Essays on Macro Financial Linkages

Angela Abbate

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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*Department of Economics*

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I confirm that chapter <2> was jointly co-authored with Massimiliano Marcellino and I contributed to 70% of the work

I confirm that chapter <4> was jointly co-authored with Sandra Eickmeier, Wolfgang Lemke and Massimiliano Marcellino, and I contributed to 25% of the work.

I confirm that chapter <4> draws upon an article forthcoming in the Journal of Money, Credit and Banking

Signature and Date:
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e a Dario, che mi spinge sempre nella direzione giusta
Abstract

The first chapter, joint with Dominik Thaler, is a New Keynesian model of how monetary policy can influence the risk-taking behaviour of banks. Lower interest rates change bank incentives, making them prefer riskier investments. This mechanism alters the tradeoff faced by the monetary authority, affecting optimal policy conduct. After estimating the model, we find that the monetary authority should react less aggressively to inflation, trading off more inflation volatility in exchange for less financial market distortions.

The second chapter, written with Prof. Massimiliano Marcellino, investigates whether modelling parameter time variation and stochastic volatility improves the forecasts of three major exchange rates vis-a-vis the US dollar. We find that modelling time-varying volatility significantly refines the estimation of forecast uncertainty through an accurate calibration of the entire forecast distribution at all forecast horizons.

Similar empirical tools are employed in the third chapter, where I show that the inclusion of default risk and risk aversion measures improves the forecasts of key activity and banking indicators. The bulk of forecast improvement takes place during the 2001 and 2008 recessions, when credit constraints were arguably binding. A structural VAR further reveals that an unexpected credit spread increase in 2010 causes an output contraction that lasts for about two years, and explains up to 35% percent of output variation.

The final project, joint with Sandra Eickmeier, Prof. Massimiliano Marcellino and Wolfgang Lemke, investigates the changing international transmission of financial shocks over 1971-2012. A time-varying parameter FAVAR shows that global financial shocks, measured as unexpected changes in a US financial condition index, strongly impact growth in the nine countries considered. In addition, financial shocks in 2008 explain approximately 20% of the GDP growth variation in the 9 countries, as opposed to an average of 5% percent before the crisis.
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Chapter 1

Monetary policy effects on bank risk taking

1.1 Introduction

The recent financial crisis has marked the importance of monitoring the different types of risks to which the financial sector, and ultimately the real economy, are exposed. A relevant aspect is whether interest rates, and therefore monetary policy, can influence the risk-taking behavior of financial intermediaries. This transmission mechanism, known as the risk-taking channel of monetary policy\footnote{The term was first coined by Borio and Zhu\cite{BorioZhu2008}.}, could have contributed to the excessive levels of financial sector balance-sheet risk which lead to the 2008 financial crisis. In the aftermath of the crisis interest rates have fallen considerably in many countries, raising concerns on whether financial market participants might be once again induced to reallocate portfolios towards riskier investments, creating the risk of yet another crisis\footnote{See for instance: (2015, 26 September). Repeat Prescription, The Economist.}

This paper addresses these concerns from a theoretical point of view, motivated by structural VAR evidence that expansionary monetary policy shocks increase bank asset risk in the US. We build a monetary DSGE model, where investment in capital is intermediated by a banking sector and is furthermore risky. Building on Dell’Ariccia et al.\cite{DellAriccia2014}, we assume that banks can choose from a continuum of investment projects, each defined by different risk-return characteristics. Every project has a certain probability of yielding capital in the next period. The safer the project is, however, the lower is the return in case of success. Since depositors cannot observe the investment choice, and because bank owners are protected by limited liability, an agency problem emerges: banks are partially isolated from the downside risk of their investment and hence choose
a risk level that exceeds what would be chosen if these frictions were absent. This problem could be mitigated if bankers held more equity. Yet, banks optimally rely on both types of funding and hence the agency problem persists, because equity is relatively more costly than deposits due to deposit insurance and a friction in the equity market. Since the importance of these distortions is proportional to the real rate of return, lower levels of the real risk-free rate induce banks to increase leverage and choose riskier investment projects. This implies that the investment intermediated by banks becomes less efficient, leading to a sizeable decline in the capital stock both directly, as a lower fraction of capital projects is successful, and indirectly, as households will not save as much. Overall, a monetary policy expansion worsens the financial market distortions, which in turn attenuate the positive output effects of the interest rate cut.

This connection between interest rates and asset risk raises the question of whether the monetary authority should take this channel into account when setting the interest rate. Since the answer to this question is of quantitative nature, we embed the banking sector in a medium scale Smets and Wouters [2007]-type DSGE model, known to fit the data well along many dimensions, and estimate it on US data with Bayesian techniques. We find that the inclusion of this additional channel improves the in-sample fit of the model, yields impulse responses that are broadly in line with the results of our VAR analysis, and predicts a path of risk taking for the estimation period that matches survey evidence.

We then analyze optimal monetary policy in the estimated model using simple rules and find that, if the risk-taking channel is active, monetary policy should be less responsive to inflation and output fluctuations. In this way, the monetary authority allows more inflation volatility in exchange for stabilizing the real interest rate, which in turn reduces the welfare detrimental volatility of the banks’ risk choice. The welfare gains from taking the risk-taking channel into account are significant.

Our work relates to a small but growing theoretical literature that links monetary policy to financial sector risk in a general equilibrium framework. Most of the existing works focus on funding risk, associating risk with leverage, and build on the financial accelerator framework of Bernanke et al. [1999]. The mechanism in these models relies on the buffer role of equity and therefore leverage is found to be counter-cyclical with respect to the balance sheet size. Our model on the contrary gives rise to pro-cyclical leverage, which is in line with the empirical evidence reported in Adrian and Shin [2014] and Adrian et al. [2015]. Following a different strategy, Angeloni and Faia [2013] and Angeloni et al. [2013] augment the financial accelerator framework and construct a model where higher leverage induced by expansionary monetary policy does not just amplify other shocks but also translates into a higher fraction of inefficient bank runs.

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3For example in Gertler et al. [2012] and de Groot [2014] a monetary expansion increases banking sector leverage, which in turn amplifies the financial accelerator and strengthens the propagation of shocks to the real economy.
In this paper by contrast we model asset risk, i.e. the riskiness of the assets on the banks’ balance sheets, another aspect of financial sector risk that seems to have played an important role in the lead-up to the 2008 financial crisis. This type of risk has so far mainly been discussed in the literature on optimal regulation such as Christensen et al. [2011] and Collard et al. [2012]. In these papers however, either the depositors or the financial regulator ensure that risk is always chosen optimally, so monetary policy has no influence on risk taking. In contrast to the previous two papers we provide microfoundations for the asset risk-taking channel and focus on monetary policy while abstracting from regulation. Adding to the literature on both types of risk taking, we furthermore systematically explore how this novel channel affects optimal monetary policy in an estimated medium scale model, where the policy maker needs to trade off several conflicting frictions.

The lack of theoretical papers on the asset risk-taking channel is not mirrored by a lack of empirical evidence. Several studies find a causal link between monetary policy and risk taking. Most of the existing research relies on loan or bank level panel data and identification is based on the assumption that monetary policy is exogenous. Jimenez et al. [2014] use micro data of the Spanish Credit Register from 1984 to 2006 and find that lower interest rates induce banks to make relatively more loans to firms that qualify as risky ex ante (firms with a bad credit history at time of granting the loan) as well as ex post (firms that default on the granted loan). They argue that this effect is economically significant and particularly strong for thinly capitalized banks. These findings are confirmed for Bolivia using credit register data in Ioannidou et al. [2015], and for the US using confidential loan level data from the Terms of Business Lending Survey in Dell’Ariccia et al. [2013]. For the US, these findings are furthermore corroborated by evidence from aggregate time series data, where identification is obtained through restrictions on the dynamic responses. Angeloni et al. [2013] and Afanasyeva and Guentner [2014] find that monetary policy shocks increase asset risk, respectively proxied by the debt stock of households and non-financial corporations, and by the net percentage of banks reporting tighter lending standards in the Fed survey of business lending. These results are confirmed for small banks by the FAVAR analysis of Buch et al. [2013], who use a more direct measure of bank risk from the Terms of Business Lending Survey. We complement these results in the following section, where we show that further evidence on the risk-taking channel for the whole banking sector can be obtained using a more parsimonious setup. All these findings can be summarized by the stylized fact that interest rate cuts increase bank asset risk.

A second stylized fact that used in the theoretical model is found by Buch et al. [2013] and Ioannidou et al. [2015]. Both show that the increase in risk taking induced by low interest rates

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4Both papers feature ad-hoc extension that relate risk to the amount of lending and hence indirectly to monetary policy.

5One could reinterpret our model as applying to an economy where regulation is not able to fully control risk taking.
is not accompanied by an offsetting increase in the risk premium on loans, indicating that the additional risk might be priced inefficiently.

Motivated by this comprehensive empirical evidence and the VAR analysis in section 2, we develop in section 3 a DSGE model of asset risk-taking, where banks respond to low interest rates by inefficiently taking more risk. Section 4 presents the results from the estimation of the model and discusses the steady-state and dynamic implications of bank risk taking. Section 5 analyzes how monetary policy should be conducted if a risk-taking channel is present and section 6 concludes.

1.2 The asset risk-taking channel in the US

To motivate our subsequent theoretical analysis, we provide additional empirical evidence on the existence of the asset risk-taking channel in the US. We employ a classical small-scale VAR that includes inflation, output, a measure of bank risk-taking and the effective federal funds rate, taken as the monetary policy instrument. Output is measured by real GDP growth, while inflation is defined as the log change in the GDP deflator.

Measuring risk taking is less straightforward. There are many notions of asset risk. One can distinguish between ex-ante, ex-post and realized asset risk. The former is the risk perceived by the bank when making a loan or buying an asset. Banks can influence this class of risk directly, when making their investment decisions (the ex-ante risk choice). On the other hand, the ex-post risk of a bank’s balance sheet is also affected by unforeseen changes in asset riskiness, that take place after origination and are largely outside the banks’ influence. Lastly, the payoff ultimately paid by an asset is a materialization of the former two types of uncertainty (realized asset risk). In this paper we focus on active risk taking, that is the level of ex-ante risk that intermediaries choose, which is however difficult to observe directly\(^6\). Therefore, we use a survey-based proxy for bank risk-taking as in [Dell’Ariccia et al. 2014] given by the weighted average of the internal risk rating assigned by banks to newly issued loans, provided by the US Terms of Business Lending Survey. See appendix 1A for a plot of this risk index\(^7\).

\(^6\)&Inferring it from realized risk (e.g. loan losses) is hardly possible with aggregate data. Inference from the spread between bank funding costs and loan rates neglects the fact that this spread not only reflects default risk but also incorporates a liquidity premium and the markup, which are likely to be affected by the same variables that influence the risk choice.

\(^7\)&The average loan risk is a perfect measure for bank risk taking if we assume that the volume of loans is constant. Else, banks could also minimize their risk exposure by reducing the quantity of loans as their average quality goes down. While the correlation between risk and loan volume growth is slightly negative, it is not significant at a 10% significance level. For a more in-depth discussion of the data we refer to [Buch et al. 2013].
Figure 1.1: Monetary policy shock on bank risk-taking: Impulse responses over a 3-year horizon, identified through the sign restriction scheme in Table 1. Error bands correspond to 90% confidence intervals reflecting rotation uncertainty. Loan safety is defined as the inverse of the average loan risk rating, standardized to take values between 0 and 100. The remaining variables are annualized. See text for further details.

We estimate a VAR over the period 1997:Q2, the start of the survey-based proxy for risk taking, to 2007:Q3. The lag length is chosen to be 1, as indicated by the BIC information criterion. We identify an unexpected monetary policy shock by using a conventional set of sign restrictions that are robust across a variety of general-equilibrium models. In particular, we assume that an expansionary monetary policy shock decreases the nominal risk-free interest rate, and increases inflation and output, both at the time of the shock and in the quarter immediately after. Risk is left unrestricted. Note that the response of inflation ensures that this shock is identified separately from a productivity shock, which increases output but decreases both the interest rate and inflation.

The response of bank asset risk to an expansionary monetary policy shock is shown in figure 1.1. An unexpected decrease in the monetary policy interest rate is followed by a moderate macroeconomic expansion: output growth increases for less than a year, while inflation displays a longer reaction of about two years. The results are compatible with the existence of a risk-taking channel in the US: a fall in the nominal interest rate leads in fact to a decrease in the ex-ante proxy for the safety of banks’ assets, i.e. banks issue riskier loans. Interestingly, the implied responses of the nominal interest rate and the risk measure are approximately proportional. These results are robust to using a recursive identification scheme, as shown in appendix 1B.

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8We have decided to cut the zero-lower bound period, but our results still hold when the latest available data are used.
9We tested two orderings (output, inflation, interest rate, risk) and (output, inflation, risk, interest rate).
1.3 A Dynamic New Keynesian model with a bank risk-taking channel

In this section we build a general-equilibrium model where monetary policy can influence the risk-taking behavior of banks, thus providing an explanation for the risk-taking channel observed in the data. As a starting point we use a standard New Keynesian model with imperfect competition and price stickiness in the goods market, which implies a role for monetary policy. We augment this basic framework with an intermediation sector based on Dell’Ariccia et al. [2014]: competitive banks obtain funds from depositors and equity holders, which they invest into capital projects carried out by capital producers. Every bank chooses its investment from a continuum of available capital production technologies, each defined by a given risk-return characteristic. The risk choice of the representative bank is affected by the level of the real interest rate, and can be shown to be suboptimal. This model reproduces two features found in the data: risk taking depends on the contemporaneous interest rate and is priced inefficiently.

While the aforementioned blocks are the necessary ingredients, in order to obtain a quantitatively more realistic model we add further elements, which are typically used in the DSGE literature. In particular we follow Smets and Wouters [2007] and allow for internal habits, investment adjustment costs and imperfect competition and wage stickiness in the labor market. Our model therefore features seven agents that are typical for DSGE models (households, unions, labor packers, capital producers, intermediate goods producers, final goods producers, and a central bank) and two agents that we introduce to model risk taking (banks, funds). Eight structural shocks hit the economy: these affect productivity, investment, time preferences, the equity premium, wage and price markups, as well as monetary and fiscal policy.

1.3.1 Households

The representative household chooses consumption $c_t$, working hours $L_t$ and savings in order to maximize its discounted lifetime utility. Saving is possible through three instruments: government bonds $s_t$, deposit funds $d_t$, and bank equity funds $e_t$. The nominal return on government bonds is safe and equal to the nominal interest rate $R_t$. The two funds enable the representative household to invest into the banking sector, and pay an uncertain nominal return of $R_{d,t+1}$ and $R_{e,t+1}$.\footnote{Note that in our notation the time index refers to the period when a variable is determined.}

Households maximize their lifetime utility function:

$$\max_{c_t, e_t, s_t, d_t, L_t} \sum_{t=0}^{\infty} \beta^t E \left[ \left( c_t - \delta c_{t-1} \right)^{1-\sigma_C} \exp \left( \varphi L_t^{1+\sigma_L} \sigma_C - 1 \right) \right], \quad (1.1)$$
subject to the per-period budget constraint in real terms:

\[ c_t + d_t + e_t + s_t + T_t = L_t w_t + d_{t-1} \frac{R_{d,t}}{\pi_t} + e_{t-1} \frac{R_{e,t}}{\pi_t} + s_{t-1} \frac{R_{s,t}}{\pi_t} + \Pi_t , \]  

where \( \pi_t \) is the inflation rate, while \( T_t \) and \( \Pi_t \) are taxes and profits form firm ownership, expressed in real terms. We allow for habits in consumption \( (\iota) \) and a time preference shock \( \varepsilon_t^B \). This shock is assumed to be persistent with log-normal innovations, like all following shocks unless otherwise specified. The household’s optimality conditions are given by the usual Euler equation and two no-arbitrage conditions:

\[
A_t = \beta E_t \left[ A_{t+1} \frac{R_t}{\pi_{t+1}} \right], \tag{1.3}
\]

\[
E_t \left[ A_{t+1} \frac{R_{d,t+1}}{\pi_{t+1}} \right] = E_t \left[ A_{t+1} \frac{R_t}{\pi_{t+1}} \right], \tag{1.4}
\]

\[
E_t \left[ A_{t+1} \frac{R_{e,t+1}}{\pi_{t+1}} \right] = E_t \left[ A_{t+1} \frac{R_t}{\pi_{t+1}} \right], \tag{1.5}
\]

where \( A_t = \varepsilon_t^B (c_t - \iota c_{t-1})^{-\sigma_c} - \beta E_t \left[ \varepsilon_t^B (c_{t+1} - \iota c_t)^{-\sigma_c} \right] \) is the marginal utility of consumption.

### 1.3.2 Labor and goods sectors

The labor and goods sectors feature monopolistic competition and nominal rigidities, which allow for a role for monetary policy. Since the modeling of these sectors follows the canonical New Keynesian model, we discuss them only briefly and refer to Smets and Wouters [2007] and Adjemian et al. [2008] for further details. The corresponding equilibrium conditions are listed in appendix 1C.

Final good producers assemble different varieties of intermediate goods through a Kimball [1995] aggregator with elasticity of substitution \( \epsilon_p \) and Kimball parameter \( k_p \), taking as given both the final good price and the prices of intermediate goods. Their optimization problem yields demand functions for each intermediate good variety as a function of its relative price.

A continuum of firms produces differentiated intermediate goods using capital \( K_{t-1} \) and “packed” labor \( l_t^d \) as inputs. The production function is Cobb-Douglas and is affected by a total factor productivity shock \( \varepsilon_t^A \). Firms use their monopolistic power to set prices, taking as given their demand schedule. As in Calvo [1983], they can reset their prices in each period with probability \( \lambda_p \), otherwise they index their prices to past inflation with degree \( \gamma_p \) and to steady state inflation with
degree \((1 - \gamma^p)\). Furthermore they are subject to a time-varying revenue tax \(\varepsilon^p_t\) that is equivalent to a markup shock, up to a first-order approximation.

The labor market resembles the product market: Packed labor is produced by labor packers, who aggregate differentiated labor services using a Kimball [1995] aggregator with elasticity of substitution \(\epsilon_w\) and Kimball parameter \(k_w\).

Differentiated labor services are produced by a continuum of unions from the households labor supply. They use their monopolistic power to set wages. Wages are reset with probability \(\lambda^w\), otherwise they are indexed to past inflation (with degree \(\gamma^w\)) and steady state inflation. Like intermediate firms, unions are subject to a stochastic wage tax \(\varepsilon^w_t\).\(^{11}\)

### 1.3.3 Equity and deposit funds

As we explain in detail below, there is a continuum of banks which intermediate the households’ savings using deposits and equity. Each bank is subject to a binary idiosyncratic shock which makes a bank fail with probability \(1 - q_{t-1}\), in which case equity is wiped out completely and depositors receive partial compensation from the deposit insurance. We assume that households invest into bank equity and deposits through two funds. The function of the equity (deposit) fund is to eliminate the idiosyncratic bank default risk by buying a perfectly diversified portfolio of 1 period equity (deposits) of all banks.

The deposit fund works without frictions, and represents the depositors’ interests perfectly. The deposit fund raises money from the households and invests it into \(d_t\) units of deposits.\(^{12}\) In the next period, the fund receives the nominal deposit rate \(r_{d,t}\) from each bank that does not fail. Deposits of failing banks are partially covered by deposit insurance. Most deposit insurance schemes around the world, including the US, guarantee all deposits up to a certain maximum amount per depositor.\(^{13}\) We represent this capped insurance model by assuming that the deposit insurance guarantees deposits up to a fraction \(\psi\) of total bank liabilities \(l_t\), which are the sum of deposits \(d_t\) and equity \(e_t\). We assume that the deposit insurance cap is inflation adjusted, to avoid complicating the monetary policy trade-off by allowing an interdependence between monetary policy and deposit insurance. As we will show later, the deposit insurance cap is always binding in equilibrium, i.e. the bank’s liabilities exceed the cap of the insurance \(r_{d,t}d_t > \psi(d_t + e_t)\pi_{t+1}\).

Defining the equity ratio \(k_t = \frac{e_t}{d_t + e_t}\), the deposit fund therefore receives a real return of \(\psi / (1 - k_t)\) per unit of deposits from each defaulting bank at \(t\). The deposit fund hence pays a nominal return

\(^{11}\) Both \(\varepsilon^p_t\) and \(\varepsilon^w_t\) follow the standard shock process augmented by an moving average component, as in Smets and Wouters [2007].

\(^{12}\) We use deposits to refer to both units of deposit funds and units of bank deposits since they are equal. We do the same for equity.

\(^{13}\) For a comprehensive documentation see, for instance, Demirguc-Kunt et al. [2005].
of:

\[ R_{d,t+1} \equiv q_t r_{d,t} + (1 - q_t) \frac{\psi}{1 - k_t} \pi_{t+1} . \]  

(1.6)

Unlike the deposit fund, which is managed frictionlessly, the equity fund is subject to a simple agency problem. In particular, we assume that the fund manager faces two options. He can behave diligently and use the funds \( e_t \), raised at \( t \), to invest into \( e_t \) units of bank equity. A fraction \( q_t \) of banks will pay back a return of \( r_{e,t+1} \) next period while the defaulting banks pay back nothing. Alternatively the fund manager can abscond with the funds, in which case he consumes a fraction \( \xi_t \) of the funds in the subsequent period and the rest is lost. Since he is a small member in the big family of the representative household his utility from doing so is \( \Lambda_{t+1} \xi_t e_t \). To prevent the fund manager from absconding funds, the equity providers promise to pay him a premium \( p_t \) at time \( t + 1 \) conditional on not absconding. This premium is rebated to the household in a lump-sum fashion and the associated utility for the fund manager is \( \Lambda_{t+1} p_t \). Once absconding is ruled out in equilibrium, the equity fund manager perfectly represents the interests of the equity providers. The nominal return on the bank equity portfolio is \( q_t r_{e,t+1} \) per unit of equity, hence one share of the equity fund pays:

\[ R_{e,t+1} \equiv q_t r_{e,t+1} - \xi_t \pi_{t+1} . \]  

(1.7)

We allow the equity premium \( \xi_t \) to vary over time.\(^1\) Note that, since bank equity is the residual income claimant, the return of the equity fund is affected by all types of aggregate risk that influences the surviving banks’ returns.

The two financial distortions introduced so far have important implications. The agency problem implies an equity premium, i.e. a premium of the risk-adjusted return on equity over the risk-free rate. Deposit insurance on the other hand acts as a subsidy on deposits, which implies a discount on the risk-adjusted return on deposits. As explained below, the difference in the costs of these two funding types induces a meaningful trade-off between bank equity and bank deposits under limited liability.

1.3.4 Capital producers

We assume that the capital production process is risky in a way that nests the standard capital production process in the New Keynesian model. In particular, capital is produced by a continuum of capital producers indexed by \( m \). At period \( t \) they invest \( i^m_t \) units of final good into a capital

\(^{14}\)This shock, driving a wedge between deposit and safe rates on one hand and equity rates on the other is similar to the risk premium shock often found in medium scale DSGE models (e.g. Smets and Wouters [2007]).
project of size \( q^m_t \). This project is successful with probability \( q^m_t \), in which case the project yields 
\[
(\omega_1 - \frac{\omega_2}{2} q^m_t) o^m_t
\]
units of capital at \( t + 1 \). Else, the project fails and only the liquidation value of 
\( \theta o^m_t \) units of capital can be recovered (where \( \theta \ll a - \frac{b}{2} q^m_t \)). Each capital producer has access to a 
continuum of technologies with different risk-return characteristics indexed by \( q^m \in [0,1] \). Given a 
chosen technology \( q^m_t \), the output of producer \( m \) therefore is:

\[
K^m_t = \begin{cases} 
(\omega_1 - \frac{\omega_2}{2} q^m_t) o^m_t & \text{with probability } q^m_t \\
\theta o^m_t & \text{else}
\end{cases}
\]

This implies that the safer the technology (higher \( q^m_t \)), the lower is output in case of success.

The bank orders the capital projects and requires the capital producer to use a certain 
technology, but this choice cannot be observed by any third party. Given the technology choice \( q_t \) 
and assuming that the projects of individual producers are uncorrelated, we can exploit the law of 
large numbers to derive aggregate capital:

\[
K_t = o_t \left( q_t \left( \omega_1 - \frac{\omega_2}{2} q_t \right) + (1 - q_t) \theta \right) .
\] (1.8)

Furthermore we assume that capital, which depreciates at rate \( \delta \), becomes a project (of undefined 
\( q_t \)) at the end of every period. That is, existing capital may be destroyed due to unsuccessful reuse, 
and it can be reused under a different technology than it was originally produced.\(^\text{15}\)

The total supply of capital projects by the capital producers is the sum of the existing capital 
projects \( o^{old}_t = (1 - \delta) K_{t-1} \), which they purchase from the owners (the banks) at price \( Q_t \), and 
the newly created projects \( o^{new}_t \), which are created by investing \( i_t \) units of the final good. We 
allow for investment adjustment costs and investment efficiency shocks, i.e. we assume that \( i_t \) 
units of investment yield 
\[
\varepsilon^I_t (1 - S(i_t/i_{t-1})) \text{ units of project, where } S = \kappa \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 .
\]

Hence \( o_t = o^{new}_t + o^{old}_t \) and \( o^{new}_t = \varepsilon^I_t \left( 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right) i_t \). Capital producers maximize their expected 
discounted profits taking as given the price \( Q_t \) and the households stochastic discount factor\(^\text{16}\):

\[
max_{i_t, o^{old}_t} E_t \sum_0^\inf \beta^t A_t \left[ Q_t \varepsilon^I_t \left( 1 - S \left( \frac{i_t}{i_{t-1}} \right) \right) i_t + Q_o o^{old}_t - i_t - Q_i o^{old}_t \right]
\]

\(^\text{15}\)This assumption ensures that we do not have to keep track of the distribution of different project types. Think 
of a project as a machine that delivers capital services and that can be run at different speeds (levels of risk). 
In case it is run at higher speed, the probability of an accident that destroys the machine is higher. After each 
period the existing machines are overhauled by the capital producers and at this point the speed setting can be 
changed.

\(^\text{16}\)Their out of steady state profits are rebated lump sum to the household.
While the old capital projects are always reused, the marginal capital project is always a new one.\footnote{We abstract from a non-negativity constraint on new projects.} Hence, the price of projects $Q_t$ is determined by new projects according to the well-known Tobin’s $q$ equation.

### 1.3.5 The Bank

The bank is the central agent of our model and is modeled similarly as in Dell’Ariccia et al. [2014]. Banks raise resources through deposits and equity and invest them into a risky project. Since depositors cannot observe the banks’ risk choice and banks are protected from the downside risk of their investment by limited liability, an agency problem arises between them when the banks choose the risk level. The less equity a bank has, the higher the incentives for risk taking. Yet, since deposit insurance and the equity premium drive a wedge between the costs of deposits and equity, the banks’ optimal capital structure comprises both equity and deposits, balancing the agency problem associated with deposits with the higher costs of equity. We will show that the equilibrium risk chosen by the banks is excessive, and that the interest rate influences the degree of its excessive.

We assume that there is a continuum of banks who behave competitively so that there is a representative bank (we therefore omit the bank’s index in what follows). The bank is owned by the equity providers, and hence maximizes the expected discounted value of profits\footnote{Profits in excess of the opportunity costs of equity.} using the household’s stochastic discount factor. Every period the bank raises deposits $d_t$ and equity $e_t$ from the respective funds (optimally choosing its liability structure). These resources are then invested into $q_t$ capital projects, purchased at price $Q_t$. When investing into capital projects, the bank chooses the risk characteristic $q_t$ of the technology applied by the capital producer. This risk choice is not observable for depositors. Each bank can only invest into one project and hence faces investment risk\footnote{The assumption that the bank can only invest into one project and can not diversify the project risk might sound stark. Yet three clarifications are in place: First, our setup is isomorphic to a model where the bank invests into an optimally diversified portfolio of investments, but is too small to perfectly diversify its portfolio. The binary payoff is then to be interpreted as the portfolio’s expected payoff conditional on default or repayment respectively. Second, if the bank was able to perfectly diversify risk, then limited liability would become meaningless and we would have a model without financial frictions. Third, we don’t allow the bank to buy the safe asset. Yet this assumption is innocuous: since the banks demand a higher return on investment than the households due to the equity premium, banks wouldn’t purchase the safe asset even if they could.} with probability $q_t$ the bank receives a high pay-off from the capital project; with probability $1 - q_t$ the investment fails and yields only the liquidation value. Assuming a sufficiently low liquidation value $\theta$, a failed project implies the default of the bank. In this case, given limited liability, equity providers get nothing and depositors get the deposit insurance benefit. In case of success the bank can repay its investors: depositors receive their promised return $r_{d,t}$ and equity providers get the state contingent return $r_{e,t+1}$. 

\begin{thebibliography}{9}
\bibitem{} Dell’Ariccia et al. [2014].
\end{thebibliography}
It is useful to think of the bank’s problem as a recursive two-stage problem. At the second stage, the bank chooses the optimal risk level \( q_t \) given a certain capital structure and a certain cost of deposits. At the first stage, the bank chooses the optimal capital structure, anticipating the implied solution for the second-stage problem. Note that not only the bank but also the bank’s financiers anticipate the second-stage risk choice and price deposits and equity accordingly, which is understood by the bank. Below we derive the solution for this recursive problem.

Before we do so, we establish the bank’s objective function. Per dollar of nominal funds raised (through deposits and equity) in period \( t \) the bank purchases \( \frac{Q_t}{P_t} \) units of the capital project from the capital producer, choosing a certain riskiness \( q_t \). If the project is successful it turns into \( (\omega_1 - \frac{\omega_2}{2}q_t)/(Q_tP_t) \) capital goods. In the next period \( t+1 \), the bank rents the capital to the firm, who pays the real rental rate \( r_{k,t+1} \) per unit of capital. Furthermore the bank receives the depreciated capital, which becomes a capital project again, with real value of \((1-\delta)Q_{t+1}\) per unit of capital. The bank’s total nominal income, per dollar raised, conditional on success is therefore:

\[
\left(\omega_1 - \frac{\omega_2}{2}q_t \right) \frac{r_{k,t+1} + (1-\delta)Q_{t+1} P_{t+1}}{Q_t P_t}
\]

At the same time, the bank has to repay the deposit and equity providers. Using the equity ratio \( k_t \), the total nominal repayment per dollar of funds due in \( t+1 \) in case of success is \( r_{e,t+1}k_t + r_{d,t}(1-k_t) \).

The bank maximizes the expected discounted value of excess profits, i.e. revenues minus funding costs, using the stochastic discount factor of the equity holders, i.e. the household. Given the success probability of \( q_t \) and the fact that the equity providers receive nothing in case of default, the bank’s objective function is:

\[
\max_{q_t, k_t} \beta E \left[ \frac{\Lambda_{t+1}}{\pi_{t+1}} q_t \left( \left(\omega_1 - \frac{\omega_2}{2}q_t \right) \frac{r_{k,t+1} + (1-\delta)Q_{t+1} P_{t+1}}{Q_t \pi_{t+1}} - r_{d,t}(1-k_t) - r_{e,t+1}k_t \right) \right].
\]

(1.9)

Note that we did not multiply the per-unit profits by the quantity of investment. In doing so we anticipate the equilibrium condition that the bank, whose objective function is linear in the quantity of investment, needs to be indifferent about the quantity of investment. The quantity will be pinned down together with the return on capital by the bank’s balance sheet equation \( e_t + d_t = Q_t\omega_t \), the market clearing and zero profit conditions.

The bank’s problem can be solved analytically, yet the expressions get fairly complex. Therefore we derive here the solution for \( \psi = \theta = 0 \), that is without deposit insurance and with a liquidation value of 0. This simplifies the expressions but the intuition remains the same. Allowing \( \psi \) and \( \theta \)
to be nonzero on the other hand is necessary to bring the model closer to the data. The solution for the general case is discussed in section 3.5.5.

To make notation more tractable we rewrite the bank’s objective function (1.9) in real variables expressed in marginal utility units:

$$\omega_1 q_t \tilde{r}_{l,t} - \frac{\omega_2}{2} q_t^2 \tilde{r}_{l,t} - q_t \tilde{r}_{d,t}(1 - k_t) - q_t \tilde{r}_{e,t} k_t ,$$  \hspace{1cm} (1.10)

For later use we rewrite the household’s no-arbitrage conditions (1.3) and (1.5) combined with the definition of the funds’ returns (1.6) and (1.7) as $\tilde{r}_{d,t} = \frac{\tilde{R}}{q_t}$ and $\tilde{r}_{e,t} = \frac{\tilde{R} + \tilde{\xi}}{q_t}$. Let us now solve the bank’s problem recursively.

**Second-stage problem:**

At the second stage, the bank has already raised $e_t + d_t$ funds and now needs to choose the risk characteristic of the investment $q_t$, such that equity holders’ utility is maximized. As already mentioned, we assume that the bank cannot write contracts conditional on $q_t$ with the depositors at stage one, since $q_t$ is not observable to them. Therefore at the second stage the bank takes the deposit rate as given. Furthermore, since the capital structure is already determined, maximizing the excess profit coincides with maximizing the profit of equity holders. The bank’s objective function is therefore:

$$\max_{q_t} \quad \omega_1 q_t \tilde{r}_{l,t} - \frac{\omega_2}{2} q_t^2 \tilde{r}_{l,t} - q_t \tilde{r}_{d,t}(1 - k_t) .$$  \hspace{1cm} (1.11)

Deriving problem (1.11) with respect to $q_t$ yields the following first-order condition:

$$q_t = \frac{\omega_1 \tilde{r}_{l,t} - \tilde{r}_{d,t} (1 - k_t)}{w_2 \tilde{r}_{l,t}} .$$  \hspace{1cm} (1.12)

---

20 That is we use the following definitions: $\tilde{r}_{l,t} = E_t \left[ \Lambda_{t+1} \left( \frac{\tilde{r}_{k,t+1} + (1-\delta)Q_{t+1}}{\tilde{Q}_t} \right) \right]$, $\tilde{r}_{d,t} = E_t \left[ \Lambda_{t+1} \frac{\tilde{r}_{d,t}}{\tilde{Q}_t} \right]$, $\tilde{r}_{e,t} = E_t \left[ \Lambda_{t+1} \tilde{\xi} \right]$.  

21 We focus on interior solutions and choose the larger of the two roots, which is the closest to the optimum, as we will see below.
**First-stage problem:**

At the point of writing the deposit contract at stage one, depositors anticipate the bank’s choices at stage two and therefore the depositors’ no arbitrage condition $\tilde{r}_{d,t} = \frac{\tilde{R}_t}{q_t}$ must hold in equilibrium. Using this equation together with the previous first-order condition (1.12) we can derive the optimal $q_t$ as a function of $k_t$ and $\tilde{r}_{l,t}$:

$$\hat{q}_t = q_t(k_t) = \frac{1}{2\omega_2 \tilde{r}_{l,t}} \left( \omega_1 \tilde{r}_{l,t} + \sqrt{(\omega_1 \tilde{r}_{l,t})^2 - 4\omega_2 \tilde{r}_{l,t} \tilde{R}_t (1 - k_t)} \right).$$  (1.13)

We can now solve the first-stage problem of the banker. The bank chooses the capital structure $k_t$ to maximize her excess profits, anticipating the $q_t(k_t)$ that will be chosen at the second stage:

$$\max_{k_t} \hat{q}_t \omega_1 \tilde{r}_{l,t} - \omega_2 \tilde{r}_{l,t} \hat{q}_t^2 - q_t \tilde{r}_{d,t} (1 - k_t) - q_t \tilde{r}_{e,t} k_t,$$  (1.14)

subject to the no-arbitrage condition for depositors ($\tilde{r}_{d,t} = \frac{\tilde{R}_t}{q_t}$) and for equity providers ($\tilde{r}_{e,t} = \frac{\tilde{R}_t + \tilde{\xi}_t}{q_t}$). Plugging these in and deriving, we obtain the first-order condition for $k_t$:

$$\omega_1 \tilde{r}_{l,t} \frac{\partial \hat{q}_t}{\partial k_t} - \tilde{\xi}_t - \frac{\omega_2}{2} \tilde{r}_{l,t} \frac{\partial \hat{q}_t^2}{\partial k_t} = 0.$$  (1.15)

which (assuming an interior solution) can be solved for $k_t$ as:

$$\hat{k}_t \equiv k_t(\tilde{r}_{l,t}) = 1 - \frac{\tilde{\xi}_t (\tilde{R}_t + \tilde{\xi}_t)(\omega_1 \tilde{r}_{l,t})^2}{\omega_2 \tilde{R}_t \tilde{r}_{l,t} \left( \tilde{R}_t + 2\tilde{\xi}_t^2 \right)}.$$  (1.16)

**Closing the bank model: the zero-profit condition**

Since there is a continuum of identical banks, each bank behaves competitively and takes the return on investment $\tilde{r}_{l,t}$ as given. Perfect competition and free entry imply that banks will enter until there are no expected excess profits to be made. In the presence of uncertainty it is natural to focus on the case that banks make no excess profit in any future state of the world:

$$\left( \omega_1 - \frac{\omega_2}{2} q_{t-1} \right) \left( r_{k,t} + \frac{1 - \delta}{\pi_t} Q_t \right) - \frac{r_{d,t-1}}{\pi_t} (1 - \hat{k}_{t-1}) - \frac{r_{e,t}}{\pi_t} \hat{k}_{t-1} = 0.$$  (1.17)

Using the equity and deposit supply schedules and taking expectation over this equation we get:

---

22Note that the agency problem arises from the fact that the bank does not consider this as a constraint of its maximization problem.
\[
\hat{q}_t \omega_1 \hat{r}_{t,t} - \frac{\omega_2}{2} \hat{r}_{t,t} \hat{q}_t^2 - \hat{k}_t \hat{\xi}_t - \hat{R}_t .
\] (1.18)

Combining (1.18) with the optimality conditions (1.13) and (1.16), we can derive analytical expressions for the equity ratio \( k_t \), riskiness choice \( q_t \) (the last term in each row is an approximation under certainty equivalence and \( R^*_t \equiv R_t / E [\pi_{t+1}] \)):

\[
k_t = \frac{\tilde{R}_t}{\tilde{R}_t + 2 \xi_t} \approx \frac{R^*_t}{R^*_t + 2 \xi_t} \quad \text{(1.19)}
\]
\[
q_t = \frac{\omega_1 (\tilde{\xi}_t + \tilde{R}_t)}{\omega_2 (2 \tilde{\xi}_t + \tilde{R}_t)} \approx \frac{\omega_1 (\xi_t + R^*_t)}{\omega_2 (2 \xi_t + R^*_t)} \quad \text{(1.20)}
\]

Properties of the banking sector equilibrium

These results for the banking sector risk choice have five interesting implications that we first summarize in a proposition, before intuitively discussing them in turn.

**Proposition 1:** Be \([\hat{r}_{t,t}, q_t, k_t] \) an equilibrium in the banking sector with interior bank choices under perfect competition. Denote the expected return on investment in capital units by \( f(q_t) \equiv (\omega_1 - \frac{\omega_2}{2} q_t) q_t \). Then:

1. Risk decreases in the real interest rate: \( \frac{\partial q_t}{\partial \tilde{R}_t} > 0 \)
2. The equity ratio increases in the real interest rate: \( \frac{\partial k_t}{\partial \tilde{R}_t} > 0 \)
3. Risk taking is excessive: \( q_t < \text{argmax} \ f(q_t) \)
4. The expected return of an investment increases in the real interest rate: \( \frac{\partial f(q_t)}{\partial \tilde{R}_t} > 0 \)
5. The expected return of an investment is a concave function of the real interest rate \( \frac{\partial^2 f(q_t)}{\partial \tilde{R}_t^2} < 0 \)

The proof can be found in appendix 1D.

The first two results can be easily seen from equations (1.19) and (1.20). As the real risk-free rate \( R^*_t \) decreases, the equity ratio \( k_t \) falls as banks substitute equity with deposits and the riskiness of the bank increases (\( q_t \) falls). The intuition behind this result is as follows: On one hand, a lower risk-free rate decreases the rate of return on capital projects, reducing the benefits of safer investments, conditional on repayment. This induces the bank to adopt a riskier investment.

\[23\text{At least under certainty equivalence or up to a first order approximation, when the } \Lambda_{t+1} \text{ terms contained in the tilde variables cancel out.}\]
technology. On the other hand, the lower risk-free rate reduces the cost of funding, leaving more resources available to the bank’s owners in case of repayment: this force contrasts the first one, making safer investments more attractive. There is a third force: a lower risk-free interest rate means that the equity premium becomes relatively more important. As a result the bank shifts from equity to deposits, internalizing less the consequences of the risk decision and choosing a higher level of risk. The first and third effects dominate, and overall a decline in the real interest rate induces banks to choose more risk. Notice that these two results depend on the assumption that the (discounted) equity premium is independent of the (discounted) real interest rate. If we allowed the equity premium to be a function \( \xi_t(\tilde{R}_t) \) of the real interest rate, the result would continue to hold under the condition that \( \xi_t(\tilde{R}_t) > \xi_t'(\tilde{R}_t) \tilde{R}_t \), which rules out proportionality. This mechanism provides a rationalization of the empirical finding in section 2: that a decline in the nominal interest rate causes an increase in bank risk taking behavior.

The third result implies that the bank’s investment could have a higher expected return (in units of capital) if the bank chose a higher level of safety. In other words, risk taking is excessive, i.e. suboptimally high. This is due to the agency problem, which arises from limited liability and the lack of commitment/contractability of the banker regarding his risk choice. The importance of this friction can be assessed by comparing the solution of the imperfect markets bank model to the solution of the model without any frictions. The frictionless risk choice can be derived under any of the following alternative scenarios: Either both equity premium and deposit insurance are zero (which eliminates the cost disadvantage of equity and leads to 100% equity finance), or contracts are complete and deposit insurance is zero (which eliminates the agency problem and leads to 100% deposit finance), or liability is not limited and deposit insurance is zero (as before), or household invests directly into a diversified portfolio of capital projects (which eliminates the financial sector all together). Since in a frictionless model \( q_t \) is chosen to maximize the consumption value of the expected return:

\[
max_{q_t} R_{t,t}(\omega_1 - \frac{\omega_2}{2} q_t) q_t
\]

the optimal level of \( q_t \) trivially is \( q^*_t = \frac{\omega_1}{\omega_2} \). Comparing the frictionless risk choice \( q^*_t \) and the choice given the friction \( q^f_t \)

\[
q^f_t = q^*_t \frac{\xi_t + \tilde{R}_t}{2 \xi_t + \tilde{R}_t} \approx q^*_t \frac{\xi_t + R^r_t}{2 \xi_t + R^r_t}
\]

\[24\text{In a monetary model, a cut in the real interest rate, the standard monetary policy tool, is followed by a decline in the real interest rate due to price stickiness.}\]
we observe that the agency friction drives a wedge between the frictionless and the actually chosen risk level. This wedge has two important features. First, it is smaller than one implying that under the agency problem the probability of repayment is too low, and hence banks choose excessive risk. Second, note that the wedge depends on $R_t^r$ and that the derivative of the first order approximation of the wedge w.r.t. $R_t^r$ is positive. This implies that the wedge increases, i.e. risk taking gets more excessive, as the real interest rate falls. As we move further away from the optimal level of risk the expected return on investments necessarily falls, which is the fourth result above.

Note that this feature of the model is consistent with the empirical finding of Ioannidou et al. [2015] and Buch et al. [2013] that the additional loan risk taking spurred by low interest rates is not fully compensated by a sufficient increase in the return on loans: As $q_t$ decreases, $(\omega_1 - \frac{\omega_2}{2} q_t)$ increases but not sufficient to avoid a drop in $(\omega_1 - \frac{\omega_2}{2} q_t) q_t$.

But not only the bank risk choice is suboptimal. Also the capital structure is chosen suboptimally, given the equity premium. If banks could commit to choose the optimal level of risk, they would not need any skin in the game. Hence they would avoid costly equity and would finance themselves fully by deposits: $k_t^o = 0$. Instead they choose $k_t^f = \frac{R_t^e}{R_t^e + 2\xi_t}$. The equity ratio resembles the two features of the risk taking. First, there is excessive use of equity funding. Second, the equity ratio is increasing in $R_t^r$ up to a first order approximation.

Both the risk and the capital structure choice have welfare implications. A marginal increase in $q_t$ means a more efficient risk choice, i.e. a higher expected return, and hence should be welfare improving, ceteris paribus. At the same time a marginal increase in $k_t$ implies, due to the equity premium, a higher markup in the intermediation process, which distorts the consumption savings choice and hence lowers welfare, ceteris paribus. Since both $q_t$ and $k_t$ are increasing functions of the real interest rate, this begs the question of whether an increase in the real rate alleviates or intensifies the misallocation due to the banking friction. The answer to this question depends on the full set of general equilibrium conditions. Given the estimated model, we will later numerically verify that the positive first effect dominates, i.e. an increase in $R_t^r$ has welfare improving consequences on the banking market.

The existence of these opposing welfare effects motivates our optimal policy experiments in section 1.5.

Finally, the last statement of the proposition implies that a mean preserving increase in the variance of the real interest rate decreases the mean of the expected return of the banks investment. This has implications for optimal monetary policy. As we discuss in detail later, the monetary

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25 This is true under certainty equivalence, i.e. up to first order approximation.

26 These two opposing forces are well known from the literature on bank capital regulation, where a raise in capital requirements hampers efficient intermediation but leads to a more stable banking sector.

27 The dominance of the risk-taking effect is intuitive for two reasons: First, while risk taking entails a real cost, the equity premium just entails a wedge but no direct real costs. Second, as the real interest rate increases the equity premium becomes less important, so a more efficient allocation is intuitive.
authority cannot affect the nonstochastic steady state of the real rate, but it can influence its volatility. The policy maker therefore has an incentive to keep the real interest rate stable, at least as long as the opposing effect of the equity premium is negligible.

**Full model with deposit insurance and liquidation value**

The simplified version of the bank’s problem presented so far is useful to explain the basic mechanism. Yet deposit insurance and a non zero liquidation value are important to improve the quantitative fit of our model to the data.

The assumptions made about deposit insurance and the liquidation value imply that depositors get the maximum of the amount covered by deposit insurance and the value of the capital recovered from a failed project. That means that their return in case of default is:

$$
\min \left( \frac{r_{d,t}}{\pi_{t+1}}, \max \left( \frac{r_{k,t+1} + (1 - \delta)Q_{t+1}}{Q_t(1 - k_t)} \frac{\theta}{1 - k_t}, \frac{\psi}{1 - k_t} \right) \right).
$$

To make deposit insurance meaningful we assume that the liquidation value $\theta$ is small enough such that $\frac{r_{k,t+1} + (1 - \delta)Q_{t+1}}{Q_t(1 - k_t)} \frac{\theta}{1 - k_t} < \frac{\psi}{1 - k_t}$, which eliminates the inner maximum.\(^{28}\) As the following lemma, proven in appendix 1D, states, the outer maximum is unambiguous in equilibrium.\(^{29}\)

**Lemma:** There can be no equilibrium such that the insurance cap is not binding, i.e. $\frac{r_{d,t}}{\pi_{t+1}} > \frac{\psi}{1 - k_t}$.

Deposits therefore pay $\frac{\psi}{1 - k_t}$ in case of default. Combining the nominal return on the the deposit funds \(^{[1.6]}\) with the households no-arbitrage condition, and defining $\tilde{\psi}_t = E [\Lambda_{t+1}] \psi$, we can write the deposit supply schedule as:

$$
q_t \tilde{r}_{d,t} + (1 - q_t) \frac{\tilde{\psi}_t}{1 - k_t} = \tilde{R}_t.
$$

\(^{28}\)In principle the fact that the return on capital is determined only one period later implies that we could have cases where this inequality is satisfied for some states of the world and violated for others. Since we will later approximate our model locally around the steady state, which allows us to consider only small shocks, we abstract from this complication. Note that this simplification is quantitatively unimportant if shocks are small and the difference between the LHS and the RHS is big in steady state.

\(^{29}\)For this result we again abstract from the effect of uncertainty. See the previous footnote.
capital that is set ex post each period such that the insurance scheme breaks even. The return on loans \( \tilde{r}_{t,t} \) can then be rewritten as:

\[
\tilde{r}_{t,t} \equiv E_t \left[ r_{k,t+1} + (1 - \delta) Q_{t+1} - \tau_{t+1} \right]
\]

where \( \tau_t = \frac{Q_{t-1}^{-1} q_{t-1} (\psi - \theta r_{k,t} + (1 - \delta) Q_t) \omega_1 - \omega_2}{\omega_2 q_{t-1}} \).

This way the tax also perfectly offsets the distortion on the quantity of investment caused by the deposit insurance. Deposit insurance therefore influences only the funding decision of the bank and, through that, the risk choice. Hence, if \( q_t \) was chosen optimally (or was simply a parameter) the deposit insurance would not have any effect.

The same procedure as outlined above can be applied to obtain closed-form solutions\(^{30}\) for the risk choice and the equity ratio. The solutions can be found in appendix 1C. As state below in proposition 2, the equilibrium characterizations in subsection 1.3.5 remain valid. In particular note that the deviation of the chosen risk (equity ratio) from the optimal level decreases (increases) in the real interest rate. Given our estimation, the risk effect dominates in terms of welfare implications. The intuition for the risk-taking channel is similar as before.

Deposit insurance makes deposits cheaper relative to equity: As a result, the bank will demand more deposits and choose a riskier investment portfolio. Deposit insurance furthermore strengthens the risk-taking channel, which is now affected not only by the importance of the equity premium relative to the real interest rate, but also by the importance the the deposit insurance cap relative to the real interest rate. On the other hand, the efficient risk level is not affected by the deposit insurance.

The liquidation value on the other hand is irrelevant for the banks’ and investors’ choice since it is assumed to be smaller than deposit insurance. Yet it eases the excessiveness of risk taking since it increases the optimal level of risk: \( q^* = \frac{\omega_1 - \theta}{\omega_2} \).

Finally, we would like to point out that none of the results in proposition 1 is due to the functional form that we have assumed for the risk return trade-off \( f(q_t) \). The statement holds even for a generic function \( f(q_t) \)\(^{31}\) under relatively weak assumptions, some of which are sufficient but non necessary. For a proof and a discussion of these assumptions see appendix 1D.

**Proposition 2:** Consider proposition 1, but replace \( f(q_t) \) by the expected return taking into account the liquidation value of failed projects: \( f(q_t) + (1 - q_t) \theta \).

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\(^{30}\)In this case, one needs to apply the adjusted deposit supply schedule \((1.21)\) and to make sensible assumptions about the relative size of parameters and about the root when solving the zero-profit equation.

\(^{31}\)Given the recovery value \( f(q_t) \) now describes the expected return *conditional on success.*
(1) Given this adjustment, all statements of proposition 1 hold for the full bank model with deposit insurance and a small enough liquidation value as well.

(2) Given this adjustment, statements (1)-(4) of proposition 1 hold for a generic conditional expected return function \( f(q_t) \) with deposit insurance and a small enough liquidation value under the additional assumptions that \( f(q_t) \) satisfies \( f(q_t) \geq 0, f''(q_t) < 0, f'''(q_t) \leq 0, f''''(q_t) \leq 0 \). Statement (5) holds if furthermore either the default probability is low relative to the parameters \( \frac{q_t}{1-q_t} \tilde{\xi}_t \geq \tilde{R}_t - \tilde{\psi}_t \) or there is no deposit insurance \( \tilde{\psi}_t = 0 \).

1.3.6 Monetary and fiscal policy

The central bank follows a nominal interest rate rule, targeting inflation and output deviations from the steady state:

\[
R_t - \bar{R} = (1 - \rho) \left( \phi_x \hat{\pi}_t + \phi_y \hat{y}_t \right) + \rho (R_{t-1} - \bar{R}) + \varepsilon^R_t, \tag{1.22}
\]

where \( \rho \) is a smoothing parameter, the hat symbol denotes percentage deviations from the steady state values, \( \bar{R} = \frac{\pi_{ss}}{\beta} \) is the steady state nominal interest rate, and \( \varepsilon^R_t \) is a monetary policy shock. In addition, the fiscal authority finances a stochastic expenditure stream \( g_y \tilde{\varepsilon}^G_t \):

\[
\ln (\varepsilon^G_t) = \rho_g \ln (\varepsilon^G_{t-1}) + u^G_t + \rho_{GA} u^A_t,
\]

where we are allowing for a correlation between exogenous spending and innovations to total factor productivity.\(^{32}\) For simplicity we rule out government debt \( (s_t = 0) \), implying that all expenditures are financed by lump sum taxes; i.e. \( g_y \tilde{\varepsilon}^G_t = T_t \).

1.4 Steady-state and dynamic implications of excessive risk taking in the estimated model

We have embedded our risk-taking channel in a medium-scale model which closely resembles the non-linear version\(^ {33}\) of Smets and Wouters [2007] and we next estimate the model parameters using Bayesian techniques. This serves two purposes. First, in order to perform a sound monetary policy evaluation we need a quantitative model that is able to replicate key empirical moments of the

:\(^{32}\)This is a shortcut to take exports into account. Productivity innovations might rise exports in the data, and a way to capture it in a closed-economy model such as ours is to allow for \( \rho_{GA} \neq 0 \) as in Smets and Wouters [2007].

:\(^ {33}\)Our model deviates from Smets and Wouters [2007] only to the extent that we abstract (for simplicity) from capital utilization, shown by the authors to be of secondary importance once wage stickiness is taken into account, and growth. Furthermore, since we use one additional time series we have added a time preference shock and reinterpreted it as an equity premium shock that affects only bank equity.
data. Second, it helps to understand whether our channel is quantitatively important compared to other monetary and real frictions that affect the monetary policy trade-off.

In this section we first discuss the estimation, and then examine the steady-state and dynamic macroeconomic implications of the risk-taking channel, before we turn to optimal monetary policy in section 1.5.

1.4.1 Model estimation

We estimate a linearized version of the model with Bayesian techniques using eight US macroeconomic time series covering the period of the great moderation from 1984q1 to 2007q3. These include the seven series used by Smets and Wouters [2007], i.e. the federal funds rate, the log of hours worked, inflation and the growth rates in the real hourly wage and in per-capita real GDP, real consumption, and real investment. To identify the banking sector parameters we add a series of the banking sector equity ratio, which we construct from aggregate bank balance-sheet data provided by the FDIC. For a full description of the data we refer to appendix 1A and to the supplementary material of Smets and Wouters [2007]. The observation equations, linking the observed time series to the variables in the model, as well as the prior specifications can be found in appendix 1C. While the priors of the non-bank parameters follow Smets and Wouters [2007], the priors for the banking sector parameters are motivated by historical averages and external estimates for the US. Note that, instead of forming priors directly about $\omega_2$ (risk return trade-off) and $\psi$ (deposit insurance), we rewrite these parameters as functions of the steady state equity ratio $\bar{k}$ and default rate $\bar{q}$. The prior mean of the steady state equity premium $\xi$ is centered around an annualized value of 6%, in line with the empirical estimates of Mehra and Prescott [1985], while the prior distribution for $\bar{k}$ is diffuse and centered around the historic mean of 12%. The prior for the liquidation value is set such that the prior value is contained between 0.3 and 0.7 with 95% probability, in line with the evidence provided by Altman et al. [2003]. The default rate $\bar{q}$ is not well identified and is therefore fixed to 0.99, which implies an annual default rate of 4%, roughly in line with the historical average of delinquency rates on US business loans. Sensitivity tests have moreover shown that this parameter is only of small quantitative relevance. Lastly, we normalize the units of capital versus final goods by setting $\omega_1$ (return of most risky asset) such that one unit of final good is expected to produce one unit of capital good in steady state.

Table 1.1 summarizes the posterior parameter values, which are broadly in line with existing empirical estimates for the US. The steady state inflation rate is estimated to be about 2.5% on an annual basis, while the posterior mean of the discount factor implies an annual steady-state real interest rate of around 1.7%. Wages are slower moving than prices: wages are reoptimized every

\[34\] In particular, the implications for optimal monetary policy behavior are very robust to the value of the steady state default rate. What matters is the importance of the channel over the business cycle, determined by the liquidation value and the extent of deposit insurance.
Table 1.1: Model estimation: prior and posterior values

<table>
<thead>
<tr>
<th>parameter</th>
<th>prior shape</th>
<th>prior mean</th>
<th>prior std</th>
<th>post. mean</th>
<th>90% HPD interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_y$</td>
<td>trend growth</td>
<td>norm 0.4</td>
<td>0.1</td>
<td>0.4264</td>
<td>0.3908 0.4618</td>
</tr>
<tr>
<td>$\mu_l$</td>
<td>labor normalization</td>
<td>norm 0</td>
<td>2</td>
<td>-0.0938</td>
<td>-1.6569 1.4777</td>
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<tr>
<td>$\alpha$</td>
<td>output share</td>
<td>norm 0.3</td>
<td>0.05</td>
<td>0.2001</td>
<td>0.1662 0.2395</td>
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<tr>
<td>$\beta$</td>
<td>real rate in %</td>
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<td>0.1</td>
<td>0.427</td>
<td>0.2992 0.5445</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>price markup</td>
<td>norm 1.25</td>
<td>0.12</td>
<td>1.5068</td>
<td>1.3621 1.6523</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>inflation in %</td>
<td>gamma 0.62</td>
<td>0.1</td>
<td>0.6263</td>
<td>0.4893 0.7616</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>TR weight on inflation</td>
<td>beta 1.5</td>
<td>0.25</td>
<td>1.8723</td>
<td>1.5489 2.2003</td>
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<tr>
<td>$\phi_p$</td>
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<td>0.8057 0.8768</td>
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<td>$\kappa$</td>
<td>investment adj. costs</td>
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<td>5.5992 9.3376</td>
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<td>gamma 2</td>
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<td>0.9726 3.0566</td>
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<tr>
<td>$\sigma_p$</td>
<td>price calvo parameter</td>
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<td>0.1</td>
<td>0.6206</td>
<td>0.5429 0.701</td>
</tr>
<tr>
<td>$\lambda_p$</td>
<td>wage calvo parameter</td>
<td>beta 0.5</td>
<td>0.1</td>
<td>0.8476</td>
<td>0.7099 0.8864</td>
</tr>
<tr>
<td>$\gamma_p$</td>
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<td>beta 0.5</td>
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<td>0.1533</td>
<td>0.0357 0.2479</td>
</tr>
<tr>
<td>$\gamma_p$</td>
<td>wage indexation</td>
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<td>0.15</td>
<td>0.448</td>
<td>0.2066 0.6829</td>
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<tr>
<td>$\lambda^{\text{equity}}$</td>
<td>equity premium</td>
<td>norm 0.015</td>
<td>0.01</td>
<td>0.0213</td>
<td>0.0054 0.0348</td>
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<tr>
<td>$\theta$</td>
<td>liquidation value</td>
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<td>0.7416</td>
<td>0.6425 0.8385</td>
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<tr>
<td>$\kappa$</td>
<td>equity ratio</td>
<td>norm 0.12</td>
<td>0.05</td>
<td>0.1231</td>
<td>0.1208 0.1254</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>structural shock processes</th>
<th>structural parameters</th>
</tr>
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<tbody>
<tr>
<td>$\sigma_A$</td>
<td>atdev TFP</td>
</tr>
<tr>
<td>$\sigma_B$</td>
<td>atdev preference</td>
</tr>
<tr>
<td>$\sigma_G$</td>
<td>atdev govt. spending</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>atdev investment</td>
</tr>
<tr>
<td>$\sigma_P$</td>
<td>atdev price markup</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>atdev monetary</td>
</tr>
<tr>
<td>$\sigma_W$</td>
<td>atdev wage markup</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>atdev equity premium</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>persistence TFP</td>
</tr>
<tr>
<td>$\rho_B$</td>
<td>persistence preference</td>
</tr>
<tr>
<td>$\rho_G$</td>
<td>persistence gov. spending</td>
</tr>
<tr>
<td>$\rho_I$</td>
<td>persistence investment</td>
</tr>
<tr>
<td>$\rho_P$</td>
<td>persistence price markup</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>persistence monetary</td>
</tr>
<tr>
<td>$\rho_W$</td>
<td>persistence wage markup</td>
</tr>
<tr>
<td>$\rho_L$</td>
<td>persistence equity premium</td>
</tr>
<tr>
<td>$\rho_{G,A}$</td>
<td>correlation gov. spending &amp; TFP</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>MA component of price markup</td>
</tr>
<tr>
<td>$\omega_w$</td>
<td>MA component of wage markup</td>
</tr>
</tbody>
</table>

A year and a half, while prices are reoptimized approximately every three quarters. The coefficient of relative risk aversion $\sigma_c$ is estimated to be 1.7, above its prior mean. The posterior estimates of the Taylor rule parameters show a strong response to inflation (1.87), a small response to output (0.02), and a high degree of interest rate smoothing (0.84).

The key banking sector parameters that determine the importance of the risk-taking channel are well identified by the data. The steady state equity ratio has a tight posterior around 12%, the posterior mean of the equity premium is around an annualized value of 9%, and the liquidation value is about 74%.[^35] For the following quantitative analysis we set the parameters to their posterior means.

[^35]: The implied mean value for deposit insurance cap $\psi$ of about 88% implies that 99% of deposits are insured in steady state. Demirgüç-Kunt et al. [2005] report that the explicit deposit insurance scheme in the US is estimated to cover between 60% and 65% of deposits. The divergence can be interpreted as implicit deposit guarantees resulting from the expectation of bailouts. The implied mean values of $\omega_1$ (1.13) and $\omega_2$ (0.2561) yield a corner solution for $\phi^{\text{opt}}$ at 1.
Table 1.2: Steady state comparison: The model without banking sector frictions features an undetermined equity ratio and risk equal to the socially optimal level; i.e. $q^o = 1$. Parameters are fixed to the posterior mean estimates of the bank model reported in table 1.1.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MODEL WITH BANKING FRICTIONS</th>
<th>MODEL WITHOUT BANKING FRICTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>q loan safety</td>
<td>0.99</td>
<td>1</td>
</tr>
<tr>
<td>k equity ratio</td>
<td>0.1231</td>
<td>0</td>
</tr>
<tr>
<td>Y output</td>
<td>0.9484</td>
<td>0.9803</td>
</tr>
<tr>
<td>C consumption</td>
<td>0.6289</td>
<td>0.6376</td>
</tr>
<tr>
<td>I investment</td>
<td>0.1488</td>
<td>0.1662</td>
</tr>
<tr>
<td>K capital</td>
<td>5.9517</td>
<td>7.0227</td>
</tr>
<tr>
<td>L labor</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>π inflation</td>
<td>0.0063</td>
<td>0.0063</td>
</tr>
<tr>
<td>R interest rate</td>
<td>0.0106</td>
<td>0.0106</td>
</tr>
</tbody>
</table>

1.4.2 Steady-state and dynamic implications of excessive risk taking

In Table 1.2 we compare the non-stochastic steady state of the model with banking frictions (henceforth bank model) with that of the model without banking frictions. In the latter model the capital structure is undetermined and risk is equal to the socially optimal level. For the given set of estimated values, the optimum is a corner solution: $q^o = 1$. In the bank model, the capital ratio is below one, implying that banks do not fully internalize the implications of their risk choice, and hence choose an excessive level of risk. This implies that the capital production technology is inefficient. Consequently, the bank economy is under-capitalized in the steady state, and output, consumption and welfare are inefficiently low.

To understand the dynamic effects of the risk-taking channel, we assess how the propagation mechanism of the model differs if a risk-taking channel is present. For illustration, we discuss an expansionary monetary policy shock. As we have just seen, the economy without financial frictions and the bank economy have different steady states. This makes dynamic comparisons of the two models difficult, since both the different behaviors of $q_t$ and $k_t$ as well as the different steady states imply different dynamics. In order not to mix the two effects, we focus on comparing models with the same steady state. For this purpose we alter the model without financial frictions by treating the risk choice $q_t$ and the equity ratio $k_t$ as parameters, which we set to the steady state values of the bank model. This model, henceforth benchmark model, not only has the same steady state as the bank model but also corresponds to a standard New Keynesian model with a small markup in capital markets.

In figure 1.2 we compare the dynamic responses in the bank model (solid red lines) and in benchmark model (dashed blue lines) to an expansionary monetary policy shock. A monetary policy expansion triggers a set of standard reactions, which are evident in the benchmark model. An unexpected fall in the nominal risk-free rate causes a drop in the real interest rate, since prices
Figure 1.2: Monetary policy shock in the bank and benchmark models: dynamic responses in the bank model (solid red lines) and in the benchmark model (dashed blue lines) to an expansionary monetary policy shock, at the mean of the posterior distribution. Shaded areas denote the highest posterior density interval at 90% for the bank model impulse responses and the black line the steady state level. Inflation and interest rates are quarter on quarter rates.

are sticky. Consequently, consumption is shifted forward, firms that can adjust the price do so causing an increase in inflation, while the remaining firms increase production. The risk-taking channel adds two further elements as both the risk level and the capital structure chosen by the bank respond to the real interest rate movement. On impact, the drop in the real interest rate cause banks to substitute equity for deposits, since the relative cost advantage of deposits increases. Consequently, banks have less skin in the game and hence take more risk (lower loan safety). The risk choice therefore moves further away from the optimal level and the expected return on aggregate investment $f(q_t) = q_t(\omega_1 - \omega_2/2q_t) + (1 - q_t)\theta$ drops. To maintain the same path of capital as in the benchmark case, households would have to invest more and consume less. Yet this would not be optimal because of consumption smoothing and because of the lower expected return on investment. Therefore investment rises by less than what would be needed to compensate the loss in investment efficiency, which makes the capital stock decline considerably. Overall, agents are worse off (in terms of welfare) in the bank model than in the benchmark economy.

We conclude this section with a few remarks on the fit of the estimated model. Comparing our bank model to the benchmark Smets and Wouters [2007]-type model we find that the parameter

\[\text{Note that the decline in the equity ratio diminishes the distortion due to the equity premium, which reduces the cost of capital. Yet this effect is tiny relative to the increase in the cost of capital due to lower investment efficiency.}\]
estimates are rather similar and the fit of the two models is comparable. The posterior odds ratio of exp(2.86) favors the bank model, though it is close but not above the value of exp(3), which, according to Jeffreys [1961], can be interpreted as conclusive evidence.

To evaluate the fit particularly with respect to the risk-taking channel we look at three statistics that were not targeted by the estimation. First, note that the responses shown in figures 1.2 are in accordance with the structural VAR results in section 2.3, in particular with the finding that the response of risk is proportional to that of the interest rate, even though our model displays a higher degree of persistence. Second, we compare the model-implied series for the risk variable $q_t$ with the risk-taking index used in the VAR analysis. Figure 1.3 shows that the model implies a cyclical pattern of risk that is roughly in line with the survey measure (the correlation is 60%). Third, the responses in figure 1.2 also show that, conditional on the monetary policy shock, leverage (the inverse of the equity ratio $k_t$) is pro-cyclical with respect to the the size of the bank balance sheet $e_t + d_t$. Conditional on the full set of shocks we find a correlation of 43% which is in line with the evidence for US data provided by Adrian and Shin [2014] and distinguishes our model from canonical financial accelerator models that build on Bernanke et al. [1999].

Lastly, we find that the introduction of the banking frictions reduces the role of the investment efficiency shock. In particular, we find that the forecast variance of the output level drops by a third, while the variance decomposition share of the investment shock drops from around 49% (estimated benchmark model) to 34% (estimated bank model) for horizons between 3 and 8 quarters. This relates to the argument of Justiniano et al. [2011], who find that the large role of this shock in explaining GDP volatility in the canonical medium-scaled DSGE model could be a spurious result that captures unmodeled financial frictions. In reducing

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37 Recall that the Smets and Wouters [2007] model is obtained by turning off the banking sector frictions. Hence bank leverage is no longer defined. For the comparison we therefore estimate the two versions of the model (with and without the banking frictions) using only the seven macro aggregates used by Smets and Wouters [2007], and calibrate the banking parameters in the bank model to the posterior estimates in table 1.1.

38 See, for instance, the discussion in Adrian et al. [2015].
the importance of this shock, the risk-taking channel proves to be capable of capturing at least some of this missing mechanism. This is intuitive because both the investment shock $\varepsilon_I^t$ and the expected return of the banks’ investment $q_t (\omega_1 - \frac{\omega_2}{2} q_t) + (1 - q_t) \theta$ enter the capital accumulation equation multiplicatively:

$$K_t = \left[ \varepsilon_I^t (1 - S(i_t/i_{t-1})) i_t + (1 - \delta) K_{t-1} \right] \left[ q_t \left( \omega_1 - \frac{\omega_2}{2} q_t \right) + (1 - q_t) \theta \right],$$

Yet they are not perfectly isomorphic, since the shock affects only net investment (new capital), while the expected return on investment affects gross investment (all capital). Moreover the path of $\varepsilon_I^t$ backed from the estimated benchmark model is strongly correlated with the path of the return on investment in the estimated bank model.

Overall, we interpret these findings as suggestive that the additional dynamics implied by the risk-taking channel are not rejected by the data and, on the contrary, help to reduce the mismatch between the benchmark model and the data.

1.5 Monetary policy with a risk-taking channel

We have seen that the risk-taking channel has both static and dynamic effects. While monetary policy does not affect the non-stochastic steady state, it can influence the dynamics of the economy. In particular, it can influence the real rate and hence affect bank risk taking. But are these additional mechanisms implied by the risk-taking channel actually quantitatively significant for monetary policy? To answer this question we determine the optimal simple implementable monetary policy rule in the risk-taking channel model. We then compare this policy to the optimal policy in the benchmark economy with the same steady state but without the risk-taking channel. This comparison has an interesting interpretation. Suppose that the actual economy features the risk-taking channel (the bank model), but that the central bank is unaware of this channel and believes that risk is an irrelevant constant from her point of view. The central bank would then implement optimal policy based on a wrong model (the benchmark model). Our comparison then answers the question of how important understanding the risk-taking channel is, in terms of optimal policy and welfare.

Notice that in this paper we consider a central bank that has no policy tools besides the interest rate. With a second instrument, such as capital regulation, the central bank could do better or

---

39For this exercise we use only the 7 nonfinancial series. Notice that the specification of our model is not exactly the same as Smets and Wouters [2007] and Justiniano et al. [2011] since we have abstracted from capital utilization. This means that the numbers are not directly comparable.
even eliminate the friction. Exploring optimal macroprudential regulation is however beyond the scope of the present paper.\footnote{For a thorough analysis of macroprudential policy in an economy with bank risk-taking see Collard et al. [2012].}

In what follows, we first discuss the concept of the optimal simple implementable monetary policy rule, and then present our results.

1.5.1 The central bank problem

We follow Schmitt-Grohe and Uribe [2007] and characterize optimal monetary policy as the policy rule that maximizes welfare among the class of simple, implementable interest-rate feedback rules\footnote{The implementability criterion requires uniqueness of the rational expectations equilibrium, while simplicity requires the interest rate to be a function of readily observable variables. For a complete discussion, see Schmitt-Grohe and Uribe [2007]. Notice that we drop their second requirement for implementability which is that an implementable rule must avoid regular zero lower bound violations.} given by:

\[
R_t - \bar{R} = \phi_{\pi} \hat{\pi}_{t+s} + \phi_{\gamma} \hat{\gamma}_{t+s} + \phi_{k} \hat{k}_{t+s} + \rho \left( R_{t-1} - \bar{R} \right). \tag{1.23}
\]

where the hat symbol denotes percentage deviations from the steady state, and the index \( s \) allows for forward- or contemporaneous-looking rules (respectively by setting \( s = 1 \) or \( s = 0 \)). The policy rule specification (1.23) is chosen for its generality, as it encompasses both standard Taylor-type rules (setting \( \phi_k = 0 \)), and the possibility that the central bank reacts to banking sector leverage, the inverse of the equity ratio \( k \) (\( \phi_k \neq 0 \)). A fall in the equity ratio implies that banks increase their debt financing, i.e. they increase leverage. As a consequence banks internalize less the downside risk of their investments, and choose loans with a higher default probability. Hence, a fall in the equity ratio signals an increase in risk taking, to which the central bank may want to respond by increasing the interest rate. We choose not to let the interest rate depend on risk taking directly, because the latter is not a readily observable variable. We furthermore impose that the inertia parameter \( \rho \) has to be non-negative. Since we are interested in the effect of systematic monetary policy, we switch off the monetary policy shock for this experiment.

The welfare criterion that defines the optimal parameter combination for rule (1.23) is the household’s conditional lifetime utility:

\[
V \equiv E_0 \sum_{t=0}^{\infty} \beta_t^t c_t u(c_t, L_t). \tag{1.24}
\]

This measure is commonly used in the literature and yields the expected lifetime utility of the representative household, conditional on the economy being at the deterministic steady state. To
Table 1.3: Optimal simple rules: optimal parameters for policy rules of the class \( R_t - \bar{R} = \phi_t \pi_{t+s} + \phi_y y_{t+s} + \phi_k k_{t+s} + \rho \left( R_{t-1} - \bar{R} \right) \). The hat symbol denotes percentage deviations from the steady state. Current-(forward-) looking rules let the interest rate react to current (future) deviations of variables from their steady state values. The second column describes the restrictions we enforced. The first line for example corresponds to the standard taylor rule with no smoothing. The last two lines are somewhat different. The penultimate line reproduces line 2 for the benchmark model and in the bank model shows the optimal rule given that the first three parameters are fixed to their benchmark values. The last line shows the estimated taylor rule. \( V \) is the welfare level associated with each policy in the bank model. \( \Omega \) is the welfare cost (in % of the consumption stream) associated to implementing in the bank model the optimal policy rule of the benchmark model (given the same restrictions on parameters). For the benchmark model the restriction \( \phi_k = 0 \) is irrelevant, since the equity ratio is a constant in the benchmark model. Entries in italics indicate restricted parameters.

<table>
<thead>
<tr>
<th>( s )</th>
<th>( \text{rule} )</th>
<th>( \text{benchmark model} )</th>
<th>( \text{bank model} )</th>
<th>( V )</th>
<th>( \Omega )</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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<td>0</td>
<td>7.100</td>
<td>0.115</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
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<td>7.100</td>
<td>0.115</td>
<td>1.059</td>
</tr>
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<td>0</td>
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<td>7.100</td>
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<td>0</td>
</tr>
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<tr>
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<td>12.084</td>
<td>0.124</td>
<td>1.114</td>
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<td>0</td>
<td>choose ( \phi_k )</td>
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<td>7.100</td>
<td>0.115</td>
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</tr>
<tr>
<td>0</td>
<td>estimated</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.841</td>
</tr>
</tbody>
</table>

be able to make meaningful comparisons of welfare levels we furthermore define the measure \( \Omega \) as the fraction of the consumption stream that a household would need to receive as a transfer under the suboptimal rule to be equally well off as under the optimal rule. If \( o \) denotes the optimal and \( s \) another suboptimal rule, this fraction \( \Omega \) is implicitly defined by the equation:

\[
V^o = E_0 \sum_{t=0}^{\infty} \beta^t c^B_t u((1 + \Omega)c^s_t, L^s_t).
\]

1.5.2 Findings

Using the welfare criterion just described we numerically determine the coefficients of the optimal simple implementable rules in the benchmark and in the bank model using second order approximations around the non-stochastic steady state. The first 5 rows of table 1.3 report the optimal coefficients for 5 different specifications of the monetary policy rule: contemporaneous and forward-looking, without inertia and with optimal inertia, without and with reaction to current leverage. The coefficients of the optimal rules generally vary greatly between the two models. A set of results, which are robust across policy rule and estimation specifications, are worth noticing.

\(^{42}\)We have experimented with different estimation samples and calibrated parameter values: while the optimized parameters and transfers slightly change, the qualitative results discussed in the text are very robust.
Table 1.4: Differences in moments associated to the optimal simple rules in the benchmark and in the bank model: This table shows the % differences in the mean and standard deviation associated to applying the different optimal rules in the bank model. The first entry, for example, indicates that under the optimal bank policy rule average risk would be 0.15% lower than if the rule optimal for the benchmark model had been applied.

<table>
<thead>
<tr>
<th>s</th>
<th>rule</th>
<th>mean q</th>
<th>$R^r$</th>
<th>$\pi$</th>
<th>y</th>
<th>c</th>
<th>standard deviation q</th>
<th>$R^r$</th>
<th>$\pi$</th>
<th>y</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\phi_k, \rho = 0$</td>
<td>0.151</td>
<td>0.002</td>
<td>-0.051</td>
<td>0.311</td>
<td>0.499</td>
<td>-43.880</td>
<td>-47.975</td>
<td>52.470</td>
<td>-0.843</td>
<td>-4.108</td>
</tr>
<tr>
<td>0</td>
<td>$\phi = 0$</td>
<td>0.214</td>
<td>0.007</td>
<td>-0.038</td>
<td>0.439</td>
<td>0.701</td>
<td>-67.949</td>
<td>-77.760</td>
<td>64.393</td>
<td>-9.545</td>
<td>-9.566</td>
</tr>
<tr>
<td>0</td>
<td>$\rho = 0$</td>
<td>0.152</td>
<td>0.003</td>
<td>-0.015</td>
<td>0.323</td>
<td>0.506</td>
<td>-41.666</td>
<td>-47.248</td>
<td>53.194</td>
<td>-0.773</td>
<td>-3.800</td>
</tr>
<tr>
<td>1</td>
<td>$\phi_k, \rho = 0$</td>
<td>0.194</td>
<td>0.011</td>
<td>-0.037</td>
<td>0.413</td>
<td>0.652</td>
<td>-50.536</td>
<td>-55.417</td>
<td>57.719</td>
<td>-2.781</td>
<td>-6.906</td>
</tr>
<tr>
<td>1</td>
<td>$\phi_k = 0$</td>
<td>0.195</td>
<td>0.004</td>
<td>-0.054</td>
<td>0.458</td>
<td>0.724</td>
<td>-65.839</td>
<td>-76.3112</td>
<td>71.906</td>
<td>-10.373</td>
<td>-11.737</td>
</tr>
<tr>
<td>0</td>
<td>choose $\phi_k$</td>
<td>0.130</td>
<td>0.001</td>
<td>-0.070</td>
<td>0.244</td>
<td>0.417</td>
<td>-41.691</td>
<td>-41.948</td>
<td>31.838</td>
<td>3.323</td>
<td>-0.345</td>
</tr>
<tr>
<td>0</td>
<td>estimated</td>
<td>0.005</td>
<td>0.001</td>
<td>-0.024</td>
<td>0.114</td>
<td>0.158</td>
<td>-15.742</td>
<td>-31.419</td>
<td>-2.9113</td>
<td>-12.908</td>
<td>-6.045</td>
</tr>
</tbody>
</table>

First, the optimal coefficients on inflation deviations are smaller in the bank model compared to the benchmark model. For any given change in inflation, the nominal interest rate should move less if a risk-taking channel is present. Furthermore, if the central bank can optimize over its smoothing parameter, then full interest rate smoothing is optimal in the bank model. Given that the optimal output coefficient is close to zero, the optimal rule is closer to a stable real interest rate rule in the bank model than in the benchmark model. In doing so, the central bank limits fluctuations in the real interest rate and hence in risk taking and slightly raises the average level of $q$ towards the efficient value, as it can be seen in table 1.4. At the same time inflation is significantly more volatile under the optimal rule. If a risk-taking channel is present, the central bank should accept higher inflation fluctuations in order to reduce the distortion stemming from risk taking.

This is because monetary policy cannot affect the deterministic steady state, but it can control the real interest rate and therefore the fluctuations in excessive risk. Upward movements of the real interest rate are welfare enhancing since they lower the level of risk taking towards the efficient level, whereas downward changes of the real interest rate lead to even more excessive risk taking. But this does not mean that movements in the interest rate are irrelevant. Since the expected return on investments $q_t (\omega_1 - \frac{2\phi}{\tau} q_t) + (1 - q_t)\theta$ is concave in the real interest rate, as we have shown above, a mean preserving increase in the volatility of the real rate reduces the average expected return of investments. Therefore the risk-taking channel provides a motive for keeping the real interest rate constant. This adds a third dimension to the central bank problem: besides trading off inflation versus output stabilization the central bank would now also like to stabilize the real interest rate. As a result, the optimal policy is tilted away from inflation stabilization.

To understand *how different* the equilibria associated to the two optimal rules are, and therefore how important it is for the central bank to take the risk-taking channel into account, we compute
the cost Ω of applying the rule that is optimal for the benchmark model in the bank model. These costs, expressed in % of the lifetime consumption stream, are reported in the last column of table 1.3. Though the costs vary a lot across policy specifications, they are always non significant. For the best performing policy (fifth row of table 1.3), the costs of applying the benchmark policy in the bank model is around 0.81% of the lifetime consumption stream. Hence, internalizing the feedback effect that the nominal interest rate has on bank risk taking pays off in terms of welfare.

Second, including an explicit reaction to banking sector leverage, in addition to inflation and output, improves welfare only marginally (compare the last column of the first and third row of table 1.3). Recall that leverage depends on both the nominal interest rate and expected inflation. By setting the nominal rate optimally as a function of current inflation, the central bank can already steer risk taking, to the extent that current and expected future inflation are highly correlated. The fact that this correlation is not perfect, and that our approximation allows for nonlinearities, accounts for the small improvement in welfare obtained by allowing a response to leverage to the policy function. To further illustrate this point, in line 6 we fix the coefficients of current inflation and output to the values optimal in the benchmark economy, and allow the central bank to respond optimally only to leverage. In this case, it is optimal to strongly raise the interest rate in response to higher leverage (lower equity ratio k). Thereby the central bank again stabilizes the real interest rate and does not much worse in terms of welfare than when the responses to inflation and output are chosen optimally (compare the last column of the third and sixth row of tables 1.3 and 1.4).

Finally, we also compare the estimated policy rule to the corresponding optimized rule. We find that the estimated rule is associated with more volatility in output and in the real rate, and entails a welfare cost of 0.3% of lifetime consumption compared to the optimized rule. Thus, a greater stabilization of the real interest rate over the analyzed period would have yielded sizable welfare gains.

1.6 Conclusion

The recent financial crisis has highlighted the importance of monitoring the level of risk to which the financial sector is exposed. In this paper we focus on one aspect of financial sector risk, ex-ante bank asset risk, and on how the latter can be influenced by monetary policy.

First, we provide new empirical evidence of the impact of monetary policy on bank risk taking. We document that unexpected monetary policy shocks, identified through sign restrictions in a classical VAR framework, increase a measure for ex-ante bank risk taking in the US. This conclusion, robust to using a recursive identification scheme, is compatible with the monetary
policy transmission mechanism in the theoretical model that we build to explain the effects of monetary policy on risk taking.

For this purpose, we extend the work of Dell'Ariccia et al. [2014] and build a dynamic general-equilibrium model where low levels of the risk-free interest rate induce banks to make riskier investments. At the core of this mechanism is an agency problem between depositors and equity providers: the latter choose the level of risk but are protected by limited liability. In general equilibrium, this friction leads to a steady state with excessive risk taking, and inefficiently low levels of capital, output and consumption. Furthermore, risk taking alters the dynamic response of the economy to shocks. In particular, an expansionary monetary policy shock has non-standard consequences: because banks choose a riskier and less efficient investment strategy, the growth of capital, output and consumption will be lower than in the model without the risk-taking channel.

In order to assess the importance of the risk-taking channel and to study optimal monetary policy, we estimate the model on US data using Bayesian methods. Including this additional channel improves the in-sample fit, yields a path for risk taking that matches survey evidence for the US and implies a pro-cyclical behavior of leverage with respect to total assets which is in line with US evidence documented by Adrian and Shin [2014]. Our policy experiments using optimal simple rules suggest that, if a risk-taking channel is present and the interest rate is the only instrument available to the monetary authority, the optimal rule should stabilize the path of the real interest rate more than without the risk-taking channel. This implies that the central bank should tolerate higher inflation volatility in order to reduce welfare detrimental fluctuations in risk taking. The welfare gains of taking the channel into account are found to be significant. Nevertheless, these results do not rule out that an alternative instrument could perform better at maximizing consumer welfare, an issue that deserves to be investigated in future work.

1.7 Appendix 1A: Data description

1.8 Appendix 1B: Empirical motivation - recursive identification scheme

1.9 Appendix 1C: The full model - Equilibrium and estimation details

Model summary: We report here the equations that enter the non-linear model, grouped by sector. Note that following Smets and Wouters [2007] we assume that different varieties of
intermediate goods and of labor are assembled through a Kimball [1995] aggregator, rather than a Dixit-Stiglitz one. This latter assumption is introduced in order to obtain estimates of price and wage rigidity that are closer to micro estimates, but we do not derive the recursive formulation here (see e.g. Adjemian et al. [2008]). Also note that the variables regarding the equity and deposit funds $R_d$ and $R_e$ have been substituted out.

**Competitive equilibrium:**  The competitive equilibrium is a path of 41 variables ($\Lambda$, $K$, $L$, $y$, $l$, $c$, $q$, $k$, $d$, $e$, $p$, $r_k$, $r_d$, $r_e$, $R$, $W$, $m_c$, $o^{new}$, $o$, $p^*$, $Z_{p1}$, $Z_{p2}$, $Z_{p3}$, $Z_{w1}$, $Z_{w2}$, $Z_{w3}$, $\Delta_{p1}$, $\Delta_{p2}$, $\Delta_{p3}$, $\Delta_{p4}$, $\Delta_{w1}$, $\Delta_{w2}$, $\Delta_{w3}\Delta_{p4}$, $W^*$, $i$, $\bar{R}$, $\bar{\xi}$, $\bar{\psi}$, $\tau$) that satisfy the following 41 equations at each point in time given initial conditions and the exogenous shock processes $\varepsilon^A$, $\varepsilon^B$, $\varepsilon^G$, $\varepsilon^I$, $\varepsilon^P$, $\varepsilon^R$, $\varepsilon^W$, $\varepsilon^\xi$.

**Household**

$$A_t = \varepsilon^B c_t^{1-\sigma C} - \beta E_t \left[ \varepsilon^{B}_{t+1} (c_{t+1} - \iota c_{t+1})^{\sigma C} \right] \quad (1.25)$$

$$E_t \left[ A_{t+1} \frac{q_t r_{d,t+1} + (1-q_t) \psi_{t+1}}{\pi_{t+1}} \right] = E_t \left[ A_{t+1} \frac{R_t}{\pi_{t+1}} \right] \quad (1.26)$$

$$E_t \left[ A_{t+1} \frac{q_t r_{e,t+1} - \xi_{t+1}}{\pi_{t+1}} \right] = E_t \left[ A_{t+1} \frac{R_t}{\pi_{t+1}} \right] \quad (1.27)$$

$$A_t = \beta E_t \left[ A_{t+1} \frac{R_t}{\pi_{t+1}} \right] \quad (1.28)$$

$$y_t = c_t + i_t + g_y \bar{\varepsilon}_{t}^G \quad (1.29)$$

**Goods sector**

$$\frac{L^d_t}{K_{t-1}} \frac{\alpha}{1-\alpha} = \frac{r_{k,t}}{w_t} \quad (1.30)$$
\[ mc_t = \frac{1}{A_t} \alpha^{-\alpha} \epsilon_r \alpha w_t^{1-\alpha} (1 - \alpha)^{\alpha - 1} \]  
\( \tag{1.31} \)

\[ \pi_t^* = \frac{\epsilon_p (1 + k_p)}{\epsilon_p (1 + k_p) - 1} \frac{Z_{p1,t}}{Z_{p2,t}} - \frac{k_p}{\epsilon_p - 1} (\pi_t^*)^{1+\epsilon_p(1+k_p)} \frac{Z_{p3,t}}{Z_{p2,t}} \]  
\( \tag{1.32} \)

\[ Z_{p1,t} = (1 - \tau_{p,t}) A_t m c_t y_t \Delta_{p1,t}^{\epsilon_p(1+k_p)/(1-\epsilon_p(1+k_p))} + \beta \lambda_p E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p \frac{1-\gamma_p}{\gamma_p}}} \right)^{\epsilon_p(1+k_p)} \right] Z_{p1,t+1} \]  
\( \tag{1.33} \)

\[ Z_{p2,t} = (1 - \tau_{p,t}) A_t y_t \Delta_{p1,t}^{\epsilon_p(1+k_p)/(1-\epsilon_p(1+k_p))} + \beta \lambda_p E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p \frac{1-\gamma_p}{\gamma_p}}} \right)^{\epsilon_p(1+k_p)-1} \right] Z_{p2,t+1} \]  
\( \tag{1.34} \)

\[ Z_{p3,t} = A_t y_t + \beta \lambda_p E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p \frac{1-\gamma_p}{\gamma_p}}} \right)^{-1} \right] Z_{p3,t+1} \]  
\( \tag{1.35} \)

\[ \Delta_{p1,t} = (1 - \lambda_p) (\pi_t^*)^{1-\epsilon_p(1+k_p)} + \lambda_p \Delta_{p1,t-1} \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p \frac{1-\gamma_p}{\gamma_p}}} \right)^{\epsilon_p(1+k_p)-1} \]  
\( \tag{1.36} \)

\[ 1 = \frac{1}{1 + k_p} \Delta_{p1,t}^{1/(1-\epsilon_p(1+k_p))} + \frac{k_p}{1 + k_p} \Delta_{p2,t} \]  
\( \tag{1.37} \)

\[ \Delta_{p2,t} = (1 - \lambda_p) \pi_t^* + \lambda_p \Delta_{p2,t-1} \left( \frac{\pi_{t+1}}{\pi_t^{\gamma_p \frac{1-\gamma_p}{\gamma_p}}} \right)^{-1} \]  
\( \tag{1.38} \)

\[ \Delta_{p3,t} = \frac{1}{1 + k_p} \Delta_{p1,t}^{\epsilon_p(1+k_p)/(1-\epsilon_p(1+k_p))} \Delta_{p4,t} + \frac{k_p}{1 + k_p} \]  
\( \tag{1.39} \)
\[ \Delta p_{4,t} = (1 - \lambda_p) \left( \frac{\pi_t^p}{\pi_t^p \bar{n}^{1-\gamma_p}} \right)^{-\epsilon_p(1+k_p)} + \lambda_p \Delta p_{4,t-1} \left( \frac{\pi_{t+1}^p}{\pi_t^p \bar{n}^{1-\gamma_p}} \right)^{\epsilon_p(1+k_p)} \] (1.40)

\[ A_t K_{t-1}^\alpha \left( \frac{L_t}{\Delta p_{3,t}} \right)^{1-\alpha} = \Delta p_{3,t} y_t \] (1.41)

**Labor sector**

\[ w_t^* = \frac{\epsilon_w}{\epsilon_w(1+k_w) - 1} Z_{w1,t} + \frac{k_w}{\epsilon_w - 1} (w_t^*)^{1+\epsilon_p(1+k_p)} \frac{Z_{w3,t}}{Z_{w2,t}} \] (1.42)

\[ Z_{w1,t} = \frac{\epsilon L_t}{1+\sigma_w} w_t^{1+\epsilon_w(1+k_w)} (C_t - C_{t-1})^{1-\sigma_w} \exp \left( \frac{L_t \sigma_c - 1}{1 + \sigma_L} L_t^{1+\sigma_L} \right) \Delta w_{1,t}^{(1+k_w)/(1-\epsilon_w(1+k_w))} \] (1.43)

\[ Z_{w2,t} = (1 - \tau_{w,t}) A_t L_t \left[ w_t^{1/(1-\epsilon_w(1+k_w))} \right]^{\epsilon_w(1+k_w)} + \beta \lambda_w E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{1-w} \bar{n}^{1-\gamma_w}} \right)^{\epsilon_w(1+k_w)-1} Z_{w1,t} \right] \] (1.44)

\[ Z_{w3,t} = (1 - \tau_{w,t}) A_t L_t + \beta \lambda_w E_t \left[ \left( \frac{\pi_{t+1}}{\pi_t^{1-w} \bar{n}^{1-\gamma_w}} \right) Z_{w3,t+1} \right] \] (1.45)

\[ \Delta w_{1,t} = (1 - \lambda_w) \left( \frac{w_t^*}{w_t} \right)^{1-\epsilon_w(1+k_w)} + \lambda_w \Delta w_{1,t-1} \left( \frac{w_{t-1}}{w_t} \right)^{1-\epsilon_w(1+k_w)} \left( \frac{\pi_{t+1}}{\pi_t^{1-w} \bar{n}^{1-\gamma_w}} \right)^{\epsilon_w(1+k_w)-1} \] (1.46)

\[ 1 = \frac{1}{1 + k_w} \Delta w_{1,t}^{1/(1-\epsilon_w(1+k_w))} + \frac{k_w}{1 + k_w} \Delta w_{2,t} \] (1.47)

\[ \Delta w_{2,t} = (1 - \lambda_w) \left( \frac{w_t^*}{w_t} \right) + \lambda_w \Delta w_{2,t-1} \left( \frac{w_{t-1}}{w_t} \right) \left( \frac{\pi_{t+1}}{\pi_t^{1-w} \bar{n}^{1-\gamma_w}} \right)^{-1} \] (1.48)
\[
\Delta w_{3,t} = \frac{1}{1 + k_w} \Delta w_{w1,t}\Delta w_{w1,t} + \frac{k_w}{1 + k_w} \Delta w_{4,t}
\] (1.49)

\[
\Delta w_{4,t} = (1 - \lambda_w) \left( \frac{w_t^*}{w_t} \right)^{-\epsilon_w(1+k_w)} + \lambda_w \Delta w_{w4,t-1} \left( \frac{w_t}{w_{t-1}} \frac{\pi_{t+1}}{\pi_t^{\gamma_w}} \frac{\pi_{t+1}}{\pi_t^{1-\gamma_w}} \right)^{\epsilon_w(1+k_w)}
\] (1.50)

**Government**

\[
R_t - \bar{R} = \phi_x \frac{\pi_{t+s}}{\pi} + \phi_y \frac{y_{t+s}}{y} + \phi_k \frac{k_{t+s}}{k} + \rho (R_{t-1} - \bar{R})
\] (1.51)

**Capital producer**

\[
K_t = q_t \left( \omega_1 - \frac{\omega_2}{2} q_t \right) \theta + (1 - q_t) \theta
\] (1.52)

\[
o_t = o^{new}_t + (1 - \delta) K_{t-1}
\] (1.53)

\[
o^{new}_t = \varepsilon_t^{I_t} \left( 1 - \frac{\kappa}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \right)
\] (1.54)

\[
Q_t \varepsilon_t^{I_t} \left[ 1 - S \left( \frac{i_t}{i_{t-1}} \right) - S' \left( \frac{i_t}{i_{t-1}} \right) \frac{i_t}{i_{t-1}} \right] - 1 = \beta E_t \left[ \frac{\Lambda_{t+1}}{A_t} \varepsilon_t^{I_t} \eta_{t+1} Q_{t+1} S' \left( \frac{i_{t+1}}{i_t} \right) \left( \frac{i_{t+1}}{i_t} \right)^2 \right].
\] (1.55)

**Bank**

\[
q_t = 1 - \frac{\tilde{R}_t - \tilde{\psi}_t}{\tilde{\psi}_t} + \frac{\sqrt{\omega_2 \left( \tilde{R}_t - \tilde{\psi}_t \right) \left( \tilde{R}_t + 2 \xi_t \right) \left( 2 \omega_1 \tilde{\psi}_t \left( \tilde{R}_t + \tilde{\xi}_t \right) + \omega_2 \left( \tilde{R}_t - \tilde{\psi}_t \right) \left( \tilde{R}_t + 2 \xi_t \right) \right)}}{\omega_2 \tilde{\psi}_t \left( \tilde{R}_t + 2 \xi_t \right)}
\] (1.56)

\[
k_t = \frac{\tilde{R}_t - \tilde{\psi}_t}{\tilde{R}_t + 2 \xi_t}
\] (1.57)
\[
\left(\omega_1 - \frac{\omega_2}{2} q_{t-1} \right) \frac{r_{k,t} + (1 - \delta) Q_t - \tau_r}{Q_{t-1}} - \frac{r_{d,t}}{\pi_{t+1}} (1 - k_t) - \frac{r_{e,t+1}}{\pi_{t+1}} k_t = 0 \quad (1.58)
\]

\[
\tau_t = \frac{Q_{t-1}^{1-q_{t-1}} \left( \psi - \theta \frac{r_{c,t+1} + (1-\delta) Q_t}{Q_{t-1}} \right)}{\omega_1 - \frac{\omega_2}{2} q_{t-1}} \quad (1.59)
\]

\[
\tilde{\xi}_t = \xi_t E_t [A_{t+1}] \quad (1.60)
\]

\[
\xi_t = \xi \xi_t^\xi \quad (1.61)
\]

\[
\tilde{R}_t = E_t \left[ A_{t+1} \frac{R_t}{\pi_{t+1}} \right] \quad (1.62)
\]

\[
\tilde{\psi}_t = \psi E_t [A_{t+1}] \quad (1.63)
\]

\[
o_t Q_t = e_t + d_t \quad (1.64)
\]

\[
k_t = e_t / (e_t + d_t) \quad (1.65)
\]

**Shock processes**

\[
\log \left( \epsilon_t^B \right) = \rho P \log \left( \epsilon_{t-1}^B \right) + \sigma^B u_t^B \quad (1.66)
\]
\[
\log \left( \varepsilon_t^Q \right) = \rho_l \log \left( \varepsilon_{t-1}^Q \right) + \sigma_u^Q \varepsilon_t^Q \quad (1.67)
\]

\[
\log \left( \varepsilon_t^G \right) = \rho_l \log \left( \varepsilon_{t-1}^G \right) + \sigma_u^G \varepsilon_t^G
\]

\[
\log \left( \varepsilon_t^P \right) = (1 - \rho_P) \log \left( \varepsilon_{t-1}^P \right) + \rho_P \log \left( \varepsilon_{t-1}^P \right) + \sigma \left( u_t^P + m_u u_{t-1}^P \right) \quad (1.69)
\]

\[
\log \left( \varepsilon_t^W \right) = (1 - \rho_W) \log \left( \varepsilon_{t-1}^W \right) + \rho_W \log \left( \varepsilon_{t-1}^W \right) + \sigma \left( u_t^W + m_u u_{t-1}^W \right) \quad (1.70)
\]

\[
\log \left( \varepsilon_t^A \right) = \rho_A \log \left( \varepsilon_{t-1}^A \right) + \sigma_u^A \varepsilon_t^A \quad (1.71)
\]

\[
\log \left( \varepsilon_t^R \right) = \rho_R \log \left( \varepsilon_{t-1}^R \right) + \sigma_u^R \varepsilon_t^R \quad (1.72)
\]

\[
\log \left( \varepsilon_t^G \right) = \rho_G \log \left( \varepsilon_{t-1}^G \right) + \sigma_u^G \varepsilon_t^G + \rho_G A \sigma_u^A \varepsilon_t^A \quad (1.73)
\]

**Observational equations:** The observation equations, linking the observed time series (left hand-side) to the variables in the non-linear model (right hand-side) are the following:

\[
100 \Delta \log \left( \frac{Y_t}{Y_{t-1}} \right) = 100 \Delta \log \left( \frac{y_t}{y_{t-1}} \right) + 100 \mu_y
\]

\[
100 \Delta \log \left( \frac{C_t}{C_{t-1}} \right) = 100 \Delta \log \left( \frac{c_t}{c_{t-1}} \right) + 100 \mu_y
\]
\[
100 \Delta \log \left( \frac{I_t}{I_{t-1}} \right) = 100 \Delta \log \left( \frac{i_t}{i_{t-1}} \right) + 100 \mu_y
\]

\[
100 \Delta \log \left( \frac{W_t}{W_{t-1}} \right) = 100 \Delta \log \left( \frac{w_t}{w_{t-1}} \right) + 100 \mu_y
\]

\[
100 \Delta \log \left( \frac{P_t}{P_{t-1}} \right) = 100 \pi_t
\]

\[
100 \log \left( \frac{H_t}{\bar{H}} \right) = 100 \log \left( \frac{L_t}{L} \right) + 100 \mu_l
\]

\[
\left( \frac{R_t}{4} \right) = 100 R
\]

\[
\tilde{E}_t = 100 k_t
\]

where \( \bar{H} \) are hours worked in 2009 and \( \mu_l \) is a shift parameter. Since there is no growth in the model, we estimate the mean growth rate in the data \( \mu_y \). The equity ratio in the data \( \tilde{E}_t \) is transformed by taking deviations from its linear trend and adding back the mean.

**Prior specifications:** We fix parameters that are not identified to values commonly used in the literature. In particular, we choose a depreciation rate \( \delta \) of 0.025, a steady-state wage markup \( \bar{\varepsilon}_W \) of 1.05, a steady-state spending to GDP ratio \( g_y \) of 18%, a weight of labor in the utility function \( \bar{L} \) such that steady-state hours are equal to 1, and curvatures of the Kimball aggregator for goods and labor varieties of 10.

For all structural shocks, we employ a non-informative uniform distribution. The persistences of the shock processes are assumed to have a beta prior distribution centered at 0.5, and with standard deviation of 0.2. Following [Sunets and Wouters, 2007], we further assume that the two markup shows have a moving average component.
The priors of the Taylor rule parameters are centered around very common values: the smoothing parameter has a Beta distribution with a mean of 0.75, while the responses to inflation and output are assumed to follow a Normal distribution with a mean of 1.5 and of 0.5/4 = 0.125.

Since we use level data of the inflation rate and of the nominal interest rate, we choose the priors for the steady state of the inflation rate $\bar{\pi}$ and the real interest rate $1/\beta - 1$ to match the mean in the data, i.e. we assumed they follow a gamma distribution respectively centered around annualized values of 2.5% and 0.9.

The parameters affecting price and wage stickiness have a beta distribution centered at 0.5 with standard deviation of 0.1. Our prior is that prices and wages are reoptimized on average every 6 months, and that the degree of indexation to past inflation is only up to 50%. The steady-state price markup is assumed to be centered around 1.25, slightly above the steady-state wage markup.

We employ very common priors for all the parameters of the utility function. Habits are centered around 0.7, the intertemporal elasticity of substitution $\sigma_c$ has a prior mean of 1.5, while the elasticity of labor supply $\sigma_l$ has a prior mean of 2. The capital share in production has a prior mean of 0.3 while the investment adjustment costs parameter has a loose prior around 4.

For the discussion on the priors for the banking sector parameters, we refer to section 4.1 in the main text.

1.10 Appendix 1D: Proofs

The risk-taking channel for a generic expected return function

Consider the bank problem discussed in section 3 with deposit insurance partial recovery but replace the expression for the expected return conditional on success $q_t (\omega_1 - \omega_2/2q_t)$ by the generic function $f(q_t)$.

Assume there exists an equilibrium $[\tilde{r}_{t}, q_t, k_t]$ under perfect competition that satisfies the following conditions: (1) the bank’s choices are interior, i.e. $[k_t, q_t] \in [0, 1]^2$, (2a) the default probability is low relative to the parameters $\frac{q_t}{(1-q_t)}\tilde{e}_t \geq \tilde{R}_t - \tilde{\psi}_t$ or (2b) there is no deposit insurance $\tilde{\psi}_t = 0$, the conditional expected return function $f(q_t)$ satisfies (3) $f(q_t) \geq 0$, $f''(q_t) < 0$ and (4) $f'''(q_t) \leq 0$, $f''''(q_t) \leq 0$.

Notice that assumption 2a), which is sufficient but by no means necessary and only needed for claim (e), is weak if we consider the empirically relevant section of the parameter space with a low equity premium (around 0.0x), a real rate just above 1 (1.0x) and high deposit insurance (0.x) and high repayment probabilities (0.9x). Assumption 3 is straightforward as it guarantees a
meaningful risk turn trade-off with an interior solution. Assumption 4 is another a sufficient but non necessary condition.

We prove that, if such a solution exists, then:

(a) risk taking is excessive: \( q_t < \arg\max f(q_t) \)

(b) the safety of assets \( q_t \) is a positive function of \( \tilde{R}_t \): \( \frac{\partial q_t}{\partial \tilde{R}_t} > 0 \)

(c) the equity ratio \( k_t \) is a positive function of \( \tilde{R}_t \): \( \frac{\partial k_t}{\partial \tilde{R}_t} > 0 \)

(d) the expected return of an investment is a positive function of \( \tilde{R}_t \): \( \frac{\partial f(q_t(\tilde{R}_t)) + (1 - q_t(\tilde{R}_t))\theta}{\partial \tilde{R}_t} > 0 \)

(e) the expected return of an investment is a concave function of \( \tilde{R}_t \): \( \frac{\partial^2 f(q_t(\tilde{R}_t)) + (1 - q_t(\tilde{R}_t))\theta}{\partial \tilde{R}_t^2} < 0 \)

For a generic return function \( f(q_t) \) the bank’s objective function at the second stage is:

\[
\max_{q_t} f(q_t) \tilde{r}_{l,t} - q_t \tilde{r}_{d,t}(1 - k_t)
\]

Deriving this problem with respect to \( q_t \) yields the following first-order condition, which by concavity is necessary and sufficient:

\[
f'(q_t)\tilde{r}_{l,t} = \tilde{r}_{d,t}(1 - k_t)
\]  

(1.74)

Notice that this condition implies \( f'(q_t) > 0 \) \((k_t \in (0, 1] \text{ by assumption}, \tilde{r}_{d,t} > 0 \text{ by the deposit supply schedule, and } \tilde{r}_{l,t} > 0 \text{ by the zero profit condition})\). Notice further that in a frictionless world, e.g. without limited liability, the banks risk choice would satisfy \( q_t^{opt} = \arg\max f(q_t) + (1 - q_t)\theta \), i.e. \( f'(q_t^{opt}) = \theta \). Since we have assumed above that the recovery value is smaller than the deposit insurance cap, which in turn is smaller than the cost of deposits by lemma 1, we have: \( \tilde{r}_{l,t}\theta < \tilde{r}_t < \tilde{r}_{d,t}(1 - k_t) \). Combining this with equation (1.74) and the frictionless optimality condition and rearranging, we obtain \( f'(q_t) > f'(q_t^{opt}) \). Given \( f''(q_t) < 0 \) this implies excessive risk taking, i.e. \( q_t < q_t^{opt} \text{ (claim(a)).} \)

Since the deposit supply schedule must hold in equilibrium, we can rewrite this condition as

\[
f'(q_t)\tilde{r}_{l,t} - \frac{\tilde{R}_t(1 - k_t) + (1 - q_t)\tilde{\psi}_t}{q_t} = 0
\]  

(1.75)

Equation (1.75) implicitly defines \( \hat{q}_t(k_t) \). Using the implicit function theorem we find that, \( f'(q_t^{opt}) = \theta < \tilde{r}_{d,t}(1 - k_t) = f'(q_t)\tilde{r}_{l,t} \)
\[ \frac{\partial q_t}{\partial k_t} = \frac{-q_t \hat{R}_t}{(1 - k_t) \hat{R}_t - \hat{\psi}_t + q_t^2 \hat{r}_{l,t} f''(q_t)} \]

At the first stage the maximization problem is

\[ \max_{k_t} f(q_t) \hat{r}_{l,t} - q_t \hat{r}_{d,t} (1 - k_t) - q_t k_t \hat{r}_{e,t} . \]

which, using the deposit and equity supply schedules \( \hat{r}_{d,t} = \frac{\hat{R}_t - \frac{1 - q_t}{q_t} \hat{\psi}_t}{q_t} \) \( \hat{r}_{e,t} = \frac{\hat{R}_t + \hat{\xi}_t}{q_t} \), can be written as

\[ \max_{k_t} f(\hat{q}_t) \hat{r}_{l,t} + (1 - q_t) \hat{\psi}_t - k_t \hat{\xi}_t - \hat{R}_t . \]

The corresponding FOC is

\[ \left( f'(\hat{q}_t) \hat{r}_{l,t} - \hat{\psi}_t \right) \frac{\partial q_t}{\partial k_t} - \hat{\xi}_t . \] (1.76)

Finally, the zero profit condition can in expectations be written as

\[ f(\hat{q}_t) \hat{r}_{l,t} + (1 - q_t) \hat{\psi}_t - k_t \hat{\xi}_t - \hat{R}_t . \] (1.77)

Equations (1.75), (1.76), (1.77) implicitly define \( q_t, k_t \) and \( \hat{r}_{l,t} \). Solving the latter two equations for \( k_t \) and \( \hat{r}_{l,t} \) we obtain:

\[ k_t = \frac{\left( -\hat{\xi}_t \hat{R}_t + q_t \hat{R}_t \hat{\psi}_t + \hat{\xi}_t \hat{\psi}_t \right)}{-\hat{\xi}_t \left( \hat{R}_t f(q) - q_t \left( \hat{R}_t - (1 - q_t) \hat{\psi}_t \right) \right) \left( \hat{R}_t f'(q) + q_t \hat{\xi}_t f''(q) \right)} \left( f(q_t) - q_t \left( \hat{R}_t - (1 - q_t) \hat{\psi}_t \right) \left( \hat{R}_t f'(q_t) + q_t \hat{\xi}_t f''(q_t) \right) \right) \] (1.78)
\[
\tilde{r}_{l,t} = \frac{\left( \tilde{R}_t + \tilde{\xi}_t \right) \left( \tilde{R}_t - \tilde{\psi}_t \right)}{\tilde{R}_t f(q_t) - q_t \left( \tilde{R}_t f'(q_t) + q_t \tilde{\xi}_t f''(q_t) \right)}
\]

(1.79)

Plugging these equations into \((1.75)\) and rearranging we obtain the following equation, which implicitly defines \(q_t\)

\[
\left( \tilde{R}_t + \tilde{\xi}_t \right) \tilde{R}_t \tilde{\psi}_t f(q_t) - \left( \tilde{R}_t \left( \tilde{R}_t + \tilde{\xi}_t \right) - \left( 1 - q_t \right) \tilde{R}_t + \tilde{\xi}_t \right) \tilde{\psi}_t f'(q_t) - q_t \tilde{\xi}_t \left( \tilde{R}_t - (1 - q_t) \tilde{\psi}_t \right) f''(q_t)
\]

\[
= 0
\]

(1.79)

We can simplify this condition further by multiplying with the denominator and dividing by \(\left( \tilde{R}_t + \tilde{\xi}_t \right) \tilde{R}_t\)

\[
\frac{\tilde{R}_t \tilde{\psi}_t f(q_t) - \left( \tilde{R}_t \left( \tilde{R}_t + \tilde{\xi}_t \right) - \left( 1 - q_t \right) \tilde{R}_t + \tilde{\xi}_t \right) \tilde{\psi}_t f'(q_t) - q_t \tilde{\xi}_t \left( \tilde{R}_t - (1 - q_t) \tilde{\psi}_t \right) f''(q_t)}{\tilde{R}_t} = 0
\]

(1.80)

Using the implicit function theorem on equation \((1.80)\) we find that

\[
\frac{\partial q_t}{\partial R_t} = - \frac{\partial F}{\partial R_t} \frac{\partial F}{\partial q_t}
\]

(1.81)

where

\[
\frac{\partial F}{\partial R_t} = \frac{\left( \tilde{R}_t^2 + \tilde{\xi}_t \tilde{\psi}_t \right) f'(q_t) + (1 - q_t) q_t \tilde{\xi}_t \tilde{\psi}_t f''(q_t)}{-\tilde{R}_t^2}
\]

\[
\frac{\partial F}{\partial q_t} = \frac{\tilde{R}_t - (1 - q_t) \tilde{\psi}_t}{{\tilde{R}_t} \left( \left( \tilde{R}_t + 2 \tilde{\xi}_t \right) f''(q_t) + q_t \tilde{\xi}_t f''(q_t) \right)}
\]

Using our assumptions on \(f\), the parameters and assuming an interior solution it is obvious that \(\frac{\partial F}{\partial q_t} > 0\). How about the \(\frac{\partial F}{\partial R_t}\)?

To get at the sign of \(\frac{\partial F}{\partial R_t}\), we solve \((1.80)\) for \(f(q_t)\)
\[ f(q_t) = \frac{\left( \tilde{R}_t \left( \tilde{R}_t + \tilde{\xi}_t \right) - (1 - q_t)\tilde{R}_t + \tilde{\xi}_t \right) \psi_t}{\tilde{R}_t \psi_t} f'(q_t) + q_t \tilde{\xi}_t \left( \tilde{R}_t - (1 - q_t)\psi_t \right) f''(q_t) \]

and plug this expression into the equations (1.78) and (1.79) for \( k_t \) and \( \tilde{r}_{l,t} \):

\[
\begin{align*}
k_t &= \frac{f'(q_t)(\tilde{R}_t + \tilde{\xi}_t)(\tilde{R}_t - \tilde{\psi}_t) + f''(q_t)q_t\tilde{\xi}_t(\tilde{R}_t - (1 - q_t)\tilde{\psi}_t)}{\tilde{R}_t \left( \left( \tilde{R}_t + \tilde{\xi}_t \right) f'(q_t) + q_t\tilde{\xi}_t f''(q_t) \right)^2} \quad (1.82) \\
\tilde{r}_{l,t} &= \frac{\left( \tilde{R}_t + \tilde{\xi}_t \right) \tilde{\psi}_t}{\left( \tilde{R}_t + \tilde{\xi}_t \right) f'(q_t) + q_t\tilde{\xi}_t f''(q_t)} \quad (1.83)
\end{align*}
\]

Since in equilibrium \( \tilde{r}_{l,t} > 0 \) and since the numerator of \( \tilde{r}_{l,t} \) is obviously positive it must hold that its denominator is also positive:

\[
\left( \tilde{R}_t + \tilde{\xi}_t \right) f'(q_t) + q_t\tilde{\xi}_t f''(q_t) > 0 \quad (1.84)
\]

Similarly, since \( k_t > 0 \) and since the denominator of \( k_t \) is obviously positive, the numerator must be positive too:

\[
f'(q_t)(\tilde{R}_t + \tilde{\xi}_t)(\tilde{R}_t - \tilde{\psi}_t) + f''(q_t)q_t\tilde{\xi}_t(\tilde{R}_t - (1 - q_t)\tilde{\psi}_t > 0 \quad (1.85)
\]

Since \( f' > 0 \) and \( f'' < 0 \) we can conclude from the previous inequality that for any \( [x_1, x_2] \in \mathbb{R}^2 \) it must hold that \( f'(q_t)x_1 + f''(q_t)x_2 > 0 \) if

\[
\frac{x_1}{x_2} \geq \frac{\left( \tilde{R}_t + \tilde{\xi}_t \right) (\tilde{R}_t - \tilde{\psi}_t)}{q_t\tilde{\xi}_t(\tilde{R}_t - (1 - q_t)\tilde{\psi}_t)} \quad (1.86)
\]

We now test this condition for the numerator of \( \frac{\partial F}{\partial \tilde{R}_t} \):

\[
\frac{\tilde{R}_t^2 + \tilde{\xi}_t\tilde{\psi}_t}{(1 - q_t)q_t\tilde{\xi}_t\tilde{\psi}_t} \leq \frac{\left( \tilde{R}_t + \tilde{\xi}_t \right) (\tilde{R}_t - \tilde{\psi}_t)}{q_t\tilde{\xi}_t(\tilde{R}_t - (1 - q_t)\tilde{\psi}_t)}
\]

Rearranging, multiplying only with positive values, yields

45
\[ 0 \leq -\tilde{R}_t (\tilde{R}_t - (1 - q_t)\tilde{\psi}_t) - q_t \tilde{\psi}_t \tilde{\xi}_t - \left( \tilde{R}_t (1 - q_t)\tilde{\psi}_t + (1 - q_t)\tilde{\psi}_t \tilde{\xi}_t \right) \frac{q_t \tilde{\psi}_t}{\tilde{R}_t - (1 - q_t)\tilde{\psi}_t} \]

The RHS is obviously negative since from the proposition that every deposit insurance cap will be exceeded it follows that \( \tilde{R}_t > \tilde{\psi}_t \). Hence the condition \( \frac{\tilde{R}_t^2 + \tilde{\xi}_t \tilde{\psi}_t}{(1 - q_t)q_t \tilde{\xi}_t \tilde{\psi}_t} \geq \frac{\tilde{R}_t + \tilde{\xi}_t}{q_t \tilde{\psi}_t} \) is satisfied and we can conclude the the numerator of \( \frac{\partial F}{\partial \tilde{R}_t} \) is positive. Hence \( \frac{\partial F}{\partial \tilde{R}_t} < 0 \) and therefore \( \frac{\partial k_t}{\partial \tilde{R}_t} > 0 \) (claim (b)).

Equation (1.82) defines \( k_t = \tilde{R}(q_t, \tilde{R}_t) \). Its derivative is given by

\[ \frac{\partial k_t}{\partial \tilde{R}_t} = \frac{\partial \tilde{R}}{\partial \tilde{R}_t} + \frac{\partial \tilde{R}}{\partial q_t} \frac{\partial q_t}{\partial \tilde{R}_t} \]

where

\[ \frac{\partial \tilde{R}}{\partial \tilde{R}_t} = \left( (f'(q_t))^2 \tilde{R}_t^2 + 2f'(q_t) f'(q_t) (1 - q_t)q_t \tilde{R}_t \tilde{\xi}_t + (f'(q_t) + f''(q_t)q_t) (f'(q_t) - f''(q_t)(1 - q_t)q_t) \tilde{\xi}_t \right) \tilde{R}_t^2 \left( f'' \tilde{\xi}_t + f'(\tilde{R}_t + \tilde{\xi}_t) \right)^2 \]

\[ \frac{\partial \tilde{R}}{\partial q_t} = \frac{q_t \tilde{\xi}_t f''(q_t) \tilde{\psi}_t}{\tilde{R}_t} \left( \frac{2 \left( \tilde{R}_t + \tilde{\xi}_t \right) f'(q_t) + q_t \tilde{\xi}_t f''(q_t)}{\tilde{R}_t \left( f'' \tilde{\xi}_t + f'(\tilde{R}_t + \tilde{\xi}_t) \right)^2} \right) \]

From (1.86) it is immediately obvious that the numerator of \( \frac{\partial \tilde{R}}{\partial q_t} \) is negative, hence \( \frac{\partial \tilde{R}}{\partial q_t} < 0 \). After division by \( \tilde{\psi}_t \), the numerator of \( \frac{\partial \tilde{R}}{\partial \tilde{R}_t} \) can be rewritten as

\[ \left( \left( \tilde{R}_t + \tilde{\xi}_t \right) f'(q_t) + q_t \tilde{\xi}_t f''(q_t) \right)^2 - (f''(q_t))^2 q_t^3 \tilde{\xi}_t^2 - f''(q_t) f'(q_t) q_t^2 \tilde{\xi}_t^2 (2\tilde{R}_t + \tilde{\xi}_t) \]

Since the first term is positive and bigger then the absolute value of the second term we can see that \( \frac{\partial k_t}{\partial \tilde{R}_t} > 0 \). Hence we have shown that \( \frac{\partial k_t}{\partial \tilde{R}_t} > 0 \) (claim (c)).

Applying the implicit function theorem a second time on equation (1.80) we can find the following expression for the second derivative of \( q_t \)

\[ \frac{\partial^2 q_t}{\partial \tilde{R}_t^2} = \left( \frac{\partial^2 F}{\partial \tilde{R}_t \partial q_t} + \frac{\partial^2 F}{\partial q_t^2} \frac{\partial q_t}{\partial \tilde{R}_t} \right) \frac{\partial F}{\partial \tilde{R}_t} - \left( \frac{\partial^2 F}{\partial \tilde{R}_t \partial q_t} \frac{\partial q_t}{\partial \tilde{R}_t} + \frac{\partial^2 F}{\partial q_t^2} \right) \frac{\partial F}{\partial q_t} \left( \frac{\partial^2 F}{\partial q_t} \right)^2 \]

(1.87)
where

\[
\frac{\partial^2 F}{\partial \tilde{R}_t \partial q_t} = \frac{\left( \tilde{R}_t^2 + 2(1 - q_t)\tilde{\xi}_t \tilde{\psi}_t \right) f''(q_t) + q_t (1 - q_t)\tilde{\xi}_t \tilde{\psi}_t f'''(q_t)}{-\tilde{R}_t}
\]

\[
\frac{\partial^2 F}{\partial q_t^2} = \tilde{\psi}_t \left( q_t \tilde{\xi}_t f'''(q_t) + \left( \tilde{R}_t + 2\tilde{\xi}_t \right) f''(q_t) \right) + \left( f'''(q_t) q_t \tilde{\xi}_t + f''(q_t) \left( \tilde{R}_t + 3\tilde{\xi}_t \right) \right) \left( \tilde{R}_t - (1 - q_t)\tilde{\psi}_t \right)
\]

\[
\frac{\partial^2 F}{\partial R_t^2} = 2 \left( f'(q_t) + f''(q_t)(1 - q)q \right) \tilde{\xi}_t \tilde{\psi}_t
\]

since \( f'' < 0 \) and \( f''' \leq 0 \) and all parameters are positive it is obvious that \( \frac{\partial^2 F}{\partial R_t \partial q_t} > 0 \) and \( \frac{\partial^2 F}{\partial q_t^2} > 0 \). The term \( \frac{\partial^2 F}{\partial R_t^2} \) is less straightforward. A sufficient condition for \( \frac{\partial^2 F}{\partial R_t^2} > 0 \) can be found using again condition (1.86)

\[
\frac{1}{(1 - q_t)q_t} \geq \frac{\left( \tilde{R}_t + \tilde{\xi}_t \right) \left( \tilde{R}_t - \tilde{\psi}_t \right)}{q_t \tilde{\xi}_t \left( \tilde{R}_t - (1 - q_t)\tilde{\psi}_t \right)}
\]

Which simplifies to

\[
\frac{q_t}{(1 - q_t)\tilde{\xi}_t} \geq \tilde{R}_t - \tilde{\psi}_t
\]

Given the signs of the terms in (1.87) we have finally verified that

\[
\frac{\partial^2 q_t}{\partial R_t^2} = \frac{((+) + (+) (+)) (-) - ((+) (+) (+)) (+)}{(+)} < 0
\]

Under alternative assumption (2b) the expression for \( \frac{\partial^2 q_t}{\partial R_t^2} \) simplifies to

\[
\frac{\partial^2 q_t}{\partial R_t^2} = - \frac{f'(q_t) \left( -2 f''(q_t)f'''(q_t)q_t \tilde{\xi}_t - 2 (f''(q_t))^2 \left( \tilde{R}_t + 2\tilde{\xi}_t \right) + f'(q_t) \left( f'''(q_t)q_t \tilde{\xi}_t + f''(q_t) \left( \tilde{R}_t + 3\tilde{\xi}_t \right) \right) \right)}{\left( f'''(q_t) \tilde{\xi}_t + f''(q_t) \left( \tilde{R}_t + 2\tilde{\xi}_t \right) \right)^3}
\]
which is negative without further conditions.

Using the signs of the derivatives of $q_t$ and the fact that $f'(q_t) - f'(q_{out}) = \theta$, we can finally determine the slope and curvature of the expected return of the bank’s investment.

$$\frac{\partial [f(q_t) + (1 - (q_t))\theta]}{\partial \tilde{R}_t} = (f'(q_t) - \theta) + \frac{\partial q_t}{\partial \tilde{R}_t} > 0$$

$$\frac{\partial^2 [f(q_t) + (1 - (q_t))\theta]}{\partial \tilde{R}_t^2} = (f'(q_t) - \theta) + \frac{\partial q_t}{\partial \tilde{R}_t} + f''(q_t) - \frac{\partial q_t}{\partial \tilde{R}_t} < 0$$

This completes the proof of claims (d) and (e).

Notice that the quadratic functional form we assumed for $f(q_t)$ in the model section satisfies assumptions (3) and (4) and we focussed on interior solutions (assumption (1)). Therefore claims (1), (2), (3) and (4) in propositions 1 and 2 hold. Furthermore, claim (5) in proposition 1 holds since assumption (2a) is satisfied. Finally, to see that claim (5) in proposition 2.1 holds independent of assumption (2a) and (2b), consider the solution for $q_t$

$$q_t = 1 - \frac{\tilde{R}_t}{\tilde{\psi}_t} + \frac{\sqrt{\omega_2 \left( \tilde{R}_t - \tilde{\psi}_t \right) (\tilde{R}_t + 2\tilde{\xi}_t) \left( 2\omega_1 \tilde{\psi}_t (\tilde{R}_t + \tilde{\xi}_t) + \omega_2 (\tilde{R}_t - \tilde{\psi}_t) (\tilde{R}_t + 2\tilde{\xi}_t) \right)}}{\omega_2 \tilde{\psi}_t (\tilde{R}_t + 2\tilde{\xi}_t)}$$

The second derivative of this expression is

$$\frac{\partial^2 q}{\partial \tilde{R}_t^2} = - \left( \left[ \tilde{R}_t + \tilde{2\xi}_t \right] \left( \omega_2 \left( \tilde{R}_t - \tilde{\psi}_t \right) (\tilde{R}_t + 2\tilde{\xi}_t) + \omega_1 \tilde{\psi}_t \ldots \right) \left( \omega_1 \omega_2 \left( 2\omega_2 \left( \tilde{R}_t - \tilde{\psi}_t \right)^3 \tilde{\xi}_t (\tilde{R}_t + 2\tilde{\xi}_t) + \omega_1 \tilde{\psi}_t \ldots \right) \left[ 2\omega_1 \tilde{\psi}_t (\tilde{R}_t + \tilde{\xi}_t) + b \left( \tilde{R}_t - \tilde{\psi}_t \right) (\tilde{R}_t + 2\tilde{\xi}_t) \right] \right) \right)^{2/3}$$

Both numerator and denominator are obviously positive, so $\frac{\partial^2 q}{\partial \tilde{R}_t^2} < 0$. Hence $\frac{\partial^2 [f(q_t) + (1 - (q_t))\theta]}{\partial \tilde{R}_t^2} < 0$. 

$\blacksquare$
Deposits in excess of insurance

The proof is by contradiction: Assume that there exists an equilibrium with no excess profits where the bank would issue so little deposits that the promised repayment $r_{d,t}$ would be lower than the cap on deposit insurance $\psi/(1 - k_t)\pi_{t+1}$ \[43\] In this case the deposit rate $r_{d,t}$ would be equal to the risk free rate $R_t$. The second stage maximization problem of the bank would then be

$$\max_{q_t \in [0,1]} f(q_t) - q_t \tilde{R}_t(1 - k_t)$$

and its solution $\hat{q}_t$ is implied by

$$f'(q_t) = \tilde{R}_t(1 - k_t)$$

The first stage maximization problem would be

$$\max_{k_t \in [0,1]} V(k) = f(\hat{q}_t) - \hat{q}\tilde{R}_t(1 - k_t) - (\tilde{\xi}_t + \tilde{R}_t)k_t$$

$\hat{q}_t$ can either be a corner or an interior solution. If $\hat{q}_t$ is a corner solution, the first stage objective function of the bank is obviously decreasing in $k_t$, hence $k_t = 0$ is optimal. If $\hat{q}_t$ is an interior solution, the first derivative of the first stage objective function is

$$\tilde{R}_t - \tilde{\xi}_t - \tilde{R}_t(1 - \hat{q}_t)$$

Since $\hat{q}_t \in [0,1]$ this derivative is negative for all $k_t \in [0,1]$, i.e. the objective function again is decreasing in $k$. Hence the solution to the first stage problem is $k_t = 0$. Optimality with full insurance therefore requires that the bank uses only deposits. This contradicts our initial assumption. This result implies that any insurance cap smaller than 100% would be exceeded by the deposit liabilities in case of default. Depositors are therefore never fully insured.

Notice that for a cap to be effective in the sense of ruling out full insurance equilibria, the cap has to be low enough. Formally speaking, it needs to hold that $r_{d,t}(1 - k_t) > \psi_t$ even under full insurance, i.e. $\tilde{R}_t > \psi_t$. \[44\]

\[43\] For simplicity, we abstract from the possibility that the cap is binding for some states of the future but not for others, which would be possible due to the inconsistency between the timing of inflation and the nominal deposit rate. Note that this distinction disappears under certainty equivalence or first order approximation.
Table 1.5: Data description: All level variables are expressed in per-capita terms (divided by \( N \)). Hours are measured as \( H_1 \cdot H_2 / N \) where \( H_1 \) is converted into an index. The nominal wage \( W \) is deflated by the GDP deflator. We define equity capital as equity plus reserves plus subordinated debt, and total liabilities as equity plus deposits. In doing so we net out two types of liabilities, since they are typically overcollateralized: federal funds purchased & repurchase agreements and federal home loan bank advances. Furthermore we omit a few categories of debt that match neither of our concepts of insured deposits and equity, or that are simply not well enough characterized: other borrowed money, uncategorized liabilities, trading book liabilities, banks liability on acceptances. All of these balance sheet positions are minor. Over the observation period, the first group accounts for roughly 11% of the balance sheet, the second for about 9%. All indexes are adjusted such that 2009 = 100. The estimation sample spans from 1984Q1 to 2007Q3 for the DSGE and from 1997Q2 to 2007Q3 for the VAR.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Series</th>
<th>Measure</th>
<th>Unit</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y )</td>
<td>real gross domestic product</td>
<td>GDP, C96</td>
<td>bn. USD</td>
<td>FRED / BEA</td>
</tr>
<tr>
<td>( P )</td>
<td>GDP deflator</td>
<td>GDPDEF</td>
<td>index</td>
<td>FRED / BEA</td>
</tr>
<tr>
<td>( R )</td>
<td>effective federal funds rate</td>
<td>FEDFUNDS</td>
<td>%</td>
<td>FRED / BOARD OF GOVERNORS</td>
</tr>
<tr>
<td>( C )</td>
<td>personal consumption expenditure</td>
<td>FCSEC</td>
<td>bn. USD</td>
<td>FRED / BEA</td>
</tr>
<tr>
<td>( I )</td>
<td>fixed private investment</td>
<td>FPI</td>
<td>bn. USD</td>
<td>FRED / BEA</td>
</tr>
<tr>
<td>( H_1 )</td>
<td>civilian employment</td>
<td>CE16OV</td>
<td>thousands</td>
<td>FRED / BLS</td>
</tr>
<tr>
<td>( H_2 )</td>
<td>nonfarm business (…) hours</td>
<td>prs85096023</td>
<td>index</td>
<td>DEPARTMENT OF LABOR</td>
</tr>
<tr>
<td>( W )</td>
<td>nonfarm business (…) hourly compensation</td>
<td>prs85006103</td>
<td>index</td>
<td>DEPARTMENT OF LABOR</td>
</tr>
<tr>
<td>( N )</td>
<td>civilian population</td>
<td>LNS1000000</td>
<td>0CE16OV</td>
<td>BLS</td>
</tr>
<tr>
<td>( q )</td>
<td>average weighted loan risk</td>
<td>own calculation</td>
<td>%</td>
<td>BOARD OF GOVERNORS</td>
</tr>
<tr>
<td>( E )</td>
<td>equity capital over liabilities</td>
<td>own calculation</td>
<td>%</td>
<td>FDIC</td>
</tr>
</tbody>
</table>

Figure 1.4: Bank risk taking and nominal interest rate: The risk measure (solid blue line, left axis) is redefined such that a decrease is associated with higher risk-taking of the banking sector, matching the definition in the theoretical model discussed later. The nominal interest rate (dashed line, right axis) is the effective federal funds rate.
Figure 1.5: An expansionary monetary policy shock - Recursive identification scheme, Federal funds rate ordered last: Error bands correspond to 90% confidence intervals obtained by bootstrap. Loan safety is defined as the inverse of the average loan risk rating, standardized to take values between 0 and 100. The remaining variables are annualized. See text for further details.

Figure 1.6: An expansionary monetary policy shock - Recursive identification scheme, Risk ordered last: Error bands correspond to 90% confidence intervals obtained by bootstrap. Loan safety is defined as the inverse of the average loan risk rating, standardized to take values between 0 and 100. The remaining variables are annualized. See text for further details.
Chapter 2

Point, interval and density forecasts of exchange rates with time-varying parameter models

*Joint with Prof. Massimiliano Marcellino*

2.1 Introduction

Exchange rates have an impact on the production decision of firms, on portfolio allocation, on a country’s prices, and more generally on its competitiveness. Hence, there is a clear need for reliable models that track the current evolution of exchange rates and predict their future behaviour, especially in times of uncertainty and financial stress. In fact, exchange rate volatility has changed over the years. It has fallen after the price shocks and inflationary pressures of the 1970s, and it has increased again in the last decade, as a consequence of the great financial crisis and possibly of the quantitative easing measures enacted by central banks around the world.

So far, a vast literature has been devoted to the construction and evaluation of the point forecasts of exchange rates. It has established, with few exceptions, that the best forecast model is a simple random walk. This puzzle, originated by the seminal work of Meese and Rogoff [1983], has not yet been solved. One explanation that has been proposed is that exchange rate unpredictability arises from the instability of the underlying stochastic process. In addition, competing models have so far been evaluated mainly on the basis of their point forecasts. Though the latter are clearly of interest, interval and density forecasts of exchange rates are also relevant for the decision making of economic agents and for the pricing of financial assets. On this latter point the literature is more limited.
Our contribution to the literature is to examine whether and to what extent the point, interval and density forecasts of three major exchange rates vis-a-vis the US dollar can be improved by assuming time variation in the coefficients of the data generating process. The exchange rates analysed are the monthly averages of the British Pound, the Japanese Yen, and the Euro, over the period 1971m1 to 2013m6. As it can be seen in figure 2.1, the volatility of these three currencies has changed over time: a constant-volatility model could therefore lead to the incorrect estimation of forecast intervals, underestimating them in periods of high volatility and overestimating them otherwise.

To model time variation, we experiment with two methods recently proposed in the literature: the time-varying parameter Bayesian vector autoregression with stochastic volatility developed by Cogley and Sargent [2005] and Primiceri [2005], and its approximation proposed by Koop and Korobilis [2013], based on forgetting factors and on an exponentially weighted moving average estimator of the shocks’ covariance matrix. Unlike the Bayesian model, the forgetting factor VAR does not take into account sampling uncertainty. On the other hand, it is considerably faster to estimate and is not as densely parametrised as its Bayesian counterpart, allowing for possible efficiency gains. The performance of these models is compared to that of two benchmarks, a Bayesian vector autoregression and a random walk (with and without GARCH innovations), by juxtaposing the respective point, interval and density forecasts.

Several results emerge. First, parameter time variation leads to smaller mean squared forecast errors, though the random walk remains the best forecast model at horizons longer than one month. Secondly, modelling time-varying volatility significantly enhances the estimation of forecast uncertainty through an accurate calibration of the entire forecast distribution. By contrast, time variation in the slope parameters is found to be modest and to offer only a negligible contribution to forecast improvement. Lastly, models with time-varying parameters typically perform better than their constant-parameter counterparts also when evaluated in terms of gains from forecast based trading strategies.

To our knowledge our comprehensive evaluation is the first of its kind in the empirical literature on exchange rate forecasting, not only for the methodology used but also for the emphasis on interval and density forecasts. A wide variety of methods has been used in the empirical literature on exchange-rate forecasting, and the consensus is that the most difficult benchmark to beat, in terms of point forecast accuracy, is the random walk. Carriero et al. [2009] and Dal Bianco et al. [2012] have improved upon the point forecasts of a random walk by relying respectively on a Bayesian vector autoregression with a large set of exchange rates, and on a mixed-frequency dynamic factor model with four weekly exchange rates and lower-frequency macroeconomic

\[^1\] As documented in Rossi [2013], the methodologies that have been shown to deliver lower mean squared forecast errors than a random walk are typically sensitive to the forecast horizon and to the sample used.
Figure 2.1: Stylized facts: Exchange rate volatility over the years, measured as the square monthly percentage change in the exchange rate.

fundamentals. Very few papers have instead focused on density forecasts. Relevant exceptions are Yongmiao et al. [2007] and Balke et al. [2013], who both show that the density forecasts of a random walk can be improved upon either with non-linear models, or with univariate Taylor-rule models with semiparametric confidence intervals. In addition, Mumtaz and Sunder-Plassmann [2013] show that a structural time-varying stochastic volatility vector autoregression outperforms its constant-parameter counterpart on the basis of the mean squared forecast error and of the Bayesian deviance information criterion. A paper closely related to ours is Della Corte et al. [2009], who establish that modelling time-varying volatility is important for the one-month ahead predictive ability of macroeconomic fundamentals. In contrast to the aforementioned works, our forecast evaluation exercise comprises forecast horizons greater than one month, a wider set of evaluation criteria, and is based on a larger forecast sample that includes the 2008 financial crisis.

The paper is organised as follows. In the next Section we describe the two time-varying parameter models used in the forecasting exercise. Section 3 compares the results delivered by the different models based on statistical criteria, while Section 4 evaluates the models through a simple trading strategy. Section 5 concludes.

Another related paper is Canova [1993] who shows, on a sample of weekly data from 1979 to 1987, that a time-varying coefficient Bayesian model with exchange rates and short-term interest rates has a higher predictive ability than a random walk. Our work stems from a similar idea, though both the methodology and the time span considered significantly differ.

Della Corte et al. [2009] employ statistical evaluation criteria, relative mean squared errors and log-likelihoods, as well as economical ones, based on the utility function of investors. In addition, the economic criterion supports an optimal combination of the model forecasts through Bayesian model averaging.
2.2 Models with time-varying parameters and changing volatility

2.2.1 The time-varying parameter stochastic volatility BVAR

We start with a short description of the time-varying parameter stochastic volatility Bayesian vector autoregression (TVP SV BVAR), developed by Cogley and Sargent [2005] and Primiceri [2005], to whom we refer for additional details. The model is written in state space form, and the measurement equation is:

\[ y_t = Z_t \beta_t + u_t, \]  

(2.1)

where \( y_t \) is a \( n \times 1 \) vector of observed variables, \( Z_t \) is a \( n \times k \) matrix of regressors, \( \beta_t \) is a \( k \times 1 \) vector of time-varying coefficients and \( u_t \) is a \( n \times 1 \) vector of innovations with covariance matrix \( \Omega_t \). Let \( Z_t \) contain a constant and \( p \) lags of each variable; it is then defined as \( Z_t = I_n \otimes [1, y'_{t-1}, \ldots, y'_{t-p}] \) with dimension \( n \times k = n \times n(1 + np) \).

Following Primiceri [2005], the covariance matrix \( \Omega_t \) can be decomposed as follows:

\[ A_t \Omega_t A'_t = \Sigma_t \Sigma'_t, \]  

(2.2)

where \( \Sigma_t \) is a diagonal matrix, with the standard deviations of the structural innovations as its elements; while \( A_t \) is a lower triangular matrix with ones on its main diagonal, which summarises the contemporaneous relationships between the variables in \( y_t \). Using the structural decomposition in 2.2, the measurement equation 2.1 can be rewritten in terms of the white-noise, homoskedastic and uncorrelated shocks \( \varepsilon_t \):

\[ y_t = Z_t \beta_t + A_t^{-1} \Sigma_t \varepsilon_t, \quad \text{with} \quad E[\varepsilon_t, \varepsilon'_t] = I_n. \]  

(2.3)

To close the model, three transition equations are specified, describing the evolution of the parameters over time:

\[ \beta_t = \beta_{t-1} + \nu_t, \]  
\[ \alpha_t = \alpha_{t-1} + \xi_t, \]  
\[ \log \sigma_t = \log \sigma_{t-1} + \eta_t, \]  

(2.4)

\[ ^4\text{The decomposition in 2.2 emphasises the two drivers of the time variation in } \Omega_t: \text{ variation in the variance of the innovations and in their correlation structure.} \]
where $\alpha_t$ is the vector of the non-zero, non-one elements of $A_t$ stacked by rows, and $\sigma_t$ is the vector of the diagonal elements in $\Sigma_t$. While the slope coefficients and those in the contemporaneous impact matrix are assumed to follow a random walk, the standard deviations of the structural innovations are modelled as geometric random walks. Finally, all the innovations of the model are posited to be distributed as a multivariate normal, with zero mean and with the following block diagonal covariance matrix:

$$V = \text{Var} \begin{bmatrix} \varepsilon_t \\ \nu_t \\ \xi_t \\ \eta_t \end{bmatrix} = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{bmatrix}. \tag{2.5}$$

The objectives of the estimation are the unobserved paths of the parameters in 2.4, indicated by $(B^T, A^T, \Sigma^T)$, and the hyperparameters in $V$. Sampling from the posterior density requires the specification of a prior distribution, as well as the use of a posterior simulator algorithm. A description of both is given below.

The covariance matrices $(Q, W, S)$ are assumed to have an inverse-Wishart distribution and to be therefore characterised by a number of degrees of freedom and a scale matrix, set to a constant fraction of the covariance matrix’s training sample estimate. The initial states of the three types of time-varying coefficients, $\beta_0, \alpha_0, \log \sigma_0$, are assumed to be normally distributed. This, together with the transition equations in 2.4, implies that conditional on $(Q, W, S)$ the prior distributions of the entire sequence of VAR coefficients, contemporaneous relationships, and log standard deviations are themselves normal. Further details on the prior specification, related to the empirical application of this paper, are provided in Section 2.3.

**Sampling from the posterior density**

To generate a sample from the posterior of $(B^T, A^T, \Sigma^T, V)$ we rely on a Gibbs-sampler, following [Primiceri 2005] and [Del Negro and Primiceri 2013].

The first step is to sample the sequence of VAR coefficients $\beta^T$, given an initial guess of the parameters. For this task, a simulation smoother like the one proposed in [Carter and Kohn 1994] can be used, exploiting the fact that the distribution of $\beta^T$, conditional on $A^T$ and $\Sigma^T$, is linear and normal. The sequence of $A^T$ can be drawn in a similar way, as its posterior distribution is normal, given $B^T$ and $\Sigma^T$.

To draw the sequence of standard errors, the model needs to be transformed, given that it is neither linear nor Gaussian in $\Sigma^T$. More specifically, at this stage of the sampler the innovations to the measurement equation are distributed as a log $\chi^2$. The transformation of the system can be achieved by using a mixture of normal approximations of the log $\chi^2$ distributions, as described
in Kim et al. [1998]. After sampling \( s^T \), the matrix of indicator variables that rules the normal approximation, the system is approximately linear and Gaussian, conditional on \( A^T, B^T, V \) and \( s^T \): a standard simulator smoother can then be applied, to recover the smoothed estimates of the volatility and of the variance of its innovations.

The last step is a draw from the inverse-Wishart distributions of the block components of \( V \), yielding a sample of the model’s covariance matrix.

### Sampling from the predictive density

Let us denote with \( y^t \) and \( \theta^t = (B^t, A^t, \Sigma^t, V) \), the history of the variables and of the coefficients from period 1 up to period \( t \). We want to forecast up to \( h \) steps ahead in the future, that is, to make predictions on the vector \( y^{t+h} = [y_{t+1}^t, \ldots, y_{t+h}^t] \). For this, we need the predictive density of the TVP SV BVAR model, which can be factored as follows, emphasising the different sources of forecast uncertainty:

\[
p(y^{t+i}, \theta^{t+i} | y^t) = p(y^{t+i} | \theta^{t+i}, y^t) \cdot p(\theta^{t+i} | \theta^t, y^t) \cdot p(\theta^t | y^t), \quad i = 1, \ldots, h .
\]  

(2.6)

To make the simulation from the predictive density \( p(y^{t+h}, \theta^{t+h} | y^t) \) less time consuming, we assume that the coefficients in \( \theta^{t+i} \) are fixed out of sample\(^5\). Conditional on each Gibbs sampler draw from \( p(\theta^t | y^t) \), we simulate a value for \( \beta^{t+1} \) by drawing the innovations \( \nu_{t+1} \) in 2.4 and for the innovations \( u_{t+1} \), drawn from a Normal distribution with variance \( \Omega_t \). A path for \( \hat{y}_{t+i}, i = 1 \ldots h \) is then generated, conditioning on \( \hat{y}_{t+i-1} \) and on \( u_{t+i} \sim N(0, \Omega_t) \). We repeat this procedure a thousand times, and store the mean and the relevant percentiles of the simulated values \( \{\hat{y}_{t+i, \kappa}, i = 1 \ldots h\}_{\kappa=1}^{1000} \). After the Gibbs sampler is completed, we take the average of these values across the posterior draws.

### 2.2.2 The time-varying parameter forgetting factor VAR

Using the stochastic volatility Bayesian VAR for a recursive forecasting exercise generally demands a high computational time, as the number of iterations required for the convergence of the Gibbs sampler is large. To reduce the computational time, Koop and Korobilis [2013] have developed a procedure which approximates the model in 2.1, by replacing the posterior draws of the covariance matrices \( Q \) and \( \Sigma_t \) with empirical estimates.

---

5The parameters used for the approximation to the log \( \chi^2 \) distribution are those of Primiceri [2005].

6This simplification is justified only if the time variation of the coefficients is moderate and, as is the case in our empirical application. Results obtained by letting \( \theta^{t+i} \) drift for \( h \geq 1 \) are indeed very similar and available upon request.
The assumptions of the TVP SV BVAR model imply that, given information up to \( t - 1 \), the slope coefficients in \( t \) are draws from a normal distribution:

\[
\beta_t | \mathcal{F}_{t-1} \sim N(\beta_{t|t-1}, P_{t|t-1}).
\]

(2.7)

The Kalman filter routine, used in the first step of the Gibbs sampler, entails a prediction for the coefficients’ covariance matrix, \( P_{t|t-1} = P_{t-1|t-1} + Q \), which involves the posterior draw of \( Q \). To circumvent this problem, the following approximation is used:

\[
P_{t|t-1} = \frac{1}{\lambda} P_{t-1|t-1}, \quad \text{with } \lambda \in (0, 1],
\]

(2.8)

where the parameter \( \lambda \) is a forgetting factor which discounts past information. A value of \( \lambda \) equal to 0.99 implies, in the case of monthly data, that observations one year ago receive 89% as much weight as current observations.

A similar approximation is used for the covariance matrix of the non-structural innovations, \( \Omega_t \). The latter is estimated as a weighted average of its past value, and of its current estimate:

\[
\hat{\Omega}_t = \kappa \hat{\Omega}_{t-1} + (1 - \kappa) \hat{\varepsilon}_t', \hat{\varepsilon}_t,
\]

(2.9)

where the weight is represented by the decay factor \( \kappa \). To summarise, the procedure developed by Koop and Korobilis [2013] is based on the Kalman filter and relies on the parametrisation of equations 2.8 and 2.9 as well as on the choice of initial conditions for the covariance matrix \( \Omega_0 \), for the slope coefficients \( \beta_0 \) and their variance \( P_0 \). Further details on the parametrisation are provided in the next Section.

There are three main differences between the TVP SV BVAR model and its approximation based on the use of forgetting factors. Firstly, the latter delivers filtered, rather than smoothed, estimates and should hence be better suited for a forecasting exercise but less suited for a full sample evaluation. Secondly, and more importantly, equations 2.8 and 2.9 in the forgetting factor model do not provide any rule for the out-of-sample evolution of the covariance matrices \( Q \) and \( \Omega_t \). Lastly, while the TVP SV BVAR embeds a structural decomposition of the innovations covariance matrix, its approximation deals solely with non-structural innovations. However, this should not be a concern, so long as the objective of the researcher lies in forecasting, rather than in a structural analysis.

**Sampling from the predictive density**  As it has been already noted, equation 2.9 is not a proper law of motion for the covariance matrix of the innovations. In order to generate samples from the predictive density, we follow Koop and Korobilis [2013] and assume \( \Omega_t \) to be fixed out of

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7 This is the Exponentially Weighted Moving Average estimator, commonly used in the finance literature.
sample. In a similar way, the out-of-sample path for the slope coefficients $\beta_{t+h}$ is assumed to be fixed out of sample and centred around the last estimated values for $\beta_{t|t}$ and for $\hat{P}_{t|t}$. Given these assumptions, we simulate 5000 values for the vector $\hat{y}_{t+h} = [\hat{y}_{t+1}, \ldots, \hat{y}_{t+h}]$, and store the mean and the relevant percentiles of the values $\{\hat{y}_{t+i,\kappa}, i = 1 \ldots h\}_{\kappa=1}^{5000}$.

### 2.3 Modelling and forecasting exchange rates

In this Section the two methodologies previously discussed are applied to jointly model and forecast three main exchange rates vis-a-vis the US dollar: the British Pound, the Japanese Yen\(^8\) and the Euro\(^9\). The currencies, defined such that an increase pertains to a depreciation, cover the period from 1971m1 to 2013m6 and have been downloaded from Datastream. Figure 2.1 shows that the volatility of the three exchange rates has changed over the years. In particular, note that after the 2008 financial crisis the volatility of the Euro and of the Pound has increased after a period of relative moderation in the previous decade.

After a description of the empirical exercise, we assess the out-of-sample forecasting performance of the time-varying parameter models relative to two benchmarks: a random walk with or without GARCH innovations, and a constant parameter BVAR estimated recursively.

#### 2.3.1 Description of the estimation and forecasting exercises

To correctly compare all forecast models, we ensure that all of them are estimated on the same data transformation and that their priors, or initial values, coincide. The three currencies are measured as percentage changes, approximated through the differences in the log levels. This transformation, common in the literature, is necessary to ensure the stability of the TVP SV BVAR forecasts. All models but the GARCH are estimated using a lag length of 12, to capture any seasonal component that might be present in the data. We use a driftless random walk prior for the slope coefficients of all the VARs, and set the priors (or initial values) for the covariance matrix of the innovations and of the slope parameters through a training sample of four years.

The remaining details of the TVP SV BVAR model prior distribution follow closely those in Primiceri [2005] and are summarised in table 2.1. The estimation coincides with the procedure

---

\(^8\)The Yen has been standardised by a factor of a 100 to avoid computational problems due to the scale of the variable.

\(^9\)As we need a long sample for the estimation, we proxy the Euro through the German Mark series provided by the Bank of England and retrieved from Datastream. The series is equal to the German Mark prior to the introduction of the Euro, and to the Euro itself afterwards, scaled by the conversion rate fixed on 1 January 1999. After January 1999 the Euro and the series used in this paper exhibit a correlation equal to 0.995. Furthermore, Bruggemann et al. [2008] show that back-casting a post-euro area series with German data works as well as back-casting it with an Euro-area average, if the series behave similarly to each other.

---
described in the theoretical section with one major modification: explosive draws of the VAR coefficients are rejected in the first step of the Gibbs sampler. This is equivalent to sampling from a restricted posterior density, with the restricted law of motion for the VAR coefficients being a truncated and renormalised version of the unrestricted one\(^\text{[10]}\).

Table 2.1: The hat notation refers to OLS estimates on a 4-year training sample. M and K are the number of variables and of parameters. S\(_1\) and S\(_2\) pertain to the non-zero blocks of S, the covariance matrix of A. For the variables with an Inverse-Wishart prior distribution, the chosen number of degrees of freedom is the lowest admissible (one more than the size of the variable), to minimise the prior weight. The shrinkage parameters k\(_Q\), k\(_S\), k\(_W\) are all set to 0.1.

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>DISTRIBUTION</th>
<th>MEAN</th>
<th>VARIANCE</th>
<th>D.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B(_0)</td>
<td>N</td>
<td>(\hat{A})(_{ols})</td>
<td>4 (\cdot) V((\hat{B})(_{ols}))</td>
<td>–</td>
</tr>
<tr>
<td>A(_0)</td>
<td>N</td>
<td>(\hat{B})(_{ols})</td>
<td>4 (\cdot) V((\hat{A})(_{ols}))</td>
<td>–</td>
</tr>
<tr>
<td>log (\sigma)_0</td>
<td>N</td>
<td>log (\hat{\sigma})(_{ols})</td>
<td>–</td>
<td>(I_M)</td>
</tr>
<tr>
<td>Q</td>
<td>IW</td>
<td>k(<em>Q) (\cdot) V((\hat{B})(</em>{ols}))</td>
<td>–</td>
<td>(K + 1)</td>
</tr>
<tr>
<td>S(_1)</td>
<td>IW</td>
<td>k(<em>S) (\cdot) 2 (\cdot) V((\hat{A})(</em>{1,ols}))</td>
<td>–</td>
<td>2</td>
</tr>
<tr>
<td>S(_2)</td>
<td>IW</td>
<td>k(<em>S) (\cdot) 3 (\cdot) V((\hat{A})(</em>{2,ols}))</td>
<td>–</td>
<td>3</td>
</tr>
<tr>
<td>W</td>
<td>IW</td>
<td>k(_W) (\cdot) 4 (\cdot) (I_M)</td>
<td>–</td>
<td>4</td>
</tr>
</tbody>
</table>

In order to explore the source and the extent of parameter time variation, we experiment with different parametrisations of the forgetting factor models. In particular, we consider forgetting factor models with only time varying slope parameters (FFVAR TV SLOPE), only time varying innovation volatility (FFVAR TV VOLA), and with both or neither sources of volatility active (respectively FFVAR TVP and FFVAR CONST). The constant parameter model is obtained by setting both forgetting factors \(\lambda\) and \(\kappa\) to 1. Time variation is the slope parameters and in the innovation volatility is obtained by setting respectively \(\lambda = 0.99\) and \(\kappa = 0.96\), as in Koop and Korobilis [2013]. In all constant volatility models we estimate the variance of the residuals recursively, in each point of time.

The flexibility of the forgetting factor models could in principle be used to study the role of macroeconomic and financial predictors of exchange rates. We consider a range of additional predictors in Appendix A and find that none improves on the performance of the model with only exchange rates over the whole forecast sample, but that adding stock prices improves the relative performance of the benchmark model with only exchange rates during the 2008 financial crisis (see figure 2.6 in Appendix A).

The two benchmark forecast models are the random walk with GARCH residuals, and the constant parameters Bayesian VAR. Point forecasts from a random walk are obtained by setting

\(^{10}\)For additional details, see Cogley and Sargent [2005].
Table 2.2: Exchange-rate prediction models

<table>
<thead>
<tr>
<th>MODELS</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 FFVAR TVP</td>
<td>forgetting factor model with time-varying slope and volatility, $\lambda = 0.99$ and $\kappa = 0.96$</td>
</tr>
<tr>
<td>2 FFVAR TV SLOPE</td>
<td>forgetting factor model with time-varying slope and constant volatility, $\lambda = 0.99$ and $\kappa = 1$</td>
</tr>
<tr>
<td>3 FFVAR TV VOLA</td>
<td>forgetting factor model with constant slope and time-varying volatility, $\lambda = 1$ and $\kappa = 0.96$</td>
</tr>
<tr>
<td>4 FFVAR CONST</td>
<td>forgetting factor model with constant slope and volatility, $\lambda = 1$ and $\kappa = 1$</td>
</tr>
<tr>
<td>5 TVP SV BVAR</td>
<td>bayesian model with time-varying parameters and stochastic volatility</td>
</tr>
<tr>
<td>6 BVAR</td>
<td>bayesian model with constant parameters</td>
</tr>
<tr>
<td>7 GARCH</td>
<td>univariate GARCH(1,1) model</td>
</tr>
</tbody>
</table>

$\hat{y}_{t+h|t} = y_{it}$, $\forall h$, while density forecasts are retrieved by estimating a GARCH(1,1) model on the transformed data and then cumulating the forecasts.\(^{11}\)

A summary of all competing models is given in table 2.2. Accounting for data transformation, lag length and training sample data, the estimation sample runs from 1976m2 to 2000m1. The estimation sample is then progressively enlarged in a pseudo-real time exercise: at each step, models are re-estimated\(^{12}\) and forecasts up to one-year ahead are computed and cumulated. Finally, to gauge how the financial crisis might have influenced the forecasting ability of the competing models, we split the forecast sample in two: a pre-crisis sample that ends in August 2008, such that the last forecasted period is always one month before the Lehman bankruptcy filing, and a crisis sample that starts in September 2008 and ends in June 2013\(^{13}\).

2.3.2 Point forecasts

Point forecasts from model a are compared with this from model b through their relative mean squared forecast error:

$$RMSFE_{i,h}^{a,b} = \frac{\sum_{t=1}^{T_f} (\hat{y}_{t+h|t}^a - y_{i,t+h})^2}{\sum_{t=1}^{T_f} (\hat{y}_{t+h|t}^b - y_{i,t+h})^2},$$

(2.10)

where $T_f$ is the number of forecasts, while $i$ and $h$ respectively index the variable and the horizon. Table 2.3 reports the mean squared forecast errors of the competing models relative to those of a random walk. Two (one) stars denote the horizon and variable for which the two mean squared

\(^{11}\)A random walk with GARCH innovations is better suited than a simple random walk to deliver density forecasts.

\(^{12}\)In the case of the time-varying stochastic volatility model of Primiceri [2005], the model is re-estimated every year as opposed to every month, due to the computational time required by the posterior simulation algorithm.

\(^{13}\)The second forecast sample is inevitably shorter due to data availability.
errors being compared are significantly different from each other, according to a Diebold and Mariano test at a 5% (10% ) significance level, modified using the small sample size correction of [Harvey et al. 1998]. We can draw three broad conclusions. First, constant parameter models deliver on average larger mean squared forecast errors, particularly in the period of the financial crisis. Second, the TVP SV BVAR has generally lower MSFE than the FFVAR TV VOLA, due to its better performance during the crisis period. In addition, both time varying parameter models improve on the point forecasts of a random walk at a one-month horizon. At longer horizons however, the assumption of parameter time variation does not lead to a significant improvement in the MSFE with respect to a random walk, with the exception of the Yen.

Table 2.3: Mean squared forecast errors of the competing models models relative to a random walk for different forecast samples and horizons. Two (one) stars denote significantly different RMSFE at a 5% (10%) significance level according to a Diebold-Mariano test, modified using the small-sample correction of [Harvey et al. 1998]. The forecasting models are described in table 2.2. The pre-crisis sample goes from 2000:m2 to 2008:m8. The crisis sample spans the period from 2008:m9 to 2013:m6.

<table>
<thead>
<tr>
<th></th>
<th>PRE-CRISIS SAMPLE</th>
<th>CRISIS SAMPLE</th>
<th>FULL SAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>h</td>
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<td>€</td>
</tr>
<tr>
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</tr>
<tr>
<td></td>
<td>h = 3</td>
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</tr>
<tr>
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</tr>
<tr>
<td></td>
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</tr>
<tr>
<td>TVP SV BVAR</td>
<td>h = 1</td>
<td>0.94*</td>
<td>0.88**</td>
</tr>
<tr>
<td></td>
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<td>0.90**</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>h = 12</td>
<td>0.88*</td>
<td>1.05</td>
</tr>
<tr>
<td>BVAR</td>
<td>h = 1</td>
<td>1.12**</td>
<td>1.02</td>
</tr>
</tbody>
</table>
|       | h = 3             | 1.13 | 1.13 | 1.26**| 1.02 | 1.25**| 1.13 | 1.08 | 1.18**| 1.19*
|       | h = 6             | 1.31**| 1.20 | 1.65**| 1.46**| 1.55**| 1.60 | 1.29**| 1.35**| 1.60**|
|       | h = 12            | 1.56**| 1.29 | 2.20**| 5.71*| 2.88**| 2.19*| 1.78**| 1.75**| 1.99**|
| FFVAR TV SLOPE | h = 1             | 1.15*| 0.93 | 1.12*| 1.17 | 1.02 | 0.81**| 1.15*| 0.98 | 0.99 |
|       | h = 3             | 1.17 | 0.94 | 1.17*| 1.66 | 1.04 | 0.83 | 1.40*| 1.01 | 0.99 |
|       | h = 6             | 0.94 | 0.94 | 1.35 | 2.46*| 1.03 | 1.02 | 1.48 | 1.04 | 1.15 |
|       | h = 12            | 1.00 | 0.88 | 1.73 | 5.98*| 1.49**| 1.23 | 1.51*| 1.13 | 1.37 |
| FFVAR CONSTANT | h = 1             | 1.15*| 0.93 | 1.12*| 1.16 | 1.03 | 0.80**| 1.15*| 0.98 | 0.99 |
|       | h = 3             | 1.17 | 0.94 | 1.17*| 1.66 | 1.04 | 0.82* | 1.40*| 1.01 | 0.98 |
|       | h = 6             | 0.95 | 0.94 | 1.34 | 2.46*| 1.03 | 1.02 | 1.48 | 1.04 | 1.14 |
|       | h = 12            | 1.02 | 0.88 | 1.74 | 5.96*| 1.49**| 1.23 | 1.52*| 1.13 | 1.37 |
2.3.3 Interval forecasts

We proceed to examine whether allowing for parameter time variation improves the calibration of the 68% and 95% forecast confidence intervals, the two most commonly used in empirical studies. The statistic we use is the coverage rate of each competing model, measured as the percentage of times in which the actual exchange rate is contained in the forecast confidence interval. As it has been previously discussed, an accurate assessment of the uncertainty surrounding point forecasts is likely to be of interest to a wide variety of forex market participants, from central banks to private investors. A model that delivers coverage rates which are significantly below their nominal counterparts underestimates forecast uncertainty. To the other extreme, coverage rates of a 100% imply that the estimated forecast confidence intervals always contain the actual values, but the confidence bands are so wide to be of little practical use. A model with correctly calibrated forecast intervals would have coverage rates which do not significantly differ from their nominal counterparts.

The empirical coverage rates of the different models, corresponding to 68% and 95% nominal coverages, are reported in table 2.4. Values in bold are not statistically different from their nominal counterparts, according to a likelihood-ratio test with 1 degree of freedom. Over the whole forecast sample, the model which has the best calibrated coverage rates is the ffvar model with time-varying volatility. The improvement over its constant volatility counterparts (BVAR, FFVAR CONST and FFVAR TV SLOPE) is evident at a 68% confidence level. Splitting the samples, we notice that the better performance of the time-varying volatility FFVAR model arises mainly in the pre-crisis forecast sample, when the competing models overestimate the variance of the series and deliver excessively large confidence intervals. The other time-varying volatility models, the TVP SV BVAR and GARCH, tend to overestimate the variance as well. One explanation for the better performance of the FFVAR TV VOLA model could lie in its parsimonious treatment of the volatility process: the losses due to its approximation of volatility through an exponentially weighted moving average process seem to be more than outweighed by its efficiency gains.

We exemplify the results of table 2.4 in figures 2.2 and 2.3, where we plot the 68% forecast confidence intervals of the FFVAR TV VOLA model together with those of the TVP SV BVAR (darker area, figure 2.2) and of the BVAR (darker area, figure 2.3), and with the actual exchange rates (in red), across forecast horizons (rows) and currencies (columns). At short and medium forecast horizon, the forgetting-factor model provides the narrowest confidence bands, which we know from table 2.4 to be accurately calibrated. Hence, while the FFVAR TV VOLA model yields an efficient, as well as correct, estimation of uncertainty, its competitors overestimate forecast uncertainty, particularly for horizons greater than 6 months.

\footnote{For details of the test we refer to Clements [2005].}

\footnote{The graphs of the remaining model’s forecast confidence intervals are available upon request.}
Figure 2.2: Forecast confidence intervals: 68% forecast confidence intervals delivered by the FFVAR TV VOLA, and by the TVP SV BVAR (darker area), by exchange rate (columns) and horizon (rows). Actual exchange rates in red.

Figure 2.3: Forecast confidence intervals: 68% forecast confidence intervals delivered by the FFVAR TV VOLA, and by the BVAR (darker area), by exchange rate (columns) and horizon (rows). Actual exchange rates in red.
2.3.4 Density forecasts

**Probability integral transforms:** Since the seminal work of by Dawid [1984] and Diebold et al. [1998], probability integral transforms have been extensively used to evaluate competing density forecasts. In particular, Diebold et al. [1998] have shown that, if the forecast model \( p(y) \) coincides with the data generating process \( f(y) \), the series \( \{z_t\}_{t=1}^{T_f} = \left\{ \int_{-\infty}^{y_t} p(y_t) dy \right\}_{t=1}^{T_f} \) of probability integral transforms is an i.i.d. sample from a \( U(0,1) \) distribution\(^{16}\).

We start with a visual inspection: figure 2.4 plots the histogram of the probability integral transforms of the one-month ahead density forecasts delivered by the competing models. The FFVAR model with time-varying volatility is the only one, together with the RW GARCH model, that delivers a uniformly distributed p.i.t. sequence. By contrast, the p.i.t. sequences of the constant volatility models, as well as of the TVP SV BVAR, are slightly hump-shaped denoting an over estimation of the variance\(^{17}\) and confirming the earlier findings of the coverage rates. A formal test of uniformity reported in table 2.5, achieved through the Kolmogorov-Smirnov test (KS), reveals that the only model for which the null of uniformity is never rejected at any horizon is the forgetting factor with time-varying volatility. For the constant volatility forgetting factor models, the null of uniformity for one-month ahead forecasts cannot be rejected only at a 10% significant level. The coverage rate results remain therefore valid also when the entire forecast distribution is taken into consideration: a parsimonious modelling of time-varying volatility such as in the FFVAR TV VOLA correctly calibrates the forecast distribution, while the competing models lead to a misspecification of the conditional variance.

**Figure 2.4: Evaluating density forecasts:** Normalised histograms of the one-step ahead p.i.t. sequences.

---

\(^{16}\)These conclusions are easily extended to a multivariate setting, such as ours. See for instance Diebold et al. [1998] and Clements [2005].

\(^{17}\)See discussion in Mitchell and Wallis [2011].
Table 2.4: Coverage rates

<table>
<thead>
<tr>
<th></th>
<th>PRE-CRISIS SAMPLE</th>
<th>CRISIS SAMPLE</th>
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<td></td>
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<td>$h$</td>
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<tr>
<td>12</td>
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</tr>
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</table>

Note: Main table values denote actual coverage rates, corresponding to nominal coverages of 68% and 95%, for different models and forecast horizons $h$. Values in bold are not significantly different from the nominal counterpart. The forecasting models are described in [22]. The pre-crisis sample goes from 2000:m2 to 2008:m8. The crisis sample spans the period from 2008:m9 to 2013:m6.
Table 2.5: Testing the probability integral transforms: Main table values are p-values for the null hypothesis of the Kolmogorov-Smirnov test that the p.i.t. sequence of the model in the column header is $U(0,1)$. The Bonferroni correction is used for horizons greater than 1; see text for details.

<table>
<thead>
<tr>
<th>MODELS</th>
<th>HORIZON</th>
<th>FFVAR TV VOLA</th>
<th>TVP SV BVAR</th>
<th>BVAR</th>
<th>FFVAR TV SLOPE</th>
<th>FFVAR TV CONST</th>
<th>RW GARCH</th>
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</table>

Log-predictive likelihoods: We now turn to an alternative evaluation of the competing predictive densities, and in particular we focus on the evolution over time of the log-predictive likelihoods, i.e. the log likelihood of observing the actual realisation of the variable, given a forecast model:

$$
\log p_{j,h,t}(y_{1,t+h} | \mathcal{F}_{j,t-1}),
$$

where $p_{j,h,t}(\cdot)$ denotes the predictive likelihood of model $j$ at horizon $h$ (possibly time-varying and thus depending on time $t$), $y_1$ is a vector of target variables (one or all of the three exchange rates), $\mathcal{F}_{t-1,j}$ is the information set of model $j$ available at $t$. Of interest is the cumulative difference between the log-predictive likelihood of the FFVAR model with time-varying volatility, $\log p_{1,h,t}$, and that of one of the alternative benchmarks, $\log g_{j,h,t}$:

$$
S_{j,h} = \sum_{t=1}^{T_J-h} \left[ \log p_{1,h,t}(y_{1,t+h} | \mathcal{F}_{1,t-1}) - \log g_{j,h,t}(y_{1,t+h} | \mathcal{F}_{g_j,t-1}) \right].
$$

This exercise is similar to what is undertaken in Amisano and Geweke [2010] and Amisano and Geweke [2013], and enables us to gauge the contribution of different observations over time in favour or against the FFVAR TV VOLA model. Moreover, the statistic in (2.12) can be interpreted as the summed difference in density forecast errors\(^{18}\) and can be justified in terms of the Kullback-Leibler distance (KLIC). The latter can be expressed as the expected difference between the true log predictive density, $f_t(\cdot)$, and the predictive density of model $j$, $p_{j,t}(\cdot)$:

$$
E[\log f_t(y_{1,t+1} | \mathcal{F}_{j,t-1}) - \log p_{j,t}(y_{1,t+1} | \mathcal{F}_{j,t-1})],
$$

where we consider the case $h = 1$ and drop the horizon subscript for expositional purposes. Under some regularity conditions, the average of the sample quantities of $f_t$ and $p_{j,t}$ yields a consistent estimator of the KLIC distance. Hence, when two different predictive densities are being compared,

\(^{18}\)See the discussion in Hall and Mitchell [2007].
\[ p_{1,t} \text{ and } p_{2,t}, \text{ the average difference between their logarithms is inherently related to their KLIC distance:} \]
\[
\frac{1}{T_f} \sum_{t=1}^{T_f} \left( \log p_{2,t}(\cdot) - \log p_{1,t}(\cdot) \right) = \frac{1}{T_f} \sum_{t=1}^{T_f} \left( \log f_t(\cdot) - \log p_{2,t}(\cdot) \right) - \left[ \frac{1}{T_f} \sum_{t=1}^{T_f} \left( \log f_t(\cdot) - \log p_{2,t}(\cdot) \right) \right],
\]
(2.14)
so that, among a class of alternative models, choosing the one with the highest average log-predictive likelihood entails selecting the model with the minimal KLIC distance.

Figure 2.5 plots the statistic \( S \) in equation 2.12 at selected forecast horizons\(^{19}\) providing further insights on the relative predictive ability of the competing models. At all forecast horizons considered, the FFVAR with time-varying volatility has a higher predictive likelihood than all competing models. The FFVAR with constant parameters and the BVAR perform very similarly to each other, and both perform relatively better than the FFVAR TV VOLA in the period of the financial crisis. Nevertheless, the relative predictive likelihood of the FFVAR TV VOLA model improves immediately after the crisis, as the model discounts the Kalman filter prediction errors through equation 2.9.

\[ \text{Figure 2.5: Cumulative differences in log-predictive likelihoods between the FFVAR with time varying volatility and various benchmarks: TVP SV BVAR (solid line), BVAR (dashed line), RW-GARCH (dashed-dotted line) and the FFVAR CONST (dotted line) at selected forecast horizons. Increases in the plotted statistics indicate whenever the FFVAR TV VOLA performs better than the alternatives.} \]

To gauge whether the differences observed in figure 2.5 are statistically significant, we use the general test of equal predictive ability proposed by Amisano and Giacomini \[2007\] and report the

\(^{19}\)The statistic obtained when the benchmark model is a simple random walk is not shown, but it is available upon request. It is always lower than that obtained when the benchmark model is a random walk with GARCH innovations.
results in table 2.6. In the forecast subsample that excludes the financial crisis, the FFVAR model with time-varying volatility has a significantly higher predictive likelihood than all competing models. Once the crisis forecast subsample is considered, the FFVAR TV VOLA model does instead slightly worse than the competing models for all horizons greater than one month, though the differences are never statistically significant. Over the whole sample modelling time-varying volatility delivers significantly smaller density forecast errors at both one-month and one-year ahead horizons, and does on average better at medium forecast horizons, though the difference is not always statistically significant.

Table 2.6: Amisano-Giacomini test statistic: The null hypothesis is that the FFVAR TV VOLA model has the same predictive ability of the forecast model indicated in the row header. Two (one) stars denote the combination model-forecast horizon at which the difference in log-predictive likelihood is significant at a 5% (10%) significance level. The pre-crisis sample goes from 2000:m2 to 2008:m8. The crisis sample spans the period from 2008:m9 to 2013:m6.

<table>
<thead>
<tr>
<th>MODELS</th>
<th>PRE-CRISIS SAMPLE</th>
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<td>FFVAR CONST</td>
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<td>3.20**</td>
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<td>RW GARCH</td>
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<td>14.84**</td>
<td>12.97**</td>
</tr>
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</table>

2.4 Economic evaluation

So far, we have relied on purely statistical criteria to evaluate exchange-rate forecasts from competing models. However, an evaluation based on economic criteria might be of interest, particularly if the statistical models are to be used in real-world applications. In what follows, we assess the performance of the competing models through a simple trading strategy, described in Carriero et al. [2009]. We take the perspective of a US-based investor who grounds her investment decisions on the predictions of a given forecast model, and has an investment horizon of one month. The investor buys foreign currency only if she expects the latter to appreciate over the period of interest; no investment is made if the currency is instead expected to depreciate. At the end of the investment period, the investor liquidates the realised gain/loss (if the currency actually appreciated/depreciated) and reinvests the initial capital. We consider trading strategies based on all competing models, as well as on a naive strategy that in each time attributes a 50% probability to a currency appreciation over the investment period. Trading strategies are evaluated on the basis of their average return $\mu(\pi)$, on the returns’ standard deviation $\sigma(\pi)$, as well as on the

\[\mu(\pi) = \frac{\sum_{i=1}^{n} \pi_i}{n}\]

\[\sigma(\pi) = \sqrt{\frac{\sum_{i=1}^{n} (\pi_i - \mu(\pi))^2}{n}}\]
Sharpe ratio ($SR$) over both the pre-crisis and crisis forecast subsamples. The three statistics are reported in table 2.7, together with the difference in the Sharpe ratio over the naive strategy ($\Delta SR$), the percentage of periods in which the currency is traded, and the percentage of times when the predicted currency change is correct.

Table 2.7 clearly shows that modelling parameter time variation seems to be important also from an economic perspective. Differently from what we observed through the statistical criteria however, the best performing model is almost always the Bayesian VAR with time-varying parameters and stochastic volatility. This is particularly evident in the case of the Pound, where it is possible to achieve a positive average return also in the subsample that includes the financial crisis. The better performance of the TVP SV BVAR can be attributed to its superior ability in point forecasting, as resulting from the MSFE comparisons.

Table 2.7: Economic evaluation: Key figures the trading strategies based on the different forecast models: average return $\mu(\pi)$, returns’ standard deviation $\sigma(\pi)$, Sharpe ratio ($SR = \frac{\mu}{\sigma}$), difference in the Sharpe ratio over the naive strategy ($\Delta SR$), percentage of periods in which the currency is traded and percentage of times when the predicted currency change is correct ($c$). In bold, best performing strategies by criterion (column) and currency.

<table>
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<tr>
<th>MODELS</th>
<th>PRE-CRISIS SAMPLE</th>
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2.5 Conclusions

A big puzzle in the foreign exchange literature is the inability to predict the future behaviour of exchange rates. In this paper we have focused on the idea that the unpredictability might be caused by time variation in the parameters of the underlying stochastic process. In particular,
we have assessed whether the modelling of time variation in the slope and volatility parameters improves the point, interval and density forecasts of three major exchange rates vis-a-vis the US dollar. We have used two state-of-the-art time-varying parameter models: the stochastic volatility BVAR of Cogley and Sargent [2005] and Primiceri [2005], and its forgetting factor approximation recently proposed by Koop and Korobilis [2013]. These models have been compared with several benchmarks: the random walk, with or without GARCH innovations, a constant-parameter BVAR, and a constant parameter forgetting factor model.

The forecast evaluation exercise provides evidence of time variation in the covariance matrix of the VAR innovations, while the contribution of time variation in the slope parameter appears to be modest. The performance of constant parameters models is in fact significantly improved only through the modelling of time-varying volatility. In particular, the FFVAR mode with time-varying volatility, though not improving point forecasts relative to a random walk, significantly refines the estimation of forecast uncertainty through an accurate calibration of the forecast confidence intervals. Analysing the forecast probability integral transforms further conveys the result that it is the entire forecast density of the three exchange rates to be correctly calibrated, and not just the 68% and 95% confidence intervals. In addition, the comparison based on density forecast errors shows that the FFVAR TV VOLA model has on average higher predictive likelihoods than all competitors at all forecast horizons, though it performs relatively worse during the financial crisis. The better performance of the forgetting factor model relative to the TVP SV BVAR is likely caused by its more parsimonious modelling of parameter time variation, such that the bias due to the approximation is outweighed by its efficiency gains. Lastly, we have evaluated the competing models on the basis of a simple trading strategy and found that modelling parameter time variation allows to achieve higher mean returns and Sharpe ratios over the whole forecast sample.
2.6 Appendix 2A: The role of macro financial predictors

Several variables qualify as potential predictors of future exchange rates. The purchasing power parity theory (PPP), first developed by Cassel [1918], postulates that the nominal exchange rate \( s_t \) should be equal to the sum of the real exchange rate \( q_t \), and the difference in the general price level between the foreign and the home country \( (p^*_t - p_t) \). Moreover, the uncovered interest rate parity (UIRP) condition suggests that exchange rate movements compensate differentials in the nominal interest rate levels \( (i^*_t - i_t) \). Empirical evidence on these models is mixed. Among others, Cheung et al. [2005] show that while the mean squared errors from PPP models are lower than those of a random walk for longer horizons, UIRP models do not significantly improve on the random walk at any horizon. On the contrary, both models are found to outperform the random walk by Della Corte et al. [2009], on the basis of statistical and economic criteria.

A relatively recent branch of exchange-rate prediction models is based on Taylor rules. These models build upon open economy frameworks, and assume that the policy rule followed by the central bank targets the country’s exchange rate, as well as output and inflation. Equating the modified Taylor rules for the home and the foreign country yields a relationship between the exchange rate and differentials in output, inflation and interest rates. The good performance of Taylor rule models has been documented, among others, by Molodtsova and Papell [2009] and Inoue and Rossi [2012], while it has been questioned by Rogoff and Stavrakeva [2008].

Finally, increasing attention is being paid to financial predictors of exchange rates. Molodtsova and Papell [2012] find that the performance of their proposed Taylor rule models can be improved, in some cases, when they are augmented with credit spreads or measures of financial conditions. In addition, Shin et al. [2010] have shown how US credit aggregates, taken as proxies for the risk appetite of financial intermediaries, can help forecasting a wide set of exchange rates.

The empirical literature shows that, contrarily to what exchange-rate determination models posit, macroeconomic fundamentals do not appear to robustly improve the prediction of future exchange rates. A possible explanation for this puzzle lies in the instability of the relationship that links exchange rates to their fundamentals. This instability has been documented, among others [22] by Rossi [2006] through a series of instability tests. We take this perspective here and how the performance of the forgetting-factor model containing only the three exchange rates (henceforth core model), varies when the set of regressors is enlarged with different macroeconomic and financial predictors, typically used in the exchange-rate prediction literature. The choice of the forgetting factor methodology is motivated not only by its computational advantages over its

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22 Using survey data, Cheung and Chinn [2001] have explained that instability might result from the behaviour of foreign exchange-rate traders, who frequently change the weight they attach to fundamentals. In addition, Bacchetta and van Wincoop [2009] show that the unstable relationship between fundamentals and exchange rates can be generated within a model whose structural parameters are unknown to economic agents, and evolve gradually over time.
Bayesian counterpart, but also by the fact that it delivers similar point forecasts to those of the TVP SV BVAR, while improving on its interval and density forecasts. The additional predictors considered are the nominal short term interest rate, the term structure slope and the growth in stock prices, measured as differentials with respect to the US. We find that none of these additional predictors improves on the performance of the core model. To exemplify this point we report the log-predictive likelihood of the macro models with respect to the core model in figure 2.6. At short horizons the performance of the macro models is very similar to that of the core model, while at larger forecast horizons the performance worsens considerably. Perhaps the most interesting result is that stock price differentials improve the performance of the core model during the financial crisis, though on average the latter displays smaller density forecast errors.

Figure 2.6: Cumulative differences in log-predictive likelihoods between the FFVAR TV VOLA with only exchange rates and the FFVAR TV VOLA model augmented with macro financial differentials: nominal short term interest rates (uirp - dotted line), stock prices growth (sp - dashed line), and term structure slope (solid line) at selected forecast horizons. Increases in the plotted statistics indicate whenever the FFVAR TV VOLA with only exchange rates performs better than the alternatives.

Note: All macro variables are monthly and expressed as differentials from the US counterpart. The term structure slope is measured as the difference between the 10-year government bond yield (source: OECD - MEI) and the money market interest rate (source: IFS - IFM). Stock prices are transformed into real through the core CPI and are taken as growth rates (source: Datastream).

We have also considered differentials in inflation, output growth, and trade balances. For brevity we do not report the results, which are available upon request.
Chapter 3

Forecasting macroeconomic indicators with default risk and risk aversion measures

3.1 Introduction

The financial and the real side of the economy are heavily intertwined: financial intermediaries provide means for firms to invest and for households to smoothen consumption with, and shocks from one sector typically transmit to the other magnifying their initial impact, as shown by the recent financial crisis in the US[1].

In this work I investigate whether measures of default risk and of risk aversion have any predictive content for key US macroeconomic quantities, such as output, prices and lending activity, and whether their predictive power has changed over time. Three different indicators are considered: two credit spread measures that have been found to be good predictors in the empirical literature, and one measure of risk aversion. The two credit spread measures are the difference between Baa and Aaa-rated bond yields, and the measure recently developed by [Gilchrist and Zakrajšek 2012] using prices of individual corporate bonds traded in the secondary market. Both measures capture variations in default risk only with a noise, as they are affected by other factors such as risk aversion and, to a lesser extent, liquidity premia and bond taxation conditions. I take this problem into account by considering a third predictor that captures precisely risk aversion: the excess bond premium, also developed by [Gilchrist and Zakrajšek 2012]. To assess time variation in the predictive power of these indicators I employ a series of time-varying forgetting factor models recently proposed by [Koop and Korobilis 2013]. These models, which can be considered a classical approximation of the time-varying Bayesian VARs of [Cogley and Sargent 2005] and [Primiceri]

[1] A recent study of the Dallas FED estimates that the output loss in the US caused by the recent financial crisis is between $6 and $14 trillions.
are considerably faster to estimate and, since they are not as densely parametrised, they have been often found to be good forecasting tools.

Several results emerge from the forecasting analysis. Controlling for the credit spread developed by Gilchrist and Zakrajšek [2012], improves the point and density forecasts of both industrial production and employment after 2000. This result complements the findings in Gilchrist and Zakrajšek [2012] by showing that the enhanced predictive performance is limited to the recession periods of 2001 and 2008, when credit constraints were arguably binding. An additional key result is that both default risk and risk aversion matter for improving the forecasts of activity indicators. These two conclusion are strengthened by a structural VAR which shows that both increased risk aversion and worsened macroeconomic fundamentals lie behind the latest US recession and subsequent slow recovery. Moreover, credit spread shocks display significant real effects only after the latest recession. In particular, an unexpected increase in the credit spread causes in 2010 an output contraction that lasts for about two years, with an annualised through of 4.8%, and explains up to 35% of the forecast error variance of industrial production.

This work is theoretically grounded in a growing number of models that rationalise the link between the financial sector and the real side of the economy, as well as the nominal one. One important transmission channel is the 'financial accelerator' mechanism, introduced in economic theory by the seminal works of Bernanke and Gertler [1989] and Bernanke et al. [1999]. Market frictions such as imperfect information create an 'external finance premium', a wedge between the actual cost of external funding faced by borrowers, and the opportunity cost of these funds. Shocks that increase the external finance premium raise the cost of borrowing, causing a suboptimal contraction in borrowing and investment, and a consequent decline in aggregate output. To the extent that the external finance premium can be proxied by credit spread measures, i.e. yield differences between securities of different credit quality, movements in credit spreads will cause future fluctuations in output. Credit spreads movements might anticipate changes in real variables in other ways. For example, as shown in Philippon [2009], worsened business fundamentals or productivity are likely to increase the default probability of both borrowers and lenders, thus increasing credit spread measures before causing fluctuations in slower moving real variables. Indicators of financial conditions might contain predictive power for prices as well. Recently Gilchrist et al. [2015] have shown, using a novel industry-level dataset, that during the great financial crisis firms with different financial positions displayed different pricing behaviours. In particular, while financially unconstrained firms decreased their prices, financially constrained ones actually increased them. They rationalize their finding through a two-sector general equilibrium model where, in response to adverse financial shocks, financially constrained firms are forced to increase prices in order not to rely on costly external finance, and overall inflation increases despite the drop in output.
There is a vast empirical literature that seeks to assess to what extent macroeconomic activity can be forecasted by credit spreads. None of it however sought to establish whether this predictive power has varied over time. Gilchrist and Zakrajšek [2012] find that their newly developed credit spread measure has a significantly higher predictive power for macroeconomic activity indicators than other measures such as the Baa-Aaa corporate bond spread, or the paper-bill spread. They moreover show that most of the predictive content of their spread measure can be attributed to its component that captures the compensation required by investors to bear default risk, and that can be considered a measure of overall risk aversion (the excess bond premium). Using a different methodology I show that, once time variation is taken into account, the predictive power of credit spreads is limited to the 2001 and 2008 recession periods, and that the excess bond premium does not exhaust the informational content of the credit spread measure. A similar conclusion is hinted at by Faust et al. [2012], without however seeking to establish the relative role of risk aversion versus default risk. In particular, the authors forecast macroeconomic activity with a Bayesian model averaging (BMA) scheme that includes, alongside with a large set of macroeconomic and financial predictors, 20 credit spreads constructed from a confidential database of corporate fixed income securities, and differentiated by risk and maturity categories. The BMA models are found to deliver lower mean squared forecast errors than an autoregressive model, particularly during NBER-dated recessions. Disentangling their results, the authors find that the highest BMA weights are assigned to the credit spread measures and it is their inclusion that significantly improves on the autoregressive point forecasts.

An assessment of the time-varying role played by financial frictions is found in Del Negro et al. [2014]. They compare the forecasting performance of two DSGE models, with and without financial frictions, and find that during the great financial crisis the model with financial frictions has on average a higher predictive density than the alternative.

The paper is organised as follows. The choice of indicators and the VAR methodology are discussed in Section 2. The in-sample fit of the competing models is presented in Section 3, while the out-of-sample evaluation is carried out in Section 4. Sections 5 performs a structural VAR analysis that establishes whether the macroeconomic effect of shocks to default risk and risk aversion measures has changed over time. Section 6 concludes.

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2The macroeconomic activity considered are the growth rates in real GDP, personal consumption expenditures, business fixed investment, industrial production, private payroll employment, civilian unemployment rate, exports and imports, over the period 1986:Q1-2011:Q3.

3The model with financial frictions uses as credit spread measure the commercial paper spread, the difference between the annualised Moody’s Seasoned Baa corporate bond yield and the 10-Year Treasury Note Yield at Constant Maturity.
3.2 Macroeconomic activity prediction models

The empirical strategy followed in this work has two main dimensions. First, I would like to investigate what is the marginal contribution of financial condition indicators in forecasting key macroeconomic variables over a large evaluation period that spans several US recessions and goes from $1980m_{1}$ to $2012m_{12}$. I consider three indicators: two measures of default risk, and one measure of risk aversion. Default risk is captured, noisily, by two credit spread measures: the difference between Baa and Aaa-rated bond yields (henceforth BA spread), and the credit measure developed by Gilchrist and Zakrajšek [2012] (henceforth GZ spread). The latter is constructed by averaging credit spreads computed for twenty different maturity and credit risk categories from a confidential database of US firms, and is found to outperform the BA spread in forecasting output and employment. Gilchrist and Zakrajšek [2012] further decomposed their spread measures in two parts: one capturing the countercyclical movements in expected default and another, the excess bond premium (henceforth EBP), that reflects deviations between the expected default risk of borrowers and the price of their debt claims. Gilchrist and Zakrajšek [2012] claim that this latter component signals the risk aversion of the financial sector, as it spikes during US recessions and is highly correlated with the leverage of highly leveraged (and high risk) agents (broker-dealers).

Standard financial accelerator theories such as in Bernanke et al. [1999], as well as recently proposed models such as Gilchrist et al. [2015] show that a shock that increases the default risk of borrowers or diminishes the risk-bearing capacity of financial intermediaries will be followed by a sharp contraction in lending and in output, and possibly by either an increase or by only a moderate decline in prices. Based on these models I look at the predictive content of these three financial indicators for key US macroeconomic aggregates: the month-on-month change in the industrial manufacturing index and in civilian employment, producer price and personal consumption expenditure inflation. I further look at the predictive content for two banking sector variables: the month-on-month change in commercial and industrial loans and the loan interest rate spread measured as the difference between the lending interest rate and the risk free rate. A summary of all the variables can be found in the lower panel of Table 3.1 below.

As a second goal, I would like to know whether the predictive power of the two credit spreads and of risk aversion have changed over time. In particular, I would like to assess whether default risk or risk aversion matter more in times of economic recessions, and especially during the latest financial crisis. To do so, I rely on a methodology recently proposed by Koop and Korobilis [2013]: the time-varying forgetting factor VAR. The latter can be considered a classical approximation of the time-varying parameter BVAR with stochastic volatility of Cogley and Sargent [2005] and Primiceri [2005], that exploits one-step ahead prediction errors to determine the degree of variation.

\footnote{In their model Gilchrist et al. [2015] show that an adverse financial shock is followed by an output contraction and an overall increase in inflation, as financially constrained firms are forced to raise prices. By contrast, a negative demand shock will decrease prices, but by a more modest amount than that posited by a standard New Keynesian model.}
This model, outlined in detail below, is estimated via the Kalman filter and it is parsimoniously parametrised. Hence, despite not taking sampling uncertainty into account, it has been shown to be a very useful forecasting tool.

To assess the marginal forecasting content of the three financial condition indicators, I first define a baseline time-varying VAR model that controls for lagged values of the macro variable being forecasted, as well as for the real interest rate and for the difference between the three-month and the ten-year constant-maturity Treasury yields. These are the same control variables used in the forecasting exercise of Gilchrist and Zakrajšek [2012]. Moreover, the slope of the Treasury yield curve is typically found to be a good predictor for macroeconomic activity. Note that I rely on a VAR rather than on a regression analysis such as in [Gilchrist and Zakrajšek 2012] as I believe that there are important dynamic relationships across the variables that should be taken into account. I then augment the benchmark VAR with one indicator at a time and assess how and to what extent the in sample and out of sample fit of the model changes. In particular, I evaluate the point and density forecasts of the indicator-based models relative to the baseline, and to an autoregressive model estimated recursively with the same lag length of the VARs. A description of all forecast models used is found in Table 3.1 below.

The spreads, the term structure slope and real interest rate measures enter the VAR in levels. Models are estimated with a lag length of 6. The VARs are initialised by shrinking the model to random walk or to a white noise process, depending on the variable transformation, while prior variance is initialised through a training sample. The estimation period is 1975\textsubscript{m}1 – 1979\textsubscript{m}12 while the forecast period is 1980\textsubscript{m}1 – 2012\textsubscript{m}12. After 1980\textsubscript{m}1 the estimation sample is progressively enlarged, one month at a time, and forecasts are computed up to one year ahead.

### 3.2.1 The time-varying parameter forgetting factor VAR

The time-varying parameter BVAR with stochastic volatility developed by the seminal works of Cogley and Sargent [2005] and Primiceri [2005] has become an important econometric tool in recent years. It features three sources of time variation: in the slope coefficients, in the volatility of the structural shocks, and in the correlation among the structural shocks. It is estimated via Bayesian methods with a 5-step Gibbs-sampler and requires a high computational time, especially to draw the covariance matrix of the innovations to the slope parameters, and the covariance matrix of the structural shocks. To reduce computational time, Koop and Korobilis [2013] have developed a procedure which approximates the Bayesian model by replacing the posterior draws of these

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5In a future version of this paper, I plan to add the comparison with the time-varying parameter BVAR with stochastic volatility of Cogley and Sargent [2005] and Primiceri [2005].

6For recent evidence see for instance Chinn and Kucko.

7If these relationships are revealed by the data not to be important, they will be automatically discounted by the Kalman filter.
Table 3.1: Macroeconomic activity prediction models: All VAR models control for past lags of the activity variable analysed, as well as for the real interest rate (R), and for the difference between the three-month and the ten-year constant-maturity Treasury yields, i.e. the slope of the Treasury yield curve (TS). TVP-VAR models are estimated using the TVP-FF-VAR methodology, with a lag length of 6. The estimation period is 1975m1 – 1979m12 while the forecast period is 1980m1 – 2012m12.

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<td>TVP-VAR EBP</td>
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<td>lending rate - risk free rate</td>
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</table>

two covariance matrices with empirical estimates. In particular, their time-varying parameter forgetting factor VAR is based on a state-space representation, just like its Bayesian counterpart. It is assumed that the $n \times 1$ vector $y_t$ of observed variables is expressed as:

$$y_t = Z_t \beta_t + u_t \quad \text{with} \quad E_t[u_t u_t'] = \Omega_t,$$

(3.1)

where $Z_t$ is a $n \times k$ matrix of regressors, and $u_t$ is a $n \times 1$ vector of innovations with covariance matrix $\Sigma_t$. $Z_t$ contains a constant and $p$ lags of each variable and is thus defined as $Z_t = I_n \otimes [1, y'_{t-1}, \ldots, y'_{t-p}]$. The measurement equation (3.1) is complemented by the transition equation for the vector of time-varying slope coefficients $\beta_t$:

$$\beta_t = \beta_{t-1} + v_t,$$

(3.2)

implying that, given information up to $t - 1$, the slope coefficients in $t$ are draws from a normal distribution:

$$\beta_t | \mathcal{F}_{t-1} \sim N(\beta_{t|t-1}, P_{t|t-1}).$$

(3.3)

In the prediction step of the Kalman filter routine, the following approximation is used:

$$P_{t|t-1} = \frac{1}{\lambda} P_{t-1|t-1}, \quad \text{with} \quad \lambda \in (0, 1],$$

(3.4)
where the parameter $\lambda$ is a forgetting factor which discounts past information. Throughout this work, a value of $\lambda$ equal to 0.99 is used, implying, in the case of monthly data, that observations one year ago receive 89% as much weight as current observations.

The second modification to the otherwise standard Kalman filter routine pertains to the estimator of the covariance matrix of the non-structural innovations, $\Omega_t$. The latter is a weighted average of its past value, and of its current estimate $\hat{u}_t$:

$$\hat{\Omega}_t = \kappa \hat{\Omega}_{t-1} + (1 - \kappa)\hat{u}_t \hat{u}_t', \quad (3.5)$$

where $\hat{u}_t$ are the Kalman filter errors and the weight is represented by the decay factor $\kappa$, set in this application to 0.96 as in Koop and Korobilis [2013].

To summarise, the procedure developed by Koop and Korobilis [2013] is based on a modified version of the Kalman filter which relies on the parametrisation of equations 3.4 and 3.5, as well as on the choice of initial conditions for the covariance matrix $\Omega_0$, for the slope coefficients $\beta_0$ and their variance $P_0$. Differently from its Bayesian counterpart, it does not disentangle the structural shocks and delivers filtered estimates, which should be better suited for forecasting purposes. More importantly, the model does not posit a proper law of motion for the covariance matrices of the slope parameters and of the non-structural errors. To circumvent this problem, the parameters are assumed to be fixed out of sample, when sampling from the predictive density. Overall, the time-varying forgetting factor model disregards sampling uncertainty but it is very parsimoniously parametrised, which could potentially yield efficiency gains big enough to compensate the bias induced by the approximations.

### 3.3 In-sample fit of financial condition indicators

As a preliminary step, I assess the marginal contribution of the three financial condition measures in improving the in-sample fit of the target variables by plotting in Figure 3.1 the in-sample log likelihood of the indicator-based models, relative to that of the baseline TVP-VAR model. Increases in the plotted statistics denote observations whose fit is improved by adding the indicator to the baseline model.

The three indicators of financial conditions do not seem to improve the in-sample fit of the baseline VARs in the 1980s and 1990s. Relevant exceptions are personal consumption expenditure (Figure 3.1d) and employment (Figure 3.1b). Particularly for the latter, the inclusion of the BA corporate bond spread marginal improves the in-sample likelihood between 1980 and 2000, suggesting that the cost of debt might have influenced hiring decisions in this period. During

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*This is the Exponentially Weighted Moving Average estimator, commonly used in the finance literature.*
the dot-com crisis in 2001 the pattern changes and controlling for the GZ spread and for the EBP improves the in-sample log likelihood of output, employment and the two banking sector variables. By contrast, the in-sample fit of prices falls. Since the in-sample fit of the BA model does not likewise increase, it is possible to conclude that risk aversion appears to have been the main contributor of the better in sample fit of activity and banking sector variables during the 2001 crisis.

The great financial crisis in 2008 alludes to a different story. All indicator-based models improve the in-sample fit of the target variables, but this time the bulk of the improvement can be attributed to countercyclical variations in default premia. Exceptions are producer prices (Figure 3.1c) and loans (Figure 3.1e), for which risk aversion seems to have played a much bigger role.

Since in-sample and out-of-sample fit are not necessarily related, I now turn to discussing the out-of-sample forecasting performance of the three indicator-based models.

3.4 Forecasting performance

3.4.1 Point forecasts

The first step of the out-of-sample analysis is to verify to what extent the addition of default risk and risk aversion indicators improves the point forecasts of the baseline TVP VAR model and of an autoregressive model estimated recursively. To assess if the forecasting ability of these models has changed during the great financial crisis period, I have divided the forecast sample into two: one that excludes the crisis period, ranging from 1980m1 to 2008m8 such that the last forecasted period is always one month before the Lehman bankruptcy filing, and a crisis forecast sample that starts in 2007m8 and ends in 2012m12. The mean squared forecast error relative to the autoregressive model are reported in Table 3.2. Values smaller than one with one (two) star denote that the model (column index) does significantly better than the autoregressive benchmark at the horizon indicated in the corresponding row at a 5% (10%) significance level, according to a Diebold-Mariano test, modified using the small-sample correction of Harvey et al. [1998]. We find several interesting results.

The TVP-VAR models perform better than the AR model at forecasting the change in industrial production for horizons of 1 to 3 months over the whole forecast sample. A similar result is found for employment, but only for the crisis subsample (and not for the core TVP VAR). The best forecast model for these two activity variables is the one augmented with the GZ spread indicator, particularly in the crisis forecast subsample. Noticeably, the forecast errors of this model are smaller than those from the model augmented with the EBP, suggesting that default risk contains informational content for the point forecasts of both industrial production and employment.
Figure 3.1: In-sample fit of the indicator-based TVP-VAR models, relative to the baseline TVP-VAR: Indicators are the GZ spread (solid red line), the BA spread (dashed blue line) and the EBP (dashed dotted red line).

(a) Industrial production

(b) Employment

(c) Producer prices

(d) Personal consumption expenditure

(e) Loans

(f) Loan interest rate spread

Note: Positive values denote that, on average, the indicator-based model provides a better in-sample fit. Increases in the statistics denote observations in which the fit of the spread-based model model is higher. Shaded areas denote NBER-dated recessions.
Table 3.2: Mean squared forecast errors of the competing models models relative to an AR model for different forecast samples and horizons. Two (one) stars denote significantly different RMSFE at a 5% (10%) significance level according to a Diebold-Mariano test, modified using the small-sample correction of Harvey et al. [1998]. The forecasting models are described in table 3.1. The pre-crisis sample goes from 1980:m1 to 2008:m8. The crisis sample spans the period from 2008:m9 to 2012:m12.

<table>
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By contrast, inflation appears to be much harder to forecast, as the autoregressive benchmark beats the VAR models over the whole forecast sample. A relevant exception is personal consumption expenditure inflation, which is forecasted well in the crisis subsample by both the BA and EBP model at horizons up to one year.

Lastly, the TVP VAR models typically forecast better better than the AR model both loans and the loan interest rate spread. However, only in the case of loans the addition of default risk and risk aversion indicators improves the performance of the core TVP VAR model.
3.4.2 Density forecasts

An alternative way to assess the forecasting performance is by examining the entire forecast distribution, rather than just the point forecasts. The aim of this Section is to gauge to what extent the inclusion of default and risk aversion measures helps to better characterise the forecast density and to predict, not only the mean, but also low and high realisations of the variables of interest. The key statistic is the evolution over time of the differences in log-predictive likelihoods between one of the three indicator-based TVP VAR models and a benchmark forecast model (either the autoregressive model or the core TVP VAR):

\[
S_{j,h}^i = \sum_{t=1}^{T_t-h} \left[ \log p_{j,h,t}(y_{t+h} | \mathcal{F}_{j,t-1}) - \log g_{i,h,t}(y_{t+h} | \mathcal{F}_{g_{i,t-1}}) \right], \quad \text{with } j = \{2,3,4\}, \ i = \{0,1\},
\]

(3.6)

where \(\log p_{j,h,i}(\cdot)\) denotes the likelihood (in log scale) of observing the realisation of the variable \(y\) at horizon \(h\) in time \(t\), given forecast model \(j\), and where \(\mathcal{F}_{t-1,j}\) is the information set of model \(j\) available at \(t\).

Figures 3.2 to 3.7 display the relevant \(S_{j,h}^i\) statistics, where \(i\) is either the autoregressive (left panels) or the baseline TVP VAR (right panels), and where increases in the statistics denote observations in favour of the indicator-based TVP VAR models. This exercise, similar to what is undertaken in Amisano and Geweke [2010] and Amisano and Geweke [2013], has a justification in terms of the Kullback-Leibler distance (KLIC). In particular, if one model has on average higher predictive likelihood than a competitor, then the forecast model is closest (in terms of the KLIC distance) to the true data generating process.

Figures 3.2 and 3.3 show that the marginal contribution of financial indicators in forecasting production and employment starts to be relevant after the 2001 recession but becomes substantial only after the great financial crisis, when credit constraints were arguably binding. Between 2000 and 2008, the EBP model does equally well or better than the GZ and BA spread models, indicating that risk aversion is the most important predictor for economic activity in this period. Default risk premia seem to have played a more important role during the 2008 financial crisis, particularly at a one-year ahead horizon. In this period in fact the GZ model yields on average the highest predictive likelihoods among the VARs. Note however that at large horizons all TVP VAR models perform worse than the autoregressive benchmark after 2008.

By contrast, financial indicators do not seem to have any predictive content for inflation measures, as shown in Figures 3.4 and 3.5. Inflation is typically very hard to forecast and it is difficult to find a model that is able to beat an autoregressive benchmark. However, controlling
for financial indicators and in particular for risk aversion, substantially worsens the forecast of producer price inflation with respect to the baseline TVP VAR. In the case of personal consumption expenditure inflation controlling for the GZ spread or the excess bond premium only marginally improves the forecasts of the baseline TVP VAR model between the mid 1980s and the mid 1990s, and during the 2008 financial crisis. The improvement is in any case much smaller than that witnessed for industrial production.

Lastly, financial indicators seem to have some predictive power only for loans and not for the loan interest rate spread. In the case of loans (Figure 3.6), the three indicator-based models deliver very similar predictive likelihoods to the baseline TVP VAR and to the autoregressive models for most of the forecast subsample. After 2001 the EBP model scores progressively better, particularly at larger horizons, and the bulk of the increase is due once again to the 2008 financial crisis. EBP offers a marginal improvement to forecasting the interest rate spread of loans (Figure 3.7), but, this is limited to longer forecast horizons and to the period 1990-2000. Overall, controlling for past values of the spread, as well as for the term structure and the real risk free rate seems to be sufficient to deliver accurate forecasts for the loan interest rate spread.

To understand what drives the better performance of the indicator-based models, and in particular of the GZ model, during the 2008 crisis period, figure 3.8 shows the one-period ahead point forecasts and 68% forecast confidence intervals of the four TVP-VAR models, together with those of the autoregressive model (dashed lines). Across the wide range of variables considered, the point forecasts of the baseline TVP-VAR model are very similar those of the autoregressive model, though the confidence intervals are narrower. Controlling for credit spreads, and in particular for the GZ spread, delivers smaller forecast errors during the financial crisis period, and shifts the entire forecast distribution to lower values, though it still over predicts the two activity indicators. A similar result holds for employment (figure 3.8b), and for personal consumption expenditure inflation, but not for producer price inflation or for loans. Conversely, controlling for the GZ spread helps to capture the increase in the compensation for lending risk in the aftermath of the 2008 financial crisis.

In addition note that all TVP VAR models deliver narrower forecast confidence intervals with respect to the AR model, suggesting that allowing for parameter time variation significantly narrows forecast uncertainty.

### 3.5 Has the importance of shocks to financial conditions changed over time?

In the previous Section I have documented time variation in the reduced form coefficients of the VARs, as well as in the predictive ability of financial condition indicators for macroeconomic
Figure 3.2: Density forecasts for industrial production

(a) Relative to the AR model

(b) Relative to baseline TVP VAR model

Figure 3.3: Density forecasts for employment

(a) Relative to the AR model

(b) Relative to baseline TVP VAR model

Note: Predictive log-likelihoods of the GZ model (solid red line), of the BA model (dashed blue line), and of the EBP model (dashed dotted red line) relative to the AR model (Figure a) and to the baseline TVP-VAR model (Figure b) at different forecast horizons. The starred black line in Figure a denotes the log-likelihood of the baseline TVP-VAR relative to the AR model. Positive values denote that, on average, the model provides a better out-of-sample fit than the benchmark. Increases in the statistics denote observations in which the fit of the TVP VAR models is higher. Shaded areas denote NBER-dated recessions.

activity. In particular, credit spreads and the excess bond premium are found to be significant regressors for macroeconomic activity during the periods of financial distress of 2001 and 2008. In this section, I seek a causal interpretation of these results and examine whether the effect of
Figure 3.4: Density forecasts for producer price inflation

(a) Relative to the AR model

(b) Relative to baseline TVP VAR model

Figure 3.5: Density forecasts for personal consumption expenditure inflation

(a) Relative to the AR model

(b) Relative to baseline TVP VAR model

Note: Predictive log-likelihoods of the GZ model (solid red line), of the BA model (dashed blue line), and of the EBP model (dashed dotted red line) relative to the AR model (Figure a) and to the baseline TVP-VAR model (Figure b) at different forecast horizons. The starred black line in Figure a denotes the log-likelihood of the baseline TVP-VAR relative to the AR model. Positive values denote that, on average, the model provides a better out-of-sample fit than the benchmark. Increases in the statistics denote observations in which the fit of the TVP VAR models is higher. Shaded areas denote NBER-dated recessions.
Figure 3.6: Density forecasts for loans

(a) Relative to the AR model

(b) Relative to baseline TVP VAR model

Figure 3.7: Density forecasts for the loan interest rate spread

(a) Relative to the AR model

(b) Relative to baseline TVP VAR model

Note: Predictive log-likelihoods of the GZ model (solid red line), of the BA model (dashed blue line), and of the EBP model (dashed dotted red line) relative to the AR model (Figure a) and to the baseline TVP-VAR model (Figure b) at different forecast horizons. The starred black line in Figure a denotes the log-likelihood of the baseline TVP-VAR relative to the AR model. Positive values denote that, on average, the model provides a better out-of-sample fit than the benchmark. Increases in the statistics denote observations in which the fit of the TVP VAR models is higher. Shaded areas denote NBER-dated recessions.
Figure 3.8: In-sample fit of the indicator-based TVP-VAR models, relative to the baseline TVP-VAR: Indicators are the GZ spread (solid red line), the BA spread (dashed blue line) and the EBP (dashed dotted red line).

Note: Positive values denote that, on average, the indicator-based model provides a better in-sample fit. Increases in the statistics denote observations in which the fit of the spread-based model model is higher. Shaded areas denote NBER-dated recessions.
credit spreads shocks on industrial production growth has changed over time. For this purpose, I rely on a modified version of the forecast models that includes industrial production, inflation, the measure of financial conditions, the nominal risk-free interest rate and the term structure slope. The VARs are identified recursively, assuming that financial condition measures are not affected by monetary policy shocks, but that the monetary authority takes into account financial conditions when setting its rate. This identification scheme is the same as the one used in Gilchrist and Zakraješek [2012] and in other works by the same authors. While this identification scheme has many caveats, I choose it in order to show that the main structural result in Gilchrist and Zakraješek [2012] is driven by the recent financial crisis.

Figures 3.9, 3.11 and 3.13 display the effects of a one-standard deviation shock in one of the three financial condition indicators at different points in time. These plots clearly show how the importance of financial condition measures has changed over time. While a GZ credit spread shock has no significant effect on industrial production before 2000, its importance gradually increases over time, and in 2010 a one-standard deviation increase in the GZ spread causes an output contraction that lasts for two years, with an annualised through of 4.8%, despite an easing of monetary policy. The forecast error variance decomposition shown in Figure 3.10 reveals that GZ spread shocks account for a negligible fraction of industrial production variance until the end of the 1990s, but that in 2010 this fraction increases to 35% after two years.

In Figure 3.13, financial conditions are instead measured with the excess bond premium, thus taking into account only risk aversion and not default risk. The same conclusions obtained with the GZ spread hold, but the magnitude of the effects are reduced. The decline in industrial production in 2010 has an annualised through of 2.4%, and the fraction of total variance explained by the financial condition shock is only 13% after 2 years (Figure 3.10). Recalling that the GZ spread controls for two components, one reflecting countercyclical default premia and one capturing the extra compensation required by investors to accept default risk, these results show that default premia are just as important as risk aversion in explaining the decline in industrial production growth in the recent years. Hence, the fall in output and slow recovery seems to have been caused by an increased risk aversion of lenders as well as by worsened macroeconomic fundamentals.

Lastly, note that results depend on how financial conditions are measured. If the BA spread replaces the GZ spread in the VAR (Figure 3.11) the retrieved financial condition shocks are never found to affect industrial production significantly.

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9In this way, no restriction on the coefficients of inflation and of the nominal interest rate are imposed. Moreover, the shock to the interest rate series can be interpreted as a pure monetary policy shock. Using the real interest rate does not alter the results.

10Differently from Gilchrist and Zakraješek [2012] I employ a smaller set of variables (the same used in the forecast exercise). In a later stage, I will expand the VAR to include the same set of variables. Results from an alternative variable ordering do not differ significantly from those reported here and are available upon request.
Figure 3.9: Responses to a one-standard deviation shock in the GZ spread over the years: Median impulse response functions (in red) with 95% confidence band (shaded area). SVAR estimated through the TVP-FF-VAR methodology, with a lag-length of 6 months, and identified through a recursive scheme, with the credit spread ordered third.

Figure 3.10: Forecast error variance decomposition: Fraction of Industrial production variance attributable to a shock in the GZ spread, for selected years.
Figure 3.11: Responses to a one-standard deviation shock in the BA spread over the years: Median impulse response functions (in red) with 95% confidence band (shaded area). SVAR estimated through the TVP-FF-VAR methodology, with a lag-length of 6 months, and identified through a recursive scheme, with the credit spread ordered third.

Figure 3.12: Forecast error variance decomposition: Fraction of Industrial production variance attributable to a shock in the BA spread, for selected years.
Figure 3.13: Responses to a one-standard deviation shock in the EBP over the years: Median impulse response functions (in red) with 95% confidence band (shaded area). SVAR estimated through the TVP-FF-VAR methodology, with a lag-length of 6 months, and identified through a recursive scheme, with the EBP ordered third.

Figure 3.14: Forecast error variance decomposition: Fraction of Industrial production variance attributable to a shock in the EBP measure, for selected years.
3.6 Conclusions

The recent financial crisis has illustrated how interrelated the financial and the real side of the economy are. Economic models tell us moreover that the existence of financial frictions strengthens the transmission of shocks and that movements in default risk or in risk aversion induce changes in the cost or in the quantity of borrowing which anticipate future movements in loans, output, and even prices. Based on these theoretical models, I verify to what extent the inclusion of financial condition indicators improves the point and density forecasts of key macroeconomic variables, relative to a baseline TVP VAR model with control variables, and to a simple autoregressive model estimated recursively.

Three financial condition indicators are considered: the spread between Baa and Aaa rated corporate bond yields, a spread developed by [Gilchrist and Zakrajšek 2012] using a richer corporate bond database, and the excess bond premium. The latter has been proposed by [Gilchrist and Zakrajšek 2012] and is defined as the residual component of the GZ spread that is not correlated to default risk but captures overall risk aversion.

Several result emerge from the forecasting analysis. First, controlling for financial condition indicators improves the forecasting performance of the baseline TVP VAR model during large recessions, when credit constraints are arguably binding. By contrast, no role is found for price measures. However, whether the forecast gain results from controlling also for default risk (credit spreads) or only for risk aversion (the excess bond premium) depends on the forecasted variable. The GZ spread delivers more accurate point forecasts for output and employment than the model that controls for the excess bond premium. This suggests that both default risk and risk aversion are important predictors for aggregate activity indicators, especially in the forecast subsample that includes the 2008 financial crisis. Controlling solely for risk aversion suffices instead to improve the point forecasts of loans and of the loan interest rate premium, relative to the autoregressive benchmark.

Secondly, the analysis of the entire forecast distribution reveals that the marginal contribution of financial indicators in forecasting output, employment and loans starts to be relevant only after the 2001 recession and becomes substantial only after the great financial crisis, when credit constraints were arguably binding. While risk aversion is the most relevant predictor for loans, default risk premia have played an important role in forecasting output and employment during the 2008 financial crisis, particularly at a one-year ahead horizon. In particular, controlling for the GZ spread helps to forecast the macroeconomic downturn seen at the end of 2008, shifting the entire distribution of industrial production towards lower values. By contrast, controlling for financial condition indicators does not improve the density forecasts of prices nor of the interest rate spread.
Lastly, I have conducted a structural analysis, to assess whether shocks to financial conditions have displayed different real effects over time. Following the same identification strategy as in Gilchrist and Zakrajšek [2012], I show that their results are mainly driven by the latest financial crisis. An unexpected worsening in financial conditions displays in fact adverse real effects only after 2008. Moreover, it appears that default risk premia matter just as much as risk aversion and that the economic recession and slow recovering after 2008 can be explained by both increased risk aversion and worsened macroeconomic fundamentals.
Chapter 4


Joint with Sandra Eickmeier, Massimiliano Marcellino and Wolfgang Lemke

4.1 Introduction

In this paper, we study the temporal evolution in the international transmission of global financial shocks. We address the following questions.

(i) How large is the impact of global financial shocks on major advanced countries, and have the shock size and its transmission changed over time?

(ii) Through what channels are global financial shocks both domestically and internationally transmitted, and can we identify changes in the transmission mechanism over time?

(iii) How strongly were the major advanced economies affected during the global financial crisis and through which channels?

While several previous papers have focused on the transmission of specific financial shocks (such as credit, stock price or house price shocks) we focus here on ‘shocks to overall financial conditions’. Specifically, we identify global financial shocks as unexpected changes in the US financial conditions index (FCI) proposed by [Hatzius et al. 2010], which summarizes 46 different US financial variables. On the one hand, this choice reflects the fact that financial markets are closely linked, a feature that has become evident during the recent financial crisis. On the other
hand, we are aware that the interpretation of results regarding the propagation of a broad financial shock is more difficult than interpreting those of more narrowly defined financial shocks. The US FCI is taken as a proxy for worldwide financial conditions, given the dominance of the US in global financial markets. We will show that indeed the US FCI is very highly correlated with other countries’ FCIs.

We use the FCI in combination with a newly compiled quarterly dataset for nine major advanced countries, namely the G7 countries as well as Australia and Spain, two additional large economies. The dataset contains 203 quarterly real activity, price, monetary, financial and trade variables, over the sample period 1971Q1-2012Q4. All our variables are jointly modeled in a factor-augmented vector autoregressive model (FAVAR). Each of the 203 international variables is then decomposed into a common component, which depends on the FCI and the (remaining) common factors, and an idiosyncratic component, which is related to variable-specific shocks. Financial shocks can affect consumption and investment, e.g. through wealth effects, changes in funding costs and financial accelerator mechanisms. A decline in demand in one country after a financial shock can then lead via trade to negative economic effects in other countries. In addition, financial shocks can spill over to other countries via integrated financial markets through foreign asset exposure and/or contagion effects which lead to highly synchronized asset prices across countries.

Factor models allow to include many international variables that can flexibly interact with each other, permitting to appropriately capture the global transmission mechanism. As a special feature, our model allows for parameter variation in the VAR for the FCI and the factors (including changes in the variance-covariance matrix of the shocks) as well as in the loadings associated with the transmission of changes in the FCI and in the factors to the international variables. This TV-FAVAR specification and associated classical (as opposed to Bayesian) estimation approach is suggested by Eickmeier et al. [2014], and extends the constant parameter FAVAR specification introduced by Bernanke et al. [2005]. Allowing parameters to change over time is important as globalization may have increased integration via trade and financial markets and also as the link between the financial and real sector has potentially become stronger. Our model can also capture potential asymmetries in transmission as, for instance, different effects over financial crisis and non-crisis periods.

The paper makes several contributions to the existing literature on international transmission. While previous studies mostly looked at the international propagation of real or monetary policy shocks, we focus instead on the international transmission of financial shocks, as done in Bagliano.

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[1] Such models are frequently used in the international business cycle literature, see, e.g., Kose et al. [2003] or Stock and Watson [2003b].


All these studies also use large models, but they focus on specific types of financial shocks (e.g. shocks to house or stock prices or credit shocks) while we focus on shocks to overall financial conditions. Also, all models employed in these three studies are based on constant parameters. This paper, by contrast, uses a fully time-varying model to assess changes in the size and transmission of financial shocks. Moreover, we look at the transmission not only via the traditional trade channel, but also via variables capturing financial and asset markets. Finally, we focus on the impact of the global financial crisis and explore whether its strong impact was due to unusually large shocks, a particularly strong transmission of that shock or a combination of the two, also by drawing comparisons to previous periods.

Our results, answering the three questions from the beginning, can be summarized as follows:

First, regarding the size and transmission of global financial shocks, we find that they have a considerable impact on the nine countries. The transmission to GDP growth in European countries and Japan has somewhat increased gradually since the 1980s, but in general changes in the transmission are not statistically significant. The size of US financial shocks also varies strongly over time, with the ‘global financial crisis shock’ being very large by historical standards.

Second, regarding the transmission channels, we find that improvements in global financial conditions, as they are reflected in an increase in asset prices and credit in most countries, trigger positive investment and (somewhat smaller) consumption reactions. Positive TFP responses probably also contribute to the rise in investment. Financial shocks are propagated internationally via financial markets and trade. Unlike financial shock volatility, the transmission of global financial shocks does not differ much across normal times and periods of financial turmoil.

Third, concerning the global financial crisis, we find that the exceptionally deep worldwide recession was mostly due to a large negative global financial shock combined with a strong propagation of that shock in the euro area and Japan. Global financial shocks explain on average approximately 20 percent of the variation in GDP growth during the crisis period, which is very large compared to an average of 5 percent over the 1971-2007 period, and larger than other turmoil periods for most countries. The transmission of the global financial crisis has been unusual in a number of other respects. Many variables, including activity variables, prices, financial variables and TFP, have reacted particularly strongly in most countries. Exports have declined, in particular in Canada, Germany and Japan, while interest rates have moved less than in the period before the global financial crisis, probably because monetary policy reached the zero lower bound in many countries. Finally, the US real effective exchange rate significantly appreciated during the global financial crisis (after a negative financial shock), while it depreciated before.

4In this respect, our analysis is most closely related to Liu et al. 2011 who analyze the transmission of global real and monetary shocks to the UK based on a Bayesian TV-FAVAR.
The paper is structured as follows. The econometric methodology is explained in section 4.2. Section 3 describes the US FCI and the large international dataset. Section 4 studies the dynamics of global financial (FCI) shocks and their evolving transmission to GDP growth in the nine countries in our panel (4.1 and 4.2). Section 4.3 explains the detected pattern of time variation in the effects of the FCI shock on growth, and pins down the main transmission channels. In Section 4.5 we report the robustness of our results with respect to the choice of the specific FCI. Section 5 concludes.

4.2 Econometric Methodology

4.2.1 The constant-parameter FAVAR model

The analysis departs from an $N$-dimensional vector $X_t$, which includes a large number of economic and financial variables for the nine countries under investigation, and is modeled with the aid of a time-invariant approximate dynamic factor model (Bai and Ng [2002], Stock and Watson [2002]):

$$X_t = \Lambda' F_t + e_t.$$  \hfill (4.1)

In equation (4.1), $F_t = (f_{1t}, \ldots, f_{rt})'$ and $e_t = (e_{1t}, \ldots, e_{Nt})'$ denote, respectively, a vector of common factors that have a major effect on all international variables and may thus be regarded as the main (common) drivers of all the countries, and a vector of variable-specific (or idiosyncratic) components. The number of common factors is generally well short of the number of variables contained in the dataset, i.e. $r << N$. In addition, $F_t$ may contain dynamic factors and their lags. To that extent, equation (4.1) is non-restrictive. Common and variable-specific components are orthogonal. The common factors are also assumed to be orthogonal to each other, and the variable-specific components can be weakly correlated with one another and also serially correlated in the sense of Chamberlain and Rothschild [1983]. The matrix of factor loadings is $\Lambda = (\lambda_1, \ldots, \lambda_N)$, where $\lambda_i$ is an $r$-dimensional vector whose elements measure the effect of each factor on variable $i$, $i = 1, \ldots, N$.

It is assumed that the dynamics of the factors can be described using a VAR($p$) model:

$$F_t = B_1 F_{t-1} + \ldots + B_p F_{t-p} + w_t, \quad E(w_t) = 0, \quad E(w_t w_t') = W.$$  \hfill (4.2)

Since the elements of $X_t$ are assumed to be zero-mean processes (and the respective data are demeaned), equations (4.1) and (4.2) do not contain intercepts.

Following Bernanke et al. [2005] we break down the $r$-dimensional vector of factors $F_t$ into an $M$-dimensional vector of observed factors $G_t$ and an $r-M$-dimensional vector of unobserved (or
latent) factors $H_t$, i.e. $F_t = (G_t', H_t')'$. For most of the analysis, $G_t$ is the US FCI published by Hatzius et al. [2010] (and $M = 1$). This FCI is an aggregate of 46 US financial/asset variables. We provide a detailed explanation of how the FCI is constructed and of the underlying series in the next section. By including the FCI, we will be able to identify global financial shocks. The ‘residual’ common factors $H_t$ consist of the other factors which drive our nine countries, most likely other global shocks or shocks that occur in one country and spill over to the other countries.

The model we have described so far can be estimated in four steps. The first step is to determine the dimension of $F_t$, i.e. the number $r$ of common (latent and observed) factors driving our large dataset; see our discussion below.

In the second step, we estimate $H_t$ by removing the observed factors from the space spanned by the $r$ factors as follows. We extract the first $r$ principal components from $X_t$ and summarize them in $\hat{F}_t$. Next, we estimate a regression of the form $G_t = \gamma' \hat{F}_t + \nu_t$. $H_t$ is then estimated as $\hat{H}_t = \gamma' \perp \hat{F}_t$ where the $r \times (r - M)$ matrix $\gamma' \perp$ denotes an orthogonal complement such that $\gamma' \perp \gamma = 0$.

The matrix of (time-invariant) factor loadings $\Lambda$ can be estimated by an OLS regression of $X_t$ on $(G_t', \hat{H}_t')'$. We should note that this very easy and fast way of cleaning the factor space from the observed factor(s) yields latent factors which are mutually orthogonal and orthogonal to the observable factor(s).

In the third step, we model the dynamics of $F_t = (G_t', \hat{H}_t')'$ with the aid of the VAR (4.2).

In a fourth step, we identify the financial shocks by applying a Cholesky decomposition to the covariance matrix of the reduced-form VAR residuals where the FCI is ordered before the international factors. Using this identification scheme, we are as flexible as possible allowing all international factors to react immediately to global financial shocks. Alternatively, we have also ordered the FCI last in the VAR. One could argue that the FCI comprises numerous fast-moving variables such as stock prices or interest rates which can react instantaneously to other disturbances while our baseline identification scheme restricts the FCI not to respond contemporaneously (although it does allow the individual financial variables which are summarized in the FCI to react immediately). Results from this alternative identification scheme, available upon request, do not considerably differ from our baseline results.

4.2.2 The time-varying parameter FAVAR model

In order to trace possible changes in the way the FCI shock affects the variables of interest in the various countries, we modify the baseline FAVAR model in (4.1) - (4.2) by allowing for time variation in the parameters. To introduce the approach, we first note that the VAR equation (4.2)

\footnote{We decided not to employ sign restrictions as opposed to contemporaneous zero restrictions due to the lack of theoretical models providing a sufficient number of meaningful and widely accepted sign restrictions.}


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can be represented as

\[ PF_t = K_1 F_{t-1} + \ldots + K_p F_{t-p} + u_t, \quad E(u_t) = 0, \quad E(u_t u'_t) = S, \]  

(4.3)

where \( P \) is lower-triangular with ones on the main diagonal, and \( S \) is a diagonal matrix. The relation to the reduced-form parameters in (4.2) is \( B_i = P^{-1} K_i \) and \( W = P^{-1} SP^{-1} \).

We relax the assumption of parameter constancy in four dimensions by allowing for time variation in: (i) the autoregressive dynamics of the factors \((K_1, \ldots, K_p)\), (ii) the contemporaneous relations captured by the matrix \( P \), (iii) the variances of factor innovations, i.e. the elements of \( S \) in (4.3), and (iv) the factor loadings in (4.1). Thus, we consider the following time-varying version of the single equations of (4.1),

\[ x_{i,t} = \Lambda_{i,t}' F_t + e_{i,t}, \quad i = 1, \ldots, N \]  

(4.4)

and the VAR (4.3),

\[ P_t F_t = K_{1,t} F_{t-1} + \ldots + K_{p,t} F_{t-p} + u_t, \quad E(u_t) = 0, \quad E(u_t u'_t) = S_t, \]  

(4.5)

where again \( P_t \) is lower-triangular with ones on the main diagonal, and \( S_t \) is diagonal. Note that we do not associate any structural interpretation to the \( P \) or \( P_t \) matrices for the moment, the decomposition of the variance covariance matrix of the residuals just serves to render the errors in (4.3) or (4.5) uncorrelated.

Let the time-varying parameters \( \{P_t, K_{1,t}, \ldots, K_{p,t}, \Lambda_{1,t}, \ldots, \Lambda_{N,t}\} \) be collected in a vector \( \alpha_t \). Note that the dimension of this vector is \( r \cdot (r-1) \cdot 0.5 + p \cdot r^2 + N \cdot r \), which can be fairly large.

As is common in time-varying parameter regression models, see e.g. Nyblom [1989], we assume the parameters to vary slowly over time, as independent random walks

\[ \alpha_t = \alpha_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, Q), \]  

(4.6)

where \( Q \) is a diagonal matrix.

In practice, the matrix \( Q \) could be non-diagonal, capturing commonality in some parameter movements. Our estimation procedure, described below, remains consistent also in this case, though not efficient. As an alternative, a specific structure could be imposed on \( Q \) (to reduce the number of free parameters), or a different model used for parameter evolution, e.g., a factor model. However, both these approaches impose precise patterns of commonality in parameter movements, which we prefer to avoid given the lack of a priori information on this issue.

It is worth mentioning that our time-varying FAVAR specification nests the standard FAVAR, since when all the elements of the \( Q \) matrix are equal to zero the former reduces to the latter.
Finally, we also allow for some persistence in the idiosyncratic components in (4.4), assuming that they follow a first-order autoregressive process:

\[ e_{i,t} = \rho_i e_{i,t-1} + \xi_{i,t}, \quad E(\xi_{i,t}) = 0, \quad E(\xi_{i,t}^2) = \sigma_i^2, \quad i = 1, \ldots, N \] (4.7)

The elements of \( \xi_t \equiv (\xi_{1,t}, \ldots, \xi_{N,t})' \) are assumed to be contemporaneously uncorrelated among themselves and over time, and uncorrelated with all the elements of \( u_t \) and \( \epsilon_t \), which are in turn assumed to be uncorrelated contemporaneously and over time.

4.2.3 Modeling volatility

A crucial point is how to model time variation in factor innovation volatility. We assume in our baseline model that the variance of each shock can be approximated by a function of three observable variables (lagged by one quarter) constructed as follows. We start with the time series of daily squared logarithmic changes in the US S&P 500 and weekly (due to data availability) squared changes of the BAA-AAA corporate bond spreads. Similar to Adrian and Rosenberg [2008] we apply an HP filter to each of the two series and obtain the HP trends at daily and weekly frequency, respectively. These trends are converted to quarterly frequency by taking averages over the days (weeks) of the respective quarters. As a third observable variable we use the dispersion of GDP growth forecasts across forecasters computed as the difference between the 75th and the 25th percentile of individual 1-quarter ahead forecasts for GDP growth (published in the Survey of Professional Forecasters and provided on the website of the Federal Reserve Bank of Philadelphia).

Stock market volatility and forecast dispersion are widely used measures of uncertainty in the economy as, e.g., pointed out by Bloom [2009]. We add to these measures the volatility of the corporate bond spread as an additional proxy.

Hence, the volatility specification of the structural shock in the \( g \)th equation has the form

\[ S_{gg,t} = c_g + b_g' Z_{t-1}, \] (4.8)

where the scalar \( c_g \) and the vector \( b_g \geq 0 \) are equation-specific, and \( Z_{t-1} \) contains the lagged observed volatility measures. We use lagged values to avoid possible endogeneity problems. Finally, the specification in (4.8) nests the homoskedastic case, which would arise from \( b_g = 0 \).

We assess the robustness of our results with respect to the modeling choice of the time-varying factor innovation volatilities. More specifically, we add other observables to our \( Z_t \) and, alternatively, assume that the volatilities follow GARCH processes. Results from alternative specifications are similar to those from the baseline specification (4.8) and are available upon request.
4.2.4 Estimation of the time-varying FAVAR model

The factors

The elements of $F_t = (G_t', \hat{H}_t')'$ are obtained by combining the principal component and the regression approaches to take care of the observable factor as in the case of the constant-parameter FAVAR model. We then treat the factors as observable and estimate the time-varying-parameter factor VAR and the loading equations. Note that, as argued by Stock and Watson [2002], Stock and Watson [2008] and Breitung and Eickmeier [2011], the factors are still estimated consistently by principal components even if there is some smooth time variation in the loading parameters (see also Banerjee et al. [2008] for finite sample simulation evidence). The intuition underlying this result is that factor estimates at time $t$ are weighted averages of the $N$ $x_i$ variables at time $t$ only.

We set the number of factors $r$ to 10. The Bai and Ng [2002] criteria indicate between 6 and 11 factors. As shown by Breitung and Eickmeier [2011], the Bai and Ng [2002] criteria over-estimate the number of factors if the loadings vary over time. This may be problematic if one is interested in individual factors. However, the factor space is consistently estimated, which is what we are after here. Moreover, Stock and Watson [1998] have shown for constant parameter factor models that the space spanned by the factors is estimated consistently when the number of factors is overestimated but not when it is underestimated. Lastly, while results are barely affected when the number of factors is increased beyond 10, they differ when a smaller number of factors is instead selected. Interestingly, there appears to be more variation in the transmission with less factors, consistent with the findings of Prieto et al. [2013]. For those reasons, we prefer a sparse parameterization, we carry out the analysis with 10 factors. We note that the 10 latent and observable factors explain a considerable fraction - 54 percent - of the variation in $X_t$ over the entire sample period.

The cross-sectional relations

Regarding the cross-sectional relations, we put each of the $N$ equations (4.4) into state space form. Since the idiosyncratic component in (4.4) follows an AR(1) process, rather than being white noise, it becomes part of the state vector besides the time-varying loading parameters. For the $i$th equation the state vector is $\tilde{\alpha}_t^{(i)} = (\lambda_t', \epsilon_t')'$. The transition equation is given by

$$\tilde{\alpha}_t^{(i)} = \Phi_i \tilde{\alpha}_{t-1}^{(i)} + \tilde{\epsilon}_t^{(i)},$$

This approach has been adopted also by Boivin et al. [2008] and Buch et al. [2014].  

Our setup does not allow the number of factors to vary over time. It would certainly be interesting to study whether this is indeed the case. However, this is beyond the scope of this paper.

Assuming AR(2) rather than AR(1) processes for the idiosyncratic components does not affect our main results. Results are available upon request.
where $\Phi_t = \text{diag}(1_r, \rho_t)$, $\tilde{\epsilon}_t^{(i)} = (\epsilon_t^{(i)}, \xi_t)'$, where $\epsilon_t^{(i)}$ are the respective elements of $\epsilon_t$ in (4.6), hence, $E(\tilde{\epsilon}_t^{(i)}) = 0$, and $E(\tilde{\epsilon}_t^{(i)}\tilde{\epsilon}_t^{(i)\prime}) = \text{diag}(q_t^{(i)}, \sigma_t^2)$. That is, $q_t^{(i)}$ contains the random-walk innovation variances of the time-varying parameters (i.e. the respective elements of $Q$ in (4.6)) and $\sigma_t^2$ is the innovation variance of the idiosyncratic component process. The measurement equation is

$$x_{i,t} = Z_t \tilde{\alpha}_t^{(i)} \tag{4.10}$$

where $Z_t = (F_t', 1)$. We estimate the $r + 2$ hyperparameters $(\rho_t, q_t^{(i)}, \sigma_t)$ of the $i$th loading equation by maximum likelihood. We then back out the path of time-varying loading parameters using the Kalman smoother $^9$

### The VAR for the factors

Since our assumptions imply independence (conditional on the factors and volatility regressors) between the $r$ equations of the VAR representation (4.5), we can likewise estimate the time-varying parameters contained in the $P_t$ and $K_{i,t}$ matrices equation by equation. For the $g^{th}$ equation in state space form, the state vector containing the time-varying parameters is given by

$$\alpha_{g,t} = (-P_{g,1,t}, \ldots, -P_{g,r-1,t}, K_{g,1,1,t}, \ldots K_{g,1,1,t}, K_{g,1,2,t}, \ldots K_{g,r-1,1,t}, K_{g,1,2,t}, \ldots K_{g,r,1,t}, \ldots K_{g,1,p,t}, \ldots K_{g,r,p,t})$$

where for $g = 1$, there are no $P$ parameters showing up. Note that due to the different number of elements coming from the triangular $P$ matrix, the dimensions of the state vectors are different for each of the $r$ equations.

The state equation is the random walk for $\alpha_{g,t}$,

$$\alpha_{g,t} = \alpha_{g,t-1} + \epsilon_{g,t}, \quad \epsilon_{g,t} \sim N(0, Q_g), \quad Q_g = \text{diag}(q_g). \tag{4.11}$$

The measurement equation is given by

$$f_{g,t} = f_{g,t}^{\prime} \alpha_{g,t} + u_{g,t}, \quad u_{g,t} \sim N(0, S_{gg,t}) \tag{4.12}$$

where

$$f_{g,t}^{\prime} = (f_{1,t}, \ldots, f_{g-1,t}, f_{g,t-1}, \ldots, f_{r,t-1}, f_{1,t-2}, \ldots, f_{r,t-2}, \ldots, f_{1,t-p}, \ldots, f_{r,t-p}),$$

and $S_{gg,t}$ is given by (4.8).

$^9$We initialise the Kalman filter through OLS estimates on a subsample that excludes the financial crisis, to avoid incurring in unstable estimates.
In a first step, we estimate for each equation the ‘hyper-parameters’ \((q_g, c_g, b_g)\) by maximum likelihood. In a second step, we filter out the time-varying parameters of each equation by the Kalman filter. We make sure that the local VAR dynamics at each time \(t\) does not imply explosive behavior. After that, the Kalman smoothing scheme is applied in the usual fashion.

We set the VAR lag length at \(p = 1\). This choice is suggested both by the need of reducing the number of parameters, and by the consideration that allowing for parameter time variation likely reduces the need of longer lags. We have carried out the analysis also with two lags instead of one. Results (available upon request) were indeed barely affected.

We note that the Monte Carlo results provided in Eickmeier et al. [2014] support a good finite sample performance of the estimation method.

**Impulse response functions and forecast error variance decompositions**

Given the estimated TV-FAVAR, the impulse response functions and forecast error variance decompositions provided in this paper are based on the (smoothed) parameter structure prevailing at the respective point in time.\(^{10}\) That is, they are computed in the standard way as with constant-parameter FAVARs but with a new parameter structure at each time \(t\). Confidence bands for the impulse response functions are computed based on a bootstrap. See Eickmeier et al. [2014] for details.

**4.2.5 Assessing the extent of parameter time variation**

One may wonder whether time variation in the parameters is really needed or a constant-parameter specification would suffice. To gauge the degree of time variation we count the number of parameters, for which the standard deviation of the Kalman-smoothed parameter path is essentially zero.\(^{11}\) It turns out that there is actual time variation (i.e. no ‘straight-line’ parameter paths) for: 30 out of the 100 parameters of the \(K\) autoregressive matrix (containing the dynamics of the VAR(1) for the 10 factors); 19 out of the 45 (= 0.5 \cdot 10 \cdot 9) parameters of the \(P\) matrix of contemporaneous relationships of the VAR; and 856 out of the 2030 loadings (since there are 10 loadings, one for each factor, for each of the 203 variables).

\(^{10}\)As a robustness check we have assessed whether results based on the filtered parameter estimates differ from those based on the smoothed estimates. This addresses the concern that sudden changes in the dynamics could be watered down by the Kalman smoother, which would bias our results, especially those regarding possible asymmetries of the transmission of financial shocks. We find that impulse responses based on filtered estimates do display more high frequency movements. Our broad picture, however, remains the same. Hence, we prefer to stick to the smoother in our baseline exercise, since some of the additional variation from the filtered estimates could just reflect small sample estimation uncertainty.

\(^{11}\)In Eickmeier et al. [2014] we argue why this should be a reasonable approach.
Finally, we have assessed whether there is indeed time variation in the volatilities of the shocks, i.e. whether the elements of $b_g$ in equation (4.8) are significant. We find that 9 out of 30 ($= 3 \cdot 10$) parameters are indeed significant at the 5% level. More specifically, the FCI shock volatility is significantly related to the stock market and corporate bond spread volatilities, but not to forecast dispersion, while the latter measure significantly enters the equations for six of the other nine (latent) factors.

Hence, as these are all sizeable fractions, we do believe that it is important to take parameter time variation into account.

4.3 Data description

4.3.1 US financial conditions index

We use in our baseline analysis the FCI for the US which has been recently constructed by Hatzius et al. [2010] and published on Mark W. Watson’s webpage. This FCI summarizes a broad set of 46 quarterly financial variables including interest rates and spreads, credit aggregates, survey measures on credit conditions, asset prices, exchange rates and the oil price. The index is based on an unbalanced dataset and is available from 1970Q2 onwards. The published FCI ends in 2009, and we update it until 2012 using the methodology by Hatzius et al. [2010].

The FCI used in our analysis is shown in Figure 4.1. An increase in the FCI can be interpreted as an improvement of ‘overall financial conditions’, while a decline reflects a worsening. The evolution of the index matches with anecdotal evidence on major financial turmoil such as the financial headwinds period in the early 1990s (see, e.g., Greenspan [1994]), the stock market crash in 1987, the burst of the dotcom bubble in 2001 and the global financial crisis in 2008-2009. The FCI is most highly positively correlated with a number of credit variables, in particular with indebtedness measures. Largest negative loadings are instead associated with various risk spreads, bank stock market volatility and tightening of lending conditions by banks. The exchange rate

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12The t-statistics for the parameters are based on the estimated standard errors obtained from the negative inverse of the Hessian of the likelihood function.

13In their paper, Hatzius et al. [2010] mainly focus on an FCI constructed as follows. Each series in the large financial dataset is purged from the influence of contemporaneous and lagged GDP growth and inflation, and then the FCI is estimated as the first principal component (PC) from the residuals. More precisely, the FCI is estimated by least squares and iterative methods since an unbalanced panel is used. Were the panel balanced, the solution to the least squares problem provides the PC of the data. In this work, we measure the FCI as the first PC of the unpurged data, an alternative discussed in Hatzius et al. [2010], as the macroeconomic influences are later removed through our empirical application. In section 4.5 we discuss the robustness of our results when the influence from US macroeconomic aggregates is instead removed prior to the analysis.
and the oil price do not appear to be major drivers of the FCI. In Eickmeier et al. [2011] we also show that most loadings barely change over time.\footnote{Overall, an increase in the FCI reflects an improvement of overall financial conditions and should lead to an increase in GDP growth in the countries under investigation. Some variables such as the oil price or the US real effective exchange rate enter the FCI positively: their increases could potentially lead to negative real effects in some countries. However, the weights of those series in the construction of the FCI (i.e. the loadings) are very small, and, hence, movements in the FCI are unlikely to mainly reflect movements in those variables.}

We refer to Hatzius et al. [2010] and Eickmeier et al. [2011] for more details on the underlying data, the classification of the variables in the groups, and a careful analysis of the statistical properties of the FCI.

### 4.3.2 Large international dataset

The dataset comprises quarterly variables over the period 1971Q1-2012Q4 for nine major advanced countries, the US, Canada, the UK, France, Italy, Germany, Spain, Japan as well as Australia. The choice of the sample period is mainly driven by data availability. We have extended the sample as far back as possible since a long period is needed to assess whether and to what extent globalization and financial deepening has changed the way US financial shocks are transmitted internationally. Another advantage is that including earlier periods of financial turmoil, reaching back up to the beginning of the 1970s helps us to better capture the most recent global financial crisis.

We include 23 variables for each country, if available for the entire sample period. These variables comprise several measures of real economic activity (GDP, personal consumption, total fixed investment, residential and non-residential investment, government consumption, government primary balance-to-GDP ratio, total factor productivity (TFP), industrial production, unemployment rate), aggregate price variables (GDP deflator, CPI), trade (activity and price) variables (real exports, real imports, export prices, import prices, the real effective exchange rate, the bilateral nominal exchange rate with the US Dollar) as well as monetary and financial variables (equity prices, residential property prices, private credit, short-term and long-term interest rates). Overall, the dataset contains $N = 203$ series.

Asset prices and credit were converted to real variables by division by the GDP deflator. Exchange rates are defined such that an increase reflects an appreciation of the respective currency.

Data are taken from various international institutions, including the BIS, the IMF, the OECD and the European Commission. These data are, in some cases, complemented with data from national sources. House prices are often not available and/or only at a biannual or annual basis. We take residential property prices from Goodhart and Hofmann [2008], who carefully constructed a quarterly dataset for 17 OECD countries for the period 1971-2006, and updated their data with recent data from the BIS. Other series such as TFP and the government balance-to-GDP
ratios were also available only on an annual basis. We converted annual to quarterly data using a cubic spline interpolation. An attractive feature of our TV-FAVAR approach is that, at least theoretically, interpolation errors and other data irregularities should only enter the idiosyncratic component of each equation, making our analysis robust since it is mostly based on the common component.

As is common practice in factor analysis, the series are transformed in a multiplicity of ways. Stationarity, where required, is obtained by differencing; all variables are entered as differences of logarithmized values, with the exception of interest rates, unemployment rates and government balance-to-GDP ratios, which are entered in levels. The series are standardized and subsequently have a zero mean and a unit variance. Finally, we remove outliers - defined here as observations of the (stationary) series with absolute deviations from the median which exceed six times the interquartile range. Following Stock and Watson [2005a], we replace them with the median of the preceding five observations.

Table 1 contains a more detailed description of the series, sources and treatment of the data.

4.4 Global financial shocks and their evolving transmission to international GDP growth

In this section we discuss the evolution of the size of FCI shocks and their transmission to the FCI and to GDP growth (as a summary measure of real activity and a key variable of interest) in the nine countries under study.

4.4.1 FCI shocks

Figure 4.2 shows the estimated time-varying standard deviation of the FCI shocks. Wide fluctuations emerge, with large values of the volatility broadly reflecting major financial turmoils in the US over the sample period under analysis, including the four postwar financial crises as dated by López-Salido and Nelson [2010], namely the ‘Bank Capital Squeeze’ in 1973-1975, the ‘LDC (less developed countries) debt crisis’ in 1982-1984, ‘the Savings and Loan Crisis’ in 1988-1991, and the

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15The means we subtract from the variables prior to the FAVAR modeling could be also time-varying. To address this potential concern, we have applied the sequential multiple breakpoint test of Bai and Perron [1998] and Bai and Perron [2003] to all series, and in case of rejection we have subtracted properly segmented rather than constant means from the series prior to estimation of our model. It turns out that the results from this alternative standardization of the variables are very similar to those presented above.

16Note that this procedure does not alter our results significantly. Results without a preliminary outlier adjustment of the data are available upon request.
global financial crisis at the end of the sample. In addition, we find high levels of the FCI shock volatility around the late 1970s/early 1980s which might also be associated with structural changes in financial markets (regulatory changes and financial innovation), the stock market crash in 1987, the Asian and Russian crisis at the end of the 1990s, and the build-up and subsequent burst of the ‘dot-com’ bubble around 2001. Finally, during the latest crisis we observe an unprecedented increase in the variance of the shock.

Next, we compute the impulse response of the FCI to its own shock, obtained as the Cholesky residual associated with the FCI equation in the TV-FAVAR. We have normalized the shock to raise the US FCI by one unit. This normalization allows us to compare further below the transmission of shocks of the same size to other variables over time.

To get a sense of the magnitude of a one-unit shock to the FCI we multiply the loadings of the main financial variables underlying the FCI by their standard deviations. For example, a (positive) one-unit shock to the FCI leads to impact increases of private nonfinancial debt by 2 percent, and of bank credit by 2 percent. It is also one that triggers impact declines of the monetary aggregate MZM, of the 30-year mortgage rate spread and of the Baa corporate bond spread by respectively 4 percent, 0.12 and 0.22 percentage points.

Figure 4.3 presents the point estimates of the impulse responses for all horizons and all points in time. The chart reveals that the effect of the shock to the FCI itself peaks on impact and turns to zero after a bit more than two years.

### 4.4.2 The changing transmission of FCI shocks to international GDP growth

Figures 4.4 and 4.5 show impulse response functions of GDP growth of the nine countries to the US financial shock. Over the whole sample period the FCI shock is positively transmitted on impact to all countries. In terms of variation over time, we find smooth changes in the effects in Europe and Japan, which would be consistent with a gradual structural change in the economies such as that implied by globalization.

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17 According to López-Salido and Nelson [2010], these three financial crises in the US fall into the pre-global financial crisis sample under investigation. The Bank Capital Squeeze was characterized by a strain on bank capital, several bank failures as well as the risk of default of the New York city government. The LDC debt crisis was characterized by elevated risk of some Latin American governments of a default on borrowings from US commercial banks. It culminated in the US government’s rescue of the Continental Illinois Bank. The Savings and Loan Crisis was reflected in bank and savings and loan failures.

18 Structural changes in financial markets are, e.g., the phasing out of regulation Q, the spreading of securitization, the creation of an interstate banking system, the introduction of risk-oriented capital adequacy requirements and the promotion of fair-value accounting and increased competition in the interbank market. See, e.g., Boivin et al. [2010]. These changes might be reflected in financial shocks but might also have led to a changing transmission.
Over the global crisis period, the impact reaction of GDP growth lies in the 0.4 to 0.8 percentage points range. The impact effect in this episode is the highest in the euro-area countries and in Japan, whereas the impact in the other countries is not high by historical standards. The effect of a FCI shock seems to have become more persistent in euro-area countries and Japan. The one-year ahead responses are typically still positive, significantly so for Japan. We will shed further light on this in the next subsection.

Another interesting issue to consider is the contribution of the financial shock in explaining the forecast error variance of GDP growth in the different countries over time. The relevant information for the one and five-year horizons is provided in Figure 4.6. The variance shares explained by FCI shocks vary notably over time, from negligible to more than 60 percent. Contributions were largest during the most recent crisis in most countries. In the US and in Canada, the shares explained in the early-1980s were similarly high compared to recently observed shares. The time-varying pattern of the variance decompositions thus resembles closely the FCI shock volatility pattern, graphed in Figure 4.2, suggesting that, for the variance decompositions, the variation in the size of the shocks dominates the changes in their transmission. The variance shares for the two horizons are quite similar, because the effects of FCI shocks on GDP growth die out relatively quickly (often within one year).

Interestingly, the contribution as well as the transmission of a same-sized shock in the US is not larger than in several other countries (despite the fact that we use a US FCI). This holds for much of the sample period and, more specifically, for the latest financial crisis episode and is consistent with other multi-country time series analyses such as Helbling et al. [2011] and Eickmeier and Ng [2011] for US credit shocks. Various explanations can be found in the literature. While Van Wincoop [2013], Bacchetta et al. [2012] and Perri and Quadrini [2011] emphasize that panic reactions and self-fulfilling expectations may have played a role for the international financial shock transmission, especially during the global financial crisis, Eickmeier and Ng [2011] argue that more fundamental channels may have been effective, such as financial institution balance-sheet effects (Devereux and Yetman [2010], Devereux and Sutherland [2011], Krugman [2008]), arbitrage (Dedola and Lombardo [2012]) and portfolio reallocation mechanisms (Van Wincoop [2013]). That line of reasoning would also be consistent in the case of the US-to-euro area transmission with Kollmann [2013] who emphasises the global banking channel. He shows in a 2-country DSGE model with a global bank, estimated on US and euro-area data from 1990 to 2010, that loan default shocks in the US have slightly larger effects on euro-area output than on US output.

To summarize, during the Great Recession we observed a negative global financial shock, which was very large by historical standards, combined with a somewhat stronger propagation of that financial shock in the US and in Japan. This finding does not contradict the observation that the overall economic downturn during the crisis was very strong in most countries. One needs to be always aware that we are looking at normalized (same-sized) FCI shocks in this exercise. Hence, even if the impact of a same-sized shock may not have a particularly large impact by historical standards, one has to recall that the standard deviation ('average size') of FCI shocks is changing over time and that it is estimated to be exceptionally large during the recent crisis, as discussed above.
shock to the euro area and Japan. However, confidence bands typically overlap, and changes in
the transmission over time and across countries tend not to be significant.

4.4.3 Understanding the transmission of global financial shocks

We now try to explain the detected pattern of the consequences of the FCI shock on growth, and
to pin down its main transmission channels by looking at the effects of the FCI shock on a variety
of other variables.

In theory, financial shocks can affect domestic consumption and investment through wealth
effects, changes in funding costs and financial accelerator mechanisms.\(^{20}\) A decline in real activity
in one country can then lead, e.g., to lower import demand, and via trade to negative economic
effects abroad. In addition, financial shocks can spill over to other countries via financial integration
and asset synchronization. The former can be achieved through countries’ portfolio exposure
to foreign assets, which might either result in a better risk sharing and help buffer shocks, or
rather reinforce the international spillovers. Asset synchronization could arise due to, for instance,
investors’ reassessment of the outlook of countries with similar fundamentals, confidence effects
or even herd behavior. Changes in financial conditions abroad would then, through the channels
presented above, affect the real sides of the foreign economies. The extent to which foreign activity
is affected depends also on the policy reaction to financial shocks implemented in foreign countries.
Our setup does not allow us to cleanly disentangle the different transmission channels, but we
will be able to assess how financial, trade and other variables capturing the different transmission
channels respond to the financial shocks.

Tables 4.2 to 4.4 present impulse responses of selected variables to the financial shocks. To
save space, we focus on the impact effect and on the effect after one year, computed as an
average over specific periods. We consider the two ‘normal’ or ‘tranquil’ times 1971Q1-1986Q4
and 1987Q3-2007Q4, from which financial turmoil periods are excluded. We choose this split
because 1987 is often seen as the beginning of financial globalization (see, e.g., Kose et al. [2007]).\(^{21}\)
Moreover, we show impulse responses on average over financial turmoil periods experienced in the
US prior to the global financial crisis, which include the financial crises as defined in López-Salido
and Nelson [2010] as well as the two stock market crashes which fall in our sample (see section 4.1).
Finally, we consider the most recent global financial crisis period (either from 2008Q1 to 2009Q2
or 2012Q4). We choose 2008 as the beginning of the global financial crisis since it broadly marks
the start of the latest recession in most countries. We consider both 2009Q2 and, alternatively,

\(^{20}\) Cecchetti et al. [2009] give a useful overview on the channels through which negative financial (crisis) shocks or a
worsening of financial conditions can have adverse effects.

\(^{21}\) In addition, other structural changes characterize the post 1986 period, namely, the growth of the financial sector
and its relation with the real economy, and the ‘Great Moderation’ (i.e. a marked decline in the volatility of
output and inflation). We will ultimately not be able to cleanly separate the effects of the various structural
changes that occurred after 1986.
2012Q4 as the end of the global financial crisis. On one hand, the US recession ends in 2009Q2, as dated by the NBER. On the other hand, the US recession was followed by the sovereign debt crisis in Europe, which was still ongoing at the end of our sample in 2012Q4.

In what follows we first establish stylised facts about the transmission mechanism of financial shocks. We then try to understand to what extent the transmission mechanism has changed over time and, more specifically, to what extent the latest crisis has been unusual.

We report impulse responses of variables related to domestic supply and demand (Table 4.2), of prices and interest rates (Table 4.3), and of variables related to the external environment (Table 4.4). The three tables show that positive FCI shocks broadly display the expected effects. They raise equity prices in all countries and credit and house prices in most countries. They also increase investment and consumption, e.g. via wealth effects, changes in funding costs and financial accelerator mechanisms. Investment increases by more than consumption. The small but positive reaction of TFP may have contributed to the positive investment reaction. A decline in the unemployment rate in most countries (not shown) may have improved the income outlook and contributed to the positive consumption response. Positive demand reactions trigger price and interest rate increases. The trade channel seems to be active, and positive import demand in some countries may have led to a rise in exports in others.

In terms of variation over time, as for GDP growth, we do not detect clear, systematic changes in the transmission to key variables between the two tranquil periods 1971-1986 and 1987-2007, perhaps with one exception. The impact and one-year ahead reactions of short-term interest rates and the one-year ahead reactions of long-term rates have declined over time, possibly because many central banks moved towards inflation targeting and reacted less to financial developments in the second half of the sample.

While we do not find systematic differences between financial turmoil periods (excluding the latest financial crisis) and tranquil periods, the global financial crisis seems to have been unusual in several respects.

First, not only GDP but also investment, prices and, to a lesser extent, consumption react more strongly in some countries compared to normal and previous financial turmoil periods. Second, the reaction of financial variables, and equity prices in particular, and the one-year ahead reaction of TFP have been stronger in many countries, which might explain the stronger and more persistent

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22 The reaction of house prices and credit are not shown in the interest of brevity. The scattered house price reactions are not surprising and could be explained by differences in local supply factors such as residential construction policies, regulation and forms of finance, as well as cross-countries differences in demand factors such as the development and the ageing of the population. Similar directional reactions of credit and house prices in most countries confirm the view that house price booms (busts) and an increase (a decrease) in leverage often coincide, which was particularly apparent before and during the crisis (e.g. Eickmeier and Hofmann [2013]).

23 The reaction of long-term interest rates follows closely that of the short-term rates: it is therefore not shown and available upon request.
activity reactions. The result for TFP can possibly be explained by postponed innovation and depressed investments in R&D due to increased risk aversion or inefficient capital allocation among firms, and a shift in the distribution of firms towards inefficient ones after a tightening in collateral constraints.\(^{24}\) The result is interesting in the light of recent discussions on whether the global financial crisis had an impact on trend growth, which tends to be strongly influenced by TFP.\(^{25}\) Our results, at least, do not stand against this hypothesis. Third, we find a decline in exports, which is particularly pronounced in Canada, Germany, and Japan. Fourth, short- and long-term interest rates react very weakly to financial shocks, which can probably be explained with monetary policy rates in many countries having hit the zero lower bound. Fifth, while the US Dollar depreciated significantly in real effective terms after negative financial shocks before 2008-2009 (consistent with standard exchange rate (UIP) theories), it appreciated significantly over the global financial crisis and possibly contributed to the negative export reaction in the US. The movement of the US Dollar can be explained by repatriation of investments to the US in the early phase of the crisis, as well as a worldwide loss in confidence and increase in risk aversion after the bankruptcy of Lehman Brothers. This triggered substantial safe-haven flows by investors, in particular towards US government bonds, despite the fact that the crisis likely originated in the US.\(^{26}\)

4.4.4 Robustness with respect to the choice of the FCI

We now try to understand to what extent our results are driven by the specific choice of the FCI. The main advantage of our FCI is that is goes back to 1970, whereas other FCIs start later. On the other hand, we use the US FCI as a proxy for global financial conditions. Hence, understanding, for example, to what extent financial developments observed in other countries influence our measure of global financial conditions will be important.

We consider two additional FCIs.\(^{27}\) First, we use the financial stress index for the US (FSI) introduced by Hubrich and Tetlow [2012], which was used by the Federal Reserve Board staff during the crisis to analyze financial conditions and their macroeconomic effects. The index has been constructed on a daily basis. It comprises 9 indicators capturing risk and uncertainty in capital markets and starts in 1975. We have converted the daily into a quarterly index by taking monthly averages. As it measures financial stress rather than financial conditions, we reverse the sign so that it is positively correlated with the baseline FCI.\(^{28}\)

\(^{24}\) For a more detailed discussion and references see Eickmeier and Ng [2011].

\(^{25}\) E.g. European Commission [2009], European Central Bank [2008], Deutsche Bundesbank [2009].

\(^{26}\) See Cecchetti et al. [2009] who describe this mechanism as well as Deutsche Bundesbank [2010] and Eickmeier and Ng [2011] for a similar finding.

\(^{27}\) We are grateful for two anonymous referees for suggesting these checks to us.

\(^{28}\) We are grateful to Kirstin Hubrich and Manfred Kremer for providing us with the US FSI and the euro-area CISS.
Second, we extract an ‘International FCI’ from a set of four FCIs: our baseline US FCI; the composite indicator of systemic stress in the financial system (CISS) for the euro area originally constructed on a daily basis from 15 measures of financial stress by Hollo et al. [2012]; as well as FCIs for the euro area and Japan published by the OECD and described in Guichard et al. [2009]. Since all indices (except for the US FCI) start well after the beginning of our sample (the euro-area CISS in 1999, the OECD FCIs in 1995 and in 1999), we use the expectation maximization algorithm to convert the unbalanced panel of FCIs into a balanced panel. The international FCI is estimated as the first PC from that panel. Up to 1994, it is identical to the baseline FCI, but then it is also influenced by international financial conditions.

Lastly we have modified our baseline FCI by removing the influence from US macroeconomic aggregates, i.e. we have regressed the baseline FCI on US GDP growth, deflator inflation and the Federal Funds rate and have used the residual as our FCI in the analysis. This allows us to account more explicitly for (US) macro influences, which are captured in the latent factors in our baseline specification.

The temporal evolution of the four FCIs is very similar, as shown in Figure 4.7. They all display the deepest troughs during the global financial crisis, with almost identical magnitudes. Replacing the baseline FCI with one of the three alternatives, in turn, leads to almost identical series for the FCI shock volatility, as shown in Figure 4.8. The main visible difference is that the financial shock volatility tends to be smaller in the early 1980s when the purged FCI is used, suggesting that our latent factors might not be adequately capturing macro and monetary policy influences in that period. The impulse responses of GDP growth and the forecast error variance shares explained by financial shocks are also quite similar across the alternative specifications. The financial shocks are very highly correlated with those retrieved from the baseline model, with correlation coefficient of 0.57 (FSI), 0.90 (International FCI), and 0.92 (baseline FCI, where US macro influence was removed). Given that results are so similar to the baseline results, we do not show them in the paper, but make them available upon request.

4.5 Concluding remarks

In this paper we derive and explain a number of stylized facts about how global financial shocks are transmitted internationally, and how the transmission has changed over time. The global financial shock is measured as an unexpected change in the Hatzius et al. [2010] US FCI. We combine the FCI with a newly compiled dataset of more than 200 variables from nine large advanced countries: Australia, Canada, France, Germany, Italy, Japan, Spain, US and UK. The large dataset is modeled

29 Note that the correlation between the baseline FCI and the euro area indices is rather high: 0.85 for the OECD FCI and 0.81 for the CISS. Conversely, the correlation between the baseline FCI and the OECD FCI for Japan is lower: 0.30.
by means of a FAVAR specification, enabling us to comprehensively analyze the (virtually) entire transmission mechanism. We exploit this feature and study not only the final effects of the financial shock on GDP growth of the nine countries but also the various transmission channels, mostly through trade and financial variables.

In order to allow for and assess the extent of time variation in the size of shocks and the transmission mechanism, we adopt the time-varying FAVAR specification introduced by Eickmeier et al. [2014], which allows for smoothly time-varying loadings, VAR coefficients and factor innovation variances and covariance. This econometric methodology therefore permits a thorough evaluation of the temporal evolution of the international transmission of the US financial shocks.

Results show that global financial shocks have a considerable impact on growth in the countries in our dataset. The time-varying approach further unveils an increase in the transmission to GDP growth in the European countries and Japan, though most changes in the transmission are not statistically significant. Moreover, the size of US financial shocks varies strongly over time, with the ‘global financial crisis shock’ being very large by historical standards and explaining approximately 20 percent on average over all countries of the variation in GDP growth during the crisis period (compared to approximately 5 percent over the 1971-2007 period). Finally, investment, exports, TFP, and financial variables display a strong decline in most countries during the crisis, as a consequence of the financial shock.
Figure 4.1: US financial conditions index (FCI)

Index computed by Hatzius et al. 2010: the underlying variables of the FCI have been retrieved from Mark Watson’s web page and updated.

Figure 4.2: Estimated sequence of the FCI shock volatility

Figure 4.3: Impulse-response functions of the FCI reacting to a one-unit FCI shock

The figure shows the time-varying impulse responses of the FCI to a one-unit FCI shock over time and over horizons (quarters).
Figure 4.4: Time-varying impulse responses of GDP growth to a one-unit FCI shock, % points

Figure 4.5: Time-varying impulse responses of GDP growth to a FCI shock at selected horizons

The figure shows point impulse responses (solid lines) together with the respective 90% confidence bands (dashed lines), expressed in % points, for selected horizons (impact, after 4 and 8 quarters).

Figure 4.6: Time-varying forecast error variance shares of GDP growth

The figure shows the time-varying fraction of unexpected changes in GDP growth attributable to FCI shocks, over horizons of 4 quarters (solid lines) and 20 quarters (dashed lines).
The reported FCIs correspond to: the baseline FCI (solid line), the US Financial Stress Index (dashed line), introduced by Hubrich and Tetlow [2012], the ‘International FCI’ (dashed-dotted line), and the baseline FCI were the influence from US macroeconomic aggregates was preliminary removed (dashed + line). See text for details.

See note to Figure 7.
Table 4.1: Data, sources, and transformations employed to achieve stationarity: 0 = levels and 1 = log-differences

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Table 4.2: Impact and one year ahead impulse responses of output (GDP), consumption (CONS), investment (INV), and total factor productivity (TFP) to FCI shocks

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Table shows impact and cumulated one year ahead impulse responses averaged across periods, expressed in percentage points. Stars denote average responses which are different from 0 at a 10% confidence level. Impulse response averages are computed over the following periods. The two ‘normal’ times of 72 – 86 and 87 – 07 refer to 1971Q1-1986Q4 and 1987Q3-2007Q4, from which financial turmoil periods are excluded. PRE-GFC aggregates all financial turmoil periods, described in Section 4.1, excluding the Great Recession. 08 – 09 and 08 – 12 respectively refer to 2008Q1-2009Q2 and to 2008Q1-2012Q4.
Table 4.3: Impact and one year ahead impulse responses of GDP deflator inflation (INFL), interest rates (R) and equity prices (EP) to FCI shocks

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Table shows impact and one year ahead impulse responses averaged across periods, expressed in percentage points. The one year ahead impulse responses of inflation and equity prices are cumulated, while those of the interest rates are kept in levels. Stars denote average responses which are different from 0 at a 10% confidence level. Impulse response averages are computed over the following periods. The two ‘normal’ times of 72 – 86 and 87 – 07 refer to 1971Q1-1986Q4 and 1987Q3-2007Q4, from which financial turmoil periods are excluded. Pre-GFC aggregates all financial turmoil periods, described in Section 4.1, excluding the Great Recession. 08 – 09 and 08 – 12 respectively refer to 2008Q1-2009Q2 and to 2008Q1-2012Q4.
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Table shows impact and one year ahead impulse responses averaged across periods, expressed in percentage points. The one year ahead impulse responses of imports and exports are cumulated, while those of the real effective exchange rates are kept in levels. Stars denote average responses which are different from 0 at a 10% confidence level. Impulse response averages are computed over the following periods. The two ‘normal’ times of 72 – 86 and 87 – 07 refer to 1971Q1-1986Q4 and 1987Q3-2007Q4, from which financial turmoil periods are excluded. PRE-GFC aggregates all financial turmoil periods, described in Section 4.1, excluding the Great Recession. 08 – 09 and 08 – 12 respectively refer to 2008Q1-2009Q2 and to 2008Q1-2012Q4.
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