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OIL FUTURES
AND STRATEGIC STOCKS AT SEA

by

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Abstract

A theoretical model explaining the determination of prices in the markets for North Sea crude oil is set up. Three markets are analysed in a three-stage game in which market concentration increases by each stage: In the first stage, the International Petroleum Exchange is modeled as a thick futures market. This market is also used to hedge against the uncertain outcome of the 15-Day forward market, modeled in the second stage. There, a small club of traders enter futures contracts knowing that this will affect the storage decision and thereby the spot price profile. The third stage models the spot market as a two-period duopoly with inventories. The strategic effect of, and interaction between, inventories and futures positions is investigated.

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1. Introduction

In his The World Price of Oil (1976), Henk Houthakker wondered why there existed no futures market for oil. Oil is a relatively standardised commodity that is stored under the ground or the sea by Mother Nature and storable above the ground and at sea. A standardised futures contract is easy to design. Let us quote (p. 2):

"For oil, there has been nothing like the Chicago wheat market or the London copper market, where prices are set daily by the offers of producers and the bids of consumers, with considerable participation by merchants and speculators.

Why does petroleum lack such a central market? In common with most commodities that are traded on futures markets, petroleum is storable. While not as homogeneous as copper, it is not more heterogeneous than wheat, and a serviceable standard contract would not be hard to design. Although transportation costs are relatively more important than for most centrally traded commodities, this would not seem an insuperable obstacle either. Perhaps the main reason for the failure of a central market to develop is that for many years the industry has been dominated by integrated companies that handle oil from the well to the gasoline pump. Merchants, brokers, and other intermediaries are relatively unimportant; as a result, arm's-length transactions have traditionally been less prevalent in petroleum than in many other raw materials. The significance of this point is that the integrated companies appear to be losing much of their control over crude oil, so that arm's-length transactions will become more common. In due course, a central market may emerge."

In the eighties, Houthakker's forecast became a reality. In the US, the Nymex crude oil contract for West Texas Intermediate (WTI) was successfully launched in 1983, as a result of the increased volatility of the price of oil after the Iranian revolution. In the meantime, North Sea oil was discovered. The need for "forward" trading of Brent oil was felt for the same reason and led (around 1986) to a standardised Contract for Brent crude oil, on what began to be called the "Brent 15-Day market". Initially, the pairwise contracts were "forward" in the strict sense of the word, that is, the particular seller had to deliver to his particular trading partner. Gradually, this market developed into a futures market, that is, a market where a large proportion of the trade was for hedging and speculation purposes only. On top of it, the IPE launched -with mixed success- a classic crude oil futures contract,¹ copied from the Nymex

¹ Its modifications are described in Philips (1991).

contract, in 1983. Today, two futures markets for Brent crude exist on top of each other: the 15-Day and the IPE. The latter has a centralised open outcry exchange, a clearing house and a growing number of participants, including the majors, oil traders and locals. The Brent 15-Day market, to the contrary, is in the hands of a club of producers and traders.

The emergence of these markets motivates the present paper, which tries to simultaneously model the oligopolistic interplay of the majors who produce North Sea oil, for whom stocks at sea have a strategic role, and the presence of two futures markets for Brent crude on which these majors are also in a strategic situation, given their size and small number. To put it simply, this paper is an attempt to combine the modeling of strategic stocks, as pioneered by Allaz (1991) and further developed by Møllgaard (1990), with an effort to give a game-theoretic explanation of how two futures markets for the same natural resource work when the corresponding spot market is controlled by a few producers. The basic approach is the one developed by Philips and Harstad (1990 and 1991).

We shall confine the analysis to the minimum oligopoly model that allows for strategic interaction between supplies, that is, a duopoly. The game proceeds in three stages. In the first stage, the producers meet with an anonymous futures market, which is supposed to mimic the characteristics of the IPE. Having determined the futures price and positions, we model the 15-Day market as the second-stage subgame, where the oil companies trade bilateral futures contracts among themselves and with a speculator. These 15-Day contracts will depend on the positions already taken on the IPE. In the third stage, the companies then play a two-period extraction game. They each have a known total to extract over the two periods, but can use stocks of crude oil at sea and the extraction profile to manipulate prices so as to render their IPE and 15-Day positions more profitable. This comes about because the maturity futures price is taken to be the second-period spot price.

Time only plays a role in the two periods, 1 and 2, of the extraction game of the third stage. The two previous stages modelling the two futures markets allow the producers to precommit themselves to certain sales (or purchases) in the second period of the third stage. One can imagine that the IPE (in the first stage) opens and closes in period -0 leaving enough time for the producers to sell or buy IPE futures as they wish. When the IPE has closed, in

period +0 the three market participants agree on their 15-Day contracts (in the second stage). When the 15-Day market closes, the extraction game of the third stage takes place. At this stage, the two producers decide on the optimal extraction, sales and storage profiles over the two periods. Only after this third stage will the stochastic demand be revealed and it is in this sense that time does not play a role in stages one and two modelling the futures markets: No relevant information is revealed during or between these two stages. This assumption allows us to focus on the strategic and speculative motives of futures market trading. We shall return to the interpretation of time in relation to the real world markets in section 7.

The rest of the article is organized as follows: Section two introduces the necessary notation for the spot market, which is described as a fairly traditional duopoly extracting an exhaustible resource. Everything is kept nicely linear or quadratic but the results carry through if these assumptions are substituted by appropriate convexity conditions. The following four sections then unravel the game backwards. Section three solves the Cournot duopoly for the extraction game. The production schedule is unaffected by the futures markets while the sales depend on the net position taken by the producers on the two futures markets. To close this gap inventories must necessarily depend on these same net positions. Section four analyses the strategic use of stocks by changing the basic model of the spot market slightly so as to highlight the strategic effect of holding inventories. These strategic inventories are also found to depend on the futures positions as well as on the producers' beliefs regarding the future spot price. Section five then recedes to the second stage, modelling the 15-Day market. The set of contracts that are mutually beneficial to the market participants (i.e. the core) is characterized. This will be a large set depending on price expectations and the futures positions taken on the IPE. Section six takes us back to the first stage where the producers' optimal positions on the IPE are modelled. Given the potential multiplicity of outcomes of the imperfectly organized 15-Day market, the IPE serves as a vehicle for the producers to speculate and hedge not only against the uncertainty of the spot market but also against the non-uniqueness of the 15-Day market.

2. Setting the Stage: Duopoly, Storage and Futures

Since the play of all three stages focuses on the cash market we shall begin our story here. Two companies, A and B, supply a homogeneous product, crude oil, to a spot market. Extraction takes place in two periods, 1 and 2, but each extractor ($i = A, B$) has a known maximum \bar{x}_i to be extracted over the two periods. We shall generally assume that cost conditions are such that it pays to pump all of \bar{x}_i . First-period production can however be stored in tankers rather than being sold immediately.

Let the first (alphabetical) subscript refer to the companies and the second (numeric) to the period. q denotes sales and x production. We impose the constraints

$$(1) \quad q_{i1} + q_{i2} = x_{i1} + x_{i2} = \bar{x}_i, \quad i = A, B$$

on total production and sales. Stocks at sea s_i are produced but not sold in the first period, that is,

$$(2.1) \quad s_i \equiv x_{i1} - q_{i1}$$

$$(2.2) \quad q_{i2} = x_{i2} + s_i, \quad (i = A, B)$$

so that the tankers have to be delivered in the second period.

Production and storage are not costless activities. We assume that production costs, C , are convex and, for simplicity, that they are quadratic in the number of barrels pumped:

$$(3) \quad C_{it} = \frac{1}{2} c_t x_{it}^2 \quad (i = A, B; \quad t = 1, 2).$$

The cost of storage, I , is taken to be linear in the number of barrels stored,

$$(4) \quad I_i = j s_i \quad (i = A, B; \quad j \geq 0),$$

and the profit of company i thus becomes

$$(5) \quad \Pi_i = p_1 q_{i1} + p_2 q_{i2} - (C_{i1} + C_{i2} + I_i) + (p^F - p_2) N_i,$$

where p_1 and p_2 are the spot prices in the two periods and p^F and N_i are the futures price and the net position resulting from the two first stages of the game.

To complete the description of the spot market, we propose linear inverse demand functions² to determine the spot prices:

$$(6.1) \quad p_t = \alpha - b q_t \quad \text{where} \quad q_t = q_{At} + q_{Bt}, \quad t = 1, 2,$$

and where the strength of demand, α , is a stochastic variable which is perceived by the agents as being distributed normally with unknown mean $\hat{\alpha}$ and known variance $\text{Var}(\alpha) = 1$:

$$(6.2) \quad \alpha \sim N(\hat{\alpha}, 1).$$

The participants on the futures markets hold different beliefs $E_i(\alpha)$ on $\hat{\alpha}$; they assign probability one to their own belief (their subjective probability distribution) and (thus) probability zero to the beliefs of the other players. We are thus in a situation of inconsistent prior beliefs in the sense of Selten (1982).³

² In earlier papers [Brianza, Philips and Richard (1990); Philips and Harstad (1991 and 1990)] the specification $q_t = \alpha' - \beta p_t$, leading to the inverse demand curve

$$p_t = \frac{\alpha'}{\beta} - \frac{1}{\beta} q_t,$$

was chosen. While this gives us a natural interpretation of α' as the level of demand, the notational simpler version with $\alpha = \alpha'/\beta$ and $b = 1/\beta$ has been chosen here. The 'strength' of demand α and the 'level' of demand α' are thus related through $\alpha = \alpha' b$.

³ It will take us too far from the main argument to discuss the origin of the differences in beliefs. We take it for an empirical fact that traders act on differences in subjective probability distributions (agreeing to disagree) even if they hold the same information (which they thus interpret differently), optimism and pessimism being inexplicable motivations for trading. Everyday futures and financial markets are crowded with busily trading agents that share the same information, thereby rejecting any zero-trade theorem. On optimistic and conservative standards of behaviour, see Greenberg (1990). On rejecting the rational expectations hypothesis, see Lovell

The risk-averse producers' ex-ante payoffs are modelled according to the mean-variance model:

$$(7) \quad W_i = E_i(\Pi_i) - \frac{K_i}{2} \text{Var}(\Pi_i),$$

where K_i measures constant absolute risk aversion.⁴

The expected profit is readily found by taking i 's expectation of (5). The variance of the profit can be shown to be

$$(8) \quad \text{Var}(\Pi_i) = (\bar{x}_i - N_i)^2.$$

We shall leave the description of the modelling of the 15-Day market and the IPE for the next section. Here we follow many a good theatre play and first offer the solution to the final stage in which the strategic effects of stocks are highlighted.

3. Stage Three: Cournot Duopoly and the Extraction Game

First note that the variability of profits arises from the unhedged part $(\bar{x}_i - N_i)$ of total extraction (see (8)) and $\text{Var}(\alpha) (= 1)$. It is unaffected by the time profile of sales or production. In order to decide on these time profiles, the companies maximize W_i with respect to x_{i1} , q_{i1} , x_{i2} , q_{i2} and s_i , which amounts to maximizing expected profits subject to the constraints (1) and (2).

Manipulation of the first-order conditions leads to the following extraction schedule:

$$(9.1) \quad x_{i1} = \frac{c_2}{c_1 + c_2} \bar{x}_i - \frac{2j}{c_1 + c_2} = \delta_1 \bar{x}_i - \frac{2j}{c_1 + c_2}, \quad i = A, B$$

(1986). On informational differences leading to different positions, see Stein (1987) and p. 27.

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This follows automatically if the underlying preferences are represented by a utility function that is exponential in profits and if profits follow the normal distribution. See Newbery and Stiglitz (1981), pp. 74-75.

$$(9.2) \quad x_{i2} = \frac{c_1}{c_1 + c_2} \bar{x}_i + \frac{2j}{c_1 + c_2} = \delta_2 \bar{x}_i + \frac{2j}{c_1 + c_2}, \quad i = A, B$$

where the first term represents cost smoothing:

$$(9.3) \quad \delta_1 = \frac{c_2}{c_1 + c_2} = \frac{c_2 x}{c_1 x + c_2 x} = \frac{\frac{1}{2} c_2 x^2}{\frac{1}{2} c_1 x^2 + \frac{1}{2} c_2 x^2}$$

$$(9.4) \quad \delta_2 = \frac{c_1}{c_1 + c_2} = \frac{c_1 x}{c_1 x + c_2 x} = \frac{\frac{1}{2} c_1 x^2}{\frac{1}{2} c_1 x^2 + \frac{1}{2} c_2 x^2}.$$

The chosen quadratic form of the cost function implies that comparing the total cost of producing a given quantity, x , in one period to the total cost of producing the same amount in both periods is equivalent to comparing the marginal cost, $c_1 x$, in this period to the sum of marginal costs and again equivalent to just comparing increments, c_i , in the marginal cost. At any rate, (9.1) has the natural interpretation that the higher the cost of production in the second period is (the higher δ_1), the more should be produced in advance in period 1. Vice-versa for second-period production: the higher the cost of first-period production is, relatively speaking, the more of it should be postponed to the cheaper second period. The second term of (9.1) and of (9.2) compares the costliness of storage to that of production. Not surprisingly, we find that the higher the cost of storage, the less should be produced in advance and the more should be postponed to the second period. Note that the optimal extraction policy does not depend on the futures positions taken.

The sales schedule for company A, say, is found to be

$$(10.1) \quad q_{A1} = \frac{1}{2} \bar{x}_A - \frac{1}{3} N_A + \frac{1}{6} N_B + \frac{1}{6} \left(\frac{j}{b} \right) \quad \text{and}$$

$$(10.2) \quad q_{A2} = \frac{1}{2} \bar{x}_A + \frac{1}{3} N_A - \frac{1}{6} N_B - \frac{1}{6} \left(\frac{j}{b} \right)$$

and its stocks of crude oil can therefore be expressed as⁵

$$(11.1) \quad s_A = \frac{1}{2} \Delta \delta \bar{x}_A + \frac{1}{3} N_A - \frac{1}{6} N_B - \frac{1}{6} \gamma j, \quad \text{where}$$

⁵ Using (2), (9.1) and (10.1).

$$(11.2) \quad \Delta \delta = \delta_1 - \delta_2 \quad \text{and}$$

$$(11.3) \quad \gamma = \frac{12b + c_1 + c_2}{b(c_1 + c_2)}.$$

That sales depend on the net short futures positions in the manner shown in equations (10) is in perfect concordance with the findings in Philips and Harstad (1991) (see equations (6) there). In addition, we find that sales depend on the term j/b which relates the marginal cost of storage to the slope of the inverse demand curve, and thus to the marginal revenue of the operation. We see that the higher the marginal cost of storage, the lower will stocks and thus second-period sales be. First-period sales will be correspondingly higher.

The size of inventory holdings is given by (11.1). The first term shows the cost-smoothing purpose of stocks. If $c_2 > c_1$ so that $\Delta \delta > 0$, a part of second-period sales should be produced in the first, less costly period and stored. This operation should take the cost of storage into account, as indicated by the fourth term, from which it is seen that higher cost of storage lowers the optimal level of stocks, as should be. The second term of (11.1) shows that if producer A has precommitted herself by taking a short position on the futures markets ($N_A > 0$), then part of this quantity will optimally be met by sales from stocks. If the rival takes a similar position ($N_B > 0$), this lowers the profitability of the operation, thereby also lowering the optimal level of stocks for company A. Aggregate stocks ($s_A + s_B$) depend positively on the aggregate net position ($N_A + N_B$).

Before discussing the strategic use of stocks, let us examine the expected spot prices:

$$(12.1) \quad E(p_1) = \hat{p} + \frac{1}{6}b(N_A + N_B) - \frac{1}{3}j$$

$$(12.2) \quad E(p_2) = \hat{p} - \frac{1}{6}b(N_A + N_B) + \frac{1}{3}j$$

$$(12.3) \quad \hat{p} = \hat{\alpha} - \frac{1}{2}b(\bar{x}_A + \bar{x}_B).$$

\hat{p} is the price that would be obtained in both periods if the futures markets and the storage facility did not exist. Net short futures positions ($N_i > 0$) represent

a binding commitment to furnish the second-period cash market with a certain amount of crude, and to the extent that this amount is already sold at the price p^F , the companies have a common interest to manipulate prices by lowering p_2 (and therefore raising p_1).

4. The Strategic Use of Stocks

The strategic use of stocks is in many ways similar to that of futures positions. Indeed, by undertaking a larger production in the first period and storing some of it for sale in the second period, the companies incur the extra cost of production and the cost of storage in the first period. In the second period these costs are sunk and the supply from stocks is costless. Stocks therefore represent a credible commitment to raise second-period sales, thereby offering a potential position as a Stackelberg leader.

In a more general model, the strategic effect of inventories would imply that total production was increased and that the average price was lower than if storage were impossible.⁶ The producers are trapped in a suboptimal Nash equilibrium because both are trying to position themselves as leaders. Most of this strategic effect is so far lost in our model since the oil companies are required to produce and sell a given total (see (1)). What is left is the direct effect of storage costs on prices, whereby higher cost of storage discourages holding inventories and leads to higher immediate sales.

To illustrate this point we change the model slightly for the sake of this subsection. It is crucial that the companies be allowed to determine the size of total production. We therefore abandon the second equality of constraint (1) and only require total production to be sold by the end of period 2. To keep things simple we also assume that demand only materializes in the second period, which implies that everything that is produced in the first period is stored:

$$(13.1) \quad x_{i1} = s_i$$

$$(13.2) \quad q_{i2} = x_{i1} + x_{i2}.$$

⁶ See Allaz (1991) and Møllgaard (1990).

The situation is more complicated than before since uncertainty now not only pertains to the average price that results from a known total production, but involves the determination of this total itself.

Maximization of W_i with respect to x_{i2} results in the following sales for company A:

$$(14) \quad q_{A2} = s_A + \frac{1}{D} \left\{ b[\Delta(\alpha) + b(s_B - s_A) - b(N_B - N_A) + K_B(s_B - N_B) - K_A(s_B - N_B)] \right. \\ \left. + (b + c_2 + K_B)[E_A(\alpha) - b(s_A + s_B) + (b + K_A)(N_A - s_A)] \right\}$$

where $D = (2b + c_2 + K_A)(2b + c_2 + K_B) - b^2 \geq 0$, $\Delta(\alpha)$ is the difference in opinion $(E_A(\alpha) - E_B(\alpha))$ on $\hat{\alpha}$, $(s_B - s_A)$ is the difference in stock levels and $(N_B - N_A)$ is similarly the difference in futures positions. $(s_i - N_i)$ represents the unhedged part of stocks, so that the two last terms in the first square brackets represent the different risk valuation of unhedged stocks.

The effect of an increase in the stocks of company A is seen to increase its own sales by

$$(15) \quad \frac{dq_{A2}}{ds_A} = \frac{1}{D} c_2 (2b + c_2 + K_B) > 0,$$

whereas it will decrease company B's sales by

$$(16) \quad \frac{dq_{B2}}{ds_A} = -\frac{1}{D} c_2 b < 0,$$

so that total production and thus total sales are increased

$$(17) \quad \frac{dq}{ds_A} = \frac{1}{D} c_2 (b + c_2 + K_B) > 0,$$

while the spot price at maturity is lower,

$$(18) \quad \frac{dp_2}{ds_A} = -\frac{1}{D} b c_2 (b + c_2 + K_B) < 0,$$

than would otherwise be the case.

The expressions for the sales quantities (14) can be substituted back into the payoff functions (7) which then can be maximized to find the subgame-perfect first-period production (or, equivalently, stocks). These will depend on the beliefs about the strength of demand and on the futures positions taken at an earlier stage:

$$(19.1) \quad s_A = v_A E_A(\alpha) + \phi_A E_B(\alpha) + \psi_A N_A + \omega_A N_B$$

$$(19.2) \quad s_B = \phi_B E_A(\alpha) + v_B E_B(\alpha) + \omega_B N_A + \psi_B N_B$$

where the coefficients v_i , ϕ_i , ψ_i and ω_i depend on the original parameters of the model, α_1 , b_1 , c_1 , c_2 , K_A , K_B , as reported in the appendix.⁷

The main difference in comparison with the solution to the model where total production is given (see (9-11)) is that the beliefs regarding the strength of demand enter the production/storage/sales decisions explicitly (see (14) and (19)). Intuition suggests that production depends positively on the agent's own expectations regarding the strength of demand (i.e. $v_i > 0$) and negatively on the rival's (i.e. $\phi_i < 0$) but this may not be true for all parameter constellations. Another difference is that risk aversion affects decision making at the production level. This was not the case when total production was given, because in that case uncertainty regarding the strength of demand only affected profits through the average price \hat{p} that pertained to the given quantity (see (12)), whereas here the total quantity can also be chosen freely. This renders the decision process much more complex. In a sense, the effect of changing the model to allow for strategic stocks has been to replace \bar{x}_A and \bar{x}_B in (9) and (10) by $E_A(\alpha)$ and $E_B(\alpha)$ in (14) and (19), and to render the corresponding coefficients more complex.

Summarizing, we have found that the production, sales and storage decisions of the two rivals of the duopoly cash market depend on the futures positions taken in advance. If there is a constraint on total production, then this enters the decisions, and storage mainly serves as a cost-smoothing device. If the restriction is not present (or not effective) then the agents' beliefs

⁷ We subsume the cost of storage in the cost of production of the first period. Since first-period production is not sold, the storage and the production decisions are essentially one and the same.

regarding the strength of demand are important, and the inventories serve a strategic purpose (in addition to the cost smoothing). The two companies are trapped in the non-cooperative Nash equilibrium of the prisoner's dilemma type: The effect of simultaneously positive stocks of crude oil will raise total production (19) and lower the spot price at maturity (18). The cooperative outcome that is obtained if both agree not to store would yield higher profits, but then each company would have an incentive to defect (15), thereby making the other firm worse off (16). Since both firms simultaneously try to act as (Stackelberg-) leaders by precommitting themselves to higher sales through inventories, a sub-optimal Nash equilibrium results.⁸

We now move on to describe the futures markets and find the futures positions that are so crucial for the sales, storage and production schedules.

5. Stage Two: The 15-Day Market

As mentioned above there are two futures markets for North Sea crude oil: the IPE and the 15-Day market. The 15-Day market, which is the subject of the present subsection, is characterised by there being only few big participants in the market. The functioning of the market is described in Mabro et al. (1986), ch. 12, and in Philips (1991). It is "an informal, self-regulating club of North Sea producers, oil traders, refiners and brokers, each of which is in the market for a variety of reasons" (Mabro et al. (1986), p. 169). These market participants bargain about standardised forward contracts via telephone and the agreed-upon contracts are then telexed. This bargaining can only be realistically modelled at the cost of a considerable increase in complexity.⁹ We shall refrain from this here and simplify the model to highlight the role played by our two producers, A and B.

To be specific we assume that only three agents take part in the 15-Day market: A, B and an oil trader, S, who is not interested in the crude oil *per se* but only in buying (selling) crude oil on the 15-Day market with the

⁸ The incentive to try to act as a leader depends crucially on the convexity of the cost functions (i.e. on c_1 and c_2) as also noted by Arvan (1985). The more convex the cost function is (the higher c_1 and c_2 are), the higher are the strategic inventories. For this reason it is difficult to separate the cost smoothing and the strategic motive for holding stocks.

⁹ This is the topic of our ongoing research.

expectation of being able to sell (buy) it on the spot market at a higher (lower) price. Such agents are referred to as arbitrageurs or speculators in the literature and we shall not deviate from this habit.

So far the two futures markets have been described crudely by an aggregate futures price p^F and the quantity, N_i , by which a producer i ($i = A, B$) is net short. We now have to be more specific. Let F_i denote the net position of player i on the IPE and f_i the net position on the 15-Day market, i.e.

$$(20) \quad N_i = F_i + f_i \quad i = A, B, S.$$

We concentrate on the 15-Day positions in this subsection. Each of these f_i 's results from potentially four different contracts

$$(21.1) \quad f_i = f'_{ij} - f'_{ji} + f'_{ik} - f'_{ki}, \quad \text{where } i = A, B, S, \quad i \neq j, \quad j \neq k, \quad i \neq k \quad \text{and all } f' \geq 0$$

where f'_{ik} signifies that i is selling to k . In our model, what is making agents trade is differences in beliefs regarding the future spot price. This being the case it will never happen that two contracts are set up where an agent i both sells to and buys from another agent at different futures prices. If the futures price is equal in the two contracts it will only be the net position that matters anyway. We shall therefore let the signed quantity

$$(22) \quad f_{ij} = f'_{ij} - f'_{ji}$$

denote the net position of i vis-à-vis j . $f_{ij} > 0$ implies that i takes a short position (sells futures) in the contract with j . We shall take this to mean that $f'_{ij} = f_{ij}$ and $f'_{ji} = 0$. Then note that $f_{ji} = -f'_{ji} = -f_{ij}$, so that net positions on the 15-Day market can be written:

$$(21.2) \quad f_A = f_{AB} + f_{AS}$$

$$(21.3) \quad f_B = -f_{AB} + f_{BS}$$

$$(21.4) \quad f_S = -(f_{AS} + f_{BS})$$

where $f_i > 0$ if the player is net short in total on the market. Note that $f_A + f_B + f_S = 0$.

The three agents, A, B, and S, trade on the expected spot price in period 2 and they all know that the spot price is formed according to (12.2 - 3). That is to say, they know that the producers' net positions on the two futures markets will influence the spot price or, to put it more polemically, that the producers will manipulate the time profile of spot prices, raising the first-period price where all sales are cash and lowering the second-period price where part of the sales have been made in advance (assuming that they are net short, i.e. that $N_a + N_b > 0$). All this is common knowledge for the market participants and it will not discourage the speculator from trading on the market. What the market participants do not agree upon is the mean value of α . Each has his firm opinion $E_i(\alpha)$ on this. They thus expect three different mean spot prices $E_i(p_2) = p_2^i$ according to their different beliefs on $\hat{\alpha}$ and according to (12.2-3). It follows from (6.2) that they perceive p_2 as being normally distributed with mean p_2^i and unit variances.

5.1 The speculators' payoff

We assume that the speculator is risk-neutral¹⁰ and thus maximizes expected profit:

$$(23.1) \quad E_S(\Pi_S) = (p_2^S - p_{AS}^{1S})f_{AS} + (p_2^S - p_{BS}^{1S})f_{BS}$$

where p_{is}^{1S} is the price of the contract that s agrees upon with i . (Such a contract is fully described by the pair (p_{ij}^{1S}, f_{ij})). The expected profit can be rewritten using (12.2 - 3):

$$(23.2) \quad -E_S(\Pi_S) = \left[\hat{p}_S - \frac{1}{6}bF + \frac{1}{3}j \right] f_S + p_{AS}^{1S} f_{AS} + p_{BS}^{1S} f_{BS}$$

where

$$\hat{p}_S = E_S(\hat{\alpha}) - \frac{1}{2}b(\bar{x}_A + \bar{x}_B)$$

and $F = F_A + F_B$ is the producers' joint net position on the IPE.

¹⁰ To assume that he is risk averse does not change the analysis substantially.

5.2 The two producers' payoffs

The producers' payoffs from trading on the futures markets are found by substituting (10) and (12) into (7) and subtracting the profit that would have been made on the spot market in the absence of the futures markets. This leaves us with the following expressions

$$(24.1) \quad W_A^{\text{FUT}} = (p^F - \hat{p}_A) N_A + \frac{1}{18} [b(N_A + N_B)^2 - j(7N_A - 2N_B)] - \frac{K_A}{2} (\bar{x}_A - N_A)^2$$

$$(24.2) \quad W_B^{\text{FUT}} = (p^F - \hat{p}_B) N_B + \frac{1}{18} [b(N_A + N_B)^2 + j(2N_A - 7N_B)] - \frac{K_B}{2} (\bar{x}_B - N_B)^2$$

The first term on the r.h.s. of these expressions illustrates the immediate gain from having a futures market: p^F is the average futures price and \hat{p}_i is the spot price that i would expect in the absence of futures markets. If $p^F > \hat{p}_A$ then A should, *ceteris paribus*, take a net short position $N_A > 0$. The third term indicates the advantage of hedging the uncertain profit. In the case of complete hedging $\bar{x}_A = N_A$ and the term is effectively optimized. The second term shows the strategic effect on the spot market profits of having a futures market. This term occurs because taking futures positions provides a credible vehicle for precommitment of spot sales or purchases. Note that, for example, A's payoff varies proportionally with

$$(24.3) \quad (N_A + N_B)^2 - \frac{j}{b} (7N_A - 2N_B)$$

and that the futures markets affect A's output even if he does not participate, namely via the rival's net position on these markets. Indeed, if $N_A = 0$, W_A^{FUT} varies in proportion to

$$(24.4) \quad N_B^2 + 2 \frac{j}{b} N_B.$$

B's net position will always affect A's payoff positively in the absence of costly storage ($j=0$). This comes about because a producer with a zero position responds to the rival's position by shifting sales to the period with a higher price. If storage is possible at a cost this is no longer true for all values of N_B , as illustrated by figure 1.

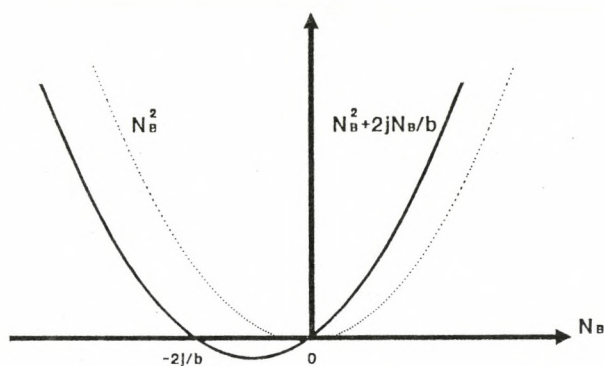


Figure 1: The effect of a unilateral futures position on the rival's profit

If B is long in the range

$$(24.5) \quad 0 > N_B > -2 \frac{j}{b} \quad (N_A = 0),$$

then A's optimal response on the spot market is to shift sales (see (10)) with N_B/b from period 1 where the price is then low (see (12.1)) to period 2 where the price is higher (12.2). Since the optimal extraction is independent of the futures positions (9), this operation can only be done via increased inventories (11.1). But increasing stocks has a cost, and so for small, long positions, B forces A to incur a loss since the cost of increasing the stocks dominates the extra expected revenue.¹¹

The producers of course realize the strategic interdependence of their futures positions - an interdependence that stems from the effect of these positions on the equilibrium spot prices and quantities. Indeed, when taking positions on the futures markets, the producers weigh the speculative, the strategic and the hedging motives according to (24).

The payoffs of the two futures markets can be split up according to the market that gives rise to them, or, put differently, we find the subgame perfect

¹¹ Note, however, that this result may hinge crucially on the assumptions of the model. In particular the unflexibility of total production (1) seems important.

equilibria by maximising the payoff of the second stage, the 15-Day market, for given positions on the IPE, before solving the first stage.

The ex-ante payoff stemming from the 15-Day market (including this market's effects on the spot market payoff) can be written:

$$(25.1) \quad W_A^{15} = (p_{AB}^{15} - \hat{p}_A) f_{AB} + (p_{AS}^{15} - \hat{p}_A) f_{AS} + \tau_1 (f_{AS} + f_{BS}) \\ + \tau_{2A} f_{AS} + \tau_3 f_{BS} + \tau_{4A} f_{AB} - \frac{K_A}{2} (f_{AB} + f_{AS})^2,$$

$$(25.2) \quad W_B^{15} = (\hat{p}_B - p_{AB}^{15}) f_{AB} + (p_{BS}^{15} - \hat{p}_B) f_{BS} + \tau_1 (f_{AS} + f_{BS}) \\ + \tau_3 f_{AS} + \tau_{2B} f_{BS} - \tau_{4B} f_{AB} - \frac{K_B}{2} (f_{BS} - f_{AB})^2,$$

where

$$(25.3) \quad \tau_1 = \frac{1}{18} b > 0$$

$$(25.4) \quad \tau_{2i} = \frac{1}{9} bF - \frac{7}{18} j + K_i (\bar{x}_i - F_i); \quad i = A, B$$

$$(25.5) \quad \tau_3 = \frac{1}{9} (bF + j)$$

$$(25.6) \quad \tau_{4i} = -\frac{1}{2} j - K_i (\bar{x}_i - F_i); \quad i = A, B.$$

5.3 The contract curves

Now, what can we say about the solutions to the 15-Day stage, without imposing further structure on the game? We require that any contract (p_{ij}^{15}, f_{ij}) belongs to the contract curve between i and j , i.e. it must be true that $MRS_i(p_{ij}^{15}, f_{ij}) = MRS_j(p_{ij}^{15}, f_{ij})$, where the marginal rates of substitution are given implicitly by (25) for the producers and (23) for the speculator. For example, the contract between the two producers should obey

$$(26) \quad MRS_A^{AB} = \frac{\frac{\partial W_A^{15}}{\partial p_{AB}^{15}}}{\frac{\partial W_A^{15}}{\partial f_{AB}}} = \frac{\frac{\partial f_{AB}}{\partial p_{AB}^{15}}}{\frac{\partial f_{AB}}{\partial W_B^{15}}} = MRS_B^{AB}.$$

These requirements are fulfilled if $f_{ij} = 0$, that is, if the two participants do not enter a contract. Less trivially, if $f_{ij} \neq 0$, (26) leads to

$$(27.1) \quad (K_A + K_B) f_{AB} + K_A f_{AS} - K_B f_{BS} = (\hat{p}_B - \hat{p}_A) + K_A (\bar{x}_A - F_A) - K_B (\bar{x}_B - F_B)$$

$$(27.2) \quad -K_A f_{AB} - \left(K_A + \frac{2}{9}b\right) f_{AS} - \frac{2}{9}b f_{BS} = (\hat{p}_A - \hat{p}_S) - K_A (\bar{x}_A - F_A) - \frac{1}{18}(bF + j)$$

$$(27.3) \quad -K_B f_{AB} + \frac{2}{9}b f_{AS} + \left(K_B + \frac{2}{9}b\right) f_{BS} = (\hat{p}_S - \hat{p}_B) - K_B (\bar{x}_B - F_B) - \frac{1}{18}(bF + j),$$

which apply to the contracts (p_{AB}^{15}, f_{AB}) , (p_{AS}^{15}, f_{AS}) and (p_{BS}^{15}, f_{BS}) , respectively. Philips and Harstad (1991) use an equivalent approach and find a similar system of equations.¹² The main differences are that the producers' positions on the IPE, F_A and F_B enter on the r.h.s. because they too can be used for hedging purposes; that their joint position on the IPE (F) enters because of the strategic effect of futures on the spot market (a feature which the two markets share); and that the cost of storage shows up since taking a futures position (on either market) changes optimal inventories (11.1).

The equational system (27) cannot be solved to obtain a unique set of three 15-Day contracts. There are two reasons for this. The first reason has to do with the fact that individual rationality points to a range of possible prices depending on the quantities. The second reason arises because (27) only determines the net positions f_A , f_B and f_S uniquely, not the decomposition on the three quantities f_{AB} , f_{AS} and f_{BS} . We discuss each point in turn.

5.4 Individual rationality

First, note that the futures prices, p_{ij}^{15} , do not appear in (27). All we can say about these prices is that they should be individually rational according to (23) and (25). Individual rationality simply states that any contract should contribute a non-negative amount to each player's payoff since a zero

¹² See their equations (13).

contribution can always be achieved by not entering the contract. This requirement puts the following bounds on the prices: For the contract (p_{AB}^{15}, f_{AB}) :

$$(28.1) \quad \begin{aligned} \hat{p}_A - K_A \left(\bar{x}_A - F_A - f_{AS} - \frac{1}{2} f_{AB} \right) + \frac{1}{2} j &\leq p_{AB}^{15} \\ &\leq \hat{p}_B - K_B \left(\bar{x}_B - F_B - f_{BS} + \frac{1}{2} f_{AB} \right) + \frac{1}{2} j \quad \text{for } f_{AB} > 0; \end{aligned}$$

for the contract (p_{AS}^{15}, f_{AS}) :

$$(28.2) \quad \begin{aligned} \hat{p}_A + \left(\frac{K_A}{2} - \frac{1}{18} b \right) f_{AS} - \frac{1}{9} b (F + f_{BS}) - K_A (\bar{x}_A - F_A - f_{AB}) + \frac{7}{18} j &\leq p_{AS}^{15} \\ &\leq \hat{p}_S - \frac{1}{6} b (F + 2 f_{AS} + f_{BS}) + \frac{1}{3} j \quad \text{for } f_{AS} > 0; \end{aligned}$$

and finally for the contract (p_{BS}^{15}, f_{BS}) :

$$(28.3) \quad \begin{aligned} \hat{p}_B - \frac{1}{9} b (F + f_{AS}) + \left(\frac{K_B}{2} - \frac{1}{18} b \right) f_{BS} - K_B (\bar{x}_B - F_B + f_{AB}) + \frac{7}{18} j &\leq p_{BS}^{15} \\ &\leq \hat{p}_S - \frac{1}{6} b (F + f_{AS} + 2 f_{BS}) + \frac{1}{3} j \quad \text{for } f_{BS} > 0. \end{aligned}$$

Note that the above three inequalities are true under the condition that f_{AB} , f_{AS} and f_{BS} be strictly positive. For each of them the inequality is reversed if the sign of the quantity is reversed. Observe also the following: The range of futures prices that are acceptable for the two players involved in a contract depends on the futures positions taken by the producers on the IPE, on the quantities of the other 15-Day contracts, on the expected spot price and on the cost of storage. This range may or may not be empty, depending on the values of these variables. In case the range is empty, this corresponds to the players agreeing on the contract $(p_{ij}^{15}, f_{ij}) = (0, 0)$ which is always a possibility. This just means that these two players do not find it profitable to enter a contract.

(28.1) says that if A is selling to B ($f_{AB} > 0$) then A will require a higher minimal sales price

- 1) the more she has already hedged on the IPE (F_A),
- 2) the more she has already hedged in a contract with the speculator (f_{AS}),
- 3) the higher is her spot price expectation (\hat{p}_A), and
- 4) the higher is the cost of storage (j).

Buyer B will accept a higher maximal buying price

- 1) the more he went short on the IPE (F_B),
- 2) the more he went short in a contract with S (f_{BS}),
- 3) the higher is his expected spot price (\hat{p}_B), and
- 4) the higher is the cost of storage (j).

Note that the cost of storage raises the minimal selling price and the maximal buying price by the same amount, thus preserving the spread. Further, observe that the minimal selling price is increasing and the maximal buying price is decreasing in the quantity f_{AB} of the contract. This ensures that the individually rational contract must have finite quantity. Indeed, it can be shown that the contract must satisfy either

$$(28.1a) \quad 0 < f_{AB} < \frac{2}{K_A + K_B} (\hat{p}_B - \hat{p}_A + K_A (\bar{x}_A - F_A - f_{AS}) - K_B (\bar{x}_B - F_B - f_{BS}))$$

in the case of A selling to B, or

$$(28.1b) \quad 0 > f_{AB} > \frac{2}{K_A + K_B} (\hat{p}_A - \hat{p}_B - K_A (x_A - F_A - f_{AS}) + K_B (\bar{x}_B - F_B - f_{BS}))$$

when B is selling to A.

The analysis of the contracts where the producers are selling to the speculator (28.2-3) follows the discussion above with the following two qualifications: Firstly, increased storage costs increase the producers' minimal selling price a bit more than the speculator's maximal buying price. Secondly, if $K_i/2 < \tau_i$, the minimal selling price will be falling in the size of the contract. However, the speculator's maximal buying price will decrease at a much faster rate in f_{iS} , thus still ensuring finite positions. The equivalents of (28.1a-b) are

$$(28.2a-b) \quad 0 < |f_{AS}| < \left[\hat{p}_S - \hat{p}_A - \tau_1 (F + f_{BS}) + K_A (\bar{x}_A - F_A - f_{AB}) - \frac{1}{18} j \right] / \left(\frac{K_A}{2} + \frac{5}{18} b \right),$$

$$(28.3a-b) \quad 0 < |f_{BS}| < \left[\hat{p}_S - \hat{p}_B - \tau_1 (F + f_{AS}) + K_B (\bar{x}_B - F_B + f_{AB}) - \frac{1}{18} j \right] / \left(\frac{K_B}{2} + \frac{5}{18} b \right).$$

Equations (28) all imply individual rationality by securing that each contract adds positively to the payoffs. Each player can, however, secure himself a minimum payoff without participating in the 15-Day market at all. Individual rationality then implies that payoffs should satisfy:

$$(29.1) \quad W_A \geq \tau_1 f_{BS}^2 + \tau_3 f_{BS}$$

$$(29.2) \quad W_B \geq \tau_1 f_{AS}^2 + \tau_3 f_{AS}^2$$

$$(29.2) \quad E_S(\Pi_S) \geq 0.$$

Note, though, that the minimal payoff of producer i may become negative if the rival k takes a position $f_{kS} \in]0, -2(F+j/b)[$.

5.5 Net 15-Day positions

The second observation on the system (27) is that it exhibits linear dependence in f_{AB} , f_{AS} and f_{BS} implying that the quantities of the contracts are indeterminate. This can be remedied partly by manipulating the system (27) using the identities (21) to find the net positions:

$$(30.1) \quad \begin{aligned} f_A^* = & \Theta \left[K_B (\hat{p}_S - \hat{p}_A) + \frac{2}{9} b (\hat{p}_B - \hat{p}_A) + (K_A K_B + \frac{2}{9} b K_A) (\bar{x}_A - F_A) \right. \\ & \left. - \frac{2}{9} b K_B (\bar{x}_B - F_B) - \frac{1}{18} K_B (bF + j) \right] \end{aligned}$$

$$(30.2) \quad \begin{aligned} f_B^* = & \Theta \left[K_A (\hat{p}_S - \hat{p}_B) - \frac{2}{9} b (\hat{p}_B - \hat{p}_A) + (K_A K_B + \frac{2}{9} b K_B) (\bar{x}_B - F_B) \right. \\ & \left. - \frac{2}{9} b K_A (\bar{x}_A - F_A) - \frac{1}{18} K_A (bF + j) \right] \end{aligned}$$

$$(30.3) \quad \begin{aligned} f_S^* = & -\Theta \left[K_A (\hat{p}_S - \hat{p}_B) + K_B (\hat{p}_S - \hat{p}_A) + K_A K_B (\bar{x}_A + \bar{x}_B - F) \right. \\ & \left. - \frac{1}{18} (K_A + K_B) (bF + j) \right] \end{aligned}$$

$$(30.4) \quad \Theta^{-1} = K_A K_B + 2b(K_A + K_B)/9.$$

The net positions are thus uniquely determined by the parameters of the spot market, the speculations about the spot price and the positions already taken on the IPE during the first stage, but the distribution on individual contracts cannot be found.

5.6 The core

The set of contracts described by equations (27) - (30) consists of those contracts that are individually and coalitionally rational at the same time and is thus basically the core. By substituting (30) into (27) the core can be described as the set of contracts $\{(p_{AB}^{15}, f_{AB}), (p_{AS}^{15}, f_{AS}), (p_{BS}^{15}, f_{BS})\}$ that satisfy the following equations simultaneously:

$$(31.1) \quad f_{AB} + f_{AS} = f_A^*$$

$$(31.2) \quad -f_{AB} + f_{BS} = f_B^*$$

$$(31.3) \quad -(f_{AS} + f_{BS}) = f_S^*$$

$$(31.4) \quad g - \frac{1}{2} K_A f_{AB} < p_{AB}^{15} < g + \frac{1}{2} K_B f_{AB} \quad \text{if } f_{AB} > 0$$

$$(31.5) \quad g' - \frac{1}{2} \left(K_A - \frac{1}{9} b \right) f_{AS} < p_{AS}^{15} < g'' + \frac{1}{6} b f_{AS} \quad \text{if } f_{AS} > 0$$

$$(31.6) \quad g' - \frac{1}{2} \left(K_B - \frac{1}{9} b \right) f_{BS} < p_{BS}^{15} < g'' + \frac{1}{6} b f_{BS} \quad \text{if } f_{BS} > 0$$

where

$$(31.7) \quad g = \Theta \left[\frac{2}{9} b (K_B \hat{p}_A + K_A \hat{p}_B) + K_A K_B \hat{p}_S \right] - \frac{2}{9} \Theta b K_A K_B (\bar{x}_A + \bar{x}_B - F) \\ + \frac{1}{18} \Theta K_A K_B (b F + j) + \frac{1}{2} j$$

$$(31.8) \quad g' = \Theta \left[\frac{1}{9} b (K_B \hat{p}_A + K_A \hat{p}_B) + \left(K_A K_B + \frac{1}{9} (K_A + K_B) \right) \hat{p}_S \right] \\ - \frac{1}{9} \Theta b K_A K_B (\bar{x}_A + \bar{x}_B - F) + \frac{1}{18} \Theta \left(K_A K_B - \frac{1}{9} (K_A + K_B) \right) (b F + j) - \frac{1}{9} b F$$

$$(31.9) \quad g'' = \Theta \left[\frac{1}{3} b (K_B \hat{p}_A + K_A \hat{p}_B) + \left(K_A K_B - \frac{1}{9} (K_A + K_B) \right) \hat{p}_S \right] \\ - \frac{1}{3} \Theta b K_A K_B (\bar{x}_A + \bar{x}_B - F) + \frac{1}{54} \Theta b (K_A + K_B) (b F + j) - \frac{1}{6} b F$$

The linear system for the quantities (31.1-3) has one degree of freedom implying that the quantities will be uniquely determined once one quantity is known. (Fix for example f_{AB} . Then (31.1) uniquely determines f_{AS} , whilst (31.2) uniquely determines f_{BS} . These quantities will be consistent with (31.1).)

Once the quantities are known, (31.4-6) determine the ranges of prices acceptable. Note that each of the inequalities is reversed if the corresponding quantity becomes negative. Also note that the lower bound appears to be decreasing and the upper bound increasing in the quantity. This somewhat surprising result has to be interpreted in the light of the constraints on quantities: If a contract becomes larger (say f_{AB}) and the price "spread" therefore increases then the other contracts become correspondingly smaller (f_{AS} decreases and f_{BS} increases one-to-one with f_{AB} positive but f_{AB} and f_{BS} have opposite signs in B's payoff function).

The three ranges that bound prices are determined by the g 's given in equations (31.7-9). The first term in each of these consists of a weighted average of the three agents' expected price, \hat{p}_A , \hat{p}_B and \hat{p}_S , the weights being functions of K_A , K_B and b . The second term depends on the degree to which the producers have hedged their production on the IPE. In case they hedged fully on the IPE ($\bar{x}_A + \bar{x}_B = F$), the term drops out. In case they hedged less on the IPE, the bounds on prices will be lower, implying that it will be more costly for the producers to hedge on the 15-Day market. The last two terms of the g 's depend on the producers' IPE position and on storage costs and basically incorporate the strategic effects of futures and storage on spot prices.

This core is never empty and always non-unique (in fact: infinitely large). We will not elaborate on the solution to the 15-Day stage here, but simply note that, a priori, there is no means to pointing out a subset of the core as being more likely as a solution.

6. Stage One: The International Petroleum Exchange

The IPE, as described in the introduction, is a formal futures market with an open outcry exchange and a clearing house. We therefore assume that a single price, p^{IPE} , will be determined on this market. The question then is what positions F_A and F_B the producers should take on this market.

The payoffs arising from the IPE are¹³

$$(32.1) \quad W_A^{IPE} = (p^{IPE} - \hat{p}_A) F_A + \tau_1 F - \frac{1}{18} (7 F_A - 2 F_B) \\ - \frac{1}{2} K_A (\bar{x}_A - F_A)^2 + E_A(W_A^{15}(F_A, F_B))$$

$$(32.2) \quad W_B^{IPE} = (p^{IPE} - \hat{p}_A) F_B + \tau_1 F + \frac{1}{18} j (2 F_A - 7 F_B) \\ - \frac{1}{2} K_B (\bar{x}_B - F_B)^2 + E_B(W_B^{15}(F_A, F_B))$$

where $E_A(W_A^{15}(F_A, F_B))$ and $E_B(W_B^{15}(F_A, F_B))$ are the payoffs that A and B expect to gain from the 15-Day market depending on what solution they expect to prevail. This could be formalised by claiming that they hold one subjective possibility distribution, h_i , on what the size of, say, f_{AB} will be and other subjective distributions, h_i^{AB} , h_i^{AS} , h_i^{BS} on what the price will be, conditional on the quantities. These probability distributions could be thought of as representing the way in which the agents think the 15-Day market works.

A's expected value of f_{AB} will then be

$$(33.1) \quad E_A(f_{AB}) = \int_{-\infty}^{\infty} f_{AB} h_A d f_{AB}$$

and the expectations with respect to the two other 15-Day quantities therefore

$$(33.2) \quad E_A(f_{AS}) = \int_{-\infty}^{\infty} (f_A^*(F_A, F_B) - f_{AB}) h_A d f_{AB}$$

and

$$(33.3) \quad E_A(f_{BS}) = \int_{-\infty}^{\infty} (f_B^*(F_A, F_B) + f_{AB}) h_A d f_{AB}.$$

The expected prices will be,

¹³ Using (24), (20) and (7).

$$(33.4) \quad E_A(p_{AB}^{15} | f_{AB}) = \int_{e - K_A t_{AB}/2}^{e + K_A t_{AS}/2} p_{AB}^{15} h_A^{AB} d p_{AB}^{15},$$

$$(33.5) \quad E_A(p_{AS}^{15} | f_{AB}) = \int_{e - (K_A - \frac{1}{9}b)(t_A^* - t_{AB})/2}^{e + b(t_A^* - t_{AB})/6} p_{AS}^{15} h_A^{AS} d p_{AS}^{15},$$

and

$$(33.6) \quad E_A(p_{BS}^{15} | f_{AB}) = \int_{e - (K_B - \frac{1}{9}b)(t_B^* - t_{AB})/2}^{e + b(t_B^* - t_{AB})/6} p_{BS}^{15} h_A^{BS} d p_{BS}^{15}.$$

Note that the net positions on the 15-Day market f_i^* depend on the position taken on the IPE, so that the limits of the integrals in (33.4-6) depend on F_A and F_B .

The expected value to A of the 15-Day transactions can be found by taking the expectation of (25.1) using (33) and (30). This will give the payoff $E_A(W_A^{15}(F_A, F_B))$ which occurs in (32.1). A similar exercise can be done for B by substituting $(h_A, h_A^{AB}, h_A^{AS}, h_A^{BS})$ by $(h_B, h_B^{AB}, h_B^{AS}, h_B^{BS})$ in (33) and taking the expectation of (25.2). This will identify (W_A^{IPE}, W_B^{IPE}) in (32.1). If the two sets of subjective probability distributions are common knowledge (to be precise: if A knows B's and B knows A's probability distributions) and if the two players take the IPE positions simultaneously, a subgame-perfect Nash equilibrium will result where A maximizes W_A^{IPE} with respect to F_A and B W_B^{IPE} with respect to F_B .

Note that when the two producers take positions on the IPE they do this for the same three motives as applies to the 15-Day market: A speculative, a strategic and a hedging motive are at play in (32) that readily compares to (25). In fact, the four first terms of (32) capture exactly these effects. But in the fifth term, the producers will realize that the position they take will have an effect on the unknown solution to the 15-Day game. So, in a sense, this adds another speculative motive: speculation with respect to the non-unique outcomes of the 15-Day game.

7. The Model and the Oil Markets

The model that was put forth above is based on several abstractions compared to the real world. One important abstraction is connected to the treatment of time. The model can be seen as a snapshot in a sense that will soon be made precise. The real world is rather a continuous series of rolling and overlapping snapshots. This section discusses how the model could be interpreted and what would be necessary to create a moving picture.

The interpretation of our game in terms of real world actions starts with the observation that the organization of the 15-Day market requires the producers to give the purchasers a 15-day notice before delivery. This notice specifies a three-day range within which delivery will take place. The oil traded thereby transits from being undated to being dated: from being traded forward to being traded spot. These fifteen days correspond to our period 1 of the extraction game since the futures markets for oil to be delivered in this period are closed. Cargoes that are lifted but not sold during this period represent an increase in stocks that can be sold in period 2.

In order to make period 2 of the extraction game correspond to the real world we adopt a strong abstraction: Assume that all cargoes of a given month are lifted within a given delivery range. In other words, the delivery month is collapsed into this range. Assume for concreteness that all September oil is to be delivered between the first and the third of September for a given year. Period 2 of the production game could be interpreted as this period (September 1-3). This then would correspond to the maturity of the 15-Day contract. The fifteen days prior to September 1st (i.e. August 16th-31st) would constitute period 1.

The two futures markets are collapsed into points in time. We can interpret this by assuming that on the 15th of August the 15-Day market opens. This is technically the last day that forward oil can be traded for delivery on September 1-3. So the market closes before August 16, and will not reopen until period 2 where, by definition, the maturity price is identical to the spot price.

The IPE closes the trading of paper barrels referring to a given delivery month in the middle of the previous month, that is to say, well before the 15-Day market stops trading this delivery month. In fact, what is the first forward month on the IPE is normally the second forward month of the 15-Day market, and the maturity price of the IPE is the 15-Day price on the closing day. The latter fact is ignored in the model and the maturity price on both futures markets is chosen to be the spot price of the second period of the extraction game. The first feature is however modelled by letting the IPE precede the 15-Day game. In other words, we assume that the IPE opens and closes only once prior to maturity, on the 14th of August.

In reality, of course, the oil markets are much more dynamic than our model allows for. First, a sequence of extraction games are played and stocks are increased or decreased between them. Stocks therefore serve as a state variable in a dynamic game. This may change the strategic effect of inventories since it is no longer true that everything that was produced but not sold in one period has to be sold in the next. Rotemberg and Saloner (1985) for example see inventories as a means to sustain high collusive prices by threatening to float the market if a rival deviates.

The two futures markets are treated as one-shot situations in our model. In the real world, these markets are open every day and trade different contracts (up to six months ahead) simultaneously. This implies that there may be much more dynamic interaction going on than here presented. For example, informational intricacies have been ignored by assuming that the subjective probability distributions and all strategic features are common knowledge. This leaves the difference in subjective probability distributions unexplained. A natural explanation of this involves differences in information (asymmetric information or incomplete knowledge) or optimistic/pessimistic behaviour as noted in footnote 3. A model of a futures market for a storable good explaining the reasons for the existence as well as the effects of asymmetric information is found in Stein (1987). This model, however, does not analyse the strategic aspects of inventories and of futures but concentrates on risk sharing and informational externalities.

Lastly it should be noted that the structure of the three markets has been simplified in the model. An obvious example of this is the quasi omission of refineries from the analysis. They are solely represented by the downward

sloping demand curves (6) which is inadequate. Similarly the problem of integrated oil companies has been ignored. These problems are at the core of our current research.

Appendix to Section 4: Strategic Stocks

This appendix is offered as a help to the reader who wants to obtain a full understanding of the model demonstrating the strategic use of stocks.

Recall that we made the following assumptions:

- (A.1) There is no exogenous upper bound to production, i.e. no \bar{x}_i , $i = A, B$.
- (A.2) There is no demand in the first period, implying that the production of the first period has to be stored, and that sales (in the second period) equal total production over the two periods, cfr. (13).

The profit functions are then given by

$$(A.3.1) \quad \Pi_A = (s_A + x_{A2} - N_A) p_2 - \frac{c_1}{2} s_A^2 - \frac{c_2}{2} x_{A2}^2 + p^F N_A,$$

$$(A.3.2) \quad \Pi_B = (s_B + x_{B2} - N_B) p_2 - \frac{c_1}{2} s_B^2 - \frac{c_2}{2} x_{B2}^2 + p^F N_B.$$

Note that the cost of storage is subsumed in the period 1 cost function: The decision to produce in period one is essentially the same as the decision to store, and we do not need two cost variables to describe the decision. All other assumptions remain unchanged, i.e. the demand curve is stochastic, linear and downward sloping (6) and the producers' utilities from profits follow the mean-variance model (7).

The last decision the producers take regards second-period production: They take this decision by simultaneously maximizing their payoffs (7) with respect to their respective decision variables (x_{A2}, x_{B2}) taking the stocks (s_A, s_B) and the net futures positions (N_A, N_B) as given. This results in the following second-period production for A (the expression for B is similar):

$$(A.4.1) \quad x_{A2} = \frac{1}{D} b \left[\Delta(\alpha) + b(s_B - s_A) - b(N_B - N_A) + K_B(s_B - N_B) - K_A(s_A - N_A) \right] \\ + \frac{1}{D} (b + c_2 + K_B) [E_A(\alpha) - b(s_A + s_B) + (b + K_A)(N_A - s_A)],$$

which is comparable to (14) and where

$$(A.4.2) \quad D = (2b + c_2 + K_A)(2b + c_2 + K_B) - b^2, \quad \text{and}$$

$$(A.4.3) \quad \Delta(\alpha) = E_A(\alpha) - E_B(\alpha).$$

The multipliers (15)-(17) follow directly from (A.4.1) and (18) follows with the additional use of (6.1).

The second-to-last decision the producers take regards how much to produce in the first period or, equivalently, how much to store. Substituting the optimized second-period productions into the payoff functions and performing a simultaneous maximization of these with respect to the stocks (taking net futures positions as given), we obtain

$$(19.1) \quad s_A = v_A E_A(\alpha) + \phi_A E_B(\alpha) + \psi_A N_A + \omega_A N_B$$

$$(19.2) \quad s_B = \phi_B E_A(\alpha) + v_B E_B(\alpha) + \omega_B N_A + \psi_B N_B$$

where

$$(A.5.1) \quad v_A = (\rho_{B1} \delta_{A\alpha} - \delta_{B1} \rho_{A\alpha}) / \mathcal{D}; \quad v_B = (\delta_{A1} \rho_{B\alpha} - \rho_{A1} \delta_{B\alpha}) / \mathcal{D};$$

$$(A.5.2) \quad \phi_A = (\rho_{B1} \delta_{B\alpha} - \delta_{B1} \rho_{B\alpha}) / \mathcal{D}; \quad \phi_B = (\delta_{A1} \rho_{A\alpha} - \rho_{A1} \delta_{A\alpha}) / \mathcal{D};$$

$$(A.5.3) \quad \psi_A = (\rho_{B1} \delta_{N_A} - \delta_{B1} \rho_{N_A}) / \mathcal{D}; \quad \psi_B = (\delta_{A1} \rho_{N_B} - \rho_{A1} \delta_{N_B}) / \mathcal{D};$$

$$(A.5.4) \quad \omega_A = (\rho_{B1} \delta_{N_B} - \delta_{B1} \rho_{N_B}) / \mathcal{D}; \quad \omega_B = (\delta_{A1} \rho_{N_A} - \rho_{A1} \delta_{N_A}) / \mathcal{D};$$

$$(A.5.5) \quad \mathcal{D} = \rho_{A1} \delta_{B1} - \rho_{B1} \delta_{A1}$$

$$(A.6.1) \quad \rho_{A1} = -(c_1 + c_2)D + c_2^2(2b + c_2 + K_B)(1 + b^2/D)$$

$$(A.6.2) \quad \rho_{B1} = (2b + K_B - c_2 b^2/D) b c_2$$

$$(A.6.3) \quad \rho_{A\alpha} = c_2(2b + c_2 + K_B)(1 + b^2/D) > 0$$

$$(A.6.4) \quad \rho_{B\alpha} = -b c_2(1 + b^2/D) < 0$$

$$(A.6.5) \quad \rho_{N_A} = c_2 (2b + c_2 + K_B)(b + K_A)(1 + b^2/D) - c_2 b^2 > 0$$

$$(A.6.6) \quad \rho_{N_B} = -b c_2 (b + K_B)(1 + b^2/D) < 0$$

$$(A.7.1) \quad \delta_{A1} = b c_2 (2b + K_A - c_2 b^2/D)$$

$$(A.7.2) \quad \delta_{B1} = -(c_1 + c_2)D + c_2^2 (2b + c_2 + K_B)(1 + b^2/D)$$

$$(A.7.3) \quad \delta_{A\alpha} = \rho_{B\alpha} < 0$$

$$(A.7.4) \quad \delta_{B\alpha} = c_2 (2b + c_2 + K_A)(1 + b^2/D) > 0$$

$$(A.7.5) \quad \delta_{N_A} = -b c_2 (b + K_A)(1 + b^2/D) < 0$$

$$(A.7.6) \quad \delta_{N_B} = c_2 (2b + c_2 + K_B)(b + K_B)(1 + b^2/D) - c_2 b^2 > 0.$$

The signs of the parameters are indicated where possible. The sign of the most important determinant, \mathcal{D} , is however undetermined but will generally be positive. Sufficient, but not necessary, conditions for non-negativity of \mathcal{D} are, for example, that simultaneously $c_2 > 1$ and $(c_1 + c_2)^2 D > b^2 c_2 (1 + b^2/D)$. But these requirements are not easily interpreted.

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