



# Macroeconomic Applications with Factor Models

Vasja Sivec

Thesis submitted for assessment with a view to obtaining the degree of  
Doctor of Economics of the European University Institute

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European University Institute  
**Department of Economics**

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# ABSTRACT

This thesis utilizes factor models to test the predictions of macroeconomic theory and introduces a new model for estimating structural relations in the economy. Factor models have proven useful in overcoming limited information bias. Limited information bias occurs because the information set of the actual decision makers in the economy is larger than the information set captured by conventional empirical models (i.e. small VARs). With the help of factors we can model a large dataset by using a small model of factors that still capture the majority of aggregate dynamics in the economy.

In the first chapter, joint work with Massimiliano Marcellino, we introduce a new empirical model: mixed frequency structural factor augmented VAR model. We show that in a mixed data frequency setting the model reduces aggregation bias and provides more precise estimates of factors and impulse responses, than competing models. We support this claim by means of a detailed Monte Carlo examination that also tests the new estimation procedure that we design. Finally we provide three empirical applications (monetary policy, oil and government expenditure shock) to show the usefulness of the model.

In the second chapter I utilize a dynamic factor model to test the predictions of the rational inattention theory as put forward by Mackowiak et al. (2009). I first estimate a time varying parameter dynamic factor model on US post-war data on macroeconomic variables and sector prices. I identify impulse responses of three macroeconomic shocks and sector specific shocks to prices. I then regress price impulse responses, void of the influences of changing variances, on the variances of the shocks, to test the predictions of the rational inattention model over time.

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# 1 Chapter

Large scale factor models have been often adopted both for forecasting and to identify structural shocks and their transmission mechanism. Mixed frequency factor models have been also used in a reduced form context, but not for structural applications, and in this paper we close this gap. First, we adapt a simple technique developed in a small scale mixed frequency VAR and factor context to the large scale case, and compare the resulting model with existing alternatives. Second, using Monte Carlo experiments, we show that the finite sample properties of the mixed frequency factor model estimation procedure are quite good. Finally, to illustrate the method we present three empirical examples dealing with the effects of, respectively, monetary, oil, and fiscal shocks.

## 1.1 Introduction

Since the pioneering work of Sims [1980], vector auto-regressions (hereafter VARs) became a dominant device to identify structural shocks and investigate their propagation mechanism. But VARs are not without flaws. To prevent the curse of dimensionality, they are estimated on a small set of macroeconomic variables. In contrast, economic agents and decision makers generally consider a large set of variables when making their decisions. This discrepancy in information sets can generate statistically biased shock responses and economically counterintuitive results. For example, a typical monetary policy VAR suffers from a price puzzle, namely, after a negative monetary policy shock (unexpected rise in the policy rate) prices initially increase.

Recently, Bernanke et al. [2005] introduced a way to overcome the curse of dimensionality in a structural VAR, see also Marcellino et al. [2005], Forni et al. [2009] and Andreou et al. [2013]. The relevant large set of economic variables are assumed to be generated by a factor model, where few common factors explain the bulk of the variation in all the variables and therefore provide an exhaustive summary of the relevant information. Factors, generally estimated by (static or dynamic) principal components, do not have a clear economic interpretation. However, they can be modeled with a VAR, possibly augmented with a few observable variables, and the VAR used to identify structural shocks. In a second step, Bernanke et al. [2005] estimate how the factors load on the macroeconomic variables, and can therefore investigate how the structural shocks affect each of the large set of variables under analysis. The combination of the factor model for the variables and the VAR for the factors is known as a factor augmented VAR (hereafter FAVAR).

Typically a FAVAR is estimated on a dataset comprised of variables of the same frequencies. For example, Bernanke et al. [2005] estimate a monetary policy VAR using only monthly variables. This implies that a monthly FAVAR leaves out potentially important variables that are observed at other than monthly frequencies. For example, it leaves out real GDP, which is accepted as the most accurate measure of economic activity but is only available at quarterly frequency. One could aggregate monthly variables to a quarterly level and estimate the model on a quarterly frequency. But then the quarterly model is subject to aggregation bias, meaning that important information gets lost in the aggregation process. This suggests to estimate FAVARs combining data at different frequencies, and various techniques are now available, see e.g., Giannone et al. [2006], Jungbacker et al. [2011], Banbura and Modugno [2010], Mariano and Murasawa [2010] and Frale et al. [2010]. All these studies on mixed frequency (MF) factor models are not of structural nature but focus on reduced form analyses, such as nowcasting and

forecasting quarterly GDP growth using monthly or higher frequency indicators.

In this paper we introduce an alternative method to estimate a large MF factor model. We start from the Doz et al. [2011] procedure for estimating plain factor models and extend it to allow for the presence of mixed frequency data, where mixed frequencies are handled along the lines of Mariano and Murasawa [2003] and Mariano and Murasawa [2010]. We then assess the finite sample performance of our procedure in a set of Monte Carlo experiments, comparing it with that of alternative estimators for MF factor or FAVAR models. It turns out the procedure performs quite well even in small samples not only in terms of factor estimation but also to recover the impulse response functions to structural shocks. Finally, it can be easily modified to allow for observable factors, in high or low frequency.

Our second contribution, as anticipated, is to show how to conduct structural economic analyses using MF FAVAR models. So far there are few examples of structural analyses based on mixed frequency data, see e.g. Giannone et al. [2010], Chiu et al. [2011], Ghysels [2012], Foroni and Marcellino [2013] and Foroni and Marcellino [2014]. However, all these papers are based on VAR or DSGE models. We present three empirical examples.

First, we add quarterly variables to the monthly dataset of Bernanke et al. [2005], in particular GDP, and investigate how monetary policy shocks identified at monthly level affect GDP and other key macroeconomic variables. Second, again using a mixed frequency dataset, we study how monthly oil price shocks propagate to quarterly GDP. Finally, in our third application, we impose quarterly government expenditure as an observable factor, governed by the sum of three latent monthly expenditure growth rates. Using this specification, we can evaluate how monthly government expenditure shocks affect the economy. In all cases we find reasonable results in economic terms from the MF FAVAR, sometimes with interesting differences with respect to the standard FAVARs and VARs.

The remainder of the paper is structured as follows. In Section 2 we introduce the MF FAVAR. We first present dynamic factor models, next explain how we suggest to estimate them in the presence of mixed frequency data, then compare our proposal with other methods suggested in the literature, and finally we discuss estimation in the presence of some observable factors. In Section 3 we use Monte Carlo experiments to analyze the performance of our estimation method when varying the cross-sectional ( $n$ ), temporal ( $T$ ) dimensions, the amount of missing observations (generated by the presence of the mixed frequency data) and the frequency of the factors. In Section 4 we present the three empirical applications, studying the effects of, respectively, monetary, oil, and

fiscal shocks. Finally, in Section 5 we summarize our main results and conclude.

## 1.2 Mixed Frequency FAVAR

### 1.2.1 The single frequency FAVAR model

We assume that an  $n$  dimensional zero mean stationary vector of variables  $y_t$  can be represented as a sum of two components, a *common component* ( $\Lambda f_t$ ) and an *idiosyncratic component* ( $e_t$ ):

$$y_t = \Lambda f_t + e_t. \quad (1)$$

$f_t$  is a  $k \times 1$  dimensional vector of factors that are common to all the variables in  $y_t$ , with the number of factors being (much) smaller than the number of variables ( $k < n$ ). Factors capture the majority of comovements in the evolution of the individual variables.  $\Lambda$  is an  $n \times k$  matrix of factor loadings. The loadings determine how the factors affect the dependent variables.  $\Lambda f_t$  is called *common component* of the factor model because it represents that part of the variability of  $y_t$  that originates from the  $k$  factors that are common to all the  $n$  variables. On the other hand,  $e_t$  is an  $n \times 1$  zero mean vector that represents the *idiosyncratic component* of the factor model. This source of variability in  $y_t$  can not be captured by the  $k$  common factors and is variable specific.

Equation (1) represents a classic factor model. If the  $n \times n$  covariance matrix of the idiosyncratic components ( $E(e_t e_t') = \Psi$ ) is a diagonal matrix, then the model in (1) becomes an *exact factor model*. A diagonal covariance matrix can be too restrictive for macroeconomic applications, so we let  $e_t$  have some limited cross correlation. Such model is called an *approximate factor model*.

Specifically, following Doz et al. [2006], we impose two conditions:

$$\text{A1) } 0 < \underline{\lambda} < \liminf_{n \rightarrow \infty} \frac{1}{n} \lambda_{\min}(\Lambda' \Lambda) \leq \limsup_{n \rightarrow \infty} \lambda_{\max} \frac{1}{n}(\Lambda' \Lambda) < \bar{\lambda} < \infty.$$

$$\text{A2) } 0 < \underline{\psi} < \liminf_{n \rightarrow \infty} \lambda_{\min}(\Psi) \leq \limsup_{n \rightarrow \infty} \lambda_{\max}(\Psi) < \bar{\psi} < \infty.$$

$\lambda_{\min}$  and  $\lambda_{\max}$  indicate the smallest and the largest eigenvalues of a matrix. Condition A1 ensures that the factors are pervasive, that is, that they affect most dependent variables. Condition A2 ensures that the variance of the idiosyncratic components is greater than zero, but limits the extent of the cross-correlation. Again as in Doz et al. [2006],  $e_t$  can be also serially correlated, see their Assumption (A3).

The common factors and the idiosyncratic components are assumed to be uncorrelated at all leads and lags,  $E(f_{jt} e_{is}) = 0$  for all  $j = 1, \dots, k$ ,  $i = 1, \dots, n$  and  $t, s \in \mathbb{Z}$ .

In equation (1),  $\Lambda$  and  $f_t$  are unobserved and need to be estimated. This poses an identification problem because there are different combinations of  $\Lambda$  and  $f_t$  that deliver

the same common component. To identify the factors we assume that the first  $k \times k$  entries of the loadings matrix form an identity matrix:  $\Lambda = \begin{bmatrix} I_k \\ \Lambda^* \end{bmatrix}$ , where  $\Lambda^*$  is an  $(n - k) \times k$  matrix of unrestricted loadings. This method of identification is also used in Bernanke et al. [2005].

Finally, we assume that the factors follow a  $p$ -th order VAR:

$$f_t = A_1 f_{t-1} + \dots + A_p f_{t-p} + u_t, \quad (2)$$

where  $p$  is finite and  $u_t$  is a  $k$  dimensional Gaussian white noise process with covariance matrix  $\Sigma$ .

The model presented in equations (1) and (2) represents a *static form* of a dynamic factor model. It is called the static form because factors enter equation (1) without lags. As shown in Stock and Watson [2005] the static form of a dynamic factor model nests the *dynamic representation*. Suppose that static factors  $f_t$  are composed of dynamic factors  $q_t$  ( $r \times 1$ ) and their lags:  $f_t = [q_t, q_{t-1}, \dots, q_{t-g}]'$ , such that  $k = r \times (g + 1)$ . We can then rewrite the model into the *dynamic form*:

$$y_t = \tilde{\lambda}(L)q_t + e_t, \quad (3)$$

$$q_t = \tilde{a}(L)q_{t-1} + v_t, \quad (4)$$

where  $\tilde{\lambda}(L)$  and  $\tilde{a}(L)$  represent lag polynomials in the dynamic representation and  $v_t$  the fundamental shocks that govern the dynamic factors. The number of fundamental shocks ( $v_t$ ) can be smaller than the number of static shocks ( $u_t$ ):  $u_t = G \times v_t$ . Where  $G$  is of dimension  $k \times r$  and  $r \leq k$ . In the static representation of the factor model we need to choose  $k$  and  $p$  high enough to capture all the effects that the dynamic factors and their lags ( $\lambda(L)q_t$ ) exert on the dependent variables. We can then use the static factor model to uncover the effects that the fundamental shocks have on the economy.

Equations (1) and (2) represent the (single frequency) FAVAR model.

### 1.2.2 Estimation of the single frequency dynamic factor model

Doz et al. [2006] propose to estimate the model in (1) and (2) using a Quasi Maximum Likelihood (QML) approach where the maximum likelihood estimates of the model are obtained using the *expectation-maximization algorithm* (hereafter EM). The EM algorithm iterates between two steps. In the first (*maximization*) step, it calculates the

maximum likelihood estimates of the factor model parameters ( $\hat{\theta} = \{\hat{\lambda}, \hat{A}, \hat{\Psi}, \hat{\Sigma}\}$ ) conditional on the estimates of the factors. In the second (*expectation*) step, conditional on the parameter estimates, it uses the Kalman filter-smoother to get the factor estimates ( $\hat{f}_t$ ) and the likelihood function of the model. The estimated factors are then used to produce another set of parameter estimates, then another set of estimated factors, and so on until convergence.

When calculating the maximum likelihood estimators, we assume that the idiosyncratic shocks  $e_t$  are not auto-correlated and cross correlated, although they often are. Hence, more properly, we obtain *QML* estimates, following White [1982]. Doz et al. [2006] show that this QML approach is valid for estimating the model parameters and the factors, even when the approximating model is mis-specified and the shocks exhibit week cross correlation and auto-correlation. They show that the QML estimators are consistent for the true factor space, with a consistency rate equal to  $\min\left\{\sqrt{T}, \frac{n}{\log(n)}\right\}$ .

The initialization of the EM algorithm requires (consistent) estimates of the factors. For this, we can use principal components (PCA), since consistency of PCA estimates of the factors results from Stock and Watson [2002b], Bai and Ng [2002], Bai [2003].

### 1.2.3 Estimation in the presence of mixed frequency data

We now extend the Doz et al. [2006] estimation procedure summarized in the previous subsection in order to handle mixed frequency data. Next, we compare our proposal with two alternative methods.

We closely follow the notation used in Mariano and Murasawa [2010], to whom we refer for additional details. We assume that we have two types of variables, low frequency and high frequency (e.g. quarterly and monthly data). Let  $y_{t,1}$  represent an  $n_1$  variate low frequency vector of variables, that are observed only every third period (e.g. only in periods  $t = 3, 6, 9, \dots$ ). Let  $y_{t,2}$  represent an  $n_2$  vector of high frequency variables that are observed in every period. As before, the total number of variables is  $n$  (where  $n = n_1 + n_2$ ) and  $y_t = [y'_{1,t}, y'_{2,t}]'$  is an  $n \times 1$  dimensional vector. We assume that underlying  $y_{1,t}$  there is a process  $y_{1,t}^*$ . Most of the time  $y_{1,t}^*$  is unobservable, except for every third period when it has the same value as the observable  $y_{1,t}$ <sup>1</sup>. We adopt the same model as in the previous section, but this time we assume that the common factors load

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<sup>1</sup>This implies that  $y_{1,t}$  is obtained via point in time sampling from  $y_{1,t}^*$ . The method can be extended for different sampling schemes and different frequency mis-matches. In fact, in the practical implementation of the mixed frequency factor model we assume that real GDP is a geometric mean of an unobserved monthly process. To preserve clarity we present the method by assuming point in time sampling.

on the (sometimes unobservable) process  $y_t^*$  instead of on (observable)  $y_t$  directly:

$$f_t = A_1 f_{t-1} + \dots + A_p f_{t-p} + u_t, \quad (5)$$

$$y_t^* = \Lambda f_t + e_t, \quad (6)$$

where  $y_t^* = [y_{1,t}^*, y_{2,t}^*]'$ . Since  $y_t^*$  is unobservable, we need to link it to the observables  $y_t$ . This is done with the following equation:

$$y_t^+ = C_t y_t^* + D_t v_t \quad (7)$$

where:

$$\begin{aligned} y_t^+ &= \begin{bmatrix} y_{1,t}^+ \\ y_{2,t}^+ \end{bmatrix} \text{ and } y_{1,t}^+ = \begin{cases} y_{1,t} & \text{when } y_{1,t} \text{ is observed} \\ v_{1,t} & \text{when } y_{1,t} \text{ is not observed} \end{cases} \\ v_t &= \begin{bmatrix} v_{1,t}^+ \\ 0 \end{bmatrix} \text{ and } v_{1,t}^+ = \begin{cases} 0 & \text{when } y_{1,t} \text{ is observed} \\ v_{1,t} & \text{when } y_{1,t} \text{ is not observed} \end{cases} \\ C_t &= \begin{bmatrix} C_{1,t} : 0_{n_2} \\ 0_{n_1} : I_{n_2} \end{bmatrix} \text{ and } C_{1,t} = \begin{cases} I_{n_1} & \text{when } y_{1,t} \text{ is observed} \\ 0_{n_1} & \text{when } y_{1,t} \text{ is not observed} \end{cases} \\ D_t &= \begin{bmatrix} D_{1,t} \\ 0_{n_2} \end{bmatrix} \text{ and } D_{1,t} = \begin{cases} 0_{n_1} & \text{when } y_{1,t} \text{ is observed} \\ I_{n_1} & \text{when } y_{1,t} \text{ is not observed} \end{cases} \end{aligned}$$

$I_{n_1}$  indicates an identity matrix of size  $n_1 \times n_1$  and  $O_{n_1}$  a matrix of zeros of size  $n_1 \times n_1$ . We assume that  $v_{1,t}$  is a normally distributed random vector of size  $n_1$ ,  $v_{1,t} \sim N(0, I_{n_1})$ . But, although  $v_t$  is a random vector, it is assumed that all the realizations of the vector  $v_t$  are simply zero. Hence, the measurement equation (7) is rewritten as if it consists of the observable variables only, and when the data is missing the missing data are replaced by a  $N(0, I_{n_1})$  random vector  $v_{1,t}$  whose realizations are zero.<sup>2</sup> Mariano and Murasawa [2010] propose this approach since it implies that equations (5) and (7) form a state space model where, from the point of view of the Kalman filter-smoother, all the variables are observed. Because the loadings of the missing data points are set to zero for the missing variables, the Kalman gain has zeros in the columns that correspond to the missing variables, so that when forecasting a new value of the state vector, the errors corresponding to the missing observation do not contribute to the new value of the state and to the new value of the state variance. The Kalman filter-smoother simply skips the influence of the missing observations when it estimates the factors and the likelihood.

It is convenient to take one last step and transform the state equation. First we

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<sup>2</sup>The value of realizations makes no difference for the method to work.

use the companion form for the factor VAR model in (5). That is, we shift the factor lags into the state vector  $s_t$ , so that the resulting model becomes a  $VAR(1)$  model in  $s_t$ . Second, we insert equation (6) into equation (7). The resulting model has the familiar state space form:

$$s_t = As_{t-1} + Bu_t \quad (8)$$

$$y_t^+ = H_t s_t + \tilde{v}_t \quad (9)$$

where

$$s_t = \begin{bmatrix} f_t (k \times 1) \\ f_{t-1:(k \times 1)} \\ \vdots \\ f_{t-p+1:(k \times 1)} \end{bmatrix}, \quad A = \begin{bmatrix} A_{1:(k \times k)} : \cdots : A_{p:(k \times k)} \\ I_{(k \times k)} \quad \cdots \quad 0_{(k \times k)} \\ \vdots \quad \ddots \quad \vdots \\ 0_{(k \times k)} \quad \cdots \quad I_k \quad 0_{(k \times k)} \end{bmatrix}, \quad B = \begin{bmatrix} \Sigma_{uu}^{\frac{1}{2}} \\ 0 : ([k(p-1)] \times k) \end{bmatrix}.$$

The matrix  $H_t$  in equation (7) is defined as:

$$H_t = \left[ \underbrace{C_t \Lambda}_{n \times k} \underbrace{0_{:(n \times k)} \cdots 0_{:(n \times k)}}_{n \times [(p-1) \times k]} \right]$$

and  $\tilde{v}_t = C_t e_t + D_t v_t$  is a *compound error term*, where  $D_t$  and  $v_t$  are defined as before.

It is clear now that the model in equations (8)-(9) represents a simple state space model that can be estimated using the Kalman filter-smoother. It can also be extended by adding a moving average component for the idiosyncratic shocks, to explicitly account for auto-correlation. This can be done by adding lags of the idiosyncratic shocks to the state vector and adjusting the matrices accordingly.

The model in equations (8)-(9) is slightly different from the model presented in Mariano and Murasawa [2010]. They plug the idiosyncratic error term into the state vector. This greatly increases the dimension of the state vector, which is not desirable in applications with large datasets since the estimation becomes very slow, or in practice infeasible. We instead form a compound error term. They estimate the model using a quasi-Newton method. We use a simpler approach based on Doz et al. [2006] QML estimator.

The starting estimate for the factors can be obtained by the EM algorithm of Stock and Watson [2002a], which is a PCA approach applied to an unbalanced dataset. We then use the EM algorithm as described in the previous section to get the maximum likelihood estimates of the parameters and the factors. Our procedure is more efficient than just



using Stock and Watson [2002a], and can more easily handle a variety of aggregation schemes.

### 1.3 Comparison with other estimation methods for MF factor models

Harvey and Pierse [1984] first handled missing data in the Kalman filter context, by modifying the updating and backdating equations of the filter. This approach can become cumbersome when handling systematically missing data, as in the mixed frequency case.

Besides the PCA based EM algorithm of Stock and Watson [2002a] mentioned above, the two computationally feasible and closest approaches to ours are those by Giannone et al. [2006] and Banbura and Modugno [2010].

Giannone et al. [2006] exploit the fact that the value for a missing data point is irrelevant if its variance is infinite. The Kalman filter puts zero weight on such points and the missing value does not affect the estimates. Banbura and Modugno [2010] use a selection matrix that modifies the Kalman filter smoother formulae so that only the available data are used in the estimation.

In practice, Giannone et al. [2006], Banbura and Modugno [2010] and our procedure induce the Kalman filter to skip the missing observations. Hence, not surprisingly, they produce numerically equal results. We believe that our procedure is easier to understand and more closely related to the approach to handle mixed frequencies in other types of models, such as VARs. Moreover, as we will see in the next section, it can be easily modified to allow for some observable factors, which is relevant for economic applications.<sup>3</sup>

### 1.4 Estimation in the presence of observed factors

Bernanke et al. [2005] assume that, in a FAVAR model for a large set of same frequency variables, one of the factors is observable. It coincides with a short term interest rate as economic theory suggests that monetary policy should affect most variables in the economy, at least in the short term, and therefore it is pervasive. This helps both structural shock identification and the interpretation of the impulse response functions.

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<sup>3</sup>Jungbacker et al. [2011] introduce a more complex procedure based on two different state space representations. A normal representation for when all data are available and a modified representation for when there are missing data. In the modified representation they add the missing data into the state vector, so that missing values are estimated together with the factors, we refer to them for additional details. Jungbacker et al. [2011] report that there are substantial computational gains with their method. We achieve similar gains because instead of putting the error term into the state vector, as in Mariano and Murasawa [2010], we form a compound error term, leaving the size of the state vector unaltered.

Hence, we now consider how observable factors can be treated in a mixed frequency context.

To ease the exposition, let us assume that we have only two factors, one latent and one observable. The model is the same as in equations (5)-(7), that we repeat for convenience:

$$f_t = A_1 f_{t-1} + \dots + A_p f_{t-p} + u_t \quad (10)$$

$$y_t^* = \Lambda f_t + e_t \quad (11)$$

$$y_t^+ = C_t y_t^* + D_t v_t \quad (12)$$

where  $f_t$  is now  $[f_{1,t}, i_t]'$ .  $f_{1,t}$  is a latent unobservable factor as before and  $i_t$  is an observable factor, in our example the interest rate. Let  $y_{N-1,t}^+$  represent all the variables in  $y_t^+$ , except the interest rate. Further, assume that the interest rate is ordered last, in the  $N^{th}$  place, in the mixed frequency vector of the dependent variables. Then the vector of dependent variables is  $y_t^+ = [y_{N-1,t}^+, i_t]'$ .

For simplicity, let us now focus on the last row of equation (11), the interest rate equation, since nothing changes for other parts of the FAVAR model. The last equation is:

$$i_t = \Lambda_N f_t + e_{N,t}, \quad (13)$$

and, since  $i_t$  coincides with the observable factor, it must be  $\Lambda_N = [0, 1]$ ,  $e_{N,t} = 0$  for all time periods. The corresponding variance and covariances of the error term of the interest rate equation are also zero ( $\Psi_{N,i} = \Psi_{i,N} = 0$ , where  $i = 1, \dots, N$ ).

The model can be then estimated using the EM procedure introduced in Section 2.3. Note that a similar procedure can be used when the observable factor is a low frequency variable.

### 1.5 A Monte Carlo Evaluation of the MF Estimation Procedure

Doz et al. [2006] prove the consistency of the QML estimator. Since our specification is nested in their model, the estimation procedure remains consistent. To verify this statement and asses the finite sample performance of the procedure when using mixed frequency data and observable factors, we set up a Monte Carlo experiment. We monitor how well the method uncovers the factors and impulse responses (hereafter IR) of the dependent variables to shocks in the observable factors, conditional on the sample size  $T$ , the size of the cross section  $n$ , the number of low frequency variables and the frequency of the factors. We focus on shocks to the observable factor since this case was not

considered in the previous literature and it is related to the empirical applications that we will present in the next section, but similar results apply for shocks to unobservable factors.

We use a modified version of the data generating process (hereafter DGP) commonly used in a same frequency setting, see among others Stock and Watson [2002b], Doz et al. [2006] and Doz et al. [2011]. Following Doz et al. [2006], we write the DGP as:

$$\begin{aligned} y_t &= \Lambda f_t + e_t, \\ f_t &= A_1 f_{t-1} + \dots + A_p f_{t-p} + u_t, \\ e_t &= d_1 e_{t-1} + \dots + d_q e_{t-q} + v_t, \end{aligned}$$

Let  $\Lambda_{ij}$  represent the  $ij^{th}$  element of  $\Lambda$ , where  $i = 1, \dots, n$  and  $j = 1, \dots, k$ . We assume  $\Lambda_{ij} \sim i.i.d.N(0, 1)$ . Let  $a_{ij}(l)$  represent the  $ij^{th}$  element of  $A_l$ , where  $l = 1, \dots, p$ . We assume:

$$a_{ij}(l) = \begin{cases} 1 - \rho & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}, i, j = 1, \dots, k,$$

and  $u_t \sim i.i.d.N(0, (1 - \rho^2)I_k)$ . Let  $d_{ij}(l)$  represent  $ij^{th}$  element of  $d_l$ , where  $l = 1, \dots, q$ . We assume:

$$d_{ij}(l) = \begin{cases} 1 - d & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}, i, j = 1, \dots, n,$$

and  $v_t \sim i.i.d.N(0, \mathcal{T})$ . And, finally, we assume that the elements of  $\mathcal{T}$  satisfy:

$$\begin{aligned} \tau_{ij} &= \sqrt{\alpha_i \alpha_j} \tau^{|i-j|} (1 - d^2), \quad i, j = 1, \dots, n, \\ \alpha_i &= \frac{\beta_i}{1 - \beta_i} \sum_{j=1}^k \Lambda_{ij}^2, \quad \beta_i \sim i.i.d.U([u, 1 - u]). \end{aligned}$$

This is a standard factor data generating process. Note that the idiosyncratic shocks are allowed to be auto-correlated and also weakly cross correlated, with cross correlation governed by the parameter  $\tau$ .  $\mathcal{T}$  is a Toeplitz matrix. When  $\tau$  is zero the model becomes an exact factor model and  $\mathcal{T}$  is a diagonal matrix. The parameter  $\beta_i$  controls for the ratio between the common component ( $\Lambda_i f_t$ ) variances and the idiosyncratic component ( $e_{it}$ ) variances (where  $\Lambda_i$  is the  $i^{th}$  row of  $\Lambda$  and  $e_{it}$  the  $i^{th}$  element of  $e_t$ ). Following Doz et al. [2006], we set it to 50%.  $u$  is a parameter that controls the cross sectional heteroscedasticity. We set it to 0.5, which implies cross correlation with the closest two adjacent time series equal to 0.5 (on average) and decays below 0.1 (on average) after the fourth closest series. Therefore, cross correlations between the idiosyncratic shocks

are clustered.

We estimate the model using the approximating specification that assumes no cross correlation. Hence, the true data generating process is an approximate factor model and we model it using an exact factor model.

We deviate from previous Monte Carlo analyses in two ways. First, we assume that some of the variables are not observed all the time. This is done simply by first simulating the model data and then deleting some of the observations in the data set. In particular, assuming that  $t$  is measured in months, some variables are only observed at the end of the quarter (so that all observations corresponding to the first two months of each quarter are deleted). These are the low frequency variables in our simulation study. Second, to align the Monte Carlo study with the MF structural FAVAR used in practice, we assume that two factors generate the data and that one factor is observable. The resulting simulated mixed frequency data set and the observable factor are then used to estimate the space spanned by factors and to produce IRs of the dependent variables to a shock in the observable factor.

We compare five different estimators for the factors. First, the PCA estimator on a data set without imposing the observable factor and without missing observations. This in practice is not feasible, but we use it as a benchmark to assess the effects of the missing observations. Second, the PCA estimator computed after dropping the series with missing observations from the data set. Third, we use the Stock and Watson [2002a] EM algorithm based approach to estimating factors from unbalanced datasets. Fourth, we use the Doz et al. [2006] two step estimator<sup>4</sup> where mixed frequencies are handled as in Mariano and Murasawa [2010]. The two step estimator of Doz et al. [2006] is called a two step estimator because in the first step they estimate the factors and the parameters by using the PCA estimator and then use the estimated parameters in the second step, where they re-estimate the factors using the Kalman filter smoother approach. The (modified) Doz et al. [2006] estimator is just the first step in our estimation procedure. Instead of stopping the estimation procedure after the first run of the Kalman filter smoother we continue with an EM algorithm<sup>5</sup>, and thereby obtain a QML estimator for a mixed frequency data set with observable factors, introduced in the previous Section.

We inspect the performance of the estimators in uncovering both the space spanned by the factors and the impulse responses of the dependent variables to a shock in the observable factor. To gain further insight, we vary the number of time observations,

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<sup>4</sup>Doz et al. [2006] estimates latent factors. We modify their method so that it can handle observable factors as proposed in subsection 2.5.

<sup>5</sup>Note that in the two step estimator approach one needs to use the same identifying restrictions that underly the PCA approach, otherwise it will produce biased estimates.

size of the cross section, the number of low frequency variables and the frequency of the observable factor.

## 1.6 Recovering the space spanned by the factors

In this section we investigate how well the alternative estimators uncover the space spanned by the factors. We base the evaluation on the trace statistic, a multivariate version of the  $R^2$  measure, also used in Stock and Watson [2002b], Giannone et al. [2006] and Banbura and Modugno [2010]. It measures how close the estimated factors are to the true factors which generated the data, and is defined as:

$$\frac{\text{Trace}(F'\hat{F}(\hat{F}'\hat{F})^{-1}\hat{F}'F)}{\text{Trace}(F'F)}, \quad (14)$$

where  $F = [F^1, F^2]$  are the true factors and  $\hat{F} = [\hat{F}^1, \hat{F}^2]$  their estimated version. The trace statistic lies between one and zero, being equal to one when the factor space is perfectly estimated.

We assume that one of the true factors is observable (i.e. we impose  $\hat{F}^2 = F^2$ ), therefore we only estimate the latent factor and the model parameters<sup>6</sup>. In Table 1 we report the average trace statistic computed over 1000 replications for different values of  $n$  and  $T$  ( $n = 50, 100, 200$ ,  $T = 50, 100, 200$ ), and a fixed number of low frequency series ( $d = 20$ ). Four main findings emerge. First, for all methods the values increase with  $n$  and/or  $T$ . Second, the values are already rather large for  $n = T = 50$ , suggesting that the procedures work well also in finite samples, notwithstanding the presence of missing observations. Third, PCA on full sample, even though based on a larger information set than the other methods, performs generally worse because the observable factor is not imposed. Finally, the DGR and our MF estimators perform comparably and slightly better than principal components, with a slight advantage in all cases for our MF estimator.

Table 2 presents results for  $n = 200, T = 200$  and a varying number of series with missing observations: 20, 100, 180. While the results naturally deteriorate when the number of missing observations increases, the average trace statistics remain quite good also when 180 series are only observable on a quarterly basis, with values in the range  $0.98 - 0.99$ <sup>7</sup>. The rationale is that the factor structure is quite strong so that few

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<sup>6</sup>For the first estimator, the unattainable PCA estimator on the full sample, we do not impose that the second factor is observable. We do this to show the merit of introducing an observable factor, even in a case when one compares it to an estimator where all the factors are latent but one observes all the data.

<sup>7</sup>The trace statistic for our method performs even better than the competitors, if we compute the

variables already contain substantial information on the factors, as indicated by the still good performance of PCA applied on the reduced sample of monthly only observations. However, adding the quarterly variables improves the trace statistic, the most so when using our MF estimator.

Table 3 presents results for  $n = 200, T = 200$  when the observable factor is of quarterly frequency and varying number of series with missing observations: 20, 100, 180. Naturally the results are slightly worse, compared to the case when the observable factor is of monthly frequency, but the estimators still perform quite well. Our estimator does slightly better than the other estimators, the more so when the number of quarterly series is high.

## 1.7 Recovering the impulse responses

In the preceding section we have seen that our MF method recovers quite well the space spanned by the factors, slightly better than the competing methods. In this section we investigate how well it uncovers the impulse responses to a shock in the observable factor. We run two experiments. In the first experiment we compare the Stock and Watson [2002a] EM algorithm to handle factor estimation in unbalanced datasets with our procedure. In the second experiment we investigate if the mixed frequency data reduces the aggregation bias that is present when one instead aggregates all the variables to a quarterly frequency.

In the first experiment we fix  $n$  and  $T$  to 200 and the number of quarterly variables to 100. We draw the factor loadings  $\Lambda$  at the first iteration and then retain the same  $\Lambda$  for the remaining replications for comparability.<sup>8</sup> We report the average estimates over 1000 replications. We explore the results along two dimensions. First, we compare the IRs of the low frequency variables (with missing data) and of the high frequency variables. Second, we investigate how the number of low frequency variables affects the IRs.

Figure 1 and Figure 2 plot the IRs of, respectively, the low frequency variables and the high frequency variables. Each figure contains IRs of the first 9 variables<sup>9</sup> Solid black lines with dots represents the true IRs. Solid red lines are the IRs obtained with the Stock and Watson [2002a] algorithm, and the solid black lines the IRs obtained using our procedure. As a measure of uncertainty, we also report the  $+/- 2$  std. dev. confidence bands obtained from the Monte Carlo experiments with our approach, and represented with dashed black lines.

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trace statistic for the latent factor only.

<sup>8</sup>We repeated the experiment several times to make sure that a specific draw of  $\Lambda$  did not affect the results.

<sup>9</sup>The variables are representative for the other variables. Note also that the loadings matrix is sampled randomly, therefore one can consider the selected IRs as being chosen randomly.

We observe from Figure 1 that the estimated IRs in general track the true IRs quite closely. Comparing the IRs obtained with our procedure (solid black line) to the IRs obtained using the Stock and Watson [2002a] procedure (solid red line) we note that there are not many systematic differences, though for most variables the IRs obtained with our procedure are closer to the true ones, but only marginally so. From Figure 2, the true and estimated IRs are in general even closer for the high frequency variables, and the differences between our procedure and that based on the Stock and Watson [2002a] factors are again small.

Figure 3 plots the IRs from an experiment where  $n = 200$ ,  $T = 200$ , there are no missing variables, but the observable factor is of quarterly frequency. In this case there can be some larger discrepancies between the true and estimated responses, but our MF FAVAR estimation method still generally outperforms the use of the Stock and Watson [2002a] factors. This is likely due to the fact that our procedure explicitly takes into account the model generating the quarterly factor whereas the Stock and Watson [2002a] approach does not.

In the second experiment we investigate if the use of mixed frequency data reduces the aggregation bias. In this experiment we fix  $n$  and  $T$  to 200. We then estimate three models. In the first model we use quarterly data, in the second we use mixed frequency data and in the last (empirically unfeasible) model we use monthly data. We set the number of quarterly series in the mixed frequency dataset to 100. We compare the IRs estimated on the monthly, mixed frequency, and quarterly datasets.<sup>10</sup> In addition, to make sure that the differences between the estimated IRs only result from the different types of datasets, we initialize the three models with the parameter values of the true DGP.

Figure 4 plots the IRs of variables that are quarterly in the mixed frequency dataset (first 9 variables) and Figure 5 plots the IRs of the variables that are monthly in the mixed frequency dataset (the last 9 variables). In each figure the black lines with dots represents the true IRs, the black solid lines the IRs estimated with the monthly dataset (dashed black lines are the  $\pm 2$  std. dev.), solid gray lines are the IRs estimated on the mixed frequency data, and the red lines the IRs estimated with the quarterly dataset.

The figures show that the IRs estimated on a monthly dataset are in general very close to those estimated on a mixed frequency dataset. In fact, the IRs estimated on the monthly dataset are often not visible because they overlap with the IRs estimated on the mixed frequency dataset almost perfectly (the solid gray line overlaps the solid black

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<sup>10</sup>To facilitate comparison of monthly IRs with the quarterly IRs we "skip sample" the monthly IRs to a quarterly frequency, namely, we record the monthly IRs only at times  $t = 1, 4, 7, \dots$

line). Both estimated IRs track the true IRs (solid black line with dots) very closely. This is natural because the sample size is quite large, the shocks belong to an always observable factor and we used the true DGP to initialize the EM algorithms. Based on this evidence, we conclude that our MF-S-FAVAR performs quite well in recovering the true IRs, even of the quarterly variables.

We next compare the IRs estimated on the mixed frequency data (solid gray lines) with the IRs estimated on the quarterly data (red lines). While the IRs estimated on the mixed frequency data track the true IRs very closely, the IRs estimated on a quarterly dataset sometimes depart from the true IRs, in particular in the short run. Two sources drive this result. The shock variances of the model estimated on the quarterly dataset are consistently overestimated, and the factor VAR parameters are consistently underestimated, with the first type of bias dominating the latter. Hence, the aggregation bias can be substantial, and the use of mixed frequency data can reduce it. Foroni and Marcellino [2013] and Foroni and Marcellino [2014] obtain similar results for, respectively, DSGE and structural VAR models.

In summary, this section shows that the MF-S-FAVAR, estimated using our method, performs quite well in recovering the space spanned by the factors and the true IRs, even in small samples. It also performs well when one of the observed factors is at quarterly frequency and it reduces the aggregation bias. To further motivate the usefulness of our method and illustrate its practical implementation, we next present three empirical applications.

## 1.8 Empirical applications

### 1.8.1 Bernanke et al. [2005] Monetary Policy Shocks

In this first application we assess the effects of monetary policy measured at the monthly level on quarterly GDP growth. We start with the original monthly FAVAR model put forward by Bernanke et al. [2005]. To bypass other influences that could affect the comparison, we use their same data set,  $X_t$ , consisting of 120 monthly variables from February 1959 to August 2001. The variables summarize all the major developments in the economy and include measures of real output, income and price indicators, interest rates, employment indices, consumption variables, housing prices, etc. To facilitate comparability, we first estimate the same monthly model as in Bernanke et al. [2005], using 3 latent factors, one observable factor (the federal funds rate) and 7 lags for the factor VAR. Bernanke et al. [2005] also use 3 latent factors and indicate that a larger number does not change the results. Next, we add quarterly GDP growth to create a mixed



frequency factor model, where quarterly GDP is modeled as a sum of three consecutive unobserved monthly growth rates.<sup>1112</sup>

Before discussing the results, it is important to consider an issue not addressed by Bernanke et al. [2005], namely, the number of dynamic factors driving  $X_t$ , given the assumed number of static factors (four in our case). We use the Stock and Watson [2005] approach to determine their number.<sup>13</sup> They suggest regressing each variable on own lags and the lags of the static factors, recover the residuals ( $\tilde{\epsilon}_{it}$ , for  $i = 1 \dots n$ ), and test how many factors drive them. The number of factors driving the estimated residuals is equal to the number of dynamic factors driving the variables  $X_t$ . Table 4 displays the values of the Bai and Ng [2002] information criteria associated with different assumptions regarding the numbers of static factors driving  $\tilde{\epsilon}_{it}$ 's ( $\hat{q}$ , with  $\hat{q} \leq 4$  where 4 is the number of static factors we use for  $X_t$ ). All criteria favor 4 static factors for  $\tilde{\epsilon}_{Xit}$ 's and hence 4 dynamic factors for  $X_t$ . This implies that static factors for  $X_t$  are equivalent to dynamic factors, and we can proceed with our structural analysis by identifying structural shocks directly on the static factor VAR residuals.

Figure 6 presents the factors estimated by PCA (solid line) and those obtained with our method (dashed line). It turns out that the first factor is smoother when estimated using our method, and the opposite holds for the second estimated factor, but overall the behavior of the three estimated factors is rather similar.

Using the monthly FAVAR model, Figure 7 reports the impulse response functions of selected variables to a monetary policy shock identified as in Bernanke et al. [2005] (together with the 90% confidence bands). In the same figure, the dashed black lines represent the IRF obtained using our estimation method for the MF-FAVAR model. Overall, the IRFs are quite similar, and those obtained with our method are most of the time statistically indistinguishable from the IRFs estimated using the Bernanke et al. [2005] approach. This result is not surprising since both methods are based on consistent parameter and factor estimators and, in addition, in this case there is only one quarterly

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<sup>11</sup>Namely,  $y_t = y_t^* + y_{t-1}^* + y_{t-2}^*$ , where  $y_t$  is the quarterly GDP growth observed only every 3rd period and  $y_t^*$  represents the latent monthly GDP growth. The results are almost identical when we assume that quarterly GDP growth is point in time sampled from monthly GDP growth. Small differences arise when we assume that quarterly GDP is modeled as a geometric mean of unobserved monthly GDP, as in Mariano and Murasawa [2010]. These alternative results are presented at the end of the section.

<sup>12</sup>It is not likely that adding a small number of quarterly series would affect the number of static factors needed to model the economy. More so because the quarterly variables can be explained with corresponding monthly variables (i.e. quarterly GDP can be explained with the factor closely related to the monthly variables that represent economic activity and the GDP deflator to the factor that predominantly explains monthly prices.). In addition the IRs of monthly variables were not affected by adding quarterly series.

<sup>13</sup>Bai and Ng [2007] test is not appropriate in our applications because the identifying assumption underlying their test ( $\Lambda\Lambda' = I$ ) is violated.

variable. However, our method can also handle mixed frequencies. Moreover, there are some interesting and significant differences. Policy rate and other interest rates show stronger increase after a monetary policy shock. As a result there is also a stronger reaction on the labor market. We estimate a stronger increase in unemployment and a stronger decrease in employment. This is possibly due to a more marked decrease in the capacity utilization rate after the monetary contraction.

In Figure 8, we report the response of the monthly (unobservable) GDP growth rate to the monetary policy shock. For comparison, we add in the same graph the response of monthly IP (with the 90% confidence bands) calculated with the Bernanke et al. [2005] method. The response of GDP has the same shape as the response of IP, although more pronounced.

Finally, Figure 9 presents the IRs obtained when we model the GDP growth rate as a geometric mean of the underlying latent monthly series,  $\ln y_t = \frac{1}{3}(\ln y_t^* + \ln y_{t-1}^* + \ln y_{t-2}^*)$  (see Mariano and Murasawa [2010] for details). Comparing Figure 7 and Figure 9, the differences are either minor or negligible.

### 1.8.2 Bernanke et al. [1997] Oil Price Shocks

In the second application we reconsider the analysis of the effects of oil price shocks by Bernanke et al. [1997]. They set up a small scale VAR for (in this order): 1) the log of real GDP, 2) the log of the GDP deflator, 3) the log of an index of spot commodity prices, 4) an indicator of the state of the oil market and 5) the level of the federal funds rate. As alternative indicators of the state of the oil market, they assess: the log of the nominal PPI for the crude oil products, Hoover-Perez's oil prices, Mork's oil prices and Hamilton's measure of oil price changes (we refer to Bernanke et al. [1997] for additional details on these measures). They estimate the model on monthly data for the period from 1965 to 1995, using interpolated data for real GDP and the GDP deflator based on a cubic spline.

Figure 10 reports the IRFs in Bernanke et al. [1997]. An oil price shock is followed by a rise in output for the first year and by a slight short-run decline of prices, when using the log level of oil prices. The other three measures produce better results, although, immediately after the oil shock one can still observe a slight increase in output. Eventually, Bernanke et al. [1997] prefer the Hamilton's measure for oil prices since it induces positive price response to an oil shock.

We now repeat their exercise but instead of estimating a VAR we estimate a MF FAVAR, for their same sample period. We combine the set of slow moving monthly variables in Bernanke et al. [2005] with quarterly GDP and GDP deflator, both modeled

as a sum of three latent monthly growth rates. We estimate a MF-FAVAR with two unobservable factors<sup>14</sup> and three observable factors,  $F_t = [f_t^1, f_t^2, P_t^{comm}, P_t^{oil}, i_t]$ . The first estimated factor turns out to be highly correlated with real measures of economic activity, and the second one with measures of prices. Hence, the VAR for the factors is similar to that by Bernanke et al. [1997], except that we use estimated factors from mixed frequency data as proxies for real variables and price movements. Using the Stock and Watson [2005] test, discussed in the previous application, we estimate that the number of dynamic factors is 5. The results are presented in Table 5. Hence, we can proceed using the static factors for the structural identification.

We then compute the IRFs to oil price shocks using a Cholesky identification, as in Bernanke et al. [1997]. The upper panel of Figure 11 reports the IRFs and the lower panel the cumulated responses (with the 90% confidence bands<sup>15</sup>). After an oil shock, real GDP immediately declines and the GDP deflator rises after a short period, in line with economic intuition, though the responses are not statistically significant. After about six months the monetary policy reacts by raising the interest rate, causing the prices to decline but also further depressing the economy.

An advantage of using a large dataset is that we can also consider the reaction of other variables. For example, in the lower panel of Figure 11, we report the responses of the CPI, IP, employment and hourly earnings. All the reactions are in line with economic theory, since IP decreases, CPI increases, and employment and earnings decrease. This provides additional support for the adopted identification scheme.

Figure 12 presents the IRs obtained when we model the GDP and GDP deflator as a geometric mean of the underlying monthly series. As in the previous application, the responses of monthly variables are more or less equivalent.

Kilian and Lewis [2011] criticize the work of Bernanke et al. [1997] on the basis that their results are driven by a specific period and a specific type of shock. Namely, they note that monetary policy response to an oil shock stems from the 1979 oil crisis period and they show that oil price shocks have little impact on interest rate and real output if one instead uses the sample from 1988 onward. In addition Kilian [2009] notes that not all oil price shocks are alike. The response of the economy depends on whether the oil shock is an oil supply shock, demand shock or oil production shock. The IRs that we obtain in our application are qualitatively similar to the IRs that Kilian [2009] obtains

<sup>14</sup>We also estimated the model using 3 and 4 latent factors. This did not affect the results significantly. The responses to oil shocks were qualitatively similar to the responses in the model presented above. We present the results from the two latent factors model to facilitate comparability with the VAR model used in Bernanke et al. [1997].

<sup>15</sup>Confidence bands were estimated using sampling with replacement in 500 bootstrap replications.

for an oil supply shock. Further investigation should address the issues raised by Kilian and Lewis [2011] and Kilian [2009]. Since this paper is primarily concerned with showing how MF-S-FAVAR can be applied to a large variety of models, we do not further pursue this issue here.

### 1.8.3 Ramey [2011] Government Expenditure Shocks

In the last application we investigate how monthly government expenditure shocks (derived from a MF FAVAR with quarterly government expenditure and a set of monthly indicators) affect several macroeconomic variables on a monthly level. Due to its novelty, this application requires a more detailed description. Hence, we describe, in turns, the related literature, the model we implement, and the results.

**Related literature** There is no consensus in the literature on the effects of government expenditure shocks. Most researchers agree that GDP and total hours worked increase (though the extent of their reaction is debated), while there is less consensus on the reaction of consumption and real wages. Among others, Fatas and Mihov [2001], Blanchard and Perotti [2002] and Pappa [2005] find that spending shocks raise consumption and real wage. This response is consistent with the new Keynesian models of Rotemberg and Woodford [1989], Devereux et al. [1996] and Gali et al. [2007].

Ramey [2011] argues that the positive response of consumption and real wages could be due to timing issues. These arise because government spending changes are announced and are therefore known in advance, before they are implemented. Hence, forward looking agents react to changes in government spending before the changes really occur. If one does not explicitly account for this timing issue in an empirical model, consumption and wages could spuriously increase in response to a government expenditure shock. For this reason, Ramey [2011] uses other variables (instead of government spending) in her study. Specifically, she uses Ramey-Shapiro war dates and shocks to government spending forecasts. Government spending forecasts are forward looking variables, therefore their sudden changes are truly unanticipated. Ramey-Shapiro war dates are instead constructed using a narrative approach. They are characterized as episodes when newspapers suddenly began to forecast large rises in government spending due to prospects of a war. Changes in these variables are less likely to be anticipated. Once controlling for expectations, Ramey [2011] finds that consumption and real wages fall as a response to a spending shock. This result is consistent with the analysis done in Ramey and Shapiro [1998], Edelberg et al. [1999] and Burnside et al. [2004], and with the response in neoclassical theoretical models (e.g., Aiyagari et al. [1992]).

We now try to shed additional light on the effects of government expenditure shocks based on our MF-S-FAVAR framework.

**Our MF-FAVAR Model** To investigate the effects of expenditure shocks we estimate a MF-FAVAR model where the majority of the dependent variables are sampled at monthly frequency but one of the observable factors, government expenditures, is quarterly. This enables us to reduce the aggregation bias that is inherent in quarterly models and to avoid the loss of information of low dimensional VARs.

We depart by replicating the MF-FAVAR model of Boivin et al. [2013], who investigate the effects of credit shocks on the economy. However, since our observable factor is the growth rate in quarterly government expenditure, to align it with the dynamics of monthly variables, we assume that the growth rate of quarterly government expenditures is the sum of three consecutive monthly growth rates. These are unobservable but can be estimated by the Kalman filter - smoother, as detailed below, and jointly modeled with the other factors summarizing the dynamics of the economy.

The dataset, kindly provided to us by Boivin et al. [2013], consists of the 124 monthly time series used in Bernanke et al. [2005], but extended to June 2009. As discussed in the first empirical application, the data consists of various nominal, financial and real indicators (such as consumer prices, producer prices, stocks, commodities and exchange rates, consumption expenditure, production indicators, interest rates and spreads, etc.), from which we extract the factors that describe the dynamics of the economy.

We extract three latent factors and impose two observable factors, the federal funds rate and the real government expenditure growth rate. The tests for the number of static factors favor excessive number of factors (over 15). We impose the same number of latent factors as in Boivin et al. [2013]<sup>16</sup>. We choose the number of dynamic factors using the Stock and Watson [2005] test, reported in Table 6. The test favors 5 dynamic factors (therefore we can proceed with our structural analysis using the shocks to static factors). The lag order  $p$  was set to 7.

We estimate the model with interest rate and government expenditure imposed as observable factors:

$$s_t = As_{t-1} + Bu_t, \quad (15)$$

$$y_t = H_t s_t + \tilde{v}_t, \quad (16)$$

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<sup>16</sup> Authors report that increasing the number of factors does not change the results qualitatively.

where

$$s_t = \begin{bmatrix} f_t \\ f_{t-1} \\ \vdots \\ f_{t-p+1} \end{bmatrix}, \quad f_t = \begin{bmatrix} GX_t^* \\ f_t^1 \\ f_t^2 \\ f_t^3 \\ i_t \end{bmatrix}, \quad (17)$$

$i_t$  is the federal funds rate,  $f_t^i$  ( $i = \{1, 2, 3\}$ ) the three latent factors, and  $GX_t^*$  the unobservable monthly real government expenditure. Monthly real government expenditure is ordered first because we assume that other fundamental shocks do not affect government expenditure in the same month. It takes longer than a month for a government to implement a change in spending decision or for the automatic stabilizers to respond. This assumption is often used in models estimated on a quarterly frequency. We believe that it is even more plausible in a monthly model.

The observable quarterly growth rate ( $GX_t$ ) is modeled as the sum of three consecutive latent monthly growth rates ( $GX_t^*$ ). Assuming that the  $i^{th}$  row of the state space model loadings matrix  $H_t$  corresponds to the quarterly growth rate, it is modified as:

$$H_{i,t} = \underbrace{\begin{bmatrix} C_{i,t}\lambda_i & C_{i,t}\lambda_i & C_{i,t}\lambda_i & \dots & 0 \end{bmatrix}}_{1 \times kp}, \quad (18)$$

where  $k = 5$  (the number of static factors) and  $p$  the number of lags in the factor VAR. This row contains three non-zero vectors ( $C_{i,t}\lambda_i, C_{i,t}\lambda_i, C_{i,t}\lambda_i$ ) that multiply the vectors of static factors ( $f_t, f_{t-1}, f_{t-2}$ ). Each  $\lambda_i = [1 \ 0 \ 0 \ 0 \ 0]$  selects only the first element of  $f_i$  (where  $i = t, t-1, t-2$ ), that is it selects the latent monthly growth rate ( $GX_i^*$ , where  $i = t, t-1, t-2$ ).  $C_{i,t}$  controls for missing observations<sup>17</sup>. The quarterly growth rate for government expenditure ( $GX_t$ ) then amounts to a sum of three consecutive latent monthly growth rates:

$$GX_t = C_{i,t}[GX_t^* + GX_{t-1}^* + GX_{t-2}^*] + \tilde{v}_{i,t}, \quad (19)$$

We also restrict the VAR dynamics for government expenditure (only). Mariano and Murasawa [2010] note that when they use higher order VARs to construct the monthly GDP series, the monthly GDP series becomes too volatile. For this reason they model it in a VAR(1) model. The reason for the excess volatility is that there are too many parameters in the model, for a variable with many missing observations. Therefore, the model

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<sup>17</sup>It is equal to 1 when quarterly government expenditures is observed and 0 when it is not.

is poorly identified. We encountered a similar issue. The resulting estimated monthly variable was too volatile and the impulse responses exhibited a volatile pattern. This is why we restrict the autoregressive dynamics of the government expenditure equation to a  $VAR(1)$ .<sup>18</sup> The model for the government expenditure growth rates then becomes similar in spirit to a Chow and Lin [1971] model for interpolating temporally aggregated series.

The blue line in Figure 13 plots the reconstructed quarterly government expenditures growth rates, for each month, calculated with equation (19). The circles represent the true quarterly government expenditure. The sum of three consecutive latent monthly growth rates adds up to observed quarterly growth rate in months when the quarterly rate is observable.

**Empirical Results** In this section we first compare the response of the core variables to a monetary policy shock in our model with the ones obtained by Boivin et al. [2013]<sup>19</sup> and then investigate how the economy responds to a latent monthly government expenditure shock.

Figure 14 plots the impulse responses of the core variables to a monetary policy shock (with the 90% confidence bands<sup>20</sup>), where the shock is identified using a Cholesky identification. Figure 15 presents the original IRFs obtained by Boivin et al. [2013] (p. 49), using their identification method. The IRFs in Figure 14 are similar to those in Figure 15, though there are a few differences. Specifically, we have a slightly stronger price puzzle than Boivin et al. [2013], but the response becomes negative earlier in our case. Monetary aggregates (M1 and M2) decline on impact after an increase in the interest rate, whereas they remain constant in Boivin et al. [2013] application, and then increase in both models. The response of the treasury bills rate (3M TB and 5Y TB) mimics the response of the federal funds rate (FFR), which is more persistent in our model than in Boivin et al. [2013]. In terms of real variables, in our model the IRF of industrial production shows a less persistent decline but somewhat stronger in magnitude. However, we have a decline in capacity utilization, while the response is positive for a few periods in Boivin et al. [2013]. The response of real personal consumption expenditure (REAL PCE and RPCE SER) is negative in both models, but the drop is persistent in Boivin et al. [2013] whereas it returns to zero in our application. For the labour market,

<sup>18</sup>The rest of the factors evolve in a  $VAR(p)$  model.

<sup>19</sup>We do so because Boivin et al. [2013] also introduce a new method to identify IRs. The focus of this section is on the response of the economy to a government expenditure shock, we only compare the IRs to a monetary policy shock in order to support the validity of our Choleski identification method.

<sup>20</sup>Confidence bands were estimated using sampling with replacement in 500 bootstrap replications.

we both have a negative impact on unemployment and employment, but in our case there is a slightly positive impact on hours worked, while the reaction in Boivin et al. [2013] is negative, followed by a persistent negative reaction in both models. Even though we stressed the differences in results, overall they are limited and generally not statistically significant. The responses we obtain are also very similar to those reported in the first empirical application (compare Figure 7 with Figure 14).

Let us now assess the effects of a government spending shock. Figure 16 plots the response of the core variables (with the 90% confidence bands<sup>21</sup>). The impact effects are generally negative but after few months prices increase while unemployment decreases and employment and average weekly hours increase. Consumer expectations improve and the number of housing starts increases. It is interesting to observe that employment reacts more than industrial production. The reason could be that a large share of government expenditure is devoted to buying services. The combination of higher prices and better economic conditions triggers a (still delayed) increase in the federal funds rate. These results are aligned with basic findings of economic theory.

The results from our model slightly differ from the results obtained in Ramey [2011]. According to Figure 16, hourly earnings fall on impact and return to zero after a few months. Variables that represent consumption fall on impact and then return to zero. Therefore the response of earnings and consumption in our model is closer to the results in Ramey [2011], than to the ones obtained by Fatas and Mihov [2001], Blanchard and Perotti [2002] and Pappa [2005]. Hence, this application further shows that using a MF-S-FAVAR can shed interesting light on relevant economic issues.

Figures 17 and 18 present the IRs to an interest rate shocks and government expenditure shock (respectively), obtained when we model the government expenditure as a geometric mean of the underlying monthly series. We observe from Figure 17 that the IRs to a monetary shock are qualitatively similar to when we model government expenditure as sum of 3 consecutive monthly growth rates. The main difference is in the confidence bands of the response of government expenditure to a monetary shock. From Figure 18 we observe that when government expenditure is modeled as a geometric mean, the responses of the variables have the same sign, but they exhibit a slightly oscillatory pattern and are less pronounced, likely due to the different type of aggregating that involves a larger number of months.

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<sup>21</sup>Confidence bands were estimated using sampling with replacement in 500 bootstrap replications.



## 1.9 Conclusions

In this paper we suggest to extend the FAVAR model to the mixed frequency case (MF-FAVAR) and use it for structural analyses, in order to better exploit all the available information, improve shock identification, and avoid temporal aggregation and variable omission biases.

We illustrate how the MF-FAVAR can be estimated using Kalman filter based techniques and show, by means of Monte Carlo experiments, that the resulting parameter and impulse response estimators work reasonably well also in finite samples.

We then use the MF-FAVAR to evaluate the effects of monetary, oil, and fiscal shocks, comparing the results with those in existing studies. Overall, we obtain reasonable responses in economic terms, sometimes with interesting differences with respect to earlier studies based on same frequency data.

The structural MF-FAVAR model can be applied in a variety of other contexts, and therefore we believe that it is an important item to be added to the standard toolbox of economists.

## Appendix

### Tables and figures

Table 1: Trace statistic from MC experiments, varying  $n$  and  $T$

n=50 d=20: Estimator\Time	T = 50	T = 100	T = 200
PCA on full sample	0.9284	0.9524	0.9608
PCA on the reduced sample	0.9402	0.9531	0.9640
SW estimator	0.9554	0.9690	0.9740
DGR estimator	0.9541	0.9693	0.9752
MF estimator	0.9678	0.9792	0.9836

This table reports trace statistic - a measure of how well the estimated factors track the true factors (eq.(14) on p.13). We fix the sample size to  $n = \{50\}$ , the number of quarterly series to  $d = \{20\}$  and vary sample length  $T = \{50, 100, 200\}$ . The DGP is described in Section 3.

n=100 d=20: Estimator\Time	T = 50	T = 100	T = 200
PCA on full sample	0.9655	0.9760	0.9802
PCA on the reduced sample	0.9781	0.9835	0.9859
SW estimator	0.9820	0.9868	0.9891
DGR estimator	0.9800	0.9862	0.9888
MF estimator	0.9838	0.9890	0.9911

This table reports trace statistic - a measure of how well the estimated factors track the true factors (eq.(14) on p.13). We fix the sample size to  $n = \{100\}$ , the number of quarterly series to  $d = \{20\}$  and vary sample length  $T = \{50, 100, 200\}$ . The DGP is described in Section 3.

n=200 d=20: Estimator\Time	T = 50	T = 100	T = 200
PCA on full sample	0.9829	0.9882	0.9902
PCA on the reduced sample	0.9895	0.9919	0.9932
SW estimator	0.9913	0.9937	0.9948
DGR estimator	0.9901	0.9932	0.9946
MF estimator	0.9913	0.9941	0.9953

This table reports trace statistic - a measure of how well the estimated factors track the true factors (eq.(14) on p.13). We fix the sample size to  $n = \{200\}$ , the number of quarterly series to  $d = \{20\}$  and vary sample length  $T = \{50, 100, 200\}$ . The DGP is described in Section 3.

Table 2: Trace statistic from MC experiments, varying number of quarterly series

T=200, n=200: Estimator \ Qrt. series	d=20	d=100	d=180
PCA on the full sample	0.9902	0.9902	0.9902
PCA on the reduced sample	0.9932	0.9882	0.9519
SW estimator	0.9948	0.9920	0.9714
DGR estimator	0.9946	0.9905	0.9746
MF estimator	0.9953	0.9932	0.9841

This table reports trace statistic - a measure of how well the estimated factors track the true factors (eq.(14) on p.13). We fix the sample size to  $n = \{200\}$ , sample length to  $T = \{200\}$  and vary the number of quarterly series  $d = \{20, 100, 180\}$ . The DGP is described in Section 3.

Table 3: Trace statistic from MC experiments, quarterly (unobservable) factor

T=200, n=200: Estimator \ Qrt. series	d=20	d=100	d= 180
PCA on full sample	0.9901	0.9901	0.9901
PCA on the reduced sample	0.9909	0.9836	0.9239
SW estimator	0.9910	0.9850	0.9350
DGR estimator	0.9913	0.9851	0.9545
MF estimator	0.9916	0.9869	0.9620

This table reports trace statistic - a measure of how well the estimated factors track the true factors (eq.(14) on p.13). We fix the sample size to  $n = \{200\}$ , sample length to  $T = \{200\}$  and vary the number of quarterly series  $d = \{20, 100, 180\}$ . The observable factor is a quarterly variable. The DGP is described in Section 3.

Table 4: Number of dynamic factors in Application 1, Stock and Watson (2005) test

$\hat{q} \setminus$ Criteria	PC1	PC2	PC3	IC1	IC2	IC3
1	0.9129	0.9144	0.9079	-0.0803	-0.0781	-0.0875
2	0.8719	0.8749	0.8620	-0.1203	-0.1159	-0.1347
3	0.8434	0.8479	0.8285	-0.1517	-0.1452	-0.1734
4	0.8193	0.8253	0.7994	-0.1836	-0.1749	-0.2124

This table reports the values of Bai and Ng (2002) information criteria used in the Stock and Watson (2005) test for selecting the number of dynamic factors ( $\hat{q}$ ). Number of dynamic factors is estimated to be the one with the smallest value of the information criteria.

Table 5: Number of dynamic factors in Application 2, Stock and Watson (2005) test

$\hat{q} \setminus$ Criteria	PC1	PC2	PC3	IC1	IC2	IC3
1	0.9097	0.9116	0.9036	-0.0792	-0.0761	-0.0890
2	0.8374	0.8412	0.8253	-0.1541	-0.1479	-0.1737
3	0.8064	0.8121	0.7882	-0.1873	-0.1780	-0.2167
4	0.7836	0.7913	0.7594	-0.2154	-0.2030	-0.2546
5	0.7719	0.7814	0.7417	-0.2321	-0.2166	-0.2810

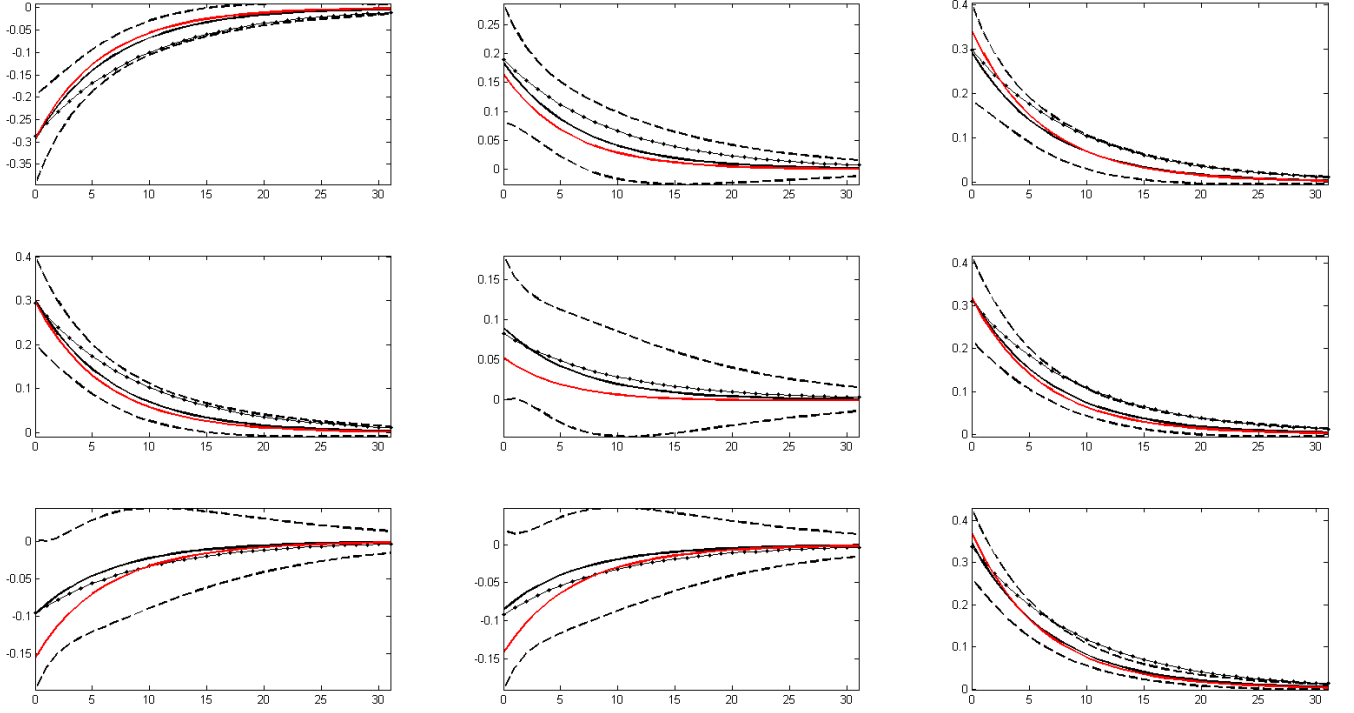
This table reports the values of Bai and Ng (2002) information criteria used in the Stock and Watson (2005) test for selecting the number of dynamic factors ( $\hat{q}$ ). Number of dynamic factors is estimated to be the one with the smallest value of the information criteria.

Table 6: Number of dynamic factors in Application 3, Stock and Watson (2005) test

$\hat{q} \setminus$ Criteria	PC1	PC2	PC3	IC1	IC2	IC3
1	0.9178	0.9189	0.9139	-0.0717	-0.0699	-0.0778
2	0.8613	0.8636	0.8537	-0.1264	-0.1228	-0.1387
3	0.8141	0.8175	0.8026	-0.1797	-0.1743	-0.1980
4	0.7897	0.7942	0.7744	-0.2095	-0.2022	-0.2339
5	0.7673	0.7729	0.7482	-0.2421	-0.2331	-0.2727

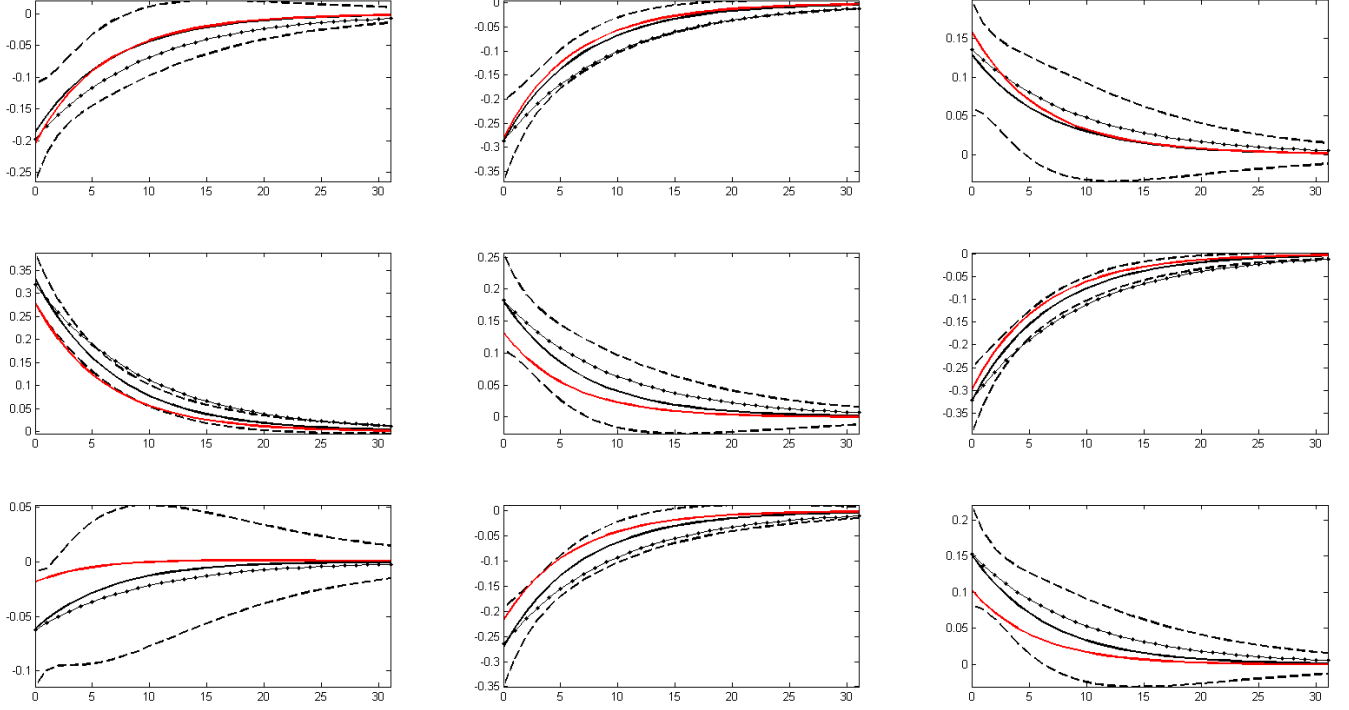
This table reports the values of Bai and Ng (2002) information criteria used in the Stock and Watson (2005) test for selecting the number of dynamic factors ( $\hat{q}$ ). Number of dynamic factors is estimated to be the one with the smallest value of the information criteria.

Figure 1: IRs to a unit shock in the observable factor of quarterly variables, comparison with Stock and Watson (2005) procedure



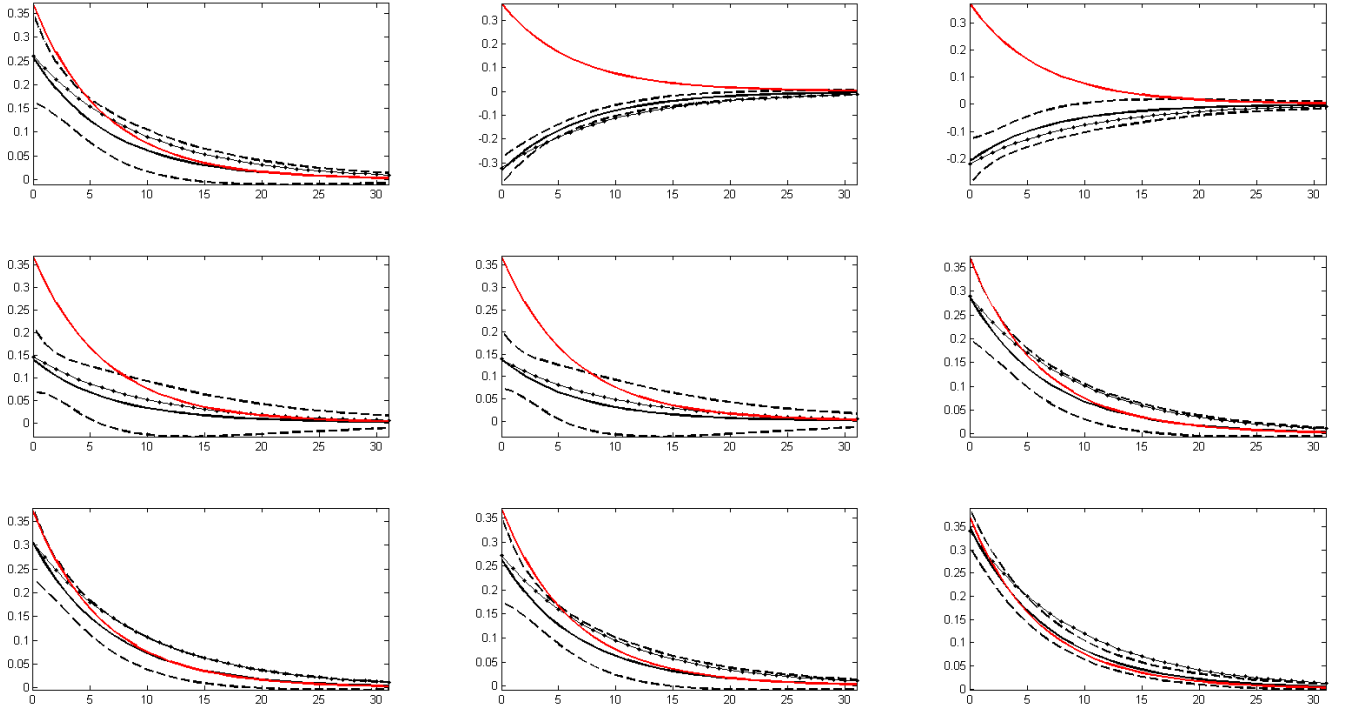
IRs of the first 9 quarterly variables: **true** - solid black with dots, **SW** - solid red, **ML** - solid black, **ML  $\pm$  2 std. dev.** - dashed black. We fix the sample size to  $n = \{200\}$ , sample length to  $T = \{200\}$  and the number of quarterly series to  $d = \{100\}$ . The DGP is described in Section 3.

Figure 2: IRs to a unit shock in the observable factor of monthly variables, comparison with Stock and Watson (2005) procedure



IRs of the first 9 monthly variables: **true** - solid black with dots, **SW** - solid red, **ML** - solid black, **ML  $\pm$  2 std. dev.** - dashed black. We fix the sample size to  $n = \{200\}$ , sample length to  $T = \{200\}$  and the number of quarterly series to  $d = \{100\}$ . The DGP is described in Section 3.

Figure 3: IRs to a unit shock in the quarterly factor



IRs of the first 9 variables: **true** - solid black with dots, **ML** - solid black, **ML  $\pm$  2 std. dev.** - dashed black. IRs are reported on a monthly frequency. We fix the sample size to  $n = \{200\}$ , sample length to  $T = \{200\}$  and the number of quarterly series to  $d = \{0\}$ . The DGP is described in Section 3.

Figure 4: (Quarterly) IRs to a unit shock in the observable factor, quarterly variables

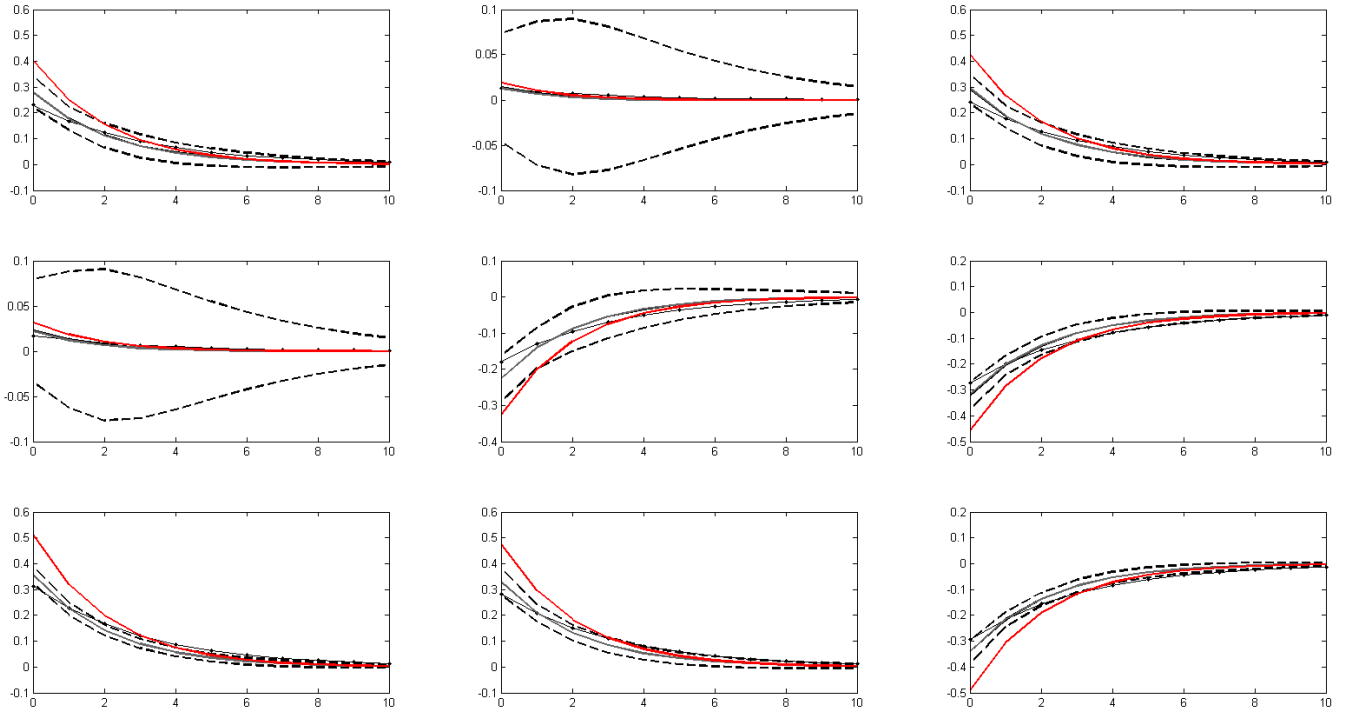


Figure displays the impulse responses of the first 9 quarterly variables to a unit shock in the observable factor, calculated using monthly dataset (solid black line, dashed black lines are the  $\pm 2$  std. dev. bands), mixed frequency dataset (solid gray line) and quarterly dataset (solid red line). The true IRs are represented with a solid black line with dots. IRs are reported on a quarterly frequency. Sample size is  $n = \{200\}$  and sample length is  $T = \{200\}$ . The number of quarterly series in a mixed frequency dataset is set to  $d = \{100\}$ . The DGP is described in Section 3.



Figure 5: (Quarterly) IRs to a unit shock in the observable factor, monthly variables

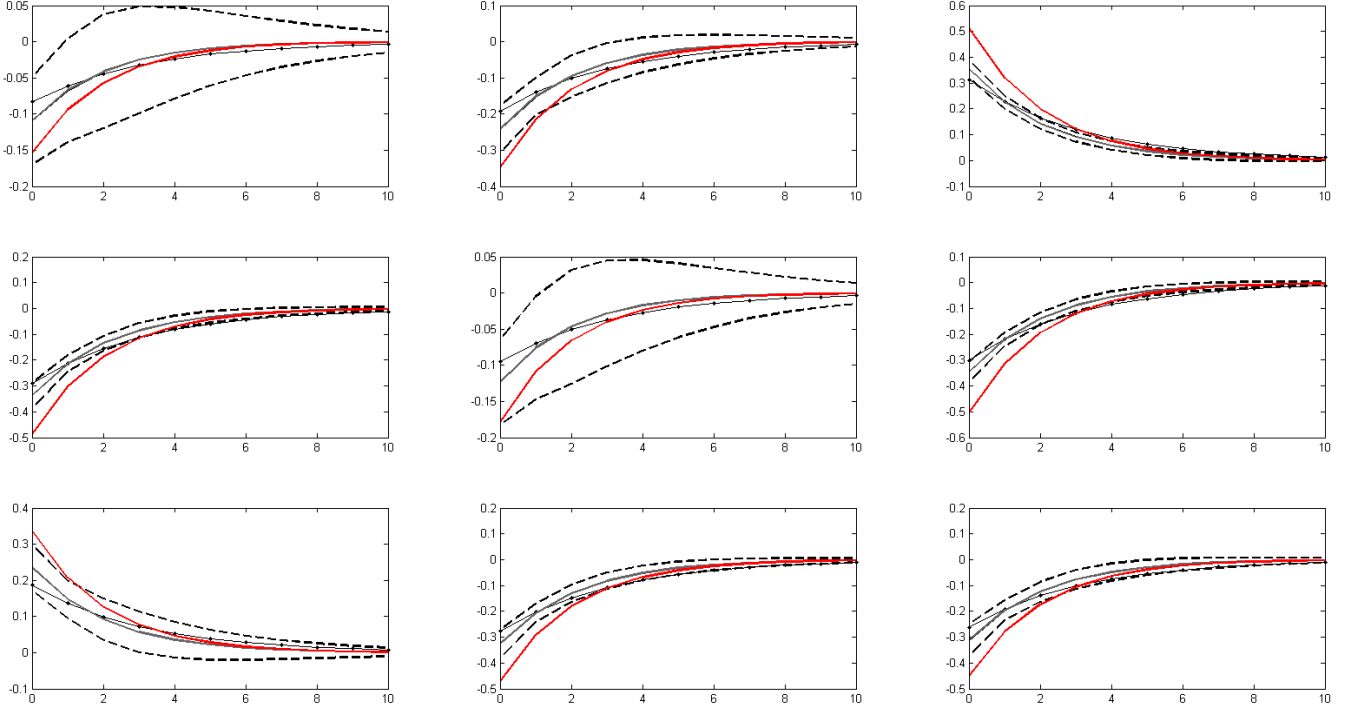


Figure displays the impulse responses of the first 9 monthly variables to a unit shock in the observable factor, calculated using monthly dataset (solid black line, dashed black lines are the  $\pm 2$  std. dev. bands), mixed frequency dataset (solid gray line) and quarterly dataset (solid red line). The true IRs are represented with a solid black line with dots. IRs are reported on a quarterly frequency. Sample size is  $n = \{200\}$  and sample length is  $T = \{200\}$ . The number of quarterly series in a mixed frequency dataset is set to  $d = \{100\}$ . The DGP is described in Section 3.

Figure 6: Comparison of the two factors estimates

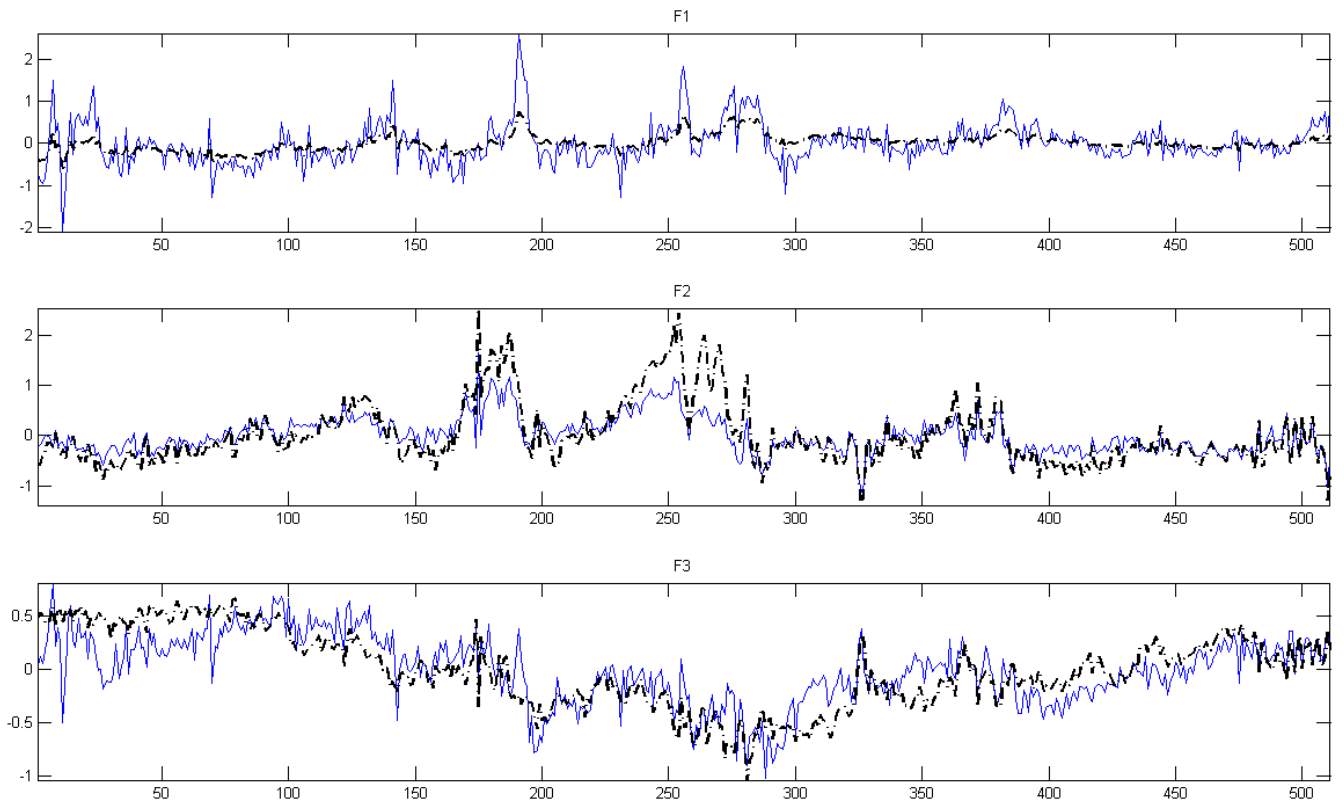


Figure displays the factors estimated using the PCA approach (solid line) and the factors estimated with MF-S-FAVAR (dashed black line), estimated on the Bernanke et al. (2005) dataset.

Figure 7: IRs of selected variables to a monetary policy shock

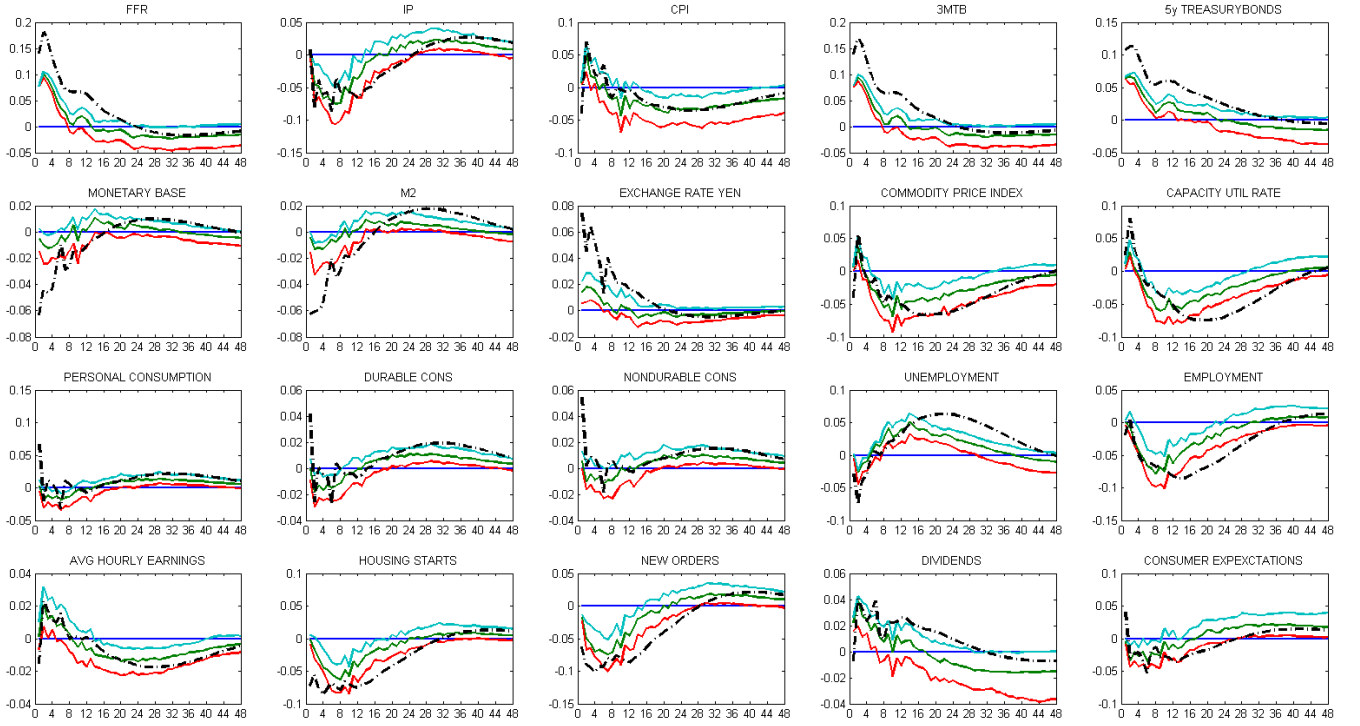


Figure displays the IRs of selected variables to a monetary policy shock, estimated on the Bernanke et al. (2005) dataset. Solid lines represent the IRs (and the confidence bands) calculated using the Bernanke et al. (2005) method and the black dashed lines the IRs estimated with MF-S-FAVAR.

Figure 8: IRs of the industrial production index and monthly GDP to a monetary policy shock

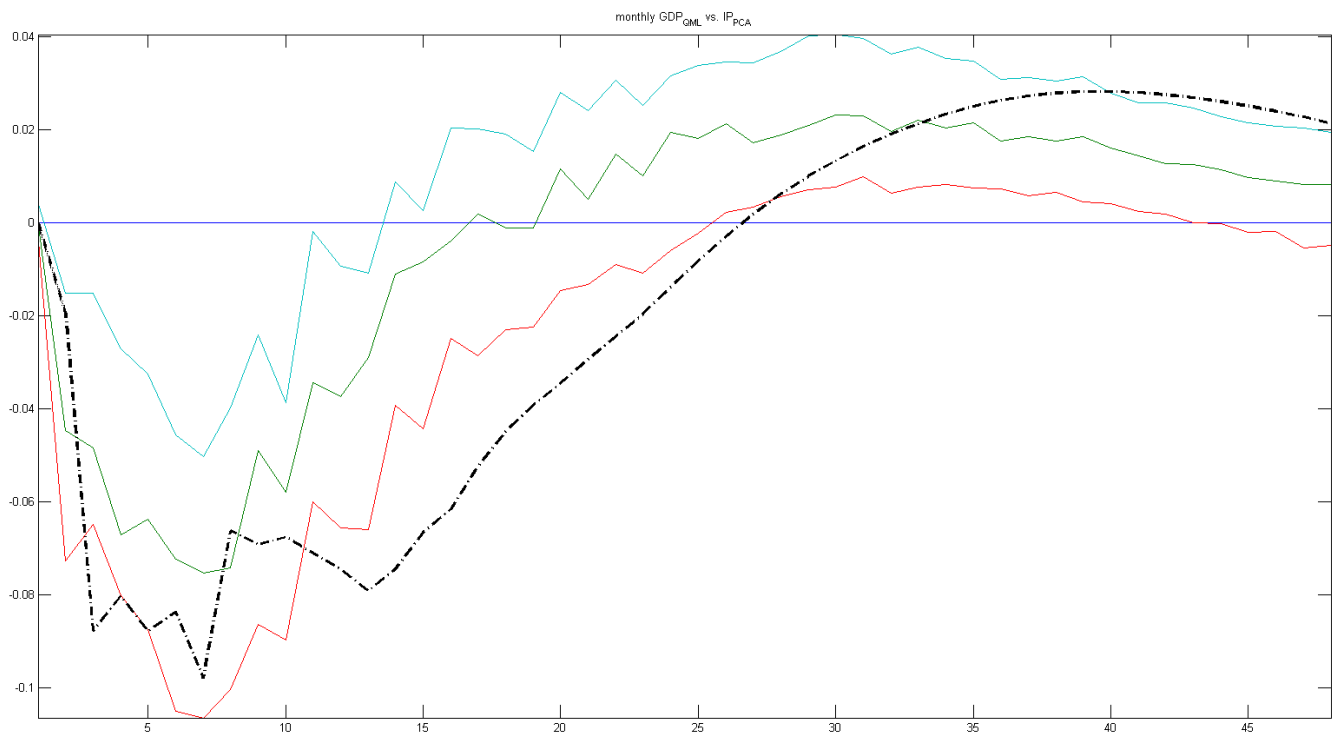


Figure displays the IR of the industrial production index (solid line), calculated using the PCA approach, and of the latent monthly GDP (dashed black line), estimated with MF-S-FAVAR. The IRs were estimated on the Bernanke et al. (2005) dataset

Figure 9: IRs of selected variables to a monetary policy shock - GDP modeled as a geometric mean

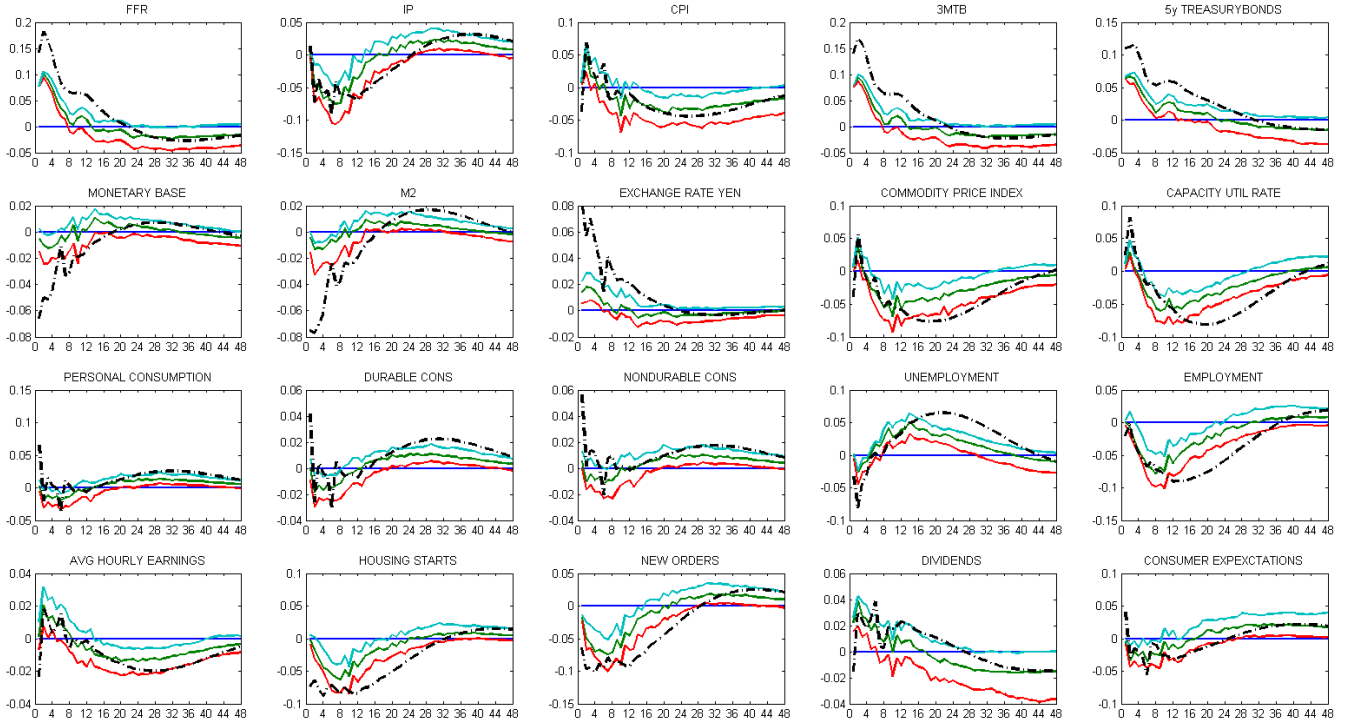
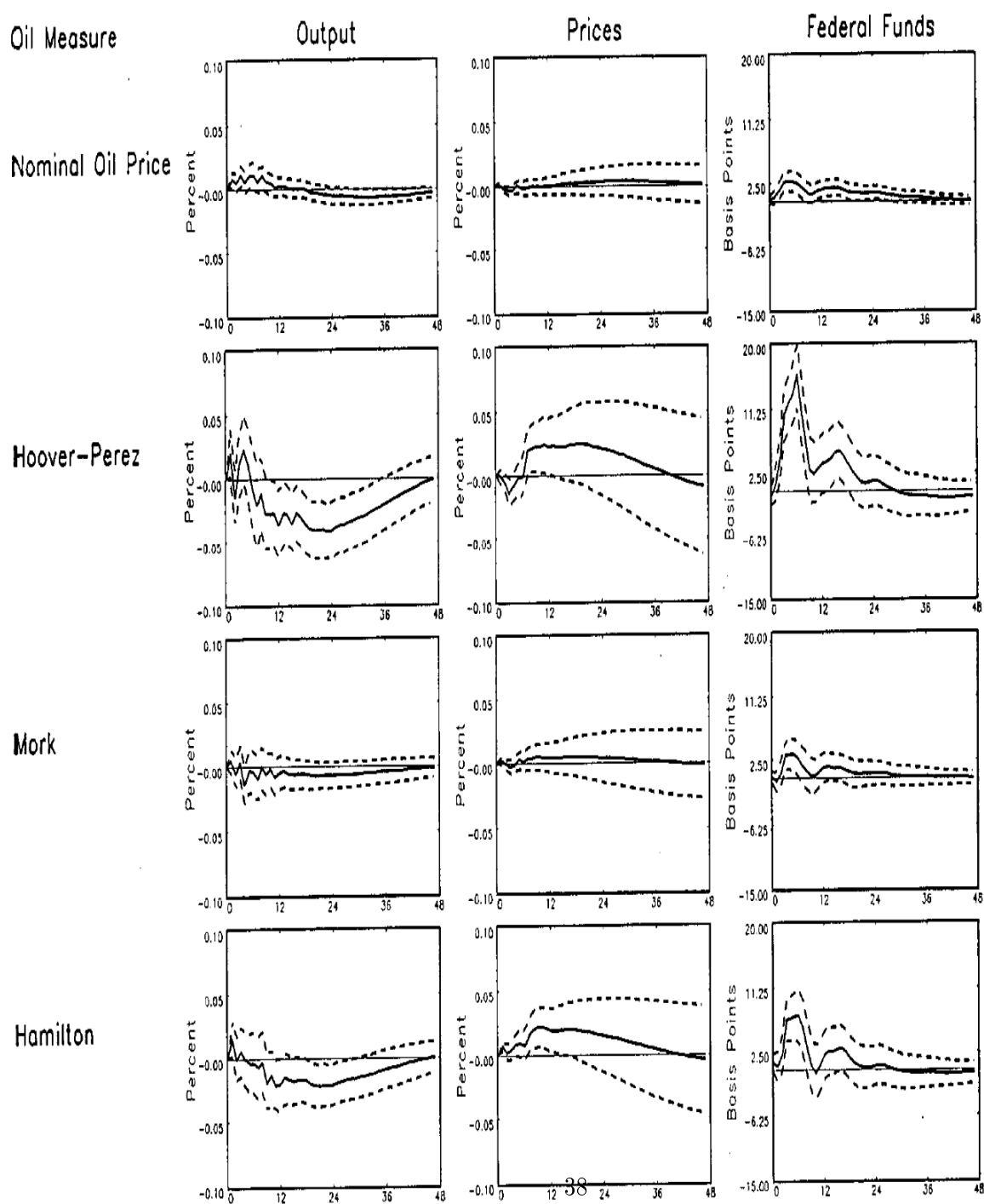


Figure displays the IRs of selected variables to a monetary policy shock, estimated on the Bernanke et al. (2005) dataset. Solid lines represent the IRs (and the confidence bands) calculated using the Bernanke et al. (2005) method and the black dashed lines the IRs estimated with MF-S-FAVAR.

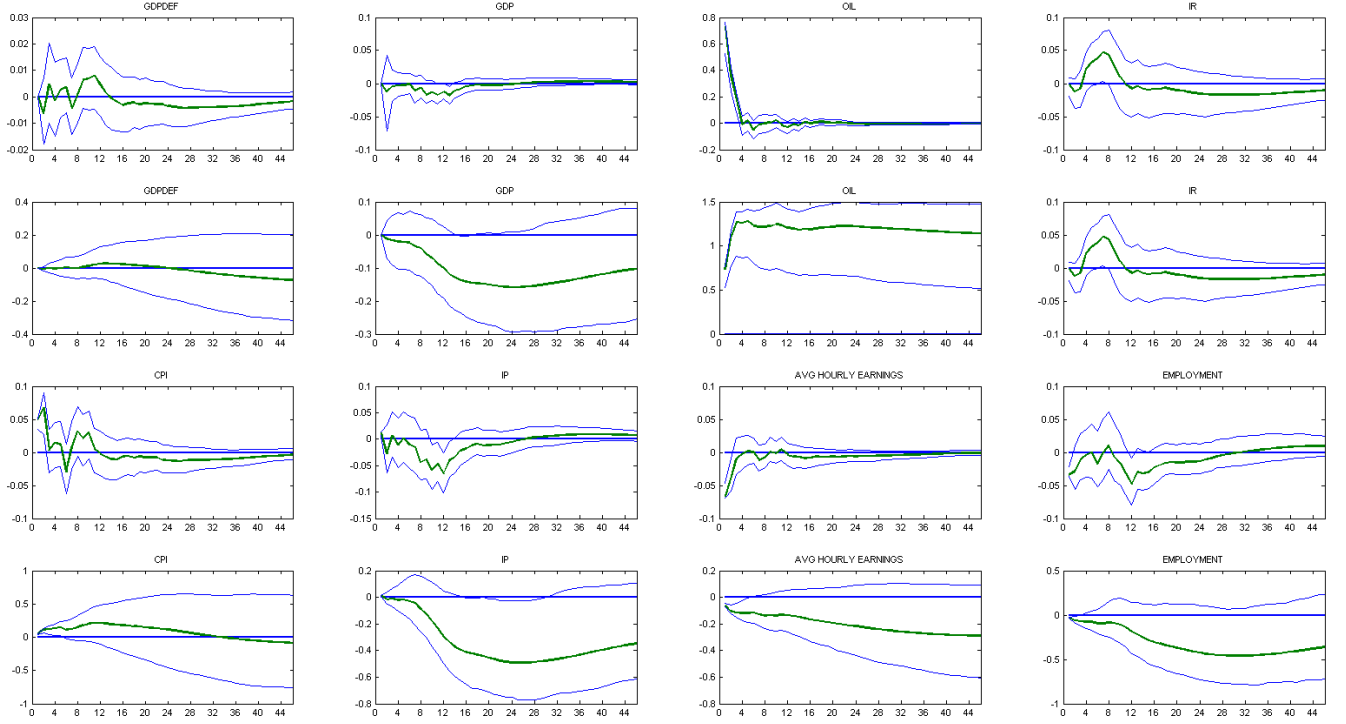
Figure 10: The effects of oil price shocks, Bernanke et al. 1997

Figure 2: Response to a 1 Percent Oil Price Shock



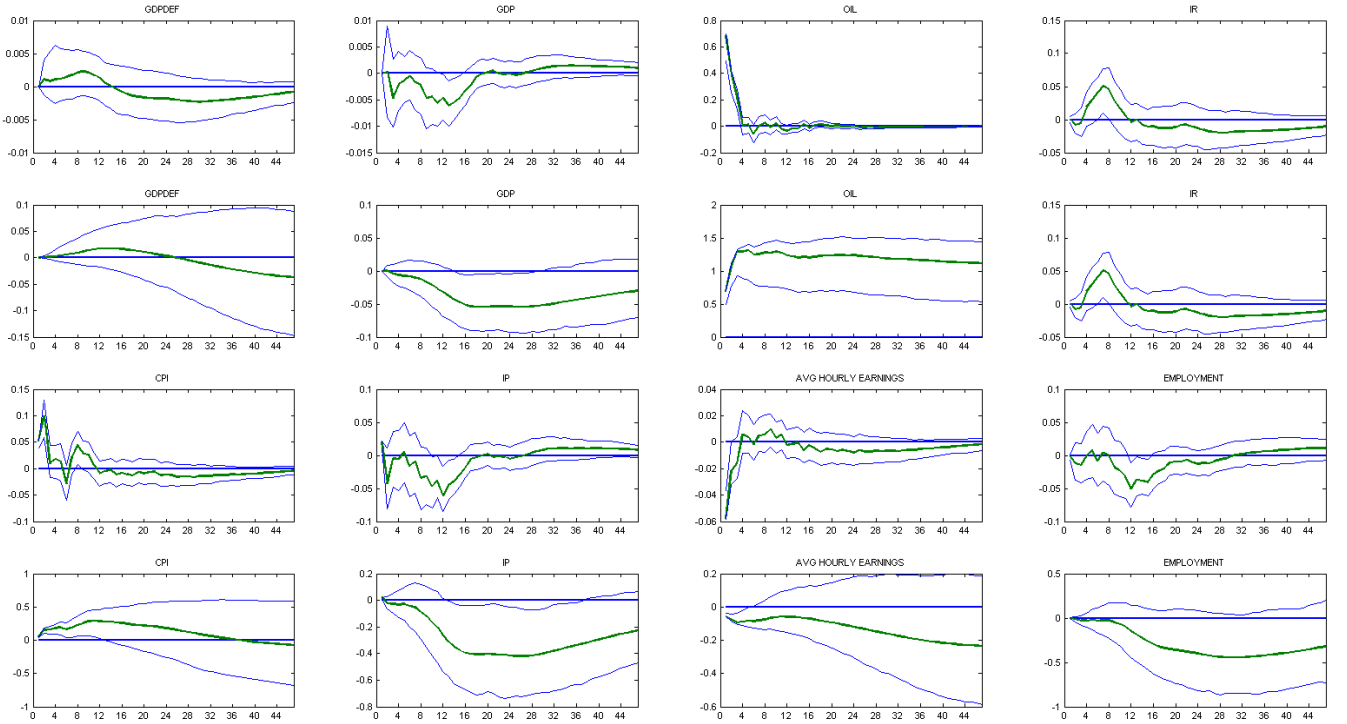
IRs to an oil price shock using different measures of oil prices, as reported in Bernanke et al. (1997).

Figure 11: MF-S-FAVAR, the effects of oil price shocks



IRs to an oil price shock using the sample sample size as in Bernanke et al. (1997). The Bernanke et al. (2005) dataset was used to extract latent factors. For each variable we plot the level (upper figure) and cumulated (lower figure) responses. Confidence bands are calculated using 500 bootstrap replications.

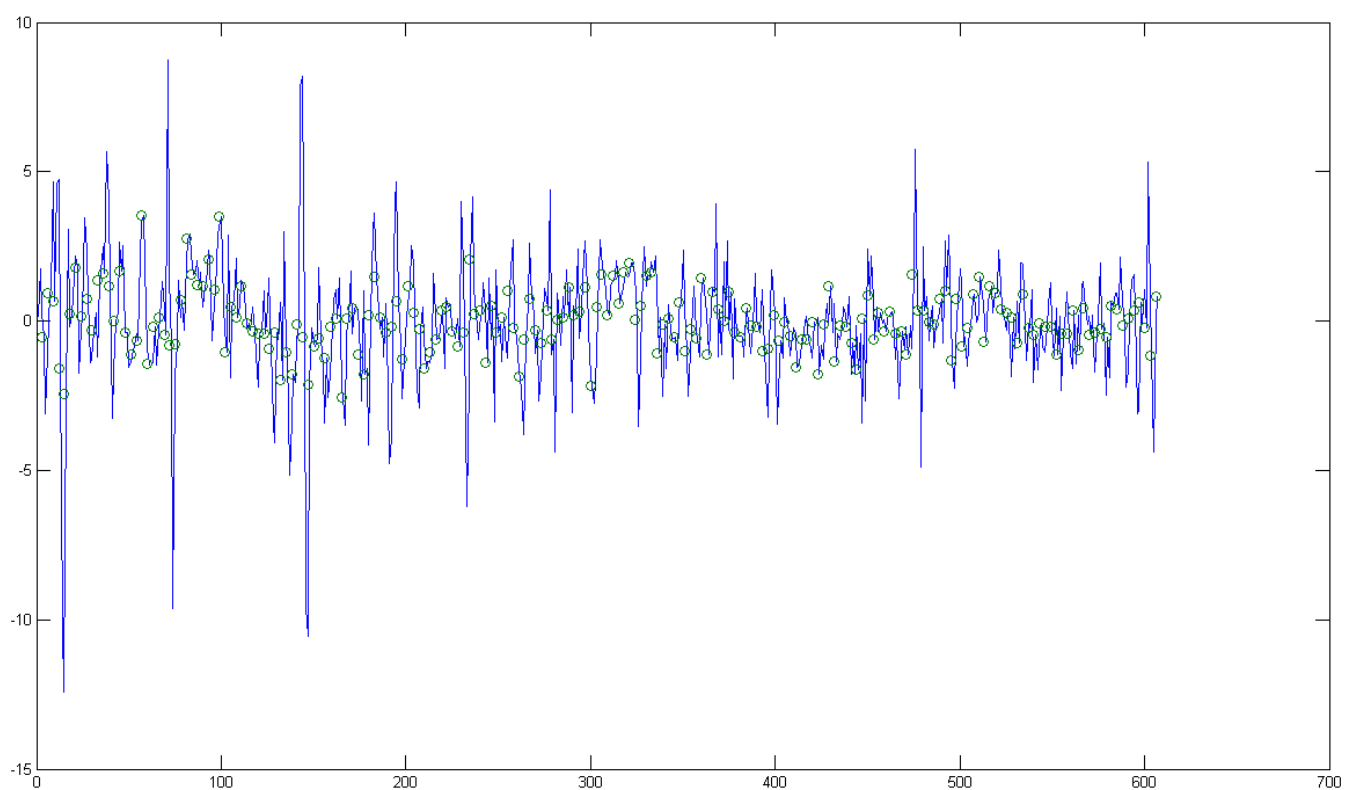
Figure 12: MF-S-FAVAR, the effects of oil price shocks - GDP and GDP delator modeled as geometric means



IRs to an oil price shock using the sample sample size as in Bernanke et al. (1997). The Bernanke et al. (2005) dataset was used to extract latent factors. For each variable we plot the level (upper figure) and cumulated (lower figure) responses. Confidence bands are calculated using 500 bootstrap replications.

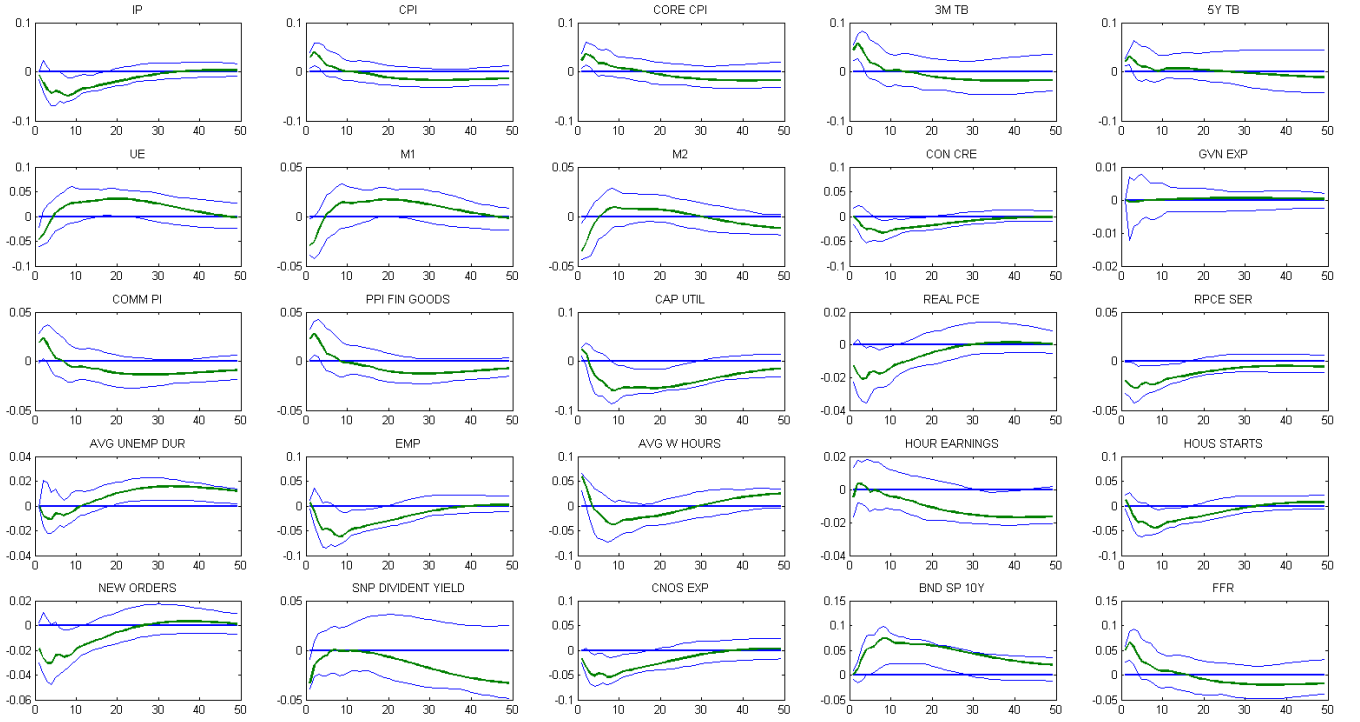


Figure 13: MF-S-FAVAR, reconstructed monthly government expenditure growth rates



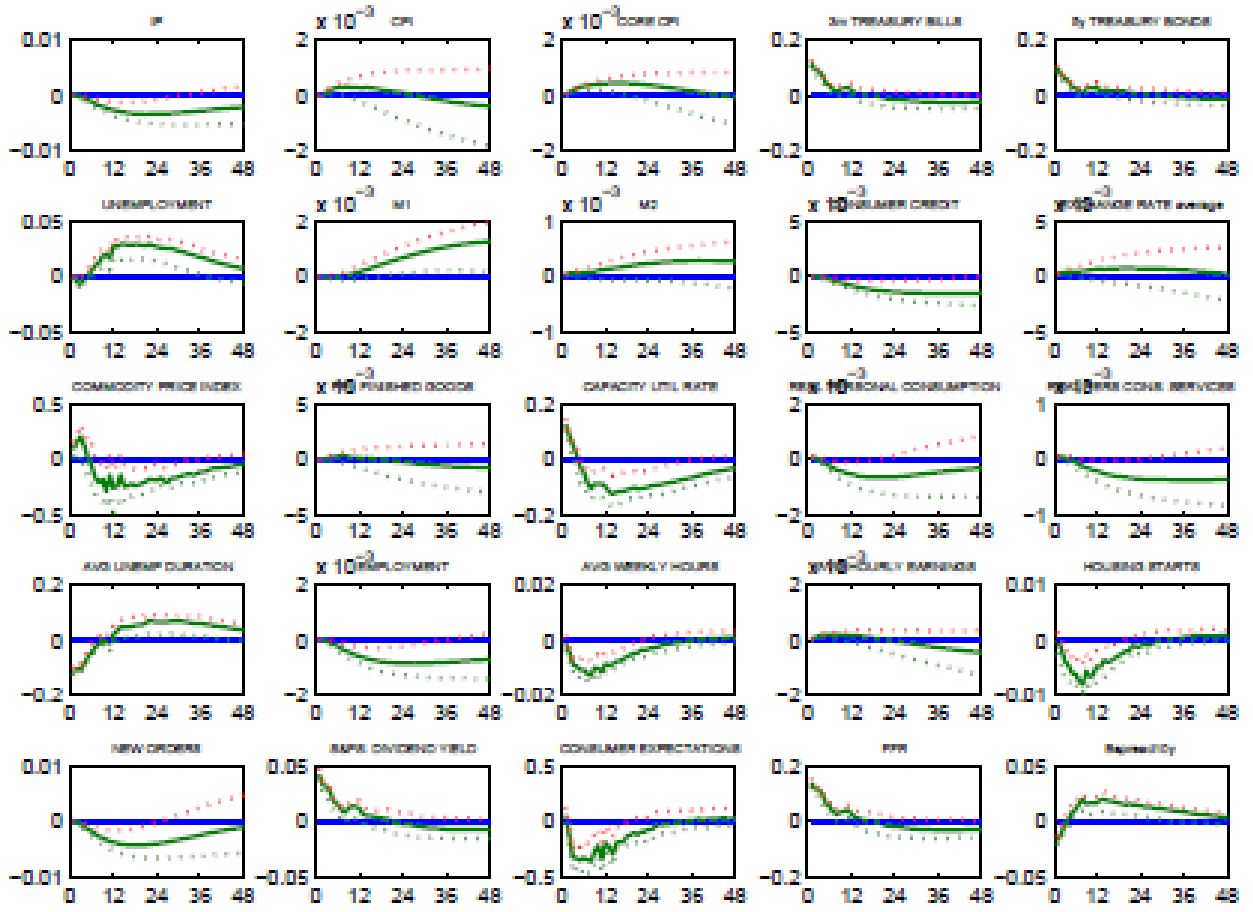
Solid blue line represents the reconstructed monthly government expenditure growth rates (Section 4.3.2, eq. (19)) and green dots the quarterly government expenditure growth rates

Figure 14: MF-S-FAVAR, IRs to a monetary policy shock



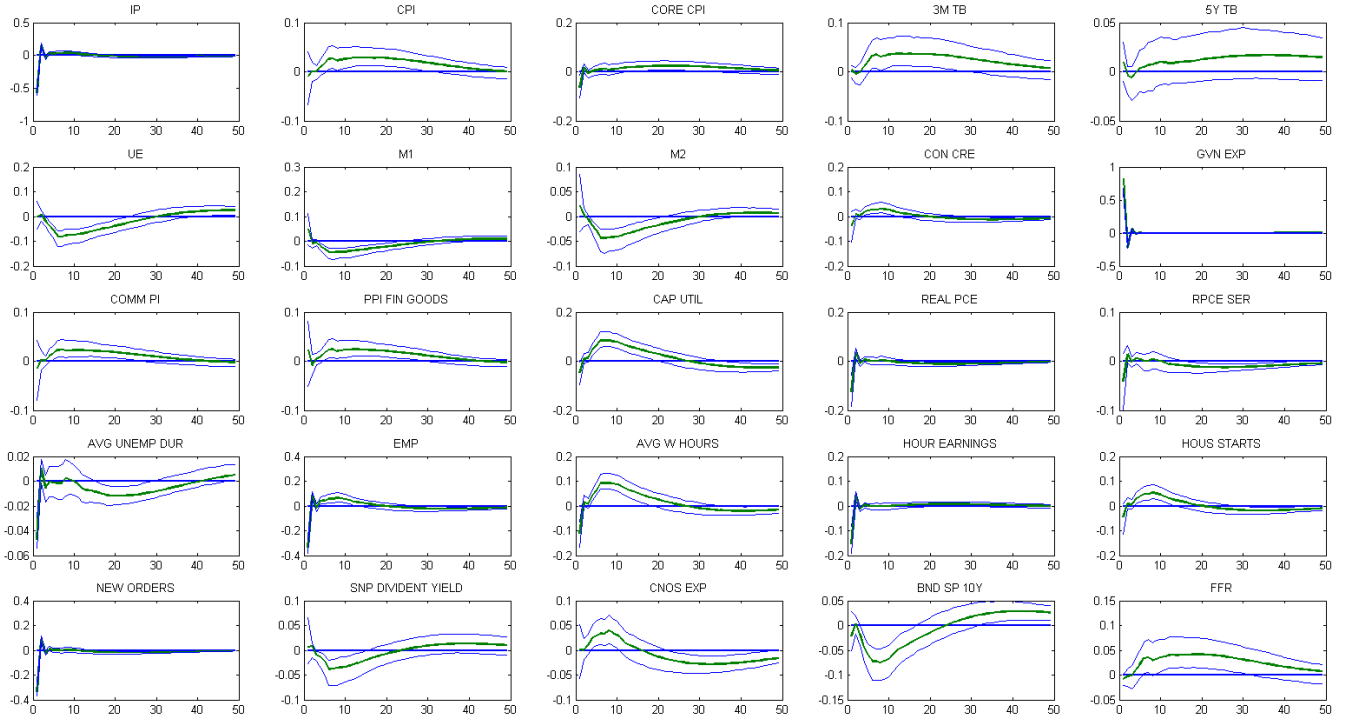
IRs of selected variables to a monetary policy shock. IRs are estimated using the Boivin et al. (2013) dataset. Blue lines are the 95% confidence bands estimated using 500 bootstrap replications.

Figure 15: IRs to a monetary policy shock, Boivin et al. (2013, p.49)



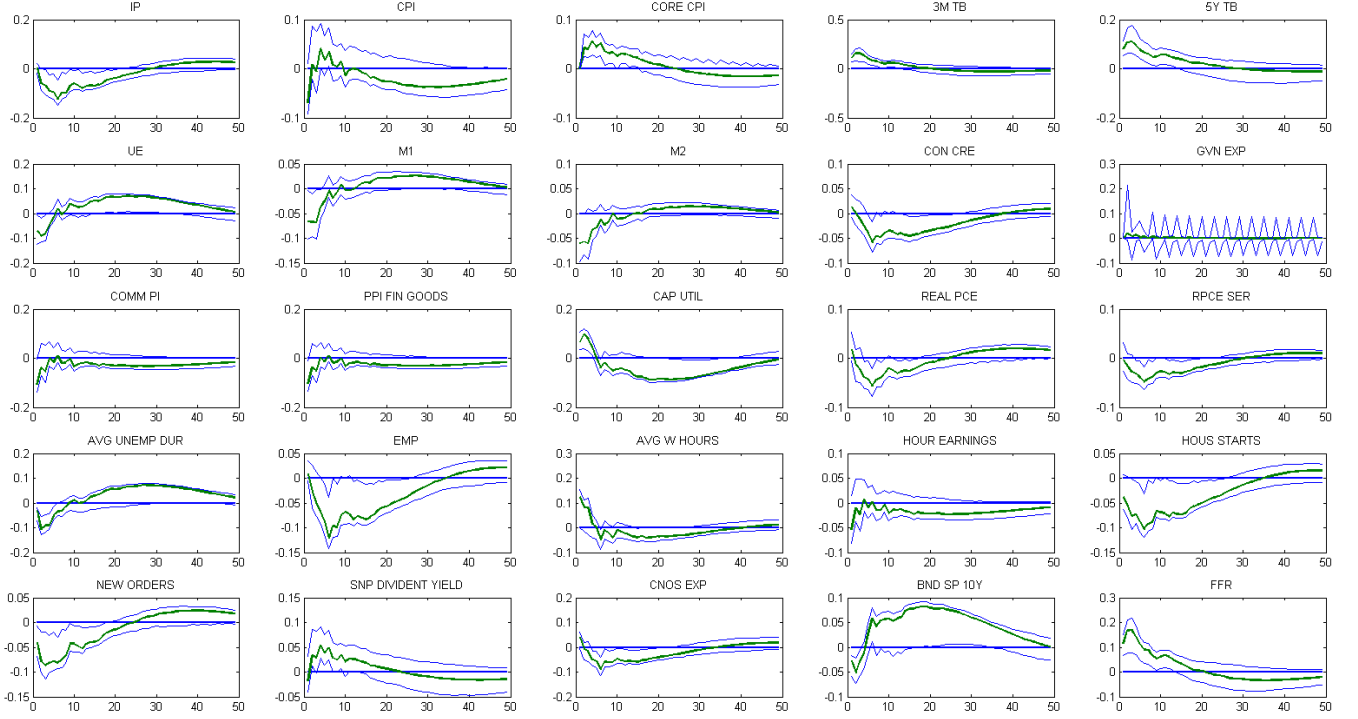
IRs of selected variables to a monetary policy shock as in Boivin et al. (2013). Dashed lines represent the 95% confidence bands.

Figure 16: MF-FAVAR, IRs to a government expenditure shock - Cholesky identification



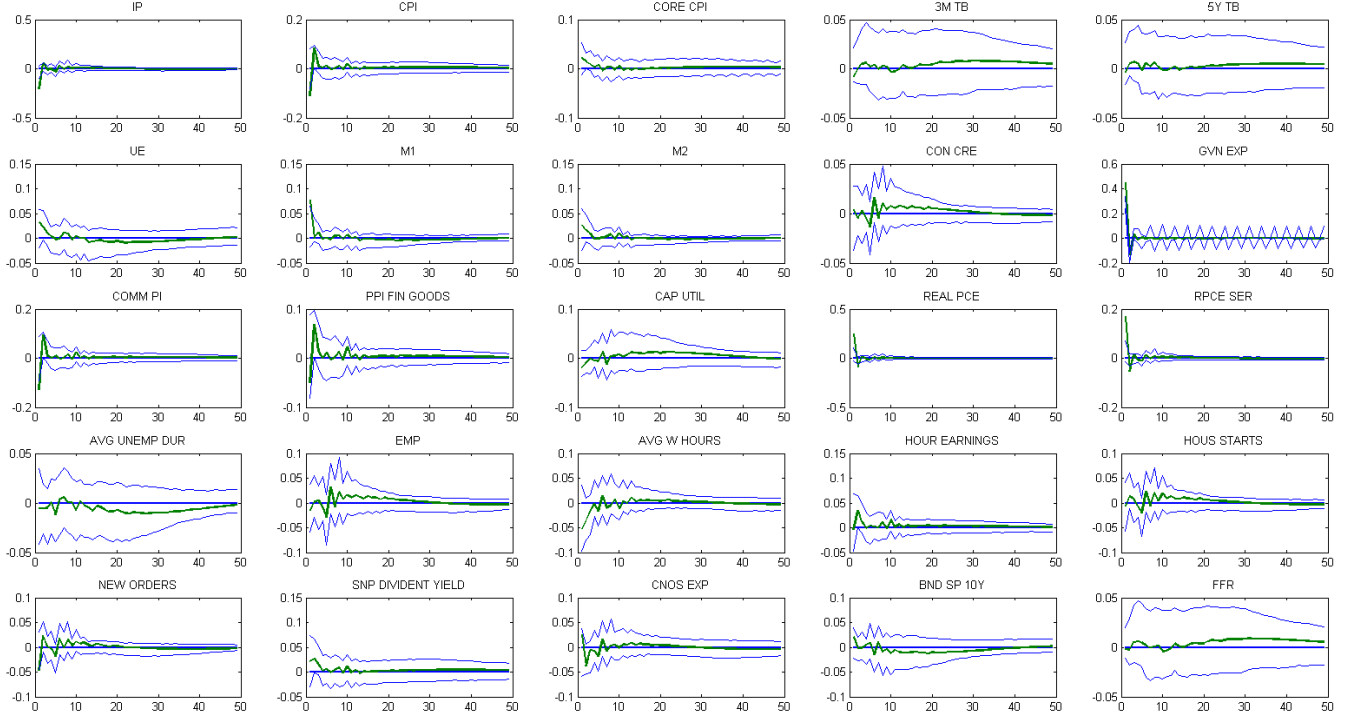
IRs of selected variables to a monthly government expenditure shock. IRs are estimated using the Boivin et al. (2013) dataset. Blue lines are the 95% confidence bands estimated using 500 bootstrap replications.

Figure 17: MF-S-FAVAR, IRs to a monetary policy shock - Cholesky identification, government expenditure modeled as a geometric mean



IRs of selected variables to a monetary policy shock. IRs are estimated using the Boivin et al. (2013) dataset. Blue lines are the 95% confidence bands estimated using 500 bootstrap replications.

Figure 18: MF-FAVAR, IRs to a government expenditure shock - Cholesky identification, government expenditure modeled as a geometric mean



IRs of selected variables to a monthly government expenditure shock. IRs are estimated using the Boivin et al. (2013) dataset. Blue lines are the 95% confidence bands estimated using 500 bootstrap replications.

## 2 Chapter

We test predictions of the rational inattention model put forward by Maćkowiak et al. [2009] in a time varying environment. Their model explains how aggregate and sector shock variances affect sector price impulse responses. We exploit the fact that variances of aggregate shocks have varied greatly over time. We estimate a time varying parameter factor model on US post-war data on macroeconomic variables and sector prices. We identify impulse responses of sector prices to macroeconomic shocks, sector shock and their respective variances. We then construct a panel of impulse responses and use fixed effects regression to test the predictions of the rational inattention model. We find empirical support for the main predictions of the model, while some are refuted by the empirical model. We find that firms do not trade off between aggregate and sector conditions in deciding which shocks to pay attention to, but do trade off between aggregate shocks, which is a new finding in the empirical literature.

### 2.1 Introduction

Recently economists became interested in peculiar differences in the behavior of aggregate and sector prices. Altissimo et al. [2009], Bils and Klenow [2002] and Clark [2006] find that inflation is more persistent at the aggregate level. Boivin et al. [2009], Maćkowiak et al. [2009] and Baumeister et al. [2010] find that prices react with a delay to an aggregate shock and with a full long run effect on impact to a sector specific shock.

Several theoretical models that try to explain these differences. Maćkowiak et al. [2009] present three models that explain them. They consider: Calvo [1983] model of staggered prices, Mankiw and Reis [2002] sticky information model and the rational inattention model of Maćkowiak and Wiederholt [2007] (hereafter the RIA model). They show that in order to produce theoretical responses to shocks that are consistent with those found in the data, one needs to impose unreasonable restrictions on the profit maximizing prices in the Calvo [1983] and the Mankiw and Reis [2002] model, but not in the RIA model<sup>22</sup>.

It is important for policy analysis and our understanding to have a good theoretical model of price setting that has been tested extensively. The predictions of the RIA model have only been tested on cross section datasets. Therefore we estimate a time varying parameter model and test the predictions of the RIA model on a panel dataset.

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<sup>22</sup>Carvalho and Lee [2011] presents a model that also produces the different sector price responses without unreasonable assumptions.

In the RIA model the decision makers have limited ability to process information. Because of their limited ability they need to decide which shocks to pay attention to, in order to minimize the losses from being hit by shocks. It turns out that to minimize the losses they allocate their attention to a shock in proportion to the variance of that shock. This prediction of the model was tested in cross section by Maćkowiak et al. [2009] and (implicitly) by Boivin et al. [2009]. They found empirical support in favor of the model. In this paper we exploit the fact that aggregate and sector shock variances do not only vary across sectors but also over time. The volatility of inflation and output has declined considerably after 70'. Several authors document this phenomena named "The Great Moderation" and convincingly argue that it was caused by a decrease in the variance of aggregate shocks. For example, Sims and Zha [2006] and Bernanke and Mihov [1995] document a decline in the volatility of monetary shocks and Gambetti et al. [2008] a decline in the volatility of demand and supply shocks. And due to the recent financial crisis we are now again faced with turbulent times. In the RIA model this has implications for how sector prices should respond to aggregate shocks over time. We exploit this fact to test the predictions of the model.

Maćkowiak et al. [2009] test some predictions of the RIA theory by using a static model. They decompose sector price variances on sector specific and aggregate variances. They then regress sector price impulse response on aggregate and sector specific variances to test if they can explain the shape of impulse responses. They find affirmative answer. But they only estimate a static model and a reduced aggregate shock. Reduced shock is a mixture of structural shocks. Therefore one can argue that should they use structural shocks the results could be different. In this paper we test the theory by using identified structural shocks and in a time varying framework.

Boivin et al. [2009] decompose sector prices into a sector specific component and component attributable to a monetary shock. They obtain similar results to Maćkowiak et al. [2009]. Both, Boivin et al. [2009] and Maćkowiak et al. [2009], assume that price responses to shocks have remained stable over time<sup>23</sup>. In contrast with them we explicitly model changes in the variances of the shocks and allow for changes in the propagation mechanism of the shocks. We do it because RIA model predicts that impulse response of a price changes when the variance of a shock changes.

Similar to our model is the model of Baumeister et al. [2010]. Like us they model changes in the transmission mechanism and the volatilities of the shocks. In contrast with us they do not allow for changes in sector specific volatilities. They estimate impulse

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<sup>23</sup>Boivin et al. [2009] perform a robustness check by estimating their model on subsample for post 1984 and find quantitative but not qualitative changes in the results.



responses of sector price responses to a monetary shock and find that the dispersion of the price responses has attenuated over time. They explain this result in a model with sticky prices. In their model the price responses have attenuated because the firms sensitivity to the marginal costs of the interest rate has increased and because wage stickiness has declined. We link the same changes to the changing variances of the shocks and provide empirical evidence for this claim.

Because we are using a time varying model we are able to test more implications of the RIA model than those that were tested by Maćkowiak et al. [2009] and Boivin et al. [2009]. Similar to them we test if the increase in the variance of the aggregate shock increases the speed and size of impulse response of sector price. Unlike them we estimate a time varying model and test this claim with a panel dataset, where we control for the individual effects. This reduces potential bias that could arise from sector specific characteristics that do not change over time<sup>24</sup>. The relation between variances of aggregate shocks and the speed of impulse responses that we find is not compatible with the RIA theory of price setting. On the other hand, we find that the variance of the aggregate shocks affect the size of impulse responses. Note that we do not test this prediction on the reduced aggregate shocks only, but also by using identified shocks. This provides us with a more detailed test of the model predictions. We then add variances of sector specific shocks to the regression and confirm that the higher is the variance of sector shock the smaller is the impulse response of price to an aggregate shock, but the effect is small and often insignificant. These results are in accordance with the predictions of the RIA model. We conclude that sector specific shocks do not seem to compete with aggregate shocks, which is not what the RIA model implies.

Next, we test if the identified aggregate shocks compete for the attention of the decision makers in firms in the same manner as idiosyncratic shocks should compete with the aggregate shocks. We find this to be the case. The RIA model also implies that the impulse response to aggregate shock should decrease if the variance of aggregate shock fell. We confirm this visually and by estimating a fixed effects model on two sub-panels. Finally, we note that the dispersion of impulse responses to aggregate shocks has decreased over time (up to the financial crisis). The RIA model is not compatible with this fact. It predicts an increase in the dispersion of impulse responses, because when firms pay less attention to aggregate shocks the impulse responses to those shocks become less uniform.

We estimated the impulses and shock variances in a dynamic factor model. They have become popular tool because they allow one to use information on a large amount

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<sup>24</sup>Such as competitiveness, openness to trade, durability of goods produced, etc...

of data in a simple, compact model and can overcome the limited information problem. There have only been a few applications of factor models in a time varying framework. Most authors apply a FAVAR approach pioneered by Bernanke et al. [2005]. A FAVAR approach is a convenient approach for identifying a monetary policy shock. One simply appends the monetary policy variable to the factors and then identifies structural shock using conventional exclusion restrictions. The FAVAR approach was used in a time varying framework by Baumeister et al. [2010], Liu et al. [2011], Mumtaz et al. [2011], Eickmeier et al. [2011b] and Bianchi et al. [2009]. This approach is elegant but it has a drawback. An assumption underlying the FAVAR approach is that the number of static factors is the same as the number of structural shocks, which was rejected in our dataset. In addition, the identification restrictions are applied on the static factors in the FAVAR approach, which is equivalent to applying them on the dynamic factors only in special cases. To avoid this issue we implemented the Forni et al. [2009] approach<sup>25</sup> to identifying structural shocks. It allows us to identify structural shocks by using sign restrictions.

The remainder of this paper is organized as follows. Section 2 briefly introduces the RIA model and its implications for the sector price behavior. In Section 3 we introduce the empirical model. Section 4 presents the data and section 5 the results. We offer final remarks in Section 6.

## 2.2 Rational inattention model of sector prices

We estimate a factor model in order to evaluate to what degree does the rational inattention model of price setting comply with reality. We investigate the model for two reasons. First, it is a relatively new and prospective model. Concept of rational inattention was first introduced by Sims [1998, 2003]. Sims argued that agents can not attend to all information perfectly and proposed to model this as a constraint on the agent's information flow. Maćkowiak and Wiederholt [2007] use this mechanism to model the trade off that firms need to make in tracking aggregate and idiosyncratic conditions. In Maćkowiak et al. [2009] they model firms to explain sector price behavior. If we are to use this model to explain reality or for policy analysis, it should be thoroughly tested. Second, Maćkowiak et al. [2009] test the implications of the model and show that their model can reproduce realistic impulse responses to aggregate and sector specific shocks. They test the model predictions in cross section. We note that the model also offers predictions over time and therefore test them.

Note that the presented does not include a micro-founded model of consumers. A fully micro-founded model is presented in Mackowiak and Wiederholt [2011]. Unfortunately

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<sup>25</sup>Hereafter FGLS

the focus of Mackowiak and Wiederholt [2011] is not on sector price behavior. Therefore we use Maćkowiak et al. [2009] to present the main ideas and only refer to Mackowiak and Wiederholt [2011] when appropriate.

This section briefly introduces the model and its implications for sector price dynamics. Due to space considerations we only present key equations and describe the intuition behind them. This is enough to set ground for the empirical part that follows. For an in depth treatment the interested reader should consult Maćkowiak et al. [2009].

Economy is populated with a continuum of sectors and in each sector there is a continuum of monopolistically competitive firms. Firms produce differentiated goods and set prices to maximize the expected discounted sum of profits. Decision makers in firms decide what to pay attention to. They can not perfectly attend to all information<sup>26</sup>. This limitation is modeled as a constraint on the information flow:

$$\underbrace{H(p_{int}^{*A}|s_{in}^{t-1}) - H(p_{int}^{*A}|s_{in}^t)}_{\kappa_A} + \underbrace{H(p_{int}^{*S}|s_{in}^{t-1}) - H(p_{int}^{*S}|s_{in}^t)}_{\kappa_S} \leq \kappa \quad (1)$$

where the LHS of the inequality represents the decision maker's information flow ( $\kappa_A + \kappa_S$ ) and the RHS his information processing capability ( $\kappa$ ).  $\kappa$  can be a fixed constant or a convex function of costs.  $\kappa_A$  and  $\kappa_S$  represent information flows concerning aggregate and sector (or idiosyncratic) conditions. The inequality states that the two information flows can not exceed the decision makers information processing capability.  $p_{int}^{*A}$  is that part of an optimal price (a profit maximizing price) of firm  $i$  in sector  $n$  at time  $t$  that is determined by aggregate conditions.  $p_{int}^{*S}$  is that part of an optimal price that is determined by sector conditions. They sum into an optimal price,  $p_{int}^* = p_{int}^{*S} + p_{int}^{*A}$ , defined as the price that maximizes firm's profits.  $s_{in}^t$  is the signal on the conditions in the economy.  $H(X|f)$  is called conditional entropy of  $X$  given information set  $f$ . It is a measure of conditional uncertainty in  $X$ . This is a standard measure of uncertainty used in information theory. The difference  $H(p_{int}^{*X}|s_{in}^{t-1}) - H(p_{int}^{*X}|s_{in}^t)$  represents the reduction of uncertainty in  $p_{int}^{*X}$ , due to arrival of the new signal  $s_{in}^t$ . The signal informs the firm on aggregate ( $s_{int}^A$ ) and idiosyncratic conditions ( $s_{int}^S$ ) and is a noisy measure of the profit maximizing price:

$$s_{int}^A = p_{int}^{*A} + \sigma_\epsilon \epsilon_t \quad (2)$$

$$s_{int}^S = p_{int}^{*S} + \sigma_\psi \psi_t \quad (3)$$

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<sup>26</sup>This is due to various reasons. There is too much information available to process them all, the decision makers have limited ability in understanding the information (it is hard to understand the consequences of rise in interest rate even for an economist). It might be too costly in terms of managerial costs to process all of the information, etc...

where  $\epsilon_t$  and  $\psi_t$  are idiosyncratic unit variance Gaussian white noise processes independent of  $u_t$  and  $v_t$ . It is assumed that the profit maximizing price follows a random walk<sup>27</sup>:

$$p_{int}^{*A} = p_{int-1}^{*A} + \sigma_A u_t \quad (4)$$

$$p_{int}^{*S} = p_{int-1}^{*S} + \sigma_S v_t \quad (5)$$

where  $u_t$  and  $v_t$  are unit variance Gaussian white noises. The firm does not observe the profit maximizing prices directly. She observes a noisy signal  $s_{in}^t = [s_{int}^A, s_{int}^S]'$  on the profit maximizing price. After observing this signal she sets the price to equal the expected optimal price:

$$p_{int} = E[p_{int}^* | s_{in}^t] \quad (6)$$

Under simplifying conditions that are not crucial for the model's results<sup>28</sup> the optimal allocation of attention can be represented by the following equation:

$$\frac{2^{\kappa_S} - 2^{-\kappa_S}}{2^{\kappa_A} - 2^{-\kappa_A}} = \frac{\sigma_S}{\sigma_A} \quad (7)$$

The division of attention to aggregate  $\kappa^A$  and sector conditions  $\kappa^S$  is proportional to the variances of aggregate and sector specific shocks. The intuition is simple. Ideally the decision maker would set the profit maximizing price, and incur no losses due to suboptimal price. In reality he is limited in his information processing capability and is not able to process all information. He must decide on how much attention to devote to aggregate and sector conditions. If the variance of sector shocks is high compared to the variance of aggregate shocks, then not paying attention to sector conditions would result in large deviations from the profit maximizing price and in high losses. Therefore the decision maker will devote more attention to sector conditions. The converse holds for the aggregate conditions. One can also show that given the distribution of the attention the following price setting behavior holds:

$$\begin{aligned} p_{int}^* - p_{int} &= \sum_{l=0}^{\infty} [(2^{-2\kappa^A})^{l+1} \sigma_A u_{t-1} - (2^{-2\kappa^A})^l (2^{-\kappa^A}) \sigma_A \epsilon_{int-l}] \\ &+ \sum_{l=0}^{\infty} [(2^{-2\kappa^S})^{l+1} \sigma_S v_{t-1} - (2^{-2\kappa^S})^l (2^{-\kappa^S}) \sigma_S \psi_{int-l}] \end{aligned} \quad (8)$$

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<sup>27</sup>This simplifying assumption is relaxed in Maćkowiak and Wiederholt [2007]

<sup>28</sup>Authors assume a normal linear profit function, Cob-Douglass production function with only labor input and that the firms are required to satisfy demand (Maćkowiak et al. [2009], p. 588).

LHS is the gap between the optimal price and an actual price. How fast the gap closes depends on how the firm distributed her attention to aggregate and sector conditions,  $\kappa^A$  and  $\kappa^S$ . If sector shock variance is substantially higher than the aggregate shock variance, then the firm will focus her attention to sector conditions, and  $\kappa_S$  will be large compared to  $\kappa_A$ . Prices will respond fast to a sector shock and the majority of the long run response will occur on impact (second part of the RHS of eq.(8)). On the other hand, price response will be dampened after an aggregate shock (first part of the RHS of eq. (8)). Next we present testable implications of the RIA model.

### 2.3 Responses to shocks under RIA

RIA model offers a set of predictions that we match to actual sector price behavior. The reader is reminded that we will decompose the variability of prices to aggregate and sector specific components and inspect how they affect the size and speed of the sector price impulse responses. The RIA model implies the following statements:

1. When the sector component of the sector price index is more volatile than the aggregate component (this happens when  $\sigma_S > \sigma_A$ ) the decision makers devote more attention to sector conditions (7) and the sector prices respond faster to a sector shock than to an aggregate shock (8).
2. The speed of response of sector price index to an aggregate shock depends positively on the aggregate component variance of the price index ( $\sigma_A$ ) and negatively on the sector component variance of the price index ( $\sigma_S$ ).
3. If the sector component of the profit maximizing price of sector price index is more volatile than the aggregate, then the cross sectional variation in the speed of response to sector shocks is smaller than cross sectional variation to aggregate shocks. Intuitively what this says is that when firms pay a lot of attention to sector shocks their responses are similar. This is because the speed of response of prices to a given shock is a concave function of the standard deviation of the shock.
4. If on average the sector component of the profit maximizing price is more volatile than the aggregate component, then the effect of a change in the aggregate component variance on the speed of responses of sector price to an aggregate shock should be higher than the effect of a change in the sector specific variance.
5. If the variance of the aggregate component of the profit maximizing price decreases, while sector component remains constant, then firms allocate more attention to

sector specific shocks and sector price indexes respond slower and less pronounced to an aggregate shock. At the same time the response to sector shocks should at least not become slower.

6. If the variance of the aggregate component decreases, while the variance of the sector component stays constant, then the cross sectional variation in the speed of responses due to aggregate shock should increase.

## 2.4 Empirical model

We estimate a semi-structural empirical model instead of a fully structural model. The reason is the following: the RIA model assumes that the decision makers have all available information at hand. In the theoretical model this reduces to two types of shocks, aggregate and sector specific shocks. The decision-makers in reality are faced with a variety of shocks. By choosing to rather estimate an empirical model we can accommodate for this fact. This also enables us to avoid a bias that could result from using reduced shocks in the analysis. Imagine that there are two aggregate shocks, one that causes a 100% of long run response on impact and one that causes a small and delayed response. By mixing the two shocks, i.e. by estimating only a response to a reduced shock, one could falsely estimate a response that has the same shape as the response predicted by the RIA model.

The choice of specific empirical model was guided by the following criteria: i) the model needs to admit a possibility that the variances of shocks are time varying, because it is a well established fact that variances of aggregate variables have changed over time, ii) the model needs to admit the possibility that other parameters are time varying because the RIA model predicts changes in the shock propagation mechanism, when shock variance changes, iii) a model needs to be able to handle a large set of information because in the RIA model the decision makers have all the available information at hand and we do not know in advance which they choose to disregard. The following two sections explain how our model can address these issues.

### 2.4.1 Necessary characteristics of the model

Because the transmission mechanism and shock variances could have been changing over time, we estimate a time varying parameter factor model. The estimated model is a synthesis of the structural model presented in Forni et al. [2009] and a time varying FAVAR model presented in Korobilis [2009]. Forni et al. [2009] proposed a structural factor model where factors are driven by a few macroeconomic shocks. They show how to estimate

the impulse responses to the structural shocks and derive the appropriate asymptotics. Korobilis [2009] estimates a factor augmented VAR model (hereafter FAVAR) of the type put forward by Bernanke et al. [2005]. He extends the model to allow for time varying coefficients and stochastic volatilities in the state and in the observation equation. We build on his time varying framework<sup>29</sup>, but add to this model a time varying autoregressive idiosyncratic shocks.

Current approaches to time variability in the structural factor models are mostly done using a FAVAR approach. FAVAR approach was used by Bianchi et al. [2009], Baumeister et al. [2010], Korobilis [2009], Liu et al. [2011], Eickmeier et al. [2011b], Eickmeier et al. [2011a] and Mumtaz et al. [2011]. This approach was (predominantly) used to identify a monetary policy shock by adding an interest rate as an observable factor. FAVAR approach has two potential shortcomings. First, it assumes that the number of structural shocks is the same as the number of static factors. Tests have rejected this in our application. Second, the restrictions are applied directly on the static factors, which is not the same as applying them on the structural shocks when the number of static factors and macroeconomic shocks differ. Using Forni et al. [2009] approach we identify the structural shocks by applying identifying restrictions directly on dynamic factors.

We extend the Korobilis [2009] model. In this paper we exploit chaining volatilities of macroeconomic and idiosyncratic shocks. If we estimated the model disregarding the autocorrelation in the idiosyncratic shocks we would have biased the estimates of the sector shock variances. Therefore we explicitly model the autocorrelation in the residuals. This was previously done by Del Negro and Otrok [2008], Liu et al. [2011] and Eickmeier et al. [2011b]. We also allow for time variability in the autocorrelations. Equation (15) shows that the shape of a price response to a sector shock depends on the sector and aggregate component variance. Since sector and aggregate variance can vary over time we need to allow for the possibility that sector prices responses to sector shocks also change over time. The resulting empirical model is thus fully capable of accommodating all the implications of the theoretical model. The next section presents the model.

## 2.5 The model

Let  $x_n^T = \{x_{it}\}_{i=1\dots n; t=1\dots T}$  represent a panel of observations, where  $n$  stands for the number of dependent variables and  $T$  for the time dimension of the panel. We assume that each dependent variable can be decomposed into two parts, a common component

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<sup>29</sup>In a sense that we allow for time variation in the parameters. We extend the model for serial correlation in the idiosyncratic component. This however is not new.

$\chi_{it}$  and an idiosyncratic component  $u_{it}$ :

$$x_{it} = \chi_{it} + u_{it} \quad (9)$$

This equation is referred to as an observation equation. Idiosyncratic component captures the effects of microeconomic shocks<sup>30</sup> and measurement errors. It is assumed to be uncorrelated with the factors (presented in the next equation) and mutually uncorrelated for all leads and lags<sup>31,32</sup>. The common component is a linear combination of  $r$  unobserved static factors:

$$\chi_{it} = a_{1it}f_{1t} + \dots + a_{rit}f_{rt} = a_{it}F_t \quad (10)$$

Static factors  $F_t = (f_{1t}, \dots, f_{rt})$  are common for all the dependent variables. They represent the aggregate state of the economy. They are modeled in a VAR with  $p$  lags:

$$F_t = d_{1t}F_{t-1} + \dots + d_{pt}F_{t-p} + e_t \quad (11)$$

$$e_t = R_tv_t \quad (12)$$

where  $e_t$  is an  $r \times 1$  vector of reduced shocks and  $v_t$  a  $q \times 1$  vector of orthogonal white noise structural shocks. The structural (macroeconomic) shocks are called dynamic factors. The reduced shocks  $e_t$  are a linear combination of unit variance dynamic factors  $v_t$ .  $r \times q$  matrix  $R_t$  defines the linear combination that translates the structural shocks into the dynamic factors (where  $VAR(e_t) = R_t * R_t' = \Xi_t$ ). The number of structural shocks is lower or equal to the number of the reduced shocks ( $q \leq r$ ). Matrix  $R_t$  is unknown. Forni et al. [2009] show that it can be estimated as the first  $q$  principal components of the covariance matrix of the reduced shocks  $\Xi_t$ . Stock and Watson [2005] show that if the true number of structural shocks is  $q$ , then the residual shocks are (asymptotically) a linear combination of the  $q$  structural shocks. Therefore we estimate  $R_t$  as:

$$R_t = K_t M_t \quad (13)$$

where  $M_t$  is a diagonal matrix with the square roots of the first  $q$  largest eigenvalues of  $\Xi_t$  on the diagonal and  $K_t$  is an  $r \times q$  matrix whose columns are the eigenvectors of  $\Xi_t$ , corresponding to the  $q$  largest eigenvalues.

We model the time varying coefficients in the VAR for the factors as in Korobilis [2014]. Korobilis [2014] proposes a Minnesota type of prior that is updated from the data.

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<sup>30</sup>Or sector specific shocks should the LHS variable be a sector price.

<sup>31</sup>That is,  $E(u_{it}F_t) = 0$  and  $E(u_{it}u_{js}) = 0$ , for all  $i, j = 1 \dots n$ ,  $j \neq i$  and  $s, t = 1 \dots T$

<sup>32</sup>This is a potential caveat in our application because we rule out sector spill over effects.



He shows that such prior is quite robust to the selection of the shrinkage parameters that affect the amount of time variation in the parameters. To facilitate estimation one has to modify the exposition of the VAR parameters in the following way:

$$d_{it} = \bar{d}_i + \tilde{d}_{it} \quad \text{for } i = 1, \dots, p \quad (14)$$

where the time varying VAR parameters  $d_{it}$  is now a sum of a constant VAR parameter  $\bar{d}_i$  and a deviation from the constant VAR parameter  $\tilde{d}_{it}$ . This does not affect the model in eq.(11). The two expositions are observationally equivalent. It does however simply the introduction of the hyper priors (further details can be found in the Appendix).

Idiosyncratic shocks in (9) can be serially correlated. If we ignore the autocorrelation, then the estimates of the idiosyncratic variances could be biased. For this reason we choose to take serial correlation explicitly into account and model it as an autoregressive process:

$$u_{it} = \theta_{1it}u_{it-1} + \dots + \theta_{qit}u_{it-q} + n_{it} \quad (15)$$

Note that we allow for time variation in the autoregressive parameters  $\theta_{jit}$  ( $j = 1, \dots, q$ ). This is necessary because the RIA model implies that a change in the idiosyncratic variance could affect how prices respond to idiosyncratic (sector specific) shocks. We restrict the AR model of the idiosyncratic shocks to be stationary.

The residuals in the observation equation  $n_{it}$  and the state equation  $e_t$  are zero mean and have time varying covariances:

$$n_{it} \sim N(0, \omega_{it}) \quad (16)$$

$$e_t \sim N(0, \Xi_t) \quad (17)$$

where  $\omega_{it}$  is a scalar and  $\Xi_t$  an  $r \times r$  matrix.  $n_{it}$  is assumed to be independent from  $e_t$  for all  $i$  and  $t$ . We parameterize  $\Xi_t$  in a standard way (as in Cogley and Sargent [2005] or Primiceri [2005]):

$$\Xi_t = B_t^{-1} \Sigma_t B_t^{-1} \quad (18)$$

$$B_t = \begin{pmatrix} 1 & 0 & \dots & 0 \\ \beta_{21t} & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{r1t} & \beta_{r2t} & \dots & 1 \end{pmatrix}$$

$$\Sigma_t = \begin{pmatrix} \sigma_{1t}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2t}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{rt}^2 \end{pmatrix}$$

Let  $A_t$  represent all the factor loading vectors stacked one on top of another:  $A_t = (a'_{1t}, \dots, a'_{nt})'$ , where  $a_{it} = (a_{1it}, \dots, a_{rit})$  for  $i = 1, \dots, n$ . Let  $\Omega_t = (\omega_{1t}, \dots, \omega_{nt})'$  be a vector of stacked idiosyncratic variances.

Let  $\tilde{D}_t = (vec(\tilde{d}_{1t})', \dots, vec(\tilde{d}_{pt})')'$  represent a stack of the time varying part of the factor VAR parameters. Let  $\Theta_t$  represent a stack of all  $\theta_{it}$ 's,  $\Theta_t = (\theta_{1t}, \dots, \theta_{nt})$ , where  $\theta_{jt} = (\theta_{1jt}, \dots, \theta_{qjt})$  for  $j = 1, \dots, n$ .  $H_t$  represents a stack of all the diagonal elements of  $\Sigma_t$ ,  $H_t = (\sigma_{1t}^2, \dots, \sigma_{rt}^2)'$  and finally let  $T_t$  represent a vector of all the entries below the diagonal of the lower triangular matrix  $B_t$ ,  $T_t = (\beta_{21t}, \dots, \beta_{r1t}, \dots, \beta_{rr-1t})'$ . We assume that the time varying parameters evolve as random walks<sup>33</sup>:

$$A_t = A_{t-1} + \eta_t^A \quad (19)$$

$$\Theta_t = \Theta_{t-1} + \eta_t^\Theta \quad (20)$$

$$\log \Omega_{it} = \log \Omega_{it-1} + \eta_{it}^\Omega \quad (21)$$

$$\tilde{D}_t = \tilde{D}_{t-1} + \eta_t^{\tilde{D}} \quad (22)$$

$$T_t = T_{t-1} + \eta_t^T \quad (23)$$

$$\log H_t = \log H_{t-1} + \eta_t^H \quad (24)$$

$$(25)$$

The random walk innovation vectors  $\eta$ 's are assumed to be independent of each other:

$$Var \begin{pmatrix} \eta_t^A \\ \eta_t^\Theta \\ \eta_t^\Omega \\ \eta_t^{\tilde{D}} \\ \eta_t^T \\ \eta_t^H \end{pmatrix} = \begin{bmatrix} Q^A & 0 & 0 & 0 & 0 & 0 \\ 0 & Q^\Theta & 0 & 0 & 0 & 0 \\ 0 & 0 & Q^\Omega & 0 & 0 & 0 \\ 0 & 0 & 0 & Q^{\tilde{D}} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q^T & 0 \\ 0 & 0 & 0 & 0 & 0 & Q^H \end{bmatrix}$$

Block diagonal structure of the covariance matrix of the  $\eta$ 's is standard. With so many parameters one needs to make simplifying assumptions. In addition we assume that the matrices  $Q^A$  and  $Q^\Theta$  are also block diagonal. This implies that the factor

<sup>33</sup>This assumption is standard in this type of models because it eases the computational cost. Cogley and Sargent [2005] and Primiceri [2005] show that it does not affect the results.

loadings in equation for  $x_{it}$  are not correlated with factor loadings in  $x_{jt}$ , where  $j \neq i$ . The same holds for  $\theta$ 's. They are only allowed to be correlated inside the  $i$ th equation. The covariance matrices of log volatilities are also assumed to be diagonal ( $Q^H$  and  $Q^\Omega$ ). In the VAR model the  $T_t$  accounts for the cross-correlation between factors. Presented model is flexible enough to capture all the potential changes in the aggregate and sector price dynamics over time.

## 2.6 Estimation

We estimate the static factors<sup>34</sup> by extracting first few principal components from the dataset. Let  $\lambda$  represent the static factor loadings. We identify the factors by assuming that  $\lambda'\lambda = I_r$ , where  $I_r$  is an identity matrix of dimension equal to the number of static factors. This approach is standard in the literature. It was proposed by Stock and Watson [2002b]. Stock and Watson [2002b] show that the principal components (hereafter PCA) consistently estimate the space spanned by the factors when  $n$  is large and the number of principal components is at least as large as the number of true factors. Moreover, Banerjee et al. [2008] and Stock and Watson [2009] show that the factors are estimated consistently even if there is some time variation in the factor loadings.

Besides possible time variation in the factor loadings, our model also admits stochastic volatility in the factor variances. It is not obvious that PCA is a consistent estimator of the factor space when time varying volatility is present. We found no reference to this issue in the literature, therefore we conducted a small Monte Carlo study. We simulated datasets of different cross section and time lengths using an approximate factor model with stochastic volatility in the residuals. Stochastic volatility was modeled as a random walk model for the variances of the residuals (as in our empirical model). Note that this is an extreme assumption because variances that evolve as random walks are explosive. Explosive behavior of variances is prevented in the empirical model because the variances are drawn conditional on the other parameters of the model, whereas in the simulation we impose explosive behavior of variances. Despite explosiveness, the PCA estimates turned out to be a consistent estimator of the factor space (details can be found in Appendix).

An alternative approach to PCA is to estimate the factors as latent variables by either using maximum likelihood methods<sup>35</sup> or by simulating them<sup>36</sup>. We chose the PCA

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<sup>34</sup>This is a slight abuse of the term. We estimate the space spanned by the static factors.

<sup>35</sup>As a robustness check we also estimated the factors using a quasi maximum likelihood approach put forward by Doz et al. [2011]. This was done on a subsample, excluding the financial crisis. This did not change our results qualitatively, which comes as no surprise, since the correlation coefficient between PCE and QML estimates of the factors was above 0.98 for the first two factors and 0.9 for the third factor.

<sup>36</sup>This approach was used by Baumeister et al. [2010], Del Negro and Otrok [2008], Liu et al. [2011],

approach for two reasons. First, it reduces the computational burden and second, it enables us to avoid some identification issues inherent in the factor models with time varying parameters. This approach was also used by Korobilis [2009] and Eickmeier et al. [2011b]. Observation equations were estimated equation by equation as in Korobilis [2009].

We used several criteria to determine the number of static and dynamic factors in the dataset. We use Deviance Information Criteria (hereafter DIC) as our main model selection criteria.

DIC is a Bayesian model selection criteria that is especially suitable when one estimates a model with time varying parameters. Conventional selection criteria (i.e. Akaike, Hannan-Quinn or Schwartz criteria) penalize the model fit with the number of parameters that are being estimated. It works well in a static model. In a time varying parameter model the number of parameters that are being estimate can dominate such criteria. On the other hand, the penalty function in DIC uses the number of *effective parameters*:

$$DIC = \bar{D} + p_D \quad (26)$$

where  $\bar{D} = E[-2\ln(L(\Delta_i))]$  is the expected likelihood evaluated at the draws of the parameters  $(\Delta_i)$  and  $p_D = E[-2\ln(L(\Delta_i)) - (-2\ln(L(\bar{\Delta})))]$  is the number of *effective parameters*, calculated as the expected difference between the likelihood evaluated at the draws and the likelihood evaluated at the expected parameter values  $(\bar{\Delta})$ . The likelihood was evaluated using a particle filter as in Ellis et al. [2014]. We use particle filter because stochastic volatility errors do not distribute in a normal distribution and therefore the model's likelihood function is not normal. With particle filter one draws  $n$  states for each draw of the parameters, calculates  $n$  likelihoods and averages over them to approximate the true likelihood (details can be found in Ellis et al. [2014]).

To reduce the computational burden we first calculate the DIC for the number of factors and then, conditioning on the number of factors, select the lag length of the factor VAR and the number of dynamic factors. For the US data Bernanke et al. [2005] estimates a model with 4 factors (with the 5-th factor being the interest rate), Korobilis [2009] estimate a model with 5 factors (with the 5-th factor being the interest rate) and Stock and Watson [2005] estimate the number of static factors to be 7. The DIC criteria for up to 7 static factors is displayed in the first column of Table 1 in Appendix. Based on the DIC a model with 5 factors is selected. Further we calculated DIC for the number of lags in the factor VAR, for up to 5 lags. Based on the second column of Table 1

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Mumtaz et al. [2011] and Bianchi et al. [2009]

we should select a VAR with only 1 lag. Because the VAR with only 1 lag produced unconventional impulse responses we decided to estimate a VAR with 2 lags, the second best DIC.

We found no reference in the literature if DIC is applicable to selection of the number of dynamic factors, therefore we used conventional tests. Both Bai and Ng [2007] test statistics favored 3 or 4 dynamic factors and Stock and Watson [2005] test favored 5 or 3 dynamic factors. Tests on the two subsamples tests strongly favored 4 or less dynamic factors. We decided to estimate 4 dynamic factors because it represents the mode of the number of factors proposed by the tests. It also enables us to identify three preferred dynamic factors and leave one dynamic factor unidentified.

Bayesian techniques were used to estimate the posterior distributions of the parameters of interest. We chose Bayesian approach because it easily handles high dimensional non-linear models by using simulation methods, rather than using maximization methods, although the classical approach was successfully applied by Eickmeier et al. [2011b]. We imposed stationarity restrictions in the VAR model and invertibility restrictions on the autocorrelated idiosyncratic shocks by rejecting the draws that implied explosive behavior.

Priors for the initial states of the coefficients, stochastic volatilities, covariances and hyper parameters in both the measurement and state equations are assumed to be independent of each other. The priors are standard, either normal-gamma or normal-Wishart. We used priors as uninformative as possible while still assuring stability of the model, though some priors are weakly informative. Cross check with the results obtained by using a training sample priors revealed a small effect on the estimated IRs and regressions, they do not affect the main conclusions of the paper, though some are changed. Each time varying parameter was sampled with the Gibbs sampler. Conditional on the static factors and the rest of the parameters each time varying parameter was casted into a standard linear state space model, to which we applied the Carter-Kohn algorithm (Carter and Kohn [1994]). We implemented a data-based Minnesota type of prior for the TVP-VAR as described Korobilis [2014]. As shown by the authors an advantage of using a data-based prior for the TVP-VAR is that it is less sensitive to the specification of the degree of time variation in the model, because the prior for the time variation is updated from the data. The convergence was verified using visual inspection and mean tests for the draws. The main algorithm is presented in the Appendix.

## 2.7 Identification of structural shocks

The method that we use to identify the structural shocks was put forward, in a time invariant framework, by Forni et al. [2009]. Examples of the method used in a non-time varying setting using similar restrictions as in this paper are provided by Pellenyi [2012], Forni et al. [2009] and Forni et al. [2010]. We found no application of the method used in a time varying setting. Using equations (9)-(12) we rewrite the factor model in its dynamic form:

$$x_{it} = b_{it}(L)v_t + u_{it} \quad (27)$$

$$b_{it}(L) = a_{it}(1 - d_{1t}L - \dots - d_{pt}L^p)R_t \quad (28)$$

where  $u_{it}$  are the idiosyncratic shocks and  $v_t$  unit variance dynamic factors. This representation is unique only up to an orthogonal transformation. Let  $H$  represent a  $q \times q$  orthonormal matrix<sup>37</sup>. We can then multiply the dynamic factors  $v_t$  with matrix  $H$  to obtain a new rotation of dynamic factors  $\epsilon_t$ :  $R_tv_t = G_t\epsilon_t$  (where  $G_t = R_tH'$  and  $\epsilon_t = Hv_t$ ); while the shock variance remains unchanged ( $Var(R_tv_t) = Var(G_t\epsilon_t) = \Xi_t$ ). This property is used to identify economically meaningful structural shocks  $\epsilon_t$ .

Economic theory implies restrictions on a set of variables in the first few periods after the shock. To get a set of admissible rotation matrices  $H$  we post-multiply the non-structural impulse responses  $b_{it}(L)$  with a candidate  $H$  matrix and then verify if the rotated responses  $b_{it}^h(L)$  satisfy theory based restrictions. If the restrictions are satisfied we retain the drawn rotation matrix  $H$ .

We chose this approach because it places the restrictions directly on the dynamic factors ( $v_t$ ) and does not require the assumption that the number of static factors is the same as the number of dynamic factors ( $r = q$ ). This was rejected in our dataset. Furthermore, cleaning the factors of the influence of the fast moving variables, as in Bernanke et al. [2005], could lead to loss of information and the underlying assumption that monetary shocks have no contemporaneous influence on the real factors could be too restrictive. We also wish to identify other shocks, besides the monetary policy shock, to verify if the predictions of the RIA model also hold for other macroeconomic shocks.

We estimate three most commonly identified shocks in the literature. The estimated shocks are: monetary policy, demand and supply shock. Sign restrictions were imposed on impact. The restrictions that we employ are compatible with a wide range of theoretical models. They can be found in the majority of DSGE models, in a New Keynesian

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<sup>37</sup>Orthonormal matrix is a matrix which when multiplied with own transpose forms an identity matrix:  $H'H = I$ .

Table 2: Table of imposed sign restrictions

	<i>MON</i>	<i>DEM</i>	<i>SUP</i>
<i>RGDP</i>	$\leq 0$	$\geq 0$	$\leq 0$
<i>PCE Q</i>	$\leq 0$	$\geq 0$	$\leq 0$
<i>CPI</i>	$\leq 0$	$\geq 0$	$\geq 0$
<i>PCE P</i>	$\leq 0$	$\geq 0$	$\geq 0$
<i>M3TB</i>	$\geq 0$	$\geq 0$	*

models and in a Real Business Cycle models. Same or similar restrictions were used in other applied work<sup>38</sup>. Table 2 summarizes the restrictions. We applied them on the following six variables<sup>39</sup>: real gross domestic product (RGDP), personal consumption expenditure quantity index (PCE Q), consumer price indices (CPI), personal consumption expenditure price index (PCE P) and the three month treasury bill rate (M3TB).

We assume that the monetary policy shock increases the 3 month treasury bill rate and decreases price indices (CPI, PCE P) and real output measures (RGDP and PCE Q), on impact and in the first two periods<sup>40</sup>.

A positive demand shocks increases both real output and prices. We assume that a central bank reacts to an increase in prices by raising the interest rate. A negative supply shock is followed by an increase in prices and a decrease in real output. Therefore the supply shock can proxy for a sudden increase in input prices<sup>41</sup>, or for a negative technology shock. The influence of a supply shock on the treasury bill rate is left unrestricted. The reason is that monetary policy may react differently to different types of supply shocks.

We estimate a time varying model therefore the impulse responses need to take into account that time varying parameters can drift over time. To deal we this issue we estimated generalized impulse responses as put forward in Koop et al. [1996]:

$$E(x_{t+j}|\Xi_i, \mu) - E(x_{t+j}|\Xi_i) \quad (29)$$

<sup>38</sup>I.e. Gambetti et al. [2008], Canova and Nicoló [2002], Forni et al. [2010],...

<sup>39</sup>RGDP, CPI and M3TB were chosen because they are one of the most commonly restricted variables. PCE P and PCE Q were chosen because they are the focus of subsequent analysis. We also experimented by replacing the M3TB with the federal funds rate (FFR), PCE Q with industrial production index (IPI) and PCE P with producer price index (PPI). The estimated impulse responses were similar.

<sup>40</sup>Since the price puzzle is sometimes present in the estimates of the impact of a monetary shock on prices, we also experimented by leaving the effect of monetary policy shock on impact and in the first quarter unrestricted. The results were qualitatively similar, but the responses were slightly more pronounced.

<sup>41</sup>In example for an oil shock.

where  $\Xi_i$  denotes the parameter of draw  $i$  and  $j$  the impulse horizon.

In addition, for each draw and time point in time we draw 100 rotation matrices. Common practice is then to calculate a mean impulse response, averaged over 100 rotation matrices, and use it as a representative impulse response. Fry and Pagan [2007] have shown that this can bias the results. Therefore we follow their recommendation and only retain the most representative rotation matrix. The most representative rotation matrix is a rotation matrix that generates the impulse responses that are closest to the mean of standardized impulse responses. The details on this procedure can be found in Fry and Pagan [2007].

## 2.8 Data

We use data on 144 U.S. macroeconomic time series that range from 1959Q1 to 2014Q2. We used similar dataset as in Korobilis [2009]. We chose it because it has been found to contain sufficient information about the state of the economy. Complete description of the series is given in the Appendix. Dataset contains time series on real variables (i.e. real GDP, industrial production index, real final sales of domestic product...), nominal variables (prices, wages, oil prices...) and financial variables (interest rates, yields, exchanges rates...). It also includes forward looking variables like commodity prices and inventories. All the data were downloaded from FRED<sup>42</sup> on-line database<sup>43</sup>. Quarterly dataset was used for practical reasons. First, if we tried to get monthly data for the selected time span, a lot of series were missing and second, the number of parameters in a time varying model grows fast with the sampling frequency of the data. We do not find this to be a shortcoming. Boivin et al. [2009] estimate the FAVAR model for sector prices using monthly and quarterly data and find minor differences. Baumeister et al. [2010] also estimate a factor model of sector prices using quarterly data. On average the 5 factors explain 58% of the variance of the 144 macroeconomic data series.

We augmented the dataset with disaggregated sector data on real personal consumption expenditure indexes (hereafter PCE) and PCE price indexes. We collected the data at the most disaggregated level and moved up to a higher aggregation level if at least one of the series in the lower category had missing observations. We ended up with 203 categories of sector prices<sup>44</sup>. We excluded some sector series from the dataset, because they

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<sup>42</sup>Federal Reserve Economic Data.

<sup>43</sup>Korobilis [2009] uses 157 variables. Six of those are not publicly available and 4 had missing values. We also excluded some variables that could not be made stationary, since they exhibit extreme behavior after the 2007 crisis.

<sup>44</sup>The difference in the number of categories in our dataset and Boivin et al. [2009] comes from the difference in the frequency of the data. More categories are available with quarterly data. Also, some



Table 3: Descriptive statistics for sector indices

	PCE P	PCE Q
Mean	0.83%	0.88%
Median	0.92%	0.90%
Std.	1.52%	3.29%
Skewness	0.75	0.12
Kurtosis	9.22	7.27
<i>Statistics apply to growth rates.</i>		

either exhibit extremely unstable behavior<sup>45</sup> or extremely implausible behavior. We excluded 7 sector price series and 4 quantity series. Sector quantity and price indexes were transformed into quarter on quarter growth rates. Table 3 contains descriptive statistics for the sector indices. On average the sector prices rose by 0.83% each quarter. Standard deviation of sector prices growth rate is a 1.52%. Quantities rose on average each quarter more than prices, by 0.88%. The standard deviation of sector quantities is almost twice the standard deviation of the prices, 3.29%. This is consistent with price stickiness of sector prices. Both distributions, of sector prices and quantities, are leptokurtic and skewed. Sector data were downloaded from BEA<sup>46</sup> on-line database.

The total dataset now includes 504 aggregate and disaggregated time series. Factors explain 43% of variation in sector prices. We find this to be quite high<sup>47</sup>. This share falls to 38%, if we estimate the factors only on the core macroeconomic variables. Since our primary focus is on sector prices we use the enlarged dataset<sup>48</sup>. Extracted factor estimates are plotted in Figure 1 in the Appendix. All variables were seasonally adjusted<sup>49</sup> and standardized before the estimation.

## 2.9 Results

To assess the plausibility of the identified shocks we first present the results for macroeconomic variables. We proceed with the analysis on the sector price dynamics and link them to the RIA model.

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categories do not match perfectly.

<sup>45</sup>Extremely unstable series is defined as series that has more than 5 outliers, where outlier is defined as an observation 5 standard deviations greater than the mean of the series (in absolute terms).

<sup>46</sup>Bureau of Economic Analysis.

<sup>47</sup>In Boivin et al. [2009] their factor model explains around 17% of the variance. The difference is due to the frequency of the data.

<sup>48</sup>The first three factors extracted from core and enlarged dataset have correlation coefficients over 0.9.

<sup>49</sup>Using X-12-ARIMA procedure for quarterly data.

### 2.9.1 Aggregate variables

Table 2 in the Appendix contains average shares of explained variances for macroeconomic variables: RGDP (real GDP), IPI (industrial production index), CPI (consumer price index), PPI (producer price index), M3TB (3 month treasury rate), UR (unemployment rate), PCE Q (real personal consumption expenditure), PCE P (personal consumption expenditure price index) and FFR (federal funds rate). On average the common component variances account for 63% of the total variability of these dependent variables. Share of explained variance averaged over the three sub-periods<sup>50</sup> does not show substantial variability. But upon closer look, presented in Figures 2-5 in the Appendix, one can observe that the shares of explained variances have varied greatly. We observe that the total variances, common component variances and idiosyncratic variances have varied for all the aggregate variables. This is especially true for PCE price index. We observe that the total variance of the PCE price index has been steadily increasing from 60' to middle of 70's, declined with the Volcker disinflation period at the beginning of the 80', started to increase again before the 2007 crisis and then decreased. Figure 5 plots the shares of explained variances over time. We observe that the share of explained variance for PCE price index increases during turbulent periods and decreases during calm periods. It increases in volatile times to 97% such as during the 1975 oil crisis, Volcker's money stock targeting and the recent financial crisis. It decreases to 91% during calm periods like after the Volcker's disinflation and in the first half of 2000. The share explained variance is especially high for the PCE price index, 95% on average. This comes as no surprise since we have included time series, that comprise this index, in the dataset from which we extracted the factors. This is important, because it assures us that the estimated factors include all aggregate influences that affect sector prices.

Figure 6 plots normalized cumulative impulse responses of selected aggregate variables (RGDP, PCE Q, CPI, PCE P and FFR), for the three shocks. The impulse responses were normalized by dividing each impulse responses with the standard deviation of the shocks. Standard deviations of the shocks were calculated as in Mumtaz et al. [2011]<sup>51</sup>. We plot the impulse responses for 5 periods. Periods were chosen to correspond to tranquil and volatile periods. Impulse responses have similar shapes in all periods, but

<sup>50</sup> Before the great moderation, the great moderation and the period of financial crisis.

<sup>51</sup> We cross-checked the results by normalizing the impulse responses with respect to a specific variable. We normalized the impulse responses of the monetary policy shock with respect to the interest rate, the supply shock, with respect to prices and the demand shock with respect to GDP. The results were similar to the ones presented in this paper for all the shocks except the monetary policy shock, which exhibited implausible behavior. We therefore decided to use the method presented in Mumtaz et al. [2011] to calculate the variances of the shocks.

they are more pronounced in the more volatile periods, such as in 70' (1973Q1) and beginning of the 80' (1981Q1) and after 2007 (2008Q1). This holds especially for price variables. Interesting, for the price variables, impulse responses shocks after 2007 are on average less pronounced. This holds especially for supply and demand shock. The responses are also (on average) smaller and flatter in less volatile times such as in 1965Q1 and 2000Q1 and more pronounced in 1974Q1 and 1981Q1. While this is a standard result for quantities there is disagreement on how the response of prices has changed due to a monetary policy shock. Some authors find no change (i.e. Sims and Zha [2006] and Primiceri [2005]) and some authors find the response to increase over time (Baumeister et al. [2010]). Like Eickmeier et al. [2011b] and Boivin et al. [2009]) we find that the responses have slightly decreased in calm periods. The RIA model predicts this for the prices. It is interesting that it also holds for the quantities.

Monetary policy shock has almost no affect on CPI and PCE P, on impact . The response is slow and builds up gradually. The effect of a monetary policy shock on price indexes diminishes after 20 quarters. Similar result holds for RGDP and PCE Q. Monetary policy shock affects the M3TB rate strongly on impact and dies out quickly. Except for a bit longer response horizons, this results are consistent with other studies (Eickmeier et al. [2011b] also find long responses). Positive demand shock causes real GDP and PCE Q to rise on impact, whereas prices respond little on impact. Note that the quantities slowly return toward its initial level whereas price level remains higher. Monetary authority responds to a demand shock with a rise in the M3TB, in a slightly delayed fashion, which could reflect lack of prompt information or reluctance to counter the shock by monetary authority. Supply shocks affects prices mildly on impact and the effect slowly builds up after the shock. Impact effect on real quantities is almost zero, but builds up gradually. Monetary authority counters the shock by immediately raising the interest rates. Figures 7-12 plot impulse responses to selected shocks for each point in time. It reconfirms our findings that they were more pronounced before 70'-80' and after the 2007 crisis. It is surprising how similar are the impulse responses to a monetary shock using the method of Forni et al. [2009] compared to the ones obtained by using the traditional Cholesky identification.

We believe that in overall our structural factor model captures the macroeconomic dynamics in the economy quite well. This is important if we want to decompose the sector prices into an aggregate and sector specific component.

### 2.9.2 Sector prices - variances

RIA model predicts that the sector price responses depend on the variances of aggregate and idiosyncratic components of prices. Therefore we first decompose the variances of sector prices on aggregate and sector components. Previous section showed that the variances of aggregate variables varying over time. Therefore also the aggregate shock variances have varying. If the sector component variances remained (approx.) stable this would imply that the ratio of common component variance to idiosyncratic variance has changed, which we can exploit in testing the RIA model.

Decomposition of sector price variances by explained and unexplained variance is presented in Table 3 in the Appendix. We note that the share of aggregate component variance in the median sector has declined in the great moderation period and rose again in the 2007 crisis. It constituted for 51% in the 70's and the beginning of 80' (47% in the full period before 1983Q4), declined to 44% in the great moderation period and increased to 56% in the 2007 crisis. Figure 13 plots the common component variances and idiosyncratic component variances for sector prices. We notice several things. First, variances of the sector prices due to aggregate shocks show similar dynamics over time, in all sectors. Second, variances of common components declined in the middle of the 80' and increased with the 2007 crisis. Inserted dashed black line presents the estimated variance of the median sector. We can see that the aggregate component variance of the median sector achieved its first peak in 1980Q4. At that point it achieved a value of 0.88, which is over 10 times the minimum median common component in 1963Q4 (0.07). It achieved it's second peak just around the beginning of the Volcker disinflation policy, after which the common component variance decreased and remained low till 2007 crisis. On the other hand sector component variances of the inflation do not show a strong pattern. Median variance of the sector component resembles a flat line implying that sector specific components have remained stable over time. The drop in sector inflation variances after the 70' is due to a drop in the variances of common shocks hitting the economy whereas sector specific variances remained stable. Figure 14 plots the total variances of sector prices and the shares of common components over time. Common component share of median sector was at its highest in 1980Q4, when it achieved 75%. It was also high in 2009Q1 (74%), presumably due to the financial crisis. It was at its lowest in 1965Q4 and 1997Q4, when it constituted less than 40% of the variance of the median sector.

### 2.9.3 Sector prices - responses to aggregate shocks

RIA predicts that sector price response to an aggregate shock depends on the sector price aggregate component variance and the idiosyncratic component variance. Aggregate shock variances have declined after the 70' and increased again with the 2007 crisis. Therefore we expect that sector prices respond more strongly to shocks before the 70', less after the 70', and more in the 2007 the crisis.

Figure 15 is composed of 9 plots. 5-th, 50-th and 95-th quantiles are sorted over columns and rows represent a response to a monetary, demand and supply shock. We begin with the description of the effect of demand and supply shock on sector prices, as presented in the second and third row. Both shocks do not affect prices much on impact (in all periods) but then slowly propagate through the economy and prices start to respond. The effect of the shocks is quite persistent. Note that the price responses are especially high around volatile years such as 1974Q1 and 1981Q1. This is consistent with the RIA model. The response in tranquiler periods is roughly half the response in the volatile periods. First row shows the median sector price response to a monetary policy shock. As before they affects prices little on impact, but the effect builds over time. The effect on impact is less than 5%, gradually builds up and stabilizes after 40 quarters. Figure 16 plots the responses of all sectors for selected periods. We note two results. First, for all the shocks and periods there are a few sectors in which responses to a shock have the opposite sign as the median responses. I.e. in some sectors prices respond positively to a monetary policy shock. This might be due to different channels of monetary policy acting in different sectors. Similar reasoning holds for other shocks. Second, we note that the dispersion of responses declines in the tranquil periods and increases in the volatile periods. This is seen as the plots get more narrow after the 80' and widens again after 2007. We calculated average standard deviations of cross sectional dispersion of sector price IRs to shocks, by periods. For every shock and for every impulse horizon the average dispersion of IRs fell after the great moderation and increased again in the 2007 crisis period. In the great moderation period (1984Q1-2007Q1) the dispersion of responses to monetary shocks fell by 25% for the impulse response in the eight period after the shock, relative to turbulent period (1970Q1-1983Q3), by 34% for a demand shock and by 35% for the supply shock. With the 2007 crisis the dispersion for a response to a monetary shock increased by 30%, by 52% for a demand shock and by 32% for the supply shocks, relative to the great moderation period. This is consistent with results in Boivin et al. [2009] and in Baumeister et al. [2010], but it is not consistent with the RIA model. RIA predicts that the magnitude of the impulse responses should decrease with a fall in the variance of the aggregate shocks, but that cross sectional variation in

the responses to aggregate shocks should increase. This is because when firms in the RIA model pay less attention to aggregate conditions their responses should be more dispersed.

#### 2.9.4 Sector prices - responses to sector shocks

We now analyze how sector prices responded to sector shocks over time. The RIA model predicts that if the idiosyncratic component of the sector prices became more important over time, as it seems to be the case, then the sector prices should respond faster (or at least not slower) and with a higher magnitude to a sector shock. Figure 17 plots cumulative impulse responses of sector prices to sector shocks for all sectors and figure 18 impulse responses for the median sector for all periods. It takes only a few quarters for the 95% of the long run impact to realize in the median sector, after a sector shock has hit the prices. Boivin et al. [2009] and Maćkowiak et al. [2009] report similar result. Note that there is very little variation in sector price responses to sector shocks over time. We also examined cross-sectional variability of prices responses to sector shocks, over time. The differences were for all practical reasons negligible (a few percent) and therefore we do not report them.

#### 2.9.5 Regressions

In this section we describe the regressions that we use to test the predictions of the RIA model. We use similar regressions as in Maćkowiak et al. [2009] and Boivin et al. [2009]. What is different is that we use a panel data-set with three identified shocks. This enables us to compare the results over time, space and shocks. Rich time variation in the variances of reduced and structural shocks enables us to draw additional conclusion that we could not if we ignored the time dimension. In addition, the identification of specific shocks assures that the effects of shocks are not mixing, which could cause false results.

Following Boivin et al. [2009] we regressed IRs on the variances by components (monetary policy, demand, supply, one non-specified component and the sector component) and following Maćkowiak et al. [2009] we regressed the speed of impulse responses on variances by components. Speed of impulse responses are defined as a ratio of average absolute response to a shock in the first two years divided by the average absolute response in the 4th-8th year:

$$\Lambda_{ti}^{Aj} = \frac{\frac{1}{8}\sum_{Q=1}^8 |\beta_{tij}|}{\frac{1}{8}\sum_{Q=24}^{32} |\beta_{tij}|} \quad (30)$$

Periods were chosen to strike a balance between short term responses and the long term response to the shocks<sup>52</sup>.  $\beta_{tij}$  presents the size of the impulse response in sector  $n$  to a shock  $j$  at time  $t$ .  $\Lambda_{tn}^{Aj}$  presents the speed of response in sector  $i$  at time  $t$  to a  $j$ -th aggregate shock.

We run several panel regressions<sup>53</sup>. We have a reasons to believe that there are sector specific missing explanatory variables that do not change over time that could bias the results. This would include sector characteristics such as: level of competition in a sector, durability of goods produced, average firm size in a sector,... We estimate fixed effects regression to avoid possible bias due to unobserved variables. Variables used in the regression and accompanying summary statistics is are in the Appendix in Table 4.

## Results

We first regressed the speed of impulse responses on common component variances due to own shocks only (i.e. we regressed the speed of impulse responses to a monetary shock on a common component variance that is attributable to the monetary policy shock). Results are in Table 5. Two regressions were performed for each shock. We regressed the speed of response to a shock on the standard deviation of the sector price that can be attributed to that specific shock (to ease expression we refer to them in the remainder of this document as own standard deviations). In the second regression we regressed the speed of response on own shock and (total) standard deviation of the remaining shocks<sup>54</sup>. The RIA model predicts that the higher is the own standard deviation the faster the price responds to that shock. The effect of the standard deviation of the remaining shocks should be negative. Our panel results do not support this prediction. Regressions on own shocks standard deviations came out significant but the results are sometimes counterintuitive. Increase in the variance due to own shock decreases the speed of response for the supply shock. Note also that the overall coefficient of determination is low for all the regressions. This is not in accordance with the RIA model. We conclude that we do not find clear support that the shocks variances affect the speed of impulse

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<sup>52</sup>Other combinations of periods did not affect the results.

<sup>53</sup>Dependent and explanatory variables are estimated. Therefore we should have used bootstrap estimates. Unfortunately the bootstrap is still running. Previous application with a time series prior included bootstrap estimates. It did not change the main conclusions of the paper, though the confidence intervals on the coefficients were wider.

<sup>54</sup>Defined as a square root of sum of common component variances of all other shocks and the sector specific shock.

responses as predicted by RIA<sup>55</sup>.

Next we regressed the size of impulse responses on the variance attributable to common factors and the variance attributable to sector specific shocks. As a measure of the size of the impulse response we took an average impulse response to a shock in the 3rd year after the impact. This period is chosen because it lies between the short run and long run response. The results, in Table 6, are partly in accordance with the RIA theory of prices. An increase in the standard deviation of the variance, due to an aggregate shock, decreases the monetary impulse response and increases the demand and supply impulse response<sup>56</sup>. This is in accordance with the RIA model. On the other hand, an increase in the standard deviation of the idiosyncratic shock increases the impulse responses of monetary, demand and supply shocks. This is not in accordance with RIA. In the RIA model idiosyncratic shocks compete with the aggregate shocks for the attention of the decision makers and should therefore dampen the impulse responses to aggregate shocks. Though these effects are small and insignificant. We also note that the coefficients of impulse responses to own standard deviations are substantially higher than the coefficients on standard deviations of the idiosyncratic shocks, as implied by the RIA model.

We next regressed the size of the impulse responses on own and other shock variance. The RIA model predicts that own shock standard deviation affects the size of the impulse response to that shock positively, whereas standard deviation of the remaining shocks should dampen them. The results are in complete accordance with the RIA model (Table 7 in the Appendix). Standard deviation of prices due to a monetary policy shock affects the size of the impulse response with a negative sign. This is in accordance with RIA, since we estimated a negative monetary shock. Other shocks standard deviation increases the impulse response to a monetary shock. Own standard deviation for supply and demand shock increase the supply and demand impulse response. When we add other shock standard deviations they dampen the impulse responses of demand and supply shock, although they can be insignificant. This leads us to believe that the decision makers in firms understand different aggregate shocks.

Both of the above regressions provide some insightful results. First they show that idiosyncratic shocks are not very important for how sector prices respond to aggregate shocks. Almost all the effects of the idiosyncratic shocks on the impulse responses (due to aggregate shocks) were insignificant or small. In addition, other shocks variances seem

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<sup>55</sup>The results did not change when we regressed the speed of responses on the standard deviation of own shock variance and other shock's variance.

<sup>56</sup>Monetary shock in our model decreases prices and the demand and supply shocks increase them. Note that in the 3D plots we plotted a negative demand shock. We did this so that the results are better visible.



to matter for impulse responses to aggregate shocks, as predicted by the RIA model. The RIA model as presented in Maćkowiak et al. [2009] only models the price variances by decomposing them to an aggregate and sector specific component. Because we identify structural aggregate shocks we can also test if the identified aggregate shocks compete for the attention of the decision makers. Table 8 presents results where we regress each of the impulse response on own standard deviation and standard deviation of other shocks. As we observe from the table the predictions of the RIA model hold fairly well. The monetary and demand shock do not compete for the attention of the decision makers in the equation for the monetary shock. Other than this result shocks compete for the attention of the decision makers, as predicted by RIA model. Note that Mackowiak and Wiederholt [2011] model rational inattention in a fully micro-founded model where they identify three structural shocks and provide some evidence for the trade-off between shocks. Unfortunately they do not yet present a detailed results on how sector specific prices respond to shocks in their model. It would be interesting to see if the results obtained in their model comply with the results presented here.

To conclude we restate the results. In our preferred specification we found that an increase in own standard deviation increases the impulse response of prices, and standard deviation due to idiosyncratic shocks decreases them, as predicted by the RIA model (though the later effect is small and sometimes insignificant). The reason could be that the decision makers in firms are very good at interpreting sector specific shocks and thus the sector specific shocks do not compete with aggregate shocks (whose effect is difficult to predict even for economists). We also tested if the aggregate shocks compete for the attention of the decision makers and concluded that they do (with one exception, monetary shock does not seem to compete with the demand shock).

### 3 Conclusion

In this paper we implement the FGLS approach into a time varying framework in order to create a panel dataset on price dynamics. We then tested the predictions of the RIA model. Based on our empirical model we confirm some of the predictions of the RIA model and refute some.

We first confirm the main prediction of the RIA model, that has been tested before, but not with a panel data and not by using data on demand and supply shocks. This prediction is that an increase in the (identified) aggregate shock price variance increases the size of impulse responses to the aggregate shock. We refute the prediction that an increase in sector component variance decreases the size of impulse responses - in any

significant manner, which does not comply with the RIA model. The reason could be that the decision makers in firms are very good at interpreting sector specific shocks and therefore do not have to devote much attention to them.

In addition we found that the impulse response to sector shocks have not changed much over time, although their share in sector price variances has increased. One narrative for this result is that managers who set prices simply understand sector shocks a lot better than aggregate shocks and so always react to sector specific shocks, but only react strongly to aggregate shocks when they are more important, that is in volatile periods. Operational macro model using the same mechanism as in the RIA a model should account for this.

Next, the RIA model predicts that with the fall in the common component variances of prices - while holding sector component variances constant -, the cross sectional dispersion of sector price responses to aggregate shocks should increase. We estimate that the common component variances of sector prices have indeed decreased over time, that the sector specific components have remained constant (on average), but contrary to the RIA model, the cross sectional dispersion of sector price responses to monetary, demand and supply shock has decreased.

In the RIA model the firms respond differently to different shocks because their importance (in setting the right price to maximize profits) is different. Prices respond more to shocks with higher variance and less to shocks with lower variance. The source of the shock is not so important, what is important is that the shocks "compete for the decision maker's attention". One would therefore also expect that there is the same trade off between identified aggregate shocks. This is indeed what we found in the data.

In this paper we focused on prices and leave detailed analysis of quantity responses to further analysis. It will be interesting to see if the results also hold for quantities. We leave this subject for future research.

## Appendix

### Tables and figures

#### Macroeconomic series

This section presents the macroeconomic data used in the application. First 144 U.S. macroeconomic time series were downloaded from the FRED (Federal Reserve Economic Data) database and the last two series from BEA (Bureau of Economic Analysis) database. We collected similar data to the dataset used in Korobilis [2009]. Some series were not available and some were excluded due to exhibiting large outliers (bank reserves namely). The core data (macroeconomic variables) consists of 144 variables. The transformation codes stand for: 1 - no transformation, 2 - first difference, 4 - logarithm and 5 - first difference in logarithms.

Num.	Mnemonic	Description	TC
1	AAA	Moody's Seasoned Aaa Corporate Bond Yield	1
2	AHECONS	Average Hourly Earnings: Construction	5
3	AHEMAN	Average Hourly Earnings: Manufacturing	5
4	AWHMAN	Average Weekly Hours: Manufacturing	1
5	AWOTMAN	Average Weekly Overtime Hours: Manufacturing	1
6	BAA	Moody's Seasoned Baa Corporate Bond Yield	1
7	BORROW	Total Borrowings of Depository Institutions from FED	5
8	BUSLOANS	Commercial and Industrial Loans	5
9	CBI	Change in Private Inventories	1
10	CIVA	Corporate Inventory Valuation Adjustment	1
11	CMDEBT	Households and NPO; Credit Market	5
12	CNCF	Corporate Net Cash Flow with IVA	5
13	COMPNFB	Nonfarm Business Sector: Comp. Per Hour	5
14	COMPRNFB	Nonfarm Business Sector: Real Comp. Per Hour	5
15	CONSUMER	Consumer Loans, All Commercial Banks	5
16	CP	Corporate Profits After Tax	5
17	CPIAUCSL	CPI for All Urban Consumers: All Items	5
18	CPIENGSL	CPI for All Urban Consumers: Energy	5
19	CPILEGSL	CPI for All Urban Consumers: All Items Less Energy	5
20	CPILFESL	CPI for All Urban Consumers: All Items Less Food & Energy	5
21	CPIUFDSL	CPI for All Urban Consumers: Food	5
22	CPIULFSL	CPI for All Urban Consumers: All Items Less Food	5
23	CURRCIR	Currency in Circulation	5
24	CURRDD	Currency Component of M1 Plus Demand Deposits	5
25	CURRSL	Currency Component of M1	5
26	DDDFCBNS	Demand Deposits Due to Foreign Commercial Banks	5
27	DDDFOINS	Demand Deposits Due to Foreign Official Institutions	5

Num.	Mnemonic	Description	TC
28	DEMDEPSL	Demand Deposits at Commercial Banks	5
29	DGI	Federal Government: Real National Defense Gross Investment	5
30	DIVIDEND	Corporate Profits after tax: Net Dividends	5
31	EXPGSC96	Real Exports of Goods & Services, 3 Decimal	5
32	FEDFUNDS	Effective Federal Funds Rate	1
33	FGCE	Federal Consumption Expenditures & Gross Investment	5
34	FGSL	Federal government transfer payments: Grants-in-aid	5
35	FINSAL	Final Sales of Domestic Product	5
36	FINSLC96	Real Final Sales of Domestic Product	5
37	FPI	Fixed Private Investment	5
38	FSDP	Final Sales to Domestic Purchasers	5
39	GDP	Gross Domestic Product	5
40	GDPC96	Real Gross Domestic Product, 3 Decimal	5
41	GDPCTPI	Gross Domestic Product: Chain-type Price Index	5
42	GDPDEF	Gross Domestic Product: Implicit Price Deflator	5
43	GGSAVE	Gross Government Saving	1
44	GS1	1-Year Treasury Constant Maturity Rate	1
45	GS10	10-Year Treasury Constant Maturity Rate	1
46	GS3	3-Year Treasury Constant Maturity Rate	1
47	GS5	5-Year Treasury Constant Maturity Rate	1
48	GSAVE	Gross Saving	5
49	HCOMPBS	Business Sector: Compensation Per Hour	5
50	HOABS	Business Sector: Hours of All Persons	5
51	HOANBS	Nonfarm Business Sector: Hours of All Persons	5
52	HOUST	Housing Starts: New Privately Owned Housing Units Started	4
53	HOUST1F	Privately Owned Housing Starts: 1-Unit Structures	4
54	HOUSTMW	Housing Starts in Midwest Census Region	4
55	HOUSTNE	Housing Starts in Northeast Census Region	4
56	HOUSTS	Housing Starts in South Census Region	4
57	HOUSTW	Housing Starts in West Census Region	4
58	IMPGSC96	Real Imports of Goods & Services, 3 Decimal	5
59	INDPRO	Industrial Production Index	1
60	INVEST	Securities in Bank Credit at All Commercial Banks	5
61	LOANINV	Bank Credit at All Commercial Banks	5
62	LOANS	Loans and Leases in Bank Credit, All Commercial Banks	5
63	M1SL	M1 Money Stock	5
64	M2MOWN	M2 Minus Own Rate	6
65	M2MSL	M2 Less Small Time Deposits	5
66	M2SL	M2 Money Stock	5
67	MANEMP	All Employees: Manufacturing	5
68	MPRIME	Bank Prime Loan Rate	1
69	MZMSL	MZM Money Stock	5
70	NAPM	ISM Manufacturing: PMI Composite Index <sup>Â©</sup>	1

71	NAPMII	ISM Manufacturing: Inventories Index <sup>Â©</sup>	1
Num.	Mnemonic	Description	TC
72	NAPMNOI	ISM Manufacturing: New Orders Index <sup>Â©</sup>	1
73	NDGI	Federal Nondefense Gross Investment	5
74	NDMANEMP	All Employees: Nondurable goods	5
75	NONREVSL	Total Nonrevolving Credit Owned and Securitized	5
76	OTHSEC	Other Securities at All Commercial Banks	5
77	PCE	Personal Consumption Expenditures	5
78	PCEPI	Personal Consumption Expenditures: Chain-type PI	5
79	PFCGEF	PPI: Finished Consumer Goods Excluding Foods	5
80	PPIACO	PPI: All Commodities	5
81	PPICPE	PPI: Finished Goods: Capital Equipment	5
82	PPICRM	PPI: Crude Materials for Further Processing	5
83	PPIENG	PPI: Fuels & Related Products & Power	5
84	PPIFCF	PPI: Finished Consumer Foods	5
85	PPIFCG	PPI: Finished Consumer Goods	5
86	PPIFGS	PPI: Finished Goods	5
87	PPIIDC	PPI: Industrial Commodities	5
88	PPIITM	PPI: Intermediate Materials: Supplies & Components	5
89	PRFI	Private Residential Fixed Investment	5
90	RCPHBS	Business Sector: Real Compensation Per Hour	5
91	REALLN	Real Estate Loans, All Commercial Banks	5
92	RENTIN	Rental Income of Persons (CCAdj)	5
93	REQRESNS	Required Reserves of Depository Institutions	5
94	RESBALNS	Total Reserve Balances Maintained with FRB	5
95	SAVINGSL	Savings Deposits - Total	5
96	SLEXPND	State & Local Government Current Expenditures	5
97	SLINV	State & Local Government Gross Investment	5
98	SRVPRD	All Employees: Service-Providing Industries	5
99	STDCBSL	Small Time Deposits at Commercial Banks	5
100	STDSL	Small Time Deposits - Total	5
101	STDTI	Small Time Deposits at Thrift Institutions	5
102	SVGCSL	Savings Deposits at Commercial Banks	5
103	SVGTI	Savings Deposits at Thrift Institutions	5
104	SVSTCSL	Savings and Small Time Deposits at Commercial Banks	5
105	SVSTSL	Savings and Small Time Deposits - Total	5
106	TB3MS	3-Month Treasury Bill: Secondary Market Rate	1
107	TB6MS	6-Month Treasury Bill: Secondary Market Rate	1
108	TCDSL	Total Checkable Deposits	5
109	TGDEF	Net Government Saving	1
110	TOTALSL	Total Consumer Credit Owned and Securitized, Outstanding	5
111	TVCKSSL	Travelers Checks Outstanding	5

Num.	Mnemonic	Description	TC
112	UEMP15OV	Num. of Civilians Unemployed for 15 Weeks & Over	5
113	UEMP15T26	Num. of Civilians Unemployed for 15 to 26 Weeks	5
114	UEMP27OV	Num. of Civilians Unemployed for 27 Weeks and Over	5
115	UEMP5TO14	Num. of Civilians Unemployed for 5 to 14 Weeks	5
121	USFIRE	All Employees: Financial Activities	5
122	USGDCB	U.S. Government Demand Deposits at Commercial Banks	5
123	USGOVT	All Employees: Government	5
124	USGSEC	Treasury and Agency Securities at All Commercial Banks	5
125	USGVDDNS	U.S. Government Demand Deposits and Note Balances	5
126	USINFO	All Employees: Information Services	5
127	USLAH	All Employees: Leisure & Hospitality	5
121	USFIRE	All Employees: Financial Activities	5
122	USGDCB	U.S. Government Demand Deposits at Commercial Banks	5
123	USGOVT	All Employees: Government	5
124	USGSEC	Treasury and Agency Securities at All Commercial Banks	5
125	USGVDDNS	U.S. Government Demand Deposits and Note Balances	5
126	USINFO	All Employees: Information Services	5
127	USLAH	All Employees: Leisure & Hospitality	5
128	USPBS	All Employees: Professional & Business Services	5
129	USPRIV	All Employees: Total Private Industries	5
130	USSERV	All Employees: Other Services	5
131	USTPU	All Employees: Trade, Transportation & Utilities	5
132	USTRIDE	All Employees: Retail Trade	5
133	USWTRADE	All Employees: Wholesale Trade	5
134	sAAA	Spread between AAA and Fedfunds	1
135	sBAA	Spread between BAA and Fedfunds	1
136	sGS1	Spread between GS1 and Fedfunds	1
137	sGS10	Spread between GS10 and Fedfunds	1
138	sGS3	Spread between GS3 and Fedfunds	1
139	sGS5	Spread between GS5 and Fedfunds	1
140	sMPRIME	Spread between Bank Prime Loan Rate and Fedfunds	1
141	sTB3MS	Spread between TB3MS and Fedfunds	1
142	sTB6MS	Spread between TB6MS and Fedfunds	1
143	PCEP	Personal consumption expenditure price index (from BEA)	5
144	PCEQ	Real Personal consumption expenditure (from BEA)	5

Table 1: DIC criteria

No. Fac.	static factors	VAR lags
1	1.3445	-4.8524
2	1.3043	-3.9215
3	1.2693	-3.4576
4	1.2386	-3.1185
5	1.2205	-3.4642
6	1.3970	/
7	2.4403	/

This table reports the DIC criteria, a Bayesian model selection criteria, for the number of static factors and number of lags. The model with the lowest DIC criteria should be selected. DIC is defined as in eq.(26) on p.13. The DIC criteria for the number of static factors is in e+005 units.

Table 2: Shares of explained variances

Period	Variable	Total	Common	Idiosyncratic	Share of Common
Full	RGDP	0.94	0.75	0.19	80.23%
	IP	0.87	0.66	0.21	76.30%
	CPI	0.72	0.66	0.06	92.17%
	PPI	0.67	0.48	0.19	74.78%
	M3TB	0.44	0.06	0.38	22.35%
	UR	0.39	0.04	0.35	11.56%
	PCE Q	0.88	0.70	0.18	78.20%
	PCE P	0.74	0.71	0.03	95.20%
1959Q3-1983Q4	RGDP	0.71	0.51	0.20	73.50%
	IP	0.77	0.41	0.36	59.41%
	CPI	0.47	0.43	0.04	90.89%
	PPI	0.41	0.34	0.07	82.64%
	M3TB	0.12	0.02	0.10	27.78%
	UR	0.39	0.02	0.37	5.60%
	PCE Q	0.73	0.49	0.25	66.75%
	PCE P	0.46	0.43	0.03	94.42%
1984Q1-2007Q2	RGDP	0.62	0.51	0.11	82.59%
	IP	0.58	0.46	0.12	79.21%
	CPI	0.49	0.44	0.05	91.63%
	PPI	0.54	0.37	0.16	71.97%
	M3TB	0.24	0.04	0.19	21.83%
	UR	0.25	0.03	0.22	12.49%
	PCE Q	0.63	0.49	0.14	78.02%
	PCE P	0.50	0.47	0.03	94.51%
2007Q3-2014Q2	RGDP	1.16	1.05	0.11	90.45%
	IP	1.12	0.97	0.15	86.95%
	CPI	0.88	0.81	0.07	92.72%
	PPI	1.01	0.53	0.49	54.16%
	M3TB	0.34	0.11	0.22	37.60%
	UR	0.53	0.08	0.45	17.61%
	PCE Q	0.97	0.91	0.06	94.10%
	PCE P	0.93	0.89	0.04	96.33%

This table displays posterior mean variance decompositions for selected aggregate variables, where: **Total** stands for total variance, **Common** for common component variance (variance explained with factor model), **Idiosyncratic** for idiosyncratic component variance (variance left unexplained with factor model), **Share of Common** for share of common component variance in total variance.



Table 3: Variance decompositions of the median sector by period

Period	Volatilities		Idiosyncratic	Share of Common
	Total	Common		
Full	0.64	0.23	0.40	46.71%
1959Q3Q1-1983Q4	0.71	0.27	0.43	46.59%
1984Q1-2007Q2	0.53	0.17	0.36	43.85%
2007Q3-2014Q2	0.74	0.31	0.43	55.83%

Table displays posterior variance decompositions for the median sector where: **Period** stands for sample, **Total** for average total variance of the median sector (averaged over time period), **Common** for average common component variance of the median sector (variance explained by the model averaged over time), **Idiosyncratic** for idiosyncratic component variance of the median sector (variance left unexplained by the model averaged over time) and **Share of Common** for the median sector average share of explained variance (averaged over time).

Table 4: Summary statistics for speed of IRs and variance decompositions

Variable	Mean	Std. Dev.	Min.	Max.
Sector	-	-	1	197
Time	-	-	1	190
sp_mon	0.0100	0.0110	0.0001	0.1292
sp_dem	0.0075	0.0059	0.0002	0.0740
sp_sup	0.0129	0.0102	0.0001	0.3540
v_mon	0.0540	0.0363	0.0025	0.2918
v_dem	0.0699	0.0558	0.0030	0.5518
v_sup	0.0885	0.0811	0.0028	0.6693
v_nspec	0.0586	0.0430	0.0027	0.3229
v_idio	0.3481	0.5113	0.0000	13.3930
ir_mon	-0.1817	0.0913	-0.5377	0.2078
ir_dem	0.3568	0.1812	-0.2220	1.2191
ir_sup	0.2344	0.1529	-0.3231	0.6593

Table 4 presents summary statistic for the variables used in the regression, where: **sp** (speed of response), **v** (variance) and **ir** (impulse response). Appended to the prefix is a suffix. Suffix can be one of the following: **mon** (monetary policy shock), **dem** (demand shock) or **sup** (supply shock). **nspec** and **idio** stand for unidentified shock and idiosyncratic shock.

Table 5: Speed of impulse responses regressed on own standard deviation and standard deviation of other shocks, 1959Q3 - 2014Q2.

Dependent \ Explanatory	const	sd(own)	sd(other)	R2 (overall)
spir_mon	.0124*	-.0109*		0.0200
	(.0113 .0135)	(-.016 -.006)		
	.0122*	-.0130*	.001	0.0218
	(.0112 .0133)	(-.019 -.007)	(-0.005 0.010)	
spir_dem	.0074 *	.0007		0.0080
	(.0071 .0077)	(-.0005 .0020)		
	.0075*	.0017*	-.0005*	0.0082
	(.0072 .0078 )	(.0002 .0031)	(-.0009 -.0001)	
spir_sup	.0140*	-.0040*		0.0234
	(.0136 .0143)	(-.0053 -.0026)		
	.0138*	-.0045*	.0004	0.002
	(.0133 .0143)	(-.0063 -.0027)	(-.0006 .0014)	

Table 5 presents regressions where we regress speed of impulse response (**spir**) of monetary (**mon**), demand (**dem**) and supply (**sup**) shocks on a constant, own standard deviation (**sd(own)**) and standard deviation of the remaining shocks (**sd(other)**).

Table 6: Size of impulse responses regressed on aggregate standard deviation and standard deviation of the idiosyncratic variance, 1959Q3 - 2014Q2

Dependent\Explanatory	const	sd(agr)	sd(idio)	R2 (overall)
ir_mon	-.101*	-.177*		.45
	(-.109 -.093)	(-.192 -.161)		
	-.100*	-.176*	-.002	.45
	(-.109 -.092)	(-.192 -.160)	(-.009 .004)	
ir_dem	.202*	.311*		.48
	(.197 .206)	(.302 .320)		
	.200*	.308*	.007	.48
	(.194 .205)	(.280 .317)	(-.002 .051)	
ir_sup	.193*	.057*		0.14
	(.180 .205)	(.033 .082)		
	.189*	.054*	.009	0.12
	(.177 .203)	(.029 .079)	(-.004 .021)	

Table 6 presents regressions where we regress size of impulse response (**ir**) of monetary (**mon**), demand (**dem**) and supply (**sup**) shocks on a constant, standard deviation of all aggregate shocks (**sd(agr)**) and standard deviation of the idiosyncratic shocks (**sd(idio)**).

Table 7: Size of impulse responses regressed on own standard deviation and standard deviation of other sock's variance, 1959Q3 - 2014Q2.

Dependent\Explanatory	const	sd(own)	sd(other)	R2 (overall)
ir_mon	-.0690*	-.528*		.49
	( -.079 -.059)	(-.573 -.485)		
	-.0697*	-.5342*	.0026	.49
	(-.0797 -.0597)	(-.5810 -.4874)	(-.0035 .0089)	
ir_dem	.187*	.666*		.53
	(.183 .191)	(0.651 0.681)		
	.189*	.676*	-.006	.53
	(.184 .193)	(.656 .693)	(-.015 .003)	
ir_sup	.152*	.248*		.24
	(.145 .159)	(.223 .272)		
	.162*	.280*	-.027*	.26
	(.153 .171)	(.253 .308)	(-.041 -.013)	

Table 7 presents regressions where we regress size of impulse response (**ir**) of monetary (**mon**), demand (**dem**) and supply (**sup**) shocks on a constant, own standard deviation (**sd(own)**) and standard deviation of the remaining shocks (**sd(other)**).

Table 8: Size of impulse responses regressed on own standard deviations of prices due to MON, DEM and SUP shock, 1970Q1-2006Q4

Dependent\Explanatory	const	sd(mon)	sd(dem)	sd(sup)	R2 (overall)
ir_mon	.281*	-1.134*	-.505*	1.218*	.22
	(.270 .293)	(-1.279 -.988)	(-.587 -.422 )	(1.154 1.283)	
ir_dem	0.235*	-.897*	1.328*	-.054	.476
	(.225 .245)	(-.982 -.813)	(1.292 1.364)	(-.1107 .0023)	
ir_sup	.281*	-1.158*	-.522*	1.206*	.224
	(.269 .293)	(-1.276 -1.039)	(-.574 -.470)	(1.127 1.285)	

Table 8 presents regressions where we regress size of impulse response (**ir**) of monetary (**mon**), demand (**dem**) and supply (**sup**) shocks on a constant, standard deviation of all aggregate shocks and standard deviation of remaining shocks (not shown due to space considerations), for the period 1970Q1 - 2006Q4.

## Figures

Figure 1: Factor estimates

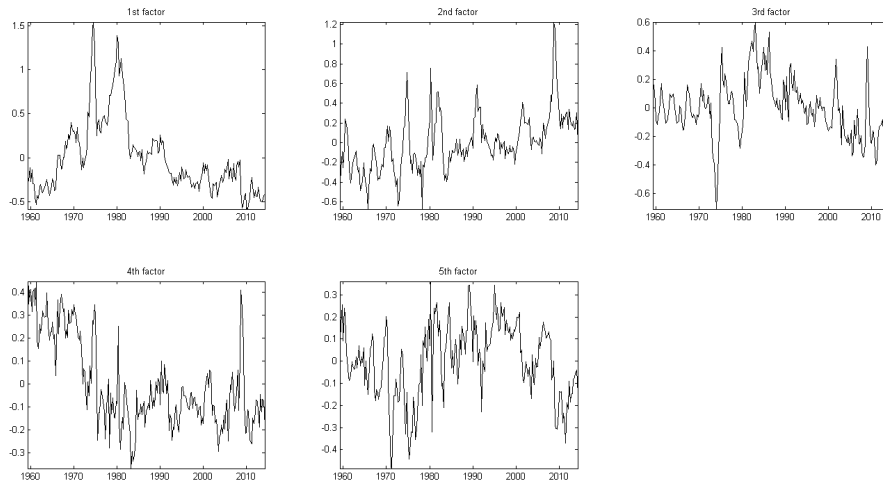


Figure displays the factors estimated on U.S. post-war macroeconomic dataset, using the PCA approach.

Figure 2: Volatility of aggregate variables

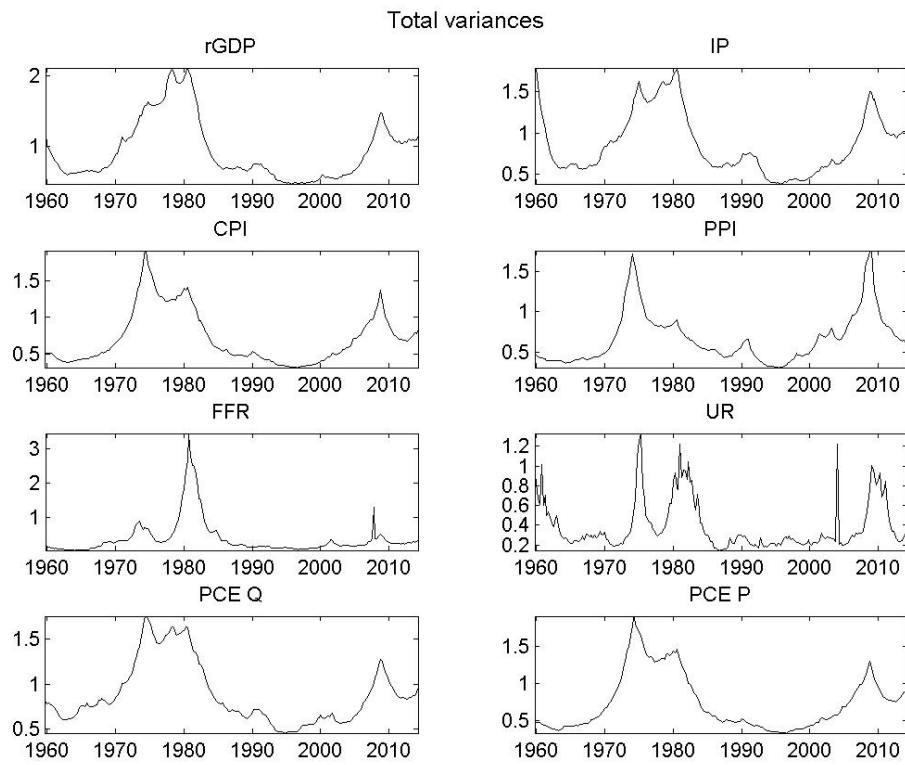


Figure displays the posterior mean of variance for selected variables.

Figure 3: Volatility of the common component of aggregate variables

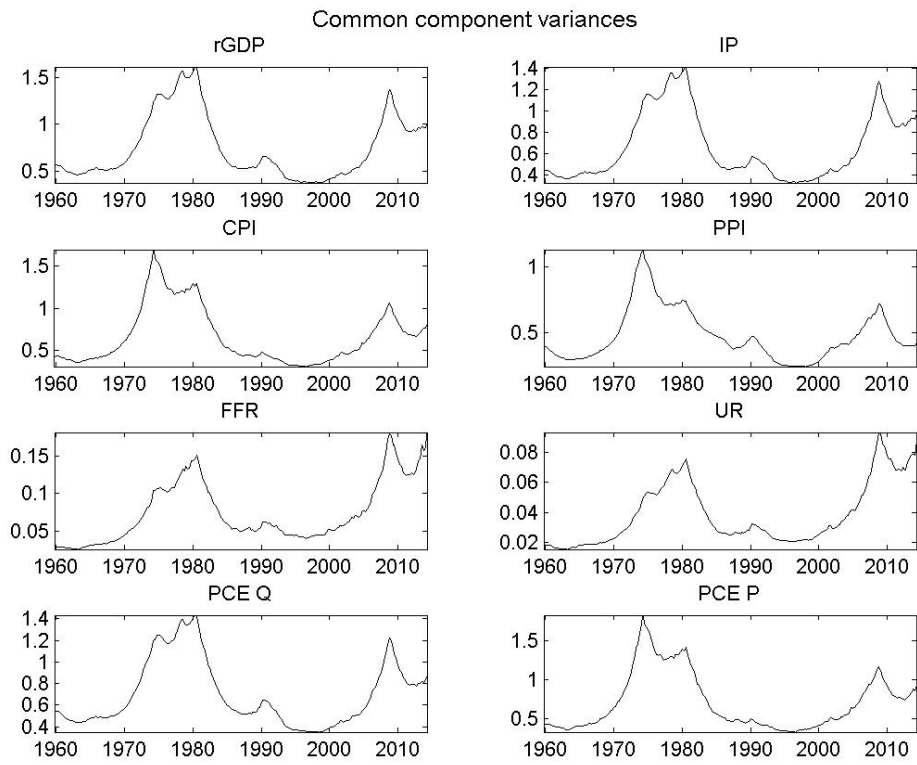


Figure displays the posterior mean of the common component variance (variance explained by aggregate factors), for selected variables.

Figure 4: Volatility of the idiosyncratic component for selected variables

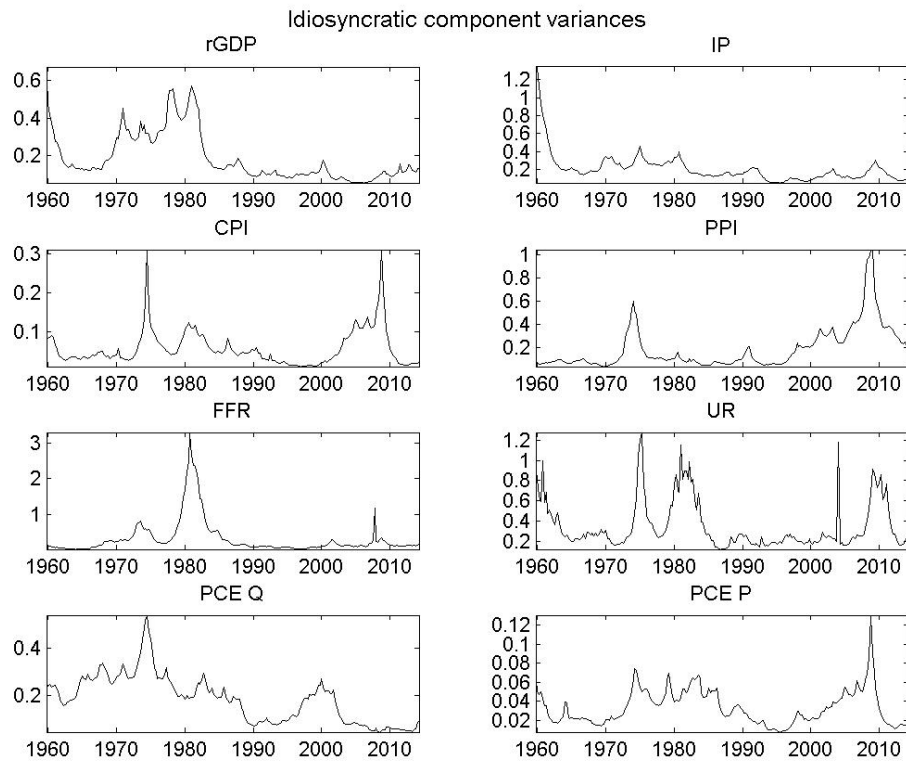


Figure displays posterior the mean of the idiosyncratic component variance (variance left unexplained by aggregate factors), for selected variables.

Figure 5: Share of common component variance in the total variance of aggregate variables

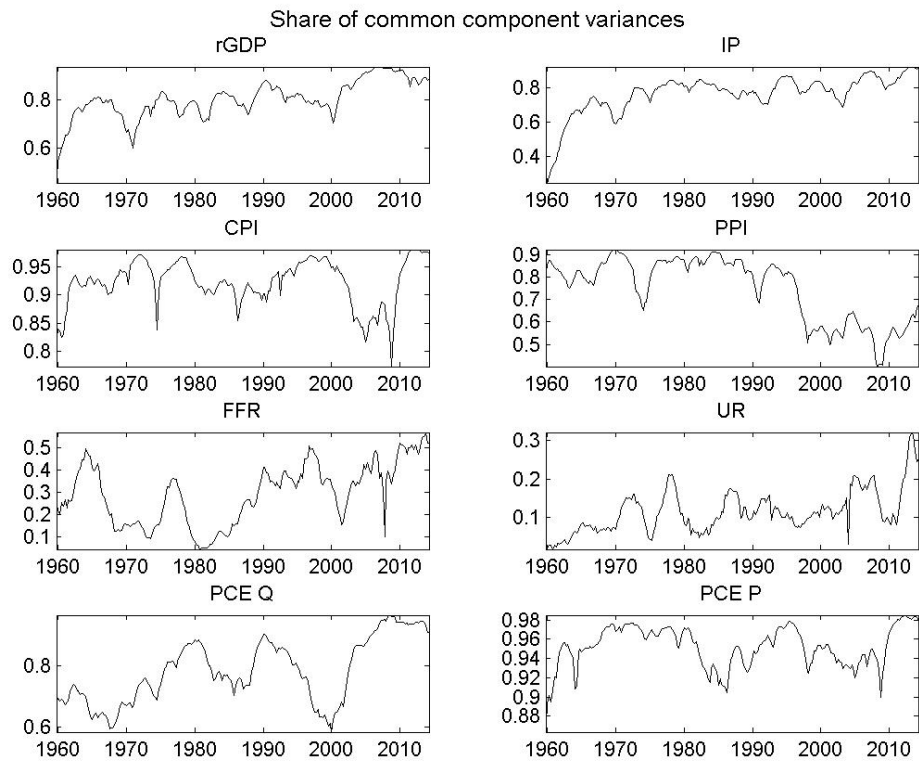


Figure displays posterior the mean of the share of common component variance in the total variance (share of variance explained by aggregate factors), for selected variables.



Figure 6: IRs of selected aggregate variables to aggregate shocks for chosen periods

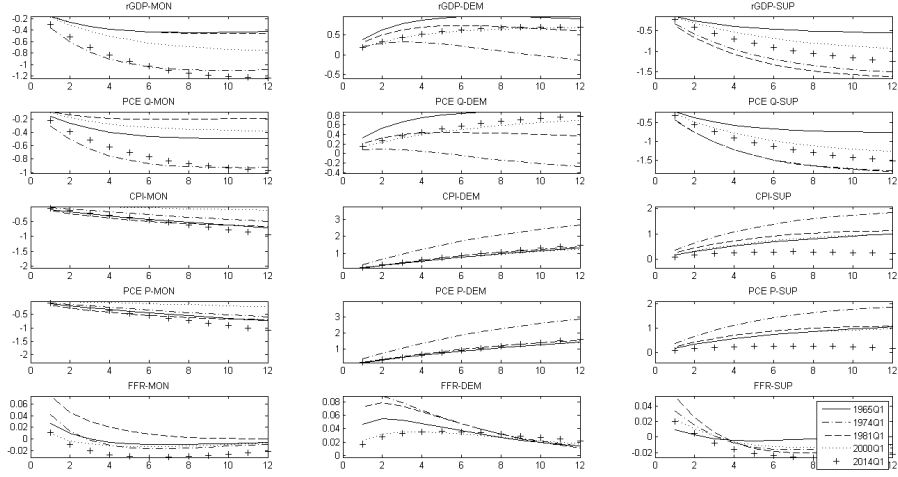


Figure displays posterior means of IRs of aggregate variables to the three identified shocks, for selected periods.

Figure 7: IRs of selected aggregate variables to a monetary policy shock

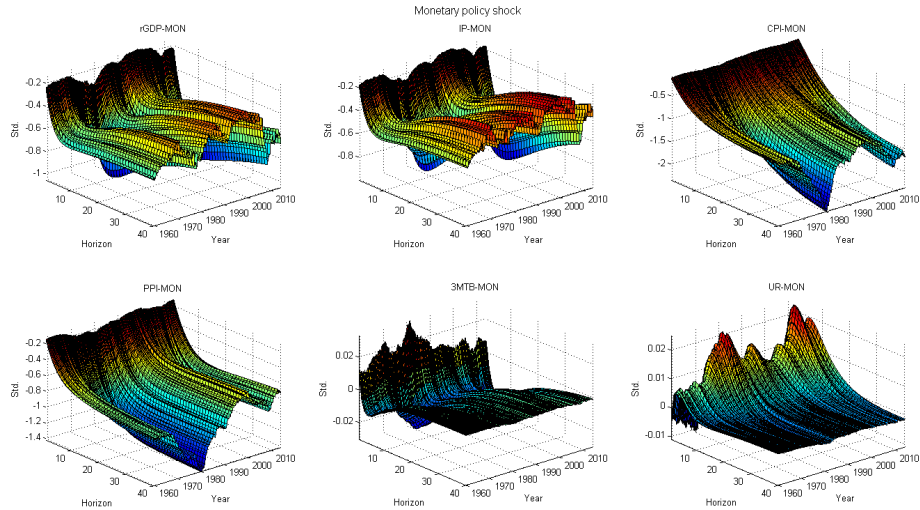


Figure 8: IRs of selected aggregate variables to a monetary policy shock

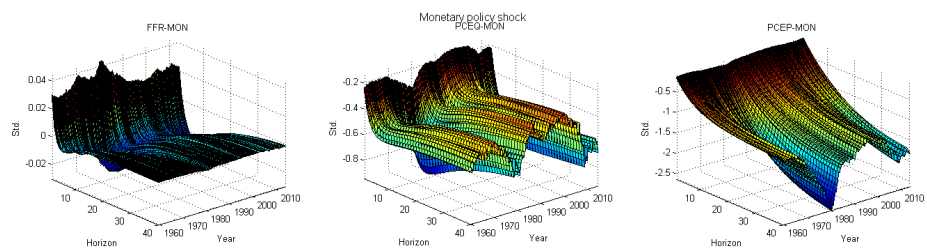


Figure 9: IRs of selected aggregate variables to a demand shock

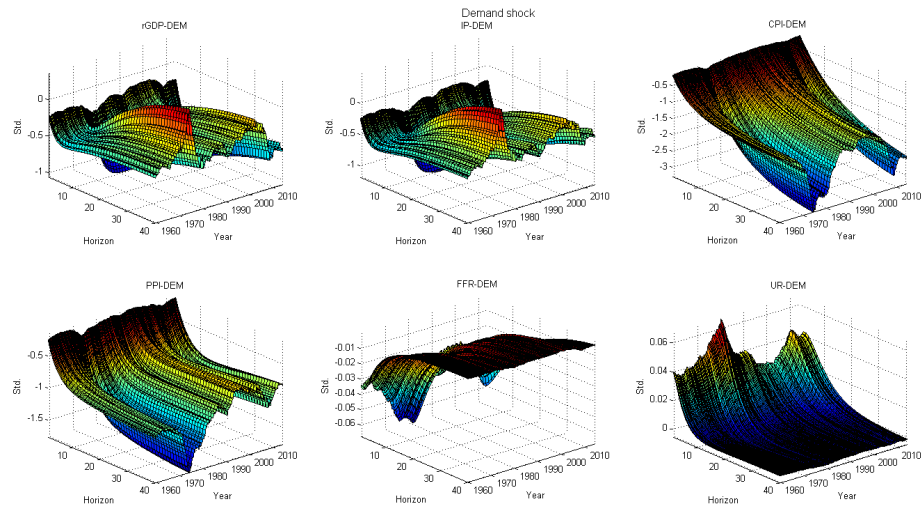


Figure 10: IRs of selected aggregate variables to a demand shock

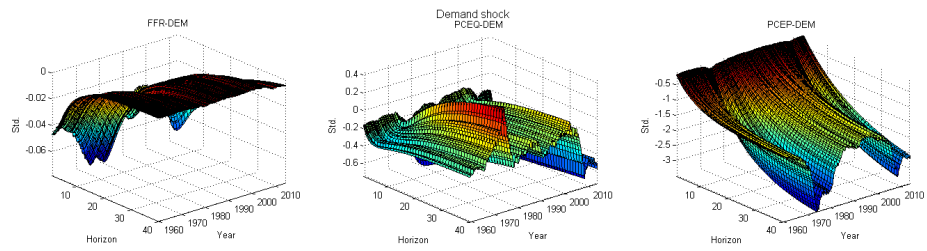


Figure 11: IRs of selected aggregate variables to a supply shock

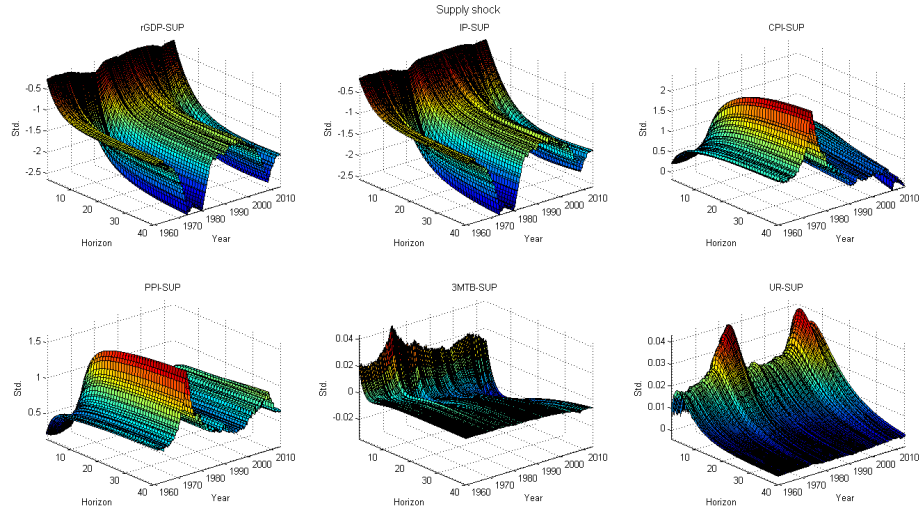


Figure 12: IRs of selected aggregate variables to a supply shock

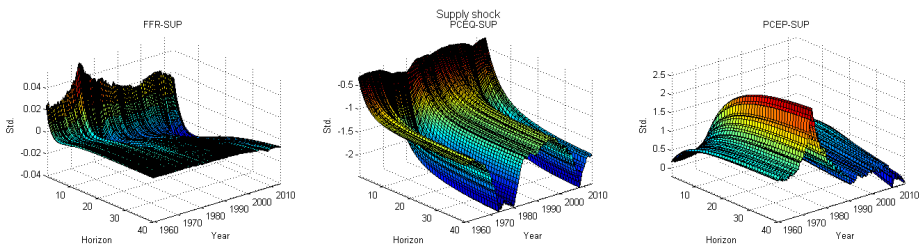


Figure 13: Aggregate and idiosyncratic component variance of sector prices

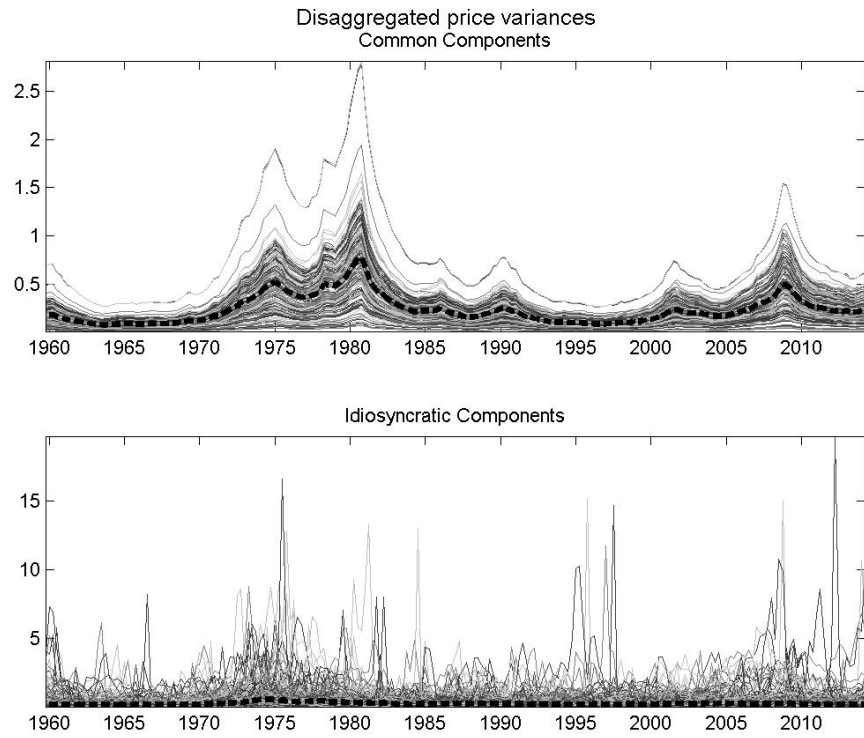


Figure displays posterior variance decompositions for sector prices. Upper panel draws posterior means of sector price common component variances (part of sector price variance explained by the model) and lower the idiosyncratic component variances (part of sector price variances left unexplained by the model). The black dashed line represents the median sector.

Figure 14: Total variances and share of common component variance of sector prices

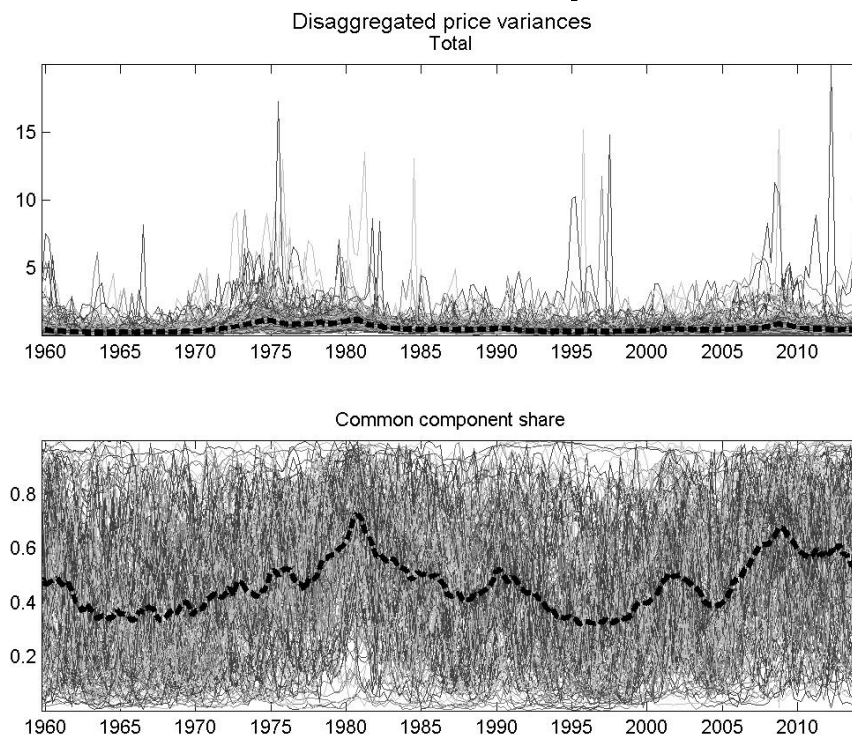


Figure displays posterior variance decompositions for sector prices. Upper panel displays posterior means of total variance of sector prices and lower the share of common component variance in total variance. The black dashed line represents the median sector.

Figure 15: Median sector impulse responses to aggregate shocks

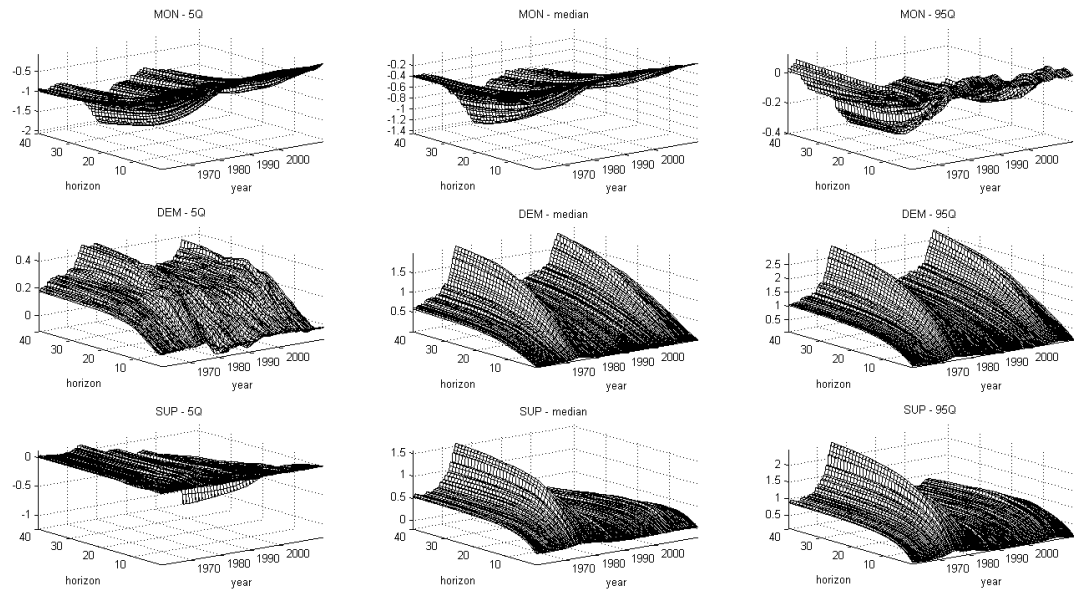


Figure 16: Sector impulse responses to aggregate shocks for selected periods

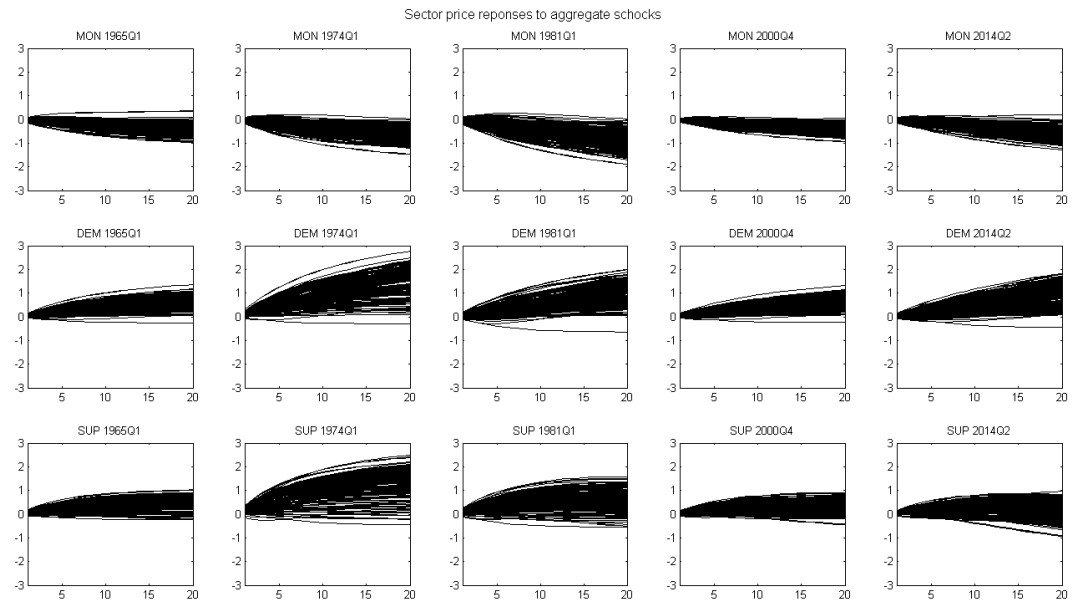


Figure 17: Cumulative sector price responses to sector shocks

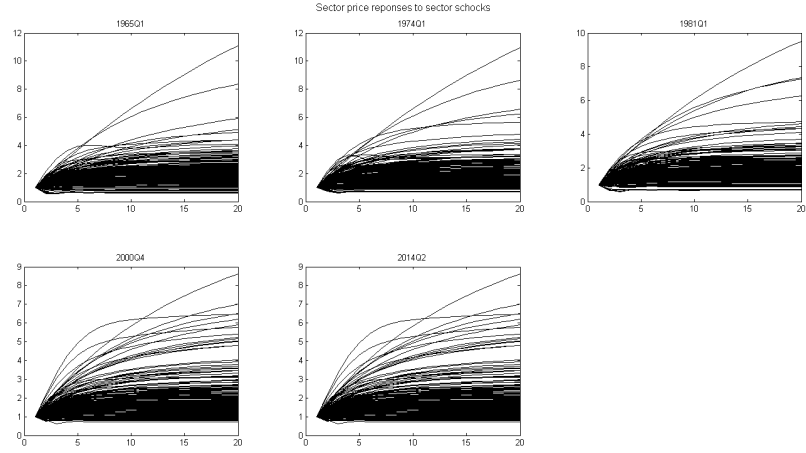


Figure 17 presents posterior means of cumulative IRs of sector prices to sector specific shocks for selected periods.

Figure 18: Median sector price response to sector shocks over time

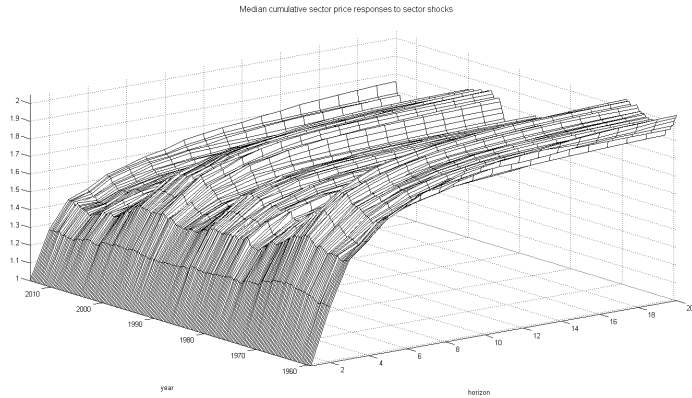


Figure 18 presents posterior mean of cumulative IRs of sector prices to specific shocks.



## Estimation

This appendix presents the estimation algorithm and the priors. We first present the estimation algorithm and then the priors. We estimate the following model:

$$x_{it} = b_{it}F_t + u_{it} \quad (31)$$

$$u_{it} = \theta_{1it}u_{it-1} + \dots + \theta_{qit}u_{it-q} + \eta_{it} \quad (32)$$

$$F_t = d_{1t}F_{t-1} + \dots + d_{pt}F_{t-p} + e_t \quad (33)$$

## Algorithm

The model in eq.(1)-(3) can be separated into two parts: a model for the observation equation (eq.(1)-(2)) and a model for the VAR of the factors (eq. (3)). Factor VAR model was estimated using the algorithm presented in Korobilis [2014]. To estimate (31) and (32) we used the following algorithm:

Time varying parameters were estimated by transforming the model into a linear state space model to which we applied the Carter-Kohn algorithm. For this reason we first present a general state space model and then the manipulations of the equations that enable us to apply the Carter-Kohn algorithm. The normal linear state space model takes the following form:

$$y_t = X_t\beta_t + \epsilon_t \quad (34)$$

$$\beta_t = \beta_{t-1}X_t + a_t \quad (35)$$

Where  $y_t$  is a  $n \times 1$  vector of dependent variables,  $X_t$  a  $k \times 1$  matrix of explanatory variables,  $\beta_t$  a  $n \times k$  matrix of time varying coefficients and  $\epsilon_t$  and  $a_t$  independent Gaussian white noise vectors. How to estimate a normal linear state space model is explained in Koop and Korobilis [2010]. We followed their approach so we only present the basic steps:

1. Initialize all the variables with arbitrary values.

2. Calculate  $u_{it} = x_{it} - b_{it}F_t$  and form a state space with:

$$y_t = u_{it} \quad (36)$$

$$X_t = [u_{it-1} \dots u_{it-q}] \quad (37)$$

$$\beta_t = [\theta_{1it} \dots \theta_{qit}]' \quad (38)$$

$$(39)$$

Draw  $[\theta_{1it} \dots \theta_{qit}]$  and the corresponding variances.

3. Given  $[\theta_{1it} \dots \theta_{qit}]$  multiply (31) with  $\theta(L) = 1 - \theta_{1it}L - \dots - \theta_{qit}L^q$ , to obtain a model with serially uncorrelated errors:

$$x_{it}^* = b_{it}F_t^* + a_{it}^* \quad (40)$$

where  $x_{it}^* = x_{it} - \theta_{1it}x_{it-1} - \dots - \theta_{qit}x_{it-q}$  and  $F_{it}^* = F_{it} - \theta_{1it}F_{it-1} - \dots - \theta_{qit}F_{it-q}$ . Form a state space model and draw  $b_{it}$ :

$$y_t = x_{it}^* \quad (41)$$

$$X_t = F_t^* \quad (42)$$

$$\beta_t = b_{it} \quad (43)$$

4. Calculate  $a_{it}^* = x_{it}^* - F_t^*$ . We assume that the errors distribute in a stochastic volatility model  $a_{it}^* = \sqrt{\omega_t}\epsilon_t$ . This is a non-linear equation that can be transformed into a linear equation. Square  $a_{it}^*$  and take logarithms to linearize the model:

$$\ln(a_{it}^{*2}) = 2\ln(\omega_t) + v_{it} \quad (44)$$

$$\ln(\omega_t) = \ln(\omega_{t-1}) + \eta_t \quad (45)$$

This is again linear state space model with:  $y_t = \ln(a_{it}^{*2})$ ,  $X_t = 2$  and  $\beta_t = \ln(\omega_t)$ . The only difference is that the residuals  $v_{it}$  are not normally distributed. Their distribution is approximated with a mixture of normals as in Koop and Korobilis [2010].

5. Repeat (2)-(4) till convergence.

## Benchmark priors

The priors used in the benchmark case are slightly informative. Given the number of time varying parameters, slightly informative priors stabilize the model. This is especially important because we restrict the model to be stationary. Too much time variation resulted in cases where virtually all the draws were rejected. We first present the priors used in the observation equation and then the priors used in the factor VAR.

We use the following priors for each observation equation:

$$A_0 \sim N(0_{r \times 1}, 4 \times I_r) \quad (46)$$

$$Q^A \sim IW(k_A^2(1+r)I_r, 50+r) \quad (47)$$

$$\Theta_0 \sim N(0_{q \times 1}, 4 \times I_q) \quad (48)$$

$$Q^\Theta \sim IW(k_\Theta^2(1+q), 50+q) \quad (49)$$

$$\log \Omega_0 \sim N(-1.3863, 4) \quad (50)$$

$$Q^\Omega \sim IW(k_\Omega^2, 2) \quad (51)$$

$$(52)$$

Because we do not know in advance what should be the sign on the period zero factor loadings and the error autoregressive coefficients ( $A_0$  and  $\Theta_0$ ) we set them to zero with variance 4. This seems reasonable considering that in the empirical applications with factor models the factor loadings of standardized variables are rarely higher than 2 and considering because we assume that the autoregressive process for the residuals is stationary. We set  $k_Q = k_\theta = 0.01$ . If we set it to  $k_Q = k_\theta = 0.1$  the algorithm had problems converging and if we set it to  $k_Q = k_\theta = 0.001$  there was almost no variability in the coefficients.  $\log \Omega_0$  was set so that the explained variance of a standardized variable is approx. 50%. In the benchmark application  $k_\Omega$  was set to 0.01.

The model for the factor VAR was taken from Korobilis [2014]. The prior the prior variances for the VAR covariance matrix and the time varying VAR coefficients are

updated from the data. We still need to set the following priors:

$$\bar{D}_0 \sim N(0_{(p \times r^2) \times 1}, V) \quad (53)$$

$$\tilde{D}_0 \sim N(0_{(p \times r^2) \times 1}, 0_{(p \times r^2) \times (p \times r^2)}) \quad (54)$$

$$Q^{\tilde{D}} \sim IW(p \times r^2 + 2, k_{Q^{\tilde{D}}} \times V) \quad (55)$$

$$V = \text{diag}(\tau_1, \dots, \tau_{(p \times r^2)}) \quad (56)$$

$$\tau_i = IG(\kappa_1, \kappa_2 \times \frac{1}{m^2}) \quad \text{for } m = 1 \dots p \quad (57)$$

$$T_0 \sim N(O_{\frac{1}{2}r(r-1) \times 1}, 10 \times I_{\frac{1}{2}r(r-1)}) \quad (58)$$

$$Q^{T_i} \sim IW(i + 1, k_{T_i} \times I_i) \quad \text{for } i = 1 \dots r - 1 \quad (59)$$

$$\log H_0 \sim N(0_{r \times 1}, 10 \times I_r) \quad (60)$$

$$Q_{i,i}^H \sim IG(8, 0.1) \quad \text{for } i = 1 \dots r \quad (61)$$

$$(62)$$

We observe that the diagonal variance matrix  $V$ , that sets the variance of the constant VAR coefficients  $\bar{D}_0$  and that of the time varying coefficients  $\tilde{D}$ , has its own prior  $\tau_i$  (for  $i = 1 \dots r$ ). This specification allows for  $\tau_i$  to be updated from the data. Note that the scale parameter for  $\tau_i$  is multiplied with an inverse of the square of the lag  $\frac{1}{m^2}$ , which shrinks the posterior estimates of parameters of distant lags toward zero.  $\kappa_1$  was set to 0.1 and  $\kappa_2$  to  $r \times p + 1$ .  $k_{Q^{\tilde{D}}}$  and  $k_{T_i}$  were set to 0.01 in benchmark specification.

Priors for the remaining time varying parameters (for  $t=1, \dots, T$ ), are implicitly defined by the structure of the model:

$$A_t \sim N(A_{t-1}, V(A_{t-1})) \quad (63)$$

$$\Theta_t \sim N(\Theta_{t-1}, kV(\Theta_{t-1})) \quad (64)$$

$$\log \Omega_t \sim N(\log \Omega_{t-1}, I_k) \quad (65)$$

$$\tilde{D}_t \sim N(\tilde{D}_{t-1}, V(\tilde{D}_{t-1})) \quad (66)$$

$$\log H_t \sim N(\log H_{t-1}, I_k) \quad (67)$$

$$(68)$$

This completes the specification of the priors. Posterior for model are presented in Korobilis [2009] for the observation equation and in Korobilis [2014] factor VAR.

## Monte Carlo simulation

This appendix describes a simulation study used to verify consistency of the PCE estimator of factors in the presence of stochastic volatility. We follow Stock and Watson [2002] in setting up the design of the experiment. Stock and Watson [2002] present a general factor model data generating process where factors evolve in a VAR and the idiosyncratic errors are allowed to be weakly cross correlated (for further details on the model the reader is referred to Stock and Watson [2002]):

$$X_t = \lambda F_t + E_{it} \quad (69)$$

$$A(L)F_t = u_t \quad \text{with} \quad u_t \quad i.i.d. \quad N(0, \Omega_t) \quad (70)$$

$$D(L)E_t = v_t \quad \text{with} \quad v_t \quad i.i.d. \quad N(0, \tau) \quad (71)$$

$$A_{ij}L = \begin{cases} 1 - \rho L & \text{if } i = j \quad i, j = 1 \dots r \\ 0 & \text{if } i \neq j \end{cases} \quad (72)$$

$$D_{ij}L = \begin{cases} \sqrt{\alpha_i}(1 - dL) & \text{if } i = j \quad i, j = 1 \dots n \\ 0 & \text{if } i \neq j \end{cases} \quad (73)$$

$$\alpha_i = \frac{\beta_i}{1 - \beta_i} \frac{1}{T} \sum_{t=1}^T (\sum_{j=1}^r \lambda_{ij} F_{jt})^2 \quad \text{with} \quad \beta_i \quad i.i.d. \quad U(u, 1 - u) \quad (74)$$

$$\tau_{ij} = \tau^{|i-j|} \frac{1}{1 - d^2} \quad i, j = 1 \dots n \quad (75)$$

$$\lambda_{ij} \quad i.i.d. \quad N(0, 1) \quad i = 1 \dots n \quad j = 1 \dots r \quad (76)$$

The only difference between the data generating process in Stock and Watson [2002] and the data generating process used in this simulation study is how we specify the covariance matrix of the factor VAR ( $\Omega_t$  in equation (2)). Stock and Watson [2002] assume that  $\Omega_t$  is an identity matrix whereas we model it as a diagonal matrix, where each element on the diagonal evolves as a random walk:

$$\Omega_{ii,t} = \sigma_{it} \quad i = 1 \dots r \quad (77)$$

$$\log(\sigma_{it}) = \log(\sigma_{it-1}) + \eta_t \quad \text{with} \quad \eta_t \quad i.i.d. \quad N(0, Q) \quad (78)$$

$Q$  and initial stochastic volatilities  $\sigma_{i0}$  were calibrated so that they correspond the values of the empirical model used in the paper. For a  $r = 5$  variate VAR, estimated in the body of the paper, the highest value of the posterior mean of  $Q$  was 0.0045 and the lowest 0.0009. We present the results for  $Q = 0.001$ . We also experimented with  $Q = 0.0001$ ,  $Q = 0.100$ , which did not affect the results. Initial volatility was set to

approximately correspond to the unconditional mean of  $\log(\sigma_{it})$ , which is  $-2.0$ . We simulated the model under the assumption of two factors,  $r = 2$ , no autocorrelation in the idiosyncratic component<sup>57</sup>,  $D(L) = I_n$ , VAR lag length was set to one,  $p = 1$ , VAR mean equations AR coefficients were set to one half,  $A(L = 1) = 0.5 \times I_r$  and finally, we set  $\tau$ , the parameter that governs the amount of cross correlation between the idiosyncratic components, to 0.5.

We simulated 500 samples under various time ( $T$ ) and cross section ( $n$ ) lengths. As in Doz et al. [2011] we calculated trace statistics for each simulated sample. The trace statistic measures how well the PCE estimates of the factors correlate with the true factors. A value of one indicates perfect correlation and a value of zero no correlation between the true factors and the estimated factors<sup>58</sup>.

Table 9: Trace statistic from MC experiments, varying  $n$  and  $T$

$tr_{pc}$	$n=10$	$n=100$	$n=1000$
$T=10$	0.27	0.34	0.35
$T=100$	0.57	0.82	0.84
$T=1000$	0.70	0.96	0.98

Table presents trace statistics from the simulation experiment. Trace statistic measures how well the estimated factor fit the true factor. It is a multivariate version of  $R^2$ . In case of perfect fit it takes value of 1 and 0 in case of no correlation. We vary cross sectional dimension (in columns) and time dimension (in rows). The simulation shows that increasing time and cross sectional dimension of the sample improves the performance of the PCE estimates of the factors, when their variances are modeled as random walks. PCE performs poorly in small samples ( $T = 10$ ). Increasing the cross sectional dimension  $n$  and time dimension  $T$  helps in improving the estimates. In large samples the PCE estimator estimates the factor space almost perfectly  $tr_{pc} = 0.98$ . Therefore the PCE estimator of factors seems to be a consistent estimator of the true factor space, even under the extreme assumption of explosive variance.

<sup>57</sup>As shown in Doz et al. [2011] ignoring the autocorrelation in the idiosyncratic component does not bias the results.

<sup>58</sup>The trace statistic is a multivariate variant of the  $R^2$  coefficient.

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