 Factor-MIDAS

Marcellino and Schumacher (2010) propose to merge factor models with the MIDAS approach, which allows to now- and forecast low frequency variables as GDP exploiting information in a large set of higher-frequency indicators. They consider three different MIDAS approaches - basic, smoothed and unrestricted - and the three alternative factor estimation methods that can account for unbalanced datasets, explained in Section 1.3.2, to have a total of nine Factor-MIDAS approaches. They then focus on German GDP and conduct now- and forecasting of quarterly GDP growth with a large set of timely monthly economic indicators. To relate Factor-MIDAS to the methods from the existing literature, the authors introduce two other approaches in the empirical comparison: a single-frequency factor model based on
quarterly aggregated data and the integrated state-space approach by Banbura and Rünstler (2011). In terms of empirical results, MIDAS with exponentially distributed lag functions performs similarly to MIDAS with unrestricted lag polynomials. In most of the cases, the simplest MIDAS with one lag of the factors is the best performing. Autoregressive dynamics plays a minor role. As far as the choice of the factor estimation technique is concerned, there are not significant differences among the different estimation methods. All Factor-MIDAS nowcasts can improve over quarterly factor forecast based on time-aggregated data, while the results compared to the state-space approach are less clear-cut and depend on the forecast horizon.

Kuzin, Marcellino and Schumacher (2012) discuss nowcast pooling versus nowcasting of German GDP growth with single models in the presence of model uncertainty, with mixed-frequency and ragged-edge data. The nowcasts are based on MIDAS regressions with few indicators and Factor-MIDAS based on large datasets. The authors compare the performance of many alternative specifications with respect to alternative estimation methods, number of factors, indicators selected for MIDAS, the role of autoregressive dynamics. In their empirical analysis, they show that indicator models tend to outperform factor models in this ex-post evaluation. It is much more difficult to beat the benchmark when the models are selected based on information criteria or past performance. As a method to avoid the specification search, all the nowcasts and forecasts can be pooled together, using different selection schemes. This approach yields additional gains with respect to the factor specification based on past performance, in particular when all single-indicator and all Factor-MIDAS forecasts are combined together using inverse MSE weights, providing full support in favour of pooling for nowcasting and short-term forecasting.

1.6 Conclusions

In this paper we reviewed the literature concerning estimation and forecasting with mixed-sampling frequency and ragged-edge data. At the moment, temporal aggregation is still the predominant technique in the empirical applications: all data are sampled at the same lower-frequency. In filtering the data so that the variables have all the same frequency, potentially useful information is discarded. Empirical studies show that mixed-frequency data matter, the use of the procedures that allow taking different frequencies and the timeliness of the data into account improve the forecasts.

One of the early approaches to deal with mixed-sampling frequency is based on use of the bridge equations, still very common in central banks, where a dynamic equation
is estimated between the low-frequency variable and time-aggregated indicators. Separate high-frequency models provide forecasts of the high-frequency indicators, and these forecasts are then aggregated and plugged into the bridge equation. Empirical studies with bridge models show that the exploitation of intra-period information reduces the forecasting error in the majority of the cases. Bridge equations result to be a useful instrument especially in nowcasting, since the more information becomes available, the more accurate they are in terms of RMSE. A comparison of this approach with the other methods to handle mixed frequency data would be interesting.

A second strand of research is based on mixed frequency regressions, where a low-frequency variable is explained by high-frequency indicators using parsimonious distributed lag models. MIDAS models are in general restricted to a limited set of variables, and estimated via NLS. Different weighting functions have been used in the literature, but which one is better to use is not clear-cut and depends on the specific analysis. Initially MIDAS models were applied to financial data, investigating stock market returns or future volatility, but recently MIDAS regressions have been employed to forecast macroeconomic variables, providing promising results for short-term forecasting.

Another, line of research relies on the state-space framework, in connection with both factors and VAR representations. The state-space setup treats the low-frequency variable as a high-frequency series with missing observations. The use of the Kalman filter allows real-time filtering, i.e., taking quarterly economic activity explained by monthly indicators as example, it is possible to obtain an estimate of GDP growth in each month. However, because of the intensive computation required by this framework and a relatively high number of parameters to be estimated, only models with few variables can be implemented. Within the class of factor models, different factor estimation techniques are described in the literature to handle the ragged-edge problem.

Recently, mixed frequency factor models and MIDAS regressions have been merged into Factor-MIDAS, which allows to forecast a low-frequency variable, exploiting the monthly information available in large datasets and handling unbalanced data as typical in real-time. Various factor estimation methods have been employed, without significant differences in their forecasting performance. Evidence shows that taking into account higher frequency information and exploiting the most recent observations pays off: Factor-MIDAS outperform quarterly factor forecasts based on time-aggregated data.

In summary, there is consensus that exploiting data at different frequencies matters, but it is not clear which method is superior. State-space models are a system approach and allow the estimation of the missing high-frequency data thanks to the use of the Kalman filter.
Within this class of models, different ways to estimate VAR parameters or factors taking into account the unbalancedness of the datasets have been proposed, but the differences don’t seem to be so pronounced. At the same time, the state-space approaches generally work only if the number of variables in the model is quite low, due to a dramatic increase in the number of parameters and associated complexity of the estimation. MIDAS models appear to be more robust to mis-specification compared both to bridge equation models and state-space approaches, and computationally simpler. There are basically no empirical assessments of the relative performance of all of these methods on the same dataset, which would instead possibly shed more light on their relative merits. Hence, we plan to undertake this exercise, with a focus on modelling and forecasting Euro area GDP and its components using a large set of monthly indicators.
BIBLIOGRAPHY


Chapter 2

A comparison of mixed frequency approaches for modelling Euro area macroeconomic variables

Joint with Massimiliano Marcellino

2.1 Introduction

In recent times, forecast models that take into account the information in unbalanced datasets have attracted substantial attention. Policy-makers, in particular, need to assess the current state of the economy in real-time, when only incomplete information is available.

In real-time, the unbalancedness of datasets arises mainly due to two features: the different sampling frequency with which the indicators are available and the so-called "ragged-edge" problem, namely publication delays of indicators cause missing values of some of the variables at the end of the sample, see Wallis (1986). As an example, one of the key indicators of macroeconomic activity, the Gross Domestic Product (GDP), is released quarterly and with a considerable publication lag, while a range of leading and coincident indicators is available more timely and at a monthly or even higher frequency.

In this paper, we focus on different classes of models which deal with an unbalanced dataset. In particular, we concentrate on three main streams in the literature: the bridge models, the state-space approach and the MIDAS approach. Bridge equations are one of the

\footnote{In this chapter I focused on the empirical analysis, collecting the data and writing the necessary codes to obtain the empirical results. The comments to the results have been jointly written by the authors.}
most used techniques, since they link monthly to quarterly variables, choosing the regressors because of their timely information content (see e.g. Baffigi et al. (2004)). State-space approaches aim at capturing the joint dynamics of indicators at different frequency. Two main models are developed within this framework: the mixed-frequency VAR and the factor models (see e.g. Zadrozny (1988) and Mariano and Murasawa (2003)). In both cases, the use of the Kalman filter allows obtaining a monthly estimate of the quarterly series. The third approach, the MIDAS method, is based on a univariate reduced form regression, which uses highly parsimonious lag polynomials to exploit the content in the higher-frequency explanatory variable and provide a high-frequency update of the quarterly frequency variable (see e.g. Ghysels et al (2004) for financial applications and Clements and Galvao (2008) for macroeconomic applications). Recently, these Factor and MIDAS approaches have been merged in the Factor-MIDAS model, augmenting the MIDAS regressions with the factors extracted from a large dataset in high frequency (see Marcellino and Schumacher (2010)).

All these approaches tackle data at different frequency and with publication delays, but at the same time they display different characteristics, and this makes it difficult to rank them a priori only on the basis of theoretical considerations. Therefore, we compare them in an extensive and detailed empirical application. Specifically, in this paper we extend the analysis in Kuzin, Marcellino and Schumacher (2011, 2012), and focus on now- and forecasting the quarterly growth rate of Euro Area GDP, using a very large set of monthly indicators (around 150 monthly series), with a wide number of forecasting methods. In particular, the main distinctive features of this analysis with respect to the one conducted in Kuzin et al. (2011, 2012) are a different sample, including the financial crisis, and the different set of indicators included. Moreover, more approaches are assessed in this analysis, including the bridge models, and small models are compared with large ones, with the introduction of factors.

In addition, to compare the different approaches, we both investigate the behavior of single indicator models and combine the forecasts within each class of models. We conduct the analysis recursively in a pseudo real-time framework, taking into account the ragged-edge structure of the dataset, and we assess the now- and forecasting performance of the models by comparing the resulting mean-squared errors (MSE). Specifically, we first investigate the performance of a large number of single indicator models. Then, since all the approaches can be subject to misspecification issues, related e.g. to indicator selection, the number of lags, etc., we propose forecast pooling as a way of dealing with this model uncertainty (see Timmermann (2006) for an extensive review of forecast pooling approaches). We consider three simple weighting schemes: the average, the median and a performance-based weighting
scheme (where the weights are equal to the inverse of the past MSE performance), and discuss whether they provide robust results against misspecification and parameter instability. We also consider the use of factor models, as a way of pooling information instead of pooling forecasts. Even in the case of factor models, we take into account the mixed-frequency nature of the dataset and compare different techniques. As additional robustness checks for our findings, we extend the forecast horizon up to four quarters ahead and compare the results obtained from recursive and rolling estimation methods. In the case of MIDAS models, we also extend the analysis to higher than monthly frequency data, incorporating in the analysis financial variables and interest rates and spreads at weekly frequency.

Moreover, in order to replicate the real-time situation in which policy makers and institutions need to assess the state of the economy, we investigate the behavior of a small group of indicators in a genuine real-time context, using data as they were at that moment, which allows taking into account data revisions.

Finally, we extend our analysis to the now- and forecasting of the economic activity components, disaggregating GDP from the output side in six branches following the NACE classification, and from the expenditure side, distinguishing among consumption, gross fixed capital formation and external balance.

We anticipate here some of the results concerning now- and forecasting of the aggregate GDP. First, there is no clear evidence of which approach to handling mixed-frequency data with ragged-edges is the best. A general finding is that looking at single indicator models, MF-VAR does not show particular improvement in terms of MSE, while generally MIDAS and bridge equations appear good methods to obtain better forecasts. Moreover, MIDAS without an AR component performs worse than the corresponding approach which incorporates it. Second, pooling forecasts is a helpful device in improving forecasting performance. Looking at the pooling results throughout different approaches, evidence seems to be in favour of a better performance of the MIDAS approach at every now- and forecast horizon. Even better results are obtained by pooling information with the use of factor models instead of pooling forecasts. Even the standard quarterly factor models can outperform the

 Andreou, Ghysels and Kourtellos (2009) also looked at large cross-section of daily financial data for forecasting quarterly real GDP growth, finding that MIDAS regression models provide substantial forecast gains against various benchmark forecasts.

 Andreou, Ghysels and Kourtellos (2009) extract information from a large daily dataset and use the factors in a MIDAS framework. Differently from what we do, they first extract a number of daily factors, then estimate MIDAS models with each one of the factors and finally pool the forecasts obtained from the different single-factor models. What they find is that the use of daily factors improves the forecasting performance compared to their random-walk benchmark. Moreover, comparing the performance of the forecasts obtained pooling factors with those obtained pooling single indicators, they obtain quite similar results, but with a slightly better performance of factor models. These results are similar to what we obtain in Section 2.7.
AR benchmark. The same general findings hold for those GDP components for which it is easy to find monthly indicators. However, checking the performance in different subsamples does not conduct to uniform results over time. In particular, in periods before the current crisis neither one of these more sophisticated models displays a clear superiority against the benchmark. However, during the first quarters of the crisis, at the end of 2008 and beginning of 2009, the models which take into account monthly information are clearly better in outperforming the benchmark. Finally, conducting the same analysis in a genuine real-time framework confirms the results found in a pseudo real-time context, allowing for the conclusion that data revisions, even though quite big in size, do not discard the reliability of the results obtained with final-vintage data.

The paper proceeds as follows. Section 2.2 describes the data. Section 2.3 discusses model specification. Section 2.4 presents the results on now- and forecasting quarterly Euro Area GDP, using single indicator models. Section 2.5 focuses on forecast pooling of the individual models, with different combination schemes. Section 2.6 summarizes the robustness analyses, including extending the forecast horizon, conducting a rolling evaluation and splitting the evaluation sample to assess stability of the findings over time, introducing several frequencies in the explanatory variables and conducting the analysis in a genuine real-time framework. Appendix A provides full details. Section 2.7 looks at the results from factor models, which pool information from a large number of time series, differently from what shown in Section 2.5 where the forecasts of individual models are pooled. Section 2.8 summarizes the results for GDP components, with more details provided in Appendix B. Section 2.9 concludes.

2.2 Data

The dataset contains Euro area quarterly and monthly series taken from the Eurostat dataset of Principal European Economic Indicators (PEEI). We consider a fixed country composition of the Euro area, as it is in 2009 at the end of the sample, with 16 countries. We collect quarterly GDP from 1996Q1 until 2009Q2, both at aggregate level and disaggregated into branches of activity and expenditure components. We also dispose of around 150 macroeconomic monthly indicators from January 1996 to August 2009, including consumer and producer price index by sector, industrial production and (deflated) turnover indexes by sector, car registrations, new orders received index, business and consumers surveys with their components, sentiment indicators, unemployment indices, monetary aggregates, interest and exchange rates. All series are seasonally adjusted. Details about the data can be found in Appendix C.
The dataset is a final dataset. However, we take into account one of the specific characteristics of the macroeconomic data in real time, the ragged-edge structure of the dataset due to different publication lags of the series. The timing of data releases is more or less the same every month, and this allows us to replicate the same pattern of missing values at the end of each recursive sample. To have an idea of the ragged-edge structure of the dataset, we show in Table 2.1 the lags of the main series in the dataset.

As outlined in the recent literature, the use of pseudo real-time datasets, which replicate the differences in the availability of data, lead to significant differences in results with respect to the use of artificially balanced datasets, see Giannone, Reichlin and Small (2008) and Breitung and Schumacher (2008) among the others. Therefore, in our paper, we replicate the ragged-edge structure of the dataset we observed at the downloaded date (31st August 2009) in each of the recursive subsamples: for each series we observe the number of missing values at the end, and we impose the same number of missing observations at each recursion, so to mimic the availability of data in real-time. As a clarifying example, at the end of August 2009, we have data on the CPI index and financial variables available until July 2009, but data on unemployment and industrial production only until June 2009, i.e. while the former variables have a delay of one month, the latter become available with a delay of two months. Therefore, when in our recursive exercise we use a subsample from January 1996 to March 2009, we impose to use the CPI index until February 2009 and unemployment until January 2009, to replicate the same data availability we would have in real-time. Moreover, we impose a similar structure of publication delays also on quarterly variables, namely GDP and its components on the supply and expenditure side. To do this, we take into account that in the Euro area, GDP and its breakdown into components are available in the third month after the end of the quarter of interest, for example the GDP figure for 2009Q1 becomes available in June 2009.

2.3 Model Specification

The aim of the experiment is to evaluate the performance of different methods available in the literature which deal with unbalanced mixed-frequency datasets, when the number of indicators is very large.

We first recursively estimate and then now- and forecast Euro area GDP growth rate, with the first evaluation quarter fixed at 2003Q1 and the last at 2009Q1, for a total of 25 recursive evaluation samples. For each quarter we compute nowcasts and forecasts, based on different information sets. In each recursion we want to predict the current GDP growth.
and the values of one and two quarters ahead. Therefore, for every single quarter in the evaluation sample we have three nowcasts and six forecasts. As an example, for 2005Q3, we have a nowcast computed in September 2005, one in August 2005 and one in July 2005; moreover, we have one-quarter ahead forecasts computed in June, May and April 2005 and two-quarters ahead forecasts computed in March, February and January 2005. Each of the nine projections we have for every realization of the GDP growth rate in the evaluation sample is based on different information, available at the point in time in which the projection is computed. Therefore we exploit the ragged-edge structure of the dataset and consider only the information available at that moment. As we have already stated in describing the data, the GDP in Euro Area is available with a delay of 65 days after the end of the quarter of interest.

In terms of notation, we denote GDP growth as $y_{t_q}$, where $t_q = 1, 2, 3, ..., T_q$ is a quarterly time index and $T_q$ is the final quarter for which GDP is available. GDP growth can be expressed also as a monthly variable with missing values, by setting $t_m = t_q, \forall t_m = 3t_q$, where $t_m$ is the monthly time index. GDP growth is observable only in $t_m = 3, 6, 9, ..., T_m^y$ where $T_m^y = 3T_q^y$. Therefore, what we want to obtain is the nowcast or forecast of the economic activity $h_q$ quarters ahead or, equivalently, $h_m = 3h_q$ months ahead. We exploit monthly stationary indicators $x_{t_m}$, with $t_m = 1, 2, 3, ..., T_m^x$, where $T_m^x$ is the final month for which the indicator is available. Usually monthly indicators are available earlier during the quarter than the GDP release, so generally we condition the forecast on the information available up to month $T_m^x$, which includes GDP information up to $T_q^y$ and indicator observations up to $T_m^x$ with $T_m^x \geq T_m^y = 3T_q^y$. The GDP growth forecast is indicated as $y_{T_m^x+h_m|T_m^x}$.

### 2.3.1 The Bridge Model approach

One of the early econometric approaches in the presence of mixed-frequency data relies on the use of bridge equations, see e.g. Baffigi, Golinelli, Parigi (2004) and Diron (2008). Bridge equations are linear regressions that link ("bridge") high frequency variables, such as industrial production or retail sales, to low frequency ones, e.g. the quarterly real GDP growth, providing some estimates of current and short-term developments in advance of the release. The "Bridge model" technique allows computing early estimates of the low-frequency variables by using high frequency indicators. They are not standard macroeconometric models, since the inclusion of specific indicators is not based on causal relations, but on the statistical fact that they contain timely updated information.

In our exercise, since the monthly indicators are usually only partially available over the projection period, the predictions of quarterly GDP growth are obtained in two steps.
First, monthly indicators are forecasted over the remainder of the quarter, on the basis of univariate time series models, and then aggregated to obtain their quarterly correspondent values. Second, the aggregated values are used as regressors in the bridge equation which allows to obtain forecasts of GDP growth.

Therefore, the bridge model to be estimated is:

\[ y_{tq} = \alpha + \sum_{i=1}^{j} \beta_i(L) x_{itq} + u_{tq} \]  

(2.1)

where \( \beta_i(L) \) is a lag polynomial of length \( k \), and \( x_{itq} \) are the selected monthly indicators aggregated at quarterly frequency.

In order to forecast the missing observations of the monthly indicators which are then aggregated to obtain a quarterly value of \( x_{itq} \), it is common practice to use autoregressive models, where the lag length is based on information criteria.

In our exercise, we use autoregressive models, where the lag length is chosen according to the BIC criterion, with the maximum lag fixed to 12. The data are then aggregated with standard methods, according to the stock/flow nature of the variables, specifically averaging over one lower-frequency period for stock variables and summing over the high-frequency indicators for flow variables. Once the data are aggregated, the number of lags of the indicators to include in the bridge model is chosen according to the BIC criterion, with a maximum lag equal to 4.

2.3.2 The MF-VAR approach

One of the most compelling approaches in the literature to deal with mixed-frequency time series at the moment is the one proposed by Zadrozny (1988) for directly estimating a VARMA model sampled at different frequencies. The approach treats all the series as generated at the highest frequency, but some of them are not observed. Those variables that are observed only at the low frequency are therefore considered as periodically missing.

Following the notation of Mariano and Murasawa (2004), we consider the state-space representation of a VAR model, treating quarterly series as monthly series with missing observations. The disaggregation of the quarterly GDP growth, \( y_{tq} \), into the unobserved month-on-month GDP growth, \( y_{tq}^{*} \), is based on the following aggregation equation:
\[ y_{tm} = \frac{1}{3} (y_{tm}^{*} + y_{tm-1}^{*} + y_{tm-2}^{*}) + \frac{1}{3} (y_{tm-1}^{*} + y_{tm-2}^{*} + y_{tm-3}^{*}) + \frac{1}{3} (y_{tm-2}^{*} + y_{tm-3}^{*} + y_{tm-4}^{*}) \]

\[ = \frac{1}{3} y_{tm}^{*} + \frac{2}{3} y_{tm-1}^{*} + y_{tm-2}^{*} + \frac{2}{3} y_{tm-3}^{*} + \frac{1}{3} y_{tm-4}^{*} \]  

(2.2)

where \( t_m = 3, 6, 9, \ldots, T_m \), since GDP growth, \( y_{tm} \), is observed the last month of each quarter, while \( y_{tm}^{*} \) is never observed.

This aggregation equation comes from the assumption that the quarterly GDP series (in log levels), \( Y_{tm} \), is the geometric mean of the latent monthly random sequence \( Y_{tm}^{*}, Y_{tm-1}^{*}, Y_{tm-2}^{*} \). Taking the three-period differences and defining \( y_{tm} = \Delta_3 Y_{tm} \) and \( y_{tm}^{*} = \Delta Y_{tm}^{*} \), we obtain eq. (2.2).

Let for all \( t_m \) the latent month-on-month GDP growth \( y_{tm}^{*} \) and the corresponding monthly indicator \( x_{tm} \) follow a bivariate VAR(\( p \)) process

\[ \phi(L_m) \begin{pmatrix} y_{tm}^{*} - \mu_y^{*} \\ x_{tm} - \mu_x \end{pmatrix} = u_{tm}, \]  

(2.3)

where \( u_{tm} \sim N(0, \Sigma) \).

**State-space representation**

In our exercise we determine the number of lags, \( p \), according to the Bayesian Information Criterion (BIC), with a maximum lag order of \( p = 4 \) months.

With \( p \leq 4 \), and defining \( s_{tm} \) and \( z_{tm} \) as

\[ s_{tm} = \begin{pmatrix} z_{tm} \\ \vdots \\ z_{tm-4} \end{pmatrix}, \quad z_{tm} = \begin{pmatrix} y_{tm}^{*} - \mu_y^{*} \\ x_{tm} - \mu_x \end{pmatrix}, \]

a state-space representation of the MF-VAR is

\[ s_{tm} = Fs_{tm-1} + Gv_{tm} \]  

(2.4)

\[ \begin{pmatrix} y_{tm} - \mu_y \\ x_{tm} - \mu_x \end{pmatrix} = Hs_{tm} \]  

(2.5)
with $\mu_y = 3\mu_y^*$ that holds, and $v_{tm} \sim N(0, I_2)$.

In the notation,

$$
F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}; \quad F_1 = \begin{bmatrix} \phi_1 & \ldots & \phi_p \\ 0_{2 \times 2(5-p)} \end{bmatrix}; \quad F_2 = \begin{bmatrix} I_8 & 0_{8 \times 2} \end{bmatrix},
$$

(2.6)

$$
G = \begin{bmatrix} \Sigma^{1/2} \\ 0_{8 \times 2} \end{bmatrix}; \quad H = \begin{bmatrix} H_0 & \ldots & H_4 \end{bmatrix}
$$

(2.7)

where $H$ contains the lag polynomial

$$
H(L_m) = \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2/3 & 0 \\ 0 & 0 \end{bmatrix} L_m + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} L_m^2 + \begin{bmatrix} 2/3 & 0 \\ 0 & 0 \end{bmatrix} L_m^3 + \begin{bmatrix} 1/3 & 0 \\ 0 & 0 \end{bmatrix} L_m^4
$$

(2.8)

**Estimation and forecasting**

The state-space representation of the mixed-frequency VAR model, described by equations (2.4) and (2.5), can be estimated by maximum-likelihood even in the presence of missing observations due to publication lags and the low-frequency nature of GDP. However, when the number of parameters is large, the ML method can fail to converge.

Therefore, we implement the EM algorithm modified to allow for missing observations. As in Mariano and Murasawa (2004), we consider the missing values as realizations of some iid standard normal random variables, i.e.

$$
y_{tm}^+ = \begin{cases} y_{tm} & \text{if } y_{tm} \text{ is observable} \\ \zeta_{tm} & \text{otherwise} \end{cases}
$$

(2.9)

where $\zeta_{tm}$ is a draw from a standard normal distribution independent of the model parameters.

The measurement equation is modified accordingly in the first two months of each quarter, where the upper row of $H$ is set to zero and a standard normal error term is added, so that the Kalman filter skips the random numbers. Since the realizations of the random numbers do not matter in practice, we replace the missing values with zeros.

We use the Kalman smoother to obtain forecasts of the economic activity. Although GDP growth for a particular month is not available, the smoother considers the monthly indicators available for the same quarter, so that nowcasting is also possible. For the months in which no observations are available also for the monthly indicators, the Kalman smoother acts exactly as the Kalman filter. What we obtain are iterative multistep forecasts and an estimate of the expected value of GDP growth in each month.
2.3.3 The MIDAS approach

MIDAS regressions are essentially tightly parameterized, reduced form regressions that involve processes sampled at different frequencies. The response to the higher-frequency explanatory variable is modelled using highly parsimonious distributed lag polynomials, to prevent the proliferation of parameters that might otherwise result, as well as the issues related to lag-order selection (see Ghysels et al. (2006), Andreou et al. (2010)).

2.3.3.1 The basic MIDAS model

MIDAS models are a direct forecasting tool, which directly rely on current and lagged indicators to estimate current and future GDP. This yields different models for different forecasting horizons. The forecast model for horizon \( h = h_m / 3 \) is:

\[
y_{t+h_q} = y_{t+h_m} = \beta_0 + \beta_1 b(L_m, \theta) x_{t_m+w} + \epsilon_{t+h_m},
\]

(2.10)

where \( y_{t_m} \) and \( x_{t_m} \) are respectively the GDP growth and the monthly indicator, \( x_{t_m}^{(3)} \) is the corresponding skip sampled monthly indicator, \( w = T_m^x - T_m^y \) and \( b(L_m, \theta) \) is the exponential Almon lag,

\[
b(L_m, \theta) = \sum_{k=0}^{K} c(k, \theta) L_m^k, \quad c(k, \theta) = \frac{\exp (\theta_1 k + \theta_2 k^2)}{\sum_{k=0}^{K} \exp (\theta_1 k + \theta_2 k^2)}.
\]

(2.11)

We estimate the MIDAS model using nonlinear least squares (NLS) in a regression of \( y_{t_m} \) on \( x_{t_m-k}^{(3)} \), yielding coefficients \( \widehat{\theta_1}, \widehat{\theta_2}, \widehat{\beta_0} \) and \( \widehat{\beta_1} \). Since the model is \( h \)-dependent, we reestimate it for multi-step forecasts and when new information becomes available. The forecast is given by:

\[
\widehat{y}_{t_m+h_m | T^x} = \widehat{\beta_0} + \widehat{\beta_1} B(L_1^{1/m}; \theta) x_{T_m}^{(3)}.
\]

(2.12)

As far as the specification is concerned, we use a large variety of initial parameter specifications, compute the residual sum of squares from equation (2.10) and choose the parameter set which gives the smallest RSS as initial values for the NLS estimation. \( K \) in the exponential Almon lag function is fixed at 12, whether the parameters are restricted to \( \theta_1 < 5 \) and \( \theta_2 < 0 \).

2.3.3.2 An extension: the AR-MIDAS model

A natural extension of the basic MIDAS model is the introduction of an autoregressive term. Including the AR dynamics is desirable but not straightforward. Ghysels, Santa-Clara and

Foroni, Claudia (2012), Econometric Models for Mixed-Frequency Data
European University Institute
DOI: 10.2870/45897
2.4. RESULTS FOR EURO AREA GDP

Valkanov (2004) show that the introduction of lagged dependent variables creates efficiency losses. Moreover, it would result in the creation of seasonal patterns in the explanatory variables.

Therefore, we follow Clements and Galvao (2008) and introduce the AR dynamics as a common factor to rule out seasonal patterns. We estimate the AR-MIDAS, defined as:

\[ y_{t+m+h} = \beta_0 + \lambda y_{t+m} + \beta_1 b(L_m, \theta) (1 - \lambda L_m^3) \tilde{x}_{t+m+w}^{(3)} + \varepsilon_{t+m+h}, \] (2.13)

where the \( \lambda \) coefficient can be estimated together with the other coefficients by NLS. Even in this case, we follow the procedure described for the MIDAS approach: first compute the RSS from (2.13), choose the parameters that minimize it, and use them as initial values for the NLS estimation.

2.4 Results for Euro area GDP

We first show the results of individual models for different now- and forecast horizons. We consider MSE as measure to compare the performance of the different models. As a benchmark, we recursively estimate an AR model of GDP growth, where the lag length is specified according to the BIC criterion. In our exercise we also considered the recursively estimated in-sample mean as a benchmark, but since the resulting MSE is greater than the MSE of the AR process, we preferred to adopt the AR in our analysis.

Table 2.2 provides evidence on the average performance of the different classes of mixed-frequency models, in order to investigate their properties and capture their differences and similarities over the full set of individual indicators. We report the average relative MSE performance for now- and forecasting quarterly GDP growth at different horizons for different classes of models, against the AR benchmark. First, we estimate every individual model and compute the relative MSE with respect to the benchmark, i.e. we calculate the MSE of every single indicator model relative to the MSE of the AR model. Then, we take the average across all the indicators of the relative MSE within a model class (Bridge, MIDAS, AR-MIDAS and MF-VAR).

Bridge models perform well and generally outperform the benchmark, though the gains are small. For most of the horizons, MIDAS cannot clearly perform better than the benchmark (the relative MSE is very close or greater than 1), whereas when we introduce the AR

---

4Since all the results are expressed in ratios with respect to the AR benchmark, in Table 2.2 we report also the absolute value of the MSE of the benchmark. Moreover, for the sake of completeness, we report also the variance of the GDP growth. As we can observe from these numbers, the AR benchmark outperform the naive variance of GDP growth, at least up to \( h_m = 4 \), that is at least for each nowcasting horizon.
component, the AR-MIDAS model beats the benchmark at all the horizons up to $h_m = 7$, behaving particularly well at short horizons. MF-VAR provides an average relative MSE larger than one, and equal to one only for larger horizons, showing therefore no particular gains in terms of forecasting performance.

A special comment on the results for $h_m = 1$ is needed: all the models except the AR-MIDAS perform very badly. This is due to the specific publication lag of the GDP in the Euro area. Since, as mentioned before, the GDP is released at the beginning of the third month of the next quarter, the one-month ahead nowcast is the only one computed with the GDP figure of the previous quarter already available. Looking at the results, therefore, this means that the information contained in the first lag for GDP matters a lot. When the performance in the previous quarter is available, it is very hard to reach a better nowcast with only the information contained in monthly indicators. On the contrary, the releases of the monthly series can improve the performance of a simple AR process, when the monthly information is added to the autoregressive component, as it is clear from the results of the AR-MIDAS.

As another way to compare the alternative mixed frequency models, we compute the relative MSE of the (AR-)MIDAS and MF-VAR to the MSE of bridge equations for each single indicator, and then average over all the relative MSEs we obtain. The results are shown in Table 2.3 (where we also report the median over all the relative MSEs).

The only model which is able to beat the bridge equations approach is the AR-MIDAS, for all the horizons up to $h_m = 7$. The inclusion of the lagged GDP therefore makes the difference in the performance of the different models. The same approach without including the autoregressive component shows worse results at every horizon. As already mentioned before, the MF-VAR performs slightly better than the (AR-)MIDAS only for relatively long horizon.

We can conclude that bridge equations and AR-MIDAS outperform MF-VAR for short now- and forecast horizons with the latter better than the former. The MF-VAR improves its performance only for $h_m = 8, 9$, but the differences with the other approaches are very limited for these longer horizons.

So far, we looked at the average performance of the models over all indicators. We can have some insights also looking at the best individual indicators. We find few monthly variables that outperform the AR benchmark up to at least horizon $h_m = 6$ and with different estimation methods. More in detail, there are three variables - the economic sentiment indicator, the production expectations for the months ahead and the unemployment rate under 25 years - which outperform the benchmark with all the four methods described above.
(bridge models, MIDAS, AR-MIDAS and MF-VAR), and two others - the manufacturing new orders received index and the general economic situation over the next 12 months - which outperform the benchmark with three methods out of four. Three of the best performing indicators are business survey components: this confirms the evidence found in the literature about the importance of the survey data as a source of timely information about the current economic situation. The two other best performing indicators are instead "hard data", that is variables on actual production and demand, which have usually more relevance in forecasting economic activity but on the other hand they are less timely. In Table 2.4, we show the performance of these best performing individual models. As it is evident from the results in the table, the gains for some horizons, especially the very short ones, are quite relevant.

2.5 Forecast pooling

The availability of many indicators leads to many forecasts of the same variable, This suggests to exploit information in the individual forecasts and combine them. Implementing forecast combination allows us to overcome misspecification bias, parameter instability and measurement errors in the datasets which may be present in the individual forecasts (see Timmermann (2006) for a detailed overview on forecast combination).

Estimating combination weights is hard since a large data sample relative to the number of the models is required to obtain appropriate estimates of the weights. Hence, here we provide results from forecast combinations within the same class of models, based on simpler combination schemes. More precisely, we consider three different combination schemes: the mean - the most exploited method in the literature, the median - the simplest rank-based weighting scheme, and a weighted mean that lets the combination weights be inversely proportional to the MSE of the previous four-quarter performance of the model (see Stock and Watson (2001)).

In Table 2.5, we provide the relative MSE performance of model pooling within a given class of models against the benchmark. The steps we conducted are the following: first, forecasts from single indicator models are computed, and means, medians and weighted means of all the forecasts within a single model are obtained. Second, the MSE of these three

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]

\[ \text{MSE} \]
different forecast combinations is calculated and divided by the MSE of the benchmark.

The results show that AR-MIDAS pooling performs pretty well: it outperforms the
benchmark at each of the nine horizons with all the three weighting combination schemes.
The weighted scheme works best, though the differences with respect to the other combina-
tion schemes are small. Pooling within MIDAS model also performs well for some horizons,
but the gains are smaller than for the AR-MIDAS. Bridge models confirm their good perfor-
ance across horizons, especially when looking at the mean and weighted mean aggregation
schemes. Pooling is instead less useful for the MF-VARs, where the gains with respect to the
benchmark are small or non existent. However, also in this case the weighted combination
performs best. Very small gains in using pooling within this class show up only for long
horizons ($h = 8, 9$). The last part of Table 2.5 contains the results of now- and forecast
combinations of all the models under consideration. The weighted average is once again the
combination scheme that provides the best results. However, the relative performance of
pooling all the models together does not behave better than all the results obtained from
pooling within single classes of models: it outperforms the pooling within the two weakest
classes of models (MIDAS and MF-VAR), but does not beat the average relative performance
of the two best approaches, the AR-MIDAS and the bridge models.

As in the case of single indicator models, in Table 2.6 we provide the relative MSE of
pooled (AR-)MIDAS and MF-VAR against pooled bridge models. First the single forecasts
are computed and aggregated with the different weighting schemes, then the MSE of the
combination is computed. The benchmark in this case is represented by the bridge equations
approach.

Contrary to the individual indicator models, pooled MIDAS shows a better performance
with respect to the bridge equations, when mean or median combinations are used. Except
for small horizons, where evidence is mixed, for the other now- and forecasting horizons the
relative MSE is below one. AR-MIDAS displays an almost uniform superior performance
with respect to the bridge equations, as in the case of individual indicator models, and
MF-VAR keeps underperforming compared to bridge models, even at longer horizons.

In summary, pooling mixed frequency models based on a large set of alternative indicators
is promising, and MSE weighted combinations of AR-MIDAS models overall produces the
best results.
2.6 Robustness analysis

We have assessed the robustness of the reported findings to a variety of modifications in the experiment design, including a longer forecast horizon (up to four quarters ahead), the role of recursive and rolling estimation, subsample analysis, several and possibly higher frequencies (adding weekly financial data to the best performing monthly indicators), and using real time data. We now summarize the main findings, with more details provided in Appendix A.

First, when extending the forecast horizon to three and four quarters ahead, the AR benchmark on average performs best. Hence, as expected, the relevance of the higher frequency indicators decreases with the forecast horizon. In terms of mixed frequency methods, the MF-VAR is slightly better than MIDAS (with and without the inclusion of the AR component), confirming the results in Section 2.4. As for pooling, for horizons beyond $h_m = 9$ combining the different MIDAS forecasts outperforms the AR benchmark, while pooling the MF-VAR forecasts is rather ineffective, in line with the findings in Section 2.5 for shorter horizons. Also in line with those results, pooling across all the individual models with a performance based weighting scheme is useful.

Second, overall in our application rolling estimation, commonly considered as a way to robustify the results in the presence of parameter instability, is not better than recursive estimation for mixed frequency models based on single indicators. Similarly, pooling forecasts obtained from recursive estimation is better than combining rolling estimation based forecasts. These findings could be due to the rather short sample size, which forces the rolling windows to be even shorter (seven years).

Third, and as another way to check for temporal stability, we have split the evaluation sample into a pre-crisis and a crisis period, where the two subsamples cover, respectively, 2003Q1 to 2006Q4 and 2007Q1 to 2009Q1. In line with other studies based on single frequency data, we find that single indicator mixed frequency models cannot on average outperform the AR benchmark prior to the crisis, at any horizon and for any class of models. However, pooling forecasts within each class still allows to obtain a better now- and forecast forecasting performance than the benchmark at least at short horizons, up to one quarter ahead. After the crisis, we find results in line to what described in Sections 2.4 and 2.5 for the entire evaluation sample. Specifically, the models that exploit the timely high frequency information have a better forecasting performance than the AR benchmark, both if we look at the average performance of the single indicator models and if we pool the forecasts within each class. Part of the better performance during the crisis is related to a major deterioration
in the AR forecasts, due to the large changes in GDP growth over these quarters. In terms of good indicators both before and during the crisis, we can list the total number of unemployed people and the total unemployment rate, some business survey components (services confidence indicators and orders placed with suppliers), financial indicators (two- and five-years interest rates) and the turnover indexes. A final interesting issue related to temporal stability is to assess whether the recovery period following the business cycle trough of 2009Q1 was more similar to the 2000-2006 sample or to the 2007Q1-2009Q1 sample. We have therefore updated the time series for GDP and a few of the best performing indicators with data up to 2010Q2, and compared the performance of a few mixed frequency models with that of the benchmark AR. We find that the best single indicators up to 2009q1 remain quite good also over 2009Q2-2010Q2, actually their performance generally further improves, but a large part of the additional gains are due to the worsening of the AR forecasts. It should be however remembered that, while interesting, evaluations based on such short samples are subject to substantial uncertainty.

Fourth, the relative performance of the models over the whole sample, and particularly over the recessionary and expansionary phases, could be driven by few large errors, whose relevance becomes even larger when squared for the computation of the MSE. To assess whether this is the case, we have repeated the analysis for the best indicators using the mean absolute error (MAE) as an evaluation criterion instead of the MSE. It turns out that the forecasting performance of the AR model still deteriorates over the recessionary phase, and even more over 2009Q2-2010Q2, but the extent of the deterioration is smaller than when measured in terms of MSE. However, the best single indicators in terms of MSE yield gains also in terms of MAE with respect to the AR, though their extent is reduced. Hence, the choice of the loss function does matter, but overall there seem to remain gains from exploiting higher frequency information in mixed frequency models.

Fifth, the MIDAS approach is flexible enough to allow for the inclusion of multiple explanatory variables at different frequencies, since each indicator is modelled with its own polynomial parameterization. The other approaches could be also modified to allow for regressors at different high frequencies but the computational costs are much higher. Hence, we have combined each of the five overall best performing monthly variables (general economic situation over the next 12 months, production expectations for the months ahead, the economic sentiment indicator, the manufacturing new orders received index and the unemployment under 25 years), with a weekly financial indicator: the three-months German interest rate, the ten-years Bund and the spread between the two. Evidence about the use of weekly data turns out to be quite mixed in our application. In general there is no clear
signal that the inclusion of data at higher frequency improves the forecasting performance, not even at very short horizons. However, the weekly spread between the three-months and the ten-years interest rate, the best of the three series considered, often reduces the MSE with respect to a model based on monthly and quarterly information only.

To conclude, when we repeat the evaluation using real time data for hard indicators, with both monthly and quarterly data revised, we obtain results similar and of the same magnitude to the ones obtained with pseudo real-time datasets, which do not take into account data revisions, confirming previous studies in the literature (see e.g. Diron (2008) and Schumacher and Breitung (2008)). Despite consistent data revisions, especially for the economic activity, the forecast results obtained with pseudo real-time datasets are reliable. As a general observation, the MIDAS approach seems to be the most sensible to data revisions, while the mixed-frequency VAR produces similar results with or without data revisions. The results do not change much when we look at the business surveys, which are generally not revised.

2.7 Large scale models

In our empirical analysis, we use a very large dataset, with many series whose aim is to capture the movements in the euro area economy (see Section 2.2). The information included in these time series can be summarized in few factors that represent the key economic driving element. Therefore, factor models, which have a long tradition in econometrics, are appealing from an economic point of view.

So far, we tried to combine the information coming from the different indicators by averaging forecasts based on different individual equations which contain only one indicator. Now, with the use of factor models, instead of pooling forecasts we pool the information contained in the dataset and summarize it into a few factors.

In what follows, we compare the results obtained from a standard quarterly factor model (see Stock and Watson (2002)) with the ones obtained from the recent approach proposed in the literature by Marcellino and Schumacher (2010), the Factor-MIDAS, which merges factor models based on large datasets with the forecast methods based on MIDAS. In the final subsection we evaluate alternative mixed frequency factor models.
### 2.7.1 Quarterly factor model

We employ the standard factor model proposed by Stock and Watson (2002). The \( h_q \)-step ahead forecast model is:

\[
y_{t+h_q} = \beta_0 + \beta (L_q) \tilde{f}_{t_q} + \lambda (L_q) y_{t_q} + \epsilon_{t_q+h_q},
\]  

where \( \beta (L_q) \) is an unrestricted lag polynomial of lag order \( P \) and \( \lambda (L_q) \) is of order \( R \). The estimation is conducted with a two-step procedure. First, the quarterly dataset, obtained by aggregating the monthly indicators over time, is used to estimate the factors by principal component analysis (PCA). Second, the estimators \( \beta_0 \), \( \beta (L_q) \) and \( \lambda (L_q) \) are obtained regressing \( y_{t+h_q} \) onto a constant, \( \tilde{f}_{t_q} \) and \( y_{t_q} \) and lags. The forecast then is formed as \( \hat{y}_{t+h_q} = \beta_0 + \beta (L_q) \tilde{f}_{t_q} + \lambda (L_q) y_{t_q} \). In our application, we choose a quarterly model with a fixed number of factors (one) and the number of lags chosen by BIC.

### 2.7.2 Factor-MIDAS models

It is possible to augment the MIDAS regressions with the factors extracted from a large dataset to obtain a richer family of models that exploit a large high-frequency dataset to predict a low-frequency variable. While the basic MIDAS framework consists of a regression of a low-frequency variable on a set of high-frequency indicators, the Factor-MIDAS approach exploits estimated factors rather than single or small groups of economic indicators as regressors.

The Factor-MIDAS model for forecast horizon \( h_q \) quarters with \( h_q = h_m / 3 \) is

\[
y_{t+h_q} = y_{t_m+h_m} = \beta_0 + \beta_1 b (L_m; \theta) \tilde{f}_{t_m+w} + \epsilon_{t_m+h_m},
\]  

where \( b (L_m; \theta) = \sum_{k=0}^{K} c (k; \theta) L_m^k \) and \( c (k; \theta) = \exp(\theta_1 k + \theta_2 k^2) / \sum_{k=0}^{K} \exp(\theta_1 k + \theta_2 k^2) \). As described above in the MIDAS models, the exponential lag function provides a parsimonious way to consider monthly lags of the factors.

The model can be estimated using nonlinear least squares in a regression of \( y_{t_m} \) onto the factors \( \tilde{f}_{t_m+w-h} \). The forecast is given by

\[
y_{T_m+h_m|T_m+w} = \beta_0 + \beta_1 b (L_m; \theta) \tilde{f}_{T_m+w}.
\]  

The projection is based on the final values of estimated factors.

MIDAS regression can be extended with the addition of autoregressive dynamics as

\[
y_{t+h_q} = y_{t_m+h_m} = \beta_0 + + \lambda y_{t_m} + \beta_1 b (L_m; \theta) \tilde{f}_{t_m+w} + \epsilon_{t_m+h_m}.
\]  

Foroni, Claudia (2012), Econometric Models for Mixed-Frequency Data
European University Institute
DOI: 10.2870/45897
The same two-step procedure described for quarterly factor models can be used also in case of mixed-frequency data. To handle the ragged-edge structure of the dataset, we follow the procedure outlined by Stock and Watson (2002), which combines the EM algorithm with PCA. Since not all observations are available, due to publication lags, the authors write the relation between observed and not fully observed variables as

\[ X_{i}^{\text{obs}} = A_{i}X_{i}, \]

where \( X_{i}^{\text{obs}} \) contains the observations available for variable \( i \), as a subset of \( X_{i} \), and \( A_{i} \) is the matrix that tackles missing values. Taking this relation into account, the EM algorithm provides an estimate of the missing values (for more details, see Stock and Watson (2002) and Marcellino and Schumacher (2010)).

In the next section we provide the results only for the case of models with \( r = q = 1 \), where \( r \) and \( q \) are respectively the number of static and dynamic factors. The EM algorithm is used to interpolate the missing values, but to avoid convergence problems the pairwise covariances are computed over the periods when both series are available. Marcellino and Schumacher (2010) compare this case with larger values of \( r \) and \( q \) in a similar application for forecasting German GDP growth, and find only small changes in the results.

**2.7.3 Results**

Nowcast and forecast results for the different kinds of factor models described in the Sections 2.7.1 and 2.7.2 are presented in Table 2.7. The numbers in the table show the relative MSE of each model to the benchmark, which once again is an AR process where the lag length is selected accordingly to the BIC criterion.

As a general result, there is evidence that the nowcasting and forecasting performance benefits a lot from the use of a large information set, summarized by factors. Factor models perform quite well up to 2 quarters ahead. They behave particularly well in nowcasting, relatively to the individual models we saw in Section 2.4. The standard quarterly factor model performs better than the benchmark for horizons up to one quarter, while there are no improvements for longer horizons. Factor-MIDAS models outperform the benchmark model at every horizon, and they also show a better performance compared to the quarterly factor model. This confirms the importance of taking into account the ragged-edge and mixed frequency structure of the dataset in terms of forecasting performance.

Since an AR process is supposed to be an appropriate benchmark, the inclusion of autoregressive dynamics in the MIDAS equation can further enhance the now- and forecasting performance. The results in Table 2.7 confirm that adding an AR term in the forecasting
equation is a good option, especially at very short horizons, while the gains are very small for longer ones. This confirms what already detected in Section 2.4.

Comparing the results in Table 2.7 with those on forecast pooling shown in Table 2.5, it turns out that generally pooling information into a factor model provides better results than pooling forecasts from different individual models, in particular up to one quarter ahead.

This evidence is confirmed even if we consider only the last part of the evaluation sample (2007Q1 - 2009Q1), as done in Section 2.6 for the individual models. There is a clear indication of a better performance of the factor models compared to the AR benchmark, and also a better performance than forecast pooling. Contrary to that, in the first part of the sample (2003Q1 - 2006Q4) it is difficult to outperform the benchmark, as in the case of individual models. Therefore, forecast pooling performs better in this first part of the sample.

### 2.7.4 Alternative mixed frequency factor models

There are alternative factor estimation methods developed in the literature to take into account unbalanced datasets. One convenient way is proposed by Altissimo et al. (2010) for estimating the New Eurocoin indicator, when each time series is realigned in order to obtain a balanced dataset and then dynamic principal components are applied to estimate the factors (vertical alignment - principal components method, VA-PCA). Another approach is the one proposed by Doz et al. (2011), based on a complete representation of the large factor model in a state-space form. The model consists of a factor representation of the monthly time series and a VAR structure which rules the behavior of the factors (Kalman filter - principal components method, KF-PCA).

As shown in Table 2.8, the differences between the three factor estimation methods are relatively small overall, confirming the evidence found for Germany by Marcellino and Schu¨macher (2010). Comparing the performances, the factors estimated with the EM algorithm and PCA (EM-PCA method) behave relatively better for most of the forecast horizons, therefore we provide the results only for this method. Only for nowcasting one and two months ahead the method proposed by Altissimo et al. (2010) has a better performance than the method proposed by Stock and Watson (2002)\(^6\).

\(^6\)For the details on the specification of these alternative factor models we refer to Marcellino and Schu¨macher (2010).
2.8 Results for euro area GDP components

From a policy perspective, it is generally important to have timely now- and forecasts not only of GDP but also of its components from the demand and supply sides. Hence, we have repeated the analysis of the forecasting performance of the mixed frequency models for the euro area GDP components, and we now summarize the main findings, with additional details provided in Appendix B.

Considering a decomposition from the output side, we follow the NACE classification of the GDP and obtain six branches of activity: agriculture, hunting, forestry and fishing; industry, excluding construction; construction; trade, hotels and restaurants, transport and communication services; financial services and business activities; other services. From the expenditure side, instead, we obtain five components: household final consumption; government final consumption; gross fixed capital formation, imports and exports.

Perhaps not surprisingly given the heterogeneity in the time series properties of the GDP components, the evidence on the relevance of the mixed frequency models is quite mixed. More specifically, starting with the GDP disaggregation from the output side, the mixed-frequency approaches outperform the benchmark for those components for which many monthly indicators are available, as in the case of the industry sector, trade and financial services. For agriculture, availability of monthly indicators is critical. The same holds for the last branch that includes a variety of economic activities (public administration and defence, compulsory social security; education; health and social work; other community, social and personal service activities; private households with employed persons) for which it is not easy to find reliable and timely monthly indicators of value added.

Looking now at the GDP components from the expenditure side, for the government final consumption it is very hard to find monthly indicators, so that it is very difficult to beat the benchmark. However, for all the other components, bridge equations and AR-MIDAS have (on average across indicators) a better performance than the benchmark, with the MF-VAR approach ranked third, since it often performs better than the benchmark, especially for longer horizons.

Now- and forecast pooling is helpful also for the components, but only for those with a sufficiently large set of indicators available, namely total industry and trade and financial services from the output side, and household final consumption, gross fixed capital formation and external balance from the expenditure side. Forecast combinations of AR-MIDAS models perform pretty well, outperforming the benchmark at several horizons. Also combinations of simple MIDAS and bridge equations allow for gains at some horizons.
To conclude, about the behavior of large scale factor models we can reach similar conclusions as for the aggregate GDP growth. Evidence is in favour of the use of factor models to predict the quarterly growth of each component for which the dataset contains enough useful information. There are significant gains especially at very short horizons: generally, exploiting the unbalanced structure of the dataset improves the performance, and the inclusion of an AR component reduces the MSE, even though not systematically. As for the case of GDP growth discussed in Section 2.7, also for the components of GDP the factor models seem to better perform relative to the benchmark than forecast pooling. This is true especially for nowcasts and short-term forecasts. However, when the forecast horizon increases, the outperformance of the factor models is no longer evident, and in many cases forecast pooling is better. Finally, for these long horizons, both methods of summarizing information (factors and forecast pooling) generally fail in beating the AR benchmark.

2.9 Conclusions

This paper extends the analysis presented in Kuzin, Marcellino, Schumacher (2011), considering a dataset of more than 150 monthly indicators to now- and forecast quarterly Euro Area GDP growth, and comparing different approaches which take into account the mixed frequency and ragged edge structure of the dataset. To start with, we compared the bridge model, the MIDAS model, with its extension incorporating an AR component, and the MF-VAR. The three approaches display some marked differences: while the bridge equations approach is a pure statistical model, where regressors are chosen by their timeliness more than by any specific economic reason, the other models presented in this paper are more sophisticated and exploit different ways to deal with an unbalanced dataset. Just as an example, while MIDAS is a single-equation approach and a direct multi-step forecast tool, MF-VAR jointly explains the indicator and GDP growth, and it is a recursive instrument to produce multi-step forecasts. Moreover, while with bridge equations and MIDAS models we obtain a monthly update of the quarterly GDP growth, with the state-space approach we can have an estimate of the monthly missing values of the GDP.

These approaches are therefore too different to have a ranking based only on theory. Hence it is preferable to compare them in empirical applications. The main results we obtained from our exercise hint at a better performance of MIDAS models, especially for the short horizons and when incorporating an AR component in the MIDAS model. Bridge models, which are less sophisticated than the other approaches, have overall a good performance. Finally, overall MF-VAR is the least promising mixed-frequency approach, at least
for very short horizons.

Pooling within each class of models results to be a good strategy to improve the performance: the MSE of the forecast combinations is smaller than the MSE of most of the individual models at every horizon. Comparing the different performance of forecast combination within each class, AR-MIDAS appears to be the best strategy.

An even better performance is obtained with the factor models, confirming that pooling information from a large number of series is useful in short-term forecasting and reduces the MSE. In particular, we looked at the factors estimated with the EM algorithm and PCA, which behave relatively better than the other models proposed in the literature, and we included these factors in a MIDAS framework. The factor-MIDAS with the inclusion of the AR component is the best in terms of relative MSE.

We assessed the robustness of the findings about Euro area GDP growth extending the forecast horizons up to four quarters ahead. While individual models hardly beat the benchmark, pooling the forecasts still allows for some gains, especially in the case of AR-MIDAS approach. We also compared recursive and rolling estimation, checking for temporal stability, but the results for individual models are not satisfactory when estimated with a rolling technique, possibly because the size of the rolling window and of our sample is still too small. Splitting the sample onto 2003-2007 and 2008-2009 evidences the difficulties in beating the benchmark before the crisis, while the mixed-frequency approaches improve their performance during quarters of dramatic drop in GDP growth. Moreover, exploiting weekly financial data in the MIDAS approach does not appear to contribute at improving the performance of the model significantly, though the spread is sometimes useful.

Since the analysis was conducted on a pseudo real-time dataset, we repeated the evaluation for a small number of best performing indicators in a genuine real-time context to check for the role of data revisions. We obtained similar ranking of the methods, and same magnitude of gains, as with pseudo real-time data. Despite consistent data revisions, especially for the economic activity measure, the findings obtained with pseudo real-time datasets are therefore reliable.

As a final contribution, we extended the analysis to the single components of the GDP, from the output side and from the expenditure side. The findings are in line with those obtained for the aggregate measure of economic activity, at least for those components for which timely monthly indicators are available.
### 2.10 Tables

Table 2.1: Main publication lags

<table>
<thead>
<tr>
<th>Main releases</th>
<th>Publishing lag</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>HICP</td>
<td>1 month</td>
<td>monthly</td>
</tr>
<tr>
<td>PPI</td>
<td>2 months</td>
<td>monthly</td>
</tr>
<tr>
<td>Industrial production</td>
<td>2 months</td>
<td>monthly</td>
</tr>
<tr>
<td>Industrial new orders</td>
<td>2 months</td>
<td>monthly</td>
</tr>
<tr>
<td>Turnover index</td>
<td>2 months</td>
<td>monthly</td>
</tr>
<tr>
<td>Hours worked</td>
<td>2 months</td>
<td>monthly</td>
</tr>
<tr>
<td>Car registrations</td>
<td>2 months</td>
<td>monthly</td>
</tr>
<tr>
<td>Retail trade</td>
<td>2 months</td>
<td>monthly</td>
</tr>
<tr>
<td>Construction output</td>
<td>2 months</td>
<td>monthly</td>
</tr>
<tr>
<td>Business survey</td>
<td>current month</td>
<td>monthly</td>
</tr>
<tr>
<td>Business climate indicator</td>
<td>current month</td>
<td>monthly</td>
</tr>
<tr>
<td>Consumer survey</td>
<td>current month</td>
<td>monthly</td>
</tr>
<tr>
<td>Money supply</td>
<td>1 month</td>
<td>monthly</td>
</tr>
<tr>
<td>Exchange rates (average)</td>
<td>1 month</td>
<td>monthly</td>
</tr>
<tr>
<td>Interest rates (average)</td>
<td>1 month</td>
<td>monthly</td>
</tr>
<tr>
<td>Stock exchange indexes (average)</td>
<td>1 month</td>
<td>monthly</td>
</tr>
<tr>
<td>Unemployment</td>
<td>2 months</td>
<td>monthly</td>
</tr>
<tr>
<td>GDP: disaggregation of sectorial value added</td>
<td>1 quarter</td>
<td>quarterly</td>
</tr>
<tr>
<td>GDP: disaggregation from expenditure side</td>
<td>1 quarter</td>
<td>quarterly</td>
</tr>
</tbody>
</table>

**Notes:** The publishing lags correspond to the number of missing observations at the end of the sample at the downloaded date.
Table 2.2: Average relative MSE performance of different classes of mixed-frequency models against AR benchmark

<table>
<thead>
<tr>
<th>model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>bridge</td>
<td>1.73</td>
<td>0.95</td>
<td>0.99</td>
<td>1.03</td>
<td>0.96</td>
<td>0.98</td>
<td>1.01</td>
<td><strong>0.99</strong></td>
<td><strong>0.98</strong></td>
</tr>
<tr>
<td>midas</td>
<td>1.69</td>
<td>0.96</td>
<td>1.03</td>
<td>1.06</td>
<td>0.96</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>ar-midas</td>
<td><strong>0.88</strong></td>
<td><strong>0.82</strong></td>
<td><strong>0.86</strong></td>
<td><strong>0.88</strong></td>
<td><strong>0.94</strong></td>
<td><strong>0.95</strong></td>
<td><strong>0.96</strong></td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td>mf-var</td>
<td>1.37</td>
<td>0.99</td>
<td>1.04</td>
<td>1.08</td>
<td>1.01</td>
<td>1.02</td>
<td>1.03</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**absolute values**

<table>
<thead>
<tr>
<th></th>
<th>MSE bm</th>
<th>variance GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.28</td>
<td>0.63</td>
</tr>
<tr>
<td>median</td>
<td>0.62</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Notes: The entries in the table are obtained as follows: first, estimate recursively every individual model and compute the relative MSE with respect to the benchmark; then, take the average all across the indicators of the relative MSE within a model class. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the best relative MSE performance for each horizon. For completeness, at the bottom of the table we report the absolute value of the MSE of the benchmark, and, as a term of comparison, the variance of the GDP growth.

Table 2.3: Average relative MSE performance of (AR-)MIDAS and MF-VAR against bridge

<table>
<thead>
<tr>
<th>model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>midas</td>
<td>1.02</td>
<td>1.05</td>
<td>1.07</td>
<td>1.05</td>
<td>1.01</td>
<td>1.01</td>
<td><strong>0.99</strong></td>
<td>1.00</td>
<td>1.02</td>
</tr>
<tr>
<td>ar-midas</td>
<td><strong>0.57</strong></td>
<td><strong>0.90</strong></td>
<td><strong>0.91</strong></td>
<td><strong>0.88</strong></td>
<td><strong>0.99</strong></td>
<td><strong>0.97</strong></td>
<td><strong>0.95</strong></td>
<td>1.03</td>
<td>1.04</td>
</tr>
<tr>
<td>mf-var</td>
<td>0.86</td>
<td>1.07</td>
<td>1.07</td>
<td>1.06</td>
<td>1.06</td>
<td>1.04</td>
<td>1.02</td>
<td>1.02</td>
<td>1.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>median</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>midas</td>
<td><strong>0.98</strong></td>
<td>1.01</td>
<td>1.02</td>
<td>1.01</td>
<td><strong>0.99</strong></td>
<td>1.01</td>
<td><strong>0.99</strong></td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td>ar-midas</td>
<td><strong>0.54</strong></td>
<td><strong>0.85</strong></td>
<td><strong>0.87</strong></td>
<td><strong>0.83</strong></td>
<td><strong>0.96</strong></td>
<td><strong>0.96</strong></td>
<td><strong>0.95</strong></td>
<td>1.01</td>
<td>1.03</td>
</tr>
<tr>
<td>mf-var</td>
<td><strong>0.80</strong></td>
<td>1.01</td>
<td>1.02</td>
<td>1.01</td>
<td>1.02</td>
<td>1.01</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Notes: The entries in the table are obtained as follows: first, estimate recursively every individual model and compute the relative MSE with respect to the benchmark; then, take the average and the median all across the indicators of the relative MSE within a model class. The benchmark is the corresponding bridge model. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the classes of models which outperform the bridge models for each horizon.
Table 2.4: Relative MSE performance of the best mixed-frequency models with different indicators against AR benchmark

<table>
<thead>
<tr>
<th>monthly indic.</th>
<th>model</th>
<th>horizon ( h_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>General economic situation over the next 12 months</td>
<td>midas</td>
<td>0.70 0.65 0.67 0.67 0.86 0.88 0.97 0.96 0.99</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.93 0.71 0.75 0.83 0.88 0.87 0.94 0.94 0.94</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.98 0.75 0.71 0.80 0.87 0.89 1.00 1.00 0.95</td>
</tr>
<tr>
<td>Production expectations for the months ahead</td>
<td>bridge</td>
<td>0.63 0.37 0.45 0.61 0.63 0.83 0.97 0.96 0.98</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.51 0.44 0.55 0.61 0.82 0.93 0.92 1.07 1.11</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.66 0.45 0.54 0.66 0.80 0.90 0.97 0.97 0.98</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.57 0.44 0.59 0.71 0.81 0.94 1.00 1.07 1.06</td>
</tr>
<tr>
<td>Economic sentiment indicator</td>
<td>bridge</td>
<td>0.67 0.40 0.38 0.46 0.58 0.63 0.92 0.92 0.89</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.59 0.42 0.47 0.57 0.82 0.86 0.93 0.93 0.94</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.60 0.54 0.60 0.70 0.81 0.88 0.98 0.97 1.06</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.77 0.57 0.63 0.77 0.81 0.88 1.00 1.02 1.03</td>
</tr>
<tr>
<td>New orders received index - Manufacturing</td>
<td>bridge</td>
<td>0.45 0.32 0.49 0.72 0.84 0.92 1.01 1.01 0.99</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.44 0.42 0.64 0.60 0.84 0.92 0.94 1.02 0.99</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.49 0.41 0.58 0.66 0.84 0.96 0.97 1.00 1.00</td>
</tr>
<tr>
<td>Unemployment - Under 25 years</td>
<td>bridge</td>
<td>0.69 0.72 0.81 0.89 0.92 0.95 0.99 0.98 0.98</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.81 0.74 0.80 0.84 0.76 0.76 0.96 0.90 0.89</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.94 0.74 0.78 0.84 0.75 0.76 0.93 0.91 0.90</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.77 0.79 0.87 0.95 0.95 0.98 1.01 1.00 1.00</td>
</tr>
</tbody>
</table>

Notes: The entries in the table are obtained as follows: first, estimate recursively every individual model, then compute the relative MSE with respect to the benchmark. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1.
Table 2.5: Relative MSE performance of model pooling within a given class of models against AR benchmark

<table>
<thead>
<tr>
<th>Model</th>
<th>$h_m$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>bridge</td>
<td>mean</td>
<td>1.53</td>
<td>0.89</td>
<td>0.94</td>
<td>0.99</td>
<td>0.94</td>
<td>0.97</td>
<td>0.99</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>weighted mean</td>
<td>1.09</td>
<td><strong>0.73</strong></td>
<td><strong>0.82</strong></td>
<td>0.89</td>
<td><strong>0.88</strong></td>
<td>0.92</td>
<td>0.95</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>median</td>
<td>1.67</td>
<td>0.98</td>
<td>1.03</td>
<td>1.07</td>
<td>1.00</td>
<td>1.03</td>
<td>1.03</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td>midas</td>
<td>mean</td>
<td>1.46</td>
<td>0.90</td>
<td>0.98</td>
<td>1.02</td>
<td>0.93</td>
<td>0.96</td>
<td>0.96</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>weighted mean</td>
<td>1.27</td>
<td>0.83</td>
<td>0.96</td>
<td>0.98</td>
<td>0.90</td>
<td>0.92</td>
<td>0.91</td>
<td><strong>0.92</strong></td>
<td><strong>0.92</strong></td>
</tr>
<tr>
<td></td>
<td>median</td>
<td>1.63</td>
<td>0.94</td>
<td>1.04</td>
<td>1.07</td>
<td>0.94</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>ar-midas</td>
<td>mean</td>
<td>0.80</td>
<td>0.78</td>
<td>0.84</td>
<td>0.86</td>
<td>0.91</td>
<td>0.92</td>
<td>0.92</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>weighted mean</td>
<td><strong>0.79</strong></td>
<td>0.76</td>
<td>0.83</td>
<td><strong>0.85</strong></td>
<td><strong>0.88</strong></td>
<td><strong>0.89</strong></td>
<td><strong>0.88</strong></td>
<td><strong>0.92</strong></td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>median</td>
<td>0.86</td>
<td>0.81</td>
<td>0.86</td>
<td>0.88</td>
<td>0.92</td>
<td>0.95</td>
<td>0.95</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>mf-var</td>
<td>mean</td>
<td>1.19</td>
<td>0.94</td>
<td>1.00</td>
<td>1.05</td>
<td>1.00</td>
<td>1.01</td>
<td>1.02</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>weighted mean</td>
<td>1.09</td>
<td>0.89</td>
<td>0.98</td>
<td>1.04</td>
<td>0.99</td>
<td>1.00</td>
<td>1.02</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>median</td>
<td>1.23</td>
<td>1.03</td>
<td>1.09</td>
<td>1.12</td>
<td>1.03</td>
<td>1.03</td>
<td>1.04</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>all</td>
<td>mean</td>
<td>1.20</td>
<td>0.87</td>
<td>0.94</td>
<td>0.98</td>
<td>0.94</td>
<td>0.96</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>weighted mean</td>
<td>1.08</td>
<td>0.82</td>
<td>0.90</td>
<td>0.95</td>
<td>0.92</td>
<td>0.92</td>
<td>0.94</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>median</td>
<td>1.19</td>
<td>0.91</td>
<td>0.95</td>
<td>1.00</td>
<td>0.97</td>
<td>1.01</td>
<td>1.01</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Notes:** The entries in the first part of the table are obtained as follows: first, forecasts from single indicator models are computed, and means, medians and weighted means of all the forecasts within a single model are obtained; second, the MSE of these three different forecast combinations is calculated and divided by the MSE of the benchmark. In the second part the entries are obtained in the same way, but the different combinations are obtained across all individual models of different classes. The estimation is conducted recursively. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the best relative MSE performance for each horizon.
Table 2.6: Relative MSE performance of model pooling within (AR-)MIDAS and MF-VAR models against bridge

<table>
<thead>
<tr>
<th>model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>midas mean</td>
<td>0.95</td>
<td>1.01</td>
<td>1.04</td>
<td>1.03</td>
<td>0.99</td>
<td>0.99</td>
<td>0.97</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td>weighted mean</td>
<td>1.16</td>
<td>1.13</td>
<td>1.16</td>
<td>1.10</td>
<td>1.02</td>
<td>1.00</td>
<td>0.96</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>median</td>
<td>0.97</td>
<td>0.96</td>
<td>1.01</td>
<td>1.00</td>
<td>0.94</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td>ar-midas mean</td>
<td>0.52</td>
<td>0.87</td>
<td>0.89</td>
<td>0.87</td>
<td>0.97</td>
<td>0.95</td>
<td>0.93</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>weighted mean</td>
<td>0.72</td>
<td>1.04</td>
<td>1.01</td>
<td>0.96</td>
<td>1.00</td>
<td>0.96</td>
<td>0.93</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>median</td>
<td>0.51</td>
<td>0.83</td>
<td>0.84</td>
<td>0.83</td>
<td>0.92</td>
<td>0.93</td>
<td>0.92</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>mf-var mean</td>
<td>0.78</td>
<td>1.05</td>
<td>1.06</td>
<td>1.06</td>
<td>1.04</td>
<td>1.03</td>
<td>1.01</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>weighted mean</td>
<td>1.00</td>
<td>1.22</td>
<td>1.19</td>
<td>1.17</td>
<td>1.12</td>
<td>1.08</td>
<td>1.06</td>
<td>1.07</td>
<td></td>
</tr>
<tr>
<td>median</td>
<td>0.74</td>
<td>1.05</td>
<td>1.06</td>
<td>1.05</td>
<td>1.02</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
<td>1.01</td>
</tr>
</tbody>
</table>

**Notes:** The entries in the table are obtained as follows: first, forecasts from single indicator models are computed, and means, medians and weighted means of all the forecasts within a single model are obtained; second, the MSE of these three different forecast combinations is calculated and divided by the MSE of the benchmark. The estimation is conducted recursively. The benchmark is the bridge equations approach. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the classes of models which outperform the bridge models for each horizon.

Table 2.7: Relative MSE performance of different classes of factor models against AR benchmark

<table>
<thead>
<tr>
<th>model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor-MIDAS (basic)</td>
<td>0.79</td>
<td>0.61</td>
<td>0.64</td>
<td>0.76</td>
<td>0.87</td>
<td>0.88</td>
<td>0.95</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>Factor-MIDAS (ar)</td>
<td>0.68</td>
<td>0.57</td>
<td>0.59</td>
<td>0.66</td>
<td>0.85</td>
<td>0.87</td>
<td>0.93</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td>Quarterly factor model</td>
<td>0.95</td>
<td>0.83</td>
<td>0.83</td>
<td>0.83</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
<td>1.03</td>
<td>1.03</td>
</tr>
</tbody>
</table>

**Notes:** The entries in the table are obtained as follows: first, estimate recursively every factor model, then compute the relative MSE with respect to the benchmark. The factors are estimated with the EM algorithm together with PCA as in Stock and Watson (2002). The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1.
Table 2.8: Relative MSE performance of different classes of factor models against AR benchmark, comparison of nowcast and forecast results for different factor estimation methods for \( r = 1 \)

<table>
<thead>
<tr>
<th>model</th>
<th>estimation method</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor-MIDAS (basic)</td>
<td>EM-PCA</td>
<td>0.79</td>
<td>0.61</td>
<td><strong>0.64</strong></td>
<td>0.76</td>
<td>0.87</td>
<td><strong>0.88</strong></td>
<td><strong>0.95</strong></td>
<td><strong>0.98</strong></td>
<td><strong>0.97</strong></td>
</tr>
<tr>
<td></td>
<td>VA-DPCA</td>
<td><strong>0.57</strong></td>
<td><strong>0.46</strong></td>
<td>0.78</td>
<td>0.88</td>
<td><strong>0.83</strong></td>
<td>0.97</td>
<td>1.00</td>
<td>1.07</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>KF-PCA</td>
<td>0.78</td>
<td>0.67</td>
<td>0.78</td>
<td>0.91</td>
<td>0.94</td>
<td>0.97</td>
<td>1.03</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>Factor-MIDAS (ar)</td>
<td>EM-PCA</td>
<td>0.68</td>
<td>0.57</td>
<td><strong>0.59</strong></td>
<td>0.66</td>
<td>0.85</td>
<td><strong>0.87</strong></td>
<td><strong>0.93</strong></td>
<td><strong>0.97</strong></td>
<td><strong>0.96</strong></td>
</tr>
<tr>
<td></td>
<td>VA-DPCA</td>
<td><strong>0.38</strong></td>
<td><strong>0.44</strong></td>
<td>0.74</td>
<td>0.75</td>
<td><strong>0.83</strong></td>
<td>0.96</td>
<td>0.97</td>
<td>1.11</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>KF-PCA</td>
<td>0.62</td>
<td>0.61</td>
<td>0.68</td>
<td>0.75</td>
<td>0.91</td>
<td>0.94</td>
<td>0.98</td>
<td>1.04</td>
<td>1.07</td>
</tr>
</tbody>
</table>

**Notes:** In the estimation method column, EM-PCA refers to the EM algorithm together with PCA as in Stock and Watson (2002), VA-DPCA is the vertical realignment and dynamic PCA used in Altissimo et al. (2006), and KFS-PCA is the Kalman smoother of state-space factors according to Doz et al. (2006). The entries in the table are obtained as follows: first, estimate recursively every factor model, then compute the relative MSE with respect to the benchmark. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the best relative MSE performance for each horizon.
BIBLIOGRAPHY


mate dynamic factor models based on Kalman filtering", Journal of Econometrics, 164(1), 188-205.


2.11 Appendix A: Robustness analysis

This Appendix assesses the robustness of the reported findings to a variety of modifications in the experiment design. In the first subsection, we extend the forecast horizon up to four quarters ahead. In the second subsection, we compare recursive and rolling estimation, checking for temporal stability. In the third subsection, we conduct a subsample analysis and look at the results for different periods. In the fourth subsection, we consider MIDAS models with several frequencies, adding weekly financial data to the best performing monthly indicators. In the final subsection, we repeat the analysis for the best indicators in a genuine real-time framework, which takes into account data revisions.

2.11.1 Extending the forecast horizon

In Sections 2.4 and 2.5, we presented the results for nowcasts and forecasts of the GDP growth up to two quarters ahead. Here, we extend the forecast horizon and look at the results for the forecasts three and four quarters ahead. We report here only the main results, and discuss the general findings.

Looking at individual models, the AR benchmark results to be the best model for horizons $h_m = 10$ up to $h_m = 15$. Table 2.9 reports the average relative MSE performance for now- and forecasting quarterly GDP growth at different horizons for different classes of models, against the AR benchmark. All the approaches, bridge, MIDAS and MF-VAR, show an average performance worse than the benchmark for three and four quarters ahead forecasts. Focusing on the relative performance of the different methods, it seems that the MF-VAR is performing slightly better than the MIDAS approach, with and without the inclusion of the AR component, confirming that the MF-VAR works better than MIDAS at longer horizons, as already highlighted in Section 2.4.

As for the horizons up to $h_m = 9$, we also consider three different forecast combination schemes: the mean, the median and a weighted mean that lets the combination weights be inversely proportional to the MSE of the previous four-quarter performance of the model.

In the first part of Table 2.10, we provide the relative MSE performance of model pooling within a given class of models against the benchmark.

Combining the different forecasts, MIDAS models outperform the benchmark. In this case, the basic model without the autoregressive component seems to work better than the correspondent model augmented by the AR dynamics. MF-VAR approach does not show any ability to beat the benchmark. Evidence is more mixed in the case of bridge models, with no clear evidence of these models outperforming the benchmark. Confirming the findings
in Section 2.5 for shorter horizons in the case of forecast pooling, MIDAS shows a better performance than MF-VAR. In the second part of Table 2.10 we look at the results of pooling across all the individual models we have: these results are quite satisfactory, especially when the weighted scheme is used, as we already noticed in Section 2.5 for shorter horizons.

2.11.2 Temporal stability

So far, we first recursively estimated and then now- and forecasted Euro area GDP growth rate. Now we compare these results with those obtained by rolling estimation, which can produce better results in the presence of parameter instability. Rolling estimation is more robust but it may increase the variance of the parameters estimates and therefore the mean square forecast error, in particular in a rather short sample as ours.

In Table 2.11, we show the average relative MSE performance for now- and forecasting quarterly GDP growth against the AR benchmark, comparing recursive and rolling estimation. Since both the individual models and the benchmark can be estimated with rolling and recursive method, we have four types of ratios: the relative MSE performance when both individual models and the benchmark are estimated recursively (these is what is presented in Section 2.4, so we do not report the results again), when both individual models and the benchmark are estimated with a rolling method, when the individual models are estimated with a rolling method and the benchmark recursively and vice versa. The size of the rolling window is equivalent to seven years.

If we look at panel A and B in Table 2.11, we see that when we do a rolling estimation of the mixed-frequency models, independently on how the benchmark is estimated, we see fewer gains compared to the results in Section 2.5. More specifically, with a rolling estimation the MIDAS class does not outperform the benchmark anymore, while the MF-VAR slightly improves its performance at longer horizons. However, when the individual models are estimated recursively, they beat not only the AR process estimated recursively but also the same process estimated with a rolling technique.

In Table 2.12, we provide the relative MSE performance of model pooling within a class against the benchmark. As in the case of individual models we combine recursive and rolling estimations of the mixed-frequency models and the benchmark. We report only the results obtained with the median, to save space. Here, we see benefits in pooling compared to the AR benchmark, either in the case of pooling obtained after recursive or rolling estimation. However the gains are bigger when the mixed-frequency models are estimated recursively. Generally, the results still hint a better performance of the AR-MIDAS compared to the simple MIDAS without autoregressive component, and good results from pooling bridge
equations. Moreover, differently from the case of recursive estimation, with a rolling estimation good results are also reached in the case of MF-VAR, especially for longer horizons. Pooling across all the models we have does not show significant gains independently on the recursive or rolling estimation method used.

To sum up, we looked at recursive and at rolling estimation of the benchmark and the mixed-frequency models. We can restrict our attention to the recursive AR benchmark because it gives a smaller MSE than the corresponding one obtained with rolling estimation (for all forecast horizons except for $h_m = 1$). Whenever the individual models are estimated recursively, we find bigger gains in terms of relative MSE than in the case of rolling estimation, especially within the MIDAS class. This can be due to the fact that our sample is still too short and we cannot have a reasonable size for the rolling window.

### 2.11.3 Subsample analysis

Another way to check for temporal stability is to split the evaluation sample and look at the results for different periods. In our analysis we consider two periods, the first from 2003Q1 to 2006Q4 and the second from 2007Q1 to 2009Q1. Here we summarize the main results.

As highlighted in different papers, many sophisticated forecast models cannot outperform a naive benchmark in periods going from the beginning of the 2000s up to the beginning of the financial crisis in 2007. This is exactly what we find with our data: the single indicator models cannot on average outperform the AR benchmark, at any horizon and for any class of models. However, pooling forecasts within each class still allows to obtain a better now- and forecast forecasting performance than the benchmark at least at short horizons, up to one quarter ahead.

Looking now at the results for the second subsample, we find results in line to what described in Sections 2.4 and 2.5 for the entire evaluation sample. The models that exploit the timely high frequency information have a better forecasting performance than the AR benchmark, both if we look at the average performance of the single indicator models and if we pool the forecasts within each class. This highlights how exploiting monthly information is useful especially in periods of crisis, as in the quarters 2008Q4 and 2009Q1 where the mixed-frequency models performed much better than an AR process. Part of the better performance is related to a major deterioration in the AR forecasts, due to the large changes in GDP growth over these quarters.

Moreover, if we look at the best performing indicators, we see that in the first subsample fewer variables contain enough information to outperform the benchmark. However, the indicators which have a MSE smaller than the benchmark up to $h_m = 9$ in the first part of the
sample usually have a good performance also in the second part. Among these outperforming indicators, we find the total number of unemployed people and the total unemployment rate, some business survey components (services confidence indicators and orders placed with suppliers), financial indicators (two- and five-years interest rates) and the turnover indexes. As a remark, those indicators who are among the best in the first subsample up to $h_m = 9$ stay among the outperforming indicators even in the second period but only for shorter horizons, generally up to $h_m = 6$.

Another interesting issue to be considered is whether the recovery period following the business cycle trough of 2009Q1 was more similar to the 2000-2006 sample or to the 2007Q1-2009Q1 sample. We have therefore updated the time series for GDP and a few of the best performing indicators with data up to 2010Q2, and compared the performance of a few mixed frequency models with that of the benchmark AR. The results are reported in Table 2.13 in terms of relative MSE of each model/indicator versus that of the AR, while the last row of the table shows the MSE of the AR. For simplicity we focus on 1- to 3-step ahead forecasts only, results for longer horizons are available upon request.

Four main findings are worth noting. First, the results for the samples 2003Q1-2009Q1, 2003Q1-2006Q4 and 2007Q1-2009Q1 are very similar to those obtained with the 2009 data vintage, suggesting that recent data revisions were not so relevant for these indicators, more on this topic in subsection 2.11.5. Second, the extent of the mentioned deterioration in the AR performance during the crisis period is evident from the table. The AR MSE increases from about 0.06 during 2003-2006 to 0.64 for $h=1$ and 1.56 for $h=2,3$ during 2007-2009. The performance further deteriorates at the beginning of the recovery phase, with values reaching 1.38 for $h=1$ and 2.40 for $h=2,3$. Third, the best single indicators up to 2009Q1 remain quite good also over 2009Q2-2010Q2, actually their performance generally further improves, but a large part of the additional gains are due to the worsening of the AR forecasts. Fourth, the ranking of the alternative mixed frequency models remains unclear, but in general bridge and mixed frequency VARs appear to behave better than over the previous periods. It should be however remembered that, while interesting, evaluations based on such short samples are subject to substantial uncertainty.

Finally, the relative performance of the models over the whole sample, and particularly over the recessionary and expansionary phases, could be driven by few large errors, whose relevance becomes even larger when squared for the computation of the MSE. To assess whether this is the case, in Table 2.14 we have repeated the analysis presented in the Table 2.13 but using the mean absolute error (MAE) as an evaluation criterion instead of the MSE. It turns out that the forecasting performance of the AR model still deteriorates over the
recessionary phase, and even more over 2009Q2-2010Q2, but the extent of the deterioration is smaller than when measured in terms of MSE. Moreover, the best single indicators in terms of MSE yield gains also in terms of MAE with respect to the AR, though their extent is reduced. Hence, the choice of the loss function does matter, but overall there seem to remain gains from exploiting higher frequency information in mixed frequency models.

2.11.4 Several frequencies

Financial variables, as interest rates and term spreads, are available even at frequencies higher than monthly. The MIDAS approach is flexible enough to allow for the inclusion of multiple explanatory variables at different frequencies, since each indicator is modelled with its own polynomial parameterization. The other approaches could be also modified to allow for regressors at different high frequencies but the computational costs are much higher.

In the case of monthly and weekly data, the MIDAS framework is extended as follows:

$$y_{tm_1} = \beta_0 + \beta_1 b(L_{m_1}; \theta_1) x_{1,tm_1+h_{m_1}}^{(m_1)} + \beta_2 b(L_{m_2}; \theta_2) x_{2,tm_2+w-h_{m_2}}^{(m_2)} + \varepsilon_{tm_1}, \quad (2.19)$$

where $m_1$ represents the monthly frequency and $m_2$ the weekly frequency.

As a robustness check, we take the five best performing monthly variables: three business survey components, general economic situation over the next 12 months, production expectations for the months ahead and the economic sentiment indicator, the manufacturing new orders received index and the unemployment under 25 years. These five variables are the only ones which beat the AR benchmark up to at least horizon $h_{m_1} = 6$, with both MIDAS and AR-MIDAS approaches (as discussed in Section 2.4). To obtain a model with monthly and weekly data, we add a weekly financial variable to each of these 5 monthly variables; we consider three weekly series: the three-months German interest rate, the ten-years Bund and the spread between the two. We end up therefore with 15 different models, each of them with a monthly and a weekly variable.

In order to compare these models with several frequencies with the ones with only monthly explanatory variables, we compute the forecasts only 1 to 9 months ahead, even though it is theoretically possible to compute a new forecast every week. Moreover, MIDAS models consider a fixed ratio between the releases of high-frequency and low-frequency data. While this is evident in the case of monthly and quarterly data, where we always have 3 months in a quarter, it is not obvious when we consider also weekly data, since the number of weeks varies during months and quarters. In our application, we consider 12 weeks per quarter, and 4 weeks per months, skipping the first week of each month in which there are five of them.
In Table 2.15, we show the ratio of the MSE of the model with monthly and weekly variable relative to the MSE of the model with only the correspondent monthly indicator. A ratio smaller than one indicates that the introduction of weekly data improves the forecasting performance. All the models are recursively estimated.

Evidence about the use of weekly data is quite mixed and there is no clear signal that the inclusion of data at higher frequency improves the forecasting performance, not even at very short horizons. More in details, weekly data appear to have some information content more in the MIDAS models than in the AR-MIDAS, where the introduction of the autoregressive component improves the forecasting performance of the models. Moreover, looking at the different weekly data considered, the spread between the three-months and the ten-years interest rate is the best of the three series considered, and it is able to reduce the MSE in almost every case. The performance of the interest rates, on the other hand, is not clear and it depends on which monthly indicator is used.

2.11.5 The role of real-time data

In the analysis conducted so far, we used a pseudo real-time dataset, taking into account the pattern of missing values due to publication lags, but using only the final vintage of data available at the downloaded date. In this way, we do not consider data revisions, which can be substantial in case of real activity. In what follows, we consider only the best performing indicators, as selected in Section 2.4. Among them, we face two types of data: the business surveys, which are not revised (or revised very few times and only marginally), and the "hard data", unemployment rate and industrial production (durable consumer goods components), which can be substantially revised. Therefore, in the following analysis we will show the results of these two group separately. In fact, while in considering unemployment rate and industrial production we have data revisions in both the explanatory variables and in the GDP growth, in considering business survey components we have revisions only in the dependent variable.

It is not straightforward to construct a real-time dataset for industrial production components, since a new version of the European standard classification of production activities has been introduced by Eurostat in January 2009, so it is hard to match the data before and after the reclassification. Since the manufacturing new orders received index is available only after January 2009, we consider another component of industrial production, the durable consumer goods production, in order to have vintages for the entire evaluation sample. Moreover, in the course of 2005 and 2006 national accounts data displayed major revisions, due to the introduction of chain-linking of real activity series. Therefore the vintages of GDP are at
constant prices up to the end of 2005 and chain-linked afterward. Finally, the same concept of Euro area has evolved over time, changing the country coverage. Two different concepts of Euro area country composition are employed in general: the fixed composition, which uses the same group of countries throughout all periods, and the changing composition, which follows the evolution of the euro area composition through time. In practice little differences are expected due to the small size in terms of GDP of the new member states. In order to conduct an analysis in real-time and to have the largest available number of vintages, we prefer to use the changing composition of Euro area, since for the fixed composition we have fewer available data: vintages of Euro area at 16 countries are available only after the enlargement to Slovakia in 2009, while for the other fixed compositions, e.g. Euro area at 12 countries, we do not have the most recent vintages after the reclassification of the industry activity.

We can have an idea of the magnitude of the revisions looking at Table 2.16, which summarizes the main statistics on the revisions.

The statistics reported in Table 2.16 refer to revisions of the series compared to the last vintage we have available at the downloaded date. Looking at the minimum and maximum, we see that the revisions can be substantial, especially in the monthly series (change in the unemployment rate and industrial production growth). On the contrary, the revisions do not appear to be so wide in the GDP growth rate. However, on average the revisions are zero for both the GDP growth, and for the two monthly series considered.

Some comments are needed also for the business survey series. Even though, as mentioned earlier, these series are not revised (or are very mildly revised due to the changing country composition of the Euro area), we can observe a change in the timing of the releases of these series: while at the beginning of our evaluation sample business surveys were re-released at the beginning of the month after the month of interest, recently the releases of the business surveys are available already in the last week of the month they refer to, therefore we can observe a change in the publication lags.

In our empirical analysis, we replicate the same forecasting exercise described in Section 2.3, with the four mixed-frequency methods (bridge models, MIDAS with and without AR component, MF-VAR) plus the quarterly autoregressive model, using real-time data. The forecasts are evaluated by comparing them to the final vintage of GDP available\(^7\).

\(^7\)In this exercise we use the data from the latest-available vintage. While in their paper, Clements and Galvao (2012) argue that the use of lightly-revised data helps improving the forecasting performance of the models, we prefer to use the latest-available vintages of data for two reasons. First, as explained in this section, there are various complications in constructing a real-time dataset for Euro area. Second, we compare the forecasts to the final vintage of GDP available, therefore the forecasting target is not first-released data. Even in the evidence found by Clements and Galvao (2012), the use of latest-available or...
When we look at hard data, with both monthly and quarterly data revised, we obtain results similar and of the same magnitude to the ones obtained with pseudo real-time datasets, which do not take into account data revisions, confirming previous studies in the literature (see e.g. Diron (2008) and Schumacher and Breitung (2008)). Despite consistent data revisions, especially for the economic activity, the forecast results obtained with pseudo real-time datasets are reliable. There is no clear evidence that the use of real-time data reduces the MSE. In Table 2.17 we compare the relative MSE obtained with a real-time dataset and the relative MSE obtained with a pseudo real-time dataset. Data revisions have no clear impact on the forecasting accuracy of the different models. The relative MSEs calculated on the two different datasets are similar in most of the cases, but the results are not uniform across the models and the horizons. As a general observation, the MIDAS approach seems to be the most sensible to data revisions, while the mixed-frequency VAR produces similar results with or without data revisions.

The results do not change much when we look at the business surveys, which are generally not revised. However, we note that the performance of the individual mixed-frequency models relative to the AR benchmark is slightly better when we use a pseudo real-time dataset than when we conduct the analysis in real-time. This can be due to the fact that the publication lag changed during the evaluation sample, as mentioned earlier in the paragraph. In constructing the pseudo real-time dataset, in fact, we impose the publication lags we see at the downloaded date in each recursion. Therefore, in the first part of the evaluation sample the pseudo real-time dataset accounts for one observation more than what we find in the genuine real-time dataset. This is reflected in the results provided in Table 2.18, where the relative MSE is slightly bigger, especially for very short horizons. However, since all in all the results are not so different depending on the dataset used, GDP growth revisions have no clear impact on forecasting performance, and the best performing indicators keep outperforming the benchmark.

lightly-revised data does not influence the results when forecasting post-revision data.
Table 2.9: Average relative MSE performance of different classes of mixed-frequency models against AR benchmark

<table>
<thead>
<tr>
<th>model</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>bridge</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
</tr>
<tr>
<td>midas</td>
<td>0.98</td>
<td>1.00</td>
<td>1.02</td>
<td>1.04</td>
<td>1.03</td>
<td>1.07</td>
</tr>
<tr>
<td>ar-midas</td>
<td>1.02</td>
<td>1.02</td>
<td>1.05</td>
<td>1.07</td>
<td>1.15</td>
<td>1.20</td>
</tr>
<tr>
<td>mfi-var</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.02</td>
<td>1.01</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Notes: The entries in the table are obtained as follows: first, estimate recursively every individual model and compute the relative MSE with respect to the benchmark; then, take the average all across the indicators of the relative MSE within a model class. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the best relative MSE performance for each horizon.
Table 2.10: Relative MSE performance of model pooling within a given class of models against AR benchmark

<table>
<thead>
<tr>
<th>Model</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>bridge</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
<td>1.00</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>weighted mean</td>
<td>0.95</td>
<td>0.95</td>
<td>0.94</td>
<td>0.96</td>
<td>0.96</td>
<td>0.98</td>
</tr>
<tr>
<td>median</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>midas</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
<td>0.97</td>
<td>0.95</td>
<td>0.97</td>
</tr>
<tr>
<td>weighted mean</td>
<td>0.92</td>
<td>0.93</td>
<td>0.93</td>
<td>0.95</td>
<td>0.94</td>
<td>0.96</td>
</tr>
<tr>
<td>median</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>ar-midas</td>
<td>0.96</td>
<td>0.97</td>
<td>0.96</td>
<td>0.98</td>
<td>1.06</td>
<td>1.09</td>
</tr>
<tr>
<td>weighted mean</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.96</td>
<td>1.04</td>
<td>1.08</td>
</tr>
<tr>
<td>median</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
<td>1.02</td>
<td>1.13</td>
<td>1.14</td>
</tr>
<tr>
<td>mfi-var</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>weighted mean</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>median</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Notes: The entries in the first part of the table are obtained as follows: first, forecasts from single indicator models are computed, and means, medians and weighted means of all the forecasts within a single model are obtained; second, the MSE of these three different forecast combinations is calculated and divided by the MSE of the benchmark. In the second part the entries are obtained in the same way, but the different combinations are obtained across all individual models of different classes. The estimation is conducted recursively. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the best relative MSE performance for each horizon.
Table 2.11: Average relative MSE performance of different classes of mixed-frequency models against AR benchmark - Rolling/Recursive comparison

<table>
<thead>
<tr>
<th>model</th>
<th>horizon ($h_m$)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. Rolling/Rolling</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bridge</td>
<td>1.89</td>
<td>0.90</td>
<td>0.94</td>
<td>0.95</td>
<td>0.96</td>
<td>0.97</td>
<td>0.98</td>
<td>1.01</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>midas</td>
<td>1.74</td>
<td>0.95</td>
<td>1.01</td>
<td>1.09</td>
<td>1.03</td>
<td>1.03</td>
<td>1.06</td>
<td>1.11</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>ar-midas</td>
<td><strong>1.01</strong></td>
<td>0.91</td>
<td>0.95</td>
<td>1.05</td>
<td>1.04</td>
<td>1.02</td>
<td>1.06</td>
<td>1.19</td>
<td>1.16</td>
<td></td>
</tr>
<tr>
<td>mf-var</td>
<td>1.53</td>
<td>0.94</td>
<td>0.96</td>
<td>0.99</td>
<td>0.98</td>
<td><strong>0.96</strong></td>
<td>0.97</td>
<td><strong>0.98</strong></td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. Rolling/Recursive</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bridge</td>
<td>1.66</td>
<td>0.97</td>
<td><strong>1.02</strong></td>
<td><strong>1.02</strong></td>
<td><strong>1.01</strong></td>
<td>1.02</td>
<td>1.03</td>
<td>1.03</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>midas</td>
<td>1.53</td>
<td>1.02</td>
<td>1.09</td>
<td>1.17</td>
<td>1.08</td>
<td>1.08</td>
<td>1.11</td>
<td>1.13</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>ar-midas</td>
<td><strong>0.89</strong></td>
<td>0.98</td>
<td><strong>1.02</strong></td>
<td>1.13</td>
<td>1.10</td>
<td>1.07</td>
<td>1.11</td>
<td>1.22</td>
<td>1.19</td>
<td></td>
</tr>
<tr>
<td>mf-var</td>
<td>1.34</td>
<td>1.01</td>
<td>1.03</td>
<td>1.07</td>
<td>1.03</td>
<td><strong>1.00</strong></td>
<td><strong>1.01</strong></td>
<td><strong>1.00</strong></td>
<td><strong>0.98</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C. Recursive/Rolling</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bridge</td>
<td>2.00</td>
<td>0.90</td>
<td>0.92</td>
<td>0.97</td>
<td>0.92</td>
<td>0.94</td>
<td>0.97</td>
<td><strong>0.97</strong></td>
<td><strong>0.97</strong></td>
<td></td>
</tr>
<tr>
<td>midas</td>
<td>1.94</td>
<td>0.90</td>
<td>0.96</td>
<td>0.99</td>
<td>0.92</td>
<td>0.95</td>
<td>0.95</td>
<td><strong>0.97</strong></td>
<td><strong>0.98</strong></td>
<td></td>
</tr>
<tr>
<td>ar-midas</td>
<td><strong>1.00</strong></td>
<td><strong>0.76</strong></td>
<td><strong>0.80</strong></td>
<td><strong>0.82</strong></td>
<td><strong>0.90</strong></td>
<td><strong>0.91</strong></td>
<td><strong>0.92</strong></td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>mf-var</td>
<td>1.55</td>
<td>0.93</td>
<td>0.96</td>
<td>1.00</td>
<td>0.96</td>
<td>0.97</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** The entries in the table are obtained as follows: first, estimate every individual model and compute the relative MSE with respect to the benchmark; then, take the average all across the indicators of the relative MSE within a model class. The benchmark is the estimate of an AR model with lag length specified accordingly to the BIC criterion. In each panel we compare estimations obtained recursively or rolling as indicated in the header of the panel: in Panel A, the individual models and the benchmark are both estimated rolling; in Panel B, the individual models are estimated rolling and the benchmark recursively; in Panel C the individual models are estimated recursively and the benchmark rolling. In each panel we also provide results of pooling across different classes of models. The rolling window has a size of 7 years. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the best relative MSE performance for each horizon in each panel.
Table 2.12: Relative MSE performance of model pooling within a given class of models against AR benchmark - Rolling/Recursive comparison (combination scheme: median)

<table>
<thead>
<tr>
<th>model</th>
<th>horizon (h_m)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A. Rolling/Rolling</td>
<td>bridge</td>
<td>1.68</td>
<td>0.90</td>
<td>0.98</td>
<td>1.00</td>
<td>0.97</td>
<td>0.98</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>midas</td>
<td>1.38</td>
<td>0.89</td>
<td>1.00</td>
<td>1.06</td>
<td>1.00</td>
<td>1.01</td>
<td>1.02</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ar-midas</td>
<td>0.87</td>
<td>0.86</td>
<td>0.89</td>
<td>0.92</td>
<td>0.99</td>
<td>0.99</td>
<td>1.00</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>mf-var</td>
<td>1.24</td>
<td>0.97</td>
<td>1.02</td>
<td>1.06</td>
<td>0.98</td>
<td>0.97</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>all</td>
<td>1.17</td>
<td>0.89</td>
<td>0.97</td>
<td>1.03</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>B. Rolling/Recursive</td>
<td>bridge</td>
<td>1.48</td>
<td>0.96</td>
<td>1.05</td>
<td>1.08</td>
<td>1.02</td>
<td>1.02</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>midas</td>
<td>1.22</td>
<td>0.95</td>
<td>1.08</td>
<td>1.14</td>
<td>1.05</td>
<td>1.06</td>
<td>1.07</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ar-midas</td>
<td>0.77</td>
<td>0.93</td>
<td>0.96</td>
<td>0.99</td>
<td>1.04</td>
<td>1.04</td>
<td>1.05</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>mf-var</td>
<td>1.09</td>
<td>1.04</td>
<td>1.10</td>
<td>1.14</td>
<td>1.03</td>
<td>1.02</td>
<td>1.02</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>all</td>
<td>1.03</td>
<td>0.96</td>
<td>1.05</td>
<td>1.11</td>
<td>1.03</td>
<td>1.03</td>
<td>1.03</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>C. Recursive/Rolling</td>
<td>bridge</td>
<td>1.90</td>
<td>0.91</td>
<td>0.96</td>
<td>0.99</td>
<td>0.96</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>midas</td>
<td>1.84</td>
<td>0.87</td>
<td>0.96</td>
<td>0.99</td>
<td>0.90</td>
<td>0.96</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ar-midas</td>
<td>0.98</td>
<td>0.76</td>
<td>0.80</td>
<td>0.81</td>
<td>0.88</td>
<td>0.90</td>
<td>0.91</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>mf-var</td>
<td>1.39</td>
<td>0.95</td>
<td>1.01</td>
<td>1.04</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>all</td>
<td>1.19</td>
<td>0.91</td>
<td>0.95</td>
<td>1.00</td>
<td>0.97</td>
<td>1.01</td>
<td>1.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Notes: The entries in the table are obtained as follows: first, forecasts from single indicator models are computed, and the median of all the forecasts within a single model is obtained; second, the MSE of these the forecast combination is calculated and divided by the MSE of the benchmark. The benchmark is the estimate of an AR model with lag length specified accordingly to the BIC criterion. In each panel we compare estimations obtained recursively or rolling as indicated in the header of the panel: in Panel A, the individual models and the benchmark are both estimated rolling; in Panel B, the individual models are estimated rolling and the benchmark recursively; in Panel C the individual models are estimated recursively and the benchmark rolling. The rolling window has a size of 7 years. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the best relative MSE performance for each horizon in each panel.
### Chapter 2

Table 2.13: Relative MSE performance of the best mixed-frequency models with different indicators against AR benchmark

<table>
<thead>
<tr>
<th>Monthly Indicator</th>
<th>Model</th>
<th>Sample: 2003q1 - 2009q1</th>
<th>Sample: 2003q1 - 2006q4</th>
<th>Sample: 2007q1 - 2009q1</th>
<th>Sample: 2009q2 - 2010q2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>horizon (h = 1)</td>
<td>horizon (h = 2)</td>
<td>horizon (h = 3)</td>
<td>horizon (h = 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>General economic situation over the next 12 months</td>
<td>bridge</td>
<td>1.26 (0.73)</td>
<td>0.73 (0.74)</td>
<td>1.08 (0.82)</td>
<td>0.79 (0.74)</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>1.03 (0.78)</td>
<td>0.73 (1.07)</td>
<td>0.94 (1.03)</td>
<td>0.71 (1.03)</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.71 (0.65)</td>
<td>0.67 (1.16)</td>
<td>0.94 (1.02)</td>
<td>0.64 (0.62)</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.95 (0.72)</td>
<td>0.75 (0.99)</td>
<td>0.74 (0.75)</td>
<td>0.95 (0.72)</td>
</tr>
<tr>
<td></td>
<td>Production expectations for the months ahead</td>
<td>bridge</td>
<td>0.65 (0.38)</td>
<td>0.46 (0.88)</td>
<td>0.71 (0.71)</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.59 (0.46)</td>
<td>0.61 (0.90)</td>
<td>0.72 (0.77)</td>
<td>0.71 (0.79)</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.52 (0.42)</td>
<td>0.54 (0.72)</td>
<td>0.65 (0.79)</td>
<td>0.49 (0.40)</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.79 (0.41)</td>
<td>0.52 (0.93)</td>
<td>0.72 (0.72)</td>
<td>0.77 (0.38)</td>
</tr>
<tr>
<td>Economic sentiment indicator</td>
<td>bridge</td>
<td>0.66 (0.39)</td>
<td>0.40 (0.69)</td>
<td>0.54 (0.72)</td>
<td>0.66 (0.38)</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.81 (0.59)</td>
<td>0.66 (0.77)</td>
<td>0.65 (0.82)</td>
<td>0.81 (0.59)</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.60 (0.54)</td>
<td>0.60 (0.78)</td>
<td>0.72 (0.88)</td>
<td>0.57 (0.52)</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.58 (0.41)</td>
<td>0.48 (0.67)</td>
<td>0.60 (0.75)</td>
<td>0.56 (0.39)</td>
</tr>
<tr>
<td>New orders received index - Manufacturing</td>
<td>bridge</td>
<td>0.78 (0.46)</td>
<td>0.64 (1.15)</td>
<td>0.87 (1.14)</td>
<td>0.72 (0.42)</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.67 (0.41)</td>
<td>0.68 (1.34)</td>
<td>1.02 (1.35)</td>
<td>0.57 (0.35)</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.55 (0.39)</td>
<td>0.63 (0.89)</td>
<td>1.04 (1.04)</td>
<td>0.49 (0.33)</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.71 (0.48)</td>
<td>0.63 (1.46)</td>
<td>0.89 (1.03)</td>
<td>0.60 (0.44)</td>
</tr>
<tr>
<td>Unemployment - Under 25 years</td>
<td>bridge</td>
<td>0.73 (0.49)</td>
<td>0.79 (1.25)</td>
<td>0.76 (0.95)</td>
<td>0.64 (0.46)</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.78 (0.52)</td>
<td>0.75 (1.21)</td>
<td>1.15 (1.53)</td>
<td>0.71 (0.46)</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.62 (0.50)</td>
<td>0.72 (1.17)</td>
<td>1.18 (1.58)</td>
<td>0.53 (0.44)</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.64 (0.57)</td>
<td>0.79 (0.98)</td>
<td>0.66 (0.95)</td>
<td>0.59 (0.56)</td>
</tr>
</tbody>
</table>

**AR: MSE**

<table>
<thead>
<tr>
<th>Sample: 2003q1 - 2009q1</th>
<th>Sample: 2003q1 - 2006q4</th>
<th>Sample: 2007q1 - 2009q1</th>
<th>Sample: 2009q2 - 2010q2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.27 (0.61)</td>
<td>0.61 (0.61)</td>
<td>0.06 (0.08)</td>
<td>0.08 (0.08)</td>
</tr>
<tr>
<td>0.64 (1.56)</td>
<td>1.56 (1.38)</td>
<td>2.40 (2.40)</td>
<td>2.40 (2.40)</td>
</tr>
</tbody>
</table>

**Notes:** The entries in the table are obtained as follows: first, estimate recursively every individual model, then compute the relative MSE with respect to the benchmark. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is indicated in the top row. The absolute value of the AR MSE (benchmark) is displayed in the bottom row of the table.
## Table 2.14: Relative MAE performance of the best mixed-frequency models with different indicators against AR benchmark

<table>
<thead>
<tr>
<th>Monthly Indicator</th>
<th>Model</th>
<th>Horizon (h_m)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic sentiment</td>
<td>bridge</td>
<td>0.90</td>
<td>0.91</td>
<td>1.01</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.91</td>
<td>0.88</td>
<td>0.89</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.98</td>
<td>0.89</td>
<td>0.90</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Notes: The entries in the table are obtained as follows: first, estimate recursively every individual model, then compute the relative MAE with respect to the benchmark. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is indicated in the top row. The absolute value of the AR MAE (benchmark) is displayed in the bottom row of the table.
Table 2.15: Relative MSE performance of models with monthly and weekly data against the correspondent model with only the monthly series

<table>
<thead>
<tr>
<th>Model</th>
<th>Monthly indic.</th>
<th>Weekly indic.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>General economic situation over the next 12 months</td>
<td>3m Fibor</td>
<td>0.84</td>
<td>0.87</td>
<td>0.89</td>
<td>1.11</td>
<td>1.02</td>
<td>0.69</td>
<td>1.16</td>
<td>1.07</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10y Bund</td>
<td>1.07</td>
<td>0.94</td>
<td>1.09</td>
<td>1.01</td>
<td>0.97</td>
<td>0.92</td>
<td>0.99</td>
<td>1.01</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Spread</td>
<td>1.00</td>
<td>0.88</td>
<td>0.88</td>
<td>0.78</td>
<td>0.74</td>
<td>0.73</td>
<td>0.82</td>
<td>0.89</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>Production expectations for the months ahead</td>
<td>3m Fibor</td>
<td>0.92</td>
<td>1.02</td>
<td>0.95</td>
<td>1.08</td>
<td>1.09</td>
<td>1.01</td>
<td>1.34</td>
<td>1.04</td>
<td>1.07</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10y Bund</td>
<td>1.02</td>
<td>0.90</td>
<td>1.03</td>
<td>0.90</td>
<td>0.94</td>
<td>0.94</td>
<td>0.97</td>
<td>0.99</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Spread</td>
<td>1.04</td>
<td>0.90</td>
<td>0.80</td>
<td>0.75</td>
<td>0.87</td>
<td>0.85</td>
<td>0.85</td>
<td>0.97</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>Economic sentiment indicator</td>
<td>3m Fibor</td>
<td>0.97</td>
<td>1.10</td>
<td>0.97</td>
<td>1.12</td>
<td>1.03</td>
<td>0.71</td>
<td>1.17</td>
<td>1.04</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10y Bund</td>
<td>1.03</td>
<td>0.91</td>
<td>1.03</td>
<td>0.97</td>
<td>0.94</td>
<td>0.94</td>
<td>0.98</td>
<td>0.99</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Spread</td>
<td>0.94</td>
<td>0.77</td>
<td>0.70</td>
<td>0.65</td>
<td>0.73</td>
<td>0.69</td>
<td>0.75</td>
<td>0.79</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>New orders received index - Manufacturing</td>
<td>3m Fibor</td>
<td>0.94</td>
<td>0.87</td>
<td>0.90</td>
<td>1.04</td>
<td>0.90</td>
<td>0.69</td>
<td>1.20</td>
<td>1.02</td>
<td>1.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10y Bund</td>
<td>1.06</td>
<td>1.15</td>
<td>1.08</td>
<td>1.03</td>
<td>1.02</td>
<td>0.91</td>
<td>0.97</td>
<td>0.99</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Spread</td>
<td>1.04</td>
<td>1.00</td>
<td>0.95</td>
<td>0.90</td>
<td>0.96</td>
<td>0.94</td>
<td>0.94</td>
<td>0.99</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>Unemployment - Under 25 years</td>
<td>3m Fibor</td>
<td>1.01</td>
<td>0.77</td>
<td>0.92</td>
<td>1.04</td>
<td>0.91</td>
<td>1.06</td>
<td>1.14</td>
<td>1.02</td>
<td>1.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10y Bund</td>
<td>1.04</td>
<td>0.92</td>
<td>1.06</td>
<td>0.98</td>
<td>0.98</td>
<td>0.92</td>
<td>1.00</td>
<td>1.07</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Spread</td>
<td>0.94</td>
<td>0.67</td>
<td>0.74</td>
<td>0.76</td>
<td>0.78</td>
<td>0.87</td>
<td>0.77</td>
<td>0.84</td>
<td>0.94</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The entries in the table are obtained as follows: estimate recursively every individual model with monthly and weekly data and compute the relative MSE with respect to the model with only the corresponding monthly indicator. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the cases in which the introduction of weekly data improves the performance.
Table 2.16: Revisions from January 2002 to March 2009: main statistics

<table>
<thead>
<tr>
<th>variable</th>
<th>average</th>
<th>sd</th>
<th>revisions</th>
<th>average</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment - under 25 years</td>
<td>-0.1</td>
<td>0.2</td>
<td>-0.7</td>
<td>1.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial production</td>
<td>0.0</td>
<td>1.9</td>
<td>-3.9</td>
<td>4.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP - market prices</td>
<td>0.6</td>
<td>0.5</td>
<td>-0.3</td>
<td>0.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** The statistics refer to the series and to their revisions compared to the last available vintage (August 2009).
Table 2.17: Relative MSE performance of individual models within a given class of models against AR benchmark, obtained with a pseudo real-time dataset and a real-time dataset. Both indicators and GDP revised

<table>
<thead>
<tr>
<th>model</th>
<th>monthly indicators</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Pseudo real-time dataset</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>midas</td>
<td>Industrial production - durable consumer goods</td>
<td>0.84</td>
<td>0.38</td>
<td>0.83</td>
<td>0.85</td>
<td>1.01</td>
<td>0.96</td>
<td>1.12</td>
<td>1.15</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>Unemployment - under 25 years</td>
<td>0.75</td>
<td>0.68</td>
<td>0.76</td>
<td>0.81</td>
<td>0.89</td>
<td>0.94</td>
<td>0.89</td>
<td>0.91</td>
<td>0.98</td>
</tr>
<tr>
<td>ar-midas</td>
<td>Industrial production - durable consumer goods</td>
<td>0.79</td>
<td>0.38</td>
<td>0.85</td>
<td>0.89</td>
<td>0.95</td>
<td>0.93</td>
<td>1.08</td>
<td>1.16</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>Unemployment - under 25 years</td>
<td>0.79</td>
<td>0.71</td>
<td>0.77</td>
<td>0.89</td>
<td>0.90</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
<td>1.00</td>
</tr>
<tr>
<td>mfd-var</td>
<td>Industrial production - durable consumer goods</td>
<td>0.78</td>
<td>0.73</td>
<td>0.97</td>
<td>1.02</td>
<td>1.01</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Unemployment - under 25 years</td>
<td>0.71</td>
<td>0.69</td>
<td>0.84</td>
<td>0.85</td>
<td>0.96</td>
<td>0.97</td>
<td>0.98</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td>bridge</td>
<td>Industrial production - durable consumer goods</td>
<td>0.52</td>
<td>0.43</td>
<td>0.76</td>
<td>0.88</td>
<td>1.01</td>
<td>0.97</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Unemployment - under 25 years</td>
<td>0.68</td>
<td>0.65</td>
<td>0.72</td>
<td>0.75</td>
<td>0.85</td>
<td>0.89</td>
<td>0.89</td>
<td>0.95</td>
<td>0.91</td>
</tr>
<tr>
<td>B. Real-time dataset</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>midas</td>
<td>Industrial production - durable consumer goods</td>
<td>0.78</td>
<td>0.50</td>
<td>0.86</td>
<td>0.90</td>
<td>1.03</td>
<td>1.01</td>
<td>1.12</td>
<td>1.10</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>Unemployment - under 25 years</td>
<td>0.65</td>
<td>0.69</td>
<td>0.76</td>
<td>0.83</td>
<td>0.90</td>
<td>0.97</td>
<td>0.92</td>
<td>0.94</td>
<td>0.97</td>
</tr>
<tr>
<td>ar-midas</td>
<td>Industrial production - durable consumer goods</td>
<td>0.71</td>
<td>0.51</td>
<td>0.87</td>
<td>0.99</td>
<td>0.96</td>
<td>1.01</td>
<td>1.11</td>
<td>1.13</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>Unemployment - under 25 years</td>
<td>0.75</td>
<td>0.71</td>
<td>0.74</td>
<td>0.88</td>
<td>0.91</td>
<td>0.94</td>
<td>1.00</td>
<td>0.97</td>
<td>0.95</td>
</tr>
<tr>
<td>mfd-var</td>
<td>Industrial production - durable consumer goods</td>
<td>0.83</td>
<td>0.77</td>
<td>0.98</td>
<td>1.01</td>
<td>1.01</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Unemployment - under 25 years</td>
<td>0.69</td>
<td>0.70</td>
<td>0.85</td>
<td>0.87</td>
<td>0.97</td>
<td>0.98</td>
<td>0.99</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>bridge</td>
<td>Industrial production - durable consumer goods</td>
<td>0.63</td>
<td>0.55</td>
<td>0.88</td>
<td>0.97</td>
<td>1.04</td>
<td>0.98</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Unemployment - under 25 years</td>
<td>0.69</td>
<td>0.74</td>
<td>0.77</td>
<td>0.80</td>
<td>0.87</td>
<td>0.90</td>
<td>0.90</td>
<td>0.96</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Notes: Panel A reports the results obtained with a pseudo real-time dataset, Panel B reports the results obtained with a genuine real-time dataset. Both the indicators and the GDP growth are subject to revisions. Forecasts are evaluated against the final vintage of GDP available at the downloaded date. The entries in the table are obtained as follows: first, estimate recursively every individual model and compute the relative MSE with respect to the benchmark; then, take the average all across the indicators of the relative MSE within a model class. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1.
Table 2.18: Relative MSE performance of individual models within a given class of models against AR benchmark, obtained with a pseudo real-time dataset and a real-time dataset. Only GDP revised

<table>
<thead>
<tr>
<th>model</th>
<th>monthly indicators</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>A. Pseudo real-time dataset</td>
<td>General economic situation over the next 12 months</td>
<td>0.45</td>
<td>0.55</td>
<td>0.52</td>
<td>0.57</td>
<td>0.74</td>
<td>0.75</td>
<td>0.93</td>
<td>0.90</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>Production expectations for the months ahead</td>
<td>0.32</td>
<td>0.34</td>
<td>0.40</td>
<td>0.50</td>
<td>0.67</td>
<td>0.81</td>
<td>0.90</td>
<td>0.94</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>Economic sentiment indicator</td>
<td>0.32</td>
<td>0.38</td>
<td>0.43</td>
<td>0.55</td>
<td>0.67</td>
<td>0.73</td>
<td>0.86</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>General economic situation over the next 12 months</td>
<td>0.45</td>
<td>0.51</td>
<td>0.53</td>
<td>0.59</td>
<td>0.73</td>
<td>0.73</td>
<td>0.97</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>ar-midas</td>
<td>Production expectations for the months ahead</td>
<td>0.33</td>
<td>0.36</td>
<td>0.43</td>
<td>0.53</td>
<td>0.68</td>
<td>0.81</td>
<td>0.89</td>
<td>0.95</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>Economic sentiment indicator</td>
<td>0.34</td>
<td>0.39</td>
<td>0.46</td>
<td>0.59</td>
<td>0.66</td>
<td>0.72</td>
<td>0.86</td>
<td>0.85</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>General economic situation over the next 12 months</td>
<td>0.59</td>
<td>0.54</td>
<td>0.57</td>
<td>0.63</td>
<td>0.74</td>
<td>0.74</td>
<td>0.83</td>
<td>0.85</td>
<td>0.86</td>
</tr>
<tr>
<td>mf-var</td>
<td>Production expectations for the months ahead</td>
<td>0.38</td>
<td>0.38</td>
<td>0.42</td>
<td>0.54</td>
<td>0.67</td>
<td>0.78</td>
<td>0.89</td>
<td>0.93</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>Economic sentiment indicator</td>
<td>0.48</td>
<td>0.34</td>
<td>0.35</td>
<td>0.50</td>
<td>0.67</td>
<td>0.73</td>
<td>0.83</td>
<td>0.86</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>General economic situation over the next 12 months</td>
<td>0.61</td>
<td>0.55</td>
<td>0.56</td>
<td>0.61</td>
<td>0.71</td>
<td>0.68</td>
<td>0.82</td>
<td>0.85</td>
<td>0.84</td>
</tr>
<tr>
<td>bridge</td>
<td>Production expectations for the months ahead</td>
<td>0.39</td>
<td>0.38</td>
<td>0.43</td>
<td>0.52</td>
<td>0.64</td>
<td>0.78</td>
<td>0.92</td>
<td>0.96</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Economic sentiment indicator</td>
<td>0.34</td>
<td>0.39</td>
<td>0.40</td>
<td>0.44</td>
<td>0.59</td>
<td>0.61</td>
<td>0.82</td>
<td>0.85</td>
<td>0.81</td>
</tr>
<tr>
<td>B. Real-time dataset</td>
<td>General economic situation over the next 12 months</td>
<td>0.62</td>
<td>0.58</td>
<td>0.59</td>
<td>0.72</td>
<td>0.76</td>
<td>0.93</td>
<td>0.89</td>
<td>0.91</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>Production expectations for the months ahead</td>
<td>0.35</td>
<td>0.34</td>
<td>0.50</td>
<td>0.64</td>
<td>0.68</td>
<td>0.90</td>
<td>0.91</td>
<td>0.94</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>Economic sentiment indicator</td>
<td>0.42</td>
<td>0.40</td>
<td>0.54</td>
<td>0.66</td>
<td>0.69</td>
<td>0.89</td>
<td>0.87</td>
<td>0.89</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>General economic situation over the next 12 months</td>
<td>0.57</td>
<td>0.54</td>
<td>0.60</td>
<td>0.72</td>
<td>0.77</td>
<td>0.96</td>
<td>0.91</td>
<td>0.89</td>
<td>0.99</td>
</tr>
<tr>
<td>ar-midas</td>
<td>Production expectations for the months ahead</td>
<td>0.36</td>
<td>0.36</td>
<td>0.51</td>
<td>0.65</td>
<td>0.68</td>
<td>0.90</td>
<td>0.91</td>
<td>0.95</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>Economic sentiment indicator</td>
<td>0.43</td>
<td>0.40</td>
<td>0.56</td>
<td>0.68</td>
<td>0.67</td>
<td>0.88</td>
<td>0.86</td>
<td>0.88</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>General economic situation over the next 12 months</td>
<td>0.61</td>
<td>0.56</td>
<td>0.65</td>
<td>0.71</td>
<td>0.75</td>
<td>0.82</td>
<td>0.82</td>
<td>0.86</td>
<td>0.91</td>
</tr>
<tr>
<td>mf-var</td>
<td>Production expectations for the months ahead</td>
<td>0.41</td>
<td>0.40</td>
<td>0.57</td>
<td>0.61</td>
<td>0.68</td>
<td>0.89</td>
<td>0.90</td>
<td>0.92</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>Economic sentiment indicator</td>
<td>0.56</td>
<td>0.49</td>
<td>0.69</td>
<td>0.58</td>
<td>0.70</td>
<td>0.85</td>
<td>0.84</td>
<td>0.87</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>General economic situation over the next 12 months</td>
<td>0.62</td>
<td>0.57</td>
<td>0.63</td>
<td>0.69</td>
<td>0.71</td>
<td>0.82</td>
<td>0.81</td>
<td>0.85</td>
<td>0.91</td>
</tr>
<tr>
<td>bridge</td>
<td>Production expectations for the months ahead</td>
<td>0.40</td>
<td>0.38</td>
<td>0.54</td>
<td>0.61</td>
<td>0.65</td>
<td>0.91</td>
<td>0.93</td>
<td>0.94</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Economic sentiment indicator</td>
<td>0.45</td>
<td>0.44</td>
<td>0.49</td>
<td>0.59</td>
<td>0.61</td>
<td>0.83</td>
<td>0.83</td>
<td>0.86</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Notes: Panel A reports the results obtained with a pseudo real-time dataset, Panel B reports the results obtained with a genuine real-time dataset. Only GDP growth is subject to revisions. Forecasts are evaluated against the final vintage of GDP available at the downloaded date. The entries in the table are obtained as follows: first, estimate recursively every individual model and compute the relative MSE with respect to the benchmark; then, take the average all across the indicators of the relative MSE within a model class. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1.
2.12 Appendix B: Results for euro area GDP components

In this Appendix, instead of focusing on the economic activity at aggregate level, we look at the disaggregate GDP components. Considering a decomposition from the output side, we follow the NACE classification of the GDP and obtain six branches of activity: agriculture, hunting, forestry and fishing; industry, excluding construction; construction; trade, hotels and restaurants, transport and communication services; financial services and business activities; other services. From the expenditure side, instead, we obtain five components: household final consumption; government final consumption; gross fixed capital formation, imports and exports.

In the Tables 2.19 and 2.20, we compare the performance of the different approaches with respect to the benchmark, which is an AR model for each singular component, where the lag length is specified accordingly to the BIC criterion. What we show is the average relative MSE performance of the different classes of mixed-frequency models against the AR benchmark. We consider nowcasts and forecasts up to two quarters ahead. In Table 2.19 we present the results for the components from the output side, while in Table 2.20 we present the results for the components from the expenditure side.

The evidence is quite mixed, depending on which component we are focusing on. More in detail, looking at the GDP disaggregation from the output side, the mixed-frequency approaches outperform the benchmark for those components for which many monthly indicators are available, as in the case of the industry sector, trade and financial services. For agriculture, availability of monthly indicators is critical. The same holds for the last branch that includes a variety of economic activities (public administration and defence, compulsory social security; education; health and social work; other community, social and personal service activities; private households with employed persons) for which it is not easy to find reliable and timely monthly indicators of value added.

As a general remark, the AR-MIDAS outperform the correspondent basic MIDAS. Moreover, this kind of models seems to work particularly well for short horizons. The evidence in favour of MF-VAR is less strong, and, as for the results on the aggregate measure of economic activity discussed in Section 2.4, this approach provides better results for longer horizons. Bridge models perform well, usually as well as MIDAS but they are outperformed by AR-MIDAS especially at very short horizons.

Looking now at the components from the expenditure side, we can reach the same kind of conclusions. Since for the government final consumption it is very hard to find monthly
indicators, it is very difficult to beat the benchmark (no gains from using monthly indicators appear at any horizon). For all the other components, bridge equations and AR-MIDAS have a better performance than the benchmark (the latter showing slightly better results than the former even in this case). The MF-VAR approach has a less clear cut performance also in this case, but it performs better than the benchmark especially for longer horizons, beating even the MIDAS (with and without AR dynamics) for $h_m = 8, 9$ forecast horizons for some components.

We now move to analyze the gains in case of forecast pooling, to assess whether there is any benefit from combining forecasts from alternative models with different explanatory variables. We calculate the forecasts with the usual three different combination schemes, but we report only the results obtained with the median, to save space.

Even with pooling, improvements are only obtained for those components for which there is a variety of indicators available, namely total industry, trade and financial services from the output side (see Table 2.21), and household final consumption, gross fixed capital formation and external balance from the expenditure side (see Table 2.22).

Generally, forecast combinations of AR-MIDAS models perform pretty well, outperforming the benchmark at several horizons. Also combinations within simple MIDAS and bridge equations allow for gains at some horizons. Pooling within the MF-VAR class also beats the benchmark but not very often, and even when the performance is better, the gains are not so big, confirming the results found for the aggregate GDP. Pooling across all the methods gives good results for some components, and generally only for very short horizons.

To conclude, as in Section 2.7, we analyze the behavior of large scale factor models in now- and forecasting each single component of the GDP, from the expenditure (Table 2.23) and supply side (Table 2.24). We can reach the same conclusions as for the aggregate GDP growth. Evidence is in favour of the use of factor models to predict the quarterly growth of each component for which the dataset contains useful information. There are significant gains especially at very short horizons: generally, exploiting the unbalanced structure of the dataset improves the performance, and the inclusion of an AR component reduces the MSE, even though not systematically. As for the case of the GDP growth discussed in Section 2.7, also for the components of GDP the factor models seem to better perform relative to the benchmark than forecast pooling. This is true especially for nowcasts and short-term forecasts. However, when the forecast horizon increases, the outperformance of the factor models is no longer evident, and in many cases forecast pooling is better. Finally, for these long horizons, both methods of summarizing information (factors and forecast pooling) generally fail in beating the AR benchmark.
Table 2.19: Average relative MSE performance of different classes of mixed-frequency models against AR benchmark - GDP components (supply side)

<table>
<thead>
<tr>
<th>Component</th>
<th>Model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, hunting, forestry and fishing</td>
<td>bridge</td>
<td>0.70</td>
<td>0.98</td>
<td>0.97</td>
<td>0.97</td>
<td>1.10</td>
<td>1.09</td>
<td>1.09</td>
<td>1.13</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.72</td>
<td>1.04</td>
<td>1.03</td>
<td>1.05</td>
<td>1.19</td>
<td>1.19</td>
<td>1.18</td>
<td>1.22</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.81</td>
<td>1.10</td>
<td>1.09</td>
<td>1.11</td>
<td>1.32</td>
<td>1.34</td>
<td>1.31</td>
<td>1.07</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.76</td>
<td>1.04</td>
<td>1.01</td>
<td>1.00</td>
<td>1.09</td>
<td>1.08</td>
<td>1.08</td>
<td>1.14</td>
<td>1.14</td>
</tr>
<tr>
<td>Total industry, excluding construction</td>
<td>bridge</td>
<td>2.06</td>
<td>0.94</td>
<td>0.98</td>
<td>1.01</td>
<td>0.93</td>
<td>0.95</td>
<td>0.96</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>2.01</td>
<td>0.95</td>
<td>1.00</td>
<td>1.04</td>
<td>0.94</td>
<td>0.96</td>
<td>0.95</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.92</td>
<td>0.89</td>
<td>0.94</td>
<td>0.97</td>
<td>0.95</td>
<td>0.96</td>
<td>0.95</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.45</td>
<td>0.94</td>
<td>0.98</td>
<td>1.02</td>
<td>0.95</td>
<td>0.96</td>
<td>0.97</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Construction</td>
<td>bridge</td>
<td>0.95</td>
<td>1.10</td>
<td>1.07</td>
<td>1.08</td>
<td>1.08</td>
<td>1.06</td>
<td>1.05</td>
<td>1.07</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.91</td>
<td>1.05</td>
<td>1.03</td>
<td>1.04</td>
<td>1.05</td>
<td>1.03</td>
<td>1.04</td>
<td>1.07</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.98</td>
<td>1.09</td>
<td>1.08</td>
<td>1.08</td>
<td>1.08</td>
<td>1.06</td>
<td>1.06</td>
<td>1.13</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.02</td>
<td>1.95</td>
<td>1.07</td>
<td>1.05</td>
<td>2.72</td>
<td>1.01</td>
<td>1.01</td>
<td>3.26</td>
<td>1.01</td>
</tr>
<tr>
<td>Trade, hotels and restaurants, transport and communication services</td>
<td>bridge</td>
<td>1.18</td>
<td>1.12</td>
<td>1.13</td>
<td>1.16</td>
<td>0.94</td>
<td>0.96</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>1.11</td>
<td>1.11</td>
<td>1.14</td>
<td>1.18</td>
<td>0.91</td>
<td>0.95</td>
<td>0.95</td>
<td>0.96</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>1.05</td>
<td>0.94</td>
<td>0.95</td>
<td>0.98</td>
<td>0.94</td>
<td>0.97</td>
<td>0.97</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.22</td>
<td>1.12</td>
<td>1.15</td>
<td>1.20</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Financial services and business activities</td>
<td>bridge</td>
<td>0.87</td>
<td>0.86</td>
<td>0.90</td>
<td>0.94</td>
<td>0.98</td>
<td>0.98</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.87</td>
<td>0.82</td>
<td>0.86</td>
<td>0.90</td>
<td>0.96</td>
<td>0.98</td>
<td>0.99</td>
<td>1.04</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.83</td>
<td>0.78</td>
<td>0.82</td>
<td>0.83</td>
<td>0.90</td>
<td>0.91</td>
<td>0.95</td>
<td>1.04</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.86</td>
<td>0.85</td>
<td>0.91</td>
<td>0.94</td>
<td>1.00</td>
<td>0.98</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Other services</td>
<td>bridge</td>
<td>1.07</td>
<td>1.07</td>
<td>1.05</td>
<td>1.04</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>1.02</td>
<td>1.15</td>
<td>1.10</td>
<td>1.10</td>
<td>1.07</td>
<td>1.06</td>
<td>1.10</td>
<td>1.18</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>1.09</td>
<td>1.17</td>
<td>1.12</td>
<td>1.12</td>
<td>1.13</td>
<td>1.13</td>
<td>1.16</td>
<td>1.40</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.15</td>
<td>1.14</td>
<td>1.11</td>
<td>1.08</td>
<td>1.03</td>
<td>1.02</td>
<td>1.02</td>
<td>1.03</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Notes: The statistics refer to the series and to their revisions compared to the last available vintage (August 2009).
Table 2.20: Average relative MSE performance of different classes of mixed-frequency models against AR benchmark - GDP components (expenditure side)

<table>
<thead>
<tr>
<th>component</th>
<th>model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final consumption -</td>
<td>bridge</td>
<td>1.08</td>
<td>0.98</td>
<td>0.95</td>
<td>0.96</td>
<td>0.95</td>
<td>0.96</td>
<td>0.98</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>households</td>
<td>midas</td>
<td>1.10</td>
<td>1.03</td>
<td>1.00</td>
<td>1.02</td>
<td>0.98</td>
<td>1.07</td>
<td>1.06</td>
<td>1.05</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>1.02</td>
<td>0.86</td>
<td>0.83</td>
<td>0.85</td>
<td>0.82</td>
<td>0.86</td>
<td>0.88</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.10</td>
<td>0.98</td>
<td>0.95</td>
<td>0.96</td>
<td>0.97</td>
<td>0.97</td>
<td>0.98</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>Final consumption -</td>
<td>bridge</td>
<td>1.10</td>
<td>1.03</td>
<td>1.02</td>
<td>1.02</td>
<td>1.03</td>
<td>1.03</td>
<td>1.03</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>Government</td>
<td>midas</td>
<td>1.15</td>
<td>1.10</td>
<td>1.03</td>
<td>1.05</td>
<td>1.11</td>
<td>1.10</td>
<td>1.18</td>
<td>1.15</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>1.10</td>
<td>1.12</td>
<td>1.07</td>
<td>1.07</td>
<td>1.23</td>
<td>1.24</td>
<td>1.29</td>
<td>1.27</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.41</td>
<td>1.63</td>
<td>1.56</td>
<td>1.45</td>
<td>1.79</td>
<td>1.66</td>
<td>1.53</td>
<td>1.85</td>
<td>1.74</td>
</tr>
<tr>
<td>Gross fixed capital</td>
<td>bridge</td>
<td>1.35</td>
<td>1.06</td>
<td>1.06</td>
<td>1.11</td>
<td>1.00</td>
<td>1.01</td>
<td>1.03</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>formation</td>
<td>midas</td>
<td>1.32</td>
<td>1.10</td>
<td>1.15</td>
<td>1.14</td>
<td>1.03</td>
<td>1.06</td>
<td>1.06</td>
<td>1.02</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.93</td>
<td>0.91</td>
<td>0.95</td>
<td>0.95</td>
<td>0.98</td>
<td>0.96</td>
<td>0.95</td>
<td>1.05</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.38</td>
<td>1.14</td>
<td>1.18</td>
<td>1.21</td>
<td>1.21</td>
<td>1.07</td>
<td>1.07</td>
<td>1.08</td>
<td>1.04</td>
</tr>
<tr>
<td>Imports</td>
<td>bridge</td>
<td>1.07</td>
<td>0.88</td>
<td>0.92</td>
<td>0.97</td>
<td>0.89</td>
<td>0.90</td>
<td>0.92</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>1.07</td>
<td>0.90</td>
<td>0.97</td>
<td>0.96</td>
<td>0.88</td>
<td>0.88</td>
<td>0.87</td>
<td>0.80</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.82</td>
<td>0.81</td>
<td>0.86</td>
<td>0.87</td>
<td>0.86</td>
<td>0.85</td>
<td>0.85</td>
<td>0.84</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.03</td>
<td>0.91</td>
<td>0.96</td>
<td>1.01</td>
<td>0.93</td>
<td>0.93</td>
<td>0.94</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>Exports</td>
<td>bridge</td>
<td>1.41</td>
<td>0.91</td>
<td>0.96</td>
<td>1.00</td>
<td>0.96</td>
<td>0.97</td>
<td>0.99</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>1.38</td>
<td>0.91</td>
<td>0.97</td>
<td>0.99</td>
<td>0.97</td>
<td>0.95</td>
<td>0.95</td>
<td>0.94</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.91</td>
<td>0.86</td>
<td>0.93</td>
<td>0.95</td>
<td>0.97</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.16</td>
<td>0.92</td>
<td>0.98</td>
<td>1.01</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
<td>0.98</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Notes: The entries in the table are obtained as follows: first, estimate recursively every individual model and compute the relative MSE with respect to the benchmark; then, take the average all across the indicators of the relative MSE within a model class. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the best relative MSE performance for each horizon and each component.
Table 2.21: Relative MSE performance of model pooling within a given class of models against AR benchmark (combination scheme: median)

<table>
<thead>
<tr>
<th>component</th>
<th>model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bridge</td>
<td>0.68</td>
<td>0.95</td>
<td>0.94</td>
<td>0.94</td>
<td>1.06</td>
<td>1.06</td>
<td>1.06</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.95</td>
<td>0.96</td>
<td>0.96</td>
<td>0.97</td>
<td>1.09</td>
<td>1.09</td>
<td>1.08</td>
<td>1.12</td>
<td>1.13</td>
</tr>
<tr>
<td>Agriculture, hunting, forestry and fishing</td>
<td>midas</td>
<td>0.75</td>
<td>1.01</td>
<td>1.01</td>
<td>1.02</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.96</td>
<td>0.96</td>
<td>0.95</td>
<td>0.94</td>
<td>1.07</td>
<td>1.07</td>
<td>1.06</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.75</td>
<td>1.01</td>
<td>1.01</td>
<td>1.02</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>1.08</td>
<td>1.08</td>
<td>1.07</td>
<td>1.09</td>
<td>1.10</td>
</tr>
<tr>
<td>Total industry, excluding construction</td>
<td>ar-midas</td>
<td>0.90</td>
<td>0.90</td>
<td>0.94</td>
<td>0.96</td>
<td>0.93</td>
<td>0.96</td>
<td>0.96</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.90</td>
<td>0.90</td>
<td>0.94</td>
<td>0.96</td>
<td>0.93</td>
<td>0.96</td>
<td>0.96</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.19</td>
<td>0.97</td>
<td>1.01</td>
<td>1.06</td>
<td>0.96</td>
<td>0.97</td>
<td>0.97</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.42</td>
<td>0.94</td>
<td>0.98</td>
<td>1.02</td>
<td>0.95</td>
<td>0.96</td>
<td>0.97</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Construction</td>
<td>bridge</td>
<td>0.83</td>
<td>0.96</td>
<td>0.96</td>
<td>0.98</td>
<td>0.99</td>
<td>0.97</td>
<td>0.96</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.87</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.93</td>
<td>1.03</td>
<td>1.03</td>
<td>1.04</td>
<td>1.02</td>
<td>1.01</td>
<td>1.02</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.88</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.85</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>bridge</td>
<td>1.14</td>
<td>1.17</td>
<td>1.17</td>
<td>1.22</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.08</td>
<td>1.04</td>
<td>1.08</td>
<td>1.12</td>
<td>0.96</td>
<td>0.98</td>
<td>0.99</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>bridge</td>
<td>0.76</td>
<td>0.82</td>
<td>0.90</td>
<td>0.95</td>
<td>1.00</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.78</td>
<td>0.70</td>
<td>0.80</td>
<td>0.83</td>
<td>0.89</td>
<td>0.95</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.73</td>
<td>0.70</td>
<td>0.78</td>
<td>0.76</td>
<td>0.84</td>
<td>0.86</td>
<td>0.90</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.71</td>
<td>0.79</td>
<td>0.92</td>
<td>0.96</td>
<td>1.00</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.74</td>
<td>0.75</td>
<td>0.85</td>
<td>0.87</td>
<td>0.94</td>
<td>0.96</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>bridge</td>
<td>0.98</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
<td>0.98</td>
<td>0.97</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.91</td>
<td>0.98</td>
<td>1.01</td>
<td>1.00</td>
<td>0.99</td>
<td>1.01</td>
<td>1.02</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.95</td>
<td>1.00</td>
<td>1.01</td>
<td>1.02</td>
<td>1.03</td>
<td>1.06</td>
<td>1.06</td>
<td>1.29</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.99</td>
<td>1.02</td>
<td>1.03</td>
<td>1.03</td>
<td>1.01</td>
<td>1.01</td>
<td>1.02</td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.94</td>
<td>0.99</td>
<td>1.02</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.02</td>
<td>1.02</td>
<td>1.02</td>
</tr>
</tbody>
</table>

**Notes:** The entries in the table are obtained as follows: first, forecasts from single indicator models are computed, and the median of all the forecasts within a single model is obtained; second, the MSE of the forecast combination is calculated and divided by the MSE of the benchmark. Only the last row of each component considers the median of all the forecasts across methods. The estimation is conducted recursively. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the best relative MSE performance for each horizon and each indicator.
Table 2.22: Relative MSE performance of model pooling within a given class of models against AR benchmark (combination scheme: median)

<table>
<thead>
<tr>
<th>component</th>
<th>model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bridge</td>
<td>1.04</td>
<td>0.99</td>
<td>1.00</td>
<td>1.01</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>1.03</td>
<td>0.93</td>
<td>0.99</td>
<td>0.99</td>
<td>0.92</td>
<td>1.01</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>Final consumption</td>
<td>ar-midas</td>
<td>0.92</td>
<td>0.75</td>
<td>0.77</td>
<td>0.79</td>
<td>0.73</td>
<td>0.75</td>
<td>0.75</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td>households</td>
<td>mf-var</td>
<td>0.95</td>
<td>0.93</td>
<td>0.96</td>
<td>0.98</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>all</td>
<td>0.95</td>
<td>0.87</td>
<td>0.91</td>
<td>0.93</td>
<td>0.91</td>
<td>0.95</td>
<td>0.95</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>bridge</td>
<td>1.04</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>1.06</td>
<td>0.99</td>
<td>0.95</td>
<td>0.98</td>
<td>1.00</td>
<td>1.02</td>
<td>1.04</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>1.01</td>
<td>0.99</td>
<td>0.97</td>
<td>0.99</td>
<td>1.08</td>
<td>1.12</td>
<td>1.12</td>
<td>1.13</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.11</td>
<td>1.04</td>
<td>1.03</td>
<td>1.03</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>all</td>
<td>1.04</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>1.00</td>
<td>1.01</td>
<td>1.01</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Gross fixed capital</td>
<td>bridge</td>
<td>1.31</td>
<td>1.07</td>
<td>1.10</td>
<td>1.16</td>
<td>1.03</td>
<td>1.05</td>
<td>1.05</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>formation</td>
<td>midas</td>
<td>1.28</td>
<td>1.06</td>
<td>1.17</td>
<td>1.15</td>
<td>1.01</td>
<td>1.04</td>
<td>1.03</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.88</td>
<td>0.90</td>
<td>0.94</td>
<td>0.94</td>
<td>0.95</td>
<td>0.94</td>
<td>0.94</td>
<td>1.01</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.27</td>
<td>1.12</td>
<td>1.20</td>
<td>1.24</td>
<td>1.07</td>
<td>1.08</td>
<td>1.08</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>all</td>
<td>1.07</td>
<td>1.00</td>
<td>1.08</td>
<td>1.11</td>
<td>1.01</td>
<td>1.04</td>
<td>1.04</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Imports</td>
<td>bridge</td>
<td>0.98</td>
<td>0.87</td>
<td>0.95</td>
<td>1.01</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>0.99</td>
<td>0.90</td>
<td>0.97</td>
<td>0.95</td>
<td>0.87</td>
<td>0.88</td>
<td>0.86</td>
<td>0.78</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.81</td>
<td>0.79</td>
<td>0.82</td>
<td>0.86</td>
<td>0.85</td>
<td>0.84</td>
<td>0.84</td>
<td>0.79</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>0.96</td>
<td>0.89</td>
<td>1.00</td>
<td>1.05</td>
<td>0.93</td>
<td>0.94</td>
<td>0.94</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>all</td>
<td>0.88</td>
<td>0.84</td>
<td>0.91</td>
<td>0.94</td>
<td>0.89</td>
<td>0.90</td>
<td>0.91</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td>Exports</td>
<td>bridge</td>
<td>1.41</td>
<td>1.05</td>
<td>1.00</td>
<td>1.05</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>midas</td>
<td>1.32</td>
<td>0.92</td>
<td>0.97</td>
<td>0.99</td>
<td>0.95</td>
<td>0.94</td>
<td>0.94</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>ar-midas</td>
<td>0.90</td>
<td>0.85</td>
<td>0.91</td>
<td>0.94</td>
<td>0.93</td>
<td>0.94</td>
<td>0.94</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>mf-var</td>
<td>1.07</td>
<td>0.93</td>
<td>1.01</td>
<td>1.04</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>all</td>
<td>1.05</td>
<td>0.90</td>
<td>0.96</td>
<td>0.99</td>
<td>0.97</td>
<td>0.97</td>
<td>0.98</td>
<td>0.97</td>
<td>0.97</td>
</tr>
</tbody>
</table>

**Notes:** The entries in the table are obtained as follows: first, forecasts from single indicator models are computed, and the median of all the forecasts within a single model is obtained; second, the MSE of the forecast combination is calculated and divided by the MSE of the benchmark. Only the last row of each component considers the median of all the forecasts across methods. The estimation is conducted recursively. The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1. The numbers in bold show the best relative MSE performance for each horizon and each indicator.
Table 2.23: Relative MSE performance of different classes of factor models against AR benchmark - GDP components (supply side)

<table>
<thead>
<tr>
<th>component</th>
<th>model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, hunting, forestry and fishing</td>
<td>Factor-MIDAS (basic)</td>
<td>0.71</td>
<td>1.04</td>
<td>1.02</td>
<td>1.06</td>
<td>1.17</td>
<td>1.22</td>
<td>1.32</td>
<td>1.38</td>
<td>1.27</td>
</tr>
<tr>
<td></td>
<td>Factor-MIDAS (ar)</td>
<td>0.81</td>
<td>1.12</td>
<td>1.08</td>
<td>1.10</td>
<td>1.22</td>
<td>1.43</td>
<td>1.34</td>
<td>1.19</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>Quarterly factor model</td>
<td>0.69</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>Total industry, excluding construction</td>
<td>Factor-MIDAS (basic)</td>
<td>1.05</td>
<td>0.69</td>
<td>0.73</td>
<td>0.82</td>
<td>0.88</td>
<td>0.90</td>
<td>0.92</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>Factor-MIDAS (ar)</td>
<td>0.57</td>
<td>0.69</td>
<td>0.71</td>
<td>0.79</td>
<td>0.89</td>
<td>0.91</td>
<td>0.94</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>Quarterly factor model</td>
<td>0.90</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>Construction</td>
<td>Factor-MIDAS (basic)</td>
<td>0.76</td>
<td>0.96</td>
<td>0.90</td>
<td>0.99</td>
<td>1.00</td>
<td>0.97</td>
<td>1.04</td>
<td>1.06</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>Factor-MIDAS (ar)</td>
<td>0.83</td>
<td>0.99</td>
<td>0.92</td>
<td>1.01</td>
<td>0.99</td>
<td>0.97</td>
<td>1.05</td>
<td>1.10</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>Quarterly factor model</td>
<td>0.98</td>
<td>1.10</td>
<td>1.10</td>
<td>1.10</td>
<td>1.05</td>
<td>1.05</td>
<td>1.05</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td>Trade, hotels and restaurants, transport and</td>
<td>Factor-MIDAS (basic)</td>
<td>0.55</td>
<td>0.66</td>
<td>0.69</td>
<td>0.84</td>
<td>0.81</td>
<td>0.84</td>
<td>0.89</td>
<td>0.93</td>
<td>0.91</td>
</tr>
<tr>
<td>activities</td>
<td>Factor-MIDAS (ar)</td>
<td>0.56</td>
<td>0.58</td>
<td>0.61</td>
<td>0.76</td>
<td>0.82</td>
<td>0.86</td>
<td>0.93</td>
<td>0.90</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>Quarterly factor model</td>
<td>0.93</td>
<td>0.89</td>
<td>0.89</td>
<td>0.89</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Financial services and business activities</td>
<td>Factor-MIDAS (basic)</td>
<td>0.42</td>
<td>0.43</td>
<td>0.46</td>
<td>0.55</td>
<td>0.74</td>
<td>0.71</td>
<td>0.81</td>
<td>0.97</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>Factor-MIDAS (ar)</td>
<td>0.40</td>
<td>0.44</td>
<td>0.46</td>
<td>0.56</td>
<td>0.71</td>
<td>0.66</td>
<td>0.78</td>
<td>1.11</td>
<td>1.18</td>
</tr>
<tr>
<td></td>
<td>Quarterly factor model</td>
<td>0.64</td>
<td>0.92</td>
<td>0.92</td>
<td>0.92</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.15</td>
<td>1.15</td>
</tr>
<tr>
<td>Other services</td>
<td>Factor-MIDAS (basic)</td>
<td>0.87</td>
<td>0.87</td>
<td>0.96</td>
<td>0.96</td>
<td>1.00</td>
<td>0.96</td>
<td>1.13</td>
<td>1.09</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>Factor-MIDAS (ar)</td>
<td>0.96</td>
<td>0.91</td>
<td>0.96</td>
<td>0.96</td>
<td>1.13</td>
<td>1.00</td>
<td>1.22</td>
<td>1.13</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>Quarterly factor model</td>
<td>0.87</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>1.13</td>
<td>1.13</td>
</tr>
</tbody>
</table>

**Notes:** The entries in the table are obtained as follows: first, estimate recursively every factor model, then compute the relative MSE with respect to the benchmark. The factors are estimated with the EM algorithm together with PCA as in Stock and Watson (2002). The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1.
Table 2.24: Relative MSE performance of different classes of factor models against AR benchmark - GDP components (expenditure side)

<table>
<thead>
<tr>
<th>component</th>
<th>model</th>
<th>model</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final consumption - households</td>
<td>Factor-MIDAS (basic)</td>
<td>0.71</td>
<td>0.72</td>
<td>0.72</td>
<td>0.78</td>
<td>0.97</td>
<td>0.93</td>
<td>0.97</td>
<td>1.21</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Factor-MIDAS (ar)</td>
<td>0.69</td>
<td>0.57</td>
<td>0.58</td>
<td>0.67</td>
<td>0.72</td>
<td>0.70</td>
<td>0.68</td>
<td>1.17</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Quarterly factor model</td>
<td>1.07</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
<td>0.83</td>
<td>0.83</td>
<td>0.79</td>
<td>0.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final consumption - Government</td>
<td>Factor-MIDAS (basic)</td>
<td>1.03</td>
<td>1.06</td>
<td>0.96</td>
<td>1.00</td>
<td>0.93</td>
<td>0.91</td>
<td>1.00</td>
<td>1.01</td>
<td>1.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Factor-MIDAS (ar)</td>
<td>0.97</td>
<td>1.06</td>
<td>1.03</td>
<td>1.01</td>
<td>1.04</td>
<td>1.00</td>
<td>1.12</td>
<td>1.09</td>
<td>1.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Quarterly factor model</td>
<td>0.96</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>1.22</td>
<td>1.22</td>
<td>1.22</td>
<td>0.93</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>Gross fixed capital formation</td>
<td>Factor-MIDAS (basic)</td>
<td>0.68</td>
<td>0.78</td>
<td>0.79</td>
<td>0.92</td>
<td>0.99</td>
<td>0.97</td>
<td>1.02</td>
<td>1.05</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Factor-MIDAS (ar)</td>
<td>0.63</td>
<td>0.68</td>
<td>0.71</td>
<td>0.78</td>
<td>0.95</td>
<td>0.94</td>
<td>0.98</td>
<td>1.09</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Quarterly factor model</td>
<td>1.11</td>
<td>1.09</td>
<td>1.09</td>
<td>1.09</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
<td>1.06</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>Imports</td>
<td>Factor-MIDAS (basic)</td>
<td>0.70</td>
<td>0.66</td>
<td>0.74</td>
<td>0.83</td>
<td>0.86</td>
<td>0.86</td>
<td>0.88</td>
<td>0.90</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Factor-MIDAS (ar)</td>
<td>0.60</td>
<td>0.59</td>
<td>0.66</td>
<td>0.59</td>
<td>0.82</td>
<td>0.84</td>
<td>0.75</td>
<td>0.83</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Quarterly factor model</td>
<td>0.50</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.83</td>
<td>0.83</td>
<td>0.83</td>
<td>0.87</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>Exports</td>
<td>Factor-MIDAS (basic)</td>
<td>0.72</td>
<td>0.64</td>
<td>0.72</td>
<td>0.79</td>
<td>0.87</td>
<td>0.88</td>
<td>0.96</td>
<td>0.93</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Factor-MIDAS (ar)</td>
<td>0.62</td>
<td>0.64</td>
<td>0.72</td>
<td>0.75</td>
<td>0.87</td>
<td>0.88</td>
<td>0.96</td>
<td>0.93</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Quarterly factor model</td>
<td>0.75</td>
<td>0.91</td>
<td>0.91</td>
<td>0.91</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.04</td>
<td>1.04</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The entries in the table are obtained as follows: first, estimate recursively every factor model, then compute the relative MSE with respect to the benchmark. The factors are estimated with the EM algorithm together with PCA as in Stock and Watson (2002). The benchmark is the recursive estimate of an AR model with lag length specified accordingly to the BIC criterion. The evaluation sample is 2003Q1-2009Q1.
2.13 Appendix C: Dataset description

MONTHLY DATA

HICP - All items excluding energy and unprocessed food
HICP - All items excluding energy, food, alcohol and tobacco
HICP - All items excluding energy and seasonal food
HICP - All items excluding energy
HICP - All items excluding tobacco
HICP - All items (HICP=Harmonized Index of Consumer Prices)
HICP - Food and non alcoholic beverages
HICP - Alcoholic beverages and tobacco
HICP - Clothing and footwear
HICP - Housing, water, electricity, gas and other fuels
HICP - Furnishings, household equipment and maintenance
HICP - Health
HICP - Transport
HICP - Communication
HICP - Recreation and culture
HICP - Education
HICP - Hotels, cafes and restaurants
HICP - Miscellaneous goods and services
HICP - Energy
HICP - Food
Producer price index - Electricity, gas, steam and air conditioning supply
Producer price index - Industry (except construction), sewerage, waste management and remediation activities
Producer price index - Mining and quarrying; manufacturing; electricity, gas, steam and air conditioning supply
Producer price index - Mining and quarrying
Producer price index - B_TO_E36
Producer price index - Manufacturing
Producer price index - Manufacturing, for new orders
Producer price index - Electricity, gas, steam and air conditioning supply
Producer price index - Water collection, treatment and supply
Producer price index - Capital goods
Producer price index - Consumer goods
Producer price index - Durable consumer goods
Producer price index - Intermediate goods
Producer price index - Non-durable consumer goods
Producer price index - Energy
Business surveys - Construction - Construction confidence indicator
Business surveys - Construction - Employment expectations for the months ahead
Business surveys - Construction - Assessment of order books
Business surveys - Construction - Price expectations for the months ahead
Business surveys - Construction - Trend of activity compared with preceding months
Business Climate Indicator
Consumer confidence indicator
Consumer surveys - Financial situation over the last 12 months
Consumer surveys - Financial situation over the next 12 months
Consumer surveys - General economic situation over the last 12 months
Consumer surveys - General economic situation over the next 12 months
Consumer surveys - Major purchases over the next 12 months
Consumer surveys - Major purchases at present
Consumer surveys - Price trends over the last 12 months
Consumer surveys - Price trends over the next 12 months
Consumer surveys - Statement on financial situation of household
Consumer surveys - Savings over the next 12 months
Consumer surveys - Savings at present
2.13. APPENDIX C: DATASET DESCRIPTION

Consumer surveys - Unemployment expectations over the next 12 months
Business surveys - Industry - Industrial confidence indicator
Business surveys - Industry - Employment expectations for the months ahead
Business surveys - Industry - Assessment of export order-book levels
Business surveys - Industry - Assessment of order-book levels
Business surveys - Industry - Expectations for the months ahead
Business surveys - Industry - Production trend observed in recent months
Business surveys - Industry - Assessment of stocks of finished products
Business surveys - Industry - Selling price expectations for the months ahead
Business surveys - Retail - Assessment of stocks
Business surveys - Retail - Retail confidence indicator
Business surveys - Retail - Expected business situation
Business surveys - Retail - Employment
Business surveys - Retail - Orders placed with suppliers
Business surveys - Retail - Present business situation
Business survey - Services - Assessment of business climate
Business survey - Services - Evolution of demand expected in the months ahead
Business survey - Services - Evolution of demand in recent months
Business survey - Services - Services Confidence Indicator
Business survey - Services - Evolution of employment in recent months
Construction confidence indicator
Consumer confidence indicator
Economic sentiment indicator
Industrial confidence indicator
Retail confidence indicator
Services Confidence Indicator
Unemployment rate according to ILO definition - Over 25 years
Unemployment rate according to ILO definition - Under 25 years
Unemployment rate according to ILO definition
Unemployment according to ILO definition - Over 25 years - Total
Unemployment according to ILO definition - Under 25 years - Total
Unemployment according to ILO definition - Total
Production index
Production index - Buildings
Production index - Civil engineering works
Production index - Construction
Production index - Mining and quarrying; manufacturing; electricity, gas, steam and air conditioning supply
Production index - Mining and quarrying; manufacturing
Production index - Mining and quarrying
Production index - Manufacturing
Production index - Manufacturing, for new orders
Production index - Electricity, gas, steam and air conditioning supply
Production index - Capital goods
Production index - Consumer goods
Production index - Durable consumer goods
Production index - Intermediate goods
Production index - Non-durable consumer goods
Turnover index - domestic market - Mining and quarrying; manufacturing
Turnover index - non-domestic market - Mining and quarrying; manufacturing
Turnover index - total - Mining and quarrying; manufacturing
Turnover index - domestic market - Manufacturing
Turnover index - non-domestic market - Manufacturing
Turnover index - total - Manufacturing
Turnover index - domestic market - Manufacturing, for new orders
Turnover index - non-domestic market - Manufacturing, for new orders
Turnover index - total - Manufacturing, for new orders
Turnover index - domestic market - Capital goods
Turnover index - non-domestic market - Capital goods
Turnover index - total - Capital goods
Turnover index - domestic market - Consumer goods
Turnover index - non-domestic market - Consumer goods
Turnover index - total - Consumer goods
Turnover index - domestic market - Durable consumer goods
Turnover index - non-domestic market - Durable consumer goods
Turnover index - total - Durable consumer goods
Turnover index - domestic market - Intermediate goods
Turnover index - non-domestic market - Intermediate goods
Turnover index - total - Intermediate goods
Turnover index - domestic market - Non-durable consumer goods
Turnover index - total - Non-durable consumer goods
Hours worked index - Construction
New orders received index - Manufacturing, for new orders
New orders received index - Manufacturing, for new orders (except heavy transport equipment)
Number of new car registrations
Deflated turnover index - Retail sale of food, beverages and tobacco
Deflated turnover index - Retail trade, except of motor vehicles and motorcycles
Deflated turnover index - Retail sale of non-food products (including fuel)
Deflated turnover index - Retail sale of non-food products (except fuel)
Deflated turnover index - Retail trade, except of motor vehicles, motorcycles and fuel
MF-M1-SA Money supply M1 - SA
MF-M2-SA Money supply M2 - SA
MF-M3-SA Money supply M3 - SA
MF-3MI-RT 3-month interest rates (average)
MF-LTGBY-RT Long term government bond yields - Maastricht definition (average)
Exchange rates US Dollar against the ECU/euro (average)
Exchange rates Yen against the ECU/euro (average)
Exchange rates Pound Sterling against the ECU/euro (average)
DAX share price index
DJ EURO STOXX 50, price index
EM government bond yield - 2 year
EM government bond yield - 3 year
EM government bond yield - 5 year
EM government bond yield - 7 year
EM government bond yield - 10 year
Germany interbank 12 month - offered rate
Germany interbank 3 month - offered rate
Germany interbank 6 month - offered rate
German yields on fully taxed bonds outstanding - all bank bonds
German yields on fully taxed bonds outstanding - corporate bonds
QUARTERLY DATA

Gross value added at constant prices (mio euro) - Agriculture, hunting, forestry and fishing
Gross value added at constant prices (mio euro) - Total industry (excluding construction)
Gross value added at constant prices (mio euro) - Construction
Gross value added at constant prices (mio euro) - Wholesale and retail trade, repair of motor vehicles, motorcycles and personal and household goods; hotels and restaurants; transport, storage and communication
Gross value added at constant prices (mio euro) - Financial intermediation; real estate, renting and business activities
Gross value added at constant prices (mio euro) - Public administration and defence, compulsory social security; education; health and social work; other community, social and personal service activities; private households with employed persons
Gross value added at constant prices (mio euro) - All NACE branches - Total
Gross domestic product at market prices - CLV2000
Gross value added - CLV2000
Final consumption expenditure: household and NPISH - CLV2000
Final consumption expenditure: general government - CLV2000
Gross fixed capital formation - total - CLV2000
Exports - total - CLV2000
Imports - total - CLV2000
Chapter 3

U-MIDAS: MIDAS regressions with unrestricted lag polynomials

Joint with Massimiliano Marcellino and Christian Schumacher

3.1 Introduction

Economic time series differ substantially with respect to their sampling frequency. For example, financial variables are observable daily or even intra-daily, whereas national accounts data such as GDP is available at quarterly frequency depending on the rules applied in statistical agencies. This raises the problem of how to conduct empirical analyses on the relationships between variables sampled at different frequencies.

The simplest solution is to work at the lowest frequency in the data, e.g. quarterly when some variables are available on a monthly basis and others on a quarterly basis. This requires time aggregation of high-frequency variables with a loss of potentially relevant high-frequency information, and a convolution of the dynamic relationships among the variables (see e.g. Marcellino (1999)).

As an alternative, mixed data-sampling (MIDAS) regressions as proposed by Ghysels, Santa-Clara, and Valkanov (2005, 2006), Ghysels, Sinko, and Valkanov (2007) and Andreou, Ghysels and Kourtellos (2010a, 2010b), amongst others, directly relate variables sampled at different frequencies without losing high-frequency information. To allow for dynamics, MIDAS regressions are typically based on distributed lag polynomials such as the exponential

---

1In this chapter I focused on the Monte Carlo experiment and on the empirical analysis, collecting the data and writing the necessary codes to obtain the empirical results. The comments to the results have been jointly written by the authors.
Almon lag to ensure a parsimonious specification (Ghysels, Sinko, and Valkanov, 2007). Due to the non-linearity of the lag polynomials, MIDAS regressions are typically estimated by non-linear least squares (NLS) following the literature on distributed lag models (Lütkepohl, 1981; Judge et al., 1985).

MIDAS regressions have been applied in the financial literature, see for example Ghysels, Rubia, and Valkanov (2009) in the context of volatility forecasting. In the macroeconomic literature, applications are often related to nowcasting and forecasting. For example, Clements and Galvao (2008, 2009) proposed to use MIDAS for forecasting quarterly GDP growth using monthly business cycle indicators, see also Kuzin, Marcellino, and Schumacher (2011), Bai, Ghysels, and Wright (2011), Marcellino and Schumacher (2010), amongst others. The recent application by Andreou, Ghysels and Kourtellos (2010b) proposes MIDAS regressions when daily financial data is used to forecast quarterly GDP.

An alternative way to handle mixed-frequency data requires writing the model in state space form with time-aggregation schemes, see e.g. Mariano and Murasawa (2003). Kuzin et al. (2011) compare mixed frequency VARs estimated with the Kalman filter with MIDAS regressions, finding an unclear ranking but confirming the good performance of MIDAS. Bai et al. (2011) compare MIDAS regressions to state space models and discuss the approximating properties of MIDAS.

In this paper, we study the performance of a variant of MIDAS which does not resort to functional distributed lag polynomials. In particular, we discuss the pros and cons of MIDAS regressions with unrestricted linear lag polynomials, which do not require NLS, but can be estimated by OLS. We will call this approach from now on unrestricted MIDAS, or U-MIDAS in brief, and compare it to the standard MIDAS approach based on the exponential Almon lag following Ghysels et al. (2005, 2006), which we simply denote as MIDAS. One reason that motivates the use of U-MIDAS in macroeconomic applications is that the difference between sampling frequencies is in many applications not so high. For example, many of the cited papers use monthly data, such as survey outcomes or industrial production, to predict quarterly GDP growth. In that case, the number of monthly lags necessary to estimate the lag polynomials might not be too large, implying that a curse of dimensionality might not be relevant. However, when financial data come into play as in Andreou, Ghysels, and Kourtellos (2010b) and Monteforte and Moretti (2010), we face more severe limits in the degrees of freedom and functional lag polynomials may be preferable.

model framework. Rodriguez and Puggioni (2010) discuss Bayesian estimation of unrestricted MIDAS equations. However, none of these papers systematically studies the role of the functional form of the lag polynomial.

We expand on the existing literature in the following respects. We discuss how U-MIDAS regressions can be derived in a general linear dynamic framework, and under which conditions the parameters of the underlying high-frequency model can be identified.

Next, we provide Monte Carlo simulations that help highlighting the advantages and disadvantages of U-MIDAS versus MIDAS. The basic design of the exercise is similar to that of Ghysels and Valkanov (2006), where a high-frequency VAR(1) is specified. We look both at the in-sample and out-of-sample nowcasting performance. We find that if the frequency mismatch is small, i.e. when mixing monthly and quarterly data, U-MIDAS is indeed better than MIDAS. With larger differences in sampling frequencies, MIDAS with exponential Almon lag polynomials is instead preferable. We also consider the case in which the restricted MIDAS model is the true DGP. Even in this favorable set up for functional lag polynomials, it turns out that U-MIDAS is still preferable when the frequency mismatch is small.

Finally, we carry out an empirical exercise, where GDP growth in the Euro area and in the US are related to different monthly indicators. Both for the Euro area and the US, we find a better in-sample performance of the U-MIDAS model, confirming the results of the Monte Carlo experiments. The evidence is more mixed when looking at the nowcasting performance, especially when the estimation sample is relatively short, as in the case of the Euro area example. Nonetheless, for several indicators U-MIDAS can outperform MIDAS also out of sample. Thus, when the frequency mismatch is small in empirical applications, in particular when mixing quarterly and monthly data, U-MIDAS can be a strong competitor for MIDAS.

The paper proceeds as follows. In Section 3.2 we provide a theoretical motivation for U-MIDAS in a linear dynamic framework and discuss its use for nowcasting and forecasting. In Section 3.3 we present the results of the Monte Carlo experiments. In Section 3.4 we discuss the empirical nowcast exercises. In Section 3.5 we summarize the main results and conclude.
3.2 The rationale behind U-MIDAS and its use in forecasting

In this section we derive the Unrestricted MIDAS (U-MIDAS) regression approach from a general dynamic linear model, consider its use as a forecasting device, and compare it with the original MIDAS specification of Ghysels et al. (2005, 2006).

3.2.1 U-MIDAS regressions in dynamic linear models

We assume that $y$ and the $N$ variables $x$ are generated by the $\text{VAR}(p)$ process

$$
\begin{pmatrix}
    a(L)_{1 \times 1} & -b(L)_{1 \times N} \\
    -d(L)_{N \times 1} & C(L)_{N \times N}
\end{pmatrix}
\begin{pmatrix}
    y_t_{1 \times 1} \\
    x_t_{N \times 1}
\end{pmatrix} =
\begin{pmatrix}
    e_{yt} \ 1 \times 1 \\
    e_{xt} \ N \times 1
\end{pmatrix},
$$

(3.1)

or

$$a(L)y_t = b_1(L)x_{1t} + \ldots + b_N(L)x_{Nt} + e_{yt}
$$

(3.2)

$$C(L)x_t = d(L)y_t + e_{xt}
$$

(3.3)

where $a(L) = 1 - a_1L - \ldots - a_pL$, $b(L) = (b_1(L), \ldots, b_N(L))$, $b_j(L) = b_{j1}L + \ldots + b_{jp}L_p$, $j = 1, \ldots, N$, $d(L) = (d_1(L), \ldots, d_N(L))^\prime$, $d_j(L) = d_{j1}L + \ldots + d_{jp}L_p$, $C(L) = I - C_1L - \ldots - C_pL_p$, and the errors are jointly white noise. For simplicity, we suppose that the starting values $y_{-p}, \ldots, y_0$ and $x_{-p}, \ldots, x_0$ are all fixed and equal to zero, which coincides with the unconditional expected value of $y$ and $x$. Different lag lengths of the polynomials in (3.2) and (3.3) can be easily handled, but at the cost of an additional complication in the notation.

We then assume that $x$ can be observed for each $t$, while $y$ can be only observed every $k$ periods. For example, $k = 3$ when $t$ is measured in months and $y$ is observed quarterly (e.g., $x$ could contain industrial production and $y$ GDP growth), while $k = 4$ when $t$ is measured in quarters and $y$ is observed annually (e.g., $x$ could contain GDP growth and $y$ fiscal variables that are typically only available on an annual basis). Let us indicate the aggregate (low) frequency by $\tau$, while $Z$ is the lag operator at $\tau$ frequency, with $Z = L^k$ and $Zy_\tau = y_{\tau-1}$. In the sequel, HF indicates high frequency ($t$) and LF low frequency ($\tau$).

Let us then introduce the operator

$$\omega(L) = \omega_0 + \omega_1L + \ldots + \omega_{k-1}L^{k-1},
$$

(3.4)

which characterizes the temporal aggregation scheme. For example, $\omega(L) = 1 + L + \ldots + L^{k-1}$ in the case of flow variables and $\omega(L) = 1$ for stock variables.
While general, this framework still imposes a few restrictions. In particular, $y$ is univariate and there are no MA components in the generating mechanism of $y$ and $x$. These restrictions simplify substantially the notation, and are helpful for the identification of the parameters of the HF model for $y$ given the LF model. The framework remains general enough to handle the majority of empirical applications, and the extensions are theoretically simple but notationally cumbersome.

The method we adopt to derive the generating mechanism for $y$ at LF is similar to that introduced by Brewer (1973), refined by Wei (1981) and Weiss (1984), and further extended by Marcellino (1999) to deal with general aggregation schemes and multivariate processes.

Let us introduce a polynomial in the lag operator, $(L)^\text{pk}$, whose degree in $L$ is at most equal to $pk - p$ and which is such that the product $h(L) = \beta(L)\alpha(L)$ only contains powers of $L^k = Z$, so that $h(L) = h(L^k) = h(Z)$. It can be shown that such a polynomial always exists, and its coefficients depend on those of $\alpha(L)$, see the above references for details.

In order to determine the AR component of the LF process, we then multiply both sides of (3.2) by $(L)^{\beta(L)}$ to get

$$h(L^k)\omega(L)y_t = \beta(L)b_1(L)\omega(L)x_{1t} + \ldots + \beta(L)b_N(L)\omega(L)x_{Nt} + \beta(L)\omega(L)e_{yt}. \quad (3.5)$$

Thus, the order of the LF AR component, $h(Z)$, is at most equal to $p$. In addition, the polynomial $h(L^k)$ can be decomposed into

$$\prod_{s=1}^{h} \prod_{i=1}^{k} (1 - \frac{1}{h_{si}}L), \quad (3.6)$$

where $h < p$ is more precisely defined in Appendix A, and at least one $h_{si}$ for each $s$ has to be such that $a(h_{si}) = 0$.

It can be shown that, in general, there is an MA component in the LF model, $q(Z)u_{yt}$. Its order, $q$, coincides with the highest multiple of $k$ non zero lag in the autocovariance function of $\beta(L)\omega(L)e_{yt}$. The coefficients of the MA component have to be such that the implied autocovariances of $q(Z)u_{yt}$ coincide with those of $\beta(L)\omega(L)e_{yt}$ evaluated at all multiples of $k$.

Let us consider now the $x$ variables, which are observable at frequency $t$. The polynomials $\beta(L)b_j(L)\omega(L) = b_j(L)\beta(L)\omega(L)$, $j = 1,\ldots, N$, are at most of order $pk + k - 1$. Each term $\beta(L)\omega(L)x_{jt}$ is a particular combination of HF values of $x_j$ that affects the LF values of $y$.

In Appendix A we show that, under certain rather strict conditions, it is possible to recover the polynomials $\alpha(L)$ and $b_j(L)$ that appear in the HF model for $y$ from the LF model, and therefore also $\beta(L)$ can be identified. In this case we can use the exact MIDAS
model

\[ h(L^k)\omega(L)y_t = b_1(L)z_{1t} + \ldots + b_N(L)z_{Nt} + q(L^k)u_{yt}, \tag{3.7} \]
\[ z_{jt} = \beta(L)\omega(L)x_{jt}, \quad j = 1, \ldots, N, \]
\[ t = k, 2k, 3k, \ldots \]

The left-hand side of this equation contains the LF variable \( y_t \), obtained from time aggregation \( \omega(L)y_t = y_r \). The LF variable is regressed on its own LF lags and on lags of \( x_{jt} \) for \( j = 1, \ldots, N \). As the polynomials \( a(L) \) and \( b_j(L) \) are identified, there is no need for a polynomial approximation.

When \( \beta(L) \) cannot be identified, we can use an approximate unrestricted MIDAS model based on a linear lag polynomial such as

\[ c(L^k)\omega(L)y_t = \delta_1(L)x_{1t-1} + \ldots + \delta_N(L)x_{Nt-1} + \epsilon_t, \tag{3.8} \]
\[ t = k, 2k, 3k, \ldots \]

where \( c(L^k) = (1 - c_1L^k - \ldots - c_vL^{kv}) \), \( \delta_j(L) = (\delta_{j,0} + \delta_{j,1}L + \ldots + \delta_{j,v}L^v) \), \( j = 1, \ldots, N \). We label this approach hereafter unrestricted MIDAS or simply U-MIDAS. ² ³

Notice that, since the polynomials \( \delta_i(L) \) operate at HF while \( c(L^k) \) in LF, the matrix of regressors in (3.8) is of the type

\[
\begin{array}{cccccccc}
  y_0 & \ldots & y_{-kc} & \delta_{1,0}x_{1,k-1} & \ldots & \delta_{1,v}x_{1,k-v-1} & \ldots & \delta_{N,0}x_{N,k-1} & \ldots & \delta_{N,v}x_{N,k-v-1} \\
  y_k & \ldots & y_{-(k-1)c} & \delta_{1,0}x_{1,2k-1} & \ldots & \delta_{1,v}x_{1,2k-v-1} & \ldots & \delta_{N,0}x_{N,2k-1} & \ldots & \delta_{N,v}x_{N,2k-v-1} \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  y_{Tk-k} & \ldots & y_{Tk-kc} & \delta_{1,0}x_{1,Tk-1} & \ldots & \delta_{1,v}x_{1,Tk-v-1} & \ldots & \delta_{N,0}x_{N,Tk-1} & \ldots & \delta_{N,v}x_{N,Tk-v-1} \\
\end{array}
\]

As an example, if \( \omega(L) = 1 \), i.e. \( y \) is a stock variable, and \( k = 3 \) (i.e., \( t \) is monthly and \( \tau \) is quarterly), the matrix of regressors becomes

\[
\begin{array}{cccccccc}
  y_0 & \ldots & y_{-3c} & \delta_{1,0}x_{1,3-1} & \ldots & \delta_{1,v}x_{1,3-v-1} & \ldots & \delta_{N,0}x_{N,3-1} & \ldots & \delta_{N,v}x_{N,3-v-1} \\
  y_3 & \ldots & y_{-2c} & \delta_{1,0}x_{1,6-1} & \ldots & \delta_{1,v}x_{1,6-v-1} & \ldots & \delta_{N,0}x_{N,6-1} & \ldots & \delta_{N,v}x_{N,6-v-1} \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  y_{3T-3} & \ldots & y_{3T-3c} & \delta_{1,0}x_{1,3T-1} & \ldots & \delta_{1,v}x_{1,3T-v-1} & \ldots & \delta_{N,0}x_{N,3T-1} & \ldots & \delta_{N,v}x_{N,3T-v-1} \\
\end{array}
\]

²The static version of U-MIDAS corresponds to the direct mixed frequency regression model of Kvedaras and Rackauskas (2010). They only consider static regressions, but allow for a larger set of aggregation schemes.

³In general, the error term \( \epsilon_t \) has an MA structure. However, for the sake of simplicity, we will work with an AR approximation throughout, since this does not affect the main points we want to make and simplifies both the notation and the estimation.
Note that if we assume that the lag orders \( c \) and \( v \) are large enough to make the error term \( \epsilon_t \) uncorrelated, then, all the parameters in the U-MIDAS model (3.8) can be estimated by simple OLS (while the aggregation scheme \( \omega(L) \) is supposed known). From a practical point of view, the lag order \( v \) could differ across variables, and \( v \) and \( c \) could be selected by an information criterion such as BIC. We will follow this approach in the Monte Carlo experiments and in the empirical applications, combining it with the use of information criterion for lag length selection.

### 3.2.2 Forecasting with U-MIDAS

To start with, let us consider the case where the forecast origin is in period \( t = T_k \) and the forecast horizon measured in \( t \) time is \( h = k \), namely, one LF period ahead. Using standard formulae, the optimal forecast (in the MSE sense and assuming that \( \epsilon_t \) is uncorrelated) can be expressed as

\[
\hat{y}_{T_k+k|T_k} = (c_1 L^k + \ldots + c_c L^{kc}) y_{T_k+k} + \delta_1(L) \hat{x}_{1T_k+k-1|T_k} + \ldots + \delta_N(L) \hat{x}_{N T_k+k-1|T_k}, \tag{3.9}
\]

where \( \hat{x}_{i|T_k} = x_{i,T_k+j|T_k} \) for \( j \leq T \).

A problem with the expression in (3.9) is that forecasts of future values of the HF variables \( x \) are also required. Following e.g. Marcellino et al. (2006), a simpler approach is to use a form of direct estimation and construct the forecast as

\[
\tilde{y}_{T_k+k|T_k} = \tilde{c}(L) y_{T_k} + \tilde{\delta}_1(L) x_{1T_k} + \ldots + \tilde{\delta}_N(L) x_{N T_k}, \tag{3.10}
\]

where the polynomials \( \tilde{c}(Z) = \tilde{c}_1 L^k + \ldots + \tilde{c}_c L^{kc} \) and \( \tilde{\delta}_i(L) \) are obtained by projecting \( y_t \) on information dated \( t - k \) or earlier, for \( t = k, 2k, \ldots, T_k \). We will use this approach in the Monte Carlo simulations and empirical applications. In general, the direct approach of (3.10) can also be extended to construct \( h k \)-step ahead forecasts given information in \( T_k \):

\[
\tilde{y}_{T_k+hk|T_k} = \tilde{\sigma}(L) y_{T_k} + \tilde{\delta}_1(L) x_{1T_k} + \ldots + \tilde{\delta}_N(L) x_{N T_k}, \tag{3.11}
\]

where the polynomials \( \tilde{\sigma}(Z) \) and \( \tilde{\delta}_i(L) \) are obtained by projecting \( y_t \) on information dated \( t - h k \) or earlier, for \( t = k, 2k, \ldots, T_k \).\(^4\)

The conditioning information set for forecasting in (3.10) contains HF information up to the end of the sample of the LF variable, namely period \( T_k \). An advantage of the MIDAS approach is that it also allows for incorporating leads of the HF variable \( x_t \) for the projections.

---

\(^4\)Marcellino and Schumacher (2010) present the details of the derivation of a direct forecasting equation for the case where the regressors are factors extracted from a large set of high frequency indicators. A similar approach can be used in this context to derive (3.10) from a given HF VAR DGP.
This is due to the fact that observations of HF indicators are much earlier available than the observations of the LF models, for example, surveys or industrial production. MIDAS with leads can exploit this early information and thus is in particular helpful for nowcasting, namely, computing and updating projections of the LF variable for the current period given all potential HF information which is available (Giannone et al., 2008; Marcellino and Schumacher, 2010; Andreou et al., 2010b; Kuzin et al., 2011). Nowcasting with HF indicators becomes politically relevant, because the publication lags for many LF variables are quite substantial. For example, quarterly GDP in Euro area is typically published around the middle of the subsequent quarter.

As a particular nowcasting example, suppose that the value of interest is still $y_{T+k}$, but that now HF information up to period $T+k+1$ is available, e.g. observations of monthly industrial production on the first month of a given quarter becomes available. Then, the expression in (3.9) can be easily modified to take the new information into account:

$$\hat{y}_{T+k|T+k+1} = \tilde{c}(L^k)y_{T+k} + \delta_1(L)\tilde{x}_{1T+k+1} + \ldots + \delta_N(L)\tilde{x}_{NT+k+1},$$

(3.12)

where $\tilde{x}_{i(T+k+j|T+k+1) = x_{iTK+j|TK+1}}$ for $j \leq T+1$. Similarly, the coefficients in (3.10) would be now obtained by projecting $y_t$ on information dated $t-k+1$ or earlier and the direct forecast becomes

$$\tilde{y}_{T+k|T+k+1} = \tilde{c}(L^k)y_{T+k} + \tilde{\delta}_1(L)x_{1T+k+1} + \ldots + \tilde{\delta}_N(L)x_{NT+k+1},$$

(3.13)

If time passes by and new HF information becomes available, say, in periods $Tk+1, Tk+2, \ldots$, the nowcast can be updated similar to the one-step ahead case.

### 3.2.3 U-MIDAS and MIDAS with exponential Almon lags

It is interesting to compare the U-MIDAS approach with the original MIDAS specification of Ghysels et al. (2005, 2006) with functional lag polynomials, see also Clements and Galvao (2008). Assuming for simplicity that $c(L^k) = 1$, $N = 1$, $\omega(L) = 1$, the U-MIDAS model in (3.8) simplifies to

$$y_t = \delta_1(L)x_{t-1} + \epsilon_t$$

(3.14)

for $t = k, 2k, \ldots, Tk$. The original MIDAS model would be

$$y_t = \beta_1 B(L, \theta)x_{t-1} + \epsilon_t,$$

(3.15)
where the polynomial $B(L, \theta)$ is the exponential Almon lag following Lütkepohl (1981) with

$$B(L, \theta) = \sum_{j=0}^{K} b(j, \theta) L^j, \quad b(j, \theta) = \frac{\exp(\theta_1 j + \theta_2 j^2)}{\sum_{j=0}^{K} \exp(\theta_1 j + \theta_2 j^2)},$$  \hspace{1cm} (3.16)

Therefore, the MIDAS specification of Ghysels et al. (2005, 2006) is nested in U-MIDAS, since it is obtained by imposing a particular dynamic pattern. The key advantage of the original MIDAS specification is that it allows for long lags with a limited number of parameters, which can be particularly useful in financial applications with a high mismatch between the sampling frequencies of $y$ and $x$, e.g. when $y$ is monthly and $x$ is daily. However, for macroeconomic applications with small differences in sampling frequencies, for example monthly and quarterly data, the specification in (3.15) can have several disadvantages. In particular, it could simply be the case that the restriction $\beta_1 B(L, \theta) = \delta_1(L)$ is not valid and that the Almon lag approximation might not be general enough. Additionally, if the impulse response function is relatively short-lived and only a few HF lags are needed to capture the weights, a linear unrestricted lag polynomial might suffice for estimation. Moreover, the model resulting from (3.15) is highly nonlinear in the parameters, so that it cannot be estimated by OLS. In summary, these considerations suggest that U-MIDAS should perform better than the original MIDAS as long as the aggregation frequency is small and U-MIDAS is not too heavily parameterized.

In general, it should be kept in mind that both MIDAS and U-MIDAS should be regarded as approximations to dynamic linear models such as the one discussed in Section 3.2.1. If we do not know the true model in practice, we cannot expect one of the approaches to dominate with empirical data. However, given a known DGP, it might be useful to identify conditions under which MIDAS or U-MIDAS do better. Thus, we will consider both approaches in the simulations below.

### 3.3 Monte Carlo experiments

This Section presents a set of Monte Carlo experiments that focus on the in-sample and forecasting performance of alternative MIDAS regressions. We discuss, in turn, the basic simulation design, the models under comparison, and the results. Next, as a robustness check, we present results for alternative simulation designs.
3.3.1 The simulation design

The simulation design is closely related to that in Ghysels and Valkanov (2006). We modify the exercise in a way to discuss its use in macroeconomic forecasting, in particular forecasting quarterly GDP. As predictors for this variable, the empirical literature typically uses monthly and daily indicators, see Clements and Galvao (2008, 2009) and Andreou, Ghysels and Kourtellos (2010b), respectively. Additionally, we consider weekly data. Thus, we end up with the sampling frequencies \( k = \{3, 12, 60\} \), which represent cases of data sampled at monthly and quarterly frequency \((k = 3)\), at weekly and quarterly frequency \((k = 12)\), or at daily and quarterly frequency \((k = 60)\).

In each case, the DGP is given by the high-frequency VAR

\[
\begin{pmatrix}
  y_t \\
  x_t
\end{pmatrix} = \begin{pmatrix}
  \rho & \delta_l \\
  \delta_h & \rho
\end{pmatrix} \begin{pmatrix}
  y_{t-1} \\
  x_{t-1}
\end{pmatrix} + \begin{pmatrix}
  e_{y,t} \\
  e_{x,t}
\end{pmatrix}.
\]

(3.17)

\( y_t \) is the LF variable and \( x_t \) is the HF variable, where \( t \) is the HF time index with \( t = 1, \ldots, (T + ES) \times k \). \( T \) defines the size of the estimation sample we use in the in-sample analysis (expressed in the low-frequency unit, e.g. quarters in our example), while for the forecasting purposes we generate an additional number of observations, which defines our evaluation sample, \( ES \) (also expressed in the low-frequency unit). \( k \) denotes the sampling frequency of the LF variable \( y_t \), whereas \( x_t \) is sampled with \( k = 1 \). We further assume that \( \omega(L) = 1 \). Thus, the LF variable \( y_t \) is available only for \( t = k, 2k, \ldots, (T + ES) \times k \).

In the VAR (3.17), the shocks \( e_{y,t} \) and \( e_{x,t} \) are sampled independently from the normal distribution with mean zero for all \( t = 1, \ldots, (T + ES) \times k \), and the variances are chosen such that the unconditional variance of \( y \) is equal to one. For the persistence parameter, we choose \( \rho = \{0.1, 0.5, 0.9\} \), and following Ghysels and Valkanov (2006), we fix \( \delta_l = \{0.1, 0.5, 1.0\} \) and \( \delta_h = 0 \). Thus, the DGP is recursive, as the HF variable affects the LF variable, but not vice versa. Later on in Section 3.3.6, we will also consider non-recursive DGPs. Overall, we cover a broad range of DGPs representing different degrees of persistence and correlation between the HF and the LF variable.

Initially, for the in-sample analysis, \( y_t \) and \( x_t \) are simulated for all \( t = 1, \ldots, T \times k \), see Ghysels and Valkanov (2006). To estimate the MIDAS regressions, the available data is defined as \( y_t \) with \( t = k, 2k, \ldots, T \times k \) and \( x_t \) with \( t = 1, 2, \ldots, T \times k \), representing mixed-frequency data which is typical in empirical applications. The number of observations for the LF variable is \( T = 100 \). To compare the in-sample fit obtained with the different methods, we look at the in-sample MSE for each simulation, defined as

\[
IS - MSE_r = \frac{1}{T} \sum_{t=1}^{T} (\hat{y}_{t,r} - y_{t,r})^2,
\]

where \( \hat{y}_{t,r} \) is the forecast of \( y_{t,r} \) at time \( t \) for horizon \( r \).
with \( r = 1, \ldots, R \). In this experiment, we fix the number of simulations at \( R = 2000 \).

In order to conduct a forecast comparison, we also simulate both variables \( ES \times k \) HF periods ahead for \( t = T \times k + 1, \ldots, T \times k + ES \times k \). \( ES \) is set equal to \( \frac{T}{2} = 50 \). The final values of the LF variable, from \( y_{T \times k + k} \) to \( y_{T \times k + ES \times k} \), will be used as the actual values to be compared with the alternative forecasts. Regarding the information set available for forecasting, we assume that we know values up to period \( \left( T + es - 1 \right) \times k \), with \( es = 1, \ldots, ES \), for the LF variable and \( \left( T + es - 1 \right) \times k + k - 1 \) for the HF variable \( x_t \). This yields forecasts of the LF variable \( k \) HF periods ahead for each date in the evaluation sample, \( y_{T \times k + es \times k} \), conditional on HF information within the LF forecast period. The corresponding forecast error is \( y_{T \times k + es \times k}^{\prime} - y_{T \times k + es \times k} \). The latter is used to compute the mean-squared error (MSE) over the evaluation sample for each replication \( r \), as \( MSE_r = \frac{1}{ES} \sum_{es=1}^{ES} \left( y_{T \times k + es \times k}^{\prime} - y_{T \times k + es \times k} \right)^2 \), where \( r = 1, \ldots, R \), and in our experiment \( R = 500 \).

For the in- and out-of-sample analyses we report summary statistics based on the empirical distribution of the IS-MSEs and MSEs over all replications, i.e., its average, median, and selected percentiles.

### 3.3.2 The models under comparison

We consider empirical MIDAS regressions that are based on estimated coefficients and possibly mis-specified functional forms. In particular, we evaluate the following two types of models

1. MIDAS with an autoregressive term as used in Clements and Galvao (2008, 2009). This follows from the fact that the HF VAR also implies an autoregressive term. In order to rule out periodic movements in the impulse response function from the HF variable \( x_t \) on the LF variable \( y_t \), a common factor specification is imposed, yielding the model

\[
y_{t \times k} = \beta_0 + \beta_1 y_{t \times k - k} + \beta_2 (1 - \beta_1 L^k) B(L, \theta) x_{t \times k - 1} + \epsilon_{t \times k},
\]

where the polynomial \( B(L, \theta) \) is the exponential Almon lag defined as in eq. (3.16).

We set \( K = 1.5 \times k \), which suffices to capture the true impulse response function as

---

\(^5\)We consider the IS-MSE to be coherent with the forecasting analysis later where, following common practice, we adopt the squared-error loss function. For the in-sample analysis, we could as well look at the \( R^2 \), a more traditional measure of goodness-of-fit, but the results would be qualitatively similar since both measures are based on the residual sum of squares.

\(^6\)The number of replications in this case is only 500 for computational issues. In fact, for each replication we need to estimate the models and then compute the forecast 50 times, one for each quarter of the evaluation sample. Therefore, even with 500 replications, we obtain 25000 forecasts.
implied by the DGP. The model is estimated using NLS, with the additional coefficient $\lambda$ and the common factor structure imposed. We apply the coefficient restrictions $-100 < \theta_1 < 5$ and $-100 < \theta_2 < 0$. As starting values might matter, we compute the residual sum of squares in each replication for alternative parameter pairs in the sets $\theta_1 = \{-0.5, 0.0, 0.5\}$ and $\theta_2 = \{-0.01, -0.1, -0.5, -1\}$. Note that when $x_t$ is available until $T \times k + es \times k - 1$, this is the final date used to estimate the coefficients of the model. Given the NLS estimates of the parameters, the forecast can be computed as

$$y_{T \times k + es \times k|T \times k + es \times k - 1} = \hat{\beta}_0 + \hat{\beta}_1 y_{T \times k + es \times k - k} + \hat{\beta}_2 (1 - \hat{\beta}_1 L_k) B(L, \hat{\theta}) x_{T \times k + es \times k - 1}. \quad (3.19)$$

2. U-MIDAS as introduced in Section 3.2. In particular, U-MIDAS is estimated without considering the Almon lag polynomial. Rather, we leave the lag polynomial of the indicator HF variable $x_t$ unrestricted. Furthermore, we do not impose the common factor restriction as in Clements and Galvao (2008) for the autoregressive term. Thus, the model becomes

$$y_{t \times k} = \mu_0 + \mu_1 y_{t \times k - k} + \psi(L) x_{t \times k - 1} + \varepsilon_{t \times k}, \quad (3.20)$$

with lag polynomial $\psi(L) = \sum_{j=0}^{K} \psi_j L^j = \psi_0 + \psi_1 L + \ldots + \psi_K L^K$. The coefficients $\mu_0$, $\mu_1$ and $\psi(L)$ are estimated by OLS. To specify the lag order, we use the BIC with a maximum lag order of $K$ to choose the lags. Given the selected BIC order $\hat{k}$ and OLS estimated parameters $\hat{\mu}_0$, $\hat{\mu}_1$ and $\hat{\psi}(L)$, the U-MIDAS forecast can be computed as

$$y_{T \times k + es \times k|T \times k + es \times k - 1} = \hat{\mu}_0 + \hat{\mu}_1 y_{T \times k + es \times k - k} + \hat{\psi}(L) x_{T \times k + es \times k - 1}. \quad (3.21)$$

### 3.3.3 Monte Carlo in-sample comparison results

We summarize the results of the in-sample evaluation in Table 3.1. As anticipated, we compare the performance of the two different methods, U-MIDAS and MIDAS, based on their respective in-sample MSEs. The table reports summary statistics for the distribution of the in-sample MSE of U-MIDAS relative to that of MIDAS as described in the previous subsection for alternative parameter values and sampling frequencies. More in detail, we first compute the ratio $IS\text{-MSE}(U\text{-MIDAS})/IS\text{-MSE}(MIDAS)$ for each replication, then report the mean, the median and selected percentiles of the distribution of these ratios. Hence, median, mean, and percentile ratio values smaller than one indicate a superior performance of U-MIDAS.

The results can be summarized as follows. When the difference in sampling frequencies is large ($k = 12, 60$), restricted MIDAS outperforms U-MIDAS for almost each value of
Table 3.1: In-sample MSE of U-MIDAS relative to in-sample MSE of MIDAS (DGP: recursive HF VAR)

<table>
<thead>
<tr>
<th>rho</th>
<th>delta l</th>
<th>delta h</th>
<th>k</th>
<th>mean</th>
<th>10th prctile</th>
<th>25th prctile</th>
<th>median prctile</th>
<th>75th prctile</th>
<th>90th prctile</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>0.00</td>
<td>3</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.00</td>
<td>1.02</td>
<td>1.04</td>
</tr>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>0.00</td>
<td>12</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.00</td>
<td>1.02</td>
<td>1.05</td>
</tr>
<tr>
<td>0.1</td>
<td>0.50</td>
<td>0.00</td>
<td>3</td>
<td>1.01</td>
<td>0.98</td>
<td>0.99</td>
<td>1.00</td>
<td>1.02</td>
<td>1.04</td>
</tr>
<tr>
<td>0.1</td>
<td>0.50</td>
<td>0.00</td>
<td>12</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.00</td>
<td>1.02</td>
<td>1.04</td>
</tr>
<tr>
<td>0.1</td>
<td>1.00</td>
<td>0.00</td>
<td>3</td>
<td>1.01</td>
<td>0.98</td>
<td>0.99</td>
<td>1.00</td>
<td>1.02</td>
<td>1.04</td>
</tr>
<tr>
<td>0.1</td>
<td>1.00</td>
<td>0.00</td>
<td>12</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.00</td>
<td>1.02</td>
<td>1.04</td>
</tr>
<tr>
<td>0.5</td>
<td>0.10</td>
<td>0.00</td>
<td>3</td>
<td>0.92</td>
<td>0.85</td>
<td>0.89</td>
<td>0.93</td>
<td>0.96</td>
<td>0.99</td>
</tr>
<tr>
<td>0.5</td>
<td>0.10</td>
<td>0.00</td>
<td>12</td>
<td>1.00</td>
<td>0.94</td>
<td>0.97</td>
<td>1.00</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>0.5</td>
<td>0.50</td>
<td>0.00</td>
<td>3</td>
<td>0.92</td>
<td>0.85</td>
<td>0.88</td>
<td>0.92</td>
<td>0.96</td>
<td>0.99</td>
</tr>
<tr>
<td>0.5</td>
<td>0.50</td>
<td>0.00</td>
<td>12</td>
<td>1.00</td>
<td>0.94</td>
<td>0.97</td>
<td>1.00</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>0.5</td>
<td>1.00</td>
<td>0.00</td>
<td>3</td>
<td>0.91</td>
<td>0.83</td>
<td>0.87</td>
<td>0.92</td>
<td>0.96</td>
<td>0.99</td>
</tr>
<tr>
<td>0.5</td>
<td>1.00</td>
<td>0.00</td>
<td>12</td>
<td>1.02</td>
<td>0.89</td>
<td>0.95</td>
<td>1.01</td>
<td>1.08</td>
<td>1.14</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.00</td>
<td>3</td>
<td>0.92</td>
<td>0.83</td>
<td>0.87</td>
<td>0.92</td>
<td>0.96</td>
<td>1.00</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.00</td>
<td>12</td>
<td>1.02</td>
<td>0.89</td>
<td>0.95</td>
<td>1.01</td>
<td>1.08</td>
<td>1.14</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.00</td>
<td>60</td>
<td>1.13</td>
<td>0.97</td>
<td>1.04</td>
<td>1.13</td>
<td>1.23</td>
<td>1.32</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.00</td>
<td>3</td>
<td>0.92</td>
<td>0.83</td>
<td>0.87</td>
<td>0.92</td>
<td>0.96</td>
<td>0.99</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00</td>
<td>0.00</td>
<td>12</td>
<td>1.02</td>
<td>0.90</td>
<td>0.95</td>
<td>1.01</td>
<td>1.08</td>
<td>1.15</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00</td>
<td>0.00</td>
<td>60</td>
<td>1.13</td>
<td>0.97</td>
<td>1.04</td>
<td>1.13</td>
<td>1.22</td>
<td>1.31</td>
</tr>
</tbody>
</table>

Notes: Columns 1 to 4 show the parameter specification for the DGP in eq. (17). The entries of columns 5 to 10 report the performance of the IS-MSE(U-MIDAS) relative to IS-MSE(MIDAS). The ratio IS-MSE(U-MIDAS)/IS-MSE(MIDAS) is computed for each replication, then in column 5 the mean of the distribution of these ratios is reported, and in columns 6 to 10 the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported.
persistence and interrelatedness. Instead, U-MIDAS clearly outperforms MIDAS for \( k = 3 \), especially when the process is persistent, that is when \( \rho = \{0.5, 0.9\} \). The distribution results tend to be very concentrated, especially when \( \rho = 0.1 \), while the values of the ratio are slightly more dispersed when the persistence is bigger. It is very interesting to notice that when \( \rho = \{0.5, 0.9\} \), U-MIDAS models outperform systematically MIDAS models, since even the values correspondent to the 90th percentile are smaller than 1.

### 3.3.4 Monte Carlo forecast comparison results

The results of the Monte Carlo forecast experiments are summarized in Table 3.2. As in the in-sample analysis, we compare the forecasting performance of U-MIDAS and MIDAS based on their (out-of-sample) MSEs, computed over the 50 periods of the evaluation sample. The Table reports the summary statistics for the distribution of the MSE of U-MIDAS relative to that of MIDAS, as described in Section 3.3.3 for the in-sample analysis. Again, a median or mean value smaller than one indicates a superior performance of U-MIDAS.

The results confirm the evidence of the in-sample analysis: U-MIDAS forecasts perform better than MIDAS for \( k = 3 \) if the process is persistent. If the difference in sampling frequencies is large (\( k = 12, 60 \)), MIDAS outperforms U-MIDAS for almost each value of persistence and interrelatedness. A likely reason for this pattern of results is that when \( k \) is large, U-MIDAS becomes heavily parameterized, notwithstanding BIC lag length selection, and imprecise estimation affects the forecasting accuracy. On the contrary, when \( k = 3 \), the number of U-MIDAS parameters is limited and their estimates precise, and the additional flexibility allowed by U-MIDAS yields a better forecasting performance than MIDAS. An alternative explanation could be that BIC selects models that are too parsimonious when \( k \) is large, and therefore omits relevant regressors. We will discuss this possibility in the next subsection.

### 3.3.5 The role of BIC for lag length selection

In Sections 3.3.3 and 3.3.4, we select the number of lags to be included in the U-MIDAS models according to the BIC criterion, which is known to prefer rather parsimonious models, at least in finite samples. We now want to check whether a different selection criterion for the number of lags can influence the results. In particular, we assess whether the use of the AIC criterion, which puts a lower loss on the number of parameters than BIC, can improve the results, and in particular those for \( k \) large.

The results in Table 3.3 imply that there are no gains from the switch in information...
### Table 3.2: Out-of-sample MSE of U-MIDAS relative to MSE of MIDAS (DGP: recursive HF VAR)

<table>
<thead>
<tr>
<th>rho</th>
<th>delta l</th>
<th>delta h</th>
<th>k</th>
<th>mean</th>
<th>10th percentile</th>
<th>25th percentile</th>
<th>median</th>
<th>75th percentile</th>
<th>90th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>0.00</td>
<td>3</td>
<td>1.01</td>
<td>0.96</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.07</td>
</tr>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>0.00</td>
<td>12</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>0.1</td>
<td>0.50</td>
<td>0.00</td>
<td>3</td>
<td>1.01</td>
<td>0.96</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.07</td>
</tr>
<tr>
<td>0.1</td>
<td>0.50</td>
<td>0.00</td>
<td>12</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>0.1</td>
<td>1.00</td>
<td>0.00</td>
<td>3</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.07</td>
</tr>
<tr>
<td>0.1</td>
<td>1.00</td>
<td>0.00</td>
<td>12</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.04</td>
<td>1.07</td>
</tr>
<tr>
<td>0.5</td>
<td>0.10</td>
<td>0.00</td>
<td>3</td>
<td>0.94</td>
<td>0.83</td>
<td>0.89</td>
<td>0.94</td>
<td>0.98</td>
<td>1.04</td>
</tr>
<tr>
<td>0.5</td>
<td>0.10</td>
<td>0.00</td>
<td>12</td>
<td>1.05</td>
<td>0.97</td>
<td>1.00</td>
<td>1.05</td>
<td>1.09</td>
<td>1.13</td>
</tr>
<tr>
<td>0.5</td>
<td>0.50</td>
<td>0.00</td>
<td>3</td>
<td>0.93</td>
<td>0.82</td>
<td>0.88</td>
<td>0.93</td>
<td>0.98</td>
<td>1.02</td>
</tr>
<tr>
<td>0.5</td>
<td>0.50</td>
<td>0.00</td>
<td>12</td>
<td>1.05</td>
<td>0.97</td>
<td>1.01</td>
<td>1.05</td>
<td>1.09</td>
<td>1.13</td>
</tr>
<tr>
<td>0.5</td>
<td>0.50</td>
<td>0.00</td>
<td>60</td>
<td>1.05</td>
<td>0.96</td>
<td>1.00</td>
<td>1.05</td>
<td>1.09</td>
<td>1.15</td>
</tr>
<tr>
<td>0.5</td>
<td>1.00</td>
<td>0.00</td>
<td>3</td>
<td>0.94</td>
<td>0.83</td>
<td>0.89</td>
<td>0.94</td>
<td>0.99</td>
<td>1.04</td>
</tr>
<tr>
<td>0.5</td>
<td>1.00</td>
<td>0.00</td>
<td>12</td>
<td>1.05</td>
<td>0.97</td>
<td>1.00</td>
<td>1.04</td>
<td>1.08</td>
<td>1.14</td>
</tr>
<tr>
<td>0.5</td>
<td>1.00</td>
<td>0.00</td>
<td>60</td>
<td>1.05</td>
<td>0.97</td>
<td>1.01</td>
<td>1.05</td>
<td>1.09</td>
<td>1.14</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.00</td>
<td>3</td>
<td>0.91</td>
<td>0.80</td>
<td>0.86</td>
<td>0.91</td>
<td>0.96</td>
<td>1.02</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.00</td>
<td>12</td>
<td>1.09</td>
<td>0.93</td>
<td>1.01</td>
<td>1.08</td>
<td>1.18</td>
<td>1.25</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.00</td>
<td>60</td>
<td>1.23</td>
<td>1.03</td>
<td>1.11</td>
<td>1.21</td>
<td>1.32</td>
<td>1.45</td>
</tr>
<tr>
<td>0.9</td>
<td>0.50</td>
<td>0.00</td>
<td>3</td>
<td>0.91</td>
<td>0.81</td>
<td>0.85</td>
<td>0.91</td>
<td>0.97</td>
<td>1.01</td>
</tr>
<tr>
<td>0.9</td>
<td>0.50</td>
<td>0.00</td>
<td>12</td>
<td>1.07</td>
<td>0.91</td>
<td>0.98</td>
<td>1.07</td>
<td>1.16</td>
<td>1.24</td>
</tr>
<tr>
<td>0.9</td>
<td>0.50</td>
<td>0.00</td>
<td>60</td>
<td>1.24</td>
<td>1.05</td>
<td>1.13</td>
<td>1.23</td>
<td>1.34</td>
<td>1.46</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00</td>
<td>0.00</td>
<td>3</td>
<td>0.92</td>
<td>0.81</td>
<td>0.86</td>
<td>0.92</td>
<td>0.98</td>
<td>1.03</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00</td>
<td>0.00</td>
<td>12</td>
<td>1.08</td>
<td>0.92</td>
<td>0.99</td>
<td>1.07</td>
<td>1.15</td>
<td>1.23</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00</td>
<td>0.00</td>
<td>60</td>
<td>1.22</td>
<td>1.04</td>
<td>1.11</td>
<td>1.21</td>
<td>1.31</td>
<td>1.44</td>
</tr>
</tbody>
</table>

**Notes:** Columns 1 to 4 show the parameter specification for the DGP in eq. (17). The entries of columns 5 to 10 report the performance of the out-of-sample MSE(U-MIDAS) relative to out-of-sample MSE(MIDAS). The MSE is calculated over an evaluation sample of 50 periods. The ratio MSE(U-MIDAS)/MSE(MIDAS) is computed for each replication, then in column 5 the mean of the distribution of these ratios is reported, and in columns 6 to 10 the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported.
Table 3.3: Out-of-sample MSE of U-MIDAS relative to out-of-sample MSE of MIDAS (DGP: recursive HF VAR). AIC selection criterion

<table>
<thead>
<tr>
<th>rho</th>
<th>delta l</th>
<th>delta h</th>
<th>k</th>
<th>mean</th>
<th>10th percentile</th>
<th>25th percentile</th>
<th>median</th>
<th>75th percentile</th>
<th>90th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>0.00</td>
<td>3</td>
<td>1.01</td>
<td>0.96</td>
<td>0.98</td>
<td>1.00</td>
<td>1.02</td>
<td>1.05</td>
</tr>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>0.00</td>
<td>12</td>
<td>1.02</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.05</td>
<td>1.09</td>
</tr>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>0.00</td>
<td>60</td>
<td>1.11</td>
<td>0.97</td>
<td>1.00</td>
<td>1.03</td>
<td>1.14</td>
<td>1.37</td>
</tr>
<tr>
<td>0.1</td>
<td>0.50</td>
<td>0.00</td>
<td>3</td>
<td>1.01</td>
<td>0.96</td>
<td>0.98</td>
<td>1.00</td>
<td>1.03</td>
<td>1.15</td>
</tr>
<tr>
<td>0.1</td>
<td>0.50</td>
<td>0.00</td>
<td>12</td>
<td>1.02</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.05</td>
<td>1.09</td>
</tr>
<tr>
<td>0.1</td>
<td>0.50</td>
<td>0.00</td>
<td>60</td>
<td>1.12</td>
<td>0.98</td>
<td>1.00</td>
<td>1.03</td>
<td>1.15</td>
<td>1.45</td>
</tr>
<tr>
<td>0.1</td>
<td>1.00</td>
<td>0.00</td>
<td>3</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>0.1</td>
<td>1.00</td>
<td>0.00</td>
<td>12</td>
<td>1.02</td>
<td>0.96</td>
<td>0.98</td>
<td>1.01</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>0.1</td>
<td>1.00</td>
<td>0.00</td>
<td>60</td>
<td>1.10</td>
<td>0.98</td>
<td>1.00</td>
<td>1.04</td>
<td>1.13</td>
<td>1.32</td>
</tr>
<tr>
<td>0.5</td>
<td>0.10</td>
<td>0.00</td>
<td>3</td>
<td>0.93</td>
<td>0.83</td>
<td>0.88</td>
<td>0.93</td>
<td>0.98</td>
<td>1.02</td>
</tr>
<tr>
<td>0.5</td>
<td>0.10</td>
<td>0.00</td>
<td>12</td>
<td>1.05</td>
<td>0.96</td>
<td>1.00</td>
<td>1.04</td>
<td>1.09</td>
<td>1.14</td>
</tr>
<tr>
<td>0.5</td>
<td>0.10</td>
<td>0.00</td>
<td>60</td>
<td>1.14</td>
<td>0.98</td>
<td>1.01</td>
<td>1.07</td>
<td>1.20</td>
<td>1.40</td>
</tr>
<tr>
<td>0.5</td>
<td>0.50</td>
<td>0.00</td>
<td>3</td>
<td>0.93</td>
<td>0.82</td>
<td>0.88</td>
<td>0.92</td>
<td>0.98</td>
<td>1.02</td>
</tr>
<tr>
<td>0.5</td>
<td>0.50</td>
<td>0.00</td>
<td>12</td>
<td>1.05</td>
<td>0.97</td>
<td>1.00</td>
<td>1.05</td>
<td>1.09</td>
<td>1.14</td>
</tr>
<tr>
<td>0.5</td>
<td>0.50</td>
<td>0.00</td>
<td>60</td>
<td>1.16</td>
<td>0.98</td>
<td>1.02</td>
<td>1.08</td>
<td>1.20</td>
<td>1.48</td>
</tr>
<tr>
<td>0.5</td>
<td>1.00</td>
<td>0.00</td>
<td>3</td>
<td>0.94</td>
<td>0.83</td>
<td>0.89</td>
<td>0.93</td>
<td>0.99</td>
<td>1.03</td>
</tr>
<tr>
<td>0.5</td>
<td>1.00</td>
<td>0.00</td>
<td>12</td>
<td>1.05</td>
<td>0.97</td>
<td>1.00</td>
<td>1.04</td>
<td>1.10</td>
<td>1.14</td>
</tr>
<tr>
<td>0.5</td>
<td>1.00</td>
<td>0.00</td>
<td>60</td>
<td>1.15</td>
<td>0.98</td>
<td>1.03</td>
<td>1.09</td>
<td>1.19</td>
<td>1.42</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.00</td>
<td>3</td>
<td>0.91</td>
<td>0.81</td>
<td>0.85</td>
<td>0.90</td>
<td>0.96</td>
<td>1.01</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.00</td>
<td>12</td>
<td>1.05</td>
<td>0.92</td>
<td>0.99</td>
<td>1.05</td>
<td>1.12</td>
<td>1.18</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.00</td>
<td>60</td>
<td>1.32</td>
<td>1.05</td>
<td>1.13</td>
<td>1.24</td>
<td>1.45</td>
<td>1.71</td>
</tr>
<tr>
<td>0.9</td>
<td>0.50</td>
<td>0.00</td>
<td>3</td>
<td>0.91</td>
<td>0.81</td>
<td>0.85</td>
<td>0.91</td>
<td>0.96</td>
<td>1.01</td>
</tr>
<tr>
<td>0.9</td>
<td>0.50</td>
<td>0.00</td>
<td>12</td>
<td>1.03</td>
<td>0.91</td>
<td>0.97</td>
<td>1.04</td>
<td>1.09</td>
<td>1.15</td>
</tr>
<tr>
<td>0.9</td>
<td>0.50</td>
<td>0.00</td>
<td>60</td>
<td>1.32</td>
<td>1.05</td>
<td>1.13</td>
<td>1.26</td>
<td>1.42</td>
<td>1.68</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00</td>
<td>0.00</td>
<td>3</td>
<td>0.92</td>
<td>0.80</td>
<td>0.86</td>
<td>0.92</td>
<td>0.97</td>
<td>1.02</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00</td>
<td>0.00</td>
<td>12</td>
<td>1.04</td>
<td>0.92</td>
<td>0.97</td>
<td>1.03</td>
<td>1.10</td>
<td>1.18</td>
</tr>
<tr>
<td>0.9</td>
<td>1.00</td>
<td>0.00</td>
<td>60</td>
<td>1.30</td>
<td>1.05</td>
<td>1.12</td>
<td>1.24</td>
<td>1.43</td>
<td>1.64</td>
</tr>
</tbody>
</table>

Notes: See Table 3.2.
3.3. MONTE CARLO EXPERIMENTS

criterion. As with BIC lag length selection, U-MIDAS outperforms MIDAS only for a small mismatch in sampling frequency \( (k = 3) \). Moreover, the results are even worse for \( k = 60 \), confirming that the main problem is the estimation of too heavily parameterized models rather than omitted regressors.\(^7\)

3.3.6 An alternative DGP with non-recursive VAR structure

To check the robustness of the results we have obtained so far, we now consider an alternative HF data generating process that allows for reverse causality from \( y \) to \( x \) by setting \( \delta_h \neq 0 \).

The values of \( \delta_l \) and \( \delta_h \) cannot be chosen freely but must be selected in order to ensure a non-explosive solution, which ensures stationarity of both \( y \) and \( x \). To ensure a stable solution, the condition

\[
\det \left[ I_2 - \left( \begin{array}{cc} \rho & \delta_l \\ \delta_h & \rho \end{array} \right) z \right] \neq 0 \text{ for } |z| \leq 1
\]  

must hold. In general, the solution is

\[
z_{1,2} = \frac{1}{\rho^2 - \delta_h \delta_l} (\rho \pm \sqrt{\delta_h \delta_l}),
\]

depending heavily on the relative size of \( \delta_l \) and \( \delta_h \). For the sake of simplicity, we assume that both processes are equally important for each other, so that \( \delta_l = \delta_h = \delta \). This implies the solutions for the characteristic roots are

\[
z_{1,2} = \frac{\rho \pm \delta}{(\rho + \delta)(\rho - \delta)}.
\]  

These roots lie outside the unit circle, if \( 1 + \delta > \rho \) and \( 1 - \delta > \rho \). Thus, if we further assume that the series \( y_t \) and \( x_t \) have a positive impact on each other, \( \delta > 0 \), only the restriction \( 1 - \delta > \rho \) is binding. Hence, depending on the selection of \( \rho = \{0.1, 0.5, 0.9\} \), we select the \( \rho, \delta \) couples

\[
\{0.1, 0.1\}, \{0.1, 0.4\}, \{0.1, 0.8\}, \\
\{0.5, 0.1\}, \{0.5, 0.2\}, \{0.5, 0.4\}, \\
\{0.9, 0.01\}, \{0.9, 0.04\}, \{0.9, 0.08\}.
\]  

with varying degrees of persistence and interrelatedness. The results on the relative forecasting performance of U-MIDAS and MIDAS can be found in Table 3.4.

Overall, the results are in line with those for the benchmark case. With a medium to high degree of persistence, the forecasting performance of U-MIDAS is better than that of MIDAS when \( k = 3 \), whereas in general MIDAS dominates for \( k = \{12, 60\} \).

\(^7\)For the sake of space, for all the robustness checks in this and the next two subsections, we only report the results on the forecasting performance. Similar conclusions however emerge from the in-sample analysis, which is available upon request.
Table 3.4: Out-of-sample MSE of U-MIDAS relative to out-of-sample MSE of MIDAS (DGP: non-recursive HF VAR)

<table>
<thead>
<tr>
<th>rho</th>
<th>delta l</th>
<th>delta h</th>
<th>k</th>
<th>mean</th>
<th>10th percentile</th>
<th>25th percentile</th>
<th>median</th>
<th>75th percentile</th>
<th>90th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>0.10</td>
<td>3</td>
<td>1.01</td>
<td>0.96</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.07</td>
</tr>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>0.10</td>
<td>12</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>0.10</td>
<td>60</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>0.1</td>
<td>0.40</td>
<td>0.40</td>
<td>3</td>
<td>1.01</td>
<td>0.96</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>0.1</td>
<td>0.40</td>
<td>0.40</td>
<td>12</td>
<td>1.01</td>
<td>0.96</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>0.1</td>
<td>0.40</td>
<td>0.40</td>
<td>60</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>0.1</td>
<td>0.80</td>
<td>0.80</td>
<td>3</td>
<td>1.02</td>
<td>0.96</td>
<td>0.99</td>
<td>1.01</td>
<td>1.04</td>
<td>1.08</td>
</tr>
<tr>
<td>0.1</td>
<td>0.80</td>
<td>0.80</td>
<td>12</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.04</td>
<td>1.07</td>
</tr>
<tr>
<td>0.1</td>
<td>0.80</td>
<td>0.80</td>
<td>60</td>
<td>1.01</td>
<td>0.97</td>
<td>0.99</td>
<td>1.01</td>
<td>1.04</td>
<td>1.07</td>
</tr>
<tr>
<td>0.5</td>
<td>0.10</td>
<td>0.10</td>
<td>3</td>
<td>0.94</td>
<td>0.83</td>
<td>0.89</td>
<td>0.94</td>
<td>0.99</td>
<td>1.04</td>
</tr>
<tr>
<td>0.5</td>
<td>0.10</td>
<td>0.10</td>
<td>12</td>
<td>1.05</td>
<td>0.97</td>
<td>1.00</td>
<td>1.05</td>
<td>1.09</td>
<td>1.13</td>
</tr>
<tr>
<td>0.5</td>
<td>0.10</td>
<td>0.10</td>
<td>60</td>
<td>1.05</td>
<td>0.97</td>
<td>1.00</td>
<td>1.04</td>
<td>1.09</td>
<td>1.14</td>
</tr>
<tr>
<td>0.5</td>
<td>0.20</td>
<td>0.20</td>
<td>3</td>
<td>0.94</td>
<td>0.83</td>
<td>0.89</td>
<td>0.94</td>
<td>0.98</td>
<td>1.03</td>
</tr>
<tr>
<td>0.5</td>
<td>0.20</td>
<td>0.20</td>
<td>12</td>
<td>1.05</td>
<td>0.97</td>
<td>1.01</td>
<td>1.05</td>
<td>1.09</td>
<td>1.12</td>
</tr>
<tr>
<td>0.5</td>
<td>0.20</td>
<td>0.20</td>
<td>60</td>
<td>1.05</td>
<td>0.96</td>
<td>1.00</td>
<td>1.05</td>
<td>1.09</td>
<td>1.14</td>
</tr>
<tr>
<td>0.5</td>
<td>0.40</td>
<td>0.40</td>
<td>3</td>
<td>0.98</td>
<td>0.90</td>
<td>0.93</td>
<td>0.98</td>
<td>1.03</td>
<td>1.07</td>
</tr>
<tr>
<td>0.5</td>
<td>0.40</td>
<td>0.40</td>
<td>12</td>
<td>1.04</td>
<td>0.96</td>
<td>1.00</td>
<td>1.04</td>
<td>1.07</td>
<td>1.12</td>
</tr>
<tr>
<td>0.5</td>
<td>0.40</td>
<td>0.40</td>
<td>60</td>
<td>1.04</td>
<td>0.96</td>
<td>1.00</td>
<td>1.03</td>
<td>1.07</td>
<td>1.11</td>
</tr>
<tr>
<td>0.9</td>
<td>0.01</td>
<td>0.01</td>
<td>3</td>
<td>0.91</td>
<td>0.81</td>
<td>0.86</td>
<td>0.91</td>
<td>0.96</td>
<td>1.02</td>
</tr>
<tr>
<td>0.9</td>
<td>0.01</td>
<td>0.01</td>
<td>12</td>
<td>1.09</td>
<td>0.93</td>
<td>1.01</td>
<td>1.08</td>
<td>1.18</td>
<td>1.25</td>
</tr>
<tr>
<td>0.9</td>
<td>0.01</td>
<td>0.01</td>
<td>60</td>
<td>1.23</td>
<td>1.03</td>
<td>1.11</td>
<td>1.21</td>
<td>1.32</td>
<td>1.45</td>
</tr>
<tr>
<td>0.9</td>
<td>0.04</td>
<td>0.04</td>
<td>3</td>
<td>0.93</td>
<td>0.83</td>
<td>0.88</td>
<td>0.92</td>
<td>0.98</td>
<td>1.03</td>
</tr>
<tr>
<td>0.9</td>
<td>0.04</td>
<td>0.04</td>
<td>12</td>
<td>1.06</td>
<td>0.91</td>
<td>0.98</td>
<td>1.06</td>
<td>1.14</td>
<td>1.22</td>
</tr>
<tr>
<td>0.9</td>
<td>0.04</td>
<td>0.04</td>
<td>60</td>
<td>1.22</td>
<td>1.04</td>
<td>1.11</td>
<td>1.20</td>
<td>1.32</td>
<td>1.44</td>
</tr>
<tr>
<td>0.9</td>
<td>0.08</td>
<td>0.08</td>
<td>3</td>
<td>0.99</td>
<td>0.93</td>
<td>0.96</td>
<td>0.99</td>
<td>1.02</td>
<td>1.05</td>
</tr>
<tr>
<td>0.9</td>
<td>0.08</td>
<td>0.08</td>
<td>12</td>
<td>0.99</td>
<td>0.92</td>
<td>0.96</td>
<td>0.99</td>
<td>1.03</td>
<td>1.06</td>
</tr>
<tr>
<td>0.9</td>
<td>0.08</td>
<td>0.08</td>
<td>60</td>
<td>1.00</td>
<td>0.93</td>
<td>0.97</td>
<td>1.00</td>
<td>1.03</td>
<td>1.06</td>
</tr>
</tbody>
</table>

Notes: See Table 3.2.
3.3.7 Using MIDAS as DGP

As a final robustness check, we now carry out Monte Carlo simulations using a MIDAS equation with exponential Almon lag as the data generating process. Thus, we are in a case that favours a priori the restricted MIDAS regression over the U-MIDAS. The DGP is

\[ y_{t \times k+k} = \beta_0 + \beta_1 B(L, \theta) x_{t \times k+k-1} + \varepsilon_{t \times k+k}, \]  

(3.25)

with the lag polynomial \( B(L, \theta) \) defined as in (3.16). We set \( k = \{3, 12, 60\} \) to mimic the design in the previous sections. When simulating \( y_{t \times k+es \times k} \), we use \( T = 100 \) and the sets \( \theta_1 = 0.7 \) and \( \theta_2 = \{-0.025, -0.05, -0.3\} \). The monthly indicator \( x_t \) is generated as an AR(1) process, with persistence equal to 0.9. Given these 9 different DGPs, we again use U-MIDAS and MIDAS as before to forecast \( y_{t \times k+es \times k} \) and evaluate their performance by MSE. For estimating MIDAS and U-MIDAS, we set \( K = 1.5 \times k \), which again suffices to capture the true impulse response functions.

To fix the starting values of \( \theta_1 \) and \( \theta_2 \), we compute the residual sum of squares in each replication for alternative parameter pairs in the sets \( \theta_1 = \{-0.5, 0.0, 0.5\} \) and \( \theta_2 = \{-0.01, -0.1, -0.5, -1\} \). As initial values, we then choose those \( \theta_1 \) and \( \theta_2 \) that minimize the residual sum of squares. Results on the forecasting performance can be found in Table 3.5.

Interestingly, the Table highlights that, even in a set up favorable to MIDAS, as long as the frequency mismatch is small (\( k = 3 \)) and therefore the number of parameters to be estimated is low, U-MIDAS still yields a better forecasting performance than MIDAS. On the other hand, MIDAS is of course strongly outperforming in the case of very large discrepancies in the sampling frequency.

As a general summary of the simulation results, we can say that as long as the dependent variable is sufficiently persistent and the frequency mismatch with the explanatory variables limited, there is strong evidence that the U-MIDAS specification is better than MIDAS. The gains are larger in in-sample analysis, where the estimation sample is longer, but are still present out-of-sample, and even when a MIDAS model is used as the data generating process.

3.4 Empirical examples

In this section, we assess the MIDAS and U-MIDAS methods in terms of their in-sample and nowcasting performance. Specifically, in the first part of our empirical analysis, we consider nowcasting quarterly Euro area GDP growth using a very large set of higher frequency
Table 3.5: Out-of-sample MSE of U-MIDAS relative to out-of-sample MSE of MIDAS, (DGP: MIDAS)

<table>
<thead>
<tr>
<th>theta1</th>
<th>theta2</th>
<th>k</th>
<th>mean</th>
<th>10th prctile</th>
<th>25th prctile</th>
<th>median</th>
<th>75th prctile</th>
<th>90th prctile</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.70</td>
<td>-0.025</td>
<td>3</td>
<td>0.95</td>
<td>0.86</td>
<td>0.91</td>
<td>0.96</td>
<td>1.01</td>
<td>1.04</td>
</tr>
<tr>
<td>0.70</td>
<td>-0.025</td>
<td>12</td>
<td>1.20</td>
<td>1.00</td>
<td>1.07</td>
<td>1.19</td>
<td>1.31</td>
<td>1.43</td>
</tr>
<tr>
<td>0.70</td>
<td>-0.025</td>
<td>60</td>
<td>1.27</td>
<td>1.06</td>
<td>1.15</td>
<td>1.25</td>
<td>1.37</td>
<td>1.48</td>
</tr>
<tr>
<td>0.70</td>
<td>-0.05</td>
<td>3</td>
<td>0.90</td>
<td>0.80</td>
<td>0.85</td>
<td>0.90</td>
<td>0.96</td>
<td>1.00</td>
</tr>
<tr>
<td>0.70</td>
<td>-0.05</td>
<td>12</td>
<td>1.09</td>
<td>0.98</td>
<td>1.03</td>
<td>1.09</td>
<td>1.15</td>
<td>1.21</td>
</tr>
<tr>
<td>0.70</td>
<td>-0.05</td>
<td>60</td>
<td>1.14</td>
<td>1.01</td>
<td>1.07</td>
<td>1.14</td>
<td>1.21</td>
<td>1.28</td>
</tr>
<tr>
<td>0.70</td>
<td>-0.3</td>
<td>3</td>
<td>0.98</td>
<td>0.91</td>
<td>0.95</td>
<td>0.98</td>
<td>1.01</td>
<td>1.04</td>
</tr>
<tr>
<td>0.70</td>
<td>-0.3</td>
<td>12</td>
<td>1.03</td>
<td>0.96</td>
<td>1.00</td>
<td>1.03</td>
<td>1.07</td>
<td>1.10</td>
</tr>
<tr>
<td>0.70</td>
<td>-0.3</td>
<td>60</td>
<td>1.03</td>
<td>0.96</td>
<td>0.99</td>
<td>1.03</td>
<td>1.07</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Notes: Columns 1 to 3 show the parameter specification for the DGP in eq. (25). The entries of columns 4 to 9 report the performance of the out-of-sample MSE(U-MIDAS) relative to out-of-sample MSE(MIDAS). The MSE is calculated over an evaluation sample of 50 periods. The ratio MSE(U-MIDAS)/MSE(MIDAS) is computed for each replication, then in column 4 the mean of the distribution of these ratios is reported, and in columns 5 to 9 the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported.
indicators. In the second part, we focus on nowcasting quarterly US GDP growth, using fewer selected monthly indicators, but with a much longer sample size. In both cases we consider a number of lags of the indicators equal to \(2k = 6\) months.

### 3.4.1 An application to Euro area GDP growth

In this section, we focus on the Euro area GDP growth rate. The dataset includes the same series of that in Foroni and Marcellino (2012), extracted from the Eurostat dataset of Principal European Economic Indicators (PEEI) and updated at the end of May 2011. The complete list of the series included is in Appendix B. Quarterly GDP is available from 1996Q1 until 2010Q4, while the roughly 140 monthly indicators from January 1996 to at most May 2011 (depending on the publication delay, there is a different number of missing observations for each series at the end of the sample). Generally, the monthly series include consumer and producer price index by sector, industrial production and (deflated) turnover indexes by sector, car registrations, new orders received index, business and consumers surveys with their components, sentiment indicators, unemployment indices, monetary aggregates, interest and exchange rates. When analyzing the nowcasting performance, we adopt a recursive approach, with the first evaluation quarter fixed at 2003Q1 and the last one at 2010Q4, for a total of 32 evaluation samples. For each quarter we compute three nowcasts, at the beginning of each month of the quarter (and including information up to the end of the previous month). The dataset is a final dataset, but we replicate the ragged-edge structure due to different publication lags of the series. Thus, we take into account the different information sets available at each point in time in which the nowcasts are computed.

To compare the alternative mixed-frequency approaches, we first look at the in-sample performance as in the Monte Carlo experiment. We first compute the in-sample ratio \(\text{MSE(U-MIDAS)}/\text{MSE(MIDAS)}\) for each monthly indicator, then report the mean, the median and the main percentiles of the distribution of these ratios across all the monthly indicators. The results are shown in Table 3.6. In the first panel we report the results for the in-sample relative MSEs computed in December 2006, in the second panel those computed in December 2010, so that we can also compare the performance before and during the financial crisis.

The ratios are always smaller than one, indicating a clear superior performance of the U-MIDAS with respect to MIDAS. In most of the cases, the in-sample MSE obtained with the unrestricted model is 10% smaller than the corresponding one obtained with the restricted polynomial. Moreover, the results are stable across the different subsamples.

Results are different in the case of the nowcasting performance.

Table 3.7 indicates that only for \(h_m = 1\) more than half of the U-MIDAS models perform
Table 3.6: Results for individual indicators. In-sample analysis for Euro area GDP growth.

<table>
<thead>
<tr>
<th>h_m</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.89</td>
<td>0.87</td>
<td>0.88</td>
</tr>
<tr>
<td>10th pctile</td>
<td>0.76</td>
<td>0.73</td>
<td>0.74</td>
</tr>
<tr>
<td>25th pctile</td>
<td>0.82</td>
<td>0.80</td>
<td>0.82</td>
</tr>
<tr>
<td>median</td>
<td>0.90</td>
<td>0.89</td>
<td>0.90</td>
</tr>
<tr>
<td>75th pctile</td>
<td>0.96</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td>90th pctile</td>
<td>1.01</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>h_m</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.86</td>
<td>0.81</td>
<td>0.82</td>
</tr>
<tr>
<td>10th pctile</td>
<td>0.69</td>
<td>0.57</td>
<td>0.58</td>
</tr>
<tr>
<td>25th pctile</td>
<td>0.80</td>
<td>0.75</td>
<td>0.74</td>
</tr>
<tr>
<td>median</td>
<td>0.89</td>
<td>0.86</td>
<td>0.87</td>
</tr>
<tr>
<td>75th pctile</td>
<td>0.95</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td>90th pctile</td>
<td>0.98</td>
<td>0.95</td>
<td>0.95</td>
</tr>
</tbody>
</table>

**Notes:** The table reports the performance of the IS-MSE(U MIDAS) relative to IS-MSE(MIDAS). The ratio IS-MSE(U MIDAS)/IS-MSE(MIDAS) is computed for each single indicator model, then in the second row the mean of the distribution of these ratios is reported, and in rows 3 to 7 the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported. Since the models change for each nowcast horizon, we report the results for each of the three $h_m$. Panel A reports the results at December 2006 (before the financial crisis), Panel B at December 2010 (with the financial crisis included in the sample).
Table 3.7: Results for individual indicators. Nowcasting performance for Euro area GDP growth.

<table>
<thead>
<tr>
<th></th>
<th>A. Sample 2003Q1 - 2006Q4</th>
<th>B. Sample 2003Q1 - 2010Q4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>h&lt;sub&gt;m&lt;/sub&gt;</td>
<td>h&lt;sub&gt;m&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td>1  2  3</td>
<td>1  2  3</td>
</tr>
<tr>
<td>mean</td>
<td>1.01 1.03 1.00</td>
<td>1.09 1.03 0.99</td>
</tr>
<tr>
<td>10th pctile</td>
<td>0.79 0.83 0.81</td>
<td>0.90 0.73 0.76</td>
</tr>
<tr>
<td>25th pctile</td>
<td>0.88 0.94 0.92</td>
<td>1.01 0.92 0.90</td>
</tr>
<tr>
<td>median</td>
<td>0.97 1.01 1.00</td>
<td>1.09 1.06 1.01</td>
</tr>
<tr>
<td>75th pctile</td>
<td>1.08 1.07 1.06</td>
<td>1.17 1.14 1.07</td>
</tr>
<tr>
<td>90th pctile</td>
<td>1.29 1.26 1.17</td>
<td>1.31 1.25 1.16</td>
</tr>
</tbody>
</table>

Notes: the table reports the performance of the MSE(U MIDAS) relative to MSE(MIDAS). The ratio MSE(U MIDAS)/MSE(MIDAS) is computed for each single indicator model, then in the second row the mean of the distribution of these ratios is reported, and in rows 3 to 7 the main percentiles (10th, 25th, 50th, 75th, 90th) of the distribution are reported. Panel A reports the results for the evaluation sample ending in December 2006 (before the financial crisis), Panel B in December 2010 (with the financial crisis included in the sample).
better than the corresponding MIDAS models, in the pre-crisis sample. If we consider also
the crisis, MIDAS performs much better. This is due to the fact that the estimates of
the parameters of the U-MIDAS are influenced substantially by the dramatic drop and
subsequent recovery of GDP in the quarters Q4 2008 and Q1 2009.

Overall the results are in line with those with simulated data. It is also worth noting that
for at least 25% of the indicators the U-MIDAS approach produces more precise nowcasts
than MIDAS, with even larger values in the pre-crisis period.

The evidence of a better performance of U-MIDAS in the pre-crisis sample emerges
also from Table 3.8, where we look at the mean and the median relative MSE performance
for nowcasting quarterly GDP growth for the two different classes of models, against a
benchmark. We consider as a benchmark an AR process, with lag length selected
according to the BIC criterion. First, we estimate every individual model and compute the relative
MSE with respect to the benchmark. Then, we take the mean and the median across all the
indicators of the relative MSE within a model class.

Table 3.8: Results for individual indicators relative to an AR benchmark. Nowcasting perfor-
mance for Euro area GDP growth

<table>
<thead>
<tr>
<th></th>
<th>Relative out of sample MSE (MSE(model) / MSE(AR benchmark))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A. Sample 2003Q1 - 2006Q4</td>
</tr>
<tr>
<td></td>
<td>( h_m ) 1 2 3</td>
</tr>
<tr>
<td>mean u-midas</td>
<td>1.14 1.11 1.16</td>
</tr>
<tr>
<td>mean midas</td>
<td>1.16 1.10 1.20</td>
</tr>
<tr>
<td>median u-midas</td>
<td>1.05 1.06 1.10</td>
</tr>
<tr>
<td>median midas</td>
<td>1.09 1.05 1.10</td>
</tr>
</tbody>
</table>

Notes: the table reports the average and the median performance of the MSE(model) relative to
MSE(benchmark). The ratio MSE(model)/MSE(benchmark) is computed for each single indicator model,
and then mean and median are computed for each class of models. Panel A reports the results for the
evaluation sample ending in December 2006 (before the financial crisis), Panel B in December 2010 (with
the financial crisis included in the sample). The benchmark is a AR model with lag length selected
according to the BIC criterion.

Table 3.8 also highlights striking differences before and after the crisis. As already noticed
in Foroni and Marcellino (2012), it is impossible to outperform a naive benchmark in the
period up to 2006. However, during the crisis, the use of monthly information becomes very important, and both MIDAS approaches clearly outperform the benchmark.

As a final remark, the worse performance out of sample than in sample of U-MIDAS might be due to the shorter estimation samples used in the recursive out of sample exercise, combined with the overall rather limited sample available for Euro area data. As small samples are more problematic for heavily parameterized models, the short sample length might be more problematic for U-MIDAS than MIDAS, even when the specification of the former is based on the BIC criterion. A second explanation could be related to the very good performance of MIDAS during the crisis.

### 3.4.2 An application to US GDP growth

Most of the empirical studies on the mixed-frequency approaches conducted so far focus on the US economy, since the available sample is much longer than for other countries. Therefore, as a robustness check on the relative performance of U-MIDAS and MIDAS, we have repeated the evaluation using US data. Specifically, we consider nowcasting quarterly US GDP growth using monthly indicators. The indicators are the ten components of the composite leading indicator provided by the Conference Board, used in the papers by Clements and Galvao (2009) and Stock and Watson (2003), starting in January 1959 and updated to July 2011. Appendix B provides a complete description of the data. As for the previous empirical application, we adopt a recursive approach in nowcasting, with the first evaluation quarter fixed at 1985Q1 and the last one at 2011Q1, for a total of 105 evaluation samples.

In this case, since the indicators are only ten, we do not report summary statistics but we compute and report the MSE of the U-MIDAS and MIDAS models using each single indicator relative to that of MIDAS. Table 3.9 shows the results of the in-sample performance, again conducting the analysis without and with the financial crisis period.

As for the Euro area, the results depict a better in-sample performance of U-MIDAS for the US GDP growth, though the gains are smaller than for the Euro area. Especially for one-month ahead nowcasts, all indicators show a relative in-sample MSE smaller or equal to one. Again as for the Euro area, qualitatively the results on the relative in sample performance of U-MIDAS and MIDAS do not change substantially before and after the crisis.

We now consider the out of sample nowcasting performance. As shown in Table 3.10, for at least half of the indicators U-MIDAS performs better than MIDAS, especially for the very short horizon \( h_m = 1 \). Moreover, the results are much more stable across different subsamples than in the case of the analysis conducted on Euro area data. In particular, the indicators for which the U-MIDAS performs very well before the crisis are the same which...
### Table 3.9: Results for individual indicators. In-sample analysis for US GDP growth

<table>
<thead>
<tr>
<th>Indicator</th>
<th>A. Sample 1959Q1 - 2006Q4</th>
<th>B. Sample 1959Q1 - 2011Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2</td>
<td>0.97 0.99 1.00</td>
<td>M2 0.96 0.98 0.99</td>
</tr>
<tr>
<td>stock</td>
<td>0.87 0.99 0.99</td>
<td>stock 0.95 0.98 0.98</td>
</tr>
<tr>
<td>hours</td>
<td>0.73 0.84 0.81</td>
<td>hours 0.74 0.84 0.79</td>
</tr>
<tr>
<td>ordersn</td>
<td>0.98 0.92 1.01</td>
<td>ordersn 0.98 1.01 1.01</td>
</tr>
<tr>
<td>ordersc</td>
<td>0.96 1.01 1.00</td>
<td>ordersc 0.96 1.01 1.00</td>
</tr>
<tr>
<td>building</td>
<td>0.82 1.00 0.98</td>
<td>building 0.95 0.99 0.98</td>
</tr>
<tr>
<td>claims</td>
<td>0.97 1.00 1.00</td>
<td>claims 0.97 0.99 1.00</td>
</tr>
<tr>
<td>vendor</td>
<td>0.92 0.91 0.90</td>
<td>vendor 0.92 0.92 0.91</td>
</tr>
<tr>
<td>spread</td>
<td>1.00 1.01 1.01</td>
<td>spread 0.99 1.00 1.01</td>
</tr>
<tr>
<td>expect</td>
<td>0.96 0.98 0.96</td>
<td>expect 0.98 0.99 0.97</td>
</tr>
</tbody>
</table>

**Notes:** The table reports the performance of the IS-MSE(U-MIDAS) relative to IS-MSE(MIDAS). The ratio IS-MSE(U-MIDAS)/IS-MSE(MIDAS) is computed for each single indicator model, and is reported in the corresponding row. Since the models change for each nowcast horizon, we report the results for each of the three $h_m$. Panel A reports the results at December 2006 (before the financial crisis), Panel B at March 2011 (with the financial crisis included in the sample).
outperform even after. The same is true for the indicators for which MIDAS outperforms the U-MIDAS. This finding confirms that a rather long estimation sample can improve and stabilize the performance of U-MIDAS.

Table 3.10: Results for individual indicators. Nowcasting performance for US GDP growth

<table>
<thead>
<tr>
<th></th>
<th>$h_m$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>M2</td>
<td>0.82</td>
<td>0.74</td>
<td>1.04</td>
</tr>
<tr>
<td>stock</td>
<td>1.10</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>hours</td>
<td>1.20</td>
<td>1.21</td>
<td>1.04</td>
</tr>
<tr>
<td>ordersn</td>
<td>1.15</td>
<td>1.19</td>
<td>1.16</td>
</tr>
<tr>
<td>ordersc</td>
<td>1.02</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>building</td>
<td>1.01</td>
<td>1.00</td>
<td>0.89</td>
</tr>
<tr>
<td>claims</td>
<td>0.95</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>vendor</td>
<td>1.00</td>
<td>0.95</td>
<td>1.11</td>
</tr>
<tr>
<td>spread</td>
<td>0.93</td>
<td>0.99</td>
<td>0.96</td>
</tr>
<tr>
<td>expect</td>
<td>0.99</td>
<td>1.03</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>M2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.80</td>
<td>0.75</td>
<td>0.97</td>
</tr>
<tr>
<td>stock</td>
<td>0.97</td>
<td>1.04</td>
<td>1.05</td>
</tr>
<tr>
<td>hours</td>
<td>1.12</td>
<td>1.11</td>
<td>0.86</td>
</tr>
<tr>
<td>ordersn</td>
<td>1.12</td>
<td>1.19</td>
<td>1.22</td>
</tr>
<tr>
<td>ordersc</td>
<td>1.03</td>
<td>1.01</td>
<td>1.02</td>
</tr>
<tr>
<td>building</td>
<td>0.91</td>
<td>0.98</td>
<td>0.91</td>
</tr>
<tr>
<td>claims</td>
<td>0.94</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>vendor</td>
<td>1.01</td>
<td>0.97</td>
<td>1.10</td>
</tr>
<tr>
<td>spread</td>
<td>0.92</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>expect</td>
<td>0.99</td>
<td>1.03</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Notes: the table reports the performance of the MSE(U MIDAS) relative to MSE(MIDAS). The ratio MSE(U MIDAS)/MSE(MIDAS) is computed for each single indicator model, and is reported in the corresponding row. Panel A reports the results for the evaluation sample ending in December 2006 (before the financial crisis), Panel B in March 2011 (with the financial crisis included in the sample).

To further compare the alternative mixed frequency approaches, we also compute the MSE of the U-MIDAS and MIDAS models using each single indicator relative to that of a benchmark, again represented by an AR model with BIC based lag length selection. The results are reported in Table 3.11.

The figures reported in Table 3.11 confirm that including or excluding the financial crisis period makes a difference. In particular, if we focus on the best performing models (in bold in Table 3.11), we see that when including the crisis period the best performer is an indicator model estimated with the restricted MIDAS, while the opposite is true for the sample without the crisis.

We can therefore conclude that, for several high frequency indicators, there is evidence that U-MIDAS performs well for nowcasting US GDP growth relative to MIDAS.
Table 3.11: Results for individual indicators relative to an AR benchmark. Nowcasting performance for US GDP growth

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>h_m</td>
<td></td>
<td></td>
<td></td>
<td>h_m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2 midas</td>
<td>1.42</td>
<td>1.40</td>
<td>1.40</td>
<td></td>
<td>M2 midas</td>
<td>1.60</td>
<td>1.57</td>
</tr>
<tr>
<td>u-midas</td>
<td>1.16</td>
<td>1.03</td>
<td>1.45</td>
<td></td>
<td>u-midas</td>
<td>1.28</td>
<td>1.18</td>
</tr>
<tr>
<td>stock midas</td>
<td>1.13</td>
<td>1.16</td>
<td>1.08</td>
<td></td>
<td>stock midas</td>
<td>1.12</td>
<td>1.03</td>
</tr>
<tr>
<td>hours midas</td>
<td>1.24</td>
<td>1.23</td>
<td>1.19</td>
<td></td>
<td>hours midas</td>
<td>1.09</td>
<td>1.08</td>
</tr>
<tr>
<td>hours u-midas</td>
<td>1.28</td>
<td>1.17</td>
<td>1.03</td>
<td></td>
<td>hours u-midas</td>
<td>1.25</td>
<td>1.16</td>
</tr>
<tr>
<td>ordersn midas</td>
<td>1.36</td>
<td>1.13</td>
<td>1.11</td>
<td></td>
<td>ordersn midas</td>
<td>1.18</td>
<td>1.06</td>
</tr>
<tr>
<td>ordersn u-midas</td>
<td>1.56</td>
<td>1.35</td>
<td>1.29</td>
<td></td>
<td>ordersn u-midas</td>
<td>1.32</td>
<td>1.26</td>
</tr>
<tr>
<td>ordersc midas</td>
<td>0.92</td>
<td>1.13</td>
<td>1.00</td>
<td></td>
<td>ordersc midas</td>
<td>0.71</td>
<td>0.91</td>
</tr>
<tr>
<td>ordersc u-midas</td>
<td>0.94</td>
<td>1.13</td>
<td>0.99</td>
<td></td>
<td>ordersc u-midas</td>
<td>0.74</td>
<td>0.92</td>
</tr>
<tr>
<td>building midas</td>
<td>1.00</td>
<td>0.98</td>
<td>1.01</td>
<td></td>
<td>building midas</td>
<td>1.02</td>
<td>0.91</td>
</tr>
<tr>
<td>building u-midas</td>
<td>1.01</td>
<td>0.98</td>
<td>0.89</td>
<td></td>
<td>building u-midas</td>
<td>0.93</td>
<td>0.90</td>
</tr>
<tr>
<td>claims midas</td>
<td>0.94</td>
<td>1.03</td>
<td>1.14</td>
<td></td>
<td>claims midas</td>
<td>0.86</td>
<td>0.93</td>
</tr>
<tr>
<td>claims u-midas</td>
<td>0.89</td>
<td>0.99</td>
<td>1.10</td>
<td></td>
<td>claims u-midas</td>
<td>0.81</td>
<td>0.90</td>
</tr>
<tr>
<td>vendor midas</td>
<td>0.97</td>
<td>0.98</td>
<td>0.94</td>
<td></td>
<td>vendor midas</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>vendor u-midas</td>
<td>0.97</td>
<td>0.93</td>
<td>1.04</td>
<td></td>
<td>vendor u-midas</td>
<td>1.02</td>
<td>0.99</td>
</tr>
<tr>
<td>spread midas</td>
<td>1.57</td>
<td>1.57</td>
<td>1.57</td>
<td></td>
<td>spread midas</td>
<td>1.55</td>
<td>1.56</td>
</tr>
<tr>
<td>spread u-midas</td>
<td>1.46</td>
<td>1.54</td>
<td>1.52</td>
<td></td>
<td>spread u-midas</td>
<td>1.43</td>
<td>1.53</td>
</tr>
<tr>
<td>expect midas</td>
<td>1.01</td>
<td>0.96</td>
<td>1.01</td>
<td></td>
<td>expect midas</td>
<td>0.94</td>
<td>0.89</td>
</tr>
<tr>
<td>expect u-midas</td>
<td>1.00</td>
<td>0.98</td>
<td>1.10</td>
<td></td>
<td>expect u-midas</td>
<td>0.93</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Notes: the table reports the performance of the MSE(model) relative to MSE(benchmark). The ratio MSE(model)/MSE(benchmark) is computed for each single indicator model, and is reported in the corresponding row. Panel A reports the results for the evaluation sample ending in December 2006 (before the financial crisis), Panel B in March 2011 (with the financial crisis included in the sample). The benchmark is a AR model with lag length selected according to the BIC criterion. The number in bold represents the best performance for each horizon.
3.5 Conclusions

In the recent literature, mixed-data sampling (MIDAS) regressions have turned out to be useful reduced-form tools for nowcasting low-frequency variables with high-frequency indicators. To avoid parameter proliferation in the case of long high-frequency lags, functional lag polynomials have been proposed. In this paper, we have discussed a variant of the MIDAS approach, which does not resort to functional lag polynomials, but rather to simple linear lag polynomials. Compared to the standard MIDAS approach in the literature, these polynomials are not restricted by a certain functional form, and we therefore call the approach unrestricted MIDAS (U-MIDAS).

We derive U-MIDAS regressions from a linear dynamic model, obtaining a simple and flexible specification to handle mixed frequency data. It can be expected to perform better for forecasting than MIDAS as long as it is not too heavily parameterized, in particular, as long as the differences in sampling frequencies are not too large. We have shown that this is indeed the case by means of Monte Carlo simulations. U-MIDAS is particularly suited to provide macroeconomic nowcasts and forecasts of quarterly variables, such as GDP growth, given timely observations of monthly indicators like industrial production. On the other hand, when daily data is available, our simulation results indicate that MIDAS with functional lag polynomials are preferable to predict quarterly variables.

In the empirical applications for Euro area and US GDP growth, we find that U-MIDAS provides a very good in-sample fit based on monthly macroeconomic indicators. The evidence is more mixed when looking at the out-of-sample nowcast performance, especially when the estimation sample is relatively short, as in the case of the Euro area example. Nonetheless, the out-of-sample evidence for the US suggests that U-MIDAS can outperform MIDAS with restricted polynomials for a rather large set of high frequency indicators.

Overall, we do not expect one polynomial specification to be dominant in every case. As U-MIDAS might be a strong competitor, we rather suggest to consider it as another alternative to the existing MIDAS approaches.
BIBLIOGRAPHY


Appendix A: Identification of the disaggregate process

Let us consider the LF exact MIDAS model for \( y \) (equation (3.7)):

\[
h(L^k)\omega(L)y_t = b_1(L)\beta(L)\omega(L)x_{1t} + \ldots + b_N(L)\beta(L)\omega(L)x_{Nt} + q(L^k)u_{yt},
\]

\( t = k, 2k, 3k, \ldots \)

We want to determine what and how many HF models are compatible with this LF model, namely, whether the parameters of the generating mechanism of \( y \) at HF can be uniquely identified from those at LF. The following discussion is based on Marcellino (1998), to whom we refer for additional details.

To start with, assuming that \( y \) follows the model in (3.26), we try and identify the \( a(L) \) polynomials that can have generated \( h(L^k) \). This requires to analyze all the possible decompositions of \( h(z) \) into \( \beta(L)a(L) \).

We have said that at least one \( h_{si} \) for each \( s \) in (3.6) has to be such that \( a(h_{si}) = 0 \). The other \( k - 1 \) \( h_{si} \)s can instead solve either \( \beta(h_{si}) = 0 \) or also \( a(h_{si}) = 0 \). Thus, for each \( s \), there are \( 2^k - 1 \) possible “distributions” of the \( h_{si} \)s as roots of \( \beta(L) \) and \( a(L) \). Hence, we obtain a total of \( (2^k - 1)^h \) potential disaggregated AR components, which can be written as

\[
\prod_m (1 - \frac{1}{h_m}L),
\]

where the \( h_m \)s are the \( h_{si} \)s which are considered as roots of \( a(l) = 0 \). The possible degree of \( a(L) \) ranges from \( h \) to \( hk \), with \( h < p \).

A simple but rather stringent sufficient condition for exact identification of the disaggregate process in our context is:

**Proposition 3.1** All the roots of \( a(l) = 0 \) are distinct and positive, or distinct and possibly negative if \( k \) is even.

**Proof.** If \( a(l) = 0 \) has distinct and positive roots, or distinct and possibly negative roots if \( k \) is even, then they coincide with those of \( h(z) = 0 \) raised to power of \( 1/k \), and this exactly identifies the AR component. Once, \( a(L) \) is exactly identified, \( \beta(L) \) is also unique. Finally, given \( \beta(L) \) and since the aggregation operator \( \omega(L) \) is known, the polynomials \( b_j(L) \) can be also recovered, \( j = 1, \ldots, N \).

To conclude, it is worth making a few comments on this result. First, Wei and Stram (1990) discuss more general sufficient a priori conditions for one disaggregate model to be
identifiable from an aggregate one. Second, the hypothesis of no MA component at the
disaggregate level can be relaxed. Marcellino (1998) shows that such an MA component can
be uniquely identified if its order is smaller than $p - 1$ and the condition in Proposition 1
holds. Third, the condition in Proposition 1 could be relaxed by imposing constraints on
the $b_i(L)$ polynomials. Fourth, when $y$ is multivariate the link between the disaggregate and
aggregate models is much more complicated, see Marcellino (1999), even though conceptually
the procedure to recover the disaggregate model is similar to the one we have proposed for the
univariate case. Finally, notice that the identification problem does not emerge clearly within
a Kalman filter approach to interpolation and forecasting, where the underlying assumption
is that the disaggregate model is known.

3.7 Appendix B: Monthly indicators

3.7.1 Monthly Euro area indicators

For the list of monthly indicators, see Appendix 2.13, in Chapter 2 of this thesis.

3.7.2 Monthly US data

<table>
<thead>
<tr>
<th>Name</th>
<th>Monthly indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2</td>
<td>Real money supply M2</td>
</tr>
<tr>
<td>stock</td>
<td>Stock price index (500 common stocks)</td>
</tr>
<tr>
<td>hours</td>
<td>Average weekly hours in manufacturing</td>
</tr>
<tr>
<td>ordersn</td>
<td>Orders: non-defence capital goods</td>
</tr>
<tr>
<td>ordesc</td>
<td>Orders: consumer goods and materials</td>
</tr>
<tr>
<td>building</td>
<td>Building permits</td>
</tr>
<tr>
<td>claims</td>
<td>New claims for unemployment insurance</td>
</tr>
<tr>
<td>vendor</td>
<td>Vendor performance diffusion index</td>
</tr>
<tr>
<td>spread</td>
<td>Term spread (10 year - Federal Funds)</td>
</tr>
<tr>
<td>expect</td>
<td>Consumer confidence index (U Michigan)</td>
</tr>
</tbody>
</table>
Chapter 4

Mixed-Frequency Structural Models: Identification, Estimation, and Policy Analysis

Joint with Massimiliano Marcellino

4.1 Introduction

Researchers typically model economic decision making processes as if conducted at fixed specified intervals of time. However, as already mentioned in the literature (see Christiano and Eichenbaum (1987)), there is no reason to believe that the frequency at which economic agents make decisions coincides with the frequency at which time series are released. Christiano and Eichenbaum (1987) evaluate the consequences of the specification error that results when agents’ true decision interval is finer than the data sampling interval. They show that the misalignment between agents’ decision intervals and the data sampling frequency is not a secondary issue, and that temporal aggregation is important in practice, having the potential to account for results considered anomalous in the literature.\footnote{The assumption that agents take decisions at a regular, fixed interval has also been questioned in the literature. Jorda (1999) analyzes the specification error that results when the agents’ decision interval is random and does not coincide with the data sampling interval, with additional results provided in Jorda and Marcellino (2004). For simplicity and analytical tractability, we proceed under the assumption that agents make decisions at fixed time intervals.}

\footnote{In this chapter I focused on the Monte Carlo experiment and on the empirical analysis, collecting the data and writing the necessary codes to obtain the empirical results. Moreover, I computed the analytical results for the aggregation issues in the case of a DSGE model. The comments to the results have been jointly written by the authors.}
Our first contribution is to provide a general treatment of the effects of time aggregation of structural VAR (SVAR) and DSGE models. With few exceptions, such as Christiano and Eichenbaum (1987), earlier literature addressed the temporal aggregation issue mostly in the context of reduced form ARMA and VARMA models (see, among the others, Brewer (1973), Wei (1981), Weiss (1984), Marcellino (1998, 1999)). Taking standard monetary models as an example, and assuming that their frequency is monthly while estimation is conducted with quarterly data, we show analytically that it is generally impossible to identify the structural parameters, and the estimated responses to the monetary shock can be rather different from the true ones.

Our main contribution is to demonstrate that the use of mixed-frequency data can improve identification, alleviate the temporal aggregation bias, and get estimated policy responses closer to the actual ones. In a monetary policy context, this means, for example, estimating models using quarterly time series of GDP but monthly data on inflation and interest rate, rather than quarterly data on all the three variables. Intuitively, the identification gains come from enlarging the information set to better match the decision timing of the central bank, and this then generates improved estimation and policy analysis.

From an econometric point of view, we also provide a general estimation method to deal with mixed-frequency estimation in a classical maximum likelihood framework. Specifically, we adapt the method of Mariano and Murasawa (2010) to a structural context. Bayesian estimation could be also considered, by combining our expression for the likelihood with the specification of prior distributions for the model parameters.

We then investigate how important these aggregation problems are in practice, and to what extent they influence the estimated parameters and structural relations across the variables. We use simulated and actual US data to estimate standard SVAR and DSGE models with quarterly aggregated data, and compare the results to those obtained with monthly or mixed-frequency data.

Overall, our empirical results support the theoretical findings and suggest that the extent of the temporal aggregation bias can be large, but substantially mitigated by the use of mixed-frequency data. Specifically, with simulated data we are able to verify that the results obtained from the mixed-frequency approach are very similar to those from the benchmark monthly model. Instead, in the SVAR example, the impulse response functions obtained with aggregated data are in some cases qualitatively and significantly different from those obtained with disaggregated data. And in the DSGE example, estimates of the Calvo parameter indicate that the average price duration is six months when estimated with mixed-frequency...
data, and twice as long when we employ quarterly data only.\footnote{The empirical results obtained from the DSGE models confirm those found in a similar experiment, but in a Bayesian framework, by Kim (2010).}

The paper is structured as follows. In Section 4.2, we discuss the main theoretical issues related to time aggregation. In Section 4.3, we look at the identification issues in the context of a simple SVAR model. In Section 4.4, we focus on the time scale issues in a basic New Keynesian DSGE model. In Section 4.5, we provide a general estimation method for structural models with data at different frequencies. In Section 4.6, we run a Monte Carlo experiment to assess the temporal aggregation issues. In Section 4.7, we present the empirical applications. In Section 4.8, we summarize our main findings and conclude.

### 4.2 Time aggregation

Typical regression models relate variables sampled at the same frequency. To ensure the same frequency, the most common solution in empirical applications is temporal aggregation of the higher-frequency observations to the lowest available frequency. The higher-frequency data are aggregated to the lowest-frequency by averaging or by taking a representative value (for example, the last month of a quarter).

Time aggregation can be essentially seen as a two-step filter. In the first step, the variable is aggregated following an aggregation scheme \( \omega (L) \), which can be seen as a one-sided filter. In the second step, the aggregated series \( \omega (L) y_t \) is skip-sampled, so that we observe the variable only every \( k \) periods.

Aggregation schemes have been introduced by Brewer (1973), refined by Wei (1981), Weiss (1984) and Marcellino (1998). Following Marcellino (1998), we now recall the main features of temporal aggregation of ARMA process, which we will then extend to a multivariate setting and use later on in the discussion.

Assume that a disaggregated process, \( y_t \), evolves accordingly to the following equation:

\[
g (L) y_t = s (L) \varepsilon_t
\]

where \( g (L) = 1 - g_1 L - g_2 L^2 - \ldots - g_g L^g \), \( s (L) = 1 - s_1 L - s_2 L^2 - \ldots - s_s L^s \), and \( \varepsilon_t \sim WN (0, \sigma^2) \) is white noise.

In aggregating the process at frequency \( \tau = k t \), we have to think at the values of the aggregated process as realizations of \( y = \{ y_{\tau} \}_{\tau=0}^{\infty} = \{ \omega (L) y_{kt} \}_{t=1}^{\infty} \), where \( \omega (L) = \omega_0 + \omega_1 L + \ldots + \omega_{k-1} L^{k-1} \) is the aggregation scheme and one-sided filter mentioned above.
Just as examples, two common aggregation schemes are average sampling, where \( \omega(L) = 1 + L + \ldots + L^{k-1} \), and point-in-time sampling, where \( \omega(L) = 1 \).

Then, we have to introduce a polynomial \( b(L) \), whose degree in \( L \) is at most equal to \( pk - p \), which is such that \( b(L)g(L) \) contains only powers of \( L^k \). It can be shown that this polynomial always exists, and its coefficients depend on those of \( g(L) \), see the above references for details. This corresponds to the second step, where the variable is skip-sampled to be observed only every \( k \) periods.

If we multiply both sides of eq. (4.1) by \( b(L) \) and \( \omega(L) \), we obtain:

\[
b(L) g(L) \omega(L) y_t = b(L) s(L) \omega(L) e_t. \tag{4.2}
\]

The left hand side of eq. (4.2) can be written as \( c(L^k) y_t \) or \( c(Z) y_t \), where \( Z = L^k \) is the lag operator at the aggregate temporal frequency.

Looking at the right hand side of eq. (4.2), \( b(L) s(L) \omega(L) e_t \) can be rewritten as \( q(Z) u_t \), where \( u_t \sim WN(0, \sigma_u^2) \). Therefore, there is an MA component in the aggregated process, and it can be shown that the order of the aggregate MA component coincides with the highest multiple of \( k \) non zero lag in the autocovariance function of \( b(L) s(L) \omega(L) e_t \), and the coefficients have to be such that the implied autocovariances coincides with those of \( b(L) s(L) \omega(L) e_t \) evaluated at multiples of \( k \), see Marcellino (1998) for details.

Marcellino (1999) extends the analysis of temporal aggregation to the multivariate case, focusing on VARIMA processes. The procedure to aggregate the variables to a low frequency is the same highlighted before, but extended to the multivariate notation. Therefore, we will have:

\[
G(L) y_t = S(L) e_t, \tag{4.3}
\]

where \( y_t \) is a \( n \)-dimensional process, \( G(L) = 1 - G_1 L - G_2 L^2 - \ldots - G_g L^g \), \( S(L) = 1 - S_1 L - S_2 L^2 - \ldots - S_s L^s \), the \( \{G_i\} \) and \( \{S_i\} \) are \( n \times n \) matrices of coefficients, and \( e_t \sim WN(0, \Sigma_e) \) is a multivariate white noise. Luetkepohl (1987) derives upper bounds for the lag length of the AR and MA components in the final form representation of the aggregated process, which are \( gn \) and \( gn + s \) respectively.

In order to obtain an aggregate process, after having chosen the aggregation scheme, we have to multiply both sides of eq. (4.3) by \( B(L) \), which is a matrix in this case. As it will become clearer when discussing identification issues, going to a multivariate process makes temporal aggregation more complicated, since in multiplying by \( B(L) \), which is a matrix and not a scalar, parameters are confounded across variables, time periods and equations.
4.3 The time scale problem in a SVAR context

Let us start analyzing the SVAR approach. To contextualize the analysis, we investigate what happens after an exogenous monetary policy shock, a strongly debated issue in macroeconomics. A huge strand of literature has attempted to explain the effects of monetary policy using VAR models. Sims (1986), Strongin (1995), Christiano, Eichenbaum and Evans (1996), Bernanke and Mihov (1998), Sims and Zha (2006), among others have analyzed the US monetary policy using VAR models at quarterly frequency, following different identification strategies.

For the purposes of our analysis, we start looking at a simple example, a SVAR model which considers three variables: the GDP growth, \( y_t \), the inflation rate, \( \pi_t \), and the policy rate, \( r_t \). While two of these variables are available at monthly frequency, one of them, \( y_t \), is released only quarterly. If the time frequency in which the agents take decisions is monthly, the best option for the econometrician would be to estimate the model also at monthly frequency. Therefore, the ideal model for estimation is the following:

\[
\begin{bmatrix}
  y_t \\
  \pi_t \\
  r_t 
\end{bmatrix} =
\begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
  y_{t-1} \\
  \pi_{t-1} \\
  r_{t-1}
\end{bmatrix} +
\begin{bmatrix}
  u_{yt} \\
  u_{\pi t} \\
  u_{rt}
\end{bmatrix},
\] (4.4)

where the covariance matrix of \( u_t = \begin{bmatrix} u_{yt} & u_{\pi t} & u_{rt} \end{bmatrix}' \) is a \( 3 \times 3 \) non-diagonal matrix \( \Sigma_u \), and \( y_t' \) is the unobservable monthly GDP growth. The econometrician only sees an aggregate GDP measure every three periods.

To recover the unobserved structural monetary policy shock \( \varepsilon_{rt} \), a researcher needs to impose some restrictions, and, one of the very popular choices is to define \( B \) as a lower triangular matrix with positive elements on the main diagonal, based on the Choleski decomposition of the covariance matrix of \( u_t \), \( \Sigma_u = BB' \).

We therefore rewrite eq. (4.4) as:

\[
\begin{bmatrix}
  y_t \\
  \pi_t \\
  r_t 
\end{bmatrix} =
\begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
  y_{t-1} \\
  \pi_{t-1} \\
  r_{t-1}
\end{bmatrix} +
\begin{bmatrix}
  b_{11} & b_{12} & b_{13} \\
  b_{21} & b_{22} & b_{23} \\
  b_{31} & b_{32} & b_{33}
\end{bmatrix}
\begin{bmatrix}
  \varepsilon_{yt} \\
  \varepsilon_{\pi t} \\
  \varepsilon_{rt}
\end{bmatrix},
\] (4.5)

where

\[
\varepsilon_t \sim (0, I_3),
\] (4.6)

---

4For an extensive review of the different strategies proposed in the literature to identify the effects of an exogenous shock to monetary policy see Christiano, Eichenbaum and Evans (1999). Other empirical studies are conducted at monthly level, see, among others, Sims (1992) for evidence on the effects of monetary policy in different countries, and Leeper, Sims and Zha (1996) for the US economy.

5A treatment of a general model is provided in Section 4.5.1.
where $\varepsilon_t = (\varepsilon_{y,t}, \varepsilon_{\pi,t}, \varepsilon_{r,t})'$.

The choice of a Choleski decomposition solves the identification problem, but it relies on the assumption that the recursive structure is chosen based on a theoretical justification. In principle, there are other zero restrictions which solve the identification problem. The triangular form is just an example, but in practice, it is the most common case (see, e.g., Eichenbaum \& Evans (1995), Christiano, Eichenbaum \& Evans (1996)). Therefore, for the purposes of our analysis we choose the Choleski decomposition as identification scheme, and note that the same analysis we conduct in the next sections can be repeated with a different ordering of variables without any major changes in our conclusions.

### 4.3.1 The common approach: aggregation at quarterly frequency

The VAR in eq. (4.5) cannot be directly estimated because one of the variables is non-observable. The common adopted solution is therefore to estimate the model at a frequency at which all the variables are available, in this case for example at a quarterly frequency.

As a first step, we need to see which is the correct representation of the agents’ model at quarterly frequency. To do that, we need to follow the steps outlined in Section 4.2 to aggregate the model described by eq. (4.5) at quarterly frequency.

Let us rewrite the VAR as:

$$Y_t^* = AY_{t-1}^* + B\varepsilon_t, \quad \varepsilon_t \sim (0, I_3), \quad (4.7)$$

where $Y_t^* = (y_{t}^*, \pi_t, r_{t})'$, or, using the lag operator $L$:

$$(I - AL)Y_t^* = B\varepsilon_t, \quad \varepsilon_t \sim (0, I_3). \quad (4.8)$$

For the sake of simplicity, we choose point-in-time sampling, i.e. $\omega(L) = 1$, meaning that the aggregate measure of GDP growth we observe corresponds to the monthly $y_{t}^*$ every third period.

Then, we introduce a polynomial $B(L)$, such that $B(L)(I - AL)$ contains only powers of $L^3$. In our case, we choose:

$$B(L) = (I + AL + A^2L^2). \quad (4.9)$$

Multiplying both sides of eq. (4.8) by the polynomial in (4.9) and $\omega(L)$, we obtain:

$$(I - A^3L^3)Y_t^* = (I + AL + A^2L^2) B\varepsilon_t, \quad (4.10)$$

or, equivalently,

$$Y_t = A^3Y_{t-1} + \xi_t, \quad (4.11)$$
4.3. THE TIME SCALE PROBLEM IN A SVAR CONTEXT

where $\tau = 3t$ indicates quarters, with

$$\xi_{\tau} \sim (0, \Omega), \quad \Omega = BB' + ABB'A' + A^2BB'A'^2.$$ (4.12)

Since we are looking at the variables only at $\tau = 3, 6, ..., 3t, ...$, all the variables in $Y_\tau$ are observable.

The econometrician estimates the aggregated process in eq. (4.11) as the following quarterly model:

$$y_{\tau} = Cy_{\tau-1} + \xi_{\tau}$$ (4.13)

with

$$\xi_{\tau} \sim (0, \Omega).$$ (4.14)

With this simple aggregation scheme, the aggregated process is still a VAR(1), where both the coefficients and the variance-covariance matrix are functions of the parameters driving the monthly process. At this point we face an identification issue: using quarterly data we obtain $\hat{C}$ and $\hat{\Omega}$, but from these matrices we cannot uniquely identify $A$ and $B$. In fact, $A^3$, which in our example is a $3 \times 3$ matrix, does not allow us to identify uniquely the parameters of $A$ (the multiplication when operated across matrices creates non-linear combinations of the original parameters, see Appendix 4.10.1 for more details).

The lack of identification of matrix $A$ translates into the lack of identification in the covariance matrix $\Omega$, which also suffers from the fact that $B$ appears in a quadratic form. Therefore, we cannot recover the underlying monthly parameters if we estimate the quarterly model.

Note that while in this specific example, starting with a VAR(1) at monthly level we still have a VAR(1) at quarterly level, this is not necessarily true either with higher order VARs or with different aggregation scheme (see Marcellino (1999) for more details). It is likely that in the aggregated process there is an MA component, which is then completely disregarded during the estimation of the aggregated process.

4.3.2 Exploiting data at different frequencies

In the analysis in Section 4.3.1 potentially useful information is discarded in aggregating series that are available at monthly frequency. In practice, we are disregarding information which is available. We show here that using the series at the frequency they are available, that is one series at quarterly frequency and keeping the other two at their monthly frequency, allows us to recover all the parameters of the original monthly model, solving the identification issue.
For convenience, let us briefly recall the monthly SVAR described by eq. (4.5) and (4.6):

\[
\begin{bmatrix}
  y_t^* \\
  \pi_t \\
  r_t
\end{bmatrix} = \begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{bmatrix} \begin{bmatrix}
  y_{t-1}^* \\
  \pi_{t-1} \\
  r_{t-1}
\end{bmatrix} + \begin{bmatrix}
  b_{11} & 0 & 0 \\
  b_{21} & b_{22} & 0 \\
  b_{31} & b_{32} & b_{33}
\end{bmatrix} \begin{bmatrix}
  \varepsilon_{yt} \\
  \varepsilon_{\pi t} \\
  \varepsilon_{rt}
\end{bmatrix},
\]

(4.15)

where

\[
\varepsilon_t \sim (0, I_3),
\]

(4.16)

with \( \varepsilon_t = (\varepsilon_{yt}, \varepsilon_{\pi t}, \varepsilon_{rt})' \).

The monthly dynamics of GDP is:

\[
y_t^* = a_{11} y_{t-1}^* + a_{12} \pi_{t-1} + a_{13} r_{t-1} + b_{11} \varepsilon_{yt},
\]

(4.17)

which aggregated at quarterly frequency becomes\(^6\):

\[
y_t^* = a_{11}^3 y_{t-3}^* + a_{12}^1 y_{t-1}^* + a_{12} a_{11} \pi_{t-2} + a_{12} a_{11}^2 \pi_{t-3} + a_{13}^1 r_{t-2} + a_{13} a_{11}^2 r_{t-3} + b_{11} \varepsilon_{yt} + a_{11} b_{11} \varepsilon_{yt-1} + a_{11}^2 b_{11} \varepsilon_{yt-2},
\]

for \( t = 3, 6, ..., T - 3, T \).

Now, let us look at the dynamics of the inflation rate:

\[
\pi_t = a_{21} y_{t-1}^* + a_{22} \pi_{t-1} + a_{23} r_{t-1} + b_{21} \varepsilon_{yt} + \varepsilon_{\pi t}.
\]

(4.19)

We see that the inflation rate at time \( t \) is influenced by the GDP at time \( t - 1 \), which of course we cannot observe. But after some algebraic manipulations, simply recursively substituting \( y_{t-1} \) with its expression in eq. (4.17), we obtain:

\[
\pi_t = a_{21} a_{11}^2 y_{t-3}^* + a_{22} y_{t-1}^* + a_{21} a_{12} \pi_{t-2} + a_{21} a_{11} a_{12} \pi_{t-3} + a_{23} r_{t-1} + a_{21} a_{13} r_{t-2} + a_{21} a_{11} a_{13} r_{t-3} + b_{22} \varepsilon_{\pi t} + b_{21} \varepsilon_{yt} + a_{21} b_{11} \varepsilon_{yt-1} + a_{21} a_{11} b_{11} \varepsilon_{yt-2},
\]

(4.20)

for \( t = 3, 6, ..., T - 3, T \), which depends only on observable values of \( y_t \).

Repeating the same steps for the third variable, the interest rate, we have:

\[
r_t = a_{31} y_{t-1}^* + a_{32} \pi_{t-1} + a_{33} r_{t-1} + b_{31} \varepsilon_{yt} + b_{32} \varepsilon_{\pi t} + b_{33} \varepsilon_{rt},
\]

(4.21)

---

\(^6\)For the details of this subsection see Appendix 4.10.2
which we can rewrite as:
\[ r_t = a_{31}y_{t-3}^* + a_{32}r_{t-1} + a_{31}a_{12}r_{t-2} + a_{31}a_{11}a_{12}r_{t-3} + 
+ a_{33}r_{t-1} + a_{31}a_{13}r_{t-2} + a_{31}a_{11}a_{13}r_{t-3} + b_{33}\varepsilon_{rt} + b_{32}\varepsilon_{rt} + 
+ b_{31}\varepsilon_{yt} + a_{31}b_{11}\varepsilon_{yt-1} + a_{31}a_{11}b_{11}\varepsilon_{yt-2}, \]
for \( t = 3, 6, ..., T - 3, T \).

Since, following our aggregation scheme, \( y_t^* \) is observed for \( t = 3, 6, ..., T - 3, T \), also \( y_{t-3}^* \) is observed at \( t = 3, 6, ..., T - 3, T \). Hence, estimating eq. (4.18), (4.20) and (4.22) is possible because all the data are available.

Eq. (4.18), (4.20) and (4.22) together uniquely identify all the parameters of the monthly SVAR. From eq. (4.18) the parameters \( a_{11}, a_{12}, a_{13} \) can be identified. Then, from eq. (4.20) we recover the parameters \( a_{21}, a_{22}, a_{23} \) and from eq. (4.68) \( a_{31}, a_{32}, a_{33} \). From the covariance matrix, we can finally obtain \( b_{11}, b_{21}, b_{22}, b_{31}, b_{32} \) and \( b_{33} \). Therefore, exploiting more information coming from data at different frequencies allows us to overcome some identification issues and recover the parameters that drive the model at the monthly frequency.

Further considerations can be made on eq. (4.18), (4.20) and (4.22). In particular, if we focus on eq. (4.18), we see how the equation relates a low-frequency variable to its lags and the lags of a high-frequency variable. This is exactly what Foroni, Marcellino and Schumacher (2012) label as "Unrestricted-MIDAS" regression, a variant of the MIDAS model proposed by Ghysels et al (2004) which does not resort to functional distributed lag polynomials. Eq. (4.20) and (4.22) represent instead the opposite case, in which the evolution of the high-frequency variable depends on its own lags and the lags of a low-frequency variable. Ghysels and Valkanov (2006) label this case as "reverse MIDAS" regression. In eq. (4.20) and (4.22), the monthly dependent variable depends on the value of the quarterly variable with a lag of three months. Therefore, these equations can be estimated only in \( t = 3, 6, ..., T - 3, T \). We could derive the equations also for \( t = 1, 4, ..., T - 5, T - 2 \) and \( t = 2, 5, ..., T - 4, T - 1 \), simply considering the lag of \( y_t^* \) available in that particular \( t \), i.e. either \( y_{t-1}^* \) or \( y_{t-2}^* \). The correct way to deal with this is to keep the value of \( y_t^* \) as a regressor constant for three consecutive months. However, this happens at the cost of changing its coefficient and the dynamic specification of the model (the number of lags of the monthly variables and their coefficients) each month. A possible solution is to estimate eq. (4.20) and (4.22) only in \( t = 3, 6, ..., T - 3, T \), which is sufficient to identify all the parameters of the model. If instead estimation in high-frequency is of interest, a first possibility is to estimate the proper set of equations separately for \( t = 1, 4, ..., T - 5, T - 2, t = 2, 5, ..., T - 4, T - 1 \) and \( t = 3, 6, ..., T - 3, T \). A second possibility

\(^7\)More details on the identification of the parameters are provided in Appendix 4.10.2.
is to group the reverse U-MIDAS equations for different values of $t$ into a single model by proper use of particular dummy variables. More details are provided in Appendix 4.10.3.

Further analysis on the relationship between MIDAS and MF-VAR models in a structural context has been recently conducted by Ghysels (2011). However, the author introduces a different MF-VAR representation from ours. In his paper, he constructs the MF-VAR process as stacked skip-sampled processes, and the model is consequently set up at the low frequency (e.g., in the case of monthly and quarterly data, the model is set up at a quarterly frequency). With this specification, the author aims at characterizing the mis-specification of a traditional low frequency VAR and the consequent mis-specifications in the impulse response functions. Moreover, he wants to show how Choleski factorizations are a more natural tool for impulse response analysis because the elements in the vector represent a sequence of time events.

4.4 The time scale problem in a DSGE model

In this section, we want to analyze the time scale problem in a different context, namely in a new Keynesian model, which is the workhorse for the monetary policy analysis in the framework of dynamic stochastic general equilibrium (DSGE) models.

Our starting point is a basic New Keynesian model (see Galí (2008) for a comprehensive derivation of it). The New Keynesian Phillips curve (NKPC, thereafter) and the dynamic IS (DIS) constitute the non-policy block, the Taylor type monetary policy rule which describes how the nominal interest rate evolves over time closes the model.

We want to show that time aggregation generates two different problems. First, since it confounds parameters across equations, it is not always possible to identify the parameters of the high frequency model, once it has been aggregated at a lower frequency. Second, even when identification is not an issue and each parameter can be uniquely identified from a quarterly model, the common approach of considering the same structural model at a different frequency leads to different interpretations of the parameters values.

We first derive the mapping from the monthly specification to the equivalent quarterly counterpart of the same model. Then, we illustrate how the temporal aggregation bias can influence the estimates of the coefficients even when the model is uniquely identified. In a second step, we use a slightly more complicated version of the model to show how time aggregation raises also identification issues. Finally, we show that the use of mixed-frequency data overcomes both the temporal aggregation bias and the identification issue, allowing to identify the parameters of the underlying monthly model even when one variable can be only
4.4. THE TIME SCALE PROBLEM IN A DSGE MODEL

observed at quarterly frequency.

An analysis on how to incorporate monthly information in estimated quarterly DSGE models has been conducted by Giannone et al. (2009). In their paper, the authors focus on how to augment the quarterly model with monthly information to obtain a better forecasting performance. In this paper, we focus instead on the identification problems and estimation bias due to the mismatch between the time scale of the DSGE model and that of the data used for the estimation.

4.4.1 A basic New Keynesian model: mapping from monthly to quarterly specification

In this subsection, we look at a very simple version of the New Keynesian model, a simplified version of the model analyzed by Clarida, Galí, Gertler (2000).

The three equations which describe the model are the following:

\[ \pi_t = \beta E_t \pi_{t+1} + ky_t^* + \varepsilon_{st}, \]  
\[ y_t^* = E_t y_{t+1}^* - \tau (R_t - E_t \pi_{t+1}) + \varepsilon_{dt}, \]  
\[ R_t = \rho_r R_{t-1} + (1 - \rho_r) (\phi_{\pi} \pi_t + \phi_y y_t^*) + \varepsilon_{rt}, \]  

where eq. (4.23) is the NKPC, eq. (4.24) the DIS and eq. (4.25) the policy rule, and \( \pi_t, y_t^* \) and \( R_t \) stand respectively for inflation rate, output growth and real interest rate. \( y_t^* \) is starred since it is not observable at a monthly frequency. For analytical tractability and without loss of generality, we assume that \( \varepsilon_{st}, \varepsilon_{dt} \) and \( \varepsilon_{Rt} \) are uncorrelated, i.i.d. and normally distributed with mean equal to zero and variance respectively equal to \( \sigma_s^2, \sigma_d^2 \) and \( \sigma_R^2 \). Finally, \( k \) is a function of the Calvo parameter \( \theta \), which describes the price rigidity, and it is defined as \( k = \frac{(1-\theta)}{\theta} \).

The model in eq. (4.23) - (4.25) can be written in matrix form as:

\[ B_0 X_t^* = C X_{t-1}^* + D E_t X_{t+1}^* + \epsilon_t, \]  

where \( X_t^* = \begin{bmatrix} \pi_t & y_t^* & R_t \end{bmatrix}' \) and \( \epsilon_t = \begin{bmatrix} \epsilon_{st} & \epsilon_{dt} & \epsilon_{rt} \end{bmatrix}' \), with \( \epsilon_t \sim N(0, I_3) \).

The unique stable solution for this model is given by

\[ A_0 X_t^* = A_1 X_{t-1}^* + \epsilon_t, \]  

with \( A_0 \) and \( A_1 \) satisfying the two following conditions:

\[ A_0 = B_0 - DA_0^{-1} A_1, \]  
\[ A_1 = C. \]
The matrices $B_0, C, D, A_0$ and $A_1$ are defined in Appendix 4.10.4.

The model is uniquely identified: all the parameters of the model in (4.23) - (4.25) appear in the data generating process defined in (4.27), and each set of parameters gives a unique value of $A_0$ and $A_1$ (for the proof, see Fucac, Waggoner and Zha (2007)).

We assume that agents’ decision interval is in months. If all the data were available at that frequency, the econometrician could simply estimate (4.27), with the restrictions determined by the structure of the economy described in the model, and obtain the estimates of all the parameters. But $y_t^i$ is not observable, therefore we cannot estimate (4.27) directly.

The common strategy adopted in the literature is to estimate the model at quarterly frequency where all the data are available. The naïf econometrician therefore simply estimates the following model:

$$A^N_0 X_t = A^N_1 X_{t-1} + \epsilon^N_t,$$

with $\epsilon^N_t \sim N(0, I)$, for $\tau = 1, 2, 3...$ where $\tau$ indicates quarters. In other words, what the econometrician does is to consider the same economy described by (4.27), and setting the agents’ decision interval equal to the sampling interval at which all the data are available. But this is obviously different than estimating (4.27) aggregated at quarterly level.

First, we consider the monthly process (4.27), and aggregate it at quarterly level, following the steps outlined in Section 4.2. Again, to keep the notation simple, we consider $\omega(L) = 1$. We obtain:

$$A^Q_0 X_t = A^Q_1 X_{t-1} + \epsilon^Q_t,$$

with $\epsilon^Q_t \sim N(0, \Sigma^Q)$, and $\Sigma^Q$ is a diagonal matrix$^8$.

With this very basic model, we can identify all the parameters which define the monthly process from $A^Q_0$ and $A^Q_1$. This allows us to isolate one of the two issues related to time aggregation, the temporal aggregation bias. The naïf estimated model obtained by considering agents acting at a quarterly frequency is different from the correct quarterly aggregated model. It will be the subject of the next subsection to show that in a slightly more complicated case, the identification of the monthly parameters from $A^Q_0$ and $A^Q_1$ is not possible anymore.

If we focus on the dynamics of the two models, (4.30) and (4.31), we see that they have the same zero restrictions in the matrices:

$$A^i_0 = \begin{bmatrix} X & X & X \\ 0 & X & X \\ X & X & X \end{bmatrix}, A^i_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & X \end{bmatrix}, \Sigma^i = \begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix}.$$  

$^8$The derivation of eq. (4.31) and the definition of all the matrices of this subsection are in Appendix 4.10.4.
Even though they have the same zero restrictions, the two models are different. First, the only non-zero element of $A_1^i$, which describes the dynamics of the model is different in the two approaches: while in the naive case $A_1^N (3, 3) = \frac{1}{\sigma_R} \rho_R^N$, in the quarterly aggregated model is $A_1^Q (3, 3) = \frac{1}{\sigma_R} \rho_R^3$, where $\Psi$ is a non-linear function of the other structural parameters. From the comparison of the two elements, we can see that $\rho_R^N$ is different from $\rho_R^3$, which is usually considered the quarterly counterpart of a monthly $\rho_r$ : the coefficient of the lagged interest rate in the quarterly aggregated model depends also on other structural parameters. The naive econometrician is therefore interpreting the coefficients in a different way from the one who considers time aggregation, and this reflects also in the interpretation of the coefficients describing the contemporaneous relations. Furthermore, the variance of the shock to the interest rate rule is obviously different in the two cases. Hence, it is not surprising that the estimation of these two different models gives rise to discrepancies in the estimated parameters.

An empirical analysis which shows a temporal aggregation bias in the estimated structural parameters has been conducted by Kim (2010) with a slightly different model setup. He found that the estimation of the Calvo parameter implies different average price duration when estimated with monthly rather than quarterly data. The analysis in this paper provides a theoretical justification for his results.

### 4.4.2 A second New Keynesian model: aggregation and loss of identification

In this section we aim at illustrating the identification issues related to time aggregation. As we have already mentioned, the aggregation of a structural process at a lower frequency always leads to non-linear combinations of the parameters, which most of the times prevent the identification of the disaggregated process.

We analyze a version of the New Keynesian model similar to that described by eq. (4.23) - (4.25). The equations which describe the model are:

\[
\begin{align*}
\pi_t &= \beta E_{t} \pi_{t+1} + k y_t^* + \varepsilon_{st}, \\
y_t^* &= E_{t} y_{t+1}^* - \tau (R_t - E_{t} \pi_{t+1}) + p y_{t-1}^* + \varepsilon_{dt}, \\
R_t &= \rho_r R_{t-1} + (1 - \rho_r) \left( \phi_n \pi_t + \phi_y y_t^* \right) + \varepsilon_{rt}.
\end{align*}
\]

The model is very similar to the one described in Section 4.4.1, with the same NKPC and the same monetary policy rule. Only the DIS changes, namely $y_t^*$ depends, among other things, not only on the expected future output but also on its value in the previous period,
In other words, the dynamic of the DIS is more complex. For more details on this DIS formulation, see Furher and Rudebusch (2004).9

Similarly to the previous example, the model can be rewritten first in a matrix form and then in a reduced form like:

\[ A_0 X_t^* = A_1 X_{t-1}^* + \epsilon_t, \quad (4.35) \]

with the same constraints on \( A_0 \) and \( A_1 \).10 We normalize \( \sigma_d \) to one, to achieve identification. The reason is that we want to start with a uniquely identified process at monthly level, in such a way that we can disentangle the identification issues coming from temporal aggregation.

Since, again, in \( X_t^* \) we have \( y_t^* \) which is not observable every month, we aggregate (4.35), in such a way that in the aggregated process we just have observations of \( y_t^* \) which are available (i.e. we have only observations at \( t, t-3, t-6, \ldots \)).

What we obtain is

\[ A_0^Q X_r = A_1^Q X_{r-1} + \epsilon_r^Q, \quad (4.36) \]

with \( \epsilon_r^Q \sim N(0, \Sigma^Q) \).

Differently from the model analyzed in Section 4.4.1, not all the parameters which describe the monthly structural model (\( \beta, k, \tau, p, \rho_r, \phi_x, \phi_y, \sigma_x, \text{and } \sigma_r \)) can be uniquely identified from \( A_0^Q, A_1^Q \) and \( \Sigma^Q \).11 This example illustrates that time aggregation creates non-linear combinations of the parameters which describe the monthly process. These non-linear combinations make recovering the original parameters impossible. Moreover, if we consider the zero restrictions, we see that they vary in the two approaches. While in the naif model we have

\[
A_0^N = \begin{bmatrix} X & X & X \\ 0 & X & X \\ X & X & X \end{bmatrix}, \quad A_1^N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix}, \quad \Sigma^N = \begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix},
\]

in the quarterly aggregated model the matrices are of the form:

\[
A_0^Q = \begin{bmatrix} X & X & X \\ 0 & X & X \\ X & X & X \end{bmatrix}, \quad A_1^Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & X & X \\ 0 & X & X \end{bmatrix}, \quad \Sigma^Q = \begin{bmatrix} X & 0 & 0 \\ 0 & X & X \\ 0 & X & X \end{bmatrix}.
\]

Comparing the restrictions of this example, we notice the second problem related to time aggregation. The naif econometrician, which estimates the same structural model setting the agents’ decision interval equal to the sampling interval at which all the data he needs

---

9Furher and Rudebusch (2004) provide also a more general, complicated version of eq. (4.33). We choose the simple version for analytical tractability.

10The description of the different matrices is in Appendix 4.10.5.

11See Appendix 4.10.5 for more details on this point.
are available, imposes zero restrictions even where in the proper aggregated model there are not, namely he assumes that $A_{1N}^N(2,3) = 0$ and $A_{1N}^N(3,2) = 0$. It is evident, therefore, that the two different models give rise to two different kinds of structural relations. While the naïf econometrician imposes no effects between output and the lag of the interest rate, if we estimate the properly aggregated model we see a dynamic relation between the same two variables. Hence, it is not surprising if the two models provide different estimates of the structural parameters. This reflects the temporal aggregation bias.

4.4.3 Exploiting mixed-frequency data to deal with identification issues

In this section we show how the parameters of the model at monthly frequency can be identified when we exploit mixed-frequency data. Taking into account the information which is available at monthly frequency (i.e. data on inflation rate and interest rate), we solve the identification issue, which we face when we aggregate all the series at quarterly level.

We first write the three equations represented in a compact form by (4.27) as:

\begin{align}
\frac{1}{\sigma_s} \pi_t + F y^*_t + GR_t &= \epsilon_{st} \\
H y^*_t + LR_t &= p y^*_{t-1} + \epsilon_{dt} \\
\frac{\phi_\pi}{\sigma_r} (\rho_r - 1) \pi_t + \frac{\phi_y}{\sigma_r} (\rho_r - 1) y^*_t + \frac{1}{\sigma_r} R_t &= \frac{1}{\sigma_r} \rho_r R_{t-1} + \epsilon_{rt}.
\end{align}

(4.37) (4.38) (4.39)

We do not have any problems in estimating eq. (4.37) and (4.39) at $t = 3, 6, 9, \ldots$ since all the data are available at this frequency. However, we cannot estimate eq. (4.38) since $y^*_{t-1}$ is not observable. Therefore, we need to modify eq. (4.38) in such a way that it contains only variables which are available at the time of estimation. If we substitute $y^*_{t-1}$ with its own expression $y^*_{t-1} = \frac{p}{H} y^*_{t-2} - \frac{L}{H} R_{t-1} + \frac{1}{H} \epsilon_{dt-1}$, and then we repeat it again for $y^*_{t-2}$, we obtain:

\begin{align}
y^*_t &= \left(\frac{p}{H}\right)^3 y^*_{t-3} - \frac{L}{H} R_t - \frac{L}{H} \left(\frac{p}{H}\right) R_{t-1} - \frac{L}{H} \left(\frac{p}{H}\right)^2 R_{t-2} + \xi_t.
\end{align}

(4.40)

From eq. (4.37), (4.40) and (4.39), we can now identify all the parameters. From eq. (4.37), we identify $\sigma_s$ and obtain $F$ and $G$; from eq. (4.39), we identify $\sigma_r, \rho_r, \phi_y$ and $\phi_\pi$, and from eq. (4.40), we obtain $\frac{L}{H}$ and $\frac{p}{H}$. These ratios, together with $F, G$ and the definition of $F, G, H, L^12$, allow us to identify all the remaining parameters, $\beta, k, \tau, p$.

From this example, we can see how the use of mixed-frequency data can solve the identification issue. In general, it is not always possible to recover all the parameters, even if $A_{1N}^N(2,3) = 0$ and $A_{1N}^N(3,2) = 0$. Foroni, Claudia (2012), Econometric Models for Mixed-Frequency Data European University Institute DOI: 10.2870/45897

\[\text{\footnotesize{Foroni, Claudia (2012), Econometric Models for Mixed-Frequency Data}}\]

\[\text{\footnotesize{European University Institute}}\]

\[\text{\footnotesize{DOI: 10.2870/45897}}\]
with mixed-frequency data. But the information conveyed in them helps in mitigating the problem.

It is worth making one final comment on the estimation of eq. (4.37) and (4.39). As already pointed out in Section 4.3.2, it would be efficient to estimate these equations not only in $t = 3, 6, 9...$ but also in $t = 1, 4, 7...$ and $t = 2, 5, 8...$, substituting $y_t^*$ with its lags which are available at each specific $t$. However, this is not necessary for the purpose of identification. We will introduce a more efficient estimation method in Section 4.5, relying on the Kalman filter.

4.5 Estimation methods for structural models with mixed-frequency data

In this section we present a general method for the estimation of a structural model with data released at different frequencies. We follow and generalize the analysis of Mariano and Murasawa (2010) and we provide the state-space representation of the models to be estimated in a maximum-likelihood framework. The low frequency series are considered as high frequency series with missing observations. In the first subsection, we provide a general way to write the state-space form for a SVAR model which takes into account data at different frequencies. In the second subsection we focus on log-linearized DSGE models, and more generally on all the models whose solution can be cast in a state-space form.

The framework could potentially be generalized to more than two frequencies. Nevertheless, this generalization complicates the notation and increases the computational effort required. Therefore, in this paper we limit our focus to the simpler case of two frequencies only.

4.5.1 Estimation of a mixed frequency SVAR model

We define $\{y_{1t}\}$ as the $N_1$-variate low frequency series observable every $m$th period, and $\{y_{2t}\}$ as the $N_2$-variate high frequency series observable every period. $\{y_{1t}^*\}$ represents the latent unobservable high frequency series underlying $\{y_{1t}\}$, such that, in the notation introduced in Section 4.2, $y_{1t} = \omega (L) y_{1t}^*$ for each $t$, where $l$ is the lag order of the polynomial $\omega (L)$. Finally, we define the $N \times 1$ vectors $y_t$ and $y_t^*$ respectively as $\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix}$ and $\begin{pmatrix} y_{1t}^* \\ y_{2t}^* \end{pmatrix}$ for all $t$, where $N = N_1 + N_2$.

To simplify the notation, let us assume that $\mu = E (y_t) = 0$ and $\mu^* = E (y_t^*) = 0$. 

Foroni, Claudia (2012), Econometric Models for Mixed-Frequency Data
European University Institute
DOI: 10.2870/45897
The VAR model we want to estimate is therefore the following:

\[ \Phi (L) y_t^* = u_t, \]  

(4.41)

where \( \Phi (L) \) is a polynomial in the lag operator of order \( p \), and \( u_t \sim N (0, \Sigma) \). Moreover, for all \( t \) the following relation must hold:

\[ y_t = H (L) y_t^*, \]  

(4.42)

where \( H (L) = \begin{pmatrix} \omega (L) & 0 \\ 0 & I \end{pmatrix} \).

The model in eq. (4.41) and (4.42) can be cast in a state-space form, and then estimated making use of the Kalman filter.

If \( p \leq l + 1 \), the state-space representation is the following:

\[ s_t = As_{t-1} + B \varepsilon_t, \]  

\[ y_t = Cs_t, \]  

(4.43)  

(4.44)

where \( \varepsilon_t \sim N (0, I_N) \), the state vector is defined as

\[ s_t = \begin{pmatrix} y_t^* & \ldots & y_{t-l}^* \end{pmatrix}', \]

and the matrices defined as

\[ A_{(l+1)N \times (l+1)N} = \begin{bmatrix} \Phi_1 & \ldots & \Phi_p & 0_{N \times (l+1-p)N} \\ I_N & 0_{lN \times N} \end{bmatrix}, \]

\[ B_{(l+1)N \times N} = \begin{bmatrix} \Sigma^{1/2} \\ 0_{lN \times N} \end{bmatrix}, \]

\[ C_{N \times (l+1)N} = \begin{bmatrix} H(0) & \ldots & H(l) \end{bmatrix}. \]

Since \( y_t \) is observable only every \( m \)th period, it has missing observations.

If \( p > l + 1 \), the state-space form is still as in eq. (4.43) and (4.44), but the state vector is now defined as

\[ s_t = \begin{pmatrix} y_t^* & \ldots & y_{t-p+1}^* \end{pmatrix}'. \]
and the matrices are the following:

\[
A_{NP \times 1} = \begin{bmatrix}
\Phi_1 & \cdots & \Phi_{p-1} & \Phi_p \\
I_{(p-1)N} & 0_{(p-1)N \times N}
\end{bmatrix},
\]

\[
B_{NP \times NP} = \begin{bmatrix}
\Sigma^{1/2} \\
0_{(p-1)N \times N}
\end{bmatrix},
\]

\[
C_{N \times NP} = \begin{bmatrix}
H \left( L \right) & 0_{N \times (p-t+1)}
\end{bmatrix}.
\]

Once the model is written in state-space form, we can estimate it by replacing the missing observations in \( y_t \) with zeros and applying the Kalman filter (see Mariano and Murasawa (2010) for details).

### 4.5.2 Estimation of a mixed frequency DSGE model

In general the solution of a log-linearized DSGE model is of the form:

\[
y_t = A(\theta) s_t + u_t, \tag{4.45}
\]

\[
s_t = B(\theta) s_{t-1} + C(\theta) \varepsilon_t, \tag{4.46}
\]

where \( s_t \) is the \( k \times 1 \) state vector, \( y_t \) is the \( N \times 1 \) vector of observables, \( \varepsilon_t \) is the \( p \times 1 \) vector of shocks, and \( u_t \) is the \( N \times 1 \) vector of possible measurement errors. All the elements depend on \( \theta \), the structural parameters of the model. Eq. (4.46) characterizes the DSGE model solution, while eq. (4.45) maps the model variables into the observable variables.

In the case we consider here, not all variables are observable at frequency \( t \). We define \( y_{1t}, y_{2t}, y_{3t}, y_{4t}^* \) as in Section 4.5.1. Following this notation, we present here a method to estimate the following system:

\[
y_t^* = A(\theta) s_t + u_t, \tag{4.47}
\]

\[
s_t = B(\theta) s_{t-1} + C(\theta) \varepsilon_t. \tag{4.48}
\]

Both \( u_t \) and \( \varepsilon_t \) are normally distributed, with \( v_t = C(\theta) \varepsilon_t \), \( E(v_tv'_t) = Q(\theta) \), \( E(u_tu'_t) = H(\theta) \). Hereafter, for simplicity, we write \( A, B, C, Q \) and \( H \) taking their dependence on \( \theta \) for given.

We need to modify the state-space form in (4.47) and (4.48) to include also the aggregation rule \( y_t = \omega(L) y_t^* \). Let us define the new state vector as:

\[
f_t \equiv \begin{bmatrix}
0_{(k+p)(l+1) \times 1}
\end{bmatrix} = \begin{bmatrix}
s_t & s_{t-1} & \cdots & u_t & u_{t-1} & \cdots & u_{t-l}
\end{bmatrix}^\prime.
\]

\( ^{13} \)For simplicity we consider \( H \) diagonal, i.e. the measurement errors are serially uncorrelated, but it can be extended to the case of serially correlated measurement errors.
The state-space representation is now

\[
y_t = G f_t, \quad (4.49)
\]

\[
f_t = M f_{t-1} + P z_t, \quad (4.50)
\]

where \( z_t \) is defined as:

\[
\begin{pmatrix} z_t \\
(\varepsilon_t) \\
(ut)
\end{pmatrix},
\]

and the matrices \( G, M, P \) are the following:

\[
G = \begin{bmatrix} H(0) & \ldots & H(l) \\ 0 & \ldots & 0 \end{bmatrix},
\]

\[
M = \begin{bmatrix} B & 0 & \ldots & 0 \\ I & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & I & 0 \end{bmatrix},
\]

\[
P' = \begin{bmatrix} C & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & I & 0 & \ldots & 0 \end{bmatrix}.
\]

Again, we can estimate the state-space form in (4.49) and (4.50) following the procedure described in Mariano and Murasawa (2010).

It is worth noticing that DSGE models may admit a VAR representation mapping economic shocks to a vector of observable variables and its lags, see Fernandez-Villaverde et al (2007) and Ravenna (2007). In the cases in which the DSGE model admits a finite VAR representation, we can also write the model in VAR form and use the estimation methods described in Section 4.5.1.

4.6 Monte Carlo experiments

This section presents Monte Carlo exercises to illustrate the theoretical analysis conducted so far, using simulated data in a controlled setup. In particular, we focus on the usefulness of
exploiting mixed-frequency data in both a SVAR framework (Section 4.6.1) and for a small-scale New Keynesian DSGE model (Section 4.6.2). In the case of a SVAR, we analyze how the estimated reaction of the different variables to a shock changes when we consider monthly, quarterly or mixed-frequency data. In the case of a DSGE monetary model, we compare the discrepancies between the estimated values of the structural parameters obtained with data at different frequencies, highlighting identification issues and temporal aggregation bias.

4.6.1 A Monte Carlo exercise within a SVAR framework

In Section 4.3, we extensively discussed the identification issues deriving from temporal aggregation in the case of a trivariate SVAR. The identification of the monthly process is generally lost with quarterly data, but it is still possible when we exploit the information contained in mixed-frequency data. We now want to see whether these issues are empirically relevant.

Consistently with the literature on VAR models, we look at the impulse response functions, which summarize the information contained in the VAR coefficients and in the covariance matrix of the residuals. We choose to orthogonalize the errors with a Choleski decomposition.

Briefly recalling the theory behind the impulse response functions to apply it to our specific case, we know that for a VAR(1) as written in eq. (4.7), the MA coefficient matrices contain the impulse responses of the system, and the $i^{th}$ coefficient of the MA representation, $\phi_i$, is equal to $A^i$. Moreover, since the covariance matrix of $u_t$, $\Sigma_u$ can be decomposed as $\Sigma_u = BB'$, the matrix that describes the orthogonalized impulse response $i$ periods after the shock is $\Theta_i = \phi_i B$.

In our case, the monthly impulse responses are therefore:

$$\Theta_0 = B$$
$$\Theta_1 = AB$$
$$\Theta_2 = A^2 B$$
$$\Theta_3 = A^3 B$$
$$\vdots$$
$$\Theta_6 = A^6 B$$

If we look instead at the impulse responses for the quarterly model defined in eq. (4.13)
and (4.14), we have:

\[
\begin{align*}
\Psi_0 &= Q \\ 
\Psi_1 &= CQ \\ 
\Psi_2 &= C^2Q \\
&\vdots
\end{align*}
\]

where \( Q \) is the matrix obtained from the decomposition of \( \Omega \). It is worth to stress that in the case of the impulse responses obtained from a quarterly model, the time horizon is in quarters and not in months. Therefore, if we want to compare the responses from quarterly and mixed-frequency models we should compare the results on the same time scale. The impact at the time of the shock is \( \Theta_0 = B \) in the monthly case and \( \Psi_0 = Q \) in the quarterly case, the impact one quarter from now is equivalent to the impact three months from now, so it is \( \Theta_3 = A^3B \) in one case and \( \Psi_1 = CQ = A^3Q \) in the other, and generally the impact \( j \) quarters from now is respectively \( \Theta_{3j} = A^{3j}B \) and \( \Psi_j = C^jQ = A^{3j}Q \). It is therefore clear that in the VAR(1) case the differences in the impulse response functions are mainly driven by the differences between the covariance matrix estimated with the two approaches and the restrictions imposed (while dynamics is also relevant for higher order high frequency VARs). The use of mixed-frequency data additionally allows to trace the dynamics of the process also intra-quarterly. This is obviously not possible when quarterly data only are used.

Our aim is to run a small exercise to compare the different impulse responses when we use only quarterly data and when we exploit mixed-frequency data. A similar exercise has been carried out by Chiu et al. (2011). Their analysis is conducted in a Bayesian framework and on a set of different monthly and quarterly variables, which include GDP, industrial production, inflation and unemployment rate. Their findings suggest the importance of taking mixed-frequency information into account in the estimation of structural VAR models.

In Section 4.6.1.1 we simulate the data, to conduct the analysis in a controlled environment. In Section 4.7.1 we implement our analysis with actual data for the US economy.

### 4.6.1.1 Simulation design and results

The simulation design is closely related to that in Foroni, Marcellino and Schumacher (2012), with the DGP given by the high-frequency VAR

\[
\begin{pmatrix}
y_t \\
x_t
\end{pmatrix}
= \begin{pmatrix}
\rho & \delta_l \\
\delta_h & \rho
\end{pmatrix}
\begin{pmatrix}
y_{t-1} \\
x_{t-1}
\end{pmatrix}
+ \begin{pmatrix}
e_{y,t} \\
e_{x,t}
\end{pmatrix},
\]

(4.53)
where $y_t$ is the low-frequency variable and $x_t$ is the high-frequency variable. With $t$ we denote the high-frequency time index with $t = 1, \ldots, T \times m$. $T$ defines the size of the estimation sample expressed in the low-frequency unit. $m$ denotes the sampling frequency of the low-frequency variable $y_t$. To be consistent with the analysis conducted in Section 4.3), we assume that $\omega(L) = 1$. Thus, the low-frequency variable $y_t$ is available only for $t = m, 2m, \ldots, T$. $m$ defines the size of the estimation sample expressed in the low-frequency unit. $m$ denotes the sampling frequency of the low-frequency variable $y_t$. To be consistent with the analysis conducted in Section 4.3), we assume that $\omega(L) = 1$. Thus, the low-frequency variable $y_t$ is available only for $t = m, 2m, \ldots, T$.

We focus our analysis on the sampling frequency $m = 3$, which represents the case of monthly and quarterly data.

In generating the variables we consider different combinations of parameters, in such a way to ensure a non-explosive solution, and therefore stationarity of both $y$ and $x$. In particular, we consider the following specifications of $\{p, \delta_l, \delta_h\}$:

$$\{0.5, 0.4, 0.4\}, \{0.5, 0.8, 0.4\}, \{0.9, 0.08, 0.08\}, \{0.9, 0.1, 0.08\}.$$ 

The shocks $e_{y,t}$ and $e_{x,t}$ are sampled independently from the normal distribution $N \sim (0, I_2)$.

The number of observations in the sample is fixed to $T = 100$ for the low-frequency variable, and therefore to 300 for the high-frequency one.

All in all, the different parameter combinations cover a broad range of DGPs with different degrees of persistence and correlation between the high-frequency and the low-frequency variable.

In our Monte Carlo analysis we look at the impulse responses obtained when we estimate (4.53) with high-frequency data, with only low-frequency data (obtained skip-sampling the series), and with mixed-frequency data. More in detail, the simulation of the data at monthly frequency allows us also to estimate the monthly VAR(1) process with standard techniques, and use it as a benchmark. Then, once the data are skip-sampled to quarterly frequency, we estimate the corresponding quarterly VAR(1) process, again by OLS\textsuperscript{15}. Finally, we consider the mixed-frequency case in which only $y$ is skip-sampled to mimic the availability of GDP at quarterly level, while $x$ is available every month. In order to estimate the mixed-frequency model, we follow Mariano and Murasawa (2010), as explained in Section 4.5.1. In each of the cases, we choose the Choleski decomposition to make the shocks orthogonal. We compute the impulse response functions to trace out what happens to the system up to 8 quarters.

\textsuperscript{14}We considered also many other specifications, in particular $\{0.1, 0.1, 0.1\}$, $\{0.1, 0.4, 0.4\}$, $\{0.1, 0.8, 0.8\}$, $\{0.5, 0.1, 0.1\}$, $\{0.5, 0.2, 0.2\}$, $\{0.5, 0.4, 0.2\}$, $\{0.9, 0.01, 0.01\}$, $\{0.9, 0.04, 0.04\}$ and $\{0.9, 0.08, 0.04\}$. The results are consistent with those described later on in the section.

\textsuperscript{15}We consider a process with one lag, because we know from theory that the quarterly aggregated process corresponding to a monthly VAR(1) is still a VAR(1). Therefore, we avoid any mis-specification issue.
ahead in the quarterly model, and up to 24 months in the monthly and mixed-frequency case.

In order to compare the effects of aggregation to the low-frequency and of the use of mixed-frequency data, we generate the data $R$ times, and for each bivariate dataset we compute the impulse responses obtained from the process estimated at monthly, quarterly and mixed frequency. Then, we report the median impulse responses, and the 10th and 90th percentiles computed across replications and for the different parameter specifications. In our experiment we fix the number of replications equal to $R = 1000$.

In Tables 4.1 to 4.4, we report the values of the impulse responses (median) and the confidence intervals, computed as the 10th and 90th percentile. We report the monthly results for the two quarters immediately after impact, which are useful not only to compare the results at monthly and quarterly level, but also to check the goodness of the mixed-frequency model in capturing the high-frequency dynamics. Furthermore, we report the response at some longer horizons (up to six quarters ahead), for the sake of conciseness only when the quarterly response is also available\(^\text{16}\).

We can summarize the main results as follows. First, if we estimate the process at the low frequency, the size of the impulse response is bigger than the one obtained at the monthly frequency, and generally outside the confidence bands of the monthly process, represented by the 10th and 90th percentile of the distribution of impulse responses in our Monte Carlo experiment. This means that if we estimate a process at a lower frequency than the true frequency of the process, we may draw wrong conclusions on the size of the impact of the shock. Second, the mixed-frequency approach works quite well in capturing the salient features of monthly process. In particular, if we look at Tables 4.2 to 4.4, in which we are considering the interaction between low and high frequency variables, or between high frequency variables only, we see how well the mixed-frequency approach captures the dynamics of the monthly process. In most of the cases, the median response computed with mixed-frequency data is very similar to the benchmark obtained when all the data are available at the higher frequency, and always inside the confidence bands of the monthly benchmark. Moreover, the same confidence bands are typically fairly similar to the monthly ones. Third, a special consideration is due to Table 4.1, in which we see the response of the low-frequency variable to the same low-frequency variable. This is the case in which both the quarterly and the mixed-frequency approaches loose most of the information (the low-frequency variable is the one which we do not observe at high-frequency level). However, while the low-frequency variable...

\(^{16}\text{Since we call period 1 the period of impact, we report the responses at 7, 10, 13, 16 months which correspond to the responses at 2, 3, 4, 5 quarters ahead.}\)
model overestimates once again the size of the response, this does not happen with the mixed-frequency approach. With the latter approach, the median impulse response remains fairly similar in size to the monthly benchmark. We can notice some differences in the confidence bands, which in this case tend to become wider, due to the less information available for estimation. Finally, a consideration has to be done referring to the different parameter specifications. Although the results summarized above are valid across specifications, the mixed-frequency approach improves its performance the higher the correlation between low- and high-frequency variables. The more the low-frequency variable depends on the high-frequency the better it is to exploit the information available at high-frequency, which is on the other side lost if we aggregate all the variables at the lowest frequency.

### 4.6.2 A Monte Carlo exercise within a DSGE framework

In this subsection we provide an illustration of the time aggregation issues in the context of a small DSGE model. The aim is to estimate the model described in Section 4.4.1, with monthly, mixed-frequency and quarterly data only, and compare the different estimates of the structural parameters. We use the standard solution methods for linear rational expectations models, and since the solution of the model has a state-space representation, we obtain maximum likelihood estimates of the structural parameters by making use of the Kalman filter. The vector of structural parameters that describes the model in eq. (4.23), (4.24) and (4.25) is \( \Theta = (\beta, \theta, \tau, \rho_r, \phi_y, \phi_x, \sigma_s, \sigma_d, \sigma_r)' \). Even though all the nine structural parameters can be identified, for the purposes of our analysis, we calibrate the values of \( \beta \) and \( \tau \), to increase the precision of the estimates of the other parameters.

#### 4.6.2.1 Simulation design and results

The simulated data are generated from the reduced form of the model described in (4.23) - (4.25). The calibrated values are: \( \beta = 0.99, \phi_y = 0.5, \phi_x = 1.5, \tau = 1, \rho_r = 0.9, k \) such that the average duration of price stickiness is equal to 10 months, which implies the Calvo parameter to be \( \theta = 0.9 \). The standard deviation of the three shocks, \( \sigma_s, \sigma_d \) and \( \sigma_r \) is fixed at 0.1. Moreover, \( \tau = 1 \) is consistent with the choice of a consumer’s logarithmic utility function. The sample size is equal to 300 monthly observations and 100 quarters to mimic the ensuing application to the US economy. The number of replications in the Monte Carlo experiment is 1000.

We want to compare the results obtained by estimating the model with mixed-frequency data to those obtained by the naif econometrician who simply disregards the aggregation...
issue and uses quarterly data. As a benchmark, we estimate the model also at monthly frequency, as if all the three series were available at a monthly level.

To estimate the model at monthly frequency, we follow the standard maximum-likelihood technique. We repeat the same estimation, using only quarterly data, as a naif econometrician would do. To follow the approach of Section 4.4.1, we consider a point-in-time aggregation scheme. Therefore, when aggregating from the monthly to the quarterly level, we simply skip-sample the series, keeping one observation every third available. Finally, to use mixed-frequency data we cast the model into the modified state-space form described in Section 4.5.2, and estimate it by means of the Kalman filter.

Table 4.5 reports the median value across replications of parameter estimates. In italics, we also report the 10th and 90th percentile. The results show that with mixed-frequency data we approximate very well the monthly structure of the economy. The estimates of the parameters are very similar to those obtained by estimating the benchmark model at monthly frequency. The estimation of the monthly process is of course possible only because we are using simulated data. Using quarterly variables, we notice that for some parameters we obtain quite different estimates and wider confidence intervals. Moreover, even for the parameters whose estimated value is similar, their interpretation can be quite different. In particular, an estimate of the Calvo parameter close to 0.9 implies an average price duration of 10 months with the monthly model, but of almost 10 quarters with quarterly data. To obtain the same implied average price duration at quarterly frequency, should be equal to 0.7. This evidence is also consistent with the findings of Kim (2010).

4.7 Empirical applications

In this section, we provide an empirical application of the analysis conducted in the previous sections, to analyse the importance of the temporal aggregation issues in practice, using of US data. As in Section 4.6, we first consider a SVAR model and then a small DSGE model.

4.7.1 SVAR model with US data

In this subsection we estimate a trivariate SVAR with data for the US economy, comparing the impulse response functions of a mixed-frequency model to the ones obtained with a

\footnote{Kim (2010) runs a similar experiment, with a modified version of our DSGE model, conducting the estimation in a Bayesian framework.}

\footnote{In computing the median and the percentiles, we excluded the replications for which we didn’t obtain convergence in the estimation process.}
standard quarterly SVAR.

We consider the real GDP quarterly growth rate, the quarterly inflation rate, measured as the growth rate of the consumer price index, and the Federal Fund rate. The sample covers the period 1985 - 2007\(^{19}\). In estimating the model with data at different frequency, we consider the quarter-on-quarter GDP growth, the quarter-on-quarter inflation rate computed at each month of the sample\(^{20}\), and the interest rate. Moving to the quarterly SVAR, we aggregate the monthly series at the quarterly level, simply skip-sampling the series and considering only one realization every third. In this way, the realizations in the last month of the quarter are considered representative of the quarter itself. A comment on the choice of the inflation variable is needed: considering the inflation rate computed as mentioned before is made in such a way that we can compare directly the impact of each variable on the other, even when different frequencies are considered. That is, we can compare the impulse response functions even in the size of the shocks.

We first estimate a trivariate quarterly VAR, where the variables are ordered as GDP growth, inflation and interest rate, following the same Choleski ordering seen before. We choose a lag order equal to 1, to follow the analytical results in Section 4.3. We compute the impulse response functions to trace out what happens to the system up to 8 quarters ahead.

Second, we estimate the mixed-frequency model with the state-space approach described in Section 4.5.1. Even in this case we consider a VAR of lag order 1, to be consistent with the aggregated results. In this case, we trace out the effects of the shock to the system for 24 months.

In Figure 4.1 we show the impulse response functions to the shocks obtained with the two different methods. The black line indicates the dynamics obtained with mixed-frequency data, the dashed red lines are the confidence bands of the mixed-frequency impulse responses\(^{21}\). The green dots are the values of the impulse response function estimated with quarterly data only, and the blue ones are the corresponding response standard errors, as well computed with a Monte Carlo method.

Figure 4.1 suggests that time aggregation plays a role in shaping the results, consistently with the results obtained with the Monte Carlo experiment. In the case of the GDP responses to shocks, we can see how including more information allows to reduce the uncertainty, which is reflected in tighter error bands when monthly information is included in the estimation.

---

\(^{19}\)Starting from 1985 reduces the problems with breaks and changes in regimes, which are out of the focus of our study.

\(^{20}\)As a clarification, we use the monthly series of the change of the CPI index from \(t - 3\) to \(t\), i.e. the series computed as follows: \(\pi_t = 100 \ast (\ln(CPI_t) - \ln(CPI_{t-3}))\).

\(^{21}\)The response standard errors are computed with a Monte Carlo method, with 1000 replications. The error bands represent the 5th and 95th percentile of the replications.
the case of the response functions of the monthly variables to a shock to a monthly variable, we can notice differences especially in the size of the reaction. It is interesting to see that for many of the periods considered, the two responses are not included in the confidence bands of the other approach, and sometimes even the standard errors do not intersect. In particular, the differences are particularly evident in the response of the interest rate. Using only quarterly data, we find a stronger a more persistent dynamic of the interest rate.

Summing up, these results confirm that choosing the temporal frequency matters also empirically, and using all the available information though in mixed-frequency is a simple and sensible choice.

### 4.7.2 Small-scale New Keynesian model with US data

We now conduct an empirical analysis using data for the US economy in a DSGE framework. Our goal is to compare the estimates of the structural parameters obtained with a mixed-frequency approach to those obtained from a standard quarterly model. As in the exercise with simulated data, we estimate the parameters of the economy described in eq. (4.23), (4.24) and (4.25).

We consider the real GDP quarterly growth rate, the inflation rate, measured as the growth rate of the consumer price index, and the yield on 3-month T-bill as interest rate. The sample covers the period 1985 - 2007. In estimating the model with data at different frequency, we consider the quarter-on-quarter GDP growth, the monthly inflation rate and the interest rate. Moving to the quarterly SVAR, we aggregate the monthly series at the quarterly level. More specifically, we construct the quarterly inflation rate as the sum of the three monthly inflation rates, and the quarterly interest rate as the average of the interest rates over the quarter.

We estimate the DSGE model within a maximum-likelihood framework, first at a standard quarterly frequency, and then with a mixed-frequency approach, rewriting the model as described in Section 4.5.2. As in the Monte Carlo, we set \( \tau = 1 \), which is equivalent to consider a logarithmic consumer utility function. Furthermore, we calibrate the value of the discount factor \( \beta \) at 0.99, the most common value in the literature.

Table 4.6 reports the estimates of the structural parameters obtained with the two approaches, mixed-frequency (Column 2) or only quarterly (Column 3) data. We concentrate in particular on the estimate of the Calvo parameter \( \theta \). While the average price duration implied by the parameter estimated with mixed-frequency data is equal to 6.5 months, the

---

22We considered also the Federal Fund Rates as more common measure of the interest rate, and the results do not change. For the use of the yield on 3-month T-bill, see Primiceri (2005) among others.
one implied by the quarterly model is equal to 4.14 quarters, which is equivalent to slightly longer than 12 months. The average price duration obtained from the estimation of a quarterly model is therefore twice as long as the one obtained incorporating monthly information by using the mixed-frequency approach.

If we look at the table, we notice that there are discrepancies also in the other parameters. In particular, the parameter which describes the persistence of the interest rate is also affected by the different frequency at which we estimate the process. Different values across estimations are also obtained in the case of the standard deviations of the shocks, but here the evidence is mixed.

The evidence obtained from this example confirms once again that the choice of the data frequency matters: it can influence the parameter estimates of the model and their interpretation, so much so that different conclusions can be reached. Therefore, the use of mixed frequency data for structural analysis, which is simple and such that the resulting information set is closer to the one of the agents, should become more common.

4.8 Conclusions

In the recent econometric literature, unbalanced datasets have attracted a substantial attention. Different methods have been proposed to deal with mixed-frequency data, but the focus has only been on improving the forecasts of key series such as GDP growth, which are usually available at a lower frequency only. In this paper, we shift the attention to the use of mixed-frequency data in the context of structural models.

The common approach in the literature is to estimate the deep parameters of the economy at a frequency such that data for all the variables are available, independently of the fact that the agents take decisions within a different time framework. Hence, structural models are typically estimated with quarterly data even if monthly or even higher frequency information on same variables is available.

We show that this practice can have important consequences when trying to give an economic interpretation to the estimated parameters and model dynamics. Using examples from the SVAR and New Keynesian DSGE literature, we derive the analytical mapping from a monthly specification to a quarterly specification of the models, showing that it is in general impossible to identify the parameters which describe the monthly process when using quarterly data only. Even when identification is possible, the naïf approach which overlooks aggregation issues can bring to misleading results, since a monthly model aggregated at quarterly level is clearly different from a quarterly model which replicates the structural
relations of the monthly model and just changes the agents’ time decision interval.

We also show that the identification issue arising from aggregation can be mitigated, and in some cases even solved, by the use of mixed frequency data. Using data at different frequencies allows us to exploit the information included in the intra-quarter lags of the monthly variables to identify more parameters than in the case we just use quarterly data.

We then provide a general classical estimation method to deal with mixed-frequency data in a structural context, based on a modified state-space framework combined with the use of the Kalman filter to deal with the missing observations in the low-frequency series.

Finally, our empirical examples, based on the estimation of small monetary SVAR and DSGE models using simulated and US data, confirm the practical importance of the aggregation issue, and that it can be alleviated by the use of mixed-frequency data.
4.9 Figures and tables

Figure 4.1: Impulse responses obtained with mixed-frequency and quarterly US data

Notes: The black lines represent the impulse responses obtained with mixed-frequency data. The red dotted lines are the confidence bands corresponding to the 5th and 95th percentile. The green dots are the values of the quarterly IRF, assigned at the corresponding first month of the quarter. The grey dots are the confidence bands corresponding to the 5th and 95th percentile. The data used cover the sample 1985-2007.
Table 4.1: Response of the low frequency variable $y$ to $y$

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\delta_\rho$</th>
<th>$\delta_h$</th>
<th>high median</th>
<th>mixed median</th>
<th>low median</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
<td>0.998</td>
<td>0.994</td>
<td>1.135</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.952</td>
<td>0.994</td>
<td>1.197</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.420</td>
<td>0.853</td>
<td>1.432</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.562</td>
<td>0.489</td>
<td>1.106</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.388</td>
<td>0.357</td>
<td>1.352</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.463</td>
<td>0.591</td>
<td>0.997</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.287</td>
<td>0.307</td>
<td>0.977</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.414</td>
<td>0.482</td>
<td>0.988</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.400</td>
<td>0.417</td>
<td>0.882</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.351</td>
<td>0.401</td>
<td>0.789</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.310</td>
<td>0.393</td>
<td>0.710</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.276</td>
<td>0.386</td>
<td>0.645</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.313</td>
<td>0.455</td>
<td>0.591</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.246</td>
<td>0.532</td>
<td>0.451</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.173</td>
<td>0.320</td>
<td>0.404</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.121</td>
<td>0.281</td>
<td>0.376</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.086</td>
<td>0.507</td>
<td>0.350</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>0.6</td>
<td>0.998</td>
<td>0.994</td>
<td>1.135</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.952</td>
<td>0.994</td>
<td>1.197</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.420</td>
<td>0.853</td>
<td>1.432</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.562</td>
<td>0.489</td>
<td>1.106</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.388</td>
<td>0.357</td>
<td>1.352</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.463</td>
<td>0.591</td>
<td>0.997</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.287</td>
<td>0.307</td>
<td>0.977</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.414</td>
<td>0.482</td>
<td>0.988</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.400</td>
<td>0.417</td>
<td>0.882</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.351</td>
<td>0.393</td>
<td>0.789</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.310</td>
<td>0.455</td>
<td>0.710</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.276</td>
<td>0.532</td>
<td>0.451</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.173</td>
<td>0.320</td>
<td>0.404</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.121</td>
<td>0.281</td>
<td>0.376</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.086</td>
<td>0.507</td>
<td>0.350</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>0.08</td>
<td>0.08</td>
<td>0.998</td>
<td>0.994</td>
<td>1.135</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.952</td>
<td>0.994</td>
<td>1.197</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.420</td>
<td>0.853</td>
<td>1.432</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.562</td>
<td>0.489</td>
<td>1.106</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.388</td>
<td>0.357</td>
<td>1.352</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.463</td>
<td>0.591</td>
<td>0.997</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.287</td>
<td>0.307</td>
<td>0.977</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.414</td>
<td>0.482</td>
<td>0.988</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.400</td>
<td>0.417</td>
<td>0.882</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.351</td>
<td>0.393</td>
<td>0.789</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.310</td>
<td>0.455</td>
<td>0.710</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.276</td>
<td>0.532</td>
<td>0.451</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.173</td>
<td>0.320</td>
<td>0.404</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.121</td>
<td>0.281</td>
<td>0.376</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.086</td>
<td>0.507</td>
<td>0.350</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>0.08</td>
<td>0.1</td>
<td>0.998</td>
<td>0.994</td>
<td>1.135</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.952</td>
<td>0.994</td>
<td>1.197</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.420</td>
<td>0.853</td>
<td>1.432</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.562</td>
<td>0.489</td>
<td>1.106</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.388</td>
<td>0.357</td>
<td>1.352</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.463</td>
<td>0.591</td>
<td>0.997</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.287</td>
<td>0.307</td>
<td>0.977</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.414</td>
<td>0.482</td>
<td>0.988</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.400</td>
<td>0.417</td>
<td>0.882</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.351</td>
<td>0.393</td>
<td>0.789</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.310</td>
<td>0.455</td>
<td>0.710</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.276</td>
<td>0.532</td>
<td>0.451</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.173</td>
<td>0.320</td>
<td>0.404</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.121</td>
<td>0.281</td>
<td>0.376</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.086</td>
<td>0.507</td>
<td>0.350</td>
</tr>
</tbody>
</table>

Notes: The table reports the value of the impulse responses obtained simulating data from the DGP in eq. (4.53), for different parameter specifications and for different periods (indicated in months in the header). For each parameter specification, we report the value of the impulse responses obtained using the data at the high frequency (high), the data skip-sampled at the low-frequency (low), and the data at mixed-frequency (mixed). We consider the median value, the 10th and 90th percentile across replications. The number of replication is 1000 for each parameter specification.
Table 4.2: Response of the high frequency variable x to y

<table>
<thead>
<tr>
<th>ρ</th>
<th>δο</th>
<th>δl</th>
<th>method</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>10</th>
<th>13</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
<td>high median</td>
<td>-0.001</td>
<td>0.395</td>
<td>0.387</td>
<td>0.348</td>
<td>0.309</td>
<td>0.275</td>
<td>0.244</td>
<td>0.172</td>
<td>0.122</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>-0.075</td>
<td>0.070</td>
<td>0.323</td>
<td>0.466</td>
<td>0.324</td>
<td>0.447</td>
<td>0.283</td>
<td>0.410</td>
<td>0.244</td>
<td>0.374</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed median</td>
<td>0.006</td>
<td>0.357</td>
<td>0.384</td>
<td>0.347</td>
<td>0.309</td>
<td>0.274</td>
<td>0.243</td>
<td>0.169</td>
<td>0.117</td>
<td>0.084</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>-0.112</td>
<td>0.153</td>
<td>0.287</td>
<td>0.512</td>
<td>0.292</td>
<td>0.468</td>
<td>0.260</td>
<td>0.429</td>
<td>0.224</td>
<td>0.387</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low median</td>
<td>0.542</td>
<td>NaN</td>
<td>NaN</td>
<td>0.657</td>
<td>NaN</td>
<td>NaN</td>
<td>0.446</td>
<td>NaN</td>
<td>0.233</td>
<td>0.157</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>0.384</td>
<td>0.718</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>0.484</td>
<td>0.812</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>0.6</td>
<td>high median</td>
<td>0.000</td>
<td>0.395</td>
<td>0.385</td>
<td>0.376</td>
<td>0.366</td>
<td>0.357</td>
<td>0.348</td>
<td>0.326</td>
<td>0.306</td>
<td>0.285</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>-0.076</td>
<td>0.074</td>
<td>0.329</td>
<td>0.456</td>
<td>0.330</td>
<td>0.438</td>
<td>0.319</td>
<td>0.429</td>
<td>0.306</td>
<td>0.426</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed median</td>
<td>0.017</td>
<td>0.391</td>
<td>0.378</td>
<td>0.372</td>
<td>0.364</td>
<td>0.353</td>
<td>0.344</td>
<td>0.319</td>
<td>0.299</td>
<td>0.279</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>-0.116</td>
<td>0.145</td>
<td>0.291</td>
<td>0.492</td>
<td>0.289</td>
<td>0.467</td>
<td>0.281</td>
<td>0.461</td>
<td>0.270</td>
<td>0.451</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low median</td>
<td>0.663</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>0.864</td>
<td>NaN</td>
<td>NaN</td>
<td>0.628</td>
<td>0.795</td>
<td>0.743</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>0.486</td>
<td>0.821</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>0.704</td>
<td>1.008</td>
<td>NaN</td>
<td>NaN</td>
</tr>
<tr>
<td>0.9</td>
<td>0.08</td>
<td>0.08</td>
<td>high median</td>
<td>-0.002</td>
<td>0.081</td>
<td>0.140</td>
<td>0.188</td>
<td>0.224</td>
<td>0.250</td>
<td>0.268</td>
<td>0.291</td>
<td>0.288</td>
<td>0.276</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>-0.074</td>
<td>0.078</td>
<td>0.001</td>
<td>0.157</td>
<td>0.052</td>
<td>0.230</td>
<td>0.087</td>
<td>0.286</td>
<td>0.114</td>
<td>0.334</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed median</td>
<td>-0.003</td>
<td>0.082</td>
<td>0.143</td>
<td>0.187</td>
<td>0.216</td>
<td>0.241</td>
<td>0.255</td>
<td>0.276</td>
<td>0.276</td>
<td>0.262</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>-0.134</td>
<td>0.165</td>
<td>-0.035</td>
<td>0.224</td>
<td>0.029</td>
<td>0.276</td>
<td>0.070</td>
<td>0.319</td>
<td>0.097</td>
<td>0.357</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low median</td>
<td>0.230</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>0.464</td>
<td>NaN</td>
<td>NaN</td>
<td>0.542</td>
<td>0.559</td>
<td>0.537</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>0.032</td>
<td>0.446</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>0.249</td>
<td>0.691</td>
<td>NaN</td>
<td>NaN</td>
</tr>
<tr>
<td>0.9</td>
<td>0.08</td>
<td>0.1</td>
<td>high median</td>
<td>-0.002</td>
<td>0.073</td>
<td>0.199</td>
<td>0.139</td>
<td>0.139</td>
<td>0.235</td>
<td>0.220</td>
<td>0.246</td>
<td>0.264</td>
<td>0.292</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>-0.075</td>
<td>0.073</td>
<td>0.003</td>
<td>0.149</td>
<td>0.057</td>
<td>0.219</td>
<td>0.095</td>
<td>0.274</td>
<td>0.120</td>
<td>0.315</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed median</td>
<td>-0.001</td>
<td>0.080</td>
<td>0.140</td>
<td>0.189</td>
<td>0.220</td>
<td>0.245</td>
<td>0.262</td>
<td>0.286</td>
<td>0.286</td>
<td>0.276</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>-0.137</td>
<td>0.129</td>
<td>-0.041</td>
<td>0.190</td>
<td>0.024</td>
<td>0.249</td>
<td>0.064</td>
<td>0.296</td>
<td>0.097</td>
<td>0.336</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low median</td>
<td>0.261</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>0.491</td>
<td>NaN</td>
<td>NaN</td>
<td>0.572</td>
<td>0.572</td>
<td>0.572</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>0.056</td>
<td>0.461</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>0.285</td>
<td>0.683</td>
<td>NaN</td>
<td>NaN</td>
</tr>
</tbody>
</table>

Notes: See notes at Table 4.1.
Table 4.3: Response of the low frequency variable y to x

<table>
<thead>
<tr>
<th>ρ</th>
<th>δh</th>
<th>δl</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>10</th>
<th>13</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
<td>high</td>
<td>0.00</td>
<td>0.397</td>
<td>0.388</td>
<td>0.350</td>
<td>0.311</td>
<td>0.277</td>
<td>0.246</td>
<td>0.174</td>
<td>0.122</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>median</td>
<td>0.00</td>
<td>0.333</td>
<td>0.458</td>
<td>0.333</td>
<td>0.445</td>
<td>0.291</td>
<td>0.408</td>
<td>0.250</td>
<td>0.372</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>0.00</td>
<td>0.286</td>
<td>0.516</td>
<td>0.299</td>
<td>0.465</td>
<td>0.264</td>
<td>0.423</td>
<td>0.229</td>
<td>0.383</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low</td>
<td>0.00</td>
<td>NaN</td>
<td>NaN</td>
<td>0.421</td>
<td>NaN</td>
<td>NaN</td>
<td>0.284</td>
<td>0.203</td>
<td>0.142</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed</td>
<td>0.00</td>
<td>0.394</td>
<td>0.381</td>
<td>0.343</td>
<td>0.306</td>
<td>0.271</td>
<td>0.241</td>
<td>0.170</td>
<td>0.119</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>0.6</td>
<td>high</td>
<td>0.00</td>
<td>0.565</td>
<td>0.578</td>
<td>0.539</td>
<td>0.588</td>
<td>0.570</td>
<td>0.556</td>
<td>0.543</td>
<td>0.509</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>median</td>
<td>0.00</td>
<td>0.528</td>
<td>0.661</td>
<td>0.521</td>
<td>0.640</td>
<td>0.503</td>
<td>0.631</td>
<td>0.462</td>
<td>0.612</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>0.00</td>
<td>0.490</td>
<td>0.744</td>
<td>0.488</td>
<td>0.691</td>
<td>0.482</td>
<td>0.621</td>
<td>0.462</td>
<td>0.612</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low</td>
<td>0.00</td>
<td>NaN</td>
<td>NaN</td>
<td>0.572</td>
<td>NaN</td>
<td>NaN</td>
<td>0.576</td>
<td>0.567</td>
<td>0.526</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed</td>
<td>0.00</td>
<td>0.404</td>
<td>0.748</td>
<td>0.485</td>
<td>0.691</td>
<td>0.482</td>
<td>0.621</td>
<td>0.462</td>
<td>0.612</td>
</tr>
<tr>
<td>0.5</td>
<td>0.08</td>
<td>0.08</td>
<td>high</td>
<td>0.00</td>
<td>0.209</td>
<td>0.538</td>
<td>0.140</td>
<td>0.456</td>
<td>0.284</td>
<td>0.397</td>
<td>0.388</td>
<td>0.391</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>median</td>
<td>0.00</td>
<td>0.099</td>
<td>0.139</td>
<td>0.085</td>
<td>0.218</td>
<td>0.243</td>
<td>0.260</td>
<td>0.284</td>
<td>0.285</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>0.00</td>
<td>0.045</td>
<td>0.117</td>
<td>0.079</td>
<td>0.206</td>
<td>0.107</td>
<td>0.269</td>
<td>0.128</td>
<td>0.316</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low</td>
<td>0.00</td>
<td>NaN</td>
<td>NaN</td>
<td>0.479</td>
<td>NaN</td>
<td>NaN</td>
<td>0.423</td>
<td>0.785</td>
<td>0.384</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed</td>
<td>0.00</td>
<td>0.089</td>
<td>0.157</td>
<td>0.204</td>
<td>0.242</td>
<td>0.268</td>
<td>0.286</td>
<td>0.305</td>
<td>0.300</td>
</tr>
<tr>
<td>0.5</td>
<td>0.08</td>
<td>0.8</td>
<td>high</td>
<td>0.00</td>
<td>0.234</td>
<td>0.277</td>
<td>0.329</td>
<td>0.333</td>
<td>0.366</td>
<td>0.369</td>
<td>0.357</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>median</td>
<td>0.00</td>
<td>0.048</td>
<td>0.162</td>
<td>0.086</td>
<td>0.269</td>
<td>0.114</td>
<td>0.335</td>
<td>0.135</td>
<td>0.370</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>0.00</td>
<td>0.045</td>
<td>0.117</td>
<td>0.079</td>
<td>0.206</td>
<td>0.107</td>
<td>0.269</td>
<td>0.128</td>
<td>0.316</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low</td>
<td>0.00</td>
<td>0.089</td>
<td>0.140</td>
<td>0.206</td>
<td>0.242</td>
<td>0.268</td>
<td>0.286</td>
<td>0.305</td>
<td>0.300</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed</td>
<td>0.00</td>
<td>0.048</td>
<td>0.162</td>
<td>0.086</td>
<td>0.269</td>
<td>0.114</td>
<td>0.335</td>
<td>0.135</td>
<td>0.370</td>
</tr>
<tr>
<td>0.9</td>
<td>0.08</td>
<td>0.8</td>
<td>high</td>
<td>0.00</td>
<td>0.191</td>
<td>0.252</td>
<td>0.140</td>
<td>0.285</td>
<td>0.234</td>
<td>0.309</td>
<td>0.333</td>
<td>0.366</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>median</td>
<td>0.00</td>
<td>0.048</td>
<td>0.162</td>
<td>0.086</td>
<td>0.269</td>
<td>0.114</td>
<td>0.335</td>
<td>0.135</td>
<td>0.370</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc)</td>
<td>0.00</td>
<td>0.045</td>
<td>0.117</td>
<td>0.079</td>
<td>0.206</td>
<td>0.107</td>
<td>0.269</td>
<td>0.128</td>
<td>0.316</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low</td>
<td>0.00</td>
<td>0.089</td>
<td>0.140</td>
<td>0.206</td>
<td>0.242</td>
<td>0.268</td>
<td>0.286</td>
<td>0.305</td>
<td>0.300</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed</td>
<td>0.00</td>
<td>0.048</td>
<td>0.162</td>
<td>0.086</td>
<td>0.269</td>
<td>0.114</td>
<td>0.335</td>
<td>0.135</td>
<td>0.370</td>
</tr>
</tbody>
</table>

Notes: See notes at Table 4.1.
<table>
<thead>
<tr>
<th>ρ</th>
<th>δν</th>
<th>δl</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>10</th>
<th>13</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4</td>
<td>0.4</td>
<td>high median</td>
<td>0.999</td>
<td>0.489</td>
<td>0.397</td>
<td>0.348</td>
<td>0.309</td>
<td>0.275</td>
<td>0.245</td>
<td>0.173</td>
<td>0.122</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc t)</td>
<td>0.945</td>
<td>1.053</td>
<td>0.427</td>
<td>0.554</td>
<td>0.346</td>
<td>0.454</td>
<td>0.294</td>
<td>0.407</td>
<td>0.251</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed median</td>
<td>0.991</td>
<td>0.487</td>
<td>0.396</td>
<td>0.346</td>
<td>0.304</td>
<td>0.270</td>
<td>0.240</td>
<td>0.169</td>
<td>0.120</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low median</td>
<td>1.185</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>0.292</td>
<td>0.199</td>
<td>0.142</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4</td>
<td>0.6</td>
<td>high median</td>
<td>0.996</td>
<td>0.483</td>
<td>0.473</td>
<td>0.460</td>
<td>0.449</td>
<td>0.438</td>
<td>0.428</td>
<td>0.400</td>
<td>0.375</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc t)</td>
<td>0.943</td>
<td>1.046</td>
<td>0.422</td>
<td>0.554</td>
<td>0.424</td>
<td>0.524</td>
<td>0.404</td>
<td>0.514</td>
<td>0.389</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed median</td>
<td>0.996</td>
<td>0.052</td>
<td>0.490</td>
<td>0.474</td>
<td>0.464</td>
<td>0.453</td>
<td>0.443</td>
<td>0.415</td>
<td>0.387</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low median</td>
<td>1.155</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>0.502</td>
<td>0.459</td>
<td>0.429</td>
</tr>
<tr>
<td>0.9</td>
<td>0.08</td>
<td>0.08</td>
<td>high median</td>
<td>1.046</td>
<td>1.267</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>0.357</td>
<td>0.698</td>
<td>0.395</td>
<td>0.648</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc t)</td>
<td>0.943</td>
<td>1.051</td>
<td>0.817</td>
<td>0.940</td>
<td>0.701</td>
<td>0.858</td>
<td>0.607</td>
<td>0.790</td>
<td>0.532</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed median</td>
<td>0.984</td>
<td>0.871</td>
<td>0.780</td>
<td>0.704</td>
<td>0.640</td>
<td>0.587</td>
<td>0.541</td>
<td>0.441</td>
<td>0.374</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low median</td>
<td>1.546</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>0.832</td>
<td>0.675</td>
<td>0.567</td>
</tr>
<tr>
<td>0.9</td>
<td>0.08</td>
<td>0.1</td>
<td>high median</td>
<td>1.402</td>
<td>1.693</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>1.093</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(10th - 90th prc t)</td>
<td>0.946</td>
<td>1.049</td>
<td>0.819</td>
<td>0.941</td>
<td>0.707</td>
<td>0.861</td>
<td>0.616</td>
<td>0.798</td>
<td>0.543</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>mixed median</td>
<td>0.987</td>
<td>0.874</td>
<td>0.782</td>
<td>0.707</td>
<td>0.646</td>
<td>0.596</td>
<td>0.545</td>
<td>0.446</td>
<td>0.374</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low median</td>
<td>1.549</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>1.092</td>
<td>NaN</td>
<td>NaN</td>
</tr>
</tbody>
</table>

Table 4.4: Response of the high frequency variable x to x

Notes: See notes at Table 4.1.
Table 4.5: Estimates of structural DSGE parameters with monthly, mixed-frequency and quarterly simulated data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DGP</th>
<th>Monthly estimates</th>
<th>Quarterly estimates</th>
<th>Mixed frequency estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>theta</td>
<td>0.9</td>
<td>0.899</td>
<td>0.893</td>
<td>0.898</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.876 0.926</td>
<td>0.857 0.935</td>
<td>0.876 0.925</td>
</tr>
<tr>
<td>phi_y</td>
<td>0.5</td>
<td>0.584</td>
<td>0.375</td>
<td>0.633</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.170 1.589</td>
<td>0.061 2.348</td>
<td>0.121 1.744</td>
</tr>
<tr>
<td>phi_pi</td>
<td>1.5</td>
<td>1.404</td>
<td>1.403</td>
<td>1.492</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.856 2.593</td>
<td>0.511 2.784</td>
<td>0.764 3.308</td>
</tr>
<tr>
<td>rho_r</td>
<td>0.9</td>
<td>0.911</td>
<td>0.887</td>
<td>0.917</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.839 0.956</td>
<td>0.803 0.967</td>
<td>0.818 0.960</td>
</tr>
<tr>
<td>sig_s</td>
<td>0.1</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.095 0.105</td>
<td>0.091 0.110</td>
<td>0.095 0.105</td>
</tr>
<tr>
<td>sig_d</td>
<td>0.1</td>
<td>0.100</td>
<td>0.099</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.094 0.105</td>
<td>0.089 0.108</td>
<td>0.090 0.108</td>
</tr>
<tr>
<td>sig_r</td>
<td>0.1</td>
<td>0.100</td>
<td>0.141</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.092 0.108</td>
<td>0.124 0.162</td>
<td>0.090 0.109</td>
</tr>
</tbody>
</table>

**Notes:** The estimates are obtained for a sample of 300 monthly observations or, equivalently 100 quarterly observations. The DGP is represented by the reduced form of the model in eq. (4.23) - (4.25). Column 2 reports the true parameters, from which we generated the data. Column 3 reports median, the 10th and 90th percentile across replications of the parameters estimated with monthly data, Column 4 with quarterly data and Column 5 with mixed-frequency data. The number of replications is fixed at 1000.

* A Calvo parameter $\theta$ equal to 0.9 at monthly frequency implies an average price duration of 10 months. To obtain the same implied average price duration at quarterly frequency $\theta$ should be equal to 0.7.
Table 4.6: Estimates of structural DSGE parameters with mixed-frequency and quarterly US data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>mixed-frequency</th>
<th>quarterly</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.8457</td>
<td>0.7584</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.2635</td>
<td>0.2530</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>1.5149</td>
<td>1.5230</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.4636</td>
<td>0.3983</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>0.2371</td>
<td>0.3840</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>0.4784</td>
<td>0.4537</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.5001</td>
<td>0.5000</td>
</tr>
</tbody>
</table>

Notes: The estimates are obtained with US data on the sample 1985-2007. Column 2 reports the parameters estimated with mixed-frequency data and Column 3 with quarterly data.
BIBLIOGRAPHY


4.10 Appendix:

4.10.1 Quarterly SVAR: identification issues

In Section 4.3.1 we aggregate the monthly SVAR at quarterly level and obtain:

\[ y_t = A^3 y_{t-1} + \xi_t, \quad (4.55) \]

with \[
\xi_t \sim (0, \Omega), \quad \Omega = BB' + ABB'A' + A^2BB'A^2. \quad (4.56)
\]

The matrix \( A^3 \) has the following form:

\[
A^3 = \begin{bmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} \\
\alpha_{21} & \alpha_{22} & \alpha_{23} \\
\alpha_{31} & \alpha_{32} & \alpha_{33}
\end{bmatrix}, \quad (4.57)
\]

\[
\alpha_{11} = a_{11}^3 + (2a_{12}a_{21} + 2a_{13}a_{31})a_{11} + a_{12}(a_{21}a_{22} + a_{23}a_{31}) + a_{13}(a_{21}a_{32} + a_{31}a_{33}) \\
\alpha_{12} = a_{12}(a_{22} + a_{21}a_{21} + a_{23}a_{32}) + a_{11}(a_{11}a_{12} + a_{12}a_{22} + a_{13}a_{32}) + a_{13}(a_{12}a_{31} + a_{22}a_{32} + a_{32}a_{33}) \\
\alpha_{13} = a_{13}(a_{13}a_{13} + a_{23}a_{32}) + a_{11}(a_{11}a_{13} + a_{12}a_{23} + a_{13}a_{33}) + a_{12}(a_{13}a_{21} + a_{22}a_{23} + a_{23}a_{33}) \\
\alpha_{21} = a_{21}(a_{11}^2 + a_{12}a_{21} + a_{13}a_{31}) + a_{22}(a_{11}a_{21} + a_{22}a_{22} + a_{23}a_{31}) + a_{23}(a_{11}a_{31} + a_{21}a_{32} + a_{31}a_{33}) \\
\alpha_{22} = a_{22}^3 + a_{11}a_{12}a_{21} + 2a_{12}a_{21}a_{22} + a_{12}a_{22}a_{31} + a_{13}a_{21}a_{32} + 2a_{22}a_{23}a_{32} + a_{23}a_{32}a_{33} \\
\alpha_{23} = a_{23}(a_{23}^2 + a_{13}a_{31} + a_{23}a_{32}) + a_{21}(a_{11}a_{13} + a_{12}a_{23} + a_{13}a_{33}) + a_{22}(a_{13}a_{21} + a_{22}a_{23} + a_{23}a_{33}) \\
\alpha_{31} = a_{31}(a_{11}^2 + a_{12}a_{21} + a_{13}a_{31}) + a_{32}(a_{11}a_{21} + a_{22}a_{22} + a_{23}a_{31}) + a_{33}(a_{11}a_{31} + a_{21}a_{32} + a_{31}a_{33}) \\
\alpha_{32} = a_{32}(a_{13}^2 + a_{12}a_{22} + a_{13}a_{32}) + a_{31}(a_{11}a_{12} + a_{12}a_{22} + a_{13}a_{32}) + a_{33}(a_{12}a_{31} + a_{22}a_{32} + a_{32}a_{33}) \\
\alpha_{33} = a_{33}^3 + a_{11}a_{13}a_{31} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} + 2a_{13}a_{31}a_{33} + a_{22}a_{23}a_{32} + 2a_{23}a_{32}a_{33}.
\]

When we work with quarterly data, we obtain the estimate of a \( 3 \times 3 \) matrix \( C = A^3 \) in eq. (4.55). In order to recover the parameters which drive the monthly process we should solve the following system for the parameters \( a_{11}, a_{12}, \ldots, a_{33} \):

\[
\begin{align*}
&c_{11} = \alpha_{11}(a_{11}, a_{12}, \ldots, a_{33}) \\
&c_{12} = \alpha_{12}(a_{11}, a_{12}, \ldots, a_{33}) \\
&\vdots \\
&c_{33} = \alpha_{33}(a_{11}, a_{12}, \ldots, a_{33})
\end{align*}
\quad , (4.58)
\]

which is a highly non linear system. This system not always admits a unique solution, and many times has to be solved numerically.
In other words, solving the system (4.58), we want to find the matrix $A$, such that $A^3 = C$, and therefore we want to find the cube roots of a matrix. We can show in the easiest case possible that this is not necessarily unique. Let us consider for simplicity $C$ as a $2 \times 2$ identity matrix\(^{23}\). It is obvious to see that $A = I_2$ is a cube root of $C$. However, it is possible to check that any matrix $A = \begin{pmatrix} d & 1 \\ 1 & d \end{pmatrix}$, $d, f \in \mathbb{R}$, $f \neq 0$, \((4.59)\) is also a cube root. This is therefore a very easy example which shows that from $C$ we cannot always uniquely identify $A$.

Moreover, since the error term is defined as 
\[
\xi_t \sim (0, \Omega), \quad \Omega = BB' + ABB'A' + A^2BB'A^2.
\] \((4.60)\) the difficulties in identifying $A$ translate into the problem of identifying $B$.

### 4.10.2 SVAR with mixed frequency data: identification issues

Let us consider the monthly process for GDP:
\[
y_t^* = a_{11}y_{t-1}^* + a_{12}\pi_{t-1} + a_{13}r_{t-1} + b_{11}\varepsilon_{yt},
\] \((4.61)\)

and rewrite it as:
\[
(1 - a_{11}L) y_t^* = a_{12}\pi_{t-1} + a_{13}r_{t-1} + \varepsilon_{yt}.
\] \((4.62)\)

To aggregate the process at a quarterly frequency we need to pre-multiply both sides of eq. (4.62) by $b(L) = (1 + a_{11}L + a_{11}^2L^2)$ to obtain
\[
y_t^* = a_{11}^3y_{t-3}^* + a_{12}^2\pi_{t-2} + a_{12}a_{11}^2\pi_{t-3} + a_{13}a_{11}^2r_{t-3} + a_{13}^2a_{11}r_{t-3} + b_{11}\varepsilon_{yt} + a_{11}b_{11}\varepsilon_{yt-1} + a_{11}^2b_{11}\varepsilon_{yt-2}.
\] \((4.63)\)

Now, let us move to the monthly process for inflation:
\[
\pi_t = a_{21}y_{t-1}^* + a_{22}\pi_{t-1} + a_{23}r_{t-1} + b_{21}\varepsilon_{yt} + b_{22}\varepsilon_{\pi t}.
\] \((4.64)\)

Since we do not observe $y_{t-1}^*$, we substitute it with its dynamics in eq. (4.61):
\[
\pi_t = a_{21} \left( a_{11}y_{t-2}^* + a_{12}\pi_{t-2} + a_{13}r_{t-2} + b_{11}\varepsilon_{yt-1} \right) + a_{22}\pi_{t-1} + a_{23}r_{t-1} + b_{21}\varepsilon_{yt} + b_{22}\varepsilon_{\pi t},
\] \((4.65)\)

\(^{23}\)The dimension of the matrix is set at $n = 2$ for simplicity, but it can be extended to any other $n$.\n
and then again, we substitute \( y_{t-2}^* \):

\[
\pi_t = a_{21} \left( a_{11} \left( a_{11} y_{t-3}^* + a_{12} r_{t-3} + a_{13} r_{t-3} + b_{11} \varepsilon_{yt-2} \right) + \right) + a_{22} \pi_{t-1} + a_{23} r_{t-1} + b_{21} \varepsilon_{yt} + b_{22} \varepsilon_{xt},
\]

(4.66)

obtaining

\[
\pi_t = a_{21} a_{11} y_{t-3}^* + a_{22} \pi_{t-1} + a_{21} a_{12} \pi_{t-2} + a_{21} a_{11} a_{12} \pi_{t-3} + a_{23} r_{t-1} + a_{21} a_{13} r_{t-2} + a_{21} a_{11} a_{13} r_{t-3} + b_{22} \varepsilon_{xt} + b_{21} \varepsilon_{yt} + a_{21} b_{11} \varepsilon_{yt-1} + a_{21} a_{11} b_{11} \varepsilon_{yt-2}.
\]

(4.67)

which depends only on observable values of \( y_t^* \).

The same can be done for \( r_t \), obtaining:

\[
r_t = a_{31} a_{11} y_{t-3}^* + a_{32} \pi_{t-1} + a_{31} a_{12} \pi_{t-2} + a_{31} a_{11} a_{12} \pi_{t-3} + a_{33} r_{t-1} + a_{31} a_{13} r_{t-2} + a_{31} a_{11} a_{13} r_{t-3} + b_{33} \varepsilon_{rt} + b_{32} \varepsilon_{xt} + b_{31} \varepsilon_{yt} + a_{31} b_{11} \varepsilon_{yt-1} + a_{31} a_{11} b_{11} \varepsilon_{yt-2}.
\]

(4.68)

From eq. (4.63) the parameters \( a_{11}, a_{12}, a_{13} \) can be identified. From eq. (4.67) we recover the parameters \( a_{21}, a_{22}, a_{23} \) and from eq. (4.68) \( a_{31}, a_{32}, a_{33} \).

From equations (4.63), (4.67) and (4.68) we obtain three series of residuals: \( \xi_{yt} = b_{11} \varepsilon_{yt} + a_{11} b_{11} \varepsilon_{yt-1} + a_{11}^2 b_{11} \varepsilon_{yt-2} \), \( \xi_{rt} = b_{22} \varepsilon_{xt} + b_{21} \varepsilon_{yt} + a_{11} b_{11} \varepsilon_{yt-1} + a_{21} a_{11} b_{11} \varepsilon_{yt-2} \) and \( \xi_{rt} = b_{33} \varepsilon_{rt} + b_{32} \varepsilon_{xt} + b_{31} \varepsilon_{yt} + a_{31} b_{11} \varepsilon_{yt-1} + a_{31} a_{11} b_{11} \varepsilon_{yt-2} \) and with covariance matrix

\[
\Sigma_\xi = \begin{bmatrix}
s_1^2 & s_{21} & s_{31} \\
s_{21} & s_2^2 & s_{32} \\
s_{31} & s_{32} & s_3^2
\end{bmatrix},
\]

(4.69)

where the elements are defined as:

\[
s_1^2 = \text{var} \left( b_{11} \varepsilon_{yt} + a_{11} b_{11} \varepsilon_{yt-1} + a_{11}^2 b_{11} \varepsilon_{yt-2} \right)
\]

(4.70)

\[
s_{21} = \text{cov} \left( b_{22} \varepsilon_{xt} + b_{21} \varepsilon_{yt} + a_{11} b_{11} \varepsilon_{yt-1} + a_{21} a_{11} b_{11} \varepsilon_{yt-2},
\begin{array}{l}
b_{11} \varepsilon_{yt} + a_{11} b_{11} \varepsilon_{yt-1} + a_{11}^2 b_{11} \varepsilon_{yt-2}
\end{array} \right)
\]

\[
s_2^2 = \text{var} \left( b_{22} \varepsilon_{xt} + b_{21} \varepsilon_{yt} + a_{11} b_{11} \varepsilon_{yt-1} + a_{21} a_{11} b_{11} \varepsilon_{yt-2} \right)
\]

\[
s_{31} = \text{cov} \left( b_{33} \varepsilon_{rt} + b_{32} \varepsilon_{xt} + b_{31} \varepsilon_{yt} + a_{31} b_{11} \varepsilon_{yt-1} + a_{31} a_{11} b_{11} \varepsilon_{yt-2},
\begin{array}{l}
b_{33} \varepsilon_{rt} + b_{32} \varepsilon_{xt} + b_{31} \varepsilon_{yt} + a_{31} b_{11} \varepsilon_{yt-1} + a_{31} a_{11} b_{11} \varepsilon_{yt-2}
\end{array} \right)
\]

\[
s_{32} = \text{cov} \left( b_{33} \varepsilon_{rt} + b_{32} \varepsilon_{xt} + b_{31} \varepsilon_{yt} + a_{31} b_{11} \varepsilon_{yt-1} + a_{31} a_{11} b_{11} \varepsilon_{yt-2},
\begin{array}{l}
b_{33} \varepsilon_{rt} + b_{32} \varepsilon_{xt} + b_{31} \varepsilon_{yt} + a_{31} b_{11} \varepsilon_{yt-1} + a_{31} a_{11} b_{11} \varepsilon_{yt-2}
\end{array} \right)
\]

\[
s_3^2 = \text{var} \left( b_{33} \varepsilon_{rt} + b_{32} \varepsilon_{xt} + b_{31} \varepsilon_{yt} + a_{31} b_{11} \varepsilon_{yt-1} + a_{31} a_{11} b_{11} \varepsilon_{yt-2} \right)
\]
Solving this system, we are able to recover the parameters $b_{21}, b_{31}, b_{32}, \sigma^2_y, \sigma^2_z$ and $\sigma^2_r$.

### 4.10.3 High-frequency dependent variables

Let us go back to the example in Section 4.3.2, and consider the dynamics of $\pi_t$ and $r_t$, which are observed monthly, but depend on $y^*_t$ which is observed only quarterly.

For $t = 3, 6, 9, \ldots$ we estimate the eq. (4.20) and (4.22), but in practice we want to estimate them also for $t = 1, 4, 7, \ldots$ and $t = 2, 5, 8, \ldots$, in which the dynamic relation is different.

For $t = 2, 5, 8, \ldots$ the equations which represent the dynamic of the processes are:

\[
\begin{align*}
\pi_t &= a_{21} a_{11} y^*_{t-2} + a_{22} \pi_{t-1} + a_{21} a_{12} \pi_{t-2} + a_{23} r_{t-1} + a_{21} a_{13} r_{t-2} + b_{22} \varepsilon_{\pi t} + b_{21} \varepsilon_{yt} + a_{21} b_{11} \varepsilon_{yt-1}, \\
r_t &= a_{31} a_{11} y^*_{t-2} + a_{32} \pi_{t-1} + a_{31} a_{12} \pi_{t-2} + a_{33} r_{t-1} + a_{31} a_{13} r_{t-2} + b_{33} \varepsilon_{rt} + b_{32} \varepsilon_{\pi t} + b_{31} \varepsilon_{yt} + a_{31} b_{11} \varepsilon_{yt-1},
\end{align*}
\]

while for $t = 1, 4, 7, \ldots$

\[
\begin{align*}
\pi_t &= a_{21} y^*_{t-1} + a_{22} \pi_{t-1} + a_{23} r_{t-1} + b_{21} \varepsilon_{yt} + \varepsilon_{\pi t}, \\
r_t &= a_{31} y^*_{t-1} + a_{32} \pi_{t-1} + a_{33} r_{t-1} + b_{31} \varepsilon_{yt} + b_{32} \varepsilon_{\pi t} + b_{33} \varepsilon_{rt}.
\end{align*}
\]

Hence, when estimating a model with the dependent variable at high frequency, we need to allow for parameter time variation.

We could also write eq. (4.20), (4.22), (4.71), (4.72), (4.73) and (4.74) in a more compact way, with the use of two dummy variables, $D2$ and $D3$, taking the value one in, respectively, each second and third month of a quarter:

\[
\begin{align*}
\pi_t &= \alpha_1 (1 - D2 - D3) y^*_{t-1} + \alpha_2 D2 y^*_{t-2} + \alpha_3 D3 y^*_{t-3} + \beta_{11} \pi_{t-1} + \beta_{12} D2 \pi_{t-1} + \beta_{13} D3 \pi_{t-1} + \beta_{22} D2 \pi_{t-2} + \beta_{23} D3 \pi_{t-2} + \beta_{33} D3 \pi_{t-3} + \\
&\quad + \gamma_{11} r_{t-1} + \gamma_{12} D2 r_{t-1} + \gamma_{13} D3 r_{t-1} + \gamma_{22} D2 r_{t-2} + \gamma_{23} D3 r_{t-2} + \gamma_{33} D3 r_{t-3} + v_t, \\
\pi_t &= \delta_1 (1 - D2 - D3) y^*_{t-1} + \delta_2 D2 y^*_{t-2} + \delta_3 D3 y^*_{t-3} + \theta_{11} \pi_{t-1} + \theta_{12} D2 \pi_{t-1} + \theta_{13} D3 \pi_{t-1} + \theta_{22} D2 \pi_{t-2} + \theta_{23} D3 \pi_{t-2} + \theta_{33} D3 \pi_{t-3} + \\
&\quad + \phi_{11} r_{t-1} + \phi_{12} D2 r_{t-1} + \phi_{13} D3 r_{t-1} + \phi_{22} D2 r_{t-2} + \phi_{23} D3 r_{t-2} + \phi_{33} D3 r_{t-3} + \eta_t.
\end{align*}
\]

In general, let us consider the following model in which $N$ high frequency variables, $x_t$, depend on values of a low a frequency variable, $y_t$:

\[
C(L)x_t = d(L)y_t + e_{xt}.
\]
The aggregation scheme is again characterized by the operator $\omega(L)$, introduced in Section 4.2. Let us introduce an $N \times N$ matrix in the lag operator, $\gamma(L)$, such that the product $g(L) = \gamma(L)d(L)$ only contains powers of $L^k = Z$, so that $g(L) = g(L^k) = g(Z)$.

Then, multiplying both sides of (4.77) by $\gamma(L)$ and $\omega(L)$, we get:

$$\gamma(L)C(L)\omega(L)x_t = \gamma(L)d(L)\omega(L)y_t + \gamma(L)\omega(L)e_{xt}, \quad (4.78)$$

$$t = k, 2k, 3k, ...$$

or

$$\tilde{C}(L)x_t = g(L^k)y_t + \xi_t, \quad (4.79)$$

$$t = k, 2k, 3k, ...$$

However, in practice, we want to estimate the model in eq. (4.79) in each high frequency period, i.e. not for $t = k, 2k, 3k, ...$ but for $t = k - i, 2k - i, 3k - i, ..., i = 1, ..., k$. Since for each $i$ the matrix $\gamma(L)$ in (4.78) is different, the dynamic relationship of $x_t$, and in general the coefficients to be estimated, change accordingly. Hence, when estimating a reverse U-MIDAS model in HF we need to allow for parameter time variation, and generalize equation (4.79) as follows:

$$\tilde{C}_i(L)x_t = g_i(L^i)y_t + \xi_{it}, \quad (4.80)$$

$$t = k - i, 2k - i, 3k - i, ..., Tk - i$$

$$i = 1, ..., k$$

For each value of $i$, (4.80) can be estimated by OLS or NLS.

4.10.4 A basic New Keynesian model: mapping and identification issues

Our basic New Keynesian model is described by the following three equations:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y^*_t + \varepsilon_{st}, \quad (4.81)$$

$$y^*_t = E_t y^*_{t+1} - \tau (R_t - E_t \pi_{t+1}) + \varepsilon_{at}, \quad (4.82)$$

$$R_t = \rho_r R_{t-1} + (1 - \rho_r) (\phi_{\pi} \pi_t + \phi_y y^*_t) + \varepsilon_{rt}. \quad (4.83)$$

Let us rewrite the model in a matrix form:

$$B_0 X^*_t = C X^*_{t-1} + D E_t X^*_t + \varepsilon_t, \quad (4.84)$$
where $X^*_t = \begin{bmatrix} \pi_t & y^*_t & R_t \end{bmatrix}'$ and $\epsilon_t = \begin{bmatrix} \epsilon_{st} & \epsilon_{dt} & \epsilon_{rt} \end{bmatrix}'$, with $\epsilon_t \sim N(0, I_3)$.

The matrices $B_0, C, D$ have the following form:

$$B_0 = \begin{bmatrix} \frac{1}{\sigma_s} & -\frac{k}{\sigma_s} & 0 \\ 0 & \frac{1}{\sigma_d} & \frac{\tau}{\sigma_d} \\ \frac{\phi_s}{\sigma_r} (\rho_r - 1) & \frac{1}{\sigma_r} \phi_d (\rho_r - 1) & \frac{1}{\sigma_r} \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sigma_r} \rho_r \end{bmatrix},$$

$$D = \begin{bmatrix} \frac{\beta}{\sigma_s} & 0 & 0 \\ \frac{\tau}{\sigma_d} & \frac{1}{\sigma_d} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The unique stable solution for this model is given by

$$A_0 X^*_t = A_1 X^*_{t-1} + \epsilon_t,$$  \hfill (4.85)

with $A_0$ and $A_1$ defined as follows:

$$A_0 = \begin{bmatrix} \frac{1}{\sigma_s} & -\frac{k}{\sigma_s} & G \\ 0 & \frac{1}{\sigma_d} & F \\ \frac{\phi_s}{\sigma_r} (\rho_r - 1) & \frac{1}{\sigma_r} \phi_d (\rho_r - 1) & \frac{1}{\sigma_r} \end{bmatrix},$$

$$A_1 = C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sigma_r} \rho_r \end{bmatrix}.$$

where

$$G = \frac{\beta}{\sigma_r \sigma_s} \rho_r G \phi_d \sigma_s + F \sigma_d \phi_y + F k \phi_d \sigma_d - G \phi_d \sigma_s \rho_r - F \sigma_d \phi_y - F k \phi_d \sigma_d \rho_r + 1$$

and

$$F = \frac{\tau}{\sigma_d} + \frac{1}{\sigma_r} \rho_r \left( \frac{F G \phi_d \sigma_s + F \sigma_d \phi_y + F k \phi_d \sigma_d - G \phi_d \sigma_s \rho_r - F \sigma_d \phi_y - F k \phi_d \sigma_d \rho_r + 1}{G \sigma_d \phi_d + F k \phi_d \sigma_r} \right).$$

It can be checked that all the parameters of this model can be easily identified.

We now provide the details for the second step of our analysis, i.e. how the monthly process can be aggregated to a quarterly level.
The process in (4.85) can be rewritten as

\[ A_0 X_t^* = A_1 A_0^{-1} A_0 X_{t-1}^* + \epsilon_t, \]  

(4.86)

and then by recursively substituting \( A_0 X_{t-i}^* \) with its equivalent \( A_1 A_0^{-1} A_0 X_{t-i}^* + \epsilon_{t-i} \), we obtain:

\[ A_0 X_t^* = A_1 A_0^{-1} A_1 A_0^{-1} A_1 X_{t-3}^* + A_1 A_0^{-1} A_1 A_0^{-1} \epsilon_{t-2} + A_1 A_0^{-1} \epsilon_{t-1} + \epsilon_t \]  

(4.87)

which we can write simply as

\[ A_0 X_t^* = A_1^Q X_{t-3}^* + u_t, \]  

(4.88)

where \( u_t \sim N \left(0, \Sigma^Q \right) \) and

\[ A_1^Q = A_1 A_0^{-1} A_1 A_0^{-1} A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{\sigma^2} \left( G_{\phi_{x} \sigma_{x} + F \sigma_{d} \phi_{y} + F k \phi_{x} \sigma_{d} - G \phi_{x} \sigma_{x} \phi_{y} - F \sigma_{d} \phi_{x} \phi_{y} - F k \phi_{x} \sigma_{d} \phi_{y} - 1 \right)^2 \\ 0 & \frac{p_3^2}{\sigma^2} \left( G_{\phi_{x} \sigma_{x} + F \sigma_{d} \phi_{y} + F k \phi_{x} \sigma_{d} - G \phi_{x} \sigma_{x} \phi_{y} - F \sigma_{d} \phi_{x} \phi_{y} - F k \phi_{x} \sigma_{d} \phi_{y} - 1 \right)^2 \end{bmatrix}. \]

We can show that from the elements of the matrices \( A_0 \) and \( A_1^Q \) defined above, we can recover all the parameters driving the monthly process. Specifically, from the matrix \( A_0 \) we identify \( \sigma_{x}, \sigma_{d}, \sigma_{r} \) and \( k \). Moreover, we obtain \( F \) and \( G \), and also the values of the combinations of parameters \( \phi_{x} (1 - \rho_{r}) \) and \( \phi_{y} (1 - \rho_{r}) \). We can rewrite the element of the matrix \( A_1^Q \), \( A_1^Q \) (3, 3) \( \frac{p_3^2}{\sigma^2} \left( G_{\phi_{x} \sigma_{x} + F \sigma_{d} \phi_{y} + F k \phi_{x} \sigma_{d} - G \phi_{x} \sigma_{x} \phi_{y} - F \sigma_{d} \phi_{x} \phi_{y} - F k \phi_{x} \sigma_{d} \phi_{y} - 1 \right)^2 \), as \( A_1^Q (3, 3) = \frac{\sigma^2}{\sigma^2} \left( G_{\phi_{x} \sigma_{x} + F \sigma_{d} \phi_{y} + F k \phi_{x} \sigma_{d} - G \phi_{x} \sigma_{x} \phi_{y} - F \sigma_{d} \phi_{x} \phi_{y} - F k \phi_{x} \sigma_{d} \phi_{y} + 1 \right)^2 \), where all the elements in the denominator are known and therefore we can recover \( \rho_{r} \) from the numerator. Having \( \rho_{r} \), we also identify \( \phi_{x} \) and \( \phi_{y} \). The last two parameters, \( \beta \) and \( \tau \) can be obtained from the definition of \( F \) and \( G \).

4.10.5 Introducing more dynamic in the Euler equation

Our New Keynesian model is described by the following three equations:

\[ \pi_t = \beta E_t \pi_{t+1} + k y_t^* + \varepsilon_{st}, \]  

(4.89)

\[ y_t^* = E_t y_{t+1}^* - \tau (R_t - E_t \pi_{t+1}) + p y_{t-1}^* + \varepsilon_{dt}, \]  

(4.90)

\[ R_t = \rho_r R_{t-1} + (1 - \rho_r) (\phi_{x} \pi_t + \phi_{y} y_t^*) + \varepsilon_{rt}, \]  

(4.91)

which can be written in matrix form as:

\[ B_0 X_t^* = C X_{t-1}^* + DE_t X_{t+1}^* + \epsilon_t, \]  

(4.92)

where \( X_t^* = [ \pi_t \ y_t^* \ R_t ]' \) and \( \epsilon_t = [ \epsilon_{st} \ \epsilon_{dt} \ \epsilon_{rt} ]' \), with \( \epsilon_t \sim N \left(0, I_3 \right) \).
We normalize $\sigma_d$ to 1.

The matrices $B_0, C, D$ have the following form:

$$B_0 = \begin{bmatrix} \frac{1}{\sigma_s} & -\frac{k}{\sigma_s} & 0 \\ \frac{\phi_k}{\sigma_r} (\rho_r - 1) & \frac{1}{\sigma_r} \phi_x (\rho_r - 1) & \frac{1}{\sigma_r} \\ 0 & 1 & \tau \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & p & 0 \\ 0 & \frac{1}{\sigma_r} \rho_r \end{bmatrix},$$

$$D = \begin{bmatrix} \frac{\beta}{\sigma_s} & 0 & 0 \\ \tau & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The unique stable solution for this model is given by

$$A_0 X_t^* = A_1 X_{t-1}^* + \epsilon_t,$$ (4.93)

with $A_0$ and $A_1$ defined as follows:

$$A_0 = \begin{bmatrix} \frac{1}{\sigma_s} & F & G \\ 0 & H & L \\ \frac{\phi_k}{\sigma_r} (\rho_r - 1) & \frac{1}{\sigma_r} \phi_x (\rho_r - 1) & \frac{1}{\sigma_r} \end{bmatrix},$$

$$A_1 = C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & p & 0 \\ 0 & \frac{1}{\sigma_r} \rho_r \end{bmatrix},$$

where

$$F = p \frac{\beta}{\sigma_s} H + L \phi_x - L \rho_r \phi_x + GH \phi_x \sigma_s - F L \phi_x \sigma_s - GH \phi_x \sigma_s \rho_r + F L \phi_x \sigma_s \rho_r \frac{k}{\sigma_s} - \frac{k}{\sigma_s},$$

$$G = \frac{\beta}{\sigma_s \sigma_r \rho_r} H + L \phi_x - L \rho_r \phi_x + GH \phi_x \sigma_s - F L \phi_x \sigma_s - GH \phi_x \sigma_s \rho_r + F L \phi_x \sigma_s \rho_r,$$

$$H = 1 - p \left( \frac{G \phi_x \sigma_s - G \phi_x \sigma_s \rho_r + 1}{H + L \phi_x - L \rho_r \phi_x + GH \phi_x \sigma_s - F L \phi_x \sigma_s - GH \phi_x \sigma_s \rho_r + F L \phi_x \sigma_s \rho_r} \right) \left( \frac{F \phi_x \sigma_s + G \phi_x \sigma_s - G \phi_x \sigma_s \rho_r}{H + L \phi_x - L \rho_r \phi_x + GH \phi_x \sigma_s - F L \phi_x \sigma_s - GH \phi_x \sigma_s \rho_r + F L \phi_x \sigma_s \rho_r} \right).$$
L = \tau + \frac{1}{\sigma_r} \rho_r \left( \frac{L H + L \phi_{\omega} - L \rho_y \phi_y + G H \phi_{\omega} \phi_{\omega} - F L \phi_{\omega} \phi_{\omega} - G H \phi_{\omega} \phi_{\omega} - F L \phi_{\omega} \phi_{\omega} + F L \phi_{\omega} \phi_{\omega}}{G H \phi_{\omega} - F L \phi_{\omega}} \right).

It can be checked that all the parameters of the model are identified.

To aggregate the process at a quarterly level, we conduct the same steps as in Appendix 4.10.4 and obtain

\[ A_0 X_t^* = A_1^Q X_{t-3}^* + u_t, \quad (4.94) \]

where \( u_t \sim N(0, \Sigma^Q) \) and

\[ A_1^Q = A_1 A_0^{-1} A_1 A_0^{-1} A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & M & N \\ 0 & P & Q \end{bmatrix}, \]

with \( M, N, P, Q \) highly non-linear functions of all the structural parameters:

\[ M = \rho \left( \frac{p^2 (G \phi_{\omega} \phi_{\omega} - G \phi_{\omega} \phi_{\omega} + 1)^2}{(H + L \phi_{\omega} - L \rho_y \phi_y + G H \phi_{\omega} \phi_{\omega} - F L \phi_{\omega} \phi_{\omega} - G H \phi_{\omega} \phi_{\omega} - F L \phi_{\omega} \phi_{\omega} + F L \phi_{\omega} \phi_{\omega})^2} \right), \]

\[ N = -\frac{1}{\sigma_r} \rho_r \left( \frac{L \rho^2 \sigma_r}{(H + L \phi_{\omega} - L \rho_y \phi_y + G H \phi_{\omega} \phi_{\omega} - F L \phi_{\omega} \phi_{\omega} - G H \phi_{\omega} \phi_{\omega} - F L \phi_{\omega} \phi_{\omega} + F L \phi_{\omega} \phi_{\omega})^2} \right), \]

\[ P = \rho \left( \frac{H \sigma_r^2}{(H + L \phi_{\omega} - L \rho_y \phi_y + G H \phi_{\omega} \phi_{\omega} - F L \phi_{\omega} \phi_{\omega} - G H \phi_{\omega} \phi_{\omega} - F L \phi_{\omega} \phi_{\omega} + F L \phi_{\omega} \phi_{\omega})^2} \right), \]

\[ Q = \frac{1}{\sigma_r} \rho_r \left( \frac{H^2}{(H + L \phi_{\omega} - L \rho_y \phi_y + G H \phi_{\omega} \phi_{\omega} - F L \phi_{\omega} \phi_{\omega} - G H \phi_{\omega} \phi_{\omega} - F L \phi_{\omega} \phi_{\omega} + F L \phi_{\omega} \phi_{\omega})^2} \right). \]

While it is still possible to easily identify \( \sigma_s \) and \( \sigma_r \), we need to solve highly non-linear equations to find the other parameters, which do not give rise to a unique solution.

### 4.10.6 Obtaining identification by exploiting mixed-frequency data

The unique stable solution for the model

\[ \pi_t = \beta E_t \pi_{t+1} + k y^*_t + \varepsilon_{st}, \quad (4.95) \]

\[ y^*_t = E_t y^*_t - \tau (R_t - E_t \pi_{t+1}) + p y^*_{t-1} + \varepsilon_{dy}, \quad (4.96) \]

\[ R_t = \rho_y R_{t-1} + (1 - \rho_y) (\phi_y \pi_t + \phi_y y^*_t) + \varepsilon_{dy}, \quad (4.97) \]
is given by

\[ A_0 X_t^* = A_1 X_{t-1}^* + \epsilon_t, \tag{4.98} \]

with \( A_0 \) and \( A_1 \) defined as follows:

\[
A_0 = \begin{bmatrix}
\frac{1}{\sigma_s} & F \\
0 & H \\
\frac{\phi_\pi}{\sigma_r} (\rho_r - 1) & \frac{1}{\sigma_r} \phi_x (\rho_r - 1) & \frac{1}{\sigma_r}
\end{bmatrix},
\]

\[
A_1 = C = \begin{bmatrix}
0 & 0 & 0 \\
0 & p & 0 \\
0 & 0 & \frac{1}{\sigma_r} \rho_r
\end{bmatrix}.
\]

We can rewrite (4.98) as a system of three equations:

\[
\frac{1}{\sigma_s} \pi_t + F y_t^* + GR_t = \epsilon_t \tag{4.99}
\]

\[
H y_t^* + LR_t = p y_{t-1}^* + \epsilon_{dt} \tag{4.100}
\]

\[
\frac{\phi_\pi}{\sigma_r} (\rho_r - 1) \pi_t + \frac{\phi_y}{\sigma_r} (\rho_r - 1) y_t^* + \frac{1}{\sigma_r} R_t = \frac{1}{\sigma_r} \rho_r R_{t-1} + \epsilon_{rt}. \tag{4.101}
\]

We then need to modify eq. (4.38) in such a way that it contains only variables which are available at the time of estimation. If we substitute \( y_{t-1}^* \) with its own expression \( y_{t-1}^* = \frac{p}{H} y_{t-2}^* - \frac{L}{H} R_{t-1} + \frac{1}{H} \epsilon_{dt-1} \), and then we repeat it again for \( y_{t-2}^* \), we obtain:

\[
y_t^* = \frac{p}{H} \left( \frac{p}{H} y_{t-2}^* - \frac{L}{H} R_{t-1} + \frac{1}{H} \epsilon_{dt-1} \right) - \frac{L}{H} R_t + \frac{1}{H} \epsilon_{dt-1} =
\]

\[
= \frac{p}{H} \left( \frac{p}{H} \left( \frac{p}{H} y_{t-3}^* - \frac{L}{H} R_{t-2} + \frac{1}{H} \epsilon_{dt-1} \right) - \frac{L}{H} R_{t-1} + \frac{1}{H} \epsilon_{dt-1} \right) - \frac{L}{H} R_t + \frac{1}{H} \epsilon_{dt-1} =
\]

\[
= \left( \frac{p}{H} \right)^3 y_{t-3}^* - \frac{L}{H} R_t - \frac{L}{H} \left( \frac{p}{H} \right) R_{t-1} - \frac{L}{H} \left( \frac{p}{H} \right)^2 R_{t-2} + \xi_t. \tag{4.102}
\]

From eq. (4.99), (4.102) and (4.101), we can now identify all the parameters. From eq. (4.99), we identify \( \sigma_s \) and obtain \( F \) and \( G \). From eq. (4.101), we identify \( \sigma_r, \rho_r, \phi_y \) and \( \phi_\pi \). From eq. (4.102), we obtain \( \frac{p}{H} \) and \( \frac{L}{H} \). Moreover, we know that \( F, G, H, L \) are defined as:

\[
F = \frac{p}{\sigma_s} \beta \frac{\sigma_s + G \sigma_s \phi_x - G \sigma_s \rho_r \phi_x}{H + L \phi_x - L \rho_r \phi_x + G \phi_x \sigma_s - FL \phi_y \sigma_s - GH \phi_y \sigma_s \rho_r + FL \phi_y \sigma_s \rho_r} - \frac{k}{\sigma_s},
\]

\[
G = \frac{\beta}{\sigma_r} \frac{\sigma_r \rho_r}{H + L \phi_x - L \rho_r \phi_x + G \phi_x \sigma_s - FL \phi_y \sigma_s - GH \phi_y \sigma_s \rho_r + FL \phi_y \sigma_s \rho_r},
\]
After some algebraic manipulations, from the definition of $G$ we obtain $\beta$, and from the definition of $F$ we identify $k$. Combining the definitions of $H$ and $L$, we obtain $\tau$. Once we have $\tau$, we have to the necessary parameters to disentangle $H$ and $L$, and as a consequence we identify also $p$. 

\[
H = 1 - p \left( \frac{G\phi_s\sigma_s-G\phi_s\sigma_s\rho_r + 1}{H+L\phi_x-L\rho_x+G\phi_x\sigma_x-FL\phi_x\sigma_x-G\phi_x\sigma_s\rho_r + FL\phi_x\sigma_s\rho_r + FL\phi_x\sigma_s\rho_r} \right)
\]

\[
L = \tau + \frac{1}{\sigma_r} \left( \frac{\sigma_r}{G\phi_x\sigma_x-FL\phi_x\sigma_x} \right)
\]