European University Institute

DEPARTMENT OF ECONOMICS

Uncertainty, Expectations, and the Business Cycle

Jan Hannes Lang

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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Dedication

To my parents, Veronika and Bob

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Abstract

This thesis adds to the recent quantitative literature that considers variations in uncertainty as impulses driving the business cycle. In chapter one a flexible partial equilibrium model that features heterogeneous firms, uncertainty shocks and various forms of capital adjustment costs is built in order to reassess whether temporarily higher uncertainty can cause recessions. It is then shown that while uncertainty shocks to demand can cause the bust, rebound and overshoot dynamics reminiscent of recessions, uncertainty shocks to total factor productivity are likely to lead to considerable and prolonged booms in economic activity. The reason for this result is that while the expectational effect of uncertainty shocks is negative and similar in magnitude for both types of uncertainty shocks, the positive distributional effect is an order of magnitude larger for total factor productivity than for demand.

Chapter two then derives and implements an identification strategy for uncertainty shocks within a Structural Vector Autoregression framework that is consistent with the way these shocks are commonly modeled in the literature. For the US it is shown that such model consistent uncertainty shocks lead to considerable booms in investment and employment and only explain a small fraction of the variation in the cross-sectional sales variance. Once uncertainty shocks are identified as the shocks that only affect dispersion upon impact, they cause a moderate drop, rebound and overshoot of investment and a large increase in the cross-sectional dispersion of revenues. The results suggest that the standard timing assumption that the expectational effect of uncertainty shocks leads the distributional effect seems questionable.

Finally, chapter three analyses endogenous variations in uncertainty and their effect on aggregate dynamics that result from imperfect information in the presence of occasional regime shifts. In a tentative model parameterization to the German manufacturing industry during the Financial Crisis it is shown that after a temporary regime shift imperfect information leads endogenously to higher forecast standard errors compared to full information, as well as higher cross-sectional dispersion of mean forecasts and forecast standard errors. It is then shown that these endogenous variations in uncertainty can lead to considerable downward amplification and some propagation of aggregate investment and revenues during a temporary downward regime shift.

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Overview of Thesis Chapters

The idea that variations in uncertainty are a cause of business cycle fluctuations has received renewed attention in recent years. This is partly based on the fact that many proxies for uncertainty such as stock market volatility, disagreement between professional forecasters, and the cross-sectional dispersion of productivity, stock returns, revenue growth and price changes all go up during recessions.[1] Based on these findings, shocks to uncertainty are commonly modeled as changes in the variance of idiosyncratic and aggregate shocks that hit agents within the quantitative macroeconomic literature .[2] This way of modeling uncertainty shocks implies that on the one hand expectations about the future get more uncertain and on the other hand the cross-sectional dispersion of fundamentals across agents increases after a positive uncertainty shock. The first channel is labeled the expectational effect, while the latter channel is labeled the distributional effect of uncertainty shocks. This thesis builds upon and adds to this line of research through three self-contained chapters.

Chapter one readdresses the question of whether temporarily higher uncertainty can lead to recessions. In particular, it is studied whether the dynamics induced by uncertainty shocks differ depending on whether they apply to demand conditions or to TFP. To answer the research question a partial equilibrium model that features heterogeneous firms, uncertainty shocks and various forms of capital adjustment costs is built and simulated. In line with the existing literature uncertainty shocks are modeled as changes in the idiosyncratic shock variance faced by firms. The central finding of the chapter is that while uncertainty shocks to demand cause the bust, rebound and overshoot dynamics reminiscent of recessions, uncertainty shocks

[1]See for example Bloom (2009), Dovern et al. (2009), Bloom et al. (2010), Kehrig (2011), Bachmann and Bayer (2011), and Berger and Vavra (2010).

[2]This way of modeling uncertainty shocks has been applied in models with labor and/or capital adjustment costs by Bloom (2009), Bloom et al. (2010), and Bachmann and Bayer (2011), in models with financial frictions by Dorofeenko et al. (2008), Gilchrist et al. (2010), Chugh (2011) and Arellano et al. (2011), in a search and matching model by Schaal (2012), and in a pricing model with adjustment costs by Vavra (2012).

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to TFP are likely to lead to large and persistent booms.

The mechanism behind this finding is as follows: While the expectational effect is negative in the presence of non-convex adjustment costs, it's magnitude does not change much between uncertainty shocks to demand and TFP. In contrast to this, the distributional effect is positive and an order of magnitude larger for uncertainty shocks to TFP than for uncertainty shocks to demand. The intuition is that for TFP shocks the revenue function is likely to have increasing returns to scale while for demand shocks the returns to scale can be constant at best. Hence, for TFP shocks higher ex-post cross-sectional dispersion is a time of opportunity which more than compensates for the negative expectational effect of uncertainty shocks, causing a prolonged boom in aggregates. For uncertainty shocks to demand the negative expectational effect dominates the distributional effect causing the recession like dynamics emphasized by Bloom (2009).

Chapter two proposes an empirical identification strategy for uncertainty shocks that is consistent with the recent vintage of quantitative models that consider variations in uncertainty as impulses driving aggregate fluctuations. The identification strategy has two parts. First, the cross-sectional variance of firm-level sales is used as a proxy for uncertainty. Second, because the expectational effect of uncertainty shocks is commonly assumed to lead the distributional effect, an uncertainty shock is identified as the shock that affects investment upon impact but not the cross-sectional variance of revenues within a Structural Vector Autoregression (SVAR) framework. This strategy for identifying uncertainty shocks is then applied to US data.

The main result from the baseline SVAR estimation is that such model consistent uncertainty shocks lead to considerable booms in investment and employment that last for around two years. Moreover, while the uncertainty shock explains most of the forecast error in investment and employment it only explains a small part of the forecast error in the cross-sectional variance of firm-level sales. Both of these results are contrary to the dynamics that are induced by these uncertainty shocks in the recent vintage of quantitative macro models. Once uncertainty shocks are identified as the shocks that only affect dispersion upon impact, the results change somewhat. An uncertainty shock in that case leads to a moderate drop, rebound and overshoot of investment and a large increase in the cross-sectional dispersion of revenues. The results suggest that the way uncertainty shocks are modeled in the quantitative literature needs to be reconsidered. In particular, the standard timing assumption that the expectational effect of uncertainty shocks leads the distributional effect seems questionable given the empirical results in this chapter.

Finally, chapter three studies endogenous variations in uncertainty and aggregate

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fluctuations that result from imperfect information and learning in an environment where regime changes in the mean happen occasionally. The idea behind this set-up is that whenever unprecedented regime shifts occur, agents become more uncertain about the true data generating process (DGP) and therefore mix different conditional distributions when forming expectations about the future. The German manufacturing industry actually experienced such an unprecedented regime shift during the Financial Crisis in mid 2008. Output collapsed by 25 % within just six months and expectations fell much more than can be explained by fundamentals. With this empirical background in mind a partial equilibrium heterogeneous firm model that features capital adjustment costs, a markov-switching driving process and imperfect information about the underlying regime is parameterized to German manufacturing data and simulated.

There are two main findings that come out of the exercises. First, after a regime shift imperfect information leads endogenously to temporarily higher uncertainty about the underlying regime. On average this leads to lower mean forecasts and higher forecast standard errors compared to full information. Moreover, during the regime shift the dispersion in beliefs increases considerably, which causes the cross-sectional dispersion of mean forecasts and forecast standard errors to increase in turn. This mechanism could be interesting in order to explain why survey responses by firms and professional forecasters get more dispersed during downturns. Second, these endogenous variations in uncertainty can lead to considerable downward amplification and some propagation of aggregate investment and revenues during a temporary downward regime shift. This is true for all types of adjustment costs, but some degree of quadratic costs are needed to match the empirical volatility of investment.

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Chapter 1

Does Higher Uncertainty Cause Recessions?

Abstract

This chapter readdresses the question of whether temporarily higher uncertainty can lead to recessions. In particular, it is studied whether the dynamics induced by uncertainty shocks differ depending on whether they apply to demand conditions or to TFP. To answer the research question a partial equilibrium model that features heterogeneous firms, uncertainty shocks and various forms of capital adjustment costs is built and simulated. The central finding of the chapter is that while uncertainty shocks to demand cause the bust, rebound and overshoot dynamics reminiscent of recessions, uncertainty shocks to TFP are likely to lead to large and persistent booms. This result can be easily understood when considering that uncertainty shocks in the model have an expectational effect as well as a distributional effect. While the expectational effect is negative in the presence of non-convex adjustment costs, it's magnitude does not change much between uncertainty shocks to demand and TFP. In contrast to this, the distributional effect is positive and an order of magnitude larger for uncertainty shocks to TFP than for uncertainty shocks to demand. The intuition is that for TFP shocks the revenue function is likely to have increasing returns to scale while for demand shocks the returns to scale can be constant at best. Hence, for TFP shocks higher ex-post cross-sectional dispersion is a time of opportunity which more than compensates for the negative expectational effect of uncertainty shocks, causing a prolonged boom in aggregates. For uncertainty shocks to demand the negative expectational effect dominates the distributional effect causing the recession like dynamics emphasized by Bloom (2009).

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1.1 Introduction

Ever since the outbreak of the financial crises in 2008, the use of the word uncertainty in relation to macroeconomic events has become increasingly popular among policy makers and the media. Statements such as 'uncertainty affects behaviour, which feeds the crisis'[1] by Olivier Blanchard and headlines such as 'Economic uncertainty drags US retail sales'[2] illustrate the widely shared view that heightened uncertainty is at least partly responsible for low economic activity. The idea that uncertainty is an important factor for driving aggregate economic outcomes also has a long tradition in economics, dating back at least to Knight (1971) and Keynes (1936). However, even though the role of uncertainty has been a major research area in the investment literature ever since the seminal paper by Bernanke (1983), the modern quantitative business cycle literature has generally abstracted from variations in uncertainty as impulses driving aggregate fluctuations.[3]

Since the publication of a recent paper by Bloom (2009), the academic interest in considering variations in uncertainty as impulses driving the business cycle has been revived again.[4] In a partial equilibrium heterogeneous firm model that features various forms of labor and capital adjustment costs, he shows quantitatively that a temporary increase in uncertainty first leads to a drop and then a subsequent rebound and overshoot of output and employment. In the paper variations in uncertainty are modeled as changes in the variance of idiosyncratic and aggregate shocks that hit the firms. This approach of modeling variations in uncertainty has since then been applied in general equilibrium models with adjustment costs by Bloom et al. (2010) and Bachmann and Bayer (2011). While the former paper confirms that the drop, rebound and overshoot dynamics are robust to general equilibrium considerations the latter paper argues that uncertainty shocks are unlikely to be a major quantitative source of business cycle fluctuations. One potential reason for these discrepancies in findings is that Bloom (2009) implicitly considers uncertainty shocks to demand, while Bachmann and Bayer (2011) consider uncertainty shocks to total factor productivity (TFP).

With this background in mind, the current chapter readdresses the question of whether temporarily higher uncertainty can lead to recessions. In particular, it is studied whether the dynamics induced by uncertainty shocks differ depending on

[1] Quote from guest article by Olivier Blanchard in the *Economist* on January 29, 2009.

[2] Article headline in the *Financial Times* on June 2, 2011.

[3] Other papers that consider the effect of uncertainty on investment include Hartman (1972), Abel (1983), Caballero (1991), Dixit and Pindyck (1994), Leahy and Whited (1996) and Guiso and Parigi (1999) among others.

[4] A discussion of the related literature can be found towards the end of the introduction.

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whether they apply to demand conditions or to TFP. To answer the research question a partial equilibrium model that features heterogeneous firms, uncertainty shocks and various forms of capital adjustment costs is built and simulated. In line with the existing literature, uncertainty shocks are modeled as changes in the variance of idiosyncratic profitability shocks. The main difference in the model set-up compared to the literature is that the specification of the revenue function allows to separately analyze the quantitative implications of uncertainty shocks to demand and TFP.

The central finding of the chapter is that while uncertainty shocks to demand cause the bust, rebound and overshoot dynamics reminiscent of recessions, uncertainty shocks to TFP are likely to lead to large and persistent booms. The difference in these dynamic effects of uncertainty shocks is caused by different degrees of returns to scale in the revenue function that are implied by demand and TFP shocks. This result can be easily understood when considering that uncertainty shocks in the model have an expectational effect as well as a distributional effect: On the one hand, expectations about the future get more uncertain and on the other hand the cross-sectional dispersion of profitability across firms increases after a positive uncertainty shock.

While the expectational effect is negative in the presence of non-convex adjustment costs, it's magnitude does not change much between uncertainty shocks to demand and TFP. In contrast to this, the distributional effect is positive and is an order of magnitude larger for uncertainty shocks to TFP than for uncertainty shocks to demand. The intuition is that for TFP shocks the revenue function is likely to have increasing returns to scale while for demand shocks the returns to scale can be constant at best. Hence, for TFP shocks higher ex-post cross-sectional dispersion is a time of opportunity which more than compensates for the negative expectational effect of uncertainty shocks, causing a prolonged boom in aggregates. For uncertainty shocks to demand the negative expectational effect dominates the distributional effect causing the recession like dynamics emphasized by Bloom (2009).

A secondary contribution of the chapter is that it is shown that uncertainty shocks have first moment implications for the distribution of profitability whenever the driving process is specified as an AR(1) process in logs, which is a common assumption in the literature. The reason for this is that an increase in the variance of a log-normal variable actually translates into an increase in the mean of the variable in levels. Adjusting for this positive first moment effect of uncertainty shocks is not easy as long as the persistence parameter of the AR(1) process is not equal to zero or one. This fact needs to be taken into account when building quantitative models that include uncertainty shocks.

There are various strands of literature that are related to this chapter. Most relevant is the recent quantitative literature on the impact of uncertainty shocks that was started by Bloom (2009). Other papers that consider uncertainty shocks in models with capital adjustment costs are Bloom et al. (2007), Bloom et al. (2010) and Bachmann and Bayer (2011). Moreover, there are various recent papers on the interaction of time-varying uncertainty with financial frictions as for example Dorofeenko et al. (2008), Gilchrist et al. (2010), Chugh (2011) and Arellano et al. (2011). The quantitative literature that was started by Bloom (2009) builds itself on an extensive literature that considers the effect of uncertainty on investment. Important contributions in this line of research include Hartman (1972), Abel (1983), Bernanke (1983), Caballero (1991) and Dixit and Pindyck (1994). Finally, this chapter builds on many papers that study the effects of capital adjustment costs such as Hayashi (1982), Abel and Eberly (1994), Abel and Eberly (1996), Caballero and Engel (1999) and Cooper and Haltiwanger (2006) among others.

The rest of the chapter proceeds along the following lines. In the next section the model set-up is presented and some necessary concepts for the analysis of uncertainty shocks are formalized. In section three analytic results are derived for the model without any capital adjustment costs. This case helps to understand the underlying dynamics in the model and in addition serves as a benchmark to which we can compare the results for models that incorporate various forms of capital adjustment costs. Section four then moves on to analyze various models with capital adjustment costs making use of numerical simulations. Finally, section five provides a brief conclusion of the chapter. All derivations of the analytical results are contained in the Appendix.

1.2 A Firm Model with Adjustment Costs and Uncertainty Shocks

In this section a partial equilibrium model with heterogeneous firms is presented.[5] As in Abel and Eberly (1994) and Cooper and Haltiwanger (2006) the firms are assumed to face a rich set of convex and non-convex adjustment costs to investment, as well as partial investment irreversibilities. Furthermore, in line with the recent uncertainty shocks literature that was started by the paper of Bloom (2009), the model features variations in uncertainty over time, which are modeled as changes in the variance of idiosyncratic profitability shocks faced by each firm.[6] Labor

[5] Throughout the chapter the terms *firm* and *plant* are used interchangeably, as it is assumed that each firm just operates one plant.

[6] See the introduction for a complete list of papers that model uncertainty shocks in this way.

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adjustment costs are disregarded to keep the model as simple as possible, while allowing for an analysis of the interaction between variations in uncertainty and aggregate investment, employment and output.[7] The main difference in the model set-up compared to the existing literature is that the specification of the revenue function allows to separately analyze the effect of uncertainty shocks to demand (Demand shifter) and supply (TFP).[8]

This set-up is mainly motivated by the fact that the existing literature has made varying implicit assumptions about the underlying structural nature of the uncertainty shocks. This might be a possible reason for why Bloom (2009) finds a considerable negative impact of uncertainty shocks, while Bachmann and Bayer (2011) find that uncertainty shocks do not alter business cycle properties a lot. The former paper implicitly assumes uncertainty shocks to demand, while the latter assumes that they are uncertainty shocks to TFP.[9] As will be shown below, the common way that uncertainty shocks are modeled implies that on the one hand expectations about the future get more uncertain upon impact and on the other hand the cross-sectional dispersion of profitability across firms increases after a positive uncertainty shock. The former channel will be labeled as the expectational effect and the latter as the distributional effect.[10] In the remainder of the chapter it will be shown that the magnitude of the distributional effect depends to a large extent on the structural nature of the uncertainty shock. Uncertainty shocks to TFP have a much higher positive distributional effect than uncertainty shocks to demand. In contrast, the expectational effect is negative and similar in magnitude for both types of uncertainty shocks in the presence of non-convex adjustment costs. Uncertainty shocks to demand therefore lead to the bust, rebound and overshoot dynamics stressed in the literature, while uncertainty shocks to supply lead to considerable booms.

[7]This approach can be justified on the grounds that Bloom et al. (2010) show that uncertainty shocks lead to a drop and rebound in output even when the model only features capital adjustment costs. Moreover, Bloom (2009) shows that disregarding labor adjustment costs affects the fit of his model by an order of magnitude less compared to disregarding capital adjustment costs.

[8]In the literature it is common to work with a reduced form shock to either revenues or profits that could be due to demand, supply (TFP) or wage movements. The flexible way the revenue function is specified in this chapter allows to study these effects separately.

[9]As will be argued below, the admissible returns to scale of the revenue function are to a large extent determined by the type of structural shock we look at. Hence, by making assumptions about the returns to scale of the revenue or profit function, we implicitly say something about the underlying shock that is assumed.

[10]Bloom (2009) refers to these two channels as the uncertainty and the volatility effect. As both are induced by the uncertainty shock it is deemed that the distinction between an expectational effect and a distributional effect seems clearer.

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1.2.1 Production, Demand and the Revenue Function

There is a continuum of risk neutral firms indexed by $i \in [0,1]$ who maximize the present discounted value of expected profit streams. Risk neutrality is assumed to isolate the effect of time-varying uncertainty that arises through the presence of non-convex adjustment costs which induce real options effects. Firms are assumed to face a constant or decreasing returns to scale (RTS) production function and a constant elasticity demand function.[11] Moreover, it is assumed that firms differ in their productivity and potentially in the demand that they face. Under these assumptions a revenue function with the following properties can be derived as shown in Appendix B:[12]

$$S(A_{i,t}, K_{i,t}, L_{i,t}) = A^c_{i,t} K^a_{i,t} L^b_{i,t} \tag{1.1}$$

In the above equation, $A^c_{i,t} = B^{1/\varepsilon}_{i,t} \tilde{A}^{(\varepsilon-1)/\varepsilon}_{i,t}$ is a reduced form shock that is comprised of TFP shocks $(\tilde{A})$ and shocks to the demand shifter of the constant elasticity demand function $(B_{i,t})$. In addition, K and L represent the capital stock and labor input, while the exponents a, b, and c determine the curvature and RTS of the revenue function. In particular, it is always the case that both $a, b \in (0,1)$ and in addition $a + b < 1$. Hence, the revenue function always displays decreasing RTS in (K, L) space. This is due to either the assumption of decreasing RTS in the production function and/or some degree of market power. However, the exact values of a and b will be determined by the specific assumptions about the demand elasticity ε and RTS of the production function.[13]

Moreover, the value of the exponent on the profitability shock c will depend on whether we look at demand or TFP shocks. In particular, for demand shocks $c = 1/\varepsilon \in (0,1)$ and in the case of supply shocks $c = (\varepsilon-1)/\varepsilon \in (0,1)$. Furthermore, the revenue function can only have constant or decreasing RTS in (A, K, L) space for demand shocks[14] while the revenue function can have either of decreasing, constant or increasing RTS for TFP shocks.[15] However, a moderate elasticity of demand

[11]The production function takes the following form $Y(\tilde{A}_{i,t}, K_{i,t}, L_{i,t}) = \tilde{A}_{i,t} K^{\nu}_{i,t} L^{\omega}_{i,t}$ while the demand function takes the form $q_{i,t} = B_{i,t} p^{-\varepsilon}_{i,t}$.

[12]The derivation of this revenue function is closely related to the one in Bloom (2009) with the difference that in his model he assumes $c = 1 - a - b$, while the specification in this chapter allows for flexible values of c. This will be of importance in the analysis further below.

[13]This connection becomes clear when looking at the formulas for those exponents: $a = \nu\frac{\varepsilon-1}{\varepsilon}$ and $b = \omega\frac{\varepsilon-1}{\varepsilon}$.

[14]This is easy to see by noting that in this case $a + b + c = [1 + (\nu + \omega)(\varepsilon - 1)]/\varepsilon \leq 1$ due to the fact that $\nu + \omega \in (0, 1]$.

[15]Here we have that $a + b + c = [\varepsilon - 1 + (\nu + \omega)(\varepsilon - 1)]/\varepsilon \leq 2(\varepsilon - 1)/\varepsilon$. Hence, whenever the demand elasticity is below two, there will be decreasing returns to scale in (A, K, L) space, while

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will usually imply increasing returns to scale in (A, K, L) space and a value of c that is close to but below one. The RTS of the revenue function will be especially important in determining how the distributional effect of uncertainty shocks impacts on aggregate investment, employment and revenues, which will be shown in section 1.3.1.

At this point it is worth to relate the specification of the revenue function in equation (1.1) to other papers on investment under uncertainty. For example Bloom (2009) works with a specification of $c = 1 - a - b$. Given his assumptions of constant returns to scale in production and some degree of market power, this implies that his shocks to A are demand shocks. In contrast, Bloom et al. (2010) and Bachmann and Bayer (2011) use a model with perfect competition so that the revenue function is equal to the assumed decreasing returns to scale production function. Their specification would therefore be equivalent to setting $c = 1$ and interpreting A as TFP shocks. Finally, Cooper and Haltiwanger (2006) and Caballero and Engel (1999) work with a reduced form profit function that would imply a value of $c = 1 - b$ if the current economic structure was assumed.[16] These seemingly minor assumptions will turn out the be important determinants for the distributional effect of uncertainty shocks. Table 1.1 summarizes the values of c that are commonly used in the investment literature along with some other common modeling assumptions that will be used further below.

Table 1.1: Common modeling assumptions in the investment literature

Paper	Exponent on A	$Z \cdot \Psi$	Form of process
Cooper and Haltiwanger (2006)	$c = 1 - b$	Yes	AR(1) in logs
Bloom (2009)	$c = 1 - a - b$	Yes	Geometric RW
Bloom et al. (2010)	$c = 1$	Yes	AR(1) in logs
Bachmann and Bayer (2011)	$c = 1$	Yes	AR(1) in logs
Khan and Thomas (2008)	$c = 1$	Yes	Markov chain
Thomas (2002)	$c = 1$	Only Z	AR(1) in logs
Caballero and Engel (1999)	$c = 1 - b$	Only Z	Geometric RW

for a demand elasticity greater than two, we can have any of the three cases of decreasing, constant or increasing returns to scale depending on the assumption made about $\nu + \omega$. As it is usually assumed that the returns to scale in the production function are not too low, a moderate demand elasticity will usually imply increasing returns to scale in the revenue function in (A, K, L) space.

[16]Their profitability shocks combine demand, TFP and factor price shocks. However, their assumptions about the exponents of the profit function imply $c = 1 - b$ within the set-up of this chapter.

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1.2.2 The Static Labor Input Decision and Profits

Throughout this chapter it is assumed that labor input can be freely adjusted within each period and becomes immediately available for production.[17] In contrast to that, new capital is assumed to take one period to be ready for use in production, so that the capital stock is taken as fixed in the current period by the firm. With these assumptions, the labor input decision of the firm becomes static and is determined by the maximization of gross profits given the capital stock and the shock to revenues. Taking the first-order condition and solving for $L_{i,t}$ gives us the following function for the optimal labor input decision:

$$L(w_t, A_{i,t}, K_{i,t}) = \kappa_t A_{i,t}^{\frac{c}{1-b}} K_{i,t}^{\frac{a}{1-b}} \tag{1.2}$$

where $\kappa_t = b^{\frac{1}{1-b}} w_t^{-\frac{1}{1-b}}$ is a parameter that depends on the wage. Moreover, plugging the labor policy function into the revenue function and into the equation for gross profits yields the following reduced form revenue and profit functions:

$$R(w_t, A_{i,t}, K_{i,t}) = \chi_t A_{i,t}^{\frac{c}{1-b}} K_{i,t}^{\frac{a}{1-b}} \tag{1.3}$$

$$\Pi(w_t, A_{i,t}, K_{i,t}) = \phi_t A_{i,t}^{\frac{c}{1-b}} K_{i,t}^{\frac{a}{1-b}} \tag{1.4}$$

where $\chi_t = b^{\frac{b}{1-b}} w_t^{-\frac{b}{1-b}}$ and $\phi_t = (1-b) b^{\frac{b}{1-b}} w_t^{-\frac{b}{1-b}}$ are again parameters that depend on the wage. Note that equations (1.2), (1.3) and (1.4) all have the same functional form in (A, K) space. Due to the assumptions of either decreasing returns to scale in production or some degree of market power, which both imply that $0 < a + b < 1$, all three functions will be concave in K. Concavity or convexity in A and whether we have increasing, constant or decreasing returns to scale in (A, K) space will however depend crucially on the value of c, i.e. on the choice of whether we consider demand or supply shocks.

Relating to the discussion above, the assumption of $c = 1 - a - b$ in Bloom (2009) implies concavity in A and constant returns to scale in (A, K). In contrast, if we set $c = 1$ as in Bloom et al. (2010) and Bachmann and Bayer (2011), the functions will be convex in A and have increasing returns to scale in (A, K). Finally, for $c = 1 - b$ as in Cooper and Haltiwanger (2006) and Caballero and Engel (1999), the functions will be linear in A and have increasing returns to scale in (A, K) space. How these different assumptions determine the effect of uncertainty shocks is shown analytically in section 1.3 for the case without capital adjustment costs and numerically in section 1.4 for the case with capital adjustment costs.

[17] This is a common assumption in most of the investment literature. See for example Hayashi (1982), Abel and Eberly (1994), Bertola and Caballero (1994), Abel and Eberly (1996), Caballero and Engel (1999), Cooper and Haltiwanger (2006) or Bloom et al. (2007) among others.

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For the rest of the chapter it is assumed that the wage rate does not vary over time. This is mainly done to keep the model as tractable as possible. Variations in wages could however be easily implemented via making them a function of the aggregate profitability shock and/or the uncertainty shock. With the assumption of a constant wage the variables κ_t, χ_t and ϕ_t become constants, which allows us to drop the wage rate from the state space of the firm. As the shock to revenues is now the only stochastic variable left in the model and it directly affects profits, it will from now on be labeled as a profitability shock.

1.2.3 The Stochastic Process for Profitability

As is standard in the investment literature with heterogeneous firms, the profitability of each firm ($A_{i,t}$) is assumed to be the product of an aggregate (Z_t) and an idiosyncratic ($\Psi_{i,t}$) component.[18] Furthermore, both the aggregate and idiosyncratic components are assumed to follow persistent AR(1) processes in logs, which is consistent with US micro data as shown by Cooper and Haltiwanger (2006). Finally, in line with the models by Bloom (2009), Bloom et al. (2010), Bachmann and Bayer (2011), Arellano et al. (2011), Gilchrist et al. (2010) and Vavra (2012) uncertainty shocks are incorporated into the model through changes in the variance of idiosyncratic shocks.[19] Given these assumptions the profitability process can be described by the following equations:

$$A_{i,t} = Z_t \Psi_{i,t} \tag{1.5}$$

$$z_t = \mu_z + \rho_z z_{t-1} + \eta_t \tag{1.6}$$

$$\psi_{i,t} = \mu_\psi + \rho_\psi \psi_{i,t-1} + \upsilon_{i,t} \tag{1.7}$$

Here, a lower case letter refers to the logarithm of the variable and $\eta_t \sim \mathcal{N}(\mu_\eta, \sigma_\eta^2)$ and $\upsilon_{i,t} \sim \mathcal{N}(\mu_{\upsilon,t-1}, \sigma_{\upsilon,t-1}^2)$ are i.i.d. innovations to aggregate and idiosyncratic profitability. Note that the idiosyncratic profitability process is specified as a markov-switching process, where it is assumed that the standard deviation of idiosyncratic shocks follows a two-point markov chain with support $\sigma_{\upsilon,t} \in \{\sigma_\upsilon^L, \sigma_\upsilon^H\}$ and transition probabilities $Pr(\sigma_{\upsilon,t+1} = \sigma_\upsilon^j | \sigma_{\upsilon,t} = \sigma_\upsilon^i) = \pi_{ij}$ between the high and low uncertainty

[18] See table 1.1 for a summary of common modeling assumptions in the investment literature.

[19] In principle, we could also assume that the variance of aggregate shocks changes over time. However, as Bloom et al. (2010) argue, this would mainly affect conditional heteroskedasticity of aggregate variables. As the main motivation of the Uncertainty Shocks literature is that cross-sectional measures of spread at the micro level vary over time, it seems sufficient to incorporate firm level uncertainty shocks. Hence, to simplify the model, the variance of shocks to the aggregate component is assumed to be constant.

state. Embedded in the above specification is the standard timing assumption that the variance of idiosyncratic shocks is known one period in advance with certainty.[20] This implies that agents always know the true variance of shocks applicable in the next period and hence all the variations in uncertainty perceived by firms are rational in the sense that they are related to more volatility in the shocks to fundamentals. Finally, this standard timing assumption for information implies that the expectational effect of uncertainty shocks leads the distributional effect by one period. I.e. firms' expectations about the future get more uncertain on impact of the uncertainty shock, but the distributional effect only starts with a one period delay once firms start drawing shocks from a more dispersed distribution. Note also that the mean of idiosyncratic profitability shocks is allowed to vary along side the variance of shocks. As is shown in Appendix C this is done in order to correct for the positive effect on mean expectations that an increase in the shock variance causes given that the process is specified in logs.

1.2.4 Specification of Capital Adjustment Costs

The firm's capital stock is fixed within each period, as it is assumed to take one period for new capital to be installed and ready for production. Moreover, capital is assumed to depreciate at the rate δ per period, so that the law of motion for the capital stock is given by the following equation, where I denotes the level of investment:

$$K_{i,t+1} = K_{i,t}(1-\delta) + I_{i,t} \tag{1.8}$$

In line with the papers by Cooper and Haltiwanger (2006) and Bloom (2009) it is assumed that the firm faces convex and non-convex costs of adjusting the capital stock, as well as partial investment irreversibilities.[21] Both, adjustment costs and partial irreversibilities bring interesting real options effects to the capital accumulation process and in particular will determine how the expectational effect of uncertainty shocks impacts on investment decisions. The adjustment cost function and the price of capital can be represented by the following equations:

$$C(A_{i,t}, K_{i,t}, I_{i,t}) = \frac{\gamma}{2}(I_{i,t}/K_{i,t})^2 K_{i,t} + (1-\lambda)\Pi(A_{i,t}, K_{i,t})1_{\{I_{i,t}\neq 0\}} + FK_{i,t}1_{\{I_{i,t}\neq 0\}} \tag{1.9}$$

$$p(I_{i,t}) = \begin{cases} p_s & , if\ I_{i,t} < 0 \\ p_b & , if\ I_{i,t} > 0 \end{cases} \tag{1.10}$$

[20]See all of the papers on uncertainty shocks mentioned above.

[21]Both papers find that all these forms of capital adjustment frictions are needed to match the micro data on investment behavior by plants/firms.

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The convex adjustment costs are assumed to be quadratic while the non-convex adjustment costs can be a fraction of current profits $(1-\lambda)$ or a fraction of the capital stock (F).[22] Finally, p_s denotes the selling price of capital and p_b the buying price and it is assumed that $p_s < p_b$ so that there are partial irreversibilities for investment.

1.2.5 The Bellman Equation of the Firm

As mentioned above, firms are assumed to be risk neutral in order to isolate the effects of uncertainty shocks that are due to adjustment frictions. Moreover, it is assumed that the discount rate, i.e. the interest rate, with which the firms discount their expected future profit streams is constant over time. In reality of course, the interest rate varies over time and should in particular be related to the business cycle. Incorporating such interest rate changes are again easily implemented as functions of the aggregate and uncertainty shock. However, to isolate the pure effects of uncertainty shocks it was decided to abstract from factor price changes. Given the objects defined in the previous subsections, the dynamic decision problem for each firm of maximizing the present discounted value of expected profits can be summarized by the following Bellman Equation. To save on notation the firm subscript i for each variable is omitted and primes denote next period variables:[23]

$$V(A,K,\sigma_v) = \max_I \Pi(A,K) - C(A,K,I) - p(I)I + \beta E_{A',\sigma_v'|A,\sigma_v}\left[V(A',K',\sigma_v')\right] \tag{1.11}$$

Here, β is the period discount factor, $\Pi(A,K)$ is the reduced form profit function, $C(A,K,I)$ captures investment adjustment costs, $p(I)$ is the effective price of newly installed or retired capital, and next period capital is given by the law of motion in equation (1.8). The solution to this Bellman Equation will yield a policy function for investment or alternatively next period capital of the form $I(A,K,\sigma_v)$ and $K'(A,K,\sigma_v)$.

[22] $1_{\{I_{i,t}\neq 0\}}$ is an indicator function that takes a value of 1 whenever investment is nonzero, and a value of 0 otherwise.

[23] In the Bellman Equation below total profitability is used as the state variable to save on notation. It should be kept in mind however that in order to solve the model, information on both the aggregate and idiosyncratic profitability is needed. Whenever total profitability is used as the state variable in this chapter it should therefore be interpreted as information on both components of total profitability.

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1.2.6 Dynamic Aggregation of the Firm Distribution

The Bellman Equation (1.11) implies that the relevant state vector for each firm consists of aggregate profitability, idiosyncratic profitability, the capital stock and the uncertainty shock. In order to determine the relevant state vector that determines the behavior of aggregate variables at each point in time, first note that firms in our model differ in two respects from each other, namely idiosyncratic profitability and capital. The aggregates in our economy will therefore be characterized by a joint distribution of the form $G_t(\Psi_{i,t}, K_{i,t})$ in every period. Let the associated density be denoted by g_t. The density g_t is indexed by t to make it explicit that it will change over time due to uncertainty shocks. In addition to this density, the aggregate profitability shock and the current variance regime will determine how aggregate variables behave. Following this logic, the aggregate of a generic endogenous variable X can be expressed as a function of the following state vector $S_t = [g_t(\Psi_{i,t}, K_{i,t}), Z_t, \sigma_{v,t}]$:

$$E[X_{i,t}|S_t] = \bar{X}_t(S_t) = \iint g_t(\Psi_{i,t}, K_{i,t}) X_{i,t}(\Psi_{i,t}, K_{i,t}, Z_t, \sigma_{v,t}) d\Psi dK \tag{1.12}$$

Moreover, with this definition of the aggregate state vector we can concisely represent the evolution of the density g_t over time. First note that g_t will depend on the joint density of idiosyncratic profitability and capital at time $t-1$. Moreover, as capital in the next period is a function of aggregate profitability and the uncertainty shock, Z_{t-1} and $\sigma_{v,t-1}$ will also affect g_t. Finally, the distribution of idiosyncratic profitability in the current period depends on last period's distribution and the uncertainty shock. But this simply amounts to saying that the joint density of idiosyncratic profitability and capital at time t depends on the aggregate state vector at time $t-1$. We can therefore express g_t as a conditional distribution of S_{t-1}:

$$g_t(\Psi_{i,t}, K_{i,t}) = g(\Psi_{i,t}, K_{i,t}|S_{t-1}) \tag{1.13}$$

1.2.7 Formal Definition of Expectational and Distributional Effects

In general, there are two different channels through which uncertainty shocks can affect aggregate investment and other endogenous variables in the model. On the one hand, the level of uncertainty can influence the investment behavior of each individual firm through it's effect on the expected dispersion of future profitability. For example, in the presence of fixed costs higher uncertainty will lead to more inaction due to real options effects. On the other hand, changes in the level of

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uncertainty can affect aggregate investment through their effect on the dispersion of idiosyncratic profitability across firms. For example, given the timing assumption implicit in equation (1.7) a positive uncertainty shock will increase the cross-sectional dispersion of idiosyncratic profitability in the subsequent period. This change in the dispersion of profitability across firms can in turn affect aggregate investment whenever investment policy functions are not linear in idiosyncratic profitability.

Similar to the terminology used by Bloom (2009), the former channel will be referred to as the expectational effect, while the latter channel will be referred to as the distributional effect of uncertainty shocks.[24] Hence, the expectational effect will be related to changes in the standard deviation of the forecast distributions of firms, while the distributional effect will be related to changes in the cross-sectional dispersion of idiosyncratic profitability. The overall influence of an uncertainty shock on endogenous aggregate variables will therefore be determined by the sum of both effects. Given the timing assumption that firms know the realization of the variance regime one period in advance, the expectational effect will lead the distributional effect by one period. I.e. when a positive uncertainty shock occurs, firms' expectations change upon impact, but the cross-sectional dispersion of idiosyncratic profitability does not change until the next period, when firms start drawing shocks from a more dispersed distribution.

In order to formally define expectational and distributional effects, it is useful to rewrite the joint density g_t as the product of a marginal and a conditional density: $g(\Psi_{i,t}, K_{i,t}|S_{t-1}) = f(\Psi_{i,t}|S_{t-1}) \cdot h(K_{i,t}|\Psi_{i,t}, S_{t-1})$. Using this in equation (1.12) we can rewrite the expression for a generic aggregate variable as follows:

$$\bar{X}_t = \iint f(\Psi_{i,t}|f_{t-1}, \sigma_{v,t-1}) \cdot h(K_{i,t}|\Psi_{i,t}, g_{t-1}, Z_{t-1}, \sigma_{v,t-1}) \cdot X_{i,t}(\Psi_{i,t}, K_{i,t}, Z_t, \sigma_{v,t}) d\Psi dK \tag{1.14}$$

Note that use has been made of the fact that the distribution of idiosyncratic profitability at time t will only depend on last period's distribution and variance regime and not on the whole state vector S_{t-1}. Taking this formulation for aggregate variables as a basis, the dynamics due to the expectational and distributional effect can be respectively defined as follows:

$$\bar{X}_t^E = \iint f(\Psi_{i,t}|f_{t-1}, \sigma_v^L) \cdot h(K_{i,t}|\Psi_{i,t}, g_{t-1}, Z_{t-1}, \sigma_{v,t-1}) \cdot X_{i,t}(\Psi_{i,t}, K_{i,t}, Z_t, \sigma_{v,t}) d\Psi dK \tag{1.15}$$

$$\bar{X}_t^D = \iint f(\Psi_{i,t}|f_{t-1}, \sigma_{v,t-1}) \cdot h(K_{i,t}|\Psi_{i,t}, g_{t-1}, Z_{t-1}, \sigma_v^L) \cdot X_{i,t}(\Psi_{i,t}, K_{i,t}, Z_t, \sigma_v^L) d\Psi dK \tag{1.16}$$

[24]The respective labels used by Bloom (2009) are uncertainty and volatility effect.

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1.3 Analytical Results in the Absence of Adjustment Costs

In order to get some intuition on how the model described in the previous section behaves, it is useful to first consider the case without adjustment costs and irreversibilities. For that case it is possible to derive analytic expressions for the capital policy function and the dynamics of aggregate variables. The insights gained from this exercise can then be used as a benchmark to which we can compare the simulation results of models with capital adjustment frictions. Particular focus is put on the analysis of the expectational and distributional effects of uncertainty shocks and how they depend on whether we consider them to demand or TFP.

1.3.1 The Capital Policy Function

In the absence of capital adjustment costs and irreversibilities, the firm's decision problem allows for an analytical solution. Taking the first-order condition of the Bellman Equation in (1.11) and combining it with the envelope condition yields the following optimal capital policy function:[25]

$$K'(A, \sigma_v) = \varphi E\left[A'^{\frac{c}{1-b}} \mid A, \sigma_v\right]^{\frac{1-b}{1-a-b}} \tag{1.17}$$

where $\varphi = [(a\beta\phi)/([1-b][p - p\beta(1-\delta)])]^{\frac{1-b}{1-a-b}}$. From this expression we can already see that even without adjustment costs there can be expectational and distributional effects of uncertainty shocks on capital accumulation. For instance, whenever c is greater (smaller) than $1-b$, the resulting convexity (concavity) of the profit function in A implies that more dispersion in future profitability shocks will increase (decrease) the desired capital stock of a single firm. Hence, the expectational effect of uncertainty shocks will be positive whenever $c > 1-b$, which is a direct implication of Jensen's inequality. The curvature of the profit function with respect to profitability is in turn determined by the returns to scale of the revenue function (1.1) in (A, L) space. To be precise, whenever c is greater (smaller) than $1-b$ there are increasing (decreasing) returns to scale in (A, L) space, the profit function is therefore convex (concave) in A and hence the expectational effect is positive (negative). Note that this result holds even though it was assumed that there are no adjustment costs.

This potentially positive effect of higher uncertainty has been first shown by

[25] A detailed derivation of this policy function and subsequent results can be found in Appendix D.

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Hartman (1972) and Abel (1983), who found that under the assumptions of constant returns to scale, perfect competition and an increasing adjustment cost function, more uncertainty in the price leads to higher investment due to the convexity of the profit function in the price.[26] In order to determine the distributional effect of uncertainty shocks without adjustment costs for the current model it is useful to derive an analytic expression for the expectation in equation (1.17):[27]

$$K'(Z, \Psi, \sigma_v) = \varphi \left(e^{\mu_z} e^{\mu_\psi} e^{\mu_\eta} e^{\frac{\sigma_\eta^2}{2} \frac{c}{1-b}} \right)^{\frac{c}{1-a-b}} \left(Z^{\rho_z} \Psi^{\rho_\psi} \right)^{\frac{c}{1-a-b}} e^{\frac{\sigma_v^2}{2} \frac{c^2 - c(1-b)}{(1-b)(1-a-b)}} \tag{1.18}$$

With this capital policy in hand it is instructive to analyze how the assumptions about the parameter c determine the sign and magnitude of the distributional effect of uncertainty shocks. As can be easily seen from the policy function, the distributional effect is positive (negative) whenever c is greater (smaller) than $1 - a - b$, which is due to the resulting convexity (concavity) of the policy function in idiosyncratic profitability and therefore more cross-sectional dispersion in profitability will increase the aggregate capital stock.[28] This in turn simply means that the distributional effect is positive (negative) whenever the revenue function (1.1) displays increasing (decreasing) returns to scale in (A, K, L) space. The intuition behind this result is that whenever there are increasing RTS in the revenue function, ex-post higher cross-sectional dispersion in the fixed factor, i.e. profitability, is a time of opportunity for firms to invest. Firms that receive large negative shocks reduce investment proportionately less than by how much firms that receive large positive shocks increase investment.

1.3.2 Expectational and Distributional Effects for Demand and TFP

With the previous general considerations in mind, we can analyze expectational and distributional effects of uncertainty shocks for the three cases of $c \in \{1, 1-b, 1-a-b\}$

[26]In their framework adjustment costs are needed to make the size of the firm determinate, which is not needed in our case due to the assumption of decreasing returns to scale. The case of $c = 1$ in the model above is qualitatively the same as their model.

[27]For the derivation it is assumed that the mean of idiosyncratic innovations changes at the same time as the variance. In particular it is assumed that $\mu_{v,t-1} = -\sigma_{v,t-1}^2/2$, which ensures that the one period ahead expected value is not affected by the increase in the innovation variance. This adjustment will be called a Jensen correction in the rest of the chapter, as the increase in the expected mean due to a higher variance is due to Jensen's inequality. See Appendix C for details.

[28]With the stochastic process for profitability assumed in this chapter, the persistence parameter ρ_ψ also affects the sign of the volatility effect. As it is common to assume highly persistent or even random walk processes for idiosyncratic profitability, the effect of the persistence parameter is ignored in this discussion.

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commonly used in the literature. First, recall that increasing RTS in the revenue function are only possible for TFP shocks and not for demand shocks.[29] For example, when we consider a calibration where the production function has constant RTS with exponents of 1/3 and 2/3 on capital and labor, and the demand elasticity is set equal to four, then $a = 0.25$, $b = 0.5$, and $c = 0.25 = 1-a-b$ for demand shocks while $c = 0.75 = 1 - a$ for supply shocks. I.e. there are constant RTS when we consider demand and there are increasing RTS when we consider supply. In fact the specification by Bloom (2009) is exactly the one just described for demand shocks. For this case the desired capital stock is now decreasing in the variance of shocks, so there is a negative expectational effect. Moreover, the distributional effect is slightly negative because the desired capital stock is concave in profitability. This is due to the fact that $c/(1 - a - b) = 1$ and ρ_ψ is smaller than one. Hence, for a random walk process, the distributional effect would be zero.

If we have instead that $c = 1-b$, as implicitly assumed in Cooper and Haltiwanger (2006), then a higher variance of shocks has no effect on the desired capital stock of each firm holding everything else equal.[30] In other words, the expectational effect is zero. However, as long as the AR(1) persistence parameter is sufficiently high there is a positive distributional effect. This can be seen by noting that the desired capital stock is convex in idiosyncratic profitability because $c/(1-a-b) > 1$. This convexity implies that more dispersion in idiosyncratic profitability across firms increases the aggregate capital stock.

Now consider the case of $c = 1$, as in Bachmann and Bayer (2011) and Bloom et al. (2010), which corresponds to perfect competition, decreasing RTS in the production function and TFP shocks. In this case, a larger variance of shocks increases the desired capital stock of each firm, i.e. the expectational effect is positive. In addition, the desired capital stock is convex in profitability, so the distributional effect is also positive. The implications of these three cases of c for expectational and distributional effects of uncertainty shocks are summarized in table 1.2.

[29] Recall from section 1.2.1 that for constant RTS in the production function and a low elasticity of demand of only two, the revenue function for TFP shocks already has constant RTS and the RTS are increasing in the value of the demand elasticity.

[30] Cooper and Haltiwanger (2006) use a profit function with an exponent of one on profitability which corresponds to $c = 1 - b$ in my framework. Hence, there are increasing RTS in the revenue function and shocks are therefore interpreted as TFP shocks, although in an environment with some degree of market power.

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Table 1.2: Distributional and expectational effects without adjustment costs

	c = 1 - a - b	**c = 1 - b**	**c = 1**
Distributional Effect	$\frac{c}{1-a-b} = 1$	$\frac{c}{1-a-b} > 1$	$\frac{c}{1-a-b} > 1$
Expectational Effect	$\frac{c^2-c(1-b)}{(1-b)(1-a-b)} < 0$	$\frac{c^2-c(1-b)}{(1-b)(1-a-b)} = 0$	$\frac{c^2-c(1-b)}{(1-b)(1-a-b)} > 0$

It has therefore been shown that even in the absence of capital adjustment costs, aggregate investment can be influenced in different ways by variations in uncertainty. We can broadly distinguish between an expectational and a distributional effect. The former has to do with the fact that an uncertainty shock affects the expected dispersion in future profitability, while the latter has to do with the fact that uncertainty shocks affect the dispersion in actual profitability with a one period delay. For the Cooper and Haltiwanger (2006) and the Bachmann and Bayer (2011) specifications of c, which can be both interpreted as TFP shocks, higher uncertainty should lead to more investment because of a positive distributional effect and a non-negative expectational effect, while in the Bloom (2009) specification the uncertainty shocks to demand will reduce aggregate investment as the expectational effect is negative and the distributional effect is zero.

1.3.3 The Dynamics of the Profitability Distribution

Before moving to the simulation of various models, it is useful to better understand the dynamics of the profitability distribution that are induced by uncertainty shocks. It has been mentioned above that the mean of idiosyncratic profitability shocks needs to be adjusted at the same time as the variance, in order for uncertainty shocks to correspond to mean preserving spreads.[31] As will be shown below, it is however not easy to avoid any type of mean effects of changes in the shock variance for a log-normal process. Given the AR(1) process in logs assumed in equation (1.7), the distribution of idiosyncratic profitability across firms will be log-normal. It can easily be shown that the dynamics of the mean and variance of log idiosyncratic

[31] In other words we are interested in pure second moment shocks without any first moment implications. The fact that the expected value of a log-normally distributed variable increases in the variance of shocks is shown in Appendix C.

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profitability are governed by the following equations:[32]

$$
\begin{aligned}
E[\psi_{i,t}] &= \frac{\mu_\psi + \mu_v^L}{1-\rho_\psi} + \sum_{j=1}^{\infty} \rho_\psi^{j-1} \Delta\mu_{v,t-j} && (1.19)\\
V[\psi_{i,t}] &= \frac{(\sigma_v^L)^2}{1-\rho_\psi^2} + \sum_{j=1}^{\infty} \rho_\psi^{2(j-1)} \Delta\sigma_{v,t-j}^2 && (1.20)
\end{aligned}
$$

Here, the variable $\Delta\mu_{v,t} = \mu_{v,t} - \mu_v^L$ captures the difference between the mean of idiosyncratic shocks under the current regime and the low regime. Analogous to this the variable $\Delta\sigma_{v,t}^2 = \sigma_{v,t}^2 - (\sigma_v^L)^2$ captures the difference between the variance of idiosyncratic shocks under the current regime and the low regime. It is obvious from the equations that the mean and variance of log idiosyncratic profitability are given by the sum of the value that would prevail under the low uncertainty regime and the accumulated effect due to occasional switches to the high uncertainty regime. In the end we are however interested in the evolution of the distribution of idiosyncratic profitability in levels and not in logs. Given that log idiosyncratic profitability is normally distributed, it is easy to establish a mapping from the moments in logs to the moments in levels:

$$
\begin{aligned}
E[\Psi_{i,t}] &= e^{E[\psi_{i,t}]+V[\psi_{i,t}]/2} && (1.21)\\
V[\Psi_{i,t}] &= e^{2E[\psi_{i,t}]+2V[\psi_{i,t}]} - e^{2E[\psi_{i,t}]+V[\psi_{i,t}]} && (1.22)
\end{aligned}
$$

If we now consider the case where the mean of shocks does not change over time, it is easy to see that the mean and variance of idiosyncratic profitability in levels will go up after an uncertainty shock. For this case an uncertainty shock also leads to an increase in the expected value of profitability for each firm, as is shown in Appendix C. In order to keep the one-period-ahead expected value of profitability unaffected by switches in the variance we can set $\mu_{v,t} = -\sigma_{v,t}^2/2$, which is equivalent to saying that $\Delta\mu_{v,t} = -\Delta\sigma_{v,t}^2/2$. But even with this so-called Jensen correction, the cross-sectional mean of idiosyncratic profitability will not stay constant over time whenever $0 < \rho_\psi < 1$.

To understand this, note that the mean of idiosyncratic profitability only stays constant if the log mean changes in exactly an off-setting way to the log variance in all periods. But from equations (1.19) and (1.20) we can immediately see that whenever $0 < \rho_\psi < 1$, the two moments do not change in offsetting ways in all periods after an uncertainty shock. To be precise, in the first period after an uncertainty shock the increase in the log variance is exactly offset by the fall in the log mean. In subsequent periods however, the log variance reverts quicker to the initial level than

[32]See Appendix E for a detailed derivation of these moments.

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the log mean due to the presence of ρ_ψ^2. Therefore, the cross-sectional mean in levels decreases compared to the low uncertainty value after a positive uncertainty shock. The dynamics of the cross-sectional mean and standard deviation for the two cases just described are illustrated in the top left panels of figures 1.1 and 1.2 for an uncertainty shock that hits in period zero and lasts for five periods. This shows that unless we work with a random-walk, it is hard to model mean-preserving spreads for driving processes in logs.

1.3.4 Simulation Results for various Parameterizations

The three different models with $c \in \{1, 1-b, 1-a-b\}$ are simulated in this section in order to illustrate quantitatively how the different assumptions about the RTS of the revenue function and hence about the nature of the underlying structural shocks affect aggregate investment, employment and revenues. The case of $c = 1$ used by Bachmann and Bayer (2011) is labeled as a pure TFP shock, while the case of $c = 1 - b$ used by Cooper and Haltiwanger (2006) is labeled as a TFP shock with market power, and the case of $c = 1 - a - b$ used by Bloom (2009) is labeled as a demand shock. Moreover, because it matters whether we apply a Jensen correction or not to the idiosyncratic profitability process, both cases are examined. The parameter values used for the simulation exercise are summarized in table 1.3 and discussed further below in section 1.4.1.

To study the effect of an uncertainty shock quantitatively the following simulation exercise is performed. Aggregate shocks are turned off and uncertainty is set to the low regime for the infinite past. In period zero an uncertainty shock hits the system that almost doubles the idiosyncratic shock variance. This uncertainty shock lasts for five consecutive periods and uncertainty is low again for all subsequent periods.[33] Figure 1.1 compares the responses of aggregate variables for the case without a Jensen correction, while figure 1.2 displays the responses for the case with a Jensen correction.

It is obvious from the figures that the higher the value of c, the higher the positive response of the variables to the uncertainty shock. This has to do with the fact that both the expectational and the distributional effects of uncertainty shocks are increasing in c, i.e. in the RTS of the revenue function. Furthermore, it can be seen that uncertainty shocks to TFP lead to considerable booms in investment, employment and revenues, while uncertainty shocks to demand lead to falls in aggregates only for the case where a Jensen correction is applied to the driving process. Quanti-

[33] The probability of an uncertainty shock lasting for five consecutive periods is slightly above 50 % for the parameterization used.

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Figure 1.1: The impact of uncertainty shocks in the model without ACs (no Jensen correction)

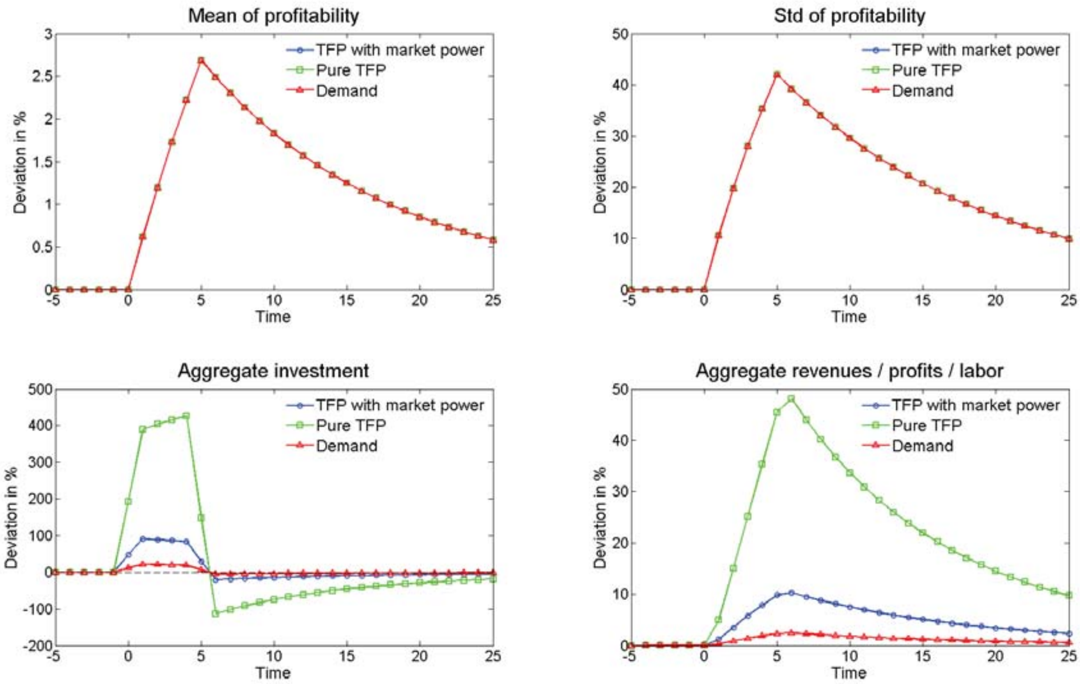
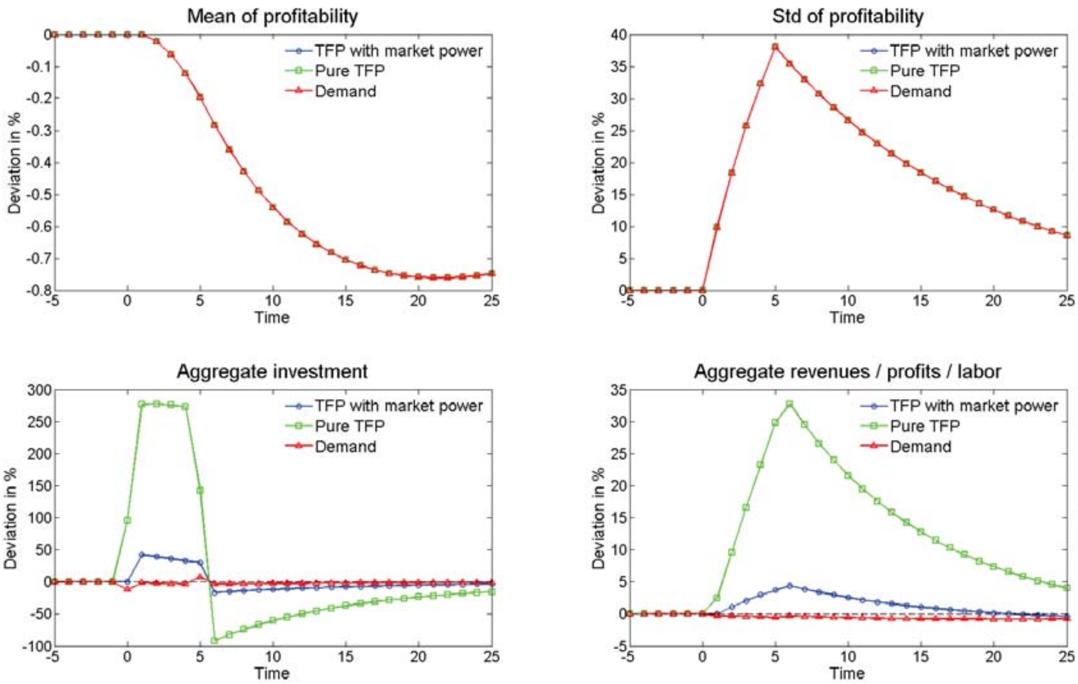


Figure 1.2: The impact of uncertainty shocks in the model without ACs (with Jensen correction)



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tatively, the burst in aggregate investment after an uncertainty shocks to pure TFP is around 250 % to 400 %, while it is 50 % to 100 % for uncertainty shocks to TFP with market power, and $\pm$ 10 % for shocks to demand.

In summary, we can say that higher uncertainty is a rather positive phenomenon when the uncertainty is to TFP, while uncertainty shocks to demand can have negative aggregate implications. The positive aggregate effects of uncertainty shocks to TFP are not only due to the convexity of profits in TFP which causes a positive expectational effect as pointed out by Hartman (1972) and Abel (1983). In addition, increasing returns to scale in (A, K, L) space for TFP shocks induce a positive distributional effect. For demand shocks, the RTS of the revenue function are always lower or equal to one and hence the distributional effect will never be positive for demand shocks when there are no adjustment costs present.

Now that it is understood how uncertainty shocks to demand and TFP affect aggregate variables in the model without adjustment costs we can move on to consider how the dynamics of aggregate variables in response to uncertainty shocks are affected by the presence of capital adjustment frictions. Due to real-options effects in the presence of non-convex adjustment costs, the expectational effect of uncertainty shocks will be altered considerably, while the distributional effect will also be somewhat affected by the presence of a discrete choice for investment.

1.4 Simulation Results for the General Model

Because it is no longer possible to derive closed form expressions for the capital policy function in the presence of capital adjustment costs, numerical methods are employed for the analysis.[34] For this purpose the same three versions of c as above are solved for three different adjustment cost parameterizations that have been estimated in the literature. The next subsection provides a discussion of the parameter values that are used to calibrate the model. This is followed by an analysis of how adjustment costs change the investment policy functions of firms in the presence of time-varying uncertainty. Finally, the aggregate dynamics induced by uncertainty shocks are analyzed. Particular focus is again put on disentangling expectational and distributional effects of uncertainty shocks.

[34] See Appendix F for more information on the algorithm and the accuracy of the approximation.

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1.4.1 Discussion of Parameters

There are two dimensions in which the parameters are changed between the different models that are solved and simulated: adjustment costs and the RTS of the revenue function in (A, K, L) space via c. A summary of all the other parameters that are not varied across the simulations can be found in table 1.3. Virtually all of these parameter values are taken from the paper by Bloom et al. (2010), who calibrate a general equilibrium business cycle model with capital adjustment costs, labor adjustment costs and uncertainty shocks. The reason for this choice of parameters is to stay as close as possible to the existing uncertainty shocks literature. In the remainder of the chapter all models will feature a Jensen correction in the mean of idiosyncratic shocks in order to allow for the biggest possible negative effect of uncertainty shocks.

The model frequency is one quarter and the depreciation rate is set to 2.6 % per period. Regarding the revenue function, it is assumed that there are decreasing returns to scale in (K, L) space, with the exponent on capital set to 0.25 and the exponent on labor set to 0.5.[35] Both the aggregate and idiosyncratic profitability processes are assumed to be highly persistent with AR(1) coefficients of around 0.96. The low standard deviation of idiosyncratic profitability shocks is set to 6.7 % and this standard deviation almost doubles in the high uncertainty state. Moreover, both uncertainty regimes are fairly persistent, with the probability of staying in the low uncertainty state being 95 % and the probability of staying in the high uncertainty state being 88.5 %. There are two parameters that are set slightly different from Bloom et al. (2010): The standard deviation of aggregate shocks is set to 1.5 % and the discount rate is set to 0.99.[36] Finally, two arbitrary normalizations are performed through setting the wage rate and the buying price of capital equal to one.

[35]As mentioned above these exponents would correspond to a constant RTS production function with exponents of 1/3 and 2/3, and a demand elasticity of four. Moreover, this implies a curvature of the profit function in capital of 0.5, which is similar to the value estimated by Cooper and Haltiwanger (2006).

[36]In the paper by Bloom et al. (2010) the aggregate profitability process is also subject to uncertainty shocks with a low standard deviation of 0.81 % and a high standard deviation of 3.49 %, while the discount rate is set to 0.996. As the high uncertainty state is less likely to occur than the low one, a constant standard deviation of 1.5 % was chosen, while the chosen discount rate of 0.99 speeds up the convergence of the value function, while it should not materially affect the results.

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Table 1.3: Common parameters across the simulations

Par	**Value**	**Description**	**Source**
β	0.99	Discount factor	Own choice
δ	0.026	Depreciation rate	Bloom et al. (2010)
w	1	Wage rate	Normalization
a	0.25	Exponent on capital	Bloom et al. (2010)
b	0.50	Exponent on labor	Bloom et al. (2010)
p_b	1	Buying price of capital	Normalization
μ_z	0	Intercept of aggregate profit.	Bloom et al. (2010)
ρ_z	0.9627	AR(1) parameter of aggregate profit.	Bloom et al. (2010)
σ_η	0.015	Std of innovations of aggregate profit.	Own choice
μ_ψ	0	Intercept of idiosyncratic profit.	Bloom et al. (2010)
ρ_ψ	0.9627	AR(1) parameter of idiosyncratic profit.	Bloom et al. (2010)
σ_υ^L	0.0671	Low Std of innovations of idio. profit.	Bloom et al. (2010)
σ_υ^H	$1.93 \cdot \sigma_\upsilon^L$	High Std of innovations of idio. profit.	Bloom et al. (2010)
π_{LL}	0.953	Probability of staying in low Std regime	Bloom et al. (2010)
π_{HH}	0.885	Probability of staying in high Std regime	Bloom et al. (2010)

The three different adjustment cost specifications that are simulated are taken from the papers by Cooper and Haltiwanger (2006) and Bloom (2009) as these papers jointly estimate the rich set of capital adjustment costs and irreversibilities that is featured in the model at hand. The estimated adjustment cost specification of Cooper and Haltiwanger (2006), which is labeled "C&H" in table 1.4, features considerable fixed costs of 20 % of profits, moderate resale losses of 1.9 % and small quadratic adjustment costs of 0.153. The adjustment cost specification labeled "Bloom" in table 1.4 refers to the estimates of Bloom (2009) from a model that features capital and labor adjustment costs, while the adjustment cost specification labeled "Bloom 2" refers to the estimates that he obtains from a model with only capital adjustment costs. Both adjustment cost specifications feature large capital resale losses of around 40 % and moderate fixed costs of 1.5 % and 1.1 % of profits. The main difference between these two adjustment cost specifications is that the former does not feature quadratic adjustment costs while the latter features moderate quadratic adjustment costs of 0.996.[37] Finally, the three different values of the curvature parameter on profitability of $c \in \{1, 1-b, 1-a-b\}$ are chosen due to their prevalence in the uncertainty shocks and capital adjustment cost literature.[38]

[37]A discussion of the adjustment cost estimates of Cooper and Haltiwanger (2006) and of Bloom (2009) can be found in Appendix G.

[38]See table 1.1 for a selective overview of assumptions that are routinely made in these two

Table 1.4: Parameters that are varied across the simulations

	Adjustment Costs			
Parameter	**C&H**	**Bloom**	**Bloom 2**	**Description**
p_s	0.981	0.661	0.573	Selling price of capital
F	0.000	0.000	0.000	Investment fixed costs
λ	0.796	0.985	0.989	% of profits kept
γ	0.153	0.000	0.996	Quadratic adjustment cost
	Curvature on Profitability			
Parameter	**Demand**	**TFP m.p.**	**TFP**	**Description**
c	$1-a-b$	$1-b$	1	Exponent on profitability

1.4.2 Characterizing the Solution of the Model

In order to understand how capital adjustment frictions change the expectational and distributional effects of uncertainty shocks, it is useful to first study the investment policy functions for the models. To this end, figure 1.3 plots investment as a function of the current capital stock for a given level of profitability under the low and high uncertainty regimes. The main feature that stands out is that a higher level of uncertainty leads to a larger investment inaction region for all three adjustment cost specifications, independent of whether we look at uncertainty shocks to demand or to TFP. This increased investment inaction in the presence of higher uncertainty is due to real-options effects that result from the presence of fixed costs and/or partial irreversibilities, which was first stressed in the seminal contribution by Bernanke (1983) and is a central feature of the models by Bloom (2009), Bloom et al. (2010) and Bachmann and Bayer (2011). Hence, the expectational effect of uncertainty shocks will be negative for all the specifications of the parameter c and the various adjustment cost specifications.

In order to better understand the mechanism driving the distributional effect of uncertainty shocks figure 1.4 plots optimal investment for the Bloom adjustment cost specification as a function of idiosyncratic profitability.[39] The important aspect to pay attention to is that the value of c affects the shape of the investment policy function outside of the inaction regions. In particular, in line with what was derived for the case without adjustment costs, the investment policies are convex for pure

literatures.

[39]The main finding that the value of c shapes the curvature of the policy function outside of the inaction region carries over to the C&H and Bloom 2 adjustment cost specifications.

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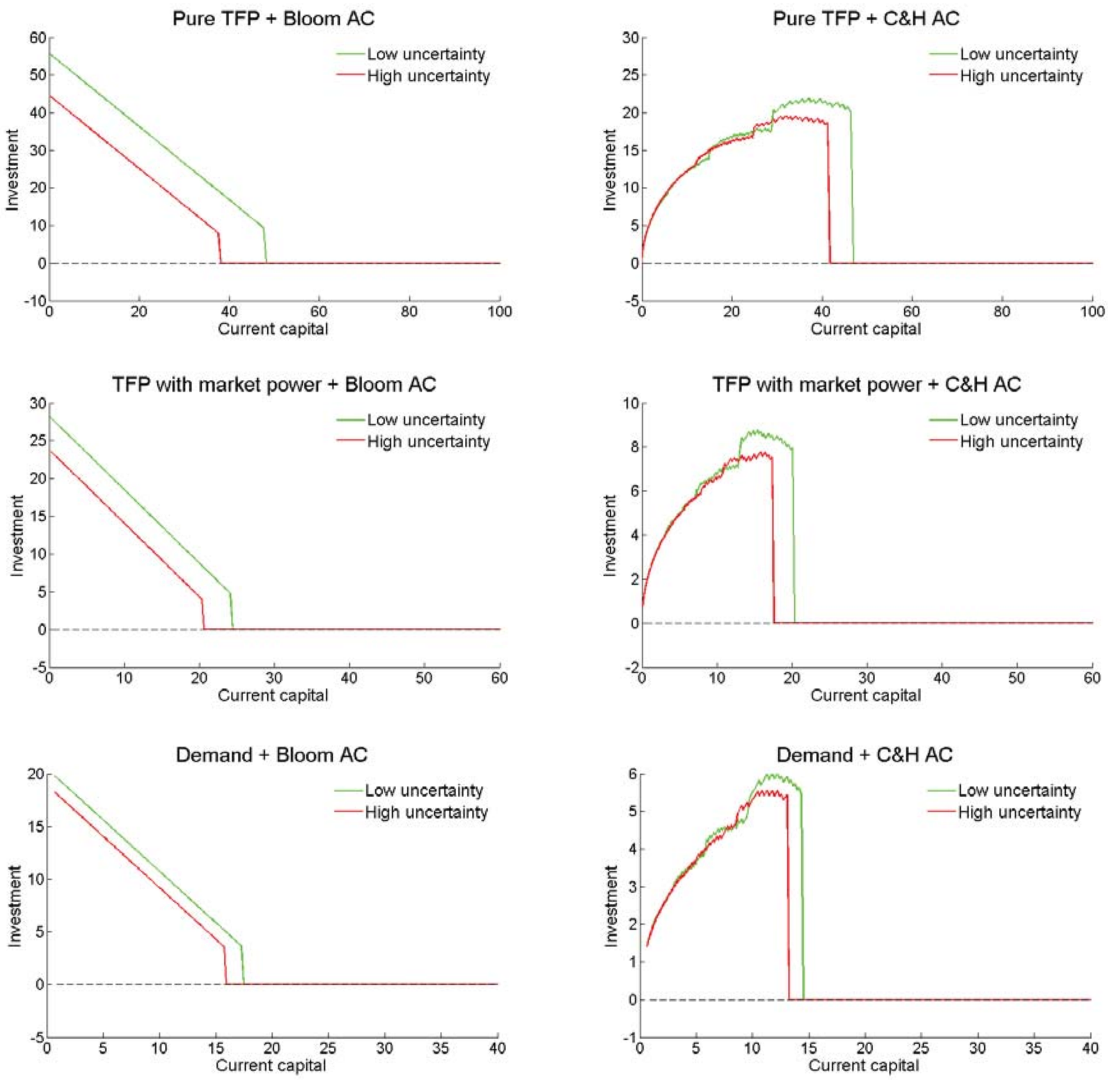
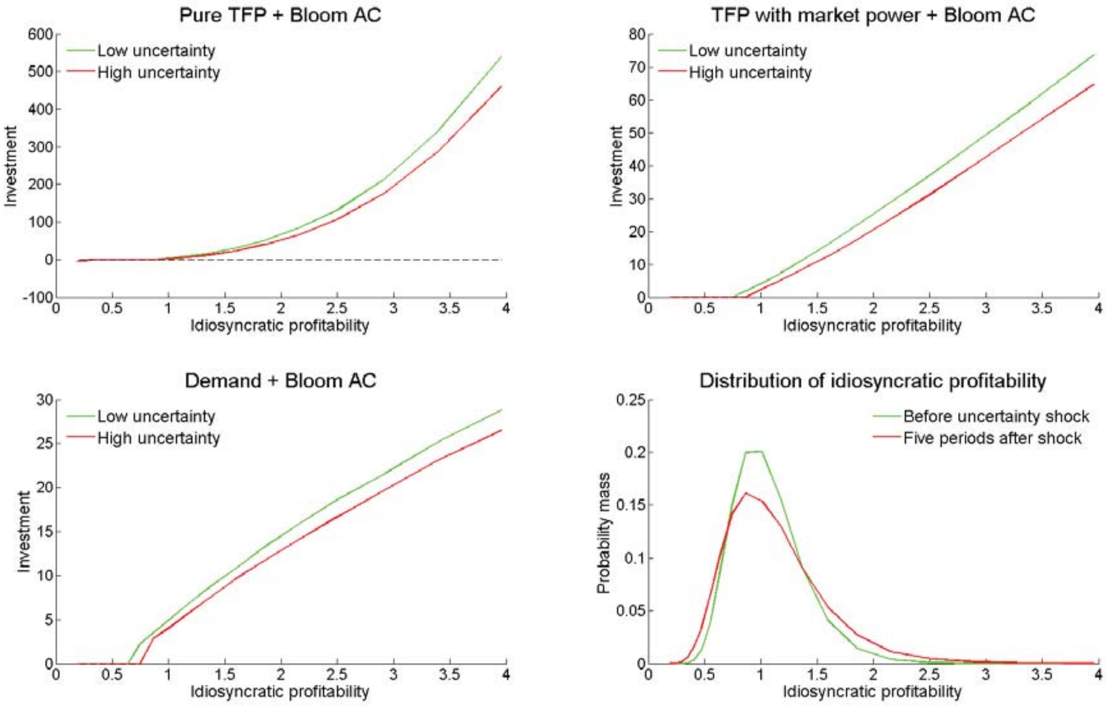
Figure 1.3: Investment policy functions under high and low uncertainty



Notes: The policy functions for the adjustment cost specification "Bloom 2" are not shown here as they are qualitatively similar to the ones of the "C&H" specification. The specific policy functions plotted are for an average sized aggregate profitability shock and an above average level of idiosyncratic profitability. The main feature that the investment inaction region is larger for the high uncertainty state carries over to other levels of profitability.

Figure 1.4: The volatility effect for the adjustment cost specification of Bloom (2009)



Notes: The specific policy functions plotted are for a medium sized capital stock and an average sized aggregate profitability shock. The distributions in the lower right panel correspond to the type of simulation performed in section 1.3.4 and 1.4.3.

TFP shocks and TFP shocks with market power and slightly concave for demand shocks outside of the inaction region. This implies that outside of the inaction regions the distributional effect is positive for $c = 1$ and $c = 1 - b$ and slightly negative for $c = 1 - a - b$. However, another important aspect to note is that due to the inaction regions, investment policies are locally convex for all three cases of c. This local convexity implies that even for $c = 1 - a - b$ the volatility effect will be positive as emphasized by Bloom (2009). Finally, the lower right panel of figure 1.4 plots the density of idiosyncratic profitability just before an uncertainty shock and after five consecutive periods of high uncertainty, which highlights the increase in dispersion of profitability that causes the distributional effect of uncertainty shocks.

1.4.3 The Effect of Uncertainty on Investment, Employment and Revenues

Now that the investment policy functions and the dynamics of idiosyncratic profitability have been analyzed, we can move on to study the impulse responses of aggregate variables to an uncertainty shock for the various models under consider-

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ation. In order to obtain the impulse responses, a similar simulation exercise as in section 1.3.4 is performed for each of the models: Aggregate shocks are shut off and the distribution of idiosyncratic profitability is set to the unconditional asymptotic distribution under the low variance regime. In period zero an uncertainty shock is simulated to hit the economy lasting for five consecutive periods.[40] Such a sequence of five consecutive periods of high uncertainty has a probability of slightly more than 50 % for the uncertainty process at hand. For all subsequent periods uncertainty is set to the low regime again. This simulation exercise is performed for a panel of 10,000 firms and repeated 100 times. The results are then averaged across the 100 simulations for each of the model specifications.

The associated impulse responses of aggregate investment and revenues for each of the model specifications can be found in figure 1.5.[41] Because the dynamics of aggregate revenues depend on the joint dynamics of profitability and the capital stock it is useful to start with the analysis of how an uncertainty shock affects investment in the various models. The first aspect to note is that the impulse responses for the C&H and Bloom adjustment cost specifications are qualitatively very similar for all three values of c. In particular, no matter whether we look at uncertainty shocks to demand or supply, there is a large negative expectational effect on investment, which is evident from the considerable investment drops in period zero.[42] Moreover, for all cases under consideration there is a subsequent rebound and overshoot in investment from period one onwards, which is due to a positive distributional effect that more than compensates for the negative expectational effect. Finally, there is a large spike in aggregate investment in period five due to the fading out of the negative expectational effect and the continued presence of the positive distributional effect.

Even though the qualitative behavior of aggregate investment is similar for uncertainty shocks to demand and TFP, the rebound and overshoot is much larger in the case of pure TFP shocks compared to demand shocks. This shows that the distributional effect of uncertainty shocks is increasing in the value of c, i.e. in the returns to scale of the revenue function, as was pointed out in the previous subsec-

[40]The first 120 periods of the simulation are dropped to avoid initialization issues with the distribution of capital across firms.

[41]Recall from section 1.2.2 that profits, revenues and labor input all have the same functional form in (A, K) space and only differ by a constant shift parameter. Therefore, the dynamics of these three variables expressed in percentage deviations from their initial value will be identical. Hence, only the dynamics of aggregate revenues are shown. Note that the dynamics reported here are for gross revenues, i.e. adjustment costs are not subtracted from revenues.

[42]Remember that period zero is the first period of the uncertainty shock so that only the expectational effect is present but not yet the distributional effect.

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tion. Finally, it is worth to look at the magnitudes of the investment changes that are induced by the uncertainty shock. The initial investment drops in period zero are mostly between 50 and 75 %, while the investment overshoots in period five are in the range of 140 to 440 %. These investment changes are very large, showing that variations in uncertainty can be of importance in models with fixed costs and irreversibilities.

The investment dynamics for the Bloom 2 adjustment cost specification are somewhat different to the ones described in the previous paragraph, due to the presence of higher quadratic adjustment costs. These quadratic adjustment costs make large adjustments of the capital stock undesirable which is evident from the fact that the drops and overshoots of investment are much smaller than for the other two adjustment cost specifications. In period zero, aggregate investment drops by around 20 to 30 % which indicates a negative expectational effect of the uncertainty shock. The overshoot of investment in subsequent periods is in the range of 10 to 90 % and again increasing in the value of c.

For the impulse responses of aggregate revenues it is more useful to group the analysis by the type of uncertainty shock rather than by the adjustment cost specification. Starting with the case of pure TFP shocks, it is straightforward to see from figure 1.5 that none of the adjustment cost models leads to a drop in aggregate revenues after an uncertainty shock. Aggregate revenues actually experience a considerable boom of around 12 to 24 % by the end of period six depending on the adjustment cost specification. In order to better understand what is driving this result it is useful to recall that the revenue function is given by $R(A_{i,t}, K_{i,t}) = \chi A_{i,t}^{c/(1-b)} K_{i,t}^{a/(1-b)}$. When we consider pure TFP shocks, i.e. when $c = 1$, more dispersion in A will increase aggregate revenues due to convexity. In period one this direct positive effect of more dispersion dominates or at least offsets the negative effect on aggregate revenues due to a lower capital stock that is induced through the investment drop in period zero. In subsequent periods this direct positive distributional effect increases in strength due to the widening dispersion in TFP as the uncertainty shock persists. In addition, the positive distributional effect causes aggregate capital to increase which reinforces the rise in aggregate revenues even further. Thus, even though uncertainty shocks cause a one period drop in aggregate investment for pure TFP shocks, they lead to considerable booms in aggregate revenues, profits and labor.[43]

Moving on to the case of TFP shocks in the presence of market power, we can see that an uncertainty shock leads to an initial drop in aggregate revenues of 0.5 to 1 % depending on the presence of moderate quadratic adjustment costs or not. Revenues

[43] Recall that for pure TFP shocks the revenue function is equivalent to the production function so that output also experiences a boom.

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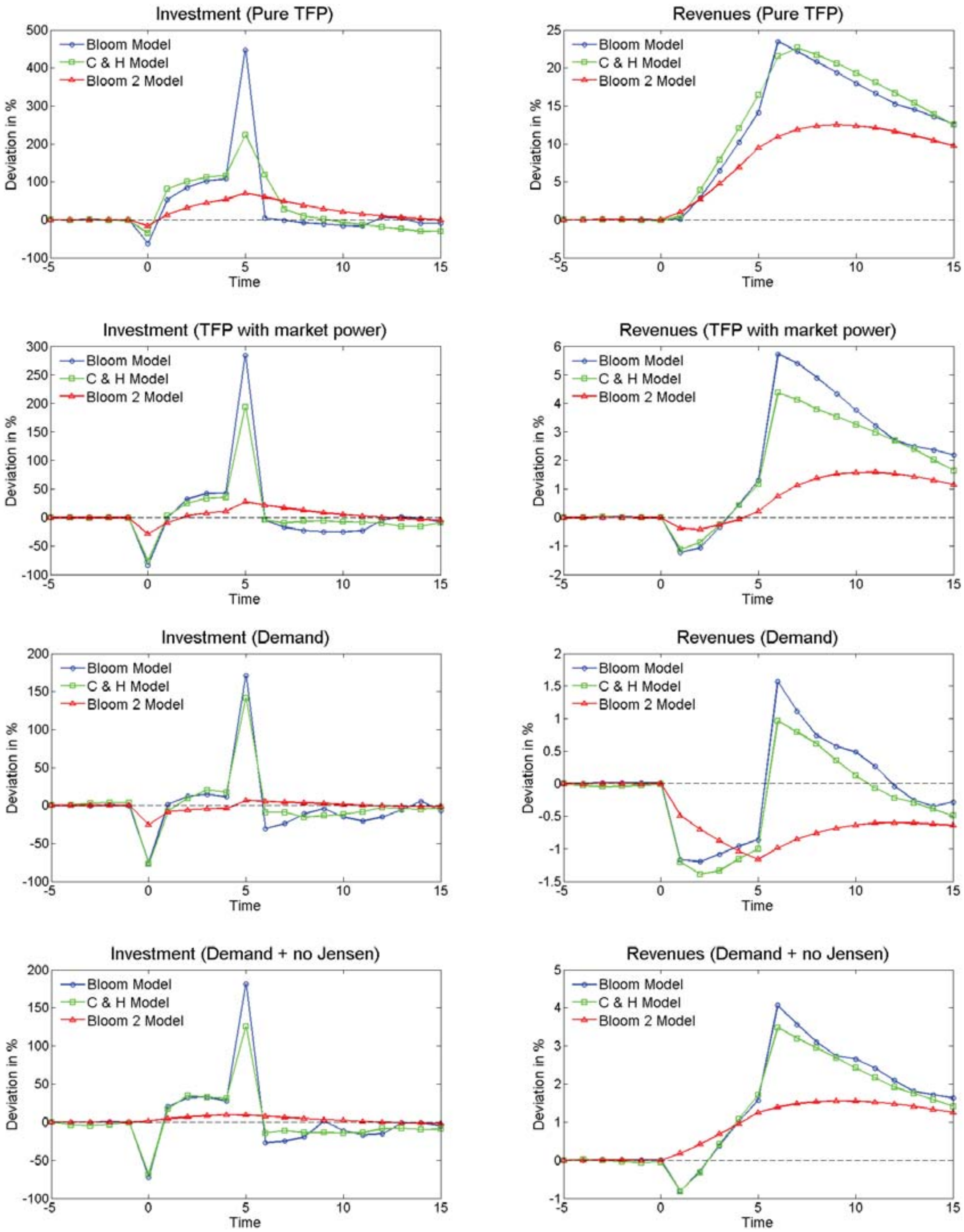
Figure 1.5: The impact of an uncertainty shock with adjustment costs



Notes: All the simulation results except for the lower two panels are for models where a Jensen correction is applied to the mean of the idiosyncratic profitability process. Aggregate labor demand and profits display the same dynamics as aggregate revenues and are therefore not shown separately. All values are specified in percentage deviations from the initial level.

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then rebound quickly and reach their intial level after around four quarters. The subsequent overshoot is much larger than the initial drop, reaching between 1.5 and 6 % from quarter six onwards, when uncertainty about the future is low again and dispersion in profitability is at its maximum. For the case at hand recall that $c = 1 - b$ which implies that revenues are linear in profitability and more dispersion should not have any effect on aggregates. Hence, the dynamics of aggregate revenues discussed are mainly driven by the dynamics of the aggregate capital stock which initially drops and subsequently rebounds and overshoots considerably. Note also that the case at hand can be seen as a maximum possible drop for TFP shocks with market power because under the assumptions of constant RTS in production with capital and labor shares of 1/3 and 2/3 and a demand elasticity of only three, the implied value of c is already higher than $1 - b$.

Finally, for the case of uncertainty shocks to demand, i.e. when $c = 1 - a - b$, a prolonged drop in aggregate revenues of around 1 to 1.5 % is induced that lasts for five consecutive quarters until uncertainty is low again. For the C&H and the Bloom adjustment cost specifications revenues subsequently overshoot the initial level by a similar magnitude and then gradually reverst back. In contrast, for the Bloom 2 adjustment cost specification the rebound is much lower so that aggregate revenues are still around 0.7 % below the initial level 15 quarters after the uncertainty shock hit the system. This is due to the fact that the mean of idiosyncratic profitability drops by around 0.75 % after the uncertainty shock and stays there for an extended period of time, as was shown in the top left panel of figure 1.2.[44] For completeness, the lowest two panels in figure 1.5 plot the impact of an uncertainty shock to demand when no Jensen correction is applied to the mean of idiosyncratic profitability. Due to the fact that in this case average profitability now increases after an uncertainty shock, the drops in aggregate revenues are lower and of shorter duration, while the overshoots are much larger and more persistent.

In summary, it has been shown that while uncertainty shocks to demand cause the type of drop, rebound and overshoot dynamics as emphasized in Bloom (2009), uncertainty shocks to TFP lead to considerable and prolonged booms. The reason for this is that the revenue function is likely to have increasing RTS in (A, K, L) space when we consider TFP shocks. This in turn leads to the fact that higher ex-post cross-sectional dispersion of TFP is a time of opportunity. In other words, the distributional effect of uncertainty shocks is highly positive.

[44]This is also the reason for the undershooting of aggregate revenues for the C&H and Bloom specifications towards the final quarters of the simulation.

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1.4.4 Analyzing Expectational and Distributional Effects

In order to further analyze the factors driving the results presented above, it is useful to consider the magnitudes of the expectational and distributional effects of uncertainty shocks for each of the models. To this end, two similar simulation exercises as in the previous subsection are performed. The difference is that once only the expectations of firms are affected by the uncertainty shock but not the actual distribution of profitability, while the other time expectations are not affected by the uncertainty shock but the distribution of profitability across firms changes. The first simulation exercise identifies the expectational effect given in equation (1.15) while the second simulation identifies the distributional effect given in equation (1.16).

Before moving to the results it is useful to visualize the driving forces at work in each of the simulations. The top two panels of figure 1.6 show how expectations of firms and the dispersion of idiosyncratic profitability behave in the case of the two simulations. From the upper left panel it can be seen that in the simulation of expectational effects firms start to expect higher dispersion in future profitability shocks when the uncertainty shock hits in period zero. In contrast, firms always expect low dispersion in the simulation of distributional effects. In the upper right panel of the figure the corresponding dynamics of the standard deviation of idiosyncratic profitability are displayed. While the dispersion of profitability does not change in the simulation of expectational effects, dispersion increases from period one to five and gradually falls back in the simulation of distributional effects. The standard simulations of an uncertainty shock from the previous subsection would feature the dynamics of expectations from the simulation of expectational effects and the dynamics of dispersion from the simulation of distributional effects.

The remaining panels in figure 1.6 display the expectational and distributional effects of aggregate revenues for each of the models under consideration. The main feature to note is that while the value of c only has minor quantitative implications for the expectational effect of uncertainty shocks, the magnitude of the distributional effect changes considerably between uncertainty shocks to demand and TFP. To be precise, the distributional effect is an order of magnitude higher for pure TFP shocks than for demand shocks.

Quantitatively, the expectational effect is negative for all models under consideration and the maximum drops in aggregate revenues are between 0.5 and 4.5 %. Moreover, the difference in the maximum drop of aggregate revenues between demand and TFP shocks is only around 1 %, indicating that the expectational effect does not change much with the value of c. In contrast, the magnitude of the positive distributional effect is 12 to 26 % for pure TFP shocks, while it is - 0.5 to 3 % for

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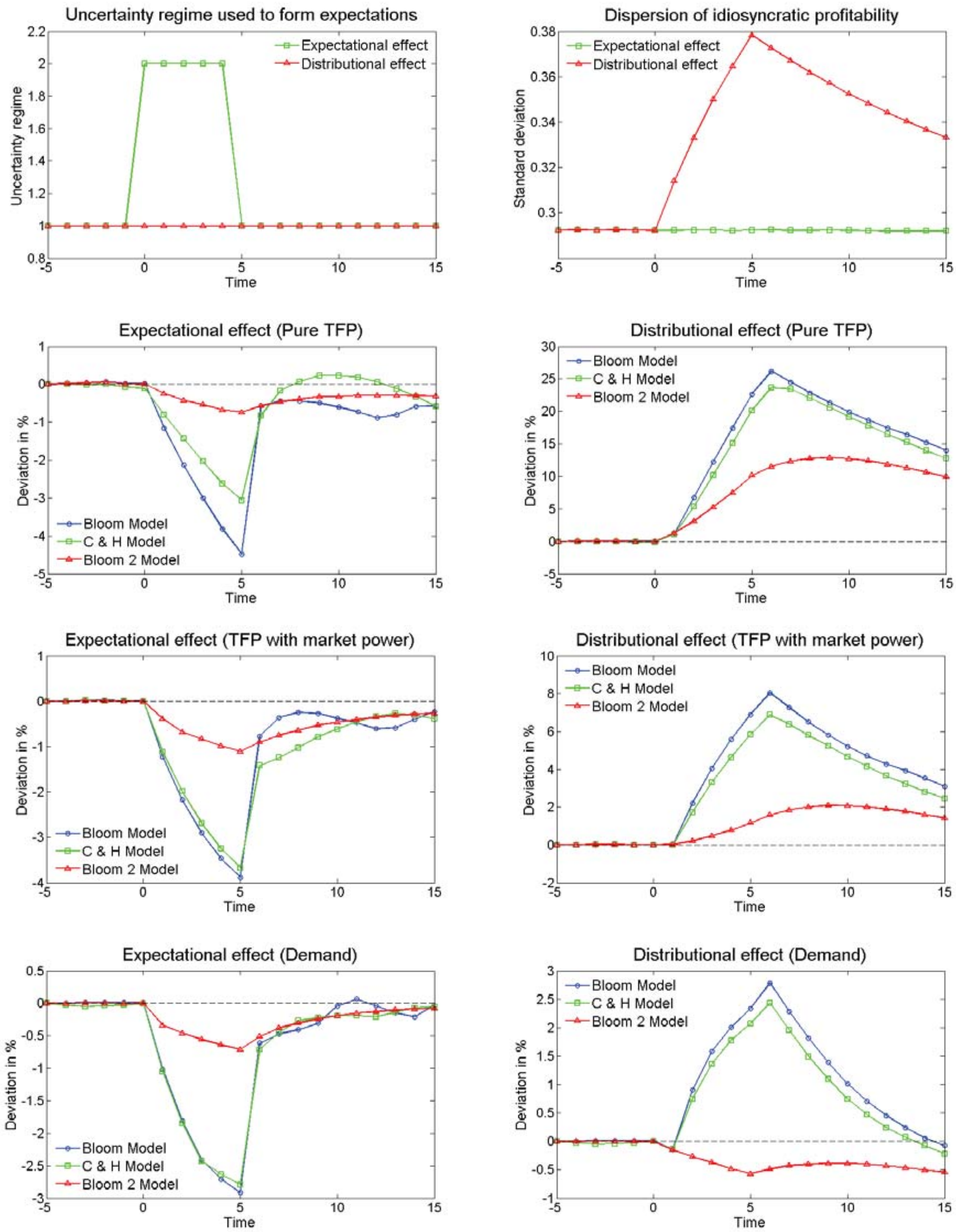
Figure 1.6: The expectational and distributional effect for aggregate revenues



Notes: All the simulation results are for models where a Jensen correction is applied to the mean of the idiosyncratic profitability process. The values are specified in percentage deviations from the initial level.

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demand shocks. Thus, the main driver behind the results found in the previous subsection is that the distributional effect of uncertainty shocks substantially increases with the RTS in the revenue function.

1.5 Conclusion

This chapter has readdressed the question of whether temporarily higher uncertainty can cause recessions. To answer the research question a partial equilibrium model with heterogeneous firms, various forms of capital adjustment costs and time-varying uncertainty was built and simulated. The main difference compared to the existing literature was that the model set-up allowed for the separate analysis of uncertainty shocks to demand and TFP.

The main finding coming out of the analysis is that while uncertainty shocks to demand lead to the drop, rebound and overshoot dynamics reminiscent of recessions, uncertainty shocks to TFP are likely to lead to considerable and prolonged booms. The main reason for these differing dynamics is that the positive distributional effect of uncertainty shocks is much larger for TFP than for demand. The intuition for this is that the revenue function is likely to have increasing RTS for TFP shocks while for demand shocks the RTS can be constant at best. Therefore, more ex-post cross-sectional dispersion in TFP is a time of opportunity for firms.

In the case of uncertainty shocks to TFP this positive distributional effect more than compensates for the negative expectational effect of uncertainty shocks that arises from real-options effects in the presence of non-convex adjustment costs, thus leading to large and persistent economic booms. In contrast, for uncertainty shocks to demand the negative expectational effect dominates until the point where uncertainty turns low again but cross-sectional dispersion is still large. Hence, uncertainty shocks to demand lead to business cycle like dynamics that feature a drop, rebound and overshoot as emphasized by Bloom (2009).

The results in this chapter where derived under the assumptions of risk-neutral firms and constant factor prices in order to isolate the effects of uncertainty shocks that work through the presence of capital adjustment costs. Even though the papers by Bloom et al. (2010) and Bachmann and Bayer (2011) consider general equilibrium effects, such as a stochastic discount factor and varying wages, it is not clear how these factors move after an uncertainty shock and affect the response due to the presence of adjustment costs. In future research it is therefore necessary to determine the empirical responses of factor prices to uncertainty shocks and how these factor prices respond in general equilibrium models with uncertainty shocks.

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Appendix A: The Demand Function

Let demand for the product of firm i at time t be given by the following constant elasticity demand function:

$$q_{i,t} = B_{i,t} \cdot p_{i,t}^{-\varepsilon} \tag{1.23}$$

Here q denotes output, p is the price, B is a potentially time-varying demand shifter, and $-\varepsilon$ is the constant price elasticity of demand. From now on firm and time subscripts will be omitted unless deemed necessary. To see that the price elasticity is given by $-\varepsilon$, recall that the price elasticity of demand is defined as:

$$E_{qp} = \frac{dq}{q} \Big/ \frac{dp}{p} = \frac{dq}{dp}\frac{p}{q} \tag{1.24}$$

Now, taking the derivative of equation (1.23) with respect to the price yields:

$$\frac{dq}{dp} = -\varepsilon B p^{-\varepsilon-1} = -\varepsilon \frac{q}{p} \tag{1.25}$$

Plugging this into the definition of the elasticity given in equation (1.24) yields:

$$E_{qp} = -\varepsilon \frac{q}{p}\frac{p}{q} = -\varepsilon \tag{1.26}$$

This constant elasticity demand function implies the following inverse demand function:

$$p_{i,t} = B_{i,t}^{\frac{1}{\varepsilon}} \cdot q_{i,t}^{-\frac{1}{\varepsilon}} \tag{1.27}$$

From this inverse demand function it is easy to see that as $\varepsilon \to \infty$, we approach the case of perfect competition as the price becomes completely unresponsive to the level of output supplied by the firm. In contrast, as $\varepsilon \to 0$, the price responds heavily to small changes in output. Thus, the higher the elasticity of demand ε, the lower the degree of market power. For this type of demand function it is important to notice that the profit maximization problem of a firm is only well defined if $\varepsilon > 1$. To see this, define revenues as $R = pq$. Remembering the fact that the price is a function of output, the change in revenues with respect to output is given by:

$$\frac{dR}{dq} = \frac{dp}{dq}q + p \tag{1.28}$$

Differentiating equation (1.27) with respect to ouput yields:

$$\frac{dp}{dq} = -\frac{1}{\varepsilon}\frac{p}{q} \tag{1.29}$$

Using this in equation (1.28) gives us:

$$\frac{dR}{dq} = -\frac{1}{\varepsilon}\frac{p}{q}q + p = p\left(1 - \frac{1}{\varepsilon}\right) = B^{\frac{1}{\varepsilon}} q^{-\frac{1}{\varepsilon}} \left(1 - \frac{1}{\varepsilon}\right) \tag{1.30}$$

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Clearly, whenever $0 < \varepsilon < 1$ total revenues will be globally decreasing in output. Assuming that costs are increasing in output, this implies that the firm would like to produce as little as possible in order to maximize profits. Therefore, we require that $\varepsilon > 1$ for the problem of the firm to be well defined and nondegenerate.

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Appendix B: Derivation of the Revenue Function

In the following a detailed derivation of the revenue function is provided in order to motivate the differences in functional forms that are implied by demand and supply shocks. It is assumed that firms differ in their productivity and potentially in the demand that they face. Let the production function of firm i at time t be given by:

$$Y(\tilde{A}_{i,t}, K_{i,t}, L_{i,t}) = \tilde{A}_{i,t} K_{i,t}^{\nu} L_{i,t}^{\omega} \tag{1.31}$$

where $\tilde{A}$ is total factor productivity, K is the capital stock, L is labor input, and ν and ω are parameters that satisfy $\nu,\ \omega \in (0,1)$ and $\nu + \omega \in (0,1]$. This specification allows for both constant and decreasing returns to scale. For example, whenever $\nu + \omega = 1$ we have constant returns to scale and the production function is given by the familiar Cobb-Douglas form. Demand for the output of firm i is given by the following constant elasticity demand function:

$$q_{i,t} = B_{i,t} p_{i,t}^{-\varepsilon} \tag{1.32}$$

Here q denotes output demanded, p is the price, B is a potentially time-varying demand shifter, and $-\varepsilon$ is the constant price elasticity of demand. It is required that $\varepsilon > 1$ for the problem of the firm to be well defined.[45] Given this production and demand function, total revenue of each firm can be expressed as:

$$\tilde{S}(B_{i,t}, \tilde{A}_{i,t}, K_{i,t}, L_{i,t}) = p_{i,t} Y_{i,t} = B_{i,t}^{\frac{1}{\varepsilon}} Y_{i,t}^{-\frac{1}{\varepsilon}} Y_{i,t} = B_{i,t}^{\frac{1}{\varepsilon}} Y(\tilde{A}_{i,t}, K_{i,t}, L_{i,t})^{\frac{\varepsilon-1}{\varepsilon}} \tag{1.33}$$

Now let us define the parameters $a = \nu\frac{\varepsilon-1}{\varepsilon}$ and $b = \omega\frac{\varepsilon-1}{\varepsilon}$. Moreover, we can combine the effects of total factor productivity and demand into one auxiliary variable. This is achieved by defining a shock to revenues as $A_{i,t}^{c} = B_{i,t}^{1/\varepsilon} \tilde{A}_{i,t}^{(\varepsilon-1)/\varepsilon}$. With these transformations we can represent the revenue function of the firm as:[46]

$$S(A_{i,t}, K_{i,t}, L_{i,t}) = A_{i,t}^{c} K_{i,t}^{a} L_{i,t}^{b} \tag{1.34}$$

[45] See Appendix A for a discussion of the properties of this kind of demand function.

[46] The derivation of this revenue function is closely related to the one in Bloom (2009) with the difference that in his model he assumes $c = 1 - a - b$, while the specification in this chapter allows for flexible values of c.

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Appendix C: Expectations with Log-Normal Profitability

To isolate the effects of time-varying uncertainty, we want an increase in uncertainty to correspond to a mean preserving spread of the relevant variable in question. Therefore, as uncertainty is equivalent to the variance of shocks in our model, we want that a change in the variance of shocks has no effect on the expected value of profitability. If the AR(1) process for idiosyncratic profitability is specified in levels, a change in the variance of shocks naturally corresponds to a mean preserving spread. However, if the AR(1) process is specified in logs, then a change in the variance of shocks will also have an effect on the expected value of the variable in levels. In other words, when we specify the AR(1) process for idiosyncratic profitability in logs, a change in the variance of shocks does not correspond to a mean preserving spread. To see this analytically, take the AR(1) process in logs for idiosyncratic profitability from equation (1.7) and transform it into levels:

$$\psi_{i,t} = \mu_\psi + \rho_\psi \psi_{i,t-1} + \upsilon_{i,t} \tag{1.35}$$

$$\Leftrightarrow e^{\psi_{i,t}} = e^{(\mu_\psi + \rho_\psi \psi_{i,t-1} + \upsilon_{i,t})} \tag{1.36}$$

$$\Leftrightarrow \Psi_{i,t} = e^{\mu_\psi} \Psi_{i,t-1}^{\rho_\psi} e^{\upsilon_{i,t}} \tag{1.37}$$

If we now take the expectation of the variable in levels and apply the fact that for a normally distributed variable, $\upsilon_{i,t} \sim \mathcal{N}(\mu_{\upsilon,t-1}, \sigma^2_{\upsilon,t-1})$, we have $E[e^{-a\upsilon_{i,t}}] = e^{-a\mu_{\upsilon,t-1} + a^2\sigma^2_{\upsilon,t-1}/2}$ we get the following result:

$$E[\Psi_{i,t}] = e^{\mu_\psi} \Psi_{i,t-1}^{\rho_\psi} E[e^{\upsilon_{i,t}}] \tag{1.38}$$

$$\Leftrightarrow E[\Psi_{i,t}] = e^{\mu_\psi} \Psi_{i,t-1}^{\rho_\psi} e^{\mu_{\upsilon,t-1} + \sigma^2_{\upsilon,t-1}/2} \tag{1.39}$$

It is obvious from this expression that the expected value of idiosyncratic profitability in levels is increasing in the variance of shocks unless the mean of the shocks changes in an offsetting manner with the variance. A natural way to avoid that the expected value of profitability rises in response to an increase in the shock variance is therefore to specify the mean of the shocks as $\mu_{\upsilon,t-1} = -\sigma^2_{\upsilon,t-1}/2$. This adjustment is labeled as a Jensen correction throughout the chapter, as the increase in the expected value in levels is a result of Jensen's Inequality.

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Appendix D: The Capital Policy Function without Adjustment Costs

In the absence of capital adjustment costs and irreversibilities, the firm's decision problem can be represented by the following Bellman Equation which is a special case of equation (1.11):

$$V(A,K,\sigma_v) = \max_{K'} \Pi(A,K) - p(K' - K(1-\delta)) + \beta E_{A',\sigma_v'|A,\sigma_v}\left[V(A',K',\sigma_v')\right] \tag{1.40}$$

Assuming that $V(A,K,\sigma_v)$ is differentiable, the first-order condition with respect to next period capital is:

$$p = \beta E_{A',\sigma_v'|A,\sigma_v}\left[V_{K'}(A',K',\sigma_v')\right] \tag{1.41}$$

By the envelope condition we have that the marginal value of capital is given by:

$$V_K(A,K,\sigma_v) = \Pi_K(A,K) + (1-\delta)p \tag{1.42}$$

Taking this condition forward one period and using it in the first-order condition yields:

$$p = \beta E_{A',\sigma_v'|A,\sigma_v}\left[\Pi_{K'}(A',K') + (1-\delta)p\right] \tag{1.43}$$

This equation states that an optimal capital choice equates the marginal cost of investment, which is given by p, to the discounted expected marginal gain, which is given by marginal profits plus the market value of the depreciated increment of investment. Using the specific functional forms assumed in the chapter, we can rewrite this as:

$$p = \beta E_{A'|A,\sigma_v}\left[\phi\frac{a}{1-b}A'^{\frac{c}{1-b}}K'^{\frac{a}{1-b}-1} + (1-\delta)p\right] \tag{1.44}$$

Solving for K', we arrive at the following explicit capital policy function:

$$K'(A,\sigma_v) = \varphi E\left[A'^{\frac{c}{1-b}} \mid A,\sigma_v\right]^{\frac{1-b}{1-a-b}} \tag{1.45}$$

where $\varphi = [(a\beta\phi)/([1-b][p-p\beta(1-\delta)])]^{\frac{1-b}{1-a-b}}$ is a constant parameter. Given the stochastic processes assumed for profitability we can derive an analytical expression for the expectation in equation (1.45). First, given the multiplicative specification of profitability and the AR(1) structure in logs we can derive the following expression:

$$\begin{aligned} K' &= \varphi E\left[(Z'\Psi')^{\frac{c}{1-b}} \mid Z,\Psi,\sigma_v\right]^{\frac{1-b}{1-a-b}} && (1.46)\\ \Leftrightarrow K' &= \varphi E\left[(e^{\mu_z}e^{\eta'}Z^{\rho_z}\cdot e^{\mu_\psi}e^{\upsilon'}\Psi^{\rho_\psi})^{\frac{c}{1-b}} \mid \sigma_v\right]^{\frac{1-b}{1-a-b}} && (1.47)\\ \Leftrightarrow K' &= \varphi(e^{\mu_z}Z^{\rho_z}e^{\mu_\psi}\Psi^{\rho_\psi})^{\frac{c}{1-a-b}}E\left[(e^{\eta'}e^{\upsilon'})^{\frac{c}{1-b}} \mid \sigma_v\right]^{\frac{1-b}{1-a-b}} && (1.48) \end{aligned}$$

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Making use of the fact that aggregate and idiosyncratic profitability shocks are independent of each other, we can separate the two terms within the expectations operator:

$$K' = \varphi(e^{\mu_z} Z^{\rho_z} e^{\mu_\psi} \Psi^{\rho_\psi})^{\frac{c}{1-a-b}} E\left[e^{\eta' \frac{c}{1-b}}\right]^{\frac{1-b}{1-a-b}} E\left[e^{\upsilon' \frac{c}{1-b}} \mid \sigma_\upsilon\right]^{\frac{1-b}{1-a-b}} \tag{1.49}$$

Furthermore, we can use the fact that for a normally distributed variable, $\epsilon \sim \mathcal{N}(\mu, \sigma^2)$, we have $E[e^{-a\epsilon}] = e^{-a\mu + a^2\sigma^2/2}$:

$$K' = \varphi(e^{\mu_z} Z^{\rho_z} e^{\mu_\psi} \Psi^{\rho_\psi})^{\frac{c}{1-a-b}} \left[e^{\mu_\eta \frac{c}{1-b} + \frac{\sigma_\eta^2}{2}\left(\frac{c}{1-b}\right)^2} e^{\mu_\upsilon \frac{c}{1-b} + \frac{\sigma_\upsilon^2}{2}\left(\frac{c}{1-b}\right)^2}\right]^{\frac{1-b}{1-a-b}} \tag{1.50}$$

$$\Leftrightarrow K' = \varphi\left(e^{\mu_z} Z^{\rho_z} e^{\mu_\psi} \Psi^{\rho_\psi} e^{\mu_\eta} e^{\frac{\sigma_\eta^2}{2}\frac{c}{1-b}} e^{\mu_\upsilon} e^{\frac{\sigma_\upsilon^2}{2}\frac{c}{1-b}}\right)^{\frac{c}{1-a-b}} \tag{1.51}$$

From this explicit capital policy function for the case without adjustment frictions we can learn some useful insights. First, unless the mean of idiosyncratic shocks is changing at the same time as the variance, the desired capital stock will be increasing in the variance of idiosyncratic shocks, independent of the value of c. This is simply a result of the fact that the expected value of profitability is increasing in the variance of idiosyncratic shocks given the assumed AR(1) process in logs.[47] We can adjust for this effect by letting $\mu_{\upsilon,t-1} = -\sigma_{\upsilon,t-1}^2/2$, so that the expected value of profitability does not change with the variance of shocks. With this structure equation (1.51) can be rewritten as:

$$K' = \varphi\left(e^{\mu_z} Z^{\rho_z} e^{\mu_\psi} \Psi^{\rho_\psi} e^{\mu_\eta} e^{\frac{\sigma_\eta^2}{2}\frac{c}{1-b}}\right)^{\frac{c}{1-a-b}} e^{\frac{\sigma_\upsilon^2}{2}\frac{c^2 - c(1-b)}{(1-b)(1-a-b)}} \tag{1.52}$$

[47] A derivation of this result can be found in Appendix C.

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Appendix E: Dynamics of the Profitability Distribution

The distribution of idiosyncratic profitability across firms is log-normal, given the AR(1) process in logs assumed in equation (1.7). Applying the expectations and variance operator to this equation, the dynamics of the cross-sectional mean and variance of idiosyncratic profitability in logs can be expressed as follows:

$$\begin{aligned} E[\psi_{i,t}] &= E[\mu_\psi + \rho_\psi \psi_{i,t-1} + \upsilon_{i,t}] && (1.53)\\ \Leftrightarrow\ E[\psi_{i,t}] &= \mu_\psi + \rho_\psi E[\psi_{i,t-1}] + E[\upsilon_{i,t}] && (1.54)\\ \Leftrightarrow\ E[\psi_{i,t}] &= \mu_\psi + \rho_\psi E[\psi_{i,t-1}] + \mu_{\upsilon,t-1} && (1.55)\end{aligned}$$

$$\begin{aligned} V[\psi_{i,t}] &= V[\mu_\psi + \rho_\psi \psi_{i,t-1} + \upsilon_{i,t}] && (1.56)\\ \Leftrightarrow\ V[\psi_{i,t}] &= \rho_\psi^2 V[\psi_{i,t-1}] + V[\upsilon_{i,t}] && (1.57)\\ \Leftrightarrow\ V[\psi_{i,t}] &= \rho_\psi^2 V[\psi_{i,t-1}] + \sigma_{\upsilon,t-1}^2 && (1.58)\end{aligned}$$

If we assume that the mean and variance of idiosyncratic shocks have been in the low state for the infinite past up to and including period t, these two moments become:

$$\begin{aligned} E[\psi_{i,t}] &= \frac{\mu_\psi + \mu_\upsilon^L}{1-\rho_\psi} && (1.59)\\ V[\psi_{i,t}] &= \frac{(\sigma_\upsilon^L)^2}{1-\rho_\psi^2} && (1.60)\end{aligned}$$

Now let us define the variable $\Delta\mu_{\upsilon,t} = \mu_{\upsilon,t} - \mu_\upsilon^L$, which captures the difference between the mean of idiosyncratic shocks under the current regime and the low regime. Analogous to this let us define the variable $\Delta\sigma_{\upsilon,t}^2 = \sigma_{\upsilon,t}^2 - (\sigma_\upsilon^L)^2$. Starting the evolution of the mean of log idiosyncratic profitability from the value in equation (1.59) we can express the dynamics as follows:

$$\begin{aligned} E[\psi_{i,t+1}] &= \frac{\mu_\psi + \mu_\upsilon^L}{1-\rho_\psi} + \Delta\mu_{\upsilon,t} && (1.61)\\ E[\psi_{i,t+2}] &= \frac{\mu_\psi + \mu_\upsilon^L}{1-\rho_\psi} + \Delta\mu_{\upsilon,t+1} + \rho_\psi\Delta\mu_{\upsilon,t} && (1.62)\\ E[\psi_{i,t+3}] &= \frac{\mu_\psi + \mu_\upsilon^L}{1-\rho_\psi} + \Delta\mu_{\upsilon,t+2} + \rho_\psi\Delta\mu_{\upsilon,t+1} + \rho_\psi^2\Delta\mu_{\upsilon,t} && (1.63)\\ &\vdots \end{aligned}$$

Similarly, starting from the variance in equation (1.60), the evolution of the

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variance of log idiosyncratic profitability can be expressed as:

$$
\begin{aligned}
V[\psi_{i,t+1}] &= \frac{(\sigma_v^L)^2}{1-\rho_\psi^2} + \Delta\sigma_{v,t}^2 && (1.64)\\
V[\psi_{i,t+2}] &= \frac{(\sigma_v^L)^2}{1-\rho_\psi^2} + \Delta\sigma_{v,t+1}^2 + \rho_\psi^2\Delta\sigma_{v,t}^2 && (1.65)\\
V[\psi_{i,t+3}] &= \frac{(\sigma_v^L)^2}{1-\rho_\psi^2} + \Delta\sigma_{v,t+2}^2 + \rho_\psi^2\Delta\sigma_{v,t+1}^2 + \rho_\psi^4\Delta\sigma_{v,t}^2 && (1.66)\\
&\vdots
\end{aligned}
$$

From these recursions, it is easy to see that we can express the mean and variance of log idiosyncratic profitability as the sum of the value that would prevail under the low regime and the accumulated effect due to occasional switches to the high regime:

$$
\begin{aligned}
E[\psi_{i,t}] &= \frac{\mu_\psi + \mu_v^L}{1-\rho_\psi} + \sum_{j=1}^{\infty}\rho_\psi^{j-1}\Delta\mu_{v,t-j} && (1.67)\\
V[\psi_{i,t}] &= \frac{(\sigma_v^L)^2}{1-\rho_\psi^2} + \sum_{j=1}^{\infty}\rho_\psi^{2(j-1)}\Delta\sigma_{v,t-j}^2 && (1.68)
\end{aligned}
$$

In the end however, we are interested in the evolution of the distribution of idiosyncratic profitability in levels. Given that log idiosyncratic profitability is normally distributed, the mean and variance of idiosyncratic profitability in levels can be derived along the following steps:

$$
\begin{aligned}
E[\Psi_{i,t}] &= E[e^{log(\Psi_{i,t})}] = E[e^{\psi_{i,t}}] && (1.69)\\
\Leftrightarrow E[\Psi_{i,t}] &= e^{E[\psi_{i,t}]+V[\psi_{i,t}]/2} && (1.70)
\end{aligned}
$$

$$
\begin{aligned}
V[\Psi_{i,t}] &= E[(\Psi_{i,t} - E[\Psi_{i,t}])^2] && (1.71)\\
\Leftrightarrow V[\Psi_{i,t}] &= E[\Psi_{i,t}^2] - 2E[\Psi_{i,t}]^2 + E[\Psi_{i,t}]^2 && (1.72)\\
\Leftrightarrow V[\Psi_{i,t}] &= E[\Psi_{i,t}^2] - E[\Psi_{i,t}]^2 && (1.73)\\
\Leftrightarrow V[\Psi_{i,t}] &= E[e^{2log(\Psi_{i,t})}] - E[e^{log(\Psi_{i,t})}]^2 && (1.74)\\
\Leftrightarrow V[\Psi_{i,t}] &= e^{2E[\psi_{i,t}]+2V[\psi_{i,t}]} - (e^{E[\psi_{i,t}]+V[\psi_{i,t}]/2})^2 && (1.75)\\
\Leftrightarrow V[\Psi_{i,t}] &= e^{2E[\psi_{i,t}]+2V[\psi_{i,t}]} - e^{2E[\psi_{i,t}]+V[\psi_{i,t}]} && (1.76)
\end{aligned}
$$

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Appendix F: Numerical Solution Technique and Accuracy

The model described in section 1.2 does not have a closed-form solution once we allow for the various forms of adjustment costs and partial irreversibilities. In the remaining parts of the chapter, the model is therefore solved numerically by discrete value function iteration in order to study the properties of investment policy functions and the models' implications for the effect of uncertainty shocks. For this procedure to be feasible, the profitability process needs to be approximated by a discrete markov chain. This approximation is done using the method proposed by Tauchen (1986), adapted to a markov-switching process. The grids for the value function iteration are chosen with 21 idiosyncratic profitability points, 10 aggregate profitability points, 2 uncertainty states, and between 350 and 1000 capital grid points depending on the model.[48] At this point it is useful to asses the accuracy of the numerical approximation that is employed. For this purpose two exercises are performed. First, the dynamics of the discretized process for idiosyncratic profitability are compared to the true dynamics of the AR(1) process in logs. Second, the dynamics of the approximated model without adjustment costs are compared to the analytical results derived above.

As mentioned above the profitability process is approximated by a discrete markov chain with 21 grid points. The question is of course whether this discretized process resembles the original AR(1) process in logs in important dimensions. To answer this question, the dynamics of the cross-sectional mean and standard deviation after an uncertainty shock are compared between the discretized process for idiosyncratic profitability and it's analytical counterpart. To this end a sample of 250,000 units is simulated for 25 periods. The initial distribution of units is set to the unconditional distribution under the low uncertainty regime. It is then assumed that in period zero an uncertainty shock hits the system lasting for 5 periods. For all subsequent periods uncertainty is set to the low regime again. Figure 1.7 displays the results of this exercise for the cases with and without a Jensen correction applied to the mean of shocks.

Regarding the accuracy of the discretized process for idiosyncratic profitability it can be observed that the dynamics of the mean and standard deviation are replicated fairly accurately in both cases. The standard deviation of idiosyncratic profitability increases considerably for five consecutive periods, and then falls back gradually as uncertainty becomes low again. The decrease in the mean with the

[48] It was checked that the results are not sensitive to an increase in the number of grid points.

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Figure 1.7: Moments of the idiosyncratic profitability distribution

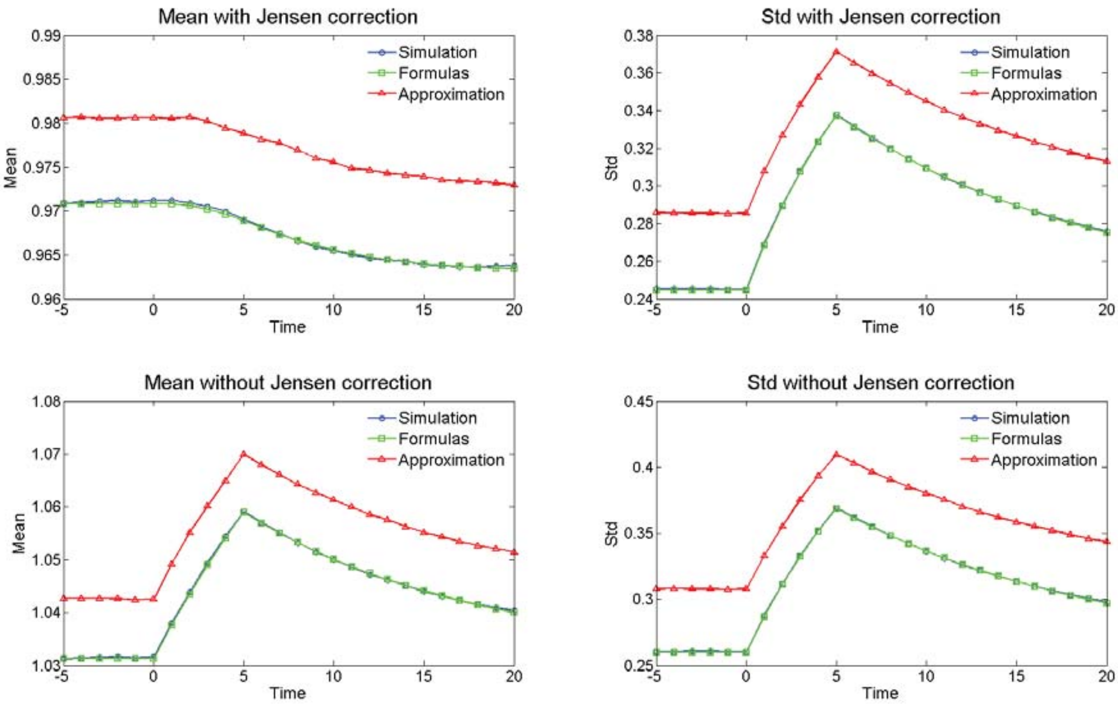
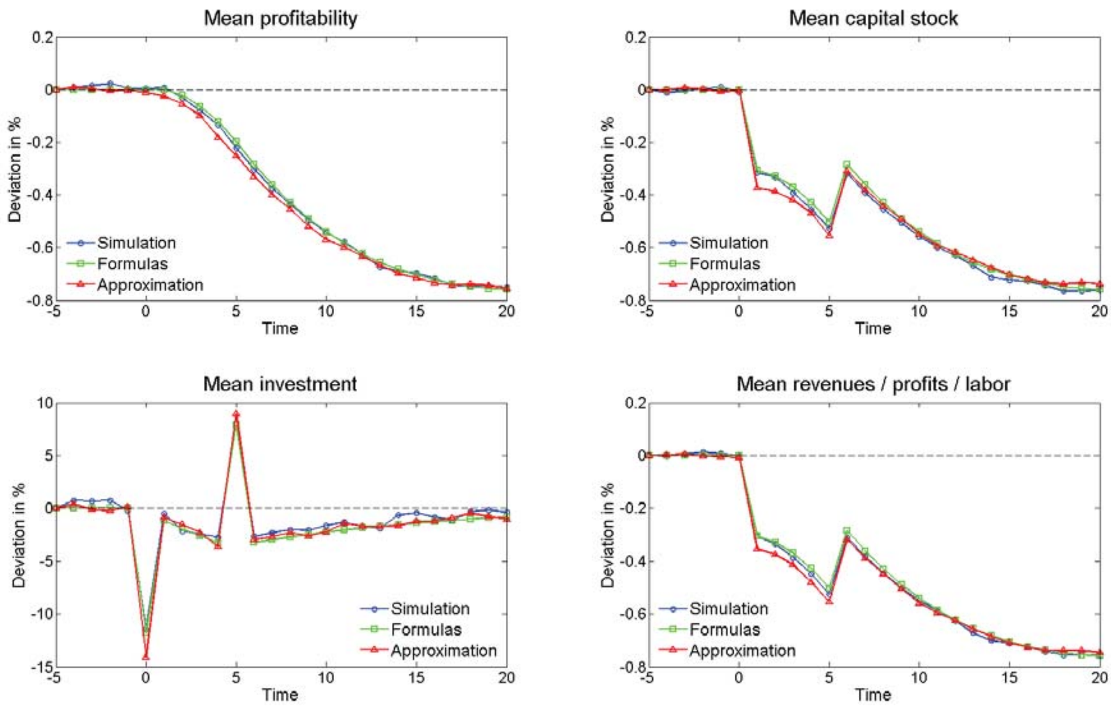


Figure 1.8: Accuracy of the approximation method without adjustment costs



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Jensen correction and the increase in the mean without the Jensen correction are also captured fairly accurately by the discretized process. However, both the mean and standard deviation are consistently higher than for the true process. In the case of the mean this difference is approximately 1 %, while for the standard deviation this difference is around 14 %. Nevertheless, the accuracy of the approximated profitability process is deemed sufficient as it replicates the dynamics of the first two moments after an uncertainty shock fairly well.

We can therefore turn to the accuracy of the aggregate dynamics produced by a discretized model without any adjustment costs. For this model we can use the analytical results developed in section 1.3 as a benchmark to compare to. The parameters are set to the values discussed in section 1.4.1 and shown in table 1.3. In addition, all adjustment costs are switched off and c is set to $1 - a - b$. Similar to the previous simulation exercise, a panel of 1,000,000 units is simulated for 25 periods. In period zero an uncertainty shock occurs for five consecutive periods and uncertainty is assumed to be low for all other periods before and after.

Figure 1.8 shows the results of this simulation. Looking at the means of idiosyncratic profitability, the capital stock, investment and revenues, profits and labor we see that an uncertainty shock leads to a prolonged drop in all variables. It is straightforward to see that the dynamics produced by the numerical approximation closely resemble the dynamics that are obtained from the analytical formulas or a simulation of the analytical model. The deviations that do occur are partly due to sampling and otherwise deemed sufficiently small. We can therefore safely move on to analyze the simulation results for more complicated models with adjustment costs.

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Appendix G: Existing Adjustment Cost Estimates

There are two main papers that estimate a rich set of capital adjustment costs for models similar to the one in section 1.2. In the following their results and differences are briefly outlined.

Cooper and Haltiwanger (2006) estimate capital adjustment costs using simulated method of moments (SMM), disregarding labor adjustment costs. They use annual data for around 7,000 large continuing manufacturing plants between 1972 and 1988 from the Longitudinal Research Database (LRD) and attempt to match the following four moments with their model: the fraction of investment bursts, the fraction of investment falls, the investment autocorrelation, and the correlation of investment with profitability.[49] Their model is specified at an annual frequency so there is no time aggregation and they estimate the profitability process and revenue function together with the adjustment costs. Their estimated adjustment cost parameters can be found in table 1.4. One caveat of their estimation is that even though they report that the inaction rate in the data is 8.1 %, they do not try to match this moment and a model with their estimated parameter values actually leads to an inaction rate of over 80 %.

Bloom (2009) jointly estimates capital and labor adjustment costs using SMM. The data he uses is a panel of 2,548 publicly traded U.S. firms from Compustat spanning the years 1981 to 2000. The firms are large (at least 500 employees and \$10m sales) and span all sectors of the economy. The model he uses is specified at a monthly frequency and at the plant level so that there is aggregation across time as well as across units in order to correspond to the data which is annual and at the firm level. The moments that he attempts to match with the model are the dynamic auto- and cross-correlations as well as the standard deviation and skewness of investment rates, employment growth rates and sales growth rates. The author does not estimate the revenue function and only estimates the variance of the profitability process.[50] The estimated adjustment cost parameters can also be found in table 1.4.

To summarize the existing estimates, Cooper and Haltiwanger (2006) find considerable fixed costs of investment and only small irreversibilities and convex adjustment costs, while Bloom (2009) finds large irreversibilities and small convex and non-convex adjustment costs. These differences in the estimates can be due to the

[49] Investment here always refers to the investment rate relative to the capital stock. Bursts and spikes are defined as investment rates exceeding plus or minus 20 %.

[50] The profitability process is assumed to follow a geometric random walk, and the variance is assumed to be the same for the plant, firm, and aggregate component.

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following reasons:

- **Data:** manufacturing vs. all industries, plant vs. firm, 72-88 vs. 81-00
- **Model:** annual vs. monthly, specification of driving process different[51]
- **Moments:** different moments they try to match

[51] Cooper and Haltiwanger (2006) estimate an AR(1) process in logs, while Bloom (2009) assumes a random walk in levels.

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Chapter 2

The Role of Uncertainty Shocks in US Business Cycles

Abstract

This chapter proposes an empirical identification strategy for uncertainty shocks that is consistent with the recent vintage of quantitative models that consider variations in uncertainty as impulses driving aggregate fluctuations. The identification strategy has two parts. First, the cross-sectional variance of firm-level sales is used as a proxy for uncertainty. Second, identifying restrictions are imposed within a Structural Vector Autoregression (SVAR) framework that are consistent with the theoretical models. This strategy for identifying uncertainty shocks is then applied to US data. The main result from the baseline SVAR estimation is that such model consistent uncertainty shocks lead to considerable booms in investment and employment that last for around two years. Moreover, while the uncertainty shock explains most of the forecast error in investment and employment it only explains a small part of the forecast error in the cross-sectional variance of firm-level sales. Both of these results are contrary to the dynamics that are induced by these uncertainty shocks in the recent vintage of quantitative macro models like Bloom (2009). Once uncertainty shocks are identified as the shocks that only affect dispersion upon impact, the results change somewhat. An uncertainty shock in that case leads to a moderate drop, rebound and overshoot of investment and a large increase in the cross-sectional dispersion of revenues. The results suggest that the way uncertainty shocks are modeled in the quantitative literature needs to be reconsidered. In particular, the standard timing assumption that the expectational effect of uncertainty shocks leads the distributional effect seems questionable given the empirical results in this chapter.

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2.1 Introduction

Heightened uncertainty is often cited as a contributing factor to the recent economic slump in the US. The basic intuition is that higher uncertainty since the outbreak of the Financial Crisis has lead firms to be more cautious in their investment and hiring decisions causing a decrease in aggregate investment and employment. With this background in mind it has become increasingly popular in the recent structural macroeconomic literature to consider variations in uncertainty as a driver for aggregate fluctuations. Such uncertainty shocks are usually modeled as a change in the innovation variance of the driving process in structural models with heterogeneous firms. For example Bloom (2009), Bloom et al. (2010) and Bachmann and Bayer (2011) use these types of uncertainty shocks in heterogeneous firm models that feature various forms of capital and labor adjustment costs. In these models, an uncertainty shock leads to a drop, rebound and overshoot in output, investment and employment due to real options effects. Moreover, Gilchrist et al. (2010), Arellano et al. (2011) and Chugh (2011) use this way of modeling uncertainty shocks in firm level models that feature financial frictions.[1]

At the same time, there has emerged an empirical literature that tries to identify the impact of uncertainty shocks from the data using Structural Vector Autoregressive Models (SVARs). Within these SVARs various proxies for uncertainty such as stock market volatility, disagreement between professional forecasters, dispersion in business survey responses and a media-based uncertainty index have been used (See Bloom (2009), Popescu and Smets (2010), Bachmann et al. (2010) and Alexopoulos and Cohen (2009) respectively). However, none of these uncertainty proxies has a direct counterpart within the recent structural macro models that feature uncertainty shocks.

In contrast to this, an observable implication of the way that uncertainty shocks are modeled in the quantitative literature is that an increase in uncertainty, i.e. in the variance of shocks, leads to an increase in dispersion of firm-level performance measures such as revenues. This is the distributional effect of uncertainty shocks. Moreover, there is an expectational effect of uncertainty shocks that results from the fact that a higher shock variance leads each firm to be more uncertain about its future profitability. Given the common timing assumption that firms know the realization of the variance regime one period in advance, the expectational effect of uncertainty shocks will lead the distributional effect by one period. Hence, it

[1] Some other papers that employ this way of modeling uncertainty shocks are for example Schaal (2012) in the context of a search and matching model and Vavra (2012) in a pricing framework with adjustment costs.

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seems natural to use the cross-sectional variance of firm-level revenues as a proxy for uncertainty and impose the identifying restriction that upon impact uncertainty shocks affect variables that immediately respond to expectations but they do not affect the sales variance.

Given the above discussion, the goal of this chapter is to study the role of uncertainty shocks in US business cycles within a SVAR framework, when uncertainty shocks are identified through a model consistent identification strategy. A model consistent identification strategy is defined as one that would identify the effect of uncertainty shocks when applied to simulated data from a structural model. Once the model consistent identification restrictions have been imposed on the VAR, the focus of the analysis is on impulse response functions and forecast error variance decompositions (FEVD). In particular, the questions addressed by this chapter are:

1. How does aggregate investment and employment respond to an uncertainty shock?
2. How much of the variation in investment and employment is due to uncertainty shocks?

The results from the baseline SVAR estimation show that such model consistent uncertainty shocks lead to considerable booms in investment and employment that last for around two years. Moreover, while the uncertainty shock explains most of the forecast error in investment and employment it only explains a small part of the forecast error in the cross-sectional variance of firm-level sales. Both of these results are contrary to the dynamics that are induced by these uncertainty shocks in the recent vintage of quantitative macro models. Various alternative SVAR specifications show that these results are qualitatively robust as long as the main identifying assumption for uncertainty shocks is imposed. However, once uncertainty shocks are identified as the shocks that only affect dispersion upon impact but not investment, the results change somewhat. An uncertainty shock in that case leads to a moderate drop, rebound and overshoot of investment of 0.5 % and a large increase in the cross-sectional dispersion of revenues. This suggests that the way uncertainty shocks are modeled in the quantitative literature needs to be reconsidered. In particular, the standard timing assumption that the expectational effect of uncertainty shocks leads the distributional effect seems questionable given the empirical results in this chapter.

As mentioned in the motivation, there are a couple of recent empirical papers that try to identify the impact of uncertainty shocks within a SVAR framework. For example, Bloom (2009) uses a monthly VAR framework with detrended US

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data where uncertainty is proxied by a stock market volatility indicator to study the impulse responses of industrial production and employment to uncertainty shocks. He finds that an uncertainty shock leads to a drop of around 1 % and 0.5 % in production and employment after around three months. Both variables rebound to their initial levels again after around 7 months and subsequently overshoot their initial levels by around 1 % and 0.5 % respectively for many months before reverting back.

Another related paper is Popescu and Smets (2010) which uses a SVAR framework to study the role of uncertainty shocks in German Business Cycles since the beginning of the 1990's. As a proxy for uncertainty they use measures of dispersion in opinions among macroeconomic forecasters. They find that uncertainty shocks lead to small temporary declines in industrial production (A drop of around 0.25 % over six months) and a more prolongued increase in unemployment (An increase of around 0.4 % over 15 months). However, using forecast error variance decompositions they also show that the overall contribution to output fluctuations is limited (slightly above 3 % of the variation in industrial production at the four year horizon).

A further paper in this line of research is Bachmann et al. (2010), who study the effect of uncertainty on manufacturing output in a SVAR framework for the US and Germany. Their uncertainty measures are constructed as the cross-sectional standard deviation of business survey responses about expected activity. In bivariate SVARs they find that innovations to their uncertainty measures are associated with slowly-building reductions in industrial production that reach a maximum of around - 1 % after two years, with no tendency to revert even after five years. When they identify an uncertainty shock as having no long-run effects, the impulse responses become statistically insignificant. They conjecture that recessions increase uncertainty rather than the other way around.

Finally, Alexopoulos and Cohen (2009) investigate empirically the role of uncertainty shocks in US business cycles. As their uncertainty measures they use the stock market volatility indicator proposed by Bloom (2009) and an indicator based on the number of New York Times articles on uncertainty or economic activity. In various SVAR specifications they find that innovations to uncertainty lead to drops and rebounds in industrial production, employment, productivity, consumption and investment that last around one to two years. Forecast error variance decompositions show that uncertainty shocks can account for 10 to 25 % of the variation in these variables.

Compared to these existing papers I use the cross-sectional standard deviation of firm-level revenues as my proxy for uncertainty, because time-variation in this measure is a direct implication of the way that uncertainty shocks are commonly

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modeled in the literature. Moreover, the identifying restrictions that are imposed within the SVAR framework are derived from a simple structural model and are not ad-hoc assumptions. In combination, these two features allow us to assess the empirical importance of the type of uncertainty shocks proposed in the literature.

The rest of the chapter is structured as follows. In section 2 the SVAR modeling framework is described. In section 3 a simple structural model that features uncertainty shocks is presented in order to test whether the proposed identification strategy is able to uncover uncertainty shocks from simulated data. In section 4 the data used for the estimation is presented and some initial statistical analysis is performed. Section 5 then presents the main results for the baseline SVAR specification. This is followed by some robustness tests with respect to the inclusion of additional variables in the SVAR and alternative identification strategies. Finally, section 7 provides a brief conclusion.

2.2 Overview of the Structural VAR Framework

This section gives a brief overview of the SVAR framework that is used in the rest of the chapter in order to identify the effect of model consistent uncertainty shocks from data for the US.

2.2.1 The General Reduced Form VAR Representation

The general framework for the analysis in this chapter is the VAR(p) model which is given by the following equation:

$$y_t = \nu + A_1 y_{t-1} + ... + A_p y_{t-p} + u_t \tag{2.1}$$

In the above system of equations $y_t = (y_{1t}, ..., y_{Kt})'$ is a random vector of size (K x 1), the A_i's are fixed coefficient matrices of size (K x K), $\nu = (\nu_1, ..., \nu_K)'$ is a vector of intercepts of size (K x 1) and finally $u_t = (u_{1t}, ..., u_{Kt})'$ is a K-dimensional white noise process with $E(u_t) = 0$, $E(u_t u_t') = \Sigma_u$ and $E(u_t u_s') = 0$ for $s \neq t$.[2] One of the issues with the model in equation (2.1) is that the components of u_t are reduced form shocks and will normally be instantaneously correlated. Therefore, no structural interpretation can be associated to them without imposing some additional assumptions regarding the structure of the data generating process.

[2]See Lütkepohl (2005) for an in depth discussion of VARs.

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2.2.2 Going from Reduced Form to Structural Representation

In order to recover the effects of structural innovations from the reduced form model given in equation (2.1) we need to specify a structural model of the form[3]:

$$Ay_t = A\nu + A_1^* y_{t-1} + ... + A_p^* y_{t-p} + B\epsilon_t \tag{2.2}$$

Here, $\epsilon_t \sim (0, I_K)$ is now a vector of structural shocks and A and B are (K x K) matrices that specify the contemporaneous influences between the endogenous variables and the impact of the structural shocks on each of the endogenous variables. Moreover, the fixed coefficient matrices are defined by $A_i^* = AA_i$. Given this structural model, the reduced form innovation vector in equation (2.1) is given by a linear combination of the structural shocks of the form $u_t = A^{-1}B\epsilon_t$. By imposing suitable restrictions on the elements of the matrices A and B it is then possible to recover the influences of the structural innovations from the estimated reduced form VAR. The strategy followed in the rest of the chapter in order to identify the impact of uncertainty shocks is to set $A = I_K$ and impose restrictions on the matrix B that are consistent with the implications of a simple structural model that features uncertainty shocks.

2.3 Model Consistent Identification of Uncertainty Shocks

In this section a simple structural model that features uncertainty shocks is presented in order to derive model consistent identifying restrictions for uncertainty shocks within a SVAR. These identifying restrictions are then tested on simulated data from the model in order to examine whether the proposed strategy to uncover uncertainty shocks would work if the real world was generated by the model. Thus, this section serves as a motivation for the kind of identifying restrictions that are applied to real world data in section 2.5 in order to study the role of uncertainty shocks in US business cycles.

2.3.1 A Simple Structural Model with Uncertainty Shocks

The model that is presented in this section is closely based on the partial equilibrium model in Lang (2012) with heterogeneous firms, uncertainty shocks and no labor

[3]See Lütkepohl (2005) chapter 9 for a detailed discussion of this structural model.

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and capital adjustment costs. The main difference in the current set-up is that the stochastic process for uncertainty is assumed to follow a continuous AR(1) process rather than a discrete Markov chain. There is a continuum of heterogeneous risk neutral firms indexed by $i \in [0,1]$ that maximize the present discounted value of expected profit streams, where profits in period t are given by:

$$\Pi(A_{i,t}, K_{i,t}) = \phi_\pi A_{i,t}^{\frac{c}{1-b}} K_{i,t}^{\frac{a}{1-b}} \tag{2.3}$$

In the above equation, ϕ_π is a constant parameter, $K_{i,t}$ is the capital stock and $A_{i,t}$ is a reduced form profitability shock that summarizes the effects of demand conditions and total factor productivity on profits. In addition, $c/(1-b) > 0$ and $a/(1-b) \in (0,1)$ are constant parameters that determine the curvature of the profit function.[4] Such a profit function can be derived under the assumption of a decreasing returns to scale (DRS) revenue function in capital and labor, a constant wage and freely adjustable labor that becomes immediately available for production.[5] Given this set-up it is easy to show that the optimal labor input and the resulting revenues have the same functional form in (A, K) space as profits and only differ by a constant:

$$L(A_{i,t}, K_{i,t}) = \phi_l A_{i,t}^{\frac{c}{1-b}} K_{i,t}^{\frac{a}{1-b}} \tag{2.4}$$

$$R(A_{i,t}, K_{i,t}) = \phi_r A_{i,t}^{\frac{c}{1-b}} K_{i,t}^{\frac{a}{1-b}} \tag{2.5}$$

As is standard in the uncertainty shocks and investment literature, the profitability of each firm is assumed to be the product of an aggregate (Z_t) and an idiosyncratic ($\Psi_{i,t}$) component.[6] Furthermore, both the aggregate and idiosyncratic components are assumed to follow persistent AR(1) processes in logs. The dynamics of profitability can therefore be represented by the following set of equations:

$$A_{i,t} = Z_t \Psi_{i,t} \tag{2.6}$$

$$z_t = \mu_z + \rho_z z_{t-1} + \eta_t \tag{2.7}$$

$$\psi_{i,t} = \mu_\psi + \rho_\psi \psi_{i,t-1} + \upsilon_{i,t} \tag{2.8}$$

Here, a lower case letter refers to the logarithm of the variable and $\eta_t \sim \mathcal{N}(\mu_\eta, \sigma_\eta^2)$ and $\upsilon_{i,t} \sim \mathcal{N}(\mu_{\upsilon,t-1}, \sigma_{\upsilon,t-1}^2)$. In line with the papers by Bloom (2009), Bloom et al. (2010) and Bachmann and Bayer (2011), uncertainty shocks are incorporated into

[4] Here, a, b and c are the exponents on capital, labor and profitability in the revenue function.

[5] A DRS revenue function in capital and labor can be either due to DRS in the production function and/or some degree of market power. See Lang (2012) for a detailed derivation of this profit function.

[6] See for example Cooper and Haltiwanger (2006), Khan and Thomas (2008), Bloom (2009), Bloom et al. (2010) or Bachmann and Bayer (2011).

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the model through changes in the variance of innovations to idiosyncratic profitability, which is implicit in the fact that $\sigma^2_{\upsilon,t-1}$ is indexed by time.[7] Moreover, embedded in the above specification is the standard timing assumption that the variance of idiosyncratic shocks is known one period in advance, which ensures that agents always know the true variance of shocks applicable in the next period. Hence, all variations in uncertainty in the model are related to fundamentals and not to imperfect information about the true state of the driving process. This way of modeling uncertainty shocks is standard in the literature and is followed in this chapter as the goal is to derive identifying restrictions for uncertainty shocks that are consistent with the recent vintage of structural models.

At this point it is useful to note that there are two channels through which uncertainty shocks affect aggregates in the model. On the one hand, there is an expectational effect that results from the fact that a higher shock variance leads each firm to be more uncertain about its future profitability. On the other hand, there is a distributional effect that results from the fact that once firms start drawing innovations from a higher variance distribution, the cross-sectional dispersion of idiosyncratic profitability increases. Given the standard timing assumption that firms know the realization of the variance regime one period in advance, the expectational effect will lead the distributional effect by one period. I.e. when an uncertainty shock occurs, firms' expectations change upon impact, but the cross-sectional dispersion of idiosyncratic profitability does not change until the next period, when firms start drawing innovations from a more dispersed distribution. This property will be exploited further below in order to derive model consistent identification restrictions of uncertainty shocks.

Given that the process for idiosyncratic profitability is specified in logs, an increase in the variance of innovations will lead to an increase in one period ahead expectations of idiosyncratic profitability in levels. To adjust for this effect, the mean of idiosyncratic innovations is assumed to adjust in an offsetting way, which is achieved by setting $\mu_{\upsilon,t} = -\sigma^2_{\upsilon,t}/2$.[8] For analytical tractability within a VAR framework, the variance of idiosyncratic profitability shocks is assumed to follow an

[7] In the above mentioned papers the variance of aggregate profitability shocks is also assumed to vary over time. For simplicity this type of uncertainty shock is omitted in the current chapter. However, this should not be an issue, for as long as the timing assumption for both types of uncertainty shocks is the same, the identifying assumptions derived below are still valid.

[8] See Lang (2012) for a detailed discussion of this result.

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AR(1) process of the form:[9]

$$\sigma_{v,t}^2 = \mu_\sigma + \rho_\sigma \sigma_{v,t-1}^2 + \varepsilon_{i,t} \tag{2.9}$$

In this equation $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ are the innovations to the idiosyncratic shock variance. Given a constant price of capital p, a constant discount factor β and the standard law of motion for capital $K_{t+1} = I_t + (1-\delta)K_t$, the decision problem of each firm can be represented by the following Bellman Equation. To save on notation the firm subscript i for each variable is omitted and primes denote next period variables:[10]

$$V(A, K, \sigma_v) = \max_{K'} \Pi(A, K) - p[K' - K(1-\delta)] + \beta E_{A', \sigma_v' | A, \sigma_v} [V(A', K', \sigma_v')] \tag{2.10}$$

Taking the first-order and envelope conditions and using the functional forms assumed above, it is easy to derive the following policy function that characterizes the optimal capital accumulation decision:

$$K_{i,t+1} = \xi \left(Z_t^{\rho_z} \Psi_{i,t}^{\rho_\psi} e^{\mu_{v,t} + \frac{\sigma_{v,t}^2}{2} \frac{c}{1-b}} \right)^{\frac{c}{1-a-b}} \tag{2.11}$$

Here, $\xi = \varphi(e^{\mu_z + \mu_\psi + \mu_\eta + c\frac{\sigma_\eta^2}{2(1-b)}})^{\frac{c}{1-a-b}}$ and $\varphi = [(a\beta\phi_\pi)/([1-b][p - p\beta(1-\delta)])]^{\frac{1-b}{1-a-b}}$ are constants that depend on the structural parameters of the model. It is easy to see that each firm's capital stock is a function of aggregate profitability, idiosyncratic profitability and the level of uncertainty.[11] In particular, the effect of uncertainty on capital accumulation by each firm is captured by the presence of $\mu_{v,t} + \frac{\sigma_{v,t}^2}{2} \frac{c}{1-b}$ in the capital policy function. With this policy function in hand we are now able to characterize the dynamics of aggregates in the model.

[9] In principle, this specification allows for negative values of the variance. However, this problem can be mitigated by setting μ_σ sufficiently high so that the probability of negative values of the variance is close to zero. In the parameterization of the model in section 2.3.3 the parameter values are chosen such that the unconditional mean of the variance process is 0.3 and its standard deviation is 0.05, so that the probability of negative variance values is virtually zero given the assumption of a normal distribution.

[10] In the Bellman Equation below total profitability is used as the state variable to save on notation. It should be kept in mind however that in order to solve the model, information on both the aggregate and idiosyncratic profitability is needed. Whenever total profitability is used as the state variable in this chapter it should therefore be interpreted as information on both components of total profitability.

[11] The fact that uncertainty can have an effect on capital accumulation even without capital adjustment costs is due to the fact of either concavity or convexity of the profit function in profitability.

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2.3.2 Deriving Model Consistent Identifying Assumptions

If we want to identify the effect of uncertainty shocks from real world data, it is necessary to devise an identification strategy that only relies on observables. Natural candidates for observable variables that come out of the model presented above are aggregate investment, aggregate employment and the cross-sectional variance of firm level revenues. The dynamics of these variables can be easily obtained by applying the expectations and variance operators to equations (2.4), (2.5), (2.11) and the law of motion for capital:

$$\begin{aligned}
E[K_{i,t}] &= \xi Z_{t-1}^{\rho_z \frac{c}{1-a-b}} E\left[\Psi_{i,t-1}^{\rho_\psi \frac{c}{1-a-b}}\right] e^{(\mu_{v,t-1}+\frac{\sigma^2_{v,t-1}}{2}\frac{c}{1-b})\frac{c}{1-a-b}} && (2.12)\\
E[I_{i,t}] &= E[K_{i,t+1}] - (1-\delta)E[K_{i,t}] && (2.13)\\
E[L_{i,t}] &= \phi_l Z_t^{\frac{c}{1-b}} E\left[\Psi_{i,t}^{\frac{c}{1-b}} K_{i,t}^{\frac{a}{1-b}}\right] && (2.14)\\
V[R_{i,t}] &= \phi_r^2 Z_t^{\frac{2c}{1-b}} V\left[\Psi_{i,t}^{\frac{c}{1-b}} K_{i,t}^{\frac{a}{1-b}}\right] && (2.15)
\end{aligned}$$

From this system of equations it is easy to see that the aggregate profitability shock affects aggregate employment, aggregate investment and the variance of revenues all contemporaneously through Z_t. In contrast, an uncertainty shock only affects aggregate investment contemporaneously, while aggregate employment and the variance of profits are not affected in the current period. This is easy to see by noting that aggregate employment and the variance of profits at time t depend on the joint distribution of idiosyncratic profitability and capital in period t. This joint distribution is however not affected by the uncertainty shock in period t, but only by past uncertainty shocks. This is a result of the standard timing assumption that the variance of idiosyncratic profitability shocks is known one period in advance. Hence the expectational effect of uncertainty shocks already materializes in period t and affects investment, while the distributional effect of uncertainty shocks only materializes in the following period, affecting the variance of profits with a one period delay.

From this discussion, it should be clear that the shocks to aggregate investment in a reduced form VAR are a linear combination of aggregate profitability shocks and uncertainty shocks, while contemporaneous shocks to aggregate employment and the variance of profits in a reduced form VAR are simply aggregate profitability shocks. Hence, we should be able to identify uncertainty shocks in a SVAR like equation (2.2) by imposing these restrictions on the matrix B and assuming that the matrix A is the identity matrix.

At this stage it is useful to point out that these identifying assumptions also hold in more complex model set-ups. For example, the identifying assumptions hold under various forms of labor and capital adjustment costs, as long as there is a time to build assumption for the respective factor of production as for example in Bloom (2009).[12] In such a set-up, uncertainty will affect hiring and investment but not the variance of current revenues due to the time to build assumption. Moreover, allowing the discount factor β to vary with uncertainty does not alter the validity of the identification strategy. However, the identification strategy will break down, whenever labor is immediately available and either the current wage is affected by uncertainty due to general equilibrium effects, or there are non-convex labor adjustment costs. This should not be of major concern though because wages are usually quite sticky at the quarterly frequency and the hiring and firing process also takes some time in most countries.

2.3.3 Test of Identifying Assumptions with Simulated Data

Now that we have derived identifying assumptions for uncertainty shocks that are consistent with recent structural models, it is instructive to test whether the proposed identification strategy actually works if it is applied to simulated data from the model. To this end, a trivariate SVAR with $y_t = (V[R_{i,t}], E[I_{i_t}], E[L_{i_t}])'$ is estimated on simulated data and the resulting impulse responses are compared to the true impulse responses from the model.

In order to uncover the impact of uncertainty shocks from the estimated reduced form VAR, the proposed identifying restrictions from above are applied. With these restrictions, uncertainty shocks are identified as the structural innovations to the investment equation when the following identifying restrictions are imposed on the B-matrix in the VAR specification:

$$B = \begin{bmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \tag{2.16}$$

To produce a sample of simulated data from the model, it is necessary to assign numerical values to the structural parameters of the model. Table 2.3 in Appendix A summarizes the parameter values that were chosen in order to generate a synthetic data set from the model consisting of 100,000 observations. The SVAR specification described above is estimated on this simulated sample using a specification with 15 lags. The resulting estimates of the impulse responses to an uncertainty shock are displayed in figure 2.1 along with the true impulse responses from the model.

[12] To be precise, this claim holds for any adjustment cost specification that does not directly affect measured revenues.

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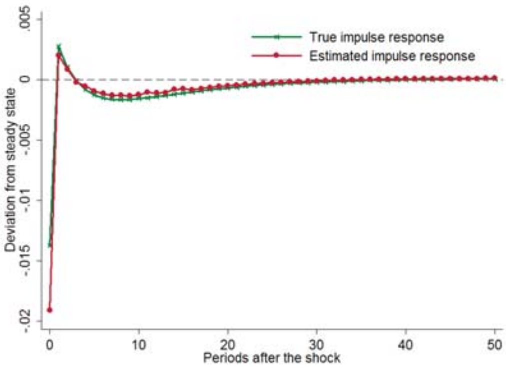
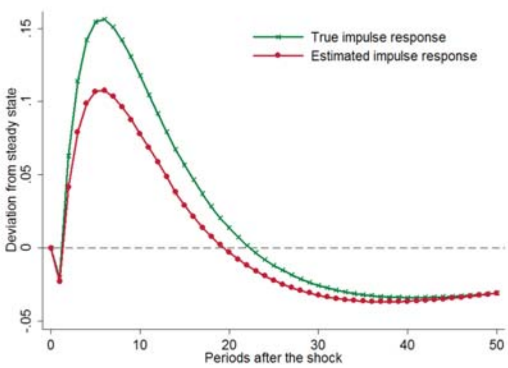
Figure 2.1: Estimated and true impulse responses from a trivariate SVAR

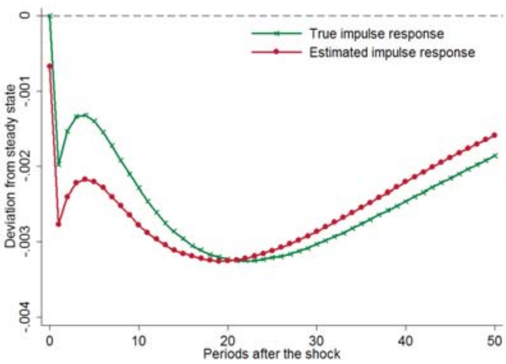
(a) Uncertainty shock → Investment



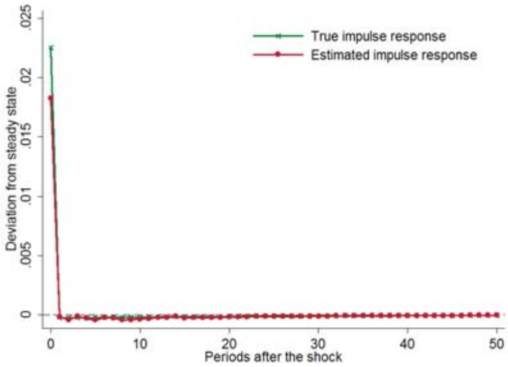
(b) Uncertainty shock → Dispersion



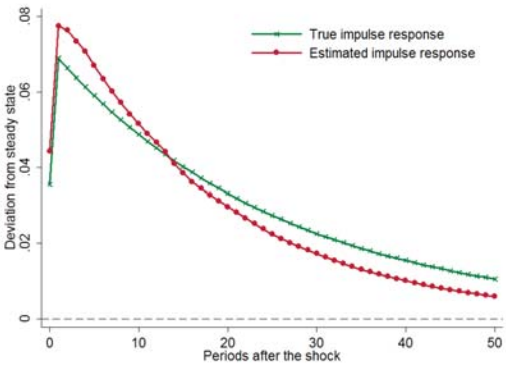
(c) Uncertainty shock → Employment



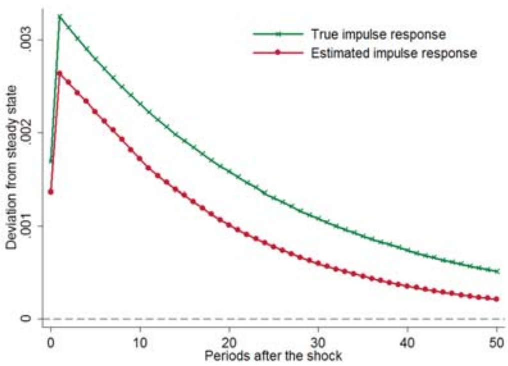
(d) Aggregate shock → Investment



(e) Aggregate shock → Dispersion



(f) Aggregate shock → Employment



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As can be seen from the figure, the estimated impulse responses to an uncertainty shock qualitatively and quantitatively resemble the true impulse responses. For completeness, the estimated impulse responses to an aggregate profitability shock are also displayed in the figure. The results here are analogous to the ones for uncertainty shocks in that the estimated impulse responses quantitatively resemble the true impulse responses. These results show that if the true data generating process was given by the simple model with uncertainty shocks and aggregate shocks described above, then the proposed identification strategy would be able to identify the effects of both types of shocks. The baseline model that is estimated in section 2.5 is therefore going to be a trivariate SVAR including the cross-sectional variance of firm-level sales, aggregate investment and aggregate employment.

2.4 Data and Preliminary Statistical Analysis

Now that the model consistent identifying assumptions for uncertainty shocks have been derived and tested, a brief discussion of the uncertainty measure and the statistical properties of the US data to which this identification strategy is applied are provided. Details of the data sources can be found in Appendix B.

2.4.1 Discussion of the Uncertainty Measure

The uncertainty proxy used in this chapter is given by the quarterly cross-sectional variance of firm-level real sales which is calculated from Compustat data for the time period 1961 Q1 to 2010 Q3.[13] The reason for using the sales variance as a proxy for uncertainty is based on the fact that the way uncertainty shocks are modeled in the literature implies that this measure should vary over time. Given that sales at the firm level are only measured and published at low frequencies, a quarterly dispersion measure is the best we can hope for. Moreover, because sales at the quarterly frequency contain a seasonal component, the cross-sectional sales variance needs to be seasonally adjusted by using the Census X-12-ARIMA method.

In order to compute the cross-sectional sales variance it is necessary to decide on how to treat entering and exiting firms. The baseline variance measure that is used in section 2.5 is constructed by restricting the sample to firms that have at least 150 quarters of observations. This basically eliminates variations in the variance that are due to cyclical variations in entry and exit. As is shown in Appendix C,

[13] Real sales are calculated by deflating nominal sales by a chain-type price index for GDP with 2005 = 100.

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Table 2.1: Business cycle properties of main variables

Variable	SD	AR(1)	Corr-lag	Corr	Corr-lead
GDP	0.016	0.85	0.32	0.46	0.43
Investment	0.052	0.90	0.36	0.45	0.43
Employment	0.013	0.93	0.45	0.49	0.43
FFR	1.591	0.83	0.44	0.45	0.38
Wage	0.006	0.91	0.19	0.12	0.00
SP 500	0.101	0.82	0.11	0.29	0.38
VXO	0.216	0.59	0.10	-0.04	-0.16
Variance	0.100	0.64	0.64	1.00	0.64
IQR	0.073	0.78	0.50	0.62	0.55

Note: SD refers to the standard deviation and AR(1) to the autocorrelation of the respective variable. Corr refers to the contemporaneous correlation of the respective variable with the cross-sectional variance of firm level sales, while Corr-lag and Corr-lead refer to the correlation when the respective variable lags or leads the sales variance by one period.

the cyclical dynamics of the cross-sectional sales variance based on different sample selections are similar to this baseline measure but contain more noise. Moreover, robustness tests in section 2.6 show that the main results are not sensitive to the sample selection.

2.4.2 Business Cycle Properties of the Main Variables

As the focus of this chapter is on the role of uncertainty shocks in US business cycles, all of the variables are logged and detrended using the HP-filter with $\lambda = 1600$, which is the common smoothing parameter used for quarterly data. Therefore, all time series used in the rest of the chapter have the interpretation of percentage deviations from trend. Figure 2.2 displays the cyclical components of all the main variables along with the dates identified as recessions by the NBER. In addition, table 2.1 summarizes the business cycle properties of the main variables such as the autocorrelation, the standard deviation and the correlation with the cross-sectional variance of sales.

The first aspect that stands out is that the cross-sectional variance of firm-level sales is quite volatile with a standard deviation of 10 % over the business cycle. In comparison, the standard deviation of the main macro variables like investment, GDP and employment is much lower with 5.2 %, 1.6 % and 1.3 % respectively. The second aspect that is apparent is that the variance of firm-level sales is less persistent than all of the main macro variables. It's AR(1) coefficient of 0.64 compares to

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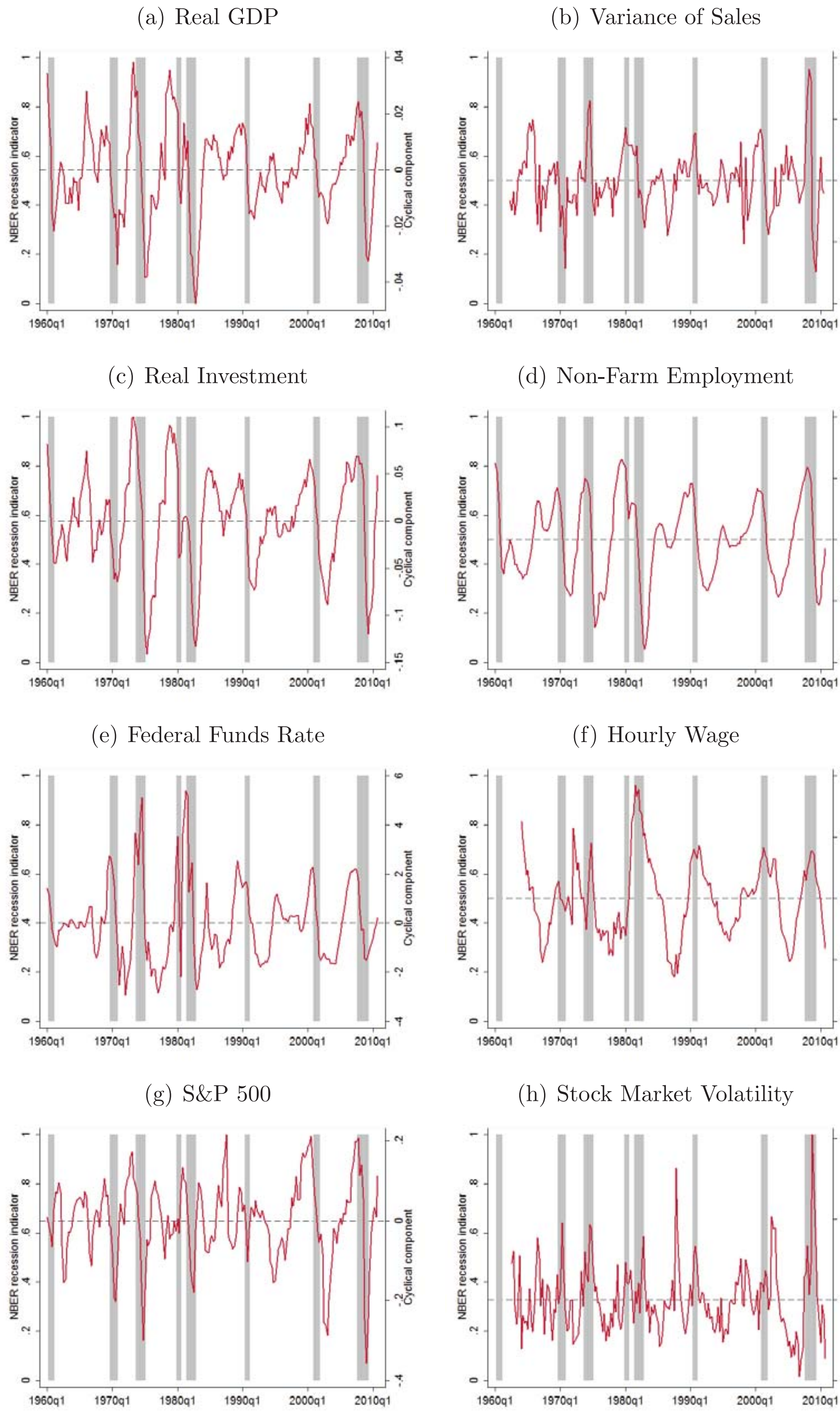
Figure 2.2: Cyclical components of main variables



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coefficients in the range of 0.82 to 0.93 for the S&P 500, the federal funds rate, wages, GDP, investment and employment.

Looking at figure 2.2 panel (b) it is apparent that the variance of firm-level sales was extraordinarily high at the beginning of the Great Recession in 2008, but has since fallen considerably again. In general, the variance of firm-level sales seems to be high during or at the beginning of recessions. In terms of co-movement with aggregate variables such as GDP, investment, employment and the federal funds rate, the cross-sectional sales variance displays a positive contemporaneous correlation of around 0.45. Moreover, the cross-sectional sales variance appears to be coincident with these variables.

2.5 The Effect of Uncertainty on Investment and Employment

In this section the baseline specification of the SVAR that includes the cross-sectional variance of firm-level sales, aggregate investment and aggregate employment is estimated. The resulting impulse responses and forecast error variance decompositions to an uncertainty shock are then analyzed.

2.5.1 The Baseline SVAR Specification

As argued in section 2.3 a trivariate SVAR that includes the cross-sectional variance of firm level sales, aggregate investment and aggregate employment is able to recover the effect of model consistent uncertainty shocks with the assumption that the B-matrix in equation (2.2) is lower triangular. This identification strategy reflects the fact that an uncertainty shock only affects aggregate investment upon impact, while an aggregate profitability shock affects aggregate investment, aggregate employment and the cross-sectional variance of sales. The aggregate profitability shock is therefore identified as the structural shock to the variance equation, while the uncertainty shock is identified as the structural shock to the investment equation. The shock to the employment equation does not have a structural interpretation given the model written down in section 2.3. This trivariate SVAR specification is estimated on the cyclical components of the three variables for the US using data from 1962 Q2 up to 2010 Q3. The SVAR is specified with 2 lags, as this is suggested by all the various information criteria.

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2.5.2 Impulse Responses to an Uncertainty Shock

The central result of this chapter can be found in figure 2.3 which displays the estimated impulse responses to an uncertainty shock and an aggregate shock for the baseline trivariate SVAR specification. The most striking feature about these estimated impulse responses is that the identified uncertainty shock actually leads to considerable booms in aggregate investment and employment of about 2 % and 0.4 % respectively that last approximately two years.[14] After this period the two aggregates undershoot their initial levels by around 1 % and 0.2 % and settle down again after around 5 years. In addition, the cross-sectional variance of sales increases moderately after the uncertainty shock by around 1.5 % and slightly undershoots its initial level after around two years.

In contrast, the identified aggregate shock leads to much smaller and shorter lived increases in aggregate investment and employment. The respective increases are 0.5 % and 0.1 %, which is about a quarter of the magnitudes induced by the identified uncertainty shock and they only last for four quarters. After that point aggregate investment and employment start to undershoot their initial levels by around the same magnitudes as the initial increases. Both aggregates settle down again after roughly five years. On the other hand, the identified aggregate shock leads to a large increase in the cross-sectional variance of sales of almost 8 % upon impact. This increase is fairly short lived so that the initial level is reached again after only five quarters.

To summarize, while the identified uncertainty shocks leads to considerable booms in investment and employment and moderate increases in the sales variance, the identified aggregate shock leads to small increases in the respective aggregates and a large burst in the sales variance. Both of these results are not in line with the dynamics that are induced by uncertainty shocks in the structural models like Bloom (2009). First, in these models uncertainty shocks lead to drop-rebound-overshoot dynamics in aggregates. Moreover, the uncertainty shock should be responsible for most of the variation in the cross-sectional sales variance.

2.5.3 Forecast Error Variance Decompositions

To further explore the role of the identified uncertainty and aggregate shocks in shaping aggregate dynamics, table 2.2 summarizes the associated forecast error variance decomposition. As can be seen, the uncertainty shock explains most of the forecast error of aggregate investment and employment, while it only explains a small

[14] These impulse responses are for an uncertainty shock of size one standard deviation.

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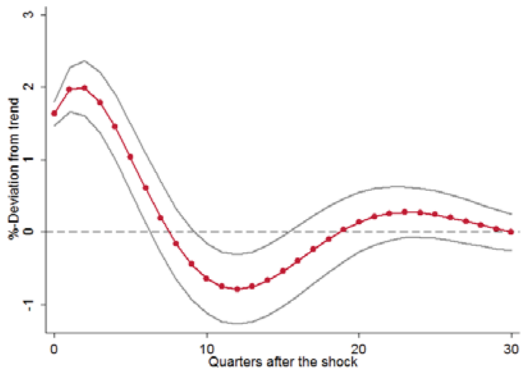
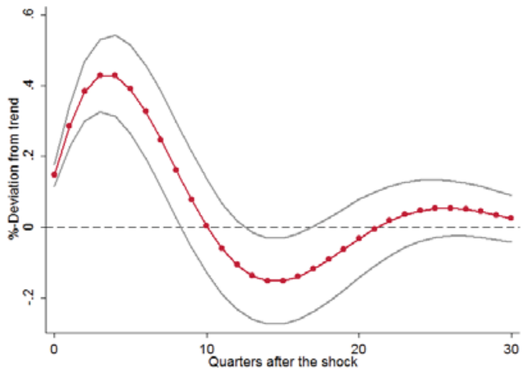
Figure 2.3: Estimated impulse responses from the baseline model

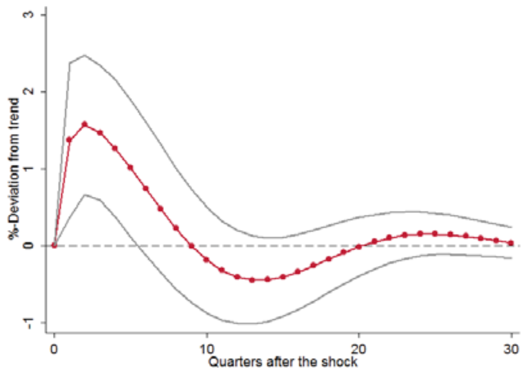
(a) Uncertainty shock → Investment



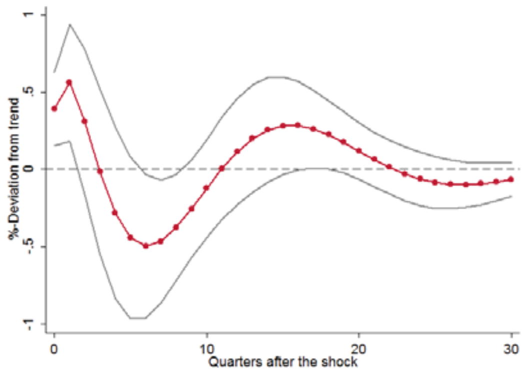
(b) Uncertainty shock → Employment



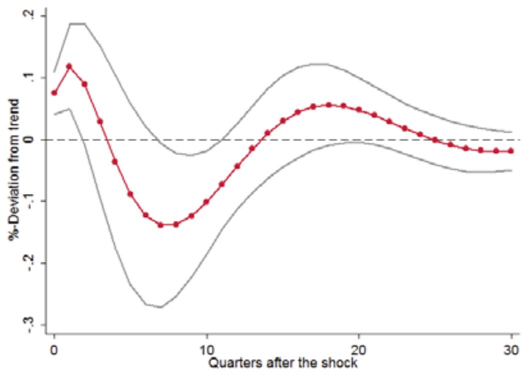
(c) Uncertainty shock → Variance



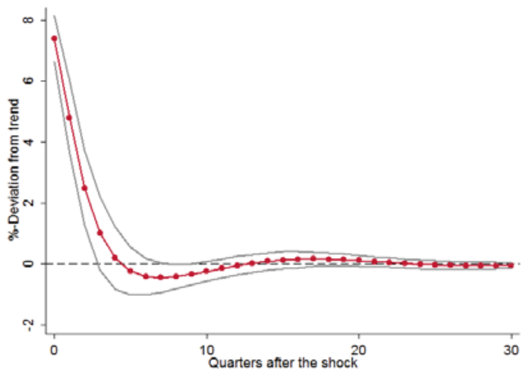
(d) Aggregate shock → Investment



(e) Aggregate shock → Employment



(f) Aggregate shock → Variance



Notes: The impulse responses are for a shock size of one standard deviation. The gray lines indicate the 95 % confidence intervals constructed by using the asymptotic standard errors.

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fraction of the variation in the cross-sectional variance of firm-level sales. In contrast, the aggregate profitability shock explains most of the forecast error of the cross-sectional variance of firm-level sales, and not much of aggregate investment and employment. These findings hold for all time horizons between one and five years.

2.6 Robustness Tests

In this section some robustness tests for the estimated effects of model consistent uncertainty shocks are performed. In particular, the robustness of the estimated impulse responses with respect to the dimension of the SVAR, the use of alternative uncertainty measures and different identifying assumptions is explored.

2.6.1 Varying the Dimension of the SVAR

Because the omission of relevant variables from the SVAR can bias the estimated response to structural shocks, a bivariate SVAR with the cross-sectional variance of firm-level sales and aggregate investment as well as a multivariate SVAR including the S&P 500, the federal funds rate and the wage in addition to the baseline variables are estimated. The identifying assumption for uncertainty shocks is the same as for the baseline estimation, i.e. an uncertainty shock affects investment upon impact but not the sales variance. For the multivariate SVAR, the uncertainty shock is also allowed to affect the federal funds rate and the wage rate contemporaneously. The S&P 500 is placed second in the VAR after the sales variance in order to control for news shocks that affect the stock market.

The results of this exercise can be found in figure 2.4. As can be seen the dynamics of investment and the cross-sectional variance of sales are qualitatively similar to the baseline results. In particular, the identified uncertainty shock still leads to a considerable boom in investment and a moderate increase in the sales variance. The main quantitative difference to the baseline specification is that for the multivariate model, the uncertainty shocks leads to slightly lower responses in investment and the sales variance.

2.6.2 Using Alternative Uncertainty Measures

Another source that could affect the estimated impulse responses to an uncertainty shock is the sample selection of firms when constructing the cross-sectional variance

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Table 2.2: Forecast Error Variance Decomposition in the baseline SVAR

Horizon	**Investment**	**Employment**	**Dispersion**
		Uncertainty shock	
1	0.95	0.35	0.00
4	0.94	0.58	0.07
8	0.89	0.67	0.10
12	0.79	0.64	0.10
16	0.78	0.62	0.11
20	0.77	0.63	0.11
		Aggregate shock	
1	0.05	0.09	1.00
4	0.04	0.04	0.92
8	0.07	0.05	0.88
12	0.07	0.08	0.87
16	0.07	0.07	0.86
20	0.07	0.08	0.86
		Employment shock	
1	0.00	0.56	0.00
4	0.02	0.38	0.01
8	0.04	0.28	0.02
12	0.14	0.28	0.02
16	0.16	0.30	0.03
20	0.16	0.30	0.03

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Figure 2.4: Robustness to the dimension of the VAR

(a) Uncertainty → Investment (b) Uncertainty → Variance

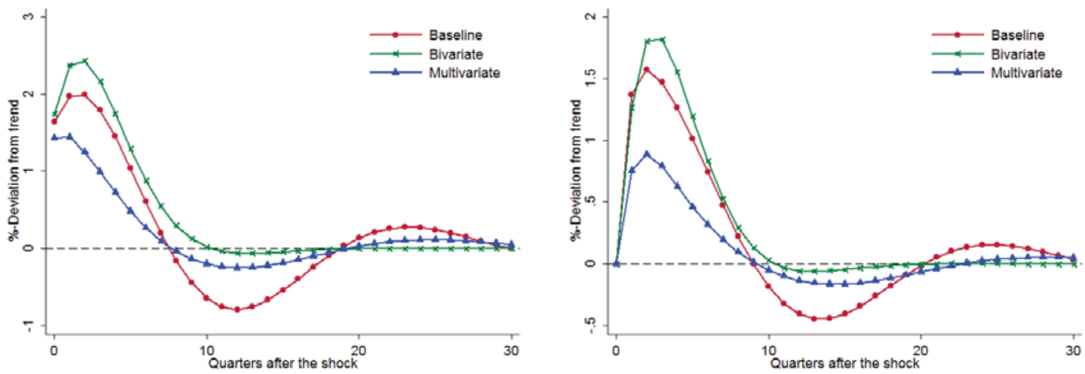
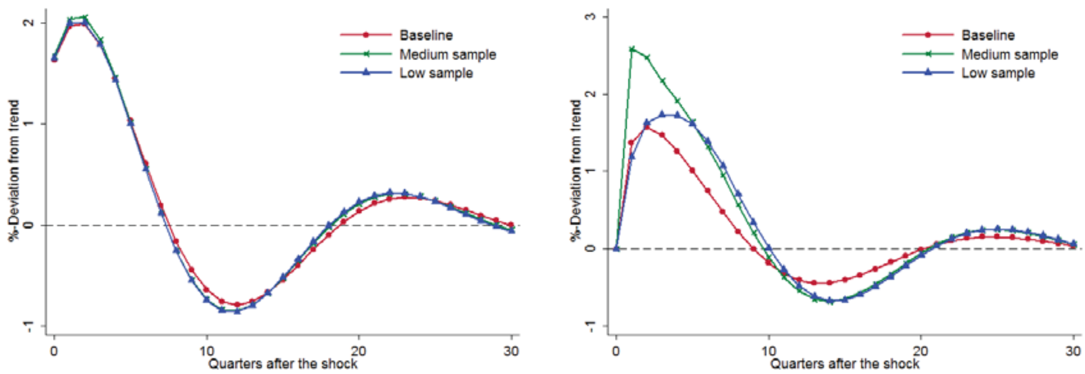


Figure 2.5: Robustness to sample selection of firms

(a) Uncertainty → Investment (b) Uncertainty → Variance



of firm-level sales. In order to explore this, figure 2.5 compares the estimated impulse responses to an uncertainty shock when the sales variance is constructed from a sample of firms with more than 100 quarterly observations (medium sample) and more than 60 quarters of observations (low sample).[15] It is evident that variations in the sample of firms do not affect the estimated impulse responses much. Moreover, it was tested whether using the interquartile range (IQR) instead of the variance of firm-level sales as the uncertainty measure changes the results. However, as figure 2.6 shows the dynamics induced by the identified uncertainty shock do not change qualitatively when using this alternative uncertainty measure.

[15]Recall that the baseline sales variance is constructed using data on all firms with more than 150 quarters of observations.

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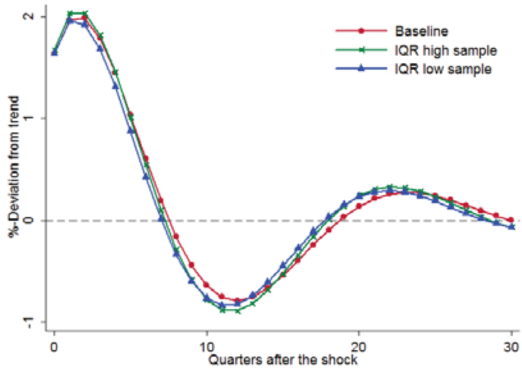
Figure 2.6: Robustness to using the IQR instead of the variance

(a) Uncertainty → Investment



(b) Uncertainty → Dispersion

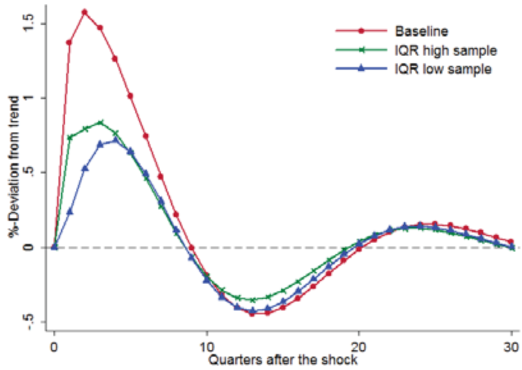
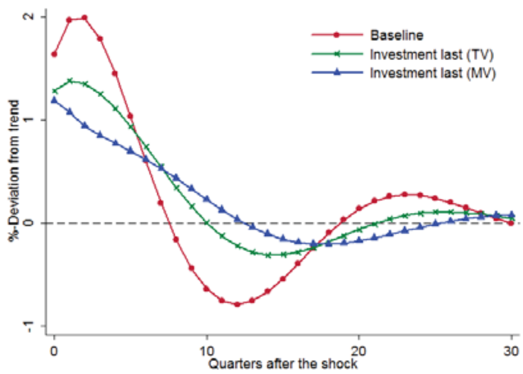
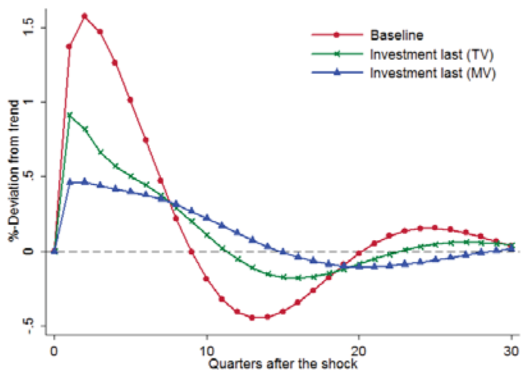


Figure 2.7: Robustness to the ordering of investment in the VAR

(a) Uncertainty → Investment



(b) Uncertainty → Variance



2.6.3 Alternative Identifying Assumptions

Because the ordering of variables within the SVAR will affect the estimated impulse responses, a trivariate and multivariate SVAR where investment is placed last in the system are also estimated for robustness purposes. The uncertainty shock is still identified as the structural shock to the investment equation, which corresponds to an identifying assumption that an uncertainty shock only affects investment upon impact but none of the other variables. The results of this exercise can be found in figure 2.7. Again, the dynamics induced by an uncertainty shock are qualitatively similar as in the baseline SVAR.

As the estimated impulse responses to model consistent uncertainty shocks are

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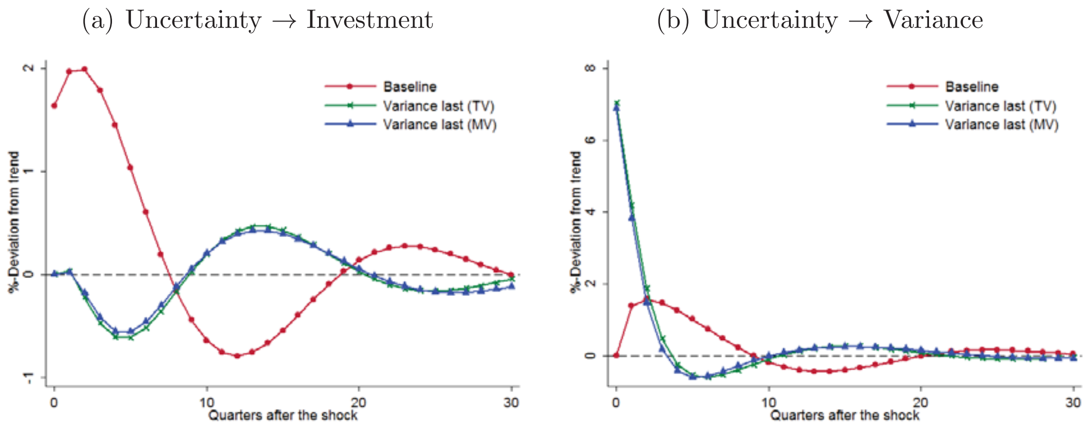
Figure 2.8: Robustness to the identifying assumptions for uncertainty shocks



not in line with the notion that higher uncertainty leads to recessions, an alternative identification strategy for uncertainty shocks appears interesting to explore. In particular, it seems questionable whether in reality firms already know the applicable innovation variance one period in advance, as is implied by the way uncertainty shocks are commonly modeled. A more realistic assumption is probably that firms need to observe realizations of more dispersed shocks to realize that they are in a state of heightened uncertainty. Therefore, an identification strategy where uncertainty shocks are identified as shocks that only affect the cross-sectional variance of firm-level sales but none of the other variables is explored. The corresponding results are displayed in figure 2.8.

The figure shows that this type of uncertainty shock actually leads to a fall of aggregate investment that reaches a maximum after 5 quarters. Moreover, aggregate investment rebounds to the initial level after around two and a half years and reaches a maximum overshoot after around three and a half years. This type of drop-rebound-overshoot behavior is more in line with standard intuition and the effect of uncertainty shocks in the paper by Bloom (2009). Furthermore, this type of uncertainty shock actually leads to a large increase of the cross-sectional sales variance of around 7 % upon impact. One important fact to note though is that the maximum drop of aggregate investment is only around - 0.5 % for a one standard deviation uncertainty shock. Given that the business cycle component of aggregate investment has a standard deviation of 5 %, this type of uncertainty shock does not explain much of the variation in investment.

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2.7 Conclusion

This chapter has proposed an empirical identification strategy for uncertainty shocks that is consistent with the way these types of shocks are modeled in the recent quantitative macro literature. The proposed identification strategy has two parts. First, the cross-sectional variance of firm-level sales is used as a proxy for uncertainty. Second, consistent with the theoretical literature, uncertainty shocks are identified in a SVAR framework as the shocks that affect investment upon impact but do not affect the cross-sectional variance of firm-level sales contemporaneously. This identifying restriction is a direct result of the standard timing assumption in the theoretical models that the applicable variance of innovations is known one period in advance. Thus, an uncertainty shock affects expectations and therefore investment upon impact but does not change the distribution of idiosyncratic profitability across firms until the next period.

This identification strategy was then applied to US data in order to study the role of model consistent uncertainty shocks in US business cycles. The main result from the baseline SVAR estimation is that these uncertainty shocks lead to considerable booms in investment and employment that last for around two years. Moreover, while the uncertainty shock explains most of the forecast error in investment and employment it only explains a small part of the forecast error in the cross-sectional variance of firm-level sales. Both of these results are contrary to the dynamics that are induced by these uncertainty shocks in the recent vintage of quantitative macro models. In addition, these dynamics do not correspond to the conventional wisdom that higher uncertainty leads to a slump in investment, employment and aggregate activity.

Various robustness tests have shown that these results do not change qualitatively when including additional variables in the SVAR or assuming that the uncertainty shock only affects investment upon impact but none of the other variables. However, imposing the identifying assumption that an uncertainty shock only affects the cross-sectional variance of firm-level sales upon impact but none of the other variables changes the dynamics considerably. When an uncertainty shock is identified in this way, higher uncertainty actually leads to a drop, rebound and overshoot of investment and a large increase in the firm-level sales variance. Nevertheless, the drop in investment is quantitatively small reaching a maximum of around - 0.5 % after 5 quarters for a one standard deviation uncertainty shock. Given that the business cycle component of investment has a standard deviation of 5 %, this type of uncertainty shock does not explain much of the variation in aggregate investment.

The above results suggest that the way uncertainty shocks are modeled in the

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quantitative macro literature needs to be reconsidered. In particular, the standard timing assumption that the expectational effect of uncertainty shocks leads the distributional effect seems questionable given the empirical results in this chapter. The impulse responses derived from the alternative identification strategy suggests that a timing assumption where firms need to observe realizations from a more dispersed distribution before they realize that they are in a state of heightened uncertainty could be promising.

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Appendix A: Supplementary Tables

Table 2.3: Parameter values used to simulate data from the model

Parameter	**Value**	**Description**
β	0.99	Discount factor
δ	0.026	Depreciation rate
w	1	Wage rate
a	0.25	Exponent on capital
b	0.50	Exponent on labor
c	$1 - a - b$	Exponent on profitability
p	1	Price of capital
μ_z	0	Intercept of aggregate profit.
ρ_z	0.9627	AR(1) parameter of aggregate profit.
σ_η	0.015	Std of innovations of aggregate profit.
μ_η	0	Mean of aggregate innovations
μ_ψ	0	Intercept of idiosyncratic profit.
ρ_ψ	0.9627	AR(1) parameter of idiosyncratic profit.
ρ_σ	0.7	AR(1) parameter of idiosyncratic innovation variance
μ_σ	0.09	Intercept of idiosyncratic innovation variance
σ_ε	0.0357	Std of innovations to idiosyncratic shock variance

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Appendix B: Description of the Data Sources

Macro Variables: All the macro variables were downloaded from Datastream. The frequency of observation is quarterly and where available the series cover the time period 1950 Q1 to 2011 Q2. Moreover, all series are seasonally adjusted. The real series are chain-type quantity indices where the base year is 2005 = 100. The series of interest are real GDP, real investment, non-farm employment, the federal funds rate and the hourly wage of private industry production workers.

NBER Recession Index: The official NBER business cycle dates were downloaded from the NBER website. Based on these dates an index was created that takes a value of one to indicate a quarter in recession and it takes a value of zero to indicate a quarter in expansion. Note that both peak and trough quarters are counted as part of a recession.

S&P 500 Stock Market Index: The series was downloaded from the FRED Database of the St. Louis Fed. The series is at a quarterly frequency and spans the time 1957Q1 to 2011Q2. The quarterly data was constructed as the average of daily data within the quarter.

Stock Market Volatility: The stock market volatility index is taken from the paper by Bloom et al. (2010), which can be downloaded from the homepage of Nick Bloom. This index is a combination of actual stock returns volatility of the S&P 500 index (for time periods before 1986) and the CBOE VXO volatility index for the S&P 100 (for time periods after 1986).[16]

Firm Level Sales: The sales figures at the firm level are taken from Compustat and span the time period 1961 Q1 to 2010 Q3. The nominal sales figures are then deflated by the price index for GDP to arrive at real sales figures, from which cross-sectional measures of dispersion are computed.

[16]The exact description of the stock market volatility series in Bloom et al. (2010) is: "CBOE VXO index of % implied volatility, on a hypothetical at the money S&P 100 option 30 days to expiration, from 1986 to 2009. Pre 1986 the VXO index is unavailable, so actual monthly returns volatilities calculated as the monthly standard-deviation of the daily S&P 500 index normalized to the same mean and variance as the VXO index when they overlap (1986-2006). Actual and VXO are correlated at 0.874 over this period. The market was closed for 4 days after 9/11, with implied volatility levels for these 4 days interpolated using the European VX1 index, generating an average volatility of 58.2 for 9/11 until 9/14 inclusive."

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Appendix C: Different Measures of Cross-Sectional Dispersion

All the dispersion measures discussed below refer to the cyclical component when logging the series and detrending it using the HP-filter with $\lambda = 1,600$. The dispersion measures are computed for various firm samples in Compustat. The different samples are defined as follows: High sample - Only uses data for firms with more than 150 quarters of observations; Medium sample - Only uses data for firms with more than 100 quarters of observations; Low sample - Only uses data for firms with more than 60 quarters of observations; Full sample - All available firm observations are used in each time period.

From figure 2.9 it is apparent that the cross-sectional standard deviation of real and nominal sales are almost the same. For consistency with the macro series the dispersion in real sales will therefore be used. From figure 2.10 it is evident that the cross-sectional standard deviation of real sales has a seasonal component. Therefore, the original dispersion series is seasonally adjusted using the Census X12-Arima method before detrending.

From figure 2.11 it can be seen that the seasonally adjusted standard deviation of real sales is somewhat affected by how we restrict the sample of firms. It appears that the less we restrict the sample, the more volatile is the dispersion measure. However, the basic dynamics of the cross-sectional standard deviation are similar across the different sample selections. The main difference is that between the mid 1980's and mid 1990's the standard deviation for the larger samples is very volatile. It therefore seems appropriate to use the sample that only considers firms with more than 150 quarters of observations.

Figure 2.12 shows the cyclical properties of other measures of spread such as the variance, the interquartile range and the coefficient of variation. Logically, the variance has the same properties as the standard deviation just scaled. The interquartile range has similar dynamics, but seems to contain more noise than the standard deviation. The coefficient of variation is less volatile and seems to have somewhat different dynamics.

Figures 2.13 and 2.14 compare the interquartile range and the coefficient of variation for different sample sizes. We can see that the interquartile range gets more volatile as we increase the number of firms in the sample and at the same time the noise gets reduced. The coefficient of variation gets more noisy and volatile as we increase the sample size, similar to what was found for the standard deviation.

Given the considerations above, the baseline dispersion measure that is used in

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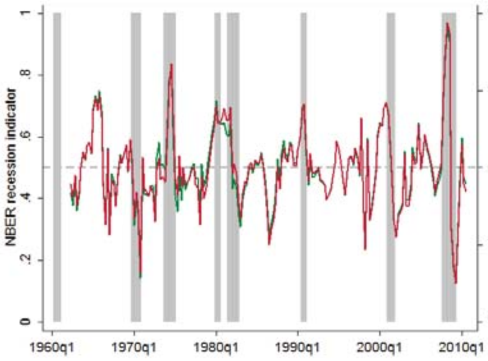
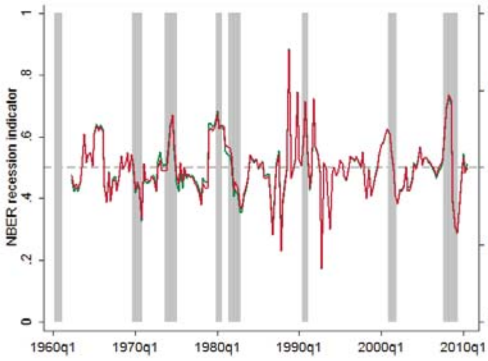
Figure 2.9: Comparison of the standard deviation between real and nominal sales

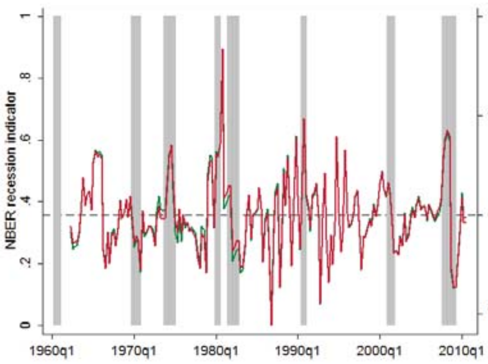
(a) High sample



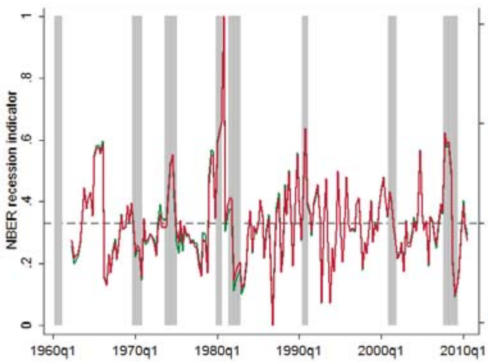
(b) Medium sample



(c) Low sample



(d) Full sample



the SVAR estimation is the cyclical component of the seasonally adjusted cross-sectional variance of real sales for the high sample.

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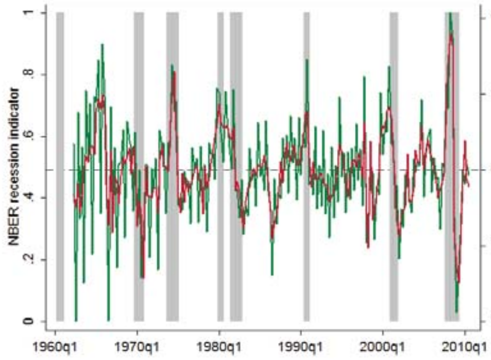
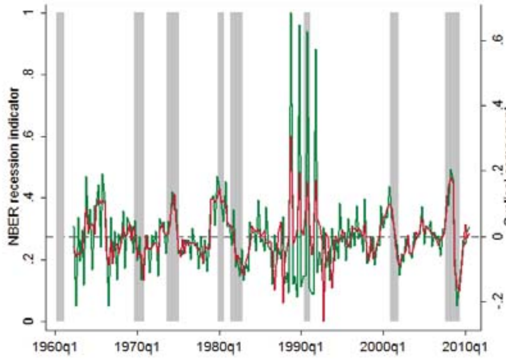
Figure 2.10: Comparison of the seasonally adjusted and unadjusted standard deviation

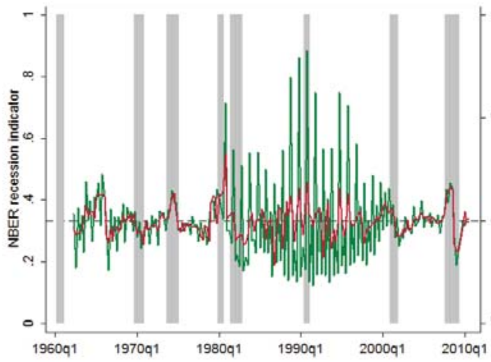
(a) High sample



(b) Medium sample



(c) Low sample



(d) Full sample

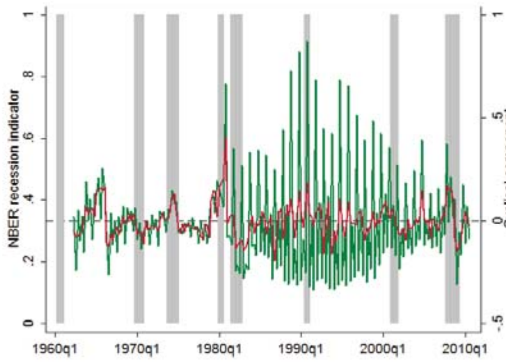


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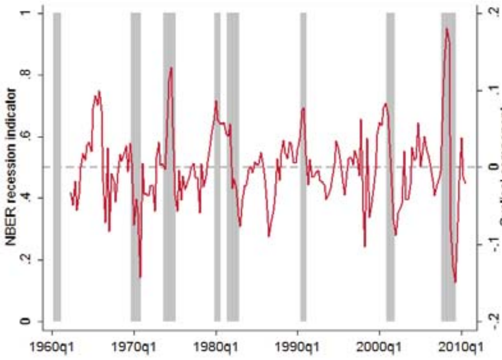
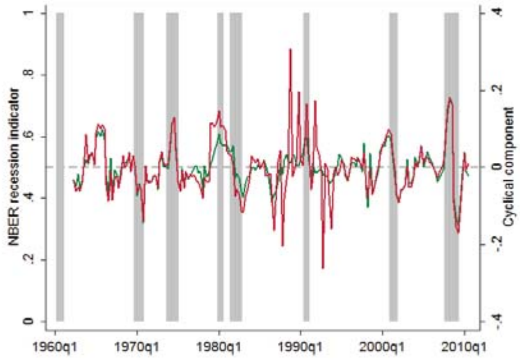
Figure 2.11: Comparison of the standard deviation depending on the sample

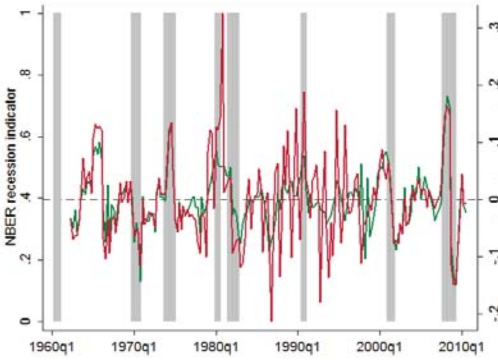
(a) High sample



(b) Medium sample



(c) Low sample



(d) Full sample

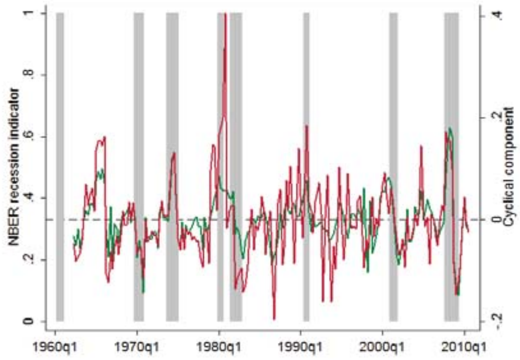


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Figure 2.12: Comparison of different measures of spread

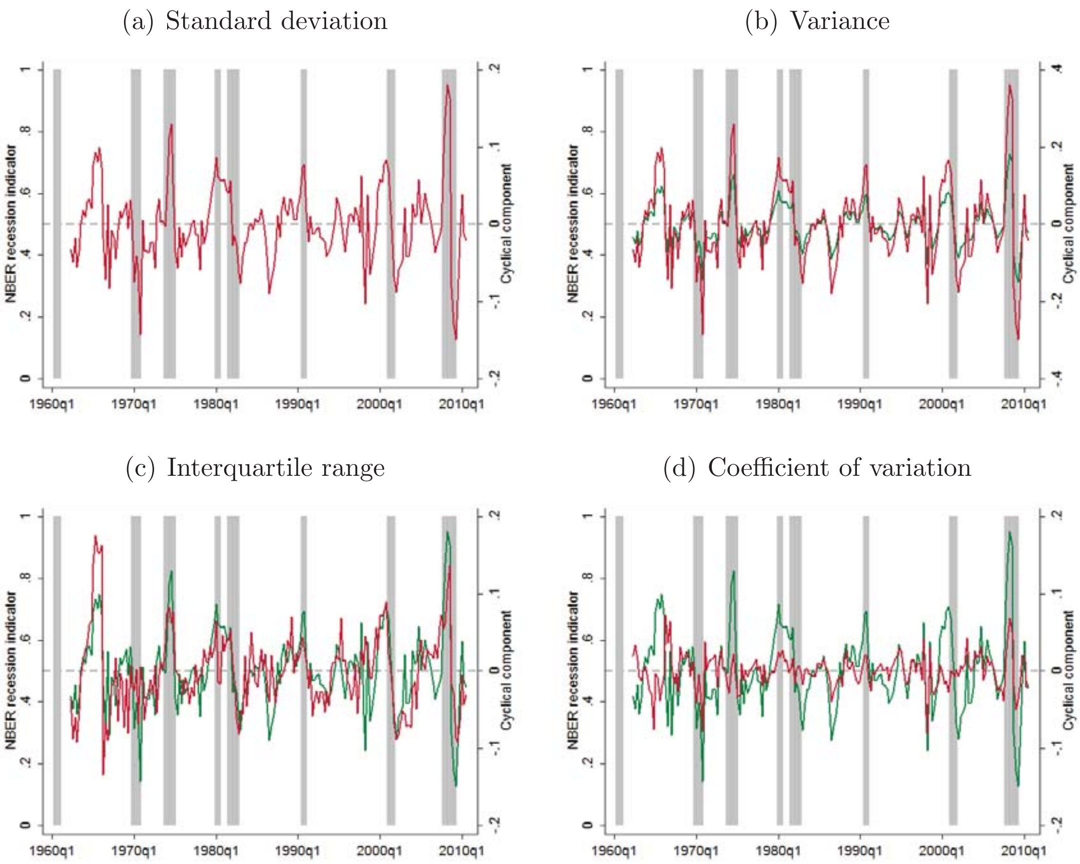

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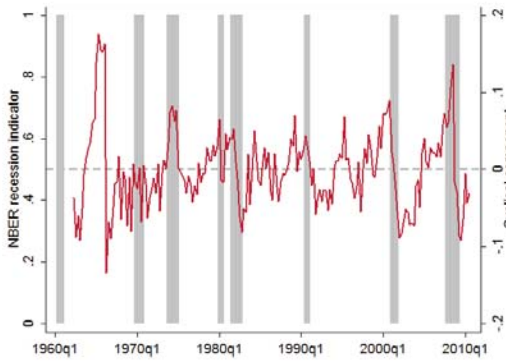
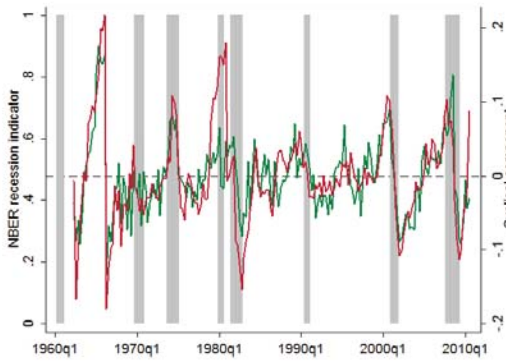
Figure 2.13: Comparison of the interquartile range depending on the sample

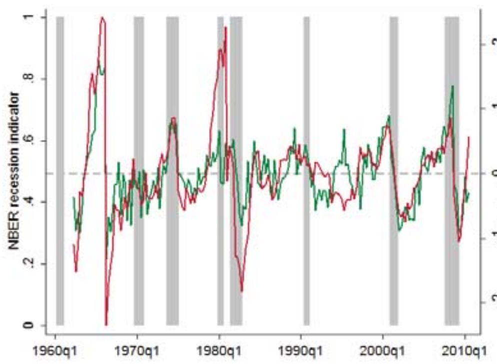
(a) High sample



(b) Medium sample



(c) Low sample



(d) Full sample

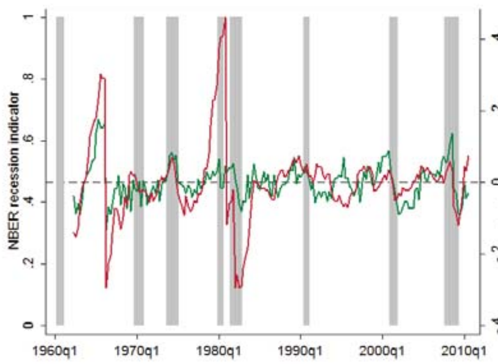


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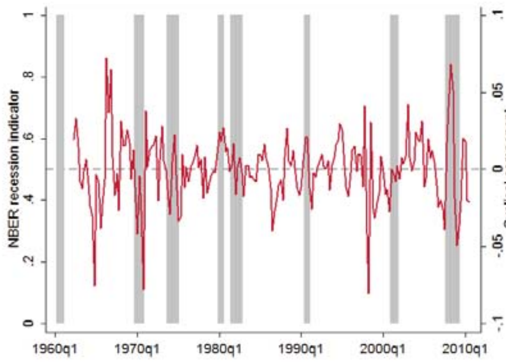
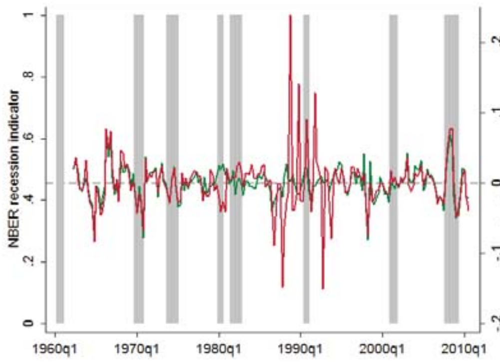
Figure 2.14: Comparison of the coefficient of variation depending on the sample

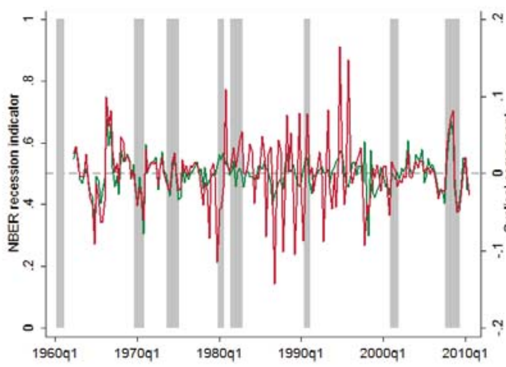
(a) High sample



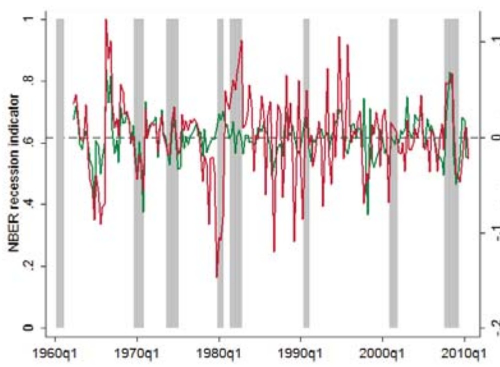
(b) Medium sample



(c) Low sample



(d) Full sample



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Chapter 3

Endogenous Variations in Uncertainty and Aggregate Investment in German Manufacturing

Abstract

This chapter studies endogenous variations in uncertainty and aggregate fluctuations that result from imperfect information and learning in an environment where regime changes in the mean happen occasionally. The idea behind this set-up is that whenever unprecedented regime shifts occur, agents become more uncertain about the true data generating process (DGP) and therefore mix different conditional distributions when forming expectations about the future. The German manufacturing industry actually experienced such an unprecedented regime shift during the Financial Crisis in mid 2008. Output collapsed by 25 % within just six months and expectations fell much more than can be explained by fundamentals. With this empirical background in mind a partial equilibrium heterogeneous firm model that features capital adjustment costs, a markov-switching driving process and imperfect information about the underlying regime is parameterized to German manufacturing data and simulated. There are two main findings that come out of the exercises. First, after a regime shift imperfect information leads endogenously to temporarily higher uncertainty about the underlying regime. On average this leads to lower mean forecasts and higher forecast standard errors compared to full information. Moreover, during the regime shift the dispersion in beliefs increases considerably, which causes the cross-sectional dispersion of mean forecasts and forecast standard

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errors to increase in turn. This mechanism could be interesting in order to explain why survey responses by firms and professional forecasters get more dispersed during downturns. Second, these endogenous variations in uncertainty can lead to considerable downward amplification and some propagation of aggregate investment and revenues during a temporary downward regime shift. This is true for all types of adjustment costs, but some degree of quadratic costs are needed to match the empirical volatility of investment.

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3.1 Introduction

The topic of time-varying uncertainty and aggregate fluctuations has become increasingly popular since the outbreak of the Financial Crisis in 2008. This is partly due to the fact that various measures of uncertainty like stock market volatility, firm level dispersion of revenues, profitability and price changes, forecaster disagreement and dispersion in firm level survey responses all go up during recessions[1]. The way that uncertainty shocks are usually modeled in the quantitative macro literature is via changes in the innovation variance of the driving process. In that sense they are an exogenous shock much like an aggregate productivity shock. There are various recent papers that explore this type of uncertainty in settings with capital and labor adjustment costs, financial frictions and costly price adjustment[2]. Most papers find a significant impact of uncertainty, although varying some of the assumptions such as the returns to scale of the revenue function or general equilibrium considerations do affect the results somewhat.

In reality uncertainty is usually elevated after major negative first moment shocks as shown in Bloom (2009) and reproduced in figure 3.1. For example, Oil crises, 9/11 and the recent Financial Crisis were all associated with high uncertainty as measured through stock market volatility. Moreover, the VDAX stock market volatility index for Germany increased to unprecedented levels after the outbreak of the Financial Crisis. It therefore seems reasonable to consider the possibility that major first moment shocks cause uncertainty to increase endogenously rather than simply assuming that there is a correlation between the aggregate shock and the uncertainty shock[3]. The underlying idea is that whenever large first moment shocks of unprecedented character hit the economy agents become more uncertain about the underlying data generating process (DGP). This uncertainty about the true underlying regime of the economy should in turn affect forecast distributions and dynamic decisions such as investment and hiring.

For example, when we look at the Financial Crisis in Germany the majority of the output collapse materialized in the manufacturing industry which experienced an unprecedented collapse of 25 % within just a few months between late 2008 and early 2009. This temporary regime shift is clearly visible in figure 3.1 panel (c), which plots the cyclical component of aggregate manufacturing revenues against

[1]See Bloom (2009), Bloom et al. (2010), Bachmann and Bayer (2011), Kehrig (2011), Berger and Vavra (2010), Dovern et al. (2009) and Bachmann et al. (2010) respectively.

[2]See Bloom et al. (2007), Bloom (2009), Bloom et al. (2010), Bachmann and Bayer (2011), Lang (2012), Arellano et al. (2011), Gilchrist et al. (2010), Chugh (2011), Dorofeenko et al. (2008) and Vavra (2012).

[3]This is for example done in Bloom (2009) and Bachmann and Bayer (2011)

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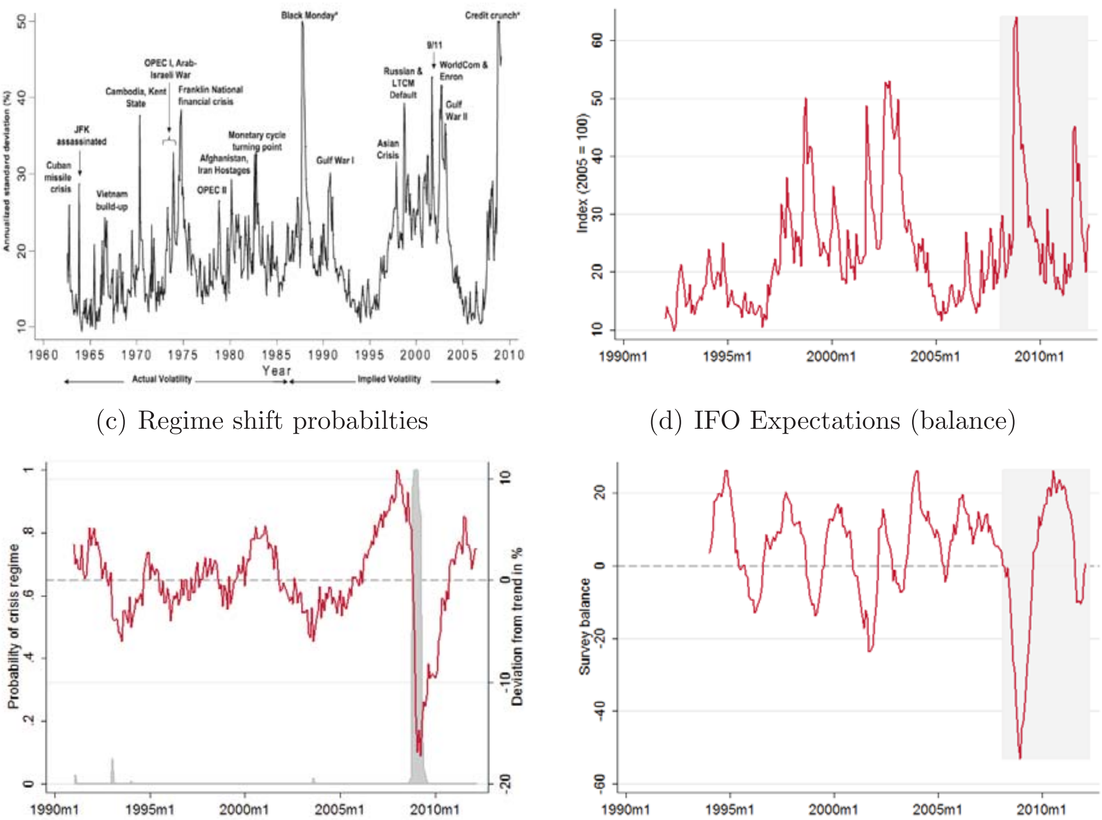
Figure 3.1: Stock market volatility for the U.S. and Germany



(a) VXO stock market volatility

(b) VDAX stock market volatility

(c) Regime shift probabilties

(d) IFO Expectations (balance)

Notes: Panel (a) is a copy of figure 1 from Bloom (2009) for the U.S. The VDAX is the stock market volatility index for Germany. The gray shaded areas in panel (b) indicate the time period since 2008m1. The Manufacturing revenues in panel (c) are in volume terms and refer to the cyclical component of the HP-filtered data when the standard smoothing parameter of 129,600 for monthly data is applied. Moreover, the smoothed probabilities in panel (c) are obtained from the estimated markov-switching model that is presented in section 3.2.2. Finally, panel (d) plots the IFO expectations balance for the German manufacturing sector. Source: Bloom (2009), German Federal Statistical Office, Datastream, Own calculations.

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the estimated probabilities of the Financial Crisis regime. The cause for the large output drop was the collapse in demand from the export and investment side of GDP, which are overrepresented in manufacturing.[4] Given such a sudden and large collapse in demand it seems reasonable to assume that there was some uncertainty regarding the depth and/or persistence of the Financial Crisis. Ex-post expectations actually turned out to fall by more than would have been warranted by developments in fundamentals during normal times, which is shown in section 3.2[5]. This could potentially indicate that firms were uncertain about the persistence of the regime shift and attached some probability that the Financial Crisis might last longer than it did in retrospect.[6]

With this empirical background in mind, this chapter addresses the following research questions. First, it is analyzed how imperfect information and Bayesian Learning affect the dynamics of expectations and uncertainty when the driving process is markov-switching and agents do not observe the underlying regime directly. Second, it is analyzed how these belief dynamics interact with capital adjustment costs in shaping the response of aggregate investment during a temporary large negative regime shift, as seen in German manufacturing during the Crisis. In order to answer these questions, a partial equilibrium, heterogeneous firm model is built that features various forms of capital adjustment costs, a markov-switching driving process, and imperfect information about the underlying regime of the process. Beliefs about the current regime are assumed to be updated by Bayesian Learning. Various indicative model parameterizations for the German manufacturing industry are then explored through simulations of a large negative temporary regime shift.[7]

There are two main findings in this chapter. First, imperfect information and Bayesian Learning lead endogenously to temporarily higher uncertainty in the sense that agents attach non-negligible probabilities to all of the possible regimes. I.e. there is more uncertainty regrading the underlying regime after a temporary regime shift. This higher endogenous uncertainty in turn affects the forecast densities used by firms considerably. In particular, on average the mean forecast and the forecast

[4]See section 3.2 for details.

[5]Panel (d) in figure 3.1 plots this expectations index for the manufacturing industry in Germany. As can be seen, expectations fell dramatically during the Financial Crisis but rebounded fairly quickly.

[6]Manufacturing revenues started their rebounded already after a few months of large drops indicating a short lived regime shift of around two quartes.

[7]A full fledged calibration or estimation using Simulated Method of Moments (SMM) is at this point not feasible due to the considerable computing power needed to solve and simulate the model with learning and time constraints of this thesis. In future research it is planned to use plant and firm level data for the German manufacturing industry in order to estimate adjustment costs along side the role of imperfect information and learning in shaping aggregate investment dynamics.

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standard error are amplified downwards and upwards respectively. The increase in the forecast standard error is mainly due to imperfect information and the resulting mixing of forecast densities under the different regimes and not due to a higher variance of shocks, which is assumed to stay constant throughout. Therefore, a temporary regime shift from a high mean regime to a low mean regime causes the forecast standard error to increase endogenously in the presence of learning. Moreover, during the regime shift the dispersion in beliefs increases considerably, as every firm gets in effect an idiosyncratic signal about the underlying regime shift in the form of idiosyncratic profitability. The increase in belief dispersion across firms in turn causes the cross-sectional dispersion of mean forecasts and forecast standard errors to increase. This mechanism could be interesting in order to explain why survey responses by firms and professional forecasters get more dispersed during downturns (See Bachmann et al. (2010) and Bloom (2009), Bloom et al. (2010), Dovern et al. (2009) respectively).

Second, these endogenous variations in uncertainty can lead to considerable downward amplification of aggregate investment and revenues during a temporary downward regime shift. This is true for all types of adjustment costs. However, in order to match the empirical volatility of aggregate investment some degree of quadratic adjustment costs is necessary. For such a parameterization with moderate fixed and quadratic adjustment costs and considerable irreversibilities aggregate investment drops more than twice as much under Bayesian Learning compared to full information. In addition, the persistence of the investment drop gets increased. This shows that endogenous variations in uncertainty that are due to imperfect information can have interesting amplification and propagation implications for aggregate investment and therefore also for revenues.

This chapter builds on various strands of literature. Most related is the literature on investment under time-varying uncertainty which was started in its modern form by the seminal paper of Bernanke (1983).[8] Interest in this topic has been revived again since the outbreak of the Financial Crisis and the publication of Bloom (2009). Compared to the recent quantitative literature on time-varying uncertainty which is based on exogenous shocks to the innovation variance, the current chapter considers variations in uncertainty as an endogenous response to regime shifts in the presence of imperfect information. The idea that information can play a role for aggregate fluctuations was already present in the early papers by Lucas (1972) and Kydland and Prescott (1982) and has recently been revived by Edge et al. (2007), Veldkamp (2005), Van Nieuwerburgh and Veldkamp (2006), Cogley and Sargent (2008),

[8]Other major references in this field include Abel (1983), Romer (1990), Caballero (1991), Demers (1991), Dixit and Pindyck (1994) and Hassler (1996).

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Lorenzoni (2009), Collard et al. (2009), Boz and Mendoza (2010), Bachmann and Moscarini (2011) and Moore and Schaller (2002). However, to my knowledge none of these papers have explored first as well as second moment implications of imperfect information. Moreover, the fact that I consider learning within a markov-switching model implies that there are occasional bursts of uncertainty rather than a permanent amplification of uncertainty that results from a recurring signal extraction problem with constant parameters. Finally, the chapter builds on the large literature on capital adjustment costs such as Hayashi (1982), Abel and Eberly (1994), Bertola and Caballero (1994), Abel and Eberly (1996), Caballero and Engel (1999), Thomas (2002), Veracierto (2002), Cooper and Haltiwanger (2006) and Khan and Thomas (2008). One of the findings of this chapter is that in order to match the aggregate investment volatility, some degree of quadratic adjustment costs is needed.

The remainder of the chapter is structured as follows. In the next section the empirical background for this chapter is provided though an analysis of the developments in the German manufacturing industry with a focus on the recent Financial Crisis. Section three then outlines the structural model of investment that is used as the analytical framework in this chapter. This is followed by an analysis of how imperfect information and learning induce endogenous variations in uncertainty in the presence of regime shifts. Building on these results, section five analyses the effects of endogenous variations in uncertainty on aggregate investment in an indicative parameterization of the model to the German manufacturing industry. Finally, section six offers a brief conclusion.

3.2 The German Manufacturing Industry and the Financial Crisis

The goal of this section is to give an overview of the developments in the German manufacturing industry with a particular focus on the Financial Crisis that started in 2008. This section therefore provides the motivating facts for analyzing a structural model of investment that features regime shifts and potentially imperfect information in the rest of the chapter. First, a brief overview of the general macroeconomic background is given, followed by evidence that the collapse in manufacturing activity was of unprecedented magnitude and can therefore be modeled as a regime shift. This is followed by some evidence that expectations fell more than usual during the worst months of the Financial Crisis. Finally, the response of aggregate investment after the Financial Crisis is analyzed.

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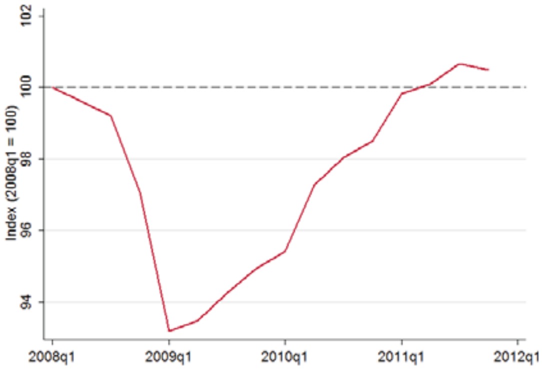
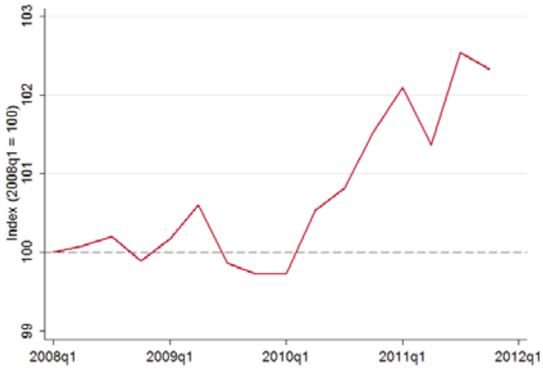
Figure 3.2: Developments in real GDP and it's components since 2008

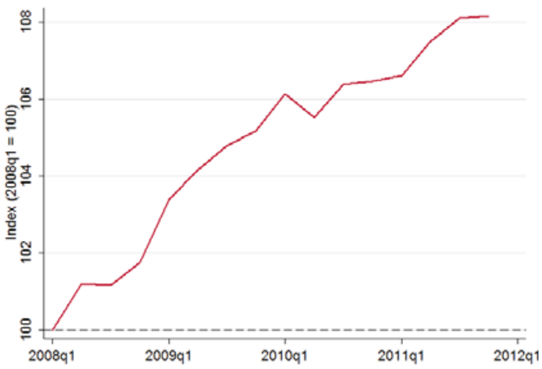
(a) Real GDP



(b) Real consumption



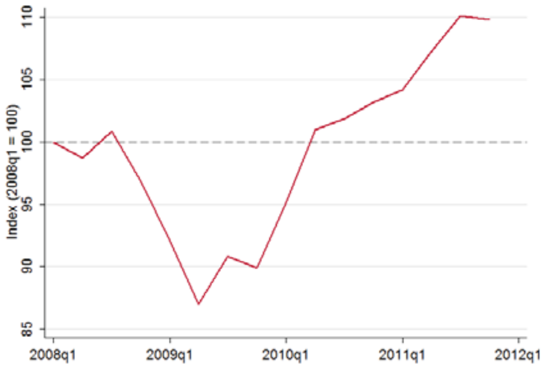
(c) Real government expenditure



(d) Real investment

(e) Real exports

(f) Real imports



Source: German Federal Statistical Office

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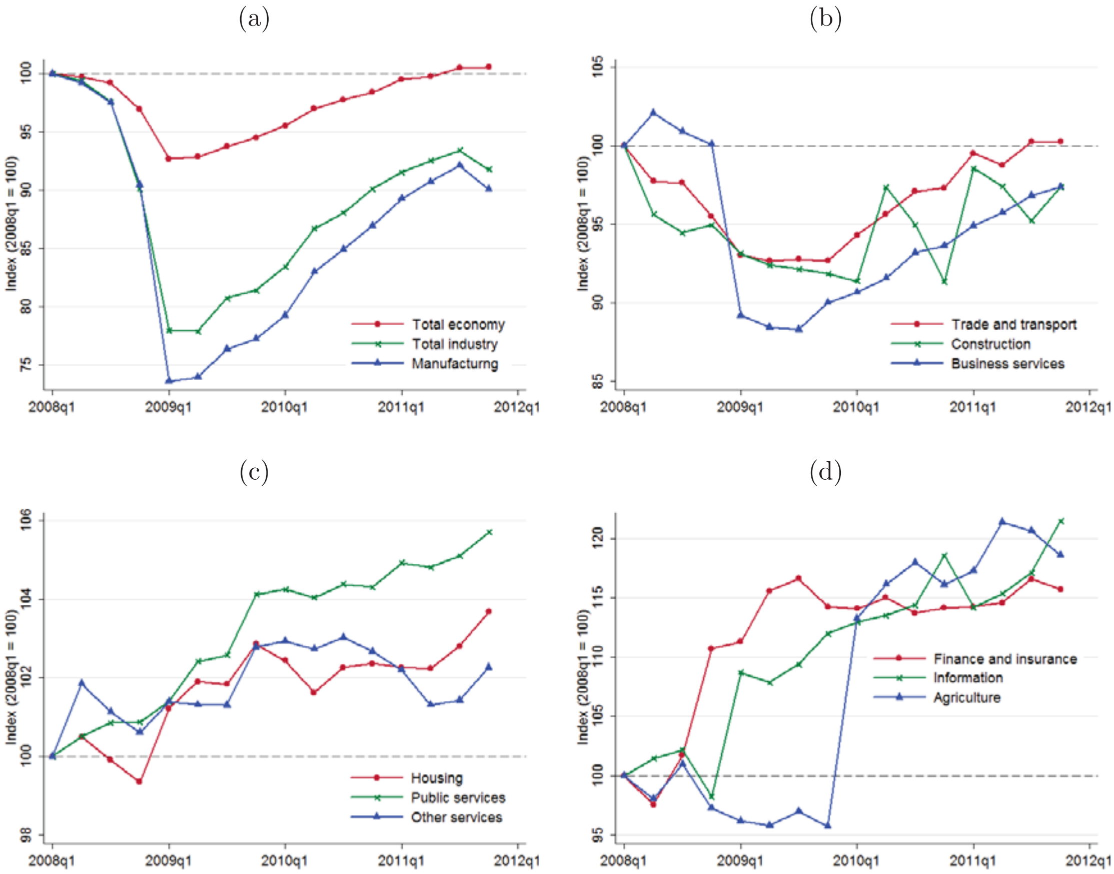
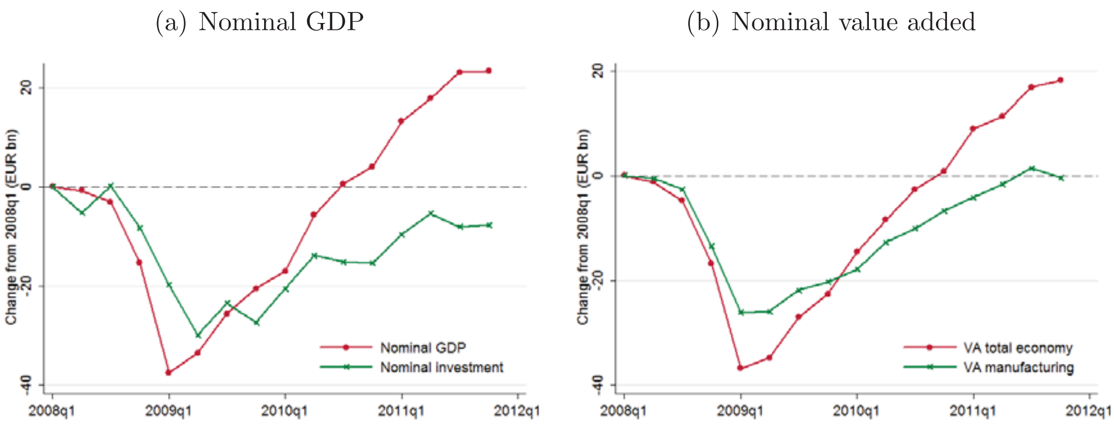
Figure 3.3: Developments in real value added by sector since 2008



Source: German Federal Statistical Office

Figure 3.4: Contributions to the fall in nominal GDP and value added



Source: German Federal Statistical Office

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3.2.1 General Macroeconomic Background

The German economy was hit extremely hard by the global Financial Crisis that started in 2008. Real GDP contracted by 6.8 % between it's cyclical peak in the first quarter of 2008 and it's cyclical trough in the first quarter of 2009. Furthermore, it took until the second quarter of 2011 for real GDP to reach it's previous peak again and at the end of 2011 real GDP was only 0.5 % above the level four years earlier. Figure 3.2 illustrates this development along with the dynamics of the main demand components of GDP. A detailed description of the data sources used throughout the chapter can be found in the Appendix.

On the demand side, the large drop in real GDP was caused by huge declines in investment and in exports, which fell by a maximum of 22.3 % and 17.3 % respectively. While real investment in the fourth quarter of 2011 was still 3.9 % below it's previous peak, real exports surpassed that peak in the first quarter of 2011 and were 5.3 % higher at the end of 2011. In contrast to this, private consumption and government expenditure did not contribute at all to the fall in real GDP. Private consumption did not change much between 2008 and 2010 and subsequently rose by 2.3 % until the end of 2011. Finally, government expenditure rose steadily since the outbreak of the Financial Crisis and was 8.2 % higher at the end of 2011 than in 2008.[9]

Naturally, the developments in aggregate demand components during the Crisis are reflected in the dynamics of industry value added (VA). As figure 3.3 shows, manufacturing was by far the hardest hit sector of the German economy, contracting by more than one fourth in the year after 2008q1. This large drop is not surprising given that the manufacturing industry produces many of Germany's exports and investment goods. Moreover, at the end of 2011 real VA was still almost 10 % below it's pre-crisis level. Other industries that were negatively affected by the crisis were trade and transport, constrcution and business services. However, their relative declines were much smaller and only ranged from around 7 % to 12 % at their peak. In contrast, the rest of the industries in Germany experienced considerable growth since the outbreak of the crisis.[10]

In summary, we can therefore say that on the demand side most of the action during the financial crisis in Germany happened in exports and aggregate investment, while on the supply side manufacturing witnessed by far the greatest decline. Figure 3.4 illustrates that the absolute changes in nominal investment and man-

[9] The dynamics of nominal GDP are not shown here but they are qualitatively the same.

[10] Again, the dynamics of nominal VA are similar to those of real VA but are not shown here. The only major difference is in the dynamics of agriculture and finance and insurance, but both industries only account for a very small share of total VA.

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ufacturing VA indeed go a long way in accounting quantitatively for the drops in aggregate GDP and VA. To be precise, in the second quarter of 2009 the absolute drops in investment and manufacturing VA were 89 % and 75 % of the drops in the respective aggregates.

3.2.2 Evidence of a Regime Shift during the Financial Crisis

The depth and speed of the decline in the German manufacturing industry during the Financial Crisis was of unprecedented character. Between the cyclical peak in January 2008 and the cyclical trough in April 2009 the volume of aggregate revenues in the manufacturing industry declined by 25 %. The majority of this decline of 22 % took place within just six months between August 2008 and February 2009. Since April 2009 aggregate revenues have been on a slow but steady rebound, but were still 6 % below the previous peak in March 2012. This abrupt collapse in revenues is clearly visible in figure 3.5 which plots the development of revenues in the German manufacturing sector since 1991. It is also evident that this drop in activity was much larger than any previous downturn no matter whether we look at revenues in levels, month-on-month growth rates, year-on-year growth rates or at the cyclical component.[11]

Table 3.1: Parameter values of the estimated markov-switching model

Parameter	Estimated Value	Description
μ_1	0.001 (0.23)	Intercept in regime 1
μ_2	-0.056 (0.00)	Intercept in regime 2
ρ_1	0.908 (0.00)	AR(1) parameter in regime 1
ρ_2	0.725 (0.00)	AR(1) parameter in regime 2
σ_1	0.014 (0.00)	Std of innovations in regime 1
σ_2	0.026 (0.12)	Std of innovations in regime 2
ω_{11}	0.995 (0.00)	Transition probability 1 to 1
ω_{12}	0.005 (0.33)	Transition probability 1 to 2
ω_{21}	0.195 (0.33)	Transition probability 2 to 1
ω_{22}	0.805 (0.08)	Transition probability 2 to 2

Notes: Values in parenthesis refer to the p-value of the respective parameter. The model was estimated on HP-filtered data of the volume of revenues, where the standard smoothing parameter of 129,600 for monthly data was used.

[11]This extraordinarily large and quick drop is also evident in the value of reveneus, revenues abroad, domestic revenues, output, orders, productivity and producer prices which can be seen from figures 3.16 in the Appendix.

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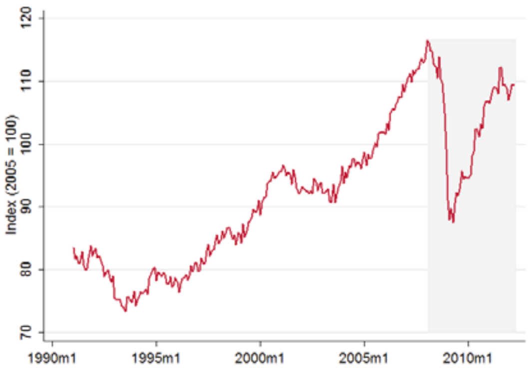
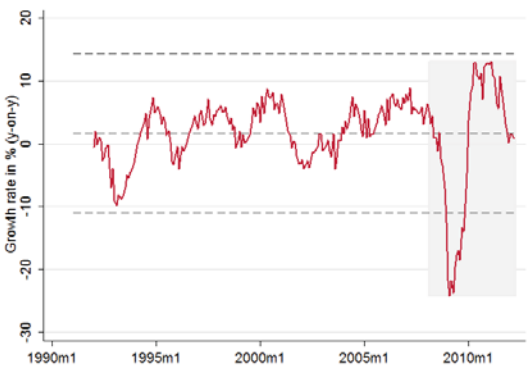
Figure 3.5: Volume of revenues in German manufacturing since 1991

(a) Level

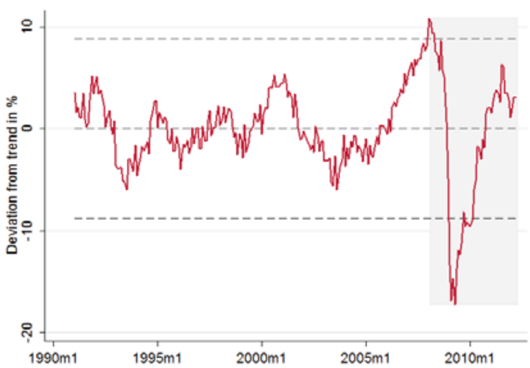


(b) m-on-m growth

(c) y-on-y growth



(d) Cyclical component



Notes: The y-on-y growth rate refers to the growth rate in each month with respect to the same month in the previous year. The cyclical component was constructed with the HP-filter using the standard smoothing parameter of 129,600 for monthly data. The dashed lines in panels (b), (c) and (d) represent the sample mean and two standard deviation bands calculated over the entire sample from 1991m1 to 2012m3. The gray shaded areas indicate the time period since 2008m1. Source: German Federal Statistical Office, Own calculations.

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In order to formally investigate the presence of a regime shift in the German manufacturing industry during the Financial Crisis a markov-switching AR(1)-model is estimated for the period 1991m1 to 2012m3. As aggregate revenues contain a unit root and because the model in section 3.3 is specified with a stationary driving process, the model is estimated on HP-filtered data, where the standard smoothing parameter of 129,600 for monthly data is used.[12] The model is specified so as to have two possible regimes and all the parameters are allowed to switch between the regimes. The results of this estimation can be found in table 3.1.[13]

As can be seen the estimation results show that there is one highly persistent regime with a zero intercept and a moderate variance, and a transitory regime with a significantly negative intercept and a higher variance. The transitory regime with the negative intercept can be interpreted as the Financial Crisis which can be seen from figure 3.6 panel (a) which plots the smoothed probabilities of the second regime. The probability of this regime jumps up to almost one in late 2008 and early 2009 when the large drops in manufacturing activity materialized in Germany. Hence, the formal estimation of a markov-switching model confirms the qualitative evidence presented above that the manufacturing industry in Germany experienced an unprecedented fall in revenues during the Financial Crisis.

3.2.3 The Response of Expectations during the Financial Crisis

One of the questions that immediately comes to mind when looking at the regime shift in manufacturing during the Financial Crisis is how it affected firms' expectations regarding future demand and revenues. Given that such a large and sudden drop in revenues had never happened before it seems reasonable to assume that there was some uncertainty regarding the depth and persistence of this regime shift. In retrospect it is fairly safe to say that the regime shift during the Financial Crisis was a temporary phenomenon, which is confirmed by the estimated markov-switching model in the previous subsection. However, it is at least conceivable that in real-time firms did not know this for sure. In principle, the more persistent the regime shift was perceived to be, the lower expectations about the future should have been.

Luckily, the IFO Institute publishes an index of expectations about the business

[12]All three versions of the augmented Dickey-Fuller test with a drift term, a trend term and without any additional term do not reject the null hypothesis of a unit root at the 5 % level of significance.

[13]The estimation was performed in MATLAB with the MS-Regress toolbox provided by Perlin (2012).

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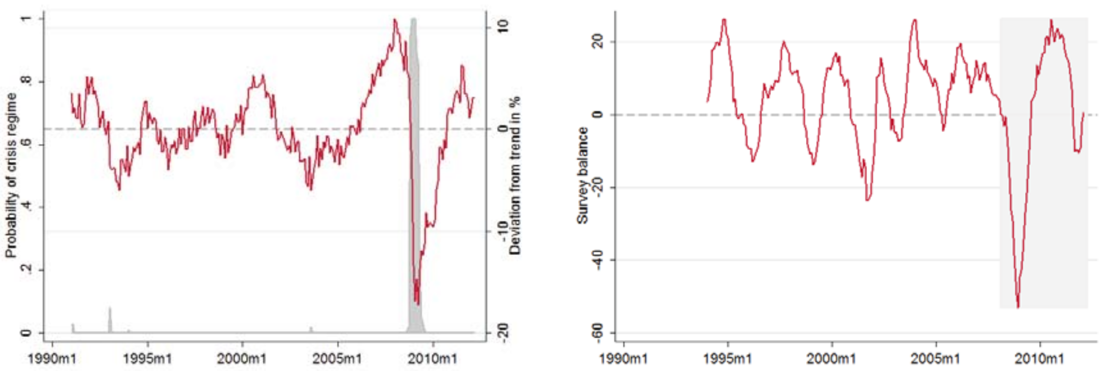
Figure 3.6: Evidence of a regime shift and the response of expectations

(a) Regime shift probabilties (b) IFO Expectations (balance)



Notes: The gray shaded regime shift probabilities in panel (a) are the smoothed probabilities from the estimated markov-switching process. The gray shaded area in panel (b) indicates the time period since 2008m1. Source: German Federal Statistical Office, Datastream, Own calculations.

development in the next six months for the German manufacturing industry. This index is based on the balance of firms responding that they expect business to improve minus the balance of firms that expect business to contract. As can be seen from figure 3.6 panel (b), this expectations index witnessed an unprecedented drop during the Financial Crisis, which is not surprising given the collapse in manufacturing revenues. In order to explore whether expectations dropped by more than usual during the Financial Crisis, the IFO expectations index was regressed on various fundamental variables and a dummy variable that takes a value of one during the worst crisis months.

The results of this exercise can be found in table 3.2. The estimated model for expectations has a fairly high R^2 of 0.958 and diagnostic tests show no indication of autocorrelation, heteroskedasticity, non-normality or non-linearity. As is evident expectations in the manufacturing industry were significantly lower during the Financial Crisis than what would have been the case if the historical relationship between fundamentals and expectations had prevailed. This is a possible indication that firms were expecting the regime shift to be quite persistent in the initial months of the Financial Crisis.

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Table 3.2: Results of regressing expectations on fundamentals

Variable	Estimated Coefficient	Description
$Intercept$	2.069 (0.001)	Constant term
IFO_{t-1}	0.994 (0.000)	First lag of IFO expectations
IFO_{t-4}	-0.182 (0.000)	Fourth lag of IFO expectations
g_orders_t	0.270 (0.000)	y-on-y growth rate of orders
$g\_revenues_t$	-0.296 (0.001)	y-on-y growth rate of revenues
$vdax_t$	-0.060 (0.010)	DAX stock volatility index
$crisis_t$	-5.905 (0.000)	Dummy for worst crisis months

Notes: The model was estimated on monthly data for the time period 1994m5 - 2011m12. The estimated model has an R^2 value of 0.958. Lags one and four of the IFO expectations variable are needed in order to avoid autocorrelation of the error term. Lags two and three are both insignificant and were therefore omitted. Other diagnostic tests showed no indication of heteroskedasticity, non-normality or non-linearity. The dummy variable for the Financial Crisis takes a value of one for the period 2008m7 to 2008m12. Values in parenthesis refer to the p-value of the respective coefficient. Data sources: Datastream, German Federal Statistical Office.

3.2.4 The Behavior of Aggregate Investment

Finally, it is instructive to look at how investment in the manufacturing industry has behaved during the Financial Crisis. Unfortunately quarterly investment data is only available for the economy as a whole. However, yearly investment data is available from 1995 to 2010 for 26 manufacturing sub-industries. Based on this data aggregate time series were constructed for total investment and machinery investment which makes up the major part of total investment in the manufacturing industry. As can be seen from figure 3.7 total investment in the manufacturing industry dropped by 22 % between 2008 and 2009. Moreover, total investment hardly recovered in 2010 and was still 21 % below its previous peak in 2008. The dynamics for machinery investment are very similar with a drop of 23 % between 2008 and 2009.

If we compare the yearly investment data in the manufacturing industry to the quarterly data for the total economy in figure 3.2 it is evident that the huge drop in 2009 is similar in magnitude. However, manufacturing investment remained subdued in 2010 whereas aggregate investment rebounded somewhat. In general, the speed and magnitude of the investment drop in the manufacturing industry during the Financial Crisis was unprecedented. In the majority of cases the year to year changes in investment were well below 10 %, with the exception of the investment bust after the stock market collapse between 2001 and 2002 and the investment boom in 2007 which saw investment changes of 13 % and 15 % respectively.

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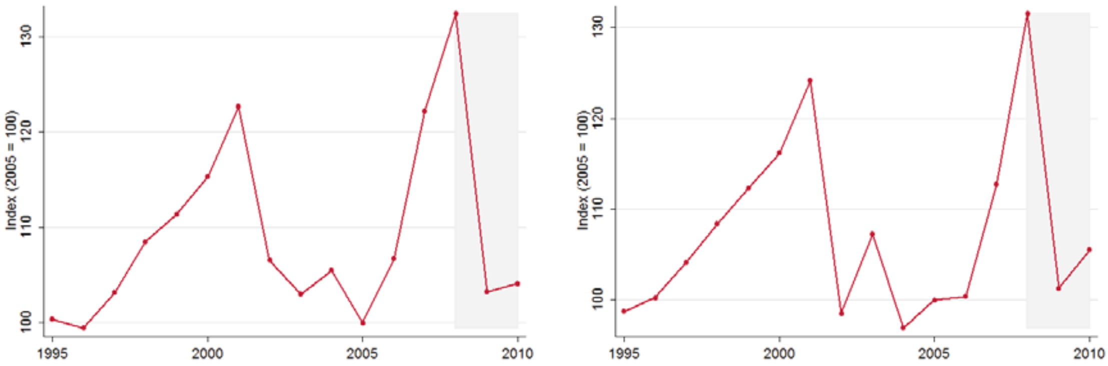
Figure 3.7: Aggregate investment in the manufacturing industry since 1995

(a) Total Investment (b) Machinery Investment



Notes: The time series were constructed from aggregate data of plant level investment activity for 26 manufacturing sub-industries. The source data has a break in 2008 due to a change in industry classifications. Figures after 2008 are therefore rescaled so as to provide a continuous series at the break point in 2008. Data sources: German Federal Statistical Office, Own calculations.

3.3 Outline of the Basic Model

In this section I present a partial equilibrium heterogeneous firm model of investment similar to Cooper and Haltiwanger (2006) and Bloom (2009). The firm faces convex, and non-convex adjustment costs to investment, as well as partial investment irreversibilities. Compared to the model of Bloom (2009), labor adjustment costs are disregarded which makes the labor input decision static and allows us to disregard it in the analysis. This is mainly done to keep the model simple and focus on the core mechanism at work. Such a model specification seems like a natural starting point as Bloom (2009) has shown that disregarding investment adjustment costs significantly biases the model, while disregarding labor adjustment costs has only second order effects. The main difference of my model compared to the literature on uncertainty shocks is that I make different assumptions about the profitability process that firms face and regarding the information agents have when they make their investment decisions.[14] To be more precise, profitability is assumed to follow a markov switching process with a switching mean and a constant variance of the innovations. Moreover, firms are assumed not to observe the current regime of the markov switching process directly. They simply observe the current realization of

[14]Profitability in this chapter is conceptually equivalent to what Bloom (2009) refers to as business conditions. This difference in labeling is due to the fact that Bloom (2009) works with a revenue function that takes the employment decision as an input whereas in this chapter the firm faces a reduced form profit function, where the optimal decision of employment has already been incorporated.

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profitability and need to form beliefs about the underlying regime of the markov switching model.

This model set-up is motivated by the type of developments that were witnessed in the German manufacturing industry during the Financial Crisis. The basic underlying idea of this specification is that variations in uncertainty are mainly driven by variations in perceived uncertainty rather than by variations in the actual variance of the driving process as is done in the existing uncertainty shocks literature. Perceived uncertainty in this model will be elevated whenever agents are not sure which regime has produced the realization of profitability, so that they mix the different conditional densities of the regimes when making forecasts of future profitability.

In the following subsections, I will first lay out the details of the firm's decision problem, and then describe the profitability process and the updating of beliefs in more detail. Finally, the Bellman Equation of the firm will be presented, which is used for the value function iteration in later sections in order to solve the model.

3.3.1 Specification of the Firm's Profit Function

There is a continuum of risk neutral firms indexed by $i \in [0, 1]$ who maximize the present discounted value of expected profit streams. In each period, the only dynamic decision that the firm faces is to decide on the capital stock for the next period. Labor can be adjusted freely in every period so that it is not part of the dynamic optimization problem. The only source of uncertainty that firms face is the profitability of capital in the future. The profit function takes the following functional form:

$$\Pi(A_{i,t}, K_{i,t}) = \phi_\pi A_{i,t}^{\frac{c}{1-b}} K_{i,t}^{\frac{a}{1-b}} \tag{3.1}$$

In the above equation, $\phi_\pi = (1-b)b^{[b/(1-b)]}w^{[-b/(1-b)]}$ is a constant parameter, $K_{i,t}$ is the capital stock and $A_{i,t}$ is a reduced form profitability shock that summarizes the effects of demand conditions and total factor productivity on profits. In addition, $c/(1-b) > 0$ and $a/(1-b) \in (0, 1)$ are constant parameters that determine the curvature of the profit function.[15] Such a profit function can be derived under the assumption of a decreasing returns to scale (DRS) revenue function in capital and labor, a constant wage (w) and freely adjustable labor that becomes immediately available for production.[16] Given this set-up it is easy to show that the optimal labor input and the resulting revenues have the same functional form in (A, K) space as

[15] Here, a, b and c are the exponents on capital, labor and profitability in the revenue function.

[16] A DRS revenue function in capital and labor can be either due to DRS in the production function and/or some degree of market power. See Lang (2012) for a detailed derivation of this profit function.

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profits and only differ by a constant:[17]

$$L(A_{i,t}, K_{i,t}) = \phi_l A_{i,t}^{\frac{c}{1-b}} K_{i,t}^{\frac{a}{1-b}} \tag{3.2}$$

$$R(A_{i,t}, K_{i,t}) = \phi_r A_{i,t}^{\frac{c}{1-b}} K_{i,t}^{\frac{a}{1-b}} \tag{3.3}$$

3.3.2 Specification of Capital Adjustment Costs

The firm's capital stock is fixed within each period, as it is assumed to take one period for new capital to be installed and ready for production. Moreover, capital is assumed to depreciate at the rate δ per period, so that the law of motion for the capital stock is given by the following equation, where I denotes the level of investment:

$$K_{i,t+1} = K_{i,t}(1-\delta) + I_{i,t} \tag{3.4}$$

In line with the papers by Cooper and Haltiwanger (2006) and Bloom (2009) it is assumed that the firm faces convex and non-convex costs of adjusting the capital stock, as well as partial investment irreversibilities.[18] Both, adjustment costs and partial irreversibilities bring interesting non-linear dynamics to the capital accumulation process and in particular will determine how variations in uncertainty affect investment decisions. The adjustment cost function and the price of capital are represented by the following equations:

$$C(A_{i,t}, K_{i,t}, I_{i,t}) = \frac{\gamma}{2}(I_{i,t}/K_{i,t})^2 K_{i,t} + (1-\lambda)\Pi(A_{i,t}, K_{i,t})1_{\{I_{i,t}\neq 0\}} \tag{3.5}$$

$$p(I_{i,t}) = \begin{cases} p_s & , if\ I_{i,t} < 0 \\ p_b & , if\ I_{i,t} > 0 \end{cases} \tag{3.6}$$

The convex adjustment costs are assumed to be quadratic while the non-convex adjustment costs are specified as a fraction of current profits $(1-\lambda)$.[19] Finally, p_s denotes the selling price of capital and p_b the buying price and it is assumed that $p_s < p_b$ so that there are partial irreversibilities for investment.

[17]The constants are given by $\phi_l = b^{[1/(1-b)]}w^{[-1/(1-b)]}$ and $\phi_r = b^{[b/(1-b)]}w^{[-b/(1-b)]}$ respectively.
[18]Both papers find that all these forms of capital adjustment frictions are needed to match the micro data on investment behavior by plants/firms.
[19]$1_{\{I_{i,t}\neq 0\}}$ is an indicator function that take a value of 1 whenever investment is nonzero, and a value of 0 otherwise.

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3.3.3 The Profitability Process

The profitability of each firm i is assumed to follow an AR(1) markov-switching process in logs with an aggregate and an idiosyncratic component:

$$A_{i,t} = Z_t \Psi_{i,t} \tag{3.7}$$

$$z_t = \mu_z + \rho_z z_{t-1} + \sigma_\eta \eta_t \tag{3.8}$$

$$\psi_{i,t} = \mu_{\psi,s_t} + \rho_{\psi,s_t} \psi_{i,t-1} + \sigma_{\upsilon,s_t} \upsilon_{i,t} \tag{3.9}$$

Here, a lower case letter refers to the logarithm of the variable. Moreover, $s_t \in S = \{1, ..., n\}$ denotes the current regime of the idiosyncratic profitability process, n is the total number of regimes, and $\eta_t \sim \mathcal{N}(0,1)$ and $\upsilon_{i,t} \sim \mathcal{N}(0,1)$. The dynamic evolution of the regimes is assumed to follow a markov process with transition matrix Ω with generic element $Pr(s_{t+1} = j | s_t = k) = \omega_{kj}$. The markov-switching process is specified in a general way so that in principle all parameters could switch between the different regimes. However, in the applied sections further below a parameterization is used that features only a switch in the mean of the process from a high state to a low state. Compared to the existing literature[20] I therefore do not include any exogenous variations in uncertainty via the variance of shocks. There are hence no changes in the cross-sectional dispersion of profitability across firms as is normally the case for uncertainty shocks. This channel is usually referred to as the volatility effect that materializes as a higher shock variance leads to more dispersed fundamentals. However, there will be an effect on forecast densities of firms during regime shifts due to the fact that it is assumed that firms do not observe the underlying regime of the markov-switching process directly. This imperfect information induces learning dynamics that cause endogenous variations in uncertainty and expectations.

3.3.4 Bayesian Updating of Beliefs

Agents in the model are assumed to know the true data generating process (DGP) given in equations (3.7) - (3.9) including all it's parameter values. However, it is also assumed that agents do not directly observe the true underlying regime that has produced the current realization of idiosyncratic profitability. Therefore, in every period they need to form beliefs about the underlying regime, after they have observed the realization of profitability. Knowing the current regime of the idiosyncratic profitability process matters for the decision of the firm because the

[20] See for example Bloom (2009), Bloom et al. (2010), Bachmann and Bayer (2011), Arellano et al. (2011), Gilchrist et al. (2010) or Vavra (2012).

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forecast density for next period's profitability will depend on it. In this chapter it is assumed that firms update their beliefs about the current regime of the idiosyncratic profitability process using Bayes' Law.

In order to work out an analytical representation for this updating process, it is useful to first define a couple of objects. Let $\mathcal{B}_{i,t}(j)$ be the subjective probability at date t that firm i assigns to being in regime $s_t = j$ after observing the realization of current idiosyncratic profitability $\psi_{i,t}$. The associated probability mass function at time t over all possible regimes is denoted by $\mathcal{B}_{i,t}$. By definition of probabilities, we have that $\sum_{j=1}^{n} \mathcal{B}_{i,t}(j) = 1$ and $\mathcal{B}_{i,t}(j) \geq 0$. In the language of Bayesian updating, $\mathcal{B}_{i,t}(j)$ is the posterior belief of firm i about being in regime j after observing the signal $\psi_{i,t}$. Using Bayes' Law this can be expressed as:

$$\mathcal{B}_{i,t}(j) = Pr(s_t = j|\psi_{i,t}, \psi_{i,t-1}, \mathcal{B}_{i,t-1}) = \frac{f(\psi_{i,t}|\psi_{i,t-1}, s_t = j)Pr(s_t = j|\mathcal{B}_{i,t-1})}{f(\psi_{i,t}|\psi_{i,t-1}, \mathcal{B}_{i,t-1})} \tag{3.10}$$

In the equation above, $f(\cdot)$ denotes the density or likelihood of idiosyncratic profitability, while $Pr(\cdot)$ denotes the probability of a particular regime. Given the transition matrix Ω for the regimes and a prior belief vector $\mathcal{B}_{i,t-1}$, the probability of moving to a particular regime j is simply given by the sum of all the transition probabilities that lead to this state, weighted by their respective subjective prior probability:

$$Pr(s_t = j|\mathcal{B}_{i,t-1}) = \sum_{k=1}^{n} \mathcal{B}_{i,t-1}(k)\omega_{kj} \tag{3.11}$$

Moreover, given the process for idiosyncratic profitability defined in equation (3.9), the likelihood of current profitability conditional on regime j prevailing is given by:

$$\lambda_{i,t|j} = f(\psi_{i,t}|\psi_{i,t-1}, s_t = j) = \frac{1}{\sqrt{2\pi\sigma^2_{\upsilon,j}}} \exp\left[-\frac{(\psi_{i,t} - \mu_{\psi,j} - \rho_{\psi,j}\psi_{i,t-1})^2}{2\sigma^2_{\upsilon,j}}\right] \tag{3.12}$$

Finally, the likelihood of current profitability conditional on the subjective prior beliefs about the underlying regime can be expressed as the sum of the conditional likelihoods under each regime j weighted by the respective subjective probability of moving to that regime. This implies that the perceived likelihood of idiosyncratic profitability in period t, given information up to and including period $t-1$, can be expressed as:

$$\lambda_{i,t} = f(\psi_{i,t}|\psi_{i,t-1}, \mathcal{B}_{i,t-1}) = \sum_{j=1}^{n}\sum_{k=1}^{n} \mathcal{B}_{i,t-1}(k)\omega_{kj}\lambda_{i,t|j} \tag{3.13}$$

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With these objects in hand, it is easy to rewrite the Bayesian updating process defined in equation (3.10) as:

$$\mathcal{B}_{i,t}(j) = \frac{\sum_{k=1}^{n} \mathcal{B}_{i,t-1}(k)\omega_{kj}\lambda_{i,t|j}}{\lambda_{i,t}} \tag{3.14}$$

This updating equation simply states that the posterior belief about regime $s_t = j$, after observing current idiosyncratic profitability $\psi_{i,t}$, is equal to the likelihood of observing $\psi_{i,t}$ conditional on regime j multiplied by the prior subjective probability of moving to that regime divided by the unconditional likelihood of observing $\psi_{i,t}$.[21] Moreover, equation (3.13) at lead one is used by firms to construct the forecast density for idiosyncratic profitability in the next period. At this point it is useful to note that $\mathcal{B}_{i,t} = h(\psi_{i,t}, \psi_{i,t-1}, \mathcal{B}_{i,t-1})$, i.e. the posterior beliefs about the underlying regime are a function of past and present idiosyncratic profitability and last period's beliefs.

3.3.5 The Bellman Equation of the Firm

In order to isolate the effects of uncertainty due to imperfect information from the effects due to changes in valuations of profit streams, firms are assumed to be risk neutral and discount future profits with a constant discount factor β. In principle it would be desirable to incorporate stochastic changes in the wage and the interest rate in order to account for general equilibrium effects of large first moment shocks. Due to computational constraints this has not been incorporated into the model yet, but is planned to be included in future research. Given the objects defined in the previous subsections, the dynamic decision problem of each firm can therefore be summarized by the following Bellman Equation, where the firm subscript i for each variable is omitted to save on notation, and primes denote next period variables:

$$V(A, K, \mathcal{B}) = \max_{I} \Pi(A, K) - C(A, K, I) - p(I)I + \beta E_{A',\mathcal{B}'|A,\mathcal{B}}\left[V(A', K', \mathcal{B}')\right] \tag{3.15}$$

$\Pi(A, K)$ is the reduced form profit function, $C(A, K, I)$ captures investment adjustment costs, and $p(I)$ is the effective price of newly installed or retired capital. $\mathcal{B}$ is a vector of size n that summarizes the distribution of beliefs about which regime has produced the current realization of profitability. Finally, conditional expectations are formed according to equation (3.13) at lead one.

[21] Unconditional on the current regime, but conditional on prior beliefs and idiosyncratic profitability.

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3.3.6 Aggregate Dynamics

The cross-section of firms in the model is characterized by a time-varying density $g_t(\Psi_{i,t}, K_{i,t}, \mathcal{B}_{i,t})$ across idiosyncratic profitability, capital and beliefs.[22] In addition, the aggregate profitability shock determines the aggregate dynamics of the model. We can therefore define an aggregate state vector $\mathcal{S}_t = [g_t(\Psi_{i,t}, K_{i,t}, \mathcal{B}_{i,t}), Z_t]$. The aggregate of a generic variable $X_{i,t}$ can therefore be expressed as:

$$E[X_{i,t}|\mathcal{S}_t] = \iiint g_t(\Psi_{i,t}, K_{i,t}, \mathcal{B}_{i,t}) X_{i,t}(Z_t, \Psi_{i,t}, K_{i,t}, \mathcal{B}_{i,t}) d\Psi dK d\mathcal{B} \tag{3.16}$$

3.4 The Effect of Learning on Uncertainty and Expectations

Now that the structural framework for the analysis has been outlined, it is instructive to understand how imperfect information and Bayesian learning influence uncertainty and expectations. To this end it is first necessary to define the exact meaning of uncertainty within the context of the model. Moreover, it is necessary to understand the factors that influence the dynamics of beliefs about the underlying regime of the markov-switching process and how these beliefs affect expectations.

3.4.1 Definition of Uncertainty within the Model

Before moving to the analysis of the effect of learning on uncertainty and expectations, it is necessary to define what is meant by uncertainty within the context of the model. In the existing uncertainty shocks literature, uncertainty is usually defined as the variance of shocks hitting the system.[23] This concept of uncertainty is therefore equivalent to the volatility of fundamental shocks and there are generally two channels through which variations in uncertainty affect outcomes. On the one hand, higher volatility leads to more dispersion in the realizations of shocks across agents. On the other hand, higher volatility leads to more dispersed forecast densities when agents form expectations about the future.

In contrast to this concept, uncertainty within the current chapter is defined as confusion about the true underlying regime of the markov-switching process. Hence, uncertainty in the model stems from incomplete information and is not necessarily

[22] Note that the dimensionality of the belief vector varies with the number of possible regimes. In general, we require the belief vector to be of dimension $n-1$ to summarize all available information.

[23] See for example Bloom (2009), Bloom et al. (2010), Bachmann and Bayer (2011), Arellano et al. (2011), Gilchrist et al. (2010) or Vavra (2012).

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related to more volatility of the fundamental shocks. Whenever there is uncertainty regarding the true underlying regime, agents will mix the conditional forecast densities under each regime according to their perceived probabilities. Depending on the parameter values of the markov-switching process in each of the regimes, this mixing of conditional densities can have implications for both first and second moments of the forecast density used by agents. In contrast, in the existing literature uncertainty shocks only have second moment implications for forecast densities as they are equivalent to mean preserving spreads.

3.4.2 Analytical Results for The Dynamics of Expectations

Because beliefs about the underlying regime of the markov-switching process play such a paramount role in determining the forecast densities used by agents, it is necessary to understand which factors influence their dynamics. By the mechanics of Bayesian learning, for a given prior belief vector $\mathcal{B}_{i,t-1}$ the posterior beliefs will depend on the overlap of the conditional densities under each regime and on the transition probabilities between the regimes. When the overlap of conditional densities is large, it is hard to distinguish which regime produced the current observation and agents will often be confused about the true underlying regime. On the other hand, when the overlap of conditional densities is low, agents will know most of the time in which regime they are. In addition, the transition probabilities determine the relative frequencies with which the different regimes occur. Hence, lower posterior probabilities are put on regimes that are less likely. In order to see how the dynamics of beliefs affect the forecast densities used by agents, it is useful to derive an expression for the expectation of an arbitrary function $m(\cdot)$ of the logarithm of next period idiosyncratic profitability:

$$E\left[m(\psi_{i,t+1})|\psi_{i,t},\mathcal{B}_{i,t}\right] = \sum_{j=1}^{n}\sum_{k=1}^{n}\mathcal{B}_{i,t}(k)\omega_{kj}E\left[m(\mu_{\psi,j}+\rho_{\psi,j}\psi_{i,t}+\sigma_{\upsilon,j}\upsilon_{i,t+1})\right] \quad (3.17)$$

As can be seen, the expectation is simply a weighted average of the expectation under each regime, where the weights are the subjective probabilities of each regime to occur. Hence, uncertainty about the underlying regime will influence the moments of the forecast density whenever the densities under some of the regimes differ. In most cases there will be a trade-off between the magnitude and persistence of the influence of learning on forecast densities. When the overlap of the densities under the different regimes is low, learning dynamics will normally be of short duration but the impact on forecast densities will be considerable due to the large difference in conditional densities. In contrast, when the overlap of the conditional densities is large, learning dynamics will be persistent but have a limited impact on forecast

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densities. However, this need not always be the case. The impact of learning on forecast densities can still be large even if two regimes have a perfect overlap of densities. This will be the case when there exists at least one other regime with a different DGP and the transition probabilities to this other regime differ considerably between the regimes with the same DGP. This set-up will actually be employed further below when an indicative parameterization for the German manufacturing industry during the Financial Crisis is explored.

Given the general expression for expectations in equation (3.17) it is easy to derive formulas for the mean and the variance of the forecast density of idiosyncratic profitability in levels:

$$E\left[\Psi_{i,t+1}|\Psi_{i,t},\mathcal{B}_{i,t}\right] = \sum_{j=1}^{n}\sum_{k=1}^{n}\mathcal{B}_{i,t}(k)\omega_{kj}\left[\Psi_{i,t}^{\rho_{\psi,j}}e^{\mu_{\psi,j}+\frac{\sigma_{\upsilon,j}^{2}}{2}}\right] \tag{3.18}$$

$$\begin{aligned} V\left[\Psi_{i,t+1}|\Psi_{i,t},\mathcal{B}_{i,t}\right] &= \sum_{j=1}^{n}\sum_{k=1}^{n}\mathcal{B}_{i,t}(k)\omega_{kj}\left[\Psi_{i,t}^{2\rho_{\psi,j}}e^{2\mu_{\psi,j}+2\sigma_{\upsilon,j}}\right] \\ &\quad -\left[\sum_{j=1}^{n}\sum_{k=1}^{n}\mathcal{B}_{i,t}(k)\omega_{kj}\left[\Psi_{i,t}^{\rho_{\psi,j}}e^{\mu_{\psi,j}+\frac{\sigma_{\upsilon,j}^{2}}{2}}\right]\right]^{2} \end{aligned} \tag{3.19}$$

3.4.3 Simulating A Regime Shift and Learning Dynamics

The specification of the markov-switching process has been very general so far, allowing an arbitrary number of regimes and potential switches in all the possible parameters. Given this general set-up the introduction of imperfect information and Bayesian learning can have very different effects depending on the parameters of the markov-switching process and the type of regime shift. Imperfect information could mute, amplify or propagate aggregate dynamics compared to full information. As the goal of this chapter is to explore the potential role of learning in the German manufacturing industry during the Financial Crisis, a parameterization that is broadly in line with the observed developments is explored in the following.

The main features of the chosen specification are that there are three possible regimes and the only parameter that switches between the regimes is the intercept term. The first regime is very persistent and features a high mean. The other two regimes have exactly the same DGP with a low mean, but one of the regimes is transitory while the other regime is highly persistent. This is reflected in the transition probabilities back to the high mean regime. This set-up is motivated by the fact that the German manufacturing industry was in a normal (high) state from the beginning of the 1990's until the middle of 2008 and in late 2008 and early 2009 there was a fairly temporary regime shift to a crisis (low) state. The introduction

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of a third crisis regime that is highly persistent is motivated by the fact that the drop in manufacturing activity was unprecedented and it is conceivable that at the time firms were therefore highly uncertain about the persistence of the collapse in demand. The fact that only the intercept term is chosen to switch is done in order to isolate the type of uncertainty due to imperfect information from the standard way of modeling uncertainty shocks through changes in the innovation variance. The specific parameter values that were chosen can be found in table 3.3.[24]

Table 3.3: Parameter values for the simulation of the markov-switching process

Parameter	**Value**	**Description**
$\mu_{\psi,1}$	0.00	Intercept of idiosyncratic profit.
$\mu_{\psi,2/3}$	-0.13	Intercept of idiosyncratic profit.
$\rho_{\psi,1/2/3}$	0.95	AR(1) parameter of idiosyncratic profit.
$\sigma_{\upsilon,1/2/3}$	0.05	Std of idiosyncratic innovations
ω_{11}	0.97	Transition probability high - high
ω_{12}	0.015	Transition probability high - low transitory
ω_{13}	0.015	Transition probability high - low persistent
ω_{21}	0.93	Transition probability low transitory - high
ω_{22}	0.07	Transition probability low transitory - low transitory
ω_{33}	0.97	Transition probability low persistent - low persistent
ω_{31}	0.03	Transition probability low persistent - high

In order to study the dynamics of beliefs and expectations for this process, the following simulation exercise is performed: After a long sequence of high regimes there is a one time switch to the low transitory state for two periods, after which the process switches back to the high regime. Such a sequence of regimes is roughly consistent with the regime shift that took place in the German manufacturing industry during the Financial Crisis that lasted for approximately six months (i.e. two quarters). This sequence of regimes is simulated for a panel of 50,000 firms in order to generate cross-sectional distributions of idiosyncratic profitability, beliefs and forecast densities.

The top left panel in figure 3.8 displays the underlying regime switch from the first state to the second state in period one that lasts for two periods. The remaining panels show the average response of beliefs for the simulated cross-section of firms in response to this regime shift. Before the regime shift takes place, firms put a very high probability of 0.98 on being in the high mean state which is the true underlying

[24]See section 3.5.1 for a discussion of the chosen parameter values.

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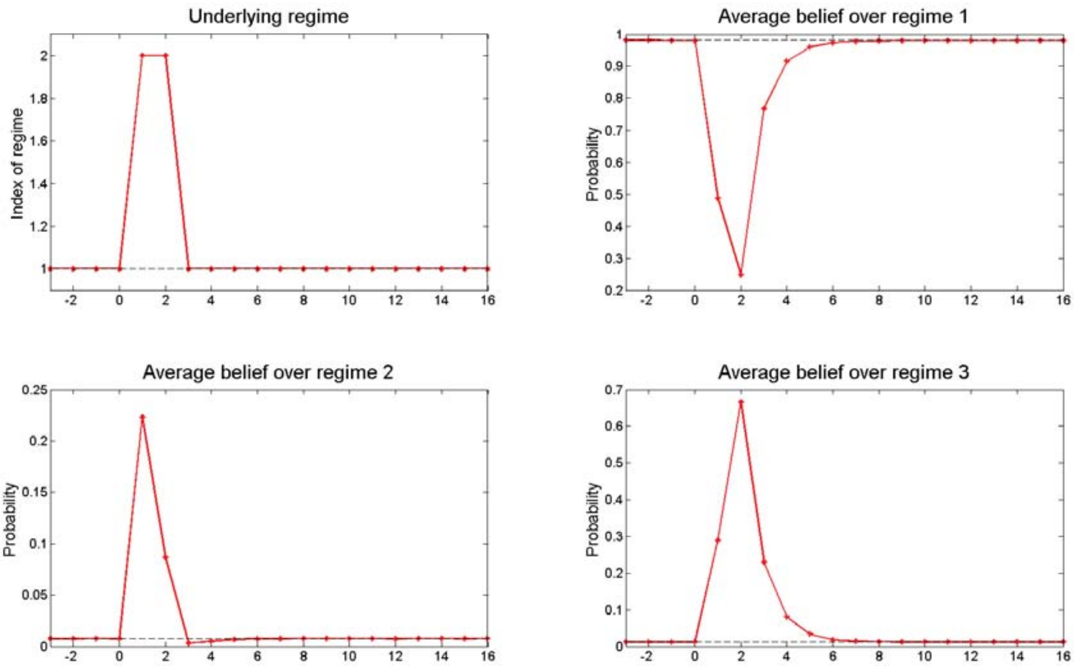
Figure 3.8: The dynamics of beliefs after a transitory regime shift



regime. In the two periods of the regime shift the average probability of still being in the high regime drops to 0.49 and 0.25 and subsequently quickly converges back to the initial level as the high regime prevails again. At the same time, the beliefs about regimes two and three jump up from around 0.01 to 0.22 and 0.29 respectively when the regime shift takes place. In the second period of the regime shift, firms put a much higher probability on being in the low persistent state of 0.66 compared to a probability of being in the low transitory state of 0.09. This is simply due to the fact that a second period of low realizations is much more likely under the persistent regime. Once the underlying state switches back to the high regime, both beliefs quickly converge back to their initial levels. In that sense, firms get more pessimistic about the future the longer the transitory regime shift remains.

The main question of interest is how these variations in uncertainty affect the expectations of firms compared to the case of perfect information where the true underlying regime is observed. The answer to this can be found in figure 3.9. While the dynamics of the distribution of profitability across firms is exactly the same for perfect information and Bayesian learning, the dynamics of mean forecasts and forecast standard deviations differ considerably. In particular, on average mean forecasts drop by much more and forecast standard deviations increase by much more under learning compared to perfect information. The mechanism behind this is that in the case of learning, firms attach some weight on being in the low persistent

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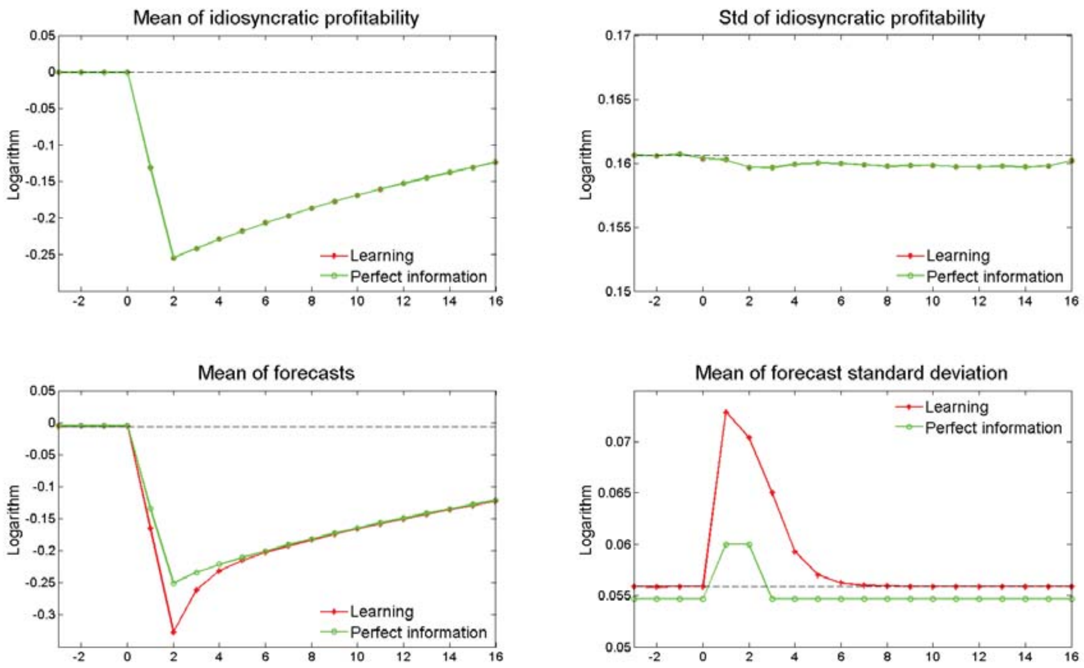
Figure 3.9: The dynamics of the profitability distribution and forecast densities



regime, even though they are in the low transitory regime. This results in a higher probability being attached to still being in the low regime in the next period, which causes mean forecasts to be lower and forecast standard deviations to be higher.

One additional aspect to note is that even though the average forecast standard deviation is affected by the regime shift, the cross-sectional dispersion of profitability stays constant throughout.[25] However, even though the dispersion of profitability across firms stays constant over time, there is more cross-sectional dispersion of expectations during the regime shift in the case of imperfect information, which can be seen from figure 3.10. In particular, the cross-sectional standard deviation of mean forecasts and forecast standard errors increases considerably for the case of learning while they are unaffected when there is perfect information.

The mechanism behind this is that beliefs become more dispersed across firms during the regime shift, which translates into more cross-sectional dispersion in the forecast densities used by firms. Before the regime shift takes place all firms are fairly certain about being in the high mean regime. However, when the regime shift occurs the idiosyncratic shocks in effect act like heterogeneous signals about the underlying regime and cause beliefs and expectations to become more dispersed. Of course, if firms could observe the mean of the cross-sectional distribution of prof-

[25]The very small variations in the cross-sectional standard deviation of idiosyncratic profitability are solely due to sampling.

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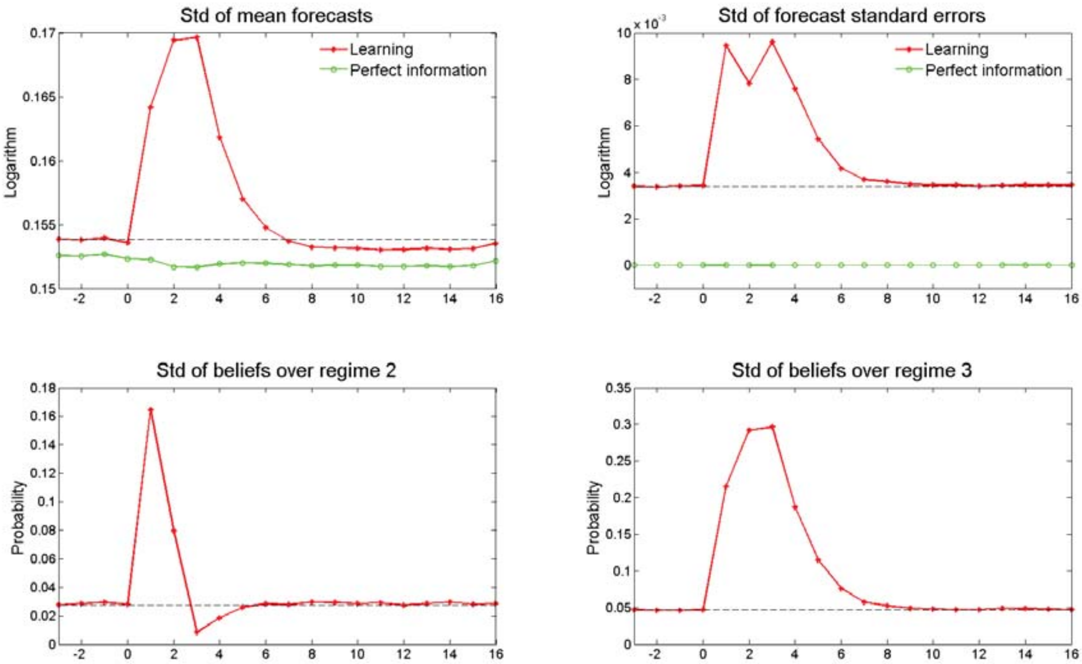
Figure 3.10: The dynamics of cross-sectional dispersion in beliefs and forecasts



itability this heterogeneity in beliefs would disappear. However, in reality aggregate variables are published with a time lag and are hence not immediately observable. As a first approximation it therefore seems reasonable to assume that firms only observe their own realization of profitability.[26] This increase in cross-sectional dispersion of expectations for the case with Bayesian learning could be an interesting mechanism to explain why survey responses by firms and professional forecasters get more dispersed during downturns (See Bachmann et al. (2010) and Bloom (2009), Bloom et al. (2010), Dovern et al. (2009) respectively).

3.5 The Effect of Time-Varying Uncertainty on Investment

Now that the dynamic implications of Bayesian learning for uncertainty and expectations have been explored, it is possible to move on to the analysis of how

[26]In principle it would be possible to condition beliefs on past aggregate variables. Nevertheless, this would still result in dispersion of beliefs to increase directly after a regime shift and simply reduce the persistence of the increase in the dispersion of beliefs. In order to keep the model as simple as possible, it is therefore assumed that firms only observe their own realization of profitability.

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this type of time-varying uncertainty affects aggregate investment. As a full-fledged estimation or calibration of the model with German manufacturing data is not feasible at this stage due to time constraints, a number of different illustrative model parameterizations are explored in this section.

3.5.1 Parameterization and Outline of the Simulation

Throughout the various simulations the parameter values in table 3.3 are used for the markov-switching process of idiosyncratic profitability. Moreover, all other parameters except for the adjustment costs are kept constant across the different simulations, the values of which can be found in table 3.4. Even though the parameters are not calibrated, they are chosen to take on 'reasonable' values for a model of the German manufacturing industry at a quarterly frequency. Hence, the discount rate is set to 0.99 and the depreciation rate to 0.025. The revenue function is chosen to have constant returns to scale in (A, K, L) space which is consistent with the assumption of a Cobb-Douglas production function and some degree of market power.[27] The exponents on capital and labor are chosen to be 0.25 and 0.50, which is equivalent to saying that the production function has exponents of 1/3 and 2/3 on capital and labor and the demand elasticity is equal to 4.[28] This also implies that the profit function has an exponent of 0.5 on capital.[29] Moreover, with this set-up the profitability shocks have the interpretation of demand shocks[30], which seems appropriate for studying the German manufacturing industry during the Financial Crisis.

The aggregate shock is chosen to have an AR(1) parameter of 0.77 and the standard deviation of shocks is set to 1.94 %. Finally, the idiosyncratic markov-switching process is parameterized with one high and two low regimes. The persistence parameter is set to 0.95 and the standard deviation of shocks to 5 % in all three regimes. The only parameter that switches between the regimes is the intercept which takes a value of 0 in the high state and a value of -0.13 in the two low states. Recall at this stage that the regime shift in German manufacturing lasted for about two quarters and revenues fell by 25 %. This specification of the markov-switching process will therefore lead to a similarly sized drop in the mean of the profitability distribution for a two period regime shift from the high to the low transitory regime, as can be

[27] This specification is in line with Bloom (2009) which is the leading recent paper on uncertainty shocks and investment.

[28] These assumptions are in line with Bloom (2009) and Bloom et al. (2010).

[29] For comparison, Cooper and Haltiwanger (2006) estimate a value of 0.592 for the US manufacturing industry.

[30] See Lang (2012) for an exposition of this result.

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seen in the top left panel of figure 3.9. Finally, the high regime is chosen to persist with a probability of 0.97 as is the low peristent regime. The low transitory regime is chosen to persist with only a 0.07 probability and switch back to the high regime with a 0.93 probability.

Table 3.4: Common parameter values across the various model simulations

Parameter	**Value**	**Description**
β	0.99	Discount factor
δ	0.025	Depreciation rate
a	0.25	Exponent on capital
b	0.50	Exponent on labor
c	$1-a-b$	Exponent on profitability
w	1	Wage rate
p_b	1	Price of capital
μ_z	0	Intercept of aggregate profit.
ρ_z	0.77	AR(1) parameter of aggregate profit.
σ_η	0.0194	Std of innovations to aggregate profit.

The simulation exercise for the various models closely follows the one performed in section 3.4.3 for the markov-switching process. Starting from a long sequence of high regimes there is a one time switch to the low transitory regime that lasts for two quarters after which the high regime prevails again. Throughout the exercise aggregate shocks are turned off and set to the unconditional mean of the AR(1) process. This sequence of shocks is simulated for a panel of 1,000 firms and repeated 100 times. The reported results are then averaged over the 100 simulation runs. The dynamics of the cross-section of beliefs, forecast densities and idiosyncratic profitability are therefore the same as discussed above and will not be presented again in this section.

3.5.2 The Case without Adjustment Costs

For the case where there are no capital adjustment costs, the model presented in section 3.3 allows for an analytical solution. Taking the first-order and envelope conditions of the Equation (3.15) and solving for next period's capital stock yields the following capital policy function:

$$K'(Z_t, \Psi_{i,t}, \mathcal{B}_{i,t}) = \varphi E\left[(Z_{t+1}\Psi_{i,t+1})^{\frac{c}{1-b}} \mid Z_t, \Psi_{i,t}, \mathcal{B}_{i,t}\right]^{\frac{1-b}{1-a-b}} \tag{3.20}$$

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In the above policy function $\varphi = [(a\beta\phi_\pi)/([1-b][p-p\beta(1-\delta)])]^{\frac{1-b}{1-a-b}}$ is a constant that depends on the structural parameters of the model. As is common for a model without adjustment costs, the current capital stock does not influence the optimal choice of next period's capital stock. The only factor that determines the optimal accumulation of capital is the expectation of future profitability. However, this expectation is now influenced by the current beliefs about the underlying regime of the markov-switching process. Applying the formula derived in equation (3.17) it is easy to rewrite the capital policy function as:

$$
\begin{aligned}
K'(Z_t, \Psi_{i,t}, \mathcal{B}_{i,t}) \quad = \quad & \left(\sum_{j=1}^{n} \sum_{k=1}^{n} \mathcal{B}_{i,t}(k)\omega_{kj} \left[\Psi_{i,t}^{\rho_{\psi,j}\frac{c}{1-b}} e^{\mu_{\psi,j}\frac{c}{1-b} + \frac{\sigma_{\upsilon,j}^2}{2}\left(\frac{c}{1-b}\right)^2} \right] \right)^{\frac{1-b}{1-a-b}} \\
& \varphi \left(e^{\mu_z} Z_t^{\rho_z} e^{\frac{\sigma_\eta^2}{2}\frac{c}{1-b}} \right)^{\frac{c}{1-a-b}} \qquad (3.21)
\end{aligned}
$$

With this policy function in hand, it is easy to simulate the model for a panel of firms and construct aggregates for investment and revenues.[31] The results of this can be found in figure 3.11. There are three main results that stand out. First, Bayesian learning and the endogenous variations in uncertainty associated with it amplify the temporary regime shift compared to the case of perfect information. In particular, the investment drop in the two periods of the regime shift is considerably larger under learning due to the fact that on average firms put a considerable weight on being in the low persistent regime. This in turn leads to a lower capital stock in subsequent periods and causes a larger drop in aggregate revenues compared to perfect information. Quantitatively, the maximum additional drops in investment and revenues caused by the endogenous uncertainty are around 125 % and 2.8 %.

This leads us to the second main result. For both models with perfect information and learning the investment response is orders of magnitude larger than what is observed in the data. Compared to the initial level under the high regime, aggregate investment falls by 480 % and 580 % respectively in the first period of the regime change. I.e. disinvestment takes place on a huge scale, which is definately not consistent with what we saw in the data as was shown above. This investment response is not surprising though, given the large drop in average profitability that is induced by the regime change and the fact that capital can be sold at no loss. Hence, as is common for models without capital adjustment costs, investment is much too volatile and in particular negative values of aggregate investment occur which does not fit the data. Hence, this shows that some form of capital adjustment costs are needed to match the data no matter whether a model with or without

[31] The dynamics of aggregate profits and labor are equivalent to the ones for revenues in the model at hand.

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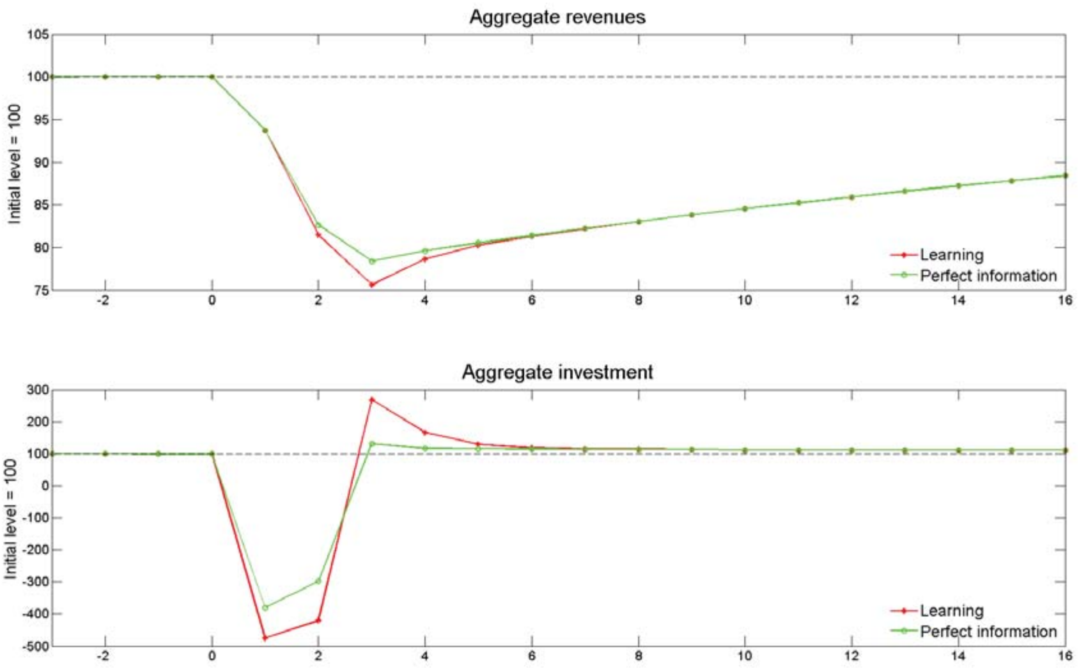
Figure 3.11: The dynamics of aggregates for the case without adjustment costs



learning is considered.

Finally, the third result is that the drop and rebound in revenues associated with the regime change is quantitatively roughly consistent with developments in the German manufacturing industry during the Financial Crisis. In particular, aggregate revenues drop by a maximum of 24.4 % and 21.6 % with and without learning three quarters into the crisis. Subsequently reveneus start to rebound in both cases but are still 11.6 % below the previous peak four years after the start of the crisis. Both the drop and the rebound are approximately in line with the dynamics in the German manufacturing industry which are displayed in figure 3.3.

3.5.3 Various Models with Adjustment Costs

The simulation results for the model without adjustment costs show that some form of capital adjustment cost is needed in order to match aggregate investment data in the German manufacturing industry during the Financial Crisis. Unfortunately an analytical solution to the model no longer exists once we incorporate various forms of adjustment frictions. It is therefore necessary to solve the model using numerical methods in order to study the role of endogenous variations in uncertainty for aggregate investment and revenues. The solution method employed to solve the model was discrete value function iteration with simplicial interpolation. Details of

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the numerical algorithm can be found in the Appendix.

Table 3.5: Adjustment cost parameters for the various model simulations

Model	p_s	λ	γ	**Description**
quad-2	1.0	1.0	2	Only quadratic ACs
quad-6	1.0	1.0	6	Only quadratic ACs
fix-70	1.0	0.7	0	Only fixed ACs
fix-95	1.0	0.95	0	Only fixed ACs
irr-70	0.7	1.0	0	Only partial irreversibilities
irr-95	0.95	1.0	0	Only partial irreversibilities
all-1	0.7	0.95	6	All forms of ACs
all-2	0.9	0.99	6	All forms of ACs

In order to study how adjustment costs change the response of aggregate investment to a regime shift with endogenous variations in uncertainty a number of different illustrative model parameterizations are solved and simulated. The different combinations of adjustment cost parameters for each of the models can be found in table 3.5. Due to the fact that various degrees of convex and non-convex adjustment costs have been estimated in the literature, each of the three adjustment cost mechanisms is solved once for a moderate and once for a high value.[32] The associated investment impulse responses to a temporary regime shift with and without Bayesian learning can be found in figure 3.12.

The first thing to note is that the introduction of investment adjustment costs, independent of the specific form, makes the response of aggregate investment to the regime shift much more persistent compared to the case without any adjustment costs. No matter whether we look at the case of Bayesian learning or perfect information, aggregate investment stays below the pre-crisis level for many quarters after the initial shock. However, the magnitude and persistence of the fall in aggregate investment differs considerably between the various model parameterizations. Moreover, the degree to which the investment responses differ between perfect information and Bayesian learning depends to a large extent on the parameter values used.

As can be seen from the top two panels in figure 3.12 learning plays a major role in shaping the investment response when only quadratic adjustment costs are

[32] Cooper and Haltiwanger (2006) for example estimate high fixed costs, low quadratic adjustment costs and low irreversibilities. Bloom (2009) in contrast estimates high irreversibilities, and low fixed and quadratic adjustment costs. Moreover, Hayashi (1982) estimates a fairly high quadratic adjustment cost.

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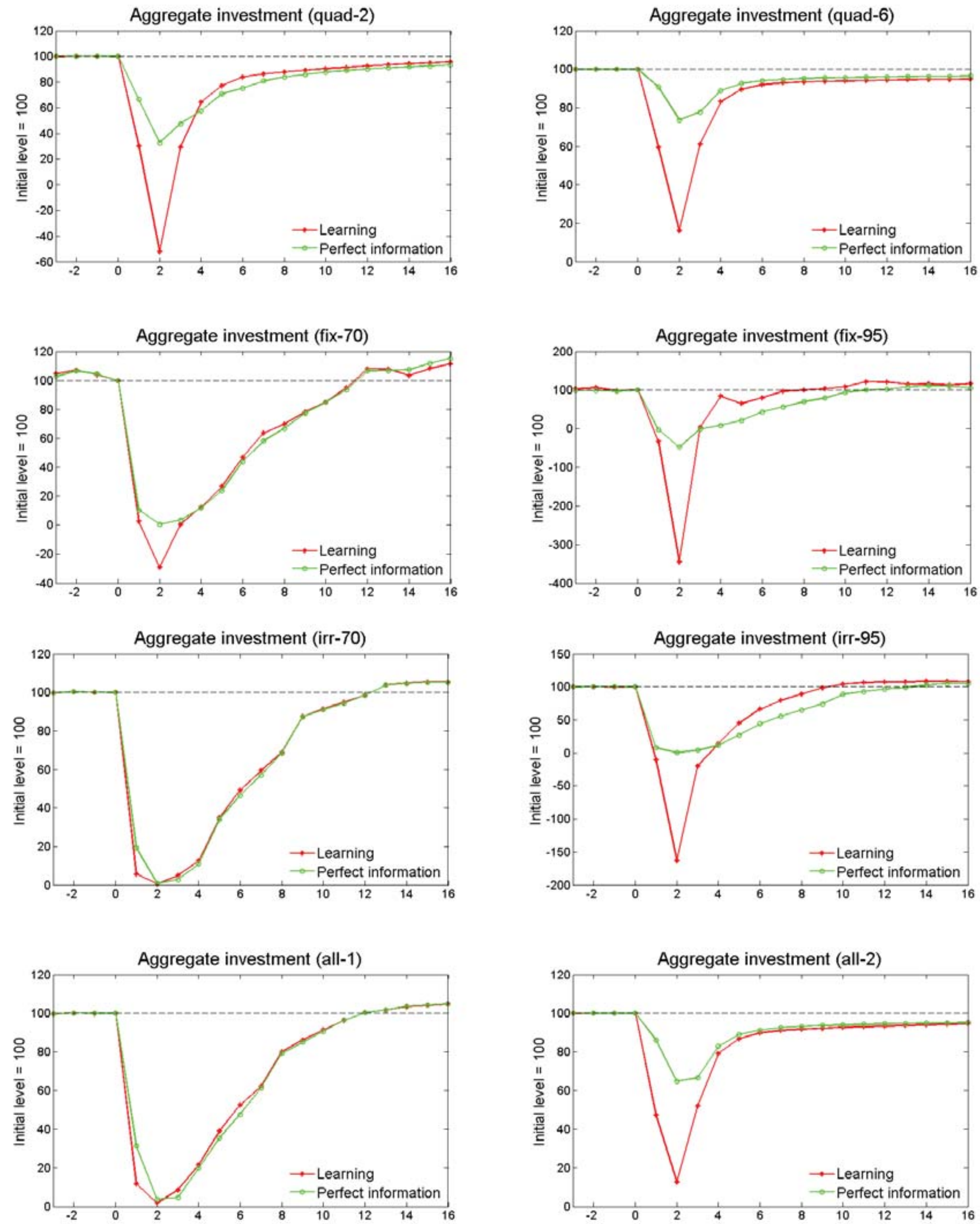
Figure 3.12: The dynamics of aggregate investment with adjustment costs



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present. In particular, learning amplifies the investment drop considerably in the first three periods after the regime shift, because firms are on average uncertain about the underlying regime and attach considerable weight on being in the low persistent one. Hence, they are more pessimistic about the future and invest less. The qualitative response of aggregate investment is more or less the same for a moderate and a high value of quadratic adjustment costs. Quantitatively though a higher value necessarily mutes the investment response as it is more costly to make large adjustments to the capital stock. With $\gamma = 2$ aggregate investment under learning drops by a maximum of over 150 % while with $\gamma = 6$ the maximum drop is only around 80 %.

Moving on the the case of partial irreversibilities only, it is apparent that the effect of imperfect information depends to a large extent on the magnitude of the irreversibility. For large irreversibilities the investment responses under perfect information and learning are very similar with only a slight amplification under learning in the first period of the regime shift. The intuition for this surprising result is that even under perfect information, aggregate investment activity virtually stalls when the regime shift takes place. In addition, due to the presence of large irreversibilities aggregate investment does not cross into the negative territory under learning, even though firms are much more pessimistic about future prospects. For a lower value of partial irreversibilities aggregate investment still drops to almost zero during the regime shift under perfect information, but this time it collapses to a maximum of minus 250 % of the initial investment level under learning. The reason is that with the lower irreversibility more firms now hit their lower investment triggers, i.e. they sell capital.

For the case of fixed costs only, the results are very similar to the case of partial irreversibilities only. For large fixed costs learning does not make a big difference for aggregate investment due to the virtual collapse even under perfect information. However, with moderate fixed costs learning amplifies the investment drop considerably as more firms hit their disinvestment triggers due to the more pessimistic beliefs and a smaller inaction region caused by the lower fixed cost.

Finally, when all three forms of adjustment frictions are incorporated, learning only plays a major role for the investment response whenever the fixed adjustment costs are not too high.[33] Even when the fixed cost is only 5 % of profits, the investment responses under learning and perfect information are very similar, dropping to almost zero after the regime shift. This fact is not due to the considerable ir-

[33]Of course this statement only applies to the type of parameterizations considered here. In particular, for another markov-switching process results could be different if aggregate investment would not fall down to zero under full information.

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reversibilities of a 30 % resale loss in this parameterization, as the same version of the model without fixed costs produces a considerable difference in investment responses.[34] Once the fixed cost is quite low, there is again a considerable downward amplification of the investment reponse under imperfect information.[35] Moreover, the qualitative dynamics are mainly governed by the quadratic adjustment cost parameter, while the level of the partial irreversibilities seems to play only a minor role and the fixed cost simply shifts down the aggregate investment responses.

For completeness, figure 3.13 also plots the dynamics of aggregate revenues for the models where learning plays the biggest role in shaping the response of aggregate investment. By definition, the difference between aggregate revenues under learning and perfect information are solely due to differences in the accumulation of capital. Therefore, the downward amplification of aggregate revenues is greatest where the downward amplification of investment is greatest. Naturally, the amplification is much smaller for revenues than for investment, ranging from 1 % to 2 %. One exception is the case with only moderate fixed costs of 5 % of profits. For that case the maximum downward amplification of revenues due to learning is around 5 %, which comes from the large incvestment drop of more than 400 % after the regime shift. Quantitatively, the two period regime shift causes drops in aggregate revenues of around 12 % to 18 % depending on the parameterization and informational assumptions. This is still considerably smaller than the drop of around 25 % witnessed during the Financial Crisis in the German manufacturing industry.

In order to assess which of the adjustment cost parameterizations is most appropriate for the German manufacturing industry, it is useful to examine the overall volatility of aggregate investment and aggregate revenues as well as some of the micro investment moments when the models are simulated with aggregate shocks only and regime shifts are shut off. As can be seen from table 3.6 some degree of quadratic adjustment costs are necessary, otherwise aggregate investment is way to volatile compared to the data. Moreover, it is hard to match both the inaction rate and the rate of investment bursts at the same time. Interestingly these failures of structural models of investment have not been sufficiently discussed in the literature. In particular, only including non-convex adjustment costs induces too much investment volatility in the aggregate. As the two leading papers on the estimation of capital adjustment costs by Cooper and Haltiwanger (2006) and Bloom (2009) find only small quadratic adjustment costs when trying to match micro investment moments, this indicates that both micro and macro moments need to be considered

[34]This second version of the model is not shown here, but investment under full information only falls by a maximum of around 30 % and under learning by a maximum of around 80 %.

[35]For values of λ between 0.97 and 0.99 and $\gamma = 6$ learning plays a major role.

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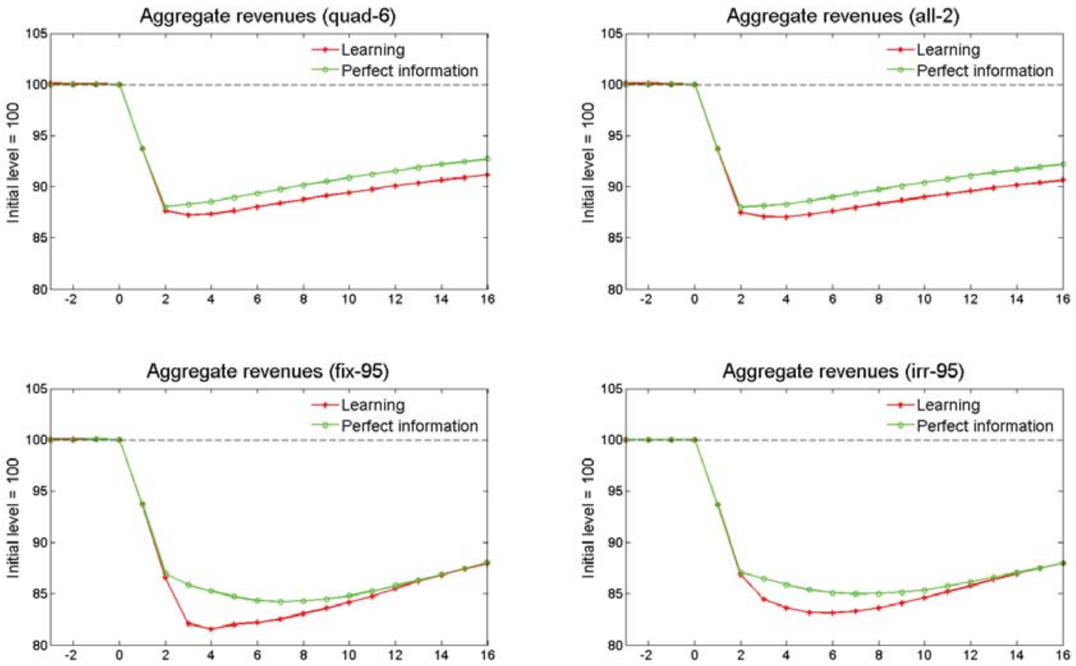
Figure 3.13: The dynamics of aggregate revenues with adjustment costs



when taking these models to the data.

Table 3.6: Volatility and investment moments for the various model simulations

Model	**Inact**	**Burst**	**Pos**	**Neg**	**CV-I**	**CV-R**
Data	0.114	0.127	0.736	0.023	0.055	0.043
quad-2	0.061	0.000	0.939	0.000	0.036	0.018
quad-6	0.010	0.000	0.990	0.000	0.005	0.017
fix-70	0.966	0.035	0.000	0.000	0.374	0.025
fix-95	0.937	0.063	0.000	0.000	0.375	0.024
irr-70	0.424	0.000	0.576	0.000	0.337	0.024
irr-95	0.457	0.000	0.543	0.000	0.341	0.025
all-1	0.500	0.000	0.499	0.000	0.115	0.019
all-2	0.018	0.000	0.982	0.000	0.006	0.017

Notes: All the micro investment moments refer to the investment rate. Inact refers to the inaction rate defined as ± 1 %, Burst refers to investment bursts of more than 20 %, Pos refers to positive investment between 1 % and 20 % and Neg refers to investment rates of below - 1 %. CV-I and CV-R represent the coefficient of variation for aggregate time series of investment and revenues. In the data the standard deviation of the cyclical components is taken so that both measures can be interpreted as percentage deviations from the mean. The micro investment moments are for the German manufacturing industry and are taken from the data appendix of the paper by Bachmann and Bayer (2011).

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3.6 Conclusion

Ever since the outbreak of the Financial Crisis in 2008 uncertainty has received renewed interest in the quantitative macro literature as a shock that drives aggregate fluctuations. However, these variations in uncertainty are usually modeled as changes in the shock variance of fundamentals and in that sense they are exogenous and similar in spirit to first-moment productivity shocks. In contrast, this chapter has studied endogenous variations in uncertainty and aggregate fluctuations that result from imperfect information and learning in an environment where regime changes in the mean happen occasionally. The idea behind this set-up is that whenever unprecedented regime shifts occur, agents become more uncertain about the true data generating process (DGP) and therefore mix different conditional distributions when forming expectations about the future. In fact, the German manufacturing industry during the Crisis was shown to fit such a set-up quite well. Since 1991 manufacturing experienced considerable growth, but in middle 2008 output collapsed by 25 % within six months. This regime shift was unprecedented and expectations fell much more than can be explained by fundamentals.

With this empirical background in mind a partial equilibrium heterogeneous firm model that features capital adjustment costs, a markov-switching driving process and imperfect information about the underlying regime was built and simulated. There are two main findings that come out of the exercises. First, imperfect information leads endogenously to temporarily higher uncertainty about the underlying regime after a regime shift. On average this leads to lower mean forecasts and higher forecast standard errors compared to full information. Moreover, during the regime shift the dispersion in beliefs increases considerably, as every firm gets in effect an idiosyncratic signal about the underlying regime shift. The increase in belief dispersion across firms in turn causes the cross-sectional dispersion of mean forecasts and forecast standard errors to increase. This mechanism could be interesting in order to explain why survey responses by firms and professional forecasters get more dispersed during downturns[36].

Second, these endogenous variations in uncertainty can lead to considerable downward amplification of aggregate investment and revenues during a temporary downward regime shift. This is true for all types of adjustment costs. However, in order to match the empirical volatility of aggregate investment some degree of quadratic adjustment costs is necessary. For such a parameterization with moderate fixed and quadratic adjustment costs and considerable irreversibilities aggregate in-

[36] See Bachmann et al. (2010) and Bloom (2009), Bloom et al. (2010), Dovern et al. (2009) respectively.

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vestment drops more than twice as much under Bayesian Learning compared to full information. In addition, the persistence of the investment drop gets increased. This shows that endogenous variations in uncertainty that are due to imperfect information can have interesting amplification and propagation implications for aggregates. Due to computational and time constraints a full fledged calibration or estimation of the model using SMM was not performed. However, in future research it is planned to estimate adjustment costs along side the role of imperfect information and learning with German firm and plant level manufacturing data.

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Appendix A: The Dynamics of Aggregate Investment Components

Because aggregate investment fell so much during the Financial Crisis in Germany, it is useful to consider it's dynamics in greater detail. In order to put the drop in aggregate investment in 2008 and 2009 into a broader picture, figure 3.14 panel (a) plots aggregate investment since the beginning of the 1990's. As can be seen, aggregate investment experienced a similarly large drop after the stock market collapse at the beginning of the 2000's although spread out over a longer time period. Moreover, investment increased fairly rapidly by more than 20 % between 2005 and the beginning of the financial crisis. Overall, aggregate investment went through three large swings since the middle of the 1990's.

When relating nominal investment to GDP though, as done in panel (b) of figure 3.14, it becomes evident that the investment share has been on a steady decline since the beginning of the 1990's until 2005 from almost 24 % to around 17 %. Since then the investment share has fluctuated around 18 %. The slump and rebound in investment during the financial crisis also manifested itself in a decline and subsequent rise again in the investment share.

As mentioned above, the fall in real investment during the financial crisis was 22.3 % and had not yet regained it's pre-recession level at the end of 2011. Figure 3.15 panel (a) shows that this fall in real investment was mainly due to a steep fall of 24.5 % in equipment investment, while investment in structures fell by slightly less than 9 %. Other investment actually increased by 17.3 % between 2008 and 2011. Panel (b) shows that similar dynamics hold for nominal investment figures. Finally, panels (c) and (d) show that the subcomponents of investment in equipment and structures display roughly similar dynamics as the aggregates.

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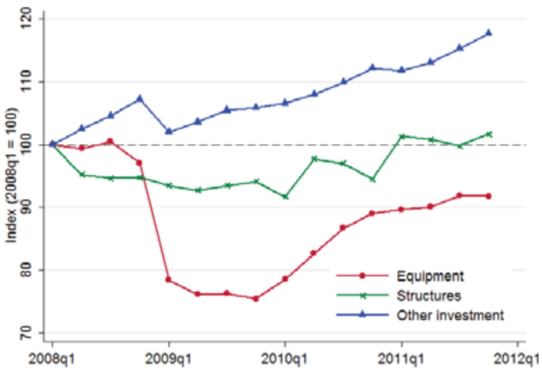
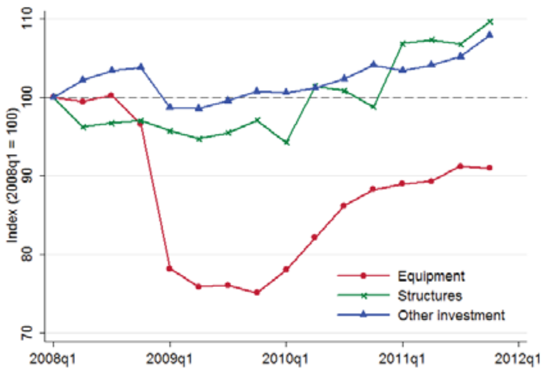
Figure 3.15: Developments in aggregate investment and it's components since 2008

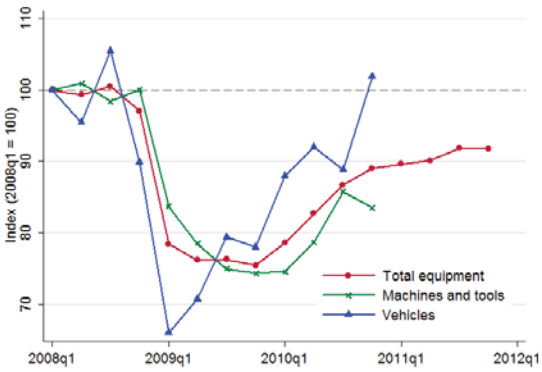
(a) Real investment



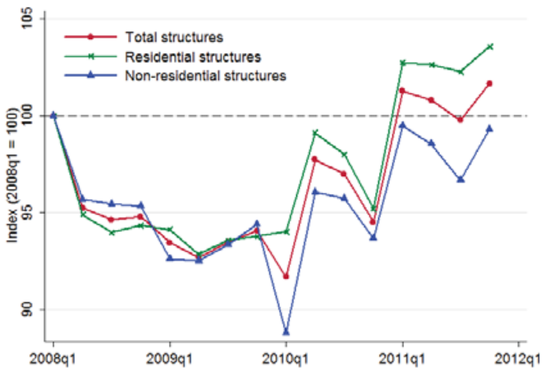
(b) Nominal investment



(c) Real investment in equipment



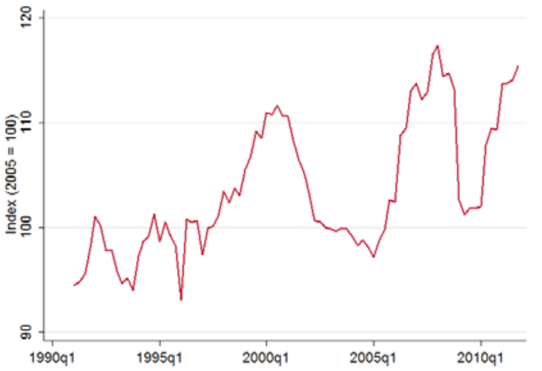
(d) Real investment in structures



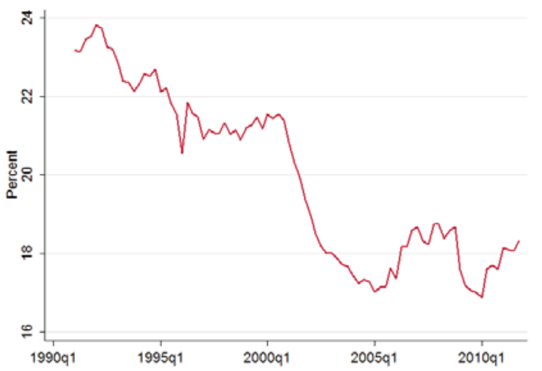
Source: German Federal Statistical Office

Figure 3.14: Developments in aggregate investment since 1991

(a) Real investment



(b) Investment share in GDP



Source: German Federal Statistical Office

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Appendix B: Supplementary Figures

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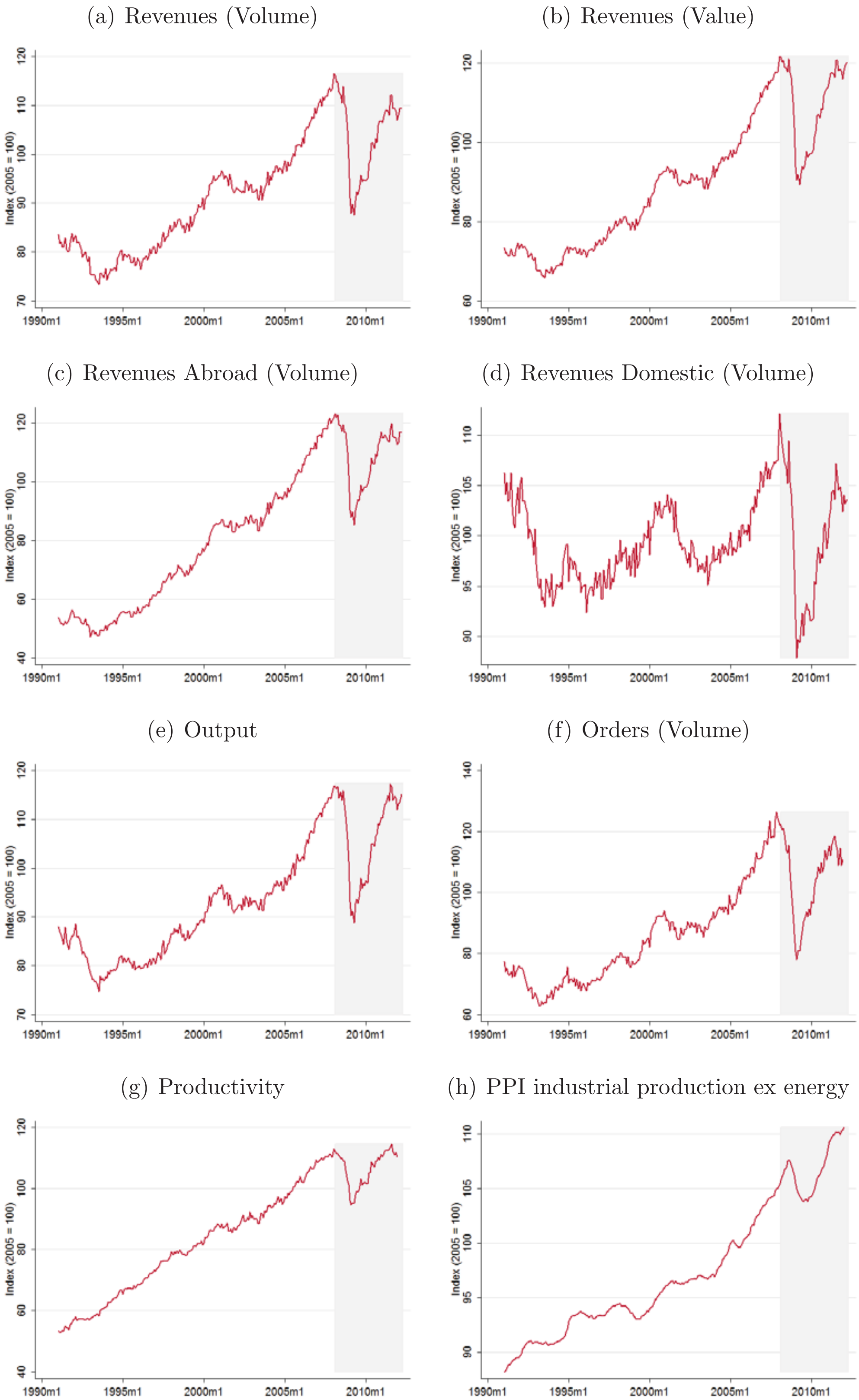
Figure 3.16: Developments in the manufacturing industry since 1991



Source: German Federal Statistical Office, Datastream

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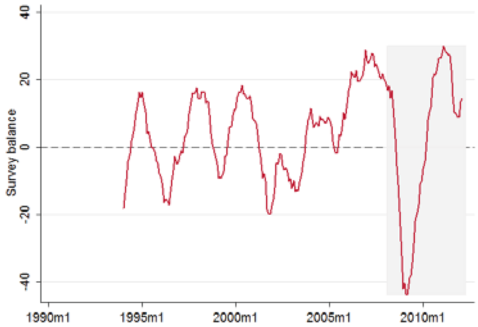
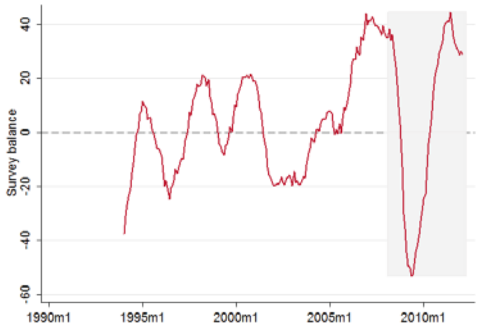
Figure 3.17: Developments of survey indices in the manufacturing industry

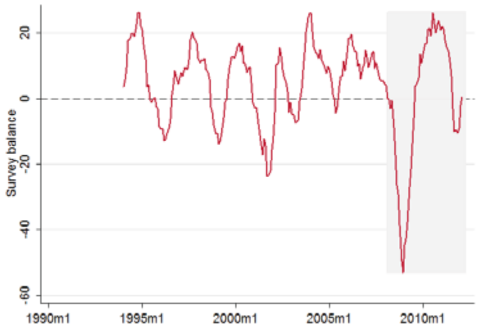
(a) IFO Index (balance)



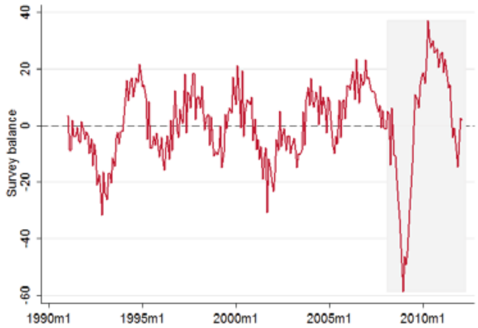
(b) IFO Situation (balance)



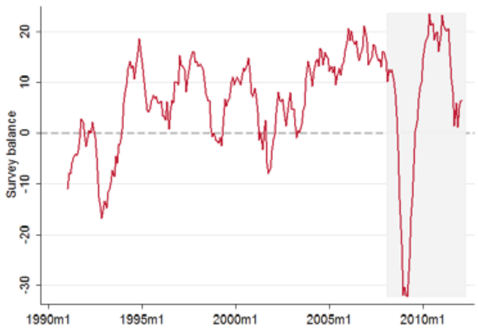
(c) IFO Expectations (balance)



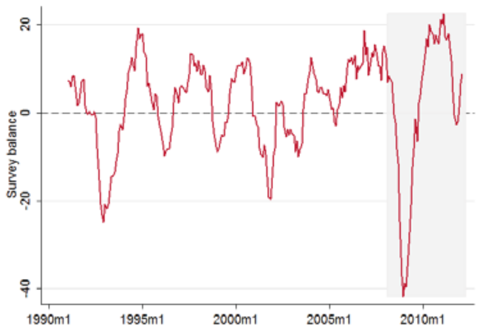
(d) IFO Demand (balance)



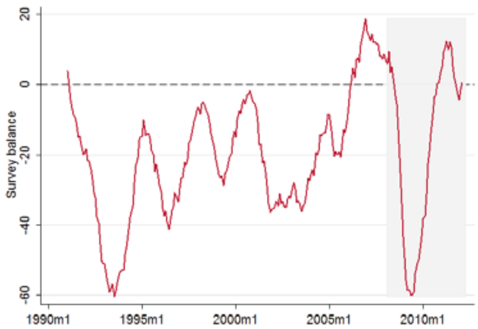
(e) IFO Trade (balance)



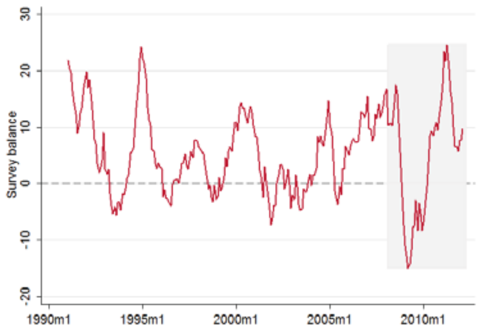
(f) IFO Production (balance)



(g) IFO Orders (balance)



(h) IFO Prices (balance)



Source: Datastream

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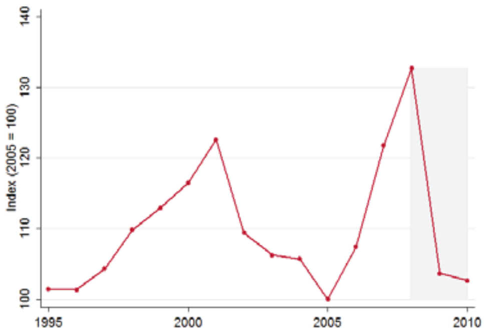
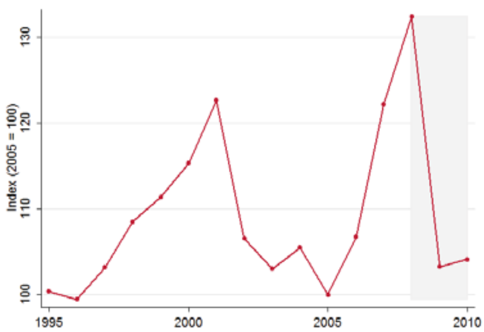
Figure 3.18: Developments of investment in the manufacturing industry

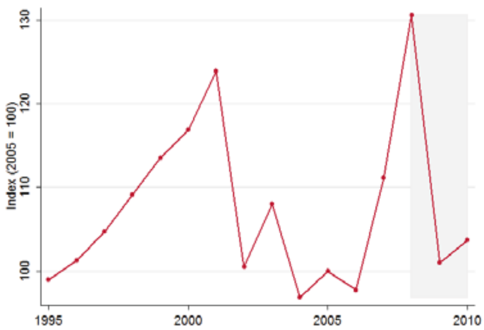
(a) Total Investment (Firms)



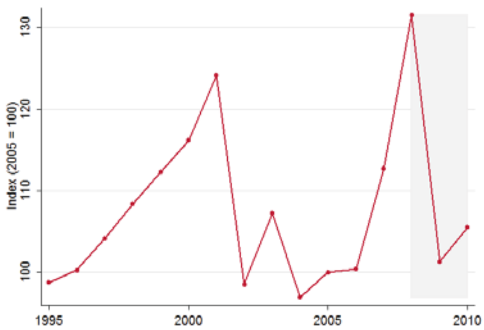
(b) Total Investment (Plants)



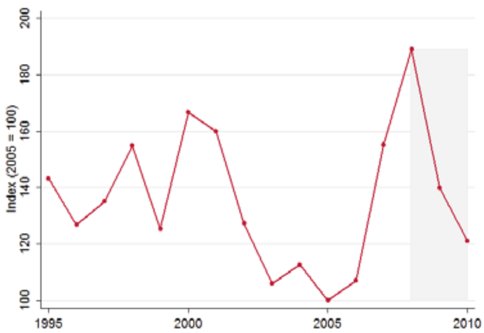
(c) Machinery Investment (Firms)



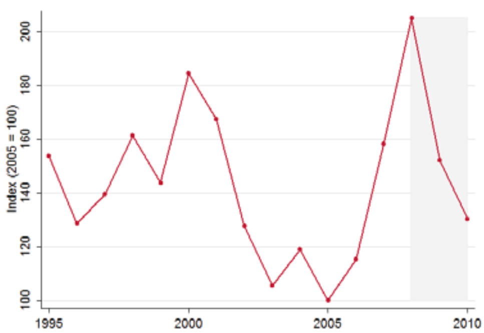
(d) Machinery Investment (Plants)



(e) Structures Investment (Firms)

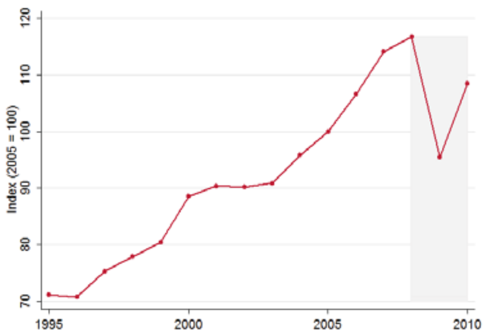


(f) Structures Investment (Plants)



(g) Total Revenues (Firms)

(h) Total Revenues (Plants)



Source: German Federal Statistical Office, Own calculations

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Appendix C: Description of Data Sources

The data being used in this chapter was obtained from three different sources: The GENESIS database of the German Federal Statistical Office, the online database of the Deutsche Bundesbank, and Thomson Reuters Datastream. Most of the series are at a monthly or quarterly frequency and span the time period from 1991 to 2012. Where applicable, the time series are seasonally adjusted with the X-12-ARIMA method of the US Census, unless explicitly stated otherwise. The source and frequency for each of the series are listed below.

German Federal Statistical Office:

- National income and product accounts (quarterly)
- Value added by industry (quarterly)
- Employment, hours and wages by industry (quarterly)
- Productivity and unit labor costs by industry (quarterly, NSA)
- Components of aggregate investment (quarterly)
- Population, employment, unemployment and self-employed (quarterly)
- Disposable income of households and components (quarterly)
- Incoming orders by manufacturing sub-industry (monthly)
- Establishments, employment, hours, wages and revenues by manufacturing sub-industry (monthly, NSA)
- Firms, plants, revenues, employment and investment by manufacturing sub-industry (yearly)

Deutsche Bundesbank:

- Stock prices, exchange rates, prices (monthly, NSA)
- Central bank policy interest rates (monthly, NSA)
- Money market interest rates for different maturities (monthly, NSA)
- Government debt interest rates for different maturities (monthly, NSA)
- Interest rates on debt of residents (monthly, NSA)

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- Monetary aggregates (monthly)
- Lending to enterprises, households and government (monthly)

Thomson Reuters Datastream:

- IFO index by sector (monthly)
- IFO business situation index by sector (monthly)
- IFO expectations index by sector (monthly)
- Industrial production (monthly)
- Current and capital account (monthly, NSA)
- Imports, exports, trade balance and terms of trade (monthly)
- Unemployment and vacancies (monthly)
- Consumer confidence and productivity (monthly)
- Wages and insolvencies (monthly, NSA)
- VDAX volatility index (monthly)

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Appendix D: Outline of the Numerical Solution Technique

As the model with adjustment costs has no analytical solution, it was solved via discrete value function iteration. To this end a grid for each of the state variables needed to be created. The points on the capital grid were chosen in increments of the depreciation rate δ so that the inaction option for investment always lies on the grid. Moreover, the aggregate and idiosyncratic profitability processes were discretized using the method proposed by Tauchen (1986) which was adapted to the case of a markov-switching process in the latter case. Finally, the belief grid was chosen to be equidistant between zero and one.

Starting from an initial value function guess the Bellman Equation then needs to be iterated until the value function has converged. One of the problems associated with the model at hand is that the posterior beliefs that are produced by Bayesian updating for a given combination of prior beliefs, and current and future idiosyncratic profitability do not usually lie on the discrete belief grid. Therefore, when taking expectations of the value function next period in the maximization step of the Bellman Equation it is necessary to use interpolation in the belief dimension(s). For a markov-switching process with just two regimes this is straightforward. However, once there are more than two regimes a further complication arises.

With just one regime, it is sufficient to include one belief dimension in the state vector of the value function that can take on any value between zero and one. Linear interpolation is then straightforward. However, with three regimes already two belief dimensions need to be included in the state vector. What makes this and higher-dimensional cases tricky is that not all belief combinations between zero and one are admissible, due to the fact that beliefs always need to sum to one and cannot be negative. I.e. only the simplex of two belief states is well defined. Therefore, multidimensional linear interpolation is no longer feasible.

A different interpolation method that can be applied for such a state space is simplicial linear interpolation. With this method, the state space is divided into triangles of points via a delaunay triangulation which are then used to produce a truly linear interpolation using three points. This method is described in more detail in Brumm and Grill (2010) in the context of a general equilibrium model. Even though this makes the interpolation feasible, it has some costs in terms of speed. Once the interpolation needs to be performed in more than two dimensions and for large numbers of grid points the interpolation stage can quickly become a bottleneck.

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In addition to the two-dimensional simplicial interpolation that is necessary to solve the model, a three-dimensional simplicial interpolation in belief and capital space is needed when simulating the model. The reason for this is that in the simulation beliefs do not lie on the grid and therefore the future choice of capital needs to be interpolated. This interpolation usually leads to a capital stock that in turn also does not lie on the capital grid. Hence, the next capital choice needs to be found by interpolating in three dimensions. This makes the simulation of the model with learning quite slow.

The models with learning that are presented in section 3.5 were solved with 30 grid points for idiosyncratic profitability, 7 grid points for aggregate profitability, 5 grid points for each of the belief states and a capital grid with 188 points. It took between four and eight hours to solve such a model and the simulation of a regime shift took again a similar amount of time. For comparison, the same model without learning took around ten to twenty minutes to solve!

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