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Abstract

The paper considers stochastic linear trends in series with a higher than annual frequency of observation. Using an approach based on ARIMA models, some of the trend models (or the model interpretation of trend estimation filters) most often found in statistics and econometrics are analysed and compared. The properties of the trend optimal estimator are derived, and the analysis is extended to seasonally adjusted and/or detrended series. It is seen that, under fairly general conditions, the estimator of the unobserved component is noninvertible, and will not accept a convergent autoregressive representation. This has implications concerning unit root testing and VAR model fitting.

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Introduction:
The Concept of a Trend: Model versus Estimator

The concept of a trend component (in short, a trend) in an economic time series is far from having a precise, universally accepted, definition. Trends have been modeled as deterministic functions of time [see, for example, Fellner (1956)], as purely stochastic processes [in economics, a standard reference is Nelson and Plosser (1982)], or as a mixture of the two [Pierce (1978)]. This paper centers on strictly stochastic trends and the stochastic process generating the trend will be assumed linear. [Nonlinear extensions, such as the one in Hamilton (1989), will not be considered.]

The trend is associated with the underlying smooth evolution of a series, free from transitory or cyclical (seasonal or not) effects. In the frequency domain, this long-term evolution is, in turn, associated with the low frequencies of the spectrum. Let the frequency be measured in radians; the zero frequency, with a cycle of infinite length, undoubtedly should be part of the trend. A frequency $\omega = (.6)10^{-5}$ implies a period of 10^6 time units, and hence to all practical effects, indistinguishable from a trend. For $\omega = .006$, the associated period of 1000 years should probably still be considered part of the trend. As ω increases and the associated period decreases, there will come a value which, clearly, should not be included in the trend. Since all economic series contain some degree of additive noise (with a flat spectrum), perhaps the most natural way to define a trend is, thus, by the spectral peak at the low frequencies; see Granger (1978) and Nerlove, Grether and Carvalho (1976).

In so far as the trend represents mostly the variation in some frequency interval $(0, \omega_0)$, where ω_0 is small, it is possible to construct bandpass filters with a close to 1 gain in that interval, and close to zero gain for other frequencies. These types of filters are often used to estimate trends. Important examples are the Henderson filter used by the program X11 [see, for example, Gourieroux and Monfort (1990, p. 102-3)], and the Hodrick and Prescott (1980) filter. Both can be seen as the solution to a constrained least squares problem, where the constraints impose some degree of smoothness. Both provide linear moving average filters, similar to those obtained when the trend is estimated by approximating smooth functions with local polynomials in time [see Kendall (1976)]. These Moving Average filters have the advantage of computational and conceptual simplicity. They provide point estimates of the trend, but there is no underlying stochastic model for the component. As Prescott (1986) states, trend is thus "defined by the computational procedure used to fit the smooth curve through the data."

The price paid for conceptual and computational simplicity can be, however, large. The designed filters always require fixing some arbitrary parameter, and this is done typically by judgement. The asymmetry of the filters implies a phase shift, which can be misleading, in particular for detecting and dating turning points. The fact that the filters are always the same, and do not depend on the stochastic properties of the series, simplifies matters, at the cost, though, of risking spurious detrending: in the limit, trends could be extracted from white-noise series. Moreover, since the estimate is, by definition, the trend, nothing in the procedure would detect those spurious trends or situations in which a given filter is not appropriate.

Finally, the procedure does not allow for proper statistical inference; for example, one cannot obtain forecasts of the trend, let alone standard errors of the forecast.

Possibly fostered by the explosion in the use of ARIMA models [see Box and Jenkins (1970)] for economic time series, the last ten years have experienced a growing interest in modeling trends. Since the work of Beveridge and Nelson (1981), Nelson and Plosser (1982), Watson (1986), and many others, stochastic models for the trend have become widespread in economics [see Stock and Watson (1988)]. In statistics, several modeling approaches have been suggested. Within the context of linear stochastic processes, we shall mostly focus on two general ones. First, the so-called ARIMA-Model-Based (AMB) approach, in which the model for the trend is derived from the ARIMA model identified for the observed series [see Box, Hillmer and Tiao (1978) and Burman (1980)]. The other approach starts by directly specifying a model for the trend; it has been denoted the Structural Time Series (STS) approach, and basic references are Harvey and Todd (1983), and Harvey (1985). Both approaches are closely related, and the models for the trend are also related, as we shall see, to those used by econometricians.

Since the trend component is never observed, one always works with estimators. In the context of ARIMA models for the trend and for the observed series, (optimal) Minimum Mean Squared Error (MMSE) estimators are easily obtained. These estimators are also moving averages, similar to the ones encountered in the design of filters; in fact, often these latter filters can be interpreted as optimal estimators for some particular models. Since the model-based approach offers a powerful tool for analysis, diagnosis, and inference, in the rest of the paper it will be used to analyse the models for the trends, and the properties of their MMSE estimators. First, section 1 presents the basic framework and some notation. Then, in sections 2, 3 and 4, some models often used for the trend component are analysed and discussed, and related to the stochastic structure of the observed series. Trends, of course, are never observed, and section 5 looks at the properties of their estimators. Finally, section 6 presents some implications for applied econometric work.

1 The General Statistical Framework

Let x_t be a time series which is the sum of a trend, p_t, and a nontrend component, n_t,

$$x_t = p_t + n_t, \tag{1}$$

where the two components are uncorrelated and each follows an ARIMA model, which we write in short as

$$\phi_p(B)p_t = \theta_p(B)b_t, \tag{2}$$

$$\phi_n(B)n_t = \theta_n(B)c_t; \tag{3}$$

B denotes the lag operator, $\phi(B)$ and $\theta(B)$ are finite polynomials in B that may contain unit roots, and b_t and c_t are orthogonal white-noise variables, with variances V_b and V_c. (Throughout the paper, a white–noise variable denotes a variable that is normally, identically, and independently distributed.) It is assumed that the roots of the autoregressive (AR) polynomials $\phi_p(B)$ and $\phi_n(B)$ are different; since AR roots for the same frequency should belong to the same component, this is not a restrictive assumption. Of course, the two polynomials in (2) and in (3) are prime.

The paper is mostly aimed at quarterly or monthly data, in which case n_t can often be written as the sum of a seasonal component, s_t, and an irregular one, u_t, both the outcome of linear stochastic processes. Then (1) becomes

$$x_t = p_t + s_t + u_t, \tag{4}$$

where the seasonal component follows the model

$$\phi_s(B)\, s_t = \theta_s(B)\, e_t \tag{5}$$

with $\phi_s(B)$ typically nonstationary, while u_t is assumed a stationary process, uncorrelated with b_t and e_t.

Combining (1), (2), and (3), it is obtained that

$$\phi_p(B)\, \phi_n(B)\, x_t = \phi_n(B)\, \theta_p(B)\, b_t + \phi_p(B)\, \theta_n(B)\, c_t,$$

and hence x_t also follows an ARIMA model of the type

$$\phi(B)\, x_t = \theta(B)\, a_t, \tag{6}$$

where $\phi(B) = \phi_p(B)\, \phi_n(B)$, and $\theta(B)\, a_t$ is the moving average (MA) process such that

$$\theta(B)\, a_t = \phi_n(B)\, \theta_p(B)\, b_t + \phi_p(B)\, \theta_n(B)\, c_t, \tag{7}$$

with a_t a white–noise variable with variance V_a [see Anderson (1971, p. 224)]. Without loss of generality, V_a is set equal to 1, so that the variances of the component innovations will be implicitly expressed as fractions of V_a, the variance of the one–period–ahead forecast error for the observed series. Since the sum of two uncorrelated MA processes [as in (7)] can only be noninvertible when the same unit root is shared by both MA polynomials, if we further assume that $\theta_p(B)$ and $\theta_n(B)$ have no common unit root, it follows that model (6) will be invertible. However, given that the concept of a trend or a seasonal component is intimately linked to nonstationary behavior, models (2) and (3) will typically be nonstationary [see Hillmer, Bell, and Tiao (1983)]. We shall still use the representation

$$\psi(B) = \theta(B)/\phi(B) \tag{8}$$

when the series is nonstationary, and similarly for $\psi_p(B)$. Further, letting ω denote frequency, the Fourier transform of $\psi(B)\,\psi(F)\,V_a$, where $F = B^{-1}$, will be referred to as the spectrum of x_t, $g_x(\omega)$ [for nonstationary series, it is often called the "pseudospectrum"; see Hillmer and Tiao (1982) or Harvey (1990)]. In a similar way, $g_p(\omega)$ will denote the spectrum of the trend.

Since observations are only available on x_t, the AMB approach starts with model (6), which can be identified and estimated directly from the data using Box–Jenkins techniques; then, looks at which models for the trend, that capture the spectral low frequency peak, are compatible with (6). From the set of all admissible models, some additional requirements permit the selection of a unique one. The STS approach proceeds in an inverse manner, by identifying a–priori models (2) and (3) for the components. Ultimately, since (2) and (3) imply a model of the type (6), both approaches are closely linked. The models, however, are different, since the identification restrictions are not the same.

Given that the components are never observed, one can only use estimators. For known models (an assumption that will be made throughout the paper), both methods obtain the trend estimator as $E(p_t/X_T)$, where $X_T = [x_1, \ldots, x_T]$ represents the available sample. This conditional expectation can be efficiently computed with the Kalman or the Wiener-Kolmogorov (WK) filters. If the first one has the appeal of its programming easiness, the WK filter, as will become apparent, is particularly suited for statistical analysis. For the models we consider, the two filters provide MMSE estimators of the components; these estimators are considered in section 5.

2 Some Models for the Trend Component

I shall consider some well–known ARIMA models, often encountered in practice when modeling monthly or quarterly economic time series. It will be seen what type of trend model is implicit in the overall ARIMA model, and the stochastic properties thereof. The trend model will be compared with several other statistical trend models, contained in well–known statistical packages. The comparison will also include the standard linear stochastic trend models used in econometrics.

Consider the general class of models for the observed series:

$$\nabla \nabla_s \, x_t = \theta(B)\, a_t, \tag{9}$$

where $\nabla = 1 - B$, $\nabla_s = 1 - B^s$, and s denotes the number of observations per year. Since ∇^2 causes a peak for $\omega = 0$ and $S(B)$ generates the peaks for the seasonal frequencies in $g_x(\omega)$, let us factorize the polynomial $\nabla\nabla_s$ into

$$\begin{aligned} \phi_p(B) &= (1-B)^2 = \nabla^2 \\ \phi_n(B) &= 1 + B + B^2 + \cdots + B^{s-1} = S, \end{aligned} \tag{10}$$

The decomposition (1) can be expressed as

$$\frac{\theta(B)}{\nabla\nabla_s}\, a_t = \frac{\theta_p(B)}{\nabla^2}\, b_t + \frac{\theta_n(B)}{S}\, c_t, \tag{11}$$

or, removing denominators,

$$\theta(B)\, a_t = S\, \theta_p(B)\, b_t + \nabla^2\, \theta_n(B)\, c_t. \tag{12}$$

Let q_p and q_n denote the orders of the MA polynomials $\theta_p(B)$ and $\theta_n(B)$, respectively. Equation (12) implies that q, the order of $\theta(B)a_t$, will be equal to

$$q = \max(q_p + s - 1,\ q_n + 2).$$

From (12), $(q+1)$ covariance equations can be derived; in the AMB approach $\theta(B)$ and V_a are assumed known, and the parameters of $\theta_p(B)$, $\theta_n(B)$, as well as the variances V_b and V_c, have to be derived from them. Since the number of unknowns is $(q_p + q_n + 2)$, it follows that, when

$$q_p + q_n + 1 > q,$$

there will be an infinite number of solutions to the system of covariance equations, and the decomposition (11) will be underidentified. It is straightforward to find that

$$q_p > 1 \qquad \text{and} \qquad q_n > s - 2,$$

are necessary and sufficient conditions for the decomposition (11) to be underidentified. The AMB decomposition restricts the order of the AR and MA polynomials in model (2) for the trend to be of the same order. Therefore, $q_p = 2$, and the trend model becomes an IMA (2,2), say

$$\nabla^2 p_t = (1 - \alpha_1\, B - \alpha_2\, B^2)\, b_t. \tag{13}$$

The decomposition (11) — or the models underlying it — is still underidentified, and a further condition is then imposed. Since the randomness of p_t in (13) is caused by the variable b_t, with variance V_b, of all the models of the type (13) that are compatible with the stochastic structure of the observed series (i.e., with model (9)), the one with smallest V_b is chosen. This yields the most stable trend, given the observed ARIMA model. As shown by Hillmer and Tiao (1982), minimizing V_b is equivalent to the requirement that it should not be possible to further decompose p_t into $p_t^* + u_t^*$, where u_t^* is white–noise, orthogonal to p_t^*. When a component satisfies this "noise–free" requirement it is termed "canonical". The canonical trend is, therefore, uncontaminated by noise, and hence its spectrum should contain a zero, since otherwise, setting $Var(u_t^*) = min\ g_p(\omega)$, a further "trend plus noise" decomposition could be achieved. The zero in the spectrum represents a unit root in the MA polynomial of the trend. Since the spectrum of the trend should be monotonically decreasing with ω, the zero should happen for $\omega = \Pi$, i.e., the unit root of the MA is $B = -1$. The model for the trend can then be rewritten

$$\nabla^2 p_t = (1 - \alpha B)(1 + B)\, b_t, \tag{14}$$

which contains two parameters, α and V_b. Now, the number of unknowns in the system of covariance equations is $(q_n + 3)$, and the number of equations $\geq q_n + 3$. The decomposition becomes, thus, identified, and there will be a unique model (14), which will represent the trend component contained in model (9).

The model for the trend in the basic Structural Model is a random walk with drift, with the drift being generated also by a random walk. In particular,

$$\begin{aligned} \nabla p_t &= \mu_t + u_t, \\ \nabla \mu_t &= v_t, \end{aligned} \tag{15}$$

where u_t and v_t are mutually orthogonal white–noise variables. This trend model is also considered by Harrison and Stevens (1971) and, within a filter design approach, by Ng and Young (1990). It is immediately seen that the above model can be expressed as an IMA(2,1) model

$$\nabla^2 p_t = (1 - \beta B)\, b_t, \tag{16}$$

where β and V_b depend on the variances of u_t and v_t. Model (16) represents an integrated of order 2, or I(2), model, with two parameters. Notice, however, that the trend given by (16) does not have the canonical property, so that orthogonal white–noise can still be extracted from it, and the trend can be expressed as p_t in (14) plus white noise [Maravall (1985)]. This difference between the two models is a consequence of the different assumptions used to reach identification. While the AMB approach uses the canonical (noise–free) condition, the STS approach, by imposing $q_p = 1$, sets a priori some additional parameters equal to zero. If in simultaneous econometric models zero parameter constraints reflect, in principle, a priori information derived from economic theory, in the context of unobserved component models, no similar rationalization for the zero parameter constraint holds.

There is another difference between the STS trend and the canonical model–based one which is worth mentioning. Writing (15) as

$$\nabla^2 p_t = v_t + \nabla u_t,$$

it follows that the lag–1 autocovariance of $\nabla^2 p_t$ in the STS case is always negative, so that β in (16) has to be positive. As a consequence, a model such as

$$\nabla^2 x_t = (1 + \theta B)\, a_t,$$

with $\theta > 0$, in the STS approach would not be a trend, nor would it be possible to extract a trend from it. (This would still be true if, more generally, u_t is allowed to be colored noise.) In the AMB approach, x_t above can be decomposed into orthogonal trend and noise, as in (1); for example, for $\theta = .2$, the trend is given by

$$\nabla^2 p_t = (1 - .092B)\,(1 + B)\, b_t, \qquad (V_b = .436),$$

and n_t is white–noise with $V_n = .040$. Therefore, the canonical trend in the AMB approach is less restrictive than that in the STS approach.

Moving on to other statistical procedures to estimate stochastic trends, consider first the well–known X11 procedure. Cleveland and Tiao (1976) showed how it could be interpreted (approximately) as a MMSE estimator in an AMB approach, where the model for the trend is given by

$$\nabla^2 p_t = (1 + .49B - .49B^2)\, b_t. \tag{17}$$

Therefore, the trend follows again an IMA(2, 2); moreover, for $B = -1$, the MA polynomial is close to zero, and hence the model is not far from a canonical trend, with a zero in the spectrum for $\omega = \Pi$. [For a discussion of Henderson's 13-term filter, see also Tiao and Hillmer (1978).] In a similar way, Tiao (1983) has shown how Akaike's BAYSEA seasonal adjustment method can also be roughly interpreted as an AMB method, with the model for the trend given now by

$$\nabla^2 p_t = b_t. \tag{18}$$

This is, in fact, the same trend model obtained by Gersch and Kitagawa (1983) using an alternative STS formulation, and the trend implied in the model–based interpretation of the Hodrick–Prescott filter [Hodrick and Prescott (1980)]. Model (18) does not satisfy the canonical property, but it can be expressed as the sum of a canonical trend, given by

$$\nabla^2 p_t^* = (1 - .172B)\,(1 + B)\, b_t^*,$$

with $Var(b_t^*) = .364V_b$, and an orthogonal white–noise variable, u_t, with variance $V_u = V_b/16$.

The four trend models (14), (16), (17), and (18), can be seen as particular cases of the IMA(2, 2) model. These models and/or associated filters are routinely used on many hundreds of thousands of series (mostly for forecasting and seasonal ajdustment); they all represent trends that are I(2) variables. This is apparently in sharp contrast with the standard linear stochastic model used to represent trends in econometrics, typically an I(1) process, most often the random walk plus drift model:

$$\nabla p_t = b_t + \mu, \tag{19}$$

where μ, the drift, is a constant [see, for example, Stock and Watson (1987) and the many references they contain]. While the statistical models that have been mentioned are mostly aimed at monthly and quarterly (unadjusted) series, the attention of econometricians when modeling trends has been directed to annual or quarterly seasonally adjusted data. Be that as it may, in so far as neither time aggregation, nor seasonal adjustment, should change the order of integration (at the zero frequency), the differences in the type of data used do not explain the different order of integration typically used by statisticians and econometricians.

3 A Frequently Encountered Class of Models

It has been shown that a variety of statistical trend models can be expressed as (14) — perhaps with some added noise. The two parameters α and V_b will allow for some flexibility, and will depend, of course, on the overall model for the series. To get a closer look at that dependence, consider the particular case of (9), for which

$$\theta(B) = (1 - \theta_1 B)\,(1 - \theta_s B^s), \tag{20}$$

with $s = 12$. This is the so–called Airline Model, discussed in Box and Jenkins (1970), which has been found appropriate for many monthly macroeconomic series. The range for the parameters is given by $|\theta_1| < 1$ and $0 \leq \theta_{12} < 1$, in which case the model is invertible and accepts a decomposition as in (4), where the components have nonnegative spectra [Hillmer and Tiao (1982)].

Table 1 presents the values of the trend parameters α and V_b as functions of the parameters of the overall ARIMA. Since V_a is set equal to one, V_b is expressed as a fraction of V_a.

Table 1: Trend Model Parameters (Monthly Series)

a) Root α of the MA				b) Variance V_b of the innovation	
		θ_{12}		θ_{12}	
		.25	.75	.25	.75
	-.75	.892	.976	.255	.592
	-.25	.892	.976	.130	.302
θ_1	.25	.892	.976	.047	.109
	.75	.899	.976	.006	.012

A first striking result is the relative constancy of the MA trend root α. Its value is close to one, so that it nearly cancels out with one of the differences in model (14). It follows that the canonical trend implicit in the Airline Model is broadly similar to the model

$$\nabla p_t = (1 + B)\,b_t + \mu, \tag{21}$$

the difference being that μ changes very slowly in time. The interpretation of (21) is straightforward. Let x_t be the random walk

$$\nabla x_t = a_t,$$

with $V_a = 1$. Then it is easily seen that x_t can be expressed as in (1), with p_t given by (21), with $\mu = 0$ and $V_b = .25$, and n_t orthogonal white–noise with $V_n = .25$ (see fig. 1). Thus (21) represents the canonical trend in a random walk, in the sense that the latter is the former plus some orthogonal white–noise. (Notice that the random walk plus drift model has the same number of parameters as model (21).)

Therefore, for many series, the I(2) trend model (14) turns out to be surprisingly close to the I(1) model (21), closely related in turn to the random walk plus drift structure. It would be unlikely that sample information could distinguish between the roots $(1 - B)$ and $(1 - .95B)$. This indeed explains the fact that, when STS models, such as (15), are fitted, the estimator of the variance of v_t is quite frequently not significantly different from zero [see Harvey and Todd (1983), or Harvey and Peters (1990)]. In this case, β of equation (16) becomes 1, and the STS trend model yields directly the random walk plus drift model. Therefore, the I(2) versus I(1) paradox can be reasonably explained in many cases.

Be that as it may, there still remains the question of which specificatin [(I(1) versus I(2))] for the trend model should be used. The difference amounts to comparing the effect of adding a constant to the I(1) model versus imposing an additional difference and an MA factor with a root very close to 1. If the trend is directly estimated, the I(1) model plus constant is likely to be obtained, since estimation would treat the two roots as overdifferencing. The specification (21) is simpler than (14), but if the overall ARIMA is efficiently estimated, derivation of the I(2) trend is straightforward, and the close–to–one trend MA root brings no special analytical or numerical problems. Conceptually, model (14) is slightly more flexible, since it allows for a slow adaptive behavior of μ, without increasing the number of parameters. Yet, ultimately, the pretension of finding a unique, universally accepted, solution to the problem of modeling a trend seems unrealistic and possibly unnecessary. What is important is that the particular model used in an application be well specified, so that it can be properly understood and analysed, and that it agrees with the overall structure of the series.

An additional remark seems worth making: The paper is mostly concerned with monthly or quarterly data, and the models used are fundamentally short–term models (in fact, Box–Jenkins ARIMA models were meant for short–term analysis). While it is true that, in the short run, model (14) with $\alpha = .95$ and model (21) can be indistinguishable, in the very long run the differences can be large. The implication is that models built to capture the short–term behavior of a series will likely be unreliable tools for analysing the very long term [a similar point is made by Diebold and Rudebusch (1991)].

Back to the results of table 1, since the value of α varies little for different values of the θ-parameters, differences in the trend model (14) will be due to differences in the variance in the trend innovation, V_b. Table 1 shows that V_b is in fact very sensitive to changes in the θ–parameters. Since b_t generates the stochastic variability in the trend, small values of V_b are associated with stable trends, while large values will produce unstable ones. Given that the spectrum of p_t will be proportional to

V_b, a stable trend will denote a trend with a thin spectral peak, and hence a close to deterministic behavior. An unstable trend will display, on the contrary, a wide spectral peak, and hence more stochastic variability. It is seen in table 1 that, as θ_1 moves from -1 to 1, V_b decreases and the trend becomes closer to being deterministic, as could be expected. Figure 2 presents the spectra of x_t for the two extreme cases considered in table 1. For $\theta_1 = .75$, $\theta_{12} = .25$, the trend is very stable and the seasonal component is strongly stochastic, as can be seen from the width of the spectral peaks. For $\theta_1 = -.75$, $\theta_{12} = .75$, the stochastic character of x_t is dominated by the trend variation, while seasonality becomes more stable. As for the white-noise irregular, the minimum of $g_x(\omega)$ is larger for the first model, and hence the irregular component will be more important in the first case. Table 2 evidences the behavior of the two models. The stable trend–unstable seasonal case presents a small trend innovation variance and a large variance of the seasonal component innovation. Also, the MA root $(1 + .75B)$ implies, in the unstable trend–stable seasonal case, a very small irregular component.

Table 2: Innovation Variances: Two Extreme Cases

	$\theta_1 = .75$ $\theta_{12} = .25$	$\theta_1 = -.75$ $\theta_{12} = .75$
Variance of trend innovation	.006	.592
Variance of seasonal innovation	.222	.027
Variance of irregular	.249	.012

Figure 3 displays the trend spectra for the two extreme cases mentioned above. They have similar shapes and what varies considerably is the area under the curve. Comparison of fig. 2 and fig. 3 illustrates one of the reasons in favor of a flexible model–based approach: the model used for the trend, and the estimation filter this model implies, should depend on the particular series under consideration. It would clearly be inappropriate, for example, to use the filter implied by the stable trend model to capture the low frequency peak in the spectrum of the series with unstable trend; the trend would be grossly underestimated.

4 Extensions and Examples

When dealing with quarterly observations, $s = 4$ in expression (9), the identification conditions remain the same, and the canonical trend is, again, given by (14). When the MA polynomial of the overall model is of the type (20), computing the trend parameters α and V_b, table 1 is replaced by table 3. Both tables roughly tell the same story: the value of α is close to 1, and, if cancelled with one of the differences, the trend component in a random walk [model (21)] is obtained. The fact that the smallest seasonal frequency in monthly data $(\Pi/6)$ is closer to 0 than the corresponding one for quarterly data $(\Pi/2)$ constrains the trend spectrum of the monthly series to be closer to the ordinate axe; this explains the larger values of α

obtained in the monthly case. The values of V_b in table 3 are slightly larger than those of table 1, in accordance with the obvious fact that a quarterly series allows for less seasonal variation than a monthly series, and hence the relative contribution of the trend increases.

Table 3: Trend Model Parameters (Quarterly Series)

a) Root α of the MA			b) Variance V_b of the innovation	
	θ_4		θ_4	
	.25	.75	.25	.75
-.75	.709	.931	.318	.621
-.25	.710	.931	.163	.317
.25	.712	.931	.106	.203
.75	.718	.931	.062	.114

As mentioned before, the model for the trend should depend on the overall model for the observed series, which can be directly identified from the data. If (9) is replaced by

$$\nabla_s x_t = \theta(B)\, a_t, \tag{22}$$

or by

$$\nabla^2 \nabla_s x_t = \theta(B)\, a_t, \tag{23}$$

then, similar derivations to that in section 2 yield the canonical trend models

$$\nabla p_t = (1+B)\, b_t, \tag{24}$$

$$\nabla^3 p_t = (1-\alpha_1 B - \alpha_2 B^2)\,(1+B)\, b_t, \tag{25}$$

respectively. Therefore, the trend in (22) is the same as the trend in a random walk, an I(1) process, while the trend in (23) is I(3), with an MA(3) polynomial. To see an example of the latter, the model

$$\nabla^2 \nabla_{12} x_t = (1-.825B)\,(1-.787B^{12})\, a_t \tag{26}$$

explained well the monthly series of the Spanish consumer price index net of the energy component (for the period 1978–1988). Deriving the implied model (25) for the trend, $V_b = .204$ and the MA polynomial $(1-\alpha_1 B - \alpha_2 B^2)$ factorizes into $(1-.98B)\,(1-.825B)$. Both roots are close to 1, and hence model (26) can be expressed as

$$\nabla p_t = (1+B)\, b_t + \mu_0 + \mu_1 t,$$

where, both, μ_0 and μ_1 are parameters that change very slowly in time. Therefore, although the trend is theoretically I(3), again, it may be difficult to distinguish it from an I(1) model on the basis of sample information.

Figure 4 displays the spectra of the trend models for several examples; these are the following:

(a) The component of the consumer price index, i.e., the I(3) trend in model (26).

(b) The monthly series of Spanish tax revenues in the period 1979–1990, for which a model of the type (22) is appropriate. The I(1) trend is as in (24), with $V_b = .001$.

(c) The monthly series of the Spanish monetary aggregate for the period 1973–1985, discussed in Maravall (1988b). The overall model is of the type (9), with an I(2) trend given by

$$\nabla^2 p_t = (1 - .96B)(1 + B)\, b_t, \qquad V_b = .234.$$

(d) The consumer durable series in Harvey and Peters (1990). For this example, the trend estimated by Harvey and Peters with a frequency domain method is equivalent to an IMA(2,1) model of the type (16), with $\beta = .884$ and innovation variance .244. Removing the noise from this model yields a canonical trend with parameters $\alpha = .884$ and $V_b = .061$.

(e) The model–based approximation to the X11 trend of Cleveland and Tiao [model (17)]. From their results, it is straightforward to find that $V_b = .020$.

Although the shapes are somewhat similar, they certainly represent different trends, whose stochastic nature is strongly linked to the variance of the trend innovation. These different models may capture, thus, different stochastic properties of the series having to do with different low frequency spectral peaks (as fig. 2 clearly illustrated). It is worth noticing in fig. 4 the relative proximity of the trend models in X11, in the canonical trend hidden in the STS example, and in the monetary aggregate series example. The three spectra, moreover, lie in between those of the more stable I(1) trend of example (b), and the more stochastic I(3) trend of example (a).

5 The MMSE Estimator of the Trend

In practice, the trend component is unobserved, and we are always forced to work with estimators. Once the models have been specified, for stationary series the MMSE estimator of p_t can be obtained with the Wiener–Kolmogorov (WK) filter [see Whittle (1963)]. The filter extends easily to nonstationary series [Bell (1984) or Maravall (1988a)], and to finite realizations of the series by replacing unknown future and past observations with their forecasts and backcasts [Cleveland and Tiao (1976)]. Numerical computation, moreover, can be greatly simplified by using the Wilson algorithm described in Burman (1980). The WK filter provides a method to obtain the conditional expectation of the trend given the available series, equivalent to the usual Kalman filter [Harvey 1989)]. Both are computationally efficient; the WK formulation will allow us to derive easily the theoretical properties of the MMSE estimator.

Using the notation of section 2, and in particular the symbolic representation (8), the WK filter, for the case of a complete realization of the series (from $-\infty$ to ∞), is given by

$$\hat{p}_t = V_b \, \frac{\psi_p(B)\, \psi_p(F)}{\psi(B)\, \psi(F)} \, x_t, \tag{27}$$

where $F = B^{-1}$ is the forward operator. Replacing the ψ–polynomials by their rational expressions, after cancelling common factors, (27) becomes

$$\hat{p}_t = \nu(B,\, F)\, x_t = V_b \, \frac{\theta_p(B)\, \phi_n(B)}{\theta(B)} \, \frac{\theta_p(F)\, \phi_n(F)}{\theta(F)} \, x_t. \tag{28}$$

The filter $\nu(B,\, F)$ is seen to be centered at t, symmetric, and convergent [due to the invertibility of $\theta(B)$]. In fact, (28) shows that the filter is equal to the autocovariance–generating function of the stationary process

$$\theta(B)\, z_t = \theta_p(B)\, \phi_n(B)\, b_t. \tag{29}$$

(For a long enough series, since $\nu(B,\, F)$ is convergent in B and in F, in practice, the results for the infinite realization will apply to the central years of the series.)

To illustrate what the filter (28) does, and how it adapts itself to the series, fig. 5 plots the frequency domain representation of the trend filters $\nu(B,\, F)$ for the two extreme examples of section 3 and fig. 2. For $\omega = 0$ both present a gain of 1, and for all seasonal frequencies they present a gain of 0, associated with the seasonal unit roots in the MA part of (29). For the stable trend–unstable seasonal model, the filter is very close to the ordinate axe, captures very low frequency variations and little else. For the unstable trend–stable seasonal model, since most of the stochastic variation in the series is accounted for by the trend, the filter does not have to remove much, besides the zeros at the seasonal frequencies.

If, in (28), x_t is replaced with (6), the MMSE estimator can be expressed as a linear filter in the series innovations (a_t):

$$\hat{p}_t = \xi(B, F)\, a_t = V_b \, \frac{\theta_p(B)}{\phi_p(B)} \, \frac{\theta_p(F)\, \phi_n(F)}{\theta(F)} \, a_t,$$

and hence the theoretical trend estimator follows the model

$$\phi_p(B)\, \hat{p}_t = \theta_p(B)\, \eta_p(F)\, a_t, \tag{30}$$

where $\eta_p(F)$ is the forward filter:

$$\eta_p(F) = V_b \frac{\theta_p(F)\,\phi_n(F)}{\theta(F)}$$

Comparing (30) with model (2) for the trend, it is seen that:

(1) As has been often pointed out, the model for the theoretical component and that for its theoretical MMSE estimator are not the same (even for an infinite realization of the series). The dynamic properties of the component and of its MMSE estimator are structurally different.

(2) From (30) or (29) it is seen that the trend estimator depends not only on the trend model, but also on the nontrend component . For example, due to the (nontrend) roots in $\eta(F)$, the trend estimator may display oscillatory behavior when the trend component model contains none.

(3) Both, the component and its estimator, share the same stationarity inducing transformation.

(4) The models for the component and for the estimator contain the same polynomials in B.

(5) The difference between the two models is due to the presence of the forward polynomial $\eta_p(F)$ in the model for the estimator. This polynomial expresses the dependence of $\hat{p}_t$ on future values of x, and will cause revisions in preliminary estimators as new observations (and hence new innovations $a_{t+k},\ k > 0$) become available.

(6) When some of the nontrend AR roots of the overall model have unit modulus (i.e., when $\phi_n(B)$ contains some unit roots), then the filter (28) and the model (30) for the trend estimator will be noninvertible. This will be, for example, the case whenever the series presents nonstationary seasonality. Thus, in particular, the class of models (9), (22), and (23), as well as the basic STS model and the model version of X11, all contain the nonstationary seasonal AR polynomial S given by (10), so that the corresponding trend estimator will be noninvertible.

For example, for the class of models given by (9) with $s = 12$, the MMSE estimator (30) becomes

$$\begin{aligned} \nabla^2 \hat{p}_t &= (1-\alpha B)\,(1+B)\,\eta_p(F)\,a_t, \\ \eta_p(F) &= V_b\,\frac{(1-\alpha F)\,(1+F)\,(1+F+\ldots+F^{11})}{\theta(F)}. \end{aligned}$$

When $\theta(B)$ is given by (20), figs. 6 and 7 present the spectra of the models for the trend and its estimator, for the two extreme examples we have been considering.

In the first case (stable trend–unstable seasonal), fig. 6a shows that the spectrum of the trend estimator is similar to that of the component, with a slightly narrower band width for the low frequency peak. In the unstable trend–stable seasonal case in fig. 7, the spectrum of the trend follows closely that of the theoretical component, except for noticeable dips at the seasonal frequencies (as shown in fig. 6b, when fig. 6a is amplified, similar dips are found).

The departures from the theoretical component model implied by MMSE estimation are easily interpreted in the frequency domain. From (27), the spectrum of the estimator, $g_{\hat{p}}(\omega)$, is given by

$$g_{\hat{p}}(\omega) = R^2(\omega)\, g_x(\omega), \tag{31}$$

where

$$R(\omega) = \frac{g_p(\omega)}{g_x(\omega)} = \frac{1}{1 + 1/r(\omega)}, \tag{32}$$

and $r(\omega) = g_p(\omega)/g_n(\omega)$. Since the trend is the signal of interest, $r(\omega)$ represents the signal–to–noise ratio. Therefore, when estimating the trend, what the MMSE method does is, for each ω, to look at the relative contribution of the theoretical trend to the spectrum of the series. If this relative contribution is high, then $r(\omega)$, and hence $R(\omega)$, will also be high, and the frequency will be mostly used for trend estimation. For example, for $\omega = 0$, $r(0)$ goes to ∞ and $R(0) = 1$. This implies that the zero frequency will only be used for estimation of the trend. If the nontrend component contains seasonal nonstationarity, then, for ω equal to the seasonal frequencies, $r(\omega)$ and $R(\omega)$ become both zero, and these frequencies are ignored when estimating the trend. Considering (31), these zeroes produce the dips in the spectra of figs. 6 and 7.

From (31) and (32), the relationship between the spectrum of the theoretical trend component and that of its MMSE estimator is found to be

$$g_{\hat{p}}(\omega) = R(\omega)\, g_p(\omega) = \frac{1}{1 + 1/r(\omega)}\, g_p(\omega).$$

Since $r(\omega) \geq 0$, it follows that $0 \leq R(\omega) \leq 1$, and hence $g_{\hat{p}}(\omega) \leq g_p(\omega)$ for every frequency, so that the trend estimator will always underestimate the theoretical trend. This is clearly seen in figs. 6 and 7, and in table 4, which compares the variance of the stationary transformation of the trend and of its estimator, for different values of the θ–parameters. The estimator always has a smaller variance, and the ratio of the two variances get further away from 1 as the trend becomes more stable. Therefore, the more stochastic the trend is, the less will its variance be underestimated (in relative terms). On the contrary, the variations of a very stable trend will be grossly underestimated. In summary, the estimator provides a more stable trend than the one implied by the theoretical model, and this effect will be particularly strong when the theoretical trend is already stable.

The difference between the dynamic properties of the theoretical component and its MMSE estimator can also be assessed in the time domain by comparing the

Table 4: Variance of the Stationary Transformation:
Trend Component and Estimator

θ_{12}	0.25				0.75			
θ_1	-0.75	-0.25	0.25	0.75	-0.75	-0.25	0.25	0.75
Trend Component	0.460	0.235	0.085	0.011	1.157	0.590	0.213	0.024
Estimator	0.212	0.078	0.014	<0.001	0.956	0.349	0.058	0.001

correlations. In section 3 it was seen how, for reasonable values of the parameter θ_1 and θ_{12}, the model for the trend derived from the Airline Model is an IMA(2, 2), with MA roots $B = -1$ and $B \doteq 1/.9$, or, approximately

$$\nabla^2 p_t = (1 + .1B - .9B^2)\, b_t.$$

The Autocorrelation Function (ACF) of $\nabla^2 p_t$ is, thus, $\rho_1 \doteq .01$, $\rho_2 \doteq -.49$, and $\rho_k = 0$ for $k > 2$. Table 5 compares this ACF with that of the estimator for different values of the θ-parameters, and for a few selected lags. The departures from the component ACF can be substantial, particularly for large values of θ_1 (associated with small values of V_b). Therefore, more stochastic trends (with large values of V_b) will have estimators more in agreement with the ACF of the component. It is worth noticing that, although for the theoretical trend $\rho_{12} = 0$, the MMSE always displays a negative value which can be quite substantial.

Table 5: Autocorrelations of the Stationary Transformation:
Trend Component and Estimator

Estimator ($\nabla^2 \hat{p}_t$)					
θ_{12}	θ_1	ρ_1	ρ_2	ρ_3	ρ_{12}
0.25	-0.75	0.04	-0.50	-0.01	-0.37
	-0.25	0.18	-0.52	-0.14	-0.37
	0.25	0.37	-0.37	-0.30	-0.37
	0.75	0.61	0.05	-0.11	-0.36
0.75	-0.75	0.03	-0.52	-0.01	-0.13
	-0.25	0.16	-0.54	-0.15	-0.13
	0.25	0.35	-0.41	-0.33	-0.13
	0.75	0.56	-0.07	-0.23	-0.14
Theoretical Component ($\nabla^2 p_t$)		0.01	-0.49	—	—

When tables 4 and 5 are put together, the following result emerges: If the series is the sum of two components with varying degrees of stability, the distortion in the variance and ACF of the components induced by MMSE estimation is stronger for the more stable one. For example, when most of the stochastic variation in the series is attributable to the presence of a stochastic trend, the underestimation of the variance and the induced autocorrelations in the estimator will be relatively minor; when the stochastic trend accounts for little, the distortions can be remarkably large. This result is somewhat comforting: the distortions are large when the component matters little.

The underestimation of the component variance when using the MMSE estimator implies that, although the components are uncorrelated, their estimators will not be so, and crosscorrelations between them will appear. Table 6 displays the contemporaneous crosscorrelations between the trend estimator on the one hand, and the seasonal and irregular estimators on the other, for the case of the Airline Model with different values of the θ–parameters. Although none of the correlations is large, they are not negligible for relatively large values of θ_1.

Table 6: Crosscorrelation Between Estimators of Different Components

	Trend and Seasonal Estimators		Trend and Irregular Estimators	
θ_{12} =	0.25	0.75	0.25	0.75
θ_1 = -0.75	-0.14	-0.08	0.17	0.17
-0.25	-0.15	-0.08	0.08	0.08
0.25	-0.17	-0.10	-0.10	-0.09
0.75	-0.22	-0.12	-0.31	-0.30

One of the by–products of a model–based approach is that theoretical values of the estimator auto and crosscorrelations can be easily derived from the models. Comparison between these values and the ones obtained empirically in an application can provide a useful element for diagnosis [Maravall (1987)].

6 Some Implications for Econometric Modeling

The distinction between theoretical component and estimator, and the different dynamic properties they exhibit, has been on occasion a source of confusion in applied econometric work. Since the component is never observed, we can only use estimators; this has some important implications, which we proceed to discuss.

Consider the full decomposition into trend, seasonal, and irregular components given by (4). Denote by x_t^a the seasonally adjusted (SA) series, equal to

$$x_t^a = x_t - s_t = p_t + u_t.$$

If, in (1), p_t is replaced by x_t^a, and n_t by s_t, most of the previous discussion remains valid. Since x_t^a is obtained by adding a stationary component to p_t, if

$$\phi_a(B)\, x_t^a = \theta_a(B)\, d_t$$

denotes the model for the SA series, it follows that $\phi_a(B)$ and $\phi_p(B)$ have the same unit roots, and this will also be true of $\phi_s(B)$ and $\phi_n(B)$. In fact, the models for p_t and x_t^a often are similar, although the addition of u_t will remove the canonical property from the SA series [it is indeed the case that x_t^a tends to be closer to the "random walk plus drift" structure than p_t; see Maravall (1988b)].

The estimator of the SA series, $\hat{x}_t^a$, is obtained, in a manner similar to that described in section 5 for estimation of the trend, with a centered, symmetric, and convergent filter. Expressing $\hat{x}_t^a$ in terms of the series innovations, results (1) to (6) in section 5 remain basically unchanged. I shall continue to center the discussion on the trend estimator, bearing in mind that it is also valid for the estimator of the SA series.

Let $X_T = (x_1, \ldots, x_T)$ denote the observed series. Since the filter $\nu(B, F)$ in (28) is convergent in both directions, we may assume that it can be safely truncated at some point. Let the number of terms in the truncated filter be $(2\,k+1)$. When $T-t < k$, the estimator of p_t (obtained by extending the series with forecasts) is not the estimator (30), but a preliminary one, which will be revised as new observations become available. As a consequence, the last k values of the series $[\hat{p}_1, \ldots, \hat{p}_T]$, computed with X_T, will be preliminary estimators; in a symmetric manner, the first k values require backcasts of the series, and will also differ from the estimator (28), with stochastic structure given by (30). The final estimator (28) will be denoted the "historical" estimator.

The preliminary estimators $(\hat{p}_1, \ldots, \hat{p}_k, \hat{p}_{T-k}, \ldots, \hat{p}_T)$ have different stochastic structures [see, for example, Pierce (1980) and Burridge and Wallis (1984)], different in turn from that of the historical estimator. Only the central values $(\hat{p}_{k+1}, \ldots, \hat{p}_{t-k})$ can properly be assumed to be homogenous, being generated by the same process (30). The model–based approach permits us to compute the variance of any revision in the estimator of p_t, and hence the value of k after which the revision can be neglected. For the range of models considered in section 3, it is found that, after three years of additional data, between 82 and 100% of the revision in the concurrent estimator of the trend has been completed; for X11 the revision variance decreases by 95%. [It can be seen that slower rates of convergence to the historical estimator (i.e., large values of k) are associated with smaller revisions; when the revision is large, the estimator tends to converge fast.] Although the trend estimator converges always faster, a similar result holds for the SA series: After 3 years of additional data, for the models of section 3, between 80 and 100% of the revision variance has disappeared; for X11 the proportion is roughly 80%.

Therefore, when fitting models to unobserved components, care should be taken to use an homogenous series of historical estimators. Although well–known, this is an often neglected correction. But, even if a homogenous series of historical estimators is used, there remain problems in some standard econometric practices having to do with fitting models to series of unobserved component estimators. In order to analyse these problems, consider again the decomposition (4), with nonstationary trend and seasonal components, and stationary irregular component. Thus, $\phi_p(B)$ will contain the factor ∇^d, with $d \geq 1$, and $\phi_s(B)$ will contain the factor S with seasonal unit roots, typically given by (10). Thus we can write

$$\nabla^d p_t = \lambda_p(B)\, b_t, \tag{33}$$

$$S s_t = \lambda_s(B)\, e_t, \tag{34}$$

where $\lambda_p(B)\, b_t$ and $\lambda_s(B)\, e_t$ are stationary processes. Finally, let

$$u_t = \lambda_u(B)\, v_t \tag{35}$$

represent the stationary and invertible irregular component (in macroeconomics, often called the cyclical component). It is easily seen that (33), (34) and (35) imply that the overall model for the series can be expressed as

$$\nabla^d S\, x_t = \lambda(B)\, a_t, \tag{36}$$

where $\lambda(B)\, a_t$ is also a stationary and invertible process. (Invertibility is guaranteed by the fact that the component u_t is invertible.)

Applying the WK filter and replacing x_t with (36), the model for the trend estimator can be finally expressed as

$$\nabla^d \hat{p}_t = \alpha_p(B,F)\, S(F)\, a_t, \tag{37}$$

where $\alpha_p(B,F) = V_b\, \lambda_p(B)\, \lambda_p(F)/\lambda(F)$ is a convergent polynomial in F and in B.

Consider now the decomposition of the series:

$$x_t = x_t^a + s_t,$$

Since $x_t^a = p_t + u_t$, and u_t is stationary, the model for the SA series can be expressed as

$$\nabla^d x_t^a = \lambda_a(B)\, d_t, \tag{38}$$

where $\lambda_a(B)\, d_t$ is a stationary and invertible process. Proceeding as in the case of the trend estimator, the MMSE estimator of x_t^a can be expressed as

$$\nabla^d \hat{x}_t^a = \alpha_a(B,F)\, S(F)\, a_t, \tag{39}$$

where $\alpha_a(B,F) = V_d\, \lambda_a(B)\, \lambda_a(F)/\lambda(F)$ is also a polynomial convergent at both extremes.

Consider, finally, the estimator of u_t, the seasonally adjusted and detrended seris (i.e., the irregular, or cyclical, component). From (35) and (36) it is obtained that the MMSE historical estimator $\hat{u}_t$, expressed as a function of the series innovations, can be written as

$$\hat{u}_t = \alpha_u(B, F)\,(1-F)^d\,S(F)\,a_t, \tag{40}$$

where $\alpha_u(B, F) = V_v\,\lambda_u(B)\,\lambda_u(F)/\lambda(F)$. Expressions (37), (39), and (40) imply that the historical estimators of the trend, of the SA series, and of the irregular component are all noninvertible series. The model for the three series contains the unit seasonal roots in the MA polynomial; for the series $\hat{u}_t$, the noninvertible factor $(1-F)^d$ is also present.

More generally, it will be the case that the estimator of an unobserved component will be noninvertible whenever at least one of the other components present in the series is nonstationary: the unit AR roots of this other component will appear in the MA polynomial of the model for the estimator. This result is valid for AMB methods, independently of whether the components are canonical or not. It applies equally to unobserved components obtained with STS methods, and, in general, to the estimators obtained with any filter that can be given an MMSE model-based interpretation (for symmetric filters, used for historical estimation, this requirement is relatively minor). As an example, for the model version of X11, fig. 8 presents the spectrum of the SA series estimator, which contains the zeros at seasonal frequencies; fig. 9 displays the nonconvergent weights of the (one-sided) AR representation of the SA series estimator.

In summary, whenever some nonstationarity is removed from the series in order to estimate a component, the estimator will be noninvertible. How restrictive can we expect this condition to be? Consider the estimators of p_t, x_t^a, and u_t. In order to obtain the three, the seasonal component has been removed. If seasonality is stochastic, the sum of the seasonal component over a year span (i.e., $S\,s_t$) cannot be expected to be always exactly zero, since seasonality is moving. Still, the very concept of seasonality implies that its annual aggregate effect should not be too far from zero. This can be formally expressed as

$$S\,s_t = w_t, \tag{41}$$

where w_t is a zero–mean, stationary process (with small variance). Expression (41) is equivalent to (34); unit AR roots are thus a sensible requirement for a stochastic seasonal component. In fact, expression (41), with different assumptions for the stationary process w_t, is overwhelmingly used as the specification of the seasonal component in model–based methods or model–based interpretation of filters [see Maravall (1989)].

From the previous discussion, two important implications follow:

A) Testing for unit roots in economic time series is often performed on seasonally adjusted data [examples can be found in Campbell and Mankiw (1987), Cochrane (1988), Perron and Phillips (1987), Schwert (1989), and Stock and Watson (1986)]. Comparing (33) with (37), or (38) with (39), it is seen that the distributions of the (stationary transformation of the) component and of its estimator are different, and this will have an effect on unit root testing, as detected by Ghysels (1990) and Ghysels and Perron (1992) for the case of the seasonally adjusted series. The lag polynomials in the model for the estimator are more complex and of larger orders than those in the component model; their converging properties will be different, and so will be, for example, the spectral density at the zero frequency. In particular, for the important case in which nonstationary stochastic seasonality has been removed from the series (and that is what most moving average seasonal adjustment methods do), the estimator of the trend and SA series are noninvertible, and will not accept an AR representation. Therefore, unit root tests that rely on AR representations, such as the Augmented Dickey Fuller type [Dickey and Fuller (1979)], should not be carried on SA series, nor on trends, nor, in general, on any unobserved component estimator when the series contains another component which is nonstationary.

As for the Phillips (1987) type test, it is worth noticing that, as reported by Phillips and Perron (1988) and by Schwert (1989), the test performs very poorly when the MA part of the series model contains a root close to -1. Since $B = -1$ is a root of $S(F)$, the noninvertible MA polynomial in the estimator model (37) or (39), one could expect a poor performance of the test when SA series or trends are used. For the first case, this is in fact what Ghysels and Perron (1992) find.

B) In the same way that a noninvertible series does not accept a univariate AR representation, since the minimum of a VAR spectrum is strictly positive, no individual series in a VAR representation can be noninvertible. Again, thus, when the series contains nonstationary stochastic seasonal and trend components, expressions (37), (39) and (40) imply that VAR models would not be appropriate to model SA series, trends, or irregular (cyclical) components, not even as approximations. It follows, moreover, that these filtered series should not be used either in a Johansen–type test for cointegration. Fitting VAR models to SA series is, however, a standard practice, often aimed at reducing dimensionality of the VAR model. Among many examples, some important ones are Blanchard and Quah (1989), Lütkepohl (1991), Hendry and Mizon (1990), Sims (1980), and Stock and Watson (1991).

Notice that, when the series is simply the sum of a stochastic trend and a stationary component, the same conclusion applies to the detrended series (obtained, for example, with the Hodrick and Prescott filter). Due to the presence of the MA factor $(1 - F)^d$, the detrended series will accept no convergent AR (or VAR) representation.

In the previous discussion, noninvertibility (i.e., the lack of a convergent AR representation, or, equivalently, the presence of a zero in the spectral density) is a property of the theoretical model that generates the estimator of the unobserved component. Of course, a particular finite realization from that model will not exactly satisfy it. This does not invalidate, however, the result that AR and VAR models, when applied to series from which a nonstationary stochastic component has been filtered out, contain a specification error. This error is due to the finite truncation of a nonconvergent filter. The departure from noninvertibility due to sample variation, mixed with the fact that (often) the SA series used is not the homogenous historical estimator, but contains preliminary estimators at both ends (as discussed at the beginning of section 6), may well explain why, in practice, the specification error may easily pass undetected.

7 Summary and Conclusions

The paper deals with modeling and estimation of linear stochastic trends in economic time series, with a higher than annual frequency of observation (the series may, thus, contain seasonality). A model-based approach is followed, and section 1 describes the general framework. The observed series is the sum of a trend and an orthogonal nontrend component, where the components (and hence the series) follow ARIMA models. The model for the trend is aimed at capturing the low-frequency peak in the series spectrum; since different series will exhibit different peaks, the model for the trend should depend on the particular series being analysed, i.e., on the overall ARIMA model. (This point is illustrated throughout the paper.)

A fairly general class of models is considered: The one for which a transformation of the type $\nabla^d \nabla_s$ produces a stationary and invertible process ($d = 0, 1, 2$; s denotes the number of observations per year.) The nonstationary part of the trend model is then ∇^{d+1}, and the stationary part depends, on the rest of the overall model, and on some additional assumptions, needed to identify a unique decomposition. It is seen how different identification assumptions yield different models for the trend. Section 2 presents some of the trend models most frequently encountered in the statistics literature (or the model interpretation of some trend estimation filters), looks at their properties and performs some comparisons. All the trend models considered can ultimately be expressed as the canonical trend obtained in an ARIMA-model-based method, plus some orthogonal white-noise.

An apparent paradox is that, while most statistical trend models (or filters) imply I(2) trends, I(1) trends are typically used in econometrics. Section 3 looks in more detail at a particular class of models for the observed series (with $\nabla \nabla_{12}$ as the stationary transformation), often reasonably appropriate for monthly macroeconomic series. The trend models obtained for different values of the parameters are compared; they are IMA(2,2) models and present the common feature that the MA part always contains a root very close to 1. Thus, from an estimation point of view, the I(2) trend would be indistinguishable from an I(1) model with drift. Section 4 extends the analysis in several directions: first, to quarterly data, and, second, to different values of d. It is concluded that the "order of integration" paradox is more apparent than real. Finally, some of the advantages and disadvantages of using the "I(1) plus drift" versus the I(2) specification are discussed.

In the model-based approach there is an important distinction between the theoretical (unobserved) component and its estimator. The MMSE estimator is obtained with the Wiener-Kolmogorov filter applied to the observed series, and its properties are discussed in section 5. It is seen that the dynamic properties of the component and of its MMSE estimator are structurally different, and that the estimator will follow a model different from that of the theoretical component. The stationary transformation is preserved, but the spectrum and ACF will be different; for the estimator, they will depend on the structure of the nontrend component.

The estimator always yields a more stable trend, and this effect is more pronounced when the theoretical trend is already stable.

It is further seen that, when the nontrend component contains some nonstationarity, the estimator of the trend and of the SA series is always noninvertible. Thus, for example, seasonal nonstationarity in the series implies that no convergent AR representation for the estimator of the trend, the SA series, or the irregular (or cyclical) component will exist. Heuristically, noninvertibility of these estimators will appear whenever the series has a stochastic seasonal component with the (most reasonable) property that the seasonal effect aggregated over a year is not far from zero. The result extends to estimators obtained with (noninvertible) "ad hoc" filters, such as the ones most often used to detrend or seasonally adjust economic series.

Since, in practice, we are forced to always work with estimators, section 6 presents two important implications for econometric work: First, due to the symmetry of the filters, care should be taken to use an homogenous series of historical estimators. Perhaps more importantly, when the original series contains stochastic nonstationary trend and seasonal components, first, contrary to common practice, some popular unit root tests cannot be carried on SA series, nor on trends; second, contrary to standard practice, too, VAR models are not appropriate for modeling SA series, nor trends, nor detrended series.

Thanks are due to Fabio Canova, Grayham Mizon, Annalisa Fedelino, the Editor and four referees for their helpful comments. Some referees showed interest in computational details. All calculations have been made with a program I have baptized SEATS, which stands for "Signal Extraction in Arima Time Series". The program originated from one developed by J. Peter Burman, for seasonal adjustment, at the Bank of England (1982 version); to him, thus, very special thanks are due. The program (jointly with the user manual) can be made available upon request.

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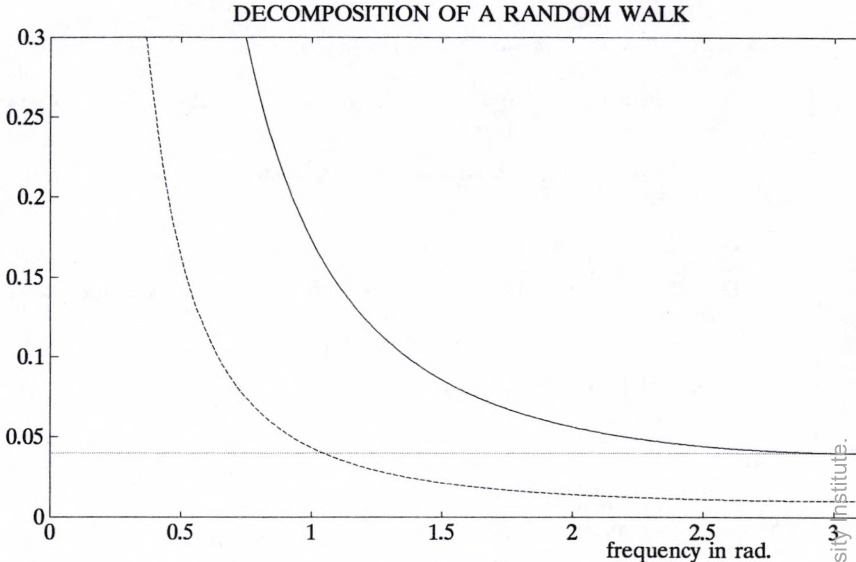

Figure 1

——— random walk
---- trend
.... noise

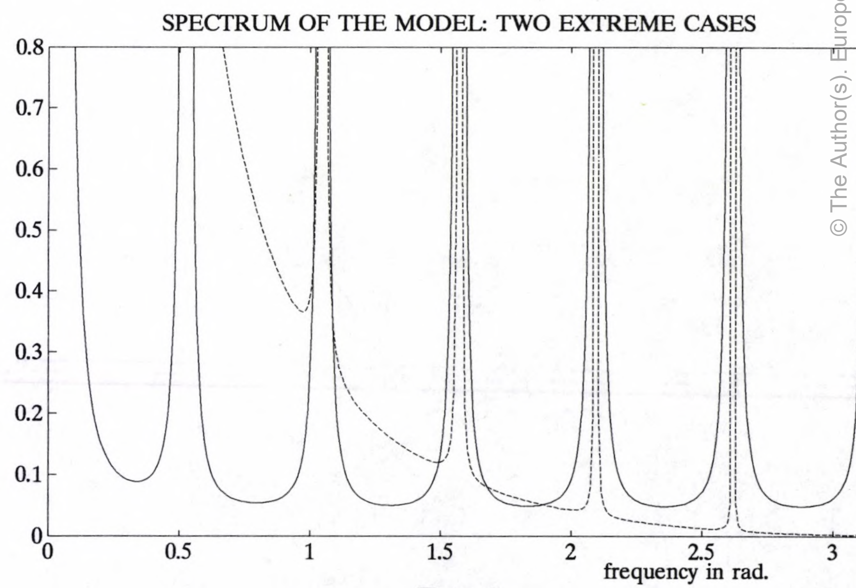

Figure 2

——— stable trend-unstable seasonal
---- unstable trend-stable seasonal

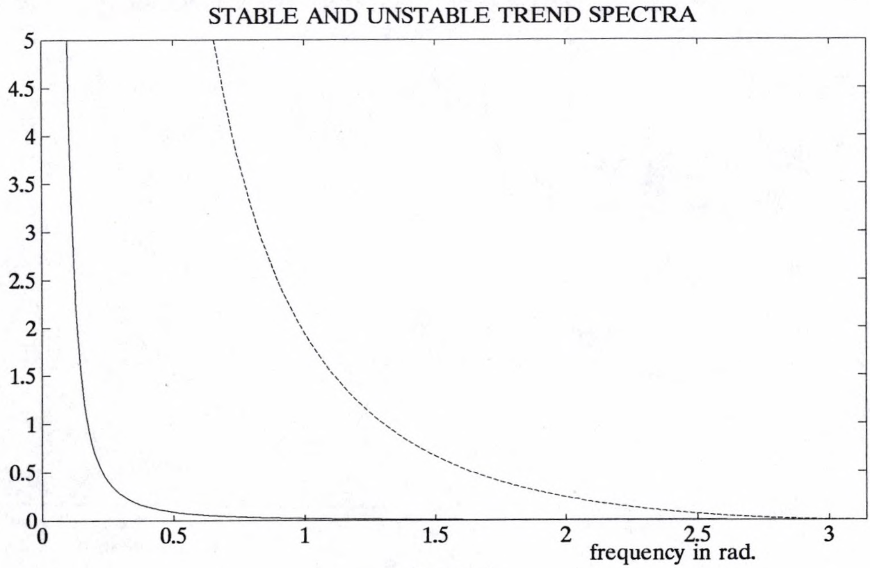

Figure 3

____ stable trend

---- unstable trend

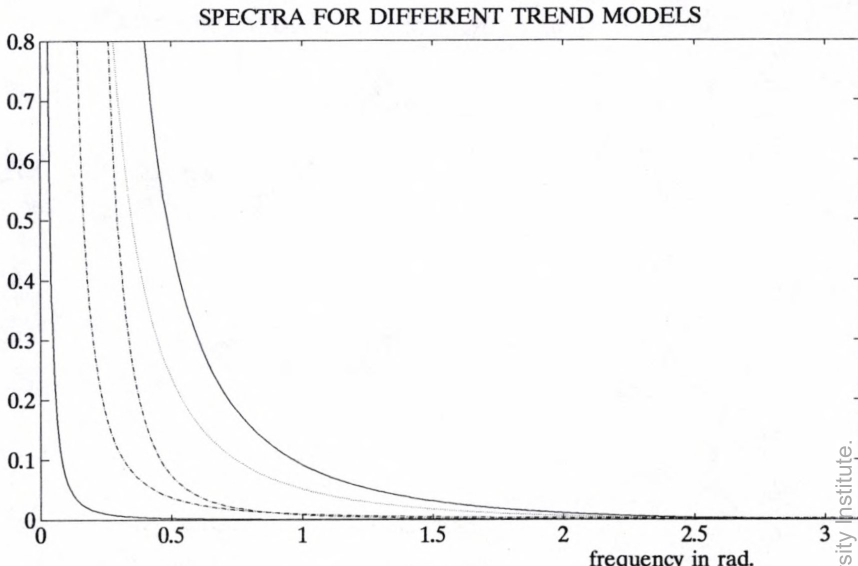

Figure 4

——— component of the sonsumer price index series [example (a)]

— — tax revenue series [example (b)]

..... monetary aggregate series [example (c)]

-.-.- consumer durable series [example (d)]

----- model approximation to X11 [example (e)]

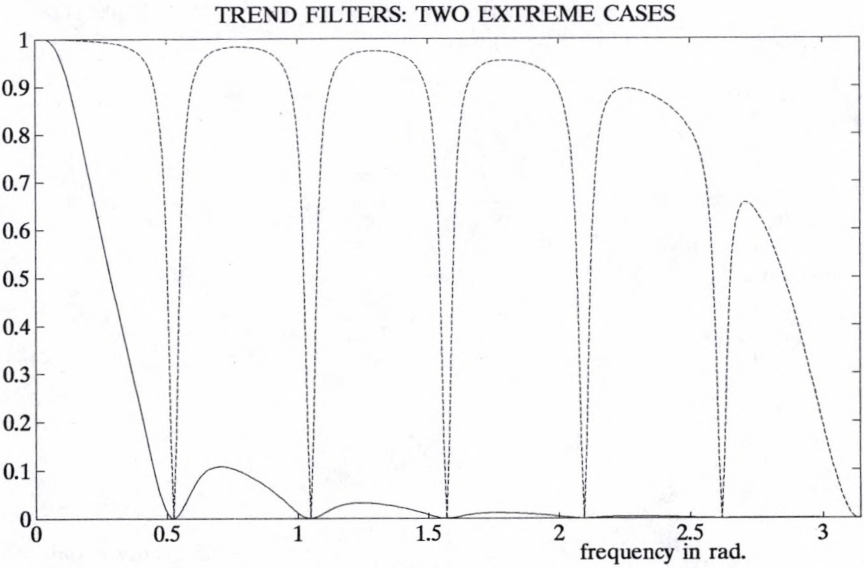

Figure 5

——— stable trend-unstable seasonal
----- unstable trend-stable seasonal

STABLE TREND SPECTRUM: MODEL AND ESTIMATOR

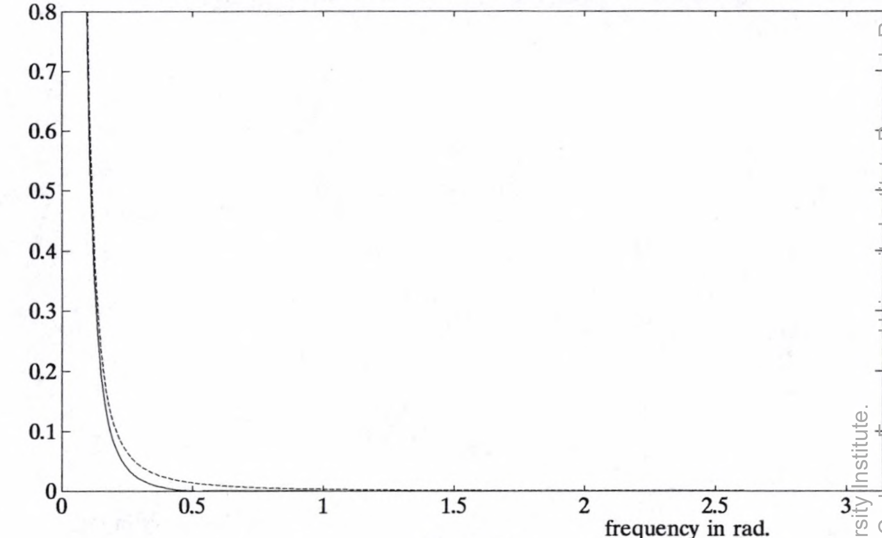

Figure 6.a

——— trend estimator
----- trend component model

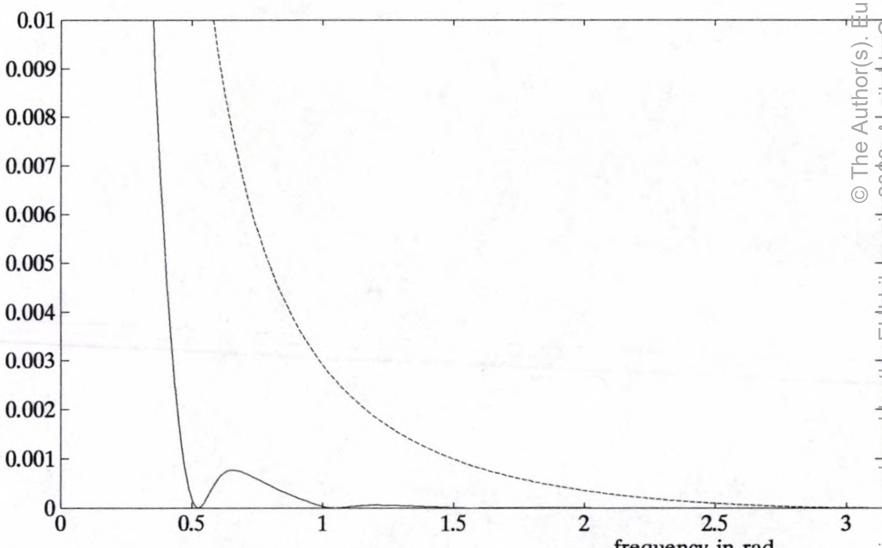

Figure 6.b

——— trend estimator
----- trend component model

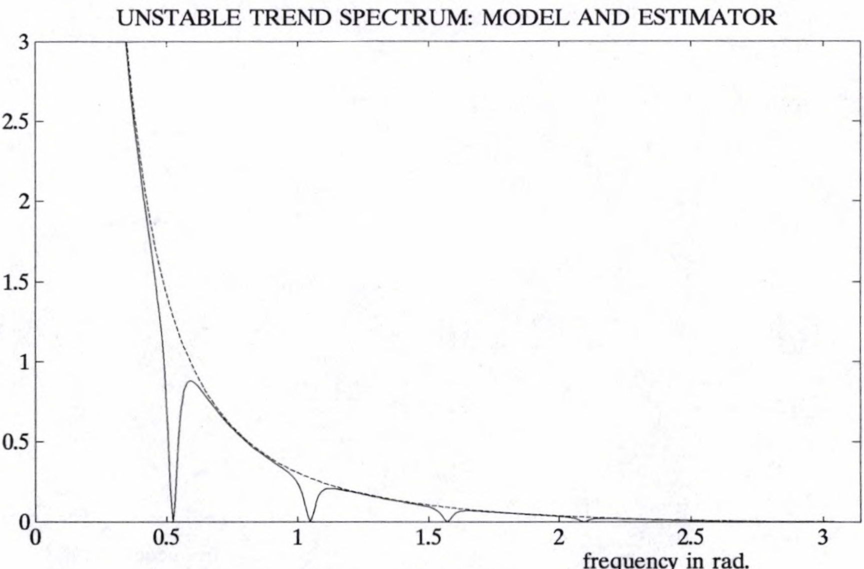

Figure 7

```
———— trend estimator
----- trend component model
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X11 SA SERIES SPECTRUM: MODEL AND ESTIMATOR

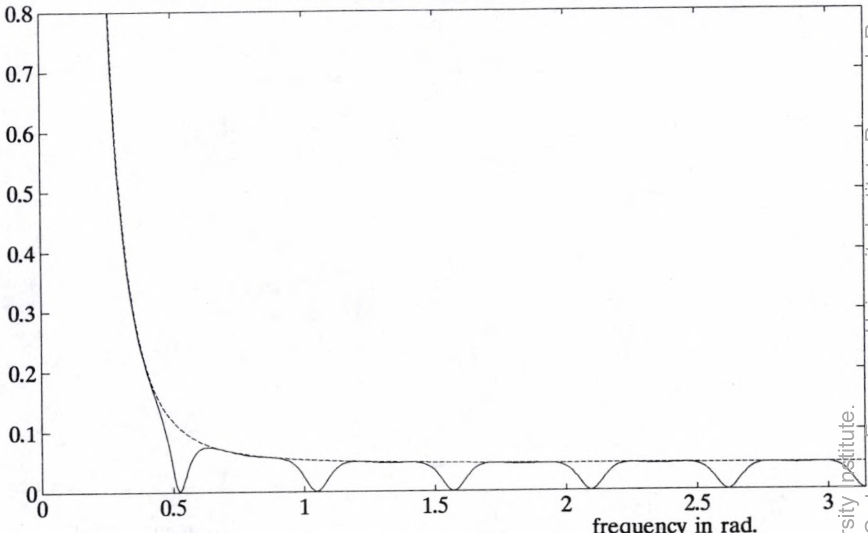

Figure 8

X11 SA SERIES ESTIMATOR: ONE-SIDED AR WEIGHTS

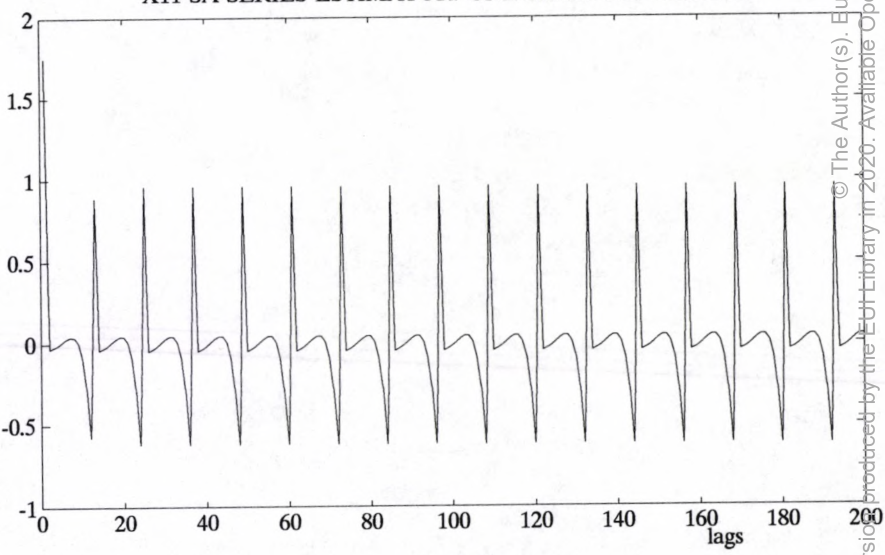

Figure 9

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