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DEPARTMENT OF ECONOMICS

Three Essays on Open Economy Macroeconomics

Guilherme de Almeida Bandeira

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, 10 March 2016

European University Institute
Department of Economics

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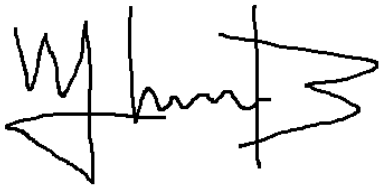

8th January 2016

para África

Only change is permanent.
Allama Iqbal

para a minha mãe, o meu pai,
e a minha mana

Abstract

This thesis consists of three papers on open economy macroeconomics.

The first paper investigates the welfare implications of a fiscal transfer rule between members of a monetary union. I build a two-country model where banks are exposed to sovereign risk. I show that the costs of expansionary fiscal policy can outweigh the benefits due to the added pressure on sovereign spreads. This is magnified when the fiscal stance is weak. On the contrary, inter-governmental transfers mitigate the fiscal strain and provide stimulus to the economy. Yet, I find that sound fiscal stances are crucial both for governments to countervail shocks on their own, and also to grant support for the implementation of the transfer scheme.

The second paper lays out an empirical model to compare the propagation of exogenous shocks in a small open economy subject to structural change. As the identification of shocks is time invariant, I am able to investigate the differences that pertain only to structural change. I estimate the model to Australia and find a regime transition occurring in 1990. The responses of domestic variables to the shocks are less exacerbated and adjust quicker after the transition. I find that increases in commodity prices are only inflationary after 1990 and as a reaction to world demand shocks, whereas commodity-market shocks are recessionary.

In the last paper, I study the implications of relative wage rigidities for monetary policy in a small commodity-exporting economy. I present a model where wages in the non-commodity sector are indexed to wages in the commodity sector, generating relative wage rigidities. For a high degree of indexation, I find some inflation is desirable. As wages are set in nominal terms, inflation partially offsets the effects of indexation on real wages. Nevertheless, as indexation is itself inflationary, the response of monetary policy to inflation still needs to be stronger. I show that policy misspecification is more problematic for higher degrees of indexation.

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São já vários os anos em que me expresso diariamente numa outra língua que não a minha. Mas é em Português que o faço melhor. Por isso, e como não faz sentido agradecer em língua estrangeira, deixo estas palavras na língua em que sinto.

Quando comecei esta empreitada, impus-me um calendário que, na altura, me parecia sensato e relativamente fácil de alcançar: acabar o doutoramento antes do meu 28º aniversário. Com uma inesgotável paciência e um inabalável optimismo, é à Evi Pappa quem primeiro agradeço por me ter ajudado a cumprir este propósito. Como supervisora, esteve sempre disponível para discutir o progresso da tese, sugerindo novos caminhos e novas perspectivas quando eu entrava em becos sem saída, motivando-me e fazendo-me acreditar no meu trabalho nos momentos em que as dúvidas pareciam intransponíveis. Como amiga, agradeço-lhe a confiança que depositou em mim ao deixar-me tomar as minhas opções geográficas, compreendendo que mais do que economistas, somos humanos.

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Spirito...

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Esta tese é dedicada à minha mãe Célia, ao meu pai Eduardo e à minha irmã Leonor. Por tudo! Por estarem ao meu lado em permanência, por acreditarem em mim, por me terem dado todas as condições materiais e psicológicas para que este trabalho fosse concluído com sucesso. Muita água correu durante estes quatro anos, mas juntos alcançámos sempre bom porto. A eles, agradeço do fundo do coração.

Byron Bay - Austrália, Janeiro de 2016

Contents

Summary

This thesis consists of three autonomous research papers on open economy macroeconomics. In common, they study the policy responses of fiscal and monetary authorities in small open economies exposed to exogenous shocks. Yet, the particular questions addressed and the methodology used differ substantially. The thesis is the result of individual research mostly carried out while I was at the European University Institute, in Florence. It also greatly benefited from my visits to the Central Bank of Colombia, the Reserve Bank of Australia, and the University of New South Wales.[1]

Chapter 1:

Fiscal Transfers in a Monetary Union with Sovereign Risk

In this paper, I study the welfare implications of a fiscal transfer rule between member states of a monetary union. In the aftermath of the sovereign debt crisis in Europe, I provide a contribution to the ongoing debate on how to reform the institutional framework of the Euro Area and make it more resilient to idiosyncratic shocks.

I build a two-country model where banks serve as financial intermediaries between depositors, on the one hand, and firms and the government, on the other. Because banks hold government bonds in their portfolios, they are exposed to sovereign credit risk. As such, an increase in sovereign spreads affects banks' leverage and leads to a disruption in the quantity and price of credit to firms. At the same time, the increase in interest payments forces the government to consolidate, amplifying the negative pressures on the economy.

I introduce an inter-governmental transfer scheme and show that, for welfare to improve in both countries, national fiscal profiles and population sizes need to be

[1] This thesis does not necessarily reflect the views of the Central Bank of Colombia or of the Reserve Bank of Australia or their staff.

nearly symmetric. Small differences between countries affect the distribution of welfare gains and deplete the support to implement the scheme. For instance, because transfers are raised *per capita*, smaller countries secure larger welfare gains, whereas big countries lose from the scheme.

I then propose a scheme under which the losses incurred by big countries are a minor fraction of the benefits they derive from trade integration due to the common currency. Under such scheme, small countries can reap substantial welfare gains, with the transfers effectively reducing the pass-through of sovereign spread shocks to the economy.

This paper shows that the costs of expansionary policy can outweigh the benefits from providing stimulus to the economy. Although counter-cyclical fiscal policy can sustain demand during a downturn, it adds pressure on sovereign spreads which feeds back to higher borrowing costs to firms. The negative effects are magnified when the fiscal stance is weak. In particular, when public debt is high and sovereign spreads are more sensitive to a deterioration of the public budget, consolidation becomes crucial to attenuate the pass-through of sovereign spread shocks.

On the contrary, inter-governmental transfers are an efficient mechanism to address idiosyncratic shocks. They mitigate the fiscal strain caused by higher interest payments and provide a stimulus to the economy. Yet, policy makers need to enforce fiscal discipline at the country level. I find that when one country conduces looser fiscal policy, it skews the welfare gains away from countries with stronger fiscal conditions. Hence, sound fiscal stances are crucial not only to add scope for governments to countervail sovereign spread shocks on their own, but also to grant wider support for the implementation of the fiscal transfer scheme.

Chapter 2:

Exogenous Shocks and Endogenous States

This paper lays out an empirical model to compare the propagation of exogenous shocks in a small open economy subject to structural change. The model allows me to disentangle the variation in the propagation of shocks that is due to exogenous factors from that due to endogenous structural change.

The econometric framework has three main features: First, the model is composed of two blocks, with one block begin exogenous with respect to the other. Second, the endogenous block is assumed to follow a Markov-switching process evolving

according to a latent variable dictating structural changes. Third, the structural shocks are identified through sign-restrictions on the responses of the exogenous block. As the identification of structural shocks is time invariant, I am able to single out specific shocks and compare their transmission between different dates. Importantly, in doing so I am able to investigate the differences that pertain only to structural change.

As an illustration, I estimate the model to Australia. A small commodity-exporting economy, Australia is highly exposed to commodity prices. I therefore identify three distinct commodity price shocks based on their underlying effects on the country's terms of trade: a world demand shock, a commodity-market shock, and a globalization shock. With regards to structural change, I find a once-and-for-all regime transition occurring in 1990. The transition appears to match relatively well the consequences of major institutional reforms in Australia, such as the float of the exchange rate, the decentralisation of the wage-setting system, and inflation targeting.

Comparing the impulse responses of real GDP growth, inflation, and changes to the nominal exchange rate, I find that structural change alters the way the shocks propagate. The responses of these variables to the three shocks are less exacerbated and adjust quicker after the transition. This is generally in line with the economic predictions associated with the interpretation of the two regimes. Notably, the changes in the responses of real GDP growth and inflation are consistent with the implementation of labour market reforms and with the move to inflation targeting.

I find that increases in commodity prices are only inflationary after 1990 and as a reaction to world demand shocks. However, as these shocks are not always associated with increases in the terms of trade, monetary policy may have been failing to address them properly. In fact, and as world demand shocks represent the largest share of terms-of-trade shocks to the Australian economy, these results show that the terms of trade are not a sufficient statistic for foreign conditions and therefore are not enough to guide policy.

In regard to commodity-market shocks, there also seems to be scope for improvement in terms of the policy response. Despite improving the terms of trade, I find evidence that these shocks are recessionary, with the negative effects on growth observed in both regimes. Nevertheless, as commodity-market shocks have no impact on inflation after the structural change, some policy loosening might be warranted.

Chapter 3:

Monetary Policy and Sectoral Relative Wage Rigidity

In the last paper, I study the implications of relative wage rigidities for the conduct of monetary policy in a small commodity-exporting economy. I propose a two-sector model of an open economy subject to commodity price shocks. The two sectors, consisting of a non-commodity and a commodity sector, use labour as input for production and are exposed to foreign demand. Labour markets are assumed to be segmented, with every unemployed household member directing his job-search to a specific sector. New hires occur according to a matching function, which depends on the number of job-seekers and of vacancies posted is each sector. Nominal wages in the non-commodity sector are assumed to be sticky and can be indexed to the evolution of wages in the commodity sector. This link creates sectoral relative wage rigidity.

I analyse the propagation of a positive commodity price shock to shed light on the mechanisms behind relative wage rigidity. The shock causes an appreciation of the exchange rate and leads to the contraction of the non-commodity sector. In order to attract workers, commodity firms raise wages, shifting job-seekers away from the non-commodity sector. As demand for non-commodity goods is weaker, firms find it difficult to retain their workers. If relative wages are rigid, non-commodity output drops further. As non-commodity firms expect wages to increase following the expansion of the commodity sector, they reduce vacancies more. As a result, total output drops, while unemployment and inflation increase.

I calibrate the model to Australia and use GMM to match the second moments of a set of relevant variables to their empirical counterpart. I find that strict inflation targeting is detrimental to welfare compared to a simple feedback rule, with the relative welfare losses being larger the higher the degree of indexation. However, I also find that optimal monetary policy is more responsive to inflation the higher the relative wage rigidity. To unveil this puzzle, I use different loss functions to describe the preferences of the monetary authority. I find that, for a higher degree of indexation, some inflation is desirable in order to smooth real wage volatility. As wages are set in nominal terms, inflation partially offsets the effects of indexation on real wages. Nevertheless, as indexation itself is inflationary, the response of monetary policy to inflation needs to be stronger.

I show that policy misspecification is more problematic for higher degrees of relat-

ive wage rigidity. Additionally, the optimal policy prescription is highly sensitive to the type of loss function the monetary authority chooses to follow. For instance, if monetary policy cares about inflation and non-commodity output volatilities, response to inflation must be stronger the more rigid relative wages are. If policy care about inflation and unemployment instead, the optimal response to inflation becomes milder for higher degrees of indexation. All things considered, although monetary policy is found to be an effective macroeconomic policy instrument, its response needs to be calibrated carefully.

Chapter 1

Fiscal Transfers in a Monetary Union with Sovereign Risk

Keywords: sovereign risk, banks, monetary union, fiscal transfers.
JEL Classification: E62, F15, F41, F42, F45.

Firenze, December 2015

1.1 Introduction

The debate over the architecture of a robust monetary union between countries attracted renewed interest during the recent sovereign debt crisis in Europe. The asymmetrical nature of sovereign interest rate shocks, coupled with the inherent constraints they pose on domestic fiscal policy, exposed a painful fault in the design of the European Monetary Union (EMU). This fault concerns the lack of adequate risk sharing mechanisms to facilitate the economic adjustments of individual member states facing idiosyncratic shocks. As seen during the crisis, soaring sovereign spreads forced a number of countries, including Greece, Ireland,

Portugal, Italy and Spain, to undertake sudden fiscal consolidation while implementing deep structural reforms. For the first three cases, the extent of the crisis required them to resort to institutional rescue programmes put in place by the International Monetary Fund (IMF) and the European Institutions. The dramatic economic toll of the crisis and the dubious response from within the EMU called into question the reversibility of the common currency.

In this paper I propose a two-country model of a monetary union where sovereign spreads affect private borrowing costs due to financial frictions. My contribution is twofold. First, I provide a consistent narrative of the events during the sovereign debt crisis, illustrating how domestic fiscal policy is constrained by the responsiveness of sovereign spreads to the fiscal conditions and by the ratio of public debt to GDP. Second, I show that a simple fiscal transfer scheme between governments is an efficient buffer to sovereign spread shocks and discuss the conditions under which such a scheme can be implemented.

During the sovereign debt crisis, banks were pivotal in passing the rise in sovereign spreads to the real economy. The fall in government bond prices and the downgrading of these assets by credit rating agencies severely weakened banks' balance sheets. As a consequence of their direct exposure to sovereign credit risk, banks' ability to raise market-based funding was adversely affected. The increase in funding costs forced them to strengthen their equity ratios and to sharply reduce overall credit provision to firms, which ultimately ignited the recession.

I capture the role of banks during the crisis by introducing a banking sector similar to that proposed in Gertler and Karadi (2011) into a two-country general equilibrium model of a monetary union. Banks serve as financial intermediaries between households, from which they take short-term deposits, and firms, to which they make long-term loans. Due to agency problems between banks and their depositors, banks are forced to moderate their leverage in order to attract deposits from households. I extend the banking sector by assuming that banks also lend to the government. In good times, the sovereign obtains funds at the risk-free interest rate. However, a spread can arise on top of the risk-free rate reflecting the credit worthiness of the government. Because banks hold sovereign bonds in their portfolios, their net worth is exposed to sovereign credit risk. Therefore, a shock to sovereign spreads deteriorates the equity value of banks and forces them to contract credit supply and to raise lending rates at the same time as they retain funds to build up the value of their net worth.

In the model, when the ratio of public debt to GDP is calibrated to 60%, I find

that a 10% increase in sovereign spreads leads to an increase of about 3.5% in the borrowing costs for firms. However, when the ratio of public debt to GDP equals 120%, the increase in private spreads is more than three quarters of the initial rise in sovereign spreads. The drop in the supply of credit to firms and the increase in borrowing costs adversely impacts investment and ignites the recession. At the trough, real GDP falls between 2% and nearly 6%, depending on the size of the ratio of public debt to GDP. These effects are magnified when sovereign spreads respond to the fiscal outlook. After the initial shock, an increase in the public deficit feeds back to sovereign spreads and further increases firms' borrowing rates. The size of the feedback loop also has implications for fiscal policy. For instance, for a public debt to GDP ratio of 120%, it is impracticable for the government to engage in counter-cyclical fiscal policy as it is forced to consolidate in order to stabilize public debt and to prevent sovereign spreads from rising further. As seen in the periphery of the EMU during the crisis, when sovereign spreads are sensitive to the fiscal outlook, there is no leeway for the government to provide a stimulus to the economy in order to countervail the recession.

The idea that the EMU should be completed with a federal fiscal arrangement is hardly a novelty in policy and academic circles. When its design was being discussed, it was clear that a system of fiscal transfers crafted to countervail idiosyncratic shocks would be crucial for the success of the single currency.[1] The argument behind a federal-like transfer mechanism drew directly on the literature of optimal currency areas.[2] With the creation of the EMU, member-states would no longer be able to use monetary policy or the exchange rate to buffer country-specific shocks. Moreover, to the extent that production factors are immobile across countries and movements in nominal prices and wages are slow, fiscal policy would become a key instrument to fuel the necessary adjustments. With this in view, the Maastricht Treaty incorporated limits on budget deficits and public debts in order to preserve sound domestic fiscal stances capable of reacting if required.

Yet, the political process aimed at endowing the EMU with an area-wide fiscal capacity lay dormant for decades until the sovereign debt crisis when domestic fiscal policies failed to operate the required adjustments. In response the severe consequences left by the crisis and the inability of the EMU to respond adequately

[1] See, for instance, the MacDougall report (European Commission, 1977) as well as the Delors (1989) report.

[2] See the seminal articles by Mundell (1961), McKinnon (1963), and Kenen (1969).

and promptly, the leaders of the European Institutions drew up a road map to create an area-wide fiscal stabilization capacity. The proposed mechanism, to be implemented before 2025, would be deployed when domestic fiscal policy cannot, on its own, counteract large asymmetric shocks.[3] In this paper, I examine the design and implementation of such a federal fiscal capacity. I investigate the extent to which fiscal transfers can effectively smooth the effects of sovereign spread shocks by considering a mechanism that affects the fiscal stance of the country. Because sovereign spreads constrain domestic fiscal policy, foreign transfers can step in and both support economic activity and mitigate the fiscal burden.

I use the model to quantify the effects on welfare of a cross-country fiscal transfer scheme that is actioned in response to a widening in sovereign spreads. I first show that in a monetary union with equal-sized regions, there are unambiguous welfare gains from implementing the scheme. Because transfers are processed between governments, they alleviate the fiscal burden directly. The scheme proves to be particularly important in bad times when the public debt to GDP ratio is high and sovereign spreads are highly responsive to the fiscal outlook. However, the distribution of welfare gains is very sensitive to asymmetries between the two regions. Notably, I find that, in order to obtain positive welfare gains for all regions, the minimum relative size for the smaller region is still higher than 48% of the entire union. This is an important challenge for the implementation of the scheme: if some countries incur welfare costs, they will likely not participate.

In order to provide a relevant and representative case study, I estimate the parameters of the model for Portugal and the Eurozone. I limit the set of schemes I consider to those under which potential welfare costs cannot exceed the welfare benefits generated by the introduction of the single currency. In other words, the alternative scenario to the transfers countries can compare to is the *status quo* pre EMU. Considering the impact of the scheme in isolation, I show that Portugal can secure welfare gains in the range of $1.44 - 7.80\%$ of lifetime consumption, while the Eurozone incurs welfare losses of $0.03 - 0.15\%$. Because the scheme is designed in a way in which it excludes net losses from entering the EMU, these results render strong support for its implementation. Regarding the role of the transfers in mitigating the real effects of sovereign spread shocks, I show that for a level of transfers that reduces the pass-through of sovereign spreads in about 1/2

[3] The 5 Presidents Report (Juncker et al., 2015) is the last high level policy contribution. It draws on and updates earlier proposals, namely by Van Rompuy et al. (2012). See also IMF (2013) for discussion.

percentage points, the trough of the recession is reduced by at least 1%. In bad times, the effects generated by the fiscal transfer scheme are considerably larger and, therefore, the dimension of the recession can be effectively reduced.

Literature: This paper is related to two strands of the literature. On the one hand, it relates to a number of papers investigating the implications of sovereign spreads for economic stability. Schabert and van Wijnbergen (2011) and Bonam and Lukkezen (2013), for instance, focus on the interactions between fiscal, monetary, and exchange rate policies, in an environment where sovereign spreads are introduced as a preemptive game between the government and speculators. The parsimonious way they model sovereign spreads is also used in the present paper. Corsetti et al. (2012), who study how the sovereign risk channel exacerbates cyclical shocks in an environment where monetary policy can be constrained at the zero-lower bound, analyse the effects of fiscal retrenchment in alleviating macroeconomic fluctuations. Bocola (2013) and Pancrazi et al. (2014) also investigate the pass-through of sovereign risk to private borrowing costs and evaluate the effectiveness of asset purchases by the central bank in stabilizing real activity. Kollmann et al. (2013) introduce a banking sector with capital requirements into an open economy model and investigate whether government provision of support to banks can stabilize the economy. The present paper draws on this literature of the pass-through of sovereign risk, but diverts from it by focusing on the implications it has on fiscal policy itself and by considering instead cross-country fiscal transfers as a means to smooth shocks.

On the other hand, this paper contributes to the literature on federal fiscal arrangements within monetary union. There is a growing literature on optimal policy and international coordination using domestic fiscal instruments for countries sharing a common currency.[4] However, less attention has been given to federal fiscal schemes. Among the exceptions, Farhi and Werning (2012) show that fiscal transfers can improve risk sharing in an environment with complete asset markets. Costain and de Blas (2012) compare fiscal policy rules that stabilize public debt through either income taxation or spending on wages and unemployment benefits and find that a policy of pro-cyclical spending on wages and transfers decided by a federal agency brings the market economy closer to the planner's solution. Kletzer and von Hagen (2000), Evers (2012) and Kim and Kim (2013) investigate different federal transfers schemes and their potential to achieve welfare gains for members of a monetary union. I expand this literature by focusing

[4] Evers (2012) and Pappa and Vassilatos (2007) provide references.

on asymmetric shocks that not only cause real fluctuations, but also constrain domestic fiscal policy. Besides presenting an actual scenario where federal fiscal arrangements can act as a stabilization mechanism, this paper adds to the literature by quantifying and discussing the welfare trade-off such policies entail in a realistic set-up.

The remainder of the paper is structured as follows. In the next section I describe the model and motivate the main extensions I have introduced. Section 3 discusses the estimation strategy and the results I obtain. I dedicate section 4 to the policy analysis. I begin by investigating the dynamic effects of sovereign spread shocks in a baseline scenario. I then discuss how domestic fiscal policy is constrained by the fiscal stance and by the behaviour of sovereign spreads. Finally, I investigate the welfare consequences of the proposed fiscal transfers scheme and discuss its dynamic impacts during episodes of sudden increases in sovereign spread. The last section concludes.

1.2 Model

In this section I lay out a general equilibrium model of a monetary union. The union is composed of two small open economies with habits in consumption, sticky prices and wages, financial frictions, and investment adjustment costs. The model presented here is an extension of the one used by Lama and Rabanal (2014). The two countries, which I call home and foreign, are of sizes n and $1-n$, respectively. Households in each country deposit their savings in domestic banks and provide labour to domestic producer firms. Households in one country can also trade bonds with households in the other country, having, however, to account for the real exchange rate. Banks serve as intermediaries between households and borrowers. They sell long-term loans to wholesale firms and to the government. Each country produces a continuum of tradeable intermediate goods that are aggregated into a final non-tradeable good. The latter is consumed by households, the government, and used for investment. Governments can raise taxes and issue long-term bonds to finance public expenditure, while the area-wide central bank sets the nominal interest rate according to a feed-back rule targeting aggregate inflation and output growth.

The following subsections describe the home economy in more detail. The description of the foreign economy is omitted for brevity since its structure is analogous to the home country, except for the government which is assumed to run zero fiscal

deficits every period.[5] All variables are in *per capita* terms, the conventional $\star$ denotes foreign variables or parameters, and the subscript h (f) denotes goods produced in the home (foreign) country and respective prices.

1.2.1 Households

There is a continuum of infinitely lived households and within each household there are two types of members: a fraction $1-f$ are workers and a fraction f are bankers. The former supply labour to non-financial firms and receive wages, while the latter manage a financial intermediary for profits. Household members switch between the two occupations but keep the relative proportion of each type constant. Hence, with probability λ_f a banker remains active in the following period, which implies that each period a fraction $(1-\lambda_f)\,f$ bankers retire and become workers. Conversely, each period the same number of workers randomly become bankers. Bankers' limited tenure avoids overaccumulation of retained earnings and ensures the financial frictions remain operative, as explained below.

Household members are assumed to pool consumption risk perfectly. Their lifetime utility is given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}, l_{i,t}\right) \qquad \text{for } i \in [0, n]$$

with

$$u\left(c_{i,t}, l_{i,t}\right) \;=\; log\left(c_{i,t} - \varrho c_{t-1}\right) - \zeta \frac{\left(l_{i,t}\right)^{1+\varphi}}{1+\varphi}$$

and where E_0 denotes the rational expectations operator conditional on the information available up to $t = 0$ and $\beta \in (0,1)$ is the household's subjective discount factor. Households derive utility from consumption, which is subject to external habit formation $\varrho \in (0,1)$, and disutility from labour, where $\varphi > 0$ is the inverse elasticity of labour supply and $\zeta > 0$ its relative weight. The consumption good is an aggregate good composed of domestic and foreign intermediate goods, as explained below. Households can deposit their savings with domestic banks and can trade foreign bonds in international financial markets. The budget

[5] Without loss of generality, I impose zero fiscal deficits for the foreign economy for simplicity. When I compare two symmetric regions in section 4, I mean total symmetry, that is, the foreign government is also allowed to issue debt.

constraint of home households in real terms is given by

$$(1+\tau_t^c)\, c_{i,t} + b_{i,b,t} + e_t r_{f,t-1} b_{i,f,t-1} \quad \leq \quad w_{i,t} l_{i,t} + r_{h,t-1} b_{i,b,t-1} + e_t b_{i,f,t} + V_t - T_t \tag{1}$$

where $b_{i,b}$ denotes deposits with domestic banks which pay the real interest rate $r_{h,t-1}$, and $b_{i,f}$ denotes bonds traded with households abroad and which pay the real interest rate $r_{f,t-1}$. For ease of exposition, the budget constraint is written such that $b_{i,b} > 0$ implies positive savings from the households, while $b_{i,f} > 0$ implies that the household is a net borrower in international markets. As a consequence of being in a monetary union, the nominal exchange rate between the two countries is fixed and therefore the real exchange rate, e_t, is simply equal to the ratio of consumer prices in both countries. Households receive labour income at the real wage rate $w_{i,t}$ and real profits from firms denoted by V_t. Finally, they pay lump-sum and distortionary taxes, T_t and $\tau_t^c c_{i,t}$ respectively, to the government.

The first-order conditions for consumption and for financial asset holdings are

$$\varsigma_t \quad = \quad \frac{1}{(1+\tau_t^c)} \frac{1}{c_t - \varrho c_{t-1}} \tag{2}$$

$$1 \quad = \quad \beta \Lambda_{t,t+1} r_{h,t} \tag{3}$$

$$1 \quad = \quad \beta \Lambda_{t,t+1} \frac{e_{t+1}}{e_t} r_{f,t} \tag{4}$$

where $\Lambda_{t,t+1} = \varsigma_{t+1}/\varsigma_t$ is the ratio of marginal utilities of consumption between t and $t+1$, and ς_t is the multiplier on the budget constraint.

I introduce nominal rigidities in wages as in Erceg et al. (2000) by assuming that households are monopolistic suppliers of differentiated labour services. As such, each household has market power to negotiate wages with intermediate good producers. In turn, intermediate good producers use a composite labour input in production, l_t, which they obtain by aggregating differentiated labour services according to

$$l_t \quad = \quad \left(\int_0^n (l_{i,t})^{\frac{\tau_w - 1}{\tau_w}} di \right)^{\frac{\tau_w}{\tau_w - 1}}$$

The demand curve for labour services from household i is thus given by

$$l_{i,t} = \left(\frac{w_{i,t}}{w_t}\right)^{-\mu_w} l_t \tag{5}$$

where $w_{i,t}$ is the real wage household i charges in order to supply $l_{i,t}$, and $w_t = \left(\int_0^1 (w_{i,t})^{1-\tau_w} di\right)^{1/(1-\tau_w)}$ is the real price index of the composite labour input. The elasticity of substitution between labour services supplied by different households is given by μ_w.

In each period, only a fraction $1-\lambda_w$ of households can re-optimize their posted nominal wage. When able to adjust its wage, household i solves

$$\underset{w_{i,t}}{\text{Max}} \ \text{E}_t \sum_{s=0}^{\infty} (\beta\lambda_w)^s \left[log\left(c_{i,t+s} - \varrho c_{t+s-1}\right) - \zeta \frac{\left(l_{i,t+s|t}\right)^{1+\varphi}}{1+\varphi} \right]$$

subject to the respective demand curve for labour services and the budget constraint. The first-order condition with respect to the optimal nominal wage w_t^* is given by

$$\text{E}_t \sum_{s=0}^{\infty} (\beta\lambda_w)^s \varsigma_{t+s} l_{i,t+s|t} \left[\frac{w_t^*}{P_{t+s}} - \frac{\tau_w}{\tau_w - 1} \frac{\zeta \left(l_{i,t+s|t}\right)^{\varphi}}{\varsigma_{t+s}} \right] = 0 \tag{6}$$

where ς_t is the multiplier on the budget constraint and $l_{i,t+s|t} == (w_t^*/w_{t+s})^{-\tau_w} l_{t+s}$ is the labour supplied in period $t+s$ for those households that last negotiated their nominal wage at t.

1.2.2 Banks

As described earlier, every period a fraction f of household members are bankers who run a domestic financial intermediary. I extend the banking sector described in Gertler and Karadi (2011) by allowing banks to provide funds to the government. Hence, banks raise deposits from domestic households and lend to domestic non-financial firms and to the government. As in Lama and Rabanal (2014), bankers do not engage in cross-border deposits or investment activities.[6] I also assume that the domestic banking sector holds the total amount of public debt issued by the government.

[6]Dedola et al. (2013) extend the framework of Gertler and Karadi (2011) to allow banks to take deposits from foreign households and to lend to foreign firms.

I motivate these two assumptions with the following stylized facts. In 2011, around 80% of the sovereign debt claims on countries in the periphery of the Eurozone was held in the balance sheets of national banks. In these same countries, domestic government bond holdings accounted for 93% of bank's equity. On the other hand, domestic banks represented roughly 75% of external financing to private firms. As a result, from 2008 to 2013, the lending volume of newly issued loans fell by more than 50% in the periphery of the EMU.[7]

Therefore, each period a continuum of banks indexed by $i \in [0, f]$ obtain deposits $b_{i,b,t}$ from households and lend funds to wholesale producers and to the government, $a_{i,x,t}$ and $a_{i,b,t}$ respectively. Denoting by $n_{i,t}$ the net worth of financial intermediary i and by $\mathcal{W}_{i,t}$ the total value of its assets, the balance sheet of bank i is then given by

$$\mathcal{W}_{i,t} = q_{x,t}a_{i,x,t} + q_{b,t}a_{i,b,t} \quad = \quad n_{i,t} + b_{i,b,t} \tag{7}$$

where $q_{j,t}$ is the relative price of claims $a_{i,j,t}$. The cost of deposits is given by the interest rate $r_{h,t}$, whereas banks require a return of $r_{x,t}$ on the loans they make to firms. The interest rate on government bonds, $r_{b,t}$, is assumed to equal the risk-free rate adjusted by a default risk premium δ_t. Expanding (7) forward, I obtain the evolution of equity capital as the difference between earnings on assets and interest payments on liabilities

$$\begin{aligned} n_{i,t} \quad = \quad & (r_{x,t-1} - r_{h,t-1})\, q_{x,t-1}a_{i,x,t-1} + ((1-\delta_t)\, r_{b,t-1} - r_{h,t-1})\, q_{b,t-1}a_{i,b,t-1} \\ & + r_{h,t-1}n_{i,t-1} \end{aligned} \tag{8}$$

Growth in equity above the risk-free return $r_{h,t}$ depends on the premium $(r_{x,t} - r_{h,t})$ earned on the loans to firms and on the return on sovereign debt.

The objective of bankers is to maximize their expected terminal net worth

$$\mathcal{N}_{i,t} \quad = \quad \mathrm{E}_0 \sum_{s=0}^{\infty} (1-\lambda_f)\, \lambda_f^s \beta^{s+1} \Lambda_{t,t+1+s} n_{i,t+1+s} \tag{9}$$

To the extent that the expected discounted returns on his assets are higher than the risk-free rate, the banker will want to raise deposits and build its net-worth indefinitely. Gertler and Karadi (2011) introduce a moral hazard problem in

[7] The figures were taken from Uhlig (2013), Acharya et al. (2014), and Bocola (2013). A report by the Bank for International Settlements, BIS(2011), provides a comprehensive discussion on the link between sovereign credit risk and banks funding conditions.

order to limit overaccumulation of retained earnings by assuming that at any given period bankers can divert a fraction ι of available assets. Having knowledge of this, depositors can force the bank into bankruptcy, but can only recover the remaining $1-\iota$ of funds. Hence, depositors will only supply funds to the bank if the following incentive-compatibility constraint is satisfied

$$\mathcal{N}_{i,t} \geq \iota \mathcal{W}_{i,t} \tag{10}$$

that is, the value of carrying on doing business must be higher than the value of diverting funds. Absent financial frictions, the risk premium on non-financial firms would be zero. With imperfect capital markets, however, the premium may be positive due to constraints on the ability of banks to raise external funds.

I solve the banker's problem by defining the leverage ratio of financial intermediaries, $\phi_{i,t}$, as

$$\mathcal{W}_{i,t} = \phi_{i,t} n_{i,t} \tag{11}$$

and by making an educated guess over the functional form of bankers' net worth. In particular, I guess that $\mathcal{N}_{i,t} = \nu_t \mathcal{W}_{i,t} + \eta_t n_{i,t}$, where ν_t is the marginal value of expanding assets, holding $n_{i,t}$ constant, and η_t is the marginal value of the bank's net worth, holding its portfolio $\mathcal{W}_{i,t}$ constant. The expressions for ν_t and η_t are given by

$$\eta_t = \mathrm{E}_0 \Omega_{t,t+1} r_{h,t} \tag{12}$$

$$\nu_t = \Omega_{t,t+1} \left((r_{x,t} - r_{h,t}) - (r_{x,t} - r_{b,t} (1 - \delta_{t+1})) \alpha_t^{\mathcal{W}} \right) \tag{13}$$

where $\alpha_t^{\mathcal{W}} = q_{b,t} a_{i,b,t} / \mathcal{W}_{i,t}$ is the share of government debt in the bank's portfolio. $\Omega_{t,t+1}$ is the banker effective discount factor which is given by

$$\Omega_{t,t+1} = \beta \Lambda_{t,t+1} \left\{ 1 + \theta \left[\eta_{t+1} + \nu_{t+1} \phi_{i,t+1} - 1 \right] \right\} \tag{14}$$

The effective discount rate of bankers differs from that of the households due to the financial friction.

As Gertler and Karadi (2011) show, when (10) binds the leverage ratio is common

to all bankers and equal to

$$\phi_t = \frac{\eta_t}{\iota - \nu_t} \tag{15}$$

That is, the amount of funds banks can intermediate is limited by their net worth due to the borrowing constraint. For positive values of net worth, the constraint binds only if $0 < \nu_t < \iota$. With $\nu_t > 0$, it is profitable to expand $\mathcal{W}_{i,t}$. However, if $\nu_t > \iota$, the incentive constraint does not bind since the value from intermediation exceeds the gain from diverting funds. In the equilibria studied below, the incentive-compatibility constraint always binds within a neighbourhood of the steady state.

Finally, aggregate net worth in any given period is the sum of the net worth of existing banks plus the start-up funds of entering banks. Surviving banks carry their total net-worth into the next period, whereas new banks receive a fraction $\epsilon/(1-\lambda_f)$ of the assets of exiting banks in order to start business. Aggregate net worth is then given by

$$\begin{aligned} n_t &= \lambda_f \left\{ \left[(r_{x,t-1} - r_{h,t-1}) - (r_{x,t-1} - r_{b,t-1}(1-\delta_t)) \alpha^{\mathcal{W}}_{t-1} \right] \phi_{t-1} + r_{h,t-1} \right\} n_{t-1} \\ &\quad + \epsilon \left\{ q_{x,t} a_{x,i,t-1} + q_{b,t} \delta_t a_{b,i,t-1} \right\} \end{aligned} \tag{16}$$

In the set up just presented, the share of government bonds in the balance sheets of banks, $\alpha^{\mathcal{W}}_t$, is not an optimizing variable for bankers. I assume instead that the banking sector provides funds to the government as the latter requires each period, without entering into optimal portfolio choices.[8] The appeal of this approach is that it gives me the flexibility to introduce sovereign risk in a transparent and parsimonious way. In particular, because I model sovereign default risk as a preemptive game between the government and speculators, the pricing of government bonds is not pinned down by banks.[9] Hence, government bonds are priced

[8]For some references, Devereux and Sutherland (2007) describe how to implement optimal portfolio choice in an open economy setting, while Dedola et al. (2013) apply this method to their model of banks with cross-border linkages. Kollmann et al. (2013) assume that banks bear real costs on government and private bond holdings in order to pin down their portfolio composition.

[9]Bocola (2013) develops a model similar to mine where the government can actually default on its debt, generating a pass-through of sovereign risk to private borrowing rates. The strategic default literature is growing rapidly after the seminal work by Eaton and Gersovitz (1981), and includes Aguiar and Gopinath (2006), Arellano (2008), Cuadra and Sapriza (2008), among many others. Two recent papers that expand this literature by including a banking sector are Gennaioli et al. (2013) and Sosa Padilla (2014).

according to

$$r_{h,t} = \mathrm{E}_t r_{b,t} \left(1 - \delta_{t+1}\right) \tag{17}$$

that is, the sovereign interest rate is equal to the risk-free rate adjusted by the default risk premium, which I describe shortly.

1.2.3 Production

Capital producers: At the end of each period, perfectly competitive capital producers buy undepreciated capital from wholesale firms and repair it. At the same time, they also invest in new capital by purchasing and transforming domestic final goods. The repaired and newly created capital is then sold to wholesalers as an input to production. The discounted real profits of capital producers, Π_{CP}, are given by

$$\underset{z_t}{\mathrm{Max}} \;\; \mathrm{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \Lambda_{t,t+s} \left\{ q_{x,t+s} \left(k_{t+s} - \left(1 - \sigma\left(u_t\right)\right) k_{t-1+s} \right) - z_{t+s} \right\}$$

where $q_{x,t}$ is the value of one unit of new capital, z_t denotes the amount of final goods that is invested to generate new capital, and $\sigma\left(u_t\right)$ denotes the rate of capital depreciation, which depends on capital utilization.

Capital producers are assumed to incur adjustment costs when investing in new capital. The law of motion of capital is thus given by

$$k_t = \left[1 - \frac{\psi}{2} \left(\frac{z_t}{z_{t-1}} - 1 \right)^2 \right] z_t + \left(1 - \sigma\left(u_t\right)\right) k_{t-1} \tag{18}$$

whit ψ governing investment adjustment costs. Substituting (18) in the objective function of capital producers, the optimal level of investment is given by

$$\begin{aligned} 1 = \; & q_{x,t} \left(1 - \frac{\psi}{2} \left(\frac{z_t}{z_{t-1}} - 1 \right)^2 - \psi \left(\frac{z_t}{z_{t-1}} - 1 \right) \frac{z_t}{z_{t-1}} \right) \\ & + \beta \Lambda_{t,t+1} q_{x,t+1} \psi \left(\frac{z_{t+1}}{z_t} - 1 \right) \frac{z_{t+1}^2}{z_t^2} \end{aligned} \tag{19}$$

Wholesale firms: Perfectly competitive wholesale firms use the composite labour input and capital in order to produce a homogeneous good. They purchase

capital from capital producers at the real price $q_{x,t}$, and finance their capital acquisition by borrowing from domestic banks. Banks thus need to issue claims $a_{x,t}$ equal to the number of units of capital acquired k_t, pricing each claim at the price of a unit of capital. After production, wholesalers sell their capital to capital producers and pay the return $r_{x,t}$ over their loans. The homogeneous good is sold to domestic retailers at the real price $p_{x,t}$.

The production function of wholesale firms is given by

$$x_t = \xi_t^s \left(u_t k_{t-1}\right)^{\alpha} \left(\xi_t^u l_t\right)^{1-\alpha} \tag{20}$$

where ξ_t^s is the total factor productivity at home, ξ_t^u a drifting labour-augmenting technology common to both countries and α is the weight of capital in production. Following the discussion in Albonico et al. (2014), I allow wholesalers to vary the effective rate of capital utilization in production, u_t. However, a higher effective use of capital increases its depreciation rate, as I assume that $\sigma'(u_t) \geq 0$. The optimal utilization rate of capital satisfies

$$p_{x,t}\alpha\frac{x_t}{u_t} = \sigma'(u_t)\, k_t \tag{21}$$

whereas the demand curve for composite labour services can be expressed as

$$w_t = p_{x,t}(1-\alpha)\frac{x_t}{l_t} \tag{22}$$

Perfect competition imposes zero profits and therefore the ex-post real return paid to banks is given by

$$r_{x,t-1} = \frac{p_{x,t}\alpha x_t/k_{t-1} + q_{x,t}\left(1-\sigma(u_t)\right)}{q_{x,t-1}} \tag{23}$$

Retail firms: A continuum of retail firms indexed by $i \in [0, n]$ purchase the homogeneous good produced by wholesalers at the price $p_{x,t}$ and differentiate it into a continuum of domestic and foreign retail goods. Retailers follow a type of local currency pricing, so that prices vary depending on the destination market. The differentiated goods they produce are sold to final good firms at home and abroad at the price $p_{i,h,t}$ and $p^{\star}_{i,h,t}$, respectively. Hence, retailer i faces two demand

curves

$$y_{i,h,t} = \left(\frac{p_{i,h,t}}{p_{h,t}}\right)^{-\mu_p} y_{h,t} \quad \text{and} \quad y^{\star}_{i,h,t} = \left(\frac{p^{\star}_{i,h,t}}{p^{\star}_{h,t}}\right)^{-\mu^{\star}_p} y^{\star}_{h,t} \tag{24}$$

from home and foreign final good producers, respectively. Retail firms are subject to Calvo price stickiness. Every period, a retailer is able to adjust prices in both markets with probability $1-\lambda_p$. When retail firms do not reoptimize prices, they simply update them to lagged inflation in the destination market. Retail prices follow

$$\begin{aligned} p_{i,h,t+s} &= \begin{cases} p^{*}_{i,h,t+s} & \text{with prob. } 1-\lambda_p \\ p^{*}_{i,h,t}\left(\Pi^{s}_{k=1}\pi_{h,t+k-1}\right)^{\vartheta} & \text{with prob. } \lambda_p \end{cases} \\ p^{\star}_{i,h,t+s} &= \begin{cases} p^{**}_{i,h,t+s} & \text{with prob. } 1-\lambda_p \\ p^{**}_{i,h,t}\left(\Pi^{s}_{k=1}\pi^{\star}_{h,t+k-1}\right)^{\vartheta} & \text{with prob. } \lambda_p \end{cases} \end{aligned} \tag{25}$$

where indexation is governed by $\vartheta \in [0,1]$, which measures the extent to which prices fully adjust to past inflation. When allowed to adjust prices, retailer i maximizes the stream of real discounted profits, $\Pi_{\mathrm{R}}(i)$, given by

$$\underset{p_{i,h,t}\, p^{\star}_{i,h,t}}{\text{Max}} \; \mathrm{E}_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \Lambda_{t,t+s} \left\{ \left[\frac{p_{i,h,t}}{p_{t+s}} - \frac{p_{x,t+s}}{p_{t+s}}\right] y_{i,h,t+s} + \left[\frac{e_{t+s}p^{\star}_{i,h,t}}{p_{t+s}} - \frac{p_{x,t+s}}{p_{t+s}}\right] y^{\star}_{i,h,t+s} \right\}$$

subject to (24) and (25). Due to differences in consumer price inflation at home and abroad, the price of retail goods sold to foreigners needs to be adjusted by the real exchange rate e_t. The numeraire p_t is the consumer price index. Solving for the optimal prices retailer i quotes in the two markets yields

$$\frac{p^{*}_{i,h,t}}{p_{h,t}} = \frac{\mu_p}{\mu_p - 1} \frac{\mathrm{E}_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \Lambda_{t,t+s} y_{h,t+s} \frac{p_{x,t+s}}{p_{t+s}} \left(\frac{p_{h,t}}{p_{h,t+s}} \left(\Pi^{s}_{k=1}\pi_{h,t+k-1}\right)^{\vartheta}\right)^{-\mu_p}}{\mathrm{E}_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \Lambda_{t,t+s} y_{h,t+s} \frac{p_{h,t+s}}{p_{t+s}} \left(\frac{p_{h,t}}{p_{h,t+s}}\right)^{1-\mu_p} \left(\Pi^{s}_{k=1}\pi_{h,t+k-1}\right)^{\vartheta(1-\mu_p)}} \tag{26}$$

and

$$\frac{p^{\star\star}_{i,h,t}}{p^{*}_{h,t}} = \frac{\mu^{\star}_{p}}{\mu^{\star}_{p}-1}\frac{\mathrm{E}_t\sum_{s=0}^{\infty}(\beta\lambda_p)^s\Lambda_{t,t+s}y^{\star}_{h,t+s}\frac{p_{x,t+s}}{p_{t+s}}\left(\frac{p^{\star}_{h,t}}{p^{\star}_{h,t+s}}\left(\Pi_{k=1}^{s}\pi^{\star}_{h,t+k-1}\right)^{\vartheta}\right)^{-\mu^{\star}_{p}}}{\mathrm{E}_t\sum_{s=0}^{\infty}(\beta\lambda_p)^s\Lambda_{t,t+s}y^{\star}_{h,t+s}\frac{e_{t+s}p^{\star}_{h,t+s}}{p_{t+s}}\left(\frac{p^{\star}_{h,t}}{p^{\star}_{h,t+s}}\right)^{1-\mu^{\star}_{p}}\left(\Pi_{k=1}^{s}\pi^{\star}_{h,t+k-1}\right)^{\vartheta\left(1-\mu^{\star}_{p}\right)}} \tag{27}$$

Although the elasticities of substitution between retail goods consumed domestically and exported, μ and $\mu^{\star}$, can vary, the parameters reflecting the degree of nominal rigidity λ_p and φ are common to domestic and export inflation.

Final good producers: Perfectly competitive firms produce a non-tradeable final good by aggregating a continuum of domestic and foreign intermediate goods. The aggregation technology for the final good is given by

$$y_t = \left[(\varpi)^{\frac{1}{\gamma}}(y_{h,t})^{\frac{\gamma-1}{\gamma}} + (1-\varpi)^{\frac{1}{\gamma}}(\overline{\tau}y_{f,t})^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} \tag{28}$$

where $\overline{\tau} \equiv (1-n)/n$ normalizes the amount of imported goods into *per capita* terms. In the above CES aggregator, the home-bias parameter ϖ denotes the fraction of goods produced at home that is used in the production of the final good. The elasticity of substitution between home-produced and imported intermediate goods is given by γ.

The two composite goods, $y_{h,t}$ and $y_{f,t}$, are an ensemble of domestic and foreign retail goods which are aggregated using a technology given by

$$y_{h,t} = \left(\int_0^n (y_{i,h,t})^{\frac{\mu_p-1}{\mu_p}}\,di\right)^{\frac{\mu_p}{\mu_p-1}} \quad \text{and} \quad y_{f,t} = \left(\int_n^1 (y_{i,f,t})^{\frac{\mu_p-1}{\mu_p}}\,di\right)^{\frac{\mu_p}{\mu_p-1}}$$

where μ_p denotes the elasticity of substitution between intermediate goods produced in each country. These two expressions give rise to the price indices $p_{h,t}$ and $p_{f,t}$ of the composite goods.

Final good producers maximize profits $p_t y_t - p_{h,t}y_{h,t} - p_{f,t}\overline{\tau}y_{f,t}$ each period, subject to (28). The resulting optimal demand functions are given by

$$y_{h,t} = \varpi\left(\frac{p_{h,t}}{p_t}\right)^{-\gamma} y_t \tag{29}$$

$$y_{f,t} = (1-\varpi)\left(\frac{p_{f,t}}{p_t}\right)^{-\gamma}\frac{n}{1-n}y_t \tag{30}$$

The consumer price index, p_t, is obtained by replacing $y_{h,t}$ and $y_{f,t}$ in (28) with the respective demand function, which implies

$$p_t = \left[\varpi\left(p_{h,t}\right)^{1-\gamma} + (1-\varpi)\left(p_{f,t}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \tag{31}$$

1.2.4 Government

The government levies lump-sum and consumption taxes, T_t and τ_t^c, and issues sovereign bonds $d_{g,t}$ to finance exogenous non-productive government consumption g_t of the domestic final good. Government debt is entirely held by domestic financial intermediaries, which are assumed to provide the government with the amount of funds it requires. Hence, in the aggregate, the number of claims held by banks must equal the total amount borrowed by the government, $a_{b,t} = d_{g,t}$. Government expenditure is given by the following rule

$$g_t = (\bar{g})^{1-\rho_g}\left(g_{t-1}\right)^{\rho_g}\left(\frac{gdp_t}{\overline{gdp}}\right)^{\kappa_g}\varepsilon_t^g \tag{32}$$

where k_g governs the response of public expenditures to the cycle. In turn, lump-sum taxes are set according to

$$T_t = \overline{T}\left(\frac{d_{g,t-1}/gdp_{t-1}}{\tilde{d}_g}\right)^{\kappa_\tau} \tag{33}$$

where κ_τ characterises the government's preferences between tax- and debt-financed expenditures and $\tilde{d}_g$ is the target level for the stock of debt as a percentage of GDP. The tax rule embedded in (33) represents the effort the government needs to make, via taxes, to maintain public debt away from an explosive path. In order to induce a direct cost in terms of welfare derived from raising taxes, I follow the discussion in Kim and Kim (2013) and let the tax rate on consumption vary depending on the effort the government makes to control public debt. Hence, distortionary taxation is defined as

$$\tau_t^c c_t = \kappa_c T_t \tag{34}$$

where κ_c is the share of consumption taxes in the total tax revenue of the govern-

ment.

I follow Chatterjee and Eyigungor (2013) and Bocola (2014), and assume that the government issues long-term securities. Each period, government bonds mature with probability λ_b, which implies an average duration of bonds of $1/\lambda_b$ periods. When bonds reach maturity, the government pays back the principal; otherwise investors receive the coupon μ_b and retain the right to obtain the principal in the future. The government's ex post budget constraint is therefore given by

$$(\lambda_b + (1-\lambda_b)\,\mu_b)\,d_{g,t-1} + g_t \;=\; T_t + q_{b,t}\,(d_{g,t} - (1-\lambda_b)\,d_{g,t-1}) \tag{35}$$

where $q_{b,t}$ is the price of loans to the government. Conversely, the return on government bonds is given by

$$r_{b,t-1} \;=\; \frac{\lambda_b + (1-\lambda_b)\,(\mu_b + q_{b,t})}{q_{b,t-1}} \tag{36}$$

I define sovereign default in a manner similar to Schabert and Wijnbergen (2011) and Corsetti et al. (2012) by assuming that the government's decision to default depends on a *fiscal limit* above which the fiscal burden is deemed to be politically unacceptable.[10] Sovereign spreads are generated as the result of a preemptive game between the government and speculators. Agents know the distribution $f(\cdot)$ of the *fiscal limit* and form their expectations on that basis. Our modelling choice is not innocuous however. On the one hand, I abstain from a complete characterization of strategic default, which is beyond the scope of this paper, and instead assume that the *fiscal limit* is stochastically determined.[11] On the other hand, I abstract from any distributional consequences of default, including its effects on the fiscal stance. In fact, actual default is neutral, as can be deduced from expression (35), in the sense that I do not consider *de facto* asset losses in the model. Instead, the probability of default is crucial for the dynamics of sovereign bond prices and, consequently, for the net worth of banks. Hence, the model attempts to provide a consistent characterization of asset dynamics, but is mute with regards to the decision of actually declaring default and its consequences.[12]

[10] Davig and Leeper (2010) introduced the notion of "fiscal limit" used here.

[11] Corsetti et al. (2012) provide some motivation for this assumption by appealing to political considerations surrounding the decision to declare default. A previous note already made useful references to the literature on strategic default.

[12] Gertler and Karadi (2011), Dedola et al. (2013) and Lama and Rabanal (2014), just to name a few recent works, explore the effects of capital shocks that affect the *actual* quantity of assets in general equilibrium models with banks. The crucial difference between shocks to the

Every period the *fiscal limit*, or the politically bearable maximum level of the tax burden or of the public debt, is drawn from $f(s_t)$. The probability of default is equal to the probability the fiscal stance exceeds the *fiscal limit*. Let $\Delta(s_t)$ be a default indicator equalling 1 when the fiscal stance goes beyond the *fiscal limit*, and zero otherwise. As shown in Schabert and Wijnbergen (2011) and Bonam and Lukkezen (2013), I can approximate the expectation over the probability of default by

$$\hat{\delta}_t = \left(\Theta/\overline{\delta}\right)\hat{s}_t + \varepsilon_t^d \tag{37}$$

where $\hat{}$ denotes first-order log-linear approximations, s_t is the fiscal stance, ε_t^d is an exogenous shock that captures the market's perception regarding the possibility of a sovereign default and $\overline{s}$ is pinned down in the steady state. The parameters Θ denotes the elasticity of the probability of default with respect to changes in the fiscal stance, that is $\partial\Delta(s_t)/\partial s_t$. I intentionally left the fiscal stance s_t undefined in (37) for there are various potential candidates for the most adequate measure. The expressions *fiscal stance* and *fiscal outlook*, which I use interchangeably in this paper, refer not only to the present fiscal conditions (as measured by the public deficit, the tax burden or the share of government expenditures to GDP, to name a few), but also, and probably more importantly, to the future sustainability of current fiscal policy (as measured, for instance, by the ratio of public debt to GDP). I have experimented with the ratio of public debt to GDP, as in Schabert and Wijnbergen (2011), and with a measure of the fiscal strain, as in Corsetti et al. (2012). Both produce similar outcomes and here I show the results for $s_t = d_{g,t}/gdp_t$.

1.2.5 Central Bank

The single central bank in the monetary union is assumed to follow a Taylor-type rule where the nominal interest rate responds to the aggregate consumer price index and to the area-wide real GDP growth according to

$$i_t^\star = \left(\bar{i}^\star\right)^{1-\rho_i}\left(i_{t-1}^\star\right)^{\rho_i}\left(\left(\frac{\tilde{\pi}_t}{\bar{\pi}}\right)^{\rho_\pi}\left(\frac{\tilde{gdp}_t}{\tilde{gdp}_{t-1}}\right)^{\rho_g}\right)^{1-\rho_i}\varepsilon_t^i \tag{38}$$

stock of capital and shocks to its *price* lies on the real effects of reducing effective capital in production.

where $\rho_i \in (0,1)$ is the smoothing parameter, ρ_π and ρ_g are the usual response coefficients. The nominal interest rate is given by the Fisher equation

$$r_t^\star = \frac{i_t^\star}{\pi_{t+1}^\star}$$

I have assumed the foreign nominal interest rate to be the policy instrument given the small size of the home country I consider in the next sections. The aggregate variables in the Taylor rule are denoted with a $\sim$ and are the sum of the respective country variables weighted by their population size.

1.2.6 Market Clearing

There are two types of markets for goods in each country that must clear in equilibrium. For intermediate goods, production by the wholesaler firms equals demand by retailers

$$x_t = \Upsilon_{h,t} y_{h,t} + \Upsilon_{h,t}^\star y_{h,t}^\star \tag{39}$$

Note that, due to price dispersion, retailers incur real losses during price setting. On the other hand, the non-tradeable domestic final good is sold to households, the government and to capital producers

$$y_t = c_t + z_t + g_t \tag{40}$$

From the aggregate budget constraint of households I obtain the following law of motion for net foreign assets

$$e_t \left(r_{f,t-1} b_{f,t-1} - b_{f,t} \right) = e_t p_{h,t}^\star y_{h,t}^\star - p_{f,t} \frac{1-n}{n} y_{f,t} \tag{41}$$

where $y_{h,t}^\star$ are exports of the home-produced intermediate composite good and $y_{f,t}$ are imports of the foreign-produced intermediate goods.

Because financial markets are incomplete, I follow Schmitt-Grohé and Uribe (2003) and ensure the model is stationary by setting $r_{f,t}$ equal to the real interest rate abroad plus a risk premium that is sensitive to the total net foreign asset position as a percentage of GDP

$$r_{f,t} = r_t^\star \Xi exp \left\{ \Gamma \left(e_t \frac{b_{f,t}}{gdp_t} - \tilde{b}_f \right) \right\} \tag{42}$$

and where GDP is defined as

$$gdp_t = p_{h,t}y_{h,t} + e_t p^{\star}_{h,t} y_{h,t} \tag{43}$$

1.3 Bayesian Estimation

In this section I estimate the model for Portugal and the Eurozone. Portugal is an illustrative example of a country that has been subject to considerable shocks to its sovereign interest rates. In the spring of 2011, Portugal became the third EMU member to request external finance assistance, after Greece and Ireland. At the time, the Portuguese government was facing a sharp increase in the costs to finance public debt, while Portuguese banks, heavily dependent on external financing, were being cut-off from market-based funding. When the assistance programme was signed in April, the 10-year yield of Portuguese government bonds were rapidly approaching the 10% mark, public debt to GDP was around 110%, and the fiscal deficit had reached 11.2% the previous year. With the program, Portugal received €78 billion, or about 43% of GDP, under the conditionality of implementing measures towards fiscal consolidation and pursuing structural reforms.[13]

I estimate the model using standard Bayesian techniques. First, the equilibrium conditions are log-linearised around a deterministic, zero-inflation steady state. As I explain in more detail, I reduce the number of parameters to estimate by calibrating some that are weakly identified by data. For the remaining parameters, I specify the priors for estimation based on previous studies. I then employ the Metropolis-Hastings algorithm with two chains of 125.000 draws to obtain the posterior distributions.[14]

1.3.1 Calibrated Parameters

The parameters I calibrate can be arranged into four different groups. The first group includes those usually calibrated in the literature and for which I pick con-

[13]Figures and further discussion about the Portuguese adjustment program can be found in a report by the European Commission of 2014.

[14]The non-linear equilibrium conditions of the model where coded in Dynare 4.4.2, with the model's solution, estimation and welfare analysis being performed using Dynare's interface. Estimation was performed under a first-order log-linear approximation, whereas the welfare analysis was done on a second-order log-linear approximation to the model's equilibrium conditions.

sensual values. The second group contains the parameters related to the banking sector, which are not estimated because of the lack of long and reliable data series I could use to identify them. Regarding these two sets of parameters, I further impose their values to be equal across countries. The parameters that pin down steady state ratios constitute the third group and their values are chosen to match long-run averages in the data. Finally, the parameters at the core of the policy analysis in section 4 form the forth group.

Table 1 reports the values for the calibrated parameters. Hereafter, the home country represents Portugal, the Euro Area is the foreign country, and one period in the model corresponds to one quarter. The values for the first set of parameters are mostly taken from Lama and Rabanal (2014). The exception is the elasticity of capital depreciation with respect to utilization, for which I use the estimate obtained by Albonico et al. (2014). The values for the parameters related to the banking sector are taken from Gertler and Karadi (2011). Lama and Rabanal (2014) and Bocola (2013) estimate some of these parameters and obtain very close estimates to the values used here. On the other hand, the spread on the sovereign interest rate is only meant to be illustrative and therefore I assume a relatively small value, below the one used by Schabert and Wijnbergen (2011) and more in line with what the data from before 2009 suggests.

Regarding the third group, I set the share of the population living in Portugal to 3% of the total of the Eurozone; the ratio of *per capita* GDP between the EMU and Portugal to 1.7; and the share of imports to GDP in Portugal, which corresponds to $1-\omega$ in the model, to 30%. Plugging these figures into the steady state version of the demand equations for final goods in both countries and using the aggregate resource constrain, I obtain an extremely high degree of home bias in the Euro Area ($\omega^{\star} = 0.995$). Hence, while Portugal is relatively sensitive to shocks pertaining to the currency area, the Eurozone is almost immune to shocks originating in Portugal. Although the degree of openness of the Eurozone is undoubtedly higher than the one implied by my calibration, I nevertheless decided to stick to these values to guarantee the consistency of the estimates for Portugal.[15]

[15]Some notes are in order. First, the value for $\omega^{\star}$ is perfectly consistent with the way I model the monetary union: Portugal represents indeed a very small share of Eurozone trade. Second, the mismatch of $\omega^{\star}$ with the data has two sources. On the one hand, I do not model countries outside the EMU, despite the large share they represent in terms of trade flows with the Eurozone. On the other hand, aggregate trade data for the Eurozone includes exports and imports within member states, magnifying the final values of net-exports. Third, because I use aggregate data for the Eurozone, parameter estimates need to be analysed with caution. There

For the policy parameters, I decided to be rather conservative and followed the standard calibration used in the literature. The ratio of public debt to annual GDP is set equal to the upper limit imposed by the Maastricht Treaty of 60%, which is close to the sample average for Portugal when I exclude the last half decade. I assume a standard $AR(1)$ process for government expenditures and set $\kappa_g = 0$. For the share of consumption taxes in the total taxation, I set κ_c equal to 40% based on Eurostat (2014). I then obtain a steady state effective tax rate of $\tau^c_{ss} = 16.58\%$, which is slightly below the estimates computed in Eurostat (2014), but in line with the estimate used by Kim and Kim (2013).

1.3.2 Data and Priors

I use a sample of 14 quarterly time series - 7 for each region - spanning between the first quarter of 1995 and the last quarter of 2014. I use nominal GDP, household consumption, investment, government expenditures, compensation of employees, the consumer price index and a nominal interest rate I define shortly. National accounts data for Portugal is taken from the Eurostat, whereas for the Euro Area I use the ECB Area Wide Model. Because Portugal accounts for just 3% of the currency area, it seems unlikely that using aggregate data for the entire Eurozone, including Portugal, constitutes a significant source of estimation bias. I obtain consumer prices from the ECB (I use the HICP indices). I use the 10-year government bond yield from the Eurostat for Portugal[16] and choose the Euribor 3-month series from the ECB for the Euro Area. All variables are already seasonally adjusted from the source except for consumer prices, which are adjusted using the X-13ARIMA procedure developed by the US Census Bureau.

To be consistent with the model, I convert the national account aggregates into *per capita* quantities using quarterly population series from the Eurostat. The same is done for wages, which I obtain dividing compensation of employees by the number of employees, also from the Eurostat. The reason behind using nominal

are a number of studies running Bayesian estimation for the Eurozone and using the same data set, which allows me to compare and evaluate the results I obtain here. On the contrary, given that previous estimates for Portugal are rare, I decided to use a calibration that is as consistent as possible with Portuguese time series in order to minimize the chances of obtaining blurred estimates. Fourth, as I discuss later, I add measurement errors to the net exports of both countries to minimize the potential bias caused by the calibration and study the robustness of the estimates I obtain.

[16] Accordingly, I set $\lambda_b = 0.025$, which implies an average maturity of government bonds of 40 quarters, and calibrate the value of the coupon, μ_b, such that in the steady state the price of government bonds equals the price of loans to firms, $\overline{q_b} = \overline{q_x}$.

variables relates to model consistency as well. Given that all aggregates have the same deflator in the model, I ensure the resource constraints in each region are met by using the consumer price index to convert all nominal quantities into real variables. Lastly, I take logs and first differences of real quantities and wages in order to render them stationary. With one exception though: Portuguese government expenditures remain non-stationary after these transformations. I therefore use the share of government expenditure to GDP and implement the corresponding changes in the model. Regarding the nominal variables, I obtain consumer price inflation by taking logs and first differences of the price level series and divide the nominal interest rates by 400 to convert them to quarterly series. I use nominal interest rates in levels because they are stationary both in the data and in the model. Finally, all variables are demeaned before estimation.

Due to the inclusion of a world technology shock with a unit root, real quantities and wages are also non-stationary in the model. Consequently, I divide these variables by the level of world technology and match actual variables to their model counterpart by noting that $\Delta y_t^o = \Delta \tilde{y}_t + \varepsilon_t^u$, where Δy_t^o corresponds to the first-difference of the log of observable real variables, $\Delta \tilde{y}_t$ is the growth of its counterpart in the model ($\tilde{y}_t$ denotes the detrended log-deviations from the steady state), and ε_t^u the innovation to the stochastic trend in logs. In total, I match the following 14 variables: Δgdp_t, Δgdp_t^*, Δc_t, Δc_t^*, Δz_t, Δz_t^*, Δg_t, Δg_t^*,Δw_t, Δw_t^*, π_t, π_t^*, $i_{b,t}$, and i_t^*.

I define the prior distributions based on the literature preforming Bayesian estimation of DSGE models of the Euro Area. In particular, I focus on studies that use the same dataset for the Eurozone as the one used here. Given that the literature on Portugal is comparatively less proficuous, I decided to have prior distributions for Portuguese parameters identical to their Euro Area counterparts. Nevertheless, due to the significantly higher volatility of Portuguese time-series, I let the priors for the standard deviations to be generally more diffuse than in previous studies. Prior distributions are shown in Table 2 to Table 4.

I use the gamma distribution for parameters assumed to be positive. Priors for the habit parameters and for the labour disutility coefficient are taken from Lama and Rabanal (2014). I let investment adjustment costs to vary across regions and set its prior mean to 2. For parameters bounded between 0 and 1, I use the beta distribution. I use the same prior distribution for price and wage lotteries as Smets and Wouters (2002). They set the prior mean to 0.75, which implies average contract durations of one year. For the price indexation coefficient, I

set prior means of 0.20, which is in line with the estimates found in previous studies. The prior for the inflation coefficient in the Taylor rule follows a normal distribution centred at 1.7 as in Smets and Wouters (2002), while the prior mean for the coefficient on output growth is set at 0.20, which is within the range of values typically used. I proceed in the same way and set the prior means of the smoothing coefficient in the Taylor rule and the persistence of shocks to 0.75, which lies between the 0.5 and 0.85 found in the literature. The prior distributions for the standard deviations of the shocks are again based on Lama and Rabanal (2014), although relatively more diffuse for the reason mentioned above. The prior mean for the standard deviations of intratemporal preference shocks is significantly higher than for the remaining shocks, which is also the case in Smets and Wouters (2002). Also worth noting that technology and cost-push shocks are assumed to be less volatile than investment specific shocks, but more volatile than intertemporal preference shocks.

1.3.3 Estimation Results

Table 2 and Table 3 show the posterior means and the 90% credible set of the estimated parameters.[17] The baseline estimates can be found under *spec.* 1. Looking first at the estimates for the Eurozone, the posterior mean for the habit persistence and the labour disutility parameters are identical to those found by Smets and Wouters (2002). Regarding nominal rigidities, I find that price contracts are, on average, shorter than wage contracts. The mean estimates are very close to those in Lama and Rabanal (2014), with prices adjusting every 3 quarters on average while wages take 5 quarters. In general, the estimates for Portugal differ by little from their Euro Area counterparts. Among the exceptions is ψ, found to be significantly higher, and the survival rates of nominal contracts, with prices adjusting more slowly than wages. The estimates for price indexation are small for both regions and around 10%, which is in line with what Lama and Rabanal (2014) obtain. Finally, our estimates for the area-wide Taylor rule are also very similar to those in the literature.

Regarding the shock processes, I estimate intertemporal preference shocks to be more persistent compared to intratemporal (or labour supply) shocks, a result also obtained by Adolfson et al. (2007) and Lama and Rabanal (2014). On the

[17] The estimation results shown here were obtained holding capital utilization and consumption taxes fixed and equal to their steady state values. Further work is being undertaken to allow these features to vary during estimation.

contrary, the persistence coefficients of stationary technology, investment specific technology, and cost-push shocks are relatively lower than what is found in the literature. Government expenditure shocks both in Portugal and in the Eurozone appear to be quite persistent and very similar to the values estimated by Smets and Wouters (2002), while the coefficient on the risk premium is in line with Adolfson et al. (2007). The estimates for the standard deviations reported in Table 4 are generally in line with our prior expectations. Comparing both regions, Portuguese shocks are systematically more volatile than Euro Area ones, and this is particularly visible for investment specific technology and cost-push shocks.

While I only model trade between the two regions, Eurozone countries have multiple trading partners and, inclusively, trade with regions outside the monetary union. Hence, the aggregate resource constraint in the model is inconsistent with actual national accounts for it excludes exports and imports vis-à-vis regions outside the model. This is particularly troublesome given that Portugal accounts for only a slim fraction of total net exports originating in the Euro Area. In order to account for trade other than between the two regions, I added measurement errors to the net exports in the model. I compare this methodological choice to the approach taken by Lama and Rabanal (2014), who estimate the model without measurement error and without including government expenditures in the set of observables. The results, reported under *spec.* 2, show virtually no changes in parameter estimates except for a smaller persistence of Portuguese government expenditures and a higher volatility of Portuguese and Euro Area government expenditure shocks. It thus seems that government expenditures are not only capturing actual shocks to public spending, but also residual volatility coming from trade outside the model. I also explore the impact of misspecification when I include government expenditures to the set of observables without adding measurement error to net exports. Not only the parameter estimates deliver very different results, as can be seen under *spec.* 3, I also find a striking mismatch between the volatility of the observable time series implied by the model and actual figures.

Turning to the second moments, the first two columns of Table 5 report the standard deviations the data and those implied by the model evaluated at the posterior mean under to the baseline estimation. The match is satisfactory for most variables, with two exceptions. On the one hand, Eurozone GDP is predicted to be more volatile, a result that is also obtained by Lama and Rabanal (2014). On the other hand, the model delivers a smaller standard deviation of Portuguese wages

despite the high estimated volatility of labour supply shocks. Note also that, although in the data the volatility of Portuguese GDP is only slightly smaller than the volatility of consumption, the model delivers the inverse ordering, with GDP predicted to be more volatile.

Table 5 also presents the unconditional variance decomposition of the variables I use for estimation. I have aggregated some shocks in order to make the presentation neat.[18] Similarly to Lama and Rabanal (2014), the international transmission of shocks appears negligible for most variables, apart from Portuguese inflation and the sovereign interest rate. Regarding the former, this finding indicates that shocks in the Eurozone feed mostly through prices and do not have a significant direct impact in real quantities. On the other hand, as sovereign spreads are exogenous in the baseline scenario, the sovereign rate is mostly explained by spread shocks themselves and by foreign shocks which feed through the common Taylor rule. Interestingly, sovereign spread shocks have negligible effects in the real economy, a result that does not seem to have been influenced by the events taking place in the very last part of the sample. In line with Ratto et al. (2008), I also find that monetary policy shocks explain only a small fraction of the volatility of Euro Area variables. All in all, and similarly to previous studies, preference and technology shocks represent the main source of fluctuations in both regions.

1.4 Sovereign Spreads and Fiscal Transfers

In this section, I start by analysing the transmission mechanism of sovereign spread shocks in the model and by assessing its conformity with actual events during the sovereign debt crisis in Portugal. In the context of asymmetric shocks within a currency area, as have been sovereign risk shocks in the Eurozone, fiscal policy becomes a crucial tool to stabilize the economy. I show, however, that sovereign risk and the fiscal outlook of a country constrains the set of actions of the government. I then run a number of policy experiments exploring the possibility of a new fiscal architecture within the EMU. In particular, I analyse the potential benefits of implementing a fiscal transfers scheme (FTS) among Eurozone member-states. Although still exotic, fiscal federalism has been subject

[18] Preference shocks include both inter- and intratemporal shocks, whereas technology shocks include the stationary and the unit root technology shocks. The two measurement errors are also shown together. Moreover, for each variable the table reports the decomposition with respect to local shocks. For instance, Portuguese variables are decomposed across different shocks originating in Portugal. All the remaining shocks are aggregated under the banner *Abroad*.

of previous academic research. Importantly, however, it now appears to be a matter of serious consideration within policy circles as well.

1.4.1 Inspecting the Mechanism

Figure 1 presents the impulse responses of selected variables to a shock that raises the sovereign spread by 10% in annual terms, as seen in Portugal during 2011.[19] The increase in uncertainty regarding the ability of the government to service its debt lowers the value of government securities and, therefore, raises the return on government bonds required by investors. As interest payments become heavier, the government incurs a budget deficit and the stock of public debt increases. Under the baseline specification, government expenditures do not respond to the cycle[20], whereas lump-sum taxes track the ratio of public debt to GDP. As such, taxes are automatically raised and the government is induced to run a primary surplus. Comparing to the actual deficit of 7.4% for Portugal in 2011, the jump in the budget deficit predicted by the model seems small. Note however that between 2010 and 2013, taxes and social contributions fell by more than 2%, while unemployment benefits, pensions, and other financial liabilities all increased (European Commission, 2014). Therefore, the baseline scenario serves as a lower bound in what respects the deteriorating effects of sovereign spread shocks on the fiscal stance.

As the price of government bonds plunges, bankers, who hold these securities in their portfolios, see their total net worth contract. This triggers a jump in the leverage ratios of banks that persists over time. In terms of magnitudes and recovery time, the model compares well with reality. Using the loan-to-deposits ratio as a measure of leverage, the figure for Portuguese banks at the beginning of 2011 was equal to 157%. It took 15 quarters to reach 117%, a fall of about 25% and similar to Figure 1. Banks' equity also went through a slow recovery, with the average Core Tier 1 adjusting from 8.1% to 12% over the same period.[21] Because of the leverage constraint, banks are forced to reduce lending and to increase the premium on loans to private firms in order to build up the value of their equity. In terms of the pass-through of sovereign spreads to firm's borrowing costs, an

[19] Figure 6 plots the 10-year government bond spreads against the German bunds for some of the countries at the centre of the European sovereign debt crisis.

[20] As a matter of fact, government expenditures as a share of GDP are constant. As GDP falls, total government expenditures will fall as well.

[21] Figures taken from European Commission (2014).

increase of 10% in the former leads to a 3.5% increase in the latter.

The drop in credit supplied by banks and the increase in borrowing costs induce a collapse in investment (of more than 10% at the trough). As firms face higher costs of capital, labour demand also contracts and total employment falls. Consequently, real output falls, dropping more than 2% at the trough. The marked contraction in domestic demand due to the fall in investment induces prices to fall. However, given the small size of Portugal relative to the EMU, the nominal interest rate is cut by less than 10 basis points. Clearly, monetary policy is not designed to address country-specific shocks, with the negligible policy loosening doing nearly nothing to buffer the recession in Portugal.

Figure 1 also shows that higher ratios of public debt intensify the magnitude of the recession. In fact, doubling of the stock of public debt leads to a fall in GDP more than twofold. When domestic banks hold a larger stock of government securities in their balance sheets, a fall in the price of sovereign bonds generates a relatively higher loss in their portfolio. As a consequence, the premium between the risk free rate and the interest rate on loans to private firms can reach more than 3/4 of the spread originally generated on the sovereign rate. This number represents quite a substantial pass-through. When public debt reaches 120% of GDP, the collapse in investment and the drop in labour demand are sizeable too. Actual figures for Portugal were not any less impressive: from 2011 to 2013, investment fell nearly 30%, while the unemployment rate went from 12.2% to 17.3%.

1.4.2 Constraints on Domestic Fiscal Policy

During the sovereign debt crisis, there was no room for counter-cyclical policy. European governments were forced to run sharp fiscal consolidation to avoid rampant sovereign interest rates, despite the economic outlook remaining weak. In this respect, the model provides informative insights on how sovereign interest rates and the fiscal outlook constrain the set of fiscal responses. Figure 2 shows the determinacy regions of the model for a range of parameter values governing fiscal policy, given the ratio of public debt to annual GDP and the elasticity of sovereign spreads to the fiscal outlook. In the figure, the values for κ_τ are within the range used in the literature (e.g. see Pappa, 2009); $\kappa_g < 0$ corresponds to the government running counter-cyclical policy; and regarding Θ, the elasticity of sovereign spreads to the fiscal stance, I consider the range of values computed by Corsetti et al. (2012). The white areas in the figure correspond to regions in the

parameter space for which public debt grows unbounded.

As shown in the left panel, as Θ increases, taxes need to react more swiftly to changes in public debt in order to keep it away from an explosive path. Since higher spreads imply higher deficits, and deficits lead to widening spreads, the government needs to raise taxes rapidly to avoid further increases in sovereign interest rates.

The government can either raises taxes or lower public expenditure in order to control public debt. The last panel to the right shows the trade-off between how government expenditure can respond to the cycle and how taxes are used to control public debt. Firstly, counter-cyclical policy is only possible when taxes are sufficiently firm in targeting public debt. On the other hand, pro-cyclical public spending is not enough, *per se*, to stabilize public debt either. In fact, the two fiscal tools work through different channels. While taxes reduce public debt directly, public spending affects GDP via a demand effect.[22] Consequently, spending alone might fail to bring sustainability to the ratio of public debt to GDP.

Both grey areas in Figure 2 represent the determinacy regions of the model when the ratio of public debt to annual GDP equals 60%. When this ratio equals 120%, determinacy only occurs within the dark grey areas. As all panels attest, a higher stock of public debt requires fiscal discipline to be stricter. Importantly, the scope for counter-cyclical government expenditures is reduced dramatically, as shown in the central panel. In particular, when Θ increases, the feedback effects of counter-cyclical expenditure on sovereign spreads dwarf any attempts to stimulate production via public spending. In effect, in these cases, counter-cyclical spending raises the ratio of public debt to GDP unambiguously, therefore failing to keep it on a sustainable path.

Clearly, the ratio of public debt to annual GDP is key to determine the range of sustainable fiscal policies that can be implemented by the government. As low debt countries are better placed to use domestic fiscal policy as a tool to absorb idiosyncratic shocks, it is not surprising the emphasis put on public debt and budget deficit figures since the early stages of the EMU. Nevertheless, the question remains: how should the EMU respond when countries experiencing fiscal strain cannot use domestic fiscal policy to countervail the recessionary effects of large

[22] In the case of taxes, the effects on GDP are of second order and depend of households' consumption smoothing.

asymmetric shocks?[23]

1.4.3 A Scheme of Fiscal Transfers: Symmetric Regions

In this section I use the estimated model to assess the welfare implications of a federal transfers scheme (FTS) that has both countries operating transfers across the border when sovereign spreads widen. Transfers from foreign to home are determined by the following simple rule

$$S_t = \kappa_s \left(\log\left(\delta_t\right) - \log\left(\overline{\delta}\right)\right) \tag{44}$$

An equivalent expression defines the transfers to be made the opposite way.[24] Importantly, the parameter governing the magnitude of the transfers, κ_s, is equal for both countries. As all variables in the model, including S_t and $S_t^\star$, are defined in *per capita* units, an equal κ_s implies an equal *per capita* burden for home and foreign households. Transfers are collected by the government and are made between governments.[25] Hence, the expressions for the government budget in both countries and for the net foreign assets have to be adjusted accordingly. The FTS proposed here addresses directly the problem of fiscal strain due to sovereign spreads. As there is no direct transfers to households or firms, the feedback to the real economy will be through taxation and public spending. Importantly, the FTS will also feed-back to the real economy through its potential effects on the pass-through of sovereign spread shocks.

To contextualize my results, I start by considering a model where both regions have symmetric governments and are both subject to sovereign spread shocks.

[23]The model presented in the previous section, although providing an accurate illustration of how fiscal policy can run into indeterminacy, it is not especially gifted to analyse optimal government spending. On the one hand, government spending is not productive nor utility enhancing in the model. On the other hand, automatic stabilizers, such as unemployment benefits, are absent. That is partly the reason why pro-cyclical public expenditure might be welfare improving for some parametrization in Figure 4. Integrating these elements in the model is left for future research.

[24]By definition, transfers are only temporary, being equal to zero in the long-run.

[25]In this paper I assume δ_t is observable and, therefore, can be used to guide policy. In reality, however, sovereign spread shocks might be difficult to measure. Importantly, it might also be the case that optimal transfers do not respond to all swings in sovereign spreads as measured, for instance, by the differentials in government bond yields in the secondary market. It is also not clear that targeting a more *fundamental* measure, such as public debt to annual GDP, solves the problem. I leave these questions for future research.

Crucially, I consider the case when both regions have equal sizes and *per capita* GDP. In Table 6, the parameter values used for the region labelled Periphery correspond to those estimated for Portugal, whereas the estimates for the Eurozone are used for the region labelled Core. The FTS is defined by the value of κ_s that maximizes the aggregate welfare of the monetary union, that is, the sum of each region's welfare weighted by its population size. I follow Schmitt-Grohé and Uribe (2007) and express welfare gains in terms of certainty-equivalent consumption. First, I compute each country's welfare for a given set of allocations $\left\{c_t^k, l_t^k\right\}_{t=0}^{\infty}$, where k corresponds to a particular value of $\tau_s \in \mathbb{R}_0^+$. I then compare it to the case of no fiscal transfers, defining the welfare gain $\tilde{\lambda}$ as

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(\left(1+\tilde{\lambda}\right) c_t^0, l_t^0\right) \quad = \quad \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^k, l_t^k\right) \tag{45}$$

For positive values of $\tilde{\lambda}$, there are gains from implementing the FTS. Welfare is computed up to a second order of approximation from the unconditional expected lifetime utility.

Table 6 reports the welfare gains from both regions engaging in the FTS. In the first column I report the baseline case where the two regions differ only in terms of the estimated parameters. The results show that the FTS is welfare improving for each region individually. This is an important finding because it states clearly the mutual benefits of both members entering the FTS. Interestingly, the Core is the region benefiting the most, with a 4% increase in permanent consumption. The difference in welfare gains between the two regions is largely explained by the difference in the set of estimated parameters.[26]

However, the distribution of welfare gains and their magnitude can vary easily depending on small asymmetries between the two regions, and in particular when the fiscal outlooks differ. For example, the second column shows that when public debt to GDP is twice as big in the Periphery as it is in the Core, welfare gains fall for the former, whereas they increase for the latter. Note that, up to the value of κ_s, the transfers are identical to the baseline scenario given that they only depend upon the sovereign spread shocks. Moreover, using the same $\kappa_s = 7.27$ as in the baseline, the Core still benefits more from entering the FTS with a Periphery with higher debt. Inspecting the reasons behind these results, I find

[26]The other factors behind the discrepancy between welfare gains are the risk premium on the interest rate on foreign bonds charged to home households and the asymmetry caused by the fact that the policy rate is the foreign nominal rate.

that transfers do not do enough to counteract the magnifying effects of public debt on real fluctuations in the Periphery, whereas the additional gains to the Core stem from the feedback effects on the real exchange rate, as seen in Figure 1.

The third column reports the case when sovereign spreads respond to the fiscal outlook in the Periphery. For a given κ_s, setting $\Theta \neq 0$ increases the persistence of sovereign spread shocks. As a result, transfers between countries become asymmetric, with those incoming to the Periphery being more prolonged in time than those incoming to the Core. This, together with the fact that spread shocks have a greater impact on real activity when $\Theta \neq 0$, explain the substantial welfare gains of the FTS to the Periphery. On the contrary, the gains for the Core disappear, clearly driven by the disproportionate costs of outgoing transfers to the Periphery relative to the benefits of incoming transfers. If I assume instead that $\Theta^\star = \Theta$, the FTS becomes again welfare improving for both regions (results not shown in the table).

The next two columns inspect the consequences of fiscal policy in the Periphery. When taxes respond less to public debt, fiscal deficits and public debt fluctuate more. Hence, after a sovereign spread shock, as banks accommodate the increase in government debt, which in turn becomes less valuable, the pass-through is magnified. As transfers stabilize public debt, the welfare gains from the FTS for the Periphery increase. On the contrary, counter-cyclical government expenditures narrow the benefits in both regions. In the Periphery, the impact of incoming transfers in stimulating output is marginal when the government is already carrying counter-cyclical policy (even when considering their positive impact in stabilizing debt). On the other hand, the losses caused by outgoing transfers due to the FTS are further magnified by fiscal policy.[27]

Finally, the last column in Table 6 investigates the case when the volatility of sovereign spreads in the Core is reduced to 95% of that seen in the Periphery. The welfare effects are strikingly clear: the Core has no advantage in joining the FTS, whereas the Periphery has additional gains. The results are not surprising; but the fact that a relatively small drop in the volatility of spread shocks produces such an antagonistic result is symptomatic of the challenges posed to the implementation of a FTS between different regions. The discussion that follows is dominated by this difficulty in supporting a FTS that causes welfare losses for some of its

[27] Note that public spending in our model is not utility enhancing, as discussed in a previous note.

participants.

1.4.4 A Scheme of Fiscal Transfers: Asymmetric Regions

Aggregate welfare is a good measure to assess the potential benefits of international fiscal transfers. However, it might be politically (and socially) impracticable to convince one country to participate in a FTS that reduces its own welfare. Therefore, rather than searching for the FTS that maximizes aggregate welfare, it is advisable to look at the welfare effects for each country individually. In this light, it turns out that modelling two countries with different sizes constitutes a challenge. In a nutshell, when the two countries differ in size, equal *per capita* transfers imply necessarily an asymmetric aggregate flow of transfers between countries. The greater is the discrepancy in relative sizes, the more (less) impact fiscal transfers have for welfare in the small (big) country.

To clarify the importance of relative sizes, I use again the model with symmetric regions Core and Periphery and run the following exercise: First, I consider only FTS for which the value of κ_s maximizes aggregate welfare. I then compute the minimum relative size of the Periphery, n, for which entering the FTS has no negative effects to the Core. Figure 3 illustrates this exercise. The minimum value I obtain is $n = 48.72\%$, which plainly shows how easy the support for a FTS can break down due to asymmetries between countries. As transfers are calculated in *per capita* terms, their aggregate levels change one-to-one with n. Although the *per capita* burden of engaging in a FTS with a smaller country diminish with n, the *per capita* benefits vanish more rapidly.

Conversely, one important aspect conveyed in the previous exercise concerns the potentially large gains small countries can secure from entering a FTS. In fact, if I were interested more broadly in FTS that generate a positive gain in aggregate welfare, despite reducing welfare in one region in particular, the minimum value of n sustaining a positive κ_s would be substantially lower. I therefore return to the model I have estimated and conduct another experiment. Suppose that to implement a FTS, all countries have to benefit from welfare gains derived from being part of a monetary union with fiscal transfers. That is, suppose first that entering the monetary union implies a gain of $\tilde{\alpha}$ in terms of lifetime consumption to all its members. A FTS can then be implemented as long as its welfare costs are smaller than $\tilde{\alpha}$. In other words, the alternative is not between implementing a FTS or not, but rather between a monetary union with a FTS and leaving the

union altogether.

Welfare costs and benefits from entering a monetary union have been studied in the literature and I take a passive stance here by simply adopting existing estimates. On the negative side, the costs associated with entering a monetary union relate to the lack of synchronization between individual countries' business cycles. Among others, the costs arise from asymmetric shock to technology and fiscal policy, home bias in consumption, and incomplete financial markets. However, a growing literature is quantifying the extent to which trade and financial integration can offset these losses. For instance, Lama and Rabanal (2014) show that a fall in trade costs, which they consider to be of a conservative magnitude, is responsible for a 1.2% increase in permanent consumption. However, if they include the business cycle costs of the common currency, they obtain a welfare loss. Auray et al. (2010) study the welfare effects of an increase in trade flows between member countries of around 10%. They show that trade integration can account for an increase of more than 7% in permanent consumption in an economy with incomplete financial markets, and that the benefits from trade could reach more than 10% of lifetime consumption if financial markets are complete. Also focusing on the level of financial markets integration, Lama and Rabanal (2014) run a rough experiment and assume that the EMU induces a sharp reduction in the volatility of private risk premium due an increase banks' risk pooling. Under this scenario, they calculate the welfare gains from entering the union to be higher than 2% of permanent consumption.

For the purpose of my experiment, I focus on two scenarios for which entering a monetary union brings welfare benefits to its members due to gains from trade. The more conservative scenario assumes a 1% increase in lifetime consumption, whereas the second scenario has a more optimistic conjecture of a 5% increase in permanent consumption. I make two more assumptions. First, I conjecture that welfare gains are identical across all member countries. Second, I suppose that the gains from trade are proportional to the size of each country entering the union. With these two assumptions, a country of size n is responsible for a permanent consumption increase of $(n/(1-n)) \times \tilde{\alpha}$ to all the remaining member countries of the union.

Table 7 shows the welfare effects of the FTS under the two scenarios. Under the assumption of a 1% gain derived from trade integration, Portugal could secure a 1.44% increase in lifetime consumption from the implementation of the FTS. In the optimistic scenario, the gain jumps to 7.8%. Table 7 also reports an approximation

to the potential benefits bigger countries could secure from the FTS. Using the estimated parameters for Portugal, I recalibrate the size n of the home country and the ratio of *per capita* GDP to match Spain and Italy. This is a simple conjecture since the parameter estimates are likely to differ across countries. Notwithstanding this caveat, the results show that for a fraction of the benefits derived from trade integration, welfare gains for the Periphery of the EMU are large.

The experiments reported in Table 7 serve to illustrate the magnitude of welfare changes involved with the implementation of the FTS. In particular, it shows that the smaller the recipient country is, the higher the potential gains it can obtain. In fact, even if the benefits from trade integration linked to the inclusion of a small country in the union were smaller than the conservative scenario, the positive impact on its welfare would still be substantial. The scheme has its limitations however. For instance, big countries like Germany, which represents less than 48% of the union, but significantly more than the 18% of Italy, would be unable to secure gains of the same magnitude as those reported in the table. Germany falls in a *grey area*: it is too big to benefit from sizeable welfare gains at the expense of the rest of the union, and too small to engage in a FTS that improves welfare everywhere as seen in Table 5.

As shown in Table 7, the welfare benefits for Portugal change modestly regardless of its domestic fiscal stance and policy. In Table 8, the value of κ_s is computed such that the Eurozone loses $(n/(1-n)) \times 1\%$ in permanent consumption. Clearly, the small size of Portugal explains the negligible variations in the values of κ_s. Notwithstanding that Tables 6 and 8 were built under different assumption, both sets of results are coherent. For instance, when $\Theta \neq 0$, κ_s falls so that the welfare losses for the Eurozone remain constant. On the other hand, welfare benefits are maximized when Portuguese fiscal policy is less strict, with taxes responding more weakly to public debt.

I have focused on the substantial welfare gains a federal transfer mechanism can generate. However, it is important to acknowledge that the implementation of the FTS requires some countries to forego a fraction of the initial gains obtained from entering the EMU. That is, the political support for the implementation of a transfers arrangement can not, by any means, be taken for granted. Yet, important considerations linked to spillover effects of sovereign spread shocks are, at least partially, absent from the model. In reality, the destabilizing effects of the European sovereign debt crisis were also felt in the Core of the EMU, where contagion was addressed seriously. For instance, in the model banks do not engage

in international intermediation. As such, considerations regarding the systemic risk one country's banking sector poses to area-wide stability are mute. Dedola et al. (2013) show that country specific shocks affecting the domestic banking sector are transmitted to foreign banks when there is financial integration, thus requiring policy coordination to buffer shocks efficiently. Clearly, these channels have direct implications for the welfare of current net losers from the FTS. Including such considerations in the cost-benefit analysis of the FTS could induce wider support for a transfer scheme in these countries.

On the other hand, the push for the implementation of a Fiscal Union in Europe is faced with concerns over the risk of moral hazard and free riding.[28] Some steps to mitigate these fears have been alluded to in the 5 Presidents Report (Juncker et al., 2015), where the authors defend three important prerequisites for the implementation of the Fiscal Union: (*i*) the economic convergence of the member states, which will increase the synchronization of business cycles, (*ii*) the enactment of fiscal rules that guarantee the sustainability of domestic fiscal accounts, which as a by-product will enable domestic fiscal policy to react to asymmetric shocks, and (*iii*) the guarantee that the interventions under the FTS have only a temporary nature.

Regarding the first point, as member states' business cycles become more synchronized, the lower are the costs of a single monetary policy with fixed exchange rates (see, for instance, Rose 2008). As such, and as discussed above, the higher the benefits from being part of the union, the easier it will be to grant support for the implementation of a FTS. Considering point (*ii*), I have rationalized how domestic fiscal policy is endogenously constrained by the fiscal outlook and how fiscal conditions can compromise the leeway needed for domestic policy to buffer spread shocks. One important dimension that remains to be addressed concerns how domestic policy should be conducted in an environment with transfers. As I have shown, the political support for a FTS between countries rests on how asymmetric fiscal conditions are. It is beyond the scope of this paper to draw on potential conditionalities involved with a FTS to enforce fiscal prudence. The model can, nevertheless, provide an accurate benchmark to think about the design of a Fiscal Union, and certainly is a good starting point for further research.[29]

[28] Although most times not made explicit, these concerns are nonetheless evident in, for instance, Juncker et al. (2015).

[29] Two aspects are particularly relevant. One the one hand, time inconsistent behaviour can severely affect the implementation of inter-governmental transfers. This can be due to the impossibility to enforce structural reforms and fiscal prudence on a sovereign nation facing fiscal

Finally, the FTS proposed in this paper satisfies point (*iii*) by construction.

1.4.5 Dynamics and Fiscal Policy

In this section, I compare the effects of domestic counter-cyclical policy, on the one hand, and of international fiscal transfers, on the other, to the transmission of sovereign spread shocks. I construct Figure 4 in the following way: I begin by assuming that Portugal and the Eurozone engage in a FTS defined by $\kappa_s = 0.05$. Secondly, I calculate the magnitude of the pass-through of sovereign shocks to the private risk premium under the FTS. I then compute the value of κ_g that, in the absence of the FTS, results in having an equal pass-through as the one calculated in the second step. In other words, I match the effects of sovereign spread shocks on the borrowing costs of private firms between the two policies.

As Figure 4 shows, both policies reduce the pass-through of sovereign spread shocks to private spreads. Although the fall in the price of government bonds is equal regardless of having any policy in place, the net worth of banks falls less in the presence of counter-cyclical policy and the FTS. This effect is due to the stimulus in aggregate demand generated by both policies, which feeds into an increasing demand for capital from private firms. Despite the fact that banks increase the risk premium in order to rebuild their net worth, the stimulus moderates the fall in the demand for credit and allows banks to reduce the pass-through. Consequently, the fall in investment is lessened as well as is the recession, which at the trough becomes nearly 1% milder.

Interestingly, the response of GDP differs between the two policies. Under counter-cyclical government expenditures the trough is more pronounced but the recovery is faster. With the FTS, the fall in GDP and its eventual recovery are more gradual. The same applies to employment, with the recovery under the FTS taking even longer than what would otherwise happen without any policy at place.

stress. If reforms and/or consolidation are not properly executed, the need for foreign transfers might persist, the positive spillovers from improving the fiscal stance might not materialize and the distribution of welfare gains from the FTS can skew easily. The same is true with respect to the *donating* country, which might be better-off not making a transfer when its fellow union member requires. On the other hand, but still related, alternative transfer rules to (44) might altogether improve the potential to implement a FTS. For instance, a mechanism that encompasses *automatic* reforms as well as transfers has the potential to maximize the positive externalities of risk-sharing at the same time as reducing the risks of moral hazard and free riding. These aspects, although extremely relevant, are left for future research.

Clearly, the two policies are not equivalent. With counter-cyclical policy, the deficit grows further and stays higher for longer. With the FTS, the magnitude of the initial jump is very similar to the baseline scenario, as is its evolution towards the steady state. The similar dynamics of the deficit under the FTS and the baseline are due to the fact that, as GDP contracts less under the FTS, the increase in taxes needed to control public debt is also smaller. In effect, fiscal transfers substitute, at least partially, the need for domestic taxation.

In terms of aggregate demand, the FTS generates an indirect stimulus through the need for less taxation, whereas with counter-cyclical policy there is a reshuffle in domestic demand, with government expenditure leading the stimulus. Instead of crowding out other domestic sources of aggregate demand, the FTS appears to be a more efficient stimulus measure. One indicator supporting this claim is inflation, which reacts considerably less compared to the other two scenarios. This is due to the impact of fiscal transfers on the real exchange rate, which mitigates the need for a domestic devaluation and reduces the inefficiencies caused by price changes.

The differences in the operating mechanisms of counter-cyclical policy and the FTS are made patently clear when I make sovereign spreads elastic to the fiscal outlook. In Figure 5 I take the same values for κ_s and κ_g as before, but assume instead that $\Theta = 0.05$, that is, that sovereign spreads react to the ratio of public debt to GDP. In this scenario, sovereign spread shocks vanish relatively slower. Considering counter-cyclical policy, the spread banks charge to firms actually increases when compared to the baseline. Yet, employment and GDP still perform better when compared to the no policy scenario, with the demand effect lead by the government supporting real activity despite the negative impact on banks. The key for this apparently counter-intuitive result lies in the behaviour of asset prices. The price of government bonds falls more abruptly when the government runs an expansion, causing the net-worth of banks to contract more than in the absence of policy. The evolution in the price of government bonds explains why banks are unable to reduce the pass-through of sovereign spreads to the private sector. As before, the FTS outperforms domestic counter-cyclical policy and, most importantly, does not cause the perverse effects on the supply of credit to the economy.

Figure 4 presents, at least partially, the trade off of domestic counter-cyclical policy. For some parametrization of the model, counter-cyclical policy can be welfare improving, insofar as it supports economic activity, reduces inflation and

the miss-allocation of resources. However, the elasticity of sovereign spreads can invert these results and potentially cause a deeper recession. There are, however, important features missing in the model. On the one hand, public expenditures have no productive nor utility enhancing use. On the other hand, taxation is modelled in a very reduced form. A study of optimal domestic public policy in an environment with sovereign spreads and international transfers has to address these aspects. It is left for future research.

1.5 Conclusion

The recent sovereign debt crisis in Europe has tested the resilience of the most ambitious supra-national endeavour seen in the old continent. The viability of the common currency, and of European integration itself, has been openly threatened. The central question has concerned the type and extent of the response the monetary union should give to asymmetric shocks to its member countries. Related to this is yet the question of what level of solidarity can be reasonably expected between members. The answers so far have been in the direction of more integration and discipline, with the Banking Union and the Fiscal Compact being just some examples. Looking ahead, however, the completion of a fully fledged monetary union requires some form of fiscal arrangement at the federal level as well. After all, it was the inability of domestic fiscal policy to tackle sovereign spread shocks in the countries most affected by the crisis that sparked the severe tensions seen within the EMU.

In this paper, I set up and estimate a model capable of providing a consistent narrative of the crisis. The model features financial frictions due to leverage constraints on banks which link the availability of credit to productive firms to the value of bank's net worth. Domestic banks are also the suppliers of credit to the government, therefore being exposed to sovereign credit risk. I illustrate the mechanisms at work during a sovereign spread shock and compare its dynamic effects to the case of Portugal in 2011. I show that the ratio of public debt to GDP and the elasticity of sovereign spreads to the fiscal outlook can substantially magnify the pass-through of sovereign spreads to private borrowers. I also show that counter-cyclical policy is not feasible when sovereign spreads react sharply to a deterioration in public finances and the debt burden is at the levels seen in the periphery of the Eurozone during the crisis.

I contribute to the debate about a future fiscal capacity at the EMU level. I

propose a simple fiscal transfer scheme between member countries triggered when sovereign spreads widen. The scheme acts at the root of the transmission mechanism of spread shocks by alleviating the fiscal strain on the government. At the same time, it provides a stimulus to real activity and reduces the impact of sovereign spreads on private lending rates. The fiscal arrangement I propose improves welfare when countries have symmetric structures, and in particular when the relative size of their economies and the profile of their fiscal stances is almost identical. However, asymmetries across countries induce welfare losses for some members. As a result, the proposed transfers scheme can easily lose political support for its implementation. Nevertheless, I demonstrate through a simple exercise that the welfare gains for a small country, like those at the core of the recent crisis, can be large. Importantly, I show that these gains can be sustained through a scheme under which the costs for the remaining members of the union is significantly smaller than the benefits they secure by sharing the common currency.

This paper provides a realistic set up where asymmetric shocks to a currency union are addressed via a supra-national scheme of fiscal transfers. It shows the large potential gains derived from a simple, reduced form scheme and highlights the fragilities regarding its implementation. Further research needs to investigate the mechanisms by which these fragilities can be reduced. Namely it should explore the spillover consequences of localized asymmetric shocks and understand the role of policy coordination and enforceability at the national level.

1.6 Tables

Table 1: Calibrated parameters

Parameter		Value
Discount factor	β	0.99
Elasticity of substitution Home and Foreign goods	γ	1.00
Capital share on production	α	0.36
Steady state depreciation rate	$\overline{\sigma}$	0.025
Elasticity of capital utilization	$\imath$	1.71
Elasticity of substitution across types of goods	μ_p	11
Elasticity of substitution across types of labour	μ_w	6
Private firms' risk premium	$\overline{r}_x - \overline{r}_h$	0.01
Steady state leverage ratio	$\overline{\phi}$	4
Fraction of divertable assets	ι	0.35
Start-up funds of new banks	ϵ	0.0038
Banker survival probability	λ_f	0.975
Steady state sovereign spread	$\overline{\delta}$	0.002
Home's population share	n	0.03
Foreign to Home per capita GDP	$\overline{gdp}^\star/\overline{gdp}$	1.7
Degree of home bias in Home	ω	0.7
Degree of home bias in Foreign	$\omega^\star$	0.9945
Steady state Government Expenditure to GDP	$\tilde{g}$	0.2
Steady state labour supply	$\overline{l}$	0.33
Weight on labour disutility	ζ	18.86
Steady state Government Debt to GDP	$\tilde{d}_g$	0.6
Sovereign spread elasticity	Θ	0
Fiscal response to public debt	κ_τ	0.15
Government Expenditure response to GDP	κ_g	0
Fiscal transfer scheme	κ_s	0
Share of consumption taxes in revenue	κ_c	0.4

Table 2: Estimation: model parameters

parameter		prior pdf	prior mean	prior s.d.	posterior spec. 1	posterior spec. 2	posterior spec. 3
Habit persistence	ϱ	*gamma*	0.50	0.10	0.51	0.52	0.76
					(0.39 , 0.63)	(0.40 , 0.63)	(0.69 , 0.84)
Habit persistence	$\varrho^\star$	*gamma*	0.50	0.10	0.54	0.55	0.86
					(0.39 , 0.67)	(0.40 , 0.70)	(0.80 , 0.91)
Inv. elast. of labour supply	φ	*gamma*	1.00	0.25	0.82	0.81	0.80
					(0.46 , 1.15)	(0.47 , 1.16)	(0.46 , 1.11)
Inv. elast. of labour supply	$\varphi^\star$	*gamma*	1.00	0.25	1.02	0.99	1.39
					(0.64 , 1.42)	(0.60 , 1.38)	(0.96 , 1.81)
Investment adjust. costs	ψ	*gamma*	2.00	1.00	3.45	3.20	4.59
					(2.11 , 4.79)	(1.89 , 4.56)	(3.22 , 5.92)
Investment adjust. costs	$\psi^\star$	*gamma*	2.00	1.00	2.57	2.57	10.64
					(1.11 , 4.05)	(1.14 , 3.92)	(7.51 , 13.68)
Price stickiness	λ_p	*beta*	0.75	0.05	0.78	0.78	0.76
					(0.73 , 0.83)	(0.73 , 0.83)	(0.70 , 0.83)
Price stickiness	$\lambda_{p^\star}$	*beta*	0.75	0.05	0.71	0.71	0.81
					(0.63 , 0.79)	(0.63 , 0.78)	(0.78 , 0.85)
Indexation to past inflation	ϑ	*beta*	0.20	0.10	0.10	0.10	0.08
					(0.02 , 0.18)	(0.02 , 0.17)	(0.01 , 0.15)
Indexation to past inflation	$\vartheta^\star$	*beta*	0.20	0.10	0.12	0.12	0.02
					(0.02 , 0.21)	(0.02 , 0.20)	(0.01 , 0.03)
Wage stickiness	λ_w	*beta*	0.75	0.05	0.69	0.69	0.75
					(0.63 , 0.75)	(0.64 , 0.78)	(0.69 , 0.78)
Wage stickiness	$\lambda_{w^\star}$	*beta*	0.75	0.05	0.82	0.81	0.54
					(0.76 , 0.89)	(0.76 , 0.87)	(0.49 , 0.60)
Interest rate smoothing	ρ_i	*beta*	0.75	0.10	0.83	0.83	0.95
					(0.80 , 0.86)	(0.80 , 0.86)	(0.94 , 0.96)
Taylor rule inflation	ρ_π	*normal*	1.70	0.20	1.69	1.69	1.69
					(1.44 , 1.93)	(1.44 , 1.92)	(1.40 , 1.98)
Taylor rule GDP growth	ρ_y	*normal*	0.20	0.10	0.52	0.51	0.22
					(0.40 , 0.64)	(0.39 , 0.63)	(0.06 , 0.38)
Cost of foreign position	Γ	*inv. gamma*	0.001	0.005	0.0008	0.0008	0.0008
					(0.0002 , 0.001)	(0.0003 , 0.001)	(0.0003 , 0.001)

Parameters with $\star$ are for the Euro Area, the remaining are for Portugal. Note that there is a common Taylor rule for both regions. The table reports the posterior mean estimates and the 90% credible set.

Table 3: Estimation: persistence parameters

parameter		prior pdf	prior mean	prior s.d.	posterior spec. 1	posterior spec. 2	posterior spec. 3
Intratemporal preferences	ρ_l	*beta*	0.75	0.10	0.34 (0.23 , 0.47)	0.34 (0.22 , 0.46)	0.39 (0.26 , 0.53)
Intratemporal preferences	$\rho_{l^\star}$	*beta*	0.75	0.10	0.66 (0.49 , 0.83)	0.68 (0.54 , 0.82)	0.95 (0.89 , 0.99)
Intertemporal preferences	ρ_c	*beta*	0.75	0.10	0.89 (0.83 , 0.96)	0.89 (0.83 , 0.96)	0.90 (0.82 , 0.98)
Intertemporal preferences	$\rho_{c^\star}$	*beta*	0.75	0.10	0.85 (0.79 , 0.93)	0.85 (0.77 , 0.93)	0.54 (0.37 , 0.70)
Stationary technology	ρ_s	*beta*	0.75	0.10	0.73 (0.56 , 0.90)	0.72 (0.56 , 0.90)	0.97 (0.96 , 0.99)
Stationary technology	$\rho_{s^\star}$	*beta*	0.75	0.10	0.66 (0.46 , 0.86)	0.67 (0.47 , 0.86)	0.82 (0.73 , 0.91)
Investment technology	ρ_z	*beta*	0.75	0.10	0.44 (0.30 , 0.59)	0.44 (0.29 , 0.58)	0.74 (0.65 , 0.84)
Investment technology	$\rho_{z^\star}$	*beta*	0.75	0.10	0.64 (0.53 , 0.75)	0.64 (0.54 , 0.74)	0.75 (0.62 , 0.89)
Cost-push	ρ_{λ_p}	*beta*	0.75	0.10	0.72 (0.57 , 0.87)	0.72 (0.58 , 0.86)	0.74 (0.58 , 0.90)
Cost-push	$\rho_{\lambda_{p^\star}}$	*beta*	0.75	0.10	0.69 (0.50 , 0.87)	0.69 (0.52 , 0.87)	0.72 (0.57 , 0.89)
Government expenditure	ρ_g	*beta*	0.75	0.10	0.94 (0.91 , 0.97)	0.79 (0.70 , 0.87)	0.94 (0.91 , 0.98)
Government expenditure	$\rho_{g^\star}$	*beta*	0.75	0.10	0.94 (0.90 , 0.98)	0.92 (0.88 , 0.96)	0.90 (0.85 , 0.95)
Sovereign risk premium	ρ_δ	*beta*	0.75	0.10	0.92 (0.89 , 0.96)	0.92 (0.89 , 0.96)	0.93 (0.90 , 0.97)
Measurement error (h)	ρ_e	*beta*	0.75	0.10	0.84 (0.77 , 0.90)	-	-
Measurement error (f)	$\rho_{e^\star}$	*beta*	0.75	0.10	0.90 (0.86 , 0.95)	-	-

Parameters with $\star$ are for the Euro Area, the remaining are for Portugal. Note that there is a common Taylor rule for both regions. The table reports the posterior mean estimates and the 90% credible set.

Table 4: Estimation: standard deviations

parameter		prior pdf	prior mean	prior s.d.	posterior spec. 1	posterior spec. 2	posterior spec. 3
Intratemporal preferences	σ_l	*gamma*	0.200	0.150	0.470	0.465	0.676
					(0.248 , 0.677)	(0.258 , 0.667)	(0.382 , 0.962)
Intratemporal preferences	$\sigma_{l^\star}$	*gamma*	0.200	0.150	0.304	0.242	0.048
					(0.066 , 0.714)	(0.081 , 0.420)	(0.033 , 0.066)
Intertemporal preferences	σ_c	*gamma*	0.010	0.0075	0.022	0.022	0.051
					(0.016 , 0.027)	(0.016 , 0.028)	(0.037 , 0.065)
Intertemporal preferences	$\sigma_{c^\star}$	*gamma*	0.010	0.0075	0.013	0.013	0.035
					(0.009 , 0.017)	(0.009 , 0.018)	(0.023 , 0.047)
Stationary technology	σ_s	*gamma*	0.020	0.015	0.020	0.020	0.064
					(0.001 , 0.039)	(0.001 , 0.039)	(0.043 , 0.086)
Stationary technology	$\sigma_{s^\star}$	*gamma*	0.020	0.015	0.010	0.009	0.172
					(0.001 , 0.017)	(0.002 , 0.016)	(0.128 , 0.215)
Investment technology	σ_z	*gamma*	0.050	0.045	0.146	0.134	0.127
					(0.093 , 0.200)	(0.082 , 0.187)	(0.094 , 0.160)
Investment technology	$\sigma_{z^\star}$	*gamma*	0.050	0.045	0.041	0.041	0.059
					(0.025 , 0.059)	(0.025 , 0.057)	(0.025 , 0.094)
Cost-push	σ_{λ_p}	*gamma*	0.020	0.015	0.052	0.052	0.015
					(0.012 , 0.085)	(0.015 , 0.085)	(0.001 , 0.029)
Cost-push	$\sigma_{\lambda_{p^\star}}$	*gamma*	0.020	0.015	0.008	0.007	0.013
					(0.001 , 0.016)	(0.001 , 0.014)	(0.001 , 0.025)
Government expenditure	σ_g	*gamma*	0.010	0.0075	0.013	0.053	0.013
					(0.011 , 0.014)	(0.046 , 0.059)	(0.011 , 0.015)
Government expenditure	$\sigma_{g^\star}$	*gamma*	0.010	0.0075	0.008	0.023	0.024
					(0.007 , 0.010)	(0.020 , 0.026)	(0.020 , 0.027)
Monetary shock	σ_i	*gamma*	0.001	0.0005	0.001	0.001	0.002
					(0.001 , 0.001)	(0.001 , 0.001)	(0.002 , 0.003)
Sovereign risk premium	σ_δ	*gamma*	0.020	0.010	0.016	0.017	0.016
					(0.014 , 0.019)	(0.014 , 0.019)	(0.014 , 0.018)
Trend technology	σ_u	*gamma*	0.020	0.010	0.008	0.008	0.02
					(0.006 , 0.011)	(0.006 , 0.010)	(0.015 , 0.025)
Measurement error (h)	σ_e	*gamma*	0.005	0.0015	0.008	-	-
					(0.007 , 0.009)		
Measurement error (f)	$\sigma_{e^\star}$	*gamma*	0.005	0.0015	0.004	-	-
					(0.003 , 0.005)		
marginal likelihood (Laplace)					3678.76	3214.72	2817.20
marginal likelihood (Harmonic mean)					3681.30	3216.74	2819.07
average acceptance rate					0.29	0.32	0.34

Parameters with $\star$ are for the Euro Area, the remaining are for Portugal. Note that there is a common Taylor rule for both regions. The table reports the posterior mean estimates and the 90% credible set.

Table 5: Variance Decomposition

	data	model	Pref.	Tech.	Inv.	Gov.	C.P.	Mon.	Def.	Abroad	M.E.
Δgdp	1.03	1.16	19.2	17.2	45.1	2.2	1.4	9.7	1.0	3.5	0.7
$\Delta gdp^\star$	0.68	1.05	6.5	35.4	38.1	2.0	7.1	10.5	0.0	0.0	0.6
Δc	1.14	1.09	84.2	9.0	0.7	0.1	0.2	2.4	0.0	3.3	0.0
$\Delta c^\star$	0.55	0.60	45.9	27.8	15.7	1.2	2.2	7.0	0.0	0.0	0.2
Δz	4.44	4.03	2.4	3.1	83.8	0.0	1.1	4.1	2.1	3.2	0.0
$\Delta z^\star$	2.31	2.81	2.6	22.5	62.8	0.1	5.6	6.5	0.0	0.0	0.0
Δg	1.24	1.01	0.0	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
$\Delta g^\star$	0.84	0.76	0.0	0.0	0.0	100.0	0.0	0.0	0.0	0.0	0.0
Δw	1.37	0.68	72.2	21.3	1.3	0.0	3.1	0.2	0.0	1.9	0.0
$\Delta w^\star$	0.29	0.37	15.5	51.4	12.2	0.2	17.8	2.8	0.0	0.0	0.1
$\Delta \pi$	0.44	0.40	18.6	24.5	5.1	0.2	3.6	13.9	0.2	33.8	0.1
$\Delta \pi^\star$	0.29	0.34	9.7	20.8	35.3	0.7	16.8	16.4	0.0	0.0	0.2
i_b	0.60	0.58	0.0	4.4	0.0	0.0	0.0	1.5	60.6	33.3	0.1
$i^\star$	0.43	0.36	8.9	12.6	70.6	1.4	1.9	4.3	0.0	0.1	0.3

Standard deviations are in percent. Standard deviations implied by the model and the unconditional variance decomposition are performed at the posterior mean estimates of the model's parameters. Shocks are aggregated as explained in the main text.

Table 6: Two Equal-sized Regions

d_g	0.6	1.2	0.6	0.6	0.6	0.6
Θ	0	0	0.01	0	0	0
κ_t	0.2	0.2	0.2	0.15	0.2	0.2
κ_g	0	0	0	0	-0.05	0
$\sigma_\delta^\star$	σ_δ	σ_δ	σ_δ	σ_δ	σ_δ	$0.95 \times \sigma_\delta$
			welfare gains (% CE consumption)			
κ_s	7.27	7.88	8.99	7.58	4.04	7.27
Core	4.08	4.50	0.00	4.08	1.49	-3.95
Periphery	0.72	0.49	5.76	0.88	0.33	8.91

The table reports unconditional welfare gains measured as % of certainty equivalent consumption. The values of κ_s reported correspond to the maximizers of aggregated welfare, assuming $n = 0.5$ and $gdp^\star/gdp = 1$. Unless otherwise stated, $d_g^\star = 0.6$, $\Theta^\star = 0$, $\kappa_t^\star = 0.2$, $\kappa_g^\star = 0$.

Table 7: One Small Open Country in a Wider Monetary Union

n	3%	12%	18%
$gdp^\star/gdp$	1.7	1.2	1
d_g	0.6	0.6	0.6
Θ	0	0	0
κ_t	0.2	0.2	0.2
κ_g	0	0	0
welfare gains (% CE consumption)			
trade gains of 1% CE consumption			
κ_s	0.006	0.018	0.023
Eurozone	-0.03	-0.14	-0.22
Periphery	1.44	1.05	0.86
trade gains of 5% CE consumption			
κ_s	0.032	0.091	0.115
Eurozone	-0.15	-0.68	-1.09
Periphery	7.80	5.38	4.44

The table reports unconditional welfare gains measured as % of certainty equivalent consumption. The values of κ_s are such that the welfare losses of the Eurozone are no bigger that the trade gains corresponding to the Periphery entering the union (assuming trade gains are equal across all union members).

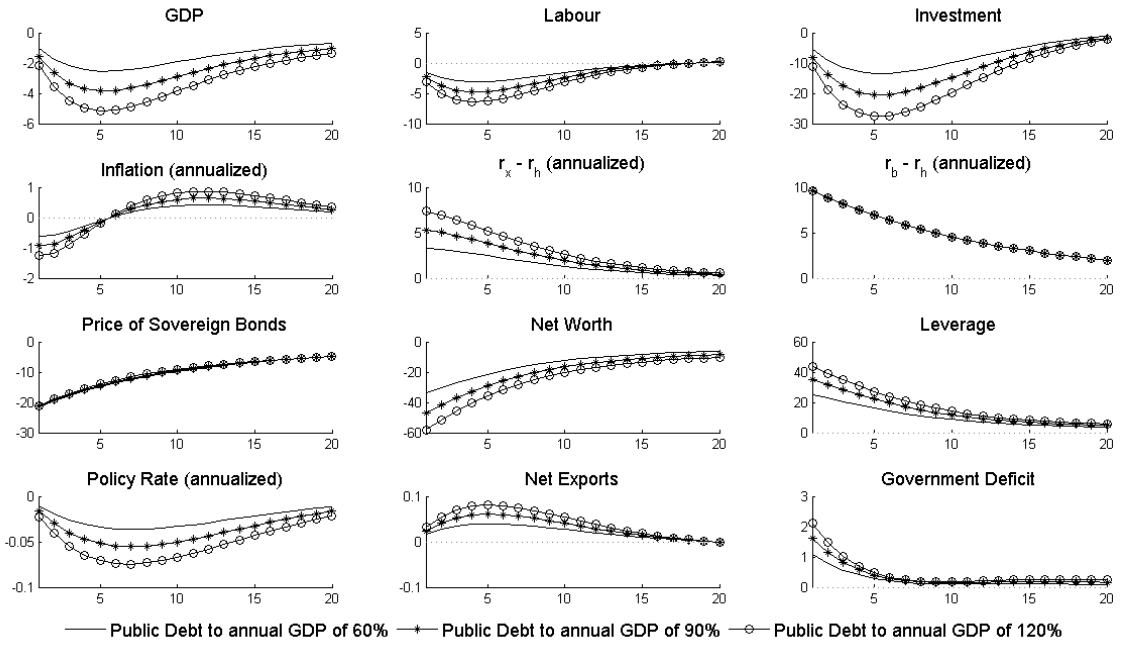
Table 8: The Case of Portugal

n	3%	3%	3%	3%	3%
$gdp^\star/gdp$	1.7	1.7	1.7	1.7	1.7
d_g	0.6	1.2	0.6	0.6	0.6
Θ	0	0	0.01	0	0
κ_t	0.2	0.2	0.2	0.1	0.2
κ_g	0	0	0	0	-0.05
welfare gains (% CE consumption)					
κ_s	0.0064	0.0064	0.0060	0.0064	0.0064
Eurozone	-0.031	-0.031	-0.031	-0.031	-0.031
Portugal	1.515	1.521	1.521	1.535	1.575

The table reports unconditional welfare gains measured as % of certainty equivalent consumption. The values of κ_s are such that the welfare losses of the Eurozone are no bigger that the trade gains corresponding to Portugal entering the union.

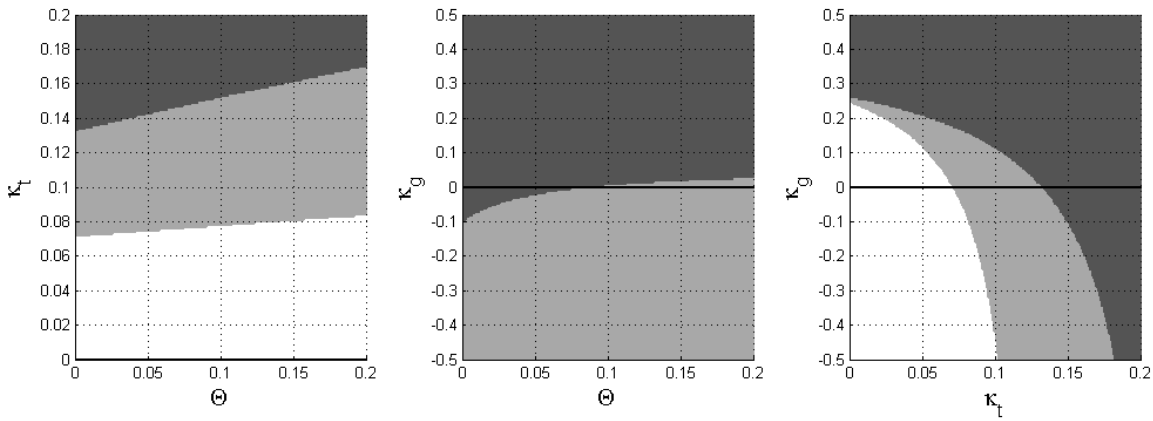
1.7 Figures

Figure 1: Responses to a Sovereign Spread Shock



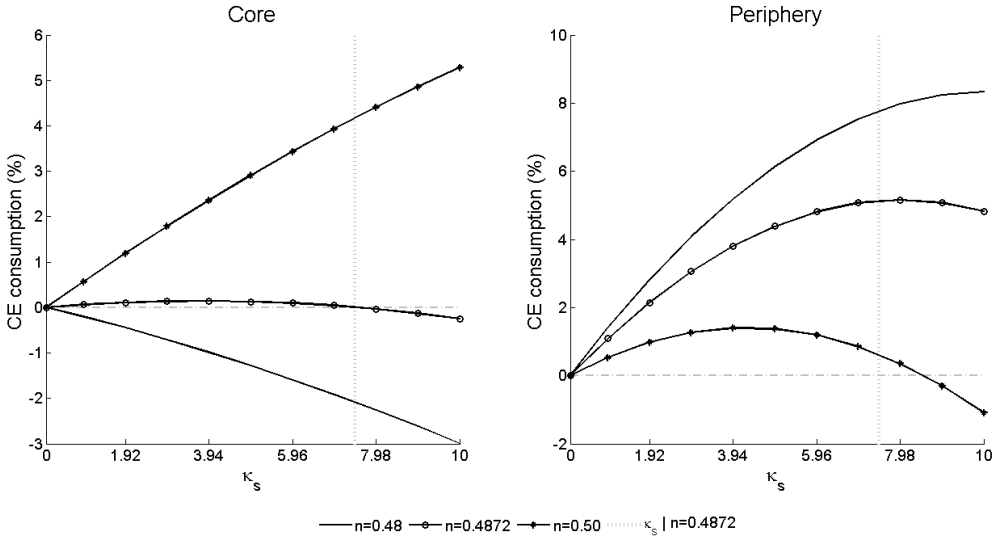
Impulse responses are expressed in terms of percet deviations from the steady state, except for the governmnet deficit and net exports, which are expressed in levels. Default values of $\kappa_t = 0.15$, $\kappa_g = 0$ and $\Theta = 0$.

Figure 2: Determinacy Regions



κ_t: elasticity of taxes to government debt to GDP, κ_g: responce of government expenditures to GDP and Θ: elasticity of sovereign spread to the fiscal outlook. Grey areas represent determinacy regions. Both areas represent determinacy for government debt to annual GDP of 60%. For government debt to annual GDP of 120%, determinacy occurs only for the dark grey areas. Default values of $\kappa_t = 0.15$, $\kappa_g = 0$ and $\Theta = 0$.

Figure 3: Welfare in a (almost) Symmetric World*



*Estimated parameters and standard deviations differ across regions. Besides assuming $n = 0.5$ and $gdp^{\star}/gdp = 1$, the Core is assumed to have a government sector identical to the Periphery, including sovereign spread shocks.

Figure 4: Dynamic Impact of Fiscal Policy ($\Theta = 0$)

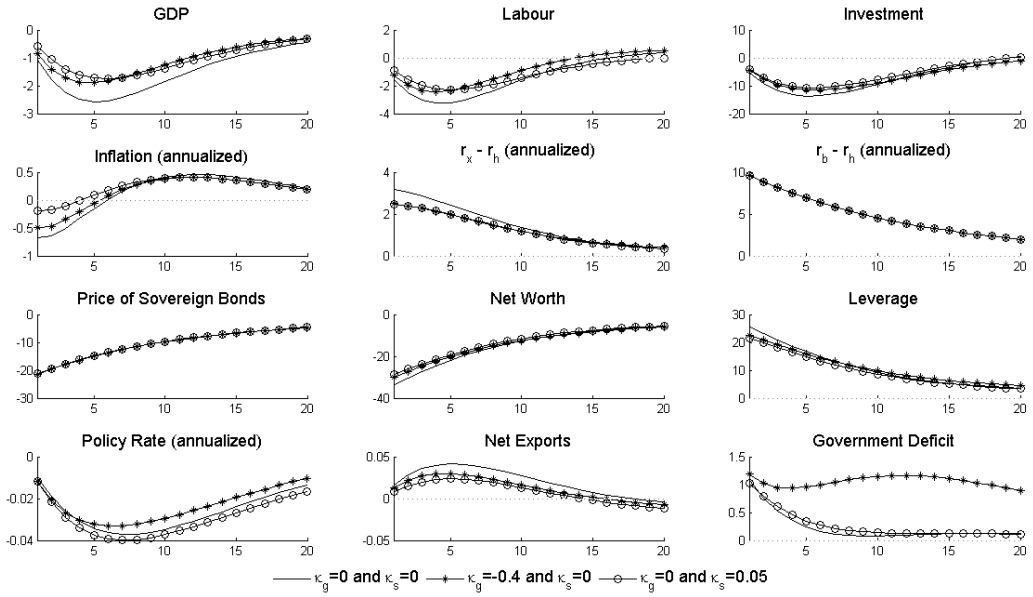
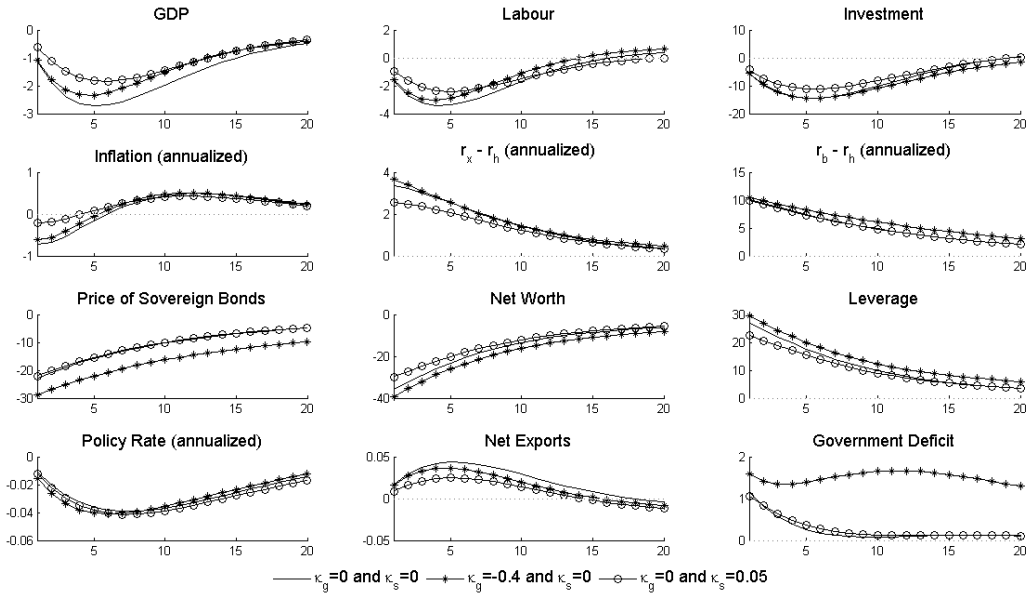
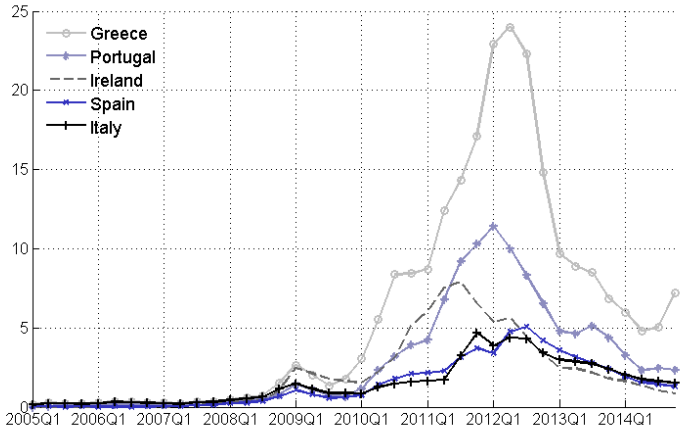


Figure 5: Dynamic Impact of Fiscal Policy ($\Theta = 0.05$)



Impulse responses are expressed in terms of percet deviations from the steady state, except for the governmnet deficit, which is expressed in levels. Default values of $\kappa_t = 0.2$ and public debt to annual GDP of 60%.

Figure 6: 10-year Sovereign Spreads (%)



Spreads computed against the 10-year yields of German bunds. Source: Eurostat.

Chapter 2

Exogenous Shocks and Endogenous States

Keywords: Markov-switching VAR; sign-restricted SVAR; terms-of-trade shocks.
JEL Classification: C32, E32, F41.

Firenze, November 2013

2.1 Introduction

It is plausible to suspect that the reaction of a small open economy to terms-of-trade shocks differs across time. These differences can be due to a number of factors, ranging from the characteristics of the country's economic structure and institutional framework, to factors largely determined by exogenous conditions. In this paper, I set up an empirical model to investigate the existence of these differences in the context of a commodity exporter. The model is specifically designed to achieve two objectives. Firstly, it accounts for structural changes pertaining to the domestic economy. These changes are deemed endogenous insofar

as they reflect changes in the stochastic processes of domestic variables. Secondly, it distinguishes terms-of-trade shocks according to their foreign causes, which are exogenous to the small open economy. The careful identification of terms-of-trade shocks is particularly important because different shocks can produce different outcomes within the domestic economy. Hence, by accurately distinguishing different foreign shocks, the model I propose is able to investigate the differences in the transmission of terms-of-trade shocks due to endogenous changes alone. As I show through a case study featuring Australia, isolating these differences not only has empirical relevance, it also has important policy implications.

Countries in which the resource sector represents a large share of total output or of total exports are particularly dependent on how commodity prices evolve in world markets. Consider, for instance, an increase in international commodity prices. For a small commodity-exporting economy, this triggers a shift in investment towards the resource sector. Production factors, including labour, are transferred to the commodity sector, which expands at the pace of the bonanza generated by the increasing value of commodity revenues. At the same time, an income effect, both reflected in terms of private earnings and in terms of tax collection, will induce an increase in aggregate demand. As the price of tradeable goods and services is fixed to global prices, the exchange rate is forced to appreciate. Consequently, foreign-produced tradeables become more competitive compared to those produced domestically. This prompts a sectoral transformation, whereby the production of non-tradeable goods expands and imports increase at the expense of a contraction of the domestic tradeable sector.

The simple economic trends outlined above are, however, dependent on various domestic characteristics, including structural and related to policy. Regarding the former, suppose that wage setting is centralized, with contracts and bargaining determined and enforced homogeneously across different sectors. Labour demand pressures in the commodity sector would then spillover to the remaining industries, pushing wages upwards economy-wide. This would increase production costs and further harm the competitiveness of the tradeable sector. It could inclusively lead to total output losses, generate higher inflation and a rise in unemployment. On the other hand, consider monetary policy being mandated to keep the exchange rate fixed. In this circumstance, the purchasing power of domestic households vis-à-vis foreign-produced tradeables does not improve. As the commodity sector expands and absorbs production factors, the supply of domestically-produced goods might not be able to respond to the increase in demand by domestic agents.

As a result, prices wil grow. With monetary policy tied to keep the nominal exchange rate pegged, inflation will rise until the real exchange rate adjusts fully to the shock.

These examples illustrate how multiple factors within a small open economy can generate disparate results in terms of the transmission of commodity price shocks. Yet, domestic characteristics are not the only source behind these differences. Shocks to commodity prices that have distinct causes can also have different consequences to a commodity exporter. For example, a shock to commodity prices caused by an expansion in world demand comes together with a general increase in world prices, notably of tradeable goods. In this scenario, the terms of trade do not improve necessarily, insofar as the increase in both prices is of the same magnitude. The implications of such a shock are significantly different from those relative to a shock caused by a drop in the production of commodities elsewhere or due to an increase in precautionary demand. These shocks do not generate an increase in the price of tradeables and therefore do not contribute directly to an increase in domestic inflation.

In this paper, I propose an empirical model consisting of a structural VAR with two blocks: one with variables describing the world market for commodities and one containing domestic variables. Importantly, I impose an exogeneity restriction with regards to the world block, isolating world variables from developments within the small open economy. In order to disentangle the variation in the transmission of shocks accruing to endogenous and exogenous factors, I combine two econometric tools for time-series analysis. First, to account for changes within the small open economy, the domestic block is assumed to follow a Markov-Switching (MS) process. As such, the parameters of the VAR for the domestic variables can change according to a latent variable describing which regime the economy is in. Furthermore, the estimated probabilities of each regime occurring at a given point in time are entirely driven by the data.

Second, the identification of the structural shocks is done through Sign-Restrictions (SR), which are imposed on the world block only. SR are a flexible method to extract an economic interpretation of structural shocks as they can be implemented on the impulse response of relevant variables upon the impact of a shock, on any following period, and on the sign and magnitude of the response. Moreover, identification through SR does not rely on possibly unlikely zero restrictions implied by traditional methods. On the downside, SR do not provide an exact identification of structural shocks as there is a continuum of structural models complying

with the same SR. Notwithstanding this, I am able to isolate the changes in the transmission of structural shocks across different domestic states by imposing SR only on the world block and assuming that the world block is time-invariant. In this way, I am able to measure the effects of state changes for each structural model individually.

I provide an illustration of the empirical model through an application to Australia. Rich in mineral resources, notably iron ore and coal, commodity exports account for around 40% of total exports. During the last decade, the rapid growth of emerging markets, and of China in particular, caused a boom in commodity prices that positioned Australia at the forefront of a large and lasting economic expansion. However, positive shocks to commodity prices did not always result in successful outcomes, with the episodes in the late 1970's being rather disastrous. The macro policy framework and the economic structure of the Australian economy also changed considerably between these two experiences. I therefore use the model to account for these changes within Australia and assess the transmission of commodity price shocks.

Using a sample ranging between 1971 and 2010, I find a clear change in regime in late 1990. This change is consistent with the transition to inflation targeting that was officially announced two years later. It also coincides with the implementation of a number of gradual reforms aimed at decentralizing wage negotiations. In terms of impulse responses, I find that after the change in regime, real GDP and inflation react less abruptly and adjust quicker to terms-of-trade shocks. I also confirm that not all terms-of-trade shocks are alike. In particular, commodity price shocks that are driven by supply disruptions in world markets have a negative impact on output, a result also described for Canada by Charnavoki and Dolado (2014). Moreover, as the estimated impact on inflation appears to be modest, my findings support the view that, during these shocks, monetary policy can provide further support to the economy.

Literature: There is a growing literature exploring time-variation of VAR parameters to trace the differences in impulse responses across time. Using a Bayesian MS-VAR, Rubio-Ramírez, Waggoner and Zha (2005) study whether the beginning of the European Monetary Union changed the conduct of monetary policy. They use different identification strategies (including SR) to obtain structural impulse responses. Sims and Zha (2006), and Sims, Waggoner and Zha (2008) investigate whether monetary policy in the US has changed using a MS mechanism as well. In turn, Baumeister and Peersman (2010) estimate a VAR with time-varying

parameters on oil market variables and impose SR to identify structural shocks. Camacho and Pérez-Quirós (2013) use Cholesky decomposition in a MS-VAR to study the impact of commodity prices on the GDP of Latin American commodity exporters. They show that commodity price shocks are procyclical and that positive shocks are more expansionary when these economies are in recession. Apart from those using exactly-identified strategies, like Cholesky, most of these works focus on state-dependent structural models for the identification of structural shocks. The majority also imposes identification restrictions on the impulse responses of relevant variables. In this paper, I argue that a better comparison of time differences in objects like impulse responses and variance decompositions should only take into account the differences arising from the data, and not those imposed by time-dependent structural identification strategies.

In a paper investigating the impacts of oil shocks, Kilian (2009) is the first to highlight the differences in the response of US variables to shocks with distinct underlying causes. Using Cholesky decomposition, he shows that an increase in oil prices caused by an expansion in the world economy does not cause the typical economic disruptions associated with increases in oil prices prompted by supply shortages. Peersman and Van Robays (2009) defend that SR achieve a more plausible identification of different oil shocks. They focus on the Euro-area and demonstrate that although all oil shocks push inflation up, oil shocks caused by an increase in world economic activity produce a transitory expansion, whereas oil shocks caused by a disruption in oil supply lead to a permanent drop in output. In a closely related paper, Jääskelä and Smith (2011), JS11 henceforth, use SR to identify terms-of-trade shocks to the Australian economy and find that these are not necessarily inflationary. In particular, they find that shocks specific to the commodity market tend to lower inflation and to expand total output. I follow this strand of the literature to motivate the identification scheme I use in the application to Australia.

Isolating differences in impulse responses due to regime changes from those due to the structural identification of shocks is particularly important when using SR. As Fry and Pagan (2011) point out, even in time-invariant models, SR can generate potential inconsistency problems due to the non-uniqueness of structural models complying with a unique set of SR. In particular, computing objects like impulse responses using the median across different models, as is typically done in the literature, can deliver misleading results insofar as the "median response" is not consistent with any single structural model. This problem is clearly exacerbated

when structural models are state-dependent. For instance, Lane, Lütkepohl and Maciejowska (2010), Herwartz and Lütkepohl (2011), and Lütkepohl and Netšunajev (2012) show that the number of identification restrictions needed to be verified increases for models with time-varying volatility. This is because the set of identification restrictions that is satisfied in a given state may not be in other states. These additional restrictions greatly complicate the identification of structural models complying with SR in all states. By successfully addressing these inconsistency problems, the model I propose is well equipped to the use of SR to study changes in the transmission of foreign shocks to a small open economy.

The rest of the paper is organized as follows. The next section describes the econometric model and explains how comparison of impulse responses and variance decompositions across different periods can be done in a coherent manner. In section 3, I present the empirical application. I start by testing the main assumptions implied by the model. I then interpret the estimated MS states and discuss the differences I find in the time-varying impulse responses and variance decompositions. The last section concludes.

2.2 Model

This section lays out the empirical model. The econometric framework has three main features: First, the model contains two blocks of variables, with one block begin exogenous with respect to the other. This restriction makes the exogenous block immune from development occurring in the endogenous block, while maintaining the inverse channel open: the process for the endogenous block is still affected by the exogenous variables. Second, the endogenous block is assumed to follow a MS process, with the VAR parameters (including the variance matrix) evolving according to a latent variable dictating regime changes. The probabilities of being in a given regime are estimated and, therefore, entirely data-dependent. Third, the structural shocks are identified through SR on the exogenous block. Due to the exogeneity restriction, the identification of structural shocks is time invariant and, as a result, the transmission of shocks through the endogenous block can be compared across regimes in a consistent manner.

The flexibility of the model proposed in this paper allows for its application to a wide range of empirical questions. Apart from the one investigated here, there are a number of other examples where it is relevant to understand the differences in the propagation of exogenous shocks due to structural change. The model has

the advantage of combining a flexible and economically sound identification of structural shocks with a model structure that guarantees a consistent comparison of objects like impulse responses or variance decompositions across different time periods. In other words, it is able to isolate the differences in the propagation of shocks that are due to structural change alone.

2.2.1 A Two-Block VAR

The empirical model can be succinctly represented in the form of a standard VAR, which I write as

$$Y_t = \alpha + \sum_{i=1}^{p} A_i Y_{t-i} + e_t \tag{1}$$

The VAR is composed of two blocks of variables, one containing variables describing the exogenous conditions in the world markets for commodities and one describing the domestic conditions of the small commodity-exporting economy. There are n world variables and d domestic variables. I can partition the VAR into $Y_t = \begin{bmatrix} W_t & D_t \end{bmatrix}'$, where W_t is the $n \times 1$ vector of world variables, and D_t represents the $d \times 1$ domestic block. Finally, α is a $(n+d) \times 1$ vector of intercepts, $e_t \sim (0, \Sigma_e)$ the $(n+d) \times 1$ vector of residuals, the sample size is T, and p is the number of lags in the VAR.

The exogenous block: The world block is assumed not to be influenced by developments occurring within the domestic block. Additionally, the world bock is also assumed not to be subject to state changes. Therefore, its dynamic behaviour can be represented by a standard linear VAR.

The assumption that the world block does not depend on the domestic variables implies that the coefficient matrices in (1) can be defined as

$$A_i = \begin{bmatrix} A_i^{WW} & 0_{n \times d} \\ A_i^{WD} & A_i^{DD} \end{bmatrix} \tag{2}$$

with A_i^{WW}, A_i^{WD}, and A_i^{DD} of dimensions $n \times n$, $d \times n$, and $d \times d$, respectively. Given (2), the world block can be represented as a VAR of the form

$$W_t = \alpha^W + \sum_{i=1}^{p} A_i^{1,W} W_{t-i} + e_t^W \tag{3}$$

with $e_t^W \sim \mathcal{N}(0, \Sigma_{e^W})$. The expression (3) can be directly estimated by OLS.

The endogenous block: On the contrary, the process for the domestic time series can involve regime changes[1]. To account for this, the domestic block is modeled as a MS-VAR. While allowing for non-linearities, the MS mechanism is an attractive device to identify economically meaningful states across the sample time. J. Hamilton was the first to apply the MS framework to the study of non-linearities in economic time-series. Hamilton (1989) studies the behaviour of the US real GDP and shows that MS states deliver a good description of US recessions and expansions. Hamilton (1990) presents an algorithm to obtain maximum likelihood estimates of the parameters following a MS process. This algorithm is also used here and details can be found in the Appendix.[2]

With regards to the domestic block, its VAR representation is of the form

$$D_t = \alpha_s^D + \sum_{i=1}^{p} \left[A_{s,i}^{WD} W_{t-i} + A_{s,i}^{DD} D_{t-i} \right] + e_{t,s}^D \tag{4}$$

with $e_{t,s}^D \sim \mathcal{N}\left(0, \Sigma_{e^D,s}\right)$. Note the inclusion of exogenous regressors in (4). These are the lags of the world variables through which the exogenous shocks impact the domestic economy.[3]

The subscript s in (4) denotes the MS state, which depends on a latent variable $s_t = \{1, \ldots, s, \ldots, S\}$. The unobservable s_t evolves according to an ergodic Markov-chain process, with s_t taking a particular value with conditional probability

$$P\{s_t = j | s_{t-1} = i, s_{t-2} = k, \mathcal{Y}_{t-1}, W_t\} = P\{s_t = j | s_{t-1} = i\} = p_{ji} \geq 0$$

These probabilities are collected in the transition matrix P. Each element of P located at the intersection of row j with column i corresponds to the transition probability that state i will be followed by state j. Obviously, $\sum_{j=1}^{S} p_{ij} = 1$. The state probabilities are entirely driven by the data during the estimation routine. Consequently, the econometrician does not impose *a priori* choices regarding the

[1] For simplicity, the terms *regime* and *state* are used interchangeably in this paper.

[2] An excellent textbook exposition of MS and the EM algorithm can be found in Krolzig (1997).

[3] While (3) is a standard VAR, (4) is a VAR with additional exogenous regressors. Throughout the paper, whenever the domestic block is mentioned, it is meant to refer to the specification in (4).

timing of structural breaks or state switches. Instead, the only task left is to interpret the economic content behind the different states and the timing of their transition. Finally, I assume that the model is stable and with white-noise Gaussian residuals $e^D_{t,s}$.

The notation in (4) is not meant to imply necessarily that all the parameters differ between states. As in Krolzig (1997), some parameters might be assumed to remain constant. In Lütkepohl and Netšunajev (2012), for instance, only $\Sigma_{e^D,s}$ is state dependent, whereas in Sims, Waggoner and Zha (2008) all parameters are allowed to change. An important aspect regarding the choice of which parameters can change has to do with the economic interpretation of the various regimes. For instance, allowing $A^{WD}_{s,i}$ to change might result in having the MS states to follow a pattern more in line with how the domestic block is influenced by world variables, rather than identifying states specifically related to the interaction of domestic variables. I check the robustness of the interpretation I give to the MS states by estimating (4) under different restrictions on the parameters that are allowed to change .

2.2.2 Extracting Structural Shocks

To draw conclusions about how domestic variables respond to a shock, it is first necessary to identify what the shock in question exactly represents. A simple procedure to achieve identification is to rewrite (1) in such a way that the structural shocks become orthogonal. This first step makes it possible to analyze one shock at the time, given that each shock is now uncorrelated with the remaining ones. A second step consists in giving an economic interpretation to the structural shocks. Due to the existence of a continuum of normalizations delivering orthogonal shocks, I can constrain the set of normalizations to those for which the shocks induce a particular dynamic behaviour of the time series. Insofar as these dynamics are consistent with sound economic predictions, I can derive an economic interpretation to each shock.

Once the system (1) is normalized, it gives rise to a structural VAR of the form

$$\mathcal{A}_0 Y_t = \sum_{i=1}^{p} \mathcal{A}_i Y_{t-i} + \varepsilon_t \tag{5}$$

with $\varepsilon_t \sim (0, \Sigma_\varepsilon)$ representing the structural shocks. The normalization of (1) is done through the matrix $\mathcal{A}_0$, with $\mathcal{A}_i = A_i \mathcal{A}_0,\ i > 0$ and $\varepsilon_t = \mathcal{A}_0 e_t$. For

simplicity, it is useful to make $\Sigma_\varepsilon = I$, imposing that the structural shocks are not only uncorrelated but also with standard deviations equal to 1. Lastly, one further restriction needs to be imposed on $\mathcal{A}_0$: in order to maintain the independence of the world block from domestic variables in (5), $\mathcal{A}_0$ has to be such that, for any i,

$$\mathcal{A}_i = \begin{bmatrix} \mathcal{A}_i^{WW} & 0_{n\times d} \\ \mathcal{A}_i^{WD} & \mathcal{A}_i^{DD} \end{bmatrix} \tag{6}$$

This restriction is satisfied if, for instance, $\mathcal{A}_0$ is lower triangular. In the literature, the default choice for $\mathcal{A}_0$ is the Cholesky decomposition of Σ_e, $\mathcal{C}$. Cholesky is appealing because, as $\Sigma_e = \mathcal{C}\mathcal{C}'$, setting $\mathcal{A}_0 = \mathcal{C}^{-1}$ not only renders the structural shocks orthogonal, it also makes $\Sigma_\varepsilon = I$. More importantly still, as $\mathcal{C}$ is a lower triangular matrix, (6) is also satisfied.

However, setting $\mathcal{A}_0 = \mathcal{C}^{-1}$ might not be sufficient to achieve a satisfactory identification of structural shocks. As has been discussed in the literature[4], the dynamics associated with the shocks obtained through the Cholesky normalization might fail to match the predictions provided by economic theory. I therefore propose the use of SR on the impulse responses of the world variables in order to obtain a valid identification.

SR amount to imposing further restrictions on $\mathcal{A}_0$ such that the impulse responses of selected variables to a given structural shock comply with economic theory in terms of sign, magnitude, and persistence. To implement SR, note first that, for any orthogonal matrix $\tilde{P}$, $\Sigma_e = \mathcal{C}\tilde{P}\tilde{P}'\mathcal{C}' = \mathcal{C}\mathcal{C}'$.[5] As such, $\mathcal{C}\tilde{P}$ is also a valid candidate to obtain (5). Moreover, because $\mathcal{A}_0^{-1} = \mathcal{C}\tilde{P}$ and $\mathcal{A}_i = A_i\mathcal{A}_0,\ i > 0$, different $\tilde{P}$ imply different structural dynamics of (5). Therefore, I can parametrize $\tilde{P}$ in such a way that the responses of the world variables to the structural shocks comply with the chosen SR scheme. Finally, $\tilde{P}$ also must comply with (6), which is the case if[6]

$$\tilde{P} = \begin{bmatrix} \tilde{P}^{WW} & 0_{n\times d} \\ \tilde{P}^{WD} & \tilde{P}^{DD} \end{bmatrix} \tag{7}$$

Assuming that the world variables are independent from developments within

[4]Peersman and Van Robays (2009) provide a discussion on how SR seem more appropriate than zero restrictions in order to identify structural shocks in a context closely related to the one here.

[5]By definition, if $\tilde{P}$ is an orthogonal matrix, $\tilde{P}\tilde{P}' = I$.

[6]Refer to the Appendix for the technical details regarding the implementation of the sign restrictions.

the domestic block implies that the coefficients and the covariance matrix in (3) are state independent. This simplifies the implementation of the SR and the computation of impulse responses. Consider again Σ_e, the covariance matrix of model (1). Its top-left $n \times n$ block contains Σ_{eW}, which is state independent. Moreover, due to the parametrization in (2) and (3), and to the fact that the Cholesky decomposition of Σ_e is a lower triangular matrix, the dynamics of the world variables depend only on the reduced form shocks e_t^W. Hence, searching for the decompositions of Σ_e that comply with a given set of SR is equivalent to searching for the decompositions of Σ_{eW} that deliver the same result. Hence, to extract structural shocks it is not necessary to consider the domestic block.[7]

Apart from easing the computational burden, imposing SR only on the responses of world variables is also crucial from an empirical point of view. On the one hand, if the impulse responses of the domestic variables were restricted, the conclusions about the effects of endogenous regime changes would be biased by construction. On the other hand, if the structural shocks were extracted within the MS-VAR block, it would have to be necessary to check the SR for each state at a time. Two approaches would then be possible: First, to accept only the decompositions that respect the SR in all states. Second, to accept the decompositions that respect SR for each state individually.[8] On the contrary, in the model proposed here, the SR need to be validated only once, as the same decomposition is valid for all S states.

As discussed by Fry and Pagan (2011), each $\tilde{P}_k$ can be seen as a particular structural model.[9] Hence, comparing the dynamics associated with different decompositions amounts to comparing different structural models. My objective, however, is to compare the dynamics associated with regime changes, and not due to different decompositions. Hence, because all decompositions that satisfy the SR are valid for all states, I guarantee that only the data is responsible for the differences

[7] The Cholesky decomposition of Σ_e is an upper triangular matrix for which its top-left $n \times n$ block corresponds to the Cholesky decomposition of Σ_{eW}.

[8] Suppose that the SR scheme involves κ restrictions. The first approach then implies checking $S \times \kappa$ restrictions in total, which is computationally demanding and practically unfeasible (refer to Lane, Lütkepohl and Maciejowska, 2010; Herwartz and Lütkepohl, 2011; and Lütkepohl and Netšunajev, 2012 for further discussion). The second option is the approach followed by Rubio-Ramírez, Waggoner and Zha (2005), Sims and Zha (2006), and Sims, Waggoner and Zha (2008), and amounts to finding $\tilde{P}(s)$, for $s = \{1, \ldots, S\}$, with $\tilde{P}(s = i) \neq \tilde{P}(s = j)$. Here, the aim is to find a $\tilde{P}$ such that $\tilde{P}(s = i) = \tilde{P}(s = j)$ for $\forall s$.

[9] Suppose that each decomposition $\mathcal{A}_0$ represents a different theoretical model depicting different impulse responses. These possible differences are denoted by model uncertainty in this paper.

in the impulse responses. In other words, the dynamic differences across time are only due to the system being in a different state, and not because a different structural model was imposed by the econometrician. This is particularly important given that the relevant comparisons are to be made about the domestic variables, which have been left unrestricted and can, as a result, behave very differently depending on the structural model.

2.2.3 Impulse Responses, Variance Decomposition and a note on Model Uncertainty

A number of recent works focuses on the derivation of impulse responses in the context of a MS-VAR model. State-dependent impulse responses, unlike those obtained from linear VARs, depict non-linearities that depend on the moment the structural model is hit by a shock, and on the size and sign of the shock. By comparing the impulse responses across regimes, it is possible to determine the magnitude and sign of the these non-linearities. Moreover, to the extent that the MS regimes are clearly identified, these differences have important policy implications. Ehrmann, Ellison and Valla (2003) derive state-dependent impulse responses conditional on the state the system is in when the shock occurs. In addition, they assume that the regime remains unchanged throughout the duration of the response. This amounts to computing impulse responses within each MS state. For this method to be valid, the duration of the response must not be longer than the expected duration of the state. Karamé (2010) also assumes that the state is known when the structural shock occurs, but allows the regime to switch along the duration of the response, with the regime change being itself influenced by the shock. In turn, Camacho and Pérez-Quirós (2013) allow the sate to be endogenously chosen also at the moment the shock hits the economy. Without the shocks, the information about which state is prevailing can be backed from the estimated smooth probabilities delivered by the EM algorithm. However, when a shock hits the economy, it is necessary to compute these probabilities again.[10]

Computing time-varying impulse responses to sign-restricted structural shocks is not trivial. Instead, because the Cholesky decomposition of Σ_e is unique, one can simply compute $\mathcal{C}_s$ for each state. Importantly, although $\mathcal{C}_s$ necessarily differ across states, these differences are only driven by the data. This is the approach

[10]Refer to the Appendix for a description of the EM algorithm and on how to compute non-linear impulse responses.

taken by Camacho and Pérez-Quirós (2013). With SR however, as there are multiple decompositions complying with the same restrictions, one $\tilde{P}_k$ must be chosen between all decompositions satisfying the SR scheme. In this paper I follow the proposal by Fry and Pagan (2011) and select the $\tilde{P}_l$ that produces the impulse responses that are the closest to the point-wise median responses across all $\tilde{P}_k$ complying with the SR. This is obtained by minimizing the normalized distance between the impulse responses associated with a given $\tilde{P}_k$ and the point-wise median response, across all shocks and all responses, for both world and domestic variables.[11] The normalization is performed with the point-wise standard deviation. I denote this decomposition by $\tilde{P}_{mt}$. If $\tilde{P}_{mt}$ is unique, non-linear impulse responses and variance decompositions can be easily computed. Because only one structural model is used (the one associated with $\tilde{P}_{mt}$) differences in impulse responses across time are only due to regime changes.[12]

Measuring uncertainty within a sign-restricted MS structural VAR is not trivial either. There are two *sources* of uncertainty: *model* uncertainty, due to the multiplicity of $\tilde{P}_k$ satisfying the SR, and *statistical* uncertainty, relating to the data sample used for estimation. Considering the former, and taking the reduced form estimates of the MS-VAR as given, confidence bands around impulse responses can be computed by ranking the impulse response of variable i to a structural shock j, at horizon h after impact, across all $\tilde{P}_k$ satisfying the SR, and selecting the $x\%$ and $100 - x\%$ values. However, regarding *statistical* uncertainty, matters are more complex. A method proposed by Gonçalves and Kilian (2004), and adapted by Herwartz and Lütkepohl (2011), consists of a fixed-design wild-bootstrap that preserves the heteroskedasticity structure of the data. This is done for a given structural model, say for $\tilde{P}_{mt}$. Given that parameter changes depend on the MS states, Herwartz and Lütkepohl (2011) estimate the confidence intervals conditional on the transition probabilities. However, confidence intervals computed this way tend to be biased, and this remains even after implementing the bootstrap-after-bootstrap method suggested by Kilian (1998). Another problem has to do with the fact that for each bootstrapped sample, the SR might not be satisfied by $\tilde{P}_{mt}$ anymore. In these situations, one could drop the bootstrap samples that do not comply with the SR under $\tilde{P}_{mt}$. In practice, however, confid-

[11] The point-wise median response is obtained by ranking the response of variable i to stroctural shock j at time h after impact, across all decompositions k that satisfy the SR, and selecting the median value.

[12] For more details regarding the computation variance decompositions in a MS-VAR setting refer to Bianchi (2013).

ence intervals computed using these different methods appeared unsatisfactory in the context of the case study presented in the next section. This is clearly a topic of further research.

2.3 Case-study: Australia

In this section, I present an application of the empirical model just described. The section starts with a brief description of Australia's recent economic history. I highlight the importance of terms-of-trade shocks for Australia's economic performance and of institutional reforms that could be behind structural changes. Secondly, I present the data used for estimation, test the main specification assumptions implied by the empirical model, and discuss the structural identification scheme used. Thirdly, I estimate the model and investigate the potential economic interpretations of the MS regime probabilities and parameter estimates. Finally, I assess whether state changes across time have had an impact on the transmission of exogenous shocks to the Australian economy and discuss the policy implications of my findings.[13]

2.3.1 Background

As reported in Figure 1, during most part of the last decade Australia has benefited from an unprecedented rise in commodity prices. From 2000 to 2010, global prices for Australia's resource exports, in Australian dollar terms, increased by 9% *per annum*.[14] By 2010, exports of natural resources accounted for more than half of total exports, with iron ore, coal, oil and gas representing the biggest shares. Table 1 and Figure 2 report the composition and the relative size commodity exports represent for the Australian economy. The rapid growth in industrial production and housing construction in Southeast Asian and China was the leading cause behind the jump in commodity prices. Given its proximity, Australia was a privileged supplier to the region. In 2010, mining revenues were about 15% of Australia's GDP, while investment accounted for 4% . These figures, about

[13] OLS and MS estimation, including the EM algorithm, were coded in Matlab. Different MS specifications were nested within the main code, whereas specification tests were coded separately. I also developed the code for all the statistical tests but the QP tests, which were performed using existing Gauss code.

[14] If not stated otherwise, the figures presented in here are taken from Connolly and Orsmond (2011), who provide a comprehensive survey on the Australian mining industry during the 2000s, and from the Australian Bureau of Statistics (ABS) and the Reserve Bank of Australia (RBA).

twice their value since the year 2000, continued to grow. Kent (2013) estimates that the resource sector has accounted for 18% of GDP in 2012, while mining investment has reached a record 8% of GDP.

The growth rate of direct employment, of 10% per year, is also remarkable. According to estimates by Rayner and Bishop (2013), total employment generated by the mining sector, both directly and owing to resource-related activities, has represented 10% of total employment in 2012. This is again twice the figure of 2000. Overall, unemployment remained low throughout the boom, with manufacturing being the only sector with a net reduction in the number of employees. Although wages in the mining industry have typically been higher than in the rest of the economy, the gap increased throughout the decade. According to Lowe (2012), since 2004 wages in the resource sector have increased by around 10% relative to the average of the remaining sectors.

With GDP growing at an average rate of 3% per year, as shown in Figure 3, Australia preformed well above major developed economies around the globe. Domestic prices remained relatively stable, with inflation under the target band of around 2 to 3% *per annum* on average, as reported in Figure 4. This is particularly notable in light of the sharp rise in Australia's terms of trade, which jumped by around 80% since the beginning of the boom. The nominal exchange rate, free to adjust to market pressures, appears to have worked as an efficient buffer to the rise in the terms of trade. Indeed, the appreciation of the real exchange rate was mostly driven by the appreciation of the nominal rate, which allowed domestic inflation to remain within the official target.

The successful experience just described is in sharp contrast with the commodity boom that occurred in Australia nearly three decades earlier. In the late 1970s/early 1980s (Battellino, 2010, dates the boom from 1978 to 1983), the price of energy commodities, such as steaming coal, oil and gas, went up in the wake of the second oil price shock. However, as described by Connolly and Orsmond (2011) , the rise in commodity prices, of 40% on average, was not accompanied by an improvement in the terms of trade. As import prices also rose at a considerable speed, Australia's terms of trade remained fairly stable. Figures collected from Freebairm (1987) show that resource-based industries accounted for 80% of total export receipts at that time. In the peak of the boom, the mining sector represented around 5% of total national income.

Pagan (1987) provides a detailed account of the events. At the beginning of the boom, mining investment went up fast, following the belief that commodity prices

would remain high. Unemployment fell and output grew at an average of 3.6% between 1978 to 1980. Optimism about the duration of the boom allowed trade unions to obtain real wage increases. At the peak, nominal wage growth reach almost 30% . As collective wage bargaining was encouraged by the government, wage inflation spread through all sectors of the economy. However, the euphoria lasted shorter than expected. By the end of the mining boom, Australia entered a severe recession. At the bottom, which occurred in 1983, the decline in output was the greatest since the great depression and unemployment reached a record high of 11%. At the time, as the Australian dollar was pegged to a trade-weighted index of foreign currencies, the slow appreciation of the exchange rate was not enough to prevent inflation from jumping to more than 10%.

Among the causes for such contrasting outcomes are institutional factors. In between the two booms, important reforms took place in Australia, among which the reforms to the labour market and to the conduct of monetary policy seem particularly relevant. Considering the former, the decentralization of wage setting, which aimed at preventing sectoral wage pressures to spillover across the economy, was initiated during the beginning of the 1980s. In a response to the rapid degradation of the economic outlook, wage indexation ceased formally in 1981. As this was not enough to curb wage inflation, wages of public employees and private workers covered by federal awards were frozen by the end of 1982. Not long after, the government and trade unions agreed on further wage moderation across other sectors of the economy. Other reforms were gradually implemented also during the 1990s. As a result, instead of widespread wage inflation of the late 1970s, during the recent boom the gap between commodity and non-commodity wages was allowed to widen. In particular, as wages in the tradeable sector were no longer indexed, wage moderation could sustain export's competitiveness during the boom.

Monetary policy was also subject to important reforms. In 1983, the exchange rate was let to float, and ten years later, in 1993, the Reserve Bank of Australia (RBA) was officially given the mandate to pursue inflation targeting. Not surprisingly, and unlike previous occasions, the nominal exchange rate accommodated most of the adjustment owing to the increase in the terms of trade during the recent boom. The appreciation effectively shifted demand from Australian tradeables at the same time as making imports cheaper in Australian dollar terms. Both effects reduced the inflationary pressures induced by the boom, helping the RBA to achieve the mandated target for inflation. Together with the reforms in the

labour market, these changes have allowed the RBA to anchor inflation and wage expectations, letting supply and demand pressures in specific sectors not to spread to the whole economy.

The focus of the next subsections is on the following questions: First, to what extent have these reforms altered the way term-of-trade shocks propagate to the Australian economy? Second, and following Kilian (2009), I investigate whether different term-of-trade shocks produce similar outcomes? When comparing the differences between two booms in commodity prices, it is important to first clarify whether the two episodes represent the same shock. If the two shocks are different, their transmission to the economy, all else equal, will most likely differ as well. For instance, Australia's terms of trade reached record high levels during the most recent boom, whereas they barely moved during the previews one. Is this difference the reason behind the contrasting outcomes described above? On the contrary, if I compare the same shock across different periods in time, differences in the reaction of the economy, all else equal, will most likely be due to different domestic circumstances. I employ the empirical model described above to shed light on these questions.

2.3.2 Model Specification

Data: The terms of trade of a country are defined as the ratio of the price of exports to the price of imports. The terms of trade inform how a country benefits from trading internationally, with an improvement implying that the revenues from exporting domestically-produced goods and services can afford the purchase of a larger volume of imports. In the context of a small commodity-exporting economy, commodity prices assume a significant role in the evolution of the terms of trade, as commodities represent a large share of total exports.

In order to capture the exogenous conditions relevant to Australia's terms of trade, and following JS11, the world block is composed of the following three variables: the world prices of Australian exports and imports, and an aggregate indicator of economic activity for Australia's major trading partners. Australian export (import) prices are obtained from the seasonally-adjusted implicit price deflator for expenditures on exports (imports) of goods and services. These deflators, which are denominated in Australian dollars, are multiplied by the quarterly average of the nominal trade-weighted index. The conversion to world prices is made in order to abstract from fluctuations in export (imports) prices caused by move-

ments in the exchange rate. The economic activity indicator is obtained from the seasonally adjusted export-weighted real GDP of Australia's major trading partners.[15]

Regarding the domestic block, the main objective is to study the impact of terms-of-trade shocks on real GDP. However, interpreting the MS states is also of particular relevance. Consequently, and following the discussion presented in the previous subsection, I add to the domestic block a measure of consumer price inflation and of changes in the nominal exchange rate. A more detailed explanation regarding this choice of variables is found in the next subsection. Real output growth is obtained from the seasonally adjusted chain volume measure of Australia's GDP, while consumer price inflation (CPI) is derived from the seasonally adjusted all-groups consumer price index, excluding the tax changes of 1999-2000. The growth rate of the nominal exchange rate is obtained from the trade-weighted index.

The data was collected from the Australian Bureau of Statistics (ABS) and the RBA, in levels and already seasonally adjusted. The sample used for estimation runs from the first quarter of 1971 to the last quarter of 2010. The sample period purposely includes years of major policy changes, notably the float of the Australian dollar in December 1983 and the announcement of inflation targeting in the first quarter of 1993. Unlike JS11, the sample period used here also includes the least recent boom mentioned in the previous section.

Augmented Dickey-Fuller tests indicate that all variables are stationary at a 1% significance level. Moreover, standard criteria suggest the use of one lag in the VAR, which is what is assumed for the estimations.[16] These results are also obtained for the sub-samples obtained by dividing the sample in two at two different dates: in the last quarter of 1990 and in the first quarter of 1993. The reason to check stationarity and the lag length of the VAR within these shorter samples is the following. The last quarter of 1990 is suggested by tests for structural breaks in the data, whereas 1993Q1 is suggested by the discussion in the previous subsection, for it was the first quarter under official inflation targeting. These are two probable dates for state changes, and the results are reassuring about the validity of the specification choices made here.[17]

[15]Major trading partners comprise Canada, China, France, Germany, Hong Kong, Indonesia, Italy, Japan, Malaysia, New Zealand, Philippines, Singapore, South Korea, Taiwan, Thailand, UK, and US.

[16]I used the AIC, AICC, and SIC criteria to select the lag length p.

[17]The choice of the lag length p of the VAR and the number of MS states S follows the

Model assumptions: One important assumption embedded in the model outlined in the previous section concerns the composition of matrix A.[18] Table 2 presents tests for the null that $A^{DW} = 0_{n\times d}$, i.e. , that the domestic variables have no significant impact on the world variables. The table reports this test under different specifications. In particular, given that the world block is constructed with the purpose of identifying terms-of-trade shocks, and specially commodity price shocks, I also test whether international commodity prices are exogenous with respect to domestic variables. This is shown in the first panel of the table. The commodity price index is from the RBA and is computed using the prices for the main commodities exported by Australia.[19] Using the commodity price index, the bivariate test for Granger causality shows that real GDP growth, Δy^d, and domestic inflation, π^d, do not have significant explanatory power with regards to the price index. Δy^d also does not seem to Granger-cause the world block composed of the three variables described in the previous section. On the contrary, the change in the nominal exchange rate, Δs, seems to have statistical significance in describing both commodity prices and the world block. In the multivariate context using the three domestic variables, block exogeneity of both the the commodity price index and of the world block cannot be rejected at the standard 5% significance level.[20]

The other implicit assumption in Section 2 is the presence of structural changes within the small open economy. Table 3 provides evidence of the existence of changes on the intercepts, coefficients and variance of the domestic block. The tests reported in the table do not address specifically the existence or type of the non-linearities present in the domestic block. Instead, they assess the significance

strategy used, for instance, in Herwartz and Lütkepohl (2011). Other strategies are suggested in Krolzig (1997) or in Psaradakis and Spagnolo (2006).

[18]For ease of notation, and given that $p = 1$, the subscript i is dropped.

[19]The index is plotted in Figure 1. Table 1 reports the shares of the main commodities used to build the index.

[20]Measured in terms of commodity exports, Australia is indeed a major player in world markets, where Australian exports can account for more than half of total exports of some commodities. However, in terms of total production, Australian shares are significantly lower. For instance, in 2007, Australia accounted for 57% of world exports of coking coal and 32% of world exports of iron ore. Yet, Australia's shares of world production were 18% for both commodities. In fact, a large share of commodity production is consumed domestically in countries like China, India and the US. This, together with the fact that several mining companies operate within Australia in a competitive environment, render support to the assumption that Australia is rather a price taker in world commodity markets. Refer to Andrews (2009), Atkin and Connolly (2013) and references therein for further discussion and figures.

and timing of parameter changes. For each domestic variable at a time, I first perform Chow tests for parameter changes for every date in the $15-85\%$ window of the sample period. The maximum test value is then compared to the critical values presented in Stock and Watson (2002). Moreover, for each variable and for the date the Chow test reaches the highest value, I again preform standard Chow tests to investigate which parameter changes are significant. This approach allows me to identify both the most likely timing and type of the structural change. To test for breaks in the variance of each variable, I use the maximum likelihood tests described in Qu and Perron (2007), henceforth QP tests. These tests are also used to investigate the timing and type of parameter changes for the multivariate case. Finally, all the VAR specifications include either the commodity price index or the world variables as exogenous regressors, as described in (4). For the QP tests, only the specification with the world variables is shown.

Univariate Chow tests with unknown date fail to show a significant change in parameters for the univariate specification with only real GDP growth. On the contrary, QP tests do find a break around the end on 1983, notably in the standard deviation. That year, as Australia suffered a sever recession in the aftermath the of commodity boom of the late 1970s/early 1980s, a number of labour market reforms aiming at curbing wage rise demands was set forth. These, together with the float of the exchange rate, can explain the date of the break.

Regarding CPI inflation, the three sets of tests detect a significant change in all parameters around the last quarter of 1990. Inflation targeting was officially announced in the beginning of 1993. But as Macfarlane (1997) explains, the transition to inflation targeting had begun much earlier. In fact, the RBA initiated the practice of announcing changes to the cash rate in as early as January 1990, with the inflation target of $2-3\%$ starting to appear in official statements already in 1992. Considering the changes in the intercepts and VAR coefficients for the nominal exchange rate, Chow tests find a break between 1993 and 1995, depending on the VAR specification. However, QP tests find a significant break in the univariate VAR already during the third quarter of 1984, a few quarters after the float was officially announced.

Interestingly, when the QP test is done for the multivariate VAR, which includes all domestic variables, a structural break is detected in the last quarter of 1990. Instead, if only breaks in Σ_e are allowed, then the date is anticipated to the beginning of 1984. One interpretation, consistent with the differences in parameter estimates for each of the two sub-samples determined by the QP tests, is that the

float of the nominal exchange rate was determinant for the stabilization of the Australian economy. By buffering foreign shocks more efficiently and, therefore, reducing the volatility of domestic variables, the impact of the float is seen directly on second moments more than on the structure of the domestic economy. On the other hand, the parameter changes caused by the implementation of inflation targeting appear to be more noted in terms of VAR coefficients, i.e., on the economic inter-relations within the domestic block. All things considered, the end of the commodity price boom in the early 1980's together with the float of the exchange rate in 1983, on the one hand, and the beginning of inflation targeting in the early 1990's, on the other hand, appear to be highly relevant in explaining the findings reported in Table 2. As I discuss in the next subsection, the timing of these events is also relevant for the interpretation of the MS regimes.

Identification of structural shocks: The scheme I use to identify terms-of-trade shocks follows that of JS11. In particular, I decompose movements in Australia's terms of trade into three different types of shocks. To achieve this, I impose SR to the impulse responses of the variables in the world block on the quarter the shock impacts these variables. The SR scheme is depicted in Table 4. In the table, a + (−) denotes a positive (negative) response of the respective variable, whereas *na* implies that no restriction has been imposed. π^x stands for export price inflation, π^m for import price inflation, and Δy^w is the growth rate of Australia's trading partners.

Following the order in Table 3, a *world demand shock* is assumed to represent an aggregate demand shock from Australia's major trading partners, which pushes up prices and economic activity. Given that the bundle of exported and imported goods and services might differ, the price index for exports and imports need not move by the same magnitude. In any case, both prices move upwards in response to an increase in aggregate demand. As in Australia commodities represent a large share of total exports, this shock captures the evolution of commodity prices associated with the global business cycle.

On the other hand, a *commodity-market shock* intends to capture an increase in the price of Australian exports resulting from an increasing demand for Australian natural resources. Unlike a world demand shock, the causes for the rise in commodity prices can be traced to, for instance, a decrease in the supply of commodities elsewhere or an increase in financial investment in commodities. The shock can also be due to an increase in precautionary demand for commodities

caused, for example, by an increase in the perceived uncertainty regrading the forecasted evolution of commodity prices. World economic activity is assumed not to pick up, excluding this to be the source of the increase in commodity prices.

Finally, a *globalization shock* is interpreted as an expansion of Emerging Markets' economic activity. This is characterized by an increase in Australian export prices, at the same time as the price of imports falls. JS11 interpret this type of shocks to be linked to the increasing role these countries have come to play in world trade. First, as these countries grow, their demand for bulk commodities increases, pushing up prices. On the contrary, the increase in industrial production at relatively competitive prices, puts downward pressure on prices of a range of non-commodity tradeable goods. Given the composition of Australian exports and imports, the combination of these price movements results in an improvement in the terms of trade.

As discussed, and although the notation may be misleading, all three shocks imply an increase in commodity prices.[21] Clearly, movements in Australia's terms of trade alone, or their absence, can fail to reveal important developments in the commodity market. Distinguishing the underlying causes for these developments can, in turn, be relevant both empirically and for policy. It is therefore of paramount importance to identify the causes for the swings in commodity prices and compare the impacts of different shocks on the Australian economy.

2.3.3 MS States and Parameter Estimates

The final task before turning to the estimation of (1) is to decide the number of MS states S. Table 5 presents the values for the AIC, BIC and HQC criteria for a linear VAR and MS-VARs with two to four MS states. For each criteria, the minimum value is preferred.[22] The values associated with the linear VAR are merely illustrative.[23] Considering the univariate specification with only real GDP growth, all criteria give preference to the most parsimonious MS-VAR. However, when the complete block of domestic variables is considered, only the BIC supports the choice of $S = 2$, whereas $S = 3$ is preferred by the AIC and HQC criteria.

[21] The labeling of the shocks was kept unchanged for ease of comparison with the literature.

[22] Refer to footnote 16 and references therein regarding the use of penalty criteria to guide the choice of S.

[23] Krolzig (1997) explains why comparing a MS-VAR with $S > 1$ to a linear VAR is not a simple task due to the existence of the nuisance parameter under the null of $S = 1$.

The results for the univariate specifications for domestic inflation and the nominal exchange rate, which can be found in Table 6, are useful to explain these findings. For instance, for the case of inflation, the AIC prefers 4 states, whereas the other two criteria choose 2 states. For the exchange rate, the AIC and the HQC choose 4, with the BIC picking 3 states. Hence, the reason behind the preference for $S = 3$ in the multivariate specification seems to be due to the inclusion of inflation and the exchange rate. This is particularly obvious for the AIC and HQC criteria, which are more sensitive and tend to pick a higher number of states for these two variables. For the BIC, the choice of $S = 2$ clearly dominates.

Notwithstanding the fact that Table 5 only partially supports the choice of $S = 2$, it seems wise to opt for the most parsimonious of the MS models for three rather practical reasons. The first regards the number of parameters to be estimated. With some specifications allowing all parameters (including the variance-covariance matrix) to be state-dependent, a smaller number of states greatly improves the quality of the estimates. Secondly, and related to this, is the relatively short sample available. This is due to the world block, and in particular to the indicator for economic activity of Australia's major trading partners, for which a larger sample is not available. Again, the higher the number of states, estimation tends to be poorer when the sample size remains fixed. The third reason is due to interpretation. Having more than two states greatly complicates the derivation of an economic interpretation to each state.

After choosing the lag-length, $p = 1$, and the number of MS states, $S = 2$, I estimate model (1) through maximum likelihood performed using the EM algorithm described in the Appendix. Figure 5 and Figure 6 present the estimated smoothed probabilities of state 1, along the sample period, for various specifications. The first panel of Figure 5 shows the smoothed probabilities for the univariate specification using real GDP growth only. The specification in the second panel includes the complete domestic block. For the each specification, I first experiment with all parameters of the VAR to be time-dependent. These include the intercept α, the VAR coefficients A^{WD} and A^{DD}, and the variance-covariance matrix Σ_e. I then investigate the drivers for the smoothed probabilities. Firstly, by only allowing Σ_e to be state-dependent. Secondly, by allowing α and Σ_e to change in time. Figure 6 explores the effects on the estimated smoothed probabilities of including inflation and the nominal exchange rate in the domestic block.

Across all specifications, state 1 clearly prevails during the first part of the sample, giving way to state 2 at different dates and sometimes with episodic recurrences.

The date when state 2 replaces state 1, which I denote by transition date, and the occurrence of episodic recurrences of state 1 depend on which variables are included in the domestic block and which set of parameters are state-dependent.

Considering first the univariate specification for real GDP growth, the transition date is found to occur in late 1983. This is broadly consistent with the evidence reported in Table 3. When only Σ_e is time-dependent, there are no recurrences, whereas if α or all parameters are state-dependent as well, state 1 peaks around 1990 and 1997. This renders support to the claim that Σ_e is behind the transition in 1983. Looking at the estimated variance of the residuals on the real GDP equation, I find that Σ_e is around 5 times higher in state 1 compared to state 2. Two tentative hypothesis seem plausible. First, the float of the nominal exchange rate in 1983. As explained above, a floating exchange rate tends to isolate domestic activity from foreign shocks, thereby stabilizing total output. Second, the labour market reforms that begun being introduced in the early 1980s. In particular, the phasing out of wage indexation in 1980, the accord on wages and competitiveness of 1983, and the implementation of a more decentralized wage-setting system in the early 1990s. These reforms may have rendered output more resilient to sectoral wage swings and, consequently, also more resilient to foreign shocks. Yet, it is worth investigating the remaining panels before conclusions are drawn.

The implications of adding consumer price inflation and changes in the nominal exchange rate to the domestic block are twofold. Firstly, the transition date tends to be postponed to the end of 1990. When other parameters besides Σ_e are state-dependent, the transition date occurs in late 1990. However, when Σ_e is the only state-dependent parameter, the transition date falls back to 1983, with state 1 emerging again around 1990. This leads me to conclude that the transition in 1983 is mostly driven by the real GDP series. The same can be said about the recurrence in 1990. In fact, and analyzing Figure 3 once more, it seems that state 1 is absorbing the periods of higher GDP volatility, which can be broadly matched to the probabilities displayed in specification 5) in Figure 5. In addition, the transition in 1983 does not seem to be driven by the nominal exchange rate either. In fact, whenever real GDP growth is included in the domestic block, the transition happens in late 1983, as can be seen in Figure 6, specifications 2) and 3). On the contrary, as depicted in specification 4), when only the two nominal variables are included, 1990 clearly dominates over 1983.

Secondly, the two states become more stable and with less recurrences. State recurrences only tend to happen when real GDP growth is added to the domestic

block. However, as shown in Figure 6, they are slightly different depending on which nominal variable pairs real GDP. Together with the time-series of the nominal variables, Figure 4 marks the episodes the inflation rate fell outside the RBA official target band and when the RBA intervened in the foreign exchange market after 1983.[24] When inflation pairs GDP growth in specification 2), some recurrences match the dates when inflation was outside of the official band (around 1997, 2001 and late 2008, although the peak in 2001 appears barely unnoticed). Regarding specification 3), the recurrences after 1983, with the exception of the one around 2005, coincide with official interventions in the foreign exchange market. Interestingly, none of these recurrences remain when the two nominal variables enter the domestic block, as in specification 4) in Figure 5 and 4) in Figure 6.

The inclusion of π^d and Δs partly justifies the composition of the domestic block. On the one hand, including these two variables avoids state 1 recurrences. This facilitates the economic interpretation of the MS regimes, since interpreting a once-and-for-all transition is clearly easier than interpreting multiple state transitions along the sample period. On the other hand, however, the new transition date does not render such a clear cut interpretation as the one in 1983. As adding more variables necessarily makes interpretation more complex, I chose to keep only these three and rely on the state-dependent parameter estimates to gather a better sense of the economics behind the 1990 transition date.

A brief note regarding the choice of the inflation series I use before turning to the parameter estimates. Unlike the remaining specifications, the inflation series used under specification 1) in Figure 6 does not account for the tax reform package introduced in 2000. As a result, and comparing with specification 4) in Figure 5, using this measure leads to the emergence of a recurrence of state 1 around the year 2000. If the tax reform is taken into consideration, and the measure of inflation is cleared from the GST hike, this peak vanishes completely. This is why I chose not to use the original CPI inflation measure.

Table 7 presents the estimates for α, A^{WD}, A^{DD}, and Σ_e, for both MS states, under specification 4) in Figure 5. Starting with the analysis of α and Σ_e, I find that the average of real GDP growth is lower and the variance of the respective residuals considerably higher in state 1. After 1993, Australia has been experiencing a long cycle of continuous growth, immune to the major global slumps such as the Asian financial crises of 1997 or the dot-com crash of 2000. In turn, CPI

[24] From the float in 1983 to 1989, there are no episodes officially dated, although interventions occurred often.

inflation is, on average, higher in state 1 and with more volatile residuals. This can be explained by the fact that, since the drop in 1990, as Figure 4 shows, inflation has remained low compared to historical levels and floating around the RBA's target band. Lastly, although the volatility of the residuals of the exchange rate equation does not change much, its intercept changes sign. As depicted in Figure 4, during the period state 1 dominates, the nominal exchange rate traces a prolonged depreciation. On the contrary, it appreciates, at a lower pace, in state 2. It is important to note that the nominal exchange rate used in the estimation is a trade-weighted index of bilateral exchange rates and that the Australian dollar was pegged to the US dollar alone from 1971 to 1974. Moreover, although the Australian dollar was pegged to the trade-weighted index in 1974, as shown in Figure 4, the *hard* peg was replaced by a *crawling* peg in 1976. This can shed light to the small differences found in the volatility of the residuals of the exchange rate equation across states.

Regarding the estimates for A^{WD} and A^{DD}, a complete interpretation of state differences is less straight-forward, specially in a multivariate context. Nevertheless, most estimate changes, and notably those relating to the persistence coefficients and to the coefficients on lagged domestic inflation, corroborate the discussion held so far. It is also worth noting that the state probabilities, and hence their interpretation, do not depend on having A^{WD} and A^{DD} changing across states, as shown in Figure 5.

These findings are all consistent with a transition to a more flexible regime, where inflation is maintained lower and both output growth and prices remain more stable. Clearly, associating the beginning of inflation targeting, which as discussed earlier started in the early 1990s, with the transition to state 2 is tempting. However, I prefer to complete this association with the two other reforms I have alluded to. In fact, both the transition from an exchange rate peg to inflation targeting and the gradual implementation of labour market reforms provide, together, a more complete and coherent description of the smoothed state probabilities and parameter estimates just discussed.

2.3.4 Impulse Responses and Variance Decomposition

Once model (1) is estimated, I draw a number of orthogonal matrices $\tilde{P}_k$ and compute the corresponding impulse responses. If the SR described in Table 4 are satisfied, $\tilde{P}_k$ is stored. When 2.000 draws are stored, the algorithm stops.

Under the various specifications I have experimented with, the acceptance rate was around 10% of all draws. In the following figures, I report the point-wise median response of the accepted draws and the $16^{th} - 84^{th}$ percentiles. As proposed by Fry and Pagan (2011), I also show the median target model. In general, I find that the latter does not differ much from the median response.

Figure 7 presents the impulse responses of the world variables to the three terms-of-trade shocks. The figure makes the SR from Table 4 notorious. Apart from the response of import price inflation, π^m, to a commodity-market shock, which is left unrestricted, all the remaining responses are significantly different from zero as imposed by the SR. The responses of export price inflation, π^x, to world demand and globalization shocks, and of foreign economic growth, Δy^w, to a commodity-market shock take longer to vanish. Nevertheless, after the shock all variables adjust within no more than 5 quarters.

Recall from (3) that the world block is a standard linear VAR. Consequently, the responses of the world variables are time-invariant. I investigate the robustness of this assumption by estimating model (3) for a sub-sample running from 1981 to 2010, as in JS11. Comparing Figure 7 and Figure 8, which plots the impulse responses under this sub-sample, unveils virtually no differences. Apart from the response of import price inflation to a commodity-market shock, which inverts its sign abruptly around the second quarter after impact, the homogeneity between the two figures is reassuring of the time-invariability of the world block.[25]

Figure 9 and Figure 10 present the impulse responses of the domestic variables for each of the two MS states. After a world demand shock, Australia's real GDP growth rises. This is true in both states, despite the response in state 2 appearing considerably milder. On the contrary, an increasing demand from Australia's trading partners has opposite effects on domestic inflation depending on the state. Before 1990, a world demand shock reduces inflation, while after that date it becomes inflationary. As discussed below, these findings point to an increasing coordination between Australian and global business cycles. Lastly, I find that the nominal exchange rate appreciates around 2% in both states, a figure consistent with JS11.

Considering commodity-market shocks, I find they are detrimental to real output. The contraction is more accentuated in state 1, reaching its trough 3 quarters after impact and causing a rise in inflation. In state 2, the effects are again substantially

[25] Figure 7 and the responses obtained after estimating (3) for a sub-sample from 1971 to 1980, not shown in this paper, look even more alike.

milder, with almost no impact on inflation. The dismal performance of output is not contradicted by JS11, and is also found in Charnavoki and Dolado (2014) for the case of Canada.[26] These effects can be due to the existence of adjustment costs following an increase in commodity prices. Notably, the contraction in state 1 is in line with the discussion regarding labour market rigidities. In particular, due to the wage-setting framework before 1990, Australia found it difficult to gain competitiveness in world markets and to avoid the recession. Alternatively to this interpretation, the slowdown of real output growth can be the result of spillover effects from the deceleration of Australia's major trading partners.

This shock has particular relevance for policy. In fact, despite the improvement in the terms of trade, policy needs to monitor carefully the underlying causes of the rise in commodity prices. With a global contraction, an increase in commodity prices seems not to be enough to sustain output. In these circumstances, and in particular during state 2, for which inflation remains nearly unchanged after the shock, some policy loosening might be warranted to stimulate growth.

Finally, a globalization shock happening before 1990 has a clear positive effect on output and an equally clear negative effect on inflation. Both effects last for around three quarters. With export prices picking up while import prices fall (which include prices of intermediaries used by Australian industries), output growth is stimulated. On the contrary, as the fall in import prices is detrimental to domestic tradeable producers, they struggle to thrive against foreign competition. Hence, the positive effect on total output of higher export prices might be eclipsed by the negative impact of cheaper imports. This can explain the very little effects on domestic variables observed in state 2. The nearly mute response of inflation is also in line with empirical evidence of a declining pass-through of import prices to domestic inflation.[27]

The previous figures have shown impulse responses computed under the assumption that the shocks do not affect state probabilities. In other words, I have assumed that there is a probability equal to 1 of a given state being prevalent on impact and throughout the time of the response. This is the approach used by Ehrmann, Ellison and Valla (2003). However, and following the discussion in Camacho and Pérez-Quirós (2013), it might be that the estimated state probabilities are influenced by the shocks. Figure 11 reports these non-linear impulse responses. The responses are computed at the end of the transition from state 1

[26] Charnavoki and Dolado (2014) link these effects to the Dutch disease literature.

[27] Refer to Chung et al. (2011) and references therein.

to state 2 in 1992. For ease of comparison, I report the previous (linear) impulse responses as well. All impulse responses are for the median target model, $\tilde{P}_{mt}$.

As shown in Figure 5, state 2 occurs with probability 1 in 1992. Considering first a world demand shock, it appears that the shock has no significant impact on the estimated state probabilities. Therefore, the non-linear impulse responses are essentially the same as the ones under the assumption state 2 prevails with probability 1. On the other hand, the responses to a commodity market shock and to a globalization shock present noticeable differences. Although displaying roughly the same shapes as the responses under state 2 being fixed, their magnitudes are higher. This seems counter-intuitive. The explanation simply lies on the impact the shocks have on state probabilities. As these shocks increase the probability of state 1, which would be nearly zero otherwise, the differences between the expected value of the domestic variables with and without the shock become larger. As a result, both the commodity-market and the globalization shocks have more pronounced effects around 1992, when the small open economy may be induced to return to state 1.

Table 8 presents the last set of results. The table reports the variance decomposition of the domestic variables under the median target model and allows me to investigate whether the explanatory significance of each shock, in terms of the volatility of the domestic variables, changes between states. Recall that, although model (5) has $n + d$ equations, I only identify n structural shocks, leaving the remaining d shocks without an economic interpretation. These unspecified shocks are grouped under the banner *domestic shocks* in Table 8.

Before 1990, more than 10% of real GDP growth volatility is due to the identified shocks to Australia's terms of trade. Among these, globalization shocks appear to be the most relevant. The same is true for domestic inflation. Interestingly, world demand and commodity-market shocks explain more than half of the volatility of the nominal exchange rate, with globalization shocks accounting for less than 10%. Comparatively, in state 2 the share of the variance of real GDP growth explained by the same shocks is more than halved, in line with evidence from the impulse responses. Instead, world demand shocks account for a considerably larger share of the fluctuations in inflation and in the nominal exchange rate. The case for inflation is particularly striking, with a change from 4.5% in state 1 to 23% in state 2. Also worth mentioning is the fact that the identified shocks account for a smaller share of the volatility of the nominal exchange rate in state 2 compared to state 1.

These results deserve a few notes. First, world demand shocks represent the bulk of terms-of-trade shocks to the Australian economy. This finding, although not contradicting JS11, is more obvious here. The increasing importance of these shocks can be attributed to the relative reduction in the importance of the remaining shocks, as the overall volatility is lower in sate 2 (see Table 7). However, its considerable importance after 1990 deserves a careful examination. On the one hand, the increasing sensitivity of inflation to exogenous pressures can be due to the number of free-trade agreements and other trade accords Australia started to promote since the 2000s. Less tariffs and restrictions on trade imply less control over foreign market conditions, and therefore could explain the significant increase in Australia's synchronization to global business cycles. On the other hand, it could point out to some lack of control by the RBA when facing world demand shocks. As pointed out by Plumb et al. (2013), forecasts of Australia's terms of trade made by the RBA tended to under-predict the extent of the commodity price boom since 2005. These under-predictions of Australia's terms of trade can explain a relatively poorer management of domestic inflation, specially when compared to a relative improvement in dealing with other inflationary pressures.

Second, commodity-market shocks have lost importance. One interpretation is that demand for Australian commodities has been driven by the expansion of foreign economies and not due to precautionary factors or disruption in the supply of commodities by other exporting countries. In fact, the expansion of China and of Southeast Asia has been reflected in the markedly increase in these countries' shares of Australia's commodity exports. Therefore, after 1990, it appears to be economic growth, rather than precautionary motives, the driver behind the increase in demand for Australian commodities and the consequent rise in commodity prices.

Third, the idea that globalization shocks are a recent reality with the opening of China and India to world trade finds no empirical support. At least, shocks that assume a reduction in import price inflation while export prices grow are relatively less important after 1990. In fact, in state 2, and in line with JS11, globalization shocks are the less important of the three identifies shocks.

2.4 Conclusion

In this paper, I design an empirical model suitable to compare the transmission of exogenous shocks to a small open economy subject to structural change. In particular, I contribute to the empirical literature by proposing a framework that guarantees the comparability of different structural shocks across time. This consistency gain is of paramount importance. On the one hand, comparing the transmission of the same shock between different dates allows me to explore the implications of structural change. In the literature, structural change within an economy is found to be an important factor accounting for differences in the transmission of exogenous shocks. On the other hand, a careful identification of shocks avoids misleading conclusions. A growing strand in the literature is unveiling the distinct consequences of seemingly similar shocks. By using a robust identification, I am able to single out specific shocks and make sure that the differences across time are indeed compared under the same shock. I therefore combine two powerful econometric tools into a consistent framework that isolates the differences in the transmission of shocks caused by endogenous structural change from those implied by different exogenous shocks.

I provide an application of the model using Australia as an illustrative example. As a small commodity-exporting economy, Australia is highly exposed to commodity price shocks. I use SR to distinguish different commodity price shocks based on their underlying causes. I identify three types of shocks: a world demand shock, a commodity-market shock, and a globalization shock. With regards to structural change, I find a once-and-for-all regime transition occurring around 1990. Moreover, I find that this transition appears to match relatively well the consequences of major institutional reforms in Australia, such as the float of the exchange rate, the flexibilisation of the wage-setting system, and inflation targeting.

Comparing the impulse responses of real GDP growth, inflation, and changes to the nominal exchange rate, I find that structural change alters the way the identified shocks propagate to the economy. Before 1990, real GDP growth reacted more to the three shocks and had a slower adjustment, while it has a milder and quicker response to the same shocks after the transition. Moreover, positive terms-of-trade shocks are only inflationary after 1990 and as a reaction to a world demand shock. Otherwise, inflation appears almost immune to shocks after that date. This is in clear contrast with the effects found before the transition.

The less exacerbated responses to the shocks and the quicker adjustments of real GDP growth after the transition are in line with the flexibilisation of the labour market, notably the wage-setting decentralization that occurred during the 1990s. Also, the more moderate responses of inflation are consistent with the move to inflation targeting, as opposed to the nominal exchange rate management practised before the transition. Yet, it seems that the RBA has not been able to curb inflation hikes due to world demand shocks.

The results found in this paper point out significant differences in the way the Australian economy has reacted to shocks to its terms of trade. These differences illustrate how changes at the policy and institutional levels can affect the propagation of foreign shocks to the domestic economy. Moreover, they also inform policy-making by identifying how the economy reacts to commodity price shocks that have different underlying causes.

2.5 Tables

Table 1: Weights used to build the Price Indices

Iron ore	32,7%
Metallurgical coal	16,4%
Thermal coal	8,4%
Gold	8,0%
LNG	6,0%
Crude oil	6,0%
Wheat	3,2%

Source: RBA.

Table 2: Granger Tests of Non-Causality

ComPI*	
Δy_t^d	cannot reject H0
π_t^d	cannot reject H0
Δs_t	significance at 10%
$[\ \Delta y_t^d \quad \pi_t^d \quad \Delta s_t\ ]$	cannot reject H0
World Block**	
Δy_t^d	cannot reject H0
π_t^d	significance at 1%
Δs_t	significance at 1%
$[\ \Delta y_t^d \quad \pi_t^d \quad \Delta s_t\ ]$	significance at 10%

*SDR. Using a nominal or a real index does not alter the results. **Includes: Export and Import prices in world prices, and an index of economic activity of major trading partners. The inverse non-causality is rejected.

Table 3: Testing for Structural Breaks

	Stock and Watson		
	ComPI*		**World Block****
Δy_t^d	1993Q4 (not)		1979Q2 (not)
π_t^d	1990Q4 (1%)		1990Q4 (1%)
Δs_t	1995Q3 (10%)		1993Q3 (1%)
	Chow		
	ComPI*		**World Block****
Δy_t^d	beta (1%)		beta (1%)
π_t^d	all (1%)		all (1%)
Δs_t	const (1%)		all (1%)
	Qu and Perron***		
	all	**beta**	**variances**
Δy_t^d	1983Q3 (1%)	1983Q3 (not)	1984Q1 (1%)
π_t^d	1990Q4 (1%)	1990Q4 (1%)	1991Q1 (1%)
Δs_t	1984Q3 (1%)	1984Q3 (not)	1984Q3 (1%)
$[\ \Delta y_t^d \quad \pi_t^d \quad \Delta s_t\ ]$	1990Q4 (1%)	1990Q4 (1%)	1984Q1 (1%)

*Commodity Price Index in SDR (using a nominal or a real index does not alter the results). **Includes export and import prices in world prices, and an index of economic activity of major trading partners. ***Estimation and testing using Gauss code provided by the authors. Specification using World Block.

Table 4: Sign Restrictions

	π_t^x	π_t^m	Δy_t^w	D
world demand shock	+	+	+	na
commodity-market shock	+	na	-	na
globalization shock	+	-	+	na

Table 5: Number of States

		[Δy_t^d]		
	log L	**AIC**	**BIC**	**HQC**
VAR(1)	-232,77	477,55	496,00	485,04
MS(2)-VAR(1)	-197,92	**423,83**	**466,89**	**441,31**
MS(3)-VAR(1)	-188,60	425,20	499,00	455,16
MS(4)-VAR(1)	-176,90	425,80	536,50	470,75
		[Δy_t^d π_t^d Δs_t]		
	log L	**AIC**	**BIC**	**HQC**
VAR(1)	-682,24	1418,49	1501,52	1452,20
MS(2)-VAR(1)	-777,63	1667,26	**1839,47**	1737,19
MS(3)-VAR(1)	-707,06	**1588,13**	1855,67	**1696,77**
MS(4)-VAR(1)	-705,47	1650,94	2019,96	1800,78

$AIC = -2 \times logL + 2 \times K$, $BIC = -2 \times logL + K \times log(T)$, and $HQC = -2 \times logL + 2 \times K \times log(log(T))$; with $K = S \times (S - 1 + l \times k \times (1 + k) + k \times (k + 1)/2)$.

Table 6: Number of States (cont.)

		[π_t^d]		
	log L	**AIC**	**BIC**	**HQC**
MS(2)-VAR(1)	-142,00	312,01	**355,06**	**329,49**
MS(3)-VAR(1)	-127,71	303,42	377,22	333,38
MS(4)-VAR(1)	-113,38	**298,77**	409,47	343,72
		[Δs_t]		
	log L	**AIC**	**BIC**	**HQC**
MS(2)-VAR(1)	-432,84	893,68	936,73	911,16
MS(3)-VAR(1)	-407,32	862,64	**936,45**	892,61
MS(4)-VAR(1)	-387,26	**846,52**	957,22	**891,47**

Table 7: Estimates

			A^{WD}			A^{DD}		
		α	π^x_{t-1}	π^m_{t-1}	Δy^w_{t-1}	Δy^d_{t-1}	π^d_{t-1}	Δs_{t-1}
	Δy^d_t	0,16	0,02	0,00	0,52	-0,13	0,03	-0,01
State 1	π^d_t	1,73	0,06	0,08	-0,14	-0,13	0,20	-0,09
	Δs_t	-1,03	0,26	0,30	-0,02	0,07	-0,26	-0,29
	Δy^d_t	0,96	-0,01	0,02	0,04	-0,03	-0,17	-0,01
State 2	π^d_t	0,60	0,00	0,06	0,14	-0,11	0,00	-0,01
	Δs_t	0,29	-0,50	0,43	0,58	0,68	-1,21	0,23

		Σ_e		
	Δy^d_t	1,38		
State 1	π^d_t	-0,07	0,99	
	Δs_t	0,52	-1,35	14,99
	Δy^d_t	0,34		
State 2	π^d_t	-0,02	0,12	
	Δs_t	0,46	0,26	15,18

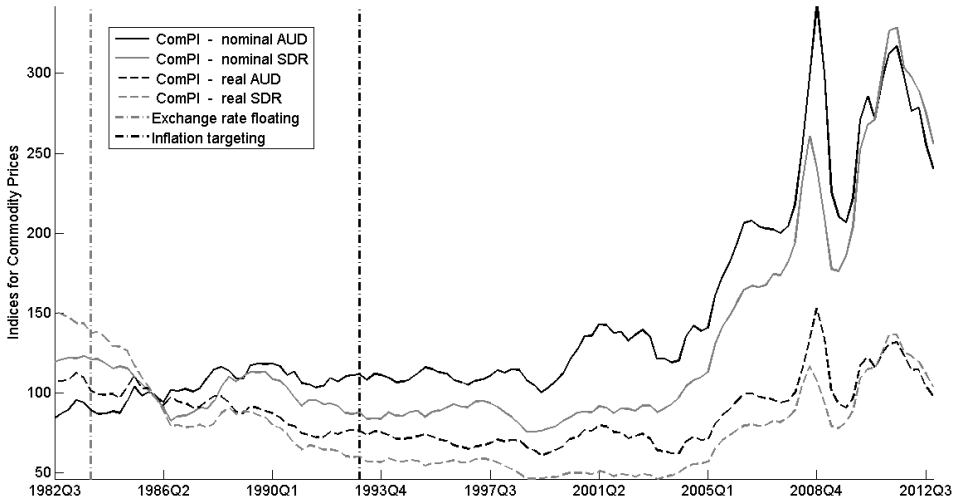
Table 8: Variance Decomposition

	State 1		
	Δy^d_t	π^d_t	Δs_t
World Demand	3,66%	4,26%	32,36%
Commodity Market	2,56%	1,31%	24,53%
Globalization	5,94%	7,48%	9,68%
Domestic shocks	87,84%	86,95%	33,42%

	State 2		
	Δy^d_t	π^d_t	Δs_t
World Demand	3,12%	22,85%	35,13%
Commodity Market	1,20%	0,60%	8,30%
Globalization	1,68%	1,78%	5,78%
Domestic shocks	94,00%	74,77%	50,79%

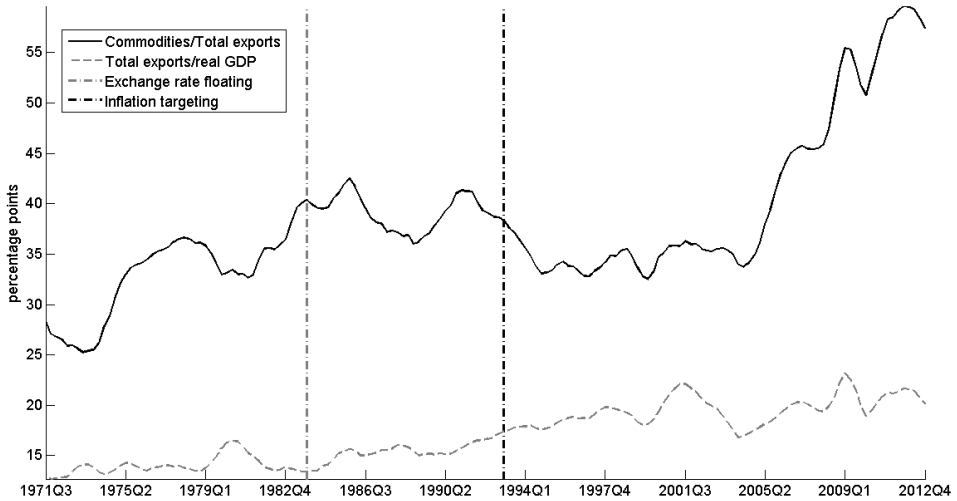
2.6 Figures

Figure 1: Commodity Price Indices



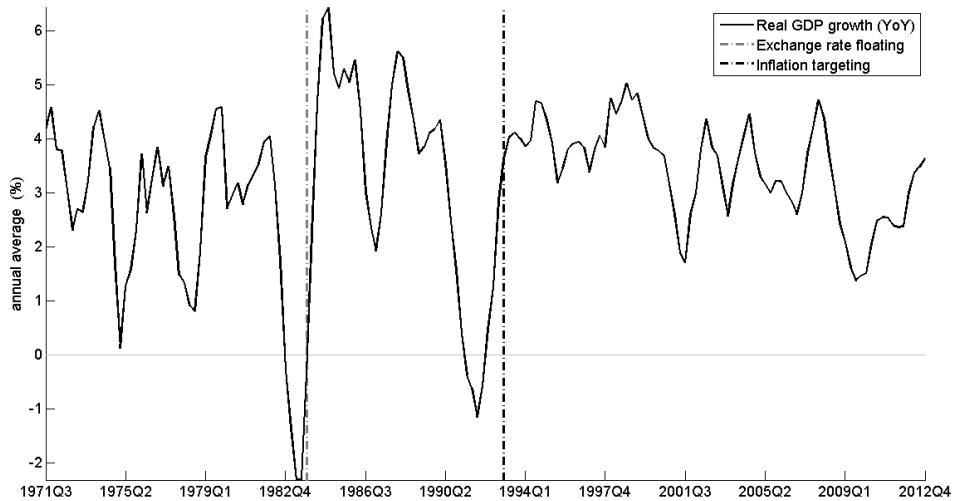
Source: RBA. Base = 100 in January 1986; quarterly averages.

Figure 2: Export Shares



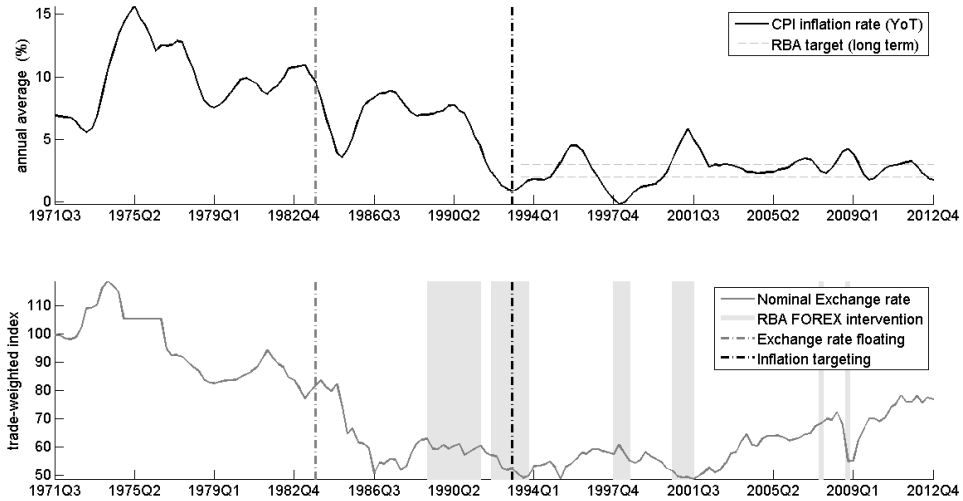
Source: ABS and author's calculations. Current values, free on board.

Figure 3: Real GDP Growth



Source: ABS. Chain volume measures.

Figure 4: CPI Inflation and Nominal Exchange Rate



Source: ABS and RBA.

Figure 5: Smoothed Probabilities of state 1

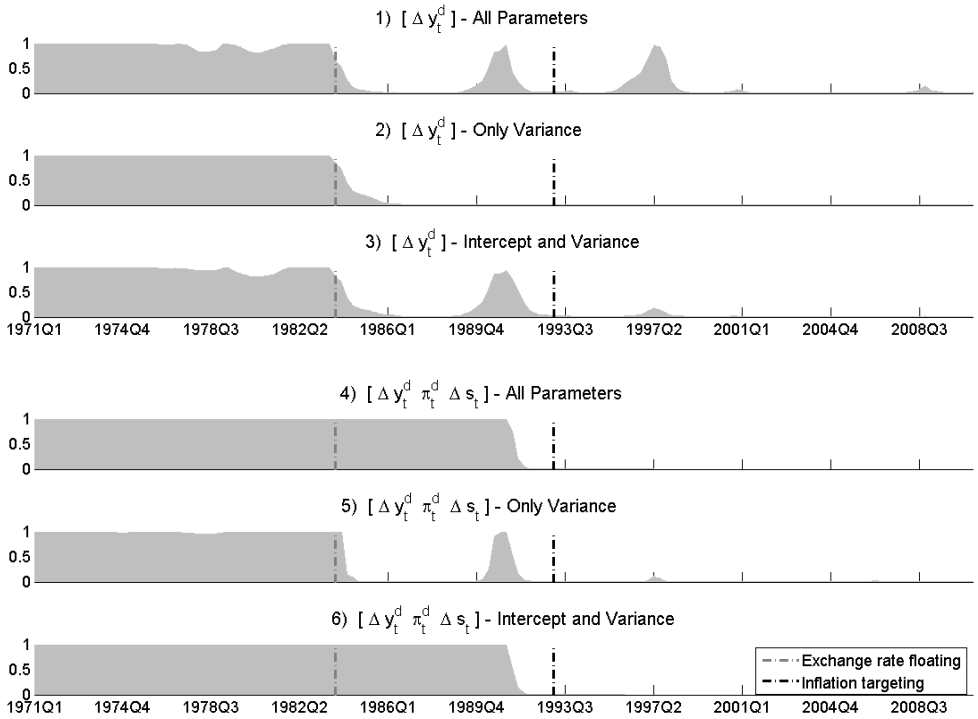


Figure 6: Smoothed Probabilities of state 1, cont.

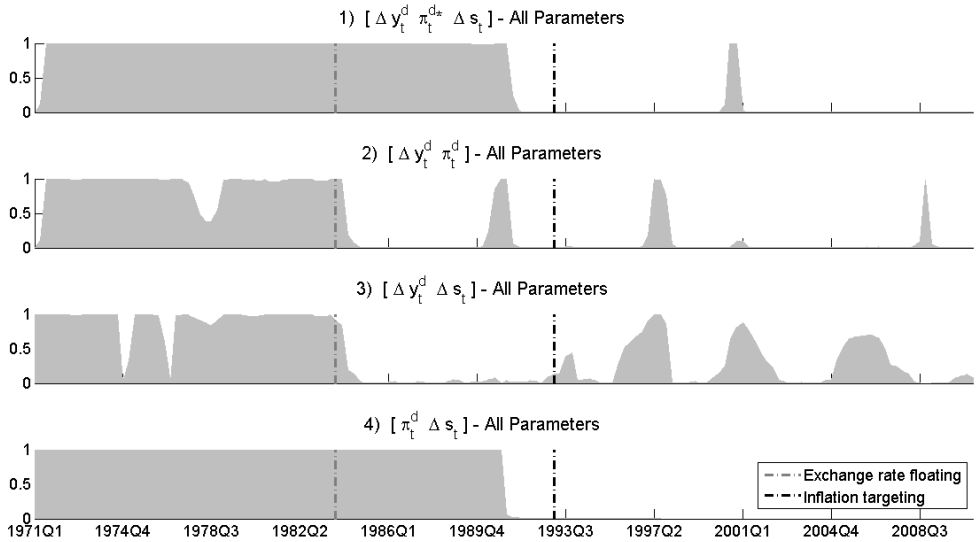
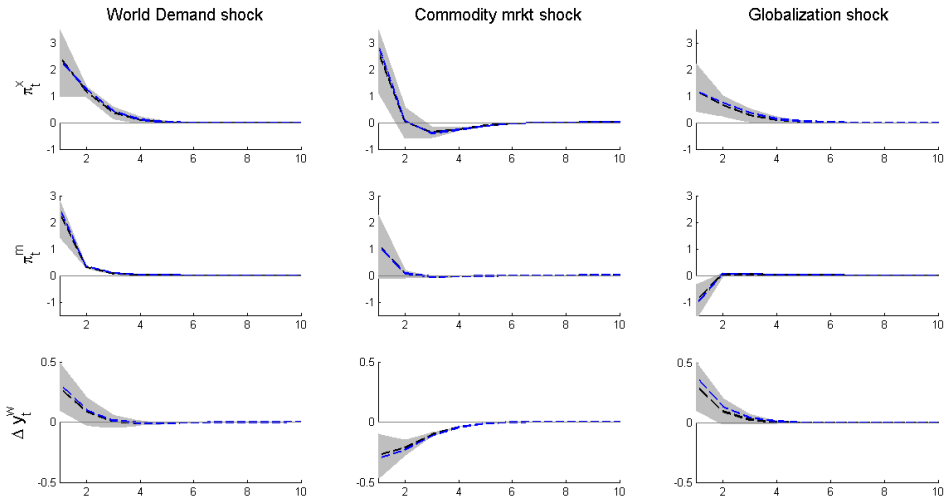
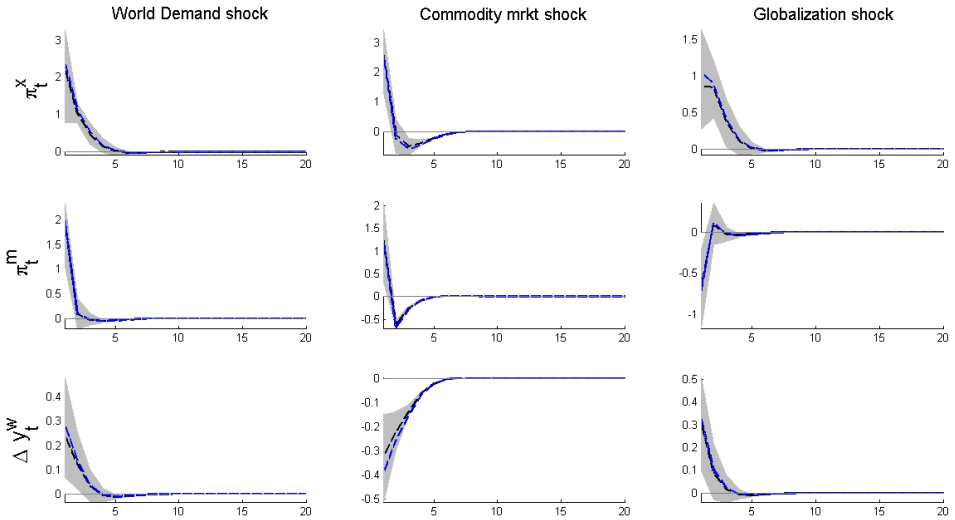


Figure 7: Impulse responses: World Block full sample



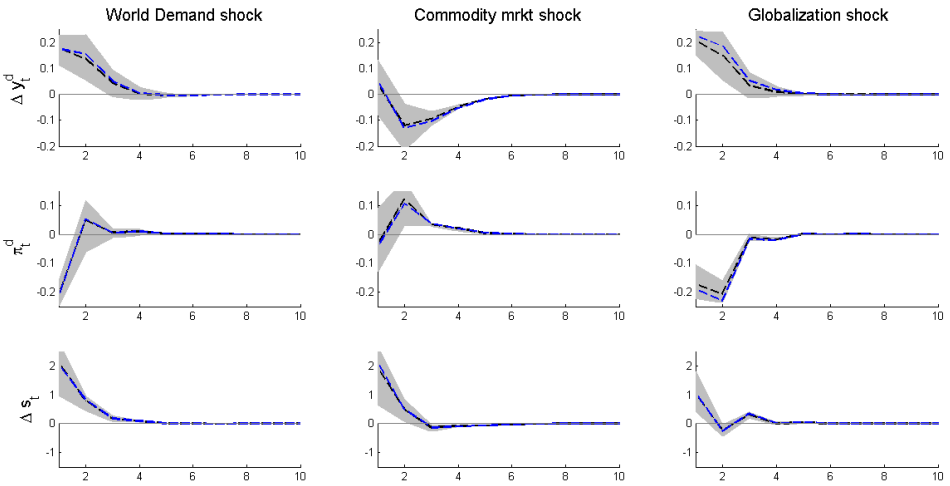
Median response (black line); Median target response (blue line), 16% – 84% percentile range (grey area).

Figure 8: Impulse responses: World Block from 1981Q1



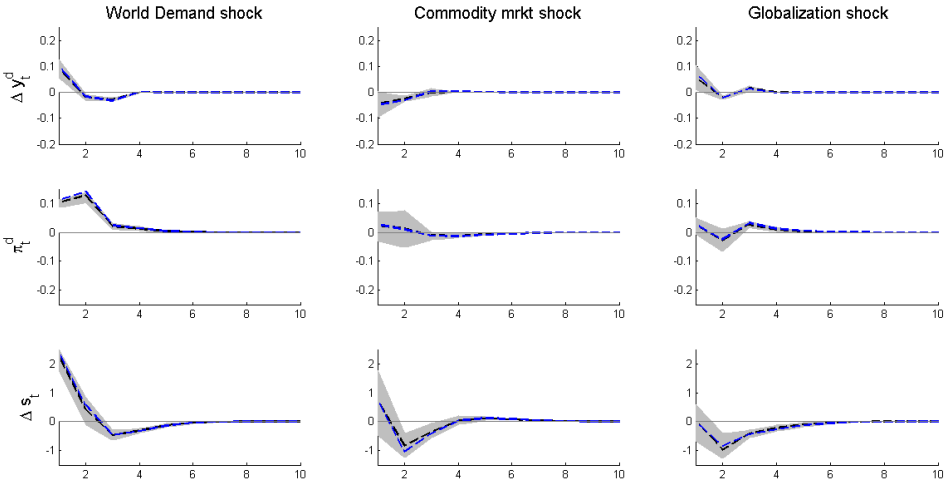
Median response (black line); Median target response (blue line), 16% – 84% percentile range (grey area).

Figure 9: Impulse responses: State 1



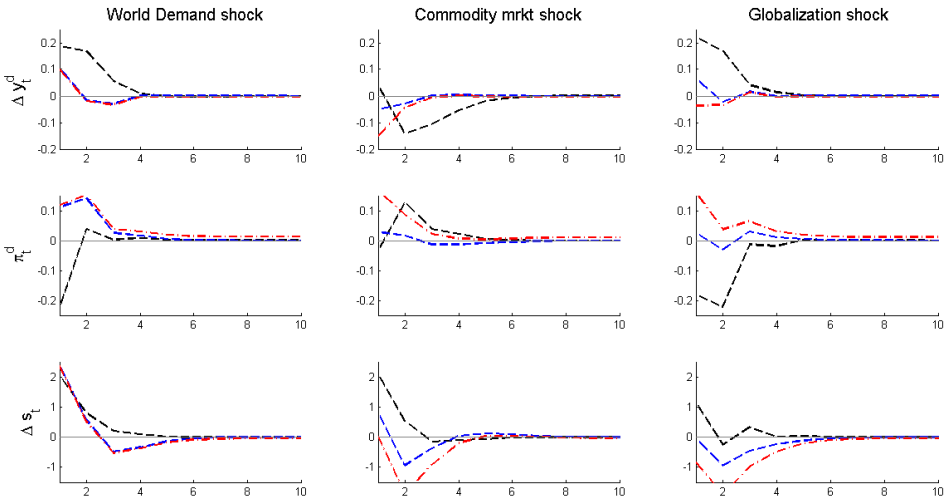
Median response (black line); Median target response (blue line), 16% – 84% percentile range (grey area).

Figure 10: Impulse responses: State 2



Median response (black line); Median target response (blue line), 16% – 84% percentile range (grey area).

Figure 11: Non-linear impulse responses, 1992Q1



State 1 (black line); State 2 (blue line), Non-linear (red line).

2.7 Appendix

Implementation of sign restrictions: Σ_e is a real symmetric matrix with positive entries. Using Cholesky decomposition, Σ_e can be expressed as $\Sigma_e = \mathcal{C}\mathcal{C}'$, where $\mathcal{C}$ is lower triangular. Therefore, making $\mathcal{A}_0^{-1} = \mathcal{C}$ ensures that the structural shocks are orthogonal and with standard deviations normalized to unity. Nevertheless, this decomposition might not satisfy *Table* 1. A way to overcome this is to obtain an orthogonal matrix $\tilde{P}$ and setting $\mathcal{A}_0^{-1} = \mathcal{C}\tilde{P}$. Given that $\tilde{P}\tilde{P}' = I$, it must also be true that $\mathcal{C}\tilde{P}\tilde{P}'\mathcal{C}' = \mathcal{A}_0^{-1}\mathcal{A}_0'^{-1} = \Sigma_e$.

In order to search over possible $\tilde{P}$ satisfying both the exogeneity restrictions for the world block and the SR depicted in *Table* 1 is to proceed as follows. Define $\mathcal{H}$ a random matrix with non-negative real entries of the form

$$\mathcal{H} = \begin{bmatrix} \mathcal{H}^1_{n\times n} & 0_{n\times d} \\ 0_{d\times n} & \mathcal{H}^2_{d\times d} \end{bmatrix} \tag{8}$$

For any given random draw of $\mathcal{H}$ (the entries of $\mathcal{H}^1_{n\times n}$ and $\mathcal{H}^2_{d\times d}$ are randomly drawn from a $\mathcal{N}(0,1)$ distribution), employ a QR decomposition and define $\mathcal{H} = QR$, with R a lower triangular matrix and Q an orthogonal matrix. Make $\tilde{P} = \mathcal{Q}$, compute the impulse response functions and check whether the SR are satisfied.

The EM algorithm: The EM algorithm is a two steps process. First, it estimates the conditional probabilities of the latent state to take a value at each point in time. Second, it maximizes the likelihood of the time series (using the probabilities just estimated) and finds the maximum likelihood estimates of the model's parameters Θ (the VAR coefficients Γ_j, the covariance matrices $\Sigma_{,j}$, and the transition probabilities). It is an iterative algorithm, repeating the two steps until the maximum likelihood estimates are constant between two consecutive iterations. Here, the first step is presented. For the computation of the maximum likelihood estimates refer to Krolzig (1997).

From (4), the conditional density the observation D_t is

$$f\left(D_t|s_t = j,\, \mathcal{Y}_{t-1};\, \Theta\right) = (2\pi)^{-\frac{d}{2}} \left|\Sigma_j^{-1}\right|^{\frac{1}{2}} exp\left\{-\frac{1}{2}\left(D_t - \Gamma_j Z_t\right)' \Sigma_j^{-1}\left(D_t - \Gamma_j Z_t\right)\right\} \tag{9}$$

which is stack in η_t for each state s_t

$$\eta_t = \begin{bmatrix} f\left(D_t|s_t=1,\,\mathcal{Y}_{t-1};\,\Theta\right) \\ \vdots \\ f\left(D_t|s_t=s,\,\mathcal{Y}_{t-1};\,\Theta\right) \\ \vdots \\ f\left(D_t|s_t=S,\,\mathcal{Y}_{t-1};\,\Theta\right) \end{bmatrix} \tag{10}$$

Denote $\xi_{t|t-1}$ the vector of conditional state probabilities

$$\xi_{t|t-1} = \begin{bmatrix} P\left\{s_t=1|\mathcal{Y}_{t-1};\,\Theta\right\} \\ \vdots \\ P\left\{s_t=s|\mathcal{Y}_{t-1};\,\Theta\right\} \\ \vdots \\ P\left\{s_t=S|\mathcal{Y}_{t-1};\,\Theta\right\} \end{bmatrix} \tag{11}$$

which are the inferences about the value of s_t based on data obtained through date $t-1$ and knowledge of Θ. With these two definitions, Hamilton (1994) shows that

$$\xi_{t|t} = \frac{\left(\xi_{t|t-1}\odot\eta_t\right)}{\mathbf{1}_S'\left(\xi_{t|t-1}\odot\eta_t\right)} \tag{12}$$

$$\xi_{t+1|t} = \mathrm{P}\xi_{t|t} \tag{13}$$

where $\odot$ denotes the element-by-element multiplication and $\mathbf{1}_S$ a $S\times 1$ vector of ones. The sample log likelihood can be obtained as

$$\begin{aligned} \mathcal{L}\left(\theta\right) &= \sum_{t=p+1}^{T} log\left(f\left(D_t|\mathcal{Y}_{t-1};\,\theta\right)\right) \\ with \quad & f\left(D_t|\mathcal{Y}_{t-1};\,\theta\right) = \mathbf{1}_S'\left(\xi_{t|t-1}\odot\eta_t\right) \end{aligned} \tag{14}$$

A relevant quantity is $\xi_{t|T}\left(j\right) = P\left(s_t=j|\mathcal{Y}_T;\,\Theta\right)$, the smoothed inference about the state at t given the information of the whole sample. This is obtained through iterating

$$\xi_{T-j|T} = \xi_{T-j|T-j}\odot\left\{\mathrm{P}'\left[\xi_{T-j+1|T}\oslash\xi_{T-j+1|T-j}\right]\right\} \tag{15}$$

for $t=T-1,\,T-2,\,\ldots,\,p+1$, and is started with $\xi_{T|T}$ obtained from (15) at $t=T$.

One other quantity of interest is $\xi_{t-1,t|T}$ which can be computed as

$$\xi_{t-1,t|T}(i,\,j) \;=\; \frac{p_{ji}\xi_{t-1|t-1}(i)\,\xi_{t|T}(j)}{\xi_{t|t-1}(j)} \tag{16}$$

where $\xi_{t-1,t|T} = P(s_t = j, s_{t-1} = i|\mathcal{Y}_T;\,\theta)$ is the probability that state j occurred at t after state i had occurred at $t-1$, given data observed through the full sample of observations.

The starting value $\xi_{1|0} = \rho$ is set to $\rho = \frac{1}{S}\mathbf{1}_S$, together with random values for Θ. To initiate iteration g, the initial values are set as

$$\rho \;=\; \xi_{1|T} \tag{17}$$

$$p_{ij} \;=\; \frac{\sum_{t=p+2}^{T}\xi_{t-1,t|T}(i,\,j)}{\sum_{t=p+2}^{T}\xi_{t-1|T}(i)} \tag{18}$$

$$\Gamma_s' \;=\; \hat{\Gamma}_s' \tag{19}$$

$$\Sigma_s \;=\; \hat{\Sigma}_s \tag{20}$$

obtained from iteration $g-1$ ($\hat{\Gamma}_s'$ and $\hat{\Sigma}_s$ are, respectively, the estimated coefficients and covariance matrices).

MS impulse responses: When the domestic block can transit between more than one state, that is, when $S > 1$, the computation of state-dependent impulse responses becomes more sophisticated. Following Camacho and Pérez-Quirós (2013), a naïve approach is taken by considering that only one shock hits the system, at time t, and compute the responses for an horizon H. That is, apart from the shock at time t, no other shocks occur before nor after. Denote $\xi_{t|k}$ a $S \times 1$ vector with the inferred probabilities of state s to prevail at time t, given the information available up to period k, and $D_{t+h|t}$ a $d \times S$ matrix whose entries are the estimated values of the domestic variables for each state (computed directly from the estimated version of (4)). The unconditional h-period ahead forecast of the domestic variables, given the information available at time t, is denoted by $D^f_{t+h|t}$ and equal to $D_{t+h|t}\xi^f_{t+h|t}$, where $\xi^f_{t+h|t} = \mathrm{P}^h\xi_{t|t}$.[28]
Therefore, it is crucial to compute the vector $\xi_{t|t}$: the probabilities of each state at

[28] $D^f_{t+h|t}$ is an $d \times 1$ vector with the expected values of the domestic variables, computed as the sum of the within state forecasts weighted by the probability of the corresponding state to occur.

the beginning of response. When no shocks hit the system, $\xi_{t|t}$ coincides with $\xi_{t|T}$, the smoothed inference about the state at t given the information of the whole sample. When a shock hits the system at time t, the vector of probabilities is denoted by $\xi^*_{t|t}$ and has to be inferred from η^*_t and $\xi_{t|t-1}$. The latter can be obtained directly from the EM algorithm, since it is assumed that no shock occurred before time t, while η^*_t is computed using the estimated value of the domestic variables $D^f_{t|t}$ plus the shock for each state. Finally, $\xi^*_{t|t}$ is obtained from

$$\xi^*_{t|t} = \frac{\left(\xi_{t|t-1} \odot \eta^*_t\right)}{\mathbf{1}'_S \left(\xi_{t|t-1} \odot \eta^*_t\right)} \tag{21}$$

with $\mathbf{1}'_S$ a vector of ones of dimension S. Finally, the impulse responses are obtained as

$$\Upsilon_t = D^{f*}_{t+h|t} - D^f_{t+h|t} \tag{22}$$

Chapter 3

Monetary Policy and Sectoral Relative Wage Rigidity

Keywords: labour market rigidity; search-and-matching; terms-of-trade shocks; commodity prices; monetary policy.
JEL Classification: E24, E52, F41, J64.

Sydney, December 2014

3.1 Introduction[§]

As with any other price, wages are a fundamental piece of market construction. They match demand and supply, signal changes in market conditions to economic agents and promote the efficient allocation of labour services. Wages are particularly important. They are a direct determinant of households' disposable income,

[§]Parts of this chapter were completed during my PhD internship at the Reserve Bank of Australia (RBA) in the last quarter of 2014. I am grateful to Dan Rees for his support during my stay at the Bank. The views I express here do not necessarily represent those of the RBA or its staff.

thereby affecting households purchasing power and living standards. Moreover, as labour services are indispensable in virtually any economic activity, wages are also a key determinant for production and economic competitiveness. Unlike most prices, however, wages are typically prevented from being formed purely by market forces. Most current arguments for mediation of wage formation, usually supervised by governmental entities, lie on the exact same causes that make wages a special price: their importance for household's living standards and for economic competitiveness. Contrary to market forces, the argument goes, centralized wage setting smooths undesirable wage variation, both in time and across segmented sectors of the economy, balances asymmetric negotiation powers between employers and employees and internalizes equity considerations ignored in decentralized systems.

In this paper, I study the implications of sectoral relative wage rigidity for the conduct of optimal monetary policy in a small commodity-exporting economy. When wages in one sector increase relative to wages elsewhere, all else equal, labour supply tends to shift to the former. This reallocation of resources across the economy occurs insofar as sectoral labour services are close substitutes. In an economy with no slack, these movements cause a contraction elsewhere. Moreover, as long as relative wages remain disparate, households' income also varies across sectors. To avoid these outcomes, which might be deemed socially or politically undesirable, wage-setting mediation may find appropriate to curb movements in sectoral relative wages.

For a small commodity-exporting economy, commodity prices have a considerable influence on economic performance. The sizeable share the resource sector represents both in terms of total output and of total exports makes these countries highly dependant on swings of commodity prices. Figure 1 reports the deviations from trend of commodity-price indexes for four commodity-exporters, illustrating how large and volatile commodity price shocks are. As with any country, commodity price shocks affect real activity and inflation insofar as commodities constitute intermediate goods in production or are used for consumption. In addition, commodity-exporters are also affected through the resource reallocation spurred by commodity price shocks. Crucially, this last effect depends on the wage-setting framework in place.

Consider an increase in commodity prices which triggers an expansion of the commodity sector. This induces a rise in relative wages awarded in the resource industry and leads to the reallocation of labour across the economy. At the same

time, the shock generates an income effect and induces an appreciation of the exchange rate. As the increase in relative wages shifts supply away from the non-commodity sectors and the appreciation of the exchange rate reduces competitiveness of domestic tradeables both at home and abroad, the non-commodity sector contracts. If labour markets are flexible and labour is perfectly substitutable between sectors, relative wages adjust and labour is efficiently allocated. However, if labour markets are segmented and wage-setting is centralised, commodity price shocks can cause inefficient labour market movements across the economy. Several commodity-exporters have experienced the consequences of centralized wage-setting arrangements.[1] Table 1 presents a number of indices that characterize the wage-setting framework for the same group of countries in Figure 1. In Canada, wage formation is highly decentralized, generally taking place at industry or firm level, whereas in Norway wages are negotiated at the national level. In Australia and New Zealand, labour market reforms have reduced the extent of wage-setting centralization, with sectoral and firm level negotiations being progressively encouraged. In these countries, sectoral relative wage differentials have grown since.

Do these differences in wage formation affect employment, production and inflation? Do they matter for the conduct of monetary policy? Assume that wages in the non-commodity sectors are forced to accompany the rise in wages in the commodity sector. For non-commodity industries, a rise in commodity prices and wages raises total costs and shrinks profit margins. As non-commodity tradeables face increasing competition from foreign producers, the rise in production costs is felt particularly painfully. Moreover, if labour markets are segmented and relative wages sticky, low labour mobility together with sectoral upward wage pressures cause severe losses in output and employment. On the other hand, as production falls and firms pass wage pressures on to prices, inflation starts going up. If these effects are strong enough, the shock becomes inflationary despite the impact the appreciation of the exchange rate has in reducing import prices. Therefore, understanding the mechanisms behind the transmission of commodity price shocks is of paramount importance to the design of optimal monetary policy. Should these inflationary pressures be curbed by the monetary authority? Can monetary policy mitigate the effects of sectoral relative wage rigidity?

[1] For Australia, which constitutes the illustrative example used in this paper, refer to the descriptions of the commodity boom in late 1970s/early 1980s in, for instance, Pagan (1987), Battellino (2010), and Connolly and Orsmond (2011).

In order to address these questions, I construct a general equilibrium model of a two-sector, small commodity-exporting economy with segmented labour markets. The commodity sector is directly exposed to exogenous commodity price shocks, as the bulk of commodity output is exported. The remaining small share of domestically-extracted commodities is used in the production of non-commodity tradeable goods. Labour services can be transferred between the two sectors. However, labour mobility is subject to frictions due to market segmentation. In particular, new hires are subject to matching frictions which depend on the number of existing vacancies and the number of job seekers. These frictions can generate a wedge in sectoral wages. I introduce relative wage rigidity in the non-commodity sector by assuming that wages there are staggered and linked to the evolution of wages in the commodity sector.

I calibrate the model to the Australian economy and show that, for a positive commodity price shock that raises wages in the commodity sector, non-commodity firms reduce output and their demand for labour more the higher the degree of sectoral relative wage rigidity. As firms internalise the degree of relative wage rigidity, they anticipate upward wage pressures due to the expansion of the commodity sector, downsizing their hiring schedule in order to sustain profit margins. Although relative wage rigidity magnifies the appreciation of the nominal exchange rate, the increase in nominal wages forces inflation to rise. As nominal wages in different sectors are tightly related, sectoral shocks amplify movements of labour. With segmented labour markets, labour reallocation is slow and inefficient, generating unemployment and inflation. As a result, I find that for economies where relative wages are slow to adjust, both inflation and unemployment are more volatile.

I derive a micro-founded welfare measure and study how monetary policy should respond to inflation under different levels of sectoral relative wage rigidity. I study simple feedback rules to the nominal interest rate and show that a stronger response to inflation for higher degrees of relative wage rigidity is optimal. Interestingly, however, comparing the optimal feedback rule with a zero-inflation policy, I find that the latter is comparatively worse when relative wages are less flexible. As sectoral nominal wages become less flexible, price inflation smooths the evolution of real wages by partially cancelling out nominal changes. Hence, although still being optimal to respond more aggressively to price inflation, strict inflation targeting is more detrimental the higher the degree of relative wage rigidity. To shed light on these apparently opposing findings, I complement the analysis using general loss functions to describe the preferences of the monetary

authority. I show that, if the monetary authority's objective is to minimize price and non-commodity output volatilities alone, the response to inflation needs to be more aggressive for higher degrees of sectoral relative wage rigidity. Instead, if the objective is to reduce price and unemployment volatilities, the response to inflation should be milder for higher degrees of relative wage rigidity. This clearly shows the trade-off between price and wage inflation volatility.

Literature: Erceg et al. (2000) provide a seminal contribution to the study of wage stickiness in a general equilibrium framework and study the trade-off between price and wage inflation for monetary policy. They establish that strict price inflation targeting is not optimal, as I do here, and show that a simple rule targeting both price and wage inflation performs nearly as well as the optimal rule. Gertler and Trigari (2006) introduce staggered wage bargaining in a search-and-matching framework, while Gertler et al. (2008) further include wage indexation to past inflation. These features allow their model to fit labour market data relatively well. In a contemporaneous contribution, Thomas (2008) also introduces staggered wages and search-and-matching frictions to a single-sector, closed economy model to investigate its implications for monetary policy. He shows that optimal monetary policy puts more weight on wage inflation as nominal wages become stickier. Yet, monetary policy is unable to fully offset the detrimental effects of wage stickiness on welfare. My paper complements this literature by generalizing its finding to a multi-sector, small open economy. In addition, I extend the literature by focusing on the implications of relative wage rigidity for the conduct of monetary policy.

The hump-shape hypothesis conjectured by Calmfors and Driffill (1988) constitutes a central piece within the literature on wage-setting centralization.[2] They establish that both extremes of no and full centralisation are conducive to real wage moderation and lower unemployment in a closed economy. Their argument also highlights the role of price inflation. For lower levels of centralisation, firms are reluctant to accept wage rises because they are price takers and cannot adjust margins. At industry level, wage rises become easier to secure as the whole industry can pass the extra costs on to prices. This is true insofar as the elasticity of substitution between goods of different industries is low. Finally, at the national level trade unions internalise the effects of nominal wage rises on prices, thereby leaving real wages unchanged.

Driffill and van der Ploeg (1993) extend the analysis to an open economy and

[2] Calmfors (1993) provides a thorough review of the literature on the hump-shape hypothesis.

find that centralisation raises wages and unemployment. In their model, as wage rises cause an appreciation of the exchange rate, the overall effect on inflation is negative. In a world where firms set prices as a mark-up over marginal costs, this effect seems contradictory. Instead, in the model presented here wage rises are inflationary for higher degrees of sectoral relative wage rigidity. Cukierman and Lippi (1999) study the implications of monetary policy in a model that preserves the hump-shape hypothesis, and show that the response of monetary policy to inflation depends on the degree of wage-setting centralization. If the monetary authority cares about inflation and unemployment volatility, they find that the optimal response to inflation is milder for higher degrees of centralisation. In the model used here, I extend these findings in a model of sectoral relative wage rigidity.

The rest of the paper is structured as follows. In the next section, I describe the model with a particular emphasis on the structure of the labour market and of wage formation. In section 3, I derive the log-linear conditions that link wage-setting to employment and output. Section 4 reports the calibration of the model. I use Australia as an illustrative case study and base the parametrization on previous studies. In addition, I use GMM to match the second moments of key variables that are particularly relevant for the analysis of monetary policy. In section 5, I study optimal monetary policy by comparing the welfare implications of various policy rules under different degrees of sectoral relative wage rigidity. Section 6 complements this analysis by focusing on loss functions that depend on inflation, non-commodity output, and unemployment volatilities. Section 7 concludes.

3.2 Model

The model described in this section extends the two-sector model of Hevia and Nicolini (2013) by including segmented labour markets and search and matching frictions. These features allow sectoral wages to differ. Households direct their job-search efforts between the two sectors and sectoral wages are set through Nash bargaining. In the non-commodity sector, besides search and matching frictions, wages are sticky as in Thomas (2008). Each period, only a fraction of wage contracts are optimally renegotiated. The remaining wage contracts follow an exogenously determined indexation rule. This rule determines how sensitive wages in the non-commodity sector are to the evolution of wages in the commodity

sector.

3.2.1 Households

A large representative household composed of a continuum of measure 1 of agents chooses the optimal bundle of domestic and foreign consumption goods and how much labour services to supply to the two production sectors. As in Merz (1995), household members are assumed to pool their income so that consumption is equal regardless of work status. The household's lifetime utility is given by

$$\tilde{\mathcal{U}} \;=\; \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u\left(c_t\right) - l\left(n_t^h,\, n_t^x\right) \right\}$$

where $0 < \beta < 1$ is the discount factor and $c_t = h\left(c_t^h,\, c_t^f\right)$ is a composite good made of domestic and foreign-produced goods. I assume $h\left(c_t^h,\, c_t^f\right)$ is a function homogeneous of degree one and increasing in both arguments, while $u\left(c_t\right)$ is increasing and concave. Household members can be either working or unemployed

$$n_t + u_t \;=\; 1$$

where n_t denotes total employment, and u_t the number of unemployed members. In turn, employed members work either in the non-commodity sector or in the commodity sector, $n_t = n_t^h + n_t^x$, and derive disutility from work according to $l\left(n_t^h,\, n_t^x\right)$, which I assume is increasing in both arguments and convex. Finally, unemployed members are also assumed to be directing their job-searching efforts in either one of the two sectors, $u_t = u_t^h + u_t^x$.

Household members can trade two risk-less nominal bonds, $\tilde{b}_t$ and $\tilde{b}_t^\star$, denominated in domestic and foreign currency, respectively. They receive nominal wages W_t^h and W_t^x from the non-commodity and commodity sectors, respectively, and aggregate real net profits $\tilde{\Pi}_t$ from firms. The household's budget constraint can therefore be written as

$$c_t + \frac{\pi_{t+1}}{i_t}\tilde{b}_t + \frac{\pi_{t+1}^\star}{i_t^\star} q_t \tilde{b}_t^\star \;\leq\; \frac{W_t^h}{P_t} n_t^h + \frac{W_t^x}{P_t} n_t^x + \tilde{b}_{t-1} + q_t \tilde{b}_{t-1}^\star + \tilde{\Pi}_t$$

where q_t is the real exchange rate, π_t and $\pi_t^\star$ are domestic and foreign gross inflation rates, and i_t and $i_t^\star$ are the gross nominal interest rates between periods $t-1$ and

t. P_t is the consumer price index and is obtained by maximizing c_t subject to $P_t c_t = P_t^h c_t^h + P_t^f c_t^f$. The solution to the households problem implies an Euler equation of the form

$$u'(c_t) = \beta \mathrm{E}_t \left[\frac{i_t}{\pi_{t+1}} u'(c_{t+1}) \right] \tag{1}$$

and the non-arbitrage between domestic and foreign bonds, which determines the uncovered interest parity condition, is given by

$$i_t = i_t^\star \mathrm{E}_t \left[\frac{\pi_{t+1}}{\pi_{t+1}^\star} \frac{q_{t+1}}{q_t} \right] \tag{2}$$

3.2.2 Non-commodity Sector

The structure of the non-commodity sector comprises three types of firms. At the start of the production chain, perfectly competitive wholesale firms produce an intermediate homogeneous good that they sell to retail firms for the real price φ_t^h. Wholesalers choose the optimal ratio of factor inputs and participate in the bargaining process with workers in order to set wages. In turn, retailers are monopolistic competitors who transform the homogeneous good into differentiated goods. This gives them market power, which they use to fix the price of their product. However, I introduce price setting frictions and assume that not all retailers are able to set prices optimally every period. Finally, perfectly competitive final firms buy the differentiated goods from retailers and produce a final composite good that is sold for consumption domestically and abroad.

Final firms: Final firms produce the final good by aggregating non-tradeable intermediate goods sold by a continuum of measure 1 of retailer firms indexed by $i \in [0, 1]$. They do so using the following technology

$$Y_t^h = \left[\int_0^1 y_{it}^{h \frac{\mu^h - 1}{\mu^h}} di \right]^{\frac{\mu^h}{\mu^h - 1}} \tag{3}$$

where $\mu^h > 1$ is the elasticity of substitution between intermediate goods. Final firms take as given the price of the final good, P_t^h, as well as the prices of each intermediate good, P_{it}^h. Solving for the optimal composition of intermediate goods

yields the demand function for retailer firm i

$$y_{it}^{h} = Y_{t}^{h}\left(\frac{P_{it}^{h}}{P_{t}^{h}}\right)^{-\mu^{h}} \tag{4}$$

Substituting these into (4), I obtain the price of the final good

$$P_{t}^{h} = \left[\int_{0}^{1} P_{it}^{h\,1-\mu^{h}} di\right]^{1/\left(1-\mu^{h}\right)}$$

which is a composite index of the intermediate prices.

Retail firms: Retailers buy intermediate goods produced by wholesale firms at the perfectly competitive real price φ_{t}^{h}. In turn, as they have market power, they can set the price P_{it}^{h} for which they sell their goods to the final good producers. However, they face price adjustment rigidities as in Calvo (1983), only being able to adjust prices with probability $(1-\delta^{p})$. As such, retailers solve

$$\max \mathrm{E}_{t}\sum_{s=0}^{\infty}\beta_{t,t+s}\left(\delta^{p}\right)^{s}\left[\frac{P_{it}^{h}}{P_{t+s}}-\varphi_{t+s}^{h}\right]y_{it+s}^{h}$$

subject to (4), and where $\beta_{t,t+s} \equiv \beta^{s}u'\left(c_{t+s}\right)/u'\left(c_{t}\right)$ is the stochastic discount factor between t and $t+s$. The first-order condition with respect to P_{it}^{h} can be expressed as

$$\frac{P_{t}^{h*}}{P_{t}^{h}} = \frac{\mu^{h}}{\mu^{h}-1}\frac{\mathrm{E}_{t}\sum_{s=0}^{\infty}\beta_{t,t+s}\left(\delta^{p}\right)^{s}\varphi_{t+s}^{h}Y_{t+s}^{h}\left(\frac{P_{t}^{h}}{P_{t+s}^{h}}\right)^{-\mu^{h}}}{\mathrm{E}_{t}\sum_{s=0}^{\infty}\beta_{t,t+s}\left(\delta^{p}\right)^{s}Y_{t+s}^{h}\frac{P_{t+s}^{h}}{P_{t+s}}\left(\frac{P_{t}^{h}}{P_{t+s}^{h}}\right)^{1-\mu^{h}}} \tag{5}$$

Note that price setters behave symmetrically. Furthermore, using the expression for the price index, I can write the evolution of P_{t}^{h} as

$$\left(P_{t}^{h}\right)^{1-\mu^{h}} = \left(1-\delta^{p}\right)\left(P_{t}^{h*}\right)^{1-\mu^{h}}+\delta^{p}\left(P_{t-1}^{h}\right)^{1-\mu^{h}} \tag{6}$$

where I used the fact that firms are randomly chosen to adjust their price. With price dispersion, there is a real loss in terms of final production. To see this, I define $y_{t}^{h} \equiv \int_{0}^{1} y_{it}^{h}di$ and, using the demand functions for each retailer, obtain $y_{t}^{h} = \Upsilon_{t}^{h}Y_{t}^{h}$, where $\Upsilon_{t}^{h} \equiv \int_{0}^{1}\left(P_{it}^{h}/P_{t}^{h}\right)^{-\mu^{h}}di \geq 1$ is an index of price dispersion.[3]

[3] Υ_{t}^{h} equals unity when all retailers practice the same price $P_{it}^{h} = P_{t}^{h}$, $\forall i$.

Wholesale firms: At the core of the non-commodity sector is a continuum of measure 1 of wholesale firms, which I index by $j \in [0,1]$. Each of them produces an homogeneous good with technology given by

$$y_{jt}^h = \alpha^h Z_t \left(n_{jt}^h\right)^{\alpha_1^h} \left(x_{jt}^h\right)^{1-\alpha_1^h}$$

where $\alpha^h = \left(\alpha_1^h\right)^{-\alpha_1^h} \left(1-\alpha_1^h\right)^{-\left(1-\alpha_1^h\right)}$, and Z_t is a stationary technology shock common to all firms. Wholesalers use domestic commodities, x_{jt}^h, and labour, n_{jt}^h, as inputs to production. In order to hire new workers, they post vacancies, v_{jt}^h, for which they incur a cost in terms of units of consumption. Hence, wholesale firms solve

$$\begin{aligned}\mathcal{F}_{jt} &= \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \frac{u'\left(c_{t+1}\right)}{u'\left(c_0\right)} \left(\varphi_t^h \alpha^h Z_t \left(n_{jt}^h\right)^{\alpha_1^h} \left(x_{jt}^h\right)^{1-\alpha_1^h} - \frac{P_t^x}{P_t} x_{jt}^h - \frac{W_{jt}^h}{P_t} n_{jt}^h \right. \\ &\quad \left. - \frac{\varsigma^h}{1+\psi^h} \frac{1}{u'\left(c_t\right)} \left(\frac{v_{jt}^h}{n_{jt}^h}\right)^{1+\psi^h} n_{jt}^h \right)\end{aligned}$$

with $\psi^h > 0$.[4]

The first-order condition with respect to the domestic commodity can be expressed in aggregate terms as

$$\varphi_t^h \left(1-\alpha_1^h\right) \alpha^h Z_t \left(\frac{n_t^h}{x_t^h}\right)^{\alpha_1^h} = \frac{P_t^x}{P_t} \tag{7}$$

Due to constant returns to scale in production, the input ratio is equalized across firms.[5] As a result, aggregate production from wholesalers is given by

$$y_t^h = \alpha^h Z_t \left(n_t^h\right)^{\alpha_1^h} \left(x_t^h\right)^{1-\alpha_1^h} \tag{8}$$

where $n_t^h = \int_0^1 n_{jt}^h dj$ and $x_t^h = \int_0^1 x_{jt}^h dj$.

[4]The choice of the functional form of hiring costs follows Thomas (2008) and is essentially due to the fact that convex costs of vacancy posting prevent corner solutions. With nominal wage stickiness, wage dispersion will induce dispersion in vacancy posting across firms. If hiring costs were linear, marginal costs of posting vacancies would be equal across firms and only the firm paying the lowest wage, and therefore having the highest marginal benefit of posting vacancies, would post vacancies.

[5]Defining $y_t^h \equiv \int_0^1 y_{jt}^h dj$, I can write $y_t^h = \alpha^h Z_t \left(n_t^h / x_t^h\right)^{\alpha_1^h} \int_0^1 x_{jt}^h dj$.

3.2.3 Commodity Sector

I follow the approach in Rayner and Bishop (2013) and take a broad definition of the commodity sector. As such, I assume it to include not only the extracting industry, but also the manufacturing industry that transforms raw commodities into products that are traded as commodities, as well as all the supporting services and industries closely related to mining activity. Moreover, because commodity prices are largely determined in international markets, I assume that commodity firms behave as price takers and have no market power over price setting. I denote by $P_t^{x\star}$ the foreign currency price of domestically-produced commodities and by P_t^x the producer nominal price. Assuming the law of one price holds, the real price domestic firms face is then given by $P_t^x/P_t = q_t P_t^{x\star}/P_t^\star$.

For simplicity, and given the discussion above, I consider a single representative commodity firm producing commodity x_t with the following technology

$$x_t = A_t \left(n_t^x\right)^{\alpha_1^x} \tag{9}$$

where A_t is a stationary technology shock specific to the commodity sector, and $0 < \alpha_1^x \leq 1$. As in Hevia and Nicolini (2013), labour is the only explicit factor in production. The assumption behind $\alpha_1^x < 1$ relates to the existence of an implicit factor in the mining industry, which is assumed to be fixed and can be interpreted as land.

The commodity firm posts vacancies in order to hire new workers in the next period. However, as with wholesale firms, vacancy posting is costly, depending on the ratio of vacancy posts, v_t^x, to total employees. Hence, the intertemporal problem of the representative commodity firm can be expressed as

$$\mathcal{C}_t = \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \frac{u'\left(c_{t+1}\right)}{u'\left(c_0\right)} \left(\frac{P_t^x}{P_t} A_t \left(n_t^x\right)^{\alpha_1^x} - \frac{W_t^x}{P_t} n_t^x - \frac{\varsigma^x}{1+\psi^x} \frac{1}{u'\left(c_t\right)} \left(\frac{v_t^x}{n_t^x}\right)^{1+\psi^x} n_t^x \right)$$

with $\psi^x > 0$ governing the elasticity of vacancy posting. Finally, domestic commodities are sold to non-commodity producers and to export

$$x_t = x_t^h + x_t^f$$

3.2.4 Labour Market

In Hevia and Nicolini (2013), labour is fully mobile since labour services are perfectly substitutable between the two sectors. Consequently, the labour market is homogeneous and a single nominal wage equalizes marginal productivities of labour in both sectors. On the contrary, I assume here that the labour market is segmented, allowing wages to differ between sectors. To achieve this, I introduce sectors specific search-and-matching technology and different wage-setting systems. In the non-commodity sector, wages are assumed to be sticky and to follow an exogenous indexation rule linking wages there to the evolution of wages in the commodity sector. Instead, in the commodity sector wages are flexibly set each period.[6]

The evolution of the labour force employed in sector $s = h, x$ follows a law of motion governed by two forces. First, an exogenous separation rate $0 < \lambda^s < 1$, determining the average duration of contracts between firms and workers. Second, an endogenous matching rate $m^s\left(v_t^s, u_t^s\right)$, which depends on a constant returns to scale technology over aggregate sectoral vacancies, v_t^s, and job-seekers, u_t^s. I assume that new matches only become productive one period after being generated. Furthermore, when deciding about the number of vacancy posts and in which sector to search for a job, firms and the household take as given the sector-wide matching rates. In turn, matching rates are a function of the aggregate labour market tightness in each sector, which is given by the ratio of vacancies to job-seekers. Hence, defining the aggregate sectoral job-finding rate by $\iota_t^s \equiv m^s/u_t^s$, the evolution of household's employment in each sector can be expressed as

$$n_{t+1}^s = \iota_t^s u_t^s + \left(1 - \lambda^s\right) n_t^s$$

Equivalently, defining by $\nu_t^s \equiv m_t^s/v_t^s$ the hiring rate of each firm j in the non-commodity sector and of the representative commodity firm, I can write the law of motion of wholesale firm j's employment as

$$n_{jt+1}^h = \nu_t^h v_{jt}^h + \left(1 - \lambda^h\right) n_{jt}^h$$

whereas the law of motion of employment in the representative commodity firm

[6] To motivate this assumption, refer to the discussion of the environment characterizing the commodity sector had earlier and to D'Arcy et al. (2012) for evidence respecting Australia.

is given by

$$n_{t+1}^{x} = \nu_t^x v_t^x + (1-\lambda^x)\, n_t^x$$

Households: Under the assumption of labour market segmentation, the representative household has to choose how many of its unemployed members to direct to a specific sector in search for a job. In sector s, the *worker surplus* is given by $H_{jt}^s = \mathcal{W}_{jt}^s - \mathcal{U}_t^s$, where $\mathcal{W}_{jt}^s$ is the asset value of being matched to firm j in sector s and $\mathcal{U}_t^s$ is the value of being unemployed in sector s. H_{jt}^s is then given by

$$H_{jt}^s = \frac{W_{jt}^s}{P_t} - \frac{\kappa^s}{c_t^{-\sigma}} - \iota_t^s \mathrm{E}_t \left\{ \beta_{t,t+1} \int_0^1 \frac{v_{jt}^s}{v_t^s} H_{jt+1}^s dj \right\} + (1-\lambda^s)\, \mathrm{E}_t \left\{ \beta_{t,t+1} H_{jt+1}^s \right\} \tag{10}$$

The *worker surplus* is the difference between the real wage earned and the disutility derived by being employed, adjusted to the future probability of being hired elsewhere and the continuation value of remaining employed with firm j in sector s. As in Gomes (2012), optimality implies that there are no additional gains from searching in one sector *vis-à-vis* the other. That is, $\mathcal{U}_t^h = \mathcal{U}_t^x$ must hold in equilibrium, which is true if

$$\iota_t^h \mathrm{E}_t \left\{ \beta_{t,t+1} \int_0^1 \frac{v_{jt}^h}{v_t^h} \left[H_{jt+1}^h \right] dj \right\} = \iota_t^x \mathrm{E}_t \left\{ \beta_{t,t+1} H_{t+1}^x \right\} \tag{11}$$

Note that in (11), the expression with respect to the commodity sector is simplified by the fact that only one representative firm is considered, whereas the future value of being hired in the non-commodity sector depends on the distribution of vacancies across the j firms.

Non-commodity sector: In the non-commodity sector, I assume that firms can renegotiate wages with probability $(1-\delta^w)$ each period. As a result, wages are staggered and the bargaining between workers and firms becomes dynamic, requiring present discounted valuations of future surpluses. This set-up follows Thomas (2008) closely. However, I extend his model of staggered wages by introducing wage indexation. When wages are not renegotiated, they are updated

according to an exogenous indexation rule. Therefore, nominal wages in firm j in period $t+k$ evolve according to

$$W_{jt+k}^{h} = \begin{cases} W_{jt+k}^{h*} & \text{with probability } 1-\delta^{w} \\ W_{jt}^{h*}\left[\prod_{i=1}^{k}\left(\pi_{t+i-1}^{W}\right)\right]^{\gamma_w} & \text{with probability } \delta^{w} \end{cases} \tag{12}$$

where W_{jt+k}^{h*} is the nominal wage that results from renegotiation in period $t+k$, and the term in square brackets is the indexation rule. As I discuss below, in this paper the indexation rule is a proxy for sectoral relative wage rigidity.

With staggered wages, the *worker surplus* in a renegotiating firm can be written as

$$\begin{aligned} H_{jt}^{h*} &= \frac{W_{jt}^{h*}}{P_t} - \frac{\kappa^h}{u'(c_t)} - \iota_t^h \mathrm{E}_t\left\{\beta_{t,t+1}\int_0^1 \frac{v_{it}^h}{v_t^h} H_{it+1}^h di\right\} \\ &\quad + \left(1-\lambda^h\right)\mathrm{E}_t\left\{\beta_{t,t+1}\left[\delta^w H_{jt+1|t}^h + \left(1-\delta^{hw}\right)H_{jt+1}^{h*}\right]\right\} \end{aligned}$$

where the subscript $|t$ indicates that firm j hasn't been able to renegotiate wages since period t, whereas the superscript $*$ indicates that wages at time t in firm j have been renegotiated. Integrating H_{it}^{h*} forward yields

$$\begin{aligned} H_{jt}^{h*} &= \mathrm{E}_t\sum_{k=0}^{\infty}\beta_{t,t+k}\left(1-\lambda^h\right)^k\left(\delta^w\right)^k\left[\frac{W_{jt}^{h*}\left[\prod_{i=1}^{k}\left(\pi_{t+i-1}^{W}\right)\right]^{\gamma_w}}{P_{t+k}} - \tilde{w}_{t+k}^{hw}\right] \\ &\quad + \left(1-\lambda^h\right)\left(1-\delta^w\right)\mathrm{E}_t\sum_{k=0}^{\infty}\beta_{t,t+k}\left(1-\lambda^h\right)^k\left(\delta^w\right)^k\left[H_{jt+1+k}^{h*}\right] \end{aligned} \tag{13}$$

with

$$\tilde{w}_{t+k}^{hw} = \frac{\kappa^h}{u'(c_{t+k})} + \iota_{t+k}^h \mathrm{E}_t\left\{\beta\frac{u'(c_{t+k+1})}{u'(c_{t+k})}\int_0^1 \frac{v_{jt+k}^h}{v_{t+s}^h} H_{jt+k+1}^h dj\right\}$$

On the other hand, the *firm surplus* for each j wholesaler is defined as $J_{jt}^h \equiv \partial\mathcal{F}_{jt}^h/\partial n_{jt}^h$, which can be written as

$$J_{jt}^h = \varphi_t^h \alpha_1^h \frac{y_t^h}{n_t^h} - \frac{W_{jt}^h}{P_t} + \frac{\psi^h \varsigma^h}{1+\psi^h}\frac{1}{u'(c_t)}\left(\frac{v_{jt}^h}{n_{jt}^h}\right)^{1+\psi^h} + \left(1-\lambda^h\right)\mathrm{E}_t\beta_{t,t+1}J_{jt+1}^h$$

where the first term is the marginal product of labour and, as I have shown before, is constant across firms. Firm j's surplus from employing one more worker is

given by the difference between the marginal product of labour and the real wage, adjusted by the marginal ease in hiring costs, which comes from relaxing labour market tightness, and the continuation value of having one extra worker employed in the future. When renegotiation occurs, the firm surplus can be expressed as

$$\begin{aligned} J_{jt}^{h*} &= \varphi_t^h \alpha_1^h \frac{y_t^h}{n_t^h} + \frac{\psi^h \varsigma^h}{1+\psi^h} \frac{1}{u'(c_t)} \left(\frac{v_{jt}^h}{n_{jt}^h}\right)^{1+\psi^h} - \frac{W_{jt}^{h*}}{P_t} \\ &\quad + \left(1-\lambda^h\right) \mathrm{E}_t \left\{\beta_{t,t+1} \left[\delta^w J_{jt+1|t}^h + (1-\delta^w) J_{jt+1}^{h*}\right]\right\} \end{aligned}$$

and integrating it forward yields

$$\begin{aligned} J_{jt}^{h*} &= \mathrm{E}_t \sum_{k=0}^{\infty} \beta_{t,t+k} \left(1-\lambda^h\right)^k (\delta^w)^k \left[\tilde{w}_{t+k|t}^{fw} - \frac{W_{jt}^{h*} \left[\prod_{j=1}^k \left(\pi_{t+j-1}^W\right)\right]^{\gamma_w}}{P_{t+k}}\right] \\ &\quad + \left(1-\lambda^h\right)(1-\delta^w) \mathrm{E}_t \sum_{k=0}^{\infty} \beta_{t,t+k} \left(1-\lambda^h\right)^k (\delta^w)^k \left[J_{jt+1+k}^{h*}\right] \end{aligned} \tag{14}$$

with

$$\tilde{w}_{t+k|t}^{fw} = \varphi_{t+k}^h \alpha_1^h \frac{y_{t+k}^h}{n_{t+k}^h} + \frac{\psi^h \varsigma^h}{1+\psi^h} \frac{1}{u'(c_{t+k})} \left(\frac{v_{jt+k|t}^h}{n_{jt+k|t}^h}\right)^{1+\psi^h}$$

For simplicity, I assume that new contracts follow a Nash-type bargaining rule by which workers and firms share the combined surpluses according to $\eta^h H_{jt}^{h*} = \left(1-\eta^h\right) J_{jt}^{h*}$, where $\eta^h \in (0,1)$ is the bargaining power of firms. Using expressions (13) and (14), W_{jt}^{h*} is determined by

$$0 = \mathrm{E}_t \sum_{k=0}^{\infty} \beta_{t,t+k} \left(1-\lambda^h\right)^k (\delta^w)^k \left[\frac{W_{jt}^{h*} \left[\prod_{i=1}^k \left(\pi_{t+i-1}^W\right)\right]^{\gamma_w}}{P_{t+k}} - w_{jt+k|t}^{h\,tar}\right] \tag{15}$$

where

$$w_{jt+k|t}^{h\,tar} = \left(1-\eta^h\right) \tilde{w}_{t+k|t}^{fw} + \eta^h \tilde{w}_{t+k}^{hw} \tag{16}$$

is the target wage. If wages were renegotiated every period, firms would do so in a symmetric fashion. The resulting real wage would then be equal to the target wage in (16), with the additional simplification coming from the fact that all firms would set an equal vacancy ratio v_{jt}^h / n_{jt}^h. However, due to wage dispersion, firms

find it optimal to adjust their vacancy posting according to the wage bill of existing contracts. From the definition of $\mathcal{F}_j$ and the law of motion for employment in firm j , the first-order condition with respect to v_{it}^h is given by

$$\varsigma^h \left(v_{jt}^h / n_{jt}^h\right)^{\psi^h} = \nu_t^h \mathrm{E}_t \beta u'\left(c_{t+1}\right) J_{jt+1}^h$$

Using the definition of J_{jt}^h, I can write firm j's job-creation condition as

$$\frac{\varsigma^h}{u'\left(c_t\right)}\left(\frac{v_{jt}^h}{n_{jt}^h}\right)^{\psi^h} = \nu_t^h \mathrm{E}_t \beta_{t,t+1} \left\{ \begin{array}{c} \varphi_{t+1}^h \alpha_1^h \frac{y_{t+1}^h}{n_{t+1}^h} - \frac{W_{jt+1}^h}{P_{t+1}} + \frac{\psi^h \varsigma^h}{1+\psi^h} \frac{1}{u'(c_{t+1})} \left(\frac{v_{jt+1}^h}{n_{jt+1}^h}\right)^{1+\psi^h} \\ + \left(1-\lambda^h\right) \frac{\varsigma^h}{\nu_{t+1}^h} \frac{1}{u'(c_{t+1})} \left(\frac{v_{jt+1}^h}{n_{jt+1}^h}\right)^{\psi^h} \end{array} \right\} \tag{17}$$

This expression demonstrates how wage dispersion across firms leads to dispersion in vacancy rates as well. Consequently, employment is also variable across wholesalers.

Commodity sector: As commodity producers are perfectly competitive and wages are flexibly set each period, there is no wage dispersion within the commodity sector. As such, expression (10) collapses to

$$H_t^x = \frac{W_t^x}{P_t} - \frac{\kappa^x}{c_t^{-\sigma}} + \left(1-\lambda^j-\iota_t^x\right) \mathrm{E}_t \left\{\beta_{t,t+1} H_{t+1}^x\right\} \tag{18}$$

On the other side of the labour market, the real value of an existing match for the representative commodity firm, the *firm surplus*, is defined as $J_t^x \equiv \partial \mathcal{C}_t / \partial n_t^x$. This can be written as

$$J_t^x = \frac{P_t^x}{P_t} \alpha_1^x \frac{x_t}{n_t^x} - \frac{W_t^x}{P_t} + \frac{\psi^x \varsigma^x}{1+\psi^x} \frac{1}{u'\left(c_t\right)} \left(\frac{v_t^x}{n_t^x}\right)^{1+\psi^x} + \left(1-\lambda^x\right) \mathrm{E}_t \beta_{t,t+1} J_{t+1}^x \tag{19}$$

and has a similar interpretation to J_{jt}^h.

Every period, the combined surpluses of workers and firms is maximized through Nash bargaining over the nominal wage. This implies a sharing rule given by $\eta^x H_t^x = \left(1-\eta^x\right) J_t^x$, where $\eta^x \in (0,1)$ is the bargaining share of firms. Using the

expressions (18) and (19), the real wage in the commodity sector is given by

$$\begin{aligned}\frac{W_t^x}{P_t} &= (1-\eta^x)\left(\frac{P_t^x}{P_t}\alpha_1^x\frac{x_t}{n_t^x}+\frac{\psi^x\varsigma^x}{1+\psi^x}\frac{1}{u'(c_t)}\left(\frac{v_t^x}{n_t^x}\right)^{1+\psi^x}\right)\\ &\quad+\eta^x\left(\frac{\kappa^x}{u'(c_t)}+\iota_t^x\mathrm{E}_t\left\{\beta_{t,t+1}H_{t+1}^x\right\}\right)\end{aligned}$$

Note how W_t^x/P_t is simply a weighted average of both the firm's and household's surpluses, where the weight on each surplus is given by the respective bargaining share η^x and $1-\eta^x$.

The representative commodity firm also has to choose how many vacancies to post each period. From the definition of $\mathcal{C}$ and the law of motion of labour for firms, the first-order condition with respect to v_t^x writes

$$\varsigma^x\left(v_t^x/n_t^x\right)^{\psi^x} = \nu_t^x\mathrm{E}_t\beta u'(c_{t+1})J_{t+1}^x \tag{20}$$

Replacing J_t^x in (20), I obtain the job-creation condition in the commodity sector

$$\frac{\varsigma^x}{u'(c_t)}\left(\frac{v_t^x}{n_t^x}\right)^{\psi^x} = \nu_t^x\mathrm{E}_t\beta_{t,t+1}\left\{\begin{array}{c}\frac{P_{t+1}^x}{P_{t+1}}\alpha_1^x\frac{x_{t+1}}{n_{t+1}^x}-\frac{W_{t+1}^x}{P_{t+1}}+\frac{\psi^x\varsigma^x}{1+\psi^x}\frac{1}{u'(c_{t+1})}\left(\frac{v_{t+1}^x}{n_{t+1}^x}\right)^{1+\psi^x}\\ +(1-\lambda^x)\frac{\varsigma^x}{\nu_{t+1}^x}\frac{1}{u'(c_{t+1})}\left(\frac{v_{t+1}^x}{n_{t+1}^x}\right)^{\psi^x}\end{array}\right\} \tag{21}$$

As matched vacancies in period t only become productive in $t+1$, at the margin, the cost of posting a vacancy equals the expected benefit of that vacancy being matched. Moreover, using (20) together with the bargaining rule over nominal wages, I can rewrite the equilibrium real wage as

$$\begin{aligned}\frac{W_t^x}{P_t}\equiv w_t^{x\ nash} &= \eta^x\frac{\kappa^x}{u'(c_t)}+(1-\eta^x)\frac{\varsigma^x}{u'(c_t)}\frac{v_t^x}{u_t^x}\left(\frac{v_t^x}{n_t^x}\right)^{\psi^x}\\ &\quad+(1-\eta^x)\left(\frac{P_t^x}{P_t}\alpha_1^x\frac{x_t}{n_t^x}+\frac{\psi^x}{1+\psi^x}\frac{\varsigma^x}{u'(c_t)}\left(\frac{v_t^x}{n_t^x}\right)^{1+\psi^x}\right)\end{aligned} \tag{22}$$

3.2.5 Foreign Sector and Market Clearing

Final firms in the non-commodity sector sell part of their production to households at home and export the rest. In equilibrium, market clearing in the non-

commodity sector implies

$$Y_t^h = c_t^h + c_t^{h\star} \tag{23}$$

where c_t^h denotes domestic consumption of the domestically-produced final good, and $c_t^{h\star}$ denotes foreign demand, which I define as

$$c_t^{h\star} = K_t^{h\star}\left(\frac{P_t^{h\star}}{P_t^\star}\right)^{-\gamma^h} \tag{24}$$

with $\gamma^h > 1$. Foreign demand is a decreasing function of the foreign-currency price of domestically-produced final goods and is subject to exogenous demand shocks, which I denote by $K_t^{h\star}$. As with commodity prices, I assume the law of one price holds for non-commodity final goods. Hence, domestic final firms export price is given by

$$\frac{P_t^{h\star}}{P_t^\star} = \frac{1}{q_t}\frac{P_t^h}{P_t} \tag{25}$$

As discussed in Schmitt-Grohé and Uribe (2003), equilibrium dynamics in open economy models with imperfect international financial markets are not stationary. As household members only have access to a risk-less bond denominated in foreign currency, I induce stationarity by assuming that the interest rate on domestic foreign assets is subject to a risk premium, which I define as

$$i_t^* = \bar{i}^* \exp\left(-\psi_b\left(\frac{q_t b_t^\star}{rgdp_t} - \overline{B}\right) + \xi_t^b\right) \tag{26}$$

where $rgdp_t$ denotes total real output

$$rgdp_t = \frac{P_t^h}{P_t}Y_t^h + \frac{P_t^x}{P_t}x_t^f \tag{27}$$

The foreign risk premium is a decreasing function of total net foreign assets held by domestic agents, which I denote by $b_t^\star$. The law of motion of net foreign assets evolves according to

$$q_t b_{t-1}^\star + tb_t = q_t \frac{\pi_{t+1}^*}{i_t^*} b_t^\star \tag{28}$$

where tb_t is the trade balance of the small commodity-exporter

$$tb_t = \frac{P_t^h}{P_t} c_t^{h\star} + \frac{P_t^x}{P_t} x_t^f - \frac{P_t^f}{P_t} c_t^f \tag{29}$$

One other feature of standard small open economy models is their inability to match the volatility of the exchange rate presented in the data. Following Smets and Wouters (2002) and Adolfson et al. (2007), I mitigate this problem by assuming incomplete exchange rate pass-through in the import sector. In particular, I introduce a continuum of measure 1 of monopolistic competitor import firms indexed by $i \in [0,1]$. As with non-commodity retailers, importers have market power to set prices but are subject to nominal rigidity as in Calvo (1983). As such, they import foreign goods at price $q_t P_t^{f\star}$ and sell their imperfectly substitutable intermediate goods to a final aggregator firm at price P_{it}^f. Each period, optimizing firms set their price according to

$$\frac{P_t^{f*}}{P_t^f} = \frac{\mu^f}{(\mu^f - 1)} \frac{\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \frac{c_{t+s}^{-\sigma}}{c_t^{-\sigma}} \left(\delta^f\right)^s q_{t+s} c_{t+s}^f \left(\frac{P_t^f}{P_{t+s}^f}\right)^{-\mu^f}}{\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \frac{c_{t+s}^{-\sigma}}{c_t^{-\sigma}} \left(\delta^f\right)^s \frac{P_{t+s}^f}{P_{t+s}} c_{t+s}^f \left(\frac{P_t^f}{P_{t+s}^f}\right)^{1-\mu^f}}$$

where $\mu^f > 1$ is the elasticity of substitution between differentiated intermediate imported goods, and δ^f is the probability that import firms are not be able to reset prices. As with the non-commodity price index P_t^h, the law of motion for P_t^f is given by

$$\left(P_t^f\right)^{1-\mu^f} = \left(1 - \delta^f\right) \left(P_t^{f\star}\right)^{1-\mu^f} + \delta^f \left(P_{t-1}^f\right)^{1-\mu^f} \tag{30}$$

where $P_t^{f\star}$ is the exogenous price of foreign-produced goods. Note from (30) that incomplete pass-through implies a departure from the law of one price with respect to import prices.

3.3 Implications of Nominal Wage Dispersion

Only a share of wholesale firms in the non-commodity sector is able to negotiate wages with workers in a given quarter. Consequently, wages in each firm are history dependent and vary according to the period they were last renegotiated.

Furthermore, as shown above, wage dispersion causes vacancy posting to vary across wholesalers as well, despite the fact that all wholesalers are able to adjust their vacancy rates every period.

Dispersion of nominal wage and vacancy posting are particularly problematic when it comes to derive equilibrium aggregate conditions. Although there is no closed-form expression for all aggregate conditions in the set-up described in the previous section, it is obviously intractable to account for a set of equilibrium conditions for the continuum of wholesalers. Instead, in this section, I derive first-order log-linear approximations to the equilibrium conditions regarding the non-commodity labour market and investigate the consequences of wage dispersion in the aggregate. I show that each firm finds it optimal to adjust vacancies in a tight relation to the wages they pay. Moreover, given that wage stickiness is inefficient, dispersion in vacancy rates is detrimental to labour allocation and output.

3.3.1 Functional Forms

For ease of exposition of the derivations below and to fully acknowledge the implications of the parametrization I chose, I first report the functional forms I consider throughout the remainder of the paper. The immediate utility function is assumed to be given by

$$u\left(c_t\right) \quad = \quad \frac{c_t^{1-\sigma}}{1-\sigma}$$

where σ is the inverse of the intertemporal elasticity of substitution. The consumption bundle $c_t = h\left(c_t^h,\, c_t^f\right)$ is a CES function

$$h\left(c_t^h,\, c_t^f\right) \quad = \quad \left[\left(\alpha^c\right)^{\frac{1}{\gamma^c}} \left(c_t^h\right)^{\frac{\gamma^c-1}{\gamma^c}} + \left(1-\alpha^c\right)^{\frac{1}{\gamma^c}} \left(c_t^f\right)^{\frac{\gamma^c-1}{\gamma^c}}\right]^{\frac{\gamma^c}{\gamma^c-1}}$$

where α^c denotes the home bias in consumption and γ_c is the elasticity of substitution between domestic and foreign-produced consumption goods. Maximizing c_t subject to $P_t c_t = P_t^h c_t^h + P_t^f c_t^f$ yields the demand for each type of consumption good

$$c_t^h = \alpha^c \left(\frac{P_t^h}{P_t}\right)^{-\gamma^c} c_t \quad \text{and} \quad c_t^f = \left(1-\alpha^c\right) \left(\frac{P_t^f}{P_t}\right)^{-\gamma^c} c_t$$

Substituting c_t^h and c_t^f in $h\left(c_t^h,\, c_t^f\right)$ for the respective demands yields the definition of the consumer price index

$$(P_t)^{1-\gamma^c} \;=\; \alpha^c \left(P_t^h\right)^{1-\gamma^c} + (1-\alpha^c)\left(P_t^f\right)^{1-\gamma^c} \tag{31}$$

In turn, I assume that the disutility from work is linear but let the disutility from working in each particular sector to differ

$$l\left(n_t^h,\, n_t^x\right) \;=\; \kappa^h n_t^h + \kappa^x n_t^x$$

Finally, the matching function in sector $s = h,\ x$ is given by

$$m_t^s \;=\; \chi^s \left(v_t^s\right)^{\epsilon^s} \left(u_t^s\right)^{1-\epsilon^s}$$

where $\chi^s > 0$ commands the matching efficiency, and $0 < \epsilon^s < 1$ is the elasticity of matches with respect to vacancies. The job-finding rates can then be expressed as $\iota_t^s = \chi^s \left(\theta_t^s\right)^{\epsilon^s}$ and the hiring rates as $\nu_t^s = \chi^s \left(\theta_t^s\right)^{\epsilon^s - 1}$, where $\theta_t^s \equiv v_t^s / u_t^s$ can be interpreted as an index of labour market tightness. Also, denote by $\phi_{jt}^s \equiv v_{jt}^s / n_{jt}^s$ the vacancy rate in firm j. In what follows, I perform a change of variables from v_t^s/u_t^s and v_t^s/n_t^s to θ_t^s and ϕ_t^s to simplify the algebra.

3.3.2 Steady State

One important decision concerns the point around which to perform the log-linear approximation. In this paper, I choose to log-linearise the model around the non-stochastic efficient steady state. Besides rendering the log-linear approximation simpler, the appealing property of the efficient steady state is that it allows me to derive of a micro-founded quadratic welfare measure as well. As I discuss later, some features of the model complicate the full derivation of the welfare criterion, even around the efficient steady state. Yet, it is useful to derive a benchmark criterion.

There are three sources of nominal rigidities and four real rigidities that cause the decentralised steady state to depart from its efficient equilibrium. On the one hand, price stickiness in the non-commodity sector, imperfect exchange rate pass-through, and staggered wages all lead to real efficiency losses. As a result, I perform the log-linearisation around a zero-inflation (both price- and wage-inflation) non-stochastic steady state. On the other hand, monopolistic competition in the

non-commodity and import sectors and wage bargaining in the non-commodity and commodity sectors also generate an inefficient steady-state allocation. The following two assumptions offset these inefficiencies.

Assumption 1: Non-commodity and importer retailers have their sales subsidised at a rate $\tau^l = 1/(\mu^l - 1)$ for $l = h,\ f$. These subsidies are raised through lump-sum taxation. ■

Assumption 2: The Hosios condition is satisfied in both sectors: $\eta^s = \epsilon^s$ for $s = h,\ x$. ■

Monopolistic competition introduces a mark-up between prices and marginal costs equal to $\mu^l/\left(\mu^l - 1\right)$, which is offset by τ^l since $\left(\mu^l/\left(\mu^l - 1\right)\right)\left(1/\left(1+\tau^l\right)\right) = 1$. On the other hand, and following Hoisos (1990), equalizing the bargaining share of firms to the elasticity of matches with respect to vacancies delivers efficient job creation.[7]

3.3.3 Log-linear Conditions

For any variable h_{it}, let $\hat{h}_{it} \equiv log\left(h_{it}/\overline{h}\right)$ be the log deviation of h_{it} from its efficient steady state $\overline{h}$. For variables with cross-sectional dispersion, define $\hat{h}_t \equiv \int_0^1 \hat{h}_{it} di$ its cross-sectional average. For prices and wages, small letters denote real variables and caps denote nominal, that is $p_t^i = P_t^i/P_t$. The proofs of the propositions below can be found in the appendix.

In what concerns the nominal rigidity in the non-commodity and import sectors, the following Phillips curves summarize the aggregate behaviour of price setters.

Proposition 1: Price inflation in the non-commodity sector is given by

$$\hat{\pi}_t^h = \frac{\left(1-\beta\delta^p\right)\left(1-\delta^p\right)}{\delta^p}\left\{\hat{\varphi}_t^h - \hat{p}_t^h\right\} + \beta \mathrm{E}_t \hat{\pi}_{t+1}^h \tag{32}$$

■

[7] These two assumptions are standard in the New Keynesian and the search and matching literatures, respectively.

Proposition 2: Price inflation in the import sector is given by

$$\hat{\pi}_t^f = \frac{\left(1-\beta\delta^f\right)\left(1-\delta^f\right)}{\delta^f}\left\{\hat{q}_t + \hat{p}_t^{f*} - \hat{p}_t^f\right\} + \beta \mathrm{E}_t \hat{\pi}_{t+1}^f \tag{33}$$

■

The wedge between the real price and marginal costs induced by nominal rigidity in prices is made explicit by expressions (32) and (33) . Without nominal frictions, all firms would be able to adjust their price. That would be the case if $\delta^l \longrightarrow 0$, $l = p,\ \ f$, and would give rise to $\hat{\varphi}_t^h = \hat{p}_t^h$ and $\hat{p}_t^f = \hat{q}_t + \hat{p}_t^{f*}$.

Considering the equilibrium conditions for wage-setting and vacancy posting in the non-commodity sector, recall expressions (13) to (17) in the previous section. In the next four propositions, I derive a log-linear approximation for the average real wage and vacancy rate in the non-commodity sector. I start by deriving the relationship between vacancy rates and wages for each wholesale firm:

Proposition 3: The vacancy rate in each wholesale firm j is given by

$$\hat{\phi}_{jt}^h = \hat{\phi}_t^h - \frac{\beta\delta^w\lambda^h\upsilon^h}{\left(1-\beta\delta^h\right)\psi^h\left(1+\psi^h\right)\varrho^h}\left(\hat{w}_{jt}^h - \hat{w}_t^h\right) \tag{34}$$

where $\varrho^h \equiv \varsigma^h\left(\bar{\phi}^h\right)^{1+\psi^h}\bar{n}^h/\left(\left(1+\psi^h\right)\bar{c}^{-\sigma}\overline{rgdp}\right)$ is the total hiring costs in the non-commodity sector $\upsilon^h \equiv \overline{w}^h\overline{n}^h/\overline{rgdp}$ is the share of non-commodity labour. ■

Expression (34) shows that the relative vacancy rate posted by firm j is negatively correlated with the relative wage it pays. In other words, firms post higher vacancy rates when the wages they pay their workers is below the average wage. In aggregate terms, the log-linear approximation of the job-creation condition, expression (17) in the previous section, is given in the following proposition:

Proposition 4: The aggregate job-creation condition in the non-commodity sec-

tor is given by

$$\frac{\left(1+\psi^h\right)\varrho^h}{\beta\lambda^h}\left(\psi^h\hat{\phi}_t^h+\left(1-\epsilon^h\right)\hat{\theta}_t^h\right) = \mathrm{E}_t\left\{\begin{array}{c}\left(\upsilon^h-\alpha_1^hQ_1\right)\sigma\hat{c}_{t+1}-\upsilon^h\hat{w}_{t+1}^h\\ +\alpha_1^hQ_1\left(\hat{\varphi}_{t+1}^h+\hat{y}_{t+1}^h-\hat{n}_{t+1}^h\right)\\ +\left(1+\psi^h\right)\psi^h\frac{\varrho^h}{\lambda^h}\hat{\phi}_{t+1}^h\\ +\left(1+\psi^h\right)\left(1-\lambda^h\right)\left(1-\epsilon^h\right)\frac{\varrho^h}{\lambda^h}\hat{\theta}_{t+1}^h\end{array}\right\} \tag{35}$$

where $Q_1 \equiv \overline{p}^h\overline{Y}^h/\overline{rgdp}$ is the share of non-commodity gross value added to real GDP. ■

Expression (35) is similar to the log-linear approximation of the job-creation condition for the commodity sector, except for the fact that $\hat{w}_{t+1}^h \neq \hat{w}_{t+1}^{h\ nash}$, as I show below.[8] Up to a first-order approximation, wage stickiness distorts efficient job creation by introducing a gap between the average real wage and the efficient Nash real wage, as in Thomas (2008).

Before deriving an approximation for the average real wage in the non-commodity sector, suppose that all renegotiating firms strike the same wage agreement $W_{it}^{h*} = W_t^{h*}$. From (17), wage symmetry implies symmetry in vacancy rates ϕ_{it}^h, which further implies symmetric wage targets $w_{it+s|t}^{h\,tar} = w_{t+s|t}^{h\,tar}$ and verifies my initial guess that $W_{it}^{h*} = W_t^{h*}$. With this, and since renegotiating firms are chosen randomly, the average nominal wage across firms evolves according to

$$W_t^h = \left(1-\delta^w\right)W_t^{h*}+\delta^wW_{t-1}^h\left(\pi_{t-1}^W\right)^{\gamma_w} \tag{36}$$

The similarity between the wage target $\hat{w}_t^{h\,tar}$ and the flexible Nash wage is easily noticed in expression (16). Denoting by $w_t^{h\ nash}$ the real wage that results from a flexible Nash equilibrium, the wage target is given in the next proposition:

Proposition 5: The target wage in the non-commodity sector is given by

$$\hat{w}_t^{h\,tar} = \hat{w}_t^{h\,nash}+\frac{\bar{\iota}^h\left(1+\Lambda\right)\beta}{\left(1-\left(1-\lambda^h\right)\beta\delta^w\right)}\frac{\delta^w}{\left(1-\delta^w\right)}\left(\gamma_w\hat{\pi}_t^W-\mathrm{E}_t\hat{\pi}_{t+1}^{wh}\right) \tag{37}$$

where $\Lambda \equiv \left(1-\epsilon^h\right)\beta\delta^w\lambda^h/\left(1-\beta\delta^w\right)$ and $\hat{w}_t^{h\,nash}$ is the log-linear equivalent of

[8] The Nash wage in the non-commodity sector is analogous to the Nash wage in the commodity sector, and would be obtained if wage bargaining in the non-commodity sector occurred every period.

expression (22) for the non-commodity sector. ■

In expression (37), $\hat{\pi}_{t+1}^{wh}$ denotes wage inflation, which is given by

$$\hat{\pi}_{t+1}^{wh} = \hat{w}_{t+1}^{h} - \hat{w}_{t}^{h} + \hat{\pi}_{t+1}$$

where $\hat{w}_t^h$ is the average wage in the non-commodity sector and $\hat{\pi}_{t+1}$ is the consumer price inflation rate. As Proposition 5 demonstrates, the gap between $\hat{w}_t^{h\,tar}$ and $\hat{w}_t^{h\,nash}$ comes from the dispersion in wages, which arises in expression (16) through the dispersion in vacancy rates. This is a crucial insight. Due to the dispersion in vacancy posting, the nominal wage firms negotiate with workers departs from the efficient wage rate that would otherwise arise if all firms could negotiate wages every period. From (37), if $\delta^w \longrightarrow 0$, both wages become equal. Moreover, as shown in (37), the gap between $\hat{w}_t^{h\,tar}$ and $\hat{w}_t^{h\,nash}$ is due to two factors. On the one hand, the dispersion in wages, which comes about through the expected nominal wage inflation term $\mathrm{E}_t\hat{\pi}_{t+1}^{wh}$. On the other hand, the indexation rule, which shows up through expression $\gamma_w\hat{\pi}_t^W$.

Finally, using (36) and (37), the log-linear approximation of expression (15) is given in the proposition below:

Proposition 6: The average wage in a staggered wage economy is given by

$$\begin{aligned}\hat{\pi}_t^{wh} - \gamma_w\hat{\pi}_{t-1}^W &= \frac{1-\beta\left(1-\lambda^h\right)\left(\delta^w\right)}{1+\Lambda}\frac{1-\delta^w}{\delta^w}\left(\hat{w}_t^{h\,nash} - \hat{w}_t^h\right) - \beta\left(1-\lambda^h\right)\delta^w\gamma_w\hat{\pi}_t^W \\ &\quad +\beta\left(1-\lambda^h\left(1-\frac{\bar{\theta}^h}{\bar{\phi}^h}\right)\right)\left(\mathrm{E}_t\hat{\pi}_{t+1}^{wh} - \gamma_w\hat{\pi}_t^W\right) \qquad (38)\end{aligned}$$

■

As all non-commodity firms would choose the Nash wage $w_t^{h\,nash}$ were they able to renegotiate wages every period, the actual average wage in the non-commodity sector can also be expressed as a deviation from the Nash wage $w_t^{h\,nash}$. Note, however, that this deviation is now not due to expected future wage inflation only. As clearly shown in (38), the average wage in the non-commodity sector depends on both current and future wage inflation, as well as past and present indexation.

Nash-bargaining over wages implies that workers and firms share the combined

surplus of the newly created match. As such, both parties will only agree to a contract such that each receives a non-negative surplus. Otherwise, one of the parties is better-off walking away. With flexible wages, the sharing rule guarantees positive surpluses to both parties, as the resulting flexible wage is simply a weighted average of both surpluses. However, this is not necessarily true when wages are staggered. In order to have both surpluses non-negative, the real wage in any given firm j at any given period t must lie between the reservation wages of workers and firms $w_{jt}^h \in \left[r_{jt}^w, r_{jt}^f\right]$, with

$$\begin{cases} r_{jt}^w = w_{jt}^h - H_{jt}^h \\ r_{jt}^f = J_{jt}^h + w_{jt}^h \end{cases}$$

If w_{jt}^h is to fall outside the bargaining set, the economic rational for the employment contract in firm j is broken, distorting the equilibrium in the non-commodity labour market. Following the approach in Thomas (2008), I inspect whether wages fall outside the bargaining set by analysing a subset of firms that have not been able to renegotiate wages for a relatively long period of time. In particular, I focus on the 1% of firms that have the oldest wage contracts, which implies those with wage contracts last negotiated at least 9 quarters ago.[9] I then simulate the model under the calibration discussed in the next section, and check whether $w_{jt|t-9}^h \in \left[r_{jt}^w, r_{jt}^f\right]$, where $w_{jt|t-9}^h$ is the updated wage paid at time t to workers in firms that last negotiated wages 9 periods ago.

Figure 2 plots $w_{jt|t-9}^h$ and the respective reservation wages for the cases of zero and full indexation, $\gamma_w = 0, 1$. When there is no wage indexation, employment relationships are never broken for at least 99% of wholesale firms. However, with full wage indexation, real wages fall short of the reservation wage of workers for 2% of the simulated quarters. Although the occurrence of negative surpluses in continuing employment contracts represents a caveat of the model, the number of times this occurs is relatively small and only for large enough values of γ_w . The increase in the volatility of reservation and real wages for higher values of γ_w, as shown in Figure 2, provides a good insight on the transmission of sectoral shocks. As I explain below, when $\gamma_w = 1$, wages in the non-commodity sector

[9] The share of firms that last bargained over wages k periods ago is given by $(\delta^w)^k (1-\delta^w)$. The share of firms with wage contracts older than T periods is then given by $1 - \sum_{k=0}^{T} (\delta^w)^k (1-\delta^w)$. With $\delta^w = 0.60$, the minimum T such that the share of firms with contracts older than T is less than 1% is $T = 9$.

are updated based on the wage inflation in the commodity sector. As a result, sectoral relative wage rigidity raises the volatility of wages in the non-commodity sector. Crucially, it also raises the deviations between average wages and their efficient level, resulting in inefficient real output and unemployment volatilities.

Wage indexation is not the only link between the two sectoral labour markets. As labour markets are segmented, workers need to consider the labour market tightness and the future benefit of being employed in a given sector when deciding where to search for a job. As such, directed search implies a non-arbitrage condition, expression (11) derived above, for which Proposition 6 derives a first-order approximation:

Proposition 7: The no-arbitrage condition that results from directed search is given by

$$\hat{\theta}_t^h + \psi^h \hat{\phi}_t^h - \Xi \frac{\delta^w}{(1-\delta^w)} \left(\mathrm{E}_t \hat{\pi}_{t+1}^{wh} - \gamma_w \hat{\pi}_t^W \right) \quad = \quad \hat{\theta}_t^x + \psi^x \hat{\phi}_t^x \tag{39}$$

with $\Xi = \lambda^h \beta \left(1+\Lambda\right) \upsilon^h / \left(1 - \left(1-\lambda^h\right) \beta \delta^w\right) \left(1-\eta^h\right) \left(1+\psi^h\right) \varrho^h$. ■

Expression (39) shows that the choice of where to search for a job depends on the difference in sectoral labour market tightness, corrected by the respective vacancy rates. In fact, when wages are fully flexible (that is $\delta^w \longrightarrow 0$) and the curvature of hiring costs is symmetric ($\psi^h = \psi^x$), a relatively higher vacancy rate in one sector implies a lower market tightness, with household members intensifying their directed search in that sector. Importantly, wage stickiness also creates a wedge in (39), distorting the allocation of labour between the two sectors.

3.4 Calibration and GMM

3.4.1 Calibration

I calibrate the model to a quarterly frequency and report the parameter values in Table 2. I first assign values to important steady state ratios to match Australian long-run averages and pick carefully the parameters governing the labour market based on previous studies for Australia. In some cases, this first step pins down important parameters. Finally, I base the calibration of the nominal rigidities on previous literature.

For simplicity, and without loss of generality, net foreign assets are set equal to zero. This implies $Q_3 \equiv \overline{c}/\overline{rgdp} = 1$. According to estimates by Rayner and Bishop (2003), the resource sector accounted for 18% of total nominal gross value added and 9.75% of employment in 2011/12, the year commodity prices peaked. In terms of final demand, they estimate that around 70% of production of commodities was exported. However, this is net of commodity imports, which are absent from the model. Based on these figures, I set $Q_1 \equiv \overline{p}^h\overline{y}^h/\overline{rgdp} = 0.85$ and $\overline{x}^f/\overline{x} = 0.80$, implying $Q_2 \equiv \overline{p}^x\overline{x}/\overline{rgdp} = 0.19$. Moreover, I set $\overline{rgdp} = 1$, and obtain $\overline{Z} = 0.92$ and $\overline{A} = 0.38$, which are the scale parameters in each sector's production function. The price of commodities is set to $\overline{p}^x = 1.2$, the real exchange rate is normalized to 1, and so is $\overline{p}^f$. From (31), it follows that $\overline{p}^h = 1$ as well. I follow Jääskela and Nimark (2011) and set the share of domestically-produced goods in consumption, α^c, to 0.8, the elasticity of substitution between domestic and foreign goods, γ^c, to 1.3, and the price elasticity of exports, γ^h, to 1.4.

I consider an unemployment rate of 6% in steady state, which is broadly in line with the Australian long-run trend since the adoption of inflation targeting in 1993. Moreover, I assume that employment in the commodity sector accounted for 5% of total employment, which implies $\overline{n}^h = 0.89$. From the time series of vacancies reported by the Australian Bureau of Statistics (ABS) since 2009, I set the vacancy rate to $\overline{\phi}^h = 0.02$ and $\overline{\phi}^x = 0.03$. Also following evidence for Australia, wages in the commodity sector are assumed to be 20% higher than in the non-commodity sector. Having set the relative weight of both sectors and the relative employment shares, the share of labour in production in the non-commodity sector is obtained from (6) and equals $\alpha_1^h = 0.96$.

Following Thomas (2008) and assuming symmetry between sectors, I set the bargaining power of firms to $\eta^s = 0.60$, the curvature parameter of hiring costs to $\psi^s = 1$, for $s = h,\ x$, and set hiring costs to represent 1% of the gross value added in each sector. As in Sheen and Wang (2012), the separation rate in the non-commodity sector is $\lambda^h = 0.1$, implying an average job tenure of 2.5 years. D'Arcy et al. (2012) calculate the average job tenure in Australia to be around 7 years. However, the distribution of job tenures has a very long tail, with the mode being closer to 2.5 years. This number seems to be a more appropriate benchmark. D'Arcy et al. (2012) also show that the job turnover is significantly higher in the commodity sector compared to the rest of the economy. For that reason, I assume $\lambda^x = 0.25$. Using the two wage equations and equation (11), evaluated at the steady state, it follows that sectoral labour disutilities differ, with $\kappa^x = 0.82$

and $\kappa^h = 0.66$. Hence, wages are higher in the commodity sector not only because job tenure there is shorter, but also because disutility is higher.[10]

Matching efficiencies are obtained from the two job-creation conditions, resulting in $\chi^h = 3.33$ and $\chi^x = 3.78$. Using these values together with the aggregate law of motion of employment in each sector, I obtain $\overline{\theta}^h = 0.36$ and $\overline{\theta}^x = 0.14$. Finally, I rearrange the wage equation in the commodity sector and obtain $\alpha_1^x = 0.30$. The marginal productivity of labour in each sector can then be computed from $\alpha_1^h Q_1/\overline{n}^h$ and $\alpha_1^x Q_2/\overline{n}^x$, resulting in 0.92 and 1.13, respectively. Although the matching efficiency in the commodity sector is higher, the labour market is tighter, with relatively less vacancies to job-seekers. In fact, commodity firms seem to find optimal to post less vacancies and hire less workers in order to sustain higher productivities. By doing so, they can afford paying higher wages compared to the non-commodity sector.

In the context of the model, the high volatilities of some international variables, namely the exchange rate and commodity prices, tend to contaminate the volatility of domestic variables as well. The simplicity of the model is partly to blame. In fact, Jääskela and Nimark (2011) and Sheen and Wang (2012) use habits in consumption, capital accumulation and investment adjustment costs to slow down the transmission of foreign shocks. As none of those real rigidities are included here, I rely on stronger nominal rigidities to match domestic volatilities empirically. For instance, regarding the parameter governing price stickiness in the non-commodity sector, I assume that prices are updated once a year on average, as opposed to 3 quarters as in Jääskela and Nimark (2011) and Sheen and Wang (2012). Both studies also estimate import prices to be updated in less than a year on average. However, to prevent the volatility of the exchange rate from spreading to the domestic variables, I set $\delta^f = 0.9$.[11] Finally, I set $\delta^w = 0.65$ so that wages adjust around every 3 quarters.[12]

[10] Jobs in the mining industry are typically located in remote areas, require relatively long periods of absence from home, include shift-work and can be physically and emotionally demanding.

[11] Matching the volatility of the exchange rate is important for the policy analysis preformed in the next sections. With $\delta^f = 0.75$, and keeping the volatility of consumption and domestic inflation unchanged, the standard deviation of the exchange rate drops from 5.6 to 3.5.

[12] Adolfson et al. (2007) estimate wages to adjust every 10 months, while Thomas (2008) assumes one year based on evidence for the US. On the other hand, Jääskela and Nimark (2011) and Sheen and Wang (2012) estimate adjustments to take place every 5.5 to 7.5 months on average. Hence, my calibration lies in between these values.

3.4.2 Matching Second Moments

There are 6 exogenous shocks in the model: two sectoral technological shocks, a risk premium shock, a monetary policy shock, an exogenous process for foreign inflation and one for the foreign price of commodities. Except for monetary policy shocks, which are i.i.d. innovations, all remaining shocks are assumed to follow an $AR(1)$ process. I follow Hevia and Nicolini (2013) and assume that the technology shocks have the same persistence but are uncorrelated. Therefore, I set to $\rho_z = \rho_a = 0.96$. For the persistence of the foreign shocks, I use the HP-filter on the logarithm of the real commodity price index for Australia, demean the foreign inflation series and run two univariate auto-regressions, obtaining $\rho_x = 0.80$ and $\rho_{\pi^\star} = 0.38$. For the persistence of the risk premium, I choose $\rho_b = 0.75$, which falls within the rage of values used in the literature. Similarly to Adolfson et al. (2007), I find that ρ_b is determinant to control the impact of exchange rate volatility on the volatility of consumption. The value chosen here allows me to match both volatilities relatively well.

I pin down the values for the standard deviation of the shocks by matching the theoretical standard deviations of important domestic variables to their empirical counterpart. As the study of optimal monetary policy in the next section relies on the impact of policy on the second moments of the model's variables, these should compare reasonable well to actual figures. I use a GMM approach to find the values of the standard deviations of the shocks for which I obtain the best match between model and data volatilities. I use quarterly employment and national accounts series from the ABS, from 1993Q1 to 2014Q4. These series are already seasonally adjusted. The inflation rate is from the RBA and excludes the interest and tax changes of 1999. The real exchange rate is the real trade-weighted index computed by the RBA. Foreign inflation is also obtained from the RBA. For the international prices of Australian commodities, I use the RBA commodity price index denominated in US dollars. Except for the inflation rates, which were simply demeaned, I filtered the logarithm of the remaining series using the HP-filter.[13]

Table 2 presents the results.[14] I use both the entire sample, which starts after

[13] The results do not change significantly using a BP-filter or a one-sided HP-filter instead.

[14] The first-order log-linear approximations to the equilibrium conditions of the model were coded in Matlab. I used Klein's toolkit to obtain the model's solution. The GMM routine has also been coded on Matlab, for which I have used parts of existing code developed by Ruge-Murcia (2007).

inflation targeting was officially announced in the first quarter of 1993, and a smaller sample truncated in 2003 to exclude the effects of the recent mining boom. Overall, the matches seem reasonably good. For each sample, I use two different weighting schemes.[15] In general, using optimal weights allows me to obtain better matches for the domestic variables. This is done at the expense of the volatility of the exchange rate and of commodity prices, which are better matched when I use equal weights. Clearly, since the mining boom increased the volatility of the exchange rate and of commodity prices, the results for the longer sample are more dependent on the weighting chosen. Although there are only 6 shocks in the model, I choose to match 7 moments. In particular, I include both the foreign price of commodities and the gross value added of the commodity sector. This is done to improve the match of commodity price volatility, as including these two variables induces a higher weight on commodity price shocks in the GMM.

In terms of the standard deviations of the shocks, using equal weights almost mutes domestic shocks. Instead, optimal weighting reduces the volatility of the foreign shocks while raising the volatility of the domestic shocks. In any case, the volatility of the commodity-specific technology shock is consistently near zero due to the high volatility of commodity price shocks. The volatility of non-commodity technology shocks is used to match the volatility of non-commodity output. In turn, the innovations in foreign inflation pin down the volatility of that variable alone, whereas the volatility of the risk premium shock is used to match the volatility of the exchange rate. The high volatility of the exchange rate tends to raise the volatility of price inflation and consumption. On the one hand, a stronger nominal rigidity in the domestic prices and a low volatility of monetary policy shocks allow me to reduce the volatility of domestic inflation. On the other hand, the calibrated value of ρ_b allows me to obtain a sensible match for the standard deviation of consumption.

3.5 Monetary Policy

The institutional framework of the model is characterized by a monetary policy rule and a wage indexation rule. Both are exogenous and convey different objectives. Monetary policy is assumed to follow a feedback rule to the nominal interest

[15] The GMM procedure minimizes a weighted average of the differences between model and data moments.

rate of the form

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1 - \rho_i)\, \omega_\pi \hat{\pi}_t + \xi_t^i \tag{40}$$

where ξ_t^i is the i.i.d. monetary policy innovation, ρ_i is the interest rate smoothing parameter, which I set to 0.85, and ω_π is the long-run response of the monetary authority to inflation. One of the advantages of considering feedback rules such as (40) is their simplicity and easiness of interpretation. In the remainder of this paper, I will investigate what values of ω_π maximize a given loss function that represents the objective of the monetary authority.

I interpret the wage indexation rule as an institutional feature of the labour market, possibly part of the legal framework of the country. The rule is set independently of monetary policy. All else constant, higher values of γ_w in (12) imply a stronger influence of the indexation rule. In this paper, I assume that π^W equals wage inflation in the commodity sector

$$\hat{\pi}_t^W \quad = \quad \hat{\pi}_t^{wx} \tag{41}$$

For values of $\gamma_w \neq 0$, wage inflation in the commodity sector is passed on to wages in the non-commodity sector. This creates a wedge between the average wage in the non-commodity sector and the efficient wage. As a result, vacancies and employment in the non-commodity sector also deviate from their efficient levels. In the next subsections, I investigate the relevance of these inefficiencies in terms of welfare.[16]

3.5.1 Commodity Price Shocks and Sectoral Relative Wage Rigidity

Before moving to the analysis of optimal monetary policy, I study the propagation of commodity price shocks through the economy and investigate the impact of sectoral relative wage rigidity on the transmission of those shocks. Figure 3 plots the impulse responses of key variables to a 1% increase in the foreign price of

[16] Expression (41) is clearly a reduce form approach to relative wage rigidities. Although not predominantly the case in the literature, the study of wage bargaining centralization and of sectoral wage setting should not neglect the potential interactions and game theoretical considerations involved at the level of trade unions. Their bargaining power, scope for coordination, individual interests and sectoral idiosyncrasies can have important implications in terms of wage setting outcomes. In this paper, my objective is to understand the implication of sectoral rigidities for monetary policy, rather than the rationale behind these rigidities *per se*. Nevertheless, why these inefficiencies arise is an extremely relevant question for further research.

Australian commodities. All responses are represented in terms of percentage deviations from steady state.

Focusing first on the thick black line, which assumes no sectoral relative wage rigidity, an increase in the foreign price of domestically-produced commodities triggers the expansion of the commodity sector. Since new hires only become productive one quarter after being matched, an unexpected commodity price shock has no immediate impact on commodity output. As commodity firms post more vacancies and raise wages, more workers are encouraged to search for a job in the resource sector. On the contrary, because the increase in commodity exports is immediate, the appreciation of the real exchange rate is felt on impact. This direct effect of the increase in commodity prices induces a prompt contraction in non-commodity output, which is magnified in the quarters that follow. Two reasons contribute to this. On the one hand, as workers are attracted to higher wages in the commodity sector, non-commodity employment falls. Note, however, that despite the fall in non-commodity employment, overall employment in the economy increases.[17] On the other hand, the appreciation of the exchange rate makes domestically-produced tradeables relatively more expensive than foreign tradeables, reducing aggregate demand. Finally, although the appreciation of the exchange rate lowers the price of imported consumption goods, as domestic retailers set prices as a mark-up over marginal costs, the shock raises consumer price inflation.

The effect of sectoral relative wage rigidity on the propagation of commodity price shocks is better captured by the response of wage inflation in the non-commodity sector. When $\gamma_w = 0$, wages in the non-commodity sector barely change. However, for $\gamma_w > 0$, the response is swift. As non-commodity firms internalise the consequences of sectoral wage movements, they anticipate the effects of a rise in wages in the commodity sector. As such, those firms that are able to negotiate wages, reduce them immediately. In addition, they cut vacancy posting further in an attempt to mitigate the impact of pay rises in the future. After this anticipation effect, movements in average non-commodity wages become dictated by the indexation rule. In turn, commodity firms also anticipate the effect of relative wage rigidity. Knowing that non-commodity firms attempt to cut wages and stop hiring workers, commodity firms raise wages by less, hire more workers, and reach

[17] The responses of sectoral employment are led one period, since decisions about directed search or vacancy posting only have an effect one quarter later. On the contrary, the response of total employment is contemporaneous and, as a result, does not move on impact.

the peak of production earlier. Combined, the effects on total employment are two-fold. At first, the abrupt reduction in non-commodity employment leads to a timid increase in total employment, below the increase observed when $\gamma_w = 0$. Two quarters later, the increase in employment in the commodity sector takes the lead and push total employment northwards. Finally, the impact of sectoral relative wage rigidity is also observed in the response of price inflation. With non-commodity wages rising substantially one quarter after the shock, and since retailers set prices as a mark-up over marginal costs, inflation jumps twice as high as compared to when $\gamma_w = 0$.

3.5.2 Welfare criterion

Log-linearising the model around the efficient steady state permits me to derive a quadratic welfare measure directly from the equilibrium conditions. I follow the LQ methodology pioneered by Rotemberg and Woodford (1997) and used in Benigno and Woodford (2006) and de Paoli (2009), among others, and obtain a quadratic measure of welfare by computing a second-order approximation to the household lifetime utility criterion and to some equilibrium conditions. I use the LQ methodology to study monetary policy for three reasons. The first reason is due to the fact that price and wage dispersion have an impact on the second moments of the model. Therefore, it is crucial to have a welfare approximation that is accurate at least to second-order. Second, the LQ methodology only requires first-order approximations of the equilibrium conditions. Because of the history dependence of non-commodity firm's wages and vacancy rates, I am only able to obtain aggregate conditions by performing first-order log-linear approximations. Moreover, and for the same reason, the derivation of higher-order approximations becomes infeasible. Third, a model-based welfare measure is attractive because it serves as a benchmark to the study of *ad hoc* loss functions that stylise the preferences of the monetary authority.

Proposition 8 provides the conditions under which I can derive an utility-based quadratic welfare approximation. The complete derivation can be found in the appendix.

Proposition 8: With $Q_3 = 1$ and assuming $\alpha^c = Q_1$ and perfect exchange rate

pass-through, the welfare criterion can be approximated by

$$\tilde{\mathcal{W}} = -\frac{\bar{c}^{-\sigma}}{2}\sum_{t=0}^{\infty}\beta^t\mathcal{L}_t + t.i.p. + \mathcal{O}^3 \tag{42}$$

with $\mathcal{L}_t$ given by

$$\mathcal{L}_t = \left\{\begin{array}{c} \Lambda^{\pi}\left(\hat{\pi}_t^h\right)^2 + \Lambda^{w}\left(\hat{\pi}_t^{wh}\right)^2 + \Lambda^{q}\left(\hat{q}_t\right)^2 \\ \sigma\left(\hat{c}_t\right)^2 + 2\sigma\hat{c}_t\hat{tb}_t + \left(1-\gamma^c\right)Q_1\left(\hat{p}_t^h\right)^2 - \left(1-Q_1\right)\left(\hat{p}_t^x\right)^2 \\ +\alpha_1^h Q_1\left(\hat{n}_t^h\right)^2 + \alpha_1^x\left(1-\alpha_1^x\right)Q_2\left(\hat{n}_t^x\right)^2 - 2\alpha_1^x Q_2\hat{n}_t^x\hat{A}_t \\ -\left(Q_1\right)^3\left(\hat{y}_t^h\right)^2 + \left(Q_2+Q_1-1\right)\left(\hat{x}_t^h\right)^2 - 2\left(1-Q_1\right)\hat{p}_t^x\hat{x}_t^f \\ +\varrho^h\left(1+\psi^h\right)\left(\left(1-\epsilon^h\right)\left(\hat{\theta}_t^h\right)^2 + \psi^h\left(\hat{\phi}_t^h\right)^2\right) \\ +\varrho^x\left(1+\psi^x\right)\left(\left(1-\epsilon^x\right)\left(\hat{\theta}_t^x\right)^2 + \psi^x\left(\hat{\phi}_t^x\right)^2\right) \end{array}\right\} \tag{43}$$

and

$$\Lambda^{\pi} = Q_1\mu^h\delta^p/\left(\left(1-\delta^p\right)\left(1-\delta^p\beta\right)\right)$$

$$\Lambda^{w} = \left(\delta^w\right)^3\left(\beta\lambda^h\upsilon^h\right)^2/\left(\left(1-\delta^w\right)\left(1-\beta\delta^w\right)^3\varrho^h\left(1+\psi^h\right)\psi^h\right)$$

$$\Lambda^{q} = \left(1-\gamma^c\right)\left(1-\alpha^c\right)$$

where $t.i.p. + \mathcal{O}^3$ stands for terms independent of policy and of third or higher order. ■

The terms in the welfare expression of Proposition 8 can be grouped into four categories according to their source within the model. First, the term in $\hat{\pi}^h$ comes directly from the resource constraint of non-commodity goods. It accounts for the detrimental effects of price stickiness in terms of real output losses. Second, the term in $\hat{\pi}^{wh}$ stems from the wage dispersion in the non-commodity sector. As wholesalers are unable to set wages efficiently, vacancy posting, employment and output also depart from their optimum levels. Third, last two lines of expression (42) capture the effects of the search-and-matching technology in the allocation of labour across sectors. These terms are also present in Thomas (2008). On the contrary, the fourth category, which includes most of the remaining terms, is due

to the novel features of my model: the existence of two sectors in the economy, together with the fact that the economy is open to trade.[18] These two features are also motivate the conditionality in Proposition 8. In particular, by imposing no exports of non-commodity output (i.e., setting $\alpha^c = Q_1$) and assuming perfect exchange rate pass-through, I can eliminate the linear terms on the exchange rate and derive a purely quadratic measure of welfare.

The first panel of Figure 4 investigates the impact of sectoral relative wage rigidity on welfare. I compare three monetary policy rules. One consists of strict inflation targeting, which sets consumer price inflation to zero at all dates. The second policy rule is based on Thomas (2008), and consists of setting a weighted average of price and wage inflation to zero every period. Lastly, I implement rule (40), choosing the value of ω_π that maximizes welfare measure (42) for a given value of γ_w. As shown in the figure, regardless of the policy rule followed by the monetary authority, sectoral relative wage rigidity is undoubtedly detrimental to welfare, with full indexation causing a welfare loss of 60% when compared to no indexation. However, this does not mean the monetary authority should be indifferent to the the degree of relative wage rigidity.

On the second panel of Figure 4, I explore the welfare implications of different policy rules against the optimized version of rule (40), for $\gamma_w \in [0, 1]$. A zero-inflation policy is more detrimental to welfare regardless of the degree of indexation. Moreover, zero inflation is worse for higher levels of indexation, with a loss of 26% compared to the optimal feedback rule. Likewise, the optimized feedback rule policy outperforms the rule studied by Thomas (2008), with the latter implying a loss between 9% and 15%. In Figure 4, I also compare the optimal policy rule to one for which ω_π is held constant regardless of the degree of indexation. In particular, I study two scenarios: first, I consider a rule with a constant ω_π set equal to the optimal ω_π when $\gamma_w = 0$; second, a rule for which the optimal long-term response to inflation is always held equal to the optimal ω_π when $\gamma_w = 1$. A mismatched value of ω_π can be detrimental to welfare, with the worst case being a choice of ω_π consistent with no indexation in an economy where non-commodity wages are fully responsive to the evolution of commodity wages. Hence, non-optimal monetary policy tends to be more of a concern for higher levels of relative wage rigidity.

The negative performance of strict inflation targeting when compared to the optimized version of (40) is particularly evident from Figure 4. Another way to

[18] The term in $\hat{c}_t$ comes directly from the utility function and is standard in the literature.

understand the differences between the two rules is to compare the impulse responses associated with each policy. Figure 5 shows the impulse responses to a commodity price shock under the two rules and for no and full indexation, $\gamma_w = 0, 1$.[19] The most notorious difference between the two policies concerns the response of total employment. Independently of the degree of indexation, a zero-inflation policy reduces employment, whereas under the feedback rule total employment increases. As shown in the figure, this effect is more pronounced the higher the value of γ_w is. Behind the drop in total employment is the accentuated fall in non-commodity employment. In fact, the reaction of non-commodity employment is at the core of the transmission mechanism of monetary policy. With inflation equal to zero, nominal wages equal real wages and non-commodity firms that are unable to renegotiate wages cannot enjoy the effects of inflation in reducing real wages over time. In other words, with zero inflation, firms can only reduce their wage bill in real terms by reducing the number of hires. Moreover, the effects of indexation in real wage is also magnified when $\hat{\pi}_t = 0$. With wages in the commodity sector rising after a commodity price shock, non-commodity firms face higher expected real costs, which can only be mitigated by contracting vacancy posting further and reducing employment more.

The trade-off between inflation and unemployment faced by the monetary authority is evident in Figure 5, with a policy of zero-inflation causing unemployment when the economy is hit by a commodity price shock. Furthermore, the trade-off becomes more stringent for higher values of γ_w. These results qualify the findings of Driffill and van der Ploeg (1993). For higher levels of indexation and with a zero-inflation policy, real wages rise more and induce a bigger contraction of employment. The hump-shaped hypothesis in the open economy context is, therefore, preserved. However, with an optimized feedback rule, wage rises are inflationary and it is not necessarily true that real wages and unemployment rise more for higher degrees of sectoral relative wage rigidity. In fact, for higher values of γ_w, wage rises following a commodity price shock cause more inflation and, although delayed, generate a more pronounced increase in total employment.

Despite begin dependent on the type of monetary policy conducted, Figure 4 provides strong evidence of the detrimental effects sectoral relative wage rigidity has on welfare. However, interpreting these results under the hump-shaped hypo-

[19] For ease of comparison, all impulse responses in this paper are for the complete model, that is, the model with imperfect exchange rate pass-through and with exports of non-commodity goods.

thesis framework might be misleading. First, the welfare criterion is an aggregate measure which includes all household members, both employed and unemployed. Hence, unlike trade unions who typically maximize worker's aggregate welfare, here I also account for the effects of relative wage rigidity on those left without a labour contract. Second, mediation of wage bargaining rarely consists of a linear process of dividing surpluses between the parties directly involved. A number of other considerations are often involved in the bargaining process which escape the particular nature and economic conditions of the labour contract under negotiation. For instance, according to Pagan (1987), during the early 1980s, optimism about the duration of a commodity boom allowed trade unions to obtain sizeable real wage increases. Although the boom was a sectoral shock, potentially harmful to other sectors exposed to foreign competition, real wages were increased throughout the economy. One consideration that might be behind sectoral relative wage rigidity relates to equity. As Figure 6 reports, for high levels of relative wage rigidity, the volatility of the differential between sectoral wages decreases substantially. Hence, social preferences that prioritize greater equality in wages can explain why sectoral relative wages are fixed despite the detrimental effects caused to the aggregate welfare criterion I have considered.

3.6 Different Policy Scenarios

The strong assumptions imposed in Proposition 8 constrain the model in two important dimensions. On the one hand, excluding exports of non-commodity goods is not only unrealistic, it also closes one important channel of the transmission of commodity price shocks. If non-commodity goods are not exposed to foreign competition, the appreciation of the exchange rate has little effects on profit margins and on production. On the other hand, assuming perfect exchange rate pass-through affects the match of second moments, as discussed above. As welfare analysis relies on second moments, a mismatch to their empirical counterpart might generate important distortions. For these reasons, in this subsection I propose simple loss functions to evaluate monetary policy and investigate different scenarios with respect to policy objectives and policy instruments.

3.6.1 Policy Evaluation using Loss Functions

Loss functions are interpreted as representing the preferences of the monetary authority. To the extent a given loss function behaves closely enough to the welfare criterion, it can be argued that the preferences of the monetary authority are aligned with aggregate welfare. Moreover, besides not requiring special assumptions to be imposed on the model, the use of simple loss functions also enables me to explore the trade-offs faced by the monetary authority and the impact sectoral relative wage rigidity has on the conduct of monetary policy.

In the remainder of this section, I consider the following loss function

$$L^1 \quad = \quad \lambda^\pi \sigma^\pi + (1 - \lambda^\pi)\, \sigma^{y^h} \tag{44}$$

where σ^π and σ^{y^h} are the standard deviations of consumer price inflation and non-commodity output, respectively, and λ^π is the relative importance the monetary authority gives to inflation volatility. The functional form of (44) is standard in the New-Keynesian literature.[20] The value of λ^π is calibrated such that the policy predictions associated with minimizing (44) approximate those associated with maximizing (42). In other words, I pin down λ^π by minimizing the difference between the optimal monetary policy's long-run reactions to inflation under L^1, for different values of γ_w, and those under the welfare criterion. I proceed as follows. First, I search for the values of ω_π that maximize the welfare criterion in a model consistent with the conditions outlined in Proposition 8. As a result, I obtain $\omega_\pi^{\mathcal{W}}\left(\gamma_w|\mathcal{M}^{\mathcal{W}}\right)$: the values of ω_π that maximize (42) for different values of γ_w, in the model consistent with Proposition 8, which I denote by $\mathcal{M}^{\mathcal{W}}$. Second, under the same *welfare-consistent* model, $\mathcal{M}^{\mathcal{W}}$, I obtain $\omega_\pi^{L^1}\left(\gamma_w|\mathcal{M}^{\mathcal{W}}, \lambda^\pi\right)$: the values of ω_π that minimize (44) for different values of γ_w. Finally, I obtain λ^π by minimizing the relative changes between $\omega_\pi^{\mathcal{W}}\left(\gamma_w|\mathcal{M}^{\mathcal{W}}\right)$ and $\omega_\pi^{L^1}\left(\gamma_w|\mathcal{M}^{\mathcal{W}}, \lambda^\pi\right)$.[21]

On the left panel of Figure 7, the dotted black line corresponds to the changes of $\omega_\pi^{\mathcal{W}}\left(\gamma_w|\mathcal{M}^{\mathcal{W}}\right)$ relative to $\omega_\pi^{\mathcal{W}}\left(\gamma_w = 0|\mathcal{M}^{\mathcal{W}}\right)$. The welfare-consistent policy response to inflation is increasing with the degree of sectoral relative wage rigidity. In comparison, the dotted grey line shows the relative changes of $\omega_\pi^{L^1}\left(\gamma_w|\mathcal{M}^{\mathcal{W}}\right)$ for $\lambda^\pi = 1/3$, which is the value of λ^π that minimises the differences between the

[20] Refer to Woodford (2003).

[21] By relative changes I mean $\omega_\pi(\gamma_w)/\omega_\pi(\gamma_w = 0)$. I match the relative changes because the absolute values of the optimal ω_π under the two objectives are very different. Moreover, for policy evaluation, the relative changes in the conduct of policy are more relevant than the absolute values of ω_π.

two lines.[22] Minimizing (44) for increasing degrees of relative wage rigidity also requires a stronger response to inflation. However, the magnitude of the relative changes is different. Under the *welfare-consistent* model, $\mathcal{M}^{\mathcal{W}}$, maximizing welfare requires a response to inflation 2.4 times higher when $\gamma_w = 1$ relative to when $\gamma_w = 0$. Instead, the relative difference is about 1.6 when the objective is to minimise the loss function. In other words, the relative importance of curbing inflation volatility under L^1 is lower for higher degrees of indexation when compared to the welfare-maximizing policy. Hence, the effects of relative wage rigidity on inflation appears to be comparatively more detrimental to welfare than they are for the loss function.

The changes of $\omega_\pi^{L^1}\left(\gamma_w|\mathcal{M}^{\mathcal{C}}\right)$ relative to $\omega_\pi^{L^1}\left(\gamma_w = 0|\mathcal{M}^{\mathcal{C}}\right)$ are also plotted on the left panel of Figure 7. These are the relative values of ω_π obtained by minimising (44) under model $\mathcal{M}^{\mathcal{C}}$ and with $\lambda^\pi = 1/3$. Model $\mathcal{M}^{\mathcal{C}}$ is the complete model calibrated as reported in Table 2, i.e., not subject to the conditions of Proposition 8. The relative change in the long-run response to inflation between no and full indexation is around 1.3, as shown by the thick black line. The drop from the 1.6 obtained under $\mathcal{M}^{\mathcal{W}}$ is due to the conditions of Proposition 8. On the one hand, with imperfect exchange rate pass-through, changes in the price of imported consumption goods are less abrupt, implying lower inflation volatility. On the other hand, allowing for exports of non-commodities exacerbates the volatility of non-commodity output through the effects of the exchange rate on foreign demand. Together, these two effects require a relatively more modest response to inflation.

Finally, the right panel of Figure 7 presents some sensitivity analysis of the policy prescription under L^1 for model $\mathcal{M}^{\mathcal{C}}$. As expected, for higher values of λ^π, the required long-run response to inflation increases. I also investigate the effects of wage stickiness on monetary policy. As in Thomas (2008), the longer the average duration of wage contracts is in the non-commodity sector, the more important controlling wage inflation becomes.[23] Consequently, the strength put in responding to price inflation falls. These results are related to the discussion of the impulse responses of Figure 5. As the probability of adjusting wages falls, with more non-commodity firms unable to renegotiate wages each period, more vacancy posts are cut and, consequently, the drop in employment and total output

[22] For instance, matching the values, rather than the relative changes, of both set of responses yields $\lambda^\pi = 0.40$, whereas matching both responses for $\gamma_w = 0$ yields $\lambda^\pi = 0.37$. Besides the differences in the values of the optimised ω_π, results do not change much. For the remainder of this paper, I will consider $\lambda^\pi = 1/3$.

[23] The average duration of wage contracts is given by $1/\left(1-\delta^w\right)$.

increases.

3.6.2 Inflation Targeting under Different Objectives

Producer price inflation: The previous sections emphasise the trade-off between price and wage inflation for optimal monetary policy. In this subsection, I start by investigating the robustness of the results found earlier using alternative policy scenarios. In particular, I modify the framework characterizing monetary policy in the following of ways. First, I change the preferences of the monetary authority. Instead of (44), I consider a loss function described by

$$L^2 = \lambda^\pi \sigma^{\pi^h} + (1 - \lambda^\pi)\, \sigma^{y^h} \tag{45}$$

where consumer price inflation is replaced by non-commodity price inflation. In this scenario, the rule describing monetary authority is given by (40) still. Second, I change the policy rule. I consider L^1 again to describe the preferences of the monetary authority, but assume instead that policy targets $\hat{\pi}_t^h$ instead of $\hat{\pi}_t$. These two scenarios explore the role of producer price inflation for monetary policy.

Figure 8 reports the optimal values of ω_π obtained under these new scenarios. As with the benchmark case, optimal policy under the two new scenarios requires a stronger response to inflation for higher degrees of relative wage rigidity, as shown by the two dashed grey lines. However, for a given degree of indexation, the optimal magnitude of the response differs. Holding the price of foreign tradeable consumption goods constant, consumer price inflation is equal to a weighted sum of non-commodity price inflation and changes in the nominal exchange rate.[24] As such, targeting $\hat{\pi}_t$ in order to control $\hat{\pi}_t^h$ requires larger reactions of the policy instrument. Instead, targeting movements in $\hat{\pi}_t^h$ directly affects non-commodity output and, consequently, the nominal exchange rate. Therefore, the optimal response of the nominal interest rate becomes smaller. Table 4 presents a range of comparisons across these policy scenarios. To the extent the monetary authority responds correctly to the inflation measure being targeted, the question of which inflation to use appears irrelevant, as the differences in terms of L^1 of targeting one or the other measure are insignificant.

Unemployment: The discussion in previous subsections identifies a puzzle

[24] This is shown in condition (31).

concerning the effects of sectoral relative wage rigidity on monetary policy. On the one hand, Figure 4 and Figure 5 seem to warrant a milder response to price inflation for higher values of γ_w in order to mitigate the effects of higher wage volatility on unemployment and output. On the other hand, the policy prescription in Figure 7 suggests a stronger response to inflation for higher degrees of relative wage rigidity. In order to conciliate both claims, I construct a fourth scenario where the preferences of the monetary authority are represented by

$$L^3 = \lambda^\pi \sigma^\pi + (1 - \lambda^\pi)\, \sigma^u \tag{46}$$

and policy targets $\hat{\pi}_t$ as in (40). In L^3, σ^u denotes the volatility of total unemployment.[25] This change has a significant impact in terms of policy prescription. In contrast to the previous results, the dash blue line in Figure 8 shows a softening of the response to inflation for higher values of γ_w. This result is a crucial insight into the transmission mechanism of monetary policy in an economy with sectoral relative wage rigidities.

As described before, a higher degree of indexation exacerbates the spillover effects of commodity price shocks into wages in the non-commodity sector. This induces non-commodity firms to adjust their demand for labour more frequently. As a result, total unemployment also becomes more volatile. These effects are shown on the left panel of Figure 9. Although sectoral relative wage rigidity increases the volatility of inflation and non-commodity output, the effects on unemployment are overwhelming. On the contrary, on the right panel of Figure 9, unemployment and non-commodity output volatilities are only marginally affected by monetary policy when compared to the effects it has on inflation. Hence, due to the powerful effects relative wage rigidity has in raising unemployment volatility, the monetary authority is forced to compromise its commitment to price stability. By allowing some inflation, monetary policy is able to moderate the fluctuation in real wages and, as a consequence, to smooth unemployment volatility.

In a game-theoretic model, Cukierman and Lippi (1999) show that when trade unions internalise the degree of inflation aversion of the monetary authority, they claim higher nominal wage rises when the central bank is more inflation averse. As real wages rise more, unemployment also tends to grow higher. Here, the mechanism is similar in the sense that with a monetary authority less inflation averse, nominal wage swings are mitigated by inflation. As a result, real wages

[25] Minimizing the volatility of non-commodity unemployment, u_t^h, does not change the results.

become less volatile and unemployment variability drops as well.

Targeting the nominal exchange rate: In a small commodity-exporting economy with floating exchange rates, commodity price shocks induce an appreciation. For consumers, the appreciation is positive, as imports become cheaper. For workers, the change in the nominal exchange rate contributes to the contraction of the non-commodity tradeable sector and might dictate the loss of jobs. These effects question whether is justifiable to manage the exchange rate in order to lessen the detrimental effects of commodity price shocks. Hevia and Nicolini (2013) investigate this possibility in a model where the only source of inefficiencies comes from price stickiness in the non-commodity sector. They find no support against letting the exchange rate to float freely.

With sectoral relative wage rigidity, the dire consequences of commodity price shocks are exacerbated. In order to understand whether managing exchange rate movements directly improves welfare, I extend the feedback rule as follows

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1 - \rho_i)\left(\omega_\pi \hat{\pi}_t + \omega_q \hat{q}_t\right) + \xi_t^i \tag{47}$$

Figure 10 reports the optimized values for both ω_π and ω_q. The blue lines correspond to a welfare maximizing policy, whereas the grey lines correspond to the optimal policy consistent with L^1. In terms of the optimal response to inflation, the augmented rule implies little differences relative to (40) when the objective is to maximize welfare. However, the magnitude of the response to inflation increase 50% when the objective is to minimize L^1. Regarding the response to the real exchange rate, the monetary authority finds it optimal to raise the nominal interest rate in face of an appreciation. Moreover, policy tightens more for higher levels of sectoral relative wage rigidity.[26] Therefore, it appears that the positive effects on consumption outweigh the negative impact of an appreciation on the non-commodity tradeable sector. Nevertheless, the gains from augmenting the feedback rule are very small, as reported in Table 4.

[26] As domestic inflation is restrained through ω_w and foreign inflation is held constant, a real appreciation implies the appreciation of the nominal exchange rate.

3.7 Conclusion

Rigidity in the adjustment of relative wages is a feature of several economies. As wages play a central role in the allocation of labour between different sectors and are a determinant component of households income, managing their evolution has important consequences. In this paper, I study the implications of sectoral relative wage rigidity for the conduct of monetary policy in a small commodity-exporting economy. The study of relative wage rigidity in the context of commodity-exporters is particularly relevant for at least three reasons. First, shocks to commodity prices are highly volatile and unpredictable. For resource-rich countries, they represent sectoral shocks to the commodity sector, which spillover to the rest of the economy through changes in relative prices and through the reallocation of factors of production. Second, the institutional characteristics of labour markets in these countries vary according to geography and over time. With different degrees of sectoral relative wage rigidity, a one size-fits-all monetary policy prescription might be inappropriate. Third, as the price of tradeables is fixed in world markets, the impact commodity price shocks have on these countries' exchange rate affects directly their non-commodity tradeable sector. As a result, the tradeable sector becomes especially sensitive to spillover effects of commodity price shocks, namely in terms of production costs.

In order to investigate the implications of relative wage rigidity, I propose a two-sector model of an open economy subject to commodity price shocks. The two sectors consist of a non-commodity and a commodity sector. Both use labour as input for production. The non-commodity sector produces a tradeable good used for consumption domestically and abroad, therefore being exposed to exchange rate fluctuations. It also uses a small share of the domestically-produced commodity good as an intermediary for production. The remaining commodity output is exported. Labour markets are assumed to be segmented, with every unemployed household member directing his job-search to a specific sector. New hires occur according to a matching function, which depends on the number of job-seekers and of vacancies posted is each sector. Nominal wages in the non-commodity sector are assumed to be sticky and can be indexed to the evolution of wages in the commodity sector. This link creates sectoral relative wage rigidity.

Because every period a random share of non-commodity firms is unable to negotiate wages with workers, wages in each individual firm are history dependent. As a result, vacancies also vary across firms. As I demonstrate in the paper,

dispersion in wages and vacancy posting affects resource allocation. To shed light on the mechanisms behind relative wage rigidity, I analyse the impulse response of a set of variables to a commodity price shock. I first assume no sectoral relative wage rigidity. The shock causes an appreciation of the exchange rate. As domestically-produced goods become relatively more expensive than foreign goods, the non-commodity sector contracts. This effect is reinforced by the expansion of the commodity sector. In order to attract workers, commodity firms raise wages, shifting job-seekers away from the non-commodity sector. As demand for non-commodity goods is weaker, firms find it difficult to retain their workers. Alternatively, if wages in the non-commodity sector are linked to the evolution of wages in the commodity sector, non-commodity output drops further. As non-commodity firms expect wages to increase following the expansion of the commodity sector, they reduce vacancies more. The effects on total employment are illustrative of the consequences of relative wage rigidity. With no indexation, total employment increases pushed by the expansion of the commodity sector. With full indexation, employment drops following the rapid reduction in non-commodity vacancies.

I calibrate the model to Australia and use GMM to match the second moments of a set of relevant variables to their empirical counterpart. I then study optimal monetary policy for different degrees of relative wage rigidity. I obtain two seemingly opposing results. First, I find that strict inflation targeting is detrimental to welfare compared to a simple feedback rule to the nominal interest rate. Moreover, the welfare losses are larger the higher the degree of indexation. Second, I find that optimal monetary policy has the monetary authority being more responsive to inflation the higher the relative wage rigidity. In order to understand this apparent puzzle, I use different loss functions to describe the preferences of the monetary authority. I find that, for a higher degree of indexation, some inflation is desirable in order to smooth real wage volatility. As wages are set in nominal terms, inflation partially offsets the effects of indexation on real wages. Nevertheless, as indexation itself is inflationary, the response of monetary policy to inflation needs to be stronger.

This paper unveils important policy implications. In particular, I show that policy misspecification is more problematic for higher degrees of relative wage rigidity, as an excessive response to inflation when non-commodity wages are highly linked to commodity wages leads to higher unemployment volatility. On the other hand, the optimal policy prescription is highly sensitive to the type of loss function the

monetary authority chooses to follow. For instance, if monetary policy cares about price inflation and non-commodity output volatilities, the strength of the response to inflation increases for higher degrees of indexation. However, if instead of non-commodity output the monetary authority is interested in curbing unemployment volatility, then the optimal response to inflation becomes milder for higher degrees of indexation. All things considered, although monetary policy is found to be an effective macroeconomic policy instrument, its response needs to be calibrated carefully.

3.8 Tables

Table 1: Wage Bargaining

	80's	90's	00's		80's	90's	00's
Australia				**Canada**			
coordination	3.5	2.4	2	coordination	1	1	1
level	4.4	2.4	2	level	1	1	1
articulation	4	4	4	articulation	0	0	0
σ_π	0.6	0.26	0.15	σ_π	-	0.04	0.04
σ_u	1.84	2.01	0.72	σ_u	-	1.67	0.52
New Zealand				**Norway**			
coordination	2.6	1	1	coordination	4	4.1	4.1
level	3.6	1.2	1	level	4.3	3.6	3.2
articulation	5	1	0	articulation	4.2	3.4	3
σ_π	2.61	0.22	0.22	σ_π	-	0.1	0.48
σ_u	2.36	2.65	0.93	σ_u	-	1.72	0.47

Source:average of annual labour market indicators from 1980 to 2009, from Visser (2013), ICTWSS. Coordination refers to the degree of wage-setting centralisation; Level refers to the predominant ($> 2/3$) level at which bargaining occurs; Articulation refers to the existence of scope for additional bargaining. Country data from national statistical agencies.

Table 2: Parameter Values and Steady State Ratios

	household			**foreign conditions**	
σ	interterporal elast. subst.	1	γ^h	export elasticity	1.40
β	discount factor	0.99	$K^{h\star}$	scale exports	0.05
γ^c	subst. home and foreign	1.3	ψ_b	risk premium	0.001
α^c	home bias	0.80		**persistence**	
κ^h	labour disutility	0.66	ρ_z	non-commodity technology	0.96
κ^x	labour disutility	0.82	ρ_a	commodity technology	0.96
	production		ρ_b	risk premium	0.75
μ^h	elasticity of substitution	1.5	ρ_x	foreign commodity price	0.80
α_1^h	labour share	0.96	$\rho_{\pi^\star}$	foreign inflation	0.39
α_1^x	labour share	0.30		**steady state**	
Z	scale non-commodity	0.92	Q_1	non-commodity to GDP	0.85
A	scale commodity	0.38	Q_2	commodity to GDP	0.1875
	Calvo lottery		Q_3	consumption to GDP	1
δ^p	non-commodity retailers	0.75	$\bar{n}$	employment rate	0.94
δ^f	import firms	0.90	$\bar{n}^h$	- in non-commodity	0.89
δ^w	non-commodity wage	0.65	$\bar{n}^x$	- in commodity	0.05
	labour market		$\bar{u}$	unemployment rate	0.06
λ^h	separation rate	0.10	$\bar{\phi}^h$	vacancy rate	0.02
λ^x	separation rate	0.25	$\bar{\phi}^x$	vacancy rate	0.03
η^h	bargaining power	0.60	$\bar{\theta}^h$	labour market tightness	0.36
η^x	bargaining power	0.60	$\bar{\theta}^x$	labour market tightness	0.14
ψ^h	elasticity of hiring costs	1	ϱ^h	hiring costs to GDP	0.01
ψ^x	elasticity of hiring costs	1	ϱ^x	hiring costs to GDP	0.00
χ^h	matching efficiency	3.33	υ^h	wage mass to GDP	0.80
χ^x	matching efficiency	3.78	υ^x	wage mass to GDP	0.05
	baseline policy		$\bar{q}$	real exchange rate	1
ω_i	interest rate smoothing	0.85	$\bar{p}^{x\star}$	foreign commodity price	1.20
ω_π	long-run inflation response	1.75	$\bar{p}^{f\star}$	price of imports	1

Table 3: Maching Second Moments

	1993Q1 - 2002Q4				**1993Q1 - 2013Q4**			
	σ		I	O	σ		I	O
c	0.79	(0.09)	0.87	0.84	0.97	(0.15)	0.99	0.97
y^h	0.85	(0.07)	0.81	0.85	0.77	(0.06)	0.76	0.77
x	1.99	(0.24)	1.99	1.93	1.83	(0.16)	2.14	1.83
q	3.96	(0.25)	3.95	3.15	5.57	(0.52)	5.53	5.16
π	0.29	(0.03)	0.29	0.29	0.36	(0.03)	0.36	0.36
$p^{x\star}$	3.88	(0.80)	3.88	4.87	8.10	(1.67)	8.04	6.62
$\pi^\star$	0.35	(0.03)	0.35	0.35	0.32	(0.03)	0.32	0.32
n^h	0.43	(0.05)	0.22	0.23	0.39	(0.04)	0.37	0.32
n^x	0.14	(0.03)	4.16	4.34	0.15	(0.02)	7.13	6.13

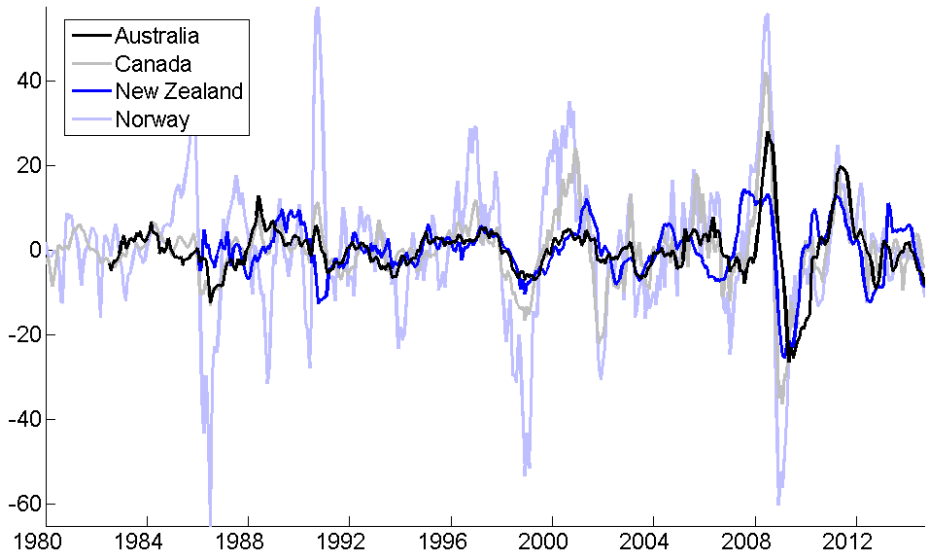
Note: GMM approach described in Ruge-Murcia (2007). For each sample, the first column reports the empirical standard deviations, with GMM-based standard errors between brackets. The remaining columns report the matched synthetic standard deviations, where I denotes equal weighting and O denotes optimal weighting using the Newey-West estimator with a Barlett kernel.

Table 4: Policy Comparison

		$\boldsymbol{\lambda^\pi = 1/3}$						
		$\gamma_w = 0$			$\gamma_w = 1$			%Δ
Loss Function	**Target**	*loss*	ω_π	ω_q	*loss*	ω_π	ω_q	*loss*
$\lambda\sigma_\pi + (1-\lambda)\sigma_y$	π	0.41	3.63	-	0.45	4.77	-	10.36
$\lambda\sigma_\pi + (1-\lambda)\sigma_y$	π^h	0.41	3.27	-	0.45	4.31	-	10.19
%Δ relative to benchmark		0.05	-9.92		-0.11	-9.64		
$\lambda\sigma_\pi + (1-\lambda)\sigma_y$	π and q	0.41	5.25	-0.07	0.45	7.59	-0.14	10.12
%Δ relative to benchmark		-0.62	44.63		-0.84	59.12		
$\lambda\sigma_{\pi^h} + (1-\lambda)\sigma_y$	π	0.41	4.27	-	0.45	5.47	-	10.06
$\lambda\sigma_\pi + (1-\lambda)\sigma_u$	π	0.29	12.34	-	0.85	2.99	-	192.91
		$\boldsymbol{\lambda^\pi = 1/2}$						
		$\gamma_w = 0$			$\gamma_w = 1$			%Δ
Loss Function	**Target**	*loss*	ω_π	ω_q	*loss*	ω_π	ω_q	*loss*
$\lambda\sigma_\pi + (1-\lambda)\sigma_y$	π	0.32	5.21	-	0.35	6.55	-	9.68
$\lambda\sigma_\pi + (1-\lambda)\sigma_y$	π^h	0.32	4.87	-	0.35	6.07	-	9.47
%Δ relative to benchmark		0.25	-6.53		0.06	-7.33		
$\lambda\sigma_\pi + (1-\lambda)\sigma_y$	π and q	0.32	7.17	-0.07	0.35	8.71	-0.09	9.73
%Δ relative to benchmark		-0.46	37.62		-0.41	32.98		
$\lambda\sigma_{\pi^h} + (1-\lambda)\sigma_y$	π	0.32	6.13	-	0.35	7.41	-	9.59
$\lambda\sigma_\pi + (1-\lambda)\sigma_u$	π	0.22	12.48	-	0.66	3.13	-	197.48

3.9 Figures

Figure 1: Commodity Price Shocks



Source: apart from Norway, for which I used the price of crude oil (Brent) alone, for the remaining countries I used indices of commodity prices based on export volumes for each country. Quarterly data in US dollars. Deviations from trend using HP filter on the logarithm of the series. Data from RBA, Statistics Canada, ANZ Commodities and Bloomberg.

Figure 2: The bargaining set

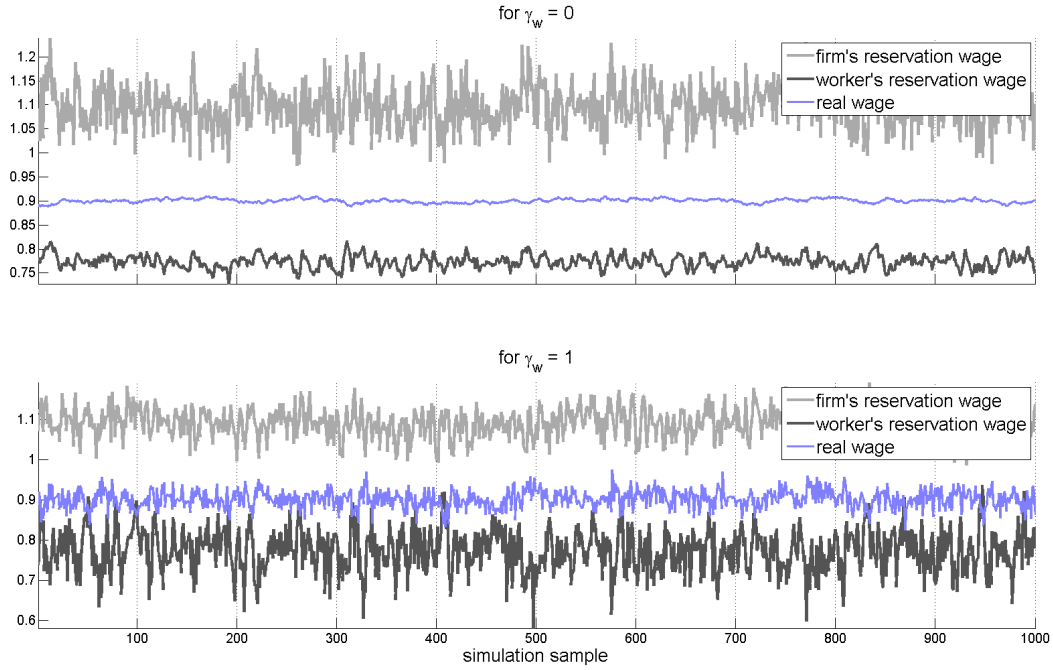


Figure 3: IRFs for a range of γ_w

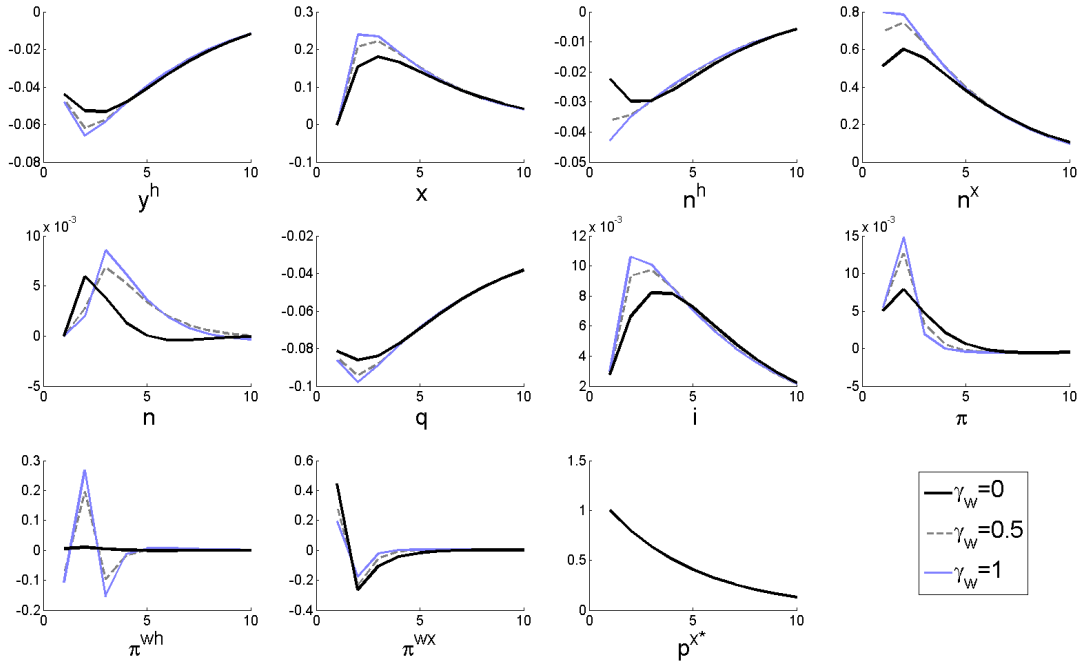
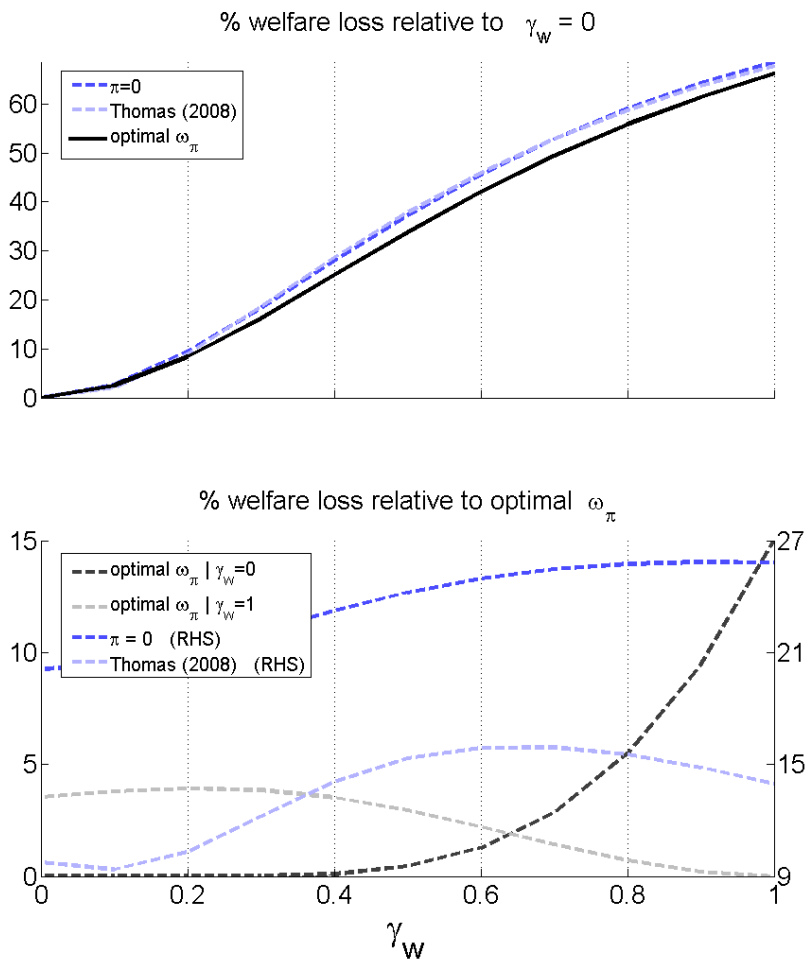


Figure 4: Behind Inflation Targeting



Note: Thomas (2008) shows that setting a weighted average of price and wage inflation to zero approximates relatively well optimal monetary policy in his model. Here, simple feedback rules outperform his weighted average rule.

Figure 5: IRFs for a range of policies

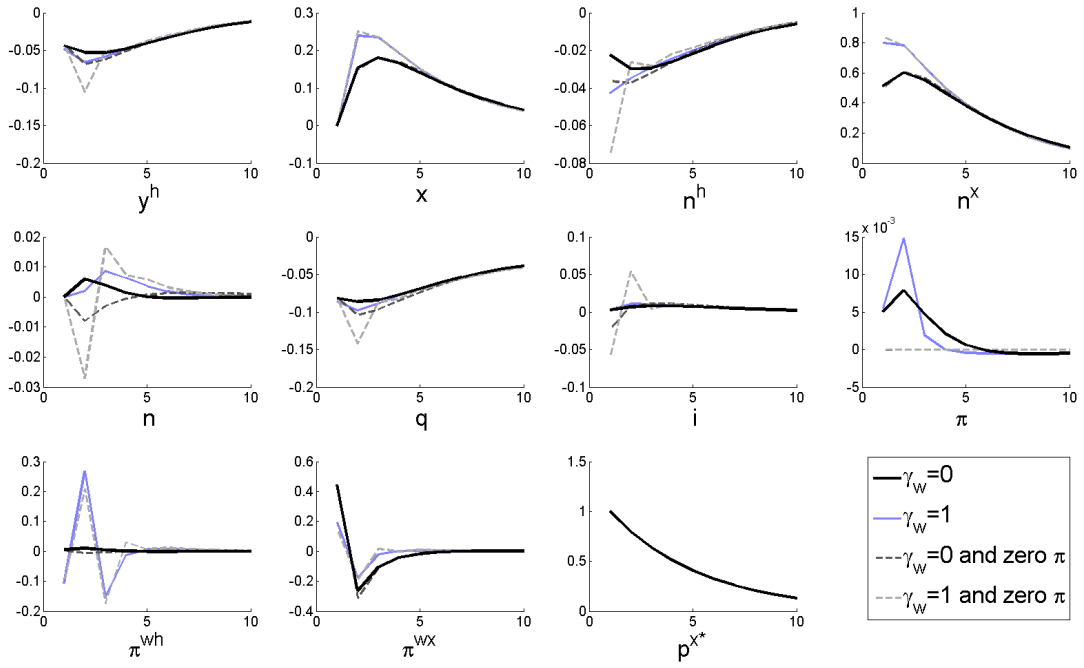


Figure 6: Relative wage volatility

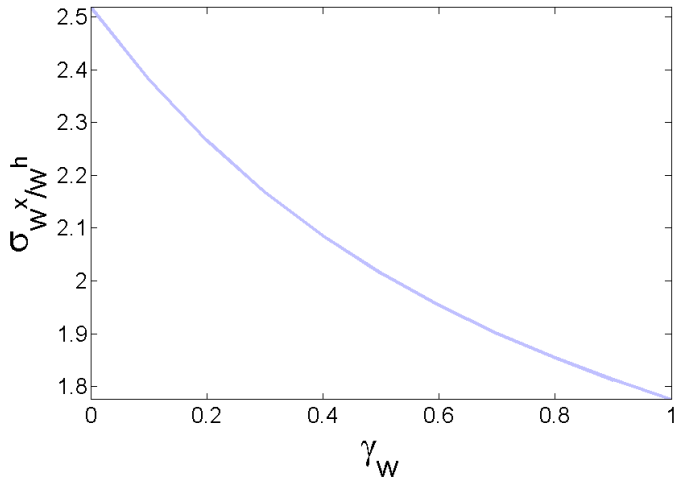


Figure 7: Loss consistent ω_π under inflation targeting

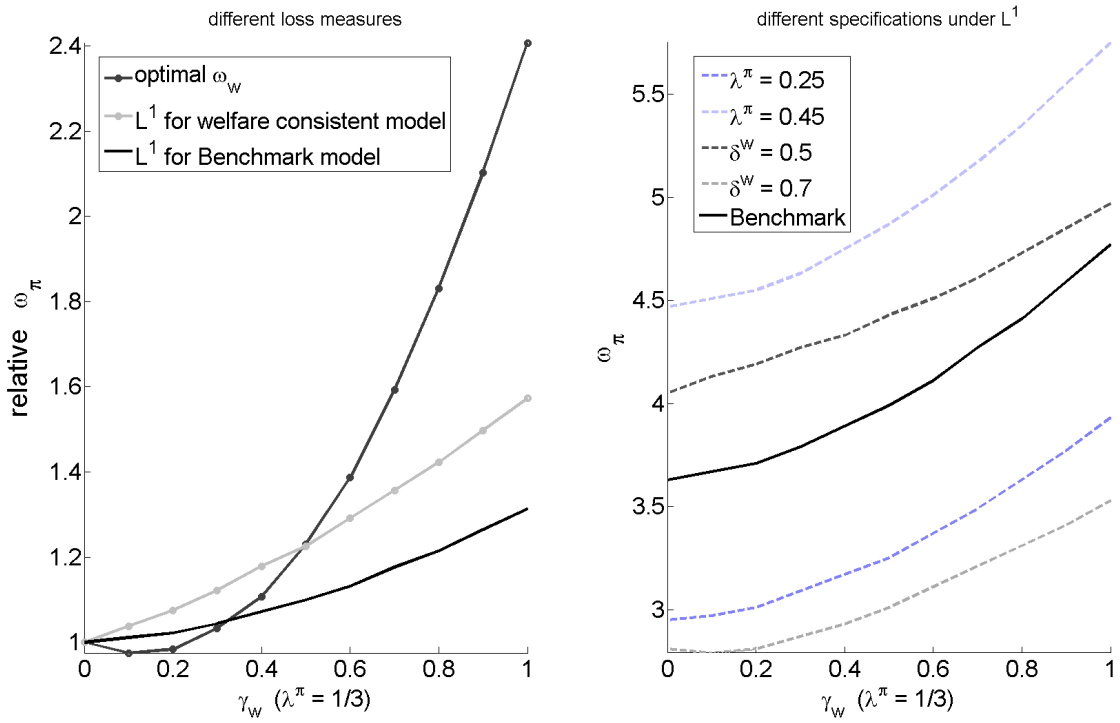


Figure 8: Different Policy Scenarios

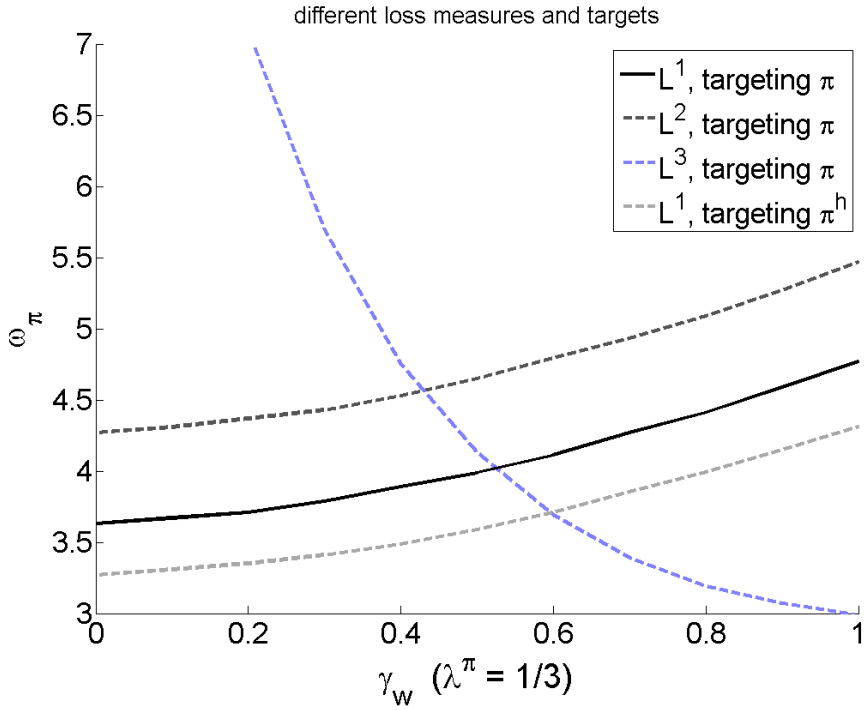


Figure 9: Volatilities

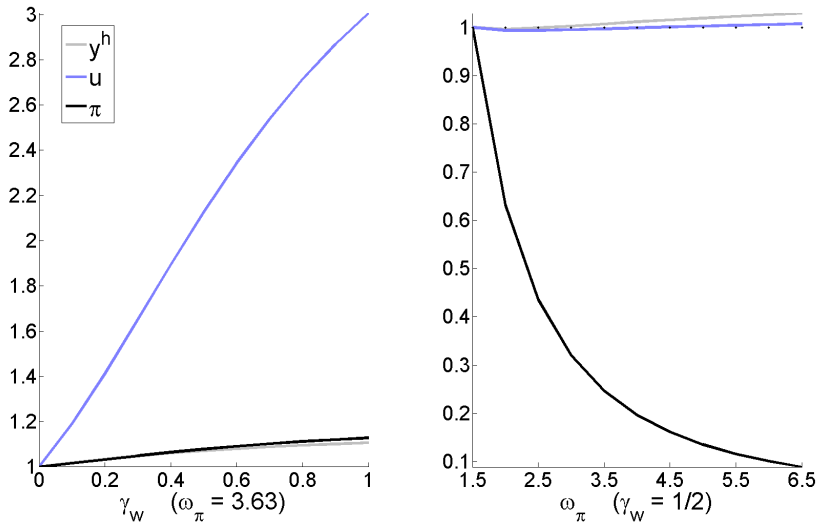
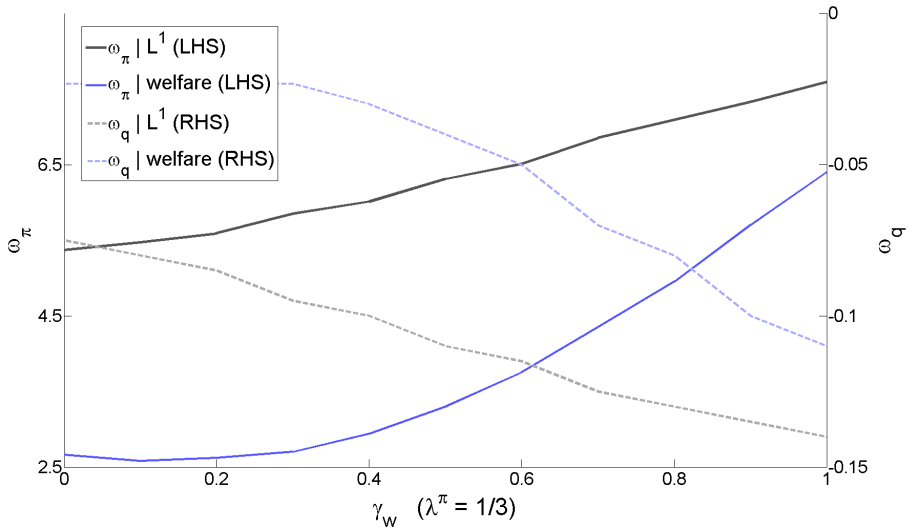


Figure 10: Monetary Policy and the Exchange rate



3.10 Appendix

For all propositions, I consider a zero-inflation, effcicent steady state, as discussed in the main text. The derivation of Proposition 1 follows Woodford (2003). For the remaining Propositions, similar derivations can be found in Thomas (2008). However, opening the economy to foreign trade and introducing two production sectors and sectoral relative wage indexation adds extra complexity to the algebra.

Propositions 1 and 2

As the derivations of expressions (31) and (32) are identical, I omit the latter for brevity. The price index P_t^h is an average of the price charged by the fraction $(1-\delta^p)$ of firms setting their prices in period t and the prices of the remaining fraction δ^p of firms that are unable to change their price. Given that ajusting firms are selected randomly, the average of the price of all firms not adjusting is P_{t-1}^h. Hence, from $P_t^h = \left[\int_0^1 P_{it}^{h\,1-\mu^h} di\right]^{1/\left(1-\mu^h\right)}$ I obtain

$$1 \quad = \quad (1-\delta^p)\left(P_t^{*h}/P_t^h\right)^{1-\mu^h} + \delta^r\left(P_{t-1}^h/P_t^h\right)^{1-\mu^h}$$

For ease of notation, I denote $Q_t = P_t^{*h}/P_t^h$. Note that, in the steady state, $\bar{Q} = 1$. Log-linearizing the expression above yields

$$\hat{Q}_t \quad = \quad \frac{\delta^p}{(1-\delta^p)}\hat{\pi}_t^h$$

Rearranging the first-order condition with respect to the nominal price, equation (5) in the main text,

$$Q_t E_t \sum_{s=0}^{\infty} \beta^s c_{t+s}^{-\sigma} (\delta^p)^s Y_{t+s}^h \frac{P_{t+s}^h}{P_{t+s}} \left(\frac{P_t^h}{P_{t+s}^h}\right)^{1-\mu^h} \quad = \quad E_t \sum_{s=0}^{\infty} \beta^s c_{t+s}^{-\sigma} (\delta^p)^s \varphi_{t+s}^h Y_{t+s}^h \left(\frac{P_t^h}{P_{t+s}^h}\right)^{-\mu^h}$$

The left side can be approximated by

$$\frac{\bar{c}^{-\sigma}\bar{Y}^h\bar{p}^h}{1-\beta\delta^p} + \frac{\bar{c}^{-\sigma}\bar{Y}^h\bar{p}^h}{1-\beta\delta^p}\hat{Q}_t + \bar{c}^{-\sigma}\bar{Y}^h\bar{p}^h E_t \sum_{s=0}^{\infty} \beta^s (\delta^p)^s \left\{-\sigma\hat{c}_{t+s} + \hat{Y}_{t+s}^h + \hat{p}_{t+s}^h + \left(\mu^h - 1\right)\left(\hat{P}_{t+s}^h - \hat{P}_t^h\right)\right\}$$

whereas the right hand side can be approximated by

$$\frac{\bar{c}^{-\sigma}\bar{\varphi}^h\bar{Y}^h}{1-\beta\delta^p} + \bar{c}^{-\sigma}\bar{\varphi}^h\bar{Y}^h E_t \sum_{s=0}^{\infty} \beta^s (\delta^p)^s \left\{-\sigma\hat{c}_{t+s} + \hat{\varphi}_{t+s}^h + \hat{Y}_{t+s}^h + \mu^h\left(\hat{P}_{t+s}^h - \hat{P}_t^h\right)\right\}$$

Setting the two equal and cancelling the terms that appear on both sides

$$\hat{Q}_t + \hat{P}_t^h = (1-\beta\delta^p)\, E_t \sum_{s=0}^{\infty} \beta^s (\delta^p)^s \left\{ \hat{\varphi}_{t+s}^h - \hat{p}_{t+s}^h + \hat{P}_{t+s}^h \right\}$$

and expanding forward

$$\hat{Q}_t + \hat{P}_t^h = (1-\beta\delta^p) \left\{ \hat{\varphi}_t^h - \hat{p}_t^h + \hat{P}_t^h \right\} + \beta\delta^p \left(\hat{Q}_{t+1} + \hat{P}_{t+1}^h \right)$$

and rearranging

$$\hat{Q}_t = (1-\beta\delta^p) \left\{ \hat{\varphi}_t^h - \hat{p}_t^h \right\} + \beta\delta^p \left(\hat{Q}_{t+1} + \hat{\pi}_{t+1}^h \right)$$

Finally, using the approximation to Q_t yields

$$\hat{\pi}_t^h = \frac{(1-\beta\delta^p)(1-\delta^p)}{\delta^p} \left\{ \hat{\varphi}_t^h - \hat{p}_t^h \right\} + \beta\hat{\pi}_{t+1}^h$$

■

Propositions 3 and 4

Proposition 3 and Proposition 4 are derived together.

Defining $mpl_t^h = \alpha_1^h y_t^h / n_t^h$ and unsing $\phi_{jt}^h = v_{jt}^h / n_{jt}^h$, firm j's job-creation condition can be approximated by

$$\frac{\varsigma^h}{\bar{\theta}^h} \frac{1}{\beta} \left(\bar{\phi}^h \right)^{\psi^h} \left(\psi^h \hat{\phi}_{jt}^h - \hat{\theta}_t^h \right) = \mathrm{E}_t \left\{ \begin{array}{c} \bar{c}^{-\sigma} \sigma \left(\bar{w}^h - \bar{\varphi}^h \overline{mpl}^h \right) \hat{c}_{t+1} \\ + \bar{c}^{-\sigma} \bar{\varphi}^h \overline{mpl}^h \left(\hat{\varphi}_{t+1}^h + \hat{mpl}_{t+1}^h \right) \\ - \bar{c}^{-\sigma} \bar{w}^h \hat{w}_{jt+1}^h - \left(1-\lambda^h\right) \frac{\varsigma^h}{\bar{\theta}^h} \left(\bar{\phi}^h \right)^{\psi^h} \hat{\theta}_{t+1}^h \\ + \psi^h \varsigma^h \left(\bar{\phi}^h \right)^{\psi^h} \left(\bar{\phi}^h + \frac{\left(1-\lambda^h\right)}{\bar{\theta}^h} \right) \hat{\phi}_{jt+1}^h \end{array} \right\}$$

Given that renegotiating firms are randomly chosen, the real wage in firm j next period is given by

$$\hat{w}_{jt+1}^h = \delta^w \hat{w}_{jt+1|t}^h + (1-\delta^w)\, \hat{w}_{t+1}^{h*}$$

and, equivalently,

$$\hat{\phi}_{jt+1}^h = \delta^w \hat{\phi}_{jt+1|t}^h + (1-\delta^w)\, \hat{\phi}_{t+1}^{h*}$$

Hence, I can rewrite the above approximation as

$$\frac{\varsigma^h}{\bar{\theta}^h}\frac{1}{\beta}\left(\bar{\phi}^h\right)^{\psi^h}\left(\psi^h\hat{\phi}^h_{jt}-\hat{\theta}^h_t\right) = \mathrm{E}_t\left\{\begin{array}{c}\bar{c}^{-\sigma}\sigma\left(\bar{w}^h-\bar{\varphi}^h\overline{mpl}^h\right)\hat{c}_{t+1}\\ +\bar{c}^{-\sigma}\bar{\varphi}^h\overline{mpl}^h\left(\hat{\varphi}^h_{t+1}+\hat{mpl}^h_{t+1}\right)\\ -\bar{c}^{-\sigma}\bar{w}^h\left(\delta^w\hat{w}^h_{jt+1|t}+\left(1-\delta^w\right)\hat{w}^{h*}_{t+1}\right)\\ -\left(1-\lambda^h\right)\frac{\varsigma^h}{\bar{\theta}^h}\left(\bar{\phi}^h\right)^{\psi^h}\hat{\theta}^h_{t+1}\\ +\psi^h\varsigma^h\left(\bar{\phi}^h\right)^{\psi^h}\left(\bar{\phi}^h+\frac{\left(1-\lambda^h\right)}{\bar{\theta}^h}\right)\delta^w\hat{\phi}^h_{jt+1|t}\\ +\psi^h\varsigma^h\left(\bar{\phi}^h\right)^{\psi^h}\left(\bar{\phi}^h+\frac{\left(1-\lambda^h\right)}{\bar{\theta}^h}\right)\left(1-\delta^w\right)\hat{\phi}^{h*}_{t+1}\end{array}\right\}$$

In order to obtain an aggregate job creation condition, I guess that the vacancy rate in firm j depends negatively on its relative wage $\tilde{w}^h_{jt}\equiv logW^h_{jt}-logW^h_t$, that is, guess that

$$\hat{\phi}^h_{jt} = \hat{\phi}^h_t-\tau_z\tilde{w}^h_{jt}$$

which implies

$$\hat{\phi}^h_{jt+1|t} = \hat{\phi}^{h0}_{t+1}-\tau_z\tilde{w}^h_{jt}$$

where $\hat{\phi}^{h0}_{t+1}$ is the average vacancy rate across non-renegotiating firms. Moreover, note that $\hat{w}^h_{jt+1|t}=\hat{w}^{h0}_{t+1}+\tilde{w}^h_{jt}$, where $\hat{w}^{h0}_{t+1}=\hat{w}^h_t+\gamma_w\hat{\pi}^W_t$ is the average real wage across non-negotiating firms. As a result, I have

$$\frac{\varsigma^h}{\bar{\theta}^h}\frac{1}{\beta}\left(\bar{\phi}^h\right)^{\psi^h}\left(\psi^h\hat{\phi}^h_{jt}-\hat{\theta}^h_t\right) = \mathrm{E}_t\left\{\begin{array}{c}\bar{c}^{-\sigma}\sigma\left(\bar{w}^h-\bar{\varphi}^h\overline{mpl}^h\right)\hat{c}_{t+1}\\ +\bar{c}^{-\sigma}\bar{\varphi}^h\overline{mpl}^h\left(\hat{\varphi}^h_{t+1}+\hat{mpl}^h_{t+1}\right)\\ -\bar{c}^{-\sigma}\bar{w}^h\left(\delta^w\left(\hat{w}^{h0}_{t+1}+\tilde{w}^h_{jt}\right)+\left(1-\delta^w\right)\hat{w}^{h*}_{t+1}\right)\\ -\left(1-\lambda^h\right)\frac{\varsigma^h}{\bar{\theta}^h}\left(\bar{\phi}^h\right)^{\psi^h}\hat{\theta}^h_{t+1}\\ +\psi^h\varsigma^h\left(\bar{\phi}^h\right)^{\psi^h}\left(\bar{\phi}^h+\frac{\left(1-\lambda^h\right)}{\bar{\theta}^h}\right)\delta^w\left(\hat{\phi}^{h0}_{t+1}-\tau_z\tilde{w}^h_{jt}\right)\\ +\psi^h\varsigma^h\left(\bar{\phi}^h\right)^{\psi^h}\left(\bar{\phi}^h+\frac{\left(1-\lambda^h\right)}{\bar{\theta}^h}\right)\left(1-\delta^w\right)\hat{\phi}^{h*}_{t+1}\end{array}\right\}$$

Finally, recall that $\hat{\phi}^h_{t+1}=\delta^w\hat{\phi}^{h0}_{t+1}+\left(1-\delta^w\right)\hat{\phi}^{h*}_{t+1}$ and $\hat{w}^h_{t+1}=\delta^w\hat{w}^{h0}_{t+1}+\left(1-\delta^w\right)\hat{w}^{h*}_{t+1}$. Using these,

$$\frac{\varsigma^h}{\bar{\theta}^h}\frac{1}{\beta}\left(\bar{\phi}^h\right)^{\psi^h}\left(\psi^h\hat{\phi}^h_{jt}-\hat{\theta}^h_t\right) = \mathrm{E}_t\left\{\begin{array}{c}\bar{c}^{-\sigma}\sigma\left(\bar{w}^h-\bar{\varphi}^h\overline{mpl}^h\right)\hat{c}_{t+1}+\bar{c}^{-\sigma}\bar{\varphi}^h\overline{mpl}^h\left(\hat{\varphi}^h_{t+1}+\hat{mpl}^h_{t+1}\right)\\ -\bar{c}^{-\sigma}\bar{w}^h\left(\hat{w}^h_{t+1}+\delta^w\tilde{w}^h_{jt}\right)-\left(1-\lambda^h\right)\frac{\varsigma^h}{\bar{\theta}^h}\left(\bar{\phi}^h\right)^{\psi^h}\hat{\theta}^h_{t+1}\\ +\psi^h\varsigma^h\left(\bar{\phi}^h\right)^{\psi^h}\left(\bar{\phi}^h+\frac{\left(1-\lambda^h\right)}{\bar{\theta}^h}\right)\left(\hat{\phi}^h_{t+1}-\delta^w\tau_z\tilde{w}^h_{jt}\right)\end{array}\right\}$$

Averaging across firms and noting that $\mathrm{E}_i \tilde{w}_{it}^h = 0$

$$\frac{\varsigma^h}{\bar{\theta}^h}\frac{1}{\beta}\left(\bar{\phi}^h\right)^{\psi^h}\left(\psi^h\hat{\phi}_t^h - \hat{\theta}_t^h\right) = \mathrm{E}_t\left\{\begin{array}{c}\bar{c}^{-\sigma}\sigma\left(\bar{w}^h - \bar{\varphi}^h\overline{mpl}^h\right)\hat{c}_{t+1} + \bar{c}^{-\sigma}\bar{\varphi}^h\overline{mpl}^h\left(\hat{\varphi}_{t+1}^h + \hat{mpl}_{t+1}^h\right) \\ -\bar{c}^{-\sigma}\bar{w}^h\hat{w}_{t+1}^h - \left(1-\lambda^h\right)\frac{\varsigma^h}{\bar{\theta}^h}\left(\bar{\phi}^h\right)^{\psi^h}\hat{\theta}_{t+1}^h \\ +\psi^h\varsigma^h\left(\bar{\phi}^h\right)^{\psi^h}\left(\bar{\phi}^h + \frac{\left(1-\lambda^h\right)}{\bar{\theta}^h}\right)\hat{\phi}_{t+1}^h\end{array}\right\}$$

which, after rearranging, gives rise to expression (35) in Proposition 4. Subtracting this expression from the first-order approximation to firm's j job creation condition, and using $\hat{\phi}_{jt}^h - \hat{\phi}_t^h = -\tau_z\tilde{w}_{jt}^h$, I obtain

$$\tau_z = \frac{\beta\delta^w\bar{w}^h\lambda^h\bar{c}^{-\sigma}}{\psi^h\varsigma^h\left(\bar{\phi}^h\right)^{1+\psi^h}\left(1-\beta\delta^w\right)}$$

which is used in expression (34) of Proposition 3.■

Proposition 5

I first derive an approximation to $\hat{H}_t^h$. With the definitions used in the demostrations of Proposition 3 and Proposition 4, the log-linear approximation to the worker surplus can be expressed as

$$\bar{H}^h\hat{H}_{it}^h = \bar{w}^h\hat{w}_{it}^h - \bar{\bar{w}}_t^{hw}\hat{\bar{w}}_t^{hw} + \left(1-\lambda^h\right)\beta\bar{H}^h\mathrm{E}_t\left\{\sigma\hat{c}_t - \sigma\hat{c}_{t+1} + \delta^w\hat{H}_{it+1}^{h0} + \left(1-\delta^w\right)\hat{H}_{t+1}^{h*}\right\}$$

As before, I guess that $\hat{H}_{it}^h = \hat{H}_t^h + \tau_{hw}\tilde{w}_{it}^h$.[27] Hence,

$$\bar{H}^h\hat{H}_{it}^h = \bar{w}^h\left(\hat{w}_t^h + \tilde{w}_{it}^h\right) - \bar{\bar{w}}_t^{hw}\hat{\bar{w}}_t^{hw} + \left(1-\lambda^h\right)\beta\bar{H}^h\mathrm{E}_t\left\{\sigma\hat{c}_t - \sigma\hat{c}_{t+1} + \delta^w\tau_{hw}\tilde{w}_{it}^h + \hat{H}_{t+1}^h\right\}$$

Averaging over all firms and subtracting, I obtain

$$\bar{H}^h\tau_{hw}\tilde{w}_{it}^h = \bar{w}^h\tilde{w}_{it}^h + \left(1-\lambda^h\right)\beta\bar{H}^h\delta^w\tau_{hw}\tilde{w}_{it}^h$$

with

$$\tau_{hw} = \frac{\bar{w}^h}{\left(1-\left(1-\lambda^h\right)\beta\delta^w\right)\bar{H}^h}$$

By the same token, log-linearizing the firm surplus yields

$$\bar{J}^h\hat{J}_{it}^h = \bar{\bar{w}}^{fw}\hat{\bar{w}}_{it}^{fw} - \bar{w}^h\hat{w}_{it}^h + \left(1-\lambda^h\right)\beta\bar{J}^h\mathrm{E}_t\left\{\sigma\hat{c}_t - \sigma\hat{c}_{t+1} + \delta^w\hat{J}_{it+1}^{h0} + \left(1-\delta^w\right)\hat{J}_{t+1}^{h*}\right\}$$

[27] This implies $\hat{H}_{it}^{h0} = \hat{H}_t^{h0} + \tau_{hw}\tilde{w}_{it-1}^h$ and $\hat{H}_{t+1}^h = \delta^w\hat{H}_{t+1}^{h0} + \left(1-\delta^w\right)\hat{H}_{t+1}^{h*}$. Note that $\hat{w}_{it}^h = \hat{w}_t^h + \tilde{w}_{it}^h$.

while the log-linearized the expression for $\bar{w}_{it}^{fw}$ is

$$\bar{\bar{w}}^{fw}\hat{\bar{w}}_{it}^{fw} = \bar{\varphi}^h \bar{mpl}_t^h \left(\hat{\varphi}_t^h + \hat{mpl}_t^h\right) + \frac{\psi^h \varsigma^h}{1+\psi^h}\frac{1}{\bar{c}^{-\sigma}}\left(\bar{\phi}^h\right)^{1+\psi^h}\left(\sigma\hat{c}_t + \left(1+\psi^h\right)\hat{\phi}_{it}^h\right)$$

which I can write as

$$\bar{\bar{w}}^{fw}\hat{\bar{w}}_{it}^{fw} = \bar{\bar{w}}^{fw}\hat{\bar{w}}_t^{fw} - \frac{\psi^h \varsigma^h}{\bar{c}^{-\sigma}}\left(\bar{\phi}^h\right)^{1+\psi^h}\tau_z \tilde{w}_{it}^h$$

Again, guessing that $\hat{J}_{it}^h = \hat{J}_t^h - \tau_{fw}\tilde{w}_{it}^h$,[28] I obtain

$$\begin{aligned}\bar{J}^h \hat{J}_{it}^h &= \bar{\bar{w}}^{fw}\hat{\bar{w}}_t^{fw} - \frac{\psi^h \varsigma^h}{\bar{c}^{-\sigma}}\left(\bar{\phi}^h\right)^{1+\psi^h}\tau_z \tilde{w}_{it}^h - \bar{w}^h\left(\hat{w}_t^h + \tilde{w}_{it}^h\right) \\ &\quad + \left(1-\lambda^h\right)\beta\bar{J}^h \mathrm{E}_t\left\{\sigma\hat{c}_t - \sigma\hat{c}_{t+1} - \delta^w\tau_{hf}\tilde{w}_{it}^h + \hat{J}_{t+1}^h\right\}\end{aligned}$$

Averaging over all firms and subtracting, I obtain

$$-\bar{J}^h\tau_{fw}\tilde{w}_{it}^h = -\frac{\psi^h \varsigma^h}{\bar{c}^{-\sigma}}\left(\bar{\phi}^h\right)^{1+\psi^h}\tau_z \tilde{w}_{it}^h - \bar{w}^h\tilde{w}_{it}^h - \left(1-\lambda^h\right)\beta\bar{J}^h\delta^w\tau_{fw}\tilde{w}_{it}^h$$

with

$$\tau_{fw} = \frac{\frac{\psi^h \varsigma^h}{\bar{c}^{-\sigma}}\left(\bar{\phi}^h\right)^{1+\psi^h}\tau_z + \bar{w}^h}{\left(1-\left(1-\lambda^h\right)\beta\delta^w\right)\bar{J}^h}$$

The log-linear approximation of the bargaining rule $\eta^h H_t^{h*} = \left(1-\eta^h\right)J_t^{h*}$ writes $\hat{H}_t^{h*} = \hat{J}_t^{h*}$. Moreover, in steady state, $\eta^h\bar{H}^h = \left(1-\eta^h\right)\bar{J}^h$. Using the guesses from above, I obtain

$$\hat{H}_{t+1}^h = \hat{J}_{t+1}^h - \left(\tau_{hf} + \tau_{hw}\right)\left(logW_{t+1}^{h*} - logW_{t+1}^h\right)$$

On the other hand, the log-linear approximation of the first-order condition with respect to vacancies writes

$$\left(1-\epsilon^h\right)\hat{\theta}_t^h + \psi^h\hat{\phi}_{it}^h = \mathrm{E}_t\left\{-\sigma\hat{c}_{t+1} + \hat{J}_{it+1}^h\right\}$$

Using $\hat{J}_{it}^h = \hat{J}_t^h - \tau_{hf}\tilde{w}_{it}^h$ and averaging yields

$$\left(1-\epsilon^h\right)\hat{\theta}_t^h + \psi^h\hat{\phi}_t^h = \mathrm{E}_t\left\{-\sigma\hat{c}_{t+1} + \hat{J}_{t+1}^h\right\}$$

[28] This implies $\hat{J}_{it}^{h0} = \hat{J}_t^{h0} - \tau_{fw}\tilde{w}_{it-1}^h$ and $\hat{J}_{t+1}^h = \delta^w\hat{J}_{t+1}^{h0} + \left(1-\delta^w\right)\hat{J}_{t+1}^{h*}$.

Using these expresions and $logW_{t+1}^{h*} - logW_{t+1}^{h}$ derived above, I write

$$\mathrm{E}_t\left\{-\sigma\hat{c}_{t+1} + \hat{H}_{t+1}^h\right\} = \left(1-\epsilon^h\right)\hat{\theta}_t^h + \psi^h\hat{\phi}_t^h - \left(\tau_{fw}+\tau_{hw}\right)\frac{\delta^w}{(1-\delta^w)}\left(\hat{\pi}_{t+1}^{wh} - \gamma_w\hat{\pi}_t^W\right)$$

Finally, expression (16) for the target wage can be approximated by

$$\begin{aligned}\bar{w}^h\hat{w}_{jt}^{h\,tar} &= \frac{\eta^h\kappa^h\sigma}{\bar{c}^{-\sigma}}\hat{c}_t + \eta^h\beta\bar{\iota}^h\bar{H}^h\left(\epsilon^h\hat{\theta}_t^h + \sigma\hat{c}_t - \sigma\hat{c}_{t+1} + \hat{H}_{t+1}^h\right)\\ &+\left(1-\eta^h\right)\left(\bar{\varphi}^h\overline{mpl}^h\left(\hat{\varphi}_t^h + \hat{mpl}_t^h\right) + \frac{\psi^h\varsigma^h}{1+\psi^h}\frac{1}{\bar{c}^{-\sigma}}\left(\bar{\phi}^h\right)^{1+\psi^h}\left(\sigma\hat{c}_t + \left(1+\psi^h\right)\hat{\phi}_{jt}^h\right)\right)\end{aligned}$$

Averaging and replacing $\hat{H}_{t+1}$, I obtain[29]

$$\bar{w}^h\hat{w}_t^{h\,tar} = \left(1-\eta^h\right)\left(\begin{array}{c}\left(\left(\frac{\bar{\theta}^h}{\bar{\phi}^h} + \frac{\psi^h}{1+\psi^h}\right)\varsigma^h\left(\bar{\phi}^h\right)^{1+\psi^h} + \frac{\eta^h\kappa^h}{\left(1-\eta^h\right)}\right)\frac{\sigma}{\bar{c}^{-\sigma}}\hat{c}_t\\ +\bar{\varphi}^h\overline{mpl}^h\left(\hat{\varphi}_t^h + \hat{mpl}_t^h\right) + \left(1+\frac{\bar{\theta}^h}{\bar{\phi}^h}\right)\frac{\psi^h\varsigma^h}{\bar{c}^{-\sigma}}\left(\bar{\phi}^h\right)^{1+\psi^h}\hat{\phi}_t^h\\ +\frac{\varsigma^h}{\bar{c}^{-\sigma}}\bar{\theta}^h\left(\bar{\phi}^h\right)^{\psi^h}\left(\hat{\theta}_t^h - \left(\tau_{fw}+\tau_{hw}\right)\frac{\delta^w}{(1-\delta^w)}\left(\hat{\pi}_{t+1}^{wh} - \gamma_h\hat{\pi}_t^W\right)\right)\end{array}\right)$$

Lastly, using a first-order approximation to the Nash wage, I can write[30]

$$\hat{w}_t^{h\,tar} = \hat{w}_t^{h\,nash} - \frac{\left(1-\beta\delta^w\left(1+\left(1-\eta^h\right)\lambda^h\right)\right)\bar{\iota}^h\beta}{\left(1-\beta\delta^w\right)\left(1-\left(1-\lambda^h\right)\beta\delta^w\right)}\frac{\delta^w}{(1-\delta^w)}\left(\hat{\pi}_{t+1}^{wh} - \gamma_w\hat{\pi}_t^W\right)$$

which is equivalent to expression (37) in Proposition 5■

Proposition 6

A log-linear approximation to the wage equation (15) can be expressed by

$$0 = \mathrm{E}_t\sum_{s=0}^{\infty}\beta^s\left(1-\lambda^h\right)^s(\delta^w)^s\left[logW_{it}^{h*} - logP_{t+s} - log\bar{w}^h - \hat{w}_{it+s|t}^{h\,tar} + \gamma_h log\left(\prod_{j=1}^{s}\left(\pi_{t+j-1}^W\right)\right)\right]$$

[29] And using $\bar{H}^h = \bar{J}^h\left(1-\eta^h\right)/\eta^h$ and $\bar{J}^h = \varsigma^h\left(\bar{\phi}^h\right)^{\psi^h}/\beta\bar{q}^h\bar{c}^{-\sigma}$.

[30] The Nash wage is given by

$$w_t^{h\,nash} = \eta^h\left(\frac{\kappa^h}{c_t^{-\sigma}} + b\right) + \left(1-\eta^h\right)\left(\varphi_t^h mpl_t^h + \frac{\psi^h\varsigma^h}{1+\psi^h}\frac{1}{c_t^{-\sigma}}\left(\phi_t^h\right)^{1+\psi^h} + \frac{\varsigma^h}{c_t^{-\sigma}}\theta_t^h\left(\phi_t^h\right)^{\psi^h}\right)$$

which to a first-order log-linear approximation writes

$$\hat{w}_t^{h\,nash} = \frac{\left(1-\eta^h\right)}{\bar{w}^h}\left(\begin{array}{c}\left(\left(\frac{\bar{\theta}^h}{\bar{\phi}^h} + \frac{\psi^h}{1+\psi^h}\right)\varsigma^h\left(\bar{\phi}^h\right)^{1+\psi^h} + \frac{\eta^h\kappa^h}{\left(1-\eta^h\right)}\right)\frac{\sigma}{\bar{c}^{-\sigma}}\hat{c}_t\\ +\bar{\varphi}^h\bar{mpl}^h\left(\hat{\varphi}_t^h + \hat{mpl}_t^h\right) + \left(1+\frac{\bar{\theta}^h}{\bar{\phi}^h}\right)\frac{\psi^h\varsigma^h}{\bar{c}^{-\sigma}}\left(\bar{\phi}^h\right)^{1+\psi^h}\hat{\phi}_t^h\\ +\frac{\varsigma^h}{\bar{c}^{-\sigma}}\bar{\theta}^h\left(\bar{\phi}^h\right)^{\psi^h}\hat{\theta}_t^h\end{array}\right)$$

From the wage target equation (16), the only term which is not common to all firms is the vacancy rate ϕ_{it}^h. Using the derivations in Proposition 5, for a firm that has not changed its nominal wage since period t, $\hat{w}_{it+s|t}^{h\,tar}$ is given by

$$\bar{w}^h \hat{w}_{it+s|t}^{h\,tar} = \bar{w}^h \hat{w}_{t+s}^{h\,tar} + \left(1-\eta^h\right)\psi^h \varsigma^h \frac{1}{\bar{c}^{-\sigma}} \left(\bar{\phi}^h\right)^{1+\psi^h} \left(\hat{\phi}_{it+s|t}^h - \hat{\phi}_{t+s}^h\right)$$

By the definition of $\hat{\phi}_{it}^h = \hat{\phi}_t^h - \tau_z \left(logW_{it}^h - logW_t^h\right)$, I can write

$$\hat{\phi}_{it+s|t}^h = \hat{\phi}_{t+s}^h - \tau_z \left(logW_{it}^{h*} + \gamma_w log \left(\prod_{j=1}^{s} \left(\pi_{t+j-1}^W\right) \right) - logW_{t+s}^h \right)$$

and hence

$$\hat{w}_{it+s|t}^{h\,tar} = \hat{w}_{t+s}^{h\,tar} - \phi \left(logW_{it}^{h*} + \gamma_w log \left(\prod_{j=1}^{s} \left(\pi_{t+j-1}^W\right) \right) - logW_{t+s}^h \right)$$

where $\phi = \left(1-\eta^h\right)\psi^h \varsigma^h \left(\bar{\phi}^h\right)^{1+\psi^h} \tau_z / \bar{w}^h \bar{c}^{-\sigma}$. Inserting this expression in the log-linear wage equation yields

$$0 = \mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \left(1-\lambda^h\right)^s (\delta^w)^s \left[\begin{array}{c} (1+\phi)\left(logW_{it}^{h*} - logW_{t+s}^h\right) + \hat{w}_{t+s}^h - \hat{w}_{t+s}^{h\,tar} \\ + (1+\phi)\,\gamma_w log\left(\prod_{j=1}^{s}\left(\pi_{t+j-1}^W\right)\right) \end{array} \right]$$

As all firms negotiating a new contract strike the same nominal wage, solving for W_t^{h*} and expading forward

$$\begin{aligned} \left(logW_t^{h*} - logW_t^h\right) &= \frac{1-\beta\left(1-\lambda^h\right)(\delta^w)}{1+\phi}\left(\hat{w}_t^{h\,tar} - \hat{w}_t^h\right) \\ &\quad -\beta\left(1-\lambda^h\right)(\delta^w)\,\gamma_w \hat{\pi}_t^W + \beta\left(1-\lambda^h\right)(\delta^w)\left(logW_{t+1}^{h*} - logW_t^h\right) \end{aligned}$$

The final step consists of log-linearizing expression (36) and replacing $logW_t^{h*} - logW_t^h$, which gives

$$\begin{aligned} \hat{\pi}_t^{wh} - \gamma_w \hat{\pi}_{t-1}^W &= \frac{1-\beta\left(1-\lambda^h\right)(\delta^w)}{1+\phi}\frac{1-\delta^w}{\delta^w}\left(\hat{w}_t^{h\,tar} - \hat{w}_t^h\right) \\ &\quad -\beta\left(1-\lambda^h\right)(1+\delta^w)\,\gamma_w \hat{\pi}_t^W + \beta\left(1-\lambda^w\right)\hat{\pi}_{t+1}^{wh} \end{aligned}$$

To obtain expression (38), I replace $\hat{w}_t^{h\,tar}$ using Proposition 5. ■

Proposition 7

A first-order approximation of expression (11) yields

$$\epsilon^h \hat{\theta}_t^h + \mathrm{E}_t \left\{ -\sigma \hat{c}_{t+1} + \hat{H}_{t+1}^h \right\} \quad = \quad \epsilon^x \hat{\theta}_t^x + \mathrm{E}_t \left\{ -\sigma \hat{c}_{t+1} + \hat{H}_{t+1}^x \right\}$$

From before, the left-hand side of this expression is equal to

$$\mathrm{E}_t \left\{ -\sigma \hat{c}_{t+1} + \hat{H}_{t+1}^h \right\} \quad = \quad \left(1 - \epsilon^h\right) \hat{\theta}_t^h + \psi^h \hat{\phi}_t^h - \left(\tau_{fw} + \tau_{hw}\right) \frac{\delta^w}{(1 - \delta^w)} \left(\hat{\pi}_{t+1}^{wh} - \gamma_w \hat{\pi}_t^W \right)$$

Using the fact that $\mathrm{E}_t \left\{ \beta c_{t+1}^{-\sigma} H_{t+1}^x \right\} = \varsigma^x \left(\theta_t^x\right)^{1-\epsilon^x} \left(\phi_t^x\right)^{\psi^x} \varsigma^x \left(1 - \eta^x\right) / \eta^x$, the right-hand side of (11) can be approximated by

$$\mathrm{E}_t \left\{ -\sigma \hat{c}_{t+1} + \hat{H}_{t+1}^x \right\} \quad = \quad \left(1 - \epsilon^x\right) \hat{\theta}_t^x + \psi^x \hat{\phi}_t^x$$

Using these expressions and replacing $\left(\tau_{fw} + \tau_{hw}\right)$, I obtain

$$\hat{\theta}_t^h + \psi^h \hat{\phi}_t^h - \frac{\chi^h \left(\theta^h\right)^{\epsilon^h - 1} \beta \bar{c}^{-\sigma} \left(1 + \Lambda\right) \bar{w}^h}{\left(1 - \left(1 - \lambda^h\right) \beta \delta^w\right) \left(1 - \eta^h\right) \varsigma^h \left(\bar{\phi}^h\right)^{\psi^h}} \frac{\delta^h}{\left(1 - \delta^h\right)} \left(\hat{\pi}_{t+1}^{wh} - \gamma_w \hat{\pi}_t^W \right) \quad = \quad \hat{\theta}_t^x + \psi^x \hat{\phi}_t^x$$

■

Proposition 8

Welfare criterion: The welfare criterion is given by

$$\tilde{\mathcal{W}} \quad = \quad \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1 - \sigma} - \kappa^h n_t^h - \kappa^x n_t^x - \frac{\varsigma^h}{1 + \psi^h} \int_0^1 \left(\phi_{it}^h\right)^{1+\psi^h} n_{it}^h di - \frac{\varsigma^x}{1 + \psi^x} \left(\phi_t^x\right)^{1+\psi^x} n_t^x \right\}$$

Second order approximations: The second-order log-linear approximation to the first term of $\tilde{\mathcal{W}}$ can be expressed by

$$\frac{c_t^{1-\sigma}}{1 - \sigma} \quad = \quad \bar{c}^{1-\sigma} \left(\hat{c}_t + \frac{1 - \sigma}{2} \hat{c}_t^2 \right) + t.i.p. + \mathcal{O}^3$$

whereas for labour disutility I write

$$\kappa^h n_t^h + \kappa^x n_t^x \quad = \quad \kappa^h \bar{n}^h \left(\hat{n}_t^h + \frac{1}{2} \left(\hat{n}_t^h\right)^2 \right) + \kappa^x \bar{n}^x \left(\hat{n}_t^x + \frac{1}{2} \left(\hat{n}_t^x\right)^2 \right) + t.i.p. + \mathcal{O}^3$$

For the aggregate costs of vacancy posting, I first write

$$\frac{\varsigma^h}{1+\psi^h}\left(\phi_{it}^h\right)^{1+\psi^h} n_{it}^h = \frac{\varsigma^h}{1+\psi^h}\left(\bar{\phi}^h\right)^{1+\psi^h} \bar{n}^h \left(\begin{array}{c} \left(1+\psi^h\right)\hat{\phi}_{it}^h + \hat{n}_{it}^h \\ +\left(1+\psi^h\right)\hat{n}_{it}^h\hat{\phi}_{it}^h \\ +\frac{1}{2}\left(\left(1+\psi^h\right)^2\left(\hat{\phi}_{it}^h\right)^2 + \left(\hat{n}_{it}^h\right)^2\right) \end{array} \right) + t.i.p. + \mathcal{O}^3$$

Moreover, I define $n_t^h \equiv \int_0^1 n_{it}^h di$, which can be approximated by[31]

$$\hat{n}_t^h = \mathrm{E}_i\hat{n}_{it}^h + \frac{1}{2}\mathrm{Var}_i\hat{n}_{it}^h + \mathcal{O}^3$$

as well as $\phi_t^h \equiv \int_0^1 \frac{n_{it}^h}{n_t^h}\phi_{it}^h di$, for which I write[32]

$$\hat{\phi}_t^h = \mathrm{E}_i\hat{\phi}_{it}^h + \frac{1}{2}\mathrm{Var}_i\hat{\phi}_{it}^h - \hat{n}_t^h\hat{\phi}_t^h + \mathrm{E}_i\hat{n}_{it}^h\hat{\phi}_{it}^h + \mathcal{O}^3$$

Finally, note that $\hat{\phi}_t^h = \hat{v}_t^h - \hat{n}_t^h$. Combining these expressions, I obtain

$$\frac{\varsigma^h}{1+\psi^h}\int_0^1\left(\phi_{it}^h\right)^{1+\psi^h} n_{it}^h di = \frac{\varsigma^h}{1+\psi^h}\left(\bar{\phi}^h\right)^{1+\psi^h} \bar{n}^h \left(\begin{array}{c} \left(1+\psi^h\right)\hat{v}_t^h - \psi^h\hat{n}_t^h \\ +\frac{1}{2}\left[\left(1+\psi^h\right)\hat{v}_t^h + \psi^h\hat{n}_t^h\right]^2 \\ +\frac{1}{2}\psi^h\left(1+\psi^h\right)\mathrm{Var}_i\hat{\phi}_{it}^h \end{array} \right) + t.i.p. + \mathcal{O}^3$$

The last term in $\tilde{\mathcal{W}}$ can be approximated by

$$\frac{\varsigma^x}{1+\psi^x}\left(\phi_t^x\right)^{1+\psi^x} n_t^x = \frac{\varsigma^x}{1+\psi^x}\left(\bar{\phi}^x\right)^{1+\psi^x} \bar{n}^x \left(\begin{array}{c} \left(1+\psi^x\right)\hat{v}_t^x - \psi^x\hat{n}_t^x \\ +\frac{1}{2}\left[\left(1+\psi^x\right)\hat{v}_t^x - \psi^x\hat{n}_t^x\right]^2 \end{array} \right) + t.i.p. + \mathcal{O}^3$$

Inserting all the above derivations into the period welfare flow and normalizing by $\overline{rgdp}$ yields

$$\frac{\tilde{\mathcal{W}}}{\overline{rgdp}\bar{c}^{-\sigma}} = \sum_{t=0}^{\infty}\beta^t \left\{ \begin{array}{c} Q_3\hat{c}_t + \left(\psi^h\varrho^h - \zeta^h\right)\hat{n}_t^h - \left(1+\psi^h\right)\varrho^h\hat{v}_t^h \\ +\left(\psi^x\varrho^x - \zeta^x\right)\hat{n}_t^x - \left(1+\psi^x\right)\varrho^x\hat{v}_t^x \\ -\frac{1}{2}\left[\begin{array}{c} \zeta^h\left(\hat{n}_t^h\right)^2 + \zeta^x\left(\hat{n}_t^x\right)^2 - Q_3\left(1-\sigma\right)\hat{c}_t^2 \\ +\varrho^h\left(\left[\left(1+\psi^h\right)\hat{v}_t^h + \psi^h\hat{n}_t^h\right]^2 + \psi^h\left(1+\psi^h\right)\mathrm{Var}_i\hat{\phi}_{it}^h\right) \\ +\varrho^x\left[\left(1+\psi^x\right)\hat{v}_t^x - \psi^x\hat{n}_t^x\right]^2 \end{array} \right] \end{array} \right\}$$
$$+t.i.p. + \mathcal{O}^3$$

To replace the linear terms in the above expression for quadratic terms, I use second-order approximations of some of the equilibrium conditions.

[31] With $\mathrm{E}_i\hat{n}_{it}^h = \int_0^1 \hat{n}_{it}^h di$, $\left(\mathrm{E}_i\hat{n}_{it}^h\right)^2 = \left(\hat{n}_t^h\right)^2 + \mathcal{O}^3$ and $\mathrm{Var}_i\hat{n}_{it}^h = \mathrm{E}_i\left(\hat{n}_{it}^h\right)^2 - \left(\mathrm{E}_i\hat{n}_{it}^h\right)^2$.

[32] With $\mathrm{E}_i\hat{\phi}_{it}^h = \int_0^1 \hat{\phi}_{it}^h di$, $\left(\mathrm{E}_i\hat{\phi}_{it}^h\right)^2 = \left(\hat{\phi}_{it}^h\right)^2 + \mathcal{O}^3$ and $\mathrm{Var}_i\hat{\phi}_{it}^h = \mathrm{E}_i\left(\hat{\phi}_{it}^h\right)^2 - \left(\mathrm{E}_i\hat{\phi}_{it}^h\right)^2$.

Labour market: A second-order approximation to the law of motion of sectoral employment writes

$$\begin{aligned}\hat{n}_{t+1}^{s}+\frac{1}{2}\left(\hat{n}_{t+1}^{s}\right)^{2} &= \lambda^{s}\epsilon^{s}\hat{v}_{t}^{s}+\lambda^{s}\left(1-\epsilon^{s}\right)\hat{u}_{t}^{s}+\frac{\lambda^{s}}{2}\left[\epsilon^{s}\hat{v}_{t}^{s}+\left(1-\epsilon^{s}\right)\hat{u}_{t}^{s}\right]^{2} \\ &\quad +\left(1-\lambda^{s}\right)\left(\hat{n}_{t}^{s}+\frac{1}{2}\left(\hat{n}_{t}^{s}\right)^{2}\right)+\mathcal{O}^{3}\end{aligned}$$

for $s=h,\ x$. Multiplying by β^{t}, summing across t, and noting that $\hat{n}_{0}$ is independent of policy

$$\begin{gathered}\sum_{t=0}^{\infty}\beta^{t}\left[\left(\tfrac{1}{\beta}-\left(1-\lambda^{s}\right)\right)\hat{n}_{t}^{s}-\lambda^{s}\epsilon^{s}\hat{v}_{t}^{s}-\lambda^{s}\left(1-\epsilon^{s}\right)\hat{u}_{t}^{s}\right] \\ = \\ \sum_{t=0}^{\infty}\beta^{t}\tfrac{1}{2}\left[\lambda^{s}\left[\epsilon^{s}\hat{v}_{t}^{s}+\left(1-\epsilon^{s}\right)\hat{u}_{t}^{s}\right]^{2}-\left(\tfrac{1}{\beta}-\left(1-\lambda^{s}\right)\right)\left(\hat{n}_{t}^{s}\right)^{2}\right]+t.i.p.+\mathcal{O}^{3}\end{gathered}$$

A second-order approximation of the labour market aggregate constraint, $1=n_{t}^{s}+n_{t}^{-s}+u_{t}^{s}+u_{t}^{-s}$, writes

$$\begin{aligned}\hat{u}_{t}^{s} &= -\frac{\bar{n}^{s}}{\bar{u}^{s}}\hat{n}_{t}^{s}-\frac{\bar{n}^{-s}}{\bar{u}^{s}}\hat{n}_{t}^{-s}-\frac{\bar{u}^{-s}}{\bar{u}^{s}}\hat{u}_{t}^{-s} \\ &\quad -\frac{1}{2}\left[\frac{\bar{n}^{s}}{\bar{u}^{s}}\left(\hat{n}_{t}^{s}\right)^{2}+\frac{\bar{n}^{-s}}{\bar{u}^{s}}\left(\hat{n}_{t}^{-s}\right)^{2}+\left(\hat{u}_{t}^{s}\right)^{2}+\frac{\bar{u}^{-s}}{\bar{u}^{s}}\left(\hat{u}_{t}^{-s}\right)^{2}\right]\end{aligned}$$

I use these three approximations, the Hoisos condition, $\eta^{s}=\epsilon^{s}$, the exact conditions $\hat{\theta}_{t}^{s}=\hat{v}_{t}^{s}-\hat{u}_{t}^{s}$ and $\hat{\phi}_{t}^{s}=\hat{v}_{t}^{s}-\hat{n}_{t}^{s}$, the sectoral job-creation conditions in the steady state

$$\frac{1}{\beta}-\left(1-\lambda^{s}\right)+\left(1-\eta^{s}\right)\lambda^{s}\frac{\bar{\theta}^{s}}{\bar{\phi}^{s}} = \frac{\eta^{s}\lambda^{s}}{\left(1+\psi^{s}\right)}\left(\varrho^{s}\right)^{-1}\left[\alpha_{1}^{s}Q_{2}+\psi^{s}\varrho^{s}-\zeta^{s}\right]$$

and the no-arbitrage condition in the steady state

$$\frac{1-\eta^{h}}{\eta^{h}}\left(1+\psi^{h}\right)\frac{\varrho^{h}}{\bar{u}^{h}} = \frac{1-\eta^{x}}{\eta^{x}}\left(1+\psi^{x}\right)\frac{\varrho^{x}}{\bar{u}^{x}}$$

to obtain

$$\begin{aligned}\frac{\tilde{\mathcal{W}}}{\overline{rgdp}\bar{c}^{-\sigma}} &= \sum_{t=0}^{\infty}\beta^{t}\left\{\begin{array}{c} Q_{3}\hat{c}_{t}-\alpha_{1}^{h}Q_{1}\hat{n}_{t}^{h}-\alpha_{1}^{x}Q_{2}\hat{n}_{t}^{x} \\ -\frac{1}{2}\left[\begin{array}{c}\alpha_{1}^{h}Q_{1}\left(\hat{n}_{t}^{h}\right)^{2}+\alpha_{1}^{x}Q_{2}\left(\hat{n}_{t}^{x}\right)^{2}-Q_{3}\left(1-\sigma\right)\hat{c}_{t}^{2} \\ +\varrho^{h}\left(1+\psi^{h}\right)\left(\left(1-\epsilon^{h}\right)\left(\hat{\theta}_{t}^{h}\right)^{2}+\psi^{h}\left(\hat{\phi}_{t}^{h}\right)^{2}+\psi^{h}\mathrm{Var}_{i}\hat{\phi}_{it}^{h}\right) \\ +\varrho^{x}\left(1+\psi^{x}\right)\left(\left(1-\epsilon^{i}\right)\left(\hat{\theta}_{t}^{i}\right)^{2}+\psi^{x}\left(\hat{\phi}_{t}^{x}\right)^{2}\right)\end{array}\right]\end{array}\right\} \\ &\quad +t.i.p.+\mathcal{O}^{3}\end{aligned}$$

Resource Constraints: Combining the aggregate resource constraint and the definition of trade-balance, I have

$$\Upsilon_t^h \left(tb_t - p_t^x x_t^f + c_t \right) = p_t^h \alpha^h Z_t \left(n_t^h \right)^{\alpha_1^h} \left(x_t^h \right)^{1-\alpha_1^h}$$

with Υ_t^h the index of price dispersion. Defining $\Delta_t = tb_t - p_t^x x_t^f + c_t$, the following approximation

$$Q_1 \hat{\Upsilon}_t^h + Q_1 \hat{\Delta}_t = Q_1 \hat{p}_t^h + \alpha_1^h Q_1 \hat{n}_t^h + \left(1 - \alpha_1^h\right) Q_1 \hat{x}_t^h + t.i.p.$$

is exact. In turn, a second-order approximation of Δ_t yields

$$\begin{aligned} Q_1 \hat{\Delta}_t &= \hat{tb}_t^h - (1 - Q_1) \left(\hat{p}_t^x + \hat{x}_t^f \right) + Q_3 \hat{c}_t \\ &\quad + \frac{1}{2} \left[\left(\hat{tb}_t^h \right)^2 - (1 - Q_1) \left(\hat{p}_t^x + \hat{x}_t^f \right)^2 + Q_3 \left(\hat{c}_t \right)^2 - (Q_1)^3 \left(\hat{y}_t^h \right)^2 \right] + \mathcal{O}^3 \end{aligned}$$

where I used the fact that $Q_1 \hat{y}_t^h = Q_1 \hat{p}_t^h + \alpha_1^h Q_1 \hat{n}_t^h + \left(1 - \alpha_1^h\right) Q_1 \hat{x}_t^h$ is an exact approximation.

The resource constraint in the commodity sector, $A_t \left(n_t^x\right)^{\alpha_1^x} = x_t^h + x_t^f$, admits the following approximation

$$\begin{gathered} \alpha_1^x Q_2 \hat{n}_t^x + \tfrac{1}{2} \left[Q_2 \left(\alpha_1^x\right)^2 \left(\hat{n}_t^x\right)^2 + 2\alpha_1^x Q_2 \hat{A}_t \hat{n}_t^x \right] \\ = \\ (Q_2 + Q_1 - 1) \hat{x}_t^h + (1 - Q_1) \hat{x}_t^f + \tfrac{1}{2} \left[(Q_2 + Q_1 - 1) \left(\hat{x}_t^h\right)^2 + (1 - Q_1) \left(\hat{x}_t^f\right)^2 \right] + t.i.p. + \mathcal{O}^3 \end{gathered}$$

In the steady state $\left(1 - \alpha_1^h\right) Q_1 = Q_2 + Q_1 - 1$. Combining these expressions, the approximation to the aggregate resource constraint can be written as

$$\begin{aligned} Q_3 \hat{c}_t - \alpha_1^h Q_1 \hat{n}_t^h - \alpha_1^x Q_2 \hat{n}_t^x &= Q_1 \hat{p}_t^h + (1 - Q_1) \hat{p}_t^x - \hat{tb}_t^h - Q_1 \hat{\Upsilon}_t^h \\ &\quad + \frac{1}{2} \begin{bmatrix} Q_2 \left(\alpha_1^x\right)^2 \left(\hat{n}_t^x\right)^2 + 2\alpha_1^x Q_2 \hat{A}_t \hat{n}_t^x \\ - (Q_2 + Q_1 - 1) \left(\hat{x}_t^h\right)^2 - Q_3 \left(\hat{c}_t\right)^2 \\ + (Q_1)^3 \left(\hat{y}_t^h\right)^2 - \left(\hat{tb}_t^h\right)^2 \\ + (1 - Q_1) \left(\left(\hat{p}_t^x + \hat{x}_t^f\right)^2 - \left(\hat{x}_t^f\right)^2 \right) \end{bmatrix} + t.i.p. + \mathcal{O}^3 \end{aligned}$$

Therefore I can write

$$\frac{\tilde{\mathcal{W}}}{\overline{rgdp}\bar{c}^{-\sigma}} = \sum_{t=0}^{\infty}\beta^t \left\{ -\frac{1}{2}\left[\begin{array}{c} Q_1\hat{p}_t^h + (1-Q_1)\,\hat{p}_t^x - \hat{tb}_t^h - Q_1\hat{\Upsilon}_t^h \\ \alpha_1^h Q_1\left(\hat{n}_t^h\right)^2 + \alpha_1^x\left(1-\alpha_1^x\right)Q_2\left(\hat{n}_t^x\right)^2 - 2\alpha_1^x Q_2\hat{n}_t^x\hat{A}_t \\ +\sigma Q_3\hat{c}_t^2 + \left(\hat{tb}_t^h\right)^2 - (1-Q_1)\left(\left(\hat{p}_t^x+\hat{x}_t^f\right)^2 - \left(\hat{x}_t^f\right)^2\right) \\ -(Q_1)^3\left(\hat{y}_t^h\right)^2 + (Q_2+Q_1-1)\left(\hat{x}_t^h\right)^2 \\ +\varrho^h\left(1+\psi^h\right)\left(\left(1-\epsilon^h\right)\left(\hat{\theta}_t^h\right)^2 + \psi^h\left(\hat{\phi}_t^h\right)^2 + \psi^h \mathrm{Var}_i\hat{\phi}_{it}^h\right) \\ +\varrho^x\left(1+\psi^x\right)\left(\left(1-\epsilon^i\right)\left(\hat{\theta}_t^i\right)^2 + \psi^x\left(\hat{\phi}_t^x\right)^2\right) \end{array}\right] \right\} + t.i.p. + \mathcal{O}^3$$

NFA and exchange rate: Using the law of motion of net foreign assents, $s_t b_{t-1} = -tb_t + s_t b_t \pi_{t+1}^*/i_t^*$, and the UIP condition, I obtain $s_t c_t^{-\sigma} b_{t-1} = -tb_t c_t^{-\sigma} + \beta s_{t+1} c_{t+1}^{-\sigma} b_t$. Integrating in time and performing a second-order approximation yields

$$\sum_{t=0}^{\infty}\beta^t\left[-\hat{tb}_t\right] = \sum_{t=0}^{\infty}\beta^t\left[-\sigma\left(1-Q_3\right)\hat{c}_t + \frac{1}{2}\left[\sigma^2\left(1-Q_3\right)\left(\hat{c}_t\right)^2 - 2\sigma\hat{c}_t\hat{tb}_t + \left(\hat{tb}_t\right)^2\right]\right] + t.i.p. + \mathcal{O}^3$$

Assuming complete exchange rate pass-through, and given that open economy is small, I define $P_t^{f*} = P_t^*$. This allows me to use the definition of the CPI index to express p_t^h as a function of the real exchange rate, which has the following second-order approximation

$$\hat{p}_t^h = -\frac{1-\alpha^c}{\alpha^c}\hat{s}_t - \left(1-\gamma^c\right)\frac{1}{2}\left[\frac{1-\alpha^c}{\alpha^c}\left(\hat{s}_t\right)^2 + \left(\hat{p}_t^h\right)^2\right] + \mathcal{O}^3$$

Note also that $\hat{p}_t^x = \hat{p}_t^{x*} + \hat{s}_t$ is an exact approximation to the price of commodities in domestic currency. Replacing $\hat{tb}_t$, $\hat{p}_t^h$ and $\hat{p}_t^x$ in the approximation to the welfare criteria

yields

$$\frac{\tilde{\mathcal{W}}}{\overline{rgdp}\bar{c}^{-\sigma}} = \sum_{t=0}^{\infty}\beta^t \left\{ -\frac{1}{2}\left[\begin{array}{c} \left(1-\left(1+\frac{1-\alpha^c}{\alpha^c}\right)Q_1\right)\hat{s}_t - \sigma\left(1-Q_3\right)\hat{c}_t - Q_1\hat{\Upsilon}_t^h \\ \left(1-\gamma^c\right)\frac{1-\alpha^c}{\alpha^c}Q_1\left(\hat{s}_t\right)^2 \\ -\left(\sigma-\left(1+\sigma\right)Q_3\right)\sigma\hat{c}_t^2 + 2\sigma\hat{c}_t\hat{tb}_t + \left(1-\gamma^c\right)Q_1\left(\hat{p}_t^h\right)^2 \\ +\alpha_1^h Q_1\left(\hat{n}_t^h\right)^2 + \alpha_1^x\left(1-\alpha_1^x\right)Q_2\left(\hat{n}_t^x\right)^2 - 2\alpha_1^x Q_2\hat{n}_t^x\hat{A}_t \\ -\left(1-Q_1\right)\left(\left(\hat{p}_t^x+\hat{x}_t^f\right)^2 - \left(\hat{x}_t^f\right)^2\right) \\ -\left(Q_1\right)^3\left(\hat{y}_t^h\right)^2 + \left(Q_2+Q_1-1\right)\left(\hat{x}_t^h\right)^2 \\ +\varrho^h\left(1+\psi^h\right)\left(\left(1-\epsilon^h\right)\left(\hat{\theta}_t^h\right)^2 + \psi^h\left(\hat{\phi}_t^h\right)^2 + \psi^h \mathrm{Var}_i\hat{\phi}_{it}^h\right) \\ +\varrho^x\left(1+\psi^x\right)\left(\left(1-\epsilon^i\right)\left(\hat{\theta}_t^i\right)^2 + \psi^x\left(\hat{\phi}_t^x\right)^2\right) \end{array}\right] \right\}$$
$$+t.i.p. + \mathcal{O}^3$$

Furthermore, assuming $Q_3 = 1$ and $\alpha^c = Q_1$, I obtain expression (42) in Proposition 8, except for the quadratic terms in price and wage inflation. These are derived next.

Quadratic terms in price and wage inflation: These derivations follow Woodford (2003) and Thomas (2008). As defined in the main text,$\Upsilon_t^h = \int_0^1 \left(P_{it}^h/P_t^h\right)^{-\mu} di$. Up to a second order of approximation, I can write

$$\hat{\Upsilon}_t^h = -\mu\left(\mathrm{E}_i\tilde{p}_{it}^h - \frac{\mu}{2}\mathrm{E}_i\left(\tilde{p}_{it}^h\right)^2\right) + \mathcal{O}^3$$

where $\tilde{p}_{it}^h = log\left(P_{it}^h/P_t^h\right)$ and noting that $\left(\hat{\Upsilon}_t^h\right)^2$ is $\mathcal{O}^4$. In turn, approximating $\left(P_t^h\right)^{1-\mu} = \int_0^1 \left(P_{it}^h\right)^{1-\mu} di$ up to second order yields

$$\mathrm{E}_i\tilde{p}_{it}^h = -\frac{1-\mu}{2}\mathrm{E}_i\left(\tilde{p}_{it}^h\right)^2 + \mathcal{O}^3$$

and therefore

$$\hat{\Upsilon}_t^h = \frac{\mu}{2}\mathrm{E}_i\left(\tilde{p}_{it}^h\right)^2 + \mathcal{O}^3$$

Moreover, from $\left(P_t^h\right)^{1-\mu} = \int_0^1 \left(P_{it}^h\right)^{1-\mu} di$, and $\hat{P}_t^h = \mathrm{E}_i log\left(P_{it}^h\right)$, it holds true that

$$\hat{P}_t^h - \hat{P}_{t-1}^h = \left(1-\delta^p\right)\left(log\left(P_t^{h*}\right) - \hat{P}_{t-1}^h\right)$$

and hence[33]

$$\begin{aligned}
\mathrm{E}_i\left(\tilde{p}_{it}^h\right)^2 &= \mathrm{Var}_i\left[log\left(P_{it}^h\right)-\hat{P}_{t-1}^h\right]+\mathcal{O}^3 \\
&= \delta^p \mathrm{E}_i\left(\tilde{p}_{it-1}^h\right)^2+\frac{\delta^p}{(1-\delta^p)}\left(\hat{P}_t^h-\hat{P}_{t-1}^h\right)^2+\mathcal{O}^3
\end{aligned}$$

Multiplying both side by $\mu/2$ and inserting the expression for $\hat{\Upsilon}_t^h$ yields

$$\hat{\Upsilon}_t^h = \delta^p \hat{\Upsilon}_{t-1}^h+\frac{\mu}{2}\frac{\delta^p}{(1-\delta^p)}\left(\hat{\pi}_t^h\right)^2+\mathcal{O}^3$$

Finally, multiplying by β^t, integrating across t, and noting that $\hat{\Upsilon}_{-1}^h$ is independent of policy at time 0 yields

$$\sum_{t=0}^{\infty}\beta^t\hat{\Upsilon}_t^h = \frac{\mu}{2}\frac{\delta^p}{(1-\delta^p)(1-\delta^p\beta)}\sum_{t=0}^{\infty}\beta^t\left(\hat{\pi}_t^h\right)^2+t.i.p.+\mathcal{O}^3$$

Regarding the term $\mathrm{Var}_i\hat{\phi}_{it}^h$, from the definition of $\hat{\phi}_{it}^h=\hat{\phi}_t^h-\tau_z\tilde{w}_{it}^h$ it follows that $\mathrm{Var}_i\hat{\phi}_{it}^h=(\tau_z)^2\,\mathrm{Var}_i\tilde{w}_{it}^h$, where $\tilde{w}_{it}^h=log\left(W_{it}^h/W_t^h\right)$. Hence, I can write

$$\begin{aligned}
\mathrm{Var}_i\tilde{w}_{it}^h &= \mathrm{Var}_i\left[log\left(W_{it}^h\right)-\hat{W}_{t-1}^h\right]+\mathcal{O}^3 \\
&= \delta^w\mathrm{Var}_i\tilde{w}_{it-1}^h+\frac{\delta^w}{(1-\delta^w)}\left(\hat{W}_t^h-\hat{W}_{t-1}^h\right)^2+\mathcal{O}^3
\end{aligned}$$

Multiplying by β^t and interating forward yields

$$\sum_{t=0}^{\infty}\beta^t\mathrm{Var}_i\hat{\phi}_{it}^h = \frac{\delta^w(\tau_z)^2}{(1-\delta^w)(1-\beta\delta^w)}\sum_{t=0}^{\infty}\beta^t\left[\left(\hat{\pi}_t^{wh}\right)^2\right]+t.i.p.+\mathcal{O}^3$$

[33] I used the fact that as $\tilde{p}_{it}^h$ is $\mathcal{O}^2$, $\mathrm{E}_i\left(\tilde{p}_{it}^h\right)^2=\mathrm{var}_i log\left(P_{it}^h\right)+\mathcal{O}^4$.

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