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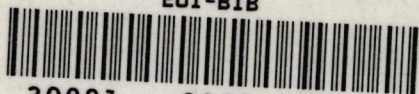
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ECONOMICS DEPARTMENT

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TESTING FOR NONLINEAR DYNAMICS IN HISTORICAL UNEMPLOYMENT SERIES

by

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Abstract

In this paper we have used historical unemployment series for the US and the UK to test the dynamics of a model that has attracted interest in explaining the different unemployment experiences between industrialized countries. The model is an extension of the one used by Alogoskoufis and Manning (1988a, b) and includes most features advanced in the literature to explain these differences.

In addition to the usual diagnostics that look at linear deviations from the null of an i.i.d. process we have used the test proposed by Brock, Dechert and Scheinkman (1987) that has power against nonlinear alternatives, including chaotic-deterministic ones. This paper provides an example where this diagnostic can be of value in assessing the adequacy of economic models.

Our findings support the idea that the US unemployment and the UK one follow different dynamic specifications with the US unemployment being adequately described by a simple AR(2) process with ARCH errors, as the model suggests. On the other hand the UK unemployment does not seem to follow the above specification. Attempts to correct for the presence of alternative linear specifications did not produce any results. We also found that the nonlinearities present in the unemployment equation residuals do not seem to be of the chaotic variety. It seems to us that more theoretical work is needed to identify the sources of the nonlinear behavior in the U K series.

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The different unemployment experiences between industrialized countries, notably the USA and European countries, have attracted a lot of recent interest among researchers and policymakers alike. One of the main differences relates to persistence. For example, although the US unemployment rate displays relatively low persistence, many of its European counterparts persist a lot more.

The recent literature (surveyed by Nickell 1990) has seen a number of attempts to explain these differences in the persistence of unemployment between Europe and the United States. However, the overwhelming majority of the literature is based on models that result in linear equilibrium processes for the unemployment rate. This is because many of the papers in the literature are specified in log-linear form, which means that they end up with linear ARMA processes for the reduced form of the unemployment rate. To the extent that all the roots of these processes are inside the unit circle, unemployment converges to a unique equilibrium or natural rate, otherwise it does not and it displays "hysteresis".

Sluggishness in labour market adjustment is the main focus of empirical studies by Blanchard and Summers (1986) and Alogoskoufis and Manning (1988). Blanchard and Summers concentrate on a linear model with insiders and outsiders, in which the only source of unemployment sluggishness is the extent to which the unemployed get disenfranchised from the active labour force. On the other hand, Alogoskoufis and Manning also allow for the effects of sluggish adjustment in real wages and labour demand. The model of Blanchard and Summers results in an AR(1) process for the reduced form of the unemployment rate, while the more general model of Alogoskoufis and Manning, which incorporates many of the features of some of the other models in the literature, implies an AR(2) process for the

unemployment rate.¹

In view of the recent interest by economists in non-linear dynamics, and in particular the possibility of chaotic dynamics (see for example Grandmont 1985), it would be interesting to know whether these linear models of unemployment persistence are satisfactory representations of reality, or whether there is a need for taking non-linearities explicitly into account.

In this paper we investigate a log-linear model that incorporates a number of features of the recent unemployment models surveyed by Nickell (1990). However, in contrast to most of the models that have been used in the empirical literature, our model allows for "forward-looking" behaviour on the part of wage setters and firms. In all other respects the model is very close to the spirit of the 1980s literature on unemployment, and results in a AR(2) equilibrium unemployment process.

We use historical unemployment series of the USA and the United Kingdom, and concentrate on testing whether the implied linear dynamics represent a satisfactory model of unemployment persistence.

We apply some recently developed techniques to test for the presence of nonlinear structure in the series under investigation. The methodology that we employ was originally developed in order to test for the presence of "chaos" in economic time series. Subsequently, these techniques have been used more as diagnostic procedures to test the specification of linearity with i.i.d. errors against a variety of nonlinear alternatives, including "chaotic" ones.

The early literature in the area of applied nonlinear chaotic dynamics, was attempting to provide a test of the theoretical models that were predicting "chaos" at the macro level, especially in the context of

¹Lindbeck and Snower (1989) discuss models of insiders and outsiders in which the effects of positive and negative shocks are asymmetric.

the business cycle (Grandmont 1985). Papers by Brock and Sayers (1988) and Frank and Stengos (1988 b) fall in that category. However, the paper by Frank, Gencay and Stengos (1988 a) uses the techniques more in a diagnostic sense to test the linear dynamic specification against nonlinear alternatives in general.

In this paper we use a test developed by Brock, Dechert and Sheinkman (1987), hereafter referred to as the BDS test, as a generalization of the methods proposed by natural scientists in the field of nonlinear dynamics. We try to identify the possible sources of nonlinearity that are present in our data. Interestingly, we find that the source of the nonlinear behavior present in the USA unemployment rate can be captured in a satisfactory manner by a simple ARCH process (see Engle 1982). However, this is not the case for the UK series, where ARCH effects are unable to clean the error structure. We find that while according to the usual diagnostics the error structure looks clean, according to the BDS statistic it does not. In short, the methods prove useful in identifying a source of difference in the dynamics of the USA and UK unemployment rate.

The rest of the paper is as follows: In section I we present the theoretical model. The model consists of forward-looking wage setters and competitive firms, who face quadratic costs of adjusting the real wage and employment respectively. We derive the equilibrium unemployment rate which is shown to follow an AR(2) process. In section II we present an overview of the diagnostic methods that we employ in the empirical section of the paper. In section III we present the empirical results, and our concluding remarks are contained in section IV.

I. A Model of Wage Setting and the Persistence of Unemployment

The economy consists of a large number of competitive firms each facing a firm specific monopoly union that sets wages.

I.1 *Wage Setting*

The objective of wage setters is to achieve a target real wage. Its logarithm is denoted by w^* . However, wage setters also face costs of adjusting real wages from period to period. We thus model them as setting wages in order to minimize the following intertemporal quadratic cost function,

$$\min_{\{w_t\}} E_t \sum_{i=0}^{\infty} \delta^i \left[\frac{1}{2}(w_{t+i} - w_{t+i}^*)^2 + \frac{\theta}{2}(w_{t+i} - w_{t+i-1})^2 \right] \quad (1)$$

where E is the expectations operator, w is the logarithm of the real wage, δ the discount factor and θ a parameter measuring the intensity of the adjustment costs.

From the FOC for a minimum we get,

$$w_t - w_t^* + \theta(w_t - w_{t-1}) - \delta\theta(E_t w_{t+1} - w_t) = 0 \quad (2)$$

The above can be rewritten as,

$$E_t w_{t+1} - [(1 + \theta(1 + \delta))/\theta\delta]w_t + (1/\delta)w_{t-1} = -(1/\delta\theta)w_t^* \quad (3)$$

We can use the forward shift operator F and the lag operator L to

rewrite (3) as,

$$(F^2 - (\rho_1 + \rho_2)F + \rho_1\rho_2)Lw_t = -(1/\delta\theta)w^* \quad (4)$$

where $\rho_1 + \rho_2 = (1 + \theta(1 + \delta))/\delta\theta > 1$, $\rho_1\rho_2 = 1/\delta$ and ρ_1 and ρ_2 are the two roots of the difference equation (3). It is straightforward to show that the roots are real, distinct and lie on either side of unity. Thus (3) is saddlepoint stable. To arrive at the saddlepoint solution, we can factorize (4) as,

$$(F - \rho_1)(F - \rho_2)Lw_t = -(1/\delta\theta)w^* \quad (5)$$

Assuming that ρ_1 is the smaller root, we can rewrite (5) as,

$$w_t = \rho_1 w_{t-1} - (1/\delta\theta)(1/(F-\rho_2))w^* = \rho_1 w_{t-1} + \{\rho_1 / [\theta(1-\rho_1\delta)]\}w^* \quad (6)$$

In (6) we have used the extra assumption that the target real wage is constant, and have made use of the fact that $\rho_2 = 1/\delta\rho_1$.

1.2 Employment Setting

Firms are competitive and determine employment by minimizing a loss function that penalizes both deviations from the usual marginal conditions and adjustment of employment from the previous period. Thus the objective is,

$$\min_{\{l_t\}} \sum_{i=0}^{\infty} \delta^i [1/2(l_{t+i} - l_{t+i}^*)^2 + \phi/2(l_{t+i} - l_{t+i-1})^2] \quad (7)$$

$$l_{t+i}^* = \alpha - \beta w_{t+i} \quad (8)$$

where l is the logarithm of employment, and l^* is the logarithm of target employment.

(8) can be derived by assuming a Cobb-Douglas production function, and taking the logarithm of both sides of the first-order condition that the marginal product of labour must equal the real wage. In (7), ϕ measures the intensity of the costs of adjustment of employment.

From the FOC for a minimum of (7) subject to (8),

$$l_{t+1} - \{[1 + \phi(1 + \delta)]/\phi\delta\}l_t + (1/\delta)l_{t-1} = - (1/\delta\phi)(\alpha - \beta w_t) \quad (9)$$

We can rewrite (9) as,

$$(F^2 - (\lambda_1 + \lambda_2)F + \lambda_1\lambda_2)Ll_t = -(1/\delta\phi)(\alpha - \beta w_t) \quad (10)$$

where λ_1 and λ_2 are the two roots, which are again real, distinct and lie on either side of the unit circle. We can factorize (10) as,

$$(F - \lambda_1)(F - \lambda_2)Ll_t = -(1/\delta\phi)(\alpha - \beta w_t) \quad (11)$$

Assuming that λ_1 is the smaller root and making use of the fact that $\lambda_1\lambda_2 = 1/\delta$, we can solve (11) as,

$$l_t = \lambda_1 l_{t-1} + (\lambda_1/\phi)E_t \sum_{i=0}^{\infty} (\lambda_1 \delta)^i (\alpha - \beta w_{t+i}) \quad (12)$$

In order to get a closed form solution we can use (6) as a forecasting

equation for real wages. From (6),

$$E_t w_{t+s} = \rho_1^s w_t + (\sum_{j=1}^s \rho_1^j w^*) / [\theta(1-\rho_1 \delta)] , \quad s \geq 1 \quad (13)$$

Substituting (13) in (12) yields

$$l_t = \lambda_1 l_{t-1} + (\lambda_1 / \phi) \sum_{i=0}^{\infty} (\lambda_1 \rho_1 \delta)^i (\alpha - \beta w_t) - \beta (\lambda_1 / \phi) \sum_{i=1}^{\infty} (\lambda_1 \delta)^i \sum_{j=1}^i \rho_1^j w^* / [\theta(1-\rho_1 \delta)] \quad (14)$$

Equation (14) above implies that,

$$l_t = \lambda_1 l_{t-1} - (\beta \lambda_1) / (1 - \lambda_1 \rho_1 \delta) w_t + l_0 \quad (15)$$

where,

$$l_0 = (\alpha \lambda_1 / \phi) / (1 - \lambda_1 \rho_1 \delta) - \{(\beta \lambda_1^2 \delta \rho_1) / [\theta(1-\rho_1 \delta)(1-\lambda_1 \rho_1 \delta)(1-\rho_1)]\} w^* \quad (16)$$

1.3 Equilibrium

To derive the equilibrium dynamics of employment and unemployment we must use (6) to substitute for w in the labour demand equation (15). From (6) we have,

$$w_t = \{(\rho_1 w^*) / \theta(1-\rho_1 \delta)\} \{1 / (1-\rho_1 L)\} \quad (17)$$

where L is the lag operator. Substituting (17) in (15) and multiplying both sides by $1-\rho_1 L$ we get,

$$l_t(1-\rho_1 L) = \lambda_1(1-\rho_1 L)l_{t-1} - \{(\beta \lambda_1 \rho_1) / [\phi \theta(1-\rho_1 \delta)(1-\lambda_1 \rho_1 \delta)]\} w^* + (1-\rho_1)l_0 \quad (18)$$

The above can be written as,

$$l_t = (\rho_1 + \lambda_1)l_{t-1} - \lambda_1 \rho_1 l_{t-2} + \tilde{l} \quad (19)$$

$$\text{where } \tilde{l} = (1 - \rho_1)l_0 - \{(\beta \lambda_1 \rho_1) / [\phi \theta (1 - \rho_1 \delta)(1 - \lambda_1 \rho_1 \delta)]\} w^* \quad (20)$$

Subtracting both sides of (19) from the log of the labour force n , and using the approximation $u \cong n - l$, where u is the unemployment rate, we obtain,

$$u_t = (\rho_1 + \lambda_1)u_{t-1} - \lambda_1 \rho_1 u_{t-2} + \tilde{u} \quad (21)$$

where $\tilde{u} = \{1 - (\rho_1 + \lambda_1) + \lambda_1 \rho_1\} n - \tilde{l}$.

Equation (21) will be the basic unemployment equation to be tested in the empirical analysis that follows.²

It would be interesting to investigate the conditions under which (21) would have a unit root. In that case the unemployment rate would display hysteresis.

The unemployment process (21) would have one unit root if either λ_1 or ρ_1 were equal to unity, i.e if either the wage adjustment equation (6) or the dynamic labour demand curve had a unit root. The root of the wage equation will tend to one as θ , the intensity of wage adjustment costs, tends to infinity. On the other hand, the root of the labour demand equation will tend to one as ϕ , the intensity of employment adjustment

²A similar linear second-order process for unemployment is derived by Alogoskoufis and Manning (1988), who use a model that extends the Blanchard and Summers (1986) model with insiders-and-outsiders, and allows for persistence in real wages and labour demand. Obviously the interpretation of the persistence parameters is different in that model.

costs, tends to infinity. If the costs of adjusting wages and employment are finite, then the unemployment rate will not have a unit root, as both λ_1 and ρ_1 will be less than one.

One could analyze the difference equation (21) in more detail. For example, if both the wage adjustment equation and the dynamic labour demand curve have unit roots, then the unemployment rate will have two unit roots. In general, the persistence of unemployment can be shown to depend positively on the degree of sluggishness of real wages and labour demand. This is a feature not only of this particular model, but also of the other log-linear models explored in the literature.

However, what is important for our purposes here is not whether the specific model that has been put forward is superior or not vis-a-vis the other models that result in a linear difference equation for unemployment, but whether this class of linear models is satisfactory as a representation of the persistence of unemployment. Thus, we will subject equation (21) to the usual diagnostics that are associated with linear alternatives and then proceed also to test against nonlinear and chaotic ones. Since the methods for testing against nonlinear alternatives that we use are relatively new, we will briefly review them in the next section.

II. Testing for the presence of nonlinear effects

The usual linear parametric techniques are not well equipped to test for the presence of nonlinear dynamics. The methods that we will present below were initially conceived to test for the presence of chaotic dynamics in time series data. For a more complete discussion, one can refer to Brock (1986) and Frank and Stengos (1988 a).

The basic concept is the correlation dimension due to Grassberger and Procaccia (1983). The notion of dimension for simple objects corresponds to the usual notion of dimension, the cardinality of the set of basis vectors. For highly irregular objects the correlation dimension may not be integer-valued. Dimensionality measures how orderly or disorderly an object is. A fixed point has no disorder and therefore has dimension zero. A line is one dimensional. A white noise is completely disordered (stochastic) and so it has infinite dimension. If the dimension is not integer-valued then the system is termed "fractal", see Mandelbrot (1977).

Start by assuming that the system is on an attractor. An attractor is a compact set S with a neighborhood such that all initial conditions in the neighborhood have S itself as their forward-limit set. Furthermore, assume that the system starts on an orbit which is dense on the the attractor. We also assume ergodicity to allow for intertemporal averages to be representative of the system. Attractors can be distinguished according to whether they are nice manifolds or they are "strange". For ordinary attractors a small change to the initial conditions remains small as time tends toward infinity. A strange attractor has "sensitive dependence on initial conditions" so that small deviations get magnified as time proceeds. Strangeness is a dynamic concept, whereas the notion of "fractal"

is geometric.

Formally one may define strangeness as follows. Let Ω be a space with a metric d and let $f:\Omega \rightarrow \Omega$ be a continuous mapping defined on Ω . A discrete dynamical system (Ω, f) is said to be strange (or chaotic) if there exists a $\delta > 0$, such that for all $\omega \in \Omega$ and all $\epsilon > 0$, there is a $\omega' \in \Omega$ and a k such that $d(\omega, \omega') < \epsilon$ but $d(f^k \omega, f^k \omega') \geq \delta$. Here $f^k \omega$ denotes the k -fold iteration of point ω by the map f . This definition follows Eckman and Ruelle (1985), but it is not the only approach, see Devaney (1986) and Guckenheimer and Holmes (1986) for alternative definitions. In this latter definition a chaotic process is defined as one with a positive Lyapunov exponent, where nearby points get stretched apart over time in such a way that nearby trajectories diverge exponentially.

Consider a time series of observations x_t , $t=1,2,3,\dots,T$ which are assumed to have been generated by an orbit that is dense on an attractor. Use this series of scalars to create an "embedding". This means constructing a series of "M-histories" as $x_t^M = (x_t, x_{t+1}, \dots, x_{t+M-1})$. This converts the series of scalars into a slightly shorter series of vectors with overlapping entries. One uses this stack of vectors to carry out the analysis. Suppose that the true but unknown system which generated the observations is n -dimensional. Then provided that $M > 2n+1$ generically the M -histories recreate the dynamics of the underlying system. This result is due to Takens (1981) and permits one to use the M -histories to analyze the system's dynamics.

Next we measure the spatial correlation amongst the points (M -histories) on the attractor by calculating the correlation integral $C^M(\epsilon)$. For a given embedding dimension M , the correlation integral is given as

$$C^M = \lim_{T \rightarrow \infty} (1/T^2) \{ \text{The number of } (i,j) \text{ for which } \|x_i^M - x_j^M\| < \epsilon \} \quad (22)$$

Here $\| \cdot \|$ denotes the distance induced by the selected norm. for small values of ϵ one has that $C^M(\epsilon) \sim \epsilon^D$ and D is the dimension of the system, see Grassberger and Procaccia(1983). Then to calculate the correlation dimension D^M

$$D^M = \lim_{\epsilon \rightarrow 0} (\ln C^M(\epsilon) / \ln \epsilon) \quad (23)$$

If the values of D^M stabilizes at some value D as M increases then D is the correlation dimension estimate. If as M increases D^M continues to rise then the system is regarded as high dimensional or stochastic. For such a system as one increases the available degrees of freedom by increasing M , the system uses the extra freedom. If instead a stable low value for D^M is obtained, then there is evidence that the system is essentially deterministic, even if fairly complicated. To actually calculate D one plots $\ln C^M(\epsilon)$ against $\ln \epsilon$ for various values of ϵ . One then calculates an intermediate range over which a straight segment is found for this plot (if such a linear segment exists).

The main drawback with the D estimates obtained by the above method is that they are point estimates. To circumvent this problem Brock, Dechert and Sheinkman (1987) developed a test statistic, the BDS test, that is based on the correlation integral. Using the statistical theory of U-statistics they derived the asymptotic distribution of the BDS statistic under the null hypothesis of an i.i.d. data generating process.

If x_t is i.i.d. then for fixed M and ϵ we have that

$$C^M(\epsilon, T) \rightarrow [C^1(\epsilon, T)]^M \text{ a.s. as } T \rightarrow \infty$$

Then,

$$\sqrt{T}(C^M(\epsilon, T) - [C^1(\epsilon, T)]^M) \longrightarrow N(0, V^M(\epsilon))$$

Hence the normalized version of the above yields the BDS statistic

$$W^M(\epsilon, T) = \sqrt{T}(C^M(\epsilon, T) - [C^1(\epsilon, T)]^M) / \sqrt{V^M(\epsilon)} \quad (24)$$

is distributed as a standard normal variate. The intuition behind this statistic is as follows. $C^M(\epsilon, T)$ is an estimate of the probability that the distance between any two M -histories x_t^M and x_s^M is less than ϵ . If the x_t 's are independent then for $t \neq s$ the probability of this joint event equals the product of the individual probabilities. Furthermore, if the x_t 's are also identically distributed then all the M probabilities under the product sign are the same.

Hsieh and Lebaron (1988) have conducted a series of monte carlo simulations to examine the small sample properties of the BDS statistic. It is found to have very good power against a number of nonlinear alternatives including ARCH, nonlinear MA, threshold AR and deterministic-chaotic tent maps.

Given the advantage of the BDS statistic we will employ it as the main diagnostic testing the null i.i.d. hypothesis against nonlinear alternatives. It is however important to note from the start that a significant BDS statistic does not necessarily imply deterministic chaos, since as we mentioned above the test has power against a wide range of stochastic alternatives as well.

The strategy that we will employ then is to estimate a lower bound to the largest Lyapunov exponent as a way of checking that the nonlinearity that the BDS might pick up is of the chaotic variety. In other words if the BDS statistic suggest the presence of nonlinear structure in the residuals of equation (21) of the previous section, then we will calculate the lower bound to the largest Lyapunov exponent. This is the strategy adopted by Frank, Gencay and Stengos (1988) to test for nonlinear behavior of the GNP

series of some industrialized countries.

The existence of a positive Lyapunov exponent is often used as a definition of chaos. It quantifies the notion of local instability, since it measures whether adjacent trajectories converge, or diverge. If they converge then the system will be stable reacting to small perturbations. If they separate then the system will be chaotic. The Lyapunov exponent measures the rate at which disorder is generated in the system, if any at all. The algorithm we employ is due to Kurths and Herzel (1987).

Start by using the M-histories to reconstruct the system. Among the M-histories select the ones that are within a certain distance and define

$$r_0(M,i,j) = ||x_i^M - x_j^M|| < \epsilon$$

Now ϵ is a small positive number and $|| \cdot ||$ is the Euclidean distance function. Once we select the nearby points in the M-space, we follow them further n -steps forward in time and calculate

$$r_n(M,i,j) = ||x_{i+n}^M - x_{j+n}^M||$$

Now take the ratio

$$d_n(M,i,j) = r_n(M,i,j)/r_0(M,i,j)$$

If the nearby points have separated then $d_n(M,i,j)$ will be positive.

Finally one aggregates over the $d_n(M,i,j)$ to get

$$L(M,n) = \sum_{i \neq j} \ln d_n(M,i,j) / N(N-1).$$

Certain features of $L(M,n)$ are discussed in Frank, Gencay and Stengos (1988). In general if the economy is stable we expect negative values for $L(M,n)$ and if the economy is unstable-chaotic we anticipate positive values for $L(M,n)$.

III. Estimation and Testing

In this section we will present the empirical evidence for equation (21) of the model that we analyze. The unemployment series that we investigate are annual series for the USA and the UK. In the case of the USA we utilize three alternative series which run between 1892 and 1987. The UK series runs between 1857 and 1987. The three US unemployment series are based on the original Department of Commerce data, the amendments suggested by Romer (1986), and the latter amendments together with the amendments suggested by Darby (1976). Table 1 contains details. We test for the presence of a unit root by using the Augmented Dickey-Fuller test, see Dickey and Fuller (1979). The evidence suggests that the series are stationary at the 5 percent level. Table 1 contains the complete set of results.

We then proceed to estimate an AR(2) unemployment equation for the series at our disposal. The results are presented in Table 2. All the diagnostics, except the one for ARCH errors in the US series, suggest that the residuals are "clean". In other words on the basis of the evidence as presented in Table 2, one could conclude that the dynamics of the unemployment series as captured by equation (21) of section II are adequately described by a linear specification. Although the model that we analyze does not call for an MA structure in the residuals, given the level of aggregation in the data, one might have expected the presence of some dependence in the residuals. However, as it is evident from the results of the Breush-Godfrey test such residual dependence was not diagnosed. The test statistic for an ARCH(2) process, suggests the presence of ARCH effects in the error structure of the US series, especially US(C). The UK

data did not show any strong ARCH effects.

Next we proceed to the application of the BDS test on the residuals of the estimated equation from Table 2. The results are presented in Table 3. All series examined display strong nonlinear behavior, by rejecting the null hypothesis of an i.i.d. process. When we use the standardized ARCH residuals, the BDS test statistics show little evidence of nonlinear structure in the US series. However, the UK series still displays strong nonlinear behavior. Table 4 contains the results for the standardized ARCH residuals. The results of Table 4 suggest that the AR(2) specification with ARCH errors seems to capture adequately the nonlinear behavior displayed in the US series. This is in contrast with the results of the UK data, where the BDS statistic values suggest that the (stochastic) linear difference equation specification does not entirely capture the dynamics of the unemployment series in question. We also tried to fit an ARMA(2,2) process to the series above in order perhaps to capture some residual MA that might be present. The results of the BDS statistics remain unchanged. Similarly higher-order linear specifications also failed to clean the UK residuals from nonlinear dependence. One might then be inclined to believe that the dynamics of that series are governed by some nonlinear difference equation. Note that the nonlinearity is in the variables not in the parameters of the model.

The next question that we want to address is whether the nonlinearities uncovered in the UK data are of the chaotic variety. In Table 5 we present the results from calculating the stretching factor $L(M,n)$ as a lower bound to the largest Lyapunov exponent for the UK series. As a benchmark we use a series of normal (pseudo) random numbers constructed with the same mean and variance as the UK series. It can be seen that the estimates from the random number series are positive and

hence more unstable than the estimates from the actual UK unemployment series. The evidence suggests that chaos although not entirely ruled out seems to be highly unlikely as an explanation of the nonlinear structure. These results are similar to the ones of Frank, Gencay and Stengos (1988) for Japanese GDP. In that case it was found that the above GNP series, although highly nonlinear, was also the most stable of all the GNP series examined in that study.

Our results suggest that the US and UK unemployment series are governed by different dynamics. In the case of the US, an AR(2) specification as suggested by the model we presented in section II, seems to provide an adequate description for the unemployment series, provided that we account for an ARCH error process. However, in the case of the UK, the model we presented in section 2 does not provide a satisfactory description of the dynamics. The behavior of the UK unemployment series seems to be governed by a nonlinear, yet not necessarily chaotic, process. Attempts to capture parts of this nonlinear process through ARCH effects or MA components in the error structure did not succeed. There is scope for further theoretical research to identify the sources of this nonlinearity in the UK unemployment rate.³

³There have been some attempts to explore the possibility of non-linearities for UK unemployment. Layard and Nickell (1986) report a wage equation in which the logarithm of unemployment affects real wages. Manning (1989, 1990) has utilized a version of this model, coupled with increasing returns to scale to explore multiple equilibria in the UK. Pissarides (1986, 1990) explores a search model with thin market externalities which results in multiplicity of equilibria and non-linear unemployment dynamics.

IV. Conclusion

We have used historical unemployment series for the US and the UK to test whether the linear dynamics largely implied by recent models that have been used to explain different unemployment experiences between industrialized countries are in accordance with the evidence. The model used, which is based on forward looking wage setters and firms includes most of the factors advanced to explain these differences.

In addition to the usual diagnostics that look at linear deviations from the null, we have used the BDS statistic that has power against nonlinear alternatives, including chaotic-deterministic ones. This paper provides an example where this diagnostic can of value in assessing the adequacy of economic models.

Our findings support the idea that the US unemployment and the UK one follow different dynamic specifications with the US unemployment being adequately described by a simple AR(2) process with ARCH errors, something that would be consistent with the model. On the other hand the UK unemployment does not seem to follow such a linear specification. Attempts to correct for the presence of alternative linear specifications did not produce any results. We also found that the nonlinearities present in the unemployment equation residuals do not seem to be of the chaotic variety. It seems to us that more theoretical work is needed to identify the sources of the nonlinear behavior in the UK series.

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Table 1
Testing for Unit Roots in UK and US Unemployment Processes
 Dependent Variable Δu_t
 Sample UK:1857-1987, USA: 1892-1987.

	United Kingdom	USA(A)	USA(B)	USA(C)
Constant	0.007 (0.004)	0.018 (0.010)	0.012 (0.005)	0.016 (0.005)
Δu_{t-1}	0.194 (0.088)	0.318 (0.099)	0.464 (0.093)	0.443 (0.094)
u_{t-1}	-0.163 (0.047)	-0.191 (0.053)	-0.147 (0.042)	-0.211 (0.051)
t	0.2×10^{-4} (0.4×10^{-4})	-0.5×10^{-4} (0.9×10^{-4})	-0.3×10^{-4} (0.7×10^{-4})	-0.3×10^{-4} (0.6×10^{-4})
s	0.018	0.026	0.018	0.018
DW	2.003	1.916	1.921	1.892
$z_1(1)$	0.023	0.952	0.461	0.995
$z_2(1)$	1.165	2.660	2.659	1.033
$z_3(1)$	2.530	0.763	7.571	0.784
τ	-3.457	-3.610	-3.500	-4.145

Notes: The numbers in parentheses are the standard errors. The z 's are Langrange Multiplier tests for misspecification. z_1 is a test for first-order residual autocorrelation, z_2 is Ramsey's RESET test for nonlinearity using the squares of the fitted values and z_3 is a test of heteroskedasticity. τ is the Augmented Disckey-Fuller test statistic with -3.51, -2.89 and -2.58 are the critical values for one, five and ten percent levels of significance. For the US we have the following series: (A) based on the original Lebergott data for 1980-1930, (B) is based on the new data of Romer(1986) for 1890-1930 and (C) is based on data from Romer for 1890-1930 and Darby (1976) for 1931-1943. The rest of the US data are from the Economic Report of the President (1988). The UK data are from Feinstein (1972) and Economic Trends, Annual Supplement (1988).

Table 2
Unemployment Processes in Level Form
Dependent Variable: u_t

Sample: UK :1857-1987, US: 1892-1987.

	United Kingdom	USA(A)	USA(B)	USA(C)
Constant	0.008 (0.003)	0.013 (0.005)	0.010 (0.003)	0.014 (0.004)
u_{t-1}	1.033 (0.087)	1.129 (0.098)	1.318 (0.092)	1.234 (0.093)
u_{t-2}	-0.195 (0.088)	-0.318 (0.098)	-0.464 (0.092)	-0.443 (0.093)
R^2	0.748	0.761	0.851	0.785
s	0.018	0.026	0.019	0.019
DW	2.004	1.916	1.921	1.892
$z_1(1)$	0.026	1.039	0.494	1.097
$z_2(1)$	0.086	2.776	0.746	0.072
$z_3(1)$	0.527	2.600	3.385	9.359
$z_4(2)$	1.935	13.822	7.663	25.819

Notes : The z 's are as in Table 1, with the numbers in parentheses being the degrees of freedom for the χ^2 variate in question. The z_4 statistic is a test for ARCH(2) errors, see Engle (1982).

Table 3

Results on the residuals of the estimated equations in Table 2

BDS statistics for various combinations of M and ϵ

$\epsilon = 1 \times \text{SD}$.

M	United Kingdom	USA(A)	USA(B)	USA(C)
3	4.087	3.965	2.937	3.923
4	4.472	4.391	3.247	4.097
5	5.293	4.612	3.130	4.036
6	6.296	5.037	3.000	3.455

$\epsilon = 1.25 \times \text{SD}$

3	3.442	4.844	2.975	4.124
4	3.208	4.924	2.941	4.229
5	3.264	4.986	2.688	4.042
6	3.417	5.113	2.299	3.653

$\epsilon = 1.5 \times \text{SD}$

3	3.362	5.327	2.897	4.389
4	2.857	5.270	2.784	4.279
5	2.746	5.202	2.630	4.109
6	2.658	5.156	2.239	3.747

Notes : The SD denotes the standard deviation of the series. All the statistics are distributed as $N(0,1)$ with a critical value of 1.96 at the 5 percent level.

Table 4
Results on the standardized ARCH residuals
BDS statistics for various combinations of M and ϵ
 $\epsilon = 1 \times \text{SD}$

M	United Kingdom	USA(A)	USA(B)	USA(C)
3	3.644	1.408	1.967	0.238
4	4.180	2.257	2.521	0.859
5	5.172	2.520	2.551	1.036
6	6.468	2.700	2.432	0.735

$\epsilon = 1.25 \times \text{SD}$

3	3.153	1.031	1.478	0.891
4	3.188	1.198	1.740	0.425
5	3.428	1.179	2.137	0.302
6	3.790	0.913	2.040	0.596

$\epsilon = 1.5 \times \text{SD}$

3	2.854	1.340	1.595	0.358
4	2.547	1.740	1.838	0.116
5	2.476	1.870	2.127	0.183
6	2.407	1.759	2.049	0.628

Table 5
 Stability Results: Estimates of $L(M,n)$ for different choices of M
 and n . The ϵ is set to $1 \times SD$.

United Kingdom			
M	n=1	n=2	n=3
3	-0.010	-0.010	-0.011
6	-0.008	-0.008	-0.009
9	-0.006	-0.008	-0.008
Random Numbers			
3	0.0089	0.0089	0.0090
6	0.0083	0.0084	0.0084
9	0.0076	0.0077	0.0076



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