



# Fiscal Policy and the Labor Market in a New Keynesian Framework

Zsuzsa Munkacsi

Thesis submitted for assessment with a view to obtaining the degree of  
Doctor of Economics of the European University Institute

Florence, 17 June 2016



European University Institute  
**Department of Economics**

Fiscal Policy and the Labor Market  
in a New Keynesian Framework

Zsuzsa Munkacsi

Thesis submitted for assessment with a view to obtaining the degree of  
Doctor of Economics of the European University Institute

**Examining Board**

Prof. Fabio Canova, EUI, Supervisor

Prof. Alessia Campolmi, University of Verona

Prof. Evi Pappa, EUI

Prof. Shu-Chun Susan Yang, National Sun Yat-Sen University, Taiwan

© Zsuzsa Munkacsi, 2016

No part of this thesis may be copied, reproduced or transmitted without prior  
permission of the author







### **Researcher declaration to accompany the submission of written work**

I Zsuzsa Munkacsi certify that I am the author of the work Fiscal Policy and the Labor Market in a New Keynesian Framework I have presented for examination for the PhD thesis at the European University Institute. I also certify that this is solely my own original work, other than where I have clearly indicated, in this declaration and in the thesis, that it is the work of others.

I warrant that I have obtained all the permissions required for using any material from other copyrighted publications.

I certify that this work complies with the *Code of Ethics in Academic Research* issued by the European University Institute (IUE 332/2/10 (CA 297)).

The copyright of this work rests with its author [quotation from it is permitted, provided that full acknowledgement is made]. This work may not be reproduced without my prior written consent. This authorisation does not, to the best of my knowledge, infringe the rights of any third party.

### **Statement of inclusion of previous work (if applicable):**

I confirm that chapter <1> is solely my work. Also, it was published as a Discussion Paper by the Deutsche Bundesbank in May 2015 (No. 6).

I confirm that chapter <2> was jointly co-authored with Rahul Anand, Purva Khera and Magnus Saxegaard. Namely, the model used in chapter <2> was developed by the four of us. Then, using the same model, Rahul Anand and Purva Khera worked on India (they already published an International Monetary Fund Working Paper in March 2016, No. 16/47), while with Magnus Saxegaard we worked on South Africa. I confirm that I contributed to 60% of the work on chapter <2>.

I confirm that chapter <3> was jointly co-authored with Daniel Baksa and I contributed to 50% of the work.

### **Signature and Date:**

Zsuzsa Munkacsi, 8 April 2016



# Abstract

In the first chapter I calculate unemployment multipliers of fiscal policies. As an innovation, I include family firms in a New Keynesian model with search and matching frictions; they behave differently in the labor market and are differently managed. Based on European data I find that both at peak and cumulatively, unemployment reacts least when consolidation is done by increasing the value-added tax. However, this policy results in the steepest decline in consumption. Also, ignoring sectoral heterogeneity might lead to incorrect conclusions.

Next, with Magnus Saxegaard we investigate the macroeconomic impacts of deregulating the labor and product markets. The novelty of the model, which was jointly developed with Rahul Anand and Purva Khera, is the inclusion of an underground sector in an open-economy model. It is a major determinant of the sign and the magnitude of reactions. We show that in South Africa both reforms increase long-run output, although labor market reforms are more successful in decreasing unemployment. Nevertheless, there are short-term costs; which can be mitigated reform packages. Finally, we find that it is usually better to start with a labor market reform.

The last chapter focuses on southern Europe where high levels of government debt are coupled with rapid population aging. With Daniel Baksa we examine the macroeconomic effects of public old-age pension reforms and other policies under conditions of aging. As a novelty, we incorporate a shadow economy into an overlapping generations model. We find that a retirement age increase implies the lowest reduction in long-run GDP, although there are doubts about its feasibility. Impacts, in general, depend on the type of pension plan. Furthermore, when moving away from the PAYG towards a fully funded regime the pension-wage replacement rate temporarily sharply drops. The presence of informality and unemployment are important, in particular for responses of labor income tax hikes.

# Acknowledgements

Special thanks, first of all, to Fabio Canova and Evi Pappa for teaching, supervising and guiding me, and supporting me in the job market; to my co-authors Rahul Anand, Purva Khera, Magnus Saxegaard and Daniel Baksa for making our ideas to become a reality; to Martin Kliem, Laszlo Bruszt and Gabor Oblath for their (job market) support; to Sara Tropper for language correction; to Jessica Spataro, Julia Valerio, Lucia Vigna, Sarah Simonsen, Riccardo Mongiu, Thomas Bourke, Titas Budrys, Anne-Sophie Barbe and Jonas Striaukas for excellent assistance; to the Luxembourg Grant Authority and the European University Institute for financial aid; to Zsuzsa Martinovszky, Istvan Martinovszky, Mesi Papp, Moni Fejes, Vali Zsiga, Gyongyi Kormendi, Daniel Kabdebo, Daniela Vintila, Joanna Franaszek, Lyuba Gramcheva, Andi Talaber, the Kovari family, the Kiss family, Angela Abbate, Asta Kuniyoshi, Szabolcs Deak, Nikolai Staehler, Dmitrij Celov, Marius Jurgilas, Mihnea Constantinescu, Erno Szivek, Etienne Yehoue, Nir Klein, Slavi Slavov Trifonov, Sita Slavov, Tamas Briglevics, Stephane Moyen, Philippe Aghion, Ronaldo Carpio, Patrick Gruening, Virmantas Kvedaras, Endre Morvay, Igor Fedotenkov, Maurizio Fiaschetti, Louis Jaeck, Olivier Pierrard and Balazs Vilagi for our chats, discussions and comments; and, to my parents and Ida Csernyak for everything.

# Contents

|   |           |
|---|-----------|
| <b>Preface</b>  | <b>1</b>  |
| <b>1 Fiscal Austerity, Unemployment and Family Firms</b>                    | <b>3</b>  |
| 1.1 Introduction . . . . .  | 3         |
| 1.2 Modeling framework . . . . .  | 9         |
| 1.2.1 Representative household . . . . .                                    | 10        |
| 1.2.2 Intermediate good producing firms and wage bargaining . . . . .       | 13        |
| 1.2.3 Final good producing firms . . . . .                                  | 18        |
| 1.2.4 Monetary authority . . . . .  | 19        |
| 1.2.5 Government . . . . .  | 19        |
| 1.2.6 Market clearing . . . . .   | 21        |
| 1.3 Calibration . . . . .   | 22        |
| 1.4 Results . . . . .   | 26        |
| 1.4.1 Baseline results . . . . .  | 26        |
| 1.4.2 The role of sectoral heterogeneity . . . . .                          | 28        |
| 1.5 Discussion . . . . .  | 29        |
| Appendix . . . . .  | 31        |
| A Steady-state calculation of the model . . . . .                           | 31        |
| B Loglinearized model . . . . .   | 34        |
| C Robustness of multipliers . . . . .                                       | 41        |
| D Impulse response functions . . . . .                                      | 42        |
| Bibliography . . . . .  | 60        |
| <b>2 Structural Reforms, Openness and the Shadow Economy</b>                | <b>61</b> |
| 2.1 Motivation and literature . . . . .                                     | 61        |
| 2.2 Model . . . . .   | 65        |
| 2.2.1 Households . . . . .  | 66        |
| 2.2.2 Capital producer . . . . .  | 69        |
| 2.2.3 Wholesale good producers . . . . .                                    | 70        |
| 2.2.4 Retailers . . . . .   | 74        |
| 2.2.5 Rest of the world, government, monetary authority and market clearing | 76        |

|              |   |            |
|--------------|---|------------|
| 2.2.6        | The role of informality and openness . . . . .  | 79         |
| 2.3          | Calibration and Bayesian estimation . . . . .   | 82         |
| 2.4          | Long- and short-run effects of deregulation policies in South-Africa . . . . .                    | 86         |
| 2.4.1        | Long-run effects of single reforms . . . . .  | 87         |
| 2.4.2        | Short-run effects of single reforms . . . . .   | 89         |
| 2.4.3        | Reform packages . . . . .   | 91         |
| 2.4.4        | Policy sequence . . . . .   | 92         |
| 2.5          | Conclusion . . . . .  | 93         |
| Appendix     | . . . . .   | 95         |
| A            | Steady state of the model . . . . .   | 95         |
| B            | The role of shadow economy . . . . .  | 106        |
| C            | Bayesian estimation . . . . .   | 110        |
| D            | Reform packages . . . . .   | 112        |
| E            | Policy sequence . . . . .   | 117        |
| Bibliography | . . . . .   | 127        |
| <b>3</b>     | <b>Aging, Pension Reforms and the Shadow Economy</b>  | <b>127</b> |
| 3.1          | Motivation and literature . . . . .   | 127        |
| 3.2          | Model . . . . .   | 134        |
| 3.2.1        | Demography . . . . .  | 135        |
| 3.2.2        | Overlapping Generations . . . . .   | 135        |
| 3.2.3        | Firms . . . . .   | 140        |
| 3.2.4        | Monetary policy . . . . .   | 145        |
| 3.2.5        | Fiscal policy . . . . .   | 145        |
| 3.2.6        | Market clearing . . . . .   | 148        |
| 3.3          | Calibration . . . . .   | 150        |
| 3.3.1        | Steady-state and dynamic parameters . . . . .   | 150        |
| 3.3.2        | Model performance . . . . .   | 152        |
| 3.4          | Results . . . . .   | 153        |
| 3.4.1        | Macroeconomic impacts in a PAYG regime in the long- and short-run                                 | 154        |
| 3.4.2        | Comparing long- and short-run macroeconomic impacts in PAYG and<br>fully funded regimes . . . . . | 157        |
| 3.4.3        | Pension regime switch . . . . .   | 162        |
| 3.4.4        | The role of informality and unemployment . . . . .  | 164        |
| 3.5          | Conclusion and policy discussion . . . . .  | 167        |
| Appendix     | . . . . .   | 172        |
| A            | Overlapping generations' optimization . . . . .   | 172        |
| B            | The public pension systems . . . . .  | 187        |
| C            | Normalized equations . . . . .  | 190        |
| D            | Steady state of the model . . . . .   | 194        |

|                        |   |             |
|------------------------|---|-------------|
| E                      | Robustness . . . . .                            | 201         |
| F                      | Additional results on Portugal . . . . .        | 205         |
| G                      | Additional results on Italy and Spain . . . . . | 212         |
|                        | Bibliography . . . . .                          | ccxlvii     |
| <b>List of Figures</b> |   | <b>ccli</b> |
| <b>List of Tables</b>  |   | <b>cclv</b> |





# Preface

This thesis investigates the macroeconomic impacts, especially those on the labor market, of fiscal policies in a New Keynesian framework. All aspects of public policies are studied: expenditure and revenue-side policies, as well structural reforms. Furthermore, not only unemployment in general, but also shadow employment in particular are examined. So is aging which is one of the key challenges governments face today.

In the first chapter I calculate unemployment multipliers of fiscal consolidation policies in a New Keynesian framework with search and matching frictions, and, as an innovation, in the presence of sectoral heterogeneity. Family and non-family firms behave differently in the labor market and are differently managed; this latter is modeled by the inclusion of intangible capital in the family sector. The model is calibrated to match European data on countries with a large percentage of family firms in employment. I find that fiscal austerity raises unemployment. Both at peak and cumulatively, unemployment reacts least when the budget is consolidated by increasing the rate of value-added tax. At peak, the highest increase in unemployment is induced by a cut in government consumption, but, cumulatively, a hike in employees' labor income tax is just as costly. There are trade-offs, however, as the value-added tax increase results in the steepest decline in consumption. Sectoral heterogeneity is crucial; multipliers of labor income taxes and government consumption are usually biased downwards, while the consumption-tax multipliers are often larger without it.

Next, with Rahul Anand, Purva Khera and Magnus Saxegaard, we treat a topic upon which the literature is strangely silent: what are the macroeconomic impacts of deregulating the labor and product markets, incorporating a shadow economy into an open-economy dynamic general equilibrium model? We examine both the long-run effects and the transition towards the new long-run equilibrium. Furthermore, we study mixed policies and the role of policy sequence, too. The unofficial sector is a major determinant of the sign and, in particular, the magnitude of impulse responses. South Africa, an example of the emerging countries, is considered when Bayesian estimating our model. Regarding the long run, we show that both labor and product market reforms considerably increase output, although labor market reforms are more successful in decreasing unemployment. Nevertheless, there are short-term costs, for example, a decrease in household consumption, net exports or output, or a decrease in competition. Combining structural reforms, however, tends to mitigate the short-run costs that inevitably accompany all policies. Finally, we find that it is usually better to start with a labor market reform than with a product market reform.

In the last chapter, with Daniel Baksa, we turn our attention towards the fiscal consequences of aging. Southern Europe is experiencing currently a double-whammy: high levels of government debt coupled with a rapidly aging population. So, the consolidation of (pension) budgets seems inevitable. In this paper we examine the short- and long-run macroeconomic effects of public old-age pension reforms and other fiscal policies under conditions of population aging. Filling in a major gap in the literature, we incorporate a shadow economy into a New Keynesian model with overlapping generations, demography and unemployment. The model is calibrated to match annual data on Portugal, Italy and Spain. We find that if population is aging in the long-run the lowest reduction in GDP happens after a retirement age increase, but we raise some doubts about the feasibility of this policy. Macroeconomic impacts, in general, depend on the type of pension plan. Moving away from the PAYG towards a fully funded regime reduces informality in the long-run, however, during the transition the implied pension-wage replacement rate sharply drops. The presence of informality and unemployment are important, in particular for responses of labor income tax hikes.

# Chapter 1

## Fiscal Austerity, Unemployment and Family Firms

### 1.1 Introduction

Since the onset of the recent crisis, many countries have experienced a rise in government debt, thus making urgent fiscal consolidation. In particular, in more than half of the EU countries the debt-to-GDP ratio exceeds 60 percent today.<sup>1</sup> At the same time, in many countries unemployment has risen to historic heights. Fiscal consolidation further affects unemployment. The question which I address in this paper is how. Notably, I calculate unemployment fiscal multipliers that show the effects of temporary fiscal policies on the unemployment rate.

The paper contributes to the literature in three ways. First, there is a debate on the sign and the magnitude of expenditure-side unemployment fiscal multipliers, while theoretical and empirical multipliers also differ. Second, as yet there are only a few examples of revenue-side unemployment fiscal multipliers. Third, in all studies firms are homogenous. In this paper, however, multipliers are based on a New-Keynesian model with search and matching frictions, and, as an innovation, with family and non-family firms. Because empirical firm-level evidence indicates that family firms behave differently in the labor market and are differently managed than non-family firms.

First, regarding the expenditure side of the budget, several models, Monacelli et al. (2010), Edelberg et al. (1999), Fatas-Mihov (2001), Gali et al. (2007), Mayer et al. (2010) and Staehler-Thomas (2012)<sup>2</sup>, find that loosening fiscal policy implies an increase in hours worked or employment. An empirical paper which finds the same is, for example, Forni-Gambetti

---

<sup>1</sup>Eurostat data.

<sup>2</sup>In a two-country DSGE model with monetary union and with government investment, employment and wages permanent shocks are investigated.

(2010).<sup>3</sup> Furthermore, Monacelli et al. (2010)<sup>4</sup> claim that an increase of 1 percent of GDP government consumption decreases unemployment by 0.6 percentage points at peak<sup>5</sup>, but their theoretical multiplier is, even though it is still positive, much lower (around 0.2 pp).

In contrast, Bruckner-Pappa (2012) suggest that, empirically, an increase in government consumption raises unemployment<sup>6</sup>, and a model with price rigidity, labor force participation, as well as short- and long-term unemployment reproduces their empirical findings. At the same time, Gomes (2011) claims that a shock to government consumption - without the inclusion of government employment in the model - implies a close-to-zero effect on unemployment. Also, based on European countries' data Dallari (2014) finds that the sign of the impact multiplier of a cut in government consumption varies country-by-country.<sup>7</sup>

Most papers concentrate solely on government consumption, although different government expenditure items might imply different employment effects.<sup>8</sup> Bermpertoglou et al. (2013) show that a decrease of 1 percent of GDP in government consumption, government investment and public vacancies increase unemployment by 0.8, 0.8 and 1.8 percentage points respectively, while a decrease of 1 percent of GDP in public wages reduces unemployment by 0.1 percentage points.<sup>9</sup> A New Keynesian model with labor force participation, short- and long-term unemployment and public employment provides similar theoretical responses. However, Gomes (2011) shows that an increase in public vacancies or public wages results in higher unemployment. Moreover, Pappa (2009) finds that a positive shock to public employment decreases total employment in several US federal states.<sup>10</sup> Also, Dallari (2014) demonstrates that after cuts in government investment, unemployment multipliers of different countries have different signs and magnitudes.<sup>11</sup> Campolmi et al. (2011) show that the unemployment multipliers of government spending are low, while those of a hiring subsidy policy are large. Lastly, Staehler-Thomas (2012) point out that while a cut in government consumption, investment or employment increase unemployment, a reduction in public wages decreases it.

The aforementioned studies rely on aggregate government expenditure data. At the same time, the narrative approach achieves identification using historical records of the policy

---

<sup>3</sup>They provide empirical evidence based on a structural, large dimensional, dynamic factor model.

<sup>4</sup>A model with search and matching frictions, real wage rigidity and distortionary taxes.

<sup>5</sup>Based on a VAR of the US.

<sup>6</sup>A 10 percent increase in government consumption results in an increase of 0.2-0.5 pp in unemployment at peak in structural VARs of several OECD countries.

<sup>7</sup>Multipliers vary from  $-4.5$  to  $8.7$ , using a panel structural VAR.

<sup>8</sup>Some specifically focus on military spending, see e.g. Rotemberg-Woodford (1992), Ramey-Shapiro (1998), Burnside et al. (2004), Ramey (2011) or Ben Zeev-Pappa (2015).

<sup>9</sup>Based on a SVAR with sign restrictions of the US.

<sup>10</sup>Using a structural VAR with sign restrictions on US aggregate and state level data.

<sup>11</sup>On impact  $-1.7$ - $4.1$  in Europe.

decision-making process. Both Guajardo et al. (2011) and Ball et al. (2013) suggest that fiscal tightening increases unemployment.<sup>12</sup> Hernandez de Cos-Moral-Benito (2011) also report similar results.<sup>13</sup>

Turning to the revenue side, I am aware of two papers only. Staehler-Thomas (2012) find that a 1 percent of GDP size increase in labor income taxes implies a 0.5 pp hike in unemployment, which is larger than the 0.3 pp multiplier of a government consumption cut (no distinction is made between employee and employer taxes, however). Then, Cogan et al. (2013) investigate the impacts of the US 2013 Budget Resolution, a reform package of cutting government purchases and transfers and lowering labor income taxes, and they find that with a labor or with capital and labor tax reductions hours worked significantly go up compared to a reduction in government purchases. Additionally, Ball et al. (2013) claim that spending-based adjustments have a more pronounced effect than tax-based adjustments, but because they adopt a narrative approach they are not able to compare separate tax policies. Moreover, Canova-Pappa (2007)<sup>14</sup> and Caldara-Kampas (2008)<sup>15</sup> study a single tax shock, too.<sup>16</sup>

Now turning to sectoral heterogeneity in particular, first I need to define family firms: a family firm is a firm that is owned and/or managed by a family. As Anderson-Reeb (2003) specify, a family firm is a firm where the fraction of equity owned by a (founding) family is above a threshold, or one in which family members sit on the board of directors.

The share of family firms in European employment is remarkable (Table 1.1)<sup>17</sup>. Almost every second worker in Europe is employed by a family firm. Family firms are more prevalent in

---

<sup>12</sup>Both use a sample of OECD countries. Guajardo et al. (2011) claim that two years after the shock a fiscal consolidation of 1 percent of GDP implies a 0.3 pp increase in unemployment, while Ball et al. (2013) find that fiscal consolidation implies an increase in long-term unemployment of about 0.5 pp in the medium-term.

<sup>13</sup>Based on a panel of OECD countries.

<sup>14</sup>Based on a dataset of US states and EU countries, identifying the shocks by sign restrictions.

<sup>15</sup>Based on a VAR of the US.

<sup>16</sup>Other streams of literature study the effects of taxes on employment, too. A consensus has not been reached either and it is unusual for the expenditure side of the budget to be compared to the revenue side of the budget. Concerning the effects of labor taxation on (cost of) employment, usually based on aggregate data and econometric methodologies, some researchers find an effect (Alesina-Perotti (1997), Blanchard-Wolfers (2000)), while others do not (Bean (1994)) or report mixed results (Daveri et al. (2000)). While a significant share of public finance literature highlights the equivalence between consumption and labor income taxes (e.g. Auerbach (2006)), others claim the opposite (Blumkin et al. (2012), Sumpson (1986)). Regarding tax shifts, in particular, microsimulations also show an ambiguous picture, Thomas-Picos-Sanchez (2012) claim that a shift from social security contribution to consumption taxes only slightly increases hours, which is not in line with Pestel-Sommer (2013). As regards other effects (growth, efficiency, inequality, reform implementation), see among others Auerbach (2006) or Pestel-Sommer (2013).

<sup>17</sup>Sources: Mandl (2008), IEF (2009), IFB (2011), Bjuggren et al. (2011) and Lindow (2013).

some countries than in others, employing 54.6 and 35.2 percent of labor force, respectively.<sup>18</sup> In addition, the level of unemployment is, on average, lower in countries where there are fewer family firms (Table 1.2), 5.0 percent compared to 8.1 percent otherwise.<sup>19</sup>

| Percentage of family firms in employment (%) |       |                         |       |
|--|-------|-------------------------|-------|
| Countries above average                      |       | Countries below average |       |
| Austria                                      | 72.50 | Finland                 | 45.50 |
| Estonia                                      | 50.00 | Netherlands             | 34.78 |
| France                                       | 49.00 | Norway                  | 40.00 |
| Germany                                      | 50.50 | Romania                 | 19.00 |
| Hungary                                      | 55.00 | Slovenia                | 26.00 |
| Ireland                                      | 50.00 | Sweden                  | 38.57 |
| Italy  | 52.00 | UK                      | 37.97 |
| Spain  | 72.50 |                         |       |
| Average 56.44                                |       | Average 33.87           |       |
| Average 46.22                                |       |                         |       |

Table 1.1: Percentage of family firms in European employment

| Unemployment (%)   |       |  |      |
|--|-------|--|------|
| Countries with above-average family firm share in employment |       | Countries with below-average family firm share in employment |      |
| Austria  | 3.71  | Finland  | 6.62 |
| Estonia  | 9.05  | Netherlands  | 3.35 |
| France   | 7.68  | Norway   | 2.49 |
| Germany  | 8.04  | Romania  | 5.43 |
| Hungary  | 6.86  | Slovenia   | 5.48 |
| Ireland  | 7.46  | Sweden   | 7.02 |
| Italy  | 8.19  | UK   | 4.32 |
| Spain  | 13.69 |  |      |
| Average 8.09   |       | Average 4.96   |      |
| Average 6.63   |       |  |      |

Table 1.2: Unemployment rate by share of family firms in European employment

Despite their remarkable labor market share, family firms have not attracted much attention in macroeconomic research yet. I am aware of a single paper, Caselli-Gennaioli (2013), which treats this topic. Based on a simple growth model, they claim a relationship between family management and the health of financial markets; that is, the worse a financial market is functioning, the greater number of family-managed firms there will be. This and the claim that family managers are less talented at running firms imply that the share of family firms

<sup>18</sup>Regarding the number of firms, the share of family firms is even larger, see for example La Porta et al. (1999) and Mandl (2008).

<sup>19</sup>Calculation is based on Eurostat data between 2000 and 2012, and Mandl (2008), IEF (2009), IFB (2011), Bjuggren et al. (2011) and Lindow (2013).

is important in explaining cross-country income differences. However, they do not talk about the labor market. Recently, Epstein-Shapiro (2014) studied labor market policies in a model with small and large firms. However, they did not consider fiscal consolidation policies, and they focused only on firm size, rather than including other firm characteristics as well.

At the same time, many of the family firms' characteristics are documented in the corporate finance literature, based on firm-level data. First, family firms behave differently in the labor market, compared to non-family firms. On the one hand, job security is stronger among family firms.<sup>20</sup> Bassanini et al. (2011) use matched employer-employee data of French companies, and find that the dismissal rate of family firms is significantly lower, and so is the subjective risk of dismissal perceived by the workers.

Sraer-Thesmar (2007), using a French sample of stock exchange-listed companies, point out that family firms pay lower wages, even after controlling for the skill and age structure of the workers. Similarly, Bassanini et al. (2011) show that family firms pay a lower wage on average than do non-family firms, and this is due to differences in unobserved characteristics. Furthermore, when a firm becomes non-family owned, its wages drop. Also, worker influence on wage setting is lower in family firms. Still, employees of family firms, compared to their counterparts in the non-family sector, are more loyal to their employers; Siebert et al. (2011) claim that this greater loyalty stems from long-term employment.

Second, as Caselli-Gennaioli (2013) point out, "inter-generational transmission of managerial responsibilities", i.e. dynastic management, is crucial for family firms. The current owner or manager ("the son") of the firm inherits managerial know-how - related to customers, suppliers and other market operators - from the previous owner or manager ("the father").

There is no consensus in the literature, however, as to whether family firms are more or less productive than non-family firms. Bennedsen et al. (2007) claim that family management has a negative impact on firm performance. However, Maury (2006) and others, Anderson-Reeb (2003), Villalonga-Amit (2006) or King-Santor (2007), find that the relationship between family management and firm performance is not monotonic.

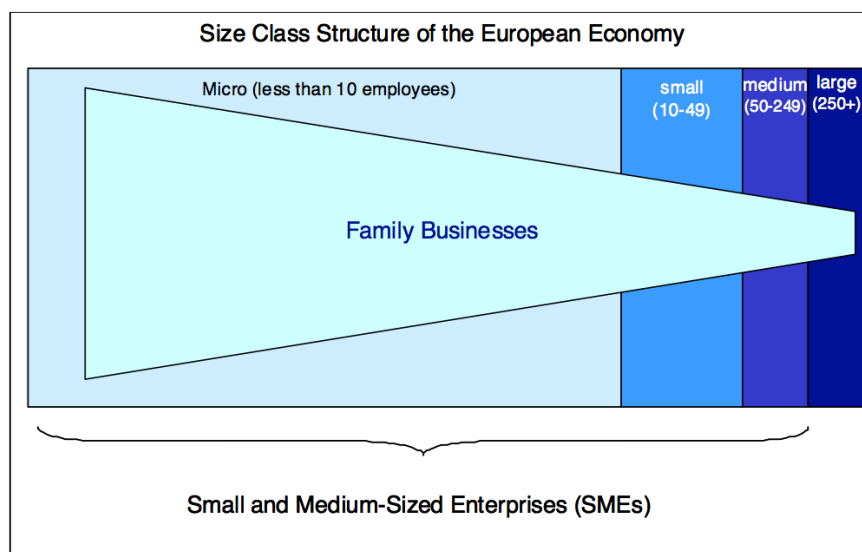
Furthermore, a general misunderstanding about family firms is that all of them are small (Figure 1.1).<sup>21</sup> As Mandl (2008) highlights, the family business sector is mainly dominated by small- and medium-sized companies. However, some of the largest firms are also family

---

<sup>20</sup>Furthermore, not only is the job security of family firms greater, but these firms are also less likely to exit the market (Nunes et al. (2014)).

<sup>21</sup>A micro or small firm employs fewer than 50 people, a medium-sized firm employs between 50 and 249 people, while a large enterprise has at least 250 employees (Eurostat terminology).

**Graph 3      Size Class Considerations of the European Economy and the European Family Business Sector**



Source: Austrian Institute for SME Research

Figure 1.1: Size structure of European (family) firms (Mandl (2008), page 50)

firms. Examples of these are Volkswagen, Metro or Bosch.<sup>22</sup> Also, as IFERA (2003) claims, not only is Wal-Mart, one of the largest companies of the world, a family company, but further 35 percent of the 500 biggest US companies are family firms.

When considering employment or turnover instead of number of firms, the dominance of small firms among family firms is even lower. According to Mandl (2008), the share of family firms with an annual turnover higher than EUR 50 million is 34 percent in Germany. Also, in Ireland 27.1 percent and in Finland 22 percent of large firms are family firms. In the UK, the share of large firms in the family sector is also notable, although it is lower (15.6 percent is reported by IFB (2011)). In Italy, the average number of employed people is 68 in the family sector and 305 in the non-family sector, based on Navaretti et al. (2008).

Another common misperception is that family firms are not export-oriented. According to IFB (2011), in the UK 19 percent of family firms sold exports last year, while the relevant non-family value was 15 percent. In Spain, both the export propensities and the export intensities of family and non-family firms are very similar (71.1 and 68.8, and 20.9 and 25.3 percent, respectively), shown by Merino et al. (2012). Similar conclusions can be drawn for Italy (Navaretti et al. (2008) and Minetti et al. (2013)).

Thus, the New Keynesian model with search and matching frictions of this paper, as an innovation, contains sectoral heterogeneity, i.e. family and non-family firms. I concentrate

<sup>22</sup><http://www.campdenfb.com/article/top-100-family-businesses-europe-1>



on the different labor market behavior of family firms, as well as introduce an intangible capital<sup>23</sup> in the family sector, which enables me to model the dynastic management of family firms. Also, the presence of the two sectors creates the possibility of sectoral movements due to changes in relative sectoral prices and wages. For the purpose of illustration, the model is calibrated to match data of those European countries where family firms have an above-average presence in the labor market.

The model predicts that all fiscal austerity policies raise unemployment. At peak, the highest increase is implied by a cut in government consumption. Nevertheless, an increase in employees' labor income tax, cumulatively, implies the same size increase in unemployment as does the government consumption cut. A higher employer social security contribution is, however, less costly in terms employment than an increase on the same scale in the tax on employees' labor income. Both at peak and cumulatively, unemployment reacts least when the budget is consolidated by increasing the rate of value-added tax. Yet, a policymaker must manage trade-offs, as the increase in value-added tax results in the steepest decline in consumption.

Sectoral heterogeneity seems to play a crucial role: unemployment fiscal multipliers are very different with and without it. With homogeneous firms multipliers of labor income tax policies and government consumption multipliers are usually smaller, while consumption tax multipliers are often biased upwards. Thus, ignoring sectoral heterogeneity might lead to incorrect policy conclusions.

The structure of the paper is as follows. The next section describes the model, while calibration is presented in Section 1.3. Results appear in Section 1.4. Section 1.5 concludes, while the Appendix provides more detail on the model's steady state, the loglinearized equations and impulse response functions.

## 1.2 Modeling framework

My model builds on a standard, closed-economy dynamic stochastic general equilibrium (DSGE) framework with price stickiness (Rotemberg (1982)) and search and matching frictions (Gertler et al. (2008) and Staehler-Thomas (2012)).

As an innovation, there is sectoral heterogeneity on the firm side, ie. family and non-family firms are distinguished. Family firms behave differently in the labor market than non-family

---

<sup>23</sup>In this paper the terms 'organisational', 'family', 'family organisational' and 'intangible capital' are used interchangeably. The author is aware of the fact that, for instance, research and development is also intangible, but in this paper only the so called 'family' capital, capital related to special firm management, is taken into account.

firms by providing greater job security, but lower wages and less bargaining power to their employees. As well, dynastic management of family firms is modeled by the inclusion of an intangible capital in the family sector, following Danthine-Jin (2007).<sup>24</sup> The presence of the two sectors creates the possibility of sectoral movements due to changes in relative sectoral prices and wages.

### 1.2.1 Representative household

A representative household maximizes expected discounted lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \quad (1.1)$$

Here,  $\beta$  is the usual deterministic discount factor. For simplicity's sake, I assume that the household only derives utility from aggregate consumption  $C_t$ .<sup>25</sup>

The household's contemporaneous utility is a constant relative risk aversion (CRRA) function of aggregate consumption:

$$u(C_t) = \frac{(C_t - hC_{t-1})^{1-\sigma_C} - 1}{1 - \sigma_C} \quad (1.2)$$

where  $h$  is the external habit parameter and  $\sigma_C$  is the relative risk aversion parameter.

There is unemployment due to search and matching frictions, which I will describe in more detail later on. A member of the household might work in the intermediate family sector ( $L_{F,t}$ ) or in the intermediate non-family sector ( $L_{NF,t}$ ). If the household member is not employed in any of these sectors, he/she is unemployed. I define the beginning of period unemployment by  $U_t$ :

$$U_t = 1 - L_{F,t-1} - L_{NF,t-1} \quad (1.3)$$

Here, I normalise the total number of labor force to one, meaning I do not take into account labor force participation decision.<sup>26</sup>

When working in the family or in the non-family sector, the household receives labor income,  $W_{F,t}$  and  $W_{NF,t}$ , respectively. These are sectoral real wages, expressed in the economy-wide price level  $P_t$ . Labor income is taxed in both sectors by  $\tau_{LEE,t}$ , which is the sum of personal labor income tax and the employees' social security contribution. While the social security contribution is deducted, modeling retirement is beyond the scope of this paper. When unemployed, the household member receives  $W_U$  unemployment benefit, also expressed in the economy-wide price level.

---

<sup>24</sup>Other examples of intangible capital include McGrattan-Prescott (2010), Ai et al. (2013), Gourio-Rudanko (2014) or McGrattan et al. (2014).

<sup>25</sup>See, for example, Gertler et al. (2008).

<sup>26</sup>A recent example of a model with labor force participation is Bermperoglou et al. (2013).

Because the household owns the firms, the household receives the dividends. The household owns and rents physical capital to intermediate family and non-family firms,  $K_{F,t}$  and  $K_{NF,t}$ , so he/she receives rental rate of capital,  $RK_{F,t}$  and  $RK_{NF,t}$ , respectively. The rental rates differ in the two sectors, and are also expressed in terms of the economy-wide price level. The household can save either in a risk-free government bond  $B_t$ , after which he/she gets  $i_{t-1}$  nominal interest rate, deflated by  $\pi_t$  quarterly inflation rate, or she can invest into physical capital. To avoid jumps, investment is subject to an adjustment cost following Christiano et al. (2005):

$$K_{F,t} = (1 - \delta_F)K_{F,t-1} + I_{F,t} - \frac{\phi^{IF}}{2} \left( \frac{I_{F,t}}{I_{F,t-1}} - 1 \right)^2 I_{F,t} \quad (1.4)$$

Here,  $I_{F,t}$  denotes investment into family physical capital,  $\delta_F$  is the family depreciation rate, and  $\phi^{IF}$  is the investment adjustment cost related to family physical capital. Investment adjustment cost is zero in steady state.<sup>27</sup>

The household consumes an aggregate consumption bundle  $C_t$ , which will be described in more detail shortly, and after her consumption she pays value-added tax  $\tau_{C,t}$ , and a lump sum tax  $T_t$  to close the model. For simplicity, I assume that income related to bonds or physical capital renting are not taxed.<sup>28</sup>

Thus, the period-by-period household budget constraint is

$$\begin{aligned} & (1 - \tau_{LEE,t})(W_{F,t}L_{F,t} + W_{NF,t}L_{NF,t}) + W_U(1 - L_{F,t} - L_{NF,t}) + \\ & + \frac{1 + i_{t-1}}{\pi_t} B_{t-1} + RK_{F,t}K_{F,t-1} + RK_{NF,t}K_{NF,t-1} + Prof_{F,t}^I + Prof_{NF,t}^I + \\ & + \frac{P_{F,t}}{P_t} Prof_{F,t}^F + \frac{P_{NF,t}}{P_t} Prof_{NF,t}^F = \\ & = (1 + \tau_{C,t})C_t + \frac{P_{F,t}}{P_t} I_{F,t} + \frac{P_{NF,t}}{P_t} I_{NF,t} + B_t + T_t \end{aligned} \quad (1.5)$$

Here,  $P_{F,t}$  ( $P_{NF,t}$ ) is the price level of the goods produced in the family (non-family) sector. As the budget constraint is expressed in terms of the aggregate price level  $P_t$ , the relative sectoral prices are considered regarding sectoral investments (the same is true of final firms' profits).

The household maximises its expected discounted lifetime utility subject to its budget constraint and the two physical capital laws of motion with respect to  $B_t$ ,  $C_t$ ,  $I_{F,t}$ ,  $I_{NF,t}$ ,  $K_{F,t}$  and  $K_{NF,t}$ . The household takes wages and labor as given, as these are determined in the labor market when bargaining with intermediate firms.

---

<sup>27</sup>Similarly in the non-family sector with  $I_{NF,t}$  as investment into non-family physical capital,  $\delta_{NF}$  as the non-family depreciation rate and  $\phi^{INF}$  as the non-family investment adjustment cost.

<sup>28</sup>A counterexample is e.g. Staehler-Thomas (2012).

Optimization yields the usual Euler equation:

$$E_t \left[ \beta \frac{1+i_t}{\pi_{t+1}} \frac{1}{1+\tau_{C,t+1}} \frac{1}{C_{t+1}} \right] = \frac{1}{1+\tau_{C,t}} \frac{1}{C_t} \quad (1.6)$$

Because there is a value-added tax, the current and next period tax levels affect the intertemporal consumption choice of the household.

The family physical capital and investment decisions of the household can be expressed by two equations, a Tobin-Q and an arbitrage condition:<sup>29</sup>

$$\begin{aligned} \frac{P_{F,t}}{P_t} &= Q_{F,t} - Q_{F,t} \frac{\phi^{IF}}{2} \left( 3 \frac{I_{F,t}^2}{I_{F,t-1}^2} - 4 \frac{I_{F,t}}{I_{F,t-1}} + 1 \right) + \\ &+ E_t \left[ Q_{F,t+1} \phi^{IF} \frac{\pi_t}{i_{t+1}} \left( \frac{I_{F,t+1}^3}{I_{F,t}^3} - \frac{I_{F,t+1}^2}{I_{F,t}^2} \right) \right] \end{aligned} \quad (1.7)$$

$$E_t \left[ \frac{1+i_t}{\pi_{t+1}} \right] = E_t \left[ \frac{RK_{F,t} + (1-\delta_F)Q_{F,t+1}}{Q_{F,t}} \right] \quad (1.8)$$

$Q_{F,t}$  is the price level of family physical capital, which is related to the Lagrangian multiplier of the relevant law of motion. A Tobin-Q of a standard, one-sector model does not contain any relative prices. However, because there are two sectors here, the relative sectoral prices appear in the Tobin-Q. Thus, in steady state, the price level of family physical capital is equal to the relative price of family goods, instead of 1 of the usual one-sector framework. Furthermore, the arbitrage condition is also affected by the sectoral price level; in steady state the rental rate of capital is thus not equal to the real interest rate net depreciation rate, but it is also affected by the sectoral Tobin-Q.

Finally,  $I_t$  aggregate investment is defined as follows:

$$I_t = \frac{P_{F,t}}{P_t} I_{F,t} + \frac{P_{NF,t}}{P_t} I_{NF,t} \quad (1.9)$$

Aggregate household consumption is a composite of goods produced by family and non-family firms:

$$C_t = \left[ \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} C_{NF,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (1.10)$$

$\gamma$  is the share of family-firm produced goods in the consumption basket, while  $\eta$  is the elasticity of substitution between family and non-family firm produced goods.

---

<sup>29</sup>Here, I describe the family sector, the non-family sector is similar.

The household minimizes its expenditure spent on consumption goods, taking into account the composite consumption function above. As a result, the demand functions for consumption of family and non-family produced goods are

$$C_{F,t} = \gamma \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \quad (1.11)$$

$$C_{NF,t} = (1 - \gamma) \left( \frac{P_{NF,t}}{P_t} \right)^{-\eta} C_t \quad (1.12)$$

The economy-wide price level  $P_t$ , which is the Lagrangian multiplier of the above maximization problem, can be expressed as a composite of the sectoral price levels:

$$P_t = [\gamma P_{F,t}^{1-\eta} + (1 - \gamma) P_{NF,t}^{1-\eta}]^{\frac{1}{1-\eta}} \quad (1.13)$$

Finally, the stochastic discount factor is, as usual,  $\beta_{t,t+1} = E_t \left[ \beta \frac{\frac{\partial u(C_{t+1})}{\partial C_{t+1}}}{\frac{\partial u(C_t)}{\partial C_t}} \right]$ .

### 1.2.2 Intermediate good producing firms and wage bargaining

In each sector, there is a continuum of intermediate goods-producing firms, which are perfectly competitive, ie. they take prices as given. They produce goods hiring labor (as well as the employer social security contribution  $\tau_{LE R,t}$ ) and renting physical capital from the household. Hiring labor requires posting vacancies, which is costly, and so induces unemployment.

Also, intermediate firms bargain over wages with workers. Modeling the labor market is similar to Gertler et al. (2008) and Staehler-Thomas (2012). Based on empirical, firm-level evidence, family firms behave differently in the labor market than non-family firms: i) their dismissal rate is lower, ii) their workers obtain a lower wage (in steady state) and iii) their workers have less bargaining power in wage setting. Moreover, family firms invest in family capital, which non-family firms do not do. This capital, an intangible one in accordance with Danthine-Jin (2007), represents the dynastic management of family firms. At this point, I will describe the intermediate firms in detail.<sup>30</sup>

Sectoral goods  $Y_{F,t}^I$  and  $Y_{NF,t}^I$  are produced based these production functions:

$$Y_{F,t}^I = A_{F,t} L_{F,t}^{\alpha_F} K_{F,t-1}^{1-\alpha_F-\mu} K_{OF,t-1}^{\mu} \quad (1.14)$$

$$Y_{NF,t}^I = A_{NF,t} L_{NF,t}^{\alpha_{NF}} K_{NF,t-1}^{1-\alpha_{NF}} \quad (1.15)$$

---

<sup>30</sup>I focus on the family sector; the non-family sector, except for the organisational capital, is parallel.

which are, for simplicity's sake, Cobb-Douglas functions with constant returns to scale.<sup>31</sup>  $\alpha_F$  and  $\alpha_{NF}$  denote labor income shares in the two sectors, respectively.  $A_{F,t}$  and  $A_{NF,t}$  are exogenous productivity levels, which are assumed to be equal in the steady state:

$$\hat{A}_{F,t} = \rho_{AF} \hat{A}_{F,t-1} + \epsilon_{AF,t} \quad (1.16)$$

$$\hat{A}_{NF,t} = \rho_{ANF} \hat{A}_{NF,t-1} + \epsilon_{ANF,t} \quad (1.17)$$

In the family sector the family capital, which represents dynastic management, is denoted by  $K_{OF,t-1}$ . Investment in family organisational capital follows a law of motion:

$$K_{OF,t} = (1 - \delta_{OF})K_{OF,t-1} + \theta I_{OF,t} - \frac{\phi^{IOF}}{2} \left( \frac{I_{OF,t}}{I_{OF,t-1}} - 1 \right)^2 I_{OF,t} \quad (1.18)$$

This is similar to the physical capital law of motion described above (here,  $\delta_{OF}$  is the family intangible depreciation rate, while  $\phi^{IOF}$  is the relevant adjustment cost). Nevertheless, parameter  $\theta$  appears, which represents the effectiveness of family organisational investment. In the baseline scenario, this parameter is set to 1. Family organisational investment  $I_{OF,t}$  is intangible, so it is not part of the final output. This creates a trade-off, namely, investment in family capital reduces goods sold today as well as profit today, but it increases the next period's family organisational capital stock, thus bringing about future production and profit.

Family firms post vacancies,  $v_t^F$ . The number of new hires (matches),  $m_t^F$ , depends on the number of vacancies posted and the number of people searching for a job,  $U_t^s$ . Searching can be described by a matching function:

$$m_t^F = \sigma_{F,m} (U_t^s)^{\sigma_F} (v_t^F)^{1-\sigma_F} \quad (1.19)$$

Here,  $\sigma_{F,m}$  is the matching efficiency, while  $\sigma_F$  denotes the matching elasticity.

Similarly to Staehler-Thomas (2012), but in contrast to Gertler et al. (2008) I assume that the number of people searching for a job in period  $t$  equals the number of people who are unemployed at the end of period  $t-1$  ( $U_t$ ) plus the number of people losing their job at the beginning of period  $t$ :

$$U_t^s = U_t + (1 - \rho^F)L_{F,t-1} + (1 - \rho^{NF})L_{NF,t-1} \quad (1.20)$$

Those who are fired can immediately start to search for a new job. Firing is exogenous: the sectoral dismissal rates are  $\rho^F$  and  $\rho^{NF}$ . Furthermore, everyone can search for a job in any of the sectors, not only in the sector in which one was working previously.

---

<sup>31</sup>Following the intangible capital literature, such as McGrattan-Prescott (2010) and McGrattan et al. (2014), constant returns to scale mean constant returns to scale in all production inputs, including the intangible capital.

Those who find a job in period  $t$  start to work immediately. The same assumption is made by Gertler et al. (2008) and Staehler-Thomas (2012), but Bermperoglou et al. (2013) assume that those who find a job in period  $t$  start to work in period  $t + 1$  only. Given the above functional forms and assumptions, the family labor law of motion is

$$L_{F,t} = \rho^F L_{F,t-1} + m_t^F \quad (1.21)$$

Finally, family vacancy filling and job finding probabilities are defined as

$$q_t^F = \frac{m_t^F}{v_t^F} \quad (1.22)$$

$$p_t^F = \frac{m_t^F}{U_t^s} \quad (1.23)$$

Regarding vacancy posting,  $\kappa^F$  and  $\kappa^{NF}$  denote per-vacancy costs. Total vacancy cost is linear in the number of vacancies posted. Also, this is the only cost in my framework. In contrast, Staehler-Thomas (2012) consider a training cost as well. Regarding the functional form, Gertler et al. (2008) do not use a linear function. This is because they assume nominal wage rigidity, which requires a quadratic function. I follow most of the literature when considering a linear function. Hence, my framework is the closest to that of Bermperoglou et al. (2013).

Intermediate profits are

$$\begin{aligned} Prof_{F,t}^I &= MC_{F,t} Y_{F,t}^I - (1 + \tau_{LER,t}) W_{F,t} L_{F,t} - RK_{F,t} K_{F,t-1} - \\ &- \kappa^F \frac{P_{F,t}}{P_t} v_t^F - MC_{F,t} I_{OF,t} \end{aligned} \quad (1.24)$$

$$\begin{aligned} Prof_{NF,t}^I &= MC_{NF,t} Y_{NF,t}^I - (1 + \tau_{LER,t}) W_{NF,t} L_{NF,t} - RK_{NF,t} K_{NF,t-1} - \\ &- \kappa^{NF} \frac{P_{NF,t}}{P_t} v_t^{NF} \end{aligned} \quad (1.25)$$

Because intermediate firms are perfectly competitive, the price of goods is equal to the real marginal cost,  $MC_{F,t}$  and  $MC_{NF,t}$  in the two sectors, respectively. Profit equals revenue net wages and rental rate of capital. Vacancy posting costs are deducted, as well. Because there are two sectors and the profits are expressed in the economy-wide price level  $P_t$ , relative sectoral prices are taken into account when calculating total vacancy posting costs. The last term in the family profit is related to family organisational investment. Specifically, some part of family production,  $I_{OF,t}$ , is not sold in the market, but it is used by the firm itself as investment in organisational capital to enhance future production. Because the family firm invests in organisational capital itself, the price of investment equals the price of goods produced.

Intermediate firms maximize their expected discounted lifetime profit by choosing labor, number of vacancies and physical capital, taking into account the production functions and labor laws of motion above:

$$E_0 \sum_{j=0}^{\infty} \beta_{t,t+j} Prof_{F,t+j}^I \quad (1.26)$$

$$E_0 \sum_{j=0}^{\infty} \beta_{t,t+j} Prof_{NF,t+j}^I \quad (1.27)$$

$\beta_{t,t+j}$  denotes the stochastic discount factor of the household between periods  $t$  and  $t+j$ .

Optimization implies a usual physical capital demand:

$$RK_{F,t} = MC_{F,t}(1 - \alpha_F - \mu) \frac{Y_{F,t}^I}{K_{F,t-1}} \quad (1.28)$$

Demand for labor, however, differs from the standard one without labor market frictions, namely, current and next period firm values affect the real wage:

$$(1 + \tau_{LER,t})W_{F,t} = MC_{F,t}\alpha_F \frac{Y_{F,t}^I}{L_{F,t}} - F_{F,t} + E_t [\beta_{t,t+1}\rho^F F_{F,t+1}] \quad (1.29)$$

where the current firm value  $F_{F,t}$  is related to the vacancy posting cost:

$$F_{F,t} = \kappa^F \frac{P_{F,t}}{P_t} \frac{1}{q_t^F} \quad (1.30)$$

Combining these two yields the wage setting equation:

$$(1 + \tau_{LER,t})W_{F,t} = MC_{F,t}\alpha_F \frac{Y_{F,t}^I}{L_{F,t}} - \kappa^F \frac{P_{F,t}}{P_t} \frac{1}{q_t^F} + E_t \left[ \beta_{t,t+1}\rho^F \kappa^F \frac{P_{F,t+1}}{P_{t+1}} \frac{1}{q_{t+1}^F} \right] \quad (1.31)$$

Additionally, only in the family sector, there is a demand for organisational capital:

$$Q_{OF,t} - E_t [\beta_{t,t+1}(1 - \delta_{OF})Q_{OF,t+1}] = E_t \left[ \beta_{t,t+1}MC_{F,t+1}\mu \frac{Y_{F,t+1}^I}{K_{OF,t}} \right] \quad (1.32)$$

In contrast to the family physical capital demand, rather than the current period's production, the next period's production is relevant. This is because the firm decides about the next period's organisational capital today, taking this period's organisational capital as given. Also, this is the reason that both the current period's and also the next period's capital prices appear.

Similarly to physical capital, there is also a Tobin-Q for family organisational capital:



$$\begin{aligned}
MC_{F,t} = & \theta Q_{OF,t} - Q_{OF,t} \frac{\phi^{IOF}}{2} \left( 3 \frac{I_{OF,t}^2}{I_{OF,t-1}^2} - 4 \frac{I_{OF,t}}{I_{OF,t-1}} + 1 \right) + \\
& + E_t \left[ \beta_{t,t+1} Q_{OF,t+1} \phi^{IOF} \left( \frac{I_{OF,t+1}^3}{I_{OF,t}^3} - \frac{I_{OF,t+1}^2}{I_{OF,t}^2} \right) \right]
\end{aligned} \tag{1.33}$$

$\theta$ , the effectiveness of family organisational investment has an impact on the price of family organisational capital. In steady state, the price of family capital is equal to  $\frac{MC_F}{\theta}$ , so the higher the effectiveness of organisational investment, the lower the price of family organisational capital. Again, the price of family capital is related to the price of production, as the family firm sacrifices its own goods to invest in this inheritable special knowlegde.

Intermediate firms and workers bargain over gross wages; in period  $t$  they bargain over wages paid in period  $t$ . Bargaining happens after matching is over. My framework closely follows that of Staehler-Thomas (2012). Rearranging labor demand gives the firm value ( $F_{F,t}$ ):

$$F_{F,t} = MC_{F,t} \alpha_F \frac{Y_{F,t}^I}{L_{F,t}} - (1 + \tau_{LER,t}) W_{F,t} + E_t [\beta_{t,t+1} \rho^F F_{F,t+1}] \tag{1.34}$$

The current firm value depends on the difference between the marginal revenue of the firm net wage (affected by employer social security contribution), while it is also related to next period's firm value, taking into account the dismissal rate of workers.

Worker value of working in the family sector ( $V_{F,t}$ ) at the end of period  $t$  is equal to the wage received by the worker in period  $t$  (affected by employee labor income tax rate) and the discounted worker value in period  $t + 1$ . This latter is a sum of remaining employed in the family sector with probability  $\rho^F$ , plus the value of being unemployed at the beginning of period  $t + 1$  with probability of losing the job:

$$V_{F,t} = (1 - \tau_{LEE,t}) W_{F,t} + E_t [\beta_{t,t+1} (\rho^F V_{F,t+1} + (1 - \rho^F) UV_{b,t+1})] \tag{1.35}$$

The value of searching for a job at the beginning of period  $t$ :

$$UV_{b,t} = p_t^F V_{F,t} + p_t^{NF} V_{NF,t} + (1 - p_t^F - p_t^{NF}) UV_{e,t} \tag{1.36}$$

With probability  $p_t^F$  the unemployed person finds a job in the family sector, with probability  $p_t^{NF}$  he/she finds a job in the non-family sector, while with probability  $1 - p_t^F - p_t^{NF}$  at the end of period  $t$  he/she is still unemployed. Those who are unemployed at the end of period  $t$  receive unemployment benefits from the government, and can search again in the next period:

$$UV_{e,t} = W_U + E_t [\beta_{t,t+1} UV_{b,t+1}] \tag{1.37}$$

Workers and intermediate firms bargain over the net surplus in the two sectors separately, family bargaining means maximizing the following expression with respect to the gross wage:

$$\max (V_{F,t}(W_{F,t}) - UV_{e,t})^{\lambda_F} (F_{F,t}(W_{F,t}))^{1-\lambda_F} \quad (1.38)$$

where  $\lambda_F$  is the bargaining powers of workers in the family sector. Optimization implies

$$\lambda^F(1 - \tau_{LEE,t})F_{F,t} = (1 - \lambda^F)(1 + \tau_{LER,t})(V_{F,t} - UV_{e,t}) \quad (1.39)$$

### 1.2.3 Final good producing firms

Similarly to Gertler et al. (2008), in each sector there is a continuum of  $(0, 1)$  final goods-producing firms which set final goods prices. I present the family sector only, as the non-family sector is parallel.

Final firm  $s$  in the family sector sells  $Y_{F,t}^F(s)$  amount of final goods at price  $P_{F,t}(s)$ . Total final output is a Dixit-Stiglitz aggregator (Dixit-Stiglitz (1977)) of  $s \in (0, 1)$  final goods with family markup equal to  $\frac{\epsilon_F}{\epsilon_F - 1}$ :

$$Y_{F,t}^F = \left( \int_0^1 Y_{F,t}^F(s)^{\frac{\epsilon_F - 1}{\epsilon_F}} ds \right)^{\frac{\epsilon_F}{\epsilon_F - 1}} \quad (1.40)$$

Then, optimization yields a demand function for each final good  $s$ :

$$Y_{F,t}^F(s) = \left( \frac{P_{F,t}}{P_{F,t}(s)} \right)^{\epsilon_F} Y_{F,t}^F \quad (1.41)$$

while the total final good price is a function of  $s \in (0, 1)$  final good prices:

$$P_{F,t} = \left( \int_0^1 P_{F,t}(s)^{1-\epsilon_F} ds \right)^{\frac{1}{1-\epsilon_F}} \quad (1.42)$$

As prices are sticky, firms must pay a quadratic cost when changing prices, following Rotemberg (1982). This cost is zero in the steady state, but around the steady state it varies depending on the ratio of the current price level to the previous period's price level of final firm  $s$ . Thus, the profit of final firm  $s$  expressed in  $P_{F,t}$  price level is

$$Prof_{F,t}^F(s) = \frac{P_{F,t}(s) - MC_{F,t}P_t}{P_{F,t}} Y_{F,t}^F(s) - \frac{\phi^F}{2} \left( \frac{\frac{P_{F,t}(s)}{P_{F,t-1}(s)}}{\pi} - 1 \right)^2 Y_{F,t}^F \quad (1.43)$$

$\phi^F$  is the price rigidity parameter, and  $\pi$  is the economy wide steady state quarterly inflation rate.

Final firms maximize expected discounted lifetime profit with respect to  $P_{F,t}(s)$  given the demand function above:

$$E_0 \sum_{j=0}^{\infty} \beta_{t,t+j} Prof_{F,t+j}^F(s) \quad (1.44)$$

Then, the optimal pricing decision is

$$\begin{aligned} \phi^F \left( \frac{\pi_t^F}{\pi} - 1 \right) \frac{\pi_t^F}{\pi} &= 1 - \epsilon_F + \epsilon_F \frac{MC_{F,t}}{\frac{P_{F,t}}{P_t}} + \\ &+ E_t \left[ \beta_{t,t+1} \phi^F \left( \frac{\pi_{t+1}^F}{\pi} - 1 \right) \frac{\pi_{t+1}^F}{\pi} \frac{Y_{F,t+1}^F}{Y_{F,t}^F} \right] \end{aligned} \quad (1.45)$$

where  $\pi_t^F = \frac{P_{F,t}}{P_{F,t-1}}$  is the sectoral inflation rate.

After loglinearising and rearranging the pricing decisions, the sectoral New-Keynesian Philips curves are

$$\hat{\pi}_t^F = \frac{\epsilon_F MC_F}{\phi_F \frac{P_F}{P}} \left( M\hat{C}_{F,t} - P\hat{F}P_t \right) + E_t \left[ \beta \pi_{t+1}^{\hat{F}} \right] \quad (1.46)$$

$$\pi_t^{\hat{N}NF} = \frac{\epsilon_F MC_{NF}}{\phi_{NF} \frac{P_{NF}}{P}} \left( M\hat{C}_{NF,t} - P\hat{N}FP_t \right) + E_t \left[ \beta \pi_{t+1}^{\hat{N}NF} \right] \quad (1.47)$$

with  $PFP_t = \frac{P_{F,t}}{P_t}$  and  $PNFP_t = \frac{P_{NF,t}}{P_t}$ . These Philips curves are similar to the standard Philips curve, apart from the fact that they contain relative sectoral prices. Substituting  $P_{F,t} = P_t$  and  $P_{NF,t} = P_t$  (ie.  $P\hat{F}P_t = 0$  and  $P\hat{N}FP_t = 0$ ) into the sectoral Philips-curves, one can immediately see that we get back the standard Philips curve.

### 1.2.4 Monetary authority

The central bank sets the next period's interest rate based on the current period inflation, following a simple Taylor rule:

$$\hat{i}_t = \rho_{\pi} \hat{\pi}_t + \epsilon_t^i \quad (1.48)$$

where  $\rho_{\pi}$  is the weight on inflation in the Taylor rule and  $\epsilon_t^i$  is an exogenous monetary policy shock.

### 1.2.5 Government

The government collects taxes: labor income taxes (personal labor income tax and social security contribution of employees, and social security contribution of employers), a value-added tax and a lump-sum tax. For purposes of simplicity, I assume that interest income of

bond holdings and income on physical capital renting are not taxed. Taxes finance government consumption expenditure  $G_t$  and unemployment benefit expenditure. Revenues  $Rev_t$  and expenditures  $Exp_t$  are

$$Rev_t = (\tau_{LEE,t} + \tau_{LER,t})(W_{F,t}L_{F,t} + W_{NF,t}L_{NF,t}) + \tau_{C,t}C_t + T_t \quad (1.49)$$

$$Exp_t = G_t + W_U(1 - L_{F,t} - L_{NF,t}) \quad (1.50)$$

Then, government deficit  $DEF_t$  is defined as the difference between expenditures and revenues. The government issues bonds ( $B_t$ ) to finance its deficit, which are bought by the household.

$$DEF_t = Exp_t - Rev_t \quad (1.51)$$

$$DEF_t = B_t - \frac{1 + i_{t-1}}{\pi_t} B_{t-1} \quad (1.52)$$

In order to avoid an explosive solution, there is a lump-sum tax rule which depends on the government debt-output ratio, following Bermperoglou et al. (2013):

$$T_t = T(T_{t-1})^{\rho_T} \left( \frac{\frac{B_{t-1}}{Y_t}}{\frac{B}{Y}} \right)^{(1-\rho_T)\xi_B} exp(\epsilon_{T,t}) \quad (1.53)$$

Here,  $\rho_T$  is the autocorrelation parameter,  $\xi_B$  is the debt rule parameter representing the sensitivity of lump-sum taxes to the government debt-output ratio and  $\epsilon_{T,t}$  is the shock. If the government debt to output ratio goes up compared to its steady state value, lump-sum tax increases.

Similarly to aggregate household consumption, aggregate government consumption is also a composite of goods produced by family and non-family firms:

$$G_t = \left[ \gamma^{\frac{1}{\eta}} G_{F,t}^{\frac{\eta-1}{\eta}} + (1-\gamma)^{\frac{1}{\eta}} G_{NF,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (1.54)$$

Demand functions follow

$$G_{F,t} = \gamma \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} G_t \quad (1.55)$$

$$G_{NF,t} = (1-\gamma) \left( \frac{P_{NF,t}}{P_t} \right)^{-\eta} G_t \quad (1.56)$$

For purposes of simplification, the share of family goods and the elasticity of substitution between family and non-family goods is the same as for household consumption, so the price levels are the same, too.

Finally, there is an exogenous shock process for each tax and expenditure item:

$$\tau_{LEE,t}^{\hat{}} = \rho_{\tau LEE} \tau_{LEE,t-1}^{\hat{}} + \epsilon_{\tau LEE,t} \quad (1.57)$$

$$\tau_{LER,t}^{\hat{}} = \rho_{\tau LER} \tau_{LER,t-1}^{\hat{}} + \epsilon_{\tau LER,t} \quad (1.58)$$

$$\tau_{C,t}^{\hat{}} = \rho_{\tau C} \tau_{C,t-1}^{\hat{}} + \epsilon_{\tau C,t} \quad (1.59)$$

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \epsilon_{G,t} \quad (1.60)$$

with  $\rho_{\tau LEE}$ ,  $\rho_{\tau LER}$ ,  $\rho_{\tau C}$ ,  $\rho_G$  autocorrelation parameters and  $\epsilon_{\tau LEE,t}$ ,  $\epsilon_{\tau LER,t}$ ,  $\epsilon_{\tau C,t}$ ,  $\epsilon_{G,t}$  shock error terms.

### 1.2.6 Market clearing

In equilibrium all markets clear. Physical capital markets clear, ie. physical capital supplied by the household is equal to physical capital demanded by intermediate firms. Similarly, labor markets clear.

Also, goods markets clear. Total final output is equal to total intermediate output:

$$Y_{F,t}^F = Y_{F,t}^I - I_{OF,t} \quad (1.61)$$

$$Y_{NF,t}^F = Y_{NF,t}^I \quad (1.62)$$

Concerning family goods, family organisational investment must be subtracted from intermediate output, as it is not distributed, but used by the firm itself. This is in line with Danthine-Jin (2007), who point out that, from an accounting point of view, GDP does not contain intangible investment due to the fact that it is treated as an expense. (This is true despite the fact that from an economical point of view it is not an expense.)

Also, output in each sector equals demand in each sector. Final output of family firms is equal to the sum of household and government consumption demand, and for physical investment demand for family firm-produced goods, too, there is a deadweight loss related to vacancy posting and price stickiness (non-family market clearing is similar).

$$Y_{F,t}^F = C_{F,t} + G_{F,t} + I_{F,t} + \kappa_F v_t^F + \frac{\phi^F}{2} \left( \frac{\pi_t^F}{\pi} - 1 \right)^2 Y_{F,t}^F \frac{P_t}{P_{F,t}} \quad (1.63)$$

Finally, total output (GDP) is defined as:

$$P_t Y_t = P_{F,t} Y_{F,t}^F + P_{NF,t} Y_{NF,t}^F \quad (1.64)$$

### 1.3 Calibration

For the purpose of illustration, the model is calibrated to match data of European countries with an above-average share of family firms in employment (Table 1.3).<sup>32</sup>

First, the long-run unemployment rate is 8.1 percent (Eurostat), and, based on Mandl (2008) and others, the share of family firms in employment is 56.4 percent. Then, Bassanini et al. (2011) is the sole paper that estimates sectoral dismissal rates. They find that the dismissal rate of family firms is 0.16 pp lower than that of the non-family sector. Separation rates in the literature vary between 1.8 percent of Bermperoglou et al. (2013), which is also close to the estimated rates of Hobijn-Sahin (2007), and the 10.5 percent of Gertler et al. (2008), which is similar to the 8-10 percent reported by Hall (1995). Following Staehler-Thomas (2012), I set the non-family dismissal rate to 6 percent, which is the middle point, so, taking into account Bassanini et al. (2011), the family rate is 5.84 percent.

Next, Bassanini et al. (2011) also show evidence that the bargaining power of workers, i.e. the importance of unions, is higher in the non-family sector (0.807), compared to the family sector (0.495). Hosios (1990) claims that an efficient solution requires that the bargaining power of workers is equal to the matching elasticity in the matching function; therefore, I set the sectoral matching elasticities accordingly. The non-family bargaining power is higher than the usual values of 0.3-0.5 of the literature (Mortensen-Nagypal (2007)), apart from Gertler et al. (2008) who use a value of slightly more than 0.9. Also, most papers, following Hosios (1990), set the matching elasticity equal to the bargaining power of workers, except Gertler et al. (2008) who fix the matching elasticity to 0.5. Moreover, Christoffel et al. (2009) consider a somewhat higher matching elasticity than bargaining power of workers (0.6 and 0.5, respectively).

Similarly to dismissal rates, there is a wide range of values found in the literature regarding the ratio of vacancy costs to wages. Bruckner-Pappa (2012) and Bermperoglou et al. (2013) use 4.5 percent following Hagedorn-Manovski (2008). But others, such as Christoffel et al. (2009) and Staehler-Thomas (2012), consider higher values, around 6-7 percents, while the highest value is used by Gertler et al. (2008) (almost 9 percent). I calibrate these ratios to 7 percent in both sectors, which is in the middle of the range.

As a next step, the share of family firm produced goods in the household and government consumption basket is 60 percent following Mandl (2008) and others. Also, price elasticity

---

<sup>32</sup>In this section we detail parameters related to the labor and goods markets and family capital, also the great ratios and parameters affecting the public budget, but the table contains all other parameter values as well.

| Notation                          | Name  | Value  | Source   |
|-----------------------------------|---|--------|--|
| $\beta$                           | discount factor   | 0.99   | standard   |
| $\pi$                             | 1 + quarterly inflation rate in SS  | 1      | standard   |
| $\epsilon_F/(\epsilon_F-1)$       | F price markup  | 1.1    | standard   |
| $\epsilon_{NF}/(\epsilon_{NF}-1)$ | NF price markup   | 1.1    | standard   |
| $\delta_F$                        | F physical capital depreciation rate  | 0.025  | standard   |
| $\delta_{NF}$                     | NF physical capital depreciation rate   | 0.025  | standard   |
| $\delta_{OF}$                     | F organisational capital depreciation rate  | 0.025  | standard   |
| $\alpha_F$                        | labor income share in F   | 0.7    | standard   |
| $\alpha_{NF}$                     | labor income share in NF  | 0.7    | standard   |
| $A_F$                             | SS productivity level in F  | 1      | simplifying assumption   |
| $A_{NF}$                          | SS productivity level in NF   | 1      | simplifying assumption   |
| $\gamma$                          | share of family firm-produced goods in aggregate household/government consumption                                     | 0.6    | Mandl (2008), IEF (2009), IFB (2011), Bjuggren et al (2011) and Lindow (2013), own calculation   |
| $\theta$                          | effectiveness of F organisational investment  | 1      | simplifying assumption   |
| $C/Y$                             | SS ratio of household consumption to output (%)   | 59.16  | own calculation based on Eurostat data between 2000-2012   |
| $DEF/Y$                           | SS ratio of government deficit to output (%)  | 3      | Maastricht Treaty  |
| $L_F$                             | SS F workforce (%)  | 51.88  | own calculation based on Eurostat data between 2000-2012 and Mandl (2008), IEF (2009), IFB (2011), Bjuggren et al (2011) and Lindow (2013) |
| $L_{NF}$                          | SS NF workforce (%)   | 40.04  |  |
| $U$                               | SS unemployment rate (%)  | 8.09   |  |
| $\tau_c$                          | SS effective value added tax rate (%)   | 12.78  |  |
| $\tau_{L,IE}$                     | SS effective labor income tax rate (employee, includes personal income tax and social security contribution rate) (%) | 28.81  | calculated based on OECD data between 2000-2012  |
| $\tau_{L,ER}$                     | SS effective labor income tax rate (employer, includes social security contribution rate) (%)                         | 24.16  |  |
| $b$                               | ratio of unemployment benefit to SS average gross real wage (%)   | 28.03  |  |
| $P_F/P_{NF}$                      | ratio of F price level to NF price level in SS  | 0.95   | simplifying assumption, Bassanini et al (2011)   |
| $\rho_F$                          | F dismissal rate  | 0.0584 | Stahler-Thomas (2012) and Bassanini et al (2011)   |
| $\rho_{NF}$                       | NF dismissal rate   | 0.06   | Stahler-Thomas (2012)  |
| $K_{OF}/K_F$                      | ratio of F organisational capital to F physical capital in SS   | 0.11   | simplifying assumption, Gourio-Rudanko (2014)  |
| $\lambda_F$                       | F bargaining power of workers   | 0.495  | Bassanini et al (2011)   |
| $\sigma_F$                        | F matching elasticity   | 0.495  | Bassanini et al (2011) and Hosios (1990)   |
| $\lambda_{NF}$                    | NF bargaining power workers   | 0.807  | Bassanini et al (2011)   |
| $\sigma_{NF}$                     | NF matching elasticity  | 0.807  | Bassanini et al (2011) and Hosios (1990)   |
| $\tau_K$                          | SS effective capital income tax rate (%)  | 20     | Mendoza et al (2014)   |
| $\delta_K$                        | tax allowance for a fraction of depreciation cost   | 0.22   | Mendoza et al (2014)   |
| $\eta$                            | price elasticity of demand  | 1.5    | assumption   |
| $\kappa_F/W_F$                    | ratio of F vacancy cost to SS F gross real wage   | 0.07   | assumption   |
| $\kappa_{NF}/W_{NF}$              | ratio of NF vacancy cost to SS NF gross real wage   | 0.07   | assumption   |

Table 1.3: Steady-state parameters

of demand between family and non-family goods is assumed to be 1.5 and I check the robustness of the results with respect to this assumption.<sup>33</sup>

Again, I am unaware of any information concerning the ratio of sectoral price levels. Nevertheless, Bassanini et al. (2011) find that wages in the family sector are 5 percent lower than wages in the non-family sector. Sraer-Thesmar (2007) also find a similar wage penalty (about 4.5 percent), even though they did not consider non-listed companies as did Bassanini et al. (2011). Hence, I consider a 5 percent gap between the sectoral price levels.

The effectiveness of family organisational investment is normalized to 1, which, following McGrattan-Prescott (2010), assumes that the accumulation process of intangible capital is the same as the usual accumulation process of physical capital.

Dynastic management connotes firm knowledge about customers, but also about suppliers. It is a latent variable that is difficult to measure, and I am unaware of any empirical estimates. Gourio-Rudanko (2014) claim that in general about 11 percent of employment is related to sales. This is the way they calibrate the weight of customer capital, an intangible capital that represents the relationship between firms and customers. By assumption, I calibrate the level of organisational to physical capital in the family sector to 11 percent. By doing so, the relationship between firms and suppliers is not fully captured, and the importance of family capital might therefore be even higher.

Now, turning to great ratios, the household consumption to GDP ratio is 59.1 percent (Eurostat).<sup>34</sup> As regards the public budget, the steady-state deficit to GDP ratio is 3 percent pursuant to the Maastricht Treaty. Following the OECD and taking into account the great ratio of household consumption, the steady-state value of the effective value-added tax rate is 12.4 percent. Then, the steady state effective labor income tax rate of employees is 28.8 percent (including both personal labor income tax and social security contribution), while the employer social security contribution rate is 24.2 percent. These tax rates are similar to those of Staehler-Thomas (2012).<sup>35</sup>

Then, the gross steady-state replacement rate of unemployment benefit is 28 percent (OECD). For this parameter, there is a very wide range of values in the literature. The implied value of Monacelli et al. (2010) is only slightly above 10 percent, but Christoffel et

---

<sup>33</sup>This and all other robustness checks are available in the Appendix.

<sup>34</sup>When calculating this ratio, GDP was modified by net exports and government investment, as the model is a closed economy and does not contain public investment.

<sup>35</sup>Other papers usually consider only a lump sum tax, apart from Bermperoglou et al. (2013).



al. (2009) set this rate to 65 percent, while Esser et al. (2013) suggest 50 percent. The most common value is around 30-40 percent; usually lower for the US than for the euro area.<sup>36</sup>

Calibration implies that the job-finding rates are 22.4 and 17.8 percent in the family and non-family sectors, respectively. This is in line with unemployment duration in the euro area where between 2000 and 2012 about 22.7 percent of unemployed people found a job within one to two months, while about 37.5 percent found a job in less than five months (Eurostat). My values are also similar to those of Christoffel et al. (2009) or Staehler-Thomas (2012). Nevertheless, for the US Shimer (2005) reports 45 percent and while there is 83 percent in Bermperoglou et al. (2013) and in Bruckner-Pappa (2012).

As regards the job-filling probabilities, they are lower than those in the literature: 15.4 and 9.9 percent in the family and non-family sectors, respectively. For the US, Bermperoglou et al. (2013) and Bruckner-Pappa (2012) consider  $\frac{2}{3}$ , while for the euro area Christoffel et al. (2009) and Staehler-Thomas (2012) use 0.7. The reason for this is that in this paper, unlike in others, the job-filling probability is linearly and positively related to the vacancy cost.<sup>37</sup> Thus, there is a trade-off in setting both the vacancy cost and the job-filling probability close to values in the literature. I calibrated the model such that the vacancy-posting cost is as similar as possible to the literature, at the cost of accepting lower job-filling probability rates. Too, the implied share of total vacancy costs in GDP is 1.7 percent, while family and non-family matching efficiencies are 0.19 and 0.16.

Dynamic parameters are based on others studies and summarized in Table 1.4.

| Notation     | Name  | Value |
|--------------|---|-------|
| $\sigma_C$   | relative risk aversion  | 1.38  |
| $h$          | external habit in consumption   | 0.71  |
| $\phi_F$     | price rigidity parameter, when share of firms which do not set prices in F is 66 %  | 56.01 |
| $\phi_{NF}$  | price rigidity parameter, when share of firms which do not set prices in NF is 66 % | 56.01 |
| $\gamma_\pi$ | inflation parameter in Taylor rule  | 2.04  |
| $\chi_B$     | debt rule parameter in lump sum tax rule  | 2     |
| $\phi_{IF}$  | F physical investment adjustment cost   | 2.48  |
| $\phi_{INF}$ | NF physical investment adjustment cost  | 2.48  |
| $\phi_{IOF}$ | F organisational investment adjustment cost   | 2.48  |
|              | all autocorrelation parameters  | 0.75  |

Table 1.4: Dynamic parameters

<sup>36</sup>It is not straightforward how to compare the replacement rates, as some models only contain a lump-sum tax. This means that there is no clear distinction between gross and net wages and, thus, between gross and net unemployment benefit replacement rates.

<sup>37</sup>See the first-order conditions with respect to the number of posted vacancies in Section 1.2.

## 1.4 Results

### 1.4.1 Baseline results

The model is loglinearized and stochastic simulations are done by Dynare 4.4.2. Four fiscal austerity shocks are studied, all of them are 1 percent of GDP size, so they are directly comparable: i) a decrease in government consumption, ii) an increase in the rate of value-added tax, iii) an increase in employees' labor income tax rate (personal labor income tax and social security contribution) and iv) an increase in employer social security contribution. Table 1.5 shows the unemployment multipliers: i) at peak and ii) cumulatively. A peak multiplier is the largest response after the shock, while the cumulative multipliers are the sum of the multipliers in the first one, two and four years, respectively. This is similar to Spilimbergo et al. (2009), Uhlig (2010) or Campolmi et al. (2011). All multipliers are presented in percentage point deviations from the steady state.<sup>38</sup>

To begin with, all fiscal consolidation policies raise unemployment. At peak, the highest increase, 0.52 percentage points, is implied by a cut in government consumption. At peak, all other policies imply a considerably lower unemployment hike.

Nevertheless, the cumulative multipliers show a somewhat different picture. On the whole, during the four years, cutting government consumption or increasing employees' labor income taxes cause the highest, around 0.5 pp increase in the unemployment rate. At the same time, over a shorter time horizon, the government consumption multiplier still exceeds any other multiplier. Furthermore, an increase in the employer social security contribution rate is less harmful for employment than an increase in the employees' labor income tax. Cumulatively, also, consolidating the budget by increasing the value added tax revenue is the least costly in terms of employment.

|                      | Government<br>consumption cut | Value added<br>tax increase | Labor income<br>tax increase<br>(employee) | Labor income<br>tax increase<br>(employer) |
|----------------------|-------------------------------|-----------------------------|--|--|
| Peak                 | 0.52                          | 0.02                        | 0.12                                       | 0.07                                       |
| Cumulative (1 year)  | 0.71                          | 0.04                        | 0.34                                       | 0.19                                       |
| Cumulative (2 years) | 0.67                          | 0.06                        | 0.55                                       | 0.31                                       |
| Cumulative (4 years) | 0.48                          | 0.03                        | 0.51                                       | 0.29                                       |

Table 1.5: Unemployment fiscal multipliers (in percentage points)

Regarding the economic driving forces, a higher consumption tax implies a decline in consumption, however, investment goes up, so total demand does not significantly change,

<sup>38</sup>Stochastic impulse response functions are shown in Figures 1.A1-1.A4 of the Appendix.

similarly to total employment. As for the cut in government consumption, even though household consumption and investment both increase, total demand considerably declines. In spite of the lower wages, firms therefore decrease employment owing to a lower demand for labor. At the same time, concerning the labor income tax shocks, the main driving force is related to wage bargaining. Increasing employees' labor income tax results in a decline in worker values, which means that being employed becomes less favourable than the outside option (unemployment benefit). The increase in the employer social security contribution rate has a negative effect on the firm values, however. The fact that increasing the employees' labor income tax is more harmful for employment than increasing the employer social security contribution highlights the relatively more important role of worker values over other channels of the model. Moreover, the greater the bargaining power of workers, the more truth there is to this claim.

Thus, the results suggest that, if the aim of the government is to consolidate the budget at the lowest price in terms of employment, an increase in the value-added tax rate is preferred. Moreover, a cut in government consumption usually implies a higher increase in unemployment than do labor income tax hikes, while an increase in employees' labor income tax is more harmful for employment than increasing the employers' social security contribution rate.

With regard to the existing literature, these results are in line with studies that suggest that tightening fiscal policy on the expenditure side increases unemployment. As regards the size of the unemployment multiplier of government consumption, it is close to that of Monacelli et al. (2010). Further, results are, at peak, consistent with Ball et al. (2013), who claim that spending-based adjustments have a more pronounced effect on unemployment than tax-based adjustments. Nonetheless, as they employ a narrative approach, they were unable to compare different tax policies, thus, another contribution of this paper is the ability of doing that. In contrary to Staehler-Thomas (2012), though, I find that cumulatively a labor income tax hike implies a similar rise in unemployment than a government consumption tax does.

Nevertheless, unemployment is clearly not the sole concern of the government. Regarding output, all policies, apart from the value-added tax policy, induce a decline in it. The largest decline occurs after a cut in government consumption; output goes down on impact by more than 0.4 percent. Also, not only the reaction of GDP, but also that of household consumption is important. The only policy that raises household consumption is a cut in government consumption (household consumption goes up by more than 0.9 percent at peak). At the same time, it goes down considerably when the rate of value-added tax is increased (at peak by 0.25 percent). Hence, there are trade-offs to consider.

Combining policies might be a way to manage these trade-offs. Given the debate on shifting the focus of taxation from labor income toward value-added taxes, two policy packages are investigated: an increase of 1 percent of GDP in revenue from value-added tax and the same size of decrease in the employees' or employers' labor income tax revenue, keeping the budget deficit unchanged.<sup>39</sup> These policies, at peak, induce a 0.1 and a 0.05 percentage points decline in unemployment, respectively. At the same time, household consumption goes down somewhat less than after an increase in value-added tax and output goes up compared to a decline following the hikes in labor income taxes.

### 1.4.2 The role of sectoral heterogeneity

Sectoral heterogeneity on the firm side - besides labor force participation and short- and long-term unemployment suggested by Bermperoglou et al. (2013) - might also explain the gap between theoretical and empirical multipliers reported but not clarified by Monacelli et al. (2010). In this section, three alternative scenarios are compared to the baseline scenario: i) only family firms, ii) only non-family firms, iii) and a two-sectoral framework without family capital; the model closest to the literature is version ii).

|                                      | BASELINE             | Only family firms | Only non-family firms | Both, but no organisational capital |
|--------------------------------------|----------------------|-------------------|-----------------------|-------------------------------------|
|                                      | Peak                 |                   |                       |                                     |
| Government consumption cut           | 0.52                 | 0.15              | 0.12                  | 0.25                                |
| Value added tax increase             | 0.02                 | 0.03              | 0.01                  | 0.02                                |
| Labor income tax increase (employee) | 0.12                 | 0.03              | 0.01                  | 0.04                                |
| Labor income tax increase (employer) | 0.07                 | 0.02              | 0.01                  | 0.02                                |
|                                      | Cumulative (1 year)  |                   |                       |                                     |
| Government consumption cut           | 0.71                 | 0.22              | 0.19                  | 0.36                                |
| Value added tax increase             | 0.04                 | 0.09              | 0.02                  | 0.06                                |
| Labor income tax increase (employee) | 0.34                 | 0.08              | 0.04                  | 0.12                                |
| Labor income tax increase (employer) | 0.19                 | 0.05              | 0.02                  | 0.07                                |
|                                      | Cumulative (2 years) |                   |                       |                                     |
| Government consumption cut           | 0.67                 | 0.25              | 0.20                  | 0.37                                |
| Value added tax increase             | 0.06                 | 0.16              | 0.03                  | 0.09                                |
| Labor income tax increase (employee) | 0.55                 | 0.15              | 0.07                  | 0.21                                |
| Labor income tax increase (employer) | 0.31                 | 0.02              | 0.04                  | 0.12                                |
|                                      | Cumulative (4 years) |                   |                       |                                     |
| Government consumption cut           | 0.48                 | 0.20              | 0.18                  | 0.31                                |
| Value added tax increase             | 0.03                 | 0.17              | 0.03                  | 0.08                                |
| Labor income tax increase (employee) | 0.51                 | 0.19              | 0.08                  | 0.23                                |
| Labor income tax increase (employer) | 0.29                 | 0.11              | 0.05                  | 0.13                                |

Table 1.6: Unemployment fiscal multipliers and sectoral heterogeneity (in percentage points)

Sectoral heterogeneity seems to play a crucial role (Table 1.6)<sup>40</sup>; unemployment multipliers

<sup>39</sup>Impulse responses are presented in Figures 1.A5-1.A6 of the Appendix.

<sup>40</sup>See also Figure 1.A7 of the Appendix.

are very different with and without it. When firms are homogeneous, multipliers of labor income tax policies and government consumption multipliers are usually biased downwards, while the consumption tax multipliers are often larger than with sectoral heterogeneity. One can also conclude that about half of the difference is due to the number of sectors, while the other half is related to the inclusion of intangible capital in the family sector.

## 1.5 Discussion

The aim of this paper was to calculate unemployment multipliers of expenditure and revenue-side fiscal austerity policies. This research contributes to the body of knowledge as there is a debate on both the sign and the size of unemployment multipliers of government expenditure items. Moreover, the literature is very narrow regarding unemployment multipliers of tax policies. Lastly, to the best of my knowledge, sectoral heterogeneity was not studied in this context before, although firms are obviously not homogeneous.

The framework was based on a standard DSGE model with sticky prices and search and matching frictions, and, as a novelty, on the firm side there was a distinction between family and non-family firms. Family firms employ a notable share of people in Europe, and the differently managed family and non-family firms behave differently in the labor market. The model was calibrated to match data of European countries with a large percentage of family firms in employment, while the characteristics of family firms were based on empirical, micro-level evidence documented in the corporate finance literature.

The model predicts that all fiscal austerity policies raise unemployment. At peak, the highest increase in unemployment is implied by a cut in government consumption. Nevertheless, a hike in employees' labor income tax, cumulatively, implies the same size of increase in unemployment as does the government consumption cut. A higher employer social security contribution is always less costly in terms employment than an increase of the same size in employees' labor income tax, however. Both at peak and cumulatively, unemployment reacts least when the budget is consolidated by increasing the rate of value-added tax. Nonetheless, a policymaker will need to deal with trade-offs, as the increase in value-added tax results in the highest decline in consumption.

Regarding the existing literature, the results are in line with those who suggest that tightening fiscal policy on the expenditure side increases unemployment. As regards the size of the unemployment multiplier of government consumption, it is close to that of Monacelli et al. (2010). Further, results are consistent with Ball et al. (2013), who show that spending-based adjustments have a more pronounced effect on unemployment than tax-based adjustments (only at peak, however). Additionally, in contrary to Staehler-Thomas (2012) I do not find

that four years after the shock the rise in unemployment, if the budget is consolidated by a cut in government consumption, is lower than after a labor income tax hike.

Sectoral heterogeneity seems to play a crucial role: unemployment fiscal multipliers are very different with and without it. When homogeneous firms are considered, multipliers of labor income tax policies and government consumption multipliers are usually lower, while the consumption tax multipliers are often higher than with sectoral heterogeneity. So ignoring it might lead to incorrect policy conclusions. Also, sectoral heterogeneity on the firm side - besides labor force participation and short- and long-term unemployment suggested by Bermperoglou et al. (2013) - might be another explanation of the gap between theoretical and empirical multipliers reported but unexplained by Monacelli et al. (2010).

Does this mean that the government should consolidate the budget by increasing the role of consumption taxation? This depends on the goals of the government; in particular, how each goal is weighted. Clearly, increasing the rate of value-added tax is least costly in terms of employment, at any time-horizon considered. However, there are trade-offs. Specifically, household consumption declines considerably after this policy, and this policy is the most harmful for household consumption. Should the government care more about the number of unemployed or about the amount of consumption of the society as a whole? This raises further questions. Does inequality increase more when more people are unemployed, or does inequality increase more when household consumption declines more? Inequality is related to the progressivity of the tax system; while labor income taxation is often progressive, consumption taxation is always regressive. A drawback of this paper's model is that - due to the representative agent assumption - no inequality measure can be defined, leaving me unable to answer these important questions. Also, do we think that, as time goes by, it is more difficult to leave unemployment? If so, then the less time that passes since losing a job, the higher the probability of finding another one. In this case, a decline in consumption might be less harmful than an increase in unemployment from a longer-term point of view.

As I pointed out earlier, sectoral heterogeneity has not been considered before in the unemployment fiscal multiplier literature. Did we find evidence of its importance in this paper? I think yes, and we can conclude that sectoral heterogeneity, and a distinction between family and non-family firms, in particular, has a crucial role. Multipliers are very different with and without it. Here we are not concerned with determining the “best” fiscal austerity policy for employment, which, according to my results, is always an increase in consumption tax, but rather, with regard to the relative employment cost of the “second-best” policies.<sup>41</sup> Still, further research is urged in this field as regards the remaining questions.

---

<sup>41</sup> “Best” policy here does not mean an optimal policy, but means the policy that causes the lowest rise in unemployment.

# Appendix

## A Steady-state calculation of the model

In this section I describe the steps of calibrating the model's steady state. First, based on data,  $\pi$ ,  $A_F$ ,  $A_{NF}$ ,  $L_F$ ,  $L_{NF}$ ,  $\tau_C$ ,  $\tau_{LEE}$  and  $\tau_{LER}$  are fixed. Then, following the steps presented below, the steady-state values of all the other variables are calculated given the parameter values and the values of the already fixed variables.  $XX$ ,  $YY$ ,  $ZZ$ , and  $AA$ ,  $BB$ ,  $CC$ ,  $DD$ ,  $EE$ ,  $FF$ ,  $GG$ ,  $HH$ ,  $II$ ,  $JJ$  are not model variables, but a combination of variables, only used for the purpose of illustration. Vacancy costs and matching efficiencies are fixed to their calibrated values when checking the robustness of the model.

$$i = \frac{\pi}{\beta} - 1 \quad (1.65)$$

$$PFP = (\gamma + (1 - \gamma)PFP PNF P^{\eta-1})^{\frac{1}{\eta-1}} \quad (1.66)$$

$$PNFP = \left( \frac{1 - \gamma PFP^{1-\eta}}{1 - \gamma} \right)^{\frac{1}{1-\eta}} \quad (1.67)$$

$$Q_F = PFP \quad (1.68)$$

$$Q_{NF} = PNFP \quad (1.69)$$

$$MC_F = PFP \frac{\epsilon_F - 1}{\epsilon_F} \quad (1.70)$$

$$MC_{NF} = PNFP \frac{\epsilon_{NF} - 1}{\epsilon_{NF}} \quad (1.71)$$

$$Q_{OF} = \frac{MC_F}{\theta} \quad (1.72)$$

$$U = 1 - L_F - L_{NF} \quad (1.73)$$

$$U_s = U + (1 - \rho^F)L_F + (1 - \rho^{NF})L_{NF} \quad (1.74)$$

$$RK_F = Q_F \left( \frac{1+i}{\pi} - 1 + \delta_F \right) \quad (1.75)$$

$$RK_{NF} = Q_{NF} \left( \frac{1+i}{\pi} - 1 + \delta_{NF} \right) \quad (1.76)$$

$$XX = \frac{Q_{OF} - (1 - \delta_{OF})\beta Q_{OF}}{\beta MC_F} \quad (1.77)$$

$$YY = \frac{RK_F}{MC_F} \quad (1.78)$$

$$ZZ = \frac{XX}{YY} KOFKF \quad (1.79)$$

$$\alpha_{OF} = (1 - \alpha_F) \frac{ZZ}{ZZ + 1} \quad (1.80)$$

$$K_F = \left( (1 - \alpha_F - \alpha_{OF}) A_F L_F^{\alpha_F} KOFKF^{\alpha_{OF}} \frac{1}{YY} \right)^{\frac{1}{\alpha_F}} \quad (1.81)$$

$$K_{OF} = KOFKF K_F \quad (1.82)$$

$$I_{OF} = \delta_{OF} K_{OF} \quad (1.83)$$

$$Y_F^I = A_F L_F^{\alpha_F} K_F^{1-\alpha_F-\alpha_{OF}} K_{OF}^{\alpha_{OF}} \quad (1.84)$$

$$Y_F^F = Y_F^I - I_{OF} \quad (1.85)$$

$$YNFIKNF = \frac{RK_{NF}}{MC_{NF}(1-\alpha_{NF})} \quad (1.86)$$

$$YNFILNF = YNFIKNF^{1-\frac{1}{\alpha_{NF}}} A_{NF}^{\frac{1}{\alpha_{NF}}} \quad (1.87)$$

$$K_{NF} = A_{NF}^{\frac{1}{\alpha_{NF}}} L_{NF} YNFIKNF^{-\frac{1}{\alpha_{NF}}} \quad (1.88)$$

$$Y_{NF}^I = A_{NF} L_{NF}^{\alpha_{NF}} K_{NF}^{1-\alpha_{NF}} \quad (1.89)$$

$$Y_{NF}^F = Y_{NF}^I \quad (1.90)$$

$$I_F = \delta_F K_F \quad (1.91)$$

$$I_{NF} = \delta_{NF} K_{NF} \quad (1.92)$$

$$Y = PFP Y_F^F + PNFP Y_{NF}^F \quad (1.93)$$

$$C = CY Y \quad (1.94)$$

$$C_F = \gamma C PFP^{-\eta} \quad (1.95)$$

$$C_{NF} = (1-\gamma) C PNFP^{-\eta} \quad (1.96)$$

$$m^F = L_F(1-\rho^F) \quad (1.97)$$

$$m^{NF} = L_{NF}(1-\rho^{NF}) \quad (1.98)$$

$$p^F = \frac{m^F}{U_s} \quad (1.99)$$

$$p^{NF} = \frac{m^{NF}}{U_s} \quad (1.100)$$

$$AA = \frac{p^F \frac{1-\tau_{LEE}}{1-\beta\rho^F} + (1-p^F-p^{NF}) \frac{bL_F}{L_F+L_{NF}}}{1-p^F\beta \frac{1-\rho^F}{1-\beta\rho^F} - p^{NF}\beta \frac{1-\rho^{NF}}{1-\beta\rho^{NF}} - (1-p^F-p^{NF})\beta} \quad (1.101)$$

$$BB = \frac{p^{NF} \frac{1-\tau_{LEE}}{1-\beta\rho^{NF}} + (1-p^F-p^{NF}) \frac{bL_{NF}}{L_F+L_{NF}}}{1-p^F\beta \frac{1-\rho^F}{1-\beta\rho^F} - p^{NF}\beta \frac{1-\rho^{NF}}{1-\beta\rho^{NF}} - (1-p^F-p^{NF})\beta} \quad (1.102)$$

$$CC = \beta AA + \frac{bL_F}{L_F+L_{NF}} \quad (1.103)$$

$$DD = \beta BB + \frac{bL_{NF}}{L_F+L_{NF}} \quad (1.104)$$

$$EE = \lambda^F(1-\tau_{LEE})MC_F\alpha_F \frac{Y_F^I}{L_F} \quad (1.105)$$

$$FF = (1+\tau_{LER}) \left( (1-\tau_{LEE}) + (1-\lambda^F)\beta(1-\rho^F)AA \right) - (1+\tau_{LER})(1-\lambda^F)(1-\beta\rho^F)CC \quad (1.106)$$

$$GG = (1+\tau_{LER}) \left( (1-\lambda^F)\beta(1-\rho^F)BB - (1-\lambda^F)(1-\beta\rho^F)DD \right) \quad (1.107)$$

$$HH = \lambda^{NF}(1-\tau_{LEE})MC_{NF}\alpha_{NF} \frac{Y_{NF}^I}{L_{NF}} \quad (1.108)$$



$$II = (1 + \tau_{LER}) \left( (1 - \lambda^{NF})\beta(1 - \rho^{NF})AA - (1 - \lambda^{NF})(1 - \beta\rho^{NF})CC \right) \quad (1.109)$$

$$JJ = (1 + \tau_{LER}) \left( (1 - \tau_{LEE}) + (1 - \lambda^{NF})\beta(1 - \rho^{NF})BB \right) - (1 + \tau_{LER})(1 - \lambda^{NF})(1 - \beta\rho^{NF})DD \quad (1.110)$$

$$W_{NF} = \frac{HH - II \frac{EE}{FF}}{JJ - II \frac{GG}{FF}} \quad (1.111)$$

$$W_F = \frac{EE - GG W_{NF}}{FF} \quad (1.112)$$

$$W_U = \frac{b(L_F W_F + L_{NF} W_{NF})}{L_F + L_{NF}} \quad (1.113)$$

$$UV_b = AA W_F + BB W_{NF} \quad (1.114)$$

$$UV_e = W_U + \beta UV_b \quad (1.115)$$

$$F_F = \frac{MC_F \alpha_F \frac{Y_F^I}{L_F} - (1 + \tau_{LER})W_F}{1 - \beta\rho^F} \quad (1.116)$$

$$F_{NF} = \frac{MC_{NF} \alpha_{NF} \frac{Y_{NF}^I}{L_{NF}} - (1 + \tau_{LER})W_{NF}}{1 - \beta\rho^{NF}} \quad (1.117)$$

$$V_F = \frac{(1 - \tau_{LEE})W_F + \beta(1 - \rho^F)UV_b}{1 - \beta\rho^F} \quad (1.118)$$

$$V_{NF} = \frac{(1 - \tau_{LEE})W_{NF} + \beta(1 - \rho^{NF})UV_b}{1 - \beta\rho^{NF}} \quad (1.119)$$

$$\kappa^F = c^F W_F \quad (1.120)$$

$$\kappa^{NF} = c^{NF} W_{NF} \quad (1.121)$$

$$q^F = PFP \frac{\kappa^F}{F_F} \quad (1.122)$$

$$q^{NF} = PNFP \frac{\kappa^{NF}}{F_{NF}} \quad (1.123)$$

$$v^F = \frac{m^F}{q^F} \quad (1.124)$$

$$v^{NF} = \frac{m^{NF}}{q^{NF}} \quad (1.125)$$

$$\sigma_{F,m} = \frac{m^F}{(U_s)^{\sigma_F} (v^F)^{1-\sigma_F}} \quad (1.126)$$

$$\sigma_{NF,m} = \frac{m^{NF}}{(U_s)^{\sigma_{NF}} (v^{NF})^{1-\sigma_{NF}}} \quad (1.127)$$

$$G = Y - C - PFP I_F - PNFP I_{NF} - \kappa^F PFP v^F - \kappa^{NF} PNFP v^{NF} \quad (1.128)$$

$$G_F = \gamma G PFP^{-\eta} \quad (1.129)$$

$$G_{NF} = (1 - \gamma) G PNFP^{-\eta} \quad (1.130)$$

$$DEF = DEF Y \quad (1.131)$$

$$B = \frac{DEF}{1 - \frac{1+i}{\pi}} \quad (1.132)$$

$$T = G + W_U U - DEF - (\tau_{LEE} + \tau_{LER})(W_F L_F + W_{NF} L_{NF}) - \tau_C C \quad (1.133)$$

$$\begin{aligned} Prof_F^I &= MC_F Y_F^I - (1 + \tau_{LER}) W_F L_F - R K_F K_F - \\ &- \kappa^F PFP v^F - MC_F I_{OF} \end{aligned} \quad (1.134)$$

$$\begin{aligned} Prof_{NF}^I &= MC_{NF} Y_{NF}^I - (1 + \tau_{LER}) W_{NF} L_{NF} - R K_{NF} K_{NF} - \\ &- \kappa^{NF} PNFP v^{NF} \end{aligned} \quad (1.135)$$

$$Prof_F^F = (PFP - MC_F) Y_F^F \quad (1.136)$$

$$Prof_{NF}^F = (PNFP - MC_{NF}) Y_{NF}^F \quad (1.137)$$

$$I = PFP I_F + PNFP I_{NF} \quad (1.138)$$

**Notation:**

- ratio of family to non-family prices

$$PFPPNFP = \frac{PFP}{PNFP} \quad (1.139)$$

- ratio of organisational capital to physical capital in the family sector

$$KOFKF = \frac{K_{OF}}{K_F} \quad (1.140)$$

- non-family output-physical capital ratio

$$YNFIKNF = \frac{Y_{NF}^I}{K_{NF}} \quad (1.141)$$

- non-family output-labor ratio

$$YNFILNF = \frac{Y_{NF}^I}{K_{NF}} \quad (1.142)$$

- great ratios (consumption to GDP and government deficit to GDP)

$$CY = \frac{C}{Y} \quad (1.143)$$

$$DEFY = \frac{DEF}{Y} \quad (1.144)$$

- ratios of vacancy costs to wages

$$c^F = \frac{\kappa^F}{W_F} \quad (1.145)$$

$$c^{NF} = \frac{\kappa^{NF}}{W_{NF}} \quad (1.146)$$

## B Loglinearized model

Now, I summarize the loglinearized equations. Symbol  $\hat{\cdot}$  means a percentage deviation from the steady state, while any variable without a time index is the steady-state value.

### Aggregation:

- Household consumption demand for family firm produced goods

$$\hat{C}_{F,t} = \hat{C}_t - \eta P \hat{F} P_t \quad (1.147)$$

- Household consumption demand for non-family firm produced goods

$$\hat{C}_{NF,t} = \hat{C}_t - \eta P N \hat{F} P_t \quad (1.148)$$

- Government consumption demand for family firm produced goods

$$\hat{G}_{F,t} = \hat{G}_t - \eta P \hat{F} P_t \quad (1.149)$$

- Government consumption demand for non-family firm produced goods

$$\hat{G}_{NF,t} = \hat{G}_t - \eta P N \hat{F} P_t \quad (1.150)$$

- Price level

$$0 = \gamma P F P I_F (P \hat{F} P_t + \hat{I}_{F,t}) + (1 - \gamma) P N F P I_{NF} (P N \hat{F} P_t + \hat{I}_{NF,t}) \quad (1.151)$$

- Definition of private physical investment

$$\hat{I} \hat{I}_t = P F P I F (\hat{I} \hat{F}_t + P \hat{F} P_t) + P N F P I N F (P N \hat{F}_t + P N \hat{F} P_t) \quad (1.152)$$

### Household:

- Euler equation

$$\frac{\tau_C}{1 + \tau_C} (\tau_C \hat{C}_{t,t} - E_t [\tau_C \hat{C}_{t+1}]) + \sigma_C \frac{1+h}{1-h} \hat{C}_t - \frac{h}{1-h} \hat{C}_{t-1} - \frac{1}{1-h} E_t [\hat{C}_{t+1} + \hat{i}_t - \pi_{t+1}] = 0 \quad (1.153)$$

- Family physical capital accumulation

$$K_F \hat{K}_{F,t} = (1 - \delta_F) K_F \hat{K}_{F,t-1} + I_F \hat{I}_{F,t} \quad (1.154)$$

- Family physical capital Tobin-Q

$$Q_F \hat{Q}_{F,t} - \phi^{IF} Q_F (\hat{I}_{F,t} - I_{F,t-1}) + \phi^{IF} Q_F \beta (\hat{I}_{F,t} - I_{F,t-1}) = P F P P \hat{F} P_t \quad (1.155)$$

- Family physical capital arbitrage condition

$$\frac{1+i}{\pi} E_t [\hat{i}_t - \pi_{t+1}] = E_t \left[ \frac{R K_F}{Q_F} R K_{F,t+1} + (1 - \delta_F) Q_{F,t+1} \right] - \frac{R K_F + (1 - \delta_F) Q_F}{Q_F} \hat{Q}_{F,t} \quad (1.156)$$

- Non-family physical capital accumulation

$$K_{NF} \hat{K}_{NF,t} = (1 - \delta_{NF}) K_{NF} \hat{K}_{NF,t-1} + I_{NF} \hat{I}_{NF,t} \quad (1.157)$$

- Non-family physical capital Tobin-Q

$$Q_{NF}Q_{\hat{NF},t} - \phi^{INF}Q_{NF}(I_{\hat{NF},t} - I_{\hat{NF},t-1}) + \\ + \phi^{INF}Q_{NF}\beta(I_{\hat{NF},t} - I_{\hat{NF},t-1}) = PNFPPN\hat{F}P_t \quad (1.158)$$

- Non-family physical capital arbitrage condition

$$\frac{1+i}{\pi}E_t[\hat{i}_t - \pi_{t+1}] = E_t\left[\frac{RK_{NF}}{Q_{NF}}RK_{\hat{NF},t+1} + (1 - \delta_{NF})Q_{NF,t+1}\right] - \\ - \frac{RK_{NF} + (1 - \delta_{NF})Q_{NF}}{Q_{NF}}Q_{\hat{NF},t} \quad (1.159)$$

### Intermediate firms and wage bargaining:

- Family production function

$$Y_{F,t}^I = A_{F,t} + \alpha_F L\hat{F}_t + (1 - \alpha_F - \alpha_{OF})K\hat{F}_{t-1} + \alpha_{OF}K_{OF,t-1} \quad (1.160)$$

- Non-family production function

$$Y_{NF,t}^I = A_{NF,t} + \alpha_{NF} L\hat{N}F_t + (1 - \alpha_{NF})K\hat{N}F_{t-1} \quad (1.161)$$

- Demand for family physical capital

$$M\hat{C}_{F,t} + Y_{F,t}^I - K_{F,t-1} = R\hat{K}_{F,t} \quad (1.162)$$

- Demand for non-family physical capital

$$M\hat{C}_{NF,t} + Y_{NF,t}^I - K_{NF,t-1} = R\hat{K}_{NF,t} \quad (1.163)$$

- Family organisational capital accumulation

$$K_{OF}K_{\hat{OF},t} = (1 - \delta_{OF})K_{OF}K_{\hat{OF},t-1} + \theta I_{OF}I_{\hat{OF},t} \quad (1.164)$$

- Family organisational capital Tobin-Q

$$MC_F M\hat{C}_{F,t} = \theta Q_{OF}Q_{\hat{OF},t} - \phi^{IOF}Q_{OF}(I_{\hat{OF},t} - I_{\hat{OF},t-1}) + \phi^{IOF}Q_{OF}\beta(E_t[I_{\hat{OF},t+1}] - I_{\hat{OF},t}) \quad (1.165)$$

- Demand for family organisational capital

$$\left(M_{CF}\alpha_{OF}\frac{Y_F^I}{K_{OF}} + (1 - \delta_{OF})Q_{OF}\right)\frac{\beta\sigma_C}{1-h}\left((1+h)\hat{C}_t - h\hat{C}_{t-1} - E_t[\hat{C}_{t+1}]\right) + \\ + \beta M_{CF}\alpha_{OF}\frac{Y_F^I}{K_{OF}}E_t\left[M\hat{C}_{F,t+1} + Y_{F,t+1}^I - K_{\hat{OF},t}\right] + (1 - \delta_{OF})Q_{OF}\beta E_t\left[Q_{\hat{OF},t+1}\right] = \\ = Q_{OF}Q_{\hat{OF},t} \quad (1.166)$$

- Definition of unemployment

$$(1 - LF - LNF)\hat{U}_t + LFL\hat{F}_{t-1} + LNFLN\hat{F}_{t-1} = 0 \quad (1.167)$$

- Definition of search pool

$$U_s\hat{U}_{s,t} = U\hat{U}_t + (1 - \rho^F)L_FL_{F,t-1} + (1 - \rho^{NF})L_{NF}L_{NF,t-1} \quad (1.168)$$

- Family matching function

$$\hat{m}_t^F = \sigma_F\hat{U}_{s,t} + (1 - \sigma_F)\hat{v}_t^F \quad (1.169)$$

- Non-family matching function

$$\hat{m}_t^{\hat{N}F} = \sigma_{NF}\hat{U}_{s,t} + (1 - \sigma_{NF})\hat{v}_t^{\hat{N}F} \quad (1.170)$$

- Job filling probability in the family sector

$$\hat{q}_t^F = \hat{m}_t^F - \hat{v}_t^F \quad (1.171)$$

- Job filling probability in the non-family sector

$$\hat{q}_t^{\hat{N}F} = \hat{m}_t^{\hat{N}F} - \hat{v}_t^{\hat{N}F} \quad (1.172)$$

- Job finding probability in the family sector

$$\hat{p}_t^F = \hat{m}_t^F - \hat{U}_{s,t} \quad (1.173)$$

- Job finding probability in the non-family sector

$$\hat{p}_t^{\hat{N}F} = \hat{m}_t^{\hat{N}F} - \hat{U}_{s,t} \quad (1.174)$$

- Labor law of motion in the family sector

$$\hat{L}_{F,t} = \rho^F L_{F,t-1} + m^F \hat{m}_t^F \quad (1.175)$$

- Labor law of motion in the non-family sector

$$\hat{L}_{NF,t} = \rho^{NF} L_{NF,t-1} + m^{NF} \hat{m}_t^{\hat{N}F} \quad (1.176)$$

- Intermediate family firm profit

$$\begin{aligned} Prof_F^I \hat{Prof}_{F,t}^I &= MC_F Y_F^I (M\hat{C}_{F,t} + \hat{Y}_{F,t}^I) - \tau_{LER} W_F L_F \tau_{LER,t} \hat{L}_{F,t} \\ &- (1 + \tau_{LER}) W_F L_F (\hat{W}_{F,t} + \hat{L}_{F,t}) - RK_F K_F (R\hat{K}_{F,t} + \hat{K}_{F,t-1}) - \\ &- \kappa^F P F P v^F (P\hat{F}P_t + \hat{v}_t^F) - MC_F I_{OF} (M\hat{C}_{F,t} + \hat{I}_{OF,t}) \end{aligned} \quad (1.177)$$

- Intermediate non-family firm profit

$$\begin{aligned} Prof_{NF}^I Prof_{NF,t}^I &= MC_{NF} Y_{NF}^I (M\hat{C}_{NF,t} + Y_{NF,t}^{\hat{I}}) - \\ &- \tau_{LER} W_{NF} L_{NF} \tau_{LE} \hat{E}_{R,t} - (1 + \tau_{LER}) W_{NF} L_{NF} (W_{NF,t}^{\hat{I}} + L_{NF,t}^{\hat{I}}) - \\ &- RK_{NF} K_{NF} (R\hat{K}_{NF,t} + K_{NF,t-1}^{\hat{I}}) - \kappa^{NF} PNF P v^{NF} (P\hat{N}F P_t + v_t^{\hat{N}F}) \end{aligned} \quad (1.178)$$

- Vacancy decision in the family sector

$$F_{F,t}^{\hat{I}} = P\hat{F}P_t - q_t^{\hat{F}} \quad (1.179)$$

- Vacancy decision in the non-family sector

$$F_{NF,t}^{\hat{I}} = P\hat{N}F P_t - q_t^{\hat{N}F} \quad (1.180)$$

- Firm value in the family sector

$$\begin{aligned} F_F F_{F,t}^{\hat{I}} &= MC_F \alpha_F \frac{Y_F^I}{L_F} \left( M\hat{C}_{F,t} + Y_{F,t}^{\hat{I}} - L_{F,t}^{\hat{I}} \right) - (1 + \tau_{LER}) W_F W_{F,t}^{\hat{I}} + \\ &+ \beta F_F \rho^F \frac{\sigma_C}{1-h} \left( (1+h)\hat{C}_t - hC_{t-1}^{\hat{I}} - E_t \left[ C_{t+1}^{\hat{I}} \right] \right) + E_t [F_{F,t+1}] - \tau_{LER} W_F \tau_{LE} \hat{E}_{R,t} \end{aligned} \quad (1.181)$$

- Firm value in the non-family sector

$$\begin{aligned} F_{NF} F_{NF,t}^{\hat{I}} &= MC_{NF} \alpha_{NF} \frac{Y_{NF}^I}{L_{NF}} \left( M\hat{C}_{NF,t} + Y_{NF,t}^{\hat{I}} - L_{NF,t}^{\hat{I}} \right) - (1 + \tau_{LER}) W_{NF} W_{NF,t}^{\hat{I}} + \\ &+ \beta F_{NF} \rho^{NF} \frac{\sigma_C}{1-h} \left( (1+h)\hat{C}_t - hC_{t-1}^{\hat{I}} - E_t \left[ C_{t+1}^{\hat{I}} \right] \right) + E_t [F_{NF,t+1}] - \tau_{LER} W_{NF} \tau_{LE} \hat{E}_{R,t} \end{aligned} \quad (1.182)$$

- Worker value in the family sector

$$\begin{aligned} V_F \hat{V}_{F,t} &= (1 - \tau_{LEE}) W_F \hat{W}_{F,t} - \tau_{LEE} W_F \tau_{LE} \hat{E}_{R,t} + \\ &+ \beta \left( \rho^F V_F + (1 - \rho^F) U V_b \right) \frac{\sigma_C}{1-h} \left( (1+h)\hat{C}_t - hC_{t-1}^{\hat{I}} - E_t \left[ C_{t+1}^{\hat{I}} \right] \right) + \\ &+ \beta \rho^F V_F E_t \left[ \hat{V}_{F,t+1} \right] + \beta (1 - \rho^F) U V_b E_t \left[ \hat{V}_{b,t+1} \right] \end{aligned} \quad (1.183)$$

- Worker value in the non-family sector

$$\begin{aligned} V_{NF} \hat{V}_{NF,t} &= (1 - \tau_{LEE}) W_{NF} \hat{W}_{NF,t} - \tau_{LEE} W_{NF} \tau_{LE} \hat{E}_{R,t} + \\ &+ \beta \left( \rho^{NF} V_{NF} + (1 - \rho^{NF}) U V_b \right) \frac{\sigma_C}{1-h} \left( (1+h)\hat{C}_t - hC_{t-1}^{\hat{I}} - E_t \left[ C_{t+1}^{\hat{I}} \right] \right) + \\ &+ \beta \rho^{NF} V_{NF} E_t \left[ \hat{V}_{NF,t+1} \right] + \beta (1 - \rho^{NF}) U V_b E_t \left[ \hat{V}_{b,t+1} \right] \end{aligned} \quad (1.184)$$

- Beginning of period unemployment value

$$\begin{aligned} U V_b U \hat{V}_{b,t} &= p^F V_F (p_t^{\hat{F}} + \hat{V}_{F,t}) + p^{NF} V_{NF} (p_t^{\hat{N}F} + \hat{V}_{NF,t}) + \\ &+ (1 - p^F - p^{NF}) U V_e U \hat{V}_{e,t} - p^F U V_e p_t^{\hat{F}} - p^{NF} U V_e p_t^{\hat{N}F} \end{aligned} \quad (1.185)$$

- End of period unemployment value

$$UV_e U\hat{V}_{e,t} = \beta UV_b \left( \frac{\sigma_C}{1-h} \left( (1+h)\hat{C}_t - h\hat{C}_{t-1} - E_t \left[ \hat{C}_{t+1} \right] \right) + E_t \left[ UV_{b,t+1} \right] \right) \quad (1.186)$$

- Family bargaining condition

$$-\frac{\tau_{LEE}}{1-\tau_{LEE}}\tau_{L\hat{E}E,t} + F_{F,t}^{\hat{}} = \frac{V_F}{V_F - UV_e}V_{F,t}^{\hat{}} - \frac{UV_e}{V_F - UV_e}U\hat{V}_{e,t} + \frac{\tau_{LER}}{1+\tau_{LER}}\tau_{L\hat{E}R,t} \quad (1.187)$$

- Non-family bargaining condition

$$-\frac{\tau_{LEE}}{1-\tau_{LEE}}\tau_{L\hat{E}E,t} + F_{NF,t}^{\hat{}} = \frac{V_{NF}}{V_{NF} - UV_e}V_{NF,t}^{\hat{}} - \frac{UV_e}{V_{NF} - UV_e}U\hat{V}_{e,t} + \frac{\tau_{LER}}{1+\tau_{LER}}\tau_{L\hat{E}R,t} \quad (1.188)$$

### Final firms:

- Final family firm profit

$$Prof_F^F Prof_{F,t}^{\hat{}} = PFPY_F^F P\hat{F}P_t - MC_F Y_F^F M\hat{C}_{F,t} + (PFP - MC_F)Y_F^F Y_{F,t}^{\hat{}} \quad (1.189)$$

- Final non-family firm profit

$$Prof_{NF}^F Prof_{NF,t}^{\hat{}} = PNFPY_{NF}^F P\hat{N}FP_t - MC_{NF} Y_{NF}^F M\hat{C}_{NF,t} + (PNFP - MC_{NF})Y_{NF}^F Y_{NF,t}^{\hat{}} \quad (1.190)$$

- Family pricing decision

$$\epsilon_F \frac{MC_F}{PFP} \left( M\hat{C}_{F,t} - P\hat{F}P_t \right) + \beta \phi^F E_t [\pi_{F,t+1}^{\hat{}}] = \phi^F \pi_{F,t}^{\hat{}} \quad (1.191)$$

- Non-family pricing decision

$$\epsilon_{NF} \frac{MC_{NF}}{PNFP} \left( M\hat{C}_{NF,t} - P\hat{N}FP_t \right) + \beta \phi^{NF} E_t [\pi_{NF,t+1}^{\hat{}}] = \phi^{NF} \pi_{NF,t}^{\hat{}} \quad (1.192)$$

### Central bank:

- Taylor rule

$$E_t \left[ \hat{i}_t \right] = \gamma_\pi \hat{\pi}_t + \epsilon_{i,t}^{\hat{}} \quad (1.193)$$

### Government:

- Budget constraint

$$\begin{aligned} G\hat{G}_t + WUUU_{t+1}^{\hat{}} &= DEF D\hat{E}F_t + (\tau_{LEE}\tau_{L\hat{E}E,t} + \tau_{LER}\tau_{L\hat{E}R,t})(W_F L_F + W_{NF} L_{NF}) + \\ &+ (\tau_{LEE} + \tau_{LER})W_F L_F (W_{F,t}^{\hat{}} + L_{F,t}^{\hat{}}) + (\tau_{LEE} + \tau_{LER})W_{NF} L_{NF} (W_{NF,t}^{\hat{}} + L_{NF,t}^{\hat{}}) + \\ &+ \tau_C C(\tau_{\hat{C},t} + \hat{C}_t) + T\hat{T}_t \end{aligned} \quad (1.194)$$

- Definition of government debt

$$DEF D\hat{E}F_t + \frac{1+i}{\pi} B \left( \hat{i}_{t-1} + B_{t-1} - \hat{\pi}_t \right) = B\hat{B}_t \quad (1.195)$$

**Market clearing and other equations:**

- Family goods' market clearing

$$Y_F^F \hat{Y}_{F,t}^F = C_F \hat{C}_{F,t} + G_F \hat{G}_{F,t} + I_F \hat{I}_{F,t} + \kappa^F v^F \hat{v}_t^F \quad (1.196)$$

- Non-family goods' market clearing

$$Y_{NF}^F \hat{Y}_{NF,t}^{\hat{F}} = C_{NF} \hat{C}_{NF,t} + G_{NF} \hat{G}_{NF,t} + I_{NF} \hat{I}_{NF,t} + \kappa^{NF} v^{NF} \hat{v}_t^{\hat{N}F} \quad (1.197)$$

- Family final and intermediate output

$$Y_F^F \hat{Y}_{F,t}^F = Y_F^I \hat{Y}_{F,t}^I - I_{OF} \hat{I}_{OF,t} \quad (1.198)$$

- Non-family final and intermediate output

$$Y_{NF,t}^{\hat{F}} = Y_{NF,t}^{\hat{I}} \quad (1.199)$$

- GDP equation

$$Y \hat{Y}_t = P F P Y_F^F (P \hat{F} P_t + \hat{Y}_{F,t}^F) + P N F P Y_{NF}^F (P \hat{N} F P_t + \hat{Y}_{NF,t}^{\hat{F}}) \quad (1.200)$$

- Definition of family inflation

$$\pi_t^F = P \hat{F} P_t + \hat{\pi}_t - P F \hat{P}_{t-1} \quad (1.201)$$

- Definition of non-family inflation

$$\pi_t^{\hat{N}F} = P \hat{N} F P_t + \hat{\pi}_t - P N \hat{F} P_{t-1} \quad (1.202)$$

**Shocks:**

- Technology shock in the family sector

$$\hat{A}_{F,t} = \rho_{AF} \hat{A}_{F,t-1} + \epsilon_{AF,t} \quad (1.203)$$

- Technology shock in the non-family sector

$$\hat{A}_{NF,t} = \rho_{ANF} \hat{A}_{NF,t-1} + \epsilon_{ANF,t} \quad (1.204)$$

- Value-added tax shock

$$\hat{\tau}_{C,t} = \rho_{\tau C} \hat{\tau}_{C,t-1} + \epsilon_{\tau C,t} \quad (1.205)$$



- Employees' labor income tax shock

$$\tau_{LEE,t}^{\hat{}} = \rho_{\tau LEE} \tau_{LEE,t-1}^{\hat{}} + \epsilon_{\tau LEE,t} \quad (1.206)$$

- Employers' labor income tax shock

$$\tau_{LER,t}^{\hat{}} = \rho_{\tau LER} \tau_{LER,t-1}^{\hat{}} + \epsilon_{\tau LER,t} \quad (1.207)$$

- Government consumption expenditure shock

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \epsilon_{G,t} \quad (1.208)$$

- Lump-sum tax rule

$$\hat{T}_t = \rho_T \hat{T}_{t-1} + (1 - \rho_T) \xi_B (\hat{B}_{t-1} - \hat{Y}_t) + \epsilon_{T,t} \quad (1.209)$$

#### Notation:

- Relative price of family firm produced goods

$$PFP_t = \frac{P_{F,t}}{P_t} \quad (1.210)$$

- Relative price of non-family firm produced goods

$$PNFP_t = \frac{P_{NF,t}}{P_t} \quad (1.211)$$

## C Robustness of multipliers

Tables 1.A1 and 1.A2 show the robustness of unemployment fiscal multipliers with respect to i) the unemployment benefit replacement rate, ii) the bargaining power of workers in the non-family sector, iii) the dismissal rate of workers in the family sector, iv) the elasticity of substitution between family and non-family goods, v) the effectiveness of family organisational investment and vi) the level of price rigidity in the family sector.

I can conclude that, in general, the signs of the multipliers are always robust, and the magnitudes of the multipliers are also highly robust.

Except those of price rigidity, which significantly affects the magnitude of the government consumption multiplier, and also that of the value-added tax. There is no evidence in the literature as to whether family or non-family firms set prices more often, so in the baseline calibration sectoral price rigidities are equal. At the same time, Goldberg-Hellerstein (2011) show that small firms set prices more often than large firms, so in an alternative scenario a larger price rigidity is considered in the family sector. When 70 percent of family firms do

not set prices compared to the baseline 66 percent, the government consumption multiplier, at peak, goes up to 0.67 pp instead of the baseline 0.52 (other multipliers do not significantly change). Concerning cumulative multipliers, both the government consumption multiplier and the consumption tax multiplier considerably increase. Still, the main policy conclusions remain the same. The importance of the degree of price rigidity for unemployment fiscal multipliers was also highlighted by Bruckner-Pappa (2012), stressing the role of the demand effect.<sup>42</sup>

Sometimes, multipliers are sensitive to the unemployment benefit replacement rate and the dismissal rate. Regarding the unemployment benefit replacement rate, there is a wide range of values in the literature (see Section 1.3 of the paper). As already pointed out by Gertler et al. (2008), this ratio is crucial for the impulse responses of a model with search and matching frictions. When decreasing it, at peak, unemployment increases less after a cut in government consumption. This might be because a lower replacement rate means a lower outside option, so being unemployed becomes relatively less attractive. Cumulatively, not only the government consumption multiplier, but also the employees' labor income tax multipliers are affected, albeit to a lesser extent. Similar findings hold for the dismissal rate.

## D Impulse response functions

Figures 1.A1-1.A4 show baseline stochastic impulse response functions of the following fiscal consolidation policies (all 1 percent of GDP in size): i) increase in value added taxes, ii) cut in government consumption, iii) increase in the employees' labor income tax (personal labor income tax and social security contribution) and iv) increase in the employer social security contribution rate.

Then, Figures 1.A5-1.A6 present stochastic impulse response functions of tax shifts from consumption to labor income taxation (1 percent of GDP size), while keeping the government deficit at its steadystate level.

Lastly, Figure 1.A7 shows the effects of sectoral heterogeneity on stochastic impulse response functions. The solid line is the baseline model. The dotted line is a one-sector model assuming that all firms are family firms, while the line with round markers is another one-sector model where all firms are non-family firms. The line with squared markers is a two-sector model without organisational capital in the family sector.<sup>43</sup>

---

<sup>42</sup>Generally, Woodford (2011) points out that sticky prices imply a larger output government expenditure multiplier.

<sup>43</sup>Impulse responses of other variables are available upon request.

|                                      | Peak     |                     |                                 |                   |
|--------------------------------------|----------|---------------------|---------------------------------|-------------------|
|                                      | BASELINE | b=0.25              | $\lambda_{NF}=\sigma_{NF}=0.75$ | $1-\rho_F=0.0576$ |
| Government consumption cut           | 0.52     | 0.46                | 0.48                            | 0.47              |
| Value added tax increase             | 0.02     | 0.02                | 0.02                            | 0.02              |
| Labor income tax increase (employee) | 0.12     | 0.10                | 0.10                            | 0.10              |
| Labor income tax increase (employer) | 0.07     | 0.06                | 0.06                            | 0.06              |
|                                      |          | $\eta=1$            | $\theta=0.975$                  | $\phi_F=75\%$     |
| Government consumption cut           |          | 0.51                | 0.53                            | 0.67              |
| Value added tax increase             |          | 0.02                | 0.02                            | 0.03              |
| Labor income tax increase (employee) |          | 0.11                | 0.12                            | 0.12              |
| Labor income tax increase (employer) |          | 0.06                | 0.07                            | 0.07              |
|                                      |          | Cumulative (1 year) |                                 |                   |
|                                      | BASELINE | b=0.25              | $\lambda_{NF}=\sigma_{NF}=0.75$ | $1-\rho_F=0.0576$ |
| Government consumption cut           | 0.71     | 0.63                | 0.65                            | 0.64              |
| Value added tax increase             | 0.04     | 0.04                | 0.05                            | 0.05              |
| Labor income tax increase (employee) | 0.34     | 0.27                | 0.29                            | 0.27              |
| Labor income tax increase (employer) | 0.19     | 0.16                | 0.17                            | 0.16              |
|                                      |          | $\eta=1$            | $\theta=0.975$                  | $\phi_F=70\%$     |
| Government consumption cut           |          | 0.68                | 0.73                            | 0.88              |
| Value added tax increase             |          | 0.05                | 0.04                            | 0.07              |
| Labor income tax increase (employee) |          | 0.32                | 0.34                            | 0.33              |
| Labor income tax increase (employer) |          | 0.18                | 0.20                            | 0.19              |

Table 1.A1: Unemployment fiscal multipliers - robustness with respect to parameter values (in percentage points)

|                                      | Cumulative (2 years) |          |                                 |                   |
|--------------------------------------|----------------------|----------|---------------------------------|-------------------|
|                                      | BASELINE             | b=0.25   | $\lambda_{NF}=\sigma_{NF}=0.75$ | $1-\rho_F=0.0576$ |
| Government consumption cut           | 0.67                 | 0.61     | 0.62                            | 0.62              |
| Value added tax increase             | 0.06                 | 0.06     | 0.07                            | 0.07              |
| Labor income tax increase (employee) | 0.55                 | 0.45     | 0.48                            | 0.45              |
| Labor income tax increase (employer) | 0.31                 | 0.26     | 0.28                            | 0.26              |
|                                      |                      | $\eta=1$ | $\theta=0.975$                  | $\phi_F=75\%$     |
| Government consumption cut           |                      | 0.63     | 0.68                            | 0.83              |
| Value added tax increase             |                      | 0.06     | 0.06                            | 0.09              |
| Labor income tax increase (employee) |                      | 0.51     | 0.56                            | 0.53              |
| Labor income tax increase (employer) |                      | 0.29     | 0.32                            | 0.31              |
|                                      | Cumulative (4 years) |          |                                 |                   |
|                                      | BASELINE             | b=0.25   | $\lambda_{NF}=\sigma_{NF}=0.75$ | $1-\rho_F=0.0576$ |
| Government consumption cut           | 0.48                 | 0.45     | 0.46                            | 0.46              |
| Value added tax increase             | 0.03                 | 0.03     | 0.05                            | 0.04              |
| Labor income tax increase (employee) | 0.51                 | 0.43     | 0.47                            | 0.44              |
| Labor income tax increase (employer) | 0.29                 | 0.25     | 0.27                            | 0.25              |
|                                      |                      | $\eta=1$ | $\theta=0.975$                  | $\phi_F=70\%$     |
| Government consumption cut           |                      | 0.43     | 0.48                            | 0.61              |
| Value added tax increase             |                      | 0.03     | 0.03                            | 0.06              |
| Labor income tax increase (employee) |                      | 0.45     | 0.53                            | 0.50              |
| Labor income tax increase (employer) |                      | 0.26     | 0.30                            | 0.29              |

Table 1.A2: Unemployment fiscal multipliers - robustness with respect to parameter values (in percentage points) cont.

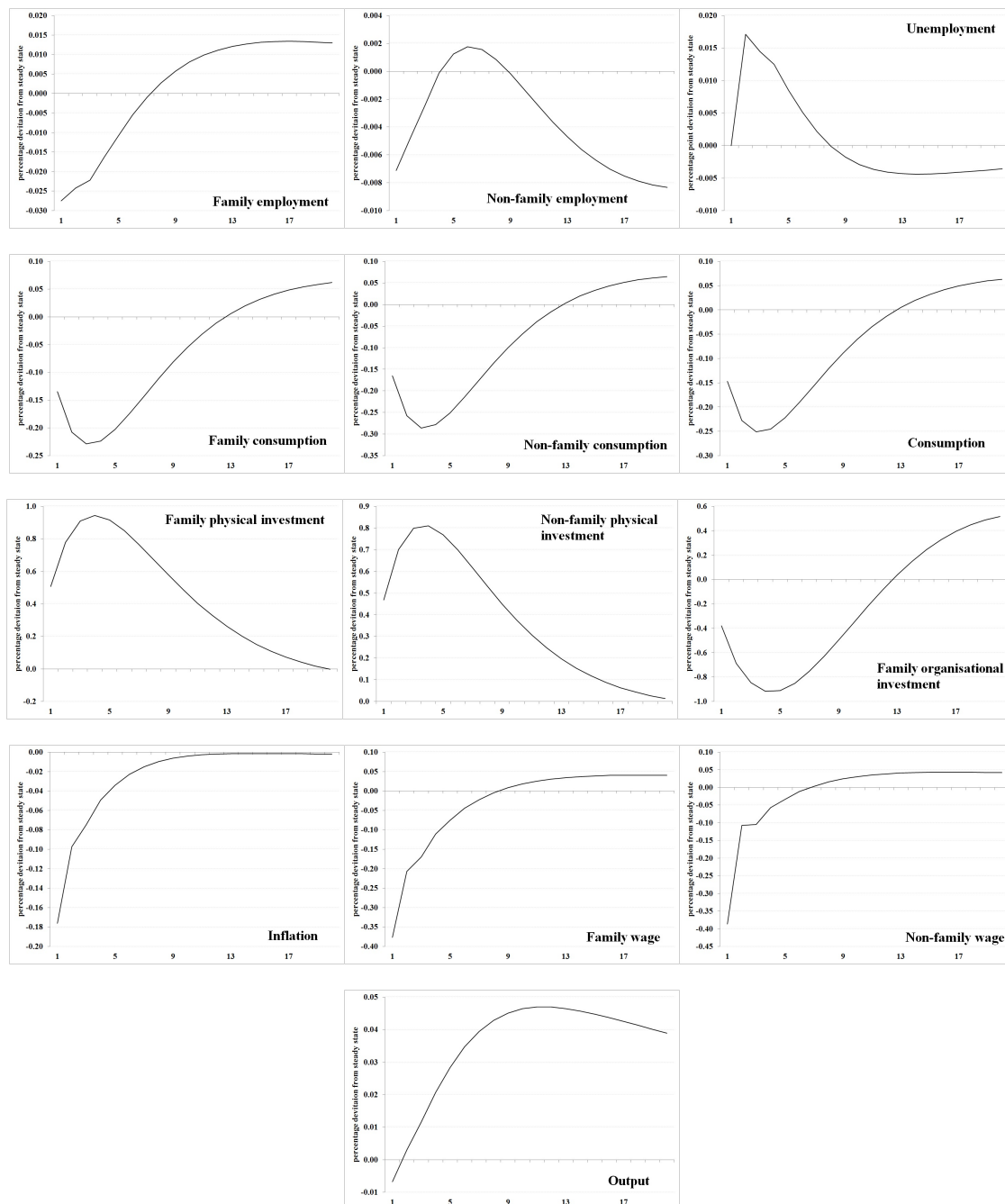


Figure 1.A1: An increase in value added tax revenue equal to 1 percent of GDP

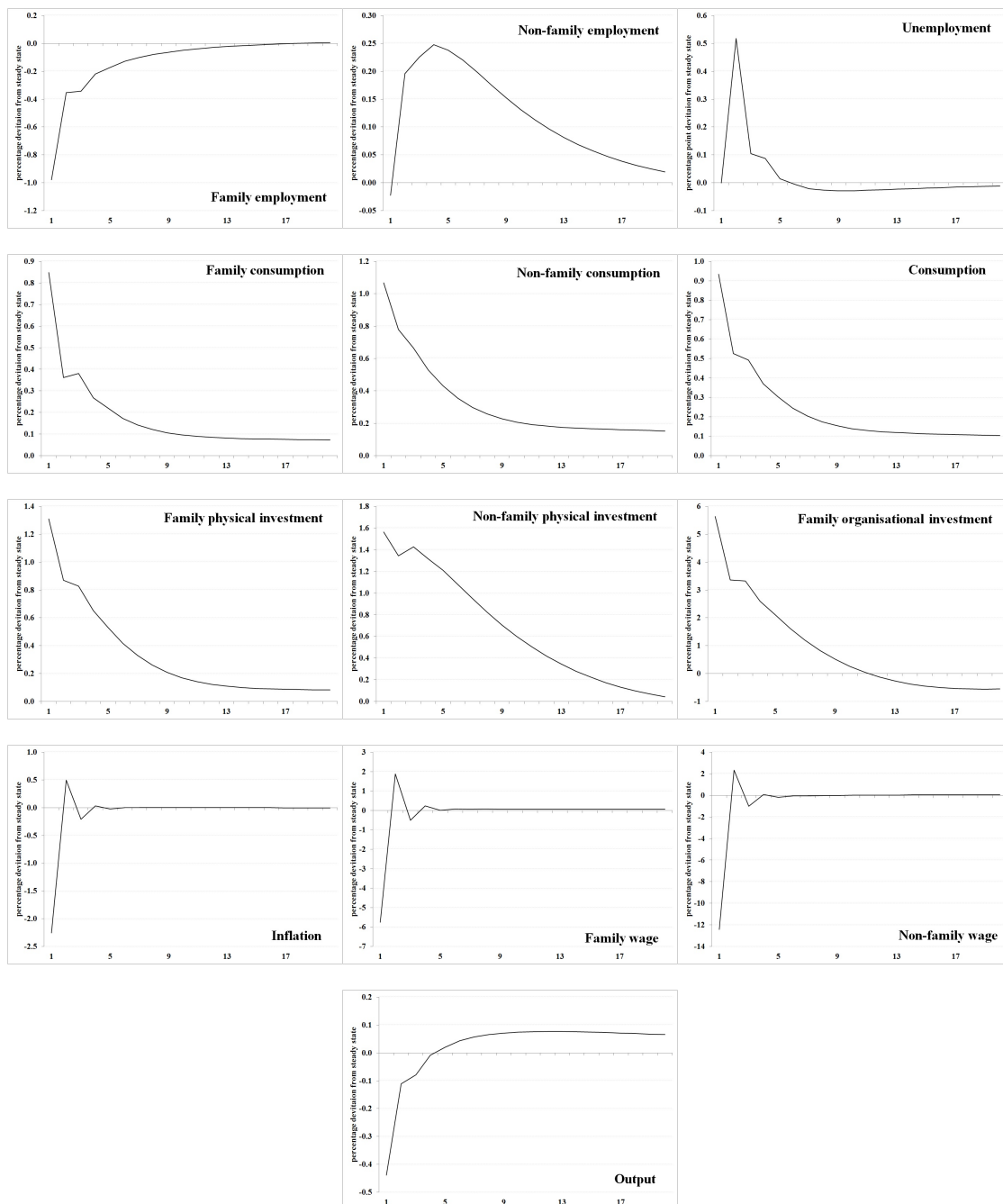


Figure 1.A2: A cut in government consumption equal to 1 percent of GDP

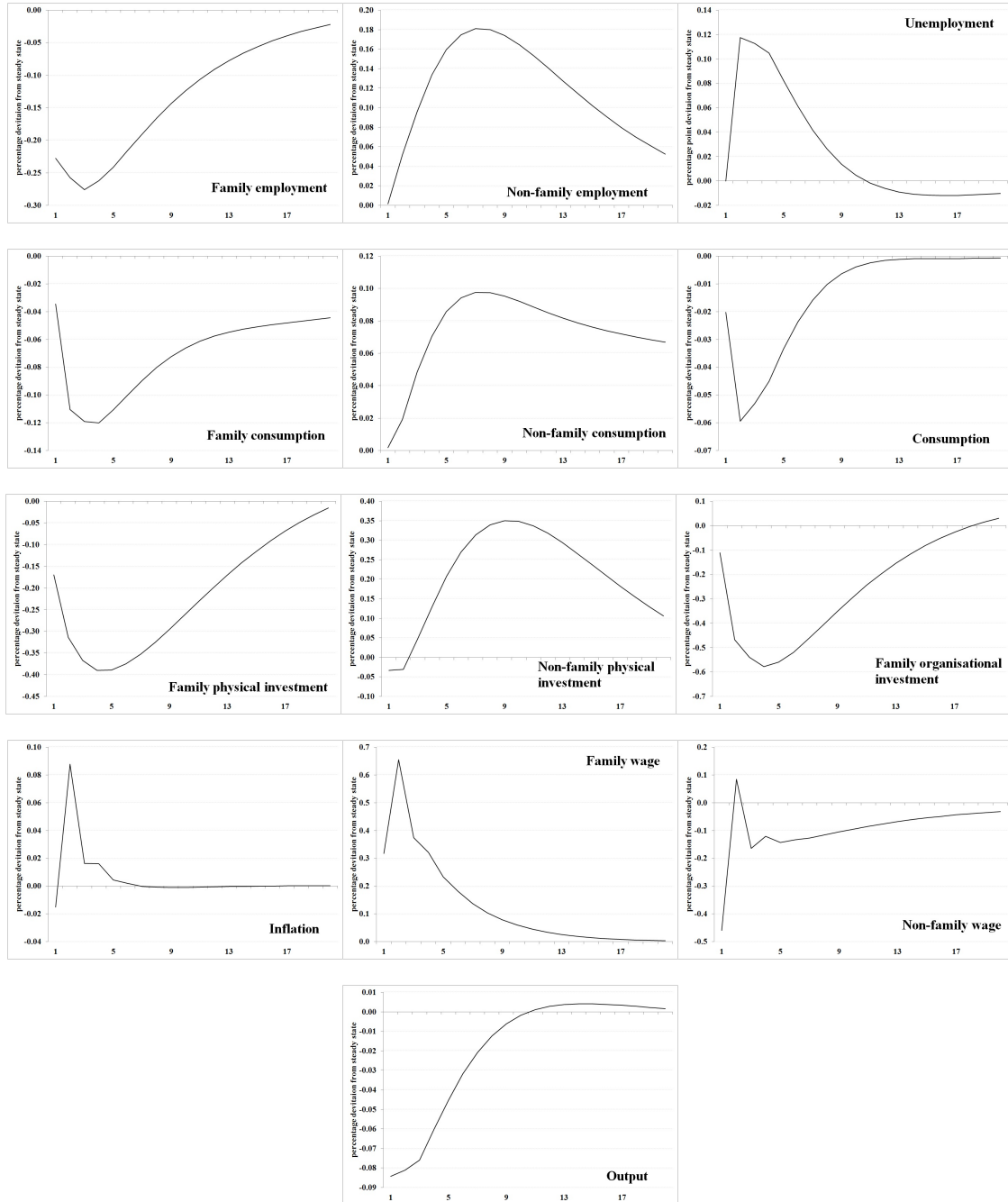


Figure 1.A3: An increase in employees' labor income tax (personal labor income tax and social security contribution) revenue equal to 1 percent of GDP

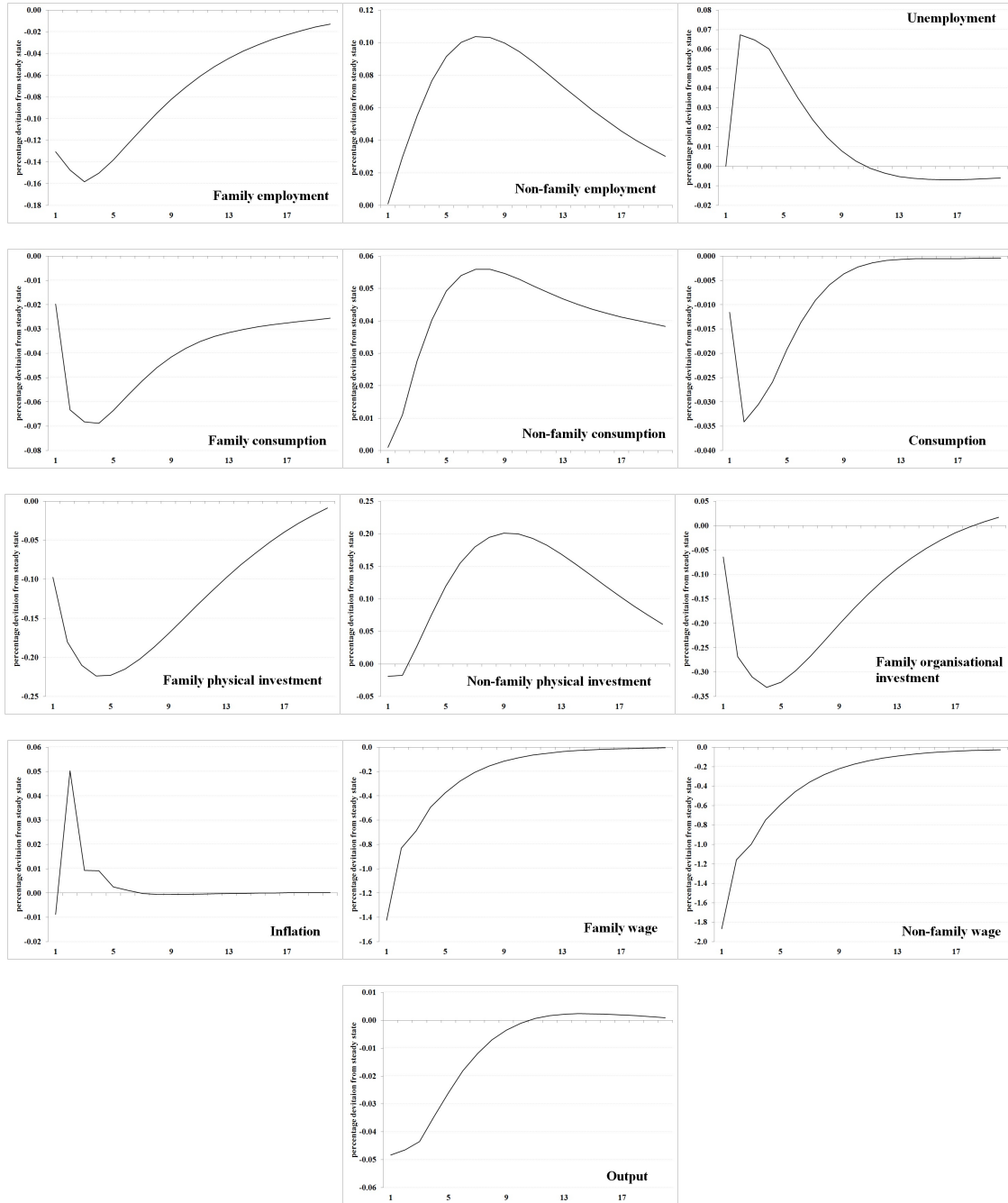


Figure 1.A4: An increase in employer social security contribution revenue equal to 1 percent of GDP



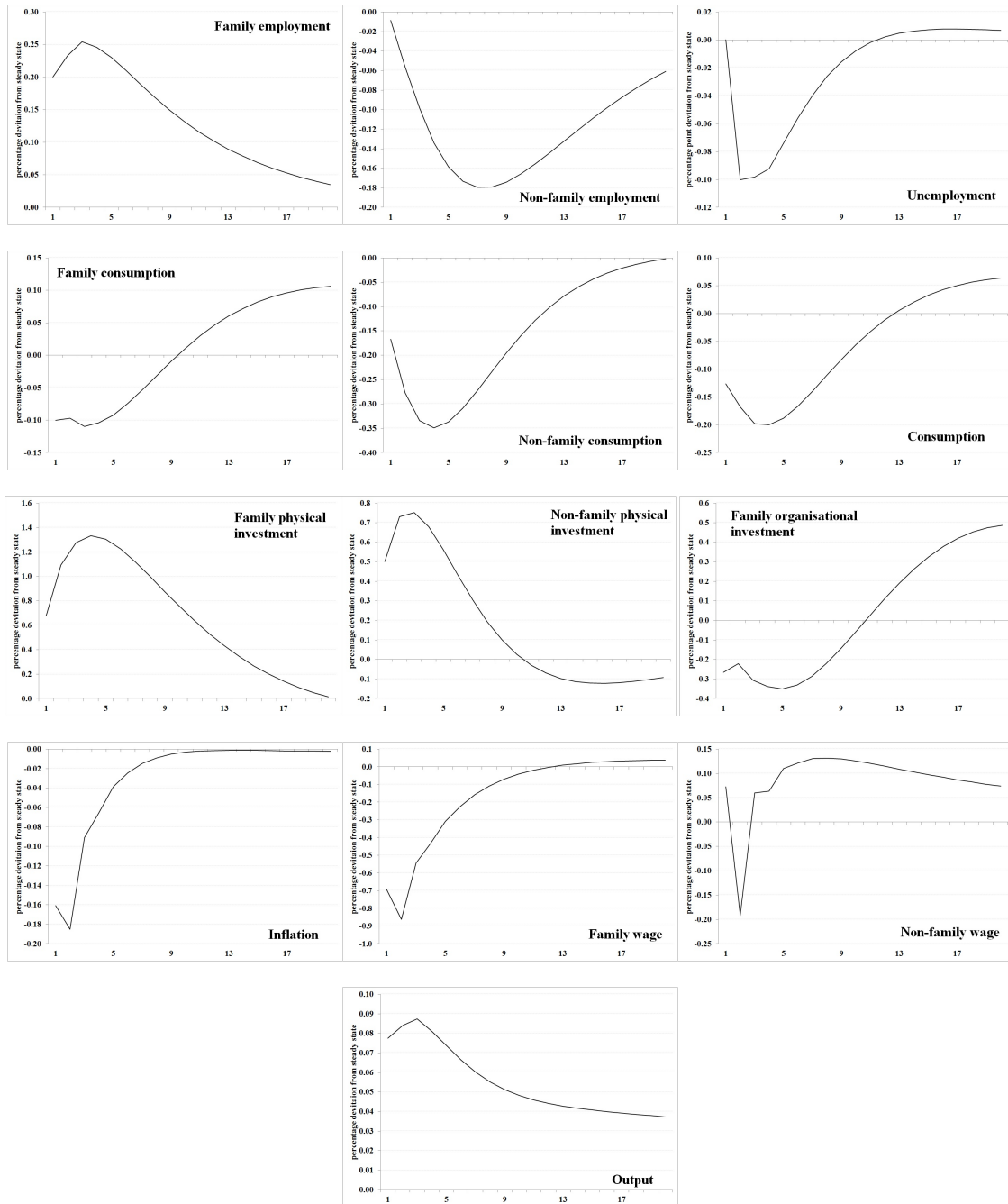


Figure 1.A5: Tax shift (a 1-1 percent of GDP decrease in employees' labor income tax revenue and increase in value added tax revenue)

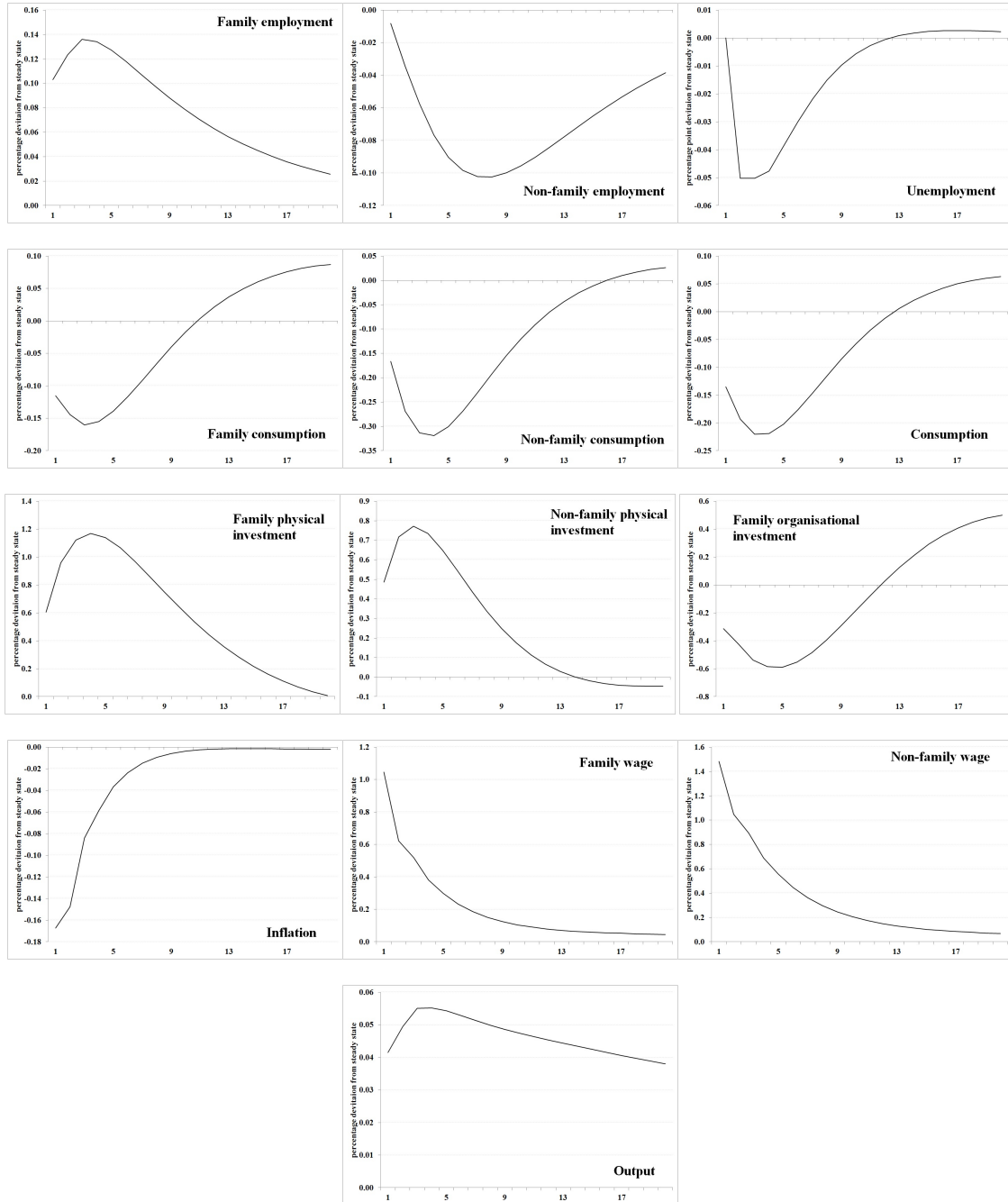


Figure 1.A6: Tax shift (a 1-1 percent of GDP decrease in employer social security contribution revenue and increase in value added tax revenue)

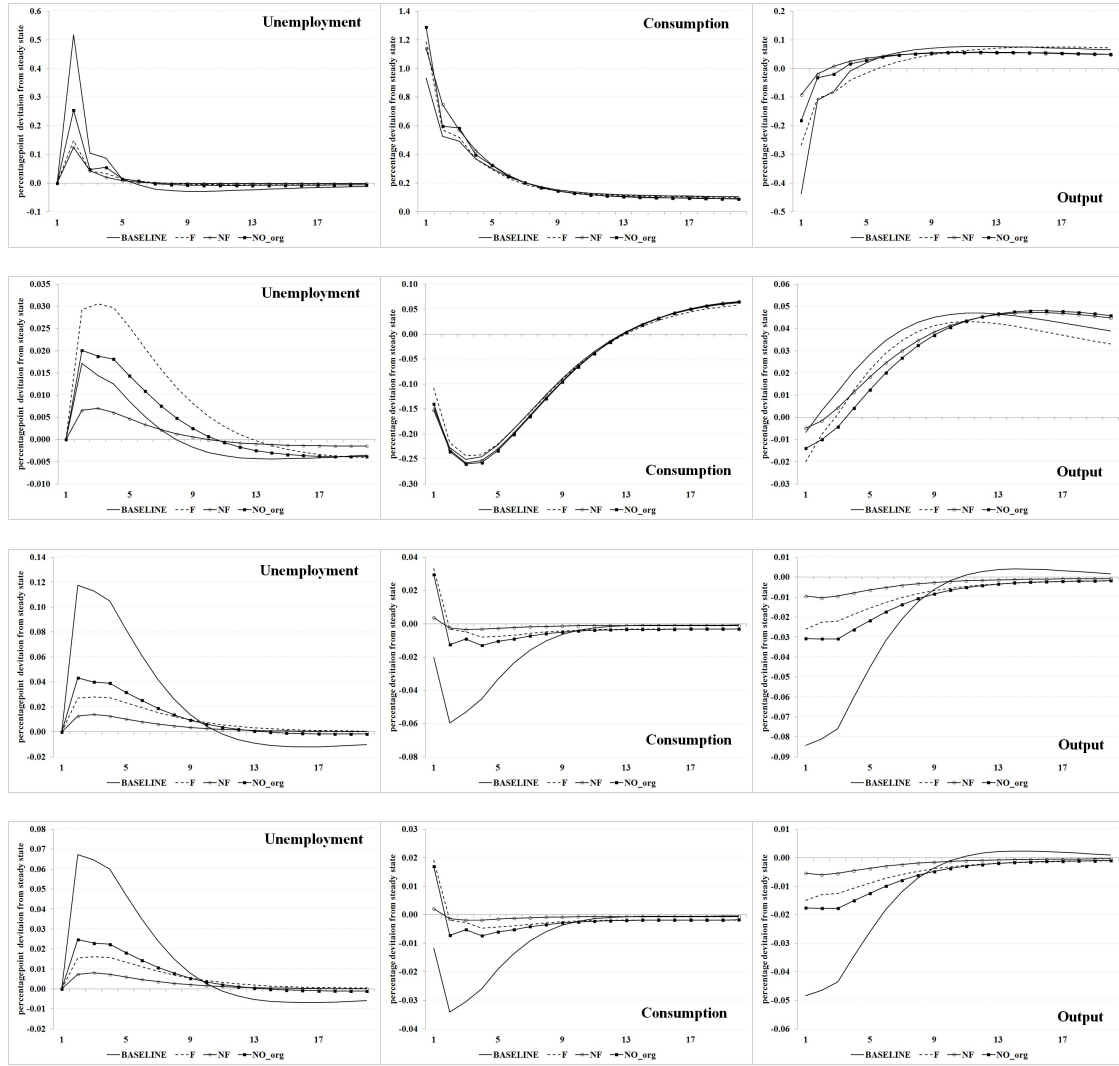


Figure 1.A7: First column: a cut in government consumption. Second column: an increase in value added tax revenue. Third column: an increase in employees' labor income tax. Fourth column: an increase in the employer social security contribution. All shocks correspond to 1 percent of GDP.

# Bibliography

- [Ai et al. (2013)] Ai, H. - Croce, M. M. - Li, K. (2013): Toward a Quantitative General Equilibrium Asset Pricing Model with Intangible Capital. *The Review of Financial Studies*, Vol. 26, No. 2, 2013.
- [Alesina-Perotti (1997)] Alesina, A. - Perotti, R. (1997): The Welfare State and Competitiveness. *The American Economic Review*, Vol. 87, No. 5 (Dec., 1997), pp. 921-939.
- [Anderson-Reeb (2003)] Anderson, R. C. - Reeb, D. M. (2003): Founding-Family Ownership and Firm Performance: Evidence from the SP 500. *The Journal of Finance*, Vol. 58, Issue 3, pp. 1301-1327, June 2003.
- [Auerbach (2006)] Auerbach, A. J. (2006): The choice between income and consumption taxes: A primer. NBER Working Paper 12307, June 2006. <http://www.nber.org/papers/w12307.pdf>
- [Ball et al. (2013)] Ball, L. - Furceri, D. - Leigh, D. - Loungani, P. (2013): The Distributional Effects of Fiscal Consolidation. IMF Working Paper WP/13/151, June 2013. <https://www.imf.org/external/pubs/ft/wp/2013/wp13151.pdf>
- [Bassanini et al. (2011)] Bassanini, A. - Caroli, E. - Reberlioux, A. - Breda, T. (2011): Working in Family Firms Less Paid but More Secure? Evidence from French Matched Employer-Employee Data. IZA Discussion Paper No. 5842, July 2011. <http://ftp.iza.org/dp5842.pdf>
- [Bean (1994)] Bean, C. R. (1994): European Unemployment: A Survey. *Journal of Economic Literature*, Vol. 32, No. 2 (Jun., 1994), pp. 573-619.
- [Ben Zeev-Pappa (2015)] Ben Zeev, N. - Pappa, E. (2015): Chronicle of a War Foretold: The Macroeconomic Effects of Anticipated Defense Spending Shocks. *The Economic Journal*, forthcoming.
- [Bennedsen et al. (2007)] Bennedsen, M. - Nielsen K. M. - Perez-Gonzalez, F. - Wolfenzon, D. (2007): Inside the Family Firm: The Role of Families in Succession Decisions and Performance. *The Quarterly Journal of Economics*, Vol. 122, No. 2 (May, 2007), pp. 647-691.

- [Bermperoglou et al. (2013)] Bermperoglou, D. - Pappa, E. - Vella, E. (2013): Spending cuts and their effects on output, unemployment and the deficit. Manuscript, December 2013. [http://apps.eui.eu/Personal/Pappa/Papers/DimEugEvi\\_17DEC13.pdf](http://apps.eui.eu/Personal/Pappa/Papers/DimEugEvi_17DEC13.pdf)
- [Bjuggren et al. (2011)] Bjuggren, C. M. - Johansson, D. - Sjogren, H. (2011): A Note on Employment and Gross Domestic Product in Swedish Family-Owned Businesses: A Descriptive Analysis. *Family Business Review*, 24(4) 362-371.
- [Blanchard-Wolfers (2000)] Blanchard, O. - Wolfers, J. (2000): The role of shocks and institutions in the rise of European unemployment: the aggregate evidence. *The Economic Journal*, 110 (March), C1-C33.
- [Blumkin et al. (2012)] Blumkin, T. - Ruffle, B. J. - Ganun, Y. (2012): Are income and consumption taxes ever really equivalent? Evidence from a real-effort experiment with real goods. *European Economic Review* 56 (2012) 1200-1219.
- [Bruckner-Pappa (2012)] Bruckner, M. - Pappa, E. (2012): Fiscal Expansions, Unemployment, and Labor Force Participation: Theory and Evidence. *International Economic Review*, Vol. 53, No. 4, November 2012.
- [Burnside et al. (2004)] Burnside, C. - Eichenbaum, M. - Fisher, J. D. M. (2004): Fiscal shocks and their consequences. *Journal of Economic Theory* 115 (2004) 89-117.
- [Caldara-Kampas (2008)] Caldara, D. - Kamps, C. (2008): What are the effects of fiscal policy shocks? A VAR-based comparative analysis. ECB Working Paper Series NO 877, March 2008. <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp877.pdf?b671fdff757ed5d1da6cd7104d694ea6>
- [Campolmi et al. (2011)] Campolmi, A. - Faia, E. - Winkler, R. (2011): Fiscal Calculus and the Labor Market. *The B.E. Journal of Macroeconomics*: Vol. 11: Iss. 1 (Contributions), Article 38.
- [Canova-Pappa (2007)] Canova, F. - Pappa, E. (2007): Price Differentials in Monetary Unions: The Role of Fiscal Shocks. *The Economic Journal*, 117 (March), 713-737.
- [Caselli-Gennaioli (2013)] Caselli, F. - Gennaioli, N. (2013): Dynastic Management. *Economic Inquiry*, Vol. 51, No. 1, Jan. 2013, pp. 971-996.
- [Christiano et al. (2005)] Christiano, L. J. - Eichenbaum, M. - Evans, C. L. (2005): Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 2005, Vol. 113, No. 1.
- [Christoffel et al. (2009)] Christoffel, K. - Kuester, K. - Linzert, T. (2009): The Role of Labor Markets for Euro Area Monetary Policy. *European Economic Review*, Vol. 53, Issue 8, pp. 908-936, Nov. 2009.

- [Cogan et al. (2013)] Cogan, J. F. - Taylor, J. B. - Wieland, V. - Wolters, M. H. (2013): Fiscal consolidation strategy. *Journal of Economic Dynamics and Control* 37 (2013) 404-421.
- [Danthine-Jin (2007)] Danthine, J. P. – Jin, X. (2007): Intangible capital, corporate valuation and asset pricing. *Economic Theory*, Vol. 32, No. 1., Symposium in Honor of Edward C. Prescott (July, 2007), pp. 157-177.
- [Dallari (2014)] Dallari, P. (2014): The labor market outcomes of austerity. Evidence for Europe. Manuscript, April 2014. <http://emmpa2014.rimir.ro/attachments/article/20/dallari-pietro-paper2.pdf>
- [Daveri et al. (2000)] Daveri, F. - Tabellini, G. - Bentolila, S. - Huizinga, H. (2000): Unemployment, growth and taxation in Industrial countries. *Economic Policy*, Vol. 15, No. 30 (Apr., 2000), pp. 47-104.
- [Dixit-Stiglitz (1977)] Dixit, A. K. – Stiglitz, J. E. (1977): Monopolistic Competition and Optimum Product Diversity. *The American Economic Review*, Vol. 67, NO. 3. (Jun., 1977), pp. 297-308.
- [Edelberg et al. (1999)] Edelberg, W. - Eichenbaum, M. - Fisher, J. D. M. (1999): Understanding the Effects of a Shock to Government Purchases. *Review of Economic Dynamics* 2, 166-206 (1999).
- [Epstein-Shapiro (2014)] Epstein, B. - Shapiro, A. F. (2014): Employment and Firm Heterogeneity, Capital Allocation and Countercyclical Labor Market Policies. Board of Governors of the FED, International Finance Discussion Paper 1115, Aug. 2014. <https://www.federalreserve.gov/pubs/ifdp/2014/1115/ifdp1115.pdf>
- [Esser et al. (2013)] Esser, I. - Ferrarini, T. - Nelson, K. - Palme, J. - Sjoberg, O. (2013): Unemployment Benefits in EU Member States. European Commission, July 2013. [https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0ahUKEwi3k0Srw\\_7LAhWC6CwKHSxdC44QFggcMAA&url=http%3A%2F%2Fec.europa.eu%2Fsocial%2FblobServlet%3FdocId%3D10852%26langId%3Den&usq=AFQjCNEJV0gQnDkVbqLXsNz3iavQ\\_jL1tA&sig2=NswUEnz9Dj0V2QB5e-M3VA&bvm=bv.118817766,d.bGg](https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0ahUKEwi3k0Srw_7LAhWC6CwKHSxdC44QFggcMAA&url=http%3A%2F%2Fec.europa.eu%2Fsocial%2FblobServlet%3FdocId%3D10852%26langId%3Den&usq=AFQjCNEJV0gQnDkVbqLXsNz3iavQ_jL1tA&sig2=NswUEnz9Dj0V2QB5e-M3VA&bvm=bv.118817766,d.bGg)
- [Fatas-Mihov (2001)] Fatas, A. - Mihov, I. (2001): The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence. CEPR Discussion Papers No 2760. [http://cepr.org/active/publications/discussion\\_papers/dp.php?dpno=2760](http://cepr.org/active/publications/discussion_papers/dp.php?dpno=2760)
- [Forni-Gambetti (2010)] Forni, M. - Gambetti, L. (2010): Fiscal Foresight and the Effects of Government Spending. CEPR Discussion Papers 7840. [http://cepr.org/active/publications/discussion\\_papers/dp.php?dpno=7840](http://cepr.org/active/publications/discussion_papers/dp.php?dpno=7840)

- [Gali et al. (2007)] Gali, J. - Lopez-Salido, J. D. - Valles, J. (2007): Understanding the Effects of Government Spending on Consumption. *Journal of the European Economic Association*, MIT Press, vol. 5(1), pages 227-270, 03.
- [Gertler et al. (2008)] Gertler, M. - Sala, L. - Trigari, A. (2008): An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining. *Journal of Money, Credit and Banking*, Vol. 40, No. 8 (Dec., 2008), pp. 1713-1764.
- [Goldberg-Hellerstein (2011)] Goldberg, P. - Hellerstein, R. (2011): How Rigid are Producer Prices? FRB of New York Staff Report No. 407, October 2011. [https://www.newyorkfed.org/medialibrary/media/research/staff\\_reports/sr407.pdf](https://www.newyorkfed.org/medialibrary/media/research/staff_reports/sr407.pdf)
- [Gomes (2011)] Gomes, P. (2011): Fiscal policy and the labour market: the effects of public sector employment and wages. EC Economic Papers 439, February 2011. [http://ec.europa.eu/economy\\_finance/publications/economic\\_paper/2011/pdf/ecp439\\_en.pdf](http://ec.europa.eu/economy_finance/publications/economic_paper/2011/pdf/ecp439_en.pdf)
- [Gourio-Rudanko (2014)] Gourio, F. - Rudanko, L. (2014): Can Intangible Capital Explain Cyclical Movements in the Labor Wedge? *American Economic Review*, 104(5): 183-88.
- [Guajardo et al. (2011)] Guajardo, J. - Leigh, D. - Pescatori, A. (2011): Expansionary Austerity: New International Evidence. IMF Working Paper WP/11/158, July 2011. <https://www.imf.org/external/pubs/ft/wp/2011/wp11158.pdf>
- [Hagedorn-Manovski (2008)] Hagedorn, M. - Manovski, I. (2008): The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited. *The American Economic Review*, Vol. 98, No. 4 (Sep., 2008), pp. 1692-1706.
- [Hall (1995)] Hall, R. (1995): Lost Jobs. *Brookings Papers on Economic Activity*, 1:1995.
- [Hernandez de Cos-Moral-Benito (2011)] Hernandez de Cos, P. - Moral-Benito, E. (2011): Endogenous fiscal consolidations. Documentos de Trabajo No 1102, Banco de Espana. <http://www.bde.es/f/webbde/SES/Secciones/Publicaciones/PublicacionesSeriadas/DocumentosTrabajo/11/Fich/dt1102e.pdf>
- [Hobijn-Sahin (2007)] Hobijn, B. - Sahin, A. (2007): Job-Finding and Separation Rates in the OECD. FED of New York Staff Report no. 298, August 2007. [https://www.newyorkfed.org/medialibrary/media/research/staff\\_reports/sr298.pdf](https://www.newyorkfed.org/medialibrary/media/research/staff_reports/sr298.pdf)
- [Hosios (1990)] Hosios, A. J. (1990): On the Efficiency of Matching and Related Models of Search and Unemployment. *Review of Economic Studies*, 59, pp. 309-327.
- [IEF (2009)] IEF (2009): The Family Business. Instituto de la Empresa Familiar, 2009.

- [IFB (2011)] IFB (2011): The UK Family Business Sector. Working to grow the UK economy. *Institute for Family Business, Oxford Economics*, November 2011.
- [IFERA (2003)] International Family Enterprise Research Academy (IFERA) (2003): Family Businesses Dominate. *Family Business Review*, vol. 16, no. 4., December 2003, pp. 235 - 239.
- [King-Santor (2007)] King, M. R. - Santor, E. (2007): Family Values: Ownership Structure, Performance and Capital Structure of Canadian Firms. Bank of Canada WP 2007-40, July 2007. <http://www.bankofcanada.ca/wp-content/uploads/2010/02/wp07-40.pdf>
- [La Porta et al. (1999)] La Porta, R. - Lopez-de-Silanes, F. - Shleifer, A. (1999): Corporate Ownership around the World. *The Journal of Finance*, Vol. 54, No. 2 (Apr., 1999), pp. 471-517.
- [Lindow (2013)] Lindow, C. M. (2013): A Strategic Fit Perspective on Family Firm Performance. *Springer Fachmedien Wiesbaden*, 2013.
- [Lombardo-Vestin (2007)] Lombardo, G. - Vestin, D. (2007): Welfare Implications of Calvo vs. Rotemberg Pricing Assumptions. ECB Working Paper Series, NO 770, June 2007. <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp770.pdf?78b9fd3e2ad1020b18eb3f4f43882a52>
- [Mandl (2008)] Mandl, I. (2008): Overview of Family Business Relevant Issues. European Commission, KMU Forschung Austria, 2008. [https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&ved=0ahUKEwiFpdS8xf7LAhXJBiwKHds3De8QFggiMAA&url=http%3A%2F%2Fec.europa.eu%2FDocsRoom%2Fdocuments%2F10389%2Fattachments%2F1%2Ftranslations%2Fen%2Frenditions%2Fnative&usg=AFQjCNGsrc7Rmi9QKlwRDKlVIOL\\_b0KEkw&sig2=P017LRfawID2jrEYQ3WsjA](https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&ved=0ahUKEwiFpdS8xf7LAhXJBiwKHds3De8QFggiMAA&url=http%3A%2F%2Fec.europa.eu%2FDocsRoom%2Fdocuments%2F10389%2Fattachments%2F1%2Ftranslations%2Fen%2Frenditions%2Fnative&usg=AFQjCNGsrc7Rmi9QKlwRDKlVIOL_b0KEkw&sig2=P017LRfawID2jrEYQ3WsjA)
- [Mayer et al. (2010)] Mayer, E. - Moyen, S. - Staehler, N. (2010): Government expenditures and unemployment: a DSGE perspective. Deutsche Bundesbank Discussion Paper Series 1: Economic Studies No 18/2010. [http://www.wiwiiss.fu-berlin.de/forschung/veranstaltungen/rse/Past\\_Programs/Winter\\_2010\\_11/RSE-Mayer.pdf](http://www.wiwiiss.fu-berlin.de/forschung/veranstaltungen/rse/Past_Programs/Winter_2010_11/RSE-Mayer.pdf)
- [Maury (2006)] Maury, B. (2006): Family ownership and firm performance: Empirical evidence from Western European corporations. *Journal of Corporate Finance*, 12 (2006) 321-341.
- [McGrattan-Prescott (2010)] McGrattan, E. R. - Prescott, E. C. (2010): Unmeasured Investment and the Puzzling US Boom in the 1990s. *American Economic Journal: Macroeconomics*, American Economic Association, vol. 2(4), pp. 88-123, October 2010.



- [McGrattan et al. (2014)] McGrattan, E. R. - Prescott, E. C. (2014): A Reassessment of Real Business Cycle Theory. *American Economic Review*, 104(5): 177-82.
- [Merino et al. (2012)] Merino, F. - Monreal-Perez, J. - Sanchez-Marin, G. (2012): Family Firm Internationalization: Influence of Familiness on the Spanish Firm Export Activity. Kiel Institute for the World Economy, Kiel Working Papers No. 1770, April 2012. [https://www.ifw-members.ifw-kiel.de/publications/family-firm-internationalization-influence-of-familiness-on-the-spanish-firm-export-activity/kwp\\_1770.pdf](https://www.ifw-members.ifw-kiel.de/publications/family-firm-internationalization-influence-of-familiness-on-the-spanish-firm-export-activity/kwp_1770.pdf)
- [Minetti et al. (2013)] Minetti, R. - Murro, P. - Zhu, S. C. (2013): Family Firms, Corporate Governance, and Export. CASMEF Working Paper Series No. 2, February 2013. <http://static.luiss.it/RePEc/pdf/casmef/1302.pdf>
- [Monacelli et al. (2010)] Monacelli, T. - Perotti, R. - Trigari, A. (2010): Unemployment Fiscal Multipliers. *Journal of Monetary Economics* 57 (2010) pp. 531-553.
- [Mortensen-Nagypal (2007)] Mortensen, D. T. - Nagypal, E. (2007): More on unemployment and vacancy fluctuations. *Review of Economic Dynamics* 10 (2007) 327-347.
- [Navaretti et al. (2008)] Navaretti, G. B. - Faini, R. - Tucci, A. (2008): Does Family Control Affect Trade Performance? Evidence for Italian Firms. Centro Studia Luca S'Agliano Development Studies Working Papers N. 260, October 2008. [http://www.dagliano.unimi.it/media/wp2008\\_260.pdf](http://www.dagliano.unimi.it/media/wp2008_260.pdf)
- [Nunes et al. (2014)] Nunes, P. M. - Serrasqueiro, Z. - da Silva, J. V. (2014): Family-owned and non family-owned SMEs: emirical evidence of survival determinants. *Economics and Business Letters* 3(1), 68-76, 2014.
- [Pappa (2009)] Pappa, E. (2009): The Effects of Fiscal Shocks on Employment and the Real Wage. *International Economic Review*, Vol. 50, No. 1 (Feb., 2009), pp. 217-244.
- [Pestel-Sommer (2013)] Pestel, N. - Sommer, E. (2013): Shifting Taxes from Labor to Consumption: Efficient, but Regressive? IZA Discussion Paper No. 784, December 2013. <http://ftp.iza.org/dp7804.pdf>
- [Ramey (2011)] Ramey, V. A. (2011): Identifying Government Spending Shocks: It's all in the Timing. *The Quarterly Journal of Economics*, Oxford University Press, vol. 126(1), pages 1-50.
- [Ramey-Shapiro (1998)] Ramey, V. A. - Shapiro, M. D. (1998): Costly capital reallocation and the effects of government spending. *Carnegie-Rochester Conference Series on Public Policy* 48 (1998) 145-194.

- [Rotemberg (1982)] Rotemberg, J. J. (1982): Sticky Prices in the United States. *Journal of Political Economy*, Vol. 90, No. 6 (Dec., 1982), pp. 1187-1211.
- [Rotemberg-Woodford (1992)] Rotemberg, J. J. - Woodford, M. (1992): Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity. *The Journal of Political Economy*, Vol. 100, No. 6, Centennial Issue. (Dec., 1994), pp. 1153-1207.
- [Shimer (2005)] Shimer, R. (2005): The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *The American Economic Review*, Vol. 95, NO. 1, March 2005, pp. 25-48.
- [Siebert et al. (2011)] Siebert, W. S. - Peng, F. - Maimaiti, Y. (2011): HRM Practices and Performance of Family-Run Workplaces: Evidence from the 2004 WERS. IZA Discussion Paper No. 5899, August 2011. <http://ftp.iza.org/dp5899.pdf>
- [Smets-Wouters (2007)] Smets, F. - Wouters, R. (2007): Shocks and Frictions in US Business Cycles A Bayesian DSGE Approach. ECB Working Paper NO 722, February 2007. <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp722.pdf?038767f7a645096eccd34a8058c562e0>
- [Spilimbergo et al. (2009)] Spilimbergo, A. - Symansky, S. - Schindler, M. (2009): Fiscal Multipliers. IMF Staff Position Note, SPN/09/11, May 2009. <https://www.imf.org/external/pubs/ft/spn/2009/spn0911.pdf>
- [Sraer-Thesmar (2007)] Sraer, D. - Thesmar, D. (2007): Performance and Behavior of Family Firms: Evidence from the French Stock Market. *Journal of the European Economic Association*, Vol. 5, No. 4 (Jun., 2007), pp. 709-751.
- [Staehler-Thomas (2012)] Staehler, N. - Thomas, C. (2012): FiMod - A DSGE Model for fiscal policy simulations. *Economic Modelling*, 29 (2012), pp. 239-261.
- [Sumpson (1986)] Sumpson, A. A. (1986): The Shift to Indirect Taxation in a Unionized Economy. *Bulletin of Economic Research* 38:1, 87-91.
- [Thomas-Picos-Sanchez (2012)] Thomas, A. - Picos-Sanchez, F. (2012): Shifting from Social Security Contributions to Consumption Taxes: The Impact on Low-Income Earner Work Incentives. OECD Taxation Working Papers, No. 11, OECD Publishing. [http://www.oecd-ilibrary.org/taxation/shifting-from-social-security-contributions-to-consumption-taxes\\_5k95qw921521-en?crawler=true](http://www.oecd-ilibrary.org/taxation/shifting-from-social-security-contributions-to-consumption-taxes_5k95qw921521-en?crawler=true)
- [Uhlig (2010)] Uhlig, H. (2010): Some fiscal calculus. *American Economic Review: Papers and Proceedings* 100 (May 2010): 30-34.

- [Villalonga-Amit (2006)] Villalonga, B. - Amit, R. (2006): How do family ownership, control and management affect firm value? *Journal of Financial Economics*, Vol. 80, pp. 385-417.
- [Woodford (2011)] Woodford, M. (2011): Simple Analytics of the Government Expenditure Multiplier. *American Economic Journal: Macroeconomics*, 3(1): 1-35.



# Chapter 2

## Structural Reforms, Openness and the Shadow Economy

*with Rahul Anand, Purva Khera and Magnus Saxegaard*

### 2.1 Motivation and literature

Since the recent economic crisis, the interest of policymakers has reverted (back) to structural reforms, tools for enhancing growth and employment. A recent example is Mario Draghi's speech delivered in December 2015<sup>1</sup>; the President of the ECB stressed that, alongside monetary policy, "structural reforms are key" to achieve prosperity. Also, IMF (2015) points out that reforms related to market deregulation (promotion of competition, hiring and firing regulations) particularly boost the economy (Table 3 on page 31).

An early example of academic research on structural reforms is Blanchard-Giavazzi (2003), who were followed by Kugler-Pica (2003), Berger-Danninger (2005), Boken-Hallett (2008), Fernandez-Villaverde et al. (2014) or Cacciatore-Fiori (2016), among others.<sup>2,3</sup> They all

---

<sup>1</sup>Monetary policy and structural reforms in the euro area, speech by Mario Draghi in Bologna on 14 December 2015. Link: <https://www.ecb.europa.eu/press/key/date/2015/html/sp151214.en.html>

<sup>2</sup>Some papers which solely consider product or labor market deregulation are Nicoletti-Scarpetta (2003), Besley-Burgess (2004), Conwey et al. (2005), Ebell-Haefke (2009) and Gomes et al. (2013). Furthermore, we mainly investigate impacts on output and unemployment, but other macroeconomic impacts are also interesting; see e.g. Alesina et al. (2005) or Estevao (2005). Additionally, Gerali et al. (2014) study effects on productivity, not of mixed deregulation reforms, but of simultaneous implementation of fiscal consolidation and an increase in competition; the latter constitutes a decrease in mark-up. Furthermore, Cacciatore et al. (2016) examine optimal monetary policy in a monetary union in the presence of product and labor market regulation. Finally, Gnocchi et al. (2015) find that labor market flexibility (e.g. wage bargaining reforms), while controlling for openness, matters for business cycle fluctuations.

<sup>3</sup>In general, shadow economy is not rarely ignored. Recent exceptions are Batini et al. (2011) who explore optimal monetary policy taking a shadow sector into account and Pappa et al. (2015) who revisit fiscal multipliers in the presence of tax evasion and corruption.

developed closed-economy models; however, there are several examples of open-economy frameworks as well, for instance Lusinyan-Muir (2013), Andres et al. (2014), Vogel (2014), Eggertson et al. (2014) or Cacciatore et al. (2015). Concerning the importance of openness, policies of the home country obviously affect the foreign country, and vice versa, as goods can move across borders; and it is more true as the country becomes more open. Notably, products can be sold not only in the home country, but also abroad, similarly, an increase in demand can be fulfilled by imports as well; while exchange rate movements and price differentials between home and foreign goods also matter.

Additionally, there is a lot of attention on tax evasion. The last “T” in the “three Ts” that were discussed by the Group of Eight at their summit in 2013 was tax<sup>4</sup>; in particular, they emphasized the importance of fighting tax avoidance. Also, G20 leaders in November 2015 endorsed some steps in order to crack down on avoiding taxes.<sup>5</sup>

Tax evasion is the major part of shadow economy<sup>6</sup>, in company with avoiding regulations. Notably, as Schneider (2012) points out, shadow economy is mostly related to avoiding taxation or legal requirements in general.<sup>7</sup> Still, we are only aware of one paper on deregulation policies, Charlot et al. (2011), where a shadow economy is taken into account. And, we are not aware of any work combining informality with openness.

Multiple points support the conviction that the underground sector and its interactions with openness must be considered when studying the impacts of structural reforms.

As, by definition, the shadow side of the economy is “in the shadow”, the government can only affect directly the official side of the economy. For instance, lowering the tax rate cannot directly influence the informal economy, as taxes are not paid there. Thus, the larger the unofficial sector, the smaller will be the fraction of the economy impacted by the government. If there were no interactions between the formal and informal economies (but there are), the macroeconomic effects of deregulation policies would always increase in magnitude, as the size of the shadow economy decreased (“composition effect”).

Additionally, the formal and informal markets interact. First, the higher the absolute level of informality, potentially the more workers and goods can move out of the shadow (“substitution effect”). Second, working in the shadow has been theorized to function as

---

<sup>4</sup>E.g. The Economist published an article on 22 June 2013 with the title of The G8 summit T time.

<sup>5</sup>See this note on OECD’s contribution for instance: <http://www.oecd.org/g20/meetings/antalya/g20-leaders-endorse-oecd-measures-to-crackdown-on-tax-evasion-reaffirm-its-role-in-ensuring-strong-sustainable-and-inclusive-growth.htm>

<sup>6</sup>In our paper the terms shadow, informal, underground and unofficial are interchangeable.

<sup>7</sup>See the Appendix for precise definitions.

a kind of “insurance policy”; the “shadow employment is tolerated because its repression increases unemployment” (Boeri-Garibaldi (2007), page 125). Accounts of any such synergy fall by the wayside when the underground economy is neglected.

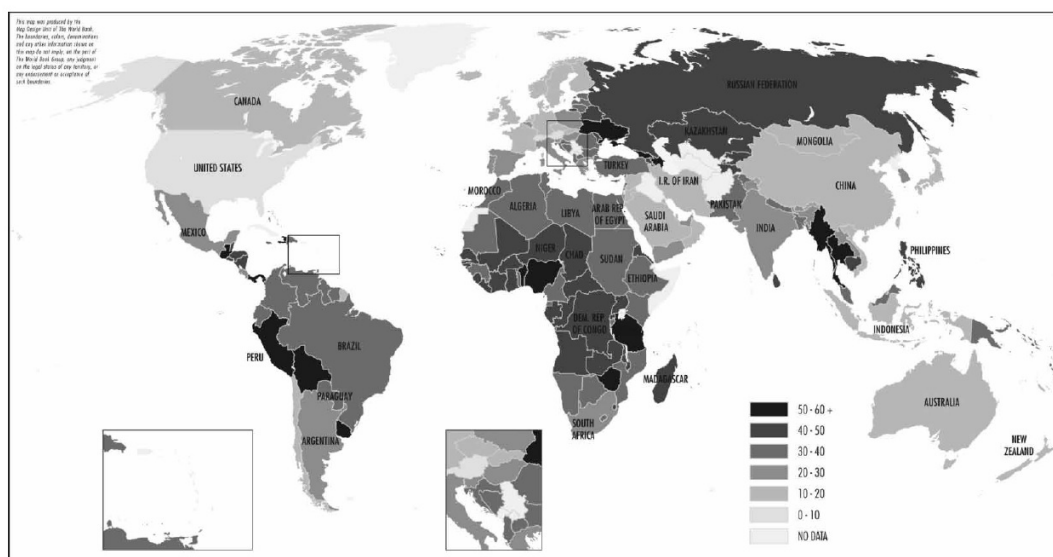


Figure 2.1: The fraction of shadow output in total output around the world. Source is Schneider et al. (2010) page 458).

How large is the shadow economy? This is a central question. In fact, in most countries it is rather sizable (Figure 2.1). Even though it is larger in developing countries, there is a great deal of informality in the developed world as well. Notably, 16.3 percent of OECD countries’ GDP and 43.2 percent of African countries’ GDP is in the shadow (Schneider (2005)). Obviously, informality is also present in employment (Schneider (2012)). Thus, the very size of the shadow economy renders vital the incorporation of it into our framework.

Regarding the relation of openness and the underground sector, legal duties are only met in the formal sector, enabling access to foreign markets, whereas unofficial firms have no such access. Consequently, a critical trade-off takes place: functioning in the formal sector creates new markets abroad, as well as higher costs in the form of legal obligations. Empirically, based on a simple OLS regression, if the size of the underground sector in GDP increases by 1 pp, openness measured as the sum of nominal exports and imports in nominal GDP, goes down by 0.8 pp (p-value is 0.01) (Figure 2.2).<sup>8</sup>

As already mentioned, we are aware of only one study that investigates the macroeconomic impacts of deregulation policies in the presence of an informal sector. We contend that

---

<sup>8</sup>We do not claim at all that this regression is enough to describe the empirical relationship between the shadow economy and openness, it is only provided with the purpose of illustration.

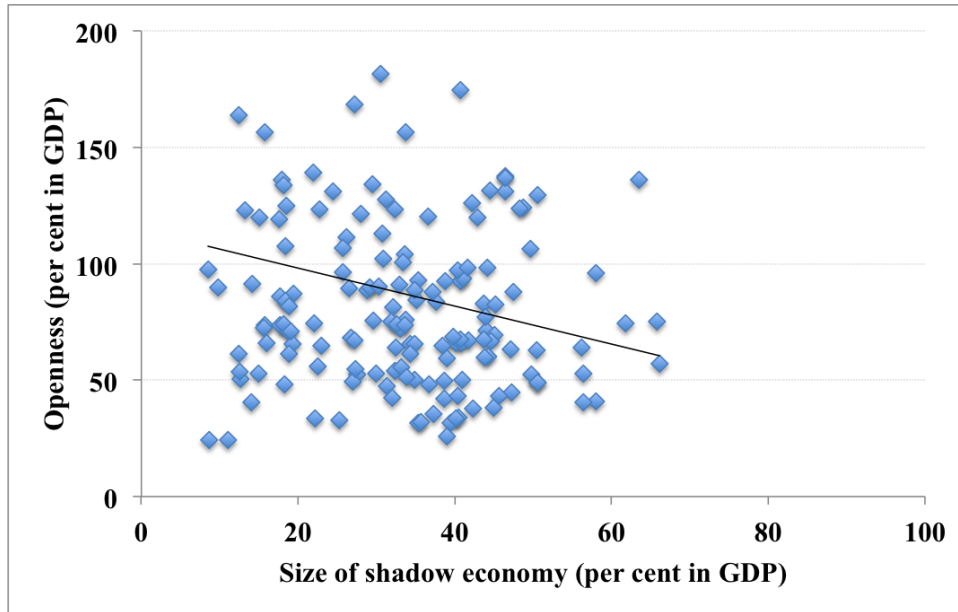


Figure 2.2: The relation between the shadow economy and openness. Openness is calculated by the authors as the ratio of the sum of nominal exports and imports in nominal GDP; data source is World Bank. Data sources of the size of the shadow economy are Schneider et al. (2010) and Schneider (2012). Both openness and the size of the shadow economy are calculated between 1999 and 2007.

Charlot et al. (2011) is incontrovertably an important step in the literature, but we can also identify some further important steps. First, the model is not dynamic; the authors study short- and long-run reforms independently of each other, rather than of examining the short-run transition towards the new long-run equilibrium. As well, reform interactions or sequencing are not evaluated. Furthermore, the model is a closed-economy, and firms in the formal and informal sectors mainly differ because there is tax evasion in the latter.

Thus, in our paper we revisit the macroeconomic impacts of structural reforms, i.e. deregulating the labor and product markets, in an open-economy framework with an underground sector.<sup>9</sup> Alongside the long-run effects, we examine the path to the new long-run equilibrium, as short-term costs might emerge even if the long-run implications were positive. Both single and parallel policies are studied, and the implementation order as well, because it might prove pivotal in policy blends.

As regards defining deregulation policies more precisely, they are permanent and unexpected reductions in the level of regulation in the formal economy. So, another property that distinguishes this paper from those without a shadow economy is that here reforms are

---

<sup>9</sup>We mainly focus on impacts on GDP and unemployment; it is beyond the scope of this paper to study in detail the reasons behind a shadow economy. Among others, the tax burden and the level of regulation are important factors, see e.g. Schneider (2005).



solely implemented in the formal sector. Specifically, we consider a decrease in the cost of hiring, bargaining power of workers and entry cost of firms. The hiring cost represents both training and administrative costs, while the bargaining power of workers is mainly related to the level of unionization. The entry cost consists of cost of registration and the time spent on bureaucracy when setting up a new business.

South Africa, an example of the emerging countries, is considered when Bayesian estimating the model.<sup>10</sup> Economic growth in emerging economies, in general, was considerable in the 1990s, and after the recent crisis many of them recovered sooner than the developed countries did (Dabla-Norris et al. (2013), IMF (2015)). Still, they usually have a larger shadow economy, hence, we focus on them.

Regarding the mechanisms, we find that the unofficial economy is a major determinant of the sign, and, particularly, the magnitude of the macroeconomic effects of structural reforms, in contrary to openness. We show that, in the long run, both labor and product market reforms considerably increase output in South Africa, although labor market reforms are somewhat more successful in decreasing unemployment. At the same time, the level of shadow employment does not decrease in the long run, although it does so relatively. Nevertheless, there are short-term costs, for example, a decrease in household consumption, net exports or output, or a decrease in competition. Reform packages often mitigate short-term costs, though, which accompany all policies we consider. Also, we find that it is usually better to start with a labor market reform than with a product market reform.

The rest of the paper is as follows. First, we describe the model and its economic channels in detail. Then, we present the process of calibration and Bayesian estimating the dynamic parameters. Later, we turn to showing and interpreting our results: long- and short-term effects of single policies, mixed policies to mitigate short-run costs and reform sequencing. The Appendix provides more information on the steps of calculating the model's steady-state and its robustness, also on the role of shadow economy, Bayesian estimation, reform packages and policy sequence.

## 2.2 Model

The model is a small open-economy dynamic general equilibrium model with sticky prices a la Rotemberg (1982), unemployment due to hiring costs and wage bargaining following Blanchard-Gali (2010) and endogenous firm entry like Bilbie et al. (2012). Figure 2.3 shows an overview of its structure.<sup>11</sup>

---

<sup>10</sup>A similar model is used to look at the impacts of deregulation reforms in India in Anand-Khera (2016).

<sup>11</sup>Notation is as follows: F for formal, I for informal, H for home, f for import/foreign, W for wholesaler (intermediate) and R for retailer (final).

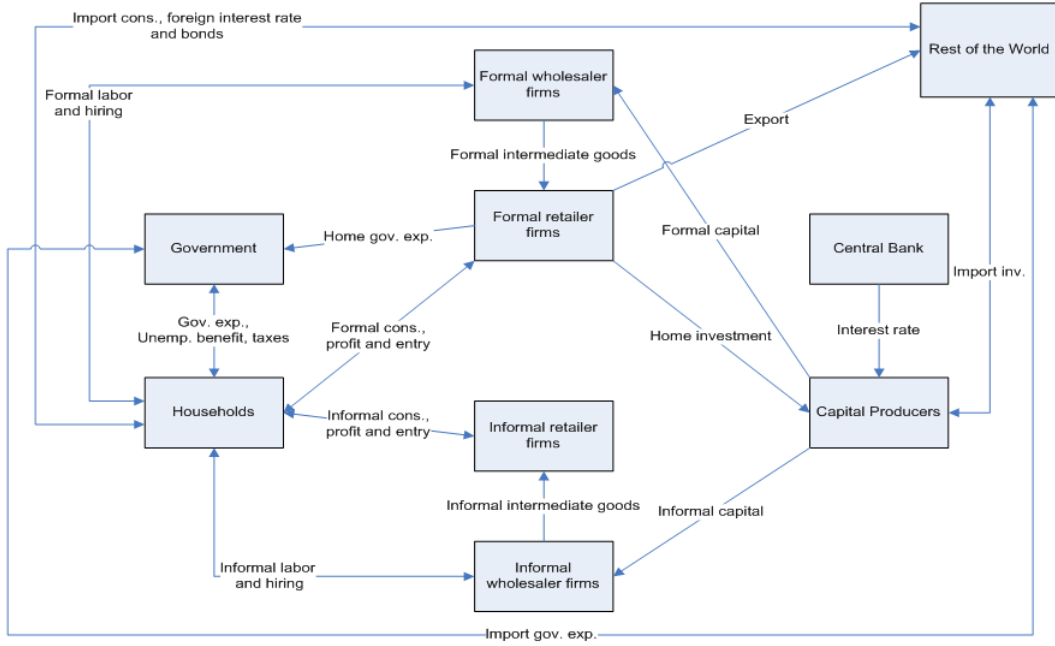


Figure 2.3: The structure of the model

The main novelty is the distinction between formal and informal sectors in the labor and goods markets. The major difference is that taxes are avoided in the underground economy. Furthermore, in line with (Williamson (1975)), the level of regulation is lower in the informal sector; hiring and producer entry manifest differently in the shadow and non-shadow sectors. Also, openness is only a consideration in the formal sector, while the government and capital producers can only buy formal goods.

At this point, we start to describe the model in detail<sup>12</sup>; it is called **STRESS** that stands for Studying **Str**uctural **R**eforms in a Small Open-Economy Dynamic General Equilibrium Model with Unemployment, Regulation and a **S**hadow **S**ector. It also reflects the need for (stress of) structural reforms that each country faces today, especially in the presence of a (large) shadow sector.

### 2.2.1 Households

There is a representative infinitely living household with perfect foresight that maximizes expected discounted lifetime utility of consumption  $C_t$ :

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \zeta_{C,t} U[C_t] \quad (2.1)$$

The contemporaneous utility is given by  $U[C_t] = (1 - hc) \ln(C_t - C_{t-1})$ , where  $\beta$  is the discount factor,  $\zeta_{C,t}$  is the preference shock and  $hc \in (0, 1)$  is the external consumption habit

<sup>12</sup>Whenever the formal maximization is parallel to the informal one, we only detail the first one.

parameter. This functional form ensures that habit persistence does not affect the long-run equilibrium.

The household faces a budget constraint in every period:

$$\begin{aligned}
& C_t + RER_t B_t + D_t + N_{F,t}^E \text{entry}_{F,t} + N_{I,t}^E \text{entry}_{I,t} + Tax_t = \\
& = DEP_t \frac{1 + i_{t-1}^*}{\pi_t} RER_{t-1} B_{t-1} + \frac{1 + i_{t-1}}{\pi_t} D_{t-1} + N_{F,t} Prof_{F,t}^R + N_{I,t} Prof_{I,t}^R + \\
& + Prof_{F,t}^W + Prof_{I,t}^W + (1 - \tau_{F,t}) W F_t L_{F,t} + W I_t L_{I,t} + W U_t U_t + Y_{HP,t}
\end{aligned} \tag{2.2}$$

As usual, the household consumes an aggregate consumption bundle  $C_t$  and saves. Savings can be in the form of foreign bonds  $B_t$  or in home bonds  $D_t$ ; all markets are complete.  $RER_t$  is the real exchange rate and  $DEP_t$  is the depreciation rate of the nominal exchange rate. Nominal interest rate on home bonds  $i_t$  is determined by the central banks' Taylor rule, while nominal interest rate on foreign bonds  $i_t^*$  depends on the exogenous foreign interest rate, on the one hand, and on an interest rate premium related to the relative amount of foreign debt holdings, on the other hand, following Schmitt-Grohe-Urbe (2003).

The household owns wholesaler and retailer firms, denoted by W and R, respectively, so the firms' respective profits return to the household. Because the retailer profit means one firm's profit, it is multiplied by the number of retailer firms. At the same time, there is no endogenous entry at the wholesaler level, or, to put it another way, the number of wholesaler firms is normalized to one. Endogenous firm entry at the retailer level also explains why the household finances entry costs ( $\text{entry}_{F,t}$  and  $\text{entry}_{I,t}$ ) of new firms  $N_{F,t}^E$  and  $N_{I,t}^E$ . Firm exit is exogenous with sectoral bankruptcy rates  $\delta_F$  and  $\delta_I$ . Thus, the laws of motion for the number of retailer firms are:

$$N_{F,t} = (1 - \delta_{F,t})(N_{F,t-1} + N_{F,t}^E) \tag{2.3}$$

$$N_{I,t} = (1 - \delta_{I,t})(N_{I,t-1} + N_{I,t}^E) \tag{2.4}$$

Here, we assume that a firm, which enters in period  $t$ , starts to operate in period  $t$ .

Besides profit, the household earns labor income from working in the formal sector ( $L_{F,t}$ ) or in the informal sector ( $L_{I,t}$ ), or it receives social benefits  $WU_t$ , which is an exogenous shock, if it is unemployed.  $WF_t$  and  $WI_t$  are the sectoral real wages; although only the formal sector's wage is subject to income  $\tau_{F,t}$  which is an exogenous variable. The household also pays a lump-sum tax  $Tax_t$  to close the model. Finally,  $Y_{HP,t}$  denotes home production.

Given the utility function and the budget constraint, two Euler equations follow, one for each bond holdings:

$$\beta(C_t - hcC_{t-1})E_t \left[ \frac{(1+i_t)\zeta_{C,t+1}}{C_{t+1} - hcC_t} \right] = E_t [\pi_{t+1}\zeta_{C,t}] \quad (2.5)$$

$$\beta(C_t - hcC_{t-1})E_t \left[ \frac{(1+i_t^*)\zeta_{C,t+1}}{C_{t+1} - hcC_t} DE P_{t+1} \right] = E_t [\pi_{t+1}\zeta_{C,t}] \quad (2.6)$$

In the second Euler equation, which refers to foreign savings, the depreciation rate of the nominal exchange rate is also taken into account.

The aggregate consumption bundle  $C_t$  consists of home-produced goods  $C_{H,t}$  and foreign-produced (imported) goods  $C_{f,t}$ , and is given by:

$$C_t = \left[ \alpha^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (2.7)$$

where  $\alpha \in (0, 1)$  can be interpreted as a measure of home bias, or one could think of  $1 - \alpha$  as the import ratio which captures the degree of openness.  $\eta > 0$  is the elasticity of substitution between home and foreign produced goods.

Total cost spent on home- and foreign-produced goods is minimised. As a result, the demand functions and the aggregate price level  $P_t$  – a composite of home price  $P_{H,t}$  and foreign (import) price  $P_{f,t}$  – are:

$$C_{H,t} = \alpha C_t \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} \quad (2.8)$$

$$C_{f,t} = (1 - \alpha) C_t \left( \frac{P_{f,t}}{P_t} \right)^{-\eta} \quad (2.9)$$

$$P_t = [\alpha P_{H,t}^{1-\eta} + (1 - \alpha) P_{f,t}^{1-\eta}]^{\frac{1}{1-\eta}} \quad (2.10)$$

The relevant inflation rates are defined as follow:

$$\pi_t = \frac{P_t}{P_{t-1}} \quad (2.11)$$

$$\pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} \quad (2.12)$$

$$\pi_{f,t} = \frac{P_{f,t}}{P_{f,t-1}} \quad (2.13)$$

Similarly to aggregate consumption, home consumption  $C_{H,t}$  is also a composite, namely, it is a composite of goods produced in the formal sector  $C_{F,t}$  and goods produced in the informal sector  $C_{I,t}$ :

$$C_{H,t} = \left[ \omega^{\frac{1}{\mu}} C_{F,t}^{\frac{\mu-1}{\mu}} + (1 - \omega)^{\frac{1}{\mu}} C_{I,t}^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \quad (2.14)$$

where  $\omega \in (0, 1)$  represents the weight of formal sector goods in the basket, and  $\mu > 0$  is the elasticity of substitution between sectoral goods.

Total cost spent on formal and informal goods is minimised, which implies the demand functions:

$$C_{F,t} = \omega C_{H,t} \left( \frac{P_{F,t}}{P_{H,t}} \right)^{-\mu} \quad (2.15)$$

$$C_{I,t} = (1 - \omega) C_{H,t} \left( \frac{P_{I,t}}{P_{H,t}} \right)^{-\mu} \quad (2.16)$$

Also, as a result of optimization, we can specify the price of home goods  $P_{H,t}$  as a function of  $P_{F,t}$  formal and  $P_{I,t}$  informal price levels:

$$P_{H,t} = [\omega P_{F,t}^{1-\mu} + (1 - \omega) P_{I,t}^{1-\mu}]^{\frac{1}{1-\mu}} \quad (2.17)$$

Relevant inflation rates are:

$$\pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}} \quad (2.18)$$

$$\pi_{I,t} = \frac{P_{I,t}}{P_{I,t-1}} \quad (2.19)$$

## 2.2.2 Capital producer

The capital producer owns physical capital, and, by investing, produces new physical capital. Investment is subject to a capital adjustment cost. This set-up follows that of Bernanke et al. (1999).

The capital law of motion is standard, except that the price of investment is not equal to the general economy-wide price level:

$$K_t = (1 - \delta) K_{t-1} + \frac{P_{INV,t}}{P_t} I_t - \frac{\phi_{INV}}{2} \left( \frac{\frac{P_{INV,t}}{P_t} I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} \quad (2.20)$$

This is because only goods produced in the formal sector can be used for investment, by assumption. The  $\frac{\phi_{INV}}{2} \left( \frac{\frac{P_{INV,t}}{P_t} I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}$  capital adjustment cost is zero in the long run, but varies around the steady state, with  $\phi_{INV}$  being the capital adjustment cost parameter and  $\delta$  the depreciation rate of physical capital.

The capital producer invests such that its profit is maximized:

$$\max Q_t \left\{ \frac{P_{INV,t}}{P_t} I_t - \frac{\phi_{INV}}{2} \left( \frac{\frac{P_{INV,t}}{P_t} I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} - \frac{P_{INV,t}}{P_t} I_t \right\} \quad (2.21)$$

where  $Q_t$  is the price of physical capital.

As a result, the Tobin-Q equation follows:

$$Q_t \left( 1 - \phi_{INV} \left( \frac{P_{INV,t}}{P_t} \frac{I_t}{K_{t-1}} - \delta \right) \right) = 1 \quad (2.22)$$

Also, because no arbitrage is possible, the following condition holds:

$$E_t \frac{1 + i_t}{\pi_{t+1}} = E_t \frac{RK_{t+1} + (1 - \delta)Q_{t+1}}{Q_t} \quad (2.23)$$

This condition means that the real return on saving in home bonds must equal the net return on saving in physical capital, which is the ratio of the sum of the current period's physical capital price level and real rental rate of capital  $RK_t$  and previous period's physical capital price level.

Finally, aggregate investment, like aggregate consumption, is a composite good. Notably, it is a composite of home produced and imported goods:

$$I_t = \left[ \alpha^{\frac{1}{\eta}} I_{H,t}^{\frac{\eta-1}{\eta}} + (1 - \alpha)^{\frac{1}{\eta}} I_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (2.24)$$

where  $\alpha \in (0, 1)$  and  $\eta > 0$  represent the same parameters as before. Nevertheless, we would like to stress that home investment is a function of formal goods only, which explains the fact that the home demand function depends on the formal goods' price level and the price of investment does not depend on the informal goods' price level either:

$$I_{H,t} = \alpha I_t \left( \frac{P_{F,t}}{P_{INV,t}} \right)^{-\eta} \quad (2.25)$$

$$I_{f,t} = (1 - \alpha) I_t \left( \frac{P_{f,t}}{P_{INV,t}} \right)^{-\eta} \quad (2.26)$$

$$P_{INV,t} = [\alpha P_{F,t}^{1-\eta} + (1 - \alpha) P_{f,t}^{1-\eta}]^{\frac{1}{1-\eta}} \quad (2.27)$$

A similar assumption is made regarding government consumption; thus, the price of investment is equal to the price of government consumption, and home demand for government consumption depends on the formal goods' price level only.

### 2.2.3 Wholesale good producers

A continuum of entrepreneurs of  $(0, 1)^{13}$  in each sector use labor ( $L_{F,t}$  and  $L_{I,t}$ ) and physical capital ( $K_{F,t}$  and  $K_{I,t}$ ) to produce intermediate goods ( $Y_{F,t}$  and  $Y_{I,t}$ ), following a constant

---

<sup>13</sup>Because in equilibrium all  $i \in (0, 1)$  intermediate firms follow the same optimization process, for the sake of simplicity we disregard the symbol  $i$  when describing the intermediate firms' optimization in most of this section.

returns to scale technology:

$$Y_{F,t} = \theta_{F,t}(K_{F,t-1})^{\psi_F}(L_{F,t})^{1-\psi_F} \quad (2.28)$$

$$Y_{I,t} = \theta_{I,t}(K_{I,t-1})^{\psi_I}(L_{I,t})^{1-\psi_I} \quad (2.29)$$

$\theta_{F,t}$  and  $\theta_{I,t}$  are exogenous sectoral productivities, and  $\psi_F$  and  $\psi_I$  are the sectoral capital income shares.

From now on, we only describe the formal sector, as the informal sector's optimization is a mirror image of it. The one-period profit of the formal intermediate firm is equal to revenue net costs spent on labor and physical capital; the cost of hiring new workers is deducted, too:

$$Prof_{F,t}^W = MC_{F,t}Y_{F,t} - WF_tL_{F,t} - RK_tK_{F,t-1} - HC_{F,t}H_{F,t} \quad (2.30)$$

$MC_{F,t}$  is the real marginal cost of production, which is equal to the price of intermediate goods as we assume that intermediate firms are price takers. The hiring cost is denoted by  $HC_{F,t}$ , while  $H_{F,t}$  is the number of hired people.

The number of employed people follows a law of motion:

$$L_{F,t} = (1 - probf_{F,t})L_{F,t-1} + H_{F,t} \quad (2.31)$$

At the beginning of period  $t$   $L_{F,t-1}$  people are employed. Then, at the beginning of period  $t$   $probf_{F,t}L_{F,t-1}$  people are fired, where the exogenous firing probability is  $probf_{F,t}$ . During period  $t$ , firms hire new workers. After firing and hiring is over, the end of period  $t$  employment will be  $L_{F,t}$ , which is also the level of employment at the beginning of period  $t + 1$ .

Regarding labor market frictions, unemployment is induced by the above-noted hiring costs. The set-up of these labor market frictions closely follows that of Blanchard-Gali (2010). The hiring cost and the search and matching frictions differ mainly in that the hiring itself is costly. Thus, unemployment costs mostly reflect not the search itself or the possible inefficiency of the match, but rather the cost – above the wage – of hiring a new worker. This is the training cost: if the educational level or expertise of the worker does not meet the needs of the firm, it must train the worker. Alternatively, one could think of this hiring cost as a cost the firm pays to the government or another company (head-hunter, recruiter), or it can proxy the administration “costs” incurred by the firm when hiring (i.e. social security paperwork, etc.)

In our model, hiring cost is a function of hiring probability:

$$HC_{F,t} = \beta_{HCF,t}(probh_{F,t})^{\alpha_{HCF}} \quad (2.32)$$

where the hiring probability is:

$$probh_{F,t} = \frac{H_{F,t}}{U_{t-1} + probf_{F,t}L_{F,t-1} + probf_{I,t}L_{I,t-1}} \quad (2.33)$$

Thus, the probability of hiring depends on the number of hired people  $H_{F,t}$  (the higher the number of hired people, the higher the probability of hiring) and on the number of people – potentially – available to hire. We assume that not only those who were unemployed at the beginning of period  $t$  can be hired, but also those who have just lost their jobs in any of the sectors. The exogenous term  $\beta_{HCF,t}$  represents the per capita hiring cost, and this is the labor market deregulation variable, too. Finally,  $\alpha_{HCF}$  is the elasticity of hiring cost with respect to the hiring probability.<sup>14</sup>

Formal sector firms maximize the expected present value of future profits:

$$\max E_t \sum_{j=0}^{\infty} Q_{t,t+j} Prof_{F,t+j}^W \quad (2.34)$$

subject to the employment law of motion discussed above. The discount rate is consistent with the Euler equation of the household  $Q_{t,t+j} = \beta \frac{U_{c,t+j}}{U_{c,t}}$ .

Capital and labor demand functions follow:

$$MC_{F,t} \psi_F \frac{Y_{F,t}}{K_{F,t-1}} = RK_t \quad (2.35)$$

$$\begin{aligned} MC_{F,t} (1 - \psi_F) \frac{Y_{F,t}}{L_{F,t}} - WF_t = \\ = HC_{F,t} - \beta \frac{C_t - hcC_{t-1}}{\zeta_{C,t}} E_t \left[ \frac{\zeta_{C,t+1}}{C_{t+1} - hcC_t} (1 - probf_{F,t+1}) HC_{F,t+1} \right] \end{aligned} \quad (2.36)$$

The capital demand function is standard. Nevertheless, the labor demand function becomes dynamic due to the presence of labor market frictions, that is, not only the wage is taken into account, but also the current and next period's hiring cost.

Wage setting is sectoral and follows a Nash bargaining process between workers and wholesaler firms, with exogenous sectoral bargaining power of the workers  $\lambda_{F,t}$  and  $\lambda_{I,t}$ .

In period  $t$  a worker is either employed in the formal sector, or she is employed in the informal sector, or she is unemployed. If a worker is employed in the formal sector, her current wage is  $(1 - \tau_{F,t})WF_t$ . In the next period, she might keep her job with probability  $1 - probf_{F,t+1}$ , or she might be fired with probability  $probf_{F,t+1}$ . If she is fired, she might find another job either in the formal sector, with probability  $probf_{F,t+1}probh_{F,t+1}$ , or in

---

<sup>14</sup>Hiring cost today might be interpreted as firing cost tomorrow. The reason is that when hiring today takes place the probability of firing tomorrow is taken into account.



the informal sector, with probability  $prob f_{F,t+1} prob h_{I,t+1}$ , or she might stay unemployed, with probability  $prob f_{F,t+1}(1 - prob h_{I,t+1} - prob h_{F,t+1})$ . Hence, the value function of being employed in the formal sector in period  $t$  is:

$$V_t^F = (1 - \tau_{F,t})WF_t + E_t \{Q_{t,t+1} [(1 - prob f_{F,t+1} + prob f_{F,t+1} prob h_{F,t+1})V_{t+1}^F] \} + \\ + E_t \{Q_{t,t+1} [prob f_{F,t+1} prob h_{I,t+1}V_{t+1}^I + prob f_{F,t+1}(1 - prob h_{I,t+1} - prob h_{F,t+1})V_{t+1}^U] \} \quad (2.37)$$

The value function of being employed in the informal sector is similar, except that the worker does not pay labor income tax today:

$$V_t^I = WI_t + E_t \{Q_{t,t+1} [(1 - prob f_{I,t+1} + prob f_{I,t+1} prob h_{I,t+1})V_{t+1}^I] \} + \\ + E_t \{Q_{t,t+1} [prob f_{I,t+1} prob h_{F,t+1}V_{t+1}^F + prob f_{I,t+1}(1 - prob h_{I,t+1} - prob h_{F,t+1})V_{t+1}^U] \} \quad (2.38)$$

If a worker is currently unemployed, she receives social security benefits today, and in the next period she might be unemployed still, with probability  $1 - prob h_{I,t+1} - prob h_{F,t+1}$ , or she might find a job in any of the two sectors:

$$V_t^U = WU_t + \\ + E_t \{Q_{t,t+1} [(1 - prob h_{I,t+1} - prob h_{F,t+1})V_{t+1}^U + prob h_{F,t+1}V_{t+1}^F + prob h_{I,t+1}V_{t+1}^I] \} \quad (2.39)$$

Workers' surplus of being employed in the formal or in the informal sector are  $V_t^F - V_t^U$  and  $V_t^I - V_t^U$ , respectively.

Sectoral firm value functions are given by the hiring costs (see Blanchard-Gali (2010) for more detail):

$$J_t^F = HC_{F,t} \quad (2.40)$$

$$J_t^I = HC_{I,t} \quad (2.41)$$

The underlying intuition is that a firm can always replace a worker by paying the hiring cost, as there is no search time required.

Workers and firms bargain over real wages, given the above value functions:

$$\max (V_t^F - V_t^U)^{\lambda_{F,t}} (J_t^F)^{1-\lambda_{F,t}} \quad (2.42)$$

$$\max (V_t^I - V_t^U)^{\lambda_{I,t}} (J_t^I)^{1-\lambda_{I,t}} \quad (2.43)$$

Two first-order bargaining conditions follow:

$$\frac{\lambda_{F,t}}{1 - \lambda_{F,t}} (1 - \tau_{F,t}) J_t^F = V_t^F - V_t^U \quad (2.44)$$

$$\frac{\lambda_{I,t}}{1 - \lambda_{I,t}} J_t^I = V_t^I - V_t^U \quad (2.45)$$

Only the formal sector' bargaining is affected by labor income taxes, as taxes are avoided in the informal sector.

### 2.2.4 Retailers

At the retailer level, there are  $N_{F,t}$  monopolistic competitive firms which distribute at no cost wholesale goods purchased from the intermediate firms. Total demand for formal goods is denoted by  $QD_{F,t}$ . We assume that the total final demand for formal goods is a Dixit-Stiglitz (1977) aggregator of the formal goods distributed by all final firms in the formal sector, where  $s$  denotes one of those firms:

$$QD_{F,t} = \left( \int_0^{N_{F,t}} QD_{F,t}(s)^{\frac{\epsilon_{F,t}-1}{\epsilon_{F,t}}} ds \right)^{\frac{\epsilon_{F,t}}{\epsilon_{F,t}-1}} \quad (2.46)$$

Here, the number of formal firms  $N_{F,t}$  enters the aggregation because there is endogenous entry and the number of firms is not normalised to one.  $\epsilon_{F,t}$  is the elasticity of substitution between goods in the formal sector. As usual, this elasticity is related to the price mark-up, which is  $\frac{\epsilon_{F,t}}{\epsilon_{F,t}-1}$ . We assume that this mark-up is endogenous and depends on the number of firms, i.e.  $\epsilon_{F,t} = \alpha_F N_{F,t}$ .

As a result of the retailer maximization problem, the aggregate price level is a function of the firms' price levels:

$$P_{F,t} = \left( \int_0^{N_{F,t}} P_{F,t}(s)^{1-\epsilon_{F,t}} ds \right)^{\frac{1}{1-\epsilon_{F,t}}} \quad (2.47)$$

and the relevant demand function is:

$$QD_{F,t}(s) = \left( \frac{P_{F,t}(s)}{P_{F,t}} \right)^{-\epsilon_{F,t}} QD_{F,t} \quad (2.48)$$

There is price stickiness at the retailer level; retailer firms set prices a la Rotemberg (1982). This means that there is a quadratic cost of price adjustment:

$$R(P_{F,t}(s)) = \frac{\phi_F}{2} \left( \frac{P_{F,t}(s)}{P_{F,t-1}(s)} \frac{1}{\pi} - 1 \right)^2 QD_{F,t}(s) \quad (2.49)$$

where the degree of price stickiness is  $\phi_F$ , and  $\pi$  is the steady-state economy-wide inflation rate. Price adjustment positively depends on final demand as well.

Retailer  $s$  maximises its expected discounted stream of future profits:

$$\max E_t \sum_{k=t}^{\infty} Q_{t,k} Prof_{F,k}^R(s) \quad (2.50)$$

where  $Q_{t,k}$  is the stochastic discount factor as before.

The one-period profit is given by:

$$Prof_{F,t}^R(s) = \left( \frac{P_{F,t}(s)}{P_t} - MC_{F,t}(s) \right) \left( \frac{P_{F,t}(s)}{P_{F,t}} \right)^{-\epsilon_{F,t}} QD_{F,t} - R(P_{F,t})(s) \quad (2.51)$$

Here,  $MC_{F,t}(s)$  is the price final firm  $s$  pays when purchasing the wholesale goods. This is not equal to  $MC_{F,t}$ , but it is related to it through the number of final firms. Specifically:

$$MC_{F,t} = \left( \int_0^{N_{F,t}} MC_{F,t}(s)^{1-\epsilon_{F,t}} ds \right)^{\frac{1}{1-\epsilon_{F,t}}} \quad (2.52)$$

This is a similar, Dixit-Stiglitz-type, expression that holds for  $P_{F,t}$  as well.

As a result, the formal sector's pricing rule is:

$$\begin{aligned} 1 - \epsilon_{F,t} \left( \frac{P_{F,t}}{P_t} - MC_{F,t} \right) \left( \frac{P_{F,t}}{P_t} \right)^{-1} - \phi_F \left( \frac{P_{F,t}(s)}{P_{F,t-1}(s)} \frac{1}{\pi} - 1 \right) \frac{1}{\pi} \frac{P_t}{P_{F,t-1}(s)} + \\ + \beta \frac{C_t - hcC_{t-1}}{\zeta_{C,t}} E_t \left[ \frac{\zeta_{C,t+1}}{C_{t+1} - hcC_t} \phi_F \left( \frac{P_{F,t+1}(s)}{P_{F,t}(s)} \frac{1}{\pi} - 1 \right) \frac{P_{F,t+1}}{P_{F,t}} \frac{N_{F,t}}{N_{F,t+1}} \frac{QD_{F,t+1}}{QD_{F,t}} \right] = 0 \end{aligned} \quad (2.53)$$

Then, product market regulation is modeled at the retailer level, and it follows Bilbie et al. (2012). A mass of firms potentially and endogenously enter the retailer market in every period. Entry is subject to an exogenous entry cost  $entry_{F,t}$ . This entry cost consists of the costs to set up a new business, e.g. company registration, but also the administrative burden a firm assumes when starting a new company. This cost is the reason that the number of firms is not normalized to one, in contrast to a standard set-up without entry decision. Also, this is why the number of firms follows a law of motion described in the household sector.

New firms enter the market until the entry cost is equal to the firm value, which is the expected discounted value of future profits, taking into account the probability of going bankrupt:

$$\begin{aligned} entry_{F,t} &= E_t \sum_{k=t}^{\infty} Q_{t,k} (1 - \delta_F)^{k-t+1} Prof_{F,k}^R(s) = \\ &= (1 - \delta_F) Prof_{F,t}^R(s) + E_t Q_{t,t+1} (1 - \delta_F) entry_{F,t+1} \end{aligned} \quad (2.54)$$

Here,  $1 - \delta_F$  enters the present value of future profits because the timing of retailers' decisions is as follow: first they decide whether to enter or not, then  $1 - \delta_F$  share of the new entrants does not go bankrupt and operates, finally all firms operating in period  $t$  decide on prices.

Deregulating the product market means that the fixed formal entry cost is decreased by the government.

## 2.2.5 Rest of the world, government, monetary authority and market clearing

The home economy is a small open-economy. Consequently, it cannot affect foreign price level or foreign interest rate, which are exogenously given.

For the sake of simplicity, we assume that the aggregate export demand is:

$$Q_t^X = \left( \frac{P_{F,t}}{e_t P_t^* \alpha_X} \right)^{-\varsigma_X} \quad (2.55)$$

where  $\varsigma_X > 0$  is the price elasticity and  $\alpha_X$  is a parameter that captures factors other than the export price, which affect the export demand, e.g. the international economic environment. Because only formal goods are exported, the price of exports in foreign currency is  $P_{F,t}^* = \frac{P_{F,t}}{e_t}$ ; here  $e_t$  is the nominal exchange rate, and we assume that the law of one price holds.

The exogenous foreign inflation is defined as  $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$ .

Also, we assume that the import price of each company  $s$  (by assumption the number of companies which import is equal to the number of formal goods producers) is related to the real exchange rate where  $\varsigma_M$  captures the exchange rate pass-through to import prices:

$$\frac{\frac{P_{f,t}(s)}{P_t}}{\frac{P_{f,t}(s)}{P}} = \left( \frac{RER_t}{RER} \right)^{\varsigma_M} \quad (2.56)$$

Then, the final import price is a Dixit-Stiglitz-type aggregator of the individual import prices, namely:

$$P_{f,t} = \left( \int_0^{N_{F,t}} P_{f,t}(s)^{1-\epsilon_{F,t}} ds \right)^{\frac{1}{1-\epsilon_{F,t}}} \quad (2.57)$$

Finally, the interest rate on foreign bond holdings  $i_t^*$  depends not only on the exogenous foreign interest rate  $if f_t$ , but also on a premium ( $\chi$ ), following Schmitt-Grohe-Urbe (2003), whereby holders of foreign debt are assumed to face an interest rate that is increasing in the country's relative net foreign debt:

$$i f f_t = i_t^* - \chi \frac{B_t - B}{e P_t^* Q^X} \quad (2.58)$$

The government sector is relatively simple, as the focus is on product and labor market regulations and not on the public sector. There is an exogenous stream of government consumption  $G_t$  which is purchased both from the home formal sector and abroad:

$$G_t = \left[ \alpha^{\frac{1}{\eta}} G_{H,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} G_{f,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (2.59)$$

Cost spent on government consumption goods is minimized, and this implies the usual demand functions:

$$G_{H,t} = \alpha G_t \left( \frac{P_{F,t}}{P_{INV,t}} \right)^{-\eta} \quad (2.60)$$

$$G_{f,t} = (1 - \alpha) G_t \left( \frac{P_{f,t}}{P_{INV,t}} \right)^{-\eta} \quad (2.61)$$

The price level of government consumption goods is the same as that of investment goods, and differs from that of household consumption goods, as the government does not buy informal goods.

Also, the government is responsible for financing social benefit expenditure to those who are not employed.

These government expenses are financed by a labor income tax – in the formal sector only – and a lump-sum tax. For simplicity's sake, we neglect consumption, capital and bond taxes. Also, we do not assume that the government has access to domestic or international bond markets.

So, the period-by-period government budget constraint is:

$$\frac{P_{F,t}}{P_t} G_t + W U_t U_t = T a x_t + \tau_{F,t} W F_t L_{F,t} \quad (2.62)$$

Then, we consider a Taylor-type monetary policy rule following Smets-Wouters (2007):

$$\frac{1 + i_t}{1 + i} = \left( \frac{1 + i_{t-1}}{1 + i} \right)^{\rho_i} \left[ \left( \frac{\pi_t}{\pi} \right)^{\rho_\pi} \left( \frac{GDP_t}{GDP} \right)^{\rho_{GDP}} \right]^{1-\rho_i} e^{\epsilon_{i,t}} \quad (2.63)$$

where  $\rho_i$  captures interest rate smoothing, and the central bank responds to current inflation and GDP.  $\epsilon_{i,t}$  is the monetary policy shock intended to capture unanticipated increases in the nominal interest rate.  $\rho_\pi$  and  $\rho_{GDP}$  are the weights on inflation and GDP, respectively.

Concerning market clearing, employment in the formal and in the informal sector adds up to total employment, which is one minus unemployment:

$$L_{F,t} + L_{I,t} = L_t \quad (2.64)$$

$$U_t = 1 - L_t \quad (2.65)$$

Formal and informal capital demand add up to total capital supplied by capital producers. Then, there is equilibrium in the home bond market.

$$K_{F,t} + K_{I,t} = K_t \quad (2.66)$$

$$D_t = Q_t K_t \quad (2.67)$$

The resource constraint for the formal sector is given by:

$$Y_{F,t} = QD_{F,t} + HC_{F,t}H_{F,t}\frac{P_t}{P_{F,t}} + N_{F,t}^E \text{entry}_{F,t}\frac{P_t}{P_{F,t}} + \frac{\phi_F}{2} \left( \frac{P_{F,t}(s)}{P_{F,t-1}(s)} \frac{1}{\pi} - 1 \right)^2 QD_{F,t}(s)N_{F,t}\frac{P_t}{P_{F,t}} \quad (2.68)$$

where  $QD_{F,t} = C_{F,t} + I_{H,t} + G_{H,t} + Q_t^X$ , which means that the total demand for formal goods is equal to the sum of home demand (demand by consumers, firms and the government) and export demand. This constraint means that the amount of goods produced in the formal sector is equal to the amount of formal goods demanded by the home and foreign economy. There are three deadweight losses, too: first, labor market frictions; second, entry cost; and third, the Rotemberg price adjustment cost.

A similar constraint holds for the informal sector as well:

$$Y_{I,t} + Y_{HP,t} = QD_{I,t} + HC_{I,t}H_{I,t}\frac{P_t}{P_{I,t}} + N_{I,t}^E \text{entry}_{I,t}\frac{P_t}{P_{I,t}} + \quad (2.69)$$

$$+ \frac{\phi_I}{2} \left( \frac{P_{I,t}(s)}{P_{I,t-1}(s)} \frac{1}{\pi} - 1 \right)^2 QD_{I,t}(s)N_{I,t}\frac{P_t}{P_{I,t}} \quad (2.70)$$

where  $QD_{I,t} = C_{I,t}$  is the total demand for informal goods.

The main difference between formal and informal constraints is that home production  $Y_{HP,t}$  enters the informal resource constraint. This is necessary to close the model, while the intuitive reason behind it is the fact that people also work at home. Here, we follow Cacciatore-Fiori (2016). Also, in line with the aforementioned paper, we assume that the value of home production is equal to a fraction  $\alpha_{HP}$  of the product of average wage and the number of unemployed people in each period:

$$Y_{HP,t} = \alpha_{HP} \frac{WF_t L_{F,t} + WI_t L_{I,t}}{L_t} U_t \quad (2.71)$$

This means that we assume that those who work at home (and for the sake of simplicity we equalize the number of unemployed to the number of working at home) “earn” the average economy-wide wage level.

Total import is a sum of imported consumption, investment and government goods:

$$Q_t^M = C_{f,t} + I_{f,t} + G_{f,t} \quad (2.72)$$

Finally, total home output and GDP are given by:

$$P_{H,t}Y_t = P_{F,t}Y_{F,t} + P_{I,t}Y_{I,t} \quad (2.73)$$

$$GDP_t + \frac{P_{f,t}}{P_t} Q_t^M = C_t + \frac{P_{INV,t}}{P_t} (I_t + G_t) + \frac{P_{F,t}}{P_t} Q_t^X \quad (2.74)$$

## 2.2.6 The role of informality and openness

As noted before, there are several differences between the formal and informal sectors (Table 2.1). First, the level of regulation, i.e. rigidities related to hiring, firing and entry, are lower in the shadow economy, in line with Williamson (1975). On the one hand, formal sector firms incur greater hiring costs than do firms in the informal sector. These costs can be associated with training to make up for educational or experiential deficits on the worker's part, but they might reflect as well administrative costs such as the time spent on hiring. Then too, the bargaining power of workers in wage setting is higher in the formal than in the informal sector. The strength of unions in the formal sector might be at play here, but this relative bargaining clout can also be related to the sector's legal environment, which provides more rights for workers than firms in setting wages. The strength of unions might also be reflected in the fact that the probability of firing is relatively higher in the informal sector. Furthermore, registering a new company is costly, in terms of both money and time. Hence, formal entry costs are larger than informal ones. Finally, price mark-up is higher and firm exit rate, similarly to the dismissal rate of workers, is lower in the formal sector than in the underground economy.

|                                      | Formal sector | Informal sector |
|--------------------------------------|---------------|-----------------|
| Entrepreneurs' labor hiring costs    | High          | Low             |
| Workers' bargaining power over wages | High          | Low             |
| Probability of firing workers        | Low           | High            |
| Retailers' entry costs               | High          | Low             |
| Retailers' price markup              | High          | Low             |
| Retailers' exit rate                 | Low           | High            |
| Taxation                             | Yes           | No              |
| Traded                               | Yes           | No              |
| Government consumption               | Yes           | No              |
| Physical capital production          | Yes           | No              |

Table 2.1: Differences between the formal and informal sectors

Alongside differences in regulation levels, other features serve to distinguish the two sectors. Notably, only formal sector's labor income falls under the taxation umbrella. Also, the government can only purchase formal goods, and, by virtue of administrative regulations, physical capital investment is made solely on the basis of formal goods. Finally, formal goods are traded abroad, but informal goods are not. This is likely explained by the fact that entering the foreign markets requires the meeting of certain legal obligations. The fact that only formal goods are traded abroad constitutes the main example in our model of interactions between the shadow economy and openness.

Due to the model's complexity, we are unable to examine analytically the mechanisms induced by the presence of informality and openness. Nevertheless, we provide simulation

results in three parametrized economies: a baseline open-economy model, an alternative closed-economy model and an another alternative model where the size of the shadow output is 1/3 of that of the baseline value.<sup>15</sup> Table 2.2 shows the long-run (25-year) effects of structural reforms; deterministic simulations were carried out by Dynare 4.4.3. The following reforms are studied: a 10 percent permanent and unexpected reduction in the formal i) hiring cost, ii) bargaining power of workers and iii) entry cost.<sup>16</sup>

First, in the long-run, macroeconomic impacts in an open-economy and in a closed-economy model are quite similar. The only exceptions are the labor market reactions of the bargaining power of workers policy.<sup>17</sup>

On the other hand, the size of the shadow economy crucially influences the reactions of the macroeconomy. This is especially true for labor market policies, but also responses of product market deregulation are affected. Notably, GDP goes up by 1 or 1.8 per cent in the long-run, if hiring costs or bargaining power of workers in the formal sector are lowered, respectively; while the same effects are 1.7 and 2.8 per cent if the size of the shadow economy is 1/3 of the original level. As regards unemployment, the presence of informality is not less important. A 10 per cent reduction in hiring costs implies a 1.7 pp drop in the rate of unemployment with low informality, while in the baseline scenario unemployment only goes down by 1.2 pp. Also, a bargaining power policy induces 3.3 and 2.4 pp decreases in unemployment, respectively. Similarly, formal employment is also essentially affected, for instance a bargaining power policy induces a 3.2 pp drop in formal employment with low informality, but only a 2 pp drop with high informality.

The main reason is that the lower the size of the shadow economy, the larger the share of the economy directly affected by structural reforms; and, this outweighs other channels in the model. Thus, in a model that does not (or does not fully) incorporate the underground sector, macroeconomic impacts seem to be larger larger than they really are. Because the model behaves as if some part of the macroeconomy was formal, even though it is informal.

---

<sup>15</sup>Tables 2.A1 and 2.A2 in the Appendix show the steady-state and dynamic parameters of the three parametrized economies.

<sup>16</sup>Fiscal consolidation policies are easy to compare in the sense that all of them can be expressed in GDP size. The size of deregulation policies, however, is less straightforward to compare. A 10 percent decrease in hiring cost means that the hiring cost is 10 percent lower compared to wages. A 10 percent decrease in workers' bargaining power means that workers have 10 percent less power in wage setting. While entry cost is typically calculated as a share in production. Hence, in terms of GDP, the sizes of these shocks might differ. The same note holds for Section 2.4.

<sup>17</sup>The model is a small open-economy model with a simple trade sector that largely depends on exogenous processes. In other open-economy models openness might be more relevant, but it is beyond the scope of this paper to further investigate this issue.



| Long-run effects of lowering the formal |                 | GDP, %           | Unemployment, %point      | Formal employment, %point   | Informal employment, %point | Formal wage, % |
|---|-----------------|------------------|---------------------------|-----------------------------|-----------------------------|----------------|
| hiring cost                             | Baseline        | 1.0              | -1.2                      | 1.0                         | 0.1                         | -0.4           |
|   | Low informality | 1.7              | -1.7                      | 1.7                         | 0.0                         | -0.2           |
| bargaining power of workers             | Baseline        | 1.8              | -2.4                      | 2.0                         | 0.4                         | -1.5           |
|   | Low informality | 2.8              | -3.3                      | 3.2                         | 0.0                         | -1.4           |
| entry cost                              | Baseline        | 0.5              | -0.2                      | 0.2                         | 0.0                         | 0.4            |
|   | Low informality | 0.8              | -0.4                      | 0.4                         | 0.0                         | 0.5            |
| Long-run effects of lowering the formal |                 | Informal wage, % | Number of formal firms, % | Number of informal firms, % | Net exports, %point         | Investment, %  |
| hiring cost                             | Baseline        | 0.2              | 0.6                       | 0.3                         | 0.3                         | -0.1           |
|   | Low informality | 0.5              | 0.9                       | 0.2                         | 0.4                         | -0.1           |
| bargaining power of workers             | Baseline        | 0.0              | 1.0                       | 0.7                         | 0.4                         | 0.4            |
|   | Low informality | 0.8              | 1.5                       | 0.3                         | 0.5                         | 0.4            |
| entry cost                              | Baseline        | 0.4              | 5.7                       | 0.2                         | 0.1                         | 0.2            |
|   | Low informality | 0.2              | 5.8                       | 0.2                         | 0.1                         | 0.2            |

| Long-run effects of lowering the formal |                | GDP, %           | Unemployment, %point      | Formal employment, %point   | Informal employment, %point | Formal wage, % |
|---|----------------|------------------|---------------------------|-----------------------------|-----------------------------|----------------|
| hiring cost                             | Baseline       | 1.0              | -1.2                      | 1.0                         | 0.1                         | -0.4           |
|   | Closed economy | 1.1              | -1.0                      | 0.8                         | 0.2                         | -0.3           |
| bargaining power of workers             | Baseline       | 1.8              | -2.4                      | 2.0                         | 0.4                         | -1.5           |
|   | Closed economy | 1.9              | -2.0                      | 1.5                         | 0.6                         | -1.2           |
| entry cost                              | Baseline       | 0.5              | -0.2                      | 0.2                         | 0.0                         | 0.4            |
|   | Closed economy | 0.4              | -0.1                      | 0.2                         | 0.0                         | 0.3            |
| Long-run effects of lowering the formal |                | Informal wage, % | Number of formal firms, % | Number of informal firms, % | Net exports, %point         | Investment, %  |
| hiring cost                             | Baseline       | 0.2              | 0.6                       | 0.3                         | 0.3                         | -0.1           |
|   | Closed economy | 0.3              | 0.7                       | 0.4                         | -                           | 0.0            |
| bargaining power of workers             | Baseline       | 0.0              | 1.0                       | 0.7                         | 0.4                         | 0.4            |
|   | Closed economy | 0.3              | 1.1                       | 0.7                         | -                           | 0.4            |
| entry cost                              | Baseline       | 0.4              | 5.7                       | 0.2                         | 0.1                         | 0.2            |
|   | Closed economy | 0.3              | 5.7                       | 0.1                         | -                           | 0.2            |

Table 2.2: The role of shadow economy and openness in long-run (25-year) effects of labor and product market deregulation policies. All deregulation policies are implemented in the formal sector, and mean a permanent and unexpected 10 percent reform size. Low informality means that the size of shadow economy in output is 1/3 of the baseline value.

Short-run dynamics are also influenced by the presence of informality; impulse responses with low informality are about 0.5-1 pp higher (Figures 2.A1 and 2.A2 in the Appendix). Additionally, labor market reforms do not raise informal employment; while informal wages go up instead of down if the bargaining power of workers is lowered. Also, inflation goes up more with low informality. Then, as regards deregulating the product market, in the baseline scenario GDP increases immediately after the shock, with low informality, however, during the first couple of quarters it decreases. It means that there is a severe cost during the transition. It happens because, on the one hand, investment declines, on the other hand, unemployment drops more than before. So, in the short-run, the size of the shadow economy also crucially affects the signs of the reactions of the macroeconomy.

On the whole, we can conclude that informality is a crucial determinant of the sign, and, in particular, the magnitude of macroeconomic responses to structural reforms.

## 2.3 Calibration and Bayesian estimation

In this section, we describe the calibration of the steady-state parameters and the Bayesian estimation of the dynamic parameters. We consider quarterly data on South Africa, which is a small open-economy and serves as an example of the emerging world where the underground economy plays, in general, a stronger role than it does in developed countries. When calibrating the steady state, our main goal is to match, as accurately as possible, data on unemployment, shadow employment, informality in GDP and openness.

First, according to the Labor Force Survey (LFS) data of the South African Reserve Bank, unemployment is pretty stable; during the last two decades it hovered around 28.7 percent, with a low dispersion around this mean (Table 2.3).<sup>18</sup> As regards formal and informal employment, the available empirical information is mixed. According to the University of Stellenbosch, the share of informal employment in the South African labor force is fairly low (15.8 percent). At the same time, ILO (2002) reports that informality in the labor market was 34 percent in 2000. Finally, DTI (2008) claims that the fraction of informal employment was on average 18.3 percent between 1996 and 2007. In terms of GDP, the size of the shadow economy is close to the value reported by ILO, namely, it is 27.3 percent (Schneider et al. (2010)).

| Variable (%)                                     | Data | Data source and year                      | Model |
|--|------|---|-------|
| Private investment as a share of GDP             | 17.8 | South African Reserve Bank, 2000Q3-2012Q2 | 22.6  |
| Openness as a share of GDP                       | 54.3 |   | 59.2  |
| Unemployment rate                                | 28.7 |   | 25.5  |
| Shadow economy as a share of GDP                 | 27.3 | Schneider et al. (2010, Table 2)          | 27.2  |
| Shadow employment as a share of total employment | 15.8 | University of Stellenbosch, 1996-2012     | 37.2  |
|  | 34.0 | International Labor Organization (2012)   |       |
|  | 18.3 | Department of Trade and Industry (2008)   |       |

Table 2.3: Evaluating the steady state of the model

First of all, we aim to match the size of the shadow economy in GDP, and the model does a very good job at it. Regarding shadow employment, its share in total employment is closest to that of ILO (2002). At the same time, unemployment is slightly underestimated and openness is slightly overestimated.<sup>19</sup> As a cross-check, we also have a look at investment to GDP ratio (the only big ratio which is not fixed during calibration); which is slightly higher in the model than in the data.

<sup>18</sup>In LFS, by design, those who are unemployed do not work in parallel (not even in the shadow economy). Hence, there is no significant overlap between the number of unemployed and the number of informally employed people in the survey.

<sup>19</sup>The Appendix shows evidence on the robustness of the long-run macroeconomic impacts of structural reforms with respect to the assumptions we made during calibrating the steady state of the model.

Now, we turn to describing the process of calibration; Table 2.4 provides an overview of it.<sup>20</sup>

First, we estimate sectoral firing probabilities using LFS data on the number of job losers and the number of new entrants among the unemployed, as well as data on formal and informal employment published by the Statistics of South Africa (SSA). Based on our own estimates, the probability of firing is 14.5 percent in the formal sector and 84.8 percent in the informal sector.

Regrettably, we are not aware of any empirical information on hiring costs. Nevertheless, the Global Competitiveness Report of the World Economic Forum provides data on firing costs; since 2006 redundancy costs were 21.9 weeks of salary on average. This means that the ratio of hiring costs to quarterly wages was 1.7. Assuming that hiring costs are at a similar magnitude, we use this ratio to calibrate them compared to wages in the formal sector. In light of the dearth of data regarding the informal sector, we assume that the fraction of hiring costs to wages is half as much in the informal as in the formal sector.

Furthermore, information regarding bargaining power of workers in South Africa is also unknown to us. In the literature, it is usually between 0.3 and 0.5 (Mortensen-Nagypal (2007)). We assume that workers in the formal sector have more bargaining power in wage setting than do workers in the informal sector. Thus, we set the formal bargaining power of workers to 0.6 and the relevant informal value to 0.4 to match the characteristics of the labor market as accurately as possible.

Regarding sectoral bankruptcy rates, we use data on the number of new and total firms; datasources are i) the Companies and Intellectual Property Commission's dataset and ii) SSA (2006), SSA (2010) and SSA (2014). Based on these datasets, we estimate the bankruptcy rates to be 10.9 percent in the formal sector and 17.3 percent in the informal sector.

As regards the level of entry costs, we are not aware of data for South Africa, either. However, Cacciatore-Fiori (2016) claim that, in the euro area, the entry cost of new companies is equal to about 2.1 months of yearly output. We use this value as a proxy to calibrate the steady-state level of formal entry costs, while we assume that the entry cost to output ratio in the informal sector is half of the relevant formal sector's value.

---

<sup>20</sup>In this section we discuss parameters related to regulation, informality and openness, but Table 2.4 shows all parameter values.

| Name of fixed or calibrated parameter or variable              | Sign of         | Value  | Source  | Year          |
|--|-----------------|--------|---|---------------|
| Discount factor  | $\beta$         | 0.99   |   |               |
| Physical capital depreciation rate                             | $\delta$        | 0.025  |   |               |
| Formal capital income share                                    | $\psi_F$        | 0.3    |   |               |
| Informal capital income share                                  | $\psi_I$        | 0.2    |   |               |
| Markup in formal sector (%)                                    | $\varepsilon_F$ | 10     |   |               |
| Markup in informal sector (%)                                  | $\varepsilon_I$ | 5      |   |               |
| Formal productivity level                                      | $\theta_F$      | 1      |   |               |
| Informal productivity level                                    | $\theta_I$      | 1      |   |               |
| Formality bias   | $\mu_F$         | 0.5    |   |               |
| Home inflation (% QoQ)   | $\pi$           | 1.26   | Statistics South Africa   | 2000Q3-2012Q2 |
| Exchange rate depreciation rate (% QoQ)                        |                 | 0.19   | South African Reserve Bank  | 2000Q3-2012Q2 |
| Exit rate of formal retailers                                  | $\delta_F$      | 0.1093 | Authors' calculation based on data of Companies and Intellectual Property Commission and Statistics South Africa (2006, | 2000-2012     |
| Exit rate of informal retailers                                | $\delta_I$      | 0.1728 |   |               |
| Household consumption expenditure to GDP ratio (%)             |                 | 63.97  | South African Reserve Bank  | 2000Q3-2012Q2 |
| Government consumption expenditure to GDP ratio (%)            |                 | 19.80  | South African Reserve Bank  | 2000Q3-2012Q2 |
| Exports to GDP ratio (%)                                       | $a_X$           | 26.40  | South African Reserve Bank  | 2000Q3-2012Q2 |
| Public social protection to GDP ratio (%)                      |                 | 5.06   | International Labor Organization  | 1995-2005     |
| Firing probability in the formal sector                        | $\text{prob}_F$ | 0.1446 | Authors' calculation based on LFS data of Statistics South Africa   | 2008Q1-2012Q4 |
| Firing probability in the informal sector                      | $\text{prob}_I$ | 0.848  |   |               |
| Ratio of hiring cost in wage in formal sector                  |                 | 1.6849 | World Economic Forum's Global Competitiveness Index Historical Dataset  | 2006-2013     |
| Ratio of hiring cost in wage in informal sector                |                 | 0.8425 |   |               |
| Elasticity of substitution between home and foreign goods      | $\mu$           | 0.6    | Steinbach et al (2009)  |               |
| Home bias  | $\alpha$        | 0.8    | Steinbach et al (2009)  |               |
| Export price elasticity  | $\zeta_X$       | 4.5    | Behar-Edwards (2004)  |               |
| Share of income tax revenue in GDP (%)                         | $\tau_F$        | 9      | Badaoui-Magnani (2013)  |               |
| Exchange rate pass-through to import prices                    | $\zeta_M$       | 0.812  | Karoro et al (2008)   |               |
| Entry cost in the formal sector (in months of production)      |                 | 2.07   |   |               |
| Entry cost in the informal sector (in months of production)    |                 | 1.04   | Cacciatore-Fiori (2016)   |               |
| Share of home production in average wage                       |                 | 25     | Cacciatore-Fiori (2016)   |               |
| Elasticity of hiring cost wrt to hiring probability (formal)   | $\alpha_{HCF}$  | 0.5    |   |               |
| Elasticity of hiring cost wrt to hiring probability (informal) | $\alpha_{HCI}$  | 0.5    |   |               |
| Elasticity of substitution between formal and informal goods   | $\eta$          | 1.5    |   |               |
| Formal bargaining power of workers                             | $\lambda_F$     | 0.6    |   |               |
| Informal bargaining power of workers                           | $\lambda_I$     | 0.4    |   |               |

Table 2.4: Calibration of steady-state parameters

In the absence of data, the formality bias, i.e. the share of formal goods in household and government consumption, is normalized to 50 percent. Furthermore, we assume that the elasticity of substitution between formal and informal goods is 1.5, which is in the range of usual values of the literature. We do not set this elasticity to one, because we believe that when purchasing informal goods personal relations between sellers and buyers also matter (but we check the robustness of our results with a unit elasticity).

Finally, concerning openness, home bias is 0.8, which means that the share of imported goods is 20 percent, and the elasticity of substitution between home- and foreign-produced goods is 0.6. Both values follow Steinbach et al. (2009). Then, export price elasticity is 4.5, in line with Behar-Edwards (2004), and the exchange rate pass-through to import prices is slightly above 0.8 according to Karoro et al. (2008).

Now, we turn to presenting the dynamic structural parameters, which are Bayesian estimated using Dynare 4.4.3. Priors and posteriors are shown in Table 2.5.<sup>21</sup>

Bayesian estimation was done by using the following quarterly time series; the time span was between 2000Q3 and 2012Q2 (data sources are in parenthesis):

- gross domestic product (International Monetary Fund, IMF)
- household consumption expenditure (South African Reserve Bank, SARB)
- government consumption expenditure (SARB)
- private investment (SARB)
- exports of goods and services (SARB)
- imports of goods and services (SARB)
- real exchange rate (SARB)
- consumer price index (SSA)
- treasury bill rate (IMF)
- compensation of employees in the formal sector (SARB)
- formal and informal employment (SSA and own estimates)

---

<sup>21</sup>Also, see Figure 2.A3 in the Appendix. Because we run deterministic simulations, we do not show priors and posteriors of autocorrelations and standard deviations of shocks here, however, they are available in Tables 2.A8-2.A9 in Appendix A. The priors of the Rotemberg price adjustment costs are based on the priors of Calvo price rigidities (equal to 50 percent) and the levels of price mark-ups, following Lombardo-Vestin (2007).

| Structural parameters              | distribution  | prior |                    | posterior |                     |       |
|------------------------------------|---------------|-------|--------------------|-----------|---------------------|-------|
|                                    |               | mean  | standard deviation | mean      | confidence interval |       |
| $h$ consumption habit              | beta          | 0.7   | 0.2                | 0.89      | 0.89                | 0.90  |
| $\gamma_i$ interest rate smoothing | beta          | 0.7   | 0.2                | 0.16      | 0.04                | 0.28  |
| $\gamma_\pi$ inflation response    | gamma         | 2     | 0.5                | 3.05      | 2.42                | 3.64  |
| $\gamma_Y$ output response         | gamma         | 0.1   | 0.1                | 0.35      | 0.24                | 0.48  |
| $dF$ formal Rotemberg cost         | gamma         | 19.80 | 10                 | 1.80      | 0.79                | 2.77  |
| $dI$ informal Rotemberg cost       | gamma         | 36.60 | 20                 | 5.05      | 1.73                | 8.78  |
| $\chi$ interest rate premium       | inverse gamma | 0.002 | 0.001              | 0.002     | 0.001               | 0.002 |
| $\zeta$ investment adjustment cost | gamma         | 0.5   | 0.2                | 0.60      | 0.37                | 0.84  |

Table 2.5: Prior and posterior distributions of structural dynamic parameters. The priors of consumption habit and interest rate smoothing are maximized to 0.9, while the prior of interest rate premium is maximized to 0.005.

Three caveats apply. First, all national accounts' time series are expressed in real terms, in 2005 prices and in South African RAND. Next, all time series are seasonally adjusted and divided by the number of population (if necessary). Finally, all time series are official data, except formal and informal employment, which are estimated by the authors before 2008Q2.<sup>22</sup>

## 2.4 Long- and short-run effects of deregulation policies in South-Africa

Permanent and unexpected reforms are carried out in the labor and product markets; and, we do not only study their long-run (25-year) impacts, but also the transition (first 5 years) between the pre-reform and the post-reform equilibria. All shocks are implemented in the formal sector, while in the papers without informality policies are defined at the aggregate level. Regarding the size of the reforms, the level of regulation (hiring cost, bargaining power of workers and entry cost) is reduced by 10 percent.<sup>23</sup> First, single policies are studied, then reform packages are investigated to see to what extent and in which parts of the economy they can mitigate short-term costs. Finally, we focus on the role of policy sequence, with a special interest in the speed of adjustment. Deterministic simulations were carried out by Dynare 4.4.3.

<sup>22</sup>Quarterly formal and informal employment data published by SSA is available since 2008Q2. First, based on yearly data, ordinary least-squares regressions of formal and informal employment (data source is DTI) on GDP, (formal) compensation of employees and inflation are estimated. Then, using these coefficients and quarterly data on GDP, (formal) compensation of employees and inflation between 2003Q3 and 2012Q2 and on formal and informal employment between 2008Q2 and 2012Q2, we estimate quarterly formal and informal employment for 2003Q3 and 2008Q1.

<sup>23</sup>See footnote 16.

### 2.4.1 Long-run effects of single reforms

In the long-run, GDP increases in the wake of all deregulation policies (Table 2.6). In particular, permanently lowering the hiring cost in the formal sector by 10 percent results in a 2.9 percent increase in GDP 25 years post-policy implementation. A reduction in the formal bargaining power of workers increases GDP by 3.3 percent, while decreasing the formal entry cost implies a 1.1 percent rise. Regarding unemployment, lowering the hiring cost, the bargaining power of workers and the entry cost in the formal sector brings about a 2.8, 4.6 and 0.5 pp decline in the rate of unemployment, respectively. Hence, product market deregulation seems to be somewhat less efficient at lowering unemployment.

Furthermore, while all policies increase both formal and informal employment, formal employment increases much more. That is to say, previously unemployed people begin working in the formal economy. The level of underground employment is largely unaffected, though. However, as a consequence, its share still considerably drops. Also, formality in the labor market increases more with a labor market, as compared to a product market, reform.

In terms of wage reactions, not only the magnitude of the responses but also the signs differ among the policies. Notably, a labor market reform decreases formal wages - wages decline more if the bargaining power of workers goes down -, while a product market reform implies the opposite. Informal wages always go up, but they go up less after a product market reform. Similarly, the level of competition (number of firms) also reacts differently; it increases in both sectors, but more in the formal sector, especially if the product market is liberalized.

Regarding the intuition, the starting point is the reaction of wages. A lower formal bargaining power of workers results in a lower formal wage, because firms have more power to control wages. Similarly, when formal hiring costs go down, firms want to cut down wages, too; for example because a lower hiring cost means that it is less important to keep the workers today. As regards reforming the product market, formal wages go up; larger demand and larger production requires to attract new workers. Informal wages always go up; workers tend to move out of informality – although to a small extent – and firms try to keep them this way. The employment reaction is, obviously, strongly related to that of wages. Notably, unemployment decreases more after a labor market reform, consistently with the decline in formal wages there. Reducing hiring costs in the formal sector not only reduces the relative cost of labor to capital, but it also encourages workers to move to the more productive formal sector. The latter is less true when formal entry costs are lowered.

As both reforms are effective in decreasing unemployment, both reforms are suitable to increase competition. Still, as a labor market reform is more successful in lowering unem-

| Long-run effects<br>when lowering<br>the ... | GDP, %              | Unemployment,<br>%point      | Formal<br>employment,<br>%point   | Informal<br>employment,<br>%point | Formal<br>wage, % |
|--|---------------------|------------------------------|-----------------------------------|-----------------------------------|-------------------|
| hiring cost                                  | 2.9                 | -2.8                         | 2.2                               | 0.6                               | -0.9              |
| bargaining<br>power of                       | 3.3                 | -4.6                         | 3.8                               | 0.8                               | -5.3              |
| entry cost                                   | 1.1                 | -0.5                         | 0.3                               | 0.2                               | 0.8               |
| Long-run effects<br>when lowering<br>the ... | Informal<br>wage, % | Number of<br>formal firms, % | Number of<br>informal<br>firms, % | Net exports,<br>%point            | Investment,<br>%  |
| hiring cost                                  | 1.8                 | 1.6                          | 1.1                               | 0.8                               | 2.9               |
| bargaining<br>power of                       | 2.5                 | 1.8                          | 1.1                               | 0.5                               | 5.4               |
| entry cost                                   | 0.5                 | 6.0                          | 0.5                               | 0.2                               | 1.3               |

Table 2.6: Long-run (25-year) effects of single structural reforms in South Africa. All deregulation policies are implemented in the formal sector, and mean a permanent and unexpected 10 percent reform size.

ployment because it directly affects the labor market (and only indirectly other markets), a product market reform is more successful in achieving higher competition because it is carried out in the product market.

Finally, as formal firms can have access to foreign markets, a decrease in the size of the shadow economy results in a positive response of net exports, too, and it is true for any policy we consider.

Our conclusions sometimes converge with and sometimes diverge from those found in the literature.<sup>24</sup> First, we find that a product market reform seems to be less successful in increasing employment. This is consistent with Charlot et al. (2011), but in contrary to Cacciatore et al. (2015) and Lusinyan-Muir (2013). The reason behind this might be that the latter do not take informality into account, hence, the fact that after a labor market reform workers move to the more productive formal sector is not considered; in particular, Lusinyan-Muir (2013) find a sizeable increase in wages when competition increases, which we do not find.

Our findings are in line with Charlot et al. (2011) in the sense that they claim that both deregulations imply a decrease in the size of the informal sector and unemployment. However, contrary to them we cannot generally conclude that a labor market deregulation has a negative effect on wages. This is only true for formal wages, while informal wages

<sup>24</sup>Importantly, comparisons are not always straightforward, because different policies, or the same policies but with different dimensions, are implemented.



always go up. At the same time, we find that product market deregulation has a positive impact on wages in both sectors.

### 2.4.2 Short-run effects of single reforms

While there are considerable long-term gains of deregulation in the form of an increase in output or a decrease in unemployment, also, shadow employment becomes less significant, short-term costs emerge during the transition (Figure 2.4).

First, GDP declines during the first year when the bargaining power of workers is lowered, while it stagnates with a hiring cost decrease. Nevertheless, thanks to a greater increase in formal employment, unemployment goes a bit more down if the bargaining power of workers is decreased. Also, shadow employment goes down after decreasing the bargaining power of workers, but it goes up if the hiring cost is cut down. Furthermore, net exports go up if the bargaining power of workers declines, but they go down if the cost of hiring declines. Also, less inflationary pressure accompanies a hiring cost policy, while formal wages decrease much more with a decline in workers' bargaining power. The competition response is also different, that is, the number of formal firms goes down first if the bargaining power of workers goes down, but otherwise it stagnates.

The intuition behind the decline and stagnation of output after the shock – in spite of the inevitable long-run gain – is that formal wages decline more in the short-term than in the long-run following the bargaining power reform, while both labor market reforms imply a smaller increase in informal wages right after the shock compared to a later point in the transition path. Hence, demand starts to increase with a lag, namely, both investment and consumption decline or increase less at the beginning. Also, it takes some time until new firms enter the market.

As regards reforming the product market, GDP declines in the first two quarters after the shock. Unemployment immediately starts to go down, though less than with a labor market reform. As with the hiring cost policy, not only formal employment, but also unofficial employment goes up. However, informal workers are less productive, hence, it negatively affects output. Although net exports increase in the long run, there is a fall in the short run. Both sectoral wages increase. The number of firms in the informal sector first declines, then starts to rise, while the number of formal firms rises considerably immediately upon policy implementation.

Compared to the labor market reforms, the speed of adjustment is larger. In particular, the level of output is already close to the post-reform level one year after the shock (it takes about two or four years to achieve the same with a labor market reform). The reason is

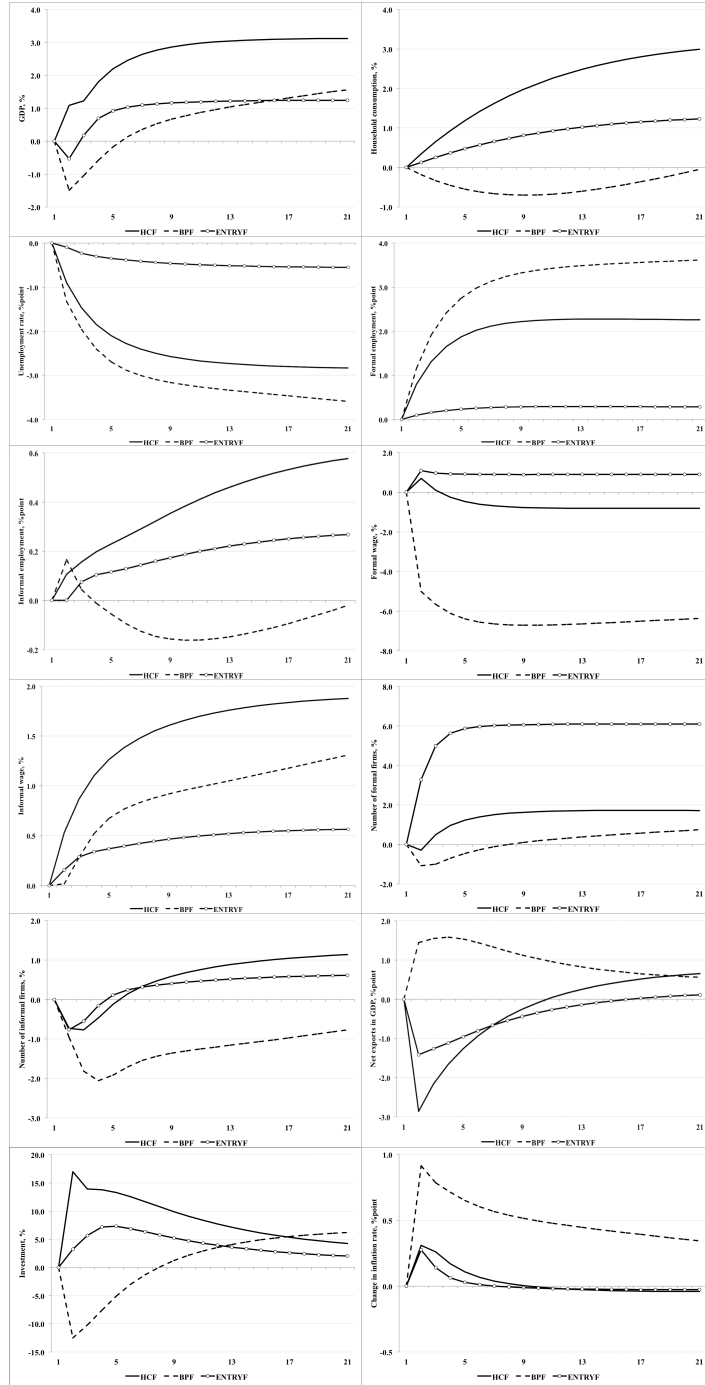


Figure 2.4: Short-run effects of single labor and product market reforms in South Africa. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal hiring cost (HCF), formal bargaining power of workers (BPF) and formal entry cost (ENTRYF).

that competition in the formal sector increases quite rapidly when the market entry barriers are reduced. In connection with this, wages also adjust quickly. Still, because entering the market takes some time, on the whole, there is a small decline in GDP after the shock.

Again, our results do not always coincide with those in the literature. Cacciatore et al. (2015) claim that short-run costs last longer with a product market reform than with a labor market reform. We find that GDP declines in the first two quarters after a product market reform, but if the bargaining power of workers is lowered, GDP decreases in the first four quarters. Also, there are other costs, such as the decline in household consumption and investment following a decrease in the bargaining power of workers, or a fall in net exports when the entry cost or the hiring cost is lowered. Thus, we can conclude that all reforms entail short-term costs, and that the diffusion (how many agents, sectors, etc. are affected), the magnitude and the duration of these negative effects are not always greater with one reform than with another one.

### 2.4.3 Reform packages

We showed that some reforms have short-run costs, while others do not; also, some workers or firms gain, while others lose. Thus, combining reforms is a potentially useful tool to mitigate overall short-term costs, at least in some cases (Table 2.A5 and Figures 2.A4-2.A7 in the Appendix).<sup>25</sup> We study the following combinations: i) a decrease in hiring cost and bargaining power of workers, ii) a decrease in hiring cost and entry cost, iii) a decrease in bargaining power of workers and entry cost, and iv) a combination of all three single policies.

When combining the hiring cost policy with the bargaining power of workers policy, output does not decrease during the first year compared to the single bargaining power of workers policy. This is also true for household consumption and investment. At the same time, net exports, formal wages and the number of firms in the formal sector still go down.

Then, if not only the formal hiring cost is lowered, but also the formal entry cost, we can avoid a decrease in GDP in the short-run. Also, if we both decrease the bargaining power of workers and the formal entry cost, neither household consumption nor net exports fall.

In line with Cacciatore et al. (2015), we can conclude that parallel reforms might mitigate short-run costs. Still, mixed policies might be costly too, depending on which policies we combine or which variables we consider.

---

<sup>25</sup>Combining reforms can also reinforce the positive impacts of single policies, but here we focus on mitigating the negative effects.

#### 2.4.4 Policy sequence

If several policies are parallel-implemented, the importance of policy sequence becomes a vital question (Figures 2.A8-2.A11 in the Appendix). Does the choice of reform sequence determine the transition path? In an effort to answer this crucial query, we will discuss now how policy sequencing affects adjustment, with a special interest in the speed of adjustment.

The first policy package we study is a decrease both in bargaining power of workers and in hiring cost. If first the hiring cost is decreased and afterward the bargaining power of workers is decreased, output goes up faster in the first half of the transition, and then it increases gradually. If the opposite order is chosen, the speed of output convergence is the opposite, too. However, if first the level of unionization is lowered, formal employment response results in unemployment falling more quickly and staying at lower levels for a longer time. Regarding underground employment, if we start with a workers' bargaining power reform, informal employment is lower in the first half of the transition and higher later, compared to starting with the hiring cost policy. Not only the speed of adjustment is affected by the sequence of policies, but sometimes the signs are as well. Particularly, net exports decrease after a decrease in hiring cost, but increase after a decrease in bargaining power of workers. A drawback of the bargaining power policy is, however, that wages in the formal sector quickly decline and do not recover, while inflation pressure is also higher. Hence, on the whole, no particular labor market policy seems better than another.

If a decrease in workers' bargaining power is combined with a decrease in entry cost, not only for unemployment, but also for output it is better to start with the bargaining power policy. Under these circumstances, output goes up faster and stays at high levels, while unemployment goes down faster and stays at low levels; once again, the latter can be explained mainly by the reaction of formal employment. As in the hiring cost policy, net exports decrease with a product market reform, while they increase otherwise. However, to start with a bargaining power policy is quite controversial; formal wages quickly and permanently decline, while informal wages soar. Regarding product market competition, to induce a considerable increase in the number of firms, the entry cost must be cut.

Similarly, as regards a combination of lowering hiring and entry costs, it seems to be more efficient to start with the hiring cost policy, as GDP immediately and permanently increases, while unemployment decreases accordingly. Now, the latter effect is due not only to the reaction of formal employment, but is related as well to the response of unofficial employment. Net exports fall regardless of policy-implementation sequence, although they decrease less after a product market reform. As regards wages, formal wages increase after the product market reform, but they go down if the hiring cost is lowered; informal wages always increase, but more if we reduce the hiring costs first. Again, product market reform

is much more successful than labor market reform in increasing market competition.

Finally, we can also conclude that it is better to start with a labor market reform, if we have a look at a mix all three policies. First, output rises faster and remains at higher levels during most of the transition path. Also, unemployment falls more quickly and steadily. Regarding net exports, there is always a decrease in net exports, but the recovery is quicker. A drawback, however, is that formal wages decline; and competition in the formal economy increases more with a product market reform.

Generally, we do not concur with Blanchard-Giavazzi (2003), who claim that it is better to start with a product market reform because they find that it increases wages, while a labor market reform decreases them. On the one hand, if there is informality in the model, this claim is only true for formal wages, as informal wages always go up after deregulation. On the other hand, short-run costs do not only emerge in connection with wages. Output, consumption, net exports are also significantly affected, and we can not conclude that starting with a product market reform is less painful for the economy in all aspects.

On the contrary, we believe that it is usually better to start with a labor market reform. This is true for output, unemployment and several other aspects of the macroeconomy, with the exception of market competition and formal wages. Beginning with a product market reform typically results in a later starting point in the transition for increase in output and decrease in unemployment, and they stay at a lower and higher level for a longer time, respectively.

## 2.5 Conclusion

In this paper we took up the topic of structural reforms, and we investigated their macroeconomic impacts, in particular on output and employment; the main innovation was the incorporation of the shadow economy into an open-economy framework. Both the long-run effects and the transition path towards the new long-run equilibrium were studied. As well, we examined both single and mixed reforms, and also the sequence of the mixed reforms. Neither mixed policies, nor the importance of implementation order, has been much addressed in the literature. We considered South Africa as an example of emerging countries.

We found that the informal economy is a crucial determinant of the sign and, in particular, the magnitude of macroeconomic effects of deregulation policies. Also, in the long run, we showed that both labor and product market reforms considerably increase output, although labor market reforms are more successful in decreasing unemployment. Then, while the level of underground employment does not decrease in the long run, its share in total employment

does. Nevertheless, there are short-term costs, for example, decreasing household consumption, investment, net exports or output, or a decrease in competition. Combining structural reforms often mitigates the short-run costs, though, which accompany all single reforms we studied. Finally, we found that it is usually better to start with a labor market reform than with a product market reform.

# Appendix

## A Steady state of the model

Driven by the complexity of our model as well as the lack of information regarding the relative price levels of goods produced in the formal and informal sectors, we calculate the steady state numerically.

First, we choose initial values for  $L_F$ ,  $L_I$ ,  $PIPFs$ ,  $PfPH$ ,  $N_F$  and  $N_I$  (see Notations). Then, we fix  $\epsilon_F$ ,  $\epsilon_I$ ,  $\pi$ ,  $DEP$ ,  $\Theta_F$ ,  $\Theta_I$ ,  $\lambda_F$ ,  $\lambda_I$ ,  $probF_F$  and  $probF_I$  (see Section 2.3 of the paper). Finally, the steady state is calculated by Matlab, using function *fsolve* following the later-listed steps. Six equations' error terms are minimized: i) formal sector's first-order condition of bargaining, ii) informal sector's first-order condition of bargaining, iii) formal sector's demand equation, iv) foreign market clearing equation, v) free-entry condition in the formal sector and vi) free-entry condition in the informal sector.

After calibrating the steady state, the values of the exogenous variables are fixed to their calibrated values, and modified during deterministic simulations:  $\Theta_F$ ,  $\Theta_I$ ,  $\tau_F$ ,  $WU$ ,  $G$ ,  $\zeta$ ,  $\beta_{HCF}$ ,  $\beta_{HCI}$ ,  $probF_F$ ,  $probF_I$ ,  $\lambda_F$ ,  $\lambda_I$ ,  $entry_F$ ,  $entry_I$ ,  $\pi^*$ ,  $1^*$  and  $\epsilon_i$ . Also, some calibrated structural parameter values ( $\alpha_F$ ,  $\alpha_I$  and  $\alpha_X$ ) are also fixed during simulations.

At the end, we provide some evidence on the model's robustness. First, we study the long-run impacts of structural reforms if the steady-state informal bargaining power of workers is reduced to 0.3 or increased to 0.5 from the original 0.4. Then, the baseline steady-state ratio of informal hiring costs and entry costs to the relevant formal value is 50 per cent; first, we increase it to 2/3, then we lower it to 40 per cent. Finally, the original elasticity of substitution between formal and informal goods is 1.5, we show the long-run effects with 1 and 2 as well.

We can conclude that the macroeconomic impacts of reforming the product market are highly robust with respect to any changes. Nevertheless, the reactions of the labor market reforms are, for some variables, affected by the level of bargaining power of workers. To a less extent, the same is true for hiring costs. Lastly, variables in the informal sector are a bit influenced by the elasticity of substitution, but on the whole the results are robust.

### Steps of calculating the model's steady state:

$$L = L_F + L_I \quad (2.75)$$

$$U = 1 - L \quad (2.76)$$

$$N_F^E = \frac{\delta_F}{1 - \delta_F} N_F \quad (2.77)$$

$$N_I^E = \frac{\delta_I}{1 - \delta_I} N_I \quad (2.78)$$

$$\pi_H = \pi \quad (2.79)$$

$$\pi_f = \pi \quad (2.80)$$

$$\pi^* = \frac{\pi}{DEP} \quad (2.81)$$

$$i^* = \frac{\pi^*}{\beta} - 1 \quad (2.82)$$

$$\pi_X = \pi^* \quad (2.83)$$

$$\pi_F = \pi_H \quad (2.84)$$

$$\pi_I = \pi_H \quad (2.85)$$

$$iff = i^* \quad (2.86)$$

$$PHP = \frac{1}{(\alpha + (1 - \alpha)PfPH^{1-\eta})^{\frac{1}{1-\eta}}} \quad (2.87)$$

$$PfPP = \frac{1}{(\alpha PfPH^{\eta-1} + (1 - \alpha))^{\frac{1}{1-\eta}}} \quad (2.88)$$

$$PfP = \frac{PfPP}{N_F^{\frac{1}{1-\epsilon_F}}} \quad (2.89)$$

$$RER = PfP^{\frac{1}{\epsilon_M}} \quad (2.90)$$

$$Q = 1 \quad (2.91)$$

$$i = \frac{\pi}{\beta} \quad (2.92)$$

$$RK = \delta + \frac{1 + i}{\pi} - 1 \quad (2.93)$$

$$DEP = \frac{\pi}{\pi^*} \quad (2.94)$$

$$PFP = \frac{1}{\left( \omega \left( N_F^{\frac{1}{1-\epsilon_F}} \right)^{1-\mu} + (1 - \omega) \left( PIPFs N_I^{\frac{1}{1-\epsilon_I}} \right)^{1-\mu} \right)^{\frac{1}{1-\mu}}} \quad (2.95)$$

$$PIP = PIPFs PFP \quad (2.96)$$

$$PFPP = PFP N_F^{\frac{1}{1-\epsilon_F}} \quad (2.97)$$

$$PIPP = PIP N_I^{\frac{1}{1-\epsilon_I}} \quad (2.98)$$

$$MC_F = PFPP PHP \frac{\epsilon_F - 1}{\epsilon_F} \quad (2.99)$$

$$MC_I = PIPP PHP \frac{\epsilon_I - 1}{\epsilon_I} \quad (2.100)$$

$$MC_F(s) = \frac{MC_F}{N_F^{\frac{1}{1-\epsilon_F}}} \quad (2.101)$$

$$MC_I(s) = \frac{MC_I}{N_I^{\frac{1}{1-\epsilon_I}}} \quad (2.102)$$



$$KILI = \frac{\psi_I MC_I \Theta_I}{RK} \quad (2.103)$$

$$Y_I = \Theta_I KILI^{\frac{\psi_I}{1-\psi_I}} L_I \quad (2.104)$$

$$K_I = KILI \frac{Y_I}{\Theta_I} \quad (2.105)$$

$$KFLF = \frac{\psi_F MC_F \Theta_F}{RK} \quad (2.106)$$

$$Y_F = \Theta_F KFLF^{\frac{\psi_F}{1-\psi_F}} L_F \quad (2.107)$$

$$K_F = KFLF \frac{Y_F}{\Theta_F} \quad (2.108)$$

$$K = K_F + K_I \quad (2.109)$$

$$\zeta = 1 \quad (2.110)$$

$$PX^* = PHP \frac{PFPP}{RER} \quad (2.111)$$

$$PHPP_I = \frac{1}{\left( \alpha + (1 - \alpha) \left( \frac{PfPH}{PFPP} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}}} \quad (2.112)$$

$$I = \frac{\delta K}{PFPP PHP} PHPP_I \quad (2.113)$$

$$I_H = \alpha PHPP_I^{-\eta} I \quad (2.114)$$

$$I_f = (1 - \alpha) PHPP_I^{-\eta} I \quad (2.115)$$

$$D = QK \quad (2.116)$$

$$H_I = L_I - (1 - probf_I) L_I \quad (2.117)$$

$$probh_I = \frac{H_I}{U + probf_I L_I + probf_F L_F} \quad (2.118)$$

$$WI = \frac{MC_I (1 - \psi_I) \frac{Y_I}{L_I}}{1 + HCIWI - \beta (1 - probf_I) HCIWI} \quad (2.119)$$

$$HC_I = HCIWIWI \quad (2.120)$$

$$H_F = L_F - (1 - probf_F) L_F \quad (2.121)$$

$$probh_F = \frac{H_F}{U + probf_I L_I + probf_F L_F} \quad (2.122)$$

$$WF = \frac{MC_F (1 - \psi_F) \frac{Y_F}{L_F}}{1 + HCFWF - \beta (1 - probf_F) HCFWF} \quad (2.123)$$

$$HC_F = HCFWFWF \quad (2.124)$$

$$\beta_{HCF} = \frac{HC_F}{probh_F^{\alpha_{HCF}}} \quad (2.125)$$

$$\beta_{HCI} = \frac{HC_I}{probh_I^{\alpha_{HCI}}} \quad (2.126)$$

$$Prof_F^W = MC_F Y_F - WF L_F - RK K_F - HC_F H_F \quad (2.127)$$

$$Prof_I^W = MC_I Y_I - WI L_I - RK K_I - HC_I H_I \quad (2.128)$$

$$entry_F = entry_FY PFPP PHPY_F \quad (2.129)$$

$$entry_I = entry_I Y \ PIPP \ PHP Y_I \quad (2.130)$$

$$Y_{HP} = \alpha_{HP} \frac{WF \ L_F + WI \ L_I}{L_F + L_I} U \quad (2.131)$$

$$C_I = Y_I + Y_{HP} - \frac{HC_I H_I + N_I^E entry_I}{PIPP \ PHP} \quad (2.132)$$

$$QD_I = C_I \quad (2.133)$$

$$QD_I(s) = \frac{QD_I}{N_I^{\frac{\epsilon_I}{\epsilon_I-1}}} \quad (2.134)$$

$$C_H = \frac{C_I}{(1-\omega)PIPP^{-\mu}} \quad (2.135)$$

$$C_F = \omega C_H PFPP^{-\mu} \quad (2.136)$$

$$C = \frac{C_H}{\alpha PHP^{-\eta}} \quad (2.137)$$

$$C_f = (1-\alpha)C \ PfPP^{-\eta} \quad (2.138)$$

$$GDP = \frac{C}{CY} \quad (2.139)$$

$$WU = WUY \frac{GDP}{U} \quad (2.140)$$

$$Q^X = QXY \frac{GDP}{PFPP \ PHP} \quad (2.141)$$

$$G = GY \frac{GDP}{PFPP \ \frac{PHP}{PHPP_I}} \quad (2.142)$$

$$G_H = \alpha PHPP_I^{-\eta} G \quad (2.143)$$

$$G_f = (1-\alpha) PHPP_I^{-\eta} G \quad (2.144)$$

$$Q^M = \frac{C + PFPP \ \frac{PHP}{PHPP_I} (I + G) + PFPP \ PHP \ Q^X - GDP}{PfPP} \quad (2.145)$$

$$V_F^R = entry_F \quad (2.146)$$

$$V_I^R = entry_I \quad (2.147)$$

$$QD_F = Y_F - \frac{HC_F H_F + N_F^E entry_F}{PFPP \ PHP} \quad (2.148)$$

$$QD_F(s) = \frac{QD_F}{N_F^{\frac{\epsilon_F}{\epsilon_F-1}}} \quad (2.149)$$

$$Prof_F^R = (PFPP \ PHP - MC_F(s)) \ QD_F(s) \quad (2.150)$$

$$Prof_I^R = (PIP \ PHP - MC_I(s)) \ QD_I(s) \quad (2.151)$$

$$Y = PFPP \ Y_F + PIPP \ Y_I \quad (2.152)$$

$$\tau_F = taxY \frac{GDP}{WFL_F} \quad (2.153)$$

$$Tax = PFPP \ \frac{PHP}{PHPP_I} G + WU \ U - \tau_F WF \ L_F \quad (2.154)$$

$$B = \frac{Tax + C + D + N_F^E entry_F + N_I^E entry_I - N_F Prof_F^R - Prof_F^W - N_I Prof_I^R}{RER^{\frac{1+iff}{\pi^*-1}}} + \frac{-Prof_I^W - (1 - \tau_F)WF L_F - WI L_I - Y_{HP} - WU U - \frac{1+i}{\pi}D}{RER^{\frac{1+iff}{\pi^*-1}}} \quad (2.155)$$

$$\alpha_F = \frac{\epsilon_F}{N_F} \quad (2.156)$$

$$\alpha_I = \frac{\epsilon_I}{N_I} \quad (2.157)$$

$$\alpha_X = exp\left(\frac{\log(Q^X)}{\varsigma_X} + \log(PXPstar)\right) \quad (2.158)$$

**Notations:**

- ratio of home-produced goods' price level to economy-wide price level

$$PHP = \frac{P_H}{P} \quad (2.159)$$

- ratio of foreign-produced goods' price level to economy-wide price level

$$PfPP = \frac{P_f}{P} \quad (2.160)$$

- ratio of formal sector's price level to home-produced goods' price level

$$PFPP = \frac{P_F}{P_H} \quad (2.161)$$

- ratio of informal sector's price level to home-produced goods' price level

$$PIPP = \frac{P_I}{P_H} \quad (2.162)$$

- ratio of one foreign firm's-produced goods' price level to economy-wide price level

$$PfP = \frac{P_f(s)}{P} \quad (2.163)$$

- ratio of one firm's price level in the formal sector to home-produced goods' price level

$$PFP = \frac{P_F(s)}{P_H} \quad (2.164)$$

- ratio of one firm's price level in the informal sector to home-produced goods' price level

$$PIP = \frac{P_I(s)}{P_H} \quad (2.165)$$

- ratio of foreign-produced goods' price level to home-produced goods' price level

$$PfPH = \frac{P_f}{P_H} \quad (2.166)$$

- ratio of one firm's price level in the informal sector to one firm's price level in the formal sector

$$PIPF_s = \frac{P_I(s)}{P_F(s)} \quad (2.167)$$

- price of exports in foreign currency

$$PX = \frac{P_F}{e} \quad (2.168)$$

- ratio of price of exports in foreign currency to foreign economy-wide price level

$$PX^* = \frac{PX}{P^*} \quad (2.169)$$

- ratio of formal sector's price level to price level of investment goods

$$PHPP_I = \frac{P_F}{P_{INV}} \quad (2.170)$$

- ratio of foreign-produced goods' price level to price level of investment goods

$$PHPP_I = \frac{P_f}{P_{INV}} \quad (2.171)$$

- physical capital-labor ratio in the formal sector

$$KFLF = \frac{K_F}{L_F} \quad (2.172)$$

- physical capital-labor ratio in the informal sector

$$KILI = \frac{K_I}{L_I} \quad (2.173)$$

- ratio of hiring cost to wages in the formal sector

$$HCFWF = \frac{HC_F}{WF} \quad (2.174)$$

- ratio of hiring cost to wages in the informal sector

$$HCIWI = \frac{HC_I}{WI} \quad (2.175)$$

- formal entry cost as a share of formal production

$$entryFY = \frac{entry_F}{PFPP \ PHP \ Y_F} \quad (2.176)$$

- informal entry cost as a share of informal production

$$entryIY = \frac{entry_I}{PIPP \ PHP \ Y_I} \quad (2.177)$$

- share of home production

$$\alpha_{HP} = \frac{Y_{HP}}{\frac{WF \ L_F + WI \ L_I}{L_F + L_I} \ U} \quad (2.178)$$

- household consumption to GDP ratio

$$CY = \frac{C}{GDP} \quad (2.179)$$

- share of public social protection expenditure in GDP

$$WUY = \frac{WU \ U}{GDP} \quad (2.180)$$

- exports to GDP ratio

$$QXY = \frac{PFPP \ PHP \ Q^X}{GDP} \quad (2.181)$$

- government consumption expenditure to GDP ratio

$$GY = \frac{PFPP \ \frac{PHP}{PHP_I} \ G}{GDP} \quad (2.182)$$

| Long-run effects of lowering the formal |   | GDP, %           | Unemployment, %point      | Formal employment, %point   | Informal employment, %point | Formal wage, % |
|---|---|------------------|---------------------------|-----------------------------|-----------------------------|----------------|
| hiring cost                             | Baseline  | 2.91             | -2.76                     | 2.20                        | 0.56                        | -0.94          |
|   | Lower informal bargaining power of workers in SS  | 3.12             | -3.07                     | 2.25                        | 0.82                        | -1.01          |
|   | Higher informal bargaining power of workers in SS | 2.64             | -2.49                     | 2.20                        | 0.30                        | -0.84          |
| bargaining power of workers             | Baseline  | 3.33             | -4.61                     | 3.84                        | 0.78                        | -5.33          |
|   | Lower informal bargaining power of workers in SS  | 3.73             | -5.09                     | 3.92                        | 1.17                        | -5.41          |
|   | Higher informal bargaining power of workers in SS | 2.68             | -4.24                     | 3.81                        | 0.44                        | -5.30          |
| entry cost                              | Baseline  | 1.11             | -0.49                     | 0.25                        | 0.24                        | 0.81           |
|   | Lower informal bargaining power of workers in SS  | 1.15             | -0.60                     | 0.27                        | 0.33                        | 0.79           |
|   | Higher informal bargaining power of workers in SS | 1.08             | -0.36                     | 0.25                        | 0.12                        | 0.84           |
| Long-run effects of lowering the formal |   | Informal wage, % | Number of formal firms, % | Number of informal firms, % | Net exports, %point         | Investment, %  |
| hiring cost                             | Baseline  | 1.80             | 1.56                      | 1.07                        | 0.79                        | -0.02          |
|   | Lower informal bargaining power of workers in SS  | 1.43             | 1.66                      | 1.19                        | 0.82                        | 0.04           |
|   | Higher informal bargaining power of workers in SS | 2.36             | 1.43                      | 0.87                        | 0.73                        | -0.01          |
| bargaining power of workers             | Baseline  | 2.47             | 1.80                      | 1.14                        | 0.50                        | 2.03           |
|   | Lower informal bargaining power of workers in SS  | 1.80             | 2.00                      | 1.37                        | 0.56                        | 2.08           |
|   | Higher informal bargaining power of workers in SS | 3.57             | 1.50                      | 0.71                        | 0.36                        | 2.20           |
| entry cost                              | Baseline  | 0.50             | 6.00                      | 0.54                        | 0.20                        | 0.21           |
|   | Lower informal bargaining power of workers in SS  | 0.44             | 6.02                      | 0.56                        | 0.21                        | 0.22           |
|   | Higher informal bargaining power of workers in SS | 0.57             | 5.98                      | 0.51                        | 0.20                        | 0.19           |

Table 2.A1: The robustness of long-run (25-year) impacts of structural reforms with lower and higher informal bargaining power of workers

| Long-run effects of lowering the formal |                                   | GDP, %           | Unemployment, %point      | Formal employment, %point   | Informal employment, %point | Formal wage, % |
|---|-----------------------------------|------------------|---------------------------|-----------------------------|-----------------------------|----------------|
| hiring cost                             | Baseline                          | 2.91             | -2.76                     | 2.20                        | 0.56                        | -0.94          |
|   | Lower informal hiring cost in SS  | 3.06             | -2.94                     | 2.20                        | 0.73                        | -0.97          |
|   | Higher informal hiring cost in SS | 2.71             | -2.58                     | 2.22                        | 0.36                        | -0.87          |
| bargaining power of workers             | Baseline                          | 3.33             | -4.61                     | 3.84                        | 0.78                        | -5.33          |
|   | Lower informal hiring cost in SS  | 3.65             | -4.88                     | 3.84                        | 1.04                        | -5.33          |
|   | Higher informal hiring cost in SS | 2.85             | -4.37                     | 3.86                        | 0.51                        | -5.33          |
| entry cost                              | Baseline                          | 1.11             | -0.49                     | 0.25                        | 0.24                        | 0.81           |
|   | Lower informal hiring cost in SS  | 1.14             | -0.56                     | 0.26                        | 0.30                        | 0.80           |
|   | Higher informal hiring cost in SS | 1.09             | -0.40                     | 0.25                        | 0.15                        | 0.83           |
| Long-run effects of lowering the formal |                                   | Informal wage, % | Number of formal firms, % | Number of informal firms, % | Net exports, %point         | Investment, %  |
| hiring cost                             | Baseline                          | 1.80             | 1.56                      | 1.07                        | 0.79                        | -0.02          |
|   | Lower informal hiring cost in SS  | 1.60             | 1.63                      | 1.17                        | 0.82                        | -0.02          |
|   | Higher informal hiring cost in SS | 2.19             | 1.46                      | 0.92                        | 0.74                        | 0.02           |
| bargaining power of workers             | Baseline                          | 2.47             | 1.80                      | 1.14                        | 0.50                        | 2.03           |
|   | Lower informal hiring cost in SS  | 2.12             | 1.95                      | 1.34                        | 0.56                        | 2.00           |
|   | Higher informal hiring cost in SS | 3.21             | 1.58                      | 0.82                        | 0.38                        | 2.19           |
| entry cost                              | Baseline                          | 0.50             | 6.00                      | 0.54                        | 0.20                        | 0.21           |
|   | Lower informal hiring cost in SS  | 0.47             | 6.01                      | 0.55                        | 0.21                        | 0.21           |
|   | Higher informal hiring cost in SS | 0.55             | 5.99                      | 0.52                        | 0.20                        | 0.20           |

Table 2.A2: The robustness of long-run (25-year) impacts of structural reforms with lower and higher informal hiring cost

| Long-run effects of lowering the formal |                                  | GDP, %           | Unemployment, %point      | Formal employment, %point   | Informal employment, %point | Formal wage, % |
|---|----------------------------------|------------------|---------------------------|-----------------------------|-----------------------------|----------------|
| hiring cost                             | Baseline                         | 2.91             | -2.76                     | 2.20                        | 0.56                        | -0.94          |
|   | Lower informal entry cost in SS  | 2.91             | -2.76                     | 2.20                        | 0.56                        | -0.94          |
|   | Higher informal entry cost in SS | 2.91             | -2.76                     | 2.20                        | 0.56                        | -0.94          |
| bargaining power of workers             | Baseline                         | 3.33             | -4.61                     | 3.84                        | 0.78                        | -5.33          |
|   | Lower informal entry cost in SS  | 3.33             | -4.61                     | 3.84                        | 0.78                        | -5.33          |
|   | Higher informal entry cost in SS | 3.33             | -4.61                     | 3.84                        | 0.78                        | -5.33          |
| entry cost                              | Baseline                         | 1.11             | -0.49                     | 0.25                        | 0.24                        | 0.81           |
|   | Lower informal entry cost in SS  | 1.11             | -0.49                     | 0.25                        | 0.24                        | 0.81           |
|   | Higher informal entry cost in SS | 1.11             | -0.49                     | 0.25                        | 0.24                        | 0.81           |
| Long-run effects of lowering the formal |                                  | Informal wage, % | Number of formal firms, % | Number of informal firms, % | Net exports, %point         | Investment, %  |
| hiring cost                             | Baseline                         | 1.80             | 1.56                      | 1.07                        | 0.79                        | -0.02          |
|   | Lower informal entry cost in SS  | 1.80             | 1.56                      | 1.07                        | 0.79                        | -0.02          |
|   | Higher informal entry cost in SS | 1.80             | 1.56                      | 1.07                        | 0.79                        | -0.02          |
| bargaining power of workers             | Baseline                         | 2.47             | 1.80                      | 1.14                        | 0.50                        | 2.03           |
|   | Lower informal entry cost in SS  | 2.47             | 1.80                      | 1.14                        | 0.50                        | 2.03           |
|   | Higher informal entry cost in SS | 2.47             | 1.80                      | 1.14                        | 0.50                        | 2.03           |
| entry cost                              | Baseline                         | 0.50             | 6.00                      | 0.54                        | 0.20                        | 0.21           |
|   | Lower informal entry cost in SS  | 0.50             | 6.00                      | 0.54                        | 0.20                        | 0.21           |
|   | Higher informal entry cost in SS | 0.50             | 6.00                      | 0.54                        | 0.20                        | 0.21           |

Table 2.A3: The robustness of long-run (25-year) impacts of structural reforms with lower and higher informal entry cost



| Long-run effects of lowering the formal |   | GDP, %           | Unemployment, %point      | Formal employment, %point   | Informal employment, %point | Formal wage, % |
|---|---|------------------|---------------------------|-----------------------------|-----------------------------|----------------|
| hiring cost                             | Baseline  | 2.91             | -2.76                     | 2.20                        | 0.56                        | -0.94          |
|   | Lower elasticity of substitution between formal and informal goods  | 2.83             | -2.87                     | 2.11                        | 0.75                        | -1.00          |
|   | Higher elasticity of substitution between formal and informal goods | 2.98             | -2.66                     | 2.27                        | 0.39                        | -0.89          |
| bargaining power of workers             | Baseline  | 3.33             | -4.61                     | 3.84                        | 0.78                        | -5.33          |
|   | Lower elasticity of substitution between formal and informal goods  | 3.21             | -4.78                     | 3.72                        | 1.06                        | -5.43          |
|   | Higher elasticity of substitution between formal and informal goods | 3.44             | -4.46                     | 3.93                        | 0.54                        | -5.25          |
| entry cost                              | Baseline  | 1.11             | -0.49                     | 0.25                        | 0.24                        | 0.81           |
|   | Lower informal entry cost in SS                                     | 1.10             | -0.50                     | 0.24                        | 0.26                        | 0.81           |
|   | Higher informal entry cost in SS                                    | 1.12             | -0.48                     | 0.26                        | 0.22                        | 0.82           |
| Long-run effects of lowering the formal |   | Informal wage, % | Number of formal firms, % | Number of informal firms, % | Net exports, %point         | Investment, %  |
| hiring cost                             | Baseline  | 1.80             | 1.56                      | 1.07                        | 0.79                        | -0.02          |
|   | Lower elasticity of substitution between formal and informal goods  | 1.99             | 1.40                      | 1.44                        | 0.79                        | -0.05          |
|   | Higher elasticity of substitution between formal and informal goods | 1.65             | 1.70                      | 0.77                        | 0.80                        | 0.00           |
| bargaining power of workers             | Baseline  | 2.47             | 1.80                      | 1.14                        | 0.50                        | 2.03           |
|   | Lower elasticity of substitution between formal and informal goods  | 2.74             | 1.57                      | 1.66                        | 0.49                        | 2.01           |
|   | Higher elasticity of substitution between formal and informal goods | 2.25             | 2.01                      | 0.73                        | 0.51                        | 2.04           |
| entry cost                              | Baseline  | 0.50             | 6.00                      | 0.54                        | 0.20                        | 0.21           |
|   | Lower elasticity of substitution between formal and informal goods  | 0.53             | 5.98                      | 0.59                        | 0.20                        | 0.20           |
|   | Higher elasticity of substitution between formal and informal goods | 0.48             | 6.02                      | 0.49                        | 0.20                        | 0.21           |

Table 2.A4: The robustness of long-run (25-year) impacts of structural reforms with lower and higher elasticity of substitution between formal and informal goods

## B The role of shadow economy

Because shadow output and employment are, by definition, in the shadow, it is not straightforward how to define or measure them.

Concerning output, the broad definitions usually incorporate all monetary and non-monetary transactions and all legal and illegal activities (Table 2.A5). Hence, not only tax evasion or tax avoidance, but also, for example, drugs and prostitution are included. In a narrow approach, however, informality is mostly related to avoiding taxation or legal requirements. As Schneider (2012) (page 6) claims: “The shadow economy includes all market-based legal production of goods and services that are deliberately concealed from public authorities for the following reasons: 1. to avoid payment of income, value added or other taxes, 2. to avoid payment of social security contributions, 3. to avoid having to meet certain legal labor market standards, such as minimum wages, maximum working hours, safety standards, etc., and 4. to avoid complying with certain administrative obligations, such as completing statistical questionnaires or other administrative forms.” Regarding employment, the “shadow labor market includes all cases, where the employees or the employers, or both, occupy a shadow economy position” (Schneider (2012), page 28).

|                    | Monetary Transactions   |   | Non-Monetary Transactions           |   |
|--------------------|---|---|-------------------------------------|---|
| Illegal Activities | Trade in stolen goods, drugs; manufacture of drugs; prostitution, gambling, fraud |   | Barter, drugs, stolen goods, etc.   | Produce or grow drugs for own use. Theft for own use. |
|                    | Tax Evasion   | Tax Avoidance   | Tax Evasion                         | Tax Avoidance   |
| Legal Activities   | Unreported income from self-employment, wages, salaries, and assets               | Employee discounts, fringe benefits (cars, subsidized food, etc.) | Barter of legal services and goods. | Do-it-yourself work                                   |

Table 2.A5: A taxonomy of underground economic activities (broad concept). Source: Mirus-Smith (1977) (page 5).

As regards estimation methodologies, Schneider-Enste (2000) point out that the most common ones are the currency demand method (the size of the shadow economy is equal to the unexplained increase in the currency demand) and the MIMIC method (a factor model). Nevertheless, they also provide a detailed description about other approaches.

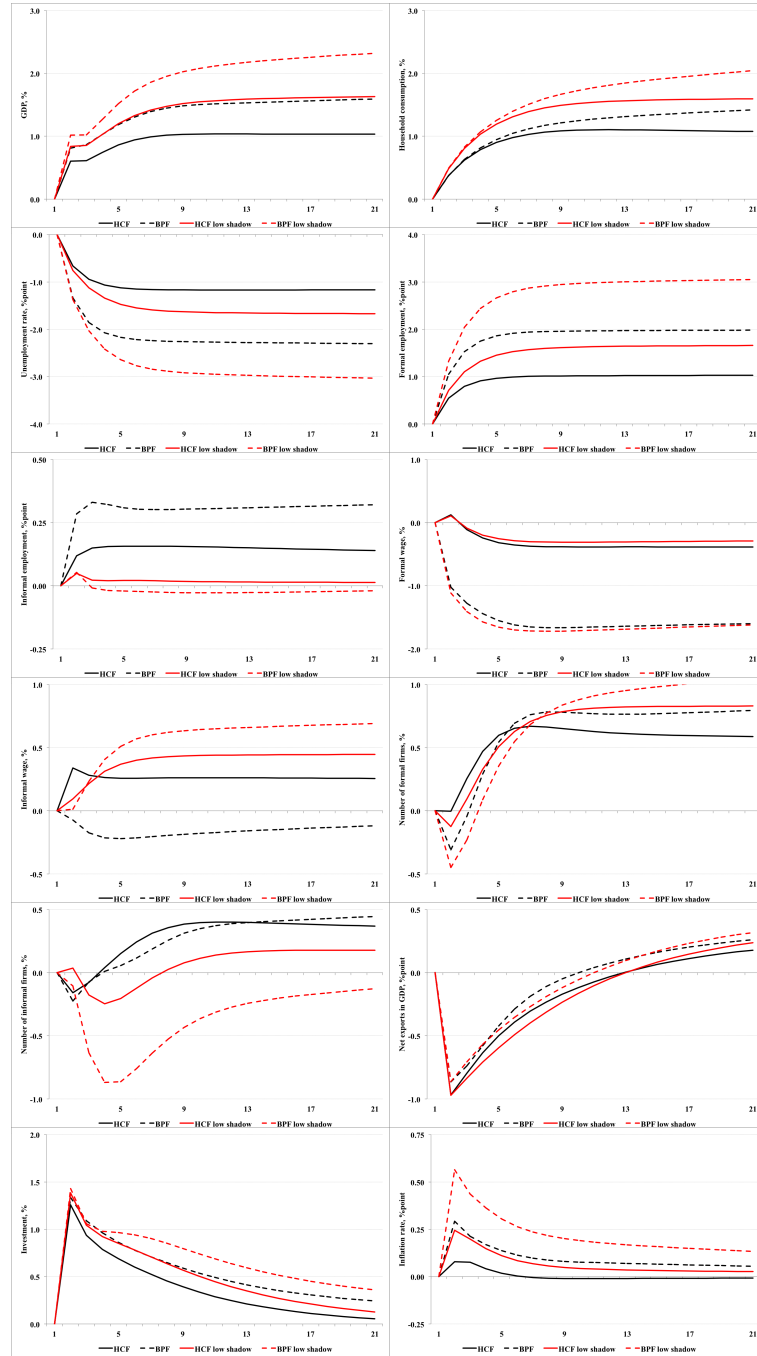


Figure 2.A1: The role of shadow economy in short-run effects of a decrease in formal hiring cost and bargaining power of workers. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal hiring cost and bargaining power of workers. Low informality (red lines) means that the size of the shadow economy in output is 1/3 of the original value.

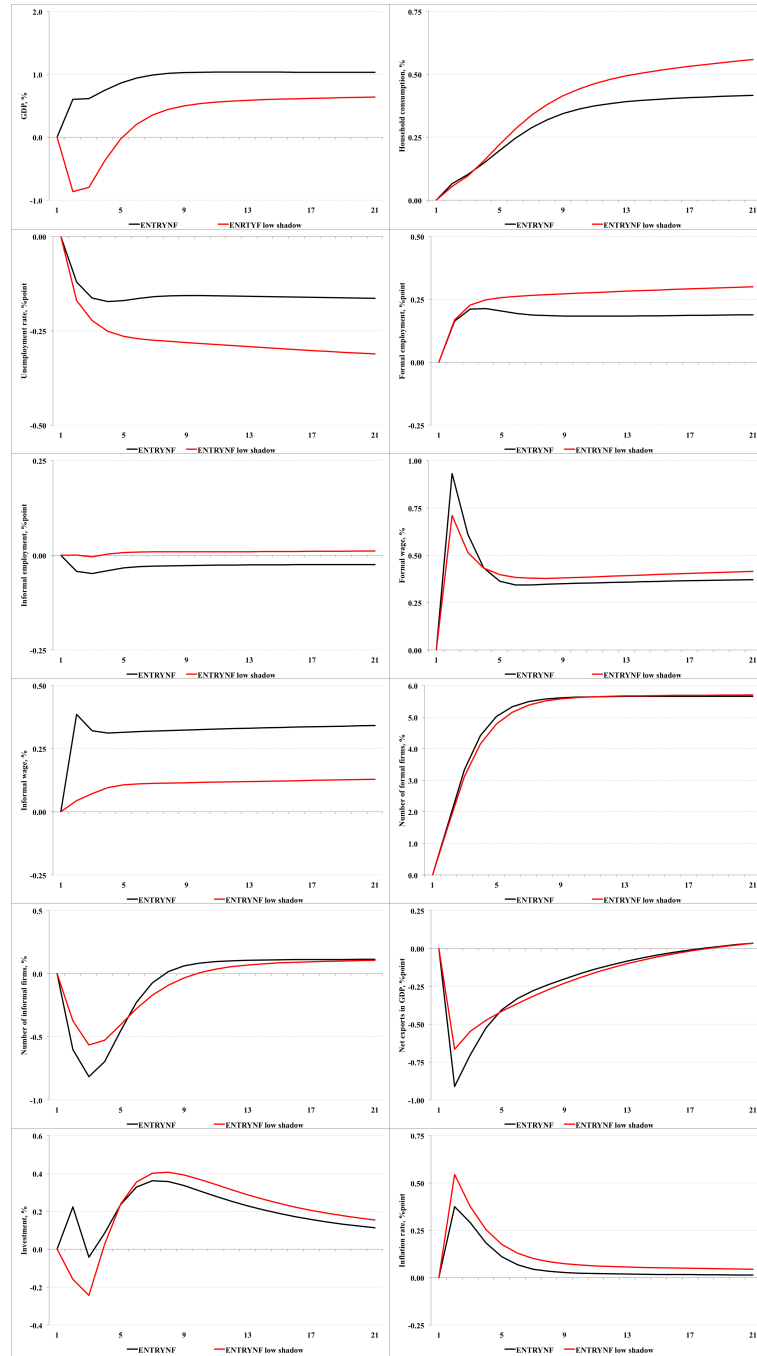


Figure 2.A2: The role of shadow economy in short-run effects of a decrease in formal entry cost. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal entry cost. Low informality (red line) means that the size of the shadow economy in output is 1/3 of the original value.

| Name of<br>fixed or calibrated parameter or variable           | Sign of          | Baseline | Alternative 1 | Alternative 2 |
|--|------------------|----------|---------------|---------------|
| Discount factor  | $\beta$          | 0.99     |               |               |
| Physical capital depreciation rate                             | $\delta$         | 0.02     |               |               |
| Formal capital income share                                    | $\psi_F$         | 0.3      |               |               |
| Informal capital income share                                  | $\psi_I$         | 0.2      |               |               |
| Markup in formal sector (%)                                    | $\varepsilon_F$  | 10       |               |               |
| Markup in informal sector (%)                                  | $\varepsilon_I$  | 10       |               |               |
| Formal productivity level                                      | $\theta_F$       | 1        |               |               |
| Informal productivity level                                    | $\theta_I$       | 1        |               |               |
| Formality bias   | $\mu_F$          | 0.4      | 0.8           |               |
| Home inflation (% QoQ)   | $\pi$            | 1        |               |               |
| Exchange rate depreciation rate (% QoQ)                        |                  | 0.5      |               | -             |
| Exit rate of formal retailers                                  | $\delta_F$       | 0.05     |               |               |
| Exit rate of informal retailers                                | $\delta_I$       | 0.15     |               |               |
| Household consumption expenditure to GDP ratio (%)             |                  | 70       |               |               |
| Government consumption expenditure to GDP ratio (%)            |                  | 20       |               |               |
| Exports to GDP ratio (%)                                       | $\alpha_X$       | 40       |               | 0             |
| Public social protection to GDP ratio (%)                      |                  | 5        |               |               |
| Firing probability in the formal sector                        | $\text{prob}f_F$ | 0.1      |               |               |
| Firing probability in the informal sector                      | $\text{prob}f_I$ | 0.15     | 0.9           |               |
| Ratio of hiring cost in wage in formal sector                  |                  | 0.5      |               |               |
| Ratio of hiring cost in wage in informal sector                |                  | 0.5      |               |               |
| Elasticity of substitution between home and foreign goods      | $\mu$            | 0.7      |               | -             |
| Home bias  | $\alpha$         | 0.7      |               | 1             |
| Export price elasticity  | $\zeta_X$        | 3        |               | -             |
| Share of income tax revenue in GDP (%)                         | $\tau_F$         | 10       |               |               |
| Exchange rate pass-through to import prices                    | $\zeta_M$        | 0.7      |               | -             |
| Entry cost in the formal sector (in months of production)      |                  | 0.67     |               |               |
| Entry cost in the informal sector (in months of production)    |                  | 0.67     |               |               |
| Share of home production in average wage                       |                  | 5        |               |               |
| Elasticity of hiring cost wrt to hiring probability (formal)   | $\alpha_{HCF}$   | 0.5      |               |               |
| Elasticity of hiring cost wrt to hiring probability (informal) | $\alpha_{HCI}$   | 0.5      |               |               |
| Elasticity of substitution between formal and informal goods   | $\eta$           | 2        |               |               |
| Formal bargaining power of workers                             | $\lambda_F$      | 0.6      |               |               |
| Informal bargaining power of workers                           | $\lambda_I$      | 0.25     |               |               |

Table 2.A6: Steady state parameters of the three parametrized economies

| Dynamic parameters |                            |       |
|--------------------|----------------------------|-------|
| $h$                | consumption habit          | 0.7   |
| $\gamma_i$         | interest rate smoothing    | 0.5   |
| $\gamma_\pi$       | inflation response         | 3     |
| $\gamma_Y$         | output response            | 0.5   |
| $dF$               | formal Rotemberg cost      | 5     |
| $dI$               | informal Rotemberg cost    | 5     |
| $\chi$             | interest rate premium      | 0.002 |
| $\zeta$            | investment adjustment cost | 2     |

Table 2.A7: Dynamic parameters of the three parametrized economies

## C Bayesian estimation

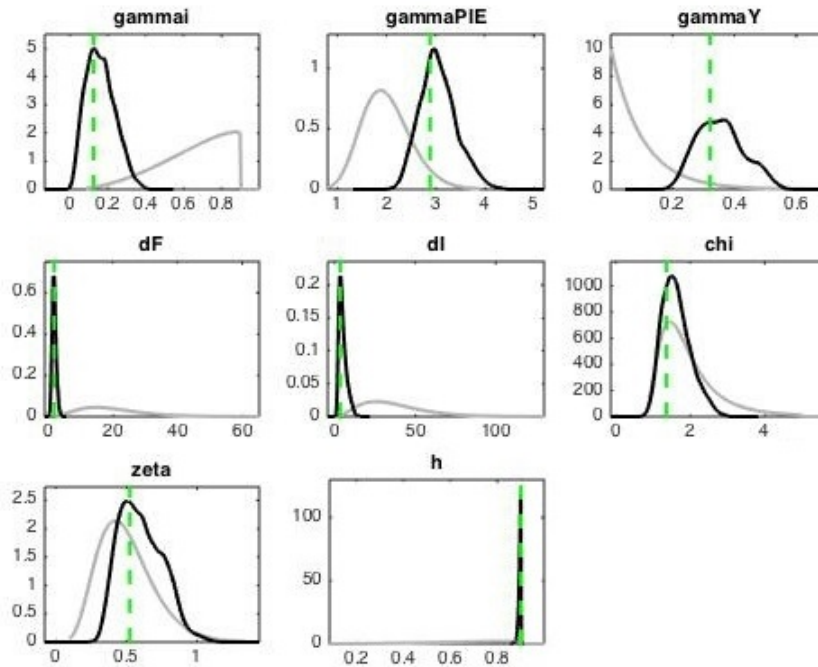


Figure 2.A3: Prior and posterior distributions of structural parameters based on Bayesian estimation using South African data. The parameters are (in order): interest rate smoothing, inflation weight and output weight in Taylor rule, formal and informal Rotemberg price rigidity costs, interest rate premium, investment adjustment cost and consumption habit.

| Shock autocorrelations                             | distribution | prior |                    | posterior |                     |      |
|--|--------------|-------|--------------------|-----------|---------------------|------|
|  |              | mean  | standard deviation | mean      | confidence interval |      |
| $\rho_{\theta F}$ formal productivity              | beta         | 0.8   | 0.2                | 0.15      | 0.05                | 0.25 |
| $\rho_{\theta I}$ informal productivity            | beta         | 0.8   | 0.2                | 0.89      | 0.88                | 0.90 |
| $\rho_{\tau F}$ income tax                         | beta         | 0.8   | 0.2                | 0.71      | 0.47                | 0.90 |
| $\rho_{WU}$ social benefit                         | beta         | 0.8   | 0.2                | 0.69      | 0.42                | 0.90 |
| $\rho_G$ government consumption                    | beta         | 0.8   | 0.2                | 0.82      | 0.73                | 0.90 |
| $\rho_{\zeta c}$ preference                        | beta         | 0.8   | 0.2                | 0.80      | 0.67                | 0.90 |
| $\rho_{\beta HCF}$ formal hiring cost              | beta         | 0.8   | 0.2                | 0.45      | 0.31                | 0.58 |
| $\rho_{\beta HCI}$ informal hiring cost            | beta         | 0.8   | 0.2                | 0.49      | 0.14                | 0.86 |
| $\rho_{\text{probf}f}$ formal firing probability   | beta         | 0.8   | 0.2                | 0.81      | 0.74                | 0.90 |
| $\rho_{\text{probf}i}$ informal firing probability | beta         | 0.8   | 0.2                | 0.84      | 0.74                | 0.90 |
| $\rho_{\lambda F}$ formal bargaining power         | beta         | 0.8   | 0.2                | 0.09      | 0.01                | 0.16 |
| $\rho_{\lambda I}$ informal bargaining power       | beta         | 0.8   | 0.2                | 0.51      | 0.20                | 0.89 |
| $\rho_{\text{entry}f}$ formal entry cost           | beta         | 0.8   | 0.2                | 0.85      | 0.81                | 0.90 |
| $\rho_{\text{entry}i}$ informal entry cost         | beta         | 0.8   | 0.2                | 0.80      | 0.68                | 0.90 |
| $\rho_{\pi^*}$ foreign inflation                   | beta         | 0.8   | 0.2                | 0.24      | 0.12                | 0.35 |
| $\rho_{i^*}$ foreign interest rate                 | beta         | 0.8   | 0.2                | 0.74      | 0.50                | 0.90 |

Table 2.A8: Prior and posterior distributions of autocorrelations of shocks based on Bayesian estimation using South African data. All priors are maximized to 0.9.

| Shock standard deviations                                 | distribution  | prior |                    | posterior |                     |      |
|---|---------------|-------|--------------------|-----------|---------------------|------|
|   |               | mean  | standard deviation | mean      | confidence interval |      |
| $\varepsilon_{\theta F}$ formal productivity              | inverse gamma | 0.2   | 0.1                | 0.43      | 0.36                | 0.49 |
| $\varepsilon_{\theta I}$ informal productivity            | inverse gamma | 0.2   | 0.1                | 0.15      | 0.12                | 0.18 |
| $\varepsilon_{\tau F}$ income tax                         | inverse gamma | 0.2   | 0.1                | 0.27      | 0.08                | 0.54 |
| $\varepsilon_{WU}$ social benefit                         | inverse gamma | 0.2   | 0.1                | 0.13      | 0.09                | 0.18 |
| $\varepsilon_G$ government consumption                    | inverse gamma | 0.2   | 0.1                | 0.07      | 0.06                | 0.08 |
| $\varepsilon_{\zeta c}$ preference                        | inverse gamma | 0.2   | 0.1                | 0.08      | 0.07                | 0.10 |
| $\varepsilon_{\beta HCF}$ formal hiring cost              | inverse gamma | 0.2   | 0.1                | 0.53      | 0.43                | 0.62 |
| $\varepsilon_{\beta HCI}$ informal hiring cost            | inverse gamma | 0.2   | 0.1                | 0.10      | 0.07                | 0.13 |
| $\varepsilon_{\text{probf}f}$ formal firing probability   | inverse gamma | 0.2   | 0.1                | 0.32      | 0.19                | 0.44 |
| $\varepsilon_{\text{probf}i}$ informal firing probability | inverse gamma | 0.2   | 0.1                | 0.17      | 0.09                | 0.24 |
| $\varepsilon_{\lambda F}$ formal bargaining power         | inverse gamma | 0.2   | 0.1                | 0.17      | 0.14                | 0.20 |
| $\varepsilon_{\lambda I}$ informal bargaining power       | inverse gamma | 0.2   | 0.1                | 0.10      | 0.07                | 0.13 |
| $\varepsilon_{\text{entry}f}$ formal entry cost           | inverse gamma | 0.2   | 0.1                | 0.79      | 0.59                | 1.01 |
| $\varepsilon_{\text{entry}i}$ informal entry cost         | inverse gamma | 0.2   | 0.1                | 0.16      | 0.08                | 0.23 |
| $\varepsilon_{i^*}$ foreign interest rate                 | inverse gamma | 0.2   | 0.1                | 0.08      | 0.06                | 0.10 |
| $\varepsilon_{\pi^*}$ foreign inflation                   | inverse gamma | 0.2   | 0.1                | 0.26      | 0.12                | 0.38 |
| $\varepsilon_i$ monetary policy                           | inverse gamma | 0.2   | 0.1                | 0.07      | 0.06                | 0.09 |

Table 2.A9: Prior and posterior distributions of standard deviations of shocks based on Bayesian estimation using South African data

## D Reform packages

| Long-run effects when lowering the ...                  | GDP, %           | Unemployment, %point      | Formal employment, %point   | Informal employment, %point | Formal wage, % |
|---|------------------|---------------------------|-----------------------------|-----------------------------|----------------|
| hiring cost and bargaining power of workers             | 6.1              | -7.2                      | 6.0                         | 1.2                         | -5.9           |
| hiring cost and entry cost                              | 4.0              | -3.2                      | 2.4                         | 0.8                         | -0.1           |
| bargaining power of workers and entry cost              | 4.4              | -5.1                      | 4.1                         | 1.0                         | -4.6           |
| hiring cost, bargaining power of workers and entry cost | 7.2              | -7.7                      | 6.2                         | 1.5                         | -5.2           |
| Long-run effects when lowering the ...                  | Informal wage, % | Number of formal firms, % | Number of informal firms, % | Net exports, %point         | Investment, %  |
| hiring cost and bargaining power of workers             | 4.3              | 3.3                       | 2.1                         | 1.2                         | 8.1            |
| hiring cost and entry cost                              | 2.3              | 7.6                       | 1.6                         | 1.0                         | 4.2            |
| bargaining power of workers and entry cost              | 3.0              | 7.9                       | 1.7                         | 0.7                         | 6.7            |
| hiring cost, bargaining power of workers and entry cost | 4.8              | 9.4                       | 2.6                         | 1.4                         | 9.4            |

Table 2.A10: Long-run effects of structural reform packages in South Africa. The table shows the long-run (25 years) effects of permanent and unexpected policies of size 10 percent implemented in the formal sector.



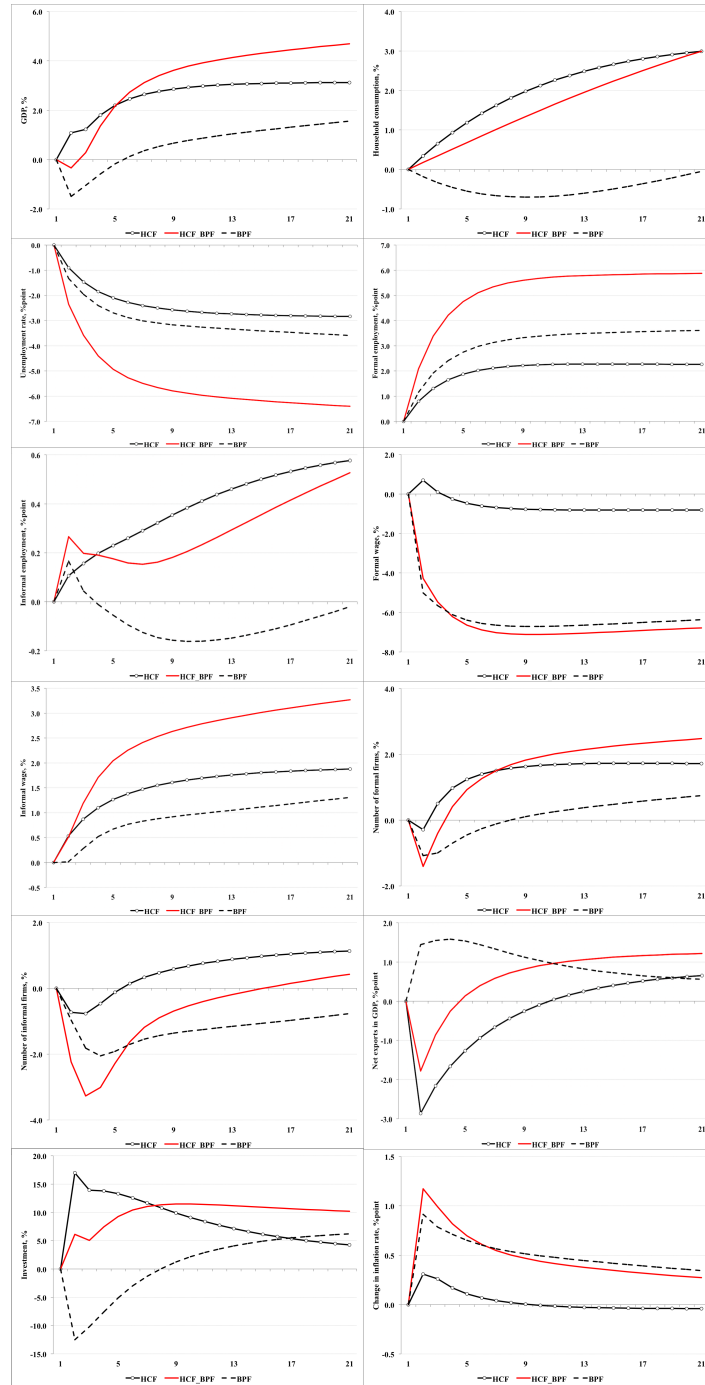


Figure 2.A4: Short-run effects of a combined decrease in formal hiring cost and bargaining power of workers in South Africa. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal hiring cost (line with marker), formal bargaining power of workers (dotted black line) and their combination (red line).

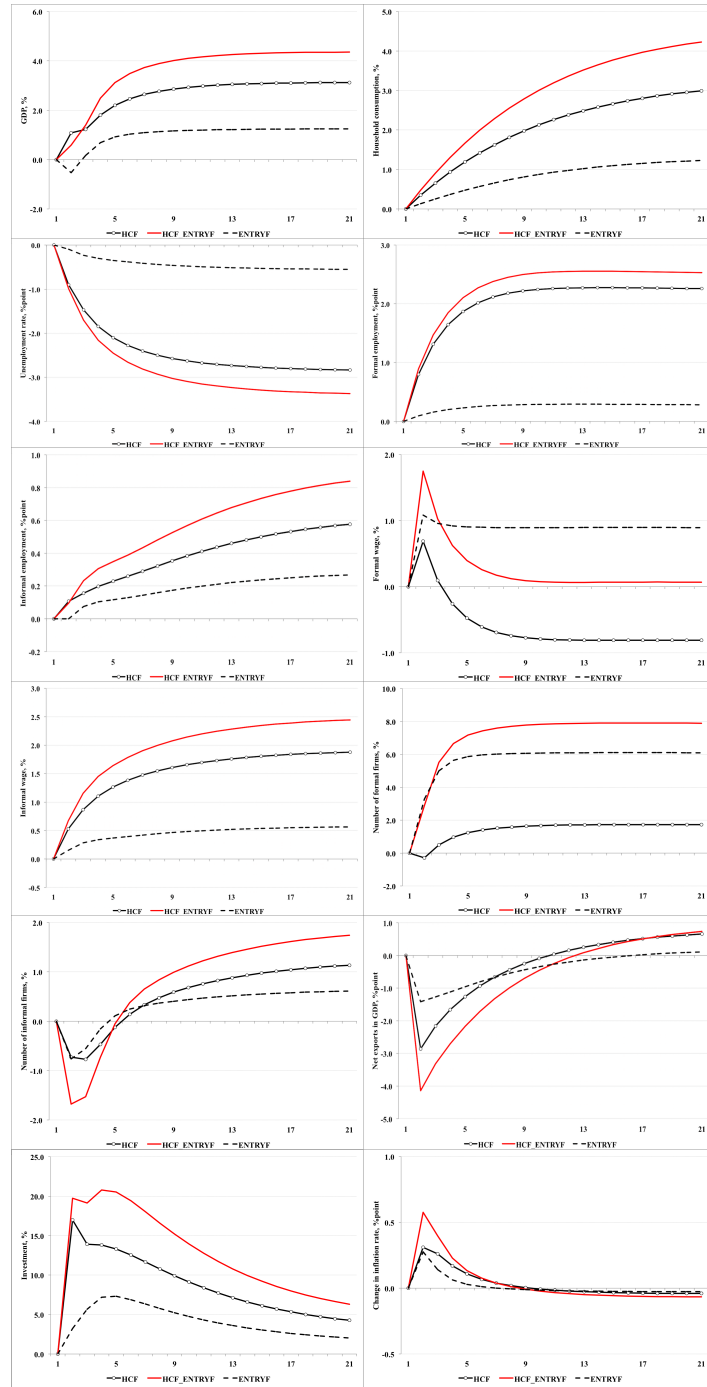


Figure 2.A5: Short-run effects of a combined decrease in formal hiring cost and entry cost in South Africa. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal hiring cost (line with marker), formal entry cost (dotted black line) and their combination (red line).

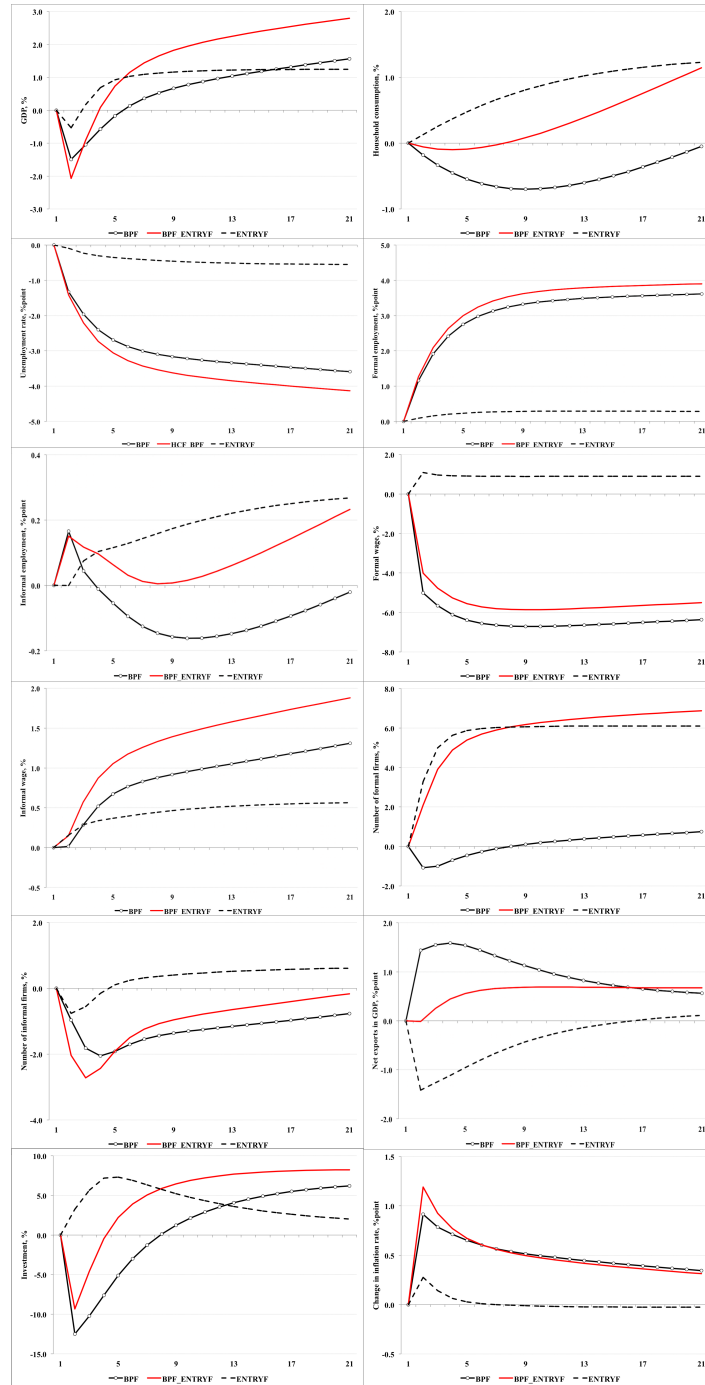


Figure 2.A6: Short-run effects of a combined decrease in formal bargaining power of workers and entry cost in South Africa. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal bargaining power of workers (line with marker), formal entry cost (dotted black line) and their combination (red line).

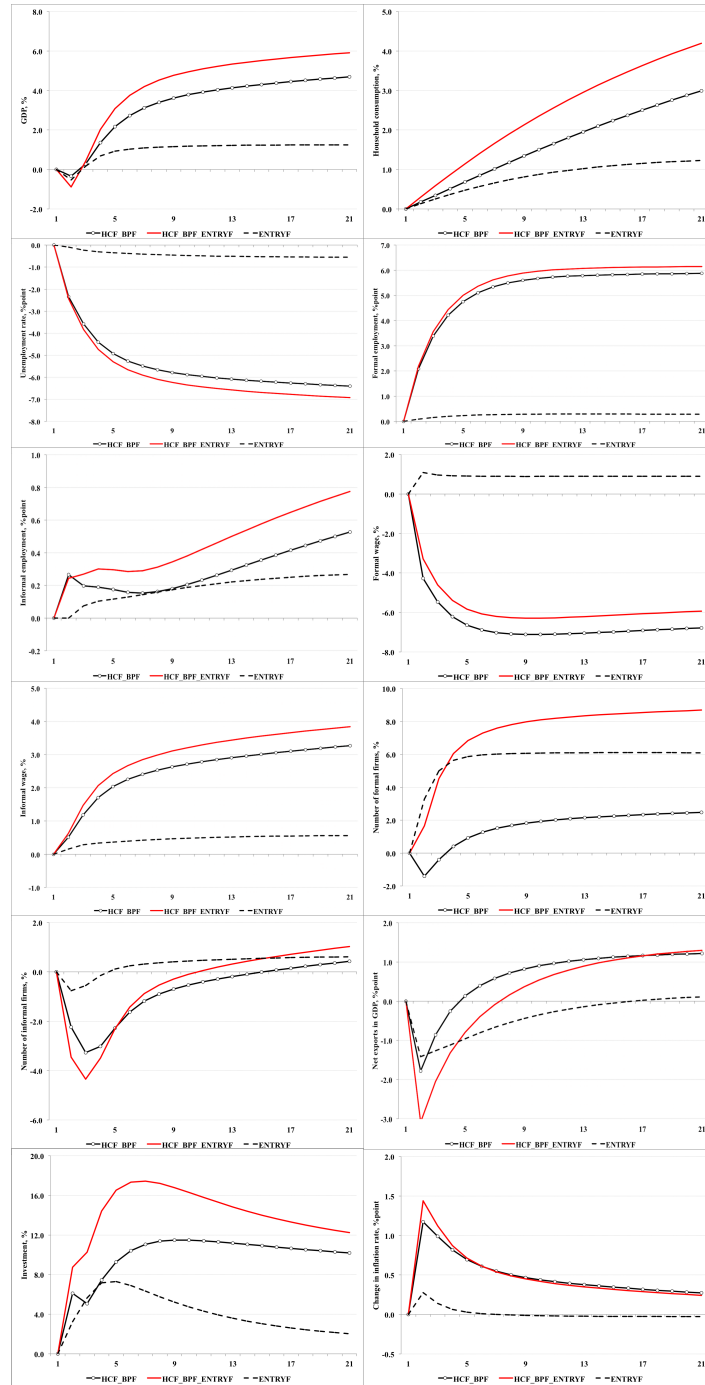


Figure 2.A7: Short-run effects of a combined decrease in formal hiring cost and bargaining power of workers and entry cost in South Africa. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal hiring cost and bargaining power of workers (line with marker), formal entry cost (dotted black line) and their combination (red line).

## E Policy sequence

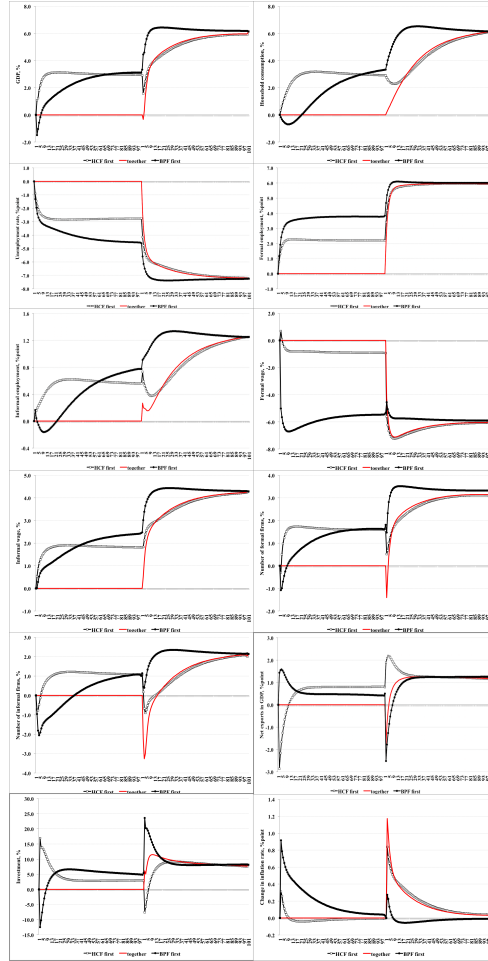


Figure 2.A8: Sequencing of a permanent decrease in formal hiring cost and bargaining power of workers in South Africa. The lines show the long-run (25 years + 25 years) effects of permanent and unexpected policies of size 10 percent. Line with squared marker: first a decrease in formal hiring cost, then a decrease in formal bargaining power of workers. Line with round marker: the opposite. Red line: both policies are implemented during the second 25 years.

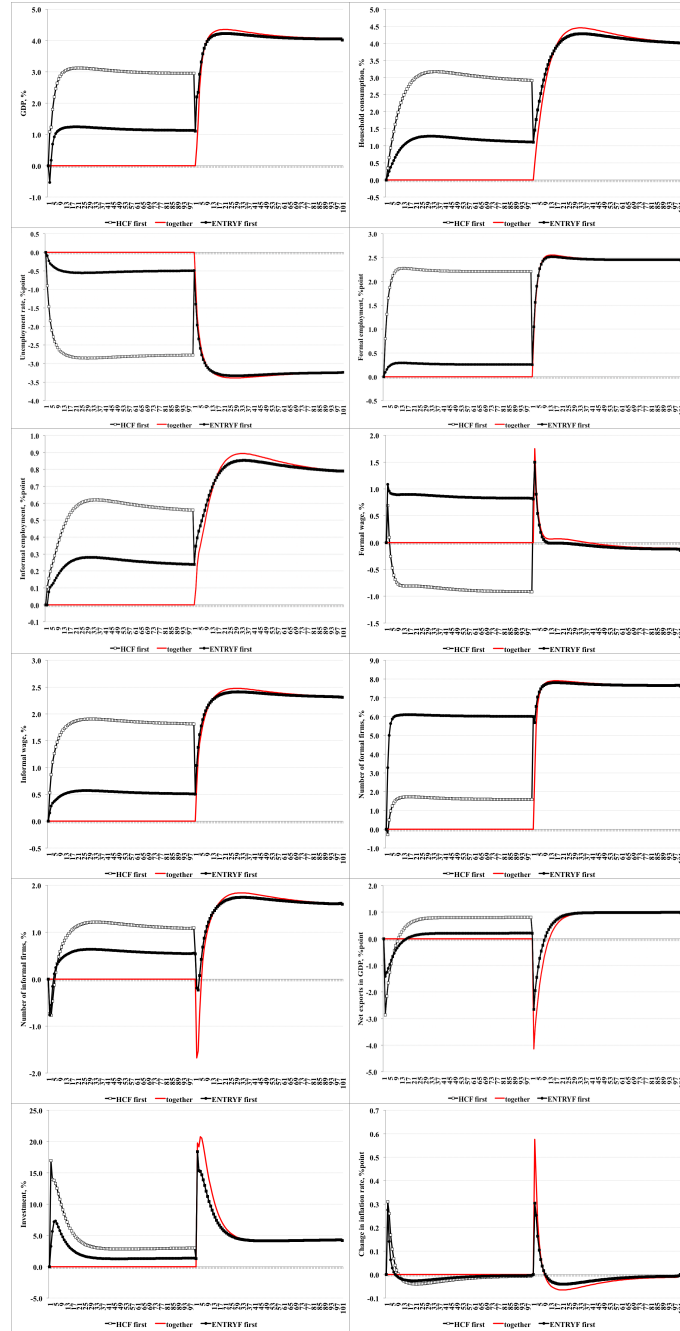


Figure 2.A9: Sequencing of a permanent decrease in formal hiring cost and entry cost in South Africa. The lines show the long-run (25 years + 25 years) effects of permanent and unexpected policies of size 10 percent. Line with squared marker: first a decrease in formal hiring cost, then a decrease in formal entry cost. Line with round marker: the opposite. Red line: both policies are implemented during the second 25 years.

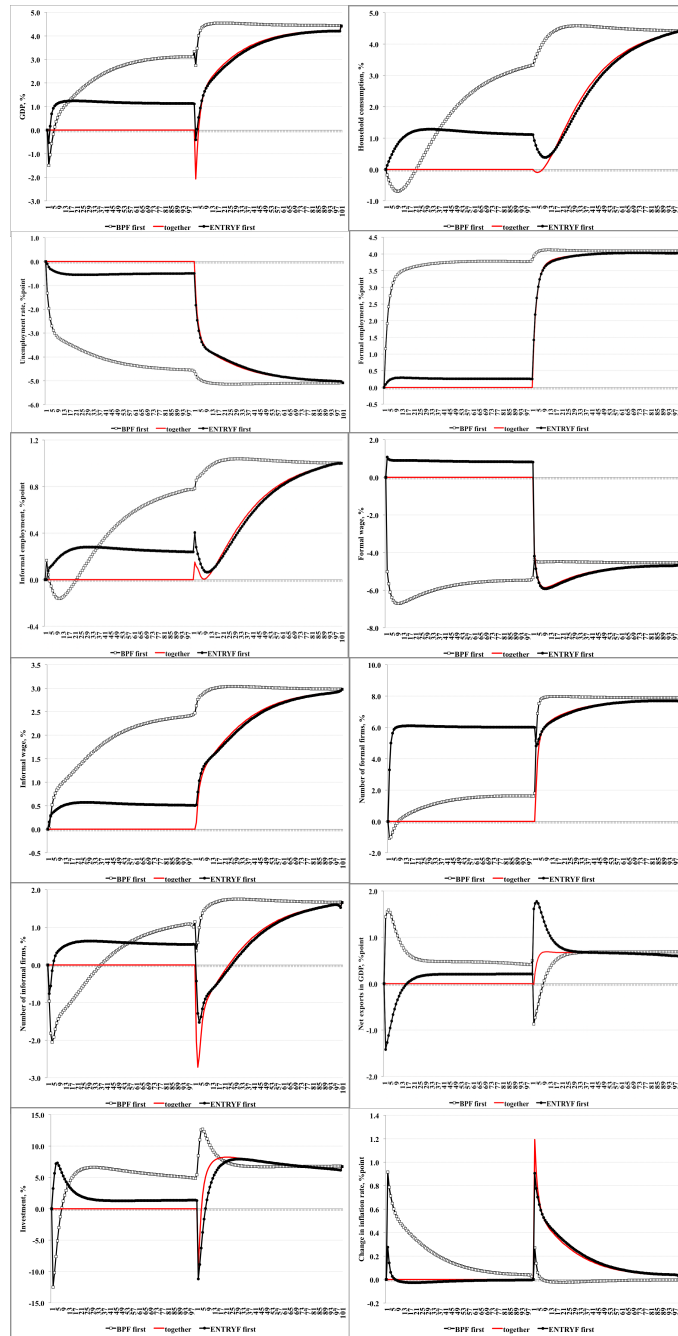


Figure 2.A10: Sequencing of a permanent decrease in formal bargaining power of workers and entry cost in South Africa. The lines show the long-run (25 years + 25 years) effects of permanent and unexpected policies of size 10 percent. Line with squared marker: first a decrease in formal bargaining power of workers, then a decrease in formal entry cost. Line with round marker: the opposite. Red line: both policies are implemented during the second 25 years.

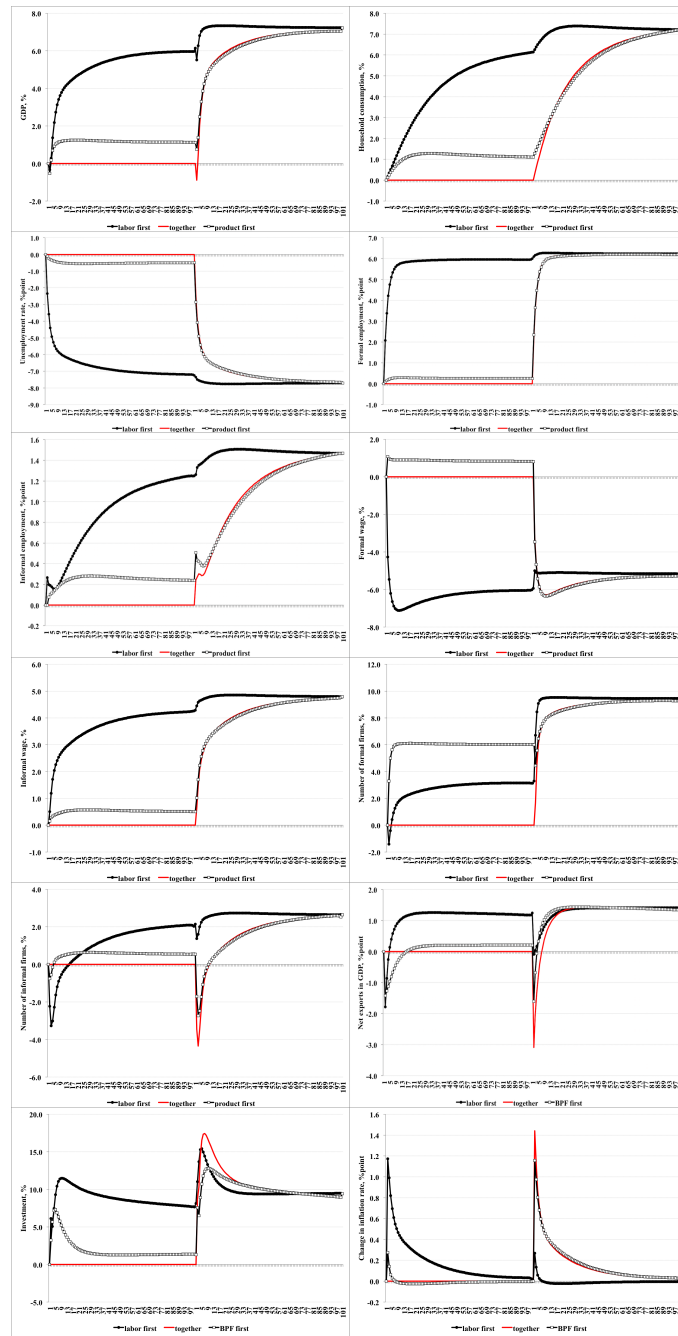


Figure 2.A11: Sequencing of a decrease in formal hiring cost and bargaining power of workers and entry cost in South Africa. The lines show the long-run (25 years + 25 years) effects of permanent and unexpected policies of size 10 percent. Line with squared marker: first a decrease in formal hiring cost and bargaining power of workers, then a decrease in formal entry cost. Line with round marker: the opposite. Red line: both policies are implemented during the second 25 years.



# Bibliography

- [Alesina et al. (2005)] Alesina, A. – Ardagna, S. – Nicoletti, G. – Schiantarelli, F. (2005): Regulation and investment. *Journal of the European Economic Association*, Vol. 3, No. 4 (Jun., 2005), pp. 791-825.
- [Anand-Khera (2016)] Anand, R. – Khera, P. (2016): Macroeconomic Impact of Product and Labor Market Reforms on Informality and Unemployment in India. IMF Working Paper WP/16/47, March 2016. <https://www.imf.org/external/pubs/ft/wp/2016/wp1647.pdf>
- [Andres et al. (2014)] Andres, J. – Arce, O. – Thomas, C. (2014): Structural reforms in a debt overhang. Banco de Espana Working Paper Series No. 1421, 2014. <http://www.bde.es/f/webbde/SES/Secciones/Publicaciones/PublicacionesSeriadas/DocumentosTrabajo/14/Fich/dt1421e.pdf>
- [Batini et al. (2011)] Batini, N. – Levine, P. – Lotti, E. – Yang, B. (2011): Informality, Frictions and Monetary Policy. University of Surrey Discussion Paper 07/11, June 2011. <https://www.surrey.ac.uk/economics/files/dpaperspdf/DP07-11.pdf>
- [Behar-Edwards (2004)] Behar, A. – Edwards, L. (2004): Estimating elasticities of demand and supply for South African manufactured exports using a vector error correction model. Centre for the Study of African Economies Series Working Paper 2004-04. [https://www.researchgate.net/publication/46469498\\_Estimating\\_Elasticities\\_of\\_Demand\\_and\\_Supply\\_for\\_South\\_African\\_Manufactured\\_Exports\\_using\\_a\\_Vector\\_Error\\_Correction\\_Model](https://www.researchgate.net/publication/46469498_Estimating_Elasticities_of_Demand_and_Supply_for_South_African_Manufactured_Exports_using_a_Vector_Error_Correction_Model)
- [Berger-Danninger (2005)] Berger, H. – Danninger, S. (2005): Labor and product market deregulation: partial, sequential, or simultaneous reform? IMF Working Paper WP/05/227, IMF, Dec 2005. <https://www.imf.org/external/pubs/ft/wp/2005/wp05227.pdf>
- [Bernanke et al. (1999)] Bernanke, B. S. – Gertler, M. – Gilchrist, S. (1999): The financial accelerator in a quantitative business cycle framework. In Taylor, J. B. – Woodford, M. (eds): *Handbook of Macroeconomics*, Elsevier, Volume 1, 1999.

- [Besley-Burgess (2004)] Besley, T. – Burgess, R. (2004): Can labor regulation hinder economic performance? Evidence from India. *The Quarterly Journal of Economics*, Febr 2004.
- [Bilbie et al. (2012)] Bilbie, F. O. – Ghironi, F. – Melitz, M. J. (2012): Endogenous entry, product variety, and business cycles. *Journal of Political Economy*, Vol. 120, No. 2 (Apr 2012), pp. 304-345.
- [Blanchard-Gali (2010)] Blanchard, O. – Gali, J. (2010): Labor markets and Monetary Policy: A New Keynesian Model with Unemployment. *American Economic Journal: Macroeconomics*: Vol. 2 No. 2 (April 2010), pp. 1-30.
- [Blanchard-Giavazzi (2003)] Blanchard, O. – Giavazzi, F. (2003): Macroeconomic effects of regulation and deregulation in goods and labor markets. *The Quarterly Journal of Economics*, Vol. 188 (2003), Issue 3 (Aug), 879-907.
- [Boken-Hallett (2008)] Boken, N. – Hallett, A. H. (2008): The Impact of Tax and Market Distortions on the Phillips Curve and the Natural Rate of Unemployment. *Economics*, Vol. 2, 2008-27, September 2008.
- [Boeri-Garibaldi (2007)] Boeri, T. – Garibaldi, P. (2007): Shadow sorting. NBER International Seminar on Macroeconomics 2005, eds. Frankel, J. and Pissarides, C., National Bureau of Economic Research, May 2007. <http://www.nber.org/chapters/c0339.pdf>
- [Cacciatore et al. (2015)] Cacciatore, M. – Duval, R. – Fiori, G. – Ghironi, F. (2015): Short-Term Pain for Long-Term Gain: Market Deregulation and Monetary Policy in Small Open Economies. NBER Working Paper No. 21784, December 2015. <http://www.nber.org/papers/w21784>
- [Cacciatore-Fiori (2016)] Cacciatore, M. – Fiori, G. (2016): The Macroeconomic Effects of Goods and Labor Markets Deregulation. *Review of Economic Dynamics*, Volume 20, April 2016, pages 1–24.
- [Cacciatore et al. (2016)] Cacciatore, M. – Fiori, G. – Ghironi, F. (2016): Market Deregulation and Optimal Monetary Policy in a Monetary Union. *Journal of International Economics*, Volume 99, March 2016, pages 120–137.
- [Charlot et al. (2011)] Charlot, O. – Malherbet, F. – Terra, C. (2011): Product market regulation, firm size, unemployment and informality in developing economies. Institute for the study of labor, IZA Discussion Paper No. 5519, Febr 2011. <http://ftp.iza.org/dp5519.pdf>
- [Christiano et al. (2005)] Christiano, L. J. - Eichenbaum, M. - Evans, C. L. (2005): Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 2005, Vol. 113, No. 1.

- [Conway et al. (2005)] Conway, P. – Janod, V. – Nicoletti, G. (2005): Product market regulation in OECD countries: 1998 to 2003. OECD Economics Department Working Papers, No. 419, OECD Publishing. [http://www.oecd-ilibrary.org/economics/product-market-regulation-in-oecd-countries\\_783417550348?crawler=true](http://www.oecd-ilibrary.org/economics/product-market-regulation-in-oecd-countries_783417550348?crawler=true)
- [Dabla-Norris et al. (2013)] Dabla-Norris, E. – Ho, G. – Kochhar, K. – Kyobe, A. – Tchaidze, R. (2013): Anchoring Growth: The Importance of Productivity-Enhancing Reforms in Emerging Market and Developing Economies. IMF Staff Discussion Note 13/08, December 2013. <https://www.imf.org/external/pubs/ft/sdn/2013/sdn1308.pdf>
- [Dixit-Stiglitz (1977)] Dixit, A. K. – Stiglitz, J. E. (1977): Monopolistic Competition and Optimum Product Diversity. *The American Economic Review*, Vol. 67, No. 3. (Jun., 1977), pp. 297-308.
- [DTI (2008)] DTI (2008): Annual Review of Small Business of South Africa 2005-2007. Department of Trade and Industry, South Africa. [http://www.dti.gov.za/sme\\_development/docs/3%20Annual%20Review%20Final%20Report%2011%20Aug%2008.pdf](http://www.dti.gov.za/sme_development/docs/3%20Annual%20Review%20Final%20Report%2011%20Aug%2008.pdf)
- [Ebell-Haefke (2009)] Ebell, M. – Haefke, C. (2009): Product market deregulation and the US unemployment miracle. *Review of Economic Dynamics* 12 (2009) 479-504.
- [Eggertson et al. (2014)] Eggertson, G. – Ferrero, A. – Raffo, A. (2014): Can structural reforms help Europe? *Journal of Monetary Economics* 61 (2014) 2-22.
- [Estevao (2005)] Estevao, M. M. (2005): Product market regulation and the benefits of wage moderation. IMF Working Paper WP/05/191, Sept 2005. <http://www.imf.org/external/pubs/cat/longres.aspx?sk=18576.0>
- [Fernandez-Villaverde et al. (2014)] Fernandez-Villaverde, J. – Guerron-Quintana, P. A – Rubio-Ramirez, J. (2014): Supply-side policies and the zero lower bound. *IMF Economic Review* (2014) 62, 248–260, July 2014.
- [Gerali et al. (2014)] Gerali, A. – Notarpietro, A. – Pisani, M. (2014): Macroeconomic effects of simultaneous implementation of reforms after the crisis. Bank of Italy Working Paper Number 997, November 2014. [https://www.bancaditalia.it/pubblicazioni/temi-discussione/2014/2014-0997/en\\_tema\\_997.pdf](https://www.bancaditalia.it/pubblicazioni/temi-discussione/2014/2014-0997/en_tema_997.pdf)
- [Gali (2011)] Gali, J. (2011): The Return of The Wage Philips Curve. *Journal of the European Economic Association*, 9(3), 436-461.
- [Gomes et al. (2013)] Gomes, S. – Jacquinot, P. – Mohr, M. – Pisani, M. (2013): Structural Reforms and Macroeconomic Performance in the Euro Area Countries: A Model-Based Assessment. *International Finance* 16:1, 2013: pp. 23-44.

- [Gnocchi et al. (2015)] Gnocchi, S. – Lagerborg, A. – Pappa, E. (2015): Do labor market institutions matter for business cycles? *Journal of Economic Dynamics and Control* 51 (2015) 299-317.
- [ILO (2002)] ILO (2002): Women and men in the informal economy: A statistical picture. International Labor Organisation, 2002. [http://www.ilo.org/wcmsp5/groups/public/---dgreports/---stat/documents/publication/wcms\\_234413.pdf](http://www.ilo.org/wcmsp5/groups/public/---dgreports/---stat/documents/publication/wcms_234413.pdf)
- [IMF (2015)] IMF (2015): Structural Reforms and Macroeconomic Performance: Initial Considerations for the Fund. International Monetary Fund Staff Report, November 2015. <http://www.imf.org/external/np/pp/eng/2015/101315.pdf>
- [Karoro et al. (2008)] Karoro, T. D. – Aziakpono, M. J. – Cattaneo, N. (2008): Exchange rate pass-through to import prices in South Africa: Is there asymmetry? Rhodes University, Working Paper Number 79, June 2008. [http://www.econrsa.org/papers/w\\_papers/wp79.pdf](http://www.econrsa.org/papers/w_papers/wp79.pdf)
- [Kugler-Pica (2003)] Kugler, A. – Pica, G. (2003): Effects of employment protection and product market regulations on the Italian labor market. Institute for the study of labor, IZA Discussion Paper No. 948, Dec 2003. <http://ftp.iza.org/dp948.pdf>
- [Lombardo-Vestin (2007)] Lombardo, G. - Vestin, D. (2007): Welfare Implications of Calvo vs. Rotemberg Pricing Assumptions. European Central Bank Working Paper Series, NO 770, June 2007. <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp770.pdf?78b9fd3e2ad1020b18eb3f4f43882a52>
- [Lusinyan-Muir (2013)] Lusinyan, L. – Muir, D. (2013): Assessing the Macroeconomic Impact of Structural Reforms: The Case of Italy. IMF Working Paper 13/22, January 2013. <https://www.imf.org/external/pubs/ft/wp/2013/wp1322.pdf>
- [Mirus-Smith (1977)] Mirus, R. – Smith, R. S. (1997): Canada's Underground Economy: Measurement and Implications, in The Underground Economy: Global Evidence of its Size and Impact, edited by Lippert, O. and Walker, M., *The Fraser Institute*, 1997.
- [Mortensen-Nagypal (2007)] Mortensen, D. T. – Nagypal, E. (2007): More on unemployment and vacancy fluctuations. *Review of Economic Dynamics* 10 (2007) 327-347.
- [Nicoletti-Scarpetta (2003)] Nicoletti, G. – Scarpetta, S. (2003): Regulation, productivity, and growth: OECD evidence. World Bank Policy Research Working Paper 2944, Jan 2003. <http://elibrary.worldbank.org/doi/abs/10.1596/1813-9450-2944>
- [Pappa et al. (2015)] Pappa, E. – Sajedi, R. – Vella, E. (2015): Fiscal consolidation with tax evasion and corruption. *Journal of International Economics* 96 (2015) S56-S75.

- [Rotemberg (1982)] Rotemberg, J. J. (1982): Sticky prices in the United States. *Journal of Political Economy*, Vol. 90, No. 6 (Dec., 1982), pp. 1187-1211.
- [Schmitt-Grohe-Urbe (2003)] Schmitt-Grohe, S – Uribe, M. (2003): Closing small open economy models. *Journal of International Economics*, Elsevier, vol. 61(1), pp. 163-185, Oct 2003.
- [Schneider (2005)] Schneider, F. (2005): Shadow economies around the world: what do we really know? *European Journal of Political Economy*, Vol. 21 (2005), 598-642.
- [Schneider (2012)] Schneider, F. (2012): The shadow economy and work in the shadow: what do we (not) know? Institute for the study of labor, IZA Discussion Paper No. 6423, March 2012. <http://ftp.iza.org/dp6423.pdf>
- [Schneider et al. (2010)] Schneider, F. – Buehn, A. – Montenegro, C. E. (2010): New estimates for the shadow economies all over the world. *International Economic Journal*, 24:4, 443-461.
- [Schneider-Enste (2000)] Schneider, F. – Enste, D. H. (2000): Shadow economies: size, causes, and consequences. *Journal of Economic Literature*, Vol. XXXVIII, March 2000.
- [Smets-Wouters (2007)] Smets, F. - Wouters, R. (2007): Shocks and Frictions in US Business Cycles A Bayesian DSGE Approach. Working Paper NO 722, European Central Bank, February 2007. <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp722.pdf?038767f7a645096eccd34a8058c562e0>
- [SSA (2006)] SSA (2006): Survey of Employers and the Self-Employed. *Statistics South Africa*, November 2006. <http://www.statssa.gov.za/publications/P0276/P02762005.pdf>
- [SSA (2010)] SSA (2010): Survey of Employers and the Self-Employed. *Statistics South Africa*, October 2010. <http://www.statssa.gov.za/publications/P0276/P02762009.pdf> <https://www.statssa.gov.za/publications/P0276/P02762009.pdf>
- [SSA (2014)] SSA (2014): Survey of Employers and the Self-Employed. *Statistics South Africa*, August 2014. <http://www.statssa.gov.za/publications/P0276/P02762013.pdf>
- [Steinbach et al. (2009)] Steinbach, R. – Mathuloe, P. – Smit, B. (2009): An open economy New Keynesian DSGE model of the South African economy. *South African Journal of Economics*, Volume 77, Issue 2, pp. 207-227, June 2009.

- [Vogel (2014)] Vogel, L. (2014): Structural reforms at the zero lower bound. Economic Papers 537, European Commission, November 2014. [http://ec.europa.eu/economy\\_finance/publications/economic\\_paper/2014/pdf/ecp537\\_en.pdf](http://ec.europa.eu/economy_finance/publications/economic_paper/2014/pdf/ecp537_en.pdf)
- [Williamson (1975)] Williamson, O. E. (1975): Markets and Hierarchies, Analysis and Antitrust Implications: A Study in the Economics of Internal Organization. *The Free Press*, December 1975.

# Chapter 3

## Aging, Pension Reforms and the Shadow Economy

*with Daniel Baksa*

### 3.1 Motivation and literature

Today, several countries suffer from a high level of public debt (Figure 3.1). Indeed, by now, government debt reached more than 60 per cent of GDP in about half of the European countries. Moreover, in some countries, e.g. in southern Europe, it reached more than 100 per cent. Old-age pensions play a pivotal part in this story (Figure 3.2).<sup>1</sup> Pension expenditure accounts for about 5-10, but often even more, per cent of GDP, hence, it exceeds spending on defence, economic affairs, environmental protection, health or education (1.5, 4.5, 0.8, 7 and 5.1 per cent, respectively).<sup>2</sup>

Looking ahead, the share of old people is projected to increase significantly in the next decades (Figure 3.3). Southern Europe will be especially strongly affected. By 2065, the proportion of people above age 65 is predicted to rise to 34.7, 29.9 and 28.4 per cent from the current 19.8, 21.3 and 18.2 per cent in Portugal, Italy and Spain, respectively. Portugal will have the second-highest ratio among the European economies. Thus, the burden on working-age people will increase, specifically, the old-age dependency ratio will more or less double (Figure 3.4). As such, each working person will be supporting 0.6 old people. An expected longevity boom is the driving factor (Figure 3.5); in five decades, sixty-five year-olds will live about 5 years longer than they do today.<sup>3</sup> Meanwhile, the fertility rate in southern Europe will slightly increase during the period (OECD (2011)).

---

<sup>1</sup>Data on Figure 3.2 shows the sum of public and private old-age pension spending.

<sup>2</sup>According to Eurostat data.

<sup>3</sup>Textboxes on chart show the exact change in additional life expectancy at age 65.

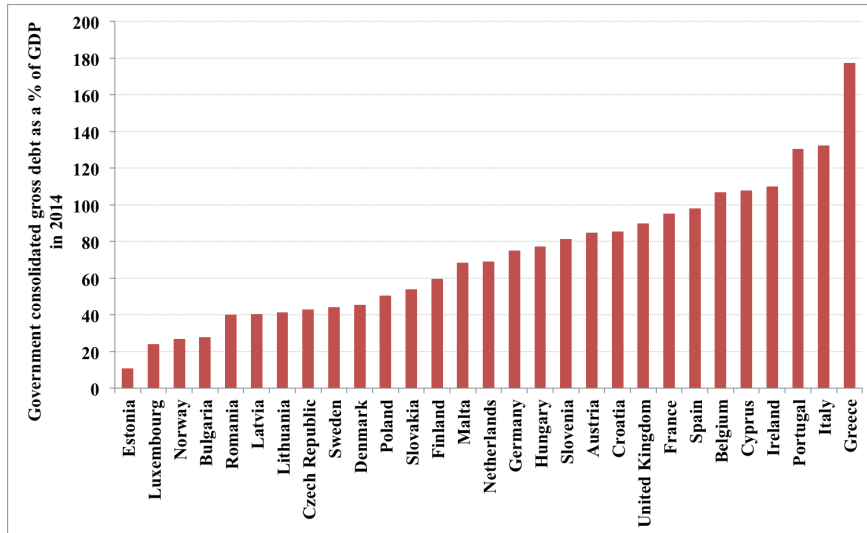


Figure 3.1: Public debt today (Eurostat)

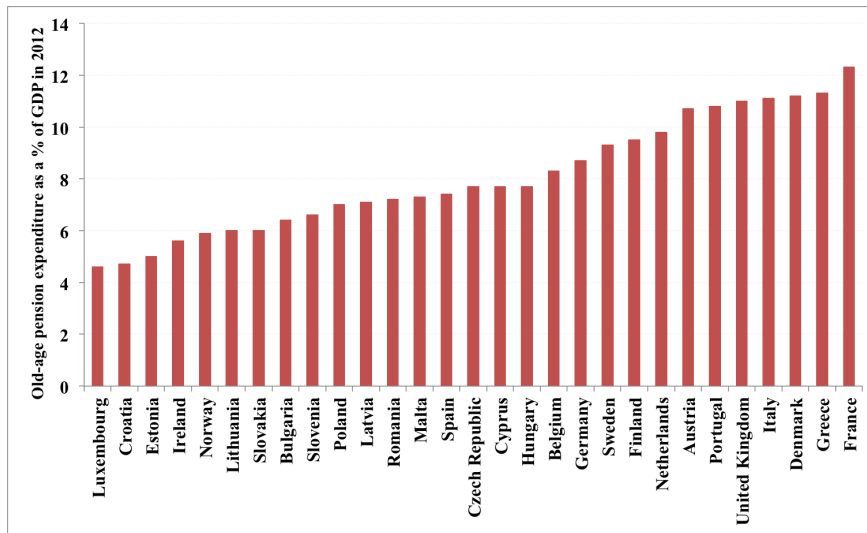


Figure 3.2: Old-age pension spending today (Eurostat)

As Gora (2014) signals, originally, the public old-age pension system was introduced to reduce old-age poverty. Technical constraints made it impossible to implement individualized participation; hence, taxation was used to finance pensions. As such, most southern European countries have a pay-as-you-go (PAYG) pension system<sup>4</sup>, although some elements of the Portuguese and the Spanish regimes could be considered as fully funded (FF). Political convenience also played a role at the time of the introduction, as tax rates were relatively

<sup>4</sup>There are two types of PAYG systems (World Bank (1994)). A 'defined benefit scheme' fixes pensions and allocates taxes accordingly, while a 'defined contribution scheme' fixes taxes that are then redistributed. Our model is closer to the first one, in practice, most systems are a combination of these two.



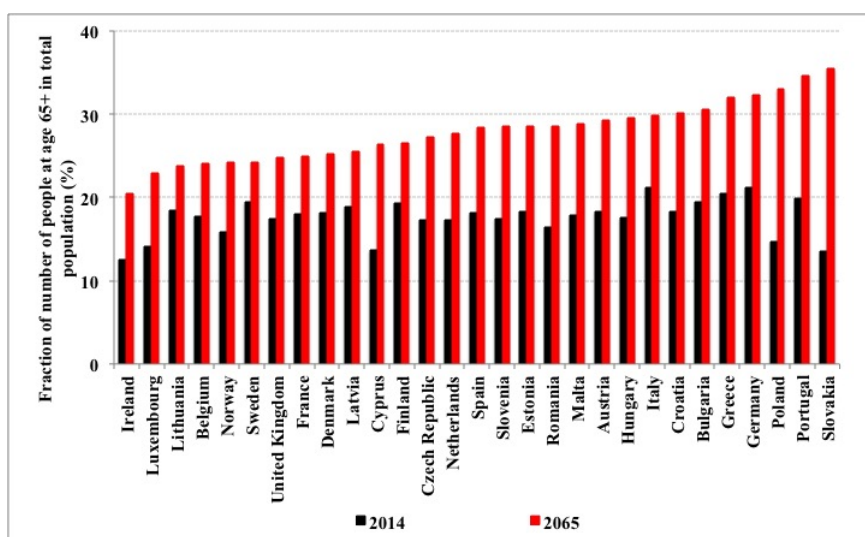


Figure 3.3: Share of elderly people today and 50 years ahead (Eurostat)

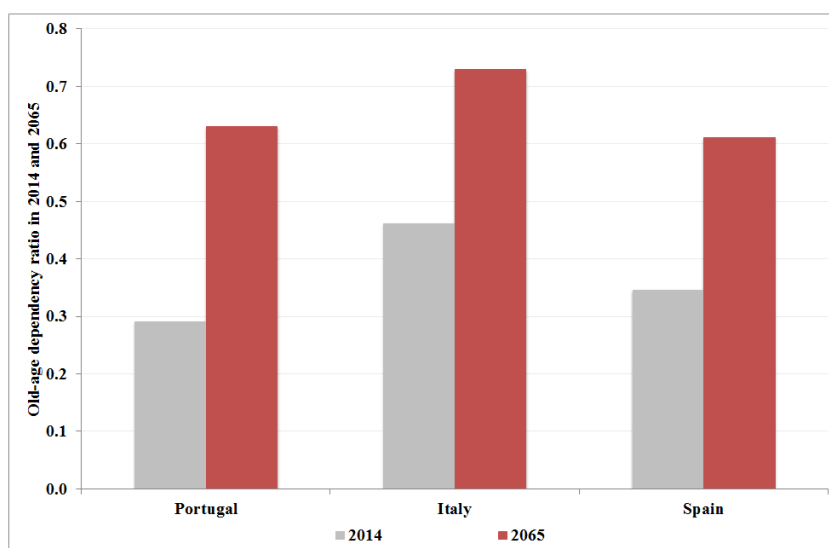


Figure 3.4: Old-age dependency ratio today and 50 years ahead (Eurostat)

low. Moreover, because life expectancy was lower than it is today, most workers died before retiring. Thanks to an increase in longevity, however, pension spending is increasing now. With relatively high debt levels, handling the fiscal consequences of aging is becoming more urgent every day. Because Portugal, Italy and Spain are among those with the highest public debt level and are projected to experience the most severe aging in the next decades, we focus on them. By doing so, we aim to contribute to the literature on macroeconomic impacts of public old-age pension reforms<sup>5</sup> and other public policies when population is aging.

<sup>5</sup>Hence, we focus on the first pillar and disregard the second (contributory forced savings) and the third (contributory voluntary savings) pillars; the terminologies follow World Bank (1994). Also, we do not study the impacts of strengthening tax collection, we leave it for further research.

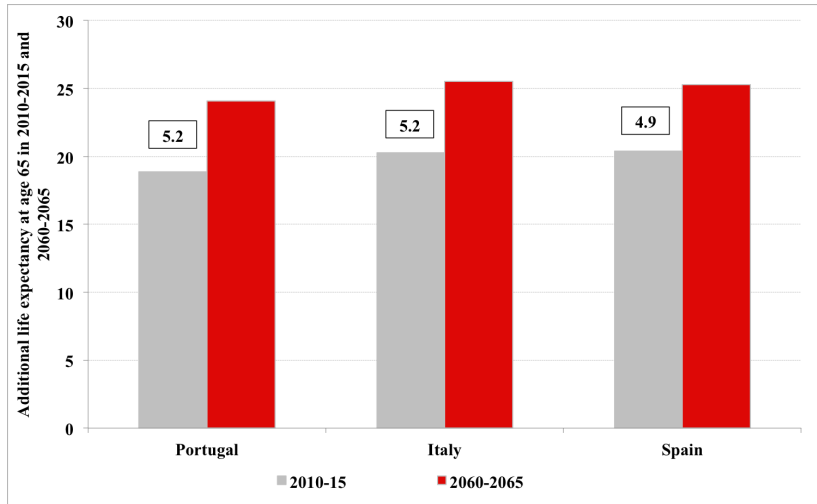


Figure 3.5: Additional life expectancy at age 65 today and 50 years ahead (OECD)

Nevertheless, other streams of the literature are not less important. Some investigate the political feasibility of such reforms, which is beyond the scope of our paper. So are optimal pension plan or optimal retirement age studies. Some examples are, without providing a detailed review, Galasso (2008), Heijdra-Romp (2009) or Beetsma et al. (2013). Also, for example, Conesa-Krueger (1999) study the political implications of moving from an unfunded to an FF system. Then, regarding the impacts of a retirement-age increase on inequality, one example is Baker-Rosnick (2012). Additionally, our model is a closed economy, while several studies focus on international spillovers, for instance Borsch-Supan et al. (2006), Razin-Sadka (1999), Storesletten (2000) or Adema et al. (2009).

By macroeconomic impacts we mainly mean output and unemployment reactions; the latter are frequently neglected in this context. Some studies without unemployment are Borsch-Supan et al. (2006), Kilponen et al. (2006), Nickel et al. (2008), Karam et al. (2010), Braz et al. (2013) or McGrattan-Prescott (2015). Furthermore, Corneo-Marquardt (1999) conclude that unemployment is independent of social security systems. In our view, however, it is somewhat misleading to assume full employment, or, to consider unemployment to be independent of pension systems. The obvious rigidities in the labor market (job hunting requires money and time on both the employer and the employee side) imply that not everyone seeking employment can find a job (at least not immediately). It is especially true for the short-run, for pension and in general for fiscal policies. Brauning (2005), Ono (2007), Ono (2010), Marchiori et al. (2011) and de la Croix et al. (2013) claim, however, that there is a relation between unemployment and social security. Also, Pierrard-Snessens (2009), Marchiori-Pierrard (2012) and Marchiori-Pierrard (2015) include unemployment into their framework. And so do we; that is, we incorporate unemployment into a New Keynesian model with overlapping generations (OLG) and demography, and, we study the importance of this channel in our model. According to our knowledge, OGRE is the first Blanchard-

Yaari-type model with unemployment. Regarding the data, unemployment is already at high levels in southern Europe, first of all in Spain where it is around 16 per cent, but also in Portugal and Italy (9.2 and 9.4 percent, respectively).<sup>6</sup>

The main novelty is extending the model with a shadow<sup>7</sup> economy, meaning tax evasion and a lower level of regulation in the underground sector.<sup>8</sup> To the best of our knowledge, this paper is the first to present an OLG and demography model with informality. Also, we are not aware of any (quantitative) papers that investigate the macroeconomic effects of public old-age pension reforms in the presence of informality, nor one that examines the impacts of such reforms on the underground sector.<sup>9</sup> There are not too many papers on/with informality in related literatures, either. We believe that, with informality, Charlot et al. (2011) is the only quantitative model to study the macroeconomic impacts of deregulation policies, while Ihrig-Moe (2004) and Pappa et al. (2015) are the only papers as regards the impacts of fiscal consolidation. As such, our work fills a major lacuna in the literature.

We contend that neglecting the shadow economy leaves a framework markedly incomplete because of several reasons. First, as the shadow economy is by definition in the shadow, public policies can only affect directly the non-shadow side of the economy. Consequently, the larger the underground economy, the smaller will be the fraction of the economy directly influenced by the government. The reason is that only formal-sector firms and workers pay taxes which can be modified by the government, or, only they are affected by the level of regulation (unionization e.g.) which, again, can be influenced by the public sector. Similarly, only formal sector workers and firms contribute to social security.

Second, the official and unofficial sectors interact. For instance, workers move between them. And, the higher the size of informality, potentially the more workers can move out of the shadow. The same is also true for goods. Furthermore, some studies claim that working in the shadow has been theorized to function as a kind of “insurance policy”; the “shadow employment is tolerated because its repression increases unemployment” (Boeri-Garibaldi (2007), page 125). Neglecting these interactions might bias the macroeconomic impacts of

---

<sup>6</sup>See Table 3.3.

<sup>7</sup>We use the terms ‘shadow’, ‘informal’, ‘underground’ and ‘unofficial’ interchangeably.

<sup>8</sup>As Schneider (2012) page 6) claims: “The shadow economy includes all market-based legal production of goods and services that are deliberately concealed from public authorities for the following reasons: 1. to avoid payment of income, value added or other taxes, 2. to avoid payment of social security contributions, 3. to avoid having to meet certain legal labor market standards, such as minimum wages, maximum working hours, safety standards, etc., and 4. to avoid complying with certain administrative obligations, such as completing statistical questionnaires or other administrative forms.”

<sup>9</sup>Regarding the latter, two empirical papers (Kumler et al. (2013), Mao et al. (2013)) are closest to ours. Also, we do not aim to study the macroeconomic and pension effects of reducing the share of the informal economy; that is an interesting question, but it is beyond the scope of our paper.

pension reforms or generally those of fiscal policy.

Third, working in the shadow is relevant not only because it affects public revenues in general, but also because it has an effect on social security, in particular. In a PAYG regime, revenues collected today are used to finance public old-age pension spending today, while in a fully funded system the current social security contributions deducted from firms and workers finance future public old-age pension spending. And, those who work informally do not contribute to social security. People work in the shadow which, in a pay-as-you-go system, reduces pension benefits or increases public debt, while in a fully funded regime it results in no pension savings for the period spent in the shadow. Hence, even though in the short-run underground employment might function as an “insurance”, in the long-run it might be costly for both the people and the state.

Lastly, in most countries the shadow economy is rather sizable (Figure 3.6). This is true for many developed economies as well as emerging countries. Specifically, 23, 27 and 22.5 per cent of output is in the shadow in Portugal, Italy and Spain, respectively (Schneider et al. (2010), Table 2 on page 454).<sup>10</sup>

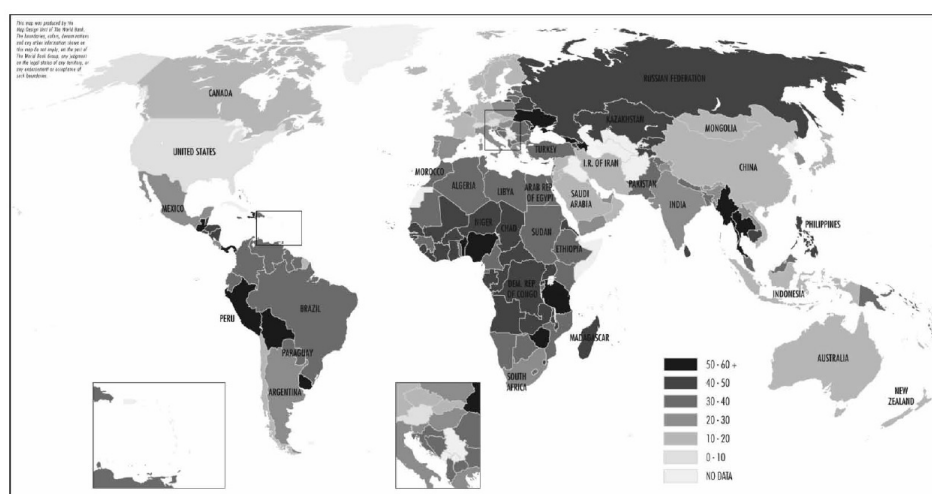


Figure 3.6: The size of shadow output (Schneider et al. (2010), page 458)

Additionally, according to our knowledge, with the exception of Fehr (2000), Kilponen et al. (2006), Diaz-Gimenez-Diaz-Saavedra (2009) and Goraus et al. (2014)<sup>11</sup>, the macroeconomic impacts of a retirement age increase are not studied, the focus is usually on social security contribution rates and pension-wage replacement rates. It is rather surprising, given

<sup>10</sup>Informality can also be measured in terms of employment, and the share of informal employment might differ from the share of informal output (Schneider (2012)).

<sup>11</sup>Only impacts on output are investigated, but not those on unemployment in ? and ?.

the fact that according to OECD (2012) 28 out of 34 OECD countries planned or already started to raise the level of retirement age. Nevertheless, all the papers that investigate the impacts of pension reforms, do not compare them with those of other fiscal policies. However, we do study a retirement age policy as well and we compare the impacts of pension reforms with those of other public policies, which is a further contribution of us towards the literature.

Finally, regarding the types of pension plans, the majority of research has been done either on reforming the PAYG plan (e.g. Nickel et al. (2008) and Karam et al. (2010), also the AINO, PESSOA and LOLA models<sup>12</sup>, or, on a switch from the PAYG towards an FF system (e.g. Borsch-Supan et al. (2006) and McGrattan-Prescott (2015)). We, like Marchiori et al. (2011) or de la Croix et al. (2013), examine both, and, we also study reform policies in a fully funded regime like Borsch-Supan-Ludwig (2011). We study regime switches, even though they are rare (examples are Columbia, Poland and Hungary; although in the latter two instances implementation was not finalized), because the incentives of workers only change if we move towards an FF plan, reforming the current PAYG system only might not be enough to handle the challenges of aging. Also, it is crucial to compare the impacts of the same policies in the two regimes in order to see not only the consequences of introducing a fully funded regime but also those of fiscal policies in the fully funded regime.

One of our main findings is that with aging population a retirement age increase causes the smallest decline in long-run GDP in all three countries we study. In general, following any policy, the majority of the GDP loss happens over the medium-term. The opposite is true for unemployment; the long-run cost is usually low (except labor income tax policies), but the short-run cost is more severe. Also, informality somewhat increases, especially after tax hikes.

Regarding the differences in the macroeconomic impacts of fiscal policies between the two pension regimes, we can conclude that they are sizeable. It is especially true for the personal income tax and social security contributions revenue<sup>13</sup> and for the long-run outcomes, but, following other policies and in the short-run, too, there are distinctions.

Furthermore, as concerns the pension regime switch, in the long-term, informality declines, both in terms of output and employment. At the same time, there is a crucial cost over the

---

<sup>12</sup>AINO: Kilponen et al. (2006), PESSOA: Braz et al. (2013) and Almeida et al. (2013), LOLA: Pierrard-Snessens (2009), Marchiori-Pierrard (2012) and Marchiori-Pierrard (2015), which are summarized in Dieppe et al. (2015)

<sup>13</sup>In the paper we often refer to this as personal income tax revenue only, because the two imply the same effects.

transition, namely, a temporary but sharp drop in the implied pension-wage replacement rate.

Unemployment and the underground sector prove to be vital. In particular, we are not only able to investigate the impacts of pension and fiscal policies on them, also, their presence modifies the reactions of the macroeconomy in general. Impulse response functions of labor income tax hikes highly depend on their presence; reactions to other policies and the pension regime switch are also affected, but to a less extent.

Finally, we can identify some policy-related concerns. To neutralize the negative fiscal impacts of the projected aging in Portugal (i.e. 34 pp increase in the old-age dependency ratio according to Eurostat data by 2065), the retirement age should be raised by 2 years. The effective retirement age is already quite high compared to other countries (66.6 years), thus, the fiscal space is quite narrow. Still, it is feasible given the Portugal life expectancy. However, it might not be feasible from a political point of view. There is a loose connection between statutory and effective retirement ages which might be explained by the incentives of employees. Additionally, there is a large cost in the form of a lower replacement rate while introducing a fully funded regime.

In the next section we describe the modeling framework. Then, we present the process of calibration, and, evaluate the performance of the model's steady state in matching the data. After that, we provide the results: i) long- and short-run impacts in a PAYG regime, ii) long- and short-run impacts with PAYG and FF plans, iii) pension regime switch and iv) the role of informality and unemployment. Finally, we close our paper with some policy discussion.

## 3.2 Model

The model is a dynamic general equilibrium (DGE) model with overlapping generations (OLG) and demography by Blanchard (1985) and Yaari (1965), unemployment following Blanchard-Gali (2010), price stickiness à la Rotemberg (1982) and a shadow economy. The model is called **OGRE** which stands for Dynamic General Equilibrium Model with **O**verlapping **G**enerations, **D**emography, **U**nemployment and a **S**hadow **E**conomy to **S**tudy **R**etirement (Public Old-Age Pension) and other Fiscal Policies. The main novelty is the distinction between formal and informal labor and goods markets in an OLG setup.

Considering in detail the OLG setup, we can distinguish two cohorts. The young (the workers) either work and pay labor income taxes, or are unemployed and receive unemployment benefits from the government. The old (the retired) do not work, but receive a public

old-age pension from the government.<sup>14</sup> Concerning the demographic environment, population is not constant over time, as young people are born and old people pass away with some probability. Also, young people retire with a given probability, and the pension of the new retired people is based on their past average wage, which is then indexed by inflation.

With regard to informality, taxes are avoided in the informal labor and goods markets. Specifically, i) labor income is only taxed in the formal sector (both on the employer and the employee side) and ii) value-added taxes are only paid after goods purchased from formal sector-producers. Another major difference between the formal and informal sectors is that the formal sector is more regulated than the informal one, meaning that labor and product market rigidities are higher in the formal sector (Williamson (1975)). Finally, the government can only buy formal goods, and the same is true for private investment.

### 3.2.1 Demography

The total number of population is  $N_t$  which is the sum of young/worker ( $N_t^Y$ ) and old/retired ( $N_t^O$ ) people. Young people retire with a probability of  $\omega_{t-1}^Y$ , while old people die with a probability of  $\omega_{t-1}^O$ . Furthermore,  $n_t$  is the fertility rate which shows the birth rate of new young (worker) people. This is a net rate, we do not model those who do not work due to their age (students), neither the mortality of the young, nor migration. So, the relevant demographic equations are:

$$N_t = N_t^Y + N_t^O \quad (3.1)$$

$$N_t^Y = (1 - \omega_{t-1}^Y)N_{t-1}^Y + n_t N_{t-1}^Y \quad (3.2)$$

$$N_t^O = (1 - \omega_{t-1}^O)N_{t-1}^O + \omega_{t-1}^Y N_{t-1}^Y \quad (3.3)$$

### 3.2.2 Overlapping Generations

The setup, which is also called the Blanchard-Yaari framework, follows Blanchard (1985) and Yaari (1965). There are two overlapping generations: the young (worker) and the old (retired). First, we describe the latter cohort.

#### The retired cohort

'Retired' agent  $i$  of retired cohort  $a$  is one individual who retired  $a$  years ago. She maximises the following Bellman equation:

$$\begin{aligned} V^O(B_{a-1,t-1}^O(i)) &= \\ &= \max \left\{ (1 + \epsilon_t^C) \left[ \frac{1}{1 - \gamma} \left\{ C_{a,t}^{O,F}(i) \right\}^{1-\gamma} + \frac{\chi}{1 - \gamma} \left\{ C_{a,t}^{O,I}(i) \right\}^{1-\gamma} \right] + \right. \\ &\quad \left. + \beta E_t(1 - \omega_t^O) V^O(B_{a,t}^O(i)) \right\} \end{aligned} \quad (3.4)$$

---

<sup>14</sup>We use the terms 'young' and 'worker' interchangeably in our paper, as we do the terms 'old' and 'retired'.

subject to this budget constraint:

$$\begin{aligned} (1 + \tau_t^C)C_{a,t}^{O,F}(i) + p_t^I C_{a,t}^{O,I}(i) + (1 - \omega_t^O)B_{a,t}^O(i) = \\ = (1 + r_{t-1})B_{a-1,t-1}^O(i) + TR_{a,t}^{PG,YO}(i) + TR_{a,t}^{FF,YO}(i) + Profit_{a,t}^O(i) - T_{a,t}^O(i) \end{aligned} \quad (3.5)$$

The retired agent does not work, but receives public pension benefits:  $TR_{a,t}^{PG,YO}(i)$  in a PAYG regime and  $TR_{a,t}^{FF,YO}(i)$  in a fully funded regime (where  $YO$  denotes pension benefits of those who just retired (were young one period before)). Pension benefits of the newly retired are determined in the period when they get retired and are indexed by inflation.<sup>15</sup> Also, some share of profits minus lump-sum taxes are received by the retired. The agent consumes formal and informal goods. Specifically, its utility depends on consuming goods produced by formal firms ( $C_{a,t}^{O,F}(i)$ ) and consuming goods produced by informal firms ( $C_{a,t}^{O,I}(i)$ ).  $\chi$  parameter shows that formal and informal goods are differently valued. Namely, formal and informal goods are not perfect substitutes, because there is no warranty for informal goods. So, one unit of an informal good implies a lower utility than one unit of a formal good. The agent pays value-added taxes (VAT) after purchasing formal goods; VAT is denoted by  $\tau_t^C$ . Because informal goods are hidden, no VAT is paid upon their purchase.  $p_t^I$  is the relative price of informal goods (expressed in formal goods' price level),  $\epsilon_t^C$  is the preference (demand) shock and  $\gamma$  is the relative risk aversion parameter. Finally, besides consuming, the agent saves in  $B_{a,t}^O(i)$  risk-free bonds and receives  $r_{t-1}$  real interest rate (nominal interest rate is denoted by  $i_{t-1}$ ) on the previous period's bond holdings. The retired agent optimizes with respect to  $C_{a,t}^{O,F}(i)$ ,  $C_{a,t}^{O,I}(i)$  and  $B_{a,t}^O(i)$ , and, when optimizing, he or she also takes into account that with probability  $\omega_t^O$  by the beginning of the next period he or she will pass away.

As a result, the Euler-equation for formal goods is:

$$E_t C_{a+1,t+1}^{O,F}(i) = E_t C_{a,t}^{O,F}(i)(1 + r_t)^{\frac{1}{\gamma}} \Lambda_{t+1} \quad (3.6)$$

where

$$E_t \Lambda_{t+1} = E_t \left\{ \beta \frac{1 + \epsilon_{t+1}^C}{1 + \epsilon_t^C} \frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} \right\}^{\frac{1}{\gamma}} \quad (3.7)$$

The Euler-equation shows the usual intertemporal substitution between periods  $t$  and  $t + 1$ ; it is also affected by value-added tax rates.

Then, optimization also implies that informal consumption can be expressed in terms of formal consumption as follows:

$$C_{a,t}^{O,I}(i) = \Upsilon_t C_{a,t}^{O,F}(i) \quad (3.8)$$

---

<sup>15</sup>More detail is available in Section 3.2.5.



where

$$\Upsilon_t = \left\{ \chi \frac{1 + \tau_t^C}{p_t^I} \right\}^{\frac{1}{\gamma}} \quad (3.9)$$

The higher the parameter  $\chi$  and the level of VAT, the larger the relative value of informal consumption to formal consumption. The opposite is true regarding the relative price of informal goods.

As noted above, the retired agent knows that he or she will die with a probability  $\omega_t^O$  by the beginning of the next period. As a consequence, the agent changes his or her behavior because his or her optimization horizon becomes finite. In contrast to a representative household framework, where, in the long run, the stochastic discount factor is equal to the inverse of one plus the real interest rate, in an overlapping generation setup this is not the case. Rather, the stochastic discount factor is a weighted sum of adjacent periods' real interest rates, where the weights are related to the demographic environment, namely, to the probabilities of death (and the retirement probability for the young).

For instance, in period  $t$  the probability of death today is  $\omega_t^O$ ; thus, the survival probability is  $1 - \omega_t^O$ . Then, the same agent's  $t + 1$ -period survival rate today is  $(1 - \omega_t^O)(1 - \omega_{t+1}^O)$ . As a consequence, the future stream of income must be discounted with the survival probabilities, and not only with the usual real interest rates, because the agent might die in any period. The same is true for retirement. Both death and retirement change the current state of the agent, and so they must be taken into account during the optimization process. As such, discount rates are affected. Because discount rates are affected, so is aggregation. In a representative agent framework there is only and always one state, thus making discounting, and so aggregation, simple. Nevertheless, in an overlapping generations setup, both discounting and aggregation are affected by demography.

We can show that the current-period individual formal consumption level of retired agent (i) can be expressed as follows:

$$\mathcal{H}_t^O C_{a,t}^{O,F}(i) = (TR_{a,t}^{PG,YO}(i) + TR_{a,t}^{FF,YO}(i))\Omega_t^O + \mathcal{I}_{a,t}^O(i) + (1 + r_{t-1})B_{a-1,t-1}^O(i) \quad (3.10)$$

where

$$\mathcal{I}_{a,t}^O(i) = Profit_{a,t}^O(i) - T_{a,t}^O(i) + E_t \frac{1 - \omega_t^O}{1 + r_t} \mathcal{I}_{a+1,t+1}^O(i) \quad (3.11)$$

$$\mathcal{H}_t^O = (1 + \tau_t^C) + p_t^I \Upsilon_t + E_t (1 - \omega_t^O)(1 + r_t)^{\frac{1}{\gamma}-1} \Lambda_{t+1} \mathcal{H}_{t+1}^O \quad (3.12)$$

$$\Omega_t^O = 1 + E_t \frac{1 - \omega_t^O}{1 + r_t} \Omega_{t+1}^O \quad (3.13)$$

and  $\Upsilon_t$  and  $\Lambda_{t+1}$  are the same as before.

Then, after aggregation, the retired generation consumes:<sup>16</sup>

$$\mathcal{H}_t^O C_t^{O,F} = (TR_t^{PG} + TR_t^{FF})\Omega_t^O + \mathcal{I}_t^O + (1 + r_{t-1})(\omega_{t-1}^Y B_{t-1}^Y + B_{t-1}^O) \quad (3.14)$$

$$C_t^{O,I} = \Upsilon_t C_t^{O,F} \quad (3.15)$$

where

$$\mathcal{I}_t^O = Profit_t^O - T_t^O + E_t \frac{1 - \omega_t^O}{(1 + r_t)(1 + g_{t+1}^{N,O})} \mathcal{I}_{t+1}^O \quad (3.16)$$

$$Profit_t^O - T_t^O = (1 - \xi)(Profit_t - T_t) \quad (3.17)$$

and  $\xi$  is the fraction of profits and lump-sum taxes that goes to the young.

Hence, the current level of formal consumption of the old equals the sum of the discounted stream of current and future pension benefits and other income, and the current level of savings. Discounting not only depends on the usual real interest rates, though, but also on mortality rates and the number of people in the retired cohort. A final crucial caveat is that  $\omega_{t-1}^Y B_{t-1}^Y$  is included in the consumption function because  $\omega_{t-1}^Y$  share of the young retired in the previous period. Also,  $g_{t+1}^{N,O}$  shows the growth rate of the number of old people.

### The young cohort

'Young' agent  $i$  of young cohort  $b$  is one individual of its cohort who started to work (was born)  $b$  years ago. The Bellman-equation of a young individual is:

$$\begin{aligned} V_t^Y(B_{b-1,t-1}^Y(i)) &= \\ &= \max \left\{ (1 + \epsilon_t^C) \left[ \frac{1}{1 - \gamma} \left\{ C_{b,t}^{Y,F}(i) \right\}^{1-\gamma} + \frac{\chi}{1 - \gamma} \left\{ C_{b,t}^{Y,I}(i) \right\}^{1-\gamma} \right] + \right. \\ &\quad \left. + \beta E_t \left( (1 - \omega_t^Y) V_{t+1}^Y(B_{b,t}^Y(i)) + \omega_t^Y V_{t+1}^O(B_{b,t}^{YO}(i)) \right) \right\} \end{aligned} \quad (3.18)$$

while the budget constraint is:

$$\begin{aligned} (1 + \tau_t^C) C_{b,t}^{Y,F}(i) + p_t^I C_{b,t}^{Y,I}(i) + (1 - \omega_t^Y) B_{b,t}^Y(i) + \omega_t^Y B_{b,t}^{YO}(i) &= \\ = (1 + r_{t-1}) B_{b-1,t-1}^Y(i) + (1 - \tau_t^{LW}) w_t^F L_{b,t}^F(i) + w_t^I L_{b,t}^I(i) + \\ + w_t^U U_{b,t}(i) + Profit_{b,t}^Y(i) - T_{b,t}^Y(i) \end{aligned} \quad (3.19)$$

By definition, a young person does not collect old-age pension benefits. Rather, he or she either works and receives labor income, or is unemployed ( $U_{b,t}(i)$  denotes unemployment) and receives  $w_t^U$  unemployment benefits. The agent can either work in the formal sector or in the informal sector;  $L_{b,t}^F(i)$  and  $L_{b,t}^I(i)$  denote formal and informal employment, and  $w_t^F$

---

<sup>16</sup>The aggregate informal consumption of the retired can be calculated by the informal-formal substitution equation.

and  $w_t^I$  are formal and informal wages. A crucial difference between the formal and informal sectors is that only income earned in the formal sector is subject to taxation; those who work in the formal sector pay a sum of personal income tax and employees' social security contributions of  $\tau_t^{LW}$ . Also, a given share of  $Profit_{b,t}^Y(i)$  profits minus  $T_{b,t}^Y(i)$  lump-sum taxes are earned by the young, where the share is equal to the fraction of young people in the whole population. The probability of retiring by the next period is  $\omega_t^Y$ . A young agent saves for two possible future states: first, for staying young ( $B_{b,t}^Y(i)$ ), second, for retiring ( $B_{b,t}^{YO}(i)$ ).<sup>17</sup> The young optimize with respect to both savings, i.e.  $B_{b,t}^Y(i)$  and  $B_{b,t}^{YO}(i)$ , and, as usual,  $C_{b,t}^{Y,F}(i)$  and  $C_{b,t}^{Y,I}(i)$ .

As a result, the Euler-equations are:

$$E_t C_{b+1,t+1}^{Y,F}(i) = E_t C_{b,t}^{Y,F}(i)(1+r_t)^{\frac{1}{\gamma}} \Lambda_{t+1} \quad (3.20)$$

$$E_t C_{0,t+1}^{O,F}(i) = E_t C_{b,t}^{Y,F}(i)(1+r_t)^{\frac{1}{\gamma}} \Lambda_{t+1} \quad (3.21)$$

where  $\Lambda_{t+1}$  is the same as before. The young agent saves for two potential future states; hence, there are two Euler-equations. These show the intertemporal substitution, as usual. However, one of them relates current-period young consumption to next-period young consumption, if the agent is still young in the next period, while the other one does the same but in relation to future old consumption.

Also, the relation between informal and formal consumptions is:

$$C_{b,t}^{Y,I}(i) = \Upsilon_t C_{b,t}^{Y,F}(i) \quad (3.22)$$

where  $\Upsilon_t$  is the same as before. As with regard to the old, informal consumption is positively affected by the weight on informal consumption in the utility function and the rate of value-added tax, but it is negatively affected by the relative price of informal to formal goods.

Then, individual formal consumption of the young are:

$$\mathcal{H}_t^Y C_{b,t}^{Y,F}(i) = \mathcal{I}_{b,t}^Y(i) + \frac{\mathcal{I}_{b,t}^{YO}(i)}{1+r_t} + (1+r_{t-1})B_{b-1,t-1}^Y(i) \quad (3.23)$$

where

$$\mathcal{I}_{b,t}^Y(i) = Inc_{b,t}(i) + E_t \frac{1-\omega_t^Y}{1+r_t} \mathcal{I}_{b,t+1}^Y(i) \quad (3.24)$$

$$Inc_{b,t}(i) = (1-\tau_t^{LW})w_t^F L_{b,t}^F(i) + w_t^I L_{b,t}^I(i) + w_t^U U_{b,t}(i) + Profit_{b,t}^Y(i) - T_{b,t}^Y(i) \quad (3.25)$$

$$\begin{aligned} \mathcal{I}_{b,t}^{YO}(i) = & E_t \omega_t^Y \left( (TR_{0,t+1}^{PG,YO}(i) + TR_{0,t+1}^{FF,YO}(i)) \Omega_{t+1}^O + \mathcal{I}_{0,t+1}^O(i) \right) + \\ & + E_t \frac{1-\omega_t^Y}{1+r_{t+1}} \mathcal{I}_{b,t+1}^{YO}(i) \end{aligned} \quad (3.26)$$

$$\mathcal{H}_t^Y = (1+\tau_t^C) + p_t^I \Upsilon_t + E_t (1+r_t)^{\frac{1}{\gamma}-1} \Lambda_{t+1} \left( (1-\omega_t^Y) \mathcal{H}_{t+1}^Y + \omega_t^Y \mathcal{H}_{t+1}^O \right) \quad (3.27)$$

and  $\Upsilon_t$  and  $\Lambda_{t+1}$  are the same as before.

---

<sup>17</sup>This is just a technical distinction; at the end of the day, all young persons' savings are denoted by  $B_b^Y$ .

Aggregate consumption is as follows:

$$\mathcal{H}_t^Y C_t^{Y,F} = \mathcal{I}_t^Y + \frac{\mathcal{I}_t^{YO}}{1+r_t} + (1+r_{t-1})(1-\omega_{t-1}^Y)B_{t-1}^Y \quad (3.28)$$

$$C_t^{Y,I} = \Upsilon_t C_t^{Y,F} \quad (3.29)$$

where

$$\mathcal{I}_t^Y = Inc_t + E_t \frac{1-\omega_t^Y}{(1+r_t)(1+g_{t+1}^{N,Y})} \mathcal{I}_{t+1}^Y \quad (3.30)$$

$$Inc_t = (1-\tau_t^{LW})w_t^F L_t^F + w_t^I L_t^I + w_t^U U_t + Profit_t^Y - T_t^Y \quad (3.31)$$

$$Profit_t^Y - T_t^Y = \xi(Profit_t - T_t) \quad (3.32)$$

$$\begin{aligned} \mathcal{I}_t^{YO} = E_t & \left( (TR_{t+1}^{PG,YO} + TR_{t+1}^{FF,YO})\Omega_{t+1}^O + \frac{\omega_t^Y}{(1+g_{t+1}^{N,Y})s_{t+1}} \mathcal{I}_{t+1}^O \right) + \\ & + E_t \frac{1-\omega_t^Y}{(1+r_{t+1})(1+g_{t+1}^{N,Y})} \mathcal{I}_{t+1}^{YO} \end{aligned} \quad (3.33)$$

and  $\Upsilon_t$ ,  $\Lambda_{t+1}$  and  $\xi$  are the same as before.

Similarly to retired consumption, the current level of young formal consumption is equal to the sum of the discounted stream of current and future income, and current savings. Nevertheless, there are two future incomes, one for being young in the future and one for being old in the future. Moreover, income denotes labor income and other income (but not pensions). Finally, discounting takes into account not only mortality rates and the number of retired people, but also the probability of retirement and the number of young people.

Finally, the aggregate young budget constraint is:

$$(1+\tau_t^C)C_t^{Y,F} + p_t^I C_t^{Y,I} + B_t^Y = Inc_t + (1+r_{t-1})(1-\omega_{t-1}^Y)B_{t-1}^Y \quad (3.34)$$

### 3.2.3 Firms

There are two types of firms: physical-capital producing firms and goods-producing firms; both types are owned by the household sector, and, among both types there are formal and informal firms. Informal firms avoid paying taxes and the level of regulation is lower in the shadow sector (Williamson (1975)).

Physical-capital producing firms use beginning-of-period physical capital and invest to produce end-of-period physical capital. Investment is subject to an investment-adjustment cost following Christiano et al. (2005).

Goods-producing firms rent physical capital from capital producers and hire labor from young households. They produce a good which is either consumed by the households or purchased by the government (only formal sector goods), or one that is used for capital production (only formal sector goods). Also, only formal firms pay social security contributions. All goods-producing firms pay an extra cost of hiring following Blanchard-Gali (2010); this hiring cost depends on the number of newly hired people. The hiring cost is higher in the formal sector, reflecting both administration and advertising costs. While hiring is endogenous, firing is exogenous. Furthermore, goods producers are monopolistically competitive and set the price level taking into account a price adjustment cost à la Rotemberg (1982). Finally, goods-producing firms bargain with workers over wages, similarly to Blanchard-Gali (2010). Formal workers have a stronger bargaining power over wages than do informal workers, and, thanks to unions, the firing probability is lower in the formal sector.

### Physical capital producers

The Bellman equation of the formal physical capital producer is:<sup>18</sup>

$$V(Inv_{t-1}^F; K_{t-1}^F) = \max \left\{ r_t^{K,F} K_{t-1}^F - Inv_t^F + E_t \frac{1}{1+r_t} V(Inv_t^F; K_t^F) \right\} \quad (3.35)$$

while it also takes into account the physical capital accumulation equation:

$$Inv_t^F = K_t^F - (1 - \delta) K_{t-1}^F - Inv_t^F S \left( \frac{Inv_t^F}{Inv_{t-1}^F (1 + g_t)} \right) \quad (3.36)$$

Here,  $r_t^{K,F}$  denotes the real rental rate of capital ( $R_t^{K,F}$  is the nominal rental rate of capital and  $r_t^{K,F} = \frac{R_t^{K,F}}{P_t^F}$ ),  $K_{t-1}^F$  is the physical capital stock,  $Inv_t^F$  denotes investment,  $Q_t^F$  is the shadow price of capital and  $\delta$  is the depreciation rate of capital. Also,  $g_t$  is the sum of technology and population growth.

Function  $S$  is the investment adjustment cost function, following Christiano et al. (2005), it looks like:

$$S \left( \frac{Inv_t^F}{Inv_{t-1}^F (1 + g_t)} \right) = \frac{\phi_{Inv}^F}{2} \left( \frac{Inv_t^F}{Inv_{t-1}^F (1 + g_t)} \right)^2 \quad (3.37)$$

where  $\phi_{Inv}^F$  is the investment adjustment cost parameter.

As a result, optimization can be summarized by two equations:

$$E_t \left( r_{t+1}^{K,F} + Q_{t+1}^F (1 - \delta) \right) = Q_t^F (1 + r_t) \quad (3.38)$$

---

<sup>18</sup>The optimization problem of the informal physical capital producer is parallel, so we only describe the formal one.

$$1 = E_t \frac{1}{1+r_t} \left[ Q_{t+1}^F S' \left( \frac{Inv_{t+1}^F}{Inv_t^F(1+g_{t+1})} \right) \left( \frac{Inv_{t+1}^F}{Inv_t^F(1+g_{t+1})} \right)^2 \right] + \\ + Q_t^F \left[ 1 - S \left( \frac{Inv_t^F}{Inv_{t-1}^F(1+g_t)} \right) - S' \left( \frac{Inv_t^F}{Inv_{t-1}^F(1+g_t)} \right) \frac{Inv_t^F}{Inv_{t-1}^F(1+g_t)} \right] \quad (3.39)$$

The first equation is the usual no-arbitrage condition. It claims that, taking into account the shadow price of capital  $Q_t^F$ , investing in capital and saving in risk-free bonds yields the same return. Without this condition a sub-optimal arbitrage could be achieved in the economy. The second equation is the usual Tobin-Q, which describes the optimal investment strategy of the firm.

### Good producers

Now, we describe goods producers in the formal sector.<sup>19</sup> One of them is denoted by  $j$ .<sup>20</sup> The (nominal) Bellman-equation of the formal sector goods producer is:

$$V(P_{t-1}^F(j), L_{t-1}^F(j)) = \max \left\{ profit_t^F(j) + E_t \frac{V(P_t^F(j), L_t^F(j))}{1+i_t} \right\} \quad (3.40)$$

which is maximized subject to the profit function, production function, demand function and the labor law of motion:

$$profit_t^F(j) = P_t^F(j) Y_t^F(j) - R_t^{K,F} K_{t-1}^F(j) - (1 + \tau_t^{SCF}) W_t^F L_t^F(j) - \\ - HC_t^F H_t^F(j) - P_t^F(j) Y_t^F R \left( \frac{P_t^F(j)}{P_{t-1}^F(j)} \right) \quad (3.41)$$

$$Y_t^F(j) = K_{t-1}^F(j)^{\alpha^F} (A_t^F L_t^F(j))^{1-\alpha^F} \quad (3.42)$$

$$Y_t^F(j) = \left( \frac{P_t^F(j)}{P_t^F} \right)^{-\varphi} Y_t^F \quad (3.43)$$

$$L_t^F(j) = (1 - pr_t^{F,F}) L_{t-1}^F(j) + H_t^F(j) \quad (3.44)$$

$$R \left( \frac{\frac{P_t^F(j)}{P_{t-1}^F(j)}}{\left( \frac{P_{t-1}^F}{P_{t-2}^F} \right)^\gamma} \right) = \frac{\phi_P}{2} \left( \frac{\frac{P_t^F(j)}{P_{t-1}^F(j)}}{\left( \frac{P_{t-1}^F}{P_{t-2}^F} \right)^\gamma} - 1 \right)^2 = \frac{\phi_P}{2} \left( \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)^\gamma} - 1 \right)^2 \quad (3.45)$$

$$HC_t^F = \kappa^F \left( pr_t^{H,F} \right)^{\alpha_{HC}} \quad (3.46)$$

$$pr_t^{H,F} = \frac{H_t^F}{U_{t-1} + pr_t^{F,F} L_{t-1}^F + pr_t^{F,I} L_{t-1}^I} \quad (3.47)$$

<sup>19</sup>The optimization problem of informal goods producers is parallel.

<sup>20</sup>Because the number of firms is equal to 1, index  $j$  is only used for the sake of mathematical convenience, at the end, a variable with  $j$  and without  $j$  will be the same.

First, regarding production, the production function is the usual Cobb-Douglas with a labor-augmenting technological process based on labor and physical capital. Here,  $A_t^F$  denotes technology level and  $1 - \alpha^F$  shows the share of labor income in total factor income. Total amount of production  $Y_t^F$  is related to firm  $j$ -production  $Y_t^F(j)$  by a usual demand function, where  $P_t^F(j)$  is the price set by firm  $j$  ( $P_t^F$  is the general price level) and  $\varphi$  denotes price elasticity of demand.

Second, the extra cost of hiring a new worker is denoted by  $HC_t^F$  (the real hiring cost is  $hc_t^F = \frac{HC_t^F}{P_t^F}$ ). This depends on the probability of hiring  $pr_t^{H,F}$ , which depends on the number of newly hired people  $H_t^F$ .  $\kappa^F$  denotes the hiring cost parameter and  $\alpha_{HC,F}$  shows the elasticity of hiring cost with respect to the probability of hiring. While hiring is endogenous, as the labor law of motion shows, firing is exogenous, and the probability of firing is  $pr_t^{F,F}$ . Also, firms pay  $\tau_t^{SSCF}$  social security contributions.

Finally, regarding price setting, function  $R$  is the Rotemberg price adjustment cost. Also, the price adjustment cost parameter is  $\phi_P^F$  and  $\gamma$  is the indexation parameter; if  $\gamma = 0$ , there is no indexation, if  $\gamma = 1$ , there is full indexation.

Although sectoral optimizations are similar, there are some differences. First and foremost is tax evasion; that is, only formal firms pay social security contributions. Moreover, formal hiring costs and bargaining power of workers are higher, while the dismissal rate is lower. These differences can be attributed to the higher degree of regulation in the formal as compared to the informal sector (Williamson (1975)).

Optimization results in two demand (physical capital and labor) and a pricing decision, while the last equation is real marginal cost ( $mc_t^F = \frac{MC_t^F}{P_t^F}$ ):

$$K_{t-1}^F = \alpha^F \frac{mc_t^F}{r_t^{K,F}} Y_t^F \quad (3.48)$$

$$mc_t^F (1 - \alpha^F) \frac{Y_t^F}{L_t^F} - (1 + \tau_t^{SSCF}) w_t^F = hc_t^F - E_t \frac{hc_{t+1}^F (1 - pr_{t+1}^{F,F})}{1 + r_t} \quad (3.49)$$

$$1 + \frac{1}{\varphi - 1} \left\{ R \left( \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)^\gamma} \right) + R' \left( \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)^\gamma} \right) \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)^\gamma} \right\} - E_t \frac{1}{\varphi - 1} \frac{Y_{t+1}^F}{Y_t^F} \frac{R' \left( \frac{1 + \pi_{t+1}^F}{(1 + \pi_t^F)^\gamma} \right) \left( \frac{1 + \pi_{t+1}^F}{(1 + \pi_t^F)^\gamma} \right)}{1 + r_t} = \frac{\varphi}{\varphi - 1} mc_t^F \quad (3.50)$$

$$mc_t^F = \left( \frac{r_t^{K,F}}{\alpha^F} \right)^{\alpha^F} \left( \frac{(1 + \tau_t^{SSCF}) w_t^F}{A_t^F (1 - \alpha^F)} \right)^{(1 - \alpha^F)} \quad (3.51)$$

Two important caveats follow. First, due to the presence of labor market frictions, labor demand becomes an intertemporal as well as an intratemporal decision. This is because labor

market rigidities create a link in employment today and tomorrow, a link that is described by the labor law of motion. Second, the pricing decision is sectoral; hence, there are two New Keynesian Philips curves, one for each sector.

Goods producers do not only produce goods and set prices, but they also bargain over wages with their workers. Because an agent either works in the formal sector, or in the informal sector or is unemployed, there are three worker value functions:

$$V_t^F = (1 - \tau_t^{LW})w_t^F + E_t \frac{1}{1 + r_t} \left[ (1 - pr_{t+1}^{F,F} + pr_{t+1}^{F,F} pr_{t+1}^{H,F}) V_{t+1}^F + pr_{t+1}^{F,F} pr_{t+1}^{H,I} V_{t+1}^I + pr_{t+1}^{F,F} (1 - pr_{t+1}^{H,F} - pr_{t+1}^{H,I}) V_{t+1}^U \right] \quad (3.52)$$

$$V_t^I = w_t^I + E_t \frac{1}{1 + r_t} \left[ (1 - pr_{t+1}^{F,I} + pr_{t+1}^{F,I} pr_{t+1}^{H,I}) V_{t+1}^I + pr_{t+1}^{F,I} pr_{t+1}^{H,F} V_{t+1}^F + pr_{t+1}^{F,I} (1 - pr_{t+1}^{H,F} - pr_{t+1}^{H,I}) V_{t+1}^U \right] \quad (3.53)$$

$$V_t^U = w_t^U + E_t \frac{1}{1 + r_t} \left[ (1 - pr_{t+1}^{H,I} - pr_{t+1}^{H,F}) V_{t+1}^U + pr_{t+1}^{H,F} V_{t+1}^F + pr_{t+1}^{H,I} V_{t+1}^I \right] \quad (3.54)$$

Today, a worker in the formal sector earns real wage  $w_t^F$  and pays personal income tax and social security contributions  $\tau_t^{LW}$ . In the next period, she might keep this job or might be fired. If fired, she could find another job either in the formal or in the informal sector. Alternatively she might stay unemployed. A similar argument holds for the informal worker value function. Being unemployed, nevertheless, means that  $w_t^U$  unemployment benefits are received today, and tomorrow she either starts to work in any of the two sectors or stays unemployed for one more period.

Firm value functions are equal to the hiring costs themselves because searching does not take time (see Blanchard-Gali (2010) for more detail).

Bargaining happens separately in each sector with bargaining powers  $\sigma^F$  and  $\sigma^I$ ; workers and firms make a common decision about wages, taking into account the value of the job over that of the outside option (not working):

$$\max_{w_t^F} (V_t^F - V_t^U)^{\sigma^F} (hc_t^F)^{1-\sigma^F} \quad (3.55)$$

$$\max_{w_t^I} (V_t^I - V_t^U)^{\sigma^I} (hc_t^I)^{1-\sigma^I} \quad (3.56)$$

As a result, the formal bargaining condition is:

$$\begin{aligned} \frac{\sigma^F}{1 - \sigma^F} hc_t^F \frac{1 - \tau_t^{LW}}{1 + \tau_t^{SSCF}} &= (1 - \tau_t^{LW}) w_t^F - w_t^U + \\ &+ E_t \frac{1}{1 + r_t} \left[ (1 - pr_{t+1}^{F,F}) (1 - pr_{t+1}^{H,F}) \left( \frac{\sigma^F}{1 - \sigma^F} hc_{t+1}^F \frac{1 - \tau_{t+1}^{LW}}{1 + \tau_{t+1}^{SSCF}} \right) - \right. \end{aligned}$$



$$-(1 - pr_{t+1}^{F,F})pr_{t+1}^{H,I} \left( \frac{\sigma^I}{1 - \sigma^I} hc_{t+1}^I \right) \Big] \quad (3.57)$$

Note, that both employer and employee taxes affect bargaining. Also, bargaining is an intertemporal decision, similarly to labor demand, due to the presence of hiring costs.<sup>21</sup>

### 3.2.4 Monetary policy

In the short run, price rigidity matters and the central bank follows a Taylor-type rule, similar to that of Smets-Wouters (2007):

$$1 + i_t = (1 + i_{t-1})^{\rho_i} E_t \left( (1 + r)(1 + \pi_{t+1}^F)^{\rho_\pi} \right)^{(1-\rho_i)} e^{\epsilon_t^i} \quad (3.58)$$

Here,  $\rho_i$  is the interest rate smoothing parameter,  $\rho_\pi$  is the weight on inflation and  $e^{\epsilon_t^i}$  is the monetary policy shock. We assume, for the sake of simplicity, that the central bank sets the interest rate based solely on the formal inflation rate.

Also, the Fisher equation is:

$$1 + i_t = (1 + r_t) E_t (1 + \pi_{t+1}^F) \quad (3.59)$$

### 3.2.5 Fiscal policy

The fiscal side of our model is very rich, especially regarding pensions. First, we present the pay-as-you-go-plan, then we move on to the fully funded regime. Also, we highlight how we model the switch between these two.

#### Pay-as-you-go pension system

The government collects revenues ( $Rev_t$ ) of value-added taxes ( $\tau_t^C$ ), labor income taxes ( $\tau_t^L$ ) and  $T_t$  lump-sum taxes:

$$Rev_t = \tau_t^C C_t^F + \tau_t^L w_t^F L_t^F + T_t \quad (3.60)$$

$$\tau_t^L = \tau_t^{PI} + (1 - \Xi)(\tau_t^{SSCW} + \tau_t^{SSCF}) \quad (3.61)$$

$$\tau_t^{LW} = \tau_t^{PI} + \tau_t^{SSCW} \quad (3.62)$$

Labor income taxes are paid by young households and firms; the former pay personal income taxes ( $\tau_t^{PI}$ ) and social security contributions ( $\tau_t^{SSCW}$ ) ( $\tau_t^{LW}$  denotes the sum of these two), while firms are only subject to social security contributions ( $\tau_t^{SSCF}$ ).  $\Xi$  is an indicator, which is 1 in a fully funded regime and 0 in a PAYG regime, while it can be time-variant as well. Obviously, value added taxes are only collected after goods purchased from formal sector producers, and, only firms and workers in the formal sector pay labor income taxes.

---

<sup>21</sup>There is a similar expression for the informal sector.

Revenues finance government consumption expenditure ( $Gov_t$ ), unemployment benefits expenditure and old-age pension expenditure ( $TR_t^{PG}$ ):

$$Exp_t = Gov_t + w_t^U U_t + TR_t^{PG} \quad (3.63)$$

As the budget might not be balanced in each period, the government issues bonds denoted by  $B_t$ . These can be purchased by all households. Thus, the government budget constraint is:

$$B_t + Rev_t = (1 + r_{t-1})B_{t-1} + Exp_t \quad (3.64)$$

Also, primary government balance ( $PB_t$ ) is defined as the difference between public revenues and expenditures, while total government balance ( $GB_t$ ) also incorporates interest payments/receipts on bond holdings:

$$PB_t = Rev_t - Exp_t \quad (3.65)$$

$$GB_t = PB_t - r_{t-1}B_{t-1} \quad (3.66)$$

Then, the government budget constraint becomes:

$$B_t = B_{t-1} - GB_t \quad (3.67)$$

Because of the presence of bonds, in order to avoid an explosive solution, similarly to Leeper (1991) or Anderson et al. (2013), a rule is introduced for lump-sum taxes:

$$T_t = \eta T + (1 - \eta) \left[ \rho T_{t-1} + (1 - \rho)(GB_t^{Target} - GB_t) \right] \quad (3.68)$$

Here,  $\eta$  shows the speed of adjustment; it is a 0-1 indicator;  $\eta = 1$  if lump-sum taxes stay at the steady-state level and 0 otherwise. A similar rule is valid for all other fiscal instruments as well (value-added tax rate, personal income tax rate, employer and employee social security contributions and government consumption), and, all rules target total government balance ( $GB_t^{Target}$ ).<sup>22</sup>

The starting point of the pay-as-you-go plan is that the pension of each newly retired person depends on a pension-wage replacement rate, and, on the individual's previous wage stream:

$$TR_{0,t}^{PG,YO}(i) = \nu_t IB_{b-1,t}^Y(i) \quad (3.69)$$

$TR_t^{PG,YO}(i)$  denotes pension received by individual ( $i$ ) who just got retired in period  $t$ .<sup>23</sup>  $\nu_t$  is the gross pension-wage replacement rate (ratio of pension to gross wages), while  $IB_{b-1,t}^Y(i)$

---

<sup>22</sup>Calibration implies that  $GB^{Target} - GB = T$ .

<sup>23</sup>YO refers to just-retired, and, is an abbreviation for young-old.

is the individual's previous years' wage stream. To be more precise, it is a simple average of wages received in the previous  $Y$  years:

$$IB_{b-1,t}^Y(i) = \frac{1}{Y}w_{t-1}^FL_{b-1,t-1}^F(i) + \frac{Y-1}{Y}IB_{b-2,t-1}^Y(i) \quad (3.70)$$

Then, total wage stream of all employees follows:

$$IB_t^Y = \frac{1}{Y}w_{t-1}^FL_{t-1}^F + \frac{Y-1}{Y}(1 - \omega_{t-2}^Y)IB_{t-1}^Y \quad (3.71)$$

Hence, total pension of those old people who just got retired is:

$$TR_t^{PG,YO} = \nu_t \omega_{t-1}^Y IB_{t-1}^Y \quad (3.72)$$

Lastly, total pension expenditure is the sum of pension of the just-retired and of those who got retired in any of the previous periods and are still alive:

$$TR_t^{PG} = TR_t^{PG,YO} + (1 - \omega_{t-1}^O)TR_{t-1}^{PG} \quad (3.73)$$

### Fully funded pension system

The crucial difference between the two pension regimes is that in a fully funded regime  $TR_t^{PG}$  is not included into public expenditures anymore:

$$Exp_t = Gov_t + w_t^U U_t \quad (3.74)$$

Period- $t$  social security contributions directly go to a pension fund, and, future individual pension benefits are related to individual pension savings. The starting point is that individual ( $i$ ) saves her social security contributions on a separate account:

$$B_{b,t}^{Y,*}(i) = \Xi(\tau_t^{SSCW} + \tau_t^{SSCF})w_t^FL_{b,t}^F(i) + (1 + r_{t-1})B_{b-1,t-1}^{Y,*}(i) \quad (3.75)$$

Here,  $B_t^{Y,*}(i)$  denotes individual ( $i$ )'s pension savings. As before, only employees and employers in the formal sector pay social security contributions, hence, informal income does not contribute to retirement savings.

Assuming that initial wealth is zero, total pension savings look like:

$$B_t^{Y,*} = \Xi(\tau_t^{SSCW} + \tau_t^{SSCF})w_t^FL_t^F + (1 + r_{t-1})(1 - \omega_{t-1}^Y)B_{t-1}^{Y,*} \quad (3.76)$$

When someone gets retired, in that period, based on her pension savings and the life expectancy, an initial pension level is set by the government. Later, it will be adjusted by inflation only. It can be shown that  $TR_{0,t}^{FF,YO}(i)$  which is a just-retired individual ( $i$ )'s pension in period  $t$  fulfills:

$$(1 + r_{t-1})B_{b-1,t-1}^{Y,*}(i) = TR_{0,t}^{FF,YO}(i)\Omega_t^O \quad (3.77)$$

Then, total pension expenditure of those who retired in period  $t$  ( $TR_t^{FF,YO}$ ) and of all retired people ( $TR_t^{FF}$ ), in the FF regime, are:

$$(1 + r_{t-1})\omega_{t-1}^Y B_{t-1}^{Y,*} = TR_t^{FF,YO} \Omega_t^O \quad (3.78)$$

$$TR_t^{FF} = TR_t^{FF,YO} + (1 - \omega_{t-1}^O) TR_{t-1}^{FF} \quad (3.79)$$

Lastly, as people get older, their pension savings shrink. So, the following holds:

$$(1 + r_{t-1})\omega_{t-1}^Y B_{t-1}^{Y,*} + (1 + r_{t-1})B_{t-1}^{O,*} = TR_t^{FF} + B_t^{O,*} \quad (3.80)$$

where  $B_t^{O,*}$  is pension savings of previously retired people in the fully funded regime.

Finally, total pension savings are:

$$B_t^* = B_t^{Y,*} + B_t^{O,*} \quad (3.81)$$

### 3.2.6 Market clearing

In equilibrium, all markets clear. First, the labor market clears.<sup>24</sup>

First, total number of workers in the formal and underground sector, and the number of unemployed people are:

$$L_t^F = \sum_{b=0}^{\infty} L_{b,t}^F = \sum_{b=0}^{\infty} N_{b,t}^Y L_{b,t}^F(i) \quad (3.82)$$

$$L_t^I = \sum_{b=0}^{\infty} L_{b,t}^I = \sum_{b=0}^{\infty} N_{b,t}^Y L_{b,t}^I(i) \quad (3.83)$$

$$U_t = \sum_{b=0}^{\infty} U_{b,t} = \sum_{b=0}^{\infty} N_{b,t}^Y U_{b,t}(i) \quad (3.84)$$

Then, the total number of young people is equal to the sum of the number of people employed in either the formal or the informal sector and the number of unemployed people:

$$U_t = N_t^Y - L_t^F - L_t^I \quad (3.85)$$

Total profits is the sum of sectoral profits:

$$Profit_t = profit_t^F + profit_t^I \quad (3.86)$$

---

<sup>24</sup>Also, the physical capital market clears: capital rented by firms equals capital produced by capital producers (this is why both are denoted by  $K$ ).

Then, there is an equilibrium in the assets market:

$$B_t + Q_t^F K_t^F + Q_t^I K_t^I = B_t^Y + B_t^O + B_t^* \quad (3.87)$$

This means that the young and the retired save in risk-free bonds, and this aggregate savings is equal to the sum of government bonds and physical capital.

Regarding the goods market, in each sector, supply must equal the relevant demand and deadweight losses due to labor and product market frictions:

$$\begin{aligned} Y_t^F = & C_t^F + Inv_t + Gov_t + hc_t^F H_t^F + R \left( \frac{P_t^F}{P_{t-1}^F} \right) + \\ & + Inv_t^F S \left( \frac{Inv_t^F}{Inv_{t-1}^F (1 + g_t)} \right) + Inv_t^I S \left( \frac{Inv_t^I}{Inv_{t-1}^I (1 + g_t)} \right) \end{aligned} \quad (3.88)$$

$$Y_t^I = C_t^I + hc_t^I H_t^I + R \left( \frac{P_t^I}{P_{t-1}^I} \right) \quad (3.89)$$

where

$$C_t^F = C_t^{O,F} + C_t^{Y,F} \quad (3.90)$$

$$C_t^I = C_t^{O,I} + C_t^{Y,I} \quad (3.91)$$

$$Inv_t = Inv_t^F + Inv_t^I \quad (3.92)$$

Formal sector goods are either consumed by households or invested to produce physical capital. Also, the government purchases goods from formal firms. Hiring costs and price rigidity constitute deadweight losses, and, there are also investment adjustment costs; these create a gap between production and demand. Concerning the informal side, the difference is that the government does not buy informally produced goods and private investment only pertains to formal goods.

Finally, GDP is, as usual, the sum of household consumption, private investment and government consumption, where total consumption is the sum of formal and informal consumption:

$$GDP_t = C_t + Inv_t + Gov_t \quad (3.93)$$

$$C_t = C_t^F + p_t^I C_t^I \quad (3.94)$$

Also, production is the sum of formal and informal production:

$$Y_t = Y_t^F + p_t^I Y_t^I \quad (3.95)$$

## 3.3 Calibration

### 3.3.1 Steady-state and dynamic parameters

In this section we describe the steady-state and dynamic parameters which are calibrated based on yearly data since 1995<sup>25</sup> of three southern European countries: Portugal, Italy and Spain; more detail on calculating the steady-state is available in the Appendix.<sup>26</sup> In doing so, our main goal is to match, as accurately as possible, the model with the data, in particular, household consumption and private investment great ratios, the level of unemployment, share of informality in output and employment, and, the share of public old-age pension spending in GDP. First, we describe the process of calibrating the steady-state parameters, which are summarized in Table 3.1.

Regarding data on the expenditure side of the budget, the government consumption expenditure<sup>27</sup> to GDP ratio is 18.5 per cent, the share of unemployment benefits expenditure in GDP is 1.1 per cent, while the gross pension-wage replacement rate is set such that the public (old-age) pension spending to GDP ratio matches the data counterpart as close as possible.

Second, as regards the revenue side of the budget, all tax rates are endogenously calibrated by fixing the relevant tax revenue shares in GDP, rather than the tax rates themselves. It is crucial as even though the fiscal side of our model is quite rich, it is still far away from the richness of the budget itself, hence, only effective tax rates are appropriate to be considered. As a share of GDP, the value-added tax revenue is 8.0, the personal income tax and employee social security contributions revenue is 8.5, while the employer social security contribution revenue is 4.6 per cent, respectively. Also, the share of government debt in GDP is 75.5 per cent.

Third, the formal and informal labor markets are characterized by several frictions: firing probabilities, hiring costs and bargaining powers of workers. The formal dismissal rate is calculated by the authors based on Eurostat Labour Force Survey (LFS) data on labor flows. As a result, 13.6 per cent of the workers are dismissed in the formal sector each year; then, we assume the informal dismissal rate to be twice as high as that. Then, we set hiring costs such that we match data on unemployment. Furthermore, the formal bargaining power of workers is the standard 0.5, while the informal value is assumed to be half of that. We check

---

<sup>25</sup>Or later, if there is no data back to 1995.

<sup>26</sup>We present calibration on Portugal in detail; tables show calibrated values for Italy and Spain as well, and, the process of calibration for them is parallel to that of Portugal.

<sup>27</sup>Following SNA notation, it includes P3S13 final consumption expenditure of general government, but it does not include D63 social transfers in kind (mainly education and health expenditure).

| NAME   | SIGN          | VALUE    |        |       | SOURCE  | YEAR      |
|--|---------------|----------|--------|-------|---|-----------|
|  |               | Portugal | Italy  | Spain |   |           |
| Discount factor  | $\beta$       |          | 0.99   |       | to match real interest rate standard                              |           |
| Physical capital depreciation rate                               | $\delta$      |          | 0.1    |       |   |           |
| Formal physical capital income share                             | $\alpha^F$    | 0.23     | 0.21   | 0.24  | to match household consumption and private investment great ratio |           |
| Informal physical capital income share                           | $\alpha^I$    | 0.23     | 0.21   | 0.24  |   |           |
| Markup (%)   | $\varphi$     | 20       |        |       | standard  |           |
| Government consumption expenditure to GDP ratio (%)              |               | 18.52    | 19.23  | 18.00 | Eurostat  | 1995-2014 |
| Unemployment benefit expenditure to GDP ratio (%)                |               | 1.09     | 0.60   | 2.65  | Eurostat  | 1995-2012 |
| Gross pension-wage replacement rate (%)                          | $\nu$         | 85       |        |       | to match public pension expenditure to GDP ratio                  |           |
| Government debt as a share of GDP (%)                            |               | 75.49    | 110.69 | 59.19 | Eurostat  | 1995-2014 |
| Firing probability in the formal sector (%)                      | $pr^{FF}$     | 13.61    | 10.91  | 22.39 | Eurostat LFS  | 1995-2014 |
| Firing probability in the informal sector (%)                    | $pr^{FI}$     | 27.22    | 21.82  | 44.78 | twice as formal   | 1995-2014 |
| Ratio of hiring cost in wage in the formal sector                |               | 1.66     | 1.81   | 1.36  |   |           |
| Ratio of hiring cost in wage in the informal sector              |               | 1.66     | 1.81   | 1.36  | to match unemployment   |           |
| Bargaining power of workers in the formal sector                 | $\sigma_F$    | 0.5      |        |       | standard  |           |
| Bargaining power of workers in the informal sector               | $\sigma_I$    | 0.25     |        |       | half as formal  |           |
| Elasticity of hiring cost wrt to hiring probability              | $\alpha_{HC}$ | 0.5      |        |       | quadratic function  |           |
| Value added tax revenue in GDP (%)                               | $\tau^C$      | 7.98     | 6.08   | 5.76  | Eurostat (2014)   | 2000-2012 |
| Share of personal income tax and employee SSC revenue in GDP (%) |               | 8.53     | 12.80  | 8.74  | OECD  | 1990-2013 |
| Share of employer SSC revenue in GDP (%)                         |               | 4.59     | 8.57   | 8.23  | OECD  | 1990-2013 |
| Probability of becoming retired (%)                              | $\omega^Y$    | 1.52     | 1.88   | 1.47  | own calculation based on Eurostat                                 | 1990-2013 |
| Mortality rate of the retired (%)                                | $\omega^O$    | 5.60     | 3.66   | 3.84  | own calculation based on Eurostat                                 | 1990-2013 |
| Net fertility rate (%)   | $n$           | 2.00     | 2.05   | 2.32  | own calculation based on Eurostat                                 | 1990-2014 |
| Technology growth rate (%)                                       | $g_A$         | 1.5      |        |       | Kumhof et al. (2010)  |           |
| Weight of informal goods in the utility function                 | $\chi$        | 0.8      | 0.7    | 0.8   | to match Schneider et al. (2010) and Schneider (2012) assumption  |           |
| Fraction of young households' profit and lump-sum taxes          | $\xi$         | 1        |        |       |   |           |

Table 3.1: Steady-state parameters

the robustness of our results with respect to the assumptions we make, and, we find that our results are highly robust; details are provided in the Appendix.

Lastly, as regards demographics, first, we calculate the size of each cohort, based on Eurostat data, then using the demographic equations we estimate the transition probabilities: the fertility rate, the mortality rate and the probability of retirement. For Portugal, people between 20 and 66.6 are considered to be 'young', while those who are 66.6 year old or older are 'old'. 66.6 is the effective retirement age published by OECD (2015).<sup>28</sup>

| NAME   | SIGN       | VALUE | SOURCE                   |
|--|------------|-------|--------------------------|
| Inverse of intertemporal elasticity of substitution      | $\gamma$   | 1     | Klyuev-Snudden (2011)    |
| Adjustment cost of physical capital investment           | $\phi_I$   | 2.5   | Christiano et al. (2005) |
| Autoregressive parameter of conventional fiscal policies |            | 0.52  | Kliem-Kriwoluzky (2010)  |
| Autoregressive parameter of pension policies             |            | 0.99  |                          |
| Autoregressive parameter of pension regime switch        |            | 0.95  |                          |
| Rotemberg price adjustment cost                          |            | 60    | Klyuev-Snudden (2011)    |
| Price indexation   | $\gamma_P$ | 0.5   |                          |
| Interest rate smoothing                                  | $\rho_i$   | 0.343 | Klyuev-Snudden (2011)    |
| Reaction to inflation in Taylor rule                     | $\rho_\pi$ | 1.483 | Klyuev-Snudden (2011)    |

Table 3.2: Dynamic parameters

Dynamic parameters are based on other studies, and, are in the range of usual values of the literature (Table 3.2).

### 3.3.2 Model performance

Table 3.3 presents some empirical stylized facts and their model counterparts.

The (household consumption and private investment) great ratios and the level of unemployment are particularly well matched.<sup>29</sup> Nevertheless, we are less good at matching the size of the shadow economy; informal employment is slightly overestimated, while the shadow output is underestimated. However, it is important to note that data on informality is quite uncertain, as, in contrary to national accounts data, for example, they are estimates themselves. Finally, the model is not fully able to replicate the share of public (old-age) pension spending in GDP, either. It is true even though data shows total old-age pension spending, while our model only contains public old-age pension spending (the share of private old-age pension spending is quite low in southern Europe).

<sup>28</sup>Because we do not incorporate the non-working young, all individuals below 20 are disregarded. Also, for Italy and Spain effective retirement ages are 61.3 and 62.7 years, respectively.

<sup>29</sup>In the Labor Force Survey, those who are unemployed do not work in parallel (not even in the shadow economy). Hence, we believe that in the survey there is no significant overlap between the number of unemployed and the number of informally employed people.



| Variable (%)  | Data     |       |       | Data source and year             | Model with PAYG |       |       |
|---|----------|-------|-------|----------------------------------|-----------------|-------|-------|
|   | Portugal | Italy | Spain |                                  | Portugal        | Italy | Spain |
| Household consumption expenditure as a share of GDP | 59.9     | 60.6  | 57.3  | Eurostat, 1995-2014              | 59.9            | 61.5  | 57.7  |
| Private investment as a share of GDP                | 21.5     | 20.2  | 23.5  | Eurostat, 1995-2014              | 21.6            | 19.2  | 24.3  |
| Unemployment rate                                   | 9.2      | 9.4   | 15.9  | Eurostat, 1995-2014              | 9.2             | 7.0   | 15.7  |
| Shadow economy as a share of GDP                    | 23.0     | 27.0  | 22.5  | Schneider et al. (2010, Table 2) | 20.6            | 19.0  | 15.4  |
| Shadow economy as a share of employment             | 36.8     | 39.7  | 41.5  | Schneider (2012, Table 4.11)     | 42.5            | 46.0  | 45.5  |
| (Public) Old-age pension spending as a share of GDP | 8.0      | 10.4  | 6.4   | Eurostat, 1995-2012              | 5.5             | 9.3   | 5.7   |

Table 3.3: Evaluating the steady-state calibration for Portugal, Italy and Spain in a PAYG regime

### 3.4 Results

Now, we turn to presenting our findings. Population aging is modeled by a decline in the mortality rate such that the old-age dependency ratio gradually increases by 5 percentage points in the next 30 years, then, it remains at that level. Because of aging, on the one hand, relatively less people work, hence, GDP per capita declines because (human) resources go down. On the one other hand, the government spends more on pension, while tax revenues decline (because less people work). This, evidently, negatively affects the budget, namely, public debt increases, so consolidation is needed by the government to keep it at a sustainable level.

Several policies are investigated (in brackets we show abbreviations used on charts). First, we study two labor income tax policies, particularly, an increase in personal income tax and employee social security contributions (PIT) and an increase in employer social security contributions (Employer SSC). Then, we focus on two pension reforms, notably, a decrease in the gross pension-wage replacement rate (Repl.) and a decrease in the retirement probability (Ret. prob.)<sup>30</sup> which is a proxy for a retirement age increase. Lastly, some other policies are also studied: an increase in the value added tax revenue (VAT) and a decrease in government consumption expenditure (GC). Additionally, we consider an increase in the net fertility rate as well (Fert.). Because we focus on southern Europe where the pension plan is pay-as-you-go, this is the regime we consider in our baseline simulations.<sup>31</sup>

<sup>30</sup>Meanwhile, the fertility rate changes such that the number of new young people remains stable. In our paper fertility is exogenous, which simplifies the framework, also, it is a reasonable assumption for developed countries.

<sup>31</sup>Also, we do not study the impacts of strengthening tax collection, we leave it for further research.

We study both the long- and the short-run macroeconomic outcomes. By long-run we mean 100 years which coincides with the working life span of two generations. The first 10 years are also detailed, because costs and benefits might differ over the long- and short-run. Then, we compare both the long- and short-term impacts with a PAYG and an FF plan, and, we also investigate the transition towards a fully funded plan. In particular, a full and a partial reform are examined; a partial reform is a combination of PAYG and FF elements (50-50 per cent). After all, we concentrate on the role of informality and unemployment in our framework.

Simulations are carried out by Dynare 4.4.3 and Iris, and, mean (permanent) deterministic simulations, moving from the original steady-state to a new long-run equilibrium.

### **3.4.1 Macroeconomic impacts in a PAYG regime in the long- and short-run**

Table 3.4 shows the long-run (100-year) macroeconomic impacts of a mortality rate decline corresponding to a gradual 5 percentage point increase in the old-age dependency ratio in the next 30 years in a PAYG regime, while Figure 3.7 demonstrates the short-run (10-year) consequences during the transition. We find that output in the long-run goes down by 3.8 per cent in Portugal, 3 per cent in Italy and 3.5 per cent in Spain. Regarding the long-run unemployment reactions, in all countries unemployment stays stable. However, aging implies a considerable rise in the government debt to GDP ratio, in particular, over the next 100 years, in Portugal, Italy and Spain, respectively, it rises by 111.1, 83.1 and 71.5 pp. Thus, the negative fiscal impacts of aging must be handled, otherwise fiscal sustainability might be questioned.

First of all, the lowest decline in GDP when the population is aging is induced by lowering the probability of retirement; all other fiscal policies imply more severe GDP losses. Increasing the retirement age, *ceteris paribus*, induces a small increase in output (for example 0.8 per cent for Portugal, because aging implied a 3.8, while aging with a retirement age increase a 3 per cent reduction in output). If the personal income tax and social security contributions revenue goes up, the Portuguese per capita GDP goes down by 5.9 per cent in the long-run; and, this policy causes the largest GDP loss. In Italy and Spain, following a similar shock, GDP declines by 5.6 and 8.2 per cent, respectively.

Similar conclusions can be made about household consumption. Regarding the labor market, however, we can conclude that aging does not significantly imply a rise in long-run unemployment, while the size of shadow economy somewhat reduces (by about 0.3-0.4 pp both in GDP and employment). Then, consolidating the budget with a labor income tax hike implies a significant rise in unemployment, especially in Spain (unemployment goes up

| <b>Long-run effects of a 5 pp increase in the old-age dependency ratio in a PAYG regime</b> |  |       |       |  |       |       |
|---|--|-------|-------|--|-------|-------|
| If the government consolidates with   | GDP per capita (%)                         |       |       | Share of formal GDP in total GDP (%point)                          |       |       |
|   | Portugal                                   | Italy | Spain | Portugal   | Italy | Spain |
| No consolidation  | -3.8                                       | -3.0  | -3.5  | 0.4  | 0.3   | 0.4   |
| Personal income tax and employee SSC  | -5.9                                       | -5.6  | -8.2  | 0.6  | 0.4   | 0.5   |
| Employer SSC  | -5.3                                       | -4.4  | -6.2  | 0.6  | 0.4   | 0.5   |
| Pension-wage replacement rate   | -3.6                                       | -2.9  | -3.3  | 0.5  | 0.3   | 0.5   |
| Retirement probability  | -3.0                                       | -2.3  | -2.7  | 0.4  | 0.3   | 0.4   |
| Value added tax   | -3.6                                       | -3.1  | -3.3  | -0.3   | -0.2  | -0.2  |
| Government consumption exp./GDP   | -3.6                                       | -3.1  | -3.3  | -0.3   | -0.2  | -0.2  |
| Fertility rate  | 0.0  | 0.0   | 0.0   | 0.1  | 0.0   | 0.1   |
| If the government consolidates with   | Total household consumption per capita (%) |       |       | Share of young household consumption in total consumption (%point) |       |       |
|   | Portugal                                   | Italy | Spain | Portugal   | Italy | Spain |
| No consolidation  | -4.9                                       | -3.8  | -4.6  | -2.8   | -2.1  | -1.7  |
| Personal income tax and employee SSC  | -7.9                                       | -7.4  | -10.9 | -1.4   | -1.3  | -1.3  |
| Employer SSC  | -7.2                                       | -5.9  | -8.3  | -0.9   | -0.5  | -0.5  |
| Pension-wage replacement rate   | -4.9                                       | -3.9  | -4.6  | -0.9   | -0.7  | -0.6  |
| Retirement probability  | -4.1                                       | -3.1  | -3.8  | -0.8   | -0.6  | -0.5  |
| Value added tax   | -5.0                                       | -4.2  | -4.5  | -0.9   | -0.5  | -0.6  |
| Government consumption exp./GDP   | -2.9                                       | -2.4  | -2.8  | -0.9   | -0.5  | -0.6  |
| Fertility rate  | -0.1                                       | -0.1  | -0.1  | 1.1  | 0.9   | 1.0   |
| If the government consolidates with   | Unemployment rate (%point)                 |       |       | Share of informal employment in total employment (%point)          |       |       |
|   | Portugal                                   | Italy | Spain | Portugal   | Italy | Spain |
| No consolidation  | 0.1  | 0.1   | 0.1   | -0.3   | -0.3  | -0.3  |
| Personal income tax and employee SSC  | 0.6  | 0.5   | 2.4   | 0.3  | 0.7   | 0.4   |
| Employer SSC  | 0.4  | 0.3   | 1.4   | 0.2  | 0.2   | 0.1   |
| Pension-wage replacement rate   | 0.1  | 0.0   | 0.1   | -0.4   | -0.3  | -0.4  |
| Retirement probability  | 0.1  | 0.1   | 0.1   | -0.4   | -0.3  | -0.3  |
| Value added tax   | -0.1                                       | -0.1  | -0.2  | 0.2  | 0.2   | 0.1   |
| Government consumption exp./GDP   | -0.1                                       | -0.1  | -0.2  | 0.2  | 0.2   | 0.1   |
| Fertility rate  | 0.0  | 0.0   | 0.0   | 0.0  | 0.0   | 0.0   |
| If the government consolidates with   | Gov. debt as a share of GDP (%point)       |       |       | Instrument (%point)  |       |       |
|   | Portugal                                   | Italy | Spain | Portugal   | Italy | Spain |
| No consolidation  | 111.1                                      | 83.1  | 71.5  | 0.0  | 0.0   | 0.0   |
| Personal income tax and employee SSC  | 1.3  | 2.1   | 2.0   | 5.2  | 4.9   | 7.7   |
| Employer SSC  | 1.0  | 1.3   | 1.1   | 6.1  | 5.7   | 8.5   |
| Pension-wage replacement rate   | 0.3  | 0.3   | 0.2   | -2.7   | -1.7  | -2.5  |
| Retirement probability  | -0.2                                       | -0.4  | -0.1  | -0.04  | -0.03 | -0.04 |
| Value added tax   | 0.5  | 0.6   | 0.4   | 5.6  | 4.0   | 4.4   |
| Government consumption exp./GDP   | 0.4  | 0.5   | 0.3   | -0.6   | -0.5  | -0.4  |
| Fertility rate  | -3.9                                       | -3.9  | -2.2  | -0.2   | -0.1  | -0.2  |

Table 3.4: Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal, Italy and Spain

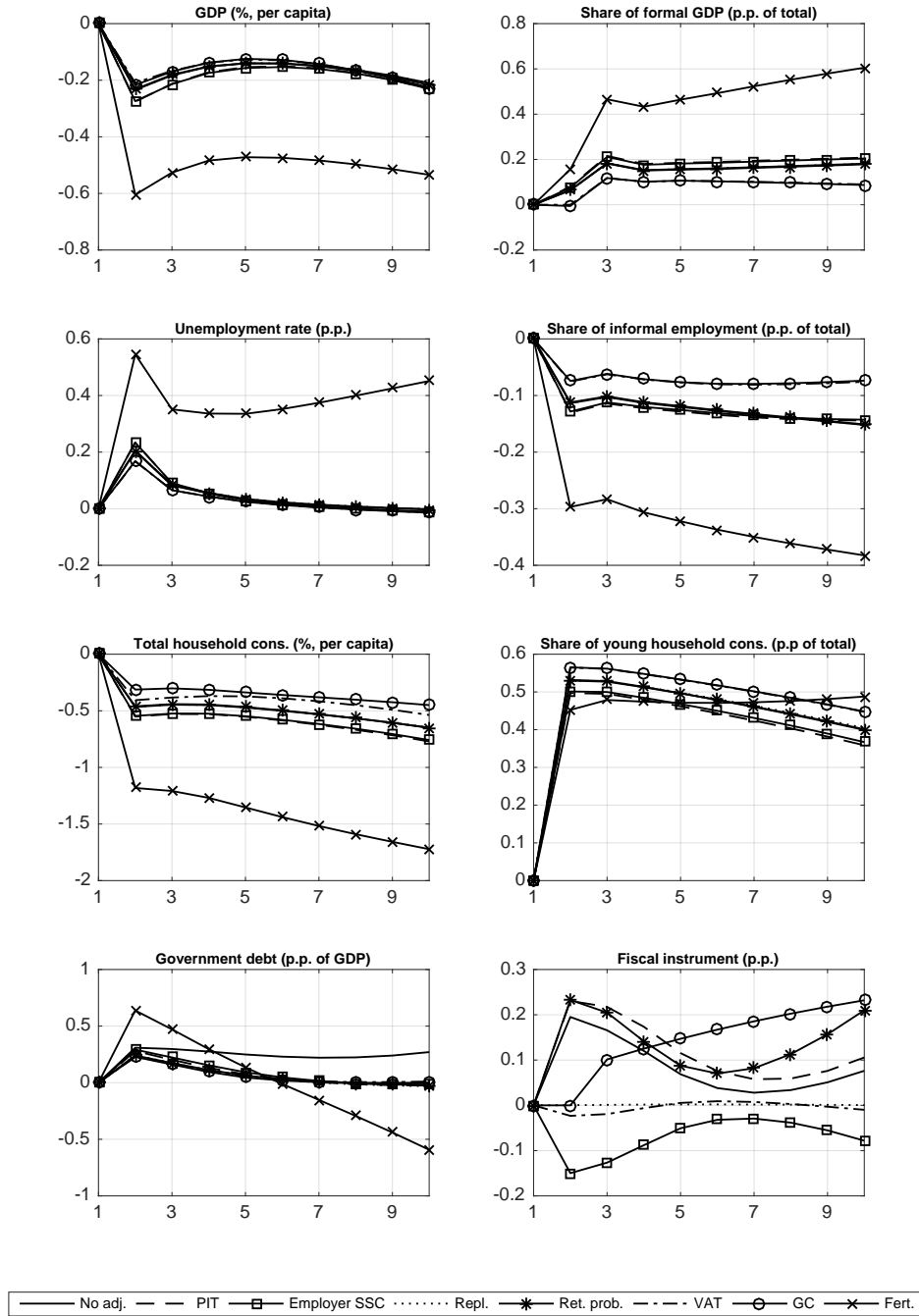


Figure 3.7: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal

by 2.4 and 1.4 pp if employees or employers' tax revenues are raised), while in Portugal and Italy the response of unemployment is about 1/4 of that of Spain. As regards informal employment, all tax increases and also an increase in government consumption, have a positive impact. Italy is the country which is the most negatively affected if personal income taxes go up, i.e. the share of informal employment rises by 1 pp more compared to the no consolidation scenario. Nevertheless, regarding the share of formal GDP in total GDP, value added taxes and government consumption are the tools that cause the largest harm, for instance in Portugal a value added tax hike implies a 0.7 pp increase in the size of the shadow economy. However, the effective value added tax rate must go up by 5.6 pp to induce that effect, while Ihrig-Moe (2004) conclude that even small tax reductions considerably lower informal employment.

So, because Portugal is the country where aging implies the largest loss in GDP (3 per cent if the old-age dependency ratio goes up by 5 pp), and, because Portugal is also the country where, according to the Eurostat's projections, the rise in the old-age dependency ratio will be the highest (34 pp by 2065), from now on we will focus on Portugal. However, additional information is shown on Italy and Spain in the Appendix.

In the short-run, with the exception of a fertility rate policy, we find similar effects across the policies. However, the size of the short-run impacts differs from that of the relevant long-run impacts. While per capita GDP declines by 5.9 per cent in the long-run, in the first 10 years the decline is less severe, around 0.2-0.3 per cent only. It means that the majority of the GDP loss happens over the medium- and long-term. Regarding unemployment, on the other hand, it goes up more in the short- than in the long-run (except after a labor income tax hike). In contrary to GDP, as concerns informality in employment and output, the vast majority of the, otherwise smaller, changes materialize immediately after the shock.

On the whole, taking into account both the long- and short-term effects, the least harmful policy, especially for output, is to increase the retirement age level. However, later we will discuss some doubts about the feasibility of this policy.

### **3.4.2 Comparing long- and short-run macroeconomic impacts in PAYG and fully funded regimes**

Now, we direct our attention to differences between the two pension regimes. The southern European countries' pension plan is PAYG. Still, along with reforming the current PAYG regime, another way to cope with the fiscal effects of aging might be a regime switch. Also, with a different pension plan, the same fiscal consolidation policy, over any time horizon, might have a different effect on the macroeconomy; this is what we will address in this section. Then, in the next section, we will move on to study the transition path of the

regime switch itself, both in the long- and in the short-run. Table 3.5 presents the long-run, while Figures 3.8-3.9 show the short-run macroeconomic impacts of consolidation policies in a PAYG and an FF plan.

First, both in the long- and short-run, consolidating the budget with a personal income tax hike hits the macroeconomy much less negatively with a fully funded regime than with a pay-as-you-go plan; and, the size of the tax hike required to fill in the gap in the budget due to aging is also lower.

Specifically, in the long-run, with a PAYG plan, GDP declines by 5.9 per cent; while the same size of shock implies a reduction in GDP of 4.3 per cent only with an FF plan. Also, the same is true for consumption. Additionally, unemployment increases by 0.2 percentage points in an FF regime, but by 0.6 pp in a PAYG regime. Furthermore, regarding informal employment, the macroeconomic impacts of fiscal consolidation in the two plans are even more different, namely, the share of informal employment in total employment goes up by 0.3 pp in a PAYG regime and goes down by 0.2 pp in an FF regime. Although neither effects are too sizeable, it is important to notice that the signs are different. Also, the share of young household consumption in total consumption increases in an FF regime, while it decreases in a PAYG regime. Lastly, the personal income tax rate must be raised by only 1.7 pp in an FF regime, while a 5.2 pp raise is required in a PAYG regime.

Then, as regards the short-run, on the one hand, quite similar conclusions can be made. GDP, household consumption, unemployment and informal employment change less if the pension regime is fully funded. At the same time, the increase in formal GDP is lower in an FF regime. Regarding the share of young consumption in total consumption, not only the size of the impacts, but also the signs differ: after the shock, in an FF regime it declines, while in a PAYG regime it goes up. Moreover, it is exactly the opposite of the long-run outcomes. So, intuitively a higher personal income tax rate implies a shift from old consumption towards young consumption in an FF regime, but the reverse is seen in a PAYG system.

Regarding other policies<sup>32</sup>, the long-run macroeconomic responses to a VAT hike or a government consumption cut differ less among the two pension regimes. The main difference is the share of young household's consumption in total consumption, which goes up in an FF regime, but goes down in a PAYG regime, as before. Again, it means that these policies hit the old people more in an FF regime than with a PAYG plan, which coincides with the

---

<sup>32</sup>Note that in an FF regime, by design, it is not possible to run simulations of modifying the employer social security contributions, the pension-wage replacement rate of the retirement age. Hence, no chart on pension reforms is presented.

| <b>Long-run effects of a 5 pp increase in the old-age dependency ratio</b> |  |      |  |      |
|--|--|------|--|------|
| If the government consolidates with  | GDP per capita (%)                         |      | Share of formal GDP in total GDP (%point)                          |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | -3.8                                       | -3.7 | 0.4  | 0.5  |
| Personal income tax and employee SSC                                       | -5.9                                       | -4.3 | 0.6  | 0.5  |
| Employer SSC   | -5.3                                       | -    | 0.6  | -    |
| Pension-wage replacement rate  | -3.6                                       | -    | 0.5  | -    |
| Retirement probability   | -3.0                                       | -    | 0.4  | -    |
| Value added tax  | -3.6                                       | -3.6 | -0.3   | 0.2  |
| Government consumption exp./GDP  | -3.6                                       | -3.6 | -0.3   | 0.2  |
| Fertility rate   | 0.0  | -0.1 | 0.1  | 0.0  |
| If the government consolidates with  | Total household consumption per capita (%) |      | Share of young household consumption in total consumption (%point) |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | -4.9                                       | -4.9 | -2.8   | -0.4 |
| Personal income tax and employee SSC                                       | -7.9                                       | -5.9 | -1.4   | 0.0  |
| Employer SSC   | -7.2                                       | -    | -0.9   | -    |
| Pension-wage replacement rate  | -4.9                                       | -    | -0.9   | -    |
| Retirement probability   | -4.1                                       | -    | -0.8   | -    |
| Value added tax  | -5.0                                       | -4.9 | -0.9   | 0.0  |
| Government consumption exp./GDP  | -2.9                                       | -4.2 | -0.9   | 0.0  |
| Fertility rate   | -0.1                                       | -0.1 | 1.1  | 0.8  |
| If the government consolidates with  | Unemployment rate (%point)                 |      | Share of informal employment in total employment (%point)          |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | 0.1  | 0.1  | -0.3   | -0.4 |
| Personal income tax and employee SSC                                       | 0.6  | 0.2  | 0.3  | -0.2 |
| Employer SSC   | 0.4  | -    | 0.2  | -    |
| Pension-wage replacement rate  | 0.1  | -    | -0.4   | -    |
| Retirement probability   | 0.1  | -    | -0.4   | -    |
| Value added tax  | -0.1                                       | 0.0  | 0.2  | -0.2 |
| Government consumption exp./GDP  | -0.1                                       | 0.0  | 0.2  | -0.2 |
| Fertility rate   | 0.0  | 0.1  | 0.0  | 0.0  |
| If the government consolidates with  | Gov. debt as a share of GDP (%point)       |      | Instrument (%point)  |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | 111.1                                      | 31.0 | 0.0  | 0.0  |
| Personal income tax and employee SSC                                       | 1.3  | 0.4  | 5.2  | 1.7  |
| Employer SSC   | 1.0  | -    | 6.1  | -    |
| Pension-wage replacement rate  | 0.3  | -    | -2.7   | -    |
| Retirement probability   | -0.2                                       | -    | -0.04  | -    |
| Value added tax  | 0.5  | 0.2  | 5.6  | 2.0  |
| Government consumption exp./GDP  | 0.4  | 0.1  | -0.6   | 0.2  |
| Fertility rate   | -3.9                                       | -3.8 | -0.2   | -0.2 |

Table 3.5: Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in Portugal, comparing PAYG and fully funded regimes

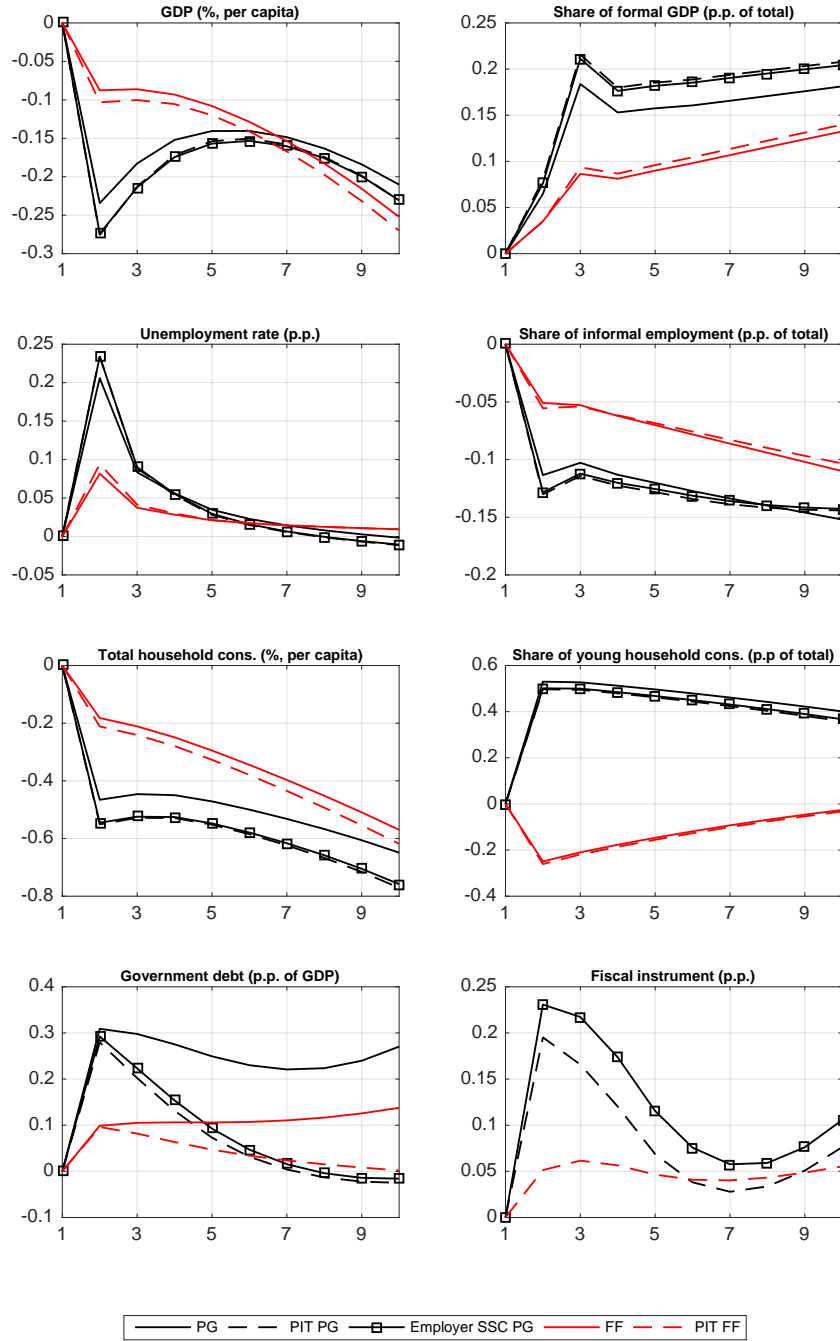


Figure 3.8: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by modifying labor income taxes in Portugal, comparing PAYG and fully funded regimes



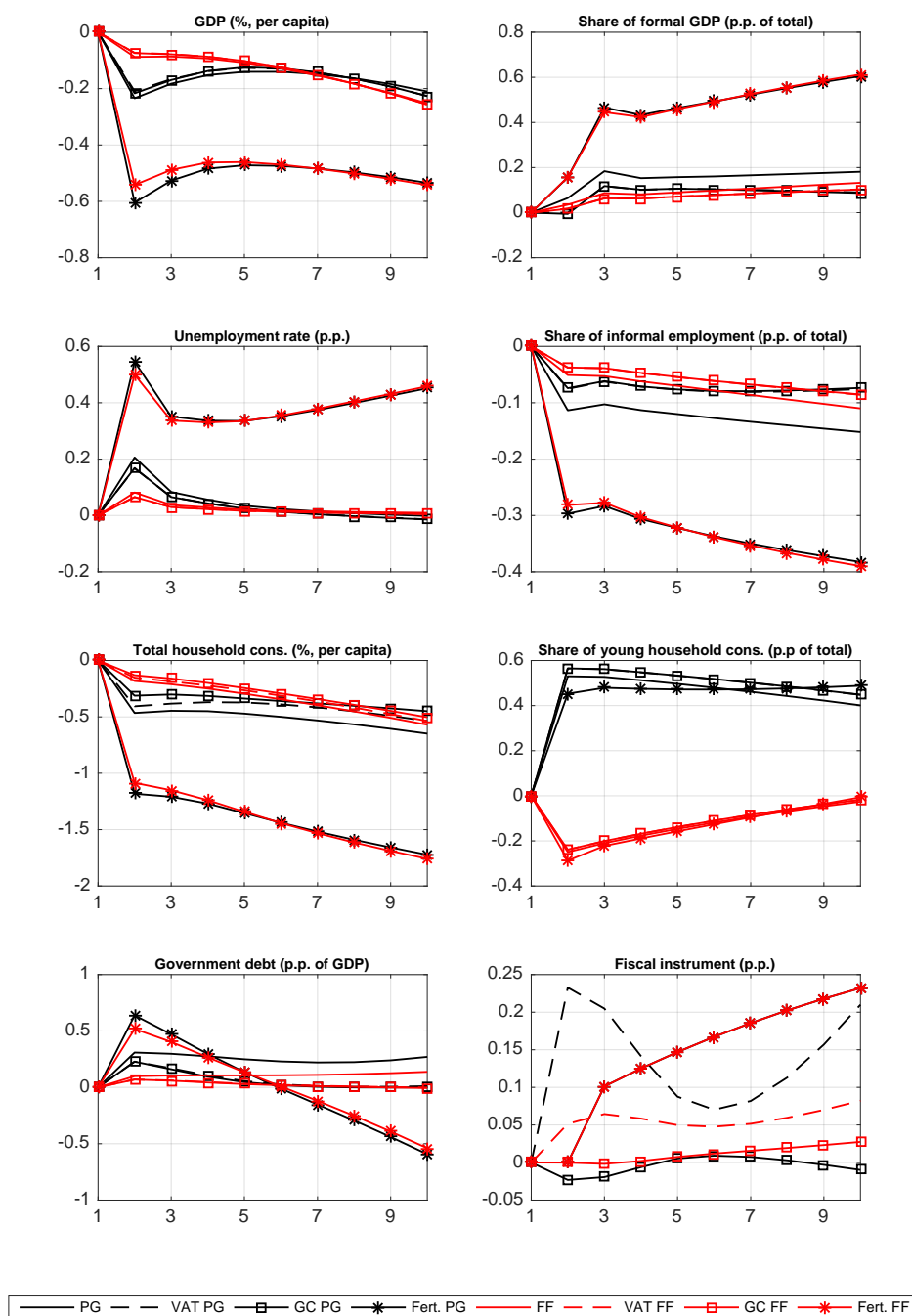


Figure 3.9: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in Portugal, comparing PAYG and fully funded regimes

fact that in a PAYG regime less self-caring is less crucial. Additionally, total household consumption goes down more in an FF regime if government consumption is reduced.

Finally, also the size of the change in the fiscal instruments are markedly different in the two pension regimes. For example, the value added tax rate must increase by 2 percentage points in an FF regime, while in a PAYG regime a much higher VAT hike (5.6 percentage points) is necessary. Then, in the short-run, both policies' impulse responses are highly different (and differences are similar to those of a personal income tax hike already described above).

To sum up, in the long-run, the macroeconomic reactions of a personal income tax hike largely depend on the type of pension regime. Also, there are crucial differences in the long-run reactions of other policies. Meanwhile, in the short-run, all policies imply noticeably unlike macroeconomic effects.

### 3.4.3 Pension regime switch

After studying how the macroeconomic impacts of fiscal policies differ in the two pension regimes, we turn our attention towards the effects of the pension regime switch itself. Two scenarios are distinguished. First, a full reform means that instead of the PAYG plan a fully funded plan is introduced. Second, a partial reform is a mixed reform, PAYG and FF elements are implemented (50-50 per cent). Beetsma et al. (2013), for instance, show that a two-tier system (first tier PAYG, second tier fully funded) is the first best, i.e. provides optimal intergenerational risk-sharing without distorting the labour supply.

| <b>Long-run effects of a switch from PAYG to fully-funded</b>      |             |                |
|--|-------------|----------------|
|  | Full reform | Partial reform |
| GDP per capita (%)   | 0.9         | 0.5            |
| Share of formal GDP in total GDP (%point)                          | 0.5         | 0.2            |
| Unemployment rate (%point)   | -0.2        | -0.1           |
| Share of informal employment in total employment (%point)          | -0.4        | -0.2           |
| Total household consumption per capita (%)                         | -0.1        | 0.0            |
| Share of young household consumption in total consumption (%point) | 7.8         | 3.9            |
| Gov. debt as a share of GDP (%point)                               | -0.3        | -0.2           |
| Implied replacement rate (%point)                                  | 2.6         | 4.9            |

Table 3.6: Long-run (100-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Portugal

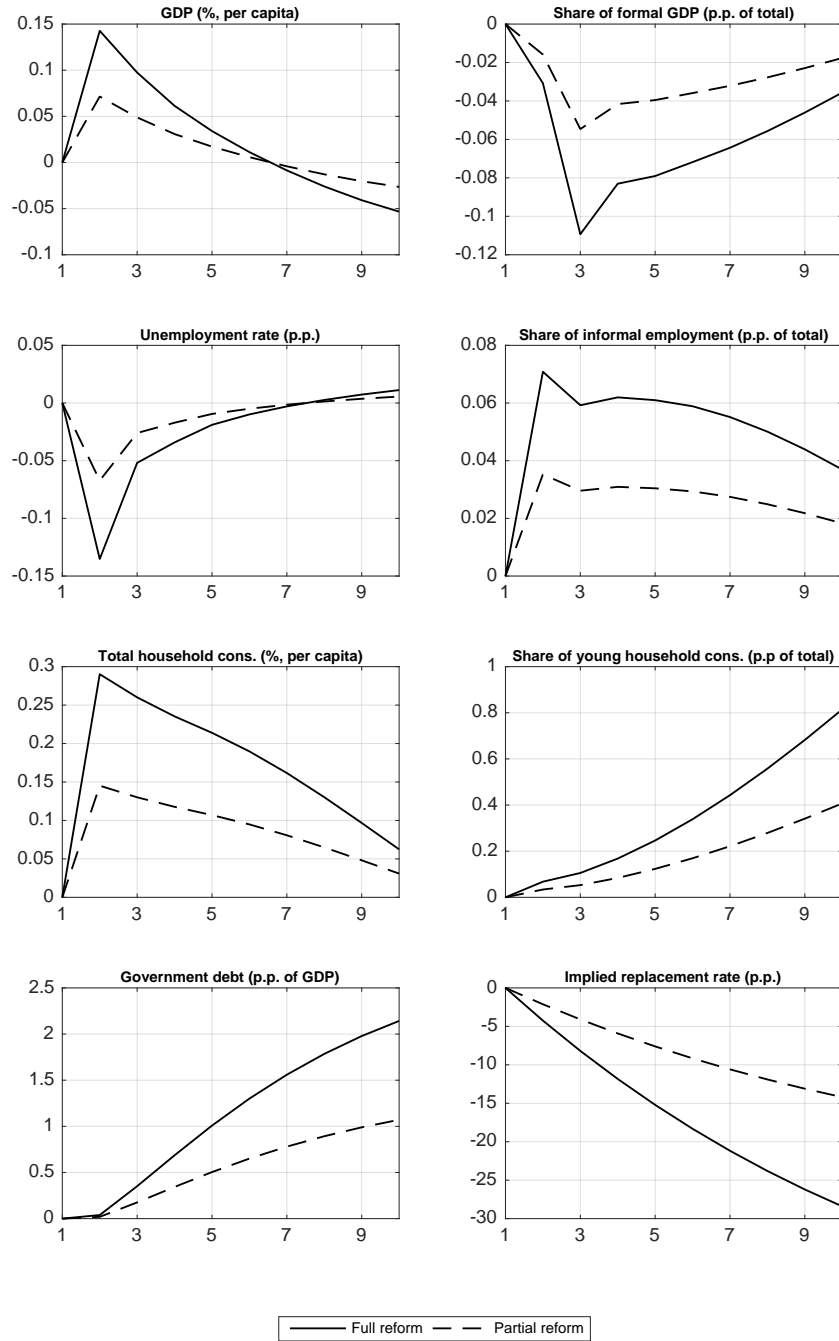


Figure 3.10: Short-run (10-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Portugal

As regards the long-run (Table 3.6), if a fully funded regime is introduced, GDP goes up; by 0.9 per cent with the full reform and 0.5 per cent with the partial reform. Also, the shadow economy shrinks. Notably, the share of formal GDP goes up by 0.5 pp, while the share of informal employment in total employment goes down by 0.4 pp in the long-run (if a full reform is implemented).

In the short-term (Figure 3.10), there are some costs. First of all, the share of formal output declines, and, the share of informal employment rises during the first couple of years after the reform. Hence, it takes some time until the above gain, i.e. the reduction in informality can materialize. A temporary benefit, however, is that household consumption goes up.

The main issue, though, is related to the replacement rate. In an FF regime there is no true replacement rate because individual pension does not depend on the previous wage stream, but on pension savings instead. But, we calculate an implied replacement rate. Although this rate, in the long-run, is higher in an FF regime, in the short-run there is a huge drop. In particular, it goes down by about 25 pp which is about one-third of the original level. We consider this to be a serious drawback which questions the feasibility of this reform, and, we will address it in more detail in the last section of the paper.

### 3.4.4 The role of informality and unemployment

A vital novelty of our model framework is the inclusion of a shadow economy into an overlapping generations framework. According to our knowledge, it was done for the first time in the literature. Informality, on the one hand, means that taxes are avoided in the shadow, on the other hand, the level of regulation is lower in the underground sector (Table 3.7). Because of several reasons listed in the introduction, we believe that its presence can reshape the responses of the macroeconomy. So can that of unemployment. While there is an additional obvious advantage of incorporating them into the framework, namely, that we can study the impacts of fiscal policies on informality and unemployment.

Because of the complexity of our model, we are unable to provide comparative statics analysis, hence, in this section, we show some simulations. Namely, two alternative scenarios are compared to the baseline scenario presented so far. First, we simulate a model without a shadow economy but with unemployment, second, we simulate a model without shadow economy and with full employment.<sup>33</sup> Table 3.8 shows the long-run effects in a PAYG regime,

---

<sup>33</sup>Some model parameters are modified in order to disregard informality and/or unemployment. These are the weight on informal household consumption in the utility function, the firing probability, the hiring cost parameter and the share of capital income in the production function (all in the informal sector). To reduce the size of informality to zero, both in output and in employment, the first parameter is reduced,

while Table 3.9 presents the long-run effects of the pension plan switch.<sup>34</sup>

|                                      | Formal sector | Informal sector |
|--------------------------------------|---------------|-----------------|
| Hiring costs                         | High          | Low             |
| Workers' bargaining power over wages | High          | Low             |
| Firing probability                   | Low           | High            |
| Taxation                             | Yes           | No              |
| Government consumption               | Yes           | No              |

Table 3.7: Rigidities in the formal and informal sectors of the model

In a PAYG regime, labor-income tax policies imply different macroeconomic impacts in the alternative model scenarios than in the baseline model. Regarding GDP, in the baseline, it goes down by 5.9 per cent, but, without a shadow economy, the decline is only 4.6 per cent. The loss in GDP is even lower if unemployment is not present; it is only 4.1 per cent. Similarly, raising the employer social security contribution rate also induces a considerably lower loss, both without a shadow economy and without a shadow economy and unemployment.

Not only the reaction of output, but also that of other variables are unlike if either informality or unemployment are not included. For instance, household consumption declines by 2 pp or 2.9 pp less in the alternative scenarios than in the baseline case (where the drop is 7.9 per cent). However, the relative size of young households' consumption is much less affected. Interestingly, the response of unemployment itself is not affected by the presence of shadow economy, either. It seems that workers, rather, move between the official and the unofficial economies, than out of or into unemployment.

The underlying intuition is related to the fact that the necessary raise in the tax rates is higher in the baseline model. Specifically, in the baseline model, the personal income tax and employee social security contribution rate is required to go up by 5.2 pp, while the employer social security contribution rate needs to be increased by 6.1 pp. Then, without an underground economy or unemployment the tax hikes are lower; 4.7 and 5.7 or 4.7 and 5.9 pp, respectively. In Section 3.1 we described two opposite channels implied by the presence of a shadow economy, namely, that the larger the size of the shadow economy, the smaller the share of the economy directly affected by the government, but the larger the amount of people and goods that can potentially move out of the shadow. It seems that the first effect dominates, ignoring the shadow side of the economy biases the tax rates downwards due to

---

while all the others are increased. Then, additionally, in the model without unemployment, also the formal hiring cost parameter is lowered.

<sup>34</sup>Figures 3.A1-3.A7 in the Appendix show the relevant short-run dynamics.

| Long-run effects of a 5 pp increase in the old-age dependency ratio in PAYG regime |  |                   |                                    |                                      |                   |                                    |  |                   |                                    |
|--|--|-------------------|------------------------------------|--------------------------------------|-------------------|------------------------------------|--|-------------------|------------------------------------|
| If the government consolidates with  | GDP per capita (%)   |                   |                                    | Unemployment rate (%point)           |                   |                                    | Total household consumption per capita (%) |                   |                                    |
|  | Baseline   | No shadow economy | No shadow economy, no unemployment | Baseline                             | No shadow economy | No shadow economy, no unemployment | Baseline                                   | No shadow economy | No shadow economy, no unemployment |
|  |  |                   |                                    |                                      |                   |                                    |  |                   |                                    |
| No consolidation   | -3.8   | -4.0              | -3.9                               | 0.1                                  | 0.0               | -                                  | -4.9                                       | -4.7              | -4.5                               |
| Personal income tax and employee SSC   | -5.9   | -4.6              | -4.1                               | 0.6                                  | 0.5               | -                                  | -7.9                                       | -5.9              | -5.0                               |
| Employer SSC   | -5.3   | -4.4              | -4.0                               | 0.4                                  | 0.4               | -                                  | -7.2                                       | -5.7              | -5.0                               |
| Pension-wage replacement rate  | -3.6   | -3.8              | -3.7                               | 0.1                                  | -0.1              | -                                  | -4.9                                       | -5.0              | -4.7                               |
| Retirement probability   | -3.0   | -3.1              | -3.0                               | 0.1                                  | 0.0               | -                                  | -4.1                                       | -4.1              | -3.8                               |
| Value added tax  | -3.6   | -3.5              | -3.6                               | -0.1                                 | -0.1              | -                                  | -5.0                                       | -4.8              | -4.6                               |
| Government consumption exp./GDP  | -3.6   | -3.5              | -3.6                               | -0.1                                 | -0.1              | -                                  | -2.9                                       | -2.3              | -1.8                               |
| Fertility rate   | 0.0  | 0.0               | 0.0                                | 0.0                                  | 0.0               | -                                  | -0.1                                       | -0.2              | -0.1                               |
| Instrument (%point)  |  |                   |                                    |                                      |                   |                                    |  |                   |                                    |
| If the government consolidates with  | Share of young household consumption in total consumption (%point) |                   |                                    | Gov. debt as a share of GDP (%point) |                   |                                    |  |                   |                                    |
|  | Baseline   | No shadow economy | No shadow economy, no unemployment | Baseline                             | No shadow economy | No shadow economy, no unemployment | Baseline                                   | No shadow economy | No shadow economy, no unemployment |
|  |  |                   |                                    |                                      |                   |                                    |  |                   |                                    |
| No consolidation   | -2.8   | -3.5              | -3.6                               | 111.1                                | 154.9             | 186.4                              | 0.0  | 0.0               | 0.0                                |
| Personal income tax and employee SSC   | -1.4   | -1.5              | -1.4                               | 1.3                                  | 0.9               | 0.8                                | 5.2  | 4.7               | 4.7                                |
| Employer SSC   | -0.9   | -1.0              | -0.9                               | 1.0                                  | 0.9               | 0.9                                | 6.1  | 5.7               | 5.9                                |
| Pension-wage replacement rate  | -0.9   | -1.1              | -1.0                               | 0.3                                  | 0.5               | 0.6                                | -2.7                                       | -2.9              | -3.1                               |
| Retirement probability   | -0.8   | -0.9              | -0.9                               | -0.2                                 | -0.1              | -0.1                               | -0.04                                      | -0.04             | -0.05                              |
| Value added tax  | -0.9   | -1.0              | -1.0                               | 0.5                                  | 0.6               | 0.7                                | 5.6  | 4.4               | 4.1                                |
| Government consumption exp./GDP  | -0.9   | -1.0              | -1.0                               | 0.4                                  | 0.4               | 0.5                                | -0.6                                       | -0.9              | -1.3                               |
| Fertility rate   | 1.1  | 1.3               | 1.1                                | -3.9                                 | -4.0              | -3.8                               | -0.2                                       | -0.2              | -0.2                               |

Table 3.8: Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment

the assumption that all economic agents pay taxes.<sup>35</sup> Tax evasion is actually the most significant part of informality.<sup>36</sup> For other policies, the presence of informality and unemployment are less crucial, except the size of the fiscal instrument that very much depends on them.

Additionally, the presence of the unofficial economy also affects the long-run outcomes of a pension regime switch, although to a less extent (Table 3.9). Unemployment declines more, if the underground sector is ignored, still, the decrease in unemployment is low. Total household consumption, however, declines without shadow economy, while it was stable in the baseline model. At the same time, on the whole, informality and unemployment are less relevant for the pension plan switch compared to reforming the PAYG regime itself.

| <b>Long-run effects of a full switch from PAYG to fully-funded</b> |          |                   |                                    |
|--|----------|-------------------|------------------------------------|
|  | Baseline | No shadow economy | No shadow economy, no unemployment |
| GDP per capita (%)   | 0.9      | 0.9               | 0.9                                |
| Share of formal GDP in total GDP (%point)                          | 0.5      | -                 | -                                  |
| Unemployment rate (%point)   | -0.2     | -0.5              | -                                  |
| Share of informal employment in total employment (%point)          | -0.4     | -                 | -                                  |
| Total household consumption per capita (%)                         | -0.1     | -0.8              | -0.4                               |
| Share of young household consumption in total consumption (%point) | 7.8      | 9.4               | 9.4                                |
| Gov. debt as a share of GDP (%point)                               | -0.3     | -0.2              | -0.2                               |
| Implied replacement rate (%point)                                  | 2.6      | 2.4               | 0.3                                |

Table 3.9: Long-run (100-year) effects of a full pension reform in Portugal, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment

### 3.5 Conclusion and policy discussion

We studied the macroeconomic impacts of pension reforms and other public policies to handle the negative fiscal consequences of aging. By doing so, we developed an overlapping generations model with demography, unemployment, and, according to our knowledge for the first time in the literature, with a shadow economy. Also, in contrary to many papers, we investigated the impacts of a retirement age increase, and, we focused on both a pay-as-you-go and fully funded regimes. The model was calibrated on annual data on Portugal, Italy and Spain, because these countries are among those that have the highest public debt level, and, are also projected to experience the most severe aging in the next decades in the

<sup>35</sup>In particular, in a model without informality, not only the long-run level of unemployment higher, but also that of formal employment, so mistakenly it is assumed that more people pay taxes than actually do, hence, a lower tax increase is required to reduce budget deficit.

<sup>36</sup>See Schneider (2012) for example.

region. We focused on Portugal, in particular, where the Eurostat projects that the old-age dependency ratio will increase by 34 pp by 2065.

Based on both the long-run outcomes and the short-run dynamics, we concluded that the best policy to cope with the negative fiscal impacts of aging is to increase the retirement age. Best here means that this is the policy that causes the lowest macroeconomic, especially output, losses in the long-run.

Based on our calculations, in Portugal, the negative impacts on the public budget of the above aging can be neutralized by a 2.1-year increase in the retirement age level (Table 3.10).<sup>37</sup> According to OECD (2015) the current level of effective retirement age in Portugal is 66.6 years. Hence, it means that Portuguese workers should retire at the age of 68.7 in 2065. As regards Italy and Spain, on the one hand, a smaller increase in the retirement age would be enough (1.5 and 1.9 years, respectively) to handle the negative fiscal impacts of aging<sup>38</sup>, on the other hand, the current retirement age levels are lower than the Portuguese one. As a consequence, in 2065, the effective retirement age should be 62.8 years in Italy and 64.6 years in Spain; these are about 6 and 4 years lower than the relevant Portuguese value.<sup>39</sup>

|  | Portugal | Italy  | Spain |
|--|----------|--------|-------|
| Retirement age increase (years)  | 2.1      | 1.5    | 1.9   |
| New effective retirement age (years)                                     | 68.7     | 62.8   | 64.6  |
| Number of people affected by retirement age increase (thousands)         | 212.5    | 1227.3 | 945.7 |
| Loss of after retirement-years in 2065 after retirement age increase (%) | -9.2     | -5.2   | -7.0  |
| Gain in after retirement-years in 2065 compared to 2015 (%)              | 18.2     | 15.6   | 12.9  |

Table 3.10: Retirement age increase and its consequences in southern Europe

Having a look at life expectancy in Portugal, it is feasible to raise the retirement age to 68.7 years. Because, for 65-year-old women and men, life expectancy on average will be 89.1 years in 2065 (OECD (2013)).

<sup>37</sup>Without shadow economy a 2.2, without unemployment a 2.3-year increase is projected.

<sup>38</sup>As a comparison, *Díaz-Giménez-Díaz-Saavedra* (2009) find that, in Spain, 3 years of retirement age increase is necessary to maintain fiscal sustainability.

<sup>39</sup>Although, the number of people affected by this policy would be significantly higher in Italy and Spain, due to the fact that more people live in those countries.



Nevertheless, raising the statutory retirement age might not result in a higher (or high enough) effective retirement age. As IMF (2014) points out there is no significant relation between the statutory and the effective retirement age levels. Hence, even if the government raises the official retirement age level, it might not modify, or might not modify enough, the effective retirement age level (Figure 3.11). Similarly, Fehr et al. (2012) claim that a 2-year increase in the official retirement age implies a 1-year increase in the effective retirement age in Germany.

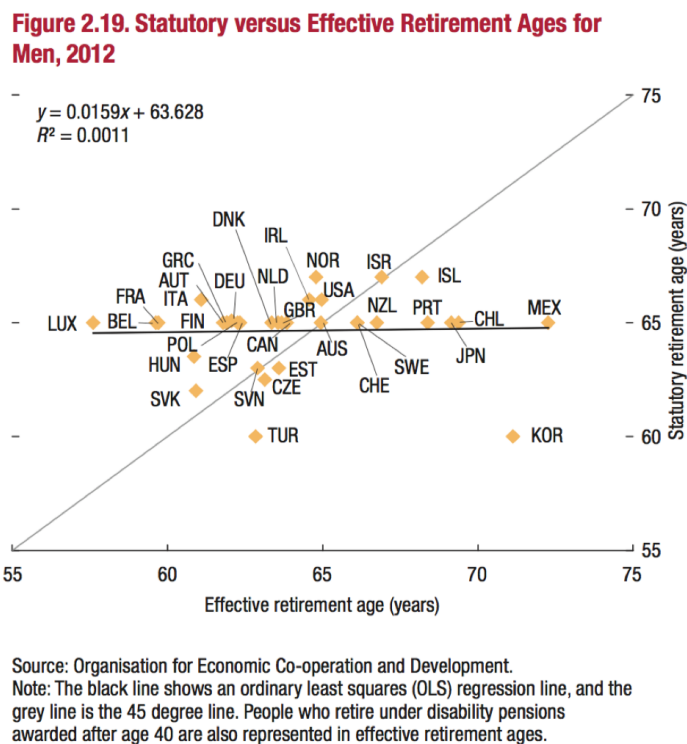


Figure 3.11: Official versus effective retirement age levels (IMF, 2014, page 39)

Several reasons might be behind this. As regards employees, a recent survey by Aegon (2014) points out that “41 per cent of employees are resistant to increasing the official retirement age taking the view that people are already expected to work long enough”. To put it otherwise, 4 people out of 10 are not willing to work longer even if they are required to. And, because of aging, while today about one-third of the people are old, in 2065 two-third of the population will be elderly. Hence, in an absolute value a large number of people seem to be reluctant to continue work at this age. In accordance with this, Galasso (2008) show, based on Eurobarometer data, that 70 per cent of EU citizens do not want a retirement age increase. The fact that a 2-year increase in the retirement age would imply that Portuguese people lose 9.2 per cent of their after-retirement time might explain why they protest. At the same time, if we also take into account the increase in life expectancy, they would still gain 18 per cent of lifetime (comparing after-retirement years in 2065 with the higher retirement

age level to after-retirement years in 2015 with the lower retirement age level).<sup>40</sup>

Regarding the firm side, World Bank (2015) claims that old people are not less productive than young people.<sup>41</sup> Also, Munnell et al. (2006) (based on US data) find that employers are not less willing to employ old people than young people, although they consider the fact that old people might not work too long which could imply a negative impact on their productivity. Thus, on the whole, it seems that whether an official retirement age increase implies an increase in the effective retirement age level as well lies on the employees' side.<sup>42</sup>

Alongside increasing the retirement age, there might be other options. First, other public policies can be also implemented. But, they decrease output or increase unemployment more than the retirement age policy does. Second, demographic changes might be needed. Notably, an increase in the fertility rate could compensate for a decrease in the mortality rate at a much lower long-run output loss (practically output stays stable in the long-run). However, in the short-run this is the policy that induces the largest increase in unemployment which raises some concerns regarding its feasibility. Also, according to OECD (2011) the fertility rate will even slightly increase in the next decades, hence, aging is mainly a consequence of a longevity boom, rather than a fertility failure. Nevertheless, the relationship between pensions and endogenous fertility remains beyond the scope of the present study.<sup>43</sup>

Now, turning our attention towards the pension regime switch, it reduces the size of the shadow economy. Particularly, introducing a fully funded pension plan implies a 0.4 pp reduction in the share of informal employment in total employment and a 0.5 pp reduction in the share of underground GDP in total GDP. At the same time, there is a sharp, about 25 pp drop in the pension-wage replacement rate, although this drop is only temporary. Still, it questions the feasibility of this policy from a political point of view. Without some additional

---

<sup>40</sup>Of course, we are aware of the fact that the health condition worsens as people get older.

<sup>41</sup>Although people below the age of 64 were studied, while in Portugal the retirement age would be almost 69 years.

<sup>42</sup>It is beyond the scope of this paper to further investigate the weak relation between official and effective retirement age levels. IMF (2014), for instance, also points out that, besides the official retirement age, there are several other factors that influence the retirement decision, among others taxes on continuing work. Thus, we do not aim to draw conclusions regarding this issue, rather, we urge further research.

<sup>43</sup>There is a great deal of extant research on this. Regarding the relation of pensions and fertility, two main streams can be distinguished in the (theoretical) literature. Barro-Becker (1989) and others argue that the fact that children tend to take care of their elderly parents is at the root of a positive relationship between fertility and pensions. As against this, Boldrin-Jones (2002) and others claim that because children tend to take care of their elderly parents, pensions and fertility are negatively related. The empirical evidence is mixed (Hohm (1975), Cigno (1993), Cigno-Rosati (1992), Boldrin et al. (2005), Wang (2015) and many others). Related to fertility, migration and social security has been studied by Razin-Sadka (1999), Storesletten (2000) and Kilponen et al. (2006), among many others.

policy, for example providing transfers for those who suffer from the negative effects of the lower pension replacement rate, implementation might fail.

We are convinced that the inclusion of a shadow economy into an OLG framework was a vital step in studying the fiscal consequences of aging. Of course, our framework still lacks some features that might constitute interesting lines of future research. First, there is no labor force participation decision. Hence, someone is either employed or unemployed when young, and, retired when old. Nevertheless, choosing to exit the labor force is also a valid option. Too, someone previously not in the labor force might enter it. Second, for the sake of simplicity we assume that retired people do not work. In practice, however, some retired people work (the extent depends on the country), so retired people have multiple sources of income as well. Finally, individuals close to retirement might have (more) trouble finding a job because of skill-mismatch; extra time might be required for skill-update. Future work might consider these important elements as well.

## Appendix<sup>44</sup>

### A Overlapping generations' optimization

First, we focus on solving the optimizing problems of the young and old generations. Then, we describe the pay-as-you-go and fully funded pension systems in detail. At the end, we list all the normalized equations, i.e. equations detrended by technology and population growth, provide the steady state calculation of the model and show evidence on the model's robustness.<sup>45</sup>

#### Demography

Total population ( $N_t$ ) is equal to the sum of the number of old (retired) ( $N_t^O$ ) and young (worker) people ( $N_t^Y$ ):

$$N_t = N_t^O + N_t^Y \quad (3.96)$$

$$N_t^Y = (1 - \omega_{t-1}^Y)N_{t-1}^Y + n_t N_{t-1}^Y \quad (3.97)$$

$$N_t^O = (1 - \omega_{t-1}^O)N_{t-1}^O + \omega_{t-1}^Y N_{t-1}^Y \quad (3.98)$$

Similarly to most of the general equilibrium models, we focus on the relative shares and not on the levels.  $s_t$  denotes the ratio of the number of old and young people, while  $s_t^Y$  denotes the share of young people in the whole population:

$$\begin{aligned} s_t &= \frac{N_t^O}{N_t^Y} = \frac{(1 - \omega_{t-1}^O)N_{t-1}^O + \omega_{t-1}^Y N_{t-1}^Y}{N_t^Y} = (1 - \omega_{t-1}^O) \frac{N_{t-1}^O}{N_{t-1}^Y} \frac{N_{t-1}^Y}{N_t^Y} + \\ &+ \omega_{t-1}^Y \frac{N_{t-1}^Y}{N_t^Y} = \frac{(1 - \omega_{t-1}^O)}{(1 - \omega_{t-1}^Y + n_t)} s_{t-1} + \frac{\omega_{t-1}^Y}{(1 - \omega_{t-1}^Y + n_t)} \end{aligned} \quad (3.99)$$

$$s_t^Y = \frac{N_t^Y}{N_t} = \frac{N_t^Y}{N_t^Y + N_t^O} = \frac{1}{1 + \frac{N_t^O}{N_t^Y}} = \frac{1}{1 + s_t} \quad (3.100)$$

Then, we can express the growth rate of each cohort:

$$1 + g_t^{N,Y} = \frac{N_t^Y}{N_{t-1}^Y} = (1 - \omega_{t-1}^Y)N_{t-1}^Y + n_t N_{t-1}^Y = 1 - \omega_{t-1}^Y + n_t \quad (3.101)$$

$$1 + g_t^{N,O} = \frac{N_t^O}{N_{t-1}^O} = \frac{(1 - \omega_{t-1}^O)N_{t-1}^O + \omega_{t-1}^Y N_{t-1}^Y}{N_{t-1}^O} = (1 - \omega_{t-1}^O) + \frac{\omega_{t-1}^Y}{s_{t-1}} \quad (3.102)$$

---

<sup>44</sup>On Figures 3.A2 and 3.A5, on the subchart of fiscal instruments retirement probabilities are shown on the right scale.

<sup>45</sup>Regarding any other technical detail, further information is available from the authors upon request.

Finally, population growth follows as:

$$\begin{aligned}
1 + g_t^N &= \frac{N_t^Y + N_t^O}{N_{t-1}^Y + N_{t-1}^O} = \frac{\frac{N_t^Y}{N_{t-1}^Y} + \frac{N_t^O}{N_{t-1}^O}}{\frac{N_{t-1}^Y}{N_{t-1}^Y} + \frac{N_{t-1}^O}{N_{t-1}^O}} = \frac{1 + g_t^{N,Y} + \frac{N_t^O}{N_{t-1}^O}}{1 + s_{t-1}} = \\
&= \frac{1 + g_t^{N,Y} + \frac{N_t^O}{N_{t-1}^O} \frac{N_t^Y}{N_t^Y}}{1 + s_{t-1}} = \frac{1 + g_t^{N,Y} + s_t(1 + g_t^{N,Y})}{1 + s_{t-1}} = (1 + g_t^{N,Y}) \frac{1 + s_t}{1 + s_{t-1}} \quad (3.103)
\end{aligned}$$

## Retired generation

### First order conditions of a retired agent

'Retired' agent  $i$  of retired cohort  $a$  is one individual who retired  $a$  years ago. Each agent maximises the following Bellman equation:

$$\begin{aligned}
V^O(B_{a-1,t-1}^O(i)) &= \\
&= \max \left\{ (1 + \epsilon_t^C) \left[ \frac{1}{1 - \gamma} \left\{ C_{a,t}^{O,F}(i) \right\}^{1-\gamma} + \frac{\chi}{1 - \gamma} \left\{ C_{a,t}^{O,I}(i) \right\}^{1-\gamma} \right] + \right. \\
&\quad \left. + \beta E_t(1 - \omega_t^O) V^O(B_{a,t}^O(i)) \right\} \quad (3.104)
\end{aligned}$$

subject to this budget constraint:

$$\begin{aligned}
(1 + \tau_t^C) C_{a,t}^{O,F}(i) + p_t^I C_{a,t}^{O,I}(i) + (1 - \omega_t^O) B_{a,t}^O(i) &= (1 + r_{t-1}) B_{a-1,t-1}^O(i) + \\
+ TR_{a,t}^{PG,YO}(i) + TR_{a,t}^{FF,YO}(i) + Profit_{a,t}^O(i) - T_{a,t}^O(i) \quad (3.105)
\end{aligned}$$

First order conditions:

$$C_{a,t}^{O,F}(i) : (1 + \epsilon_t^C) \left\{ C_{a,t}^{O,F}(i) \right\}^{-\gamma} + \lambda_{a,t}^O (1 + \tau_t^C) = 0 \quad (3.106)$$

$$C_{a,t}^{O,I}(i) : (1 + \epsilon_t^C) \chi \left\{ C_{a,t}^{O,I}(i) \right\}^{-\gamma} + \lambda_{a,t}^O p_t^I = 0 \quad (3.107)$$

$$B_{a,t}^O(i) : \beta E_t(1 - \omega_t^O) V_{B_{a,t}^O(i)}^O + E_t(1 - \omega_t^O) \lambda_{a,t}^O = 0 \quad (3.108)$$

One-period-ahead Envelope theorem:

$$E_t V_{B_{a,t}^O(i)} = -E_t \lambda_{a+1,t+1}^O (1 + r_t) \quad (3.109)$$

The first order conditions imply the Euler-equation:

$$\beta E_t \frac{(1 + \epsilon_{t+1}^C) \left( C_{a,t}^{O,F}(i) \right)^\gamma}{(1 + \epsilon_t^C) \left( C_{a+1,t+1}^{O,F}(i) \right)^\gamma} (1 + r_t) \frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} = 1 \quad (3.110)$$

which can be rearranged:

$$C_{a,t}^{O,F}(i) E_t \left\{ \beta \frac{1 + \epsilon_{t+1}^C}{1 + \epsilon_t^C} (1 + r_t) \frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} \right\}^{\frac{1}{\gamma}} = E_t C_{a+1,t+1}^{O,F}(i) \quad (3.111)$$

$$E_t C_{a+1,t+1}^{O,F}(i) = E_t C_{a,t}^{O,F}(i) (1 + r_t)^{\frac{1}{\gamma}} \Lambda_{t+1} \quad (3.112)$$

where  $E_t \Lambda_{t+1} = E_t \left\{ \beta \frac{1 + \epsilon_{t+1}^C}{1 + \epsilon_t^C} \frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} \right\}^{\frac{1}{\gamma}}$ .

Based on the Euler-equation all future retired consumptions follow:

$$E_t C_{a+n,t+n}^{O,F}(i) = C_{a,t}^{O,F}(i) E_t \prod_{k=1}^n (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} \quad (3.113)$$

Then, the substitution between formal and informal goods also follows from the first order conditions:

$$(1 + \epsilon_t^C) \left\{ C_{a,t}^{O,F}(i) \right\}^{-\gamma} = (1 + \epsilon_t^C) \chi \left\{ C_{a,t}^{O,I}(i) \right\}^{-\gamma} \frac{1 + \tau_t^C}{p_t^I} \quad (3.114)$$

which can be rewritten as:

$$C_{a,t}^{O,I}(i) = \Upsilon_t C_{a,t}^{O,F}(i) \quad (3.115)$$

where  $\Upsilon_t = \left\{ \chi \frac{1 + \tau_t^C}{p_t^I} \right\}^{\frac{1}{\gamma}}$ .

### Individual consumption of a retired agent

First, we derive the intertemporal budget constraint from the one-period budget constraint:

$$\begin{aligned} E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^O) \left( (1 + \tau_{t+n}^C) C_{a+n,t+n}^{O,F}(i) + p_{t+n}^I C_{a+n,t+n}^{O,I}(i) \right)}{\prod_{k=1}^n (1 + r_{t+k-1})} = \\ = E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^O) \left( TR_{a+n,t+n}^{PG,YO}(i) + TR_{a+n,t+n}^{FF,YO}(i) + Profit_{a+n,t+n}^O(i) - T_{a+n,t+n}^O(i) \right)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ + (1 + r_{t-1}) B_{a-1,t-1}^O(i) \end{aligned} \quad (3.116)$$

if  $k > n$  and  $r_{t+k} = 0$ .

Then, we plug in the formal-informal substitution equation:

$$\begin{aligned} E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^O) \left( (1 + \tau_{t+n}^C) C_{a+n,t+n}^{O,F}(i) + p_{t+n}^I C_{a+n,t+n}^{O,F}(i) \Upsilon_{t+n} \right)}{\prod_{k=1}^n (1 + r_{t+k-1})} = \\ = E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^O) \left( TR_{a+n,t+n}^{PG,YO}(i) + TR_{a+n,t+n}^{FF,YO}(i) + Profit_{a+n,t+n}^O(i) - T_{a+n,t+n}^O(i) \right)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ + (1 + r_{t-1}) B_{a-1,t-1}^O(i) \end{aligned} \quad (3.117)$$

After some rearranging:

$$\begin{aligned}
& E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^O) C_{a+n,t+n}^{O,F} (i) ((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n})}{\prod_{k=1}^n (1 + r_{t+k-1})} = \\
& = E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^O) \left( TR_{a+n,t+n}^{PG,YO} (i) + TR_{a+n,t+n}^{FF,YO} (i) + Profit_{a+n,t+n}^O (i) - T_{a+n,t+n}^O (i) \right)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\
& + (1 + r_{t-1}) B_{a-1,t-1}^O (i) \tag{3.118}
\end{aligned}$$

Now, we can use the Euler equation for future consumptions:

$$\begin{aligned}
& E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^O) C_{a,t}^{O,F} (i) \prod_{k=1}^n (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} ((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n})}{\prod_{k=1}^n (1 + r_{t+k-1})} = \\
& = E_t \sum_{n=0}^{\infty} \frac{(1 - \omega_{t+k-1}^O)^n \left( TR_{a+n,t+n}^{PG,YO} (i) + TR_{a+n,t+n}^{FF,YO} (i) + Profit_{a+n,t+n}^O (i) - T_{a+n,t+n}^O (i) \right)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\
& + (1 + r_{t-1}) B_{a-1,t-1}^O (i) \tag{3.119}
\end{aligned}$$

Finally, if we rearrange we get consumption of agent  $i$  of cohort  $a$  at time  $t$  as a function of present value of pension and other income and initial wealth:

$$\begin{aligned}
C_{a,t}^{O,F} (i) & = \frac{E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^O) (TR_{a+n,t+n}^{PG,YO} (i) + TR_{a+n,t+n}^{FF,YO} (i) + Profit_{a+n,t+n}^O (i) - T_{a+n,t+n}^O (i))}{\prod_{k=1}^n (1 + r_{t+k-1})}}{E_t \sum_{n=0}^{\infty} ((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n}) \prod_{k=1}^n (1 - \omega_{t+k-1}^O) (1 + r_{t+k-1})^{\frac{1}{\gamma}-1} \Lambda_{t+k}} + \\
& + \frac{(1 + r_{t-1}) B_{a-1,t-1}^O (i)}{E_t \sum_{n=0}^{\infty} ((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n}) \prod_{k=1}^n (1 - \omega_{t+k-1}^O) (1 + r_{t+k-1})^{\frac{1}{\gamma}-1} \Lambda_{t+k}} \tag{3.120}
\end{aligned}$$

After some simplification and by introducing some additional variables, the final version of consumption of agent  $i$  of cohort  $a$  at time  $t$  is:

$$\begin{aligned}
\mathcal{H}_t^O C_{a,t}^{O,F} (i) & = (TR_{a,t}^{PG,YO} (i) + TR_{a,t}^{FF,YO} (i)) \Omega_t^O + \mathcal{I}_{a,t}^O (i) + \\
& + (1 + r_{t-1}) B_{a-1,t-1}^O (i) \tag{3.121}
\end{aligned}$$

$$\Omega_t^O = 1 + E_t \frac{1 - \omega_t^O}{1 + r_t} \Omega_{t+1}^O \tag{3.122}$$

$$\mathcal{I}_{a,t}^O (i) = Profit_{a,t}^O (i) - T_{a,t}^O (i) + E_t \frac{1 - \omega_t^O}{1 + r_t} \mathcal{I}_{a+1,t+1}^O (i) \tag{3.123}$$

$$\mathcal{H}_t^O = (1 + \tau_t^C) + p_t^I \Upsilon_t + E_t (1 - \omega_t^O) (1 + r_t)^{\frac{1}{\gamma}-1} \Lambda_{t+1} \mathcal{H}_{t+1}^O \tag{3.124}$$

Here, we would like to note that  $TR_{n,t+n}^{PG,YO} (i) = TR_{0,t}^{PG,YO} (i) \forall n > 0$ , and, the same is true for fully funded pensions.

## Aggregate consumption of the retired cohort

Aggregate consumption is equal to the sum of total pension and other income and initial wealth:

$$\begin{aligned} \sum_{a=0}^{\infty} N_{a,t}^O(i) C_{a,t}^{O,F}(i) \mathcal{H}_t^O &= \sum_{a=0}^{\infty} N_{a,t}^O(i) \left( TR_{a,t}^{PG,YO}(i) + TR_{a,t}^{FF,YO}(i) \right) \Omega_t^O + \\ &+ \sum_{a=0}^{\infty} N_{a,t}^O(i) \mathcal{I}_{a,t}^O(i) + (1 + r_{t-1}) \sum_{a=0}^{\infty} N_{a,t}^O(i) B_{a-1,t-1}^O(i) \end{aligned} \quad (3.125)$$

First, the number of old people declines over time:

$$N_{a+1,t}^O = (1 - \omega_{t-1}^O) N_{a,t-1}^O \quad (3.126)$$

$$N_{a+2,t}^O = (1 - \omega_{t-1}^O)(1 - \omega_{t-2}^O) N_{a,t-2}^O \quad (3.127)$$

$$\vdots \quad (3.128)$$

and

$$N_t^O = \sum_{a=0}^{\infty} N_{a,t}^O \quad (3.129)$$

Now, we can express aggregate pension income in period  $t$  of those who got retired today, one period before, etc.:

$$TR_t^{PG,YO} + TR_t^{FF,YO} = N_{0,t}^O (TR_{0,t}^{PG,YO}(i) + TR_{0,t}^{FF,YO}(i)) \quad (3.130)$$

$$\begin{aligned} (1 - \omega_{t-1}^O) (TR_{t-1}^{PG,YO} + TR_{t-1}^{FF,YO}) &= (1 - \omega_{t-1}^O) N_{0,t-1}^O (TR_{0,t-1}^{PG,YO}(i) + \\ &+ TR_{0,t-1}^{FF,YO}(i)) = N_{1,t}^O (TR_{1,t}^{PG,YO}(i) + TR_{1,t}^{FF,YO}(i)) \end{aligned} \quad (3.131)$$

$$\vdots \quad (3.132)$$

using  $TR_{n,t+n}^{PG,YO}(i) = TR_{0,t}^{PG,YO}(i) \forall n > 0$  again.

Then, adding up all pensions implies:

$$\begin{aligned} TR_t^{PG} + TR_t^{FF} &= TR_t^{PG,YO} + TR_t^{FF,YO} + (1 - \omega_{t-1}^O) (TR_{t-1}^{PG,YO} + TR_{t-1}^{FF,YO}) + \\ &+ (1 - \omega_{t-1}^O)(1 - \omega_{t-2}^O) (TR_{t-2}^{PG,YO} + TR_{t-2}^{FF,YO}) + \dots \end{aligned} \quad (3.133)$$

Similarly, we can express the aggregate value of all other incomes:

$$\begin{aligned} \mathcal{I}_t^O &= \sum_{a=0}^{\infty} N_{a,t}^O \mathcal{I}_{a,t}^O(i) = \sum_{a=0}^{\infty} N_{a,t}^O (Profit_{a,t}^O(i) - T_{a,t}^O(i)) + \\ &+ E_t \frac{1 - \omega_t^O}{1 + r_t} \sum_{a=0}^{\infty} N_{a,t}^O(i) \mathcal{I}_{a+1,t+1}^O(i) = \\ &= Profit_t^O - T_t^O + E_t \frac{1}{1 + r_t} \sum_{a=0}^{\infty} N_{a+1,t+1}^O \mathcal{I}_{a+1,t+1}^O(i) \end{aligned} \quad (3.134)$$



Where we note, because  $\mathcal{I}_t^O$  refers to those who are already retired in period  $t$ , that

$$E_t \sum_{a=0}^{\infty} N_{a+1,t+1}^O(i) \mathcal{I}_{a+1,t+1}^O(i) = E_t \mathcal{I}_{t+1}^O - E_t N_{a,t+1}^O \mathcal{I}_{a,t+1}^O(i) \quad (3.135)$$

Plugging this back to the recursive formula results in:

$$\mathcal{I}_t^O = Profit_t^O - T_t^O + E_t \frac{1}{1+r_t} (\mathcal{I}_{t+1}^O - N_{a,t+1}^O \mathcal{I}_{a,t+1}^O(i)) \quad (3.136)$$

Then, the law of large numbers implies:

$$\mathcal{I}_t^O = Profit_t^O - T_t^O + E_t \frac{1}{1+r_t} \mathcal{I}_{t+1}^O \left( 1 - \frac{N_{a,t+1}^O}{N_{t+1}^O} \right) \quad (3.137)$$

We rearrange the last term:

$$\begin{aligned} 1 - E_t \frac{N_{a,t+1}^O}{N_{t+1}^O} &= 1 - E_t \frac{\omega_t^Y N_t^Y}{N_{t+1}^O} = 1 - E_t \frac{\omega_t^Y N_t^Y}{N_{t+1}^O} \frac{N_t^O}{N_t^O} = \\ &= 1 - E_t \frac{\omega_t^Y}{s_t} \frac{1}{1+g_{t+1}^{N,O}} = 1 - E_t \frac{\omega_t^Y}{s_t} \frac{1}{1-\omega_t^O + \frac{\omega_t^Y}{s_t}} = 1 - E_t \frac{\frac{\omega_t^Y}{s_t}}{1-\omega_t^O + \frac{\omega_t^Y}{s_t}} = \\ &= E_t \frac{1-\omega_t^O + \frac{\omega_t^Y}{s_t} - \frac{\omega_t^Y}{s_t}}{1-\omega_t^O + \frac{\omega_t^Y}{s_t}} = E_t \frac{1-\omega_t^O}{1-\omega_t^O + \frac{\omega_t^Y}{s_t}} = E_t \frac{1-\omega_t^O}{1+g_{t+1}^{N,O}} \end{aligned} \quad (3.138)$$

Finally, total other income is:

$$\mathcal{I}_t^O = Profit_t^O - T_t^O + E_t \frac{1-\omega_t^O}{(1+r_t)(1+g_{t+1}^{N,O})} \mathcal{I}_{t+1}^O \quad (3.139)$$

where we can define a rule how to divide the aggregate profits and lump-sum taxes among the retired and young cohorts:

$$Profit_t^O - T_t^O = (1-\xi)(Profit_t - T_t) \quad (3.140)$$

and  $\xi$  is a parameter between zero and one, that shows the fraction of profit minus lump-sum taxes that goes to young cohort.

Now, aggregate consumption of the retired cohort is defined as:

$$C_t^{O,F} = \sum_{a=0}^{\infty} N_{a,t}^O C_{a,t}^{O,F}(i) \quad (3.141)$$

while total savings of the retired is:

$$\sum_{a=0}^{\infty} N_{a,t}^O B_{a,t-1}^O(i) = N_{0,t}^O B_{0,t-1}^O(i) + \sum_{a=1}^{\infty} N_{a,t}^O B_{a,t-1}^O(i) \quad (3.142)$$

Here, we need to be careful with the just-retired agents, they were young one period before, and, had different savings then. We can use the law of large numbers, and the following expression:  $N_{0,t}^O = \omega_{t-1}^Y N_{t-1}^Y$ :

$$N_{0,t}^O B_{0,t-1}^O(i) = N_{0,t}^O \sum_{b=1}^{\infty} B_{b,t-1}^{Y,last}(i) \simeq \omega_{t-1}^Y N_{t-1}^Y \frac{B_{t-1}^Y}{N_{t-1}^Y} \quad (3.143)$$

where the *last* refers to the fact those who get retired today spent their last year in the young cohort in the previous year.

Then, from  $t-1$  to  $t$  it is easy to see that:  $\sum_{a=1}^{\infty} N_{a,t}^O = \sum_{a=1}^{\infty} (1 - \omega_{t-1}^O) N_{a-1,t-1}^O$  which implies that

$$\sum_{a=0}^{\infty} N_{a,t}^O B_{a,t-1}^O(i) = \omega_{t-1}^Y B_{t-1}^Y + \sum_{a=1}^{\infty} (1 - \omega_{t-1}^O) N_{a-1,t-1}^O B_{a,t-1}^O(i) \quad (3.144)$$

Here, the second term means that only those retired agents cumulate savings who expect to survive the next period. Hence, the amount of aggregate old-age savings from the previous period is  $B_{t-1}^O = \sum_{a=1}^{\infty} (1 - \omega_{t-1}^O) N_{a-1,t-1}^O B_{a,t-1}^O(i)$ . Then, overall savings of the retired cohort in period  $t$  can be expressed easily by adding just-retired savings from the previous period's young cohorts:

$$\sum_{a=0}^{\infty} N_{a,t}^O B_{a,t-1}^O(i) = \omega_{t-1}^Y B_{t-1}^Y + B_{t-1}^O \quad (3.145)$$

As a last step, we put together all parts of the equation, so, aggregate consumption of formal goods of the retired cohort is:

$$\mathcal{H}_t^O C_t^{O,F} = (TR_t^{PG} + TR_t^{FF}) \Omega_t^O + \mathcal{I}_t^O + (1 + r_{t-1}) (\omega_{t-1}^Y B_{t-1}^Y + B_{t-1}^O) \quad (3.146)$$

Also, informal consumption is:

$$C_t^{O,I} = \Upsilon_t C_t^{O,F} \quad (3.147)$$

## Young generation

### First order conditions of a young agent

'Young' agent  $i$  of young cohort  $b$  is one individual of its cohort who started to work (was born)  $b$  years ago. The Bellman-equation of a young individual is:

$$\begin{aligned} V_t^Y(B_{b-1,t-1}^Y(i)) &= \\ &= \max \left\{ (1 + \epsilon_t^C) \left[ \frac{1}{1 - \gamma} \left\{ C_{b,t}^{Y,F}(i) \right\}^{1-\gamma} + \frac{\chi}{1 - \gamma} \left\{ C_{b,t}^{Y,I}(i) \right\}^{1-\gamma} \right] + \right. \\ &\quad \left. + \beta E_t \left( (1 - \omega_t^Y) V_{t+1}^Y(B_{b,t}^Y(i)) + \omega_t^Y V_{t+1}^O(B_{b,t}^{YO}(i)) \right) \right\} \end{aligned} \quad (3.148)$$

while the budget constraint is:

$$(1 + \tau_t^C)C_{b,t}^{Y,F}(i) + p_t^I C_{b,t}^{Y,I}(i) + (1 - \omega_t^Y)B_{b,t}^Y(i) + \omega_t^Y B_{b,t}^{YO}(i) =$$

$$= (1 + r_{t-1})B_{b-1,t-1}^Y(i) + (1 - \tau_t^{LW})w_t^F L_{b,t}^F(i) + w_t^I L_{b,t}^I(i) +$$

$$+ w_t^U U_{b,t}(i) + Profit_{b,t}^Y(i) - T_{b,t}^Y(i) \quad (3.149)$$

$$+ w_t^U U_{b,t}(i) + Profit_{b,t}^Y(i) - T_{b,t}^Y(i) \quad (3.150)$$

First order conditions:

$$C_{b,t}^{Y,F}(i) : (1 + \epsilon_t^C) \left\{ C_{b,t}^{Y,F}(i) \right\}^{-\gamma} + \lambda_{b,t}^Y (1 + \tau_t^C) = 0 \quad (3.151)$$

$$C_{b,t}^{Y,I}(i) : (1 + \epsilon_t^C) \chi \left\{ C_{b,t}^{Y,I}(i) \right\}^{-\gamma} + \lambda_{b,t}^Y p_t^I = 0 \quad (3.152)$$

$$B_{b,t}^Y(i) : \beta E_t (1 - \omega_t^Y) V_{B_{b,t}^Y}^Y + E_t (1 - \omega_t^Y) \lambda_{b,t}^Y = 0 \quad (3.153)$$

$$B_{b,t}^{YO}(i) : \beta E_t \omega_t^Y V_{B_{b,t}^{YO}}^Y + E_t \omega_t^Y \lambda_{b,t}^Y = 0 \quad (3.154)$$

One-period-ahead Envelope theorem:

$$E_t V_{B_{b,t}^Y} = -E_t \lambda_{b+1,t+1}^Y (1 + r_t) \quad (3.155)$$

Also, from the retired agent's optimization we know that:

$$E_t V_{B_{b,t}^{YO}} = -E_t \lambda_{0,t+1}^O (1 + r_t) = -E_t \lambda_{b+1,t+1}^O (1 + r_t) \quad (3.156)$$

where  $E_t \lambda_{0,t+1}^O = E_t \lambda_{b+1,t+1}^O$  because someone who was young in  $t$  gets retired in  $t + 1$ .

Thus, the Euler equations of the young individual are:

$$\beta E_t \frac{(1 + \epsilon_{t+1}^C) \left( C_{b,t}^{Y,F}(i) \right)^\gamma}{(1 + \epsilon_t^C) \left( C_{b+1,t+1}^{Y,F}(i) \right)^\gamma} (1 + r_t) \frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} = 1 \quad (3.157)$$

$$\beta E_t \frac{(1 + \epsilon_{t+1}^C) \left( C_{b,t}^{Y,F}(i) \right)^\gamma}{(1 + \epsilon_t^C) \left( C_{0,t+1}^{O,F}(i) \right)^\gamma} (1 + r_t) \frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} = 1 \quad (3.158)$$

Rearranging:

$$E_t C_{b+1,t+1}^{Y,F}(i) = C_{b,t}^{Y,F}(i) (1 + r_t)^{\frac{1}{\gamma}} E_t \Lambda_{t+1} \quad (3.159)$$

$$E_t C_{0,t+1}^{O,F}(i) = E_t C_{b,t}^{Y,F}(i) (1 + r_t)^{\frac{1}{\gamma}} \Lambda_{t+1} \quad (3.160)$$

Also, we can express each period's consumption as a function of period- $t$  consumption and the discount rate:

$$E_t C_{b+n,t+n}^{Y,F}(i) = C_{b,t}^{Y,F}(i) E_t \prod_{k=1}^n (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} \quad (3.161)$$

Furthermore, the first order conditions also imply a substitution between formal and informal goods:

$$(1 + \epsilon_t^C) \left\{ C_{b,t}^{Y,F}(i) \right\}^{-\gamma} = (1 + \epsilon_t^C) \chi \left\{ C_{b,t}^{Y,I}(i) \right\}^{-\gamma} \frac{1 + \tau_t^C}{p_t^I} \quad (3.162)$$

or with more simple notations:

$$C_{b,t}^{Y,I}(i) = \Upsilon_t C_{b,t}^{Y,F}(i) \quad (3.163)$$

### Individual consumption of a young agent

First of all, we would like to stress that one needs to be careful when deriving the young agent's individual consumption because old-age incomes and expenditures must be taken into account, too. Moreover, the young agents also consider the probability of retirement, for instance, in period  $t$  the probability that a young agent becomes retired in period  $t+1$  is  $\omega_t^Y$ , while the probability that the same agent becomes retired in period  $t+2$  is  $(1 - \omega_t^Y)\omega_{t+1}^Y$ . So, the first term of the left-hand side of this equation shows the stream of lifetime consumption if the agent stays young, then, from the second term onwards she retires with some probability in each period:

$$\begin{aligned} & E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^Y) ((1 + \tau_{t+n}^C) C_{b+n,t+n}^{Y,F}(i) + p_{t+n}^I C_{b+n,t+n}^{Y,I}(i))}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + E_t \omega_t^Y \left( \sum_{n=1}^{\infty} \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O) (((1 + \tau_{t+n}^C) C_{n-1,t+n}^{O,F}(i) + p_{t+n}^I C_{n-1,t+n}^{O,I}(i)))}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \\ & + E_t (1 - \omega_t^Y) \omega_{t+1}^Y \left( \sum_{n=2}^{\infty} \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O) (((1 + \tau_{t+n}^C) C_{n-2,t+n}^{O,F}(i) + p_{t+n}^I C_{n-2,t+n}^{O,I}(i)))}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \dots \\ & = E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^Y) \left[ (1 - \tau_{t+n}^L) w_{b+n,t+n}^F L_{b+n,t+n}^F(i) + w_{b+n,t+n}^I L_{b+n,t+n}^I(i) + w_{b+n,t+n}^U U_{b+n,t+n}(i) \right]}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + (1 + r_{t-1}) B_{b-1,t-1}^Y(i) + \\ & + E_t \omega_t^Y \sum_{n=1}^{\infty} \left( TR_{n-1,t+n}^{PG,YO}(i) + TR_{n-1,t+n}^{FF,YO}(i) + Profit_{n-1,t+n}^O(i) - T_{n-1,t+n}^O(i) \right) \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + E_t (1 - \omega_t^Y) \omega_{t+1}^Y \sum_{n=2}^{\infty} \left( TR_{n-2,t+n}^{PG,YO}(i) + TR_{n-2,t+n}^{FF,YO}(i) + Profit_{n-2,t+n}^O(i) - T_{n-2,t+n}^O(i) \right) \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + E_t (1 - \omega_t^Y) (1 - \omega_{t+1}^Y) \omega_{t+2}^Y \sum_{n=3}^{\infty} \left( TR_{n-3,t+n}^{PG,YO}(i) + TR_{n-3,t+n}^{FF,YO}(i) + Profit_{n-3,t+n}^O(i) - T_{n-3,t+n}^O(i) \right) \frac{\prod_{k=4}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + \dots \end{aligned} \quad (3.164)$$

It is easier to express all consumptions in terms of formal goods

$$\begin{aligned} & E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^Y) ((1 + \tau_{t+n}^C) C_{b+n,t+n}^{Y,F}(i) + p_{t+n}^I C_{b+n,t+n}^{Y,F}(i) \Upsilon_{t+n})}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + E_t \omega_t^Y \left( \sum_{n=1}^{\infty} \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O) (((1 + \tau_{t+n}^C) C_{n-1,t+n}^{O,F}(i) + p_{t+n}^I C_{n-1,t+n}^{O,F}(i) \Upsilon_{t+n}))}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \\ & + E_t (1 - \omega_t^Y) \omega_{t+1}^Y \left( \sum_{n=2}^{\infty} \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O) (((1 + \tau_{t+n}^C) C_{n-2,t+n}^{O,F}(i) + p_{t+n}^I C_{n-2,t+n}^{O,F}(i) \Upsilon_{t+n}))}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \dots \\ & = E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^Y) \left[ (1 - \tau_{t+n}^L) w_{b+n,t+n}^F L_{b+n,t+n}^F(i) + w_{b+n,t+n}^I L_{b+n,t+n}^I(i) + w_{b+n,t+n}^U U_{b+n,t+n}(i) \right]}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + (1 + r_{t-1}) B_{b-1,t-1}^Y(i) + \\ & + E_t \omega_t^Y \sum_{n=1}^{\infty} \left( TR_{n-1,t+n}^{PG,YO}(i) + TR_{n-1,t+n}^{FF,YO}(i) + Profit_{n-1,t+n}^O(i) - T_{n-1,t+n}^O(i) \right) \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + E_t (1 - \omega_t^Y) \omega_{t+1}^Y \sum_{n=2}^{\infty} \left( TR_{n-2,t+n}^{PG,YO}(i) + TR_{n-2,t+n}^{FF,YO}(i) + Profit_{n-2,t+n}^O(i) - T_{n-2,t+n}^O(i) \right) \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \dots \end{aligned} \quad (3.165)$$

Based on the Euler-equations, we can express expected future consumptions. Let's consider an agent who is young in period  $t$ , then her consumption functions in the next periods after getting retired are:

$$E_t C_{n,t+n+1}^{O,F}(i) = E_t C_{0,t+1}^{O,F}(i) \prod_{k=2}^n (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} \quad (3.166)$$

On the other hand, if the agent stays young in period  $t + 1$  and gets retired after that, then her future old-age consumptions look like

$$E_t C_{n,t+n+2}^{O,F}(i) = E_t C_{0,t+2}^{O,F}(i) \prod_{k=3}^n (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} \quad (3.167)$$

Now, we plug them into the intertemporal budget constraint:

$$\begin{aligned} & E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^Y) ((1 + \tau_{t+n}^C) C_{b+n,t+n}^{Y,F}(i) + p_{t+n}^I C_{b+n,t+n}^{Y,F}(i) \Upsilon_{t+n})}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + E_t \omega_t^Y \left( \sum_{n=1}^{\infty} \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O) C_{0,t+1}^{O,F}(i) (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} (((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n}))}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \\ & + E_t (1 - \omega_t^Y) \omega_{t+1}^Y \left( \sum_{n=2}^{\infty} \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O) C_{0,t+2}^{O,F}(i) (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} (((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n}))}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \dots \\ & = E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^Y) [(1 - \tau_{t+n}^L) w_{b+n,t+n}^F L_{b+n,t+n}^F(i) + w_{b+n,t+n}^I L_{b+n,t+n}^I(i) + w_{b+n,t+n}^U U_{b+n,t+n}(i)]}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + (1 + r_{t-1}) B_{b-1,t-1}^Y(i) + \\ & + E_t \omega_t^Y \sum_{n=1}^{\infty} \left( T R_{n-1,t+n}^{PG,YO}(i) + T R_{n-1,t+n}^{FF,YO}(i) + Profit_{n-1,t+n}^O(i) - T_{n-1,t+n}^O(i) \right) \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + E_t (1 - \omega_t^Y) \omega_{t+1}^Y \sum_{n=2}^{\infty} \left( T R_{n-2,t+n}^{PG,YO}(i) + T R_{n-2,t+n}^{FF,YO}(i) + Profit_{n-2,t+n}^O(i) - T_{n-2,t+n}^O(i) \right) \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \dots \quad (3.168) \end{aligned}$$

After that, we use the other Euler equation (the one that shows the substitution between period  $t$  young and period  $t + 1$  old-age consumption):

$$\begin{aligned} & E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^Y) ((1 + \tau_{t+n}^C) C_{b+n,t+n}^{Y,F}(i) + p_{t+n}^I C_{b+n,t+n}^{Y,F}(i) \Upsilon_{t+n})}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + E_t \omega_t^Y \left( \sum_{n=1}^{\infty} \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O) C_{b,t}^{Y,F}(i) (1 + r_t)^{\frac{1}{\gamma}} \Lambda_{t+1} (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} (((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n}))}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \\ & + E_t (1 - \omega_t^Y) \omega_{t+1}^Y \left( \sum_{n=2}^{\infty} \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O) C_{b+1,t+1}^{Y,F}(i) (1 + r_{t+1})^{\frac{1}{\gamma}} \Lambda_{t+2} (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} (((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n}))}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \dots = \\ & = E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^Y) [(1 - \tau_{t+n}^L) w_{b+n,t+n}^F L_{b+n,t+n}^F(i) + w_{b+n,t+n}^I L_{b+n,t+n}^I(i) + w_{b+n,t+n}^U U_{b+n,t+n}(i)]}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + (1 + r_{t-1}) B_{b-1,t-1}^Y(i) + \\ & + E_t \omega_t^Y \sum_{n=1}^{\infty} \left( T R_{n-1,t+n}^{PG,YO}(i) + T R_{n-1,t+n}^{FF,YO}(i) + Profit_{n-1,t+n}^O(i) - T_{n-1,t+n}^O(i) \right) \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\ & + E_t (1 - \omega_t^Y) \omega_{t+1}^Y \sum_{n=2}^{\infty} \left( T R_{n-2,t+n}^{PG,YO}(i) + T R_{n-2,t+n}^{FF,YO}(i) + Profit_{n-2,t+n}^O(i) - T_{n-2,t+n}^O(i) \right) \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=1}^n (1 + r_{t+k-1})} + \dots \quad (3.169) \end{aligned}$$

Concentrating on consumptions:

$$\begin{aligned}
& E_t \sum_{n=1}^{\infty} \frac{\prod_{k=1}^n (1 - \omega_{t+k-1}^Y) C_{b+n,t+n}^{Y,F}(i) ((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n})}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\
& + E_t \omega_t^Y \left( \sum_{n=1}^{\infty} \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O) C_{b,t}^{Y,F}(i) (1 + r_t)^{\frac{1}{\gamma}} \Lambda_{t+1} (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} ((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n})}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \\
& + E_t (1 - \omega_t^Y) \omega_{t+1}^Y \left( \sum_{n=2}^{\infty} \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O) C_{b+1,t+1}^{Y,F}(i) (1 + r_{t+1})^{\frac{1}{\gamma}} \Lambda_{t+2} (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} ((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n})}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \\
& + \dots
\end{aligned} \tag{3.170}$$

We rearrange

$$\begin{aligned}
& C_{b,t}^{Y,F}(i) ((1 + \tau_t^C) + p_t^I \Upsilon_t) + \\
& + C_{b,t}^{Y,F}(i) E_t \omega_t^Y \left( \sum_{n=1}^{\infty} \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O) (1 + r_t)^{\frac{1}{\gamma}} \Lambda_{t+1} (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} ((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n})}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \\
& + E_t C_{b+1,t+1}^{Y,F}(i) \frac{(1 - \omega_t^Y) ((1 + \tau_{t+1}^C) + p_{t+1}^I \Upsilon_{t+1})}{(1 + r_t)} + \\
& + E_t C_{b+1,t+1}^{Y,F}(i) (1 - \omega_t^Y) \omega_{t+1}^Y \left( \sum_{n=2}^{\infty} \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O) (1 + r_{t+1})^{\frac{1}{\gamma}} \Lambda_{t+2} (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} ((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n})}{\prod_{k=1}^n (1 + r_{t+k-1})} \right) + \\
& + \dots
\end{aligned} \tag{3.171}$$

Simplifying before recursive substitution:

$$\begin{aligned}
& C_{b,t}^{Y,F}(i) ((1 + \tau_t^C) + p_t^I \Upsilon_t) + \\
& + C_{b,t}^{Y,F}(i) E_t \frac{\omega_t^Y (1 + r_t)^{\frac{1}{\gamma}} \Lambda_{t+1}}{1 + r_t} \left( \sum_{n=1}^{\infty} \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O) (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} ((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n})}{\prod_{k=2}^n (1 + r_{t+k-1})} \right) + \\
& + E_t C_{b+1,t+1}^{Y,F}(i) \frac{(1 - \omega_t^Y) ((1 + \tau_{t+1}^C) + p_{t+1}^I \Upsilon_{t+1})}{(1 + r_t)} + \\
& + E_t C_{b+1,t+1}^{Y,F}(i) (1 - \omega_t^Y) \frac{\omega_{t+1}^Y (1 + r_{t+1})^{\frac{1}{\gamma}} \Lambda_{t+2}}{(1 + r_t)(1 + r_{t+1})} \left( \sum_{n=2}^{\infty} \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O) (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} ((1 + \tau_{t+n}^C) + p_{t+n}^I \Upsilon_{t+n})}{\prod_{k=3}^n (1 + r_{t+k-1})} \right) + \\
& + \dots
\end{aligned} \tag{3.172}$$

Now, we can use  $\mathcal{H}_{t+1}^O$  from retired agents' optimization:

$$\begin{aligned}
& C_{b,t}^{Y,F}(i) \left[ (1 + \tau_t^C) + p_t^I \Upsilon_t + E_t \omega_t^Y (1 + r_t)^{\frac{1}{\gamma}-1} \Lambda_{t+1} \mathcal{H}_{t+1}^O \right] + \\
& + E_t C_{b+1,t+1}^{Y,F}(i) \frac{(1 - \omega_t^Y)}{(1 + r_t)} \left[ (1 + \tau_{t+1}^C) + p_{t+1}^I \Upsilon_{t+1} + \omega_{t+1}^Y (1 + r_{t+1})^{\frac{1}{\gamma}-1} \Lambda_{t+2} \mathcal{H}_{t+2}^O \right] + \dots
\end{aligned} \tag{3.173}$$

And, using the Euler-equation again (to have period- $t$  consumption only):

$$E_t C_{b+n,t+n}^{Y,F}(i) = C_{b,t}^{Y,F}(i) E_t \prod_{k=1}^n (1 + r_{t+k-1})^{\frac{1}{\gamma}} \Lambda_{t+k} \tag{3.174}$$

Lastly

$$\begin{aligned}
& C_{b,t}^{Y,F}(i) \left[ (1 + \tau_t^C) + p_t^I \Upsilon_t + E_t \omega_t^Y (1 + r_t)^{\frac{1}{\gamma}-1} \Lambda_{t+1} \mathcal{H}_{t+1}^O \right] + \\
& C_{b,t}^{Y,F}(i) E_t (1 + r_t)^{\frac{1}{\gamma}} \Lambda_{t+1} \frac{(1 - \omega_t^Y)}{(1 + r_t)} \left[ (1 + \tau_{t+1}^C) + p_{t+1}^I \Upsilon_{t+1} + \omega_{t+1}^Y (1 + r_{t+1})^{\frac{1}{\gamma}-1} \Lambda_{t+2} \mathcal{H}_{t+2}^O \right] + \\
& C_{b,t}^{Y,F}(i) E_t (1 + r_t)^{\frac{1}{\gamma}} \Lambda_{t+1} (1 + r_{t+1})^{\frac{1}{\gamma}} \Lambda_{t+2} \frac{(1 - \omega_t^Y)(1 - \omega_{t+1}^Y)}{(1 + r_t)(1 + r_{t+1})} \left[ (1 + \tau_{t+2}^C) + p_{t+2}^I \Upsilon_{t+2} + \omega_{t+2}^Y (1 + r_{t+2})^{\frac{1}{\gamma}-1} \Lambda_{t+3} \mathcal{H}_{t+3}^O \right] + (3.175)
\end{aligned}$$

which is equal to

$$C_{b,t}^{Y,F}(i) \mathcal{H}_t^Y \quad (3.176)$$

where

$$\mathcal{H}_t^Y = (1 + \tau_t^C) + p_t^I \Upsilon_t + E_t (1 + r_t)^{\frac{1}{\gamma}-1} \Lambda_{t+1} \left( (1 - \omega_t^Y) \mathcal{H}_{t+1}^Y + \omega_t^Y \mathcal{H}_{t+1}^O \right) \quad (3.177)$$

Similarly to consumption above, the young agent's budget constraint contains old-age income items, i.e. expected revenue from the pension fund, profits from firms and lump-sum taxes.

$$\begin{aligned}
& \mathcal{I}_{b,t}^{YO}(i) = E_t \omega_t^Y \left( (TR_{0,t+1}^{PG,YO}(i) + TR_{n,t+n}^{FF,YO}(i)) \Omega_{t+1}^O + \sum_{n=1}^{\infty} \frac{\prod_{k=2}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=2}^n (1 + r_{t+k-1})} (Profit_{n-1,t+n}^O(i) - T_{n-1,t+n}^O(i)) \right) + \\
& + E_t \frac{(1 - \omega_t^Y) \omega_{t+1}^Y}{(1 + r_{t+1})} \left( (TR_{0,t+2}^{PG,YO}(i) + TR_{0,t+2}^{FF,YO}(i)) \Omega_{t+2}^O + \sum_{n=2}^{\infty} \frac{\prod_{k=3}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=3}^n (1 + r_{t+k-1})} (Profit_{n-2,t+n}^O(i) - T_{n-2,t+n}^O(i)) \right) + \\
& + E_t \frac{(1 - \omega_t^Y)(1 - \omega_{t+1}^Y) \omega_{t+2}^Y}{(1 + r_{t+1})(1 + r_{t+2})} \left( (TR_{0,t+3}^{PG,YO}(i) + TR_{0,t+3}^{FF,YO}(i)) \Omega_{t+3}^O + \right. \\
& \left. + E_t \frac{(1 - \omega_t^Y)(1 - \omega_{t+1}^Y) \omega_{t+2}^Y}{(1 + r_{t+1})(1 + r_{t+2})} \left( \sum_{n=3}^{\infty} \frac{\prod_{k=4}^n (1 - \omega_{t+k-1}^O)}{\prod_{k=4}^n (1 + r_{t+k-1})} (Profit_{n-3,t+n}^O(i) - T_{n-3,t+n}^O(i)) \right) + \dots \right) \quad (3.178)
\end{aligned}$$

Again, we use that  $TR_{n,t+n}^{PG,YO}(i) = TR_{0,t}^{PG,YO}(i) \forall n > 0$ , and, the same is true for fully funded pensions.

Then, using the definition of  $\mathcal{I}_{a,t}^O(i)$  we can rewrite total old-age income as

$$\begin{aligned}
& \mathcal{I}_{b,t}^{YO}(i) = E_t \omega_t^Y \left( (TR_{0,t+1}^{PG,YO}(i) + TR_{0,t+1}^{FF,YO}(i)) \Omega_{t+1}^O + \mathcal{I}_{a,t+1}^O(i) \right) + \\
& + E_t \frac{(1 - \omega_t^Y) \omega_{t+1}^Y}{1 + r_{t+1}} \left( (TR_{0,t+2}^{PG,YO}(i) + TR_{0,t+2}^{FF,YO}(i)) \Omega_{t+2}^O + \mathcal{I}_{a,t+2}^O(i) \right) + \\
& + E_t \frac{(1 - \omega_t^Y)(1 - \omega_{t+1}^Y) \omega_{t+2}^Y}{(1 + r_{t+1})(1 + r_{t+2})} \left( (TR_{0,t+3}^{PG,YO}(i) + TR_{0,t+3}^{FF,YO}(i)) \Omega_{t+3}^O + \mathcal{I}_{a,t+3}^O(i) \right) + \dots \quad (3.179)
\end{aligned}$$

which in a recursive way looks as

$$\begin{aligned}
& \mathcal{I}_{b,t}^{YO}(i) = E_t \omega_t^Y \left( (TR_{0,t+1}^{PG,YO}(i) + TR_{0,t+1}^{FF,YO}(i)) \Omega_{t+1}^O + \mathcal{I}_{0,t+1}^O(i) \right) + \\
& + E_t \frac{(1 - \omega_t^Y)}{(1 + r_{t+1})} \mathcal{I}_{b+1,t+1}^{YO}(i) \quad (3.180)
\end{aligned}$$

Furthermore, young-age income is (using a new variable  $Inc_t(i)$ )

$$\begin{aligned}
& \mathcal{I}_{b,t}^Y(i) = E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^{\infty} (1 - \omega_{t+k-1}^Y)^n \left[ (1 - \tau_{t+n}^{LW}) w_{t+n}^F L_{b+n,t+n}^F(i) + w_{t+n}^I L_{b+n,t+n}^I(i) + w_{t+n}^U U_{b+n,t+n}(i) \right]}{\prod_{k=1}^n (1 + r_{t+k-1})} + \\
& + E_t \sum_{n=0}^{\infty} \frac{\prod_{k=1}^{\infty} (1 - \omega_{t+k-1}^Y)^n \left[ Profit_{b+n,t+n}^Y(i) - T_{b+n,t+n}^Y(i) \right]}{\prod_{k=1}^n (1 + r_{t+k-1})} = \\
& = (1 - \tau_t^{LW}) w_t^F L_{b,t}^F(i) + w_t^I L_{b,t}^I(i) + w_t^U U_{b,t}(i) + Profit_{b,t}^Y(i) - T_{b,t}^Y(i) + E_t \frac{1 - \omega_t^Y}{1 + r_t} \mathcal{I}_{b+1,t+1}^Y(i) = \\
& = Inc_{b,t}(i) + E_t \frac{1 - \omega_t^Y}{1 + r_t} \mathcal{I}_{b+1,t+1}^Y(i) \quad (3.181)
\end{aligned}$$

Thus, the individual consumption function of agent  $i$  of cohort  $b$  in period  $t$  is

$$\mathcal{H}_t^Y C_{b,t}^{Y,F}(i) = \mathcal{I}_{b,t}^Y(i) + \frac{\mathcal{I}_{b,t}^{YO}(i)}{1+r_t} + (1+r_{t-1})B_{b-1,t-1}^Y(i) \quad (3.182)$$

### Aggregate consumption of the young cohort

In the first step we need to express the total number of young people. If  $N_{b,t}^Y$  is the number of  $b$ -year old workers, the total number of workers is

$$N_t^Y = \sum_{b=0}^{\infty} N_{b,t}^Y \quad (3.183)$$

Following the previous idea, we sum up all consumptions, incomes and savings:

$$\begin{aligned} \mathcal{H}_t^Y \sum_{b=0}^{\infty} N_{b,t}^Y C_{b,t}^{Y,F}(i) &= \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b,t}^Y(i) + \frac{1}{1+r_t} \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b,t}^{YO}(i) + \\ &+ (1+r_{t-1}) \sum_{b=0}^{\infty} N_{b,t}^Y B_{b-1,t-1}^Y(i) \end{aligned} \quad (3.184)$$

where we note that the new young workers in time  $t$  have zero savings from the previous period.

$$\begin{aligned} \mathcal{H}_t^Y \sum_{b=0}^{\infty} N_{b,t}^Y C_{b,t}^{Y,F}(i) &= \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b,t}^Y(i) + \frac{1}{1+r_t} \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b,t}^{YO}(i) + \\ &+ (1+r_{t-1}) \sum_{b=1}^{\infty} N_{b,t}^Y \frac{N_{b-1,t-1}^Y}{N_{b-1,t-1}^Y} B_{b-1,t-1}^Y(i) \end{aligned} \quad (3.185)$$

Rearranging

$$\begin{aligned} \mathcal{H}_t^Y \sum_{b=0}^{\infty} N_{b,t}^Y C_{b,t}^{Y,F}(i) &= \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b,t}^Y(i) + \frac{1}{1+r_t} \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b,t}^{YO}(i) + \\ &+ (1+r_{t-1})(1-\omega_{t-1}^Y) \sum_{b=1}^{\infty} N_{b-1,t-1}^Y B_{b-1,t-1}^Y(i) \end{aligned} \quad (3.186)$$

Aggregate values are defined as

$$C_t^{Y,F} \equiv \sum_{b=0}^{\infty} N_{b,t}^Y C_{b,t}^{Y,F}(i) \quad (3.187)$$

$$B_{t-1}^Y \equiv \sum_{b=1}^{\infty} N_{b-1,t-1}^Y B_{b-1,t-1}^Y(i) \quad (3.188)$$

$$\mathcal{I}_t^Y \equiv \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b,t}^Y(i) \quad (3.189)$$



$$\mathcal{I}_t^{YO} \equiv \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b,t}^{YO}(i) \quad (3.190)$$

It is important to note that in each period, independently from the survival probabilities, each young agent saves for the next period, hence, the overall savings  $B_{t-1}^Y = \sum_{b=1}^{\infty} N_{b-1,t-1}^Y B_{b-1,t-1}^Y(i)$  is divided among those who remain young and get retired.

As a result, the aggregate consumption functions are

$$\mathcal{H}_t^Y C_t^{Y,F} = \mathcal{I}_t^Y + \frac{\mathcal{I}_t^{YO}}{1+r_t} + (1+r_{t-1})(1-\omega_{t-1}^Y)B_{t-1}^Y \quad (3.191)$$

$$C_t^{Y,I} = \Upsilon_t C_t^{Y,F} \quad (3.192)$$

Now we need to aggregate the supporting variables as well. First of all, we rename individual contemporary income

$$Inc_{b,t}(i) = (1-\tau_t^{LW})w_t^F L_{b,t}^F(i) + w_t^I L_{b,t}^I(i) + w_t^U U_{b,t}(i) + Profit_{b,t}^Y(i) - T_{b,t}^Y(i) \quad (3.193)$$

Aggregating

$$Inc_t = (1-\tau_t^{LW})w_t^F L_t^F + w_t^I L_t^I + w_t^U U_t + Profit_t^Y - T_t^Y \quad (3.194)$$

where

$$Profit_t^Y - T_t^Y = \xi(Profit_t - T_t) \quad (3.195)$$

Aggregating and rearranging

$$\begin{aligned} \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b,t}^Y(i) &= \sum_{b=0}^{\infty} N_{b,t}^Y Inc_{b,t}(i) + E_t \frac{(1-\omega_t^Y)}{1+r_t} \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b+1,t+1}^Y(i) \\ &= \sum_{b=0}^{\infty} N_{b,t}^Y Inc_{b,t}(i) + E_t \frac{1}{1+r_t} \sum_{b=0}^{\infty} N_{b+1,t+1}^Y \mathcal{I}_{b+1,t+1}^Y(i) \end{aligned} \quad (3.196)$$

Because  $\mathcal{I}_{t+1}^Y$  contains the income of the new-born people as well, the last term can be rearranged, using the law of large numbers, as:

$$\begin{aligned} E_t \sum_{b=0}^{\infty} N_{b+1,t+1}^Y \mathcal{I}_{b+1,t+1}^Y(i) &= E_t \mathcal{I}_{t+1}^Y - E_t N_{b,t+1}^Y \mathcal{I}_{b,t+1}^Y(i) = \\ &= E_t \mathcal{I}_{t+1}^Y \left( 1 - \frac{N_{b,t+1}^Y}{N_{t+1}^Y} \right) = E_t \mathcal{I}_{t+1}^Y \left( 1 - \frac{n_t N_t^Y}{N_{t+1}^Y} \right) \end{aligned} \quad (3.197)$$

Then, total young income is:

$$\mathcal{I}_t^Y = Inc_t + E_t \frac{1-\omega_t^Y}{(1+r_t)(1+g_{t+1}^{N,Y})} \mathcal{I}_{t+1}^Y \quad (3.198)$$

A similar exercise can be done for pension benefits. First, we define  $\mathcal{I}_t^{YO}$  which can be rearranged

$$\begin{aligned}
\mathcal{I}_t^{YO} &= \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b,t}^{YO}(i) = E_t \omega_t^Y \sum_{b=0}^{\infty} N_{b,t}^Y \left( (TR_{0,t+1}^{FF,YO}(i) + E_t TR_{0,t+1}^{PG,YO}(i)) \Omega_{t+1}^O + \mathcal{I}_{0,t+1}^O(i) \right) + \\
&+ \frac{(1 - \omega_t^Y)}{(1 + r_{t+1})} \sum_{b=0}^{\infty} N_{b,t}^Y \mathcal{I}_{b+1,t+1}^{YO}(i) = \\
&= E_t N_{0,t+1}^O \left( (TR_{0,t+1}^{FF,YO}(i) + TR_{0,t+1}^{PG,YO}(i)) \Omega_{t+1}^O + \mathcal{I}_{0,t+1}^O(i) \right) + \\
&+ E_t \frac{1}{(1 + r_{t+1})} \sum_{b=0}^{\infty} N_{b+1,t+1}^Y \mathcal{I}_{b+1,t+1}^{YO}(i) \tag{3.199}
\end{aligned}$$

Now, similarly to total young income, the last term can be expressed as

$$E_t \sum_{b=0}^{\infty} N_{b+1,t+1}^Y \mathcal{I}_{b+1,t+1}^{YO}(i) = E_t \frac{1 - \omega_t^Y}{1 + g_{t+1}^{N,Y}} \mathcal{I}_{t+1}^{YO} \tag{3.200}$$

Also we know that

$$E_t N_{0,t+1}^O \left( (TR_{0,t+1}^{FF,YO}(i) + TR_{0,t+1}^{PG,YO}(i)) \Omega_{t+1}^O \right) = E_t (TR_{t+1}^{FF,YO} + TR_{t+1}^{PG,YO}) \Omega_{t+1}^O \tag{3.201}$$

$$E_t N_{0,t+1}^O \mathcal{I}_{0,t+1}^O(i) = \frac{\omega_t^Y N_t^Y}{N_{t+1}^O} \mathcal{I}_{t+1}^O = E_t \frac{\omega_t^Y}{(1 + g_{t+1}^{N,Y}) s_{t+1}} \mathcal{I}_{t+1}^O \tag{3.202}$$

Finally, the expected income of the young after getting retired is

$$\mathcal{I}_t^{YO} = E_t (TR_{t+1}^{PG,YO} + TR_{t+1}^{FF,YO}) \Omega_{t+1}^O + E_t \left( \frac{\omega_t^Y}{(1 + g_{t+1}^{N,Y}) s_{t+1}} \mathcal{I}_{t+1}^O + \right) E_t \frac{1 - \omega_t^Y}{(1 + r_{t+1})(1 + g_{t+1}^{N,Y})} \mathcal{I}_{t+1}^{YO} \tag{3.203}$$

### Aggregating the young households' budget constraints

The individual budget constraint of a young agent is

$$\begin{aligned}
(1 + \tau_t^C) C_{b,t}^{Y,F}(i) + p_t^I C_{b,t}^{Y,I}(i) + (1 - \omega_t^Y) B_{b,t}^Y(i) + \omega_t^Y B_{b,t}^{Y*}(i) = \\
= Inc_{b,t}(i) + (1 + r_{t-1}) B_{b-1,t-1}^Y(i) \tag{3.204}
\end{aligned}$$

Aggregating

$$\begin{aligned}
\sum_{b=0}^{\infty} N_{b,t}^Y ((1 + \tau_t^C) C_{b,t}^{Y,F}(i) + p_t^I C_{b,t}^{Y,I}(i)) + \sum_{b=0}^{\infty} N_{b,t}^Y (1 - \omega_t^Y) B_{b,t}^Y(i) + \sum_{b=0}^{\infty} N_{b,t}^Y \omega_t^Y B_{b,t}^{Y*}(i) = \\
= \sum_{b=0}^{\infty} N_{b,t}^Y Inc_{b,t}(i) + (1 + r_{t-1}) \sum_{b=1}^{\infty} N_{b,t}^Y B_{b-1,t-1}^Y(i) \tag{3.205}
\end{aligned}$$

where the definition of aggregate savings is:

$$\sum_{b=1}^{\infty} N_{b,t}^Y B_{b-1,t-1}^Y(i) = \sum_{b=1}^{\infty} (1 - \omega_{t-1}^Y) N_{b-1,t-1}^Y B_{b-1,t-1}^Y(i) \tag{3.206}$$

After aggregation, there is no difference between the  $B_t^Y$  and  $B_t^{Y*}$ . So, we can easily express the aggregate budget constraint:

$$(1 + \tau_t^C)C_t^{Y,F} + p_t^I C_t^{Y,I} + B_t^Y = Inc_t + (1 + r_{t-1})(1 - \omega_{t-1}^Y)B_{t-1}^Y \quad (3.207)$$

## B The public pension systems

### Pay-as-you-go pension system

In a PAYG regime, all public revenues finance all public expenditures (also pension benefits):

$$Rev_t = \tau_t^C C_t^F + \tau_t^L w_t^F L_t^F + T_t \quad (3.208)$$

$$\tau_t^L = \tau_t^{PI} + (1 - \Xi)(\tau_t^{SSCW} + \tau_t^{SSCF}) \quad (3.209)$$

$$Exp_t = Gov_t + w_t^U U_t + TR_t^{PG} \quad (3.210)$$

where  $\Xi$  is 0 in a PAYG regime, 1 in a fully funded regime, but it can be time-variant as well.

The number of the just-retired agents (those who were young one period before) is

$$N_{0,t}^O = \sum_{b=1}^{\infty} \omega_{t-1}^Y N_{b-1,t-1}^Y \quad (3.211)$$

Individual ( $i$ )'s pension in the year of retirement  $t$  is based on replacement rate  $\nu_t$  and the average of the last  $Y$  years' income:

$$TR_{0,t}^{PG,YO}(i) = \nu_t IB_{b-1,t}^Y(i) \quad (3.212)$$

where

$$IB_{b-1,t}^Y(i) = \frac{1}{Y} w_{t-1}^F L_{b-1,t-1}^F(i) + \frac{Y-1}{Y} IB_{b-2,t-1}^Y(i) \quad (3.213)$$

Aggregating the last expression implies

$$\begin{aligned} IB_t^Y &= \sum_{b=1}^{\infty} N_{b-1,t-1}^Y IB_{b-1,t}^Y(i) = \\ &= \frac{1}{Y} w_{t-1}^F \sum_{b=1}^{\infty} N_{b-1,t-1}^Y L_{b-1,t-1}^F(i) + \frac{Y-1}{Y} \sum_{b=2}^{\infty} N_{b-1,t-1}^Y IB_{b-2,t-1}^Y(i) \end{aligned} \quad (3.214)$$

It is also true that

$$IB_t^Y = \frac{1}{Y} w_{t-1}^F L_{t-1}^F + \frac{Y-1}{Y} \sum_{b=2}^{\infty} N_{b-1,t-1}^Y \frac{N_{b-2,t-2}^Y}{N_{b-2,t-2}^Y} IB_{b-2,t-1}^Y(i) \quad (3.215)$$

which can be rearranged

$$IB_t^Y = \frac{1}{Y} w_{t-1}^F L_{t-1}^F + \frac{Y-1}{Y} (1 - \omega_{t-2}^Y) \sum_{b=2}^{\infty} N_{b-2,t-2}^Y IB_{b-2,t-1}^Y(i) \quad (3.216)$$

Then

$$IB_t^Y = \frac{1}{Y} w_{t-1}^F L_{t-1}^F + \frac{Y-1}{Y} (1 - \omega_{t-2}^Y) IB_{t-1}^Y \quad (3.217)$$

We need this to aggregate the just-retired pension benefits

$$N_{0,t}^O TR_{0,t}^{PG,YO}(i) = \nu_t N_{0,t}^O IB_{b-1,t}^Y(i) = \nu_t \omega_{t-1}^Y \sum_{b=1}^{\infty} N_{b-1,t-1}^Y IB_{b-1,t}^Y(i) \quad (3.218)$$

Total pension expenditure of the just retired is simple with the new notations

$$TR_t^{PG,YO} = \nu_t \omega_{t-1}^Y IB_t^Y \quad (3.219)$$

Furthermore, total pension expenditure of all retired people is

$$TR_t^{PG} = TR_t^{PG,YO} + (1 - \omega_{t-1}^O) TR_{t-1}^{PG,YO} + (1 - \omega_{t-2}^O) TR_{t-2}^{PG,YO} + \dots \quad (3.220)$$

which can be rewritten as

$$TR_t^{PG} = TR_t^{PG,YO} + (1 - \omega_{t-1}^O) TR_{t-1}^{PG} \quad (3.221)$$

### Fully funded pension system

In the fully funded regime, pension benefits are not financed from the public budget, but separately based on individual savings:

$$Exp_t = Gov_t + w_t^U U_t \quad (3.222)$$

Agent ( $i$ ) has a private account, where she can accumulate her own pension wealth:

$$B_{b,t}^{Y*}(i) = \Xi(\tau_t^{SSCW} + \tau_t^{SSCF}) w_t^F L_{b,t}^F(i) + (1 + r_{t-1}) B_{b-1,t-1}^{Y*}(i) \quad (3.223)$$

Aggregating

$$B_t^{Y*} = \sum_{b=0}^{\infty} N_{b,t}^Y B_{b,t}^{Y*}(i) = \Xi(\tau_t^{SSCW} + \tau_t^{SSCF}) w_t^F \sum_{b=0}^{\infty} N_{b,t}^Y L_{b,t}^F(i) + (1 + r_{t-1}) \sum_{b=1}^{\infty} N_{b,t}^Y B_{b-1,t-1}^{Y*}(i) \quad (3.224)$$

Then the last term can be rearranged

$$\sum_{b=1}^{\infty} N_{b,t}^Y B_{b-1,t-1}^{Y*}(i) = (1 - \omega_{t-1}^Y) \sum_{b=1}^{\infty} N_{b-1,t-1}^Y B_{b-1,t-1}^{Y*}(i) = (1 - \omega_{t-1}^Y) B_{t-1}^{Y*} \quad (3.225)$$

Hence

$$B_t^{Y*} = \Xi(\tau_t^{SSCW} + \tau_t^{SSCF}) w_t^F L_t^F + (1 + r_{t-1}) (1 - \omega_{t-1}^Y) B_{t-1}^{Y*} \quad (3.226)$$

In the moment of getting retirement the just-retired benefits are calculated based on the so far accumulated pension wealth, expected life expectancy and the discount rates. In each period the pension fund transfers this amount, while the rest remains on the account.

$$(1 + r_{t-1})B_{b-1,t-1}^{Y*}(i) = TR_{0,t}^{FF,YO}(i) + E_t \frac{1 - \omega_t^O}{1 + r_t} TR_{1,t+1}^{FF,YO}(i) + E_t \frac{(1 - \omega_t^O)(1 - \omega_{t+1}^O)}{(1 + r_t)(1 + r_{t+1})} TR_{2,t+2}^{FF,YO}(i) + \dots = TR_{0,t}^{FF,YO}(i)\Omega_t^O \quad (3.227)$$

Aggregating and rearranging

$$(1 + r_{t-1})N_{0,t}^O B_{b-1,t-1}^{Y*}(i) = N_{0,t}^O TR_{0,t}^{FF,YO}(i)\Omega_t^O = TR_t^{FF,YO}\Omega_t^O \quad (3.228)$$

$$(1 + r_{t-1}) \sum_{b=1}^{\infty} \omega_{t-1}^Y N_{b-1,t-1}^Y B_{b-1,t-1}^{Y*}(i) = TR_t^{FF,YO}\Omega_t^O \quad (3.229)$$

$$(1 + r_{t-1})\omega_{t-1}^Y B_{t-1}^{Y*} = TR_t^{FF,YO}\Omega_t^O \quad (3.230)$$

Overall total pension expenditure in the fully funded regime can be given in a recursive way, similarly to that of the PAYG regime, as

$$TR_t^{FF} = TR_t^{FF,YO} + (1 - \omega_{t-1}^O)TR_{t-1}^{FF} \quad (3.231)$$

Finally, we can express savings in each period after pension benefits were deducted

$$(1 + r_{t-1})B_{a-1,t-1}^{O,*}(i) = TR_{a,t}^{FF,YO}(i) + (1 - \omega_t^O)B_{a,t}^{O,*}(i) \quad (3.232)$$

Aggregating and rearranging

$$(1 + r_{t-1}) \sum_{a=0}^{\infty} N_{a,t}^O B_{a-1,t-1}^{O,*}(i) = \sum_{a=0}^{\infty} N_{a,t}^O TR_{a,t}^{FF,YO}(i) + (1 - \omega_t^O) \sum_{a=0}^{\infty} N_{a,t}^O B_{a,t}^{O,*}(i) = TR_t^{FF} + B_t^{O,*} \quad (3.233)$$

The left-hand side of the above equation can be rearranged as

$$\begin{aligned} \sum_{a=0}^{\infty} N_{a,t}^O B_{a-1,t-1}^{O,*}(i) &= N_{0,t}^O B_{-1,t-1}^{O,*}(i) + \sum_{a=1}^{\infty} N_{a,t}^O B_{a-1,t-1}^{O,*}(i) = \\ &= N_{0,t}^O B_{-1,t-1}^{O,*}(i) + (1 - \omega_{t-1}^O) \sum_{a=1}^{\infty} N_{a-1,t-1}^O B_{a-1,t-1}^{O,*}(i) \end{aligned} \quad (3.234)$$

Lastly, we use that  $B_{-1,t-1}^{O,*}(i) = B_{b-1,t-1}^{Y,last,*}(i)$  because the initial old pension wealth is equal to pension savings accumulated over lifetime. Also, we rewrite  $N_{a,t}^O$ . So the first term of the above equation becomes

$$\omega_{t-1}^Y \sum_{b=1}^{\infty} N_{b-1,t-1}^Y B_{b-1,t-1}^{Y,last,*}(i) + (1 - \omega_{t-1}^O) \sum_{a=1}^{\infty} N_{a-1,t-1}^O B_{a-1,t-1}^{O,*}(i) \quad (3.235)$$

Finally the market clearing equations for old-age pension savings are

$$(1 + r_{t-1})\omega_{t-1}^Y B_{t-1}^{Y*} + (1 + r_{t-1})B_{t-1}^{O,*} = TR_t^{FF} + B_t^{O,*} \quad (3.236)$$

$$B_t^* = B_t^{Y*} + B_t^{O,*} \quad (3.237)$$

## C Normalized equations

Each variable must be detrended; individual variables are normalized with technology ( $A_t$ ) and aggregate variables are normalized with technology and population ( $N_t$ ), because there are technology and population growth in the model. This section lists all the final equations of the model, detrended variable  $x_t$  is denoted by  $\tilde{x}_t$ .

Demography:

$$s_t = \frac{(1 - \omega_{t-1}^O)}{(1 - \omega_{t-1}^Y + n_t)} s_{t-1} + \frac{\omega_{t-1}^Y}{(1 - \omega_{t-1}^Y + n_t)} \quad (3.238)$$

$$s_t^Y = \frac{1}{1 + s_t} \quad (3.239)$$

$$1 + g_t^{N,Y} = 1 - \omega_{t-1}^Y + n_t \quad (3.240)$$

$$1 + g_t^{N,O} = (1 - \omega_{t-1}^O) + \frac{\omega_{t-1}^Y}{s_{t-1}} \quad (3.241)$$

$$1 + g_t^N = (1 + g_t^{N,Y}) \frac{1 + s_t}{1 + s_{t-1}} \quad (3.242)$$

Overlapping generations:

$$\mathcal{H}_t^O C_t^{\tilde{O},F} = (T\tilde{R}_t^{PG} + T\tilde{R}_t^{FF})\Omega_t^O + \tilde{\mathcal{I}}_t^O + \frac{(1 + r_{t-1})}{1 + g_t} \left( \omega_{t-1}^Y B_{t-1}^{\tilde{Y}} + B_{t-1}^{\tilde{O}} \right) \quad (3.243)$$

$$\tilde{\mathcal{I}}_t^O = (1 - \xi) \left( Profit_t - \tilde{T}_t \right) + E_t \frac{1 + g_{t+1}}{1 + r_t} \frac{1 - \omega_t^O}{1 + g_{t+1}^{N,O}} \tilde{\mathcal{I}}_{t+1}^O \quad (3.244)$$

$$\mathcal{H}_t^O = (1 + \tau_t^C) + p_t^I \Upsilon_t + E_t (1 - \omega_t^O) (1 + r_t)^{\frac{1}{\gamma} - 1} \Lambda_{t+1} \mathcal{H}_{t+1}^O \quad (3.245)$$

$$\mathcal{H}_t^Y C_t^{\tilde{Y},F} = \tilde{\mathcal{I}}_t^Y + \frac{\tilde{\mathcal{I}}_t^O}{1 + r_t} + \frac{(1 + r_{t-1})(1 - \omega_{t-1}^Y)}{1 + g_t} B_{t-1}^{\tilde{Y}} \quad (3.246)$$

$$\Omega_t^O = 1 + E_t \frac{1 - \omega_t^O}{1 + r_t} \Omega_{t+1}^O \quad (3.247)$$

$$C_t^{\tilde{O},I} = \Upsilon_t C_t^{\tilde{O},F} \quad (3.248)$$

$$E_t \Lambda_{t+1} = E_t \left\{ \beta \frac{1 + \epsilon_{t+1}^C}{1 + \epsilon_t^C} \frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} \right\}^{\frac{1}{\gamma}} \quad (3.249)$$

$$\Upsilon_t = \left\{ \chi \frac{1 + \tau_t^C}{p_t^I} \right\}^{\frac{1}{\gamma}} \quad (3.250)$$

$$\mathcal{H}_t^Y = (1 + \tau_t^C) + p_t^I \Upsilon_t + E_t (1 + r_t)^{\frac{1}{\gamma} - 1} \Lambda_{t+1} \left( (1 - \omega_t^Y) \mathcal{H}_{t+1}^Y + \omega_t^Y \mathcal{H}_{t+1}^O \right) \quad (3.251)$$

$$Inc_t = (1 - \tau_t^{LW}) \tilde{w}_t^F \tilde{L}_t^F + \tilde{w}_t^I \tilde{L}_t^I + \tilde{w}_t^U \tilde{U}_t + \xi \left( Profit_t - \tilde{T}_t \right) \quad (3.252)$$

$$\tilde{\mathcal{I}}_t^Y = \tilde{I}nc_t + E_t \frac{(1 - \omega_t^Y)(1 + g_{t+1})}{(1 + r_t)(1 + g_{t+1}^{N,Y})} \tilde{\mathcal{I}}_{t+1}^Y \quad (3.253)$$

$$\begin{aligned} \tilde{\mathcal{I}}_t^{YO} &= E_t(1 + g_{t+1}) \left( (TR_{t+1}^{\tilde{P}G,YO} + TR_{t+1}^{\tilde{F}F,YO}) \Omega_{t+1}^O + \frac{\omega_t^Y}{(1 + g_{t+1}^{N,Y})_{s_{t+1}}} \tilde{\mathcal{I}}_{t+1}^O \right) + \\ &+ E_t \frac{(1 - \omega_{t+1}^Y)(1 + g_{t+1})}{(1 + r_{t+1})(1 + g_{t+1}^{N,Y})} \tilde{\mathcal{I}}_{t+1}^{YO} \end{aligned} \quad (3.254)$$

$$C_t^{\tilde{Y},I} = \Upsilon_t C_t^{\tilde{Y},F} \quad (3.255)$$

$$(1 + \tau_t^C) C_t^{\tilde{Y},F} + p_t^I C_t^{\tilde{Y},I} + \tilde{B}_t^Y = \tilde{I}nc_t + \frac{(1 + r_{t-1})}{1 + g_t} (1 - \omega_{t-1}^Y) \tilde{B}_{t-1}^Y \quad (3.256)$$

Formal firms - except labor market:

$$R \left( \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)^\gamma} \right) = \frac{\phi_P}{2} \left( \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)^\gamma} - 1 \right)^2 \quad (3.257)$$

$$\begin{aligned} 1 + \frac{1}{\varphi - 1} R \left( \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)^\gamma} \right) + \frac{1}{\varphi - 1} R' \left( \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)^\gamma} \right) \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)^\gamma} - \\ - E_t \frac{1}{\varphi - 1} \frac{(1 + g_{t+1}) Y_{t+1}^F}{\tilde{Y}_t^F} \frac{R' \left( \frac{1 + \pi_{t+1}^F}{(1 + \pi_t^F)^\gamma} \right) \left( \frac{1 + \pi_{t+1}^F}{(1 + \pi_t^F)^\gamma} \right)}{1 + r_t} = \frac{\varphi}{\varphi - 1} mc_t^F \end{aligned} \quad (3.258)$$

$$mc_t^F = \left( \frac{r_t^{K,F}}{\alpha^F} \right)^{\alpha^F} \left( \frac{(1 + \tau_t^{SSCF}) \tilde{w}_t^F}{\tilde{A}_t^F (1 - \alpha^F)} \right)^{(1 - \alpha^F)} \quad (3.259)$$

$$\frac{1}{1 + g_t} K_{t-1}^{\tilde{F}} = \alpha^F \frac{mc_t^F}{r_t^{K,F}} \tilde{Y}_t^F \quad (3.260)$$

$$mc_t^F (1 - \alpha^F) \frac{\tilde{Y}_t^F}{\tilde{L}_t^F} - (1 + \tau_t^{SSCF}) \tilde{w}_t^F = h \tilde{c}_t^F - E_t \frac{h \tilde{c}_{t+1}^F (1 - p r_{t+1}^{F,F})}{(1 + r_t)} (1 + g_{t+1}^A) \quad (3.261)$$

$$profit_t^F = \tilde{Y}_t^F - r_t^{K,F} K_{t-1}^{\tilde{F}} \frac{1}{1 + g_t} - (1 + \tau_t^{SSCW}) \tilde{w}_t^F \tilde{L}_t^F - R(\cdot) \tilde{Y}_t^F - h \tilde{c}_t^F \tilde{H}_t^F \quad (3.262)$$

$$E_t(r_{t+1}^{K,F} + Q_{t+1}^F(1 - \delta)) = Q_t^F(1 + r_t) \quad (3.263)$$

$$\begin{aligned} 1 &= Q_t^F \left( 1 - S \left( \frac{In \tilde{v}_t^F}{In \tilde{v}_{t-1}^F} \right) - S' \left( \frac{In \tilde{v}_t^F}{In \tilde{v}_{t-1}^F} \right) \frac{In \tilde{v}_t^F}{In \tilde{v}_{t-1}^F} \right) + \\ &+ E_t \frac{1}{1 + r_t} Q_{t+1}^F S' \left( \frac{In \tilde{v}_{t+1}^F}{In \tilde{v}_t^F} \right) \left( \frac{In \tilde{v}_{t+1}^F}{In \tilde{v}_t^F} \right)^2 \end{aligned} \quad (3.264)$$

$$In \tilde{v}_t^F \left( 1 - S \left( \frac{In \tilde{v}_t^F}{In \tilde{v}_{t-1}^F} \right) \right) = \tilde{K}_t^F - (1 - \delta) \frac{K_{t-1}^{\tilde{F}}}{1 + g_t} \quad (3.265)$$

Informal firms - except labor market:

$$R \left( \frac{1 + \pi_t^I}{(1 + \pi_{t-1}^I)^\gamma} \right) = \frac{\phi_P}{2} \left( \frac{1 + \pi_t^I}{(1 + \pi_{t-1}^I)^\gamma} - 1 \right)^2 \quad (3.266)$$

$$1 + \frac{1}{\varphi - 1} R \left( \frac{1 + \pi_t^I}{(1 + \pi_{t-1}^I)^\gamma} \right) + \frac{1}{\varphi - 1} R' \left( \frac{1 + \pi_t^I}{(1 + \pi_{t-1}^I)^\gamma} \right) \frac{1 + \pi_t^I}{(1 + \pi_{t-1}^I)^\gamma} -$$

$$- E_t \frac{1}{\varphi - 1} \frac{(1 + g_{t+1}) \tilde{Y}_{t+1}^I}{\tilde{Y}_t^I} \frac{R' \left( \frac{1 + \pi_{t+1}^I}{(1 + \pi_t^I)^\gamma} \right) \left( \frac{1 + \pi_{t+1}^I}{(1 + \pi_t^I)^\gamma} \right)}{1 + r_t} = \frac{\varphi}{\varphi - 1} \frac{mc_t^I}{p_t^I} \quad (3.267)$$

$$mc_t^I = \left( \frac{r_t^{K,I}}{\alpha^I} \right)^{\alpha^I} \left( \frac{\tilde{w}_t^I}{\tilde{A}_t^I (1 - \alpha^I)} \right)^{(1 - \alpha^I)} \quad (3.268)$$

$$\frac{1}{1 + g_t} K_{t-1}^I = \alpha^I \frac{mc_t^I}{r_t^{K,I}} \tilde{Y}_t^I \quad (3.269)$$

$$mc_t^I (1 - \alpha^I) \frac{\tilde{Y}_t^I}{\tilde{L}_t^I} - \tilde{w}_t^I = \tilde{h}c_t^I - E_t \frac{\tilde{h}c_{t+1}^I (1 - pr_{t+1}^{F,I})}{(1 + r_t)} (1 + g_{t+1}^A) \quad (3.270)$$

$$profit_t^I = p_t^I \tilde{Y}_t^I - r_t^{K,I} K_{t-1}^I \frac{1}{1 + g_t} - \tilde{w}_t^I \tilde{L}_t^I - R(\cdot) \tilde{Y}_t^I - \tilde{h}c_t^I \tilde{H}_t^I \quad (3.271)$$

$$E_t (r_{t+1}^{K,I} + Q_{t+1}^I (1 - \delta)) = Q_t^I (1 + r_t) \quad (3.272)$$

$$Q_t^I \left( 1 - S \left( \frac{In \tilde{v}_t^I}{In \tilde{v}_{t-1}^I} \right) - S' \left( \frac{In \tilde{v}_t^I}{In \tilde{v}_{t-1}^I} \right) \frac{In \tilde{v}_t^I}{In \tilde{v}_{t-1}^I} \right) +$$

$$+ E_t \frac{1}{1 + r_t} Q_{t+1}^I S' \left( \frac{In \tilde{v}_{t+1}^I}{In \tilde{v}_t^I} \right) \left( \frac{In \tilde{v}_{t+1}^I}{In \tilde{v}_t^I} \right)^2 = 1 \quad (3.273)$$

$$In \tilde{v}_t^I \left( 1 - S \left( \frac{In \tilde{v}_t^I}{In \tilde{v}_{t-1}^I} \right) \right) = \tilde{K}_t^I - (1 - \delta) \frac{K_{t-1}^I}{1 + g_t} \quad (3.274)$$

Labor market with wage bargaining:

$$hc_t^F = \kappa^F (pr_t^{H,F})^{\alpha_{HC}} \quad (3.275)$$

$$pr_t^{H,F} = \frac{\tilde{H}_t^F}{\frac{\tilde{U}_{t-1}}{1 + g_t^N} + pr_t^{F,F} \frac{L_{t-1}^{\tilde{F}}}{1 + g_t^N} + pr_t^{F,I} \frac{L_{t-1}^{\tilde{I}}}{1 + g_t^N}} \quad (3.276)$$

$$\tilde{L}_t^F = (1 - pr_t^F) \frac{L_{t-1}^{\tilde{F}}}{1 + g_t^N} + \tilde{H}_t^F \quad (3.277)$$

$$\frac{\sigma_F}{1 - \sigma_F} \tilde{h}c_t^F \frac{1 - \tau_t^{LW}}{1 + \tau_t^{SSCF}} = (1 - \tau_t^{LW}) \tilde{w}_t^F - \tilde{w}_t^U +$$

$$+ E_t \frac{1 + g_{t+1}^A}{1 + r_t} \left[ (1 - pr_{t+1}^{F,F}) (1 - pr_{t+1}^{H,F}) \left( \frac{\sigma_F}{1 - \sigma_F} \tilde{h}c_{t+1}^F \frac{1 - \tau_{t+1}^{LW}}{1 + \tau_{t+1}^{SSCF}} \right) \right] -$$

$$- E_t \frac{1 + g_{t+1}^A}{1 + r_t} \left[ (1 - pr_{t+1}^{F,F}) pr_{t+1}^{H,I} \left( \frac{\sigma_I}{1 - \sigma_I} \tilde{h}c_t^I \right) \right] \quad (3.278)$$

$$hc_t^I = \kappa^I (pr_t^{H,I})^{\alpha_{HC}} \quad (3.279)$$



$$pr_t^{H,I} = \frac{\tilde{H}_t^I}{\frac{U_{t-1}^N}{1+g_t^N} + pr_t^{F,F} \frac{L_{t-1}^{\tilde{F}}}{1+g_t^N} + pr_t^{F,I} \frac{L_{t-1}^{\tilde{I}}}{1+g_t^N}} \quad (3.280)$$

$$\tilde{L}_t^I = (1 - pr_t^{F,I}) \frac{L_{t-1}^{\tilde{I}}}{1+g_t^N} + \tilde{H}_t^I \quad (3.281)$$

$$\begin{aligned} \frac{\sigma_I}{1-\sigma_I} \tilde{h}c_t^I &= \tilde{w}_t^I - \tilde{w}_t^U + E_t \frac{1+g_{t+1}^A}{1+r_t} \left[ (1 - pr_{t+1}^{F,I})(1 - pr_{t+1}^{H,I}) \frac{\sigma_I}{1-\sigma_I} \tilde{h}c_{t+1}^I \right] - \\ &- E_t \frac{1+g_{t+1}^A}{1+r_t} \left[ pr_{t+1}^{H,F} (1 - pr_{t+1}^{F,I}) \frac{\sigma_F}{1-\sigma_F} \tilde{h}c_{t+1}^{\tilde{F}} \frac{1 - \tau_{t+1}^{LW}}{1 + \tau_{t+1}^{SSC}} \right] \end{aligned} \quad (3.282)$$

Government<sup>46</sup>

$$R\tilde{e}v_t = \tau_t^C \tilde{C}_t^F + \tau_t^L \tilde{w}_t^F \tilde{L}_t^F + \tilde{T}_t \quad (3.283)$$

$$\tau_t^{LW} = \tau_t^{PI} + \tau_t^{SSCW} \quad (3.284)$$

$$\tilde{B}_t = \frac{1}{1+g_t} \tilde{B}_{t-1} - G\tilde{B}_t \quad (3.285)$$

$$P\tilde{B}_t = R\tilde{e}v_t - E\tilde{x}p_t \quad (3.286)$$

$$G\tilde{B}_t = P\tilde{B}_t - r_{t-1} \frac{1}{1+g_t} \tilde{B}_{t-1} \quad (3.287)$$

$$\tilde{T}_t = \eta T + (1 - \eta) \left[ \rho \tilde{T}_{t-1} + (1 - \rho)(G\tilde{B}_t^{\tilde{T}arget} - G\tilde{B}_t) \right] \quad (3.288)$$

PAYG pension system:<sup>47</sup>

$$\tau_t^L = \tau_t^{PI} + (1 - \Xi)(\tau_t^{SSCW} + \tau_t^{SSCF}) \quad (3.289)$$

$$E\tilde{x}p_t = G\tilde{o}v_t + \tilde{w}_t^U \tilde{U}_t + T\tilde{R}_t^{PG} \quad (3.290)$$

$$I\tilde{B}_t^Y = \frac{1}{Y} \tilde{w}_{t-1}^{\tilde{F}} L_{t-1}^{\tilde{F}} + \frac{Y-1}{Y} \frac{(1 - \omega_{t-2}^Y)}{1+g_{t-1}} I\tilde{B}_{t-1}^Y \quad (3.291)$$

$$T\tilde{R}_t^{\tilde{P}G,YO} = \nu_t \frac{\omega_{t-1}^Y}{1+g_t} I\tilde{B}_t^Y \quad (3.292)$$

$$T\tilde{R}_t^{PG} = T\tilde{R}_t^{\tilde{P}G,YO} + \frac{(1 - \omega_{t-1}^O)}{1+g_t} T\tilde{R}_{t-1}^{PG} \quad (3.293)$$

Fully funded pension system:

$$E\tilde{x}p_t = G\tilde{o}v_t + \tilde{w}_t^U \tilde{U}_t \quad (3.294)$$

$$\tilde{B}_t^{Y*} = \Xi(\tau_t^{SSCW} + \tau_t^{SSCF}) \tilde{w}_t^F \tilde{L}_t^F + \frac{(1+r_{t-1})}{1+g_t} (1 - \omega_{t-1}^Y) \tilde{B}_{t-1}^{Y*} \quad (3.295)$$

$$\frac{(1+r_{t-1})}{1+g_t} \omega_{t-1}^Y \tilde{B}_{t-1}^{Y*} = T\tilde{R}_t^{\tilde{F}F,YO} \Omega_t^O \quad (3.296)$$

$$T\tilde{R}_t^{\tilde{F}FF} = T\tilde{R}_t^{\tilde{F}F,YO} + (1 - \omega_{t-1}^O) \frac{1}{1+g_t} T\tilde{R}_{t-1}^{\tilde{F}FF} \quad (3.297)$$

<sup>46</sup>There is a similar rule for all other fiscal instruments, not only for lump-sum taxes.

<sup>47</sup> $I\tilde{B}_t^Y$  in the third and fourth equations is normalized by  $A_{t-1}N_{t-1}$ .

$$\frac{(1+r_{t-1})}{1+g_t} \omega_{t-1}^Y B_{t-1}^{\tilde{Y}*} + \frac{(1+r_{t-1})}{1+g_t} B_{t-1}^{\tilde{O},*} = T \tilde{R}_t^{FF} + B_t^{\tilde{O},*} \quad (3.298)$$

$$\tilde{B}_t^* = B_t^{\tilde{Y}*} + B_t^{\tilde{O},*} \quad (3.299)$$

Monetary policy:

$$1+i_t = (1+i_{t-1})^{\rho_i} E_t \left( (1+r) (1+\pi_{t+1}^F)^{\phi_\pi} \right)^{1-\rho_i} e^{\epsilon_t^i} \quad (3.300)$$

$$1+i_t = (1+r_t) E_t (1+\pi_{t+1}^F) \quad (3.301)$$

Market clearing:

$$\tilde{U}_t = s_t^Y - \tilde{L}_t^F - \tilde{L}_t^I \quad (3.302)$$

$$\tilde{Profit}_t = profit_t^F + profit_t^I \quad (3.303)$$

$$\tilde{B}_t + Q_t^F \tilde{K}_t^F + Q_t^I \tilde{K}_t^I = \tilde{B}_t^Y + \tilde{B}_t^O + \tilde{B}_t^* \quad (3.304)$$

$$\begin{aligned} \tilde{Y}_t^F &= \tilde{C}_t^F + \tilde{Inv}_t + \tilde{Gov}_t + \tilde{hc}_t^F \tilde{H}_t^F + R \left( \frac{P_t^F}{P_{t-1}^F} \right) + \tilde{Inv}_t^F S \left( \frac{\tilde{Inv}_t^F}{\tilde{Inv}_{t-1}^F} \right) + \\ &+ \tilde{Inv}_t^I S \left( \frac{\tilde{Inv}_t^I}{\tilde{Inv}_{t-1}^I} \right) \end{aligned} \quad (3.305)$$

$$\tilde{Y}_t^I = \tilde{C}_t^I + \tilde{hc}_t^I \tilde{H}_t^I + R \left( \frac{P_t^I}{P_{t-1}^I} \right) \quad (3.306)$$

$$\tilde{C}_t^F = \tilde{C}_t^{\tilde{Y},F} + \tilde{C}_t^{\tilde{O},F} \quad (3.307)$$

$$\tilde{C}_t^I = \tilde{C}_t^{\tilde{Y},I} + \tilde{C}_t^{\tilde{O},I} \quad (3.308)$$

$$\tilde{Inv}_t = \tilde{Inv}_t^F + \tilde{Inv}_t^I \quad (3.309)$$

$$G \tilde{D} P_t = \tilde{C}_t + \tilde{Inv}_t + \tilde{Gov}_t \quad (3.310)$$

$$\tilde{C}_t = \tilde{C}_t^F + p_t^I \tilde{C}_t^I \quad (3.311)$$

$$\tilde{Y}_t = \tilde{Y}_t^F + p_t^I \tilde{Y}_t^I \quad (3.312)$$

## D Steady state of the model

The steady state is solved numerically. First, we specify initial guesses for the following variables:  $r$ ,  $p^I$ ,  $\frac{Y^F}{Y}$ ,  $pr^{H,F}$  and  $pr^{H,I}$ . Then, as a function of initial guesses, we can determine the variables of production, labor market and bargaining and those of the government and pension systems. Finally, we turn to the consumption and savings functions. At the end, using the market clearing equations and some other leftover equations, we can check whether our initial guesses are correct. First, we calculate each variable in terms of production, then, after calculating  $\tilde{Y}$  itself, we can find the steady-state levels of the variables. Now, we describe the steps of calculating the model's steady state in detail.

First, the demographic equations are:

$$s = \frac{\frac{\omega^Y}{(1-\omega^Y+n)}}{1 - \frac{(1-\omega^O)}{(1-\omega^Y+n)}} \quad (3.313)$$

$$s^Y = \frac{1}{1+s} \quad (3.314)$$

$$1 + g^{N,Y} = 1 - \omega^Y + n \quad (3.315)$$

$$1 + g^{N,O} = (1 - \omega^O) + \frac{\omega^Y}{s} \quad (3.316)$$

$$1 + g^N = (1 + g^{N,Y}) \frac{1+s}{1+s} \quad (3.317)$$

Additionally, the balanced growth trend can be given by:

$$(1 + g) = (1 + g^A)(1 + g^N) \quad (3.318)$$

where  $g^A$  is the exogenous productivity growth.

Then, we need to guess an initial value for  $r$ ,  $p^I$ ,  $\frac{\tilde{Y}^I}{\tilde{Y}}$ ,  $pr^{H,F}$  and  $pr^{H,I}$  which are verified by the Newton-Raphson algorithm. Assuming  $\pi^F = 0$  in the steady state implies:

$$i = r \quad (3.319)$$

In the steady state the quadratic adjustment costs are zero, so:

$$Q^F = 1 \quad (3.320)$$

$$Q^I = 1 \quad (3.321)$$

and from the no-arbitrage conditions:

$$r^{K,F} = r + \delta \quad (3.322)$$

$$r^{K,I} = r + \delta \quad (3.323)$$

Then, we can calculate the marginal costs from the Phillips-curves:

$$mc^F = \frac{\varphi - 1}{\varphi} \quad (3.324)$$

$$mc^I = \frac{\varphi - 1}{\varphi} p^I \quad (3.325)$$

Because we have a guess for  $\frac{\tilde{Y}^I}{\tilde{Y}}$  and  $p^I$ , formal production share is given by:

$$\frac{\tilde{Y}^F}{\tilde{Y}} = 1 - p^I \frac{\tilde{Y}^I}{\tilde{Y}} \quad (3.326)$$

Also, we can express the capital-output ratios from the capital demands:

$$\frac{\tilde{K}^F}{\tilde{Y}} = (1 + g)\alpha^F \frac{mc^F}{r^{K,F}} \frac{\tilde{Y}^F}{\tilde{Y}} \quad (3.327)$$

$$\frac{\tilde{K}^I}{\tilde{Y}} = (1 + g)\alpha^I \frac{mc^I}{r^{K,I}} \frac{\tilde{Y}^I}{\tilde{Y}} \quad (3.328)$$

and then the investment-output ratios are:

$$\frac{I\tilde{n}v^F}{\tilde{Y}} = \frac{\tilde{K}^F}{\tilde{Y}} \left( 1 - (1 - \delta) \frac{1}{1 + g} \right) \quad (3.329)$$

$$\frac{I\tilde{n}v^I}{\tilde{Y}} = \frac{\tilde{K}^I}{\tilde{Y}} \left( 1 - (1 - \delta) \frac{1}{1 + g} \right) \quad (3.330)$$

$$\frac{I\tilde{n}v}{\tilde{Y}} = \frac{I\tilde{n}v^F}{\tilde{Y}} + \frac{I\tilde{n}v^I}{\tilde{Y}} \quad (3.331)$$

As a next step, from the marginal cost functions we calculate the gross wages (assuming that  $\tilde{A}^F = 1$  and  $\tilde{A}^I = 1$ ):

$$(1 + \tau^{SSCF})\tilde{w}^F = (1 - \alpha^F) \left[ \frac{mc^F}{\left( \frac{r^{K,F}}{\alpha^F} \right)^{\alpha^F}} \right]^{\frac{1}{1 - \alpha^F}} \quad (3.332)$$

$$\tilde{w}^I = (1 - \alpha^I) \left[ \frac{mc^I}{\left( \frac{r^{K,I}}{\alpha^I} \right)^{\alpha^I}} \right]^{\frac{1}{1 - \alpha^I}} \quad (3.333)$$

Now, we know the ratios of hiring costs to wages ( $Wage\_Ratio^F$  and  $Wage\_Ratio^I$ ), so we can endogenously determine  $\kappa^F$  and  $\kappa^I$ :

$$hc^F = Wage\_Ratio^F (1 + \tau^{SSCF})\tilde{w}^F \quad (3.334)$$

$$hc^I = Wage\_Ratio^I \tilde{w}^I \quad (3.335)$$

$$\kappa^F = \frac{hc^F}{pr^{H,F}} \quad (3.336)$$

$$\kappa^I = \frac{hc^I}{pr^{H,I}} \quad (3.337)$$

Then, using the labor demand equations we can calculate the labor-output ratios:

$$\frac{\tilde{L}^F}{\tilde{Y}} = \frac{\tilde{Y}^F}{\tilde{Y}} \frac{mc^F(1 - \alpha^F)}{(1 + \tau^{SSCF})\tilde{w}^F + \tilde{hc}^F - \frac{\tilde{hc}^F(1 - pr^{F,F})}{(1 + r)}(1 + g^A)} \quad (3.338)$$

$$\frac{\tilde{L}^I}{\tilde{Y}} = \frac{\tilde{Y}^I}{\tilde{Y}} \frac{mc^I(1 - \alpha^I)}{\tilde{w}^I + \tilde{hc}^I - \frac{\tilde{hc}^I(1 - pr^{F,I})}{(1 + r)}(1 + g^A)} \quad (3.339)$$

Now, the hiring-output ratios are:

$$\frac{\tilde{H}^F}{\tilde{Y}} = \frac{\tilde{L}^F}{\tilde{Y}} \left( 1 - (1 - pr^F) \frac{\tilde{L}^F}{1 + g^N} \right) \quad (3.340)$$

$$\frac{\tilde{H}^I}{\tilde{Y}} = \frac{\tilde{L}^I}{\tilde{Y}} \left( 1 - (1 - pr^{F,I}) \frac{\tilde{L}^I}{1 + g^N} \right) \quad (3.341)$$

And, the unemployment-output ratio can be given by using the  $pr^{H,F}$  equation:

$$\frac{\tilde{U}}{\tilde{Y}} = (1 + g^N) \frac{\frac{\tilde{H}^F}{\tilde{Y}}}{pr^{H,F}} - pr^{F,F} \frac{\tilde{L}^F}{\tilde{Y}} - pr^{F,I} \frac{\tilde{L}^I}{\tilde{Y}} \quad (3.342)$$

The remaining three labor market equations ( $pr^{H,F}$  and the two wage bargaining equations) are used by the Newton-Raphson algorithm to verify the initial guesses for  $\frac{\tilde{Y}^I}{\tilde{Y}}$ ,  $pr^{H,F}$  and  $pr^{H,I}$ .

Next, we can calculate profits in total production:

$$\frac{\tilde{profit}^F}{\tilde{Y}} = p^F \frac{\tilde{Y}^F}{\tilde{Y}} - r^{K,F} \frac{\tilde{K}^F}{\tilde{Y}} \frac{1}{1 + g} - (1 + \tau^{SSCW}) \tilde{w}^F \frac{\tilde{L}^F}{\tilde{Y}} - \tilde{h}c^F \frac{\tilde{H}^F}{\tilde{Y}} \quad (3.343)$$

$$\frac{\tilde{profit}^I}{\tilde{Y}} = p^I \frac{\tilde{Y}^I}{\tilde{Y}} - r^{K,I} \frac{\tilde{K}^I}{\tilde{Y}} \frac{1}{1 + g} - \tilde{w}^I \frac{\tilde{L}^I}{\tilde{Y}} - \tilde{h}c^I \frac{\tilde{H}^I}{\tilde{Y}} \quad (3.344)$$

Using the goods market clearing conditions, we can express formal and informal consumption as a share of total production:

$$\frac{\tilde{C}^F}{\tilde{Y}} = \frac{\tilde{Y}^F}{\tilde{Y}} - \frac{\tilde{I}nv}{\tilde{Y}} - \frac{\tilde{G}ov}{\tilde{Y}} - \tilde{h}c^F \frac{\tilde{H}^F}{\tilde{Y}} \quad (3.345)$$

$$\frac{\tilde{C}^I}{\tilde{Y}} = \frac{\tilde{Y}^I}{\tilde{Y}} - \tilde{h}c^I \frac{\tilde{H}^I}{\tilde{Y}} \quad (3.346)$$

As a result, total consumption-output ratio is:

$$\frac{\tilde{C}}{\tilde{Y}} = \frac{\tilde{C}^F}{\tilde{Y}} + p^I \frac{\tilde{C}^I}{\tilde{Y}} \quad (3.347)$$

Because all variabls are expressed as a share of production so far, we need the GDP-production ratio:

$$\frac{G\tilde{D}P}{\tilde{Y}} = \frac{\tilde{C}}{\tilde{Y}} + \frac{\tilde{I}nv}{\tilde{Y}} + \frac{\tilde{G}ov}{\tilde{Y}} \quad (3.348)$$

$$(3.349)$$

We observe all distortionary tax revenues as a share of GDP, hence, we can calculate the effective tax rates which are consistent with the model's labor and goods markets.

Now we can express all other fiscal variables (the initial calibration is done with a PAYG pension plan):

$$\frac{I\tilde{B}^Y}{\tilde{Y}} = \frac{\frac{1}{\tilde{Y}}w^F\frac{\tilde{L}^F}{\tilde{Y}}}{1 - \frac{Y-1}{Y}\frac{(1-\omega^Y)}{1+g}} \quad (3.350)$$

$$\frac{TR^{\tilde{P}G,YO}}{\tilde{Y}} = \nu \frac{\omega^Y}{1+g} \frac{I\tilde{B}^Y}{\tilde{Y}} \quad (3.351)$$

$$\frac{T\tilde{R}^{PG}}{\tilde{Y}} = \frac{\frac{TR^{\tilde{P}G,YO}}{\tilde{Y}}}{1 - \frac{(1-\omega^O)}{1+g}} \quad (3.352)$$

$$\frac{Exp}{\tilde{Y}} = \frac{Gov}{\tilde{Y}} + w^U \frac{\tilde{U}}{\tilde{Y}} + \frac{T\tilde{R}^{PG}}{\tilde{Y}} \quad (3.353)$$

Taking into account the data on the government debt to GDP ratio, we calculate the lump-sum tax to GDP ratio:

$$\frac{\tilde{G}B}{\tilde{Y}} = \left( \frac{1}{1+g} - 1 \right) \frac{\tilde{B}}{\tilde{Y}} \quad (3.354)$$

$$\frac{\tilde{P}B}{\tilde{Y}} = \frac{\tilde{G}B}{\tilde{Y}} + r \frac{1}{1+g} \frac{\tilde{B}}{\tilde{Y}} \quad (3.355)$$

$$\frac{\tilde{R}ev}{\tilde{Y}} = \frac{\tilde{P}B}{\tilde{Y}} + \frac{Exp}{\tilde{Y}} \quad (3.356)$$

$$\frac{\tilde{T}}{\tilde{Y}} = \frac{\tilde{R}ev}{\tilde{Y}} - \tau^C \frac{\tilde{C}^F}{\tilde{Y}} - \tau^L w^F \frac{\tilde{L}^F}{\tilde{Y}} \quad (3.357)$$

Now we express all supporting variables of the households:

$$\frac{\tilde{\mathcal{I}}^O}{\tilde{Y}} = \frac{(1-\xi) \left( \frac{\tilde{Profit}}{\tilde{Y}} - \frac{\tilde{T}}{\tilde{Y}} \right)}{1 - \frac{1+g}{1+r} \frac{1-\omega^O}{1+g^{N,O}}} \quad (3.358)$$

$$\Omega^O = \frac{1}{1 - \frac{1-\omega^O}{1+r}} \quad (3.359)$$

$$\Lambda = \beta^{\frac{1}{\gamma}} \quad (3.360)$$

$$\Upsilon = \left\{ \chi \frac{1+\tau^C}{p^I} \right\}^{\frac{1}{\gamma}} \quad (3.361)$$

$$\mathcal{H}^O = \frac{(1+\tau^C) + p^I \Upsilon}{1 - (1-\omega^O)(1+r)^{\frac{1}{\gamma}-1} \Lambda} \quad (3.362)$$

$$\mathcal{H}^Y = \frac{(1+\tau^C) + p^I \Upsilon + (1+r)^{\frac{1}{\gamma}-1} \Lambda \omega^Y \mathcal{H}^O}{1 - (1+r)^{\frac{1}{\gamma}-1} \Lambda (1-\omega^Y)} \quad (3.363)$$

$$\frac{\tilde{Inc}}{\tilde{Y}} = (1-\tau^{LW})w^F \frac{\tilde{L}^F}{\tilde{Y}} + \tilde{w}^I \frac{\tilde{L}^I}{\tilde{Y}} + \tilde{w}^U \frac{\tilde{U}}{\tilde{Y}} + \xi \left( \frac{\tilde{Profit}}{\tilde{Y}} - \frac{\tilde{T}}{\tilde{Y}} \right) \quad (3.364)$$

$$\frac{\tilde{\mathcal{I}}^Y}{\tilde{Y}} = \frac{\frac{\tilde{Inc}}{\tilde{Y}}}{1 - \frac{(1-\omega^Y)(1+g)}{(1+r)(1+g^{N,Y})}} \quad (3.365)$$

$$\frac{\tilde{\mathcal{I}}^{YO}}{\tilde{Y}} = \frac{(1+g) \left( \left( \frac{TR^{PG,YO}}{\tilde{Y}} + \frac{TR^{FF,YO}}{\tilde{Y}} \right) \Omega^O + \frac{\omega^Y}{(1+g^{N,Y})_s} \frac{\tilde{\mathcal{I}}^O}{\tilde{Y}} \right)}{1 - \frac{(1-\omega^Y)(1+g)}{(1+r)(1+g^{N,Y})}} \quad (3.366)$$

Using the young agents' consumption function, budget constraint and the remaining first order conditions we can express the savings to total production ratio:

$$\mathcal{H}^Y \frac{C^{\tilde{Y},F}}{\tilde{Y}} = \frac{\tilde{\mathcal{I}}^Y}{\tilde{Y}} + \frac{\frac{\tilde{\mathcal{I}}^{YO}}{\tilde{Y}}}{1+r} + \frac{(1+r)(1-\omega^Y)}{1+g} \frac{\tilde{B}^Y}{\tilde{Y}} \quad (3.367)$$

$$\frac{C^{\tilde{Y},I}}{\tilde{Y}} = \Upsilon \frac{C^{\tilde{Y},F}}{\tilde{Y}} \quad (3.368)$$

$$(1+\tau^C) \frac{C^{\tilde{Y},F}}{\tilde{Y}} + p^I \frac{C^{\tilde{Y},I}}{\tilde{Y}} + \frac{\tilde{B}^Y}{\tilde{Y}} = \frac{\tilde{Inc}}{\tilde{Y}} + \frac{(1+r)}{1+g} (1-\omega^Y) \frac{\tilde{B}^Y}{\tilde{Y}} \quad (3.369)$$

where we plug the informal consumption to production ratio in, in order to express the formal consumption to production ratio:

$$\frac{C^{\tilde{Y},F}}{\tilde{Y}} = \frac{\frac{\tilde{Inc}}{\tilde{Y}} + \left( \frac{(1+r)}{1+g} (1-\omega^Y) - 1 \right) \frac{\tilde{B}^Y}{\tilde{Y}}}{(1+\tau^C) + p^I \Upsilon} \quad (3.370)$$

Then, plugging this back in the consumption function:

$$\mathcal{H}^Y \frac{\frac{\tilde{Inc}}{\tilde{Y}} + \left( \frac{(1+r)}{1+g} (1-\omega^Y) - 1 \right) \frac{\tilde{B}^Y}{\tilde{Y}}}{(1+\tau^C) + p^I \Upsilon} = \frac{\tilde{\mathcal{I}}^Y}{\tilde{Y}} + \frac{\frac{\tilde{\mathcal{I}}^{YO}}{\tilde{Y}}}{1+r} + \frac{(1+r)(1-\omega^Y)}{1+g} \frac{\tilde{B}^Y}{\tilde{Y}} \quad (3.371)$$

$$(3.372)$$

So, the steady-state savings as a share of production are:

$$\frac{\tilde{B}^Y}{\tilde{Y}} = \frac{\frac{(1+\tau^C)+p^I \Upsilon}{\mathcal{H}^Y} \left( \frac{\tilde{\mathcal{I}}^Y}{\tilde{Y}} + \frac{\frac{\tilde{\mathcal{I}}^{YO}}{\tilde{Y}}}{1+r} \right) - \frac{\tilde{Inc}}{\tilde{Y}}}{\left( \frac{(1+r)}{1+g} (1-\omega^Y) - 1 \right) - \frac{(1+\tau^C)+p^I \Upsilon}{\mathcal{H}^Y} \frac{(1+r)(1-\omega^Y)}{1+g}} \quad (3.373)$$

As a next step we can express the steady-state formal and informal consumptions:

$$\frac{C^{\tilde{Y},F}}{\tilde{Y}} = \frac{\frac{\tilde{\mathcal{I}}^Y}{\tilde{Y}} + \frac{\frac{\tilde{\mathcal{I}}^{YO}}{\tilde{Y}}}{1+r} + \frac{(1+r)(1-\omega^Y)}{1+g} \frac{\tilde{B}_{t-1}^Y}{\tilde{Y}}}{\mathcal{H}^Y} \quad (3.374)$$

$$\frac{C^{\tilde{Y},I}}{\tilde{Y}} = \Upsilon \frac{C^{\tilde{Y},F}}{\tilde{Y}} \quad (3.375)$$

and, using the formal goods market clearing condition, the old households' formal and informal consumption functions are:

$$\frac{C^{\tilde{O},F}}{\tilde{Y}} = \frac{\tilde{C}^F}{\tilde{Y}} - \frac{C^{\tilde{Y},F}}{\tilde{Y}} \quad (3.376)$$

$$\frac{C^{\tilde{O},I}}{\tilde{Y}} = \Upsilon \frac{C^{\tilde{O},F}}{\tilde{Y}} \quad (3.377)$$

$$(3.378)$$

Finally, the retired savings to production look like:

$$\frac{\tilde{B}^O}{\tilde{Y}} = \frac{\mathcal{H}^O \frac{C^{\tilde{O},F}}{\tilde{Y}} - (\frac{TR^{PG}}{\tilde{Y}} + \frac{TR^{FF}}{\tilde{Y}}) \Omega^O - \frac{\tilde{T}^O}{\tilde{Y}} - \frac{(1+r)}{1+g} \omega^Y \frac{\tilde{B}^Y}{\tilde{Y}}}{\frac{(1+r)}{1+g}} \quad (3.379)$$

In the last step we calculate total production based on the unemployment equation:

$$\frac{\tilde{U}}{\tilde{Y}} = \frac{s^Y}{\tilde{Y}} - \frac{\tilde{L}^F}{\tilde{Y}} - \frac{\tilde{L}^I}{\tilde{Y}} \quad (3.380)$$

$$\tilde{Y} = \frac{s^Y}{\frac{\tilde{U}}{\tilde{Y}} + \frac{\tilde{L}^F}{\tilde{Y}} + \frac{\tilde{L}^I}{\tilde{Y}}} \quad (3.381)$$

Now, the levels of all variables can be lastly expressed. The five remaining equations (see below) are used to verify the initial guesses for  $r$ ,  $p^I$ ,  $\frac{\tilde{Y}^I}{\tilde{Y}}$ ,  $pr^{H,F}$  and  $pr^{H,I}$ . If the guesses are correct, the left- and right-hand sides of the remaining equations are equal. Otherwise the Newton-Raphson method chooses new initial values for these five variables; and, this process goes on until no new initial values must be chosen.

$$\begin{aligned} res_1 = & \frac{\sigma_F}{1 - \sigma_F} \tilde{h} \tilde{c}^F \frac{1 - \tau^{LW}}{1 + \tau^{SSCF}} - \left\{ (1 - \tau^{LW}) \tilde{w}^F - \tilde{w}^U + \right. \\ & + \frac{1 + g^A}{1 + r} \left[ (1 - pr^{F,F})(1 - pr^{H,F}) \left( \frac{\sigma_F}{1 - \sigma_F} \tilde{h} \tilde{c}^F \frac{1 - \tau^{LW}}{1 + \tau^{SSCF}} \right) \right] - \\ & \left. - \frac{1 + g^A}{1 + r} \left[ (1 - pr^{F,F}) pr^{H,I} \left( \frac{\sigma_I}{1 - \sigma_I} \tilde{h} \tilde{c}^I \right) \right] \right\} \end{aligned} \quad (3.382)$$

$$res_2 = pr^{H,I} - \left( \frac{\tilde{H}^I}{\frac{\tilde{U}}{1+g^N} + pr^{F,F} \frac{\tilde{L}^F}{1+g^N} + pr^{F,I} \frac{\tilde{L}^I}{1+g^N}} \right) \quad (3.383)$$

$$\begin{aligned} res_3 = & \frac{\sigma_I}{1 - \sigma_I} \tilde{h} \tilde{c}^I - \left\{ \tilde{w}^I - \tilde{w}^U + E \frac{1 + g^A}{1 + r} \left[ (1 - pr^{F,I})(1 - pr^{H,I}) \frac{\sigma_I}{1 - \sigma_I} \tilde{h} \tilde{c}^I \right] - \right. \\ & \left. - \frac{1 + g^A}{1 + r} \left[ pr^{H,F} (1 - pr^{F,I}) \frac{\sigma_F}{1 - \sigma_F} \tilde{h} \tilde{c}^F \frac{1 - \tau^{LW}}{1 + \tau^{SSCF}} \right] \right\} \end{aligned} \quad (3.384)$$

$$res_4 = \tilde{B} + Q^F \tilde{K}^F + Q^I \tilde{K}^I - (\tilde{B}^Y + \tilde{B}^O) \quad (3.385)$$

$$res_5 = \tilde{C}^I - C^{\tilde{Y},I} - C^{\tilde{O},I} \quad (3.386)$$



## E Robustness

As we discussed in the paper, three assumptions were made when calibrating the steady state of the model. First, the bargaining power of workers was to be assumed half in the informal than in the formal sector. Then, the level of informal firing cost was equal to twice the value of that of the official economy. Finally, we assumed that all profits minus lump-sum taxes went to the young.

Table 3.A1 shows the long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal in the baseline model and with lower and higher levels of informal bargaining powers of workers (0.25 and  $\frac{1}{3}$  respectively). Then, Table 3.A2 shows similar effects when modifying the informal firing probability by  $\pm 2.5$  pp. Lastly, Table 3.A3 shows the long-run reactions when only half of the profits and lump-sum taxes go to the young, and, the other half goes to the old. We can conclude that the results are highly robust with respect to the assumptions we made.



| Long-run effects of a 5 %points-increase in the old-age dependency ratio in PAYG regime |  |                                   |                                    |  |                                   |                                    |                                      |                                   |                                    |   |                                   |                                    |
|---|--|-----------------------------------|------------------------------------|--|-----------------------------------|------------------------------------|--------------------------------------|-----------------------------------|------------------------------------|---|-----------------------------------|------------------------------------|
| If the government consolidates with   | GDP per capita (%)                         |                                   |                                    | Share of formal GDP in total GDP (%point)                          |                                   |                                    | Unemployment rate (%point)           |                                   |                                    | Share of informal employment in total employment (%point) |                                   |                                    |
|   | Baseline                                   | Lower informal firing probability | Higher informal firing probability | Baseline   | Lower informal firing probability | Higher informal firing probability | Baseline                             | Lower informal firing probability | Higher informal firing probability | Baseline  | Lower informal firing probability | Higher informal firing probability |
| No consolidation  | -3.81                                      | -3.80                             | -3.82                              | 0.39   | 0.39                              | 0.38                               | 0.11                                 | 0.11                              | 0.11                               | -0.33   | -0.34                             | -0.33                              |
| Personal income tax and employee SSC  | -5.90                                      | -5.91                             | -5.90                              | 0.59   | 0.60                              | 0.57                               | 0.57                                 | 0.51                              | 0.62                               | 0.35  | 0.38                              | 0.32                               |
| Employer SSC  | -5.32                                      | -5.33                             | -5.32                              | 0.57   | 0.58                              | 0.56                               | 0.43                                 | 0.39                              | 0.47                               | 0.15  | 0.17                              | 0.13                               |
| Pension-wage replacement rate   | -3.63                                      | -3.61                             | -3.64                              | 0.48   | 0.49                              | 0.48                               | 0.07                                 | 0.07                              | 0.07                               | -0.43   | -0.44                             | -0.42                              |
| Retirement probability  | -3.02                                      | -3.01                             | -3.03                              | 0.42   | 0.42                              | 0.42                               | 0.08                                 | 0.08                              | 0.08                               | -0.36   | -0.37                             | -0.36                              |
| Value added tax   | -3.65                                      | -3.65                             | -3.64                              | -0.26  | -0.25                             | -0.26                              | -0.12                                | -0.12                             | -0.12                              | 0.21  | 0.21                              | 0.21                               |
| Government consumption exp./GDP   | -3.65                                      | -3.65                             | -3.64                              | -0.26  | -0.25                             | -0.26                              | -0.12                                | -0.12                             | -0.12                              | 0.21  | 0.21                              | 0.21                               |
| Fertility rate  | 0.04                                       | 0.05                              | 0.04                               | 0.06   | 0.06                              | 0.06                               | 0.00                                 | 0.01                              | 0.00                               | -0.05   | -0.05                             | -0.04                              |
| Total household consumption per capita (%)  |  |                                   |                                    |  |                                   |                                    |                                      |                                   |                                    |   |                                   |                                    |
| If the government consolidates with   | Total household consumption per capita (%) |                                   |                                    | Share of young household consumption in total consumption (%point) |                                   |                                    | Gov. debt as a share of GDP (%point) |                                   |                                    | Instrument (%point)                                       |                                   |                                    |
|   | Baseline                                   | Lower informal firing probability | Higher informal firing probability | Baseline   | Lower informal firing probability | Higher informal firing probability | Baseline                             | Lower informal firing probability | Higher informal firing probability | Baseline  | Lower informal firing probability | Higher informal firing probability |
| No consolidation  | -4.87                                      | -4.87                             | -4.87                              | -2.78  | -2.78                             | -2.78                              | 111.12                               | 110.77                            | 111.44                             | 0.00  | 0.00                              | 0.00                               |
| Personal income tax and employee SSC  | -7.89                                      | -7.93                             | -7.86                              | -1.41  | -1.42                             | -1.41                              | 1.26                                 | 1.26                              | 1.25                               | 5.17  | 5.16                              | 5.18                               |
| Employer SSC  | -7.16                                      | -7.19                             | -7.14                              | -0.91  | -0.91                             | -0.90                              | 1.04                                 | 1.05                              | 1.04                               | 6.07  | 6.06                              | 6.09                               |
| Pension-wage replacement rate   | -4.91                                      | -4.91                             | -4.92                              | -0.94  | -0.94                             | -0.94                              | 0.33                                 | 0.34                              | 0.33                               | -2.71   | -2.70                             | -2.71                              |
| Retirement probability  | -4.13                                      | -4.13                             | -4.14                              | -0.81  | -0.81                             | -0.81                              | -0.24                                | -0.24                             | -0.25                              | -0.04   | -0.04                             | -0.04                              |
| Value added tax   | -4.98                                      | -5.00                             | -4.97                              | -0.86  | -0.86                             | -0.86                              | 0.52                                 | 0.52                              | 0.52                               | 5.64  | 5.64                              | 5.64                               |
| Government consumption exp./GDP   | -2.94                                      | -2.95                             | -2.92                              | -0.86  | -0.86                             | -0.86                              | 0.39                                 | 0.39                              | 0.39                               | -0.57   | -0.56                             | -0.58                              |
| Fertility rate  | -0.08                                      | -0.08                             | -0.09                              | 1.11   | 1.11                              | 1.12                               | -3.85                                | -3.84                             | -3.86                              | -0.24   | -0.24                             | -0.24                              |

Table 3.A2: Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal, comparing the baseline scenario with lower and higher informal firing probabilities

| Long-run effects of a 5 %points-increase in the old-age dependency ratio in PAYG regime |  |   |  |   |                                      |   |
|---|--|---|--|---|--------------------------------------|---|
| If the government consolidates with   | GDP per capita (%)                         |   | Share of formal GDP in total GDP (%point)                          |   | Unemployment rate (%point)           |   |
|   | Baseline                                   | Lower fraction of profits and lump-sum taxes to the young | Baseline   | Lower fraction of profits and lump-sum taxes to the young | Baseline                             | Lower fraction of profits and lump-sum taxes to the young |
| No consolidation  | -3.81                                      | -3.82   | 0.39   | 0.38  | 0.11                                 | -0.33   |
| Personal income tax and employee SSC  | -5.90                                      | -5.90   | 0.59   | 0.59  | 0.57                                 | 0.35  |
| Employer SSC  | -5.32                                      | -5.31   | 0.57   | 0.58  | 0.43                                 | 0.15  |
| Pension-wage replacement rate   | -3.63                                      | -3.56   | 0.48   | 0.51  | 0.07                                 | 0.13  |
| Retirement probability  | -3.02                                      | -2.98   | 0.42   | 0.45  | 0.08                                 | -0.43   |
| Value added tax   | -3.65                                      | -3.62   | -0.26  | -0.23   | -0.12                                | -0.36   |
| Government consumption exp./GDP   | -3.65                                      | -3.62   | -0.26  | -0.23   | -0.12                                | 0.21  |
| Fertility rate  | 0.04                                       | 0.12  | 0.06   | 0.08  | -0.12                                | 0.18  |
|   |  |   |  |   | 0.00                                 | 0.21  |
|   |  |   |  |   |                                      | -0.05   |
|   |  |   |  |   |                                      | -0.07   |
| If the government consolidates with   | Total household consumption per capita (%) |   | Share of young household consumption in total consumption (%point) |   | Gov. debt as a share of GDP (%point) |   |
|   | Baseline                                   | Lower fraction of profits and lump-sum taxes to the young | Baseline   | Lower fraction of profits and lump-sum taxes to the young | Baseline                             | Lower fraction of profits and lump-sum taxes to the young |
| No consolidation  | -4.87                                      | -4.86   | -2.78  | -2.89   | 111.12                               | 0.00  |
| Personal income tax and employee SSC  | -7.89                                      | -7.89   | -1.41  | -1.44   | 1.26                                 | 5.17  |
| Employer SSC  | -7.16                                      | -7.15   | -0.91  | -0.94   | 1.04                                 | 5.14  |
| Pension-wage replacement rate   | -4.91                                      | -4.91   | -0.94  | -0.11   | 0.33                                 | 6.03  |
| Retirement probability  | -4.13                                      | -4.16   | -0.81  | -0.04   | -0.24                                | -2.71   |
| Value added tax   | -4.98                                      | -4.97   | -0.86  | -0.66   | 0.52                                 | -0.04   |
| Government consumption exp./GDP   | -2.94                                      | -2.99   | -0.86  | -0.66   | 0.39                                 | 5.64  |
| Fertility rate  | -0.08                                      | -0.07   | 1.11   | 0.86  | -3.85                                | -0.57   |
|   |  |   |  |   |                                      | -0.24   |
|   |  |   |  |   |                                      | -0.24   |

Table 3.A3: Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal, comparing the baseline scenario with a lower fraction of profits and lump-sum taxes to the young

## F Additional results on Portugal

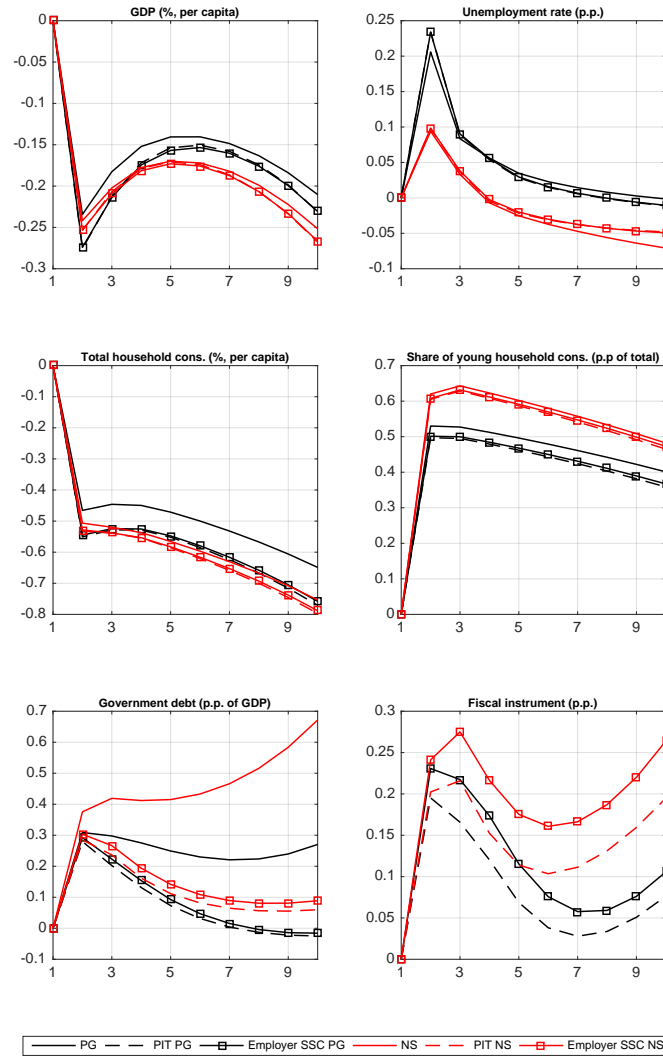


Figure 3.A1: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy in Portugal

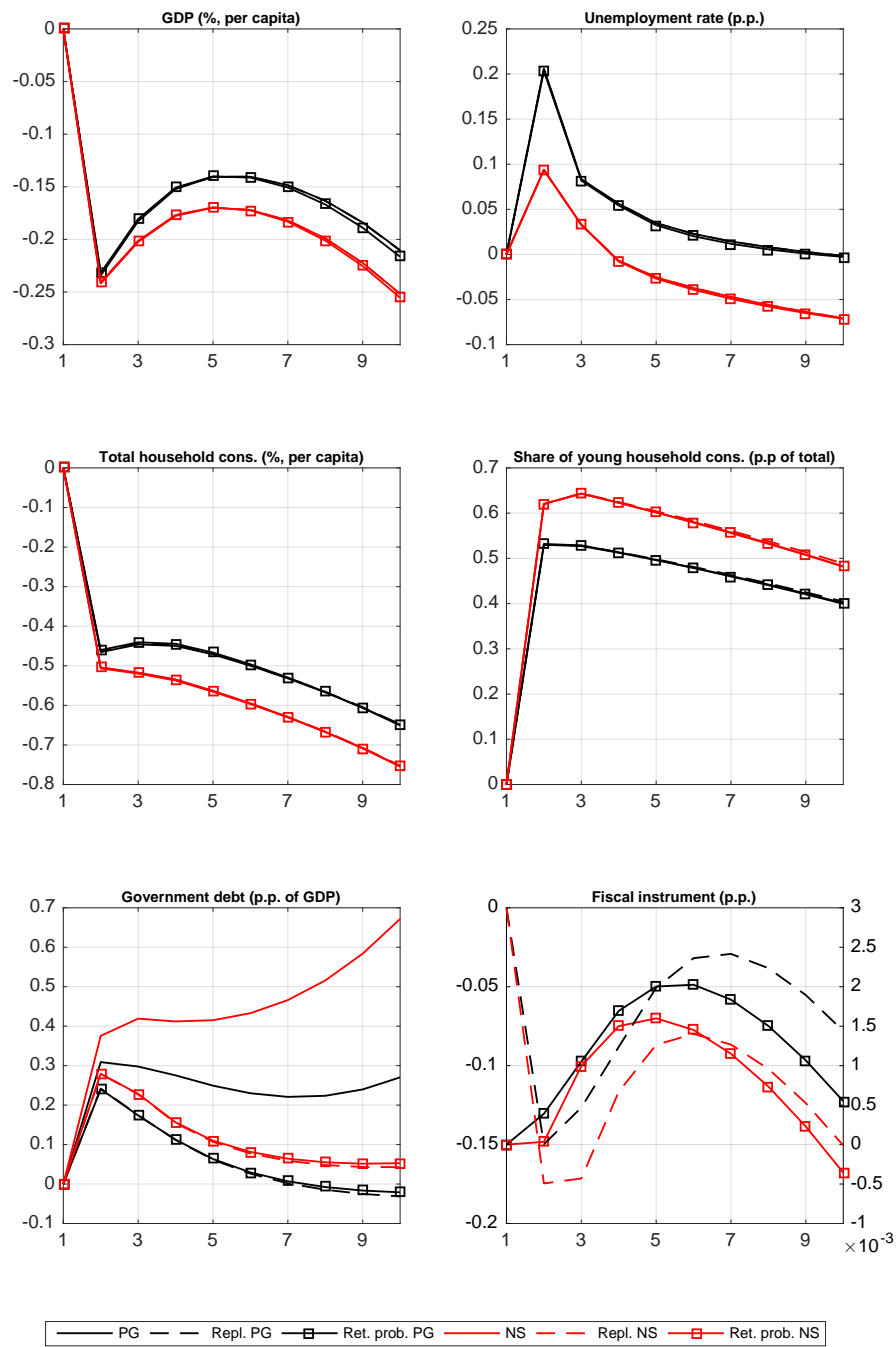


Figure 3.A2: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy in Portugal

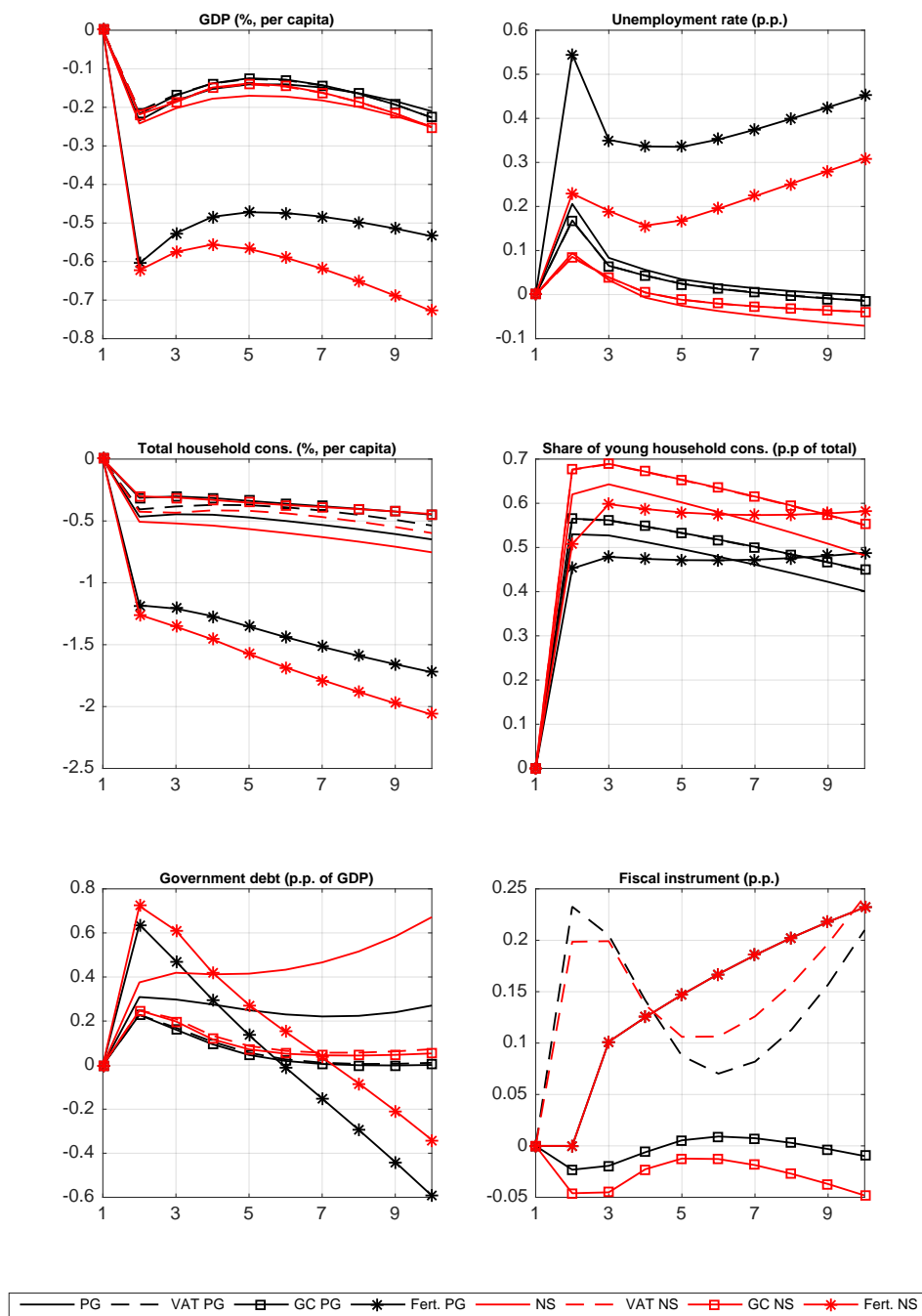


Figure 3.A3: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy in Portugal

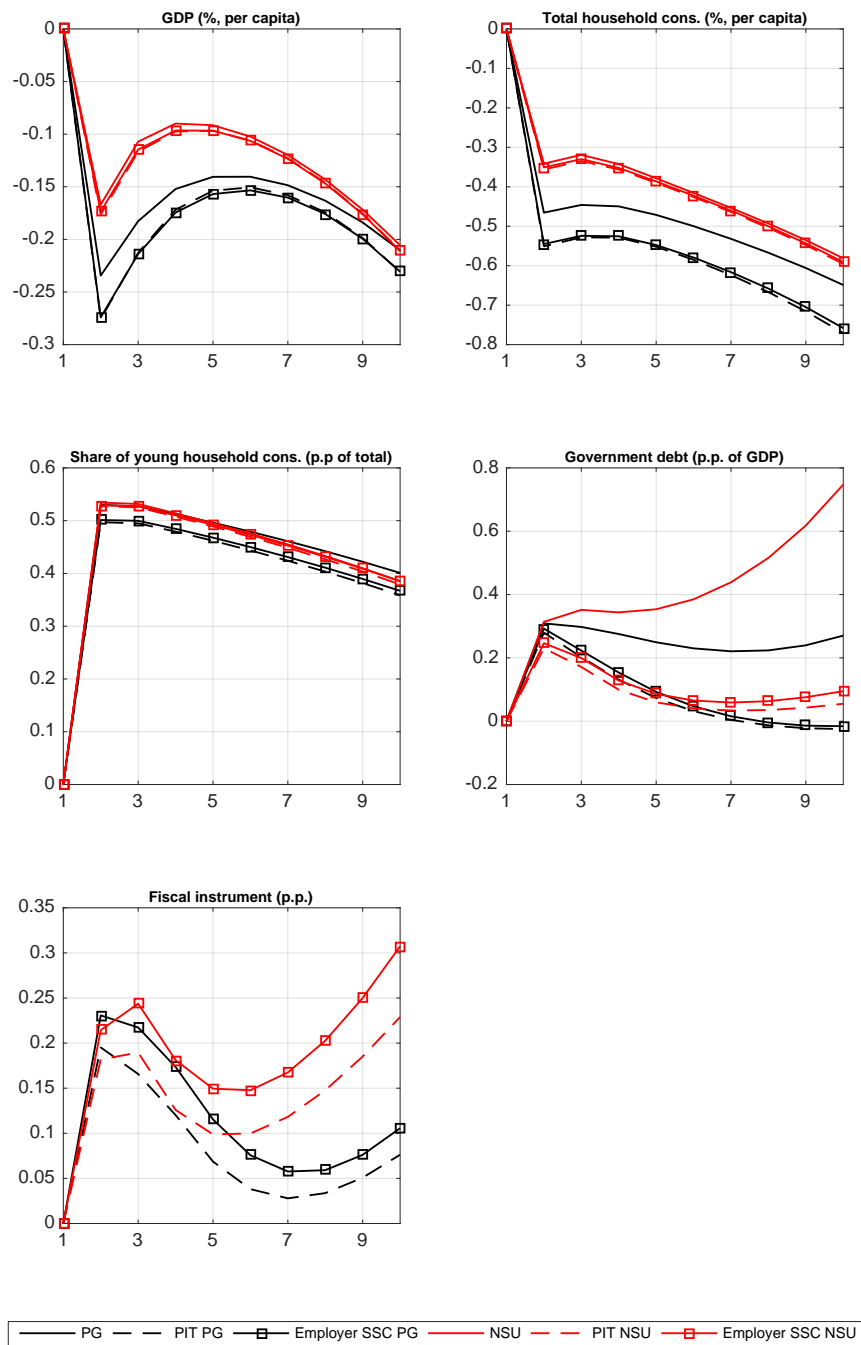


Figure 3.A4: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy and unemployment in Portugal



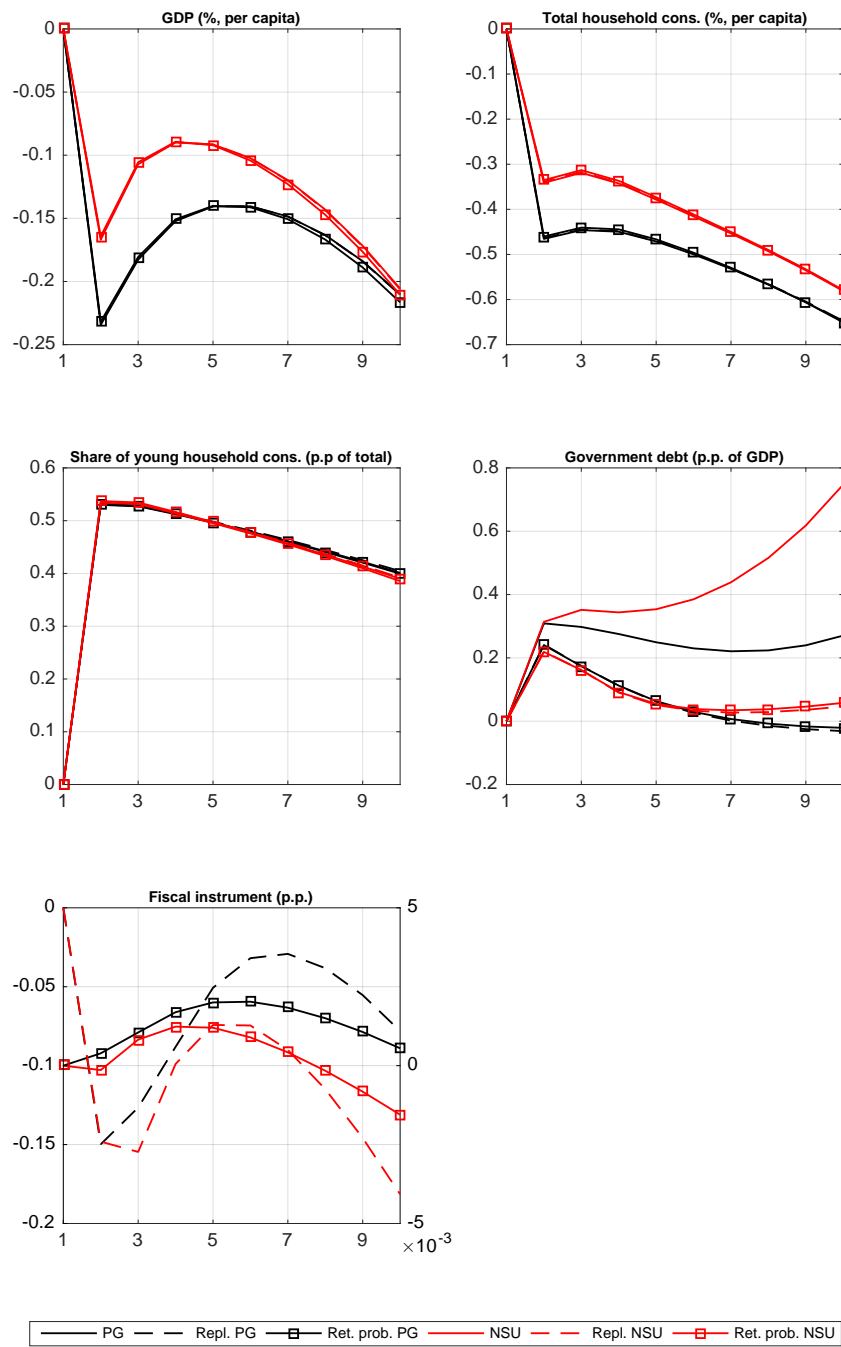


Figure 3.A5: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy and unemployment in Portugal

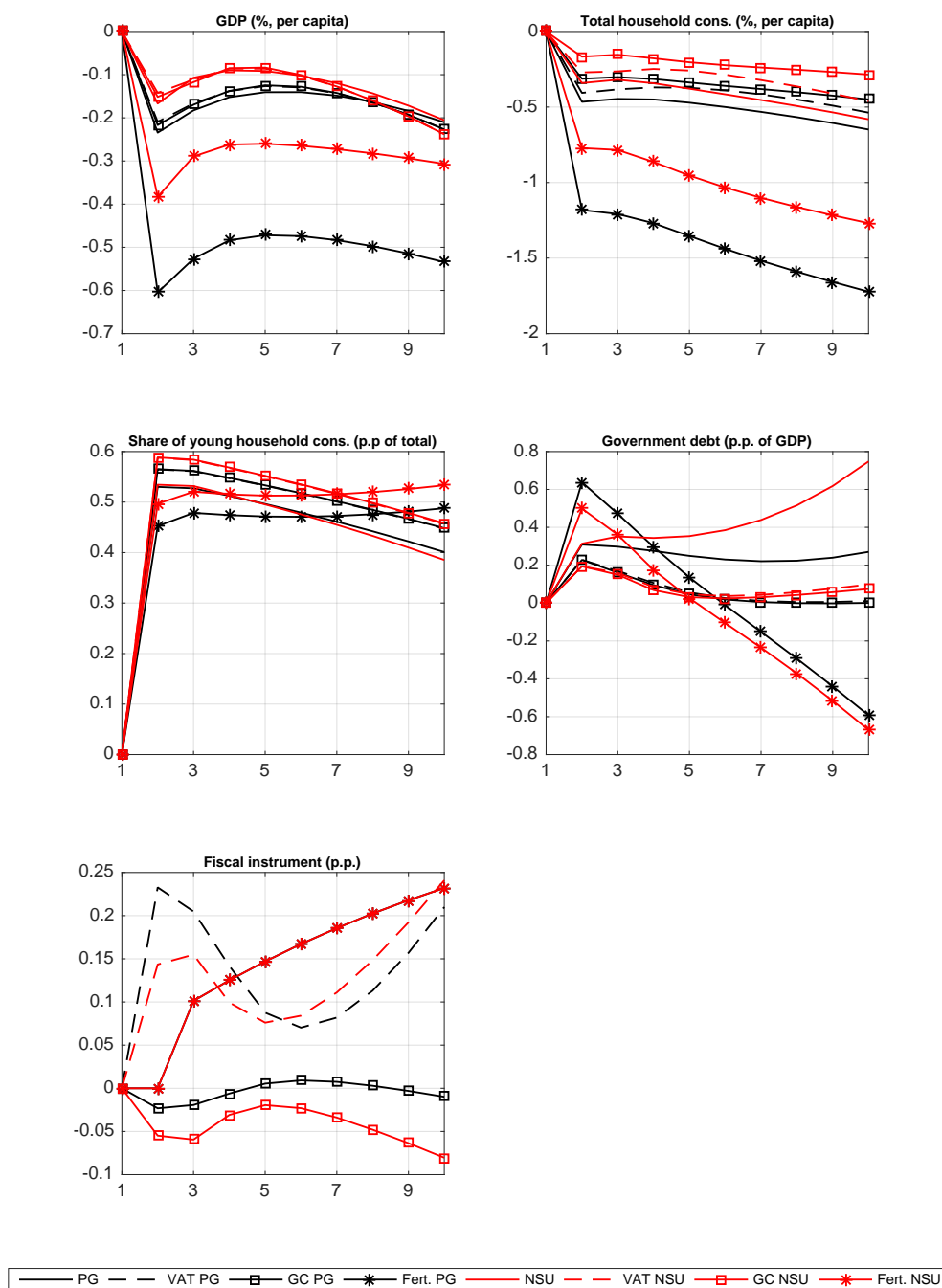


Figure 3.A6: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy and unemployment in Portugal

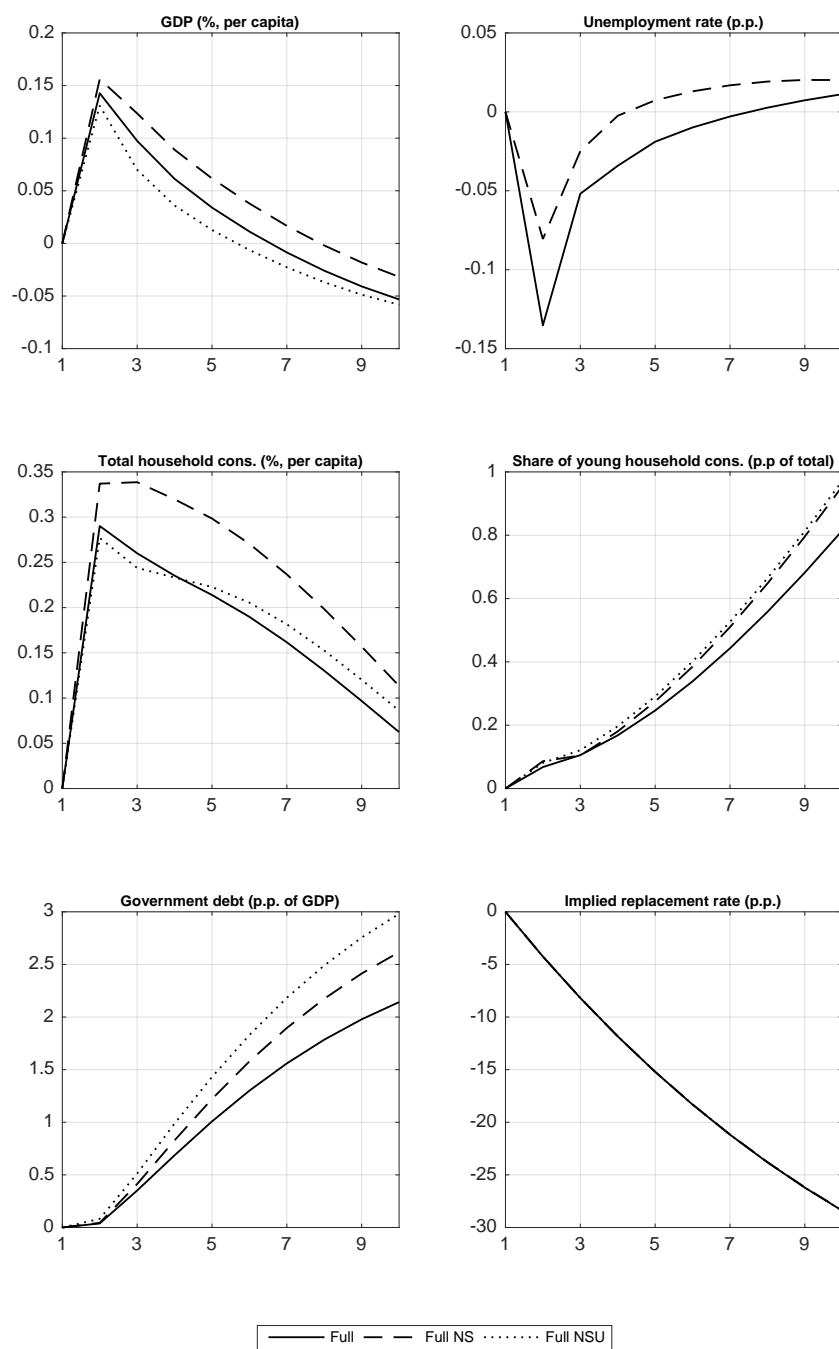


Figure 3.A7: Short-run (10-year) effects of a full pension reform in Portugal, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment

## G Additional results on Italy and Spain

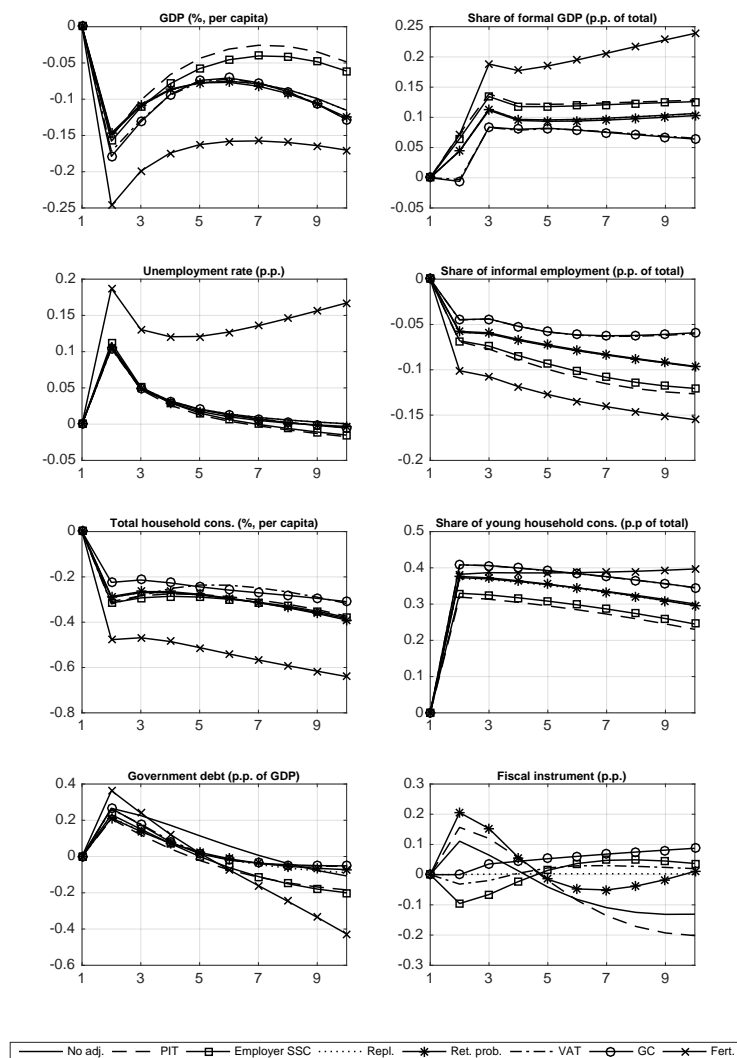


Figure 3.A8: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Italy

| <b>Long-run effects of a 5 pp increase in the old-age dependency ratio</b> |  |      |  |      |
|--|--|------|--|------|
| If the government consolidates with  | GDP per capita (%)                         |      | Share of formal GDP in total GDP (%point)                          |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | -3.0                                       | -2.9 | 0.3  | 0.3  |
| Personal income tax and employee SSC                                       | -5.6                                       | -3.8 | 0.4  | 0.4  |
| Employer SSC   | -4.4                                       | -    | 0.4  | -    |
| Pension-wage replacement rate  | -2.9                                       | -    | 0.3  | -    |
| Retirement probability   | -2.3                                       | -    | 0.3  | -    |
| Value added tax  | -3.1                                       | -3.0 | -0.2   | 0.1  |
| Government consumption exp./GDP  | -3.1                                       | -3.0 | -0.2   | 0.1  |
| Fertility rate   | 0.0  | -0.2 | 0.0  | 0.0  |
| If the government consolidates with  | Total household consumption per capita (%) |      | Share of young household consumption in total consumption (%point) |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | -3.8                                       | -3.9 | -2.1   | -0.4 |
| Personal income tax and employee SSC                                       | -7.4                                       | -5.1 | -1.3   | -0.1 |
| Employer SSC   | -5.9                                       | -    | -0.5   | -    |
| Pension-wage replacement rate  | -3.9                                       | -    | -0.7   | -    |
| Retirement probability   | -3.1                                       | -    | -0.6   | -    |
| Value added tax  | -4.2                                       | -4.0 | -0.5   | 0.0  |
| Government consumption exp./GDP  | -2.4                                       | -3.3 | -0.5   | 0.0  |
| Fertility rate   | -0.1                                       | -0.1 | 0.9  | 0.4  |
| If the government consolidates with  | Unemployment rate (%point)                 |      | Share of informal employment in total employment (%point)          |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | 0.1  | 0.1  | -0.3   | -0.3 |
| Personal income tax and employee SSC                                       | 0.5  | 0.2  | 0.7  | 0.0  |
| Employer SSC   | 0.3  | -    | 0.2  | -    |
| Pension-wage replacement rate  | 0.0  | -    | -0.3   | -    |
| Retirement probability   | 0.1  | -    | -0.3   | -    |
| Value added tax  | -0.1                                       | 0.0  | 0.2  | -0.1 |
| Government consumption exp./GDP  | -0.1                                       | 0.0  | 0.2  | -0.1 |
| Fertility rate   | 0.0  | 0.1  | 0.0  | 0.0  |
| If the government consolidates with  | Gov. debt as a share of GDP (%point)       |      | Instrument (%point)  |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | 83.1                                       | 25.0 | 0.0  | 0.0  |
| Personal income tax and employee SSC                                       | 2.1  | 0.7  | 4.9  | 1.7  |
| Employer SSC   | 1.3  | -    | 5.7  | -    |
| Pension-wage replacement rate  | 0.3  | -    | -1.7   | -    |
| Retirement probability   | -0.4                                       | -    | -0.03  | -    |
| Value added tax  | 0.6  | 0.2  | 4.0  | 1.6  |
| Government consumption exp./GDP  | 0.5  | 0.2  | -0.5   | 0.1  |
| Fertility rate   | -3.9                                       | -3.7 | -0.1   | -0.1 |

Table 3.A4: Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in Italy, comparing PAYG and fully-funded regimes

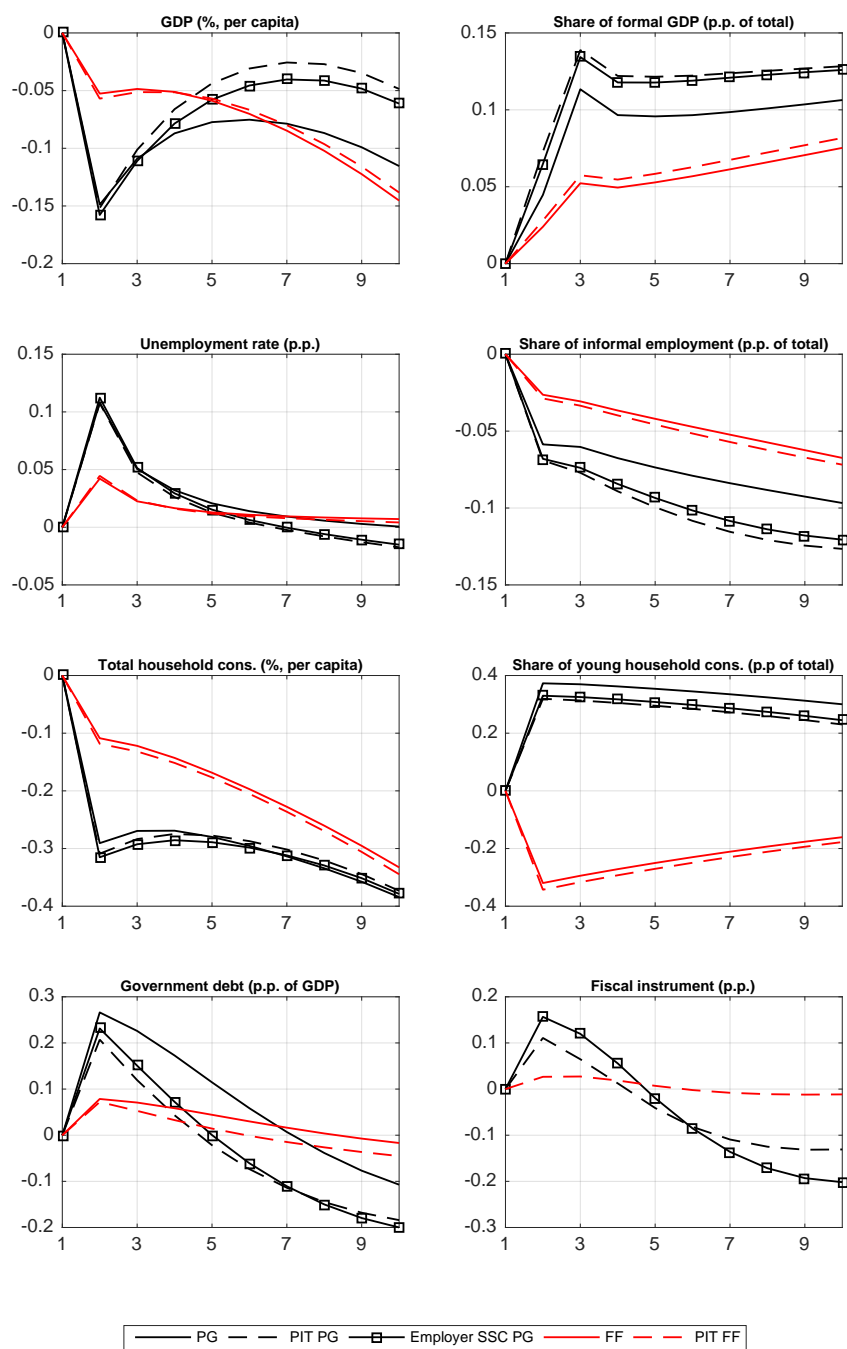


Figure 3.A9: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by modifying labor income taxes in Italy, comparing PAYG and fully-funded regimes

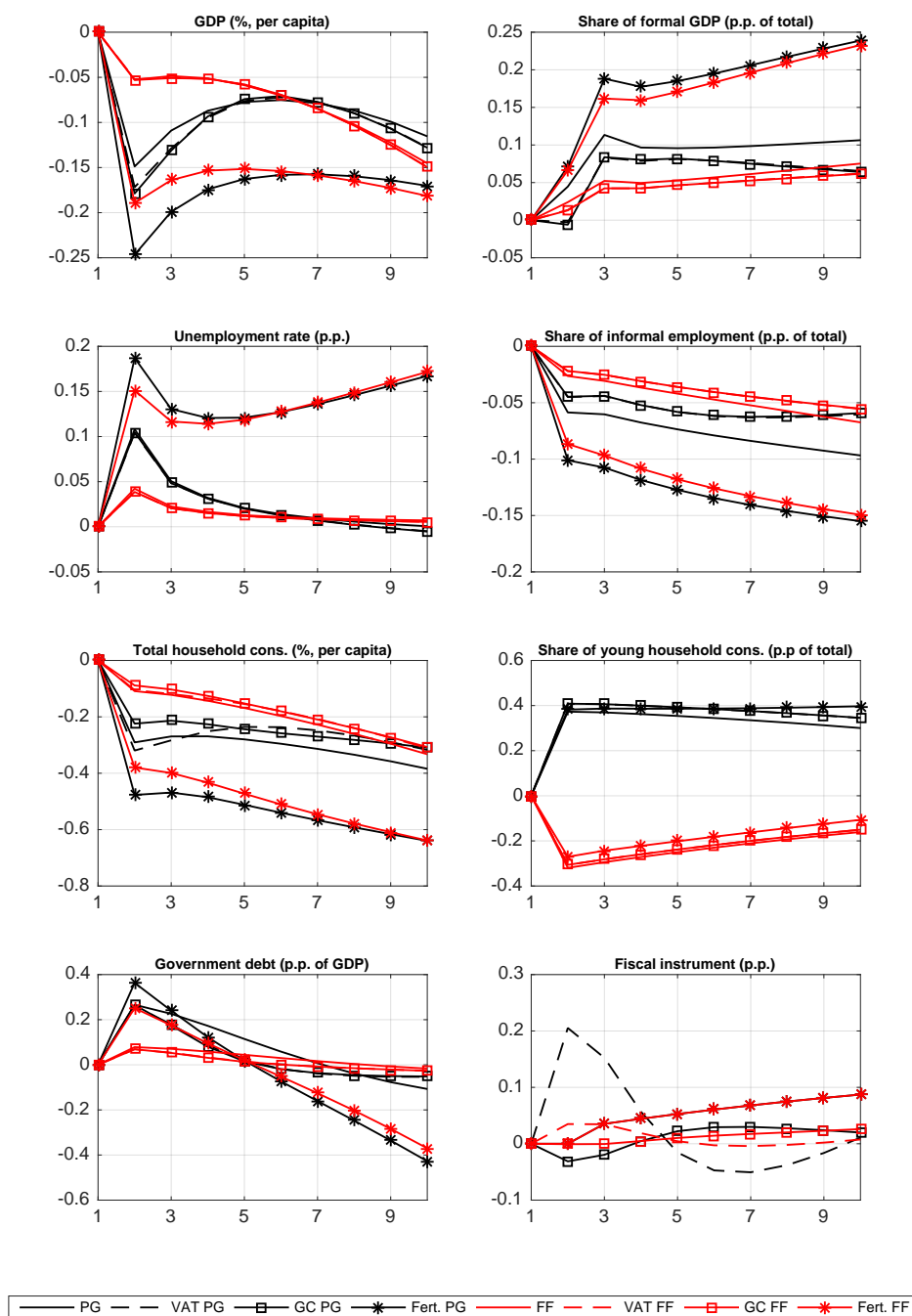


Figure 3.A10: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in Italy, comparing PAYG and fully-funded regimes

| Long-run effects of a 5 pp increase in the old-age dependency ratio in PAYG regime |  |                   |                                    |                                      |                   |                                    |  |                   |                                    |
|--|--|-------------------|------------------------------------|--------------------------------------|-------------------|------------------------------------|--|-------------------|------------------------------------|
| If the government consolidates with  | GDP per capita (%)   |                   |                                    | Unemployment rate (%point)           |                   |                                    | Total household consumption per capita (%) |                   |                                    |
|  | Baseline   | No shadow economy | No shadow economy, no unemployment | Baseline                             | No shadow economy | No shadow economy, no unemployment | Baseline                                   | No shadow economy | No shadow economy, no unemployment |
|  |  |                   |                                    |                                      |                   |                                    |  |                   |                                    |
| No consolidation   | -3.0   | -3.2              | -3.2                               | 0.1                                  | 0.0               | -                                  | -3.8                                       | -3.7              | -3.6                               |
| Personal income tax and employee SSC   | -5.6   | -3.6              | -3.2                               | 0.5                                  | 0.4               | -                                  | -7.4                                       | -4.6              | -4.0                               |
| Employer SSC   | -4.4   | -3.5              | -3.2                               | 0.3                                  | 0.2               | -                                  | -5.9                                       | -4.4              | -3.9                               |
| Pension-wage replacement rate  | -2.9   | -3.1              | -3.0                               | 0.0                                  | 0.0               | -                                  | -3.9                                       | -3.9              | -3.7                               |
| Retirement probability   | -2.3   | -2.4              | -2.4                               | 0.1                                  | 0.0               | -                                  | -3.1                                       | -3.1              | -2.9                               |
| Value added tax  | -3.1   | -2.9              | -2.9                               | -0.1                                 | 0.0               | -                                  | -4.2                                       | -3.9              | -3.7                               |
| Government consumption exp./GDP  | -3.1   | -2.9              | -2.9                               | -0.1                                 | 0.0               | -                                  | -2.4                                       | -1.8              | -1.5                               |
| Fertility rate   | 0.0  | 0.0               | 0.0                                | 0.0                                  | 0.0               | -                                  | -0.1                                       | -0.2              | -0.1                               |
| If the government consolidates with  | Share of young household consumption in total consumption (%point) |                   |                                    | Gov. debt as a share of GDP (%point) |                   |                                    | Instrument (%point)                        |                   |                                    |
|  | Baseline   | No shadow economy | No shadow economy, no unemployment | Baseline                             | No shadow economy | No shadow economy, no unemployment | Baseline                                   | No shadow economy | No shadow economy, no unemployment |
|  |  |                   |                                    |                                      |                   |                                    |  |                   |                                    |
| No consolidation   | -2.1   | -2.7              | -2.7                               | 83.1                                 | 118.8             | 138.0                              | 0.0  | 0.0               | 0.0                                |
| Personal income tax and employee SSC   | -1.3   | -1.1              | -1.1                               | 2.1                                  | 1.0               | 0.9                                | 4.9  | 4.6               | 3.8                                |
| Employer SSC   | -0.5   | -0.5              | -0.4                               | 1.3                                  | 1.0               | 1.0                                | 5.7  | 5.3               | 5.3                                |
| Pension-wage replacement rate  | -0.7   | -0.7              | -0.7                               | 0.3                                  | 0.5               | 0.6                                | -1.7                                       | -1.9              | -2.0                               |
| Retirement probability   | -0.6   | -0.6              | -0.6                               | -0.4                                 | -0.3              | -0.3                               | -0.03                                      | -0.03             | -0.04                              |
| Value added tax  | -0.5   | -0.6              | -0.6                               | 0.6                                  | 0.6               | 0.7                                | 4.0  | 3.1               | 2.9                                |
| Government consumption exp./GDP  | -0.5   | -0.6              | -0.6                               | 0.5                                  | 0.5               | 0.5                                | -0.5                                       | -0.8              | -1.1                               |
| Fertility rate   | 0.9  | 1.0               | 0.9                                | -3.9                                 | -4.0              | -3.9                               | -0.1                                       | -0.1              | -0.1                               |

Table 3.A5: Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Italy, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment



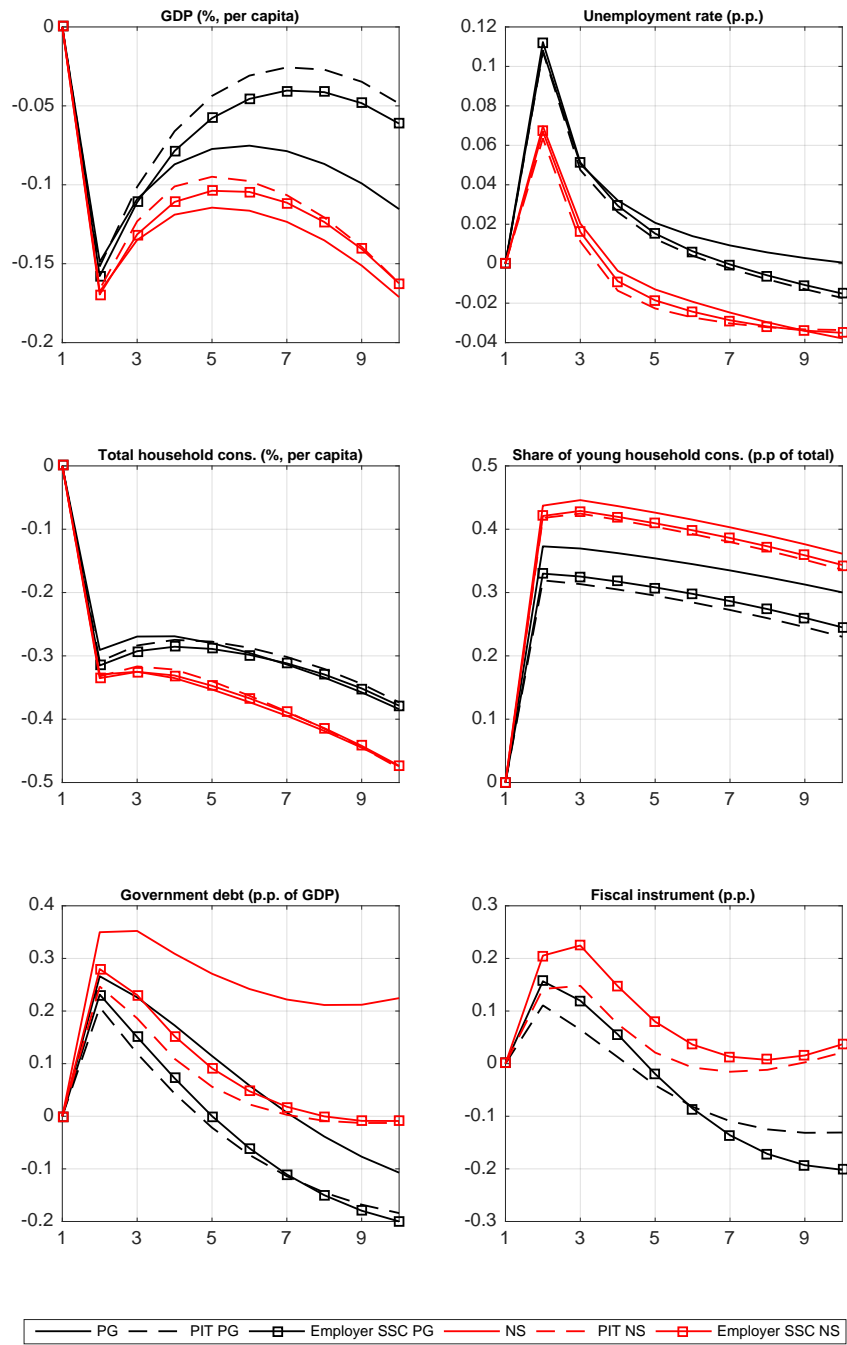


Figure 3.A11: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy in Italy

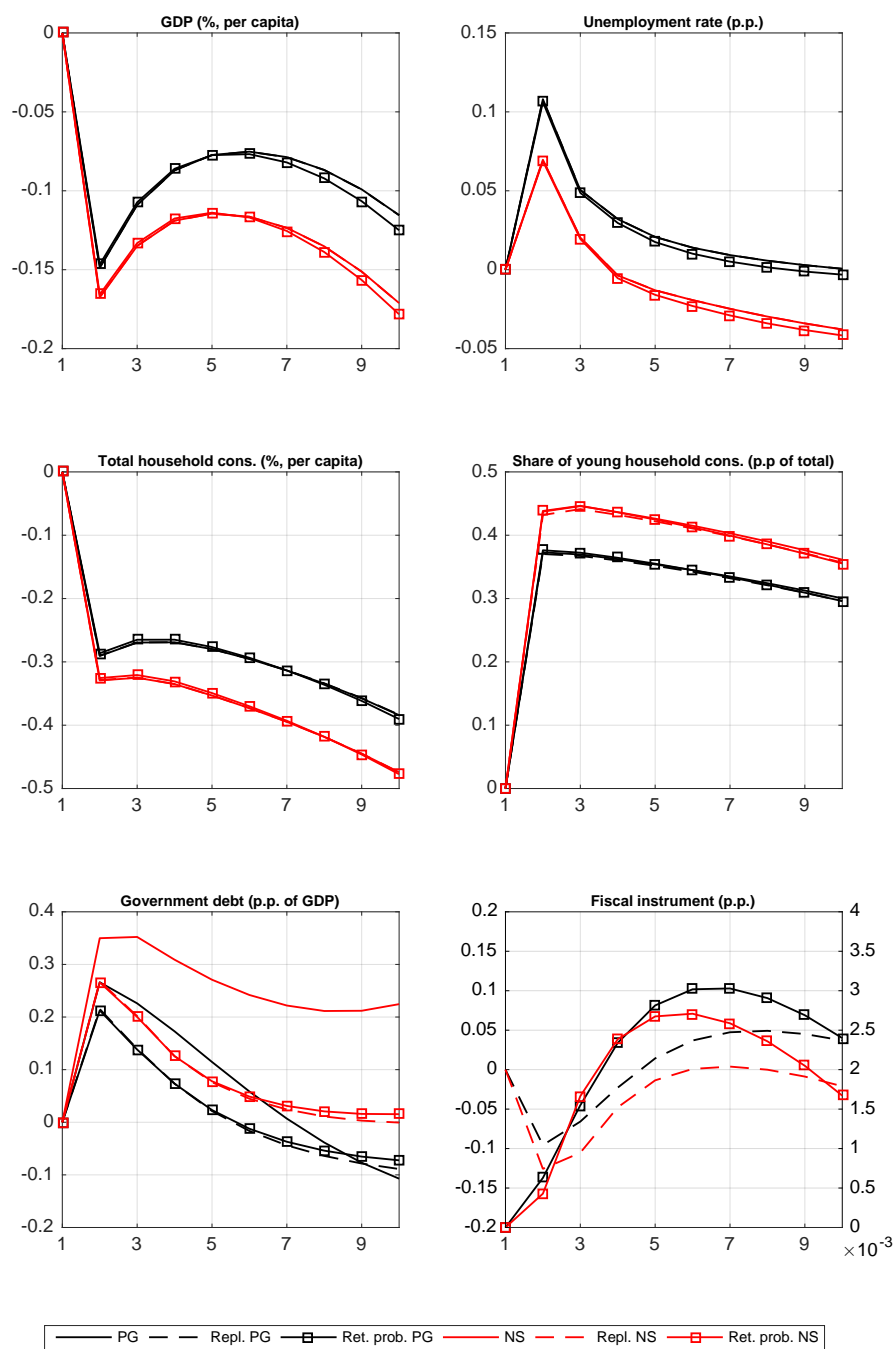


Figure 3.A12: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy in Italy

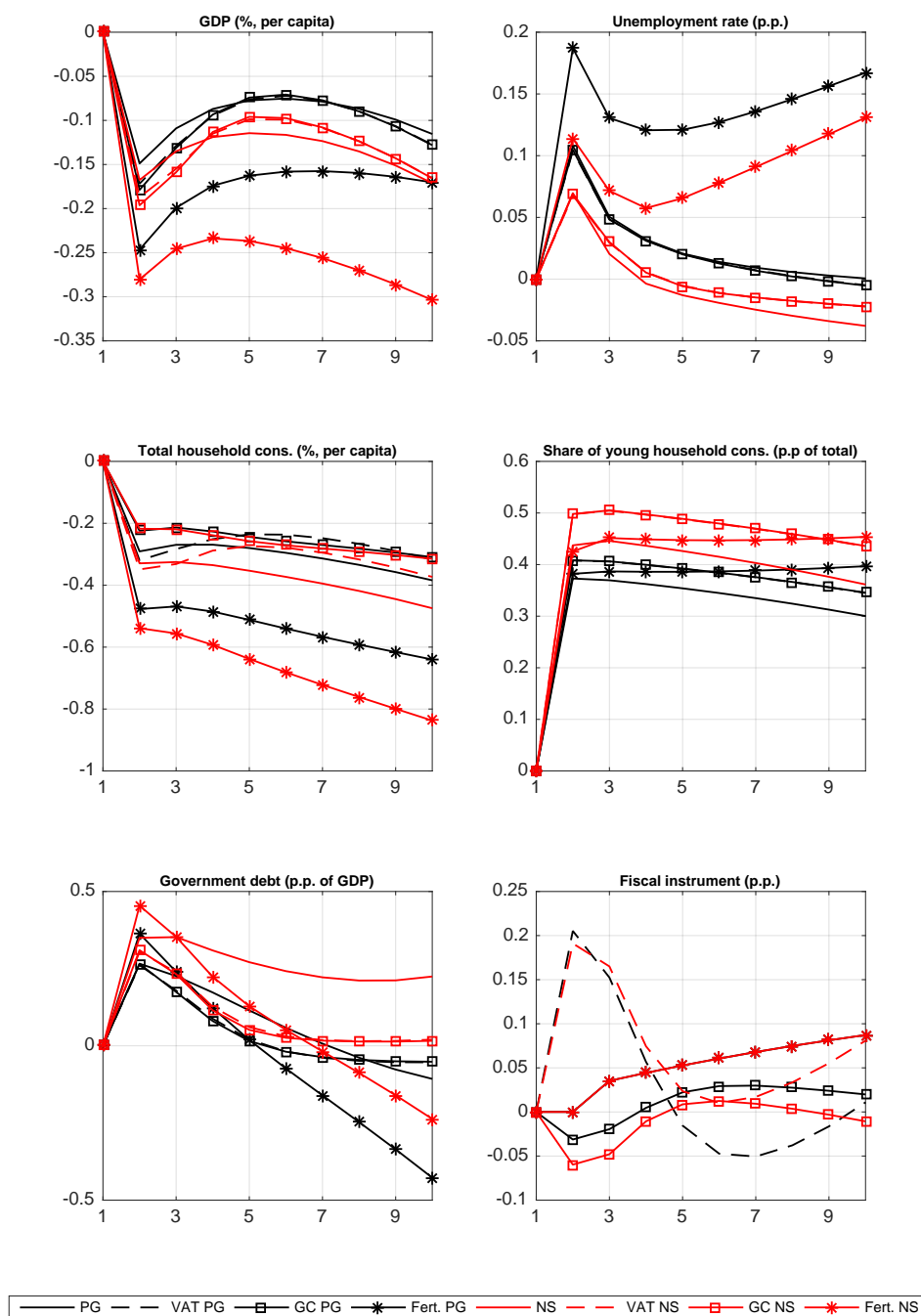


Figure 3.A13: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy in Italy

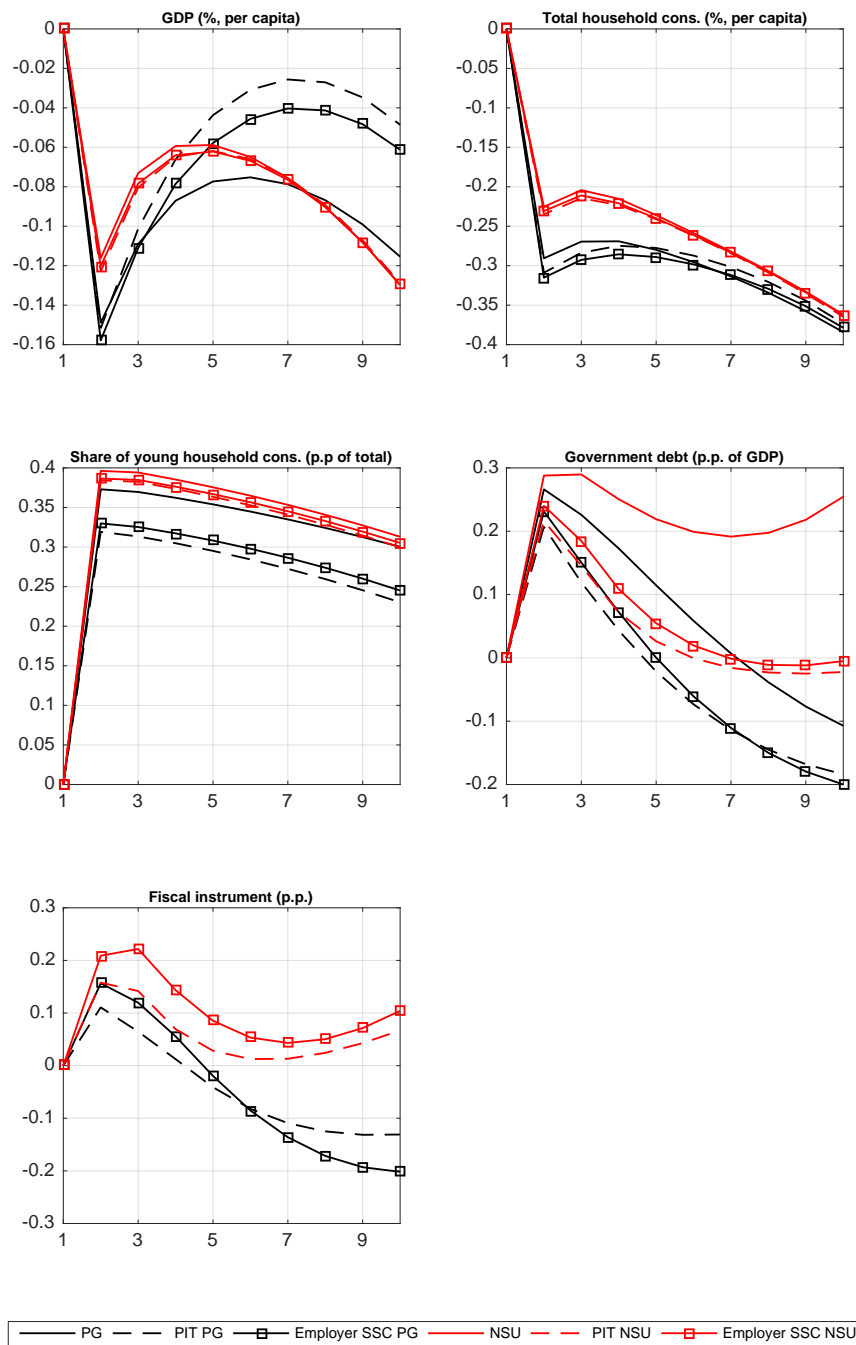


Figure 3.A14: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy and unemployment in Italy

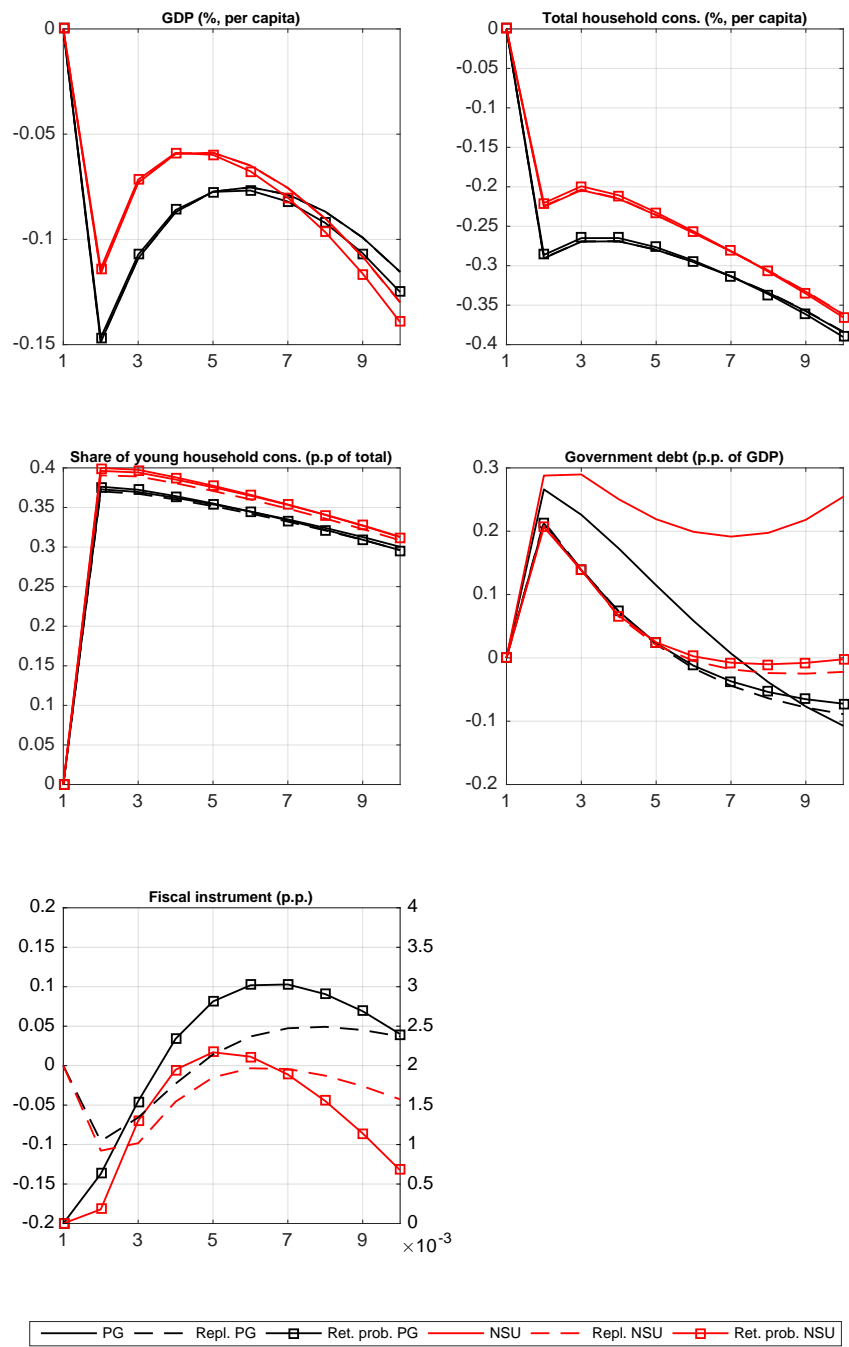


Figure 3.A15: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy and unemployment in Italy

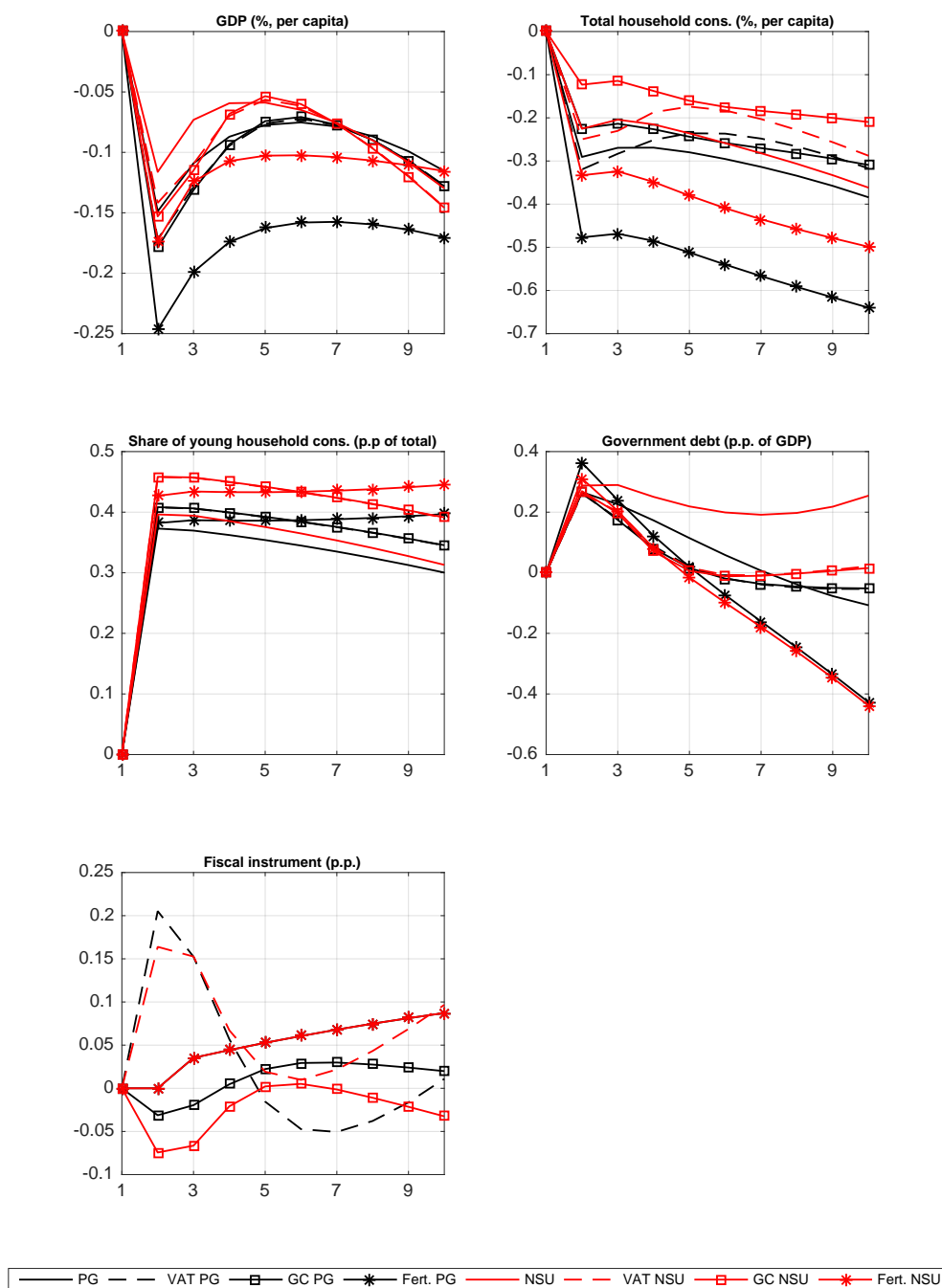


Figure 3.A16: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy and unemployment in Italy

| <b>Long-run effects of a switch from PAYG to fully-funded</b>      |             |                |
|--|-------------|----------------|
|  | Full reform | Partial reform |
| GDP per capita (%)   | 1.1         | 0.6            |
| Share of formal GDP in total GDP (%point)                          | 0.6         | 0.3            |
| Unemployment rate (%point)   | -0.2        | -0.1           |
| Share of informal employment in total employment (%point)          | -0.6        | -0.3           |
| Total household consumption per capita (%)                         | -0.2        | -0.1           |
| Share of young household consumption in total consumption (%point) | 13.0        | 6.4            |
| Gov. debt as a share of GDP (%point)                               | -1.4        | -0.8           |
| Implied replacement rate (%point)                                  | -7.9        | 1.2            |

Table 3.A6: Long-run (100-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Italy

| <b>Long-run effects of a full switch from PAYG to fully-funded</b> |          |                   |                                    |
|--|----------|-------------------|------------------------------------|
|  | Baseline | No shadow economy | No shadow economy, no unemployment |
| GDP per capita (%)   | 1.1      | 0.7               | 0.9                                |
| Share of formal GDP in total GDP (%point)                          | 0.6      | -                 | -                                  |
| Unemployment rate (%point)   | -0.2     | -0.5              | -                                  |
| Share of informal employment in total employment (%point)          | -0.6     | -                 | -                                  |
| Total household consumption per capita (%)                         | -0.2     | -1.6              | -0.9                               |
| Share of young household consumption in total consumption (%point) | 13.0     | 15.2              | 15.4                               |
| Gov. debt as a share of GDP (%point)                               | -1.4     | -1.2              | -1.4                               |
| Implied replacement rate (%point)                                  | -7.9     | -8.0              | -9.7                               |

Table 3.A7: Long-run (100-year) effects of a full pension reform in Italy, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment

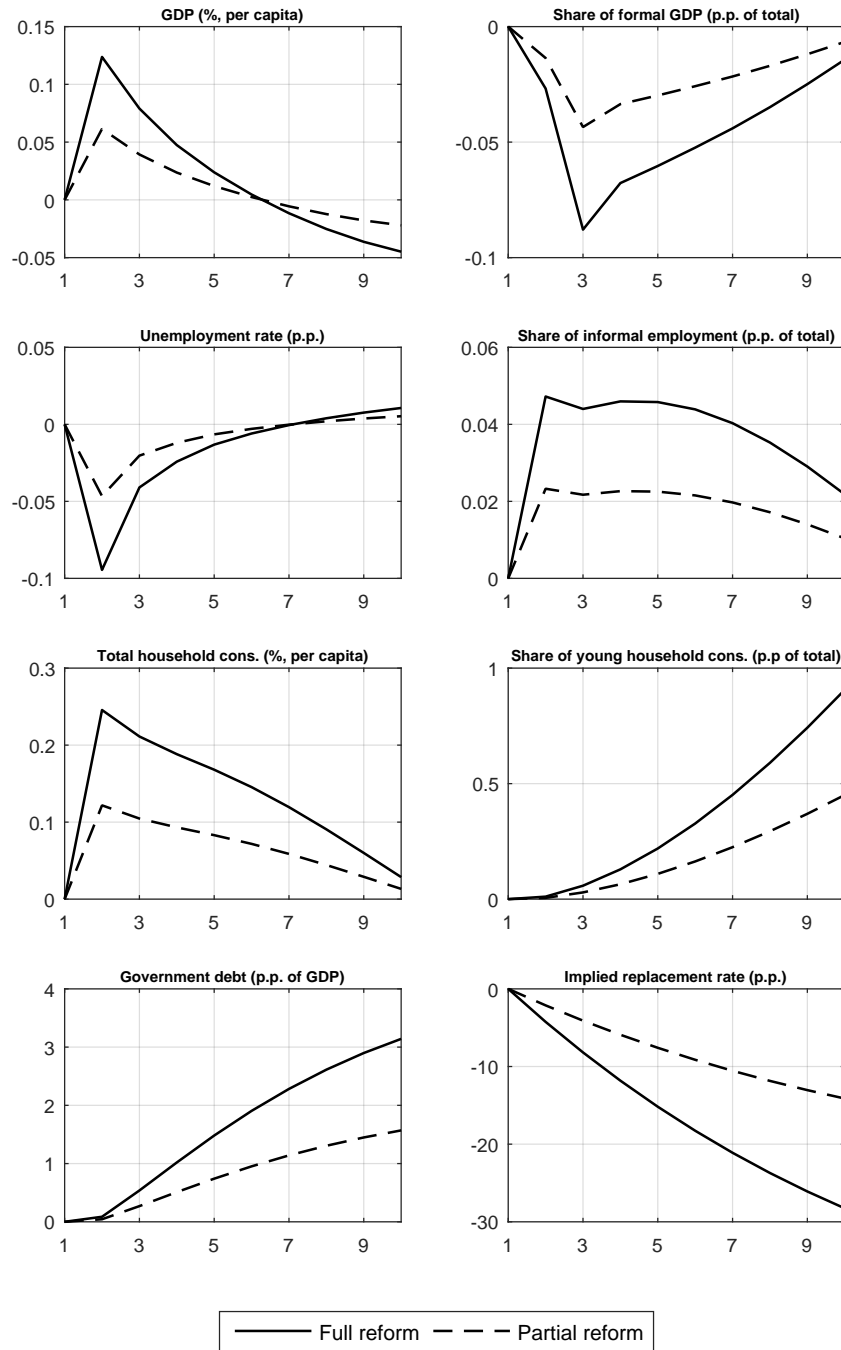


Figure 3.A17: Short-run (10-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Italy



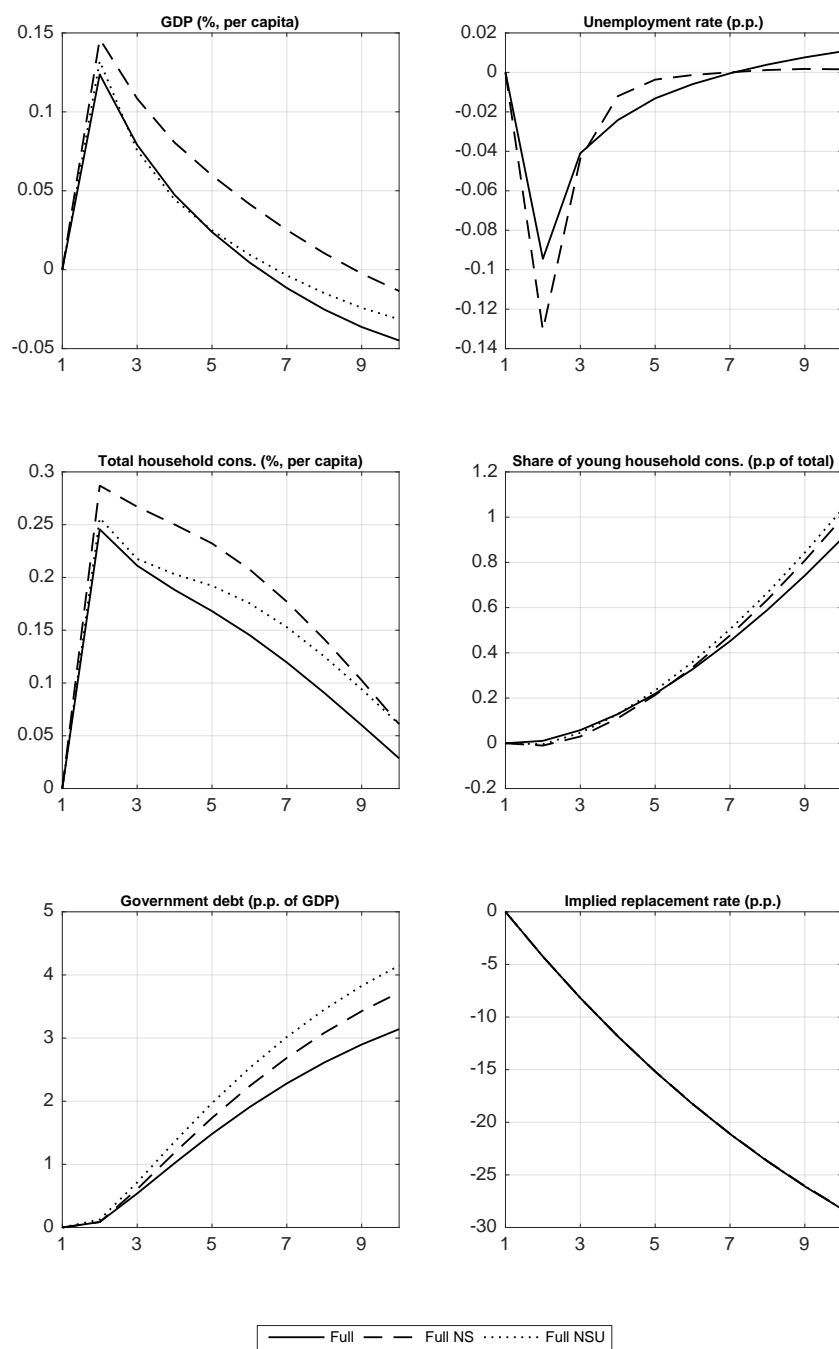


Figure 3.A18: Short-run (10-year) effects of a full pension reform in Italy, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment

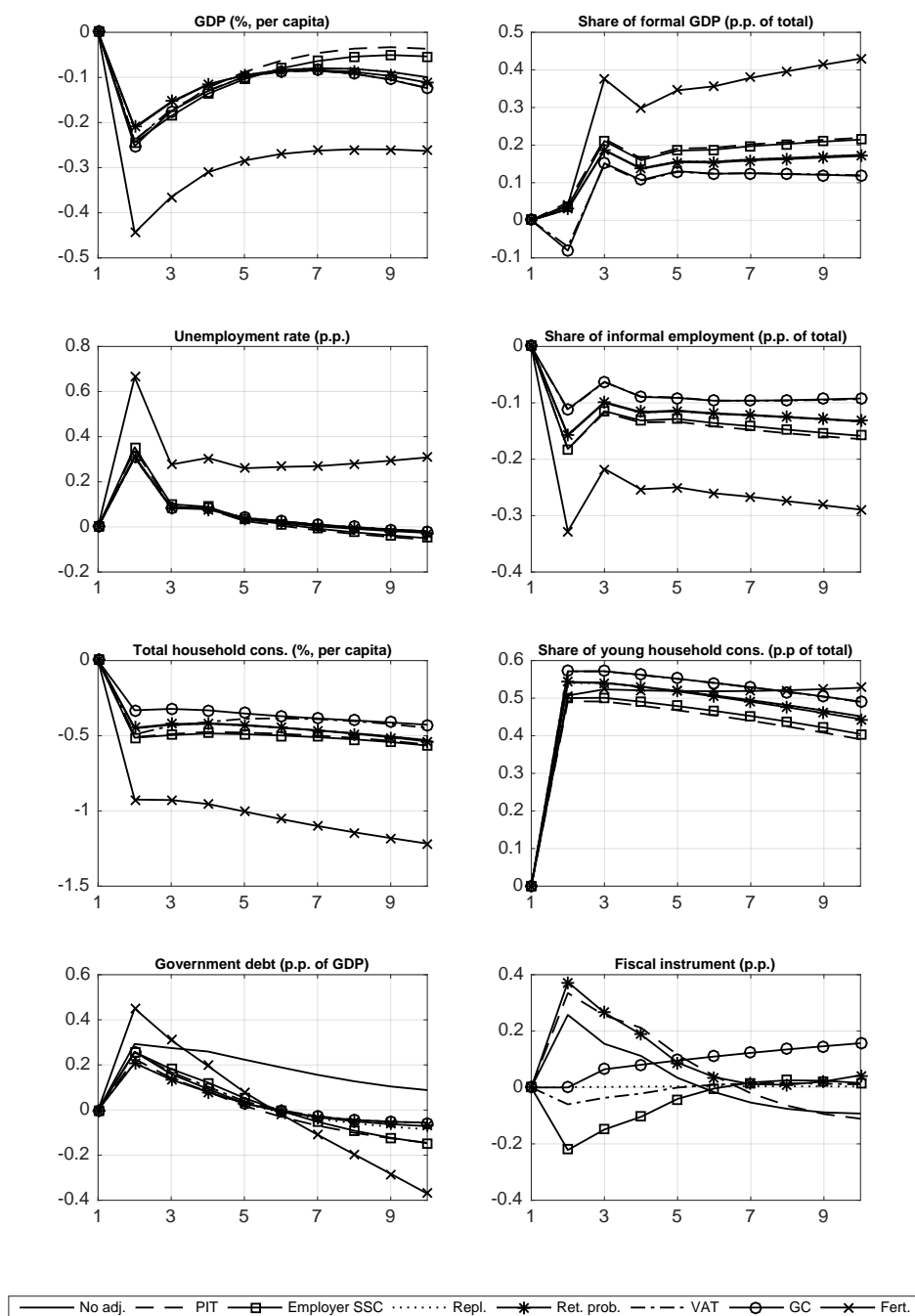


Figure 3.A19: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Spain

| <b>Long-run effects of a 5 pp increase in the old-age dependency ratio</b> |  |      |  |      |
|--|--|------|--|------|
| If the government consolidates with  | GDP per capita (%)                         |      | Share of formal GDP in total GDP (%point)                          |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | -3.5                                       | -3.4 | 0.4  | 0.4  |
| Personal income tax and employee SSC                                       | -8.2                                       | -4.3 | 0.5  | 0.5  |
| Employer SSC   | -6.2                                       | -    | 0.5  | -    |
| Pension-wage replacement rate  | -3.3                                       | -    | 0.5  | -    |
| Retirement probability   | -2.7                                       | -    | 0.4  | -    |
| Value added tax  | -3.3                                       | -3.3 | -0.2   | 0.2  |
| Government consumption exp./GDP  | -3.3                                       | -3.3 | -0.2   | 0.2  |
| Fertility rate   | 0.0  | -0.2 | 0.1  | 0.0  |
| If the government consolidates with  | Total household consumption per capita (%) |      | Share of young household consumption in total consumption (%point) |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | -4.6                                       | -4.6 | -1.7   | -0.1 |
| Personal income tax and employee SSC                                       | -10.9                                      | -5.9 | -1.3   | 0.1  |
| Employer SSC   | -8.3                                       | -    | -0.5   | -    |
| Pension-wage replacement rate  | -4.6                                       | -    | -0.6   | -    |
| Retirement probability   | -3.8                                       | -    | -0.5   | -    |
| Value added tax  | -4.5                                       | -4.6 | -0.6   | 0.1  |
| Government consumption exp./GDP  | -2.8                                       | -4.0 | -0.6   | 0.1  |
| Fertility rate   | -0.1                                       | -0.2 | 1.0  | 0.6  |
| If the government consolidates with  | Unemployment rate (%point)                 |      | Share of informal employment in total employment (%point)          |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | 0.1  | 0.1  | -0.3   | -0.4 |
| Personal income tax and employee SSC                                       | 2.4  | 0.5  | 0.4  | -0.2 |
| Employer SSC   | 1.4  | -    | 0.1  | -    |
| Pension-wage replacement rate  | 0.1  | -    | -0.4   | -    |
| Retirement probability   | 0.1  | -    | -0.3   | -    |
| Value added tax  | -0.2                                       | 0.0  | 0.1  | -0.2 |
| Government consumption exp./GDP  | -0.2                                       | 0.0  | 0.1  | -0.2 |
| Fertility rate   | 0.0  | 0.1  | 0.0  | 0.0  |
| If the government consolidates with  | Gov. debt as a share of GDP (%point)       |      | Instrument (%point)  |      |
|  | PAYG                                       | FF   | PAYG   | FF   |
| No consolidation   | 71.5                                       | 18.0 | 0.0  | 0.0  |
| Personal income tax and employee SSC                                       | 2.0  | 0.4  | 7.7  | 1.8  |
| Employer SSC   | 1.1  | -    | 8.5  | -    |
| Pension-wage replacement rate  | 0.2  | -    | -2.5   | -    |
| Retirement probability   | -0.1                                       | -    | -0.04  | -    |
| Value added tax  | 0.4  | 0.1  | 4.4  | 1.4  |
| Government consumption exp./GDP  | 0.3  | 0.1  | -0.4   | 0.3  |
| Fertility rate   | -2.2                                       | -2.1 | -0.2   | -0.2 |

Table 3.A8: Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in Spain, comparing PAYG and fully-funded regimes

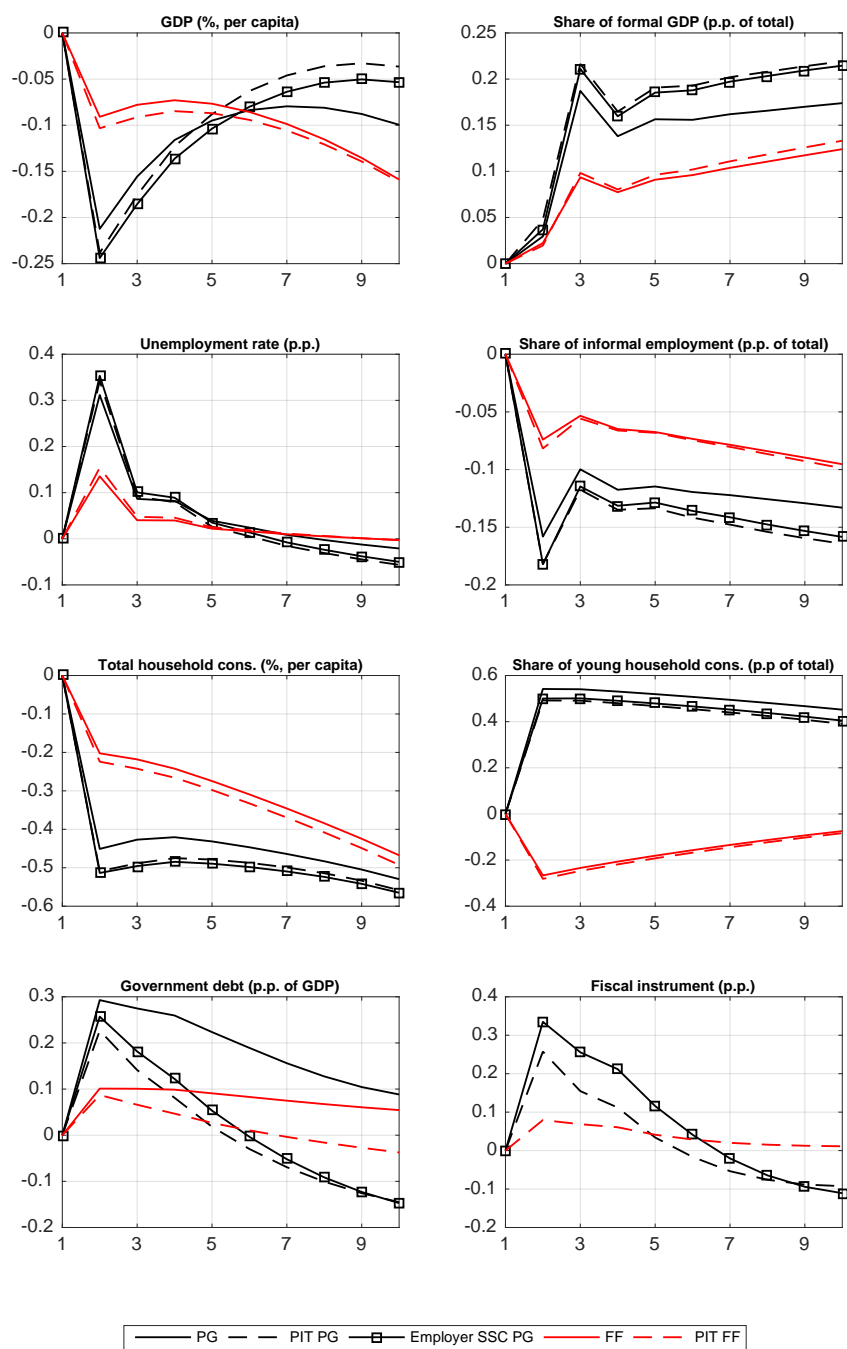


Figure 3.A20: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by modifying labor income taxes in Spain, comparing PAYG and fully-funded regimes

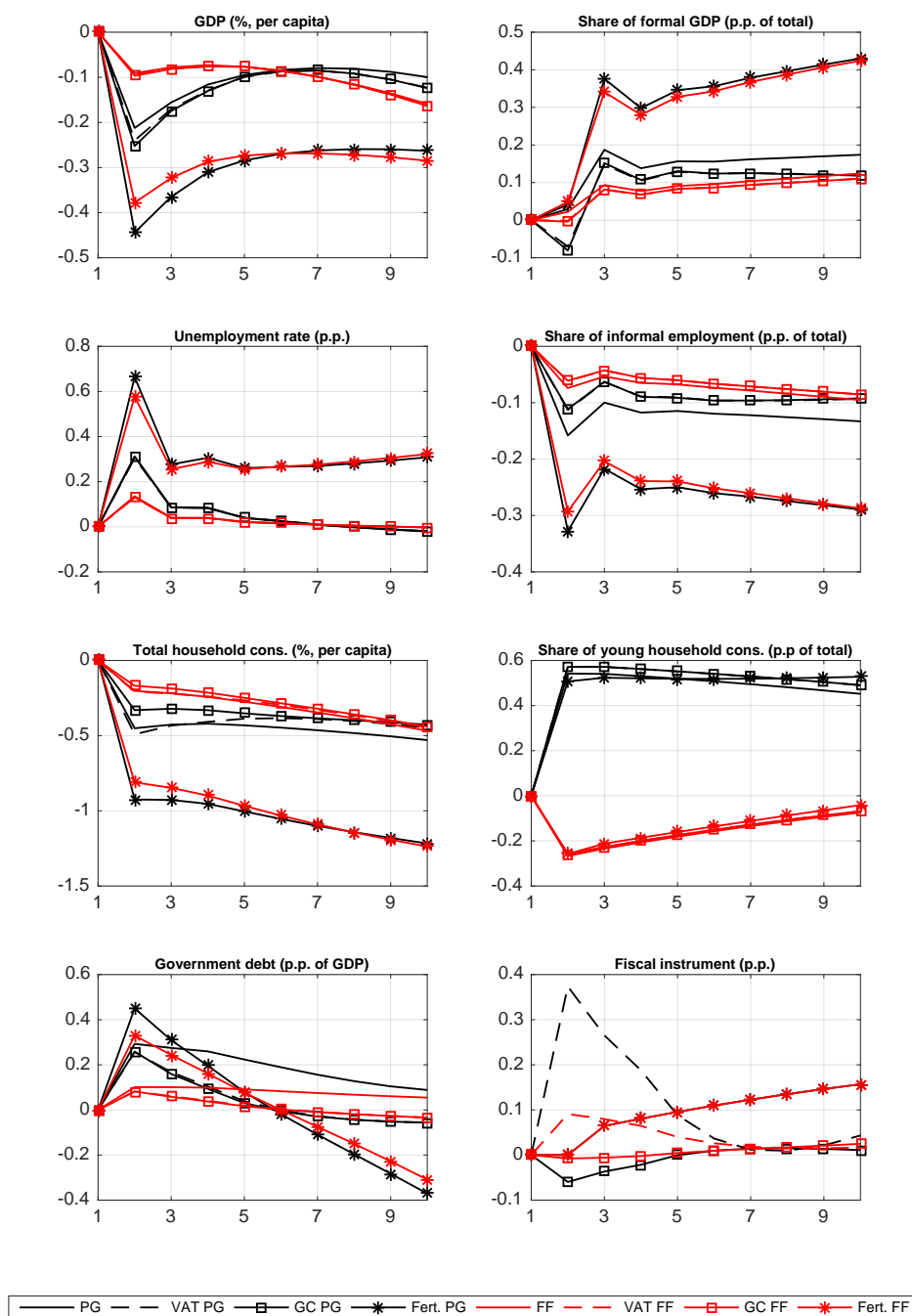


Figure 3.A21: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in Spain, comparing PAYG and fully-funded regimes

| Long-run effects of a 5 pp increase in the old-age dependency ratio in PAYG regime |  |                   |                                    |                                      |                   |                                    |  |                   |                                    |
|--|--|-------------------|------------------------------------|--------------------------------------|-------------------|------------------------------------|--|-------------------|------------------------------------|
| If the government consolidates with  | GDP per capita (%)   |                   |                                    | Unemployment rate (%point)           |                   |                                    | Total household consumption per capita (%) |                   |                                    |
|  | Baseline   | No shadow economy | No shadow economy, no unemployment | Baseline                             | No shadow economy | No shadow economy, no unemployment | Baseline                                   | No shadow economy | No shadow economy, no unemployment |
| No consolidation   | -3.5   | -3.7              | -3.6                               | 0.1                                  | 0.0               | -                                  | -4.6                                       | -4.5              | -4.1                               |
| Personal income tax and employee SSC   | -8.2   | -6.4              | -4.6                               | 2.4                                  | 1.5               | -                                  | -10.9                                      | -8.2              | -5.5                               |
| Employer SSC   | -6.2   | -5.3              | -4.3                               | 1.4                                  | 0.9               | -                                  | -8.3                                       | -6.9              | -5.2                               |
| Pension-wage replacement rate  | -3.3   | -3.4              | -3.4                               | 0.1                                  | -0.1              | -                                  | -4.6                                       | -4.6              | -4.2                               |
| Retirement probability   | -2.7   | -2.8              | -2.6                               | 0.1                                  | -0.1              | -                                  | -3.8                                       | -3.8              | -3.3                               |
| Value added tax  | -3.3   | -3.2              | -3.3                               | -0.2                                 | -0.1              | -                                  | -4.5                                       | -4.5              | -4.2                               |
| Government consumption exp./GDP  | -3.3   | -3.2              | -3.3                               | -0.2                                 | -0.1              | -                                  | -2.8                                       | -2.5              | -1.8                               |
| Fertility rate   | 0.0  | 0.1               | 0.0                                | 0.0                                  | 0.0               | -                                  | -0.1                                       | -0.2              | -0.1                               |
| If the government consolidates with  | Share of young household consumption in total consumption (%point) |                   |                                    | Gov. debt as a share of GDP (%point) |                   |                                    | Instrument (%point)                        |                   |                                    |
|  | Baseline   | No shadow economy | No shadow economy, no unemployment | Baseline                             | No shadow economy | No shadow economy, no unemployment | Baseline                                   | No shadow economy | No shadow economy, no unemployment |
| No consolidation   | -1.7   | -2.1              | -2.2                               | 71.5                                 | 84.7              | 122.4                              | 0.0  | 0.0               | 0.0                                |
| Personal income tax and employee SSC   | -1.3   | -1.2              | -1.0                               | 2.0                                  | 1.3               | 0.8                                | 7.7  | 5.9               | 5.3                                |
| Employer SSC   | -0.5   | -0.6              | -0.4                               | 1.1                                  | 0.9               | 0.8                                | 8.5  | 7.2               | 7.2                                |
| Pension-wage replacement rate  | -0.6   | -0.7              | -0.6                               | 0.2                                  | 0.3               | 0.4                                | -2.5                                       | -2.5              | -3.0                               |
| Retirement probability   | -0.5   | -0.6              | -0.5                               | -0.1                                 | 0.0               | 0.0                                | -0.04                                      | -0.04             | -0.04                              |
| Value added tax  | -0.6   | -0.7              | -0.6                               | 0.4                                  | 0.4               | 0.5                                | 4.4  | 3.7               | 3.4                                |
| Government consumption exp./GDP  | -0.6   | -0.7              | -0.6                               | 0.3                                  | 0.3               | 0.4                                | -0.4                                       | -0.6              | -1.2                               |
| Fertility rate   | 1.0  | 1.2               | 0.9                                | -2.2                                 | -2.3              | -2.2                               | -0.2                                       | -0.2              | -0.2                               |

Table 3.A9: Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Spain, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment

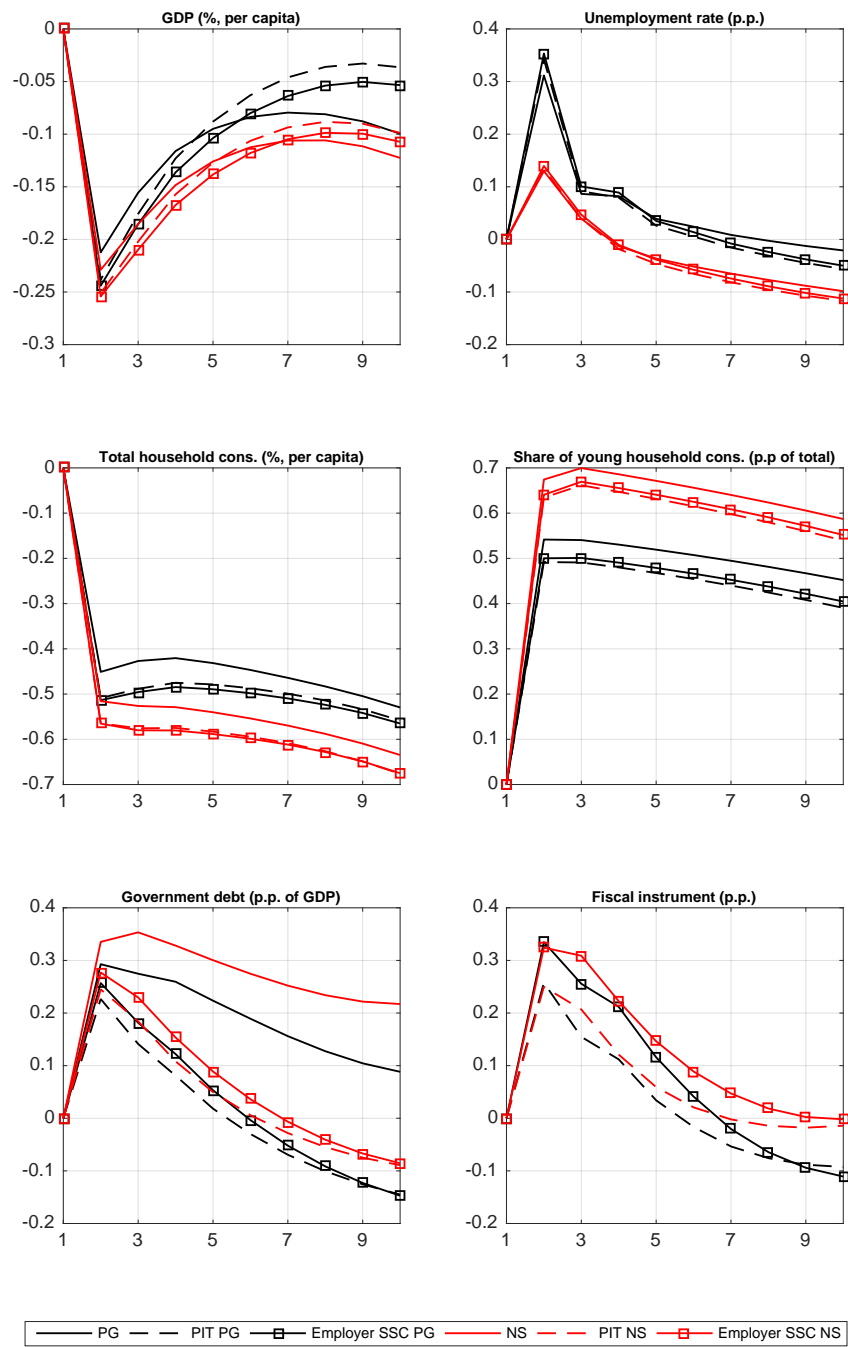


Figure 3.A22: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy in Spain

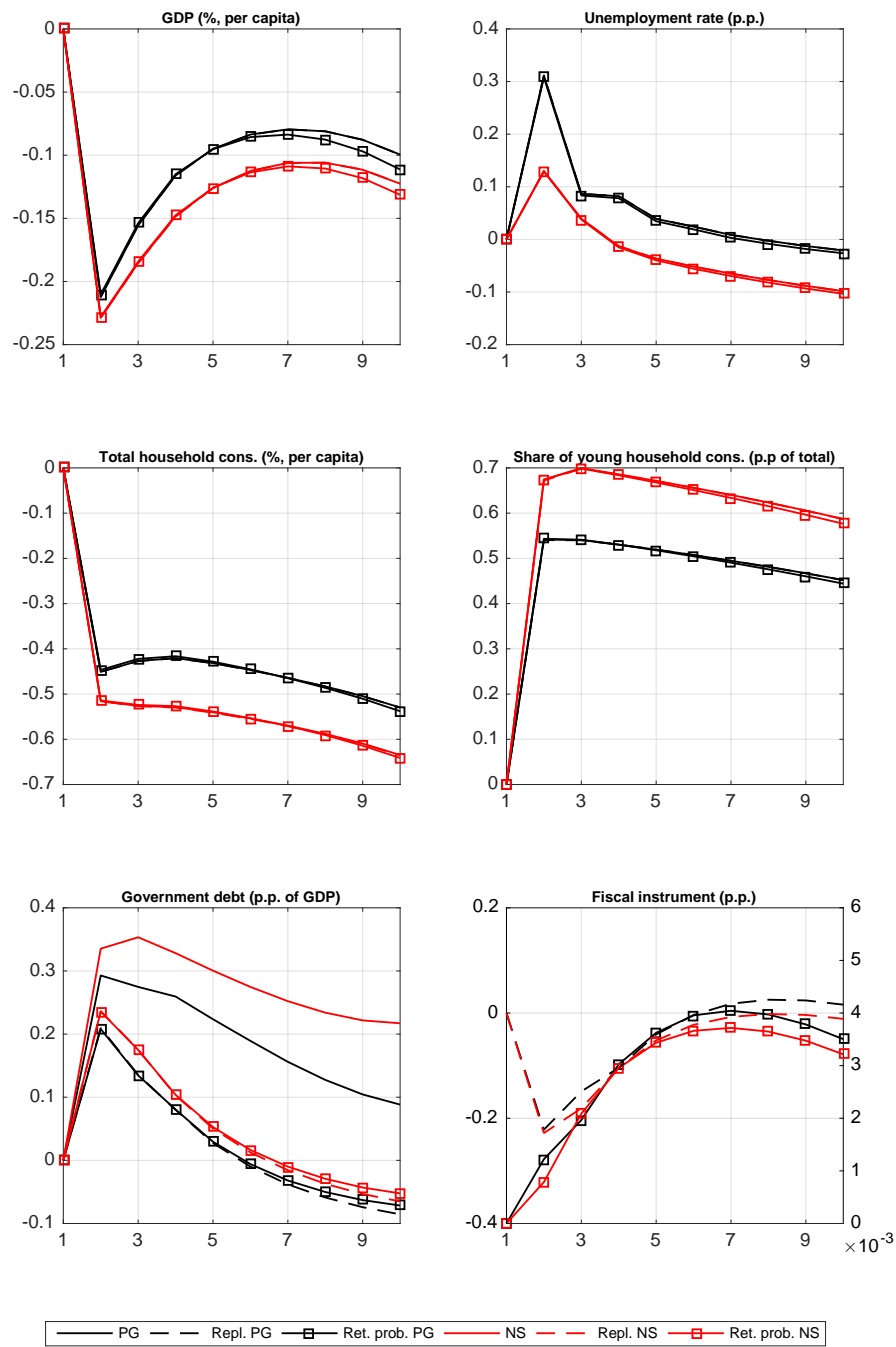


Figure 3.A23: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy in Spain



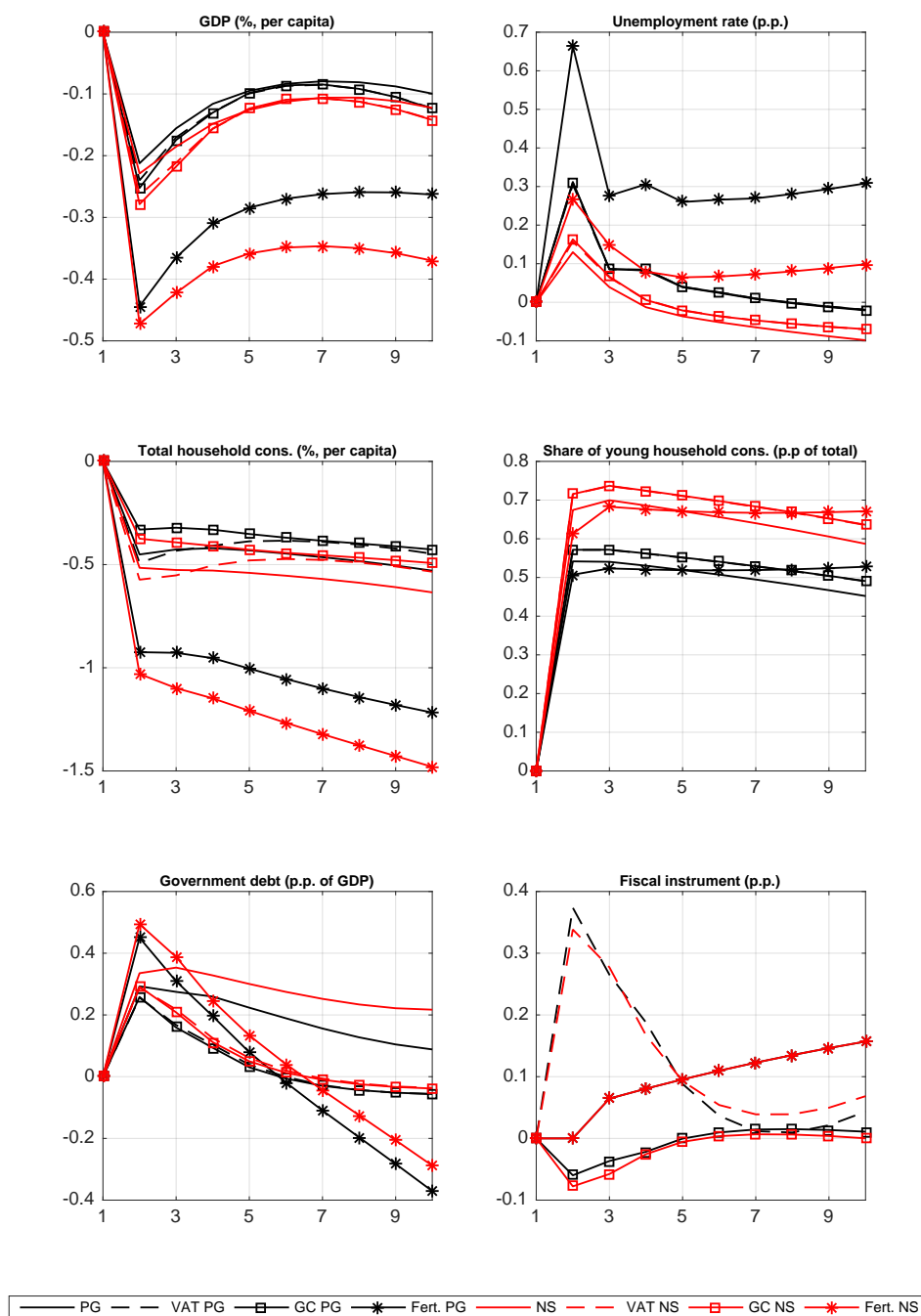


Figure 3.A24: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy in Spain

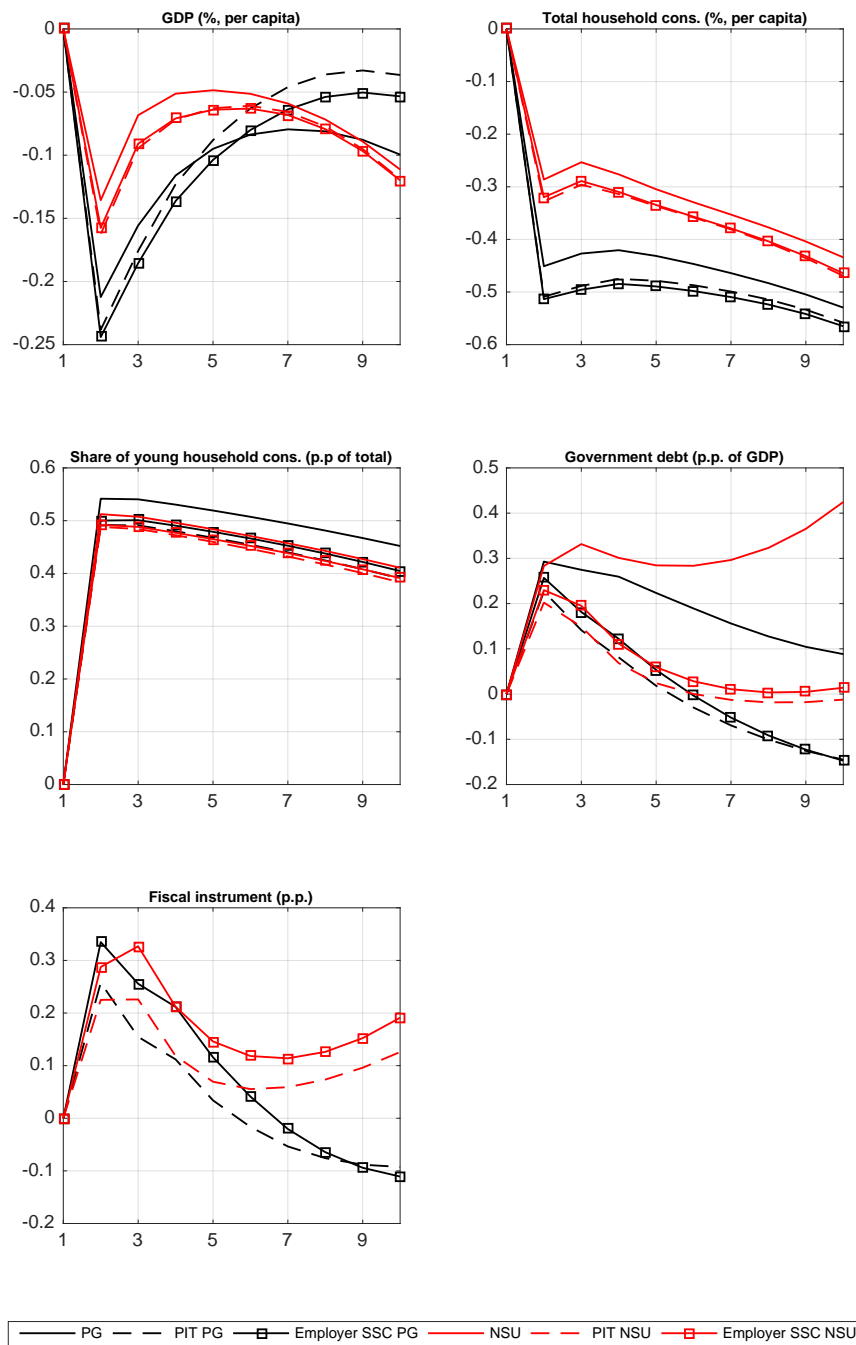


Figure 3.A25: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy and unemployment in Spain

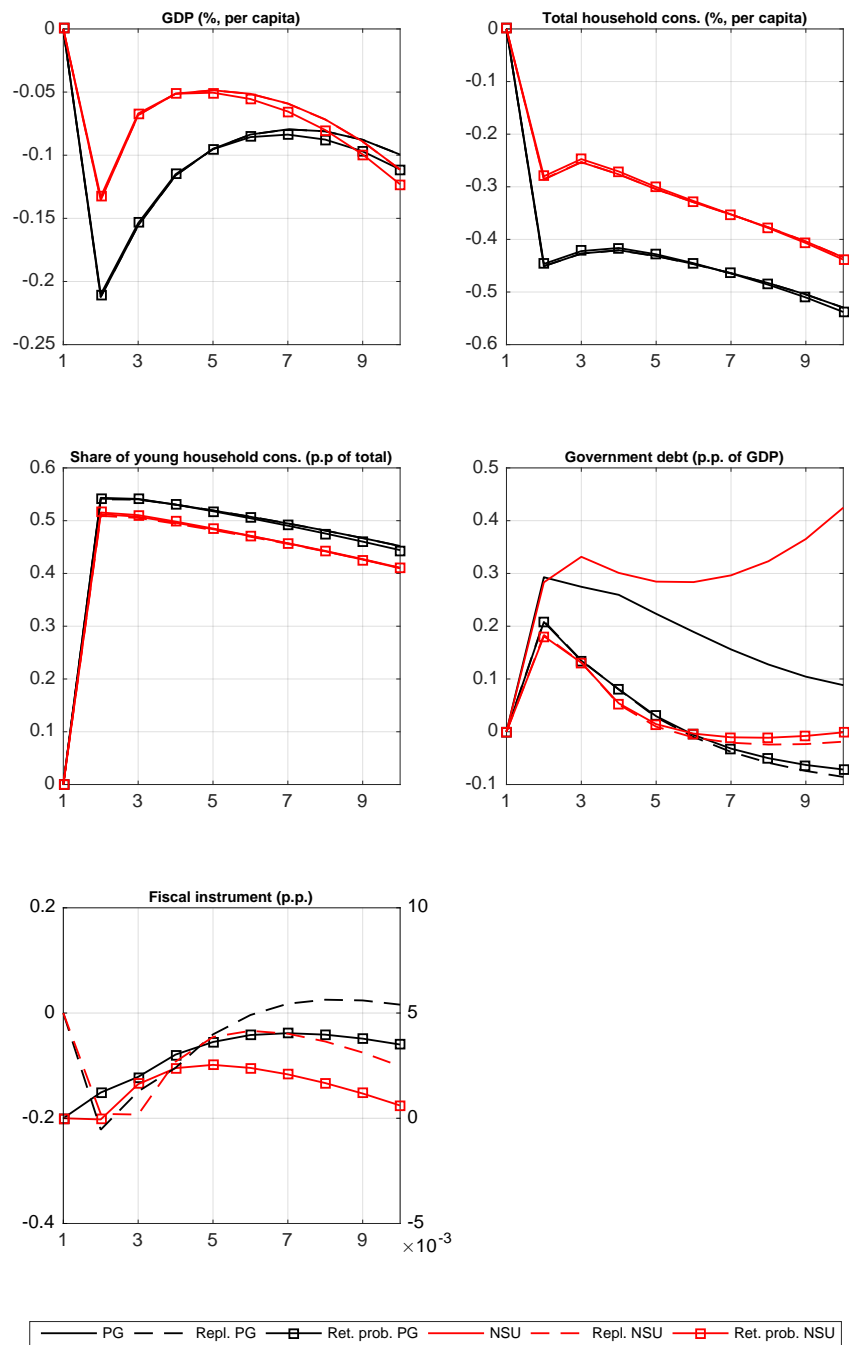


Figure 3.A26: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy and unemployment in Spain

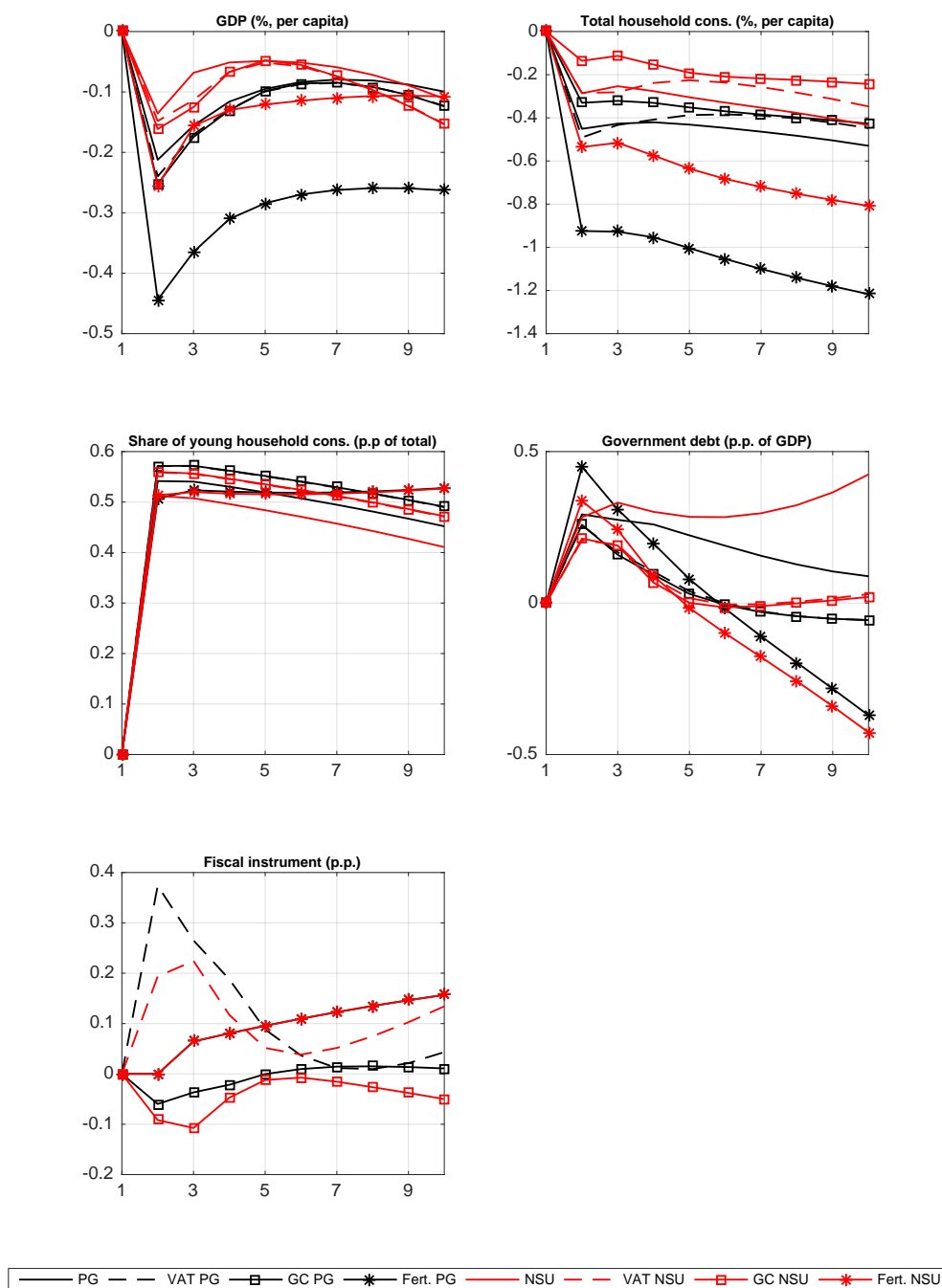


Figure 3.A27: Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy and unemployment in Spain

| <b>Long-run effects of a switch from PAYG to fully-funded</b>      |             |                |
|--|-------------|----------------|
|  | Full reform | Partial reform |
| GDP per capita (%)   | 1.8         | 0.9            |
| Share of formal GDP in total GDP (%point)                          | 0.6         | 0.3            |
| Unemployment rate (%point)   | -0.6        | -0.3           |
| Share of informal employment in total employment (%point)          | -0.5        | -0.2           |
| Total household consumption per capita (%)                         | 0.4         | 0.3            |
| Share of young household consumption in total consumption (%point) | 8.4         | 4.1            |
| Gov. debt as a share of GDP (%point)                               | -0.2        | -0.2           |
| Implied replacement rate (%point)                                  | -1.1        | 3.8            |

Table 3.A10: Long-run (100-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Spain

| <b>Long-run effects of a full switch from PAYG to fully-funded</b> |          |                   |                                    |
|--|----------|-------------------|------------------------------------|
|  | Baseline | No shadow economy | No shadow economy, no unemployment |
| GDP per capita (%)   | 1.8      | 2.1               | 1.4                                |
| Share of formal GDP in total GDP (%point)                          | 0.6      | -                 | -                                  |
| Unemployment rate (%point)   | -0.6     | -1.0              | -                                  |
| Share of informal employment in total employment (%point)          | -0.5     | -                 | -                                  |
| Total household consumption per capita (%)                         | 0.4      | -0.2              | -0.4                               |
| Share of young household consumption in total consumption (%point) | 8.4      | 10.1              | 9.7                                |
| Gov. debt as a share of GDP (%point)                               | -0.2     | -0.2              | 0.0                                |
| Implied replacement rate (%point)                                  | -1.1     | -0.8              | -3.4                               |

Table 3.A11: Long-run (100-year) effects of a full pension reform in Spain, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment

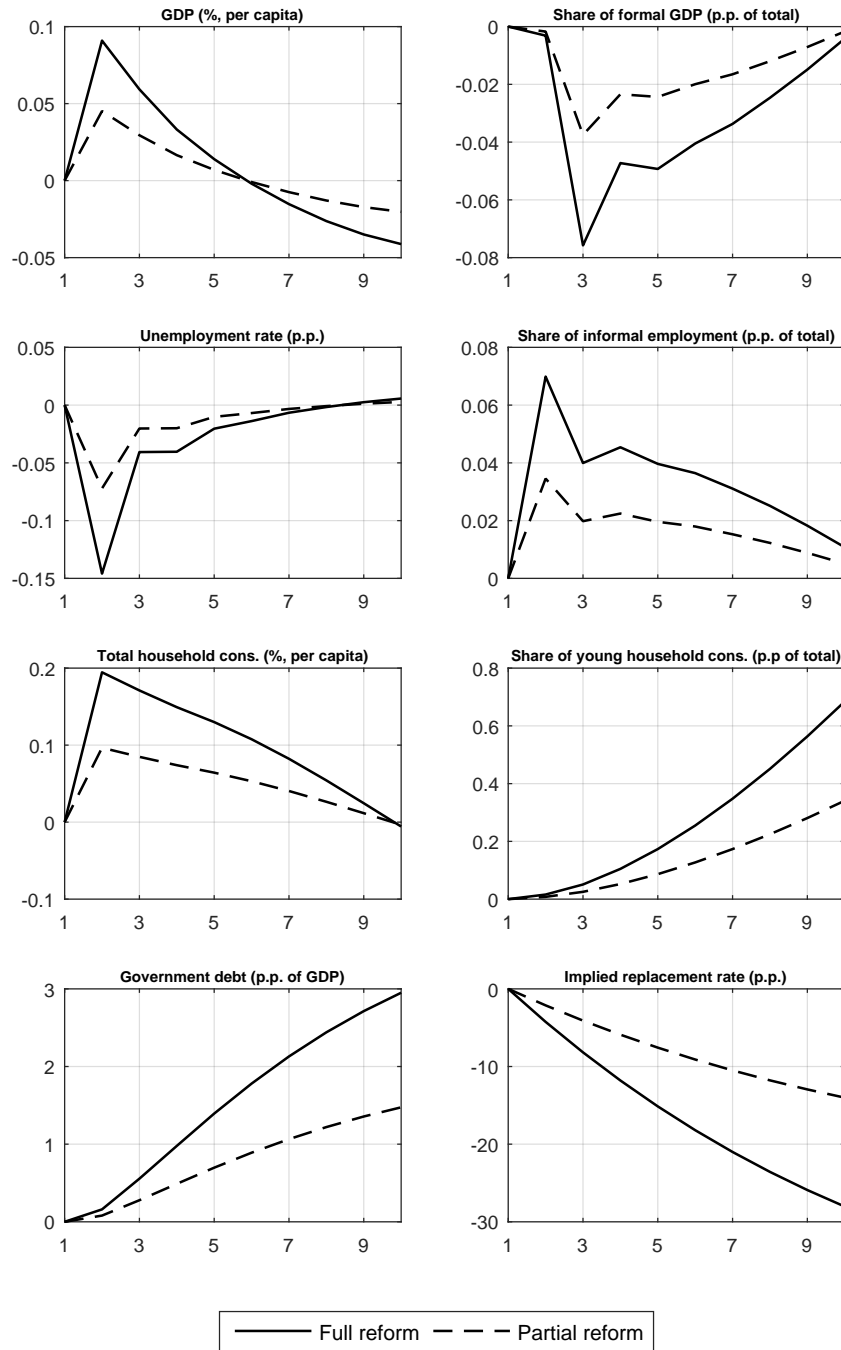


Figure 3.A28: Short-run (10-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Spain

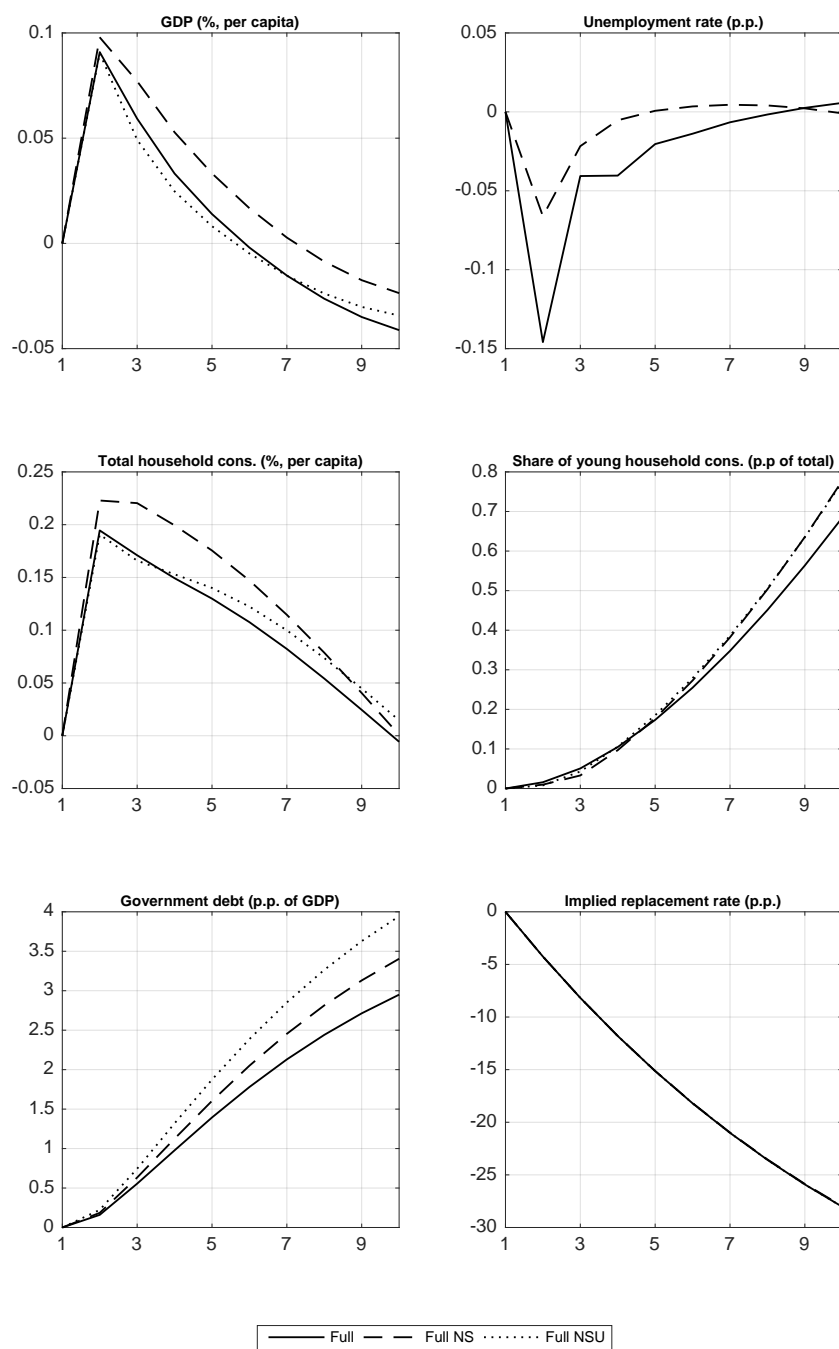


Figure 3.A29: Short-run (10-year) effects of a full pension reform in Spain, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment

# Bibliography

- [Adema et al. (2009)] Adema, Y. - Meijdam, L. - Verbon, H. A. A. (2009): The international spillover effects of pension reforms. *International Tax and Public Finance* (2009) 16: 670-696.
- [Aegon (2014)] Aegon (2014): The Changing Face of Retirement. The Aegon Retirement Readiness Survey 2014. <http://www.aegon.com/Documents/aegon-com/Research/2014-Retirement-Survey/Aegon-Retirement-Survey-2014.pdf>
- [Almeida et al. (2013)] Almeida, V. - Castro, G. - Felix, R. M. - Julio, P. - Maria, J. R. (2013): Inside PESSOA - A detailed description of the model. Banco de Portugal Working Papers 16/2013, October 2013. <http://www.bportugal.pt/en-US/BdP%20Publications%20Research/wp201316.pdf>
- [Anderson et al. (2013)] Anderson, D. - Hunt, B. - Kortelainen, M. - Kumhof, M. - Laxton, D. - Muir, D. - Mursula, S. - Snudden, S. (2013): Getting to Know GIMF: The Simulation Properties of the Global Integrated Monetary and Fiscal Model. International Monetary Fund WP 13/55, February 2013. <https://www.imf.org/external/pubs/ft/wp/2013/wp1355.pdf>
- [Baker-Rosnick (2012)] Baker, D. - Rosnick, D. (2012): The Impact on Inequality of Raising the Social Security Retirement Age. Center for Economic and Policy Research, April 2012. <http://cepr.net/documents/publications/ss-2012-04.pdf>
- [Barro-Becker (1989)] Barro, R. J. - Becker, G. S. (1989): Fertility choice in a model of economic growth. *Econometrica* 57: 481-501.
- [Beetsma et al. (2013)] Beetsma, R. M. W. J. - Romp, W. E. - Vos, S. J. (2013): Intergenerational Risk Sharing, Pensions, and Endogenous Labour Supply in General Equilibrium. *Scandinavian Journal of Economics* 115(1), 141-154, 2013.
- [Blanchard (1985)] Blanchard, O. J. (1985): Debt, Deficits, and Finite Horizons. *The Journal of Political Economy*, Vol. 93, No. 2., pp. 223-247, April 1985.



- [Blanchard-Gali (2010)] Blanchard, O. J. - Gali, J. (2010): Labor markets and Monetary Policy: A New Keynesian Model with Unemployment. *American Economic Journal: Macroeconomics*: Vol. 2 No. 2 (April 2010), pp. 1-30.
- [Boeri-Garibaldi (2007)] Boeri, T. – Garibaldi, P. (2007): Shadow sorting. NBER International Seminar on Macroeconomics 2005, eds. Frankel, J. and Pissarides, C., National Bureau of Economic Research, May 2007. <http://www.nber.org/chapters/c0339.pdf>
- [Boldrin-Jones (2002)] Boldrin, M. - Jones, L. E. (2002): Mortality, fertility, and saving in a Malthusian economy. *Review of Economic Dynamics* 5: 775–814.
- [Boldrin et al. (2005)] Boldrin, M. - De Nardi, M. - Jones, L. E. (2005): Fertility and social security. NBER Working Paper Series 11146. <http://www.micheleboldrin.com/Papers/fert&socsec.pdf>
- [Borsch-Supan-Ludwig (2011)] Borsch-Supan, A. - Ludwig, A. (2011): Old Europe ages: Reforms and Reform Backlashes, in *Demography and the Economy*, edited by John B. Shoven, The University of Chicago Press, 2011.
- [Borsch-Supan et al. (2006)] Borsch-Supan, A. - Ludwig, A. - Winter, J. (2006): Ageing, Pension Reform and Capital Flows: A Multi-Country Simulation Model. *Economica*, New Series, Vol. 73, No. 292 (Nov., 2006), pp. 625-658.
- [Brauninger (2005)] Brauninger, M. (2005): Social security, unemployment, and growth. *International Tax and Public Finances* 12: 423–434.
- [Braz et al. (2013)] Braz, C. R. - Castro, G. - Felix, R. M. - Maria, J. R. (2013): Ageing and Fiscal Sustainability in a Small Euro Area Economy. Banco de Portugal Working Papers 04/2013, March 2013. <http://www.bportugal.pt/en-US/BdP%20Publications%20Research/wp201304.pdf>
- [Charlot et al. (2011)] Charlot, O. - Malherbet, F. - Terra, C. (2011): Product market regulation, firm size, unemployment and informality in developing economies. Institute for the study of labor, IZA Discussion Paper No. 5519, Febr 2011. <http://ftp.iza.org/dp5519.pdf>
- [Christiano et al. (2005)] Christiano, L. J. - Eichenbaum, M. - Evans, C. L. (2005): Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*, 2005, Vol. 113, No. 1.
- [Cigno (1993)] Cigno, A. (1993): Intergenerational transfers without altruism: Family, market and state. *European Journal of Political Economy* 9: 505–518.

- [Cigno-Rosati (1992)] Cigno, A. - Rosati, F. C. (1992): The Effects of Financial Markets and Social Security on Saving and Fertility Behaviour in Italy. *Journal of Population Economics*, Vol. 5, No. 4 (Nov., 1992), pp. 319-341.
- [Conesa-Krueger (1999)] Conesa, J. C. - Krueger, D. (1999): Social Security Reform with Heterogeneous Agents. *Review of Economic Dynamics* 2, 757-795 (1999).
- [Corneo-Marquardt (1999)] Corneo, G. - Marquardt, M. (1999): Public pensions, unemployment insurance, and growth. *Journal of Public Economics* 75: 293-311.
- [de la Croix et al. (2013)] de la Croix, D. - Pierrard, O. - Sneessens, H. (2013): Ageing and Pensions in General Equilibrium: Labor Market Imperfections Matter. *Journal of Economic Dynamics and Control*, Elsevier, vol. 37(1), pages 104-124.
- [Diaz-Gimenez-Diaz-Saavedra (2009)] Diaz-Gimenez, J. - Diaz-Saavedra, J. (2009): Delaying retirement in Spain. *Review of Economic Dynamics* 12 (2009) 147-167.
- [Dieppe et al. (2015)] Dieppe, A. - Guarda, P. (2015), editors: Public debt, population ageing and medium-term growth. European Central Bank Working Paper No 165, August 2015. <https://www.ecb.europa.eu/pub/pdf/scpops/ecbop165.en.pdf>
- [Eurostat (2014)] Eurostat (2014): Taxation trends in the European Union. Data for the EU Member States, Iceland and Norway. [http://ec.europa.eu/taxation\\_customs/resources/documents/taxation/gen\\_info/economic\\_analysis/tax\\_structures/2014/report.pdf](http://ec.europa.eu/taxation_customs/resources/documents/taxation/gen_info/economic_analysis/tax_structures/2014/report.pdf)
- [Fehr (2000)] Fehr, H. (2000): Pension Reform during the Demographic Transition. *The Scandinavian Journal of Economics*, Vol. 102, No. 3, Social Security in the 21st Century (Jun., 2000), pp. 419-443.
- [Fehr et al. (2012)] Fehr, H. - Kallweit, M. - Kindermann, F. (2012): Pension reform with variable retirement age: a simulation analysis for Germany. *Journal of Pension Economics and Finance*, Vol. 11, Issue 3, July 2012, pp. 389-417.
- [Galasso (2008)] Galasso, V. (2008): Postponing retirement: the political effect of aging. *Journal of Public Economics* 92 (2008) 2157-2169.
- [Gora (2014)] Gora, M. (2014): Redesigning pension systems. The institutional structure of pension systems should follow population developments. IZA World of Labor 2014: 51. <http://wol.iza.org/articles/redesigning-pension-systems-1.pdf>
- [Goraus et al. (2014)] Goraus, K. - Makarski, K. - Tyrowicz, J. (2014): Does social security reform reduce gains from increasing the retirement age? University of Warsaw Working Papers No. 3/2014 (120). [http://www.wne.uw.edu.pl/files/3013/9637/0657/WNE\\_WP120\\_2014.pdf](http://www.wne.uw.edu.pl/files/3013/9637/0657/WNE_WP120_2014.pdf)

- [Heijdra-Romp (2009)] Heijdra, B. J. - Romp, W. E. (2009): Retirement, pensions, and ageing. *Journal of Public Economics* 93 (2009) 584-604.
- [Hohm (1975)] Hohm, C. F. (1975): Social Security and Fertility: An International Perspective. *Economics*, Vol. 12, No. 4 (Nov., 1975), pp. 629-644.
- [Ihrig-Moe (2004)] Ihrig, J. - Moe, K. S. (2004): Lurking in the shadows: the informal sector and government policy. *Journal of Development Economics* 73 (2004) 541-557.
- [IMF (2014)] IMF (2014): Fiscal Monitor October 2014 Back to Work How Fiscal Policy Can Help. <http://www.imf.org/external/pubs/ft/fm/2014/02/fmindex.htm>
- [Karam et al. (2010)] Karam, P. - Muir, D. - Pereira, J. - Tuladhar, A. (2010): Macroeconomic Effects of Public Pension Reforms. IMF Working Paper, WP/10/297, International Monetary Fund, December 2010. <http://www.imf.org/external/pubs/ft/wp/2010/wp10297.pdf>
- [Kilponen et al. (2006)] Kilponen, J. - Kinnunen, H. - Ripatti, A. (2006): Population ageing in a small open economy - some policy experiments with a tractable general equilibrium model. Bank of Finland Research Discussion Papers 28/2006. <https://core.ac.uk/download/files/153/6678097.pdf>
- [Kliem-Kriwoluzky (2014)] Kliem, M. - Kriwoluzky, A. (2014): Toward a Taylor Rule for Fiscal Policy. Review of Economic Dynamics, Elsevier for the Society for Economic Dynamics, vol. 17(2), pages 294-302, April.
- [Klyuev-Snudden (2011)] Klyuev, V. - Snudden, S. (2011): Effects of Fiscal Consolidation in the Czech Republic. IMF Working Paper WP/11/65. <https://www.imf.org/external/pubs/ft/wp/2011/wp1165.pdf>
- [Kumhof et al. (2010)] Kumhof, M. - Laxton, D. - Muir, D. - Mursula, S. (2010): The Global Integrated Monetary and Fiscal Model (GIMF) - Theoretical Structure. International Monetary Fund Working Paper 10/34, February 2010. <http://www.imf.org/external/pubs/cat/longres.aspx?sk=23615.0>
- [Kumler et al. (2013)] Kumler, T. - Verhoogen, E. - Frias, J. A. (2013): Enlisting employees in improving payroll-tax compliance: Evidence from Mexico. NBER Working Papers 19385. <http://www.nber.org/papers/w19385.pdf>
- [Leeper (1991)] Leeper, E. (1991): Equilibria under active and passive monetary and fiscal policies. *Journal of Monetary Economics*, 27 (1991), 129-147.
- [Mao et al. (2013)] Mao, J. - Zhang, L. - Zhao, J. (2013): Social Security Taxation and Compliance: The Chinese Evidence. Manuscript. [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=2151695](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2151695)

- [Marchiori-Pierrard (2012)] Marchiori, L. - Pierrard, O. (2012): LOLA 2.0: Luxembourg Overlapping Generation Model for Policy Analysis. Banque Centrale du Luxembourg Working Paper No 76, July 2012. <http://www.bcl.lu/en/publications/Working-papers/76/BCLWP076.pdf>
- [Marchiori-Pierrard (2015)] Marchiori, L. - Pierrard, O. (2015): LOLA 3.0: Luxembourg Overlapping Generation Model for Policy Analysis Introduction of a Financial Sector in LOLA. Banque Centrale du Luxembourg Working Paper No 100, November 2015. <http://www.bcl.lu/en/publications/Working-papers/100/BCLWP100.pdf>
- [Marchiori et al. (2011)] Marchiori, L. - Pierrard, O. - Sneessens, H. (2011): Demography, Capital Flows and Unemployment. Banque Centrale du Luxembourg, Working Paper No. 69, October 2011. <http://www.bcl.lu/en/publications/Working-papers/69/BCLWP069.pdf>
- [McGrattan-Prescott (2015)] McGrattan, E. R. - Prescott, E. C. (2015): On Financing Retirement with an Aging Population. Federal Reserve Bank of Minneapolis Research Department Staff Report 472, April 2015. <https://www.minneapolisfed.org/research/sr/sr472.pdf>
- [Munnell et al. (2006)] Munnell, A. H. - Sass, S. A. - Soto, M. (2006): Employer Attitudes Towards Older Workers: Survey Results. Center for Retirement Research, Boston College, June 2006. [http://crr.bc.edu/wp-content/uploads/2006/07/wob\\_3.pdf](http://crr.bc.edu/wp-content/uploads/2006/07/wob_3.pdf)
- [Nickel et al. (2008)] Nickel, C. - Rother, P. - Theophilopoulou, A. (2008): Population ageing and public pension reforms in a small open economy. ECB Working Paper Series NO 863, European Central Bank, February 2008. <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp863.pdf?20a1b8586c28db71e1541d48da40f2e7>
- [OECD (2011)] OECD (2011): Pensions at a Glance 2011: Retirement-Income Systems in OECD and G20 Countries. OECD Publishing, 2011. <http://www.oecd-ilibrary.org/docserver/download/8111011e.pdf?expires=1460110959&id=id&accname=guest&checksum=0EE11C34D01FF9E52CEB35AC35B19131>
- [OECD (2012)] OECD (2012): OECD Pensions Outlook 2012. OECD Publishing, 2012. [http://www.oecd-ilibrary.org/finance-and-investment/oecd-pensions-outlook-2012\\_9789264169401-en](http://www.oecd-ilibrary.org/finance-and-investment/oecd-pensions-outlook-2012_9789264169401-en)
- [OECD (2013)] OECD (2013): OECD Pensions Outlook 2013: OECD and G20 indicators. OECD Publishing, 2013. <http://www.oecd.org/pensions/public-pensions/OECDPensionsAtAGlance2013.pdf>

- [OECD (2015)] OECD (2015): Pensions at a Glance 2015: OECD and G20 Indicators. OECD Publishing, 2015. <http://www.oecd.org/publications/oecd-pensions-at-a-glance-19991363.htm>
- [Ono (2007)] Ono, T. (2007): Unemployment dynamics in an OLG economy with public pensions. *Economic Theory* 33: 549–577.
- [Ono (2010)] Ono, T. (2010): Growth and unemployment in an olg economy with public pensions. *Journal of Population Economics* 23: 737–767.
- [Pappa et al. (2015)] Pappa, E. - Sajedi, R. - Vella, E. (2015): Fiscal Consolidation with Tax Evasion and Rent Seeking, *Journal of International Economics*, Volume 96, pp. S56-S75, July 2015.
- [Pierrard-Snessens (2009)] Pierrard, O. - Sneessens, H. R. (2009): LOLA 1.0: Luxembourg Overlapping Generation Model for Policy Analysis. Banque Centrale du Luxembourg Working Paper No 36, March 2009. [http://www.bcl.lu/fr/Recherche/publications/cahiers\\_etudes/36/BCLWP036.pdf](http://www.bcl.lu/fr/Recherche/publications/cahiers_etudes/36/BCLWP036.pdf)
- [Razin-Sadka (1999)] Razin, A. - Sadka, E. (1999): Migration and pension with international capital mobility. *Journal of Public Economics* 74 (1999) 141-150.
- [Rotemberg (1982)] Rotemberg, J. J. (1982): Sticky Prices in the United States, *Journal of Political Economy*, Vol. 90, No. 6, pp. 1187-1211.
- [Schneider (2012)] Schneider, F. (2012): The Shadow Economy and Work in the Shadow: What Do We (Not) Know? IZA Discussion Paper Series No. 6423, March 2012. <http://ftp.iza.org/dp6423.pdf>
- [Schneider et al. (2010)] Schneider, F. - Buehn, A. - Montenegro, C. E. (2010): New Estimates for the Shadow Economies all over the World. *International Economic Journal*, 24:4, 443-461.
- [Smets-Wouters (2007)] Smets, F. - Wouters, R. (2007): Shocks and Frictions in US Business Cycles A Bayesian DSGE Approach. Working Paper NO 722, European Central Bank, February 2007. <https://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp722.pdf?038767f7a645096eccd34a8058c562e0>
- [Storesletten (2000)] Storesletten, K. (2000): Sustaining Fiscal Policy through Immigration. *Journal of Political Economy*, Vol. 108, No. 2 (April 2000), pp. 300-323.
- [Wang (2015)] Wang, L. (2015): A policy Analysis on Endogenous Fertility and Unemployment. Manuscript. [https://www.researchgate.net/publication/279243211\\_A\\_Policy\\_Analysis\\_on\\_Endogenous\\_Fertility\\_and\\_Unemployment](https://www.researchgate.net/publication/279243211_A_Policy_Analysis_on_Endogenous_Fertility_and_Unemployment)

- [Williamson (1975)] Williamson, O. E. (1975): Markets and Hierarchies, Analysis and Antitrust Implications: A Study in the Economics of Internal Organization. *The Free Press*, December 1975.
- [World Bank (1994)] World Bank (1994): Averting the Old Age Crisis. Policies to Protect the Old and to Promote Growth. A World Bank Policy Research Report. Oxford University Press.
- [World Bank (2015)] World Bank (2015): Golden Ageing. Prospects for Healthy, Active, and Prosperous Aging in Europe and Central Asia. <https://openknowledge.worldbank.org/handle/10986/22018>
- [Yaari (1965)] Yaari, M. E. (1965): Uncertain Lifetime, Life Insurance, and the Theory of the Consumer. *The Review of Economic Studies*, Vol. 32, No. 2, pp. 137-150, April 1965.

# List of Figures

|      |   |    |
|------|---|----|
| 1.1  | Size structure of European (family) firms (Mandl (2008), page 50) . . . . .   | 8  |
| 1.A1 | An increase in value added tax revenue equal to 1 percent of GDP . . . . .  | 45 |
| 1.A2 | A cut in government consumption equal to 1 percent of GDP . . . . .   | 46 |
| 1.A3 | An increase in employees' labor income tax (personal labor income tax and social security contribution) revenue equal to 1 percent of GDP . . . . .   | 47 |
| 1.A4 | An increase in employer social security contribution revenue equal to 1 percent of GDP . . . . .  | 48 |
| 1.A5 | Tax shift (a 1-1 percent of GDP decrease in employees' labor income tax revenue and increase in value added tax revenue) . . . . .  | 49 |
| 1.A6 | Tax shift (a 1-1 percent of GDP decrease in employer social security contribution revenue and increase in value added tax revenue) . . . . .  | 50 |
| 1.A7 | First column: a cut in government consumption. Second column: an increase in value added tax revenue. Third column: an increase in employees' labor income tax. Fourth column: an increase in the employer social security contribution. All shocks correspond to 1 percent of GDP. . . . .   | 51 |
| 2.1  | The fraction of shadow output in total output around the world. Source is Schneider et al. (2010) page 458). . . . .  | 63 |
| 2.2  | The relation between the shadow economy and openness. Openness is calculated by the authors as the ratio of the sum of nominal exports and imports in nominal GDP; data source is World Bank. Data sources of the size of the shadow economy are Schneider et al. (2010) and Schneider (2012). Both openness and the size of the shadow economy are calculated between 1999 and 2007. . . . . | 64 |
| 2.3  | The structure of the model . . . . .  | 66 |
| 2.4  | Short-run effects of single labor and product market reforms in South Africa. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal hiring cost (HCF), formal bargaining power of workers (BPF) and formal entry cost (ENTRYF). . . . .   | 90 |

|      |   |     |
|------|---|-----|
| 2.A1 | The role of shadow economy in short-run effects of a decrease in formal hiring cost and bargaining power of workers. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal hiring cost and bargaining power of workers. Low informality (red lines) means that the size of the shadow economy in output is 1/3 of the original value. . . . . | 107 |
| 2.A2 | The role of shadow economy in short-run effects of a decrease in formal entry cost. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal entry cost. Low informality (red line) means that the size of the shadow economy in output is 1/3 of the original value. . . . .  | 108 |
| 2.A3 | Prior and posterior distributions of structural parameters based on Bayesian estimation using South African data. The parameters are (in order): interest rate smoothing, inflation weight and output weight in Taylor rule, formal and informal Rotemberg price rigidity costs, interest rate premium, investment adjustment cost and consumption habit. . . . .   | 110 |
| 2.A4 | Short-run effects of a combined decrease in formal hiring cost and bargaining power of workers in South Africa. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal hiring cost (line with marker), formal bargaining power of workers (dotted black line) and their combination (red line). . . . .  | 113 |
| 2.A5 | Short-run effects of a combined decrease in formal hiring cost and entry cost in South Africa. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal hiring cost (line with marker), formal entry cost (dotted black line) and their combination (red line). . . . .  | 114 |
| 2.A6 | Short-run effects of a combined decrease in formal bargaining power of workers and entry cost in South Africa. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal bargaining power of workers (line with marker), formal entry cost (dotted black line) and their combination (red line). . . . .  | 115 |
| 2.A7 | Short-run effects of a combined decrease in formal hiring cost and bargaining power of workers and entry cost in South Africa. The chart shows the short-run effects (first 5 years after the shock) of a permanent and unexpected 10 percent decrease in formal hiring cost and bargaining power of workers (line with marker), formal entry cost (dotted black line) and their combination (red line). . . . .          | 116 |



|       |   |     |
|-------|---|-----|
| 2.A8  | Sequencing of a permanent decrease in formal hiring cost and bargaining power of workers in South Africa. The lines show the long-run (25 years + 25 years) effects of permanent and unexpected policies of size 10 percent. Line with squared marker: first a decrease in formal hiring cost, then a decrease in formal bargaining power of workers. Line with round marker: the opposite. Red line: both policies are implemented during the second 25 years. . . . .                     | 117 |
| 2.A9  | Sequencing of a permanent decrease in formal hiring cost and entry cost in South Africa. The lines show the long-run (25 years + 25 years) effects of permanent and unexpected policies of size 10 percent. Line with squared marker: first a decrease in formal hiring cost, then a decrease in formal entry cost. Line with round marker: the opposite. Red line: both policies are implemented during the second 25 years. . . . .   | 118 |
| 2.A10 | Sequencing of a permanent decrease in formal bargaining power of workers and entry cost in South Africa. The lines show the long-run (25 years + 25 years) effects of permanent and unexpected policies of size 10 percent. Line with squared marker: first a decrease in formal bargaining power of workers, then a decrease in formal entry cost. Line with round marker: the opposite. Red line: both policies are implemented during the second 25 years. . . . .                       | 119 |
| 2.A11 | Sequencing of a decrease in formal hiring cost and bargaining power of workers and entry cost in South Africa. The lines show the long-run (25 years + 25 years) effects of permanent and unexpected policies of size 10 percent. Line with squared marker: first a decrease in formal hiring cost and bargaining power of workers, then a decrease in formal entry cost. Line with round marker: the opposite. Red line: both policies are implemented during the second 25 years. . . . . | 120 |
| 3.1   | Public debt today (Eurostat) . . . . .  | 128 |
| 3.2   | Old-age pension spending today (Eurostat) . . . . .   | 128 |
| 3.3   | Share of elderly people today and 50 years ahead (Eurostat) . . . . .   | 129 |
| 3.4   | Old-age dependency ratio today and 50 years ahead (Eurostat) . . . . .  | 129 |
| 3.5   | Additional life expectancy at age 65 today and 50 years ahead (OECD) . .  | 130 |
| 3.6   | The size of shadow output (Schneider et al. (2010), page 458) . . . . .   | 132 |
| 3.7   | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal . . .  | 156 |
| 3.8   | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by modifying labor income taxes in Portugal, comparing PAYG and fully funded regimes . . . . .  | 160 |
| 3.9   | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in Portugal, comparing PAYG and fully funded regimes . . . . .   | 161 |

|       |   |     |
|-------|---|-----|
| 3.10  | Short-run (10-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Portugal . . . . .  | 163 |
| 3.11  | Official versus effective retirement age levels (IMF, 2014, page 39) . . . . .  | 169 |
| 3.A1  | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy in Portugal . . . . .                | 205 |
| 3.A2  | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy in Portugal . . . . .                  | 206 |
| 3.A3  | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy in Portugal . . . . .             | 207 |
| 3.A4  | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy and unemployment in Portugal .       | 208 |
| 3.A5  | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy and unemployment in Portugal . . . . . | 209 |
| 3.A6  | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy and unemployment in Portugal .    | 210 |
| 3.A7  | Short-run (10-year) effects of a full pension reform in Portugal, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment . . . . .                        | 211 |
| 3.A8  | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Italy . . . . .   | 212 |
| 3.A9  | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by modifying labor income taxes in Italy, comparing PAYG and fully-funded regimes . . . . .                 | 214 |
| 3.A10 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in Italy, comparing PAYG and fully-funded regimes . . . . .                        | 215 |
| 3.A11 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy in Italy . . . . .                   | 217 |
| 3.A12 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy in Italy . . . . .                     | 218 |

|       |   |     |
|-------|---|-----|
| 3.A13 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy in Italy . . . . .              | 219 |
| 3.A14 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy and unemployment in Italy . . .    | 220 |
| 3.A15 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy and unemployment in Italy . . . . .  | 221 |
| 3.A16 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy and unemployment in Italy . . . | 222 |
| 3.A17 | Short-run (10-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Italy . . . . .   | 224 |
| 3.A18 | Short-run (10-year) effects of a full pension reform in Italy, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment . . . . .                         | 225 |
| 3.A19 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Spain . . . .   | 226 |
| 3.A20 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by modifying labor income taxes in Spain, comparing PAYG and fully-funded regimes . . . . .               | 228 |
| 3.A21 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in Spain, comparing PAYG and fully-funded regimes . . . . .                      | 229 |
| 3.A22 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy in Spain . . . . .                 | 231 |
| 3.A23 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy in Spain . . . . .                   | 232 |
| 3.A24 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy in Spain . . . . .              | 233 |
| 3.A25 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by labor income taxes in a PAYG regime with and without shadow economy and unemployment in Spain . . .    | 234 |

|       |   |     |
|-------|---|-----|
| 3.A26 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by pension policies in a PAYG regime with and without shadow economy and unemployment in Spain . . . . .  | 235 |
| 3.A27 | Short-run (10-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio by other fiscal policies in a PAYG regime with and without shadow economy and unemployment in Spain . . . | 236 |
| 3.A28 | Short-run (10-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Spain . . . . .   | 238 |
| 3.A29 | Short-run (10-year) effects of a full pension reform in Spain, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment . . . . .                         | 239 |

# List of Tables

|      |  |     |
|------|--|-----|
| 1.1  | Percentage of family firms in European employment . . . . .  | 6   |
| 1.2  | Unemployment rate by share of family firms in European employment . . .  | 6   |
| 1.3  | Steady-state parameters . . . . .  | 23  |
| 1.4  | Dynamic parameters . . . . .   | 25  |
| 1.5  | Unemployment fiscal multipliers (in percentage points) . . . . .   | 26  |
| 1.6  | Unemployment fiscal multipliers and sectoral heterogeneity (in percentage points) . . . . .  | 28  |
| 1.A1 | Unemployment fiscal multipliers - robustness with respect to parameter values (in percentage points) . . . . .   | 43  |
| 1.A2 | Unemployment fiscal multipliers - robustness with respect to parameter values (in percentage points) cont. . . . .   | 44  |
| 2.1  | Differences between the formal and informal sectors . . . . .  | 79  |
| 2.2  | The role of shadow economy and openness in long-run (25-year) effects of labor and product market deregulation policies. All deregulation policies are implemented in the formal sector, and mean a permanent and unexpected 10 percent reform size. Low informality means that the size of shadow economy in output is 1/3 of the baseline value. . . . . | 81  |
| 2.3  | Evaluating the steady state of the model . . . . .   | 82  |
| 2.4  | Calibration of steady-state parameters . . . . .   | 84  |
| 2.5  | Prior and posterior distributions of structural dynamic parameters. The priors of consumption habit and interest rate smoothing are maximized to 0.9, while the prior of interest rate premium is maximized to 0.005. . . . .  | 86  |
| 2.6  | Long-run (25-year) effects of single structural reforms in South Africa. All deregulation policies are implemented in the formal sector, and mean a permanent and unexpected 10 percent reform size. . . . .   | 88  |
| 2.A1 | The robustness of long-run (25-year) impacts of structural reforms with lower and higher informal bargaining power of workers . . . . .  | 102 |
| 2.A2 | The robustness of long-run (25-year) impacts of structural reforms with lower and higher informal hiring cost . . . . .  | 103 |

|       |  |     |
|-------|--|-----|
| 2.A3  | The robustness of long-run (25-year) impacts of structural reforms with lower and higher informal entry cost . . . . .   | 104 |
| 2.A4  | The robustness of long-run (25-year) impacts of structural reforms with lower and higher elasticity of substitution between formal and informal goods . .  | 105 |
| 2.A5  | A taxonomy of underground economic activities (broad concept). Source: Mirus-Smith (1977) (page 5). . . . .  | 106 |
| 2.A6  | Steady state parameters of the three parametrized economies . . . . .  | 109 |
| 2.A7  | Dynamic parameters of the three parametrized economies . . . . .   | 110 |
| 2.A8  | Prior and posterior distributions of autocorrelations of shocks based on Bayesian estimation using South African data. All priors are maximized to 0.9. . . .  | 111 |
| 2.A9  | Prior and posterior distributions of standard deviations of shocks based on Bayesian estimation using South African data . . . . .   | 111 |
| 2.A10 | Long-run effects of structural reform packages in South Africa. The table shows the long-run (25 years) effects of permanent and unexpected policies of size 10 percent implemented in the formal sector. . . . .  | 112 |
| 3.1   | Steady-state parameters . . . . .  | 151 |
| 3.2   | Dynamic parameters . . . . .   | 152 |
| 3.3   | Evaluating the steady-state calibration for Portugal, Italy and Spain in a PAYG regime . . . . .   | 153 |
| 3.4   | Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal, Italy and Spain . . . . .  | 155 |
| 3.5   | Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in Portugal, comparing PAYG and fully funded regimes . . . . .   | 159 |
| 3.6   | Long-run (100-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Portugal . . . . .   | 162 |
| 3.7   | Rigidities in the formal and informal sectors of the model . . . . .   | 165 |
| 3.8   | Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment . . . . . | 166 |
| 3.9   | Long-run (100-year) effects of a full pension reform in Portugal, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment . . . . .   | 167 |
| 3.10  | Retirement age increase and its consequences in southern Europe . . . . .  | 168 |

|       |   |     |
|-------|---|-----|
| 3.A1  | Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal, comparing the baseline scenario with lower and higher informal bargaining powers of workers . . . . .                               | 202 |
| 3.A2  | Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal, comparing the baseline scenario with lower and higher informal firing probabilities   | 203 |
| 3.A3  | Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal, comparing the baseline scenario with a lower fraction of profits and lump-sum taxes to the young . . . . .                          | 204 |
| 3.A4  | Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in Italy, comparing PAYG and fully-funded regimes . . . . .   | 213 |
| 3.A5  | Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Italy, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment . . . . . | 216 |
| 3.A6  | Long-run (100-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Italy . . . . .   | 223 |
| 3.A7  | Long-run (100-year) effects of a full pension reform in Italy, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment . . . . .   | 223 |
| 3.A8  | Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in Spain, comparing PAYG and fully-funded regimes . . . . .   | 227 |
| 3.A9  | Long-run (100-year) effects of neutralizing the budgetary impact of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Spain, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment . . . . . | 230 |
| 3.A10 | Long-run (100-year) effects of a full and a partial (50 per cent) move away from the PAYG plan towards the fully funded regime in Spain . . . . .   | 237 |
| 3.A11 | Long-run (100-year) effects of a full pension reform in Spain, comparing the baseline scenario with a model without shadow economy and with a model without shadow economy and unemployment . . . . .   | 237 |