EUI WORKING PAPERS IN ECONOMICS

EUI Working Paper ECO No. 92/88

Encompassing Univariate Models in Multivariate Time Series: A Case Study

AGUSTÍN MARAVALL
and
ALEXANDRE MATHIS

European University Institute, Florence

Please note
As from January 1990 the EUI Working Paper Series is divided into six sub-series, each sub-series is numbered individually (e.g. EUI Working Paper **LAW** No. 90/1).

EUROPEAN UNIVERSITY INSTITUTE, FLORENCE

ECONOMICS DEPARTMENT

EUI Working Paper **ECO** No. 92/88

Encompassing Univariate Models in Multivariate Time Series: A Case Study

AGUSTÍN MARAVALL
and
ALEXANDRE MATHIS

BADIA FIESOLANA, SAN DOMENICO (FI)

Printed in Italy in July 1992
European University Institute
Badia Fiesolana
I – 50016 San Domenico (FI)
Italy

May 1992

ENCOMPASSING UNIVARIATE MODELS IN MULTIVARIATE TIME SERIES: A CASE STUDY

Agustín Maraval
European University Institute
Florence

Alexandre Mathis
O.F.C.E.
Paris

Abstract

Through the encompassing principle, univariate ARIMA analysis could provide an important tool for diagnosis of VAR models: The univariate ARIMA models implied by the VAR should explain the results from univariate analysis. This comparison is seldom performed, possibly due to the paradox that, while the implied ARIMA models typically contain a large number of parameters, univariate analysis yields highly parsimonious models.

Using a VAR application to six French macroeconomic variables, it is seen how the encompassing check is straightforward to perform, and surprisingly accurate. The VAR model explains univariate analysis, and the gain from multivariate modelling can be properly attributed to relationships among the variables. Finally, the univariate and VAR models are used to measure the persistence (or long-term effect) of shocks on the macro variables considered. Again, inferences based on univariate models are encompassed by the VAR, although, on occasion, inference based on univariate analysis can be misleading.

Please address correspondence to
Professor A. Maravall, Department of Economics, European University Institute, Badia Fiesolana, I-50016 San Domenico di Fiesole (FI), Italy. (FAX no.: +39-55-50.92.202)

1 Introduction

After the crisis of traditional structural econometric models, a particular multivariate time series specification, the Vector Autoregression or VAR model, has become a standard tool used in testing macroeconomic hypotheses. Zellner and Palm (1974, 1975) showed that the reduced form of a dynamic structural econometric model has a multivariate time series model expression, and that this relationship could be exploited empirically as a diagnostic tool in assessing the appropriateness of a structural model. As Hendry and Mizon (1990) state, a well specified structural model should encompass the results obtained with a VAR model; similar analyses are also found in Monfort and Rabemananjara (1990), Clements and Mizon (1991), and Palm (1986).

It is also well known that a multivariate time series model implies a set of univariate models for each of the series. Thus, as argued by Palm (1986), univariate results can, in turn, provide a benchmark for multivariate models, and should be explained by them. When done, the comparison usually takes the form of comparing the forecasting performances of the multivariate model versus the set of univariate models, identified with Box-Jenkins (1970) techniques [see Palm (1983)]. More generally, however, since the multivariate model implies a set of univariate models, these should be derived from the fitted multivariate one, and then compared with the models obtained through univariate analysis. If the two sets of univariate models are clearly different, then there is reason to suspect specification error in some of the models. Given that, in general, direct identification of the univariate model is simpler than identification of the multivariate one, lack of agreement between the two sets of univariate models may well indicate misspecification of the multivariate model and invalidate, as a consequence, its use in testing economic hypotheses.

Therefore, the use of univariate models as a diagnostic tool should include the comparison between the univariate models derived from the multivariate one and those obtained with univariate analysis (we shall refer to them as "implied" and "estimated" univariate models). This comparison, however, is seldom done. Univariate analysis is used (often wrongly) in identification of multivariate models [see, for example, Jenkins (1979), and Maravall (1981)]; it is hardly ever used (as it rightly should) in the diagnostics stage. Perhaps this is due to what Rose (1986) has termed "the autoregressivity paradox", which can be described as follows:

It is a well-known fact that the immense majority of ARIMA models fitted to economic series are parsimonious, including few parameters. Yet even relatively small multivariate models imply univariate models with a very large number of parameters. Therefore, if the world is multivariate (as it is), ARIMA models should be highly unparsimonious, and hence of little practical use. Yet we know that this is not the case. How can the two facts be reconciled? Rose (1986, 1987) suggests an explanation: macroeconomic variables are basically contemporaneously correlated and there are few dynamic relationships among them. The explanation is a bit drastic, and it seems sensible to seek for some alternative one. As pointed out by Wallis (1977), two possibilities come to mind: First, it may happen that the autoregressive (AR) and moving average (MA) polynomials of the implied ARIMA model have roots in common. Cancelling them out, the order of the model would be reduced. Second, some of those two polynomials may contain a large number of small coefficients, that would be undetectable for the sample size used. The first possibility will be denoted the "root effect", and the second, the "coefficient effect".

Although both effects are certainly possible, the question remains of whether they can be measured with enough accuracy in actual applications. For example, the autoregressive coefficient estimates in VAR models are, on occasion, unstable, and the roots of the polynomials are sensitive to small variations in those coefficients. That factor might have an effect on the detection of common roots. Furthermore, it is an empirical fact that often the factorization of the determinant of the AR matrix in VAR models yields roots with relatively large modulus. This might affect the presence of small coefficients in the implied univariate representation.

Yet the issue of whether the root and coefficient effect can be actually detected, so as to simplify an ARMA model with perhaps 40 or 50 parameters to an ARMA model with (at most) 2 or 3 parameters, is ultimately an empirical issue. Therefore, we shall look at an example consisting of a standard VAR model, for 6 quarterly macroeconomic variables. We shall see whether, in practice, despite the Autoregressivity Paradox, univariate analysis (a relatively familiar tool) can be of practical help in checking the adequacy of a multivariate model. Finally, we consider what the comparison says in terms of an economic application: the measurement of the persistence of macroeconomic shocks.

2 Univariate Models Implied by a Vector Autoregressive Model

Let $z_t = (z_{1t}, ..., z_{kt})'$ be a stationary stochastic vector process which follows the VAR model

$$\Phi(L)\, z_t = a_t + \mu \,, \tag{1}$$

where L is the usual lag operator, $\Phi(L)$ is a $(k \times k)$ matrix with finite polynomials in L as elements, a_t is a k-dimensional white noise variable with zero mean vector and contemporaneous covariance matrix Ω, and μ is a vector of constants. If the (i, j)-th element of $\Phi(L)$ is a polynomial with coefficients Φ_{ijk}, $k = 0, 1, ...,$ we adopt the standardization $\Phi_{iio} = 1$, and $\Phi_{ijo} = 0$ for $i \neq j$. Finally, the stationarity of z_t implies that the roots of the equation $|\Phi(L)| = 0$ (where $| \bullet |$ denotes the determinant of a matrix) lie outside the unit circle. Following Zellner and Palm (1974), to obtain the univariate representation of z_{it} $(i = 1, ..., k)$, we simply need to express (1) as

$$z_t = [\Phi(L)]^{-1}(a_t + \mu) = |\Phi(L)|^{-1}\, \Phi^*(L)(a_t + \mu), \tag{2}$$

where $\Phi^*(L)$ is the adjoint matrix of $\Phi(L)$. For the i-th element of z_t, expression (2) becomes

$$|\Phi(L)|\, z_{it} = \sum_{j=1}^{k} \Phi^*_{ij}(L)\, a_{jt} + c_i \tag{3}$$

where c_i is a constant, equal to the i-th element of $\Phi^*(1)\mu$. Since (ignoring the constant) the right-hand side (r.h.s.) of (3) is the sum of k finite moving averages, it can also be represented as a finite moving average $\theta_i(L)\, u_{it}$, where u_{it} is a white-noise variable, such that

$$\theta_i(L)\, u_{it} = \sum_{j=1}^{k} \Phi^*_{ij}(L)\, a_{jt} \,, \tag{4}$$

[see Anderson (1971)]. Considering (3) and (4), the univariate models implied by (1) are given by

$$\phi(L)\, z_{it} = \theta_i(L)\, u_{it} + c_i \qquad (i = 1, ..., k)\,, \tag{5}$$

where the autoregressive (AR) polynomial is equal to

$$\phi(L) = \left|\Phi(L)\right| , \tag{6}$$

and each moving average polynomial, together with the variance of the univariate innovation u_{it}, can be obtained through (4) as detailed in Appendix A. It is worth noting that:

a) A VAR process implies univariate ARMA (not simply AR) models, and that all univariate models share the same AR polynomial (6).
b) As shown in Appendix A, the univariate MA polynomials are always invertible, with the orders (q_i) depending on the elements $\Phi_{ij}(L)$.
c) When $\mu \neq 0$, the univariate model for z_{it} will always contain a constant.

3 A Case Study: The Series and Univariate Analysis

We consider six quarterly macroeconomic series of the French economy, taken from Deniau et al. (1989). Each series has 84 observations and starts in the first quarter of 1963. The series are the following:

d_t = Public Debt
y_t = Gross Domestic Product (GDP)
p_t = GDP Deflator
r_t = Interest Rate (on first class bonds)
n_t = Balance of the Current Account (exports-imports)
m_t = Monetary Aggregate (M1).

The sources of the series, as well as some (minor) modifications performed on them, are described in the above reference. Figure 1 plots the six series; all, except n_t (which can take negative values), have been log transformed. They seem to exhibit, in all cases, a nonconstant mean, and, as seen in figure 2, Autocorrelation Functions (ACF) that converge very slowly. The Augmented Dickey-Fuller (ADF) tests, allowing for a constant (according to result c above), are presented in the first row of table 1. At the 5% size, the critical value - taken from MacKinnon (1991) - is 2.90, and hence in no case is the unit root hypothesis rejected. (All regressions were run with 9 lags, enough to whiten all series.)

First differences of all series were, thus, taken, and the following ARIMA models were identified and estimated (all differenced series were centered around the mean).

$$\left(1 - \underset{(.11)}{.74}\, L^4\right) \nabla \log d_t = \left(1 - \underset{(.16)}{.32}\, L^4\right) u_{1t}\,, \tag{7a}$$

$$\nabla \log y_t = u_{2t}\,, \tag{7b}$$

$$\nabla^2 \log p_t = \left(1 - \underset{(.07)}{.74}\, L)\right) u_{3t}\,,$$

$$\left(1 - \underset{(.10)}{.35}\, L\right) \nabla \log r_t = u_{4t}\,, \tag{7d}$$

$$\nabla n_t = \left(1 - \underset{(.10)}{.47}\, L^4\right) u_{5t}\,, \tag{7e}$$

$$\left(1 + \underset{(.09)}{.19}\, L - \underset{(.10)}{.50}\, L^4\right) \nabla \log m_t = u_{6t}\,. \tag{7f}$$

(The numbers in parentheses below the parameter estimates are the associated standard errors.) The ACF of the residuals are displayed in figure 3, and in all cases they are seen to be close to the ACF of white noise. The Box-Ljung-Pierce Q statistics for the first 27 autocorrelations are displayed in the second row of table 1, and for the six series they are smaller than the corresponding χ^2 (5%) critical value. The residual standard deviations are displayed in the first column of table 2. Three comments are in order:

a) Since our aim is to confront the parsimony of these estimated univariate models with the lack of parsimony of univariate ARIMAs derived from a VAR model, an important model selection criterion was to minimize the number of parameters. Although alternative specifications are certainly possible, the models in (7) passed all diagnostics and, besides the innovation variance, no model contains more than two parameters.

b) All variables are integrated of order 1 [or I(1)], except for the GNP deflator p_t . However, estimation of an ARIMA (1, 1, 1) model without imposing the second unit root yields:

$$(1 - .94\,L)\,\nabla \log p_t = (1 - .70\,L)\,u_{3t}\,, \qquad (7c)$$
$$\qquad\;(.04) \qquad\qquad\qquad\qquad (.09)$$

with slightly smaller values of Q_{27} and of the residual variance. Since there are no compelling reasons to impose the second unit root, to preserve the order of integration, we shall use as the estimated univariate model for p_t that given by (7c).

c) Finally, concerning the model for m_t, factorization of the AR polynomial produced the following roots

Modulus	Frequency
.80	0
.84	$\pi/2$
.89	π

The first root $(1 - .8\,L)$ is associated with the trend, and the last two with the once and twice a year seasonal frequencies; all roots display a relatively large modulus.

4 Testing for Cointegration

Before proceeding to estimation of a multivariate model for the six variables, we need to test for the presence of cointegration relationships among them. Let $x_t = (x_{1t}, \cdots, x_{6t})$ denote the vector of the six undifferenced variables; two procedures will be applied. First, following Engle and Granger (1987), we compute the six regressions

$$x_{jt} = \alpha_o + \alpha_j\, t + \sum_{i=1;\, i\neq j}^{6} \alpha_i\, x_{it} + e_{jt}, \qquad (8)$$

$(j = 1, \cdots, 6)$. Then, ADF tests are run on the series of estimated residuals $\hat{e}_{jt}$. If in no case the null hypothesis of a unit root is rejected, the series are not cointegrated.

For the six regressions of the type (8), the first row of table 3 shows the Q (27) statistics associated with the autocorrelation function of the residuals obtained in the Dickey-Fuller regression on $\hat{e}_{jt}$ (using up to 4 lagged values). The second and third rows present the Dickey-Fuller t-statistics to test for the hypothesis that there is a unit root, and its corresponding 5% critical value; these last values have been computed using the response surface regression of MacKinnon (1991). It is seen that in no case the unit root hypothesis is rejected. If the term $\alpha_j t$ is removed from (8), the results remain basically unchanged, except when the variable n_t is the regressand, in which case the t-statistic becomes marginally significant.

The second type of cointegration test performed is that proposed by Johansen (1988), based on the rank of the matrix Π in the multivariate regression

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \cdots + \Gamma_{p-1} \Delta x_{t-p+1} + \Pi x_{t-p} + \mu + \varepsilon_t .$$

For our set of series, $p = 2$ was enough in order to obtain white-noise residuals. Let r denote the number of cointegrating vectors. Table 4 presents the lambda-max and trace tests for the sequential testing of $H_0 : r \leq j$ $(j = 5, \cdots, 1, 0)$, where the 5% critical values have been taken from Gardeazabal and Regulez (1990). Both tests indicate that the six series can be safely assumed to be non-cointegrated.

5 The Vector Autoregressive Model

Since there are no cointegration relationships, the VAR model can be specified in first differences of the variables. Such a VAR model, for the six variables we consider, was estimated by Deniau et al. (1989) in order to analyze the effect of the public debt on several macroeconomic variables of the French economy. The model was identified in a manner similar to that proposed by Hsiao (1981) and Caines et al. (1981). In a first step, the VAR structure is determined, equation by equation, according to the results of "causality tests" between variables; the maximum lags are found with an MFPE information criterion. The model thus specified, is estimated as a SURE model. We reestimated the same VAR with the rates of growth replaced by the differences in logs. Also, a few parameters that were not significant were removed. (The effect of these modifications was minor.)

The estimated model has a total of 20 autoregressive coefficients, and hence, for a 6-variate VAR, it is considerably parsimonious (an average of 3.3 parameters per equation). In terms of equation (1), letting z_t denote the vector:

$$z_t = \left(\nabla \log d_t, \nabla \log y_t, \nabla \log p_t, \nabla \log r_t, \nabla n_t, \nabla \log m_t\right)' ,$$

the estimated $\Phi(L)$ matrix is given by

$$\Phi(L) = \begin{bmatrix} \phi_{11} & 0 & \phi_{13} & 0 & 0 & \phi_{16} \\ \phi_{21} & \phi_{22} & 0 & \phi_{24} & 0 & 0 \\ 0 & 0 & \phi_{33} & \phi_{34} & 0 & 0 \\ 0 & 0 & 0 & \phi_{44} & \phi_{45} & 0 \\ 0 & \phi_{52} & 0 & \phi_{54} & \phi_{55} & 0 \\ \phi_{61} & \phi_{62} & 0 & 0 & 0 & \phi_{66} \end{bmatrix} ,$$

where 0 denotes the null polynomial, and the nonzero elements are the following polynomials in L:

$$\phi_{11} = 1 - \underset{(2.07)}{.202} L^4 ; \quad \phi_{13} = \underset{(3.06)}{-1.08} L \underset{(2.78)}{- .987} L^4 ; \quad \phi_{16} = \underset{(-3.46)}{.548} L^2 ;$$

$$\phi_{21} = \underset{(3.13)}{- .055} L^8 ; \quad \phi_{22} = 1; \quad \phi_{24} = \underset{(3.62)}{- .065} L^7 ;$$

$$\phi_{33} = 1 - \underset{(1.98)}{.137} L - \underset{(4.3)}{.299} L^2 - \underset{(4.3)}{.303} L^5 ; \quad \phi_{34} = \underset{(3.76)}{- .058} L^2 ;$$

$$\phi_{44} = 1 - \underset{(3.48)}{.34} L ; \quad \phi_{45} = \underset{(-2.36)}{.3} L ;$$

$$\phi_{52} = \underset{(-2.9)}{.964} L ; \quad \phi_{54} = \underset{(-2.58)}{.192} L ; \quad \phi_{55} = 1 + \underset{(-4.1)}{.4} L^4 ;$$

$$\phi_{61} = \underset{(1.94)}{- .097} L + \underset{(-2.96)}{.15} L^2 ; \quad \phi_{62} = \underset{(6.62)}{- .73} L ; \quad \phi_{66} = 1 + \underset{(-2.1)}{.18} L - \underset{(3.65)}{.35} L^4 ;$$

the t-statistics of the parameter estimates are given in parentheses. In terms of its economic interpretation, the model implies that a positive shock in the public debt increases, in the short to medium term, aggregate demand (with a limited crowding-out effect), which in turn increases imports. As a consequence, the balance of the current account deteriorates, and there is an increase in interest rates associated with foreign capital inflows. Economic interpretation, however, is not our present concern, and we refer to the Deniau et al. paper.

An important element in the diagnosis of a VAR model is the behavior of the vector of estimated residuals $\hat{a}_t$. Table 5 summarizes the correlation functions among the components of $\hat{a}_t$. The "+", "-", and "." signs indicate, respectively, a positive significant correlation, a negative significant correlation, and a correlation that can be assumed to be zero [see Tiao and Box (1981)]. The distribution of the significant correlations appears to be random; the largest positive and negative values are .31 and -.30, and the number of significant ones is 17, or approximately 5% of the total number of computed correlations. The residuals obtained behave, thus, as a white-noise vector.

The residual variances are given in the second column of table 2. Compared to the ones obtained in the univariate ARIMA fit, it is seen that the innovations in the multivariate model have smaller variances. The percent reduction varies between 2% (variable n_t) and 23% (variable y_t), with an average reduction of approximately 11%.

To further validate the models, an out-of-sample forecasting exercise was performed. Some of the series were modified after 1985, and the last observation available on our complete set of series is for 1985/4. In order to increase the number of out-of-sample forecasts, the ARIMA and VAR models were estimated with data up to 1983/4. Then, one-period-ahead forecasts were computed for the four quarters of 1984 and of 1985. Table 6 presents the Root Mean Squared Error of the out-of-sample forecasts for the six series. The results for y_t, p_t, r_t, and m_t are clearly close to the in-sample values given in table 2, and for d_t the out-of-sample forecast is better. For n_t, the out-of-sample forecast deteriorates and an F-test for the equality of variances in the case of the VAR model yielded the value 3.4, and hence equality could be marginally rejected. (For the other series, the corresponding F values were not significant.) As for the relative performance of ARIMAs and VAR models in out-of-sample forecasting, for four variables the VAR yields better forecasts, while in two cases the ARIMA models perform better. In no case, however, the difference between the two forecasts is large. Considering the improvement in in-sample fit and the overall better performance in out-of-sample forecasting of the multivariate model, the univariate ARIMA models do not seem to "parsimoniously encompass" the VAR one [see Hendry and Mizon (1992)].

In summary, both the set of estimated ARIMA models and the VAR model behave reasonably. The multivariate structure does not bring spectacular improvement, but it does bring some. Altogether, considering the simplicity of the models identified, the results represent sensible applications of univariate ARIMA and VAR modeling.

6 Implied Univariate Models in the VAR and Comparison with the Estimated ARIMA Models: Ad-hoc Comparison

Following the derivation of section 2 and Appendix A, the univariate ARIMA models implicit in the VAR model have been obtained. The third column of table 2 contains the innovation variances of the implied univariate models. They are similar to those obtained with the estimated ARIMA models, and slightly closer to the innovation variances of the VAR model.

Concerning the autoregressive and moving average coefficients of the implied ARIMAs, the common AR polynomial, $\phi(L)$ of (5), is of order 22. The order q_i of the six moving average polynomials are those in the first row of table 8, and hence, despite the parsimony of our VAR model, the example provides a good illustration of the autoregressivity paradox referred to earlier: While the univariate models implied by the VAR contain an average of 42 AR and MA parameters, the univariate models estimated in section 3 have an average number of 1.3 parameters. We mentioned before two simple reasons that might explain the apparent paradox; let us see how they operate in practice.

It is well known that when the matrix $\Phi(L)$ in (1) has a block triangular or block diagonal structure, exact cancellation of roots between the AR and MA polynomials in some of the implied univariate models will occur [see Goldberger (1959), Wallis (1977), and Palm (1986)]. The matrix $\Phi(L)$, in our case, does not have that type of structure, and hence no such root cancellations can be done. For each of the six series, the 22 roots of $\phi(L)$ have to be compared with the roots of the corresponding polynomial $\theta_i(L)$. Computation of the 144 roots shows that the VAR model is indeed stationary, although a high proportion of the roots are relatively large and, for example, only 7 of them are smaller than .5 in modulus. The MA polynomials are invertible, and they also display roots that are relatively large in modulus.

When comparing, for the six series, the AR and MA roots, in order to decide which of them cancel out, a criterion is needed. Let ω and h denote the frequency (in radians) and modulus, respectively, of a complex root, and consider, for example, the implied ARMA model for the variable d_t. Table 7 displays the roots of the AR and MA polynomials (to facilitate interpretation, the roots displayed are those of L^{-1}). It is easy to accept that the MA root with $\omega = 2.59$ and $h = .78$ will cancel out with the root with the same frequency and modulus in $\phi(L)$. But, what about the pair of roots ($\omega = 1.62$, $h = .78$) and ($\omega = 1.54$, $h = .85$)? Since, on the one hand, the roots of the polynomials in the implied ARIMA models are complicated functions of 41 parameters (those in $\Phi(L)$ and in Ω) and, on the other hand, the comparison involves 144 roots, computed from 84 observations on the vector of variables, we had a-priori doubts as to whether formal testing could be of help, and hence proceeded in two ways. First, a simple ad-hoc criterion is used, which is fairly restrictive and biased towards undercancellation. Second, a formal test, adapted from Gourieroux, Monfort and Renault (1989) is performed.

The ad-hoc criterion, discussed in Appendix B, is as follows: The root $(\hat{h}_1, \hat{\omega}_1)$ will cancel with the root $(\hat{h}_2, \hat{\omega}_2)$ if

a) $|\hat{h}_1 - \hat{h}_2| \leq .05$,

b) $|\hat{\omega}_1 - \hat{\omega}_2| \leq .05$,

c) $|\hat{h}_1 - \hat{h}_2| + |\hat{\omega}_1 - \hat{\omega}_2| \leq .07$.

Applying this criterion to our example, to get an insight into the proximity of the cancelled roots, we consider the two that are most distant, in terms of the sum of the two absolute deviations. These are the root ($\omega = .73$, $h = .81$) in the AR polynomial, and the root ($\omega = .78$, $h = .79$) in the MA polynomial, of the univariate model for r_t. The two roots generate the AR and MA polynomials $(1 - 1.21\,L + .66\,L^2)$ and $(1 - 1.12\,L + .63\,L^2)$, respectively. Assume the AR polynomial is the result of estimating an AR(2) model with T =

84 observations (our sample size), and that we perform the test $\phi_1 = 1.12$ and $\phi_2 = -.63$ (i.e., the AR polynomial is equal to the MA one). Then, denoting by M the asymptotic covariance matrix of the autoregressive parameter estimators,

$$S = \left(\hat{\Phi} - \Phi\right)' M^{-1} \left(\hat{\Phi} - \Phi\right) \;\dot{\sim}\; \chi^2_2 ,$$

where $\hat{\Phi}' = (1.21, -.66)$ and $\Phi' = (1.12, -.63)$. Using the expression for M in Box and Jenkins (1970, p. 244), it is found that $S = 1.61$, certainly below the 95% critical value of 5.99. Since this result holds for the pair of cancelled roots that are most distant, it is clear that the criterion favors undercancellation.

Using the above criterion, roots were cancelled in the implied ARMA model; remultiplying the remaining ones, new models are obtained for the six variables. Their orders are indicated in the second row of table 8: They have been considerably reduced (the average number of parameters per model drops from 42 to 17), but the models are still far from the parsimony of the ARMA models from univariate analysis.

The implied ARMA models obtained after removing common roots are displayed in table 9. Since (as shown in Appendix B) the standard deviation of $\hat{\omega}$ and of $\hat{h}$ are larger for roots with smaller modulus, cancellation will be likely to affect the roots with relatively large modulus. Thus the remaining ARMA models will mostly contain the smaller roots, which are estimated with less precision. In considering whether the ARMA models of table 9 can be made more parsimonious by removing small coefficients (undetectable in estimation), again a criterion is needed. Considering that most of the standard errors of the parameters in the estimated ARMA models of expression (7a to f) are in the order of .09 or larger, a reasonable criterion is to remove coefficients that are below .18 in absolute value. Proceeding in this way, the following models are obtained.

a) Series d_t : $$(1 - .65L^4)\, \nabla \log d_t = (1 - .31\, L^4)\, u_{1t} , \qquad (9a)$$

which is quite close to (7a). In this case, the VAR model certainly explains the model obtained in univariate analysis.

b) Series y_t : $$(1 - .92L)\, \nabla \log y_t = (1 - .98\, L)\, u_{2t} , \qquad (9b)$$

or, approximately, the random walk model of equation (7b). Again, the VAR model explains well the estimated univariate ARIMA model. Considering the relatively large decrease in the residual variance of y_t in the VAR model, the variable GDP seems particularly suited for multivariate analysis.

c) Series p_t : $$\left(1 - .15\,L - .30L^2 - .30L^5\right) \nabla \log p_t = u_{3t}, \tag{9c}$$

Model (9c) appears to be quite distant from (7c), yet if in the latter the MA polynomial is inverted and approximated up to the fourth power, the product $(1 - .94\,L)(1 - .70)^{-1}$, after deleting small coefficients, yields

$$\left(1 - .24\,L - .17L^2 - .23L^5\right) \nabla \log p_t = u_{3t},$$

more in line with (9c). However, in so far as a fourth order approximation to $(1 - .70\,L)^{-1}$ is a poor approximation, the series p_t illustrates how, when the univariate model contains a relatively large MA root, VAR models will have trouble capturing that behavior.

d) Series r_t : $$\left(1 + .18\,L - .25L^2\right) \nabla \log r_t = (1 + .50\,L)\, u_{4t},$$

somewhat different from the model (7d). However, expressing the model in pure autoregressive form, it is obtained

$$(1 - .32\,L)\ \nabla \log r_t = u_{4t}, \tag{9d}$$

with all other parameters smaller than .10 and converging fast towards zero. Models (7d) and (9d) are obviously very close.

e) Series n_t $$\left(1 + .21\,L - .25L^2 + .48L^4\right) \nabla n_t = \left(1 + .21\,L - .20B^2\right) u_{5t}.$$

Expressing this time the model in pure MA form, it is obtained that

$$\nabla n_t = \left(1 - .47\,L^4\right) u_{5t}, \tag{9e}$$

with all other MA parameters smaller than .15. Model (9e) is the same as (7e).

f) Series m_t : $$\left(1 - .65L^4\right) \nabla \log m_t = \left(1 - .29\,L - .18\,L^4\right) u_{6t}.$$

The autoregressive expression is found to be

$$\left(1 + .29\,L \;-\; .46\,L^4\right)\,\nabla \log m_t \;=\; u_{6t}\,, \tag{9f}$$

with all other parameters smaller than .10. Again, model (9f) is reasonably close to model (7f).

In conclusion, when the (conservative) ad-hoc criterion is used, careful analysis of the multivariate VAR model explains well the models obtained with univariate analysis. We turn next to the results of formal testing.

7 Comparison of the Implied and Estimated Univariate ARIMA Models: A Test Procedure

Despite the large number of roots we wish to compare, it is straightforward to adapt to our case an ingenious testing procedure developed by Gourieroux, Monfort, and Renault (1989). Let the AR and MA polynomials of one of the implied ARIMA models be, respectively,

$$\phi\,(L) = 1 + \phi_1\,B + \cdots + \phi_p\,B^p\,,$$
$$\theta\,(L) = 1 + \theta_1\,B + \cdots + \theta_q\,B^q\,,$$

and assume there are r common roots shared by the two polynomials. Then, there exists a polynomial $\lambda\,(L)$, of order r, formed by the product of all the common roots, such that

$$\phi\,(L) = \lambda(L)\,\alpha(L)\,,$$
$$\theta\,(L) = \lambda(L)\,\beta(L)\,.$$

Let $\alpha(L) = 1 + \alpha_1 L + \cdots + \alpha_a B^a$, $a = p - r$ and $\beta(L) = 1 + \beta_1 L + \cdots + \beta_b\,B^b$, $b = q - r$ Removing $\lambda\,(L)$ from the previous two equations yields

$$\phi\,(L)\,\beta\,(L) = \theta\,(L)\,\alpha\,(L)\,, \tag{10}$$

an identity between two polynomials of order $k = p + q - r$. Denote by ψ the vector of coefficients of the implied ARMA, and by δ the vector of coefficients after the common polynomial $\lambda\,(L)$ has been removed, i.e.

$$\psi = \left[\phi_1, \cdots, \phi_p, \theta_1, \cdots, \theta_q\right]' \tag{11}$$

$$\delta = \left[\alpha_1, \cdots, \alpha_a, \beta_1, \cdots, \beta_b\right]' . \tag{12}$$

Equating the coefficients of $L^j (j = 1, \cdots, k)$ in (10) yields a system of k equations. Conditional on ψ, the system is linear in δ, and can be written as

$$h = H\,\delta \tag{13}$$

where h is a k-dimensional vector, and H a $k \times (k - r)$ matrix. Conditional on δ, the system is linear in ψ, and can be expressed as

$$e = E\,\psi ,$$

where e is a k-dimensional vector, and E a $k \times (p + q)$ matrix. The test consists of the following procedure:

1) Run OLS on (13) to obtain $\hat{\delta}$, and with this estimator construct the matrix $\hat{E}$. Compute, then, the $(k \times k)$ matrix $\xi = \hat{E}\,\Sigma\,\hat{E}'$, where Σ denotes the covariance matrix of the estimators of the parameters in ψ.

2) Run GLS on (13), using ξ as the covariance matrix of the error term, and denote by SSR the sum of squares of the residuals in this regression. For the test consisting of

H_0: $\phi(L)$ and $\theta(L)$ have exactly r common roots;

H_A: $\phi(L)$ and $\theta(L)$ have, at most, r common roots;

the statistics $(T \times SSR)$ is distributed as a χ^2 variable with r degrees of freedom. In order to proceed sequentially, we start with $r = \min(p, q)$, i.e., with r equal to its maximum possible value. If H_0 is rejected, we then set $r' = r - 1$ and redo the test, until H_0 is not rejected.

Implementation of the test requires computation of the matrices H, h, E, e, and Σ. A simple procedure is described in Appendix C. Table 10 presents the results from the test (for a 5% size) and it is seen that the orders of the ARMA models obtained after removal of the common roots are much smaller than the ones obtained with the ad-hoc (restrictive) criterion of section 7. The coefficients of the ARMA model after removal of the common roots are the elements of δ, consistently estimated when running the test; they are displayed in table 11. Ignoring small parameters, table 11 yields the following models

$$\left(1 - .40\,L^4\right) \nabla \log d_t = u_{1t} \tag{14a}$$

$$\nabla \log y_t = u_{2t} \tag{14b}$$

$$\left(1 - .16\,L - .28\,L^2 - .30\,L^5\right) \nabla \log p_t = u_{3t} \tag{14c}$$

$$(1 - .31\,L)\, \nabla \log r_t = u_{4t} \tag{14d}$$

$$\nabla n_t = u_{5t} \tag{14e}$$

$$\left(1 - .91\,L + .33\,L^2 - .45\,L^3\right) \nabla \log m_t = \left(1 + .79\,L + .17\,L^2\right) u_{6t} \tag{14f}$$

The first model is similar to model (7a), since the AR representation of the latter is, approximately, $\left(1 - .42\,L^4 - .13\,L^8\right) \nabla \log d_t = u_{1t}$. Model (14b) is the same as model (7b), and models (14d) and (7d) are practically identical. As for the series p_t, model (14c) is very close to the implied ARIMA model obtained with the ad-hoc criterion [i.e., model (9c)], which was seen to be a rough approximation to (7c). For the first four series, thus, the test gives results that are in close agreement with the results of direct univariate analysis, and with the implied univariate models obtained with the ad-hoc criterion.

For the last two series, however, models (14e) and (14f) ae markedly different from models (7e) and (7f). In both cases it happens that significant coefficients at seasonal lags are missing. This is due to the fact that the test yields a value of r which is too large, so that the AR and MA polynomials in the simplified ARMA models are not long enough to reach the seasonal lags. Setting $r = 18$ for n_t, and $r = 17$ for m_t, so as to allow for seasonal coefficients, the model for n_t can be expressed as (once small coefficients have been removed)

$$\nabla n_t = \left(1 - .40\,L^4\right) u_{5t}\,,$$

and that for m_t as

$$\left(1 + .15\,L - .53\,L^4\right) \nabla \log m_t = u_{6t}\,.$$

These two models are now very similar to model (7e) and (7f). Therefore, our example shows that, when using the Gourieroux-Monfort-Renault test for cancelling common roots, care should be taken with seasonal models. Blind application of the test may overestimate the value of r, with the consequence that seasonal coefficients may be left out from the derived model. Once this fact is taken into account, the test is seen to perform surprisingly well. In summary, it seems safe to conclude that the ARMA models obtained from univariate analysis are quite in agreement with the univariate models derived from the VAR. This result is true whether the comparison is made with an ad-hoc criterion or with a testing procedure.

8 An Economic Application

The comparison of the VAR model with the ARIMA models estimated with univariate techniques has shown how the results obtained in the latter can be reasonably explained by the VAR. Be that as it may, since the comparison implies cancelling many roots and removing many small coefficients, it is of interest to see how, when those models are used in economic applications, inferences may be affected by the type of model used.

The application we chose is related to the effort by macroeconomists at explaining the permanent changes in aggregate output, as well as the fluctuations around this "permanent component". From an early period when the permanent component (or trend) of the series was assumed deterministic, economists have moved towards modeling trends as stochastic components. When a variable contains a stochastic trend, a shock in the series will not only affect the so-called cyclical component, but will also have an impact on the permanent one. The measurement of this long-term effect (or "persistence") of shocks has been the subject of attention by macroeconomists. In a univariate world, for I(1) series with Wold representation

$$\nabla x_t = \psi(B)\, u_t\,, \tag{15}$$

the impact of a shock u_t on x_{t+k} is given by $(1 + \psi_1 + \cdots + \psi_k)\, u_t$. Following Campbell and Mankiw (1987), the persistence of a standardized shock $u_t = 1$ can be defined as its very long-run impact on the series or, more formally, as

$$m = \lim_{k \to \infty} \sum_{i=0}^{k} \psi_k = \psi(1).$$

There has been considerable interest in estimating persistence, in particular for the case of aggregate output, where different values of $\psi(1)$ have been assigned to different theories of the business cycle. If $\psi(1) > 1$, "real factors", typically associated with supply (such as changes in productivity), would account for, both, economic growth, and most of the business cycle. On the contrary, if $\psi(1) < 1$, the business cycle would be more likely to be associated with transitory (typically demand) shocks; see, for example, the discussion in Lippi and Reichlin (1991).

Of the several approaches to the estimation of persistence, we shall select three that are relevant to our example. First, following Campbell and Mankiw (1987), $\psi(1)$ can be obtained from the univariate ARIMA estimation of (15) using Box-Jenkins methods. Second, since additional variables may provide information in explaining deviations of a variable with respect to its trend level, Evans (1989) computes the measure using the parameters from a VAR estimation. Specifically, he proposes to use $\psi(1)$ in the univariate ARIMA model implied by the VAR one. These two measures are based, in theory, on the same set of univariate innovations. Moreover, since the ARIMA models implied by the VAR should be in agreement with the ACF of the series, and this function is the basic identification tool in univariate analysis, the two measures of persistence should not be too distant. Discrepancies between them would be likely to indicate misspecification in some of the models.

Evans finds, however, that persistence of GNP, measured with the ARIMA model implied by his VAR model, is considerably different from the measures obtained by Campbell and Mankiw with univariate analysis. In order to see whether this discrepancy flags some problem with the model specification, we reestimated the bivariate VAR model of Evans (who kindly supplied us with the data). The equation for GNP is given by

$$y_t = \underset{(.38)}{-.62} + \underset{(.11)}{.13}\, y_{t-1} + \underset{(.11)}{.18}\, y_{t-2} + \underset{(.10)}{.02}\, y_{t-3} - \underset{(.33)}{.48}\, x_{t-1} + \underset{(.50)}{1.32}\, x_{t-2} - \underset{(.33)}{.59}\, x_{t-3} - \underset{(.26)}{.85}\, d_t + a_t\,, \tag{16}$$

where $y_t = \nabla \log \mathrm{GNP}$, x_t is the unemployment rate, and d_t a step dummy variable capturing a structural break; the numbers in parenthesis denote standard errors.

To judge the validity of the equation it is not possible to perform a proper out-of-sample forecast exercise because the series y_t has been subsequently revised (partly because of revisions in seasonal factors). We split the sample period used by Evans into two subperiods, one with the first 100 observations, and the other with the last 40 observations. His VAR model was reestimated for the first subperiod, and one-period-ahead forecasts were computed for the second subperiod. Figure 4 compares the associated one-period-ahead forecast errors with those obtained with a simple "AR(1) + constant" structure (with no structural break), estimated also for the first 100 observations. The two series of errors are very close, and hence the large number of parameters in (16) does not improve upon the naive AR(1) specification. The equation is overly parametrized, and this is reflected in the large standard errors of the parameter estimates. The difference between the VAR and univariate measures of persistence does not seem, thus, the result of a more efficient multivariate estimation; on the contrary, the VAR model obtained seems an unreliable tool for inference.

Finally, Pesaran et al. (1992) suggest a multivariate measure of persistence, with the innovations defined with respect to the multivariate information set. In the univariate case, if $g(\omega)$ denotes the spectrum of ∇x_t in (15), using a well-known result, $g(0) = \psi(1)^2 \sigma_u^2$. The multivariate extension of this result, for the case of the VAR model given by (1), is

$$g(0) = \left[\Phi(1)^{-1}\right] \Omega \left[\Phi(1)^{-1}\right]' .$$

The measure of persistence proposed by Pesaran et al. is given by the squared root of the elements of the main diagonal of this matrix, standardized by the variance of the appropriate multivariate innovation. (For a vector with only one variable, the multivariate measure becomes the univariate one.)

Table 12 presents the three measures of persistence for the 6 variables we consider. The first two measures refer to the response to the univariate innovation, which is a function of all the innovations of the multivariate model, as shown in expression (4). Therefore, the two measures are not strictly comparable to the one obtained with the VAR model, which reflects the response to the innovation defined in a multivariate information set. It is seen, however, that, given the precision of the measurements, for four variables d_t, p_t, r_t and m_t the three measures are reasonably close. A unit innovation in public debt has a large permanent effect on the level of debt, and a similar result is obtained for the price level variable. In this later case, the discrepancy between the two ARIMA measures may reflect the limitations of the VAR model in capturing a series with a relatively large MA root, as mentioned in section 7. For the interest rate and the monetary aggregate series the persistence is slightly larger than 1, although for m_t it could be easily accepted as equal to 1.

For the series y_t and n_t the univariate and multivariate results are more distant. For the GDP series the univariate measure of persistence is 1, while the multivariate measure is 1.7 and, considering the standard errors, they cannot be accepted as equal. According to the interpretation mentioned above, this could be seen as evidence that, when the innovation is cleaned of the effects due to other correlated shocks (i.e., when the information set is enlarged), the real business cycle theory gains support. For the balance of trade series, the univariate measure is below one, while the multivariate measure is 1. An economic interpretation of the persistence measures is beyond the scope of this paper. Relevant to our discussion are the following two results:

a) The proximity of the measures of persistence between the estimated and implied ARIMA models shows that inferences drawn from the VAR (concerning persistence) explain well the ones obtained from univariate analysis. Altogether, it is somewhat striking that the measurement is not more affected by the numerous cancellations of roots and removal of coefficients.

b) The univariate measure of persistence may be a reasonable approximation to the persistence measured in a wider information set. But there are cases when this is clearly not true.

9 Summary and Conclusions

It is well known that a linear dynamic structural econometric model has a reduced form with a multivariate linear time series model expression, which in turn implies univariate ARIMA models for each of the series. An important way to evaluate a structural econometric model, thus, is by checking for whether it encompasses the appropriate VAR model. Since the univariate models implied by VAR models have ARIMA expressions, in a similar manner, an important way to evaluate a VAR model is to see if the results obtained with univariate analysis can be explained by the VAR, i.e., if the ARIMA models implied by the VAR are close to the ones found in univariate analysis. Since identification of univariate models is easier than identification of (not too small) VAR models, if an implied ARIMA model is substantially different from the ARIMA model that fits the univariate series, the difference may well reflect misspecification of the multivariate model (an example is provided in section 8).

Although the idea is simple, it is however rarely put into practice. This may be partly due to the fear that the comparison may be worthless because of the so-called autoregressivity paradox: while the ARIMA models from univariate analysis typically have very few parameters, the implied ARIMA models, even for relatively small VAR models, have a very large number of parameters. Can we reasonably expect to bring, for example, a 45-parameter ARIMA model down to a 1- or 2-parameter one? More generally, can we expect univariate models to be useful as diagnostic tools for VAR models?

The question is general, but the answer is ultimately empirical. Thus we consider a particular application: A VAR model for six quarterly macroeconomic variables. First, using univariate analysis, ARIMA models are fit to each one of the series. Not counting the innovation variances, all models have at most two parameters. Then, after testing for cointegration, a parsimonious VAR model is estimated; the model is a slight modification of the one used by some French economists to analyze the effect of public debt on several macroeconomic variables. It is seen that, both, the set of univariate ARIMA models and the VAR model provide good fits and perform reasonably well in out-of-sample forecasting.

Next, the univariate models implied by the VAR are derived (following a procedure described in Appendix A). All have an AR polynomial of order 22, and the orders of the MA polynomials vary between 18 and 24. The application considered provides thus a good example for the autoregressivity paradox: ignoring the innovation variances, the average number of parameters is 42 for the implied ARIMA models and 1.3 for the ones estimated in univariate analysis.

In order to compare the two types of models, the roots of the common AR and of the six MA polynomials of the implied models are computed (a total of 144 roots). To determine which ones should cancel out, two approaches are followed. First, we use a simple ad-hoc criterion (discussed in Appendix C), biased towards undercancellation. Once the common roots are removed, careful analysis of the simplified models shows that the ARIMA models from univariate estimation are remarkably close to the ones implied by the VAR. The comparison also evidences the gain from multivariate modelling for some variables (in particular, GDP) and, in the case of the price variable, the difficulties of the VAR specifications in handling series with a large moving average root. Second, a formal test is applied to determine the roots that could be cancelled. The test is seen to be biased towards overcancellation, in particular when the series contains seasonality. Careful application of the test, however, yields finally implied ARIMA models that are in agreement with those obtained with the ad-hoc criterion and with univariate analysis.

In summary, the VAR model explains reasonably well the results from univariate analysis and passes, thus, the encompassing test. All considered, it seems safe to conclude that the improvement obtained with the multivariate model is not very large, but that it can be properly attributed to having captured some relationships among the macroeconomic variables.

Although the differences between the implied and estimated ARIMA models are relatively small, it is still of interest to see what effect they may have when the models are used for economic inference. As an example, we consider the problem of measuring the so-called persistence, or long-term effect, of shocks on macroeconomic variables. Persistence has been estimated in different ways, three of which are relevant to our discussion: First, it has been measured using ARIMA models from univariate analysis. Second, it has been measured using implied ARIMA models (derived from VAR ones). Third, we consider a multivariate measure based directly on the VAR model.

The three measures of persistence are computed for the six series. The first two measures are close, and hence the VAR model again explains well the inference obtained in univariate analysis. The comparison also shows how, although for some variables the inference based on univariate analysis may approximate the one based on multivariate models, on occasion, it can be misleading. This is clearly the case for the GDP variable.

APPENDIX A: Univariate ARIMA Models Implied by a Vector Autoregressive Model

As seen in section 2, the univariate model for the i-th series implied by the multivariate VAR is given by (5), where $\phi(L)$ is straightforward to obtain through (6), and the moving average part $\theta_i(L)\, u_{it}$ satisfies (4). We proceed to summarize a procedure (easy to implement in most available softwares) to obtain $\theta_i(L)$ and the variance of u_{it}, σ_i^2.

The adjoint matrix $\Phi^*(L)$ is directly obtained from $\Phi(L)$, and hence the elements $\Phi^*_{ij}(L)$ and the matrix Ω (with the contemporaneous covariances of the vector a_t) are assumed known. Let q denote the order of the polynomial $\theta_i(L)$. The autocovariances of the r.h.s. of (4), say $\gamma_0, \gamma_1, \cdots, \gamma_q$, can be obtained through the Autocovariance Generating Function (ACGF)

$$\gamma_i(L) = f_i(L)\, \Omega\, f_i\left(L^{-1}\right)',$$

where $f_i(L)$ is the i-th row of $\Phi^*(L)$. The ACGF $\gamma_i(L)$ is an expression of the type

$$\gamma_i(L) = \gamma_0 + \sum_{i=1}^{q} \gamma_i\left(L^i + L^{-i}\right), \tag{A.1}$$

and our aim is to find the moving average process $\theta_i(L)\, u_{it}$ that generates this set of autocovariances. We proceed as follows (for notational simplicity the subscript i is dropped). Write (A.1) as

$$\gamma(L) = L^{-q}\left(\gamma_q + \cdots + \gamma_0 L^q + \cdots \gamma_q L^{2q}\right) = L^{-q}\, \Gamma(L).$$

Since the polynomial $\Gamma(L)$ is symmetric around L^q, the 2q roots of the equation $\Gamma(z) = 0$ can be expressed as the two sets $(r_1 \cdots r_q)$ and $\left(r_1^{-1} \cdots r_q^{-1}\right)$, with $|r_i| \geq 1 \geq |r_i^{-1}|$, $i = 1, \cdots q$. In practice, however, there is no need to compute the 2q roots of $\Gamma(z)$. Using the transformation $y = z + 1/z$, the polynomial $\Gamma(z)$ is transformed into a polynomial in y of order q, say

$$A(y) = a_0 + a_1 y + \cdots + a_q y^q, \tag{A.2}$$

where the vector of coefficients $a = (a_0, a_1, \cdots, a_q)'$ is obtained as follows. Let

$b_0 = 2$.
$b_1 = (0,\ 1)$,
$b_j = (0,\ b_{j-1}) - (b_{j-2},\ 0,\ 0)$, $\quad j = 2, \cdots, q$,

and build the $(q+1)\times(q+1)$ matrix $S=[s_1, \cdots, s_{q+1}]$, with the columns given by:

$$s_1 = (1, O_q)',$$
$$s_j = (b_{j-1}, O_{q-j+1})', \quad j = 2, \cdots, q,$$
$$s_{q+1} = b_q',$$

where O_k denotes a k-dimensional row vector of zeros. Then, $a = S\gamma$, where $\gamma = (\gamma_0, \gamma_1, \cdots, \gamma_q)'$. Let $y_1, \cdots, y_q$ denote the q roots of (A.2). In each of the equations

$$z^2 - y_j z + 1 = 0, \quad j = 1, \cdots, q, \tag{A.3}$$

selecting the root z_j such that $|z_j| \geq 1$, the polynomial $\theta(L)$ is found through

$$\theta(L) = (1 - z_1 L) \cdots (1 - z_q L), \tag{A.4}$$

and σ_i^2 can be obtained from $\sigma_i^2 = \gamma_0 \left(1 + \sum_{i=1}^{q} \theta_i^2\right)^{-1}$.

Notice that the coefficient y_j in (A.3) can be complex. In this case, the solution is found in the following way: Let $y_j = a + b\,i$, and define $k = a^2 - b^2 - 4$, $m = 2ab$, and $h^2 = [(|k| + (k^2 + m^2)^{\frac{1}{2}})/2]$. Then, if $z_j = z_j^r + z_j^i\, i$ is a solution of (A.3), its real and imaginary parts are given by:

$$z_j^r = (-a \pm c)/2; \quad z_j^i = (-b \pm d)/2,$$

where, when $k \geq 0$, $c = h$, $d = m/2h$, and when $k < 0$, $d = [\text{sign}(m)]\, h$, $c = m/2d$.

The derivation of $\theta_i(B)\, u_{it}$ is valid for invertible as well as noninvertible moving averages. (In the latter case, the unit root would appear twice in $\Gamma(L)$.) But, as we proceed to show, the moving average part of the implied ARIMA will always be invertible, thus $|z_i| > 1$ for $i = 1, \cdots, q$.

A univariate finite order autoregressive model, by construction, is invertible. But, as seen in Section 2, the univariate models implied by multivariate VAR ones are not finite autoregressive models, but full ARMAS, where the moving average part can be long and complex. There is thus the question of whether, for some values of the ϕ-parameters in the AR matrix, the MA part of an implied univariate model may include a unit root.

Consider the VAR model given by (1). We have seen in Section (A.1) that, in the factorization of $\gamma(L)$, we can always choose $\theta_i(L)$ so as to have all roots on or outside the unit circle. Thus we only have to prove that no root of $\theta_i(L)$ will be on the unit circle. If $\theta_i(L)$ has a unit root, this implies a zero in the spectrum for an associated frequency. If the spectrum of the l.h.s. of (4) has a zero, all components in the r.h.s. of (4) have a spectrum with a zero for that particular frequency [see Teräsvirta (1977)], and hence the polynomials $\Phi^*_{ij}(L)$ $(j = 1 \cdots k)$ will share the same unit root. Considering the expansion of the determinant of $\Phi(L)$ by the elements of the i-th row:

$$\left|\Phi(L)\right| = \sum_{j=1}^{k} \Phi_{ij}(L)\,\Phi^*_{ij}(L)\,,$$

and factorizing the unit root common to $\Phi^*_{i1}(L) \cdots \Phi^*_{ik}(L)$, the same unit root will have to appear in $\left|\Phi(L)\right|$. The root would thus be present in the AR and MA polynomials of the implied ARIMA model, and hence it would cancel out. It follows that the univariate models implied by the VAR model are always invertible.

APPENDIX B: A Comment on the Precision of the Frequency and Modulus of the Roots in an Estimated Autoregressive Model

When using models with AR expressions, it is often of interest to look at the roots of the AR polynomials, where the roots are expressed in terms of the frequency ω and the modulus h. Since ω and h are computed as functions of the AR parameters, it is important to know how errors in the estimators of the latter induce imprecision in the measurements of ω and h. In our case, the interest is due to the need to select a criteria to determine when two roots can be safely assumed to be close enough for cancellation. Since our comparison involves 144 roots, where the modulus and frequency of each one are nonlinear functions of the 41 parameters in the matrices $\Phi(L)$ and Ω, we seek a simple ad-hoc criterion, such that only roots that are clearly close in a probabilistic sense will be cancelled. In order to do that, we consider the case of an AR(2) model for series of the same length as ours ($T = 84$). One could expect perhaps a precision somewhat similar to that of our VAR model, with 3.3 parameters per equation: the slight gain from the multivariate fit could compensate for the small increase in the number of parameters.

Let the AR(2) model be given by

$$z_t - \phi_1 z_{t-1} - \phi_2 z_{t-2} = a_t\,. \tag{B.1}$$

Expressing the roots of $x^2 - \phi_1 x - \phi_2 = 0$ in terms of frequency and modulus, for $0 < \omega < \pi$ (i.e., when the roots are not real), it is obtained that

$$h = \sqrt{-\phi_2}\,; \qquad \omega = \operatorname{acos} \frac{\phi_1}{2\sqrt{-\phi_2}}\,. \tag{B.2}$$

It is then possible to approximate the functions that relate the estimation errors in h and ω (to be denoted δ_h and δ_ω) to the estimation errors in ϕ_1 and ϕ_2 (denoted ε_1 and ε_2 , respectively). Since, for the relevant range $0 > \phi_2 > -1$ and $0 < \omega < \Pi$, the functions given by (B 2) are continuous in ϕ_1 and ϕ_2 , it is straightforward to obtain the linear approximation that relates $\delta = \left(\delta_h, \delta_\omega\right)$ to $\varepsilon = (\varepsilon_1, \varepsilon_2)$. The estimators of h and ω are consistent (becoming super-consistent when $h = 1$), and the asymptotic covariance matrix of δ , V_δ , can be linearly approximated by

$$V_\delta = D\, V_\varepsilon\, D' , \tag{B.3}$$

where V_ε is the asymptotic covariance matrix of ε , equal to

$$V_\varepsilon = \frac{1 - h^4}{T} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}; \qquad \rho = -\frac{2h\cos\omega}{1 + h^2} ,$$

and D is the matrix of derivatives

$$D = \left(\frac{\partial\delta}{\partial\varepsilon}\right) = \begin{bmatrix} 0 & -(2h)^{-1} \\ -(2h\sin\omega)^{-1} & -\left(2\,h^2\tan\omega\right)^{-1} \end{bmatrix}.$$

In our example, the vast majority of the roots have modulus in the range .6 to .9. For h = .6, .75, and .9, table 13 presents the standard deviations of the estimation errors for h and ϕ_2, obtained with the asymptotic approximation (they do not depend on ω). The larger the modulus, the smaller the estimation error becomes for both parameters. For the values in table 13, despite its larger numerical value (in absolute terms), the modulus is estimated with more precision than ϕ_2 .

In order to assess the accuracy of the linear approximation, 10.000 simulations of 84 observations each from model (B.1) were made, for the 3 values of h in table 13 and 50 partitions of the interval $\omega \in (0, \Pi)$. In each case, the AR coefficients were estimated, and the estimators of h and ω were obtained through (B.2). For the pairs (h, ω), figures 5 and 6 compare the standard errors of the modulus and frequency estimators, respectively, obtained with the simulation and with the linear approximation. For δ_m , the approximation works reasonably well; for δ_ω , except for relatively large modulus, the approximation is less reliable. From the figures it is seen that, for complex roots with values of h between .6 and .9 (our range of concern), the standard error of δ_h varies between .03 and .09, while that of δ_ω varies between .04 and .3. Considering the positive correlations between the two errors for low values of h and ω (figure 7), we adopted the following simple criterion: for the two roots (h_1, ω_1) and (h_2, ω_2) to cancel, we require that the differences $h_1 - h_2$ and $\omega_1 - \omega_2$ be smaller than .05 (in absoute value). We require further that the sum of the two absolute differences be smaller than .07.

The criterion seems safe in the following sense: Consider a pair of cancelled roots in one of the implied ARIMA models, and let δ and d denote the errors in the estimators of (h, ω) in the AR and MA roots, respectively. Assume δ is distributed normally, with zero mean vector, and covariance matrix (B.3), and that we wish to test $\delta = d$. For all roots actually cancelled, the p-value of the test would be smaller than .5. In this way, the criterion will tend towards undercancellation, and will avoid cancelling roots measured with imprecision.

APPENDIX C: Common Roots Test: Computation of the Matrices

To carry out the test described in section 7, the matrices H, h, E, e, and Σ need to be computed. For the first four, this can easily be done in the following way: Let O_j denote a column vector of j zeros, and define the vector $c = (c_1, \cdots, c_d)'$, the $(m \times n)$ matrix

$$A(c) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ c_1 & 1 & \ddots & 0 \\ \vdots & & \ddots & 1 \\ c_d & & & c_1 \\ 0 & c_d & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_d \end{bmatrix},$$

with $n < d < m$, and the m-dimensional vector $b(c) = \left(1, c', O'_{m-d-1}\right)'$. Then,

$$h = b(\phi) - b(\theta), \qquad e = b(\alpha) - b(\beta),$$

and the matrices H and E can be obtained through

$$H = [\, H_1 \;\vdots\; -H_2 \,], \qquad E = [\, -E_1 \;\vdots\; E_2 \,],$$

where H_1, H_2, E_1 and E_2 are given by

$$\begin{aligned} H_1 &= A(\theta), & n &= p - r \\ H_2 &= A(\phi), & n &= q - r \\ E_1 &= A(\beta), & n &= p \\ E_2 &= A(\alpha), & n &= q, \end{aligned}$$

and $m = p+q-r$ in all cases.

Finally, we need an estimator of $\Sigma = \text{cov}(\psi)$, where ψ contains the parameters of the implied univariate model. These parameters are functions of the VAR model parameter estimates, as indicated by equations (4) and (6). The VAR model parameters are the AR coefficients in $\Phi(L)$ and the elements of Ω, the residual error covariance matrix. Let Φ denote the vector of AR coefficient estimators, and σ the vector containing the estimators of the elements in Ω. (In order to simplify notation, we delete the symbol "^" to denote an estimator.) Then, a linear approximation to Σ yields

$$\Sigma \doteq JMJ',$$

where

$$J = \left[\frac{\partial \psi}{\partial \Phi} \vdots \frac{\partial \psi}{\partial \sigma}\right], \qquad M = \text{Cov}(\Phi, \sigma)' = \begin{bmatrix} M_\Phi & M_{\Phi\sigma} \\ M_{\sigma\Phi} & M_\sigma \end{bmatrix}.$$

The derivatives in J have been computed numerically. As for the matrix M, the first submatrix $M_\Phi = \text{Cov}(\Phi)$ is available from the VAR estimation results; also, asymptotically, $M_{\Phi\sigma} = M_{\sigma\Phi} = 0$. In order to obtain $M_\sigma = \text{Cov}(\sigma)$, its elements are expressions of the form $\text{Cov}(\sigma_{ij}\sigma_{kh})$, where $\Omega = (\sigma_{ij})$, and

$$\sigma_{ij} = T^{-1} \sum_t a_{it} a_{jt}.$$

Since the vector $a_t \sim N_6(0, \Omega)$, from its moment generating function, it is straightforward to find that, for all values of i, j, k, and h,

$$\text{Cov}(\sigma_{ij}\sigma_{kh}) \doteq (\sigma_{ik}\sigma_{jh} + \sigma_{jk}\sigma_{ih}) / T,$$

and hence M_σ can be easily computed.

Thanks are due to Stephania Fabrizio, Grayham Mizon, Franz Palm, the Associate Editor and two Referees for their valuable comments.

References

Anderson, T.W., 1971, The statistical analysis of time series (John Wiley, New York).

Box, G.E.P. and G.M. Jenkins, 1970, Time series analysis: forecasting and control (Holden Day, San Francisco).

Caines, P.E., C.W. Keng and S.P. Sethi, 1981, Causality analysis and multivariate autoregressive modelling with an application to supermarket sales analysis, Journal of Economic Dynamics and Control 3, 267-298.

Campbell, J.Y. and N.G. Mankiw, 1987, Permanent and transitory components in macro-economic fluctuations, American Economic Review Proceedings 1987, 111-117.

Clements, M.P. and G.E. Mizon, 1991, Empirical analysis of macroeconomic time series: VAR and structural models, European Economic Review 35, 887-932.

Deniau, C., G. Fiori and A. Mathis, 1989, Impact de la dette publique sur quelques variables macroéconomiques françaises, Economie et Prévision 90, 87-95.

Engle, R.F. and C.W.J. Granger, 1987, Cointegration and error correction: representation, estimation and testing, Econometrica 55, 251-276.

Evans, G.W., 1989, Output and unemployment dynamics in the United States: 1950-1985, Journal of Applied Econometrics 4, 213-237.

Gardeazabal, J., and M. Regulez, 1990, Long run exchange rate determination in the light of cointegration theory, mimeo, University of Pennsylvania, Sept. 1990.

Goldberger, A.S., 1959, Impact multipliers and dynamic properties of the Klein-Goldberger model (North-Holland Publ. Co, Amsterdam).

Gourieroux, C., A. Monfort and E. Renault, 1989, Testing for common roots, Econometrica 57, 171-185.

Hendry, D.F. and G.E. Mizon, 1992, Evaluating dynamic econometric models by encompassing the VAR", in P.C.B. Phillips and V.B. Hall, eds., Models, Methods and Applications of Econometrics, Essays in Honor of Rex Bergstrom (Basil Blackwell, Oxford).

Hsiao, C., 1981, Autoregressive modelling and money-income causality detection, Journal of Monetary Economics 7, 85-106.

Jenkins, G.M., 1979, Practical experiences with modelling and forecasting time series, G.J.P. publ.; also in O.D. Anderson, ed., Forecasting (North-Holland, Amsterdam).

Johansen, S., 1988, Statistical analysis of cointegration vectors, Journal of Economic Dynamics and Control 12, 231-254.

Lippi, M. and L. Reichlin, 1991, Permanent and temporary fluctuations in macro-economics, in N. Thygesen, K. Velupillai and S. Zambelli, eds., Recent developments in business cycle theory, Proceedings of an IEA Conference (MacMillan, forthcoming).

MacKinnon, J.G., 1991, Critical values for cointegration tests, in R.F. Engle and C.W.J. Granger, eds., Long-run economic relationships; readings in cointegration (Oxford University Press, Oxford).

Maravall, A., 1981, A note on identification of multivariate time series models, Journal of Econometrics 16, 237-247.

Monfort, A. and R. Rabemananjara, 1990, From a VAR model to a structural model, with an application to the wage-price spiral, Journal of Applied Econometrics 5, 203-227.

Palm, F.C., 1986, Structural econometric modeling and time series analysis, Applied Mathematics and Computation 20, 349-364.

______, 1983, Structural econometric modeling and time series analysis: an integrated approach, in A. Zellner, ed., Applied Time Series Analysis of Economic Data (U.S. Department of Commerce - Bureau of the Census, 199-233).

Pesaran, M.H., R.G. Pierse and K.C. Lee, 1990, Persistence, cointegration and aggregation: a disaggregated analysis of output fluctuations in the U.S. economy, Journal of Econometrics (forthcoming).

Rose, A.K., 1986, Four paradoxes in GNP, Economics Letters 22, 137-141.

______, 1985, The autoregressivity paradox in macroeconomics, Ph.D. Dissertation, Department of Economics, MIT.

Teräsvirta, T., 1977, The invertibility of sums of discrete MA and ARMA processes, Scandinavian Journal of Statistics 4, 165-170.

Tiao, G.C. and G.E.P. Box, 1981, Modeling multiple time series with applications, Journal of the American Statistical Association 71, 802-816.

Wallis, K.F., 1977, Multiple time series analysis and the final form of econometric models, Econometrica 45, 1481-1497.

Zellner, A. and F.C. Palm, 1975, Time series and structural analysis of monetary models of the U.S. economy, Sankhyà Ser. C 37, 12-56.

______, 1974, Time series analysis and simultaneous equation econometric models, Journal of Econometrics 2, 17-54.

Table 1

Tests on the Univariate Series

	d_t	y_t	p_t	r_t	n_t	m_t
ADF-t	.62	-.89	1.12	-1.61	-1.90	.67
Q_{27}	19.9	20.8	24.3	22.8	22.9	24.0

Table 2

Residual Standard Deviation

	Estimated ARIMA	Implied ARIMA	VAR
d_t	.0359	.0359	.0319
y_t	.0112	.0092	.0086
p_t	.0079	.0073	.0070
r_t	.0394	.0374	.0361
n_t	.0288	.0311	.0283
m_t	.0199	.0205	.0186

Table 3

Engle-Granger Cointegration Test

	d_t	y_t	p_t	r_t	n_t	m_t
Q_{27}	34.4	28.2	26.4	23.9	36.5	30.1
ADF-t	-2.93	-1.79	-2.67	-3.38	-4.42	-3.16
Critical value (5%)	-5.22	-5.22	-5.22	-5.22	-5.22	-5.23

Table 4

Johansen Cointegration Test

	Number of Cointegrating Vectors					
	$r \leq 5$	$r \leq 4$	$r \leq 3$	$r \leq 2$	$r \leq 1$	$r = 0$
Lambda-max test	3.45	7.99	10.14	13.33	22.29	35.59
Critical value (5%)	10.25	14.17	22.30	26.58	37.76	42.04
Trace test	3.45	11.44	21.59	34.92	57.21	92.80
Critical value (5%)	10.25	16.94	32.59	47.11	72.48	98.20

Table 5

VAR Estimation: Auto and Crosscorrelations of Residuals

	d_t	y_t	p_t	r_t	n_t	m_t
d_t	••••••••••••	••••••••••••	••••••••••••	••Θ•••••••••	•••⊕••••••••	••••••••••••
y_t	••••••••••••	••••••••••••	•••••⊕••••••	•⊕••Θ•••••••	•••••••••⊕••	••••••••••••
p_t	••••••••••••	••••••••••••	••••••••••••	••••Θ•••••••	••••••••••••	••••••••••••
r_t	••••••••••••	••••••••••••	••••••••••••	••••••••••••	•••••••••⊕••	••••••••••••
n_t	•⊕••••••••••	••••••••••••	••Θ•••••••••	•••••Θ••••••	•••••••Θ••••	Θ•••••••⊕•••
m_t	•••••••••••⊕	••••••••⊕•••	••••••••••••	••••••••••••	•••⊕••••••••	••••••••••••

⊕ = Sigificant positive correlation

Θ = Significant negative correlation

• = Insignificant correlation

Table 6

Out-of-Sample Forecast: RMSE

	Estimated ARIMA	VAR
d_t	.0148	.0155
y_t	.0133	.0119
p_t	.0081	.0078
r_t	.0496	.0435
n_t	.0502	.0529
m_t	.0175	.0154

Table 7

Series d_t : AR and MA Roots of the
Univariate Model Implied by the VAR

a) Roots of the Autoregressive Polynomial

Root	Modulus	Frequency
- 0.92	0.92	3.14
0.91	0.91	0
0.03 ± 0.85i	0.85	1.54
0.61 ± 0.54i	0.81	0.73
- 0.67 ± 0.41i	0.78	2.59
- 0.56 ± 0.55i	0.78	2.36
0.77 ± 0.10i	0.77	0.13
0.29 ± 0.68i	0.73	1.17
- 0.15 ± 0.67i	0.69	1.79
- 0.55 ± 0.25i	0.61	2.71
0.28 ± 0.53i	0.60	1.08
0.50	0.50	0
- 0.26	0.26	3.14

b) Roots of the Moving Average Polynomial

Root	Modulus	Frequency
0.85	0.85	0
0.60 ± 0.54i	0.81	0.73
- 0.67 ± 0.41i	0.78	2.59
- 0.04 ± 0.78i	0.78	1.62
- 0.55 ± 0.55i	0.77	2.36
- 0.76	0.76	3.14
0.30 ± 0.67i	0.73	1.15
0.66 ± 0.07i	0.66	0.10
0.23 ± 0.50i	0.56	1.14
- 0.37 ± 0.33i	0.49	2.41
- 0.19	0.19	3.14

Table 8

Order of ARMA (p, q) Model

Series	d_t		y_t		p_t		r_t		n_t		m_t	
	p	q	p	q	p	q	p	q	p	q	p	q
Implied ARMA	22	19	22	24	22	17	22	20	22	21	22	21
Implied ARMA after ad-hoc root cancellation	13	10	10	12	5	0	6	4	10	9	13	12
Estimated ARMA	1	1	0	0	1	1	1	0	0	1	2	0

Table 9

Implied ARMA Models After Cancellation of Roots: Ad-hoc Criterion

Variable	d_t	y_t	p_t	r_t	n_t	m_t
AR coefficients						
Lag 1	.05	.92	.15	-.18	-.21	.02
Lag 2	.04	-.08	.30	.25	.25	.04
Lag 3	-.03	.09	-.01	-.14	-.08	-.03
Lag 4	.65	.02	.01	-.08	-.48	.65
Lag 5	-.06	-.08	.30	.02	-.06	-.06
Lag 6	-.00	-.00	—	-.01	.08	-.00
Lag 7	-.02	.03	—	—	-.04	-.02
Lag 8	-.07	-.04	—	—	-.03	-.07
Lag 9	-.01	.01	—	—	.02	-.01
Lag 10	.00	.00	—	—	.01	.00
Lag 11	-.02	—	—	—	—	-.02
Lag 12	.01	—	—	—	—	.01
Lag 13	.00	—	—	—	—	.00
MA coefficients						
Lag 1	.03	.98	—	-.50	-.21	.29
Lag 2	-.04	-.18	—	.06	.20	-.09
Lag 3	.02	.12	—	-.10	-.07	.05
Lag 4	.31	.03	—	-.11	-.03	.18
Lag 5	-.01	-.17	—	—	.00	-.02
Lag 6	.00	.05	—	—	-.01	.00
Lag 7	.00	-.09	—	—	-.01	.00
Lag 8	.00	.10	—	—	-.00	-.02
Lag 9	-.01	-.00	—	—	.00	.01
Lag 10	-.00	-.00	—	—	—	-.01
Lag 11	—	.01	—	—	—	-.00
Lag 12	—	.00	—	—	—	.00

Table 10

Test for Common Roots

Series	Order of Implied ARMA	Number of Common Roots	Order of Simplified ARMA
d_t	(22, 19)	18	(4, 1)
y_t	(22, 24)	22	(0, 2)
p_t	(22, 17)	17	(5, 0)
r_t	(22, 20)	20	(2, 0)
n_t	(22, 21)	21	(1, 0)
m_t	(22, 21)	19	(3, 2)

Table 11

Implied ARMA Models After Cancellation of Roots: Results from the Test

Variable	d_t	y_t	p_t	r_t	n_t	m_t
AR coefficients:						
Lag 1	.03	—	.16	.31	-.12	.91
Lag 2	.14	—	.28	.09	—	-.33
Lag 3	-.01	—	-.02	—	—	.45
Lag 4	.40	—	-.01	—	—	—
Lag 5	—	—	.30	—	—	—
MA coefficients:						
Lag 1	.02	-.03	—	—	—	-.79
Lag 2	—	-.00	—	—	—	-.17

Table 12

Measures of Persistence

	Estimated ARIMA	Implied ARIMA	VAR
d_t	2.61 (.80)	2.57	2.95 (.77)
y_t	1 (.00)	1.10	1.69 (.28)
p_t	5.01 (1.89)	3.80	4.56 (1.35)
r_t	1.54 (.16)	1.61	1.75 (.19)
n_t	.53 (.10)	.78	1.01 (.15)
m_t	1.45 (.31)	1.18	1.24 (.19)

The standard errors of the estimators (computed using linear approximations) are provided in parentheses.

Table 13

Modulus Estimator for a Complex Root in an AR(2) Model:
Asymptotic Results
(T = 84)

Modulus h	S.E. of $\hat{h}$	AR coeff. ϕ_2	S.E. of $\hat{\phi}_2$
.600	.085	.360	.102
.750	.060	.562	.090
.900	.036	.810	.064

Figure 1

SERIES

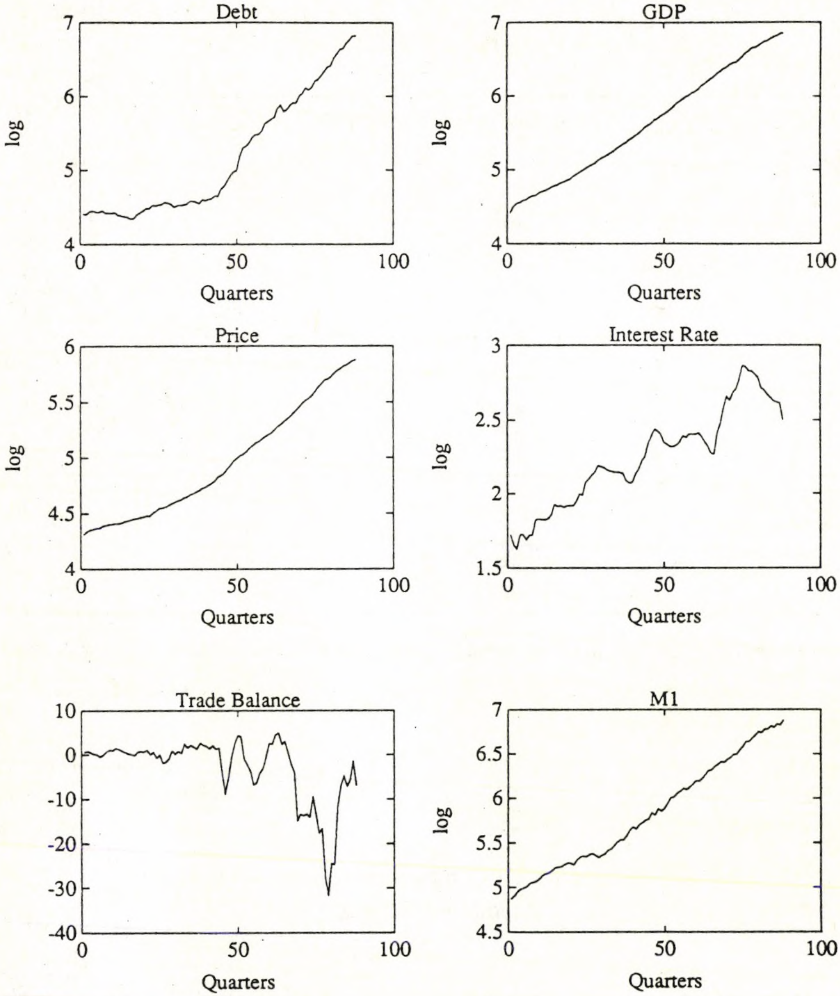


Figure 2

ACF: SERIES

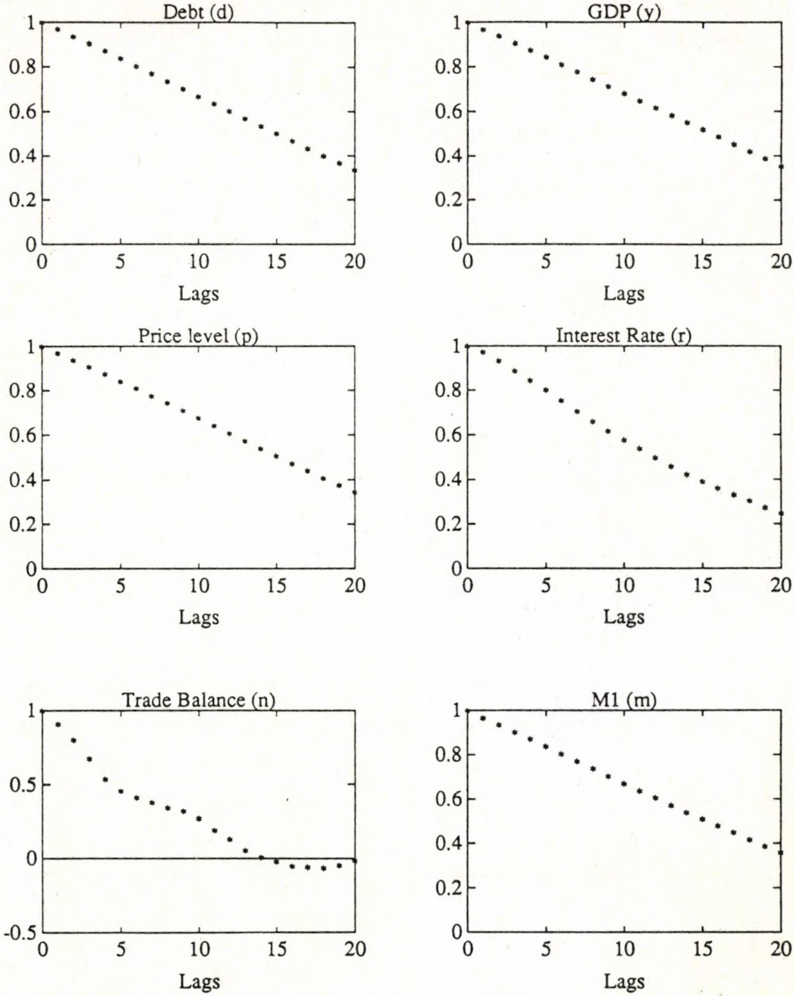


Figure 3

ACF: RESIDUALS

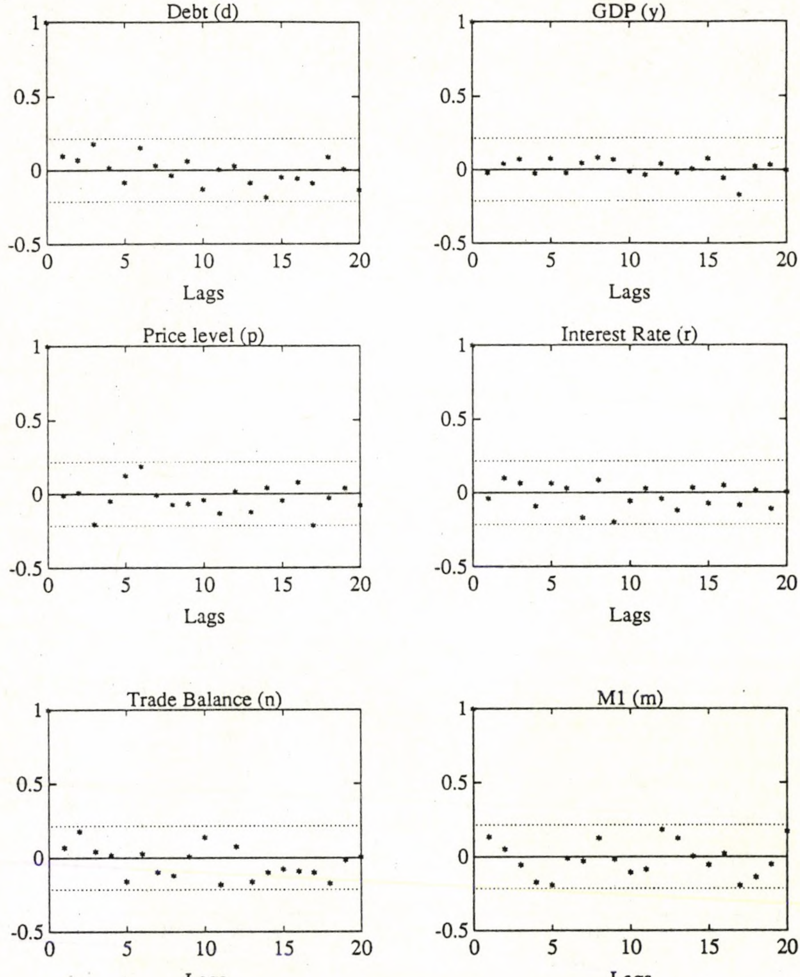


Figure 4

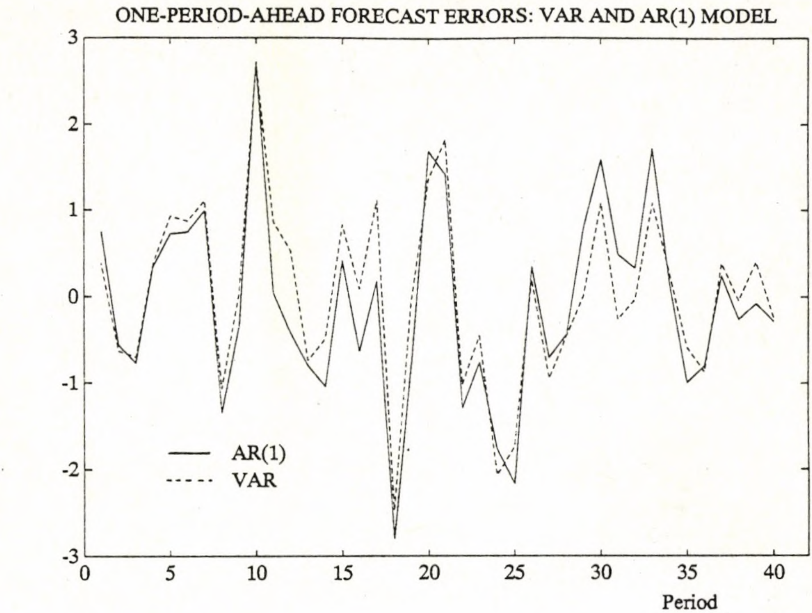


Figure 5

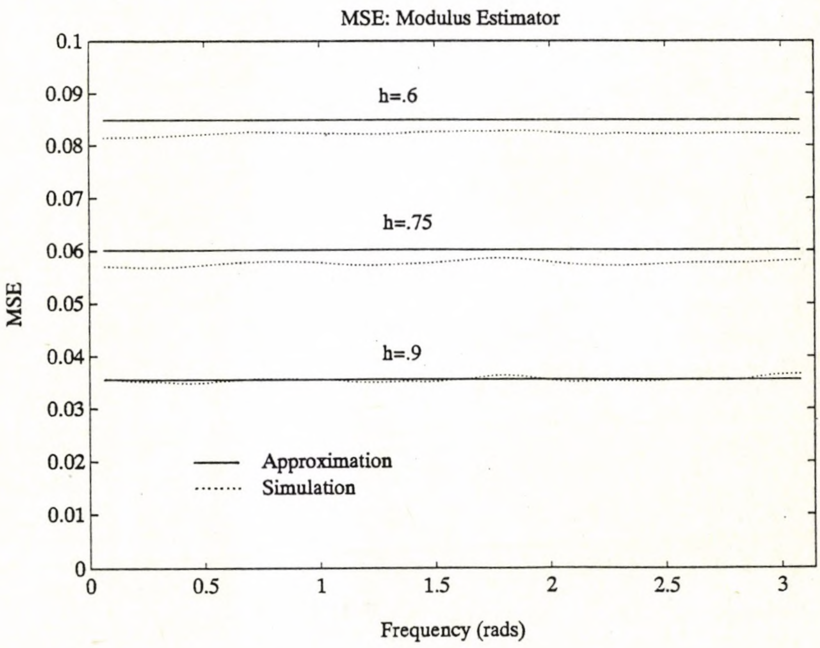


Figure 6

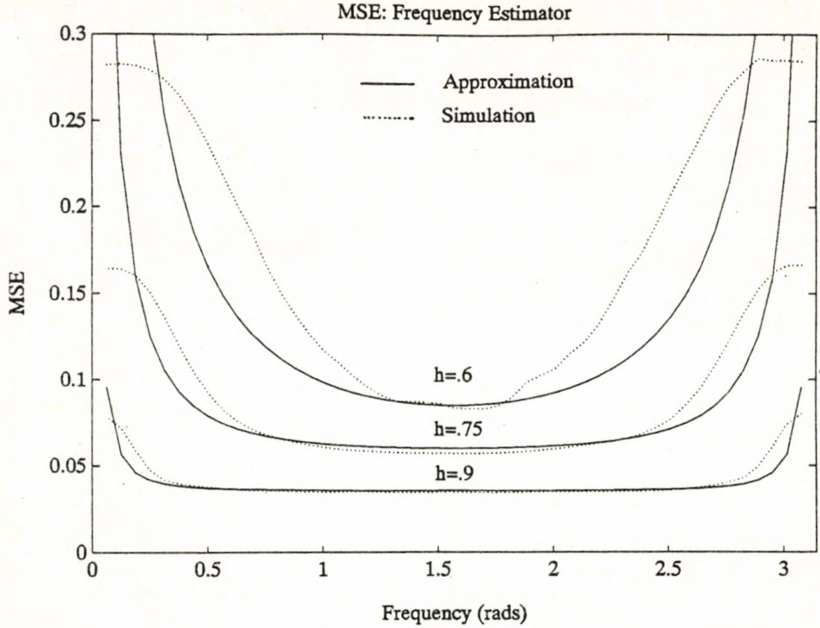
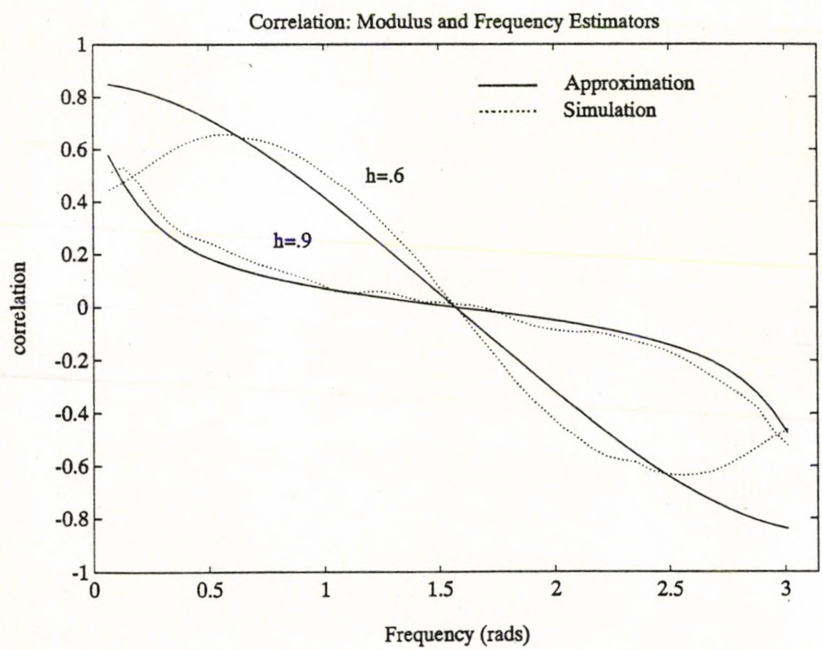


Figure 7



EUI WORKING PAPERS

EUI Working Papers are published and distributed by the European University Institute, Florence

Copies can be obtained free of charge – depending on the availability of stocks – from:

The Publications Officer
European University Institute
Badia Fiesolana
I-50016 San Domenico di Fiesole (FI)
Italy

Please use order form overleaf

Publications of the European University Institute

Economics Department Working Paper Series

To Economics Department **WP**
European University Institute
Badia Fiesolana
I-50016 San Domenico di Fiesole (FI)
Italy

From Name . . . .

Address. . . . .

. . . .

. . . .

. . . .

. . . .

(Please print)

❒ Please enter/confirm my name on EUI Economics Dept. Mailing List
❒ Please send me a complete list of EUI Working Papers
❒ Please send me a complete list of EUI book publications
❒ Please send me the EUI brochure Academic Year 1992/93

Please send me the following EUI **ECO** Working Paper(s):

No, Author . . . .
Title: . . . .
No, Author . . . .
Title: . . . .
No, Author . . . .
Title: . . . .
No, Author . . . .
Title: . . . .

Date . . . . Signature . . . .

Working Papers of the Department of Economics Published since 1990

ECO No. 90/1
Tamer BASAR and Mark SALMON
Credibility and the Value of Information Transmission in a Model of Monetary Policy and Inflation

ECO No. 90/2
Horst UNGERER
The EMS – The First Ten Years Policies – Developments – Evolution

ECO No. 90/3
Peter J. HAMMOND
Interpersonal Comparisons of Utility: Why and how they are and should be made

ECO No. 90/4
Peter J. HAMMOND
A Revelation Principle for (Boundedly) Bayesian Rationalizable Strategies

ECO No. 90/5
Peter J. HAMMOND
Independence of Irrelevant Interpersonal Comparisons

ECO No. 90/6
Hal R. VARIAN
A Solution to the Problem of Externalities and Public Goods when Agents are Well-Informed

ECO No. 90/7
Hal R. VARIAN
Sequential Provision of Public Goods

ECO No. 90/8
T. BRIANZA, L. PHLIPS and J.F. RICHARD
Futures Markets, Speculation and Monopoly Pricing

ECO No. 90/9
Anthony B. ATKINSON/ John MICKLEWRIGHT
Unemployment Compensation and Labour Market Transition: A Critical Review

ECO No. 90/10
Peter J. HAMMOND
The Role of Information in Economics

ECO No. 90/11
Nicos M. CHRISTODOULAKIS
Debt Dynamics in a Small Open Economy

ECO No. 90/12
Stephen C. SMITH
On the Economic Rationale for Codetermination Law

ECO No. 90/13
Elettra AGLIARDI
Learning by Doing and Market Structures

ECO No. 90/14
Peter J. HAMMOND
Intertemporal Objectives

ECO No. 90/15
Andrew EVANS/Stephen MARTIN
Socially Acceptable Distortion of Competition: EC Policy on State Aid

ECO No. 90/16
Stephen MARTIN
Fringe Size and Cartel Stability

ECO No. 90/17
John MICKLEWRIGHT
Why Do Less Than a Quarter of the Unemployed in Britain Receive Unemployment Insurance?

ECO No. 90/18
Mrudula A. PATEL
Optimal Life Cycle Saving With Borrowing Constraints: A Graphical Solution

ECO No. 90/19
Peter J. HAMMOND
Money Metric Measures of Individual and Social Welfare Allowing for Environmental Externalities

ECO No. 90/20
Louis PHLIPS/
Ronald M. HARSTAD
Oligopolistic Manipulation of Spot Markets and the Timing of Futures Market Speculation

* Working Paper out of print

ECO No. 90/21
Christian DUSTMANN
Earnings Adjustment of Temporary Migrants

ECO No. 90/22
John MICKLEWRIGHT
The Reform of Unemployment Compensation:
Choices for East and West

ECO No. 90/23
Joerg MAYER
U. S. Dollar and Deutschmark as Reserve Assets

ECO No. 90/24
Sheila MARNIE
Labour Market Reform in the USSR: Fact or Fiction?

ECO No. 90/25
Peter JENSEN/
Niels WESTERGÅRD-NIELSEN
Temporary Layoffs and the Duration of Unemployment: An Empirical Analysis

ECO No. 90/26
Stephan L. KALB
Market-Led Approaches to European Monetary Union in the Light of a Legal Restrictions Theory of Money

ECO No. 90/27
Robert J. WALDMANN
Implausible Results or Implausible Data? Anomalies in the Construction of Value Added Data and Implications for Estimates of Price-Cost Markups

ECO No. 90/28
Stephen MARTIN
Periodic Model Changes in Oligopoly

ECO No. 90/29
Nicos CHRISTODOULAKIS/
Martin WEALE
Imperfect Competition in an Open Economy

***

ECO No. 91/30
Steve ALPERN/Dennis J. SNOWER
Unemployment Through 'Learning From Experience'

ECO No. 91/31
David M. PRESCOTT/Thanasis STENGOS
Testing for Forecastible Nonlinear Dependence in Weekly Gold Rates of Return

ECO No. 91/32
Peter J. HAMMOND
Harsanyi's Utilitarian Theorem:
A Simpler Proof and Some Ethical Connotations

ECO No. 91/33
Anthony B. ATKINSON/
John MICKLEWRIGHT
Economic Transformation in Eastern Europe and the Distribution of Income

ECO No. 91/34
Svend ALBAEK
On Nash and Stackelberg Equilibria when Costs are Private Information

ECO No. 91/35
Stephen MARTIN
Private and Social Incentives to Form R & D Joint Ventures

ECO No. 91/36
Louis PHLIPS
Manipulation of Crude Oil Futures

ECO No. 91/37
Xavier CALSAMIGLIA/Alan KIRMAN
A Unique Informationally Efficient and Decentralized Mechanism With Fair Outcomes

ECO No. 91/38
George S. ALOGOSKOUFIS/
Thanasis STENGOS
Testing for Nonlinear Dynamics in Historical Unemployment Series

ECO No. 91/39
Peter J. HAMMOND
The Moral Status of Profits and Other Rewards:
A Perspective From Modern Welfare Economics

* Working Paper out of print

ECO No. 91/40
Vincent BROUSSEAU/Alan KIRMAN
The Dynamics of Learning in Mis-Specified Models

ECO No. 91/41
Robert James WALDMANN
Assessing the Relative Sizes of Industry- and Nation Specific Shocks to Output

ECO No. 91/42
Thorsten HENS/Alan KIRMAN/Louis PHLIPS
Exchange Rates and Oligopoly

ECO No. 91/43
Peter J. HAMMOND
Consequentialist Decision Theory and Utilitarian Ethics

ECO No. 91/44
Stephen MARTIN
Endogenous Firm Efficiency in a Cournot Principal-Agent Model

ECO No. 91/45
Svend ALBAEK
Upstream or Downstream Information Sharing?

ECO No. 91/46
Thomas H. McCURDY/
Thanasis STENGOS
A Comparison of Risk-Premium Forecasts Implied by Parametric Versus Nonparametric Conditional Mean Estimators

ECO No. 91/47
Christian DUSTMANN
Temporary Migration and the Investment into Human Capital

ECO No. 91/48
Jean-Daniel GUIGOU
Should Bankruptcy Proceedings be Initiated by a Mixed Creditor/Shareholder?

ECO No. 91/49
Nick VRIEND
Market-Making and Decentralized Trade

ECO No. 91/50
Jeffrey L. COLES/Peter J. HAMMOND
Walrasian Equilibrium without Survival: Existence, Efficiency, and Remedial Policy

ECO No. 91/51
Frank CRITCHLEY/Paul MARRIOTT/
Mark SALMON
Preferred Point Geometry and Statistical Manifolds

ECO No. 91/52
Costanza TORRICELLI
The Influence of Futures on Spot Price Volatility in a Model for a Storable Commodity

ECO No. 91/53
Frank CRITCHLEY/Paul MARRIOTT/
Mark SALMON
Preferred Point Geometry and the Local Differential Geometry of the Kullback-Leibler Divergence

ECO No. 91/54
Peter MØLLGAARD/
Louis PHLIPS
Oil Futures and Strategic Stocks at Sea

ECO No. 91/55
Christian DUSTMANN/
John MICKLEWRIGHT
Benefits, Incentives and Uncertainty

ECO No. 91/56
John MICKLEWRIGHT/
Gianna GIANNELLI
Why do Women Married to Unemployed Men have Low Participation Rates?

ECO No. 91/57
John MICKLEWRIGHT
Income Support for the Unemployed in Hungary

ECO No. 91/58
Fabio CANOVA
Detrending and Business Cycle Facts

ECO No. 91/59
Fabio CANOVA/
Jane MARRINAN
Reconciling the Term Structure of Interest Rates with the Consumption Based ICAP Model

ECO No. 91/60
John FINGLETON
Inventory Holdings by a Monopolist Middleman

* Working Paper out of print

ECO No. 92/61
Sara CONNOLLY/John MICKLEWRIGHT/Stephen NICKELL
The Occupational Success of Young Men Who Left School at Sixteen

ECO No. 92/62
Pier Luigi SACCO
Noise Traders Permanence in Stock Markets: A Tâtonnement Approach. I: Informational Dynamics for the Two-Dimensional Case

ECO No. 92/63
Robert J. WALDMANN
Asymmetric Oligopolies

ECO No. 92/64
Robert J. WALDMANN /Stephen C. SMITH
A Partial Solution to the Financial Risk and Perverse Response Problems of Labour-Managed Firms: Industry-Average Performance Bonds

ECO No. 92/65
Agustín MARAVALL/Víctor GÓMEZ
Signal Extraction in ARIMA Time Series Program SEATS

ECO No. 92/66
Luigi BRIGHI
A Note on the Demand Theory of the Weak Axioms

ECO No. 92/67
Nikolaos GEORGANTZIS
The Effect of Mergers on Potential Competition under Economies or Diseconomies of Joint Production

ECO No. 92/68
Robert J. WALDMANN/ J. Bradford DE LONG
Interpreting Procyclical Productivity: Evidence from a Cross-Nation Cross-Industry Panel

ECO No. 92/69
Christian DUSTMANN/John MICKLEWRIGHT
Means-Tested Unemployment Benefit and Family Labour Supply: A Dynamic Analysis

ECO No. 92/70
Fabio CANOVA/Bruce E. HANSEN
Are Seasonal Patterns Constant Over Time? A Test for Seasonal Stability

ECO No. 92/71
Alessandra PELLONI
Long-Run Consequences of Finite Exchange Rate Bubbles

ECO No. 92/72
Jane MARRINAN
The Effects of Government Spending on Saving and Investment in an Open Economy

ECO No. 92/73
Fabio CANOVA and Jane MARRINAN
Profits, Risk and Uncertainty in Foreign Exchange Markets

ECO No. 92/74
Louis PHLIPS
Basing Point Pricing, Competition and Market Integration

ECO No. 92/75
Stephen MARTIN
Economic Efficiency and Concentration: Are Mergers a Fitting Response?

ECO No. 92/76
Luisa ZANCHI
The Inter-Industry Wage Structure: Empirical Evidence for Germany and a Comparison With the U.S. and Sweden

ECO NO. 92/77
Agustín MARAVALL
Stochastic Linear Trends: Models and Estimators

ECO No. 92/78
Fabio CANOVA
Three Tests for the Existence of Cycles in Time Series

ECO No. 92/79
Peter J. HAMMOND/Jaime SEMPERE
Limits to the Potential Gains from Market Integration and Other Supply-Side Policies

* Working Paper out of print

ECO No. 92/80
Víctor GÓMEZ and Agustín MARAVALL
Estimation, Prediction and Interpolation for Nonstationary Series with the Kalman Filter

ECO No. 92/81
Víctor GÓMEZ and Agustín MARAVALL
Time Series Regression with ARIMA Noise and Missing Observations Program TRAM

ECO No. 92/82
J. Bradford DE LONG/ Marco BECHT
"Excess Volatility" and the German Stock Market, 1876-1990

ECO No. 92/83
Alan KIRMAN/Louis PHLIPS
Exchange Rate Pass-Through and Market Structure

ECO No. 92/84
Christian DUSTMANN
Migration, Savings and Uncertainty

ECO No. 92/85
J. Bradford DE LONG
Productivity Growth and Machinery Investment: A Long-Run Look, 1870-1980

ECO NO. 92/86
Robert B. BARSKY and J. Bradford DE LONG
Why Does the Stock Market Fluctuate?

ECO No. 92/87
Anthony B. ATKINSON/John MICKLEWRIGHT
The Distribution of Income in Eastern Europe

ECO No.92/88
Agustín MARAVALL/Alexandre MATHIS
Encompassing Unvariate Models in Multivariate Time Series: A Case Study

* Working Paper out of print