Economic curtailment of intermittent renewable energy sources

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Abstract

In a power system featuring a large share of intermittent renewable energy sources (RES) and inflexible thermal generators, efficiency gains on generation costs could be achieved by curtailing the production of RES. However, as RES feature very low variable production costs, over-curtailment can be costly. In this article, we use a stylised analytical model to assess this trade-off. We show that while curtailing RES when their variability is high and the system flexibility is low can reduce generation costs, the different stakeholders (consumers, dispatchable generators, RES) will not necessarily benefit from such measures. As a consequence, generators will opt for a sub-optimal level of curtailment, and this level of curtailment should rather be set by the TSO. Either incentive to provide the TSO with accurate forecasts of RES availability, or alternatively centralised forecasting by the TSO, should then be put into place to solve the resulting problem of asymmetry of information.

Keywords

Market design, Curtailment, Large-scale renewables, Intermittency

Classification codes: Q42, L94
1. Introduction

In order to foster the development of renewable energy sources (RES) in Europe, RES benefit from priority of dispatch. Following European directive 2009/28/EC, priority should be given to RES as long as the safety of the power system is not threatened. The curtailment of electricity, i.e. the use of less RES generation than potentially available, should therefore be minimised and should occur only when needed to ensure security of supply.

However, such a priority should be questioned at times when intermittent RES constitute a significant share of the generation mix. The variability of RES and the limited flexibility of the conventional thermal units constitute a challenge for the operation of power systems. This inflexibility is reflected for instance through the occurrence of significantly negative prices in Germany ((Mayer, 2013; Nicolosi (2010)). Such prices reveal that while the variable-cost of electricity generated by RES is equal to zero, releasing the constraints on RES dispatch could lead to benefits. Economic curtailment of RES should then be considered as an additional tool to the technical curtailment of RES.

The optimal level of RES curtailment is the result of a trade-off. On the one hand, not using fully “free” (i.e. with a zero marginal-cost) RES energy may result in higher generation costs, as the substitutes are more expensive. On the other hand, it allows releasing part of the binding technical constraints for inflexible thermal power plants. This trade-off is hence impacted by the marginal costs and the flexibility of the thermal power plants, as well as the variability of RES generation. An additional issue is the very different consequences for the stakeholders involved: consumers, thermal power plants, and RES power plants. The level of curtailment maximising the social welfare might result in losses for the stakeholders offering the RES energy. In the absence of compensations, this optimal level of curtailment will then not be reached. The literature on RES curtailment is still in its infancy, and most studies have been focusing on curtailment of RES in order to solve local congestions or to ensure security of supply: curtailment for higher economic efficiency has seldom been studied. Moreover, existing quantitative studies do not deal with variations in the key parameters such as system flexibility or RES variability, and do not assess the impact on each category of stakeholders. In this article, we build a stylised model of energy production in order to study the mechanisms of RES curtailment for economic reasons. The analysis of the aforementioned trade-off and the consequences on the stakeholders are at the core of our reflection.

First, as we want to focus on the efficiency of operations for a given generation mix, our model is a short-term model and the installed capacity of RES and thermal units are exogenous fixed parameters. It is also considered that consumers do not react to prices and that demand for energy is fixed and inelastic to prices. This demand is met by energy supplied by RES generators and thermal generators. Note that the generators do not adopt any strategic behaviour and offer energy at their marginal generation cost. Second, in order to take into account the impact of the variability of production by RES, we consider two successive production time-periods. Availability of RES is stable within each period but can vary significantly between the two periods. Availability of thermal units can also evolve between two periods as units that have not been generating in the first period are limited in the

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1 The term “variable” is sometimes considered to describe the nature of RES behaviour more accurately. However, the term “intermittent” is commonly employed and will be used in this paper, referring mainly to wind and solar PV technologies.

2 All through this paper we employ the term “economic curtailment” as opposed to “technical curtailment”, i.e. required to ensure safety of operations. It does not mean that technical curtailment has no economic rationale or that economic curtailment is not grounded in technical fundamentals.
second time-period due to technical ramping or start-up constraints. Third, it is possible to curtail RES generation in first period. The trade-off is then the one described previously: curtailing RES generation in the first-period leads to higher generation costs in the first-period but allows reducing costs and prices in the second period. Finally, the optimal level of curtailment is established as the one maximising the social welfare, and the impact of a given level of curtailment on each categories of stakeholders is obtained by measuring the variation of their surplus compared to a situation without any curtailment.

Our results confirm that potential savings will be achieved by adopting an optimal level of curtailment, and we describe the relationship between the key parameters driving these benefits. We then show that depending on the level of RES installed capacity and the system flexibility, the price-impact and the volume-impact of RES curtailment can lead to gains or losses for each stakeholders. Interestingly enough, RES can benefit from curtailment even without compensation. In addition, we argue that if decisions to curtail RES are taken by generators, it will result in a sub-optimal level of curtailment. Note that this will be especially the case if thermal generators and RES generators belong to the same utilities. At last, the quality and transparency of data on wind availability will be crucial to ensure that efficient decisions are taken, while RES generators will have significant incentives to manipulate these data.

Our paper is organised as follows: We first review the existing literature in section 0, and highlight the complementarity of our stylised approach with the existing quantitative studies. We then describe the framework of our model and the main assumptions made in section 0. Analytical results are detailed in section 0, while their policy implications are discussed in section 0.

2. Previous Works

The topic of economic RES curtailment has not been dealt with extensively so far, as the share of intermittent RES in the generation mix was not significant, and priority was given to a fast development of these resources.

Most existing works on RES curtailment are empirical studies identifying best practices among the curtailment mechanisms put into place worldwide. This is for instance the case of a collection of reports by the National Renewable Energy Laboratory (Fink, Mudd, Porter, & Morgenstern, 2009; Lew et al., 2013; Rogers, Fink, & Porter, 2010). These studies highlight the fact that curtailment occurs mainly for technical reasons, when the system encounters transmission or operational constraints. An analysis of different policies for principles of access, including best practices of interruptible connections for wind generation, can also be found in studies by Currie et al. (2011) and Anaya and Pollitt (2013). Yet their focus is the connection of distributed generation at lower costs for network operators. Note that an interesting exception is a study realised for the Public Service Company of Colorado, revealing that curtailing wind to reduce the cycling costs of coal units would lead to significant benefits (Xcel Energy, 2011).

The concept of economic wind curtailment in a context of large-scale integration of electricity from RES is discussed in depth in a qualitative analysis by Brandstädt, Brunekreeft, and Jahnke (2011). Through the example of Germany, they argue that removing the restrictions on RES curtailment will be necessary as the system would otherwise feature too much inflexibility both on supply and demand side. They also present a compensation scheme leading to a reduction of total system costs without deteriorating RES revenues. Lastly, the authors argue that such a policy would not conflict with climate policies as higher investments in RES would compensate for the curtailed low-carbon energy.

A few quantitative studies can also be found. Ela (2009) argues that curtailing wind generation can be economically advantageous, using the example of a simple three-bus system. Yet in his model these benefits result from the existence of congested lines, with wind generation at a given bus preventing
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the dispatch of cheaper generators. The constraints resulting from the limited flexibility of thermal generators are not taken into account.

Finally, in a recent paper, Wu and Kapuscinski (2013) built a highly detailed power system stochastic optimisation model, and identified a series of efficiency gains thanks to a policy of wind curtailment. They show that the flexibility provided by curtailling RES allows the use of cheap and inflexible thermal units instead of more expensive flexible thermal units. The major components of the savings identified by Wu and Kapuscinski result from avoided cycling costs. According to their study, by curtailting intermittent RES, it is not only possible to lower operation costs but it is also possible to achieve system emission reductions.

Despite these quantitative studies, we believe there is room for further investigation. A limit of the existing numerical quantitative studies is that key parameters such as the system flexibility or the variability of RES are either not considered or set to a single value. Hence, a first significant contribution of our approach based on a stylised model is that we are able to describe the relationship between the pivotal parameters and the optimal level of curtailment. Moreover, existing works only assess the variations of overall generation costs, while the impacts on each stakeholder can be quite different. By using a tailor-made stylised model we are able to focus on optimal curtailment policy for different values of these parameters. Therefore, a second significant contribution of our study is that we are able to analyse how the efficiency gains achieved thanks to curtailment would be shared between the different stakeholders.

3. Model

3.1 Modelling framework

Our analytical model solves a two-period unit-commitment problem. During each of these periods, a constant fixed demand is to be met by generation from RES and a set of thermal generators. We consider that generators bid their marginal cost and that the price is set as the marginal cost of the marginal unit. Note that our problem is a short-term one, and that the installed capacities are fixed parameters. We assume that the available capacity of RES is lower than the demand so that the price will be set by the marginal cost of the marginal thermal generator.³

RES generation is variable and uncertain. RES are available for sure in the first period A. When curtailment decisions are taken in period A, RES availability in period B is still uncertain. In the case when RES are not available in period B, thermal units will have to adapt their production to meet the demand for energy.⁴

We consider that all the thermal units available in period A are also available in period B, as their availability should not vary over such a short lapse of time. However, we assume that thermal generators have limited flexibility. The least flexible units not generating in period A will not be able to start-up or to ramp-up to full production between the two periods. These inflexible producers will therefore withdraw their offers from the supply function, and the resulting inverse supply function will hence feature a steeper slope in period B than in period A. This is illustrated in Figure 1.

³ We ignore the case of scarcity when the available thermal capacity is lower than the demand. As a result of the fast development of RES generation, most power systems dealing with a high share of intermittent renewables also typically feature over-supply. Moreover, as the cost of scarcity is quite high, it is very likely that these reserve margins will be preserved.

⁴ Note that even if there is no uncertainty regarding the availability of RES, our approach remains relevant as RES generators might be unavailable for sure in period B.
As RES units have a marginal cost equal to zero, they should be dispatched first. However, the production is optimised over both time-periods simultaneously. It is possible to curtail RES generation in period $A$, which will lead to higher costs and a higher electricity price in period $A$. Inflexible units generating in period $A$ will then be available in period $B$, leading to lower costs and a lower electricity price in period $B$. The optimal level of RES curtailment will be a result of this trade-off between generation costs in period $A$ and generation costs in period $B$.

As of today, the remuneration of RES generators is not purely based on wholesale electricity prices. RES can for instance receive a premium on top of the market-price. When curtailed, the RES generators can also receive compensation, as described for instance by Brandstätter et al. (2011). In the absence of demand elasticity, the total welfare is only affected by generation costs. The optimal level of curtailment is therefore not affected by the remuneration and compensation schemes. However, in order to calculate the impact on each stakeholder (i.e. consumers, RES generators, thermal generators), we defined a set of remuneration and compensation schemes: feed-in premium or pure market-based remuneration; full compensation or no compensation.

For simplification, we do not consider the carbon emission costs in our discussion, as we assume these costs could be easily internalised in the variable generation costs of thermal producers.

### 3.2 Model implementation

**RES availability**

We assume the generation mix features intermittent RES with an available capacity $K_r < D$. In the first period $A$, RES can generate any amount of energy $q^A_r \leq K_r$ at a marginal cost equal to zero. We consider two states of nature in period $B$. The first state “availability of RES” is denoted by the superscript $\bar{w}$ and occurs with probability $\nu$; RES can then generate any amount of energy $q^B_r \leq K_r$ at a marginal cost equal to zero. The second state “unavailability of RES” is denoted by the superscript $\bar{w}$ and occurs with probability $1 - \nu$; in this case RES are unable to deliver any energy at all in period $B$. 

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**Figure 1: Evolution of the merit-order of thermal units due to start-up and ramping constraints**

![Merit-Order in period A: all units are available](image1)

![Merit-Order in period B: part of the not-fully flexible units (shadow) is unavailable](image2)
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For simplification, we assume in this paper that there are no significant constraints for thermal plants to ramp down. Therefore, when available in period $B$, RES will be generating at full potential and $q_t^B = K_r$ in the first state. For the sake of simplicity we can then denote $q_t^B$ as $q_r$.

**Thermal generation and price formation**

As the RES available capacity $K_r$ is not sufficient to meet the demand $D$, the remaining energy must be delivered by thermal generators. We assume that the market is perfectly competitive and that generators bid their marginal cost of generating energy as described by Stoft (2002). We consider that generators are fully available in period $A$ and that the marginal cost $C_A(q_t^A)$ of generating the quantity of energy $q_t^A$ with thermal generators in period $A$ is linear:

$$C_A(q_t^A) = a + b \cdot q_t^A$$

The parameters $a$ and $b$ are inputs that depend on the power system properties. The variable $b$ will be higher when the range of marginal costs of the different generation units will be higher.

The price $p_A$ is then set as the marginal cost of the most expensive unit needed to meet demand $D$. The resulting aggregated inverse supply function in period $A$ when RES generate the quantity $q_r$ is then the following:

$$p_A(q_r) = a + b \cdot (D - q_r)$$

We also assume, as described in section 0, that due to the start-up and ramp-up constraints, part of the thermal generators not delivering any energy in period $A$ will not be available in period $B$. This will result in a steeper cost function and the marginal cost $C_B(q_t^w, q_t^A)$ for thermal generators of delivering $q_t^w$ in case RES are unavailable will then obey to the following equation graphically described in Figure 2:

$$\begin{align*}
C_B(q_t^w, q_t^A) &= a + b \cdot q_t^w & \text{for } q_t^w \leq q_t^A \\
C_B(q_t^w, q_t^A) &= a + b \cdot q_t^w + b \cdot \varphi \cdot (q_t^w - q_t^A) & \text{for } q_t^w > q_t^A
\end{align*}$$

$\varphi$ is a penalty parameter that reflects the inflexibility of the thermal generators.

**Figure 2: Evolution of the inverse supply function of thermal generators**

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5 The MIT energy initiative (2012) has for instance enlightened us to the fact that modern nuclear plants ramp asymmetrically: it takes them one hour to ramp-down 20%, while they might need up to 8 hours to ramp-up to full potential. Moreover, most thermal units feature significant start-up time.
As demand is equal to $D$ in both periods, the resulting price $p^w(q_r)$ in period $B$ when RES are unavailable and RES have generated the quantity $q_r$ in period $A$ will therefore be equal to:

$$p^w(q_r) = a + b.D + b.q_r$$

When RES are available in period $B$, RES will be generating at full potential $K_r$. The amount of energy generated by thermal generators $D - K_r$ will hence be lower or equal to the amount of energy generated by thermal generators in period $A$. The price will then be equal to:

$$p^w = a + b.(D - K_r)$$

**Remuneration and compensation scheme for RES**

In period $A$, RES receive for the energy generated $q_r$ remuneration $R_g$ per unit of energy, based on the market price $p_A(q_r)$ and possibly a premium $Z$. In the case of a remuneration based on market prices only, the premium $Z$ is equal to 0. In the case of a remuneration that is made of both market-revenue and a premium (e.g. Feed-in premium), $Z > 0$.

RES generators can also receive compensation for the energy curtailed $K_r - q_r$. The remuneration $R_c$ per unit of energy curtailed is then a by definition a share $(1 - \alpha)$ of the market price component, and a share $(1 - \beta)$ of the premium component. Depending on the compensation schemes, $\alpha$ and $\beta$ are equal to 0 (full compensation of the related component) or 1 (no compensation of the related component). In this paper we focus on two extreme cases. Under case #1, RES generators do not receive any compensation at all: $\alpha = \beta = 1$. Under case #2, RES generators receive full compensation when curtailed: $\alpha = \beta = 0$.

$$\begin{align*}
R_g &= p_A(q_r) + Z \\
R_c &= (1 - \alpha)p_A(q_r) + (1 - \beta)Z
\end{align*}$$

Where $Z \geq 0$ and $(\alpha, \beta) \in \{0,1\}^2$

In period $B$, RES do not receive any remuneration if unavailable, and receive a remuneration $R^w$ per unit of energy generated if available, with $R^w = p^w(q_r) + Z$.

**4. Analytical Results**

**4.1 Optimal level of curtailment**

**Optimal level of curtailment**

In this section, we determine the level of curtailment maximising the social welfare. The optimal level of curtailment is defined as the one in which the production $q_r$ by RES in period $A$ maximises the social welfare $S(q_r)$ across period $A$ and period $B$, for both states of nature. This problem can be simplified as demand is fixed and inelastic in both time-periods. Any variation of the consumer surplus is then automatically compensated by a variation of generators surplus; social welfare is maximised when the generation costs across both time periods $C(q_r)$ are minimised.

The optimal level of production $\tilde{q}_r$ is therefore defined as:

$$\tilde{q}_r = \arg \min_{q_r} C(q_r)$$

Subject to $0 \leq q_r \leq K_r$

Where $C(q_r) = \int_{q_r}^D [C_A(x - q_r)] \, dx + \nu. \int_{K_r}^D [C_B(x - K_r, D - q_r)] \, dx + (1 - \nu). \int_0^D [C_B(x, D - q_r)] \, dx$
**Proposition 1.** The optimal level of curtailment is independent of the remuneration and compensation schemes for RES generators, and is such that RES generate in period A the quantity \( \bar{q}_r \):

\[
\begin{align*}
\bar{q}_r &= \frac{a + bD}{b \cdot (1 + \varphi \cdot (1 - v))} & \text{for } K_r > \frac{a + bD}{b \cdot (1 + \varphi \cdot (1 - v))} \\
\bar{q}_r &= K_r & \text{for } K_r \leq \frac{a + bD}{b \cdot (1 + \varphi \cdot (1 - v))}
\end{align*}
\]

Savings \( \Delta S \) are such that:

\[
\Delta S = S(\bar{q}_r) - S(K_r) = \frac{1}{2} \left( 1 + \varphi \cdot (1 - v) \right) \times (K_r - \bar{q}_r)^2
\]

When the available capacity is high and flexibility is low, savings can be achieved by curtailing RES generation in period A. The incentives to curtailment decrease with the flexibility of the system (represented by factor \( \varphi \)) and increase with the variability of the system (represented by \( 1 - v \)).

Note that in the simple case in which the cheapest thermal units have very low marginal costs (i.e. \( a \approx 0 \)) the threshold is equal to \( \frac{D}{1 + \varphi \cdot (1 - v)} \). Without inflexibility costs (\( \varphi = 0 \)) or variability (\( 1 - v = 0 \)), RES would generate as much as available, until demand \( D \) is met. Yet, as there are flexibility issues, only a smaller share of the demand \( \frac{D}{1 + \varphi \cdot (1 - v)} \) should be generated by RES.

The level of curtailment \( \bar{q}_r \) maximising the social welfare is represented in Figure 3.

**Curtailment level maximising the profits of RES generators**

In this section we determine the curtailment level maximising the profits of RES generators. These profits are only impacted in period A; in period B RES are either unavailable (and hence do not receive anything) or are available, fully generating, and receiving a market price \( p^w \) and potentially a premium \( Z \) that are independent from the curtailment level.

**Proposition 2.** The level of curtailment maximising the profits of RES generators is such that RES generate the quantity \( q^*_r \) in period A.

2.1) **Case #1: Feed-in Premium / No compensation**

\[
\begin{align*}
q^*_r &= \frac{Z + a + bD}{2 \cdot b} & \text{for } K_r \geq \frac{Z + a + bD}{2 \cdot b} \\
q^*_r &= K_r & \text{for } K_r \leq \frac{Z + a + bD}{2 \cdot b}
\end{align*}
\]

2.2) **Case #2: Feed-in premium /Full compensation**

\[ q^*_r = 0 \quad \forall Z \geq 0 \]

In the absence of compensation, the optimal level of curtailment for RES generators is not null when the available capacity is higher than a given threshold. RES are willing to reduce their volume of production, as they benefit from the consequent rise of wholesale prices. The production level is independent from the system flexibility and the RES variability, and is only affected by the premium and the nature of the costs of thermal generators. RES generators tend to over-curtail their production when flexibility is high and variability low; they tend to under-curtail their production when flexibility is low and variability is high. A higher premium leads to lower curtailment as the gains from higher prices are partially offset by the loss of the premium. A steeper curve of marginal costs of thermal generators gives more incentives to curtailment as the price effect will be higher for a given volume of curtailment.

In the case RES generators get full compensation when curtailed, they will have an incentive to over-curtail, as they will benefit from the resulting higher prices.
The level of curtailment maximising the profit of RES generators is represented in Figure 3 for the two compensation schemes.

Curtailment level maximising the profits of all generators
In this section we determine the curtailment level maximising the profits of both thermal and RES generators, for instance when they are integrated within a large utility.

**Proposition 3.** The level of curtailment maximising the profits of both RES and thermal generators is such that RES generate in period A the quantity \( q_{R+T}^* \)

\[
\begin{align*}
q_{R+T}^* = D + \frac{Z + a - bD}{b(1 - \nu). \varphi + 1} & \quad \text{for } K_r \geq D + \frac{Z + a - bD}{b((1 - \nu). \varphi + 1)} \\
q_{R+T}^* = K_r & \quad \text{Otherwise}
\end{align*}
\]

3.1) Case #1: Feed-in Premium / No compensation

3.2) Case #2: Feed-in premium / Full compensation

\[
\begin{align*}
q_{R+T}^* = D - \frac{K_r}{(1 - \nu). \varphi - 1} & \quad \text{for } (1 - \nu). \varphi \in \left[ 0; 1 + \frac{K_r}{D} \right] \\
q_{R+T}^* = K_r & \quad \text{for } (1 - \nu). \varphi \in \left[ 1 + \frac{K_r}{D}; 1 + \frac{K_r}{D - K_r} \right] \\
q_{R+T}^* = K_r & \quad \text{for } (1 - \nu). \varphi \geq 1 + \frac{K_r}{D - K_r}
\end{align*}
\]

Once again, in the absence of compensation, the level of curtailment maximising the profits of generators is not null. Moreover, when the available RES capacity is high and the curve of the marginal costs of thermal generators steep, the incentives to curtail wind generation are then higher when the system flexibility is high or the variability is low, as both RES generators and thermal generators benefit from higher prices in period A. Note that the social welfare increases in case of curtailment when the system flexibility is low and the variability high: producers therefore have incentives to curtail RES that go against the system benefits. Incentives to over-curtailment when flexibility is high and variability is low can be partially offset by the existence of a premium, as this premium is lost in case of over curtailment. Yet such a premium also leads to under-curtailment when the system flexibility is low and the variability is high.
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Figure 3: Optimal level of curtailment and level of curtailment maximising the profits of generators, with and without compensation

In the case RES generators get compensation when curtailed, the incentives to over-curtail are even higher than without compensation since the RES generators keep receiving the premium, and the price-impact occurs without impacting the volume impact. For high flexibility and low variability, the generators would then rather withhold their whole RES production.

The level of curtailment maximising the profit of both kinds of generators is represented for the two compensation schemes in Figure 3.

**Proposition 4.** In the case when RES do not receive any premium and when no compensation is provided to curtailed RES, the level of curtailment maximising the profits of both RES and thermal generators is further from the optimal level of curtailment than the level of curtailment maximising the profits of RES only.

\[
Z = 0, \quad \alpha = 1 \implies |\tilde{q}_r - q_R^*| \leq |\tilde{q}_r - q_{R+T}^*| \\
\text{And more specifically} \begin{cases} (1 - \nu), \varphi \leq 1 \implies \tilde{q}_r \geq q_R \geq q_{R+T}^* \\ (1 - \nu), \varphi \geq 1 \implies \tilde{q}_r \leq q_R \leq q_{R+T}^* \end{cases}
\]

When no compensation is provided to RES generators and when they do not receive any premium for the energy generated on top of wholesale prices, integrated RES generators and conventional generators will have higher incentives to deviate from the optimal level of generation than RES alone.

On-the-side payments to RES, such as feed-in premium and compensation schemes affect the behaviour of RES generators, and integration with thermal generators can then be beneficial.
4.2 Impact of curtailment on each stakeholder

In this section, we look at the impact of optimal curtailment on the three categories of stakeholders identified in this study: consumers, RES generators and thermal generators. Indeed, even though the optimal level of curtailment increases the total social welfare, whether these stakeholders will benefit or lose depends highly upon the system flexibility, available RES capacity, and generation volatility.

We denote $\Delta S_c$ (respectively $\Delta S_R$ and $\Delta S_T$) the variations in the surplus of consumers (respectively RES generators and thermal generators) resulting from the switch from no-curtailment policy to optimal-curtailment policy.

**Proposition 5.1.** In the case when RES generators do not receive any compensation when curtailed, i.e. $\alpha = \beta = 1$ then:

$$\Delta S_C \geq 0 \Leftrightarrow (1 - \nu) \cdot \varphi \geq 1 - \frac{Z}{D \cdot b}$$

$$\Delta S_R \geq 0 \Leftrightarrow K_r \geq \frac{b}{Z + (a + bD)} \cdot \frac{\varphi (1 - \nu)}{1 + \varphi (1 - \nu)}$$

$$\Delta S_T \geq 0 \Leftrightarrow (1 - \nu) \cdot \varphi \leq 1$$

Consumers will benefit from curtailment if flexibility is low and variability is high, while thermal generators will benefit from optimal curtailment if flexibility is high and variability is low. This will not be affected by the available capacity of RES.

However, RES benefit from an optimal level of curtailment, even without compensation, when available capacity is high: the higher-price impact will then offset the reduced-volume impact.

When RES generators receive a higher premium, curtailment leads to further benefits for consumers and higher losses for RES generators.

This result is illustrated in Figure 4, in the simple case when RES do not get any premium on top of wholesale prices (i.e. $Z = 0$). Note than in the area $D$, only consumers will benefit from optimal curtailment.

**Figure 4: Impact of optimal curtailment on the different stakeholders in case no compensation and no premium are paid to RES generators**

![Figure 4: Impact of optimal curtailment on the different stakeholders in case no compensation and no premium are paid to RES generators](image-url)
**Proposition 5.2.** In the case when RES generators receive full compensation when curtailed, i.e. $\alpha = \beta = 0$ then:

\[
\Delta S_C \geq 0 \iff (1 - \nu).\varphi \geq \frac{1}{2} \times \left[ \frac{a + b.D}{b.D} + \sqrt{\left(\frac{a + b.D}{b.D}\right)^2 + 4} \right]
\]
\[
\Delta S_R \geq 0
\]
\[
\Delta S_T \geq 0 \iff (1 - \nu).\varphi \leq 1
\]

Once again, consumers will benefit from curtailment if flexibility is low and variability is high, while thermal generators will benefit from optimal curtailment if flexibility is high and variability is low. This will not be affected by the available capacity of RES or by the premium value. As they receive compensation when curtailed, RES generators will always benefit from optimal curtailment. These results are illustrated in Figure 5.

Note that in this case, the sign of the impact on the different stakeholders is not affected by the available RES capacity or by the premium paid to RES generators.

**Figure 5: Impact of optimal curtailment on the different stakeholders in case full compensation is paid to RES generators**

4.3 Extra costs as a result of lack of information

In this section, we consider that the level of curtailment is set by an agent (e.g. the Transmission System Operator) aiming at maximising the social welfare based on variability information provided by RES generators. RES will have incentives to manipulate this information in order to increase their profits.
Proposition 6. Similarly, in case the optimal level of curtailment is set based on an incorrect estimation \(1 - \hat{v}\) of the variability \(1 - v\), variation of the social welfare compared to the optimal curtailment level will be equal to:

\[
\Delta(v, \hat{v}) = S(\hat{q}_r) - S(q_r) = -\frac{(a + b . D)^2}{2 . b} \cdot \frac{\varphi^2}{(1 + (1 - v) . \varphi)^3} . (\hat{v} - v)^2
\]

In case RES do not get any compensation, variation of the profits of RES generators will be equal to:

\[
\Delta_R(v, \hat{v}) = S_R(\hat{q}_r) - S_R(q_r) = \frac{(a + b . D)}{b} \cdot \frac{\varphi . (v - \hat{v})}{(1 + (1 - v) . \varphi)^2} \cdot (\beta . Z + (a + b . D) . \frac{1 - \varphi . (1 - v)}{1 + \varphi . (1 - v)})
\]

In particular, for \(Z = 0\), when RES do not receive any premium,

\[
\Delta_R(v, \hat{v}) = \frac{(a + b . D)^2}{b} . \frac{\varphi . (1 - \varphi . (1 - v))}{(1 + (1 - v) . \varphi)^3} . (v - \hat{v})
\]

\(\{\text{For } (1 - v) . \varphi \leq 1, \quad (1 - \hat{v}) \geq (1 - v) \Rightarrow \Delta_R(v, \hat{v}) \geq 0 \text{ and } \Delta(v, \hat{v}) \leq 0\}
\]

\(\{\text{For } (1 - v) . \varphi \geq 1, \quad (1 - \hat{v}) \leq (1 - v) \Rightarrow \Delta_R(v, \hat{v}) \geq 0 \text{ and } \Delta(v, \hat{v}) \leq 0\}\)

When variability is low and flexibility is high, RES generators will tend to provide overestimations of variability (i.e. higher values for \(1 - v\)) leading to over-curtailment of RES generation. Oppositely, when variability is high and flexibility is low, RES generators will tend to provide underestimations of variability (i.e. lower values for \(1 - v\)) leading to under-curtailment of RES generation.

A penalty imposed on RES generators equal to \(-\Delta_R(v, \hat{v})\) could correct these incentives when the forecasts delivered differ from the realised output.

5. Results Discussion

5.1 Optimal level of curtailment and distributional impacts

Conclusion 1: It is rationale to curtail RES generation if flexibility is low and available RES capacity high.

From proposition 1, we are able to identify an optimal level of curtailment when the available RES capacity is higher than a threshold decreasing as the flexibility of the system and as the variability of the RES generation increases. For a stable thermal generation mix, curtailment will hence become beneficial as the penetration of RES becomes significant. Curtailment policies will then become increasingly relevant in a context of large-scale development of renewables. The priority of dispatch to RES as it exists today in Europe should then be reassessed. In case variability is high and flexibility low, savings can be significant.

This level of curtailment does not depend on the nature of RES remuneration, nor on whether RES get compensated, as it only reflects the generation costs, and the trade-off between making the most of available RES with zero marginal cost and allowing cheaper inflexible thermal units to generate energy.

Conclusion 2: The impact of curtailment is different for each stakeholder and varies with the available RES capacity and the system flexibility. In particular RES can benefit from curtailment even without a compensation scheme.

From propositions 5.1 and 5.2, we see that the surplus of the main stakeholders is affected in very diverse ways by curtailing RES generation. The benefits of generators are subject to a price-impact, as prices initially increase when RES generation is curtailed, and a volume-impact. Thermal generators tend to benefit from curtailment when flexibility is high and volatility is low, while consumers benefit from curtailment when flexibility is low and volatility is high. Interestingly enough, RES can benefit
from curtailment even in the case when they do not receive any compensation. This is the case when the available capacity is important enough so that the losses from lower generation are offset by higher prices.

While the optimal level of curtailment is not impacted by the remuneration and compensation schemes of RES, it drives the redistribution of the resulting benefits. When RES do not receive any compensation, the sign of the impact of curtailment switches as more generating units are available. The sign of the impact on the surplus of consumers and RES generators is also affected as the premium paid to RES on top of the wholesale price is affected. It implies that any curtailment scheme will have to be versatile enough to adapt to changing circumstances.

Providing compensation to curtailed RES generators allows them to benefit from curtailment whatever the available capacity, and hedge the different stakeholders against the variability of the premium. Note that even in the case of compensation to RES, consumers (who pay this compensation) can benefit from the reduced generation costs due to an optimal level of curtailment.

5.2 Delivering the optimal level of curtailment

Conclusion 3: Leaving curtailment decisions to generators will lead to sub-optimal levels of curtailment. This will especially be the case if RES and thermal generators are integrated within a single company.

Generators (either RES or thermal generators) can lose from curtailment and the level of curtailment maximising their profits can be substantially different from the optimal level of curtailment, as shown in proposition 2 and 3. Generators tend to over-curtail generation when the system flexibility is high and RES variability is low, while they tend to under-curtail generation when the system flexibility is low and RES variability is high. There are cases, as illustrated in Figure 4 in which only consumers will benefit from an optimal level of curtailment. It is then unlikely this optimal level of curtailment could be reached through decentralised decisions in a market from which consumers are absent. These results suggest that the decision regarding the amount of RES energy to be curtailed should be taken by an agent such as the transmission system operator.

Proposition 4 reveals that, in the absence of compensation for curtailed generation, the level of curtailment maximising the profits of integrated RES and thermal generators is even further from the optimal level of curtailment than the one maximising the profits of RES alone. We can conclude that when both kinds of generators are concentrated within a single utility, special attention should be paid to the level of curtailment implemented.

Conclusion 4: If the decision regarding the level of curtailment is taken by the system operator, a problem of asymmetry of information will occur. Incentives should be put into place to ensure the quality of production forecasts communicated by producers. Alternatively centralised forecasting should be implemented.

Even when a decision is decentralised to the system operator, RES could manipulate the information they provide to the system operator so as to influence its decision on the curtailment level. Proposition 6 shows that when variability is slow and flexibility is high, RES generators will have incentives to provide estimates of variability (i.e. in our context, the likelihood of a rapid reduction of RES availability) that are too high. On the other hand, RES generators will have incentives to provide too low estimates of variability when variability is high and flexibility is low.

This problem can be solved by exposing intermittent RES to the costs resulting from deviations from their declared schedule. Measures similar to the EU regulation 1227/2011 on wholesale energy market integrity and transparency (REMIT) can also be implemented. This regulation compels participants to disclose any insider information that could significantly affect wholesale power prices,
such as the unavailability of generation units. However, in the case of REMIT, only plants with an installed capacity higher than 100 MW will be concerned, which excludes most of the RES installations. Such shortcomings will be an obstacle to the efficient management of RES production.

Alternatively, the TSO can centralise forecasting activities to make sure that it has access to quality forecast. Perez-Arriaga and Batlle (2012) already argued that the benefits of aggregating data justify centralisation of wind forecasting activities.
Appendixes

A.1 Nomenclature

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Demand for energy in both periods</td>
</tr>
<tr>
<td>$q_r$</td>
<td>Quantity of energy generated by RES in period A</td>
</tr>
<tr>
<td>$K_r$</td>
<td>Potential for energy generation by RES in period A and B when available</td>
</tr>
<tr>
<td>$C_A(q)$</td>
<td>Marginal cost for thermal generators of generating the quantity of energy $q$ in period A</td>
</tr>
<tr>
<td>$C_B(q_2, q_1)$</td>
<td>Marginal cost for thermal generators of generating the quantity of energy $q_2$ in period B when they generated $q_1$ in period A</td>
</tr>
<tr>
<td>$C(q)$</td>
<td>Total expected generation costs when RES generated $q$ during period A</td>
</tr>
<tr>
<td>$p_A(q)$</td>
<td>Price in period A when RES generated $q$ during period A</td>
</tr>
<tr>
<td>$p^w(q)$</td>
<td>Price in period B when RES generated $q$ during period A and RES are not available in period B.</td>
</tr>
<tr>
<td>$p^w$</td>
<td>Price in period B when RES are available in period B.</td>
</tr>
<tr>
<td>$a$</td>
<td>Constant parameter of the inversed supply-function for thermal generators</td>
</tr>
<tr>
<td>$b$</td>
<td>Slope of the inversed supply-function for thermal generators</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Flexibility penalty for non-committed RES generators</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Probability that RES are available in period B</td>
</tr>
<tr>
<td>$S(q)$</td>
<td>Total economic surplus when RES generated $q$ during period A</td>
</tr>
<tr>
<td>$S_A(q)$</td>
<td>Economic surplus of thermal generators in period A when RES generated $q$ during period A</td>
</tr>
<tr>
<td>$S_B(q)$</td>
<td>Economic surplus of thermal generators in period B when RES generated $q$ during period A</td>
</tr>
<tr>
<td>$S_R(q)$</td>
<td>Economic surplus of RES generators when RES generated $q$ during period A</td>
</tr>
<tr>
<td>$S_T+R(q)$</td>
<td>Economic surplus of all generators when RES generated $q$ during period A</td>
</tr>
<tr>
<td>$\bar{q}_r$</td>
<td>Level of production of RES in period A when curtailment is optimal</td>
</tr>
<tr>
<td>$\tilde{q}_r$</td>
<td>Level of production of RES in period A maximising RES surplus</td>
</tr>
<tr>
<td>$q_{R+T}^*$</td>
<td>Level of production of RES in period A maximising generators surplus</td>
</tr>
</tbody>
</table>
A.2. Proof of Proposition 1

As demand is not flexible, social welfare is maximised when the generation costs are minimum. The optimal level of production $\bar{q}_r$ is then defined as:

$$\bar{q}_r = \arg \min_{q_r} C(q_r)$$
subject to $0 \leq q_r \leq K_r$

where $C(q_r) = \int_{q_r}^{D} [C_A(x - q_r)] \, dx + v. \int_{0}^{D-K_r} [C_B(x - K_r, D - q_r)] \, dx + (1-v). \int_{0}^{D} [C_B(x, D - q_r)] \, dx$

$$C(q_r) = \int_{0}^{D-q_r} [C_A(x)] \, dx + v. \int_{0}^{D-K_r} [C_B(x, D - q_r)] \, dx + (1-v). \int_{0}^{D} [C_B(x, D - q_r)] \, dx \quad (1)$$

By definition, $K_r \geq q_r$ and $D - K_r \leq D - q_r$

$$C(q_r) = \int_{0}^{D-q_r} [a + b.x] \, dx + v. \int_{0}^{D-K_r} [a + b.x + b. \varphi.(x - (D-q_r))] \, dx$$

$$C(q_r) = \int_{0}^{D-q_r} [a + b.x] \, dx + v. \int_{0}^{D-K_r} [a + b.x] \, dx + (1-v). \left( \int_{0}^{D} [a + b.x] \, dx + \int_{0}^{q_r} [b. \varphi.x] \, dx \right)$$

$$=> \frac{dC(q_r)}{dq_r} = - \left( a + b.(D-q_r) \right) + (1-v). b. \varphi.q_r \quad (2)$$

$$=> \begin{cases} \bar{q}_r = \frac{a + bD}{b \left( 1 + \varphi(1-v) \right)} & \text{for } K_r > \frac{a + bD}{b \left( 1 + \varphi(1-v) \right)} \\ \bar{q}_r = K_r & \text{for } K_r \leq \frac{a + bD}{b \left( 1 + \varphi(1-v) \right)} \end{cases} \quad (3)$$

Moreover, savings when curtailing RES generation from $K_r$ to $\bar{q}_r$ will then be equal to:

$$\Delta S = S(\bar{q}_r) - S(K_r) = C(K_r) - C(\bar{q}_r) = \int_{\bar{q}_r}^{K_r} \frac{dC(q_r)}{dq_r} \, dq_r$$

$$=> \Delta S = \left( 1 + \varphi(1-v) \right) \times (K_r - \bar{q}_r)^2 \quad (4)$$

A.3. Proof of Proposition 2

The surplus of wind generators in period $B$ is not impacted by the curtailment level, as the output is equal to 0 in case these resources are not available, and as it is always fully dispatched at the same prices in case these resources are available.

Therefore, the production $q^*_r$ of RES in period $A$ maximising the profits of RES generators is defined as:

$$q^*_r = \arg \max_{q_r} S^A_R(q_r)$$
subject to $0 \leq q_r \leq K_r$

where $S^A_R(q_r)$ is the surplus of RES generators in period $A$:

$$S^A_R(q_r) = \int_{0}^{q_r} [p_A(q_r) + Z] \, dq + \int_{q_r}^{K_r} [(1-\alpha).p_A(q_r) + (1-\beta).Z] \, dq \quad (5)$$

And $p_A(q_r)$ is the marginal cost of generating of the most expensive thermal unit called:

$$p_A(q_r) = a + b.(D - q_r)$$
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\[ \Rightarrow \frac{dS^A_r(q_r)}{dq_r} = \beta. Z + \alpha. (a + b. D) - (1 - \alpha). b. K_r - 2. \alpha. b. q_r \]  \hspace{1cm} (6.1)

**Case #1: Feed-in Premium, with no compensation**

\[ \Rightarrow \begin{cases} 
q^*_B = \frac{Z + a + bD}{2. b} & \text{for } K_r \geq \frac{Z + a + bD}{2. b} \\
q^*_B = K_r & \text{for } K_r \leq \frac{Z + a + bD}{2. b} 
\end{cases} \]  \hspace{1cm} (6.2)

**Case #2: Feed-in Premium, with full compensation**

\[ \alpha = 0 ; \beta = 0 \]

\[ \Rightarrow \forall q_r \in [0, q_r], \frac{dS^A_r(q_r)}{dq_r} = -b. K_r < 0 \]  \hspace{1cm} (6.3)

\[ \Rightarrow q^*_r = 0 \]

**A.4. Proof of Proposition 3**

For a RES generation equal to \( q_r \) in period A, the surplus of thermal generators \( S^A_r(q_r) \) in period A is equal to:

\[ S^A_r(q_r) = \int_0^{D-q_r} [p_A(q_r) - C_A(q)] dq \]

where \( p_A(q_r) = a + b. (D - q_r) \) and \( C_A(q) = a + b. (D - q) \)

\[ \Rightarrow \frac{dS^A_r(q_r)}{dq_r} = -b. (D - q_r) \]  \hspace{1cm} (7.1)

The profit of conventional generators in phase B only depends on the curtailment level in period A in the case in which there is no wind. For a RES generation equal to \( q_r \) in period A, the surplus of thermal generators \( S^B_r(q_r) \) in period B is equal to:

\[ S^B_r(q_r) = (1 - \nu) \left[ D . p^\omega(q_r) - \int_0^D C_B(q, D - q_r) dq \right] + \nu. \lambda(q_r) \]

with \( \frac{d\lambda(q_r)}{dq_r} = 0 \)

and where by definition:

\[ p^\omega(q_r) = a + b. D + b. \varphi. q_r \]

\[ C_B(q^w, q^A) = a + b. q^w \]  \hspace{1cm} for \( q^w \leq q^A \)

\[ C_B(q^w, q^A) = a + b. q^w + b. \varphi. (q^w - q^A) \]  \hspace{1cm} for \( q^w > q^A \)

\[ \Rightarrow \frac{dS^B_r(q_r)}{dq_r} = (1 - \nu). \varphi. b. (D - q_r) \]  \hspace{1cm} (7.2)

The surplus \( S_{T+R}(q_r) \) of both thermal generators and RES generators is such that:

\[ \frac{dS_{T+R}(q_r)}{dq_r} = \frac{dS^A_r(q_r)}{dq_r} + \frac{dS^B_r(q_r)}{dq_r} \]

From equations 6.1, 7.1 and 7.2, \[ \frac{dS_{T+R}(q_r)}{dq_r} = \left( (1 - \nu). \varphi - 1 \right). b. (D - q_r) + \beta. Z + \alpha. (a + b. D) - (1 - \alpha). b. K_r \]
\[ - 2. \alpha. b. q_r \]  \hspace{1cm} (8)
\textbf{Case #1: Feed-in Premium, with no compensation} \\
\[ \alpha = 1 \; ; \; \beta = 1 \]
\[ \frac{dS_{T+R}(q_r)}{dq_r} = \left((1 - \nu).\varphi - 1\right) b (D - q_r) + \beta Z + (a + bD) - 2b q_r \]
\[ \Rightarrow \begin{cases} q_{R+T}^* = D + \frac{Z + a - bD}{b \left((1 - \nu).\varphi + 1\right)} & \text{for } K_r \geq D + \frac{Z + a - bD}{b \left((1 - \nu).\varphi + 1\right)} \\ q_{R+T}^* = K_r & \text{Otherwise} \end{cases} \tag{9.1} \]

\textbf{Case #2: Feed-in Premium, with full compensation} \\
\[ \alpha = 0 \; ; \; \beta = 0 \]
\[ \frac{dS_{T+R}(q_r)}{dq_r} = \left((1 - \nu).\varphi - 1\right) b (D - q_r) - b K_r \]
\[ \Rightarrow \begin{cases} q_{R+T}^* = D - \frac{K_r}{(1 - \nu).\varphi} & \text{for } (1 - \nu).\varphi \in \left[0; 1 + \frac{K_r}{D}\right] \\ q_{R+T}^* = 0 & \text{for } (1 - \nu).\varphi \in \left[1 + \frac{K_r}{D}; 1 + \frac{K_r}{D - K_r}\right] \\ q_{R+T}^* = K_r & \text{for } (1 - \nu).\varphi \geq 1 + \frac{K_r}{D - K_r} \end{cases} \tag{9.2} \]

\textbf{A.5. Proof of Proposition 4}

In the case when RES do not receive any premium (i.e. \( Z = 0 \)) and when no compensation is provided to curtailed RES (i.e. \( \alpha = 1 \)), the level of curtailment \( q_R^* \) maximising the profit of RES generators according to equation 6.2 is:
\[ \begin{cases} q_R^* = \frac{a + bD}{2b} & \text{for } K_r \geq \frac{a + bD}{2b} \\ q_R^* = K_r & \text{for } K_r \leq \frac{a + bD}{2b} \end{cases} \tag{10.1} \]

Similarly, the level of curtailment \( q_{R+T}^* \) maximising the profit of both thermal and RES generators is, according to equation 9.1:
\[ \begin{cases} q_{R+T}^* = D + \frac{a - bD}{b \left((1 - \nu).\varphi + 1\right)} & \text{for } K_r \geq D + \frac{a - bD}{b \left((1 - \nu).\varphi + 1\right)} \\ q_{R+T}^* = K_r & \text{Otherwise} \end{cases} \tag{10.2} \]

Finally, the optimal level of curtailment \( \tilde{q}_r \) is according to equation 3 such that:
\[ \begin{cases} \tilde{q}_r = \frac{a + bD}{b(1 + \varphi(1 - \nu))} & \text{for } K_r > \frac{a + bD}{b(1 + \varphi(1 - \nu))} \\ \tilde{q}_r = K_r & \text{for } K_r \leq \frac{a + bD}{b(1 + \varphi(1 - \nu))} \end{cases} \]

We distinguish three possibilities:

If \( \frac{a + bD}{2b} \leq K_r \):

By assumption, \( K_r \leq D \) and as a consequence \( \frac{a + bD}{2b} \leq D \) which implies that \( a \leq bD \). 
In this case, \( q_R^* = \frac{a + bD}{2b} \leq K_r \)
Economic curtailment of intermittent renewable energy sources

\[ \tilde{q}_r = \frac{a+bd}{b(1+\varphi(1-v))} \Rightarrow \tilde{q}_r \geq \frac{a+bd}{2b} \text{ for } \varphi(1-v) \leq 1 \text{ and } \tilde{q}_r \leq \frac{a+bd}{2b} \text{ for } \varphi(1-v) \geq 1 \]

\[ q_{R+T}^* = D + \frac{a-bD}{b((1-v)\cdot 1 + 1)} \]

as \( b \cdot D \) , \( q_{R+T}^* \) increases as \( (1-v) \cdot \varphi \) increases.

Moreover, for \( \varphi(1-v) = 1 \), \( q_{R+T}^* = \frac{a+bd}{2b} \).

As a result, \( q_{R+T}^* \leq \frac{a+bd}{2b} \) for \( \varphi(1-v) \leq 1 \) and \( q_{R+T}^* \geq \frac{a+bd}{2b} \) for \( \varphi(1-v) \geq 1 \)

\[ \Rightarrow \left\{ \begin{array}{l}
(1-v) \cdot \varphi \leq 1 \Rightarrow \tilde{q}_r \geq q_R^* \geq q_{R+T}^* \\
(1-v) \cdot \varphi \geq 1 \Rightarrow \tilde{q}_r \leq q_R^* \leq q_{R+T}^*
\end{array} \right. \] and proposition 4 is proved in case \( \frac{a+bd}{2b} \leq K_r \).

If \( \frac{a}{b} \geq K_r \):

Then \( a \geq K_r \cdot b \), and by assumption \( K_r \leq D \). As a result \( \frac{a+bd}{2b} \geq K_r \) and \( q_R^* = K_r \).

\[ \tilde{q}_r = K_r \text{ for } K_r \leq \frac{a+bd}{b(1+\varphi(1-v))} \text{ and therefore at least as long as } \varphi(1-v) \leq 1 \text{ since } \frac{a+bd}{2b} \geq K_r \]

\[ \tilde{q}_r = \frac{a+bd}{b(1+\varphi(1-v))} \text{ for } K_r > \frac{a+bd}{b(1+\varphi(1-v))} \]

\[ q_{R+T}^* = K_r \text{ for } K_r \leq D + \frac{a-bD}{b((1-v)\cdot 1 + 1)} \]

If \( \frac{a}{b} \geq D \) then: \( D + \frac{a-bD}{b((1-v)\cdot 1 + 1)} \) decreases as \( (1-v) \cdot \varphi \) increases and

\[ \lim_{(1-v)\cdot 1 + 1 \to \infty} D + \frac{a-bD}{b((1-v)\cdot 1 + 1)} = D \geq K_r \]

\[ \Rightarrow \text{ If } \frac{a}{b} \geq D, \forall (1-v) \cdot \varphi \geq 0, D + \frac{a-bD}{b((1-v)\cdot 1 + 1)} \geq K_r \]

If \( \frac{a}{b} \leq D \) then:

\( D + \frac{a-bD}{b((1-v)\cdot 1 + 1)} \) increases as \( (1-v) \cdot \varphi \) increases and \( D + \frac{a-bD}{b((1-v)\cdot 1 + 1)} = \frac{a}{b} \geq K_r \) when \( (1-v) \cdot \varphi = 0 \)

\[ \Rightarrow \text{ If } \frac{a}{b} \geq D, \forall (1-v) \cdot \varphi \geq 0, D + \frac{a-bD}{b((1-v)\cdot 1 + 1)} \geq K_r \]

\[ \Rightarrow \forall (1-v) \cdot \varphi \geq 0, q_{R+T}^* = K_r \]
If $\frac{a}{b} \leq K_r \leq \frac{a+bD}{2b}$:

From equation 10.1 we have:

$q_R^* = K_r$

From equation 3 we have:

$\bar{q}_r = K_r \text{ for } K_r \leq \frac{a+bD}{b(1+\varphi(1-\nu))}$ and therefore at least as long as $\varphi(1-\nu) \leq 1$ since $\frac{a+bD}{2b} \geq K_r$

$\bar{q}_r = \frac{a + bD}{b(1 + \varphi(1-\nu))}$ for $K_r > \frac{a + bD}{b(1 + \varphi(1-\nu))}$

From equation 10.2 and as $\leq b, K_r \leq b, D, q_{R+T}^*$ increases as $(1-\nu)\varphi$ increases.

Moreover, for $\varphi(1-\nu) = 1, D + \frac{a-bD}{b((1-\nu)\varphi+1)} = \frac{a+bD}{2b} \geq K_r$

As a result, $q_{R+T}^* = K_r \text{ for } \varphi(1-\nu) \geq 1$

$\Rightarrow \{ (1-\nu) \varphi \leq 1 \Rightarrow \bar{q}_r \geq q_R^* \geq q_{R+T}^* \text{ and proposition } 4 \text{ is proved in case } \frac{a}{b} \leq K_r \leq \frac{a+bD}{2b} \}$

Proposition 4 is therefore demonstrated in all cases.

A.6. Proof of proposition 5.1 and 5.2

Variation in the surplus of consumers

As demand is inelastic, the variation of the surplus of consumers in period $A$ $\Delta S_C^A$ when generation is curtailed to the optimal level of curtailment is equal to the variation of costs charged to consumers. Energy generated is remunerated $p_A(K_r)$ when RES generation is not curtailed. In addition, RES generators receive a premium $Z$. Energy generated is remunerated $p_A(\bar{q}_r)$ when generation is curtailed to the optimal curtailment level, RES generators receive a premium $Z$ when generating and compensation $(1-\alpha).p_A(\bar{q}_r)$ + $(1-\beta).Z$ when curtailed.

The surplus of consumers in period $A$ when generation is curtailed to the optimal level of curtailment is therefore equal to:

$$\Delta S_C^A = \int_0^{K_r} [p_A(K_r) + Z] dx + \int_{K_r}^{D} [p_A(K_r)] dx$$

$$- \int_0^{\bar{q}_r} [p_A(\bar{q}_r) + Z] dx - \int_{\bar{q}_r}^{K_r} [(1-\alpha).p_A(\bar{q}_r) + (1-\beta).Z] dx - \int_{\bar{q}_r}^{D} [p_A(\bar{q}_r)] dx$$

$$\Delta S_C^A = -D.b.(K_r - \bar{q}_r) - (1-\alpha).(K_r - \bar{q}_r).[(a + bD).\frac{\varphi(1-\nu)}{1+\varphi(1-\nu)}] + \beta.Z.(K_r - \bar{q}_r)$$

In period $B$, the surplus of consumers is only impacted when there is no wind, with probability $1-\nu$, as the price paid to thermal generators decreases when RES generation has been curtailed in period $A$. 


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\[ \Delta S_R = S^A_R (\bar{q}_l) - S^A_R (K_r) \]
\[ \Delta S_C^B = (1 - \nu) \left( \int_0^D [p^w (K_r)] dx - \int_0^D [p^w (\bar{q}_r)] dx \right) \]
\[ \Delta S_C^B = (1 - \nu) . D. b. \phi. (K_r - \bar{q}_r) \]

The total variation of the consumer surplus when RES generation is curtailed from \( K_r \) to \( \bar{q}_r \) is therefore equal to:

\[ \Delta S_C = \Delta S_C^A + \Delta S_C^B \]
\[ \Delta S_C = \left[ (1 - \nu) \cdot \phi \cdot (K_r - \bar{q}_r) - (1 - \alpha). (K_r - \bar{q}_r) \right] \frac{\varphi(1-\nu)}{1+\varphi(1-\nu)} + \beta. Z. (K_r - \bar{q}_r) \] \hspace{1cm} (11.1)

**Variation in the surplus of thermal generators**

The variation \( \Delta S_T^A \) of the surplus of thermal generators in period \( A \) when RES generation is curtailed from \( K_r \) to \( \bar{q}_r \) is:

\[ \Delta S_T^A = \int_{\bar{q}_r}^D [p_A (\bar{q}_r) - C_A (x - \bar{q}_r)] dx - \int_{K_r}^D [p_A (K_r) - C_A (x - K_r)] dx \]
\[ \Delta S_T^A = b. (K_r - \bar{q}_r). \left( D - \frac{\bar{q}_r + K_r}{2} \right) \]

In period \( B \), the surplus of thermal generators is only impacted when there is no wind, with probability \( 1 - \nu \):

\[ \Delta S_T^B = (1 - \nu) \left( \int_0^D [p^w (\bar{q}_r) - C_B (x, D - \bar{q}_r)] dx - \int_0^D [p^w (K_r) - C_B (x, D - K_r)] dx \right) \]
\[ \Delta S_T^B = -(1 - \nu) . b. \phi. (K_r - \bar{q}_r). \left( D - \frac{\bar{q}_r + K_r}{2} \right) \]

The total variation of the surplus of thermal generators when RES generation is curtailed from \( K_r \) to \( \bar{q}_r \) is therefore equal to:

\[ \Delta S_T = \Delta S_T^A + \Delta S_T^B \]
\[ \Delta S_T = (1 - (1 - \nu) \cdot \phi) . b. (K_r - \bar{q}_r). \left( D - \frac{\bar{q}_r + K_r}{2} \right) \] \hspace{1cm} (11.2)

**Variation in the surplus of RES generators**

The surplus of thermal generators when RES generation is curtailed from \( K_r \) to \( \bar{q}_r \) is only impacted in phase \( A \).

\[ \Delta S_R = \int_0^{\bar{q}_r} [p_A (\bar{q}_r) + Z] dx + \int_{\bar{q}_r}^{K_r} [(1 - \alpha). p_A (\bar{q}_r) + (1 - \beta) . Z] dx \]
\[ - \int_0^{K_r} [p_A (\bar{q}_r) + Z] dx \]
\[ \Delta S_R = - (K_r - \bar{q}_r). \left( (a + bD). \frac{\varphi(1-\nu)}{1+\varphi(1-\nu)} - b. K_r \right) + (1 - \alpha). (K_r - \bar{q}_r). \frac{\varphi(1-\nu)}{1+\varphi(1-\nu)} \]
\[ - \beta. Z. (K_r - \bar{q}_r) \] \hspace{1cm} (11.3)

**Proof of Proposition 5.1**

In the case when RES generators do not receive any compensation when curtailed, i.e. \( \beta = 1 \), then:

According to 11.1:
\[ \Delta S_C = \left( (1 - \nu) . \phi - 1 \right) . D. b. + Z \cdot (K_r - \bar{q}_r) \]
and, as $K_r \geq \bar{q}_r$ :

$$\Delta S_C \geq 0 \Leftrightarrow (1 - \nu) \cdot \varphi \geq 1 - \frac{Z}{D \cdot b}$$

According to 11.2 :

$$\Delta S_r = (1 - (1 - \nu) \cdot \varphi) \cdot b \cdot (K_r - \bar{q}_r) \cdot \left(D - \frac{\bar{q}_r + K_r}{2}\right)$$

and, as $D \geq K_r \geq \bar{q}_r$, therefore:

$$\Delta S_r \geq 0 \Leftrightarrow (1 - \nu) \cdot \varphi \leq 1$$

According to 11.3:

$$\Delta S_r = -(K_r - \bar{q}_r) \cdot \left[Z + (a + bD) \cdot \frac{\varphi(1 - \nu)}{1 + \varphi(1 - \nu)} - b \cdot K_r\right]$$

And, as $K_r \geq \bar{q}_r$ :

$$\Delta S_r \geq 0 \Leftrightarrow K_r \geq \frac{1}{b} \left[Z + (a + bD) \cdot \frac{\varphi(1 - \nu)}{1 + \varphi(1 - \nu)}\right]$$

**Proof of Proposition 5.2**

In the case when RES generators receive full compensation when curtailed, i.e. $\beta = 0$, then:

According to 11.1:

$$\Delta S_C = (K_r - \bar{q}_r) \cdot \left[((1 - \nu) \cdot \varphi - 1) \cdot D \cdot b - \left[(a + bD) \cdot \frac{\varphi(1 - \nu)}{1 + \varphi(1 - \nu)}\right]\right]$$

And, as $K_r \geq \bar{q}_r$ :

$$\Delta S_C \geq 0 \Leftrightarrow ((1 - \nu) \cdot \varphi - 1) \cdot D \cdot b - \left[(a + bD) \cdot \frac{\varphi(1 - \nu)}{1 + \varphi(1 - \nu)}\right] \geq 0$$

$$\Delta S_C \geq 0 \Leftrightarrow ((1 - \nu) \cdot \varphi)^2 - \frac{a + bD}{b \cdot D} \cdot ((1 - \nu) \cdot \varphi) - 1 \geq 0$$

This quadratic equation admits only one positive root, and:

$$\Delta S_C \geq 0 \Leftrightarrow (1 - \nu) \cdot \varphi \geq \frac{1}{2} \times \left[\frac{a + bD}{b \cdot D} + \sqrt{\left(\frac{a + bD}{b \cdot D}\right)^2 + 4}\right]$$

According to 11.2:

$$\Delta S_r = (1 - (1 - \nu) \cdot \varphi) \cdot b \cdot (K_r - \bar{q}_r) \cdot \left(D - \frac{\bar{q}_r + K_r}{2}\right)$$

and, as $D \geq K_r \geq \bar{q}_r$, therefore:

$$\Delta S_r \geq 0 \Leftrightarrow (1 - \nu) \cdot \varphi \leq 1$$

According to 11.3:

$$\Delta S_r = (K_r - \bar{q}_r) \cdot b \cdot K_r$$

and, as $K_r \geq \bar{q}_r$ :

$$\Delta S_r \geq 0$$

**A.7. Proof of Proposition 6**

For two level of curtailment resulting in RES generation $\bar{q}_r$ and $\hat{q}_r$, the variation of the social welfare is equal to:

$$S(\bar{q}_r) - S(\hat{q}_r) = C(\bar{q}_r) - C(\hat{q}_r) = \int_{\bar{q}_r}^{\hat{q}_r} \frac{dC(q)}{dq} dq$$

From equation (2) :

$$S(\bar{q}_r) - S(\hat{q}_r) = (a + bD) \cdot (\hat{q}_r - \bar{q}_r) + \frac{1 + \varphi(1 - \nu)}{2} \cdot b \cdot (\hat{q}_r^2 - \bar{q}_r^2)$$

(12.1)

Similarly, the variation of the surplus of RES generators is equal to:
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\[ S_R(\tilde{q}_r) - S_R(\tilde{q}_r) = \int_{\tilde{q}_r}^{\tilde{q}_r} \frac{dS_R(q_r)}{dq_r} \, dq_r \]

According to equation (6.1), and in case RES do not receive any compensation (i.e. \( \alpha = \beta = 1 \)):

\[ S_R(\tilde{q}_r) - S_R(\tilde{q}_r) = (Z + (a + b \cdot D))(\tilde{q}_r - \tilde{q}_r) + b.(\tilde{q}_r^2 - \tilde{q}_r^2) \quad (12.2) \]

We then define \( \tilde{q}_r \) as the optimal production level in period \( A \) corresponding to a variability equal to \((1 - \nu)\) and \( \tilde{q}_r \) as the optimal production level in period \( A \) corresponding to a variability equal to \((1 - \bar{\nu})\).

According to equation (3):

\[ \tilde{q}_r = \frac{a + bD}{b \cdot (1 + \varphi(1 - \nu))} \quad \text{for} \quad K_r > \frac{a + bD}{b \cdot (1 + \varphi(1 - \nu))} \]

\[ \tilde{q}_r = \frac{a + bD}{b \cdot (1 + \varphi(1 - \bar{\nu}))} \quad \text{for} \quad K_r > \frac{a + bD}{b \cdot (1 + \varphi(1 - \bar{\nu}))} \]

We also assume that there is a rational for curtailment in both cases:

\[ K_r > \frac{a + bD}{b \cdot (1 + \varphi(1 - \nu))} \quad \text{and} \quad K_r > \frac{a + bD}{b \cdot (1 + \varphi(1 - \bar{\nu}))} \]

According to equation (12.1):

\[ S(\tilde{q}_r) - S(\tilde{q}_r) = (a + b \cdot D)(\tilde{q}_r - \tilde{q}_r) + \frac{1 + \varphi(1 - \nu)}{2} \cdot b \cdot (\tilde{q}_r^2 - \tilde{q}_r^2) \]

\[ S(\tilde{q}_r) - S(\tilde{q}_r) = \frac{(a + b \cdot D)^2}{2 \cdot b} \cdot \varphi^2 \cdot (1 - \nu)^2 \cdot \left(1 + \varphi(1 - \nu)\right) \left(1 + \varphi(1 - \nu)\right)^2 \]

For \( \nu - \bar{\nu} \ll \nu \) then \( S(\tilde{q}_r) - S(\tilde{q}_r) \approx \frac{(a + b \cdot D)^2}{2 \cdot b} \cdot \varphi^2 \cdot (1 - \nu)^2 \cdot \left(1 + \varphi(1 - \nu)\right)^2 \quad (13.1) \]

Similarly, according to equation (12.2):

\[ S_R(\tilde{q}_r) - S_R(\tilde{q}_r) = (Z + (a + b \cdot D))(\tilde{q}_r - \tilde{q}_r) + b.(\tilde{q}_r^2 - \tilde{q}_r^2) \]

For \( \nu - \bar{\nu} \ll \nu \), then \( S(\tilde{q}_r) - S(\tilde{q}_r) \approx \frac{(a + b \cdot D) \varphi(1 - \nu)^2}{b} \cdot \left(1 + \varphi(1 - \nu)\right)^2 \cdot (Z + (a + b \cdot D).\frac{(\varphi(1 - \nu) - 1)}{1 + \varphi(1 - \nu)}) \quad (13.2) \]

And in particular, for \( Z = 0 \):

\[ S(\tilde{q}_r) - S(\tilde{q}_r) \approx \frac{(a + b \cdot D)^2}{b} \cdot \varphi(1 - \nu)^2 \cdot \frac{(1 - \nu) - 1}{(1 + \varphi(1 - \nu))^2} \quad (13.3) \]
References


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