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Abstract

The paper discusses a two-period model where unit costs are private information. The firms are quantity setters and each firm can only produce in one of the two periods. The market clears after period two. If both firms always produce in the same period, a Cournot-Nash "situation" arises. If one firm always produces in the first period and the other in the second, a Stackelberg situation results. The analysis shows that Cournot-Nash can never be a Perfect Bayesian Equilibrium of this more general model, while Stackelberg situations can always be supported as PBEs. The results are similar if firms are price setters.

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1. Introduction.

An often advanced criticism of the well-known Stackelberg leadership model is that it provides no endogenous explanation of why one firm is the leader and another the follower. It is a simple textbook exercise to check that in a duopoly model where the goods are substitutes, the firms quantity setters, and the inverse demand functions are linear, both firms prefer being leader to being follower. Furthermore, both firms prefer playing Cournot-Nash to being the follower in the Stackelberg game. So why should we ever see a situation where one duopolist chooses its quantity after the other? One possible explanation could be to look at the historical evolution in the industry. Perhaps a specific firm has dominated for decades and tradition has lead other firms to accept its leadership. However superficially plausible this explanation may seem in some cases, it is nevertheless not satisfying from a theoretical perspective. Simply pointing to history cannot answer the question of why the follower accepts the situation today. Other factors must be at work. An alternative explanation could be that one firm moving first is a way of reaching a semi-cooperative agreement, providing a kind of focal point. But then the textbook Stackelberg model is not relevant, since it assumes non-cooperative behaviour. Finally, the Stackelberg situation could be the outcome of a complicated dynamic game. This is an interesting argument which deserves attention. However, in this paper I shall concentrate on a "single-round" model where each firm only produces once.

The approach in this paper is to analyse a model where both quantity setting firms are allowed to choose in which of two periods they will produce. Hence, a Stackelberg situation would evolve if the firms in equilibrium produce in different periods. A priori, one of several other possibilities is that the firms in equilibrium both choose to produce in the same period, and the Cournot-Nash situation would result. Two crucial assumptions of the model are that each firm can only produce in one of the two periods, and that the market clears after the second period. This seems to be the most appropriate avenue to take if one wants to take seriously the Stackelberg model. Note, however, that the two periods are not necessarily separated by any significant physical time. What is necessary is that a firm producing in the first period makes some irreversible production decision which the other firm can observe before making its production decision in the second period.

Another distinguishing feature of the model is that the duopolists know their own unit cost but not that of the rival firm. Thus, the model is analysed as a game of imperfect information, where the unit costs are private information. There is then an incentive for high cost firms to defer production until the second period in order to hide their weak competitive situation, which would be exploited by the rival firm if a high cost firm produced in the first period while the rival deferred production. On the other hand, low cost firms would want to separate themselves from these high cost firms by producing in the first period. The analysis of these conflicting motives of hiding and separating, and of reaping the first mover advantage versus reacting optimally to the rival's action, is the focus of this paper.

The main result is that the Stackelberg solution comes out better than the Cournot-Nash in this set-up. Since my interpretation of these two concepts is slightly unconventional they ought to be explained at this point. In this model a Stackelberg situation is one where one firm always produces in the first period, no matter its unit cost, while the other firm always produces in the second period, no matter its unit cost. In contrast, in a Cournot-Nash situation both firms always produce in the same period (be it 1 or 2), no matter their unit costs. Interpreted this way, the analysis reveals that Stackelberg situations are

Perfect Bayesian Equilibria (PBEs) of this model, while a Cournot-Nash situation can never be a PBE. Actually, the statement can be made slightly stronger, namely that if one firm always produces in one period, all PBEs have the other firm always producing in the other period. This leaves me so far with two types of equilibrium, that is, the two Stackelberg situations where each firm can either be leader or follower. This, however, is not necessarily the whole story. There may be additional sets of PBEs which unfortunately are not very tractable. In these types of PBEs some types (identified by unit costs) of each firm will produce in period 1 and others in period 2. I give an example of such an equilibrium (an "odd" equilibrium) in Section 5.

The next section relates the work in this paper to the existing literature. In Section 3 the quantity setting model is described in more detail, and the analysis of the equilibria follows in Section 4. Section 5 presents a simple example of an equilibrium which is neither of the Nash nor the Stackelberg type, and it is shown that this equilibrium does not survive a small change in the parameters of the model. Section 6 briefly describes the results from repeating the analysis under the alternative assumption that price is the strategic variable. The final section offers some concluding remarks and reflections on directions for further research.

2. Related Literature.

The improved game theoretic tools for analysing models with imperfect information have recently been applied to a number of two-period oligopoly models. The two closest in spirit to this paper are Mailath [1988] and Albaek [1990].

Mailath assumes that one of two duopolists receives a private signal about a random demand parameter. The firm with the private information then decides whether to engage in simultaneous quantity competition (to play Nash) or to make its quantity decision before the other firm (to be a Stackelberg leader). Mailath finds that the only equilibrium (after applying an equilibrium refinement) has the informed firm moving first, no matter what the observed signal is. Thus, his main result, that the firms will never move simultaneously, is similar to mine, although in my model the statement is weaker, namely that there is no equilibrium where both firms always move at the same time. In an "odd" equilibrium the firms may in fact end up producing in the same period. Mailath does not allow both firms to receive information and decide on when to produce. If I adopted his setting I would also find a single equilbrium, and if Mailath adopted mine, I conjecture that his model could also contain an odd equilibrium. Due to the additional complications arising from the signalling aspects of his model, this equilibrium would probably be extremely difficult to analyse.

In Albaek [1990] I pursue a question very similar to the one posed in this paper, namely to what extent private cost information can lead to endogenous explanations of the distribution of Stackelberg roles. However, in that paper the firms decide before they know their own cost whether they will play Stackelberg or Nash, and who shall be leader and follower. The main result is that if quantity is the strategic variable and cost variances sufficiently different, the firms can sometimes agree on such an endogenous distribution of Stackelberg roles. However, this is not possible if price is the strategic variable.

Gal-Or [1987] also analyses a Stackelberg game with imperfect information. However, in her model the emphasis is not on endogenous timing decisions, but rather on how the introduction of private information about a random demand parameter can alter the advantage of being a Stackelberg leader. In contrast to Mailath's model, both quantity setting firms receive private signals. If the leader uses a separating strategy, that is, chooses different quantities for different values of its signal, the follower can perfectly infer the leader's signal through the output choice. Hence, the follower can pool the two signals and always base its output decision on better information than the leader. Gal-Or shows that this informational effect may be sufficiently strong to render the follower's position more favourable than the leader's, contrary to the perfect information situation.

A common characteristic of the models described so far is that each duopolist can only produce in one of the two periods. In Mailath [1988] and Albaek [1990] one or both of the firms can decide in which period to produce, while in Gal-Or [1987] the issue of who produces when is an exogenous feature of the model. I will now briefly mention two two-period models with private information about costs where firms produce in <u>both</u> periods.

In Zachau [1986] two duopolists are uncertain about the rival's unit cost which can take on one of two possible values. A separating equilibrium is one where each firm chooses a different first period output level for different unit costs. Zachau finds that of many sequential equilibria only a unique separating equilibrium survives the strong requirement of stability. In this equilibrium the duopolists will try to signal low costs through choosing high first period outputs.

Mailath [1989] lets n firms simultaneously choose prices in each of the two periods. Again, each firm knows its own cost, but not the cost of any competitor. Although there are also semipooling equilibria, Mailath concentrates on separating equilibria and show that these all induce the same equilibrium path. If all goods are substitutes, all the first period prices in the signalling equilibrium are higher than in a benchmark nonsignalling equilibrium where the costs for some reason become common knowledge at the end of the first period.

While the papers discussed above assume some form of asymmetric information, there is another strand of recent literature analysing Stackelberg models in full information environments. For my purpose the most interesting of these are Hamilton and Slutsky [1990], Robson [1990] and Simon [1987]. In the section of their paper most relevant for my analysis Hamilton and Slutsky extend a model of Dowrick [1986]. They show that if attention is restricted to undominated strategies in a complete information model similar to mine (although with more general payoff functions), then the two Stackelberg equilibria (that is, either firm can be the leader) are the only pure strategy equilibria if the basic duopoly game has a unique equilibrium in the interior of the action space (Theorem VIII, p. 43). Hence, their result confirms that the Stackelberg equilibrium tends to perform better than (simultaneous) Nash if games are extended to model the timing decision endogenously. A similar conclusion is reached by Robson [1990], who analyses a price-setting duopoly in which the firms can choose and commit to a price at any of a countable set of dates before the fixed market-clearing date. However, the cost of setting a price is increasing in the difference between the price-setting and the market-clearing dates. Robson's analysis shows that "the only subgame perfect equilibria are more reminiscent of Stackelberg than of Bertrand". Simon [1987] is the only paper modelling the timing decision in continuous time. The general conclusion carries over to this setting, since he finds that in a Stackelberg/Cournot model in continuous time the two firms will never choose output simultaneously.

Although not directly related to the work presented in this paper, the papers by d'Aspremont and Gérard-Varet [1980] and Robson [1989] ought also to be mentioned. d'Aspremont and Gérard-Varet call an n-person game Stackelberg-solvable if it possesses at least one Stackelberg equilibrium defined as an n-tuple of strategies where each player's strategy is a best choice given that the other (n-1) players will play a Nash equilibrium of the remaining game taking the "leader's" strategy as given. Note that this does not correspond to the standard definition of a Stackelberg equilibrium in a duopoly; indeed, in a simple Cournot model with linear demand there is no Stackelberg equilibrium according to the definition of d'Aspremont and Gérard-Varet, hence the game is not Stackelberg-solvable (d'Aspremont and Gérard-Varet [1980], Example C, pp. 205-207). Robson [1989] extends this work by allowing for explicit timing decisions, that is, in order to be a leader, a firm has to choose to move in the first period which, however, involves a cost not present in moving in the second period. Robson then explores the relationship between the Stackelberg equilibria in the model of d'Aspremont and Gérard-Varet and subgame perefect equilibria in his own extended model. He shows that if an n-tuple of strategies forms a Stackelberg equilibrium in the static model, there exists a subgame perfect equilibrium of the extended model which has all firms moving in the second period and playing the same strategies as in the static Stackelberg equilibrium. Furthermore, in a two-person game where all Nash equilibria of the static game are Stackelberg equilibria, all subgame perfect equiibria of the extended game involve a Stackelberg equilibrium strategy vector, chosen by both firms in the second period.

Finally, a number of recent papers have analysed the attractiveness of the different roles in a Stackelberg duopoly in various environments (Ono [1982], Gal-Or [1985], Boyer and Moreaux [1987a, 1987b]). However, since none of these papers addresses the question of endogenous timing I will not go into the details of their analyses.

3. The Model.

The duopolists face two decisions: when to produce and how much to produce. If a firm produces in period one, it cannot produce in period two, and vice versa. A firm producing in period two can, before making its output decision, observe the output of a firm producing in period one. However, if both firms produce in the same period, neither of them observes the other's output before choosing its own. The market clears after period two. To keep the analysis tractable, the inverse demand function is assumed to be linear in total output,

$$p = d - (q_a + q_b)$$

where q_a and q_b are the outputs of the two firms A and B.[1] Note that the products of the two firms are assumed to be homogeneous, and that the slope, without loss of generality, has been normalized to unity.

The firms know their own costs but are uncertain about the rival's cost. This uncertainty is modelled by assuming each firm to have a subjective probability measure over the possible levels of the rival's costs. To be precise, the production technologies are such that unit costs are constant, that is, the total costs of firm i is $C_i(q_i) = c_i q_i$, i = A, B, where c_i is firm i's unit cost (its "type"). Hence, there

[1] A firm will be identified by upper or lower case letters, whatever is most convenient.

are no fixed costs. Firm A believes that c_b belongs to a closed interval $I_b = [l_b, h_b]$, with $0 < l_b < h_b$, and h_b finite. The actual value of c_b does in fact belong to this interval. Furthermore, for technical reasons, I assume that $d + l_i - 2h_j > 0$; $i, j = A, B; i \neq j$.[2] The prior beliefs of A about B's unit cost is described by the probability space (I_b, B_b, μ_b) where B_b is the Borel σ-algebra on I_b and μ_b a probability measure. The assumptions on B's prior beliefs about A's unit cost are defined in a similar way and summarized by the probability space (I_a, B_a, μ_a) where the measure μ_a is independent of μ_b. Note that the subscript a refers to B's prior beliefs about A's cost and vice versa. The two probability spaces are common knowledge, a standard assumption in this type of model.

The model is solved by deriving a plan for what firm B (and correspondingly firm A) would do for each value of c_b in I_b, although it of course already knows its own cost. A strategy for player B is therefore a quadruple of measurable functions, $\sigma_b = (\theta_b, q_b^1, q_b^{2n}, q_b^{2f})$ where θ_b: $I_b \rightarrow [0, 1]$ gives the probability that firm B with cost c_b will produce in the first period; q_b^1: $I_b \rightarrow \mathbf{R}_+$ is the quantity, as a function of c_b, it will produce in period one if it decides to produce in that period. If it instead decides to produce in period two, there are two possibilities: q_b^{2n}: $I_b \rightarrow \mathbf{R}_+$ is, again as a function of c_b, the quantity firm B will produce in period 2, if firm A also produces in period 2 (n is for "Nash"); and finally q_b^{2f}: $I_b \times \mathbf{R}_+ \rightarrow \mathbf{R}_+$ gives as a function of c_b and q_a the quantity firm B will produce in the second period when A has produced in the first period (f for "follower").[3]

In the specification above I have not allowed for mixed strategies over quantities,

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[2] This assumption ensures that quantities always are strictly positive.

[3] The word "strategy" may at times be used not only for σ_a and σ_b, but also for their component functions.

only over the decision on when to produce. The use of mixed strategies when there is an uncountable number of types poses some technical problems (see Aumann [1964], Milgrom and Weber [1985]) which there seems to be no reason to introduce here.

The profit to firm A with unit cost c_a from producing q_a units of output when firm B produces q_b is $\pi_a(q_a,q_b;c_a) = (d-q_a-q_b-c_a)q_a$. The firms are risk neutral, and no side payments are allowed. Furthermore, there is no discounting between the two periods. Hence each firm maximizes its individual expected profit. Given a strategy profile (σ_a, σ_b) the expected profit to firm A with marginal cost c_a is

$$\begin{aligned}
P_a(\sigma_a,\sigma_b;c_a) \quad = \quad & \int_{I_b} \theta_a(c_a)\theta_b(c_b)\pi_a(q_a^1(c_a),q_b^1(c_b);c_a)d\mu_b \\
& + \int_{I_b} \theta_a(c_a)[1-\theta_b(c_b)]\pi_a(q_a^1(c_a),q_b^{2f}(c_b,q_a);c_a)d\mu_b \\
& + \int_{I_b} [1-\theta_a(c_a)]\theta_b(c_b)\pi_a(q_a^{2f}(c_a,q_b),q_b^1(c_b);c_a)d\mu_b \\
& + \int_{I_b} [1-\theta_a(c_a)][1-\theta_b(c_b)]\pi_a(q_a^{2n}(c_a),q_b^{2n}(c_b);c_a)d\mu_b
\end{aligned}$$

At several points in the paper firm A will be assumed always to produce in one of the two periods, while some types of firm B with positive probability produce in period 1 and some in period 2. The firm B types who produce in period 1 are then identified by a measure ν_b^1 and those who produce in period 2 by ν_b^2. Consistency requires that $\nu_b^1(C) + \nu_b^2(C) = \mu_b(C)$ for all $C \in B_b$. Furthermore, if a proposed strategy for firm B is $\sigma_b = (\theta_b, q_b^1, q_b^{2n}, q_b^{2f})$ then

$$\nu_b^1(C) \quad = \quad \int_C \theta_b(c_b)d\mu_b \qquad\qquad \text{for all } C \in B_b$$

$$\nu_b^2(C) = \int_C [1-\theta(c_b)]d\mu_b \qquad \text{for all } C \in B_b$$

Define the mass of firm B types who produce in period i by

$$m_b^i = \int_{I_b} d\nu_b^i \qquad i = 1, 2$$

where, naturally, $m_b^1 + m_b^2 = 1$.

The average unit cost of each of the two firms is

$$\bar{c}_i = \int_{I_i} c_i d\mu_i \qquad i = A, B$$

If $m_b^i > 0$, the conditional average unit cost for the firm B types who produce in period i is

$$\bar{c}_b^i = \int_{I_b} \frac{c_b d\nu_b^i}{m_b^i} \qquad i = 1, 2$$

Obviously, for consistency, it must be true that $\bar{c}_b = m_b^1\bar{c}_b^1 + m_b^2\bar{c}_b^2$.

4. Equilibria.

The equilibrium notion employed in this paper is Perfect Bayesian Equilibrium (PBE). Readers interested in a general definition, and related concepts such as Trembling Hand Perfect Equilibrium and Sequential Equilibrium, may consult Fudenberg and Tirole [1989]. It should be noted that all these equilibrium concepts formally are defined for finite games, while the firms in this model have infinite action spaces since they are allowed to choose from a continuum of quantity levels.

In the present model a strategy profile (σ_a, σ_b) is part of a Perfect Bayesian Equilibrium if

(i) $P_i(\sigma_i,\sigma_j;c_i) \geq P_i(\sigma'_i,\sigma_j;c_i)$ for all c_i and σ'_i; $i,j = A, B$; $i \neq j$ [4]

(ii) $q_i^{2f}(c_i,q_j) \in \operatorname*{argmax}_{q_i \geq 0} \pi_i(q_i,q_j;c_i)$ for all c_i and q_j; $i,j = A, B$; $i \neq j$

(iii) there exists a probability measure ψ_j defined over the measurable space (I_j, B_j) such that

$$q_i^{2n}(c_i) \in \operatorname*{argmax}_{q_i \geq 0} \int_{I_j} \pi_i(q_i,q_j^{2n}(c_j);c_i)d\psi_j \text{ for all } c_i; \; i,j = A, B; \; i \neq j$$

Condition (i) states that the strategy profile has to be a Nash equilibrium, while (ii) ensures that a "follower" will always choose the quantity which maximizes its profit. The third condition says that the quantity choice in a second period Nash-like situation can be rationalized by some belief ψ_j over which types of the rival would wait and produce in the second period. Of course, given a particular strategy profile, one or both of the conditions (ii) and (iii) may be implied by (i). However, for some strategy profiles certain events may happen with zero probability. In those situations conditions (ii) and (iii) guarantee that responses to out-of-equilibrium behaviour are rational.

[4] "For all c_i and σ'_i" means "for all $c_i \in I_i$ and all $\sigma'_i \in \Sigma_i$", where Σ_i is the set of all permissible strategies for firm i.

Note that no belief-measure about a leader's cost is specified in condition (ii). A follower cares only about the quantity choice of the leader. The rival's cost matters only to a firm in so far that it indicates something about the quantity choice the rival wil make. When the leader has already chosen its output level the follower simply maximizes profit taking the leader's output as given. What marginal cost the leader has is in this situation of no interest to the follower.

This also means that signalling questions do not arise in this model, as it would if the asymmetric information was about a common value parameter. For instance, Mailath [1988] analyses signalling equilibria in a similar model where the information is about the strength of demand. Another example would be if the costs of the two firms were correlated and the firms do not know their own cost perfectly. Then a first mover would have an incentive to try to persuade the second mover that costs are higher than they in fact are. Signalling issues would also arise if the firms could produce in both periods. Then a firm would in the first period try to signal that it has a low cost in order to make the rival restrain its output in the second period. As a consequence of the above arguments there are in the present model no pooling equilibria in quantity choices: different cost types always choose different output levels. The only "pooling" going on is that types with different costs may produce in the same period and hence cannot be distinguished by the rival before the output levels are chosen (unless, of course, a firm is a leader).

In full, the equilibrium concept specifies a strategy profile and beliefs at each information set. These beliefs may implicitly be defined from the strategy profile via Bayes' rule. In this model the only interesting case with beliefs not specified by Bayes' rule is when one of the firms always produces in period one. Then the other player's response to out-of-equilibrium behaviour must be rationalized by some belief as required by condition (iii). In the following the word "equilibrium" means a strategy profile and associated beliefs, although for expositional ease these beliefs will not be specified explicitly when they can be deduced from the strategy profile.

This section focuses on the possible types of equilibrium in the quantity game. The analysis proceeds through two propositions stating what can not be equilibria.

Proposition 1. *There is no Perfect Bayesian Equilibrium in which* $\theta_i(c_i) = 1$ *for all* $c_i \in I_i$ *and* $\theta_j(c_j) > 0$ *for some* $c_j \in I_j$, $i \neq j$.

Proof. See Appendix 1.

The intuition behind this result is very simple. Imagine firm A always produces in period 1.[5] Suppose there is an equilibrium where some firm B type with positive probability also produces in period 1. Then this B type will in general be on its mean, but not its exact, reaction function, since it does not know the exact quantity A will produce. However, by deviating to period 2 our B type would observe A's output and hence be on its exact reaction function. Proposition 1 thus simply states that it is always better to be on the exact, rather than the mean, reaction function.

Note that Proposition 1 says there can be no equilibrium where both firms always produce in period 1. If a situation where both firms always produce in the same period is interpreted as a Cournot-Nash solution for this model, Proposition

[5] In this context "always" means "with probability one for all possible unit costs".

1 shows that no such solution can exist in the first period. The next proposition rules out such a situation in the second period as well.

Proposition 2. *There is no Perfect Bayesian Equilibrium in which* $\theta_i(c_i) = 0$ *for all* $c_i \in I_i$ *and* $\theta_j(c_j) < 1$ *for some* $c_j \in I_j$, $i \neq j$.

Proof. See Appendix 1.

Again, the intuition is quite straightforward. Imagine now that firm A always produces in period 2. If a subset of I_b produces in period 2 with a positive probability, A will, using Bayes' rule, form a belief about who it is playing against. Due to the linearity of the inverse demand function, all A really cares about is the average unit cost, say $\bar{c}_b^2$, of the B types who produce in period 2. Some of these types will have costs lower than $\bar{c}_b^2$, but will be treated as if their cost were $\bar{c}_b^2$. By deviating to period 1 these types reveal their stronger competitive situation, and simultaneously reap the first mover advantage a leader has when quantity is the strategic variable and goods are substitutes. Clearly, the proposition must be true.

Proposition 2 shows that a Cournot-Nash like situation in the second period cannot be a Perfect Bayesian Equilibrium. Indeed, Propositions 1 and 2 together show that, in this model, there is no PBE where both firms always produce in the same period.

Two possible types of equilibrium remain. One can be thought of as a kind of Stackelberg equilibrium, where one firm (the leader) always produces in period 1 while the other (the follower) always produces in period 2. The other possibility is an "odd" equilibrium where both firms have some types producing in the first period and some in the second. While in general both types of equilibrium may exist, I shall concentrate on the Stackelberg equilibria. However, in the next section I will briefly discuss an "odd" equilibrium.

Proposition 3. *There always exist Perfect Bayesian Equilibria in which* $\theta_i(c_i) = 0$ *for all* $c_i \in I_i$ *and* $\theta_j(c_j) = 1$ *for all* $c_j \in I_j$, $i \neq j$.

Proof. See Appendix 1.

The intuition behind the proof is as follows. The firm which in equilibrium is supposed always to produce in period two (say, firm B) will never deviate since the other firm's (A's) output decision is not changed by the deviation, and being on the exact reaction function is always better than being on the mean. I then concentrate on the belief used by firm B if it unexpectedly finds itself called upon to play Bayesian-Nash in period two rather than being the Stackelberg follower. If I can find a belief which keeps all A types from deviating, I have proved the proposition. Now, for an A type considering a deviation the worst B can think is that A for sure has cost h_a, since B then will behave aggressively and drive down the price. In this case, it is easy to show that the higher the c_a, the higher the temptation to deviate since there is a larger gain to be had from hiding one's cost. Thus firm h_a has the most to gain from deviating. However, if B thinks for sure that it is playing against h_a, then h_a will simply be switching from being a Stackelberg leader to playing Nash, which can never be a profitable move. But if deviating is not profitable for h_a it is not profitable for any $c_a < h_a$. Hence, I have found a belief which supports the Stackelberg situation as a PBE. Since the choice of A as leader was arbitrary, the converse Stackelberg situation can also be supported as a PBE.

Note that the proposition only describes the equilibrium path, not the equilibrium itself, since the associated beliefs are not specified. In fact, each of the two Stackelberg outcomes can be supported by any belief belonging to a connected set in the space of possible beliefs: all beliefs which generate a sufficiently high marginal cost will do.[6]

As mentioned above, there may be more than the two sets of equilibria already found. These additional equilibria would be characterized by some types of each firm producing in the first period and others in the second, perhaps even with some types using mixed strategies to determine when to produce. While it is relatively easy to construct examples in which such equilibria exist, it has so far proved difficult to solve for them, or just determine some qualitative features, in the general model. Since the primary focus of the paper is contrasting the merits of the Cournot-Nash and the Stackelberg solutions within this more general framework, I have decided to proceed without a full analysis of all the model's equilibria. However, in the next section I present a simple example of an equilibrium which is neither of the Nash nor the Stackelberg type.

5. A Simple Example.

Assume that the marginal cost of each of the two firms only can take on two values. With probability 0.5, $c_i = l$ and with probability 0.5, $c_i = h$, i = A, B. Suppose a firm with probability 1 produces in period 1 if it has a low marginal cost, and with probablity 1 in period 2 if the marginal cost is high. Framed in

[6] Kohlberg and Mertens [1986] make extensive use of the concept of a connected set of equilibria.

the language of Section 2 , $\theta_i(l) = 1$ and $\theta_i(h) = 0$, i = A, B. In this section I calculate the expected profits from timing production in this way, using the associated optimal quantity strategies. I then show that this is an equilibrium for some parameter values, but not for others.

The expected profit to a low cost firm from this situation is (calculations are shown in Appendix 2)

$$P(l) = \tfrac{3}{256}\,[3d+h-4l]^2$$

while the expected profit to a high cost firm is

$$P(h) = \tfrac{1}{512}\,[5d-9h+4l]^2 + \tfrac{1}{18}\,[d-h]^2$$

By deviating to period 2, and making the optimal output choices, the low cost firm would alternatively have expected profits of

$$P'(l) = \tfrac{1}{72}\,[2d+h-3l]^2 + \tfrac{1}{512}\,[5d-h-4l]^2$$

Similarly, the maximal expected profits to a high cost firm from deviating to period 1 is

$$P'(h) = \tfrac{1}{768}\,[9d-13h+4l]^2$$

Suppose the parameters have the specific values d = 120, l = 10, and h = 20. Then P(l) = 1355, P(h) = 969, P'(l) = 1304, and P'(h) = 963. Since $P(l) \geq P'(l)$ and $P(h) \geq P'(h)$, the strategies form an equilibrium.[7] It is obvious that the low

[7] That is, the timing functions $\theta_i(c_i)$ given above, and the associated quantity functions $q_i^1(c_i)$, $q_i^{2n}(c_i)$ and $q_i^{2f}(c_i)$, derived in Appendix 2, form an equilibrium.

cost leader will not want to deviate. Since the cost difference is big, there is a large benefit to the high cost follower of knowing exactly which type it is playing against, and it will forego the first mover advantage to get this information. Hence, this example has shown that there can be equilibria of other types than the ones discussed in the previous section.

Now, change the value of h to 12. Then $P(l) = 1292$, $P(h) = 1201$, $P'(l) = 1271$, $P'(h) = 1210$. In this case the high cost firm will deviate to period 1, and the situation is not an equilibrium. Since the high and low cost firms are so similar, the value to the high cost firm of knowing which type it is playing against is small, while it will gain substantially if it ends up being a leader to a high cost follower.

This section has given a simple example of a model in which there is at least one more equilibrium than the two Stackelberg equilibria, which the analysis in Section 3 showed will always exist.[8] However, it was also shown that a change in the value of one of the parameters made the equilibrium disappear. This means that there is no hope of a general proposition stating that there always exists "odd" equilibria in which low cost firms produce in the first period and high cost firms in the second. I have not yet been able to find a general way of determining when these additional equilibria do exist and what they look like. At the moment this question is left for future research.

[8] I do not state these equilibria here, as they can easily be found from the derivations in Appendix 1.

6. Price as Strategic Variable.

It is a straightforward exercise to repeat the preceding analysis under the alternative assumption that price is the strategic variable.[9] If both firms always produce in the same period a Bertrand-Nash situation would arise while a Stackelberg situation would result if they always produce in separate periods. As was the case for the model with quantity as strategic variable, Bertrand-Nash situations can never be Perfect Bayesian Equilibria while Stackelberg situations can always be supported by PBEs.

There are some fundamental strategic differences between quantity and price competition with complete information. First, under quantity competition the leader's position brings greater profit to a firm than the Cournot-Nash equilibrium, which again is better than the follower's position (assuming that the rival has the same cost in each of the three situations). Under price competition the follower's position is preferred to the leader's, which again is preferred to the Bertrand-Nash equilibrium. Second, under quantity competition the higher the rival's cost the higher one's quantity will be. Since reaction functions are downwards sloping high cost firms will in incomplete information situations want to pretend to be low cost firms, while low cost firms will want to separate themselves from high cost firms. Under price competition, the higher the rival's cost the higher one's price, and since reaction functions are upwards sloping it is now the low cost firms who will hide, and the high cost firms who will want to separate themselves. While one initially may think these differences could change the results of the preceding analysis since the strategic environment is changed,

[9] Of course the products would have to be less than perfect substitutes for the analysis to have any interest.

this turns out not to be the case. In the following paragraphs some heuristic arguments for this result are presented while rigorous proofs are left to the reader.

A Bertrand-Nash situation in the first period cannot be a PBE by the same argument that applied in Section 4. It is always preferable to be on the exact, rather than the mean reaction curve. Hence all types of both firms would have an incentive to deviate from this arrangement. A Bertrand-Nash situation in the second period would be broken by high cost types since they can reveal that they indeed are of high cost by deviating to period one, and at the same time position themselves as leader, which is at least as good as playing Bertrand-Nash.

The Stackelberg situations can be supported as PBEs in almost the same way as in Section 4. However, now the worst a follower can think about a deviating leader's cost is that it is the lowest possible. Hence, if the follower holds a belief with a sufficiently low mean, all leader types will be deterred from deviating. And, since obviously no follower type would want to deviate, the Stackelberg situation can be sustained as a PBE. However, as in Section 4, there may be additional PBEs of a more complicated nature which will not be discussed here.

7. Conclusion.

The paper has shown that if one, in an imperfect information model, tries to model both when two firms will make their strategic decision (quantity or price) and what the value of that decision variable will be, then the familiar one-period (Bayesian) Cournot-Nash and Bertrand-Nash equilibria cannot be rationalized as Perfect Bayesian Equilibria of this more general two-period model. However, the

Stackelberg model, which in this context could be interpreted as a situation where one firm always produces in period one and the other in period two, can always be supported as a Perfect Bayesian Equilibrium of the two-period model.

These results underline the problems of concentrating on simultaneous-choice models in a world where most choices probably are made in what would be most accurately described by continuous time models. Modelling this explicitly may prove very difficult.[10] This paper has at least shown that using simple two-period models can give drastically different results from the one-period simultaneous-choice model. Thus careful examination of the particular market to be analysed is necessary before choosing the appropriate model. One important consideration is then to what extent a "follower" can observe a "leader's" choice. If there is no way a leader can make its choice observable (or maybe just make it clear that a choice has been made), the Stackelberg model of course loses its attraction.

The analysis has given some indication of half an answer to a question posed in the introduction. The question is: why should a firm accept being follower in the quantity game when the follower's position yields less profit than even the Cournot-Nash situation? The half-answer is: because the Stackelberg situation is an equilibrium while the Cournot-Nash equilibrium is not. This is only a half-answer, because it does not determine how firms decide who should be the leader and who should be the follower. Or, in other words, how do firms choose between the two sets of equilibria that were discussed above, that is, the two converse Stackelberg configurations? Although a general study of this problem for the

[10] Simon [1987] has, as an example of a more general theory, analysed a Stackelberg/Cournot model without uncertainty in continuous time. His efforts show that such analysis indeed does become very involved.

moment is left for future research, the analysis in Albaek [1990] points to some situations where the solution is fairly obvious: one of the two Stackelberg configurations may Pareto dominate the other, in which case it seems reasonable to predict that the firms will use this outcome as a focal point.

Appendix 1: Proof of Propositions.

Proof of Proposition 1.

Assume firm A always produces in the first period. From the definition of a PBE, clearly

$$q_b^{2f}(c_b,q_a) \;=\; \frac{d}{2} - \frac{q_a}{2} - \frac{c_b}{2}$$

Then firm A with unit cost c_a will find q_a^1 as

$$q_a^1(c_a) \;\in\; \operatorname*{argmax}_{q_a\geq 0} \left\{ \int_{I_b}[d-q_a-q_b^1(c_b)-c_a]q_a d\nu_b^1 + \right.$$

$$\left. \int_{I_b}[d-q_a-q_b^{2f}(c_b,q_a)-c_a]q_a d\nu_b^2 \right\}$$

The first order condition of this problem is

$$(d-2q_a-c_a)m_b^1 \;-\; \int_{I_b}q_b^1(c_b)d\nu_b^1 \;+\; (\frac{d}{2}-q_a-c_a+\frac{\bar{c}_b^2}{2})m_b^2 \;=\; 0$$

or

$$\text{(A1)} \qquad q_a^1 \;=\; \frac{d}{2} \;-\; \frac{c_a}{2m_b^1+m_b^2} \;-\; \frac{\int_{I_b}q_b^1(c_b)d\nu_b^1}{2m_b^1+m_b^2} \;+\; \frac{\frac{1}{2}\bar{c}_b^2 m_b^2}{2m_b^1+m_b^2}$$

To find q_b^1, solve

$$q_b^1 \;\in\; \operatorname*{argmax}_{q_b\geq 0} \int_{I_a}[d-q_a^1(c_a)-q_b-c_b]q_b d\mu_a$$

The first order condition gives

$$\text{(A2)} \qquad q_b^1 = \frac{d}{2} - \frac{\int_{I_a} q_a^1(c_a)d\mu_a}{2} - \frac{c_b}{2}$$

From (A1) and (A2), $q_a^1(c_a)$ and $q_b^1(c_b)$ are affine in c_a and c_b, say

$$\text{(A3)} \qquad q_a^1(c_a) = \alpha_a - \beta_a c_a$$

$$\text{(A4)} \qquad q_b^1(c_b) = \alpha_b - \beta_b c_b$$

It is immediate from (A1) and (A2) that

$$\beta_a = \frac{1}{2m_b^1+m_b^2}$$

and

$$\beta_b = \frac{1}{2}$$

Taking expectations over (A3) and (A4) yields

$$\int_{I_a} q_a^1(c_a)d\mu_a = \alpha_a - \frac{\bar{c}_a}{2m_b^1+m_b^2}$$

$$\int_{I_b} q_b^1(c_b)d\nu_b^1 = m_b^1(\alpha_b - \frac{\bar{c}_b^1}{2})$$

Using this and (A1) and (A2) to solve for α_a and α_b in (A3) and (A4) leads to

$$\alpha_a = \frac{d}{(2+m_b^1)} + \frac{\bar{c}_b+m_b^1(\bar{c}_b-\bar{c}_a)}{(2+m_b^1)(1+m_b^1)}$$

$$\alpha_b = \frac{(1+m_b^1)d}{2(2+m_b^1)} + \frac{\bar{c}_a-\frac{1}{2}\bar{c}_b}{2+m_b^1}$$

The expected profit to firm B with unit cost c_b if it produces in period 1 is then

$$\int_{I_a} \pi_b(q_a^1(c_a), q_b^1(c_b); c_b) d\mu_a = \int_{I_a} [d-(\alpha_a-\beta_a c_a)-(\alpha_b-\beta_b c_b)-c_b][\alpha_b-\beta_b c_b] d\mu_a$$

$$= \left(\frac{(1+m_b^1)d}{2(1+m_b^1)} + \frac{\bar{c}_a - \frac{1}{2}\bar{c}_b}{2+m_b^1} - \frac{c_b}{2} \right)^2$$

If the same firm produces in period 2 its expected profit is

$$\int_{I_a} \pi_b(q_a^1(c_a), q_b^{2f}(c_b); c_b) d\mu_a$$

$$= \int_{I_a} \left(d-(\alpha_a-\beta_a c_a) - \frac{d-(\alpha_a-\beta_a c_a)-c_b}{2} - c_b \right) \left(\frac{d-(\alpha_a-\beta_a c_a)-c_b}{2} \right) d\mu_a$$

$$= \int_{I_a} \left(\frac{(1+m_b^1)d}{2(2+m_b^1)} - \frac{\frac{1}{2}\bar{c}_b}{2+m_b^1} + \frac{m_b^1 \bar{c}_a}{2(2+m_b^1)(1+m_b^1)} + \frac{c_a}{2(1+m_b^1)} - \frac{c_b}{2} \right)^2 d\mu_a$$

$$= \left(\frac{(1+m_b^1)d}{2(2+m_b^1)} + \frac{\bar{c}_a - \frac{1}{2}\bar{c}_b}{2+m_b^1} - \frac{c_b}{2} \right)^2 + \frac{V_a}{2(1+m_b^1)}$$

where

$$V_a = \int_{I_a} \left(c_a - \bar{c}_a \right)^2 d\mu_a$$

Since $V_a > 0$, $\int_{I_a} \pi_b(q_a^1(c_a), q_b^1(c_b); c_b) d\mu_a > \int_{I_a} \pi_b(q_a^1(c_a), q_b^{2f}(c_b, q_a); c_b) d\mu_a$ for all $c_b \in I_b$. Hence, $\theta_b(c_b) = 0$ for all $c_b \in I_b$, or, in words, no firm B type will ever produce in period 1 with positive probability if firm A always produces in period 1. Since the choice of A as the period 1 producer was arbitrary, the proposition is proved.

□

Proof of Proposition 2.

Assume now that firm A always produces in the second period. Clearly,

$$q_a^{2f}(c_a,q_b) = \frac{d}{2} - \frac{q_b}{2} - \frac{c_a}{2}$$

Firm B with unit cost c_b will then find $q_b^1(c_b)$ as

$$q_b^1(c_b) \in \underset{q_b \geq 0}{\operatorname{argmax}} \int_{I_a} [d - q_a^{2f}(c_a,q_b) - q_b - c_b] q_b \, d\mu_a$$

The first order condition gives

$$q_b^1(c_b) = \frac{d}{2} + \frac{\bar{c}_a}{2} - c_b$$

which means that

$$q_a = \frac{d}{4} + \frac{c_b}{2} - \frac{\bar{c}_a}{4} - \frac{c_a}{2}$$

The expected profit to firm B with unit cost c_b from producing in period 1 is therefore

$$\int_{I_a} \pi_b(q_a^{2f}(c_a,q_b), q_b^1(c_b); c_b) \, d\mu_a$$

$$\text{(A5)} \qquad = \int_{I_a} \Big(d - \frac{d - \bar{c}_a + 2c_b - 2c_a}{4} - \frac{d + \bar{c}_a - c_b}{2} - c_b\Big)\Big(\frac{d + \bar{c}_a - 2c_b}{2}\Big) d\mu_a$$

$$= \Big(\frac{d + \bar{c}_a - 2c_b}{4}\Big)\Big(\frac{d + \bar{c}_a - 2c_b}{2}\Big)$$

The firm B types who produce in period 2 are identified by the measure ν_b^2. Then the first order conditions for expected profit maximizing output choices in period 2 for firm A and firm B with unit costs c_a and c_b are

$$\text{(A6)} \qquad q_a = \frac{d}{2} - \frac{1}{2}\int_{I_b} \frac{q_b^{2n}(c_b)}{m_b^2} d\nu_b^2 - \frac{c_a}{2}$$

$$\text{(A7)} \qquad q_b = \frac{d}{2} - \frac{1}{2}\int_{I_a} q_a^{2n}(c_a) d\mu_a - \frac{c_b}{2}$$

Clearly, the strategies are affine in own costs, say

$$\text{(A8)} \qquad q_a^{2n}(c_a) = \gamma_a - \delta_a c_a$$

$$\text{(A9)} \qquad q_b^{2n}(c_b) = \gamma_b - \delta_b c_b$$

Comparing (A6), (A7), (A8) and (A9) immediately gives

$$\delta_a = \delta_b = \frac{1}{2}$$

Taking expectations over (A8) and (A9) yields

$$\int_{I_a} q_a^{2n}(c_a) d\mu_a = \gamma_a - \frac{\bar{c}_a}{2}$$

$$\int_{I_b} q_b^{2n}(c_b) d\nu_b^2 = m_b^2(\gamma_b - \frac{\bar{c}_b^2}{2})$$

Using this and (A6) and (A7) to solve for γ_a and γ_b in (A8) and (A9) leads to

$$\gamma_a = \frac{d}{3} + \frac{\bar{c}_b^2}{3} - \frac{\bar{c}_a}{6}$$

$$\gamma_b \;=\; \frac{d}{3} + \frac{\bar{c}_a}{3} - \frac{\bar{c}_b^2}{6}$$

Then the expected profit to firm B with cost c_b from producing in period 2 is

$$\int_{I_a} \pi_b(q_a^{2n}(c_a), q_b^{2n}(c_b); c_b)\, d\mu_a$$

$$= \int_{I_a} [d-(\gamma_a-\delta_a c_a)-(\gamma_b-\delta_b c_b)-c_b][\gamma_b-\delta_b c_b]\, d\mu_a$$

$$\text{(A10)} \qquad = \left(\frac{d}{3} + \frac{\bar{c}_a}{3} - \frac{\bar{c}_b^2}{6} - \frac{c_b}{2}\right)^2$$

In order to compare (A5) and (A10) it is useful to introduce two easily checked extra profit expressions. If firm B with cost c_b is a Stackelberg leader against a follower with cost $\bar{c}_a$ for certain, the profit to firm B is

$$\pi_b'(\bar{c}_a, c_b) \;=\; \left(\frac{d+\bar{c}_a-2c_b}{4}\right)\left(\frac{d+\bar{c}_a-2c_b}{2}\right)$$

If the same two firms played Cournot-Nash, the profit to firm B would be

$$\pi_b''(\bar{c}_a, c_b) \;=\; \left(\frac{a+\bar{c}_a-2c_b}{3}\right)^2$$

Since a Stackelberg leader can always choose the Cournot-Nash outcome,

$$\int_{I_a} \pi_b(q_a^{2f}(c_a, q_b), q_b^1(c_b); c_b)\, d\mu_a \;=\; \pi_b'(\bar{c}_a, c_b) \;\geq\; \pi_b''(\bar{c}_a, c_b)$$

Now consider firm B with cost $c_b < \bar{c}_b^2$. Then $\frac{2c_b}{3} < \frac{\bar{c}_b^2}{6} + \frac{c_b}{2}$, and therefore

$$\pi_b''(\bar{c}_a, c_b) \;>\; \int_{I_a} \pi_b(q_a^{2n}(c_a), q_b^{2n}(c_b); c_b)\, d\mu_a$$

Hence, for all $c_b < \bar{c}_b^2$,

$$\int_{I_a} \pi_b(q_a^{2f}(c_a,q_b),q_b^1(c_b);c_b)d\mu_a \;>\; \int_{I_a} \pi_b(q_a^{2n}(c_a),q_b^{2n}(c_b);c_b)d\mu_a$$

and all these types will therefore deviate to period 1. Clearly, for any suggested equilibrium, where ν_b^2 is not concentrated at a single c_b, all types with $c_b < m_b^2$ would rather produce in period 1. If ν_b^2 is concentrated at a single point, say c_b', then firm A will know for certain that it is playing against c_b' in the second period. But then firm B with cost c_b' will obviously prefer to be the leader instead of playing Nash in period 2. This breaks the suggested equilibrium. Since the choice of A as the period 2 producer was arbitrary, the proposition is proved.

□

Proof of Proposition 3.

Suppose firm A is the leader and B the follower. The proposed equilibrium strategies of the two firms are easily found from the preceding analysis as

$$q_a^1(c_a) \;=\; \frac{d}{2} + \frac{\bar{c}_b}{2} - c_a$$

$$q_b^{2f}(c_b,\, q_a) \;=\; \frac{d}{2} - \frac{q_a}{2} - \frac{c_b}{2}$$

No B type will ever deviate, since firm A's output decision is not changed by B's deviation, and being on the exact reaction function is always better than being on the mean. (See proof of Proposition 1 for a rigorous argument.)

From (A5) the expected equilibrium profit to firm A will be

$$\int_{I_b} \pi_a(q_a^1(c_a), q_b^{2f}(c_b,q_a); c_a)d\mu_b = \left(\frac{d+\bar{c}_b-2c_a}{4}\right)\left(\frac{d+\bar{c}_b-2c_a}{2}\right)$$

A deviation to period 2 by an A type is a zero probability event. Hence, Bayes' rule poses no restriction on the beliefs of firm B about which A type (in I_a) has deviated. Let this belief be summarized by the probability measure ψ_a. The average c_a according to ψ_a is

$$\bar{c}_a^{\psi} = \int_{I_a} c_a d\psi_a$$

From the proof of Proposition 2 it is easily seen that the period 2 strategies if firm A deviates are

$$q_a^{2n}(c_a) = \frac{d}{3} + \frac{\bar{c}_b}{3} - \frac{\bar{c}_a^{\psi}}{6} - \frac{c_a}{2}$$

$$q_b^{2n}(c_b) = \frac{d}{3} + \frac{\bar{c}_a^{\psi}}{3} - \frac{\bar{c}_b}{6} - \frac{c_b}{2}$$

and from (A10) the expected profit to firm A with cost c_a from deviating to period 2 is

$$\int_{I_b} \pi_a(q_a^{2n}(c_a), q_b^{2n}(c_b); c_a)d\mu_b = \left(\frac{d}{3} + \frac{\bar{c}_b}{3} - \frac{\bar{c}_a^{\psi}}{6} - \frac{c_a}{2}\right)^2$$

Denote now by $T_a(c_a, \bar{c}_a^{\psi})$ the temptation to deviate from period 1 period 2 for firm A with cost c_a, when firm B's belief specifies an average marginal cost of $\bar{c}_a^{\psi}$.

$$T_a(c_a,\bar{c}_a^{\psi}) = \int_{I_b} \pi_a(q_a^{2n}(c_a), q_b^{2n}(c_b); c_a)d\mu_b - \int_{I_b} \pi_a(q_a^1(c_a), q_b^{2f}(c_b,q_a); c_a)d\mu_a$$

$$= \left(\frac{d}{3} + \frac{\bar{c}_b}{3} - \frac{\bar{c}_a^{\psi}}{6} - \frac{c_a}{2}\right)^2 - \left(\frac{d+\bar{c}_b-2c_a}{4}\right)\left(\frac{d+\bar{c}_b-2c_a}{2}\right)$$

I need to show that there exists a belief ψ_a such that $T_a(c_a,\bar{c}_a^{\psi}) \leq 0$ for all $c_a \in I_a$. Now, concentrate the belief on h_a (such that $\bar{c}_a^{\psi} = h_a$). Differentiating T_a with respect to c_a gives

$$\frac{dT_a(c_a,h_a)}{dc_a} = \frac{1}{6}(d + \bar{c}_b + h_a - 3c_a) > 0 \qquad \text{for all } c_a \in I_a$$

where the inequality follows from the assumption that $d + l_b - 2h_a > 0$. Now, $T_a(h_a,h_a)$ is the difference between the profit to a firm with cost h_a playing Cournot-Nash against a firm with $\bar{c}_b$ and the profit to the same A firm from being Stackelberg leader against firm $\bar{c}_b$. Clearly, $T_a(h_a,h_a)$ must be non-positive, that is $T_a(h_a,h_a) \leq 0$. Since $dT_a(c_a,h_a)/dc_a < 0$, $T_a(c_a,h_a) < 0$ for all $c_a < h_a$. Hence, there will always exist a belief ψ_a such that no A type will deviate to period two, and the PBE holds. Since A was chosen arbitrarily as leader, there is a symmetric equilibrium with A as follower.

□

Appendix 2: Profit Expressions in Section 5.

The follower strategy is found directly from the first order condition of $\max_{q_i}[120-q_i-q_j-c_i]q_i$. Hence, $q_i^{2f}(c_i,q_j) = d - \frac{1}{2}q_j - \frac{1}{2}c_i$. A low cost firm will with probability 0.5 produce simultaneously in period 1 with another low cost firm, and with probability 0.5 have a high cost follower reacting to its output. Hence, a low cost firm will solve

$$\max_{q_i} \tfrac{1}{2}\, [d-q_i-q_j^1(l)-l]\, q_i + \tfrac{1}{2}\, [d-q_i-q_j^{2f}(h,q_i)-l]\, q_i$$

Inserting the follower's reaction to q_i, the first order condition gives the following reaction function for the low cost firm:

$$q_i^1(l) = \tfrac{1}{2}d - \tfrac{1}{3}q_j^1(l) + \tfrac{1}{6}h - \tfrac{2}{3}l$$

Firm j has a similar reaction function, and the equilibrium strategies can be solved to be

$$q_i^1(l) = \tfrac{3}{8}d + \tfrac{1}{8}h - \tfrac{1}{2}l$$

The output of a high cost firm acting as a follower to this output is then

$$q_i^{2f}(h) = \tfrac{1}{2}d - \tfrac{1}{2}q_j^1(l) - \tfrac{1}{2}h$$

$$= \tfrac{5}{16}d - \tfrac{9}{16}h + \tfrac{1}{4}l$$

I also need to find the equilibrium strategies if two high cost firms are facing each other in period 2, that is, to find $\max_{q_i} [d-q_i-q_j^{2n}(h)-h]q_i$. Solving this yields

$$q_i^{2n}(h) = \tfrac{1}{3}d - \tfrac{1}{3}h$$

The expected profit to a low cost firm in the proposed equilibrium is then

$$\begin{aligned}
P(l) &= \tfrac{1}{2}\,[d-\tfrac{3}{8}d-\tfrac{1}{8}h+\tfrac{1}{2}l-\tfrac{3}{8}d-\tfrac{1}{8}+\tfrac{1}{2}l-l]\,[\tfrac{3}{8}d+\tfrac{1}{8}h-\tfrac{1}{2}l] \\
&\quad + \tfrac{1}{2}\,[d-\tfrac{3}{8}d-\tfrac{1}{8}h+\tfrac{1}{2}l-\tfrac{5}{16}d+\tfrac{9}{16}h-\tfrac{1}{4}l-l]\,[\tfrac{3}{8}d+\tfrac{1}{8}h-\tfrac{1}{2}l] \\
&= [\tfrac{9}{16}d+\tfrac{3}{32}h-\tfrac{3}{8}l]\,[\tfrac{3}{8}d+\tfrac{1}{8}h-\tfrac{1}{2}l] \\
&= \tfrac{3}{256}\,[3d+h-4l]^2
\end{aligned}$$

Similarly, the expected profit to a high cost firm is

$$\begin{aligned}
P(h) &= \tfrac{1}{2}\,[d-\tfrac{5}{16}d+\tfrac{9}{16}h-\tfrac{1}{4}l-\tfrac{3}{8}d-\tfrac{1}{8}h+\tfrac{1}{2}l-h]\,[\tfrac{5}{16}d-\tfrac{9}{16}h+\tfrac{1}{4}l] \\
&\quad + \tfrac{1}{2}\,[d-\tfrac{1}{3}d+\tfrac{1}{3}h-\tfrac{1}{3}d+\tfrac{1}{3}h-h]\,[\tfrac{1}{3}d-\tfrac{1}{3}h] \\
&= \tfrac{1}{2}\,[\tfrac{5}{16}d-\tfrac{9}{16}h+\tfrac{1}{4}l]^2 + \tfrac{1}{2}\,[\tfrac{1}{3}d-\tfrac{1}{3}h]^2 \\
&= \tfrac{1}{512}\,[5d-9h+4l]^2 + \tfrac{1}{18}\,[d-h]^2
\end{aligned}$$

Notice that in the proposed equilibrium Bayes' rule yields beliefs for all information sets. Hence, there is no need for specifying out-of-equilibrium beliefs. To check whether the strategies form an equilibrium I must calculate the profits from a (best) deviation for both high and low cost firms.

If a low cost firm deviates to period 2 and ends up playing as a follower to another low cost firm it will choose output $q_i^{2f}(l,q_j^1(l)) = \frac{1}{2}d - \frac{1}{2}q_j^1(l) - \frac{1}{2}l = \frac{5}{16}d - \frac{1}{16}h - \frac{1}{4}l$. With equal probability it will find itself playing against a high cost firm, which thinks that its opponent also is has high cost. Then the deviating low cost firm will solve $\max_{q_i} [d-q_i-q_j^{2n}(h)-l]\, q_i$. The solution is $q_i = \frac{1}{2}d - \frac{1}{2}q_j^{2n}(h) - \frac{1}{2}l = \frac{1}{3}d + \frac{1}{6}h - \frac{1}{2}l$.

The expected profit from this deviation, is then

$$P'(l) = \tfrac{1}{2}\,[d-\tfrac{5}{16}d+\tfrac{1}{16}+\tfrac{1}{4}l-\tfrac{3}{8}d-\tfrac{1}{8}h+\tfrac{1}{2}l-l]\,[\tfrac{5}{16}d-\tfrac{1}{16}h-\tfrac{1}{4}l]$$

$$+ \tfrac{1}{2}\,[d-\tfrac{1}{3}d-\tfrac{1}{6}h+\tfrac{1}{2}l-\tfrac{1}{3}d+\tfrac{1}{3}h-l]\,[\tfrac{1}{3}d+\tfrac{1}{6}h-\tfrac{1}{2}l]$$

$$= \tfrac{1}{2}\,[\tfrac{5}{16}d-\tfrac{1}{16}h-\tfrac{1}{4}l]^2 + \tfrac{1}{2}\,[\tfrac{1}{3}d+\tfrac{1}{6}h-\tfrac{1}{2}l]^2$$

$$= \tfrac{1}{512}\,[5d-h-4l]^2 + \tfrac{1}{72}\,[2d+h-3l]^2$$

Finally, a high cost firm deviating to produce in period 1 instead of period 2 will with probability 0.5 face a low cost opponent in period 1 and with probability 0.5 have a high cost firm following in period 2. Hence, it's problem for deciding the optimal deviation is

$$\max_{q_i}\ \tfrac{1}{2}\,[d-q_i-\tfrac{3}{8}d-\tfrac{1}{8}h+\tfrac{1}{2}l-h]\,q_i + \tfrac{1}{2}\,[d-q_i-q_j^{2f}(h,q_i)-h]\,q_i$$

Inserting the reaction function $q_j^{2f}(h,q_i)$ and solving for q_i gives $q_i = \frac{3}{8}d + \frac{1}{6}l - \frac{13}{24}h$.

The expected profit from the deviation is therefore

$$
\begin{aligned}
P'(h) = {} & \tfrac{1}{2}\,[d-\tfrac{3}{8}d-\tfrac{1}{6}l+\tfrac{13}{24}h-\tfrac{3}{8}d-\tfrac{1}{8}h+\tfrac{1}{2}l-h]\,[\tfrac{3}{8}d+\tfrac{1}{6}l-\tfrac{13}{24}h] \\
& + \tfrac{1}{2}\,[d-\tfrac{3}{8}d-\tfrac{1}{6}l+\tfrac{13}{24}h-(\tfrac{1}{2}d-\tfrac{3}{16}d-\tfrac{1}{12}l+\tfrac{13}{48}h-\tfrac{1}{2}h)-h]\,[\tfrac{3}{8}d+\tfrac{1}{6}l-\tfrac{13}{24}h] \\
& = [\tfrac{9}{32}d+\tfrac{1}{8}l-\tfrac{13}{32}h]\,[\tfrac{3}{8}d+\tfrac{1}{6}l-\tfrac{13}{24}h] \\
& = \tfrac{1}{768}\,[9d+4l-13h]^2
\end{aligned}
$$

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