# EUI WORKING PAPERS IN ECONOMICS

EUI Working Paper ECO No. 92/62

**Noise Traders Permanence in Stock Markets: A Tâtonnement Approach. I: Informational Dynamics for the Two-Dimensional Case**

Pier Luigi Sacco

European University Institute, Florence

*Please note*
As from January 1990 the EUI Working Paper Series is divided into six sub-series, each sub-series is numbered individually (e.g. EUI Working Paper **LAW** No. 90/1).

**EUROPEAN UNIVERSITY INSTITUTE, FLORENCE**

ECONOMICS DEPARTMENT

EUI Working Paper **ECO** No. 92/62

# Noise Traders Permanence in Stock Markets: A Tâtonnement Approach. I: Informational Dynamics for the Two-Dimensional Case

PIER LUIGI SACCO

**BADIA FIESOLANA, SAN DOMENICO (FI)**

Printed in Italy in February 1992
European University Institute
Badia Fiesolana
I–50016 San Domenico (FI)
Italy

# Noise traders permanence in stock markets: A tâtonnement approach. I: Informational Dynamics for the two–dimensional case

Revised November 6th, 1991


Pier Luigi Sacco

Department of Economics
European University Institute
Badia Fiesolana, 50016 S. Domenico di Fiesole (Fi)
and
Department of Economic Sciences
University of Florence
Via Montebello, 7, 50123 Florence (Italy)*



**Abstract**

This paper studies a model of a stock market with noise traders, building on earlier work of De Long et al.; the market is open at discrete points in time; when the market is closed, fictional trade takes place. As fictional trade takes place, traders learn the returns to rational and noise trading as depending on the distribution of rational and noise traders in the population. This in turn changes the distribution of types; the process goes on until a stationary distribution is reached. Conditions for the eventual permanence of noise traders in equilibrium are given, both in a static (evolutionary stability) and in a dynamic (replicator dynamics) evolutionary framework. It is found that a dynamic evolutionary analysis leads to substantial improvements on existing results.



*Address: Pier Luigi Sacco, Dept. of Economics, European University Institute, Badia Fiesolana, Via dei Roccettini, 9, 50016 S. Domenico di Fiesole (Fi), Italy.

Noise traders permanence in stock markets: A tâtonnement approach. I: Informational Dynamics for the two–dimensional case*

## 1. Introduction.

In a series of papers, De Long, Shleifer, Summers and Waldmann (DSSW henceforth) study a model of a stock market in which there are both rational traders, who form their price expectations accurately according to fundamentals, and noise traders, who misperceive fundamental values [see DSSW (1989, 1990a,b, 1991)]. Among other things, DSSW investigate the conditions under which noise traders can survive in the stock market. Surprisingly enough, they show that there are cases in which noise traders do survive and even prosper, contrary to the classical thesis of Friedman (1953) that only the rational stay in the market in the long run. DSSW (1991) is in particular entirely devoted to the question of the survival of noise traders. The pathbreaking contributions of DSSW, however, are admittedly just a scratch on the surface of this question, which has several facets and is technically very demanding. The central point is the following: If noise traders misperceive fundamentals and constitute a relatively 'large' part of the market, they create distortions on market prices. Therefore, a dynamic model of price determination for a stock market in which a substantial number of noise traders is active must explain how the effect of the noise traders' misperceptions on prices interacts with the changes in the wealth and (possibly) in the number of noise traders as time unfolds. A satisfactory theoretical treatment of the issue requires that the model be embedded in a full–blown dynamic general equilibrium framework. In the absence

*I would like to thank David Easley and Mark Salmon for useful conversations and Richard Day, Alan Kirman, Robert Waldmann and two anonymous referees of this journal for very effective comments and suggestions on earlier drafts. Partial financial support from a "G. Mortara" scholarship, Banca d' Italia, is gratefully acknowledged. The usual disclaimer applies.

of such a framework, DSSW must limit their attention to the analysis of the long run distribution of wealth under the assumption that noise traders do not affect market prices.

As DSSW (1991) themselves suggest, an evolutionary analysis is one of the most promising routes to take in the quest for the best theoretical model of noise trading. The easiest move in this direction is to adopt a static approach, namely to study the evolutionary stability of noise trading. An evolutionary stable state (ESS) is a distribution of types across the population that cannot be successfully invaded by any small proportion of 'mutant' individuals. These types are identified by the strategies adopted by their members when interacting with other members of the population. Invoking evolutionary stability as a solution concept has an obvious advantage but also an obvious shortcoming if compared to a fully dynamic evolutionary analysis. The advantage is that, given the static nature of the notion of evolutionary stability, one can dispense with the difficult problem of the impact of noise traders on the price dynamics. On the other hand, this kind of analysis says little about the existence of a dynamic process through which an ESS can be reached starting from a certain set of initial conditions. Moreover, the real trouble with evolutionary stability is that, in the polymorphic case in which two or more population types coexist in equilibrium (as it is in the DSSW model if noise traders do not disappear but also do not invade the market completely), the ESS is quite fragile w.r.t. perturbations that involve *equilibrium*, rather than *mutant*, strategies: All those population types which are observed at the ESS must receive the same payoff; therefore, if a slight perturbation of the equilibrium proportion of types across the population occurs for whatever reason, there is no force pushing the population back to the ESS, since no individual in the population suffers a loss from abandoning his(her) equilibrium strategy for any other one which is played with positive frequence at the ESS. On the other hand, if the distribution of types is not the one dictated by the ESS, there is now the possibility that some 'mutant' type can successfully invade the population. These two facts together call for a dynamic analysis that explicitely derives the conditions under which a given distribution of types is a stable stationary state[1].

[1]This criticism of the notion of an ESS in the polymorphic case is made

When the static framework of evolutionary stability is substituted by a dynamic one, based for example on the so called replicator dynamics (according to which a type' s size within the population increases if and only if the type earns a return that is greater than the average one), one can sometimes find that there are attractors for the dynamics that do not correspond to ESSs, and, conversely, that ESSs may not be stable stationary states for the dynamics[2]. When this occurs, it is reasonable to expect that a dynamic evolutionary analysis provides richer insights than a static one. Therefore, once having committed oneself to an evolutionary justification of noise trading, there may be a large return to a dynamic evolutionary analysis; although technically more demanding, a dynamic analysis is likely to provide more complete answers to the issue of the permanence of noise traders than a static one.

This is in fact the case in the present paper. We adopt an evolutionary dynamic framework, based on the replicator dynamics, which has a tractable level of analytical difficulty and nevertheless allows to make clear cut predictions on the equilibrium distribution of rational and noise traders. This is of course done at the cost of some simplifications, that seem however to retain many important features of a realistic stock market dynamics.

The plan of the paper is the following: In section 2 we provide a brief informal survey of the results of DSSW and of related work, provide a detailed description of our approach and explain to what extent our results improve upon, or differ from, the available ones. In section 3 we give a detailed exposition of the DSSW (1990b), which will be the basis of our own model. In section 4 we give an analytical discussion of evolutionary stability as applied to the DSSW model. In section 5 our dynamic model, as well as our main results, are presented. Section 6 is devoted to the interpretation of our results. Section 7 concludes. Proofs of the results are given in the Appendix.

## 2. Comparison with related work.

e.g. in Vega Redondo (1988).

[2]More on this in sections 4 and 7 below.

The issue of the relative performance of noise and rational traders is explicitely tackled in DSSW (1990b), (1991). In DSSW (1990b), the authors study an overlapping generations model where agents live two periods; when young, they must make a portfolio choice between a safe and a risky asset (say, a stock). This is the only decision they must make during their lifetime; when old, traders simply consume their wealth. The two assets yield the same dividend; however, while the former is in perfectly elastic supply, the latter is in fixed supply. Moreover, the price of the stock is subject to misperception by noise traders; this is in fact the only reason why the stock is risky: In the absence of misperceptions, it would be a safe one as well. The agents' horizon is short; therefore, they cannot wait to recover a capital loss. Noise traders may have higher expected returns than rational traders if they are 'bullish' (i.e. if their misperception of the price has positive sign), but at the cost of lower expected utility because of their riskier portfolios. This means that a noise trader may end up wealthier than a rational one, but also that (s)he runs a substantial risk to end up quite poor. In order to determine how the proportion of noise traders changes over time, DSSW assume that, although in principle a new rational and a new noise trader, respectively, is born for each one dead, in each period a *small* fraction of newly born traders changes its type to imitate the strategy that looks more successful in terms of expected returns. It is crucial that the number of switchers be small in any period because traders should otherwise take into account how the change in the distribution of types affects returns, and this would make the solution of the model considerably more difficult. Under this assumption, it can be shown that either the proportion of noise traders tends to zero in the long run, or the market is eventually all made up of noise traders, according to whether the initial proportion of noise traders is above or below a certain critical value. If one assumes that the stock has fundamentally risky returns, it can be shown that the eventual proportion of noise traders has a lower bound which is strictly positive —that is, noise traders are never driven out of the market completely. It must be emphasized, however, that this model is not able to determine how the distribution of wealth between the two types of traders changes over time.

DSSW (1991) tackles this latter problem, in the context of a model with infinitely lived agents who make consumption and portfolio choices. Portfolio choices concern a safe asset and a continuum of risky assets (say, stocks). There is a continuum of different types of noise traders, each one misperceiving the *return* of a single stock. The subject of the misperception may be mean returns, the variance of returns, or both; the magnitude of the misperception is the same across all types of traders. Misperceptions of the variance turn out to be the crucial element of the model. It is moreover assumed that the presence of noise traders does not affect asset prices; this implies that in this model noise traders do not exhibit the typical 'buy high–sell low' pattern of behavior, but merely tend to invest an excessive share of their wealth in the asset which is subject to misperception. Under these assumptions, it turns out that noise traders who misperceive only the variance of returns of one asset have higher expected returns than rational investors. Moreover, for any configuration of the parameters of the distribution of asset returns, there is a range of values of the variance misperception parameter centered around one (i.e., no variance misperception) for which noise traders' aggregate wealth is larger than rational traders' aggregate wealth; in other words, noise traders not only survive in the long run, but also dominate rational traders in wealth, in spite of the fact that they tend to overestimate the returns of their portfolios and therefore to consume too much. It is also found, however, that if rational traders have logarithmic utility, then no type of noise trader may dominate rational traders in wealth.

There are other interesting models in the literature that study the relative performance of rational and nonrational traders in asset markets. Day and Huang (1991) propose a deterministic model of a stock market where three types of agents are active: sophisticated investors ($\alpha$–investors in their terminology), naive investors ($\beta$–investors) and market makers. $\alpha$–investors exploit information optimally in order to estimate the long–run fundamental value of the stock, which is their reference price. They also estimate a topping and a bottoming price for the stock; these are 'no trespassing' points for the realization of the speculative gains of $\alpha$–investors. Beyond these points, the likelihood that the upward or downward drift, respectively, of the stock price is reverted is too high; a sophisticated investor will therefore liquidate his(her) position before price enters the danger zone. As to $\beta$–investors, they base their decisions on simple extrapolative rules based on past price trends. They will therefore tend to buy high and sell low like noise traders of DSSW (1990b). On the other hand, their expectations will be grossly confirmed unless the market price is near a turning point, i.e., a topping or bottoming price. Market makers are the third type of agents. They are ready to sell to or buy from other market participants any quantity of the stock at a price set by them and updated at discrete intervals. Market makers adjust price much like a Walrasian auctioneer, raising it when there is excess demand and lowering it when there is excess supply. This is a realistic description of market makers' behavior in actual asset markets. On the basis of these assumptions, Day and Huang show that their deterministic model can display any kind of complex behavior, from cycles to bifurcations to chaos; as a result, the generated sequences of stock prices may be so erratic to be indistinguishable from series generated by a stochastic process. Of course, the kind of dynamics generated by the model depends on the behavioral parameters of the three types of agents; the crucial conditions for the appearance of complex dynamics are that $\beta$–investors must be very active in the proximity of the long run fundamental value of the stock (in order to prevent convergence) and that $\alpha$–investors must be very reactive in the proximity of the topping and bottoming values (this gives rise to intermittent phases of bear and bull markets; typically, erratic price behavior will be observed within each phase).

Kirman (1991a,b) presents a model of an asset market in which, following Frankel and Froot (1986), there are two types of agents: Fundamentalists, who forecast next period' s price on the basis of market fundamentals, and chartists, who use simple adaptive extrapolation rules; there is a clear analogy with the rational/noise traders dichotomy of DSSW. Kirman assumes that any individual in the population of traders may change its type with a given (small) probability. Moreover, traders meet at random several times between two market sessions; at each meeting, one of the traders can be 'converted' to the other trader' s type with a given probability. As a result of these assumptions, the proportion of traders' types in the market evolves as a Markov chain and the best forecast of the asset' s price for the next period is the one dictated by the majority view. In other words, if most traders are chartists it pays to be a chartist, etc. As a consequence, traders are induced to change their type if they believe that the majority of traders is of the other type, and this fact is common knowledge; the proportion of types is only imperfectly observable, however. It can be shown that with a large number of traders, for suitable choices of the transition probabilities, the density of the proportion of traders' types tends to a symmetric, bimodal beta distribution; consequently, one can observe intermittent phases of 'fundamentalist' and 'chartist' markets. Simulations of the model show that the presence of chartists does not necessarily imply an increase in the instability of the price of the asset around the fundamental value, and that there are even cases in which it has a stabilizing effect on the price.

Finally, Blume and Easley (1989) consider a model of an asset market where there are $H$ assets whose relative dividends are state dependent. In spite of the fact that assets' payoffs have a stochastic nature, relative payoffs are deterministic. Each trader has a subjective probability distribution on the states of nature. The model is defined over an infinite sequence of periods. At each period traders choose how to allocate their wealth between consumption and assets. Blume and Easley are interested in the long run distribution of wealth across traders, much like DSSW (1991). They show that, if all traders have identical savings rates, the trader who (almost surely) eventually dominates in wealth all other traders is the one whose beliefs are closest in relative entropy to the true probability distribution of states. In other words, with equal savings rates the market selects for the 'most rational' investor. If, however, savings rates differ across traders, eventual dominance in wealth depends not only on traders' rationality (as defined above) but also on their relative savings ratios. More specifically, it may be the case that the 'most rational' trader is eventually driven out of the market by a less rational trader if the highest relative entropy of the latter' s beliefs w.r.t. the true probability distribution is more than compensated by a substantially lower savings rate. In other words, the market may select for (relatively) nonrational traders if these are substantially stingier than (relatively) more rational traders and if their beliefs do not depart too much from rational beliefs.

Our brief survey clearly shows that each of the models listed above is concerned with a potentially important facet of stock market dynamics. For example, DSSW (1990b) and Kirman emphasize the variability in the proportion of behavioral types and the role of imitative behavior; Day and Huang point out the importance of a realistic description of market institutions for the understanding of the price dynamics and focus on dynamic complexity and intermittent phases of bull and bear markets, a characteristic partly explored also by Kirman; DSSW (1991) and Blume and Easley study the long run distribution of wealth between rational and nonrational traders. It is also apparent, however, that the theoretical frameworks under which the various models are built are too different to allow for a meaningful comparison of results. These two considerations point in the same direction, namely, the necessity of a more general model which can address the issues posed by the today available models within a common (necessarily broader) framework.

The model presented in this paper aims at being a first step in this direction. It has been conceived as a part of a more general model which should allow one to study the dynamics of the proportion of behavioral types, of wealth shares and of stock market prices all together. We give an idea of our general model below. Subsequently, we identify the part of the model which is actually introduced in the present paper and show how our results improve upon some of the already available ones.

Our approach to the modelling of stock market dynamics could be labelled a 'tâtonnement' one. Market dynamics is defined over an infinite sequence of periods. We focus our attention on a given couple of periods, say periods $t$ and $t+1$, and investigate how the observed information on the returns to rational and noise trading between these two periods affects the proportion of types. The mechanism works as follows. Market outcomes for period $t$ are given, as well as the realization of the noise traders' misperception for period $t$. Then a new generation of traders, who will trade at time $t+1$, is born, whereas old traders die (after having consumed their wealth). It is assumed that, at the birth time, traders belong to a certain type. There is no necessary relationship between the type of old traders dying and that of the newly born traders; the number of births and deaths are equal for simplicity, however. The birth of new traders and the disappearance of the old ones will bring about a perturbation in the distribution of types (as well as in the distribution of the noisy stock between types), thus altering the returns to rational and noise trading, respectively. Traders will now generally have an incentive to trade. Markets are still not open, however. Traders are free to undertake off–the–market transactions at fictional prices, which are set by an auctioneer in the standard way. Off–the–market transactions cannot be binding, i.e., traders cannot sign contracts if the market is closed. In spite of this, off–the–market transactions can be very useful to traders, who use them to learn how the relative performance of rational and noise trading depends on the current distribution of types and exploit this information during market transactions. The informational dynamics generated by fictional trade is the following: By observing the dynamics of (expected) returns, traders may decide to change their type; this changes the distribution of types and therefore affects (expected) returns, and so on. This dynamic process is modelled as a replicator dynamics, i.e., the rate of change of the proportion of traders of a given type is equal to the difference between the expected return to the corresponding strategy and the average expected return[3]. The process goes on until no trader has an incentive to change his(her) decision through the observation of returns, i.e., until the dynamics converges to a stationary point. To emphasize its notional character, the informational dynamics happens in continuous time.

There is a basic difference between our replicator dynamics and the imitative dynamics of DSSW (1990b), namely, that in the DSSW model population dynamics is driven by the difference between types' expected returns, independently of the types' size in the population, whereas in our model it is driven by the difference between the expected return to each type and the *average* expected return, which clearly depends on the types' size; this latter fact creates a more complex relationship between types' returns and their size in the population than in the DSSW model, and accounts for the richer dynamic behavior of our model. Another

[3]This aspect of our model has clear connections with the Kirman model. As in the Kirman model, in our model there is no necessary link between noise trading and bounded rationality. The same trader may switch from noise to rational trading and vice versa according to the dynamics of returns. On the other hand, the assumptions of our model clash with the efficient markets paradigm and more generally with the approach that sees individuals as exclusively concerned with the information about fundamental values [see e.g. Lucas (1986) and the critical remarks in Sacco (1991a,b)].

important difference is that in our model we do not impose any exogenous restriction on the rate at which traders may switch their type. Finally, it must be reminded that ours is a continuous time dynamic model, whereas DSSW (1990b) is a discrete time one.

Once the informational dynamics has come to rest, the noise traders' misperception at time $t+1$, $\rho_{t+1}$, is observed and the market is opened for period $t+1$. The actual proportion of rational and noise traders at this stage is the result of the informational dynamics that took place previously. It is an *ex ante* equilibrium proportion: When the market opens, every trader is happy about his(her) type[4] and transactions take place; the market clearing price is found and the equilibrium holdings of the stock for the two types are determined. The same story can now be told again for periods $t+1$, $t+2$ and so on. To get a concise picture of the timing of traders' actions, one can imagine that markets are opened only on 'Monday', a second after the current noise traders' misperception has been observed; on 'Tuesday', old traders consume their wealth; on 'Wednesday', old traders die and new traders are born; fictitious trade happens all over the other days of the week[5].

The fictitious nature of off–the–market transactions explains our reference to tâtonnement. Three important characteristics of our model need to be stressed. First, the distribution of types and market prices are determined separately. This allows us to study the changes in the proportion of traders' types without being concerned with their effect on stock prices; conversely, stock prices are determined once the equi-

[4]The observation of $\rho_{r+1}$ could in principle create new incentives for traders to change their type. As the market opens, however, transactions take place very rapidly and traders have no time to exploit this new piece of information. Of course, the noise traders' misperception at $t+1$ has an effect on *realized* returns at $t+1$, the observation of which may disappoint traders of a certain type, but cannot modify their portfolio choices, which have already been made. In a more realistic model where traders live several periods and make several portfolio choices during their lifetime, realized returns would affect their portfolio decisions in the *next* period.

[5]This aspect of the model is reminescent of the Hicksian temporary general equilibrium framework [Hicks (1946)].

librium proportion of types for the current period has been determined. We can therefore in principle avoid the restrictive assumptions of DSSW (1990b) and (1991) (respectively, that only a small number of traders changes type in each period and that noise traders do not affect stock prices) and determine both the proportion of traders' types and the distribution of wealth across types within the same model. Second, our model has both a deterministic and a stochastic nature. The main stochastic element of the model is the evolution of noise traders' misperceptions with their effect on prices and therefore on the distribution of wealth; the main deterministic element is the informational dynamics which takes place between market periods. This characteristic of the model allows us a joint treatment of the stochastic and deterministic sources of dynamic complexity of stock market prices. Third, the contraposition of actual market transactions and off–the–market notional transactions allows us to introduce an element of realism in the model, which, however, still scores poorly as a credible description of market institutions. This is in our opinion one of the weaknesses of our model; the introduction of traders who play the role of market makers, as in the Day and Huang model, would make it much more general.

In the present paper, we study only one aspect of the general model, namely, the informational dynamics between a given couple of market sessions. Our model exploits the basic architecture of DSSW (1990b). Given the role played by the informational dynamics within the more general model, that is, the determination of the equilibrium proportion of traders' types, the results we obtain are meaningfully comparable only with those of DSSW (1990b) and, although quite partially, with those of Kirman, i.e., with the papers which address more or less the same issue[6]. A more global evaluation of our model must be left for future research.

Our results are the following.

A first set of results, actually the most important, concerns our dynamic evolutionary model. It is found that both rational and noise

---

[6]It is interesting to observe that the actual behavior of our model is the product of the interplay of its deterministic and stochastic parts; although the dynamics is deterministic, its parameters are stochastically determined and differ from time to time.

traders may be the only type observed in the market in equilibrium, the actual type observed depending on the *current* noise traders' misperception; however, there is also the possibility that rational and noise traders coexist in equilibrium. This is an improvement w.r.t. DDSW (1990b) where the two types could coexist in equilibrium only in the presence of a fundamentally risky asset. Moreover, it is also shown that the stability of a given stationary proportion of types crucially depends on the relative shares of the stock owned by the two types, and therefore, on the distribution of wealth. Interestingly enough, there are ranges of values of noise traders' misperceptions for which all members of the type which initially holds the majority of the stock are converted to the other type in equilibrium[7]. Finally, whereas in DSSW (1990b) noise traders cannot be observed in equilibrium if their misperceptions are bearish, in our dynamic model bearish noise traders will indeed be the only type observed in equilibrium if they initially hold the majority of the noisy stock.

The second set of results concerns what we could call the (comparative) statics of our model, and more specifically the evolutionary stability of the equilibrium distributions of types. It is found that the stable stationary equilibria of our dynamic model are not necessarily ESSs, although they are the only feasible candidates for an ESS. In fact, it turns out that our dynamic analysis provides much finer predictions on the actual proportion of rational and noise traders that will be observed in equilibrium than the ESS conditions do. As a matter of fact, a static evolutionary analysis does not lead to any substantial improvement upon the original results of DSSW (1990b).

Lastly, as far as the Kirman model is concerned, our results suggest that traders of different types can coexist *in equilibrium* even when the proportion of types is perfectly observable; in the Kirman model, this latter assumption would imply that, *in equilibrium*, all traders are converted to the majority type. It must however be emphasized once again that our assumptions are not close enough to those of Kirman to

[7]In fact, when the stock is equally distributed between the two types, the tâtonnement dynamics does not operate and the equilibrium proportion of types is trivially equal to the initial one.

make our results really comparable[8].

## 3. Basic architecture of the model: The DSSW (1990b) model.

Our model builds on DSSW (1990b). The basic assumptions of the model have been described in section 2. We consider for simplicity the version of the model without fundamental risk. Thus we have two stocks, both of which have a fundamental value equal to one and pay a fixed real dividend equal to $r$ in the absence of misperception. No stock has intrinsic risk. One of the stocks, however, is subject to the misperceptions of noise traders (the noisy stock). Misperceptions are assumed to be i.i.d and to have a normal distribution, with mean $\overline{\rho}$ and variance $\sigma_{\rho}^{2}$[9]

The price at time $t$ of the noisy stock is $p_t$. The share of the noisy stock held by noise traders at time $t$ is $\beta_t^n$, the share held by rational traders at $t$ is $\beta_t^r = 1 - \beta_t^n$.

Each trader maximizes his(her) expected utility and has constant absolute risk aversion[10]. Stationarity of $\rho_t$ implies prices are stationary.

---

[8]More specifically, traders in the Kirman model need to make more sophisticated calculations than traders in our model do.

[9]In a really satisfactory model of stock market dynamics, the evolution of noise traders' misperceptions should be endogenously determined in order to derive a truly realistic correlation pattern across periods. We do not pursue this point in the current formulation of the model to keep technical difficulties at a manageable level; this is however an important extension of the model that should eventually be made. We are grateful to a referee for pointing this out.

[10]It must be emphasized here that the assumption of constant absolute risk aversion implies that our tâtonnement dynamics does not depend on the distribution of wealth across the economy. This would of course no longer be the case with different utility functions; traders would have to maximize a nonlinear function of the returns, and this would change the dynamics considerably, at the cost of a decrease in the analytical tractability of the model.

Of course, individual calculations are conditioned on next period' s expected price. Let $W^*$ be the expected final wealth for a trader, let $\sigma_W^2$ be the associated variance and let $\delta$ be the coefficient of absolute risk aversion. Then the individual maximization problem boils down to [see DSSW (1990b), p. 708]

$$\max_{W} \quad W^* - \delta\sigma_W^2 \tag{1}$$

The return for both noise traders, $R_t^n$, and rational traders, $R_t^r$, is given by

$$R_t = \beta_t[r + p_{t+1} - (1+r)p_t] \tag{2}$$

where $\beta_t = \beta_t^n$ for noise traders, $\beta_t = \beta_t^r$ for rational traders, and $(1+r)p_t$ is the opportunity cost of giving up holding the safe stock.

The maximization problem for a noise trader is

$$\max_{\beta_t^n} \quad \tilde{W} + E_t(R_t^n) + \beta_t^n \rho_t - \delta(\beta_t^n)^2\sigma_p^2 \tag{3}$$

where $E_t(\cdot)$ denotes the expectation taken at time $t$, $\tilde{W}$ is the amount of wealth that is independent of portfolio decisions (i.e., labour income earned when young), and $\sigma_p^2$ is the expected variance of price, which is equal across periods given the stationarity assumption.

The $\beta_t^n \rho_t$ term in (3) is due to the noise trader' s misperceptions of the return on noisy stock. For each unit of the noisy stock the noise trader believes (s)he will receive an extra return of $\rho_t$. $\rho_t$ may be negative.

For a rational trader one has

$$\max_{\beta_t^r} \quad \tilde{W} + E_t(R_t^r) - \delta(\beta_t^r)^2\sigma_p^2 \tag{4}$$

The first order conditions for (3) and (4) yield

$$\beta_t^n = \frac{r + E_t(p_{t+1}) - (1+r)p_t + \rho_t}{2\delta\sigma_p^2} \text{ and } \beta_t^r = \frac{r + E_t(p_{t+1}) - (1+r)p_t}{2\delta\sigma_p^2} \tag{5}$$

respectively[11]. The misperceptions of noise traders lead them to buy more, or less, of the noisy stock than rational traders at $t$ depending on the sign of $\rho_t$.

The proportion of noise traders in the economy at the end of time $t$ is $\mu$. Imposition of the market clearing condition $\beta_t^r + \beta_t^n = 1$ implies

$$p_t = \frac{r + E_t(p_{t+1}) - 2\delta\sigma_p^2 + \mu\rho_t}{1+r} \tag{6}$$

Now, since

$$\sigma_p^2 = \frac{\mu^2\sigma_\rho^2}{(1+r)^2} \tag{7}$$

[see e.g. DSSW (1990b), p. 711], one has that

$$p_t = 1 + \frac{\mu(\rho_t - \overline{\rho})}{1+r} + \frac{\mu\overline{\rho}}{r} - \frac{2\delta\mu^2\sigma_\rho^2}{r(1+r)^2} \tag{8}$$

There are three factors that affect how noise traders cause the price of the noisy stock to deviate from its fundamental value: a) the variability of noise traders' misperceptions, whose effect on price depends on the proportion of noise traders in the economy; b) the direct impact of the average misperception on the price (i.e. the systematic price distortion caused by noise trading); note that, things being equal, a high price for the noisy stock raises the opportunity cost from not holding the safe asset; and c) the risk premium for holding the noisy stock when noise traders are present. The presence of noise traders increases the expected return from holding the noisy stock, although at the cost of increased price risk. This situations reminds somewhat the inception of bubbles in asset markets. For a detailed discussion of the interpretation of the various components of the pricing function (8) see [DSSW (1990b; pp. 711–13)]. We will return on the interpretation of the DSSW model in section 6 where we discuss our results.

## 4. ESSs for the DSSW (1990b) model.

[11]Note that under the assumption of constant absolute risk aversion traders' portfolio choices do not depend on the level of their wealth.

Consider a 'large' population of individuals who are to play a certain game $\mathcal{G}$. Let $\Sigma$ be the set of available strategies (the same for all individuals) and let $L(\Sigma)$ be the corresponding set of frequence distributions over $\Sigma$. Let us identify each strategy in $\Sigma$ with a population type; that is, individual $i$ is of type $h$ if (s)he plays strategy $h$. The expected payoff function $\pi(\cdot, \gamma[\cdot]) : \Sigma \times L(\Sigma) \rightarrow \Re$ of $\mathcal{G}$ is defined as follows. Let $i$ be a player drawn from the population and let $\gamma \in L(\Sigma)$ be the strategy profile played by the various members of the population other than $i$. Since the population is 'large', the strategy choice of $i$ has a negligible impact on the overall distribution of strategies. If, say, only two distinct strategies $f$, $g$ are played in the population with positive frequence (equivalently, only two types $f$, $g$ are present in the population), the notation $\gamma[(1-\epsilon) \cdot f \oplus \epsilon \cdot g]$ means that a proportion $1-\epsilon$ of the population is playing $f$, while a proportion $\epsilon$ is playing $g$ (i.e., a fraction $1-\epsilon$ of the population is of type $f$ and a fraction $\epsilon$ is of type $g$). The expected payoff from playing strategy $h$ when $\gamma[\cdot]$ is the distribution of strategies (types) across the population is defined as a function of $h$ and $\gamma$ and is given by $\pi(h, \gamma[\cdot])$[12].

A distribution $\tilde{\lambda} \in L(\Sigma)$ is an evolutionary stable state if, for any $f \in \Sigma$ for which $\tilde{\lambda}(f) = 0$, there exists an interval $\mathcal{E} = (0, \epsilon^*]$ such that, for all $\epsilon \in \mathcal{E}$, and for all $h$, $h'$ such that $\tilde{\lambda}(h) > 0$, $\tilde{\lambda}(h') > 0$, one has[13]

E1) $\pi(h, \gamma[(1-\epsilon) \cdot \tilde{\lambda} \oplus \epsilon \cdot f]) = \pi(h', \gamma[(1-\epsilon) \cdot \tilde{\lambda} \oplus \epsilon \cdot f])$;

E2) $\pi(h, \gamma[(1-\epsilon) \cdot \tilde{\lambda} \oplus \epsilon \cdot f]) > \pi(f, \gamma[(1-\epsilon) \cdot \tilde{\lambda} \oplus \epsilon \cdot f])$.

In other words, all strategies which are played with positive probability in the ESS must yield the same expected payoffs (otherwise it would pay for individuals adopting less profitable strategies to switch to the more profitable strategy), and all strategies which are played with zero probability in the ESS must yield strictly inferior payoffs[14]. The rationale

[12] In the terminology of Maynard Smith (1982), these assumptions correspond to a 'playing the field' interaction structure.

[13] If at the ESS all strategies in $\Sigma$ are played with positive frequence, condition E2 has to be suitably modified. See section 1 of the Appendix.

[14] If we allow for mixed strategies, the definition of an ESS can be made more compact; see Crawford (1991). It is however scarcely attractive in

behind the formal definition is the following. An ESS is a distribution of types across the population such that no 'mutant' type can successfully invade the population. In other words, in an ESS no 'small' fraction of individuals has an incentive to abandon their equilibrium strategies for any other strategy which is played with zero frequence at the equilibrium, because any such strategy yields strictly inferior payoffs[15].

As emphasized in section 1, however, an ESS is a potentially fragile situation; since all the strategies which have a positive frequence in the ESS yield the same expected payoff, every individual can abandon his(her) equilibrium strategy for any other *equilibrium* strategy (i.e., for any other strategy which is played with *positive* frequence at the ESS) without suffering from any loss. As long as this actually happens, the distribution of types is no longer the equilibrium one and there are now 'mutant' types that can successfully invade the population.

In spite of its deficiencies, the ESS solution concept is an useful reference point for the analysis of issues of survival like the one we are interested in. One should look at the ESSs of the model as a sort of 'comparative statics', with all the caveats of this kind of analysis [see Samuelson (1947)].

In this spirit, we study in this section the existence of ESSs for our version of the DSSW (1990b) model. Keeping in mind the description of the timing of our model as given in section 2, we must emphasize that since our fictional trade takes place *after* the end of the market session at time $t$, the current noise traders' misperception $\rho_t$ is known when fictional trade begins; moreover, every trader knows it[16]. Let now $\psi$ be the strategy 'play as a noise trader at $t$' and $\phi$ be the strategy 'play as

our context to allow for players randomizing between rational and noise trading; therefore we confine our attention to pure strategies.

[15]ESS was introduced as a game-theoretic solution concept in theoretical biology by Maynard Smith (1982). See Crawford (1991) for a comprehensive discussion of its relevance for economic analysis.

[16]Leahy (1989) finds the conditions for ESSs for the DSSW (1990b) model under a different assumption, namely, that current noise traders' misperceptions $\rho_t$ are not fixed when traders interact; rather, they are stochastic. This assumption implies a different timing of events than in our model, one which is closer to the original setup of DSSW (1990b).

a rational trader at $t$'.

We have the following results:

PROPOSITION 1. *Consider our version of the DSSW (1990b) model without systematic risk as presented in section 3. Then:*

*a)* $\mu = 0$ *is always an ESS for every possible value of current noise traders' misperceptions* $\rho_t$*;*

*b)* $\mu = 1$ *is an ESS if and only if* $0 < \rho_t < 2\delta\sigma_\rho^2/(1+r)^2$*;*

*c) no 'mixed' ESS* $\hat{\mu}$ *exists,* $0 < \hat{\mu} < 1$.

The proof of Proposition 1 is reported in section 1 of the Appendix.

Part a) of the Proposition says that rational trading is always evolutionarily stable. This means that no small fraction of noise traders can successfully invade a population of rational traders. The rationale behind the result is that if the population is made almost entirely of rational traders, the influence of noise traders on prices is too small to make noise trading profitable. Although interesting, this result does *not* by any means imply that noise trading will never be observed. In fact, the dynamic analysis of the next section will show that, for a wide range of parameter values, noise traders are observed in equilibrium, or even they are the only type observed, in spite of the intuitions provided by our comparative static analysis. Part b) of the Proposition says that noise trading is evolutionarily stable if noise traders are moderately bullish. If noise traders' misperceptions are bearish, a population of noise traders can be invaded by rational traders because bearish noise traders tend to hold a relatively small amount of the noisy stock w.r.t. rational traders and the returns to risk bearing are therefore mainly collected by rational traders [see also DSSW (1990b), sec. IIA]. If noise traders' misperceptions are too bullish, a population of noise traders can be invaded by rational traders because the price of the noisy stock is now so high and so volatile that the (expected) return to noise trading is very small and nevertheless bullish noise traders tend to hold large amounts of the noisy stock and therefore to bear a considerable amount of risk, which is in itself very large. Part c) is self evident. It is worth to stress, however, that on the grounds of a purely static analysis there is no clue that rational and noise traders may coexist in equilibrium.

## 5. Tâtonnement dynamics.

A complete verbal description of the model, and in particular of its dynamic structure, has been given in section 2. The basic equations were given in section 3. Section 4 provided a static analysis of the model. We are now ready for the technical analysis of the dynamics.

From equation (2), we know that the return to noise traders at time $t$ may be written as $\beta_t^n(r + E_t(p_{t+1}) - p_t - rp_t)$. Advancing (8) one period yields

$$E_t(p_{t+1}) = 1 + \frac{\mu\overline{\rho}}{r} - \frac{2\delta\mu^2\sigma_\rho^2}{r(1+r)^2} \tag{9}$$

Given (8), this leads to

$$E_t(p_{t+1}) - p_t = \frac{\mu(\overline{\rho} - \rho_t)}{1+r} \tag{10}$$

Substituting (10) and (8) into (2), one gets

$$R_t^n = \beta_t^n \mu[\frac{2\delta\mu\sigma_\rho^2}{(1+r)^2} - \rho_t \tag{11}$$

while, similarly,

$$R_t^r = \beta_t^r \mu[\frac{2\delta\mu\sigma_\rho^2}{(1+r)^2} - \rho_t] \tag{12}$$

The average return at time $t$, $\overline{R}_t$, is given by

$$\overline{R}_t = \mu R_t^n + (1-\mu)R_t^r \tag{13}$$

Let $\tilde{\Gamma}(\mu) \equiv \mu[\frac{2\delta\mu\sigma_\rho^2}{(1+r)^2} - \rho_t]$. It is easy to check that

$$\overline{R}_t = \tilde{\Gamma}(\mu)\mu(\beta_t^n - \beta_t^r) + \tilde{\Gamma}(\mu)\beta_t^r \tag{14}$$

The replicator dynamics for our model is given by [compare e.g. Hofbauer and Sygmund (1988)]

$$\dot{\mu} = \mu(R_t^n - \overline{R}_t) \equiv \eta(\mu) \tag{15}$$

where $\mu$ belongs to the one–dimensional (unit) simplex.

Substituting (11) and (14) into (15) and noting that $\beta_t^r = 1 - \beta_t^n$ one finds that

$$\dot{\mu} = \mu(1-\mu)\tilde{\Gamma}(\mu)(2\beta_t^n - 1) \qquad (16)$$

Letting now $\Gamma(\mu) \equiv \frac{1}{\mu}\tilde{\Gamma}(\mu)$, we have

$$\dot{\mu} = \mu^2(1-\mu)(2\beta_t^n - 1)\Gamma(\mu) \qquad (17)$$

By imposing the condition $\dot{\mu} = 0$ on (17), it follows that the replicator dynamics (15) has $\mu = 0$ as a stationary point with multiplicity two and $\mu = 1$ as a stationary point with multiplicity one. Therefore, under the replicator dynamics both rational and noise trading can predominate in equilibrium. Moreover, since $\Gamma(\mu)$ is linear in $\mu$, the replicator dynamics has at most one more stationary point in the interior of the one–dimensional simplex, a point where one generically finds a mixed proportion of rational and noise traders, who therefore coexist in the market in period $t+1$.

Setting $\Gamma(\mu) = 0$ one easily finds that the interior stationary point is given by

$$\hat{\mu} = \frac{(1+r)^2\rho_t}{2\delta\sigma_\rho^2} \qquad (18)$$

where of course it must be that $0 \leq \hat{\mu} \leq 1$[17]. It is easy to check that this restriction implies $0 < \rho_t < 2\delta\sigma_\rho^2/(1+r)^2$. In particular, if $\rho_t = (2\delta\sigma_\rho^2)/(1+r)^2$, $\hat{\mu} = 1$; if $\rho_t = 0$, $\hat{\mu} = 0$.

[17] As a technical note, we observe that although our replicator dynamics is given by a polynomial of degree 4 in $\mu$, this does *not* belong to a versal unfolding of $\mu^4$. The *uni*versal unfolding of $\mu^4$ is in fact $U(\mu) = \mu^4 + a\mu^2 + b\mu$ [see e.g. Golubitsky and Guillemin (1973)]. This is an exoteric but technically precise way of saying, in the language of the analysis of structural stability, that one of the roots of our polinomial, more specifically $\mu = 0$, is not a simple root but a root of multiplicity two, and that therefore our model has potentially one more stationary point $\tilde{\mu}$, other than $\hat{\mu}$, belonging to the interior of the one–dimensional simplex; under the dynamics (17), it is however permanently 'captured' by $\mu = 0$. For any small perturbation of the functional form of (17) that

Outside this range of values, $\hat{\mu}$ lies outside the one–dimensional simplex and no interior stationary point exists.

The dynamic behavior of the model can be summarized as follows:

PROPOSITION 2. *Let $\beta_t^n < 1/2$. Then:*

a) *if $\rho_t \leq 0$, $\mu = 0$ and $\mu = 1$ are the only stationary points; moreover, $\mu = 0$ is a global attractor for the dynamics, i.e. $\mu \to 0$ for every initial condition $\mu_0 \neq 1$;*

b) *if $0 < \rho_t \leq 2\delta\sigma_\rho^2/(1+r)^2$, $\hat{\mu}$ lies in the interior of the one–dimensional simplex and is a global attractor, i.e., $\mu \to \hat{\mu}$ for every $\mu_0 \neq 0, 1$;*

c) *if $\rho_t > 2\delta\sigma_\rho^2/(1+r)^2$, $\mu = 0$ and $\mu = 1$ are again the only stationary points and $\mu = 1$ is a global attractor for the dynamics, i.e., $\mu \to 1$ for every $\mu_0 \neq 1$.*

*Let now $\beta_t^n > 1/2$. Then*

d) *if $\rho_t \leq 0$, $\mu = 1$ is now the global attractor, i.e.$\mu \to 1$ for every $\mu_0 \neq 0$;*

e) *if $0 < \rho_t < 2\delta\sigma_\rho^2/(1+r)^2$, $\hat{\mu}$ is a repellor and $\mu = 0$, $\mu = 1$ are attractors; moreover, $\mu \to 0$ whenever $\mu_0 \in [0, 2\delta\sigma_\rho^2/(1+r)^2)$ and $\mu \to 1$ whenever $\mu_0 \in (2\delta\sigma_\rho^2/(1+r)^2, 1]$;*

f) *if $\rho_t \geq 2\delta\sigma_\rho^2/(1+r)^2$, $\mu = 0$ is a global attractor, i.e., $\mu \to 0$ for every $\mu_0 \neq 1$.*

g) *finally, if $\beta_t^n = 1/2$, $\mu$ will stay at its initial value $\mu_0$, whatever $\mu_0 \in [0, 1]$.*

The proof of Proposition 2 is reported in section 2 of the Appendix.

## 6. Interpretation of the results.

In this section we analyze in detail Proposition 2 as well as its implications for the permanence of noise traders. A first important

---

puts it in the interior of the one–dimensional simplex, $\tilde{\mu}$ is separated from $\mu = 0$ and we have one more stationary point in which rational and noise traders coexist. In this case, either $\mu = 0$ or $\mu = 1$ must be an attractor for the replicator dynamics, *but not both.* Moreover, there is also one attracting and one repelling stationary point where rational and noise traders coexist.

insight into our results comes from the geometry of our model, which we now discuss.

Let us begin from the case $\beta_t^n > 1/2$.

For $\rho_t > 2\delta\sigma_\rho^2/(1+r)^2$, $\hat{\mu}$ lies outside the one–dimensional simplex, to the right of $\mu = 1$, which is a repellor. At $\rho_t = 2\delta\sigma_\rho^2/(1+r)^2$, $\hat{\mu}$ crosses the frontier of the one–dimensional simplex and coincides with $\mu = 1$ (see Figure 1a).

For $0 < \rho_t < 2\delta\sigma_\rho^2/(1+r)^2$, $\hat{\mu}$ belongs to the interior of the one–dimensional simplex. A new stationary point which was previously 'captured' by $\mu = 1$ now emerges. Moreover, since in this range both $\mu = 1$ and $\mu = 0$ are attractors, $\hat{\mu}$ must be a repellor (see Figure 1b). In other words, $\hat{\mu}$ is the separatrix between the attraction basins of the two stable stationary points $\mu = 0$ and $\mu = 1$.

[insert Figure 1 about here]

As $\rho_t$ comes closer to 0, $\hat{\mu}$ comes closer to $\mu = 0$. This means that the basin of attraction of $\mu = 0$ is shrinking and that of $\mu = 1$ is stretching. At $\rho_t = 0$, $\hat{\mu}$ is now 'captured' by $\mu = 0$, whose basin of attraction has completely disappeared; $\mu = 0$ is therefore now a repellor (see Figure 1c).

We come now to the case $\beta_t^n < 1/2$.

Again as before, for $\rho_t > 2\delta\sigma_\rho^2/(1+r)^2$, $\hat{\mu}$ lies outside the unit simplex, at the right of $\mu = 1$, which, however, is now an attractor. At $\rho_t = 2\delta\sigma_\rho^2/(1+r)^2$, $\hat{\mu}$ crosses $\mu = 1$ (see Figure 2a).

As $0 < \rho_t < 2\delta\sigma_\rho^2/(1+r)^2$, $\hat{\mu}$ enters the interior of the unit simplex. $\hat{\mu}$ is now an attractor, whose basin of attraction coincides with the interior of the unit simplex; accordingly, $\mu = 0$ and $\mu = 1$ are repellors (see Figure 2b). In other words, as $\hat{\mu}$ crosses $\mu = 1$, it 'steals' the stability of the latter and turns it into a repellor.

[insert Figure 2 about here]

Finally, as $\rho_t$ crosses 0, $\hat{\mu}$ crosses $\mu = 0$, which now becomes an attractor; i.e., for $\rho_t < 0$ not only $\mu = 0$ absorbs $\hat{\mu}$ but also 'steals' in turn the basin of attraction of the latter, thus becoming an attractor itself (see Figure 2c).

To sum up, as $\rho_t$ decreases along the real line two different patterns of behavior emerge for the cases $\beta_t^n > 1/2$ and $\beta_t^n < 1/2$. In the former, we observe the continuous stretching of the basin of attraction of $\mu = 1$ and, consequently, the shrinking of that of $\mu = 0$ (the opposite is of course true if $\rho_t$ increases); in the latter, we observe two exchanges of stability, first from $\mu = 1$ to $\hat{\mu}$ and, subsequently, from $\hat{\mu}$ to $\mu = 0$ (once again, the order is reversed if $\rho_t$ increases).

So far for the technical meaning of our results. As to their interpretation, the first thing to remark is that Proposition 2 draws a basic distinction between the case where rational traders *initially* hold the majority of the noisy stock and the case where noise traders do. When noise traders initially hold the minority of the noisy stock, it is required that the misperception in period $t$ be bullish if noise traders are to be observed in equilibrium at the beginning of period $t+1$. More specifically, if it is only moderately bullish in the sense made precise by Proposition 2, rational and noise traders coexist in equilibrium; if they are very bullish, the market is eventually made up of noise traders. When noise traders initially hold the majority of the noisy stock, it is instead required that the misperception be bearish or only moderately bullish. In the former case, only noise traders are observed in equilibrium; in the latter, the same happens if the initial proportion of noise traders is high enough; otherwise, only rational traders are observed. Finally, if the misperception is very bullish, noise traders are never observed in equilibrium.

The economic sense of these conditions is rather clear [see also the discussion in DSSW (1990b; sections IC, IIA)]. Assume that noise traders initially hold the majority of the noisy stock ($\beta_t^n > 1/2$). If noise traders are currently very bullish and hold the majority of the stock, its price rises considerably and therefore reduces noise traders' return to risk bearing substantially. This is related to what DSSW call the 'price pressure' effect. Moreover, if noise traders are currently very bullish and hold a lot of the stock, they run a substantial risk of suffering from a large capital loss; in the DSSW terminology, this is the result of the 'buy high sell low' effect. As a result, no noise traders will be observed in equilibrium. If on the other hand the current misperception is only moderately bullish, and if the initial proportion of noise traders is large enough, the so called 'create space' effect prevails: Price risk is not as high as before, and it is shared among a number of traders that is large enough to make noise trading attractive. If on the other hand the initial proportion of noise traders is small and nevertheless they hold the majority of the noisy stock, even with moderately bullish misperceptions noise trading is still a very risky business because price risk must be shared among a small number of traders. Consequently, no noise traders are observed in equilibrium; i.e., the 'create space' effect is still overwhelmed by the 'price pressure' and 'buy high sell low' effects. Finally, if noise traders are currently bearish and nevertheless hold the majority of the stock, noise trading becomes very attractive because there is now the possibility of large capital gains at a relatively low risk. This possibility accounts for the disappearance of rational traders in equilibrium; it conflicts somewhat, however, with the fourth and last effect considered by DSSW in their paper, namely the 'hold more' effect. Equation (5) above suggests that, on the average, noise traders hold relatively more of the noisy stock when their average misperceptions are bullish than when they are bearish; a consequence of this fact is that the difference in expected returns between noise and rational trading can never be positive if noise traders have bearish expectations on the average, because in this case they hold a relatively low amount of the noisy stock and the returns to risk bearing accrue more to rational than to noise traders [see DSSW (1990b), pp. 714–5]. This of course suggests that noise traders should eventually disappear when their misperceptions are bearish. In our dynamic model, however, noise traders may 'hold more' of the noisy stock than rational traders despite their *currently* bearish misperception; it is in fact true that whenever this happens, *rational* traders disappear in equilibrium. To understand this difference between our results and those of DSSW, it is important to stress how average misperceptions, and therefore the 'hold more' effect itself, play no role in the determination of the equilibrium distribution of types in our model; only the *current* misperceptions matter. As a consequence, the fact that 'on the average' noise traders buy relatively less of the noisy stock when their expectations are bearish than when they are bullish has no clear implication in terms of the final outcome of each single round of fictitious trade. Another reason for the difference between our results and those of DSSW is that, as we have already remarked, the driving force of our dynamical model is not the difference in expected returns, but the difference between the expected return to each type and the average expected return, which gives rise to a more complex dynamic interaction than that of the DSSW model.

On the basis of the above discussion, the interpretation of the stability conditions for the case $\beta_t^n < 1/2$ is not difficult. When noise traders initially hold the minority of the noisy stock, if their misperception is very bullish noise trading may create its own space without increasing price risk excessively; this makes noise trading very attractive and therefore no rational traders are observed in equilibrium. If the misperception is only moderately bullish, it is necessary that the expected returns to noise trading, which are now relatively modest, are shared among a relatively small number of traders for it to be attractive. Hence, rational and noise traders coexist in equilibrium. Finally, if the misperception is bearish, noise trading is not profitable because traders now face a low chance of a capital gain while still facing a non-negligible amount of price risk; hence, no noise traders are observed in equilibrium.

## 7. Conclusion: Dynamics matters.

It is a standard result that if the payoff function is linear in the distribution of population types, any ESS is an asymptotically stable state for the replicator dynamics [see Hofbauer, Schuster and Sygmund (1979)]. Thus, whenever the above linearity assumption is met, static analysis based on the ESS solution concept tells us a good deal about the dynamic behavior of types under a replicator dynamics. This assumption is unfortunately not met in our model, as it can be readily checked from equations (11), (12): The variability of noise traders' misperceptions causes a quadratic dependence of types' returns on the distribution of types. As a consequence, on an a priori basis there is no reason to suspect that a static evolutionary analysis based on the ESS solution concept may be a good substitute for a dynamic one. Our results prove that this suspect is indeed a reality. The predictions of the static and the dynamic analysis about the eventual disappearance of noise traders in equilibrium are in fact very different; in fact, a static evolutionary analysis merely replicates some of the findings of DSSW (1990b).

Let us see this point in more detail. We have shown that $\mu = 0$ is an ESS whatever the noise traders' misperceptions; however, it is not always an attractor for the replicator dynamics. Also, we have seen that $\mu = 1$ is an ESS for $0 < \rho_t < 2\delta\sigma_\rho^2/(1+r)^2$; dynamic analysis reveals, however, that when the current noise traders' misperception is in the above mentioned range $\mu = 1$ is an attractor for the replicator dynamics only if a) noise traders hold the majority of the noisy stock; and b) the initial proportion of noise traders is large enough (more specifically, larger than $\hat{\mu}$)[18]. Moreover, whereas static analysis rules out the possibility that noise traders are observed in equilibrium if their misperceptions are bearish, dynamic analysis shows that *only* noise traders, however bearish, will be observed in equilibrium if noise traders initially hold the majority of the noisy stock. Finally, although no ESS where rational and noise traders coexist can be found, coexistence of rational and noise trading can emerge as a stable equilibrium of the replicator dynamics for a certain range of values of noise traders' misperception and for a certain initial distribution of the noisy stock between rational and noise traders. (It is worth remarking that, when rational and noise traders coexist in equilibrium, the actual proportion of the two types varies with the current noise traders' misperception $\rho_t$.) In fact, the very same distribution of types $\hat{\mu}$ which can be an attractor for the replicator dynamics was also a candidate for an ESS, but it did not pass the test.

The answers we get from a dynamic evolutionary analysis are, apparently, much more precise and insightful than the ones provided by a static one. We therefore conclude that only a dynamic analysis allows us to appreciate fully the conditions for noise traders' permanence in a stock market. Our ambition is now that of framing the results of this paper into a broader context where the difficult question of the eventual survival of noise traders in the long run is tackled in full generality, along the lines described in section 2. This hard task we leave for future research.

[18]This second requirement is also present, even if somewhat loosely, in the static ESS analysis, which considers *small* perturbations of $\mu = 1$.

## Appendix

### 1. Proof of Proposition 1.

*Part a).*

From equation (8), one has

$$r + E_t(p_{t+1}) - (1+r)p_t = \frac{2\delta\mu^2\sigma_\rho^2}{(1+r)^2} - \mu\rho_t \tag{A1}$$

It is clear that when the population is all made up of rational traders, $\mu = 0$ and, from (A1), one has that $E_t[\pi(\phi, \gamma[1 \cdot \phi])] = 0$ (remember that the population is large). On the other hand, as to $E_t[\pi(\psi, \gamma[1 \cdot \phi])]$ one has that

$$E_t[\pi(\psi, \gamma[1 \cdot \phi])] = \lim_{\mu \to 0} E_t\{\beta_t^n \cdot [r + E_t(p_{t+1}) - (1+r)p_t]\} \tag{A2}$$

Remembering that $r + E_t(p_{t+1}) - (1+r)p_t = 2\delta\mu^2\sigma_\rho^2/(1+r)^2 - \mu\rho_t$ [this is equation (16) of DSSW(1990b)], we obtain through simple algebraic manipulations

$$E_t[\pi(\psi, \gamma[1 \cdot \phi])] =$$

$$\lim_{\mu \to 0} \frac{2\delta\sigma_\rho^2\mu^2}{(1+r)^2} + (1-2\mu)\rho_t - \frac{(1-\mu)(1+r)^2\rho_t^2}{2\delta\sigma_\rho^2\mu} \tag{A3}$$

In order to prove part a) of the Proposition, recalling conditions E1–E2, we must simply prove that $E_t[\pi(\psi, \gamma[1 \cdot \phi])] - E_t[\pi(\phi, \gamma[1 \cdot \phi])]$ remains strictly negative for every possible value of $\rho_t$. Given that the latter term is zero, this quantity is simply equal to (A3). As $\mu \to 0$, the first term of (A3) tends to zero, whereas the second tends to $\rho_t$. The third term is more complex to analyze. It is however easy to check that

$$\lim_{\mu \to 0} \quad -\frac{(1-\mu)(1+r)^2\rho_t^2}{2\delta\sigma_\rho^2\mu} = -\infty \tag{A4}$$

Since $E_t[\pi(\psi, \gamma[1 \cdot \phi])]$ is $C^\infty$ for $\mu \in (0, 1]$, we can therefore conclude that it must stay below zero in a (right) neighborhood of $\mu = 0$ whatever the value of $\rho_t$. This completes the proof.

*Part b)*

Following the same line of reasoning of part a) of the proof, one can show that

$$E_t[\pi(\phi, \gamma[1 \cdot \psi])] = \frac{2\delta\sigma_\rho^2\mu^2}{(1+r)^2} - 2\mu\rho_t + \frac{(1+r)^2\rho_t^2}{2\delta\sigma_\rho^2} \tag{A5}$$

By the same token, it turns out that

$$E_t[\pi(\psi, \gamma[1 \cdot \psi])] = \frac{2\delta\sigma_\rho^2}{(1+r)^2} - \rho_t \tag{A6}$$

Thus, one can write the quantity $E_t[\pi(\phi, \gamma[1 \cdot \psi])] - E_t[\pi(\psi, \gamma[1 \cdot \psi])]$ as

$$\frac{2\delta\sigma_\rho^2}{(1+r)^2}(\mu^2 - 1) - (2\mu - 1)\rho_t + \frac{(1+r)^2\rho_t^2}{2\delta\sigma_\rho^2} \tag{A7}$$

Now, from the definition of an ESS it follows that the values of $\rho_t$ for which $\mu = 1$ is an ESS are those values for which $E_t[\pi(\phi, \gamma[1 \cdot \psi])] - E_t[\pi(\psi, \gamma[1 \cdot \psi])]$ is less then zero for $\mu(\rho_t)$ in a (left) neighborhood of one. The limiting values $\rho_t^*$ of $\rho_t$ for which this holds true will be those for which $\mu(\rho_t^*) = 1$. Clearly, given the notation adopted in the definition of an ESS, one has $\epsilon^* = \mu(\rho_t^*)$. Setting $\mu = 1$ in (A7) gives

$$-\rho_t + \frac{(1+r)^2\rho_t^2}{2\delta\sigma_\rho^2} < 0 \tag{A8}$$

It is easy to check that (A8) is verified whenever

$$0 < \rho_t < \frac{2\delta\sigma_\rho^2}{(1+r)^2} \tag{A9}$$

This completes the proof.

*Part c)*

Let us assume that there is an ESS at $\hat{\mu}$, $0 < \hat{\mu} < 1$. Then a necessary condition for the existence of a 'mixed' ESS [also known as the Bishop–Cannings theorem; see Maynard Smith (1982)] is that, for every couple of strategies $\phi$, $\psi$ which are played with positive frequence at the ESS one must have

$$E_t[\pi(\phi, \gamma[\hat{\mu} \cdot \psi \oplus (1-\hat{\mu}) \cdot \phi])] = E_t[\pi(\psi, \gamma[\hat{\mu} \cdot \psi \oplus (1-\hat{\mu}) \cdot \phi])] \quad \text{(A10)}$$

Simple algebraic manipulations show that this implies in our case that

$$\frac{(1+r)^2 \rho_t^2}{2\delta\sigma_\rho^2} + \frac{(1-\hat{\mu})(1+r)^2\rho_t^2}{2\delta\sigma_\rho^2\hat{\mu}} - \rho_t = 0 \quad \text{(A11)}$$

from which it easily follows that

$$\hat{\mu} = \frac{(1+r)^2 \rho_t}{2\delta\sigma_\rho^2} \quad \text{(A12)}$$

is the unique candidate to a 'mixed' ESS. To decide whether or not it is an ESS we have to test its stability w.r.t. small perturbations of the distribution of population types. This amounts to imposing some sort of stability condition on expected payoffs in a neighborhood of $\hat{\mu}$. Condition E2 in the standard definition of an ESS will not do, however, because all feasible strategies are played with positive frequency at $\hat{\mu}$. The choice of the stability condition is therefore somewhat arbitrary; a reasonable candidate is the following:

$$\frac{dE_t[\pi(\phi, \gamma[\hat{\mu} \cdot \psi \oplus (1-\hat{\mu}) \cdot \phi])]}{d\mu} > 0 \quad \text{(A13)}$$

$$\frac{dE_t[\pi(\psi, \gamma[\hat{\mu} \cdot \psi \oplus (1-\hat{\mu}) \cdot \phi])]}{d\mu} < 0 \quad \text{(A14)}$$

The interpretation of these conditions is as follows: If a type of trader tends to be in excess of the equilibrium proportion in a small neighborhood of $\hat{\mu}$, the return from adopting that type descreases and the return from adopting the other type increases; this should restore the equilibrium proportion.

As to (A13), the condition is equivalent to

$$\frac{4\delta\sigma_\rho^2\hat{\mu}}{(1+r)^2} - 2\rho_t > 0 \tag{A15}$$

Plugging (A12) into (A15), the condition boils down to $2\rho_t - 2\rho_t = 0$, i.e., condition (A13) is satisfied only weakly.

On the other hand, (A14) implies

$$\frac{4\delta\sigma_\rho^2\hat{\mu}}{(1+r)^2} - 2\rho_t + \frac{(1+r)^2\rho_t^2}{2\delta\sigma_\rho^2\hat{\mu}^2} < 0 \tag{A16}$$

Plugging (A12) into (A16), the condition becomes

$$\frac{2\delta\sigma_\rho^2}{(1+r)^2} < 0 \tag{A17}$$

which is never met.

We can therefore conclude that $\hat{\mu}$ fails the stability test for an ESS. This completes the proof.

## 2. Proof of Proposition 2.

Our replicator dynamics takes place on the one–dimensional simplex. This rules out the possibility of cyclic and of chaotic behavior that require, respectively, a two–dimensional and a three–dimensional state space. Besides limiting considerably the number of possible behaviors of the dynamics, the fact that the state space is one–dimensional greatly simplifies the analysis. It is easy to understand that the global behavior of the dynamics on a one–dimensional state space is easily reconstructed by 'glueing' together local behaviors in the neighborhoods of the stationary points. This feature of the model is a consequence of the fact that only one type of noise traders exists. With more than one type of noise traders, the state space would have a dimension larger than one and the global analysis of the dynamics would be considerably more difficult.

It is moreover easy to check that our dynamics is smooth, i.e., it is differentiable up to any order. This implies that a complete characterization of local behavior is provided by the value of the gradient at the stationary points [see e.g. Hirsch and Smale (1974)]. The gradient $J_\eta(\mu)$ of the dynamics (15) is given by:

$$J_\eta(\mu) \equiv \frac{\partial \eta}{\partial \mu}(\mu) = (2\beta_t^n - 1)\{(2\mu - 3\mu^2)[\frac{2\delta\mu\sigma_\rho^2}{(1+r)^2} - \rho_t] + (\mu^2 - \mu^3)\frac{2\delta\sigma_\rho^2}{1+r}\} \tag{A18}$$

$\mu = 0$ is a critical point for the dynamics; i.e., $J_\eta(0) = 0$. It is however a nondegenerate critical point, i.e., the second derivative of $\eta$ evaluated in $\mu = 0$ is nonzero. This means that the local properties of $\eta$ at zero are determined by the second derivative of $\eta$ at $\mu = 0$, that we write $J_\eta^2(\mu)$ and is found to be

$$J_\eta^2(0) \equiv \frac{\partial^2 \eta}{\partial \mu}(0) = (2\beta_t^n - 1)\rho_t \tag{A19}$$

As a consequence, one has that $J_\eta^2(0) > 0$, and thus $\mu = 0$ is a repellor, if either

$$\begin{cases} \beta_t^n > \frac{1}{2} \\ \rho_t < 0 \end{cases} \tag{A20}$$

or

$$\begin{cases} \beta_t^n < \frac{1}{2} \\ \rho_t > 0 \end{cases} \tag{A21}$$

If $J_\eta^2(0) > 0$, $\mu = 0$ is locally unstable; otherwise, it is locally stable. If $J_\eta^2(0) < 0$, $\mu = 0$ is locally stable[19].

When $\rho_t = 0$, however, $J_\eta^2(0) = 0$, $\mu = 0$ is a degenerate critical point and we have to look at the third derivative of $\eta$ at zero. Routine calculations show that

---

[19]A geometric intuition for these standard mathematical conditions is the following. If $J_\eta^2(0) > 0$, the graph of $\eta$ cuts the horizontal axis 'from below'; in the reverse case, it cuts the axis 'from above'. See also Figures 1–2 in the text.

$$J_\eta^3(0) = 4(2\beta_t^n - 1)\frac{\delta\sigma_\rho^2}{1+r}[\frac{2}{1+r} + 1] \tag{A22}$$

As before, $\mu = 0$ is locally unstable if $J_\eta^3(0) > 0$, and locally stable in the opposite case[20]. Therefore, we have that if $\rho_t = 0$, $\mu = 0$ is locally stable whenever $\beta_t^n > 1/2$, and locally unstable whenever $\beta_t^n < 1/2$.

So far for $\mu = 0$. We have now to calculate the gradient of $\eta$ at $\mu = 1$. One obtains

$$J_\eta(1) = (1 - 2\beta_t^n)[\frac{2\delta\sigma_\rho^2}{(1+r)^2} - \rho_t] \tag{A23}$$

from which it follows that if $\rho_t \neq 2\delta\sigma_\rho^2/(1+r)^2$, $J_\eta(1) \neq 0$ and $\mu = 1$ is a regular point of $\eta$; more specifically, $J_\eta(1) > 0$ if either

$$\begin{cases} \beta_t^n > \frac{1}{2} \\ \rho_t > \frac{2\delta\sigma_\rho^2}{(1+r)^2} \end{cases} \tag{A24}$$

or

$$\begin{cases} \beta_t^n < \frac{1}{2} \\ \rho_t < \frac{2\delta\sigma_\rho^2}{(1+r)^2} \end{cases} \tag{A25}$$

If however $\rho_t = 2\delta\sigma_\rho^2/(1+r)^2$, $\mu = 1$ is a critical point and we have to check the sign of $J_\eta^2(1)$. One obtains

$$J_\eta^2(1) = 2(1 - 2\beta_t^2)\frac{\delta\sigma_\rho^2}{1+r}[\frac{1}{1+r} + 1] \tag{A26}$$

which implies that $J_\eta^2(1) > 0$ for $\beta_t^n < 1/2$.

Finally, we should compute the gradient of $\eta$ at $\hat{\mu}$. This computation is rather cumbersome; it is however not necessary since it is easily checked that the local geometry of the dynamics at $\hat{\mu}$ can be reconstructed from the information we already possess when $\hat{\mu}$ it lies in the interior of the one–dimensional simplex.

As to the special case $\beta_t^n = 1/2$, a simple inspection of equation (17) shows that in this case $\eta(\mu) \equiv 0$, i.e., there is actually no dynamics

[20] Geometric considerations run as before.

at all and the system never leaves its initial position $\mu_0$, whatever it is. This completes the proof of Proposition 2.

## References

Blume, Lawrence and David Easley, 1989, Wealth dynamics and the market selection hypothesis, mimeo, Cornell University.

Crawford, Vincent P., 1991, An 'evolutionary' interpretation of Van huyck, Battalio and Beil' s experimental results on coordination, Games and Economic Behavior 3, 25–59.

Day, Richard H. and Weihong Huang, 1990, Bulls, bears and market sheep, Journal of Economic Behavior and Organization 14, 299–329.

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers and Robert J. Waldmann, 1989, The size and incidence of the losses from noise trading, Journal of Finance 44, 681–696.

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers and Robert J. Waldmann, 1990a, Positive feedback investment strategies and destabilizing rational speculation, Journal of Finance 45, 379–395.

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers and Robert J. Waldmann, 1990b, Noise trader risk in financial markets, Journal of Political Economy 98, 703–738.

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers and Robert J. Waldmann, 1991, The survival of noise traders in financial markets, Journal of Business 64, 1–19.

Frankel, J.A. and K.A. Froot, 1986, The dollar as an irrational speculative bubble: The tale of fundamentalists and chartists, Marcus Wallenberg Papers on International Finance 1, 51–62.

Friedman, Milton, 1953, The case for flexible exchange rates, in: Essays in positive economics (The University of Chicago Press, Chicago).

Golubitsky, Martin and Victor Guillemin, 1973, Stable mappings and their singularities (Springer Verlag, Berlin).

Hicks, John R., 1946, Value and capital, second edition (Clarendon Press, Oxford).

Hirsch, Morris W. and Stephen Smale, 1974, Differential equations, dynamical systems and linear algebra (Academic Press, New York).

Hofbauer, Josef and Karl Sygmund, 1988, Dynamical systems and the theory of evolution (Cambridge University Press, Cambridge).

Hofbauer, Josef, Peter Schuster and Karl Sygmund, 1979, A note on evolutionary stable strategies and game dynamics, Journal of Theoretical Biology 81, 609–612.

Kirman, Alan, 1991a, Epidemics of opinions and speculative bubbles in financial markets, in M. Taylor (ed.), Money and financial markets (Macmillan, London).

Kirman, Alan, 1991b, Ants, rationality and recruitment, mimeo, European University Institute.

Leahy, John, 1989, On asset market behavior: The implications and evolutionary stability of noise trading, unpublished M.A. dissertation, University of Warwick.

Lucas, Robert E. Jr., 1986, Adaptive behavior and economic theory, The Journal of Business 59, S401–S426.

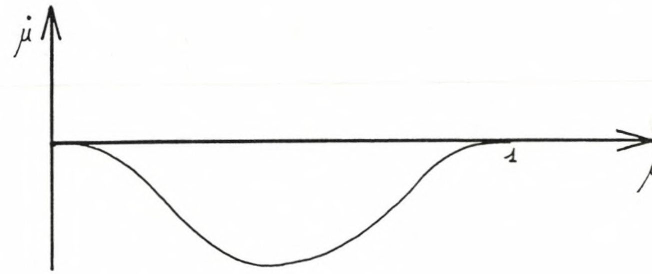
Maynard Smith, John, 1982, Evolution and the theory of games (Cambridge University Press, Cambridge).

Sacco, Pier Luigi, 1991a, Projectual vs. adaptive behavior: A comment on Heiner, Journal of Economic Behavior and Organization, forthcoming.

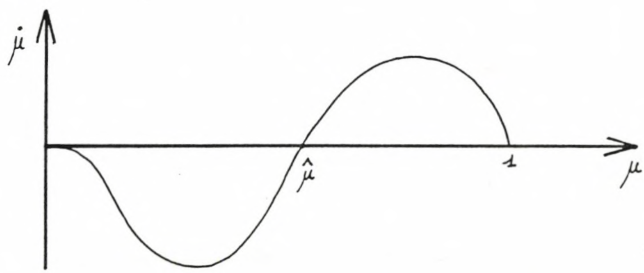
Sacco, Pier Luigi, 1991b, Stock market behavior: What theoretical framework (if any?), International Economic Journal, forthcoming.

Samuelson, P.A., 1947, Foundations of economic analysis (Harvard University Press, Cambridge).

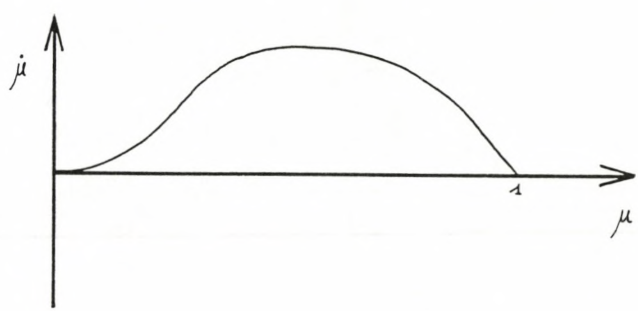
Vega Redondo, Fernando, 1988, Economics from an evolutionary viewpoint. Part I: Models of evolution in theoretical biology and game theory, unpublished lecture notes, University of Alicante.




(a) $\rho_t = 2\delta\sigma_\rho^2/(1+r)^2$

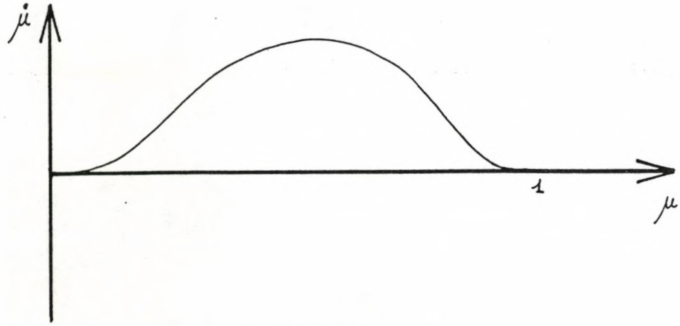



(b) $0 < \rho_t < 2\delta\sigma_\rho^2/(1+r)^2$

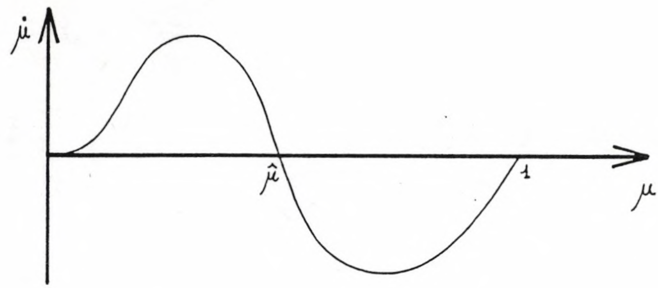



(c) $\rho_t = 0$

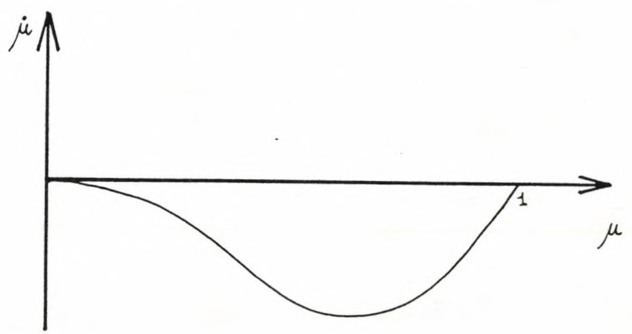
Fig. 1. $\beta_t^n > 1/2$




(a) $\rho_t = 2\delta\sigma_\rho^2/(1+r)^2$




(b) $0 < \rho_t < 2\delta\sigma_\rho^2/(1+r)^2$




(c) $\rho_t = 0$

Fig. 2. $\beta_t^n < 1/2$

# EUI WORKING PAPERS

EUI Working Papers are published and distributed by the
European University Institute, Florence

Copies can be obtained free of charge – depending on the availability of stocks – from:

The Publications Officer
European University Institute
Badia Fiesolana
I-50016 San Domenico di Fiesole (FI)
Italy

**Please use order form overleaf**

# Publications of the European University Institute

To The Publications Officer
European University Institute
Badia Fiesolana
I-50016 San Domenico di Fiesole (FI)
Italy

From Name . . . . . . . . . . . . . . . . . . . . . . .

Address . . . . . . . . . . . . . . . . . . . . . .
. . . . . . . . . . . . . . . . . . . . . . . . . . .
. . . . . . . . . . . . . . . . . . . . . . . . . . .
. . . . . . . . . . . . . . . . . . . . . . . . . . .
. . . . . . . . . . . . . . . . . . . . . . . . . . .

❒ Please send me a complete list of EUI Working Papers
❒ Please send me a complete list of EUI book publications
❒ Please send me the EUI brochure Academic Year 1992/93

Please send me the following EUI Working Paper(s):

No, Author . . . . . . . . . . . . . . . . . . . . . . . . . . .
*Title:* . . . . . . . . . . . . . . . . . . . . . . . . . . .
No, Author . . . . . . . . . . . . . . . . . . . . . . . . . . .
*Title*: . . . . . . . . . . . . . . . . . . . . . . . . . . .
No, Author . . . . . . . . . . . . . . . . . . . . . . . . . . .
*Title:* . . . . . . . . . . . . . . . . . . . . . . . . . . .
No, Author . . . . . . . . . . . . . . . . . . . . . . . . . . .
*Title:* . . . . . . . . . . . . . . . . . . . . . . . . . . .

Date . . . . . . . . . . . . . .

Signature . . . . . . . . . . . . . . . . . . . . . . . .

## Working Papers of the Department of Economics Published since 1989

**89/370**
B. BENSAID/ R.J. GARY-BOBO/
S. FEDERBUSCH
The Strategic Aspects of Profit Sharing in the Industry

**89/374**
Francisco S. TORRES
Small Countries and Exogenous Policy Shocks

**89/375**
Renzo DAVIDDI
Rouble Convertibility: A Realistic Target

**89/377**
Elettra AGLIARDI
On the Robustness of Contestability Theory

**89/378**
Stephen MARTIN
The Welfare Consequences of Transaction Costs in Financial Markets

**89/381**
Susan SENIOR NELLO
Recent Developments in Relations Between the EC and Eastern Europe

**89/382**
Jean GABSZEWICZ/ Paolo GARELLA/
Charles NOLLET
Spatial Price Competition With Uninformed Buyers

**89/383**
Benedetto GUI
Beneficiary and Dominant Roles in Organizations: The Case of Nonprofits

**89/384**
Agustín MARAVALL/ Daniel PEÑA
Missing Observations, Additive Outliers and Inverse Autocorrelation Function

**89/385**
Stephen MARTIN
Product Differentiation and Market Performance in Oligopoly

**89/386**
Dalia MARIN
Is the Export-Led Growth Hypothesis Valid for Industrialized Countries?

**89/387**
Stephen MARTIN
Modeling Oligopolistic Interaction

**89/388**
Jean-Claude CHOURAQUI
The Conduct of Monetary Policy: What have we Learned From Recent Experience

**89/390**
Corrado BENASSI
Imperfect Information and Financial Markets: A General Equilibrium Model

**89/394**
Serge-Christophe KOLM
Adequacy, Equity and Fundamental Dominance: Unanimous and Comparable Allocations in Rational Social Choice, with Applications to Marriage and Wages

**89/395**
Daniel HEYMANN/ Axel LEIJONHUFVUD
On the Use of Currency Reform in Inflation Stabilization

**89/400**
Robert J. GARY-BOBO
On the Existence of Equilibrium Configurations in a Class of Asymmetric Market Entry Games *

**89/402**
Stephen MARTIN
Direct Foreign Investment in The United States

**89/413**
Francisco S. TORRES
Portugal, the EMS and 1992: Stabilization and Liberalization

**89/416**
Joerg MAYER
Reserve Switches and Exchange-Rate Variability: The Presumed Inherent Instability of the Multiple Reserve-Currency System

**89/417**
José P. ESPERANÇA/ Neil KAY
Foreign Direct Investment and Competition in the Advertising Sector: The Italian Case

* Working Paper out of print

**89/418**
Luigi BRIGHI/ Mario FORNI
Aggregation Across Agents in Demand Systems

**89/420**
Corrado BENASSI
A Competitive Model of Credit Intermediation

**89/422**
Marcus MILLER/ Mark SALMON
When does Coordination pay?

**89/423**
Marcus MILLER/ Mark SALMON/
Alan SUTHERLAND
Time Consistency, Discounting and the Returns to Cooperation

**89/424**
Frank CRITCHLEY/ Paul MARRIOTT/
Mark SALMON
On the Differential Geometry of the Wald Test with Nonlinear Restrictions

**89/425**
Peter J. HAMMOND
On the Impossibility of Perfect Capital Markets

**89/426**
Peter J. HAMMOND
Perfected Option Markets in Economies with Adverse Selection

**89/427**
Peter J. HAMMOND
Irreducibility, Resource Relatedness, and Survival with Individual Non-Convexities

* * *

**ECO No. 90/1****
Tamer BASAR and Mark SALMON
Credibility and the Value of Information Transmission in a Model of Monetary Policy and Inflation

**ECO No. 90/2**
Horst UNGERER
The EMS – The First Ten Years
Policies – Developments – Evolution

**ECO No. 90/3**
Peter J. HAMMOND
Interpersonal Comparisons of Utility: Why and how they are and should be made

** Please note: As from January 1990, the EUI Working Papers Series is divided into six sub-series, each series will be numbered individually (e.g. EUI Working Paper LAW No. 90/1).

**ECO No. 90/4**
Peter J. HAMMOND
A Revelation Principle for (Boundedly) Bayesian Rationalizable Strategies

**ECO No. 90/5**
Peter J. HAMMOND
Independence of Irrelevant Interpersonal Comparisons

**ECO No. 90/6**
Hal R. VARIAN
A Solution to the Problem of Externalities and Public Goods when Agents are Well-Informed

**ECO No. 90/7**
Hal R. VARIAN
Sequential Provision of Public Goods

**ECO No. 90/8**
T. BRIANZA, L. PHLIPS and J.F. RICHARD
Futures Markets, Speculation and Monopoly Pricing

**ECO No. 90/9**
Anthony B. ATKINSON/ John MICKLEWRIGHT
Unemployment Compensation and Labour Market Transition: A Critical Review

**ECO No. 90/10**
Peter J. HAMMOND
The Role of Information in Economics

**ECO No. 90/11**
Nicos M. CHRISTODOULAKIS
Debt Dynamics in a Small Open Economy

**ECO No. 90/12**
Stephen C. SMITH
On the Economic Rationale for Codetermination Law

**ECO No. 90/13**
Elettra AGLIARDI
Learning by Doing and Market Structures

**ECO No. 90/14**
Peter J. HAMMOND
Intertemporal Objectives

**ECO No. 90/15**
Andrew EVANS/Stephen MARTIN
Socially Acceptable Distortion of Competition: EC Policy on State Aid

* Working Paper out of print

**ECO No. 90/16**
Stephen MARTIN
Fringe Size and Cartel Stability

**ECO No. 90/17**
John MICKLEWRIGHT
Why Do Less Than a Quarter of the Unemployed in Britain Receive Unemployment Insurance?

**ECO No. 90/18**
Mrudula A. PATEL
Optimal Life Cycle Saving With Borrowing Constraints: A Graphical Solution

**ECO No. 90/19**
Peter J. HAMMOND
Money Metric Measures of Individual and Social Welfare Allowing for Environmental Externalities

**ECO No. 90/20**
Louis PHLIPS/
Ronald M. HARSTAD
Oligopolistic Manipulation of Spot Markets and the Timing of Futures Market Speculation

**ECO No. 90/21**
Christian DUSTMANN
Earnings Adjustment of Temporary Migrants

**ECO No. 90/22**
John MICKLEWRIGHT
The Reform of Unemployment Compensation: Choices for East and West

**ECO No. 90/23**
Joerg MAYER
U. S. Dollar and Deutschmark as Reserve Assets

**ECO No. 90/24**
Sheila MARNIE
Labour Market Reform in the USSR: Fact or Fiction?

**ECO No. 90/25**
Peter JENSEN/
Niels WESTERGÅRD-NIELSEN
Temporary Layoffs and the Duration of Unemployment: An Empirical Analysis

**ECO No. 90/26**
Stephan L. KALB
Market-Led Approaches to European Monetary Union in the Light of a Legal Restrictions Theory of Money

**ECO No. 90/27**
Robert J. WALDMANN
Implausible Results or Implausible Data? Anomalies in the Construction of Value Added Data and Implications for Estimates of Price-Cost Markups

**ECO No. 90/28**
Stephen MARTIN
Periodic Model Changes in Oligopoly

**ECO No. 90/29**
Nicos CHRISTODOULAKIS/
Martin WEALE
Imperfect Competition in an Open Economy

* * *

**ECO No. 91/30**
Steve ALPERN/Dennis J. SNOWER
Unemployment Through 'Learning From Experience'

**ECO No. 91/31**
David M. PRESCOTT/Thanasis STENGOS
Testing for Forecastible Nonlinear Dependence in Weekly Gold Rates of Return

**ECO No. 91/32**
Peter J. HAMMOND
Harsanyi's Utilitarian Theorem: A Simpler Proof and Some Ethical Connotations

**ECO No. 91/33**
Anthony B. ATKINSON/
John MICKLEWRIGHT
Economic Transformation in Eastern Europe and the Distribution of Income

**ECO No. 91/34**
Svend ALBAEK
On Nash and Stackelberg Equilibria when Costs are Private Information

**ECO No. 91/35**
Stephen MARTIN
Private and Social Incentives to Form R & D Joint Ventures

**ECO No. 91/36**
Louis PHLIPS
Manipulation of Crude Oil Futures

**ECO No. 91/37**
Xavier CALSAMIGLIA/Alan KIRMAN
A Unique Informationally Efficient and Decentralized Mechanism With Fair Outcomes

* Working Paper out of print

**ECO No. 91/38**
George S. ALOGOSKOUFIS/
Thanasis STENGOS
Testing for Nonlinear Dynamics in Historical Unemployment Series

**ECO No. 91/39**
Peter J. HAMMOND
The Moral Status of Profits and Other Rewards: A Perspective From Modern Welfare Economics

**ECO No. 91/40**
Vincent BROUSSEAU/Alan KIRMAN
The Dynamics of Learning in Mis-Specified Models

**ECO No. 91/41**
Robert James WALDMANN
Assessing the Relative Sizes of Industry- and Nation Specific Shocks to Output

**ECO No. 91/42**
Thorsten HENS/Alan KIRMAN/Louis PHLIPS
Exchange Rates and Oligopoly

**ECO No. 91/43**
Peter J. HAMMOND
Consequentialist Decision Theory and Utilitarian Ethics

**ECO No. 91/44**
Stephen MARTIN
Endogenous Firm Efficiency in a Cournot Principal-Agent Model

**ECO No. 91/45**
Svend ALBAEK
Upstream or Downstream Information Sharing?

**ECO No. 91/46**
Thomas H. McCURDY/
Thanasis STENGOS
A Comparison of Risk-Premium Forecasts Implied by Parametric Versus Nonparametric Conditional Mean Estimators

**ECO No. 91/47**
Christian DUSTMANN
Temporary Migration and the Investment into Human Capital

**ECO No. 91/48**
Jean-Daniel GUIGOU
Should Bankruptcy Proceedings be Initiated by a Mixed Creditor/Shareholder?

**ECO No. 91/49**
Nick VRIEND
Market-Making and Decentralized Trade

**ECO No. 91/50**
Jeffrey L. COLES/Peter J. HAMMOND
Walrasian Equilibrium without Survival: Existence, Efficiency, and Remedial Policy

**ECO No. 91/51**
Frank CRITCHLEY/Paul MARRIOTT/
Mark SALMON
Preferred Point Geometry and Statistical Manifolds

**ECO No. 91/52**
Costanza TORRICELLI
The Influence of Futures on Spot Price Volatility in a Model for a Storable Commodity

**ECO No. 91/53**
Frank CRITCHLEY/Paul MARRIOTT/
Mark SALMON
Preferred Point Geometry and the Local Differential Geometry of the Kullback-Leibler Divergence

**ECO No. 91/54**
Peter MØLLGAARD/
Louis PHLIPS
Oil Futures and Strategic Stocks at Sea

**ECO No. 91/55**
Christian DUSTMANN/
John MICKLEWRIGHT
Benefits, Incentives and Uncertainty

**ECO No. 91/56**
John MICKLEWRIGHT/
Gianna GIANNELLI
Why do Women Married to Unemployed Men have Low Participation Rates?

**ECO No. 91/57**
John MICKLEWRIGHT
Income Support for the Unemployed in Hungary

**ECO No. 91/58**
Fabio CANOVA
Detrending and Business Cycle Facts

**ECO No. 91/59**
Fabio CANOVA/
Jane MARRINAN
Reconciling the Term Structure of Interest Rates with the Consumption Based ICAP Model

**ECO No. 91/60**
John FINGLETON
Inventory Holdings by a Monopolist Middleman

* Working Paper out of print

**ECO No. 92/61**
Sara CONNOLLY/John MICKLEWRIGHT/Stephen NICKELL
The Occupational Success of Young Men Who Left School at Sixteen

**ECO No. 92/62**
Pier Luigi SACCO
Noise Traders Permanence in Stock Markets: A Tâtonnement Approach.
I: Informational Dynamics for the Two-Dimensional Case

* Working Paper out of print