Economic Crises and Government Policy

Andrew R. Gimber

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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Abstract

This thesis consists of two chapters exploring how even benevolent governments may struggle to convince their citizens that they will stick to the policies that ensure the best outcomes in equilibrium. If people believe that the government will optimally choose a different policy in the event of a crisis, their reaction to that belief may in fact bring about just such a crisis. This thesis investigates the circumstances in which these kinds of commitment problems can be overcome.

The first chapter is on bank resolution, where the choice between resolving insolvent banks and bailing them out creates a time inconsistency problem. To deter banks from taking excessive risks, governments want to convince them that they will choose resolution. However, when facing the costs of liquidating banks, governments may be tempted to bail them out instead. By strengthening their bank resolution regimes, governments reduce these costs, thus credibly committing themselves to choosing resolution over bailouts. Governments with greater resources face a more severe commitment problem. When banks interact strategically, improving the resolution regime can eliminate equilibria in which they coordinate on risky investment strategies.

In the second chapter, Antoine Camous and I present a theory linking the cyclicality of fiscal policy to inherited public debt. When debt is low, fiscal policy is countercyclical, in the sense that the government responds to reductions in output by cutting the tax rate. Above a threshold level of debt, however, optimal fiscal policy becomes procyclical. This creates the possibility of self-fulfilling crises, in which output is low because workers expect high taxes, and the government sets high taxes because output is low. Our model suggests why highly indebted governments might implement procyclical fiscal policy during recessions, even without facing high sovereign risk premia.
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I am grateful to all the faculty and staff of the EUI Economics Department who make it such a great place in which to study and do research. For financial support during my PhD, I thank the taxpayers of the United Kingdom and the Marcia Whitney Scholarship Fund.

Most of all, I thank my parents, Judith and Peter Gimber, who have always been there for me and asked for nothing in return except that I do my best. I dedicate this thesis to them.

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Contents

Abstract

Acknowledgements

1 Bank Resolution, Bailouts and the Time Inconsistency Problem

1.1 Introduction

1.1.1 Related Literature

1.1.2 Examples of Bank Resolution Regimes

1.2 A Model of Financial Intermediation with Moral Hazard

1.2.1 The Government’s Choice of Bailouts vs. Resolution

1.2.2 Banks’ Temptation to Gamble with Investors’ Funds

1.2.3 Investors’ Choice Between Gambling and Prudent Banks

1.2.4 Optimal Investment in the Bank Resolution Authority

1.2.5 Comparison with the Case of Costless Pre-Commitment

1.3 Robustness

1.3.1 Strategic Interactions Among Banks

1.3.2 Fire Sale Effects

1.3.3 Limited Government Resources

1.3.4 Production Sector with Distortionary Taxation

1.4 Conclusion

2 Public Debt and the Cyclicality of Fiscal Policy

2.1 Introduction

2.2 A Model of Taxpayer Coordination Failure

2.2.1 Static Economy with a Balanced Budget

2.2.2 Two-Period Economy with Taxes and Debt Issuance

2.3 Analysis

2.3.1 Properties of the Labour Supply Function

2.3.2 Properties of the Tax Policy Function

2.3.3 Equilibria

2.3.4 Tax-Base and Consumption-Smoothing Effects

2.4 Example with Closed-Form Solutions

2.4.1 Inherited Debt and the Cyclicality of Fiscal Policy
2.5 Robustness .............................................. 46
  2.5.1 Endogenous Government Spending ................. 46
  2.5.2 Allowing for Default on Newly Issued Debt .... 47
  2.5.3 Private Access to International Markets .......... 50
2.6 Conclusion ............................................. 52
Chapter 1

Bank Resolution, Bailouts and the Time Inconsistency Problem

1.1 Introduction

In the aftermath of the 2007–2008 financial crisis, new bank resolution regimes have been enacted or proposed in several countries. In the United Kingdom, the Banking Act 2009 established a Special Resolution Regime for failing banks. In the United States, the Dodd–Frank Act of 2010 created a new federal receivership process for failing financial companies deemed to pose a systemic risk. The European Union adopted the Bank Recovery and Resolution Directive in 2014, along with a Single Resolution Mechanism for the euro area as a key pillar of the banking union.

Such regimes allow for banks (and often other financial institutions) that are in imminent danger of failure to be dealt with outside the scope of normal corporate insolvency laws, on the grounds that this reduces the systemic risk such failures pose to the financial system. Under standard bankruptcy procedures, coordination failures among a bank’s creditors might prevent them from maximizing the value of its remaining assets (see, for example, Morris and Shin (2004).) Furthermore, creditors might force the bank to sell off its assets at fire sale prices, without taking into account the effect of lower asset prices on other institutions’ balance sheets. Disorderly liquidation might therefore lead to financial contagion. A normal bankruptcy procedure could also interrupt a bank’s ability to provide payment services and other “critical functions” to its customers, with potentially far-reaching implications for the economy as a whole. The intended effect of bank resolution regimes, therefore, is to reduce the ex post social costs of financial institutions becoming insolvent.

Another way of preventing a failing institution from posing a systemic risk is simply to prevent it from failing ex post. Thus, the use of bank resolution regimes may be a partial substitute for forms of direct and indirect public assistance such as capital injections, special liquidity facilities, asset purchase schemes and liability guarantees. This implies that efficient bank resolution regimes can serve to reduce governments’ ex post incentives to engage in bailouts. Governments might wish to commit to a no-bailout policy in order to
discourage financial institutions from taking excessive risks, but they may find themselves unable to relinquish their discretion. A government unable to tie its hands with respect to future bailouts might therefore choose to improve its bank resolution regime in order to solve its time inconsistency problem.

Such improvements come at a cost, however. The process of establishing or reforming a bank resolution regime requires the passage of legislation, which may delay other items on the government’s legislative agenda or expend its political capital. Although bank resolution regimes may reduce the workload of normal bankruptcy courts, they may place additional burdens on firms and the authorities responsible for implementing them. Such regimes may call for additional monitoring of financial institutions by regulators to ensure that they are resolvable, and for more extensive cooperation between different government agencies, both within and across national borders.

This paper presents a simple model that demonstrates the potential role of bank resolution regimes in reducing moral hazard in the financial sector. Bank resolution reform is modelled as a costly investment that raises the fraction of the value of a bank’s assets that can be recovered in resolution. If this fraction is high enough, the government’s time inconsistency problem disappears, meaning it in fact prefers to resolve insolvent banks instead of bailing them out. This in turn will deter banks from taking socially inefficient risks. Bank resolution reform thus serves as a costly commitment device against bailouts. Indeed, in the model presented below, this is its only role. A government with full and costless commitment would simply announce that it will never bail banks out. This would induce banks to invest prudently, and in fact (given the structure of asset returns in the model) no bank would ever need to be bailed out or resolved.

When the model is extended to introduce strategic interactions among banks, banks may wish to coordinate their investment strategies. During a financial crisis, in which many banks become insolvent at the same time, the government may be more reluctant to resolve banks than in normal times. However, a sufficiently robust resolution regime can still commit the government to choosing resolution over bailouts, thus eliminating equilibria in which banks coordinate on risky investment strategies.

The greater a government’s fiscal capacity relative to the size of its banking sector, the more bailouts it can afford. This means well-resourced governments will find it more difficult to convince banks that they will not receive bailouts in the event of insolvency. These governments will require particularly efficient bank resolution regimes if they are to credibly commit themselves to avoiding bailouts. A further implication is that efforts to shrink the banking sector may result in a loss of anti-bailout credibility, and that such efforts are best combined with bank resolution reform to restore this credibility.

The rest of the paper is organized as follows: Section 1.1.1 surveys some of the related literature on moral hazard in the financial sector and the time consistency of government financial policy; Section 1.1.2 discusses the bank resolution regimes currently in place in the UK and the US; Section 2.2 presents a simple model of optimal government investment in bank resolution regimes; Section 1.3 shows that the key results of this simple model are preserved when the model is extended in various directions; and Section 2.6 concludes.
1.1.1 Related Literature

Hellmann et al. (2000) consider a model with deposit insurance in which banks face a portfolio decision between a “prudent” asset and a “gambling” asset. Competition between banks erodes their franchise values, thus encouraging socially inefficient gambling. Capital requirements force banks to internalize the adverse effects of gambling, and so sufficiently strict capital requirements can induce them to invest prudently. However, forcing banks to hold more capital is costly and reduces their franchise values (which, ceteris paribus, increases their incentives to gamble). This means gambling can be prevented more efficiently by a combination of capital requirements and deposit rate ceilings, which limit interbank competition and thereby preserve bank franchise values. Cooper and Ross (2002) study a similar environment in which Diamond and Dybvig (1983)-style runs are also possible. Since there are no special costs associated with holding bank capital in their model, they demonstrate that the first-best allocation can be achieved by a combination of full deposit insurance (to prevent runs) and capital requirements (to prevent gambling).

Optimal contracts can require ex post inefficiency in some states of the world, and therefore be time inconsistent. For example, a credible threat of bankruptcy might mitigate moral hazard problems, but if bankruptcy is costly then renegotiation will be optimal ex post. Chari and Kehoe (2013) argue that governments face stronger incentives than private agents to avoid bankruptcies ex post because they take fire sale effects into account. As a result, the time inconsistency problem is more severe for governments than for private agents. They study a dynamic contracting model incorporating reputation effects, and find that introducing a bailout authority without commitment reduces welfare in equilibrium. By reducing the government’s incentive to intervene ex post, ex ante regulation limiting the size and leverage of firms can be welfare improving.

Farhi and Tirole (2012) show that when bailouts are non-targeted and involve fixed costs there are strategic complementarities in banks’ liquidity-hoarding and risk-taking decisions. When other banks hold fewer liquid (more toxic) assets, the expected size of the bailout increases and so any particular bank will want to become less liquid (more risky), too. In addition, because the incentives for a bailout are increasing in the number of banks that fail, banks will prefer to fail together than to fail alone and so will choose to correlate their assets. The authors find that, by imposing ex ante liquidity requirements (or, equivalently, leverage limits) on banks, a regulator can eliminate the central bank’s temptation to pursue a low interest rate policy ex post. (In this model, the bailout takes the form of a subsidized interest rate.)

Grochulski (2011) presents a framework in which debt default by a large financial firm is assumed to have spillover effects on the wider economy, creating incentives for the government to provide bailouts ex post. Even though equity is wiped out whenever the firm defaults on its debt, government bailouts still provide an implicit subsidy to the firm’s shareholders. This is because the prospect of a bailout removes creditors’ incentives to monitor the firm’s investment decisions, thus allowing it to choose a riskier project that yields higher expected returns for shareholders. The paper considers a number of possible
means of addressing the government’s time inconsistency problem, including improvements in resolution policy, taxes on extraordinary profits, and various forms of ex ante regulation. Whereas in the model of Grochulski (2011) the social costs of financial failure take a general form, in the present paper they will be modelled explicitly in terms of inefficient liquidation of banks’ assets.

Acharya and Yorulmazer (2008) posit that outsiders cannot realize the full value of banking assets, and that banks are therefore the most efficient users of banking assets (provided they can be incentivized to avoid the bad projects that yield them private, non-pecuniary benefits). This means that there will be a loss of allocative efficiency whenever the banking sector as a whole lacks sufficient liquidity to purchase the assets of failed banks. This in turn creates an incentive for the authorities to bail out failed banks or provide liquidity to surviving banks. Whereas the former policy provides incentives for banks to herd (since intervention is more likely when many banks are failing at once), the latter policy encourages banks to diversify (since surviving banks can benefit by purchasing assets at “fire sale” or “cash-in-the-market” prices). The authors demonstrate that ex ante welfare is higher when banks diversify, and that a policy of providing liquidity to surviving banks is time consistent.

The model presented below will differ by assuming asset specificity at the individual bank level: only the bank that originated an asset can realize its full value, and so liquidation of bank assets is always costly. This means providing liquidity to surviving banks will not be an effective substitute for bailouts here. Although the present paper will not model cash-in-the-market pricing explicitly, Section 1.3.2 will allow the ex post costs of resolution to depend upon the volume of bank assets being resolved at once, thus mirroring the type of fire sale concerns present in Acharya and Yorulmazer (2008).

DeYoung et al. (2013) model a repeated game between a utility-maximizing resolution authority and a profit-maximizing banking sector. Unlike in most of the literature (including the present contribution), the riskiness of banks’ portfolios is taken as given. Instead, banks choose the complexity of their portfolios, and this determines whether or not the resolution authority is able to resolve them. An improvement in the resolution technology raises the complexity threshold below which failed banks can be resolved, whereas in the present paper, improvements in the resolution technology increase the fraction of the value of a bank’s assets that are recovered in resolution.

Unlike many of the papers just discussed, the model presented below will abstract away from ex ante regulation. Farhi and Tirole (2012) argue that if regulation is costly it should be confined to those institutions the authorities are most tempted to bail out ex post. They suggest that this corresponds to large retail banks and “other large financial institutions that are deeply interconnected with them through opaque transactions”. However, the very opacity of these connections is likely to make identifying systemically important institutions ex ante very difficult. Chari and Kehoe (2013) focus on regulations that reduce firms’ abilities to become too big or too leveraged to fail. However, financial innovation and the growth of the shadow banking system may undermine the effectiveness of regulatory capital requirements.
Regulations that aim to prevent financial institutions from becoming too systemically important to fail do nothing to reduce the government’s bailout incentives in the event that they are circumvented. As a result, firms will face strong temptations of their own to evade the regulatory requirements. This line of argument implies the desirability of mechanisms that reduce ex post incentives for bailouts. Such mechanisms would complement regulatory restrictions on firms: if the authorities are more sanguine about the prospect of resolving banks, then the expectation of bailouts will be reduced and the incentive to evade regulations will be smaller.

1.1.2 Examples of Bank Resolution Regimes

United Kingdom

In the United Kingdom, the Banking Act 2009 permanently established a Special Resolution Regime (SRR), which provides the Bank of England (in consultation with the other UK authorities) with a number of tools for dealing with failing banks.\(^1\) The special resolution objectives, which “are to be balanced as appropriate in each case”, are as follows: to protect and enhance the stability of the financial systems of the United Kingdom (in particular the continuity of banking services); to protect and enhance public confidence in the stability of the banking systems of the United Kingdom; to protect depositors; to protect public funds; and to avoid interfering with property rights in contravention of a Convention right (within the meaning of the Human Rights Act 1998).\(^2\)

The SRR provides three “stabilisation tools”: sale of all or part of a firm’s business to a private sector buyer, transfer of the same to a bridge bank, and bail-in.\(^3\) In exceptional circumstances, the failing firm can also be taken into temporary public ownership by HM Treasury. The bank insolvency procedure can be used instead of or alongside these tools. In the case of partial sale or transfer, the bank administration procedure is invoked to ensure continuity of essential services from the residual bank. The prudential regulator (either the Prudential Regulation Authority or the Financial Conduct Authority) is responsible for determining whether a firm meets the criteria for being put into the SRR. The Bank of England is then responsible for deciding which of the tools to use and for carrying out the resolution transaction, except in the case of temporary public ownership.

Since the Act’s passage in February 2009, two institutions have been resolved under the SRR. In March 2009, the core parts of Dunfermline building society were transferred to Nationwide building society. Dunfermline’s social housing loans and related deposits were held temporarily in a bridge bank, and then sold to Nationwide in July of that year. In June 2011, Southsea Mortgage and Investment Company Limited was placed into the bank insolvency procedure. The Financial Services Compensation Scheme covered deposits up to the insured limit (then £85,000 per depositor), and all other creditors’ claims were to be

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1The Financial Services Act 2012 expanded the scope of the UK resolution regime to bank holding companies, central counterparties and certain investment firms and their group companies.


3In a bail-in, creditors’ claims are written down in order to absorb the firm’s losses and recapitalize it. This power was added as part of the UK’s implementation of the EU Bank Recovery and Resolution Directive.
handled by the bank liquidator. The Banking (Special Provisions) Act 2008, the emergency precursor to the Banking Act 2009, was used to authorize: the nationalization of Northern Rock in February 2008, the nationalization of Bradford & Bingley’s mortgage assets and the sale of its savings deposits and branch network to Abbey in September 2008, and the transfer to ING Direct of Heritable Bank and Kaupthing Edge in October of the same year.

United States

Title II of the Dodd–Frank Act of 2010 is titled Orderly Liquidation Authority, and its stated purpose is “to provide the necessary authority to liquidate failing financial companies that pose a significant risk to the financial stability of the United States in a manner that mitigates such risk and minimizes moral hazard.” A failing financial company can be put into receivership under the provisions of the Act if the relevant regulatory authorities and the Secretary of the Treasury deem that it would pose a systemic risk to the financial system.

Once the authorities determine that a company poses a systemic risk, the Federal Deposit Insurance Corporation (FDIC) is appointed as receiver (except in the case of insurance companies, which are dealt with under state law). The directors and officers of the company have a right to contest the decision, but unless the US District Court for the District of Columbia determines within 24 hours that the Secretary’s determination was “arbitrary and capricious”, the appointment of the FDIC as receiver will go through.

Once it assumes control over a company, the FDIC may act without consulting or giving notice to the company’s creditors, counterparties or shareholders. The powers granted to the FDIC as receiver are similar to those granted to the UK authorities under the SRR. It may sell off the company’s assets, or arrange for the company as a whole to be acquired by private buyers. It can also create a bridge company to hold some of the company’s assets and liabilities while a buyer is being found. The FDIC has the power to review claims on the company, and may deviate from the principle of equal treatment for similarly situated claimants in order to maximize the value of the company’s assets (as long as all claimants receive at least as much as they would have under the usual bankruptcy procedures).

1.2 A Model of Financial Intermediation with Moral Hazard

Consider an infinite-horizon economy populated by a unit measure of risk-neutral investors, some of whom are also bankers, and a government that may resolve failed banks or bail them out. Each investor is endowed with a single unit of the consumption good at the start of every period. A subset of investors are also endowed with banking licences, which permit them to offer non-state-contingent debt contracts to other investors. Banker $i$ promises to pay a gross interest rate $r_i$ at the end of the period for each unit of the consumption good he raises at the start. Investors can choose either to accept such a debt contract or to store their consumption endowment until the end of the period, and

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4 Dodd–Frank Wall Street Reform and Consumer Protection Act, Public Law 111-203, 124 Stat. 1454 (2010), Section 204(a).
5 See ibid., Sections 210(b)(4)(A), 210(d)(2), 210(d)(3) and 210(h)(5)(E).
they will choose whichever option yields them the highest expected return. If multiple banks offer the same interest rate, customers are divided evenly between them. There is no inter-period storage technology, so all goods are consumed at the end of each period.

As in Hellmann et al. (2000), bankers can invest their customers’ funds in two assets: a risky (“gambling”) asset and a safe (“prudent”) asset. The risky asset pays a gross return of \( R_H \) at the end of the period with probability \( \theta \), and \( R_L \) with probability \( 1 - \theta \). The safe asset yields a gross return of \( S > 1 \) with certainty. The risky asset has a higher return than the safe asset in the good state of the world, but a lower expected return than storage, that is, \( \theta R_H + (1 - \theta)R_L < 1 < S < R_H \). Since all agents in this economy are risk-neutral, this means that bank-intermediated investment is potentially welfare-enhancing, but gambling is inefficient. Nevertheless, the prospect of bailouts may induce investors to supply funding to banks they know will gamble.

At the end of each period, a bank may or may not be able to repay its investors the full amount it promised them. If the realized return from its investments exceeds the promised interest rate \( r_i \), then the bank pays its investors in full and the banker keeps and consumes any profits. If the bank’s investments yield less than \( r_i \) then the bank is insolvent. In equilibrium, insolvent banks will be those that choose the risky asset and get the low return \( R_L \). Let \( \sigma \) denote the fraction of gambling bankers whose gamble succeeds and yields the high return \( R_H \). For now, we assume that bankers’ asset returns are independent, so \( \sigma = \theta \), but in Section 1.3.1 below we will also consider the case of perfect correlation.

When banks become insolvent, the government can either bail them out or put them through resolution. In a bailout, the government makes up the difference between the amount owed to investors and the value of the bank’s assets, investors are paid in full, and the bank is allowed to continue trading. When the government opts to resolve a bank, the bank resolution authority strips the banker of his licence, liquidates the bank’s assets, and distributes the proceeds proportionally to investors.

There is asset specificity at the individual bank level: only if a banker is allowed to continue running his bank can he engage in the monitoring necessary to realize the full value of its assets. In a bailout, this full “going concern” value of the bank’s assets is preserved. In contrast, when the bank resolution authority liquidates an insolvent bank’s assets, it can only recover a fraction \( \lambda(h) < 1 \) of their full value.

The government’s objective function depends on the total welfare of the agents in the economy. Since all agents are risk-neutral and there is only one consumption good, this is equivalent to the total value of the economy’s resources. However, if it bails out a bank, the government’s objective function is reduced by an amount \( \eta < R_L \), which represents the costs of bailouts that go beyond the moral hazard problem they create. These costs include: the need in practice to use distortionary taxation to pay for bailouts; the fact that bailouts could involve redistribution from poor to rich, as in Cooper and Kempf (2013); and

---

6Bankers cannot invest their own endowments in these assets directly, but like all other investors they may entrust their personal funds to another bank or use intra-period storage. This corresponds to the so-called Volcker Rule (Section 619 of the Dodd–Frank Act), which prohibits banks from proprietary trading.

7Bank resolution authorities’ transfer, bridge bank and bail-in powers are not modelled explicitly here.
the prospect of angry taxpayers punishing governments for bailouts at the ballot box. The
distortionary effects of the taxation needed to pay for bailouts will be modelled explicitly
in Section 1.3.4, but for now the reduced-form parameter \( \eta \) allows us to focus on the moral
hazard and time consistency aspects of our model. Initially we will assume that \( \eta \) is a
per-bank cost, but Section 1.3.1 will consider the implications of a fixed bailout cost that
is paid whenever the government bails out a strictly positive fraction of banks.

We abstract away from all forms of ex ante regulation of banks. In particular, the gov-
ernment cannot impose capital requirements or deposit-rate ceilings. The only sanction
for a gambling banker is that he may lose his licence. Confiscated banking licences are
randomly reassigned at the beginning of the next period. Since there is a continuum of
agents in the economy, the probability of a banker having a particular licence returned to
him is zero. However, all agents have the same probability of receiving a licence, including
those who already have a licence and those who have recently lost one. This means some
bankers may end up with multiple licences, but since market share is split evenly across
banks offering the same interest rate, each additional licence is potentially valuable.

We model bank resolution reform as a costly investment by the government that im-
proves the recovery fraction \( \lambda(h) \). The government’s investment \( h \) in its bank resolution
authority is made at the very beginning of each period, and depreciates fully between peri-
ods. The UK special resolution objective of protecting taxpayers, and the FDIC’s discretion
under the Dodd–Frank Act to prioritize asset value over equal treatment, demonstrate that
recovering a high fraction of the value of a bank’s assets is indeed a key objective of reso-
lution. Another important priority for bank resolution authorities is the mitigation of fire
sale effects, which will be introduced in Section 1.3.2 below.

We will solve the model backwards, starting from the government’s choice between
bailouts and resolution, and then compare the model’s equilibrium outcome to that of a
model in which the government can costlessly pre-commit to a policy for insolvent banks.

1.2.1 The Government’s Choice of Bailouts vs. Resolution

We begin our analysis at the final stage of a period, when banks’ returns have been realized
and the government must decide what to do with those that are insolvent. Due to our
assumption that funds are split evenly between banks offering the same interest rate, all
insolvent banks will be of the same size in equilibrium. This means there is no distinction
between the government’s decision about what fraction of insolvent banks to bail out and
what fraction of insolvent banks’ total liabilities to cover (we rule out partial bailouts of
individual banks).

The government’s objective function is:

\[
V = X - h + g \left( \theta R_H + (1 - \theta) \left( b(R_L - \eta) + (1 - b) \lambda(h) R_L \right) \right) + (1 - g)S,
\]

where \( X \) is an exogenous source of revenue, \( h \) is the up-front investment the government
made in its bank resolution authority, \( g \) is the proportion of banks that chose to gamble,
and \( S \) is the return on the safe asset. Only a fraction \( \theta \) of gambling banks will obtain the
high return \( R_H \) (recall that we are assuming independent asset returns, so \( \sigma = \theta \)), and in equilibrium the remaining \( 1 - \theta \) of them that get the low return \( R_L \) will all be insolvent. For the fraction \( b \) of (randomly chosen) insolvent banks that the government chooses to bail out, the full asset value is preserved, but the government suffers the economic and political cost \( \eta \) of bailouts. The remaining proportion \( 1 - b \) of insolvent banks are shut down by the bank resolution authority, which can only recover a fraction \( \lambda(h) \) of the value of their assets in resolution.

The part of the government’s objective function that depends on the bailout fraction \( b \) is

\[
g(1 - \theta) b \left( (1 - \lambda) R_L - \eta \right). \]

This expression is linear in \( b \), which means that the government’s optimal choice of the bailout fraction will be as follows:

\[
b^* = \begin{cases} 
0 & \text{if } (1 - \lambda) R_L \leq \eta, \\
1 & \text{otherwise}. 
\end{cases} \]

This means the government will not bail out any banks as long as the resources that would be saved by avoiding resolution are less than or equal to the economic and political costs of the bailout itself.

The government’s ex post no-bailout condition is therefore

\[
\lambda(h) \geq 1 - \frac{\eta}{R_L}. \tag{1.1} \]

We can now see that the government’s ex post decision between bailouts and resolution depends on its initial investment in the bank resolution authority. The ex post no-bailout condition (1.1) will be more easily satisfied when the recovery fraction \( \lambda \) is higher, and this is a function of the up-front investment \( h \).\(^8\) This means the government can effectively commit itself to a no-bailout policy by making a sufficiently large investment in its bank resolution authority. This is the crucial mechanism through which bank resolution reform can reduce moral hazard.

By observing the government’s investment in its bank resolution authority, banks and their investors will be able to infer whether they will be bailed out or resolved in the event of insolvency. To see how this affects their behaviour, we turn now to the portfolio choice problem banks face once they have raised their funds.

### 1.2.2 Banks’ Temptation to Gamble with Investors’ Funds

Consider a bank that has promised its investors the prevailing market interest rate \( r \), and has thus raised a proportional share (which for convenience we normalize to one) of all the available funds. If it always invests prudently, it will earn profits of \( S - r \) each period.

---

\(^8\)For comparison, an increase in \( \lambda \) in the present framework is analogous to a reduction in \( M \) (the ratio of social to private costs arising from losses on financial firm debt) in the framework of Grochulski (2011).
so its franchise value is just

\[ \sum_{t=0}^{\infty} \beta^t (S - r) = \frac{S - r}{1 - \beta}, \]

where \( \beta < 1 \) is the per-period discount factor.

Now suppose instead that the bank gambles for \( T \) periods and returns to prudence thereafter. In each period that it gambles, the bank will make a profit of \( R_H - r \) with probability \( \theta \). The bank will be allowed to continue operating in the next period if its gamble is successful (with probability \( \theta \)), or if it is unsuccessful but receives a bailout (with probability \( (1 - \theta)b \)). (In equilibrium, all insolvent banks are symmetric, and insolvent banks are chosen at random to make up the government’s chosen bailout proportion \( b \), so the probability of a given insolvent bank receiving a bailout is just \( b \)). If the bank survives all \( T \) periods of gambling, it will earn profits of \( S - r \) in each future period. Its franchise value will therefore be:

\[
\frac{1 - \left( \left( \theta + (1 - \theta)b \right) \beta \right)^T}{1 - \left( \theta + (1 - \theta)b \right) \beta} \theta (R_H - r) + \left( \left( \theta + (1 - \theta)b \right) \beta \right)^T \frac{S - r}{1 - \beta}. \]

Banks will prefer not to gamble ex post if the franchise value associated with gambling for \( T \geq 1 \) periods is less than or equal to the franchise value of investing prudently forever:

\[
\frac{1 - \left( \left( \theta + (1 - \theta)b \right) \beta \right)^T}{1 - \left( \theta + (1 - \theta)b \right) \beta} \theta (R_H - r) + \left( \left( \theta + (1 - \theta)b \right) \beta \right)^T \frac{S - r}{1 - \beta} \leq \frac{S - r}{1 - \beta}.
\]

Solving for the interest rate \( r \) yields the following ex post no-gambling condition:

\[
\frac{1 - \left( \left( \theta + (1 - \theta)b \right) \beta \right)^T}{1 - \left( \theta + (1 - \theta)b \right) \beta} (S - (1 - \beta) \theta R_H) \leq \frac{S - r}{1 - \beta} \equiv r^P(b),
\]

where \( r^P \) is the maximum interest rate compatible with prudent investment, which depends on the anticipated bailout fraction \( b \). This means that a bank can commit itself to investing prudently by announcing an interest rate less than or equal to \( r^P \). Note that \( T \), the number of periods to gamble, drops out of this expression: if it is not profitable to gamble for one period, it will not be profitable to gamble for more periods.\(^9\)

\textbf{1.2.3 Investors’ Choice Between Gambling and Prudent Banks}

Since banks are competitive, they will effectively attempt to maximize investors’ utility subject to a non-negative profit constraint. Investors’ utility is increasing in the promised interest rate \( r \), but this interest rate will also determine whether banks will be tempted to gamble ex post. If the promised interest rate exceeds \( r^P \), the maximum interest rate compatible with prudence (that is, if the ex post no-gambling condition (1.2) is violated),

\(^9\)This is an application of the one-shot deviation principle (see, for example, Fudenberg and Tirole (1991, 108–110)).
then investors will anticipate that the bank will gamble with their funds.

A bank that is content to reveal that it will gamble can promise an interest rate as high as $R_H$ without making losses. Competition among gambling banks will drive their interest rates up to this level.\textsuperscript{10} If a bank’s gamble succeeds, it pays out the full return $R_H$ to its investors. If the gamble fails, the bank shuts down, and investors’ payoffs depend on the bailout probability $b$ and the recovery fraction $\lambda$. In contrast, a bank that wishes to commit itself to prudence can offer an interest rate no higher than $r_P$. Competition among prudent banks will drive the interest rate up to this level but no higher, so they will earn a profit of $S - r_P$ each period.

In a competitive equilibrium, banks will offer whichever interest rate maximizes investors’ expected utility. The expected return from investing one unit with a prudent bank is just the gross interest rate $r_P$, since there is no possibility of failure when a bank invests prudently. The expected return from a bank that promises the interest rate $R_H$ (and thereby reveals that it will gamble) is $(\theta + (1-\theta)b)R_H + (1-\theta)(1-b)\lambda(h)R_L$. If investors’ expected returns from a prudent bank weakly exceed those from a gambling bank, then all banks will offer the prudent interest rate, $r_P$. We will have a prudent equilibrium whenever the following ex ante no-gambling condition is satisfied:

$$
(\theta + (1-\theta)b)R_H + (1-\theta)(1-b)\lambda(h)R_L \leq \frac{(1-(\theta + (1-\theta)b)\beta)S - (1-\beta)\theta R_H}{(1-\theta)(1-b\beta)}.
$$

In a prudent equilibrium, competition between banks does not drive interest rates above $r_P$, since any bank offering a higher rate will gamble and thus yield investors a lower expected return. This means that bankers earn positive profits in a prudent equilibrium. However, the existence of these profits does not mean that the equilibrium is inefficient: indeed, it is these profits that induce bankers to choose the asset with the highest expected return for society.

Holding the bailout probability $b$ constant, an increase in the recovery fraction $\lambda$ makes gambling banks more attractive to investors, since it increases their payoff in the event that their bank is allowed to fail. (However, as condition (1.1) shows, the bailout probability $b$ depends negatively upon the recovery fraction $\lambda$.)

An increase in the bailout probability $b$ has two effects, both of which make gambling banks more attractive to investors. First, it means investors are more likely to be made whole in the event that their gambling bank gets into trouble. Second, it reduces the maximum interest rate that prudent banks can offer. Since a higher bailout probability reduces the risk of a bank losing its franchise, the franchise value of prudent banks must rise in order to deter them from gambling. This in turn requires higher profits each period, which means paying a lower interest rate to investors.

Let $b^P$ be the cutoff value of the bailout probability, at or below which investors prefer

---

\textsuperscript{10}If all banks are offering $r < R_H$, it will be a profitable deviation for a given bank to increase its interest rate. However, once all banks are offering $r = R_H$, this is no longer the case. Thus, even though investors might benefit from higher interest rates (since they would receive transfers from the government in the event of a bailout), we stipulate that banks will offer at most $r = R_H$. 

11
prudent banks, and above which investors prefer gambling banks. Investors will always prefer gambling banks if they are guaranteed to be bailed out. Under certain parameter restrictions, investors will prefer prudence whenever insolvent banks are guaranteed to be resolved. The following Proposition establishes this formally:

**Proposition 1.** If
\[ \theta \leq \frac{S - 1}{(1 - \beta)R_H + \beta S - 1} \in (0, 1), \]
then \( \exists ! b_P \in [0, 1) \).

**Proof.** Differentiating both sides of (1.3) with respect to the bailout probability \( b \) yields
\[
(1-\theta)(R_H - \lambda(h)R_L) > 0 \text{ and } -(1-\beta)\beta \theta (R_H - S)/ \left( (1-\theta) (1-b\beta)^2 \right) < 0 \text{ respectively,}
\]
confirming that gambling banks become relatively more attractive to investors as the bailout probability increases. This tells us that the cutoff bailout probability \( b_P \) will be unique. If \( b = 0 \), that is, if the government is expected to resolve all insolvent banks, then the ex ante no-gambling condition (1.3) becomes:
\[
\theta R_H + (1 - \theta) \lambda(h)R_L \leq \frac{(1 - \beta \theta)S - (1 - \beta) \theta R_H}{1 - \theta}.
\]
Since \( \lambda(h) < 1 \) for all \( h \), our assumption that the expected return on the risky asset is less than that on storage ensures that the left-hand side of this condition is less than one, and assumption (1.4) ensures that the right-hand side is greater than or equal to one. This means that prudence is preferred when \( b = 0 \), so we must have \( b_P \leq 0 \). (Assumption (1.4) also ensures that prudent banking pays investors a return greater than or equal to that of storage when \( b = 0 \), thus satisfying investors’ participation constraints.)

On the other hand, if \( b = 1 \), that is, if all banks will be bailed out in the event of insolvency, then condition (1.3) becomes \( R_H \leq S \). Since the risky asset’s high return is greater than that of the safe asset, this condition is always violated. This tells us that \( b_P < 1 \).

---

### 1.2.4 Optimal Investment in the Bank Resolution Authority

We saw in Section 1.2.1 above that, in this basic version of the model, the government will choose either to resolve all insolvent banks or to bail them all out. Proposition 1 tells us that there will be a prudent equilibrium if the government is expected to resolve all insolvent banks. From condition (1.1), we can see that the government will find it optimal to do so ex post as long as the recovery fraction \( \lambda \) is sufficiently high, which in turn depends on the up-front investment \( h \) in the bank resolution authority.

The minimum value of the recovery fraction that satisfies condition (1.1) is \( \lambda(h) = 1 - \eta/R_L \). Here it will be more convenient to think of the government as choosing \( \lambda \) directly, and making whatever investment \( h(\lambda) \) is necessary to achieve this. We can think of \( h(1 - \eta/R_L) \) as the cost the government must pay to establish its commitment to a no-bailout policy. Since paying this cost will ensure that banks invest prudently, and by assumption the prudent asset never fails, there is no benefit to investing more than this.
amount in the bank resolution authority. Making any lesser investment will guarantee bailouts ex post, and thus ensure that the authority is never used. Therefore, if the government decides not to invest enough to establish its commitment, then there is no reason for it to invest anything at all.

If the government decides to pay the commitment cost, the expected value of its objective function will be

$$\mathbb{E}(V) = X - h\left(1 - \frac{\eta}{R_L}\right) + S.$$ 

If instead it decides to invest zero, the expected value will be

$$\mathbb{E}(V) = X + \theta R_H + (1 - \theta)(R_L - \eta).$$ 

Comparing these two values tells us that as long as

$$h\left(1 - \frac{\eta}{R_L}\right) \leq S - \left(\theta R_H + (1 - \theta)(R_L - \eta)\right),$$

the government will choose to pay the commitment cost and thus ensure that banks invest prudently in equilibrium.

Since the prudent asset has a guaranteed payoff, the only role of the strengthened bank resolution authority is to convince investors that insolvent banks will be resolved instead of bailed out. As with deposit insurance in the Diamond and Dybvig (1983) model, the bank resolution authority is beneficial despite not being used in equilibrium. However, in contrast with Diamond and Dybvig (1983), the government policy required to implement the preferred equilibrium is costly. Indeed, if condition (1.5) were not satisfied, the cost of commitment would be too high, and so the government would prefer to make no investment in its bank resolution authority, let banks gamble, and bail them out in the event of insolvency.

### 1.2.5 Comparison with the Case of Costless Pre-Commitment

To emphasize that the role of the bank resolution authority here is to establish the government’s commitment to choosing resolution over bailouts, we now compare the outcome of our basic model to that of a variant in which the government can costlessly pre-commit to a policy for insolvent banks.

Suppose now that at the beginning of each period, the government simultaneously chooses its investment $h$ in the bank resolution authority, and pre-commits to bail out a fraction $b$ of insolvent banks (and to resolve the rest). From Proposition 1, we know that if the government commits to $b \leq b^P$, then in equilibrium all banks will invest prudently and the expected value of the government’s objective function will be

$$\mathbb{E}(V) = X - h(\lambda) + S.$$ 

Since by assumption prudent banks never become insolvent, the bank resolution authority...
will never be used, and so the recovery fraction is absent from the government’s objective function. This means the government’s optimal policy is to announce \( b \leq b^P \) and to invest nothing in bank resolution reform.

If we were to make the return on the prudent asset stochastic, then a government with costless commitment might also wish to invest a positive amount in its bank resolution authority. Nevertheless, there would still exist regions of the parameter space in which it would optimally invest less than a government with discretion.

1.3 Robustness

Having shown in a simple model that governments can commit themselves to choosing resolution over bailouts by improving their bank resolution regimes, we now examine the robustness of this result to various extensions that relax the model’s assumptions.

1.3.1 Strategic Interactions Among Banks

We assumed in the discussion above that the economic and political cost of bailouts, \( \eta \), was a per-bank cost. Since the recovery fraction \( \lambda \) is the same for all banks, and invariant to the measure of banks being resolved, this leaves no room for strategic interactions among banks.

Suppose now that the cost \( \eta \) is paid whenever the government engages in bailouts, regardless of how many banks are bailed out or what fraction of total industry liabilities those banks represent.\(^\text{11}\)

Let \( 1_{b > 0} \) be an indicator variable that takes the value one if the government engages in bailouts and zero otherwise. Then the ex post value of the government’s objective function when it bails out a fraction \( b \) of insolvent banks will be

\[
V = X - h + g\left(\sigma R_H + (1 - \sigma)\left(bR_L + (1 - b)\lambda(h)R_L\right)\right) + (1 - g)S - 1_{b > 0}\eta,
\]

where \( \sigma \) is the fraction of gambling banks whose assets yield the high return \( R_H \) and are therefore solvent (which above was always equal to \( \theta \) since we assumed independent asset returns). The part of this expression that depends on the bailout fraction \( b \) is

\[
g(1 - \sigma)b\left(1 - \lambda(h)\right)R_L - 1_{b > 0}\eta.
\]

Inspecting this expression, we can see that the relevant choice for the government is either to bail out all \( g(1 - \sigma) \) insolvent banks, or to abstain from bailouts completely. Since the cost \( \eta \) does not increase as further banks are bailed out, the government has no reason to limit the scope of bailouts once it has paid the cost (since bailouts avoid the loss of resources associated with liquidation). The government therefore compares the fixed

\(^{11}\)This undermines the interpretation of \( \eta \) as the cost of distortionary taxation or undesirable redistribution, and instead encourages its interpretation as a political cost associated with a loss of reputation. In this sense it is akin to the fixed cost of abandoning a currency peg in “second generation” models of speculative attack such as Obstfeld (1996).
cost $\eta$ of bailouts to the resources $g(1 - \sigma)(1 - \lambda(h))R_L$ that would be saved by bailing out all insolvent banks rather than resolving them, and its ex post no-bailout condition becomes:

$$\lambda(h) \geq 1 - \frac{\eta}{g(1 - \sigma)R_L}. \quad (1.6)$$

We now consider how knowledge of this ex post condition will affect the behaviour of banks and investors. Banks’ ex post and ex ante no-gambling conditions will be unchanged from (1.2) and (1.3) above. However, unlike condition (1.1) above, whether or not condition (1.6) is satisfied may depend on the proportion $g$ of banks that choose to gamble, and the fraction $1 - \sigma$ of those that become insolvent. For a given value of the recovery fraction $\lambda$, the more banks that gamble ex ante (and the more of them that become insolvent), the more tempting it is for the government to bail insolvent banks out ex post. This means that whether or not a given bank anticipates a bailout depends on how many other banks it expects to gamble. This introduces the prospect of strategic interactions among banks (what Acharya and Yorulmazer (2007) refer to as the “too many to fail” problem) and of multiple equilibria.

As above, we assume that (1.4) is satisfied, so investors will prefer prudent banks to gambling ones if they anticipate that insolvent banks will be resolved rather than bailed out. Rearranging the government’s new ex post no-bailout condition (1.6), we can see that this expectation will be justified as long as

$$g \leq \frac{\eta}{(1 - \sigma)(1 - \lambda(h))R_L} \equiv g^P(h, \sigma) > 0, \quad (1.7)$$

whereas if this condition is violated, investors will anticipate that the government will provide universal bailouts.

We introduce the binary variable $g_i$ to denote a given bank’s portfolio decision, with one representing gambling and zero representing prudent investment. Then from condition (1.7) and Proposition 1, we can see that the bank’s reaction function will be

$$g_i = \begin{cases} 
0 & \text{if } \mathbb{E}(g) \leq g^P(h, \sigma), \\
1 & \text{otherwise}. 
\end{cases} \quad (1.8)$$

In equilibrium, expectations about $g$ will be correct, and so we will have $g = \mathbb{E}(g)$. Since banks are symmetric, in equilibrium we will also have $g = g_i$. Either all banks will invest prudently ($g = 0$) or all banks will gamble ($g = 1$), depending on which strategy yields the highest expected payoff for investors.

**Proposition 2.** Suppose that the cost $\eta$ of bailouts does not depend on the proportion $g(1 - \sigma)b$ of banks the government bails out, and let condition (1.4) be satisfied. Then an equilibrium in which all banks invest prudently ($g = 0$) always exists, regardless of the government’s investment $h$ in the bank resolution authority.

**Proof.** Observe from condition (1.7) that $g^P(h, \sigma) > 0$ for all $h \geq 0$ and for all $\sigma \in [0, 1]$. Suppose that all agents expect that no banks will gamble in equilibrium, that is, $\mathbb{E}(g) = 0$. 
Then from the reaction function (1.8), we can see that banks will optimally invest prudently (since this is what investors prefer), and this validates the expectation that no banks will gamble ($g = 0$).

**Proposition 3.** Suppose that $\lim_{h \to \infty} \lambda(h) = 1$, and let condition (1.4) be satisfied. Then a sufficiently large investment $h \geq h^P$ in the bank resolution authority can eliminate the equilibrium in which all banks gamble ($g = 1$).

**Proof.** Consider the ex ante no-bailout condition (1.7). Since $\eta < R_L$, $\lim_{h \to \infty} \lambda(h) = 1$ implies $\lim_{h \to \infty} g^P(h, \sigma) = \infty$ for all $\sigma \in [0, 1]$. The recovery fraction $\lambda(h)$ is monotonically increasing in the investment $h$ in the bank resolution authority, so if $\lim_{h \to \infty} g^P(h, \sigma) = \infty$, then for all $\sigma \in [0, 1]$ there must exist some $h^P(\sigma)$ such that $g^P(h^P(\sigma), \sigma) \geq 1$. When $g^P(h, \sigma) \geq 1$, agents will anticipate from condition (1.6) that the government will resolve all insolvent banks, regardless of the proportion $g(1 - \sigma)$ that are insolvent. Since condition (1.4) is satisfied, we know from Proposition 1 that when the government is expected to resolve all insolvent banks, investors will prefer prudent banks to gambling ones.

Propositions 2 and 3 are illustrated in Figure 1.1, which plots an individual bank’s optimal strategy $g_i$ as a function of the proportion $g$ of its competitors it expects to gamble, for a given correlation of asset returns. As the investment $h$ in the bank resolution authority (and hence the recovery fraction $\lambda$) increases, the threshold value $g^P$ above which gambling is optimal also increases, shifting the dotted vertical line to the right. When $h$ (and thus $\lambda$) is high enough such that $g^P \geq 1$, the gambling equilibrium no longer exists, since no matter how many other banks are expected to gamble, any individual bank will be more attractive to investors if it invests prudently.

In order to eliminate the gambling equilibrium, the government would have to make an investment in the bank resolution authority sufficient to raise the recovery fraction to $\lambda \geq 1 - \eta/(1 - \sigma)R_L$ for all possible values of $\sigma$. In the independent returns case, we always have $\sigma = \theta$, whereas if returns were perfectly correlated, $\sigma$ would be equal.
to either zero or one. Since the bailout fraction $b$ only matters for a given bank when its asset yields the low return $R_L$, the relevant case is the one in which $\sigma = 0$. Looking at condition (1.6), we see that the right-hand side is decreasing in $\sigma$. This implies that it is more costly for the government to commit to a no-bailout policy (and hence induce banks to invest prudently) when banks’ asset returns are more highly correlated.

### 1.3.2 Fire Sale Effects

Fire sale effects are another potential source of policy-induced strategic interactions among banks. If the recovery fraction $\lambda$ is a function not only of the government’s investment $h$ in the bank resolution authority, but also the proportion $\rho \equiv g(1 - \sigma)(1 - b)$ of banks that are resolved in a given period, then the government’s optimal bailout policy ex post may depend on the number of insolvent banks, as in the version of the model with a fixed bailout cost $\eta$ in Section 1.3.1 above.

Let $\eta$ be a per-bank cost as in Section 2.2, and specify

$$\lambda = \lambda(h, \rho),$$

with $\rho \equiv g(1 - \sigma)(1 - b)$ and $\lambda(\rho) < 0$. Then for a given investment $h$ in the bank resolution authority and a given proportion $\nu \equiv g(1 - \sigma)$ of insolvent banks, the government will choose the bailout fraction $b$ ex post to maximize the value of insolvent banks’ assets, net of the bailout costs:

$$b^*(h, \nu) = \arg\max_b b(R_L - \eta) + (1 - b)\lambda(h, \rho)R_L \quad \text{s.t.} \quad b \in [0, 1]. \quad (1.9)$$

In Section 1.3.1 above, the government faced a fixed cost of bailouts, and so found it optimal either to bail all insolvent banks out or to resolve them all. With fire sale effects and per-bank bailout costs, the government trades off the marginal cost of an additional bailout with the marginal benefits in terms of assets not liquidated and a higher recovery fraction $\lambda$. This means it may choose to bail out an interior fraction of insolvent banks.

Assuming an interior solution for the optimal bailout fraction $b^*$, by totally differentiating the first-order condition associated with (1.9) and rearranging, we can show that $b^*$ will depend on the fraction $\nu \equiv g(1 - \sigma)$ of banks that become insolvent as follows:

$$b^*_\nu(h, \nu) = \frac{1 - b^*(h, \nu)}{\nu} \geq 0.$$

Using this expression, it is straightforward to calculate that the optimal bailout fraction $b^*$ will depend on the proportion $g$ of banks that choose to gamble, and the fraction $\sigma$ of those whose assets yield the high return $R_H$ (and hence are solvent) as follows:

$$\frac{\partial b^*(h, \nu)}{\partial g} = \frac{1 - b^*(h, \nu)}{g} \geq 0,$$

\footnote{We use $\lambda(\rho)$ to denote the partial derivative of the function $\lambda(h, \rho)$ with respect to the argument $\rho$.}
and
\[ \frac{\partial b^*(h, \nu)}{\partial \sigma} = -\frac{1 - b^*(h, \nu)}{1 - \sigma} \leq 0. \]

This tells us that, as above, there will be strategic complementarities in banks’ decisions about whether or not to gamble, and that greater correlation of banks’ asset returns will make gambling more attractive.

With a fixed bailout cost, the size of the government’s ex ante investment \( h \) could affect whether or not its no-bailout condition would hold ex post. With fire sale effects and per-bank bailout costs, the government can still constrain its ex post behaviour by investing in its bank resolution authority.

**Proposition 4.** Assume \( \lambda_{\rho}(h, \rho) \leq 0 \) (marginal fire sale effects are not ameliorated as more assets are liquidated) and \( \lambda_{\rho,h}(h, \rho) \geq 0 \) (investment in the bank resolution authority does not exacerbate fire sale effects). Then the government’s optimal ex post bailout fraction \( b^* \) is decreasing in its up-front investment \( h \) in the bank resolution authority.

**Proof.** The first-order condition associated with (1.9) is
\[ (1 - \lambda(h, \rho) - \rho \lambda(h, \rho)) R_L - \eta = 0. \]

Totally differentiating this with respect to \( h \) and rearranging shows us that the government’s optimal bailout fraction \( b^*_h \) depends on \( h \) as follows:
\[ b^*_h(h, \rho) = \frac{\lambda(h, \rho) + \rho \lambda_{h,h}(h, \rho)}{2 \lambda(h, \rho) + \rho \lambda_{\rho}(h, \rho)}. \]

Since \( \lambda(h, \rho) > 0, \lambda_{h,h}(h, \rho) < 0, \rho \geq 0 \) and \( v \geq 0 \), our assumptions on \( \lambda(h, \rho) \) and \( \lambda_{h,h}(h, \rho) \) are sufficient to ensure that the sign of this expression is unambiguously negative. \( \blacksquare \)

Let \( R_i \) denote the return on bank \( i \)’s investment portfolio. In order to prevent gambling, the government must ensure that the following inequality holds:
\[ \left( \theta + (1 - \theta) \hat{b} \right) R_H + (1 - \theta) (1 - \hat{b}) \lambda(h, \rho) R_L \leq \frac{\left( 1 - \beta \left( \theta + (1 - \theta) \hat{b} \right) \right) S - (1 - \beta) \theta R_H}{(1 - \theta) (1 - \beta \hat{b})}, \]
(1.10)

where \( \hat{b} \equiv E(b \mid R_i = R_L) \) and \( \lambda(h, \rho) \equiv E(\lambda(h, \rho) \mid R_i = R_L) \). The above expression states that the expected return from investing with a bank that promises \( R_H \) and gambles must not exceed that from investing with a bank that promises the highest interest rate compatible with prudence.

In order to completely eliminate gambling equilibria, condition (1.10) must be satisfied even in the government’s worst-case scenario, which is when all banks choose to gamble \( (g = 1) \) and perfectly correlate their asset returns \( (\sigma = 0 \text{ when } R_i = R_L) \). In the worst-case scenario, we will have
\[ \rho = 1 - b. \]
(1.11)

Therefore, if the government wishes to prevent gambling, its optimal investment \( h^* \) in its bank resolution authority will be the minimum necessary to ensure that condition (1.10) holds subject to equations (1.9) and (1.11). This will be desirable for the government when-
ever  

\[ S - h^* \geq \theta R_H + (1 - \theta)(R_L - \eta). \]

The requirement that the bank resolution authority be effective enough to handle even
the most severe financial crises without the government being tempted to resort to bailouts
is a demanding one indeed. However, as we shall see in the following section, limits to the
government’s resources can ameliorate its time inconsistency problem in precisely these
scenarios, taking the burden of establishing a credible no-bailout commitment off the bank
resolution authority.

1.3.3 Limited Government Resources

Thus far we have assumed that the government is deep-pocketed, that is, that its exoge-
nous pool of resources \( X \) (raised through taxation or the sale of state-owned resources) is
sufficiently large that it can afford to bail out the entire banking system. The purpose of
this section is to relax this assumption and draw out the implications for banks’ behaviour
and optimal government policy.

With a continuum of symmetric banks, we can define the maximum fraction of banks
the government can afford to bail out, \( \bar{b} \), as follows:

\[
\bar{b} \equiv \min \left\{ \frac{X - h}{g(1 - \sigma)(r - R_L)}, 1 \right\}.
\]

Ex post, the government will bail out a fraction \( b(h, \nu) = \min\{b^*(h, \nu), \bar{b}(h, \nu)\} \) of insol-
vent banks, where \( b^*(h, \nu) \) is the optimal bailout fraction defined in (1.9) above. We have
seen already that

\[
b^*_\nu(h, \nu) = \frac{1 - b^*(h, \nu)}{\nu} \geq 0.
\]

Differentiating with respect to \( \nu \) yields:

\[
b^*_\nu(h, \nu) = -\frac{2 \left(1 - b^*(h, \nu)\right)}{\nu^2} \leq 0,
\]

which tells us that \( b^*(h, \nu) \) is increasing and concave.

Focusing on the region in which \( \bar{b} < 1 \), we can see immediately the intuitive result that
the more banks are insolvent, the lower the fraction of them the government can afford to
bail out:

\[
\bar{b}_\nu(h, \nu) = -\frac{X - h}{\nu^2(r - R_L)} \leq 0.
\]

Taking the second derivative yields:

\[
\bar{b}_{\nu^2}(h, \nu) = \frac{2(X - h)}{\nu^3(r - R_L)} \geq 0,
\]

which implies that \( \bar{b}(h, \nu) \) is decreasing and convex.

We can plot curves for \( b^*(h, \nu) \) and \( \bar{b}(h, \nu) \) in \((\nu, b)\) space to illustrate how the opti-
mal and maximum bailout fractions depend on the proportion of banks that are insolvent.
Figure 1.2: Ex Post Bailout Fraction \( b \) as a Function of the Proportion \( \nu \) of Insolvent Banks.

Since \( b^*(h,0) = 0 \) and \( \bar{b}(h,0) = 1 \) for all values of \( h \), and \( b^*_\nu(h,\nu) \geq 0 \) and \( \bar{b}_\nu(h,\nu) \leq 0 \) for all values of \( h \) and \( \nu \), there can be at most one crossing of these curves. If they do not cross, then the government’s resource constraint is never binding and so \( b(h,\nu) = b^*(h,\nu) \) regardless of how many banks are insolvent. In this case, \( b(h,\nu) \) will be concave everywhere.

If the curves do cross, as in Figure 1.2, then there will be a kink in the \( b(h,\nu) \) curve at \( \nu^*(h) \equiv \arg \max \nu b(h,\nu) \), the insolvency fraction at which the ex post bailout fraction is maximized. To the left of this kink, \( b(h,\nu) \) will take on the concavity of \( b^*(h,\nu) \); to the right of the kink, \( b(h,\nu) \) will take on the convexity of \( \bar{b}(h,\nu) \). These properties of \( b(h,\nu) \) will be important for the determination of equilibrium.

We have seen in Section 1.3.2 above that the ex post optimal bailout fraction is decreasing in the government’s ex ante investment in its bank resolution authority: \( b^*_\nu(h,\nu) \leq 0 \). By again focusing on the region in which \( \bar{b} < 1 \), we can show that the same is true of the maximum affordable bailout fraction:

\[
\hat{b}_h(h,\nu) = \frac{1}{\nu(r - R_L)} < 0.
\]

This shows that it is still the case when the government’s resources are limited that it can commit to a lower ex post bailout fraction by investing more in its bank resolution authority.

For the special cases in which banks’ risky asset returns are exogenously fixed as either perfectly correlated or independent, the equilibrium outcomes can be determined straightforwardly. The no-gambling condition in these cases is the same as (1.10) above, but now \( \hat{b} \equiv \mathbb{E} \left( b \, | \, R_i = R_L \right) \) and \( \hat{\lambda}(h,\rho) \equiv \mathbb{E} \left( \lambda(h,\rho) \, | \, R_i = R_L \right) \) must take into account the possibility that the government may be budget constrained, i.e. that the ex post bailout fraction \( \hat{b} \) may not be the optimal fraction \( b^* \) but instead be the maximum affordable fraction \( \bar{b} \).

In the former case, a given bank will be insolvent if and only if all banks are insolvent. Therefore we will have \( \nu = 0 \) for a fraction \( \theta \) of the states of the world and \( \nu = 1 \) for
the remaining $1 - \theta$ states. The expected probability of a given insolvent bank receiving a bailout is therefore $b(h, 1)$, and the associated recovery fraction (for those banks that do not receive a bailout) is $\lambda(h, 1 - b(h, 1))$.

When asset returns are independent, the existence of a continuum of banks guarantees that the proportion that become insolvent is $1 - \theta$ in every state of the world. The expected probability of receiving a bailout conditional on being insolvent is therefore $b(h, 1 - \theta)$, and the associated recovery fraction is $\lambda(h, (1 - \theta)(1 - b(h, 1 - \theta)))$. For both these special cases, the government can use these expressions along with equation (1.10) to find the critical investment level $h^P$ at which gambling ceases to be profitable, and determine whether this investment is worthwhile.

1.3.4 Production Sector with Distortionary Taxation

In this section we will incorporate into the model a production sector with distortionary taxation, allowing us to microfound the reduced-form parameter $\eta$ that represented the cost of bailouts in the basic model. We will show that the key results derived above continue to hold in this more explicit version of the model.

Suppose now that all investors are also workers, who supply labour to competitive firms. Workers’ period utility functions are increasing and linear in consumption, $c$, and decreasing and strictly concave in labour supply, $n$:

$$u(c, n) = c - v(n)$$

with $v'(n) > 0$, $v''(n) > 0$ and $v'''(n) \geq 0$.

Workers’ lifetime utility is given by $U = \sum_{t=0}^{\infty} \beta u(c, n)$, where $\beta \in (0, 1)$ is the discount factor. Since utility is linear in consumption, there will be no wealth effects on labour supply. Furthermore, since there is no inter-period storage technology and utility is additively separable across time, there is no intertemporal substitution to consider. This means the optimal labour supply, $n^*$, will be that which satisfies the first-order condition equating the post-tax real wage, $(1 - \tau)w$, to the marginal disutility of labour supply, $v'(n)$.

All firms have access to the same constant-returns-to-scale technology $f(n) = zn$, which is increasing and linear in labour supply, $n$: $f'(n) = z > 0$ and $f''(n) = 0$. The capital stock is fixed, identical across firms and normalized to one. Since the production sector is perfectly competitive, the pre-tax real wage, $w$, will equal the marginal product of labour, $f'(n) = z$. Combining this with the first-order condition above gives us the following optimality condition for workers:

$$(1 - \tau)z = v'(n^*)$$

(1.12)

Since $z$ is a constant, we can see that the optimal labour supply, $n^*$, will depend only upon the contemporaneous tax rate, $\tau$. Moreover, since $v''(n) > 0$, the optimal labour supply must be strictly decreasing in the tax rate: $n^*'(\tau) < 0$.

The government sets the income tax rate after banks’ investment returns have been realized and before workers choose their labour supply. It must pay for its up-front in-
vestment in the bank resolution authority, \( h \), and for the cost of any bank bailouts. We assume that the government can borrow at a risk-free gross interest rate of one within each period, but is subject to a balanced-budget requirement at the end of each period. Since income taxes are distortionary, the government will choose the minimum tax rate compatible with raising the necessary level of revenue, \( X \), that is, it will choose the tax rate on the left of the Laffer curve:

\[
\tau \equiv \min \tau \quad \text{s.t.} \quad \tau z n'(\tau) = X \equiv h + g(1 - \theta)b(r - R_L).
\]

The government’s chosen tax rate, \( \tau \), depends on its level of expenditure, \( X \), which in turn depends on the investment in the bank resolution authority, \( h \), and the bailout fraction, \( b \).

As above, the government cares about the total value of resources under its jurisdiction, but now it also accounts for the disutility associated with labour effort. The government’s objective function becomes:

\[
V = -h + g\left(\theta R_H + (1 - \theta)\left(b + (1 - b)\lambda(h)\right)R_L\right) + (1 - g)S + z n'(\tau) - \nu(n'(\tau)).
\]

Letting \( \tilde{X} \equiv \max_{\tau} \tau z n'(\tau) \) denote the maximum expenditure the government can afford, we have the following expression for the maximum fraction of insolvent banks the government can bail out:

\[
\tilde{b} \equiv \min_{\tilde{b} \leq b} \left\{ \frac{\tilde{X} - h}{g(1 - \theta)(r - R_L)} \right\}.
\]  

(1.13)

Once investment decisions have been made and banks’ returns have been realized, the government’s ex post maximization problem is:

\[
\max_{b \leq \bar{b}} \quad g(1 - \theta)\left(1 - \lambda(h)\right)R_L + (r - R_L) \left(z - \nu'(\tau)\right) n''(\tau) \tilde{X}'(X)
\]

\[
- \nu(n'(\tau)) \right) \left(1 + g(1 - \theta)(r - R_L)b^{\ast}(h) - R_L \lambda'(h)\right) = 0.
\]

(1.14)

\textbf{Proposition 5.} \textit{In the extended model with production and distortionary taxation, the ex post bailout fraction} \( b^{\ast}(h) \) \textit{is weakly decreasing in the government’s up-front investment in its bank resolution authority,} \( h \).

\textit{Proof.} We proceed by first demonstrating that \( b^{\ast\ast}(h) < 0 \), and then showing that \( \tilde{b}'(h) \leq 0 \). The first-order condition associated with (1.14) is:

\[
g(1 - \theta)\left(1 - \lambda(h)\right)R_L + (r - R_L) \left(z - \nu'(\tau)\right) n''(\tau) \tilde{X}'(X)
\]

\[
= 0,
\]

which implicitly defines the optimal bailout fraction, \( b^{\ast} \). Totally differentiating this condition with respect to the up-front investment, \( h \), gives:

\[
g(1 - \theta)\left(r - R_L\right)\left(n''(\tau) \tilde{X}'(X) + n'''(\tau) \tilde{X}'(X)^2\right) \left(z - \nu'(\tau)\right) \ldots
\]

\[
\ldots - \left(n''(\tau) \tilde{X}'(X)\right)^2 \nu''(n^{\ast}) \left(1 + g(1 - \theta)(r - R_L)b^{\ast}(h) - R_L \lambda'(h)\right) = 0.
\]
Rearranging, we have:

\[ b^*(h) = \frac{A - 1}{g(1 - \theta)(r - R_L)}, \]

where

\[ A \equiv \frac{R_L \lambda'(h)}{(r - R_L)\left(\left(n^{**}(\tau) \lambda''(X) + n^{*''}(\tau) \lambda'(X)^2\right)\left(z - v'(n^*)\right) - \left(\left(n^{*'}(\tau) \lambda'(X)\right)^2 v''(n^*)\right)} \]

We wish to show that \( b^*(h) < 0 \). We have \( g(1 - \theta) > 0 \), since the bailout fraction \( b^* \) is only defined when the fraction of insolvent banks is strictly positive. We also have \( r - R_L > 0 \), since \( R_L \) is the lowest possible return, and competition in the banking sector will drive the interest rate, \( r \), above this. We have already assumed that \( \lambda'(h) \geq 0 \), that is, investments in the bank resolution authority always weakly improve the recovery fraction. From the representative worker’s optimality condition (1.12), it follows that \( z - v'(n^*) > 0 \). Furthermore, the optimal labour supply is strictly decreasing in the tax rate, so \( n^{**}(\tau) < 0 \). The marginal disutility of labour effort is increasing by assumption, so \( v''(n^*) > 0 \). The squared terms must clearly be positive, so all that remains in order to show that \( b^*(h) < 0 \) is to show that \( n^{*''}(\tau) \leq 0 \) and \( \lambda''(X) \leq 0 \).

Totally differentiating the representative worker’s optimality condition (1.12) twice with respect to the tax rate, \( \tau \), and rearranging gives us:

\[ n^{***}(\tau) = -\frac{\left(n^{*'}(\tau)\right)^2 v''''(n^*)}{v''(n^*)} \leq 0. \]

Now consider the government’s balanced-budget constraint \( \tau z n^*(\tau) = X \). Totally differentiating this twice with respect to the level of expenditure, \( X \), and rearranging gives us:

\[ z''(X) = \frac{(2n^{*'}(\tau) + \tau n^{*''}(\tau))z'(X)}{z\left(n^*(\tau) + \tau n^{*''}(\tau)\right)^2} < 0. \]

(Since the government chooses the tax rate on the left of the Laffer curve, we must have \( z'(X) > 0 \).)

Having shown that \( b^*(h) < 0 \), all that remains is to show that \( \bar{b}'(h) \leq 0 \). By inspecting (1.13), we can see that \( \bar{b} \) is strictly decreasing in \( h \) whenever \( \bar{X} - h < g(1 - \theta)(r - R_L) \), and invariant to \( h \) elsewhere.

\[ \square \]

1.4 Conclusion

The model presented in this paper demonstrates that an efficient bank resolution regime can indeed reduce moral hazard in the financial sector. Under certain parameter values, the government’s optimal investment in such a regime is higher than it would be if it could costlessly commit to a no-bailout policy. The purpose of this investment is to make the government’s no-bailout commitment credible, and thus deter banks from gambling. Indeed, under the maintained assumption that prudent banks never fail, the resolution
regime will never be used in equilibrium. Thus, although the direct effect of improved bank resolution is to reduce the ex post costs of bank failures, its most important effect may be the indirect one of reducing the ex ante probability of such failures.

When the likelihood of bailouts depends on how many banks become insolvent at once, there can be strategic complementarities in banks’ risk-taking decisions that give rise to multiple equilibria. In such environments, a sufficiently effective bank resolution authority may be able to eliminate equilibria in which banks gamble because they expect others to do so. However, if banks’ asset returns are highly correlated, the resolution authority may have to be able to cope with severe financial crises in order to make the government’s commitment to avoid bailouts credible. When the government’s resources are limited, the bank resolution authority is no longer needed as a commitment device in severe financial crises: the unaffordability of bailouts does the work instead.

Bibliography


Chapter 2

Public Debt and the Cyclicality of Fiscal Policy

This chapter was written jointly with Antoine Camous.

2.1 Introduction

Public debt to GDP ratios in advanced economies have been rising since the mid-1970s, and have recently reached levels not seen since just after World War II (Abbas et al., 2011). The recent financial crisis and the ensuing Great Recession exacerbated this trend through bailouts, stimulus packages, rising unemployment claims, and falling tax revenues. This has led to a heated debate over the pace of fiscal consolidation, with one side emphasizing the burden on economic growth imposed by high levels of public debt, and the other warning that pursuing austerity during a recession could be very costly or even self-defeating.

In this paper we present a new theory that provides a partial reconciliation of these two views. We show that there is a threshold level of debt above which the economy is vulnerable to self-fulfilling fiscal crises. However, the mechanism that makes such crises possible is that fiscal policy becomes procyclical, in the sense that the government’s optimal response to a reduction in output is to raise the tax rate.1 Thus, our model lends qualified support to both sides of the debate over fiscal consolidation: the proximate cause of the crisis is the government’s desire to raise the tax rate in a recession, but the source of this desire is the high level of public debt.

In Calvo (1988) and related papers,2 investors’ expectations of sovereign default cause them to charge a risk premium that makes default more likely. Corsetti et al. (2013) argue that this sovereign risk channel provides a motivation for fiscal consolidation. However, even countries that did not face an increase in sovereign risk premia have pursued fiscal consolidation in the years since the onset of the Great Recession. Our focus in this pa-

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1Since "procyclical" can be used to describe both variables that are positively correlated with output and policies that exacerbate the business cycle, there is potential for ambiguity when describing the cyclicality of tax rates. Throughout this paper, we use "procyclical" to refer to a negative correlation of tax rates and output, that is, tax policy that could exacerbate output fluctuations.

2See also Cooper (2015) and Lorenzoni and Werning (2013), for instance.
per is not on self-fulfilling expectations of sovereign default, but on another type of self-fulfilling macroeconomic crisis caused by high levels of public debt. Accordingly, we focus on cases in which investors charge the lowest risk premium compatible with the economy’s fundamentals. Our analysis can explain why a highly indebted government might adopt procyclical fiscal policy during a recession, even without facing a high sovereign risk premium. Indeed, in our baseline model debt is risk free because the government never defaults, and debt sustainability is ensured by future fiscal capacity.\footnote{We show in section 2.5.2 that our results are robust to allowing for government default.}

Unlike a committed Ramsey planner, the government in our model takes households’ current labour supply decisions and output as given when setting the contemporaneous tax rate and issuing new debt. Fiscal policy is therefore a function of current output, as well as of the inherited stock of public debt. This leads to a standard time inconsistency problem of the kind identified by Kydland and Prescott (1977). Whatever the level of public debt, the government always chooses a higher contemporaneous tax rate than a Ramsey planner would choose, because it does not internalize the distortionary effect on current output. However, the key insight of our analysis is that the government’s inability to commit to a tax rate can have even more severe consequences, because when debt is high fiscal policy becomes procyclical, thereby inducing a coordination problem among households.

When the economy suffers a fall in output, there are two countervailing effects on the government’s optimal choice of the contemporaneous tax rate. The first is that, for given tax rates, contemporaneous consumption falls relative to future consumption. This provides the government with a consumption-smoothing motive to reduce the contemporaneous tax rate relative to the future tax rate.\footnote{In our baseline model we abstract away from private-sector borrowing decisions, but this consumption-smoothing motive applies whenever consumers are not fully Ricardian. We relax this assumption in section 2.5.3.} The second effect, which we call the tax-base effect, is that the contemporaneous tax base shrinks, meaning that the government must raise tax rates at some point in order to remain solvent in the long run.

When the inherited stock of public debt is low, the consumption-smoothing effect dominates. This means fiscal policy is countercyclical: the government’s optimal response to a fall in output is to cut the tax rate and issue more debt, postponing the necessary tax collection to the future. A household that expected aggregate labour supply to be low would therefore anticipate a low tax rate, and choose a high level of labour supply itself. Under these conditions there is no scope for coordination failure, and our economy has a unique equilibrium.

However, when the inherited level of public debt is high, the tax-base effect dominates. Optimal fiscal policy then becomes procyclical, because deferring all fiscal consolidation (tax increases) when output is low would impose an unacceptable burden on future consumption. This unleashes the possibility of multiple equilibria. In the good equilibrium, labour supply is high because workers anticipate a low tax rate, and the government optimally chooses a low tax rate because output is high. In bad equilibria, which we label fiscal policy traps, workers restrict their labour supply in anticipation of a high tax rate,
and the resulting low output induces the government to fulfill their pessimistic expectations by setting a high tax rate. Welfare is lower in fiscal policy trap equilibria than in the high tax-base, low tax-rate equilibrium.

The idea that high levels of public debt can pose a threat to the economy is most famously associated with Reinhart and Rogoff (2010). In particular, they argue that countries with sovereign debt to GDP ratios above 90 percent have significantly lower rates of economic growth on average. The burden of distortionary taxation imposed by debt service could explain why high levels of debt might reduce growth, but not why there might be a discrete drop in growth above some threshold level of debt. Our model contributes a novel explanation for why there might be such a threshold effect, based on self-fulfilling beliefs about the stance of fiscal policy. In our model, a country with a level of public debt just above the threshold is exposed to the risk of a high-tax, low-output equilibrium. If this equilibrium is selected, the country’s economic performance will be significantly worse than that of a similar country with a public debt level just below the threshold.

Schmitt-Grohe and Uribe (1997) introduce similar concerns about taxpayer coordination failure into a dynamic model with income and capital taxation. They consider only balanced-budget fiscal policy: specifically, they study the undesired consequences of balanced-budget rules when labour supply and tax rates are chosen simultaneously. In addition to procyclical fiscal policy, they show that this leads to equilibrium indeterminacy. The authors stress the crucial role of capital accumulation in driving the result. Finally, they consider the role of public debt, but not as a choice variable: they maintain budget balance and a fixed stock of public debt, so what matters are the interest rates that have to be serviced. Nevertheless, high levels of debt in their analysis can lead to fiscal policy indeterminacy.

Cole and Kehoe (2000) consider a dynamic environment in which the government is prone to self-fulfilling debt rollover crises. They assume a constant tax rate, and allow the government to adjust its debt level by varying its expenditure. In their model, there is a source of domestically initiated crisis, via capital accumulation. By reducing saving, households reduce capital next period and bring the economy into the crisis zone where market shutdown is an option, hence making the initial belief that drove the reduction in saving self-fulfilling.

The rest of the paper is organized as follows. In section 2.2, we present the general framework of analysis. Section 2.3 sets out our main analytical results. Next, in section 2.4, we illustrate by way of an example the mechanism by which the cyclicality of fiscal policy depends on the inherited debt position and can lead to a self-fulfilling crisis. In section 2.5, we build on this example to investigate the robustness of our results to relaxing several of our baseline assumptions. Section 2.6 concludes.

### 2.2 A Model of Taxpayer Coordination Failure

In this section, we first outline the mechanism by which taxpayer coordination failure can arise in a static environment, and then present the general framework of our dynamic
Figure 2.1: Equilibria on Either Side of the Laffer Curve

This figure outlines the coordination problem created by the government’s inability to commit to a tax rate. For a given level of government expenditure, there are two levels of labour supply, associated with different tax rates, that satisfy the government budget constraint. The equilibrium with high labour supply and a low tax rate provides higher utility than the one with low labour supply and a high tax rate.

analysis.

2.2.1 Static Economy with a Balanced Budget

Consider a static environment, as proposed by Cooper (1999, 131–132), in which the government must finance a fixed level of expenditure $G$ through a proportional ex post tax on labour income. The economy is populated by a mass-one continuum of ex ante identical households, indexed by $i \in [0, 1]$, who derive utility from consumption, $c_i$, and disutility from labour supply, $n_i$. Production is linear, so with a proportional tax rate $\tau$, household $i$’s consumption is $c_i = (1 - \tau)n_i$. Since the pre-tax real wage is fixed at unity, households’ optimal labour supply will be a function of the tax rate: $n_i = n(\tau)$. We assume that the substitution effect dominates the income effect in the utility function, so that labour supply is decreasing in the tax rate: $dn(\tau)/d\tau < 0$.

The government’s budget balance constraint is $\tau n = G$, where $n = \int n_i di$ is aggregate labour supply. The government must pay for its fixed expenditure, so the tax rate will depend negatively on the tax base: $\tau = G/n$. This creates strategic complementarities among households: the higher is aggregate labour supply, the lower will be the tax rate, and so the higher is household $i$’s optimal labour supply.

The equilibrium condition is $\tau n(\tau) = G$. As Figure 2.1 shows, there are two Pareto-ranked equilibria: a good equilibrium with a low tax rate $\tau^G$ and high labour supply $n(\tau^G)$, and a bad equilibrium with a high tax rate $\tau^B$ and low labour supply $n(\tau^B)$. Given the presence of strategic uncertainty over the tax rate, households may coordinate on the inefficient Nash equilibrium, which lies on the downward-sloping part of the Laffer curve.

If the government could credibly commit to a tax rate, this strategic uncertainty among households would disappear. However, in a static environment with fixed expenditure, $G$ as pre-contracted expenses that do not enter into household utility directly.
the government has no choice but to respond to a revenue shortfall by raising the tax rate. The combination of discretion over the tax rate and an absolute requirement to balance the budget leads to the possibility of coordination failure. The first of these assumptions is reasonable: sovereign governments cannot in fact commit to keep tax rates constant regardless of the state of the economy.\footnote{Income tax policy can change relatively quickly, particularly during crises, and even retroactive tax increases are not unheard of. On 6 November 2012, voters in California passed Proposition 30, which included increases in top marginal tax rates that applied retroactively to income earned since 1 January 2012. The Minnesota omnibus tax bill (HF 677), signed into law on 23 May 2013, included a new top income tax bracket and an increase in the alternative minimum tax rate, both of which applied retroactively to the beginning of 2013.} However, the balanced-budget view of fiscal policy is less realistic because governments routinely borrow to cover revenue shortfalls when output is lower than expected (and even balanced-budget constitutional amendments can be overturned).

The focus of the present paper is therefore to offer the government the possibility to issue new debt rather than increase taxes in the event of a revenue shortfall. Does this allow the government to eliminate the taxpayer coordination failure and steer the economy to the more efficient outcome with a low tax rate and high labour supply? Our answer will be that this depends on the inherited debt level. If the outstanding debt burden is sufficiently low, then the government’s ability to adjust its debt position in the event of a revenue shortfall will ensure that there is a unique, low-tax equilibrium. However, if the inherited stock of debt is large enough then the government will optimally respond to lower output with higher taxes, unleashing the possibility of a fiscal policy trap.

### 2.2.2 Two-Period Economy with Taxes and Debt Issuance

We consider a two-period economy: $t = 1, 2$. The government inherits a level of debt $B_1$, owed to foreign investors. In period 1, households choose labour supply and produce accordingly. The government then sets its fiscal policy, choosing the tax rate on labour income $r_1$ and the new debt $B_2$ to be issued to foreign investors. This debt is backed by future primary fiscal surpluses and is always repaid in period 2, so the government can borrow at the risk-free rate $R$ between periods 1 and 2. We interpret the terminal period 2 as the long run.

The focus of our analysis is on the determinants of labour supply and fiscal policy in period 1. We next describe these choices.

#### Households’ Preferences and Choices

There is a unit mass of households in the economy, indexed by $i \in [0,1]$. Households live over the two periods but are hand-to-mouth consumers, meaning they can neither save nor borrow between periods 1 and 2.\footnote{We explore the implications of relaxing this assumption in section 2.5.3 below.} Moreover, since households are atomistic, they do not internalize the impact of their labour supply choices on the government’s choices of tax rate and debt issuance. In period 1, household $i$ forms a belief about the tax rate $r_1$ and
solves:

$$\max_{n_{1,i}} u(c_{1,i}) - g(n_{1,i})$$  \hspace{1cm} (2.1)

subject to

$$c_{1,i} = (1 - \tau_1)z_1f(n_{1,i}).$$  \hspace{1cm} (2.2)

Consumption utility is increasing and concave: $u'(\cdot) > 0$ and $u''(\cdot) < 0$; and labour disutility is increasing and convex: $g'(\cdot) > 0$ and $g''(\cdot) < 0$.

The individual production function is $y_{1,i} = z_1 f(n_{1,i})$, where $z_1 > 0$ is an aggregate productivity parameter and $f(\cdot)$ is an increasing function that exhibits weakly decreasing returns to scale and is unbounded above: $f'(\cdot) > 0$, $f''(\cdot) \leq 0$ and $\lim_{n \to +\infty} f(\cdot) = +\infty$.

The labour supply decision $n(\tau_1)$ is implicitly defined by the following first-order condition:

$$(1 - \tau_1)z_1 f'(n_{1,i})u''(1 - \tau_1)z_1 f(n_{1,i}) = g'(n_{1,i}).$$  \hspace{1cm} (2.3)

We assume that the curvature of the utility function is such that substitution effects dominate income effects:

$$u'(c) + cu''(c) > 0 \quad \forall c \geq 0.$$  \hspace{1cm} (2.4)

This ensures that labour supply is a decreasing function of the tax rate:

$$\frac{dn(\tau_1)}{d\tau_1} = \frac{z_1 f'(\cdot)u''(\cdot) + \left((1 - \tau_1)z_1 f'(\cdot)\right)^2 u''(\cdot) - g''(\cdot)}{(1 - \tau_1)z_1 f''(\cdot)u'(\cdot) + \left((1 - \tau_1)z_1 f'(\cdot)\right)^2 u''(\cdot) - g''(\cdot)} < 0.$$  \hspace{1cm} (2.4)

**Government’s Preferences and Choices**

The government faces an intertemporal tax-smoothing problem. It has an inherited stock of debt owed to foreign investors, $B_1$, which it is committed to repaying. In each period, the government also has to finance an exogenous amount of expenses $G_t \geq 0$, which do not enter into household utility directly.\(^8\) Given inherited debt $B_1$ and aggregate labour supply $n_1$, it optimally sets the tax rate $\tau_1$ and issues new debt $B_2$ to risk-neutral foreign investors. Importantly, the choice of $B_2$ is constrained by the requirement that all outstanding debt is repaid in period 2.\(^9\) Future fiscal capacity is defined by the maximum amount of debt $\tilde{B}_2$ that can be issued in period 1.

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\(^8\)In section 2.5.1, we endogenize short-run public expenditure $G_1$.

\(^9\)This assumption is introduced to highlight the fact that the mechanism at play in our analysis, namely the link between inherited debt, the cyclicality of fiscal policy and the possibility of taxpayer coordination failure, is not driven by self-fulfilling increases in sovereign risk premia. In section 2.5.2, we relax this assumption and show that our results still hold.
The government’s maximization problem is as follows:

\[
\begin{align*}
\max_{\tau_1, B_2} & \quad u(c_1) - g(n_1) + \beta V(B_2) \\
\text{subject to} & \quad B_1 + G_1 \leq \tau_1 z_1 f(n_1) + \frac{B_2}{R} \\
& \quad B_2 \leq \bar{B}_2.
\end{align*}
\] (2.5)

The function \(V(\cdot)\) captures the continuation utility of the economy when in period 1 the government issues bonds with face value \(B_2\) to be repaid in period 2. The government budget constraint (2.6) states that debt service and government expenditure in period 1 must be financed by proportional taxes on output and new debt issuance. Expression (2.7) states that, because of the long-run solvency requirement, the government also faces a borrowing limit \(\bar{B}_2\).

The continuation utility function \(V(\cdot)\) satisfies the following concavity assumptions:

\[
V'(\cdot) < 0, \quad V''(\cdot) < 0.
\] (2.8)

In addition, we impose

\[
\lim_{B_2 \to \bar{B}_2} V'(\cdot) = -\infty.
\] (2.9)

This condition ensures that the government’s borrowing limit (2.7) will not bind in equilibrium (see Lemma 1 below).\(^\text{10}\)

Since \(V(\cdot)\) is decreasing in \(B_2\), the government budget constraint (2.6) will be satisfied with equality. Substituting this into the government’s objective function (2.5) and differentiating with respect to the short-run tax rate \(\tau_1\) yields the following first-order condition:

\[
u'(1 - \tau_1) z_1 f(n_1) = -\beta RV'(R(B_1 + G_1 - \tau_1 z_1 f(n_1)))
\] (2.10)

Equation (2.10) implicitly defines the tax policy function \(\tau(n_1, B_1)\).\(^\text{11}\) We will demonstrate below that the optimal short-run tax rate is unambiguously increasing in the inherited debt level \(B_1\), but that the sign of its derivative with respect to short-run labour supply \(n_1\) is ambiguous. When \(d\tau(\cdot)/dn_1 > 0\), we say that fiscal policy is \textit{countercyclical}, meaning a drop in output induces the government to lower the tax rate; when \(d\tau(\cdot)/dn_1 < 0\), we say that fiscal policy is \textit{procyclical}, meaning a drop in output induces the government to raise the tax rate. We will also show that the cyclicality of fiscal policy and the number of equilibria in this economy depend on the inherited level of debt.

\(^{10}\)Condition (2.9) states that the marginal utility of a reduction in the future debt burden approaches infinity as the government approaches its debt limit. We demonstrate below that this condition is satisfied for natural specifications of \(V(\cdot)\) in which the cost of issuing additional debt in period 1 is higher taxes and lower consumption in period 2.

\(^{11}\)For comparison, a Ramsey planner with the ability to commit to a tax rate would solve (2.5) subject to (2.6), (2.7) and the additional constraint \(n_1 = n(\tau_1)\), implicitly defined by (2.3).
**Equilibrium Definition**

The relevant choices of households and the government are both made in period 1. The government inherits an amount of debt $B_1$. Households form expectations about fiscal policy, supply labour and produce accordingly. Given its outstanding debt and the economy’s tax base, the government sets fiscal policy to maximize the lifetime utility of the population.

The relevant variables for the government’s decisions are aggregate labour supply, $n_1$, and the inherited amount of debt $B_1$. Given $(n_1, B_1)$, the government sets $\tau_1$ and issues new bonds $B_2$. We denote the policy functions $\tau(n_1, B_1)$ and $B(n_1, B_1)$. In the long run, i.e. in period 2, debt is fully repaid.

Accordingly, an equilibrium in this environment is defined as follows:

**Definition.** A subgame-perfect rational expectations equilibrium is a labour supply decision $n_1$, a tax rate $\tau_1$ and debt issuance $B_2$ such that:

- Given outstanding debt $B_1$, households form rational expectations about fiscal policy, and supply labour $n_1$ to maximize their intratemporal utility (2.1).

- Given $(n_1, B_1)$, the government sets the tax rate $\tau_1$ and issues debt $B_2$ to maximize aggregate lifetime utility (2.5) subject to its budget constraint (2.6) and borrowing limit (2.7).

Some comments are in order. First, we spell out the game and equilibrium definition as sequential actions, where households supply labour and then the government sets taxes. Similar economic interactions would prevail if moves were simultaneous. On the other hand, it is essential that the government does not move first. Indeed, if the government had a way to act as a Stackelberg leader and commit to its policy, it would naturally solve the coordination problem by choosing a tax rate on the left-hand side of the Laffer curve.

Second, although the government takes labour supply as given and therefore does not face a Laffer curve, Nash equilibrium requires consistency between the tax rate the private sector expects and the tax rate the government chooses. All equilibria must therefore be on the labour income Laffer curve, but not all points on the Laffer curve will be equilibria.

More importantly, the analysis will unveil conditions under which the equilibrium is unique or not. If the policy functions of households and the fiscal authority exhibit substitutability, which we interpret as fiscal policy being *countercyclical*, then there will be a unique equilibrium. If instead they exhibit complementarity, i.e. if fiscal policy is *procyclical*, then there may be multiple equilibria.\(^\text{12}\)

The next section is dedicated to deriving conditions on the inherited level of debt that give rise to complementarities and create the possibility of fiscal policy traps.

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\(^{12}\)Formally, since $dn(\cdot)/d\tau < 0$, the policy functions exhibit complementarities if and only if $d\tau(\cdot)/dn \leq 0$.  

34
2.3 Analysis

This section establishes the key result of the paper, namely that the level of debt is critical to the cyclicality of fiscal policy and can induce complementarities that give rise to fiscal policy traps. The argument is built on a geometric interpretation of the model in \((n_1, \tau_1)\) space.\(^\text{13}\) Equilibria in this environment can be represented by intersections of the labour supply function \(n(\tau_1)\) and the tax policy function \(\tau(n_1, B_1)\). We will show that there are three threshold levels of inherited debt, \(B^*_1 \leq \hat{B}_1 < \bar{B}_1\), such that when \(B_1 < B^*_1\) a unique equilibrium is guaranteed, when \(\hat{B}_1 < B_1 < \bar{B}_1\) there will be multiple equilibria, and when \(B_1 > \bar{B}_1\) there will not be any equilibria. This result will support our key idea that the level of debt is critical in creating the potential for self-fulfilling fiscal crises.

We begin by characterising the labour supply function, which is everywhere downward sloping and invariant to the inherited debt stock \(B_1\). We then characterize the government’s tax policy function, starting with the limits imposed by the government’s budget constraint and borrowing limit. Unlike the labour supply function, the tax policy function’s position and slope does depend on the inherited debt stock \(B_1\).

We then show that when inherited debt is sufficiently low \((B_1 < B^*_1)\), the tax policy function will be upward sloping (countercyclical) at least until it crosses the labour supply function, thereby ensuring a unique equilibrium. Then, we demonstrate that when the inherited amount of debt is high enough that the repayment of newly issued debt cannot be met without tax revenues in period 1 \((\hat{B}_1 < B_1 < \bar{B}_1)\), then the tax policy function will cross the labour supply function at least twice. This situation gives rise to multiple equilibria.

We conclude this section with an economic explanation of why the slope of the tax policy function is ambiguous and depends on the inherited debt stock \(B_1\). We decompose the government’s optimal response to a change in labour supply into two countervailing effects: a tax-base effect and a consumption-smoothing effect.

2.3.1 Properties of the Labour Supply Function

From (2.4) we know that labour supply is a monotonically decreasing function of the tax rate, so the labour supply function \(n(\tau_1)\) is downward sloping in \((n_1, \tau_1)\) space. Optimal labour supply is zero when the tax rate is 100 percent, and \(n(0) > 0\) when the tax rate is zero. The labour supply function starts at \((0, 1)\) and cuts the horizontal axis at \((n(0), 0)\). It continues below the horizontal axis, because greater effort can be induced by negative tax rates (i.e. labour income subsidies).

Optimal labour supply depends only on the tax rate \(\tau_1\), so the labour supply function will be unaffected by changes in the inherited debt stock \(B_1\) or in the government’s debt issuance \(B_2\). Figure 2.2 summarizes the properties of \(n(\tau_1)\), the reaction function of households.

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\(^\text{13}\)The geometric approach is very convenient, both for preserving generality of the results and for conveying the main intuitions underlying our analysis.
2.3.2 Properties of the Tax Policy Function

The number of intersections (and hence the number of equilibria) will therefore depend on the shape of the tax policy function, which, as we will show in this section, does depend on the debt stock $B_1$ as well as on the quantity of labour supplied, $n_1$. We will show that changes in $B_1$ both shift the tax policy function and alter its slope, thereby affecting the number of equilibria.

Constraints on the Government’s Choice of Tax Rate

Let us first consider the constraints the government faces. The borrowing limit $\hat{B}_2$ in (2.7) is the highest level of debt that the government can feasibly repay in period 2 (often referred to in the literature as the “natural” borrowing limit). This of course depends on the government’s fiscal capacity in period 2. Let the maximum rollover threshold debt level,

$$\hat{B}_1 = \frac{\bar{B}_2}{R - G_1}, \quad (2.11)$$

be the inherited debt level at which the government is exactly solvent in period 2 if it collects zero revenue in period 1. For debt levels strictly above this threshold, the government cannot repay its debts in period 2 without collecting some tax revenue in period 1. For debt levels strictly below this threshold, on the other hand, the government can in fact afford to subsidize labour supply in period 1 by setting a negative income tax rate $\tau_1 < 0$ and still be solvent in period 2.\footnote{Note that for large enough $G_1$, the threshold $\hat{B}_1$ will be negative, meaning that the government must inherit net claims on foreign wealth in order to be able to afford not to collect any revenue in period 1.}

For now, we only consider equilibria in which the government repays its debts in period 2. Accordingly, we define the lower bound on short-run labour supply $n(B_1)$ as the level of short-run labour supply at or below which the government’s fiscal policy is not
well defined because repayment of the debt is not feasible. Formally, we have:

\[ n(B_1) = \begin{cases} f^{-1}\left( \frac{B_1 - \hat{B}_1}{z_1} \right) & \text{if } B_1 > \hat{B}_1, \\ 0 & \text{if } B_1 \leq \hat{B}_1. \end{cases} \]

Since it is the short-run tax rate that matters for labour supply decisions, it will be convenient to rewrite the government’s constraints in terms of this tax rate. We define the minimum short-run tax rate \( \tau(n_1, B_1) \) as the tax rate in period 1 that, given the inherited debt level \( B_1 \), the economy’s tax base \( y_1 = z_1 f(n_1) \) and the government’s budget constraint (2.6), requires the government to issue debt up to its borrowing limit \( \bar{B}_2 \). The tax rate \( \tau(\cdot) \) is therefore the lowest tax rate in period 1 such that full repayment of the public debt is feasible in period 2. As the borrowing limit depends on the government’s long-run fiscal capacity, so will the minimum short-run tax rate. Formally, using the government budget constraint, \( \tau(\cdot) \) is given by:

\[ \tau(n_1, B_1) = \frac{B_1 - \hat{B}_1}{z_1 f(n_1)}, \quad n_1 > 0, n_1 \geq n(B_1). \]

Figure 2.3 illustrates the characterisation of the minimum short-run tax rate \( \tau(\cdot) \). As the inherited debt level \( B_1 \) increases, for a given labour supply \( n_1 \), the tax rate must rise to ensure long-run solvency, so the curve shifts up. If \( B_1 > \hat{B}_1 \), positive short-run tax revenue is needed to ensure long-run solvency, but the higher is the short-run labour supply \( n_1 \), the lower is the minimum tax rate. If \( B_1 < \hat{B}_1 \), the government can afford to set negative rates \( \tau_1 < 0 \) (i.e. to subsidize labour), but the higher is the short-run labour supply, the smaller this subsidy has to be. For \( B_1 = \hat{B}_1 \), no short-run revenue is needed to ensure long-run solvency, but the government cannot afford subsidies, either.

Of course, if the inherited level of debt \( B_1 \) is too high, the government will be unable to raise enough revenue to remain solvent, and there will be no equilibrium. Clearly, if the required revenue in period 1 exceeds that which would be raised at the peak of the Laffer curve, repayment will not be feasible. However, the maximum inherited debt level that can be sustained in equilibrium is less than this level. The government’s lack of commitment reduces the amount of tax revenue it can raise in equilibrium.\(^{15}\)

Accordingly, we define \( \hat{B}_1 \) as the upper limit on the amount of inherited debt \( B_1 \) that the government can sustain in equilibrium. It is derived as follows. In equilibrium, household’s expectations of the tax rate in period 1 must be correct, and labour supply must be optimal: \( n_1 = n(\tau_1) \). Equilibrium also requires that the tax rate is set optimally given the level of output and the inherited debt level, that is, \( \tau_1 = \tau(n_1, B_1) \). Equilibrium tax revenue in period 1 will therefore be given by the Laffer curve \( \tau_1 z_1 f\left(n(\tau_1)\right) \). Therefore, the maximum inherited debt level \( B_1 \) is such that, by raising the maximum tax revenue and issuing the maximum amount of debt \( \bar{B}_2 \), the government has just enough resources to finance its spending \( G_1 \) in period 1. It is the highest level of inherited debt \( B_1 \) that satisfies the

\(^{15}\)If workers were to supply the amount of labour consistent with the peak of the Laffer curve, the government would optimally choose to raise the tax rate.
following two equations:

\[ R \left( \bar{B}_1 + G_1 - \tau(n_1, \bar{B}_1)z_1f(n(\tau(n_1, \bar{B}_1))) \right) = \bar{B}_2, \]

\[ \tau_1 = \tau(n_1, \bar{B}_1). \]

**Borrowing Limit Does Not Bind in Equilibrium**

We have now defined all the ingredients necessary to prove that the borrowing limit (2.7) does not bind in equilibrium. This justifies restricting our attention to interior solutions of the government’s maximization problem. This point is formalized in Lemma 1.

**Lemma 1.** For all \( B_1 < \bar{B}_1 \) and for all \( n_1 > n(B_1) \), we have \( B(n_1, B_1) < \bar{B}_2 \) and \( \tau(n_1, B_1) > \bar{\tau}(n_1, B_1) \). That is, the borrowing limit (2.7) does not bind, and the optimal short-run tax rate is strictly greater than what is required for long-run solvency.

**Proof.** Suppose, on the way to a contradiction, that there exist \( B_1 < \bar{B}_1 \) and \( n_1 > n(B_1) \) such that the optimal debt issuance is \( B(n_1, B_1) = \bar{B}_2 \) and the optimal short-run tax rate is \( \tau(n_1, B_1) = \bar{\tau}(n_1, B_1) \). From (2.9), we have \( V'(\bar{B}_2) = -\infty \). Given \( n_1 > n(B_1) \) and our curvature assumptions on the utility function, for all \( \tau_1 < 1 \) we have \( u'(1 - \tau_1)z_1f(n_1) < +\infty \). The combination of \( V'(\cdot) = -\infty \) and \( u'(\cdot) < +\infty \) violates the government’s first-order condition (2.10). Given \( B_1 < \bar{B}_1 \) and \( n_1 > n(B_1) \), it is feasible for the government to raise the short-run tax rate to \( \bar{\tau} \in (\bar{\tau}(n_1, B_1), 1) \) and reduce debt issuance to \( \bar{B}_2 < \bar{B}_1 \).

Relative to the candidate policy, this alternative policy produces an arbitrarily large long-run marginal benefit at a strictly finite short-run marginal cost, and so the candidate policy \( B(n_1, B_1) = \bar{B}_2 \) and \( \tau(n_1, B_1) = \bar{\tau}(n_1, B_1) \) cannot be optimal. ■
This Lemma tells us that the optimal short-run tax rate $\tau(n_1, B_1)$ will be the interior solution implicitly defined by the first-order condition (2.10). Since the government’s budget constraint (2.6) will be satisfied with equality, the debt issuance decision $B(n_1, B_1)$ will be given by:

$$B_2 = R\left(B_1 + G_1 - \tau(n_1, B_1)z_1f(n_1)\right).$$

Since households’ decisions depend only on the tax rate $\tau_1$, we are interested mainly in the properties of the tax policy function $\tau(n_1, B_1)$.

**Optimal Tax Rate Is Increasing in Inherited Debt**

We first show that an increase in $B_1$ induces an increase in the tax rate $\tau_1$ for any level of labour supply $n_1$.\(^ {16}\)

**Lemma 2.**

$$\frac{d\tau(n_1, B_1)}{dB_1} = \frac{\beta R^2 V''(\cdot)}{z_1 f(n_1) \left(u''(\cdot) + \beta R^2 V''(\cdot)\right)} > 0. \quad (2.12)$$

*Proof.* The expression is derived by totally differentiating the government’s first-order condition (2.10) with respect to $B_1$ and rearranging. Standard assumptions on the curvature of the utility functions, $u''(\cdot) < 0$ and $V''(\cdot) < 0$, guarantee that the expression is positive. \(\blacksquare\)

The economic intuition behind this result is straightforward. An increase in the inherited debt stock $B_1$ means the government is poorer overall. In order to remain solvent, it must raise taxes in period 1, period 2, or both. Given that the marginal utility of consumption is decreasing in both periods, optimality requires the government to spread the pain of an increase in $B_1$ over both periods, meaning the short-run tax rate $\tau_1$ must rise.

**Tax Policy Function Is Upward Sloping Whenever Negative**

We are mainly interested in the slope of the tax policy function, that is, how the optimal tax rate responds to changes in labour supply. Taking the total derivative of (2.10) with respect to $n_1$, we get:

$$\frac{d\tau(n_1, B_1)}{dn_1} = \frac{f'(n_1)}{f(n_1)} \left(\frac{(1 - \tau_1)u''(\cdot) - \tau_1 \beta R^2 V''(\cdot)}{u''(\cdot) + \beta R^2 V''(\cdot)}\right). \quad (2.13)$$

In general the sign of this expression will be ambiguous, and will depend on the inherited debt stock $B_1$.\(^ {17}\) However, the expression is unambiguously positive (and therefore the tax policy function is upward sloping, i.e. countercyclical) whenever the short-run tax rate $\tau_1$ is negative:

\(^{16}\)In $(n_1, \tau_1)$ space, this feature is represented by an upward shift of the tax policy function as the inherited debt stock $B_1$ increases.

\(^{17}\)In section 2.3.4 below, we provide some economic analysis of this ambiguity by decomposing the government’s response to a change in labour supply into a tax-base effect and a consumption-smoothing effect.
Lemma 3. \[ \frac{d\tau(n_1, B_1)}{dn_1} > 0 \quad \forall \tau(n_1, B_1) < 0. \]

Proof. Totally differentiating the government’s first-order condition (2.10) with respect to \(n_1\) and rearranging yields:

\[ \frac{d\tau(n_1, B_1)}{dn_1} = \frac{f'(n_1)}{f(n_1)} \left( \frac{u''(c_1)}{u''(c_1) + \beta R^2 V''(B_2)} - \tau(n_1, B_1) \right). \]

Since \(\beta \geq 0\), \(u''(\cdot) < 0\) and \(V''(\cdot) < 0\), we have

\[ \frac{u''(c_1)}{u''(c_1) + \beta R^2 V''(B_2)} \in [0, 1]. \]

Since \(f(\cdot) > 0\) and \(f'(\cdot) > 0\), the whole expression must be positive whenever \(\tau(n_1, B_1) < 0\).

2.3.3 Equilibria

Combining the analysis of households’ labour supply function and the government’s tax policy function, we are now ready to derive conditions under which the equilibrium of the economy is unique or not, i.e. conditions under which fiscal policy traps can arise.

Unique Equilibrium When Debt Is Low

Combining Lemmas 2 and 3, we can show that there will be a unique equilibrium whenever the inherited debt stock \(B_1\) is sufficiently low.

**Proposition 6.** Let \(B_1^*\) be such that \(\tau(n(0), B_1^*) = 0\). Then for all \(B_1 < B_1^*\), there will be a unique equilibrium.

Proof. From Lemma 2 we know that \(\tau(n(0), B_1) < \tau(n(0), B_1^*) = 0\) for all \(B_1 < B_1^*\), that is, the optimal tax rate will be negative whenever labour supply is \(n(0)\) and inherited debt is less than the threshold value \(B_1^*\). Then from Lemma 3 we know that whenever \(B_1 < B_1^*\) the tax policy function will be negative valued and upward sloping for all values of labour supply \(n_1 \leq n(0)\), and indeed will continue to slope upwards at least until it cuts the horizontal axis. Before it does so, it will cut the (downward-sloping) labour supply function exactly once.

Multiple Equilibria When Debt Is High

The government’s budget constraint (2.6) means that if it inherits a sufficiently large stock of debt \(B_1\), it will be forced to collect tax revenue in period 1 in order to stay within its borrowing limit (2.7). Whenever the inherited debt level \(B_1\) is high enough that the government must collect taxes in period 1 (but not so high that repayment becomes infeasible), the economy will exhibit multiple equilibria.
Proposition 7. Let $\hat{B}_1 = \hat{B}_2/R - G_1$, where $\hat{B}_2$ is the natural borrowing limit, and let $\bar{B}_1$ be the highest inherited debt level for which an equilibrium exists. Then for all $B_1 \in (\hat{B}_1, \bar{B}_1)$, the economy exhibits multiple equilibria.

Proof. For all $B_1 \in (\hat{B}_1, \bar{B}_1)$, we have $\tau(n(B_1), B_1) = \tau(n(B_1), B_1) = 1 > n^{-1}(n(B_1))$. That is, when inherited debt is above the maximum rollover threshold $\hat{B}_1$ and short-run labour supply is at its minimum value $n(\cdot)$, the government’s optimal short-run tax rate is 100 percent, because this is the only feasible choice. We know that 100 percent is higher than the tax rate that would induce labour supply of $n(\cdot) > 0$, because labour supply is decreasing in the tax rate and it is optimal not to work when the tax rate is 100 percent.

For all $B_1 \in (\hat{B}_1, \bar{B}_1)$, we have $\tau(n(0), B_1) > \tau(n(0), B_1) > n^{-1}(n(0)) = 0$. This says that, when inherited debt is above the maximum rollover threshold $\hat{B}_1$ and labour supply is at the value that would optimally be chosen if the tax rate were zero, the optimal tax rate is in fact strictly positive.

These two pieces tell us that the optimal tax curve lies above the labour supply curve at two points: when $n_1$ is at the minimum level consistent with solvency, $n(B_1)$, and when $n_1$ is at the point consistent with zero taxes, $n(0)$. There can’t be an equilibrium to the left of (i.e. with a lower labour supply than) $n(B_1)$, because solvency would be violated whatever fiscal policy the government chose. We also know that there can’t be an equilibrium to the right of (i.e. with a higher labour supply than) $n(0)$, because the labour supply curve is negative valued after that point, and $\tau(n_1, B_1)$ is strictly positive for all $n_1$ whenever $B_1 > \hat{B}_1$. So if an equilibrium exists, it must be between $n(B_1)$ and $n(0)$. Apart from the special case of tangency (with $B_1 = \bar{B}_1$), if the optimal tax curve crosses below the labour supply curve somewhere to the right of $n(B_1)$, it must cross it again in order to be above it at $n(0)$.

Figure 2.4: Existence of Fiscal Policy Traps
Welfare Ordering of Equilibria

**Proposition 8.** The equilibria in Proposition 7 with higher labour supply \( n_1 \) Pareto dominate those with lower labour supply.

**Proof.** Since all households are ex ante identical and all equilibria are symmetric, the welfare ordering of equilibria depends on the utility of the representative household.

All equilibria must lie on the labour supply curve \( n(\tau_1) \), which is downward sloping, so equilibria featuring higher short-run labour supply \( n_1 \) must also feature a lower short-run tax rate \( \tau_1 \). The short-run tax rate \( \tau_1 \) enters into the household budget constraint (2.2), and since labour supply cannot be negative, a reduction in \( \tau_1 \) expands the household’s choice set, meaning the household is (weakly) better off in period 1.

All that remains to be shown is that in equilibria with higher short-run labour supply, the representative household is also better off in period 2. Since \( V'(B_2) < 0 \), we need to show that the government’s optimal debt issuance \( B_2 \) is lower in equilibria featuring higher short-run labour supply \( n_1 \). To see this, note that optimal fiscal policy must satisfy the first-order condition (2.10):

\[
u'(1 - \tau_1)z_1 f(n_1)) = -\beta V'(B_2).
\]

Consider two equilibria, one “good” and one “bad”, with \( n^G_1 > n^B_1 \) and \( \tau^G_1 < \tau^B_1 \). Now suppose (on the way to a contradiction) that the good equilibrium features higher debt issuance: \( B^G_2 > B^B_2 \). Then from \( V''(B_2) < 0 \) we have \( V'(B^G_2) < V'(B^B_2) \), meaning \( -\beta V'(B^G_2) > -\beta V'(B^B_2) \). In order for the government’s first-order condition to be satisfied in both equilibria, we would therefore need \( u'(1 - \tau^G_1)z_1 f(n^G_1)) > u'(1 - \tau^B_1)z_1 f(n^B_1)) \). However, given that output is increasing in labour supply and \( u''(\cdot) < 0 \), this would require \( \tau^G_1 > \tau^B_1 \), which cannot be the case because by hypothesis the good equilibrium features a lower tax rate.

A lower tax rate in period 1 means households are wealthier in period 1. Since substitution effects dominate income effects, their response is to increase their labour supply, which increases the government’s tax base. This induces a reduction in the government’s optimal debt issuance, so households are wealthier in period 2 as well.

### 2.3.4 Tax-Base and Consumption-Smoothing Effects

In this subsection we provide some economic intuition for our main result that optimal fiscal policy is procyclical when the burden of inherited debt is large. We do so by providing a decomposition of the effect of a change in labour supply on the optimal tax rate. We identify two countervailing effects at play, which we label tax-base and consumption-smoothing effects.

Consider a reduction in period 1 labour supply \( n_1 \). Ceteris paribus, this reduces period 1 consumption relative to period 2 consumption, thereby providing the government with a consumption-smoothing motive to reduce the period 1 tax rate relative to the period 2 tax
rate. On the other hand, when the period 1 tax rate is positive, a reduction in period 1 labour supply shrinks the overall tax base. In order for the government to remain solvent, therefore, the average tax rate across periods 1 and 2 must rise.

Similarly to how the effect of a price change on demand can be decomposed into a substitution and an income effect, we can decompose the effect of a change in labour supply on the optimal tax rate by rewriting the slope of the tax policy function (2.13) as follows:

\[
\frac{d\tau(n_1,B_1)}{dn_1} = (1 - \tau_1) \frac{f''(n_1)u''(\cdot)}{f(n_1)\left(u''(\cdot) + \beta R^2 V''(\cdot)\right)} - \tau_1 z_1 f'(n_1) \frac{d\tau(n_1,B_1)}{dB_1}.
\]

The first term captures the consumption-smoothing effect, which is unambiguously positive (meaning a reduction in labour supply prompts a reduction in the tax rate i.e. that fiscal policy is countercyclical). The second term captures the tax-base effect, which operates through the impact of a change in labour supply on the total fiscal resources available to the government. It is therefore no accident that the size of the tax-base effect is linked to the effect of a change in the inherited debt stock on the optimal tax rate, \(d\tau(n_1,B_1)/dB_1\).

The relative strength of the tax-base and consumption-smoothing effects will determine the cyclicity of the government’s optimal fiscal policy. When the consumption-smoothing effect dominates, fiscal policy will be countercyclical and the tax policy function will be upward sloping. Noting that the size of both effects depends on the short-run tax rate \(\tau_1\), which itself depends positively on the inherited debt level \(B_1\) as per Lemma 2, we can see that the cyclicity of fiscal policy will depend on the inherited debt level.

However, the effect of inherited debt on the cyclicity of fiscal policy is not guaranteed to be monotonic in all cases. This potential non-monotonicity means that there may not necessarily be a cut-off level of debt above which fiscal policy switches from being countercyclical to procyclical. Nevertheless, Proposition 6 guarantees that fiscal policy will always be countercyclical over the relevant range of labour supply when inherited debt is below the threshold \(B_1^*\), ensuring a unique equilibrium. Similarly, Proposition 7 guarantees that there will be multiple equilibria (which requires that fiscal policy is at least locally procyclical) whenever inherited debt exceeds the maximum rollover threshold \(\hat{B}_1\).

Note that the sign of the tax-base effect depends on whether the period 1 tax rate is positive or negative. This provides the intuition behind the result in Lemma 3 that the tax policy function is upward sloping whenever the tax rate is negative. With a negative tax rate (i.e. a labour subsidy), a reduction in labour supply actually reduces the fiscal burden on the government. This reverses the usual sign of the tax-base effect, meaning it reinforces rather than counteracts the consumption-smoothing effect. With both effects acting in the same countercyclical direction, the government’s fiscal policy will be unambiguously countercyclical whenever the short-run tax rate is negative.
2.4 Example with Closed-Form Solutions

In this section we present an analytical example of the class of economies described previously, and clearly highlight the general result of section 2.3.

We adopt the following specification. In period 1, self-employed households convert labour effort into output using the following production function:

\[ y_1 = z_1 n_1^\alpha, \quad \alpha > 0, \]

where \( \alpha \) captures returns to scale. The government inherits a stock of debt \( B_1 \) owed to foreigners, chooses a proportional income tax rate \( \tau_1 \) and issues an amount of bonds \( B_2 \) (again to foreigners) at the risk-free interest rate \( R \).

Period 2, the long run, is an endowment economy in which the government levies lump-sum taxes. The per-capita endowment of output is \( y_2 \), and to economize on notation we normalize period 2 government expenditure, \( G_2 \), to zero.\(^{18}\)

The representative household’s lifetime utility is given by:

\[ U(c_1, n_1, c_2) = u(c_1) - g(n_1) + \beta u(c_2), \]

where instantaneous consumption utility is given by

\[ u(c_t) = \frac{c_t^{1-\sigma}}{1 - \sigma}, \quad \sigma \in (0, 1) \]

in both periods, and the disutility from labour effort in period 1 is given by

\[ g(n_1) = \frac{n_1^\gamma}{\gamma}, \quad \gamma > 0. \]

Substituting the budget constraint \( c_1 = (1 - \tau_1)y_1 \) and the production function into the objective function and solving the household’s first-order condition yields the following expression for optimal labour supply:

\[ n(\tau_1) = \left( \alpha ((1 - \tau_1)z_1)^{1-\sigma} \right)^\frac{1}{1-\sigma}. \]

The government faces the usual budget constraint (2.6). With lump-sum taxation in period 2, the natural borrowing limit \( \bar{B}_2 \) in (2.7) is given by the long-run endowment \( y_2 \), since long-run consumption \( c_2 = y_2 - B_2 \) cannot be negative. The government’s continuation utility \( V(B_2) \) from issuing an amount of debt \( B_2 \) is simply households’ utility \( u(y_2 - B_2) \) of consuming the amount left over after lump-sum taxes are levied on the endowment to pay off the debt. It follows immediately that conditions (2.8) and (2.9) on the continuation utility function are satisfied.\(^{19}\)

\(^{18}\)This normalization is innocuous because with lump-sum taxes in period 2, an increase in \( G_2 \) is equivalent to a decrease in \( y_2 \).

\(^{19}\)Formally, \( V'(B_2) = -u'(y_2 - B_2) = -(y_2 - B_2)^{-\sigma} < 0, \quad V''(B_2) = u''(y_2 - B_2) = -\sigma(y_2 - B_2)^{-1-\sigma} < 0 \) and \( \lim_{B_2 \to \bar{B}_2} V'(B_2) = \lim_{B_2 \to y_2} u'(y_2 - B_2) = \lim_{c_2 \to 0} c_2^{-\sigma} = +\infty. \)
The maximum rollover threshold level of inherited debt, above which the government must collect revenue in period 1 in order to remain solvent, is given by:

\[ \hat{B}_1 = \bar{B}_2 / R - G_1 = y_2 / R - G_1. \]  \hspace{1cm} (2.14)

### 2.4.1 Inherited Debt and the Cyclicality of Fiscal Policy

Solving the government’s optimization problem yields the following tax policy function:

\[ \tau(n_1, B_1) = \frac{(\beta R)^{1/\sigma}}{R + (\beta R)^{1/\sigma}} - \frac{R(\hat{B}_1 - B_1)}{R + (\beta R)^{1/\sigma}} \frac{n_1}{z_1 n_1^\alpha}. \]  \hspace{1cm} (2.15)

This solution allows us to characterize precisely how the cyclicality of fiscal policy depends on the inherited level of debt.

**Proposition 9.** The cyclicality of fiscal policy depends on the inherited debt level \( B_1 \) as follows:

\[
\frac{d\tau(n_1, B_1)}{dn_1} = \frac{R(\hat{B}_1 - B_1)\alpha}{(R + (\beta R)^{1/\sigma}) z_1 n_1^{1+\alpha}} \begin{cases} 
> 0 & \text{(countercyclical) if } B_1 < \hat{B}_1 \\
= 0 & \text{(acyclical) if } B_1 = \hat{B}_1 \\
< 0 & \text{(procyclical) if } B_1 > \hat{B}_1.
\end{cases}
\]

Accordingly, the equilibrium of the economy is unique if and only if \( B_1 < \hat{B}_1 \), and fiscal policy traps may emerge for high levels of inherited debt \( B_1 \).

**Proof.** Differentiation of (2.15) and application of Propositions 6 and 7.

In the proof of Proposition 7, we saw the general result that when public debt is above the maximum rollover threshold level \( \hat{B}_1 \), the government’s tax policy function must be at least *locally* procyclical. Proposition 9 shows that there is a starker relationship between the level of public debt and the cyclicality of fiscal policy in this particular case. For levels of debt above \( \hat{B}_1 \), fiscal policy is procyclical for *all* values of labour supply.

Since for any given value of inherited debt \( B_1 \) the government’s tax policy function is monotonic, we can guarantee that there is a unique cutoff value of \( B_1 \), below which there will be a unique equilibrium and above which there will be two equilibria.\(^{20}\) The three cases are illustrated in Figure 2.5. In panel (a), debt is below the threshold \( \hat{B}_1 \) and so the tax policy function is upward sloping for all values of labour supply. It therefore crosses the labour supply function just once, ensuring a unique equilibrium. Panel (b) shows that the equilibrium is also unique when inherited debt is equal to the threshold \( \hat{B}_1 \) and the tax policy function is horizontal. Whenever inherited debt exceeds this threshold, as in panel (c), the tax policy function is downward sloping for all values of labour supply and there are two equilibria.

Looking at equation (2.14) we can see that the threshold value of debt does not depend on contemporaneous parameters, such as productivity \( z_1 \), but only on future variables.

\(^{20}\)This is true whenever the tax policy function is monotonic for all values of \( B_1 \), not just for the particular example we consider here.
such as fiscal capacity $y_2$. Although an increase in productivity reduces the optimal tax rate for a given level of labour supply, it cannot eliminate the possibility of fiscal policy traps. No matter how high is productivity, if debt is above the maximum rollover threshold then fiscal policy will be procyclical. This supports the idea that future fiscal capacity is essential in steering the economy away from fiscal policy traps.

2.5 Robustness

So far, we have constrained the choice set of the government by assuming exogenous government spending and not allowing the possibility of defaulting on debt. Further, we ruled out the possibility for households to smooth consumption themselves by accessing international capital markets. These assumptions imposed strong restrictions on the fiscal capacity of the country. These elements gave rise to the key result of the analysis, namely that the level of inherited debt is decisive in inducing procyclical fiscal policy and paving the way for a self-fulfilling fiscal crisis. In this section we relax these assumptions and investigate the robustness of our result.

2.5.1 Endogenous Government Spending

Consider a government with a high inherited level of debt. Facing a low value of labour supply, would the government rather increase its tax rate or reduce government expenditure?

To endogenize the choice of public expenditure, we assume that households derive instantaneous utility $v(G_1)$ from public expenditure $G_1$ by the government. The next Proposition shows that the key result of the baseline model still holds, even if the possibility of adjusting government expenditure provides the government with some “breathing room”:
the threshold level of debt is higher, but above this threshold, fiscal policy is procyclical and there is still the risk of fiscal policy traps. We adapt the analytical specification introduced in section 2.4 above and assume that \( \nu(G_1) = \frac{G_1^{1-\sigma}}{1-\sigma} \). Formally, given \((n_1, B_1)\), the government solves:

\[
\max_{\tau, G_1, B_2} u\left((1 - \tau)z_1 f(n_1)\right) - g(n_1) + \nu(G_1) + \beta u(y_2 - B_2)
\]

subject to the usual government budget constraint (2.6) and borrowing constraint (2.7).

Note that with endogenous \( G_1 \), the maximum rollover threshold level of debt becomes:

\[
\hat{B}_1 = \frac{\hat{B}_2}{R} = \frac{y_2}{R},
\]

because the government has the option of setting \( G_1 = 0 \).

The solution to the government’s maximization problem gives the following tax policy function:

\[
\tau(n_1, B_1) = \frac{R + (\beta R)^{1/\sigma}}{2R + (\beta R)^{1/\sigma}} \frac{R(\hat{B}_1 - B_1)}{(2R + (\beta R)^{1/\sigma}) z_1 n_1^{1+\sigma}}.
\]

(2.16)

**Proposition 10.** The cyclicity of fiscal policy depends on the inherited debt level \( B_1 \) as follows:

\[
\frac{d\tau(n_1, B_1)}{dn_1} = \frac{R(\hat{B}_1 - B_1)\alpha}{(2R + (\beta R)^{1/\sigma}) z_1 n_1^{1+\sigma}} > 0 \quad \text{(countercyclical) if } B_1 < \hat{B}_1
\]

\[
= 0 \quad \text{(acyclical) if } B_1 = \hat{B}_1
\]

\[
< 0 \quad \text{(procyclical) if } B_1 > \hat{B}_1.
\]

Accordingly, the equilibrium of the economy is unique if and only if \( B_1 < \hat{B}_1 \), and fiscal policy traps may emerge for high levels of inherited debt \( B_1 \).

**Proof.** Differentiation of (2.16) and application of Propositions 6 and 7. □

Intuitively, when \( G_1 \) and \( c_1 \) are complements, the government will optimally choose to reduce \( G_1 \) in proportion with \( c_1 \) when labour supply \( n_1 \) decreases and the country is poorer. This allows the government to raise the tax rate by less than in the case with exogenous government expenditure. Nevertheless, once the government is above its short-run borrowing limit, it will have to raise the tax rate, preserving the risk of fiscal policy traps.

### 2.5.2 Allowing for Default on Newly Issued Debt

In our baseline model, we assume that the government is committed to repaying its debts in full in period 2. This commitment implies the limit \( \hat{B}_2 \) to the amount of debt the government can issue in period 1 (see equation (2.7)). This debt issuance limit, together with the tax-base effect that becomes stronger as the government approaches it, causes optimal fiscal policy to be procyclical when the inherited debt level is high.
We now relax the hard solvency constraint and allow the government to choose strategically in period 2 whether or not to repay its debts. We show that this does not eliminate the possibility of self-fulfilling fiscal crises. In fact, the lack of commitment to debt repayment, i.e. the prospect of default in period 2, tightens the borrowing constraint in period 1. This in turn decreases the threshold level of debt \( \hat{B}_1 \) above which the economy is susceptible to fiscal policy traps.

**Stochastic Long-Run Output and Strategic Default**

To develop this idea, we amend the model as follows. Let long-run output \( y_2 \) be stochastic, distributed uniformly on \([\hat{y}_2, \bar{y}_2]\). Denote \( F(\cdot) \) the cumulative distribution function of \( y_2 \). As in section 2.4 above, the government can use lump-sum taxes in period 2 to repay its debt \( B_2 \), in which case period 2 consumption will be \( c_2 = y_2 - B_2 \). If instead the government chooses to default in period 2, the economy suffers a proportional output loss \( \delta_2 \), so period 2 consumption becomes \( c_2 = (1 - \delta_2)y_2 \).

The default cost \( \delta_2 \) can be interpreted as the government’s degree of commitment to repaying its debts in period 2. The extreme case of \( \delta_2 = 1 \) induces a strong commitment to repay and captures the hard solvency constraint assumed up to now. At the opposite extreme of \( \delta_2 = 0 \), default is costless, so the government would always default. Given outstanding bonds \( B_2 \), it is optimal for the government to repay its debts in period 2 whenever output \( y_2 \) satisfies:

\[
y_2 - B_2 \geq (1 - \delta_2)y_2.
\]

This relation gives the threshold \( \hat{y}_2(B_2) \), realizations of \( y_2 \) below which the government defaults on its bonds \( B_2 \):

\[
\hat{y}_2(B_2) = \frac{B_2}{\delta_2}.
\]

Risk-neutral foreign investors anticipate the strategic default decision of the government. Accordingly, the price schedule \( q(B_2) \) satisfies the following no-arbitrage condition:

\[
q(B_2) = \frac{1 - F(\hat{y}_2(B_2))}{R},
\]

where \( R \) is the risk-free interest rate. In this expression, the credit risk associated with the issuance of bonds \( B_2 \) is captured by \( F(\hat{y}_2(B_2)) \), the probability that long-run output will be below the default threshold \( \hat{y}_2(B_2) \). The possibility of strategic default can lead to indeterminacy in the price schedule (2.18), as studied in Calvo (1988) and Cooper (2015). As our focus is on the occurrence of fiscal policy traps rather than self-fulfilling increases in sovereign risk premia, whenever several prices satisfy the price schedule (2.18) we assume that investors select the “fundamental” outcome with the lowest risk premium. In this case, the price of debt \( q(B_2) \) is decreasing in the amount of bonds \( B_2 \) issued, reflecting the increased probability of default.
Lack of Commitment to Repay Reduces Borrowing Limit

We are now ready to prove that despite its capacity to default on debt in period 2, the government may still be susceptible to fiscal policy traps. As in our baseline model, government borrowing between period 1 and 2 is constrained. This in turn induces a maximum level of inherited debt $\hat{B}_1$ such that the government can roll over its obligations without having to collect tax revenue in period 1. As in the baseline model, the economy is under the threat of fiscal policy traps whenever inherited debt $B_1$ is above this threshold. Interestingly, this threshold is increasing in the commitment parameter $\delta_2$. In other words, the less committed a country is to repaying its debt, the lower is the debt threshold at which it becomes vulnerable to fiscal policy traps.

**Proposition 11.** Whenever the government can default on its debt in period 2, there is a debt rollover threshold $\hat{B}_1$ above which the country is subject to fiscal policy traps. The threshold is increasing in the output loss parameter $\delta_2$ (i.e. an increase in commitment increases debt capacity).

**Proof.** We first demonstrate that there is a maximum amount of revenue that the government can raise in period 1, and that this amount is decreasing in $\delta_2$. The revenue raised in period 1 by issuing $B_2$ bonds is $q(B_2)B_2$, where the price schedule $q(B_2)$ satisfies (2.18). Using the default threshold (2.17), resources from debt issuance are:

$$q(B_2)B_2 = \frac{1 - F(B_2/\delta_2)}{R} B_2.$$  

The right-hand side is equal to 0 for $B_2 = 0$ and for $B_2 = \delta_2 \bar{y}_2$, and is strictly positive for any value of $B_2$ in between. Hence the right-hand side reaches a maximum for $B_2 = \bar{B}_2 = \big(0, \delta_2 \bar{y}_2\big)$. The maximum period 1 revenue from debt issuance is therefore $q(\bar{B}_2)\bar{B}_2$. Since the price of debt is strictly increasing in $\delta_2$, the revenue collected $q(B_2)B_2$ is also increasing in $\delta_2$, and so is the maximum amount that can be collected.

As in (2.11) above, there is a maximum amount of debt $\hat{B}_1$ that can be rolled over without raising any tax revenue in period 1. This threshold is increasing in the maximum amount of revenue that can be raised by issuing new debt, and therefore in $\delta_2$:

$$\hat{B}_1 = q(\bar{B}_2)\bar{B}_2 - G_1.$$  

If the stock of inherited debt $B_1$ exceeds the maximum rollover threshold $\hat{B}_1$, then as in our baseline model the government will have to gather revenue in period 1. Indeed, to remain within this limit the government must set a short-run tax rate at least equal to

$$\tau(n_1, B_1) = \frac{B_1 - \hat{B}_1}{z_1 f(n_1)}.$$  

It follows that, as before, when $B_1 > \hat{B}_1$ the optimal tax policy function is at least locally procyclical, and Proposition 7 applies.  

\[\blacksquare\]
The intuition behind this result is as follows. As the default cost $\delta_2$ falls, investors know that the government will default in more states of the world in period 2 because it faces a lower penalty for doing so. This causes them to charge a higher risk premium, thereby reducing the amount of revenue the government can raise in period 1 by issuing new debt.

Overall, allowing the government to default on its debts in period 2 does not, therefore, eliminate the possibility of self-fulfilling fiscal crises.

2.5.3 Private Access to International Markets

In our benchmark model, we made the simplifying assumption that households lived hand-to-mouth and could neither save nor borrow between periods 1 and 2. Here we consider the polar opposite case in which households have full access to international capital markets at the risk-free rate $R$.\textsuperscript{21} Allowing households to borrow and save across time allows them to smooth consumption when taxes change. Are fiscal policy traps still possible in this environment?

Whereas in section 2.4 we modelled period 2 as an endowment economy with lump-sum taxation, here we model period 2 as a production economy with endogenous labour supply. Under this set-up, labour taxation in period 2 influences the labour supply decision. This modelling approach is aimed at avoiding redundancy between public and private intertemporal decisions. Further, we assume that the government cannot tax saving or consumption directly. As we shall demonstrate below, when households can privately smooth consumption across time, the combination of an inelastic short-run tax base and an elastic long-run tax base leads the government to tax the short-run tax base at 100 percent. Fiscal policy is determinate but the outcome is unambiguously worse, since households use international markets to avoid taxes, which in turn induces the government to set the highest tax rate possible.

The timing is as follows. In period 1, households choose their borrowing, denoted $a$, and their short-run labour supply $n_1$.\textsuperscript{22} Having observed $n_1$ and $a$, the government then sets a short-run tax rate $\tau_1$ and issues new debt $B_2$. In period 2, households supply labour $n_2$, clear their borrowing position and consume. The government sets a tax rate $\tau_2$ to meet its budget constraint and clear its debt position. The solution is derived by backward induction.

Period 2 consumption is given by $c_2 = (1 - \tau_2)z_2f(n_2) - Ra$. With the disutility of labour captured by the function $g(\cdot)$, households solve the following problem:

$$V_H(a) = \max_{n_2} \left( (1 - \tau_2)z_2f(n_2) - Ra \right) - g(n_2).$$

Optimal period 2 labour supply $n_2(\tau_2, a)$ is implicitly defined by households’ intratem-

\textsuperscript{21}We thank Russell Cooper for hinting at this extension.

\textsuperscript{22}Saving is denoted by a negative value of $a$. 50
poral first-order condition:

\[(1 - \tau_2)z_2f(n_2)u'(c_2) = g'(n_2).\]  

(2.20)

The government’s period 2 budget constraint implicitly defines \(\tau_2(a, B_2)\):²³

\[\tau_2 z_2 f(n_2) = B_2,\]  

(2.21)

where \(n_2 \equiv n_2(\tau_2, a)\) is given by (2.20).

Now, in period 1, given \(\tau_1\) and \(\tau_2\), households solve the following problem:²⁴

\[
\max_{n_1, a} u\left( (1 - \tau_1)z_1 f(n_1) + a \right) - g(n_1) + \beta V_H(a),
\]

where \(V_H(a)\) is the value of the period 2 problem (2.19) when the household has borrowed \(a\). The household intertemporal first-order condition is therefore given by:

\[
u'( (1 - \tau_1)z_1 f(n_1) + a ) = -\beta V_H'(a)
\]

\[u'(c_1) = \beta Ru'(c_2).\]

(2.22)

Unlike households, the government internalizes the effect of its period 1 choices on the tax rate in period 2. When the government issues debt \(B_2\), its continuation value is given by:

\[V_G(a, B_2) = u\left( (1 - \tau_2(a, B_2))z_2 f(n_2(\tau_2(a, B_2))) \right) - Ra - g(n_2(\tau_2(a, B_2))).\]

The government’s period 1 problem is therefore:

\[
\max_{\tau_1} u\left( (1 - \tau_1)z_1 f(n_1) + a \right) - g(n_1) + \beta V_G(a, B_2),
\]

where government debt issuance is given by the budget constraint \(B_2 = R\left( B_1 - \tau_1 z_1 f(n_1) \right)\). The government’s intertemporal first-order condition is:

\[
u'( (1 - \tau_1)z_1 f(n_1) + a ) = -\beta R \frac{dV_G(a, B_2)}{dB_2}
\]

\[u'(c_1) = \beta Ru'(c_2) z_2 f(n_2) \frac{d\tau_2(a, B_2)}{dB_2}.\]  

(2.23)

Comparing the household intratemporal first-order condition (2.22) with that of the government (2.23), we see that they differ by the term \(z_2 f(n_2) \frac{d\tau_2(a, B_2)}{dB_2}\), which captures the distortionary effects of taxation in period 2, internalized by the government. This difference leads to the striking implication of this extension of our model:

²³As in Section 2.5.2, in order to abstract away from other sources of coordination failures, we select the tax rate on the left-hand side of the Laffer curve in period 2. As above, period 2 government spending is normalized to zero.

²⁴Although the tax rates \(\tau_1\) and \(\tau_2\) are determined by aggregate household borrowing and labour supply decisions, each individual household takes them as given when making its own decisions.
Proposition 12. When households have access to international capital markets and taxation is distortionary in period 2, there is a unique equilibrium with \( \tau_1 = 1, n_1 = 0 \) and \( a > 0 \).

Proof. Comparing the household intertemporal first-order condition (2.22) and the government’s intertemporal first-order condition (2.23), we see that the right-hand side of the latter contains an additional term \( z_2 f(n_2) \frac{dz_2(a,B)}{dB_2} > 0 \). This means that only one of these expressions can hold in equilibrium. Households’ period 2 labour effort \( n_2 \) is unbounded above and they are committed to repay their debts \( Ra \) in full, so unlike the government they face no borrowing constraints. This ensures that households’ intertemporal first-order condition holds in equilibrium and the government’s does not.

If the government’s intertemporal first-order condition does not hold, then the tax rate \( \tau_1 \) it sets must be one of the corner solutions. Given that the extra term on the right-hand side of the government’s intertemporal first-order condition is positive, then if the households’ first-order condition holds, the government wants to increase the term on the left-hand side, \( u'(c_1) \). Since \( u''(\cdot) < 0 \), this means the government wants to decrease \( c_1 = (1 - \tau_1)z_1 f(n_1) \), which it does by setting \( \tau_1 \) as high as possible.

Given \( \tau_1 = 1 \), it is optimal for households to supply \( n_1 = 0 \). Finally, since \( u'(0) = +\infty \), intertemporal optimization by households requires \( a > 0 \). ■

In contrast to the baseline model with hand-to-mouth households, the equilibrium outcome when households have access to perfect private capital markets is characterized by a unique tax rate in period 1, independent of the level of inherited debt \( B_1 \). Still, the equilibrium outcome is unambiguously worse, since no production occurs in period 1 and the whole burden of taxation and debt repayment is postponed to period 2.

The intuition behind this seemingly perverse result is that households’ access to capital markets undermines the government’s consumption-smoothing motive for keeping the period 1 tax rate low. Households then avoid high period 1 taxes by cutting their labour supply, and borrow to preserve their period 1 consumption level. The implication is that the absence of perfect private capital markets is beneficial in this environment, because it partially compensates for the government’s inability to commit not to tax the inelastic short-run tax base excessively.

2.6 Conclusion

The recent rise (and subsequent fall) of sovereign debt spreads in the euro area periphery has prompted renewed interest in multiple equilibria and self-fulfilling crises. Yet it was not only countries facing increased borrowing costs that pursued contractionary fiscal policies during the Great Recession.

In this paper we have proposed a potential explanation for why governments might pursue procyclical fiscal policies despite not facing increased sovereign risk premia. When the inherited stock of public debt is sufficiently high, concerns about the burden of future taxes may overweek concerns about preserving consumption in the face of a decline in output, making even optimal fiscal policy procyclical. This procyclicality unleashes the
possibility of a different kind of crisis, fuelled not by self-fulfilling fears of higher sovereign spreads but by self-fulfilling fears of a decline in output.

**Bibliography**


