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THE INFLUENCE OF FUTURES ON SPOT PRICE VOLATILITY IN A MODEL FOR A STORABLE COMMODITY

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ABSTRACT

The influence of futures on spot prices is investigated in a two-date-one-period model with storage. The characterization of storage firms as hedgers is new in the literature and sheds light on the way futures affect spot prices in the presence of storage. The paper proves, on analytical grounds, a stabilizing influence of futures on spot prices when storage decisions are taken.

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1. Introduction

Models of commodity futures have been widely used to investigate the hedging and speculative behavior and the sign and determinants of the market bias (i.e the difference between the futures and the expected spot price).

The literature has focused on the influence of futures markets on spot price stability only from the late seventies on. The need for a joint consideration of storage and futures when analyzing this issue is due to the role that both play in the intertemporal allocation of resources.

Kawai(1983b) and Turnovsky(1983) develop multiperiod models emphasizing steady-state solutions. Non-linearities introduced by storage make it impossible to explicitly compare price variabilities in the presence and in the absence of futures markets in this type of models. More restrictive assumptions or empirical methods are needed to draw conclusions.

Storage is the important feature that characterizes the model presented here. The storage firm is a firm whose income is correlated with prices and can be characterized as a hedger: it can be thought of as an agricultural producer who has to decide how much of his crop to store and how much to sell. Anderson and Danthine(1983), Kawai(1983b) and Turnovsky(1983) present the storage firm as essentially a speculator. Speculation does not properly characterize storage activities since commodity producers generally store for hedging purposes as well. Allowing for hedging gives interesting results which can be usefully compared with those presented in the existing literature .

The structure of the paper is as follows. In section 2, the basic model for the investigation of the storage decision of a risk-averse firm is set up; in section 3, futures trading is allowed for and optimal futures and storage decision are derived to investigate the interactions between the two; in section 4, the influence of futures trading on spot price volatility is assessed. Conclusions and final remarks follow.

2. A Microeconomic Model with Storage

A two-date-one-period rational expectations equilibrium model is considered to analyze the storage decision of a competitive firm and to determine equilibrium relationships.

The time pattern of trading is as follows. At date 1 the representative storage firm has a first and second period random endowment of a storable good. One can think of this endowment as a crop whose exact amount is influenced by previous decisions and by the weather. At a point in time $1+\varepsilon$ immediately following date 1, uncertainty about the first period endowment is resolved. At this point in time spot trading, storage decision and futures trading, when allowed for, take place. At date two, only spot trading takes place and the futures contract comes to maturity.

The decision problem described in this model is that of an agricultural producer that has to decide how much of its crop to store and how much to sell.

The basic model is a representative agent model with standard assumptions on aggregation. There are two agents:

- a representative consumer with deterministic demand schedule at each date i so that the aggregate demand takes the following form:

  $D_i = A - ap_i \qquad i=1,2$

  where p_i is the storable good price.

  It is assumed that the representative consumer does not store.

- a representative agricultural producer that has to make storage decisions[1] at the first date. It knows the demand schedule and the distribution of endowments, which is the same at the two dates, i=1,2:

  $\mathbf{y}_i \sim N(E(\mathbf{y}), \sigma^2_{\mathbf{y}})$ with $Corr(\mathbf{y}_1,\mathbf{y}_2)=0$.

These firms are competitive in that they take prices as given.

The following assumptions are made:

hp1: at each date there is a positive correlation between individual, $\mathbf{y}_i$, and aggregate, $\mathbf{Y}_i$, output which has the following distribution:

$\mathbf{Y}_i \sim N(E(\mathbf{Y});\sigma_{\mathbf{Y}}^2)$ with $Corr(\mathbf{Y}_1,\mathbf{Y}_2)=0$.

hp2: storage decisions are governed by a quadratic cost

(1) The storage decisions at issue here, are taken only for profit maximization motives. Convenience yield and finanacial costs considerations will not be taken into account in the maximization problem.

function:

$$c(s) = \frac{1}{2} d\, s^2 \qquad \text{with} \quad d>0$$

where s is the amount stored by the representative firm in period 1;[(2)]

hp3: firms are risk-averse;

hp4: firms have rational expectations on the price distribution at each date; randomness in prices is due to randomness in endowments.

hp5: firms have a discount factor δ.

All agents are assumed to know the model and to have identical information on the endowment distributions.

It should be emphasized that, unlike in the previous literature, in this model the firms are not pure storage companies. They produce and are simultaneously engaged in storage activities. This reflects the characterization of storage given in the more institutional type of literature. Williams (1987) introduces a convenience yield argument and transaction costs, which are exogenous in the present model, but still represent a motive for keeping inventories which characterize more a producing-storing firm than a storage firm represented as a speculator. This distinction is clear in Goss (1972), who characterizes inventories as carried by hedgers and merchants, the latter being "really speculators in spot". So far, in the theoretical literature the whole storage is done by merchants; in the model

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(2) It is worth observing that were the cost function not quadratic, a futures contract would be a substitute for storage, i.e. futures and storage levels would be linearly independent.

developed here the existence of merchants is neglected to stress the characterization of storage companies as agents who hedge as well. This feature will influence conclusions on storage levels and price volatility.

Under these assumptions the maximization problem of the firm can be expressed in mean-variance terms(3). Keeping in mind that storage decisions are taken after the observation of $\mathbf{Y}_1 = Y_1$, the maximization problem of the storage firm is:

$$(1) \qquad \max_{s} \{\pi_1 + \delta[E(\pi_2/Y_1) - \frac{1}{2} \alpha \delta \, Var\,(\pi_2/Y_1)]\}$$

$$\pi_1 = p_1 \, y_1 - p_1 \, s - \frac{1}{2} d \, s_1{}^2$$

$$\pi_2 = \mathbf{p}_2(s + \mathbf{y}_2)$$

where α is the coefficient of absolute risk-aversion which is assumed to be constant and bold characters denote random variables.

First order condition gives:

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(3) Conditions for deriving the mean-variance maximand from expected utility maximization (i.e. prices normally or at least symmetrically distributed) do not hold in this model as in most models on futures, e.g. Kawai(1983a,b;1984), Turnovsky(1983), Turnovsky and Campbell(1985). The use of a mean-variance objective function which can be justified either as an ad hoc objective, which has proved to be plausible and tractable in futures theory, or as an approximation to a more general utility function, which gives a negligible error. See Newbery(1988) for more discussion on this issue.

$$(2) \qquad s = \frac{\delta E(\mathbf{p}_2/Y_1) - p_1}{d'} - \frac{\delta^2 \alpha Cov(\mathbf{p}_2, \mathbf{p}_2\mathbf{y}_2/Y_1)}{d'}$$

where $d' \equiv d + \delta^2 \alpha Var(\mathbf{p}_2/Y_1)$ and can be thought of as the rate of change of the marginal cost of storage in a model characterized by endowment randomness and hence price uncertainty at both dates. The equilibrium value of $Cov(\mathbf{p}_2, \mathbf{p}_2\mathbf{y}_2/Y_1)$ depends on the parameters of the model and the endowments distributions as shown in Appendix A, Section AIII.

Negative storage levels can be ruled out by assuming an exogenous positive amount of storage held for, e.g., convenience yield motives which is enough to more than offset any possible negative level of speculative or hedging storage. Alternatively, one can look at equilibrium levels of storage (see (2') and (5')in section 3) to work out conditions on the endowment distributions which ensure a positive level of storage. Yet, the issue will not be analitically investigated here since the focus is on spot price variability.

The two terms on the right hand side of (2) can be addressed, respectively, as the "speculative term " and the "hedging term". The first term explains that part of storage justified by considerations on the differential between the expected and the current price. The second term is a "hedging term" since it reflects the correlation between prices and endowment in period 2. It differs from the optimal inventory rules elaborated in the literature and will play an important role in the following analysis.

Equilibrium solutions to the spot market model can be obtained substituting the storage function (2) and the demand function in the equilibrium condition:

(3) $D_1 + S = Y_1$

$\mathbf{D}_2 = \mathbf{Y}_1 + S$

where S=ns and n is the number of storage firms, and solving for prices. The derivation of equilibrium prices is in Appendix A , section AI.

3. A Microeconomic Model of Storage and Futures

The influence of futures on optimal storage has already been investigated under the standard assumption on the storage firm (e.g. Kawai (1983b) and Turnovsky(1983)).

In order to examine how firms' storage decisions are modified within the model developed here, when a competitive futures market is open, consider the following modification of the model. Assume the model is composed of three types of agents: the two already operating in the previous model, a representative consumer and a representative producer, of whom only the latter takes part in the futures market, plus a representative risk-neutral pure speculator. There are no information asymmetries among the agents.

The assumption of a risk-neutral speculator can be justified on the basis of a risk-pooling argument and implies an indeterminate level of speculators' futures demand, F^s. In fact a risk-neutral agent will be always willing to take all the randomness that the risk-averse agent is trying to escape. Hence the storage firms' (i.e. hedgers') futures demand is always met. As a consequence the market is unbiased, implying:

$\delta E(\mathbf{p}_2/Y_1) = f$

where f is the equilibrium futures price.

This assumption allows the separation of the price volatility from the market bias analysis.

To assess whether and how storage behavior is modified in the presence of futures trading, the maximization problem (1) is modified as follows:

$$(4) \qquad \max_{s^f, F^f} \{\pi_1 + \delta[E(\pi_2/Y_1) - \frac{1}{2}\delta\alpha \, Var(\pi_2/Y_1)]\}$$

$$\pi_1 = p_1{}^f y_1 - p_1{}^f s + f F^f - \frac{1}{2} d (s^f)^2$$

$$\pi_2 = \mathbf{p}_2{}^f s^f + \mathbf{p}_2{}^f \mathbf{y}_2 - \mathbf{p}_2{}^f F^f$$

where :

F^f is the storage firm's futures supply

and the other variables are defined as before with the superscript standing for the futures case.

Solving (4) the optimal solutions are:

$$(5) \qquad s^f = [f - p_1{}^f]/d$$

$$(6) \qquad F^f = \frac{[f - p_1{}^f]}{d} + \frac{[f - \delta E(\mathbf{p}_2{}^f/Y_1)]}{\delta^2 \alpha Var(\mathbf{p}_2{}^f/Y_1)} + \frac{Cov(\mathbf{p}_2{}^f, \mathbf{p}_2{}^f \mathbf{y}_2/Y_1)}{Var(\mathbf{p}_2{}^f/Y_1)}$$

Equation (5) implies the possibility of futures trading inducing important modifications in storage decisions. The optimal storage decision now depends on the futures price instead of the expected next period price.

There is also a change in the price sensitivity of the storage function. In addition there is no hedging component influencing the storage decision: when no futures trading opportunities are offered (see equation (2)), the firm must resort to the spot market for insurance purposes. In the presence of futures markets this function is performed via futures contract as formalized in (6). The first term on the right hand side is a routine hedging one (i.e. exactly matching the spot market position).The second term is the purely speculative one. The third one is a hedging term as well: it aims at reducing income variability by taking into account the correlation between income and prices and can be referred to as a hedging adjustment term (HAT).

While (5) is a well-known result, (6) still differs from what obtained in Anderson and Danthine(1983), Kawai(1983b) and Turnovsky(1983) by the covariance term. Again it is the assumption on storage as an activity performed by the producer that makes the difference. The storing agent typically has to face a variability of income which is correlated with spot prices variability. Hence the consideration of this correlation has to play a role in the determination of either the optimal storage rule (no-futures case) or the optimal futures position (futures case).

Equilibrium solutions to the spot and futures market model can be obtained substituting the storage, the futures and the demand functions in the equilibrium condition:

$$(7)\quad D_1 + S^f = Y_1$$
$$\mathbf{D}_2 = \mathbf{Y}_1 + S^f$$
$$F^f + F^s = 0$$

where $S^f = ns^f$. The derivation of equilibrium prices is in Appendix A , section AII.

4. The Effects of Futures on Spot Price Volatility

Having analyzed storage decision in both the no-futures and futures case and having solved for equilibrium prices, the stabilizing or destabilizing effect of futures on spot prices is addressed.

As a measure of price volatility the spot price variances are taken. The type of price volatility at issue in this paper is the one measured by the unconditional price variance. By focusing on the conditional price variances, only the randomness introduced in the model by period 2 random endowment is taken into account. This implies that the firm's "viewpoint" in the time axis of trading is 1 and not $1+\varepsilon$. Conditioned on the observation of $\mathbf{Y}_1=Y_1$, the period 1 price is non-stochastic and the price in period 2 is random only to the extent that $\mathbf{Y}_2$ is.

Price volatility at date 2, when the price variability is more complicated being due to randomness in both dates' endowments is first addressed.

The distribution of second period price conditional on the realization of $\mathbf{Y}_1=Y_1$ and that of first period endowment prior to $\mathbf{Y}_1$ are known. Hence the Rao-Blackwell theorem is used to find the relationship between conditional and unconditional price variances:

(8) $Var(\mathbf{p}_2) = E_{Y1}[Var(\mathbf{p}_2/Y_1)] + Var_{Y1}[E(\mathbf{p}_2/Y_1)]$

(9) $Var(\mathbf{p}_2{}^f) = E_{Y1}[Var(\mathbf{p}_2{}^f/Y_1)] + Var_{Y1}[E(\mathbf{p}_2{}^f/Y_1)]$

Since the first term on the right hand side of (8) and (9) is the conditional variance which is the same in the two cases, conclusions on price variability are determined by the relative magnitude of the second terms on the right hand side of (8) and (9).

Substituting the equilibrium value of $E(\mathbf{p}_2/Y_1)$ and $E(\mathbf{p}_2{}^f/Y_1)$ and recalling that the output distributions are not correlated, the following variances are obtained:

$$(10)\quad Var_{Y1}[E(\mathbf{p}_2/Y_1)] = \frac{n^2}{a^2(ad'+n+\delta n)^2}\left\{1 - \frac{\delta^2\alpha\sigma_Y{}^2}{a(ad'+n+\delta n)+\delta^2\alpha\sigma_Y{}^2}\right\}^2\sigma_Y{}^2$$

$$(11)\quad Var_{Y1}[E(\mathbf{p}_2{}^f/Y_1)] = \frac{n^2}{a^2(ad+n+\delta n)^2}\,\sigma_Y{}^2$$

Recalling that $d' \equiv d + \delta^2\alpha Var(\mathbf{p}_2/Y_1)$ and taking into account that the quantity in curly brackets in (10) is less than one, leads to:

$Var_{Y1}[E(\mathbf{p}_2/Y_1)] < Var_{Y1}[E(\mathbf{p}_2{}^f/Y_1)]$.

Hence we can conclude that the unconditional period 2 price variance is unambiguously greater in the presence of a futures market.

The period 2 prices' variance is greater with futures due to its dependence on output uncertainty. In fact one can think of these variances as follows:

(12) $Var(\mathbf{p}_2) \propto Var(\mathbf{Y}_2) + Var(S) + 2Cov(\mathbf{Y}_2, S_1)$

(13) $Var(\mathbf{p}_2{}^f) \propto Var(\mathbf{Y}_2) + Var(S^f) + 2Cov(\mathbf{Y}_2, S_1{}^f)$

where $\propto$ here stands for proportionality.

Since it is assumed the two output distributions are uncorrelated, the covariances in (12) and (13) are zero.

Substituting the equilibrium solutions for prices in the storage functions (2) and (5), the equilibrium storage levels are:

$$(2')\quad S = n\frac{1}{ad'+n+\delta n}[Y_1-\delta E(\mathbf{Y}_2)] + \frac{\delta - 1}{ad'+n+\delta n}A + - n\frac{a\delta}{ad'+n+\delta n}\alpha Cov(\mathbf{p}_2, \mathbf{p}_2\mathbf{y}_2/Y_1)$$

$$(5')\quad S^f = n\frac{1}{ad+n+\delta n}[Y_1 - \delta E(\mathbf{Y}_2)] - \frac{\delta - 1}{ad+n+\delta n}A$$

in the model without and in that with futures, respectively. Hence the unconditional storage variances are:

$$(14)\quad Var(S) = \frac{n^2}{(ad'+n+\delta n)^2}\left\{1 - \frac{\delta^2\alpha\sigma_Y{}^2}{a(ad'+n+\delta n)+\delta^2\alpha\sigma_Y{}^2}\right\}\sigma_Y{}^2$$

$$(15)\quad Var(S^f) = \frac{n^2}{(ad+n+\delta n)^2}\sigma_Y{}^2$$

The storage variance is greater with futures than without since storage with futures is more sensitive to the first period endowment which at date 1 is a random variable.

Price variances in (12) and (13) can now be expressed as follows:

$$(12') \quad Var(\mathbf{p}_2) \propto \sigma_Y^2 + \frac{n^2}{(ad'+n+\delta n)^2}\{1 - \frac{\delta^2\alpha\sigma_Y^2}{a(ad'+n+\delta n)+\delta^2\alpha\sigma_Y^2}\}^2 \sigma_Y^2$$

$$(13') \quad Var(\mathbf{p}_2{}^f) \propto \sigma_Y^2 + \frac{n^2}{(ad\ +n+\delta n)^2} \sigma_Y^2$$

A greater storage variance in the presence of futures determines a greater period 2 price variance with futures than without. This conclusion stems from the circumstance that storage, and hence prices, are less sensitive to the endowment variance in period 2 because of a greater marginal cost of storage (d'>d) and the existence of a covariance term in the storage function.

As for period 1, the price variance is smaller without futures since the period 1 price variance is determined by period 1 endowment and storage variances and the covariance effect. This covariance term is weighted with a negative sign since in period 1 storage is deducted from the given random endowment:

$$(16) \quad Var(\mathbf{p}_1) \propto Var(\mathbf{Y}_1) + Var(\mathbf{S}) - 2Cov(\mathbf{Y}_1,\mathbf{S})$$

$$(17) \quad Var(\mathbf{p}_1{}^f) \propto Var(\mathbf{Y}_1) + Var(\mathbf{S}^f) - 2Cov(\mathbf{Y}_1,\mathbf{S}^f).$$

Substituting the equilibrium level of storage (2') and (5') in (16) and (17) respectively, gives:

$$(16') \quad Var(\mathbf{p}_1) \propto \sigma_Y^2 + \frac{n^2}{(ad'+n+\delta n)^2}\{1 - \frac{\delta^2\alpha\sigma_Y^2}{a[a(ad'+n+\delta n)+\delta^2\alpha\sigma_Y^2]}\}^2 \sigma_Y^2$$
$$- \frac{2n}{ad'+n+\delta n}\{1 - \frac{\delta^2\alpha\sigma_Y^2}{a[a(ad'+n+\delta n)+\delta^2\alpha\sigma_Y^2]}\}\sigma_Y^2$$

$$(17') \quad Var(\mathbf{p}_1{}^f) \propto \sigma_Y{}^2 + \frac{n^2}{(ad+n+\delta n)^2} \sigma_Y{}^2 - \frac{2n}{ad+n+\delta n} \sigma_Y{}^2$$

The sum of the second and third term on the right hand side of (16') and (17') is negative but in absolute value it is greater in (17'), the futures case. Hence period 1 price variability is smaller in such a case. The covariance effect in the presence of futures, overwhelms the storage variance effect and, thus, the total effect is a smaller price variance in period 1 when futures exists. A more formal discussion of this point is contained in Appendix B.

As in the date 2 prices analysis, this conclusion relies on: (i) d' bigger than d, (ii) the existence of a covariance term in the storage function. These two instances increase the sensitivity of spot prices to the endowment variance in the futures case. Yet at date 1 they play a role in determining the magnitude (in absolute value) of the covariance between date 1 endowment and storage. This leads to the conclusion that the existence of futures lowers the variability of prices.

Summing up, the existence of futures lowers price variability at the date at which storage decision are taken, but increases price variability at the successive date when no storage decisions have to be taken. This asymmetry of the results on price volatility between the two dates results from the asymmetric nature of two-period models in the presence of storage.

It is not generally possible to extrapolate results obtained in a two-date model, because storage might introduce serial correlations that are not usually captured in such a model. Yet, since a multiperiod framework is characterized by the simultaneity of storage and futures decision at any date, period 1 result would point in the direction of a stabilizing influence of futures on spot prices. When storage decisions are taken, the covariance effect between the endowment in that period and storage offsets the endowment variance effect more heavily in the futures case. This is due to the existence of futures that changes, in the direction of an increase, the price sensitivity of the inventory demand function.

5. Conclusions

On the basis of a two-date-one-period model of futures for a storable commodity, the existence of a stabilizing influence of futures on spot prices when storage decisions are taken is proved.

The characterization of storage firms as producers (i.e. hedgers) is new in the literature. It highlights an important channel through which futures influence spot prices. In Turnovsky(1983) the introduction of a futures market increases the price responsiveness of the supply curve, but leaves the inventory demand curve unchanged. In the model presented here, the introduction of a futures market works out its effect through a modification of storage behavior. Precisely the interaction between production randomness and storage variability, which is different in magnitude in the futures and no-futures case, is the determinant of the conclusion on the futures stabilizing effect.

The investigation of the price volatility issue is done on a purely analytical ground and conclusions are consistent with those obtained by Kawai(1983b) in a numerical illustration of a similar model.

A welfare analysis of the price stabilization issue presented in this paper is not straightforward. It is therefore left for future research since the objective here was to provide a positive analysis of the stabilization issue.

Appendix A: Equilibrium Solutions for the Model

The equilibrium prices are obtained conditional on the observation of $\mathbf{Y}_1=Y_1$, since spot and futures trading take place after the uncertainty on $\mathbf{Y}_1$ is resolved.

AI - Model without futures

The equilibrium conditions are:

(A1) $D_1 + S = Y_1$

$\mathbf{D}_2 = \mathbf{Y}_1 + S \quad$ where $S=ns$

Substituting (2) in AI, solving for p_1 and substituting back in (A1), with a bit of algebra the following equilibrium prices are derived:

$$\text{(A2)}\quad E(\mathbf{p}_2/Y_1) = \frac{(ad'+2n)A}{a(ad'+n+\delta n)} - \frac{n\,Y_1}{a(ad'+n+\delta n)} - \frac{(ad'+n)E(\mathbf{Y}_2)}{a(ad'+n+\delta n)} + \frac{\delta^2 n}{ad'+\delta n+n}\alpha\mathrm{Cov}(\mathbf{p}_2,\mathbf{p}_2\mathbf{Y}_2/Y_1)$$

$$\text{(A3)}\quad p_1 = \frac{(ad'+2\delta n)A}{a(ad'+n+\delta n)} - \frac{(ad'+\delta n)Y_1}{a(ad'+n+\delta n)} - \frac{\delta n\,E(\mathbf{Y}_2)}{a(ad'+n+\delta n)} - \frac{\delta^2 n}{ad'+n+\delta n}\alpha\mathrm{Cov}(\mathbf{p}_2,\mathbf{p}_2\mathbf{Y}_2/Y_1)$$

$$\text{(A4)}\quad \mathbf{p}_2 = \frac{(ad'+2n)A}{a(ad'+\delta n+n)} - \frac{n\,Y_1}{a(ad'+n\delta n)} - \frac{1}{a}\mathbf{Y}_2 + \frac{\delta n}{a(ad'+n+\delta n)}E(\mathbf{Y}_2) + \frac{\delta^2 n}{ad'+n+\delta n}\alpha\mathrm{Cov}(\mathbf{p}_2,\mathbf{p}_2\mathbf{Y}_2/Y_1)$$

where $d' = d + \alpha\,\delta^2\,\mathrm{Var}(\mathbf{p}^2/Y_1)$.

AII - Model with Futures

The equilibrium conditions are:

$$D_1 + S^f = Y_1$$

(A5) $$\mathbf{D}_2 = \mathbf{Y}_2 + S^f$$

$$F^f + F^s = 0$$

where the last equation corresponds to the unbiasedness condition.

Substituting (5) in (A5), solving for $p_1{}^f$ and substituting back in (A5), the following equilibrium prices are derived:

(A6) $$p_1{}^f = \frac{d}{ad+n}(A-Y_1) + \frac{\delta n}{ad+n}E(\mathbf{p}_2{}^f/Y_1)$$

(A7) $$E(\mathbf{p}_2{}^f/Y_1) = \frac{(ad+2n)A}{a(ad+n+\delta n)} - \frac{(ad+n)E(Y_2)}{a(ad+n+\delta n)} - \frac{n\ Y_1}{a(ad+n+\delta n)}$$

(A8) $$p_1{}^f = \frac{(ad+2\delta n)A}{a(ad+n+\delta n)} - \frac{\delta nE(\mathbf{Y}_2)}{a(ad+n+\delta n)} - \frac{(ad+\delta n)Y_1}{a(ad+n+\delta n)}$$

(A9) $$\mathbf{p}_2{}^f = \frac{(ad+2n)A}{a(ad+n+\delta n)} - \frac{1}{a}\mathbf{Y}_2 + \frac{n}{a(ad+n+\delta n)}[\delta E(\mathbf{Y}_2) - Y_1]$$

AIII - The RE $Cov(\mathbf{p}_2,\mathbf{p}_2\mathbf{y}_2/Y_1)$

The definition of covariance implies:

(A10) $$Cov(\mathbf{p}_2,\mathbf{p}_2\mathbf{y}_2/Y_1)=E[(\mathbf{p}_2-E(\mathbf{p}_2/Y_1))(\mathbf{p}_2\mathbf{y}_2-E(\mathbf{p}_2\mathbf{y}_2/Y_1))]$$

Substituting (A2) and (A4) into (A10), carrying out the necessary calculations and recalling the uncorrelation assumption between the endowments distribution, gives:

$$\text{(A11)}\quad Cov(\mathbf{p}_2,\mathbf{p}_2\mathbf{y}_2/Y_1) = -\frac{\delta^2\alpha\sigma_Y^2}{a(ad'+n+\delta n)}Cov(\mathbf{p}_2,\mathbf{p}_2\mathbf{y}_2/Y_1)$$

$$+\frac{E(\mathbf{Y}_2^3)-E(\mathbf{Y}_2)E(\mathbf{Y}_2^2)}{a^2n} - \frac{(ad'+2n)A-nY_1+\delta nE(\mathbf{Y}_2)}{a^2(ad'+n+\delta n)}\sigma_Y^2$$

In order to get the rational expectation expression for the covariance between period 2 prices and value of endowment, (A11) is solved for $Cov(\mathbf{p}_2,\mathbf{p}_2\mathbf{y}_2/Y_1)$:

$$\text{(A12)}\quad Cov(\mathbf{p}_2,\mathbf{p}_2\mathbf{y}_2/Y_1) = \frac{ad'+n+\delta n}{an[a(ad'+n+\delta n)+\delta^2\alpha\sigma_Y^2]}\{E(\mathbf{Y}_2^3)$$

$$- E(\mathbf{Y}_2)E(\mathbf{Y}_2^2) + \frac{(ad'+2n)A-nY1+\delta nE(Y2)}{ad'+n+\delta n}\sigma_Y^2$$

or, by the properties of the moment generating function:

$$\text{(A13)}\quad Cov(\mathbf{p}_2,\mathbf{p}_2\mathbf{y}_2/Y_1) = \frac{\sigma_Y^2}{an[a(ad'+n+\delta n)+\delta^2\alpha\sigma_Y^2]}\{nY_1 +$$

$$[2(ad'+n)+\delta n]E(\mathbf{Y}_2) - (ad'+2n)A\}$$

which is just a function of the parameters of the model and the endowment distributions.

Appendix B: A More Formal Analysis of the Comparison between Period 1 Price Variances

The two relevant terms for the comparison in (16') and (17') can be represented by the following functions:

$$f(x,y) = \frac{n^2}{(ay+n+\delta n)^2} x^2 - \frac{2n}{ay+n+\delta n} x$$

$$\text{where} \quad x = \{1 - \frac{\delta^2 \alpha \sigma_Y{}^2}{a[a(ay+n+\delta n)+\delta^2 \alpha \sigma_Y{}^2}\} \quad \text{hence} \quad 0<x\leq 1.$$

In order to get the two relevant terms in (17') we have to put x=1 and y=d. For (16') instead we have that y=d'>d but x can take any value s.t. $0<x\leq 1$.

When $Cov(\mathbf{p}_2,\mathbf{p}_2\mathbf{y}_2)=0$, then x=1 and $|f(1,d')| < |f(1,d)|$. Hence conclusions on period 1 price variability hold.

In general, since f(x,y) is monotonically decreasing in $0<x\leq 1$, then

$|f(x,d')| < |f(1,d)|$ for any x s.t. $0<x\leq 1$.

Hence the conclusion holds for the non-zero covariance case too. This covariance introduces a further element of sensitivity of the period 1 price variance to $\mathbf{Y}_1$ in the no-futures case lowering the sensitivity of $Var(\mathbf{p}_1)$ to $Var(\mathbf{Y}_1)$.

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