



Essays in Economics of Education and Elections

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Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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To my parents

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Abstract

How economic agents can make sense from imperfect information is a central challenge in economic theory. In this thesis, I first explore how voters try to infer the quality of their government based not only on the information they personally receive but also on observations of their home and foreign governments' policies. Can voters learn from such information and thus any improved accountability reduce "political pandering"? Secondly, I study two models of education where the incentives of both students and firms are profoundly affected by the imperfect informativeness of education certificates and study how increases in enrolment and tuition fees affect educational and job market outcomes.

The first chapter, *Pandering Across Borders*, studies when voters can use information from foreign countries to reduce domestic political pandering, and when pandering is contagious between countries. The voters condition their electoral decisions not only on policies chosen in their home countries, but also on those implemented abroad. Since the policy decisions are driven by re-election concerns, both sources of information may be biased. As a result, informational linkages between the countries give rise to pandering externalities which lead to ambiguous welfare effects of access to international news. The model also shows that institutional harmonisation via internal synchronisation of election dates increases the parameter range in which pandering may occur.

Beliefs, Access Constraints and Voluntary Education Decisions, the second chapter of this thesis, contributes to the debate on the negative consequences of high growth rates in university enrollment with a focus on CEE countries. I propose a theory how low education supply elasticities in the short run can lead to self-fulfilling equilibria in a setting in which signalling is reduced to an effortless binary certification technology. When the agents believe that the certification precision is low they enrol at a higher rate and, due to those inelasticities, their beliefs fulfil. The opposite holds when the agents have high beliefs on the quality. The selection among these equilibria depends on students' initial beliefs about the quality of the certification technology.

The final chapter, *Tuition Fees in a Signalling Model of Education*, analyses the trade-off

between tuition fees and educational effort. Education serves purely as a signaling device and implies a non-pecuniary cost inversely proportional to students' ability, while tuition fees are independent of ability. In this framework, higher tuition fees can be beneficial for high ability students since they reduce the enrolment rates of the less able agents reducing the effort level necessary to separate. The overall effect of tuition fees is complex and is associated with non-monotonicities in actions of the players in the model

Chapter 1

Pandering Across Borders

Abstract

Economies are increasingly linked through common shocks but remain sovereign politically; this paper studies when voters can use information from foreign countries to reduce domestic political pandering, and when pandering is contagious between countries. The voters condition their electoral decisions not only on policies chosen in their home countries, but also on those implemented abroad. As both sources of information may be biased, since the policy decisions are driven by re-election concerns, the welfare implications of additional information are ambiguous. Compared to a single country model, the quality of voters information has to be significantly weaker for the pandering to appear. However, pandering in one country creates information distortion also for voters abroad. Existence of those pandering externalities leads to an increase of pandering probability in the equilibrium in which this behaviour occurs. The model also shows that institutional harmonisation, via internal synchronisation of election dates, increases the parameter range in which pandering may occur.

Keywords: Pandering, Information, Electoral Cycles

JEL Classification Numbers: D72, D78, D82.

1.1 Introduction

Representative democracies allow politicians discretion over policy. This allows them to implement even unpopular policies, if they believe them to be in the social interest. Alternatively, politicians could pander to the moods of the electorate and implement popular, but suboptimal, decisions. I study in this paper how international linkages affect politicians' in-

centives to pander in a setting with rational voters; furthermore, I study when institutional harmonisation – in terms of internally synchronised election dates – promotes or reduces political pandering. To my knowledge, this is the first study of pandering in a cross-national setting.

I study a two-country model. The two countries share the same state of the world. In each country, voters receive a noisy public signal on the state of the world. Privately informed politicians may be of high quality, in which case they observe the state of the world directly, or low quality - in this case, they receive a noisy signal, but one which is still more precise than voters' information. In each country, the incumbent politician - observing his signal and the domestic voters' prior beliefs about the optimal action - first implements a policy; in the case where the politician implements the voters' preferred policy against his superior information, I shall speak of "pandering". In the next stage, the true state of the world is revealed to voters in both countries with some probability. Voters also observe the policy decision made in the other country. The election stage follows, where voters apply Bayesian updating to their information to assess the probability of their incumbent being low-quality and vote accordingly. If their posterior belief about the quality of the incumbent exceeds the expected quality of the challenger, the incumbent secures re-election. Otherwise, he is replaced with a challenger.

When voters have access to foreign information, pandering is often attenuated. For example, if the government of one country is commonly known to be of high quality, pandering cannot arise in either country. Also, under asynchronous election dates, the government with the longer horizon towards the election, and hence a higher chance that the voters will learn the true state of the world, has a lower incentive to pander. But voters in the country with the early election also observe the foreign policy, learning from it and making it more likely that domestic pandering will be discovered by voters. This can also eliminate pandering equilibria. In such a setting, when decisions taken in one country are free of pandering, they serve the voters also in the other country as an additional, unbiased source of information.

International linkages can, however, also lead to "pandering contagion". In such equilibria, pandering is an equilibrium only if governments in both countries co-ordinate on a pandering strategy. Policy co-ordination, where governments in both countries implement the same decision, may then be wrongly interpreted by voters as good indication of truthful policy. The voters' trust in the state of the world, inflated by the policy co-ordination, gives the governments an opportunity to pander to a greater extent than if the decisions were taken without the information from the other country. This is because when pandering occurs in both countries, policy harmonization is less informative - the signal is jammed. Relatedly,

Gentzkow and Shapiro (2006) or Prat (2005), have shown in different settings that more information does not always reduce pandering.

When the state of the world underlying the incumbents' decisions is the same across the two countries, asynchronous electoral cycles are weakly dominant for voter welfare. With asynchronous cycles, voters in one of the countries have more time for learning the outcomes of implemented policies. Voters equipped with good enough information enforce truthful strategies, when the electoral race is close. In such a case, the actions of the government in that country are used by the voters in the other country as unbiased signals. The foreign policies become equivalent to unbiased media as in Ashworth and Shotts (2010), and hence a good indicator of the true state of the world. Under synchronous election cycles, voters in both countries have the same information level. If the common election date happens to be far ahead in time and the voters are likely to learn the true state of the world, pandering is unlikely. However, it may happen that elections take place when the voters in both countries have a low level of information. Although learning from actions taken abroad still takes place, pandering decisions distort this process. When the probability of voters learning the true state of the world is sufficiently low, pandering contagion occurs. The decision to pander in one country enforces pandering abroad.

Pandering is an important feature of representative democracies. Not only is pandering frequently alleged to be contagious by political observers,¹ but it has been shown that pandering can occur in an equilibrium even when voters are fully rational. Pandering in economic literature can be understood in two ways. In Canes-Wrone et al. (2001), a modelling framework for this paper and already cited Ashworth and Shotts (2010), governments and voters share preferences over the actions. They receive, however, different signals about the optimal choice. In this setting, pandering implies ignoring socially valuable information if it goes against the *ex ante* preferred action of the majority.

A different strand of the literature shows how pandering can arise when politicians' preferences are not aligned with those of voters. In this type of model the non-congruent incumbent politicians are implementing policies that are *ex post* socially harmful. If prior popular belief is in favour of such a policy, a non-congruent politician can not only implement

¹Recent British anti-migration rhetoric and steps taken by the British prime minister David Cameron to curb inflow of new workers and restrict access to social benefits for foreigners, despite ample evidence on net benefits of migrants for the British economy, has been an inspiration for growth of anti-migration sentiment in Germany. The nationalisation of Hungarian pension funds by Victor Orbán's government is said to be an inspiration for a similar decision taken by Donald Tusk's government in Poland. The transfer of resources from private pension funds to the state budget will reduce the reported public debt and allow for fiscal expansion in the election year.

privately preferred policy, but also secure re-election (*cf* Maskin and Tirole (2004)). The relationship between voters' perceptions of policymakers and policy choices has been studied in this type of environment. Fang (2008) analyses external sources of information and shows that the existence of unbiased international institutions enforces both biased and unbiased politicians to seek policy advice and hence to reduce the policy bias. Morelli and Van Weelden (2013) combine the two branches and consider how the incentive to pander relates to voters' perceptions of candidates' preferences and valence.

Pandering externalities have been studied in the context of electoral competition. Heidhues and Lagerlöf (2003) show, in a simple framework with binary actions, that electoral competition leads to pandering. In a recent, spatial model of electoral competition, Kartik *et al.* (2013) show that the direction of this effect is just the opposite and that competition leads to anti-pandering. The politicians commit to policies that put more weight on the private signals than what is subscribed by an unbiased strategy.

The Synchronisation of electoral cycles has been discussed as a reform proposal for the European Union. Sapir and Sekkat (1999) identify the optimal degree of synchronisation in a model with two open economies and cross-country spillovers from political business cycles in a model with two open economies. The authors extend the earlier literature on political business cycles to the open economy setting. They show that when voters are concerned about inflation and employment, the incumbent, in a period prior to the election, is likely to manipulate inflation which, if unanticipated, affects employment. This behaviour is likely to induce 'political business cycles'. With a high degree of economic interdependence, these cycles would tend to spill over between countries. For instance, a politically motivated expansion of aggregate demand in one country could induce effects on economic activity abroad. Such spillovers create coordination problems. The authors conclude that the impact of coordinating electoral calendars depends on the type of the international spillovers. If the spillovers are positive, the coordination on a single election date is never harmful. This paper aims to answer the same research question and analyses the welfare implications of the synchronisation of electoral calendars between two countries. However, the source of possible welfare effects is fundamentally different. In my model, the decisions taken abroad do not affect the home economy directly. They are used, only, as a source of information. Nevertheless, the coordination of electoral still cycles matters.

The paper is organised as follows. The next section outlines the model. Section 1.3 shows the equilibrium analysis of the game. Section 1.4 analyses the benefits of information availability and compares the results to the baseline model, in which the voters learn only the decisions taken in their home countries. In Section 1.5 I discuss the implications of institu-

tional harmonisation via the internal synchronisation of election dates. Finally, [Section 1.6](#) concludes.

1.2 Model

1.2.1 Setup

In each of the two periods Nature draws a single state of the world shared by two identical countries $c \in \{D, F\}$; $\omega \in \{A, B\}$ in period one and $\tilde{\omega} \in \{\tilde{A}, \tilde{B}\}$ in period two. States of the world are the same across countries and independent across time with $Pr(\omega = A) = Pr(\tilde{\omega} = \tilde{A}) = \pi > 1/2$. The governments' policy choices in each country are $x_c \in \{A, B\}$ in period one and $\tilde{x}_c \in \{\tilde{A}, \tilde{B}\}$ in period two. The correct policy for each period matches the state of the world, i.e., $x_c = \omega$ and $\tilde{x}_c = \tilde{\omega}$. Representative voters in each country observe both actions and with probability ρ learn the true state of the world. At the end of the first period representative voters choose to re-elect the incumbent or remove her from office by electing a challenger. Overall, the sequence of actions is as follows

1. Nature determines the quality θ of the incumbent I and challenger O in each country.
2. Nature determines the first period state of the world ω .
3. The incumbents observe private signals s_c^I .
4. The incumbents simultaneously select the policies x_c .
5. Nature determines whether the voters in each country learn ω before the election.
6. The voters V either re-elects the incumbent I or elects the challenger O .
7. New state of the world $\tilde{\omega}$ and signals \tilde{s}_c^I , seen by the new incumbent, are drawn by Nature and new actions \tilde{x}_c taken.

Graphically, the timeline is presented on [Figure 3.1](#).

1.2.2 Players

In each country, the incumbents and challengers are of type $\theta_c \in \Theta = \{H, L\}$, i.e. they can be either of high quality or of low quality. The quality of each is private information and is determined by Nature. In each country, both the incumbent and the challenger are

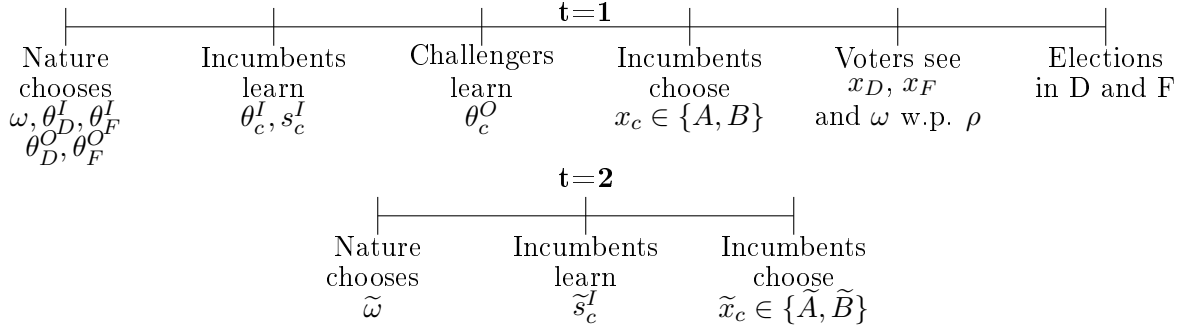


Figure 1.1: Timeline

drawn from the same pool of politicians and the probability of a politician to be of a high type is $\gamma \in (0, 1)$. The incumbents, independently in each country, receive private signals about the state of the world $s_c^I \in \{A, B\}$ in period one and $\tilde{s}_c^I \in \{A, B\}$ in period two. A high-quality politician's signal is perfect: $s_c^I = \omega$ and $\tilde{s}_c^I = \tilde{\omega}$. A low-quality incumbent's signal is informative but not perfect: $Pr(s_c^I = \omega | \omega) = Pr(\tilde{s}_c^I = \tilde{\omega} | \tilde{\omega}) = q > \pi$. The fact that $q > \pi$ assures that even for the low types, the state of the world indicated by the signal is more likely to happen.

Incumbents receive utility from their political legacy, which is built if they implement a policy that favours the citizens, i.e. they chose a policy that matches the state of the world ω in period one and $\tilde{\omega}$ in period two. The pay-off of the incumbent is, therefore:

$$U_c^I = \left\{ \begin{array}{ll} 1 & \text{if } x_c = \omega \\ 0 & \text{if } x_c \neq \omega \end{array} \right\} + \left\{ \begin{array}{ll} 1 & \text{if } \tilde{x}_c = \tilde{\omega} \text{ and re-elected} \\ 0 & \text{if } \tilde{x}_c \neq \tilde{\omega} \text{ or if not re-elected} \end{array} \right\}$$

The utility from the decision taken in period one will be referred to as the first period utility. If the incumbent is re-elected she receives a chance to leave a legacy in the subsequent term, after which the game ends. This specification assures that the high type strictly prefers not to pander in the first period, as long as there is some uncertainty about the re-election.² The subsequent period pay-off weighted by the re-election probability will be referred to as the second period utility. The challenger's utility is similar to the incumbent's second period utility. He receives 1 if elected and the policy chosen in the second period is correct and zero otherwise.

²Discounting would not qualitatively change the results.

The voters in both countries are policy motivated with utility

$$U_c^V = \left\{ \begin{array}{ll} 1 & \text{if } x_c = \omega \\ 0 & \text{if } x_c \neq \omega \end{array} \right\} + \left\{ \begin{array}{ll} 1 & \text{if } \tilde{x}_c = \tilde{\omega} \\ 0 & \text{if } \tilde{x}_c \neq \tilde{\omega} \end{array} \right\}$$

Since the voters take their actions only once the policy decisions are already taken, only the second-part of the utility function is relevant for the voters' electoral choices. As the second period utility depends on the quality of the government, the voters act prospectively and elects the candidate who he believes is more likely to be high quality.

In making this choice the voter does not directly observe the quality of the incumbent. However, he does observe the policies x_D and x_F chosen in each of the countries, which may tell him something about the incumbent's quality. Finally, I assume that with some probability $\rho \in [\frac{(1-q)(1-\pi+q-2\pi q)}{2\pi q^2}, 1]$ ³ the uncertainty about the state of the world ω resolves (in both countries at the same time). With probability ρ , the voter learns the true first period state before election day; otherwise he votes knowing only the policy choices made in each country. Formally, the voter's signal is $s^V \in \{A, B, \phi\}$, where ϕ means no information. If uncertainty resolves then $s^V = \omega$. The rationale for the restriction on ρ is explained in [Lemma 1.3.2](#). A low ρ means that either the election is imminent (so there is little time for information to be publicly revealed) or that the policy being chosen is unlikely to produce any easily assessed short run effects.

The model presented here is based on the [Canes-Wrone et al. \(2001\)](#) setting. To allow for comparisons with the baseline model I keep the original assumptions and notation whenever possible.

1.2.3 Beliefs and Strategies

For incumbents in each country, I specify strategies in four information sets: the incumbent is high or low quality and receives a signal $s_c^I = A$ or $s_c^I = B$. Let $\sigma_s^{\theta,c} \in [0, 1]$ be the probability that a type θ incumbent in a country c who observes signal s chooses policy $x_c = A$. If, upon observing the signal indicating the *ex ante* less likely action B , the politician implements the voters' preferred policy A against his superior information, it shall be defined as "pandering". The strategies depend on the politician's beliefs about the state of the world. The high-quality politician's beliefs about the state of the world are trivial since she receives a perfect signal. As in [Canes-Wrone et al. \(2001\)](#), her strategies are also simple; she always follows her signal in the first period ($\sigma_{s=A}^{H,c} = 1$ and $\sigma_{s=B}^{H,c} = 0$)

³The restriction on ρ is slack for $\pi > \frac{\sqrt{2}}{2}$.

because the potential utility gain from re-election never outweighs the sure loss incurred by choosing the wrong first-period policy. Low type's beliefs are more complicated and are given by $Pr(\omega = A|s_c^I = A) = \frac{q\pi}{q\pi+(1-q)(1-\pi)}$, $Pr(\omega = B|s_c^I = A) = \frac{(1-q)(1-\pi)}{q\pi+(1-q)(1-\pi)}$, $Pr(\omega = A|s_c^I = B) = \frac{(1-q)\pi}{(1-q)\pi+q(1-\pi)}$, $Pr(\omega = B|s_c^I = B) = \frac{q(1-\pi)}{q\pi+(1-q)(1-\pi)}$.

Voters receive information from two sources, the signal s^V and the actions of the governments. Since the countries share the state of the world, both actions taken at home and abroad convey information to the voters. Therefore, for voters in each country, I specify strategies and beliefs in twelve information sets: for each possible observable policy couplets ($\{x_c = A, x_{-c} = A\}$, $\{x_c = A, x_{-c} = B\}$, $\{x_c = B, x_{-c} = A\}$ and $\{x_c = B, x_{-c} = B\}$) for each signal observed $s^V \in \{A, B, \phi\}$.

Let the voter's strategies in these information sets be the probabilities that the voter in country c re-elects the incumbent, be denoted by ν_{x_c, x_{-c}, s^V}^c . Likewise, let the voter's beliefs about the probability that the incumbent is high quality be denoted by μ_{x_c, x_{-c}, s^V}^c .

The equilibrium concept that I employ is that of perfect Bayesian equilibrium, where this equilibrium concept is defined in the usual way: all players (governments of both types in both countries and voters in both countries) must make optimal choices at all information sets given their beliefs, and the beliefs are formed using Bayes' rule when that is defined. For the sake of brevity I will refer to a strategy profile of the players as an equilibrium if there exist beliefs of the players such that this strategy profile together with these beliefs form a perfect Bayesian equilibrium.

1.3 Analysis

Solving the game by backward induction, I start with politicians elected for the second period. Since the game ends then, they always follow their signals and a high type receives utility 1, while a low type receives, in expectations, q . The voters enter the game in their respective countries when the first period decisions are already taken. The voters' second period utilities are equal to the governments' second period utilities, therefore voters' only objective is the maximisation of the future government's quality. The voters will use the information revealed in actions taken in both countries and the observed state of the world, and re-elect the incumbent if the belief about her quality exceeds the probability that the challenger is a high type (γ). Otherwise, they replace the incumbent. Therefore, the voters will always choose cut-point strategies. The voters' beliefs are as follows.

Lemma 1.3.1. *If the incumbents always follow their signals the ordering of the voters' beliefs*

is the following:

$$0 = \mu_{A,A,B}^c = \mu_{A,B,B}^c = \mu_{B,A,A}^c = \mu_{B,B,A}^c < \mu_{B,A,\phi}^c < \mu_{A,B,\phi}^c < \gamma$$

$$\gamma < \mu_{B,B,\phi}^c < \mu_{A,A,\phi}^c < \mu_{A,A,A}^c = \mu_{A,B,A}^c = \mu_{B,A,B}^c = \mu_{B,B,B}^c < 1$$

Lemma 1.3.1 follows directly from Bayes' Rule. The lemma says that when an action taken by the home government does not match the observed state of the world, the voter is certain that the incumbent is of low quality. When the observed state of the world matches the action, the voter's posterior belief about the incumbent's quality exceeds the prior γ . When the state of the world is not observed the posterior belief depends also on the action taken abroad and hence is sensitive to the foreign incumbent's strategies. For more details and the proof see the [Appendix](#).

Lemma 1.3.2. *When $\rho > \frac{(1-q)(1-\pi+q-2\pi q)}{2\pi q^2}$ the low type incumbent always chooses action $x_c = A$ upon observing $s_c^I = A$.*

When ρ is sufficiently high, action A is the best response for the low type government receiving the signal indicating the *ex ante* more likely state of the world, $s_c^I = A$. It holds for all strategies of the low type receiving signal B and all strategies of the low type abroad. For the proof, see the [Appendix](#).

Within this range, the further analysis of the government actions can be, therefore, limited to strategies of the low types observing signal $s_c^I = B$.

The following lemma presents two relations crucial for the key results presented in the paper.

Lemma 1.3.3. *If high types and low types receiving signal A follow their signals then:*

i) *It always holds that:*

$$\mu_{A,B,\phi}^c < \gamma < \mu_{B,B,\phi}^c$$

ii) *If home incumbent of the low types receiving signal B panders with sufficiently high probability, $\sigma_B^{L,c} > \underline{\sigma_B^{L,c}}$, the ordering of beliefs between $\mu_{B,A,\phi}^c$, $\mu_{A,A,\phi}^c$ and γ changes to*

$$\mu_{A,A,\phi}^c < \gamma < \mu_{B,A,\phi}^c$$

For the proof and the explicit expression for $\underline{\sigma_B^{L,c}}$, see the [Appendix](#). **Lemma 1.3.1** and **Lemma 1.3.3** show that, given the imposed restriction on ρ , the order of posterior beliefs relative to the prior γ is sensitive to strategies of a low type receiving signal B only when

the uncertainty about the true state of the world is not resolved and the observed actions are $\{B, A\}$ or $\{A, A\}$.⁴

Canes-Wrone et al. (2001) in their single country model show that there exist two types of equilibria depending on the value of ρ . If the voters learn quickly (ρ is sufficiently high) there is no pandering in equilibrium. All types of incumbents follow their signals. However, as ρ decreases, benefits from pandering for the low type receiving signal B , when the state of the world is not observed, outweigh the losses that accrue when the voters learn the true state. In the other type of equilibrium occurring when ρ is low, when the low type receives a signal A , she always acts truthfully and when the signal is B , she panders with positive probability. In a two country setting, as in this paper, the situation is more complicated due to pandering externalities. Beliefs on the quality of home politicians are determined not only by the observed action at home, but also by actions taken abroad. To structure the analysis that follows I define two types of equilibria that will emerge.

Definition 1.3.1.

1. A *Truth Equilibrium (TE)* is an equilibrium in which all players in both countries follow their signals, i.e. $\sigma_A^{H,c} = \sigma_A^{H,-c} = \sigma_A^{L,c} = \sigma_A^{L,-c} = 1$ and $\sigma_B^{H,c} = \sigma_B^{H,-c} = \sigma_B^{L,c} = \sigma_B^{L,-c} = 0$.
2. A *Pandering Equilibrium (PE)* is an equilibrium in which low types in both countries pander with positive probability i.e. $\sigma_A^{H,c} = \sigma_A^{H,-c} = \sigma_A^{L,c} = \sigma_A^{L,-c} = 1$, $\sigma_B^{H,c} = \sigma_B^{H,-c} = 0$ and $\sigma_B^{L,c} = \sigma_B^{L,-c} \in (0, 1]$.

The conditions under which the defined equilibrium types occur are described in the proposition below.

Proposition 1.3.1. *When the probability of learning the true state of the world by the voters is $\rho < 1$, then*

1. *When the probability of learning the truth is sufficiently high, $\rho > \underline{\rho}$, the Truth Equilibrium is the only equilibrium.*
2. *For lower values of $\rho < \underline{\rho}$ there is an equilibrium multiplicity. Both the Pandering Equilibrium and the Truth Equilibrium can emerge.*

⁴Given that incumbents in all other information sets follow the truth, which holds in equilibrium (in all equilibrium types)

The detailed proof together with the threshold value of $\underline{\rho}$ and the specification of equilibrium strategies is presented in the [Appendix](#).

Intuitively, as in [Canes-Wrone et al. \(2001\)](#), pandering may bring benefits only when the true state of the world is not revealed. Conditional on uncertainty resolving, the incumbent needs to match his policy to the state, and he wants to follow his signal.

Conditional on uncertainty not resolving, policy co-ordination, where governments in both countries implement the same decision, is interpreted by voters as a good indicator for truthful policy and hence on high incumbent's quality. The home incumbent needs to match the foreign government's action.

Observation 1.3.1. *If the foreign incumbent always follows the truth, the home incumbent never panders.*

When the foreign government always follows the truth, the home incumbent, in an attempt to match the foreign policy, chooses the action most likely to be the foreign incumbent's signal, i.e. plays truth.⁵ Therefore the Truth Equilibrium exists for all values of ρ .

Since pandering abroad implies an increase in the probability that policy A is taken, it increases the returns to pandering for the home government. Given the imprecision of the low type signals, foreign pandering makes the co-ordination on A more likely. Therefore, when ρ is sufficiently low and the degree of pandering abroad sufficiently high, pandering becomes contagious. This strategy becomes the best response for the home low type government. The benefits from pandering accrue only when the uncertainty is not resolved, therefore for the Pandering Equilibrium to exist, ρ has to be sufficiently low. This equilibrium type, however, is unstable, as it is based on mixed strategies of both the voters and the low type incumbents in each country.

It has to be noted that the two types of equilibria described exist also for $\rho < \frac{(1-q)(1-\pi+q-2\pi q)}{2\pi q^2}$, ruled out by assumption. However, in this range the set of equilibria is richer than the two equilibria defined in the proposition.

Proposition 1.3.2. *From the voters' perspective a Truth Equilibrium is strictly better than a Pandering Equilibrium. All types of incumbents are better off in a Pandering Equilibrium.*

⁵The information received by the voters through the action of the truthful foreign government, is exactly the same as the information transmitted by the unbiased media in the [Ashworth and Shotts \(2010\)](#) setting when the political and media announcements are simultaneous. [Ashworth and Shotts \(2010\)](#) also analyse cases in which the voters prior beliefs about incumbent and challenger quality differ where pandering occurs despite additional unbiased information.

For the detailed proof see the [Appendix](#). Intuitively, in the Pandering Equilibrium voters re-elect the incumbents more frequently. As explained in detail in the proof of [Proposition 1.3.1](#), in both types of equilibrium, when the true state of the world is observed, the voters re-elect when the action taken by the home government matches the true state of the world. In the Truth Equilibrium, when the voters do not receive a signal they re-elect only upon observing $\{A, A, \phi\}$ and $\{B, B, \phi\}$, while in Pandering equilibrium, in addition, they also re-elect with positive probability upon observing $\{B, A, \phi\}$.

Voter welfare is composed of the first and the second period payoffs. It is immediate that when the incumbent panders, she reduces the probability of making the correct action and hence reduces the voter's first period utility. The effect on the second period utility is ambiguous. On the one hand, if the state of the world is not revealed, pandering increases the re-election probability of a low type to a greater extent than high types'. At the same time, since low type incumbents are less frequently using their signals the voters can more frequently detect the low types when the true state of the world is revealed. After some algebra, it can be shown that the first period utility effect always outweighs the second period effect.

1.4 Isolation or Integration?

When the voters do not learn about the actions taken abroad, actions in one country don't affect the beliefs of voters in the other country and the model is identical to [Canes-Wrone et al. \(2001\)](#). The comparison of the results of the two specifications sheds light on the role of access to international news on political decision making. To enhance clarity, it is useful to apply the following definition.

Definition 1.4.1.

1. *When countries are Isolated, the voters observe only actions taken in their home country*
2. *When countries are Integrated, the voters observe actions taken both in their home countries and in the foreign country.*

The results of the [Canes-Wrone et al. \(2001\)](#) model can be summarised in the following lemma:

Lemma 1.4.1. *When countries are isolated and the incumbents and challengers are drawn from the same pool of politicians, there are two types of equilibrium depending on the value*

of ρ . When $\rho > \hat{\rho} = \frac{(1-q)(\pi-q+2\pi q)}{2(1-\pi)q^2}$ there is a unique Truth Equilibrium. For lower values of ρ , the Pandering Equilibrium is the only equilibrium. If the government is of a low type and receives a signal B she panders with probability $\sigma_B^L = \sigma_B^{*,L} = 1 - \frac{1-\pi}{\pi(1-q)+(1-\pi)q}$. Otherwise the government always follows the truth.

For the proof, see [Canes-Wrone et al. \(2001\)](#).

To show the potential costs and benefits from integration, I compare the results of the two settings.

Proposition 1.4.1. *When the two countries are Integrated, the threshold level ρ above which pandering never occurs is strictly lower than when the countries are Isolated ($\underline{\rho} < \hat{\rho}$). However, the degree of pandering in the Pandering Equilibrium when the countries are Integrated is strictly higher than in the Pandering Equilibrium when the countries are Isolated.*

The result comes directly from the comparison of the expressions for the threshold values $\underline{\rho}$ and $\hat{\rho}$ presented in [Lemma 1.4.1](#) and in the proof of [Proposition 1.3.1](#). Intuitively, when countries are Integrated, actions taken in both countries convey information. As a result, the critical amount of information necessary to make following the signals the incumbents' optimal strategy in all information sets is collected for lower ρ . However, the advantages of an additional source of information turn detrimental when the governments actually decide to pander and hence bias the information source. Since the underlying state of the world is shared by the countries, a co-ordination of actions on the same policy is a stronger indicator of government quality than a choice of a popular action in an Isolated country. As a result, when countries are Integrated, a higher pandering probability is necessary for voters to change their re-election strategies. It has to be noted, however, that when the countries are Isolated, pandering is the only equilibrium when $\rho < \hat{\rho}$, while in integrated countries the Pandering Equilibrium is only one of the two equilibrium types that may emerge when $\rho < \underline{\rho}$. The ranges of existence of the two types of equilibria in an Isolated and Integrated country for select set of parameters is presented graphically in [Figure 1.2](#).

Lastly, the implications of the different equilibrium types in the two settings for voters' welfare outlined in the next proposition.

Proposition 1.4.2. *From the voters' expected utility perspective*

1. *The Truth Equilibrium when the countries are Integrated is strictly better than any equilibrium in an Isolated country.*
2. *Pandering Equilibrium when the countries are Integrated is better than the Pandering Equilibrium when the countries are Isolated when q is sufficiently low and γ is sufficiently high.*

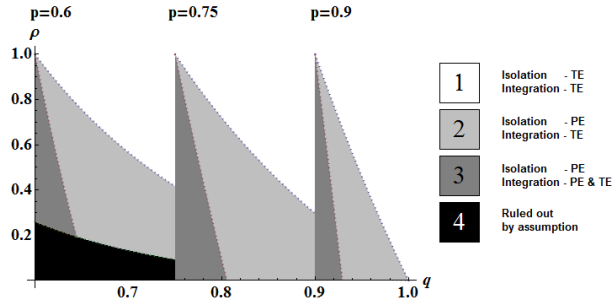


Figure 1.2: The range of existence of different equilibrium types in an Isolated and an Integrated country for $\gamma = 0.5$ and different values of p , ρ and q

1 (white area) - the Truth Equilibrium in an Integrated and an Isolated country, **2** (light gray area) - the Truth Equilibrium in an Integrated country and the Pandering Equilibrium in an Isolated country, **3** (dark gray area) - the Truth and the Pandering Equilibria in an Integrated country, the Pandering Equilibrium in an Isolated country, **4** (black area) - the area ruled out by assumption $\rho \in [\frac{(1-q)(1-\pi+q-2\pi q)}{2\pi q^2}, 1]$

For the proof, see [Appendix](#). Intuitively, in Truth Equilibrium the voters receive only unbiased information. When the countries are Integrated, voters have access to more information which leads to better electoral decisions. More surprising is **2**, which results from a trade-off between more distortion when the home or foreign incumbent is of a low type and receives a signal B and more unbiased information coming from governments in choosing to play truth. When q is low, low type governments have little informational advantage over the voters, therefore pandering leads to little loss of socially valuable information.

1.5 Extension - Different Election Dates

The outlined setting allows also for an analysis of the implications of institutional harmonisation via internal synchronisation of election dates between countries. The degree of electoral cycle synchronisation affects the difference in time to elections between the countries and hence the voters' probability of learning the true state of the world. Therefore, in this section, I allow the voters' probabilities of learning the truth to differ between the countries. More specifically, I assume that the voters receive public information about the true state of the world in two chunks. Immediately after the governments' actions, they observe the true state with probability ρ_L . After some time, if the true state of the world is not revealed, they receive another signal and learn the truth with probability $\rho_M \equiv \frac{\rho_H - \rho_L}{1 - \rho_L}$. The assumption on ρ_M implies that the *ex ante* probability of learning the truth after observing two signals is

ρ_H .

Definition 1.5.1.

1. When the cycle is *Synchronised*, elections are held on the same date in each country, hence the value of ρ is the same across the countries.
 - i) If elections are **early** after the policy decisions, the voters observe only the first signal and hence, the probability of voters learning the truth is ρ_L in each country.
 - ii) If elections are **late** after the policy decisions, the voters receive both signals and hence, the probability of voters learning the truth is ρ_H in each country.
2. When the cycle is *Asynchronous*, elections are held on different dates in each country. Therefore, there is always one country in which elections are **early** and one in which elections are **late**.⁶

The differences between the harmonisation regimes are presented graphically on [Figure 1.3](#) for an Asynchronous cycle, on [Figure 1.4\(a\)](#) for a Synchronised cycle with early elections and in [Figure 1.4\(b\)](#) for a Synchronised cycle with late elections. The figures present the sequence of actions in period $t = 1$ only. The timing in $t = 2$ is, in all cases, identical to the baseline scenario presented in the previous sections and drawn in [Figure 1.1](#).

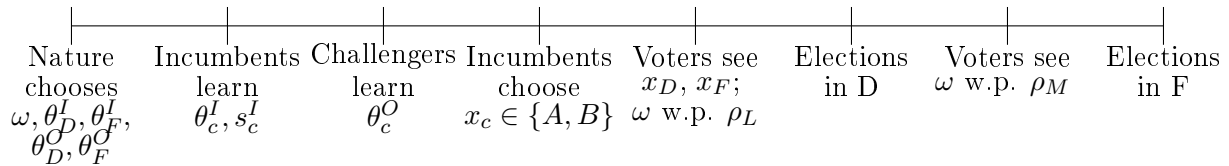


Figure 1.3: Asynchronous Electoral Cycle: Timeline ($t = 1$)

⁶One can question this assumption given the different probabilities of observing the true state of the world in each period. Alternatively we can assume that there is an *ex ante* probability $m \in (0, 1)$ of a voter in a given country receiving one signal and probability $1 - m$ of receiving also a second chance to observe the true state of the world. In such a case a synchronised cycle implies that with probability m both countries observe only one signal and with probability $1 - m$ both countries observe both signals. Asynchronous cycle would imply that with probability m^2 voters in both countries observe only one signal, with probability $2m(1 - m)$ voters in one country receive one and voters in the other country receive two signals and with probability $(1 - m)^2$ voters in both countries receive both signals before the elections. Such a setting, qualitatively, does not change the results.

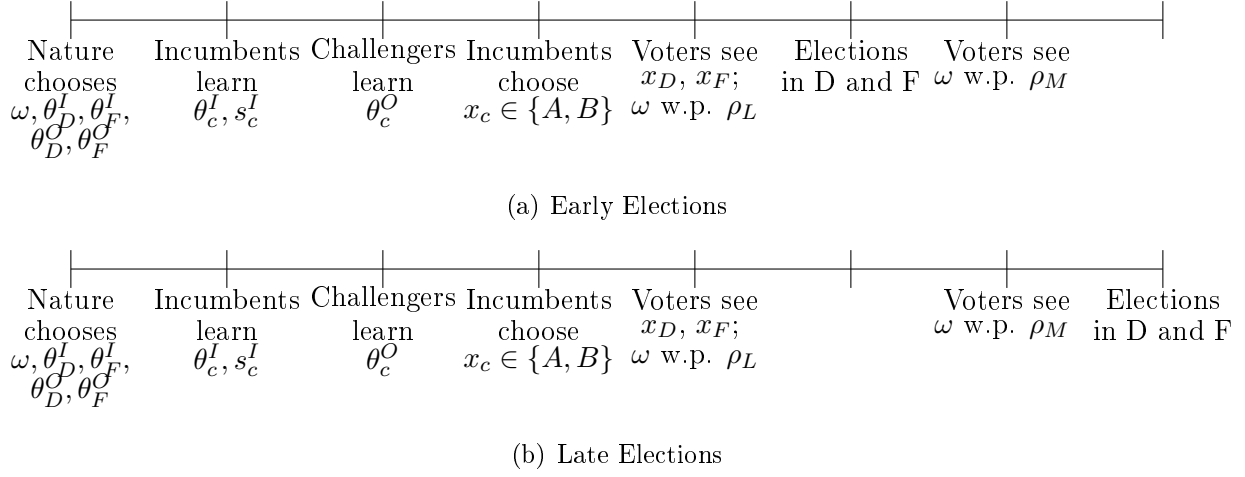


Figure 1.4: Synchronised Electoral Cycle: Timeline ($t = 1$)

Due to assumptions on the values of ρ_M and ρ_H the time between the actions taken and the elections can be reduced to a single period with a value of $\rho \in \{\rho_L, \rho_H\}$ determining the actual time until the election. Furthermore, I assume $\frac{1-\pi-\pi q-q^2+2\pi q^2}{2\pi q^2} < \rho_L < \rho_H = 1$. The latter assumption, although not inconsequential, is used for mathematical tractability.

Proposition 1.5.1. *1. When the cycle is Synchronised and elections are late, Truth Equilibrium is a unique equilibrium.*

2. When the cycle is Synchronised, elections are early and voters' information sufficiently high ($\rho_L > \underline{\rho}$) the Truth Equilibrium is a unique equilibrium.

3. When the cycle is Synchronised, elections are early and voters' information is low ($\rho_L < \underline{\rho}$) there is an equilibrium multiplicity. Both the Truth Equilibrium and the Pandering Equilibrium can emerge.

4. When the cycle is Asynchronous, the Truth Equilibrium is a unique equilibrium.

Proof. The model outlined in Section 1.2 and analysed in Section 1.3 is equivalent to the Synchronised cycle, therefore, the results presented in Proposition 1.3.1 can be applied directly in this proposition. Since, by assumption, $\rho_H > \underline{\rho}$, 1. holds. Since $\frac{1-\pi-\pi q-q^2+2\pi q^2}{2\pi q^2} < \rho_L < 1$, 2. and 3. hold.

The Asynchronous cycle allows for a kind of information sharing between the voters in the two countries. The less informed voters from the country with early elections benefit from better information in the other country. Since $\rho_H = 1$, there is no pandering in the country with informed voters. As a result, the voters in the country with early elections are

exposed to an unbiased source of information through the foreign government actions. As Observation 1.3.1 suggests, the unbiased information assures that the home government also never panders, as stated in 4.. \square

The advantage of the Asynchronous cycle is that it helps steer the incumbents off the Pandering Equilibrium. The Asynchronous cycle uniquely assures the Truth Equilibrium. This is not the case with the Synchronised cycle. In this regime, if elections are early and the probability of learning the truth is sufficiently low, both the Truth and the Pandering equilibria can emerge.

1.6 Conclusions

This paper contributes to the literature on the organisation of international politics in two ways. It discusses the consequences of enhanced knowledge about the decisions taken on other countries coming from international news. Secondly, it shows that harmonized electoral cycles can have adverse political economy effects.

I present a two country model in which the countries share the same state of the world. Governments in each of the countries receive, independently, private signals about the state and take actions observable by voters in both countries. When voters don't observe the true state of the world with certainty, a low quality politician may ignore socially valuable information if it goes against the *ex ante* preferred action of the majority. When the state of the world is not observed, international linkages allow the voters to infer about the state of the world and the incumbent's quality also from the actions taken abroad, which leads to pandering externalities. An increase in the degree of pandering in one country positively affects the returns to pandering in the other country by distorting voters' information. This results in a possibility of pandering contagion, a situation in which an increase in pandering in one country makes the best response in the other country to switch from truthfulness to pandering.

Access to information about the policies chosen abroad is beneficial for the voters as it reduces the range of parameters in which the incumbents pander. When this information is available, for the pandering to occur, the threshold probability of voters learning the true state of the world is strictly lower than when the country is isolated. However, the advantage of international linkages does not come at no cost. In contrast to the media announcements in the [Ashworth and Shotts \(2010\)](#) model, the political actions taken abroad may convey biased information. When the voters learn the policy choices of the foreign countries, they

excessively trust that policy co-ordination is a good indication of correct policy choices, which allows the governments to pander at a higher rate than when the countries are isolated. Therefore, media coverage of foreign affairs is good when the outcomes of the implemented policies are easily observable. However, for long term policies, for which the true state of the world is unlikely to be observed, e.g. climate policy or pension reforms, and for issues driven strongly by value systems such as sexual and religious minority rights or abortion the linkages between countries may accelerate the spread of incorrect policies.

The model also shows that a synchronised electoral cycle is never better for the voters than an asynchronous cycle. In the latter regime, having a long time until elections in one of the countries gives the voters a good chance of learning the outcomes of the policy choices and ensures truthful behaviour. This gives voters in the other country a reliable enough source of information about the state of the world that they also enforce truthful behaviour.

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Appendix - Proofs

Proof of Lemma 1.3.1

To show this result I apply the intuition presented in [Canes-Wrone et al. \(2001\)](#) to the extended setting. The beliefs can be considered in three groups. First, if uncertainty is resolved, the beliefs about the quality of the home government are independent of the actions taken abroad. Therefore, when $x_c \neq \omega$, the voter knows the incumbent is low quality because, as noted before, a high-quality incumbent always follows her signal, which is perfect, and hence always chooses the correct policy. Thus it holds that when $x_c \neq \omega$, $\mu_{x_c, x_{-c}, s^V}^c = 0$. Second, if uncertainty is resolved and $x_c = \omega$, the voter's belief that the incumbent is high quality, is strictly greater than γ , her ex-ante belief, because high quality politicians receive better signals than the low-quality ones. The posteriors are strictly less than one since low-quality politicians also sometimes choose $x_c = \omega_c$.

Third, if uncertainty is not resolved, the voters infer about the state of the world only from the actions taken. Since the state of the world underlying the decisions is the same in both countries, actions taken both at home and abroad are informative. The beliefs $\mu_{B,A,\phi}^c$, $\mu_{A,B,\phi}^c$, $\mu_{B,B,\phi}^c$, $\mu_{A,A,\phi}^c$ are strictly positive, since the high type always follows their signals. At the same time they are strictly lower than the beliefs formed when the true state of the world matches the action observed. Lack of resolution allows for the decision taken to be wrong, which can happen only if the incumbent is of low type.

The relations $\mu_{B,B,\phi}^c > \gamma$ and $\mu_{A,A,\phi}^c > \gamma$ while $\mu_{B,A,\phi}^c < \gamma$ and $\mu_{A,B,\phi}^c < \gamma$ come from the fact that when the home incumbent is of a high type, it is more likely that the foreign politician takes the same action than if the home incumbent is of a low type.

The relations $\mu_{B,B,\phi}^c < \mu_{A,A,\phi}^c$ and $\mu_{B,A,\phi}^c < \mu_{A,B,\phi}^c$ come from the fact that a signal A is more frequently received by high types. The probability of the low type receiving signal A is given by $\pi q + (1 - \pi)(1 - q)$ is strictly lower than π .

For the sake of completeness the posterior beliefs in each information set are presented

below:

$$\begin{aligned}\mu_{B,A,\phi}^c &= \frac{(1-p)(1-q)(1-\gamma)\gamma}{(1-p)(1-q)(1-\gamma)\gamma + (1-\gamma)((1-p)(1-q)q(1-\gamma) + p(1-q)(q(1-\gamma) + \gamma))} \\ \mu_{A,B,\phi}^c &= \frac{p(1-q)(1-\gamma)\gamma}{p(1-q)(1-\gamma)\gamma + (1-\gamma)(p(1-q)q(1-\gamma) + (1-p)(1-q)(q(1-\gamma) + \gamma))} \\ \mu_{B,B,\phi}^c &= \frac{(1-p)\gamma(q(1-\gamma) + \gamma)}{(1-p)\gamma(q(1-\gamma) + \gamma) + (1-\gamma)(p(1-q)^2(1-\gamma) + (1-p)q(q(1-\gamma) + \gamma))} \\ \mu_{A,A,\phi}^c &= \frac{p\gamma(q(1-\gamma) + \gamma)}{p\gamma(q(1-\gamma) + \gamma) + (1-\gamma)((1-p)(1-q)^2(1-\gamma) + pq(q(1-\gamma) + \gamma))} \\ \mu_{A,A,A}^c &= \frac{p\gamma(q(1-\gamma) + \gamma)}{pq(1-\gamma)(q(1-\gamma) + \gamma) + p\gamma(q(1-\gamma) + \gamma)} = \frac{\gamma}{q + \gamma - q\gamma} \\ \mu_{A,B,A}^c &= \frac{p(1-q)(1-\gamma)\gamma}{p(1-q)q(1-\gamma)^2 + p(1-q)(1-\gamma)\gamma} = \frac{\gamma}{q + \gamma - q\gamma} \\ \mu_{B,A,B}^c &= \frac{(1-p)(1-q)(1-\gamma)\gamma}{(1-p)(1-q)q(1-\gamma)^2 + (1-p)(1-q)(1-\gamma)\gamma} = \frac{\gamma}{q + \gamma - q\gamma} \\ \mu_{B,B,B}^c &= \frac{(1-p)\gamma(q(1-\gamma) + \gamma)}{(1-p)q(1-\gamma)(q(1-\gamma) + \gamma) + (1-p)\gamma(q(1-\gamma) + \gamma)} = \frac{\gamma}{q + \gamma - q\gamma}\end{aligned}$$

Proof of Lemma 1.3.2

Not following the signal can be beneficial only if the state of the world is not revealed. In such a case the voters can be in one of the four information sets, depending on the observed actions. It is easy to show that if a low type government receives a signal A and the voter plays $\nu_{A,A,\phi}^c = 1$, $\nu_{A,B,\phi}^c = 1$, $\nu_{B,A,\phi}^c = 0$, $\nu_{B,B,\phi}^c = 0$, the incumbent is re-elected whenever she plays A . For other beliefs, the situation is not so clear-cut. Nevertheless, simple algebra shows that when $\nu_{A,A,\phi}^c = 1$, $\nu_{A,B,\phi}^c = 0$, $\nu_{B,A,\phi}^c = 0$, $\nu_{B,B,\phi}^c = 1$, A is always the best response, whenever $\rho > 1 - \frac{(1+q)(\pi+q-1)}{2\pi q^2(1-\gamma)}$. When $\nu_{A,A,\phi}^c = 0$, $\nu_{A,B,\phi}^c = 0$, $\nu_{B,A,\phi}^c = 1$, $\nu_{B,B,\phi}^c = 1$, A is always the best response, whenever $\rho > \frac{(1-q)(1-\pi+q-2\pi q)}{2\pi q^2}$. Finally, when $\nu_{A,A,\phi}^c = 0$, $\nu_{A,B,\phi}^c = 1$, $\nu_{B,A,\phi}^c = 1$, $\nu_{B,B,\phi}^c = 0$, A is always the best response, whenever $\rho > 1 - \frac{(1+q)(\pi+q-1)}{2q(\pi q - (1-\pi)(1-q)\gamma)}$. The three thresholds coincide for $\gamma = 0$. For all other values of γ , $\frac{(1-q)(1-\pi+q-2\pi q)}{2\pi q^2}$ is strictly greater than the two other values.

Proof of Lemma 1.3.3

Voter's belief that the incumbent is high quality, given the strategies of the home and foreign incumbents receiving signal B $\sigma_B^{L,c}$ and $\sigma_B^{L,-c}$ and the observed action when the true state of the world is not observed are as follows:

$$\mu_{A,A,\phi}^c = \frac{p\gamma((\sigma_B^{L,-c}(1-q) + q)(1-\gamma) + \gamma)}{p\gamma((\sigma_B^{L,-c}(1-q) + q)(1-\gamma) + \gamma) + (1-\gamma)((1-p)(1-q + \sigma_B^{L,-c}q)(1-q + q\sigma_B^{L,c})(1-\gamma) + p(q + (1-q)\sigma_B^{L,c})(\sigma_B^{L,-c}(1-q) + q)(1-\gamma) + \gamma))}$$

$$\begin{aligned}\mu_{B,B,\phi}^c &= \frac{(1-p)\gamma((1-\sigma_B^{L,-c})q(1-\gamma) + \gamma)}{(1-p)\gamma((1-\sigma_B^{L,-c})q(1-\gamma) + \gamma) + (1-\gamma)\left((1-\sigma_B^{L,-c})p(1-q)^2(1-\sigma_B^{L,c})(1-\gamma) + (1-p)q(1-\sigma_B^{L,c})((1-\sigma_B^{L,-c})q(1-\gamma) + \gamma)\right)} \\ \mu_{A,B,\phi}^c &= \frac{(1-\sigma_B^{L,-c})p(1-q)(1-\gamma)\gamma}{(1-\sigma_B^{L,-c})p(1-q)(1-\gamma)\gamma + (1-\gamma)\left((1-\sigma_B^{L,-c})p(1-q)(q + (1-q)\sigma_B^{L,c})(1-\gamma) + (1-p)(1-q + q\sigma_B^{L,c})((1-\sigma_B^{L,-c})q(1-\gamma) + \gamma)\right)} \\ \mu_{B,A,\phi}^c &= \frac{(1-p)(1-q + \sigma_B^{L,-c}q)(1-\gamma)\gamma}{(1-p)(1-q + \sigma_B^{L,-c}q)(1-\gamma)\gamma + (1-\gamma)\left((1-p)q(1-q + \sigma_B^{L,-c}q)(1-\sigma_B^{L,c})(1-\gamma) + p(1-q)(1-\sigma_B^{L,c})((\sigma_B^{L,-c}(1-q) + q)(1-\gamma) + \gamma)\right)}\end{aligned}$$

To see 1. note that the more pandering there is in a given country, the more likely it is that when action B is taken the incumbent is of a high type. Therefore, the more pandering there is in the foreign country, when a pair of actions $\{A, B\}$ is observed, the more likely it is that the action A is taken by the misinformed low type. Therefore, as pandering either in the foreign country or at home increases, the value of $\mu_{A,B,\phi}^c$ decreases, i.e. stays below γ . The same mechanism makes $\mu_{B,B,\phi}^c$ increase in the level of pandering in either of the countries, keeping it above γ .

To see 2. it suffices to apply the same logic to $\mu_{B,A,\phi}^c$ and $\mu_{A,A,\phi}^c$. It is easy to show that the former probability is increasing in the low type pandering, both at home and abroad. At the same time $\mu_{A,A,\phi}^c$ is decreasing in pandering both at home and abroad. The more the politicians pander, the less likely it is that an action A is taken by a high type. At the point:

$$\sigma_B^{L,c} = \underline{\sigma_B^{L,c}} = \frac{(1-q)(\pi(1 + \sigma_B^{L,-c} - \sigma_B^{L,-c}\gamma) - (1 - (1 - \sigma_B^{L,-c})q)(1 - \gamma))}{\sigma_B^{L,-c}(\pi - 2\pi q + q^2)(1 - \gamma) + (1-q)(q(1 - \gamma) + \pi\gamma)}$$

the two probabilities are equal. If there is no pandering in the foreign country, the two beliefs are equal at $\sigma_B^{L,c} = \frac{\pi + q + \gamma - q\gamma - 1}{q + \pi\gamma - q\gamma}$. The value of $\sigma_B^{L,c}$ that makes the two beliefs equal decreases with the degree of pandering abroad.

The actions of incumbents of other type, or observing other signals, obviously, affect also these beliefs. However, as it has been stated, given the pay-off structure, high type never panders and, given the assumptions on ρ , the low type receiving signal A also never panders.

Proof of Proposition 1.3.1

Given the pay-off structure and the restriction on ρ , the high types and low types receiving signals A always follow them. Lemma 1.3.3 implies that in ten out of twelve information sets voters' actions are independent of the beliefs held. The strategies are always $\nu_{A,A,A}^c = 1$, $\nu_{A,B,A}^c = 1$, $\nu_{B,A,A}^c = 0$, $\nu_{B,B,A}^c = 0$, $\nu_{A,A,B}^c = 0$, $\nu_{A,B,B}^c = 0$, $\nu_{B,A,B}^c = 1$, $\nu_{B,B,B}^c = 1$, $\nu_{A,B,\phi}^c = 0$ and $\nu_{B,B,\phi}^c = 1$. All that remains to be identified is the strategies $\nu_{B,A,\phi}^c$ and $\nu_{A,A,\phi}^c$.

Incumbents' first period utility is strictly increasing in probability of taking a correct action. Second period utility is built of two elements that depend on voter's information.

If the uncertainty is resolved, given the beliefs held by the voter outlined in [Lemma 1.3.1](#), this component coincides with the first period utility.⁷ When voters don't learn the truth, the pay-off depends also on the actions taken in the other country. Pandering decisions need to take into account losses from a sure decrease in the first period utility, a sure decrease in the re-election prospects when the uncertainty is resolved and possible benefits from the increase in the second period utility when the uncertainty is not resolved.

Suppose, now, that a low type receives a signal B and home voter's strategy is $\nu_{B,A,\phi}^c = 1$ and $\nu_{A,A,\phi}^c = 0$. In such a case, when $s^V = \phi$ the incumbent gets surely re-elected by playing $x_c = B$.⁸ As $x_c = B$ also maximises the re-election prospects when the truth is revealed and the first period utility, it must be the best response.

When home voter plays $\nu_{B,A,\phi}^c = 0$ and $\nu_{A,A,\phi}^c = 1$, the situation is less clear for a low type receiving a signal B . When the uncertainty is not resolved, the re-election prospects depend also on the action taken abroad.⁹ However, it can be shown that when $\gamma > \frac{\pi+q-2\pi q}{2q-2\pi q}$, B is played strictly more frequently than A by the incumbent in the other country.¹⁰ Therefore, in this range, the choice of action $x_c = A$ strictly decreases the re-election prospects and no pandering is the best response. For lower levels of average quality of the political class, pandering may increase the expected pay-off, when the uncertainty is not resolved, if the degree of pandering abroad is high enough. It can be shown, however, that even if the low type abroad always chooses $x_{-c} = A$, when the probability of voters learning the truth is sufficiently high, i.e. :

$$\rho > 1 - \frac{q - \pi}{2(1 - \pi)q(1 - \gamma)}$$

pandering is never the best response. The benefits from pandering when the uncertainty is not resolved never outweigh the losses to the other components of the low type's pay-off. For lower levels of ρ there always exists a level of pandering in the other country that makes pandering the best response.

For sufficiently low γ , sufficiently low ρ and sufficiently high degree of pandering abroad, pandering could be the optimal action under condition that $\nu_{B,A,\phi}^c = 0$ and $\nu_{A,A,\phi}^c = 1$ is the voters best response. However, as [Lemma 1.3.3](#) states, when $\sigma_B^{L,c} > \underline{\sigma_B^{L,c}}$ it is not the case

⁷Up to the scaling factor, the expected second period pay-off if re-elected, i.e. 1 for high types and q for the low type.

⁸Since [Lemma 1.3.3](#), $\nu_{A,B,\phi}^c = 0$ and $\nu_{B,B,\phi}^c = 1$

⁹Since $\nu_{A,A,\phi}^c = 1$ and $\nu_{B,B,\phi}^c = 1$ the best response for $x_{-c} = A$ is $x_c = A$ and the best response for $x_{-c} = B$ is $x_c = B$

¹⁰In this range, the probability of the government in the other country being of high quality is so high, that given the signal received B is the most likely action, even if the foreign incumbent of the low type always plays A

anymore. Therefore, holding the actions abroad fixed, the only possible set of mutual best responses between the voters and the incumbent is such that the incumbent plays $\sigma_B^{L,c} = \underline{\sigma_B^{L,c}}$ and the voter plays $\nu_{A,A,\phi}^c = 1$ and sets $\nu_{B,A,\phi}^c$ to such a level that the incumbent is indifferent between $x_c = A$ and $x_c = B$.

Assume that such a value $\nu_{B,A,\phi}^c$ exists and the incumbent in c plays $\underline{\sigma_B^{L,c}}$. Since the two countries are *ex ante* identical, the same logic can be applied to $-c$. Therefore, it is possible calculate the value $\sigma_B^{*,L}$ such that $\sigma_B^{*,L} = \underline{\sigma_B^{L,c}} = \underline{\sigma_B^{L,-c}}$. Some algebra proves that:

$$\sigma_B^{*,L} = \frac{\pi(1-q)(1-2\gamma) + 2q(q+\gamma-q\gamma-1) + \sqrt{\pi(1-q)(5\pi+8q-9\pi q+4(1-\pi)(2-3q)\gamma-4(1-\pi)(1-q)\gamma^2-4)}}{2(\pi-2\pi q+q^2)(1-\gamma)}$$

It is easy to calculate that when the low type abroad chooses $\sigma_B^{L,-c} = \sigma_B^{*,L}$ the voter's action that makes the incumbent indifferent between $x_c = A$ and $x_c = B$ is $\nu_{A,A,\phi}^c = 1$ and $\nu_{B,A,\phi}^c = \nu_{B,A,\phi}^*$ such that:

$$\nu_{B,A,\phi}^* = \frac{2\left(q(1+q) + \sqrt{\pi q \sqrt{(1-q)(5\pi+8q-9\pi q+4(1-\pi)(2-3q)\gamma-4(1-\pi)(1-q)\gamma^2-4)}}(1-\rho) - \pi(1+q^2(1-\rho)+q\rho)\right)}{\sqrt{\pi q \left(\sqrt{(1-q)(5\pi+8q-9\pi q+4(1-\pi)(2-3q)\gamma-4(1-\pi)(1-q)\gamma^2-4)} - \sqrt{\pi(1-q)}\right)}(1-\rho)}$$

Given the symmetry, the same actions are chosen by the voters in the other country. For this set of strategies to be an equilibrium, it remains to assure that there exists a set of parameters in which the following holds. When the incumbent in the foreign country plays $\sigma_B^{L,-c} = \sigma_B^{*,L}$ and home voter plays $\nu_{A,A,\phi}^c = 1$ and $\nu_{B,A,\phi}^c = \nu_{B,A,\phi}^*$ pandering is the best response. Some algebra proves that a Pandering Equilibrium exists for sufficiently low ρ , i.e. for

$$\rho < \underline{\rho} = \frac{\pi - q - (1-\pi)q^2 + q\sqrt{\pi(1-q)(5\pi+8q-9\pi q+4(1-\pi)(2-3q)\gamma-4(1-\pi)(1-q)\gamma^2-4)}}{q\left(\sqrt{\pi(1-q)(5\pi+8q-9\pi q+4(1-\pi)(2-3q)\gamma-4(1-\pi)(1-q)\gamma^2-4)} - \pi(1-q)\right)}$$

The Pandering Equilibrium is, however, not unique. In the same range of parameters the Truth Equilibrium can also emerge. Since signals are informative and the true state the world is the same across the countries, if the incumbent in the foreign country always follows the truth, $x_{-c} = B$ is the most likely action, given $s_c^I = B$. Since, $\forall \sigma_B^{L,c} \forall \sigma_B^{L,-c} \mu_{A,B,\phi}^c < \gamma$, choice of $x_c = A$ when $x_{-c} = B$ is the most likely action abroad does not lead to a re-election and hence does not increase the second period utility. At the same time pandering strictly reduces the first period utility, therefore the joint effect of pandering is strictly negative.

For $\rho > \underline{\rho}$, since the voter is too likely to learn the truth and the incumbents always follow the truth; the Truth Equilibrium is a unique equilibrium;

Proof of Proposition 1.3.2

Voter's expected utility is simply the probability a correct action in each period. This probability depends on the government's quality in each of the periods and the equilibrium type. I will refer to equilibrium type as $\tau \in \{TE, PE\}$, where TE denotes a Truth Equilibrium and PE denotes a Pandering Equilibrium. The expected first period utility is, simply, the probability of a correct action in the first period. The second period utility depends on the re-election probability. In each of the equilibrium types, when the state of the world is revealed (with probability ρ) the incumbent gets re-elected when the action matches the true state of the world. When the state of the world is not revealed re-election probability varies between the equilibrium types. If the incumbent is replaced with a challenger, with probability γ the challenger is of a high type and brings utility 1 and with probability $(1 - \gamma)$ is of a low type and brings, in expectations, utility q . Formally voter's expected utility in a given equilibrium and given the incumbents' type is given by:

$$E [U_c^V | \theta_c, \tau] = (1 + \rho \mathcal{A}^{\theta_c}) \mathcal{B}_\tau^{\theta_c} + (1 - \rho)(\mathcal{A}^{\theta_c} \mathcal{C}_\tau^{\theta_c} + (\gamma + (1 - \gamma)q)(1 - \mathcal{C}_\tau^{\theta_c}) \quad (1.6.1)$$

\mathcal{A}^{θ_c} is a type dependent probability of taking a correct action in the second period - $\mathcal{A}^L = q$ and $\mathcal{A}^H = 1$. $\mathcal{B}_\tau^{\theta_c}$ is the probability of taking a correct action in the first period. It depends on the type of incumbent and the equilibrium type.

$$\begin{aligned} \mathcal{B}_T^H &= \mathcal{B}_P^H = 1 \\ \mathcal{B}_T^L &= (1 - p)q + pq \\ \mathcal{B}_P^L &= pq + (1 - p)q(1 - \sigma_B^{*,L}) + p(1 - q)\sigma_B^{*,L} \end{aligned}$$

$\mathcal{C}_\tau^{\theta_c}$ is the re-election probability. It also depends on the incumbent and equilibrium type.

$$\begin{aligned} \mathcal{C}_T^H &= (p(\gamma + (1 - \gamma)q) + (1 - p)(\gamma + (1 - \gamma)q)) \\ \mathcal{C}_P^H &= (p(\gamma + (1 - \gamma)q + (1 - q)\sigma_B^{*,L}) + (1 - p)(\gamma + (1 - \gamma)q(1 - \sigma_B^{*,L}) + (1 - \gamma)(1 - q + q\sigma_B^{*,L})\nu_{B,A,\phi}^*)) \\ \mathcal{C}_T^L &= ((1 - p)(1 - q)^2(1 - \gamma) + p(1 - q)^2(1 - \gamma) + (1 - p)q(q(1 - \gamma) + \gamma) + pq(q(1 - \gamma) + \gamma)) \\ \mathcal{C}_P^L &= \left((1 - p)q(\gamma + q(1 - \gamma)(1 - \sigma_B^{*,L})) + p(1 - q)^2(1 - \gamma)(1 - \sigma_B^{*,L}) \right) (1 - \sigma_B^{*,L}) + \\ & (1 - p)(1 - q)(1 - \gamma)(1 - q + q\sigma_B^{*,L}) + pq(\gamma + (1 - \gamma)(q + (1 - q)\sigma_B^{*,L})) + \\ & \sigma_B^{*,L}((1 - p)q(1 - \gamma)(1 - q + q\sigma_B^{*,L}) + p(1 - q)(\gamma + (1 - \gamma)(q + (1 - q)\sigma_B^{*,L}))) + \\ & \nu_{B,A,\phi}^*(1 - \sigma_B^{*,L})((1 - p)q(1 - \gamma)(1 - q + q\sigma_B^{*,L}) + p(1 - q)(\gamma + (1 - \gamma)(q + (1 - q)\sigma_B^{*,L}))) \end{aligned}$$

The expected voters utility in a given equilibrium type is given by:

$$E [U_c^V | \tau] = \gamma E [U_c^V | \theta_c = H, \tau] + (1 - \gamma) E [U_c^V | \theta_c = L, \tau]$$

The dominance of the Truth Equilibrium comes from comparison of the expected utilities in the two equilibrium types in the range of parameters in which a Pandering Equilibrium exists.

Proof of Proposition 1.4.2

When the countries are Isolated we can also write voters' expected utility in a form of Equation 1.6.1. To distinguish from the Integrated setting, I will refer to $\widehat{\mathcal{A}}^{\theta_c}$, $\widehat{\mathcal{B}}_{\tau}^{\theta_c}$, $\widehat{\mathcal{C}}_{\tau}^{\theta_c}$ as the equivalents of \mathcal{A}^{θ_c} , $\mathcal{B}_{\tau}^{\theta_c}$, $\mathcal{C}_{\tau}^{\theta_c}$ in the Isolated country. Compared to the Integrated countries, the values of $\widehat{\mathcal{A}}^{\theta_c}$, $\widehat{\mathcal{B}}_T^H$ and $\widehat{\mathcal{B}}_P^H$ are the same as under Integration, $\widehat{\mathcal{B}}_P^L = pq + (1-p)q(1 - \sigma_B^{*,L}) + p(1-q)\sigma_B^{*,L}$ and the values of $\widehat{\mathcal{C}}_{\tau}^{\theta_c}$ are given by the following:

$$\begin{aligned}\widehat{\mathcal{C}}_T^H &= p \\ \widehat{\mathcal{C}}_P^H &= p + (1-p) * \widehat{\nu}_B^* \\ \widehat{\mathcal{C}}_T^L &= pq + (1-p)(1-q) \\ \widehat{\mathcal{C}}_P^L &= (1-p)(1-q) + pq + (p(1-q) + (1-p)q)(1 - \widehat{\sigma}_B^{*,L}) + (p(1-q) + (1-p)q)\nu_B^* \widehat{\sigma}_B^{*,L}\end{aligned}$$

Where $\widehat{\nu}_B^* = \frac{p(1+q\delta-2q^2\delta(1-\rho))-q(1-q(\delta-2\delta\rho))}{q(q+p(1-2q))\delta(1-\rho)}$ is the equilibrium probability of voters re-electing the incumbent when the true state of the world is not revealed and the observed action is B . Low type's pandering probability is $\widehat{\sigma}_B^{*,L} = 1 - \frac{1-\pi}{\pi(1-q)+(1-\pi)q}$ (cf Canes-Wrone et al. (2001)).

To see 1. note that the only difference between the Truth Equilibrium in an Integrated and an Isolated country is in the re-election probability when the true state of the world is not observed. The difference between the two is given by the following

$$\mathcal{C}_T^H - \widehat{\mathcal{C}}_T^H = q + \gamma - q\gamma - p > 0$$

and

$$\mathcal{C}_T^L - \widehat{\mathcal{C}}_T^L = (1-2q)(p - q(1-\gamma) - \gamma) > 0$$

In the Truth Equilibrium when the countries are Integrated the re-election probability of both high and low types is strictly higher than under isolation. While the former positively affects the voters' expected utility, the latter has the negative effect. However, it is easy to show that the overall effect of Integration is positive as:

$$\begin{aligned}\gamma * (\mathcal{A}^H - (\mathcal{A}^H * \gamma + (1-\gamma) * \mathcal{A}^L)) (\mathcal{C}_T^H - \widehat{\mathcal{C}}_T^H) + (1-\gamma)(\mathcal{A}^L - (\mathcal{A}^H * \gamma + (1-\gamma) * \mathcal{A}^L)) (\mathcal{C}_T^L - \widehat{\mathcal{C}}_T^L) &= \\ &= 2(q-1)^2(q(1-\gamma) + \gamma - p)(1-\gamma)\gamma(1-\rho) > 0\end{aligned}$$

where $\mathcal{A}^H * \gamma + (1 - \gamma) * \mathcal{A}^L$ is the expected second period utility if the challenger is elected.

Due to algebraic complexity \mathcal{Q} . can be shown only numerically. [Table 1.1](#) presents voters' expected utility in a Truth Equilibrium and a Pandering Equilibrium for an Integrated and an Isolated country.

Table 1.1: Voter's expected utilities in different equilibrium types

		$p=3/5$						$p=4/5$					
		$q=P+\frac{(1-p)}{10}$		$q=P+\frac{5(1-p)}{10}$		$q=P+\frac{9(1-p)}{10}$		$q=P+\frac{(1-p)}{10}$		$q=P+\frac{5(1-p)}{10}$		$q=P+\frac{9(1-p)}{10}$	
$\gamma=1/3$	$\rho=1/4$	\emptyset	1.531	1.739	\emptyset	1.947	\emptyset	\emptyset	1.757	1.852	\emptyset	1.973	\emptyset
		1.538	1.524	1.740	\emptyset	1.947	\emptyset	1.766	1.756	1.869	\emptyset	1.973	\emptyset
	$\rho=1/2$	\emptyset	1.531	1.739	\emptyset	1.947	\emptyset	\emptyset	1.759	1.868	\emptyset	1.973	\emptyset
		1.542	\emptyset	1.741	\emptyset	1.947	\emptyset	1.766	1.758	1.869	\emptyset	1.973	\emptyset
	$\rho=3/4$	\emptyset	1.538	1.740	\emptyset	1.947	\emptyset	\emptyset	1.761	1.869	\emptyset	1.973	\emptyset
		1.545	\emptyset	1.741	\emptyset	1.947	\emptyset	1.767	\emptyset	1.869	\emptyset	1.973	\emptyset
$\gamma=2/3$	$\rho=1/4$	\emptyset	1.765	\emptyset	1.863	1.973	\emptyset	\emptyset	1.880	\emptyset	1.926	1.987	\emptyset
		1.784	\emptyset	1.875	\emptyset	1.974	\emptyset	1.887	1.881	1.935	\emptyset	1.987	\emptyset
	$\rho=1/2$	\emptyset	1.773	1.872	\emptyset	1.974	\emptyset	\emptyset	1.881	1.935	\emptyset	1.987	\emptyset
		1.785	\emptyset	1.875	\emptyset	1.974	\emptyset	1.887	\emptyset	1.935	\emptyset	1.987	\emptyset
	$\rho=3/4$	\emptyset	1.780	1.874	\emptyset	1.974	\emptyset	\emptyset	1.883	1.935	\emptyset	1.987	\emptyset
		1.787	\emptyset	1.875	\emptyset	1.974	\emptyset	1.887	\emptyset	1.935	\emptyset	1.987	\emptyset

Each cell of the table represents voters expected utility in one of the two equilibrium types for an Isolated and an Integrated country, top left - a Truth Equilibrium in an Isolated country; top right - a Pandering Equilibrium in an Isolated country; bottom left - a Truth Equilibrium in an Integrated country; bottom right - a Pandering Equilibrium in an Integrated country. \emptyset implies that the particular equilibrium type does not exist for a given set of parameters. A cell in which a Pandering equilibrium yields higher welfare for an Integrated than an isolated country is marked green.

Chapter 2

Beliefs, Access Constraints and Voluntary Education Decisions

Abstract

In the recent decades university enrolment has grown enormously, however, as this paper points out, this increase might not have only benign welfare consequences.

I focus on universities as certification devices, that rather than enhancing students' human capital, transmit (imperfect) information about the abilities of their graduates to potential employers. I explore the effects of perverse incentives, for agents with low innate abilities, to enrol in university only when the quality of the education sector is low. When the access to education is exogenously constrained and the number of university places does is not sufficient to meet the educational needs, some of the high types are left without quality certificates. As the access to academic resources becomes less competitive, enrolment among the high types increases. Consequently, employers become more pessimistic about the quality of the workers without a diploma, leading to a vicious circle of excessive educational enrolment.

The effect is even stronger when the supply of academic resources is inelastic i.e. when an increase in enrolment, for a fixed investment per student, leads to a decrease in quality. Multiple equilibria exist and the final outcomes depend on agents' initial beliefs. Individual beliefs on returns to education are crucial for selecting among multiple equilibria.

In so far as large scale surveys can reveal societal beliefs, empirical data from the Central and Eastern European countries are broadly consistent with the model.

Keywords: Certification, Higher Education, Access Constraints

JEL Classification Numbers: D82, C72, H42 , I23

2.1 Introduction

The last 50 years can be considered the age of university education. The enrolment rates at tertiary education institutions have increased from around 5% in the early 1950s in most developed countries to more than 50% in recent times (Ansell (2010)). There are various reasons for this change. Undoubtedly, the technological progress has increased the demand for skilled labour (cf Card and DiNardo (2002)). Population growth in the western world has significantly decreased (in some countries it is even negative). The resulting increase in the relative scarcity of labour in relation to capital justifies higher investment in human capital. Societal perception of higher education has also changed. It is no longer a consumption good enjoyed by the elite but an investment in human and social capital. Access to universities has become more universal and a reduction of entrance constraints for the socially impaired groups is an important social and political issue. This quote by Barr (2004) gives an idea of current perceptions. *"The expansion of higher education throughout the OECD and beyond is both necessary and desirable."*

However, mass university education does not necessarily imply mass success on the labour market. Recent studies show an increase in overeducation rates. An increasing number of graduates take jobs that do not utilise their qualifications (Felstead et al. (2007)). There is no agreement in the literature about what explains this phenomenon. Education quality may be insufficient and not lead to accumulation of human capital, a hypothesis that is supported by evidence on credential inflation. Another possible explanation for the limited success of mass education is the change in relative innate abilities of graduates compared to non-graduates. Or, simply, the demand for skilled labour may not go in hand with the increased supply.

The focus of the discussion has been so far on developed countries, which experienced a rapid growth of university enrolment in the 1980s and earlier. In many developing countries this change took place later and was even more rapid. Figure 2.1 depicts the relative growth of the population of students in Central and Eastern Europe (CEE) and Western Europe in the last decade.¹

In some countries, e.g. in Romania or Poland, the annual changes to the student population were often exceeding 10%. The number of enrolled students in CEE from 1999 to 2007

¹ Figure 2.1 is based on gross enrolment data, therefore it double-counts students enrolled at more than one faculty, but it represents the actual capacity use of those students. At the same time, this overestimates the effect of the students that are registered without actual use of the university services (e.g. register just in order to receive the student status).

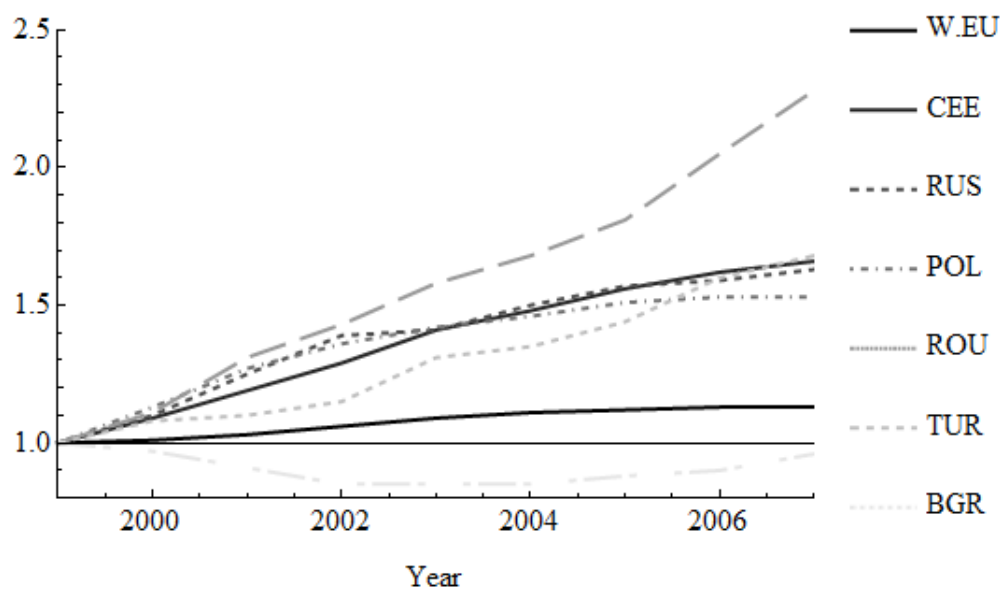


Figure 2.1: Change in student population in 1999-2007

Source: UIS UNESCO database, 1999=1

has increased, on average, by 69%, compared to only a 13% increase in Western Europe (the EU15).² CEE countries are clearly catching up with their more developed neighbours and are bridging the education attainment gap. Thanks to the growth in the last two decades the proportion of the population with higher education has reached the level in developed countries or has even exceeded it. This sudden growth certainly raises questions about capacity constraints in education system.

This rapid growth was possible due to socio-economic changes following the fall of communism. The change of the economic system has created huge changes in the demand for human capital since it restored the link between marginal productivity and wages. To meet this demand there was a large liberalisation of the higher education market which led to an expansion in the size of existing public universities and the foundation of numerous private universities. The strict government control of the curricula and student numbers was lifted. Private institutions, offering education for a fee, moved the burden of financing education from the governments towards the users and increased the capacity of the education systems.³

²Calculated using the UNESCO Institute for Statistics database.

³There were numerous other reasons for an increased demand for higher education in the region. E.g. the rapid economic transition led to a large increase in unemployment in the region in the early 1990s,

The enrolment rates in CEE have increased despite general concerns about the quality of higher education. Although empirical evidence is very scarce (Machin and McNally (2007)), press reports supported by reports commissioned by national authorities raise these issues.⁴

The main objective of this study is to analyse, from a theoretical perspective, the effects of improving access to university education, when university's sole purpose is the provision of certification. In particular, I propose a mechanism that can shed light on the strikingly high growth of tertiary education enrolment rates in CEE. More specifically, I analyse what happens when the strict state control over the higher education system (through capacity controls) is being lifted. Therefore, I focus on a change in the welfare of agents between the system with restricted access to certification and one with unrestricted access. The aspect of administrative restrictions on certification provision is, to my knowledge, unexplored in the literature. I will show that the relaxing of admission restrictions for higher education can be beneficial only if the certification quality is sufficiently high. Obviously, access to certification may be desirable on other grounds. In this paper, however, I am focusing on the link between Pareto-efficiency, access and certification quality. Finally, I show the effects of short term supply inelasticities, resulting in a decrease in certification quality, and their influence on an increase in enrolment rates. Hence, the higher the growth in enrolment rates, the lower the certification precision for a given level of per student expenditure.

In the model, I assume that agents, differing in their innate abilities, are privately informed about their type. They may, at a cost, voluntarily undergo an imperfect certification process, which is the only mechanism by which they can signal their abilities to the competitive employers. The employers offer one type of job in which the productivity of the worker is defined by his type, though two job types and supermodularities in the job-worker match are explored in the extension of the basic model. The bigger the imperfections, the higher the chances for those of low innate abilities to receive the high ability certificate. Due to the nature of certification, the concepts: education quality, certification quality and luck are

therefore university education was a way to delay entering the labour market. The opening of the EU labour markets was the driving force for the university enrolment rates' growth in the later years, at least for the new member states. Even though many graduates could not find the jobs in their fields, they hoped to capitalize on their education through emigration (Thaut (2009)). Finally, the growth in the gross number of students is caused also by the increase in the number of students that study at more than one faculty in order to signal their quality.

⁴E.g. E&Y and IBnGR (2009). This position is also backed by the World Bank (2005), which recommends undertaking substantial changes needed in tertiary education to shift the focus from quantity to quality. E.g. a study for Lithuania states the "poor quality education system is blamed for the country's failure to produce educated young people with skills and knowledge suited to Lithuania's labour market needs, thus contributing to emigration motivations and recent labour shortages in certain sectors" (Thaut (2009)).

equivalent in determining outcomes in the presented setting.

The model is designed to analyse the effects of the relaxing of the constraints on access to education. I present two regimes. In the first regime, restrictions in access are modelled through exogenous limits on certification supply. The number of university places is fixed regardless of the level of demand. In the second regime, the exogenous capacity constraints are lifted and access to education is given to all who apply.⁵

In the extension of the model I analyse the short run effects of the transition. I assume the existence of short term capacity constraints, given by a convex short term certification quality provision cost function. I assume that agents make their educational decisions based on prior beliefs on the return to education. I will show that those beliefs can be self-confirming. The imperfection of certification can be interpreted as the role of chance in examinations and, possibly, social connections in determining education outcomes.⁶ In the extended model with the convex cost function, a rapid growth of enrolment over a short period of time pushes the education/ certification quality down, increasing the role of luck in the process. If, on the contrary, agents initially believe that education is more precise in revealing their true type, low skill agents abstain from education. Differences in prior beliefs in the population of agents can lead to differences in educational outcomes due to the existence of multiple equilibria in the model. This result could be a possible explanation for differences in the growth rates of higher education in CEE.

In context of education economics, this paper contributes to our understanding of the effects of an expansion of tertiary education, particularly in transition countries. This phenomenon has been studied in the empirical literature mainly in parallel with the college-high school wage gap (see e.g. [Card and Lemieux \(2001\)](#) or [Walker and Zhu \(2008\)](#) for the US, UK and Canadian data). The effect of the higher education expansion in transition economies has been, to my knowledge, unexplored.

This paper is a contribution to the literature on quality disclosure thoroughly reviewed by [Dranove and Jin \(2010\)](#). My work draws mostly on a short paper on imperfect certification and it's implications on the market size by [De and Nabar \(1991\)](#). The authors present a simple model with two risk neutral, privately informed sellers of different types, a single buyer and an exogenous certification technology (which is costly and imperfect). The authors present a disturbing result, that the higher the certification accuracy a lower the demand for

⁵An alternative interpretation is an improvement in access due to e.g. an introduction of a large scale student loan programme or simply a baby boom.

⁶An alternative mechanism that fits into this framework is corruption. As described by [Rothstein and Uslaner \(2006\)](#) in some countries parents "buy" their children's way into good schools, especially universities, and then pay even more for good grades.

certification services. Unless certification costs are too high, all high quality and some low quality sellers desire certification. As accuracy increases, the high types continue to demand certification - if anything, they will be better off. At the same time, the chances of low type sellers securing a high ranking decrease, thus reducing their demand for certification. As a result, the total demand decreases. The same mechanism is, obviously, present also in my model.

Another recent approach to the issues related to certification is a work by [Rosar and Schulte \(2010\)](#). The authors explore the trade-off between the quality of the generated information and the participation level. They analyse the optimal certification device from a perspective of an information-loving receiver. In their setting, contrary to the papers mentioned before and this work, the agents are risk averse and imperfectly informed about their own type. The optimal device needs to balance two effects, the information quality provided by the test and self-revelation through differences in participation rates between the types. Therefore the optimal device is often imperfect. If the test is too accurate, an information-averse individual may refrain from taking the test. The authors consider only a costless technology, which significantly affects the results.

Models of imperfect certification can be viewed as a simplification of noisy signalling models. In this class of models, differential rates (between high and low type) of being seen as a high type (by the employer) in equilibrium are driven by differences in effort costs between the types satisfying the single crossing condition. In a certification setting this property is reduced to a stochastic, exogenous, type-dependant truth-revealing mechanism. The relation between signalling effort, monetary costs of education and education decisions is studied Chapter 3 of this thesis. The main focus is on the level of substitutability between the pecuniary and non-pecuniary costs of education conditional on the quality of the signal.

Finally, the last source of inspiration for this paper is the recent interest in explaining numerous aspects of policy preferences through self confirming beliefs based on World Value Survey (WVS) results. [Alesina and Angeletos \(2005\)](#) and [Benabou and Tirole \(2006\)](#) have found a correlation between one item of the questionnaire - a belief that it is luck rather than hard work that determines economic outcomes - and the share of social expenditure as a proportion of GDP. The authors have, independently, proposed a causal mechanisms in which the societal beliefs are self-confirming. Both models show that in countries where the belief in the role of luck is high, the share of public expenditure to GDP is high. Here, I show a similar effect of beliefs on the total enrolment rate in CEE.

The paper is organised as follows. In [Section 2.2](#) I present the the model and the key assumptions. [Section 2.3](#) describes the effect of the relaxation of exogenous capacity con-

straints on the education system. I first show an equilibrium with fixed enrolment quotas (administrative capacity constraints) and then the equilibrium with the constraints relaxed. Subsequently the welfare implications of the relaxation of those constraints are discussed. Differences between certification and licensing are discussed in [Subsection 2.3.4](#). Since the basic model assumes only one type of jobs, multiple job types with supermodularities in job-worker matching are explored in [Subsection 2.3.5](#). [Section 2.4](#) discusses the effects of short term supply inelasticities once the capacity constraints are lifted; the existence of equilibria multiplicity is shown. [Section 2.5](#) briefly discusses the relation between different possible equilibria and beliefs in the role of luck in determining economic outcomes in Central and Eastern Europe. Finally, [Section 2.6](#) concludes.

2.2 The model

This section introduces the setup.

2.2.1 Agents

I assume a continuum of privately informed risk neutral agents of mass 1. The agents differ in productivity, and they can be of type $\theta \in \Theta = \{\theta_H, \theta_L\}$. I standardise the values to $\theta_H = 1$ and $\theta_L = 0$. This double standardisation allows to interpret all other parameters in a convenient metric - a proportion of a difference in lifetime products between high and low types.⁷

The proportion of high types is a public information and is set to $\lambda \in [0, 1]$. The agents will be sometimes referred to as students and test takers throughout the text.

The agents can disclose their type only through voluntary enrolment at a certification institution. Throughout the text a word university will be used as a synonym, but the range of applications stretches to other forms of certification. The certification institution serves only testing purposes and provides no human or social capital gains. The agents make a strategic decision $d_i \in D = [0, 1]$, where 1 is to apply for a education at a private cost s if accepted⁸, 0 otherwise. In-between values of d_i are interpreted as mixed strategies. The pri-

⁷If one wants to incorporate employer's learning in the model, $\theta_H - \theta_L$ should be interpreted as an expected difference in product delivered between high and low type in time before the true type is learned (on the job) by the employer and the wages adjusted accordingly

⁸This cost could be interpreted as an opportunity cost of being at university. The fact that $s > \theta_L$ implicitly assumes that education is related to an experience loss relatively to those who decide not to undertake education. For other applications of the model one could interpret s as e.g. costs of preparing

vate cost is not a transfer to the certifier and is independent of certification precision. Due to access constraints, if the number of applications exceeds the certifiers' capacity, applications are randomly selected for certification.⁹ If the agents apply and get accepted, they undergo a noisy test procedure which with certain probability $p \in [0.5, 1]$ reveals their true type. I assume a transparent testing procedure with voluntary participation, hence test results are observable by the employer. However application decisions are private information, therefore an employer can't see a difference between an agent not applying and one applying but not being accepted for certification. For clarity, by enrolment I mean taking the test, which is not equivalent to application, since, due to access constraints, not all application can be accepted.¹⁰

The number of agents in the economy implies that a decision of an individual i has only an infinitesimal effect on the aggregate application rate. Therefore, I define q as an aggregate application rate of low type agents and m as the aggregate application rate of the high type. The total application rate is defined as $\Omega \equiv m * \lambda + q * (1 - \lambda)$.

2.2.2 Certifiers (Universities)

I assume a single, non strategic certification institution, which offers certification to all applicants subject to it's capacity constraints. The certification technology is binary i.e. the certifier issues only pass and fail certificates. Pass certificate can be interpreted as a university degree. The probabilities of false positive and false negative results are equal and exogenous as presented in Table 2. The symmetry condition is not required for the results, but it significantly simplifies the analysis. The exogeneity assumption may sound strict, however, universities are usually exogenously financed from public purse and the level

product documentation for certification.

⁹This assumption of lack of screening technology and the use of a lottery instead is strong, however, it fits the CEE before 1990s. Positive discrimination of working class was a part of social policy. Applicants from low income farming and working class background had their admission requirements lowered, compared to other social classes. At the same time the admission restrictions, in particular on non-technical faculties were a part of social engineering. Moreover, in every country, there are dimensions orthogonal to the innate abilities that affect the university admission but not (or to a lesser extend) the final university degree like the parents wealth and education.

¹⁰One could think of different testing procedures. The possible variations are mandatory (as opposed to voluntary) testing, which would be appropriate for analysis of compulsory education. Another possibility is opaque (as opposed to transparent) testing, i.e. strategic individual decisions on disclosure of test taking decision (possibility of hiding the negative test result). See [Rosar and Schulte \(2011\)](#) for a discussion in a world with costless certification technology. Note that their results do not always hold when agents bear certification costs.

of expenditure indirectly defines the certification quality. I assume that the government covers the exogenous cost c per each certificate issued (either pass or fail). The government subsidy is the only source of university financing. In [Section 2.4](#) I will explore the effects of endogenous quality, where, with fixed per capita financing, the quality decreases with a sudden growth in the use of educational resources.

Table 2. Certification Probabilities

	High Type	Low Type
Pr(Pass)	p	$1-p$
Pr(Fail)	$1-p$	p

The certifier's capacity is exogenously fixed to ϕ . If the total number of applications exceeds the number of places offered, the university randomly draws those to be accepted for certification. Acceptance rate $\delta(\Omega)$ is simply an inverse of an applications to slots ratio. If the number of applications increases, for a given level of ϕ , the probability of being accepted decreases. Therefore, the higher the capacity of certification system ϕ , the higher the acceptance rate for a given level of applications. If the number of applications is lower than the number of slots, all of those who apply get accepted. Formally the acceptance rate is defined as:

$$\delta(\Omega) = \begin{cases} 1 & \text{if } q = 0 \text{ and } m = 0 \\ \min(\frac{\phi}{\Omega}, 1) & \text{otherwise} \end{cases} \quad (2.2.1)$$

The capacity constraints are lifted in [Subsection 2.3.2](#). In [Section 2.4](#) the certification quality is endogenised.

2.2.3 Employers

I assume perfectly competitive employers compete in wages in a Bertrand Fashion. Employers learn the aggregate enrolment rate Ω . Their only information about the job applicants is the certificate received from the certification institute or lack of thereof. I specify strategies and beliefs in three information sets. μ_P , μ_F and μ_U represent a probability that an individual passing presenting a pass (P), fail (F) or no certificate (U) is of high quality. Similarly I define wage offers w_P , w_F and w_U paid to the workers depending on the signal received.

2.2.4 Timing

I consider the following timing of the game. First nature allocates the productivity levels among agents and the certification quality p and the public and private costs c and s are

set. Subsequently, agents make their application decisions. Once the admission decisions are made and the results of the test announced, employers perfectly observe the test results and make take-it-or-leave offers to the workers on a competitive labour market. The timeline is presented in [Figure 2.2](#). In the first stage the agents make a decision whether to apply for a certificate or not. Proportion m of high types and proportion q of low types decides to do so. Those who choose certification enter a stage in which a random selection for certification takes place. With the acceptance probability δ agents undertake a test which with probability p reveals their true type. With probability $1 - \delta$ agents are not accepted to the university and end up in a pool of uncertified agents together with those who did not apply for certification. In a scenario in which the capacity constraints are lifted ($\delta = 1$), all who apply undergo certification.

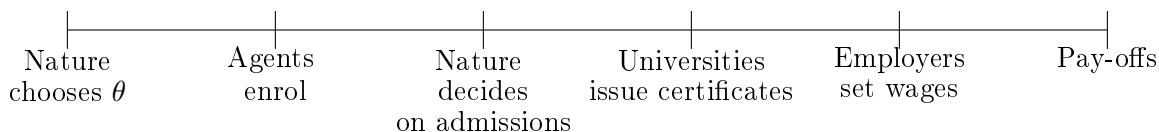


Figure 2.2: Timeline

2.2.5 Equilibrium concept

In the analysis, I employ a D1 refinement of Nash equilibrium to solve the game, such that the following sequence of actions hold. Certification financing rule is set. Students take certification decisions. Employers map their beliefs on student types into wage offers. Belief-strategy combination satisfies Bayes' rule whenever possible, and sequential rationality always. D1 refinement restricts the out-of-equilibrium beliefs. It requires that if the set of receiver's responses for which one type of senders gains from deviating is larger than the set for which a second type gains, then receiver's beliefs must assign zero probability to the second type ([Cho and Kreps \(1987\)](#)). I solve the game by backward induction and start with the strategies of the employers. For simplicity, I focus only on symmetric equilibria, in which all agents of the same type act in the same way.

2.3 Effects of Exogenous Certification Access Constraints

In this section, I will show the effect of exogenous certification access constraints on individual and social welfare in a setting with one type of job and in an extension with supermodularities in job-worker match.

2.3.1 Exogenous Access Constraints

Solving the game by backward induction I start with the employers and their wage offers. The competition between the employers sets the wage offers equal to the expected productivity. Given the assumption on the labour productivity between the types, the wages of agents who are accepted for certification and pass (subscript P) or fail (F) and those who decide not to take the test (U) are equal to the beliefs for each of the signals and are given by the following

$$\mu_P(p, q, m) = \frac{pm\lambda}{(1-p)q(1-\lambda) + pm\lambda} = w_P(p, q, m) \quad (2.3.1)$$

$$\mu_F(p, q, m) = \frac{(1-p)m\lambda}{pq(1-\lambda) + (1-p)m\lambda} = w_F(p, q, m) \quad (2.3.2)$$

$$\mu_U(p, q, m) = \frac{\lambda(1-m\delta(\Omega))}{(1-\lambda)(1-q\delta(\Omega)) + \lambda(1-m\delta(\Omega))} = w_U(p, q, m) \quad (2.3.3)$$

w_P and w_F are, respectively, equal to the proportion of high types in the pool of students who passed and failed the test. The acceptance rate $\delta(\Omega)$ is defined in [Equation 2.2.1](#). w_U is the proportion of high types in the pool of agents that did not take any test. The employer can not distinguish between agents who were not accepted and those who did not apply.

To perform the analysis it is useful to define the expected payoffs conditional on being accepted to a university for both types, given by:

$$Ew_H(q, m, p) = p * w_P(p, q, m) + (1 - p) * w_F(p, q, m) - s$$

$$Ew_L(q, m, p) = (1 - p) * w_P(p, q, m) + p * w_F(p, q, m) - s$$

Agents making their certification decisions weigh the probability of getting the pass or fail certificate (and the respective wages) at a certain cost s and compare it to their outside option, refraining from education. When the agents are indifferent between the two options they choose $d_i \in (0, 1)$. We can write students expected utilities, for both types, in the following way:

$$U_H(q, m, p, d_i) = d_i * \delta(\Omega) * Ew_H(q, m, p) + (1 - d_i * \delta(\Omega))w_U(p, q, m) \quad (2.3.4)$$

$$U_L(q, m, p, d_i) = d_i * \delta(\Omega) * Ew_L(q, m, p) + (1 - d_i * \delta(\Omega)) * w_U(p, q, m) \quad (2.3.5)$$

In words, whenever an agent makes a decision to enter the university, with probability $\delta(\Omega)$ she is admitted and gets the expected wage for a university graduate for a given type and

pays the cost s . With probability $1 - \delta(\Omega)$ she is not accepted by the university and remains uncertified. If an agent decides not to apply for the certification, she gets the wage of an uncertified worker with certainty.

I impose the following assumptions:

$$\mathbf{A1} : s < 1 - \lambda \qquad \mathbf{A2} : \phi < \lambda \qquad \mathbf{A3} : 1/2 < p \leq 1$$

A1 assures that a high type prefers perfect certification to pooling on no certification. A2 assures that access constraints are strict enough, such that not all high types can be certified under the capacity constraints. The fact that $p \neq 1/2$ assures that the certification is informative. $1/2 < p$ is assumed just for consistence with the schooling terminology, the same logic would apply if $0 \leq p < 1/2$ with the opposite interpretation of the test results.

Proposition 2.3.1. *Whenever assumptions A1, A2 and A3 are fulfilled and $p < 1$ the equilibrium is semi-pooling such that all high types ($m = 1$) and some, but not all, low types ($q \in (0, 1)$) apply. For $p = 1$, $m = 1$ and $q = 0$.*

For proof see the [Appendix](#).

Intuitively, in the described regime, the demand for university degrees exceeds the supply. The agents that apply but are not admitted to the test are mixed with those who voluntarily refrain from certification. The certification capacity constraints lead to a loss of information that could be acquired from voluntary decisions with no constraints. In the unique D1-robust equilibrium of this game all high types always apply. The application rate of the low types (denoted as q_{pre}^*), together with application rates for the case with no capacity constraints (q_{post}^*) are presented graphically in [Figure 2.3](#).

The information flow from workers to employers is negatively affected by the constraints. However, as it will be shown in the next section the relaxation of the constraints, does not necessary lead to an increase in utility of the high type agents.

2.3.2 Relaxation of Constraints

Now I will describe the economy after the relaxation of certification constraints; $\phi^{post} = 1$. Suppose that the authorities have decided to provide certification to everyone who requests it, keeping the same level of financing per student. The relaxation of the student's quotas sets $\delta(\Omega) = 1$ disregard of the application rate. All the remaining assumptions remain the same.

From the employers' perspective the problem stays the same. At the stage they enter the game all the parameters are known, therefore the pass and fail wages are given by

Equation 2.3.1 and **Equation 2.3.2**. The wage of uncertified workers is given by **Equation 2.3.3** with $\delta(\Omega) = 1$.

From the students' perspective the regime change does not change the economic problem faced by them, however, it significantly affects the possible set of outcomes. First of all, since all students can be accepted, if the results from the previous section hold, i.e. $m = 1$ the uncertified wage would decrease, lowering the low types outside option. Lowered outside option increases the pressure on the low types to apply. Keeping in mind assumption A1 I establish the following results.

Proposition 2.3.2. *When there are no constraints and $p < 1$, all the good students enrol ($m = 1$) and $q \in [0, 1]$. Equilibrium low type application rate is strictly higher than under certification constraints.*

For proof see the **Appendix**

Interestingly, for high enough s and low enough p an equilibrium with the high type pay-offs lower than λ , i.e. lower than in the pooling equilibrium, can be supported in separating equilibrium.

Figure 2.3 shows the equilibrium application rates of the low types in the two regimes for different values of s . As it can be seen, the application rate before transition is always lower than after and both rates are weakly decreasing in certification precision. The two coincide and converge to zero only when the certificate is perfectly precise. In such a case none of the low types applies. It can also be seen that for low values of p and s full participation of the low types can be achieved in the regime without administrative constraints. This happens only for $s < \lambda$ and sufficiently low p .

2.3.3 Welfare comparison

It is a standard characteristic of a certification model,¹¹ that from the welfare perspective the high certification rate equilibrium is strictly dominated by the low certification rate equilibrium. Given the assumptions, the total output and the total workers' remunerations in both cases are equal to λ , but the total level of wasteful investment in certification is lower when the certification rate is low. The total certification cost $((c + s) * \delta * \Omega)$ is, by construction, a pure social waste.¹² It is more interesting to compare the utility of both

¹¹At least in a simple standard with no modularities in a production / matching function

¹²Another potential social objective is fairness. It could be interpreted as decreasing the role of luck (e.g. lifting the constraints) or assuring the wages reflect the true productivities. As it is shown in this section, pursuing this objective does not always result in maximising welfare of the high types.

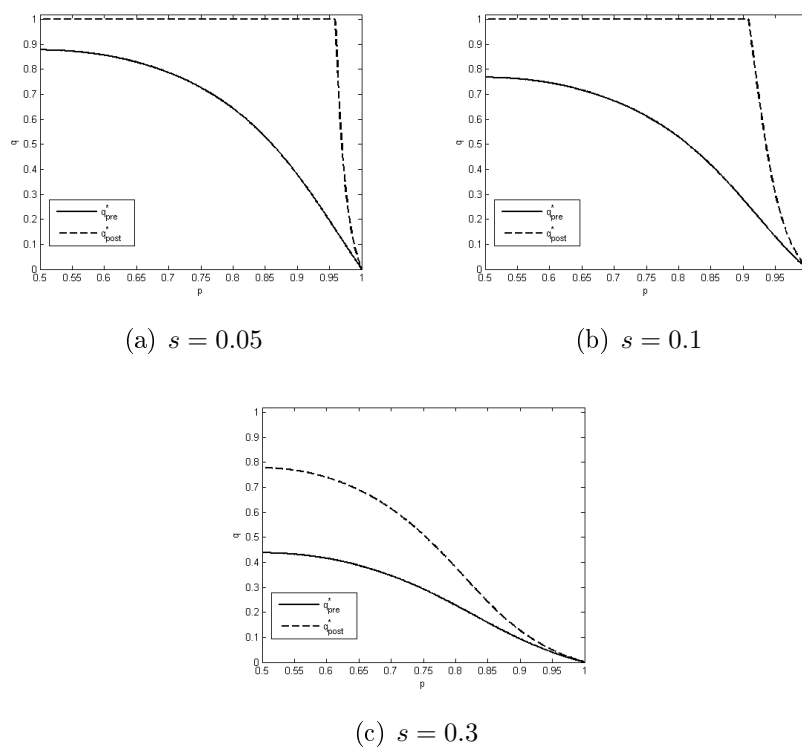


Figure 2.3: Low type equilibrium enrolment rate as a function of p for different levels of s and $\lambda = 0.25$ and $\phi = 0.15$ with (q_{pre}^*) and without (q_{post}^*) capacity constraints

types between the two regimes. The relaxation of exogenous capacity constraints clearly has an ambiguous effect. Although, it allows everyone to undergo the certification, it increases the proportion of the low types in the pool of certified agents lowering the wages.

Proposition 2.3.3. *Low types, for $1/2 < p < 1$, are always worse off when the capacity constraints are lifted. High types are also worse off if p is not sufficiently high. Since certification is a pure social waste the social welfare is maximised for $\phi = 0$.*

For the proof see the [Appendix](#).

The relaxation of the constraints implies higher number of students and hence higher amount of private resources spent on education. As a result, the total pool divided between the agents decreases. [Figure 2.4](#) presents the range of parameters for which high type benefits from the regime change (white area) in a numerical example. This happens only if the level of c and the resulting low levels of p are sufficiently high. At the same time there is a wide range of parameters where high types only lose on relaxation of the constraints (black area). In this range of parameters the crowding out effect of low types entering education (due

to the value of their outside option) has a dominant role. In other words the expansion of tertiary education can be beneficial only if sufficiently high quality level is assured.

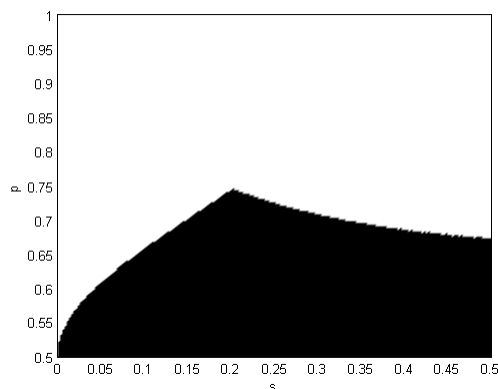


Figure 2.4: Effect of regime change on high type's welfare for $\lambda = 0.25$ and $\phi = 0.15$

2.3.4 Licensing

Licensure, although similar to professional certification, is characterized by one key difference. In contrast to a certificate, a license is a legal requirement for practising a profession. Therefore a failure of being granted a license is much more than just a bad signal about the quality. Those who do not receive a license are not allowed to practice. In the modelling framework presented above licensing implies that $w_F = w_U = 0$. This equality holds regardless of the access constraints. As a result, the relaxation of access constraints does not change the expected wage of those who fail the certification, and, therefore, the low type incentives to apply remain unchanged. The low type's application rates before and after the relaxation of the constraints remain the same, but, obviously, the number of licensed agents will change. Therefore improved access to certification is purely beneficial for both the high and the low types.

2.3.5 Licensing and multiple job types

Since signaling is considered a pure waste, any improvements in access to certification lead to a (weak) decrease in social welfare. The situation changes when we consider more than one type of jobs and supermodularities in job-worker matching. One of realistic examples is a situation that stands as a rationale behind licensing is a case in which a person of low abilities performing a job that is subject to licensing may cause significant damage to buyers

of the services (e.g. doctors, lawyers or brokers). Suppose now that there exist two types of jobs $j \in \{l, h\}$. A job-worker match will be denoted by $\theta_{i,j}$ where $i \in \{H, L\}$ denotes the worker's type. For a standard job (denoted by superscript l no certificates are required and the productivity of high and low types stays the same as in the previous parts (i.e. $\theta_{Hl} = 1$ and $\theta_{Ll} = 0$, the first subscript represents the worker's type and the second one the job's type). In order to perform the difficult job (denoted by h) a certificate is required and on the job productivity is given by θ_{Hh} and θ_{Lh} for the high and low type respectively. In order to follow more closely the doctors example I assume that the worker-job matching function exhibits the following property $\theta_{Hh} > \theta_{Hl}$ and $\theta_{Ll} > \theta_{Lh}$.¹³

Conjecture 2.3.1. *When the certification quality is low it is optimal to set the constraints to 0 ($\phi = 0$). When the quality is sufficiently high it is optimal to relax the constraints ($\phi = 1$). For lower values of certification precision there exists an intermediate level of access constraints $\phi \in (0, 1)$ that maximises social welfare.*

Although analytical proof can not be shown due to algebraical complexities, I will provide the intuition supported with a graphical example.

Social welfare is given by the following:

$$SW(\phi) = \lambda(\theta_{Hl} + p(\theta_{Hh} - \theta_{Hl})\delta(\phi, q^*(\phi))) + (1 - \lambda)(\theta_{Ll} + (1 - p)(\theta_{Lh} - \theta_{Ll})q^*(\phi)\delta(\phi, q^*(\phi))) - (\lambda + (1 - \lambda) * q^*(\phi)) * \delta(\phi, q^*(\phi)) * (c + s)$$

In words, it is the sum of all productivities for both types weighted by the probability of a given job-worker match net of the sum of private and public cost of certification. Whenever the access constraint is binding (i.e. $\phi < \lambda + q^* * (1 - \lambda)$, where q^* , the low type equilibrium enrolment rate is a function of ϕ and all other parameters) an increase in ϕ leads to an increase in q^* due to the mechanism presented in the previous section.

The impact of change in ϕ is a sum of three effects. The first one is the change in access of the high types to certification net of increase in certification costs ($c + s$). The effect of the change is positive, however, one has to keep in mind that the effect of the increase in access due to increases in the number of slots $\delta_\phi(\phi, q^*(\phi))$ is mitigated by the crowding out effect due to an increase in low types application rate $q^{*'}(\phi)\delta_q(\phi, q(\phi))$. The second effect is the decrease in social welfare due to increased number of low types accepted for difficult jobs due to increased application rate and it is clearly negative. Finally, the third effect, of an

¹³Standard supermodularity condition is given by $\theta_{Hh} + \theta_{Ll} > \theta_{Hl} + \theta_{Lh}$, however it is reasonable to assume that the productivity of the high type on an difficult job is higher than on a standard job. Moreover, to justify licensing I assume that the low type on advanced job can only do harm.

ambiguous sign, is the change to the low type acceptance rate due to joint effect of higher application rate and increased number of slots. Because of those complexities the analytical solution for q^* is not feasible.

The overall effect of change in ϕ on social welfare depends on all parameters and is presented graphically in [Figure 2.5](#) (thick line, and Y-axis on the left hand side). On the same figure a dashed line represents the equilibrium low type enrolment rate (with the Y-axis on the right hand side of the plot). For the plot, the following parameter values have been used: $\theta_{HH} = 2$, $\theta_{HL} = 1$, $\theta_{LL} = 0$, $\theta_{LH} = -1$, $s = 0.1$, $c = 0.1$, $\lambda = 0.25$.

It's immediate that when the certification is infinitesimally informative, given the assumptions, the social welfare is maximised when $\phi = 0$. This can be seen on the left panel of [Figure 2.5](#), where the social welfare is maximised when no-one is allowed to take certification. Increase in ϕ leads to an increase in q^* and resulting decrease in social welfare. When certification is perfect (the right panel), it's immediate that only high types apply and any level of access constraints $\phi \geq \lambda$ maximises social welfare. As it can be seen on the right panel, for higher levels of ϕ access constraints are not binding since the low types decide not to enter university. Situation is more interesting for intermediate levels of p (central panel). Although, the low type application rate is increasing over the whole range of ϕ , the social welfare is increasing for low values of ϕ and decreasing for high values of ϕ . When the constraints are very tight the effect of an increase in access for the high types outweighs the crowding out effect. The plot allows to make a statement that, there exists an open set of parameters in which there exists a socially optimal level of access constraints, $\phi \in (0, 1)$.

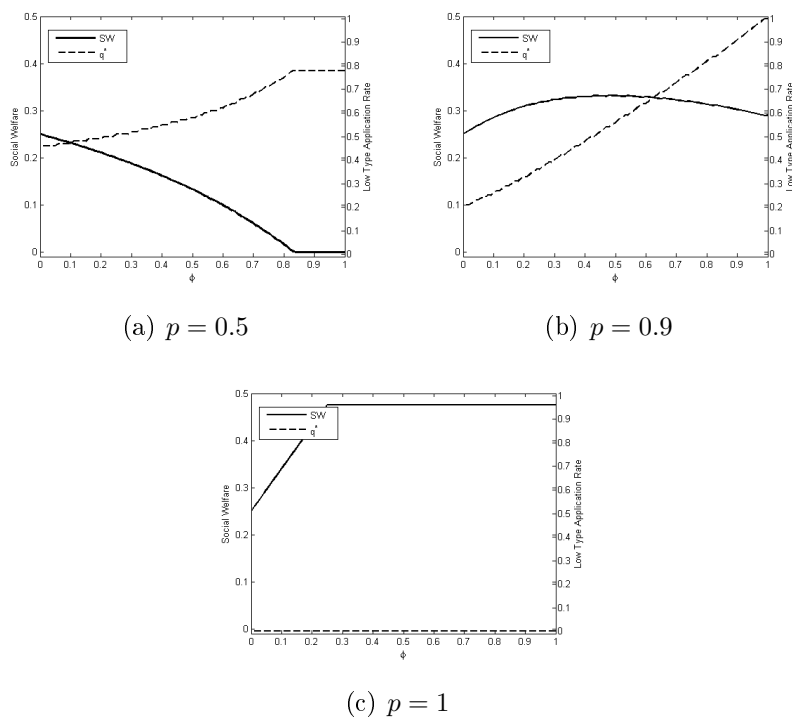


Figure 2.5: Social welfare and low type's application rate for $\theta_{HH} = 2$, $\theta_{HL} = 0$, $\theta_{LL} = 0$, $\theta_{LH} = -1$, $s = 0.1$, $\lambda = 0.25$ and different values of p as a function of access constraints ϕ

2.4 Effect of Exogenous and Endogenous Certification Access Constraints

In this section I analyse the case where, in addition to administrative access constraints, there exist endogenous constraints in form of supply inelasticities.¹⁴

As discussed in the introduction, it's natural to think of quasi-bottlenecks in response to a huge change in demand, at least in the short run.

Suppose, there exists an explicit link between the public expenditure per student s and the quality of universities. This link, that incorporates the supply inelasticities, is defined through the unit cost function $\chi(\Omega\delta(q, m), p)$ per certificate issued. The cost function is assumed to meet the following:

¹⁴Another, unfortunately, quite realistic, but less subtle approach of modelling endogenous constraints would be to assume that the government, while keeping the same aggregate level of financing increases the number of students admitted. In such a case, the increase in the number of students would result in decrease in per student expenditure.

$$\begin{aligned} \mathbf{A4} : \chi(\Omega\delta(\Omega), p = 0.5) = 0 & \quad \mathbf{A5} : \chi_1(\Omega\delta(\Omega), p) > 0 & \quad \mathbf{A6} : \chi_2(\Omega\delta(\Omega), p) > 0 \\ \mathbf{A7} : \chi(\Omega\delta(\Omega), p) \text{ is bounded} & & \end{aligned}$$

The digit in the subscript indicates the argument with respect to which a derivative is taken. A4 assures that the cost of providing a completely uninformative test is zero. A5 is the key assumption in this chapter, which implies negative externalities of enrolling students imposed on certification precision. A6 is a natural assumption that states, simply, that the cost of certification precision is increasing with precision. A7 assures that there exists a fee c that can buy perfect certification disregard of the enrolment rate. A case equivalent to $\chi_1(\Omega\delta(\Omega), p) = 0$ is analysed in [Subsection 2.3.2¹⁵](#). For graphical illustration, the following cost function is used throughout the text

$$\chi(p, \Omega) = \alpha \left(\frac{\Omega\delta(\Omega)}{\phi} \right)^\gamma (p - 0.5)^2 \quad (2.4.1)$$

where ϕ is the capacity constraint prior to the relaxation and $\gamma \geq 0$ is interpreted as a measure of short term supply inelasticities. When $\gamma = 0$ there are only exogenous access constraints, i.e. the situation is analogous to the previous section. The higher γ the lower the quality level for a given level of public education expenditure c and given raise in the enrolment rate. To highlight the short term inelasticities, I assume that the certification cost depends on the enrolment rate relative to the initial capacity (before the relaxation of the constraint). Before the transition, the total number of students ($\Omega\delta(\Omega)$) never exceeds ϕ . When the administrative constraints are lifted ($\delta(\Omega) = 1$), the cost depends on the rate of growth of number of delivered certificates relatively to initial capacity constraints. This assumption is aimed at capturing the costs of adjusting from one equilibrium to another, and can be interpreted as a short term supply curve.

Supply inelasticities play a significant role in determining the outcomes of this game. Contrary to the previous case, certification quality depends not only directly on c but also on the total number of registered agents Ω . As a result the enrolment rates of the two types affect the individual decision in two ways. The first effect is as in the baseline case, through the proportion of types in different pools of students. The second channel is via negative externalities from enrolment. Since the enrolment decisions are made only after the financing level c is announced, the certification quality is not known ex ante by the agents. Due to the supply inelasticities, the higher the number of registered students, the lower the quality. As

¹⁵In this case an exogenous cost c uniquely defines p .

an example, the effect of change in low types' application rate on low types expected utility after the relaxation of access constraints is given by (Equation 2.4.2). Analogical analysis can be performed for the changes in m and for the effect on U_H .

$$\frac{\partial U_L}{\partial q} = \underbrace{(w_F - w_P)}_{<0} \underbrace{\frac{\partial p}{\partial q}}_{<0} + p \left(\underbrace{p \frac{\partial w_F}{\partial q}}_{<0} + \underbrace{\frac{\partial w_F}{\partial p}}_{<0} \underbrace{\frac{dp}{dq}}_{<0} \right) + (1-p) \left(\underbrace{p \frac{\partial w_P}{\partial q}}_{<0} + \underbrace{\frac{\partial w_P}{\partial p}}_{>0} \underbrace{\frac{dp}{dq}}_{<0} \right) \quad (2.4.2)$$

The change of the low type enrolment rate affects those who enrol in three ways. The first one is the effect of the certification quality change due to change in enrolment rates on the expected wage of the low type. For a given fee, an increase in enrolment decreases the certification quality, which leads to an increase in the low types' expected wage. The second effect is the impact of enrolment rate on the fail wage, which is ambiguous. On the one hand an increase in q decreases it. At the same time a decrease in p caused by the supply inelasticities leads to an increase in w_F . The third effect, of change in q on the pass wage, is clearly negative, both through the higher number of low types attempting the test and the lowered quality of the test. Similar reasoning has to be undertaken for the high type and for the high types participation.

The analysis is performed in a similar fashion to the previous section. First I describe an equilibrium in a scenario when both endogenous and exogenous constraints coexist. In the second scenario, the administrative capacity constraints are lifted and only the supply inelasticities exist. This scenario will be referred to as the transition regime.

2.4.1 Both Constraints

In this case, as I will show, the introduction of endogenous capacity constraints in a form described above does not change the outcomes from the scenario without endogenous constraints. The results are still valid (given Assumptions 1 and 2).

Proposition 2.4.1. *Endogenous capacity constraints do not affect the equilibria when exogenous constraints are binding.*

For proof see the [Appendix](#)

The certification system always works, in equilibrium, at its full capacity ϕ , therefore agents can perfectly infer the quality. As a result the change of the cost function has no effect on the outcomes. However, once the exogenous constraints are lifted, the results will change, as shown in the next section.

2.4.2 Relaxation of the Exogenous Constraints

In this section I analyse the effect of endogenous capacity constraints once the exogenous constraints are lifted. In such a case they truly affect the economy. Contrary to the previous case, here, the low types can coordinate thanks to which new equilibria emerge. It's clear that the endogenous capacity constraints negatively affect the welfare of the high types. For any level of tuition subsidy c and $\Omega > \phi$ the resulting certification quality is lower than in the case without the supply inelasticity. The possibility of the low types to affect the education quality leads to an interesting result, a multiplicity of equilibria.

Conjecture 2.4.1. *When the certification provision cost function satisfies the assumptions A5 to A7, multiple equilibria with different education quality and enrolment rate exist. The final outcome depends on the prior beliefs on the education quality.*

A clear analytical proof of the proposition is not feasible, therefore a numerical analysis will be presented instead. The result follows directly from [Equation 2.4.2](#) presenting a differentiation of U_L with respect to q . The ambiguous effect may lead to multiple equilibria depending on the initial beliefs on the enrolment rates. It does not, however change the result achieved in the previous regimes that $m = 1$. For some parameters high types receive in equilibrium less than λ .

Numerical Example

I will present a numerical analysis for the cost function defined in ([Equation 2.4.1](#)). I will present, in details, all equilibria for a special case with ($\lambda = 0.25$, $\phi = 0.15$, $\alpha = 0.3$ and $\gamma = 1$).¹⁶

Analysing the change of U_L , we can identify 6 cases in which, in equilibrium, the high types always enrol and the low types optimal decision depends on the parameter range. The graphical representation of the equilibria types can be found in [Figure 2.6](#), [Figure 2.7](#) and [Figure 2.8](#). [Figure 2.6](#) and [Figure 2.7](#) depict the same result from different perspectives, as a function of low type enrolment rate or certification precision. The two, for a fixed level of c are clearly interconnected, however, the story behind the two beliefs is different. The thin line in [Figure 2.6](#) depicts the expected utility of the low type as a function of low type enrolment rate q conditional on $m = 1$ (solid line) and the certification quality for a given low types' enrolment (dashed line). The values of c and s differ between the analysed cases. Whenever,

¹⁶This set of parameters implies that perfect certification under exogenous capacity constraints costs 7.5% of future product (throughout the life, after graduation) of a high type. Under full enrolment, perfect certification is almost prohibitively high and would require the input of $c = 0.5$

for a given value of q , utility is positive, it is optimal for the low type to enrol. However, than it's optimal for all low types, therefore full low type enrolment rate is an equilibrium, whenever low types utility remains positive for $q = 1$. Whenever, for a given level of q low type's utility is negative, it's optimal not to enrol. $q = 0$ is sustained in equilibrium only, when c is sufficiently high, such that when only high types, perfect certification is assured. Another equilibria exist for such values of q that the low types are indifferent between the enrolment and staying without a certificate.

Figure 2.7 presents ex post realisation of certification quality (by construction, equivalent to beliefs on the role of luck in determining outcomes) given the ex ante beliefs (horizontal axis) and best responses of all players. If the low type agents believe that the luck plays an important role they expect other agents to enrol which drives the certification quality down. At the same time, if they believe that luck plays only a limited role, the certification precision remains high. This figure is derived directly from Figure 2.6. For the anticipated certification precision levels that give low types positive utility one can expect full enrolment and hence the resulting certification precision. For anticipated precision that leads to negative utility one can expect zero enrolment rate of the low types. The equilibria are given by intersections of the line of ex-post certification precision (thick line) and the 45° (dashed line) line. Figure 2.8 presents the area of existence of different equilibrium types on c - s space for different values of α .

A short description of the 6 cases is presented below.

1. For low values c and s (the bottom left dark gray and light gray area next to it in Figure 2.8¹⁷) the utility of the low type is decreasing in q but stays positive (Figure 2.6(a)). Hence, there is a single equilibrium with full enrolment. Similarly, when looking at Figure 2.7(a) the agents know that, disregard of the anticipated quality, all low types will enrol and drive the quality to the lowest level $p(M = 1, f)$. In the light gray area in Figure 2.8 the high types' equilibrium utility is lower than λ .
2. For moderate values of c and low values of s (dark gray and white areas in the top left corner of Figure 2.8) there exist two stable equilibria with $q = 0$ and $q = 1$ (Figure 2.6(b) and Figure 2.7(a)). In addition, there is one unstable equilibrium such that $q \in (0, 1)$. In this case, perfect certification is attainable for low levels of low type enrolment. Hence, the expected wage is zero (the flat part of the U_L curve). If agents anticipate that the enrolment rate is low, they don't enrol. For higher enrolment levels the effect of the decrease in certification quality is strong, leading to positive utility for

¹⁷The described areas are easiest to identify on the third plot, for $\alpha = 0.5$

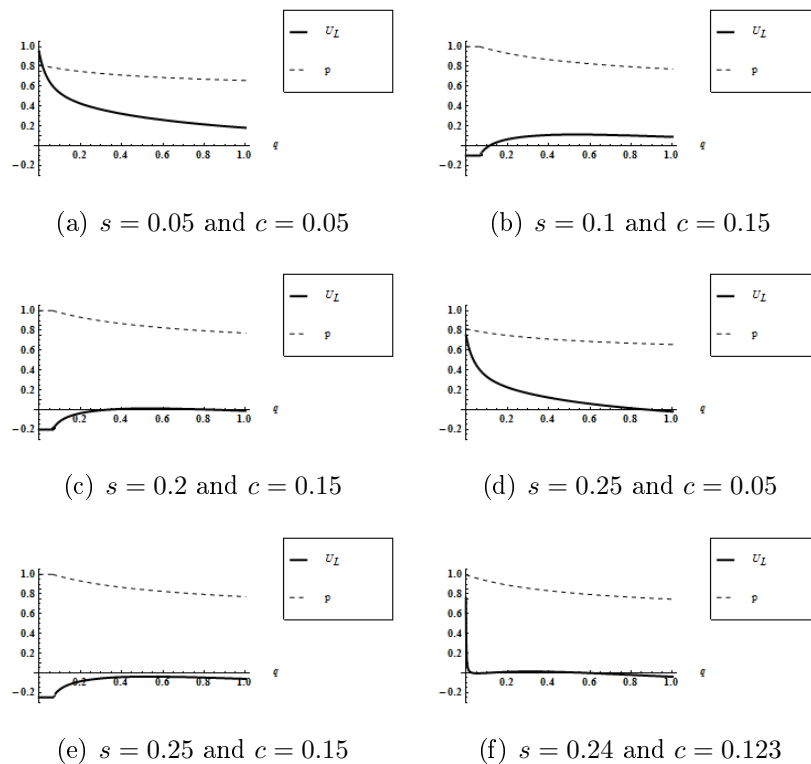


Figure 2.6: Low type utility as a function of q for $\lambda = 0.25$ and $\phi = 0.15$ and $\alpha = 0.3$

higher values of beliefs. Therefore for high enough q the optimal decision is to enrol. If the low types coordinate on $q = 1$ and the values of c and s are in the white area on the plot the high types receive in equilibrium utility lower than λ .

3. For moderate values of c and s (black and small very light grey area east of it on [Figure 2.8](#)) 3 equilibria exist with $q = 0$ and with two values of $q \in (0, 1)$ ([Figure 2.6\(c\)](#) and [Figure 2.7\(c\)](#)). The utility function is initially increasing in q and becomes positive for moderate values. For higher values of q , the negative impact of increased number of low types with certificates on the wage becomes stronger than the benefits from the decreased quality induced by the increase in enrolment rate. The expected utility starts to decrease becoming negative for high values of q . The two equilibria with positive enrolment are defined by the intersections of U_L curve and the horizontal axis. For the highest low type equilibrium enrolment rate the high types receive less than λ for c and s in the light gray area.
4. For low values of c and high values of s (light grey and gray areas in the bottom right

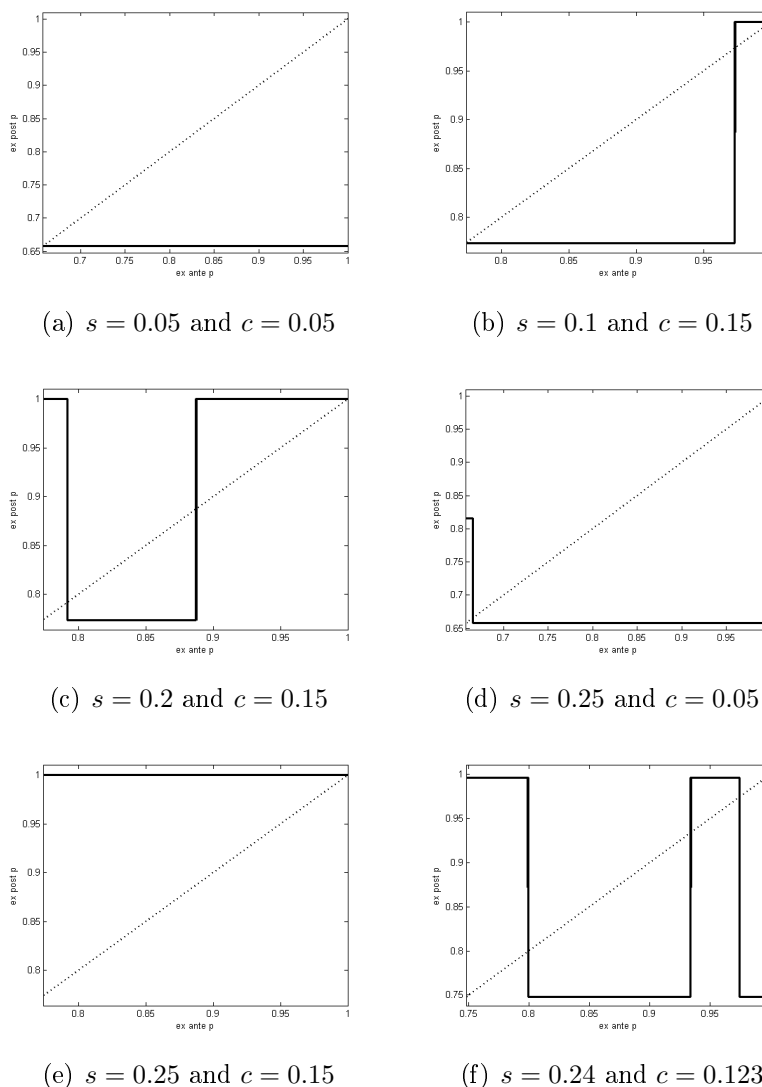


Figure 2.7: Ex post and ex ante certification precision p for q for $\lambda = 0.25$ and $\phi = 0.15$ and $\alpha = 0.3$ and different c and s

corner in Figure 2.8) there exist one equilibrium with $q \in (0, 1)$ (Figure 2.6(d) and Figure 2.7(d)). The case is analogous to the one in Figure 2.6(a). The utility is strictly decreasing in q . However the higher value of s causes that the utility is positive for low values of q and negative for high values. Therefore the only equilibrium is the intersection of the U_L curve and the horizontal axis. In the light gray area the equilibrium payoff of the high type is smaller than λ .

5. For high values of c and s (gray area in the top right corner) there exist one equilibrium

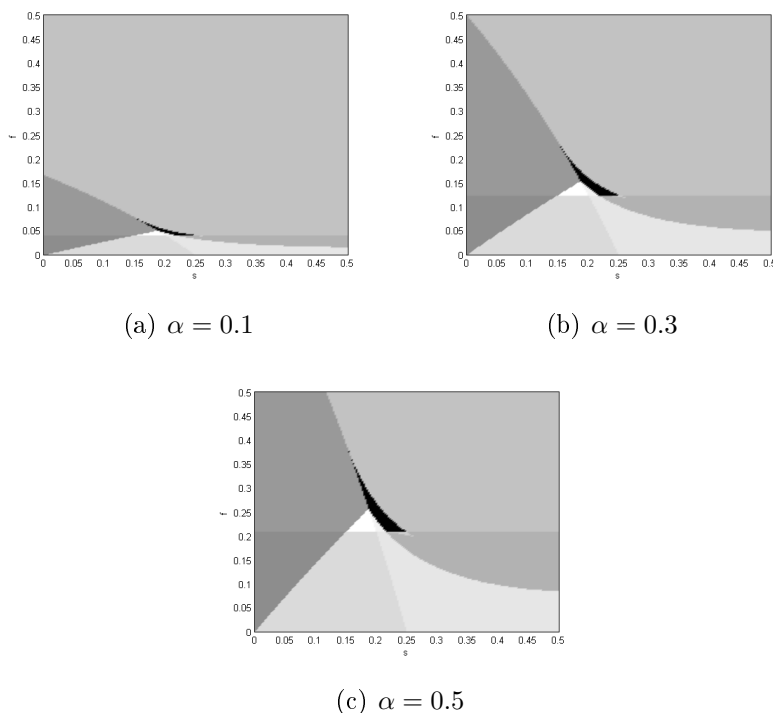


Figure 2.8: Equilibria range on $c - s$ space for $\lambda = 0.25$, $\phi = 0.15$ and different levels of α

with $q = 0$ ([Figure 2.6\(e\)](#) and [Figure 2.7\(e\)](#)). Low types utility from enrolment is always negative disregard of the enrolment rate. The inverted U-shape indicates that the effect of certification quality is strong, but due to high costs s insufficient to make the utility positive.

6. For moderate values of c and s (very small gray area at the bottom tip of the black area in [Figure 2.8](#)) three equilibria with $q \in (0, 1)$ exist ([Figure 2.6\(f\)](#) and [Figure 2.7\(f\)](#)). This case clearly shows the interplay of the two effects. Even or very low values of q the certification is imperfect. For q close to zero even the fail wage is high, therefore the low types may expect high utility. However, as the enrolment increases the fail wage goes significantly down bringing the U_L below zero. At this point the certification quality decrease effect comes into play moving the curve in the positive range again. Finally for high values of q the quantity effect dominates. The three equilibria are given by the intersections of the U_L curve and the horizontal axis.

The analysis so far shows that supply inelasticities may result in multiplicity of equilibria. This model shows that the change of the enrolment rate may be an effect of differences in

beliefs in the role of luck in determining the educational outcomes. If such beliefs are held by the society, the low types coordinate and drive the quality of education down, fulfilling their beliefs.

The welfare analysis will be skipped, the effect of $\gamma > 0$ is clear. The higher the cost of certification provision the lower the certification precision which results in decreased utility of the high types and increase in low types wasteful participation.

2.5 Discussion

The model presented suggests that there may exist a relationship between beliefs in the role of luck in determining educational outcomes and data on tertiary education enrolment rates in countries that experienced a relaxation of constraints such as those in Central and Eastern Europe in 1990s. Though the concept of beliefs is a bit vague, there have been attempts to quantify it. One such attempt is the World Value Survey, which asks the respondents to assess the following statement. "In the long run, hard work usually brings a better life. Or, hard work does not generally bring success; it's more a matter of luck and connections." The answers are coded from 1 to 10, where 10 implies the highest belief in the role of luck and connections. Since, in the presented setting, universities exist only for certification purposes, these beliefs coincide with the beliefs on the education quality. The higher the role of luck, the higher the probability of the low type of being certified as a high type. The survey question is more general than the type of beliefs analysed in the model. The beliefs on education quality do not necessary coincide with the beliefs on the role of luck in other fields, therefore direct inference from the data is not a perfect test of the model. Nevertheless, the correlation shown on the Figure below may be read as a support for the hypothesis stated in this paper.

Figure 2.9 shows positive correlation between belief in the role of luck and total tertiary education enrolment rates in 19 CEE countries¹⁸. As it can be seen, the higher the role of luck, the higher the enrolment rates.¹⁹

¹⁸All CEE countries for which the data was available.

¹⁹The enrolment rate is defined here as the number of students registered at university relatively to the population in age group 20-24.

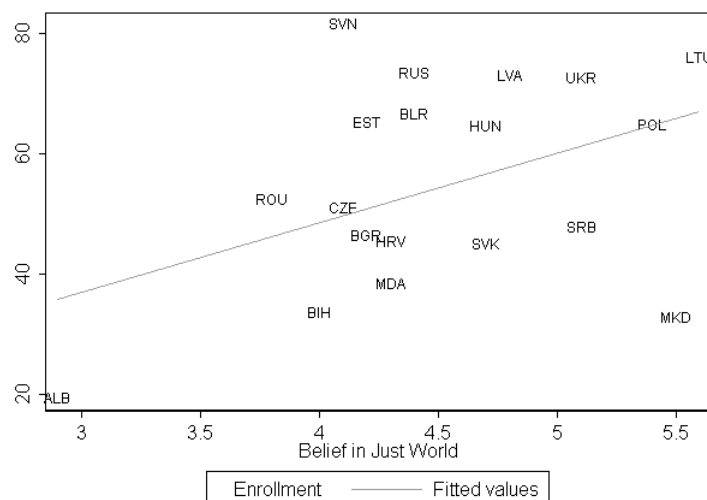


Figure 2.9: Correlation between average tertiary education gross enrolment rates in years 2002-2008 and a belief that luck and connections determine outcomes

2.6 Conclusions

The key message of this study is that relaxing access constraints to higher education can be harmful even for those who, without the growth of the university sector, would stay outside the education if a sufficiently high quality of education was not provided. Policy goals aiming solely at increasing participation in higher education such as those stated in the EU's Europe 2020 strategy²⁰ may have adverse welfare consequences.

When access constraints are binding, not all high ability individuals receive a chance to get a university degree. As a result, some of them remain in the pool of uneducated workers. Improved access decreases the proportion of agents with high abilities that don't enter the university and hence, negatively affects the expected wage of those that choose not to obtain the university education. Consequently, when university degrees are not perfect signals of student abilities, the less able face stronger incentives to game the system in order to try to get a degree. Higher low type enrolment rate reduces the self-selection component of the informativeness of university degrees.

The negative effect of improving access, when the informativeness of degrees is low, is magnified when there exist supply inelasticities and when the certification quality decreases with the number of certificates issued, when per student expenditure are fixed. This can

²⁰ The document states two key policy objectives in the field of education. These are reducing the school dropout rates to below 10 %, with at least 40 % of 30–34-year-olds completing tertiary education.

lead to a vicious circle in which an increase in enrolment rates leads to a decrease in quality of certification. As the low ability agents' chances to get a degree increase, the demand for certification also increases driving the quality further down. The inelasticities lead to a multiplicity of equilibria and the equilibrium selected depends on societal beliefs about education quality.

One of the possible applications of the model is an analysis of the changes in the higher education enrolment rates in Central and Eastern Europe after the collapse of communism. In that region, universities used to operate at low capacity. The liberalisation of the education system accompanying the economic transition, together with the change in the structure of labour demand led to a sudden jump in the demand for higher education. Enrolment rates were increasing despite growing concerns about university quality and have reached or exceeded those in Western Europe. This paper presents a self-perpetuating mechanism, in which the enrolment rates and education quality interact that can explain the phenomenon. The model shows that, when the university system operates with short term capacity constraints, an increase in demand with fixed financing rules, leads to a decrease in certification quality, which in turn increases the less able agents' demand for certification.

The results lead to some policy recommendations. First of all, when improving access to education the authorities should focus on assuring high quality of education. General availability or freedom to choose education are definitely desirable but, as the model shows, could possibly be detrimental for society if a sufficiently high quality level is not assured. Another finding is that private education costs help agents self-select and partially improve information revelation. The model, however, does not suggest that higher education should not be provided free of charge. An alternative, and widely supported model of education financing, the mean-tested loan system, would assure free education at the point of use (see e.g. [Barr \(2004\)](#) and [Jacobs and Van Der Ploeg \(2006\)](#)) and would not be pure tax-financing. However, given the economic instability in the region, aversion towards financial institutions caused by the economic turmoil during the transition and general risk aversion there are serious risks regarding the applicability of the idea.

Another possible solution for mitigating the possible adverse welfare effects of the huge increase in demand is in the regulation of admissions. Instead of a complete relaxation of admission caps, a smooth transition through a gradual increase in the number of places would be recommended. Similar measures are currently under discussion in CEE countries. For example the Polish parliament has recently passed a bill imposing a requirement that an approval from Ministry of Education is required for an increase in enrolment at a university exceeding 2%.

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Appendix - Proofs

Proof of Proposition 2.3.1

Proof. The agents optimisation problem can be written as the following

$$\max_{d_i \in [0,1]} U_i(p, q, m, d_i) \quad (2.6.1)$$

Agents of a given type choose to apply whenever the expected wage of a university student net of costs exceeds the wage of the uncertified agents.

It is immediate that, whenever certification is perfect ($p = 1$), all high types who apply and get accepted receive $1 - s$. Assumption 1 ensures that the decision to apply in such a case, dominates the possible coordination on $m = 0$. Low types, if apply and get accepted, receive $-s$ for sure, therefore $m = 1$ and $q = 0$ must be the equilibrium.

For lower levels of certification precision, the situation is more complex. Firstly, I will show that $m = 1$ and $q^* \in (0, 1)$ and consistent beliefs form an equilibrium and subsequently I will rule out all other possibilities.

Suppose $m = 1$. Since all good students apply, given the Assumption 2, the acceptance rate $\delta(\Omega) < 1$ assures that some of the good students enter the pool of untested workers driving the uncertified wage above 0. To determine the optimal application rate of low types note that if both $m = 1$ and $q = 1$, the proportions of high types and low types in the pool of certified and uncertified students are the same, hence $U_L(d_i = 0, q = 1, m = 1) = \lambda$. Since for any level of $p > 0.5$ the test is informative, $U_L(d_i = 1, q = 1, m = 1) < \lambda$. So it can't be that all the low types apply. At the same time $U_L(d_i = 1, q = 0, m = 1) = 1$, therefore it can't be that none of the low types applies. From continuity of w_U , w_P and w_F there must exist $0 < q^* < 1$ such that $U_L(d_i = 1, q = q^*, m = 1) = U_L(d_i = 0, q = q^*, m = 1)$. Assumption 2 guarantees that this value is strictly positive. In such a case an action $d_L = q^*$ and consistent beliefs $\mu_i(q)$ fully concentrated on q^* form the best response. For any level of $\mu_i(q)$ high types holding beliefs $\mu_i(m) = 1$ find it optimal to apply, therefore the belief system $\mu_i(q) = q^*$ and $\mu_i(m) = 1$ and individual strategies $d_L = q^*$ and $d_H = 1$ form an

equilibrium. The closed form solution of the equilibrium value of q^* exists but the degree of polynomial solving the indifference condition makes it difficult to present the result. From inspection of the respective expected wages, it is immediate that an increase in λ increases while the increase in s decreases the equilibrium q^* . The relation between ϕ and q^* will be explored in the next section. Since, the test is informative and $q^* < 1$, it is immediate that high type holding sufficiently high beliefs on m enrolls, therefore $d_H = 1$ is indeed the best response.

To see that no beliefs consistent with $m \in [0, 1)$ can be supported in equilibrium note the following. $\forall_{p>0.5} Ew_H > Ew_L$, while the outside option yield the same outcome for both types, therefore it can't be that both high and low type are mixing at the same time. To rule out $m = 0$, note that if $\mu_L(m) = 0$ the anticipated wage of the low type taking the test is $U_L(d_i = 1, \cdot) = 0 < U_L(d_i = 0, \cdot) = w_U(m = 0, \cdot) \geq \lambda$ hence the optimal response of a low type is $d_L = 0$.

The equilibrium in which no-one enrolls and high types hold beliefs $\mu_H(m = 0) = 1$ and $\mu_H(q = 0) < 1$ while $\mu_L(m = 0) = 1$ and $\mu_L(q = 0) < 1$, i.e. that both types belief that none of the high types and some of the low types apply is ruled out by the D1 refinement. To see it more clearly note that since the signalling cost is the same for everyone both types can potentially benefit from deviation if the wages offered by the employers are sufficiently high. However, given the informativeness of the test, the set of wages making the high types deviation profitable is larger than that of the low type, therefore, upon observing a deviation the employers assign probability 1 to the high type making the deviation profitable for him. Weaker refinements (e.g. intuitive criterion) do not rule out that equilibrium.

The argument seizes to be valid for $p = 0.5$ and the resulting non-informativeness of the test. □

Proof of Proposition 2.3.2

Proof. The logic behind the proof that $m = 1$ is exactly the same as in the case with constraints. However, since now all the high types that want to take the certification are accepted, the expected wage of non-certified workers is 0. This decreases the value of the outside option of the low type. As a result, the low types equilibrium application rate is strictly higher than in the previous case. It's clear that for sufficiently low level of private education costs s (and sufficiently low p) all low types apply. If the costs exceed the value \underline{s} such that $U_L(p(\cdot), q = 1, m = 1, d_i^L = 1, \underline{s}) = 0$ the full application rate can not be an equilibrium any more. Low types are better off by staying out of certification, in which case they receive 0 utility. However, as in the previous case, from continuity, there exists such

q that $U_L(\cdot, q, d_i^L = 1) = U_L(\cdot, q, d_i^L = 0) = 0$. Only such level of low type enrolment can be supported in equilibrium. If the payoff from enrolment is higher the individually rational decision is to enrol. The equilibrium q_{post}^* in this case can be obtained analytically and is given by:

$$q_{post}^* = \min\left(1, \frac{\left(-s - (-1 + p)p(1 + 2s) + \sqrt{(-1 + p)^2 p^2 + 2(1 - 2p)^2(-1 + p)ps + (1 - 2p)^2 s^2}\right) \lambda}{2(-1 + p)ps(-1 + \lambda)}\right)$$

Equilibrium, in which $m = 0$ and $q = 0$ is again ruled out by D1. \square

Proof of Proposition 2.3.3

Proof. Since the investment in certification, from social perspective, is a pure waste, it's immediate that the social welfare is maximised when there is no wasteful investment. For the first part of the proposition, I have shown that for $p > 0.5$ the high types always fully enrol. To see that there exists a parameter space where high types lose on the relaxation of constraints note the following. When the certification is perfect ($p = 1$), under no constraints all high types undertake certification in equilibrium and their type is truly revealed, hence $U_H = 1 - s$. When there are constraints all the high types (and none of the low types) apply, but only $\delta(m = 1, q = 0) = \phi/\lambda$ of them gets accepted and receives a payoff $1 - s$ and the rest remains uncertified receiving a strictly lower payoff $\frac{(\lambda - \phi)^2}{\lambda(1 - \phi)} < \lambda < 1 - s$.

Now suppose the certification is infinitesimally informative. All the high types apply and the proportion of the low types applying is such that their expected wage from applying and staying uncertified is the same. The level of informativeness of the test implies that in the limit $\lim_{p \rightarrow 0.5^+} U_H = \lim_{p \rightarrow 0.5^+} Ew_H = \lim_{p \rightarrow 0.5^+} Ew_L = \lim_{p \rightarrow 0.5^+} w_U = \lim_{p \rightarrow 0.5^+} U_L$. Since high and low types split the available payoff equally their expected wages are determined by the total waste due to certification costs. These costs are strictly higher when the constraints are lifted, therefore both high and low types are better off under constraints. Since high type utility is continuous there exists an open set of values p under which the high type is better off under constraints.

Low types are always worse off since the relaxation of capacity constraints implies that none of the high types remains uncertified. At the same time higher proportion of low types undertakes the test. Since the test is informative, more information on the true types is revealed which clearly negatively affects the low types. Moreover, since $m = 1$, whenever the low type's enrolment rate in post-transition regime $q_{post}^* < 1$ she receives utility of 0.

The fact that welfare is maximised in the absence of signalling technology is a standard

feature of signaling models. This result is modified if there exist supermodularities in job-employer matching as it is shown in the next section. \square

Proof of Proposition 2.4.1

Proof. To see the result note that, similarly to the previous case for any value of $p(\cdot)$ $Ew_H > Ew_L$, therefore it can't be that in equilibrium both types mix at the same time. Suppose now that $m = 1$, then the exogenous capacity constraint is reached. $\Omega\delta(\Omega) = \phi$ disregard of the low types enrolment decisions. Therefore the subsidy c uniquely pins down the certification precision p . The analysis from the previous section remains valid. All the equilibria with $m = 1$ from the previous scenario apply also here.

The certification quality changes only for the total number of applications $\Omega < \phi$, which, given that high and low types can't mix at the same time may happen only if $q = 0$ and $m < \frac{\phi}{\lambda}$, which can not be an equilibrium. \square

Chapter 3

Tuition Fees in a Signalling Model of Education

Abstract

In many developed countries, compulsory education is provided free of charge while voluntary university education is offered at a fee exogenously set by the authorities. This paper analyses the effect of tuition fees on educational decisions in education systems of different quality. I present a model of interactions between privately informed students of two types and competitive employers that play a noisy signalling game when university enrolment is costly. I show the effect of education quality on educational expenditure, enrolment decisions and educational effort exerted. I highlight the degree of substitutability between tuition fees and the non-pecuniary costs of educational effort under asymmetry of information. In the analysed game, individuals invest in education to signal their innate abilities to future employers. They decide whether to enrol in a university and choose what educational effort to exert. Universities truthfully reveal to the employers the educational effort they observe imperfectly. Tuition fees can partially substitute educational effort in revealing information about the type, reducing the necessary effort level. Furthermore they can restore a separating equilibrium which, in the absence of fees, ceases to exist for high levels of noise. The impact of tuition fees is, however, complex as it can both increase and decrease the equilibrium effort level depending on parameters.

Keywords: Signalling games, Noise, Tuition Fees

JEL Classification Numbers: D82, C72, H42 , I23

3.1 Introduction

There is a general agreement in the economic literature that it is difficult to overstate the importance of education quality for economic development. [Hanushek \(2011\)](#) claims that just replacing the bottom 5-8 percent of teachers with average teachers in American schools could bring a present value of \$100 trillion increase in additional long term productivity. The potential social gains from understanding the role of quality should be motivating enough to pursue research in this area. Given that importance, education quality, together with university tuition fees and enrolment rates, have always been in the centre of public debate around the world. Despite the importance of education systems, the channels through which tuition fees affect education quality and student's incentives to enrol in university education are not well understood in the literature. [McDuff \(2007\)](#), a notable empirical exception, shows that an increase in both tuition fees and quality at American in-state public colleges and universities increases demand among good students but decreases demand among bad students. More precisely, an increase in both quality¹ and tuition increases demand among applicants with high scores in pre-entry exams but decreases demand among low achievers, resulting in a negative net change in the number of applications.

In this paper I will present a mechanism that can rationalise this result. In a framework in which education is purely a signalling device, with no human capital effect, I analyse the relation between educational quality and tuition fees. When signals are imprecise, the monetary cost of obtaining a degree (money burning) may supplement the educational effort in revelation of innate abilities. Bad students, given the fee, find it more costly to obtain a degree than the good ones. Therefore, the tuition fee that discourages students from entering the educational system affects mostly the enrolment rate of bad students. As a result, as the fee increases, the employers face a higher proportion of good students in a pool of university graduates. This allows the employers to decrease their requirements for prospective candidates for high skill jobs necessary to select a skilled job applicant. If the level of noise is high, the utility gain from a decrease in the required educational effort may compensate the monetary cost of education. Consequently, good students want to pay tuition fees exceeding the marginal cost of education provision. Tuition fees are a better substitute for educational effort in deterring the low types from enrolment when the level of noise is moderate than when the signal is precise. When the signal is precise the fees need to be very high for the described mechanism to work.

¹The author designed a quality index based on mean SAT scores of the entering freshman class, freshman retention rate, and spending per student.

The key mechanism behind those results is the assumption of information asymmetry which is mitigated through a signalling technology (education). The signal is received with noise and the level of noise will be referred to as the quality of education. Noise, a common problem in education systems, may be caused by a number of factors. School curricula often test different qualifications than those required by the employers, the grading system is often coarse by design, the exam grades reflect not only knowledge in a given field but also the propensity to cheat. The list here is by no means complete, but it highlights the importance of including noise in a model of education signalling.

Since the seminal work of [Spence \(1973\)](#), signalling models have received a lot of attention in economics and other sciences. In particular, the addition of noise to the original model have been widely used in educational settings. Applied work in the field follows closely theoretical results obtained by [Matthews and Mirman \(1983\)](#) and [Carlsson and Dasgupta \(1997\)](#). Their developments assure uniqueness of separating equilibria and allow for smooth, and hence more interesting comparative statics with respect to prior beliefs and other model parameters. The modelling framework developed there is the basis for this paper. Drawing from the described work, [Landeras and De Villarreal \(2005\)](#) analysed the effects of noise on education expenditure. They show that noise in the education credential system may lead to an over investment in education. They show that, when the transmitted signal is not clear, agents put more effort in communicating their true type. However, those authors consider signalling technologies with only one type of cost satisfying a single crossing condition. In this paper, in addition to the effort with different marginal costs across types, I add pecuniary costs, which are the same for both student types.

An important complement to signalling is the literature on certification. In the certification setting, the separation between types takes place via an effortless certification technology (though pecuniary costs are possible) that offers different probabilities of a passing grade to different types. This paper draws from the contributions of [De and Nabar \(1991\)](#) who analyse the impact of the quality of a signalling device (a binary test) on the demand for certification and the utility of players. The authors show that the lower the certification precision, for a given price, the higher the incentives for the seller of the low quality good to attempt to obtain a certificate of quality. An increased number of applications of the low types decreases the informativeness of the certificate, which decreases the utility of the high quality sellers. In a noisy signalling setting employment probabilities, in equilibrium, differ between the types akin to a pure certification model. When the noise level is high tuition fees can substitute for certification precision by deterring low type from taking the test.

The combination of signalling and grades, i.e. a situation in which single crossing condi-

tion holds for two (possibly) independent dimensions, has been analysed in detail by [Daley and Green \(forthcoming\)](#). Their approach, although very rich and inspiring, is not necessarily the most suitable when applied to the European education systems. In many European countries tuition fees are often set by authorities to a not too high level. Consequently, the range of scholarships attracting the most able students is limited making the financial dimension of enrolment not too sensitive to student abilities. Therefore, in this paper, I analyse a model with two types of signals but with single crossing condition holding only in one of the dimensions.

Educational effort, which leads to human and social capital accumulation, that is not fully observed and adequately rewarded on the job market, rather than treated as a waste can be a policymaker's objective. Taking this perspective, [Dubey and Geanakoplos \(2010\)](#) show that in order to maximise the effort level there should be a non-zero level of noise in the grading system. However, those authors don't analyse the effect of the pecuniary costs of education and the resulting self-selection effect. I show that the effect of tuition fees on equilibrium signalling effort is non-monotonic. An increase in the pecuniary costs can lead to either an increase or a decrease in aggregate educational effort depending on parameter values.

The recent experimental work by [de Haan et al. \(2011\)](#) and [Jeitschko and Normann \(2012\)](#) demonstrate the empirical relevance of the predictions of noisy signalling games. Their laboratory experiments show that agents' actions differ between the deterministic and the stochastic environment. As predicted by the theory, there is more signalling behaviour when the signals are noisy.

The paper is organised as follows. The next section presents the basic modelling assumption and characterises the players and their objectives. In [Section 3.3](#) I define the equilibrium concept in a benchmark model without tuition fees to demonstrate the key relations between senders and receivers. In [Section 3.4](#), the effects of introducing tuition fees for participation in education are presented. In [Section 3.5](#), I conduct a welfare analysis of the economy in order to make a statement on the optimality of tuition fees. Finally, [Section 3.6](#) concludes.

3.2 Model

I study interactions between senders (students), receivers (employers) and tuition fees charged by non-strategic third party signalling institutions (universities) that play a noisy signalling game in an economy with asymmetric information. Details about players and their preferences are presented below.

3.2.1 Students

The population of students is, for simplicity, represented by a single, risk neutral, privately informed representative agent (she). With probability $\lambda \in [0, 1]$ she is of high type and has learning abilities θ_H . With probability $1 - \lambda$ she is of low type and exhibits abilities θ_L where $\theta_H > \theta_L$. W.l.o.g. I normalise $\theta_L = 1$. The variables characterising the two types are presented with respective subscripts H and L .

Before entering the labour market, the agent makes a binary decision whether to register at a university, denoted by $d_i \in [0, 1]$, where 0 means never register, 1 is always register and in-between values represent mixed strategies. If the agent registers, she pays a type independent tuition fee f and at an additional non-pecuniary cost $c(e_i, \theta_i) = \frac{e_i}{\theta_i}$ chooses educational attainment $e_i \in \mathcal{R}^+$. The sole purpose of education is signalling, it does not affect productivity. Upon leaving the university the student enters the labour market, where she receives a wage offer conditional on her academic record. I assume there are no costs other than tuition fees incurred by the student while at university,². Utility of a worker that has decided to pay a tuition fee receiving wage w is given by:

$$U_i = E(w_i | e_i, \sigma_i) - d_i * \left(f + \frac{e_i}{\theta_i} \right) \quad (3.2.1)$$

Relation between the expected wage, individual effort is explained later in the paper. If a student receives more than one identical job offers she chooses one at random.

3.2.2 University

I assume a single, non strategic university (signalling technology) that observes imperfect measures of student's educational attainment and issues credentials, which are truthfully revealed to employers. Credentials are the only available source of information on abilities of prospective workers. I assume the credentials z_i to be a sum of the student's educational effort e and a random variable $\sigma * \varepsilon_i$, where ε_i is drawn from a differentiable distribution $G(\varepsilon)$ that is symmetric and has zero mean. σ is a scaling factor that can be understood as a measure of quality of universities. The lower σ , the higher the quality. Henceforth I will refer to σ as the level of noise. The level of noise is taken as given by the university. Formally, I write z as:

$$z_i = e_i + \sigma * \varepsilon_i \quad (3.2.2)$$

² One can think of some other interpretations of this assumption e.g. that human capital returns to education compensate the opportunity cost.

I also assume that the conditional density of z given e ($z|e$) weakly (strongly on a non empty support) satisfies the monotone likelihood ratio property (MLRP). Intuitively, this implies that higher levels of true educational attainment e obtained by a student become more likely as the observed signal z increases.³ In the remaining part, I assume ε to have a triangular distribution $g(\varepsilon)$ with a support on $[-1, 1]$. The distribution has been chosen for algebraic convenience.

The cost of education provision (certification) is assumed to be equal 0. In the next section in a benchmark model I assume that university charges no tuition fees, while in the subsequent sections the fee is set to an exogenous level f .

3.2.3 Employers

I assume two types of jobs, a high skill and a low skill jobs. In a perfectly competitive market for low skill jobs, labour productivity and hence the wage for both types is the same and w.l.o.g. normalised to 0.

In the high skill job market, two employers compete in recruitment strategies (requirements) to select a high type to fill the vacancy. All workers employed in this sector are paid an exogenously set wage $w > 0$.⁴ However, the productivity of high skill jobs is affected by the worker's type. The total surplus generated by matching a high skill job with a high type is x where $x > w$, so the employer's surplus is $x - w$. A low type matched with a high skill job generates no surplus, so the cost for the employer is $-w$.

Since the receiver does not observe directly the sender's effort, she has to use statistical inference in order to update her beliefs about the action taken, and thus learn the agent's type. The employer decides to hire if the expected profit given the signal (and posterior belief about the type) of filling the vacancy is non negative. The expected profit from employing a student sending a signal z , under Bayesian learning and high type educational strategies $\mu_H(e)$ and low types' strategy $\mu_L(e)$, is given by:

$$V_E(z|\mu_H, \mu_L) = -w + \beta(z) * x \quad (3.2.3)$$

³These assumptions facilitate the equilibrium analysis, because they imply that the employer uses a cut-off strategy in any non-pooling equilibrium (cf. [de Haan et al. \(2011\)](#)).

⁴Predefined wage bands are common in many countries in publicly financed positions. Moreover, the model applies to entry-level positions, where the wage differentiation is rather limited.

where :

$$\beta(z) = \frac{\lambda * \int g\left(\frac{z-e}{\sigma}\right) * \mu_H(e) de}{\lambda * \int g\left(\frac{z-e}{\sigma}\right) * \mu_H(e) de + (1 - \lambda) * \int g\left(\frac{z-e}{\sigma}\right) * \mu_L(e) de} \quad (3.2.4)$$

$\beta(z)$ is the employers' posterior belief that the student is of the high type upon observing credentials z and given that the students use strategies $\mu_H(e)$ for high and $\mu_L(e)$ for low type respectively. More precisely, $\mu_H(e)$ ($\mu_L(e)$) denote the probability with which the high (low) type chooses signal costs e in equilibrium.

3.2.4 Timing

First nature assigns the student type and the level of noise. Subsequently, tuition fees are exogenously set (to zero in the next section). Employers commit to a rule determining which workers to employ (given the exogenous wage w). Knowing the employers' strategies the agent chooses whether to enrol a university and if she decides to do so she chooses her educational attainment. In the next stage, the education system causes noisy distortions ε to the education outcomes and payoffs are realised. Graphically the timeline is presented in [Figure 3.1](#).

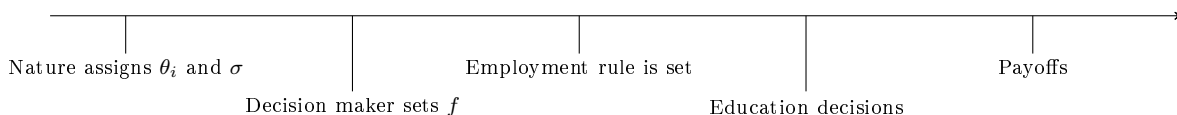


Figure 3.1: Timeline

3.2.5 Equilibrium Concept

I employ a perfect Bayesian equilibrium concept to solve the game, such that the following sequence of actions hold. Employers commit to a mapping from their beliefs on student's signal to employment decision. The representative student maps from employers' strategies to education decisions. Belief-strategy combination satisfies Bayes' rule whenever possible, and sequential rationality always.

In this type of games a pooling equilibrium in which both student types choose $e = 0$ always exists. In this trivial equilibrium employers never (always) hire when $\lambda < (>) \frac{w}{x}$. Throughout, the paper I will focus only on non trivial equilibria and limit the analysis to parameter range in which those equilibria exist.

Definition 3.2.1. *A non trivial equilibrium is an equilibrium in which employers' strategy changes with the signal observed.*

3.3 Free Education

To give a better understanding of the key elements of the model and to give intuition about the players' strategies I will first present a standard noisy signalling model with no pecuniary costs of education. Later, I analyse the effect of tuition fees on equilibrium outcomes.

Assumption 1. $f = 0$. *The university charges no tuition fee.*

To rule out trivial equilibria in which everyone is employed irrespective of the educational expenditure and effort I make the following assumption:

Assumption 2. *If $\sigma < w$, $\lambda < \frac{w(2w^2 - \sigma^2)\theta_H^2}{2w^2x\theta_H^2 - \sigma^2(x + w(\theta_H^2 - 1))}$, otherwise $\lambda < \frac{w}{x}$*

Assumption 2 allows to focus only on non trivial equilibria with employment. More detailed interpretation is presented later in the text. The assumption excludes the area above the solid line.

I solve the game in the order suggested by backward induction.

Lemma 3.3.1. *The employers' best response rule in a non trivial equilibrium is of the following form:*

$$\begin{aligned}\pi(z) &= 0 \text{ if } z < z^* \\ \pi(z) &= 1 \text{ if } z > z^*\end{aligned}$$

where $\pi(z)$ is the employment probability after observing signal z .

Due to MLRP property $\beta(z)$ is increasing in z . Since $g(\varepsilon)$ is continuous, there exists a threshold level z^* such that the expected profits from having the vacancy filled are equal to zero, and above which positive profits are observed. For a formal proof see [Carlsson and Dasgupta \(1997\)](#) or the appendix of [de Haan et al. \(2011\)](#). As a result the employer's profit is given by:

$$U_E = \int_{z^*}^{\infty} V_E(z) dz.$$

For further analysis it is useful to define the expected wage as a function of choice variables and parameters

Definition 3.3.1. *Expected wage:*

$$E(w_i|\cdot)|e_i, \sigma_i) = w * \left(1 - G \left(\frac{z^*(\cdot) - e_i(\sigma_i)}{\sigma} \right) \right)$$

A student's payoff is equal to the expected wage (the exogenous wage multiplied by the probability of sending the signal higher than the threshold conditional on educational attainment) net of the cost of the signal sent. Therefore we can rewrite Equation 3.2.1 in the following way:

$$U_i(e_i, \cdot) = \max_{e_i \geq 0} w * \left(1 - G \left(\frac{z^* - e_i}{\sigma} \right) \right) - \frac{e_i}{\theta_i} \quad (3.3.1)$$

Where z^* is the threshold level set by the receiver.

Lemma 3.3.2. *The optimal level of effort for a type i for given threshold and noise level takes the following values:*

$$e_i^* = \begin{cases} \bar{e}_i = \sigma + z^* - \frac{\sigma^2}{w\theta_i} & \text{if } z^* < \frac{\sigma^2}{2w\theta_i} + w\theta_i - \sigma \wedge 0 < \sigma \leq (2 - \sqrt{2}) w\theta_i \\ \text{or } z^* < \frac{(1+\sqrt{2})\sigma(w\theta_i - \sigma)}{w\theta_i} \wedge (2 - \sqrt{2}) w\theta_i < \sigma < w\theta_i & \\ \underline{e}_i = 0 & \text{otherwise} \end{cases} \quad (3.3.2)$$

The result comes from maximising Equation 3.3.1 with respect to the effort level for each of the types. I will call \underline{e}_i low effort level and \bar{e}_i high level of effort of the type i with a caveat that the low effort level is the same, i.e. zero, for both types, while the high effort level is higher for the high type. Zero effort makes the model similar to a pure certification model, in which the probability of receiving the certificate is determined by the employers' threshold. The condition above shows that if the level of noise σ and the employer's threshold z^* are not too high, both student types want to exert high effort \bar{e}_i . However, the understanding of what "not too high" is differs between the types. High types continue exerting high effort for higher levels of employer's threshold and higher levels of noise than the low types. The optimal effort condition implies that there exists such a threshold z^* that a high type finds it optimally to exert \bar{e}_H while the low type chooses \underline{e}_L . In particular, when $\sigma > w$ low type is exerting no effort disregard of the threshold. When σ or z^* are too high, good students find it too costly to differentiate and both types choose no effort at all.

To characterise the equilibria it is useful to define the following critical values of σ . The interpretation is presented in the description of the subsequent proposition.

Definition 3.3.2.

Given the types' optimal responses we can identify the following equilibria of the game.

$$\begin{aligned}\sigma_1 &= (2 - \sqrt{2}) w & \sigma_2 &= \frac{\sqrt{2}(1-\lambda)w^2\theta_H}{(1+\sqrt{2})(1-\lambda)w\theta_H - \lambda(x-w)} \\ \sigma_3 &= \frac{w^2\theta_H(1-\lambda)(\sqrt{2}x\lambda + w(2-\sqrt{2}-2\lambda))}{2wx\lambda - x^2\lambda^2 + w^2(1-4\lambda+2\lambda^2)}\end{aligned}$$

Proposition 3.3.1. *There exist the following non trivial equilibria in which the high type uses pure strategies:*

1. *If $\sigma \leq \sigma_1$ there exists a unique non trivial equilibrium in which the employers set the threshold to $z^* = w - \sigma + \frac{\sigma^2}{2w}$, high type always chooses effort level $e_H = \bar{e}_H$ and the low type mixes between exerting effort $e_L = \bar{e}_L$ and effort $e_L = 0$.*
2. *If $\sigma_1 \leq \sigma < \sigma_2$ there exists a unique non trivial equilibrium in which the employers set the threshold to $z^* = \frac{(1+\sqrt{2})(w-\sigma)\sigma}{w}$, high type always chooses effort level $e_H = \bar{e}_H$ and the low type mixes between exerting effort $e_L = \bar{e}_L$ and effort $e_L = 0$.*
3. *If $\sigma_2 < \sigma < \sigma_3$ there exists a non trivial pure strategy equilibrium in which the employers set the threshold to $z^* = \sigma - \frac{\sigma^2\lambda(x-w)}{(1-\lambda)w^2\theta_H}$, high type always chooses effort level $e_H = \bar{e}_H$ and the low type always chooses $e_L = 0$.*
4. *If $\sigma > \sigma_3$ non trivial equilibrium does not exist.*

There exists also an equilibrium in which the high type mixes between $e_H = \bar{e}_H$ and $e_H = 0$ while the low type always chooses $e_L = 0$.

For the formal proof see the [Appendix](#). Informally, the employers move first and in the equilibrium they are aware of the sender's incentives to manipulate the flow of information. That is, they are aware that the high type will choose an action in hopes of distinguishing herself from the low type and, similarly, that the low type will attempt to mimic the high type. It can be shown that, given the [Assumption 2](#), when both types always choose \bar{e}_i no equilibrium exist, therefore the equilibrium threshold (or the noise level) has to be sufficiently high to deter the low type from exerting high effort. At the same time the equilibrium threshold has to be high enough to assure that the employers do not make losses. When σ is low, ($\sigma < \sigma_2$) the equilibrium threshold level is driven by the deterrence reasoning. In equilibrium, the threshold is set to a level that the low type is indifferent between the two effort levels and mixes in a way that the employers receive zero expected profit on the marginal employer. When $\sigma = \sigma_1$, $z^* = \sigma$ and hence for $0 < \sigma \leq \sigma_1$ the low type's utility is zero. When $\sigma > \sigma_1$, $z^* < \sigma$ and hence even zero effort gives chances for employment. When $\sigma_1 \leq \sigma < \sigma_2$ low type's probability of choosing \bar{e}_L decreases with σ until it reaches 0

for $\sigma = \sigma_2$. For $\sigma_2 < \sigma < \sigma_3$ in a non trivial equilibrium employers set the threshold to a level that assures zero expected profit on the marginal employer, given that the high type always exerts high effort and the low type always exerts low effort. When $\sigma = \sigma_3$ high type is indifferent between exerting high and low effort. For higher values of σ (area under the dashed line in the bottom right corner in [Figure 3.2](#)), signalling becomes too costly also for the high type and no separating equilibrium exists. I will refer to the equilibrium threshold level for $0 < \sigma < \sigma_2$ as "prohibitive" threshold as it makes the high effort level prohibitively expensive for the low type. Interestingly this "prohibitive" strategy is independent of the proportion of high types in population.

The area of existence of non-trivial equilibria in $\sigma - \lambda$ space is presented graphically in [Figure 3.2](#). The equilibrium space is restricted from above by [Assumption 2](#) and from below by incentive compatibility constraint of the high type.

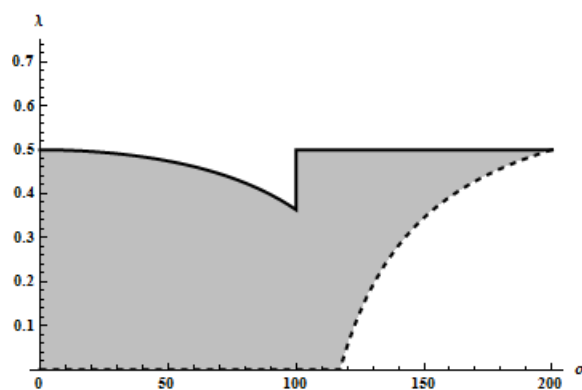


Figure 3.2: Range of existence of separating equilibrium
Other parameter values: $\theta_H = 2$, $x = 200$, $w = 100$, $\lambda = 0.2$

[Figure 3.3](#) presents educational effort of the high type \bar{e}_H , the low type effort (\bar{e}_L and \underline{e}_L) and the optimal threshold level z^* . The figure clearly shows that equilibrium threshold consists of two parts. The decreasing part in range $\sigma < \sigma_2$ makes the low type choose no educational effort. The increasing part for $\sigma_2 < \sigma < \sigma_3$ sets the employers' expected profit from employing the marginal student to 0 (given that $e_H = \bar{e}_H$ and $e_L = \underline{e}_L$). For high values of σ , signalling becomes too costly for the high type and the equilibrium collapses. As will be shown in the next section, the separating equilibrium in this range can be restored with tuition fees. Before moving to the next section presenting the effects of introduction of tuition fees on equilibrium characteristics, it is useful to make a couple of observations.

Observation 3.3.1. *For low levels of σ , in all of the non-trivial equilibria the threshold level z^* and optimal educational attainment \bar{e}_H are independent of λ . For high values of σ , z^* and*

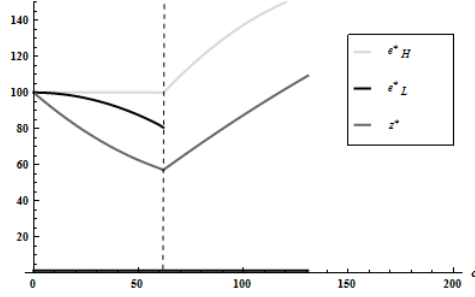


Figure 3.3: Optimal Threshold and Signal

Other parameter values: $\theta_H = 2$, $x = 200$, $w = 100$, $\lambda = 0.2$. The dashed line marks σ_2 beyond which pure strategy equilibrium exists.

\bar{e}_H are decreasing in λ .

Observation 3.3.2. For $\sigma \leq \sigma_1$ the low type receives zero utility. For larger σ , her expected pay-off increases in σ .

Both observations follow directly from the functional forms of z^* and \bar{e}_H .

3.4 Tuition fees

In this section I drop the assumption that the university charges no tuition fee (**Assumption 1**). Without the tuition fees, the agents had no reasons not to enrol. With the introduction of a pecuniary cost, it is necessary to extend the decision space to two dimensions (d_i, e_i) . The proportion of high types at university changes from λ to $\hat{\lambda}(d_H, d_L) = \frac{d_H * \lambda}{d_H * \lambda + (1 - \lambda) * d_L}$.

For an agent to register at a university, the tuition fee cannot exceed the expected benefit from education. If an agent chooses $d_i = 0$, she does not participate in the second stage of the signalling game in which she would choose her effort.

Given the multiplicity of the employers threshold-setting strategies in **Proposition 3.3.1**, the specification of all equilibria with tuition fees is tedious. To facilitate the analysis it is useful to define the following levels of tuition fees.

Definition 3.4.1.

$$\begin{aligned}
 f_A &= \frac{(x-w)^2 \lambda^2 \sigma^2}{2w^3 \theta_H^2 (1-\lambda)^2} & f_C &= w - \frac{\sigma^2}{2w} \\
 f_B &= w - (2 + \sqrt{2}) \sigma + \frac{(3+2\sqrt{2})\sigma^2}{2w} & f_D &= w - \frac{\sigma^2}{2w\theta_H^2} \\
 f_E &= \frac{2w^2 \theta_H^2 - 2(2+\sqrt{2})w\theta_H \sigma + (3+2\sqrt{2})\sigma^2}{2w\theta_H^2}
 \end{aligned}$$

Proposition 3.4.1. *There exist the following non trivial equilibria in which the high type always enrolls and exerts high effort:*

1. *If $\sigma \leq \sigma_1$ and $f < f_C$ or $\sigma_1 < \sigma < w$ and $f_B < f < f_C$ there exists a unique non trivial equilibrium in which the employers set the threshold that is decreasing in f and the low type mixes between enrolment and no education. The low type, if enrolls, exerts high effort. For $f_C < f < f_D$, low type's enrolment rate is decreasing in f , those who enrol exert zero effort.*
2. *If $\sigma_1 < \sigma < \sigma_2$ and the fee is sufficiently low ($f < f_B$), there exists a unique non trivial equilibrium in which the employers threshold does not change with f , low type always enrolls and mixes between exerting high and low effort.*
3. *If $\sigma_2 < \sigma < \sigma_3$ and $f < f_A$ low type always enrolls and exerts low effort and the employers threshold does not change with f .*
4. *If $\sigma_2 < \sigma < w$ and $f_A < f < f_B$ or when $w < \sigma < \sigma_3$ and $f_A < f < f_D$ or when $\sigma_3 < \sigma < \theta_H * w$ and $f_E < f < f_D$ low type mixes between enrolment with low effort and staying out of education. Equilibrium enrolment rate and employer's threshold are decreasing in f .*

There exist also non trivial equilibria in which the high type mixes between enrolment with high effort and enrolment with low effort while the low type enrolls and exerts low effort. Furthermore, there exists also a continuum of trivial equilibria with and without employment.

The proposition describes all equilibria with high type's full enrolment, for full characterisation and the proof see the [Appendix](#). Intuitively, the agent enrolls whenever the expected wage is higher than the tuition fee. Therefore, when the fee is lower than the low type's payoff in equilibrium without tuition fees (see [Proposition 3.3.1](#)), the fees don't affect the equilibrium strategies as in 2. and 3.⁵ Compared to equilibrium with no tuition fees the only difference is that the rents are captured by the universities.

In other cases tuition fees affect the equilibrium strategies. In 1. the introduction of tuition fees results in a decrease of the equilibrium threshold. The decrease is necessary to make the low type indifferent between enrolment with high effort and staying out of education. No one can enrol and exert no effort as this strategy would assure negative

⁵When $\sigma_1 < \sigma < \sigma_2$ and $f < f_B$, i.e. when the fee is lower than the expected wage of a student exerting no effort under the "prohibitive" threshold or when $\sigma_2 < \sigma < \sigma_3$ and $f < f_A$, i.e. when the fee is lower than the expected wage for an agent exerting zero effort given the equilibrium threshold level.

pay-off unless the threshold is sufficiently low, which happens for $f > f_C$. For $f = f_C$, $\bar{e}_L = \underline{e}_L = 0$.

Observation 3.4.1. *As long as the high type always enrolls, a fee is set to $f \in (f_A, f_D)$, when $\sigma > w$, to $f \in (f_A, f_B)$ when $\sigma_2 < \sigma < w$, or $f \in (f_C, f_D)$ when $\sigma < w$ low type's equilibrium enrolment decision d_L is a one to one mapping from tuition fees.*

Finally, when $\sigma_2 < \sigma < \theta_H * w$ and $f > f_A$ the fees exceed the low types pay-off with no tuition fees and low type's incentive compatibility condition is slack. Even if the threshold is decreased, the low types continue exerting low effort. There is a degree of substitutability between educational effort and tuition fees. Costly education works as a partially revealing self-selection mechanism. When the tuition fees are high, low type enrolls with lower probability, hence the employers are more confident about the quality of the students, which allows them to reduce the threshold (to a level not lower than the "prohibitive" threshold). For the equilibrium to exist the fee can not exceed f_D , a level above which the high type's best response is negative effort, ruled out by assumption. The same mechanism is observed for $f_C < f < f_D$ when or $\sigma < w$.

Observation 3.4.2. *When tuition fee is sufficiently high ($f \in (f_E, f_D)$) a separating equilibrium in the range in which without tuition fees effort is prohibitively expensive for the high type ($\sigma > \sigma_3$) is restored.*

Without tuition fees there exists a level of noise above which signalling becomes prohibitively expensive even for the high type and separating equilibrium does not exist. This can change with sufficiently high tuition fees. As shown before, over a certain range of parameters, conditional on high type's enrolment, tuition fees deter the low type from education. This results in a decrease in employer requirements necessary to assure a non negative profit on the marginal employee. In that range a decrease in effort costs for the high type exceeds the resulting increase in tuition fees. Therefore, when tuition fees are high enough, the threshold decreases to a level that makes separation profitable for the high type. The area in which sufficiently high tuition fees can restore separating equilibrium is marked as Area 4 in [Figure 3.4](#).

3.5 Welfare Analysis

The welfare implications of tuition fees, given the multiplicity of equilibrium types is far from being obvious. This section shows that players in this game have different preferences

with respect to tuition fees. High types is best off when the tuition fees are zero for low levels of noise while for high levels of noise he prefers positive fees. Employers' preferences are the opposite, their profits are maximised for high fees when the noise level is low and for high fees when the signal is precise. The results are formally established in the following propositions.

Proposition 3.5.1. *From high type's perspective the optimal tuition fees are equal*

$$f^* = \begin{cases} 0 & \text{if } \lambda > \frac{w}{2x-w} \text{ or } \sigma < \frac{2w^{3/2}\theta_H(1-\lambda)}{\sqrt{w}((2+\sqrt{2})\theta_H-\sqrt{2})(1-\lambda)+\sqrt{2-2\lambda}\sqrt{w+w\lambda-2x\lambda}} \\ f_B & \text{if } \frac{2w^{3/2}\theta_H(1-\lambda)}{\sqrt{w}((2+\sqrt{2})\theta_H-\sqrt{2})(1-\lambda)+\sqrt{2-2\lambda}\sqrt{w+w\lambda-2x\lambda}} < \sigma < \frac{\sqrt{2}w\theta_H}{\theta_H+\sqrt{2}\theta_H-1} \\ \frac{\sigma^2}{2w\theta_H^2} & \text{if } \lambda < \frac{w}{2x-w} \text{ and } \sigma > \frac{\sqrt{2}w\theta_H}{\theta_H+\sqrt{2}\theta_H-1} \end{cases} \quad (3.5.1)$$

For the proof, see the [Appendix](#). Intuitively, as stressed in [Observation 3.4.1](#), there exists a range of parameters in which tuition fees deter low types from enrolment, which allows the employers to reduce the threshold. In that range, in an equilibrium with $f \in (f_A, \frac{\sigma^2}{2w\theta_H^2})$ savings on educational effort cost caused by marginal increase in tuition fees exceed the pecuniary cost increment. In equilibrium with $f < f_A$ tuition fees don't affect the equilibrium strategies, therefore the positive effect of tuition fees has to be sufficiently strong to outweigh the losses accruing for low values of f . The threshold is sufficiently sensitive to changes in fees only when the proportion of high types in the pool of students is low.

The ranges of different values of optimal fee are presented graphically in [Figure 3.4](#). Without the tuition fees, employment equilibrium does not exist in Area 4 as the effort cost is prohibitively expensive. Sufficiently high tuition fees allow for a decrease of the low type enrolment and hence a decrease in the threshold to a level that does not breach the high type's incentive compatibility constraint. High type's utility, in this range, is maximised when $f = \frac{\sigma^2}{2w\theta_H^2}$. The same fee is optimal also in Area 3, though in that range an employment equilibrium exists also without tuition fees. This area is bounded from above by $\lambda = \frac{w}{2x-w}$. For higher values of λ the reduction of threshold in response to an increase in tuition fee is too small and the high types are best off without tuition fees. In Area 2, that corresponds to the second row of [Equation 3.5.1](#), the optimal tuition fee is f_B . This level of fees reduces the threshold to a level for which the low type is indifferent between exerting high and low effort. In Area 1, i.e. for low levels of σ or high values of λ the high type is best off when the fees are set to 0.

Proposition 3.5.2. *For all values of σ , social welfare is the highest in an equilibrium with tuition fees set to $f = f_D$, the highest level allowing for an employment equilibrium. For $\sigma > \sigma_2$ employers' profits are the highest when tuition fees are set the lowest level for which an*

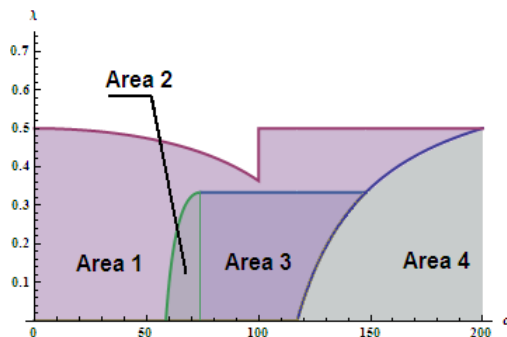


Figure 3.4: Range of existence of separating equilibrium with tuition fees

Other parameter values: $\theta_H = 2$, $x = 200$, $w = 100$, $\lambda = 0.2$

employment equilibrium exist. For lower levels of noise the profits are maximised at $f = f_D$. Both functions are non-monotonic in tuition fees. Low types never benefit from tuition fees.

For the proof, see the [Appendix](#). Intuitively, the only determinants of social welfare are high type's employment probability and total educational effort exerted by the students. Tuition fees and wages are merely transfers and affect the pay-offs only through their impact on equilibrium strategies. High type's employment probability, in employment equilibria, is independent of the threshold level, as long as the threshold is not too high and his incentive compatibility constraint is not breached. This component of total welfare depends only on noise level. The other component, effort, changes with the employer's threshold, which, in equilibrium, changes with tuition fees. When the fees are set to the highest level allowing employment equilibrium ($f = f_D$) employer's threshold reaches its minimum and hence the additive effort also reaches its minimum.

In equilibrium in which the low type student exerts high effort, the lower the low type's enrolment rate the higher the employer's pay-off. When low types exerts low effort, the higher the threshold the higher the pay-off. Employer's profit in equilibria in which the low type exerts low effort is strictly higher than in equilibria in which low type exerts high effort. For low level of noise this equilibrium type is reached only for high fees, while for higher levels this type of equilibrium exists with both high and low fees. Although the enrolment decreases in fees, the threshold decreases even faster, which makes employers best off with zero fees.

High type's utility, social welfare and employer's profit are presented graphically in [Figure 3.5](#). The observed discontinuity in employer's profit on the left panel is caused by a change of equilibrium types. To the left of the jump, all low types that register exert high effort. As a result the median of the low type effort distribution is higher than the employer's

threshold. Increase of tuition fees above f_C makes low type exert zero effort. As a result the median of the low type's signal distribution is lower than the threshold, which, given that the equilibrium enrolment rate is not higher than in the former case makes the employers profits grow. The right panel shows that when the noise level is high, for the non trivial equilibrium to exist the tuition fees can not be too low or too high.

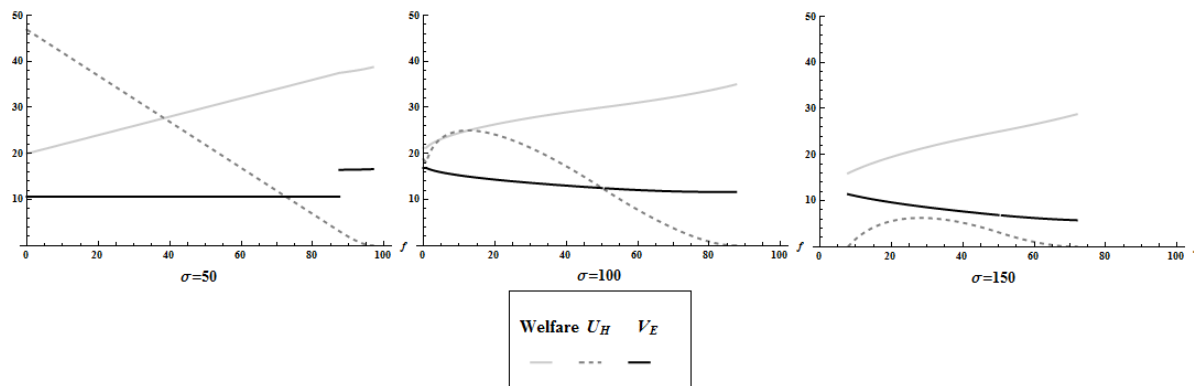


Figure 3.5: Social Welfare, High type's utility and employer's profit for different levels of tuition fee

Even though I have shown that for high values of noise the social welfare increases in f , one has to keep in mind that the surplus is now captured by the universities. The low types are worse off compared to the scenario in which certification was free, whereas the high types may benefit from introduction of fees.

3.6 Conclusions

This paper focuses on the effect of noisy signalling and tuition fees on students' education investment decisions and the subsequent employment probabilities. The paper contributes to the economic literature by characterising equilibria in a noisy signalling model in which signalling is divided into two stages. In the first stage (costly enrolment), students decide to enrol in a university at a fee. This signalling technology does not satisfy the single crossing condition, but the signal is perfectly observed by the employers. In the second stage, the enrolled students exert education effort. In this signalling technology, the high type has a cost advantage over the low type, but the signal is observed by the employers with noise. Compared to the standard noisy signalling model, an additional signalling device, even if it does not satisfy the single crossing condition, can support the educational effort in revealing

the information about the true type to the employers when the effort is observed by the employers with noise.

The paper describes in details the complex, nonmonotonic relationship between the tuition fees and students' and employers' strategies. Depending on the parameters, an increase in tuition fees can lead to either an increase or a decrease in high types' and employers' payoffs and students' educational effort. Therefore, the higher education authorities need to be very careful when setting the tuition fees in an attempt to select the most favourable equilibrium for policy objectives.

For example [Ostrovsky and Schwarz \(2010\)](#) show in a model without tuition fees that if universities' main objective is reputation defined as graduates' employment probability, the certification institutions may have incentives to reveal less than they actually know. I show that tuition fees can be also used as a tool that serves this purpose as they may increase incentives of the low types to exert effort or reduce employer's requirements. For the same reason, tuition fees could be used by the authorities if, as in [Dubey and Geanakoplos \(2010\)](#), one of the universities' objectives is the maximisation of educational effort.

As disturbing can be considered the fact that for a wide range of university quality levels high type's welfare is maximised when the certification institutions collect a significant part of surplus despite no cost incurred in the certification process. However, this happens only when the signal quality is very low which, hopefully, is a purely theoretical result. The results of the model apply only when agents are perfectly informed about their types, face no budget constraints and are risk neutral. In reality students learn their types also during the education process and differ in risk aversion and in initial endowments. As a result, one could expect the propensity to invest in education to be related not only to price and quality of education but also to self confidence and wealth.

The paper can be extended in a number of directions. First of all, in the model I abstract from human capital formation. Introduction of additional benefits would alter the welfare calculations, though the key result would qualitatively remain undistorted. Another possibility is to connect the model closer to [Daley and Green \(forthcoming\)](#) and to analyse the effects of scholarships for the best students. This scenario would be even more interesting when uncertainty about the types is introduced also on the students' side. In this scenario, the students would learn their type fully only at the university i.e. after the decision to pay tuition fees is taken.

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Appendix - Proofs

Proof of Proposition 3.3.1

Given the students' optimal response function (see Equation 3.3.2) we have 4 pairs of high and low type strategies that could possibly form a pure strategy equilibrium. They are given by $\{\bar{e}_H, \bar{e}_L\}$, $\{\bar{e}_H, \underline{e}_L\}$, $\{\underline{e}_H, \bar{e}_L\}$, $\{\underline{e}_H, \underline{e}_L\}$ where the expressions in brackets represent the optimal educational attainment of the high and the low type respectively. To find the equilibria (or more specifically to show that 3 of the 4 couplets can not form a pure strategy equilibrium) I first assume that they are equilibrium strategies and attempt to find the best response of the employers, which move first. If the triple forms a set of mutual best response it is an equilibrium.

Firstly, I show that the pair $\{\bar{e}_H, \bar{e}_L\}$ cannot be a pure strategy equilibrium. Suppose that the students choose $\{\bar{e}_H, \bar{e}_L\}$ with probability 1. The posterior belief that an agent sending signal equal to the threshold z is of high type $\beta(z)$ simplifies to:

$$\beta(z) = \frac{\lambda * \phi\left(\frac{\sigma}{w\theta_H} - 1\right)}{\lambda * \phi\left(\frac{\sigma}{w\theta_H} - 1\right) + (1 - \lambda) * \phi\left(\frac{\sigma}{w\theta_L} - 1\right)}$$

Since the high effort level function is additive, the posterior belief does not depend on threshold z^* , the terms cancel out (however it does depend on σ). Any threshold satisfying low type's could be an equilibrium if the employer's expected profit given in the equation below is non negative.

$$V_i^E(z = 0, \bar{e}_H, \bar{e}_L) = \lambda * (x - w) * \left(1 - G\left(\frac{0 - \bar{e}_H}{\sigma}\right)\right) - (1 - \lambda) * (w) * \left(1 - G\left(\frac{0 - \bar{e}_L}{\sigma}\right)\right)$$

This happens only if $\lambda > \frac{w(2w^2 - \sigma^2)\theta_H^2}{2w^2x\theta_H^2 - \sigma^2(x + w(\theta_H^2 - 1))}$ which is ruled out by Assumption 2 . Therefore the triple $\{z = 0, \bar{e}_H, \bar{e}_L\}$ can not be a pure strategy equilibrium.

Another candidate is defined by the pair of students strategies $\{\bar{e}_H, \underline{e}_L\}$. If that is the equilibrium strategy, the probability that an agent sending a signal equal to threshold z is of a high type is given by

$$\beta(z) = \frac{\lambda * \phi\left(\frac{\sigma}{w\theta_H} - 1\right)}{\lambda * \phi\left(\frac{\sigma}{w\theta_H} - 1\right) + (1 - \lambda) * \phi\left(\frac{z}{\sigma}\right)}$$

Now, the posterior beliefs (weakly) monotonically change with the observed signal. This condition is sufficient for the existence of a cut-off strategy (see [Carlsson and Dasgupta \(1997\)](#) for a formal proof). The existence of a threshold strategy of the employer is a necessary condition for the equilibrium.

When the types play $\{\bar{e}_H, \underline{e}_L\}$ the threshold solving

$$\max_{z \geq 0} \int_z^{+\infty} -w + \beta(z) * x dz$$

is given by

$$z^* = \sigma - \frac{\sigma^2 \lambda (x - w)}{(1 - \lambda) w^2 \theta_H} \quad (3.6.1)$$

For this threshold to be the equilibrium \bar{e}_H and \underline{e}_L need to be the best responses. It is the case for $\sigma \in (\sigma_2, \sigma_3)$. For lower values of σ the low type prefers \bar{e}_L . For higher values of noise, effort \bar{e}_H becomes prohibitively expensive for the high type.

The proof that $\{\underline{e}_H, \bar{e}_L\}$ can not be an equilibrium couplet of senders strategies follows similar logic. The posterior belief given by:

$$\beta(z) = \frac{\lambda * \phi\left(\frac{z}{\sigma}\right)}{\lambda * \phi\left(\frac{z}{\sigma}\right) + (1 - \lambda) * \phi\left(\frac{\sigma}{w} - 1\right)}$$

is decreasing in the observed signal, therefore it can not be an equilibrium.

Finally, if the student chooses $\{e_H = 0, e_L = 0\}$, the employment decision depends only the proportion of the high types in the population. The employers employ if $\lambda \geq \frac{w}{x}$ which is also ruled out by [Assumption 2](#), so it can not be an equilibrium.

There exist also mixed strategy equilibria. Since $\theta_H > 1$ (single crossing condition holds) there are only two candidates for mixed strategy equilibria. One in which the low type is indifferent between \underline{e}_L and \bar{e}_L while the high type chooses \bar{e}_H . And the other one in which the low type chooses \underline{e}_L while the high type is indifferent between \underline{e}_H and \bar{e}_H .

The first candidate is an equilibrium in range described in 1. and 2.. For this equilibrium to exist the employers have to set the threshold to a level for which the low type is indifferent between exerting effort $e_L = 0$ and $e_L = \bar{e}_L$, while the high type still has the incentives to

exert effort. The employer problem is then:

$$\begin{aligned} \max_{z \geq 0} \quad & \int_z^{+\infty} -w + \beta(z) * x dz \\ \text{s.t.} \quad & U_H(\underline{e}_H) \leq U_H(\bar{e}_H) \\ & U_L(\underline{e}_L) \geq U_L(\bar{e}_L) \end{aligned}$$

solved by $z^* = w - \sigma + \frac{\sigma^2}{2w}$ when $\sigma \leq \sigma_1$ and $z^* = \frac{(1+\sqrt{2})(w-\sigma)\sigma}{w}$ for higher values of σ . For these thresholds to form an equilibrium, the low type has to choose the high effort with probability q such that sets the expected profit on the marginal employee to zero.

$$\beta(z) = \frac{\lambda * \phi\left(\frac{\sigma}{w\theta_H} - 1\right)}{\lambda * \phi\left(\frac{\sigma}{w\theta_H} - 1\right) + (1 - \lambda) * \left(q * \phi\left(\frac{\sigma}{w} - 1\right) + (1 - q) * \phi\left(\frac{z}{\sigma}\right)\right)} = 0$$

It is the case for when the low type choose the high effort with probability $q = \frac{(w-x)\lambda}{w\theta_H(-1+\lambda)}$ in the range 1.. When σ is in the range 2. the low type mixes with probability $q = \frac{w((1+\sqrt{2})\theta_H(1-\lambda)+\lambda)\sigma - \sqrt{2}w^2\theta_H(1-\lambda) - x\lambda\sigma}{\sqrt{2}w\theta_H(1-\lambda)(\sigma-w)}$. This probability falls to zero (i.e. the mixed strategy equilibrium collapses to a full strategy equilibrium) as σ reaches σ_2 . For higher values of noise the employer has to apply threshold defined in 3..

There exist also an equilibrium in which high type does not exert high effort with certainty. The high type is mixing between $e_H = \bar{e}_H$ and $e_H = 0$ when the two actions give the same pay-off, which happens when the employers threshold are sufficiently high, i.e. :

$$z^* = \frac{(1 + \sqrt{2})(w\theta_H - \sigma)\sigma}{w\theta_H} \quad (3.6.2)$$

For this threshold to be sustained in equilibrium, the profit on the marginal student has to be zero. I.e. the high type has to choose high effort with such probability h that, given the threshold, the following is set to zero.

$$\beta(z) = \frac{\lambda * (h * \phi\left(\frac{\sigma}{w\theta_H} - 1\right) + (1 - h) * \phi\left(\frac{z^*}{\sigma}\right))}{\lambda * (h * \phi\left(\frac{\sigma}{w\theta_H} - 1\right) + (1 - h) * \phi\left(\frac{z^*}{\sigma}\right)) + (1 - \lambda) * \phi\left(\frac{z^*}{\sigma}\right)} = 0$$

Which is solved for

$$h = \frac{\sqrt{2}w^2\theta_H + (1 + \sqrt{2})x\lambda\sigma - w(\sqrt{2}x\theta_H\lambda + (1 + \sqrt{2} + \lambda)\sigma)}{\lambda(\sqrt{2}w^2\theta_H + \sqrt{2}x\sigma - w(\sqrt{2}x\theta_H + \sigma + \sqrt{2}\sigma))}$$

This equilibrium exists only for $2w\theta_H - \sqrt{2}w\theta_H < \sigma < \sigma_3$. For $\sigma = 2w\theta_H - \sqrt{2}w\theta_H$, $z^* = \sigma$. For lower values of σ the low types exerting low effort never gets employed, hence [Equation 3.6](#) is never 0. When $\sigma > \sigma_3$ even if the high type exerts high effort with certainty, the noise is so high that the employer makes losses on the marginal employee, the equilibrium collapses to separating equilibrium in which all the high types exert high effort defined earlier.

Proof of Proposition 3.4.1

Depending on the level of σ and f , I distinguish three types of equilibria in which the high type always enrolls and exerts high effort:

1. Type 1. An equilibrium in which the strategies stay the same as in the absence of tuition fees (defined in Proposition 3.3.1)
2. Type 2. An equilibrium in which an increase in tuition fee leads to a decrease in employers threshold by the same value. In this equilibrium type low type mixes between enrolment with exerting high effort and staying out of education.
3. Type 3. An equilibrium in which an increase in tuition fee leads to a decrease in employers threshold and low type's enrolment rate. In this equilibrium type, low type mixes between enrolment with exerting low effort and staying out of education.

When $\sigma \leq \sigma_1$, when tuition fees are set to zero, the employer's threshold is set to a level that makes the low type indifferent between exerting high and low effort and the low type's utility is zero. In this range of σ , for $f < f_C$ the equilibrium is of Type 2. The introduction of tuition fees, keeping the threshold fixed, makes the return from enrolment negative. The threshold that makes the low type indifferent between enrolment and exerting high effort and staying out of education is given by:

$$z^* = w - \sigma + \frac{\sigma^2}{2w} - f \quad (3.6.3)$$

To make this threshold an equilibrium the low type's enrolment rate has to be such that the employer earns zero profit (Equation 3.2.3) on the marginal employee given the threshold, given that:

$$\beta(z) = \frac{\lambda * \phi\left(\frac{\sigma}{w\theta_H} - 1\right)}{\lambda * \phi\left(\frac{\sigma}{w\theta_H} - 1\right) + d_L * (1 - \lambda)\phi\left(\frac{\sigma}{w\theta_L} - 1\right)}$$

This happens only when $d_L = \frac{(w-x)\lambda}{w\theta_H(-1+\lambda)}$. The low type's enrolment level is equal to the low type's probability of exerting high effort in equilibrium with zero fees. This equilibrium type is sustained only when f is not too high. When $f = f_C$ the low type's best response is $\bar{e}_L = 0$ and further decrease of the low type's effort with the decrease in the threshold is not possible.

When $\sigma < \sigma_1$ and $f > f_C$ the equilibrium is of Type 3. When high type always enrolls and chooses $e_H = \bar{e}_H$ and low type enrolls only with probability d_L and always chooses $e_L = 0$,

the employer sets a threshold that maximises [Equation 3.2.3](#) given that:

$$\beta(z) = \frac{\lambda * \phi\left(\frac{\sigma}{w\theta_H} - 1\right)}{\lambda * \phi\left(\frac{\sigma}{w\theta_H} - 1\right) + d_L * (1 - \lambda) * \phi\left(\frac{z^*}{\sigma}\right)}$$

Which is solved by

$$z^*(d_L) = \sigma - \frac{(x - w)\lambda\sigma^2}{d_L w^2 \theta_H (1 - \lambda)} \quad (3.6.4)$$

The low type enrolls whenever the expected wage given the low effort exceeds the fee.

$$w * \left(1 - G\left(\frac{z(d_L)^*}{\sigma}\right)\right) \leq f$$

It is easy to show that given the employers best response when $d_L = 1$ the expected wage is lower than the threshold and when $d_L = 0$ the threshold is such that the optimal decision is to enrol, therefore the only equilibrium candidate is such a level that sets the threshold to a level that makes the low type indifferent between the enrolment and no education. In other words the threshold is such that the tuition fee is equal to the expected wage of a student exerting zero effort, which holds for:

$$d_L(f) = \begin{cases} \frac{(x-w)\lambda\sigma}{\sqrt{2}\sqrt{f}w^{3/2}\theta_H(1-\lambda)} & \text{if } f < w/2 \\ \left(\frac{\sqrt{2}\sqrt{w-f}}{\sqrt{w}} - 1\right)\sigma & \text{if } f > w/2 \end{cases} \quad (3.6.5)$$

The change of the expression at $f = w/2$ results from the shape of the single peaked density function at the median.

Substituting [Equation 3.6.5](#) into [Equation 3.6.4](#) the employer's equilibrium threshold is given by:

$$z(f)^* = \begin{cases} \sigma - \frac{\sqrt{2}\sqrt{f}\sigma}{\sqrt{w}} & \text{if } f < w/2 \\ \frac{(2\sqrt{w} + \sqrt{2}\sqrt{-f+w})(w-x)\lambda\sigma}{2w^{3/2}(f+w)\theta_H(-1+\lambda)} & \text{if } f > w/2 \end{cases} \quad (3.6.6)$$

The result implies that in Type 3 equilibrium there is a unique monotonic relation between the tuition fees and low type's equilibrium enrolment rate.

When $\sigma_1 < \sigma < \sigma_2$ and $f < f_B$ or when $\sigma_2 < \sigma < \sigma_3$ and $f < f_A$ the equilibrium is of Type 1. Tuition fee does not affect the equilibrium strategies as the fee is lower than the expected wage of a student exerting zero effort. In this range even with tuition fees low type enrolment decisions still assure positive payoff.

When $\sigma_1 < \sigma < \sigma_2$ and $f = f_B$ the low type's utility equals zero, further increase in tuition fees requires a reduction in the employer's threshold to make the low type enrol.

Therefore, for higher values of f the only equilibrium is of the Type 2, in which $z^* = w - \sigma + \frac{\sigma^2}{2w} - f$, the low type enrolls with probability $d_L = \frac{(w-x)\lambda}{w\theta_H(-1+\lambda)}$, high type always enrolls and both types exert high effort. Further increase (when $f_C < f < f_D$), similarly as in previously described case $\sigma < \sigma_1$ leads to a Type 3 equilibrium.

When $\sigma_2 < \sigma < \sigma_3$, $f = f_A$ sets the low type's utility to zero and further increase leads to Type 3 equilibrium for $f_A < f < f_B$ when $\sigma_2 < \sigma < w$ and for $f_A < f < f_D$ when $w < \sigma < \sigma_3$. When $\sigma_2 < \sigma < w$ and $f = f_B$ the equilibrium threshold reduces to a level that the low type's best response is to exert high effort and thus for $f_B < f < f_C$ the only equilibrium is of Type 2. When $f_C < f < f_D$ similarly as in the case when $\sigma < \sigma_2$ the equilibrium is of Type 3.

When $\sigma_3 < \sigma < \theta_H * w$ there is no separating equilibrium without tuition fees. However, tuition fees may help restore it. Note that since $\sigma_3 > w$, when $\sigma > \sigma_3$ low type never exerts effort (see Equation 3.3.2) therefore the mechanism existing in Type 3 equilibrium that assures the unique relation between tuition fee and low type's enrolment, given high type's enrolment and high effort applies also here. Suppose that high type always enrolls and exerts high effort, low type never exerts high effort and employer's threshold is given by Equation 3.6.6. In this case identifying the tuition fee that restores the separation is equivalent to identifying a fee that makes the high type indifferent between the high and low effort, i.e. the fee that solves the following equation.

$$w * \left(1 - G \left(\frac{z^*(f) - e_H(z^*(f))}{\sigma} \right) \right) - \frac{e_H(z^*(f))}{\theta_H} = w * \left(1 - G \left(\frac{z^*(f)}{\sigma} \right) \right)$$

The equality holds for

$$f = \frac{2w^2\theta_H^2 - 2(2 + \sqrt{2})w\theta_H\sigma + (3 + 2\sqrt{2})\sigma^2}{2w\theta_H^2} = f_E$$

For higher fees ($f_E < f < f_D$) a separating equilibrium exists.

Similarly the the case without tuition fees there exists also an equilibrium in which high type does not exert high effort with certainty. This equilibrium, described in the proof of Proposition 3.3.1, exists as long as $f < f_E$, i.e. the fee is lower than the expected utility of an agent exerting no effort given the employer's threshold in this equilibrium type.

There exist also two types of trivial equilibria. In one type, that exists for all parameters, none of the types enrolls and the employer never employs. In the other type the fee is so high that none of the types exerts effort ($f_D < f < w$). In this equilibrium both high and low types enroll only with probability such that low type enrolment probability is $\frac{\lambda(x-w)}{(1-\lambda)w}$ times greater than high type's enrolment and the employer employs with probability f/w . In this equilibrium all players receive zero utility.

Proof of Proposition 3.5.1

Of the equilibrium types described in Proposition 3.4.1 only in Type 3 an increase in tuition fee may positively affect high type's utility. In Type 1 equilibrium, an increase in tuition fees leads only to a decrease of students utilities as the equilibrium strategies do not change with tuition fees. In Type 2 equilibrium an increase in tuition fees results in a decrease of employer's threshold. However the additional pecuniary costs exceeds the savings on educational effort. A unit increase in tuition fees leads, in equilibrium, to a unit decrease in employers' threshold, which brings a unit decrease in high type's effort. Since $\theta_H > 1$ the resulting savings on effort cost are lower than the additional pecuniary costs. In Type 3 equilibrium the substitutability of tuition fees and employer's threshold is non linear. In that range, due to unique relation between f and z^* established in Equation 3.6.6, choosing the optimal tuition fee, for $f < f/2$ is equivalent to maximising the following

$$\begin{aligned} U(f, \cdot) &= \max_f w * \left(1 - G \left(\frac{z^*(f) - e_H(z^*(f), \cdot)}{\sigma} \right) \right) - \frac{e_H(z^*(f), \cdot)}{\theta_H} - f = \\ &= \max_f w - \frac{2\sigma}{\theta_H} + \frac{\sqrt{2}\sqrt{f}\sigma}{\sqrt{w}\theta_H} + \frac{\sigma^2}{2w\theta_H^2} - f \end{aligned}$$

The optimal fee is

$$f^* = \frac{\sigma^2}{2w\theta_H^2} \quad (3.6.7)$$

which results in low type's enrolment rate $d_L^* = \frac{\lambda(x-w)}{(1-\lambda)w}$ and high type's utility $U(f^*, \cdot) = \frac{(w\theta_H - \sigma)^2}{w\theta_H^2}$.

The utility as a function of f is increasing for $f_A < f < f^*$ only in the range of parameters for which Type 3 equilibrium exist. To find the global optimum I compare the utility at f^* with utility at $f = 0$ (when the separating equilibrium with no fees exists, i.e. $\sigma < \sigma_3$) when the employer's threshold is defined in Equation 3.6.1 or with utility at $f = f_E$ with the threshold defined by Equation 3.6.6 for higher values of σ . It can be shown that when $\sigma < \sigma_3$ the utility of high types under the optimal tuition fee f^* exceeds the utility under no fees, i.e.

$$\frac{(w\theta_H - \sigma)^2}{w\theta_H^2} > w - \frac{2\sigma}{\theta_H} + \frac{(w - 3w\lambda + 2x\lambda)\sigma^2}{2w^2\theta_H^2(1-\lambda)}$$

only when the proportion of high types is not too high ($\lambda < \frac{w}{2x-w}$). When $\sigma > \sigma_3$ the utility with f^* is always higher than utility with $f = f_E$.

Furthermore, f^* can be the optimal fee only if stays within the range allowed by Type 3 equilibrium, i.e. for $\sigma_2 < \sigma < w$ when it does not exceed f_B . It is the case for $\sigma > \frac{\sqrt{2}w\theta_H}{\theta_H + \sqrt{2}\theta_H - 1}$.

For lower values of σ , $f^* = f_B$ can be the optimal fee if the resulting utility exceeds utility with no tuition fees, which happens when $\sigma < \frac{2w^{3/2}\theta_H(1-\lambda)}{\sqrt{w((2+\sqrt{2})\theta_H-\sqrt{2})(1-\lambda)+\sqrt{2}-2\lambda\sqrt{w+w\lambda-2x\lambda}}}$. Resulting low type's enrolment rate is $d_L(f_B) = \frac{\sigma\lambda(w-x)}{(1-\lambda)w(\sqrt{2}w-(1+\sqrt{2})\sigma)\theta_H}$.

Proof of Proposition 3.5.2

Total welfare, given the equilibrium actions of the players, is defined as the probability of employment of good students on high skill jobs net of the total educational effort and is given by:

$$V_{SW} = \lambda * x * \left(1 - G\left(\frac{z^*(f, \cdot) - e_H^*(f, \cdot)}{\sigma}\right)\right) - \lambda * \frac{e_H^*(f, \cdot)}{\theta_H} - (1 - \lambda) * d_L(f) * e_L^*(f, \cdot)$$

To see that the social welfare is maximised for $f = f_D$ note that the first component of the equation above, given the high type's best response is to exert high effort, is not affected by a threshold and hence does not change with tuition fees. Employer's threshold and hence the students effort in each equilibrium type is weakly decreasing in f (strictly for $f > f_B$) and reaches its minimum at $f = f_D$. Finally low type's effort for $f > f_C$ is zero.

Employer's profit is given by:

$$V_E = \lambda * (x - w) * \left(1 - G\left(\frac{z^*(f, \cdot) - e_H^*(f, \cdot)}{\sigma}\right)\right) - (1 - \lambda) * w * d_L(f) * \left(1 - G\left(\frac{z^*(f, \cdot) - e_L^*(f, \cdot)}{\sigma}\right)\right) \quad (3.6.8)$$

The first component, expected profit from employing the high type, does not change with tuition fees, as long as the high type exerts high effort. The second component, loss on employing low types, depends on the level of effort and employment probability and, if the low type exerts low effort, also on the threshold level.

Substituting Equation 3.6.5 and Equation 3.6.6 into Equation 3.6.8 one can verify that

the employer's profit is given by one of the following expressions depending on parameters:

$$\begin{aligned}
V_E(e_L = \bar{e}_L \wedge (f < f_C \wedge \sigma < \sigma_1 \vee \sigma_1 < \sigma < w \wedge f_B < f < f_C)) &= \frac{(x-w)(\theta_H-1)\lambda(2w^2\theta_H + \sigma^2)}{2w^2\theta_H^2} \\
V_E(e_L = \bar{e}_L \wedge \sigma_1 < \sigma < \sigma_2 \wedge f < f_A) &= \\
&= \frac{1}{2}(x-w)\lambda \left(2 - \frac{\sigma^2}{w^2\theta_H^2} \right) - \frac{(-\sqrt{2}w^2\theta_H(-1+\lambda) + w((1+\sqrt{2})\theta_H(-1+\lambda) - \lambda)\sigma + x\lambda\sigma) \left(1 - \frac{\sigma^2}{2w^2} \right)}{\sqrt{2}\theta_H(w-\sigma)} \\
V_E(e_L = 0 \wedge f < f_A \wedge \sigma_2 < \sigma < w) &= \frac{(x-w)\lambda(2w^3\theta_H^2(1-\lambda) - (w-2w\lambda+x\lambda)\sigma^2)}{2w^3\theta_H^2(1-\lambda)} \\
V_E\left(e_L = 0 \wedge \left(f_A < f < f_B \wedge \sigma_2 < \sigma < w \vee \sigma > w \wedge f_A < f < \frac{w}{2}\right)\right) &= \\
&= \frac{(x-w)\lambda(2w^2\theta_H^2 - \sigma(\sqrt{2}\sqrt{fw}\theta_H + \sigma))}{2w^2\theta_H^2} \\
V_E\left(e_L = 0 \wedge \left(f_C < f < f_D \wedge \sigma < w \vee \sigma > w \wedge \frac{w}{2} < f < f_D\right)\right) &= \\
&= \frac{(x-w)\lambda \left(2w^3\theta_H^2 - w\sigma^2 + f \left(2w^2\theta_H^2 - 2w\theta_H\sigma - \sigma \left(\sqrt{2}\sqrt{w(-f+w)}\theta_H + \sigma \right) \right) \right)}{2w^2(f+w)\theta_H^2}
\end{aligned}$$

Firstly, employer's profit is strictly higher in equilibria in which low type exerts zero effort. For $f < w/2$ when low type's exert high effort, the median of the low type effort distribution is higher than the employer's threshold. While when the low type choose low effort, as long as $f < w/2$, signal distribution is lower than the threshold. For $f > w/2$ median of both distributions is higher than the threshold, but when the employment rate strictly lower in equilibria in which the low type exerts zero effort.

When the tuition fees and noise level are such that low effort is the low type's optimal strategy, employer's pay-off is non-monotonic in tuition fees. In those equilibria, for $f < f_A$, as in the third row, the profits are independent of f . Employer's profit is decreasing in f for $f_A < f < 2w(\sqrt{2}-1)$ and increasing for higher values of f . An equilibrium in which profit is increasing in f exists only for $\sigma < \sqrt{6-4\sqrt{2}w}\theta_H$. For higher values of σ , $f_D < 2w(\sqrt{2}-1)$ and hence the non trivial employment equilibrium does not exist. It always holds that $\sigma_3 > \sqrt{6-4\sqrt{2}w}\theta_H$.

It always holds that, whenever equilibrium exists for both $f \in [0, f_A)$ and for $f = f_D$ employer's profit is strictly higher for $f \in [0, f_A)$. The increase in profits above $f = 2w(\sqrt{2}-1)$ does not compensate the losses accruing for $f \in (f_A, 2w(\sqrt{2}-1))$.