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MWP 2014/22
Max Weber Programme

Pundits and Quacks:
Financial Experts and Market Feedback

Jesper Rudiger and Adrien Vigier

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EUI Working Paper **MWP** 2014/22

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ISSN 1830-7728

© Jesper Rudiger and Adrien Vigier, 2014
Printed in Italy
European University Institute
Badia Fiesolana
I – 50014 San Domenico di Fiesole (FI)
Italy
www.eui.eu
cadmus.eui.eu

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Keywords

Informational Cascades; Experts; Reputation; Asset Price Bubbles.

JEL Codes: D82; D83; D84; G14; G20

The authors thank Mehmet Ekmekci, Piero Gottardi, and Johannes Hörner for particularly helpful conversations, as well as seminar audiences at the European University Institute, the University of Oslo, the University of Copenhagen, Cambridge University, and the Free University Amsterdam. All mistakes are our own.

Jesper Rudiger (corresponding author)

Department of Economics, European University Institute, Via delle Fontanelle 10, I-50014 San Domenico, Italy. Telephone: +39 055 4685 698. E-mail: jesper.rudiger@eui.eu

Adrien Vivier

Department of Economics, University of Oslo, Moltke Moes Vei 31, N-0851 Oslo, Norway. Telephone: +47 2285 4520. E-mail: a.h.vigier@econ.uio.no

PUNDITS AND QUACKS: FINANCIAL EXPERTS AND MARKET FEEDBACK*

Jesper Rudiger[†] and Adrien Vigier[‡]

Abstract

By choosing whether or not to follow a financial expert's advice, a privately informed trader implicitly screens the ability of this expert. We explore the performance of the resulting feedback mechanism. In the medium run, feedback may altogether break down, enabling experts of low ability to maintain a lasting reputation and affect prices durably. Yet in the long run, the market almost always learns experts' true type. While prices get stuck in the medium run, they thus converge in the long run to the asset's correct valuation.

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[†]Department of Economics, European University Institute, Via delle Fontanelle 10, I-50014 San Domenico, Italy. *Telephone:* +39 055 4685 698. *E-mail:* jesper.rudiger@eui.eu (corresponding author)

[‡]Department of Economics, University of Oslo, Moltke Moes Vei 31, N-0851 Oslo, Norway. *Telephone:* +47 2285 4520. *E-mail:* a.h.vigier@econ.uio.no

1 Introduction

There exists an inherent tension between financial experts and those who listen to them: analysts take risks in making recommendations, but do not fully internalize the costs of those risks. This problem is ordinarily solved by the market itself: if a pundit were consistently to give out bad advice, he would in time be driven out of his trade. Yet the recent crisis has stressed the limitations of this market mechanism. Financial experts – who were later revealed to have backed worthless assets against better knowledge – were considered highly reputable up until the point of the market collapse.¹ Why, then, did market screening apparently break down, allowing quacks to stay in the market for so long?

To address the issue, this paper develops a model of asset markets in which information is provided by a strategic expert of unknown ability and where feedback about the expert is endogenous, and occurs through the impact of expert advice on prices. We show first that prices deviate in the medium run from assets' true value because of the occurrence of so-called *reputational cascades*, whereby financial experts of low ability are able to maintain a lasting reputation and affect prices durably. We then show that reputational cascades are almost always transient, and that prices converge in the long run to their correct values.

The model works as follows. The market is made up of (a) a competitive and risk-neutral market maker, (b) a financial expert, and (c) a sequence of traders. Each period a new trader arrives and faces the option to buy and sell a given asset at the prices set by the market maker. The expert provides information about the asset. A trader is one of two types, defining the motives of his trades: an *informed trader*, who trades to maximize profits at the market maker's expense, or a *liquidity trader*, who trades for exogenous motives unrelated to profits; a trader's type is private information. The expert is either *good* or *bad*; his type too is private information. An expert of high ability receives informative signals of the asset's true value. A bad expert on the other hand knows nothing more than the publicly available information about the asset. The expert's *reputation*, which evolves over time, is defined as the public belief that he is of the good type. We assume that the expert is motivated by career concerns, and aims to maximize his reputation.²

Within each period, the timing of the game is as follows. If the expert is good, he observes his private signal of the true asset value. The expert (whichever his type) then releases a report

¹See for instance: <http://www.economist.com/news/finance-and-economics/21594358-bear-market-or-bull-analysts-give-bad-advice-consistently-wrong>.

²To keep the model tractable, we assume that the expert is myopic. Each period, he maximizes his expected reputation next period.

communicating his (possibly fake) information to the market participants. If the timing of information acquisition is symmetric, then market maker and trader observe the report *before* prices are set; we will say by contrast that the timing of information acquisition is asymmetric if the market maker observes the report with delay, and *after* setting the prices. Finally, the trader observes (a) the prices and (b) if a report was released, a private signal of the expert's ability, and decides whether to buy, sell, or abstain. The market then updates expert reputation using Bayes' rule based on all publicly available information at the end of the period, consisting of the latest report and trading outcome.³

We first analyze the benchmark case in which timing of information acquisition is symmetric, i.e. where traders and market maker observe the report before trade. By lending credibility to historical reports, positive signals of the expert type induce traders to follow past trends in prices. Conversely, negative signals of the expert type lower the credibility of historical reports, and induce traders to act as contrarians with respect to past trends. Informed traders' actions thus perfectly reflect their private information of the expert type. We say in this case that *screening of the expert is efficient*. This in turn implies that new statistical information about the expert becomes incorporated into prices each period.

We then explore the asymmetric case, i.e. where the market maker observes the report with delay. An informed trader then possesses two pieces of information with which to make profits against the market maker: the latest report, and his private signal of the expert type. When these pieces of information clash, traders are faced with a dilemma. A trader may for instance observe a positive report, but receive a negative signal about expert ability. In this example, the report suggests that the asset may currently be undervalued, while the signal about expert ability suggests that past trends in prices may have overvalued the asset. When such dilemmas occur the trader's decision to buy, sell or abstain depends in general on the entire history. However, we show that if expert reputation is high and past trends indecisive then, in equilibrium, traders ignore the private signal they receive and blindly follow the report they observe: traders' actions no longer reveal information about expert ability. We say in this case that *screening of the expert breaks down*. Information about the expert conveyed to the market in that period – if any at all – must therefore proceed from the report itself. Hence, a bad expert can fully hide his type by choosing to mimic the reporting behavior of a good expert.⁴ Indeed, we establish that mimicking of the good type is in this case optimal for

³In fact, we show that the relation between trading outcomes and prices is one-to-one. Hence, an alternative – perhaps more realistic – formulation is to say that Bayesian updating is based upon the evolution of prices.

⁴Outside a cascade, a bad expert will not copy the behavior of a good expert completely. In particular, he

a bad expert. Thus, in equilibrium, a break-down of screening ultimately prevents any new information about the expert from being incorporated into prices. We say in this case that a *reputational cascade* occurs.

The possibility of a reputational cascade hinges upon the ability of an expert of given ability to achieve reputation above a certain threshold. Moreover, this threshold varies with the history, raising two difficulties. First, attempting to approach the threshold may be self-defeating if, on the path toward it, the threshold in fact rises. Second, as the expert nears the current threshold, traders' screening of the expert gradually loses its efficiency. Acting strategically, a bad expert will in turn adopt a reporting behavior comparable to that of a good expert. This implies that the closer he gets to actually achieving the threshold, the smaller the scope for improving his reputation. We show (Theorem 1) that in spite of these difficulties reputational cascades set off with positive probability irrespective of (i) expert type, and (ii) initial reputation.

While the occurrence of cascades implies that prices do get stuck in the medium run, we show next that prices converge in the long run to the asset's correct valuation (Theorem 2): if the expert is good, prices converge to the true asset value; if the expert is bad, prices converge to the unconditional mean. Reputational cascades are therefore transient events. The intuition is as follows. When the market is in a cascade, it stops accumulating information about the expert, but continues accumulating information about the asset. The greater the information contained in the public history, the more critical the signal of the expert type. An overwhelming dominance of historical positive reports, say, will push the public valuation of the asset – and therefore prices – up toward their highest value. On the other hand, a trader with a negative signal of the expert type will see his valuation of the asset revert toward the unconditional mean. The higher the price, the more the trader believes the asset to be overvalued. When prices are sufficiently close to 1, this will induce him to trade *against* historical trends (i.e. sell the asset), independently of the report he observes within that period. Traders' screening of the expert thus becomes efficient again, and the market exits the cascade it was in. At that point, if the expert is bad his reputation will fall (on average) and drive prices back to their unconditional mean. If the expert is good, his reputation will rise (on average), and allow him to convey through his reports the information he possesses about the asset's true value.

The possible occurrence of reputational cascades has far-reaching consequences for the

will not publish reports as often as the bad expert, as publishing releases information about his ability.

functioning of financial markets, and could help shed light on events related to the recent financial crisis. First and foremost, our paper provides a tractable model explaining financial markets' difficulty in evaluating experts when feedback about ability is endogenous, and occurs through the impact of expert advice on prices. In states of the world where experts turn out to be good, the occurrence of a cascade slows down price convergence to the true asset value. More distressingly perhaps, is the observation that cascades allow experts of low ability to maintain their reputation and affect prices for a considerable amount of time. As long as price movements remain informative about expert ability, then price deviations from long-run levels are impeded on two fronts: (a) a bad expert's reputation will on average decrease, limiting his impact on prices; (b) a bad expert exposes himself by publishing reports, and will therefore avoid releasing (false) information. In a cascade, these two arguments no longer hold, and an expert with low ability inevitably exerts greater influence on prices.

After the literature review, the paper proceeds as follows. Section 2 develops a model of a financial market with advice. Section 3 characterizes equilibrium behavior within a given period of our model. Section 4 addresses the paper's central question and explores how much and what kind of information becomes incorporated into prices over time. Section 5 discusses specific aspects of our paper: herding, prices' informational efficiency, and the implications of our model for market frenzies and crashes. Section 6 concludes. All proofs are contained in the Appendix.

1.1 Relation to Literature

The present paper is, to the best of our knowledge, the first to (i) examine financial experts' impact on asset prices when feedback about ability is endogenous and occurs through prices themselves, and (ii) assess financial markets' performance in evaluating experts of unknown ability.

We first and foremost contribute to the literature exploring the role of experts in procuring information in financial markets. This literature starts with Admati and Pfleiderer (1986) who studied a strategic expert's disclosure of information, and showed that the optimal strategy typically entails the artificial addition of noise to the information possessed by the expert, in order to overcome the dilution in the value of information due to leakage through informative prices. Yet in their paper, no uncertainty remains in equilibrium about the quality of the information provided by the expert. Our paper is in this sense closer to the work of Benabou

and Laroque (1992) or Ottaviani and Sørensen (2006). Benabou and Laroque analyze the credibility of a financial expert with short-run incentives to deceive the market and resort to insider trading.⁵ However, in their model, prices play no role in evaluating the expert. By contrast we endogenize the feedback about expert ability, which in our model occurs through prices. Ottaviani and Sørensen model professional forecasters who endeavor, through their forecasts of a stock price, to convince the market that they are well informed. If the forecasts affect the price, a ‘beauty contest’ emerges among forecasters. In their model there is no issue of screening as traders have no information about experts, whereas our focus is on the dynamic revelation of traders’ information about experts, and the ultimate failure of the market to properly channel this information.⁶

Our paper also contributes to the literature on herding in financial markets: the occurrence of reputational cascades in our model is akin to herd behavior on the part of informed traders. The first traders to arrive screen the expert according to their private signal of the expert type. A trader who arrives following a sequence of reputation-enhancing events, however, believes that, with high probability, those who came before him observed positive signals of the expert type. When the sequence of reputation-enhancing events is sufficiently long, the expected information contained in the expert’s reports swamps that of the trader’s private signal about the expert. At that point, even if he observes a negative signal of the expert type, the trader will decide to trust the expert and trade according to the advice contained in his report. Our paper crucially distinguishes itself from existing models of herding in financial markets – such as for instance Bikhchandani et al. (1992), Banerjee (1992), Avery and Zemsky (1998), or Park and Sabourian (2011) – by the fact that information about the asset is provided by a strategic expert of unknown ability. This implies that:

1. the extent of the information asymmetry between traders and market maker evolves over time, according to the reputation of the expert;
2. if traders ever ignore private information about expert ability, then bad experts *will* hide their type – by mimicking the behavior of good experts.

Both implications lie at the very heart of our analysis. Taking the first point, when expert reputation is high, the information asymmetry is mainly driven by the content of the expert’s

⁵In a similar vein, Allen and Gale (1992) investigate how an uninformed manipulator can make a profit on the stock market by pretending to be an informed trader.

⁶More generally, we are connected to the broader reputational cheap talk literature (Scharfstein and Stein, 1990; Trueman, 1994; Zwiebel, 1998), which does not analyze asset markets. They find that experts tend to bias reports toward priors and that multiple experts tend to herd.

report and traders will optimally ignore private information about expert ability; this and the second point together imply that once expert reputation reaches a threshold, then new information about the expert stops reaching the market altogether: a reputational cascade sets off, allowing experts of low ability to maintain a lasting reputation.

Finally, the break-down of learning about expert type occurring in our model is related to the market break-down occurring in Ely and Välimäki (2003). In their model a series of Principals sequentially interact with a single Agent, whose type (good or bad) is unknown to the Principals. In some cases, the good Agent should take a given action, but that action is also the preferred action of a bad Agent. To distinguish himself, the good Agent then favors the inefficient action, but by doing so he kills the Principals' incentives to hire him. In both that paper and ours, the crux lies in the failure of each Principal (viz. informed traders) to internalize the benefits to others from learning about the type of the Agent (viz. the financial expert). However the break-down of learning takes opposite forms in the two papers: In Ely and Välimäki (2003) learning breaks down because the Principals stop 'trading'; by contrast learning breaks down (viz. a reputational cascade occurs) in our paper because the Principals trade with probability one.⁷

2 The Model

We model a discrete-time sequential trade market for a financial asset in the spirit of Glosten and Milgrom (1985). The traded asset has fundamental value θ which takes its value in $\{-1, 1\}$ with equal probability: $\mathbb{E}[\theta] = 0$. In each period there are three participants to the market: a market maker (MM), a financial expert (FE), and a trader. The market maker and the expert are long lived, whereas a new trader arrives each period. The market maker sets prices at which he wishes to trade the asset. The expert provides information about the asset. Finally, the trader decides whether or not to trade. We next provide the details and notation describing this market.

The Market Maker. Each period $t = 0, 1, 2, \dots$ the MM posts *ask* (p_t^a) and *bid* (p_t^b) prices at which he will sell or buy one unit of the asset, respectively. We assume that the MM is risk-neutral and competitive. Quoted prices thus equal the expected asset value conditional on all information available to the MM at the time of trade.

⁷In our model, when the market is in a reputational cascade, an informed trader buys the asset if he observes a positive report and sells the asset if he observes a negative one.

The Trader. Each period t a new trader arrives to the market and is given the opportunity to trade one unit of the asset. The trader observes the prices set by the MM and decides whether to buy at p_t^a , sell at p_t^b , or abstain from trading. We let $y_t \in \{a, b, n\}$ denote the trader's action: $y_t = a$ if he buys, $y_t = b$ if he sells, and $y_t = n$ if he abstains. He subsequently leaves the market.

The trader is of one of two types, defining the motives of his trades. The trader knows his own type, while other market participants have probabilistic beliefs about the trader's type. With probability $\mu \in (0, 1)$, the trader is an *informed trader* and trades to maximize profits at the market maker's expense. With probability $1 - \mu$ the trader is a *liquidity trader* and trades for exogenous motives, unrelated to profits. A liquidity trader buys, abstains or sells the asset with probability $1/3$ each.

The Expert. The asset's true value θ is unobserved. All supplementary information about the asset – if any – is provided by the financial expert. Let τ denote the expert's type, which defines his ability: $\tau = G$ if he is good, $\tau = B$ if he is bad. His type is drawn at the beginning of the game and known only to himself: $\tau = G$ with probability $\lambda_0 \in (0, 1)$, and $\tau = B$ with probability $1 - \lambda_0$. The parameter λ_0 thus defines the *reputation* of the expert at the beginning of the game. We assume that the expert is myopic and aims to maximize his expected end-of-period reputation.⁸

Before trade takes place, a good expert receives the private signal $x_t \in \{-1, 0, 1\}$, where $x_t = 0$ is used to indicate that the FE receives no signal in that period. Let $\theta_t^G := \mathbb{E}[\theta|G; x_1, \dots, x_t = 0]$, and $\theta_t^G(x_t) := \mathbb{E}[\theta|G; x_1, \dots, x_t \neq 0]$. We assume that the signal x_t is imperfectly but positively correlated with θ , so that $\mathbb{P}(x_t = \theta|x_t \neq 0) = \phi \in (1/2, 1)$. A bad expert has no private information concerning the asset: $\mathbb{E}_t[\theta|B] = 0$.⁹

Financial Reports. In each period the FE communicates with market participants through the report $r_t \in \{-1, 0, 1\}$, where $r_t = 0$ if the expert chooses not to release a report. We will say that the timing of information acquisition is *symmetric* if the report is observed by all market participants before price setting. By contrast, we will say that the timing of information acquisition is *asymmetric* if the MM observes the report with delay, and after setting the prices.

We assume that by reading the expert's report, the trader learns about the expert's true

⁸We later describe reputation updating.

⁹See subsection on timing and strategies for a formal definition of the operator $\mathbb{E}_t[\cdot]$.

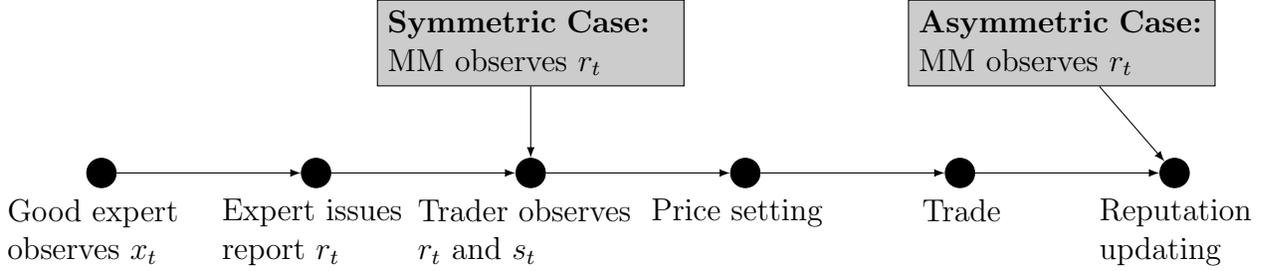


Figure 1 – Timing

type. For instance, the expert may in his reports reveal some information regarding his assessment of other auxiliary economic variables which traders know something about, and which they can then use to make inferences about the expert’s type.¹⁰ For simplicity, we assume that the trader observes the signal $s_t \in \{0, 1\}$ satisfying

$$\mathbb{P}(s_t = 1 | \tau = G, r_t = x_t \neq 0) = \pi,$$

where $\pi \in (0, 1)$, and $\mathbb{P}(s_t = 1 | \cdot) = 0$ otherwise. A good expert receiving private signal $x_t \in \{-1, 1\}$ and accurately reporting his private information thus provides evidence of his ability with probability π .¹¹

Timing and Strategies. The timing of the game, within each period, is as follows (c.f. Figure 1). If $\tau = G$, the expert first observes his private signal x_t . The expert (irrespective of his type) then releases the report r_t . If the timing of information acquisition is symmetric, the MM observes r_t and announces the ask and bid prices; if the timing is asymmetric, the MM sets the prices before observing r_t . Finally, the trader observes the report, his private signal about the expert and the prices, and decides whether to buy, sell, or abstain. The market then updates the reputation using Bayes’ rule, based on all publicly available information when the period ends, consisting of the latest report and trading outcome.

¹⁰A simple way of modeling this is to introduce an auxiliary random variable Z_t , uniformly distributed on the unit interval. The expert, if he is good, observes Z_t with probability π in each period. The period- t trader, on the other hand, always observes Z_t .

¹¹Our specification of s_t thus makes three separate assumptions. First, $r_t \neq x_t$ implies $s_t = 0$. This ensures that truth-telling is strictly dominant for a good expert (see Lemma 1). Second, $r_t = 0$ implies $s_t = 0$. This assumption is made for the sake of realism, but affects neither the results of Theorem 1 nor of those of Theorem 2. Second, $s_t = 0$ always if $\tau = B$. This assumption is for computational simplicity only. All that is required is that the signal s_t satisfies the Monotone Likelihood Ratio Property.

Let $\mathcal{R}_t := (r_0, \dots, r_t)$ denote the sequence of reports and $\mathcal{Y}_t := (y_0, \dots, y_t)$ the sequence of trades up to and including period t . \mathcal{H}_t denotes the public information at the beginning of period t , consisting of previous trades and reports: $\mathcal{H}_t := \{\mathcal{R}_{t-1}, \mathcal{Y}_{t-1}\}$. Define $\mathbb{P}_t(\cdot) := \mathbb{P}(\cdot | \mathcal{H}_t)$ as the probability operator conditional on \mathcal{H}_t . Define similarly the conditional expectation operator $\mathbb{E}_t[\cdot] := \mathbb{E}[\cdot | \mathcal{H}_t]$. The updated reputation conditional on the the report r_t and the trading outcome y_t is thus:

$$\lambda_{t+1}(r_t, y_t) = \frac{\lambda_t \mathbb{P}_t(r_t, y_t | G)}{\lambda_t \mathbb{P}_t(r_t, y_t | G) + (1 - \lambda_t) \mathbb{P}_t(r_t, y_t | B)}.$$

For each period t , we let $\underline{q}_t(x_t)$ denote the strategy of a good expert, as a function of his private signal x_t , and $\underline{\sigma}_t$ denote the strategy of a bad expert. $q_t^{r_t}(x_t)$ (resp., $\sigma_t^{r_t}$) denotes the probability with which a good expert releases r_t (resp., with which a bad expert releases r_t). $\underline{\xi}_t(r_t, s_t)$ denotes an informed trader's strategy, as a function of the report r_t and his private signal s_t of the expert. We let $\xi_t^{y_t}(r_t, s_t)$ denote the probability with which he takes action y_t . Finally, $\underline{p}_t := (p_t^a, p_t^b)$.

Equilibrium. The structure of the game described above is common knowledge. To avoid uninteresting ‘babbling’ equilibria, we focus the analysis on *truthful* equilibria, i.e. in which a good expert simply reports the signal he observes: $q_t^{r_t}(x_t) = 1$ if $r_t = x_t$.

Definition 1. *Given history \mathcal{H}_t , the triplet $(\underline{\sigma}_t, \underline{\xi}_t, \underline{p}_t)$ constitutes a (truthful) equilibrium in period t if an expert of type $\tau = G$ truthfully reports his private signal x_t of the true asset value, while:*

1. $\underline{\sigma}_t$ maximizes a bad expert's expected reputation in the beginning of period $t + 1$.
2. $\underline{\xi}_t$ maximizes an informed trader's expected profits from trade.
3. Prices are set competitively; for $y = a, b$:

$$\begin{aligned} p_t^y &= \mathbb{E}_t[\theta | y_t = y, r_t] && \text{if information acquisition is symmetric} \\ p_t^y &= \mathbb{E}_t[\theta | y_t = y] && \text{if information acquisition is asymmetric.} \end{aligned}$$

The following simple result establishes the rationality of a good expert's behavior implicit in a truthful equilibrium.

Lemma 1. *If $(\underline{\sigma}_t, \underline{\xi}_t, \underline{p}_t)$ constitutes a (truthful) equilibrium in period t then, given $(\underline{\sigma}_t, \underline{\xi}_t, \underline{p}_t)$, truthfully reporting his private signal x_t maximizes $\mathbb{E}_t[\lambda_{t+1}|G]$.*

We can now formally define our concept of equilibrium. A family of strategies for FE, informed traders, and MM is a (truthful) equilibrium of our game if for all t , and all \mathcal{H}_t , it induces a (truthful) equilibrium in period t .

Throughout, the vector \underline{q}_t summarizes public beliefs about a truthful good expert's behavior in period t . Thus, $q_t^{r_t} := \mathbb{P}_t(x_t = r_t)$.

3 Preliminary Analysis: Informed Traders and the Screening of Expert Type

We exploit in this section the recursive structure of our model, making t and the history \mathcal{H}_t into parameters of the analysis, and exploring the one-shot game played within period t among (a) the MM, (b) the current trader, and (c) the FE.

Price adjustments reflecting information of the MM typically eliminate the inefficiency arising in the classic herding models (Bikhchandani et al., 1992; Banerjee, 1992), whereby traders fail to act upon their own private information. Yet, with sufficient information asymmetry between traders and MM, traders may rationally choose to ignore part of the information they possess (Avery and Zemsky, 1998; Park and Sabourian, 2011). In our model, this occurs when (i) traders observe reports before the MM, and (ii) expert reputation is high. Section 3.1 establishes the necessity of the first condition; section 3.2 demonstrates the necessary and sufficient nature of the two conditions.

The following notation and definitions will be used throughout the paper. $v_t(r_t, s_t) := \mathbb{E}_t[\theta|r_t, s_t]$ denotes an informed trader's valuation of the asset in period t after observing the

report r_t and signal s_t . Thus:

$$\begin{aligned}
v_t(1, 1) &= \theta_t^G(1), \\
v_t(1, 0) &= \frac{\lambda_t q_t^+(1 - \pi)}{\lambda_t q_t^+(1 - \pi) + (1 - \lambda_t)\sigma_t^+} \theta_t^G(1), \\
v_t(0, 0) &= \frac{\lambda_t q_t^0}{\lambda_t q_t^0 + (1 - \lambda_t)\sigma_t^0} \theta_t^G, \\
v_t(-1, 1) &= \theta_t^G(-1), \\
v_t(-1, 0) &= \frac{\lambda_t q_t^-(1 - \pi)}{\lambda_t q_t^-(1 - \pi) + (1 - \lambda_t)\sigma_t^-} \theta_t^G(-1).
\end{aligned}$$

The equilibrium ranking of the valuations determines informed traders' actions, which in turn determine the information reflected in prices. Much of our model's interest springs from the fact that rather than being fixed, the ranking of the valuations – and, by way of consequence, the information reflected in prices – typically evolves over time. We establish in the Appendix, as an intermediary result, that if $\theta_t^G(-1) > 0$ then *in any equilibrium* $v_t(1, 1)$ is (strictly) the highest valuation and $v_t(-1, 0)$ (strictly) the lowest valuation.¹² It is easy to see that if $\theta_t^G(-1) < 0 < \theta_t^G(1)$ then $v_t(-1, 1)$ is (strictly) the lowest valuation and $v_t(1, 1)$ (strictly) the highest valuation.

Define also p_t^0 as the market maker's valuation of the asset at the time of setting the prices (but before knowing the trade order). It is sometimes useful to think of p_t^0 as the price that the MM would set if he knew the trader in period t to be a liquidity trader. This yields:

$$\begin{aligned}
p_t^0 &:= \mathbb{E}_t[\theta|r_t] && \text{if information acquisition is symmetric;} \\
p_t^0 &:= \mathbb{E}_t[\theta] && \text{if information acquisition is asymmetric.}
\end{aligned}$$

Finally, we will say, broadly speaking, that *screening is efficient* in period t when informed traders' actions perfectly reflect their private information, and that *screening breaks down* when traders' actions are independent of the signal s_t .

Definition 2. *Given trading strategy $\xi_{\underline{t}}$, say that **screening is efficient** if $\xi_t^{y_t}(r_t, 1) > 0 \Rightarrow \xi_t^{y_t}(r_t, 0) = 0, \forall y_t, r_t$.*

Definition 3. *Given trading strategy $\xi_{\underline{t}}$, say that **screening breaks down on the positive side***

¹²By symmetry, if $\theta_t^G(1) < 0$ then *in any equilibrium* $v_t(-1, 1)$ is (strictly) the lowest valuation and $v_t(1, 0)$ (strictly) the highest valuation.

(resp., negative side) if there exist $y_t \in \{a, b, n\}$ such that $\xi_t^{y_t}(1, 1) = 1 = \xi_t^{y_t}(1, 0)$ (resp., such that $\xi_t^{y_t}(-1, 1) = 1 = \xi_t^{y_t}(-1, 0)$). Say that **screening breaks down** if it does so on both the positive and negative side.

When screening is efficient then, conditional on the report r_t , an informed trader observing $s_t = 1$ never takes the action of an informed trader observing $s_t = 0$. This allows the market to learn about the expert. By contrast, when screening breaks down, an informed trader observing $s_t = 1$ always takes the action of an informed trader observing $s_t = 0$. Conditional on the report r_t , a trader's action y_t then reveals nothing about the expert.¹³

3.1 Symmetric Timing of Information Acquisition

We begin with the simplest case and show that if traders and MM observe the report before price setting then, in equilibrium, informed traders' actions always reflect their private information about the expert.

When the timing of information acquisition is symmetric the only informational asymmetry between trader and MM pertains to (a) the trader's own type (informed vs. liquidity), and (b) the trader's signal s_t of the expert type. After observing the report, but before observing the trade order, the MM updates his belief that the expert is good based on the knowledge of r_t . This yields

$$p_t^0 = \frac{\lambda_t q_t^{r_t}}{\lambda_t q_t^{r_t} + (1 - \lambda_t) \sigma_t^{r_t}} \theta_t^G(r_t).$$

As indicated earlier, one may think of p_t^0 as liquidity traders' 'valuation' of the asset. Since equilibrium prices reflect a weighted average of the valuations of trading agents, any equilibrium price can be expressed as a weighted average of $v_t(r_t, 0)$, p_t^0 , and $v_t(r_t, 1)$.

Trading behavior then depends on the sign of the valuations. First, if $\theta_t^G(r_t) > 0$ we obtain, by immediate inspection, the inequalities

$$v_t(r_t, 0) < p_t^0 < v_t(r_t, 1).$$

Thus in any equilibrium (a) informed traders observing $s_t = 1$ buy with probability 1, while (b) informed traders observing $s_t = 0$ sell with probability 1. Second, if $\theta_t^G(r_t) < 0$ the above inequalities are reversed, and so are the optimal actions of traders. Third, if $\theta_t^G(r_t) = 0$ the

¹³The qualification is important, since the report itself may reveal something about the expert type.

above inequalities become equalities, and this leaves traders indifferent between trading or abstaining. Our first proposition summarizes these results:

Proposition 1. *Consider symmetric timing of information acquisition. In equilibrium, screening is efficient, except possibly in the special case where $\theta_t^G(r_t) = 0$.*

Whenever information acquisition is symmetric, the price mechanism will force traders to reveal the only piece of private information which they possess: their signal about the expert. In other words, Proposition 1 establishes the necessity of asymmetry in the timing of information acquisition for screening to break down.¹⁴

3.2 Asymmetric Timing of Information Acquisition

We now show that when the MM observes reports with delay then, in equilibrium, traders' actions may stop reflecting their private information.

Under asymmetric timing of information acquisition, an informed trader possesses at the time of trade two pieces of information with which to make profits against the market maker: the latest report, and his private signal of the expert type. When these pieces of information clash, traders are faced with a dilemma.

Suppose to fix ideas that past trends in prices are positive, such that $\theta_t^G(r_t) > 0$, for all $r_t \in \{-1, 0, 1\}$. An informed trader now observes $(r_t, s_t) = (1, 0)$. On the one hand, the current report suggests that prices likely *undervalue* the asset's true value. The private signal of expert ability, on the other hand, suggests that the market may overestimate the expert. Since we assumed past trends in prices – driven by the expert – to be positive, this in turn suggests that prices likely *overvalue* the asset's true value.

We next show that if expert reputation is high then an equilibrium can be found where, when such dilemmas occur, an informed trader always follows the information contained in the report and ignores his private signal of the expert.

Lemma 2. *Consider asymmetric timing of information acquisition. There exists $\hat{\lambda}_t \in (0, 1)$ such that screening breaks down in some equilibrium of period t if and only if $\lambda_t \geq \hat{\lambda}_t$.*

The intuition is as follows. As λ_t approaches 1, the valuation of a trader observing r_t

¹⁴See Rudiger and Vigier (2014) for a formal study of information acquisition in sequential trading models, as well as the seminal work of Grossman and Stiglitz (1980).

approaches $\theta_t^G(r_t)$, independently of the signal s_t . Furthermore,

$$p_t^0 = \lambda_t \theta_t^G,$$

and thus liquidity traders' 'valuation' of the asset tends to θ_t^G as λ_t approaches 1. A market maker receiving, say, a buy order, sets the price to reflect the mean of the valuations from the potential buyers he may be facing. In this weighted average, the weight of θ_t^G is bounded below by the positive mass of liquidity traders. Since $\theta_t^G(-1) < \theta_t^G < \theta_t^G(1)$, it follows that we can find an equilibrium in which all informed traders observing $r_t = 1$ choose to buy the asset and where all informed traders observing $r_t = -1$ choose to sell the asset – independently of the signal s_t .

As usual in models of strategic communication, our framework generally allows multiple equilibria to exist. This puts a question mark on the scope of Lemma 2. Our next result addresses the issue, by establishing that if screening breaks down in one equilibrium, then this must be the unique equilibrium.

Lemma 3. *Consider asymmetric timing of information acquisition. If an equilibrium of period t exists where screening breaks down, then this is the unique equilibrium of period t .*

We here sketch the main arguments of the proof. We suppose that a no-screening equilibrium exists alongside with another equilibrium, and proceed to analyze the second equilibrium. Denote these two equilibria by $E1$ and $E2$, respectively, and denote by $\underline{\sigma}_1$ and $\underline{\sigma}_2$ the strategy of a bad expert in the two equilibria, respectively.

First, suppose that $\sigma_2^+ > \sigma_1^+$. Then the expected reputation of publishing a positive report must be lower in $E2$ than in $E1$: one, a positive report is more likely to be sent by a bad expert; two, there is no less screening in $E2$ than in $E1$, since $E1$ is a no-screening equilibrium. Notice also that, in any equilibrium, the expected reputation from not publishing a report is entirely determined by a bad expert's strategy, since no information is revealed by traders' actions in this case (their private signal is always zero). Thus, if $\sigma_2^+ > \sigma_1^+$ then we must have $\sigma_2^0 > \sigma_1^0$. If not, then in $E2$ the expected reputation from not publishing a report would be greater than that of publishing a positive report, which would be inconsistent with equilibrium. As a consequence, $\sigma_2^- < \sigma_1^-$.

Next, we show in the proof that the less likely it is that a report was published by a bad expert, the less traders gain from screening it. Thus: since $\sigma_2^- < \sigma_1^-$ and $E1$ is a no-screening equilibrium, screening must break down on the negative side in $E2$. But then the expected

reputation of publishing a negative report is greater in $E2$ than it is in $E1$. Combining this observation with our first remark above finally shows that in $E2$, the expected reputation of publishing a negative report is greater than that from publishing a positive report. But this is inconsistent with equilibrium.

It follows from the steps above that expert strategies must be the same in the two equilibria, and since expert strategies completely determine traders' valuations, prices are also the same. The final part of the proof then shows that if prices are the same in the two equilibria, then screening breaks down in $E2$ as well. Thus, the two equilibria are identical.

Lemma 2 establishes the necessary conditions for an equilibrium to exist in which screening breaks down. Lemma 3 shows that if screening breaks down in one equilibrium, then this equilibrium is unique. Combining these observations therefore yields this section's main result:

Proposition 2. *Consider asymmetric timing of information acquisition. Then in equilibrium screening breaks down in period t if and only if $\lambda_t \geq \hat{\lambda}_t$, with $\hat{\lambda}_t$ as defined in Lemma 2.*

Proposition 2 establishes that asymmetric timing of information acquisition and high reputation are necessary and sufficient for screening to break down in equilibrium.

3.3 Screening and Expert Behavior

We conclude Section 3 by shedding light on the expert's reporting strategy as a function of traders' screening behavior.¹⁵

A break-down of screening in period t implies that, conditional on the report r_t , a trader's action y_t reveals nothing about the expert. Information about the expert conveyed to the market in that period – if any at all – must therefore proceed from the report itself. Hence, a bad expert can fully hide his type by mimicking the behavior of a good one. We show that indeed this is the optimal course of action.

The situation is otherwise more intricate. When screening occurs, the information contained in the signal s_t is (imperfectly, due to liquidity traders) transmitted to the market and leads on average to a decrease of reputation for a bad expert. This creates an incentive for a bad expert to behave less aggressively in his reporting strategy than a good expert would (i.e. to publish fewer reports). Let in what follows $\sigma_t = \sigma_t^+ + \sigma_t^-$ (resp., $q_t = q_t^+ + q_t^-$) denote the *aggressiveness* of a bad expert (resp., a good expert). We then have:

¹⁵Naturally, in equilibrium traders' screening behavior is itself a function of the information contained in the reports of the expert.

Proposition 3. *In equilibrium, when screening breaks down, then $\underline{\sigma}_t = \underline{q}_t$. Otherwise, a bad expert is less aggressive than a good expert: $\sigma_t < q_t$.*

A break-down of screening is thus associated with an increase in the reporting activity of bad experts. In this way, markets with no screening are more likely to incorporate ‘bad information’ into prices, because experts of low ability will relish the opportunity to publish reports without the risk of being exposed.

4 Reputational Cascades

We turn with this section to the central question of our paper: how much and what kind of information becomes incorporated into prices over time?

We begin with the key definitions of our analysis. A *complete informational cascade* occurs when the distribution over observable outcomes is statistically independent of the underlying state of nature, here consisting of the fundamental value θ and the expert type τ . A *reputational cascade* on the other hand is a partial informational cascade; it requires only that the distribution over observable outcomes be independent of the expert type τ . Thus, in a reputational cascade, prices stop incorporating any new information about the expert, but may still incorporate some information about the asset’s true value.

Definition 4. A *complete informational cascade* occurs in period t when

$$\mathbb{P}_t(r_t, y_t | \theta, \tau) = \mathbb{P}_t(r_t, y_t), \quad \forall r_t, y_t.$$

Definition 5. A *reputational cascade* occurs in period t when

$$\mathbb{P}_t(r_t, y_t | \tau) = \mathbb{P}_t(r_t, y_t), \quad \forall r_t, y_t.$$

We now ask the question: do informational cascades of any kind ever occur as part of an equilibrium of our model? We begin with a simple negative result, and establish that complete informational cascades are prevented from ever taking place.

Proposition 4. *A complete informational cascade never occurs in any equilibrium.*

While complete informational cascades are prevented from ever occurring, the results of Section 3 concerning the possibility of screening breaking down suggest on the other hand the

possibility of reputational cascades actually taking place in equilibrium. First, by Proposition 2, high expert reputation will induce a break-down of screening (under asymmetric timing of information acquisition). Second, by Proposition 3, a break-down of screening will induce a bad expert to fully hide his type by mimicking the behavior of a good expert. Hence, assuming asymmetric information acquisition, the distribution over observable outcomes will be the same independently of the expert type if – but only if – the expert ever achieves a threshold level of reputation $\hat{\lambda}_t$. The question is then: starting with initial reputation λ_0 , will an expert of type τ ever gain sufficient reputation?

We show in the Appendix that the threshold $\hat{\lambda}_t$ needed to set off a break-down of screening depends on the history \mathcal{H}_t through the sequence of historical reports \mathcal{R}_{t-1} . This observation raises two difficulties. First, if the threshold increases with expert reputation, the expert's efforts to achieve the threshold level of reputation may be self-defeating. Second, screening gradually loses its efficiency as one approaches the threshold, and a bad expert optimally responds by behaving more like a good expert. This implies that the closer one gets to actually achieving the threshold, the smaller the scope for improving the reputation.

These difficulties notwithstanding, the following result establishes that whichever the initial reputation of the expert, and whichever the ability type of the expert, a period t and history \mathcal{H}_t can be found in which a reputational cascade begins.

Theorem 1.

1. ***Symmetric timing of information acquisition:*** *A reputational cascade never occurs in any equilibrium.*
2. ***Asymmetric timing of information acquisition:*** *In any equilibrium, for all $\lambda_0 > 0$ and $\tau \in \{G, B\}$, a reputational cascade occurs with strictly positive probability.*

The proof of the first part rests on Proposition 1, and the observation that screening must break down for a reputational cascade to occur. The proof of the second part contains three steps. Step 1 formalizes the argument that, in equilibrium, a break-down of screening sets off a reputational cascade. Step 2 establishes sufficient conditions inducing efficient screening, and allows us to bound below the per-period growth of reputation following a reputation-enhancing event in period t (e.g. the release of a positive report followed by an increase in the price). We show as a result that, starting with initial reputation λ_0 , any level of reputation can be reached with positive probability even by a bad expert. Step 3 then shows

that once the expert has achieved sufficient reputation, there is a path on which he maintains his reputation but the threshold $\hat{\lambda}_t$ needed to set off a break-down of screening falls. Using step 1 therefore establishes that a cascade eventually sets off on this path. Finally, since it contains a finite number of steps (each of which has non-zero probability), this path occurs with strictly positive probability.

The occurrence of reputational cascades naturally raises the question of prices' informational efficiency in the long run.¹⁶ Since the FE provides all information concerning the asset, prices' long run convergence is intimately linked to the learning of the expert type. A number of observations can be made already at this point.

First, while reputational cascades are favorable to experts of low ability, maintaining the status quo is a difficult task. If he could avoid releasing new reports, a bad expert could sustain a cascade indefinitely. However, in equilibrium, a bad expert must mimic the behavior of a good one.¹⁷ Each period a new report is therefore published with probability 1/2. Moreover, reports must eventually support either $\theta = 1$ or $\theta = -1$, since $|\theta_t^G| \rightarrow 1$ if the expert is good.¹⁸ But notice that $\theta_t^G \rightarrow 1$ yields

$$v_t(-1, 0) \simeq v_t(1, 0) < p_t^0 < v_t(-1, 1) \simeq v_t(1, 1),$$

while $\theta_t^G \rightarrow -1$ gives

$$v_t(-1, 1) \simeq v_t(1, 1) < p_t^0 < v_t(-1, 0) \simeq v_t(1, 0).$$

In both cases the inequalities imply that screening eventually becomes efficient, and cascades therefore eventually end.

Our second remark pertains to the evolution of reputation under 'normal' circumstances. By Proposition 3 a bad expert behaves less aggressively than a good one when the market is outside of a reputational cascade. His reputation thus falls if he avoids to release a report. Since in equilibrium he must be indifferent between reporting or not, he must on average lose reputation from one period to the next: $\mathbb{E}_t[\lambda_{t+1}|B] < \lambda_t$. Moreover, reputation is a martingale:

$$\lambda_t = \mathbb{E}_t[\lambda_{t+1}] = \lambda_t \mathbb{E}_t[\lambda_{t+1}|G] + (1 - \lambda_t) \mathbb{E}_t[\lambda_{t+1}|B].$$

¹⁶I.e. whether or not prices successfully aggregate private information in the long run (so-called strong-form informational efficiency).

¹⁷See Proposition 3.

¹⁸If $\tau = G$, then $|\theta_t^G| \rightarrow 1$ is immediate, by the Law of Large Numbers.

Hence $\mathbb{E}_t[\lambda_{t+1}|G] > \lambda_t$: a good expert on average gains reputation from one period to the next. Reputation thus drifts in the appropriate direction, irrespective of the expert type. However, the question of learning remains: will sufficiently much information filter through so that reputation converges to either 0 or 1? And if so, can incorrect learning ever occur, so that reputation tends to 1 when $\tau = B$ or to 0 when $\tau = G$? We establish through Theorem 2 that in the long run the market learns the expert's true type.¹⁹ While prices get stuck in the medium run (Theorem 1), they converge in the long run to the asset's correct valuation.

Theorem 2. *Consider asymmetric timing of information acquisition. In any equilibrium:*

1. *Conditional on $\tau = G$, prices converge almost surely to the true asset value θ .*
2. *Conditional on $\tau = B$, prices converge almost surely to zero.*

Our proof of Theorem 2 has two main steps. We begin by showing that $\lambda = \lim \lambda_t$ takes value in $\{0, 1\}$.²⁰ If for some realization λ tended to $\tilde{\lambda} \in (0, 1)$ then arguments similar to those used above (where we establish that cascades are transient events) would show that screening eventually becomes efficient along the induced path. But if screening is efficient then changes in reputation from one period to the next are bounded away from zero, which contradicts the convergence. Thus $\lambda \in \{0, 1\}$. We next establish in the second step that if $\tau = B$ then the ratio $\lambda/(1 - \lambda)$ must be integrable, as must be the inverse ratio if $\tau = G$. Using the first step hence shows that if $\tau = B$ then $\lambda = 0$ a.s., and similarly that $\lambda = 1$ a.s. if $\tau = G$. The market thus learns the expert's true type with probability 1. The proof is concluded by noting that $\theta_t^G \rightarrow \theta$ if the expert is in fact a good expert.

5 Discussion

5.1 Sequential Trading and Prices' Informational Efficiency

The question of prices' informational efficiency in sequential trading was first studied in the seminal paper of Glosten and Milgrom (1985).²¹ Their analysis may broadly speaking be

¹⁹We thus establish that $\mathbb{E}[\lambda_t|B]$ not only decreases but converges in fact to 0 and, similarly, that $\mathbb{E}[\lambda_t|G]$ not only increases but converges in fact to 1. The Dominated Convergence Theorem then gives $\mathbb{P}(\lambda = 0|B) = 1$ and $\mathbb{P}(\lambda = 1|G) = 1$, where λ denotes the limit of λ_t (which we know exists by the Martingale Convergence Theorem).

²⁰As noted earlier, reputation is a bounded martingale, and so the limit exists.

²¹See also Avery and Zemsky (1998) and Dasgupta and Prat (2008).

summarized as follows. Since prices follow a bounded martingale, they converge to some value p . Provided that each period the probability of a buy and a sell order are bounded below, the bid-ask spread therefore converges to zero. Adverse selection in the market must therefore disappear in the long run.

In our model all information about the asset is provided by the FE. Adverse selection in the market thus disappears when either (a) the expert's reputation tends to 0, or (b) prices tend to $-1/1$.²² Prices' long run convergence to their correct values can therefore *not* be deduced from Glosten and Milgrom (1985).²³ The question of correct convergence boils down to whether or not the market learns the expert's true type. The market we model ultimately succeeds in aggregating private information due to the self-defeating nature of the bad expert's optimal strategy: the more the latter tries to convince the market that he knows the asset's true value, the more the market wants to know whether this information is correct or not. There will therefore always be learning about expert type and, by way of consequence, about the asset's true value.

5.2 Bad Experts and Market Crashes

The possible occurrence of reputational cascades has far-reaching consequences for the functioning of financial markets, suggesting interesting avenues for research, and helping shed light on events related to the latest financial crisis. This section summarizes our main observations on the issue.

First and foremost, our paper provides a tractable model explaining financial markets' difficulty to evaluate experts when feedback about ability is endogenous, and occurs through the impact of expert advice on prices. If the expert is good, reputational cascades work to his detriment, since they slow down the market's learning about his type. If the expert is bad, cascades work in his favor, allowing him to maintain a reputation for extended lengths of time. However, in both cases cascades will cause prices to deviate from long run levels: if the expert is good, it will take longer before the market incorporates his information into prices; if he is bad, the market won't discard his advice in the medium run. These remarks are aggravated by the fact that in a cascade a bad expert no longer exposes himself by publishing reports,

²²When reputation converges to zero, nobody is expected to have superior information: adverse selection disappears. When reputation does not converge to zero, prices must contain all information about the asset (resp., the expert), since otherwise the report (resp., the ability signal) will confer an informational advantage upon the trader.

²³In sharp contrast to Avery and Zemsky (1998), for example.

and responds in turn by releasing more (false) information. Thus, in a cascade an expert with low ability will exert greater influence on prices.

Next, how is market activity affected by reputational cascades? First, as noted above, when the market enters a cascade, the rate at which a low ability expert releases information increases. Second, when driving a reputational cascade, an expert's advice induces probability 1 of trade occurring.²⁴ Reputational cascades thus maximize expected traded volumes over a given period of time. In this sense, our analysis sheds light on episodes of *frenzies* taking place in financial markets, and identifies a new channel through which they can occur.

Finally, we conclude with some key remarks regarding the possibility of using our framework and analysis to shed light on the occurrence of crashes in financial markets. The intuition is simple. When the market is in a reputational cascade, it stops accumulating information about the expert but goes on accumulating information about the asset. As the amount of information contained in the public history grows, a trader with a negative signal of the expert type stands to make increasing profits by using this signal to trade against historical trends. Cascades therefore end precisely *as prices become high*. A crash occurs when the market exits a reputational cascade driven by a low-ability expert. At this point, the market starts to evaluate the expert again and prices revert toward zero. Rather than being caused by the release of new information, crashes in our model therefore result from the *simultaneous and uniform depreciation of past accumulated information*.²⁵ These insights are, to the best of our knowledge, novel.²⁶

5.3 Herding vs. Reputational Cascades

The occurrence of reputational cascades is akin to herd behavior on the part of informed traders. The first traders to arrive screen the expert according to their private signal of the expert type. A trader who arrives following a sequence of reputation-enhancing events, on the other hand, believes that with some probability those who came before him observed positive signals of the expert type. When the sequence of reputation-enhancing events is

²⁴Formally, in a cascade $r_t \neq 0 \Rightarrow y_t \neq n$.

²⁵This is in sharp contrast to e.g. the classic model of Lee (1998) wherein transaction costs lead traders to accumulate information until a trigger occurs, at which point the sudden release of traders' information induces prices to re-adjust abruptly.

²⁶Veldkamp (2006) shows a positive empirical correlation between indicators of 'herding' (high price and high price volatility) and the quantity of news in the market. She remarks that this is inconsistent with traditional herding models: increased information should remedy the information asymmetry problem and dissolve the herd. However, as argued above, this positive correlation is exactly what our model predicts.

sufficiently long, the resulting beliefs swamp the information contained in the trader's signal of the expert type. At that point, even if he observes a negative signal of the expert type, a trader will decide to trust the expert and trade according to the advice contained in his report. 'Informational herding' (Ottaviani and Sørensen, 2000) then prevails. However, unlike classic herding models, traders do not follow the actions of those who precede them: in a reputational cascade, they buy following a positive report and sell following a negative one. Instead, they herd in the sense of blindly following expert advice after observing others doing so.

6 Conclusion

By choosing whether or not to follow expert advice, privately informed traders ordinarily allow markets to evaluate expert ability. We investigate in this paper the performance and implications of the resulting feedback mechanism. A trader receiving a 'buy' advice but observing a negative signal of the expert's ability faces a dilemma: he may choose to ignore the advice, allowing the market to learn about his assessment of the expert; if the expert turns out to be good, however, he foregoes by doing so a profitable opportunity. Expert screening thus breaks down due to traders' failure to internalize the full benefits from learning about the expert's type. Reputational cascades – in which no new information about the expert reaches the market – occur as the result of strategic experts exploiting the resulting market failure. In a reputational cascade, bad experts publish more aggressively since they can do so without the risk of being revealed. Reputational cascades thus maximize (bad) trading activity. Even though markets are informationally efficient in the limit, this may lead to significant medium run distortions: frenzies and crashes may occur. Rather than being caused by the release of new information, crashes occur in our model because of the sudden depreciation of information already incorporated in prices.

A Appendix: Proofs

To shorten notation, we will throughout the Appendix let $\gamma := \frac{\mu}{\mu+(1-\mu)/3}$.

Proof of Lemma 1: Parts of the proof uses results and definitions from later Claims and Lemmas. We here simply sketch the main arguments.

Let ${}^B\lambda_{t+1}^e(r_t) = \mathbb{E}_t[\lambda_{t+1}|B]$ (resp. ${}^G\lambda_{t+1}^e(r_t) = \mathbb{E}_t[\lambda_{t+1}|G]$) denote a bad expert's (resp. good expert's) expectedated reputation next period from playing r_t in the current period, under equilibrium $(\underline{\sigma}_t, \underline{\xi}_t, \underline{p}_t)$. Either screening breaks down in this equilibrium, or it does not. If it does, then ${}^G\lambda_{t+1}^e(r_t) = {}^B\lambda_{t+1}^e(r_t)$, for all r_t , and a good expert is indifferent between truthfully reporting his private signal or lying. If it does not then for $x_t \neq 0$ we have ${}^G\lambda_{t+1}^e(x_t) > {}^G\lambda_{t+1}^e(r_t)$, $r_t \neq x_t$, while for $x_t = 0$: ${}^G\lambda_{t+1}^e(r_t) = {}^B\lambda_{t+1}^e(r_t)$, for all r_t . ■

Before proving Lemma 2 we show the following claim, which states that we can (partially) rank valuations in equilibrium. We take t and \mathcal{H}_t as given, allowing us to drop time subscripts.

Claim 1. *Let $\theta_t^G(-1) > 0$. Then, in any equilibrium of period t :*

1. $v_t(1, 1)$ is (strictly) the highest of all valuations.
2. $v_t(-1, 0)$ is (strictly) the lowest of all valuations.
3. $v_t(-1, 0) < p_t^0 < v_t(1, 1)$.

In particular, in any equilibrium of period t : $\xi_t^a(1, 1) = 1$ and $\xi_t^b(-1, 0) = 1$.

Proof of Claim 1: Part 1 is immediate. We prove the second part. Let $(\underline{\sigma}, \underline{\xi}, \underline{p})$ denote the equilibrium considered.

Step 1: $v(-1, 0) < v(0, 0)$. Let $\beta(r, s)$ denotes a trader's updated belief of the expert type after observing (r, s) in the current period. We have:

$$\lambda^e(0, \underline{\sigma}, \underline{\xi}) = \lambda^e(-1, \underline{\sigma}, \underline{\xi}) > \beta(-1, 0).$$

Step 1 now follows since

$$v(-1, 0) = \beta(-1, 0)\theta^G(-1) < \lambda^e(0, \underline{\sigma}, \underline{\xi})\theta^G(-1) = \beta(0, 0)\theta^G(-1) < \beta(0, 0)\theta^G = v(0, 0).$$

Step 2: $v(-1, 0) < v(1, 0)$. Suppose, for the sake of contradiction, that $v(-1, 0) \geq v(1, 0)$. Then $\beta(-1, 0) > \beta(1, 0)$, and so $\sigma^-/q^- < \sigma^+/q^+$. But $v(-1, 0) \geq v(1, 0)$ also implies (using Step 1) that $v(1, 0)$ is the lowest valuation, in which case $\xi^b(1, 0) = 1$. Since, by Part 1 of the Claim, $\xi^a(1, 1) = 1$, it is now easy to see²⁷ that we must have $\lambda^e(1, \underline{\sigma}, \underline{\xi}) < \lambda^e(-1, \underline{\sigma}, \underline{\xi})$. But this is impossible, in equilibrium. ■

Proof of Lemma 2: By Proposition 3, if screening breaks down in period t then a trader observing $(r_t, s_t) = (0, 0)$ learns nothing about the asset nor about the expert. Thus $v_t(0, 0) = p_t^0$ and, by Claim 1, this trader abstains in equilibrium and *a fortiori* plays no role in the determination of prices.²⁸

We work the rest of the proof for the case where $\theta_t^G(-1) > 0$ (other cases can be treated similarly). By Claim 1, in any equilibrium a (informed) trader with $(r_t, s_t) = (1, 1)$ buys the asset with probability 1 while a trader with $(r_t, s_t) = (-1, 0)$ sells the asset with probability 1. Thus, for screening to break down, a trader with $(r_t, s_t) = (1, 0)$ must buy with probability 1 while a trader with $(r_t, s_t) = (-1, 1)$ must sell with probability 1.

The first condition is consistent with equilibrium price setting if and only if

$$\frac{\lambda_t(1-\pi)q_t^+}{\lambda_t(1-\pi)q_t^+ + (1-\lambda_t)q_t^+} \cdot \theta_t^G(1) \geq \frac{\gamma q_t^+}{\gamma q_t^+ + 1 - \gamma} \cdot \lambda_t \theta_t^G(1) + \frac{1-\gamma}{\gamma q_t^+ + 1 - \gamma} \cdot \lambda_t \theta_t^G,$$

which is satisfied for $\lambda_t \geq \hat{\lambda}_t^+$, where

$$\hat{\lambda}_t^+ = 1 - \frac{1-\pi}{\pi} \cdot \frac{(1-\gamma) \left(1 - \frac{\theta_t^G}{\theta_t^G(1)}\right)}{\gamma q_t^+ + (1-\gamma) \frac{\theta_t^G}{\theta_t^G(1)}}.$$

The second condition is consistent with equilibrium price setting if and only if

$$\theta_t^G(-1) \leq \frac{\gamma q_t^-}{\gamma q_t^- + 1 - \gamma} \cdot \lambda_t \theta_t^G(-1) + \frac{1-\gamma}{\gamma q_t^- + 1 - \gamma} \cdot \lambda_t \theta_t^G,$$

²⁷The underlying arguments are similar to those used in the proofs of Claims 3 and 5. We do not repeat them.

²⁸Notice that by part 3 of Claim 1: $p_t^b < p_t^0 < p_t^a$.

which is satisfied for $\lambda_t \geq \hat{\lambda}_t^-$, where

$$\hat{\lambda}_t^- = 1 - \frac{(1 - \gamma) \left(1 - \frac{\theta_t^G(-1)}{\theta_t^G}\right)}{\gamma q_t^- \frac{\theta_t^G(-1)}{\theta_t^G} + 1 - \gamma}.$$

Setting $\hat{\lambda}_t = \max\{\hat{\lambda}_t^+, \hat{\lambda}_t^-\}$, the previous arguments thus establish that an equilibrium exists where screening breaks down if and only if $\lambda_t \geq \hat{\lambda}_t$. ■

We next state and prove a series of claims in view of establishing Lemma 3. In these claims we take t and \mathcal{H}_t given and fixed. In particular, this allows us to drop time subscripts. Furthermore, we let $\lambda^e(r, \underline{\sigma}, \underline{\xi})$ denote a bad expert's expected reputation at the beginning of next period from playing r , when his strategy is $\underline{\sigma}$ and informed traders behave according to $\underline{\xi}$. Thus, in equilibrium:

$$\lambda^e(-1, \underline{\sigma}, \underline{\xi}) = \lambda^e(0, \underline{\sigma}, \underline{\xi}) = \lambda^e(1, \underline{\sigma}, \underline{\xi}). \quad (1)$$

Claim 2 then proves that if in two equilibria the expert has the same strategy, then prices also must be the same in the the two equilibria. Claims 3-4 remark that more screening leads to lower expected reputation, and Claim 5 that a more aggressive behavior from a bad expert lowers his corresponding expected reputation. Finally, Claim 6 links expert strategy, prices and screening.

Claim 2. *Let $(\underline{\sigma}_1, \underline{\xi}_1, \underline{p}_1)$ and $(\underline{\sigma}_2, \underline{\xi}_2, \underline{p}_2)$ be two equilibria such that $\underline{\sigma}_1 = \underline{\sigma}_2$. Then $\underline{p}_1 = \underline{p}_2$.*

Proof of Claim 2: The strategy $\underline{\sigma}$ determines the valuations of all traders. For $z \in [-1, 1]$, and assuming that *given the prices all informed traders play a best response*, the functions

$$\begin{aligned} h_{\underline{\sigma}}^a(z) &= \mathbb{E}[\theta | y = a, p^a = z, \underline{\sigma}] \\ h_{\underline{\sigma}}^b(z) &= \mathbb{E}[\theta | y = b, p^b = z, \underline{\sigma}] \end{aligned}$$

are well-defined.

Note that the function $h_{\underline{\sigma}}^a$ is strictly increasing for z such that $h_{\underline{\sigma}}^a(z) > z$, and strictly decreasing for z such that $h_{\underline{\sigma}}^a(z) < z$. Moreover, $h_{\underline{\sigma}}^a(p^0) > p^0$ while $h_{\underline{\sigma}}^a(1) < 1$. Hence there exists a unique z such that $h_{\underline{\sigma}}^a(z) = z$; this is the competitive ask price.

Similarly, the function $h_{\underline{\sigma}}^b$ is strictly decreasing for z such that $h_{\underline{\sigma}}^b(z) < z$, and strictly increasing for z such that $h_{\underline{\sigma}}^b(z) > z$. Moreover, $h_{\underline{\sigma}}^b(p^0) < p^0$ while $h_{\underline{\sigma}}^b(-1) > -1$. Hence, there exists a unique z such that $h_{\underline{\sigma}}^b(z) = z$; this is the competitive bid price. ■

Claim 3. Consider $\underline{\sigma}$ such that $\sigma^r > 0, \forall r$.

1. If $\underline{\xi}_1$ entails a break-down of screening on the positive side then $\lambda^e(1, \underline{\sigma}, \underline{\xi}_1) \geq \lambda^e(1, \underline{\sigma}, \underline{\xi}_2)$, with strict inequality unless $\underline{\xi}_2$ entails a break-down of screening on the positive side too.
2. If $\underline{\xi}_1$ entails a break-down of screening on the negative side then $\lambda^e(-1, \underline{\sigma}, \underline{\xi}_1) \geq \lambda^e(-1, \underline{\sigma}, \underline{\xi}_2)$, with strict inequality unless $\underline{\xi}_2$ entails a break-down of screening on the negative side too.

Proof of Claim 3: Let, for $\mathbb{P}(y|r, \underline{\xi}, B) > 0$:

$$L(y|r, \underline{\xi}) = \frac{\mathbb{P}(y|r, \underline{\xi}, G)}{\mathbb{P}(y|r, \underline{\xi}, B)}.$$

Note that

$$\mathbb{E}[L(y|r, \underline{\xi})|r, \underline{\xi}, B] = \sum_{y: \mathbb{P}(y|r, \underline{\xi}, B) > 0} \mathbb{P}(y|r, \underline{\xi}, B) \frac{\mathbb{P}(y|r, \underline{\xi}, G)}{\mathbb{P}(y|r, \underline{\xi}, B)} = 1.$$

Using Bayes' rule:

$$\lambda^e(r, \underline{\sigma}, \underline{\xi}) = \mathbb{E}[M_{\sigma^r}(L(y|r, \underline{\xi}))|r, \underline{\xi}, B], \quad (2)$$

where $M_{\sigma^r}(x) = \frac{\lambda q^r x}{\lambda q^r x + (1-\lambda)\sigma^r}$ is concave.

Observe next that if $\underline{\xi}_1$ entails a break-down of screening on the positive side then $L(y|r = 1, \underline{\xi}_1) = 1$ for all y . So either $\underline{\xi}_2$ entails a break-down of screening on the positive side too or, conditional on the expert being bad, the distribution of $L(y|r = 1, \underline{\xi}_2)$ is a mean-preserving spread of the distribution of $L(y|r = 1, \underline{\xi}_1)$. In the latter case we obtain, by (2) and concavity of M_{σ^r} : $\lambda^e(1, \underline{\sigma}, \underline{\xi}_2) < \lambda^e(1, \underline{\sigma}, \underline{\xi}_1)$.

The proof of Part 2 of the Claim is similar, and omitted. ■

Claim 4. Let $r, r' \in \{-1, 1\}$. If for $y \neq \hat{y}$ we have $\xi^y(r, 0) = \xi^{\hat{y}}(r, 1) = 1$ and moreover $\frac{\sigma^r}{q^r} > \frac{\sigma^{r'}}{q^{r'}}$, then $\lambda^e(r', \underline{\sigma}, \underline{\xi}) > \lambda^e(r, \underline{\sigma}, \underline{\xi})$.

Proof of Claim 4: The proof is similar to that of Claim 3 and is therefore omitted.

■

Claim 5. *If $\xi_{\underline{1}}$ entails a break-down of screening then for any $\xi_{\underline{2}}$:*

1. $\sigma_2^+ > \sigma_1^+ \Rightarrow \lambda^e(1, \underline{\sigma}_1, \xi_{\underline{1}}) > \lambda^e(1, \underline{\sigma}_2, \xi_{\underline{2}})$.
2. $\sigma_2^- > \sigma_1^- \Rightarrow \lambda^e(-1, \underline{\sigma}_1, \xi_{\underline{1}}) > \lambda^e(-1, \underline{\sigma}_2, \xi_{\underline{2}})$.

Proof of Claim 5: Using the same notation as in the proof of Claim 3:

$$\begin{aligned}
 \lambda^e(1, \underline{\sigma}_2, \xi_{\underline{2}}) &= \mathbb{E}[M_{\sigma_2^+}(L(y|r = 1, \xi_{\underline{2}})) | r = 1, \xi_{\underline{2}}, B] \\
 &< \mathbb{E}[M_{\sigma_1^+}(L(y|r = 1, \xi_{\underline{2}})) | r = 1, \xi_{\underline{2}}, B] \\
 &= \lambda^e(1, \underline{\sigma}_1, \xi_{\underline{2}}) \\
 &\leq \lambda^e(1, \underline{\sigma}_1, \xi_{\underline{1}}).
 \end{aligned}$$

The first inequality follows from the fact that $M_{\sigma_2^+}(x) < M_{\sigma_1^+}(x)$ for all x . The last inequality is an application of Claim 3.

The proof of Part 2 of the Claim is similar, and omitted.

■

Let $\Delta\sigma^r = \sigma_2^r - \sigma_1^r$. Then

Claim 6. *Consider asymmetric timing of information acquisition. Assume $\theta^G(-1) > 0$. Let $(\underline{\sigma}_1, \xi_{\underline{1}}, \underline{p}_1)$ and $(\underline{\sigma}_2, \xi_{\underline{2}}, \underline{p}_2)$ be two equilibria. If $\Delta\sigma^- < 0$, $\Delta\sigma^+ \geq 0$ and $\Delta\sigma^0 > 0$, then $p_1^b \leq p_2^b$. Furthermore, if $p_1^b = p_2^b$ then $\xi_2^b(0, 0) = \xi_2^b(1, 0) = 1$.*

Proof of Claim 6: Denote the valuations in equilibrium $(\underline{\sigma}_i, \xi_{\underline{i}}, \underline{p}_i)$ by $v_i(\cdot)$, for $i = 1, 2$. Either Equilibrium 2 preserves the ordering of traders' valuations from Equilibrium 1 or it does not. Call these Case 1 and Case 2, respectively.

Case 1: Using notation from the proof of Claim 2:

$$\begin{aligned}
p_1^b &= h_{\underline{\sigma}_1}^b(p_1^b) \\
&= \frac{\gamma\lambda \sum_{(r,s)} \xi_1^b(r,s) q^r (1-\pi)^{1-s} \pi^s \theta^G(r) + (1-\gamma)\lambda \theta^G}{\gamma[\lambda \sum_{(r,s)} \xi_1^b(r,s) q^r (1-\pi)^{1-s} \pi^s + (1-\lambda)(\xi_1^b(-1,0)\sigma_1^- + \xi_1^b(0,0)\sigma_1^0 + \xi_1^b(1,0)\sigma_1^+)] + (1-\gamma)} \\
&= \min_{z \in [-1,1]} h_{\underline{\sigma}_1}^b(z) \\
&\leq \frac{\gamma\lambda \sum_{(r,s)} \xi_2^b(r,s) q^r (1-\pi)^{1-s} \pi^s \theta^G(r) + (1-\gamma)\lambda \theta^G}{\gamma[\lambda \sum_{(r,s)} \xi_2^b(r,s) q^r (1-\pi)^{1-s} \pi^s + (1-\lambda)(\xi_2^b(-1,0)\sigma_1^- + \xi_2^b(0,0)\sigma_1^0 + \xi_2^b(1,0)\sigma_1^+)] + (1-\gamma)} \\
&\leq \frac{\gamma\lambda \sum_{(r,s)} \xi_2^b(r,s) q^r (1-\pi)^{1-s} \pi^s \theta^G(r) + (1-\gamma)\lambda \theta^G}{\gamma[\lambda \sum_{(r,s)} \xi_2^b(r,s) q^r (1-\pi)^{1-s} \pi^s + (1-\lambda)(\xi_2^b(-1,0)\sigma_2^- + \xi_2^b(0,0)\sigma_2^0 + \xi_2^b(1,0)\sigma_2^+)] + (1-\gamma)} \\
&= h_{\underline{\sigma}_2}^b(p_2^b) \\
&= p_2^b.
\end{aligned}$$

The first inequality is due to the fact that, since equilibrium 2 preserves the ordering of traders' valuations from equilibrium 1:

$$\frac{\gamma\lambda \sum_{(r,s)} \xi_2^b(r,s) q^r (1-\pi)^{1-s} \pi^s \theta^G(r) + (1-\gamma)\lambda \theta^G}{\gamma[\lambda \sum_{(r,s)} \xi_2^b(r,s) q^r (1-\pi)^{1-s} \pi^s + (1-\lambda)(\xi_2^b(-1,0)\sigma_1^- + \xi_2^b(0,0)\sigma_1^0 + \xi_2^b(1,0)\sigma_1^+)] + (1-\gamma)} = h_{\underline{\sigma}_1}^b(z),$$

for some $z \in [-1, 1]$.

The last inequality follows from $\xi_2^b(-1, 0) = 1$ (Claim 1), the observation that $\sum_r \Delta\sigma^r = 0$, and the sign assumptions on $\Delta\sigma^r$:

$$\begin{aligned}
\xi_2^b(-1,0)\Delta\sigma^- + \xi_2^b(0,0)\Delta\sigma^0 + \xi_2^b(1,0)\Delta\sigma^+ &= \Delta\sigma^- + \xi_2^b(0,0)\Delta\sigma^0 + \xi_2^b(1,0)\Delta\sigma^+ \\
&\leq \Delta\sigma^- + \xi_2^b(0,0)\Delta\sigma^0 + \Delta\sigma^+ \\
&= (\xi_2^b(0,0) - 1)\Delta\sigma^0 \\
&\leq 0.
\end{aligned}$$

In particular, a necessary condition for $p_1^b = p_2^b$ is that $\xi_2^b(0,0) = \xi_2^b(1,0) = 1$.

Case 2: Similar to Case 1, except that the first inequality in the sequence is now a strict inequality since in this case informed traders' behavior $\underline{\xi}_2$ is "inefficient" given the valuations from equilibrium 1.

■

Proof of Lemma 3: Fix t and \mathcal{H}_t . In particular, this allows us to drop time subscripts. As usual we work the proof for the case where $\theta_t^G(-1) > 0$. Other cases can be treated similarly.

Let $(\underline{\sigma}_1, \underline{\xi}_1, \underline{p}_1)$ denote the equilibrium where screening breaks down ($E1$), and $(\underline{\sigma}_2, \underline{\xi}_2, \underline{p}_2)$ some arbitrary equilibrium ($E2$). We will show that $(\underline{\sigma}_2, \underline{\xi}_2, \underline{p}_2) = (\underline{\sigma}_1, \underline{\xi}_1, \underline{p}_1)$.

Step 1: $\underline{\sigma}_2 = \underline{\sigma}_1$. We consider 3 Cases.

Case 1: $\sigma_2^+ > \sigma_1^+$. Applying Claim 5 yields $\lambda^e(1, \underline{\sigma}_2, \underline{\xi}_2) < \lambda^e(1, \underline{\sigma}_1, \underline{\xi}_1)$. Hence (1) implies $\lambda^e(r, \underline{\sigma}_2, \underline{\xi}_2) < \lambda^e(r, \underline{\sigma}_1, \underline{\xi}_1)$ for all r . Then $\sigma_2^0 > \sigma_1^0$, and thus $\sigma_2^- < \sigma_1^-$. Applying Claim 5 again, we must have $\xi_2^b(-1, 1) < 1$, since otherwise $\lambda^e(-1, \underline{\sigma}_2, \underline{\xi}_2) > \lambda^e(-1, \underline{\sigma}_1, \underline{\xi}_1)$, a contradiction.²⁹

Applying Claim 6 gives $p_2^b \geq p_1^b$. This together with $\xi_2^b(-1, 1) < \xi_1^b(-1, 1) = 1$ yields $v_2(-1, 1) \geq p_2^b \geq p_1^b \geq v_1(-1, 1)$. Since $v_2(-1, 1) = v_1(-1, 1)$, we have $p_2^b = p_1^b$. Then from Claims 1 and 6: $\xi_2^b(1, 0) = \xi_2^a(1, 1) = 1$. Furthermore: $\frac{\sigma_2^+}{q^+} > \frac{\sigma_1^+}{q^+} = 1 = \frac{\sigma_1^-}{q^-} > \frac{\sigma_2^-}{q^-}$.³⁰ Applying Claim 4 then gives $\lambda^e(1, \underline{\sigma}_2, \underline{\xi}_2) < \lambda^e(-1, \underline{\sigma}_2, \underline{\xi}_2)$, again a contradiction with (1).

Case 2: $\sigma_2^+ < \sigma_1^+$. Thus: $\lambda^e(1, \underline{\sigma}_2, \underline{\xi}_2) < \lambda^e(1, \underline{\sigma}_1, \underline{\xi}_1)$. Otherwise, (1) and Claim 5 would yield $\sigma_2^0 \leq \sigma_1^0$ and $\sigma_2^- \leq \sigma_1^-$, a contradiction. Applying Claim 5 again: $\xi_2^a(1, 0) < 1$.³¹ And by (1): $\sigma_2^0 > \sigma_1^0$.

It follows that $v_2(1, 0) \leq p_2^a$, whereas $v_1(1, 0) \geq p_1^a$. Since $v(1, 0)$ is decreasing in σ^+ , then $v_2(1, 0) > v_1(1, 0)$, implying $p_2^a > p_1^a$. Recall that p_i^a is a convex combination of $v_i(1, 1)$, $v_i(1, 0)$, $v_i(0, 0)$, $v_i(-1, 1)$ and p^0 .³² Let \tilde{p}_i^a denote the fictitious ask price obtained from removing traders observing $(r, s) = (1, 0)$. Thus: $\tilde{p}_2^a \geq p_2^a > p_1^a \geq \tilde{p}_1^a$. Notice that \tilde{p}_i^a is decreasing in σ_i^0 and does not depend on $\underline{\sigma}_i$ in any other way. Thus $\tilde{p}_2^a < \tilde{p}_1^a$, a contradiction.

Case 3: $\sigma_2^+ = \sigma_1^+$. Rule out $\sigma_2^- < \sigma_1^-$ as in Case 1. Now suppose $\sigma_2^- > \sigma_1^-$. Then $\sigma_2^0 < \sigma_1^0$, and hence $\lambda^e(0, \underline{\sigma}_2, \underline{\xi}_2) > \lambda^e(0, \underline{\sigma}_1, \underline{\xi}_1)$. But that is impossible because, applying Claim 5, we must have $\lambda^e(1, \underline{\sigma}_2, \underline{\xi}_2) \leq \lambda^e(1, \underline{\sigma}_1, \underline{\xi}_1)$, a contradiction with (1).

²⁹In particular, if $\xi_2^b(-1, 1) = 1$ then $E2$ would have a break-down of screening on the negative side. $E2$ would then play the role of the equilibrium with break-down of screening in Claim 5.

³⁰That $\sigma_1^+/q^+ = 1 = \sigma_1^-/q^-$ follows from Proposition 3 below, as screening breaks down in $E1$.

³¹If $\xi_2^a(1, 0) = 1$, screening would break down on the positive side: $\lambda^e(1, \underline{\sigma}_2, \underline{\xi}_2) > \lambda^e(1, \underline{\sigma}_1, \underline{\xi}_1)$.

³²By Claim 1, $v_i(-1, 0) < p_i^a$ and hence does not enter the ask price.

Step 2: $\underline{p}_2 = \underline{p}_1$. Follows from Step 1 together with Claim 2.

Step 3: $\underline{\xi}_2 = \underline{\xi}_1$. From Step 1, we have $\lambda^e(0, \underline{\sigma}_2, \underline{\xi}_2) = \lambda^e(0, \underline{\sigma}_1, \underline{\xi}_1)$. Applying Claim 3 then immediately establishes that in $E2$ screening must break down on both sides. That $\xi_2^n(0, 0) = \xi_1^n(0, 0) = 1$ follows from noting that $p_i^b < p^0 = v_i(0, 0) < p_i^a$, $i \in \{1, 2\}$. ■

Proof of Proposition 3: First notice that if screening breaks down in period t and a bad expert mimics a good one, then $\lambda_{t+1}^e(r_t) = \lambda_t$ for all r_t . Mimicking is thus a best response if screening breaks down in period t . In fact, by Claim 5, it is the unique best response. This establishes the first part of the Proposition.

Next, suppose that screening does *not* break down in period t and $\sigma_t \geq q_t$. Arguments similar to those developed in the proof of Claim 3 then establish that either $\lambda_{t+1}^e(1) < \lambda_t$ or $\lambda_{t+1}^e(-1) < \lambda_t$ (or both). Either way, we obtain a contradiction since $\sigma_t \geq q_t \Rightarrow \sigma_t^0 \leq q_t^0 \Rightarrow \lambda_{t+1}^e(0) \geq \lambda_t$. ■

Proof of Proposition 4: If a complete informational cascade occurs in period t then $p_t^b = p_t^0 = p_t^a$. By Claim 1 the bid-ask spread must be strictly positive unless (possibly) $\lambda_t = 0$. But this is ruled out, since in that case there would have to be $t' < t$ such that $\lambda_{t'} > 0$ and $\lambda_{t'+1} = 0$. Clearly no outcome $(r_{t'}, y_{t'})$ would ever induce this in equilibrium. ■

Proof of Theorem 1: For Part 1, notice that a break-down of screening is a necessary condition for a reputational cascade to occur. But, by Proposition 1, screening is always efficient under symmetric timing of information acquisition.

We next prove Part 2 of the Theorem. By Proposition 3, a break-down of screening in period t implies $\underline{\sigma}_t = \underline{q}_t$ and thus $\mathbb{P}_t(r_t|\tau) = \mathbb{P}_t(r_t)$, for all r_t . If the trader in period t is a liquidity trader, his action is unaffected by r_t and a fortiori also unaffected by the type τ . If he is an informed trader then his action is unaffected by s_t – by definition of screening break-down – but may be affected by r_t . Yet, since $\underline{\sigma}_t = \underline{q}_t$, an informed trader's action must be unaffected by the type τ . This shows that if screening breaks down in equilibrium in period t , then $\mathbb{P}_t(r_t, y_t|\tau) = \mathbb{P}_t(r_t, y_t)$ for all r_t, y_t . Hence, if we can show that screening breaks down

for some time t and history \mathcal{H}_t , where the probability of \mathcal{H}_t occurring is non-zero, then we are done.

Using Proposition 2, observe that for any time t and history \mathcal{H}_t there exists a threshold $\hat{\lambda}_t < 1$ such that screening breaks down in equilibrium in period t if $\lambda_t > \hat{\lambda}_t$. So we only need to show that some history \mathcal{H}_t such that $\lambda_t > \hat{\lambda}_t$ in fact occurs with positive probability.

Fix λ_t . Notice that choosing θ_t^G sufficiently close to 1 guarantees the following sequence of inequalities:

$$v_t(-1, 0) < v_t(1, 0) < p_t^0 < v_t(-1, 1) < v_t(1, 1).$$

So choosing θ_t^G sufficiently close to 1 guarantees efficient screening in equilibrium.³³ This in turn implies that we can find (r_t, y_t) such that $\mathbb{P}_t(r_t, y_t | \tau) > 0$ for $\tau = B, G$ and $\lambda_{t+1}(r_t, y_t) \geq g^1(\lambda_t)$, where

$$g^n(w) = \frac{w(\gamma\pi + (1 - \gamma))^n}{w(\gamma\pi + (1 - \gamma))^n + (1 - w)(1 - \gamma)^n}.$$

Notice that $g^n(w) \xrightarrow{n \rightarrow \infty} 1$, for all $w \in (0, 1)$. In particular, for any \underline{w} and \bar{w} , $0 < \underline{w} < \bar{w} < 1$, there exists $n(\underline{w}, \bar{w}) \in \mathbb{N}$ such that

$$g^{n(\underline{w}, \bar{w})}(\underline{w}) > \bar{w}.$$

We can now show that, starting from $\lambda_0 > 0$, in equilibrium an expert (whichever his type) can in finite time achieve any desired level of reputation $\tilde{\lambda} \in (0, 1)$ with strictly positive probability. First, accumulate positive reports inducing θ^G sufficiently close to 1 that, for all $\lambda < \tilde{\lambda}$, efficient screening is guaranteed in equilibrium. Note that this can be done whichever the type of the expert, given Claim 7, and moreover can be done all the while maintaining at least reputation $\lambda_0 > 0$. If at that point reputation is above $\tilde{\lambda}$ then we are done. Otherwise, increasing reputation during at most $n(\lambda_0, \tilde{\lambda})$ consecutive periods induces a reputation at least equal to $g^{n(\lambda_0, \tilde{\lambda})} > \tilde{\lambda}$.

Finally, let $\hat{\lambda}_0$ denote the threshold required for screening to break down given $\mathcal{R}_t = \emptyset$. If $\lambda_0 \geq \hat{\lambda}_0$ then a reputational cascade occurs at $t = 0$ and the statement of the Theorem is trivial. Consider next $\lambda_0 > \hat{\lambda}_0$. By the previous step we know that $\hat{\lambda}_0$ can be achieved in finite time T . Let \mathcal{R}_{T-1} denote the sequence of reports accumulated at that point, and \mathcal{R}_{2T-1} an arbitrary sequence of reports such that $\theta_{2T-1}^G = 0$.³⁴ Again applying Claim 7 then, starting from time T , \mathcal{R}_{2T-1} is generated with positive probability, independently of the expert's type.

³³Recall that in this case $p_t^b < p_t^0 < p_t^a$.

³⁴One such sequence is given by $r_t = r_{t-T}$, for $t \in \{T, T+1, \dots, 2T-1\}$.

Moreover – once again – this can be done while maintaining at least reputation λ_T , in which case: $\lambda_{2T-1} > \hat{\lambda}_0$. The proof is now concluded by noting that the threshold prevailing at that point is $\hat{\lambda}_{2T-1} = \hat{\lambda}_0 < \lambda_{2T-1}$. A reputational cascade thus ensues. ■

Before proving Theorem 2, we prove a series of claims. Claim 7 establishes a lower bound on a bad expert’s propensity to publish reports. Claim 8 deduces that an infinite number of reports is published almost surely. Claim 9 lower-bounds the probability that a significant jump in reputation occurs, whenever reputation has not converged to either zero or one. Claim 10 shows that reputation eventually will converge to either zero or one, almost surely. Finally, Claim 11 concludes that reputation converges to the ‘correct’ value. With these results in hand, we then prove the theorem.

Claim 7. *There exists $\delta > 0$ such that in any equilibrium and for any history \mathcal{H}_t and r then $\sigma_t^r > \delta$.*

Proof of Claim 7: Notice that due to liquidity traders we can find $\ell > 0$ such that for any r_t, y_t and $\underline{\xi}_t$ then (using notation from Claim 3):

$$\ell^{-1} < L_t(y_t|r_t, \underline{\xi}_t) < \ell.$$

Note also that for any history \mathcal{H}_t and any report r_t then $q_t^{r_t} \in [\frac{1-\phi}{2}, \frac{1}{2}]$. Thus for any \mathcal{H}_t, r_t, y_t and $\underline{\xi}_t$:

$$\frac{\mathbb{P}_t(y_t, r_t|\underline{\sigma}_t, \underline{\xi}_t, G)}{\mathbb{P}_t(y_t, r_t|\underline{\sigma}_t, \underline{\xi}_t, B)} = \frac{q_t^{r_t}\mathbb{P}_t(y_t|r_t, \underline{\xi}_t, G)}{\sigma_t^{r_t}\mathbb{P}_t(y_t|r_t, \underline{\xi}_t, B)} > \frac{(1-\phi)\ell^{-1}}{2\sigma_t^{r_t}}.$$

Hence:

$$\sigma_t^{r_t} < \frac{1-\phi}{2}\ell^{-1} \Rightarrow \frac{\mathbb{P}_t(y_t, r_t|\underline{\sigma}_t, \underline{\xi}_t, G)}{\mathbb{P}_t(y_t, r_t|\underline{\sigma}_t, \underline{\xi}_t, B)} > 1.$$

The claim now follows by setting $\delta = \frac{1-\phi}{2}\ell^{-1}$, since if a bad expert were ever to issue report r_t with less than probability δ , then he could, by publishing r_t , increase his reputation with certainty. But we know by Proposition 3 that this is impossible in equilibrium. ■

Claim 8. *In any equilibrium, an infinite number of reports is published a.s..*

Proof of Claim 8: Follows from Claim 7 and applying the first Borel-Cantelli Lemma. ■

Claim 9. For all $\tilde{\lambda} \in (0, 1)$, there exists $\varepsilon_{\tilde{\lambda}} > 0$ and $\delta_{\tilde{\lambda}} > 0$ with the following property: if $|\lambda_t - \tilde{\lambda}| < \delta_{\tilde{\lambda}}$ and $(\underline{\sigma}_t, \underline{\xi}_t, \underline{p}_t)$ denotes an equilibrium in period t where screening is efficient, then

$$\mathbb{P}_t(|\lambda_{t+1} - \lambda_t| > \varepsilon_{\tilde{\lambda}}) \geq \frac{1}{2}.$$

Proof of Claim 9: Let $\tilde{\lambda} \in (0, 1)$ and define the functions $G_{\tilde{\lambda}}^1$ and $G_{\tilde{\lambda}}^2$ by

$$\begin{aligned} G_{\tilde{\lambda}}^1(u, v; x) := & \frac{\tilde{\lambda}u}{\tilde{\lambda}u + (1 - \tilde{\lambda})} - \left[\frac{1 - \mu}{3} \cdot \frac{\tilde{\lambda}v(\gamma\pi + (1 - \gamma))}{\tilde{\lambda}v(\gamma\pi + (1 - \gamma)) + (1 - \tilde{\lambda})(1 - \gamma)} \right. \\ & + \left(\frac{1 - \mu}{3} + \mu x \right) \cdot \frac{\tilde{\lambda}v(\gamma(1 - \pi)x + (1 - \gamma))}{\tilde{\lambda}v(\gamma(1 - \pi)x + (1 - \gamma)) + (1 - \tilde{\lambda})((1 - \gamma) + \gamma x)} \\ & \left. + \left(\frac{1 - \mu}{3} + \mu(1 - x) \right) \cdot \frac{\tilde{\lambda}v(\gamma(1 - \pi)(1 - x) + (1 - \gamma))}{\tilde{\lambda}v(\gamma(1 - \pi)(1 - x) + (1 - \gamma)) + (1 - \tilde{\lambda})((1 - \gamma) + \gamma(1 - x))} \right]; \end{aligned}$$

$$\begin{aligned} G_{\tilde{\lambda}}^2(u, v; x) := & \frac{\tilde{\lambda}u}{\tilde{\lambda}u + (1 - \tilde{\lambda})} - \left[\frac{1 - \mu}{3} \cdot \frac{\tilde{\lambda}v(\gamma\pi x + (1 - \gamma))}{\tilde{\lambda}v(\gamma\pi x + (1 - \gamma)) + (1 - \tilde{\lambda})(1 - \gamma)} \right. \\ & + \frac{1 - \mu}{3} \cdot \frac{\tilde{\lambda}v(\gamma\pi(1 - x) + (1 - \gamma))}{\tilde{\lambda}v(\gamma\pi(1 - x) + (1 - \gamma)) + (1 - \tilde{\lambda})(1 - \gamma)} \\ & \left. + \left(\frac{1 - \mu}{3} + \mu \right) \cdot \frac{\tilde{\lambda}v(\gamma(1 - \pi) + (1 - \gamma))}{\tilde{\lambda}v(\gamma(1 - \pi) + (1 - \gamma)) + (1 - \tilde{\lambda})} \right]. \end{aligned}$$

For $i = 1, 2$, $G_{\tilde{\lambda}}^i(u, v; x)$ is continuous, increasing in u , decreasing in v , and using arguments similar to those of Claim 3: $G_{\tilde{\lambda}}^i(1, 1; x) > 0$, for all $x \in [0, 1]$. Let $B((u, v), \eta)$ be the η -ball around the point (u, v) . In particular, we can thus find a radius $\eta_{\tilde{\lambda}}$ such that $(u, v) \in B((1, 1), \eta_{\tilde{\lambda}}) \Rightarrow G_{\tilde{\lambda}}^i(u, v; x) > 0$, $i = 1, 2$.

Next, notice that if $\lambda_t = \tilde{\lambda}$ and $(\underline{\sigma}_t, \underline{\xi}_t, \underline{p}_t)$ denotes an equilibrium in period t where screening is efficient then equilibrium conditions imply that we can find $x^+, x^- \in [0, 1]$ with $G_{\tilde{\lambda}}^1\left(\frac{q_t^0}{\sigma_t^0}, \frac{q_t^+}{\sigma_t^+}; x^+\right) = 0$ and $G_{\tilde{\lambda}}^2\left(\frac{q_t^0}{\sigma_t^0}, \frac{q_t^-}{\sigma_t^-}; x^-\right) = 0$, or vice versa.³⁵ Thus, by the previous step: $\left(\frac{q_t^0}{\sigma_t^0}, \frac{q_t^+}{\sigma_t^+}\right) \notin B((1, 1), \eta_{\tilde{\lambda}})$ and $\left(\frac{q_t^0}{\sigma_t^0}, \frac{q_t^-}{\sigma_t^-}\right) \notin B((1, 1), \eta_{\tilde{\lambda}})$.

³⁵We place ourselves here in the case $\theta_t^G(-1) > 0$. By Claim 1, traders observing $(r_t, s_t) = (1, 1)$ buy with probability 1 and traders observing $(r_t, s_t) = (-1, 0)$ sell with probability 1. This implies that when screening

By proposition 3, either (a) one of $\frac{q_t^+}{\sigma_t^+}$ and $\frac{q_t^-}{\sigma_t^-}$ is lesser or equal than 1 or (b) both are strictly greater than 1:

Case (a): Suppose that $\frac{q_t^+}{\sigma_t^+} \leq 1$ (the case $\frac{q_t^-}{\sigma_t^-} \leq 1$ is similar). Then, for all $x \in [0, 1]$: $G_{\tilde{\lambda}}^1(\frac{q_t^0}{\sigma_t^0}, \frac{q_t^+}{\sigma_t^+}; x) \geq G_{\tilde{\lambda}}^1(\frac{q_t^0}{\sigma_t^0}, 1; x)$. In particular: $\frac{q_t^0}{\sigma_t^0} > 1 - \eta_{\tilde{\lambda}} \Rightarrow G_{\tilde{\lambda}}^1(\frac{q_t^0}{\sigma_t^0}, \frac{q_t^+}{\sigma_t^+}; x^+) \geq G_{\tilde{\lambda}}^1(\frac{q_t^0}{\sigma_t^0}, 1; x^+) > 0$. But this contradicts the equilibrium condition. Hence $\frac{q_t^0}{\sigma_t^0} \leq 1 - \eta_{\tilde{\lambda}}$.

Case (b): We have in this case $|\sigma_t^0 - q_t^0| = |\sigma_t^+ - q_t^+| + |\sigma_t^- - q_t^-|$. Since moreover $q_t^r \in [\frac{1-\phi}{2}, \frac{1}{2}]$, for all r , we can find $\eta'_{\tilde{\lambda}} > 0$ such that $(\frac{q_t^0}{\sigma_t^0}, \frac{q_t^+}{\sigma_t^+}) \notin B((1, 1), \eta_{\tilde{\lambda}}) \Rightarrow |\frac{q_t^0}{\sigma_t^0} - 1| \geq \eta'_{\tilde{\lambda}}$.

Now define $\eta''_{\tilde{\lambda}} := \min\{\eta_{\tilde{\lambda}}, \eta'_{\tilde{\lambda}}\}$. We then have, following publication of report $r_t = 0$:

$$\begin{aligned} |\lambda_{t+1} - \lambda_t| &= \left| \frac{\tilde{\lambda} \frac{q_t^0}{\sigma_t^0}}{\tilde{\lambda} \frac{q_t^0}{\sigma_t^0} + (1 - \tilde{\lambda})} - \tilde{\lambda} \right| \\ &\geq \left| \frac{\tilde{\lambda}(1 - \eta''_{\tilde{\lambda}})}{\tilde{\lambda}(1 - \eta''_{\tilde{\lambda}}) + (1 - \tilde{\lambda})} - \tilde{\lambda} \right| \\ &:= 2\varepsilon_{\tilde{\lambda}}. \end{aligned}$$

The claim now follows by continuity of $G_{\tilde{\lambda}}(u, v; x)$ in all of its variables, including $\tilde{\lambda}$. ■

The sequences (λ_t) and (θ_t^G) are bounded martingales with respect to the filtration (\mathcal{H}_t) . The Martingale Convergence Theorem thus applies. We let in what follows the random variable λ denote the (a.s.) limit of the sequence (λ_t) and θ^G the (a.s.) limit of (θ_t^G) .

Claim 10. *Let $\lambda = \lim_{t \rightarrow \infty} \lambda_t$. Then $\lambda \in \{0, 1\}$ a.s..*

Proof of Claim 10: We prove the claim for asymmetric timing of information acquisition (the proof for the symmetric case is almost identical, and therefore omitted).

Let W denote the event $\lambda \notin \{0, 1\}$. Note that by Claim 8 then θ_t^G must converge to either 1 or to -1 almost surely. Furthermore, if $\lambda \in (0, 1)$ and θ_t^G tends to 1 we obtain, for t large enough:

$$v_t(-1, 0) < v_t(1, 0) < p_t^0 < v_t(-1, 1) < v_t(1, 1).$$

is efficient traders observing $(r_t, s_t) = (1, 0)$ randomize between two actions at most (sell and abstain), as do traders observing $(r_t, s_t) = (-1, 1)$ (buy and abstain). The scalar x^+ parameterizes the strategy of traders observing $(r_t, s_t) = (1, 0)$. The scalar x^- parameterizes the strategy of traders observing $(r_t, s_t) = (-1, 1)$.

If $\lambda \in (0, 1)$ and θ_t^G tends to -1 we obtain similarly, for t large enough:

$$v_t(-1, 1) < v_t(1, 1) < p_t^0 < v_t(-1, 0) < v_t(1, 0).$$

Either way, this implies that screening eventually becomes efficient almost surely.

Now let $W(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2)$ denote the event $|\lambda - \tilde{\lambda}| < \delta_{\tilde{\lambda}}/2$, where $\tilde{\lambda} \in (0, 1)$ and $\delta_{\tilde{\lambda}}$ as defined in Claim 9. Clearly $W(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2) \subset W$. By the previous step, for any $\omega \in W(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2)$ we can (almost surely) define a smallest time $T(\omega)$ such that for all $s \geq T(\omega)$: (a) screening is efficient and (b) $|\lambda_s - \lambda| < \delta_{\tilde{\lambda}}/2$. Let $V_k = \{\omega : T(\omega) = k\}$. Then $W(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2) = \bigcup V_k$. Furthermore, applying Claim 9:

$$\mathbb{P}(|\lambda_{s+1} - \lambda_s| > \varepsilon_{\tilde{\lambda}} | V_k) \geq \frac{1}{2}, \quad s \geq k.$$

But (λ_t) converges almost surely. Hence, $\mathbb{P}(V_k) = 0$, for all k , and ultimately $\mathbb{P}(W(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2)) = 0$.

Let $B(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2)$ denote the open ball with center $\tilde{\lambda}$ and radius $\delta_{\tilde{\lambda}}/2$. For any n , the interval $[1/n, 1 - 1/n]$ has an open cover consisting of open balls $B(\tilde{\lambda}, \delta_{\tilde{\lambda}}/2)$, $\tilde{\lambda} \in [1/n, 1 - 1/n]$. By compactness, we can extract a finite sub-cover $\left(B(\tilde{\lambda}_s, \delta_{\tilde{\lambda}_s}/2)\right)_{s=1}^S$. Then

$$\mathbb{P}(\lambda \in [1/n, 1 - 1/n]) \leq \mathbb{P}\left(\lambda \in \bigcup B(\tilde{\lambda}_s, \delta_{\tilde{\lambda}_s}/2)\right) \leq \sum \mathbb{P}\left(\lambda \in B(\tilde{\lambda}_s, \delta_{\tilde{\lambda}_s}/2)\right) = 0.$$

This being true for all n , we finally obtain $\mathbb{P}(\lambda \in \{0, 1\}) = 1$. ■

Claim 11. *The market learns the expert type almost surely. Let $\lambda = \lim_{t \rightarrow \infty} \lambda_t$: if $\tau = G$ then $\lambda = 1$ a.s.; if $\tau = B$ then $\lambda = 0$ a.s.*

Proof of Claim 11: Consider first $\tau = B$. In any equilibrium:

$$\begin{aligned} \mathbb{E}_t \left[\frac{\lambda_{t+1}}{1 - \lambda_{t+1}} \middle| B \right] &= \sum_{r_{t+1}, y_{t+1}} \mathbb{P}_t(r_{t+1}, y_{t+1} | B) \cdot \frac{\lambda_t \mathbb{P}_t(r_{t+1}, y_{t+1} | G)}{(1 - \lambda_t) \mathbb{P}_t(r_{t+1}, y_{t+1} | B)} \\ &= \frac{\lambda_t}{1 - \lambda_t} \sum_{r_{t+1}, y_{t+1}} \mathbb{P}_t(r_{t+1}, y_{t+1} | G) \\ &= \frac{\lambda_t}{1 - \lambda_t}. \end{aligned}$$

Hence $\mathbb{E}\left[\frac{\lambda_t}{1-\lambda_t}\middle|B\right] = \frac{\lambda_0}{1-\lambda_0}$, for all t . Fatou's Lemma then gives $\mathbb{E}\left[\frac{\lambda}{1-\lambda}\middle|B\right] \leq \frac{\lambda_0}{1-\lambda_0}$. Hence $\mathbb{P}(\lambda = 1|B) = 0$.

Similar derivations give $\mathbb{E}\left[\frac{\lambda_t}{1-\lambda_t}\middle|G\right] = \frac{1-\lambda_0}{\lambda_0}$, for all t , and so $\mathbb{E}\left[\frac{1-\lambda}{\lambda}\middle|G\right] \leq \frac{1-\lambda_0}{\lambda_0}$. Hence $\mathbb{P}(\lambda = 0|G) = 0$.

Claim 11 now follows immediately, by application of Claim 10. ■

Proof of Theorem 2: If the expert is a good expert then the Law of Large Numbers yields $\lim_{t \rightarrow \infty} \theta_t^G = \theta$ a.s.. Both parts of the Theorem now follow from Claim 11. ■

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