Essays on Heterogeneous Agents, Occupational Choice, and Development

David Strauss

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, 23 July 2014

European University Institute
Department of Economics

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Examining Board

Prof. Piero Gottardi, Supervisor, EUI
Prof. Patrick Legros, Université Libre de Bruxelles, ECARES
Prof. Massimo Morelli, Columbia University
Prof. José V. Rodriguez Mora, University of Edinburgh

Abstract

The aim of this thesis is to contribute to our understanding of the consequences of economic development on the occupational choice of agents and its effects on macroeconomic variables, such as structural change and income inequality. The interplay between different types of agents is at the center of my research, both when it comes to matching between agents and sorting from agents to sectors.

The first chapter focuses on the role of financial development for structural change. When credit constrained, more talented agents sort into more labor-intensive sectors than less talented agents. When borrowing becomes more available, talented agents sort into capital-intensive sectors to optimally leverage their talent. Consequently, the capital rental rates rise and less talented agents sort into labor-intensive sectors. Thus, financial development reverses the sorting pattern. Furthermore, simulation results indicate that employment shares increase in relatively labor-intensive sectors. I show that the empirical data is consistent with these theoretical findings.

In the second chapter, I analyze the consequences of cooperation on inequality. I develop a heterogeneous agents model of cooperation distinguishing between two types of cooperation, between-task and within-task cooperation. The former is the opportunity to assign different tasks to different agents. The latter is the reassignment of tasks from one agent to another in cases where the first agent fails. Cooperation increases inequality at the top and decreases inequality at the bottom. Within-task cooperation is more inequality-enhancing than between-task cooperation. I also show that cooperation can lead to a greater skill premium in economies with a more dispersed talent distribution.

Acknowledgements

This PhD thesis is the product of five years at the EUI during which I received a great deal of help and support from many people.

First of all, the guidance, patience and ever-encouraging support from my adviser Piero Gottardi were invaluable for me. His rigor in matters of economics taught me a lot about what it takes to become a good researcher. I would like to thank him for all the hours spent in his office learning about the subtleties of research.

I would also like to thank Massimo Morelli for his advice and support during his time at the EUI, and for making my stay at Columbia University possible. I benefited a great deal from our discussions and I took many memories home with me from my stay in the Big Apple. Further, I want to thank Patrick Legros and Sevi Mora for joining the thesis committee; and generally all the researchers and professors that I had the chance to meet during my PhD, and who helped me shape my view on economics in general, and develop this thesis in particular. *Inter alia* I want to thank Dan Bernhardt, Wouter Dessein, Jan Eeckhout, Andrea Ichino, Gregor Jarosch, Joachim Jungherr, Tim Kehoe, Philipp Kircher, Fuhito Kojima, Patrick Legros, David K. Levine, and Markus Poschke.

During three years of my PhD studies, I was supported by a scholarship from the German Academic Exchange Service (DAAD), which also contributed to my research visit at the Columbia University in New York.

Special thanks goes to the administrative staff and the many other non-academic members of the EUI, who contribute to the functioning of this institution beyond all measure. In particular, I would like to thank in particular Jessica, Marcia, Lucia,

Giuseppe, Loredana, Sonia, Thomas and everybody else from the staff at Villa San Paolo.

During these last five years, I met a great many people, some of whom became very good friends along the way. I enjoyed meeting each one of them. Without you my time in Florence would not have been the same.

I want to thank my mother and sister for the love and support during all the years of my education. Family is always there for you when you need them most. Blood is thicker than water. My dear Sofia made the last years a rollercoaster, and showed me that life has to be enjoyed before anything else. Thank you for the ride and take care!

Last but not least, I would like to thank my dearest friends for having my back during all these years. F-town for life!

Mainz, May 26th, 2014

In memoriam patris mei.

Preface

This thesis consists of two papers that develop occupational choice models of heterogeneous agents to study the consequences of development on economic outcomes. The consequences of certain economic developments (financial development, technological progress) on occupational choice of heterogeneous agents are analyzed together with the effects on macroeconomic variables, such as structural change, income inequality and poverty. A lot of economic issues cannot be sufficiently well explained deploying canonical homogeneous agents models. In particular, the interaction between agents - either directly within a firm or via the labor market or indirectly via the demand for different goods/sectors - demands matching models in order to analyze in depth the consequences of exogenous change (e.g. technical progress, changes in policies or institutions) on macroeconomic outcomes. The interplay of agents' occupational choice and the supply and demand for different types of agents on the labor market, and its consequences for economic outcomes, can only be fully understand using heterogeneous agents models. Matching between agents and agents' sorting to different sectors stands at the center of my research.

In the first chapter, I develop a theory of structural change associated with financial development. More specifically, I build a heterogeneous agents model of sectoral choice with financial constraints. In equilibrium, when credit constrained, highly talented agents sort into labor-intensive sectors and less talented agents sort into more capital-intensive sectors. Negative assortative matching (NAM) prevails between agents' talent and sectoral capital intensity. By contrast, when borrowing becomes more available, highly talented agents sort into more capital-intensive sectors to optimally leverage their

talent. Thus, among unconstrained agents positive assortative matching (PAM) prevails between agents' talent and sectoral capital intensity. The equilibrium sorting reverses under financial constraints. As a consequence of financial development, i.e. less tight borrowing constraints, the capital rental rates rise and low talented agents sort into more labor-intensive sectors. Simulation results indicate that employment shares increase in relatively labor-intensive sectors. Using a two-step strategy, I find that financial development induces an increase in the alignment of the sectoral capital intensity with the average sectoral wage, consistent with my model. Simultaneously, it leads to a decrease in the alignment of sectoral capital intensity with sectoral employment shares. Both effects are not only statistically but also economically significant.

The second chapter analysis the consequences of cooperation among agents on income inequality. The historic increase in the amount of agents engaged in the production of any good is indisputable. In order to analyze its consequences, I develop a heterogeneous agents model of cooperation distinguishing between two types of cooperation, between-task and within-task cooperation. The former reflects the possibility to share the overall workload into different tasks and assign those to different agents. The latter represents the reassignment of tasks from one agent to another in case of failure of the first agent. I restrict attention to a particular, tractable information structure that yet allows both types of cooperation to occur in equilibrium. The equilibrium allocation is characterized, particularly the equilibrium sorting of agents into modes of cooperation and matching between agents. Cooperation leads to increasing inequality at the top and decreasing inequality at the bottom of the talent distribution. Within-task cooperation is more inequality-enhancing than between-task cooperation. This may help explain evolutions in income inequality in response to the information and communication technology revolution in recent years. Finally, I study how the information structure and talent distribution shape the returns to talent in an economy. Particularly, both a wider talent distribution and a better information structure are likely to increase the skill-premium. This sheds some light on potential differences in the skill-premium across countries, for example the United States and continental Europe.

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Chapter 1

Financial Development and Sorting Reversals - A Theory of Structural Change

In this paper I develop a theory of structural change associated with financial development. When credit constrained, high-skilled agents sort into more labor-intensive sectors than less talented agents. When borrowing becomes more available, highly talented agents sort into more capital-intensive sectors in order to optimally lever their talent. Consequently, capital rental rates rise and low-skilled agents sort into labor-intensive sectors. Financial development reverses the sorting pattern. Furthermore, simulation results indicate that employment shares increase in relatively labor-intensive sectors. Using a two-step strategy, I find that financial development induces an increase in the alignment of sectoral capital intensity with average sectoral wage and to a decrease in the alignment of sectoral capital intensity with sectoral employment shares, consistent with my model. Both effects are not only statistically but also economically significant.

Keywords: Financial Development, Structural Change, Occupational Choice, Sorting Reversals.

JEL Classification Number: O10, J24, L16, E44.

1.1 Introduction

An empirical phenomenon associated with development is structural change, that is, shifts in sectoral employment shares. In the literature, structural change is mainly interpreted as a consequence of non-homothetic preferences or heterogeneous technological growth rates across sectors.[1] Both theories try to explain the observed structural employment shifts during growth. Yet they are silent on changes in workforce composition within sectors over time.

Figures 1.1 and 1.1 indicate that this may be an important omission. They hint at differences in the sectoral allocation of labor that depend on the level of development in a country. Both figures consider how strongly sectoral wages are aligned with sectoral wages in the United States. Specifically, sectors within countries are ranked according to their average wage. The correlation of this ranking between a pair of countries is then labeled the sectoral wage alignment. Figure 1.1 shows that sectoral wage alignment with the United States is significantly higher for other developed countries compared to less developed countries. If average wages are a good proxy for average sectoral talent (see Abowd et al. (1999)), then this proxy suggests different sorting patterns for the agents into sectors across different stages of development. Figure 1.1 complements that view by indicating that this phenomenon is related to growth. It shows that the sectoral wage alignment between the United States and South Korea increased during a period of very high growth in South Korea.

I develop a theory of structural change that generates both employment shifts and changes in workforce composition within sectors over time. The driving force behind both individual sorting and the observed macroeconomic structural change is financial development.[2] As the financial system improves, the sectoral composition of agents

[1] Kongsamut et al. (2001) are an example of the former explanation for structural change and Ngai and Pissarides (2007) are an example of the latter. The distinction between these types of theories is between demand-side and supply-side-driven structural change.

[2] The importance of financial development for growth is well documented. See Levine (1997) for a review of the literature.

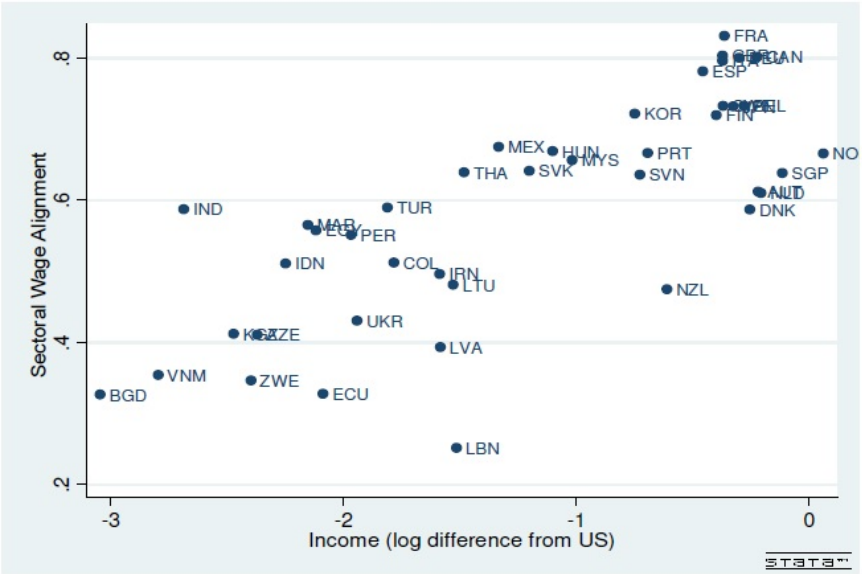

Figure 1.1: Sectoral Wage Alignment of Economies with the United States in 2000 (taken from Sampson (2011))

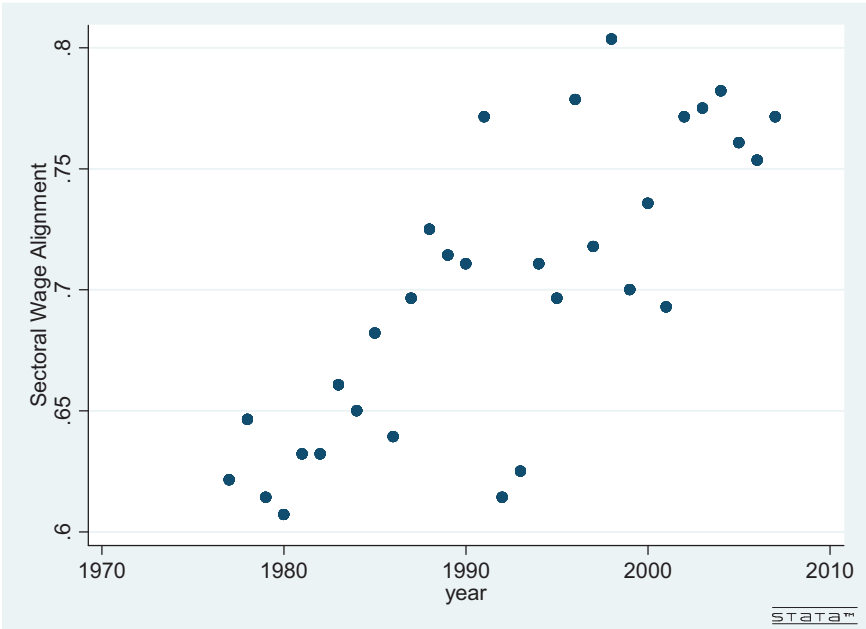

Figure 1.2: Sectoral Wage Alignment between United States and South Korea over time

changes both across and within sectors. In other words, both the number of agents and what type of agent sort into a sector depends on the level of financial development.

More specifically, I develop an occupational choice model of heterogeneous agents with financial constraints. Agents differ in their levels of talent, and sectors differ in capital productivity. In equilibrium, the sorting pattern depends on whether or not agents are financially constrained. In particular, in the set of constrained agents, the more talented ones sort into less capital productive sectors because they are less constrained in sectors where the optimal capital-labor ratio is smaller. This sorting implies that negative assortative matching (NAM) prevails for constrained agents between their talent and sectoral capital intensity. Hence, earnings are higher in labor-intensive sectors.

By contrast, for unconstrained agents, the optimal capital-labor ratio increases in talent. This implies that more talented agents sort into more capital-intensive sectors where they can obtain optimal leverage for their higher level of talent. Thus, positive assortative matching (PAM) prevails for the set of unconstrained agents, and the observed earnings are higher in capital-intensive sectors.

In equilibrium, the economy is partitioned into two convex sets. All agents below a certain cut-off level of ability are unconstrained, and PAM applies within that set. In turn, the set composed of all agents above the cut-off level of ability is characterized by NAM, and all agents are borrowing constrained. Financial development leads to an increase in the unconstrained set and a decrease in the constrained set as more agents become financially unconstrained. Thus, in the aggregate, the correlation between an agent's talent and sectoral capital intensity increases and the sorting gradually reverses. Furthermore, financial development tends to induce shifts in sectoral employment shares. The specifics of employment shifts depend on the distribution of talent and sectoral capital-intensities in an economy. Yet simulations suggest that the employment shares of labor-intensive sectors increase with financial development for a large range of specification.

I use EU-Klems data on 14 different sectors in 29 countries for up to 38 years in order to compare my theoretical predictions with empirical data. In particular, I regress sectoral capital intensity on the average sectoral wage for each country-year pair to obtain

an estimate of the alignment between sectoral capital intensity and level of talent. Then I regress these estimated alignment coefficients on the level of financial development, controlling for country and year fixed effects, as well as other candidate explanations of structural change. I carry out the same exercise for sectoral employment shares instead of average wages. I find that financial development induces both sorting reversals and structural change. The proxy for financial development, the ratio of private debt over GDP, is highly significant and has the expected sign. The debt to GDP ratio increases from an average of 0.47 to an average of 1.13 between 1970 and 2007 in the data set and is able to explain substantial parts of the increase in the observed alignment between sectoral capital intensity and the average sectoral wage in that period. In particular, the predicted change in the average alignment coefficient due to financial development is 0.09 compared to an increase on average of 0.17 over the time period. The explanatory power for structural change is smaller, albeit still significant.

The rest of the paper is organized as follows. First, I present a review of the literature. Then I explain the basic mechanism of the model in a numerical example (section 1.2). Following this, I present the general model setup in section 1.3 and characterize the competitive equilibrium (section 1.4). In section 1.5, I discuss the central sorting properties, prove the existence and uniqueness of the equilibrium, and analyze the consequences of financial development on sorting and structural change. In section 1.6, I simulate the economy to show the extent to which structural change occurs, and I also discuss the consequences on inequality. I empirically test for the two central predictions of my model in section 1.8. Finally, I conclude in section 1.9. All of the proofs are presented in the Appendix.

Review of the Literature This paper is related to several strands in the literature. There are two main explanations for structural change. On the one hand, there is a vast amount of literature on structural change based on non-homothetic preferences.[3]

[3]See Matsuyama (1992), Echevarria (1997), Laitner (2000), Caselli and II (2001), Kongsamut et al. (2001), and Gollin et al. (2002).

A similar approach is Stokey (1988) who bases her notion of structural change on a hierarchy of needs.[4] The argument is that economic growth implies that some goods in the consumer consumption basket gain in importance relative to others. These relative demand shifts lead agents to sort into sectors with increasing demand as the economy grows. On the other hand, some authors follow Baumol (1967) and explain structural change through supply-side or "technological" mechanisms.[5] The principal mechanism is that some sectors have higher exogenous growth rates in their total factor productivity (TFP). Complementarities in consumption imply that labor moves away from these sectors into sectors with slower TPF growth rates. My model is more closely related to the supply side theories of structural change. I differ in that the structural change takes place because of the decreasing supply of capital for financially unconstrained agents. The decrease in capital occurs because of financial development and leads to resorting into sectors with lower optimal capital-labor ratios.

This paper also contributes to the vast literature on the misallocation due to financial constraints in occupational choice models. The two seminal papers in the literature are Galor and Zeira (1993) and Banerjee and Newman (1993). Both papers generate misallocation from imperfect capital markets in an economy populated by agents with heterogeneous levels of wealth. In particular, they show that financial frictions cause the long-run steady state distribution of income to become dependent on the initial wealth distribution and thus generate the possibility of multiple steady states.[6] In my model the form of talent misallocation differs qualitatively from these approaches. These studies show that financial constraints imply that wealth matters for occupational

[4]For related approaches see Foellmi and Zweimüller (2002), Matsuyama (2002) and Buera and Kaboski (2012).

[5]Important contributions in that line of research are Ngai and Pissarides (2007), Zuleta and Young (2007), and Acemoglu and Guerrieri (2008).

[6]Lloyd-Ellis and Bernhardt (2000) extend this model by introducing heterogeneous talent. Other important studies exploring the consequences of financial constraints on occupational choice are Bernhardt and Backus (1990), Piketty (1997), Aghion and Bolton (1997), Galor and Moav (2000), and Gine and Townsend (2004). Ghatak et al. (2001, 2007) focus on the possibility of multiple equilibria and the political consequences of the interaction between credit markets and occupational choice.

choice, and hence misallocation occurs as some agents with higher (lower) wealth and less (higher) talent sort into industries they might not sort into without financial constraints. Nevertheless, PAM occurs conditional on wealth. By contrast, my model features sorting reversals resulting from the borrowing constraint conditional on wealth, that is, the optimal PAM pattern reverses to the NAM pattern.

Legros and Newman (2002) show that financial constraints can distort the sorting pattern. They model financial constraints by assuming that the productivity of a pair of agents has to exceed the fixed costs of production by a certain margin in order to produce output. Thus, some production cannot occur, even though the value of output exceeds fixed costs, therefore making production efficient. This inefficiency changes the surplus function of the agents and can thus distort PAM (where it was previously efficient). The authors then use a numerical example to show that PAM can actually be distorted all the way to NAM. By contrast, in my model PAM is inevitably distorted to NAM for financially constrained agents. Furhermore, they consider team production, whereas I show that sorting of agents into sectors can also be reversed without team production.

Buera et al. (2011) have recently estimated the importance of financial constraints during growth by using firm-level data for Mexico and the United States. They find that financial frictions explain substantial parts of the observed cross-country differences in income. Hsieh and Klenow (2009) illustrate empirically the importance of intra-sectoral misallocation between countries. These authors estimate the quantitative effect of misallocation in China and India compared to the United States, and find that many of the differences in income can be explained by misallocation. In a related paper Jeong and Townsend (2007) focus on inter-sectoral differences. They point out the importance of capital deepening and occupational shifts in explaining the very striking TFP growth in Thailand between 1976 and 1996. My model indicates that comparisons of very narrowly defined industries across countries may miss the point that the actual sorting pattern differs across countries. Hence, agents within the same sector may be very heterogeneous across countries.

The paper is also related to the work carried out by Sampson (2011). He observes

systematic differences in the assignment function of heterogeneous talent into sectors across countries. In contrast to my model, his aim to explain these differences through differing production functions across countries, and not financial constraints. In particular, the ranking of sectoral capital productivity differs across countries in his model, which implies different sorting of agents into sectors across countries.

The importance of financial development for growth is documented in a survey by Levine (2005). Furthermore, Beck et al. (2005) document the implications of well-developed financial systems for economic development and the alleviation of poverty.

1.2 An Example

In this section I explain the central mechanism with a numerical example before I turn to the general setup. The production function Y combines agent j's talent θ_j with capital X by using Cobb-Douglas technology, that is, $Y(j,i,X) = \theta_j^{1.2-\alpha_i} X^{\alpha_i}$ where i denotes the sector. The economy consists of a mass 1 of agents. There are two types of agents with talent $\theta_L = 1$ and $\theta_H = 4$ and two sectors with capital intensities $\alpha_1 = 0.3$ and $\alpha_2 = 0.7$. The fraction of high-skilled agents is $q = \frac{1}{6}$. The aggregate amount of capital in the economy is $\overline{X} = 15$. The goods prices are considered to be exogenous and equal to $p_1 = 0.8$ and $p_2 = 0.4$ respectively.[7]

The agents choose the amount of capital X to use and the sector i to work in in order to maximize earnings $\Pi(i,j) = p_i \theta_j^{z-\alpha_i} X(i,j)^{\alpha_i} - rX(i,j)$. The capital rental rate is denoted by r and determined in equilibrium such that the capital market clears, that is, $qX_H + (1-q)X_L = 15$ where $X_H(X_L)$ denotes the amount of capital contracted by high-skilled (low-skilled) agents.[8]

[7] I discuss the assumption of exogenous goods prices later on (see section 1.4).

[8] Agents with the same level of talent always sort into the same sector and consequently contract equal amounts of capital, except in the zero-probability event that one type of agent is indifferent to both sectors.

$X(j,i)^*$	1	2
$\theta_L = 1$	1.95	7.96
$\theta_H = 4$	11.60	80.24

Table 1.1: Optimal Capital Choices

$\Pi(j,i)$	1	2
$\theta_L = 1$	0.68	0.51
$\theta_H = 4$	4.07	5.17

Table 1.2: Sectoral Earnings

Sorting Without Financial Constraints. I consider the sectoral choice and capital allocation in an equilibrium without any financial constraints. The equilibrium capital rental rate is $r = 0.15$. The first-order condition on capital choice implies that $X(j,i)^* = \left(\frac{p_i \alpha_i}{r}\right)^{\frac{1}{1-\alpha_i}} \theta^{\frac{z-\alpha_i}{1-\alpha_i}}$ is the optimal amount of capital used in sector i by agent j. In Tables 1.1 and 1.2, I show the optimal capital choice and the earnings of the agents in both sectors respectively.

Profit maximization implies that low-skilled agents sort into the more labor-intensive sector 1 and high-skilled agents sort into the more capital-intensive sector 2. Thus, PAM prevails between agents' talent and sectoral capital intensity.

$X(j,i)^*$	1	2
$\theta_L = 1$	2.56	14.96
$\theta_H = 4$	15.20	150.78

Table 1.3: Optimal Capital Choices

Sorting with Financial Constraints. However, if there is a borrowing constraint at $\overline{x} = 20$, then no agent can use more than 20 units of capital. Table 1.1 illustrates that high-skilled agents are constrained in the more capital-intensive sector. If the interest rate remains $r = 0.15$, then their earnings are lower in sector 2 at 3.51. This causes the agents to resort into sector 1. The earnings of low-skilled agents do not change. Yet this is not an equilibrium since the capital market does not clear. In particular, there is an excess capital supply. In equilibrium, the capital rental rate falls to $r = 0.12$.

The unconstrained capital choices $X(j,i)^*$ at the new capital rental rate are shown in Table 1.3. Only high-skilled agents are capital-constrained in sector 2. The sectoral earnings are displayed taking into account that high-skilled agents only use $\overline{x} = 20$ in sector 2.

The equilibrium sorting reverses. High-skilled agents now prefer the more labor-intensive sector while low-skilled agents maximize their earnings in the more capital-intensive sector. Thus, NAM prevails between agents' talent and sectoral capital intensity. The reason for the resorting is different. High-skilled agents resort because of the borrowing constraint that is less severe in more labor-intensive sectors. Low-skilled agents, on the other hand, resort because of the equilibrium effect on the capital market. Because the capital rental rate r decreases, their potential earnings increase more in more capital-intensive sectors. This increase induces them to resort.

$\Pi(j,i)$	1	2
$\theta_L = 1$	0.74	0.80
$\theta_H = 4$	4.41	4.03

Table 1.4: Constrained Earnings

1.3 The Model

I describe now the static structure of the model:

Agents. There is a continuum of agents of unity mass. Agent j possesses wealth w in terms of the numéraire good capital and talent $\theta_j \in \mathbb{R}_{\geq 0}$.[9] Thus, the population is fully characterized by a certain level of wealth w together with a cumulative talent distribution function $\Phi(\theta)$ (the corresponding pdf is denoted $\varphi(\theta)$).[10] I assume full support for θ and label the least talented agent $\underline{\theta}$ and the most talented agent $\overline{\theta}$.

Agents choose which good to produce, how much capital to use in production, and what to consume. There is no disutility from work, and capital cannot be consumed. These two assumptions imply that all agents are active in production and that storing capital is a dominated choice.

There is a continuum of goods i. Agents' preferences are described by the utility function $u(c)$ where $u : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is twice continuously differentiable, strictly increasing, and has the following standard properties: $u'' < 0$ (i.e. the marginal utility is decreasing) and $u'(0) = \infty$. The consumption aggregator c across commodities is

$$c = \left(\int_0^1 c(i)^{\frac{\tau-1}{\tau}} \, di \right)^{\frac{\tau}{\tau-1}}$$

[9] I will use the labels talent and skills interchangeably throughout the paper.

[10] Note that the assumption of a degenerate wealth distribution is not necessary to obtain the results.

where $i \in [0,1]$ denotes the good variety. The constant elasticity of substitution (CES) between the different goods is indicated by τ.

Production. Every agent engages in the production of a single commodity. Hence there are no firms. The output of sector $i \in [0,1]$ for agent j with talent θ_j together with the amount of capital X is given by:

$$Y_i(\theta_j, X) = \theta_j^{z-\alpha_i} X^{\alpha_i}$$

with $z > 1$ and capital productivity $\alpha_i \in (0,1)$. The production function has increasing returns to scale, and the restriction on α implies that the output is concave in capital in each sector. I also assume differentiability and continuity in the function that maps the unit interval i into α, and rank the sectors according to their capital intensity, that is, $\frac{\partial \alpha_i}{\partial i} > 0$. I denote $\alpha_{min} \equiv \alpha_0$ and $\alpha_{max} \equiv \alpha_1$. Thus, the sectors differ only in terms of their capital productivity. [11]

Borrowing Constraints. Financial markets are subject to frictions. Agents can lend or borrow capital, but only up to a certain limit. The size of the borrowing limit is determined, *inter alia*, by contract enforcement rights, property rights, the strength of the judicial system, and in general the state of institutions in the economy. I define $\overline{L}(\theta_j, w)$ as the maximum amount of capital that an agent endowed with talent θ_j and wealth w can borrow. In particular, I assume that the borrowing limit depends solely on wealth w, $\overline{L}(\theta_j, w) \equiv \lambda w$ for all θ_j.[12]

[11] The qualitative result of sorting reversals is robust to the specific production function chosen. A sufficient condition is that $Y_i(\theta_j, X)$ is homothetic and exhibits increasing returns to scale between talent and capital; and further is either log-supermodular or log-submodular in capital and the sectoral characteristic. Whether the sorting reversals occur from NAM to PAM or *vice versa* depends on whether the function is log-supermodular or log-submodular.

[12] This assumption can be based on asymmetric information about borrowers' talent in the capital market that results in borrowing limits that depend solely on their wealth. However, this rather strict assumption is not necessary for the qualitative results (sorting reversals) to hold. What is required is that among constrained agents $\frac{1+\overline{L}(\theta_j, w)}{\theta_j}$ decreases in talent.

1.4 The Agents' Problem and Capital Market Clearing

In the following partial equilibrium, I assume that capital cannot be borrowed from the world market, but that goods can be bought and sold at zero transportation costs from the world economy at price $p(i)$. Thus, the economy faces exogenous goods prices $p(i)$ for $i \in [0, 1]$. I use this assumption for two reasons. The first reason is tractability. Without making any further assumptions on the distribution of agents and sectors, not all results can be proven with endogenous prices. The second reason is that the equilibrium effects on the financial markets are pivotal for the theoretical results in my model. Endogenous goods prices do not alter the results on sorting reversals a great deal as the numerical examples below show (see section 1.6).[13]

1.4.1 The Agents' Maximization Decision

The agents' maximization problem can be separated into two parts. First, the agent maximizes consumption given her income $\omega(\theta_j)$. Second, she chooses the sector and the capital investment to maximize that income.

Consumption Maximization. Because agents can buy and sell products at world market prices $p(i)$ at zero transportation cost, the consumption decision is disentangled from the production structure of the economy. In particular, the level of consumption expenditure is $\omega(\theta_j) = \int_0^1 c(i)p(i)\,di$. Given the utility function, the optimal consumption bundle is:

$$c(i) = \left(\frac{p(i)}{P}\right)^{-\tau} \omega(\theta_j) \tag{1.1}$$

where $P = \left(\int_0^1 p(i)^{1-\tau} di\right)^{\frac{1}{1-\tau}}$. For all positive prices for goods, each and every single variety of good $i \in [0, 1]$ has a strictly positive demand.

[13] Yet to some extent they do matter as regards the effects on structural change as discussed below (see 1.5.3 and 1.6).

Income Maximization. I also split the income maximization problem into two parts. First, I consider the income maximizing choice of capital conditional on the sector i chosen. Then, I turn to the income maximizing sectoral choice.

The agent chooses the capital that maximizes her income ω that is conditional on her talent and sectoral choice i:

$$\begin{aligned} \max_{X_j} \quad & \omega(\theta_j) = p(i)\,\theta_j^{z-\alpha_i} X_j^{\alpha_i} - rX_j \\ s.t. & X_j \leq (1+\lambda)w \\ & X_j \geq 0 \end{aligned}$$

where X_j is the amount of capital used in production. The loans received or given to the capital market are defined as $\tilde{X}_j = X_j - w$. If the borrowing constraint does not bind, then the optimal amount of capital $X_{j*}(i, \theta_j)$ used in the production in sector i by agent j is determined by the first-order condition:

$$p(i)\,\alpha_i\,\theta_j^{z-\alpha_i}\,X_{j*}^{\alpha_i-1} = r.$$

The amount of capital borrowed from or invested in the domestic capital market is thus $\tilde{X}_{j*} = X_{j*} - w$.[14] The marginal return to capital equals its marginal costs. The necessary condition can be rewritten as:

$$\left(\frac{X_{j*}}{\theta_j}\right)^{1-\alpha_i} = \frac{p(i)\alpha_i}{r}\,\theta_j^{z-1}. \tag{1.2}$$

The optimal capital-talent ratio is denoted $s^*(i, \theta_j) \equiv \frac{X_{j*}(i,\theta_j)}{\theta_j}$ for an agent with talent θ_j active in sector i. It measures the optimal amount of capital used per unit of skill and captures the extent to which agents optimally leverage their talent by combining it with capital. The ratio increases in the good price $p(i)$ and capital intensity α_i and decreases in the capital rental rate r. Importantly, the optimal capital-talent ratio rises with the talent of an agent because of the increasing returns to scale.[15] This rise implies that more skilled agents contract over-proportionally more capital in equilibrium.

[14] I suppress the dependencies of the choice variables on the exogenous variables for notational convenience when this does not cause confusion.

[15] If z equals to one (c.r.s.), then the ratio is identical for all agents.

Borrowing Constraints. If $X_{j*}(i, \theta_j) \geq (\lambda + 1)w$, then an agent is financially constrained in sector i and borrows up to her borrowing limit. In that case the marginal rate of capital is higher than the capital rental rate,

$$p(i)\, \alpha_i\, \theta_j^{z-\alpha_i}\, ((\lambda + 1)w)^{\alpha_i - 1} > r.$$

Whether or not the constraint binds depends on both the sector of production and the agent's talent. However, an agent's borrowing constraint is more likely to bind if she is highly talented, the sector is capital-intensive, or when general borrowing possibilities (λ) are low.

Sector of Production. Agents take into account the optimal capital choice in sector i when it comes to the sorting decision into sectors. The sectoral choice maximizes an agent's income and hence

$$i(\theta) = \arg\max_i\, p(i)\, \theta_j^{z-\alpha_i} (X_j(\theta_j, i, w))^{\alpha_i} - rX_j(\theta_j, i, w) + rw \tag{1.3}$$

where $X_j(\theta_j, i, w) \in \{X_{j*}(\theta_j, i), (\lambda+1)w\}$ depend on whether or not agent j is constrained in sector i.

1.4.2 Capital Market Clearing

Every agent is endowed with w units of the numéraire good capital. Because the economy is populated by a mass 1 of agents, the aggregate supply of capital is simply w. The aggregate demand is given by the integral over all the agents' capital choices. Every agent with the same talent sorts into the same sector in equilibrium that implies that they demand the same amount of capital. Thus, the capital market clears if and only if

$$\int_{\theta_j} X_j(\theta_j, i(\theta_j), w, r)\varphi(\theta_j)\, d\theta_j = w \tag{1.4}$$

where $i(\theta_j)$ denotes the sectoral choice of an agent with talent θ_j, and I make explicit the dependence of X_j on r.

1.4.3 Equilibrium

I can now define a competitive equilibrium in this setup as

- an allocation of agents to sectors $i(\theta_j)$,
- consumption $c(i, \omega(\theta_j))$,
- an allocation of capital to agents $X_j(i, \theta_j, w)$,
- and a capital rental price r,

such that:

1. Agents choose the capital allocation, $X_j(i, \theta_j, w)$, and the sector, $i(\theta_j, w)$, to maximize their earnings ((1.2) and (1.3)).
2. The agents choose their consumption patterns $c(i, \omega(\theta_j))$ to maximize their utility (1.1).
3. And the capital market clears (1.4).

1.5 Equilibrium Properties

Before I characterize the equilibrium, I need to make an assumption on the exogenous goods price function.

Assumption 1.1.
The price function p is continuous and differentiable.

This assumption is in accordance with the assumption of CES preferences. With endogenous goods prices, the assumption of CES preferences has two consequences. First, the price function must be continuous in i. A non-continuous price function implies that some sectors are dominated by others; that is, no agent chooses to be active in the dominated sectors. Second, $X_{j*}(i, \theta_j, w)$ is strictly monotonically increasing in i. If

$X_{j*}(i, \theta_j, w)$ were decreasing for some i, some sectors were dominated.[16] I also assume that p is differentiable.[17]

1.5.1 Sorting

The main focus of the paper is on the effect of financial development on sorting. Financial development is defined as an increase in the borrowing constraint, λ. The central theoretical result is that the equilibrium matching between agents and sectors reverses due to financial development. The reversal also depends quantitatively on other parameters of the model, but my focus is on the increases in λ because the qualitative fact of sorting reversals depends exclusively on the borrowing constraint.

Unconstrained Sorting. First, if any pair of agents is unconstrained in any pair of sectors, then the borrowing constraint does not bind for either agent in those sectors. Therefore, the marginal product of capital is equalized across both agents. In other words, the necessary condition for an interior solution (1.2) holds. In that case, the following proposition applies:

Proposition 1.1.
For any pair of agents j and j' with $\theta_j > \theta_{j'}$ sorting into two sectors i and i' with $\alpha_i > \alpha_{i'}$, PAM prevails in any equilibrium if the necessary condition for an interior solution (1.2) holds for both agents in both sectors.

The proofs of all propositions are in the Appendix. Note that not every good has to be in positive supply as I consider a small open economy with fixed goods prices $p(i)$.

The reasoning is that efficiency implies that highly talented agents use large amounts of capital relative to their talent. From the first-order condition on capital choice, we know that the optimal capital-talent ratio $\frac{X_{j*}(i,\theta_j,w)}{\theta_j}$ increases with talent under increasing

[16] In the Appendix I show that in a closed economy, all equilibrium price functions have to meet these conditions (and hence world economy prices have to meet them as well).

[17] The assumption on differentiability is only used to prove proposition 1.8 below.

returns to scale. Therefore, the most skilled agents sort into the most capital productive sectors where they can optimally leverage their higher level of talent.

Constrained Sorting. The reasoning differs when two agents are constrained in two sectors. Then, both agents borrow up to the borrowing limit irrespective of their sectoral choice. In this case, sorting is characterized by the reversed proposition.

Proposition 1.2.
For any two agents j and j' with $\theta_j > \theta_{j'}$ sorting into two sectors i and i' with $\alpha_i > \alpha_{i'}$, NAM prevails in any equilibrium if the borrowing limit binds for both agents in both sectors.

If agents are constrained, then there is no difference in capital usage between more and less skilled agents. Thus, the highly talented agents sort into the sectors with the larger marginal labor productivity.

Sorting Reversals. I expand on this result on sorting reversals by relating it to a paper by Costinot (2009) which discusses log-submodularity between agents characteristics and sectoral characteristics as a sufficient condition for PAM. In my model, the sectoral choice depends exclusively on the talent via the capital-to-talent ratio $s(i, \theta_j) \equiv \frac{X_j(i,\theta_j)}{\theta_j}$ of the agents, not θ_j directly. To see this, note that earnings for constrained agents can be rewritten as

$$p(i)\, \theta_j^z\, s(i, \theta_j)^{\alpha_i} - r\lambda w.$$

The last term is independent of the sectoral choice and thus can be discarded when it comes to sorting. Thus, the optimal choice implies that $p(i)\, s(i, \theta_j)^{\alpha_i}$ is maximized because θ_j^z affects earnings in all of the sectors equally. For unconstrained agents, the FOC (1.2) implies that the payoff is

$$p(i)\, \theta_j^z\, (1 - \alpha_i)\, s(i, \theta_j)^{\alpha_i},$$

and hence the optimal choice implies that $p(i)\, (1 - \alpha_i)\, s(i, \theta_j)^{\alpha_i}$ is maximized because again θ_j^z affects earnings in all sectors equally. Log-supermodularity prevails between s

and α_i as

$$\frac{\partial^2 \ln(\omega(\theta_j))}{\partial\alpha\, \partial s(i,\theta_j)} = \frac{1}{s(i,\theta_j)}$$

for both unconstrained and constrained agents. Thus, agents with a higher capital-to-talent ratio sort into more capital-intensive sectors.

Yet, for constrained agents, the least talented have higher capital-talent ratios because $X_j = (1+\lambda)w$ for all θ. Therefore, $s(i,\theta_j) = \frac{(1+\lambda)w}{\theta_j}$ decreases in talent. Thus, more talented agents sort into less capital-intensive sectors, and NAM holds.[18]

By contrast, the optimal capital-talent ratio $s^*(i,\theta_j) \equiv \frac{X_{j*}(i,\theta_j)}{\theta_j}$ increases in talent for unconstrained agents because of the increasing returns to scale. Therefore PAM prevails among unconstrained agents. Because more talented agents use a higher capital-talent ratio, they obtain optimal leverage for their talent if they choose the more capital-intensive sectors. With constant returns to scale ($z = 1$), the sorting between unconstrained agents is indeterminate.

Assignment Function. So far I have analyzed the sorting between any pair of agents that is either constrained or unconstrained. I turn now to the aggregate equilibrium assignment function $i(\theta)$ of agents to sectors. I first define the marginally constrained agent as $\theta^*(\lambda) \equiv \theta_j : X_{j*}(i(\theta_j),\theta_j,\lambda,w) = (1+\lambda)w$. Then, the following result holds in any equilibrium.

Proposition 1.3.

The economy can be separated into two convex sets $\theta \in [\underline{\theta}, \theta^*(\lambda)]$ and $\theta \in [\theta^*(\lambda), \overline{\theta}]$. Every agent in the set $[\underline{\theta}, \theta^*(\lambda)]$ is unconstrained, and PAM holds within the set. Every agent in $[\theta^*(\lambda), \overline{\theta}]$ is constrained, and NAM holds for these agents.

The reason is that in equilibrium, no agent can be constrained when a more talented agent is unconstrained as the marginal return to capital is higher for more talented agents. Consequently, the most talented agents are constrained. The economies rank

[18] Furthermore, NAM holds if the borrowing constraint $\overline{L}(\theta^j, w)$ depends on talent as long as $\frac{\partial\left(\frac{(1+\overline{L}(\theta^j,w))w}{\theta^j}\right)}{\partial\theta^j} < 0$.

generally somewhere between zero-borrowing constraints and infinite borrowing possibilities. Therefore, they are characterized by a mix of PAM and NAM. PAM prevails for less talented agents; while NAM prevails among more skilled agents.

Financial Development. Sorting reversals occur with financial development. As borrowing possibilities in an economy improve, PAM applies to a larger set of agents. More specifically, $\theta^*(\lambda)$ increases in λ. The following proposition characterizes the process of sorting reversals due to increases in the borrowing level λ.

Proposition 1.4.

- *For every level of wealth w, there is a $\lambda^*(w)$ such that for every $\lambda > \lambda^*(w)$ the sorting in the whole economy is characterized by PAM.*
- *For every $0 < \lambda < \lambda^*(w)$, the sorting in the economy is PAM for some agents and NAM for the others.*
- *The capital rental rate r increases in λ for all $\lambda < \lambda^*(w)$.*
- *$\theta^*(\lambda)$ increases in λ for every $\lambda < \lambda^*(w)$. $\theta^*(0) = \underline{\theta}$ and $\theta^*(\lambda^*(w)) = \overline{\theta}$.*

First, irrespective of level of wealth, there is some level of financial development for which society as a whole is characterized by PAM. In that case, further financial development has no effect on the equilibrium assignment. Second, NAM only prevails in the whole economy for zero borrowing possibilities. Because the marginal returns to capital are not equalized, any positive level of borrowing constraint implies that some amount of capital is traded. Therefore, some agents have to supply capital to the market and hence become unconstrained.

In general, financial development increases the feasible set of capital allocation to agents. An increase in λ leads to an increase in capital demand of constrained agents and consequently to an increase in the capital rental rate r. This increase implies that unconstrained agents lower the used capital-to-talent ratio. Therefore, they sort into more

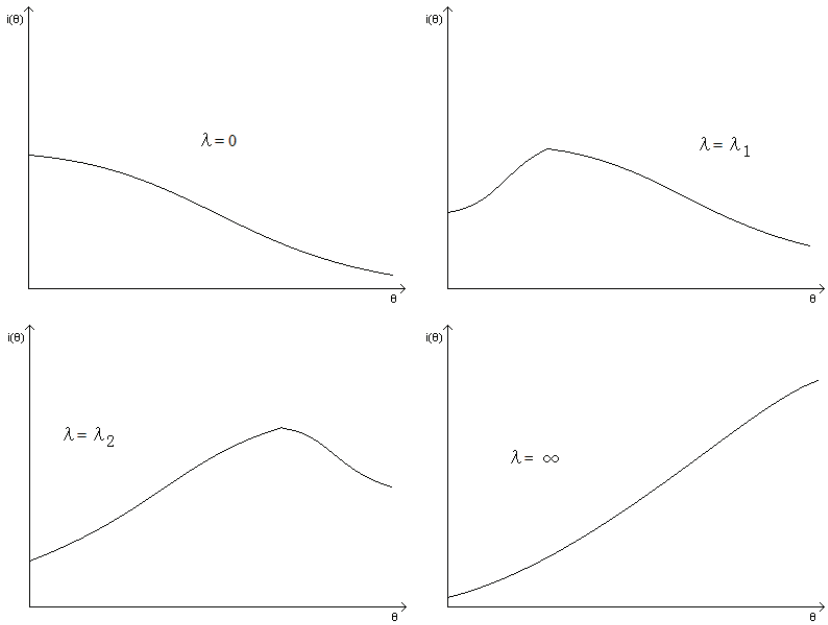

Figure 1.3: Equilibrium sorting for different λ

labor-intensive sectors as the optimal sectoral choice increases in $s(i, \theta_j)$. By contrast, constrained agents use a higher capital-talent ratio because of financial development and sort into more capital-intensive sectors. In particular, the marginally constrained agent becomes unconstrained, and the $\theta^*(\lambda)$ increases.

The process of sorting reversals is qualitatively displayed in Figure 3 which depicts the assignment function $i(\theta)$. First, $\lambda = 0$, and NAM prevails throughout the economy. The increases in λ (first to λ_1, then to λ_2) imply that some constrained agents become unconstrained and resort into less capital-intensive sectors. By contrast, the agents that remain constrained sort into more capital-intensive sectors. For the set of constrained agents, NAM prevails while the set of unconstrained agents is characterized by PAM. Note that some agents first sort upwards (while being constrained), and then sort downwards (after being unconstrained) during the process of financial development.

In particular, except $\lambda = 0$ and $\lambda > \lambda^*(w)$, there are always sectors with multiple types of agents. Specifically, constrained and very talented agents and unconstrained

agents with more modest talent sort into the same sector which maximizes earnings for both. The agent with modest talent rents out part of her capital endowment, and the highly talented agent borrows up to her constraint.[19]

1.5.2 Existence and Uniqueness.

In this subsection, I prove that the equilibrium always exists and furthermore that it is unique.

Proposition 1.5.

A unique equilibrium always exists.

Furthermore, the equilibrium is equivalent to the social planners solution given the borrowing constraint. In this sense, it is also is constrained efficient, that is, maximizing national output at given prices conditional on the borrowing constraint.[20]

Whether the equilibrium is also Pareto optimal (unconstrained efficient) or not depends on the borrowing constraint λ and the level of wealth w, as well as the distributions of talent θ and capital productivity α_i, and the degree of increasing returns to scale, z.

1.5.3 Structural Change - The Allocation of Labor.

Up to this point I have discussed the sorting reversals that occur with financial development. I will now examine the consequences of financial development on structural change, that is, the shifts in sectoral employment shares. In contrast to my results on sorting reversals, the direction of structural change depends on the openness of an economy. Hence, the assumption of exogenous goods prices is not innocent in this context. Therefore, I will first discuss the effects on employment shares in an economy with both exogenous goods prices and an exogenous capital rental rate. Then, I will turn to the qualitative predictions in the case of an endogenous capital rental rate. In the simulation

[19]The empirical observation that labor productivity dispersion within a sector is higher in developing than developed countries is consistent with this.

[20]Because complete markets exist the first fundamental theorem of welfare economics guarantees constrained efficiency (and hence existence and uniqueness).

exercise below (see section 1.6) I also use endogenous goods prices to show that the effect of financial development on structural change is heavily dependent on the openness of an economy.

It is important to note that the aggregate consequences of the resorting of constrained and unconstrained agents are not independent of the distribution of both talent and sectoral characteristics in an economy. As long as $\lambda < \lambda^*(w)$, financial development implies that constrained agents resort into more capital-intensive sectors, and unconstrained agents resort into more labor-intensive sectors. Whether or not the employment shares then increase in rather labor-intensive or capital-intensive sectors depends on the size of the change in the capital rental rate r due to financial development. This change depends quantitatively on both distributions. Thus, the relative supply of highly talented agents *vs.* less talented agents plays a role in sectoral employment shifts with financial development.

Exogenous Goods Prices and Capital Rental Rates. As a first step, I analyze employment shifts for an economy with both exogenous goods prices p and an exogenous capital rental rate r. Sorting reversals still occur in this case. Sorting for constrained agents is characterized by NAM while it is characterized by PAM among unconstrained agents. The difference is that with an exogenous capital rental rate r, unconstrained agents do not resort because their optimal capital-talent ratio does not change. Thus, because no agents resort into the more labor-intensive sectors with financial development, and constrained agents resort into more capital-intensive sectors, the following proposition holds:

Proposition 1.6.

With exogenous goods prices $p(i)$ and an exogenous capital rental rate r, the share of labor sorting into the most labor-intensive sectors decreases with financial development, that is:

$$\frac{d\left(\int_0^i \int_{\theta_j \in \Omega_i} \varphi(\theta_j) d\theta_j di\right)}{d\lambda} \leq 0 \quad \forall \quad i,$$

with the inequality being strict for every $i(\overline{\theta}) < i < i(\theta^*(\lambda))$ and $\lambda < \lambda^*(w)$.

In other words, the mass of agents that sort into sectors $[0, i]$ (weakly) decreases in the borrowing constraint, λ.

Exogenous Goods Prices and Endogenous Capital rental rates. With an endogenous capital rental rate, there are two effects. On the one hand, as λ increases, agents can obtain more leverage for their talent and thus tend to resort into more capital-intensive sectors. This direct effect is opposed by a general equilibrium effect that is only present if the capital rental rate is endogenous: As λ goes up, the capital rental rate increases, which pushes the agents towards more labor-intensive sectors.

Therefore, with an endogenous capital market, the effects of financial development on sectoral employment shares depend crucially on the initial conditions. If most agents are borrowing constrained, then the employment shares in the capital-intensive sectors increase with financial development. The reason is that for constrained agents, the direct effect dominates, and they sort into more capital-intensive sectors with financial development.

By contrast, if most agents are unconstrained, then an increase in the borrowing possibilities leads to shifts in the employment shares to more labor-intensive sectors. As the general equilibrium effect dominates for unconstrained agents, the employment shares of labor-intensive sectors increase if initially most agents are already unconstrained.

Proposition 1.7.

For low levels of financial development, a marginal increase in financial development causes employment shares to increase in relatively capital-intensive sectors. By contrast, for high levels of financial development, further increases in the financial development raise the employment shares in labor-intensive sectors. Mathematically,

$$\text{if } i(\underline{\theta}) > i(\overline{\theta}), \quad \frac{d\left(\int_0^i \int_{\theta_j \in \Omega_i} \varphi(\theta_j) d\theta_j di\right)}{d\lambda} < 0 \quad \forall \quad i < i(\underline{\theta}), \text{ and}$$

$$\text{if } i(\underline{\theta}) < i(\overline{\theta}) \text{ and } \lambda < \lambda^*(w), \quad \frac{d\left(\int_0^i \int_{\theta_j \in \Omega_i} \varphi(\theta_j) d\theta_j di\right)}{d\lambda} > 0 \quad \forall \quad i < i(\overline{\theta}).$$

In words, if $i(\underline{\theta}) > i(\overline{\theta})$ then the mass of agents that sort into the sectors $[0, i]$ decreases in the borrowing constraint λ for all $i < i(\underline{\theta})$. In contrast, if $i(\underline{\theta}) < i(\overline{\theta})$ the the mass

of agents that sort into the sectors $[0, i]$ increases in the borrowing constraint λ for all $i < i(\overline{\theta})$. This proposition analyzes shifts in sectoral employment shares for a subset of sectors and for specific initial conditions.

A second analytical result is that economies face sectoral concentration with financial development. Sectoral concentration for a set Ψ of agents is defined as a decrease in the range of the set of sectors that the set Ψ sorts into. That is, the range of $\{i \in [0,1] | i \in i(\theta) \wedge \theta \in \Psi\}$ decreases with financial development. I also define the set of unconstrained, interior agents as $\Psi_U(\lambda) \equiv \{\theta | X(\theta, \lambda) = X_*(\theta_j, i(\theta), \lambda) \wedge i(\theta) \in [0, 1)\}$. These are all unconstrained agents except for those who sort into the most capital-intensive sector. The set of constrained, interior agents is defined as $\Psi_C(\lambda) \equiv \{\theta | X(\theta, \lambda) = (1+\lambda)w \wedge i(\theta) \in (0, 1]\}$. This set contains all constrained agents except those who sort into the most labor-intensive sector.[21] Then, the following proposition holds:

Proposition 1.8.
For the sets $\Psi_U(\lambda)$ and $\Psi_C(\lambda)$ for some λ, any increase from λ to λ' leads to sectoral concentration of the sets $\Psi_U(\lambda)$ and $\Psi_C(\lambda)$.

This proposition implies that the sectoral allocation becomes more concentrated. For both the set of agents who have been unconstrained initially and the set of agents who have been constrained initially, the range of sectors the agents sort into decreases with financial development. For unconstrained agents, the reason is that earnings are most responsive to increases in r in the most capital-intensive sectors. Therefore, unconstrained agents who were initially active in sectors with higher capital intensity resort more compared to agents already active in relatively labor-intensive sectors. Therefore the set of sectors unconstrained agents sort into decreases. In much the same way, for constrained agents earnings are most responsive to improvements of borrowing possibilities in the more capital-intensive sectors. Therefore, the most talented constrained agents, who are initially active in very labor-intensive sectors, resort the most. Thus, the set of sectors constrained agents sort into also decreases. This proposition does not hold for agents that

[21] I have made this exclusion because the first-order condition for sectoral choice does not hold for excluded agents; that is, they do not choose an interior solution.

become unconstrained with financial development, but only for those who were either already unconstrained initially, or who remain constrained after financial development.

However, this result on sectoral concentration does not tell us where these agents center. In particular, either there is a concentration towards relatively intermediate sectors (neither very labor-intensive nor very capital-intensive) or sectoral polarization - both very labor-intensive and very capital-intensive sectors experience increases in sectoral employment shares. Numerically, I find that - particularly in the late stages - financial development displays concentration in form of sectoral polarization. Modestly talented agents sort into very labor-intensive sectors because of the high capital rental rates while highly talented agents sort into very capital-intensive sectors to optimally leverage their higher levels of talent. This qualitative feature of structural change depends on the assumption of exogenous goods prices in contrast to the first one discussed. In simulations, using endogenous goods prices, this polarization only exists for very high levels of elasticity of substitution in consumption.

1.6 Simulation

I simulate the model in order to show the differences between the effect of financial development on structural change with exogenous and endogenous goods prices. In particular, simulations of a closed economy indicate that sectoral employment shares in labor-intensive sectors increase with financial development.

Endogenous goods markets imply an additional force for shifts in sectoral employment shares. Financial development causes the unit of labor active in capital-intensive sectors to become more productive as compared to labor-intensive sectors. Both the talent and the capital active in capital-intensive sectors increase with financial development. Complementarities in consumption imply that labor-intensive sectoral employment shares grow.[22] This is essentially the mechanism that drives the structural change in models based on a technological structural change such as Acemoglu and Guerrieri (2008).

[22] With substitutability in consumption this would tend to increase employment shares in capital-intensive sectors.

In a closed economy, goods markets have to clear alongside the capital market. Market clearing requires that for every sector i,

$$\int_{\theta} c(i,\theta)\varphi(\theta)d\theta = \int_{\theta\in\Omega_i} Y(i,\theta)\varphi(\theta)d\theta$$

where Ω_i denotes the set of agents active in sector i. The equation can be rewritten as

$$\int_{\theta} \left(\frac{p(i)}{P}\right)^{-\tau} \omega(\theta)\varphi(\theta)d\theta = \int_{\theta\in\Omega_i} \theta^{z-\alpha_i}(X_j(\theta,i,w))^{\alpha_i}\varphi(\theta)d\theta$$

where $P = \left(\int_0^1 p(i)^{1-\tau}di\right)^{\frac{1}{1-\tau}}$. The amount of any good supplied in the economy has to equal the demand for that commodity. The optimal consumption pattern between two sectors i and k for any agent j together with goods market clearing implies that

$$\left(\frac{p(i)}{p(k)}\right)^{-\tau} = \frac{\int_{\theta\in\Omega_i} Y(i,\theta)\varphi(\theta)d\theta}{\int_{\theta\in\Omega_k} Y(k,\theta)\varphi(\theta)d\theta}.$$

Parameter Choice. Technology-driven theories of structural change such as Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008) require either complementarity or substitutability in consumption in order to generate structural change. Therefore, I simulate the model with an elasticity of consumption $\tau = 1$ to shut down this channel. Thus, the simulation also helps to highlight the additional channel of structural change present in my model and differentiates my approach from theirs.

The degree of increasing returns is $z = 1.2$. Different choices for that parameter make the results more pronounced, but do not change them qualitatively. In principle, an increase in z is equivalent to an increase in the skewness of the talent distribution. I use 85 industries whose capital intensity α is uniformly distributed between 0.3 and 0.9. The distribution of talent is log-normal. The amount of agents is $10,000$.[23] I give the results using two wealth levels, 50 and 100, and the degree of borrowing possibilities ranges from $\lambda = 0.01$ to $\lambda = 10$. This allows me to discriminate between the channels of economic and financial development.

[23] I am aware that the amounts of agents and sectors are rather low for the theoretical two-sided continuum assumption considered. These amounts also prevent the continuity result for sectoral choice from becoming visible. Neither an increase in the amount of sectors nor the agents changes the qualitative picture. Moreover, different talent distributions do not change the qualitative results.

Parameter choices	
z	1.2
α_{min}	0.3
α_{max}	0.9
τ	1
# Industries	85

Results. The main conclusion of the simulation is that financial development leads to substantial shifts in sectoral employment shares to more labor-intensive sectors in a closed economy. I have experimented with a variety of different parameter constellations and the effects of financial development on structural change do not reveal much variation.

Furthermore, financial development plays quite a different role from economic development, which is represented by increases in the wealth endowment level. Both types of development imply that the economy is able to produce capital-intensive goods at a lower cost. Hence, the price of labor-intensive goods raises. [24] The intuition for the increase in labor-intensive goods prices with financial development is that output is most responsive to a relaxation of the borrowing constraint in the most capital-intensive sectors. Furthermore, sorting reversals induces the talent of agents active in capital-intensive sectors to increase. Both forces imply that financially developed countries have a comparative advantage in the production of capital-intensive goods. Hence, the relative prices of labor-intensive goods increase with financial development.

Yet the capital rental rate decreases with economic development while it increases with financial development.[25] Therefore, economic development leaves the sorting pattern

[24] The exact amount of increase in my simulation is reported below.

[25] Here too, the respective numbers of my simulation are provided below.

largely unchanged whereas financial development induces both sorting reversals and structural change. The reason is that higher levels of wealth make capital in general more abundant, and thus lower the capital rental rate. Thus, the capital demand increases as well as its supply. In reality, financial and economic developments go hand in hand. Thus, my theory predicts the prices of labor-intensive goods to increase but whether the interest rate increases or decreases with development is *ceteris paribus* ambiguous and depends on the specifics of the economy.

In Figures 1.4 and 1.5, the assignment function $i(\theta)$ of the agents to sectors is depicted for two wealth levels $w = \{50, 100\}$ and four different borrowing limits, $\lambda = \{0.01, 0.4, 1, 10\}$. The x-axis depicts the talent percentile of the agents, and the y-axis depicts the sectors. With almost no borrowing possibilities, $\lambda = 0.01$, almost every agent is borrowing constrained. Only the very least talented agents lend capital to all other agents. Therefore, the sorting pattern is characterized by NAM for almost all agents for both levels of wealth. When borrowing opportunities increase, more agents become unconstrained. As shown above analytically, unconstrained agents form a convex set at the lower end of the talent distribution. Sorting among these agents is characterized by PAM, whereas sorting among constrained agents is characterized by NAM. The discontinuity in the sorting pattern occurs due to the discretized version of the model. Thus, agents do not smoothly resort from the most capital-intensive sectors into the more labor-intensive ones.[26]

Even with borrowing possibilities of ten times its wealth, almost 5% of the population remains constrained. This is because the log-normality of the talent distribution implies huge differences for optimal capital choices, in particular among the top percentiles of the talent distribution. As a comparison of the two figures indicates, wealth has a very small impact on the sorting pattern. No large differences exist in the sorting, although the level of wealth doubles.[27]

Figure 1.6 shows the aggregate allocation of labor.[28] In a closed economy, financial

[26] This discontinuous sorting would not occur with a continuum of sectors.

[27] Different levels of wealth do not change the result.

[28] I only show the results for wealth $w = 50$ as it is clear that the graph for $w = 100$ has to look very

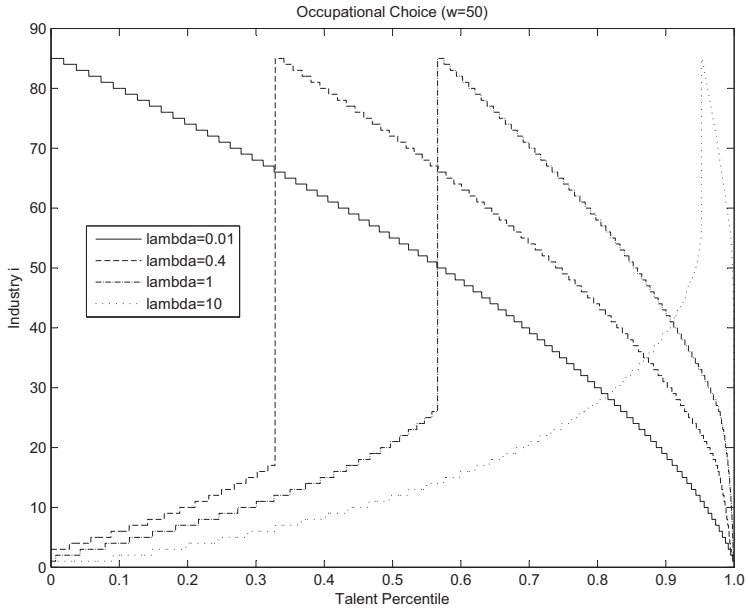

Figure 1.4: Occupational Choice

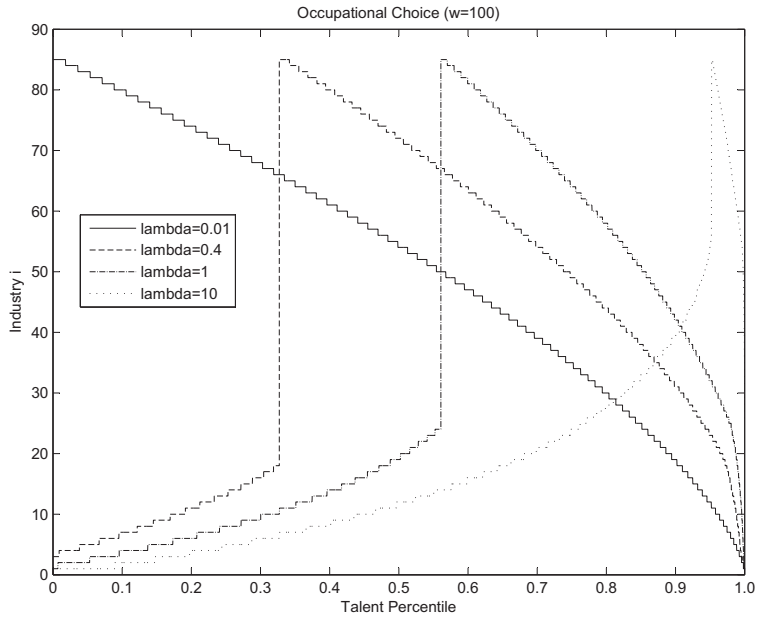

Figure 1.5: Occupational Choice

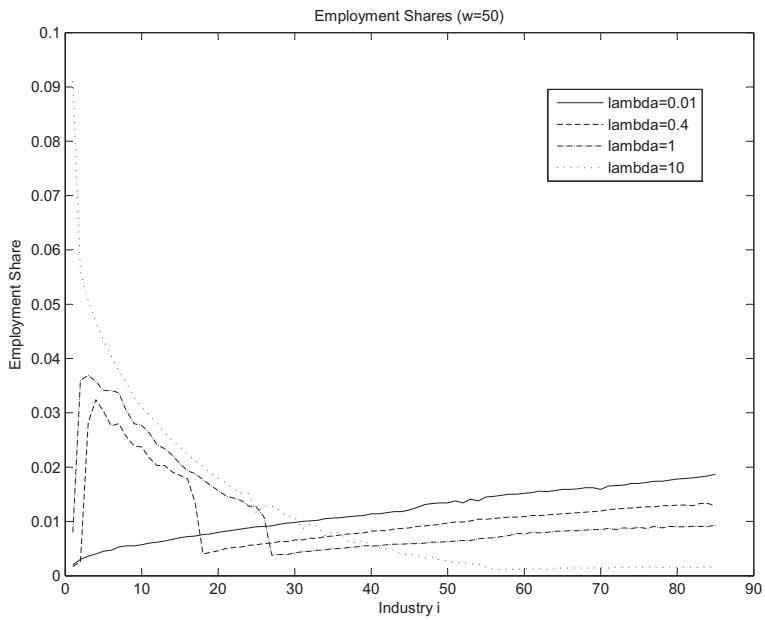

Figure 1.6: Employment Shares

development implies that sectoral employment shares of labor-intensive goods increase. This reverses the analytical result on structural change in an open economy with both fixed goods prices and capital rental rates. With endogenous goods prices, the model implies that the employment shares in the labor-intensive sectors increase with financial development. Note that this effect is not only because there are more low-skilled than high-skilled agents in the economy. Sectoral employment shares in the labor-intensive sectors for high levels of financial development far exceed those of the capital-intensive sectors at low levels of development.

The pattern of structural change is similar when I consider different levels of elasticity of substitution, although the quantitative picture looks slightly different. In particular, sorting is more responsive to changes in wealth in these cases. More specifically, in line with Acemoglu and Guerrieri (2008), I find that a high level of substitutability leads to increases in the employment shares of the capital-intensive sectors due to increases in wealth and the opposite occurs for low levels of elasticity of substitution.

similar.

The capital rental rate increases substantially with financial development. This effect is more pronounced for low levels of wealth. In the two scenarios depicted in the figures, the rate rises from $r = 0.0976$ to $r = 0.1197$ with level of wealth of $w = 50$, and from $r = 0.0598$ to $r = 0.740$ with a wealth level of $w = 100$. These numbers also show the effect that an increase in wealth reduces the capital rental rate. Financial development also leads to an icrease in the relative prices for the labor-intensive goods. As already mentioned, relative prices also increase with wealth. Considering a wealth level of 50, the relative price between the most labor-intensive and most capital-intensive good rises from $\frac{p(1)}{p(85)} = 5.1655$ to $\frac{p(1)}{p(85)} = 6.4418$. By contrast, with a wealth level of 100, the relative price increases from $\frac{p(1)}{p(85)} = 8.5358$ to $\frac{p(1)}{p(85)} = 9.7297$.

Comparison Between Open and Closed Economies. If the economy faces both an exogenous capital rental rate and exogenous goods prices, financial development causes sectoral employment shares to increase in the capital-intensive sectors. Hence, the results are reversed. With an endogenous capital rental rate, sectoral concentration occurs. Simulation results indicate that for financially developed countries this concentration takes the form of sectoral polarization. Hence, I conclude that both the strength and the direction of structural change due to financial development are highly influenced by the openness of an economy.

1.7 Earnings Inequality

This section briefly addresses the effects of financial development on income inequality. In particular, I show that financial development leads to a decrease in low-income inequality and a simultaneous increase in top-income inequality. This result holds both for exogenous and endogenous goods prices. In order to demonstrate this, I will first show the analytical results with exogenous goods prices and then turn to simulation results with endogenous goods prices.

1.7.1 Labor Income Elasticity

Analytically, the elasticity of the labor earnings can be used as a measure of inequality. A higher earnings elasticity for some level of talent implies a steeper earnings function at that point.

Unconstrained Sorting. Among unconstrained agents, the elasticity of labor earnings is largest for the most talented agents within that set. The reason is that more skilled agents contract over-proportionally more capital and sort into sectors where the output is more responsive to increases in the capital-talent ratio. Mathematically, the labor earnings of an agent j are given by

$$\nu(i(\theta_j), \theta_j, w) = p(i(\theta_j))\, \theta_j^{z-\alpha_i}\, X_j(i(\theta_j), \theta_j, w)^{\alpha_i} - r\, X_j(i(\theta_j), \theta_j, w).$$

When an agent is unconstrained, the envelope theorem implicates that the marginal labor earnings are

$$\frac{\partial \nu(i(\theta_j), \theta_j, w)}{\partial \theta_j} = \frac{z-\alpha_i}{1-\alpha_i}\, r \left(\frac{p(i)\alpha_i}{r}\right)^{\frac{1}{1-\alpha_i}} \theta_j^{\frac{z-1}{1-\alpha_i}}\, \frac{1-\alpha_i}{\alpha_i},$$

and hence the earnings elasticity reduces to

$$\epsilon_\nu(i(\theta_j), \theta_j, w) = \frac{\frac{\partial \nu(i(\theta_j), \theta_j, w)}{\partial \theta_j}\theta_j}{\nu(i(\theta_j), \theta_j, w)} = 1 + \frac{z-1}{1-\alpha_i}.$$

This equation shows that the earnings elasticity increases with the capital intensity of a sector and therefore also with talent because PAM prevails for unconstrained agents. Hence, the curvature of the earnings function is steepest for the most talented agents. This is consistent with the empirical observations on wage inequality which found that it is largest among the top percentiles of the income distribution.[29]

[29] I can also show this curvature more directly. The first derivative of the labor earnings function can be rewritten as $\frac{\partial \nu(i(\theta_j), \theta_j, w)}{\partial \theta_j} = \frac{z-\alpha_i}{(1-\alpha_i)\theta_j}\nu(i(\theta_j), \theta_j, w)$. This equation allows me to calculate the second derivative as $\frac{\partial^2 \nu(i(\theta_j), \theta_j, w)}{\partial \theta_j^2} = \frac{z-1}{1-\alpha_i}\frac{z-\alpha_i}{(1-\alpha_i)\theta_j^2}\nu(i(\theta_j), \theta_j, w) + \frac{z-1}{(1-\alpha_i)^2\theta_j}\nu(i(\theta_j), \theta_j, w)\frac{d\alpha_i}{d\theta_j}$. Because PAM prevails, the last term is positive and so is the second derivative. Hence, the labor earnings function is convex for unconstrained agents.

Constrained Sorting. This result no longer holds unambiguously for constrained agents. Their earnings can be represented by

$$\nu(i, \theta_j, w) = p(i)\, \theta_j^{z-\alpha_i}\, (w + \lambda w)^{\alpha_i} - (1 + \lambda) r w.$$

Given that $\frac{\partial \nu(i,\theta_j,w)}{\partial i} = 0$ because of the maximization problem for the agent, and $\frac{dw}{d\theta_j} = 0$ because of the talent-independent borrowing constraint; the earnings derivative is

$$\frac{d\nu(i, \theta_j, w)}{d\theta_j} = \frac{\partial \nu(i, \theta_j, w)}{\partial \theta_j} = (z - \alpha_i)\, p(i)\, \theta_j^{z-\alpha_i-1}\, (w + \lambda w)^{\alpha_i},$$

and the earnings elasticity is

$$\frac{\frac{\partial \nu(i(\theta_j),\theta_j,w)}{\partial \theta_j} \theta_j}{\nu(i(\theta_j), \theta_j, w)} = \frac{z - \alpha_i}{1 - \left(\frac{w+\lambda w}{\theta_j}\right)^{1-\alpha_i} \frac{r}{p(i)\theta_j^{z-1}}}.$$

The denominator is equal to the labor intensity of sector i populated by constrained agents with talent θ_j and therefore necessarily increases in talent. However, the numerator also increases in talent because of NAM. This implies that in general for constrained agents, whether the wage inequality is higher among low-skilled or high-skilled agents is ambiguous.[30]

Effects of Financial Development. I now turn to the effects of financial development on wage inequality. First, I consider agents who are already unconstrained before the increase in λ. Because these agents sort into more labor-intensive sectors, their wage elasticity decreases. Thus, the labor income inequality for agents with modest levels of talent falls because of financial development. Because less talented agents lend more capital to the market and the capital rental rate increases with financial development, this effect is reinforced when considering overall income. This is a potential explanation for the recent decrease in inequality among low-skilled agents in developed countries. Financial development is relatively good news for the least talented agents because they sort into the most labor-intensive sectors where the rise in the capital rental rate r does not affect them much.

[30] In the same way, the sign of the second derivative cannot be established unambiguously.

By contrast, the effects on constrained agents are generally ambiguous. Because constrained agents sort into more capital productive sectors due to financial development, both the labor intensity (denominator) and the labor productivity (numerator) decrease. But eventually, the first effect dominates the latter and the inequality increases. The reason is that the income elasticity converges with the equation for unconstrained agents in which case an increase in α_i unambiguously increases the income inequality. This is in line with the huge increases in income inequality within the top percentiles of the income distribution in recent years. The argument is that while modestly talented agents already have been unconstrained, the top agents have become more and more unconstrained in recent years, which implies an increase in wage inequality among them.

1.7.2 Simulation Results

Figure 1.7 depicts the earnings function for three different levels of borrowing constraints and a wealth level of $w = 50$. The other parameters are as discussed earlier. As in the case of the exogenous goods prices, financial development decreases the income inequality among low-skilled agents, while at the same time increasing the top income inequality. In contrast to the theoretical analysis I depict overall earnings here, i.e. labor earnings + capital earnings. The reason is that I want to discuss both effects. Nevertheless, I will also present the change in inequality for pure labor earnings.

The principal reason for the decreasing inequality among low-skilled agents is that the least talented agents profit the most from an increase in the capital rental rate r because their opportunity cost of renting out capital is very low. In Figure 1.8, I display the decrease in inequality among low-skilled agents. The $50 : 10$ earnings ratio decreases from 1.21 to 1.15 with financial development. The respective ratio for pure labor income decreases from 1.84 to 1.79.

The increase in earnings inequality in the top percentiles of income distribution occurs because the highest benefits from additional borrowing possibilities occur for the most talented individuals as their marginal return from employing more capital is the highest. The ratio of earnings between the 99th and 90th percentiles increases from 2.11 to 2.84.

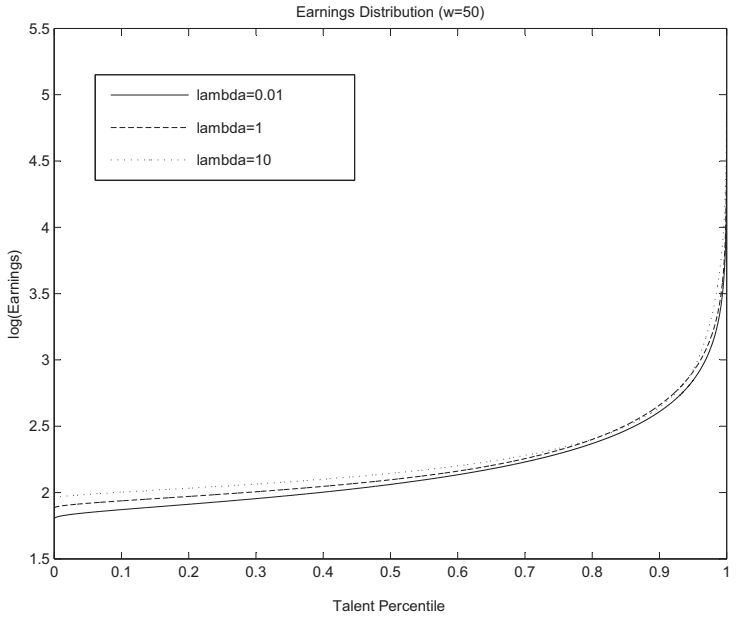

Figure 1.7: Earnings Distribution

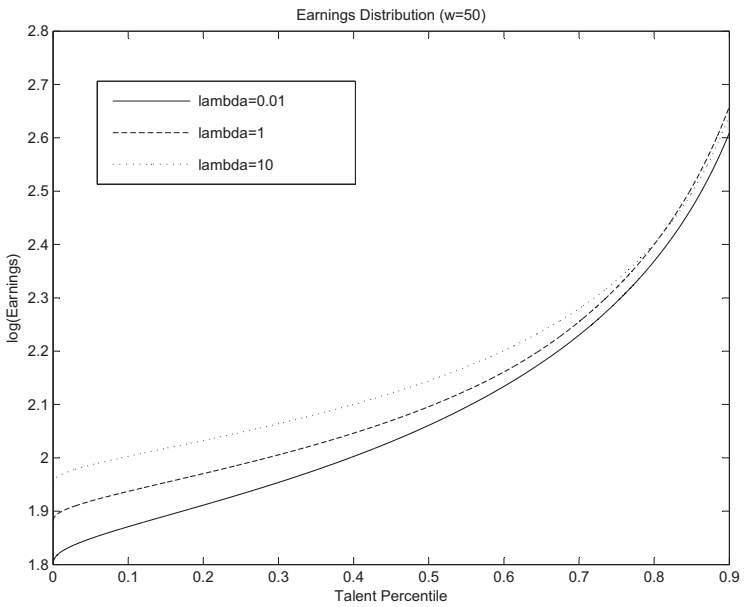

Figure 1.8: Earnings Distribution - Bottom

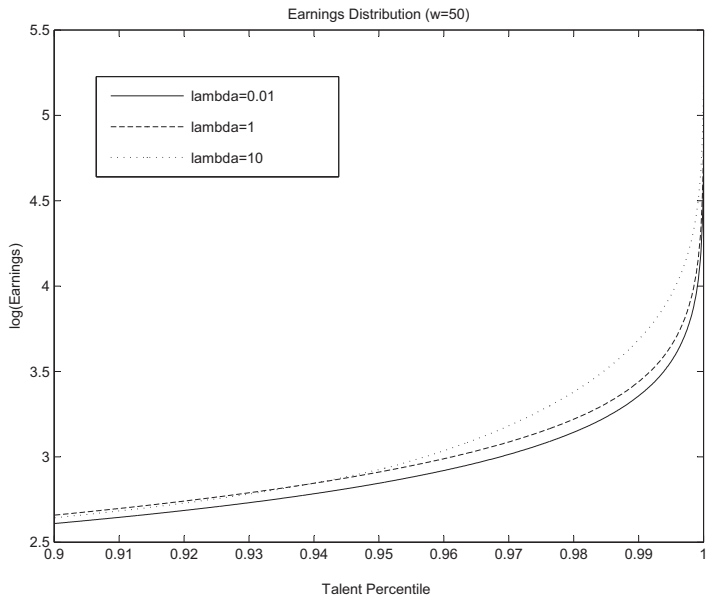

Figure 1.9: Earnings Distribution - Top

Figure 1.9 shows this huge increase in top-income inequality. The increase in pure labor income is even from 2.73 to 4.20. Furthermore, not all agents necessarily gain from financial development. The figures show that the rise from $\lambda = 1$ to $\lambda = 10$ makes some talented agents worse off when borrowing constraint increases. To summarize, I find that financial development has important impact on earings inequality. In particular, it can explain the simultaneous decrease in low-income inequality and the huge increases in top-income inequality that is empirically observed. This holds both with exogenous and endogenous goods prices and for both labor earnings and total income.

1.8 Empirics

I now compare the theoretical predictions with empirical findings. A word of caution here: in this section my aim is not to establish any causality, but rather to check whether the empirical data supports my theory. The two central propositions of my model are that financial development induces sorting reversals and structural change. The former

implies that the set of agents for which PAM holds increases. The latter implies that at least for closed economies, the set of agents that sort into labor-intensive industries increases.

So far I have used the terms capital productivity and capital intensity interchangeably. Empirically, I cannot observe the sectoral capital productivity α. Thus, I proxy it with the capital intensity of a sector; that is, the share of value added that accrues to capital. Lemma 1.1 shows that this is consistent with my model. Higher values of α are associated with higher levels of capital intensity.

Lemma 1.1. *The capital intensity increases with the capital productivity α for both constrained and unconstrained agents.*

Yet, the alignment of capital intensity with average talent is negative for constrained agents because of NAM and is positive for unconstrained agents because of PAM. This implies that I expect the alignment of the capital intensity with the average talent of a sector to increase with financial development as the set of agents characterized by PAM increases.

Furthermore, I have used the term "agent" without resorting to any notion of worker or entrepreneur in the theoretical part. To be more precise in this section I assume them to be entrepreneurs in the following as they contract capital. Yet, I do not possess data on individual entrepreneurs, and hence I am unable to test the proposition of sorting reversals directly. Instead, I test how well the sectoral capital productivity is aligned with the average sectoral wage. The latter is a proxy for entrepreneurial talent. In the Appendix 1.10.2, I provide a theoretical extension in which the entrepreneurs can employ both capital and labor to show that the paid wages are aligned with entrepreneurial talent. Furthermore, sorting reversals occur in this extension as well. Among constrained entrepreneurs, the more talented ones sort into more labor-intensive sectors. By contrast, more skilled entrepreneurs sort into more capital-intensive sectors when they are unconstrained. Therefore, the average sectoral wage is an appropriate proxy for the average entrepreneurial talent in a sector.[31]

[31] If, in turn, agents are assumed to be workers, then wages are a proxy for talent. Empirically, data

This leads to the first theoretical prediction to be tested empirically:

Prediction 1.1. *The alignment of the average sectoral wage with sectoral capital intensity increases with financial development.*

This prediction is independent of whether or not goods prices are endogenous, that is, whether the economy is closed or open.

By contrast, the theoretical predictions for the direction of structural change depend on the openness of an economy. The above simulations indicate that in a closed economy, sectoral employment shares in labor-intensive sectors increase with financial development. These numerical results are robust for various different specifications. In the extension more entrepreneurs sort into more labor-intensive sectors with financial development. Yet, the fact that more talented entrepreneurs employ more workers may counteract the robust simulation results from above. This makes the following prediction is rather suggestive.

Prediction 1.2. *The alignment of the employment share with sectoral capital intensity decreases with financial development in a closed economy.*

I use a two-step strategy to test the predictions of my model. To test for sorting reversals, I regress sectoral capital intensity on the average sectoral wage for each country-year pair. Then, in a second step, I regress the estimated β-coefficients on financial development. In the same way, I regress sectoral capital intensity on the sectoral employment share for each country-year pair to test for structural change. However, in the second step, I not only regress on the financial development indicator, but also on the interaction between financial development and economic openness to assess the effect of financial development in a closed economy.

on the skill levels of workers by sector is only available at high levels of aggregation for most countries. The evidence suggests that wages are a good proxy for talent (see, e.g. Krueger and Summers (1987) and Abowd et al. (1999)). Yet, this assumption were to imply that the borrowing constraint of the firm would depend on the number of workers.

1.8.1 Alternative Candidate Explanation - Technological Change

I am aware that there are other candidate explanations for structural change and for sorting reversals. Non-homothetic preferences and technological change are the most prominent explanations for structural change. Technological change is also a potential candidate explanation to explain sorting reversals. I will briefly present a model of technological change that can explain the change in the alignment of the average sectoral wage with sectoral capital intensity over time.[32]

The models presented by Ngai and Pissarides (2007), and Acemoglu and Guerrieri (2008) on technologically-driven structural change cannot simply be extended to incorporate sorting reversals along the growth path. Extending their models to allow for heterogeneous talent and increasing returns to scale, is not enough to generate sorting reversals. The reason is that any change in the ranking of sectors with respect to their capital intensity is accompanied by a change in the sorting behavior of heterogeneous agents. In other words, the relationship between talent and sectoral capital-intensity remains the same.

In order to observe the empirical pattern of sorting reversals, the sectors that make more intense use of highly-skilled labour have to become more capital-intensive over time. A simple production function that can generate sorting reversals is a Cobb-Douglas production function in capital, high-skilled and low-skilled labor. Output in sector i at time t is given by

$$Y_{it}(L_H, L_L, X) = K^{\alpha_{it}} \left(L_H^{\beta_{it}} L_L^{1-\beta_{it}} \right)^{1-\alpha_{it}}$$

where K is the amount of capital used in production, L_L is the amount of low-skilled labor, and L_H the amount of high-skilled labor. The capital intensity at time t is denoted by α_{it} and the high-skill intensity among labor by β_{it}. With some fixed amount of capital, high-skilled and low-skilled labor in the economy, there will be some equilibrium capital rental rate r_t and market clearing wages w_{Ht} and w_{Lt} for the high-skilled and low-skilled labor.

[32] Two key contributions in the literature that relate the sorting pattern to technological progress are Galor and Tsiddon (1997) and Caselli (1999).

If the capital intensity in the more high-skill intensive sector (with large β_{it}) is low (low α_{it}), then average wages are higher in the labor-intensive sector. High capital intensity (high α_{it}) in the more high-skill intensive sector means that average wages are relatively high in the capital-intensive sector. Hence, if technological change implies that the capital intensity of the relatively high-skill intensive sectors increases over time, then average wages increase in capital-intensive sectors as the proportion of high-skilled agents in those sectors increases. Such an *ad hoc* model of exogenous technological change can replicate my predictions for sorting reversals. I will therefore include technological change as an alternative explanation for sorting reversals in my regressions.

1.8.2 Data

The main data set is taken from the EU-Klems database that covers 29 countries over the period 1970 - 2007. In my main specification, I use 14 industries for each country-year pair.[33] In particular, the variables on sectoral capital intensity, average wage, and the employment share are taken or constructed from this database. I match the data set with the financial development indicators from the World Bank Financial Development Database. Further controls are taken from the UN database, the Penn World Table, and the CANA database.

The EU-Klems database comprises 25 European Union countries plus Australia, Japan, Korea, and the United States.

The specific advantage of this data set is that it covers the whole economy. The analytical analysis shows that the process of resorting as a result of financial development is non-monotonic for some sectors. More specifically, many sectors experience both increases and decreases in average sectoral talent during periods of financial development.

[33] Due to its specific characteristics I have excluded the agricultural sector. I also do not disaggregate the manufacturing sector any further, because otherwise the results obtained may depend heavily on one sector with an average employment share of 20% (which decreased from approximately 25% to 17% in the time period). Furthermore, McMillan and Rodrik (2011) reveal the special nature of manufacturing (unconditional convergence), and this raises further doubts about the advisability of including more disaggregated data on that sector.

Therefore, the theoretical predictions on the effect of financial development may not be clear if sectoral coverage is only for a subset of the economy.

A central question is how to proxy for financial development. Theoretically, financial development is equal to increases in the borrowing possibilities of the agents. I proxy borrowing possibilities with actual borrowing.[34] The indicator for the level of financial development is the ratio of private debt over GDP. In particular, the ratio is private credit by deposit money banks and other financial institutions over GDP.[35]

There are other candidate explanations of structural change, in particular based on non-homothetic preferences and technological change. Therefore, I control for these two channels. I use the $log(GDP)$ as a proxy for non-homothetic preferences. As income increases, the optimal consumption basket changes. This leads to changes in sectoral employment shares because of the altered demand. In particular, if consumption shifts to more labor-intensive goods with income (e.g. services), I expect that GDP per capita causes the alignment of sectoral employment shares with sectoral capital intensity to decrease.

It is more difficult to control for technology-driven structural change. The commonly used proxy for technological change is the relative price for investment goods (versus consumption goods). The data is taken from the Penn World Table and translated using investment-specific and consumption-specific PPP exchange rates. Because I am interested in the relative price of investment faced by domestic producers, I follow Restuccia and Urrutia (2001) and Karabarbounis and Neiman (2013), and divide the relative price of a country's investment by that of the United States. This ratio is then

[34] As long as any increase in borrowing constraints is accompanied by an increase in actual borrowing, this approximation holds. In my model this occurs as long as the economy is not completely unconstrained.

[35] This is a commonly used proxy for financial development taken from the World Bank Financial Development Database by Thorsten Beck, Asli Demirguc-Kunt, Ross Eric Levine, Martin Cihak and Erik H.B. Feyen, labeled "pcrdbofgdp" in the data set (see, e.g. Beck et al. (2000) and Beck et al. (2007)). For robustness checks, I also use the ratio of private credit by deposit money banks over GDP (labeled "pcrdbgdp" in their database) and total liabilities over GDP ("llgdp", for example used in King and Levine (1993)).

multiplied by the ratio of the investment price deflator to the personal consumption expenditure deflator for the United States, which is obtained from the U.S. Bureau of Economic Analysis. This gives the relative price of investment measured at domestic prices for each country.[36]

I proxy economic openness with the ratio of the sum of exports and imports over GDP. Then I add several country-year specific variables that have potential effects on the sorting pattern within a country for robustness. In particular, I control for the gross enrollment ratio in tertiary education in order to account for human capital accumulation, the number of telephones subscribers per 1,000 inhabitants as a proxy for the infrastructure in the economy, and the Gini-coefficient to help capture the variance in the distribution of talent. Moreover, I use some indicators of political stability such as the Corruption Perceptions Index, a political rights indicator, and a civil rights indicator. All of these control variables are taken from the CANA database.

1.8.3 Construction of Variables

The average hourly sectoral wage is given in the EU-Klems database. For sector i in country h at time t, I label the wage

$$\bar{\omega}_t^h(i).$$

The capital intensity of sector i in country j at time t is calculated as

$$CI_t^h(i) \equiv 1 - \frac{\sum_{j \in \Omega_t^h(i)} \omega_t^i(j)}{\sum_k VA_t^i(k)}$$

where $\omega_t^i(j)$ denotes the labor earnings of individual j in sector i at time t, $VA_t^i(k)$ the value added of firm k in sector i at time t, and $\Omega_t^h(i)$ denotes the set of individuals active in sector i in country h at time t. The latter ratio is the aggregate labor payments in a sector over the aggregate value added in that sector. Both variables can be observed in the EU-Klems database.

[36] As a robustness check, I use the ratio of the $R\&D$-expenditure to GDP in a country and the number of patents per 1,000 inhabitants as proxies for technological change.

The employment share is calculated as the proportion of individuals active in sector i in country h at time t:

$$\pi_t^h(i) \equiv \frac{N_t^{ih}}{N_t^h}$$

where N_t^h denotes the number of individuals active in economy h at time t, and N_t^{ih} denotes the number of individuals at time t active in sector i in country h.

The alignment of the average sectoral wage with the capital intensity can vary because of changes in the intensive and extensive margins. The former is an increase in the alignment of the capital intensities with the sectoral wages without any change in the ranking. This is not necessarily an indicator of sorting reversals. Therefore, I construct the ranks of the sectoral capital intensity and the average wages, respectively, in order to only observe the extensive margin. This is an indicator of sorting reversals because there have to be changes in the ranking of either the capital intensities or the average wages to have variation in the estimated alignment coefficients.[37]

I denote the capital intensity rank of sector i in country y at time t as $r(CI_t^h(i))$. The rank of the average sectoral wage is labeled $r(\bar{\omega}_t^h(i))$. Thus, to test the predictions on sorting reversals, I run the following regression for each country-year pair:

$$r(CI_t^h(i)) = \alpha + \beta_{h,t}^w r(\bar{\omega}_t^h(i)) + \epsilon_{i,j,t}. \tag{1.5}$$

I exclude the sectors with negative capital intensities in this regression. The reason is that data analysis suggests that most negative capital intensities result from governmental subventions as negative capital intensities occur particularly often in sectors such as mining or agriculture.[38]

The analogous regression for each country-year pair to test the prediction on structural change is

$$CI_t^h(i) = \alpha + \beta_{h,t}^e \pi_t^h(i) + \epsilon_{i,j,t}. \tag{1.6}$$

[37] As a robustness check I also redo the exercise without using ranks. The results do not change qualitatively.

[38] Including these sectors does not alter the results significantly. I also exclude the country-year pairs for which less than 8 data points are available. The same holds for the next regression for the sectoral capital intensity on the employment shares.

I do not use ranks in that regression as I am interested in both the extensive and intensive margin when it comes to the alignment between the sectoral capital-intensity and the employment shares.

In Figure 1.10, I show the alignment of the average sectoral wages with capital intensity over time for two countries in the sample: the United States and Korea. I chose these countries because the United States represents the most advanced economy in the data set, and South Korea because it has experienced the greatest growth. In 1970, the GDP per capita in South Korea was 284$ in US-$ at current prices. In 2007, GDP amounted to 22,090 US-$.[39] This implies that while the GDP per capita of South Korea was 5.81% of that of the United States in 1970, it rose to 47.83% in 2007.

I plot the estimated β coefficients from regressing the sectoral capital intensities on the average sectoral hourly wage for both countries over time. The figure shows that the estimated βs are positive for the United States for all years. Nevertheless, I observe a further increase in the alignment of the sectoral capital intensities with the earnings over time.[40] By contrast, the coefficient of the alignment for South Korea is negative for most years in the sample and only turns positive in the late 1990s. A negative coefficient indicates that wages are higher in more labor-intensive sectors. This is consistent with my model for the early stages of financial development.

The alignment of the sectoral capital intensity with the employment shares over time is depicted in Figure 1.11. While the β coefficient is always negative for the United States, it is significantly positive for South Korea until the early 1990s. In both figures I observe a rather steep trend in South Korea compared to the United States. This is not surprising given that the period was characterized by enormous growth for South Korea as compared to the United States.

These figures indicate that there is some relation between development and the alignment of the sectoral capital intensities with average wages and employment shares. In order to see whether this is because of financial development, I regress the β coefficients

[39] See http://data.un.org for the UN data for “Per capita GDP at current prices - US dollars".

[40] Note that the β coefficients are bounded between -1 and 1 as it is a rank regression.

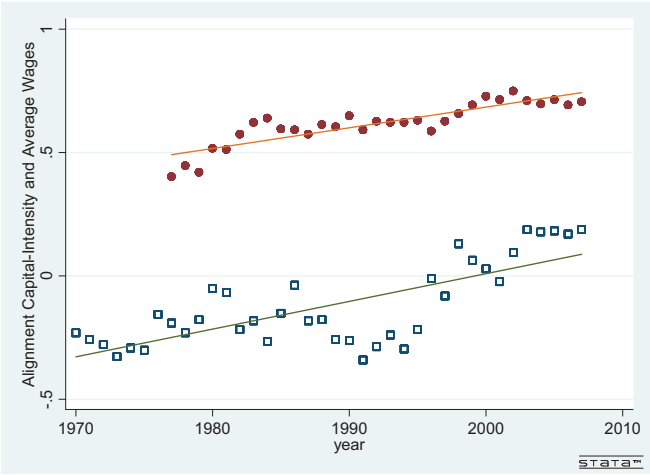

Figure 1.10: The alignment between sectoral capital intensity and wages

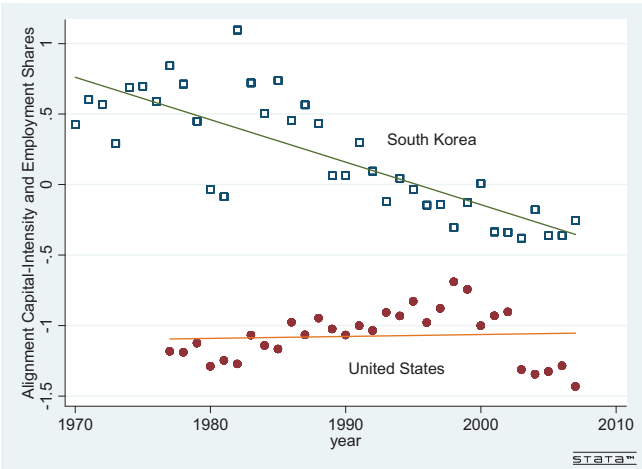

Figure 1.11: The alignment between sectoral capital intensity and employment shares

on the financial development indicator while controlling for other variables.

The central regression on testing sorting reversals is then

$$\hat{\beta}^{w}_{h,t} = \alpha + \nu_h + \nu_t + \gamma_{FD} FD_{h,t-2} + X_{h,t-2}\gamma + \epsilon_{j,t} \tag{1.7}$$

where γ_h indicates the country fixed effects, ν_t the year fixed effects, $FD_{j,t-2}$ the financial development indicator in country h at $t-2$, and $X_{h,t-2}$ stands for a vector of country time specific control variables. I use the financial development indicator and controls lagged by two years.[41] The reason is twofold. The main reason is that financial development as well as technological change require some time until they translate into changes in the sectoral wage levels. Therefore, the financial development and technological change in the past that have effects on the alignment of the sectoral wages with the capital intensities today. Furthermore, to some extent, I account for the potential reverse causality problems because the alignment of the wages with the capital intensity today should not greatly influence past financial development.

The analogous regression equation for structural change is

$$\hat{\beta}^{e}_{h,t} = \alpha + \nu_h + \nu_t + \gamma_{FD} FD_{h,t-2} + X_{h,t-2}\gamma + \gamma_I (FD * EO)_{h,t-2} + \epsilon_{j,t}. \tag{1.8}$$

It is important to include the interaction term $(FD * EO)$ between financial development and economic openness in the regression because I predict that financial development causes shifts to labor-intensive sectors in a closed economy. If I add the interaction term, then the coefficient γ_{FD} can be interpreted as the marginal change of $\hat{\beta}^{e}_{h,t}$ due to an increase in financial development conditional on the level of economic openness being zero.[42]

The average level of financial development is 0.73 with a standard deviation of 0.4. In Figure 1.12, I depict the levels of financial development for the United States and South Korea. For both countries, I observe a huge increase over time.

[41] The results do not change a great deal if I lag by one year or three years.

[42] Deriving the above equation with respect to FD shows this.

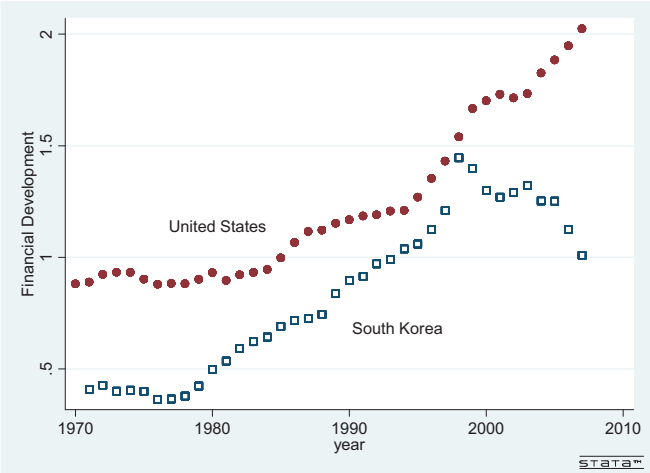

Figure 1.12: The level of financial development over time

1.8.4 Regression Results.

In all the regressions I weight observations by total employment, control both for country and year fixed effects, and use clustered standard errors. This is necessary as the errors within a country are likely to be correlated.

I am aware that including clustered standard errors can be problematic as the number of clusters included is not particularly large given that the data set only covers 29 countries. Therefore, the estimated standard errors may still be too small. On the other hand, because I use the estimated β-coefficients in my regression analysis as dependent variable the standard errors in the second regression might be too high because of measurement errors in the first regression stage.

Sorting Reversals. The regression results for my main specification are shown in Table (1.5). In the first column, I regress $\hat{\beta}^{w}_{h,t}$ on the financial development indicator alone. The coefficient is positive and highly significant. This indicates that financial development is conducive to sorting reversals, the alignment of the ranks of sectoral capital intensity with the ranks of average hourly wages increases with financial development.

In the second column I include the proxies for the alternative theories of structural change: the relative price of investment and the $log(GDP)$. I find that the coefficient on

financial development remains highly significant. The technological change also appears to be conducive to an increase in the alignment of sectoral capital intensity with wages. As the relative price of investment decreases, the β coefficient increases. The significance of the technological change channel disappears when I include the above mentioned control variables.

The average level of financial development in the first three years in the data set $(1970 - 1972)$ is 0.49. For the last three years $(2003 - 2005)$ it is on average 0.92. This implies that the predicted increase of the alignment coefficient between sectoral capital intensity and average wage during this period is $(0.92 - 0.49) * 0.218 = 0.09$, all other things remaining equal. The observed average increase in the alignment coefficient is approximately 0.17. This implies that financial development is able to explain a substantial fraction of the average increase in alignment between capital intensity and average wage over the period covered. Nevertheless, there is still a rather large degree of heterogeneity across countries, which is to be expected given the heterogeneity in countries' characteristics.[43]

Structural Change. The regression results for the alignment between employment shares and sectoral capital intensity are in Table (1.6). In the first column, I regress the estimated β coefficients on the country and year fixed effects together with the financial development indicator (again, controlling for the country clustered standard errors). I find a negative sign that indicates that higher levels of financial development lead to employment shifts towards more labor-intensive sectors. Yet, the coefficient is not significant at the conventional significance levels. In the second column, I include the economic openness indicator and interact it with financial development. Now, the financial development indicator becomes significant. This is in line with the prediction that in a closed economy employment shares increase in labor-intensive sectors due to

[43]Including financial development in the regression accounts for approximately 15% of the unexplained variation compared to only regressing on country and year fixed effects. Mathematically, $1 - \text{adj.}(R^2)$ decreases by 15%. The rather low additional explanatory power this seems to indicate is not surprising given that fixed effects already account for more than 85% of the observed variation.

Table 1.5: Sorting Reversals - in Ranks

Dependent Variable: Alignment btw			
Capital-Intensity and Average Wage ($\hat{\beta}^{w}_{h,t}$)	(1)	(2)	(3)
L2.Financial Development (FD)	0.218***	0.188***	0.174***
	(0.0565)	(0.0627)	(0.0302)
L2.Rel. Price of Investment		-0.347*	-0.159
		(0.200)	(0.203)
L2.log(GDP per capita)		0.0344	0.0766
		(0.0719)	(0.0495)
Controls	No	No	Yes
Country fixed effects	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes
N	789	789	547
R^2	0.818	0.822	0.887
adj. R^2	0.802	0.806	0.873

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

financial development. Furthermore, the interaction term has the expected sign, higher levels of economic openness imply that financial development leads to less structural change to the labor-intensive sectors. The coefficient on financial development remains significant if I add the other candidate explanations for structural change and control variables.

The observed average change in the alignment coefficient between sectoral capital intensity and employment shares is -0.35 over the period examined. The observed change in financial development predicts an average change of $(0.92 - 0.49) * 0.33 = -.14$. Thus, the relative explanatory power of financial development for structural change seems to be smaller compared to sorting reversals.[44] Again, note that there is a great deal of heterogeneity across countries, which is to be expected given the heterogeneity in country characteristics and my theoretical predictions for the effects of economic openness on the coefficient of financial development.

My theory is supported by the data which suggests that financial development leads to sorting reversals and structural change. The latter effect only appears to be significant for a closed economy: a degree of openness equal to zero. Another support for my theory is that regressing the change in the ranking of the employment shares of a sector over time on the change in the rank of the average sectoral wages over time gives a slope coefficient of -0.027 with a robust t-statistic of 5.86. Thus, the employment shares tend to increase in lower wage sectors.

I do not draw any conclusions about the importance of the other two channels for structural change. The predictions on the relation between the alignment of employment shares with sectoral capital intensities and the other candidate explanations are ambiguous. On the one hand, this is because different countries in the sample are at different stages of development. On the other hand, there is no clear-cut prediction as to why these theories should be related to the capital intensities of sectors. Neither non-homothetic preferences nor technological change necessarily imply that the employment shares

[44] Moreover, the share of unexplained variance that financial development can explain is lower for structural change. $1 - \text{adj.}(R^2)$ decreases by around 6% compared to 15% in the case of sorting reversals.

Table 1.6: Structural Change

Dependent Variable: Alignment btw Capital-Intensity and Employment Share ($\hat{\beta}^e_{h,t}$)	(1)	(2)	(3)	(4)
L2.Financial Development (FD)	-0.330	-0.622**	-0.392*	-0.505***
	(0.229)	(0.233)	(0.194)	(0.175)
L2.i.FD*EO		1.043	0.670	0.475
		(0.616)	(0.465)	(0.367)
L2.Economic Openness (EO)		-0.463	-0.548	-0.952
		(0.641)	(0.889)	(1.012)
L2.Rel. Price of Investment			1.533	1.539*
			(1.185)	(0.830)
L2.log(GDP per capita)			-0.661	-0.516
			(0.402)	(0.466)
Controls	No	No	No	Yes
Country fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
N	772	544	544	532
R^2	0.846	0.875	0.885	0.902
adj. R^2	0.832	0.862	0.872	0.889

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

increase or decrease in capital-intensive sectors.

With non-homothetic preferences, there is a great deal of debate on whether or not goods consumed relatively heavily at latter stages of development (especially services) are more or less labor intensive. Furthermore, not all countries in the sample are equally developed. Latvia and Lithuania are certainly at different stages of development than Sweden and the Unites States.

Technological change can occur in many ways that affect the sorting behavior of agents. Besides raising the TFP in relatively capital-intensive sectors (see Acemoglu and Guerrieri (2008)), technological change can also alter the relation between the capital intensity and high-skill intensity, or cause switches from log-supermodularity to log-submodularity, or from decreasing to increasing returns. Because my theory indicates that all of these things are related to sorting, it is not clear what to predict for the change in alignment of sectoral capital intensity with employment shares due to technological change.

Furthermore, even in technologically-driven models of structural change, the expected sign on the coefficient of the proxy for technological change depends on the elasticity of substitution. If the elasticity of substitution in consumption is greater (smaller) than one, I expect the sign to be negative (positive). I actually find that the coefficient on the relative price of investment is negative and significant for a number of countries, *inter alia* the United States, when I run separate regressions for the countries.[45] This suggests that in the United States, technological change tends to raise employment in capital-intensive sectors. This is in line with the finding of Karabarbounis and Neiman (2013) on decreases in the labor share. However, other countries show positive coefficients, and consequently I refrain from drawing any particular conclusions. It would be interesting to explore further the potential explanations for the differences in the signs of the coefficients across countries. This is, unfortunately, beyond the scope of this paper.

In general, I would like to test the theory more directly by using a micro-level data

[45]Clearly, I cannot control for year-fixed effects in this framework. Generally, I find some heterogeneity in the signs of the coefficients of the proxies for both technological change and non-homothetic preferences when I run the regressions for each country separately. I find much less heterogeneity in the sign on the financial development indicator.

set to test the predictions of my model; in particular, the hump shape of the sorting pattern of agents to sectors. This will be a subject for future research.

Robustness Checks. I perform the following robustness checks.[46] First, I run the same regressions including sectors with negative capital intensity. The coefficients on the effect of financial development on the alignment of the sectoral capital intensity with the average wage are slightly smaller but remain highly significant. The same holds true for the effect of financial development on the alignment of sectoral employment shares with sectoral capital intensities. I also take three-year averages of the observations in order to control for short-run fluctuations and the results do not change.

Furthermore, I run the regression of the capital intensities on the average wages for each country-year pair without taking ranks. Instead, I weight the observations by their relative employment share. The coefficients are still positive but only significant at the 5% or 10%-level. Then, I use the fraction of high-skilled agents instead of the average sectoral wage in the first step regression on sorting reversals. I have less observations as data on the share of high-skilled agents is not available for some countries. Nevertheless, the coefficient on financial development is positive and highly significant.[47]

I also use different proxies for technological change. In particular, both the ratio of $R\&D$ expenditure over GDP as well as the number of patents per 1,000 inhabitants do not alter the qualitative results. The inclusion of different sets of control variables does not vary the results much either.

1.9 Conclusion

In this paper, I propose a theoretical model of the occupational choice of financially constrained agents. My central result is that financial development implies sorting

[46] I do not give the results of the robustness checks here due to space constraints. All results are available on request.

[47] In this case, $1 - \text{adj.}(R^2)$ decreases by approximately 23% when including the financial development indicator compared to only regressing on country and year fixed effects.

reversals. Relaxing the borrowing constraint leads to a switching in the sorting behavior of agents. When financially unconstrained, more talented agents sort into more capital-intensive sectors, which reverses the sorting pattern for financially constrained agents. This feature can be observed empirically and is associated with financial development. Furthermore, financial development can induce structural change, that is, shifts in sectoral employment shares. I discuss the possible consequences of financial development on structural change analytically and numerically. Empirically, I observe that financial development induces the employment shares to increase in relatively labor-intensive sectors, which is consistent with my model.

The goal of the model is to highlight a new channel for structural change through sorting reversals. In addition to illustrating how (financial) development can be associated with changing employment shares across sectors, the model also emphasizes a changing composition of the workforce across sectors. It is important to understand the heterogeneous sorting patterns of agents towards sectors across the world. Furthermore, I show that this channel has non-trivial impacts on inequality. In particular, it is able to explain the huge increase in top-income inequality in developed countries in recent years together with the simultaneous decrease in low-income inequality.

Because governments can strengthen property rights and other institutions conducive for financial development, there is good reason to believe that they can loosen borrowing constraints.[48] My model highlights the novel features of the associated economic responses.

I plan to extend the model in order to analyze further the economic relevance of sorting reversals. More specifically, I want to allow for heterogeneous wealth in a dynamic framework and to analyze the dynamic saving incentives of heterogenous agents, in particular under endogenous human capital formation, learning-by-doing and trade. Furthermore, I plan to analyze the model's predictions empirically using a micro dataset that allows me to test directly for the hump-shaped sorting pattern predicted by my model.

[48] See for example Beck and Levine (2008) and Chinn and Ito (2006) for the importance of institutions for financial development.

An empirical phenomenon that fits the picture depicted by my theory is remigration in China. Several decades ago, borrowing conditions were low in China and many high-skilled Chinese migrated to the United States and Europe. In recent years, however, borrowing conditions have greatly improved in China and these migrants are remigrating to China as capital becomes increasingly abundant. It would be interesting to explore whether this empirical phenomenon is associated with financial development and sorting reversals.

1.10 Appendix

1.10.1 Proofs

Discussion of Assumption 1.1

Proof. I justify assumption 1.1 by showing that in any closed economy, the equilibrium price function has to possess the mentioned properties. I first prove that $p(i)$ is continuous. Suppose that it is not continuous, in particular that there is a discontinuity at i', $\lim_{i\to i'_+} p(i) \equiv p(i'_+) \neq \lim_{i\to i'_-} p(i) \equiv p(i'_-)$. Without loss of generality, I assume that $p(i'_+) > p(i'_-)$. For constrained agents, this condition implies that the earnings in i'_+ are strictly greater for every agent than in i'_-,

$$p(i'_+)\theta_j^z \left(\frac{(1+\lambda)w}{\theta_j}\right)^{\alpha_{i'_+}} > p(i'_-)\theta_j^z \left(\frac{(1+\lambda)w}{\theta_j}\right)^{\alpha_{i'_-}}$$

for all j because α_i is continuous in i.

The same holds for unconstrained agents. The earnings of every agent are greater in i'_+ than in i'_- because

$$\left(\frac{\alpha_{i'_+} p(i'_+)}{r}\right)^{\frac{1}{1-\alpha_{i'_+}}} \theta_j^{\frac{z-\alpha_{i'_+}}{1-\alpha_{i'_+}}} r(\frac{1}{\alpha_{i'_+}} - 1) > \left(\frac{\alpha_{i'_-} p(i'_-)}{r}\right)^{\frac{1}{1-\alpha_{i'_-}}} \theta_j^{\frac{z-\alpha_{i'_-}}{1-\alpha_{i'_-}}} r(\frac{1}{\alpha_{i'_-}} - 1)$$

because of the continuity of earnings in α_i and the continuity of α_i in i.

This implies that no agent chooses to be active in sector i'_-. This contradicts the clearing of the goods market because the utility function implies that every commodity is demanded to some extent at any positive price. Therefore, $p(i)$ has to be continuous in equilibrium.

A corrolary of the restriction that every sectors has to be chosen by some agents is that $X_{j*}(i, \theta_j, w)$ is continuous in i. From an agent's perspective, $X_{j*}(i, \theta_j, w)$ is a function of two continuous variables, α_i and $p(i)$. Inspection of equation (1.2) shows that the RHS of the equation has to be continuous, $\alpha_i p(i)$ is continuous if both α_i and $p(i)$ are continuous. Therefore, the LHS has to be continuous as well and implies that $X_{j*}(i, \theta_j, w)$ is continuous given that $\alpha_i \in (0, 1)$.

□

Proof of proposition 1.1

Proof. The sectoral allocation is governed by the earnings maximization of the agents. If no borrowing constraints bind the FOC; then (1.2) holds for every agent, that is, agent j with talent θ_j produces by using $X_{j*}(i, \theta_j, w)$ amounts of capital. This implies that an agent's earnings can be rewritten as

$$\begin{aligned}\omega(i, \theta_j, w) &= p(i)\, \theta_j^{z-\alpha_i} (X_{j*}(\theta_j, i, w))^{\alpha_i} - rX_{j*}(i, \theta_j, w) + rw = \\ &= r \left(\frac{p(i)\alpha_i}{r}\right)^{\frac{1}{1-\alpha_i}} \theta_j^{\frac{z-\alpha_i}{1-\alpha_i}} \left(\frac{1}{\alpha_i} - 1\right) + rw.\end{aligned}$$

Note that the last term, rw, is independent of sectoral choice.

Consider now two agents j and j' with $\theta_j > \theta_{j'}$, and two sectors i and i' with $i > i'$ (and hence $\alpha_i > \alpha_{i'}$). I proceed by contradiction. Suppose sorting is characterized by NAM. The sorting is characterized by NAM if and only if

$$\omega(i, \theta_j, w) < \omega(i', \theta_j, w)$$

and

$$\omega(i, \theta_{j'}, w) > \omega(i', \theta_{j'}, w).$$

These two conditions can be rewritten as

$$\frac{\left(\frac{\alpha_i}{r}\right)^{\frac{1}{1-\alpha_i}}}{\left(\frac{\alpha_{i'}}{r}\right)^{\frac{1}{1-\alpha_{i'}}}} \frac{\frac{1}{\alpha_i} - 1}{\frac{1}{\alpha_j} - 1} \theta_j^{\frac{(z-1)(\alpha_i - \alpha_{i'})}{(1-\alpha_i)(1-\alpha_{i'})}} < \frac{p(i')^{\frac{1}{1-\alpha_{i'}}}}{p(i)^{\frac{1}{1-\alpha_i}}}$$

and

$$\frac{\left(\frac{\alpha_i}{r}\right)^{\frac{1}{1-\alpha_i}}}{\left(\frac{\alpha_{i'}}{r}\right)^{\frac{1}{1-\alpha_{i'}}}} \frac{\frac{1}{\alpha_i} - 1}{\frac{1}{\alpha_j} - 1} \theta_{j'}^{\frac{(z-1)(\alpha_i - \alpha_{i'})}{(1-\alpha_i)(1-\alpha_{i'})}} > \frac{p(i')^{\frac{1}{1-\alpha_{i'}}}}{p(i)^{\frac{1}{1-\alpha_i}}}.$$

The ratio of the sectoral prices is exogenous. Hence, both agents may prefer one sector over the other. But for NAM suppose w.l.o.g. that both agents sort into different sectors. Then, the necessary and sufficient condition for NAM is

$$\frac{\left(\frac{\alpha_i}{r}\right)^{\frac{1}{1-\alpha_i}}}{\left(\frac{\alpha_{i'}}{r}\right)^{\frac{1}{1-\alpha_{i'}}}} \frac{\frac{1}{\alpha_i} - 1}{\frac{1}{\alpha_j} - 1} \theta_j^{\frac{(z-1)(\alpha_i - \alpha_{i'})}{(1-\alpha_i)(1-\alpha_{i'})}} < \frac{\left(\frac{\alpha_i}{r}\right)^{\frac{1}{1-\alpha_i}}}{\left(\frac{\alpha_{i'}}{r}\right)^{\frac{1}{1-\alpha_{i'}}}} \frac{\frac{1}{\alpha_i} - 1}{\frac{1}{\alpha_j} - 1} \theta_{j'}^{\frac{(z-1)(\alpha_i - \alpha_{i'})}{(1-\alpha_i)(1-\alpha_{i'})}}.$$

This simplifies to

$$\theta_j^{\frac{(z-1)(\alpha_i-\alpha_{i'})}{(1-\alpha_i)(1-\alpha_{i'})}} < \theta_{j'}^{\frac{(z-1)(\alpha_i-\alpha_{i'})}{(1-\alpha_i)(1-\alpha_{i'})}}$$

that can never be true as $\theta_j > \theta_{j'}$ and $\alpha_i > \alpha_{i'}$. Thus, PAM has to prevail for any pair of unconstrained agents.

Note that $z > 1$ is necessary for this result to be true. Constant returns to scale make the sorting indeterminate, and $z < 1$ implies that NAM prevails. □

Proof of Proposition 1.2

Proof. If the borrowing limit always binds, then the earnings of agent j in sector i are given by:

$$\omega(i, \theta_j, w) = p(i)\, \theta_j^{z-\alpha_i}((1+\lambda)w)^{\alpha_i} - r(1+\lambda)w.$$

As before, considering two agents and two sectors, the sorting is characterized by PAM if and only if

$$\omega(i, \theta_j, w) > \omega(i', \theta_j, w)$$

and

$$\omega(i, \theta_{j'}, w) < \omega(i', \theta_{j'}, w).$$

These two conditions are now

$$\frac{\theta_j^{z-\alpha_i}((1+\lambda)w)^{\alpha_i}}{\theta_j^{z-\alpha_{i'}}((1+\lambda)w)^{\alpha_{i'}}} > \frac{p(i)}{p(i')}$$

and

$$\frac{\theta_{j'}^{z-\alpha_i}((1+\lambda)w)^{\alpha_i}}{\theta_{j'}^{z-\alpha_{i'}}((1+\lambda)w)^{\alpha_{i'}}} < \frac{p(i)}{p(i')}$$

Again, suppose that both goods are in positive supply. The necessary and sufficient condition is

$$\frac{\theta_j^{z-\alpha_i}((1+\lambda)w)^{\alpha_i}}{\theta_j^{z-\alpha_{i'}}((1+\lambda)w)^{\alpha_{i'}}} > \frac{\theta_{j'}^{z-\alpha_i}((1+\lambda)w)^{\alpha_i}}{\theta_{j'}^{z-\alpha_{i'}}((1+\lambda)w)^{\alpha_{i'}}},$$

or equivalently

$$\theta_j^{\alpha_{i'}-\alpha_i} > \theta_{j'}^{\alpha_{i'}-\alpha_i}.$$

This condition can never be fulfilled because $\theta_j > \theta_{j'}$ and $\alpha_i > \alpha_{i'}$. Therefore, NAM prevails for any pair of constrained agents. Note that this result does not depend on increasing returns to scale. It resembles the standard matching results following Becker (1973). □

Proof of Proposition 1.3

Proof. First, I show that if an agent with talent $\theta^{'}$ is unconstrained in sector $i^{'}$, then $\nexists \theta < \theta^{'} : i(\theta) > i^{'}$. Hence, all agents with talent $\theta < \theta^{'}$ are unconstrained as well, and PAM prevails for all agents with talent $\theta < \theta^{'}$.

Then, I show that if an agent $\theta^{'}$ is constrained in sector $i^{'}$, then $\nexists \theta > \theta^{'} : i(\theta) > i^{'}$. Thus, all more talented agents are constrained as well, and NAM prevails for this set of agents.

These two facts combined prove proposition 1.3.

Part 1. I restrict attention to sectors active in equilibrium. In these sectors, $X_{j*}(i, \theta_j, w)$ is strictly and monotonically increasing in i. To see this, suppose that $X_{j*}(i', \theta_j, w) > X_{j*}(i, \theta_j, w)$ for some $i' < i$. The FOC on capital (1.2) allows the rewriting of the earnings $\omega(i, \theta_j, w) = rX_{j*}(i, \theta_j, w)(\frac{1}{\alpha_i} - 1)$ for unconstrained agents. Hence, if $X_{j*}(i', \theta_j, w) > X_{j*}(i, \theta_j, w)$, then

$$rX_{j*}(i', \theta_j, w)(\frac{1}{\alpha_{i'}} - 1) > rX_{j*}(i, \theta_j, w)(\frac{1}{\alpha_i} - 1)$$

as α_i increases in i. No unconstrained agent sorts into sector i. If no unconstrained agent sorts into sector i then constrained agents do not sort into that sector either. The reason is that while unconstrained agents can adjust their capital choice in different sectors, constrained agents cannot. Therefore, if the goods prices are such that no unconstrained agent sorts into the more capital-intensive sector, then neither does any constrained agent.

For proving the first part of the proposition, assume that agent $\theta^{'}$ is unconstrained in sector $i^{'} \equiv i(\theta^{'})$. As shown, $X_{j*}(i, \theta_j, w)$ is continuous and strictly and monotonically increasing.

Hence, either there is a sector $i'' : X_{j*}(i'', \theta', w, \lambda) = (1+\lambda)w$ or $X_{j*}(1, \theta', w, \lambda) < (1+\lambda)w$, that is, the agent is unconstrained in every sector.

In the latter case, there cannot be any less talented agent in $i > i'$ by PAM for unconstrained agents (because less talented agents are unconstrained *a fortiori* as capital demand increases in talent).

In the former case, there may not be any agent with talent $\theta < \theta'$ in any sector $i \in [i', i'']$. If there were this would have to imply PAM between these two agents as the lower talented agent is also unconstrained. Hence, PAM prevails between agents in $i \in [i', i'']$. But then the agent with talent θ' cannot be marginally constrained in i'' because the agent $\theta(i'')$ has to be unconstrained in order for PAM to prevail. I conclude that if an agent with talent θ' is unconstrained in sector i', then $\nexists \theta < \theta' : i(\theta) > i'$.

Part 2. For the second part, assume that agent θ' is constrained in sector $i' \equiv i(\theta')$. Any more skilled agent is constrained in sector i' as well. Furthermore, both the agent with talent θ' and any more talented agent are constrained in any sector $i > i'$.

But proposition 1.2 implies that NAM prevails for any pair of agents that are constrained in any pair of sectors. Thus $\nexists \theta > \theta' : i(\theta) > i'$.

□

Proof of Proposition 1.4

Proof. I will prove the four statements of the proposition in turn.

Statement 1. First, $\lambda = \infty$ implies that every agent is unconstrained. Because PAM prevails for any pair of unconstrained agents, the whole economy is characterized by PAM. Because a change in λ changes continuously the feasibility set of capital allocations $\exists \lambda^*(w) < \infty$ such that the whole economy is characterized by PAM, and every agent is unconstrained. Further increases in λ increase the feasible set of allocations. But because sorting is already characterized by the efficient solution at $\lambda^*(w)$, the allocation of capital does not change. Hence, for every $\lambda > \lambda^*(w)$ the economy is characterized by pure PAM.

Statement 2. Because agents are heterogeneous in talent and sectors in capital productivity, the marginal returns to capital cannot be equalized across agents at $\lambda = 0$ (conditional on efficient sorting). Hence, any $\lambda > 0$ implies that gains of trade in capital can be realized. Thus, some agents rent out capital and become unconstrained. From proposition 1.3, I know that these agents form a convex set at the bottom of the talent distribution and that PAM prevails for these agents.

Statement 3. To prove that the capital rental rate increases in the borrowing constraint, suppose that λ increases for any $\lambda < \lambda^*(w)$, and the capital rental rate r does not increase. In particular, suppose it remains constant.

The capital demand for the set of unconstrained agents does not change because neither r nor the exogenous goods prices $p(i)$ change, and thus sectoral choice does not change either. By contrast, the capital demand of the set of constrained agents increases because the marginal return to capital is larger than r before the change in λ.

This implies that there is excess demand for capital; that is, capital market clearing does not occur, since it did clear before the change of λ. If the capital rental rate decreases, then the capital demand increases for both constrained and unconstrained agents. The capital market does not clear either. Hence, this cannot represent an equilibrium. I conclude that the capital rental rate r has to increase for any $\lambda < \lambda^*(w)$.

Statement 4. Lastly, I prove that $\theta^*(\lambda)$ increases with λ. The part of an unconstrained agent's earnings that depends on the sectoral choice can be written as

$$\omega(i, \theta_j, w) - rw = r\left(\frac{p(i)\alpha_i}{r}\right)^{\frac{1}{1-\alpha_i}} \theta_j^{\frac{z-\alpha_i}{1-\alpha_i}} (\frac{1}{\alpha_i} - 1).$$

Taking derivatives implies that

$$\frac{\partial^2 \log(\omega(i, \theta_j, w) - rw)}{\partial r \partial \alpha_i} = \frac{-1}{r(1-\alpha_i)^2} \; < \; 0.$$

Hence, because the capital rental rate increases the optimal sectoral choice for any unconstrained agent decreases. As the agent with talent $\theta^*(\lambda)$ is marginally unconstrained her optimal sectoral choice also decreases.

In turn, for constrained agents the part of earnings that depends on the sectoral choice is

$$\omega(i, \theta_j, w) + r(1+\lambda)w = \; p(i)\, \theta_j^{z-\alpha_i}((1+\lambda)w)^{\alpha_i}.$$

Note that this implies that the agent is constrained irrespective of the sector chosen. This assumption is w.l.o.g because there is a continuum of sectors, and hence the marginal choice is between two sectors where in which she is constrained. Thus, any marginal change in the capital rental rate does not affect the sectoral choice. By contrast, the choice is influenced by the change in the financial constraint, λ. Again, taking derivatives implies that

$$\frac{\partial^2 \log(\omega(i, \theta_j, w) + r(1+\lambda)w}{\partial\lambda\partial\alpha_i} = \frac{1}{(1+\lambda)} \; > \; 0.$$

Thus, the optimal sectoral choice increases for any constrained agent in λ. Therefore, a more skilled agent is marginally unconstrained at a higher level of λ.

With $\lambda = 0$, proposition 1.2 applies for every agent and hence $\theta^*(0) = \underline{\theta}$. By contrast, for $\lambda = \lambda^*(w)$, the proposition 1.1 applies for every agent and thus $\theta^*(\lambda^*(w)) = \overline{\theta}$. □

Proof of Proposition 1.5

Proof. I first prove existance and then uniqueness. As proposition 1.3 shows, the economy can be partitioned into two sets, $[\underline{\theta}, \theta^*(\lambda)]$ and $[\theta^*(\lambda), \overline{\theta}]$. All agents in the latter set demand capital equal to $(1+\lambda)w$, and the capital demand of agents in the former set is characterized by $X_{j*}(i, \theta_j)^{1-\alpha_i} = \frac{p(i)\alpha_i}{r}\, \theta_j^{z-\alpha_i}$.

Thus, in order to show that the aggregate capital demand is continuous in r I need to demonstrate that both $\theta^*(\lambda)$ and $X_{j*}(i, \theta_j)$ are continuous in r.

Because α_i and $p(i)$ are continuous in i, $X_{j*}(i, \theta_j)$ is continuous in r as long as $i(\theta_j)$ is continuous in r. I know from assumption 1.1 that $X_{j*}(i, \theta_j)$ is continuously and monotonically increasing in i. Because the earnings can be rewritten using the FOC on capital as

$$(\frac{1}{\alpha_i} - 1) r X_{j*}(i, \theta_j),$$

the optimal sectoral choice $i(\theta_j)$ is continuous in r.

Furthermore, the marginal agent $\theta^*(\lambda)$ increases continuously and monotonically because $X_{j*}(i, \theta_j)$ increases continuously and monotonically in i for every agent. Thus, the aggregate capital demand is continuous in r. Furthermore it monotonically increases in r.

What remains to be shown is that if $\lim r \to 0(\infty)$, then there is an excess demand (supply) of capital. The necessary condition on capital implies that capital demand goes to infinity for $\lim r \to 0$. This means that as long as $\lambda > 0$, there is an excess demand for capital in the limit. By contrast, if $\lim r \to \infty$, then the optimal capital goes to zero, implying an excess supply for capital. By the mean value theorem, an equilibrium has to exist.

Uniqueness follows from the fact that the set of constrained agents decreases monotonically in r together with the monotonicity of $i(\theta)$ in r. □

Proof of Proposition 1.6

Proof. Towards a contradiction suppose that

$$\frac{d\left(\int_0^i \int_{\theta_j \in \Omega_i} \varphi(\theta_j) d\theta_j di\right)}{d\lambda} > 0,$$

for some i. This condition shows that some agents must have resorted into a more labor-intensive sector with financial development. Yet unconstrained agents do not resort and constrained agents sort into more capital-intensive sectors. Hence, $\frac{d\left(\int_0^i \int_{\theta_j \in \Omega_i} \varphi(\theta_j) d\theta_j di\right)}{d\lambda} \leq 0$ for all i.

The second part of the proposition is true because for every $\min\{i(\underline{\theta}), i(\overline{\theta})\} < i < i(\theta^*(\lambda))$ and $\lambda < \lambda^*(w)$, there is a constrained agent who chooses i before the development and now chooses some $i' > i$. □

Proof of Proposition 1.7

Proof. If $i(\underline{\theta}) > i(\overline{\theta})$, then in the sectors $[0, i(\underline{\theta}))$ only constrained agents are active. Because constrained agents sort unambiguously into more capital-intensive sectors, the first result is immediate.

If $i(\underline{\theta}) < i(\overline{\theta})$ and $\lambda < \lambda^*(w)$ (not all agents are unconstrained initially), all of the agents active in sectors $[0, i(\overline{\theta}))$ are unconstrained and thus sort into more labor-intensive sectors with financial development. This implies that employment shares in these labor-intensive sectors rise. □

Proof of Proposition 1.8

Proof. For constrained agents, $rX_j(\theta_j, i, w) - rw = \lambda rw$, and thus independent of sectoral choice. Taking logs of the first term in (1.3) and differentiating with respect to i implies that the earnings of a financially constrained agent are maximal at

$$\ln \theta_j - \ln w - \ln(1+\lambda) = \frac{p'(i)}{p(i)} \frac{1}{\alpha'(i)}$$

where $p'(i) \equiv \frac{\partial p(i)}{\partial i}$ and $\alpha'(i) \equiv \frac{\partial \alpha_i}{\partial i}$. This condition only holds if the agent's sectoral choice is in the interior, that is, $i(\theta_j) \in (0,1)$. Then, the difference in talent between two sectors i and k is

$$\ln \theta(i) - \ln \theta(k) = \frac{1}{\alpha'(i)} \frac{p'(i)}{p(i)} - \frac{1}{\alpha'(k)} \frac{p'(k)}{p(k)}.$$

Note that the RHS is a constant and thus does not change with financial development. Therefore, the LHS has to remain constant. Because every constrained agent sorts into more capital-intensive sectors with financial development, the talent of the agent active in sector i has to increase. Thus, $\theta(i) - \theta(k)$ has to increase with financial development.

For unconstrained agents, rw is independent of sectoral choice. By using the FOC on capital (1.2), the logarithm and differentiating imply that the necessary condition for the optimal sectoral choice for unconstrained agents reads as

$$(z-1)\ln \theta_j - \ln r = -\ln p(i) - \ln \alpha_i - \frac{1-\alpha_i}{\alpha'(i)} \frac{p'(i)}{p(i)}.$$

Note that this condition coincides with the one for constrained agents when $X_{j*}(\theta_j) = (1+\lambda)w$ as $(1-\alpha_i)\ln(w(1+\lambda)) = \ln p(i) + \ln \alpha_i + \ln r + (z-\alpha_i)\ln \theta_j$ in that case. Again, this condition does only hold if the agent's sectoral choice is in the interior, that is, $i(\theta_j) \in (0,1)$. Now, the difference in the talent sorting into sectors i and k has to be

equal to

$$(z-1)\ln\theta(i)-(z-1)\ln\theta(k)=\frac{1-\alpha_k}{\alpha'(k)}\frac{p'(k)}{p(k)}+\ln p(k)+\ln\alpha_k-\frac{1-\alpha_i}{\alpha'(i)}\frac{p'(i)}{p(i)}-\ln p(i)-\ln\alpha_i$$

Because unconstrained agents sort into more labor-intensive sectors with financial development, the talent of an agent active in sector i also increases in this case. Hence, $\theta(i)-\theta(k)$ has to increase with financial development.

As the difference in talent of agents that sort into two sectors i and k increases, the difference in the sectors that two agents with talent θ_H and θ_L sort into has to decrease. Therefore, between any two agents with talent θ_j and θ_l that were both unconstrained or both constrained before the financial development, sectoral concentration has to occur. □

Proof of Lemma 1.1

Proof. The value output can be split into capital and labor income. The capital payments $r\,X_j(i,\theta_j,w)$ include both payments for capital rented on the capital market and possessed by an agent. In the following, I denote labor payments by

$$\nu(i,\theta_j,w)\equiv p(i)\,\theta_j^{z-\alpha_i}(X_j(i,\theta_j,w))^{\alpha_i}-r\,X_j(i,\theta_j,w).$$

The capital intensity of sector i populated with agent θ_j is defined as

$$\kappa(i,w,\lambda)\equiv\frac{rX_j(i,\theta_j(i),w)}{p(i)\,\theta_j(i)^{z-\alpha_i}(X_j(i,\theta_j(i),w))^{\alpha_i}}.$$

In general, a sector i is populated by more than one type of agent because the equilibrium assignment function is non-monotone. Therefore, the inverse of the function is given only within the sets of agents $[\underline{\theta},\theta^*(\lambda)]$ and $[\theta^*(\lambda),\overline{\theta}]$, as the assignment is strictly monotone within both the set of constrained and unconstrained agents. I denote with $\theta(i)$ the multivalued function composed of the two inverse assignment functions $i^{-1}(\theta_j)$ (for the sets of constrained and unconstrained agents) evaluated at i.

In cases of unconstrained capital allocation and sectoral choice, the labor intensity of sector i can be written as

$$\xi(i,w,\infty)\equiv 1-\kappa(i,w,\infty)=1-\alpha_i.$$

This holds for any agent $\theta(i)$ that is unconstrained. Sectors with higher α possess higher capital intensity in equilibrium. Because the assignment function is monotonically increasing for unconstrained agents, the more talented agents are active in more capital-intensive sectors. Empirically, a positive relation between the capital intensity and the wages exists for the United States.[49]

By contrast, labor intensity for a constrained agent with talent θ_j in sector i is

$$\xi(i, w, 0) = 1 - \kappa(i, w, 0) = 1 - \frac{r}{p(i)\theta_j^{z-1}} \left(\frac{w(1+\lambda)}{\theta_j} \right)^{1-\alpha_i} > 1 - \alpha_i.$$

This labor intensity necessarily increases in talent.[50] The inequality stems from the fact that capital intensity in a sector populated by constrained agents is less than optimal. Since negative assortative matching prevails for constrained agents in equilibrium, the capital intensity also increases in i for constrained agents. □

[49]See, for example, Slichter (1950) and Sampson (2011).

[50]If $p(i(\theta_j))\theta^{z-\alpha_i}(w(1+\lambda))^{\alpha_i}$ were to decrease in talent, this would contradict the agent's optimal sectoral choice because the capital payments are constant for constrained agents.

1.10.2 Theoretical Extension

I extend the model by allowing agents to contract not only capital but also workers. The reason for this extension is to show that in a reasonable framework, the average wages in a sector co-move with entrepreneurial talent, and sorting reversals of entrepreneurs to sectors occur. Hence, it bridges the gap between the analytical part and the empirical part as I use average sectoral labor earnings in the latter.

Setup. In particular, I use a one-period version of the directed search model by Kaas and Kircher (2011). The output reads as a simple extension, an entrepreneur j with talent θ_j who uses capital X and the amount of labor L produces in sector i:

$$Y_i(\theta_j, X, L) = \theta_j^{z-k} \left(X^{\alpha_i} L^{1-\alpha_i} \right)^k,$$

with $z > 1$ and $k < 1$. Entrepreneurs can post wages in order to attract workers. Higher wages lead to higher vacancy filling probabilities in equilibrium, but higher pay per worker. Entrepreneurs face an iso-elastic cost function of vacancy posting, $c(v) = \frac{1}{\sigma} v^{\sigma}$.

The borrowing constraint is assumed to only apply to capital and not to labor. This represents the fact that labor payments, contrary to capital, can be paid *ex post*. Furthermore, vacancy costs should be thought of in terms of time spent by the entrepreneur searching for labor instead of productive activity, as opposed to financial costs.

Workers and Matching Technology. Workers are homogeneous and choose a labor market characterized by a wage ω. Furthermore, they face the outside option of not entering any market and earning B. The matching technology in any labor market is characterized by a Cobb-Douglas matching function. Hence, the amount of matches formed in a specific labor market is equal to

$$M(V, U) = V^{1-\gamma} U^{\gamma}$$

where V is the amount of vacancies posted in the market, and U is the amount of unemployed searching in that specific market. I define labor market tightness as $\rho \equiv \frac{V}{U}$;

then the vacancy filling probability can be written as $q(\rho) = \frac{M(V,U)}{V} = \rho^{-\gamma}$.[51]

From the assumption of the matching technology and worker homogeneity together with the outside option B, it follows that in any equilibrium

$$B = \rho q(\rho)\omega(\rho) = \rho^{1-\gamma}\omega(\rho).$$

Workers' expected earnings have to be equal across the different labor markets. This links the wage posted with labor market tightness. A higher wage implies that the labor market is less tight and that the vacancy filling probability is higher.

Entrepreneurs' Maximization Problem. The maximization problem of the entrepreneur now reads as

$$\begin{aligned}
\max_{i,X_j,v,\rho} \quad & p(i)\,\theta_j^{z-k}\left(X_j^{\alpha_i}\left(\rho^{-\gamma}v\right)^{1-\alpha_i}\right)^k - r\tilde{X}_j - \frac{1}{\sigma}v^\sigma - \frac{B}{\rho}v \\
s.t. & X_j \leq \tilde{X}_j + w \\
& \tilde{X}_j \leq \lambda w \\
& X_j \geq 0
\end{aligned}$$

where I use the fact that $L = q(\rho)v$ and $\omega(\rho) = \frac{B}{\rho q(\rho)}$. Again, I disentangle the maximization problem into two parts. First, I show the maximization decision conditional on sectoral choice, and then I turn to optimal sectoral choice for constrained and unconstrained agents.

The first-order conditions read as follows:

$$X_j: \quad \alpha_i k p(i)\theta_j^{z-k}\left(X_j^{\alpha_i}\left(\rho^{-\gamma}v\right)^{1-\alpha_i}\right)^k = rX_j \tag{1.9}$$

$$v: \quad (1-\alpha_i)kp(i)\theta_j^{z-k}\left(X_j^{\alpha_i}\left(\rho^{-\gamma}v\right)^{1-\alpha_i}\right)^k = v^\sigma + \frac{B}{\rho}v \tag{1.10}$$

$$\rho: \quad (1-\alpha_i)k\gamma p(i)\theta_j^{z-k}\left(X_j^{\alpha_i}\left(\rho^{-\gamma}v\right)^{1-\alpha_i}\right)^k = \frac{B}{\rho}v \tag{1.11}$$

Combining the last two first-order conditions implies that

$$\rho = \frac{B(1-\gamma)}{\gamma v^{\sigma-1}}.$$

[51] Normally, labor market tightness is denoted by θ. Because I have used that variable already for entrepreneurial talent, I chose ρ instead.

This equation dictates whether more talented entrepreneurs pay higher or lower wages. The complementarity between the entrepreneurial talent and labor implies that better entrepreneurs will post more vacancies. The above equation shows that if and only if $\sigma > 1$, then the entrepreneurs posting more vacancies choose tighter labor markets, that is, those with a lower ρ. And from the indifference condition for workers, tighter labor markets are only sustainable if the offered wage is higher.

The reasoning underlying this result is that with convex vacancy costs, the marginal cost of a vacancy is higher for more talented entrepreneurs. Therefore, they are willing to offer higher wages because their return from having a higher vacancy filling probability is higher compared to less-talented entrepreneurs. Note that this result is independent of whether entrepreneurs are borrowing constrained or not. In what follows I work with the assumption that $\sigma > 1$.[52]

Constrained Entrepreneurs. For borrowing constrained entrepreneurs with $X_j \equiv \overline{X} = (1+\lambda)w$, the vacancies posted as a function of the talent read as

$$v^{\sigma-(1-\alpha_i)k-(\sigma-1)\gamma(1-\alpha_i)k} = (1-\gamma)(1-\alpha_i)kp(i)\left(\frac{B(1-\gamma)}{\gamma}\right)^{-\gamma(1-\alpha_i)k}\theta_j^{z-k}\overline{X}^{\alpha_i k}.$$

This function confirms that more talented entrepreneurs post more vacancies as $\sigma-(1-\alpha_i)k-(\sigma-1)\gamma(1-\alpha_i)k > 0$. The earnings of an entrepreneur with talent θ_j in sector i can thus be written as

$$\pi(\theta_j, i) = p(i)^{\frac{\sigma}{\Sigma_i}}\theta_j^{\frac{\sigma(z-k)}{\Sigma_i}}\overline{X}^{\frac{\alpha_i k\sigma}{\Sigma_i}}[(1-\alpha_i)k]^{\frac{\sigma-\Sigma_i}{\Sigma_i}}\left(\frac{B}{\gamma}\right)^{\frac{-\gamma(1-\alpha_i)k\sigma}{\Sigma_i}}(1-\gamma)^{\frac{(1-\alpha_i)k(1-\gamma)}{\Sigma_i}}\frac{\Sigma_i}{\sigma} - r\lambda w$$

where $\Sigma_i = \sigma-(1-\alpha_i)k-(\sigma-1)\gamma(1-\alpha_i)k$.

I prove that NAM prevails between the level of entrepreneurial talent and sectoral capital intensity by contradiction. Suppose there is a constrained agent with talent θ_H who prefers sector i over j with $i > j$, and at the same time an entrepreneur with talent $\theta_L < \theta_H$ prefers j over i.

Hence, suppose that

$$\pi(\theta_H, i) > \pi(\theta_H, j)$$

[52]Empirical estimates of the vacancy costs suggest convexity, see, for example, Cooper et al. (2007).

and simultaneously

$$\pi(\theta_L, i) < \pi(\theta_L, j).$$

This implies that

$$\theta_H^{\frac{\sigma(z-k)k(1+\gamma(\sigma-1))(\alpha_j-\alpha_i)}{\Sigma_i\Sigma_j}} > \theta_L^{\frac{\sigma(z-k)k(1+\gamma(\sigma-1))(\alpha_j-\alpha_i)}{\Sigma_i\Sigma_j}},$$

which is a contradiction as $\alpha_j < \alpha_i$ and $\theta_H > \theta_L$ and $\sigma > 1$. Therefore, the sorting between entrepreneurs and sectoral capital intensity must be characterized by NAM for financially constrained entrepreneurs.

Unconstrained Entrepreneurs. This is different for unconstrained entrepreneurs. Combining the first-order conditions implies that

$$\left(X_j^*\right)^{\Sigma_i-\alpha_i k\sigma} = p(i)^\sigma\, \theta_j^{\sigma(z-k)} \left(\frac{\alpha_i}{r}\right)^{\Sigma_i} \left(\frac{B}{\gamma}\right)^{-\gamma(1-\alpha_i)k\sigma} k^\sigma\, (1-\alpha_i)^{\sigma-\Sigma_i}\, (1-\gamma)^{(1-\alpha_i)k(1-\gamma)}.$$

The necessary and sufficient condition for the capital demand to be over-proportionally increasing in talent θ_j is

$$\sigma(z-1) > (1-\alpha_i)k(1-\gamma)(\sigma-1).$$

This condition is for example fulfilled if the production function is assumed to be $Y_i(\theta_j, X, L) = \theta_j\, \left(X^{\alpha_i} L^{1-\alpha_i}\right)^k$, with $k < 1$.

The earnings of an unconstrained agent in sector i can be rewritten as

$$\pi(\theta_j, i) =$$
$$= (kp(i))^{\frac{\sigma}{\Gamma_i}}\, \theta_j^{\frac{\sigma(z-k)}{\Gamma_i}} \left(\frac{B}{\gamma}\right)^{\frac{-\gamma(1-\alpha_i)k\sigma}{\Gamma_i}} (1-\gamma)^{\frac{(1-\alpha_i)k(1-\gamma)}{\Gamma_i}} (1-\alpha_i)^{\frac{\sigma-\Sigma_i}{\Gamma_i}} \left(\frac{\alpha_i}{r}\right)^{\frac{\alpha_i k\sigma}{\Gamma_i}} \frac{\Gamma_i}{k\sigma}.$$

where $\Gamma_i = \Sigma_i - \alpha_i k\sigma$. The necessary and sufficient condition for PAM between entrepreneurial talent and sectoral capital intensity then is

$$\theta_H^{\frac{(\alpha_i-\alpha_j)(z-k)\sigma(\sigma-1)k(1-\gamma)}{(\Sigma_i-\alpha_i k\sigma)(\Sigma_j-\alpha_j k\sigma)}} > \theta_L^{\frac{(\alpha_i-\alpha_j)(z-k)\sigma(\sigma-1)k(1-\gamma)}{(\Sigma_i-\alpha_i k\sigma)(\Sigma_j-\alpha_j k\sigma)}},$$

for any two agents with $\theta_H > \theta_L$ and for any two sectors $i > j$. This is always fulfilled as $\alpha_i > \alpha_j$. Thus, for unconstrained entrepreneurs, PAM prevails and the more talented

entrepreneurs sort into more capital-intensive sectors. Hence, because the wages offered monotonically increase in entrepreneurial talent, the sorting reversals due to financial development implies that the alignment of the average sectoral wages with the sectoral capital-intensity increases.

Chapter 2

Modes of Cooperation - and the Returns to Talent

In this paper I analyze the consequences of cooperation on inequality. I develop a heterogeneous agents model distinguishing between two types of cooperation, between-task and within-task cooperation. The former reflects the chance to assign different tasks to different agents. The latter represents the reassignment of tasks from one agent to another if the first agent fails to complete the task. The equilibrium allocation is analyzed with a particular focus on the sorting of agents into modes of cooperation and matching between agents. Cooperation increases inequality at the top and decreases inequality at the bottom of the talent distribution. Within-task cooperation is more inequality-enhancing than between-task cooperation. This may help explain changes in income inequality in the wake of the information and communication technology revolution. Finally, I examine how the talent distribution shapes returns to talent in an economy. In particular, a wider talent distribution is likely to increase the skill premium. This sheds some light on differences in the skill premium across countries.

Keywords: Cooperation, Occupational Choice, Wage Differentials, Organization of the firm.

JEL Classification Number: J24, J3, L2, M5.

2.1 Introduction

From the first human settlements in around 10,000 BC to the present day, there has been a colossal increase in the division of labor. Not only has the range of consumption goods grown dramatically, but an even larger increase can be observed in the number of occupations open to people.[1] The increase in cooperation between individuals seems to be one of the most distinct features of progress. Clearly, this raises some research questions: What are the consequences of cooperation for earnings distribution? How are earnings likely to evolve over time if cooperation continues to increase? This paper takes a step towards answering such questions.

One salient difference between Europe and the U.S. is the more dispersed distribution of talent and the higher skill premium in the U.S. (see Devroye and Freeman (2002), Leuven et al. (2004)).[2] A second focus of the paper is to address the role that cooperation may play in explaining the larger skill premium in the U.S. in comparison to Europe. In particular, I find that a more dispersed talent distribution in an economy leads to a higher skill premium if the skill distribution is sufficiently right-skewed.[3] Thus, cooperation tends to act as a mechanism which amplifies the dispersion of earnings compared to talent. The reason is that the dispersion in talent affects occupational choice and the mode of cooperation chosen.

In order to address these research questions, I have developed a heterogeneous agents model that allows for different types of cooperation. In particular, I extend a version of the model used in a series of papers by Garicano (2000), and Garicano and Rossi-Hansberg (2004, 2006). In their model agents face one uncertain task and cooperation occurs when agents unable to perform the task pass it on to other (more skilled) agents. In order to

[1] As early on as the late eighteenth century the production of a single pin was divided into eighteen distinct operations (Smith and Garnier (1845)). Levine (2012) mentions that the production of a Boeing 747 requires more than 6,000,000 parts, each involving another series of operations.

[2] See Leuven et al. (2004) for a discussion of alternative attempts in the literature to explain the more dispersed earnings distribution in the U.S.

[3] Devroye and Freeman (2002) find that the difference in the skill premium explains a substantial part of the difference in earnings dispersion between Europe and the U.S.

capture the empirical fact that a great deal of cooperation takes place between tasks, I introduce a second task and allow for both cooperation between tasks and cooperation within tasks. In production, the overall workload is often bundled into different sets of tasks. I use the simplifying assumption that it is exactly partitioned into two (sets of) tasks, one containing tasks that any agent (at least in that line of work) can perform, and one containing the remaining tasks. Thus, the latter is similar to the one used by Garicano and Rossi-Hansberg.

More specifically, the economy is characterized by a continuum of heterogeneous agents who differ in a one-dimensional talent measure, i.e. agents can be ranked according to skill.[4] Any completed project can be sold at an exogenous price p and consists of two tasks A and B. The former requires time but no special knowledge, while the latter requires both time and expertise. As the task requirement of B is unknown *ex-ante*, agents face uncertainty in task B compliance. The two types of cooperation possible are between-task cooperation, i.e. one agent specializes in carrying out task A, and the other on performing task B, and within-task cooperation, where agents who fail to perform task B pass the unfinished product on to more skilled agents who may be able to perform the task.

While the form of between-task cooperation is rather specific (as task A can be exerted by any agent), it is an empirically relevant case. In most lines of works a multitude of tasks requires time but no particular skill beyond the basic skills learnt during training. For example, many architects employed carry out exclusively standard tasks in projects, whereas only the partners focus on the more creative and specialized work. The tasks performed by a nurse are generally time-consuming, but do not require any knowledge beyond that learnt during training. Running standard regressions every day for a bank is a job that any economist with a PhD can perform. Similarly, in many jobs people are exclusively engaged in tasks that are time consuming, but do not require any knowledge or skill beyond that taught during their initial training.

[4] I would like to extend this model to allow for both multi-dimensional talent and tasks but this is beyond the scope of the current paper.

I characterize the equilibrium, with a specific focus on the matching pattern between agents and equilibrium sorting into forms of cooperation. In general, cooperation leads to a decrease in inequality at the bottom of the talent distribution and an increase at the top. The reason is that cooperation takes place between high-skilled and low-skilled agents, and both skill groups gain. Thus, the agents who loose in relative terms are those in the middle of the talent distribution. Still, there are some notable differences between the consequences of an increase in between-task and within-task cooperation. Increases in between-task cooperation actually reduce inequality both at the top and the bottom of the income distribution relative to within-task cooperation. Thus, while cooperation generally enhances top-income inequality and reduces low-income inequality, within-task cooperation strengthens the first effect and weakens the latter. Furthermore, in contrast to between-task cooperation, in within-task cooperation the marginal earnings of an agent are dependent on the talent of the cooperators.

As mentioned, cooperation can help to explain the higher returns to talent in the U.S. in comparison to those in Europe. A more dispersed skill distribution implies a higher skill premium in the presence of cooperation (at least for right-skewed distributions). Moreover, better information on the difficulty of task B is equivalent to an increase in the talent dispersion. This may help to explain certain long-term trends in the skill premium.

Related Literature This paper is related to different strands of literature. First, there is the vast literature on determinants of earnings distribution.[5] A great deal of research focuses on potential explanations for observed earnings differentials. Early studies relating the earnings and talent distribution via a scale-of-operations or span-of-control effect are Tuck (1954), Mayer (1960), Lucas Jr. (1978), and Rosen (1982). Yet, in contrast to my paper they do not internalize the cooperation decision and have no explicit matching decisions.

There is an extensive literature which tries to explain the evolution of inequality over

[5]For reviews of alternative theories explaining observed wage differentials see Sattinger (1993) and Neal and Rosen (2000).

time. For example, Acemoglu (2003) analyzes the impact of international trade on wage inequality, both across countries and over time. The extensive literature on the impact of technological change on the equilibrium distribution of earnings deals with this issue, such as Galor and Tsiddon (1997), Acemoglu (1998), Galor and Moav (2000), Krusell et al. (2000), Mobius and Schoenle (1996), Saint-Paul (2001).[6] They consider technological change directly affecting the productivity of agents, whereas I target the effects caused by the decreasing costs of cooperation in order to focus on the collaboration channel in explaining changes in income inequalities.

My research also relates to the literature on cross-country differences in skill premia and wage differentials, and their determinants. Krueger et al. (2010) present an extensive analysis of the variation between countries when it comes to skill premia. Devroye and Freeman (2002) and Leuven et al. (2004) address the issue of whether differences in talent distribution can explain the observed earnings differentials. I show how cooperation plays a role in explaining these differences.

I also refer to the extensive matching literature starting with the seminal paper by Becker (1973). Important contributions examining the role of cooperation on earnings differentials in matching models are Kremer (1993), Kremer and Maskin (1996), and Legros and Newman (2002). The distinctive element in my model is that cooperation is a choice variable. Other examples of the growing literature of heterogeneous agent models with assignment or matching are Grossman and Maggi (2000), Grossman (2004), and Ohnsorge and Trefler (2007).

In its focus on cooperation among agents the paper is related to the literature on the theory of the firm. However, this sort of literature focuses more on the structure of cooperation (within the firm or via the market) and less on whether agents want to cooperate.[7] Most literature on organizational theory focuses on incentive problems between agents within cooperation.[8]

[6] For an overview of the literature on skill-biased technological change see Violante (2008).

[7] For a review of some of the central theories of the firm see Gibbons (2005).

[8] For example, Calvo and Wellisz (1978, 1979) and Williamson (1967) develop hierarchical theories of the firm based on incentives.

A less extensive strand in the literature focuses on the cognitive limits of agents as a reason for cooperation. My analysis belongs to this strand of research.[9] The work most closely related to ours is represented by Garicano (2000) and Garicano and Rossi-Hansberg (2004, 2006).[10] They consider a knowledge economy with a distribution of problems and knowledge among agents. Agents can either draw problems from the distribution and try to solve them, or they can buy unsolved problems from other agents and try to resolve them. The latter option has the downside of a lower probability of success, but the upside of less costly problems (more specifically, in their framework the latter option allows to try to solve more problems). This form of cooperation is captured by the within-task cooperation in my model.

In contrast to my work, there is no between-task cooperation as all tasks are equal *ex-ante* in their setup. Empirically, part of the tasks necessary in a job are performed by secretaries, assistants, nurses etc. These tasks do not require any specific knowledge beyond the general training undertaken in that particular occupation. As these kinds of tasks play a substantial role in all market segments, an inclusion of this form of cooperation is necessary in order to assess the effects of cooperation on the distribution of labor and earnings correctly.

The rest of the paper is organized as follows. In section 2.2 I present the model and in 2.3 the agents' maximization decisions. Section 2.4 defines and characterizes the competitive equilibrium. Central properties of the equilibrium are discussed in 2.5. I analyze the consequences of cooperation on inequality in section 2.6. The link between cooperation, the dispersion of talent and the skill premium are discussed in section 2.7, and section 2.8 concludes. All proofs are relegated to the Appendix.

2.2 Setup

The economy consists of a continuum of income-maximizing agents of measure 1. Agents need both knowledge and time to perform tasks. They are heterogeneous in knowledge

[9] For a review of this line of research, see Garicano and Prat (2011).

[10] Another paper along the same lines is Saint-Paul (2007).

t, distributed according to a probability distribution function $\phi(t)$ with full support between $\underline{t}$ and $\bar{t}$.[11] The respective c.d.f. is denoted $\Phi(t)$, is continuous and increases strictly monotonically. In turn, agents are homogeneous in both the time required to perform a task and time endowment. For expositional reasons I assume that each agent is able to perform at most two tasks per period.

Production. Agents engage in projects and each successfully exhibited project yields the same value p. It is composed of two project-specific tasks A and B combined via a Leontief technology.[12] Both tasks require time and knowledge. In particular, they require the same amount of time but differ in difficulty. Task A can be carried out by any agent, whereas task B is characterized by an idiosyncratic difficulty draw q distributed according to a continuous and strictly monotonically increasing cumulative distribution function $\Psi(q)$, with the p.d.f. denoted $\psi(q)$. Only agents with $t \geq q$ can complete their tasks successfully. Thus, each agent faces an *ex-ante* probability of accomplishing task B equal to $\Psi(t)$. It also implies that agents can be ranked by talent, i.e. agents with $t > t'$ can carry out any task an agent with talent t' can complete.

It is important to note that tasks A and B are project-specific. Agents cannot simply assemble any successfully performed task A and B to final projects, but both project-specific tasks must be carried out to complete the project.[13]

Furthermore, agents only realize whether they are able to perform a project-specific task B once they start such a task. Therefore, irrespective of compliance, each agent can try to perform at most two tasks.

Cooperation. I turn now to the cooperation possibilities among agents. First, all agents can choose to work alone and try to carry out both tasks themselves. I refer to these agents as *autarkic* (A). Moreover, agents can pay some fixed costs of cooperation and choose to collaborate with other agents. These fixed costs incorporate search and

[11] I use the terms talent, skill, knowledge, and expertise interchangeably throughout.

[12] One can also think of two sets of tasks A and B.

[13] The word "project" is a general term denoting services, goods production, etc.

matching costs, as well as other cooperation-specific transaction costs, such as contracting costs or potential legal costs. In particular, two forms of cooperation are possible: between-task cooperation and within-task cooperation.[14]

I should mention briefly that I do not take issues of financial liability into account. I assume that any fixed costs or remuneration payments can be afforded irrespective of the success or failure of a project.

Between-task Cooperation. Between-task cooperation allows agents to share the workload *ex-ante*, before knowing the difficulty of the project. In particular, at a fixed cost c_a, two agents can specialize in different tasks and assemble exerted project-specific tasks A and B afterwards. It is clear that there will always be one agent who specializes exclusively in production of task A and another who performs task B. Henceforth, I label agents specializing in task A production *workers* (W), and those who specializing in task B production *entrepreneurs* (E).

Within-task Cooperation. By contrast, within-task cooperation allows agents to pass on any task that is not completed successfully by one agent to be performed by another agent. More specifically, a fixed cost c_p is required to pass on an unfinished project, including a failed task B and completed task A, to another agent. I assume that an unfinished task can only be passed on to another agent once.[15] The likelihood of successful task completion is lower for tasks that are passed on. However, the performed project-specific task A implies that agents can acquire two unfinished projects. Agents who only acquire unfinished projects, i.e. those where other agents have failed in task B production, are referred to as *consultants* (C), and agents who sell their projects after failure to complete them are indicated by a subscript c (e.g. A_c or E_c). Clearly, agents

[14] Alternative labels are *ex-ante* and *ex-post* cooperation to highlight that some forms of cooperation are created before any information on task difficulty is revealed and other forms are created afterwards. These labels also indicate the relatively long-term relationship of between-task cooperation.

[15] This simplifies the analysis. Garicano and Rossi-Hansberg (2006) demonstrate that the qualitative results do not change when unfinished tasks are passed on several times.

may also engage in both types of cooperation.

Timing The timing of the production process takes the following form:

1. Cooperations are formed.
2. Assignment of tasks in between-task cooperation.
3. Idiosyncratic difficulty draws q.
4. All assigned tasks are attempted (and completed where possible).
5. Reallocation of tasks in within-task cooperation.
6. Reallocated tasks are attempted (and completed where possible).
7. Final projects are assembled and sold.

2.3 The Agent's Maximization Problem.

Agents choose the occupation that maximizes their earnings. In this model, occupational choice refers to whether an agent wants to cooperate and, conditional on cooperation, what role she wants to carry out within the cooperation, and with whom she wants to cooperate.

I assume that entrepreneurs and consultants choose with whom to cooperate.[16] The analysis is separated into maximization decisions for entrepreneurs and consultants and occupational choice decisions of agents.

The Entrepreneur's Maximization Problem. Agents who participate in between-task cooperation have to choose with whom to cooperate and how to distribute the tasks. As any agent can perform task A, the less talented agents in such a cooperation are

[16] This assumption is without loss of generality, i.e. both the matching and pay-off pattern are the same when workers and agents selling unfinished projects make the decision. This follows from the uniqueness of the competitive equilibrium, to be proven below (see proposition 2.4).

assigned these tasks in order to increase the likelihood of success. Furthermore, the amount of tasks A executed equals that of tasks B attempted due to the project-specificity of tasks.

Thus, entrepreneurs choose the worker with talent s to maximize their earnings $\pi_E(t)$, i.e.

$$\pi_E(t) = \max_s \, 2\,p\,\Psi(t) - w(s) - c_a. \tag{2.1}$$

As any worker is able to carry out task A, entrepreneurs choose the worker who accepts the lowest wage $w(s)$. In equilibrium a flat wage w always prevails.

The Consultant's Maximization Problem. Agents who participate in within-task cooperation have to choose on which side of the bargain they want to be. Clearly, all autarkic agents and entrepreneurs are willing to sell their unfinished projects (in case of failure) as long as they receive a positive price, and sell to the consultant offering the highest price. In turn, agents who buy unfinished projects have to choose which agent to buy from. Their earnings can be written as

$$\pi_C(t) = \max_s \, 2\,p\,Pr(q \leq t | q > s) - 2\,c_p - 2\,r(s), \tag{2.2}$$

where s indicates the talent of the seller. The price an agent with talent s receives for selling her unfinished project is labeled $r(s)$. The conditional probability of success can be rewritten as

$$Pr(t \geq q | q > s) = \frac{\Psi(t) - \Psi(s)}{1 - \Psi(s)},$$

if $t \geq s$. Clearly, consultants are more talented than the agents they buy from, as they would otherwise not face a positive success probability. This leads to the following optimality condition:

$$p\,\frac{\psi(s)(\Psi(t) - 1)}{(1 - \Psi(s))^2} = r'(s). \tag{2.3}$$

$\psi(s)$ is the marginal increase in the probability of success at point s. The marginal price for an unfinished project equals the marginal return it provides. As $\Psi(t)$ is bounded above by 1 the price $r(s)$ decreases in s. This means that *ceteris paribus* consultants prefer to

buy projects from less-skilled agents, since this increases their conditional probability of success.

The marginal price depends not only on the talent of the agent selling the unfinished project but also on the talent of the agent buying it, as the conditional probability of success depends on both agents' talent in a non-separable way. Indeed, the equilibrium price function $r(s)$ depends on the whole distribution of talent.[17]

Note that the maximization problem for consultants is independent of whether the seller is an entrepreneur or an autarkic agent as both payment and fixed costs occur per project and not per cooperator.

Occupational Choice An agent with talent t chooses the occupation that maximizes her payoff. Her earnings are thus

$$\omega(t) \equiv \max\{\pi_W(t); \pi_{A_c}(t); \pi_A(t); \pi_{E_c}(t); \pi_E(t); \pi_C(t)\}. \tag{2.4}$$

Agents who take part in cooperation must also choose with whom to cooperate as discussed above. I suppress this dependence here for notational convenience. The earnings in the respective occupations are as follows:

$$\pi_W(t) = w(t); \tag{2.5}$$

$$\pi_{A_c}(t) = p\,\Psi(t) + (1 - \Psi(t))\,r(t); \tag{2.6}$$

$$\pi_A(t) = p\,\Psi(t); \tag{2.7}$$

$$\pi_{E_c}(t) = 2\,p\,\Psi(t) + 2\,(1 - \Psi(t))\,r(t) - c_a - w(s); \tag{2.8}$$

$$\pi_E(t) = 2\,p\,\Psi(t) - c_a - w(s); \tag{2.9}$$

$$\pi_C(t) = 2\,p\,\frac{\Psi(t) - \Psi(s)}{1 - \Psi(s)} - 2\,c_p - 2\,r(s). \tag{2.10}$$

2.4 Market Clearing and Competitive Equilibrium

Before I turn to the definition of a competitive equilibrium in this model, I will discuss the markets that have to clear. In particular, the supply of workers or unfinished projects has

[17] When it does not cause any confusion, I suppress the dependence for notational convenience.

to equal the demand for workers or unfinished projects by entrepreneurs and consultants.

Labor Market Clearing. Let A_W denote the set of workers and $A_{E^T} \equiv A_E \cup A_{E_c}$ the set of all entrepreneurs. The latter also incorporates entrepreneurs who engage in within-task cooperation and sell their unfinished projects. I denote with $e(t)$ the entrepreneur matched to a worker of talent t. The labor market clears if for every $t \in A_W$

$$\int_{[0,t]\cap A_W} \phi(t')dt' = \int_{[e(0),e(t)]\cap A_{E^T}} \phi(t')dt'. \tag{2.11}$$

The left-hand side presents the supply of workers and the right-hand side gives the corresponding demand by entrepreneurs.

In the notation above I assume implicitly positive assortative matching. In section 2.5.1 I demonstrate that matching between workers and entrepreneurs is indeterminate and hence this assumption is w.l.o.g.

Task Market Clearing. Similarly, the supply of unfinished projects has to equal its demand at any point. Let A_{A_c} denote the set of autarkic agents who cooperate within-tasks and A_{E_c} the set of entrepreneurs who engage in within-task cooperation. Similarly, the set of consultants who purchase unfinished projects from autarkic agents is denoted A_{C^A} and that of consultants who purchase from entrepreneurs A_{C^E}. Furthermore, the consultant matched with an agent with talent t is labeled $c_A(t)$ if the agent is autarkic and $c_E(t)$ if she is an entrepreneur.[18] For the markets of uncompleted tasks B to clear at any point, it is necessary that for every $t \in A_{A_c}$

$$\int_{[0,t]\cap A_{A_c}} (1-\Psi(t'))\phi(t')dt' = \int_{[c_A(0),c_A(t)]\cap A_{C^A}} 2\phi(t')dt' \tag{2.12}$$

and for every $t \in A_{E_c}$

$$\int_{[0,t]\cap A_{E_c}} 2\,(1-\Psi(t'))\phi(t')dt' = \int_{[c_E(0),c_E(t)]\cap A_{C^E}} 2\phi(t')dt'. \tag{2.13}$$

The left-hand sides of equations (2.12) and (2.13) are the supply of unfinished tasks B of autarkic agents and entrepreneurs, respectively. Note that the latter have twice the

[18] Note that A_A denotes the set of autarkic agents and A_C the joint set of all consultants.

probability of failure since they are assigned task B twice. The right-hand sides represent the demand for unfinished projects by consultants, each of whom demands two unfinished tasks.

In the above formulation I implicitly assume positive assortative matching. This is proven to hold in any equilibrium in section 2.5.1.

Competitive Equilibrium I am now able to define the notion of a competitive equilibrium in this economy as

- a collection of occupational sets, A_W, A_{A_c}, A_A, A_{E_c}, A_E, A_{C^A} and A_{C^E},
- an earnings function, $\omega(t) : [\underline{t}, \overline{t}] \to \mathbb{R}_+$,[19]
- assignment functions, $e(t) : A_W \to A_E$, $c_A(t) : A_{A_c} \to A_{C^A}$ and $c_E(t) : A_{E_c} \to A_{C^E}$,

such that:

1. Agents choose occupations to maximize earnings, (2.4).
2. Entrepreneurs and consultants choose cooperators in order to maximize their profits, (2.1) and (2.2).
3. Labor and Task markets clear, (2.11), (2.12) and (2.13).[20]

2.5 Equilibrium Properties

I now turn to some central properties of equilibrium. First, I establish the matching pattern between agents (workers and entrepreneurs as well as unfinished projects and consultants) that prevails in any equilibrium, together with the equilibrium price functions $w(t)$ and $r(t)$. Then, the sorting pattern of agents into occupations is analyzed and the existence and uniqueness of the equilibrium allocation are established.

[19]The earnings function depends on the two price functions, $w(t)$ and $r(t)$. Hence I can replace the earnings function with those functions embedded in it.

[20]Note that some occupational sets may be empty in equilibrium, depending on the exogenous parameters. In this case the labor or task markets may not exist.

2.5.1 Matching

Recall that there is $1:1$ matching between workers and entrepreneurs in between-task cooperation, and workers only perform task A. As any agent assigned to this task is sure to complete it, there is a flat wage among workers. Furthermore, entrepreneurial earnings do not depend on the talent of the worker and therefore the explicit matching pattern between workers and entrepreneurs is indeterminate. These results are summarized as follows:

Proposition 2.1. *Workers earn a flat wage $w(t) = w > 0$. Matching between workers and entrepreneurs is indeterminate.*

While matching between workers and entrepreneurs is indeterminate, there is a clear matching pattern between unfinished projects and consultants. *Ceteris paribus* all consultants prefer to buy unfinished projects from less talented agents as this increases their conditional likelihood of success. Therefore, market clearing for unfinished projects implies that the price function $r(t)$ decreases monotonically.

The decrease in conditional success probability due to an increase in the expected difficulty of the unfinished project is greater for less talented consultants. Therefore, less talented consultants are willing to pay more in order to obtain projects that are expected to be easier than more talented consultants. This implies that any equilibrium displays positive assortative matching. More talented agents pass their unfinished tasks to more talented consultants. This result is summarized in the following proposition.

Proposition 2.2. *For any positive price $r(t) > 0$ for unfinished projects holds that $\frac{\partial r(t)}{\partial t} < 0$. Any equilibrium assignment exhibits positive assortative matching between the sets A_{A_c} and A_{C^A} (and also between A_{E_c} and A_{C^E}).*

Given that result I directly obtain the assignment function via

$$\frac{\partial c_A(t)}{\partial t} = \frac{1-\Psi(t)}{2}\frac{\phi(t)}{\phi(c_A(t))}, \tag{2.14}$$

by deriving the task market clearing condition. In the same way,

$$\frac{\partial c_E(t)}{\partial t} = (1 - \Psi(t))\frac{\phi(t)}{\phi(c_E(t))}.^{21} \tag{2.15}$$

These differential equations together with the boundaries of the occupational sets determine equilibrium assignment. I denote the boundary between set A_i and A_j as t_{ij}, e.g. t_{AE} denotes the cut-off between autarkic agents and entrepreneurs. Note that there is $n(t) : 1$ matching on the market for unfinished projects, with $n(t) = \frac{2}{1-\Psi(t)}$ for autarkic agents and $n(t) = \frac{1}{1-\Psi(t)}$ for entrepreneurs.

2.5.2 Earnings

Before I go on to describe equilibrium sorting, I need to discuss a few properties of any equilibrium earnings function. As discussed, a flat wage w and a continuously and decreasing price function $r(t)$ for unfinished projects prevails in any equilibrium. Together with the fact that both the probability of success $\Psi(t)$ and the c.d.f. $\Phi(t)$ increase continuously and monotonically in talent, this implies that earnings are a continuous and monotonically increasing function in talent. This is summarized in the following lemma.

Lemma 2.1. *In any equilibrium, the earnings function $\omega(t)$ is continuous and increases monotonically in t.*

2.5.3 Sorting into Occupations

Turning to the equilibrium sorting of agents into occupations an initial result is that not all occupational sets have to be non-empty in equilibrium. Depending on both the distribution of talent and task difficulty, along with cooperation costs, some form(s) of cooperation may not occur in equilibrium. This leads to empty occupational sets. Second, not all occupational sets can co-exist in equilibrium. In particular, one of the sets A_A and A_{EC} has to be empty. If there are autarkic agents in equilibrium for which the price of selling the unfinished projects is non-positive, there cannot be any entrepreneur facing

[21] We also need the occupational sets to be convex for these equations to hold for any t. This is proven below, see (2.5.3).

a positive price $r(t)$, and *vice versa*. Which of the sets (if any) is non-empty depends on the parameters of the model.

Any equilibrium displays occupational stratification, i.e. occupations form convex sets. In Kremer and Maskin (1996) and Legros and Newman (2002) non-convex occupational sets may prevail due to complementarities in production. The specific form of complementarity considered in this model does not allow for this. The equilibrium displays a clear ordering of the occupational sets by talent. The following proposition summarizes these results.

Proposition 2.3. *Any equilibrium is characterized by occupational stratification, i.e. occupational sets are convex. Ranking agents from low to high, the occupational sets can be ranked in the ordering A_W, A_{A_c}, A_A, A_{E^C}, A_E, A_{C^A} and A_{C^E}. The lowest talented agents remain workers while the most talented become consultants. The two sets A_A and A_{E^C} are mutually exclusive and not all other occupational sets have to be non-empty in equilibrium.*

Thus, assignment can be completely characterized by cut-off agents t_{ij} together with the differential equations derived from the market clearing conditions above (see 2.5.1).

2.5.4 Existence and Uniqueness

Up to this point I have characterized properties of any potential equilibrium without reference to whether existence and uniqueness of the competitive equilibrium is guaranteed. Indeed, for any set of parameters, there is always a unique equilibrium allocation characterized by a set of cut-offs t_{WA_c}, t_{A_cA}, t_{AE_c}, t_{E_cE}, t_{EC^A}, and $t_{C^AC^E}$. If $t_{ij} = t_{jk}$, the occupational set A_j is empty.

Proposition 2.4. *For any set of parameters there is always a unique competitive equilibrium allocation.*

The proof is divided into several parts. First, I partition the parameter space and show that for each set of parameters only one candidate equilibrium type - defined as the set of non-empty occupational sets - remains, and then continue to show that this

candidate equilibrium does indeed exist and is also unique. In particular, there is only one combination of prices r_0 and w that clears both the labor and task market and satisfies agents' earnings maximizing occupational choice.[22]

Different economies are likely to be characterized by different equilibrium types, depending on the distribution of talent and the difficulty of tasks, as well as the value of the projects p and the cost of cooperation c_a and c_p.

2.6 Cooperation and Earnings Inequality

The increase in collaboration between agents in production over time is an undisputed consequence of progress and specialization. Technological progress facilitates cooperation and thus reduces cooperation costs over time. This is in line with the empirical fact that average firm size increases over time.[23] However, not all technological developments benefit both modes of cooperation in the same way. For example, the standardization of production processes facilitated the separation of overall workload into different tasks, and thus constitutes a reduction in the costs of between-task cooperation, c_a. By contrast, the availability of internet and online searching mechanisms facilitated the search for experts and thus reduced the cooperation costs between buyers and sellers of unfinished projects, c_p.[24] As increases in the two types of cooperation have different consequences for earnings inequality, it is important to distinguish between them and to assess empirically which type of cooperation benefited most from a particular technological improvement or policy reform.

[22]Recall that the price r_0 together with the cut-off talents between occupational sets and consultants' necessary condition implies a unique matching pattern and equilibrium earnings $r(t)$.

[23]See Poschke (2014) for a summary on the empirical evidence of increasing average firm size over time. He also provides evidence that the number of entrepreneurs and self-employed decreases over time consistent with my model.

[24]Clearly, many technological developments cause a decrease in overall cooperation costs. Yet they almost always benefit one type of cooperation more than another. For example, online search mechanisms particularly facilitated short-term labor contracts. As within-task cooperation is more characterized by short-term labor contracts, this mode of cooperation benefited particularly from the IT revolution.

In general, cooperation leads to an increase in high-income inequality (between the 90%- and 50%-percentile) and a decrease in low-income inequality (between the 50%- and 10%-percentile) compared to cases where there is no cooperation. Yet, considering these two different types of cooperation implies additional results. In particular, while reductions in c_p increase both inequality among highly-talented and less talented agents, drops in c_a actually reduce inequality among those two groups of agents when both modes of cooperation are present. In general, within-task cooperation is more inequality-enhancing than between-task cooperation. This helps explain why there has been a increase in inequality attributed to the increase in information and communication technology in recent years (see e.g. Michaels et al. (2014)). They found that information and communication technologies (ICT) led to a polarization of markets, increasing demand for low-skilled and high-skilled agents at the expense of those with an intermediate level of talent.

Turning to the analysis of the effect of cooperation on equilibrium earnings inequality I consider both discrete and marginal changes in cooperation costs and analyze the differences and similarities between the two modes of cooperation as regards earnings inequality. First, I compare the earnings distribution where cooperation costs are prohibitively high with a situation where lower cooperation costs allow two-thirds of the population engage in cooperation. Then, I consider in turn the effects of marginal decreases in cooperation costs on sorting and earnings for the scenarios that either one or both modes of cooperation prevail in equilibrium.[25]

Before discussing the consequences of decreased cooperation costs on the earnings distribution, I need to define the three different concepts of inequality used in my model.

Definition 2.1. *I define low-income (high-income) inequality in this model as the earnings ratio between the* 50%*-percentile and* 10%*-percentile (*90%*- and* 50%*-percentile) of the income distribution.*

An increase (decrease) in within occupational group inequality is defined as an increase

[25] I exclude the case where no cooperation takes place in equilibrium as any marginal change in cooperation costs has no consequence on occupational choice in that scenario.

(decrease) in the marginal earnings of agents in an occupational set, i.e. an increase (decrease) in $\pi_i'(t)$ *for all* $t \in A_i$.

In this framework, cooperation is primarily between agents with high and low levels of knowledge. I therefore use the concepts of low-income and high-income inequality in order to analyze the evolution of earnings differentials between agents who cooperate and agents who do not. Thus, I always consider scenarios that imply that both the 90%-percentile and the 10%-percentile of the talent distribution are engaged in cooperation. The issue which agents, high-skilled or low-skilled, benefit more from cooperation is discussed briefly the Appendix (see 2.9.2).

Note that both the equilibrium allocation and earnings depend exclusively on the distribution of "effective" talent, i.e. the distribution of success probability $\Psi(t)$ in the economy. It can be written as $G(x) \equiv \Phi(\Psi^{-1}(x))$ for $x \in [0, 1]$. The marginal distribution of "effective" talent is therefore given by $g(x) \equiv \frac{\partial G(x)}{\partial x} = \frac{\phi(x)}{\psi(x)}$.[26]

2.6.1 Discrete Changes

Highly talented agents benefit most from assigning relatively simple tasks to other agents in order to use their high levels of talent most efficiently, and low-skilled agents are the first to accept these assigned simpler tasks. Therefore, the first agents to engage in cooperation are the least talented and the most talented individuals. Agents engaged in cooperation always form convex sets at the bottom and top of the talent distribution.[27]

Cooperation affects earnings of highly talented and less talented agents differently. The talent of the most talented agents is leveraged on the whole organization and therefore the marginal returns to talent increases for these agents. By contrast, the talent of low-skilled agents matters less for output within cooperation and therefore the marginal returns to talent decrease for them. In the extreme case of between-task cooperation,

[26] In the following, I use the terms "effective" talent and talent interchangeably when it does not cause any confusion.

[27] Note that this stark result depends on the specific structure of task A. If there is also a degree of uncertainty in task A the first agents to engage in between-task cooperation may not be the most skilled and least-skilled agents (see Legros and Newman (2002)).

output is independent of the talent of workers and hence marginal returns to talent of low-skilled agents are 0.

Therefore, cooperation reduces the earnings inequality among low-skilled agents and increases inequality among highly talented individuals. These results are summarized in the proposition below.

Proposition 2.5. *Cooperation reduces low-income inequality and increases high-income inequality. The marginal returns to talent are greater in within-task cooperation than in between-task cooperation.*

The second part of the proposition holds for both the agents at the top and bottom set of the talent distribution who are engaged in cooperation. Inequality is larger within the set A_C than within A_E, and it is also larger within A_{A_c} than within A_W. Therefore, inequality in general is larger due to within-task cooperation.[28]

Figure 2.1 depicts the change in earnings due to cooperation for a log-normal "effective" talent distribution. The dashed line represents expected earnings as a function of talent if no cooperation occurs. The price p is set at 1 and hence all agents earn on expectation their success probability in task B, $\Psi(t)$. Agents can always remain autarkic and thus $\Psi(t)$ represents agents' outside option. Hence, any agent engaged in cooperation earns more than she did as an autarkic agent.

The other graphs depict earnings when either one of the two type types of cooperation is feasible.[29] In both scenarios, the costs of cooperation are such that one third of agents remains autarkic. The agents most reluctant to enter any form of cooperation are those with intermediate levels of talent. They are not sufficiently talented to contract workers or to purchase unfinished projects, and they are also too expensive to contract as workers and too talented to buy their unfinished projects.[30] Therefore, high-skilled and low-skilled

[28]Yet, as within-task cooperation is characterized by $n : 1$ matching there are more less talented individuals entering into cooperation for each highly talented individual in within-task cooperation than in between-task cooperation. Therefore, within-task cooperation concentrates the gains from cooperation on a few extremely talented agents.

[29]I exclude the case where both modes of cooperation prevail for better visibility.

[30]Depending on the parameters it may be the case that all agents are engaged in some form of

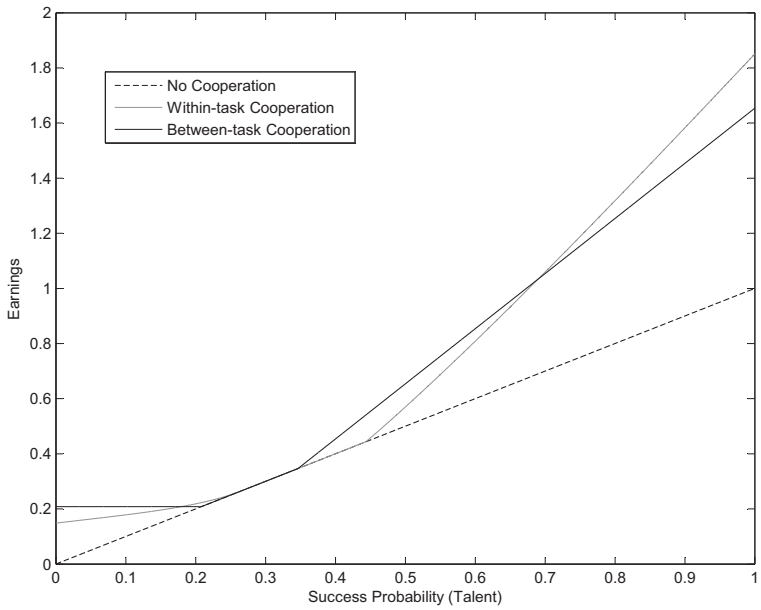

Figure 2.1: Change in Earnings due to Cooperation (log-normal talent distribution)

agents gain from cooperation relative to agents with intermediate levels of talent.

As mentioned, if only between-task cooperation occurs (the black line) earnings display lower marginal returns to talent for both highly talented and less talented agents compared to the scenario when equilibrium is characterized by within-task cooperation (the grey line). That implies that the inequality both among high-skilled and low-skilled agents is lower with between-task than within-task cooperation.

2.6.2 Marginal Changes in Cooperation Costs

Turning to the discussion of the effects of a marginal reduction in cooperation costs on sorting and inequality when some cooperation is already in place, I analyze the cases where one type of cooperation and both modes of cooperation are in place. While I focus on analytical results in the first part, in the latter part I mainly provide simulation results as some consequences of cost reductions are ambiguous when both modes of cooperation are present.

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cooperation.

One Type of Cooperation Suppose only one type of cooperation prevails in equilibrium.[31] Clearly, only marginal reductions in cooperation costs for the mode of cooperation present in equilibrium affect sorting and earnings. A decrease in c_a (c_p) causes more agents to engage in cooperation and both sets A_W and A_E (A_{A_c} and A_C) necessarily increase, whereas the occupational set A_A decreases.[32] The earnings of agents in cooperation increase, while earnings of autarkic agents remain unaffected.

Thus, autarkic agents loose relative to other occupations. As these agents are located at the center of the earnings distribution, reductions in cooperation costs generally imply that low-income inequality decreases while high-income inequality increases.[33]

As the marginal earnings of workers and entrepreneurs do not depend on their match, within occupational group inequality for these sets does not depend on cooperation costs. By contrast, the marginal earnings of both consultants and autarkic agents engaged in within-task cooperation do depend on the respective match. Therefore, any change in the matching pattern affects inequality within these occupational groups. More specifically, within occupational group inequality increases in A_{A_c} and A_C if and only if the talent of the match increases. Reductions in c_p cause any agent in A_C (A_{A_c}) to be matched with a more (less) talented agent in A_{A_c} (A_C). Hence, within-group inequality increases for consultants and decreases for agents in A_{A_c} due to a drop in c_p.[34]

The following proposition summarizes these results:

Proposition 2.6.

[31] A sufficient condition for exclusively between-task cooperation occurring in equilibrium is $p\Psi(\bar{t}) > p\Psi(\underline{t}) + c_a$ and $2p\,\frac{\Psi(\bar{t})-\Psi(\underline{t})}{1-\Psi(\underline{t})} - p\Psi(\bar{t}) < 2c_p$. The analog condition for within-task cooperation as the only mode of cooperation in equilibrium is $p\Psi(\bar{t}) < p\Psi(\underline{t}) + c_a$ and $2p\,\frac{\Psi(\bar{t})-\Psi(\underline{t})}{1-\Psi(\underline{t})} - p\Psi(\bar{t}) > 2c_p$.

[32] A special case is one where all agents are already engaged in within-task cooperation. In this case a reduction in cooperation costs does not alter any occupational sets.

[33] An exception is where all agents are engaged in within-task cooperation. Then, sorting is unaffected by a change in cooperation costs, and any reduction in c_p is equally shared among all agents. This implies that both low-income and high-income inequality decreases in that particular case.

[34] Here too, the case that all agents are engaged in within-task cooperation is an exception. As the match is not affected by a change in cooperation costs in that scenario, within occupational group inequality does not change.

- *Low-income inequality decreases due to a reduction in cooperation costs.*
- *High-income inequality increases due to a reduction in cooperation costs.*
- *Within occupational group inequality is constant in the sets A_W, A_A, and A_C. Reductions in c_p increase inequality within the set A_C and reduce inequality within the set A_{A_c}.*

Note that the result that inequality within the sets A_W and A_E is unaffected by changes in cooperation costs is a consequence of the fact that compliance is independent of talent in task A. If this were not the case, within occupational group inequality would also depend on the costs of cooperation for between-task cooperation. How it would change in response to reductions in c_a depends crucially on the equilibrium matching function, which can be a rather complicated function as shown in Legros and Newman (2002). Nevertheless, this specific case is economically relevant in a great many occupations.

I have not yet mentioned any results with respect to the evolution of the inequality between the 90%-percentile and 10%-percentile. Which agents in cooperation profit more from a drop in cooperation costs depends heavily on the "effective" distribution of talent. In particular, the more right-skewed the distribution, the larger the share of earnings that goes to more highly talented agents. This is a straightforward consequence of relative supply and demand.[35]

An important difference between reductions in c_a and c_p is that the former always imply an increase in aggregate output. Between-task cooperation produces on expectation more output than its separate individuals as autarkic agents. By contrast, reductions in c_p do not necessarily produce an increase in aggregate output. This is because consultants reduce the likelihood of unfinished projects at the expense of fewer projects initiated.

Both Cooperation Forms Occur. Most of the results from the above discussion carry over when both modes of cooperation prevail. I therefore focus on the additional

[35] In the Appendix, I derive the marginal effects on w and r_0 and discuss this in more depth.

features when both cooperation types occur and the differences between reductions in c_a and c_p when it comes to inequality, and provide simulation results to illustrate these results graphically.

First, the earnings of all agents cooperating are affected by any decrease in cooperation costs due to general equilibrium effects. Yet, earnings change more in the occupational sets directly affected (e.g. A_W and A_E if c_a drops) than in those affected via general equilibrium effects (A_{A_c} and A_C if c_a falls). Therefore, inequality among high-skilled agents decreases when c_a falls as entrepreneurs gain relative to consultants. Conversely, it increases when c_p falls as consultants gain relative to entrepreneurs, and within-group inequality increases in A_C. Similarly, inequality among less talented agents decreases when c_a falls as workers gain relative to agents in A_{A_c}, and increases when c_p falls. Thus, inequality increases (more) due to a decrease in c_p than c_a.

Furthermore, reductions in c_a now affect also inequality within occupational sets, but only within A_{A_c} and A_C due to general equilibrium effects. Simulation results indicate that for most parameter sets earnings for consultants increase and inequality within this set decreases while inequality increases within A_{A_c} and all agents in that set loose. The reason is that entrepreneurs demand less low-skilled labor than consultants do. In particular, each entrepreneurs demands one worker while consultants demand on expectation $n(t) = 2/(1-\Psi(c^{-1}(t)))$ low-skilled labor. Therefore, a drop in c_a implies less demand for low-skilled labor and therefore increases consultants' earnings at the expense of agents in A_{A_c} due to general equilibrium effects. Figures 2.2 and 2.3 display this fact for both a normal and a log-normal distribution. The solid lines indicate earnings for high costs c_a and the dashed lines earnings for low between-task cooperation costs. Still, in both cases the earnings of agents in between-task cooperations increase relative to those in within-task cooperation, and hence inequality falls both among low-talented and high-talented agents.

By contrast, earnings of workers tend to always increase due to a drop in c_p. Again, the reason is that each consultant demands more low-skilled agents than an entrepreneur ($n(t) = 2/(1 - \Psi(c^{-1}(t)))$ versus 1). Therefore, both w and r_0 increase in order to satisfy

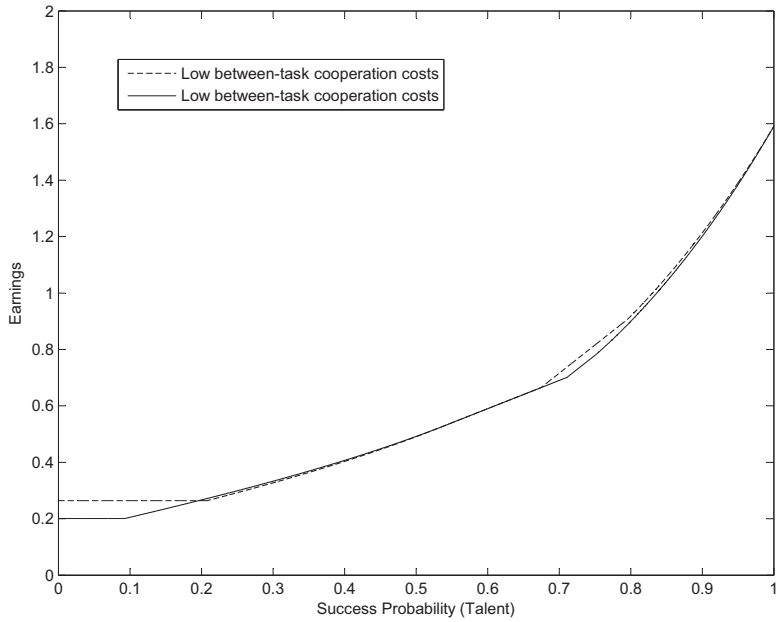

Figure 2.2: Change in between task cooperation cost for a normal talent distribution

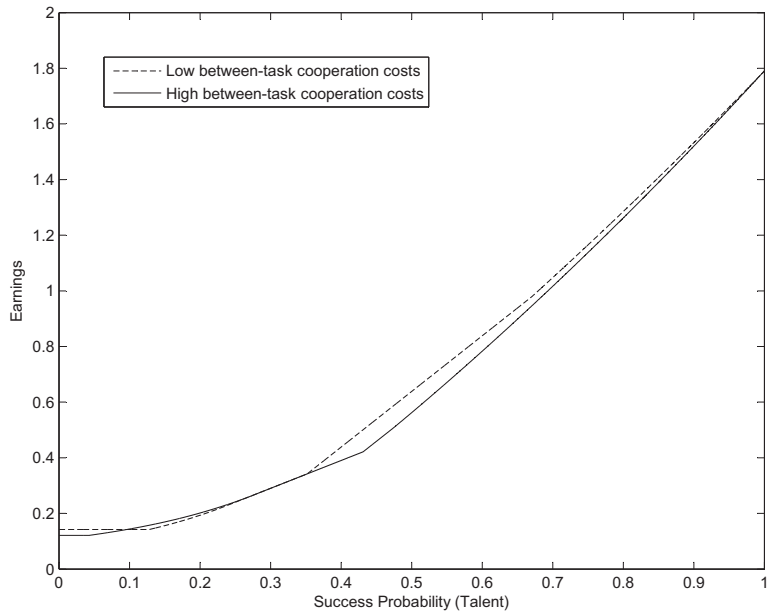

Figure 2.3: Change in between task cooperation cost for a log-normal talent distribution

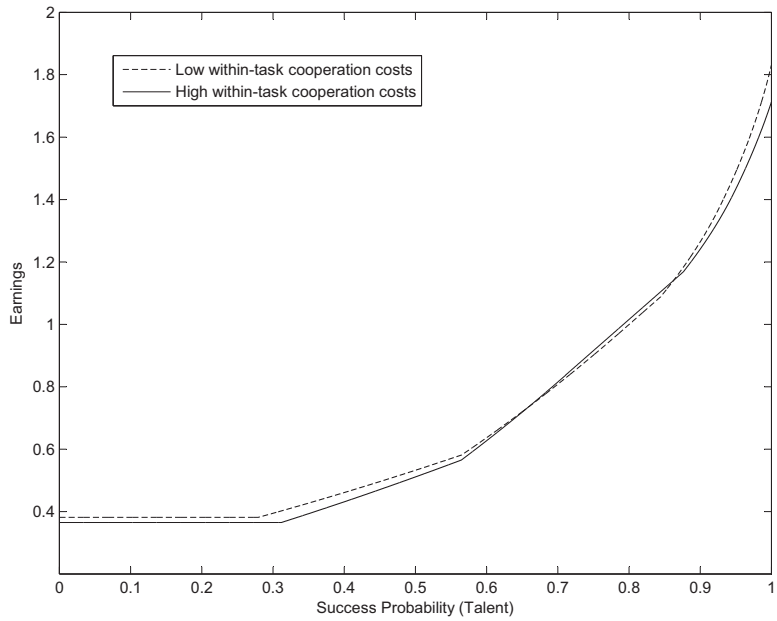

Figure 2.4: Change in within task cooperation cost for a uniform talent distribution

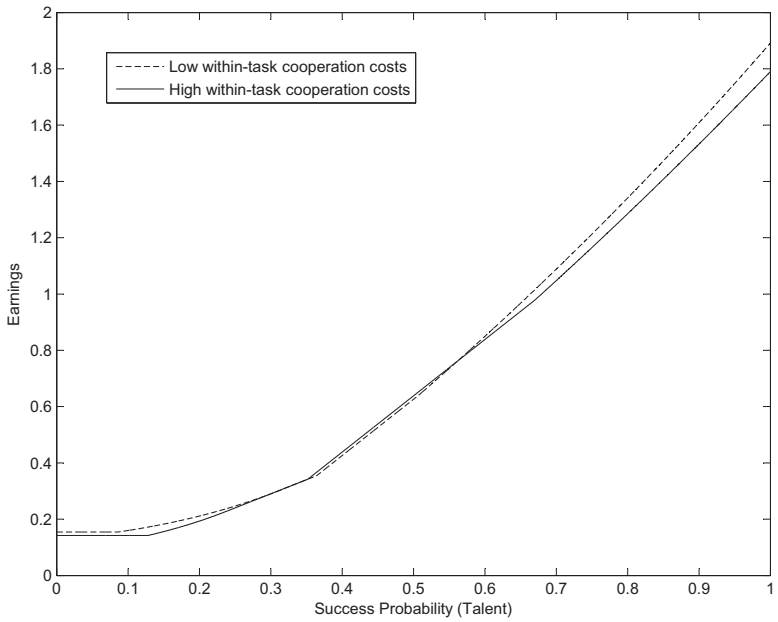

Figure 2.5: Change in within task cooperation cost for a log-normal talent distribution

the increasing demand for low-skilled labor in equilibrium. Still, earnings for agents in A_{A_c} increase more than in A_W. Therefore, inequality among less talented agents raises. As entrepreneurs loose, inequality among highly talented agents spikes as c_p falls. Thus, drops in within-task cooperation lead to higher levels of inequality compared to reductions in between-task cooperation, particularly among high-skilled agents. Figures 2.4 and 2.5 depict those results graphically for both a uniform and log-normal talent distribution. Income inequality among high-skilled agents increases both due to the decrease in earnings for entrepreneurs and the increase in within group inequality for consultants.

2.7 Information Structure, Dispersion of Talent and Returns to Talent

The distribution of skills is more dispersed in the U.S. than in most European countries (see, for example, Grossman and Maggi (2000) and Devroye and Freeman (2002)). Furthermore, the U.S. not only has a more dispersed earnings distribution, but also a higher skill premium. Devroye and Freeman (2002) argue that a large part of the difference in earnings is due to the fact that the skill premium is larger in the U.S. In what follows, I discuss whether cooperation can increase the skill premium in the more dispersed country in terms of talent.[36]

More specifically, I analyze the effect of changes in the higher moments on the returns to talent, due to cooperation. Without cooperation, the earnings function would be independent of the talent distribution. I focus on both the second and third moment of the talent distribution (dispersion and skewness). I find that a larger dispersion in talent can lead to a higher skill premium, consistent with the fact that the skill premium in U.S. is larger than in Europe.[37] This result depends on a positive third moment. In general,

[36]When discussing the dispersion of skills, I refer mainly to the dispersion in educational outcomes. This does not only depend on the distribution of innate ability, but is also heavily influenced by social and monetary inequality, and by educational policy.

[37]See Krueger et al. (2010) for an extensive discussion on cross-country differences in skill premia and

increases in the skewness of the talent distribution lead to higher returns to earnings.

As mentioned, both the equilibrium allocation and earnings depend exclusively on the distribution of "effective" talent (success probability), written as $G(x) \equiv \Phi(\Psi^{-1}(x))$ for $x \in [0,1]$ with the marginal distribution denoted $g(x) \equiv \frac{\partial G(x)}{\partial x} = \frac{\phi(x)}{\psi(x)}$. Any pair of distributions Φ and Ψ implies the same equilibrium allocation and earnings as another pair Φ^* and Ψ^* if

$$\Phi(\Psi^{-1}(x)) = \Phi^*((\Psi^*)^{-1}(x)) \quad \forall x \in [0,1].$$

This implies that there is an inherent relationship between the distributions of talent and task difficulty. In other words, any change in the distribution of task difficulty has a mirrored change in the talent distribution that has exactly the same effects.

2.7.1 The Information Structure and Talent Distribution.

The information structure of task A is fixed, i.e. all agents are able to perform it. Instead, consider a change in the information structure of task B. Suppose that the expectation of task difficulty does not alter, but the variance alters. More specifically, consider a change from distribution Ψ to Ψ^* such that Ψ^* second-order stochastically dominates Ψ, i.e.

$$\int_0^t \Psi^*(s)ds \leq \int_0^t \Psi(s)ds \quad \forall t,$$

with $\int_0^t \Psi^*(s)ds < \int_0^t \Psi(s)ds$ for some t. This can be interpreted as the economy possessing better information of the difficulty of task B under distribution Ψ^* than under Ψ. The success probability for less talented agents is lower under Ψ^* than under Ψ, while the reverse holds true for highly talented agents. Thus, a better information structure is equivalent to a more dispersed talent distribution.

Similarly, an increase in the skewness of the talent distribution is equivalent to a decrease in the skewness of the distribution of task difficulty. Hence, the effect of an increase in difficult problems (which implies a decrease in the skewness) is equal to the effect of an increase in the skewness of the talent distribution. This relates my paper to

earnings distribution.

the extensive research on the education race between high-skill demand and skill supply.[38]

2.7.2 Talent Distribution and Returns to Talent.

Here I discuss the effects of a more dispersed "effective" talent distribution, i.e. the distribution of success probability, on both the equilibrium allocation and earnings. The question is whether and under what conditions a more dispersed talent distribution implies higher returns to talent, i.e. a higher skill premium. The skewness of the distribution is vitally important for the consequences of a more dispersed talent distribution on the skill premium, and the discussion therefore distinguishes between a symmetric and right-skewed talent distribution. I will give conditions under which the more dispersed distribution of success probability gives rise to a higher skill premium, and I provide simulation results to illustrate my findings.

Symmetric Talent Distribution. First, consider a change in a normal success probability distribution function from $G(x)$ to $H(x)$ that leaves the mean unaffected but increases the variance.[39]

If only between-task cooperation occurs in equilibrium, what matters for the skill premium is whether the set of entrepreneurs increases relative to the set of workers. I define $x_{ij} \equiv x(t_{ij})$, i.e. the success probability of the agent who is indifferent between occupations i and j in equilibrium when the distribution function is $G(x)$. If $H(x_{WA}) > 1 - H(x_{AE})$, the relative supply of workers increases and causes the skill premium to rise. The reverse occurs if $H(x_{WA}) < 1 - H(x_{AE})$. The demand for workers increases more than its supply, causing a higher equilibrium wage and a decrease in the skill premium. If the talent distribution is symmetric, $H(x_{WA}) = 1 - H(x_{AE})$ as there is $1:1$ matching between entrepreneurs and workers, and they form convex sets at the top and bottom of the talent distribution, respectively. Hence, relative labor demand remains constant and the earnings function and skill premium are not affected. However, a larger variance implies

[38] For an overview of this literature see Acemoglu and Autor (2011).

[39] In what follows I use the normal distribution as a reference, but the results hold true for (most) symmetric distributions, e.g. a uniform distribution.

that more people are engaged in cooperation and that the income distribution is more dispersed.

Similarly, if only within-task cooperation prevails in equilibrium, the change in sorting and earnings depends on whether the demand for unfinished projects increases or decreases relative to its supply. Yet, the fact that the price $r(t)$ depends on talent implies that the supply and demand at any point of the talent distribution matter. This often impedes clear-cut results. More specifically, if $\int_0^x (1-s)h(s)ds < \int_{c(0)}^{c(x)} 2h(s)ds$ for all $x \in A_{A_c}$, the relative supply of unfinished projects decreases at any point in A_{A_c}. In response, $r(t)$ increases for all agents in A_{A_c} and the returns to skill decrease.[40] With a normal distribution, the relative demand for unfinished projects increases at all points. This implies that an increase in the variance of the "effective" talent distribution leads to a reduction in returns to talent.

When both modes of cooperation occur in equilibrium, two things change. On the one hand, changes in the demand for workers (unfinished projects) also influence the equilibrium prices for unfinished projects (workers) as the outside options of occupations change. On the other hand, the set of entrepreneurs is no longer the highest set, i.e. the set with the most talented agents. As the set of workers remains the bottom set, this implies that $\frac{H(x_{WA_c})}{H(x_{EC})-H(x_{AE})} > 1$ and therefore the wage w falls and earnings of entrepreneurs rise for the more dispersed normal distribution. Similarly, the set A_{A_c} is no longer the bottom set. Therefore, $\int_0^x (1-s)h(s)ds < \int_{c(0)}^{c(x)} 2h(s)ds$ for most $x \in A_{A_c}$ and the earnings of consultants decrease whilst those of agents selling unfinished projects increase.

These results can be seen in figure 2.6. Comparing the dashed line which represents an economy with a normal distribution with equal mean but higher variance than the one represented by the solid line, I find that both the top and bottom set (A_W and A_C)

[40] The reverse applies if $\int_0^x (1-s)h(s)ds > \int_{c(0)}^{c(x)} 2h(s)ds$ for all $x \in A_{A_c}$. In this case, $r(t)$ decreases for all $t \in A_{A_c}$. However, often $\exists x \in A_{A_c} : \int_0^x (1-s)h(s)ds > \int_{c(0)}^{c(x)} 2h(s)ds$ and $\exists x \in A_{A_c} : \int_0^x (1-s)h(s)ds < \int_{c(0)}^{c(x)} 2h(s)ds$. In this case, analytical results cannot be derived and often some consultants gain whilst others loose under the distribution $H(x)$.

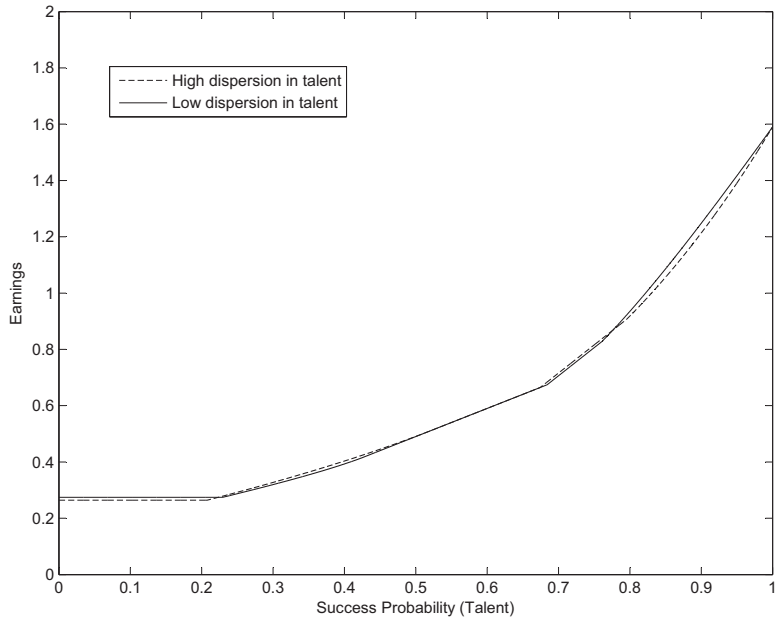

Figure 2.6: Change in variance for a normal talent distribution

loose due to the increase in variance. On the contrary, both occupational sets A_{A_c} and A_E gain.

Right-skewed Talent Distribution. If the talent distribution is right-skewed (e.g. log-normal or Pareto distribution) the general logic regarding the consequences of an increase in variance still applies, yet the results differ, when there is a change in the "effective" talent distribution from $G(x)$ to $H(x)$ that leaves the mean unaffected but increases the variance.

In particular, if only between-task cooperation prevails in equilibrium, an increase in variance holding the mean constant implies that the supply of workers becomes more abundant, i.e. $H(x_{WA}) > 1 - H(x_{AE})$. Therefore, the equilibrium wage w decreases and returns to talent increase.

If only within-task cooperation occurs, it is no longer necessarily true that an increase in the variance of the effective talent distribution leads to a decrease in the returns to talent as with the normal talent distribution. By contrast, for high levels of c_p,

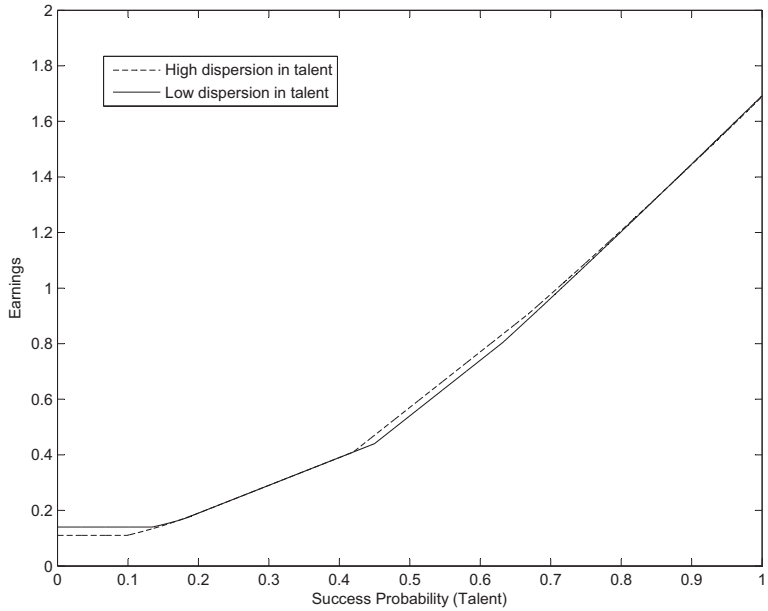

Figure 2.7: Change in variance for a log-normal talent distribution

i.e. rather small sets of both A_C and A_{A_c}, an increase in the variance is likely to make all agents in A_{A_c} worse off and all consultants better off. The reason is that in that case, $\int_0^x (1-s)h(s)ds > \int_{c(0)}^{c(x)} 2h(s)ds$ for all $x \in A_{A_c}$. For lower levels of c_p it is likely that some agents gain and others loose within both the sets A_{A_c} and A_C. While $\int_0^{x_{A_cA}} (1-s)h(s)ds < 2(1-H(x_{AC}))$, $\int_0^x (1-s)h(s)ds > \int_{c(0)}^{c(x)} 2h(s)ds$ for some $x \in A_{A_c}$.

If both modes of cooperation occur, workers always loose and entrepreneurs always gain due to the increase in variance. Furthermore, consultants may also gain if the talent distribution is sufficiently skewed. The latter part did not occur when the talent distribution was symmetric. Yet, as mentioned above, this depends on the particular parameters of the economy, and some consultants may gain while others loose. Note that the reduction in workers' earnings is more pronounced the more skewed the distribution of the probability of success.

Figure 2.7 shows these changes in earnings due to an increase in variance for a log-normal distribution. The dashed line represents the distribution with equal mean but higher variance than the solid line. In this case, entrepreneurs and most consultants gain

while workers and autarkic agents engaged in within-task cooperation loose. The drop in equilibrium wage w is substantial and the skill premium is higher in the more dispersed economy.

To sum up, where both types of cooperation occur, a more dispersed talent distribution implies larger returns to talent for most skewed distributions, whereas for normal distribution, both the least talented and the most talented loose. Thus, an economy such as the U.S. with a more dispersed talent distribution compared to continental Europe displays a larger variance in earnings both due to the higher returns to talent and the more dispersed inherent talent distribution.

2.8 Conclusion

To conclude, I have developed a heterogeneous agents model of cooperation, where agents can choose between two modes of cooperation: between-task and within-task cooperation. In the former different tasks can be assigned to different agents before production begins. In the latter tasks can be reassigned to (more skilled) agents when an agent fails to complete the task. I characterize the equilibrium, and show the equilibrium properties when it comes to matching between agents and sorting of agents into types of cooperation.

I then discuss the effects of cooperation on earnings inequality and the consequences of an increase of cooperation over time on inequality. I find that in general cooperation favors low-skilled and high-skilled agents at the expense of the medium-skilled. The effects of a decrease in cooperation costs differ depending on which form of cooperation benefits most: while reductions in between-task cooperation costs reduce inequality among both the low-skilled agents and the high-skilled agents, the reverse holds true for reductions in within-task cooperation costs. The paper highlights the importance of a detailed analysis on which form of cooperation benefit most from some labor-market policies as the consequences for inequality are critically dependent on whether between-task or within-task cooperation costs decrease.

Finally, I analyzed whether cooperation can help explain the observable cross-country

differences in skill premia. In particular, I find that a more dispersed talent distribution is likely to imply a higher skill premium if the distribution is skewed. This also shows the importance of higher moments in explaining cross-country differences in earnings. The more skewed the distribution, the larger the skill premium.

I also show that a more dispersed talent distribution is equivalent to better information on the task difficulty, i.e. less variance in the task difficulty distribution. Similarly, a higher skew in the talent distribution is equivalent to a lower skew in the task difficulty distribution. If over time the information on task difficulty is improved the skill premium is expected to increase over time. Similarly, if over time tasks either become routine and can be resolved by any agent with the help of machines and computers (task A), or they become increasingly demanding and specific skills are required (i.e. the difficulty of task B becomes more left-skewed), the "effective" skill distribution becomes more right-skewed and the skill premium increases over time.

I have left some extensions of the model for future research. In particular, I would like to extend the between-task cooperation by allowing for some uncertainty in task A and an inclusion of multi-dimensional talent, which implies that the ranking between agents can differ across tasks.

2.9 Appendix

2.9.1 Proofs

Proof of Proposition 2.1

Proof. The proof is by contradiction. Assume that a worker i with talent t earns a higher wage than another worker j with talent t', i.e. $w(t) > w(t')$. Income maximization implies that all entrepreneurs prefer to employ agent j and there is no demand for worker i, thus contradicting labor market clearing.

As all workers earn the same wage w, they are indifferent regarding who they match with. The same applies for entrepreneurs. Hence, the matching pattern is indeterminate.

Finally, each agent can remain autarkic and hence $w \geq p\Psi(t)$ for all $t \in A_W$. □

Proof of Proposition 2.2

Proof. First, consider the price $r(t)$ for unfinished projects of an agent with talent t. Suppose the price is positive and non-decreasing at some point t', i.e. $r(t') \leq r(t' + \epsilon)$ for some $\epsilon > 0$. Then, a strictly positive mass of consultants wants to contract the unfinished projects of the agent with talent t', and no agent wants to employ the agent with $t + \epsilon$, as the conditional success probability decreases in talent of the seller. This contradicts task market clearing at all points $t \in A_{A_c}(A_{E_c})$ and, hence, the price function $r(t)$ has to decrease at all points $t \in A_{A_c}(A_{E_c})$.

Second, I turn to the proof of positive assortative matching. Consider two consultants with talent $t_1 > t_2$, and two unfinished projects executed by two agents with $t_3 > t_4$. As occupational stratification prevails (to be proven below in 2.9.1), $t_2 > t_3$.

Towards a contradiction, suppose that there is negative assortative matching, i.e. the consultant with t_1 (t_2) buys the project of the agent with t_4 (t_3). Income maximization of the consultants implies that

$$p\,\frac{\Psi(t_1) - \Psi(t_4)}{1 - \Psi(t_4)} - p\,\frac{\Psi(t_1) - \Psi(t_3)}{1 - \Psi(t_3)} \geq r(t_4) - r(t_3),$$

and

$$p\,\frac{\Psi(t_2)-\Psi(t_4)}{1-\Psi(t_4)} - p\,\frac{\Psi(t_2)-\Psi(t_3)}{1-\Psi(t_3)} \leq r(t_4) - r(t_3).$$

Combining these two inequalities and manipulating the LHS leads to the following necessary condition:

$$\frac{(1-\Psi(t_1))(\Psi(t_3)-\Psi(q\leq t_4))}{(1-\Psi(t_4))(1-\Psi(t_3))} \geq \frac{(1-\Psi(t_2))(\Psi(t_3)-\Psi(q\leq t_4))}{(1-\Psi(t_4))(1-\Psi(t_3))},$$

which simplifies to the contradiction

$$\Psi(t_2) \geq \Psi(t_1),$$

as $t_1 > t_2$.

In other words, the conditional success probability is supermodular in the talent of the consultant and the agent selling the unfinished project,

$$\frac{\partial^2 \left(\frac{\Psi(t)-\Psi(s)}{1-\Psi(s)}\right)}{\partial\, t\, \partial\, s} = \frac{\psi(t)\,\psi(s)}{(1-\Psi(s))^2} > 0,$$

and thus positive assortative matching prevails. □

Proof of Lemma 2.1

Proof. Suppose earnings are not continuous at some point t', i.e. $\lim_{\epsilon\to 0}\omega(t'-\epsilon) \neq \lim_{\epsilon\to 0}\omega(t'+\epsilon)$. W.l.o.g. suppose $\lim_{\epsilon\to 0}\omega(t'-\epsilon) < \lim_{\epsilon\to 0}\omega(t'+\epsilon)$.

If both agents with talent $\omega(t'-\epsilon)$ and $\omega(t'+\epsilon)$ are in different occupations, the agent with talent $t'-\epsilon$ can earn more if she switches occupations, thus contradicting income maximization. In turn, if they are in the same occupation, the agent with talent $t'-\epsilon$ can earn more contracting the cooperator of the agent with talent $\omega(t'+\epsilon)$, again contradicting income maximization. Hence, the earnings function is continuous.

Similarly, if it were non-increasing, the more talented agent can increase earnings by switching to the occupation of the less talented agent and contracting her cooperator, contradicting income maximizing behavior. Thus, earnings are monotonically and continuously increasing in equilibrium. □

Proof of Proposition 2.3

I start to show the ordering between the (potential) occupational sets. Therefore I compare the occupational sets pairwise according to their ordering. Transitivity ensures the overall result. The arguments also hold if some of the sets are empty in equilibrium.

$\mathbf{A_W}$ **vs.** $\mathbf{A_{A_c}}$ Workers earn a flat wage, hence $\frac{\partial \pi_W(t)}{\partial t} = 0$. The marginal earnings of an autarkic agent engaged in within-task cooperation are

$$\frac{d\,\pi_{A_c}(t)}{d\,t} = \psi(t)\left(p - r(t) - p\,\frac{1-\Psi(c_A(t))}{1-\Psi(t)}\right) + (1-\Psi(t))\frac{\partial r(t)}{\partial c_A(t)}\frac{\partial c_A(t)}{\partial t}$$

from deriving (2.6) with respect to t. The last term is clearly positive, and the first term is positive if

$$p\,\frac{\Psi(c_A(t)) - \Psi(t)}{1-\Psi(t)} > r(t).$$

Towards a contradiction, suppose that

$$p\,\frac{\Psi(c_A(t)) - \Psi(t)}{1-\Psi(t)} < r(t).$$

But, for the agent with talent $c_A(t)$ to choose to become a consultant, it is necessary that

$$p\,\frac{\Psi(c_A(t)) - \Psi(t)}{1-\Psi(t)} - r(t) > c_p.$$

Hence,

$$\frac{d\,\pi_{A_c}(t)}{d\,t} > 0.$$

Towards a contradiction, suppose $\exists t' > t'' : \quad t' \in A_W \wedge t'' \in A_{A_c}$. Income maximization implies that the agent with t' earns more as a worker than in any other occupation. In particular, $\pi_W(t') = w > \pi_{A_c}(t')$. But this directly contradicts the income maximization of the agent with t'' as $\frac{d\,\pi_{A_c}(t)}{d\,t} > 0$, and hence $\pi_{A_c}(t') > \pi_{A_c}(t'')$. Therefore, in equilibrium, $\nexists t' > t'' : \quad t' \in A_W \wedge t'' \in A_{A_c}$.

$\mathbf{A_{A_c}}$ **vs.** $\mathbf{A_A}$ A comparison of the equations (2.6) and (2.7) implies that all autarkic agents choose to engage in within-task cooperation as long as $r(t) > 0$. Proposition 2.2 states that $r'(t) < 0$ in equilibrium for all $r(t) > 0$. This directly implies that if

for any agent $\pi_A(t) \geq \pi_{A_c}(t)$, for any agent with $t' > t: \quad \pi_A(t') > \pi_{A_c}(t')$. Hence, in equilibrium $\nexists t' > t'': \quad t' \in A_{A_c} \wedge t'' \in A_A$.

$\mathbf{A_{A_c}}$ vs. $\mathbf{A_{E_c}}$ The indifference condition between being an entrepreneur engaged in within-task cooperation and an autarkic agent cooperating within-tasks is given by

$$p\,\Psi(t_{A_cE^C}) + (1 - \Psi(t_{A_cE^C}))\,r(t_{A_cE^C}) = w + c_a.$$

As the LHS is strictly increasing in t, as shown above, all agents with $t > t_{A_cE^C}$ strictly prefer to be an entrepreneur engaged in within-task cooperation than an autarkic agent engaged in such cooperation. Hence, $\nexists t' > t'': \quad t' \in A_{A_c} \wedge t'' \in A_{E_c}$.

$\mathbf{A_A}$ vs. $\mathbf{A_E}$ The marginal earnings of an autarkic agent are given by

$$\pi'_A(t) = p\,\psi(t),$$

while those of an entrepreneur are equal to

$$\pi'_E(t) = 2\,p\,\psi(t).$$

The marginal earnings are greater for entrepreneurs than for autarkic agents for any t as $\psi(t) > 0$. Towards a contradiction, suppose that $\exists t' > t'': \quad t' \in A_A \wedge t'' \in A_E$. Income maximization implies that the agent with talent t' optimizes her occupational choice. In particular, $\pi_A(t') > \pi_E(t')$. This, together with the fact that $\pi'_E(t) > \pi'_A(t)$ for any t leads directly to $\pi_A(t'') > \pi_E(t'')$, contradicting the income maximizing occupational choice of the agent with talent t''. Thus, in equilibrium $\exists t' > t'': \quad t' \in A_A \wedge t'' \in A_E$.

$\mathbf{A_{E^C}}$ vs. $\mathbf{A_E}$ The argument is analogous to the comparison between the occupational sets A_{A_c} and A_A. Entrepreneurs choose within-task cooperation as long as $r(t) > 0$. As $r'(t) < 0$, if for any agent $\pi_E(t) \geq \pi_{E_c}(t)$, i.e. $r(t) \leq 0$, for all agents with $t' > t$: $\pi_E(t') > \pi_{E_c}(t')$. Hence, in equilibrium $\nexists t' > t'': \quad t' \in A_{E_c} \wedge t'' \in A_E$.

$\mathbf{A_E}$ vs. $\mathbf{A_C}$ I compare the occupational sets of entrepreneurs and any type of consultant (either of autarkic agents or entrepreneurs). The latter set is denoted A_C.

The marginal earnings of an entrepreneur are given by

$$\pi'_E(t) = 2\,p\,\psi(t),$$

while those of a consultant are equal to

$$\pi'_C(t) = 2\,p\,\frac{\psi(t)}{1-\Psi(c^{-1}(t))},$$

by the envelope theorem. The function $c^{-1}(t)$ denotes the inverse of the assignment function. As the denominator in the equation of consultants' marginal earnings is bounded above by 1, the marginal earnings of consultants are necessarily higher than those of entrepreneurs for any t. Again, towards a contradiction, suppose $\exists t' > t'' : \quad t' \in A_E \wedge t'' \in A_C$. Income maximization implies that $\pi_E(t') > \pi_C(t')$, which together with $\pi'_C(t) > \pi'_E(t)$ directly implies $\pi_E(t'') > \pi_C(t'')$, contradicting the income maximizing choice of the agent with talent t''. Hence, $\nexists t' > t'' : \quad t' \in A_E \wedge t'' \in A_C$ in equilibrium.

$\mathbf{A_{C^A}}$ vs. $\mathbf{A_{C^E}}$ This result follows directly from the proposition 2.2. Positive assortative matching prevails between consultants and the talent of agents selling unfinished projects. Furthermore, I have shown that entrepreneurs are more talented than autarkic agents engaged in within-task cooperation. Therefore, the consultants of entrepreneurs are more talented than those of autarkic agents, i.e. $\nexists t' > t'' : \quad t' \in A_{C^A} \wedge t'' \in A_{C^E}$ in equilibrium.

This completes the proof of the ordering of occupational sets. Occupational stratification follows directly by transitivity. I did not compare A_A and A_{E^C} because these two occupational sets are mutually exclusive.

To see this, suppose first that $\exists t' > t'' : \quad t' \in A_{E_c} \wedge t'' \in A_A$. Optimality of the occupational choice of the agent with talent t' implies that $r(t') > 0$. But this implies that $r(t'') > 0$ as $r'(t) < 0$ for all t. This contradicts the optimality of occupational choice of the agent with talent t''. Hence, in equilibrium $\nexists t' > t'' : \quad t' \in A_{E_c} \wedge t'' \in A_A$.

Suppose that $\exists t' > t'' : \quad t' \in A_A \wedge t'' \in A_{E_c}$. The income maximizing occupational choice of the agent with talent t'' implies that $\pi_{E_c}(t'') > \pi_{A_c}(t'')$, or equivalently

$$p\,\Psi(t'') + (1 - \Psi(t''))\,r(t'') \geq w + c_a.$$

Furthermore, I have shown above that the LHS of this equation strictly increases in talent t. Hence, *a fortiori*

$$p\,\Psi(t') + (1 - \Psi(t'))\,r(t') \geq w + c_a.$$

But this directly contradicts the optimality of the occupational choice for the agent with talent t' because in order to chose autarky over entrepreneurship it has to hold that $\pi_A(t') > \pi_E(t')$, or

$$p\,\Psi(t') \leq w + c_a.$$

Thus, I conclude that in equilibrium $\nexists t' > t'' : \quad t' \in A_A \wedge t'' \in A_{E_c}$ and thus one of the two occupational sets A_A and A_{E_c} has to be empty.

The fact that not all sets have to exist in equilibrium can be easily seen due to the exogeneity of cooperation costs c_a and c_p. If both tend towards infinity, only autarkic agents remain active in equilibrium. If the relative cost differences are too high, one form of cooperation is dominated by the other.

Proof of Proposition 2.4

Proof. The proof is divided into several parts. First I partition the parameter space into subspaces in which only one equilibrium type can prevail as all other equilibrium types contradict agents' income-maximizing behavior. Then I demonstrate that an equilibrium of this particular type always exists and is also unique.

I will now start to partition the parameter space:

1. As their is no disutility of work and all agents possess a non-negative success probability in task B, all agents will choose to work. The next question is whether any agent will enter either type of cooperation in equilibrium. The following two

conditions preclude this. In particular, if the most talented agent prefers to remain autarkic to engaging in any type of cooperation, then so will all the others.

Given that an entrepreneur has to pay the outside option of an agent to induce her to join the cooperation, the respective conditions to prefer autarky over entrepreneurship reads as follows:

$$p\,\Psi(\bar{t}) < p\,\Psi(\underline{t}) + c_a. \tag{2.16}$$

The outside option of the least talented agent is $p\,\Psi(\underline{t})$. If the additional projects sold ($p\,\Psi(\bar{t})$) as an entrepreneur compared to an autarkic agent do not compensate for the additional costs ($p\,\Psi(\underline{t})+c_a$), no agent will choose to become an entrepreneur. Clearly, the gains increase in talent, while the costs remain constant. Therefore, if the most talented agent prefers autarky over entrepreneurship, so will all the others.

Similarly, for the most talented agent to prefer autarky over consulting it is sufficient that

$$2\,p\,\frac{\Psi(\bar{t}) - \Psi(\underline{t})}{1 - \Psi(\underline{t})} - p\,\Psi(\bar{t}) < 2\,c_p. \tag{2.17}$$

The LHS represents the additional projects sold on expectation when consulting the least talented agents instead of remaining autarkic, and the RHS represents the additional costs. Note that in this case the only costs are the fixed costs $2\,c_p$, as agents are willing to sell their unfinished projects at any positive price $r(t)$. Here too, the LHS increases in talent, i.e. the first agent to enter consulting is the most talented agent.

If both conditions are met, then all occupational sets except of autarky must be empty. The existence and uniqueness of the competitive equilibrium follows directly from the fact that the success probability function $\Psi(q)$ is well-defined.

2. Let us now consider the case where one of the two conditions, (2.16) or (2.17) is met, and the other is not. This implies that necessarily one but only type of cooperation occurs in equilibrium.

(a) In particular, if (2.16) is satisfied and (2.17) is not, only within-task cooperation takes place. Note that in this case, the condition for any agent with talent t to prefer entrepreneurship over autarky reads as

$$p\,\Psi(t) + (1 - \Psi(t))\,r(t) < p\,\Psi(\underline{t}) + (1 - \Psi(\underline{t}))\,r(\underline{t}) + c_a. \tag{2.18}$$

In the proof of proposition 2.3 I have demonstrated that the LHS increases in t. Thus, the first agent to enter entrepreneurship is the most talented agent. This combined with the fact that

$$(1 - \Psi(\underline{t}))\,r(\underline{t}) - (1 - \Psi(t))\,r(t) \geq 0$$

implies that (2.16) is a sufficient condition for no agent to enter entrerpreneurship.

Thus, the only feasible equilibrium types are A_{A_c}, A_A, A_{C^A} and A_{A_c}, A_{C^A}. What remains to be demonstrated is that one, and only one, of these two equilibrium types exists and that the equilibrium is unique.

To do so, I fix a price r_0 for the least talented agent, $r(\underline{t})$, and use the indifference conditions to calculate t_{A_cA} and t_{AC^A} as

$$t_{AC^A} = \min\{\bar{t}, \max\{\underline{t}, \Psi^{-1}(\frac{2(r_0 + c_p)(1 - \Psi(\underline{t})) + 2p\Psi(\underline{t})}{p(1 + \Psi(\underline{t}))})\}\},$$

and

$$t_{A_cA} \equiv \min\{t_{AC^A}, t : r_0 = \int_{\underline{t}}^{t} \frac{\psi(s)(1 - \Psi(c(s)))}{(1 - \Psi(s))^2} ds\},$$

with $c(s) = t_{AC^A} + \int_{\underline{t}}^{s} \frac{1-\Psi(s)}{2} \frac{\phi(s)}{\phi(c(s))} ds$.[41] Note that both t_{AC^A} and t_{A_cA} are continuously and monotonically increasing in r_0 (strictly for r_0 such that $\underline{t} < t_{AC^A} < \bar{t}$, which is fulfilled given that some agents want to become consultants in equilibrium). As t_{AC^A} strictly increases in r_0, $c(t)$ monotonically

[41] If $t_{AC^A} = t_{A_cA}$ the first indifference condition must be replaced by the relevant condition for $t_{A_cC^A}$,

$$p\,\Psi(t_{A_cC^A}) + (1 - \Psi(t_{A_cC^A}))r(t_{A_cC^A}) = 2\,p\,\frac{\Psi(t_{A_cC^A}) - \Psi(\underline{t})}{1 - \Psi(\underline{t})} - 2\,c_p - 2\,r_0.$$

increases for all $t \in A_{A_c}$ and so does $r'(t)$. Hence, $r(t)$ monotonically increases for all $t \in A_{A_c}$ and thus t_{A_cA} as well. To conclude, both t_{AC^A} and t_{A_cA} increase continuously and monotonically in r_0.

The excess supply function on the task market is defined as

$$ES(r_0, t_{A_cA}, t_{AC^A}) \equiv \int_{\underline{t}}^{t_{A_cA}} (1 - Pr(s))\phi(s)ds - \int_{t_{AC^A}}^{\bar{t}} 2\phi(s)ds.$$

As $ES(r_0, t_{A_cA}, t_{AC^A})$ continuously and monotonically increases both in t_{A_cA} and t_{AC^A} (strictly for $\underline{t} \leq t_{A_cA} \leq t_{AC^A} < \bar{t}$), it strictly and monotonically increases in r_0.

Furthermore, note that

$$ES(0, t_{A_cA}, t_{AC^A}) = -\int_{t_{AC^A}}^{\bar{t}} 2\phi(s)ds < 0,$$

and

$$ES(r_0^{**}, t_{A_cA}, t_{AC^A}) = \int_{\underline{t}}^{t_{A_cA}} (1 - Pr(s))\phi(s)ds > 0,$$

where $r_0^{**} = \{r_0 : 2\,p\,\frac{\Psi(\bar{t})-\Psi(\underline{t})}{1-\Psi(\underline{t})} - 2\,r_0 - 2\,c_p = p\,\Psi(\bar{t})\}$. Hence, by the Mean Value Theorem, r_0^* exists, where r_0^* is such that

$$ES(r_0^*, t_{A_cA}, t_{AC^A}) = 0,$$

and it is also unique given that $ES(r_0, t_{A_cA}, t_{AC^A})$ is strictly monotonically increasing in the range of r_0 indicated by the algorithm.

(b) On the contrary, if (2.17) is satisfied and (2.16) is not, only between-task cooperation prevails in equilibrium. In this case, as long as $c_a > 0$, the equilibrium consists of the occupational sets $\{A_W, A_A, A_E\}$ with the wage $w > 0$ determined by the labor market clearing condition. For any positive cooperation costs some agents will remain autarkic. I still need to demonstrate that an equilibrium of type $\{A_W, A_A, A_E\}$ always exists and is unique.

Fixing a flat wage $w \geq 0$ and thresholds $t_{WA} < t_{AE}$ allows me to calculate the earnings of all agents using equations (2.7) and (2.9). In this case the equilibrium conditions (2.4) and (2.11) are not necessarily satisfied.

I will now construct an equilibrium that also satisfies these conditions. First, I fix w and calculate t_{WA} and t_{AE} as

$$t_{WA} = \Psi^{-1}(\frac{w}{p})$$

and

$$t_{AE} = \Psi^{-1}(\frac{w + c_a}{p}),$$

using both indifference conditions. The excess labor supply is defined as

$$ES(w, t^*_{WA}, t^*_{AE}) \equiv \int_{\underline{t}}^{t^*_{WA}} \phi(s)ds - \int_{t^*_{AE}}^{\bar{t}} \phi(s)ds,$$

with $t^*_{WA} = \min\{\bar{t}, \max\{\underline{t}, t_{WA}\}\}$ and $t^*_{AE} = \min\{\bar{t}, \max\{\underline{t}, t_{AE}\}\}$. Note that both t_{WA} and t_{AE} continuously and monotonically increase in w as Ψ is a strictly and monotonically increasing function. Therefore, the labor demand (supply) weakly decreases (increases) in w (strictly if $t_{AE} \in [\underline{t}, \bar{t}]$ and $t_{WA} \in [\underline{t}, \bar{t}]$, respectively). Thus in order to show both existence and uniqueness it is sufficient to show that the excess labor supply is negative (positive) for a low (high) wage w. Note that

$$ES(0, t^*_{WA}, t^*_{AE}) = -\int_{\Psi^{-1}(\frac{c_a}{p})}^{\bar{t}} \phi(s)ds < 0,$$

which is negative as $\Psi^{-1}(\frac{c_a}{p}) < \bar{t}$ due to the equilibrium type fixed by the algorithm. Moreover,

$$ES(p\Psi(\bar{t}) - c_a, t^*_{WA}, t^*_{AE}) = \int_{\underline{t}}^{\Psi^{-1}(\frac{p\Psi(\bar{t}) - c_a}{p})} \phi(s)ds > 0,$$

as $\Psi^{-1}(\frac{p\Psi(\bar{t}) - c_a}{p}) > \underline{t}$ by the algorithm. Therefore, I can conclude that the equilibrium exists by the Mean Value Theorem, and that it is also single-valued as the excess labor supply continuously and strictly monotonically increases for any w such that the equilibrium type determined by the algorithm is $\{W, A, E\}$.

3. Finally, consider the case that both conditions, (2.16) and (2.17), are not met. This does not necessarily imply that all occupational sets are active. To determine which mode(s) of cooperation prevail in equilibrium, I proceed as follows:

   (a) First, I construct the $\{A_W, A_A, A_E\}$ candidate equilibrium from above as I determine the market clearing wage w and earnings in case only between-task cooperation would prevail. This is the unique equilibrium if and only if the most talented agent does not prefer to become a consultant, i.e. iff

   $$2\,p\,\frac{\Psi(\bar{t}) - \Psi(t_{WA})}{1 - \Psi(t_{WA})} - 2\,c_p < 2\,p\,\Psi(\bar{t}) - w - c_a, \tag{2.19}$$

   where t_{WA} is the cut-off agent who is indifferent between the occupational sets A_W and A_A as calculated for the $\{A_W, A_A, A_E\}$ candidate equilibrium, and is the agent who constitutes the best match if $\bar{t}$ becomes a consultant. Furthermore, w denotes the labor market clearing wage for that equilibrium. If (2.19) is satisfied, the equilibrium is the same as if (2.17) were satisfied and existence and uniqueness is guaranteed.

   (b) If (2.19) is not satisfied, I construct the $\{A_{A_c}, (A_A), A_{C^A}\}$ candidate equilibrium from above when condition (2.16) was fulfilled, but (2.17) not. No agent wants to engage in between-task cooperation if the least talented consultant (denoted $t_{\underline{C^A}}$) does not prefer to become an entrepreneur, i.e. if

   $$2\,p\,\Psi(t_{\underline{C^A}}) - \omega(\underline{t}) - c_a < 2\,p\,\frac{\Psi(t_{\underline{C^A}}) - \Psi(\underline{t})}{1 - \Psi(\underline{t})} - 2\,r_0 - 2\,c_p, \tag{2.20}$$

   The least talented consultant is the first one who prefers to become an entrepreneur. In this case she would employ the least talented agent, and pay her the outside option $\omega(\underline{t})$. Note that r_0 is the task market clearing price from above. If the agent with talent $t_{\underline{C^A}}$ does not prefer to become an entrepreneur, no agent will and the equilibrium is the same as if (2.16) were satisfied and existence and uniqueness is guaranteed.

   (c) Lastly, if both (2.19) and (2.20) are not satisfied both types of cooperation prevail in equilibrium.

The existence and uniqueness proof when both types of cooperation are present is a combination of the two cases above. First, fix some w. Then, fix some r_0 and calculate t_{WA_c}, t_{A_cA}, and t_{EC_A} similar to above using the relevant indifference conditions. The excess supply function on the task market now reads as

$$ES(r_0, w, t_{WA_c}, t_{A_cA}, t_{EC^A}) \equiv \int_{t_{WA_c}}^{t_{A_cA}} (1 - Pr(s))\phi(s)ds - \int_{t_{EC^A}}^{\bar{t}} 2\phi(s)ds.$$

It is straightforward to verify that for low (high) levels of r_0 there is excess demand (supply). Note that t_{EC^A} strictly and monotonically increases in r_0. Therefore, $r'(t)$ increases strictly and monotonically for all $t \in A_{A_c}$. As the cdf $\Phi(t)$ is continuously and monotonically increasing, this implies that $r(t)$ increases for all $t \in A_{A_c}$ and in particular for $r(t_{A_cA})$. Hence, the excess supply function increases continuously and monotonically in r_0. Thus, there exists a unique $r_0^*(w, t_{WA_c}, t_{A_cA}, t_{AC^A})$ that clears the task market.

Clearly, this does not imply that the labor market clears alongside the task market. Therefore, I now need to demonstrate that $r_0^*(w, t_{WA_c}, t^*_{A_cA}, t_{AC^A})$ changes continuously and monotonically in w, and there is excess labor demand for low levels of w and excess labor supply for high levels of w. The former follows from the fact that any increase in w causes t_{EC^A} to decrease continuously and monotonically and t_{WA_c} to increase continuously and monotonically. Thus any agent with $t \in A_{A_c}$ is matched with a less skilled consultant, and thus $r'(t)$ decreases. Hence, the excess supply function unambiguously decreases and thus the task market clearing $r_0^*(w, t_{WA_c}, t^*_{A_cA}, t_{AC^A})$ has to increase continuously and monotonically. The latter follows straightforward from the fact that for $w = 0$, there is excess labor demand and for $w = p\Psi(\bar{t}) - c_a$, there is excess labor supply (given that in equilibrium both modes of cooperation prevail).

I conclude that there is always a unique equilibrium.

□

Proof of Proposition 2.6

Proof. As the median agent is presumed to remain autarkic, her earnings are not affected by any change in cooperation costs whereas any agent who takes part in cooperation enjoys an increase in earnings if cooperation costs fall. As both the 10%-percentile and 90%-percentile of the talent distribution are engaged in cooperation, the first two parts of the proposition follow immediately.

The marginal earnings of agents in the sets A_W, A_A, and A_C depend exclusively on the agent's talent. Therefore, changes in cooperation costs do not affect their marginal earnings. By contrast, the marginal earnings of agents in A_C and A_{A_c} depend on the talent of the match. If cooperation costs c_p fall, all agents in A_{A_c} are matched with a less skilled consultant (recall that both occupations form convex sets at the bottom and top of the talent distribution). Therefore, $\frac{dr'(t)}{dc_p} < 0$ for all t, as less talented consultants have a lower marginal willingness to pay for unfinished projects (bear in mind that $r'(t) < 0$). Thus, a reduction in cooperation costs causes the equilibrium price function $r(t)$ to become flatter and the inequality within the set A_{A_c} to decrease.

In other words, the least talented agent in A_{A_c} profits most from the reduction in c_p whereas among consultants the reverse applies. From the envelope theorem it is clear that the marginal earnings of a consultant are $\pi'_C(t) = 2\,p\,\frac{\psi(t)}{1-\Psi(c^{-1}(t))}$. As the match improves for all agents in A_C, so do marginal earnings. Hence, the most talented agent in A_C will benefit most from a reduction in c_p and inequality within the set of consultants increases. I conclude, that a reduction in within-task cooperation costs reduces the within-group inequality in the set A_{A_c} and increases it in A_C. □

2.9.2 Reductions in Cooperation Costs

Here I provide the derivation of the comparative statics results on cooperation costs, both for between-task cooperation and within-task cooperation, and discuss briefly the consequences for the inequality between the 90%-percentile and 10%-percentile. I analyze the cases that only between-task and within-task cooperation are present in equilibrium.

Between-task Cooperation Recall that the equilibrium is defined by the labor market clearing condition (2.11) and two indifference conditions for t_{WA} and t_{AE} if only between-task cooperation prevails in equilibrium. The equilibrium conditions read as follows:

$$\begin{aligned} p\,\Psi(t_{AE}) - w - c_a &= 0 \\ p\,\Psi(t_{WA}) - w &= 0 \\ 1 - \Phi(t_{AE}) - \Phi(t_{WA}) &= 0 \end{aligned}$$

The total differentials of the first condition (after plugging in the second one) and the third condition read as

$$\begin{aligned} \frac{d\,t_{WA}}{d\,t_{AE}} &= \frac{\psi(t_{AE})}{\psi(t_{WA})} - \frac{1}{p\,\psi(t_{WA})}\frac{d\,c_a}{d\,t_{AE}}, \\ \frac{d\,t_{WA}}{d\,t_{AE}} &= -\frac{\phi(t_{AE})}{\phi(t_{WA})}, \end{aligned}$$

which combined deliver

$$\frac{d\,t_{AE}}{d\,c_a} = \frac{\phi(t_{WA})}{p\,\psi(t_{AE})\phi(t_{WA}) + p\,\psi(t_{WA})\phi(t_{AE})},$$

and

$$\frac{d\,t_{WA}}{d\,c_a} = \frac{-\phi(t_{AE})}{p\,\psi(t_{AE})\phi(t_{WA}) + p\,\psi(t_{WA})\phi(t_{AE})}.$$

Similarly, the change in equilibrium wage is given by

$$\frac{d\,w}{d\,c_a} = \frac{-\psi(t_{WA})\phi(t_{AE})}{\psi(t_{AE})\phi(t_{WA}) + \psi(t_{WA})\phi(t_{AE})} = -\frac{1}{1+\xi(t_{WA},t_{AE})},$$

where $\xi(t_{WA},t_{AE}) = \frac{\frac{\phi(t_{WA})}{\psi(t_{WA})}}{\frac{\phi(t_{AE})}{\psi(t_{AE})}}$. The set of agents who choose to become entrepreneurs decreases in the cooperation cost c_a. Consequently, so does the set of workers due to labor market clearing. The indifference condition for the agent separating workers from autarkic agents implies that the wage w has to decrease in c_a as well.

Whether the inequality between entrepreneurs and workers increases or decreases hinges exclusively on the distribution of success probability. Recall that $\xi(t_{WA},t_{AE}) =$

$\frac{g(x_{WA})}{g(x_{AE})}$, the ratio of the marginal distributions of "effective" talent at t_{WA} and t_{AE}. Any change in the talent or task difficulty distribution, that increases this ratio, implies that a larger share of the gains from less costly cooperation accrues to entrepreneurs. For example, if the probability of success is characterized by a normal distribution, $\xi(t_{WA}, t_{AE}) = 1$, and the gains from less expensive cooperation are shared equally among workers and entrepreneurs. By contrast, if the "effective" talent distribution is a Pareto, greater part of the gains obtained from less costly cooperation goes to entrepreneurs. In this case, the entrepreneurs' share is largest for high levels of c_a. If the set of autarkic agents goes to 0, then the share of cost reduction that goes to workers in terms of a wage increase tends towards 0.5, independent of the talent distribution. In general, the more skewed the distribution, the larger the share of cost reductions for entrepreneurs.

Within-task Cooperation In cases where only within-task cooperation occurs in equilibrium, the equilibrium conditions read as follows if some agents remain autarkic:

$$
\begin{aligned}
p\,\frac{\Psi(t_{AC^A})(1+\Psi(\underline{t}))-2\Psi(\underline{t})}{1-\Psi(\underline{t})} - 2\,r0 - 2\,c_p =& 0\\
r_0 - p\int_{\underline{t}}^{t_{A_cA}} \frac{\psi(s)(1-\Psi(c(s)))}{(1-\Psi(s))^2}ds =& 0\\
\int_{\underline{t}}^{t}(1-\Psi(s))\phi(s)ds - \int_{c(\underline{t})}^{c(t)} 2\phi(c(s))ds =& 0 \quad \forall t \in A_{A_c}
\end{aligned}
$$

The total differentials of the first condition (after plugging in the second one) and the third condition read as

$$
\begin{aligned}
\frac{d\,t_{A_cA}}{d\,t_{AC^A}} =& \frac{\frac{p(1+\Psi(\underline{t}))\psi(t_{AC^A})}{1-\Psi(\underline{t})} + 2p\int_{\underline{t}}^{t_{A_cA}} \frac{1}{1+\int_{\underline{t}}^{s}\phi'(c(u))\frac{1-\Psi(u)}{2}\frac{\phi(u)}{(\phi(c(u)))^2}du}\frac{\psi(s)\psi(c(s))}{(1-\Psi(s))^2}ds}{2p\frac{\psi(t_{A_cA})(1-\Psi(\bar{t}))}{(1-\Psi(t_{A_cA}))^2}} -\\
& -\frac{2}{2p\frac{\psi(t_{A_cA})(1-\Psi(\bar{t}))}{(1-\Psi(t_{A_cA}))^2}}\frac{dc_p}{dt_{AC^A}},\\
\frac{d\,t_{A_cA}}{d\,t_{AC^A}} =& -\frac{2\phi(t_{AC^A})}{(1-\Psi(t_{A_cA}))\phi(t_{A_cA})},
\end{aligned}
$$

Combining these two equation implies

$$\frac{dc_p}{dt_{AC^A}} = p\frac{\psi(t_{A_cA})(1-\Psi(\bar{t}))}{(1-\Psi(t_{A_cA}))^2}\frac{2\phi(t_{AC^A})}{(1-\Psi(t_{A_cA}))\phi(t_{A_cA})} + \frac{p(1+\Psi(\underline{t}))\psi(t_{AC^A})}{2(1-\Psi(\underline{t}))} + \\ +p\int_{\underline{t}}^{t_{A_cA}} \frac{1}{1+\int_{\underline{t}}^{s}\phi'(c(u))\frac{1-\Psi(u)}{2}\frac{\phi(u)}{(\phi(c(u)))^2}du}\frac{\psi(s)\psi(c(s))}{(1-\Psi(s))^2}ds.$$

As this derivative is positive, it follows directly from the task market clearing condition that $\frac{dt_{A_cA}}{dc_p} < 0$ and $\frac{dr_0}{dc_p} < 0$. In the case of within-task cooperation, however, not only the marginal distribution at the occupational cut-off levels, but the entire distribution affects how cost reductions are shared between cooperating agents. However, it still holds that the share that accrues to consultants increases with the skewness of the talent distribution.

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