



Essays in Macroeconomic Theory: Fiscal and Monetary Policy

Antoine Camous

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Doctor of Economics of the European University Institute

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Abstract

This thesis investigates the design of appropriate institutions to ensure the good conduct of fiscal and monetary policy. The three chapters develop theoretical frameworks to address the time-inconsistency of policy plans or prevent the occurrence of self-fulfilling prophecies.

Time-inconsistency refers to a situation where preferences over policy change over time. Optimal policy plans are not credible, since agents anticipate the implementation of another policy in the future. This issue is particularly pervasive to monetary policy, since nominal quantities (price level, interest rates, etc.) are very sensitive to expected policies, but predetermined to actual policy choices.

The first chapter investigates how fiscal policy can mitigate the inflation bias of monetary policy in an economy with heterogeneous agents. Whenever there is a desire for redistribution, progressive fiscal helps to implement a policy mix less biased toward inflation. Importantly, even the richest supports some fiscal progressivity, since over their life cycle, they benefit from a more balanced policy-mix.

A self-fulfilling prophecy, or coordination failure, refers to a situation where a more desirable economic outcome could be reached, but fail to be, by the only effect of pessimistic expectations. Self-fulfilling debt crises are a classical example: pessimistic investors bid down the price of debt, which increases the likelihood of default, which in turn justifies the initial decrease in price.

The second chapter, co-authored with Russell Cooper, asks whether monetary policy can deter self-fulfilling debt crises. The analysis shows how a counter-cyclical inflation policy with commitment is effective in doing so. Importantly, it can be implemented without endangering the primary objective of monetary policy, to deliver an inflation target for instance.

The third chapter, co-authored with Andrew Gimber, revisits the classic Laffer curve coordination failure: taxes could be low, but they are high because agents anticipate high tax rates. In a dynamic environment with debt issuance, the multiplicity of equilibria critically depends on inherited debt. At high levels of public debt, fiscal policy is pro-cyclical: taxes increase when output decreases, and self-fulfilling fiscal crisis can occur. Overall, this chapter sheds light on the perils of high level of public debt.

*à mes parents,
et à mon frère, Julien.*

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Mannheim, November 12th 2015

Preface

This thesis investigates the design of appropriate institutions to ensure the good conduct of fiscal and monetary policy. The three chapters develop theoretical frameworks to study and address the time-inconsistency of optimal policy plans or prevent the occurrence of self-fulfilling prophecies.

Time-inconsistency refers to a situation where preferences over policy plans change over time. Optimal policy plans are not credible, since agents anticipate the implementation of another policy in the future. This issue is particularly pervasive to monetary policy, since nominal quantities (price level, interest rates, etc.) are very sensitive to expected policies, but predetermined to actual policy choices. A self-fulfilling prophecy, or a coordination failure, refers to a situation where a more desirable economic outcome could be reached, but fail to be, by the only effect of pessimistic expectations. A classic example is self-fulfilling debt crisis: pessimistic investors bid down the price of debt, which increases the likelihood of default, which in turn justifies the initial decrease in price.

The thesis consists in three chapters, related to fiscal and monetary policy, each asking a specific research questions: can fiscal policy contribute to mitigate the time-inconsistency of monetary policy? Can central bank interventions deter self-fulfilling debt crisis? Can fiscal crisis be avoided with active debt policy?

Sequential monetary policy is biased toward inflation since nominal quantities are pre-determined to policy decisions. In equilibrium, welfare losses stem from the anticipation of the *inflation bias*. This issue has long been studied but none of the proposed solutions, as for instance appointing a *conservative central banker*, is fully satisfactory.

The first chapter investigates how fiscal policy can address the time-inconsistency of monetary policy. The model considers an environment where agents differ in ability, hence in income level, and a public good must be financed by resorting to a mix of money seignorage and labor income tax. I introduce the possibility for fiscal policy to be *progressive*, where agents with higher income face higher marginal tax rates. Importantly, I allow the policymaker to determine the progressivity of the tax schedule one period in advance, so that it is predetermined to monetary policy choices. Then, the desirability of redistribution from rich to poor agents implements a policy-mix less biased toward inflation.

The analysis reveals the following elements. First, in the case where the policymaker does not value redistribution across the population, progressivity is not part of the policy mix, for it strengthens the welfare costs associated with labor taxation. The outcome is the most extreme form of inflation bias.

To analyze whether progressivity in conjunction with redistributive concerns is effective to mitigate the inflation bias, I design a two-stage political game. In the second stage, individuals vote over the policy mix given the progressivity of the tax schedule and predetermined money holding. Individual preferences reveal strategic choices: on the one hand, every agent weighs the

cost of distortionary taxation on their current labor supply decision; on the other hand, they consider how progressivity shifts the burden of labor taxation to richer agents. Poorer agents favor then lower inflation and higher labor taxes. The outcome of the vote is that the inflation bias is reduced, and so whenever the fiscal tax schedule displays some progressivity.

Next, I consider the first stage of the game, where progressivity is determined before agents form their saving decision. At this stage, individual demand for money is sensitive to expected inflation. The analysis reveals then that all agents, rich or poor, support some level of progressivity. Indeed, by reducing the inflation bias, progressivity helps to implement a more balanced policy mix.

Finally, I show using numerical simulations how the welfare properties of this economy depend on *inequalities*, understood here as the dispersion of individual abilities. Essentially, the capacity of fiscal progressivity to curb welfare losses from the inflation bias is strengthened by a larger dispersion of productivity.

The second chapter, co-authored with Russell Cooper, considers an environment with debt fragility, namely sovereign default driven by coordination failure among investors: negative investors' sentiment results in an increase in borrowing costs, leaving the government with a higher debt burden and a higher probability of default.

Our environment features a central bank that prints money and transfer seignorage revenue to the treasury. The fiscal authority chooses to raise complementary labor taxes and repay its debt or default on its nominal obligations. The monetary intervention has several influences. The inflation tax provides real resources to the government, thus reducing required labor taxes to service debt. Also, the realized value of inflation alters the real value of debt and consequently the debt burden left to the fiscal authority. Finally, expectations of future inflation and thus the tax base for seignorage are determined by the monetary regime.

Do monetary interventions deter self-fulfilling debt crisis? The answer depends critically on the monetary policy regime. Under strict inflation targeting, nominal debt is de facto a real non contingent asset and debt fragility persists. Whenever the central bank commits to provide as much resources as necessary to repay debt, private agents anticipate the monetary bailout, price an inflation premium into nominal interest rates and reduce their demand for money. In effect, they neutralize the capacity of the central bank to intervene and debt fragility persists.

Finally, we derive a monetary policy rule that both anchors inflation expectations and deters self-fulfilling debt crisis. This policy is reminiscent of the commitment of the European Central Bank to undertake "*whatever it takes*" to counter pessimistic self-fulfilling expectations in the Eurozone. Under this rule, no actual intervention is required and debt is uniquely valued. Moreover, this policy does not endanger the primary objective of the central bank, to anchor inflation expectations around an inflation target.

The third chapter, joint with Andrew Gimber, investigates the role of public debt in fiscal crisis.

We consider an environment with a potential Laffer curve coordination failure: in the "good" equilibrium, labor supply is high because workers anticipate a low tax rate, and the government chooses a low tax rate because output is high. In the "bad" equilibrium, labelled *fiscal trap*, workers restrict their labor supply in anticipation of a high tax rate, and the resulting low output induces the government to fulfill their pessimistic expectations with high taxes.

In a dynamic environment with debt issuance, the multiplicity of equilibria critically depends on the intertemporal liabilities of the government. The analysis isolates inherited debt as a special element of the government budget constraint, since it is generally predetermined to any fiscal decision program (income, expenses, debt issuance or default). We show that there is a threshold level of debt above which the economy is vulnerable to self-fulfilling *fiscal traps*.

Two countervailing effects drive fiscal choices: on the one hand, a decrease in output induces the government to decrease taxes and preserve current consumption; on the other hand, to ensure the solvency of its debt position, the government wishes to raise taxes. When the inherited stock of public debt is low, the former effect dominates and fiscal policy is countercyclical. The economy is characterized by a unique equilibrium. When the inherited level of public debt is high, the second effect dominates. Optimal fiscal policy then becomes pro-cyclical, because deferring tax increases when output is low would impose an unacceptable burden on future consumption.

In this case, the country is vulnerable to self-fulfilling fiscal crisis, stemming from private agents coordination failure. In this case, a "bad" equilibrium can arise, where workers restrict their labor supply in anticipation of a high tax rate, and the resulting low output induces the government optimally to fulfill their pessimistic expectations with high taxes. We further investigate the robustness of our result to alternative scenarios, allowing for adjustments in public spending, debt default and private access to international markets.

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Chapter 1

Fiscal Discipline on Monetary Policy

1.1 Introduction

Standard views on monetary-fiscal interactions state that monetary policy generates redistribution, but it cannot do much about it. Indeed, monetary policy is and ought to be a 'blunt' tool. Fiscal policy on the other hand, with the appropriate set of targeted instruments, could address these redistributive concerns. Each authority plays its score. This view is for instance supported by a former Chair of the Federal Reserve:

Policies designed to affect the distribution of wealth and income are, appropriately, the province of elected officials, not the Fed (...) Monetary policy is a blunt tool which certainly affects the distribution of income and wealth (...) Other types of policies are better suited to addressing legitimate concerns about inequality. [Bernanke (2015)]¹

This paper argues that fiscal-monetary interactions are more subtle. Fiscal policy should not be confined to undo the redistributive consequences of monetary policy. Especially, fiscal policy has the unique capacity to shape heterogeneity within the economy and influence decisively the conduct of monetary policy under discretion.

The underlying problem considered in the present analysis is the classic time-inconsistency of optimal policy plans, initially uncovered by Kydland and Prescott (1977). As time goes by, optimal policy changes. Indeed, once expectations are set and private decisions taken, the policymaker no longer factors in the expectational benefits of the optimal policy plan. This intertemporal inconsistency generates welfare losses, since private agents anticipate the conduct of future policies.

This issue is particularly pervasive in nominal economies, as shown by Calvo (1978) for instance. Indeed, nominal quantities (interest rates, money holding) are crucially sensitive to expectations, but policies are implemented once expectations are locked-in and real decisions made.

Several institutional solutions have been proposed to address this issue. The monetary authority could engage its reputation to prevent deviations from the announced policy plan, as in Barro and Gordon (1983). Alternatively, the strategic appointment of a conservative central banker could mitigate the excessive use of the inflation tax, as proposed by Rogoff (1985).²

This paper suggests a novel institutional feature to mitigate the welfare cost of monetary discretion and support time-consistent policies. Formally, the present analysis asks whether progressive fiscal policy can curb the inflation bias of a monetary authority acting under discretion. The analysis stresses that progressive fiscal policy generates redistributive conflicts over policy choices, and the resolution of this conflict limits the inflation bias.

¹Source: Brookings, *Monetary Policy and Inequality*, June 2015.

²These references do not cover the whole set of recommendations proposed by the literature.

I consider an environment with heterogeneous agents, where a public good is financed by resorting to a mix of money seignorage and labor income taxes. The source of heterogeneity across agents is lifetime productivity. The environment is structured to highlight dynamic distortions on labor supply decisions and conflicts across agents. Informational restrictions prevent the use of first-best type-specific lump sum taxes. Accordingly, any form of taxation is distortionary. Still, the elasticity of the seignorage tax base changes over time. The optimal policy is not time-consistent and an inflation bias arises when policy is implemented sequentially.

I introduce in this environment the possibility for fiscal policy to be progressive, namely that as income increases, marginal tax rates increase above average tax rates. In effect, this paper studies a three parameters taxation program. The labor tax plan is captured by a level and a progressivity parameter. The inflation tax operates as a proportional tax on money holding. As in Werning (2007), introducing progressivity on the labor tax plan introduces both redistribution within the private sector and productive efficiency considerations.

Consider as a benchmark a benevolent policymaker. With agents' linear utility in consumption, the planner has no redistributive concerns. The optimal policy plan under commitment requires to tax labor and money holding evenly, with no progressivity, and so to spread uniformly aggregate distortions on labor supply decisions - across time and across the population. Under discretion though, real money holdings are pre-determined to the tax collection decision, hence the temptation to rely predominantly on the inflation tax. The latter is then not distortionary compared to labor income taxes. Agents anticipate the willingness of the policymaker to resort to the inflation tax and reduce their demand for money accordingly. This leads in equilibrium to a classic inflation bias and welfare losses: inflation that is costless *ex post* is costly from an *ex ante* perspective. Overall, progressivity generates only further distortions on labor supply decisions and is *a priori* not desirable in an environment with only efficiency concerns.

Next, I relax the productive efficiency objective of the benevolent planner and study the determinants of policy parameters when redistributive effects are taken into account. To do so, I build a two-stage political game, where progressivity is determined one period in advance and the tax mix is determined contemporaneously to the provision of the public good by majority voting.

This approach allows me to study specific determinants of the redistributive - efficiency trade-offs induced by fiscal progressivity. The voting mechanism outlines how progressivity generates redistributive conflicts across the population over the tax mix. This stage of the analysis reveals the *equity* component, or redistributive nature of progressivity. Then, I analyze whether the resolution of these conflicts justifies to commit to progressivity.³ This step outlines the potential for progressivity to support intertemporal *efficiency* by providing the right dynamic incentives.

In the second stage of the game, agents vote over the relative mix of inflation and labor taxes, given the progressivity of the tax plan and the distribution of real money holding. Individual preferences reveal strategic choices of individuals. On the one hand, every agent weighs their

³The pre-commitment to progressivity reflects *tax inertia*, as in Farhi (2010) or Ferriere (2015): some components of the tax code, such as '*assiette*' or '*progressivity*' need time to be adjusted and are thus predetermined to the decision of the '*level*' of taxes to be collected.

individual exposure to each source of taxes. Agents are naturally biased toward the inflation tax, since predetermined money holding form an inelastic tax base at this stage. On the other hand, they consider how progressivity shifts the burden of labor taxation towards richer agent.

With proportional labor taxes (no progressivity), agents unanimously support the inflation tax to reap the inelastic tax base. With progressivity, redistributive conflicts emerge. Low productivity agents support labor taxes, whereas high productivity agents vote for inflationary policies, to contain the welfare cost induced by high tax distortions on their labor supply. Under a progressive tax plan, the decisive median agent favors moderately inflationary choices, thereby reducing the magnitude of the inflation tax.

In the first stage of the game, agents learn their type, anticipate inflation, supply labor and save. When asked about their taste for progressive tax plan, they weigh the disincentive effect of progressivity, their exposure to labor taxes and the beneficial effect of curbing the inflation tax. The central result of the analysis is that all agents favor positive level of progressivity for its dynamic incentives provision.

Still, the choice of progressivity is not monotonic in productivity. The lowest productivity agent support the level of progressivity that maximize the use of labor taxes; the highest productivity agent support progressivity just enough to equalize marginal utilities over each of its tax base.

Finally, I show using numerical simulations how the optimal level of progressivity is influenced by the distribution of productivity over the population. Essentially, the capacity of fiscal progressivity to curb welfare losses from monetary discretion is enhanced by a larger dispersion of productivity. For higher level of variance in the distribution of productivity, the induced allocation and intertemporal welfare gets closer to the full commitment allocation of the benevolent planner.

Overall, the political economy analysis allows to disentangle *equity* and *efficiency* concerns generated by fiscal progressivity, by first outlining redistributive conflicts between agents given the progressivity of fiscal policy, and then characterizing the level of progressivity to sustain intertemporal time consistent policy plans.

Albanesi (2003) investigates whether monetary and fiscal policy plans are time consistent in an economy with cash and credit goods, as in Lucas and Stokey (1983). The central result is that the policy plan can be time-consistent under a specific distribution of nominal assets across the population, but the analysis is silent on how to implement this distribution. In contrast, Camous and Cooper (2014) analyze the choices of a discretionary policy maker in an environment with heterogenous agents but absent redistributive concerns. They find that a strong inflationary bias emerges. This project aims at investigating the impact of redistributive concerns within a related environment.

The general idea of the present analysis is to shape redistributive forces within a heterogenous economy to support time consistent policy plans. A close analysis can be found in Farhi, Sleet, Werning, and Yeltekin (2012), in the context of capital taxation in an economy with imperfect commitment. Progressivity optimally emerges as part of a dynamic plan, since it mitigates inequalities

and the associated temptation to reduce them by exerting a capital levy.⁴

Cooper and Kempf (2013) investigates the credibility of deposit insurance in a heterogeneous Diamond-Dybvig economy. Redistributive concerns play a key role in the decision of the government to intervene *ex post* in case of a bank run, but the credibility of deposit insurance can be ensured with an appropriate *ex ante* tax scheme.

A similar idea is applied by Ferriere (2015) in a public debt environment with strategic default: committing to progressive fiscal policy allows to influence the default decision *ex post* and the price of debt issuance *ex ante*.

The rest of the document is organized as follow. Section 1.2 describes the economic environment and the properties of progressive fiscal plans. In Section 1.3, I characterize the optimal policy plan of a benevolent planner, to highlight the time inconsistency of monetary policy and the inflation bias. Then I define in Section 1.4 the political economy environment and study the outcome of the game. Section 3.6 concludes by discussing how the insights of the analysis would generalize to richer environments.

1.2 Economic Environment

In this section, I set up a simple economy with heterogeneous agents to analyze the influence of progressive fiscal policy on the conduct of monetary policy. Time is discrete and finite. In effect, the model is built as a real economy. Still, the analogy in notations and interpretations with a nominal economy is kept on purpose and justified along the exposition. The environment is parsimonious enough to capture the main forces at play, i.e. the distribution of taxes and distortions on labor supply decisions. Especially, prices are flexible and the only cost of inflation derives from expectations. These features allow to focus neatly on conflicts arising from tax policy choices.⁵

1.2.1 Environment

Private Economy

Consider a two-period economy, $t \in \{1, 2\}$, populated by a mass 1 of agents, and a government that needs to finance an exogenous amount of expenses g at $t = 2$. Agents differ in lifetime labor productivity z , distributed over the population according to the cumulative distribution function $F(\cdot)$ defined on the compact set $[z_l, z_h]$, with $0 < z_l < z_h \leq 1$.⁶

The preferences of an agent of type z over consumption and labor are represented by a utility function $U(z; c_2, n_2, n_1)$. Agents work and save when young ($t = 1$), work and consume when old

⁴Scheuer and Wolitzky (2014) investigates time-consistent dynamic taxation in the same environment, under the assumption that a policy is sustainable if it maintains the support of a large enough political coalition over time. The profile of capital taxation is U-shaped in their economy, so as to build a strong middle class and avoid the formation of a reforming coalition.

⁵In the concluding remarks, I explain how the main results would generalize to richer environments.

⁶The numerical illustrations provided throughout the analysis assume that z is uniformly distributed over $[0, 1]$.

($t = 2$). This structure introduces an explicit motive for saving without resorting to additional frictions.⁷ Formally, savings in this real environment are invested in a storage technology that provides no return. Still, I call this object *money*, and justify below the analogy of this environment with a nominal economy.

Throughout the analysis, I assume that preferences are linear in consumption and quadratic in labor disutility:⁸

$$U(z; c_2, n_2, n_1) = c_2 - \frac{(n_2)^2}{2} - \frac{(n_1)^2}{2}. \quad (1.1)$$

Production is linear. An agent of type z produces output $y_t = zn_t$, where n_t is labor supply at $t \in \{1, 2\}$. Agents are taxed at $t = 2$. Fiscal policy collects resources from old agents labor income, and monetary policy operates as a tax on real money holdings. The tax structure follows the informational restrictions introduced by Mirrlees (1971): only labor income is publicly observable, whereas individual productivity, money holding and labor supply are private information. Accordingly, an agent that earns labor income $y_2 = zn_2$ at $t = 2$ pays taxes according to a *tax plan* $\tau(y_2, \theta)$, where θ is the labor tax parameter. Similarly, as real money holdings are not observable, taxes on money holding are constrained to be linear. Accordingly, *monetary policy* operates as an inflation tax on money holding at $t = 2$, with the uniform rate noted π . The expected inflation rate is noted π^e .

The budget constraints of an agent of type z write:

$$\begin{aligned} m &= y_1, \\ c_2 &= y_2 - \tau(y_2, \theta) + m(1 - \pi), \end{aligned} \quad (1.2)$$

where m represents real money holding, held between $t = 1$ and $t = 2$.

The solution to individuals optimization problem is straightforward and gives to the following expressions characterizing the optimal productions decisions $y_1(\cdot)$ and $y_2(\cdot)$:⁹

$$y_1(z, \pi^e) = z^2(1 - \pi^e) \qquad y_2(z, \theta) = z^2 \left[1 - \frac{\partial \tau(y_2, \theta)}{\partial y_2} \right]. \quad (1.3)$$

Production decisions in both young and old age are driven by real return to working, defined as the product of individual productivity and marginal tax rates. Especially, high anticipated inflation induces agents to reduce labor supply and money demand when young.¹⁰ Similarly, at $t = 2$, production decisions are driven by marginal tax rates and individual productivity. Note

⁷The structure generates a demand for money similar to *money in the utility function*, as in Calvo and Guidotti (1993) for instance.

⁸Quasi-linear preferences imply that consumption absorbs all income effects, which simplify the analysis of tax distortions.

⁹These expressions will be called *implementability* or *envelope conditions* when the problem of the government is considered.

¹⁰Note that the demand for money exhibits complementarities with inflation. A seignorage Laffer curve naturally arises, and for a given level of seignorage income, two inflation rates are possible. This indeterminacy is not the focus of the present analysis. Accordingly, whenever necessary, I assume that private agents' expectations of inflation lie on the upward sloping part of the seignorage Laffer curve.

that due to the linear quadratic structure, there is no income effect: production decision at $t = 2$ is driven only by labor taxes and at $t = 1$ only by expected inflation.

Given the dynamic nature of the model, I define value functions $V_t(\cdot)$ at each point in time. At $t = 2$, given its real money holding m , an agent of type z exposed to a tax plan $\tau(\cdot, \theta)$ and inflation rate π derives utility according to:

$$V_2(z, m, \theta, \pi) = y_2(z, \theta) - \tau(y_2(z, \theta), \theta) - \frac{(y_2(z, \theta)/2)^2}{2} + m(1 - \pi). \quad (1.4)$$

Similarly, at $t = 1$, an agent of type z , considering a labor tax plan $\tau(\cdot, \theta)$ and inflation rate π^e :

$$V_1(z, \theta, \pi^e) = y_2(z, \theta) - \tau(y_2(z, \theta), \theta) - \frac{(y_2(z, \theta)/2)^2}{2} + y_1(z, \pi^e)(1 - \pi^e) - \frac{(y_1(z, \pi^e)/2)^2}{2}. \quad (1.5)$$

The central difference between these indirect utility functions is the following. At $t = 1$, private agents anticipate the disincentive effect of inflation on their labor supply decision, whereas at $t = 2$, real money holding is predetermined and inflation operates as a non distortionary tax, in contrast to labor taxation. As the analysis is conducted in a deterministic environment, perfect foresight will ensure $\pi^e = \pi$.¹¹

Note that the distribution of real money holding in the population at $t = 2$ is non degenerate. Formally, from (1.2) and (1.3), individual demand for money at $t = 1$ writes:

$$m(z, \pi^e) = z^2(1 - \pi^e). \quad (1.6)$$

With M being aggregate real money holding at $t = 2$, individual money holdings at $t = 2$ satisfy the following distribution across the population:¹²

$$\phi(z, M) = \frac{z^2}{E(z^2)}M. \quad (1.7)$$

The Government

I now turn to the description of the government and the policy tools. The only purpose of the government is to finance an exogenous public good g , by collecting taxes at $t = 2$, either from labor income or via seignorage of money. As mentioned above, I follow Mirrlees (1971) and assume that individual productivity, money holding and labor supply decisions are privately observed, only labor income is publicly observable.¹³ Accordingly, the government collects labor taxes on observable labor income and *seignorage revenue*, as a proportional tax on predetermined money

¹¹In a stochastic environment, these expressions would be modified to account for the realization of an exogenous shock at $t = 2$, and the expectations over the shock at $t = 1$. Generalization of the results to stochastic shocks to government expenditures is discussed in Section 1.4.2.

¹² M is an endogenous object derived in the analysis, but its level does not affect the relative distribution of money holding across the population.

¹³The informational restriction prevents the implementation of type-specific lump sum taxes. Further, I implicitly assume that the productivity level z_1 is low enough to prevent the implementation of a flat lump-sum tax across the population. Both reasons provide an endogenous rationale for the use of distortionary taxes.

holding.¹⁴

Given aggregate money holding M , the budget constraint of the government at $t = 2$ writes:

$$\int_z \tau(y_2(z, \theta), \theta) dF(z) + \int_z \pi \phi(z, M) dF(z) = g, \quad (1.8)$$

where the first term captures labor income tax under θ , the second seignorage income, and $\phi(z, M)$ is the distribution of real money holding, given by (1.7).

In this environment, there is no *ex post* cost of inflation and real money holdings are pre-determined to policy choices, whereas production decisions at $t = 2$ are sensitive to the tax plan parameter θ . Hence, seignorage is a very attractive source of taxation under discretion. Still, the choice of taxes affects the distribution of wealth and consumption across agents. This dimension is potentially magnified in the presence of progressive income taxation.

1.2.2 Progressive Tax Plan

Consider a two parameters labor tax plan $\tau(y, \theta) \equiv \tau(y, \alpha, \lambda)$, where y is labor income, α captures the progressivity and λ the level of labor taxes. This plan needs two key properties for the purpose of the analysis. A tax plan is progressive if and only if marginal tax rates are higher than average taxes for all level of income:¹⁵

$$\epsilon_{\tau(\cdot)}^y = \frac{\partial \tau(y, \theta) / \partial y}{\tau(y, \theta) / y} > 1 \quad \forall y > 0. \quad (1.9)$$

Second, we restrict attention to fiscal tax plans that are not redistributive *per-se*. This second property is introduced to neatly focus on redistributive conflicts between labor income tax and inflation and not redistributive conflicts driven by labor taxation.¹⁶ Formally, no agent can receive a positive fiscal transfer. This condition writes:

$$\tau(y, \theta) \geq 0 \quad \forall y > 0. \quad (1.10)$$

From now on, I assume the following isoelastic form for the tax plan.

Assumption 1. Let $\alpha \geq 0$ and $\lambda \geq 0$. The tax plan writes:

$$\tau(y, \alpha, \lambda) = \lambda y^{1+\alpha}. \quad (\mathbf{A.1})$$

This specification satisfies the desired properties (1.9) and (1.10).¹⁷ Especially, the average and

¹⁴To be clear, the informational constraint induces the tax rate on pre-determined quantities to be linear. This is the essential ingredients that support the *monetary policy* interpretation of the model: monetary policy operates as a *blunt* and *anonymous* tax on the predetermined tax base.

¹⁵This expression can also be understood as the elasticity of taxes with respect to labor income. This definition of progressivity is standard, see for instance Benabou (2002), Heathcote, Storesletten, and Violante (2014) or Holter, Krueger, and Stepanchuk (2014).

¹⁶For an analysis of redistribution via progressive labor taxation, see for instance Meltzer and Richard (1981).

¹⁷An alternative candidate could be the following quadratic form: $\tau(y, \alpha, \lambda) = \lambda(y + \alpha y^2)$.

marginal tax rates write respectively $\frac{\tau(\cdot)}{y} = \lambda y^\alpha$ and $\frac{d\tau(\cdot)}{dy} = \lambda(1 + \alpha)y^\alpha$. The ratio of marginal tax to average tax rates is $\epsilon_{\tau(\cdot)}^y = 1 + \alpha$. Accordingly, when $\alpha = 0$, the tax plan implements a flat tax rate λ , and for any $\alpha > 0$, the tax plan is progressive.¹⁸ Under Assumption 1, the production decision of an agent of type z , i.e. $y_2(z, \alpha, \lambda)$, is implicitly defined by the following expression:¹⁹

$$1 - \lambda(1 + \alpha)y_2^\alpha - \frac{y_2}{z^2} = 0. \quad (1.11)$$

Some Properties of Progressive Taxation

Note $t(z, \alpha, \lambda)$, the labor *tax function* for an agent of type z subject to the tax plan $\theta = (\alpha, \lambda)$. It is the *tax plan* evaluated at the production decision (1.11):

$$t(z, \alpha, \lambda) = \tau(y_2(z, \alpha, \lambda), \alpha, \lambda). \quad (1.12)$$

Over the population, define the *aggregate tax function* $T(\alpha, \lambda)$ as:

$$T(\alpha, \lambda) = \int_z t(z, \alpha, \lambda) dF(z). \quad (1.13)$$

In the absence of progressivity, i.e. whenever the tax plan implements a flat tax rate, the properties of these functions are well known and represented in Figure 1.1.²⁰

The following lemma establishes some useful properties for these tax functions when $\alpha > 0$.

Lemma 1. *Given $\alpha > 0$, the tax function $t(z, \alpha, \lambda)$ is defined for all $\lambda \geq 0$ with the following salient properties:*

- *Single-peaked Laffer curve, reached at $\bar{\lambda}(z, \alpha) = \frac{1}{2(1+\alpha)(\frac{z}{2})^\alpha}$.*
- *Strictly concave on the upward slopping part of the Laffer curve.*
- *Strictly increasing in productivity z : $\frac{dt(\cdot)}{dz} > 0$.*

Proof. See Appendix 1.6.2. ■

The Laffer curve shape of the tax functions reflects the classic competing *behavioral response* and *mechanical effects* of raising taxes:

$$\frac{dt(z, \alpha, \lambda)}{d\lambda} = \frac{\partial \tau(\cdot)}{\partial y_2} \frac{dy_2(\cdot)}{d\lambda} + \frac{\partial \tau(\cdot)}{\partial \lambda}. \quad (1.14)$$

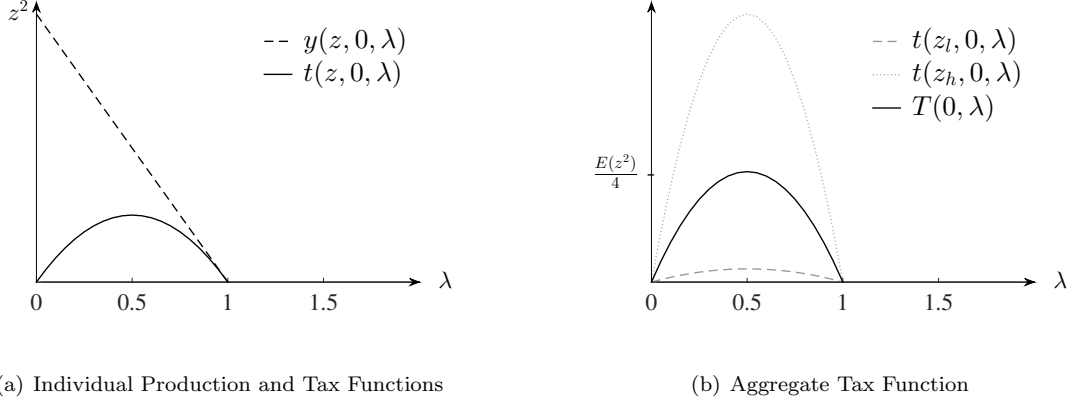
The first term is negative and called *behavioral response*: an increase in labor taxes decreases labor supply and production. The second term, labelled *mechanical effect*, is positive and captures

¹⁸Note that for any $\alpha \in [-1, 0]$, the tax plan is *regressive*. I do not consider this parameter space as it does not emerge as a candidate policy choice in the analysis.

¹⁹As represented in Figure 1.2 and established in Appendix 1.6.1, for $\alpha > 0$, $y_2(z, \alpha, \lambda)$ is positive, strictly convex and strictly decreasing for all $\lambda \geq 0$.

²⁰In the case $\alpha = 0$, the individual tax function writes $t(z, 0, \lambda) = z^2(1 - \lambda)\lambda$ and the aggregate tax function $T(0, \lambda) = E(z^2)(1 - \lambda)\lambda$. These functions are strictly concave, positive for $\lambda \in [0, 1]$ and reach a global maximum for $\lambda = 1/2$.

Figure 1.1: Production and Tax Functions without Progressivity ($\alpha = 0$)



This figure considers the case of no progressivity, i.e. $\alpha = 0$. The left panel represents the production decision (1.3) and the tax function (1.12) as a function of λ for an agent of type z . The right panel outlines how the tax functions aggregate over the population according to (1.13).

the increase in tax collected. For low levels of labor taxes, the *mechanical effect* dominates, whereas for high levels of labor taxes, the *behavioral response* dominates and the level of tax collected is decreasing in λ .

Importantly, these properties carry through to the aggregate tax function:

Lemma 2. *Given α , the aggregate tax function $T(\alpha, \lambda)$ is single-peaked and strictly concave on the upward sloping part of the Laffer curve.*

Proof. The argument considers partitions of the productivity space $[z_l, z_h]$ and shows that the properties of the individual tax functions $t(z, \alpha, \lambda)$ are preserved when these functions are sequentially added.

Lemma 1 establishes that for all z , $t(z, \alpha, \lambda)$ is single-peaked and strictly concave for all $\lambda \in [0, \hat{\lambda}(z, \alpha)]$, with $\hat{\lambda}(z, \alpha) > \bar{\lambda}(z, \alpha)$.²¹ Moreover both $\bar{\lambda}(z, \alpha)$ and $\hat{\lambda}(z, \alpha)$ are decreasing in z .

Consider $F(z^0, \alpha, \lambda) = f(z_l)t(z_l, \alpha, \lambda)$. This function satisfies the same properties as $t(z_l, \alpha, \lambda)$. Note $\bar{\lambda}(z^0, \alpha)$ the value of λ that maximizes $F(z^0, \alpha, \lambda)$. Naturally, $\bar{\lambda}(z^0, \alpha) = \bar{\lambda}(z_l, \alpha)$.

There is a productivity level $z^1 \in (z^0, z_h]$ such that $\hat{\lambda}(z^1, \alpha) = \bar{\lambda}(z^0, \alpha)$ and for all $z \in [z^0, z^1]$, $f(z)t(z, \alpha, \lambda)$ is strictly concave and single-peaked, for all $\lambda \in [0, \hat{\lambda}(z^1, \alpha)]$. Accordingly,

$$F(z^1, \alpha, \lambda) = \int_{z^0}^{z^1} f(z)t(z, \alpha, \lambda)dF(z) + F(z^0, \alpha, \lambda) \quad (1.15)$$

is also single-peaked, reached at $\lambda = \bar{\lambda}(z^1, \alpha) < \bar{\lambda}(z^0, \alpha)$ and strictly concave over $[0, \bar{\lambda}(z^1, \alpha)]$.

Similarly, there is a productivity level $z^2 \in (z^1, z_h]$ such that $\hat{\lambda}(z^2, \alpha) = \bar{\lambda}(z^1, \alpha)$, and by the

²¹See equation (1.68) in Appendix 1.6.2.

same argument,

$$F(z^2, \alpha, \lambda) = \int_{z^1}^{z^2} f(z)t(z, \alpha, \lambda)dF(z) + F(z^1, \alpha, \lambda) \quad (1.16)$$

is also single-peaked, reached at $\lambda = \bar{\lambda}(z^2, \alpha) < \bar{\lambda}(z^1, \alpha)$, and strictly concave over $[0, \bar{\lambda}(z^2, \alpha)]$.

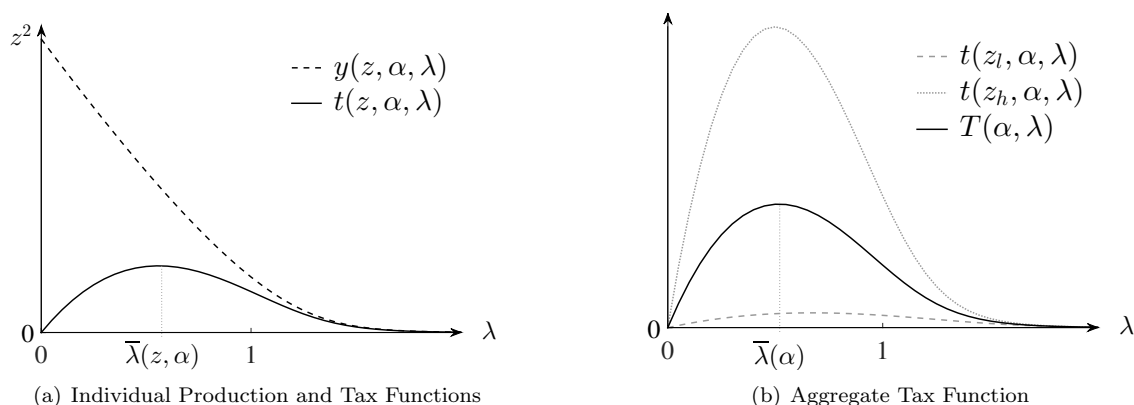
Eventually, after n iterations,

$$F(z^n, \alpha, \lambda) = \int_{z^{n-1}}^{z^n} f(z)t(z, \alpha, \lambda)dF(z) + F(z^{n-1}, \alpha, \lambda) = \int_{z_l}^{z_h} t(z, \alpha, \lambda)dF(z) = T(\alpha, \lambda), \quad (1.17)$$

reaches a global maximum for $\lambda = \bar{\lambda}(\alpha) \in (\bar{\lambda}(z_h, \alpha), \bar{\lambda}(z_l, \alpha))$, and is strictly concave on its upward slopping part. ■

Figure 1.2 represents the production functions (1.3) under a tax plan $\theta = (\alpha, \lambda)$ with $\alpha > 0$, the tax function (1.12) for an agent of type z , the aggregate tax function (1.13) and summarizes the key properties of Lemmas 1 and 2. The individual tax function reaches a maximum in $\bar{\lambda}(z, \alpha)$. By analogy, the peak of the aggregate tax function is reached for a value of λ noted $\bar{\lambda}(\alpha)$.

Figure 1.2: Production and Tax Functions with Progressivity ($\alpha > 0$)



The left panel represents the production decision (1.3) and the tax function (1.12) for an agent of type z when the tax plan features labor tax progressivity, i.e. $\alpha > 0$. The right panel outlines how the tax functions aggregate according to (1.13).

1.2.3 Assumptions

The following assumptions are used in the developments to characterize the policy outcomes. The first imposes a restriction on the distribution of agents to exhibit the usual property that the mean productivity agent has higher productivity than the median.²²

²²This is typically the case for distributions that exhibit positive skewness.

Assumption 2. Let $z_m = F^{-1}(\frac{1}{2})$ be the median productivity level. It satisfies:

$$z_m \leq E(z). \tag{A.2}$$

Further, as government expenses play no particular role in this environment, I impose the following upper limit on g .

Assumption 3. g is non stochastic and satisfies

$$0 < g < \frac{E(z^2)}{4}. \tag{A.3}$$

This restriction guarantees the existence of interior solutions to the taxation programs, namely that there are enough resources in the economy in any circumstances to finance the public good.²³

More importantly, the environment considers the presence of *tax-inertia* in fiscal policy, as in Farhi (2010) and Ferriere (2015).²⁴ The legislative process regarding fiscal policy is complex and some structural elements of the tax code, influencing fiscal incidence for instance, requires more time to be adjusted.²⁵ Formally, in the present environment:

Assumption 4. Fiscal progressivity α is set one period in advance to tax collection. (A.4)

Accordingly, an essential feature of the environment is that progressivity is set at $t = 1$ (*de facto* commitment), and predetermined to the choice of the relative mix of tax, i.e. level of labor taxes and magnitude of inflation tax.

1.3 Productive Efficiency and the Benevolent Planner

This section considers as a benchmark the policy plans of a benevolent planner, both under commitment and discretion. The planner does not have redistributive concerns, since individuals utility is linear in consumption. Accordingly, this section investigates whether progressivity can be part of an optimal policy plan and thus mitigate the aggregate welfare consequences of taxation.

1.3.1 Welfare Cost of Progressivity

This section takes a detour and consider a static labor taxation program. It derives a critical property of isoelastic tax plans (A.1): progressivity induces unambiguous welfare losses, and so both at the individual and aggregate level. Productive efficiency requires no progressivity. This is formalized in the following Lemma.

²³It is derived under the scenario of no labor taxation and top of the seignorage Laffer curve.

²⁴Farhi (2010) introduces tax inertia on capital taxation in a neoclassical growth model with incomplete markets. In his analysis, this assumption deters the replication of the complete market outcome with state-contingent capital taxes. Ferriere (2015) on progressivity of fiscal policy as in the present analysis.

²⁵Also, tax inertia in fiscal policy is often contrasted with the flexibility of monetary policy.

Lemma 3. Consider the static problem of financing a public good using labor taxes only. Both in homogenous ($z_l = z_h$) and heterogenous agent economies ($z_l < z_h$), the optimal plan requires no progressivity, i.e. $\alpha = 0$.

Proof. Consider first the case of homogenous productivity, and the following static program:

$$\max_{\alpha, \lambda} W(z, \alpha, \lambda) = y(z, \alpha, \lambda) - t(z, \alpha, \lambda) - \frac{(y(z, \alpha, \lambda)/z)^2}{2}, \quad (1.18)$$

$$\text{s.t. } t(z, \alpha, \lambda) = \tau(y(z, \alpha, \lambda), \alpha, \lambda) = g, \quad (1.19)$$

and non negativity constraints $\alpha \geq 0, \lambda \geq 0$.

The budget constraint (1.19) implicitly defines $\lambda(\alpha)$, the level of labor taxes required to finance g given progressivity α . Accordingly, the problem rewrites:

$$\max_{\alpha} \tilde{W}(z, \alpha) = \max_{\alpha} W(z, \alpha, \lambda(\alpha)). \quad (1.20)$$

Using the implementability condition (1.3), we can compute:

$$\frac{d\tilde{W}(\cdot)}{d\alpha} = -\frac{\partial\tau(\cdot)}{\partial\alpha} - \frac{\partial\tau(\cdot)}{\partial\lambda} \frac{d\lambda(\cdot)}{d\alpha}, \quad (1.21)$$

and totally differentiating (1.19) with respect to λ and α :

$$\frac{d\lambda(\cdot)}{d\alpha} = -\frac{d\tau(\cdot)/d\alpha}{d\tau(\cdot)/d\lambda}, \text{ with } \frac{d\tau(\cdot)}{dx} = \frac{\partial\tau(\cdot)}{\partial y} \frac{dy(\cdot)}{dx} + \frac{\partial\tau(\cdot)}{\partial x}, \quad x \in \{\alpha, \lambda\}. \quad (1.22)$$

These expressions allow to rewrite (1.21) as:

$$\frac{d\tilde{W}(\cdot)}{d\alpha} = \frac{\partial\tau(\cdot)/\partial y}{d\tau(\cdot)/d\lambda} \frac{\lambda y(\cdot)^{1+\alpha}}{1+\alpha} \frac{dy(\cdot)}{d\lambda}. \quad (1.23)$$

Since $d\tau(\cdot)/d\lambda > 0$ on the upward slopping part of the Laffer curve, it gives the desired result, i.e. $\tilde{W}(\alpha)$ is strictly decreasing in α and therefore is maximum for $\alpha = 0$.

Now, let's consider a similar problem, except that agents have different levels of productivity $z \sim [z_l, z_h]$. Let's note the optimal tax plan (α^*, λ^*) . This tax plan induces a distribution $\{g_z\}$ of the tax burden g across the population. Formally, for all z , $t(z, \alpha^*, \lambda^*) = g_z$.

Assume for now that type-specific flat rates are feasible. Using the previous result, since all agents dislike progressivity, they would unanimously favor a type-specific tax scheme $\{\lambda_z\}$ that replicates the distribution of the tax burden $\{g_z\}$, but with no progressivity, i.e. $\lambda_z y(z, 0, \lambda_z) = g_z$. So does the benevolent planner. Now, within this class of tax schemes, the planner prefers one that implements a flat tax rate across the population. Indeed, efficiency requires to equalize labor supply elasticities across the population.²⁶ The labor supply elasticity of an agent z to a tax rate λ_z writes $\epsilon(z, \lambda_z) = \frac{\lambda_z}{n(\cdot)} \frac{dn(\cdot)}{d\lambda_z} = -\frac{\lambda_z}{1-\lambda_z}$. Accordingly, for all $z' \neq z$, $\epsilon(z', \lambda_{z'}) = \epsilon(z, \lambda_z)$ if and only

²⁶This claim can be stated formally by solving $\max_{\{\lambda_z\}} \int_z W(z, 0, \lambda_z) dF(z)$ subject to $\int_z t(z, 0, \lambda_z) dF(z) = g$.

if $\lambda_z = \lambda_{z'}$.

Overall, within the class of isoelastic tax scheme (α, λ) , a benevolent planner implements one with no progressivity, i.e. $\alpha = 0$. ■

For a given level of taxes to be collected, progressivity is only costly, for it increases marginal tax rates, labor supply distortions and weighs on welfare. Therefore, individual agents dislike progressivity for a given tax bill. Further, in the economy with heterogenous agents, efficiency commands the equalization of labor supply elasticities across the population. This is achieved with a flat tax rate, i.e. there is no aggregate efficiency gain to progressivity in an economy with heterogenous agents.

1.3.2 Optimal Dynamic Policy Plan and Time Inconsistency

Now, I consider the dynamic economy described in Section 1.2 and characterize the optimal policy plans of a benevolent planner, both under commitment and discretion. As underlined before, with linear-quadratic preferences, the planner pursues a pure efficiency objective when choosing over the labor tax plan and the inflation rate.

Under commitment, a benevolent policymaker solves:

$$\max_{\alpha, \lambda, \pi} \int_z V_1(z, \alpha, \lambda, \pi) dF(z), \quad (1.24)$$

subject to the government budget constraint (3.6), the individual demand for money (1.6), the production decisions (1.3), and the non-negativity constraints $\alpha \geq 0$, $\lambda \geq 0$, $\pi \geq 0$.

At $t = 2$, i.e. under discretion, the policymaker no longer internalizes the impact of its policy plan on the demand for money. Given the aggregate level of money $M \geq 0$, the induced distribution of real money holding $m(z) \equiv \phi(z, M)$, and a predetermined level of progressivity $\alpha \geq 0$, it considers the following program:

$$\max_{\lambda, \pi} \int_z V_2(z, m, \alpha, \lambda, \pi) dF(z), \quad (1.25)$$

subject to the government budget constraint (3.6), the production decisions (1.3), and non negativity constraints $\lambda \geq 0$, $\pi \geq 0$. The following proposition characterizes the optimal policy plans under commitment and discretion.

Proposition 1. *The optimal dynamic policy plan prescribes no progressivity ($\alpha = 0$) and an equal sharing of tax distortions across time ($\lambda = \pi$). Under discretion, for any level of progressivity $\alpha \geq 0$, the policy plan implements the highest rate of inflation and possibly labor taxes to meet the budget constraint. Welfare is lower under discretion.*

Proof. First consider the planner acting under commitment, deriving the policy plan at $t = 1$. By Lemma 3, we can rule out the possibility of $\alpha > 0$. Indeed, if positive labor taxes are raised

as part of the optimal plan, keeping the total amount collected fixed, welfare is higher with no progressivity. Hence, the benevolent government solves:

$$\max_{\lambda, \pi} \int_z V_1(z, 0, \lambda, \pi) dF(z) = E(z^2) \frac{(1-\lambda)^2}{2} + E(z^2) \frac{(1-\pi)^2}{2} \quad (1.26)$$

subject to the government budget constraint:

$$E(z^2)(1-\lambda)\lambda + E(z^2)(1-\pi)\pi = g. \quad (1.27)$$

This problem is symmetric in the choice variables λ and π . Accordingly, any interior solution to (1.26) satisfies $\lambda = \pi$. Further note that with no progressivity, the program of the government over the heterogenous population $z \sim [z_l, z_h]$ is isomorphic to a program over a homogenous population with productivity $\sqrt{E(z^2)}$.

Next, consider the planner acting under discretion. In this case, real money holdings are predetermined to the tax decision, the inflation tax is non distortionary, whereas labor taxation is distortionary, and so for any $\alpha \geq 0$. Any optimal plan requires thus to resort to the inflation tax, and possibly complete seignorage with labor taxation if needed.²⁷ As usual, intertemporal welfare is lower under discretion than under commitment. ■

As seen in Lemma 3, progressivity raises marginal tax rates, and accordingly labor supply distortions and welfare losses. Both under commitment and discretion, a benevolent planner that is interested only in minimizing distortions would avoid any form of progressivity in labor taxation.²⁸

Under commitment, the planner wants to spread equally the burden of taxation across agents and time. This policy plan is not time consistent. Indeed, as real money holdings are predetermined to tax choices, *ex post* inflation is beneficial since it operates much like a non distortionary lump-sum tax. Inflation is higher under discretion than under commitment: this is the classic inflation bias at play. The welfare losses discretion stem from the anticipation of inflation and its negative effect on the demand for money.

Accordingly the rest of the analysis investigates whether progressive labor taxation would be desirable to support time consistent policy plans, whenever redistributive concerns are considered. To do so, the following section develops a political economy analysis of the determinants of policy parameters. This approach will prove beneficial to highlight the conflicts that arise across agents once progressivity is set. The idea to pre-commit to fiscal progressivity turns out to be decisive to curb the inflation bias and support time consistent policy plans. This is the focus of next section.

²⁷Formally, given aggregate real money holding $M \geq 0$, the government under discretion with $\alpha = 0$ solves: $\max_{\lambda, \pi} \int_z V_2(z, m, 0, \lambda, \pi) dF(z)$ subject to $E(z^2)(1-\lambda)\lambda + \pi M = g$, where $m \equiv m(z)$ is given by (1.7). One can show that any interior solution to this program requires $\lambda = 0$. For $\alpha > 0$, first recall from Lemma 3 that distortions are lower with no progressivity, i.e. welfare higher. If the government were to raise positive labor taxes with $\alpha > 0$, it would do as well for $\alpha = 0$.

²⁸This element stems essentially from the absence of redistributive concern from the policymaker perspective, related to linear utility of consumption at the individual level.

1.4 A Political Economy

The analysis seeks to highlight the potential use of progressive fiscal policy to curb the inflation bias and support time consistent policy plans in an environment with equity concerns. To do so, I modify the collective choice mechanism of Section 1.3 and design the following game. At $t = 1$, the progressivity of fiscal policy is set, and so before agents know their productivity level. At $t = 2$, given the progressivity of the tax plan, majority voting determines the mix of labor taxes and seignorage. The voting protocol here is a substitute for explicit redistributive concerns.²⁹

Intuitively, progressive fiscal policy would modify the willingness to rely exclusively on the inelastic tax base if it generates sufficient redistributive conflicts across the population. Accordingly, at $t = 2$, the voting protocol outlines the nature and the direction of these conflicts. Anticipating on the outcome of the vote, progressivity is determined at $t = 1$ to provide appropriate intertemporal incentives. The analysis outlines the key elements giving rise to progressive fiscal policy in this dynamic perspective.

1.4.1 Timing and Equilibrium Definition

I consider a political two-stage game with the following sequential decisions. In stage 1, the progressivity parameter α is set behind a *veil of ignorance*, then individual productivities are known, agents form inflation expectations, supply labor and save. In stage 2, given the progressivity of the tax plan and the distribution of real money holdings, majority voting determines the relative magnitude of labor taxes and inflation. Then agents produce, are taxed and consume. Note that the relevant state vector at $t = 2$ is $\mathcal{S}_2 = (\alpha, M)$, since the aggregate level of money M uniquely pins down the whole distribution of real money holdings (1.7).³⁰

This sequential choice mechanism reveals how fiscal progressivity can mitigate the excessive use of the inflation tax under discretion (Proposition 1). The voting mechanism at $t = 2$ stresses the direction and magnitude of potential conflicts over policy choices across the population. Especially, the analysis unveils how individual preferences for tax policy are influenced by the magnitude of fiscal progressivity.

At $t = 1$, the progressivity parameter is set behind a *veil of ignorance*, namely before agents learn their individual productivity. Still, the choice of α will reflect individual preferences for progressivity. More importantly, the choice of progressivity will be driven by its influence on the outcome of the vote and the magnitude of the inflation tax. Overall, this stage reveals whether progressivity is appropriate to mitigate the inflation bias and induce dynamic efficiency.³¹

Therefore, a politico-economic equilibrium in this environment is defined as follow.

²⁹Usually, equity or redistributive concerns are captured either by curvature in the utility function of individual agents, or concave program of the planner. This section considers the political economy as a substitute for these features.

³⁰This is a consequence of the assumption of lifetime idiosyncratic productivity.

³¹With the hypothesis of lifetime productivity, the choice of α behind the *veil of ignorance* could also be interpreted as the choice of a benevolent planner anticipating the voting outcome at $t = 2$.

Definition 1. An equilibrium is defined as a set of policy choices $(\alpha^*, \lambda^*, \pi^*)$ and private agents decisions such that:

- i. Productions and saving decisions of private agents are optimal, given the policy implemented.
- ii. Given fiscal progressivity α and aggregate real money M , the tax mix $\{\lambda^*(\alpha, M), \pi^*(\alpha, M)\}$ implemented at $t = 2$ cannot be defeated by any alternative in a majority vote.
- iii. Given perfect foresight of stage 2 vote outcome, the choice of progressivity α^* is set behind a 'veil of ignorance' at $t = 1$.

Since private decisions and policy choices at $t = 1$ internalize the outcome of the vote during stage 2, this game is solved by backward induction. The following analysis reveals the key forces that drive each policy decision.

1.4.2 Stage 2 - Vote over the Relative Mix of Taxes

As stated above, the game spelled out is solved by backward induction. This section considers the vote over the tax mix in stage 2. The formal protocol for majority voting is the following: two political candidates, only interested in being elected, offer a tax platform and commit to implement it once in office. The outcome of the vote, called Condorcet winner, must survive pairwise evaluation of all competing alternative. In other terms, the winning tax policy is preferred by a majority of voters to any other policy. It is usually the favorite policy choice of one agent in the population, called the decisive voter.³²

Accordingly, the analysis establishes the existence of a Condorcet winner and characterizes the properties of the outcome of the vote. Critically, I focus on two key elements. First, I show how the presence of progressivity induces conflicts across the populations over the relative mix of taxes. The nature of these conflicts supports the existence of a Condorcet winner. Second, I identify the decisive voter and verify whether it supports positive labor taxation.

Importantly, I restrict the analysis to values of the aggregate seignorage tax base $M \geq \frac{E(z^2)}{2}$, anticipating on the equilibrium outcome. Indeed, under Assumption 3, seignorage alone could finance the public good, with a maximum inflation rate $\pi = \frac{1}{2}$. This restriction is not essential and allows to avoid tedious discussions about corner solutions.³³

Individual Ranking of Policy Alternatives

To understand the evaluation of policy proposals, I first study the behavior of individual preferences over policy alternatives. Importantly, voters internalize the impact of policy proposals on their individual production decisions and on the aggregate behavior of the economy.

³²The outcome of majority voting is in general the solution to a modified taxation program with a social welfare function in which only the utility of the decisive voter carries positive weight. Despite the fact that almost all agents dislike the policy choice, it is usually considered as a good approximation to unveil conflicts in the population, since half of the population wants to move in one direction, the other half in the other direction.

³³These corner solutions are being discussed in the case of the benevolent planner policy plan - see Proposition 1.

Formally, an agent of type z evaluates different policies (λ, π) that belongs to the government budget constraint (3.6), given the progressivity parameter α and aggregate real money holding M . Thus, the set of policy alternatives is unidimensional. Note $\pi(\lambda, \alpha, M)$ the level of inflation required to satisfy the government budget constraint when the level of taxes is $\lambda \geq 0$. An agent of type z ranks different policies $\{\lambda, \pi(\lambda, \alpha, M)\}$ according to the following value function:

$$\tilde{V}_2(z, M, \alpha, \lambda) \equiv V_2(z, \phi(z, M), \alpha, \lambda, \pi(\cdot)). \quad (1.28)$$

The derivative of this function with respect to λ outlines the trade-offs involved when varying the level of labor taxes $\lambda \geq 0$. Using the envelope conditions (1.3), it writes:

$$\frac{d\tilde{V}_2(z, \alpha, M, \lambda)}{d\lambda} = -\frac{\partial\tau(\cdot)}{\partial\lambda} - \phi(z, M) \frac{d\pi(\cdot)}{d\lambda}. \quad (1.29)$$

This expression involves two terms that reflect the cost and benefits of raising labor taxes. On the one hand, positive labor taxation is distortionary and costly. This is captured by the marginal tax rate $\frac{\partial\tau(\cdot)}{\partial\lambda}$ of agent z . On the other hand, an increase in labor taxation decreases the magnitude of the inflation tax and preserves money holding as a source of consumption.³⁴ This is captured by the marginal consumption benefit from real money holding $m(z) = \phi(z, M)$, net of the change in inflation $\frac{d\pi(\cdot)}{d\lambda}$. This last term reflects the strategic dimension embedded in the evaluation of policy alternatives. Using the government budget constraint to derive $\frac{d\pi(\cdot)}{d\lambda}$, (1.29) rewrites:

$$\frac{d\tilde{V}_2(z, \alpha, M, \lambda)}{d\lambda} = -\frac{\partial\tau(\cdot)}{\partial\lambda} + \frac{z^2}{E(z^2)} \frac{dT(\alpha, \lambda)}{d\lambda}. \quad (1.30)$$

Accordingly, the presence of progressivity in labor taxation could induce a significant discrepancy between the individual marginal tax rate and the aggregate marginal level of taxes collected. The following Lemma establishes the monotonic ranking of policy alternatives around individual favorite policy, the so-called *single-peaked* property of the value function (1.28).

Lemma 4. *For any $\alpha \geq 0, M \geq \frac{E(z^2)}{2}$, preferences over policy choices are single-peaked.*

Proof. See Appendix 1.6.3. ■

This result is intuitive: if for a given level of labor taxes, a marginal increase in λ induces individual welfare losses, then for higher level of labor taxes, a further marginal increase in labor taxes must be welfare decreasing. Accordingly, the value function (1.28) has a unique critical point in λ over $[0, +\infty]$, which characterizes a global maximum. In other terms, all agents have a unique bliss point policy.

Note from (1.30) that the shape of agent z value function is in effect independent of M , π or g . In other terms, the ranking of levels of labor taxes λ is independent of seignorage revenue. The

³⁴Again, the analysis will stress that the relevant levels of labor taxes lie on the upward slopping part of the Laffer curve, so that $\frac{d\pi(\cdot)}{d\lambda} < 0$. Whenever λ lies on the downward slopping part of the aggregate Laffer curve, i.e. $\lambda \geq \bar{\lambda}(\alpha)$, then $\frac{d\tilde{V}_2(\cdot)}{d\lambda} < 0$. See equation (1.30).

essential force driving the willingness of an agent of type z to increase or decrease labor taxation lie in the capacity of fiscal progressivity to raise labor taxes over the whole population, while minimizing the individual cost to agent z .³⁵

Accordingly, the bliss policy of an agent of type z is noted $\lambda^*(z, \alpha)$. Whenever the favorite policy is interior, i.e. $\lambda^*(z, \alpha) > 0$, it is the solution to $\frac{d\tilde{V}_2(z, \alpha, M, \lambda)}{d\lambda} = 0$. Otherwise, it is simply $\lambda^*(z, \alpha) = 0$.

In the absence of progressivity, $\alpha = 0$, agents unanimously vote in favor of financing the public good with the inflation tax. This is formalized in the following Lemma.

Lemma 5. *Whenever $\alpha = 0$, all agents share the same bliss point policy, with $\lambda^*(z, 0) = 0$. Accordingly, their favorite policy is to finance the public good with the inflation tax only.*

Proof. When $\alpha = 0$, then $y(z, 0, \lambda) = z^2(1 - \lambda)$ and $T(0, \lambda) = E(z^2)(1 - \lambda)\lambda$. Therefore, (1.30) rewrites $\frac{d\tilde{V}_2(z, \alpha, M, \lambda)}{d\lambda} = -z^2\lambda \leq 0$. Thus, for all z , $\tilde{V}_2(\cdot)$ is decreasing in λ . All agents have single-peaked preferences with $\lambda^*(z, 0) = 0$. ■

This result is stronger than the outcome of the optimal policy plan under discretion (Proposition 1). Indeed, not only aggregate productive efficiency requires the exclusive use of the inflation tax, but agents unanimously support seignorage to take advantage of the inelastic tax base.³⁶ No agent strategically supports labor taxes. With no progressivity, there is no conflict in policy choices.

With positive progressivity $\alpha > 0$, this unanimity no longer holds and redistributive conflicts arise. Low productivity agents support positive labor taxation to strategically shift the burden of taxation to high productivity agent. Similarly, high productivity agents support inflationary policies. This result is established in the following lemma.

Lemma 6. *Whenever $\alpha > 0$, agents disagree over the policy plan. Individual bliss policies can be ordered by type: the lower productivity z , the higher the desired level of labor taxation $\lambda^*(z, \alpha)$. Formally, there is a productivity cut-off $\bar{z}(\alpha) \in (z_l, z_h)$ such that:*

- $\lambda^*(z, \alpha) > 0$ if and only if $z < \bar{z}(\alpha)$, and $\lambda^*(z, \alpha) = 0$ otherwise.
- For all $z < \bar{z}(\alpha)$, $\lambda^*(z, \alpha)$ is strictly decreasing in z .
- $\lim_{z_l \rightarrow 0} \lambda^*(z_l, \alpha) = \bar{\lambda}(\alpha)$.

Proof. See Appendix 1.6.4. ■

Lemma 6 establishes the ordering of bliss point policies by productivity type z . While the usual single-crossing property does not hold in this economy,³⁷ I show that whenever $\alpha > 0$,

³⁵Still, the level of inflation needed to clear the government budget constraint does depend on the seignorage tax base M .

³⁶Another interesting benchmark would be no heterogeneity ($z_l = z_h$) with progressivity ($\alpha > 0$). In this case, the presence of progressivity reinforces the distortionary effect of labor taxation, and the difference in tax base elasticities. Agents unanimously support of the inflation tax. See Lemma 3.

³⁷Single-crossing is a usual property of environments with majority voting: the marginal rate of substitution between policy choices over the choice domain is monotone in the ordering of voters. For an extensive analysis of single-crossing property and majority voting, see Gans and Smart (1996).

low productivity agents have a higher desire for labor taxation, and so to strategically use the *tax-shifting* opportunity allowed by progressivity: collecting a high level of labor taxes at a low individual cost. For any positive level of progressive labor tax, conflicts over policy choices arise. These redistributive conflicts may dampen the magnitude of the inflation tax.

Formally, the population is split in two, according to the cut-off value $\bar{z}(\alpha)$. Any agent with productivity $z > \bar{z}(\alpha)$ would not support any labor taxation, whereas any agent with productivity $z < \bar{z}(\alpha)$ would approve positive labor taxes. The cut-off \bar{z} satisfies:³⁸

$$\bar{z}(\alpha) = \left[\frac{E(z^{2(1+\alpha)})}{E(z^2)} \right]^{\frac{1}{2\alpha}}. \quad (1.31)$$

Figure 1.3 provides a graphical illustration of this result. The left panel represents the favorite choice of the level of labor taxes $\lambda^*(\cdot)$ as a function of productivity z . The lower individual productivity, the higher the support for labor taxation, since it collects relatively more taxes on all higher productivity agents. Naturally, the lowest productivity agent z_l has the highest desire for labor taxation. Interestingly, when the lower bound on productivity z_l gets very small, the associated bliss point policy is to collect as much taxes as the aggregate Laffer curve allows, namely set the level of labor taxes to $\bar{\lambda}(\alpha)$.³⁹ To illustrate the *tax-shifting* at play with progressivity, the right panel represents the distribution of taxes, average taxes and marginal taxes induced by the favorite policy of the median productivity agent z_m . The average tax rate $\lambda y_2(\cdot)^\alpha$ is increasing in z whenever $\alpha > 0$. In the case of no progressivity, the average tax rate over the population would be constant.

In turn, the associated level of inflation is increasing in z : higher productivity agents internalize that they would bear the burden of higher labor taxes, hence they favor more inflationary policies.

Outcome of the Vote

Lemma 4, namely *single-peaked* preferences, is sufficient to establish the existence of a Condorcet winner under majority voting. Still, to characterize the policy outcome, one needs to identify the decisive voter. Lemmas 5 and 6 identifies the median productivity agent as the median voter.

Altogether, these results allow to characterize the outcome of the vote. The following proposition formalizes the existence of a Condorcet winner $\{\lambda^*(\alpha, M), \pi^*(\alpha, M)\}$:

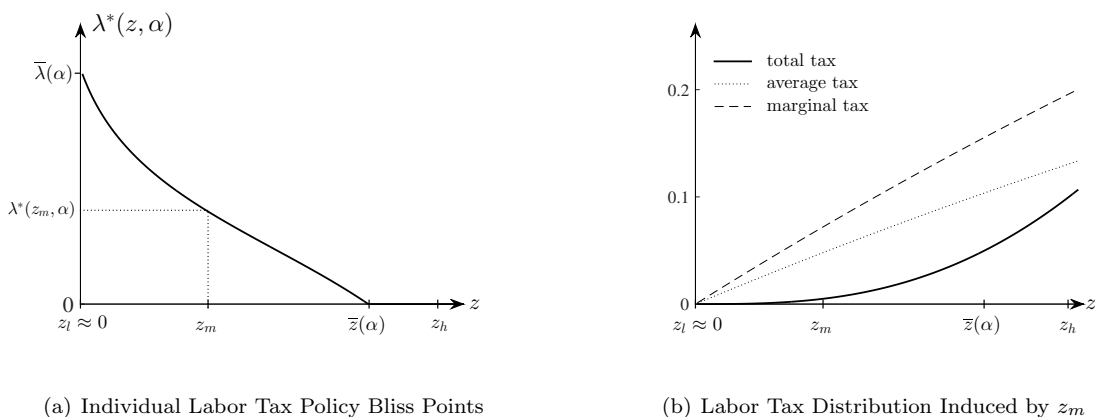
Proposition 2. *Given $M \geq \frac{E(z^2)}{2}$ and $\alpha \geq 0$, majority voting selects a unique policy choice. The decisive voter is the median productivity agent, so that $\lambda^*(\alpha, M) = \lambda^*(z_m, \alpha)$, with the following characteristics:*

- For $\alpha = 0$, the implemented policy relies exclusively on the inflation tax: $\lambda^*(\alpha, M) = 0$.
- For any $\alpha > 0$, the policy implements positive labor taxes $\lambda^*(\alpha, M) > 0$, possibly complemented with the inflation tax.

³⁸See Appendix 1.6.4 for a formal derivation of the cut-off $\bar{z}(\alpha)$.

³⁹When $z \approx 0$ and $\alpha > 0$, the average rate tends to 0 for any λ , while the average tax rate on predetermined money holding is π .

Figure 1.3: Individual Bliss Point Policies - Stage 2 Vote ($\alpha > 0$)



This figure represents the essential properties of stage 2 vote under progressive fiscal policy, i.e. $\alpha > 0$. The left panel displays the individual favorite level of labor taxes, decreasing in z . The right panel illustrates the *tax-shifting* effect that supports the desire for positive labor taxation. The average tax rate is increasing in z , while it would be constant under no progressivity.

Proof. First, by the assumption of permanent lifetime productivity, individual type z and real money holding $\phi(z, M)$ are perfectly correlated, so that agents differ *de facto* only in one dimension. Moreover, since preferences are single-peaked over a unidimensional policy choice, majority voting induces a unique Condorcet winner. The outcome of the vote is the bliss point policy of the median voter. Since bliss policies are ranked by productivity type (Lemmas 5 and 6), the decisive voter is the median productivity agent. This is a classic application of the median voter theorem.⁴⁰

Lemma 5 establishes that whenever the labor tax plan is not progressive ($\alpha = 0$), then agents unanimously vote for no labor taxes, hence the outcome of the vote is naturally one with only seignorage.

Lemma 6 establishes that this consensus is broken whenever there is some progressivity. I verify that for any $\alpha > 0$, the median voter supports strictly positive labor taxation. Formally, I verify that $z_m < \bar{z}(\alpha)$, where $\bar{z}(\alpha)$ is defined in Lemma 6 and satisfies (1.31). Using Jensen inequality:

$$\frac{E(z^{2(1+\alpha)})}{E(z^2)^{1+\alpha}} \geq 1 \Rightarrow \bar{z}(\alpha)^{2\alpha} \geq E(z^2)^\alpha \Rightarrow \bar{z}(\alpha)^2 \geq E(z^2). \quad (1.32)$$

Using the definition of the variance $E(z^2) = V(z^2) + E(z)^2$, one gets:

$$\bar{z}(\alpha) > E(z) \geq z_m, \quad (1.33)$$

where the last inequality comes from Assumption 2. For any $\alpha > 0$, the median productivity agent z_m is below the cut-off value $\bar{z}(\alpha)$. ■

As mentioned, with no progressivity, agents unanimously vote in favor of no labor taxes, since

⁴⁰For detailed references, see for instance detailed in Persson and Tabellini (2002), chapters 2 and 3.

the individual marginal cost systematically outweighs the aggregate benefits of collecting labor taxes over the whole population.

Whenever $\alpha > 0$, the outcome of the vote is one of positive labor taxes: for any distribution of productivities that satisfies Assumption 2, i.e. where the median productivity level is below the mean, the *tax-shifting* effect is strong enough that the median agent does want to collect positive labor taxes. The tax distribution induced by the choice of the median productivity agent is represented in Figure 1.7(b). Overall, any level of progressivity $\alpha > 0$ contributes to curb the inflation tax under majority voting.

Relaxing the restriction $M \geq \frac{E(z^2)}{2}$ does not modify the essence of the result, but may trigger corner solutions. Consider for instance the extreme case $M = 0$, then all agents have no choice but to implement positive labor taxes to meet the government budget constraint.⁴¹

Finally, since the bliss policy of the median productivity agent does not depend on the aggregate level of money holding M , the overall level of labor taxes collected is not sensitive to M . Accordingly, relaxing Assumption 3 by allowing stochastic shocks to government expense g would not modify the result: the level of labor taxes would not be sensitive to the realization of the shock, the inflation tax would absorb all the randomness.⁴²

Influence of Fiscal Progressivity on the Tax Mix

The previous result has established that the level of labor taxes $\lambda^*(\alpha, M)$ implemented under majority voting is positive if and only the labor tax plan is progressive. An essential element is then to understand how the implied labor tax function $T(\alpha, \lambda^*(\alpha, M))$, and conversely the inflation rate, is sensitive to progressivity.⁴³

Lemma 7. *The tax function $T(\alpha, \lambda^*(\alpha, M))$ is positive for all $\alpha \geq 0$, admits a global maximum, and is eventually converging to 0 as the level of progressivity gets to infinity.*

Proof. The proof establishes some critical properties of the tax function induced by the outcome of the vote:

$$T(\alpha, \lambda^*(\alpha, M)) = \int_z t(z, \alpha, \lambda^*(\alpha, M)) dF(z). \quad (1.34)$$

First, from Lemma 5, $T(0, \lambda^*(0, M)) = 0$. Second, its derivative when $\alpha = 0$ is strictly positive.⁴⁴ Formally, Appendix 1.6.5 derives the following inequality:

$$\left. \frac{d\lambda^*(\alpha, M)}{d\alpha} \right|_{\alpha=0} > 0. \quad (1.35)$$

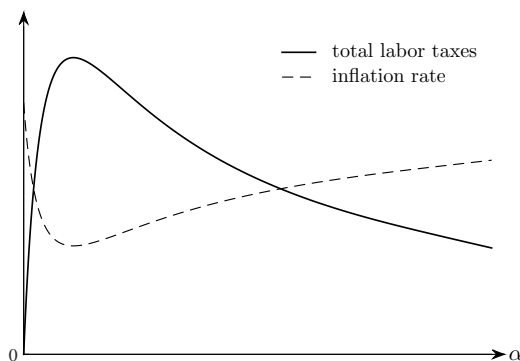
⁴¹More generally, whenever $0 \leq M \leq g - T(\alpha, \lambda^*(z_m, \alpha))$, then the policy implemented is $\pi = 1$ and a necessary level of labor taxes to meet the government budget constraint.

⁴²Further, as inflation expectations are only sensitive to the mean level of inflation, and not to any other moment, stochastic shocks to government expenses would not modify the analysis of stage 1.

⁴³Formally, $T(\alpha, \lambda^*(\alpha, M))$ is the aggregate labor tax function (1.13) evaluated at the vote outcome $\lambda^*(\alpha, M) = \lambda^*(z_m, \alpha)$.

⁴⁴Appendix 1.6.5 presents an extensive derivation of this partial result.

Figure 1.4: Policy Mix as a Function of Progressivity



This figure represents the tax mix implemented under majority voting as a function of productivity α . The plain line represents $T(\alpha, \lambda^*(\alpha, M))$, the aggregate level of labor taxes. The dashed line represents the inflation rate needed to meet the government budget constraint. These curves do not read as Laffer curves but rather reveals the trade-offs faced by the decisive voter. When $\alpha = 0$, unanimity for the inflation tax gives rise to high inflation and no labor taxes. When α increases, the median productivity agent supports higher labor taxes, up to a point where it becomes individually costly to raise more labor taxes. In the limit, no labor taxes are collected.

Further, as for any $\lambda > 0$ and $\alpha \geq 0$, $y(z, \alpha, \lambda) > 0$ and $t(z, \alpha, \lambda) > 0$, we have that $T(\alpha, \lambda^*(\alpha, M)) > 0$.

Finally, consider the level of labor taxes when progressivity gets toward infinity. Rewrite the tax function for an agent z as:

$$t(z, \alpha, \lambda) = \lambda e^{(1+\alpha) \log(y(z, \alpha, \lambda))}. \quad (1.36)$$

From this expression, we have $\lim_{\alpha \rightarrow +\infty} t(z, \alpha, \lambda) = 0$. *A fortiori*, $\lim_{\alpha \rightarrow +\infty} T(\alpha, \lambda^*(\alpha, M)) = 0$. Given these properties, the tax function $T(\alpha, \lambda^*(\alpha, M))$ has a global maximum. ■

Figure 1.4 represents the change in total labor tax collected as a function of α . As the median productivity agent is decisive, it is important to understand how his willingness to raise labor taxes are modified when α increases. When $\alpha = 0$, no labor tax is collected. Then an increase in progressivity induces the median agents to increase total labor taxes collected. As the level of progressivity increases further, the distortions induced leads him to decrease the total amount of labor taxes collected. Eventually, as α tends to infinity, the distortionary effect of progressivity is too high for any amount of labor taxes to be collected. By the government budget constraint, the level of seignorage revenue, as well as the inflation rate, is the mirror of the behavior of total labor taxes collected.⁴⁵

⁴⁵Note that these curves should not be read as standard Laffer curve. Indeed, as shown in next section, individuals turn to have favorite level of progressivity that lies on both the upward and downward sloping part of these curves.

1.4.3 Stage 1 - The Determinants of Fiscal Progressivity

The previous section has characterized the outcome of the vote and the tax mix implemented in stage 2, when the progressivity of the labor income tax plan was given. This section investigates whether progressivity arises in this environment, when it is set behind a *veil of ignorance* at $t = 1$. In the political literature, the veil of ignorance refers to a choice mechanism where parties involved in the decision process know nothing about their particular abilities, tastes, and position within the social order of society. Accordingly, they make choices based upon moral considerations, since they will not be able to make choices based on self- or class-interest.

Under this scenario in the present environment, progressivity is determined before agents learn their individual productivity level. Still, to establish the emergence of $\alpha^* > 0$ in equilibrium, I first study the preferences of an agent of type z over progressivity.

Individual Preferences over Progressive Labor Tax

As in Section 1.3, individual agents internalize the impact of policy choices on their individual production decisions and on the aggregate behavior of the economy. An agent of type z has preferences over progressivity $\alpha \geq 0$ according to the following value function:

$$\tilde{V}_1(z, \alpha) \equiv V_1(z, \alpha, \lambda, \pi), \quad (1.37)$$

where $\lambda = \lambda^*(\alpha, M)$ and $\pi = \pi^*(\alpha, M)$ are respectively the fiscal level and inflation rate, given by the outcome of the vote at $t = 2$ (Proposition 2). Further, M is the aggregate seignorage tax base. It is the sum of individual demand for money (1.3), which in turn are a function of expected inflation at $t = 1$. From (1.6) and (1.7), it satisfies:

$$M = E(z^2)(1 - \pi^*(\alpha, M)). \quad (1.38)$$

This value function has two components, associated to each labor supply decisions. Production at $t = 1$ is subject to the inflation tax, whereas the production at $t = 2$ is subject to labor income tax. The derivative of (1.37) with respect to α outlines the sources of variation of welfare for an agent of type z when changing the level of progressivity α . Formally, using the envelope conditions (1.3):

$$\frac{d\tilde{V}_1(z, \alpha)}{d\alpha} = - \underbrace{\frac{\partial \tau(\cdot)}{\partial \alpha} - \frac{\partial \tau(\cdot)}{\partial \lambda} \frac{d\lambda^*(\cdot)}{d\alpha}}_{\text{labor income tax}} - \underbrace{y_1(\cdot) \frac{d\pi^*(\cdot)}{d\alpha}}_{\text{inflation tax}} \quad (1.39)$$

The first two terms reflect the welfare losses associated to progressive labor income tax. They capture both the direct effect of progressivity and the distortions induced by individual exposure to labor taxes. The magnitude of the latter depends on the relative position of agent z within the distribution, i.e. on its exposure to the *tax-shifting effect* identified in stage 2.

The third term is the marginal cost of inflation. It refers to the dynamic incentives provided by progressivity, i.e. the capacity of progressivity to balance the tax burden over labor income and seignorage tax bases.

To understand how progressivity operates, consider an extreme case with α close to 0. The decisive voter implements at $t = 2$ a policy relying essentially on inflation to finance the public good. An increase in α would then decrease inflation and transfer some of the tax burden on $t = 2$ labor income. Accordingly, progressivity α is critical to allocate the tax burden on each labor supply decision, anticipating the outcome of the vote in stage 2. In effect, progressivity contributes to balance inevitable welfare losses on each production decision, in a time consistent way. Therefore, the last term of (1.39) is decisive in providing the dynamic incentives that support progressivity.

This intuition, and the unanimous desire for dynamic incentives, is formalized in the following Lemma.

Lemma 8. *Any agent $z \in [z_l, z_h]$ would favor a strictly positive level of progressivity, i.e. for all z , $\alpha^*(z) > 0$.*

Proof. To establish this result, I need to show that the derivative of the value function (1.37) when $\alpha = 0$ is strictly positive for any z . $\lambda^*(\alpha)$ and $\pi^*(\alpha)$ satisfies the government budget constraint:⁴⁶

$$T(\alpha, \lambda^*(\alpha)) + E(z^2)\pi^*(\alpha)(1 - \pi^*(\alpha)) = g. \quad (1.40)$$

Importantly, from the analysis in Section 1.4.2, we know that whenever $\alpha = 0$, $\lambda^*(0) = 0$ and all the public good is financed with inflation only. Therefore:

$$\frac{\partial \tau(\cdot)}{\partial y_2} = \lambda(1 + \alpha)y_2(\cdot)^\alpha \quad \Rightarrow \quad \left. \frac{\partial \tau(\cdot)}{\partial y_2} \right|_{\alpha=0} = 0, \quad (1.41)$$

$$\frac{\partial \tau(\cdot)}{\partial \alpha} = \lambda \log(y_2(\cdot))y_2(\cdot)^{1+\alpha} \quad \Rightarrow \quad \left. \frac{\partial \tau(\cdot)}{\partial \alpha} \right|_{\alpha=0} = 0, \quad (1.42)$$

$$\frac{\partial \tau(\cdot)}{\partial \lambda} = y_2(\cdot)^{1+\alpha} \quad \Rightarrow \quad \left. \frac{\partial \tau(\cdot)}{\partial \lambda} \right|_{\alpha=0} = z^2. \quad (1.43)$$

Totally differentiating the government budget constraint (1.40) with respect to α :

$$\frac{dT(\alpha, \lambda^*(\alpha))}{d\alpha} + E(z^2)(1 - 2\pi^*(\alpha))\frac{d\pi^*(\alpha)}{d\alpha} = 0. \quad (1.44)$$

The first term writes:

$$\frac{dT(\alpha, \lambda^*(\alpha))}{d\alpha} = \int_z \frac{\partial \tau(\cdot)}{\partial y_2} \frac{dy_2(\cdot)}{d\alpha} + \frac{\partial \tau(\cdot)}{\partial \alpha} + \frac{\partial \tau(\cdot)}{\partial \lambda} \frac{d\lambda^*(\cdot)}{d\alpha} dF(z). \quad (1.45)$$

⁴⁶Recall from Lemma 6 that the outcome of the vote $\lambda^*(\cdot)$ is independent of the aggregate seignorage tax base M . In (1.40), the dynamic between inflation and the seignorage tax base is captured by the quadratic term in $\pi^*(\cdot)$.

Using (1.41), (1.42) and (1.43), we can evaluate (1.44) in $\alpha = 0$ and get:

$$\left. \frac{d\lambda^*(\cdot)}{d\alpha} \right|_{\alpha=0} + (1 - 2\pi^*(0)) \left. \frac{d\pi^*(\cdot)}{d\alpha} \right|_{\alpha=0} = 0. \quad (1.46)$$

As $\left. \frac{d\lambda^*(\cdot)}{d\alpha} \right|_{\alpha=0} > 0$, from (1.35), we have $\left. \frac{d\pi^*(\cdot)}{d\alpha} \right|_{\alpha=0} < 0$. Substituting this last expression into (1.39):

$$\begin{aligned} \left. \frac{d\tilde{V}_1(\alpha, z)}{d\alpha} \right|_{\alpha=0} &= -z^2 \left. \frac{d\lambda^*(\cdot)}{d\alpha} \right|_{\alpha=0} - z^2 (1 - \pi^*(0)) \left. \frac{d\pi^*(\cdot)}{d\alpha} \right|_{\alpha=0} \\ &= -z^2 \left. \frac{d\pi^*(\cdot)}{d\alpha} \right|_{\alpha=0} \pi^*(0) > 0, \end{aligned} \quad (1.47)$$

where the last inequality uses $\left. \frac{d\pi^*(\cdot)}{d\alpha} \right|_{\alpha=0} < 0$ and $\pi^*(0) > 0$. ■

Overall, this lemma shows that any agent z would support a strictly positive level of progressivity at $t = 1$, for it provides dynamic incentives and curbs the excessive use of the inflation tax. It is essential that progressivity α is evaluated before individual demands for money are formed. Indeed, if it were not the case, the result would not hold, and especially high productivity agents would not favor positive level of progressivity.

Figure 1.5 plots $\alpha^*(z)$ as a function of z . Individual favorite choice of progressivity is not monotonic in productivity z . Indeed, individuals weigh their individual exposure to labor taxation, the deadweight loss associated with progressivity and the reduction in inflation. When the lower bound on productivity is close to 0, an agent of type z_l would anticipate that with progressivity, its average tax rate is nul.⁴⁷ Therefore, it would implement the level of progressivity that would maximize total labor taxes collected.⁴⁸ An agent with a low z would then supports a higher level of progressivity to exploit further the *tax-shifting* possibility, while minimizing his individual exposure to labor taxes.⁴⁹ An agent with a higher z would support a lower level of progressivity, since it internalizes that it would bear a large welfare cost associated to labor taxes. The highest productivity agent z_h would favor progressivity just enough to exploit the dynamic rebalancing between inflation and labor taxes.

Progressivity Behind a *Veil of Ignorance*

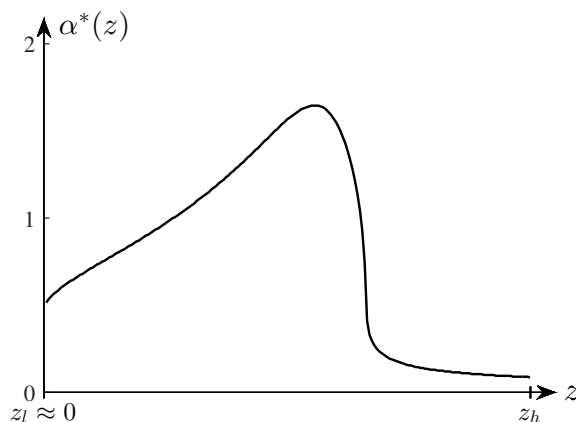
The equilibrium definition stated in Section 1.4.1 indicates that progressivity is set behind a *veil of ignorance*, namely before agents learn their productivity parameter z . Under this mechanism, the choice of progressivity is not driven by special-interests, but rather reflects individual preferences in the economy, independently of particular productivity level.

⁴⁷Recall that with $\alpha > 0$, the average tax rate writes $\frac{\tau(\cdot)}{y_2(\cdot)} = \lambda y_2(\cdot)^\alpha$.

⁴⁸Formally, agent $z_l \approx 0$ would pick α that maximizes $T(\alpha, \lambda^*(\alpha))$, the peak of the labor income tax function. See Figure 1.4.

⁴⁹Note that in this case, the choice of α would lie in the downward sloping part of the tax function $T(\alpha, \lambda^*(\alpha))$.

Figure 1.5: Stage I Individual Choice of Progressivity



This figure plots the individual favorite choice of α as a function of productivity z . The non monotonicity of $\alpha^*(z)$ stems from the interplay between *tax-shifting*, deadweight loss of progressivity and reduction in inflation. When $z_l \approx 0$, the favorite α exploits all the labor tax possibilities offered by the outcome of the vote. For a low value of z , an agent would select a higher α to take benefit of the *tax-shifting* effect. An agent with a high z would choose a lower value of α , for it internalizes that it bears a large cost of labor income taxes.

Under this scenario, the level of progressivity is set to solve the following program:

$$\max_{\alpha} \int_z \tilde{V}_1(z, \alpha) dF(z), \quad (1.48)$$

where the tax mix parameters $\lambda^*(\cdot)$ and $\pi^*(\cdot)$ are the outcome of stage 2 vote, as in (1.37).

This program involve two types of efficiency concerns. First, as outlined in Lemma 3, progressivity is not desirable when it comes to maximize aggregate production at $t = 2$. On the other hand, as shown in Lemma 8, progressivity is effective in curbing the inflation bias and balancing distortions on each labor supply decision. In this program though, the *tax-shifting* property that was motivating strategic variations in progressivity in Lemma 8 is not present. The following proposition makes clear that efficiency concerns induced by the distribution of labor taxes over the population do not deter the emergence of progressivity.

Proposition 3. *Whenever progressivity is set at $t = 1$ behind a 'veil of ignorance', then $\alpha^* > 0$.*

Proof. Note $W(\alpha) \equiv \int_w \tilde{V}_1(z, \alpha) dF(z)$ the welfare criterion of interest. Applying Lemma 8, we naturally have $W'(0) > 0$, so that the optimal level of progressivity is not zero. As when α gets very large no labor taxes are effectively collected (see Lemma 7), α^* is finite. ■

Recall that a planner under commitment - Proposition 1, would optimally seek to equalize distortions and welfare losses over the two tax bases. Here, progressivity allows to support a

similar allocation, with the burden of taxation distributed over each labor decision. Further, the tax allocation induced by the choice of progressivity is time-consistent.

Progressivity creates further source of distortions - on $t = 2$ labor supply, but these are outweighed by the reduction on the inflation bias. The political environment stresses that strategic conflicts at $t = 2$ are effective to support a time consistent reduction in inflation. Note that the *ex ante* aggregate welfare is higher under this institutional scheme than under a pure discretionary benevolent planner set-up. Indeed, the discretionary policy could be implemented with $\alpha = 0$. At $t = 2$ then, unanimity for inflation would pick the tax mix that relies on the inflation tax to finance the public good (see Lemma 5). The analysis makes clear that this allocation is dominated by one with a positive level of fiscal progressivity. Finally, as in Lemma 8, it is critical that α is set before agents from individual demand for money, since its desirability relies on its capacity to preserve the real value of money holding.

1.4.4 Progressivity, Dispersion and Welfare

As mentioned in Section 1.4.2, heterogeneity in agents productivity is essential for positive labor taxes to emerge in equilibrium. Intuitively, with progressivity, the median productivity agent strategically shift the burden of taxation to higher productivity agents. Had the median voter more agents *above* him, it would favor a higher level of labor taxes. In turn, in stage 1, the level of progressivity selected would improve the intertemporal welfare of the overall economy.

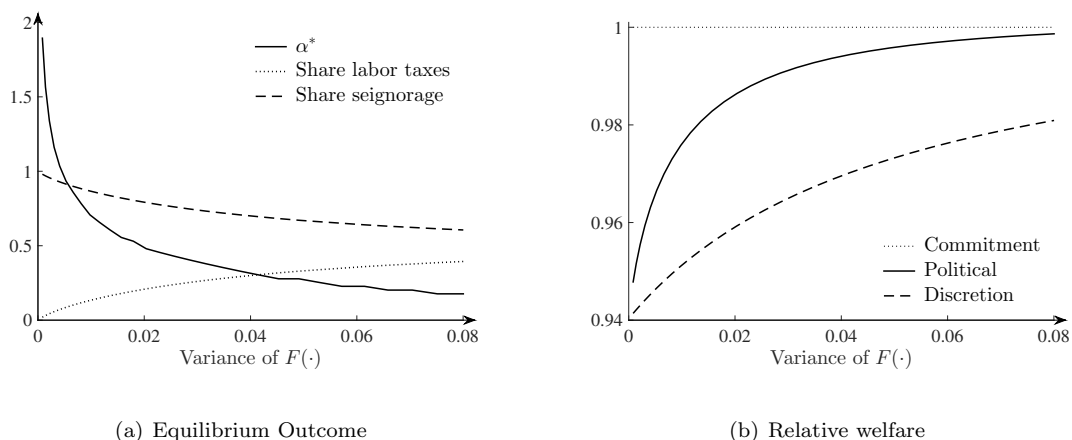
To verify this intuition, I perform the following numerical exercise. I assume that productivities are distributed uniformly on $[z_l, z_h]$ and the median productivity level is fixed at 0.5. Then, I compute the equilibrium outcome and associated welfare, increasing the variance of the distribution of productivities.

Figure 1.6 plots the outcome of this exercise. The left panel represents the level of progressivity selected behind the *veil of ignorance*, and the induced breakdown of the government resources into labor taxes and seignorage. The right panel represents the intertemporal welfare of the economy relative to the full commitment benchmark.

Two elements emerge. When the variance of productivity increases, the selected level of progressivity decreases, but the rebalancing from seignorage to labor taxes is improved, as would prescribe the optimal plan under commitment. Accordingly, as the variance of productivity increases, pre-committing to progressivity allows welfare to get closer to the commitment outcome.

Overall, the higher the variance of productivity, the lower the inflation bias and the closer is the economy to the commitment outcome. Accordingly, pre-committing to fiscal progressivity is more effective to curb the inflation bias in an economy with substantial heterogeneity.

Figure 1.6: Dispersion of Productivity and Welfare



This figure plots the numerical simulation exercise described in Section 1.4.4. As the variance of productivity increases, the level of progressivity selected under the political game decreases, and the balance of taxes improve: more labor taxes are collected, the inflation bias is reduced. Accordingly, the intertemporal welfare relative to the commitment benchmark increases.

1.5 Conclusions

This paper studies how the design of fiscal policy can address the time inconsistency of monetary policy. In a stylized environment, I showed how progressive fiscal policy generates redistributive conflicts that mitigate the excessive use of the inflation tax. Further, progressivity is desirable, despite its inner distortionary nature, since it contributes to minimize intertemporal distortions over tax bases.

The analysis is developed in a framework that embeds two key simplifications. First, the economy considered here is real, not nominal. Second, the voting mechanism is used as a substitute for the absence of explicit equity concerns. I now discuss these points.

A planner with explicit redistributive concerns would like to pre-commit to progressivity as shown in the political environment here, since the desirability of redistribution would resonate with the direction of conflicts analyzed here.⁵⁰

Further, the results presented here would still hold in a fully-fledged nominal economy, where prices and wages would be sensitive to inflation. For instance, a nominal economy could feature an *ex post* cost of inflation, stemming from a *cash-in-advance* constraint or price stickiness for instance. Alternatively, if wages were nominal, the tax plan could generate *bracket creep*, where progressive taxation increases automatically as taxpayers move into higher tax brackets due to inflation. Such features would put a natural brake on the desire of the inflation tax, but not alleviate the essential *tax-shifting* dynamics outlined in the analysis.

An interesting avenue for research would be to relax the assumption of permanent lifetime

⁵⁰Such explicit redistributive concerns would be captured by curvature in individuals utility functions.

productivity to generate an empirically plausible distribution of income and wealth. Such analysis would provide further evidences in favor of the capacity of fiscal progressivity to curb the inflation bias.

1.6 Appendix

1.6.1 Production under Progressive Tax Plan

Consider the $t = 2$ production decision of an agent of type z subject to the tax plan $\theta = (\alpha, \lambda)$.

$$\max_{y_2} y_2 - \tau(y_2, \alpha, \lambda) - \frac{(y_2/z)^2}{2}. \quad (1.49)$$

Under the isoelastic tax plan **A.1**, the first order condition characterizing $y_2(z, \alpha, \lambda)$ is given by:

$$1 - \lambda(1 + \alpha)y_2^\alpha - \frac{y_2}{z^2} = 0. \quad (1.50)$$

The derivatives of the production function with respect to the parameters are given by:

$$\frac{dy_2(\cdot)}{d\lambda} = \frac{-(1 + \alpha)y_2^\alpha}{\lambda(1 + \alpha)\alpha y_2^{\alpha-1} + 1/z^2} < 0, \quad (1.51)$$

$$\frac{dy_2(\cdot)}{d\alpha} = -\frac{\lambda y_2^\alpha (1 + (1 + \alpha) \log(y_2))}{\lambda(1 + \alpha)\alpha y_2^{\alpha-1} + 1/z^2}, \quad (1.52)$$

$$\frac{dy_2(\cdot)}{dz} = \frac{2y_2/z^3}{\lambda(1 + \alpha)\alpha y_2^{\alpha-1} + 1/z^2} > 0. \quad (1.53)$$

$y_2(\cdot)$ is decreasing in λ and strictly positive. When $\alpha > 0$, $y_2(\cdot)$ tends to 0 when λ goes to $+\infty$.

Consider the second derivative of $y_2(\cdot)$ with respect to λ . Using (1.50), rewrite (1.51) as:

$$\frac{dy_2(\cdot)}{d\lambda} = -(z^2)^{1+\alpha} (1 + \alpha) \frac{(y_2/z^2)^{1+\alpha}}{\alpha + (1 - \alpha)y_2/z^2}. \quad (1.54)$$

The second derivative writes then:

$$\frac{d^2 y_2(\cdot)}{d\lambda^2} = -(z^2)^{1+\alpha} (1 + \alpha) G'(y_2/z^2) \frac{dy_2(\cdot)}{d\lambda}, \quad (1.55)$$

with

$$G(X) = \frac{X^{1+\alpha}}{\alpha + (1 - \alpha)X} \quad G'(X) = \alpha X^\alpha \frac{1 + X + \alpha(1 - X)}{[\alpha + (1 - \alpha)X]^2} > 0 \quad \forall X \in [0, 1]. \quad (1.56)$$

Overall, the labor supply function $y_2(z, \alpha, \lambda)$ is strictly decreasing and convex for all $\lambda \geq 0$ and $\alpha > 0$.

1.6.2 Tax Function under Progressive Tax Plan (lemma 1)

Single-Peaked Laffer Curve

By definition, $t(z, \alpha, \lambda) = \tau(y_2(z, \alpha, \lambda), \alpha, \lambda)$. Taking the total derivative of $t(\cdot)$ with respect to λ :

$$\frac{dt(z, \alpha, \lambda)}{d\lambda} = \frac{\partial \tau(\cdot)}{\partial y_2} \frac{dy_2(\cdot)}{d\lambda} + \frac{\partial \tau(\cdot)}{\partial \lambda} = \lambda(1 + \alpha)y_2(\cdot)^\alpha \frac{dy_2(\cdot)}{d\lambda} + y_2(\cdot)^{1+\alpha}. \quad (1.57)$$

Using (1.51), we can rewrite (1.57) as:

$$\frac{dt(\cdot)}{d\lambda} = -\frac{1}{1 + \alpha} \frac{dy_2(\cdot)}{d\lambda} \left[\frac{2y_2(\cdot)}{z^2} - 1 \right]. \quad (1.58)$$

Hence we get that $\frac{dt(\cdot)}{d\lambda} \geq 0$ if and only if $y_2(z, \alpha, \lambda) \leq \frac{z^2}{2}$, i.e. using (1.50), if and only if:

$$0 \leq \lambda \leq \bar{\lambda}(z, \alpha) = \frac{1}{2(1 + \alpha)(z^2/2)^\alpha}. \quad (1.59)$$

Strict Concavity on the Upward Slopping Part of the Laffer Curve

Take the second derivative of the tax function w.r.t. λ :

$$\frac{d^2 t(\cdot)}{d\lambda^2} = -\frac{1}{1 + \alpha} \left[\frac{d^2 y_2(\cdot)}{d\lambda^2} \left(\frac{2y_2(\cdot)}{z^2} - 1 \right) + \frac{2}{z^2} \left(\frac{dy_2(\cdot)}{d\lambda} \right)^2 \right]. \quad (1.60)$$

Rewrite (1.51) as:

$$\frac{dy_2(\cdot)}{d\lambda} = \frac{-(1 + \alpha)y_2(\cdot)^{1+\alpha}}{\alpha + (1 - \alpha)y_2(\cdot)/z^2}, \quad (1.61)$$

and get:

$$\frac{d^2 y_2(\cdot)}{d\lambda^2} = \frac{\alpha}{y_2(\cdot)D(\cdot)} [1 + D(\cdot)] \left(\frac{dy_2(\cdot)}{d\lambda} \right)^2, \quad (1.62)$$

where $D(\cdot)$ is the denominator of (1.61): $D(\cdot) = \alpha + (1 - \alpha)\frac{y_2(\cdot)}{z^2}$. Using (1.62), we can rewrite (1.60) as:

$$\frac{d^2 t(\cdot)}{d\lambda^2} = -\frac{1}{1 + \alpha} \frac{1}{D(\cdot)y_2(\cdot)} \left(\frac{dy_2(\cdot)}{d\lambda} \right)^2 \left[\alpha(1 + D(\cdot)) \left(\frac{2y_2(\cdot)}{z^2} - 1 \right) + \frac{2y_2(\cdot)}{z^2} D(\cdot) \right]. \quad (1.63)$$

The term into brackets is critical for the sign of $\frac{d^2 t(\cdot)}{d\lambda^2}$. Posing $X = \frac{y_2}{z^2}$, we can rewrite the term into brackets as a polynomial $P(X)$, where the range of interest is $X \in [0, 1]$:

$$P(X) = \alpha(1 + D(\cdot))(2X - 1) + 2XD(\cdot), \quad (1.64)$$

with $D(\cdot) = \alpha + (1 - \alpha)X$. Further computations lead to:

$$P(X) = 2(1 - \alpha^2)X^2 + 3\alpha(1 + \alpha)X - \alpha(1 + \alpha). \quad (1.65)$$

We verify:

$$P(0) = -\alpha(1 + \alpha) < 0 \quad P(1/2) = \frac{1 + \alpha}{2} > 0 \quad P(1) = 2(1 + \alpha) > 0. \quad (1.66)$$

Hence, there is a unique $\hat{X} \in (0, 1/2)$ such that $P(X) > 0$ if and only if $X \in [\hat{X}, 1]$, i.e. there is a unique $\hat{\lambda}(z, \alpha) > \bar{\lambda}(z, \alpha)$ such that:

$$\frac{d^2 t(\cdot)}{d\lambda^2} \leq 0 \iff \lambda \leq \hat{\lambda}(z, \alpha). \quad (1.67)$$

Further note that:

$$\frac{dX}{dz} = \frac{dy_2(\cdot)/z^2}{dz} = -\lambda(1 + \alpha)\alpha y_2(\cdot)^{\alpha-1} \frac{dy_2(\cdot)}{dz} < 0, \quad (1.68)$$

which imply $\frac{d\hat{\lambda}(z, \alpha)}{dz} < 0$, i.e. the upper bound of strict concavity of the individual Laffer curves are (inversely) ordered by productivity z .

Ordering of the Tax Functions by Productivity

We are to left verify that the tax functions are ordered by productivity type z . Using (1.53):

$$\frac{dt(z, \alpha, \lambda)}{dz} = \frac{\partial \tau(\cdot)}{\partial y_2} \frac{dy_2(\cdot)}{dz} > 0. \quad (1.69)$$

1.6.3 Single-Peaked Preferences - Stage 2 (lemma 4)

I show that the value function (1.28) is single peaked for any $\lambda \geq 0$, any $\alpha \geq 0$ and any $z \in [z_l, z_h]$. Formally, the shape of this function is given by (1.30):

$$\frac{1}{z^2} \frac{d\tilde{V}_2(\cdot)}{d\lambda} = -\frac{1}{z^2} \frac{\partial \tau(\cdot)}{\partial \lambda} + \frac{1}{E(z^2)} \int_z \frac{dt(z, \alpha, \lambda)}{d\lambda} dF(z), \quad (1.70)$$

where for all z , the derivative of the tax function is driven by the *behavioral response* $\frac{\partial \tau(z, \cdot)}{\partial y_2} \frac{dy_2(z, \cdot)}{d\lambda}$ and the *mechanical response* $\frac{\partial \tau(z, \cdot)}{\partial \lambda}$.

With no progressivity - $\alpha = 0$

With $\alpha = 0$, then $y(z, 0, \lambda) = z^2(1 - \lambda)$ and $T(0, \lambda) = E(z^2)(1 - \lambda)\lambda$. Therefore (1.70) rewrites:

$$\frac{d\tilde{V}_2(z, \alpha, M, \lambda)}{d\lambda} = -z^2\lambda \leq 0. \quad (1.71)$$

Thus, for all z , $\tilde{V}_2(\cdot)$ is decreasing in λ . All agents have single-peaked preferences and the peak is reached for $\lambda^*(z, 0) = 0$.

With progressivity - $\alpha > 0$

The following developments establish the single-peaked property of value functions (1.28) for specific probability distributions $F(\cdot)$ and then uses an aggregation approach to generalize the result to any probability distribution function.

Note that for all $\lambda \geq \bar{\lambda}(\alpha)$, $\frac{d\tilde{V}_2(\cdot)}{d\lambda}$ is strictly negative (downward slopping part of the aggregate Laffer curve). Hence we are interested in the behavior of individual preferences over labor taxation on the upward slopping part of the aggregate Laffer curve, i.e. over $\lambda \in [0, \bar{\lambda}(\alpha)]$.

Two intermediate results will prove useful in the following developments:

i. Behavioral response. Consider the term $G(z, \lambda) = \frac{1}{z^2} \frac{\partial \tau(\cdot)}{\partial \lambda}$, then:

$$\frac{dG(z, \lambda)}{dz} > 0 \quad \frac{d^2G(z, \lambda)}{dzd\lambda} < 0 \quad \forall \lambda \in [0, \bar{\lambda}(z, \alpha)]. \quad (\mathbf{IR.1})$$

ii. Mechanical response. Consider the term $H(z, \lambda) = \frac{1}{z^2} \frac{\partial \tau(\cdot)}{\partial y_2} \frac{dy_2(\cdot)}{d\lambda}$. For any $\lambda \geq 0$, it is negative, initially decreasing, with at most one critical point. (IR.2)

Proof of IR.1 Let $G(z, \lambda) = \frac{1}{z^2} \frac{\partial \tau(\cdot)}{\partial \lambda} = \frac{y_2(\cdot)^{1+\alpha}}{z^2}$. The first derivative w.r.t. z writes:

$$\begin{aligned} \frac{dG(z, \lambda)}{dz} &= \frac{1}{z^4} \left[(1 + \alpha) y_2(\cdot)^\alpha \frac{dy_2(\cdot)}{dz} z^2 - 2z y_2(\cdot)^{1+\alpha} \right] \\ &= \frac{2\alpha}{z^3} \frac{y_2(\cdot)^\alpha}{\lambda(1 + \alpha) \alpha y_2(\cdot)^{\alpha-1} + 1/z^2} \left[\frac{2y_2(\cdot)}{z^2} - 1 \right], \end{aligned} \quad (1.72)$$

which is positive on the upward slopping part of the Laffer curve. Further, we can rewrite this expression as:

$$\frac{dG(z, \lambda)}{dz} = \frac{2\alpha z^{2(1+\alpha)}}{z^3} \frac{(y_2(\cdot)/z^2)^{1+\alpha}}{\alpha(1 - y_2(\cdot)/z^2) + y_2(\cdot)/z^2} \left[\frac{2y_2(\cdot)}{z^2} - 1 \right]. \quad (1.73)$$

Note $Q(X) = \frac{X^{1+\alpha}}{\alpha+(1-\alpha)X} (2X - 1)$ so that the cross second derivative of $G(\cdot)$ writes:

$$\frac{d^2G(z, \lambda)}{dzd\lambda} = \frac{2\alpha z^{2(1+\alpha)}}{z^3} Q'(y_2(\cdot)/z^2) \frac{dy_2(\cdot)}{d\lambda} \frac{1}{z^2}. \quad (1.74)$$

The sign of $Q'(\cdot)$ is critical for the sign of this expression. Formally, with $Q(X) = N(X)/D(X)$, it writes:

$$Q'(X) = \frac{N'(X)D(X) - D'(X)N(X)}{D(X)^2} \quad (1.75)$$

$$N'(X) = X^\alpha [2(2 + \alpha)X - (1 + \alpha)] \quad D'(X) = (1 - \alpha). \quad (1.76)$$

Reorganizing the numerator of $Q'(X)$:

$$Q'(X) = \frac{X^\alpha(1+\alpha)}{D(X)^2} [-\alpha + 3\alpha X + 2(1-\alpha)X^2]. \quad (1.77)$$

Now consider $P(X) = -\alpha + 3\alpha X + 2(1-\alpha)X^2$ and verify that $P(0) = -\alpha < 0$, $P(1/2) = 1/2 > 0$ and $P(1) = 2 > 0$, so that for all $X \in [1/2, 1]$, $P(X) > 0$.

Hence, for all $\lambda \in [0, \bar{\lambda}(z, \alpha)]$, as $y_2(\cdot)/z^2 \in [1/2, 1]$, we have $\frac{d^2 G(z, \lambda)}{dz d\lambda} < 0$.

Proof of IR.2 Consider now $H(z, \lambda) = \frac{1}{z^2} \frac{\partial \tau(\cdot)}{\partial y_2} \frac{dy_2(\cdot)}{d\lambda}$. It is unambiguously negative. Rewrite $H(\cdot)$ as:

$$\begin{aligned} H(z, \lambda) &= \frac{1}{z^2} \frac{\partial \tau(\cdot)}{\partial y_2} \frac{dy_2(\cdot)}{d\lambda} \\ &= -\frac{z^{2(1+\alpha)}}{z^2} \left[1 - \frac{y_2(\cdot)}{z^2} \right] \frac{(1+\alpha)(y_2(\cdot)/z^2)^{1+\alpha}}{\alpha + (1-\alpha)y_2(\cdot)/z^2} \\ &= -z^{2\alpha}(1+\alpha)Q(y_2(\cdot)/z^2), \end{aligned} \quad (1.78)$$

with $Q(X) = (1-X)\frac{X^{1+\alpha}}{\alpha+(1-\alpha)X}$. The first derivative of $H(\cdot)$ w.r.t. λ writes then:

$$\frac{dH(z, \lambda)}{d\lambda} = -z^{2\alpha}(1+\alpha)Q'(y_2(\cdot)/z^2) \frac{dy_2(\cdot)}{d\lambda} \frac{1}{z^2}. \quad (1.79)$$

The sign of $Q'(\cdot)$ is critical for the sign of $\frac{dH(z, \lambda)}{d\lambda}$. Formally, with $Q(X) = N(X)/D(X)$, we can derive:

$$Q'(X) = \frac{N'(X)D(X) - D'(X)N(X)}{D(X)^2} \quad (1.80)$$

$$N'(X) = X\alpha[1+\alpha - (2+\alpha)X] \quad D'(X) = (1-\alpha). \quad (1.81)$$

Reorganizing the numerator of $Q'(X)$:

$$Q'(X) = \frac{X^\alpha}{D(X)^2} [(1+\alpha)\alpha - X\alpha(2\alpha+1) - X^2(1-\alpha)(1+\alpha)]. \quad (1.82)$$

Now consider $P(X) = (1+\alpha)\alpha - X\alpha(2\alpha+1) - X^2(1-\alpha)(1+\alpha)$ and verify that $P(0) = (1+\alpha)\alpha > 0$ and $P(1) = -1 < 0$. Since $X = y_2(\cdot)/z^2$ and $\frac{dy_2(\cdot)}{d\lambda}$, we can conclude that $Q'(y_2(\cdot)/z^2) = -1 < 0$ when $\lambda = 0$ and over $\lambda \geq 0$, $Q'(y_2(\cdot)/z^2) = 0$ has a unique solution.

Overall, $H(z, \lambda)$ is negative, initially increasing and has a unique critical point in λ over $[0, +\infty]$.

Degenerate Distribution Consider the preferences over the tax mix of an agent of type z when the probability distribution of productivity is a degenerate distribution in h .⁵¹ For this special

⁵¹Think as population of mass 1 of agents of productivity h and mass 0 of agent of productivity z .

case, note $\tilde{V}_2(\cdot)$ the value function of an agent of type z and rewrite (1.70) as:

$$\frac{1}{z^2} \frac{d\tilde{V}_2(\cdot)}{d\lambda} = \frac{1}{h^2} \frac{\partial \tau(h, \cdot)}{\partial y_2} \frac{dy_2(h, \cdot)}{d\lambda} + \frac{1}{h^2} \frac{\partial \tau(h, \cdot)}{\partial \lambda} - \frac{1}{z^2} \frac{\partial \tau(z, \cdot)}{\partial \lambda}. \quad (1.83)$$

The first term is 0 for $\lambda = 0$, and strictly negative for all $\lambda > 0$. Let's consider the following cases:

i If $z = h$: second and third terms in (1.83) cancel out. Unambiguously, for all $\lambda \geq 0$:

$$\frac{d\tilde{V}_2(\cdot)}{d\lambda} \leq 0, \quad (1.84)$$

where the inequality is binding if and only if $\lambda = 0$. Accordingly, the value function is single peaked, where the maximum is reached for $\lambda = 0$.

ii If $z > h$: the sum of second and third terms in (1.83) is strictly negative.

Indeed, using **IR.1**, $\frac{d}{dz} \left[\frac{1}{z^2} \frac{\partial \tau(\cdot)}{\partial \lambda} \right] > 0$ on the upward slopping part of the Laffer curve. Unambiguously, for all $\lambda > 0$:

$$\frac{d\tilde{V}_2(\cdot)}{d\lambda} < 0. \quad (1.85)$$

Accordingly, the value function is single peaked, where the maximum is reached for $\lambda = 0$.

iii If $z < h$: using **IR.1**, the sum of second and third terms in (1.83) is strictly positive.

Accordingly, $\frac{d\tilde{V}_2(\cdot)}{d\lambda} \Big|_{\lambda=0} > 0$. Since for all $\lambda > \bar{\lambda}(h, \alpha)$ $\frac{d\tilde{V}_2(\cdot)}{d\lambda} \Big|_{\lambda=0} < 0$, we can conclude that $\tilde{V}_2(\cdot)$ has a critical point in $[0, \bar{\lambda}(h, \alpha)]$ that characterizes a global maximum. To ensure single-peakedness, we show that this critical point is unique. Let $\lambda^*(z, h, \alpha)$ be a solution to $\frac{d\tilde{V}_2(\cdot)}{d\lambda} = 0$. It satisfies:

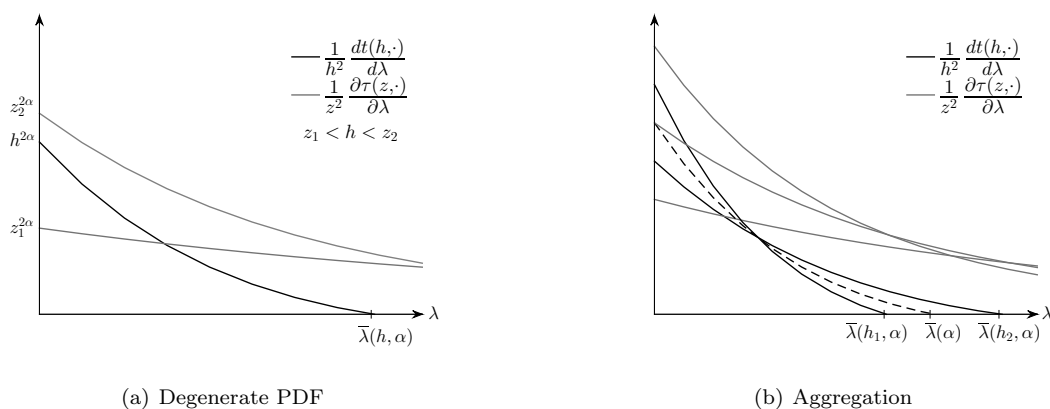
$$-\frac{1}{h^2} \frac{\partial \tau(h, \cdot)}{\partial y_2} \frac{dy_2(h, \cdot)}{d\lambda} = \frac{1}{h^2} \frac{\partial \tau(h, \cdot)}{\partial \lambda} - \frac{1}{z^2} \frac{\partial \tau(z, \cdot)}{\partial \lambda}. \quad (1.86)$$

Over $[0, \bar{\lambda}(h, \alpha)]$, the right-hand side is positive and decreasing. By **IR.2**, The left-hand side is 0 for $\lambda = 0$, positive, initially increasing, has at most one critical point, and is strictly superior to the right-hand side for $\lambda = \bar{\lambda}(h, \alpha)$.

Accordingly, $\lambda^*(z, h, \alpha)$ is the unique critical point of (1.83) for $\lambda \geq 0$. It lies on the upward slopping part of the left-hand side of (1.86). It characterizes a global maximum. The value function is single-peaked.

Figure 1.8(a) summarizes these findings by representing the first derivative of the Laffer curve in h , i.e. first two terms in (1.83), and the individual cost of taxation to agent z , i.e. third term in (1.83).

Figure 1.7: Single-Peaked Preferences



This figure represents graphically each term of the derivative w.r.t. λ of the value function, see (1.70). The left panel considers the cases of degenerate PDF, where agent z expresses its preferences when the population is made of agents of type h . The value function has exactly one critical point which characterizes a global maximum, if and only if $z < h$. Otherwise, the value function reaches a maximum for $\lambda = 0$. The right panel considers the aggregation process that leads to the single-peaked preference result for any PDF. Especially, the dashed line represents $\frac{1}{E(h^2)} \int_h \frac{dt(\cdot)}{d\lambda} dF(h)$, which as shown in Lemma 2, has the same properties as for any degenerate PDF.

Aggregation The generalization of the single-peakedness of value functions presented above relies on two elements. First, for any probability function $F(\cdot)$, (1.70) can be written as a weighted sum of the functions (1.83). Formally,

$$\begin{aligned} \frac{1}{z^2} \frac{d\tilde{V}_2(\cdot)}{d\lambda} &= \frac{1}{E(h^2)} \int_h \frac{1}{z^2} \frac{d\tilde{V}_2(\cdot)}{d\lambda} h^2 f(h) dh \\ &= -\frac{1}{z^2} \frac{\partial \tau(z, \cdot)}{\partial \lambda} + \frac{1}{E(h^2)} \frac{dT(\alpha, \lambda)}{d\lambda}. \end{aligned} \quad (1.87)$$

Second, as shown in Lemma 2, the properties of individual tax functions $t(h, \alpha, \lambda)$ carry to the aggregate tax function $\int_h t(h, \alpha, \lambda) dF(h)$ for any $F(\cdot)$. Accordingly, the single-peaked properties of $\tilde{V}_2(\cdot)$ is also preserved under additivity.⁵² Figure 1.8(b) presents a graphical argument to make this point clear, relying on the additive properties of individual tax functions.

Overall, the value function (1.28) is single peaked for any z , any α and any $F(\cdot)$. A necessary and sufficient condition for the peak to be non 0, i.e. to be reached at $\lambda > 0$, is:

$$\left. \frac{d\tilde{V}_2(\cdot)}{d\lambda} \right|_{\lambda=0} > 0. \quad (1.88)$$

1.6.4 Policy Conflicts under Progressivity - Stage 2 (lemma 6)

Consider individual policy choices under progressive labor taxes, i.e. $\alpha > 0$. By Lemma 4, the value function $\tilde{V}_2(\cdot)$ given by (1.28) is single-peaked and downward slopping $\forall \lambda \geq \bar{\lambda}(\alpha)$. If there is

⁵²Importantly, the multiplying or *weighting* terms in (1.87) are all positive and do not modify the variations of the functions considered.

an interior global maximum $\lambda^*(z, \alpha) > 0$, then it is unique and it satisfies the following conditions:

$$\frac{d\tilde{V}_2(\cdot)}{d\lambda} = -\frac{\partial\tau(\cdot)}{\partial\lambda} + \frac{z^2}{E(z^2)} \frac{dT(\alpha, \lambda)}{d\lambda} = 0 \quad \frac{d^2\tilde{V}_2(\cdot)}{d\lambda^2} \Big|_{\lambda=\lambda^*(z, \alpha)} < 0. \quad (1.89)$$

Therefore, a necessary and sufficient condition for existence of an interior global maximum is:

$$\frac{d\tilde{V}_2(\cdot)}{d\lambda} \Big|_{\lambda=0} > 0. \quad (1.90)$$

This condition induces the cut-off $\bar{z}(\alpha)$, such that $\lambda^*(z, \alpha) > 0$ if and only if $z < \bar{z}(\alpha)$. If $z \geq \bar{z}(\alpha)$, then $\lambda^*(z, \alpha) = 0$. Formally, solving (1.90), $\bar{z}(\alpha)$ is defined by:

$$\bar{z}^{2\alpha} = \frac{E(z^{2(1+\alpha)})}{E(z^2)}. \quad (1.91)$$

To verify the ordering of bliss point policy choice by productivity type, I derive the following comparative statics for all $z < \bar{z}(\alpha)$. Totally differentiating (1.89) with respect to λ and z :

$$\frac{d\lambda^*(z, \alpha)}{dz} = -\frac{\frac{d^2\tilde{V}_2(\cdot)}{d\lambda dz}}{\frac{d^2\tilde{V}_2(\cdot)}{d\lambda^2}} \Big|_{\lambda=\lambda^*(z, \alpha)}. \quad (1.92)$$

The denominator is negative since $\lambda^*(\alpha, z)$ is a global maximum.

Next, I show that the numerator is negative if and only if the marginal rate of substitution (MRS) between inflation and labor taxes is decreasing in z . The MRS is defined as:

$$\text{MRS}(z) = -\frac{dV_2(\cdot)/d\lambda}{dV_2(\cdot)/d\pi} = -\frac{\partial\tau(\cdot)/\partial\lambda}{m(z)} = -\frac{E(z^2)}{M} \frac{\partial\tau(\cdot)/\partial\lambda}{z^2}, \quad (1.93)$$

and its derivative w.r.t. z :

$$\frac{d\text{MRS}(z)}{dz} = \frac{E(z^2)}{z^2 M} \left[-\frac{d\partial\tau(\cdot)/\partial\lambda}{dz} + \frac{2}{z} \frac{\partial\tau(\cdot)}{\partial\lambda} \right]. \quad (1.94)$$

Now, taking the derivative of $\frac{d\tilde{V}_2(\cdot)}{d\lambda}$ w.r.t. z :

$$\frac{d^2\tilde{V}_2(\cdot)}{d\lambda dz} = -\frac{d\partial\tau(\cdot)/\partial\lambda}{dz} + \frac{2z}{E(z^2)} \frac{dT(\alpha, \lambda)}{d\lambda}, \quad (1.95)$$

and evaluating this expression in $\lambda^*(z, \alpha)$, using (1.89):

$$\frac{d^2\tilde{V}_2(\cdot)}{d\lambda dz} \Big|_{\lambda=\lambda^*(z, \alpha)} = -\frac{d\partial\tau(\cdot)/\partial\lambda}{dz} + \frac{2}{z} \frac{\partial\tau(\cdot)}{\partial\lambda} = \frac{z^2 M}{E(z^2)} \frac{d\text{MRS}(z)}{dz}. \quad (1.96)$$

Overall, we get:

$$\frac{d^2\tilde{V}_2(\cdot)}{d\lambda dz} \Big|_{\lambda=\lambda^*(z, \alpha)} < 0 \Leftrightarrow \frac{d\text{MRS}(z)}{dz} \Big|_{\lambda=\lambda^*(z, \alpha)} < 0. \quad (1.97)$$

Next I show that the derivative of the MRS w.r.t. z is indeed negative whenever agent z selects a value of λ on the upward slopping part of its Laffer curve, i.e. for all $\lambda \leq \bar{\lambda}(z, \alpha)$. From (1.94):

$$\begin{aligned} \frac{d\text{MRS}(z)}{dz} &= -\frac{E(z^2)}{M} \frac{1}{z^4} \left[(1+\alpha)y_2(\cdot)^\alpha \frac{dy_2(\cdot)}{dz} z^2 - 2zy_2(\cdot)^{1+\alpha} \right] \\ &= -\frac{y_2(\cdot)^\alpha}{z^3} \frac{E(z^2)}{M} \frac{2\alpha}{\lambda\alpha(1+\alpha)y_2(\cdot)^{\alpha-1} + 1/z^2} \left[\frac{2y_2(\cdot)}{z^2} - 1 \right], \end{aligned} \quad (1.98)$$

which is negative as long as $y_2(\cdot) \leq \frac{z^2}{2}$, i.e. as long as $\lambda \leq \bar{\lambda}(z, \alpha)$.

Finally, I show that $\lambda^*(z, \alpha)$ is necessarily on the upward slopping part of agent z Laffer curve.⁵³

$$\begin{aligned} \lambda^*(z, \alpha) < \bar{\lambda}(z, \alpha) &\Leftrightarrow \left. \frac{d\tilde{V}_2}{d\lambda} \right|_{\lambda=\bar{\lambda}(z, \alpha)} < 0 \\ &\Leftrightarrow \left. \frac{1}{z^2} \frac{\partial \tau(\cdot)}{\partial \lambda} \right|_{\lambda=\bar{\lambda}(z, \alpha)} > \left. \frac{1}{E(z^2)} \frac{dT(\alpha, \lambda)}{d\lambda} \right|_{\lambda=\bar{\lambda}(z, \alpha)} \\ &\Leftrightarrow \frac{1}{2} \left(\frac{z^2}{2} \right)^\alpha > \frac{1}{E(h^2)} \left[\int_{z_l}^{z_h} \left. \frac{dt(h)}{d\lambda} \right|_{\lambda=\bar{\lambda}(z, \alpha)} dF(h) \right]. \end{aligned} \quad (1.99)$$

Accordingly, if for all $h \in [z_l, z_h]$, $\left. \frac{dt(h)}{d\lambda} \right|_{\lambda=\bar{\lambda}(z, \alpha)} < \frac{h^2}{2} \left(\frac{z^2}{2} \right)^\alpha$, then we have the desired result. Using $\bar{\lambda}(z, \alpha) = \frac{1}{2(1+\alpha) \left(\frac{z^2}{2} \right)^\alpha}$, we can verify that $y_2(h, \cdot)$ evaluated in $\bar{\lambda}(z, \alpha)$ is defined by:

$$1 - \frac{y_2^\alpha}{2 \left(\frac{z^2}{2} \right)^\alpha} - \frac{y_2}{h^2} = 0. \quad (1.100)$$

The inequality (1.99) is then satisfied if and only if, for $\lambda = \bar{\lambda}(z, \alpha)$:

$$2 \left[1 - \frac{y_2(h, \cdot)}{h^2} \right] \left[\frac{2y_2(h, \cdot)}{h^2} - 1 \right] < \frac{\alpha}{2} \frac{h^2}{y_2(h, \cdot)} + \frac{1-\alpha}{2}. \quad (1.101)$$

Let $X = \frac{y_2(h, \cdot)}{h^2} \in [0, 1]$. The last expression rewrites then:

$$2 \left[1 - X \right] \left[2X - 1 \right] < \frac{\alpha}{2X} + \frac{1-\alpha}{2} \quad (1.102)$$

The right-hand side is bigger than 1/2 for all $X \in [0, 1]$, whereas the left-hand side reaches a maximum value of 1/4.

Altogether, we have the desired result: $\forall z \in [z_l, \bar{z}(\alpha)]$, $\frac{d\lambda^*(z, \alpha)}{dz} < 0$. Finally, note that for all $\alpha > 0$,

$$\lim_{z \rightarrow 0} \frac{1}{z^2} \left. \frac{\partial \tau(\cdot)}{\partial \lambda} \right|_{\lambda=0} = \lim_{z \rightarrow 0} z^{2\alpha} = 0,$$

which induces $\lim_{z \rightarrow 0} \frac{1}{E(z^2)} dT(\alpha, \lambda) \Big|_{\lambda=\lambda^*(z, \alpha)} = 0$ and therefore $\lim_{z \rightarrow 0} \lambda^*(z, \alpha) = \bar{\lambda}(\alpha)$. This property

⁵³Intuitively, if $\lambda^*(z, \alpha) > \bar{\lambda}(z, \alpha)$ and since $\bar{\lambda}(z, \alpha)$ is decreasing in z , all agents that have a higher productivity level are taxed at a level on the downward slopping part of their Laffer curve. Agent z could then increase the total tax bill on higher productivity agents by reducing the level of taxes.

outlines the clear dominance of the *tax-shifting* effect for low values of z .

1.6.5 Total Taxes as a Function of Progressivity (lemma 7)

This section derives the following partial result:

$$\left. \frac{dT(\alpha, \lambda^*(\cdot))}{d\alpha} \right|_{\alpha=0} > 0. \quad (1.103)$$

Formally, the total derivative of the tax function is given by:

$$\frac{dT(\alpha, \lambda^*(\cdot))}{d\alpha} = \int_z \frac{\partial \tau(\cdot)}{\partial y_2} \frac{dy_2(\cdot)}{d\alpha} + \frac{\partial \tau(\cdot)}{\partial \alpha} + \frac{\partial \tau(\cdot)}{\partial \lambda} \frac{d\lambda^*(\cdot)}{d\alpha} dF(z). \quad (1.104)$$

From Lemma 5, $\lambda^*(0, M) = 0$. Therefore, we can easily verify $\left. \frac{\partial \tau(\cdot)}{\partial y_2} \right|_{\alpha=0} = 0$, $\left. \frac{dy_2(\cdot)}{d\alpha} \right|_{\alpha=0} = 0$ and $\left. \frac{\partial \tau(\cdot)}{\partial \alpha} \right|_{\alpha=0} = 0$. Since $\frac{\partial \tau(\cdot)}{\partial \lambda} > 0$, (1.103) holds if and only if:

$$\left. \frac{d\lambda^*(\cdot)}{d\alpha} \right|_{\alpha=0} > 0, \quad (1.105)$$

where $\lambda^*(\cdot)$ is given by the bliss policy choice of the median productivity agent. It is the solution to:

$$\frac{d\tilde{V}_2(z_m, \alpha, M, \lambda)}{d\lambda} = 0. \quad (1.106)$$

Totally differentiating this expression with respect to λ and α gives:

$$\frac{d\lambda^*(\cdot)}{d\alpha} = - \left. \frac{d^2 \tilde{V}_2(\cdot)/d\lambda d\alpha}{d^2 \tilde{V}_2(\cdot)/d\lambda^2} \right|_{\lambda=\lambda^*(\cdot)}. \quad (1.107)$$

The denominator is negative since the value function is strictly-quasi concave. The numerator is given by:

$$\frac{d^2 \tilde{V}_2(z_m, \alpha, M, \lambda)}{d\lambda d\alpha} = - \frac{d\partial \tau(\cdot)/\partial \lambda}{d\alpha} + \frac{z_m^2}{E(z^2)} \frac{d^2 T(\alpha, \lambda)}{d\lambda d\alpha}. \quad (1.108)$$

Consider the second term in this expression:

$$\frac{d^2 T(\alpha, \lambda)}{d\lambda d\alpha} = \int_z \frac{d}{d\alpha} \left[\frac{\partial \tau(\cdot)}{\partial y_2} \frac{dy_2(\cdot)}{d\lambda} \right] + \frac{d\partial \tau(\cdot)/\partial \lambda}{d\alpha} dF(z). \quad (1.109)$$

As $\frac{\partial \tau(\cdot)}{\partial y_2} \frac{dy_2(\cdot)}{d\lambda} = -\lambda \frac{(1+\alpha)^2 y_2(\cdot)^{2\alpha}}{\lambda(1+\alpha)\alpha y_2(\cdot)^{\alpha-1} + 1/z^2}$, we can easily show using $\lambda^*(0, M) = 0$:

$$\int_z \frac{d}{d\alpha} \left[\frac{\partial \tau(\cdot)}{\partial y_2} \frac{dy_2(\cdot)}{d\lambda} \right] \Big|_{\alpha=0} = 0. \quad (1.110)$$

Further:

$$\frac{d\partial\tau(\cdot)/\partial\lambda}{d\alpha} = \left[\log(y_2(\cdot)) + \frac{1+\alpha}{y_2(\cdot)} \frac{dy_2(\cdot)}{d\alpha} \right] y_2(\cdot)^{1+\alpha} \Rightarrow \frac{d\partial\tau(\cdot)/\partial\lambda}{d\alpha} \Big|_{\alpha=0} = z^2 \log(z^2). \quad (1.111)$$

Rewrite (1.108) then as:

$$\frac{d^2\tilde{V}_2(z_m, \cdot)}{d\lambda d\alpha} \Big|_{\alpha=0} = -z_m^2 \log(z_m^2) + \frac{z_m^2}{E(z^2)} \int_z z^2 \log(z^2) dF(z). \quad (1.112)$$

Using the formula for the covariance,⁵⁴ and since z^2 and $\log(z^2)$ are both increasing of z :

$$\frac{d^2\tilde{V}_2(z_m, \cdot)}{d\lambda d\alpha} \Big|_{\alpha=0} > z_m^2 [-\log(z_m^2) + \log E(z^2)]. \quad (1.113)$$

Since $\log E(z^2) > \log E(z)^2$, and using Assumption 2, we finally get:

$$\frac{d^2\tilde{V}_2(z_m, \cdot)}{d\lambda d\alpha} \Big|_{\alpha=0} > 0, \quad (1.114)$$

so that (1.105) holds and *a fortiori* (1.103).

⁵⁴ $\text{cov}(X, Y) = E(XY) - E(X)E(Y)$.

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Chapter 2

Monetary Policy and Debt Fragility

with Russell Cooper

2.1 Introduction

But there is another message I want to tell you. Within our mandates, the ECB is ready to do whatever it takes to preserve the euro. And believe me, it will be enough.

[Mario Draghi, July 2012]¹

This paper studies the interaction of fiscal and monetary policy in the presence of strategic uncertainty over the value of government debt. In real economies, beliefs of investors about the likelihood of government default, and hence the value of its debt, can be self-fulfilling. Pessimistic investors, fearing government default, will only purchase government debt if there is a sufficient risk premium. The resulting increase in the cost of funds makes default more likely.² Pessimism can be self-fulfilling even if fundamentals are sound enough that an equilibrium without default exists as well.

These results hold for real economies, in which the intervention of a monetary authority is not considered. Does this debt fragility exist in a nominal economy? The presence of a monetary authority can provide an alternative source of revenue through an inflation tax and perhaps use its influence to stabilize real interest rates. Can the monetary authority act to eliminate strategic uncertainty over the value of sovereign debt? If so, will it have an incentive to do so? The answers to these questions are relevant for assessing the relevance of these results on strategic uncertainty in debt markets and for guidance on the conduct of monetary policy.

The overlapping generations model with active fiscal and monetary interventions provides a framework for analysis. The model is structured to highlight strategic uncertainty in the pricing of government debt stemming from the default choice of a government. By construction, there is an equilibrium without default, and in general there are other equilibria with state contingent default.

The monetary authority intervenes through transfers to the fiscal entity, financed by an inflation tax. The monetary intervention has a number of influences. First, the inflation tax delivers real resources to the government, thus reducing the debt burden from taxation. Second, the realized value of inflation alters the real value of debt and consequently the debt burden left to

¹This statement is an excerpt from the address of Mario Draghi, President of the European Central Bank, at a financial conference, in July 2012.

²This interaction between beliefs and default is central to Calvo (1988); and other contributions that followed, including Cole and Kehoe (2000), Roch and Uhlig (2012) and Cooper (2012).

the fiscal authority. Third, it may impact expectations of future inflation and thus the tax base for seignorage.

Given these transfers and its outstanding obligations, the fiscal authority chooses to default or not. Our analysis emphasizes the dependence of this default decision, and thus the extent of strategic uncertainty, on the conduct of monetary policy. As our results develop, the capacity of the monetary authority to stabilize sovereign debt markets relies on the interplay of the inflation and expectation channels, not on the collection of revenue from the inflation tax.

We undertake the analysis under alternative monetary policy regimes. If there is complete discretion in monetary policy or if the monetary authority commits to a strict inflation target, the strategic uncertainty in the real economy is present in the monetary economy.

However, if the monetary authority can commit to a particular state contingent transfer function, given an inflation target, then its intervention can stabilize debt valuations. Specifically, this policy is designed to eliminate all equilibria with state contingent default, preserving the one in which debt is risk-free. Interestingly, this desired intervention does not “bail-out” the fiscal authority. Rather, the countercyclical nature of this policy induces an accommodative fiscal stance in times of low productivity. This intervention leans against negative sentiments of investors and preserves the fundamental price of debt.

This policy is reminiscent of the commitment of the European Central Bank, reflected in the above quote of Mario Draghi, to undertake *whatever it takes* to counter pessimistic self-fulfilling expectations on Eurozone sovereign debt markets. Under the intervention we design, the central bank uses its commitment power to have a stabilizing influence on sovereign’s debt valuations. In equilibrium, no actual intervention is required and debt is uniquely valued. Moreover, this policy does not endanger the primary objective of the central bank, to anchor inflation expectations around an inflation target.

We present conditions such that this policy is credible, so that commitment to its implementation is not needed. The implementation requires a punishment: we consider the case where a deviating monetary authority returns to a strict inflation target regime. Not surprisingly, all else the same, a patient monetary authority is less likely to deviate. But there is another element in the analysis: the higher the risk of self-fulfilling debt crisis in the inflation target regime, the **more** credible is the promise of the central bank to intervene *ex post* to counter pessimistic beliefs on debt valuation.

Other analyses examine possible strategies for central banks to address self-fulfilling debt crises. Calvo (1988) extends his real economy to include a discussion of inflation as a form of partial default. He imposes an exogenous motive of money demand and an explicit cost of inflation function that affects net output.³ Calvo (1988) argues that there may exist multiple equilibria in the determination of inflation and the nominal interest rate on government debt. For this analysis, there is no interaction between fiscal and monetary debt repudiation.

Corsetti and Dedola (2013) augments Calvo’s framework to study the interaction of fiscal and

³Specifically, see equations (16) and (19) in Calvo (1988).

monetary policy. Their analysis retains some of the central features of Calvo's model, including exogenous money demand and costly *ex post* inflation. They argue that monetary interventions through the printing press will not generally resolve debt fragility. But, the central bank, through its holding of government debt, can have a stabilizing influence.

Aguiar, Amador, Farhi, and Gopinath (2013) build a nominal economy with a debt roll-over crisis, as in Cole and Kehoe (2000). They investigate the optimal degree of conservativeness of the central bank (as in Rogoff (1985)) as a tool to address inefficient debt crises. Moderate inflation aversion contains the occurrence of self-fulfilling debt crisis and restrains the inflation bias in normal times.

Our analysis differs from these others papers in a couple of fundamental ways: (i) endogenous money demand and (ii) no assumed *ex post* costs of inflation. In particular, money demand in our model comes from intertemporal savings of households and the cost of inflation occurs through the *ex ante* effect of anticipated inflation on labor supply. The endogeneity of money demand is central to our results. Our environment displays a complementarity between expected and realized inflation. Money demand reflects expected inflation. The absence of an explicit *ex post* cost of inflation provides a strong incentive for *ex post* inflation. With these two features, under discretion, the central bank can be cornered into a high inflation equilibrium, unable to either inflate nominal debt or provide additional resources via seignorage, along with a low inflation equilibrium.⁴ Accordingly, our analysis of fragility in debt markets builds upon the critical need to anchor inflation expectations.

The paper is structured as follows. Section 2.2 describes the economic environment and the fiscal problem of the government. Section 2.3 displays debt fragility in a benchmark real economy. Section 2.4 defines the relevant equilibrium concept in the nominal economy and investigates the presence of debt fragility under two monetary policy frameworks: delegation and discretion. Section 2.5 characterizes a monetary policy rule that can eliminate debt fragility and presents conditions such that this strategy is credible. Section 3.6 concludes.

2.2 Economic Environment

Consider an overlapping generation economy with domestic and foreign agents. Agents live two periods. Time is discrete and infinite.

There are a couple of key components of the model. First, agents differ in productivity in young age and form a demand for savings. Relatively poor agents hold money as a store of value rather than incurring a cost to save through an intermediary. Importantly, money demand is endogenous, thus making the tax base for seignorage dependent on inflation expectations of young agents.

Second the government issues debt each period and faces a choice on how to finance the repayment of its obligations. In particular, the government can tax labor income, print money or default on its debt.

⁴This feature of our model is similar to that explored in Chari, Christiano, and Eichenbaum (1998).

The environment is structured to highlight debt fragility: there are multiple self-fulfilling values of government debt. In this section, we describe the choices of private agents and the fiscal environment.

2.2.1 Private Agents

Every period, a continuum of mass 1 of domestic agents (households) is born and lives two periods. These agents consume only when old and their preferences are linear in consumption and labor disutility is quadratic. This restriction is introduced to neatly capture the reaction of agents to government policy choices.

Domestic agents produce a perishable good in both young and old age. Production is linear. In youth, productivity is heterogenous. A mass ν^m of agents have low productivity $z^m = 1$. A mass $\nu^l = 1 - \nu^m$ of agents have high productivity $z^l = z > 1$. In old age, productivity A is stochastic, i.i.d., and common to all old agents.⁵

Agents have access to two technologies to store value: money or financially intermediated claims. Access to the latter is costly: agents pay a participation cost Γ for access to intermediaries. Limited financial market participation sorts agents in two groups. For convenience, we will refer to *poor* agents, who will hold only *money* in equilibrium, and *rich* agents, who hold *intermediated claims* in equilibrium. Intermediated claims are invested either in nominal government bonds or in a risk-free asset, e.g. storage, that delivers a real return $R > 1$.

Poor Households

Poor households have low labor productivity $z^m = 1$ in youth. Their savings between young and old age are composed only of money holdings, whose real return is given by $\tilde{\pi}'$, the inverse of the gross inflation rate.⁶ Their labor supply decisions in young and old age solve:

$$\max_{n, n'} E[u(c') - g(n')] - g(n), \quad (2.1)$$

subject to young and old age real budget constraints:

$$m = n \quad (2.2)$$

$$c' = A'n'(1 - \tau') + m\tilde{\pi}' + t'. \quad (2.3)$$

In youth, poor agents supply labor n and have real money holdings, m , carried on from young to old age. Return on money is given by the gross inverse inflation rate $\tilde{\pi}'$. In old age, poor agents supply labor n' , which is augmented by aggregate productivity A' . τ' is the tax rate on labor income of old agents and $t' \geq 0$ a possible lump-sum transfer. Denote by n_y^m and n_o^m the optimal

⁵Formally, the distribution of A has full support on the closed and compact set $[A_l, A_h]$. $F(\cdot)$ is the associated cumulative distribution function, and $f(\cdot) = F'(\cdot)$.

⁶We verify later that these agents prefer to save via money rather than costly intermediaries in equilibrium.

labor supply decision of young and old poor agents. With $u(c) = c$ and $g(n) = \frac{n^2}{2}$, labor supply decisions are:

$$n_y^m = E(\tilde{\pi}') \quad \text{and} \quad n_o^m = A'(1 - \tau'). \quad (2.4)$$

Labor supply in both young and old age are driven by real returns to working. In youth, agents form expectations $\tilde{\pi}^e = E(\tilde{\pi}')$, and supply labor accordingly: if agents expect high inflation, i.e. a low $\tilde{\pi}^e$, they will reduce labor supply and the associated demand for real money holding. Similarly, tax on old age labor income is distortionary: a high tax rate reduces return to working and hence the labor supply of old agents.

In contrast to, for example, Calvo (1988), money demand is endogenous in our model, reflecting a labor supply and an asset market participation decision. This is important since the impact of expected monetary interventions is to influence the magnitude of the *ex post* tax base created by money holdings. This interaction between the tax base and the inflation tax rate generates an inflation Laffer curve.

Rich Households and Financial Intermediation

Rich households differ from poor agents by their productivity in youth, $z^I = z > 1$. This higher productivity induces them to pay the fixed cost Γ to access intermediated saving. A parametric restriction ensures that young rich agents save via the financial sector for any positive expected inflation rate.⁷ Formally,

Assumption 1.

$$z^2 > \frac{R\Gamma}{R^2 - 1} > 1. \quad (\text{A.1})$$

The rich solve:

$$\max_{n, n'} E[u(c') - g(n')] - g(n), \quad (2.5)$$

subject to young and old age real budget constraints:

$$m + s = zn - \Gamma \quad (2.6)$$

$$s = b^I + k \quad (2.7)$$

$$c' = A'n'(1 - \tau') + \tilde{\pi}'m + \mathbb{1}_D(1 + i')\tilde{\pi}'b^I + Rk + t'. \quad (2.8)$$

In youth, rich agents supply labor n and produce zn . After incurring the fixed cost Γ , they invest a per capita amount s in intermediated claims. These claims are invested in government bonds b^I and risk-free assets k so that $s = b^I + k$, where b^I denotes the per-capita holding of government debt of domestic rich agents. Government debt is nominal and pays an interest rate

⁷We verify this in characterizing equilibria.

i' next period if there is no default. When old, these agents supply labor n' , contingent on the realization of A' and the tax rate τ' . Consumption in old age depends on the decision $D \in \{r, d\}$ of the government to repay or default on its debt, captured here by the operator $\mathbb{1}_D$ in (2.8): $\mathbb{1}_r=1$ and $\mathbb{1}_d=0$.

Finally, given linear utility of consumption, the portfolio decision between intermediated saving s and money holding m is only driven by expected returns. As long as expected return on money holding $\tilde{\pi}^e$ is strictly inferior to the real return R on the risk-free asset, rich households do not hold money. The portfolio for intermediated savings will include both nominal government debt and risk-free asset as long as the expected return on government debt equals that on the asset:

$$(1 + i')\tilde{\pi}^e(1 - P^d) = R, \quad (2.9)$$

where P^d is the probability of default, determined in equilibrium, and $\tilde{\pi}^e$ the expected inflation over states where debt is repaid. We refer to this as the ‘no-arbitrage condition’. Denote by n_y^I and n_o^I the optimal labor supply decisions of intermediated agents in young and old age. The solution to (2.5) implies:

$$n_y^I = Rz \quad \text{and} \quad n_o^I = A'(1 - \tau'). \quad (2.10)$$

Labor supply n_y^I of young agents is determined by the expected return R on intermediated savings. In old age though, the effective return on intermediated savings will depend on the realized inverse inflation rate $\tilde{\pi}'$, the nominal interest rate i' and the default decision of the government.

Foreign Households

In addition to domestic agents, there are also foreign households who hold domestic debt. Like rich households, they save through intermediaries that hold government debt and consume the domestic good. The details of the foreign economy are not important for this analysis except that foreign households are risk neutral and have access to domestic debt as a store of value. In equilibrium, they hold a fraction $(1 - \theta)$ of domestic debt.⁸

2.2.2 The Government

The government is composed of a treasury and a central bank. Every period, it has to finance a constant and exogenous flow of real expenses g . Government expenditures do not enter into agents utility. To finance these expenses, it can raise taxes on old agent labor income, print money and issue nominal debt B' .⁹ Alternatively, it can default on its inherited debt obligation.

⁸Given the indifference of risk neutral agents regarding their portfolio of government debt and storage, θ is not determined in equilibrium. Thus equilibria will be characterized for given values of θ .

⁹The assumption of no taxation of income when young is just a simplification that allows us to neatly disentangle demand for money, for intermediated claims and labor supply driven by taxation.

Under repayment, the real budget constraint of the government is:

$$(1 + i)\tilde{\pi}b + g = \tau(\nu^m An_o^m(\tau) + \nu^I An_o^I(\tau)) + \frac{\Delta M}{P} + b'. \quad (2.11)$$

The left hand side contains the real liabilities of the government, net of realized inflation $\tilde{\pi}$, where b is real debt outstanding. On the right hand side, $n_o^j(\tau)$ is the labor supply decision of old agents of type $j \in \{I, m\}$, ΔM is the change in the total money supply (M) and P is the price level. Denote by σ the rate of money creation that implements the change in money supply ΔM .

Assume $g = b'$, so that new expenses are financed exclusively by debt. With this restriction, fiscal policy has no intergenerational element. Instead, debt created when agents are young is paid for or defaulted on when these agents are old.¹⁰ We return to this restriction in our concluding comments. So the government budget constraint under repayment becomes generation specific:

$$(1 + i)\tilde{\pi}b = \tau(\nu^m An_o^m(\tau) + \nu^I An_o^I(\tau)) + \frac{\Delta M}{P}. \quad (2.12)$$

Instead of repayment, the government can fully renege on its debt. But there are two costs of default for domestic agents. First, direct costs of default are born by old rich agents, who hold a fraction θ of government debt. Second, if the government repudiates its debt, the country suffers from a deadweight loss, as commonly assumed in the literature on strategic default.¹¹ Formally, aggregate productivity contemporaneously drops by a proportional factor γ . The model excludes punishments involving exclusion from future capital markets. This is partly to ensure that default effects are contained within a generation but also reflects the quantitative finding that the main force preventing default is the direct output loss.¹²

As the government budget constraint holds over time for a given generation, a decision to default on period t debt has no direct effect on future generations. That is, default affects only the welfare of current old agents, who otherwise are taxed via seignorage or labor tax.

The government weights the welfare burden of tax distortions against the direct costs and penalty induced by the default decision. Denote by $W^r(\cdot)$ the welfare of the economy under *repayment* and by $W^d(\cdot)$ under *default*. The decision to default is optimal whenever $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) \geq 0$.

Given aggregate productivity A , nominal interest rate i , real money tax base m_{-1} , tax rate τ , money printing rate σ that satisfy (2.12) and the induced inverse inflation rate $\tilde{\pi}$, the welfare

¹⁰This use of generational budget balance appears in Chari and Kehoe (1990) and Cooper, Kempf, and Peled (2010), for example. An alternative, as in Cole and Kehoe (2000), could add more strategic uncertainty through debt rollover. For an analysis of self-fulfilling debt crisis with active debt issuance and maturity management, see Lorenzoni and Werning (2013).

¹¹Penalties and direct sanctions are central theoretic concepts for enforcement of international asset trade. See the seminal work by Eaton and Gersovitz (1981). For an extensive review, see Eaton and Fernandez (1995).

¹²Empirical evidence regarding reputation costs of default are mixed: exclusion from international credit markets are short-lived and premium following defaults are usually found to be negligible. An extensive discussion can be found in Trebesch, Papaioannou, and Das (2012). From a theoretical point of view, Bulow and Rogoff (1989) show that reputation mechanisms cannot enforce international asset trade, if the government can buy foreign assets as an alternative source of insurance.

criterion $W^D(\cdot)$ for $D \in \{r, d\}$ is:

$$W^D(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}) = \nu^m \left(c_o^m(D) - \frac{n_o^m(D)^2}{2} \right) + \nu^I \left(c_o^I(D) - \frac{n_o^I(D)^2}{2} \right). \quad (2.13)$$

The levels of $\tilde{\pi}$ are chosen under each of the options, as a function of the monetary regime under which the economy operates.

Specifically, under repayment, $D = r$, the welfare of old agents is:

$$W^r(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}^r) = \frac{[A(1 - \tau)]^2}{2} + \nu^m m_{-1} \tilde{\pi}^r + ((1 + i)\tilde{\pi}^r - R)\theta b + \nu^I R(Rz^2 - \Gamma). \quad (2.14)$$

Here the inflation is created by the printing of money that is transferred directly to the treasury.

The option to default, $D = d$, triggers penalties but no tax need be raised. In keeping with the generational view of the budget constraint, any money creation in the current period is transferred lump-sum to the current old. The amount of this transfer will depend on the monetary regime. In this case, the welfare of old agents becomes:

$$W^d(A, i, m_{-1}, \sigma, \tilde{\pi}^d) = \frac{[A(1 - \gamma)]^2}{2} + \nu^m m_{-1} \tilde{\pi}^d - R\theta b + \nu^I R(Rz^2 - \Gamma) + T(\sigma, m_{-1}, \tilde{\pi}^d), \quad (2.15)$$

where $T(\sigma, m_{-1}, \tilde{\pi}^d)$ is the aggregate lump sum transfer to old agents that implements $\tilde{\pi}^d$.¹³

2.2.3 Assumptions

The following two assumptions are used for characterizing equilibria. The first places a lower bound on γ so that default is costly, especially when no debt is held by domestic agents.

Assumption 2.

$$\frac{A_l^2 \gamma (2 - \gamma)}{2} > \nu^m. \quad (\mathbf{A.2})$$

Under this assumption, default is not a desirable option when seignorage revenue alone could service principal and interest on debt.¹⁴ The next assumption ensures that the fundamentals of the economy are compatible with a risk-free outcome, i.e. given the real level of debt b , a real interest rate of R , the debt will be repaid for all A . That is, there is a solution to (2.9) without default. Formally,

Assumption 3. $b < \bar{b}$ where

$$\bar{b} = \frac{A_l^2 (1 - \gamma) \gamma}{R}. \quad (\mathbf{A.3})$$

Note that Assumption 3 is stated in the extreme case where there is no seignorage revenue, and all debt is held by foreigners.¹⁵ The presence of an equilibrium without default provides a

¹³Computations are detailed in Appendix 2.7.1.

¹⁴This is established in the construction of equilibria.

¹⁵This assumption is derived using the government budget constraint with no inflation, no fiscal resource from seignorage and all debt is held by foreigners ($\theta = 0$). It implies that there will be an equilibrium without default

convenient benchmark for the analysis.

2.3 Debt Fragility in a Real Economy

To explicitly illustrate debt fragility, first consider this economy without money and nominal quantities. The equilibrium in the monetary economy will be constructed on the foundation of the multiplicity in the real economy.

The government issues real debt and can raise only taxes on labor income. Given A and real interest rate i , its budget constraint under repayment is simply:

$$(1 + i)b = A^2(1 - \tau)\tau. \quad (2.16)$$

Let τ be the smallest tax rate satisfying (2.16) so that tax revenue are locally increasing in the tax rate.

Private agents can save only through intermediation. For simplicity, set $\Gamma = 0$ so that there are no costs associated to saving through the holding of government debt. Further, assume $\theta = 0$ so that all debt is held by foreigners and all domestic saving is through storage.

The labor supply choices of the rich are given by (2.10). Since the poor access the intermediary for saving, their labor supply decisions are given by

$$n_y^m = R \quad \text{and} \quad n_o^m = A'(1 - \tau'). \quad (2.17)$$

The government defaults whenever $\tilde{W}^d(\cdot) \geq \tilde{W}^r(\cdot)$, where these values for the real economy are defined by:

$$\tilde{W}^r(A, i, \tau) = \frac{[A(1 - \tau)]^2}{2} + \nu^m R^2 + \nu^I (Rz)^2, \quad (2.18)$$

and

$$\tilde{W}^d(A, i) = \frac{[A(1 - \gamma)]^2}{2} + \nu^m R^2 + \nu^I (Rz)^2. \quad (2.19)$$

For the real economy with $\theta = 0$, the government will default whenever $\tau > \gamma$. Equivalently, using (2.16), the government defaults for any realization of $A < \bar{A}$, where \bar{A} satisfies:

$$\bar{A}^2 = \frac{(1 + i)b}{\gamma(1 - \gamma)}. \quad (2.20)$$

This expression defines $\bar{A}(i)$, the default threshold of the government as a function of the real interest rate i .

risk when some of the debt is held by domestic agents and when money printing does provide resources to the fiscal authority. Indeed, domestic holding of public debt or a higher money printing rate relaxes the willingness of the fiscal authority to default rather than repay its debt.

The probability of default given i is $F(\bar{A}(i))$. Using this, the no-arbitrage condition (2.9) becomes:

$$(1 + i)\left(1 - F(\bar{A}(i))\right) = R. \quad (2.21)$$

This equation may have several solutions.¹⁶ Default arises both because of fundamental shocks (low A) and strategic uncertainty: the probability of default depends on the interest rate, and in equilibrium on the beliefs of investors which determine this probability. Hence the multiplicity.

Proposition 1. *If government debt has value, then there are multiple interest rates that solve the no-arbitrage condition (2.21).*

Proof. An equilibrium of the debt financing problem is characterized by an interest rate i and a default threshold, \bar{A} solving (2.20) and (2.21). Combining these expressions yields

$$\bar{A}^2(1 - F(\bar{A})) = \frac{Rb}{\gamma(1 - \gamma)}. \quad (2.22)$$

Any \bar{A} solving this equation is part of an equilibrium.

Denote by $G(A)$ the left side and by Z the right side of (2.22). $G(\cdot)$ is continuous on $[A_l, A_h]$, $G(A_l) = A_l^2 > 0$ and $G(A_h) = 0$.

Consistent with Assumption 3, if $Z < A_l^2 = G(A_l)$, there is a risk free interest rate: $\bar{A} = A_l$ and $(1 + i) = R$ is a solution to (2.22). Also, by continuity of $G(\cdot)$, there is $\bar{A} \in (A_l, A_h)$ such that $G(\bar{A}) = Z$. Hence there is also an interest rate that carries a risk-premium and that solves the no-arbitrage condition.

Relaxing Assumption 3, if $Z > A_l^2 = G(A_l)$, all equilibria include default risk and thus a risk premium. If b is very high, then Z will be large as well and there may be no equilibrium in which debt is valued. If debt is valued so that there is a solution to (2.22), then $G(A) > Z$ for some $A > A_l$. Again, by continuity, there is a second equilibrium. ■

The multiple equilibria of the debt financing problem identified here are welfare ordered. The *fundamental* equilibrium with certain repayment provides higher utility than any other equilibrium with higher interest rate and state contingent default. In the fundamental equilibrium, repayment is preferred to default in those states where default is optimal in the other equilibria. And in repayment states, lower interest rate in the fundamental equilibrium requires lower taxes, hence higher welfare, than under the other equilibria.

Importantly, note that Proposition 1 does not directly restrict the level of debt, b . As long as debt has value, there are multiple equilibria. The level of debt though cannot be too large. Else an equilibrium with valued debt will not exist, since the government would default for all realizations of A .

¹⁶This source of multiplicity is at the heart of Calvo (1988) and Cooper (2012).

The underlying source of strategic uncertainty is aptly captured by Proposition 1 for the real economy. It is a building block for the analysis of a monetary economy. The subsequent developments allow debt to be held both internally and externally.

2.4 Debt Fragility in a Monetary Economy

This section studies the interaction of monetary interventions and debt fragility. Intuitively, monetary policy acts via three channels. First, it can collect seignorage taxes and supplement the resources collected through labor taxation. Second, by adjusting the realized inflation rate, it can lower the real value of debt.

But, third, there are potential resource costs of funding the government through the inflation tax: young agents perceiving high inflation in the future will work less, reducing the monetary tax base. This effect though depends on the extent of discretion in the conduct of monetary policy. Also, the mean inflation rate $\tilde{\pi}^e$ is priced into the nominal interest rate, which makes attempts to deliver surprise inflation difficult.

Accordingly, this section of the paper is constructed around two polar cases, distinguished by the ability of the monetary authority to commit.

- i. *Monetary delegation*: monetary policy decisions are made by an independent central bank that pursues a known and explicit rule, independent of fiscal considerations. We consider the case of strict inflation targeting: the central bank is committed to an unconditional inflation rate.
- ii. *Monetary discretion*: monetary and fiscal decisions are linked. Given the state of the economy, money creation and taxes are set so as to minimize welfare costs and tax distortions of old agents given a budget constraint. Default is also chosen optimally *ex post*.

Before analyzing debt fragility under these monetary regimes, we formally define the equilibrium concept of the nominal economy.

2.4.1 State Variables and Equilibrium Definition

The strategic uncertainty identified in Proposition 1 is modeled through a sunspot variable, denoted s , that corresponds to confidence of domestic and foreign households about the repayment of government debt next period.

- If $s = s^o$, agents are “optimists” : they coordinate on the risk free (fundamental) price of the government debt.
- If $s = s^p$, agents are “pessimists” : they coordinate on higher risk / lower price equilibria with state contingent default.

The distribution of sunspot shocks is i.i.d. Denote by $p \in (0, 1)$ the probability of $s = s^o$. In the event there is a unique equilibrium price, then the fundamental price obtains regardless of the sunspot realization. Note that we only consider cases where debt has value.¹⁷

The state of the economy is $\mathcal{S} = (A, i, m_{-1}, s, s_{-1})$. Aggregate productivity, A , is realized and directly affects the productivity of the old. There are two endogenous predetermined state variables, m_{-1} and i , respectively real money holdings of current old agents, and the nominal interest rate on outstanding public debt. Both the sunspot shock last period, s_{-1} , and the current realization, s , may impact fiscal policy, monetary policy and the choices of private agents.

To define a Stationary Rational Expectations Equilibrium (SREE), it is necessary to be precise about market clearing conditions, the link between money printing, inflation and seignorage revenue and from these, the government budget constraint. These conditions are used in the equilibrium definition and in constructing various types of equilibria.

Market Clearing

In every state, the markets for money and bonds must clear. The condition for money market clearing is

$$\nu^m m(\mathcal{S}) = \frac{M(\mathcal{S})}{P(\mathcal{S})} \quad \forall \mathcal{S}, \quad (2.23)$$

where $P(\mathcal{S})$ is the state dependent money price of goods and $M(\mathcal{S})$ is the stock of fiat money. This equation implies that the real money demand of the current young equals the real value of the supply.

The market for government debt clears if the no-arbitrage condition (2.9) holds and the savings of the rich plus the demand from the foreigners is not less than the real stock of government debt. We assume that the foreigners' endowment is large enough to clear the market for bonds as long as (2.9) is met.

Government Budget Constraint, Inflation and Seignorage

The SREE version of the government budget constraint, (2.12), requires a couple of building blocks. The inverse inflation rate, $\tilde{\pi}$, is given by:

$$\tilde{\pi}(\mathcal{S}) = \frac{P(\mathcal{S}_{-1})}{P(\mathcal{S})} = \frac{m(\mathcal{S})}{m(\mathcal{S}_{-1})} \frac{1}{1 + \sigma(\mathcal{S})}, \quad (2.24)$$

using (2.23). Revenue from seignorage is:

$$\frac{\Delta M}{P(\mathcal{S})} = \sigma(\mathcal{S}) \nu^m m(\mathcal{S}_{-1}) \tilde{\pi}(\mathcal{S}) = \nu^m m(\mathcal{S}) \left(\frac{\sigma(\mathcal{S})}{1 + \sigma(\mathcal{S})} \right). \quad (2.25)$$

¹⁷The case of "market shutdown", where debt has no value, is not of direct interest for our analysis.

Here $m(\mathcal{S}_{-1})$ represents the real money holdings of the current old. Importantly, these equations imply a one-to-one mapping between the rate of money creation $\sigma(\mathcal{S})$ and realized inverse inflation $\tilde{\pi}(\mathcal{S})$. This reflects the fact that $m(\mathcal{S}_{-1})$ is given in (2.24) and the employment and money demand for the current generation, $m(\mathcal{S})$, is, as we verify below, independent of the current rate of money creation. Accordingly, our equilibrium definition is stated with the government setting inflation $\tilde{\pi}(\mathcal{S})$.

Embedded in (2.25) is an interaction between inflation expectations, that determines the real money holding $m(\mathcal{S}_{-1})$, and realized inflation. This element will give rise to a monetary Laffer curve and strategic interactions between expected inflation and delivered inflation, as unveiled in the rest of the analysis.

Substituting these expressions for seignorage and the inverse inflation rate into the government budget constraint:

$$(1+i)\tilde{\pi}(\mathcal{S})b = \tau(\mathcal{S})(\nu^m An_o^m(\tau(\mathcal{S})) + \nu^I An_o^I(\tau(\mathcal{S}))) + \nu^m m(\mathcal{S}) \left(\frac{\sigma(\mathcal{S})}{1+\sigma(\mathcal{S})} \right), \quad (2.26)$$

and using the labor supply policy functions of old agents:

$$(1+i)\tilde{\pi}(\mathcal{S})b = A^2(1-\tau(\mathcal{S}))\tau(\mathcal{S}) + \nu^m m(\mathcal{S}) \left(\frac{\sigma(\mathcal{S})}{1+\sigma(\mathcal{S})} \right). \quad (2.27)$$

Stationary Rational Expectations Equilibrium

Definition 1. A *Stationary Rational Expectations Equilibrium (SREE)* is given by:

- The labor supply and savings decisions of private agents, $(n_y^m(\mathcal{S}), n_o^m(\mathcal{S}), n_y^I(\mathcal{S}), n_o^I(\mathcal{S}), m(\mathcal{S}), k(\mathcal{S}), b(\mathcal{S}))$, who form rational expectations in youth, supply labor in young and old age, solve (2.1) and (2.5) subject to their respective budget constraints (2.2),(2.3) and (2.6),(2.8), given state contingent monetary and fiscal policies $(\{\tau(\mathcal{S}), \tilde{\pi}(\mathcal{S}), D(\mathcal{S})\})$, for all \mathcal{S} .
- The government maximizes its welfare criterion by choosing a policy $(\{\tau(\mathcal{S}), \tilde{\pi}(\mathcal{S}), D(\mathcal{S})\})$ subject to the government budget constraint, (2.27) for all \mathcal{S} .
- All markets clear (goods, money, bonds) for all \mathcal{S} .

The welfare criterion of the government will depend on the monetary policy framework, as detailed below. The polar cases of delegation and discretion are studied within this framework; the conduct of monetary policy determines what the government takes as given in choosing its policy.¹⁸ Also, we characterize equilibria for given θ , share of government debt held by domestic agents, as its value is not pinned down in equilibrium.

By making the sunspot binary (optimism or pessimism), we restrict attention to equilibria with potentially at most two levels of the nominal interest rate. As discussed in Section 2.3, there could

¹⁸Aguiar, Amador, Farhi, and Gopinath (2013) and Corsetti and Dedola (2013) study discretionary monetary authorities. Our analysis will also highlight particular forms of commitment by the central bank as well as the reputation effect necessary to implement this solution without commitment.

be more self-fulfilling levels of interest rates associated with different default thresholds. Allowing the sunspot variable to have more than two realizations could capture these outcomes, without changing the essential nature of the following analysis.

2.4.2 Monetary Delegation

In this institutional structure, the treasury has discretionary power over fiscal policy, choosing fiscal policy *ex post* given the monetary intervention. In contrast, the monetary authority is endowed with a commitment technology. We find that under monetary delegation to a strict inflation target, debt fragility remains. The key intuition behind this result is that strict inflation targeting turns a nominal debt contract into a real security. Hence the presence of debt fragility in real economy identified in section 2.3 persists in the nominal economy with strict inflation targeting.

One interpretation of this structure is that the government of an individual country delegates its monetary policy to an independent central bank, by joining a monetary union for instance. The central bank of the union pursues an independent policy of strict inflation targeting and the fiscal authority is left with discretionary tax policy choices (taxes or default).

Specifically, the central bank commits to an inflation target $0 < \tilde{\pi}^* \leq 1$ and delivers it by printing money. By doing so, the central bank does not accommodate productivity shocks. Revenue from seignorage is transferred to the treasury. Formally, the policy of the central bank is:

$$\tilde{\pi}(\mathcal{S}) = \tilde{\pi}^* \quad \forall \mathcal{S}. \quad (2.28)$$

As the central bank is bound to deliver its inflation target $\tilde{\pi}^*$, agents' expectations are $\tilde{\pi}^e = \tilde{\pi}^*$.¹⁹ In a stationary equilibrium, there is a stationary rate of money creation, σ^* , directly linked to the target inflation: $\frac{1}{1+\sigma^*} = \tilde{\pi}^*$.

Using (2.25), modified to reflect the equilibrium under an inflation target $\tilde{\pi}^*$, revenue obtained from seignorage is:

$$\frac{\Delta M}{P(\mathcal{S})} = \nu^m m(\mathcal{S}) \left(\frac{\sigma(\mathcal{S})}{1 + \sigma(\mathcal{S})} \right) = \nu^m \tilde{\pi}^* (1 - \tilde{\pi}^*), \quad (2.29)$$

as $m = m_{-1} = \tilde{\pi}^e = \tilde{\pi}^*$. This is maximized at $\tilde{\pi}^L \equiv \frac{1}{2}$ which is the top of the seignorage “Laffer curve”. At $\tilde{\pi}^* > \tilde{\pi}^L$, a reduction in $\tilde{\pi}^*$ (i.e. an increase in the rate of inflation) will increase revenue.²⁰ Within this monetary set-up, the government budget constraint under repayment becomes:

$$(1 + i)\tilde{\pi}^*b = A^2(1 - \tau)\tau + \nu^m \tilde{\pi}^*(1 - \tilde{\pi}^*). \quad (2.30)$$

¹⁹In particular, if there is default, the monetary authority prints money and transfers it to old agents to meet this target.

²⁰The determination of the optimal inflation target $\tilde{\pi}^*$ is not part of the present analysis. The model could provide a positive theory of inflation, where the inflation target would be set to minimize distortions associated to tax revenue. Given the Laffer curve property of seignorage, any inflation target $0 < \tilde{\pi}^* < \tilde{\pi}^L$ is inefficient, but this does not affect the essential results regarding debt fragility.

To formally derive the result that debt fragility persists in this monetary regime, we establish the existence of several interest rates that solve the no-arbitrage condition. To do so, we first verify that the default decision in the monetary economy has the same monotonicity property as in the real economy: if the government defaults for a given realization of technology \bar{A} , then it would default for any lower realization $A \leq \bar{A}$.

Lemma 1. *Under Assumption 2, given a level of real obligations $(1+i)\tilde{\pi}^*b$, there is a unique $\bar{A}(i) \in [A_l, A_h]$ such that if $A \leq \bar{A}(i)$, then the treasury defaults on its debt. Otherwise it repays its debt.*

Proof. Given a nominal interest rate i , the decision to repay or default on debt is given by $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot)$, where the relevant welfare criteria are given by (2.14) and (2.15) and the lump-sum transfer under default by $T(\tilde{\pi}^*) = \nu^m \tilde{\pi}^* (1 - \tilde{\pi}^*)$. Hence, a point of indifference between default and repayment, $\bar{A}(i)$ solves:

$$\frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} = (1+i)\tilde{\pi}^*\theta b - \nu^m \tilde{\pi}^* (1 - \tilde{\pi}^*), \quad (2.31)$$

where τ satisfies the government budget constraint (2.30). Denote by $G(A)$ the left side of (2.31). Clearly if $G(A)$ is monotonically decreasing in A , then the default decision satisfies the desired cut-off rule. Rewrite $G(A)$ as follow:

$$G(A) = \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} - \frac{A^2\tau(\tau-2)}{2}. \quad (2.32)$$

Using the government budget constraint, (2.32) rewrites:

$$G(A) = \frac{A^2\gamma(\gamma-2)}{2} - [(1+i)\tilde{\pi}^*b - \nu^m \tilde{\pi}^* (1 - \tilde{\pi}^*)] \frac{(\tau-2)}{2(1-\tau)}. \quad (2.33)$$

The first term is negative since $\gamma < 1$. If seignorage revenue is enough to service debt, then no tax need be raised and $\bar{A}(i) = A_l$, by Assumption 2.²¹ Otherwise, $(1+i)\tilde{\pi}^*b - \nu^m \tilde{\pi}^* (1 - \tilde{\pi}^*) > 0$. Finally, we need to derive the monotonicity of $\frac{\tau-2}{1-\tau}$ with respect to A . Its derivative is:

$$\frac{-1}{(1-\tau)^2} \frac{d\tau}{dA} > 0, \quad (2.34)$$

which is positive since $\frac{d\tau}{dA} < 0$ for the lowest value of τ that solves the budget constraint. Overall, we have $G'(A) < 0$. Hence, the cut-off value $\bar{A}(i)$ is unique and default occurs if and only if $A \leq \bar{A}(i)$.

Note that if $\bar{A}(i) \leq A_l$, then debt is risk free. Finally, $\bar{A}(i) = A_h$ is inconsistent with the assumption that debt has value. ■

From this result, the probability of default P^d becomes $F(\bar{A}(i))$. This probability also implicitly

²¹To see this, set $\theta = 0$, $m = 1$ and $\tilde{\pi}^r = 0$ in (2.14) and (2.15) and verify that $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) < 0$ under Assumption 2.

depends on $\tilde{\pi}^*$, which appears on the right side of (2.31). Altogether, an interest rate for the government debt solves:

$$(1+i)\tilde{\pi}^*(1-F(\bar{A}(i))) = R. \quad (2.35)$$

This expression outlines the interplay between beliefs, probability of default and best-response of the government. As in the real economy, the probability of default depends on the interest rate, and in equilibrium on the beliefs of investors which determine this probability.

Lemma 2. *Under Assumptions 2 and 3, for any inflation target $0 < \tilde{\pi}^* \leq 1$, there are multiple interest rates that solve the no-arbitrage condition (2.35).*

Proof. An equilibrium of the debt financing problem is characterized by an interest rate i and a default threshold \bar{A} . Importantly, an equilibrium is such that *beliefs* of investors are consistent with the *best response* of the government.

Investors believe that the government defaults with probability $P^d = F(\bar{A})$. This belief induces $\bar{A}^b(i)$, the default threshold consistent with P^d :

$$(1+i)\tilde{\pi}^*(1-F(\bar{A})) = R \Rightarrow \bar{A}^b(i). \quad (2.36)$$

Given i , the government decision to repay or default induces $\bar{A}^g(i)$, the realization of A for which the government is indifferent between default and repayment:²²

$$\Delta(A, i) = W^d(A, i) - W^r(A, i) = 0 \Rightarrow \bar{A}^g(i). \quad (2.37)$$

An equilibrium requires $\bar{A}^b(i) = \bar{A}^g(i)$. The nominal interest rate i can take value on $[\underline{i}, +\infty)$ where \underline{i} is the nominal interest rate consistent with risk-free debt. Formally, it satisfies $(1+\underline{i})\tilde{\pi}^* = R$. We study the monotonicity properties of $\bar{A}^b(\cdot)$ and $\bar{A}^g(\cdot)$.

The default threshold $\bar{A}^b(i)$ induced by belief of investors has the following properties. First, $\bar{A}^b(\underline{i}) = A_l$: if investors charge \underline{i} , it means that they expect no default. Second, differentiating (2.36) with respect to \bar{A} and i , one gets:

$$\frac{d\bar{A}^b(i)}{di} = \frac{(1-F(\bar{A}))}{f(\bar{A})(1+i)} > 0, \quad (2.38)$$

since $f(\cdot) > 0$. Finally, $\lim_{i \rightarrow +\infty} \bar{A}^b(i) = A_h$.

The best response of the government to i is captured by $\bar{A}^g(i)$, the default threshold. Given Assumption 3, for low values of i , debt is risk free.²³ Hence, there is $\epsilon > 0$ such that $\bar{A}^g(\underline{i} + \epsilon) = A_l$.

²²Lemma 1 established that this threshold is unique. To determine $\Delta(A, i)$ from (2.14) and (2.15), set $\tilde{\pi} = m = \tilde{\pi}^*$ and set τ from (2.30) if the government decides to repay.

²³Relaxing Assumption 3 and allowing a fundamental equilibrium with positive probability of default does not change the central result that several interest rates are compatible with the no-arbitrage condition. This is explicit in the real environment, as in Proposition 1, and can be established in the nominal economy as well.

Second, by differentiating (2.31) with respect to \bar{A} and i , one gets:

$$\tilde{\pi}^* b \left[\frac{1-\tau}{1-2\tau} - \theta \right] di + \bar{A} \left[(1-\gamma)^2 - \frac{(1-\tau)^2}{1-2\tau} \right] d\bar{A} = 0. \quad (2.39)$$

The factor of di is positive since $\frac{1-\tau}{1-2\tau} > 1$ and the factor of $d\bar{A}$ is negative since $\frac{(1-\tau)^2}{1-2\tau} > 1$. Hence:

$$\text{if } \bar{A}^g(i) \in (A_l, A_h), \text{ then } \frac{d\bar{A}^g(i)}{di} > 0. \quad (2.40)$$

Finally, there is an upper bound \bar{i} such that default occurs for all A if $i \geq \bar{i}$:

$$\forall i > \bar{i}, \bar{A}^g(i) = A_h. \quad (2.41)$$

By continuity of the functions $\bar{A}^g(\cdot)$ and $\bar{A}^b(\cdot)$, there is a value $i > \underline{i}$ that satisfies $\bar{A}^g(i) = \bar{A}^b(i)$. ■

The monotonicity properties of $\bar{A}^g(i)$ and $\bar{A}^b(i)$ are summarized in Figure 2.1. Under Assumption 3, there is always an equilibrium with certain repayment, where the nominal interest rate is \underline{i} . In addition, there will exist an equilibrium in which the debt is never repaid and, accordingly, investors place zero probability on repayment.²⁴ Lemma 2 characterizes additional interior equilibria in which default arises with a positive probability: there is $\bar{A} \in (A_l, A_h)$ and $i > \underline{i}$ that satisfy the no-arbitrage condition with state contingent default.

Figure 2.1 illustrates the multiplicity of equilibria, including three interior equilibria. The equilibrium of the debt financing problem labeled \star is a locally stable equilibrium with a positive probability of default. Here local stability refers to best-response dynamics and is used for comparative statics.²⁵

This lemma provides the basis for the existence of a SREE in which sunspots matter, i.e. the value of government debt is dependent upon the beliefs of investors. In equilibrium there are sunspot dependent variations in employment, output and consumption.

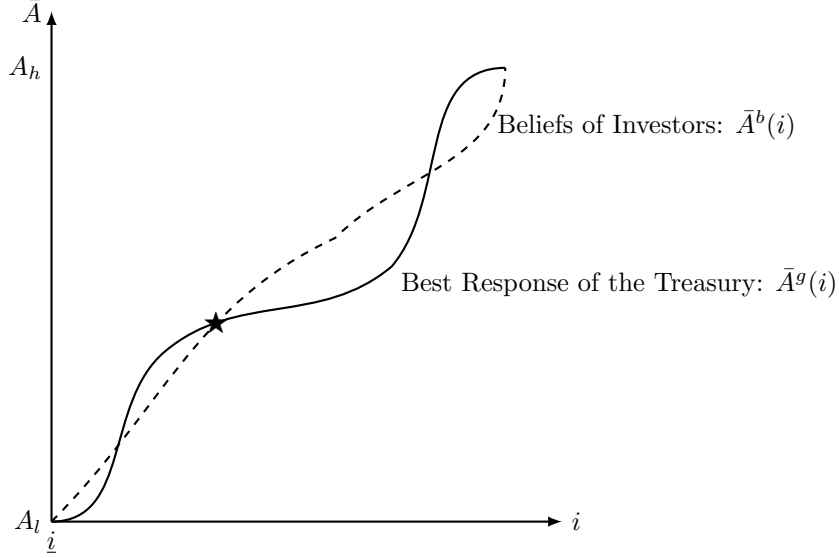
Proposition 2. *Under Assumption 2 and 3, for any $0 < \tilde{\pi}^* \leq 1$, there is a SREE with the following characteristics:*

1. *If $s_{-1} = s^o$, the government security is risk free and the treasury reimburses with probability 1.*
2. *If $s_{-1} = s^p$, the interest rate incorporates a risk-premium and the treasury defaults on its debt with positive probability.*

²⁴As mentioned previously, we discard this “market shutdown” case, which always exists.

²⁵Specifically, ‘best response dynamics’ points to the dynamics induced by investors responding to the treasury, followed by the treasury responding to investors. To see why the \star equilibrium is locally stable, suppose the interest rate i is lower than the equilibrium value. Given i , the treasury decision is captured by a threshold level for default, $\bar{A}^g(i)$, along the solid line. Given this, investors will ‘set’ an interest rate such that the no-arbitrage conditions holds, i.e. $\bar{A}^b(i)$ along the dashed line. Following this dynamic will lead to the locally stable equilibrium.

Figure 2.1: Multiplicity of Interest Rates under Monetary Delegation



This figure represents the mapping from interest rate i to default threshold \bar{A} , both for investors and the fiscal authority. Investors associate an interest rate i to a default threshold via the probability of default in the no-arbitrage condition. This is the dashed line. Given the interest rate i , the optimal decision of the fiscal authority to service its debt or default is captured by the default threshold, indicated by the solid line. An equilibrium is reached when beliefs of investors are consistent with the best-response of the fiscal authority. The figure highlights the existence of several equilibria, one of them being risk-free. The equilibrium indicated with a \star is locally stable under best response dynamics.

Proof. The characterization of the SREE directly comes from Lemma 2 and the existence of several interest rates compatible with the no-arbitrage condition in equilibrium. We describe the optimal behavior of agents consistent with the equilibrium definition.

As $\tilde{\pi}^e = \tilde{\pi}^* \in (0, 1]$, poor agents save only with money holding and rich young agents invest in intermediated claims. Indeed, consider a young household with productivity z . It can either save with money holding or via the financial sector, incurring the fixed cost Γ .

If it chooses to hold money, its labor supply when young is $n = z\tilde{\pi}^e$, its real demand for money holding is $zn = z^2\tilde{\pi}^e$ and the net expected contribution to consumption: $(z\tilde{\pi}^e)^2$. If it chooses the intermediated savings, its labor supply when young is $n = Rz$, its savings net of the intermediation cost $s = Rz^2 - \Gamma$ and the net expected contribution to consumption: $R(Rz^2 - \Gamma)$. Hence, intermediated saving dominates money holding if and only if:

$$z^2 > \frac{R\Gamma}{R^2 - (\tilde{\pi}^e)^2}, \quad (2.42)$$

which is true for any $\tilde{\pi}^e \in (0, 1]$ as long as Assumption 1 holds. An aggregate fraction $\theta \in [0, 1]$ of the government security is being held by domestic rich agents.

If $s_{-1} = s^o$, then young agents form expectations $P^d = 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. They supply labor accordingly. Consequently, the interest rate on debt satisfies the no-arbitrage condition (2.9) with $P^d = 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. Given i , seignorage revenue $\nu^m \tilde{\pi}^*(1 - \tilde{\pi}^*)$ and using Assumption 3, the

optimal policy of the treasury is then to raise labor taxes τ for all A so as to satisfy its budget constraint and repay its debt.

All markets clear. The money demand of the young poor agents is constant at $\tilde{\pi}^*$. The price level adjusts to ensure market clearing. From this, $\tilde{\pi}^* = \frac{1}{1+\sigma^*}$. In this equilibrium, inflation targeting and setting fixed money growth rate are equivalent. Given the no-arbitrage condition, the bond market clears assuming the foreign lenders have enough endowment to buy the government debt not purchased by domestic rich agents.

For the case $s_{-1} = s^p$, we outline only differences with the previous case. From Lemma 2, there is an interest rate i that carries a risk premium and satisfy the no-arbitrage condition, such that $(1+i)\tilde{\pi}^* > R$. Young agents form expectations $P^d > 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. They price the government debt according to $P^d > 0$ and $\tilde{\pi}^e = \tilde{\pi}^*$. Given i and seignorage revenue $\nu^m \tilde{\pi}^*(1 - \tilde{\pi}^*)$, there is a unique threshold $\bar{A}(i)$ such that the optimal policy of the treasury is to raise labor taxes τ for all $A \geq \bar{A}(i)$ to satisfy its budget constraint and default otherwise. Finally, expectations are consistent with the best response of the government: $P^d = F(\bar{A}(i))$. ■

Aguiar, Amador, Farhi, and Gopinath (2013) find a similar result if there is a very high perceived cost of inflation: when the central bank is very inflation averse, it never chooses to inflate the nominal value of debt, it is *de facto* committed to no inflation, converting nominal debt into real debt. Roll-over crises occur for a larger range of debt.

Overall this section, particularly Proposition 2, makes clear that debt fragility, as identified in real economies (Proposition 1), extends to economies with nominal debt. In effect, the inflation target of the monetary authority converts the nominal obligation to a real one. Seignorage does reduce the real debt burden left to the fiscal authority, but without eliminating the underlying strategic uncertainty. The choice of the inflation target does not allow the monetary authority to peg the real interest rate. Instead the real interest rate on debt continues to reflect the sentiments of investors.

In Section 2.5, this stationary equilibrium is used as a threat point to support the credibility of an alternative monetary regime. Let $V^{dlg}(\tilde{\pi}^*, p)$ be the life-time welfare of agents in a given generation under *monetary delegation*, where $\tilde{\pi}^*$ is the inflation target and p the probability of optimism. Formally, using (2.13) and (2.37), with $m_{-1} = \tilde{\pi}^e = \tilde{\pi}^*$:

$$\begin{aligned}
 V^{dlg}(\tilde{\pi}^*, p) = & p \left[\int_{A_l}^{A_h} W^r(A, \underline{i}) dF(A) \right] \\
 & + (1-p) \left[\int_{A_l}^{\bar{A}} W^d(A, i) dF(A) + \int_{\bar{A}}^{A_h} W^r(A, i) dF(A) \right] - \sum_{j \in \{m, I\}} \nu^j \frac{(n_y^j)^2}{2}. \tag{2.43}
 \end{aligned}$$

The first term corresponds to the expected welfare of old agents under optimism, when the nominal interest rate \underline{i} induces repayment for any realization of technology A . The second term is the expected welfare under pessimism, where the risk premium included in the nominal interest rate $i > \underline{i}$ leads the treasury to default for low realizations of $A \in [A_l, \bar{A}]$. Finally, as inflation

expectations are anchored, the third term captures young agents' disutility of labor is independent of the realization of the sunspot shock.²⁶

Importantly, the welfare of a generation under monetary delegation is increasing in the probability of optimism p and decreasing with the nominal interest rate i associated with pessimism. Indeed, as discussed above, the equilibrium under optimism Pareto dominates the coordination failure outcome, and the higher the risk premium under pessimism, the lower is welfare.

Finally, Proposition 2 is stated for any level of inflation target $\tilde{\pi}^*$. This does not imply though that the equilibrium is independent of the inflation target. The inflation target will influence seignorage revenue and affect the fiscal burden. The size and magnitude of these effects will depend on the target inflation relative to the peak of the "Laffer curve".

Proposition 3. *In the equilibrium characterized in Proposition 2, for $\tilde{\pi}^* \geq \tilde{\pi}^L$, an increase in the target inflation rate will increase seignorage and lower the probability of default if and only if the equilibrium of the debt financing problem is locally stable.*

Proof. The proof relies on three expressions. The point of indifference between repayment and default, (i.e. the default threshold \bar{A}) is given in (2.31), the government budget constraint given in (2.30), and the no-arbitrage condition, given in (2.35). These are all evaluated at a given inflation target and thus $\tilde{\pi}^*$. Substituting the no-arbitrage condition into (2.31) gives:

$$\frac{[\bar{A}(1-\gamma)]^2}{2} - \frac{[\bar{A}(1-\tau)]^2}{2} + \nu^m \tilde{\pi}^* (1 - \tilde{\pi}^*) - \frac{R\theta b}{1 - F(\bar{A})} = 0, \quad (2.44)$$

where τ satisfies the government budget constraint evaluated at \bar{A} .

Taking the derivative of (2.44) w.r.t. \bar{A} and π^* :

$$\left[\bar{A}(1-\gamma)^2 - \bar{A}(1-\tau)^2 + \bar{A}^2(1-\tau)\tau_A - \frac{R\theta b f(\bar{A})}{(1-F(\bar{A}))^2} \right] d\bar{A} + \left[\bar{A}^2(1-\tau)\tau_{\tilde{\pi}^*} + \nu^m(1-2\tilde{\pi}^*) \right] d\pi^* = 0, \quad (2.45)$$

where τ_A and $\tau_{\tilde{\pi}^*}$ are given by the derivative of the government budget constraint evaluated in \bar{A} . Substituting the no-arbitrage condition into the budget constraint (2.30) and taking the derivative w.r.t. τ, π^*, A , one gets:

$$\tau_A = \frac{1}{A^2(1-2\tau)} \left[\frac{Rbf(A)}{(1-F(A))^2} - 2A(1-\tau)\tau \right] \quad \tau_{\tilde{\pi}^*} = -\frac{\nu^m(1-2\tilde{\pi}^*)}{A^2(1-2\tau)}. \quad (2.46)$$

Rearranging (2.45), one gets:

$$\left[\bar{A}(1-\gamma)^2 - \bar{A} \frac{(1-\tau)^2}{1-2\tau} + \frac{Rbf(\bar{A})}{(1-F(\bar{A}))^2} \left(\frac{1-\tau}{1-2\tau} - \theta \right) \right] d\bar{A} + \left[\nu^m(1-2\tilde{\pi}^*) \left(1 - \frac{1-\tau}{1-2\tau} \right) \right] d\tilde{\pi}^* = 0. \quad (2.47)$$

²⁶Formally, $n_y^m = \tilde{\pi}^*$ and $n_y^l = Rz$ from (2.4) and (2.10).

The factor of $d\tilde{\pi}^*$ is positive since $\tilde{\pi}^* \geq \tilde{\pi}^L$ and $\frac{1-\tau}{1-2\tau} > 1$. Hence the sign of the factor of $d\bar{A}$ is critical to derive the response of the default threshold to a change in the inflation target.

This sign is determined by the condition of local stability. An equilibrium is locally stable under best response dynamics if and only if $\frac{d\bar{A}^g(i)}{di} < \frac{d\bar{A}^b(i)}{di}$. Rewriting (2.38) with the no-arbitrage condition and using (2.39), the condition for local stability becomes:

$$\bar{A}(1-\gamma)^2 - \bar{A}\frac{(1-\tau)^2}{1-2\tau} + \frac{Rbf(A)}{(1-F(A))^2} \left[\frac{1-\tau}{1-2\tau} - \theta \right] < 0. \quad (2.48)$$

Hence under local stability, $\frac{d\bar{A}}{d\tilde{\pi}^*} > 0$. ■

Proposition 3 is essentially a comparative statics result and thus holds for only a subset of equilibria of the debt financing problem, i.e. those that are locally stable under best response dynamics. A locally stable equilibrium is indicated in Figure 2.1 and refers to the determination of the interest rate on debt and the default cut-off. The relative slopes of the two curves at this point are used in the proof of Proposition 3.

This section studies debt fragility when the monetary authority commits to a strict inflation target. As the environment does not feature any ex post cost of inflation, the central bank could be tempted to deviate, inflate beyond expectations and thus steal the benefits of predetermined real money balances. Still, this policy could be sustained with a reputational mechanism that takes full demonetization of the economy as the punishment for deviating from the inflation target. Indeed, the welfare in the real economy is lower than the welfare in the monetary economy. We formally discuss this class of reputational mechanisms in Section 2.5.2.

2.4.3 Monetary Discretion

Does monetary discretion insulate against debt fragility? Intuitively, a discretionary policy maker could adjust inflation and seignorage to accommodate variations in the price of government debt driven by strategic uncertainty and avoid default.

In a monetary discretion regime, the government has full discretionary power over both monetary and fiscal policy. It designs its policy $(\tau(\mathcal{S}), \tilde{\pi}(\mathcal{S}), D(\mathcal{S}))$ in every state, as a best response to realized productivity shock A , the sunspots (s_{-1}, s) and predetermined variables of the economy m_{-1} and i . The government maximizes the welfare of home agents. This is, in effect, the same as minimizing the cost of its policy to taxpayers, hence to old agents, since they contribute to government's resources via the tax on labor income and seignorage on money holding.

In an environment with discretion, money creation provides an *ex post* source of revenue without creating any distortion. This low social cost of revenue ought to reduce the likelihood of default and stabilize debt values.

But, an essential element of this environment is the interaction between expected and realized inflation. Specifically, if agents anticipate high inflation (low $\tilde{\pi}^e$), they would reduce labor supply in youth and their real money holdings m_{-1} accordingly. To collect revenue from seignorage, the

central bank then has to deliver a higher inflation rate (low $\tilde{\pi}$), consistent with the initial beliefs of agents. Hence, under discretion, the capacity of the central bank to support a stressed fiscal authority may be compromised by the strategic complementarity between expected inflation and delivered inflation: if agents anticipate the willingness of the central bank to resort to inflation, the real money tax base would decrease, which in turn reduces the capacity of the central bank to intervene.

The formation of expectations by young agents reflects these *ex post* policy choices. Let $\tilde{\pi}^e(\mathcal{S})$ denote the expectation of future (inverse) inflation given the current state \mathcal{S} . Then the requirement of rational expectations is $\tilde{\pi}^e(\mathcal{S}) = E_{\mathcal{S}'|\mathcal{S}}\tilde{\pi}(\mathcal{S}')$ where the expectation is over the future state given \mathcal{S} . This condition will be used in the construction of equilibria under discretion.

To characterize a SREE under discretion, we first determine the policy choices of a discretionary policy maker, then analyze the debt pricing dimension of the equilibrium and the associated stationary inflation expectations. The SREE combines these essential elements.

Choice Problem of a Discretionary Government

Given the productivity shock A , real money holding m_{-1} and nominal interest rate i on debt, the government chooses the money printing rate σ and whether to default ($D = d$) or raise taxes τ and repay its debt ($D = r$). Under this regime, the government sets the money growth rate σ^D and collects seignorage given by:

$$\frac{\Delta M}{P} = \nu^m m_{-1} \sigma^D \tilde{\pi}^D, \quad (2.49)$$

where m_{-1} is the real money holding of current old agents and $\tilde{\pi}^D$ is the inverse rate of inflation for $D \in \{r, d\}$ induced by the choice of σ^D . It is given by:

$$\tilde{\pi}^D = \frac{1}{1 + \sigma^D} \frac{m}{m_{-1}}. \quad (2.50)$$

Here $m \equiv m(\mathcal{S})$ is the real money demand of current young agents. As seen in (2.4), the money demand of the young is driven by current inflation expectations that are entirely independent of the current choices of the discretionary policy maker. Further, m_{-1} , real money held by the current old, is predetermined when the monetary authority decides on σ^D . Thus (2.50) captures a direct link from money growth to inverse inflation $\tilde{\pi}^D$. The government budget constraint under repayment is:

$$(1 + i)\tilde{\pi}^r b = A^2(1 - \tau)\tau + \nu^m m_{-1} \sigma^r \tilde{\pi}^r. \quad (2.51)$$

Hence, the government solves

$$D \in \{r, d\} = \operatorname{argmax} \left[\max_{\tau, \sigma^r} W^r(A, i, m_{-1}, \tau, \sigma^r, \tilde{\pi}^r), \max_{\sigma^d} W^d(A, i, m_{-1}, \sigma^d, \tilde{\pi}^d) \right], \quad (2.52)$$

subject to its budget constraint (2.51), a non-negativity constraint on labor tax $\tau \geq 0$ and the following restriction on the realized inverse inflation rate: $\tilde{\pi}^D \in [\underline{\tilde{\pi}}, 1]$. The solution generates a default choice as well as a tax rate τ in the event of repayment and money growth rates σ^D dependent on the default decision, $D = d, r$. As mentioned above, the money growth rate induces a realization of the inflation rate, hence we describe monetary policy as the choice of $\tilde{\pi}^D(\cdot)$.

Following Calvo (1978), we assume that money printing is bounded so that the effective inverse inflation rate cannot be lower than $\underline{\tilde{\pi}} > 0$.²⁷ Importantly, our results do not hinge upon this precise bound, but rather lie in the strategic interaction between expected and realized inflation under discretion.²⁸

If the government chooses to repay the debt, the real money tax base m_{-1} is given. Its policy is naturally biased toward inflation since taxing money holdings does not distort labor supply decisions of current money holders. If this tax revenue is sufficient to cover its obligations, there is no labor tax imposed, and, using Assumption 2, repayment is preferred over default. Else, if seignorage does not generate enough revenue to cover its obligations, the government must impose a labor tax if it chooses to avoid default. Under discretion, the government relies primarily on seignorage revenue to service debt. This characterization is summarized in Lemma 3.

In the event of default, the choice of the inflation rate is welfare neutral given the specified social welfare function: when default occurs, monetary policy is implemented via lump-sum transfers which are purely redistributive, and consequently has no influence on the choices of the government. We set $\tilde{\pi}^d = \underline{\tilde{\pi}}$ in the event of default so that this rate is consistent with the inflation chosen whenever the government is indifferent between default and repayment.

The following lemma summarizes the state contingent choices of the government under discretion.

Lemma 3. *Under Assumption 2, given \mathcal{S} , the policy choices of the discretionary government are:*

1. *if the government chooses to repay its debt, then*

a. $\tilde{\pi}^r = \max \{ \underline{\tilde{\pi}}, \Pi(\mathcal{S}) \}$, where $\Pi(\mathcal{S}) = \frac{\nu^m m(\mathcal{S})}{\nu^m m_{-1} + (1+i)b}$,

b. $\tau > 0$ and solves the government budget constraint (2.51) if and only if $\tilde{\pi}^r = \underline{\tilde{\pi}}$.

2. *if the government chooses to default, then $\tau = 0$ and $\tilde{\pi}^d = \underline{\tilde{\pi}}$.*

3. *the government chooses to default if and only if*

$$\Delta(\cdot) = \frac{[A(1-\gamma)]^2}{2} - \frac{[A(1-\tau)]^2}{2} - (1+i)\underline{\tilde{\pi}}\theta b + T(\mathcal{S}, \underline{\tilde{\pi}}) > 0,$$

²⁷Chari, Christiano, and Eichenbaum (1998) impose a similar restriction on the highest inflation regime that the central bank can implement. In the appendix of that paper, this restriction is rationalized by the presence of an alternative technology such that agents can bypass the cash-in-advance constraint. In effect, the return on this alternative technology pins down the worst sustainable equilibrium and thus $\underline{\tilde{\pi}}$. In our framework, the poor could store at a return of $r < 1$ instead of holding money and a parallel argument could be made for $\underline{\tilde{\pi}}$.

²⁸Corsetti and Dedola (2013) and Aguiar, Amador, Farhi, and Gopinath (2013) adopt an *ex post* cost of inflation to limit money creation.

where τ solves (2.51) given $\tilde{\pi}^r = \tilde{\pi}$ under repayment and $T(\cdot)$ is the lump-sum transfer that implements $\tilde{\pi}$ under default.

Proof. If the government repays, it will first use the inflation tax to obtain revenue since this tax is not distortionary. It will use labor taxation only if needed to repay the debt. Hence, if the real inflation tax base is large enough to service debt, then its labor tax policy is $\tau = 0$.

We derive first the condition under which seignorage alone is enough to service debt. From the budget constraint (2.51), if $\tau = 0$, then $(1+i)\tilde{\pi}^r b = \nu^m \tilde{\pi}^r m_{-1} \sigma$ implying $\sigma = \frac{(1+i)b}{\nu^m m_{-1}}$. Using (2.50), we get that under repayment $\tilde{\pi}^r = \frac{m(\mathcal{S})}{m_{-1}} \frac{1}{1+\sigma}$, where m_{-1} is real money held by the old and $m(\mathcal{S})$ is the level of real money demand of the current young. The resulting inverse rate of inflation is given by $\Pi(\mathcal{S}) = \frac{\nu^m m(\mathcal{S})}{(1+i)b + \nu^m m_{-1}}$. Hence, resource from seignorage is enough to service debt if $\Pi(\mathcal{S}) \geq \tilde{\pi}$.

We next verify that $\Pi(\mathcal{S}) \geq \tilde{\pi}$ implies the treasury chooses to service its debt rather than default, i.e. $\Delta(\cdot) \equiv W^d(\cdot) - W^r(\cdot) < 0$. With $\tau = 0$, $\Delta(\cdot)$ is:

$$\Delta(\cdot) = \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} + \nu^m m_{-1} (\tilde{\pi} - \tilde{\pi}^r) - (1+i)\tilde{\pi}^r \theta b + T(\mathcal{S}, \tilde{\pi}). \quad (2.53)$$

Here $T(\cdot) = \nu^m m_{-1} \sigma^d \tilde{\pi}$ is the lump-sum transfer that implements $\tilde{\pi}^d = \tilde{\pi}$, with $\sigma^d = \frac{m(\mathcal{S})}{\tilde{\pi} m_{-1}} - 1$. Also, as seignorage is sufficient to service principal and interest on debt, $(1+i)b = \sigma^r \nu^m m_{-1}$, with $\sigma^r = \frac{m(\mathcal{S})}{\tilde{\pi}^r m_{-1}} - 1$. Finally, by the definition of $\tilde{\pi}^D$, $\nu^m m_{-1} \tilde{\pi}^D (1 + \sigma^D) = \nu^m m(\mathcal{S})$ for $D = r, d$. Rearranging (2.53), one gets:

$$\begin{aligned} \Delta(\cdot) &= \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} - \nu^m m_{-1} \tilde{\pi}^r \sigma^r (\theta - 1) \\ &= \frac{[A(1-\gamma)]^2}{2} - \frac{A^2}{2} - \nu^m m(\mathcal{S}) \frac{\sigma^r}{1 + \sigma^r} (\theta - 1). \end{aligned} \quad (2.54)$$

This is negative by Assumption 2 as long as $\frac{m(\mathcal{S})\sigma^r}{1+\sigma^r} (1-\theta) < 1$. With $\theta \leq 1$ and $\sigma \geq 0$ under both optimism and pessimism, $\tilde{\pi} \leq 1$ so that $m(\mathcal{S}) = \tilde{\pi}^e(\mathcal{S}) \leq 1$. Hence $\frac{m(\mathcal{S})\sigma}{1+\sigma} (1-\theta) < 1$. We get $\Delta(\cdot) < 0$, i.e. when seignorage is enough to service principal and interest, the government chooses not to default.

If resource from seignorage is not enough to service principal and interest on debt, then positive labor taxes are implemented: $\tau > 0$ if and only if $\tilde{\pi} > \Pi(\mathcal{S})$. In this case, default is possible. Using these elements together with (2.14) and (2.15), one gets the expression for $\Delta(\cdot)$ stated in the Lemma. ■

Equation (2.54) highlights an interesting aspect of redistribution in this economy. If all debt is held internally, $\theta = 1$, the default decision in this monetary economy is independent of the rate of money creation. This reflects the fact that the inflation tax is not distortionary and simply redistributes between rich and poor, both of whom are risk neutral. In this case, $\Delta < 0$ and there is surely no default. But if $\theta < 1$, then the inflation tax borne by the poor old agents is

redistributed to the foreign holders of the debt, which is welfare reducing. Still, as argued in the proof, default does not occur if the debt obligation can be financed entirely by seignorage.

Price of Government Debt

Building on this characterization, we next investigate whether debt fragility remains under monetary discretion. First, we show that the multiplicity of interest rates consistent with the no-arbitrage condition (2.9) persists and interacts with inflation expectations.

Lemma 4. *Under Assumptions 2 and 3, under monetary discretion, there are multiple interest rates that solve the no-arbitrage condition (2.9).*

Proof. Consider the debt pricing building block of the equilibrium. We show that there are several possible outcomes, and consistent with our equilibrium definition, these different outcomes are driven by the realization of the sunspot s_{-1} . Using Assumption 3 and Lemma 3, there is a risk-free equilibrium of the debt financing problem, with inflation expectations $\tilde{\pi}^e(s^o) \geq \tilde{\pi}$.²⁹ This may arise with $\tau = 0$ and $\tilde{\pi}^e(s^o) \geq \tilde{\pi}$ or, from Lemma 3, with $\tau > 0$ and $\tilde{\pi}^e(s^o) = \tilde{\pi}$.

Suppose investors believe the government will default on its debt with positive probability. If the belief is self-fulfilling, then the optimal policy of the government must be to set the inflation level to $\tilde{\pi}$ for all A whether it reimburses its debt or defaults. Otherwise, resources from seignorage would be enough to cover principal and interest on debt for all realization of A , and default would be avoided. Hence, inflation expectations of agents are consistent with the best response of the government at $\tilde{\pi}^e(s^p) = \tilde{\pi}$. The no-arbitrage condition pricing public debt becomes:

$$(1 + i)\tilde{\pi}\left(1 - F(\bar{A}(i))\right) = R, \quad (2.55)$$

where $\bar{A}(i)$, defined in Lemma 1, is the boundary of the default region given i .

From Lemma 2, we know that there are at least two interest rates i that are consistent with this equilibrium condition, one of which carries a risk-premium and induces the government to default for some realizations of A . Hence the initial pessimistic beliefs are self-fulfilling and support the existence of an interest rate that carries a positive probability of default. ■

The key is that inflation expectations and probability of default are jointly linked by the anticipation of the best response of the discretionary government. In particular, the interest rate with a risk-premium that solves the no-arbitrage condition is systematically associated with the lowest real money tax base $m = \tilde{\pi}^e(s^p) = \tilde{\pi}$, which in turn prevents the central bank from inflating away the real value of debt.

²⁹In general $\tilde{\pi}^e(S)$ denotes expected (inverse) inflation. The notation $\tilde{\pi}^e(s)$ highlights the dependence of expectations on the sunspot, s . This is the expectation held by young agents regarding the future value of $\tilde{\pi}$. This value determines the labor supply and real money demand of young poor agents. It also influences the nominal interest rate, see (2.9).

Inflation Expectations

We now turn to the third component of the equilibrium: the determinants of inflation expectations $\tilde{\pi}^e(s)$. This section establishes two results. First, there exist inflation expectations, contingent on the sunspot realization, that are consistent with the choices of the government. Second, there may be multiple such levels of inflationary expectations. This last point arises from the complementarities between monetary expectations and policy response.

As shown in the proof of Lemma 4, in the event of pessimism, young agents expect high inflation, i.e. $\tilde{\pi}^e(s^p) = \tilde{\pi}$. That is, whenever the equilibrium of the debt financing problem induces state-contingent default, the inflation rate is maximal. So, regardless of the current state \mathcal{S} , given pessimism in the previous period, $s_{-1} = s^p$, the inverse inflation rate is $\tilde{\pi}(\mathcal{S}) = \tilde{\pi}$. This is then consistent with the initial expectations of the current old, formed when they were young in the previous period, i.e. $\tilde{\pi}^e(s^p) = \tilde{\pi}$.

The issues of existence and multiplicity of inflation expectations arise when $s_{-1} = s^o$. From Lemma 4, young agents anticipate the government will service its debt obligation for all \mathcal{S} . The question is then how the government will repay its obligation. Given the bias toward inflationary financing of debt, what determines $\tilde{\pi}^e(s^o)$ is whether seignorage resource is enough to service principal and interest on debt for all (A, s) . If the debt is not too large, then the inflation tax alone is sufficient to cover debt obligations: i.e. $\tilde{\pi}^r(A, s, \cdot) > \tilde{\pi}$ for all (A, s) and $\tilde{\pi}^e(s^o) > \tilde{\pi}$. In this case, $\tau(A, s) = 0$ for all (A, s) . Else, the inflation tax will be maximal, $\tilde{\pi}^e(s^o) = \tilde{\pi}$, and supplemented by a labor tax. In both cases $D(A, s, \cdot) = r$ for all (A, s) .

Formally, Lemma 3 established that with $s_{-1} = s^o$, given the real money tax base $\nu^m m_{-1} = \nu^m \tilde{\pi}^e(s^o)$, the inflation delivered by the discretionary government satisfies:

$$\tilde{\pi}^r(A, s, \cdot) = \max \left\{ \frac{\nu^m \tilde{\pi}^e(s)}{\nu^m \tilde{\pi}^e(s^o) + (1+i)b}; \tilde{\pi} \right\} \quad \forall A \quad \forall s \in \{s^o, s^p\}, \quad (2.56)$$

where the max operator captures whether seignorage resource is enough to service principal and interest on debt, and $\tilde{\pi}^e(s) = m(\mathcal{S})$ is the real money demand of current young agents, conditional on the realization of the current sunspot s . The no-arbitrage condition gives: $(1+i)\tilde{\pi}^e(s^o) = R$. Accordingly, $\tilde{\pi}^e(s^o) \geq \tilde{\pi}$ can be part of a stationary equilibrium if and only if it satisfies:

$$\tilde{\pi}^e(s^o) = p \max \left\{ \frac{\nu^m \tilde{\pi}^e(s^o)}{\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b}; \tilde{\pi} \right\} + (1-p) \max \left\{ \frac{\nu^m \tilde{\pi}^e(s^p)}{\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b}; \tilde{\pi} \right\}, \quad (2.57)$$

where p is the stationary probability of optimism.

The following lemma establishes the existence of stationary inflation expectations under optimism that are consistent with the policy choices of the government for all $b \in (0, \bar{b})$.

Lemma 5. *Given Assumptions 2 and 3, under monetary discretion, there is a debt threshold $\hat{b} = \frac{\nu^m \tilde{\pi}(1-\tilde{\pi})}{R}$ such that:*

1. *If $0 < b < \hat{b}$, then $\tilde{\pi}^e(s^o) > \tilde{\pi}$ is consistent with the government choice $\tilde{\pi}^r(A, s, \cdot) > \tilde{\pi}$, for all*

(A, s) .

2. If $\hat{b} \leq b < \bar{b}$, then $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$ is consistent with the government choice $\tilde{\pi}^r(A, s, \cdot) = \underline{\tilde{\pi}}$, for all (A, s) .

Proof. Computation details are provided in Appendix 2.7.2. ■

In the first case, the level of debt and inflation expectations are such that seignorage is sufficient to service debt for any realization of s . In the second case, the level of debt and inflation expectations are such that seignorage is not sufficient to service debt, and must be complemented with labor taxes for any realization of s .³⁰

In fact, for some debt level $b \geq \hat{b}$, $\tilde{\pi}^e(s^o)$ can take several values. This is due to the interactions between expected inflation and delivered inflation, present in (2.57). It can give rise to a seignorage Laffer curve, where several rates of inflation in excess of $\underline{\tilde{\pi}}$ generate the same real resource.³¹ In addition, there is another possible outcome with maximum inflation, i.e. $\tilde{\pi}^e(s^o) = \underline{\tilde{\pi}}$, and positive labor taxes. These elements are summarized in Figure 2.2. To be clear, this multiplicity is an outgrowth of allowing an endogenous money demand through the overlapping generations structure.

Lemma 5 establishes the existence of $\tilde{\pi}^e(s^o)$ for all $b < \bar{b}$, but does not describe all the possible regimes. Indeed, as explained in the proof of Lemma 4, the inflation regime under optimism does not influence the existence of multiple valuations of debt. Hence, without loss of generality, we select in Lemma 5 an inflation regime for $b \geq \hat{b}$, marked in blue in Figure 2.2.

Equilibrium characterization

The analysis has established the government budget constraint under discretion, the potential for multiple solutions to the debt valuation equation and the existence of inflation expectations consistent with monetary policy. Taken together, these elements create the basis for sunspot equilibria associated with the valuation of government debt. Formally,

Proposition 4. *For any $\underline{\tilde{\pi}} > 0$, under Assumptions 2 and 3, there is a SREE under discretion with the following properties:*

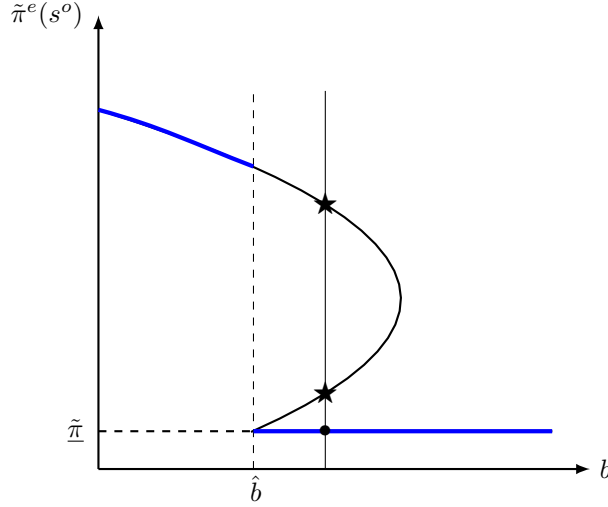
1. If $s_{-1} = s^o$, government debt is risk free as the treasury reimburses with probability 1, with either:

a. if $0 < b < \hat{b}$, then $\tilde{\pi}^e(s^o) > \underline{\tilde{\pi}}$ and for all A all s , $\tilde{\pi}(A, s, \cdot) > \underline{\tilde{\pi}}$, $\tau(A, s, \cdot) = 0$,
 $D(A, s, \cdot) = r$,

³⁰We do not impose further parametric restriction to ensure that $\hat{b} < \bar{b}$, where \bar{b} is defined by Assumption 3. This requires the lower bound on productivity A_l or the cost of default γ to be high enough or the share of money holder ν^m to be low enough. If it were the case that $\hat{b} \geq \bar{b}$, then only case 1 of Lemma 5 would apply, our results would not be affected.

³¹This is the standard textbook Laffer curve, as in Theorem 26.2 of Azariadis (1993), and discussed in Appendix 2.7.2.

Figure 2.2: Multiplicity of Inflation Regime under Optimism



The figure illustrates the possibility of multiple stationary inflation expectations compatible with the policy of the government. For $b \geq \hat{b}$, several outcomes are possible: the ones marked \star correspond to the seignorage Laffer curve, where several rates of inflation are consistent with the repayment of debt, with no labor taxes. In addition, the outcome labelled \bullet can arise: agents reduce their real money demand to $\tilde{\pi}$, which in turn induces the discretionary government to generate $\tilde{\pi}$ and complement seignorage resource with positive labor taxes to service debt. The inflation regime under optimism selected in this analysis are indicated in blue.

b. if $\hat{b} \leq b < \bar{b}$, then $\tilde{\pi}^e(s^o) = \tilde{\pi}$ and for all A all s , $\tilde{\pi}(A, s, \cdot) = \tilde{\pi}$, $\tau(A, s, \cdot) > 0$, $D(A, s, \cdot) = r$.

2. If $s_{-1} = s^p$, the interest rate incorporates a risk-premium. For all A , $\tilde{\pi}(A, \cdot) = \tilde{\pi}$. The treasury defaults on its debt for all $A < \bar{A}$ where $\bar{A} \in (A_l, A_h)$ and $\tilde{\pi}^e(s^p) = \tilde{\pi}$.

Proof. We describe the optimal behavior of agents consistent with the equilibrium definition. This proof builds on Lemma 4 and the existence of several interest rates (and associated inflation expectations) consistent with the equilibrium definition.

If $s_{-1} = s^o$, then by Assumption 3, debt is risk free. Two cases need to be distinguished, as established in Lemma 5. If $b < \hat{b}$, then inflation expectations under optimism $\tilde{\pi}^e(s^o)$ allow seignorage resource to be sufficient to service principal and interest on debt. Young agents form expectations of no default and $\tilde{\pi}^e(s^o) > \tilde{\pi}$. They supply labor accordingly, young agents with low productivity save with money, young rich agents save via intermediated claims; the interest rate i on the government security satisfies the no-arbitrage condition (2.9) with a zero probability of default, i.e. $P^d = 0$, and $\tilde{\pi}^e(s^o)$. The optimal policy of the government is then to set for all A , all s , $\tilde{\pi}(A, s, \cdot) = \tilde{\pi}$, $\tau(A, s, \cdot) = 0$ and repay the debt.

On the other hand, if $\hat{b} \leq b < \bar{b}$, then there is an equilibrium with $\tilde{\pi}^e(s^o) = \tilde{\pi}$, seignorage resource is not sufficient and taxes need be raised to service debt. Using Lemma 3 and Assumption 3, for all A , all s , $\tilde{\pi}(A, s, \cdot) = \tilde{\pi}$, $\tau(A, s, \cdot)$ solves the government budget constraint (2.51) and debt is repaid. Accordingly, young agents form expectations $P^d = 0$, $\tilde{\pi}^e(s^o) = \tilde{\pi}$, the government security is priced according to (2.9). In both cases, all markets clear.

For $s_{-1} = s^p$, we detail only the differences with the previous case. Independently of the level of b , young agents form rational expectations in which there is a positive probability of default, i.e. $P^d > 0$, and $\tilde{\pi}^e(s^p) = \tilde{\pi}$. The government security is priced accordingly. Given i and seignorage revenue $\nu^m \tilde{\pi}(1 - \tilde{\pi})$, there is a unique threshold $\bar{A}(i) > A_l$ such that the optimal policy is to raise labor taxes τ for all $A \geq \bar{A}(i)$ so as to satisfy the budget constraint (2.51) and default otherwise. Finally, expectations are consistent with the best response of the government: $P^d = F(\bar{A}(i))$. ■

Does monetary discretion provide a shield against debt fragility? Can the government inflate the real value of debt and generate additional resources to service its debt? The answer is negative. As the proposition makes clear, this result does not hinge upon a particular inflation ceiling $\tilde{\pi}$.³² Indeed, when pessimism hits the economy, the interplay between inflation expectations and real money tax base *corners* the central bank into a high inflation regime with no more capacity to inflate debt or provide additional resources to the treasury. Hence, under monetary discretion, the sunspot shock to investors confidence triggers a joint shift in inflation expectations and debt sustainability. This shift in inflation expectations is the driving force that neutralizes the strategy of the discretionary government to print money and collect seignorage to service its debt.

Finally, the strategic complementarity between expected and delivered inflation may give rise to inflation multiplicity, as discussed in sub-section 2.4.3. This element is not explicit in Proposition 4. The point of the proposition, and of this paper, is to study the interaction of debt fragility and monetary policy. Accordingly, the analysis in Proposition 4 rests on the selection of a particular equilibrium in the event of optimism, i.e. $s_{-1} = s^o$. The equilibrium selection is orthogonal to the multiple solutions of the debt pricing equation, as argued before. Of course, it is possible to add another, independent, dimension to the problem by introducing strategic uncertainty over the the inflation rate, given optimism.

2.5 Leaning Against the Winds

These results make clear that the monetary authority may be unable to prevent debt fragility. If there is a commitment to an unconditional inflation target, the environment is similar to a real economy, thus exposing the debt to multiple valuations. If the monetary authority has complete discretion, then it will use the inflation tax to raise revenue *ex post* and again fiscal tools may be needed to finance debt repayments. In both cases, when productivity is low, the tax burden can become excessive leading to default. In equilibrium, the valuation of debt will be subject to investor sentiments in a sunspot equilibrium.

This section considers a more nuanced monetary intervention which is accommodative during periods of low productivity. We show that with commitment, this intervention eliminates debt fragility. Further, we provide conditions such that reputation effects are strong enough to support this outcome without commitment.

³²Specifically, it holds for $\tilde{\pi}$ arbitrarily close to 0, i.e. an inflation ceiling arbitrarily high.

2.5.1 Stabilization through Commitment

This section returns to the commitment case. Instead of imposing an inflation target, we allow the central bank to choose a state-contingent inflation policy that alters the real debt burden and distributes resources from seignorage across states. As in Chari, Christiano, and Eichenbaum (1998) this is a one period commitment, allowing the monetary authority to commit to inflation next period, contingent on the current state. By carefully choosing the distribution of realized future inflation, the central bank can provide a shield against debt fragility.

Suppose the monetary authority commits to a rule given by $\tilde{\pi}(A, i, s_{-1})$: the rate of inflation in the current period depends on current productivity, the interest rate on outstanding debt as well as the sunspot realization from previous period.³³ This rule is devised with a couple of key properties. First, to induce agents to hold money, the rule will deliver a target rate of inflation. Second, it will support the fundamental equilibrium by using monetary tools to counter pessimistic expectations so that equilibria with strategic uncertainty no longer exist.³⁴ In this way, the monetary authority responds to variations in current beliefs, reflected in the sunspot and the interest rate, by appropriately setting policy for the future. Importantly, if investors were pessimistic in the previous period, this policy responds to variations in productivity: the rate of inflation is inversely related to current productivity. Specifically, when A is high, the rate of inflation is relatively low and fiscal policy, through the setting of tax rates, bears more of the burden of financing debt obligations. But during times of low productivity, when default is likely, the monetary authority inflates the real value of debt and generates seignorage revenue. Both effects allow the fiscal authority to set low taxes and avoid default.

We first describe the desired properties of this policy, derive its existence and properties in Lemma 6. Then, we characterize the stationary equilibrium of the economy under $\tilde{\pi}(A, i, s_{-1})$ and argue that such monetary policy rule stabilizes debt valuations.³⁵

Specifically, suppose the central bank commits to a rule in which $\tilde{\pi}(A, i, s^o) = \tilde{\pi}^*$ for all (A, i) : under optimism, there is an inflation target as in *monetary delegation*. Delivered inflation $\tilde{\pi}^*$ is independent of both current productivity A and the interest rate on debt. When $s_{-1} = s^p$, the central bank implements a state dependent (on (A, i)) monetary policy, labelled '*law*', for *leaning against the winds*. This policy satisfies two key properties.

First, given pessimism the policy rule anchors inflation expectations: $\tilde{\pi}(A, i, s^p)$ meets the inflation target *on average*:

$$\int_A \tilde{\pi}(A, i, s^p) dF(A) = \tilde{\pi}^*, \quad (2.58)$$

³³This commitment is independent of other elements of the state vector.

³⁴To be clear, the policy is designed to eliminate equilibria with state contingent default. The equilibrium with certain default remains.

³⁵As written, the intervention depends on (A, i, s_{-1}) . In the equilibrium constructed below, optimism is equivalent to an interest rate satisfying $(1 + i)\tilde{\pi}^* = R$. Hence there is only one interest rate conditional on optimism. If there is pessimism, we condition monetary policy on the interest rate on the outstanding debt in order to specify the monetary intervention both on and off the equilibrium path. An alternative would write the equilibrium conditions solely as a function of the interest rate, not the sunspots. This is used in the discussion of policy implementation.

for all i . Combined with the policy under optimism, $\tilde{\pi}(A, i, s^o) = \tilde{\pi}^*$, unconditional inflation expectations are anchored at $\tilde{\pi}^*$. Thus, the real money tax base is invariant and resources from seignorage are given by:

$$\frac{\Delta M}{P} = \nu^m \tilde{\pi}^* (1 - \tilde{\pi}(A, i, s^p)). \quad (2.59)$$

The government budget constraint under repayment becomes:

$$(1 + i)\tilde{\pi}(A, i, s^p)b = A^2(1 - \tau)\tau + \nu^m \tilde{\pi}^* (1 - \tilde{\pi}(A, i, s^p)). \quad (2.60)$$

Second, $\tilde{\pi}(A, i, s^p)$ deters state contingent default: given A and i , the treasury either reimburses its debt with probability 0 or 1. For low values of debt obligations, the fiscal authority will choose to repay its debt, for all A . For high values of these obligations, the fiscal authority will default, again for all A . Of course, the size of the debt obligations are determined in equilibrium, based upon investor beliefs and central bank policy. Formally, Lemma 6 establishes that there is a monetary policy rule 'law' that satisfies these two properties.

Lemma 6. *Given an inflation target $0 < \tilde{\pi}^* \leq 1$, there is a monetary policy rule $\tilde{\pi}(A, i, s^p)$ that satisfies the inflation target and deters state contingent default. Moreover, $\tilde{\pi}(A, i, s^p) > 0$ for all (A, i) and is increasing in A .*

Proof. We derive a state-contingent monetary policy rule $\tilde{\pi}(A, i, s^p)$ that satisfies (2.58) and deters state contingent default. Consider the case $\theta = 0$, where all debt is held abroad, and $\nu^m \approx 0$, which makes seignorage a negligible source of income for the fiscal authority. This simplified framework outlines clearly that the capacity of the central bank to influence the default decision of the treasury does not primarily rely on providing more or less resources, but rather on its capacity to alter the real return to debt across states. The proof is extended to the general case $\theta \geq 0$ and $\nu^m \geq 0$ in Appendix 2.7.3.

Given $s_{-1} = s^p$, we focus on the dependence of inflation on the interest rate on outstanding debt and the realization of the technological shock A . Consider a state contingent rule $\tilde{\pi}(A, i, s^p)$, denoted $\tilde{\pi}_A^p$ in the following analysis. This rule induces a unique interest rate cut-off i^δ such that if $i < i^\delta$ then the fiscal authority is induced to repay its debt for all A , i.e. with probability 1. If $i > i^\delta$, then the fiscal authority defaults for all A , i.e. with probability 1. For $i = i^\delta$, the fiscal authority is indifferent between repayment and default for all A . This condition for indifference is:

$$\Delta(A, i^\delta, m_{-1}, \tau, \tilde{\pi}_A^p) = W^d(\cdot) - W^r(\cdot) = 0 \quad \forall A, \quad (2.61)$$

where $m_{-1} = \tilde{\pi}^*$ using the inflation target condition (2.58), $\tilde{\pi}_A^p$ is defined below and τ satisfies the government budget constraint (2.60) given $(A, i^\delta, \tilde{\pi}_A^p)$:

$$(1 + i^\delta)\tilde{\pi}_A^p b = A^2(1 - \tau)\tau. \quad (2.62)$$

Using $\theta = 0$ and $\nu^m \approx 0$, (2.61) implies $\tau = \gamma$ for all A . From the government budget constraint:

$$\tilde{\pi}_A^p = \frac{A^2(1-\gamma)\gamma}{(1+i^\delta)b} \quad \forall A. \quad (2.63)$$

Applying the inflation target requirement (2.58), the nominal interest rate cut-off i^δ is:

$$1 + i^\delta = \frac{(1-\gamma)\gamma}{\pi^*b} \int_A A^2 dF(A), \quad (2.64)$$

which gives:

$$\tilde{\pi}_A^p = \frac{A^2 \pi^*}{\int_A A^2 dF(A)}. \quad (2.65)$$

We verify that this monetary rule deters state contingent default:

$$\frac{d\Delta(A, i, m, \tau, \tilde{\pi}_A^p)}{di} = A^2(1-\tau) \frac{d\tau}{di}, \quad (2.66)$$

where $\frac{d\tau}{di} = \frac{\tilde{\pi}_A^p b}{A^2(1-2\tau)} > 0$ from (2.62). As $\Delta(A, i^\delta, \tilde{\pi}^*, \tau, \tilde{\pi}_A^p) = 0$ for all A , we get that for all A and all $i < i^\delta$, $\Delta(\cdot) < 0$ and for all $i > i^\delta$, $\Delta(\cdot) > 0$. Hence there is no nominal interest rate $i > 0$ that induces the fiscal authority to default on its debt in a state-contingent manner. Finally, from (2.65), we get $\tilde{\pi}_A^p > 0$ and $\frac{d\tilde{\pi}_A^p}{dA} > 0$. ■

The lemma establishes two critical properties of ‘law’ - $\tilde{\pi}(A, i, s^p)$. First, for all A , $\tilde{\pi}(A, i, s^p) > 0$, which rules out any issue of demonetization of the economy and state-contingent complete default via inflation. Second, the policy rule is countercyclical: $\tilde{\pi}(A, i, s^p)$ is increasing in A , i.e. the lower the technology realization, the higher is inflation. Accordingly, the real return to debt is state contingent and increasing in A .³⁶ Also, this policy distributes resource from seignorage across states, with high seignorage revenue $\nu^m \tilde{\pi}^*(1 - \tilde{\pi}_A^p)$ for low realizations of A . Hence, even if seignorage revenue is not essential to rule out equilibria with default, the policy further contributes to lower the fiscal burden in states where fiscal needs are the highest, i.e. the induced fiscal policy is also countercyclical.³⁷

When the central bank commits to $\tilde{\pi}(A, i, s_{-1})$, there is a unique price for debt, namely the fundamental price under inflation targeting. That is, there is no sunspot equilibrium affecting the valuation of debt. Formally,

Proposition 5. *Under Assumptions 2, 3, when the monetary authority commits to $\tilde{\pi}(A, i, s_{-1})$, with $\tilde{\pi}(A, i, s^p)$ given in Lemma 6, debt is uniquely valued and risk-free. Debt fragility is eliminated.*

Proof. Under Assumption 3, there is a risk-free outcome under strict inflation target $0 < \tilde{\pi}^* \leq 1$. Hence, there is an equilibrium nominal interest rate \underline{i} under optimism that satisfies $(1 + \underline{i})\tilde{\pi}^* = R$.

³⁶The central bank uses its unique capacity to generate state-contingent inflation to turn a non-state contingent nominal bond into a state contingent real asset.

³⁷In fact, the proof in the text focuses on the case of ν^m near zero, where seignorage resource is negligible.

Now under pessimism, the monetary authority commits to $\tilde{\pi}(A, i, s^p)$ as defined in Lemma 6. As seen in the proof of this lemma, this rule delivers inflation as a function of the technological shock A . It is noted $\{\tilde{\pi}_A^p\}$ in the following developments. We verify that under this rule, the best response of the treasury is to repay its debt for all A and that the equilibrium interest rate is \underline{i} .

A central property of $\{\tilde{\pi}_A^p\}$ is that it delivers the inflation target on average. By continuity and monotonicity of $\tilde{\pi}_A^p$ in A , there is a realization \tilde{A} such that $\tilde{\pi}_{\tilde{A}}^p = \tilde{\pi}^*$. In this case, the best-response of the fiscal authority is to raise taxes and repay its debt. Second, $\{\tilde{\pi}_A^p\}$ is such that if the fiscal authority repays its debt with positive probability, it repays its debt with probability 1. Hence, under $\{\tilde{\pi}_A^p\}$, the fiscal authority repays its debt for all A : debt is risk-free. Finally, the no-arbitrage condition under inflation target $\tilde{\pi}^*$ uniquely pins down the nominal interest rate. Hence, under $\{\tilde{\pi}_A^p\}$, the nominal interest rate is \underline{i} :

$$(1 + \underline{i}) \int_A \tilde{\pi}_A^p dF(A) = (1 + \underline{i})\tilde{\pi}^* = R. \quad (2.67)$$

■

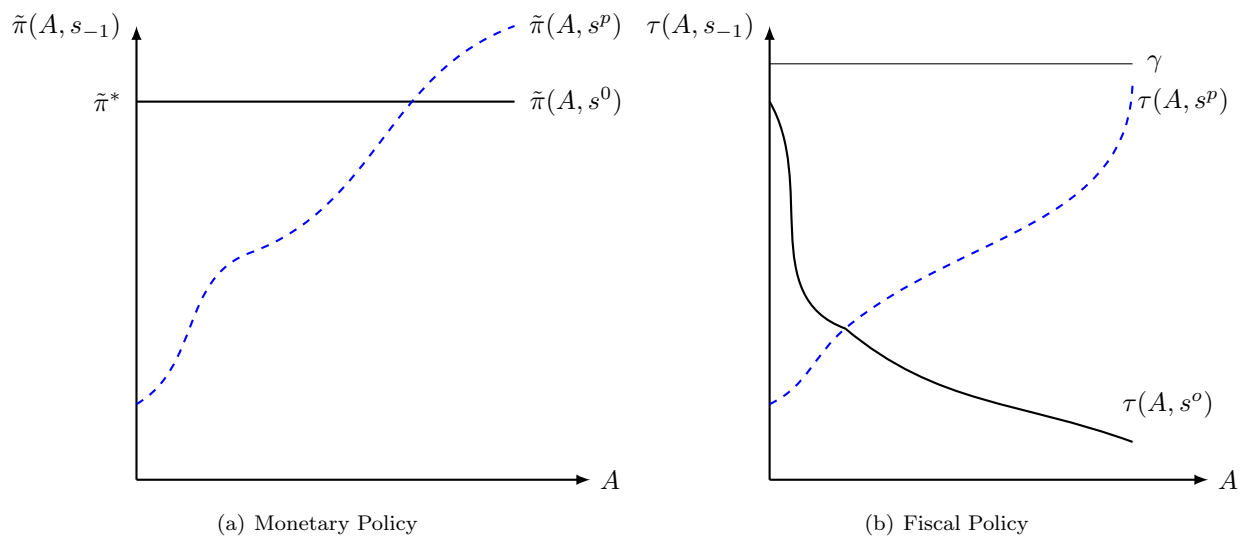
This proposition makes clear that the commitment of the central bank rules out the effect of pessimism on the value of debt. The key to this result is the relaxation of the incentive to default by the fiscal authority through the erosion of the real return to debt in low productivity states.

Figure 2.3 displays the equilibrium monetary policy rule and the induced tax policy, as described in Proposition 5.³⁸ In the case $s_{-1} = s^p$, note the distribution of inflation over realization of A : for low A , high inflation, i.e. low real value of debt and high seignorage revenue. Hence, in case of pessimism, the monetary authority implements a countercyclical policy that stabilizes the price of debt and provides fiscal relief for low values of A , compensated by lower inflation for higher realizations of A . A critical element of this policy is the commitment of the central bank so that inflation expectations are anchored and the real money tax base is not sensitive to variations in private agents sentiments. It illustrates how the central bank can alter the real value of debt, and incidentally distribute income from seignorage, so as to contain the fiscal pressure that weights on the fiscal authority. In this sense, the monetary authority *leans against the winds* of pessimism as well as those associated with low productivity.

As written, the monetary intervention depends jointly on the sunspot from the previous period as well as the interest on outstanding debt. Along the equilibrium path, from Proposition 5, only the fundamental price of debt will be observed. Though extraneous uncertainty may still exist, it will not be reflected in the equilibrium interest rates. With this in mind, it may be more natural to condition monetary interventions on interest rates so that along the equilibrium path, no actual intervention is needed. But, the monetary authority stands ready to intervene in response to higher interest rates that reflect investor pessimism. This is, in effect, a threat of the monetary authority off the equilibrium path to intervene either to support the fiscal authority or, if interest rates are too high, to allow default with probability one.

³⁸The dependence on i is not explicit as these are the policy functions along the equilibrium path.

Figure 2.3: State Dependent Monetary and Fiscal Policy



The left panel represents the state dependent monetary policy to which the central bank commits. The right panel represents the induced fiscal policy. The dependence of the policies on the sunspot and realized productivity are displayed.

Formally, in this case, the monetary authority commits to the following policy, labelled 'wit', for 'whatever it takes':

$$\begin{aligned} \text{if } i = \underline{i}, \text{ then 'dlg': } \forall A \tilde{\pi}(A, i) &= \tilde{\pi}^* \\ \text{if } i > \underline{i}, \text{ then 'law': } \forall A \tilde{\pi}(A, i) &= \tilde{\pi}_A^p \end{aligned} \quad (2.68)$$

With this implementation, the central bank commits to a strategy conditional on the nominal interest rate and ensures that private investors coordinate on the fundamental price of debt \underline{i} . In equilibrium, only the fundamental price of debt is observed and the central bank implements its unconditional inflation target. This approach is reminiscent of the analysis in Bassetto (2005). Indeed, committing to this specific strategy rather than to a policy rule allows the monetary authority to react to deviations from private agents and ensures a unique equilibrium outcome.³⁹

Under this rule, given an inflation target $\tilde{\pi}^*$, debt fragility is eliminated and the expected life-time welfare of private agents is given by:

$$V^{wit}(\tilde{\pi}^*) = \int_{A_l}^{A_h} W^r(A, \underline{i}) dF(A) - \sum_{j \in \{m, l\}} \nu^j \frac{(n_y^j)^2}{2} \geq V^{dlg}(\tilde{\pi}^*, p), \quad (2.69)$$

The inequality is strict whenever the probability of optimism p is lower than 1.

³⁹In other words, committing to a strategy allows the monetary authority a second mover-advantage while anchoring expectations in this dynamic game.

2.5.2 Stabilization without Commitment

The preceding analysis assumes that the monetary authority is granted the capacity to commit to policies and argues that this power can eliminate multiplicity. This section replaces the assumption of commitment with an equilibrium response to deviations from 'wit' that can, in principle, support this policy without commitment. The argument uses a version of the grim trigger strategy in repeated games, as in Rubinstein (1979). We focus on certain features of the economy, such as the presence of strategic uncertainty, that make deviations from policy 'wit' costly and thus support the elimination of debt fragility as an outcome without commitment.

The monetary authority has an incentive to deviate from its committed policy to take advantage of the non-distortionary nature of the inflation tax. To counter this gain, we construct an equilibrium in which any deviations from 'wit' are met by a strict application of 'dlg', the monetary delegation regime of strict inflation targeting. From Section 2.4.2, this generates lifetime welfare for each generation of $V^{dlg}(\tilde{\pi}^*, p)$, given by (2.43), and which depends positively on p , the probability of optimism.

The construction, formalized in Proposition 6, goes as follows. Along the equilibrium path the monetary authority follows 'wit' described in (2.68). This generates lifetime welfare of $V^{wit}(\tilde{\pi}^*)$. In a given state, the central bank can deviate and consider any policy whatsoever: this is modeled as complete discretion for a single period, which is the most profitable deviation. After that deviation, the 'whatever it takes' type intervention is no longer credible. The monetary authority returns to its essential mandate of a targeted inflation rate $\tilde{\pi}^*$, labelled 'delegation'.

As in Lemma 6, assume $\theta = 0$ for this presentation. We derive two expressions for the welfare of old agents under the two outcomes of following the 'wit' policy (2.68) or operating under discretion 'disc', $W^{wit}(\cdot)$ and $W^{disc}(\cdot)$ respectively. First consider the policy to be supported.

$$W^{wit}(A, i, m_{-1}) = \begin{cases} W^{dlg}(A, i, m_{-1}) = \frac{[A(1-\tau^{dlg})]^2}{2} + \nu^m m_{-1} \tilde{\pi}^{dlg} + \nu^I R(Rz^2 - \Gamma), & \text{if } i = \underline{i} \\ W^{law}(A, i, m_{-1}) = \frac{[A(1-\tau^{law})]^2}{2} + \nu^m m_{-1} \tilde{\pi}^{law} + \nu^I R(Rz^2 - \Gamma), & \text{if } i > \underline{i} \end{cases} \quad (2.70)$$

Under 'delegation', the central bank implements its strict inflation target $\tilde{\pi}^{dlg} = \tilde{\pi}^*$. Whenever the nominal interest rate is above its fundamental value, the central bank implements $\tilde{\pi}^{law} = \tilde{\pi}_A^p$, as defined in Lemma 6.⁴⁰ The associated tax rates solve the government budget constraint (2.12) given the monetary intervention. Recall that 'wit' describes policy as long as $i \in [\underline{i}, i^\delta]$. If the interest rate on debt exceeds i^δ the monetary authority cannot prevent a certain default.⁴¹ Thus we explore credibility only for $i \in [\underline{i}, i^\delta]$.

⁴⁰For the clarity of the exposition in this section, we assume that $\tilde{\pi}$ is low enough so that the feasibility of 'law' is not an issue, i.e. $\tilde{\pi}_{A_1}^p > \tilde{\pi}$. Further, we assume that b is high enough, i.e. $b > \nu^m \tilde{\pi}^*$. None of the results depend crucially on these elements.

⁴¹Again, the market shutdown case is of no interest for the present analysis.

Now consider the potential deviation. Under discretion, given individual real money holding m_{-1} , the welfare of old agents is given by:

$$W^{disc}(A, i, m_{-1}) = \frac{[A(1 - \tau^{disc})]^2}{2} + \nu^m m_{-1} \tilde{\pi}^{disc} + \nu^I R(Rz^2 - \Gamma), \quad (2.71)$$

$W^{disc}(A, i, m_{-1})$ is the welfare of the old agents when the tax rate and inverse inflation rates, $(\tau^{disc}, \tilde{\pi}^{disc})$ are determined under discretion, as in Proposition 4. In this case, the optimal monetary policy is to rely on the inflation tax: $\tilde{\pi}^{disc} = \tilde{\pi}$. The base for this tax, i.e. the money holdings of the old agents, will be determined by the inflationary expectations they held in the prior period.

The incentive for maintaining policy 'wit' is characterized by the following difference in agents' welfare:

$$\Delta(A, i, m_{-1}, p) = [W^{wit}(A, i, m_{-1}) - W^{disc}(A, i, m_{-1})] + \frac{\beta}{1 - \beta} [V^{wit}(\tilde{\pi}^*) - V^{dlg}(\tilde{\pi}^*, p)]. \quad (2.72)$$

Here $W^{wit}(A, i, m_{-1}) - W^{disc}(A, i, m_{-1})$ is the immediate gain to old agents from deviating to full discretion from 'wit'. This is clearly a gain since the non-distortionary inflation tax is desirable *ex post*, particularly when agents are holding large money balances.

The punishment arises from the second term, $\frac{\beta}{1 - \beta} [V^{wit}(\tilde{\pi}^*) - V^{dlg}(\tilde{\pi}^*, p)]$, where $\beta \in (0, 1]$ is the rate at which the monetary authority discounts successive generations.⁴² Here the punishment for deviating from policy 'wit' is the continuing operation of the monetary authority under the strict inflation target regime. The idea here is that, like the ECB, the policy 'wit' is built upon a basic commitment to an inflation target.⁴³ As constructed, that inflation target is met on average by the policy and along the equilibrium path. If policy 'wit' is not maintained, then the experiment with that form of intervention is over and the monetary authority returns to its essential goal of strict inflation target. In fact, resorting to the inflation target is a punishment precisely because of the possibility of self-fulfilling debt crisis. Indeed, since $V^{wit}(\tilde{\pi}^*) = V^{dlg}(\tilde{\pi}^*, 1)$, we have $V^{wit}(\tilde{\pi}^*) > V^{dlg}(\tilde{\pi}^*, p)$ for $p < 1$.

We evaluate the credibility of the policy 'wit' whenever the capacity of the institution to anchor inflation expectations is not challenged. Accordingly, along the deviations considered, $m_{-1} = \tilde{\pi}^*$. If $\Delta(A, i, \tilde{\pi}^*, p) \geq 0$, then policy 'wit' is incentive compatible in state (A, i, p) . Using this construction, the 'wit' policy can be supported in an equilibrium without commitment if the costs of deviating from it are sufficiently high. Formally,

Proposition 6. *If the probability of pessimism is sufficiently high and β close enough to unity, then the monetary authority will pursue the 'wit' policy in all states, i.e. $\Delta(A, i, \tilde{\pi}^*, p) \geq 0$ for all A and $i \in [i, i^\delta]$. Debt fragility is eliminated.*

Proof. Clearly $W^{wit}(A, i, \tilde{\pi}^*) - W^{disc}(A, i, \tilde{\pi}^*) \leq 0$ since the monetary authority operating under

⁴²This welfare criterion is not inconsistent with the one derived for the repayment vs. default decision on debt. As discussed in 2.2.2, this decision has welfare consequences that are contained within a generation.

⁴³The deviation from 'wit' to 'disc' could be replaced by a deviation to the inflation target. This would generate a smaller short term gain, making it easier to support 'wit'.

discretion could replicate policy 'wit'. In fact from Proposition 4, it will choose $\tilde{\pi} = \underline{\tilde{\pi}}$. This inflation rate is higher than that under policy 'wit' for any (A, i) . Higher inflation relaxes the debt burden left to be serviced with distortionary taxation and unambiguously increases welfare. Also note that by construction 'wit' deters state-contingent default. A fortiori, no default is possible under discretion.

As $V^{dlg}(\tilde{\pi}^*, p)$ is increasing in p , $\Delta(A, i, \tilde{\pi}^*, p)$ is decreasing in p . So for low enough p , $V^{wit}(\tilde{\pi}^*) - V^{dlg}(\tilde{\pi}^*, p)$ can be large. Further, for β close to unity, $\frac{\beta}{1-\beta}[V^{wit}(\tilde{\pi}^*) - V^{dlg}(\tilde{\pi}^*, p)]$ can be arbitrarily large. Hence for p sufficiently small and β close enough to unity, $\Delta(A, i, \tilde{\pi}^*, p) > 0$ for all A . ■

This proposition nests two types of deviations from the equilibrium path. First, suppose investors in period $t - 1$ believe in policy 'wit', charge the fundamental interest rate \underline{i} and expect the unconditional inflation target $\tilde{\pi}^*$ in all states. Still in period t the monetary authority operating under 'wit' can choose to deviate and resort to discretion. The gain from this is the use of the non-distortionary inflation tax, which is the highest for $A = A_l$. The cost is that 'whatever it takes' is no longer credible. But the foundation of the monetary authority as following strict inflation targeting, i.e. *delegation*, is not altered. If the conditions of Proposition 6 are satisfied, the monetary authority does not deviate along the equilibrium path.

Providing incentives for the monetary authority along the equilibrium path is necessary but not sufficient for 'whatever it takes' to be incentive compatible. Consider a deviation by investors in which they believe there is a positive probability of default implying $i > \underline{i}$. We maintain the integrity of the monetary authority and thus anchor inflationary expectations at $\tilde{\pi}^*$.⁴⁴ In this case, 'wit' prescribes to implement $\{\tilde{\pi}_A^p\}$ as described in Lemma 6. In this case, the incentive compatibility constraint will be binding in all states if it binds when $A = A_h$. Indeed, to deliver the inflation target *on average*, the central bank tightens monetary policy whenever the realization of technology is high. Still, if the conditions for Proposition 6 hold, the monetary authority will have an incentive to intervene to preserve its reputation and follow policy 'wit'. In this case, the pessimism of the investors is not warranted, whatever the realization of A .

The conditions for supporting policy 'wit' have two components. This first is the usual condition that the monetary authority does not discount the future too heavily. The second is not standard and involves the strategic uncertainty of the model. A gain from policy 'wit' is the elimination of debt crisis that do arise with probability $(1 - p)$ under the strict inflation target regime. As this probability of pessimism gets larger, the penalty associated with sticking to the inflation target regime is larger. Accordingly, the higher the risk of coordination failure under inflation targeting, the more credible it is for the central bank to promise to undertake 'whatever it takes' to counter pessimistic beliefs.

Finally, note that here we are not considering a deviation in which investors no longer trust the monetary authority to meet the inflation target on average. Otherwise, investors may hold arbitrary expectations about future inflation, which would influence the gain from discretion relative to 'wit'.

⁴⁴As discussed earlier, policy 'wit' applies only for interest rates below a level denoted i^δ . For pessimism sufficiently high so that $i > i^\delta$, 'wit' prescribes default with probability one. In that situation, there is no credibility to evaluate.

As argued in Section 2.4.2, the foundation for the credibility of the strict inflation target regime could be supported with the threat of demonetization of the economy.

2.6 Conclusions

The goal of this paper was to determine whether monetary policy enhances or mitigates fiscal fragility. Cast as a real economy, the basic environment has fragile debt: there are multiple valuations of government debt depending on the beliefs of investors.

The effects of introducing monetary interventions depends on the commitment of the central bank. If the central bank is committed to an inflation target, then debt fragility remains. If the central bank is allowed full discretion, then the presence of an inelastic source of finance through seignorage is internalized by private agents. Any temptation to inflate the real value of debt is anticipated and debt fragility remains.

Finally, we analyze how a committed central bank can deter debt fragility, by designing a specific monetary policy rule. We devise a state contingent intervention that eliminates pessimistic evaluations of government debt. The policy requires the monetary authority to implement a countercyclical policy, that erodes the real value of debt and provides resources, through seignorage, in times of low productivity and thus low revenue. By supporting the fiscal authorities in these states, the incentive for default is eliminated. Sovereign debt is no longer subject to multiple valuations driven by investors' sentiments. Interestingly, the credibility of this monetary strategy increases with the risk of self-fulfilling debt crisis.

A number of extensions are worth consideration. First, the paper studies the extremes of commitment and discretion. An interesting middle case would be stochastic commitment. A government acting in period t would be allowed to adjust its policy in period $t+1$ with a probability less than one. This partial commitment would create a cost of high inflation and thus enrich the analysis.

Second, the model is dynamic but the fiscal policy is within a generation. Thus we have assumed away the possibilities of debt turnover and intertemporal punishments for default.

Third, our analysis has underlined that the capacity of the central bank to stabilize debt valuations rely on the issuance of non contingent nominal assets labelled in domestic currency. It does not apply to real, indexed debt or debt issued in foreign currency. Allowing governments this choice would be of interest.

Finally, as in many other studies, the outcome with discretion imposes an upper bound on inflation. Providing further micro foundations for this bound remains an open area. Perhaps a political economy model that stresses the redistribution aspects of labor income vs inflation tax would be a productive approach.

2.7 Appendix

2.7.1 Welfare under Repayment and under Default

As explained in section (2.2.2), the repayment vs. default decision in this environment is a discrete choice that affects only the welfare of old agents. Hence, the welfare criteria of interest for $D \in \{r, d\}$ is:

$$W^D(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}) = \nu^m \left[c_o^m(D) - \frac{n_o^m(D)^2}{2} \right] + \nu^I \left[c_o^I(D) - \frac{n_o^I(D)^2}{2} \right]. \quad (2.73)$$

Using the labor supply policy functions from (2.4) and (2.10), we get the following consumption and labor supply vectors:

$$\begin{aligned} c_o^m(r) &= An_o^m(r)(1 - \tau) + m_{-1}\tilde{\pi}^r & c_o^m(d) &= An_o^m(d)(1 - \gamma) + m_{-1}\tilde{\pi}^d + t \\ n_o^m(r) &= A(1 - \tau) & n_o^m(d) &= A(1 - \gamma) \\ c_o^I(r) &= An_o^I(r)(1 - \tau) + (1 + i)\tilde{\pi}^r b^I + Rk & c_o^I(d) &= An_o^I(d)(1 - \gamma) + Rk + t \\ n_o^I(r) &= A(1 - \tau) & n_o^I(d) &= A(1 - \gamma). \end{aligned}$$

Using $\nu^I b^I = \theta b$, one can solve for k , the risk-free component of individual portfolio of rich agents from their budget constraint:

$$zn_y^I = Rz^2 = b^I + k + \Gamma \Rightarrow \nu^I Rk = \nu^I R(Rz^2 - \Gamma) - R\theta b. \quad (2.74)$$

We derive the expressions for $W^r(\cdot)$ and $W^d(\cdot)$:

$$W^r(A, i, m_{-1}, \tau, \sigma, \tilde{\pi}^r) = \frac{[A(1 - \tau)]^2}{2} + \nu^m m_{-1} \tilde{\pi}^r + ((1 + i)\tilde{\pi}^r - R)\theta b + \nu^I R(Rz^2 - \Gamma) \quad (2.75)$$

$$W^d(A, i, m_{-1}, \sigma, \tilde{\pi}^d) = \frac{[A(1 - \gamma)]^2}{2} + \nu^m m_{-1} \tilde{\pi}^d - R\theta b + \nu^I R(Rz^2 - \Gamma) + T(\cdot), \quad (2.76)$$

where τ solves the government budget constraint under repayment and $T(\cdot) = \nu^m m_{-1} \sigma \tilde{\pi}^d$ is a lump sum transfer that implements $\tilde{\pi}^d$ under default.

Default is optimal whenever $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot) \geq 0$.

2.7.2 Proof Lemma 5

As discussed in section 2.4.3, $\tilde{\pi}^e(s^o) \geq \tilde{\pi}$ can be part of a stationary equilibrium if and only if it satisfies:

$$\tilde{\pi}^e(s^o) = p \max \left\{ \frac{\nu^m \tilde{\pi}^e(s^o)}{\nu^m \tilde{\pi}^e(s^o) + (1 + i)b}; \tilde{\pi} \right\} + (1 - p) \max \left\{ \frac{\nu^m \tilde{\pi}^e(s^p)}{\nu^m \tilde{\pi}^e(s^o) + (1 + i)b}; \tilde{\pi} \right\}, \quad (2.77)$$

where the no-arbitrage condition gives: $(1 + i)\tilde{\pi}^e(s^o) = R$, and p is the stationary probability of optimism.

First consider the situation in which seignorage alone is not sufficient to service principal and interest on debt. In this case, the government sets $\tilde{\pi}^r(s, \cdot) = \tilde{\pi}$ for all s , and raises additional labor taxes. Agents form expectations accordingly and (2.77) writes:

$$\tilde{\pi}^e(s^o) = (1 - p)\tilde{\pi} + p\tilde{\pi} = \tilde{\pi}. \quad (2.78)$$

This case emerges whenever $\frac{\nu^m \tilde{\pi}}{\nu^m \tilde{\pi} + \frac{R}{\tilde{\pi}} b} \leq \tilde{\pi}$, which rewrites:

$$b \geq \hat{b} = \frac{\nu^m \tilde{\pi}(1 - \tilde{\pi})}{R}. \quad (2.79)$$

Next, we show that whenever $0 < b < \hat{b}$, there is a level of inflation expectation under optimism, $\tilde{\pi}^e(s^o)$, such that seignorage alone is sufficient to service principal and interest on debt for all (A, s) . (2.77) writes then:

$$\tilde{\pi}^e(s^o) = p \frac{\nu^m \tilde{\pi}^e(s^o)}{\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b} + (1 - p) \frac{\nu^m \tilde{\pi}}{\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b}. \quad (2.80)$$

Multiply both sides by $\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b$ and get:

$$\nu^m \tilde{\pi}^e(s^o)^2 - p\nu^m \tilde{\pi}^e(s^o) + Rb - (1 - p)\nu^m \tilde{\pi} = 0. \quad (2.81)$$

Hence, (2.80) has at least a positive solution if $b \leq b^\alpha$, where:

$$b^\alpha = \frac{p^2 \nu^m + 4(1 - p)\nu^m \tilde{\pi}}{4R}. \quad (2.82)$$

Under this condition, the solution to (2.81) that is necessarily positive⁴⁵ is given by:

$$\tilde{\pi}^e(s^o) = \frac{p + \sqrt{p^2 + 4(1 - p)\tilde{\pi} - 4\frac{Rb}{\nu^m}}}{2}. \quad (2.83)$$

This solution is compatible with (2.77) if it satisfies the following two conditions:

$$\frac{\nu^m \tilde{\pi}^e(s)}{\nu^m \tilde{\pi}^e(s^o) + \frac{R}{\tilde{\pi}^e(s^o)} b} \geq \tilde{\pi} \quad \forall s \in \{s^o, s^p\}, \quad (2.84)$$

We verify that $b^\alpha \geq \hat{b}$ and that for all $b < \hat{b}$, when $\tilde{\pi}^e(s^o)$ is given by (2.83), then the conditions (2.84) are satisfied.

⁴⁵The other solution to the polynomial can be both positive and feasible, hence there is possibly multiple stationary inflation regimes due to the Laffer curve property of seignorage.

Note $F(p) = 4R(b^\alpha - \hat{b})$. Substituting and rearranging:

$$F(p) = p^2\nu^m - p4\nu^m\tilde{\pi} + 4\nu^m\tilde{\pi}^2 = \nu^m(p - 2\tilde{\pi})^2 \geq 0, \quad (2.85)$$

which gives $b^\alpha \geq \hat{b}$.

Next, note $G(p, b) \equiv \tilde{\pi}^e(s^o)$, where $\tilde{\pi}^e(s^o)$ is given by (2.83). The feasibility condition (2.84) for $s = s^p$ then reads:

$$G(b, p) = \frac{p + \sqrt{p^2 + 4(1-p)\tilde{\pi} - 4\frac{Rb}{\nu^m}}}{2} \geq \sqrt{\frac{Rb\tilde{\pi}}{\nu^m(1-\tilde{\pi})}}. \quad (2.86)$$

In this expression, the left side $G(b, p)$ is decreasing in b , whereas the right side is increasing in b ; $G(0, p) > 0$ and the right side is equal to 0 for $b = 0$; $G(\hat{b}, p) \geq \tilde{\pi}$ and the right side is equal to $\tilde{\pi}$, for $b = \hat{b}$. Hence for all $b < \hat{b}$, (2.86) is satisfied.

Finally, the feasibility condition (2.84) for $s = s^o$ requires $b \leq b^\delta = \frac{\nu^m}{4R}$ and:

$$\frac{1 - \sqrt{1 - 4\frac{Rb}{\nu^m}}}{2} \leq G(b, p) \leq \frac{1 + \sqrt{1 - 4\frac{Rb}{\nu^m}}}{2}. \quad (2.87)$$

Since $\tilde{\pi}(1 - \tilde{\pi}) \leq \frac{1}{4}$, we have $\hat{b} \leq b^\delta$. Note $\mathcal{B}_l(b)$ and $\mathcal{B}_u(b)$ the lower and upper bounds of this inequality.

$\mathcal{B}_l(b)$, is increasing in b , $\mathcal{B}_l(0) = 0$, $\mathcal{B}_l(\hat{b}) = \frac{1 - \sqrt{1 - 4\tilde{\pi}(1-\tilde{\pi})}}{2} = \frac{1 - \sqrt{(1-2\tilde{\pi})^2}}{2} \leq \tilde{\pi}$ for all $\tilde{\pi} \in [0, 1]$. As $G(b, p)$ is decreasing in b and $G(\hat{b}, p) \geq \tilde{\pi}$, we have that for all $b \in [0, \hat{b}]$, $G(b, p) \geq \mathcal{B}_l(b)$.

We finally verify that $G(b, p) \leq \mathcal{B}_u(b)$ for all $b < \hat{b}$. Taking the derivatives of $G(b, p)$ w.r.t. p :

$$\frac{dG(\cdot)}{dp} = \frac{1}{2} \left(1 + \frac{p - 2\tilde{\pi}}{\sqrt{p^2 + 4(1-p)\tilde{\pi} - 4\frac{Rb}{\nu^m}}} \right). \quad (2.88)$$

If $p - 2\tilde{\pi} > 0$, then $\frac{dG(\cdot)}{dp} > 0$. If $p - 2\tilde{\pi} < 0$, then verify that $-1 \leq \frac{p - 2\tilde{\pi}}{\sqrt{p^2 + 4(1-p)\tilde{\pi} - 4\frac{Rb}{\nu^m}}} \leq 0$, so that again $\frac{dG(\cdot)}{dp} > 0$. Hence, for all $p \in [0, 1]$, all $\tilde{\pi} \in [0, 1]$, all $b \in [0, \hat{b}]$:

$$G(b, p) \leq G(b, 1) = \frac{1 + \sqrt{1 - 4\frac{Rb}{\nu^m}}}{2} = \mathcal{B}_u(b). \quad (2.89)$$

Overall, we have shown that for all $b \leq \hat{b}$, there is $\tilde{\pi}^e(s^o)$ that satisfies (2.83) and solves (2.77).

2.7.3 Existence of the "Leaning Against the Winds" Policy

This section details the proof of Lemma 6 in the general case $\theta \in [0, 1]$ and $\nu^m \geq 0$.

We adopt the following notations. Consider the central bank committing to a policy contingent on A , noted $\{\tilde{\pi}_A\}$, and such that $\int_A \tilde{\pi}_A dF(A) = \tilde{\pi}^*$. Given $m_{-1} = \tilde{\pi}^e(\cdot) = \tilde{\pi}^*$, where $\tilde{\pi}^*$ is the inflation target of the central bank, the discretionary default decision of the treasury is captured

by:

$$\begin{aligned}\Delta(A, i, \tilde{\pi}^*, \tau, \tilde{\pi}_A) &= W^d(\cdot) - W^r(\cdot) \\ &= \frac{[A(1 - \gamma)]^2}{2} - \frac{[A(1 - \tau)]^2}{2} + \nu^m \tilde{\pi}^* (1 - \tilde{\pi}_A) - (1 + i) \tilde{\pi}_A \theta b,\end{aligned}\quad (2.90)$$

where τ solves the government budget constraint given $\tilde{\pi}_A$:

$$G(A, i, \tilde{\pi}^*, \tau, \tilde{\pi}_A) = A^2(1 - \tau)\tau + \nu^m \tilde{\pi}^* (1 - \tilde{\pi}_A) - (1 + i) \tilde{\pi}_A b = 0. \quad (2.91)$$

Moreover, in the economy with $\theta > 0$, default occurs for two reasons: either it is the best response of the treasury: $\Delta(\cdot) > 0$, or the fiscal capacity of the country cannot service debt, since $\tau \leq \frac{1}{2}$.

We show that there is a unique state-dependent inflation policy $\{\tilde{\pi}_A^p\}$ and an induced interest rate cut-off i^δ such that the policy delivers the inflation target on average, and, if the central bank commits to $\{\tilde{\pi}_A^p\}$, then the fiscal authority services its obligation for all A if and only if $i < i^\delta$.

We proceed in two steps: first we show that for any i^t , there is a unique policy $\{\tilde{\pi}_A(i^t)\}$ such that the treasury reimburses its debt if and only if $i < i^t$. Second, we show that there is a unique i^δ such that $\{\tilde{\pi}_A(i^\delta)\}$ satisfies the inflation target. The desired policy is given by $\tilde{\pi}_A^p = \tilde{\pi}_A(i^\delta)$ for all A .

Part I. Consider a nominal interest rate i^t such that $1 + i^t > 0$ and a realization $A \in [A_l, A_h]$.

(i) The following elements establish that there is a unique inflation level $\tilde{\pi}_A(i^t)$ such that the fiscal authority is indifferent between repayment and default.

First, there is an inverse inflation rate $\tilde{\pi}_A^1(i^t)$ such that debt is serviced with no taxes on labor income.

$$G(A, i^t, \tilde{\pi}^*, \tau, \tilde{\pi}_A^1(i^t)) = 0 \Rightarrow \tau = 0. \quad (2.92)$$

In this case, using Assumption 2, $\Delta(\cdot) < 0$. Using the government budget constraint with $\tau = 0$, one gets:

$$\tilde{\pi}_A^1(i^t) = \frac{\nu^m \tilde{\pi}^*}{\nu^m \tilde{\pi}^* + (1 + i^t)b} > 0. \quad (2.93)$$

Similarly, the central bank can set the inverse inflation rate to $\tilde{\pi}_A^2(i^t)$ so that if the treasury desires to service its debt, it has to set $\tau = \frac{1}{2}$. Formally:

$$\tilde{\pi}_A^2(i^t) = \frac{\frac{A^2}{4} + \nu^m \tilde{\pi}^*}{\nu^m \tilde{\pi}^* + (1 + i^t)b}. \quad (2.94)$$

Importantly, for any inflation rate between these two cases, the lower the inflation, i.e. the higher $\tilde{\pi}_A$, the higher the tax rate to service debt. Formally, differentiating the government budget

constraint w.r.t. τ and $\tilde{\pi}_A$:

$$\forall \tilde{\pi}_A \in [\tilde{\pi}_A^1(i^t), \tilde{\pi}_A^2(i^t)], \quad \frac{d\tau}{d\tilde{\pi}_A} = \frac{\nu^m \tilde{\pi}^* + (1+i^t)b}{A^2(1-2\tau)} > 0. \quad (2.95)$$

Moreover, the lower the inflation, i.e. the higher $\tilde{\pi}_A$, the higher the value of $\Delta(\cdot) = W^d(\cdot) - W^r(\cdot)$:

$$\frac{d\Delta(\cdot)}{d\tilde{\pi}_A} = \frac{1-\tau}{1-2\tau} (\nu^m \tilde{\pi}^* + (1+i^t)b) - (\nu^m \tilde{\pi}^* + (1+i^t)\theta b) > 0, \quad (2.96)$$

since $\frac{1-\tau}{1-2\tau} > 1$ for $\tau \in [0, \frac{1}{2}]$.

Hence, there is a unique $\tilde{\pi}_A(i^t)$ that has the desired property to make the treasury indifferent between repayment and default. Especially,

- if $\Delta(A, i^t, \tilde{\pi}^*, \frac{1}{2}, \tilde{\pi}_A^2(i^t)) > 0$, then $\tilde{\pi}_A^1(i^t) < \tilde{\pi}_A(i^t) < \tilde{\pi}_A^2(i^t)$,
- if $\Delta(A, i^t, \tilde{\pi}^*, \frac{1}{2}, \tilde{\pi}_A^2(i^t)) \leq 0$, then $\tilde{\pi}_A(i^t) = \tilde{\pi}_A^2(i^t)$.

(ii) Next, we verify that for any $i < i^t$, the fiscal authority services its debt, otherwise for any $i > i^t$, it defaults. Given $\tilde{\pi}_A(i^t)$, we have:

$$\frac{d\Delta(\cdot)}{di} = A^2(1-\tau) \frac{d\tau}{di} - \tilde{\pi}_A(i^t)\theta b = \frac{1-\tau}{1-2\tau} \tilde{\pi}_A(i^t)b - \tilde{\pi}_A(i^t)\theta b > 0. \quad (2.97)$$

(iii) Also, we establish the following properties of $\tilde{\pi}_A(i^t)$:

$$\frac{d\tilde{\pi}_A(i^t)}{dA} > 0 \qquad \frac{d\tilde{\pi}_A^p(i^t)}{di^t} < 0. \quad (2.98)$$

If $\tilde{\pi}_A(i^t) = \tilde{\pi}_A^2(i^t)$, these properties are straightforward. In the case $\tilde{\pi}_A(i^t) < \tilde{\pi}_A^2(i^t)$, first differentiate the government budget constraint w.r.t. $(A, i, \tau, \tilde{\pi}_A)$ to get:

$$\frac{d\tau}{dA} = -\frac{2(1-\tau)\tau}{A(1-2\tau)} \qquad \frac{d\tau}{di} = \frac{\tilde{\pi}_A b}{A^2(1-2\tau)} \qquad \frac{d\tau}{d\tilde{\pi}_A} = \frac{\nu^m \tilde{\pi}^* + (1+i)b}{A^2(1-2\tau)} \quad (2.99)$$

Then differentiate $\Delta(A, i, \tilde{\pi}^*, \tau, \tilde{\pi}_A)$ w.r.t to its arguments and using the derivative of τ w.r.t $(A, i, \tilde{\pi}_A)$, one gets:

$$\left[A(1-\gamma)^2 - A \frac{(1-\tau)^2}{1-2\tau} \right] dA + \left[\frac{1-\tau}{1-2\tau} (\nu^m \tilde{\pi}^* + (1+i)b) - (\nu^m \tilde{\pi}^* + (1+i)\theta b) \right] d\tilde{\pi}_A = 0 \quad (2.100)$$

$$\left[\frac{1-\tau}{1-2\tau} \tilde{\pi}_A b - \tilde{\pi}_A \theta b \right] di + \left[\frac{1-\tau}{1-2\tau} (\nu^m \tilde{\pi}^* + (1+i)b) - (\nu^m \tilde{\pi}^* + (1+i)\theta b) \right] d\tilde{\pi}_A = 0. \quad (2.101)$$

Since $\frac{1-\tau}{1-2\tau} > \frac{(1-\tau)^2}{1-2\tau} > 1$ for all $0 \leq \tau \leq \frac{1}{2}$ and $0 \leq \theta \leq 1$, we get the desired results.

(iv) Finally, the limits behavior of $\tilde{\pi}_A(i^t)$ are derived from the inequality

$$\tilde{\pi}_A^1(i^t) < \tilde{\pi}_A(i^t) \leq \tilde{\pi}_A^2(i^t), \quad (2.102)$$

which gives $\lim_{i^t \rightarrow +\infty} \tilde{\pi}_A(i^t) = 0$ and $\lim_{i^t \rightarrow -1} \tilde{\pi}_A(i^t) > 1$.

Part II. By applying the inflation target requirement (2.58), we show that there is a unique $i^\delta > 0$ such that:

$$\int_A \tilde{\pi}_A(i^\delta) dF(A) = \tilde{\pi}^*. \quad (2.103)$$

Note $H(i) = \int_A \tilde{\pi}_A(i) dF(A)$, which is defined for all i such that $1 + i > 0$. The properties of $\tilde{\pi}_A(i)$ naturally convey to $H(i)$: $H(i)$ is strictly decreasing in i ; $\lim_{i \rightarrow +\infty} H(i) = 0$; $\lim_{i \rightarrow -1} H(i) > 1$.

Hence there is a unique i^δ such that $H(i^\delta) = \tilde{\pi}^*$.

Overall, the monetary policy rule $\{\tilde{\pi}_A^p\}$ that meets the inflation target and deters state contingent default, exists, and satisfies:

$$\forall A \tilde{\pi}_A^p = \tilde{\pi}_A^p(i^\delta). \quad (2.104)$$

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Chapter 3

Public Debt and the Cyclicity of Fiscal Policy

with Andrew R. Gimber

3.1 Introduction

Public debt to GDP ratios in advanced economies have been rising since the mid-1970s, and have recently reached levels not seen since just after World War II Abbas, Belhocine, El-Ganainy, and Horton (2011). The recent financial crisis and the ensuing Great Recession exacerbated this trend through bailouts, stimulus packages, rising unemployment claims, and falling tax revenues. This has led to a heated debate over the pace of fiscal consolidation, with one side emphasizing the burden on economic growth imposed by high levels of public debt, and the other warning that pursuing austerity during a recession could be very costly or even self-defeating.

In this paper we present a new theory that provides a partial reconciliation of these two views. We show that there is a threshold level of debt above which the economy is vulnerable to self-fulfilling fiscal crises. However, the mechanism that makes such crises possible is that fiscal policy becomes procyclical, in the sense that the government's optimal response to a reduction in output is to raise the tax rate.¹ Thus, our model lends qualified support to both sides of the debate over fiscal consolidation: the proximate cause of the crisis is the government's desire to raise the tax rate in a recession, but the source of this desire is the high level of public debt.

In Calvo (1988) and related papers,² investors' expectations of sovereign default cause them to charge a risk premium that makes default more likely. Corsetti, Kuester, Meier, and Müller (2013) argue that this sovereign risk channel provides a motivation for fiscal consolidation. However, even countries that did not face an increase in sovereign risk premia have pursued fiscal consolidation in the years since the onset of the Great Recession. Our focus in this paper is not on self-fulfilling expectations of sovereign default, but on another type of self-fulfilling macroeconomic crisis caused by high levels of public debt. Accordingly, we focus on cases in which investors charge the lowest risk premium compatible with the economy's fundamentals. Our analysis can explain why a highly indebted government might adopt procyclical fiscal policy during a recession, even without facing a high sovereign risk premium. Indeed, in our baseline model debt is risk free because the government

¹Since "procyclical" can be used to describe both variables that are positively correlated with output and policies that exacerbate the business cycle, there is potential for ambiguity when describing the cyclicity of tax rates. Throughout this paper, we use "procyclical" to refer to a negative correlation of tax rates and output, that is, tax policy that could exacerbate output fluctuations.

²See also Cooper (2015) and Lorenzoni and Werning (2013), for instance.

never defaults, and debt sustainability is ensured by future fiscal capacity.³

Unlike a committed Ramsey planner, the government in our model takes households' current labour supply decisions and output as given when setting the contemporaneous tax rate and issuing new debt. Fiscal policy is therefore a function of current output, as well as of the inherited stock of public debt. This leads to a standard time inconsistency problem of the kind identified by Kydland and Prescott (1977). Whatever the level of public debt, the government always chooses a higher contemporaneous tax rate than a Ramsey planner would choose, because it does not internalize the distortionary effect on current output. However, the key insight of our analysis is that the government's inability to commit to a tax rate can have even more severe consequences, because when debt is high fiscal policy becomes procyclical, thereby inducing a coordination problem among households.

When the economy suffers a fall in output, there are two countervailing effects on the government's optimal choice of the contemporaneous tax rate. The first is that, for given tax rates, contemporaneous consumption falls relative to future consumption. This provides the government with a *consumption-smoothing* motive to reduce the contemporaneous tax rate relative to the future tax rate.⁴ The second effect, which we call the *tax-base* effect, is that the contemporaneous tax base shrinks, meaning that the government must raise tax rates at some point in order to remain solvent in the long run.

When the inherited stock of public debt is low, the consumption-smoothing effect dominates. This means fiscal policy is countercyclical: the government's optimal response to a fall in output is to cut the tax rate and issue more debt, postponing the necessary tax collection to the future. A household that expected aggregate labour supply to be low would therefore anticipate a low tax rate, and choose a high level of labour supply itself. Under these conditions there is no scope for coordination failure, and our economy has a unique equilibrium.

However, when the inherited level of public debt is high, the tax-base effect dominates. Optimal fiscal policy then becomes procyclical, because deferring all fiscal consolidation (tax increases) when output is low would impose an unacceptable burden on future consumption. This unleashes the possibility of multiple equilibria. In the good equilibrium, labour supply is high because workers anticipate a low tax rate, and the government optimally chooses a low tax rate because output is high. In bad equilibria, which we label *fiscal policy traps*, workers restrict their labour supply in anticipation of a high tax rate, and the resulting low output induces the government to fulfil their pessimistic expectations by setting a high tax rate. Welfare is lower in fiscal policy trap equilibria than in the high tax-base, low tax-rate equilibrium.

The idea that high levels of public debt can pose a threat to the economy is most famously associated with Reinhart and Rogoff (2010). In particular, they argue that countries with sovereign debt to GDP ratios above 90 percent have significantly lower rates of economic growth on average. The burden of distortionary taxation imposed by debt service could explain why high levels of

³We show in section 3.5.2 that our results are robust to allowing for government default.

⁴In our baseline model we abstract away from private-sector borrowing decisions, but this consumption-smoothing motive applies whenever consumers are not fully Ricardian. We relax this assumption in section 3.5.3.

debt might reduce growth, but not why there might be a discrete drop in growth above some threshold level of debt. Our model contributes a novel explanation for why there might be such a threshold effect, based on self-fulfilling beliefs about the stance of fiscal policy. In our model, a country with a level of public debt just above the threshold is exposed to the risk of a high-tax, low-output equilibrium. If this equilibrium is selected, the country's economic performance will be significantly worse than that of a similar country with a public debt level just below the threshold.

Schmitt-Grohé and Uribe (1997) introduce similar concerns about taxpayer coordination failure into a dynamic model with income and capital taxation. They consider only balanced-budget fiscal policy: specifically, they study the undesired consequences of balanced-budget rules when labour supply and tax rates are chosen simultaneously. In addition to procyclical fiscal policy, they show that this leads to equilibrium indeterminacy. The authors stress the crucial role of capital accumulation in driving the result. Finally, they consider the role of public debt, but not as a choice variable: they maintain budget balance and a fixed stock of public debt, so what matters are the interest rates that have to be serviced. Nevertheless, high levels of debt in their analysis can lead to fiscal policy indeterminacy.

Cole and Kehoe (2000) consider a dynamic environment in which the government is prone to self-fulfilling debt rollover crises. They assume a constant tax rate, and allow the government to adjust its debt level by varying its expenditure. In their model, there is a source of domestically initiated crisis, via capital accumulation. By reducing saving, households reduce capital next period and bring the economy into the crisis zone where market shutdown is an option, hence making the initial belief that drove the reduction in saving self-fulfilling.

The rest of the paper is organized as follows. In section 3.2, we present the general framework of analysis. Section 3.3 sets out our main analytical results. Next, in section 3.4, we illustrate by way of an example the mechanism by which the cyclical policy depends on the inherited debt position and can lead to a self-fulfilling crisis. In section 3.5, we build on this example to investigate the robustness of our results to relaxing several of our baseline assumptions. Section 3.6 concludes.

3.2 A Model of Taxpayer Coordination Failure

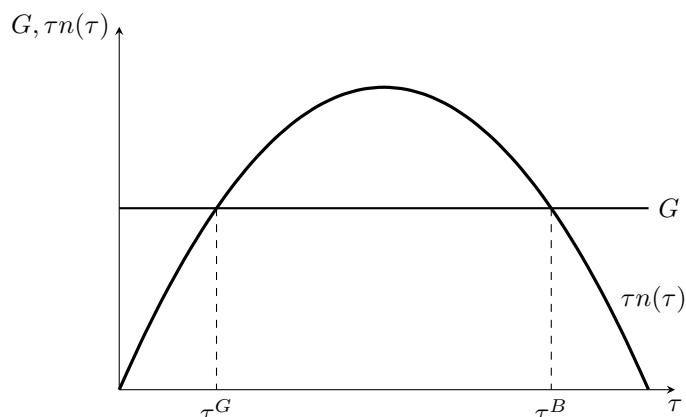
In this section, we first outline the mechanism by which taxpayer coordination failure can arise in a static environment, and then present the general framework of our dynamic analysis.

3.2.1 Static Economy with a Balanced Budget

Consider a static environment, as proposed by Cooper (1999, 131–132), in which the government must finance a fixed level of expenditure G through a proportional ex post tax on labour income.⁵ The economy is populated by a mass-one continuum of ex ante identical households, indexed by

⁵We can think of G as pre-contracted expenses that do not enter into household utility directly.

Figure 3.1: Equilibria on Either Side of the Laffer Curve



This figure outlines the coordination problem created by the government's inability to commit to a tax rate. For a given level of government expenditure, there are two levels of labour supply, associated with different tax rates, that satisfy the government budget constraint. The equilibrium with high labour supply and a low tax rate provides higher utility than the one with low labour supply and a high tax rate.

$i \in [0, 1]$, who derive utility from consumption, c_i , and disutility from labour supply, n_i . Production is linear, so with a proportional tax rate τ , household i 's consumption is $c_i = (1 - \tau)n_i$. Since the pre-tax real wage is fixed at unity, households' optimal labour supply will be a function of the tax rate: $n_i = n(\tau)$. We assume that the substitution effect dominates the income effect in the utility function, so that labour supply is decreasing in the tax rate: $dn(\tau)/d\tau < 0$.

The government's budget balance constraint is $\tau n = G$, where $n = \int_i n_i di$ is aggregate labour supply. The government must pay for its fixed expenditure, so the tax rate will depend negatively on the tax base: $\tau = G/n$. This creates strategic complementarities among households: the higher is aggregate labour supply, the lower will be the tax rate, and so the higher is household i 's optimal labour supply.

The equilibrium condition is $\tau n(\tau) = G$. As Figure 3.1 shows, there are two Pareto-ranked equilibria: a good equilibrium with a low tax rate τ^G and high labour supply $n(\tau^G)$, and a bad equilibrium with a high tax rate τ^B and low labour supply $n(\tau^B)$. Given the presence of strategic uncertainty over the tax rate, households may coordinate on the inefficient Nash equilibrium, which lies on the downward-sloping part of the Laffer curve.

If the government could credibly commit to a tax rate, this strategic uncertainty among households would disappear. However, in a static environment with fixed expenditure, the government has no choice but to respond to a revenue shortfall by raising the tax rate. The combination of discretion over the tax rate and an absolute requirement to balance the budget leads to the possibility of coordination failure. The first of these assumptions is reasonable: sovereign governments cannot in fact commit to keep tax rates constant regardless of the state of the economy.⁶ However, the

⁶Income tax policy can change relatively quickly, particularly during crises, and even retroactive tax increases are not unheard of. On 6 November 2012, voters in California passed Proposition 30, which included increases in top marginal tax rates that applied retroactively to income earned since 1 January 2012. The Minnesota omnibus tax bill (HF 677), signed into law on 23 May 2013, included a new top income tax bracket and an increase in the

balanced-budget view of fiscal policy is less realistic because governments routinely borrow to cover revenue shortfalls when output is lower than expected (and even balanced-budget constitutional amendments can be overturned).

The focus of the present paper is therefore to offer the government the possibility to issue new debt rather than increase taxes in the event of a revenue shortfall. Does this allow the government to eliminate the taxpayer coordination failure and steer the economy to the more efficient outcome with a low tax rate and high labour supply? Our answer will be that this depends on the inherited debt level. If the outstanding debt burden is sufficiently low, then the government's ability to adjust its debt position in the event of a revenue shortfall will ensure that there is a unique, low-tax equilibrium. However, if the inherited stock of debt is large enough then the government will optimally respond to lower output with higher taxes, unleashing the possibility of a fiscal policy trap.

3.2.2 Two-Period Economy with Taxes and Debt Issuance

We consider a two-period economy: $t = 1, 2$. The government inherits a level of debt B_1 , owed to foreign investors. In period 1, households choose labour supply and produce accordingly. The government then sets its fiscal policy, choosing the tax rate on labour income τ_1 and the new debt B_2 to be issued to foreign investors. This debt is backed by future primary fiscal surpluses and is always repaid in period 2, so the government can borrow at the risk-free rate R between periods 1 and 2. We interpret the terminal period 2 as the long run.

The focus of our analysis is on the determinants of labour supply and fiscal policy in period 1. We next describe these choices.

Households' Preferences and Choices

There is a unit mass of households in the economy, indexed by $i \in [0, 1]$. Households live over the two periods but are hand-to-mouth consumers, meaning they can neither save nor borrow between periods 1 and 2.⁷ Moreover, since households are atomistic, they do not internalize the impact of their labour supply choices on the government's choices of tax rate and debt issuance. In period 1, household i forms a belief about the tax rate τ_1 and solves:

$$\max_{n_{1,i}} u(c_{1,i}) - g(n_{1,i}) \tag{3.1}$$

subject to

$$c_{1,i} = (1 - \tau_1)z_1 f(n_{1,i}). \tag{3.2}$$

Consumption utility is increasing and concave: $u'(\cdot) > 0$ and $u''(\cdot) < 0$; and labour disutility is increasing and convex: $g'(\cdot) > 0$ and $g''(\cdot) < 0$.

alternative minimum tax rate, both of which applied retroactively to the beginning of 2013.

⁷We explore the implications of relaxing this assumption in section 3.5.3 below.

The individual production function is $y_{1,i} = z_1 f(n_{1,i})$, where $z_1 > 0$ is an aggregate productivity parameter and $f(\cdot)$ is an increasing function that exhibits weakly decreasing returns to scale and is unbounded above: $f'(\cdot) > 0$, $f''(\cdot) \leq 0$ and $\lim_{n \rightarrow +\infty} f(\cdot) = +\infty$.

The labour supply decision $n(\tau_1)$ is implicitly defined by the following first-order condition:

$$(1 - \tau_1)z_1 f'(n_{1,i})u'((1 - \tau_1)z_1 f(n_{1,i})) = g'(n_{1,i}). \quad (3.3)$$

We assume that the curvature of the utility function is such that substitution effects dominate income effects:

$$u'(c) + cu''(c) > 0 \quad \forall c \geq 0.$$

This ensures that labour supply is a decreasing function of the tax rate:

$$\frac{dn(\tau_1)}{d\tau_1} = \frac{z_1 f'(\cdot)(u'(\cdot) + c_1 u''(\cdot))}{(1 - \tau_1)z_1 f''(\cdot)u'(\cdot) + ((1 - \tau_1)z_1 f'(\cdot))^2 u''(\cdot) - g''(\cdot)} < 0. \quad (3.4)$$

Government's Preferences and Choices

The government faces an intertemporal tax-smoothing problem. It has an inherited stock of debt owed to foreign investors, B_1 , which it is committed to repaying. In each period, the government also has to finance an exogenous amount of expenses $G_t \geq 0$, which do not enter into household utility directly.⁸ Given inherited debt B_1 and aggregate labour supply n_1 , it optimally sets the tax rate τ_1 and issues new debt B_2 to risk-neutral foreign investors. Importantly, the choice of B_2 is constrained by the requirement that all outstanding debt is repaid in period 2.⁹ Future fiscal capacity is defined by the maximum amount of debt \bar{B}_2 that can be issued in period 1.

The government's maximization problem is as follows:

$$\max_{\tau_1, B_2} u(c_1) - g(n_1) + \beta V(B_2) \quad (3.5)$$

subject to

$$B_1 + G_1 \leq \tau_1 z_1 f(n_1) + \frac{B_2}{R} \quad (3.6)$$

$$B_2 \leq \bar{B}_2. \quad (3.7)$$

The function $V(\cdot)$ captures the continuation utility of the economy when in period 1 the government issues bonds with face value B_2 to be repaid in period 2. The government budget constraint (3.6) states that debt service and government expenditure in period 1 must be financed by proportional taxes on output and new debt issuance. Expression (3.7) states that, because of the long-run solvency requirement, the government also faces a borrowing limit \bar{B}_2 .

⁸In section 3.5.1, we endogenize short-run public expenditure G_1 .

⁹This assumption is introduced to highlight the fact that the mechanism at play in our analysis, namely the link between inherited debt, the cyclicity of fiscal policy and the possibility of taxpayer coordination failure, is not driven by self-fulfilling increases in sovereign risk premia. In section 3.5.2, we relax this assumption and show that our results still hold.

The continuation utility function $V(\cdot)$ satisfies the following concavity assumptions:

$$V'(\cdot) < 0, \quad V''(\cdot) < 0. \quad (3.8)$$

In addition, we impose

$$\lim_{B_2 \rightarrow \bar{B}_2} V'(\cdot) = -\infty. \quad (3.9)$$

This condition ensures that the government's borrowing limit (3.7) will not bind in equilibrium (see Lemma 1 below).¹⁰

Since $V(\cdot)$ is decreasing in B_2 , the government budget constraint (3.6) will be satisfied with equality. Substituting this into the government's objective function (3.5) and differentiating with respect to the short-run tax rate τ_1 yields the following first-order condition:

$$u'((1 - \tau_1)z_1 f(n_1)) = -\beta R V'(R(B_1 + G_1 - \tau_1 z_1 f(n_1))). \quad (3.10)$$

Equation (3.10) implicitly defines the tax policy function $\tau(n_1, B_1)$.¹¹ We will demonstrate below that the optimal short-run tax rate is unambiguously increasing in the inherited debt level B_1 , but that the sign of its derivative with respect to short-run labour supply n_1 is ambiguous. When $d\tau(\cdot)/dn_1 > 0$, we say that fiscal policy is *countercyclical*, meaning a drop in output induces the government to lower the tax rate; when $d\tau(\cdot)/dn_1 < 0$, we say that fiscal policy is *procyclical*, meaning a drop in output induces the government to raise the tax rate. We will also show that the cyclical policy and the number of equilibria in this economy depend on the inherited level of debt.

Equilibrium Definition

The relevant choices of households and the government are both made in period 1. The government inherits an amount of debt B_1 . Households form expectations about fiscal policy, supply labour and produce accordingly. Given its outstanding debt and the economy's tax base, the government sets fiscal policy to maximize the lifetime utility of the population.

The relevant variables for the government's decisions are aggregate labour supply, n_1 , and the inherited amount of debt B_1 . Given (n_1, B_1) , the government sets τ_1 and issues new bonds B_2 . We denote the policy functions $\tau(n_1, B_1)$ and $B(n_1, B_1)$. In the long run, i.e. in period 2, debt is fully repaid.

Accordingly, an equilibrium in this environment is defined as follows:

Definition 1. *A subgame-perfect rational expectations equilibrium is a labour supply decision n_1 , a tax rate τ_1 and debt issuance B_2 such that:*

¹⁰Condition (3.9) states that the marginal utility of a reduction in the future debt burden approaches infinity as the government approaches its debt limit. We demonstrate below that this condition is satisfied for natural specifications of $V(\cdot)$ in which the cost of issuing additional debt in period 1 is higher taxes and lower consumption in period 2.

¹¹For comparison, a Ramsey planner with the ability to commit to a tax rate would solve (3.5) subject to (3.6), (3.7) and the additional constraint $n_1 = n(\tau_1)$, implicitly defined by (3.3).

- Given outstanding debt B_1 , households form rational expectations about fiscal policy, and supply labour n_1 to maximize their intratemporal utility (3.1).
- Given (n_1, B_1) , the government sets the tax rate τ_1 and issues debt B_2 to maximize aggregate lifetime utility (3.5) subject to its budget constraint (3.6) and borrowing limit (3.7).

Some comments are in order. First, we spell out the game and equilibrium definition as sequential actions, where households supply labour and then the government sets taxes. Similar economic interactions would prevail if moves were simultaneous. On the other hand, it is essential that the government does not move first. Indeed, if the government had a way to act as a Stackelberg leader and commit to its policy, it would naturally solve the coordination problem by choosing a tax rate on the left-hand side of the Laffer curve.

Second, although the government takes labour supply as given and therefore does not face a Laffer curve, Nash equilibrium requires consistency between the tax rate the private sector expects and the tax rate the government chooses. All equilibria must therefore be on the labour income Laffer curve, but not all points on the Laffer curve will be equilibria.

More importantly, the analysis will unveil conditions under which the equilibrium is unique or not. If the policy functions of households and the fiscal authority exhibit substitutability, which we interpret as fiscal policy being *countercyclical*, then there will be a unique equilibrium. If instead they exhibit complementarity, i.e. if fiscal policy is *procyclical*, then there may be multiple equilibria.¹²

The next section is dedicated to deriving conditions on the inherited level of debt that give rise to complementarities and create the possibility of fiscal policy traps.

3.3 Analysis

This section establishes the key result of the paper, namely that the level of debt is critical to the cyclicity of fiscal policy and can induce complementarities that give rise to fiscal policy traps. The argument is built on a geometric interpretation of the model in (n_1, τ_1) space.¹³ Equilibria in this environment can be represented by intersections of the labour supply function $n(\tau_1)$ and the tax policy function $\tau(n_1, B_1)$. We will show that there are three threshold levels of inherited debt, $B_1^* \leq \hat{B}_1 < \bar{B}_1$, such that when $B_1 < B_1^*$ a unique equilibrium is guaranteed, when $\hat{B}_1 < B_1 < \bar{B}_1$ there will be multiple equilibria, and when $B_1 > \bar{B}_1$ there will not be any equilibria. This result will support our key idea that the level of debt is critical in creating the potential for self-fulfilling fiscal crises.

We begin by characterising the labour supply function, which is everywhere downward sloping and invariant to the inherited debt stock B_1 . We then characterize the government's tax policy function, starting with the limits imposed by the government's budget constraint and borrowing

¹²Formally, since $dn(\cdot)/d\tau < 0$, the policy functions exhibit complementarities if and only if $d\tau(\cdot)/dn \leq 0$.

¹³The geometric approach is very convenient, both for preserving generality of the results and for conveying the main intuitions underlying our analysis.

limit. Unlike the labour supply function, the tax policy function's position and slope does depend on the inherited debt stock B_1 .

We then show that when inherited debt is sufficiently low ($B_1 < B_1^*$), the tax policy function will be upward sloping (countercyclical) at least until it crosses the labour supply function, thereby ensuring a unique equilibrium. Then, we demonstrate that when the inherited amount of debt is high enough that the repayment of newly issued debt cannot be met without tax revenues in period 1 ($\hat{B}_1 < B_1 < \bar{B}_1$), then the tax policy function will cross the labour supply function at least twice. This situation gives rise to multiple equilibria.

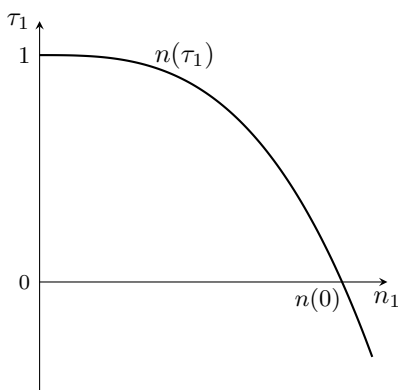
We conclude this section with an economic explanation of why the slope of the tax policy function is ambiguous and depends on the inherited debt stock B_1 . We decompose the government's optimal response to a change in labour supply into two countervailing effects: a tax-base effect and a consumption-smoothing effect.

3.3.1 Properties of the Labour Supply Function

From (3.4) we know that labour supply is a monotonically decreasing function of the tax rate, so the labour supply function $n(\tau_1)$ is downward sloping in (n_1, τ_1) space. Optimal labour supply is zero when the tax rate is 100 percent, and $n(0) > 0$ when the tax rate is zero. The labour supply function starts at $(0, 1)$ and cuts the horizontal axis at $(n(0), 0)$. It continues below the horizontal axis, because greater effort can be induced by negative tax rates (i.e. labour income subsidies).

Optimal labour supply depends only on the tax rate τ_1 , so the labour supply function will be unaffected by changes in the inherited debt stock B_1 or in the government's debt issuance B_2 . Figure 3.2 summarizes the properties of $n(\tau_1)$, the reaction function of households.

Figure 3.2: Labour Supply Function



3.3.2 Properties of the Tax Policy Function

The number of intersections (and hence the number of equilibria) will therefore depend on the shape of the tax policy function, which, as we will show in this section, does depend on the debt

stock B_1 as well as on the quantity of labour supplied, n_1 . We will show that changes in B_1 both shift the tax policy function and alter its slope, thereby affecting the number of equilibria.

Constraints on the Government's Choice of Tax Rate

Let us first consider the constraints the government faces. The *borrowing limit* \bar{B}_2 in (3.7) is the highest level of debt that the government can feasibly repay in period 2 (often referred to in the literature as the “natural” borrowing limit). This of course depends on the government's fiscal capacity in period 2. Let the *maximum rollover threshold* debt level,

$$\hat{B}_1 = \bar{B}_2/R - G_1, \quad (3.11)$$

be the inherited debt level at which the government is exactly solvent in period 2 if it collects zero revenue in period 1. For debt levels strictly above this threshold, the government cannot repay its debts in period 2 without collecting some tax revenue in period 1. For debt levels strictly below this threshold, on the other hand, the government can in fact afford to subsidize labour supply in period 1 by setting a negative income tax rate $\tau_1 < 0$ and still be solvent in period 2.¹⁴

For now, we only consider equilibria in which the government repays its debts in period 2. Accordingly, we define the *lower bound on short-run labour supply* $\underline{n}(B_1)$ as the level of short-run labour supply at or below which the government's fiscal policy is not well defined because repayment of the debt is not feasible. Formally, we have:

$$\underline{n}(B_1) = \begin{cases} f^{-1}\left(\frac{B_1 - \hat{B}_1}{z_1}\right) & \text{if } B_1 > \hat{B}_1, \\ 0 & \text{if } B_1 \leq \hat{B}_1. \end{cases}$$

Since it is the short-run tax rate that matters for labour supply decisions, it will be convenient to rewrite the government's constraints in terms of this tax rate. We define the *minimum short-run tax rate* $\underline{\tau}(n_1, B_1)$ as the tax rate in period 1 that, given the inherited debt level B_1 , the economy's tax base $y_1 = z_1 f(n_1)$ and the government's budget constraint (3.6), requires the government to issue debt up to its borrowing limit \bar{B}_2 . The tax rate $\underline{\tau}(\cdot)$ is therefore the lowest tax rate in period 1 such that full repayment of the public debt is feasible in period 2. As the borrowing limit depends on the government's long-run fiscal capacity, so will the minimum short-run tax rate. Formally, using the government budget constraint, $\underline{\tau}(\cdot)$ is given by:

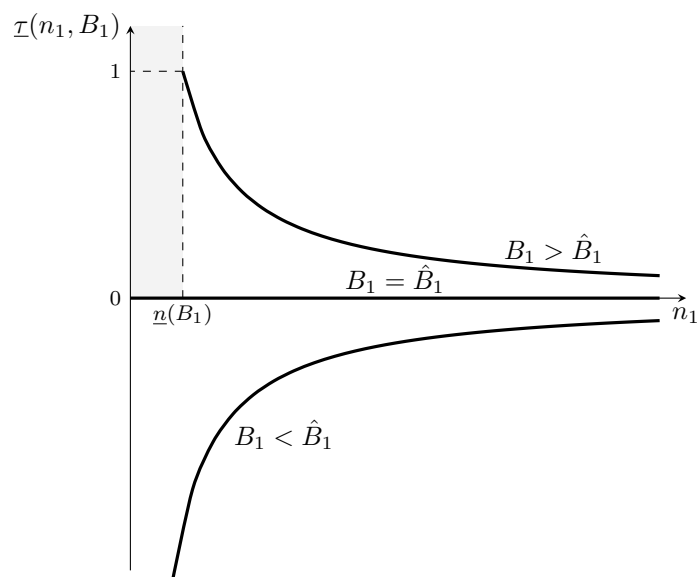
$$\underline{\tau}(n_1, B_1) = \frac{B_1 - \hat{B}_1}{z_1 f(n_1)}, \quad n_1 > 0, n_1 \geq \underline{n}(B_1).$$

Figure 3.3 illustrates the characterisation of the minimum short-run tax rate $\underline{\tau}(\cdot)$. As the inherited debt level B_1 increases, for a given labour supply n_1 , the tax rate must rise to ensure

¹⁴Note that for large enough G_1 , the threshold \hat{B}_1 will be negative, meaning that the government must inherit net claims on foreign wealth in order to be able to afford not to collect any revenue in period 1.

long-run solvency, so the curve shifts up. If $B_1 > \hat{B}_1$, positive short-run tax revenue is needed to ensure long-run solvency, but the higher is the short-run labour supply n_1 , the lower is the minimum tax rate. If $B_1 < \hat{B}_1$, the government can afford to set negative rates $\tau_1 < 0$ (i.e. to subsidize labour), but the higher is the short-run labour supply, the smaller this subsidy has to be. For $B_1 = \hat{B}_1$, no short-run revenue is needed to ensure long-run solvency, but the government cannot afford subsidies, either.

Figure 3.3: Minimum Short-Run Tax Rate $\tau(n_1, B_1)$



This figure summarizes the constraints on the government's optimization problem. It displays the minimum short-run tax rate induced by inherited public debt, labour supply and future fiscal capacity.

Of course, if the inherited level of debt B_1 is too high, the government will be unable to raise enough revenue to remain solvent, and there will be no equilibrium. Clearly, if the required revenue in period 1 exceeds that which would be raised at the peak of the Laffer curve, repayment will not be feasible. However, the maximum inherited debt level that can be sustained in equilibrium is less than this level. The government's lack of commitment reduces the amount of tax revenue it can raise in equilibrium.¹⁵

Accordingly, we define \bar{B}_1 as the upper limit on the amount of inherited debt B_1 that the government can sustain in equilibrium. It is derived as follows. In equilibrium, households' expectations of the tax rate in period 1 must be correct, and labour supply must be optimal: $n_1 = n(\tau_1)$. Equilibrium also requires that the tax rate is set optimally given the level of output and the inherited debt level, that is, $\tau_1 = \tau(n_1, B_1)$. Equilibrium tax revenue in period 1 will therefore be given by the Laffer curve $\tau_1 z_1 f(n(\tau_1))$. Therefore, the maximum inherited debt level \bar{B}_1 is such that, by raising the maximum tax revenue and issuing the maximum amount of debt \bar{B}_2 , the government has just enough resources to finance its spending G_1 in period 1. It is the highest level of inherited

¹⁵If workers were to supply the amount of labour consistent with the peak of the Laffer curve, the government would optimally choose to raise the tax rate.

debt B_1 that satisfies the following two equations:

$$\begin{aligned} R(\bar{B}_1 + G_1 - \tau(n_1, \bar{B}_1)z_1f(n(\tau(n_1, \bar{B}_1)))) &= \bar{B}_2, \\ \tau_1 &= \tau(n_1, \bar{B}_1). \end{aligned}$$

Borrowing Limit Does Not Bind in Equilibrium

We have now defined all the ingredients necessary to prove that the borrowing limit (3.7) does not bind in equilibrium. This justifies restricting our attention to interior solutions of the government's maximization problem. This point is formalized in Lemma 1.

Lemma 1. *For all $B_1 < \bar{B}_1$ and for all $n_1 > \underline{n}(B_1)$, we have $B(n_1, B_1) < \bar{B}_2$ and $\tau(n_1, B_1) > \underline{\tau}(n_1, B_1)$. That is, the borrowing limit (3.7) does not bind, and the optimal short-run tax rate is strictly greater than what is required for long-run solvency.*

Proof. Suppose, on the way to a contradiction, that there exist $B_1 < \bar{B}_1$ and $n_1 > \underline{n}(B_1)$ such that the optimal debt issuance is $B(n_1, B_1) = \bar{B}_2$ and the optimal short-run tax rate is $\tau(n_1, B_1) = \underline{\tau}(n_1, B_1)$. From (3.9), we have $V'(\bar{B}_2) = -\infty$. Given $n_1 > \underline{n}(B_1)$ and our curvature assumptions on the utility function, for all $\tau_1 < 1$ we have $u'((1 - \tau_1)z_1f(n_1)) < +\infty$. The combination of $V'(\cdot) = -\infty$ and $u'(\cdot) < +\infty$ violates the government's first-order condition (3.10). Given $B_1 < \bar{B}_1$ and $n_1 > \underline{n}(B_1)$, it is feasible for the government to raise the short-run tax rate to $\tilde{\tau} \in (\underline{\tau}(n_1, B_1), 1)$ and reduce debt issuance to $\tilde{B}_2 < \bar{B}_2$. Relative to the candidate policy, this alternative policy produces an arbitrarily large long-run marginal benefit at a strictly finite short-run marginal cost, and so the candidate policy $B(n_1, B_1) = \bar{B}_2$ and $\tau(n_1, B_1) = \underline{\tau}(n_1, B_1)$ cannot be optimal. ■

This Lemma tells us that the optimal short-run tax rate $\tau(n_1, B_1)$ will be the interior solution implicitly defined by the first-order condition (3.10). Since the government's budget constraint (3.6) will be satisfied with equality, the debt issuance decision $B(n_1, B_1)$ will be given by:

$$B_2 = R(B_1 + G_1 - \tau(n_1, B_1)z_1f(n_1)).$$

Since households' decisions depend only on the tax rate τ_1 , we are interested mainly in the properties of the tax policy function $\tau(n_1, B_1)$.

Optimal Tax Rate Is Increasing in Inherited Debt

We first show that an increase in B_1 induces an increase in the tax rate τ_1 for any level of labour supply n_1 .¹⁶

¹⁶In (n_1, τ_1) space, this feature is represented by an upward shift of the tax policy function as the inherited debt stock B_1 increases.

Lemma 2.

$$\frac{d\tau(n_1, B_1)}{dB_1} = \frac{\beta R^2 V''(\cdot)}{z_1 f(n_1)(u''(\cdot) + \beta R^2 V''(\cdot))} > 0. \quad (3.12)$$

Proof. The expression is derived by totally differentiating the government's first-order condition (3.10) with respect to B_1 and rearranging. Standard assumptions on the curvature of the utility functions, $u''(\cdot) < 0$ and $V''(\cdot) < 0$, guarantee that the expression is positive. ■

The economic intuition behind this result is straightforward. An increase in the inherited debt stock B_1 means the government is poorer overall. In order to remain solvent, it must raise taxes in period 1, period 2, or both. Given that the marginal utility of consumption is decreasing in both periods, optimality requires the government to spread the pain of an increase in B_1 over both periods, meaning the short-run tax rate τ_1 must rise.

Tax Policy Function Is Upward Sloping Whenever Negative

We are mainly interested in the slope of the tax policy function, that is, how the optimal tax rate responds to changes in labour supply. Taking the total derivative of (3.10) with respect to n_1 , we get:

$$\frac{d\tau(n_1, B_1)}{dn_1} = \frac{f'(n_1)}{f(n_1)} \left(\frac{(1 - \tau_1)u''(\cdot) - \tau_1 \beta R^2 V''(\cdot)}{u''(\cdot) + \beta R^2 V''(\cdot)} \right). \quad (3.13)$$

In general the sign of this expression will be ambiguous, and will depend on the inherited debt stock B_1 .¹⁷ However, the expression is unambiguously positive (and therefore the tax policy function is upward sloping, i.e. countercyclical) whenever the short-run tax rate τ_1 is negative:

Lemma 3.

$$\frac{d\tau(n_1, B_1)}{dn_1} > 0 \quad \forall \tau(n_1, B_1) < 0.$$

Proof. Totally differentiating the government's first-order condition (3.10) with respect to n_1 and rearranging yields:

$$\frac{d\tau(n_1, B_1)}{dn_1} = \frac{f'(n_1)}{f(n_1)} \left(\frac{u''(c_1)}{u''(c_1) + \beta R^2 V''(B_2)} - \tau(n_1, B_1) \right).$$

Since $\beta \geq 0$, $u''(\cdot) < 0$ and $V''(\cdot) < 0$, we have

$$\frac{u''(c_1)}{u''(c_1) + \beta R^2 V''(B_2)} \in [0, 1].$$

Since $f(\cdot) > 0$ and $f'(\cdot) > 0$, the whole expression must be positive whenever $\tau(n_1, B_1) < 0$. ■

¹⁷In section 3.3.4 below, we provide some economic analysis of this ambiguity by decomposing the government's response to a change in labour supply into a tax-base effect and a consumption-smoothing effect.

3.3.3 Equilibria

Combining the analysis of households' labour supply function and the government's tax policy function, we are now ready to derive conditions under which the equilibrium of the economy is unique or not, i.e. conditions under which fiscal policy traps can arise.

Unique Equilibrium When Debt Is Low

Combining Lemmas 2 and 3, we can show that there will be a unique equilibrium whenever the inherited debt stock B_1 is sufficiently low.

Proposition 1. *Let B_1^* be such that $\tau(n(0), B_1^*) = 0$. Then for all $B_1 < B_1^*$, there will be a unique equilibrium.*

Proof. From Lemma 2 we know that $\tau(n(0), B_1) < \tau(n(0), B_1^*) = 0$ for all $B_1 < B_1^*$, that is, the optimal tax rate will be negative whenever labour supply is $n(0)$ and inherited debt is less than the threshold value B_1^* . Then from Lemma 3 we know that whenever $B_1 < B_1^*$ the tax policy function will be negative valued and upward sloping for all values of labour supply $n_1 \leq n(0)$, and indeed will continue to slope upwards at least until it cuts the horizontal axis. Before it does so, it will cut the (downward-sloping) labour supply function exactly once. ■

Multiple Equilibria When Debt Is High

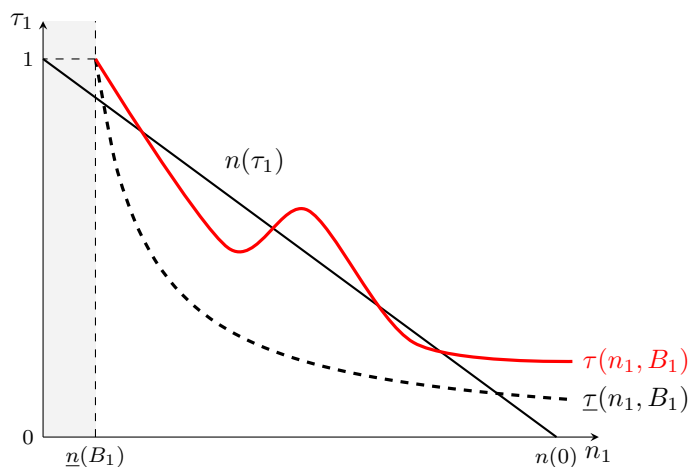
The government's budget constraint (3.6) means that if it inherits a sufficiently large stock of debt B_1 , it will be forced to collect tax revenue in period 1 in order to stay within its borrowing limit (3.7). Whenever the inherited debt level B_1 is high enough that the government must collect taxes in period 1 (but not so high that repayment becomes infeasible), the economy will exhibit multiple equilibria.

Proposition 2. *Let $\hat{B}_1 = \bar{B}_2/R - G_1$, where \bar{B}_2 is the natural borrowing limit, and let \bar{B}_1 be the highest inherited debt level for which an equilibrium exists. Then for all $B_1 \in (\hat{B}_1, \bar{B}_1)$, the economy exhibits multiple equilibria.*

Proof. For all $B_1 \in (\hat{B}_1, \bar{B}_1)$, we have $\tau(\underline{n}(B_1), B_1) = \underline{\tau}(\underline{n}(B_1), B_1) = 1 > n^{-1}(\underline{n}(B_1))$. That is, when inherited debt is above the maximum rollover threshold \hat{B}_1 and short-run labour supply is at its minimum value $\underline{n}(\cdot)$, the government's optimal short-run tax rate is 100 percent, because this is the only feasible choice. We know that 100 percent is higher than the tax rate that would induce labour supply of $\underline{n}(\cdot) > 0$, because labour supply is decreasing in the tax rate and it is optimal not to work when the tax rate is 100 percent.

For all $B_1 \in (\hat{B}_1, \bar{B}_1)$, we have $\tau(n(0), B_1) > \underline{\tau}(n(0), B_1) > n^{-1}(n(0)) = 0$. This says that, when inherited debt is above the maximum rollover threshold \hat{B}_1 and labour supply is at the value that would optimally be chosen if the tax rate were zero, the optimal tax rate is in fact strictly positive.

Figure 3.4: Existence of Fiscal Policy Traps



These two pieces tell us that the optimal tax curve lies above the labour supply curve at two points: when n_1 is at the minimum level consistent with solvency, $\underline{n}(B_1)$, and when n_1 is at the point consistent with zero taxes, $n(0)$. There can't be an equilibrium to the left of (i.e. with a lower labour supply than) $\underline{n}(B_1)$, because solvency would be violated whatever fiscal policy the government chose. We also know that there can't be an equilibrium to the right of (i.e. with a higher labour supply than) $n(0)$, because the labour supply curve is negative valued after that point, and $\underline{\tau}(n_1, B_1)$ is strictly positive for all n_1 whenever $B_1 > \hat{B}_1$. So if an equilibrium exists, it must be between $\underline{n}(B_1)$ and $n(0)$. Apart from the special case of tangency (with $B_1 = \bar{B}_1$), if the optimal tax curve crosses below the labour supply curve somewhere to the right of $\underline{n}(B_1)$, it must cross it again in order to be above it at $n(0)$. ■

Welfare Ordering of Equilibria

Proposition 3. *The equilibria in Proposition 2 with higher labour supply n_1 Pareto dominate those with lower labour supply.*

Proof. Since all households are ex ante identical and all equilibria are symmetric, the welfare ordering of equilibria depends on the utility of the representative household.

All equilibria must lie on the labour supply curve $n(\tau_1)$, which is downward sloping, so equilibria featuring higher short-run labour supply n_1 must also feature a lower short-run tax rate τ_1 . The short-run tax rate τ_1 enters into the household budget constraint (3.2), and since labour supply cannot be negative, a reduction in τ_1 expands the household's choice set, meaning the household is (weakly) better off in period 1.

All that remains to be shown is that in equilibria with higher short-run labour supply, the representative household is also better off in period 2. Since $V'(B_2) < 0$, we need to show that the government's optimal debt issuance B_2 is lower in equilibria featuring higher short-run labour

supply n_1 . To see this, note that optimal fiscal policy must satisfy the first-order condition (3.10):

$$u'((1 - \tau_1)z_1 f(n_1)) = -\beta V'(B_2).$$

Consider two equilibria, one “good” and one “bad”, with $n_1^G > n_1^B$ and $\tau_1^G < \tau_1^B$. Now suppose (on the way to a contradiction) that the good equilibrium features higher debt issuance: $B_2^G > B_2^B$. Then from $V''(B_2) < 0$ we have $V'(B_2^G) < V'(B_2^B)$, meaning $-\beta V'(B_2^G) > -\beta V'(B_2^B)$. In order for the government’s first-order condition to be satisfied in both equilibria, we would therefore need $u'((1 - \tau_1^G)z_1 f(n_1^G)) > u'((1 - \tau_1^B)z_1 f(n_1^B))$. However, given that output is increasing in labour supply and $u''(\cdot) < 0$, this would require $\tau_1^G > \tau_1^B$, which cannot be the case because by hypothesis the good equilibrium features a lower tax rate. ■

A lower tax rate in period 1 means households are wealthier in period 1. Since substitution effects dominate income effects, their response is to increase their labour supply, which increases the government’s tax base. This induces a reduction in the government’s optimal debt issuance, so households are wealthier in period 2 as well.

3.3.4 Tax-Base and Consumption-Smoothing Effects

In this subsection we provide some economic intuition for our main result that optimal fiscal policy is procyclical when the burden of inherited debt is large. We do so by providing a decomposition of the effect of a change in labour supply on the optimal tax rate. We identify two countervailing effects at play, which we label *tax-base* and *consumption-smoothing* effects.

Consider a reduction in period 1 labour supply n_1 . Ceteris paribus, this reduces period 1 consumption relative to period 2 consumption, thereby providing the government with a consumption-smoothing motive to reduce the period 1 tax rate relative to the period 2 tax rate. On the other hand, when the period 1 tax rate is positive, a reduction in period 1 labour supply shrinks the overall tax base. In order for the government to remain solvent, therefore, the average tax rate across periods 1 and 2 must rise.

Similarly to how the effect of a price change on demand can be decomposed into a substitution and an income effect, we can decompose the effect of a change in labour supply on the optimal tax rate by rewriting the slope of the tax policy function (3.13) as follows:

$$\frac{d\tau(n_1, B_1)}{dn_1} = (1 - \tau_1) \frac{f'(n_1)u''(\cdot)}{f(n_1)(u''(\cdot) + \beta R^2 V''(\cdot))} - \tau_1 z_1 f'(n_1) \frac{dB_1}{dB_1}.$$

The first term captures the consumption-smoothing effect, which is unambiguously positive (meaning a reduction in labour supply prompts a reduction in the tax rate i.e. that fiscal policy is countercyclical). The second term captures the tax-base effect, which operates through the impact of a change in labour supply on the total fiscal resources available to the government. It is therefore no accident that the size of the tax-base effect is linked to the effect of a change in the inherited

debt stock on the optimal tax rate, $d\tau(n_1, B_1)/dB_1$.

The relative strength of the tax-base and consumption-smoothing effects will determine the cyclicalities of the government's optimal fiscal policy. When the consumption-smoothing effect dominates, fiscal policy will be countercyclical and the tax policy function will be upward sloping. Noting that the size of both effects depends on the short-run tax rate τ_1 , which itself depends positively on the inherited debt level B_1 as per Lemma 2, we can see that the cyclicalities of fiscal policy will depend on the inherited debt level.

However, the effect of inherited debt on the cyclicalities of fiscal policy is not guaranteed to be monotonic in all cases. This potential non-monotonicity means that there may not necessarily be a cut-off level of debt above which fiscal policy switches from being countercyclical to procyclical. Nevertheless, Proposition 1 guarantees that fiscal policy will always be countercyclical over the relevant range of labour supply when inherited debt is below the threshold B_1^* , ensuring a unique equilibrium. Similarly, Proposition 2 guarantees that there will be multiple equilibria (which requires that fiscal policy is at least locally procyclical) whenever inherited debt exceeds the maximum rollover threshold \hat{B}_1 .

Note that the sign of the tax-base effect depends on whether the period 1 tax rate is positive or negative. This provides the intuition behind the result in Lemma 3 that the tax policy function is upward sloping whenever the tax rate is negative. With a negative tax rate (i.e. a labour subsidy), a reduction in labour supply actually reduces the fiscal burden on the government. This reverses the usual sign of the tax-base effect, meaning it reinforces rather than counteracts the consumption-smoothing effect. With both effects acting in the same countercyclical direction, the government's fiscal policy will be unambiguously countercyclical whenever the short-run tax rate is negative.

3.4 Example with Closed-Form Solutions

In this section we present an analytical example of the class of economies described previously, and clearly highlight the general result of section 3.3.

We adopt the following specification. In period 1, self-employed households convert labour effort into output using the following production function:

$$y_1 = z_1 n_1^\alpha, \quad \alpha > 0,$$

where α captures returns to scale. The government inherits a stock of debt B_1 owed to foreigners, chooses a proportional income tax rate τ_1 and issues an amount of bonds B_2 (again to foreigners) at the risk-free interest rate R .

Period 2, the long run, is an endowment economy in which the government levies lump-sum taxes. The per-capita endowment of output is y_2 , and to economize on notation we normalize

period 2 government expenditure, G_2 , to zero.¹⁸

The representative household's lifetime utility is given by:

$$U(c_1, n_1, c_2) = u(c_1) - g(n_1) + \beta u(c_2),$$

where instantaneous consumption utility is given by

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad \sigma \in (0, 1)$$

in both periods, and the disutility from labour effort in period 1 is given by

$$g(n_1) = \frac{n_1^\gamma}{\gamma}, \quad \gamma > 0.$$

Substituting the budget constraint $c_1 = (1-\tau_1)y_1$ and the production function into the objective function and solving the household's first-order condition yields the following expression for optimal labour supply:

$$n(\tau_1) = \left(\alpha ((1-\tau_1)z_1)^{1-\sigma} \right)^{\frac{1}{\gamma-\alpha(1-\sigma)}}.$$

The government faces the usual budget constraint (3.6). With lump-sum taxation in period 2, the natural borrowing limit \bar{B}_2 in (3.7) is given by the long-run endowment y_2 , since long-run consumption $c_2 = y_2 - B_2$ cannot be negative. The government's continuation utility $V(B_2)$ from issuing an amount of debt B_2 is simply households' utility $u(y_2 - B_2)$ of consuming the amount left over after lump-sum taxes are levied on the endowment to pay off the debt. It follows immediately that conditions (3.8) and (3.9) on the continuation utility function are satisfied.¹⁹

The maximum rollover threshold level of inherited debt, above which the government must collect revenue in period 1 in order to remain solvent, is given by:

$$\hat{B}_1 = \bar{B}_2/R - G_1 = y_2/R - G_1. \tag{3.14}$$

3.4.1 Inherited Debt and the Cyclicity of Fiscal Policy

Solving the government's optimization problem yields the following tax policy function:

$$\tau(n_1, B_1) = \frac{(\beta R)^{1/\sigma}}{R + (\beta R)^{1/\sigma}} - \frac{R(\hat{B}_1 - B_1)}{(R + (\beta R)^{1/\sigma})z_1 n_1 \alpha}. \tag{3.15}$$

This solution allows us to characterize precisely how the cyclicity of fiscal policy depends on the inherited level of debt.

¹⁸This normalization is innocuous because with lump-sum taxes in period 2, an increase in G_2 is equivalent to a decrease in y_2 .

¹⁹Formally, $V'(B_2) = -u'(y_2 - B_2) = -(y_2 - B_2)^{-\sigma} < 0$, $V''(B_2) = u''(y_2 - B_2) = -\sigma(y_2 - B_2)^{-1-\sigma} < 0$ and $\lim_{B_2 \rightarrow \bar{B}_2} V'(B_2) = \lim_{B_2 \rightarrow y_2} u'(y_2 - B_2) = \lim_{c_2 \rightarrow 0} c_2^{-\sigma} = +\infty$.

Proposition 4. *The cyclicity of fiscal policy depends on the inherited debt level B_1 as follows:*

$$\frac{d\tau(n_1, B_1)}{dn_1} = \frac{R(\hat{B}_1 - B_1)\alpha}{(R + (\beta R)^{1/\sigma}) z_1 n_1^{1+\alpha}} \begin{cases} > 0 & (\text{countercyclical}) \text{ if } B_1 < \hat{B}_1 \\ = 0 & (\text{acyclical}) \text{ if } B_1 = \hat{B}_1 \\ < 0 & (\text{procyclical}) \text{ if } B_1 > \hat{B}_1. \end{cases}$$

Accordingly, the equilibrium of the economy is unique if and only if $B_1 < \hat{B}_1$, and fiscal policy traps may emerge for high levels of inherited debt B_1 .

Proof. Differentiation of (3.15) and application of Propositions 1 and 2. ■

In the proof of Proposition 2, we saw the general result that when public debt is above the maximum rollover threshold level \hat{B}_1 , the government's tax policy function must be at least locally procyclical. Proposition 4 shows that there is a starker relationship between the level of public debt and the cyclicity of fiscal policy in this particular case. For levels of debt above \hat{B}_1 , fiscal policy is procyclical for all values of labour supply.

Since for any given value of inherited debt B_1 the government's tax policy function is monotonic, we can guarantee that there is a unique cutoff value of B_1 , below which there will be a unique equilibrium and above which there will be two equilibria.²⁰ The three cases are illustrated in Figure 3.5. In panel (a), debt is below the threshold \hat{B}_1 and so the tax policy function is upward sloping for all values of labour supply. It therefore crosses the labour supply function just once, ensuring a unique equilibrium. Panel (b) shows that the equilibrium is also unique when inherited debt is equal to the threshold \hat{B}_1 and the tax policy function is horizontal. Whenever inherited debt exceeds this threshold, as in panel (c), the tax policy function is downward sloping for all values of labour supply and there are two equilibria.

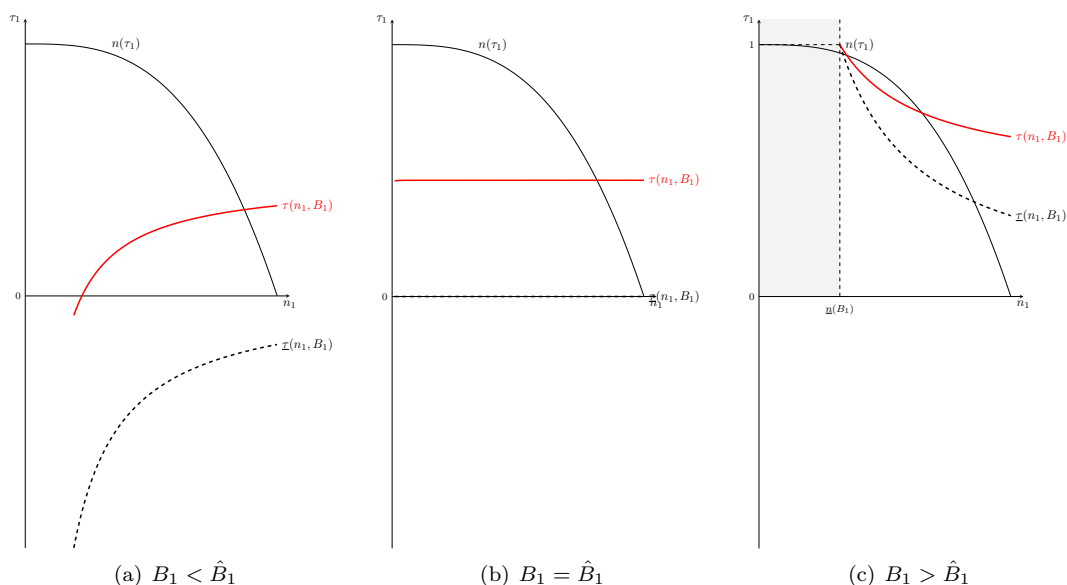
Looking at equation (3.14) we can see that the threshold value of debt does not depend on contemporaneous parameters, such as productivity z_1 , but only on future variables, such as fiscal capacity y_2 . Although an increase in productivity reduces the optimal tax rate for a given level of labour supply, it cannot eliminate the possibility of fiscal policy traps. No matter how high is productivity, if debt is above the maximum rollover threshold then fiscal policy will be procyclical. This supports the idea that future fiscal capacity is essential in steering the economy away from fiscal policy traps.

3.5 Robustness

So far, we have constrained the choice set of the government by assuming exogenous government spending and not allowing the possibility of defaulting on debt. Further, we ruled out the possibility for households to smooth consumption themselves by accessing international capital markets.

²⁰This is true whenever the tax policy function is monotonic for all values of B_1 , not just for the particular example we consider here.

Figure 3.5: Inherited Debt and the Cyclicality of Fiscal Policy



These assumptions imposed strong restrictions on the fiscal capacity of the country. These elements gave rise to the key result of the analysis, namely that the level of inherited debt is decisive in inducing procyclical fiscal policy and paving the way for a self-fulfilling fiscal crisis. In this section we relax these assumptions and investigate the robustness of our result.

3.5.1 Endogenous Government Spending

Consider a government with a high inherited level of debt. Facing a low value of labour supply, would the government rather increase its tax rate or reduce government expenditure?

To endogenize the choice of public expenditure, we assume that households derive instantaneous utility $v(G_1)$ from public expenditure G_1 by the government. The next Proposition shows that the key result of the baseline model still holds, even if the possibility of adjusting government expenditure provides the government with some “breathing room”: the threshold level of debt is higher, but above this threshold, fiscal policy is procyclical and there is still the risk of fiscal policy traps. We adapt the analytical specification introduced in section 3.4 above and assume that $v(G_1) = \frac{G_1^{1-\sigma}}{1-\sigma}$. Formally, given (n_1, B_1) , the government solves:

$$\max_{\tau_1, G_1, B_2} u((1 - \tau_1)n_1) + v(G_1) + \beta u(y_2 - B_2)$$

subject to the usual government budget constraint (3.6) and borrowing constraint (3.7).

Note that with endogenous G_1 , the maximum rollover threshold level of debt becomes:

$$\hat{B}_1 = \bar{B}_2/R = y_2/R,$$

because the government has the option of setting $G_1 = 0$.

The solution to the government's maximization problem gives the following tax policy function:

$$\tau(n_1, B_1) = \frac{R + (\beta R)^{1/\sigma}}{2R + (\beta R)^{1/\sigma}} - \frac{R(\hat{B}_1 - B_1)}{(2R + (\beta R)^{1/\sigma})z_1 n_1 \alpha}. \quad (3.16)$$

Proposition 5. *The cyclicity of fiscal policy depends on the inherited debt level B_1 as follows:*

$$\frac{d\tau(n_1, B_1)}{dn_1} = \frac{R(\hat{B}_1 - B_1)\alpha}{(2R + (\beta R)^{1/\sigma})z_1 n_1^{1+\alpha}} \begin{cases} > 0 & (\text{countercyclical}) \text{ if } B_1 < \hat{B}_1 \\ = 0 & (\text{acyclical}) \text{ if } B_1 = \hat{B}_1 \\ < 0 & (\text{procyclical}) \text{ if } B_1 > \hat{B}_1. \end{cases}$$

Accordingly, the equilibrium of the economy is unique if and only if $B_1 < \hat{B}_1$, and fiscal policy traps may emerge for high levels of inherited debt B_1 .

Proof. Differentiation of (3.16) and application of Propositions 1 and 2. ■

Intuitively, when G_1 and c_1 are complements, the government will optimally choose to reduce G_1 in proportion with c_1 when labour supply n_1 decreases and the country is poorer. This allows the government to raise the tax rate by less than in the case with exogenous government expenditure. Nevertheless, once the government is above its short-run borrowing limit, it will have to raise the tax rate, preserving the risk of fiscal policy traps.

3.5.2 Allowing for Default on Newly Issued Debt

In our baseline model, we assume that the government is committed to repaying its debts in full in period 2. This commitment implies the limit \bar{B}_2 to the amount of debt the government can issue in period 1 (see equation (3.7)). This debt issuance limit, together with the tax-base effect that becomes stronger as the government approaches it, causes optimal fiscal policy to be procyclical when the inherited debt level is high.

We now relax the hard solvency constraint and allow the government to choose strategically in period 2 whether or not to repay its debts. We show that this does not eliminate the possibility of self-fulfilling fiscal crises. In fact, the lack of commitment to debt repayment, i.e. the prospect of default in period 2, tightens the borrowing constraint in period 1. This in turn decreases the threshold level of debt \hat{B}_1 above which the economy is susceptible to fiscal policy traps.

Stochastic Long-Run Output and Strategic Default

To develop this idea, we amend the model as follows. Let long-run output y_2 be stochastic, distributed uniformly on $[y_2, \bar{y}_2]$. Denote $F(\cdot)$ the cumulative distribution function of y_2 . As in section 3.4 above, the government can use lump-sum taxes in period 2 to repay its debt B_2 , in which case period 2 consumption will be $c_2 = y_2 - B_2$. If instead the government chooses to default

in period 2, the economy suffers a proportional output loss δ_2 , so period 2 consumption becomes $c_2 = (1 - \delta_2)y_2$.

The default cost δ_2 can be interpreted as the government's degree of commitment to repaying its debts in period 2. The extreme case of $\delta_2 = 1$ induces a strong commitment to repay and captures the hard solvency constraint assumed up to now. At the opposite extreme of $\delta_2 = 0$, default is costless, so the government would always default. Given outstanding bonds B_2 , it is optimal for the government to repay its debts in period 2 whenever output y_2 satisfies:

$$y_2 - B_2 \geq (1 - \delta_2)y_2.$$

This relation gives the threshold $\hat{y}_2(B_2)$, realizations of y_2 below which the government defaults on its bonds B_2 :

$$\hat{y}_2(B_2) = B_2/\delta_2. \tag{3.17}$$

Risk-neutral foreign investors anticipate the strategic default decision of the government. Accordingly, the price schedule $q(B_2)$ satisfies the following no-arbitrage condition:

$$q(B_2) = \frac{1 - F(\hat{y}_2(B_2))}{R}, \tag{3.18}$$

where R is the risk-free interest rate. In this expression, the credit risk associated with the issuance of bonds B_2 is captured by $F(\hat{y}_2(B_2))$, the probability that long-run output will be below the default threshold $\hat{y}_2(B_2)$. The possibility of strategic default can lead to indeterminacy in the price schedule (3.18), as studied in Calvo (1988) and Cooper (2015). As our focus is on the occurrence of fiscal policy traps rather than self-fulfilling increases in sovereign risk premia, whenever several prices satisfy the price schedule (3.18) we assume that investors select the "fundamental" outcome with the lowest risk premium. In this case, the price of debt $q(B_2)$ is decreasing in the amount of bonds B_2 issued, reflecting the increased probability of default.

Lack of Commitment to Repay Reduces Borrowing Limit

We are now ready to prove that despite its capacity to default on debt in period 2, the government may still be susceptible to fiscal policy traps. As in our baseline model, government borrowing between period 1 and 2 is constrained. This in turn induces a maximum level of inherited debt \hat{B}_1 such that the government can roll over its obligations without having to collect tax revenue in period 1. As in the baseline model, the economy is under the threat of fiscal policy traps whenever inherited debt B_1 is above this threshold. Interestingly, this threshold is increasing in the commitment parameter δ_2 . In other words, the less committed a country is to repaying its debt, the lower is the debt threshold at which it becomes vulnerable to fiscal policy traps.

Proposition 6. *Whenever the government can default on its debt in period 2, there is a debt rollover threshold \hat{B}_1 above which the country is subject to fiscal policy traps. The threshold is*

increasing in the output loss parameter δ_2 (i.e. an increase in commitment increases debt capacity).

Proof. We first demonstrate that there is a maximum amount of revenue that the government can raise in period 1, and that this amount is decreasing in δ_2 . The revenue raised in period 1 by issuing B_2 bonds is $q(B_2)B_2$, where the price schedule $q(B_2)$ satisfies (3.18). Using the default threshold (3.17), resources from debt issuance are:

$$q(B_2)B_2 = \frac{1 - F(B_2/\delta_2)}{R} B_2.$$

The right-hand side is equal to 0 for $B_2 = 0$ and for $B_2 = \delta_2 \bar{y}_2$, and is strictly positive for any value of B_2 in between. Hence the right-hand side reaches a maximum for $B_2 = \bar{B}_2 \in (0, \delta_2 \bar{y}_2)$. The maximum period 1 revenue from debt issuance is therefore $q(\bar{B}_2)\bar{B}_2$. Since the price of debt is strictly increasing in δ_2 , the revenue collected $q(B_2)B_2$ is also increasing in δ_2 , and so is the maximum amount that can be collected.

As in (3.11) above, there is a maximum amount of debt \hat{B}_1 that can be rolled over without raising any tax revenue in period 1. This threshold is increasing in the maximum amount of revenue that can be raised by issuing new debt, and therefore in δ_2 :

$$\hat{B}_1 = q(\bar{B}_2)\bar{B}_2 - G_1.$$

If the stock of inherited debt B_1 exceeds the maximum rollover threshold \hat{B}_1 , then as in our baseline model the government will have to gather revenue in period 1. Indeed, to remain within this limit the government must set a short-run tax rate at least equal to

$$\tau(n_1, B_1) = \frac{B_1 - \hat{B}_1}{z_1 f(n_1)}.$$

It follows that, as before, when $B_1 > \hat{B}_1$ the optimal tax policy function is at least locally procyclical, and Proposition 2 applies.

■

The intuition behind this result is as follows. As the default cost δ_2 falls, investors know that the government will default in more states of the world in period 2 because it faces a lower penalty for doing so. This causes them to charge a higher risk premium, thereby reducing the amount of revenue the government can raise in period 1 by issuing new debt.

Overall, allowing the government to default on its debts in period 2 does not, therefore, eliminate the possibility of self-fulfilling fiscal crises.

3.5.3 Private Access to International Markets

In our benchmark model, we made the simplifying assumption that households lived hand-to-mouth and could neither save nor borrow between periods 1 and 2. Here we consider the polar

opposite case in which households have full access to international capital markets at the risk-free rate R .²¹ Allowing households to borrow and save across time allows them to smooth consumption when taxes change. Are fiscal policy traps still possible in this environment?

Whereas in section 3.4 we modelled period 2 as an endowment economy with lump-sum taxation, here we model period 2 as a production economy with endogenous labour supply. Under this set-up, labour taxation in period 2 influences the labour supply decision. This modelling approach is aimed at avoiding redundancy between public and private intertemporal decisions. Further, we assume that the government cannot tax saving or consumption directly. As we shall demonstrate below, when households can privately smooth consumption across time, the combination of an inelastic short-run tax base and an elastic long-run tax base leads the government to tax the short-run tax base at 100 percent. Fiscal policy is determinate but the outcome is unambiguously worse, since households use international markets to avoid taxes, which in turn induces the government to set the highest tax rate possible.

The timing is as follows. In period 1, households choose their borrowing, denoted a , and their short-run labour supply n_1 .²² Having observed n_1 and a , the government then sets a short-run tax rate τ_1 and issues new debt B_2 . In period 2, households supply labour n_2 , clear their borrowing position and consume. The government sets a tax rate τ_2 to meet its budget constraint and clear its debt position. The solution is derived by backward induction.

Period 2 consumption is given by $c_2 = (1 - \tau_2)z_2f(n_2) - Ra$. With the disutility of labour captured by the function $g(\cdot)$, households solve the following problem:

$$V_H(a) = \max_{n_2} u((1 - \tau_2)z_2f(n_2) - Ra) - g(n_2). \quad (3.19)$$

Optimal period 2 labour supply $n_2(\tau_2, a)$ is implicitly defined by households' intratemporal first-order condition:

$$(1 - \tau_2)z_2f(n_2)u'(c_2) = g'(n_2). \quad (3.20)$$

The government's period 2 budget constraint implicitly defines $\tau_2(a, B_2)$:²³

$$\tau_2z_2f(n_2) = B_2, \quad (3.21)$$

where $n_2 \equiv n_2(\tau_2, a)$ is given by (3.20).

Now, in period 1, given τ_1 and τ_2 , households solve the following problem:²⁴

$$\max_{n_1, a} u((1 - \tau_1)z_1f(n_1) + a) - g(n_1) + \beta V_H(a),$$

²¹We thank Russell Cooper for hinting at this extension.

²²Saving is denoted by a negative value of a .

²³As in Section 3.5.2, in order to abstract away from other sources of coordination failures, we select the tax rate on the left-hand side of the Laffer curve in period 2. As above, period 2 government spending is normalized to zero.

²⁴Although the tax rates τ_1 and τ_2 are determined by aggregate household borrowing and labour supply decisions, each individual household takes them as given when making its own decisions.

where $V_H(a)$ is the value of the period 2 problem (3.19) when the household has borrowed a . The household intertemporal first-order condition is therefore given by:

$$\begin{aligned} u'((1 - \tau_1)z_1f(n_1) + a) &= -\beta V'_H(a) \\ u'(c_1) &= \beta R u'(c_2). \end{aligned} \quad (3.22)$$

Unlike households, the government internalizes the effect of its period 1 choices on the tax rate in period 2. When the government issues debt B_2 , its continuation value is given by:

$$V_G(a, B_2) = u\left((1 - \tau_2(a, B_2))z_2f\left(n_2(\tau_2(a, B_2))\right) - Ra\right) - g\left(n_2(\tau_2(a, B_2))\right).$$

The government's period 1 problem is therefore:

$$\max_{\tau_1} u((1 - \tau_1)z_1f(n_1) + a) - g(n_1) + \beta V_G(a, B_2),$$

where government debt issuance is given by the budget constraint $B_2 = R(B_1 - \tau_1 z_1 f(n_1))$. The government's intertemporal first-order condition is:

$$\begin{aligned} u'((1 - \tau_1)z_1f(n_1) + a) &= -\beta R \frac{dV_G(a, B_2)}{dB_2} \\ u'(c_1) &= \beta R u'(c_2) z_2 f(n_2) \frac{d\tau_2(a, B_2)}{dB_2}. \end{aligned} \quad (3.23)$$

Comparing the household intratemporal first-order condition (3.22) with that of the government (3.23), we see that they differ by the term $z_2 f(n_2) \frac{d\tau_2(a, B_2)}{dB_2}$, which captures the distortionary effects of taxation in period 2, internalized by the government. This difference leads to the striking implication of this extension of our model:

Proposition 7. *When households have access to international capital markets and taxation is distortionary in period 2, there is a unique equilibrium with $\tau_1 = 1$, $n_1 = 0$ and $a > 0$.*

Proof. Comparing the household intertemporal first-order condition (3.22) and the government's intertemporal first-order condition (3.23), we see that the right-hand side of the latter contains an additional term $z_2 f(n_2) \frac{d\tau_2(a, B_2)}{dB_2} > 0$. This means that only one of these expressions can hold in equilibrium. Households' period 2 labour effort n_2 is unbounded above and they are committed to repay their debts Ra in full, so unlike the government they face no borrowing constraints. This ensures that households' intertemporal first-order condition holds in equilibrium and the government's does not.

If the government's intertemporal first-order condition does not hold, then the tax rate τ_1 it sets must be one of the corner solutions. Given that the extra term on the right-hand side of the government's intertemporal first-order condition is positive, then if the households' first-order condition holds, the government wants to increase the term on the left-hand side, $u'(c_1)$. Since $u''(\cdot) < 0$, this means the government wants to decrease $c_1 = (1 - \tau_1)z_1f(n_1)$, which it does by

setting τ_1 as high as possible.

Given $\tau_1 = 1$, it is optimal for households to supply $n_1 = 0$. Finally, since $u'(0) = +\infty$, intertemporal optimization by households requires $a > 0$. ■

In contrast to the baseline model with hand-to-mouth households, the equilibrium outcome when households have access to perfect private capital markets is characterized by a unique tax rate in period 1, independent of the level of inherited debt B_1 . Still, the equilibrium outcome is unambiguously worse, since no production occurs in period 1 and the whole burden of taxation and debt repayment is postponed to period 2.

The intuition behind this seemingly perverse result is that households' access to capital markets undermines the government's consumption-smoothing motive for keeping the period 1 tax rate low. Households then avoid high period 1 taxes by cutting their labour supply, and borrow to preserve their period 1 consumption level. The implication is that the absence of perfect private capital markets is beneficial in this environment, because it partially compensates for the government's inability to commit not to tax the inelastic short-run tax base excessively.

3.6 Conclusion

The recent rise (and subsequent fall) of sovereign debt spreads in the euro area periphery has prompted renewed interest in multiple equilibria and self-fulfilling crises. Yet it was not only countries facing increased borrowing costs that pursued contractionary fiscal policies during the Great Recession.

In this paper we have proposed a potential explanation for why governments might pursue procyclical fiscal policies despite not facing increased sovereign risk premia. When the inherited stock of public debt is sufficiently high, concerns about the burden of future taxes may overwhelm concerns about preserving consumption in the face of a decline in output, making even optimal fiscal policy procyclical. This procyclicality unleashes the possibility of a different kind of crisis, fuelled not by self-fulfilling fears of higher sovereign spreads but by self-fulfilling fears of a decline in output.

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