



Three Essays on Corruption and Auctions

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Thesis submitted for assessment with a view to obtaining the degree
of Doctor of Economics of the European University Institute

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Department of Economics

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Abstract

This thesis contributes to the understanding of corruption and auctions. It consists of three chapters focusing on diverse aspects of these two general topics as well as their combinations, from an applied microeconomic theory perspective: (i) the effects of corruption on bidding behaviour in all-pay auctions and the auctioneer's decisions, (ii) the use of central bank bond auctions as tools to measure banks' liquidity risk, and (iii) the persistence of corruption and corruption differences between similar economies.

As discussed in recent bibliography, auctions performed by an intermediary between the seller of the good and buyers can be penetrable by corruption. Furthermore, corruption can enter auctions in different forms. In the first chapter of this thesis, entitled *Corruption in All-Pay Auctions*, we compare the effects of pure pecuniary corruption and favouritism on bidding behaviour and the auctioneer's expected revenue, in the context of All-Pay Auctions, used to model lobbying, labour-market tournaments and competition for monopoly power. We provide conditions under which favouritism makes bidders more or less aggressive than in the benchmark model without corruption, and prove that bidders are always more aggressive when faced with a non favouritist corrupt auctioneer. In both cases, the revenue maximizing auctioneer deprives his collaborator of all "surplus" of corruption. Finally, we study the auctioneer's choice of corruption type, and find that his expected revenue is not necessarily monotonic in the probability that he chooses one type of corruption or the other.

In the second chapter, entitled *Bond Auctions and Financial Sector Liquidity Risk*, a joint work with Grégory Claeys, we aim to provide a tool for central banks – and in particular for the Central Bank of Chile – to measure liquidity risk in their financial sector using the bidding behaviour of banks in bond auctions. First, we build a model combining the auction literature and the financial economics literature to understand precisely the effect of the liquidity risk affecting banks on their bidding strategies in those auctions. We develop a benchmark version of the model with no insurance against the liquidity shock, and another with a lender of last resort to see how the behavior of the banks is affected by this policy. Based on the revelation principle characterizing auctions, and using a unique dataset collected at Central Bank of Chile containing all the details of its open market operation auctions (where it sells bonds to drain money from the banking sector) between 2002 and 2012, we estimate the distribution of the liquidity risk across Chilean banks and its changes over time. The evolution of the estimated distribution seems to capture well the main episodes of liquidity stress of the last decade in the Chilean banking sector. This measuring tool could be used by other central banks conducting similar open market operations and in need of evaluating in real time the

evolution of the liquidity risk affecting their financial sector.

In the third chapter, entitled *Strategic Complementarities and Corruption*, we study an environment where agents compete against each other for the acquisition of a public good procurement project, assigned by the government and handled by a possibly corrupt inspector. We find that there exists multiplicity of equilibria, and in specific both "good" equilibria without corruption, and "bad" equilibria where corruption arises. This is very useful for us to interpret why countries that are quite similar in all other characteristics, can differ a lot in the level of corruption in their economy. Our result is consistent with recent bibliography on procurement, however multiplicity of equilibria in our model arises without any particular assumptions on the preferences of individuals. In our effort to confirm the multiplicity of equilibria also in the repeated game, we find that inspectors might consider it profitable to suffer negative payoffs in the first period of the game, in order to create more fuzziness as to how much corruption there is in the economy, and thus decrease the probability of getting caught for everyone, guaranteeing themselves bigger positive payoffs in their last period in the game.

*Στη Λαμία, την Αθήνα, τη Φλωρεντία, τη Ρέν, τη Μαδρίτη
Μα περισσότερο στους γονείς μου.*

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Preface

This thesis contributes to the understanding of corruption and auctions. It consists of three chapters focusing on diverse aspects of these two general topics as well as their combinations, from an applied microeconomic theory perspective: (i) the effects of corruption on bidding behaviour in all-pay auctions and the auctioneer's decisions, (ii) the use of central bank bond auctions as tools to measure banks' liquidity risk, and (iii) the persistence of corruption and corruption differences between similar economies. The first and third chapter of this thesis focus on corruption and its effects on the economy. The past forty years, policy makers and academics alike have shown deep interest in understanding corruption, as countries of every sort- developing, transition countries as well as major economies- have suffered corruption scandals. Most economists accept that corruption can occur where rents exist, usually as a result of government regulation, however many economic activities plagued by corruption are yet to be studied, like lobbying and labour market tournaments. Assessing the mechanism of corruption in All-Pay Auctions, representing such economic activities is one of the goals of this thesis, along with examining why corruption is a persistent phenomenon. The second chapter of the thesis concentrates on liquidity risk as the main determinant of banks' bidding strategies in Central Banks' bond auctions. Recent literature, especially after the financial turmoil starting in 2007 has aimed to provide measures of liquidity risk mostly based on banks' balance-sheet data, or market-based data. A third goal of this thesis is to provide a tool for central banks to measure liquidity risk in their financial sector using the bidding behavior of banks in bond auctions.

As discussed in recent bibliography, auctions performed by an intermediary between the seller of the good and buyers can be penetrable by corruption. Furthermore, corruption can enter auctions in different forms. In the first chapter of this thesis, entitled *Corruption in All-Pay Auctions*, we compare the effects of pure pecuniary corruption and favouritism on bidding behaviour and the auctioneer's expected revenue, in the context of All-Pay Auctions, used to model lobbying, labour-market tournaments and competition for monopoly power. We provide conditions under which favouritism makes bidders more or less aggressive than in the benchmark model without corruption, and prove that bidders are always more aggressive when faced with a non favouritist corrupt auctioneer. In both cases, the revenue maximizing auctioneer deprives his collaborator of all "surplus of corruption". Finally, we study the auctioneer's choice of corruption type, and find that his expected revenue is not necessarily monotonic in the probability that he chooses one type of corruption or the other.

In our study of corruption, we chose the sealed bid All-Pay Auction as our main

setting; as far as we know there is no literature studying corruption in this setting. The basic strand of literature focuses on auctions as procurement mechanisms and consequently uses the First Price Auction. Nevertheless, we deem it important to study the effects of corruption in All-Pay Auctions for two reasons: they represent economic procedures that are not immune to corruption, and due to their modeling particularity can offer insights that First Price Auctions cannot.

When studying pure pecuniary corruption, an interesting result of our paper that is particular to All-Pay auctions is that, contrary to Second Price Auctions that are immune to corruption, in the merely theoretical concept of a Second Price All-Pay Auction this is not the case. This is due to the fact that in a Second Price All-Pay Auction players do not have a dominant strategy to follow, thus corruption alters bidding behaviour and bidders become more aggressive than without corruption. We also discuss the possibility that, within the setting of posterior corruption, the second highest bidder is approached by the auctioneer. We find that in an auction with only two participants an equilibrium does not exist however, against our original intuition, for $N > 2$ an equilibrium can exist.

In our study of bidding behaviour of agents, we find that in the presence of a favourite bidder, other bidders become more or less aggressive given some conditions on the curvature of the distribution of valuations (analogous condition to Arozamena and Weinschelbaum (2009), who study favouritism in First Price Auctions). This is not the case however when the auctioneer is only interested in maximizing his payoffs, when in the presence of corruption bidders always become more aggressive. However in both settings, the corrupt auctioneer robs bidders off all "surplus" of corruption.

In our paper, we not only consider the two types of corrupt auctioneers separately, but also attempt a comparison between the two. We find that the corrupt auctioneer does not necessarily prefer to be one type or the other with probability 1, and depending on the distribution of valuations might choose both with positive probability.

In the second chapter, entitled *Bond Auctions and Financial Sector Liquidity Risk*, a joint work with Grégory Claeys, we aim to provide a tool for central banks – and in particular for the Central Bank of Chile – to measure liquidity risk in their financial sector using the bidding behaviour of banks in bond auctions. First, we build a model combining the auction literature and the financial economics literature to understand precisely the effect of the liquidity risk affecting banks on their bidding strategies in those auctions. We develop a benchmark version of the model with no insurance against the liquidity shock, and another with a lender of last resort to see how the behavior of the banks is affected by this policy. Based on the revelation principle characterizing auctions, and using a unique dataset collected at

Central Bank of Chile containing all the details of its open market operation auctions (where it sells bonds to drain money from the banking sector) between 2002 and 2012, we estimate the distribution of the liquidity risk across Chilean banks and its changes over time. The evolution of the estimated distribution seems to capture well the main episodes of liquidity stress of the last decade in the Chilean banking sector. This measuring tool could be used by other central banks conducting similar open market operations and in need of evaluating in real time the evolution of the liquidity risk affecting their financial sector.

Our model consists of three periods: in the first period players (banks) place bids (rates) in a uniform multi-unit bond auction organized by an institution (central bank)-that wishes to sell a specific volume of bonds- given their own probability of becoming illiquid in the future, which is exogenous and private information. In the second period the probability materializes and now illiquid players must repay the creditors that endowed them with their initial wealth. Given that bonds bought in the first period take two periods to mature, illiquid players now make losses. In the third period profits (or losses) of players are calculated. We study increasing symmetric equilibria in two settings: one without any insurance for illiquid players, and one where the institution (central bank) acts as Lender of Last Resort for illiquid players. We find that although the symmetric bids (rates) in the LLR case are bounded by the LLR pay-back rate, in the no insurance case symmetric bids become much higher. After having obtained the symmetric bidding functions, we perform a structural analysis of the Bond Auctions of the Central Bank of Chile. Using the Kumaraswami distribution, and observed bids from the 30-day bond (PDBC) auctions from 2002 to 2012, we estimate the liquidity risk distribution of Chilean Banks; we find that the evolution of the estimated distribution captures well the main episodes of liquidity stress of the last decade in the Chilean banking sector.

In the third chapter, entitled *Strategic Complementarities and Corruption*, we study an environment where agents compete against each other for the acquisition of a public good procurement project, assigned by the government and handled by a possibly corrupt inspector. We find that there exists multiplicity of equilibria, and in specific both "good" equilibria without corruption, and "bad" equilibria where corruption arises. The multiplicity of equilibria arising in our simple one-shot model is very useful for us to interpret why countries that are quite similar in all other characteristics, can differ a lot in the level of corruption in their economy, like Chile and Argentina or the Italian North and South. An important feature of the multiplicity of equilibria in our simple model is that they don't depend on any strong assumptions on either the agents' or the inspectors' preferences.

In our effort to confirm the multiplicity of equilibria also in the repeated game,

we find that inspectors might consider it profitable to suffer negative payoffs in the first period of the game, in order to create more fuzziness as to how much corruption there is in the economy, and thus decrease the probability of getting caught, guaranteeing themselves bigger positive payoffs in their last period in the game. Our result provides an alternative explanation to already existing literature on the persistence of corruption. Unlike research that attributes persistence to the perception of one's environment as corrupt, or to group dynamics and reputation, our simple model explains persistence as a self induced situation aiming to personal gains, which in turn creates (positive and negative) externalities to the rest of the actors in the economy, both agents and (other) inspectors.

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Chapter 1

Corruption in All-Pay Auctions

1.1 Motivation and Literature

As mechanisms that involve an intermediary, auctions have been repeatedly used in the literature for the study of corruption, since in the presence of an intermediary, no mechanism is immune to it. In our study of corruption, we chose the sealed bid All-Pay Auction as our main setting; as far as we know there is no literature studying corruption in this setting. The basic strand of literature focuses on auctions as procurement mechanisms and consequently uses the First Price Auction. Nevertheless, we deem it important to study the effects of corruption in All-Pay Auctions for two reasons: they represent economic procedures-such as lobbying, labour market tournaments, competition for market power- that are not immune to corruption, and due to their modeling particularity can offer insights that First Price Auctions cannot.

Examples of corruption in labour market tournaments can be found in several empirical papers; in most of them, corruption takes the form of favouritism, or nepotism. For example Combes et al.(2008) using data from 1984 to 2003 find that network connections are more important than actual merits as determinants of success at the "concours d'agregation en sciences economiques", the centralized hiring procedure of economics professors in France. Although both publication records and professional network were found to be statistically significant determinants of success in the french economics departments' job market, the network effect was greater. On nepotism, Kramarz et al. (2007), using a Swedish population-wide employer-employee data set, show that it is very common for highschool graduates to work in the same plants as their parents (and especially their fathers); they also show that firms hire more graduating workers when children of their existing employees graduate, even if they have lower school grades, offering them more job stability and higher initial wages.

Although corruption and lobbying have always been faced in the literature as two separate economic phenomena, most notably argued as being substitutes (see for example Campos and Giovannoni (2008)), recent scandals of corrupt lobbyists imply that lobbying is indeed not immune to corruption itself. In 2005, Jack Abramoff, a then top lobbyist for Greenberg Traurig law firm, was accused, and later convicted for offering free meals up to 150.000 US dollars, plus 65.000 on his personal tab at his restaurant "Signatures" to republican congressmen he was lobbying to in

2002-2003, while no present over 100 US dollars per year is allowed by any lobbyist to any politician. According to New York Times (July 6th 2005), "...In the restaurant's early months, a customer list noted who could dine for free...handwritten notes next to 18 names - lawyers, lobbyists and eight current or former lawmakers...". Abramoff, who was later caught overcharging the Native American Casino Association, clients of his, on lobbying costs, up to 80 million, is not the only corrupt lobbyist to be discovered by justice. In 2013, British Energy Policy MP Tim Yeo was accused by the Sunday Times of "tutoring" businesses how to lobby the government at 7.000 pounds per day.

An interesting application on the All-Pay Auction can be found in Baye, Kovenock and de Vries(1993), who find that politicians seeking to maximize political rents, are better off excluding the lobbyists with the highest valuations. When studying corruption in auctions, recent literature focuses on specific types of corruption. For example Arozamena and Weinschelbaum (2009) model favouritism, while Menezes and Monteiro (2006) build a First Price auction where the auctioneer colludes with the highest bidder, after observing bids.

In our paper, we not only consider these two types of corrupt auctioneers separately, but also attempt a comparison between the two. We find that the corrupt auctioneer does not necessarily prefer to be one type or the other with probability 1, and depending on the distribution of valuations might chose both with positive probability.

When the auctioneer approaches the highest bidder after observing bids (a situation which henceforth we will call posterior corruption), players bid more aggressively than in the benchmark model without corruption¹. We find then that the revenue maximizing auctioneer robs his collaborating bidder of all "surplus" of corruption. An interesting result of our paper that is particular to All-Pay auctions is that, contrary to Second Price Auctions that are immune to corruption, in the merely theoretical concept of a Second Price All-Pay Auction this is not the case. This is due to the fact that in a Second Price All-Pay Auction players do not have a dominant strategy to follow, thus corruption alters bidding behaviour and bidders become more aggressive than without corruption.

Finally, we discuss the possibility that, within the setting of posterior corruption, the second highest bidder is approached by the auctioneer. We find that in an auction with only two participants an equilibrium does not exist however, against our original intuition, for $N > 2$ an equilibrium can exist.

This type of corruption where the auctioneer choses to collaborate with a bidder after observing bids, has been studied by Menezes and Monteiro (2006), Lengwiler

¹Analogous conditions for the First Price Auction can be found in Menezes and Monteiro (2006)

and Wolfstetter (2010) and Celentani and Ganuza (2002) among others. While Menezes and Monteiro investigate how corruption affects the outcome of a first-price auction (bidding behavior, efficiency and the sellers expected revenue) and argue that the auctioneer approaches only the winner to offer the possibility of a reduction in his bid in exchange for a bribe, Lengwiler and Wolfstetter (2010) argue that the assumption that the auctioneer would ask for a bribe from the winner of the auction is not the only choice, as it might be profitable to ask for a bribe from the second highest bidder, which would in turn create inefficiencies. Previous work on the possibility of inefficient outcomes was done by Burguet and Che (2004), who study competitive procurement administered by a corrupt agent who is willing to manipulate his evaluation of contract proposals in exchange for bribes. They find that if the agent is corrupt and has large manipulation power, bribery makes it costly for the efficient firm to secure a sure win, so in equilibrium it loses the contract with positive probability.

The second type of corruption we focus on is favouritism. Unlike Arozamena and Weinschelbaum (2009) that study favouritism in First Price Auctions, we built an All-Pay auction and find that in the presence of a favourite bidder, other bidders become more or less aggressive given some conditions on the curvature of the distribution of valuations; and as in the posterior corruption case, the revenue maximizing auctioneer will deprive his "favourite" of all "surplus" of corruption. One of the first papers to explore favouritism is Laffont and Tirole (1991), who study favouritism in multidimensional auctions; they find that it might appear when the auctioneer assesses product quality.

The remainder of the paper is organized as follows. Section 2 describes the benchmark All-Pay Auction model without corruption. In Section 3 we study bidding behaviour and the auctioneer's maximization problem in Posterior corruption and Favouritism. Section 4 is devoted to the auctioneer's choice of corrupt environment. We conclude in Section 5. Proofs can be found in Appendices A through D.

1.2 Benchmark Model: The All-Pay Auction

Our setting is one where the seller of an indivisible object wishes to sell it to one of N buyers. The mechanism through which the winner is chosen, is a First Price All-Pay Auction, where buyers place one-dimensional price bids; in the setting of the all pay auction, the highest bidding agent wins the object, and all agents pay their respective bids. The seller assigns the auction procedure to an agent of his, henceforth called the auctioneer, which for simplicity, and without loss of generality is assumed to gain a flat wage, which we normalize to 0.

We consider the incomplete information setting, where N risk neutral (payoff maximizing) players place bids for the indivisible object; each player i 's valuation of the object v_i is private information, however it is common knowledge that valuations are iid distributed over $[v, \bar{v}]$ with common distribution F . We assume that F has full support and positive and continuous density f .

As a first step, we discuss the agents' bidding behaviour without corruption. Under no corruption, the highest bidder wins the object, however all bidders have to pay their respective bids; in case of a tie, we assume that one of the tying bidders is randomly assigned the object. Thus, the payoffs for player i are:

$$\pi_i = \left\{ \begin{array}{l} v_i - b_i, \text{ if } b_i > b_j \forall j \neq i \\ -b_i, \text{ if } b_i < b_j \text{ for some } j \neq i \\ \frac{v_i}{\#\{m: b_m = b_i\}} - b_i, \text{ if } b_i = b_m > b_j \forall i, m \neq j \text{ and } \forall i \neq m \end{array} \right\}$$

We focus only in the symmetric equilibrium and strategies β increasing in valuations. We assume that $\beta(\underline{v}) = 0$, since otherwise the bidder with the lowest valuation would make losses with probability 1². Assume all players but player i bid given the increasing strategy $\beta(\cdot)$. Then, bidder i wins by placing bid b , if $b \geq \beta(y_1) \forall j \neq i \Rightarrow y_1 \leq \beta^{-1}(b)$, where y_1 is the highest of the remaining $N - 1$ valuations, i.e. the highest order statistic of the $N - 1$ remaining values. Thus, i maximizes his expected payoff (we drop subscript i for simplicity) :

$$\max_b \Pi = v F^{(N-1)}(\beta^{-1}(b)) - b$$

In a symmetric equilibrium $b = \beta(v)$, and thus:

$$\frac{v(N-1)F(v)^{(N-2)}f(v)}{\beta'(v)} = 1 \Rightarrow v(N-1)F(v)^{(N-2)}f(v) = \beta'(v)$$

Keeping in mind that $\beta(\underline{v}) = 0$, the increasing symmetric bidding function is given by:

$$\beta(v) = v F^{(N-1)}(v) - \int_0^v F^{(N-1)}(y) dy$$

The resulting symmetric bidding function is indeed increasing, with $\beta(\underline{v}) = 0$ and $\beta(v) < v$. Notice that because in an All-Pay Auction every player forfeits their bid, bids are smaller than in an analogous First Price Auction.

²The condition can also be derived using the property of Revenue Equivalence

1.3 The Model: Posterior Corruption and Favouritism

Always using an All-Pay Auction as our basic setting, we now look at two different types of corrupt auctioneers. First, we will consider the situation where the corrupt auctioneer **after** observing the bids approaches the highest bidder and proposes a bribe scheme, to be described later on. Second, we will study an environment where the corrupt auctioneer has a favourite bidder to whom he wishes to allocate the good **before** observing bids³. Henceforth we will refer to the first situation as Posterior Corruption and to the second situation as Favouritism.

1.3.1 Posterior Corruption

After observing bids, the auctioneer approaches the highest bidder, and proposes to lower his bid to the second highest bid, thus securing him a win and lowering his payment. In return, he asks for a bribe, proportional to the difference between the highest and second highest bid; This "surplus of corruption" is divided between the auctioneer and the winner by the sharing parameter $\alpha_{pc} \in [0, 1]$, which for now we consider exogenous. Then, the maximization problem of bidder i bidding b when all other bidders are playing given a symmetric increasing bidding function of their valuation $\beta()$ is:

$$\begin{aligned} \max_b \Pi = & F(\beta^{-1}(b))^{N-1} [v - E[\beta(y_1) \mid \beta(y_1) \leq b] - \\ & \alpha(b - E[\beta(y_1) \mid \beta(y_1) \leq b])] - b[1 - F(\beta^{-1}(b))^{N-1}] \end{aligned}$$

As in the benchmark case, bidder i will win if his bid is higher than the highest of everyone else's bid, i.e. if $b \geq \beta(y_1) \Rightarrow \beta^{-1}(b) \geq y_1$, a manipulation we are able to do since by assumption $\beta()$ is continuous and increasing and thus invertible. Furthermore, y_1 is the highest order statistic of $N - 1$ valuations distributed according to $F()$, and so its distribution is $F()^{N-1}$. Now with probability $F(\beta^{-1}(b))^{N-1}$ bidder i wins and has to pay the second highest bid to the seller, as well as a proportion of the "corruption surplus" to the corrupt auctioneer. In case he loses, he has to pay his own bid b . Since player i cannot know the highest of $N - 1$ bids, he calculates its expectation in case of a win:

$$E[\beta(y_1) \mid \beta(y_1) \leq b] = \frac{1}{F(\beta^{-1}(b))^{N-1}} \int_{\underline{v}}^{\beta^{-1}(b)} \beta(y_1) f(y_1) (N-1) F(y_1)^{N-2} dy_1$$

³For a similar application in the First Price Auction see for example Arozamena and Weinschelbaum (2009)

The resulting first order condition after imposing symmetry is:

$$\beta'_{pc}(v) = \frac{vf(v)(N-1)F(v)^{N-2}}{1-(1-\alpha)F(v)^{N-1}}$$

Integrating with initial condition $\beta(\underline{v}) = 0$ yields the increasing symmetric bidding function:

$$\beta_{pc}(v) = \int_{\underline{v}}^v \frac{y_1 f(y_1)(N-1)F(y_1)^{N-2}}{1-(1-\alpha)F(y_1)^{N-1}} dy_1$$

Setting $\varphi'(y_1) = \frac{f(y_1)(N-1)F(y_1)^{N-2}}{1-(1-\alpha)F(y_1)^{N-1}}$ we get:

$$\beta_{pc}(v) = v \frac{\text{Log}[1-(1-\alpha)F(v)^{N-1}]}{\alpha-1} - \int_{\underline{v}}^v \frac{\text{Log}[1-(1-\alpha)F(y_1)^{N-1}]}{\alpha-1}$$

Proposition 1 *The limit of $\beta_{pc}(v)$ as $\alpha \rightarrow 1$ is equal to $\beta(v)$, the increasing symmetric equilibrium bidding function in the benchmark no corruption case. Also, $\beta^{PostC}(v)$ at $\alpha = 0$ is equal to $\beta_{sp}(v)$, the increasing symmetric equilibrium bidding function in a Second Price All Pay auction, where the highest bidder wins the object and pays the second highest bid, and all other bidders forfeit their bids.*

The proof of Proposition 1 is provided in Appendix A1. Notice that since $\alpha \leq 1$, and $\text{Log}(x)$ is an increasing function of x , then $\frac{\partial \beta_{pc}(v)}{\partial \alpha} < 0 \rightarrow \beta_{pc}(v)$ decreasing in α . Thus, with a corrupt auctioneer bidders become uniformly more aggressive than without corruption, i.e. $\beta_{pc}(v) \geq \beta(v)$; also, the less they have to bribe the auctioneer, the higher their bids become.

After having obtained the bidding function for players, we turn to the auctioneer's point of view. Up until now we considered the sharing parameter α_{pc} exogenous. However, the auctioneer will choose the sharing parameter that maximizes his expected revenue.

Proposition 2 *The auctioneer will set $\alpha_{pc}^* = 1$, leaving his collaborator indifferent between accepting or rejecting the corrupt agreement.*

The formal proof of Proposition 2 can be found in Appendix A2. The auctioneer's expected revenue is nothing but the sum of the ex-ante expected payments

of players towards him. As we saw before, bids are decreasing in α_{pc} ; however the difference between the two highest bids is weakly increasing in α_{pc} . Thus, the auctioneer's expected revenue⁴

$$ER_{pc} = \alpha N \int_v^{\bar{v}} \frac{v(1-F(v))g(v)G(v)}{1-(1-\alpha)G(v)} dv$$

is strictly increasing in α_{pc} and as such, maximized at $\alpha_{pc}^* = 1$

Posterior Corruption in Second Price All-Pay Auctions

As has been discussed in several papers, the Second Price Auction is not affected by corruption. The underlying reason is that in this type of auction, players have a dominant strategy-bidding their own valuation. However, this is not the case in the context of a Second Price All-Pay Auction, where the highest bidder wins the object and pays the second highest bid, and all other players forfeit their bids. The maximization problem of an agent in a Second Price All-Pay Auction with a corrupt auctioneer is⁵:

$$\max_b \Pi = F(\beta^{-1}(b))^{N-1} [v - (1-\alpha) E[\beta(y_2) | \beta(y_1) \leq b] - \alpha E[\beta(y_1) | \beta(y_1) \leq b]] - b[1 - F(\beta^{-1}(b))^{N-1}]$$

Taking the first order condition and imposing symmetry yields the following differential equation:

$$\beta'_{spc}(v) - \beta(v) \left(\frac{f(v)(N-1)(1-\alpha)}{1-F(v)^{N-1}} \right) (F(v)^{N-2} - (N-2)f(v)F(v)^{N-3}) = \frac{v(N-1)F(v)^{N-2}f(v)}{1-F(v)^{N-1}}$$

Without corruption, the analogous differential equation resulting from the maximization problem of agents is⁶:

$$\beta'_{sp}(v) = \frac{v(N-1)F(v)^{N-2}f(v)}{1-F(v)^{N-1}}$$

⁴Where we substituted $g(v) = (N-1)f(v)F(v)^{N-2}$ and $G(v) = F(v)^{N-1}$

⁵In this setting the corruption agreement between the auctioneer is the same as before, the only difference being that the winner of the object that would normally pay the second highest bid, will now pay the third highest bid.

⁶The proof can be found in Appendix A2

Notice that the right hand sides of the two equations are equal. Thus, $\beta_{sp}(v) \neq \beta_{spc}(v)$ unless $\alpha = 1$.

Proposition 3 *Contrary to the Second Price Auction where bidders place bids, the highest bidder wins and pays the second highest bid where corruption does not affect the symmetric bidding equilibrium (see for example Menezes and Monteiro (2006)), in the Second Price All-Pay Auction, corruption alters players' symmetric bidding behaviour, making them uniformly more aggressive.*

Posterior Corruption: Second Highest Bidder Approached by the Auctioneer

In the previous parts of the paper, we looked only at situations where the highest bidder is approached by the corrupt auctioneer. Now we consider the situation where the auctioneer, after observing bids, approaches the second highest bidder; his proposal is to match his bid to the highest bid, thus securing him a win, in exchange for a proportional bribe. The bidder will accept only if his valuation is high enough to be able to afford the good and the bribe.

Proposition 4 *In the All-Pay auction where the corrupt auctioneer after observing bids approaches the second highest bidder and proposes to match his bid to the highest bid, thus securing him a win in exchange for a proportional bribe, for arbitrarily small sharing parameter α and $N = 2$, an increasing symmetric equilibrium does not exist. For $\alpha > 0$ and general N an increasing symmetric equilibrium can exist.*

In order to prove that an increasing symmetric equilibrium does not exist in All-Pay auctions where the corrupt auctioneer approaches the second highest bidder, with $N = 2$ and $\alpha = 0$, we will look at the bidding decision of player 1 with valuation $v_1 = \bar{v}$. Suppose that player 2 places a bid according to some increasing function of his valuation $\beta(v_2)$. The expected payoff for player 1 with strategy $b = \beta(\bar{v})$ is:

$$\Pi(\bar{v}, \beta(\bar{v})) = Prob(\beta(\bar{v}) \leq v_2)(-\beta(\bar{v})) + (1 - Prob(\beta(\bar{v}) \leq v_2))(\bar{v} - \beta(\bar{v}))$$

The player with the highest valuation knows that by bidding according to $\beta()$ his bid will be the highest. Given that the auctioneer will approach the second bidder, player 1 knows that he will be the eventual winner only if the second bidder's

valuation is not high enough. Thus his probability of winning is not equal to 1. If instead player 1 chooses to bid $b = 0$, his expected payoff is:

$$\Pi(\bar{v}, 0) = \bar{v} - \mathbb{E}[\beta(v_2) \mid v_2 \leq \bar{v}] = \bar{v} - \mathbb{E}[\beta(v_2)]$$

Given that $\beta(\cdot)$ is increasing, notice that $\mathbb{E}[\beta(v_2)] \leq \beta[\bar{v}]$, and thus $\Pi(\bar{v}, 0) \geq \Pi(\bar{v}, \beta(\bar{v}))$. Then, for the player with the highest valuation it is not best response to play according to the increasing bidding function used by the second player; we can conclude that an increasing symmetric equilibrium does not exist.

The intuition behind this result is that in such an environment players are torn between wishing to have the highest or second highest bid, as in both cases they win with positive probability. The player with the highest valuation knows that by placing $\beta(\bar{v})$ he loses the object with positive probability; in a setting with only 2 players however, he can choose $b = 0$ and win with probability 1, as he will have placed the second highest bid, and can afford to pay any bid $b \in [\underline{v}, \bar{v}]$. With $N > 2$, choosing any bid $b < \beta(\bar{v})$ would not guarantee him a win with probability 1, and thus an increasing symmetric $\beta(v)$ might exist.

1.3.2 Favouritism

Now we will consider an All-Pay Auction where the auctioneer has a "favourite" bidder⁷ to whom he wishes to allocate the good, before observing bids. Before the auction starts, the auctioneer and his favourite reach an agreement that the former will reveal all bids to the latter, and allow him to bid afterwards. In case the favourite's bid is a winning bid, the auctioneer asks for a compensation proportional to the difference between the favourite's valuation and his bid, with sharing parameter $\alpha_f \in [0, 1]$. Since in an All-Pay Auction all bidders forfeit their bids, the favourite bidder will then bid:

$$b_f = \begin{cases} 0 & \text{if } v_f < \bar{b} \\ \bar{b} & \text{otherwise} \end{cases}$$

, where \bar{b} is the highest of $N - 1$ bids. In case his valuation is high enough to afford paying the highest of $N - 1$ bids for the good, he will do so in order to win, whereas if his valuation is not high enough he will bid 0 in order to minimize his costs.

We assume that the rest $N - 1$ bidders are aware of the presence of a favourite, and we look at their bidding behaviour. The maximization problem of non-corrupt

⁷An example of favouritism in the literature is Arozamena and Weinschelbaum, who study the welfare effects of favouritism in a First Price Auction

bidder i bidding b if all other non corrupt bidders play according to an increasing function of their valuations, is:

$$\max_b \Pi = vF(\beta^{-1}(b))^{N-2}F(b) - b$$

In this setting player i knows that he is bidding against $N - 2$ honest and 1 corrupt player. Thus he knows that in order to win, his bid needs to be higher than $N - 2$ bids and the favourite's valuation. The resulting differential equation, after imposing symmetry, is:

$$\beta'_f(v) = \frac{vf(v)(N - 2)F(v)^{N-3}F(\beta(v))}{1 - vF(v)^{N-2}f(\beta(v))}$$

The above differential equation cannot be analytically solved unless we assume a specific distribution of valuations. Nevertheless, although we cannot guarantee uniqueness, in Appendix B we prove the existence and optimality of a symmetric increasing bidding function solving the differential equation, under the condition that $f(v)$ decreasing in v . Given the existence of an increasing symmetric $\beta_f(v)$, we are interested in comparing bidding behaviour in favouritism and the benchmark model without corruption.

Proposition 5 *If $(\beta_f^{-1}(b))' > \frac{f(\beta_f(v))}{f(v)}$ then bidders bid uniformly less aggressively than in the benchmark no corruption case. Analogously, if $(\beta_f^{-1}(b))' < \frac{f(\beta_f(v))}{f(v)}$ then bidders bid uniformly more aggressively than in the benchmark no corruption case.*

More (less) aggression in our context means that $\beta_f(v) > (<)\beta(v)$, i.e. player i with valuation v will place a higher (lower) bid in the presence of a favouritist auctioneer than in an honest environment. An alternative view is that player i will place the same bid in both settings, only if his valuation is lower (higher) in favouritism than without corruption, i.e. if $\beta_f(v_1) = \beta(v_2)$ then $v_1 < (>)v_2$. The proof of Proposition 5 can be found in Appendix C. The intuition behind this result lies in the fact that non-corrupt bidders are driven by two different forces. On one hand, they know that with their valuation there are cases where they would have won in the benchmark no corruption case, but now lose for sure; this effect drives them to bid less aggressively. However, they also know that if the valuation of the corrupt bidder is not high enough given their own bid, they can win. This effect drives bidders to bid more aggressively. Ultimately it is the curvature of $F()$ that defines whether or not players place uniformly higher or lower bids than in the benchmark model.

After having obtained the conditions for equilibrium and the properties of the bidding function, we turn to the auctioneer's point of view. As said before, if the favourite's valuation is high enough, he will set \bar{b} and win the object. Now we know that $\bar{b} = \beta_f(y_1)$, the bid of the player with the highest among $N - 1$ valuations. In this case the auctioneer and his favourite share the "surplus" of corruption which is the difference between v_f and $\beta_f(v)$, with sharing parameter $\alpha_f \in [0, 1]$. Since the auctioneer does not know his favourite's valuation, his expected revenue will be equal to the ex-ante payment of his favourite bidder towards him.

Proposition 6 *The auctioneer will set $\alpha_f^* = 1$*

The favourite's expected payment to the auctioneer is:

$$p(v) = \alpha_f \text{Prob}(v > \beta_f(y_1))(v - \mathbb{E} \beta_f(y_1)) \mid \beta_f(y_1) < v$$

The ex-ante expected payment is:

$$\mathbb{E}(p(V)) = \alpha_f \left[\int_{\underline{v}}^{\bar{v}} [v F_{Y_1}(\beta_f(v^{-1})) - \int_{\underline{v}}^{\beta_f^{-1}(v)} \beta_f(y_1) (F_{Y_1}(y_1))' dy_1] f(v) dv \right] = ER_f$$

Since $\beta_f(v)$ is constant in α_f then ER_f is strictly increasing in α_f and the auctioneer's revenue maximizing α_f^* is equal to 1.

1.3.3 The Corrupt Auctioneer

Throughout the paper we have assumed that there exist two types of corrupt auctioneers: one that strictly prefers posterior corruption, and one that strictly prefers favouritism. In this section we drop this assumption and allow the auctioneer to chose which type he prefers to be. We assume that bidders know that they are facing an auctioneer that prefers posterior corruption with probability λ , and favouritism with probability $1 - \lambda$. We assume that the auctioneer's choice is binding, and check how he will chose λ in order to maximize his expected revenue. We will solve the game backwards, first solving the bidder's maximization problem, taking into consideration our previous result that $\alpha_{pc}^* = \alpha_f^* = 1$.

For convenience we will consider $N = 2$ without loss of generality, as our qualitative results hold for any N . As before, players place their respective bids. Then, with probability λ the highest bidder wins (and pays all the surplus of corruption to

the auctioneer), and with probability $1 - \lambda$ the auctioneer asks his favourite bidder to rebid, after revealing his rival's bid. In order to be able to obtain a symmetric bidding function, we will assume that before placing bids, bidders do not know whether with probability $1 - \lambda$ they will be the favourite or not. Thus, they assign probability $\frac{1}{2}$ to both events.

Player i knows that with probability $\frac{1}{2}(1 - \lambda)$ he will be able to rebid after observing his rival's bid, and thus will set:

$$b_f = \begin{cases} 0 & \text{if } v_i \text{ smaller than his rival's bid} \\ \text{rival's bid} & \text{otherwise} \end{cases}$$

As before, if v_i is bigger than his rival's bid, the auctioneer will take away all the surplus of corruption from him. With probability λ player i is no longer the favourite, and thus will choose $b = \operatorname{argmax} E\Pi(v)$. Then, the maximization problem of player i is:

$$\max_b \Pi(v, b) = \lambda[v\operatorname{Prob}(b > \beta(y)) - b] + \frac{1}{2}(1 - \lambda)[v\operatorname{Prob}(b > y) - b]$$

Arranging the probabilities, the maximization problem becomes:

$$\max_b \Pi(v, b) = \lambda[vF(\beta^{-1}(b) > y) - b] + \frac{1}{2}(1 - \lambda)[vF(b) - b]$$

The symmetric first order condition yields the differential equation:

$$\beta'(v_h) = \frac{2\lambda v_h f(v_h)}{1 + \lambda - (1 - \lambda)v_h f(\beta(v_h))}$$

As a convex combination of the differential equations discussed before, this has an increasing solution. However notice that now $f(\cdot)$ need not be strictly decreasing. Now, the auctioneer's expected revenue is:

$$ER(\lambda) = \lambda ER_{pc}(\lambda) + (1 - \lambda)ER_f(\lambda)$$

, where $ER_{pc}(\lambda) = 2 \int_{\underline{v}}^{\bar{v}} [1 - F(y)] F(y) \frac{\partial \beta(y, \lambda)}{\partial y} dy$ and $ER_f(\lambda) = \int_{\underline{v}}^{\bar{v}} [1 - F(\beta(y, \lambda))] F(y) \frac{\partial \beta(y, \lambda)}{\partial y} dy$

Thus:

$$ER(\lambda) = \int_{\underline{v}}^{\bar{v}} F(y) \frac{\partial \beta(y, \lambda)}{\partial y} [2\lambda(1 - F(y)) + (1 - \lambda)(1 - F(\beta(y, \lambda)))] dy$$

Proposition 7 *The auctioneer’s expected revenue is not necessarily monotonic in λ . The curvature of the expected revenue depends on the curvature of the bidding function and the pdf of valuations. For example, with uniform distribution on $[0,1]$, the expected revenue is a concave function of λ and maximized at interior λ .*

The proof of Proposition 7 can be found in Appendix D. In Figure 1 we show $ER(\lambda)$ for $v_i \sim Uniform[0, 1]$; $ER(\lambda)$ is concave and maximized at interior λ .

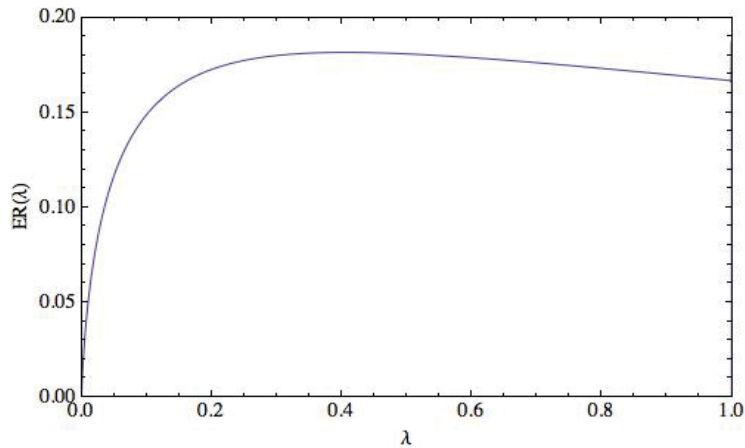


Figure 1.1: Auctioneer’s Expected Revenue as a function of λ ($v_i \sim U[0, 1]$)

1.4 Conclusions and Future Work

Throughout the paper we tried to explore symmetric equilibrium bidding behaviour and the auctioneer’s choices, in the setting of an All-Pay Auction where corruption can occur. We considered two different types of corruption schemes: one where the corrupt auctioneer colludes with the winner of the auction, and one where the corrupt auctioneer has a preferred bidder, whom he wishes to allocate the good to. We found conditions under which players bid more or less aggressively in favouritism than in the benchmark no corruption model, and showed that in posterior corruption bidders are always more aggressive.

Furthermore we argued that a Second Price All-Pay Auction is penetrable to corruption as, contrary to a Second Price Auction, the game is not dominance solvable. Coming to the auctioneer’s choices, we found that in both corruption settings he will deprive his colluding bidders of any ”surplus of corruption”. Finally, we saw

that the auctioneer's revenue is not necessarily monotonic in the probability that he chooses one type of corruption or the other.

What we did not talk about is the welfare effects of corruption both on the bidders and the seller; an interesting extension of our paper could give monitoring power to the seller, and potential punishments to the auctioneer if caught "cheating", which might lead to less/no corruption.

Appendix

Appendix A1

In order to prove that $\lim_{\alpha \rightarrow 1} \beta_{pc}(v) = \beta(v)$, we only need to apply L'Hopital's rule. Since:

$$\lim_{\alpha \rightarrow 1} \text{Log}[1 - (1 - \alpha)F(v)^{N-1}] = \lim_{\alpha \rightarrow 1} \alpha - 1 = 0$$

, and $\lim_{\alpha \rightarrow 1} \frac{\partial \text{Log}[1 - (1 - \alpha)F(v)^{N-1}]}{\frac{\partial \alpha}{\partial \alpha}} = F(v)^{N-1}$, then:

$$\lim_{\alpha \rightarrow 1} \beta_{pc}(v) = vF(v)^{N-1} - \int_{\underline{v}}^v F(y_1)^{N-1} dy_1 = \beta(v)$$

In order to conclude the proof of Proposition 1, we need to show that $\beta_{pc}(v, \alpha = 0) = \beta_{sp}$. Thus, we need to solve for the increasing symmetric equilibrium bidding function for the Second Price All Pay auction, where the highest bidder wins the object and pays the second highest bid, and all other bidders pay their respective bids. Revenue Equivalence implies:

$$[1 - F(v)^{N-1}] \beta_{sp}(v) + \int_{\underline{v}}^v \beta_{sp}(y_1) f(y_1) (N-1) F(y_1)^{N-2} dy_1 = \int_{\underline{v}}^v y_1 f(y_1) (N-1) F(y_1)^{N-2} dy_1$$

Differentiating with respect to v , we get:

$$\beta'_{sp}(v) = \frac{v f(v) (N-1) F(v)^{N-2}}{1 - F(v)^{N-1}} \Rightarrow \beta_{sp}(v) = \int_{\underline{v}}^v \frac{y_1 f(y_1) (N-1) F(y_1)^{N-2}}{1 - F(y_1)^{N-1}} dy_1$$

Setting $\varphi'(y_1) = \frac{f(y_1)(N-1)F(y_1)^{N-2}}{1-(1-\alpha)F(y_1)^{N-1}}$:

$$\beta_{sp}(v) = v[-\text{Log}(1 - F(v)^{N-1})] - \int_{\underline{v}}^v \underline{v} - \text{Log}(1 - F(y_1)^{N-1}) dy_1$$

, and thus: $\beta_{pc}(v, \alpha = 0) = \beta_{sp}(v)$, which concludes our proof.

Appendix A2

In this Appendix we prove that the auctioneer's expected revenue is strictly increasing in α_{pc} . We will begin our proof by computing player i's expected payment to the auctioneer. The expected payment is :

$$p(v) = \alpha \text{Prob}(\text{win})(\beta(v) - \mathbb{E}[\beta(y_1) | y_1 \leq v]) \Rightarrow$$

$$p(v) = \alpha [G(v) \int_{\underline{v}}^v \frac{y_1 g(y_1)}{1 - (1 - \alpha)G(y_1)} dy_1 - \int_{\underline{v}}^v \int_{\underline{v}}^{y_1} \frac{y g(y)}{1 - (1 - \alpha)G(y)} dy g(y_1) dy_1]$$

, where we substituted $g() = (N - 1)f()F()^{N-2}$ and $G() = F()^{N-1}$. With a change of variables we get:

$$p(v) = \alpha \int_{\underline{v}}^v \frac{y_1 g(y_1) G(y_1)}{1 - (1 - \alpha)G(y_1)} dy_1$$

Now, we can calculate the ex-ante expected payment to the auctioneer, ie player i's expected payment to the auctioneer before knowing his valuation v_i :

$$\mathbb{E}(p(V)) = \alpha \int_{\underline{v}}^{\bar{v}} \int_{\underline{v}}^v \frac{y_1 (1 - F(y_1)) g(y_1) G(y_1)}{1 - (1 - \alpha)G(y_1)} dy_1 f(v) dv$$

Again, with a change of variables, we have:

$$\mathbb{E}(p(V)) = \alpha \int_{\underline{v}}^{\bar{v}} \frac{v(1 - F(v)) g(v) G(v)}{1 - (1 - \alpha)G(v)} dv$$

The expected revenue of the auctioneer, is nothing but the sum of ex-ante expected payments of bidders. Thus $ER_{pc} = N \mathbb{E}(p(V))$. The auctioneer wishes to maximize

his expected revenue with respect to α . The derivative of ER_{pc} with respect to α is:

$$\frac{\partial ER_{pc}}{\partial \alpha} = N \int_{\underline{v}}^{\bar{v}} \frac{v(1-F(v))g(v)G(v)}{1-(1-\alpha)G(v)} \frac{1-G(v)}{1-(1-\alpha)G(v)} dv > 0$$

Since $\frac{\partial ER_{pc}}{\partial \alpha} > 0, \forall \alpha \in [0, 1]$, the auctioneer maximizes his expected revenue by setting $\alpha_{pc}^* = 1$. Then as we already saw, bidders will play according to the increasing symmetric function without corruption.

Appendix B

To prove existence we follow Li and Tan(2000). We wish to show that an increasing solution to the differential equation

$$\beta'_f(v) = \frac{vf(v)(N-2)F(v)^{N-3}F(\beta(v))}{1-vF(v)^{N-2}f(\beta(v))}$$

exists. With $N = 2$, the FOC becomes $f(b) = \frac{1}{v}$ which has an increasing solution, as long as $f(\cdot)$ is decreasing in its argument.

For $N > 2$, we are looking for an increasing solution $\beta_f(v)$ such that $v \in [\underline{v}, \bar{v}]$, $\beta_f(v) \in [0, \bar{v}]$ and $\beta_f(v) \leq v \forall v$, with the initial condition $\beta_f(\underline{v}) = 0$. Thus, our domain is $D = \{v \in [\underline{v}, \bar{v}], b \in [0, \bar{v}] : b \leq v\}$. However, in this domain, both the numerator and the denominator of our differential equation can be 0, so we need to restrict our domain of existence of a solution, in order to be able to apply existence theorems.

The numerator is zero for $b = \underline{v}$, thus we need to exclude such b in order for the numerator to be strictly positive. Also, the denominator can be 0 if $1 - vF(v)^{N-2}f(\beta_f(v)) = 0$. Let $k(b, v) = 1 - vF(v)^{N-2}f(b)$, and let \tilde{b} be such that $k(\tilde{b}, v) = 0$. Since $f(b)$ decreasing, then $k(b, v)$ increasing in b , and thus for any $b > \tilde{b}$, $n(b, v) > 0$.

Our new domain is:

$$D' = \{b \in (0, \bar{v}], v \in (\underline{v}, \bar{v}], \hat{b} < b \leq \bar{v}, \hat{b} = \begin{cases} \tilde{b} & \text{if } k(\tilde{b}) = 0 \text{ for } \tilde{b} \in (0, \bar{v}) \\ 0 & \text{otherwise} \end{cases}$$

On the new domain the standard existence theorems apply. Then for any $(b_0, v_0) \in D'$ there exists a unique solution $\beta_{f_0}(v)$ in a neighbourhood of v_0 such that $\beta_{f_0}(v_0) = b_0$ and that solution is continuous in v_0 . The solution can be extended to the left and right and then be defined in a larger interval. Let (V_0, V_1)

be the biggest open interval where the solution exists. Then, by the extensibility theorem, when v approaches V_1 , $(v, \beta_{f0}(v))$ approaches a point on the boundary of D' .

We wish to show that $\lim_{v \rightarrow V_1} \beta_{f0}(v) \neq \tilde{b}$. Suppose that $\lim_{v \rightarrow V_1} \beta_{f0}(v) = \tilde{b}$. Then $k(V_1, \lim_{v \rightarrow V_1} \beta_{f0}(v)) = 0$ and thus $\beta'_{f0}(v)$ can be made arbitrarily large. Since there exists V_1 such that as v approaches V_1 , $\beta_f(v)$ approaches b , then for any $\epsilon > 0$, there exist $\delta > 0$ such that if $V_1 - v < \delta$, $|\beta_f(v) - b| < \epsilon$. Since for sufficiently ϵ , $\beta'_f(v)$ is arbitrarily large, if v moves closer to V_1 by a given distance, $|\beta_f(v) - b| < \epsilon$ does not hold. Thus, $\lim_{v \rightarrow V_1} \beta_{f0}(v) = \bar{v}$. Then we have a strictly increasing solution that takes values on the interval $[v_0, \bar{v}]$ and we need to extend it again such that it takes values on the whole interval of valuations $[\underline{v}, \bar{v}]$.

Now we need to prove that the second order condition for optimality holds. In this part of the proof we follow Arozamena and Weinschelbaum (2001). The expected payoff of bidder i is:

$$\Pi(b) = vF^{(N-2)}(\beta_f^{-1}(b))F(b) - b$$

Then:

$$\Pi'(b) = -1 + v[g(\beta_f^{-1}(b))\frac{1}{\beta'_f(\beta_f^{-1}(b))}F(b) + G(\beta_f^{-1}(b))f(b)](\mathbf{a})$$

Also recall that the First Order Condition under symmetry is:

$$1 = v[g(v)\frac{1}{\beta'_f(v)}F(\beta_f(v)) + G(v)f(\beta_f(v))](\mathbf{b})$$

Plugging **(b)** into **(a)** yields:

$$\Pi'_b = v[g(\beta_f^{-1}(b))\frac{1}{\beta'_f(\beta_f^{-1}(b))}F(b) + G(\beta_f^{-1}(b))f(b) - g(v)\frac{1}{\beta'_f(v)}F(\beta_f(v)) + G(v)f(\beta_f(v))]$$

Since all the above functions are increasing, when $\beta_f^{-1}(b) < v \Rightarrow \pi'_b < 0$, and when $\beta_f^{-1}(b) > v \Rightarrow \pi'_b > 0$. Thus, it is optimal for bidder i to chose $b = \beta_f(v)$ such that $\beta_f^{-1}(b) = v$, as there $\pi'_b = 0$, which concludes our proof.

Appendix C

In order to prove that under some conditions, $\beta(v) > (<) \beta_f(v)$, we will follow Arozamena and Weinschelbaum (2009), and prove that if for some $\tilde{b} \in [0, \bar{v}]$,

$\beta^{-1}(\tilde{b}) = \beta_f^{-1}(\tilde{b})$ then $(\beta^{-1}(\tilde{b}))' > (<)(\beta_f^{-1}(\tilde{b}))'$ if $(\beta_f^{-1})' < (>)\frac{f(\tilde{b})}{f(\beta_f^{-1}(\tilde{b}))}$. For convenience we will set $\beta_f^{-1}(b) = q_f$ and $\beta^{-1}(b) = q$. Note here that since $\beta(v)$ and $\beta_f(v)$ do not appear in the differential equations without corruption and favouritism, a particularity of the All-Pay Auction, in order to be able to compare them, for the purpose of this proof we will use the equivalent differential equations:

$$q = \frac{1}{(N-1)F(q)^{N-2}q'}$$

and

$$q_f = \frac{1}{(N-2)F(q_f)^{N-3}f(q_f)F(b)q'_f + F(q_f)^{(N-2)}f(b)}$$

Suppose that for some \tilde{b} we have $q(\tilde{b}) = q_f(\tilde{b})$. Then, from the above we have:

$$q' = \frac{(N-2)F(b)}{(N-1)F(q_f)}q'_f + \frac{f(b)}{(N-1)f(q_f)}$$

We can see that if $\frac{f(b)}{f(q_f)q'_f} > \frac{(N-1)F(q_f) - (N-2)F(b)}{F(q_f)}$ then $q' > q'_f$. However notice that since $\frac{(N-1)F(q_f) - (N-2)F(b)}{F(q_f)} > 1$ always, then $\frac{f(b)}{f(q_f)q'_f} > \frac{(N-1)F(q_f) - (N-2)F(b)}{F(q_f)}$ implies that $\frac{f(b)}{f(q_f)q'_f} > 1$. Thus, we get that $q' > (<)q'_f$ iff $q'_f < (>)\frac{f(b)}{f(q_f)}$, i.e. players bid more (less) aggressively in favouritism than in the case of no corruption. In order to conclude our proof, we need to show that such \tilde{b} does not exist, ie if $q'_f < (>)\frac{f(b)}{f(q_f)}$, then $q > (<)q_f \forall b \in [0, \bar{v}]$. We will focus on the case where $q'_f < \frac{f(b)}{f(q_f)}$, as the proof for the opposite case is analogous.

Suppose towards a contradiction that for some $b_0 \in [0, \tilde{b}]$, $q(b_0) \leq q_f(b_0)$. This implies that $\frac{F(q_f(b_0))}{F(q(b_0))} > 1$. Then it should be true that $\frac{\partial \frac{F(q_f(b_0))}{F(q(b_0))}}{\partial b_0} > 0$. However:

$$\frac{\partial \frac{F(q_f(b_0))}{F(q(b_0))}}{\partial b_0} = \frac{q'_f f(q_f)F(q) - q'F(q_f)f(q)}{F(q(b_0))^2} < 0$$

, since $q'_f < \frac{f(b)}{f(q_f)}$, $f()$ is decreasing, $F()$ is increasing, and $q(b_0) < q_f(b_0)$, which is a contradiction.

Appendix D

The derivative of the auctioneer's expected revenue with respect to λ is:

$$\frac{\partial ER(\lambda)}{\partial \lambda} =$$

$$\underbrace{2 \int_{\underline{v}}^{\bar{v}} [1 - F(y)] F(y) \left[\underbrace{\frac{\partial \beta(y, \lambda)}{\partial y}}_{\mathbf{a}'} + \lambda \underbrace{\frac{\partial^2 \beta(y, \lambda)}{\partial y \partial \lambda}}_{\mathbf{a}''} \right] dy}_{\mathbf{a}} - \underbrace{\int_{\underline{v}}^{\bar{v}} [1 - F(\beta(y, \lambda))] F(y) \frac{\partial \beta(y, \lambda)}{\partial y} dy}_{\mathbf{b}} +$$

$$\underbrace{(1 - \lambda) \int_{\underline{v}}^{\bar{v}} F(y) \left[(1 - F(\beta(y, \lambda))) \underbrace{\frac{\partial^2 \beta(y, \lambda)}{\partial y \partial \lambda}}_{\mathbf{c}'} - \underbrace{\frac{\partial F(\beta(y, \lambda))}{\partial \lambda}}_{\mathbf{c}''} \frac{\partial \beta(y, \lambda)}{\partial y} \right] dy}_{\mathbf{c}}$$

Notice that \mathbf{b} is strictly positive. Also notice that \mathbf{a} is also strictly positive as long as $\mathbf{a}'' > 0$. If instead $\mathbf{a}'' < 0$, then the sign of \mathbf{a} is uncertain, and depends on the relative magnitudes of \mathbf{a}' and \mathbf{a}'' . As for \mathbf{c} , notice that its sign is also uncertain, and depends on the signs of \mathbf{c}' and \mathbf{c}'' , and their relative magnitudes. Thus $ER(\lambda)$ might be increasing, decreasing or concave, depending on the curvature of $F(v)$, which concludes our proof.

Chapter 2

Bond Auctions and Financial Sector Liquidity Risk

With Grégory Claeys

2.1 Introduction

Liquidity risk is inherent to the very nature of the banking activity which is to transform short term liabilities into long term assets. Traditionally, this was done by collecting deposits in order to make loans for long term projects. However, in the last two decades, this risk has been pretty much neglected by banks and regulators alike. Banks around the world have been relying more and more on short term wholesale funding such as the asset-backed commercial paper market, the repo market and the overnight interbank market. The counterpart of this rapid growth in wholesale funding was a parallel decrease of historically more stable¹ retail deposits in the funding of the banks. A good – although a bit extreme – example of this trend is the trajectory of the British bank Northern Rock in the years leading to the financial crisis, with its ratio of deposits to total liabilities declining from 62.7% in 1997 to only 22.4% in 2006 (Bank of England, 2007b).

However, during the financial turmoil that started in 2007 and worsened especially after Lehman Brothers' failure in September 2008 – and described at length in Brunnermeier (2009) – those crucial sources of short term funding for financial institutions all vanished at the same time. Consequently, banks relying on this kind of funding had difficulties to meet their obligations and some of them actually failed, meanwhile others like Northern Rock were nationalized to avoid bankruptcy, highlighting the fact that liquidity risk had not disappeared.

Since then, there has been a renewed interest in the issue of liquidity risk in the financial sector. This is why the main purpose of this paper is to build a tool for central banks and regulators to measure this risk in order to be able to assess the fragility of their financial sector. Note that the measuring tool described in this paper has been constructed especially for the Central Bank of Chile (BCCh)

¹At least since the implementation of deposit insurance in many countries to avoid the phenomenon of bank runs that was prevalent during the financial crises of the 1930s.

because of the availability of its very rich open market operations dataset to us, but that it could be used by any other central bank conducting similar open market operations.

Before explaining in detail how we intend to proceed, let's give first a careful definition of liquidity risk because the word liquidity has often been used to describe different concepts, and we want to be clear about what we intend to understand and measure in this paper. We define liquidity as the ability of an agent to settle her obligations with immediacy. It is clearly a binary concept: an agent is liquid or illiquid. On the contrary, liquidity risk is a continuous concept and we define it as the probability to become unable to settle obligations over a specific horizon. This is what we want to measure in this paper. Finally, it is also interesting to distinguish, as in Brunnermeier and Pedersen (2009), funding liquidity: the ease to access funding (which is agent specific), from market liquidity: the ease to sell an asset (which is asset specific), because in the paper we want to focus exclusively on funding liquidity risk.

There exist already various measures of liquidity risk in the literature. Some of them are based on data from balance-sheets of banks like the Liquidity Coverage Ratio and the Net Stable Funding Ratio introduced by Basel III after the subprime crisis (BIS, 2013), other are market-based like the very simple LIBOR-OIS spread or more complex like the composite financial market liquidity indicator introduced by the Bank of England (2007a). However, we consider that those measures of liquidity risk are imperfect either because they depend heavily on stress scenarios and are very sensitive to expert categorizations of assets and liabilities (in the case of balance-sheet based indices) or because they do not allow to disentangle liquidity risk from other risks like solvency risk (in the case of market-based indices). We want to build a new tool to measure liquidity risk that will not be subject to those flaws. We think that, although it might be difficult for central banks to assess the liquidity risk of a specific bank or of the whole banking sector using data from balance-sheets or financial markets, we can still assume two things: first, banks are aware of their own liquidity risk and, second, they may be less willing to give up their cash if this risk is high, to avoid bankruptcy. That is why the basic idea behind this paper is to try to extract this information directly from the banks using their bidding behavior in the bond auctions of the BCCh.

Why do we think that the banks' bids in those auctions reveal their liquidity risk? First, it is important to note that the BCCh sells bonds to the Chilean commercial banks in order to drain money from the banking sector to control the quantity of money in circulation in the economy and meet its inflation target. It is purely a technical open market operation by the central bank and not at all a way to finance itself or to finance the Chilean government. Therefore, those bonds issued by the central bank bear absolutely no solvency risk whatsoever: the BCCh

can always print pesos to reimburse the banks when the bonds mature. That is why, in our opinion, the trade-off faced by a bank on whether to hold cash or bonds depends only on the liquidity risk of the bank between the moment the bond is bought and the moment it matures. Consequently, our paper could potentially be of interest to any central bank conducting similar open market operations and in need to evaluate in real time the evolution of the liquidity risk affecting their financial sector in general, or their commercial banks individually.

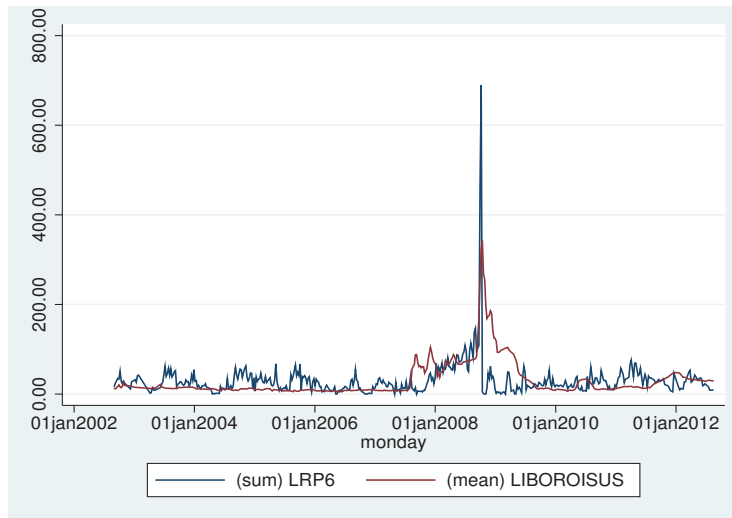


Figure 2.1: Spread between rate bids for ST Bonds and deposit rate vs LIBOR OIS spread (in basis points)

Figure 1 is quite informative. This chart depicts the evolution of two different variables from 2002 to 2012. On one side, the red curve represents the famous LIBOR-OIS spread², which we consider an imperfect measure of liquidity risk but a good first approximation for the liquidity stress in international financial markets. On the other side, the blue curve represents the difference between the average interest rate asked by Chilean commercial banks to buy the short-term bonds of the BCCh and the deposit rate at the BCCh where banks can leave their reserves

²The LIBOR-OIS spread is defined as the difference between the 3-month LIBOR rate at which banks borrow unsecured funds from each other in the London wholesale money market and the Overnight Indexed Swap which is roughly equivalent to an overnight rate rolled-over every day for 3 months.

and have access to them whenever they want. At first glance, it appears that there is correlation between periods of international liquidity stress and an increase in the premium asked by banks on those bonds, at least during the 2007-08 financial turmoil. This seems to confirm our initial intuition that banks ask for a higher liquidity premium on the BCCh bonds to give up their cash if they anticipate a higher probability to face a liquidity problem.

In practice, how can we use those auctions to measure liquidity risk in the financial sector? The first step is to build a simple model to understand, and quantify, how the possibility of liquidity shock at the bank level affects their bidding strategies in a bond auction similar to the one conducted by the BCCh. In terms of methodology, the idea is to combine the theoretical research in multi-unit auctions initiated by Wilson (1979) with the literature on liquidity crises initiated by Diamond and Dybvig (1983). By itself, this would fill a hole in the theoretical literature, as most of the articles discussing liquidity crises assume exogenous (deterministic or stochastic) asset returns. On the contrary, the auction framework will allow us to endogenize the interest rate and understand fully how it is determined. The model will therefore contribute to both the financial literature and the auction theory by showing how the possibility of a liquidity shock affects bidders' strategies and the auction outcome. To address these questions we develop a 3-period model. In the first period, there is an auction where a central bank sells bonds that mature in the third period. Before the auction takes place, banks (the buyers) discover their own private liquidity risk and the distribution of this risk across bidders. In the second period, an idiosyncratic liquidity shock materializes for some banks, which makes those holding bonds illiquid as their cash is invested in the bonds. This allows us to obtain theoretical bidding strategies as a function of the probability of having a liquidity shock. Intuitively, a bank with a higher probability to get the liquidity shock should be inclined to ask for a higher rate to insure itself against the shock if it is awarded some bonds in the auction. We develop two versions of the model, one with no insurance against the liquidity shock and another one with a Central Bank acting as a lender of last resort to see how the bidding behavior of the banks is affected by this policy.

The second step is to use the theoretical bidding strategies of the banks from the model and the dataset containing all the details of the bond auctions conducted by the BCCh between 2002 and 2012 to estimate the distribution of liquidity risk across Chilean banks and its changes over time. To perform those estimations we build on the literature on structural econometrics in auctions reviewed extensively for instance in Paarsch and Hong (2006). Besides, once the parameters of the distribution are estimated for each period, this method also allows us to retrieve for each bank participating in the auction-its probability to be hit by a liquidity

shock during the duration of the bond just by inverting its bid thanks to the theoretical bidding function obtained in the model. In the end, this helps us create an interesting measuring tool for Central Banks conducting similar open market operations and in need of evaluating in real time the evolution of the liquidity risk affecting their financial sector in general or each of its banks more precisely.

The paper is organized as follows. Section 2 explains how open market operations in general and bond auctions in particular are conducted at the BCCh. In this section we also describe our unique dataset collected with the help of the BCCh containing all the open market operations details for a whole decade. Section 3 presents the theoretical model of the auction in two different settings. The estimation of the distribution of liquidity risk in the Chilean financial sector across time is discussed in section 4. Section 5 concludes.

2.2 Open Market Operations at the Central Bank of Chile

2.2.1 Liquidity Management Framework at the BCCh

Like many central banks around the World, the BCCh has adopted an inflation targeting framework for its monetary policy. It started with a partial inflation targeting framework in 1990 and moved to a full adoption, in combination with a flexible foreign exchange regime, in September 1999. To meet its inflation target, the central bank uses the overnight nominal interest rate as its main policy instrument. It sets a notional level for the monetary policy rate (MPR), and then adjusts the quantity of money in the market to bring the overnight interbank interest rate around that level. It also offers permanent overnight deposit (FPD) and lending facilities (FPL) to Chilean commercial banks with a view to keeping the interbank lending rate close to the MPR. Like the Fed, the ECB or the Bank of England, the BCCh can inject money via repo operations. However, unlike those central banks, this is not the main tool for liquidity management³ in Chile. Indeed, because of important capital inflows in the past and the constitution of large foreign exchange reserves in the 1990s before the adoption of a fully flexible foreign exchange rate regime, the BCCh controls the quantity of money in the economy not by injecting money every week in the financial sector but by draining money from it. Thus, the adjustment operations (like the repo operations) employed to inject money in the

³Note that in this subsection, the term liquidity is used not to define the ability for an agent to settle her obligation as in the introduction or the rest of the article, but to define the management by the central bank of the quantity of money in circulation in various markets or more generally in the economy.

financial sector if necessary are used more sporadically and mainly for fine-tuning.

The main operations of the BCCh are therefore the regular structural Open Market Operations performed weekly or sometimes even bi-weekly to drain money from the banking sector. Those operations take the form of auctions where the BCCh issues different types of bonds and sells them to commercial banks. These bonds are short-term notes (PDBC) due in 30 to 360 days, nominal bonds with maturities of 2, 5 and 10 years (BCP2, BCP5 and BCP10 respectively) and inflation-indexed bonds with maturities of 5 and 10 years (BCU5 and BCU10).

Those various bonds can be purchased by agents authorized to participate in the primary market. In practice, in 2012, the participants to these auctions included twenty-three banks, four pension fund administrators, the unemployment fund administrator, three insurance companies and four stock brokers.

Among the bonds sold by the BCCh, PDBC (the short term notes) are the most heavily used to manage and regulate the quantity of money in circulation in the financial system within a given month or from one month to the next. The auction schedule for these notes is announced monthly, when the Bank operations calendar for the month is made public. The program planning takes into account the liquidity demand forecast, maturing issues from previous periods, strategies for complying with reserve requirements and seasonal factors affecting liquidity in the period.

The reader interested in more details in the liquidity management by the BCCh should consult the document published by the central bank (2012) on this topic.

2.2.2 Bond Auction Format at the BCCh

To sell short term bonds (which are the one that will be of interest in the last section of this paper), the BCCh carries out mainly multi-unit uniform auctions. The auctions are multi-unit since a fixed number of identical units of a homogenous commodity (bonds) are sold, and uniform since all winners of the auction receive the same interest rate on the bonds they buy, regardless of their actual bid.

In this type of auction, the BCCh reveals first the volume of bonds it wants to sell to the bidders. Then, each participant submits to the BCCh the minimum rate at which they would accept to buy a given volume of PDBC. In the award procedure, the BCCh ranks the bidders' offers by interest rate, ranking the bids from the smallest to the highest rates and awarding bonds to the smallest until the quantity being auctioned is totally allotted. The results are announced at the close of the auction and all the banks awarded with bonds get the cut off rate (the highest rate at which bonds have been awarded). The BCCh retains the option to award a different amount than scheduled, which in the case of bonds is $\pm 20\%$ of the amount auctioned, or to declare unilaterally the auction as deserted because

the rates asked by banks are too high. We have not noted any occurrence of such a decision in our dataset; however we believe that it is a tool for the BCCh to cancel an auction in case it suspects collusion between banks to obtain higher interest rates.

2.2.3 Dataset of Open Market Operations (2002-2012)

Our dataset contains all the details of the open market operations of the BCCh from September 2002 to august 2012. In particular, it consists of all the bidding information, in every bond auction conducted by the BCCh during that period. The information includes the total volume of bonds allotted by the central bank in each auction, the marginal (or cut-off) rate and more importantly the bidders' identities and the rates asked by each bidder. This dataset is not publicly available and usually only the information on the total volume allotted and marginal rates are available on the BCCh web site.

Year	PDBC30	PDBC90	PDBC180	PDBC360	BCP2	BCP5	BCP10
2005	335	348	0	0	153	221	235
2006	569	560	0	0	57	86	37
2007	427	406	9	13	86	90	12
2008	534	315	133	133	217	213	113
2009	574	550	316	81	137	73	0
2010	751	279	112	127	228	261	0
2011	668	446	96	6	89	216	176
2012	500	175	35	0	123	178	150
Total	4392	3095	701	360	1090	1338	723

Table 2.1: Descriptive statistics: Number of bond auctions per year

Table 1 and table 2 summarize respectively the descriptive statistics on the number of auctions performed by the BCCh and on the marginal rates for all types of bonds sold from 2005 to 2012.

2.3 Model of a Bond Auction with Liquidity Risk

Our modelling idea is to replicate the bond auctions performed by the Central Bank of Chile in the Chilean banking sector as closely as possible.

Year	PDBC30	PDBC90	PDBC180	PDBC360	BCP2	BCP5	BCP10
2005	3.83 (0.45)	4.19 (0.43)	4.04 (0.74)	4.32 (0.67)	5.05 (0.74)	5.95 (0.56)	6.63 (0.38)
2006	4.93 (0.26)	5.09 (0.16)	5.25 (0.11)	5.43 (0.12)	5.99 (0.23)	6.47 (0.33)	6.85 (0.39)
2007	5.33 (0.34)	5.41 (0.39)	5.47 (0.44)	5.48 (0.46)	5.77 (0.49)	6.10 (0.42)	6.35 (0.36)
2008	7.10 (0.86)	7.18 (0.83)	7.20 (0.83)	7.05 (0.77)	7.07 (0.73)	6.96 (0.64)	6.95 (0.56)
2009	1.96 (2.22)	1.69 (1.89)	1.57 (1.56)	1.86 (1.19)	2.91 (0.76)	4.64 (0.56)	5.35 (0.58)
2010	1.53 (1.01)	1.76 (1.11)	2.09 (1.17)	2.72 (1.03)	3.72 (0.63)	5.11 (0.19)	5.86 (0.22)
2011	4.76 (0.77)	4.78 (0.60)	4.82 (0.50)	4.88 (0.50)	5.19 (0.61)	5.56 (0.57)	5.83 (0.54)
2012	5.00 (0.07)	4.93 (0.07)	4.87 (0.13)	4.80 (0.23)	4.96 (0.31)	5.21 (0.28)	5.45 (0.23)

* Standard deviations in parentheses

Table 2.2: Descriptive statistics: marginal rates

2.3.1 The Basic Setup

There are 3 dates, $t \in \{0, 1, 2\}$. There is an institution⁴ wanting to raise an amount of money Q by selling a number Q of 2-period bonds at price $p = 1$ in an auction in $t = 0$. The money is then paid back with interest (determined in the auction) to the bond buyers in $t = 2$.

There are N potential buyers of the bonds (i.e. banks) maximizing profits. At the beginning of $t = 0$, each bank $i \in [1, N]$ gathers one unit of cash at interest rate r_0 normalized to 0. After each bank has gathered its funds, the liquidity risk of their funding is privately revealed to each of them. This risk is represented by p_i the probability for bank i to be subject to an idiosyncratic liquidity shock in $t = 1$. This p_i is a random variable independently drawn by each bank from a common distribution F defined on the interval $[0, 1]$, with positive and continuous pdf f .

The auction in $t=0$

The auction taking place in $t = 0$ is a uniform multi-unit auction. First, the bond seller announces the number Q of bonds it wants to sell. Second, the banks place their bids. A bid consists of the minimum net interest rate r_i at which the bank is willing to accept to buy a bond. The bank proposing the smallest rate gets a bond, then the bank proposing the second smallest rate gets the bond, then the third, etc. until the total volume Q allotted by the bond seller is reached. The cut-off

⁴This institution can be interpreted as a Central Bank draining liquidity temporarily from the banking sector (as it will be the case in the application in the next section).

rate r_s is the highest rate at which the supply of bonds Q is exhausted. A uniform auction is defined by the fact that all winning bidders receive the market clearing cut off interest rate r_s in $t = 2$. To say it differently, bank i is awarded 1 bond if it bids $r_i \leq r_s$ and 0 otherwise.

Finally, the banks which are not awarded any bonds in the auction invest their cash in a risk free technology yielding an interest rate r_L per period. This risk free investment can be thought as the overnight deposit facility of the central bank where the money can be withdrawn by the bank at any time (the rate r_L would be fixed by the central bank) or it can simply be thought as cash with 0 interest rate. Without loss of generality and in order to simplify the model's notation, we just assume $r_L = 0$.

The liquidity shock in $t=1$

In $t = 1$, if the idiosyncratic liquidity shock does not materialize for bank i , the bank's creditors do not withdraw the cash from the bank in the intermediate period and wait until $t = 2$ to take it back. If the idiosyncratic liquidity shock materializes, bank i has to give back the cash to its creditors immediately in $t = 1$. However, in this case, if the bank has invested in a bond, it has no cash left to finance the withdrawal.

If there is no way to raise some cash in $t = 1$ we assume that the bank goes bankrupt and makes a loss equivalent to the money it can not reimburse to its creditors. In a second step, we also explore what changes if there exists a lender of last resort policy where the bank can borrow from the central bank lending facility at a fixed high rate r_H if it becomes illiquid in $t = 1$.

Collection of profits in $t=2$

For the banks which have not suffered the liquidity shock in the intermediate period, in $t = 2$, the banks collect their profits r_s from the maturing bonds if they have some and reimburse their creditors.

2.3.2 Bidding Strategy of the Banks in the Bond Auction

Let's say that the seller wants to sell $Q = k + 1$ bonds in the auction. It means that among the N potential buyers, $k + 1$ bidders are served: one is the cut-off bidder who has proposed r_s in the auction and k bidders have proposed rates smaller than r_s . We assume that $N \geq 2$ and that $k \in [0, N - 2]$ so that there is always at least one bidder that gets nothing in the auction, otherwise all banks would have an incentive to bid a rate equal to infinity which would make the problem trivial and uninteresting.

The strategies available to the players are the rates they bid. Since we focus on symmetric equilibria, we suppose that all banks use the same bidding strategy β which is a function of their probability to get an idiosyncratic shock in $t = 1$. Let's determine this function $\beta(p_i)$.

The maximization problem of representative bank i bidding r_i when all other banks bid according to the symmetric increasing function $\beta(p_i)$ can be written as:

$$\begin{aligned} \max_{r_i} & Pr(r_i < \beta(Y_k)).[(1 - p_i)E(\beta(Y_k)/\beta(Y_k) > r_i) - p_i] \\ & + Pr(\beta(Y_k) < r_i < \beta(Y_{k+1})).[(1 - p_i)r_i - p_i] \\ & + Pr(r_1 > \beta(Y_{k+1})).0 \end{aligned} \quad (2.1)$$

where Y_k is the k^{th} order statistic attached to the random variables p_j sorted into ascending order, for $j \in [0, N - 1]$, i.e. the k^{th} highest among the $N - 1$ random variables.

The objective function maximized in equation (2.1) is easy to understand. The first line represents the expected profits if the bid of bank i is smaller than the cut off rate (i.e. the k^{th} highest rate proposed, bank i excluded) and is therefore awarded a bond in the auction: in this case, either the shock does not materialize (with probability $1 - p_i$) and the bank makes a profit equal to the expected cut off rate given that this rate is higher than its bid, or the shock materializes (with probability p_i) and it makes a loss equal to 1. Similarly, the second line represents the expected profits if the bid of bank i is the cut off rate: in this case, either the shock does not materialize and the bank makes a profit equal to its bid, or the shock does materializes and it makes a loss equal to 1 again. The third line is the expected profits if bank i is not awarded any bond in the auction. It is equal to 0 as the bank does not make any investment and is indifferent between giving back the money to its creditors in $t = 1$ or $t = 2$.

Since β is continuous and increasing in p_i , the maximization problem can be rewritten as:

$$\begin{aligned} \max_{r_i} & (1 - Pr(Y_k < \beta^{-1}(r_i))).[(1 - p_i)E(\beta(Y_k)/Y_k > \beta^{-1}(r_i)) - p_i] \\ & + Pr(Y_k < \beta^{-1}(r_i) < Y_{k+1}).[(1 - p_i)r_i - p_i] \end{aligned}$$

where all the probabilities and conditional expectations are easily computable. Taking the first order condition with respect to r_i and imposing symmetry, such that $r_i = \beta(p_i)$, we obtain after some manipulations the following differential equation:

$$\beta'(p_i) + \beta(p_i) \frac{(N - k - 1)f(p_i)}{1 - F(p_i)} = \frac{(N - k - 1)f(p_i)}{1 - F(p_i)} \frac{p_i}{1 - p_i} \quad (2.2)$$

Equation (2.2) is among the few differential equations that have closed-form solutions. Following the method proposed by Boyce and Di Prima (1977), we can obtain the following equilibrium-bidding function:

$$\beta(p_i) = \frac{-1}{(F(p_i) - 1)^{N-k-1}} \int_{p_i}^1 (F(p) - 1)^{N-k-1} \frac{p}{1-p} f(p) \frac{N-k-1}{1-F(p)} dp$$

which is indeed increasing in p_i and such that $\lim_{p_i \rightarrow 1} \beta(p_i) = +\infty$.
 With p_i distributed uniformly on $[0, 1]$ the bidding function is depicted in figure 2.

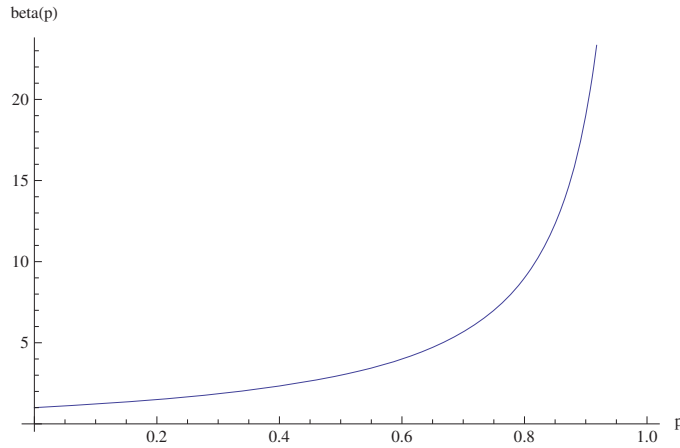


Figure 2.2: Bidding strategy with a uniform distribution, $N = 8$ and $Q = 6$

The intuition behind this result is quite simple. Because there is no other way for them to insure themselves against the shock when they hold a bond, banks compensate this risk by asking a higher rate on those bonds. The fact that $\lim_{p_i \rightarrow 1} \beta(p_i) = +\infty$ is understandable as the bank with $p_i = 1$ is sure to have the shock in $t = 1$ so it does not really want to buy the bond since it is sure to make a loss if it does.

An interesting policy implication of the model for the moment is that we have shown that an increase of the liquidity risk in the financial sector (for instance with a shift of its distribution towards a riskier one) can have a huge impact on the rates asked by banks in a bond auction even if there isn't any change in the fundamentals of those bonds or any solvency problem whatsoever.

2.3.3 Introduction of a Lender of Last Resort in the Framework

Since the concept was introduced and developed by Thornton (1802) and Bagehot (1873) in the 19th century, most central banks have been acting as lenders of last

resort in order to save illiquid but solvent banks from bankruptcy by allowing them to borrow from the discount window whenever it seems necessary for the stability of the financial sector. Given the importance taken by this central bank function, we thought it could be interesting to introduce it in our model in order to see its impact on the strategies of the banks in the auction.

In our model, a lender of last resort policy can be represented by the fact that a bank holding a bond subject to a liquidity shock in $t = 1$ can now borrow from the discount window of the central bank to reimburse its creditors and avoid bankruptcy. In this case, in $t = 2$, the bank collects its profits from the bond but it also has to repay its loan with an interest rate r_H fixed by the central bank.

Therefore, the maximization problem of bank i bidding r_i^L when every other bank plays according to an increasing symmetric bidding function β_L is:

$$\begin{aligned} \max_{r_i^L} & Pr(r_i^L < \beta_L(Y_k)).[(1 - p_i)E(\beta_L(Y_k)/\beta_L(Y_k) > r_i^L) + p_i(E(\beta_L(Y_k)/\beta_L(Y_k) > r_i^L) - r_H)] \\ & + Pr(\beta_L(Y_k) < r_i^L < \beta_L(Y_{k+1})).[(1 - p_i)r_i^L + p_i(r_i^L - r_H)] \\ & + Pr(r_i^L > \beta_L(Y_{k+1})).0 \end{aligned} \quad (2.3)$$

, where again Y_k is the k^{th} order statistic attached to the random variables p_j sorted into ascending order for $j \in [0, N - 1]$.

The main difference between objective functions (2.1) and (2.3) lays in the fact that, if bank i suffers a liquidity shock, it can avoid bankruptcy by borrowing from the central bank which modifies its expected profits as it can keep the bond until it matures. Since β_L is continuous and increasing in p_i , the maximization problem becomes:

$$\begin{aligned} \max_{r_i^L} & (1 - Pr(Y_k < \beta_L^{-1}(r_i^L))).[E(\beta_L(Y_k)/\beta_L(Y_k) > r_i^L) - p_i r_H] \\ & + Pr(Y_k < \beta_L^{-1}(r_i^L) < Y_{k+1}).[r_i^L - p_i r_H] \end{aligned}$$

where all the probabilities and conditional expectations are easily computable. Taking the first order condition with respect to r_i^L and imposing symmetry, such that $r_i^L = \beta_L(p_i)$, we obtain after some manipulations the following differential equation:

$$\beta_L'(p_i) + \beta_L(p_i) \frac{(N - k - 1)f(p_i)}{F(p_i) - 1} = p_i r_H \frac{(N - k - 1)f(p_i)}{F(p_i) - 1}$$

Using the same method as before to solve the differential equation, we obtain

the following equilibrium-bid function:

$$\beta_L(p_i) = p_i r_H + \frac{r_H}{(1 - F(p_i))^{N-k-1}} \int_{p_i}^1 (1 - F(p))^{N-k-1} dp$$

which is increasing in p_i and such that $\lim_{p_i \rightarrow 1} \beta_L(p_i) = r_H$.

With p_i distributed uniformly the strategy is depicted in figure 3.

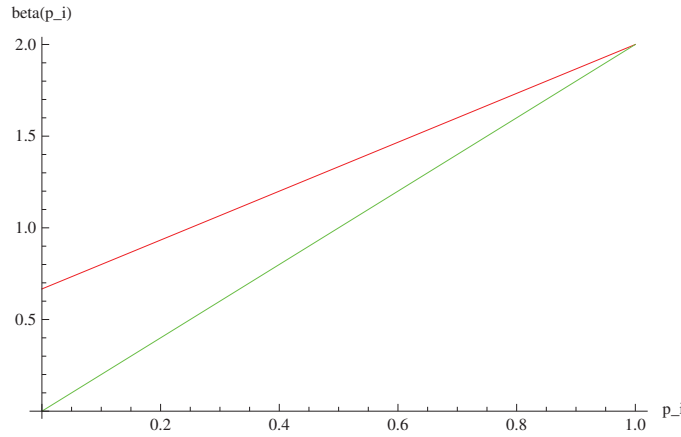


Figure 2.3: Bidding strategy (in red), $p_i \cdot r_H$ (in green), $N = 8$ and $Q = 6$, $r_H = 2$

As can be seen in figure 3, unlike the previous case, the rates are now bounded by the rate r_H charged by the central bank for its loan in $t = 1$. This makes sense because even if it is costly to use the lending facility, the lender of last resort provides an ex post insurance to the banks buying bonds. For instance, the bank with probability one to get the shock is now indifferent between getting the bond in the auction at the rate r_H and not getting the bond because in any case its profit is going to be 0.

In fact, this is an interesting side result in terms of policy implications, as it shows that the introduction of a lender of last resort willing to lend cash to banks suffering a liquidity shock can have a huge impact on the strategies of the bank and on the result of a bond auction by bounding the rate with its lending rate.

However, although the LLR extension of the model is theoretically interesting, we were unable to fit it to our data, since rates bid during the crisis exceeded the rate asked by the LLR. It is therefore not used in the following empirical part.

One possible explanation is the fact that in reality banks are not completely sure whether the central bank will intervene as an LLR and lend them money in case of illiquidity or not. A possible extension left for future research would be to introduce a non zero probability of the central bank not acting as a LLR in the model. This probability perceived by the banks could also vary through time and could be interesting to estimate.

2.4 Structural Analysis of the Bond Auctions of the BCCh

The objective of this section is to use the theoretical bidding strategies of the banks from the previous section in order to put some structure on the dataset from BCCh bond auctions by estimating the parameters of the distribution of the liquidity shock probabilities across the Chilean Banks and its changes over time.

2.4.1 Choice of the Distribution Form

In order to be able to perform this estimation, we need to assume that the distribution of the liquidity shock probabilities across banks takes a particular distribution form. We have chosen the Kumaraswamy distribution (introduced by the author of the same name in 1980), as it is characterized by several properties that are needed in our analysis.

As described in Jones (2009), the Kumaraswamy distribution is defined on $[0, 1]$ which perfectly suits our needs since we are interested in the distribution of a probability. Second, it is particularly straightforward with only two parameters a and b and has a very simple PDF: $f(x; a, b) = abx^{a-1}(1 - x^a)^{b-1}$ and CDF: $F(x; a, b) = 1 - (1 - x^a)^b$. Nonetheless, this density function is very flexible and can take various types of shapes (unimodal, uniantimodal, increasing, decreasing, monotone or even constant depending on the values of the parameters⁵) as can be seen in figure 4. That is why we believe that this distribution form does not impose too much restriction on the data.

2.4.2 Data Selection

In terms of data, in order to estimate the liquidity risk distribution, we decided to restrict ourselves to the observed bids from the 30-day bond (PDBC) auctions

⁵In that sense the Kumaraswamy distribution is quite similar to the Beta distribution but with a simple explicit formula for its distribution function not involving any special functions

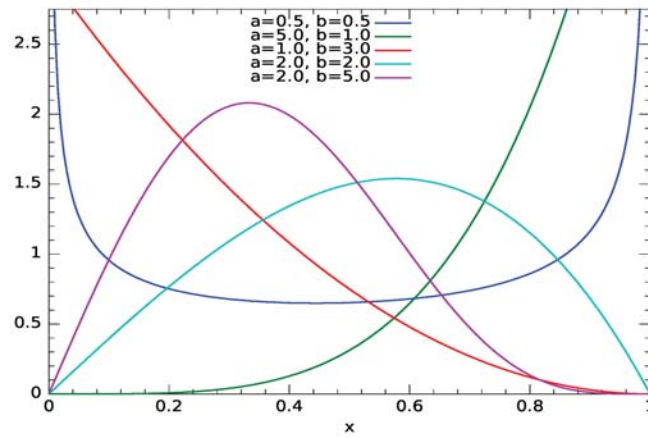


Figure 2.4: The Kumaraswamy probability density function for different parameters

from 2002 to 2012 and put aside the other auctions of the BCCh. The main reason behind this choice is simply the shorter maturity of those bonds, which makes them a better substitute for cash or reserves at the central bank, their main difference resulting from the expectation of a liquidity shock in the following month which would suit perfectly our model. Another reason to use this auction is that we believe that these short term bonds are more likely to be kept in the balance sheets of banks (as it is assumed in our model) until they mature than the long term bonds and not resold in a secondary market. Finally, given that they are really short-term, we can also assume that there is no inflation premium asked as it would be the case with longer maturity bonds.

More practical reasons on our choice of data include the fact that the 30-day PDBC auctions are the most frequent open market operations of the BCCh as can be seen in table 3 (with 4392 auctions performed between 2005 and 2012), while at the same time being the auctions with the largest number of participants (for more details, see the summary statistics of the PDBC auctions from 2002 to 2012 in table 3).

We have also decided to group auctions of the same month together to get enough data per period of estimation. We know that this is a strong assumption because it means that the liquidity risk distribution varies every month but is constant in a given month. However, within the model framework it could simply

Numbers of periods-months	118
Average Number of Bidders / period	48
Average Number of Winning Bidders / period	29
Average Rate Bid	3.62
Average Cut Off Rate	3.61
Average Standard deviation of bids / period	0.25

Table 2.3: Summary Statistics for 30-day PDBC Auctions

be interpreted with banks drawing a new liquidity risk from the new distribution at the beginning of each month.

Finally, instead of using the rate bids alone as done in the model, in the empirical section of the paper we use the difference between the rate bids and the deposit rate at the central bank at the time of the auction (remember that for the sake of simplicity we assumed $r_L = 0$ in the model). Indeed, we think that the liquidity premium should be observed between those two variables and not between the rate bids and simply cash earning 0% interest rate, because this spread takes into account the variations in the monetary policy rate (the deposit facility follows exactly the MPR with only minus 0.25 percentage point) inducing some volatility in the bids not related to the evolution of the liquidity risk.

2.4.3 Estimation of the Liquidity Risk Distribution

In this section we perform a structural estimation of the parameters of the distribution for each period (each month) of our sample using our theoretical results from the previous section. We use the maximum likelihood method to estimate the parameter vector $\theta = (a, b)$ according to equation 2.4.

$$\max_{a,b} \prod_{i=1}^N Pr(p_i = \beta^{-1}(b_i, a, b)) \quad (2.4)$$

Using the data previously described, we can estimate the parameters of the distribution for each period of our sample. Figure 5 presents the estimated density functions at various interesting points in time during the studied decade.

As can be seen in Figure 5, at the beginning of 2004 the density function has a decreasing form and is definitely skewed to the right before becoming more and more symmetric in 2005. Then, making a big jump in time, we can see that during the worst months of the financial turmoil in september and october 2008 after the bankruptcy of Lehman Brothers, the density function definitely moved to the

right revealing what appears to be a huge increase in liquidity risk in the Chilean financial sector.

In response to this risk and to the deterioration of international financial markets, the BCCh decided to increase the quantity of money in the domestic financial system at the end of 2008 and 2009. They used part of their international reserves and some swap agreements with the Fed to lend some U.S. dollars to the Chilean commercial banks in need. Aside from that the BCCh, only for a strict six-month period during the crisis, put in place regular repo operations aimed at injecting pesos every week. To encourage participation in those repo operations they expanded the eligible collaterals. At the beginning of 2009, the BCCh modified its debt schedule by suspending the issue of five-year peso bonds (BCP5), five-year and ten-year UF bonds (BCU5 and BCU10) in the primary market in order to reduce the usual money draining resulting from these open market operations⁶. All these measures seem to have worked well and resulted in an enormous decrease in the liquidity risk of the financial sector, as can be seen in density function for May 2009. All in all, it seems that the evolution of the estimated distribution captures well the main episodes of liquidity stress of the last decade in the Chilean banking sector.

Besides the estimation of PDF of liquidity risk across banks for each period, we are also able to infer the probability to be hit by a liquidity shock during the duration of the bond for all the banks participating in the auction just by inverting their bids thanks to our theoretical bidding function obtained in the previous section. So, not only the method proposed in this paper provides to the central bank a way to monitor the liquidity risk of its financial sector as a whole but also to monitor each of the Chilean commercial banks participating to its bond auctions⁷.

2.5 Conclusion

In the end, the paper proposes a ready-to-use tool for all central banks conducting similar open market operations to the BCCh and in need of evaluating in real time the evolution of the liquidity risk affecting their financial sector in general or their commercial banks individually. To do that, we modeled the bond auction

⁶This gives us another reason not to use the auctions for those bonds to do our estimations: these changes decided by the BCCh made those auctions unusable to monitor the liquidity risk when they became an instrument of the central bank to reduce this liquidity risk.

⁷For confidentiality reasons concerning the Chilean banks, it is impossible to show individual results for any particular bank in the present paper. Given the small number of banks and the structure of the banking sector in Chile, it would also be difficult to show them with hidden identities without making them easily recognizable.

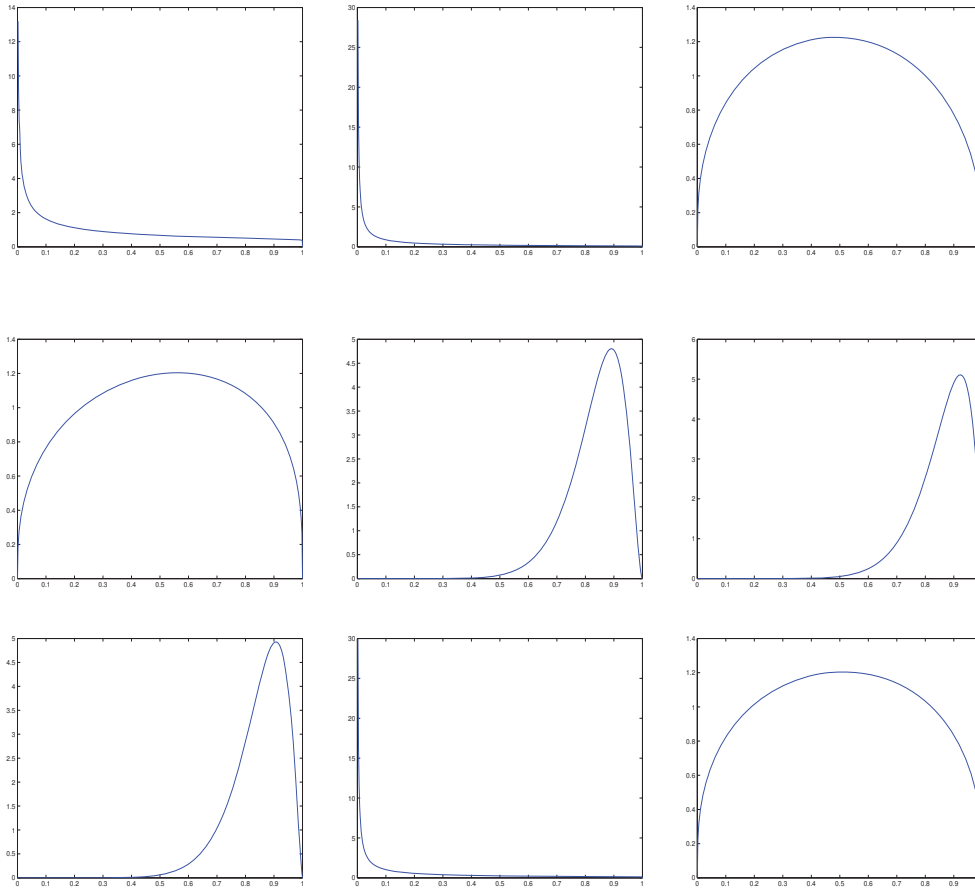


Figure 2.5: From left to right and top to bottom, estimated PDF for January, May and August 2004, January and November 2005, September and October 2008, May 2009, August 2010.

conducted by the BCCh and used the theoretical results obtained to estimate the parameters of the distribution of liquidity risk in the Chilean banking sector for each month in the period 2002 to 2012.

In addition to this central goal, we believe that this paper also contributes to the financial literature as well as to the auction theory literature by proposing a simple model of bidding behavior in a multi-unit auction where bidders are affected by liquidity risk. The model carefully explores the link between liquidity risk in the financial sector and bond rates. Although the model is designed in a very simple

way because it was constructed mainly to be used in the structural estimation, it also has some interesting policy implications. The comparison of our two different settings is particularly enlightening as it shows that the introduction of a central bank acting as a lender of last resort willing to lend cash to banks suffering a liquidity shock can have a huge impact on the bidding strategies of banks by capping their rate with the lending rate it proposes at the discount window.

Finally, we think that it could be interesting to extend the analysis of the model presented in this paper and to develop it further. A possible extension could be for instance to replace the lender of last resort in the second period by a careful modeling of a secondary market in which banks hit by the liquidity shock would be able to sell the bond to banks that were not awarded bonds in the auction in $t = 1$ the initial period. The type of insurance provided by this secondary market would depend on the market liquidity and therefore on which banks have been awarded bonds in the auction and which banks will get the shock. Given that there is a finite number of bidders, this feature would result in aggregate uncertainty in the model. This could lead to a result analogous to the cash-in-the-market concept developed in Allen and Gale (1994) where financial institutions are forced to sell assets in order to obtain liquidity, and because the supply and demand of liquidity are inelastic in the short-run, a small degree of aggregate uncertainty could cause large fluctuations in asset prices. In this case, the rates asked by investors in the auction may be distorted by their own liquidity risk but also by the aggregate uncertainty coming from the secondary market. In any case, these features will surely have some interesting implications on the bidding strategies of the banks. We leave this possible extension for future research.

Appendix

In this appendix we prove that the increasing symmetric bidding function in the LoLR case is a Nash Equilibrium. The proof for the no insurance case is analogous. In order for the bidding function obtained (necessary condition) to be a Nash Equilibrium, the second order (sufficient) condition needs to be satisfied. The

maximization problem of bank i is:

$$\begin{aligned} \max_{r_i} \Pi = & \int_{\beta^{-1}(r_i)}^1 \beta(p) \frac{(N-1)!}{(K-1)!(N-1-k)!} f(p) F(p)^{K-1} (1 - F(p))^{N-1-K} dp \\ & - [1 - \sum_{j=k}^{N-1} \binom{N-1}{j} (F(\beta^{-1}(r_i)))^j (1 - F(\beta^{-1}(r_i)))^{N-1-j}] (p_i r_h) \\ & + \frac{(N-1)!}{(N-1-K)!K!} F(\beta^{-1}(r_i))^K (1 - F(\beta^{-1}(r_i)))^{N-1-K} (r_i - p_i r_h) \end{aligned}$$

The derivative of the profit with respect to r_i is given by:

$$\begin{aligned} \Pi_{r_i} = & (r_i - p_i r_h) \frac{f(\beta^{-1}(r_i))}{\beta'(\beta^{-1}(r_i))} \frac{(N-1)!}{(N-2-K)!K!} F(\beta^{-1}(r_i))^K (1 - F(\beta^{-1}(r_i)))^{N-2-K} \\ & + \frac{(N-1)!}{(N-1-K)!K!} F(\beta^{-1}(r_i))^K (1 - F(\beta^{-1}(r_i)))^{N-1-K} \end{aligned} \quad (2.5)$$

At the symmetric solution $p_i = \beta^{-1}(r_i)$. Then when maximizing profits the first order condition when imposing symmetry yields:

$$\begin{aligned} & \frac{(N-1)!}{(N-1-K)!K!} F(\beta^{-1}(r_i))^K (1 - F(\beta^{-1}(r_i)))^{N-1-K} = (2.6) \\ (\beta^{-1}(r_i) r_h - r_i) & \frac{f(\beta^{-1}(r_i))}{\beta'(\beta^{-1}(r_i))} \frac{(N-1)!}{(N-2-K)!K!} F(\beta^{-1}(r_i))^K (1 - F(\beta^{-1}(r_i)))^{N-2-K} \end{aligned}$$

By substituting (2.6) into (2.5) we get:

$$\Pi_{r_i} = \frac{f(\beta^{-1}(r_i))}{\beta'(\beta^{-1}(r_i))} \frac{(N-1)!}{(N-2-K)!K!} F(\beta^{-1}(r_i))^K (1 - F(\beta^{-1}(r_i)))^{N-2-K} (\beta^{-1}(r_i) r_h - p_i r_h)$$

Suppose all other banks are bidding according to the symmetric bidding function $\beta(p)$. Also suppose that bank i decides not to play according to the symmetric equilibrium, but instead chose some $r_i \neq \beta(p_i)$ such that $r_i = \beta(q)$ with $q \neq p_i$. Then for any q' such that $q' > p_i$ we have $\Pi_{r_i} > 0$, and accordingly for any q'' such that $q'' < p_i$ we have $\Pi_{r_i} < 0$. We have $\Pi_{r_i} = 0$ only for $q = p_i$ i.e. when bank i bids according to the symmetric bidding function. Thus, the symmetric bidding function $\beta(p_i)$ is indeed an equilibrium.

Chapter 3

Strategic Complementarities and Corruption

3.1 Motivation and Literature

The past forty years, policy makers and academics alike have shown deep interest in understanding corruption, as countries of every sort- developing, transition countries as well as major economies- have suffered corruption scandals. Most economists accept that corruption can occur where rents exist, usually as a result of government regulation. Recent empirical research has focused on proving the theoretical hypotheses on corruption using cross-country data. Since collecting data on corruption is not an easy task most such data are country-level indices based on firm and household surveys and expert assessments, the corruption perception indices which are interpreted as measures of corruption experience. Such indices are the Transparency International's Corruption Perception Index (CPI)¹, the corruption index of the International Country Risk Guide (ICRG) and the World Bank's Control of Corruption index (WB)². Despite the subjective nature of the indices, many economists have used them, as the correlation between indices produced by different rating agencies is very high, suggesting that most observers more or less agree on how corrupt countries seem to be.

On the presence of corruption in the economic system and its consequences, Mauro (1995, 1996, 1998) argues that corruption is likely to occur where restrictions and government intervention lead to the presence of excessive profits or rents. Based on Kraay and Van Rijckeghem (1995) and Haque and Sahay (1996) Mauro argues that if civil servants are paid higher wages than similarly qualified private workers, there would be less corruption in the economic environment. On the consequences of corruption, Shleifer and Vishny (1993) go one step further, arguing that the structure of government institutions and of the political process are very important determinants of the level of corruption and that the illegality of corruption and the need for secrecy make it much more distortionary and costly than taxation. An interesting phenomenon, also discussed in the literature on corruption, is that otherwise similar countries (in terms of per capita GDP, geography, public sector

¹Can be found for years 2001-2013 at: <http://www.transparency.org/research/cpi/overview>

²Can be found at: <http://info.worldbank.org/governance/wgi/index.aspxhome>

wage structure, political history, even some cultural characteristics like language or religion) or even regions within a country, can differ greatly in the level of corruption in their economy.

Delmonte and Papagni (2007) perform a regional panel data analysis to study the determinants of corruption in Italy during 1963-2001, using statistics on crimes against the public administration at a regional level. Apart from determining the corruption inducing factors in the Italian economy (including economic variables and political and cultural influences), they research the differences of corruption levels between the Italian North and South. They find that indeed the levels of corruption in the South and North of Italy are significantly different, and claim that these differences might be attributed to cultural differences, differences in the modernization process, greater state intervention in the South as well as the degree of competition of political parties in the two regions.

An interesting example of countries sharing similar characteristics but having big corruption differences is the one of Chile and Argentina; despite their many common characteristics (comparable GDP per capita level, political history, language, public sector wages, shared borders), their Perception Corruption Indices vary greatly, with Argentina scoring 34 (high corruption) and Chile scoring 71 (low corruption)³ in 2013. Although the literature has not tried to address this interesting phenomenon to our knowledge, Boruchowicz and Wagner (2011) discuss the corruption differences between the Chilean and Argentinean police forces and argue that the longer and better training of the Chilean police, as well as its less strong bonds with the national political elite are the main reasons behind these differences. Although this paper focuses specifically on corruption in the police forces, as they represent public administration we deem it important to be mentioned here.

Taking as given the above, and mainly the "mystery" behind corruption differences between similar countries/regions-which is very often attributed to cultural reasons, as well as the fact that excessive rents can create corruption, we build a model with multiplicity of equilibria as its main feature. Thus, we build a simple model of procurement, in order to understand how economies with very common characteristics can vary greatly in their level of corruption. In our simple model where agents compete for a public good procurement by the government, managed by a potentially corrupt inspector, we find that multiple equilibria can arise, and more interestingly ones without any corruption and ones with full corruption, without imposing any assumptions or restrictions on either the agents' or the inspectors' preferences. Multiplicity of equilibria is a common feature of our paper with some other research on corruption. Andvig and Moene (1990), find multiplicity of equilibria in a dynamic model of corruption where both the agents and

³Results can be found at: <http://cpi.transparency.org/cpi2013/results/>

inspectors can propose/ask for bribes. However, the multiplicity of equilibria obtained in their model is based on inspectors' "cost of corruption", as they assume that inspectors' have different costs when involved in a corrupt act. In fact, in the extension where inspectors are identical the multiplicity of equilibria breaks down: there is either full corruption or no corruption, depending on the model's parameters. In our model however, the multiplicity of equilibria, which is found in both the one-shot and repeated game do not depend on any assumption on the agents' and inspectors' preferences.

Multiplicity of equilibria, is a common characteristic with other work on corruption as well, although used to explain questions different to ours; Emerson (2005) presents a model of the interaction between corrupt government officials and industrial firms, to show that corruption is antithetical to competition. Many equilibria arise, where one equilibrium is characterized by high corruption and low competition, and another is characterized by low corruption and high competition. However, Celentani and Ganuza (2001), derive different results. In the context of a procurement problem in which the procurement agent is supposed to allocate a project through an auction with two dimensional price-quality bids, while potential bidders have private information about their production costs, they find that it may be that corruption is higher in a more competitive environment.

In our model, the corruption monitoring power of the government depends on its monitoring efforts, however the higher the level of corruption in the economy, the harder it is for the government to trace which individuals are in fact corrupt, and subsequently punish them. This affects the equilibria of our one shot game, however it is more noticeable in the repeated game, as we find that inspectors might prefer to make loses in the initial period in order to increase the level of corruption in the economy and make it easier for themselves to be corrupt and make higher positive payoffs in the last period of the game. This can lead of course to persistence of corruption in the repeated game. As explained in Paldam (2006), this persistence is consistent with reality; using the Transparency International CPI, Paldam finds that countries transitioning from high corruption to low corruption normally take a couple of centuries to go through a corruption change of about 70%, with fast transitions of the "Asian type miracle" still taking half a century.

Theoretical explanations of this persistence phenomenon different to ours have been provided in the past; Sah (2005) attributes both big and persistent corruption differences across similar countries to individuals' perceptions of their environment. In a dynamic model where bureaucrats and citizens interact, revising their (identical) initial beliefs on the level of corruption in the economy, he finds that persistence of corruption is due to individuals' past experiences, affecting their current and future actions, which subsequently influence current and future realities. In Sah's model the reason of corruption persistence is the perception of one's environment,

whereas in ours it is the will to affect one's environment, to reach personal gains. The idea that past corrupt behaviour might be used by an agent as a tool to enforce present corrupt behaviour is also discussed in Tirole (1996), although within a very different context, the one of group reputation. The model treats group reputation as an aggregate of individual reputations, where a member's current incentives are affected by his past behavior and because his track record is observed only with noise, by the group's past behavior as well. Individual past behavior is imperfectly observed and past behavior of the member's group conditions the group's current behavior and therefore can be used to predict the member's individual behavior. The main results imply that the agents who were alive at the first period of the game have smeared their reputation, and thus have more incentives to engage in corrupt activities than if they had always behaved honestly.

In our paper, corruption is punished if found out by the government, which exerts effort to uncover corrupt individuals. However, at least for now the government's monitoring effort is considered exogenous, as we don't deal with the government's maximization problem. On the relationship between corruption and punishment, Di Tella Dal Bo E. and Dal Bo P. (2006), find that punishment lowers the returns from public office and reduces the incentives of high-ability citizens to enter public life, and that cheaper threats and more resources subject to official discretion are associated with more frequent corruption and less able politicians.

The rest of the paper is organized as follows: Section 2 presents the model and discussed the equilibria of the one-shot game. In Section 3 the repeated game is presented. Section 4 concludes our results.

3.2 The Model

A government wishes to procure a public good, and assigns the procurement procedure to inspector k , whose payoffs we normalize to 0 without loss of generality. There exist N agents in the economy, denoted by $j = 1, \dots, N$ competing for the project, and the worth of the project is exogenous and the same for everyone, denoted by Π .

The agents decide whether or not to offer a bribe to the inspector, and the inspector decides whether to accept or not, after observing all offers. The bribing offer is known only to the agent who proposes the bribe and the auctioneer. In case of corruption, the inspector receives a fraction z of the worth of the project, the remaining $1 - z$ fraction going to the corrupt agent, who gets the project with probability one. In case $m \in 1, N$ agents offer bribes, then they each win the

project with probability $\frac{1}{m}$.

The government is aware of the possibility of corruption, and thus exerts effort X to discover it. The probability of finding out a corrupt activity $Pr(\cdot, X)$ is increasing in X . For now, X is exogenous, and common knowledge to all players in the economy. In case a corrupt couple is found out, both the corrupt agent and the corrupt inspector, suffer punishment Ω , Ω exogenous, common knowledge and measured in the same units as Π .

3.2.1 One-shot Game

First, we discuss the one shot game. For simplicity, and again without loss of generality, we present here the case with one inspector and only two agents, hence $j = 1, 2$.

In order to derive the payoffs of the agents, we need first to specify two probability functions: the probability of getting the project $P(\cdot)$, and the probability of getting caught if corrupt $Pr(\cdot, X)$.

As mentioned before, in case only one agent offers a bribe and it is accepted, they win the project with probability 1. In case both offer and it is accepted, they win it with probability $\frac{1}{2}$ each. In case no one offers a bribe, they each get it with probability $\gamma < \frac{1}{2}$, which can be interpreted as the agents' talent, whereas if only one offers a bribe and it is not accepted, the one not offering receives the project with probability $\delta = 2\gamma < 1$. In case both offer a bribe and it is not accepted, no one is assigned the project. Summing up, we define the probability function of receiving the project $P(B_j, B_{-j}, B_k)$, where B is an indicator function and $B = 0$ when no bribe is offered (agents)/accepted (inspector), and $B = 1$ when a bribe is offered (agents)/accepted (inspector). Thus:

$$\left\{ \begin{array}{l} P(1, 1, 1) = \frac{1}{2}, P(1, 0, 1) = 1, P(0, 1, 1) = 0 \\ P(0, 0, 1) = \gamma < \frac{1}{2}, P(1, 0, 0) = 0, P(1, 1, 0) = 0 \\ P(0, 0, 0) = \gamma, P(0, 1, 0) = \delta < 1 \end{array} \right\}$$

The probability of getting caught will depend on the effort of the government X , the number of corrupt people in the economy, as well as the number of times an individual has been corrupt (which in our one shot environment cannot be more than one). Thus, the probability function of getting caught is $Pr_i(M_i, M, X)$, where $M_i = 0, 1$ is the number of times an individual has been corrupt, and $M \in [0, 3]$ is the sum of corrupt individuals in the economy.

The probability of getting caught is zero if the individual has not been corrupt, is increasing in the number of times one has been corrupt, and decreasing in the volume of corruption in the economy, i.e.: $Pr_i(0, M, X) = 0 \forall M \in [0, 3]$ and X , $\frac{\partial Pr_i(M_i, M, X)}{\partial M_i} > 0$, and $\frac{\partial Pr_i(M_i, M, X)}{\partial M} < 0$.

The actions available to agents are $\{B_j = 0, B_j = 1\}$, and the actions available to the inspector are $\{B_k = 0, B_k = 1\}$.

Proposition 1: *The one shot game has multiple equilibria, some of which in dominant strategies.*

We here solve the one shot game in order to prove Proposition 1. As discussed before, first the agents decide whether to offer a bribe or not, and then the inspector decides whether to accept or not; thus we will solve the game by backwards induction.

The payoffs for the inspector are:

$$\pi_k = \begin{cases} z\Pi - \Omega Pr_i(M_i, M, X) & \text{if he accepts a bribe if it is offered} \\ 0 & \text{otherwise} \end{cases}$$

It is apparent that the action chosen by the inspector will depend on the relative magnitudes of the probability of getting caught and the sharing parameter z , and thus we can distinguish three different cases for the inspector:

$$B_k = \begin{cases} 1 & \text{if } z\Pi - Pr(1, 2, X)\Omega > 0 \text{ (**Case 1**)} \\ 1 & \text{if } z\Pi - Pr(1, 2, X)\Omega < 0 \text{ and } z\Pi - Pr(1, 3, X)\Omega > 0 \text{ (**Case 2**) and **both** agents bribe} \\ 0 & \text{if } z\Pi - Pr(1, 2, X)\Omega < 0 \text{ and } z\Pi - Pr(1, 3, X)\Omega > 0 \text{ and **not both** agents bribe} \\ 0 & \text{if } z\Pi - Pr(1, 3, X)\Omega < 0 \text{ (**Case 3**)} \end{cases}$$

Thus the inspector always accepts a bribe if the probability of punishment for corruption is very low, even with few corrupt individuals in the economy, accepts a bribe only if both agents propose it if the punishment probability is relatively high with few corrupt individuals in the economy, and never accepts it if the probability of being caught is very high, even if there are two many corrupt individuals in the economy.

Now turning to the agents, their payoffs are:

$$\pi_j = \left\{ \begin{array}{l} \frac{1}{2}(1-z)\Pi - \Omega Pr(1, 3, X), \text{ if both bribe and } k \text{ accepts} \\ (1-z)\Pi - \Omega Pr(1, 2, X), \text{ if only } j \text{ bribes and } k \text{ accepts} \\ 0, \text{ if only } -j \text{ bribes and } k \text{ accepts} \\ \gamma\Pi, \text{ if no one bribes} \\ 0, \text{ if either both or only } j \text{ bribe and } k \text{ does not accept} \\ \delta\Pi, \text{ if only } -j \text{ bribes and } k \text{ does not accept} \end{array} \right\}$$

Let us state the following conditions:

Condition1: $\frac{1}{2}(1-z)\Pi - \Omega Pr(1, 3, X) > 0$

Condition2: $(1-z)\Pi - \Omega Pr(1, 2, X) > \gamma\Pi$

Condition3: $\frac{1}{2}(1-z)\Pi - \Omega Pr(1, 3, X) > \delta\Pi$

Then, we have the following equilibria:

When in Case 1, the Inspector always accepts. Then, If Conditions 1 and 2 hold, there exists a dominant strategy for the agents which is to bribe, and each agent gets the project with probability $\frac{1}{2}$. If conditions 1 and 2 do not hold, it's a dominant strategy not to bribe, and each agent gets the project with probability γ . If condition 1 holds but 2 not, there exist two Nash Equilibria: $(B_1 = 1, B_2 = 1)$ where both agents get the project with probability $\frac{1}{2}$, and $(B_1 = 0, B_2 = 0)$ where both get it with probability γ . If condition 1 doesn't hold but 2 does, we have again two NE: $(B_1 = 1, B_2 = 0)$ where 1 gets it with probability 1 and 2 gets it with probability 0, and $(B_1 = 0, B_2 = 1)$, where 1 gets it with probability 0 and 2 gets it with probability 1.

When in Case 2, the Inspector accepts only if both agents bribe. If condition 3 holds, there exist two NE: $(B_1 = 1, B_2 = 1)$ where both agents get the project with probability $\frac{1}{2}$, and $(B_1 = 0, B_2 = 0)$ where both get it with probability γ . If condition 3 doesn't hold, there exists a dominant strategy which is not to bribe, and each agent gets the project with probability γ .

When in Case 3, the equilibrium is unique: no one offers a bribe and each agent gets the project with probability γ .

As can be observed, even the simple one-shot game has multiple equilibria; especially interesting to us is the multiplicity of equilibria in Cases 1 and 2, as there the economy can end up being fully corrupt, or not corrupt at all, while the conditions that regulate corruption are exactly the same. Notice that multiple equilibria arise when the government's monitoring efforts are mild; when instead they are very strong corruption disappears, whereas when they are very low, the economy

becomes very corrupt.

It is also interesting to notice that in our one-shot game, multiplicity of equilibria does not require any assumptions on the preferences of either the agents or the inspector.

3.3 Repeated Game

We now consider an environment, where in every period one inspector and two agents are born, all living two periods each, such that every period the economy is populated by one old inspector and two old agents, one young inspector and two young agents. As before, the inspectors know which agents bribe them, as well as which ones have bribed them in the past; the agents only know their own past and present actions and their inspectors response to them. The probabilities of the agents getting the object are then the same as in the one shot game.

However, the probabilities of getting caught now change; more specifically in a game with two periods only, the potential values for M_i and M change, and now we have: $Pr_i(M_i, M, X)$, where $M_i \in [0, 2]$ is the number of times an individual has been corrupt, and $M \in [0, 9]$ is the sum of corrupt instances in the economy (corrupt cases from the first period can also be discovered in the second period of the game). The probabilities of getting caught, retain the three properties described before, and also have a non-correlation property:

$$Pr_i(2, M, X) = Pr_i(1, M, X)Pr(1, M, X) + 2Pr_i(1, M, X)(1 - Pr_i(1, M, X)) = 1 - (1 - Pr_i(1, M, X))(1 - Pr_i(1, M, X)) < 2Pr_i(1, M, X)$$

i.e. the events of getting caught being corrupt in different time periods are uncorrelated. This property garrantees that being corrupt in the first period of the game increases your likelihood to bribe (or accept a bribe) in the second.

Concluding, the probability of getting caught is decreasing in corruption in the overall economy and is underproportionally increasing in the own amount of corruption.

3.3.1 The equilibria

The inspectors have again a 0 or 1 decision. They have information on the bribes proposed to them by the agents and, if they are "old", some further information on last period. In particular, they know whether or not they were corrupt last period and, depending on the assumption whether or not they observe the age of the agent they are communicating with, some information of how corrupt the previous period's agents are. We will continue using the assumption that inspectors know

the "age" of the agents' they are communicating with. Thus, the most information is held by old inspectors, and the least information is held by young agents.

Proposition 2: *Inspectors might be willing to suffer negative payoffs in their first period in the game, in order to guarantee themselves greater positive payoffs in the second period.*

In order to understand Proposition 2, let us look at the inspectors' problem. Inspectors have to decide whether or not to accept a bribe if offered. An old agent's present action depends on his last period's action, and the amount of overall corruption. In particular, if having bribed last period, the question is whether or not

$$z\Pi - \Omega \sum_{M=0}^9 Prob_{ok}(M)Pr(2, M, X) > -\Omega \sum_{M=0}^6 Prob_{ok}(M)Pr(1, M, X).$$

, since we have assumed that an individual can be caught if being corrupt in any period of the game. We also assume that punishment Ω is the same no matter how many times one has been caught being corrupt, for simplicity and without loss of generality. In words, the question is whether the gain from bribing in his second period is greater than the additional probability of getting caught times the price to be paid, and $Prob_{ok}(M)$ is the subjective probability that an old inspector assigns to the event that there exist M corruption cases in the overall economy. For old inspectors that were not corrupt, the analogous equation is:

$$z\Pi - \Omega \sum_{M=0}^9 Prob_{ok}(M)Pr(1, M, X) > 0$$

Now, let us consider young inspectors; they want to maximize life-time pay-offs. In case a young inspector does not accept any bribe he earns zero payoff. Consider now a particular case:

$$z\Pi - \Omega \sum_{M=0}^9 Prob_{yk}(M)Pr(1, M, X) < -\Omega \sum_{M=0}^6 Prob_{yk}(M)Pr(0, M, X) = 0,$$

, where $Prob_{yk}(M)$ is the subjective probability that the young inspector assigns to the event that M corruption cases exist in the economy. However,

$$z\Pi - \Omega \sum_{M=0}^9 Prob_{yk}(M)Pr(1, M, X) > -\Omega \sum_{M=0}^6 Prob_{yk}(M)Pr(0, M, X) - \epsilon$$

It is apparent, that it can well be that

$$z\Pi - \Omega \sum_{M=0}^9 Prob_{ok}(M)Pr(2, M, X) > -\Omega \sum_{M=0}^6 Prob_{ok}(M)Pr(1, M, X) + \epsilon$$

In that case, even though the inspector is making loses when young, he is offsetting this by greater positive payoffs in his second period of the game. The intuition behind our result is that the inspector might be willing to accept a bribe risking to be caught and suffer losses in the first period, in order to increase the level of corruption in the economy-thus decreasing the probability that he will be caught next period, if this can guarantee him positive two-period payoffs.

Our result provides an alternative explanation to already existing literature on the persistence of corruption. Unlike research that attributes persistence to the perception of one's environment as corrupt, or to group dynamics and reputation, our simple model explains persistence as a self induced situation aiming to personal gains, which in turn creates (positive and negative) externalities to the rest of the actors in the economy, both agents and (other) inspectors.

3.4 Conclusions and Future Work

Within the context of an environment where agents compete against each other for the acquisition of a public good procurement project, assigned by the government and handled by a possibly corrupt inspector, we find that there exists multiplicity of equilibria, and in specific both "good" equilibria without corruption, and "bad" equilibria where corruption arises. The multiplicity of equilibria arising in our simple one-shot model is very useful for us to interpret why countries that are quite similar in all other characteristics, can differ a lot in the level of corruption in their economy, like Chile and Argentina or the Italian North and South. An important feature of the multiplicity of equilibria in our model is that they don't depend on any strong assumptions on either the agents' or the inspectors' preferences.

In our effort to confirm the multiplicity of equilibria also in the repeated game, we find that inspectors might consider it profitable to suffer negative payoffs in the first period of the game, in order to create more fuzziness as to how much corruption there is in the economy, and thus decrease the probability of getting caught, guaranteeing themselves bigger positive payoffs in their last period in the game. Our result provides an alternative explanation to already existing literature on the persistence of corruption. Unlike research that attributes persistence to the perception of one's environment as corrupt, or to group dynamics and reputation, our simple model explains persistence as a self induced situation aiming to personal gains, which in turn creates (positive and negative) externalities to the rest of the

actors in the economy, both agents and (other) inspectors.

A possibly interesting extension of the model would be its expansion to T time periods, K inspectors and J agents. Endogenizing the government monitoring effort X and solving the government's maximization problem could lead to interesting implications on the level of corruption in the economy and possibly rule out some of the equilibria we found both in the one-shot and the repeated game.

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