European University Institute
DEPARTMENT OF ECONOMICS

Three Essays in International Macroeconomics

Moritz Alexander Roth

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, 15 November 2016

European University Institute
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I confirm that chapter 2 – 'Foreign Direct Investment and the Equity Home Bias Puzzle' - was jointly co-authored with Sven Blank and Mathias Hoffmann and I contributed 65% of the work.

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Moritz Roth

Acknowledgements

I am heavily grateful and indebted to my PhD supervisors Árpád Ábrahám and Andrea Mattozzi for their constant encouragement, curiosity and critique that made it possible to create this thesis. No less I would like to thank Evi Pappa, Juan Dolado, Mathias Hoffmann, Sven Blank, Agustín Bénétrix, Harris Dellas, Roberto Pancrazi, Marek Raczko, Dominik Menno and all of my EUI classmates for valuable discussions and comments that have significantly improved the papers included in this PhD thesis. To my classmates and fellow researchers at the EUI I would like to say a special thank you. The suffering through long nights of problem sets and hunting deadlines made us stick together and created a warm and encouraging atmosphere throughout the PhD years. I am grateful to have spent this intense time with you in one of the most beautiful cities in the world and hope to see you wherever our ways cross again. Most grateful I am to my family and friends. To Marina and Michael for being caring, supportive and wonderful parents. To Max for being the best brother one can imagine, lass uns unseren Namen immer alle Ehre machen. To my grandparents, Irma and Albert, who at over 90 years of age still feverishly track my steps. To all my friends who have supported me through the years of this endeavor, through the ups and downs of completing this work and of life as it is. Especially, I would like to say thank you to Felix, Lutz, Benni, Jochen, Timo, Christian, Martin and the "Fründts", to Lisa, Natascha, Melita, John, Brais, Dominik, Noemie, Pawel, Pavel, Ilya, Paul, Iain, Altynay and Jekaterina for being friends to count on, always remember that you can count on me as well. Another warm thank you, I would like to say to the EUI ECO administrative staff for being supportive with all kinds of organizational issues and especially for helping in the final steps to submit this thesis.

Abstract

This thesis sheds light on three questions in international macroeconomics. The first chapter investigates why business cycle correlations are state-dependent and higher in recessions than in expansions. I suggest a mechanism to explain why this is the case. Therefore, I build an international real business cycle model with occasionally binding constraints on capacity utilization which can account for state-dependent cross-country correlations in GDP growth rates. Empirically, I successfully test for the presence of capacity constraints using data from the G7 advanced economies in a Bayesian threshold autoregressive (T-VAR) model. This finding supports capacity constraints as a prominent transmission channel of cross-country GDP asymmetries in recession compared to expansions. The second chapter is joint work with Mathias Hoffmann and Sven Blank of the Deutsche Bundesbank. It analyzes how foreign direct investment (FDI) influences optimal country portfolio diversification. In a DSGE model that features the endogenous choice of firms to become internationally active through either exports or foreign direct investment (FDI), we find that the optimal equity holdings of agents are more biased towards domestic firms than in a model without FDI. The third chapter explores under which circumstances member countries of a monetary union will not find it optimal to bail each other out. To investigate these circumstances, I build a model of cross-country holdings of sovereign debt with the possibility of default, as well as the possibility to negotiate a bailout for a struggling country. I show that if there is a representative household in each individual member country, a bailout solution always exists. In a second version of the model that involves heterogeneity in household wealth within the bailout providing country, the utilitarian government of this country optimally refuses a bailout in some states of the world.

Contents

Chapter 1

International Co-movements in Recessions

1.1 Introduction

This paper is motivated by the empirical finding that business cycle co-movements across countries are higher during economic contractions than during economic expansions (e.g. Yetman, 2011; Antonakakis and Scharler, 2012, as well as my own empirical evidence). In an international real business cycle framework I investigate the reasons and potential mechanisms that cause co-movements of GDP across countries to be state-dependent. In the data, I find that the average pairwise correlation of GDP growth rates between 20 OECD countries in a quarterly sample from Q1:1961 to Q4:2011 is between 5.4 and 22.7 percentage points higher during recessions compared to expansions.[1] The main purpose of the paper is to build a framework in which country-specific shocks and their spillovers to other countries, endogenously lead to higher cross-country co-movements in GDP during recessions. To achieve this asymmetry, I am introducing a friction in the form of an occasionally binding capacity utilization constraint in an otherwise standard 2-country, 2-goods large-open economy model (e.g. Heathcote and Perri, 2002). The friction can be interpreted as the maximum production capacity of the machines operated in a given country. At least in the short-run, this maximum capacity cannot be increased. The implication of the occasionally binding constraint is that following a sequence of good shocks, a given country's machines reach their maximum capacity and the increase in production is dampened compared to an unconstrained economy. After a sequence of bad shocks, machines can be left idle and the economy remains unconstrained. This introduces asymmetric responses to shocks in the sense that negative shocks to one country have stronger effects on this country's economy than positive ones. The crucial feature of the mechanism to create state-dependent cross-country correlations is that the asymmetries also spill over internationally and can even be amplified by the presence of a similar occasionally binding constraint in the other country. Countries are interlinked through trade in intermediary production goods,

[1] Since the latest financial crisis led to a historical high in cross-country GDP correlations, the magnitude of the result depends on whether this time period is included in the sample or not. Furthermore, different procedures to disentangle recessions and expansions are compared.

as each country produces one of these intermediary goods and uses the domestic as well as the foreign intermediary good in the production of a final consumption good. Therefore, a positive (negative) shock to a given economy affects the production of intermediary goods of both countries positively (negatively). Due to the fact that negative shocks have higher effects than positive ones in this model with occasionally binding capacity constraints recessions spill-over more intensively than expansions between countries and this leads to state-dependent cross-country correlations. I show that the proposed mechanism can match the differences in cross-country GDP growth correlations between expansions and recessions observed in the data if tradable intermediary goods are to a certain degree complementary. Lastly, I find empirical evidence for threshold effects in the capacity utilization rate of the US economy and use the resulting threshold estimates in a Bayesian threshold autoregression (TVAR) to obtain asymmetric empirical responses of the G7 advanced economies' variables to positive, as well as negative US TFP shocks. The resulting impulse response functions not only mirror the impulse responses of the theoretical model, but are also in line with the assumed complementarity between Home and Foreign intermediary good. The necessity of such an extension of the workhorse 2-countries, 2 goods model arises from the fact that the standard model is not capable of producing asymmetries in cross-country correlations, because it is absolutely symmetric and usually solved using linear perturbation methods. Taking these asymmetries into account explicitly is relevant for economic policy conclusions drawn from international real business cycle (RBC) models. Thus, economic policy conclusions drawn from linear and symmetric models might be misleading if in the real world agents anticipate international asymmetries in the business cycle and adjust their decisions to their expectations. The question of why business cycle co-movements are significantly higher during recessions compared to expansions has to the best of my knowledge not been investigated in the literature. Additionally, the consequences of this fact on policy-making have not been explored either. There are several empirical as well as theoretical papers in the literature related to this paper. The empirical fact that business cycle co-movements across countries are increasing in recessions has been pointed out by Yetman (2011). He takes the US cycle and US recessions as reference data and shows that the co-movement of different country groups (G7, Europe, Asia-Pacific) with the US business cycle is only positive and significantly different from zero if the US is in a recession. Using a dynamic conditional correlations (DCC) approach, Antonakakis and Scharler (2012) find that the cross-country correlations between a number of developed countries significantly increases during US recessions in the years between 1960 and 2009. Moreover, there are other references in the literature which find that business cycles in the G7 countries become more similar in recessions (for instance Canova et al., 2007) or that individual countries' cycles are more affected by the global cycle in global recessions (e.g. Claessens et al., 2013; Helbling and Bayoumi, 2003). The mechanism put forward in this paper as a cause of state-dependent co-movements in an international real business cycle model is an occasionally binding constraint on capital utilization. These types of constraints have already been used to explain within country business cycle asymmetries, i.e. the fact that recessions are usually sharper and shorter than expansions (e.g. Hansen and Prescott, 2005; Knueppel, 2014). In this paper, I show that within-country asymmetries can also be a cause for international correlations to become asymmetric between recessions and expansions. The threshold tests I perform on US utilization rates support this mechanism and the TVAR evidence obtained using these test results are in line with the theoretical model results, further strengthening the relevance of the proposed mechanism.

1.2 Empirical analysis

To establish the fact that business cycle co-movements are higher during recessions than during expansions, I obtain quarterly data on GDP for the time span of Q1:1961 to Q4:2011 from the OECD's Quarterly National Accounts database.[2] Due to data availability considerations, I restrict the analysis to 20 out of 34 OECD countries, which I aggregate to the country groups EU-13[3], G7, NAFTA and Oceania. A list of the countries included in these groups is given in section A.1 in the appendix. All of the series are at constant 2005 prices (OECD reference year), seasonally adjusted and converted to US dollar values. Furthermore, I also obtain annual population data from the OECD.Stats database and normalize all observations of a year by population to obtain per capita values and take logarithms of the series.[4] To assess the patterns in international co-movement in US recessions and expansions, I calculate different measures of correlation and co-movement in the business cycles.

1.2.1 Disentangling expansions and recessions

Most of the empirical work on business cycle correlation during expansions and recessions identify the US business cycle as a reference cycle for the analysis of co-movement in recessions and expansions. Therefore, to identify recessions of the US economy most authors in the literature use the NBER recession dates to disentangle expansions and recessions. Because in this paper, I additionally investigate the sources of asymmetries in business cycle correlations, I need a procedure that can be applied to the empirical data as well as to the model generated data in the same way. Therefore, to disentangle recessions and expansions, I use the turning point algorithm developed by Harding and Pagan (2002). The algorithm identifies turning points in the log-series of GDP. If a given observation is a maximum among the previous and the following 2 observations, the algorithm identifies this observation as a peak. Similarly, if a given observation is a minimum among the previous and the following 2 observations, the algorithm identifies this observation as a trough. Along the business cycle, expansions are defined as the time span between a trough to a peak, while recessions are the time span between a peak to a trough. The algorithm also performs validity check to ensure for instance that a trough is always followed by a peak and vice versa. To see how the algorithm compares to the NBER recession dates, table 1.1 shows the recession dates identified by the turning point algorithm, as well as the NBER recession dates.

[2]I also obtain annual data for the same time period from the Annual National Accounts database to perform some illustrations and robustness checks.

[3]Greece and Ireland had to be excluded from the EU-15 because of data availability.

[4]Since population data is only available at annual frequency and GDP data is quarterly, I use linear interpolation to obtain population size for the quarters within a given year.

Table 1.1 – Recession periods in the US, 1961-2011

NBER recessions	TP recessions
Q1:1961	
	Q2:1962 - Q4:1964
	Q2:1966 - Q4:1967
Q4:1969 - Q4:1970	Q2:1969 - Q4:1970
Q4:1973 - Q1:1975	Q3:1973 - Q1:1975
	Q2:1976 - Q4:1976
Q1:1980 - Q3:1980	Q1:1979 - Q3:1980
Q3:1981 - Q4:1982	Q2:1981 - Q4:1982
	Q4:1985 - Q1:1987
Q3:1990 - Q1:1991	Q2:1990 - Q4:1991
	Q1:1993 - Q3:1993
	Q1:1995 - Q1:1996
Q1:2001 - Q4:2001	Q3:2000 - Q1:2003
	Q4:2005 - Q1:2006
Q4:2007 - Q2:2009	Q1:2008 - Q2:2009

The algorithm matches the dates and lengths of the NBER dated recessions well. At the same time it identifies more recession periods than the dating comittee at NBER. Most likely, these episodes are downturns that the NBER did not find severe enough to term them recessions, but they do fulfill the dating criteria of the turning point algorithm.[5]

1.2.2 Conditional Correlations

First, I calculate correlations of GDP growth rates between the identified country blocks, as well as the individual G7 countries, conditional on the US being in an expansionary or recession period. I also test the difference between correlation coefficients using a Fisher r-z-transformation of the coefficients.[6] The results are shown in table 1.2. All the correlations within the country groups increase in US recessions compared to expansions. This is true for recessions identified by both NBER and the TP algorithm. These correlation differences are highly significant for the EU-13 and the G7, mainly because there are more countries in these groups and thus the sample size is larger. Moreover, the differences for expansions and recessions identified by the TP algorithm tend to be more significant since the number of expansion and recession (114 vs. 89) periods is more balanced than using the NBER dates (169 vs. 35).[7] Also for the individual countries in the G7 group all correlation

[5]To check robustness I increased the time span to identify peaks and troughs from ± 2 periods around a given observation to ± 3 and ± 4. Although, the identified recession episodes tend to get shorter, the number of identified peaks and troughs does not change. Since ± 2 is the number of periods used by Harding and Pagan (2002) and this specification cover the NBER recession dates best, I use this specification.

[6]This is necessary as correlation coefficients are defined on $[-1, 1]$, while the test statistic on the difference between coefficients is defined on $(-\infty, +\infty)$.

[7]The balance of observations is not a driver of the observed differences, in a robustness exercise I calculated the same measures for the same number of high growth periods (boom), as there are

Table 1.2 – Conditional Correlations - GDP Q1:1961 - Q4:2011

	NBER			TP		
Country	Exp	Contr	C > E	Exp	Contr	C > E
Individual G7						
Canada	0.2910	0.4007	Yes	0.2795	0.4279	Yes
France	0.0332	0.3729	Yes*	-0.0256	0.4270	Yes***
Germany	0.1244	0.2923	Yes	0.0026	0.2336	Yes
Italy	0.0031	0.2704	Yes	0.0935	0.3206	Yes*
Japan	0.1111	0.3705	Yes	0.1165	0.4382	Yes**
UK	0.0797	0.4580	Yes**	0.0742	0.4276	Yes***
Observations	169	35		114	89	
Country Blocks						
EU 13	0.1370	0.3805	Yes***	0.1318	0.3871	Yes***
G7	0.2346	0.4521	Yes***	0.2201	0.4678	Yes***
NAFTA	0.4552	0.5655	Yes	0.4209	0.5471	Yes
Oceania	0.3701	0.4620	Yes	0.3802	0.4458	Yes
Observations	C*169	C*35		C*114	C*89	
Average						
All 20	0.1177	0.3413	**0.2236*** **	0.1096	0.3562	**0.2466*** **
p-value (2s)			0.0000			0.0000
Observations	20*169	20*35		20*114	20*89	

$***$,$**$ and $*$, indicate $p < 0.01$, $p < 0.05$, $p < 0.1$, respectively. The table shows cross-country correlations of GDP growth rates between the given country or country block and the US during expansions (Exp) and contractions (Contr). Expansions and contractions are found using the NBER recession dates in the NBER columns and the Harding and Pagan (2002) turning point algorithm in the TP columns. The columns titled $C > E$ indicate if the correlation coefficient is higher during contractions compared to expansions. For the country blocks, C is the number of countries within a given country block, excluding the US. For EU13 $C = 13$, for G7 $C = 6$, for NAFTA $C = 2$ and for Oceania $C = 2$.

Table 1.3 – Conditional Correlations - GDP Q1:1961 - Q4:2006

	NBER			TP		
Country	Exp	Contr	C > E	Exp	Contr	C > E
Individual G7						
Canada	0.2890	0.2873	No	0.2332	0.3394	Yes
France	0.0263	0.2462	Yes	-0.0562	0.3222	Yes***
Germany	0.1413	0.1134	No	0.0154	0.1140	Yes
Italy	-0.0199	0.1669	Yes	0.0239	0.1905	Yes
Japan	0.0871	0.1946	Yes	0.0683	0.3172	Yes*
UK	0.0534	0.3791	Yes	-0.0127	0.3187	Yes**
Observations	155	28		99	83	
Country Blocks						
EU 13	0.1267	0.2592	Yes**	0.0994	0.2775	Yes***
G7	0.2253	0.3411	Yes	0.1817	0.3717	Yes***
NAFTA	0.4595	0.4539	No	0.4107	0.4728	Yes
Oceania	0.3658	0.4660	Yes	0.3672	0.4338	Yes
Observations	C*155	C*28		C*99	C*83	
Average						
All 20	0.1099	0.2145	**0.1046****	0.0809	0.2516	**0.1707*****
p-value (2s)			0.0215			0.0000
Observations	20*155	20*28		20*99	20*83	

$***$,$**$ and $*$, indicate $p < 0.01$, $p < 0.05$, $p < 0.1$, respectively. The table shows cross-country correlations of GDP growth rates between the given country or country block and the US during expansions (Exp) and contractions (Contr). Expansions and contractions are found using the NBER recession dates in the NBER columns and the Harding and Pagan (2002) turning point algorithm in the TP columns. The columns titled $C > E$ indicate if the correlation coefficient is higher during contractions compared to expansions. For the country blocks, C is the number of countries within a given country block, excluding the US. For EU13 $C = 13$, for G7 $C = 6$, for NAFTA $C = 2$ and for Oceania $C = 2$.

coefficients are larger in recessions compared to expansions. Calculating the average correlation difference between all countries in the sample with the US during the identified US business cycle states, I find differences of 22.4 percentage points using the NBER recession dates and 24.7 percentage points using the TP algorithm. Both of them are significantly different from zero at the 1% confidence level. As it is very likely that the global recession of 2007-2009 has a large impact on the cross-country correlations in recessions and expansions in the time period investigated here, I do the same calculations as above, excluding the years 2007-2011 from the sample. The results are shown in table 1.3. As expected, the differences decrease in general, but overall correlations in recessions are still significantly higher than in expansions. For the individual G7 countries this result is reversed for Canada and Germany when recessions are identified by NBER dates, while for the country blocks it is reversed for the NAFTA using NBER dates. For the other countries and country blocks the main effect keeps its direction. Over the whole sample the average correlation are 10.5 percentage points higher during NBER dates and 17.1 percentage points higher

recession periods in the sample. Results do not change considerably.

during TP recessions. Both are significantly different from zero at least at the 5% confidence level.

1.2.3 Yetman Synchronization

The literature has also proposed alternatives to correlation measures. For instance co-movement measures, i.e. indicators if business cycles are in the same phase, have been proposed (see for instance Yetman (2011) or de Haan et al. (2007)). Here, I am concentrating on a measure proposed by Yetman (2011). The co-movement measure of Yetman (2011) is defined as the product of z-scores of annual GDP growth rates, i.e

$$\rho_{ijt} = z_{it} z_{jt} \tag{1.1}$$

where

$$z_{it} = \frac{(y_{it} - \bar{y_i})}{\sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(y_{it} - \bar{y_i})^2}} \tag{1.2}$$

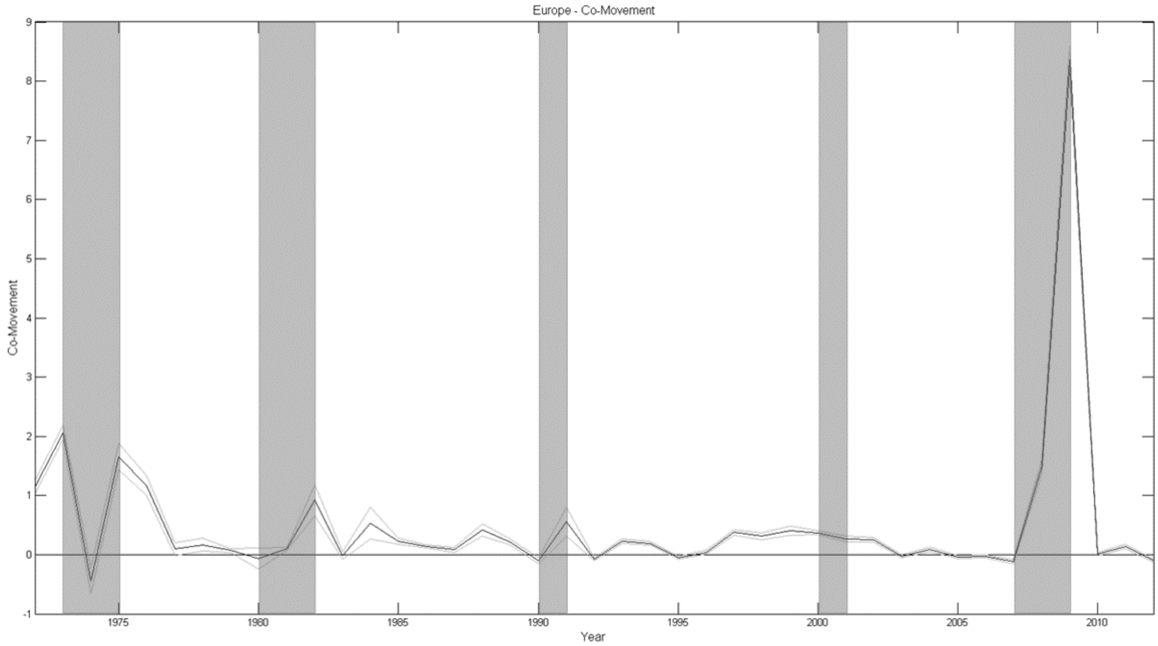
and y are GDP growth rates. The z-score normalization thus ensures that positive co-movement is indicated if both countries are growing above or below their mean growth rate and negative co-movement is indicated if one country grows above its mean growth rate while the other grows below its mean growth rate. Despite the degree of freedom adjustment used in calculating the Yetman measure, the time average across co-movements corresponds to the uncorrected correlations. Thus, the average Yetman measure is very similar to the correlations above. Therefore, I do not state the results here explicitly. The advantage of the co-movement measure is that it can be calculated at each point in time. Concentrating on the co-movement between the European aggregate and the US, and using annual data for illustrative purposes[8], I follow Yetman (2011) and regress the co-movement measure of 13 European countries with the US reference cycle on country fixed effects and time dummies such that the coefficients of the time dummies indicate an average estimator of European co-movement with the US reference cycle at a given point in time and the standard deviations of these estimators indicate their significant difference from zero. Figure 1.1 shows that significantly positive spikes in the co-movement measure in most cases coincide with the NBER recession dates of the US economy, while co-movement is moderately positive the remaining time.

1.2.4 Average correlations of country pairs in US recessions

So far, the focus has only been on correlations of the countries and country blocks in the sample with the US economy during US recessions. Now, I look at the average cross-country GDP correlations of all these countries with each other during US recession periods. To obtain a clear picture of the state-dependent correlations in recessions and expansions of all country-pairs on average, table 1.4 shows the average correlations across all country-pairs for both considered time spans, as well as both recession identification methods. It shows that cross-country correlations

[8]Observations on the measure calculated with quarterly data is too frequent to create a nice and clear plot.

Figure 1.1 – Yetman synchronization - GDP 1972 - 2011



across all countries are significantly higher during US recessions compared to US expansions. It also shows that the cross-correlations have increased due to the recent global financial crisis, but that the most conservative measure still indicates that cross-country correlations increased by at least 5.44 percentage points during US recessions compared to US expansions if we exclude this recent crisis. In fact though, the difference in correlations might have been as high as 22.7 percentage points, if the recent financial crisis is considered a part of the underlying data generating process of the global economy. Given the findings in this section, it is crucial that we understand what might be driving differences in observed cross-country correlations during expansions and contractions. In the following section, I propose a mechanism which can account for these observed differences.

1.3 Theoretical Analysis

Why do we observe that business cycle co-movements are significantly higher during recessions compared to expansions and what are the consequences of this fact on the decisions of economic agents and policy-making? These questions have to the best of my knowledge not been investigated in the literature. An understanding of these differences is important because agents that anticipate systematic differences in economic outcomes across the business cycle will adapt their economic decisions to these differences. The standard international real business cycle model (IRBC), i.e. the workhorse model with which economist model economic decisions in international macroeconomics, cannot generate asymmetries between countries by construction. It is typically solved using linear perturbations around its deterministic steady state. Policy recommendations drawn by economists who base their conclusions on linear models when in fact important non-linearities are present in the data might be misled. In this theoretical section, I am building a framework in

Table 1.4 – Correlations of country pairs

	NBER			TP		
1961-2011						
Country	Exp	Contr	Diff	Exp	Contr	Diff
Avg. Correlation	0.1910	0.3973	**0.2270*****	0.1994	0.3389	**0.1535*****
P-Value			**0.0000**			**0.0000**
Obs.	210*169	210*35		210*114	210*89	
1961-2006						
Avg. Correlation	0.1862	0.2479	**0.0679*****	0.1901	0.2395	**0.0544*****
P-Value			**0.0000**			**0.0000**
Obs.	210*155	210*28		210*99	210*83	

$***$,$**$ and $*$, indicate $p < 0.01$, $p < 0.05$, $p < 0.1$, respectively. The table shows cross-country correlations of GDP growth rates between all 20 countries during US expansions (Exp) and US contractions (Contr). Expansions and contractions are found using the NBER recession dates in the NBER columns and the Harding and Pagan (2002) turning point algorithm in the TP columns. The columns titled 'Diff' give the difference in correlation coefficient calculated for expansions and contractions. The results are calculated for the full sample of Q1:1961 to Q4:2011, as well as for a subsample from Q1:1961 to Q4:2006 which excludes the recent financial crisis.

which country-specific shocks and their spill-overs to other countries endogenously lead to higher cross-country co-movements in recessions compared to expansions. For this purpose, I am introducing a friction in the form of an occasionally binding capacity utilization constraint in an otherwise standard 2-country, 2-goods large-open economy model (e.g. Heathcote and Perri, 2002). To solve the model, I use the solution algorithm for models with occasionally binding constraints developed by Guerrieri and Iacoviello (2015), which is able to capture non-linearities arising from the occasionally binding constraints. For the model to match the observed asymmetries well, I choose to target the most conservative measure obtained by the empirical analysis. Therefore, the targeted difference in line with the data is the increase of 5.44 percentage points between expansions and recessions obtained by applying the TP algorithm on data excluding the global financial crisis. This number is also broadly in line with the findings of Antonakakis and Scharler (2012). I will show that the model produces systematically higher cross-country correlations in contractions compared to expansions. In order to match the targeted magnitude, tradable intermediate goods have to be complements to a certain degree.

1.4 International Model with Occasionally Binding Capacity Constraints

The model economy consists of Home country (1) and Foreign country (2). Despite the occasionally binding capacity constraint it follows the exposition of Heathcote and Perri (2002). Within these countries there is an identical measure of infinitely lived households. Moreover, in each country, there exists a representative producer of a final consumption good and a representative producer of intermediate goods. The intermediate goods can be traded internationally, Foreign and Home intermediate goods are imperfect substitutes in the production process of the final good. Final goods can only be invested or consumed in the country they are produced in. The model economy experiences a random event $s_t \in S$ every period t. S is a possibly infinite set of states of the world. The history of events up to and including date t is given by s^t. At date 0 $\Pi(s^t)$ denotes he probability that any particular history s^t has realized up to t. Households choose to supply capital and labor to intermediate-good-producing firms (i-firms). These firms are perfectly competitive. It is assumed that households' labor as well as their capital cannot be exchanged internationally, i.e. it is internationally immobile. The capital stock $k_i(s^t)$ of each country i is owned by that country's households at any point in time t. Moreover, they choose the intensity with which firms can operate the households' machines. Households can only save in an international uncontingent bond and therefore financial markets are incomplete. Households in each country obtain their utility from consumption, $c_i(s^t)$, and leisure $1-n_i(s^t)$. In the definition of leisure, $n_i(s^t)$ is the amount of labor supplied and total period time endowment is fixed at 1.

1.4.1 The maximization problems of the agents

In each country $i = 1, 2$ there is a representative final good producer, an intermediate good producer and a representative household.

Intermediate good firms

The intermediate good firms produce country i's intermediate good. They are termed a for country 1 and b for country 2. For the production process they hire labor and rent capital from the households, which own all the resources of the economy. Intermediate good firms operate a Cobb-Douglas production technology

$$F(z_i(s^t), k_i(s^t), n_i(s^t), u_i(s^t)) = e^{z_i(s^t)} \left(u_i(s^t)k_i(s^t)\right)^{\theta} n_i(s^t)^{1-\theta} \tag{1.3}$$

where $z_i(s^t)$ is an exogenous technology shock. The rental on capital and the wage rate in country i are given by $w_i(s^t)$ and $r_i(s^t)$. They are denoted in terms of country i's intermediate good. after history s^t, the static maximization problem an intermediate firm in country i is

$$max_{k_i(s^t), n_i(s^t)} \{F(z_i(s^t), k_i(s^t), n_i(s^t), u_i(s^t)) - w_i(s^t)n_i(s^t) - r_i(s^t)u_i(s^t)k_i(s^t)\}$$
$$\text{subject to } k_i(s^t), n_i(s^t) \geq 0. \tag{1.4}$$

Final good firms

Investment adds to country i's capital stock as follows:

$$k_i(s^{t+1}) = \left[1 - \delta(u_i(s^t))\right] k_i(s^t) + x_i(s^t). \tag{1.5}$$

Here, $\delta(u_i(s^t))$ is the depreciation rate, which depends on the degree of capital utilization in this model, and $x_i(s^t)$ is country i's investment in terms of final goods.

Final goods are produced using intermediate goods a and b as inputs. They operate a constant returns to scale technology and are perfectly competitive:

$$G_i(a_i(s^t), b_i(s^t)) = \begin{cases} [\omega_1 a_i(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\omega_1) b_i(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} & \text{if } i = 1, \\ [(1-\omega_1) a_i(s^t)^{\frac{\sigma-1}{\sigma}} + \omega_1 b_i(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} & \text{if } i = 2. \end{cases} \tag{1.6}$$

The elasticity of substitution between goods a and b is σ and $\omega_1 > 0.5$ denotes the home bias in the production of domestic final goods. The maximation problem of country i final good firm's after history s^t is

$$max_{a_i(s^t),b_i(s^t)}\{G(a_i(s^t), b_i(s^t)) - q_i^a(s^t)a_i(s^t) - q_i^b(s^t)b_i(s^t)\}$$
$$\text{subject to } a_i(s^t), b_i(s^t) \geq 0 \tag{1.7}$$

for $i = 1, 2$. $q_i^a(s^t)$ and $q_i^b(s^t)$ denote the country i prices of intermediary goods a and b in units of country i's final good.

Households

The per-period utility for the country i household after history s^t is given by the standard Cobb-Douglas utility function introduced by Heathcote and Perri (2002):

$$U\left[c_i(s^t), 1 - n_i(s^t)\right] = \frac{1}{\gamma}\left[c_i(s^t)^{\mu}(1 - n_i(s^t))^{1-\mu}\right]^{\gamma}. \tag{1.8}$$

The budget constraints of households in country i is denoted in terms of the final good produced in country i, where $i = 1, 2$. For the representative Home household this budget constraint is given by

$$c_1(s^t) + x_1(s^t) + q_1^a(s^t)Q(s^t)B_1(s^t) + \frac{\phi}{2}k_1(s^t)\left[\frac{x_1(s^t)}{k_1(s^t)} - \delta(u_1(s^t))\right]^2$$
$$= q_1^a(s^t)\left(w_1(s^t)n_1(s^t) + u_1(s^t)r_1(s^t)k_1(s^t)\right) + q_1^a(s^t)\left(B_1(s^{t-1}) - \Phi(B_1(s^t))\right). \tag{1.9}$$

For the representative Foreign household it is

$$c_2(s^t) + x_2(s^t) + q_2^a(s^t)Q(s^t)B_2(s^t) + \frac{\phi}{2}k_2(s^t)\left[\frac{x_2(s^t)}{k_2(s^t)} - \delta(u_2(s^t))\right]^2$$
$$= q_2^b(s^t)\left(w_2(s^t)n_2(s^t) + u_2(s^t)r_2(s^t)k_2(s^t)\right) + q_2^a(s^t)\left(B_2(s^{t-1}) - \Phi(B_2(s^t))\right). \tag{1.10}$$

Here, $c_i(s^t)$ denotes consumption and $x_i(s^t)$ is investment in country i. Both are denominated in i's final good. The holdings of the international bond $B_i(s^t)$ are denoted in terms of the Home intermediate good a. The wage rate $w_i(s^t)$ and the rental rate $r_i(s^t)$ are denoted in country i's final good. $n_i(s^t)$ is the amount of labor supplied by the household to intermediate firms and $k_i(s^t)$ is the amount of capital rent out to intermediate firms. $u_i(s^t)$ is the rate of capital utilization. Intermediate firms pay the rental rate for each unit of effective capital $u_i(s^t)k_i(s^t)$ in their use. $\Phi()$ is a small adjustment cost on bond holdings that ensures the determinancy of the international bond positions as for instance in Schmitt-Grohe and Uribe (2003). The functional form of the depreciation function is assumed to be

$$\delta(u_i(s^t)) = \delta u_i(s^t)^{\eta}. \tag{1.11}$$

I assume that there is an upper bound on capital utilization. The upper bound is motivated by fact that machines cannot be used over their capacity of 100%. Therefore it holds that

$$u_i(s^t) \leq 1. \tag{1.12}$$

The maximization problem of the representative country i household is

$$max_{c_i(s^t),n_i(s^t),x_i(s^t),k_i(s^{t+1}),B_i(s^t),u_i(s^t)} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} U\left[c_i(s^t), 1 - n_i(s^t)\right] \tag{1.13}$$

subject to the budget constraint (1.9) or (1.10) for country 1 or 2, the respective law of motion for capital (1.5) as well as the occasionally binding capacity utilization constraint (1.12).

1.4.2 Equilibrium Conditions

The first-order optimality conditions for the households and firms are obtained from the agents' maximization problems outlined above. They are given in the Technical Appendix. Next, I define the stochastic disturbances and the market clearing conditions.

1.4.3 Market Clearing Conditions

The bond market clearing condition states that the international bond is in zero net supply:

$$B_1(s^t) + B_2(s^t) = 0. \tag{1.14}$$

For the intermediate good market the supply has to be equal to demand from Home and Foreign:

$$a_1(s^t) + a_2(s^t) = e^{z_1(s^t)} \left(u_1(s^t)k_1(s^t)\right)^{\theta} n_1(s^t)^{1-\theta} = y_1(s^t) \tag{1.15}$$

$$b_1(s^t) + b_2(s^t) = e^{z_2(s^t)} \left(u_2(s^t)k_2(s^t)\right)^{\theta} n_2(s^t)^{1-\theta} = y_2(s^t). \tag{1.16}$$

For the final good market consumption and investment demand from households has to be equal to the supply of the final good within a given country (as final goods are not internationally traded):

$$c_i(s^t) + x_i(s^t) = G_i(a_i(s^t), b_i(s^t)) \tag{1.17}$$

for $i = 1, 2$.

1.4.4 Exogenous process

The vector of shocks $z(s^t) = [z_1(s^t), z_2(s^t)]$ follows the law of motion

$$z(s^t) = Az(a^{t-1}) + \epsilon(s^t) \tag{1.18}$$

with A being a 2×2-matrix and $\epsilon(s^t)$ being a 2×1-vector of independently distributed random variables with variance-covariance matrix Σ.

1.4.5 Equilibrium

The equilibrium of the model is given by a set of policy functions for the Home household $c_1(s^t), n_1(s^t), x_1(s^t), k_1(s^{t+1}), u_1(s^t), B_1(s^t)$, and the same policy functions for the Foreign household $c_2(s^t), n_2(s^t), x_2(s^t), k_2(s^{t+1}), u_2(s^t), B_2(s^t)$, obtained from the households' first-order conditions, a set of choice functions of the Home and Foreign intermediary and final good firms $a_1(s^t), a_2(s^t), b_1(s^t), b_2(s^t)$, $G_1(s^t), G_2(s^t)$, $y_1(s^t), y_2(s^t)$, and prices $Q(s^t), r_1(s^t), r_2(s^t), w_1(s^t), w_2(s^t), q_1^a(s^t), q_2^a(s^t), q_1^b(s^t), q_2^b(s^t)$, such that, given the realizations of the random disturbances $z_1(s^t), z_2(s^t)$ and the Lagrange multipliers on the occasionally binding constraints $\lambda_1(s^t), \lambda_2(s^t)$,

1. goods markets for intermediary and final goods clear,
2. factor markets for labor and capital clear,
3. the international bond market clears.

1.5 Solution Method

I am using the algorithm developed by Guerrieri and Iacoviello (2015) (the 'OccBin' toolkit) to solve the dynamic stochastic general equilibrium (DSGE) model with occasionally binding constraints. In essence the solution algorithm relies on the fact that a model including an occasionally binding constraint can be represented by a model with different regimes. The model under investigation is log-linearized around the same point of approximation under each of these regimes. The algorithm combines the information about which regime prevails for the model economy in a given state and the dynamics within as well as across these regimes. In this way a model with occasionally binding constraints can be solved and simulated. As Guerrieri and Iacoviello (2015) point out it is important that the algorithm does not only result in the model switching from one linear regime to another. Rather, the anticipation effects of when a certain regime prevails and for how long it is expected to prevail can create high degrees of non-linearity. In the following I briefly describe how the algorithm works. In principle it implements a piecewise-linear approximation to the agents' policy rules. Guerrieri and Iacoviello (2015) describe the algorithm mostly for an example with one occasionally binding constraint. Since in an international model there are two identical countries, the model investigated here has two occasionally binding constraints. A model with two constraints has four regimes, one in which both constraints are slack, two in which one constraint binds and the other one is slack, and one in which both constraints bind. Under each of these regimes the model is log-linearized around its non-stochastic steady state. Following the notation of Guerrieri and Iacoviello (2015), the regime that prevails at the steady state is called 'reference regime' or $(M1)$, the other regimes are called 'alternative regimes' or $(M2), (M3)$ and $(M4)$. Which combination of the constraints bind or are slack at the reference regime does not matter for the algorithm. But two conditions have to be satisfied in order for the algorithm to be applicable:

1. The Blanchard and Kahn (1980) conditions for the existence of a rational expectation solution have to be fulfilled in the reference regime (not necessarily

in the alternative regimes), and

2. the model has to return to the reference regime in a finite number of periods in case a shock moves it to one of the alternative regimes and agents expect no further shocks to occur.

Closely following Guerrieri and Iacoviello (2015), but extending their description to a model with two constraints, I will now define the piecewise-linear solution of such a model. The occasionally binding constraints are denoted $g_1(E_tX_{t+1}, X_t, X_{t-1}) \leq 0$ and $g_2(E_tX_{t+1}, X_t, X_{t-1}) \leq 0$. Assuming that under the reference regime neither of them binds $(M1)$ can be written

$$A_{11}E_tX_{t+1} + B_{11}X_t + C_{11}X_{t-1} + E_{11}\epsilon_t = 0, \tag{1.19}$$

where X is $n \times 1$ vector of all the endogenous variables in the model; E_t is the conditional expectations operator; A_{11}, B_{11}, C_{11} are $n \times n$ matrices of structural parameters for the linearized model equations; ϵ is a size $m \times 1$ vector of zero mean i.i.d. shocks and E_{11} is a $m \times n$ matrix of structural parameters. When g_1 binds and g_2 is slack, we can write $(M2)$ as

$$A_{21}E_tX_{t+1} + B_{21}X_t + C_{21}X_{t-1} + D_{21} + E_{21}\epsilon_t = 0, \tag{1.20}$$

where the notation is analogous to the one in $(M1)$, with the addition that the $n \times 1$ column vector D_{21} of structural parameters enters the system of equations because the linearization is taken around an approximation point in which $(M1)$ applies. Similarly, regimes $(M3)$ and $(M4)$ are defined as

$$A_{12}E_tX_{t+1} + B_{12}X_t + C_{12}X_{t-1} + D_{12} + E_{12}\epsilon_t = 0 \tag{1.21}$$

and

$$A_{22}E_tX_{t+1} + B_{22}X_t + C_{22}X_{t-1} + D_{22} + E_{22}\epsilon_t = 0, \tag{1.22}$$

where the notation is again analogous to the regimes above. **Definition 1 (Guerrieri and Iacoviello (2015)).** A solution of a model with two occasionally binding constraints is a function $f : X_{t-1} \times \epsilon_t \to X_t$ such that the conditions under system $(M1)$,$(M2)$,$(M3)$ or $(M4)$ apply, depending on whether the occasionally binding constraints $g_1(E_tX_{t+1}, X_t, X_{t-1}) \leq 0$ and/or $g_2(E_tX_{t+1}, X_t, X_{t-1}) \leq 0$ bind or are slack. I refer to the paper by Guerrieri and Iacoviello (2015) for a detailed description of the algorithm and to the OccBin toolkit for the codes to implement the solution procedure.

1.6 Results

1.6.1 Calibration

I am calibrating the model to produce quarterly simulated data. Therefore, I choose a discount factor of $\beta = 0.99$. Relative risk aversion is set to the standard value $1 - \gamma = 2$. The capital share in intermediate good production is set to $\theta = 0.36$ and the consumption share in utility is $\mu = 0.34$, which are also standard values in the

Table 1.5 – Calibration

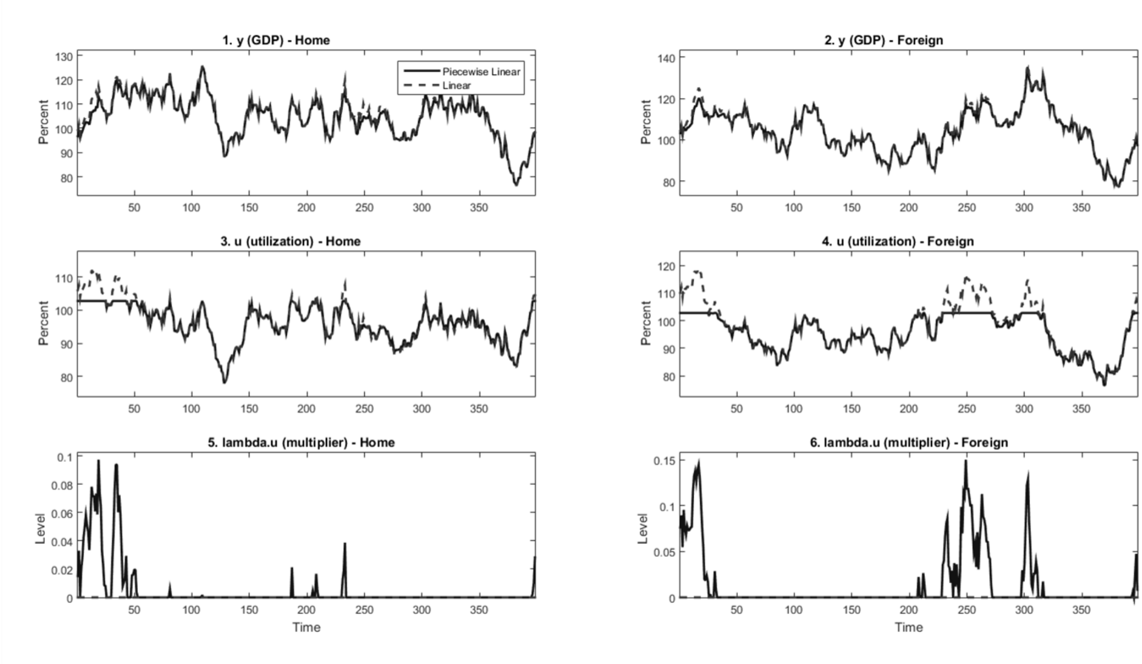
Parameter	Description	Value
β	discount rate	0.99
Φ	maximum utilization rate	1
θ	capital share	0.36
ω	home bias in production of final goods	0.7
σ	substitution elasticity in final good production	0.5
$1-\gamma$	relative risk aversion	2
μ	consumption share in utility	0.34
δ	depreciation parameter	0.025
η	depreciation parameter	1.42
ϕ	bond adjustment costs	0.0005
ϕ_k	investment adjustment costs	0.5

literature. The depreciation rate is governed by two parameter, a standard linear component which I set to $\delta = 0.025$, as well as the component varying with capacity utilization which I set to $\eta = 1.42$. The maximum capacity utilization is given by 100% which is indicated by $\Phi = 1$. The parameter for the small bond adjustment costs and capital adjustment costs are set to standard values. This calibration largely follows Heathcote and Perri (2002). Other than Heathcote and Perri (2002), I choose shock processes that are not correlated in order to clearly see the spillover effects of uncorrelated shocks in the model. Furthermore, I choose a home bias in consumption parameter of $\omega = 0.7$ in order to show the functioning of the model more clearly, as this is an important channel of international correlations in the model. For, the elasticity of substitution between home and foreign intermediate goods in the production of final good, I choose a value of $\sigma = 0.5$ which I take from Corsetti et al. (2005, 2008). I also perform a robustness check with the value chosen by Heathcote and Perri (2002) of $\sigma = 0.9$. Estimated values for σ vary quite a bit in the literature. While Taylor (1993) estimates a value of $\sigma = 0.39$ for the US, Whalley (1984) estimates a value of $\sigma = 1.5$. Thus, the chosen value are well in range with the data. The benchmark parameter values are summarized in Table 1.5. With these parameters, the steady state utilization of capital is 97.3%. Therefore, I assume that producers in the absence of shocks, use their machines to less than their maximum capacity of 100%. The occasionally binding constraint on capital utilization invokes that there is a physical limit such that machines cannot be utilized more than their full capacity. In my model, as a result of shocks the economy can be driven into situations in which the constraint binds and producers cannot increase utilization to a level that would be optimal in the absence of the constraint. In the next sections, I investigate the consequences of this physical bound on the symmetry of business cycles and the cross-correlations of international business cycles created by the model.

1.6.2 Disentangling expansions and recessions

Since I am foremost interested in whether my model can replicate the asymmetry of business cycle correlations between expansions and recessions, I need to find a

Figure 1.2 – Simulations



reasonable way of disentangling business cycle phases from the simulated model data. For comparability I use the exact same approach on the simulated data as I applied to the empirical data in section 1.2.

1.6.3 Simulations

To investigate the cross-country correlations of GDP and other macroeconomic variables I run 1000 simulations of 1400 periods each. I am dropping the first 1000 periods of each simulation, such that the simulation results are not influenced by the initial conditions. Therefore, in essence the simulation simulates 1000 world economies consisting of two countries for 100 years. Both countries' random disturbances are assumed to have a persistence parameter of $\rho = 0.95$ and a standard deviation of $\sigma_e = 0.02$ For illustration purposes I plot series of the 400 valid periods out of the last simulation for the calibration using $\sigma = 0.5$.

Figure 1.2 shows the simulations of GDP, capital utilization and the Lagrange parameter on the occasionally binding constraint for Home on the left-hand and Foreign on the right-hand side. The dashed line gives the simulations of the unconstrained model and the solid line shows the simulations with the constraint imposed. Notice that GDP in booms is decreased in comparison to the unrestricted model due to the binding capacity constraints. This creates a negative skewness of GDP of the two individual countries. Another important point is that the constraints on capacity utilization do not bind necessarily at the same time. For this example simulation the constraint for both countries bind most of the time for the first 50 periods, but after that it is only Foreign for which the constraint binds a substantial amount of time between periods 200 and 300. Moreover, one can see that the correlation of

GDP is high for both countries, but the effect of the constraint on cross-country correlations is hard to interpret in this figure.

1.6.4 International Correlations

Now, I first turn to the simulation moments in recessions and expansions overall in the unconstrained model, as well as in a model in which the constraint is occasionally binding. In table 1.6 I summarize the correlations calculated from the simulated data using the benchmark calibration in recessions and expansions. Recession and expansion episodes are determined using the turning point algorithm by Harding and Pagan (2002) and the cyclical component of the HP filter GDP data being positive or negative. The first two rows show the correlations between the economies in the unconstrained symmetric model. The first row shows the results using the turning point (TP) algorithm. The difference if Home is in recession is positive and significant at the 5% level, but very small. If Foreign is in a recession the correlation is small and not significantly different from zero. The second row shows the same numbers obtained by applying the HP filter. The correlations if Home or Foreign are in recession are small and not significantly different from zero at the 10% level. The third and fourth rows show the results in the constrained model. The third row shows the correlations obtained using the TP algorithm. The difference between correlations in Home contractions (0.7780) and Home expansions (0.7400) amounts to 3.8 percentage points and is statistically significantly different from zero at the 1% confidence level. A similar difference holds for Foreign expansions and recessions. In the fourth columns, the same numbers are calculated for the HP cyclical component. Here the difference between correlations during Home contractions (0.8333) and Home expansions (0.7419) amounts to 9.14 percentage points and is significant at the 1% confidence level. Again a similar difference holds for Foreign contractions and Foreign expansions. Now, I turn to the correlation results for calibrations with an elasticity of substitution bewteen intermediary goods of $\sigma = 0.9$. They are given in table 1.7. The differences decrease when the elasticity of substitution is increased. The numbers for the unconstrained model are not significantly different from zero. The difference between correlations in Home contractions (0.6346) and Home expansions (0.6267) calculated with the TP algorithm amounts to 0.8 percentage points and is statistically significantly different from zero at the 1% confidence level. A similar difference holds for Foreign expansions and recessions. Also the difference calculated with the HP filter gives a similar difference and levels of correlations. With in-between values of σ the obtained differences are between the two calibrations above. The model is able to replicate a difference in cross-country correlations of around 4 percentage points for the benchmark calibration of $\sigma = 0.5$. This is at the lower end of the calibration range and means that international intermediary goods have to have a certain degree of complementarity in order to make international correlations higher in recessions than in expansions. For a higher degree of substitutability producers are more flexible in their choice of inputs, output is not depressed as much and spillovers are more symmetric.

Table 1.6 – Conditional Correlations from the Model - GDP

	Home			Foreign		
	Contr		**Exp**	**Contr**		**Exp**
Unconstrained TP	0.8784		0.8743	0.8779		0.8757
Difference		0.0040**			0.0022	
P-Value		0.0138			0.1884	
Unconstrained HP	0.8988		0.8984	0.8984		0.8981
Difference		0.0005			0.0004	
P-Value		0.7193			0.7971	
Constrained TP	0.7780		0.7400	0.7758		0.7414
Difference		**0.0380*** **			**0.0344*** **	
P-Value		0.0000			0.0000	
Constrained HP	0.8333		0.7419	0.8338		0.7410
Difference		**0.0914*** **			**0.0928*** **	
P-Value		0.0000			0.0000	

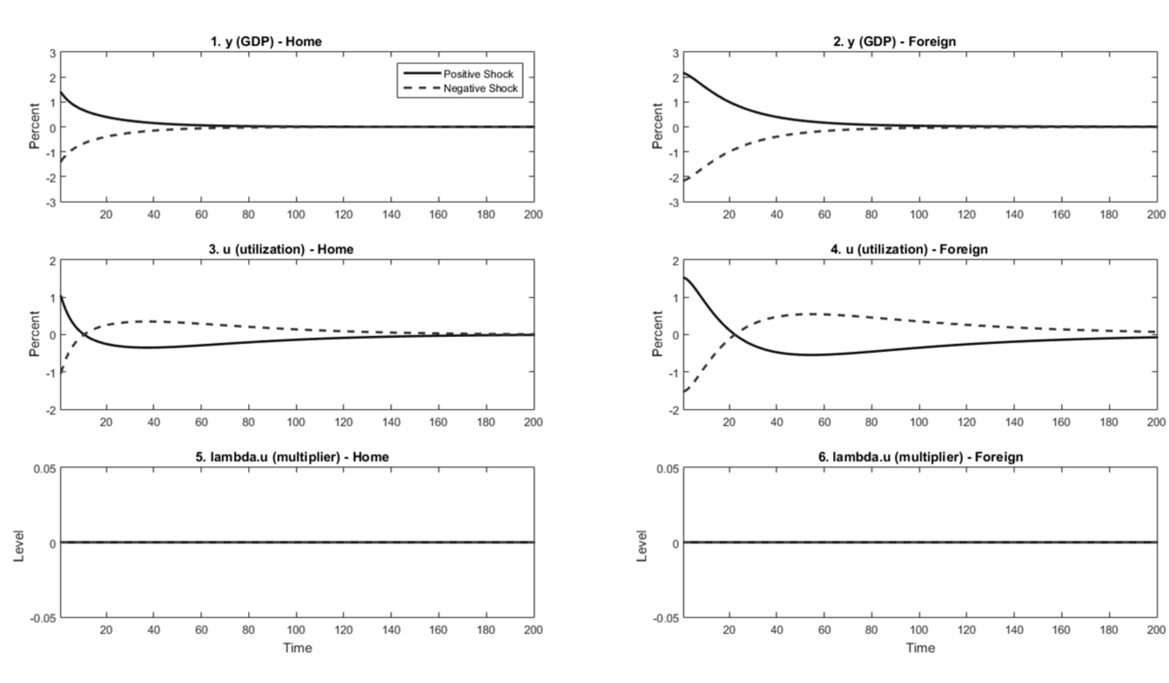
$***$,$**$ and $*$, indicate $p < 0.01$, $p < 0.05$, $p < 0.1$, respectively. The table shows cross-country correlations of GDP growth rates between Home and Foreign produced by simulations of the model. The 'Home' column shows the results for contractions (Contr) and expansions (Exp) determined using the Home GDP series. The 'Foreign' column shows the same results using the Foreign GDP series. Expansions and contractions are found using the Harding and Pagan (2002) turning point algorithm in the rows indicating 'TP' and using the HP-filtered GDP data being below or above trend in the rows indicating 'HP'. The first two rows give the results for the unconstrained symmetric model, while the last two rows show the results for the model in which the constraint is invoked.

Table 1.7 – Conditional Correlations from the Model - GDP

	Home			Foreign		
	Contr		**Exp**	**Contr**		**Exp**
Unconstrained TP	0.6413		0.6391	0.6407		0.6394
Difference		0.0023			0.0014	
P-Value		0.2932			0.5253	
Unconstrained HP	0.6857		0.6847	0.6853		0.6849
Difference		0.0010			0.0004	
P-Value		0.6054			0.8279	
Constrained TP	0.6346		0.6267	0.6349		0.6266
Difference		**0.0079*** **			**0.0083*** **	
P-Value		0.0003			0.0001	
Constrained HP	0.6801		0.6689	0.6795		0.6695
Difference		**0.0112*** **			**0.0100*** **	
P-Value		0.0000			0.0000	

$***$,$**$ and $*$, indicate $p < 0.01$, $p < 0.05$, $p < 0.1$, respectively. The table shows cross-country correlations of GDP growth rates between Home and Foreign produced by simulations of the model. The 'Home' column shows the results for contractions (Contr) and expansions (Exp) determined using the Home GDP series. The 'Foreign' column shows the same results using the Foreign GDP series. Expansions and contractions are found using the Harding and Pagan (2002) turning point algorithm in the rows indicating 'TP' and using the HP-filtered GDP data being below or above trend in the rows indicating 'HP'. The first two rows give the results for the unconstrained symmetric model, while the last two rows show the results for the model in which the constraint is invoked.

Figure 1.3 – Impulse responses - 1 std benchmark

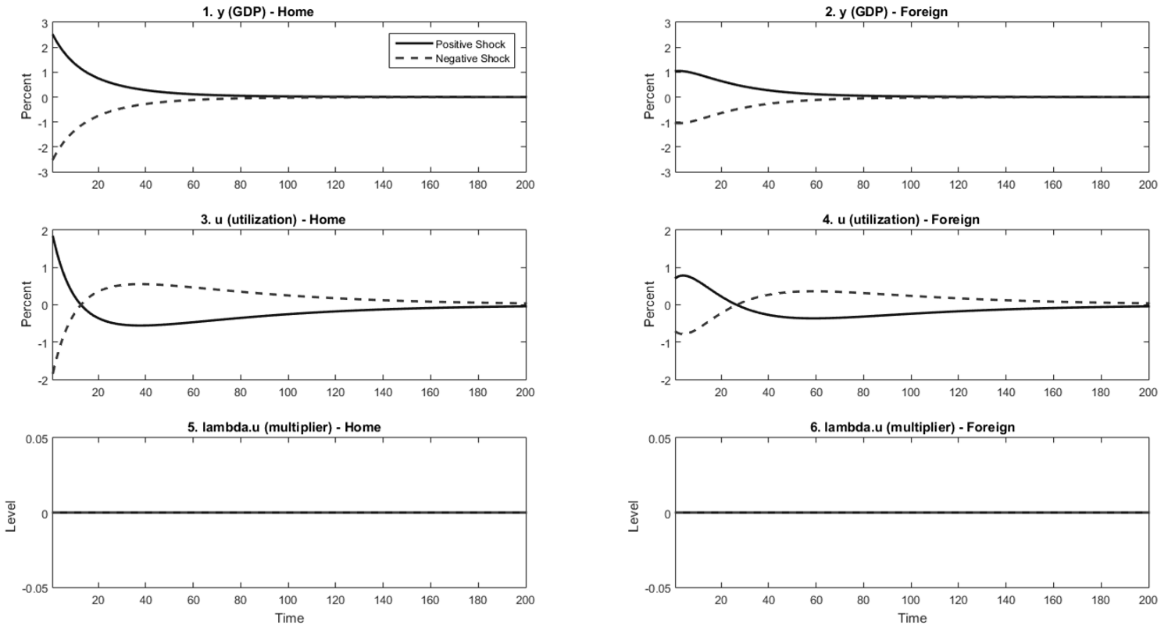


1.6.5 Impulse responses

To understand how the investigated mechanism works to create asymmetry, I compare the impulse response functions to shocks of different magnitudes. The shocks are chosen to have a standard deviation of $\sigma_e = 0.02$. In this section, I compare impulse response functions to positive and negative shocks to Home TFP, while holding Foreign TFP constant. Figure 1.3 shows the responses of GDP, capital utilization and the Lagrange multiplier on the occasionally binding constraint for a 1 standard deviation shock to Home TFP for Home variables on the left-hand side, as well as Foreign variables on the right-hand side. This is for the benchmark calibration.

For a one standard deviation shock, the occasionally binding constraints are not violated, thus the IRFs for positive and negative shocks are perfectly symmetric and the Lagrange multipliers remain at zero. Furthermore, note that there are spillover effects of a shock to the Home country to the Foreign country due to trade linkages. The assumed value for the substitution elasticity imply that the one standard deviation shock to Home leads to an increase in Home GDP by nearly 1.5%, while Foreign GDP increases by 2%. Therefore, the low elasticity of substitution implies that the Home shock has a larger effect on the Foreign country's GDP than on Home GDP. For the robustness check of $\sigma = 0.9$ this result is reversed. As can be seen in figure 1.4, the Home TFP shock has a larger effect in Home than in Foreign. Still, as the constraints are not violated, the responses are completely symmetric for a positive and a negative shock to Home TFP.

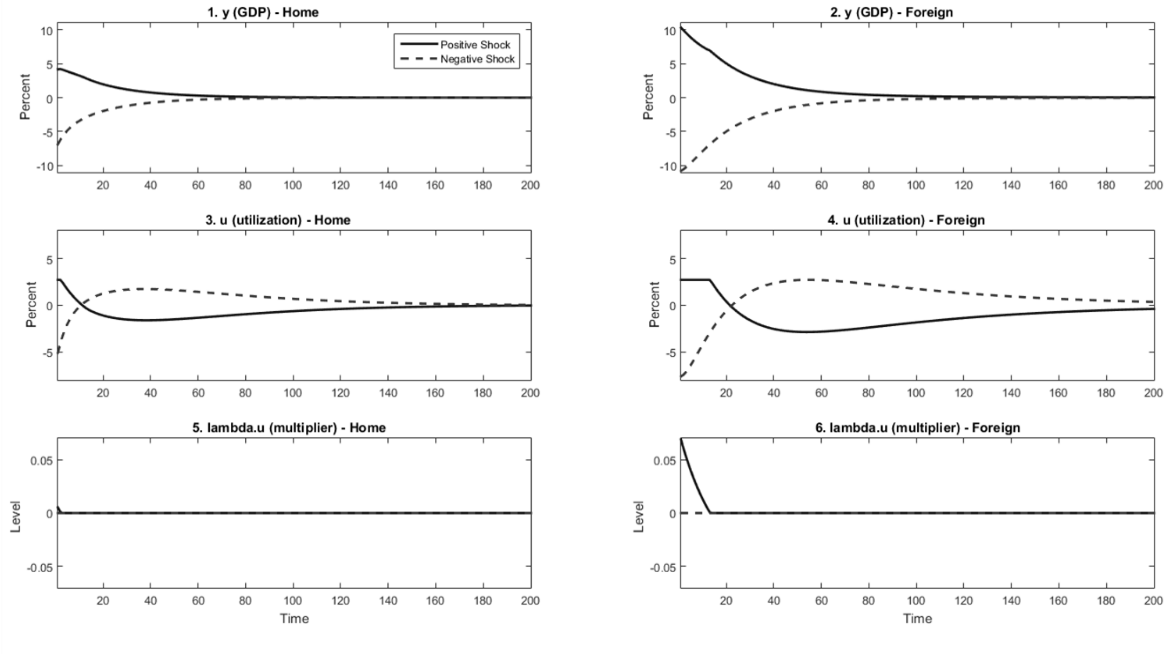
Figure 1.4 – Impulse responses - 1 std $\sigma = 0.9$



Next, I want to show a case that illustrates the workings of the model when the constraint is violated. Therefore, in figure 1.5 I show the same response picture for a five standard deviations shock to Home GDP. This is a very large shock that in practice will happen very rarely. Keep in mind that during the model simulations both countries are hit by a variety of shocks and that the constraint might become binding after several small shocks hit the economy. From the simulations we saw that the constraint binds a considerable number of periods for Home as well as Foreign. To concentrate on the effects of a single shock to the Home economy, I assume a large shock to make the workings of the constraint obvious. Technically the constraint starts to bind for the Foreign economy for a two standard deviation shock, but the effects only become visible for a larger shock as the one shown.

In figure 1.5, as the Lagrange multipliers are different from zero, the constraints bind for both countries when the shock hits, but only for the Foreign country the constraint binds longer than one period. The most important point of having this large shock is that the asymmetry in spillovers between positive and negative shocks becomes visible. In the response plots for GDP in Home and Foreign one can see that the drop in GDP is larger for a negative shock than the increase in GDP for a positive shock of the same size. This is true for the response of Home GDP to the Home TFP shock, as well as for the spillover of this shock to Foreign GDP. Therefore, the model can indeed create asymmetric international spillovers, i.e. Home recessions have larger effects on Foreign GDP than Home expansions. Note that the response of Foreign GDP to a Home TFP shock is larger than on Home GDP. As I will show with empirical impulse responses from the VAR evidence in the next section, for a US TFP shock, it is not uncommon that the responses of smaller other countries to a US TFP shock are relatively larger than on US variables themselves. For robustness,

Figure 1.5 – Impulse responses - 5 std benchmark



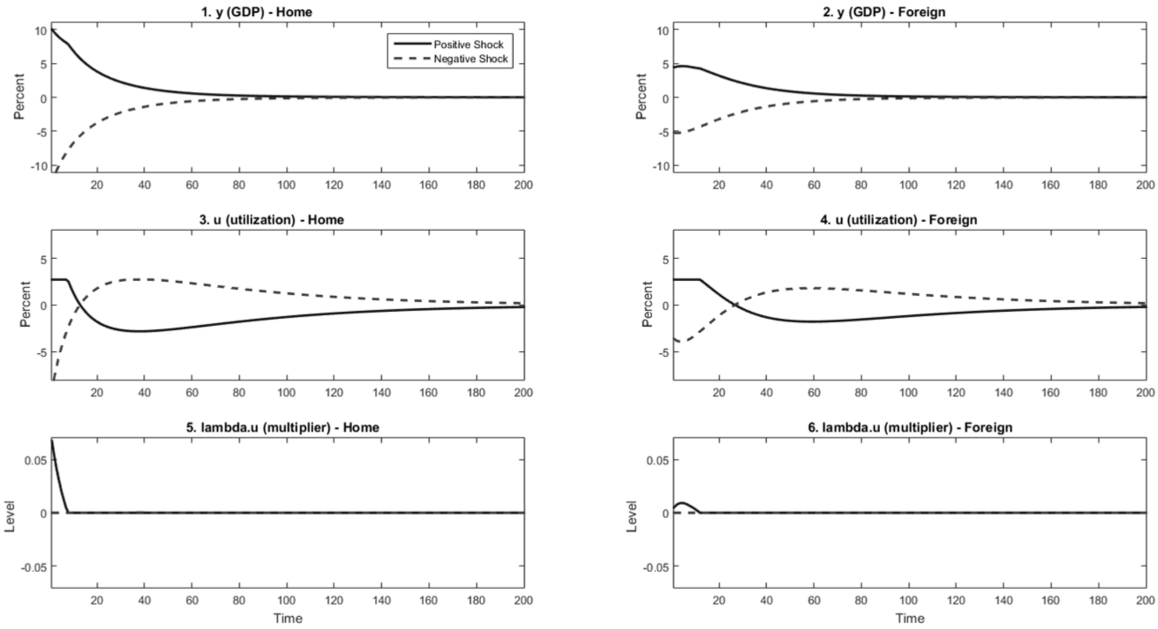
in figure 1.6 I show the same plot for the $\sigma = 0.9$ calibration.

Here we can see that the shocks to Home TFP spills over less than in the $\sigma = 0.5$ benchmark case. Furthermore, the difference in the responses of GDP to a positive and a negative shock are not as high as in the benchmark calibration, which contributes to the finding that the differences in correlations are not as high as in the benchmark calibration. Overall, the impulse response analysis shows that the model can create asymmetric spillovers and as I showed with the simulations, these asymmetric spillovers result in state-dependent cross-country correlations that can match the observed differences in the data if I assume a sufficient amount of complementarity between internationally tradable intermediary goods. Therefore, my empirical as well as theoretical findings show that we might miss a lot of interesting features of the world, if we do not take into account asymmetries in international RBC models.

1.7 VAR Evidence

After having shown that occasionally binding capacity constraints are capable of producing business cycle asymmetry and asymmetric international spillovers for sufficient complementarity between intermediary goods, in this section I motivate this channel empirically. The empirical equivalent of a regime switch due to occasionally binding constraints are threshold effects in a given time series. Therefore, the assumption of occasionally binding constraints on capacity utilization at the aggregate production level to explain state-dependent cross country GDP correlations can be justified by the existence of threshold effects in capacity utilization giving rise to

Figure 1.6 – Impulse responses - 5 std $\sigma = 0.9$



business cycle asymmetries. In this section, I investigate the empirical evidence on the presence of threshold effects in the capacity utilization rate of the US economy. Moreover, the consequences threshold effects within the US economy have on each of the remaining six G7 countries (Canada, France, Germany, Italy, Japan and UK) are explored. A complication is that the utilization series for Japan is indexed and set to 100 in Q1:2010, therefore it is not directly comparable with the other series which are given as percentage of total production capacity. The analysis proceeds as follows. In a first step, I test for threshold effects in US capacity utilization using Hansen (2000)'s threshold test. In a second step, I use the threshold estimates from Hansen's test as informative priors for the estimation of a reduced-form threshold vector autoregressive (TVAR) model. Finally, using short-run zero restrictions, I identify a US TFP shock and track the impact it has on US capacity utilization, US GDP, as well as Foreign[9] capacity utilization and Foreign GDP.

1.7.1 Data

The GDP series and capacity utilization series are gathered using Thomson Reuters Datastream from various national and international organizations (see table A.2 in appendix). All series are transformed to be seasonally adjusted using the X13 Census filter in EVIEWS and the GDP series are at constant 2010 prices and per capita, if not provided in this way. The US TFP series is from of Fernald (2012). The capacity utilization series for all investigated countries despite Japan represents the percentage utilization of all available production capacities across all industries. The data availability for each country is limited by the introduction data of a capacity utilization variable in both the respective country and the US. For the US a quarterly

[9]Foreign refers to each of the remaining G7 countries on a rotating basis.

capacity utilization measure is available from Q1:1967, so this is the earliest possible data point for the international VAR estimation.[10] For the UK a capacity utilization measure is only available from Q1:1985 onward and thus US-UK country pair has the least observations available for the estimation and testing procedures. The latest data point available for France, Germany and Italy is Q4:2015, while for Canada it is Q3:2015 and for the UK it is Q2:2015. For the VAR estimation I work with mean adjusted utilization levels to make the series fluctuate around zero. Moreover, I work with year-over-year GDP growth rates and also the TFP data of Fernald (2012) is provided in year-over-year growth rates. The hypothesis of stationarity in a standard VAR the same specifications as used in the threshold VAR later cannot be rejected for any country pair.[11] Therefore, no further adjustments to transform the variables to stationarity are necessary.

1.7.2 Hansen Threshold test

The threshold test is based on a regression of lagged dependent variables X_{t-d} on the independent variable Y_t. The independent variable in the regressions considered here is the US GDP growth rate, while the X's include 6 lags of first differences in US GDP and Foreign GDP, as well as the level of US capacity utilization and Foreign capacity utilization. The test allows for heteroscedastic residuals and is based on 1000 draws with a trimming value of 0.2 for all countries. I test for threshold effects in each of the included lags of US capacity utilization. The results are shown in table 1.8.

A first observation from the table is that with the varying availability of the capacity utilization series, even for the US the mean utilization rate varies for each country pair, though without large deviations from 80%. Moreover, the mean utilization rates vary across countries in a range of 75% to 84%. The results of the Hansen threshold test show that there are significant threshold effects in US capacity utilization that determine the behavior of the US GDP series in all specifications. For Canada there are significant differences in the effects of US capacity utilization on US GDP depending on whether US capacity utilization was more than 1.449 percentage points above its mean four quarters ago or 2.386 percentage points above its mean 6 quarters earlier. The specifications for France, Germany and Italy all show significant threshold effects in US capacity utilization with shorter delays. Germany and Italy have significant effects at delays of one, two and three quarters. For Germany they occur if the US utilization rate was 0.092 percentage points below its mean one quarter earlier, 1.308 percentage points above its mean two quarters ago or 0.808 percentage points above mean three quarters before. For the specification using Italian time series the US utilization rate had to be 0.886 percentage points above its mean one quarter earlier, 2.394 percentage points above its mean two quarters before or 0.290 percentage points below its mean three quarters ago for significant threshold effects to show. For Japan and the UK, significantly different effects occur at shorter lags (1 or 2), as well as longer lags (5 or 6). For

[10] Although for instance for Germany a measure is available from Q1:1960 onward.

[11] These test have been conducted using the standard unit root tests in EVIEWS.

Table 1.8 – Hansen (2000)'s threshold test - US utilization, mean adjusted - 6 Lags, 1000 draws

	Dep.Var:	**US GDP**				
Threshold Variable	US-CAN (adj.lvl)	US-FRA (adj.lvl)	US-GER (adj.lvl)	US-ITA (adj.lvl)	US-JPY (adj.index)	US-UK (adj.lvl)
u^{US}_{t-1}	0.886	**-2.484****	**-0.092****	**0.886****	0.074	**1.683*****
p-Value	(0.327)	(0.028)	(0.023)	(0.028)	(0.223)	(0.008)
u^{US}_{t-2}	2.067	**-1.555****	**1.308***	**2.394***	**0.235***	**1.746*****
p-Value	(0.237)	(0.046)	(0.096)	(0.085)	(0.059)	(0.005)
u^{US}_{t-3}	1.648	**-0.326****	**0.808****	**-0.290****	0.137	-1.180
p-Value	(0.287)	(0.043)	(0.027)	(0.020)	(0.105)	(0.298)
u^{US}_{t-4}	**1.449****	**-0.222***	1.308	2.422	**2.640***	-1.028
p-Value	(0.029)	(0.052)	(0.221)	(0.136)	(0.078)	(0.170)
u^{US}_{t-5}	1.058	-2.636	0.808	2.649	-0.617	**0.840***
p-Value	(0.417)	(0.397)	(0.149)	(0.211)	(0.224)	(0.081)
u^{US}_{t-6}	**2.386***	-3.363	1.208	-1.025	**0.513*****	**1.748****
p-Value	(0.080)	(0.448)	(0.447)	(0.282)	(0.011)	(0.025)
Avg u US	80.34	79.65	80.46	80.21	80.24	79.51
Avg u FOR	81.17	84.33	83.64	75.05	112.08	80.80
Observations	189	156	190	184	187	122

$***$,$**$ and $*$, indicate $p < 0.01$, $p < 0.05$, $p < 0.1$, respectively. The specification always include six lags of US TFP, US GDP, US utilization, Foreign utilization and Foreign GDP.

the specification using UK data, the US capacity utilization rate needed to be at least 1.683 percentage points above its mean one quarter earlier, 1.746 percentage points above its mean two quarters earlier, 0.84 percentage points above its mean 5 quarters ago or 1.748 percentage points above its mean 6 quarters ago in order to reveal significantly different effects in the explanation of US GDP across the two identified regimes. For the specification including Japanese data the interpretation of Japanese variables is different as the utilization series for Japan is given as an index set to 100 in Q1:2010. Because in this section, I estimate threshold effects in US capacity utilization, the interpretation of the above results is the same as for the other country pairs. For the US-Japan country pair there are significant threshold effects if US capacity utilization was at least 0.235 percentage points above its mean two quarters before, 2.64 percentage points above its mean 4 quarters ago or 0.513 percentage points above its mean 6 quarters earlier. While for Canada, Germany, Italy, Japan and the UK most threshold effects occur if the US utilization rate was above mean in the past, for France all significant threshold effects occur if the US utilization rate was below mean. There are significant threshold effects for the specification using French time series data, when US capacity utilization was between 2.484 percentage points below its mean one quarter earlier and 0.222 percentage points below its mean 4 quarters before. This indicates that for France threshold effects rather show when US utilization was low in the past, compared to the other countries examined for which they show when US capacity utilization was high in the recent past. If I find that in the high utilization regime GDP responses of the US and the rest of the G7 countries to a positive US TFP shock are dampened in comparison to US and Foreign GDP responses to a negative US TFP shock in the low utilization regime, I find that for five out of six country pairs there is statistical evidence of capacity utilization constraints being present when production capacities are already used to a high degree.[12] This is exactly what I investigate in the next section using the estimated thresholds from Hansen's test as informative priors in the estimation of a Bayesian T-VAR.

1.7.3 Bayesian Threshold VAR

In this section, I investigate the effects of the presence of the above threshold effect on the transmission of a US TFP shock through the US economy and to the other four countries investigated here, with a special interest in the asymmetries that can arise within the US economy and how they spill over internationally. For this purpose, I estimate a Bayesian vector autoregression (VAR) that can account for the presence of different regime depending on if a certain threshold variable is below or above an estimated threshold value. Therefore, this type of model is called a (Bayesian) threshold vector autoregression model (T-VAR model).

Theory

With at least 2 regimes and at least 2 lags the threshold VAR models estimated in this paper are highly parameterized. To improve inference using these models combined with the limited data availability of macroeconomic time series, it is common in the literature to estimate unrestricted VAR models by Bayesian methods. To obtain impulse response functions, short run zero-restrictions are imposed on the responses of the variables included in the VAR. The usage of Bayesian techniques in a setting like this allows the inclusion of prior information on the parameters to be estimated in a natural way and therefore improves the inference in these models. For estimation of this type of model prior distributions for all parts of the econometric model that are treated as exogenous have to be assumed. A systematic way to do this for an underlying structural VAR model is to use natural conjugate priors for which the prior, the likelihood and the posterior have the same distributional form. This assumption allows the implementation of the dummy variable approach to elicit priors for structural VARs following Sims and Zha (1998) and Banbura et al. (2010). The posterior distributions of these exogenous model parameters are derived from the prior distributions and the model's Bayes factor. To be able to obtain the posterior distributions and do inference on the model's parameters some prior specifications require the use of simulation based techniques, such as Markov chain Monte Carlo (MCMC) procedures. These MCMC procedures create a Markov chain converging to a chain of drawings from the posterior distribution and thus allow the researcher to draw inference from posteriors that are analytically intractable. Frequently used procedures to create such converging Markov chains are the Metropolis-Hastings algorithm or the Gibbs sampler. The procedures I use to estimate the threshold VAR follow the algorithm of Chen and Lee (1995), as used by Alessandri and Mumtaz (2013).[13] This procedure uses the Gibbs sampler

[12] As the analysis uses aggregate data, the economies operate well below full utilization even in the strongest boom periods they barely exceed 90%.

[13] Their codes are available under https://sites.google.com/site/hmumtaz77/code

to create the Markov chain from which drawings from the approximate posterior distributions are obtained. The threshold VAR is given by:

$$Y_t = \left[c_1 + \sum_{j=1}^{p} A_{1,j} Y_{t-j} + \epsilon_{1,t}\right] S_t + \left[c_2 + \sum_{j=1}^{p} A_{2,j} Y_{t-j} + \epsilon_{2,t}\right] (1 - S_t)\,, \qquad (1.23)$$

with $\epsilon_{i,t}$ $N(0, \Omega_i)$ and $S_t = 1 \iff Y_{1,t-d} \geq Z$ for $t = 1, ..., T$. Y_t is the $N \times 1$ vector of endogenous variables, which includes the first differences of US total factor productivity (TFP), the level of US capacity utilization and US GDP growth rates for all presented specifications and on a rotating basis the level of Foreign capacity utilization and the first difference of Foreign GDP. Here, Foreign represents each of the G7 countries excluding the US in turn. c_i, A_i and Ω_i are a constant term, the VAR-coefficient matrix and the variance-covariance matrix of the **iid** disturbance term ϵ_t for the two regimes $i = 1, 2$, respectively. These three matrices for the two VAR regimes contain all but two of the parameters of the TVAR. The other two are the threshold Z and the threshold delay d. The two latter parameters are unobserved and thus have to be estimated. For each period, the threshold variable Z determines the prevalent regime. Because I obtained estimates of the threshold value of US capacity utilization for each country pair, I use this estimated value as initial value as priors for Z.

Parameters of Bayesian Estimation

To implement the Bayesian Threshold VAR estimation parameters regarding the prior distributions on the model parameters and the number of draws, as well as the simulation horizon for the impulse response functions have to be set. I am using the mean of the significant threshold estimates of Hansen (2000)'s test for threshold effects as mean of the conjugate prior distribution on the threshold in US capacity utilization Z for each country pair. The variance of the prior distribution of the threshold parameter is set at $\sigma^2 = 0.5$ for all country pairs. This assumption and the assumptions on other estimation parameters are summarized in table 1.9. This parameterization is in line with standard values chosen in the literature. In this calibration I set the prior tightness parameter $\lambda_P = 0.5$, which is slightly looser than for instance the parameterization of Alessandri and Mumtaz (2013), while the values of the prior tightness on the sum of coefficients and the constant term are set according to those in Alessandri and Mumtaz (2013). As the threshold estimates from the Hansen test conducted above give a statistical indication of the threshold value in the data, I choose the variance on the threshold value to be 0.5.

1.7.4 Impulse Responses

I am now using the threshold estimates of Hansen (2000)'s test for threshold effects as priors and a Bayesian vector autoregression allowing for threshold effect (TVAR) for the US and the rest of the G7 country pairs. The following figures show the impulse responses for a positive standardized US total factor productivity shock initialized at the mean levels of each variable. The standard deviation of the shock is set to 0.1 and the investigated shock are a positive two standard deviation shock

Table 1.9 – Bayesian TVAR - Parameters

Parameter	Description	Value	Posterior Value
Prior Threshold Z	Canada	1.9174	3.5772
	France	-1.1467	-2.3768
	Germany	0.6747	3.9950
	Italy	0.9967	4.3436
	Japan	1.1293	-0.4056
	UK	1.2552	3.9329
σ^2	Variance on threshold	0.5	
Reps	Simulation replications	20500	
Drop	Disregarded initial draws	20000	
λ_P	Prior tightness	0.5	
τ_P	Prior tightness on sum of coefficients	$10 * \lambda_P$	
ϵ_P	prior tightness on constant	1/10000	
d	Maximum lag in threshold	2	
scalein	Prior random walk variance	0.01	
scaleex	Standard deviation of simulated shock	$[-0.1; 0.1]$	
horzir	Horizon IRF	40	

The utilization series for Japan is indexed and set to 100 in Q1:2010, therefore it is not directly comparable with the other series given in percentage of total capacity.

triggering the high utilization regime and a negative 2 standard deviation shock triggering the low utilization regime. The identification procedure is a standard Cholesky zero restriction identification scheme building on the assumption that a TFP shock contemporaneously affects all other variables in the VAR (they are first in the variable ordering), while shocks to the other variables in the VAR are not affecting US TFP contemporaneously.

US responses to a US TFP shock

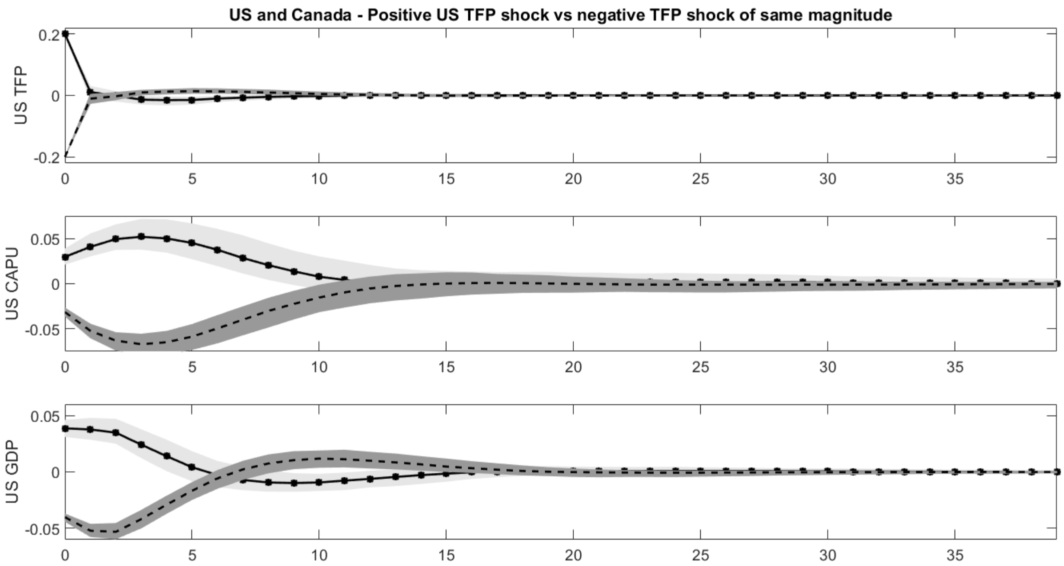
The US variables are included in the specification for each country pair. For brevity, I concentrate on the responses of these variables obtained from the US-Canada specification which are shown in figure 1.7.[14] Despite for the case of France, the responses are similar and the following interpretation is valid. [15]

The figure shows that in responses to a positive US TFP shock (dotted lines with lighter confidence bands), US capacity utilization and US GDP growth increase, while they decrease in response to a negative TFP shock (dashed lines with darker confidence bands). Furthermore, in comparison with a negative TFP shock hitting the economy at the mean of all the specified variables, i.e. a shock that does not trigger a regime switch, the responses for the positive shock are lower. Thus, I find evidence that positive TFP shocks have lower effects on US capacity utilization and

[14] The responses obtained from the other country pair specifications can be found in the appendix.

[15] For France the results indicate that a regime switch is triggered when a negative shock hits the economy, as the impulse responses are initialized at the mean levels of each variable and the threshold estimate is lower than the mean for France.

Figure 1.7 – US - Canada impulse response functions - US variables



US GDP growth than negative TFP shocks of the same magnitude, if one accounts for the estimated threshold effects in US capacity utilization. I interpret this as an occasional inability to adjust capacity utilization beyond a certain degree within a given period. This is the motivation to investigate the international transmission of TFP shocks in a model with occasionally binding constraints on capacity utilization in the theoretical section of this paper.

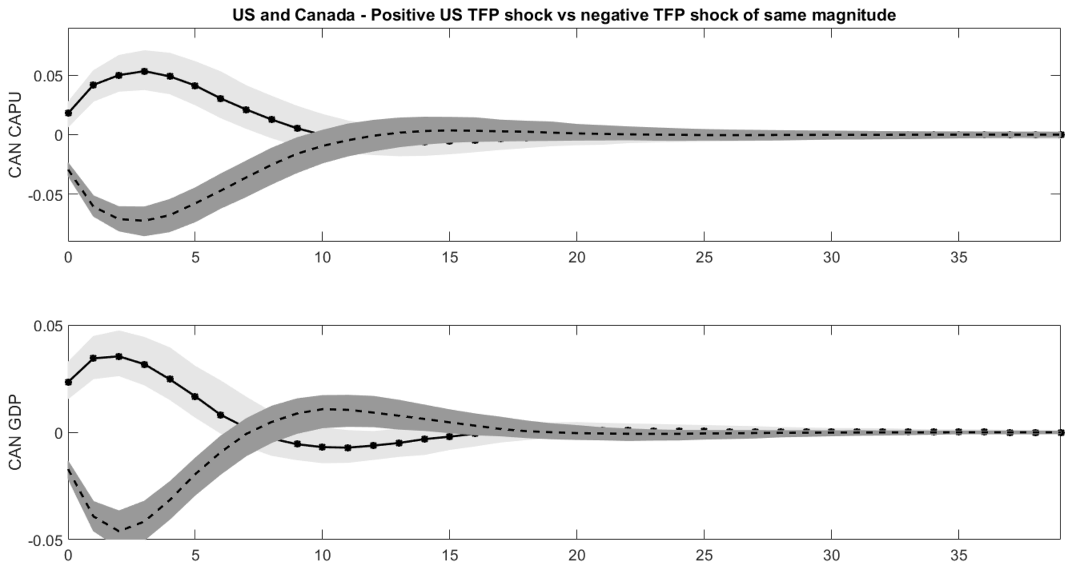
International responses to a US TFP shock

US - Canada

For the case of the US-Canada example the threshold estimation within the Bayesian VAR procedure suggests a mean threshold on US capacity utilization of 83.92 percent instead of 82.26 percent, as estimated by the threshold test above. Figure 1.8 shows the responses of Canadian capacity utilization and Canadian GDP growth to a US TFP shock.

Similarly to the domestic US responses, the Canadian responses show that there are positive international spillover effects to a positive US TFP shock on Canadian capacity utilization and Canadian GDP growth (dotted lines), while there are negative international spillover effects on the same variables in response to a negative TFP shock (dashed lines). The comparison between the responses to positive and negative US TFP shocks provide evidence that negative US TFP shocks have higher international spillover effects to the Canadian economy compared to a positive US TFP shock. Therefore, spillovers show the same asymmetry that is present in the responses to US TFP shock in the US economy. Moreover, the responses to Canadian GDP and capacity utilization are only slightly smaller in magnitude than the

Figure 1.8 – US - Canada impulse response functions - Canada variables



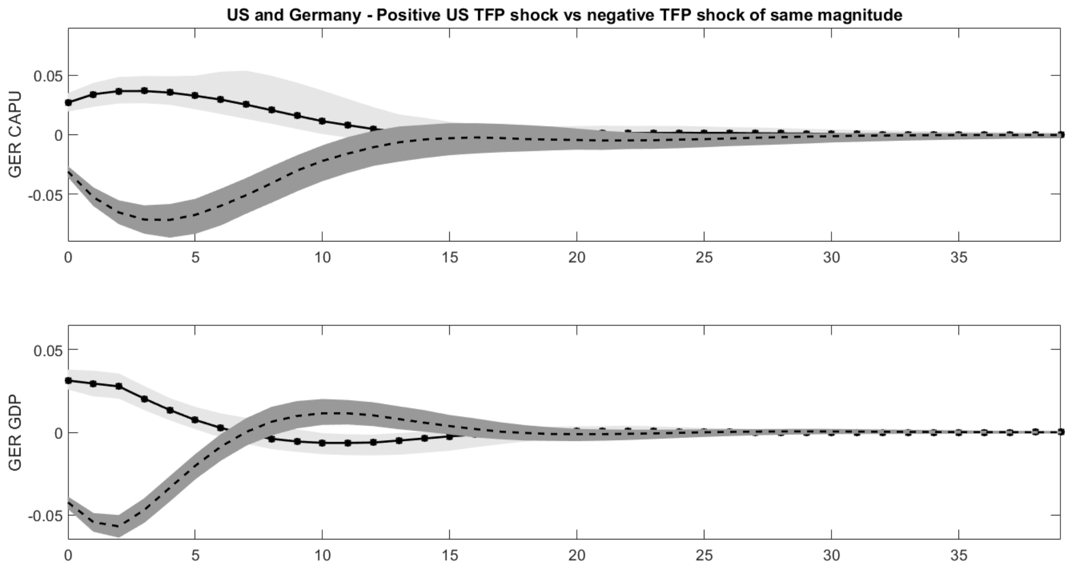
responses to the US variables themselves, indicating that spillovers of a US TFP shock are almost as high as the domestic effects on the US themselves. This, fact was also present in the theoretical model responses for the benchmark calibration and supports the model assumption of a intertemporal elasticity of substitution between intermediary goods of $\sigma = 0.5$, which was a crucial assumption for creating asymmetric business cycle correlations in the theoretical model. I also find this fact confirmed for the US-Germany specification below.

US - Germany

For Germany the prior threshold estimate is at 81.13 percent, while the Posterior threshold estimate has a mean of 84.45.

Similarly to the international spillovers of a US TFP shock to Canada, the German responses show that there are positive international spillover effects to a positive US TFP shock on German capacity utilization and German GDP growth (dotted lines), while there negative international spillover effects on the same variables in response to a negative TFP shock (dashed lines). The comparison between the responses to positive and negative US TFP shocks provide evidence that negative US TFP shocks have higher international spillover effects to the German economy compared to a positive US TFP shock. Therefore, also for Germany spillovers show the same asymmetry that is present in the responses to US TFP shock in the US economy. In this specification, astonishingly, the responses to German GDP and capacity utilization are larger in magnitude than the responses to the US variables themselves (compare to figure in the Appendix), indicating that spillovers of a US TFP shock are relatively higher as the domestic effects on the US themselves. Again, this supports the benchmark calibration of the theoretical model.

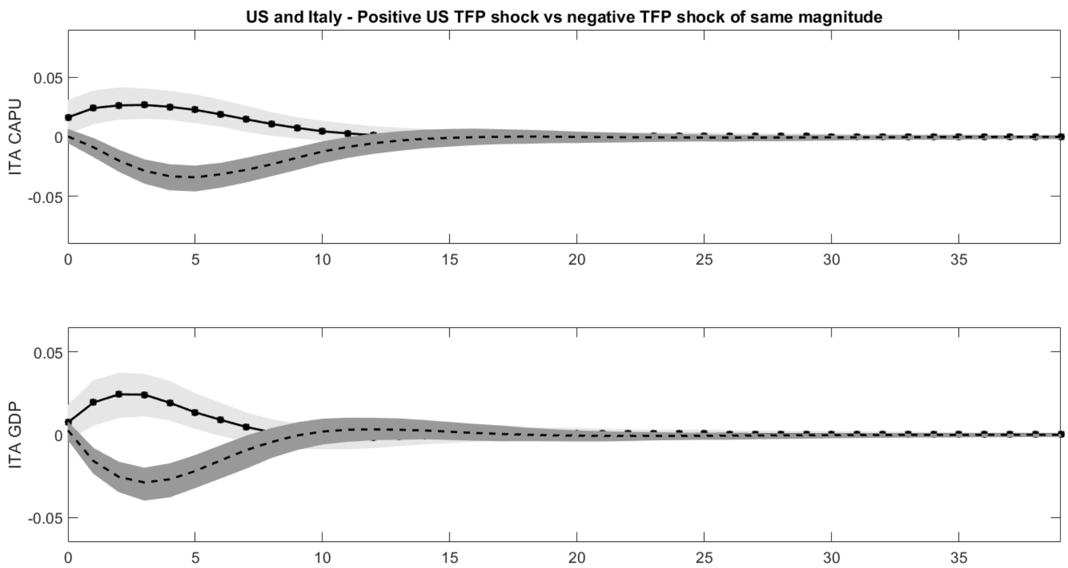
Figure 1.9 – US - Germany impulse response functions - German variables



US - Italy

For Italy the prior threshold estimate is at 81.13 percent, while the Posterior threshold estimate has a mean of 84.45.

Figure 1.10 – US - Italy impulse response functions - Italian variables



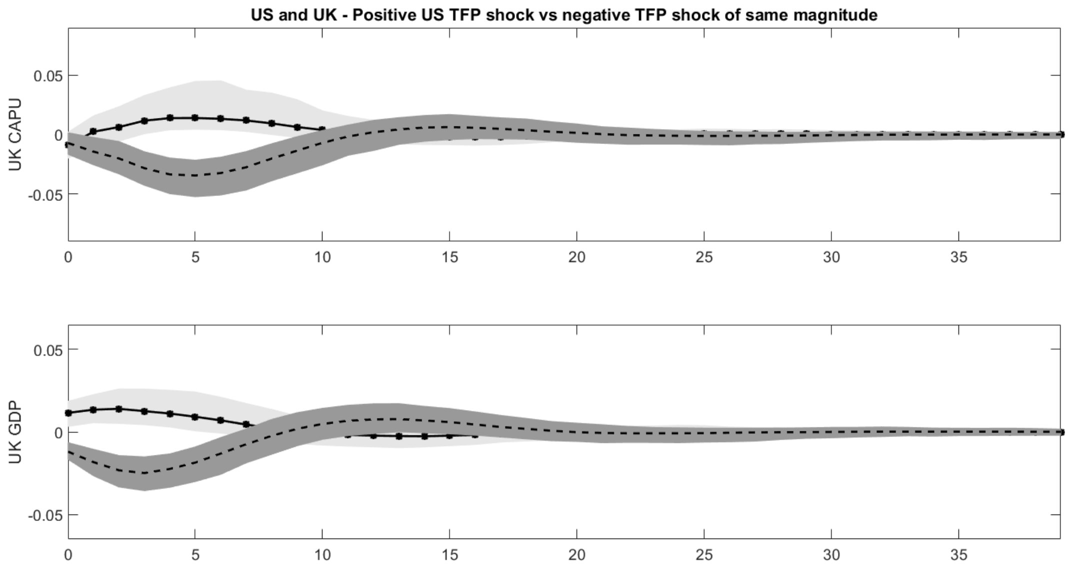
As expected, the differences in the response of the utilization rate and GDP go in the same direction as for Canada or Germany. The utilization rate changes less after a positive TFP shock in the US in the high utilization regime, than in the low utilization regime. At the same time, GDP growth increases by more.

Responses for Italy are slightly smaller in magnitude than for the US variables themselves, not directly supporting the theoretical benchmark calibration but still showing substantial asymmetry.

US - UK

Hansen's threshold estimate for the UK is 82.05 percent and the posterior mean is 84.73 percent.

Figure 1.11 – US - UK impulse response functions - UK variables



The overall pattern of the US-UK responses is similar to the ones investigated above. Similar to Italian responses, the responses for the UK are slightly smaller in magnitude compared to the US responses to a US TFP shock. I omit the responses of France and Japan here, they can be found in the appendix. For France, this is the case because the threshold estimates indicate a regime switch below average values of US capacity utilization. For Japan, the responses are not directly comparable to those of the other countries as utilization is given as an index instead of being given as percentage of total production capacity.

1.7.5 Summary of VAR evidence

The above analysis provides evidence that for five out of six country pairs, threshold effects in the US capacity utilization rate cause asymmetric responses to a US GDP shock both within the US economy and internationally. For these country pairs the data supports a positive threshold in the utilization rate, i.e. in booms when utilization is high the response to a US TFP shocks is dampened. The explanation of this phenomenon that I highlight in this paper is that in US booms, more and more individual producers hit a capacity utilization constrained where their machines work at their maximum production capacity and producers take time to expand their capacities. In this way the full potential of a positive TFP shock cannot transmit into production and thus dampens both the national and international shock responses, while a negative shock is fully transmitted into the production process, decreasing the utilization of existing machines. For France there is evidence that the threshold effect is below mean utilization. Note that the asymmetry in responses still goes in the same direction, i.e. the response to a positive shock is dampened in comparison to a negative shock. Therefore, the asymmetry in cross-country GDP correlations between France and the US in recessions and expansions can be explained using capacity constraints, but the fact that these capacity constraints are identified below mean utilization may be explained by additional channels being at work dominating the utilization channel. I leave this to further research.

1.8 Conclusion

In this paper I show that cross country correlations of GDP increase during recessions compared to expansions and that this phenomenon can in part be explained in a real business cycle model with occasionally binding capacity constraints. Intuitively, the constraints cause asymmetries in the business cycles of individual countries, which spill over to other countries. Thus, spill-overs of negative shocks are higher than spill-overs of positive shocks, thereby causing the same correlation pattern as in the data. To match the magnitude of the correlation differences in the observed data I assume a value for the elasticity of substitution between tradable intermediate goods suggesting that they are to a certain degree complementary. I successfully test the presence of capacity constraints as a mechanism which leads to business cycle asymmetries empirically using data from the G7 advanced economies in a Bayesian threshold autoregressive model. This finding does not only support capacity constraints as a prominent transmission channel of cross-country GDP asymmetries in recession compared to expansions, it also supports the benchmark calibration of the theoretical model with intermediary goods being complementary to a certain degree. So far the exact consequences of asymmetric cross-country correlations for the international economy and policy choices remain largely unexplored. Exploring the consequences on agents' choices and policy recommendations is an interesting avenue for future research.

Chapter 2

Foreign Direct Investment and the Equity Home Bias Puzzle

joint with Sven Blank[1] and Mathias Hoffmann[2][3]

2.1 Introduction

How does the degree of internationalization of a country's firms influence the equity investment choices of its households, and how does it affect the widely observed equity home bias? Focusing on an individual household, investing in an internationally active domestic firm is more attractive than investing in a firm that is solely domestically active. The reason is that the revenue stream of internationally active firms is less prone to country-specific economic conditions and thus provides a higher diversification benefit. Note that this argument only involves portfolio diversification through holding equities of different types of domestic firms, neglecting the possibility of obtaining portfolio diversification by holding equity of foreign firms. Most economic work on international portfolio choice and its empirical patterns exclusively focuses on the latter portfolio diversification motive and disregards indirect diversification through internationally active domestic firms. In this paper, we give a more diverse picture of different internationalization strategies for firms and their implications for the optimal portfolio choice of households. The internationalization strategies we model are exporting an foreign direct investment (FDI). They are fundamentally different in the sense that a firm serving foreign markets through exporting only sustains all of its production facilities in its country of residence, while a firm serving the foreign market through FDI sets up production plants in the foreign countries it serves. An FDI firm thus becomes directly prone to the economic conditions in the countries it serves, while an export firm is only indirectly prone to these conditions through the foreign countries' demand for its domestically produced products. In modelling different internationalization strategies we are es-

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pecially interested in the consequences these different strategies have for the equity home bias. It is a well established empirical fact that individuals all over the world hold large fractions of their wealth in assets of their own country (see table 2.1 for evidence on selected countries). Parts of the literature on the home equity bias regard this observation as an indication that households might not diversify optimally (e.g. Baxter and Jermann (1997)), others try to rationalize the observation through specific economic channels (e.g. Obstfeld and Rogoff (2001), Heathcote and Perri (2013)).

Table 2.1 – Equity Home Bias in 2008

Country	Domestic market in % of world market capitalization	Share of portfolio in domestic equity in %	Degree of Equity Home Bias (EHB_i)
Australia	1.8	76.1	0.76
Brazil	1.6	98.6	0.99
China	7.8	99.2	0.99
Canada	2.7	80.2	0.80
Euro Area	13.5	56.7	0.50
Japan	8.9	73.5	0.71
South Africa	1.4	87.8	0.88
South Korea	1.4	88.5	0.88
Sweden	0.7	43.6	0.43
Switzerland	2.3	50.9	0.50
United Kingdom	5.1	54.5	0.52
United States	32.6	77.2	0.66

$$EHB_i = 1 - \frac{\text{Share of Foreign Equities in Country i Equity Holdings}}{\text{Share of Foreign Equities in the World Market Portfolio}} \tag{2.1}$$

Source: Coeurdacier and Rey (2013)

In this paper, the possibility to obtain international diversification indirectly by investing in domestic multinational firms allows us to take an in-between position. On the one hand, we provide an explanation for why it might be optimal for agents to bias their equity holdings towards domestic firms, instead of diversifying their equity holding internationally. On the other hand, we argue that the empirical measure of home bias is underestimating the true international diversification of households as it does not take into account indirect diversification through internationally active firms. Empirically, Cai and Warnock (2012) show the relevance of this channel for the US. They find that indirect diversification accounts for approximately 25% of the US equity home bias. Our contribution is that we investigate the effects of internationalization strategies on optimal portfolio choice of households in a theoretical framework which enables us to shed light on the mechanisms at work for different internationalization strategies. Namely, we investigate the effects of introducing the possibility for firms to either tap foreign markets by exporting their products or by setting up foreign affiliates through foreign direct investment (FDI). We find that, due to the higher production diversification possibilities in the model with FDI, the average domestic firm is indeed less prone to country-specific shocks, giving a better hedge against these shocks than in a model with international trade only. Thus, given a standard calibration the home bias in optimal portfolio positions is higher if we allow for FDI. Our work builds upon two strands of the literature. First, the literature on the home equity bias puzzle that started with Lucas (1982). Second, the literature on international trade with heterogeneous firms building on Melitz (2003). Lucas (1982) investigates a two-country endowment economy with a single consumption good in which both countries are hit by independently and identical distributed dividend shocks. Perfect risk-sharing in this economy is achieved if all agents hold a percentage of their wealth in the stock of a given country which is equal to the relative size of its market capitalization.[4] The fact that empirical evidence suggests that agents hold a substantially higher share in domestic equity constitutes the original equity home bias puzzle and can be interpreted as evidence for a lack of international diversification. Since then, numerous attempts have been made to explain or rationalize this empirical finding. The second substantial contribution to the literature has been made by Baxter and Jermann (1997). Essentially, they add labor income to Lucas' one-good model and find that in order to hedge against labor income risk it is optimal for agents to short home equity and to take up large long positions in foreign equity. Thus in their model the discrepancy between theoretically optimal and observed diversification is even higher than in Lucas (1982), indicating that the international diversification puzzle is even worse than previously thought. A third fundamental contribution is Heathcote and Perri (2013) who investigate a two good production economy with labor and capital income and find that a standard international macroeconomic model building on Backus et al. (1992) can rationalize the equity home bias to a large extent. This is because in their framework domestic equity is a good hedge against labor income risk. An extensive review of the home bias literature is given in Coeurdacier and Rey (2013). The second relevant strand of literature on which this paper builds on is the workhorse model of international trade with heterogeneous firms by Melitz (2003), and its extension by Ghironi and Melitz (2005) who investigate macroeconomic fluctuations with heterogeneous firms. The main feature of these models is that they include endogenous selection into international trade and can therefore replicate the observed differences in firm-specific productivity between exporters and non-exporters. In these papers, firms have an individual and heterogeneous productivity which they draw from an exogenously given Pareto-distribution. If firms are productive enough they are able to afford the fixed costs to enter foreign markets as well as the shipping costs to export their product to these markets. Helpman et al. (2004) extend the Melitz (2003) model by the possibility for firms to serve the foreign market by horizontal FDI. Foreign affiliates are costly to set up, operate with the firm-specific productivity of their parent firm, but hire local production factors in the host economy. In line with the empirical evidence only the most productive firms engage in FDI. Contessi (2010) incorporates horizontal FDI in the model of Ghironi and Melitz (2005) to investigate its implications for macroeconomic fluctuations. To investigate the impact of FDI activity on international asset positions, we extend the Ghironi and Melitz (2005) model by FDI, international bonds, as well as international equity holdings. The introduction of horizontal foreign direct investment follows the style of Helpman et al. (2004) and Contessi (2010). Hamano (2015) investigates the hedging of variety risk in a Ghironi and Melitz (2005) type economy with international bond and equity markets. His paper is our orientation point for the asset structure in our model. We contribute to both strands of the literature. By introducing horizontal FDI in

[4]In Lucas (1982) there are two symmetric countries, thus 50-50 portfolios are optimal.

a model of heterogeneous firms and international portfolio choice we are able to thoroughly investigate the real link between different internationalization strategies of firms and the portfolio decisions of households. This theoretical foundation allows us to evaluate how the measure of the home equity bias should be adjusted to account for non-diversified vs. diversified investments.

2.2 The model

The model consists of two countries, Home (H) and Foreign (F). If not stated otherwise, foreign variables are indicated by an asterisk. First, we turn to the households' optimization problems in section 2.2.1, before we have a detailed look at the structure of the firms' sector in section 2.2.2.

2.2.1 Households

The representative households in Home maximizes its expected stream of utility from consumption C_t and labor L_t,

$$E_t \sum_{s=t}^{\infty} \beta_{s-t} U_t(C_t, L_t), \tag{2.2}$$

where the utility function takes the CRRA form

$$U_t(C_t, L_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \tag{2.3}$$

with the coefficient of relative risk aversion $\gamma(\leq 1)$. $\chi(>0)$ represents the dis-utility of supplying labor $L_t(\in [0,1])$ and $\varphi(\geq 0)$ is the Frisch elasticity of supplying labor.[5] β_t is the endogenous discount factor and evolves as follows

$$\beta_{t+1} = \beta_t \Upsilon(C_t) \text{ , with } \beta_0 = 1. \tag{2.4}$$

The functional form of $\Upsilon(C_t)$ is

$$\Upsilon(C_t) = \bar{\beta} C_t^{-\nu} \tag{2.5}$$

where $0 \leq \nu < \gamma$ and $0 < \bar{\beta} C^{-\nu} < 1$. Including an endogenous discount factor with $\Upsilon'(C_t) < 0$ guarantees the stationarity of the model including net foreign asset dynamics as pointed out by Schmitt-Grohe and Uribe (2003). C_t is assumed to be a nested CES aggregator of goods that are produced domestically by home firms $C_{H,t}$, imported goods that are produced abroad by Foreign firms $C_{X,t}$ and goods that are domestically produced by affiliates of Foreign firms $C_{I,t}$:

$$C_t = [\alpha_1^{\frac{1}{\omega}} C_{H,t}^{1-\frac{1}{\omega}} + \alpha_2^{\frac{1}{\omega}} C_{X,t}^{1-\frac{1}{\omega}} + (1-\alpha_1-\alpha_2)^{\frac{1}{\omega}} C_{I,t}^{1-\frac{1}{\omega}}]^{\frac{1}{1-\frac{1}{\omega}}}. \tag{2.6}$$

[5] With $\varphi = \infty$ marginal disutility of labor is constant at χ. With $\varphi = 0$ it becomes infinite and labor supply becomes inelastic. Following Hamano (2015) in the calibration disutility will be increasing in the labor supplied.

In this definition, the α's give different weights to the three distinct good categories, i.e. home produced goods, imported goods and goods produced in the domestic economy by affiliates of foreign firms. The nested CES structure is commonly used in international macroeconomic models with portfolio choice, since it gives a natural notion of home bias in consumption (see for instance Obstfeld and Rogoff (2001) and van Wincoop and Warnock (2010)). Because in these models there are only domestically produced and imported goods the weights represent the home bias in consumption directly. In the case of FDI, there are three types of goods entering the aggregator. To be able to interpret the weights in a similar way as in a two good model, one has to assume that FDI goods are interpreted by agents either as domestic or foreign goods. In the calibration later on, we will assume that FDI goods are treated as domestic goods by the households. The implicit assumption that households care more about where a good is produced than about the origin of the firm producing the good seems the more realistic one to us. Thus, in our model $(1-\alpha_2)$ is a measure of the home bias in consumption. To complete the nested preference structure over the consumption goods, $C_{H,t}$, $C_{X,t}$ and $C_{I,t}$ are themselves consumption baskets of varieties within the classified categories of domestic, imported and FDI goods. They are defined as

$$C_{H,t} = V_{H,t}\left(\int_{\zeta\in\Omega} c_{D,t}(\zeta)^{1-\frac{1}{\sigma}} d\zeta\right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

$$C_{X,t} = V^*_{X,t}\left(\int_{\vartheta\in\Omega} c_{X,t}(\vartheta)^{1-\frac{1}{\sigma}} d\vartheta\right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

$$C_{I,t} = V^*_{I,t}\left(\int_{\varphi\in\Omega} c_{I,t}(\varphi)^{1-\frac{1}{\sigma}} d\varphi\right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

where $V_{H,t} \equiv N_{D,t}^{\psi-\frac{1}{\sigma-1}}$, $V^*_{X,t} \equiv N_{X,t}^{*\psi-\frac{1}{\sigma-1}}$ and $V^*_{I,t} \equiv N_{I,t}^{*\psi-\frac{1}{\sigma-1}}$ introduce a love of variety following Benassy (1996). $N_{D,t}$, $N^*_{X,t}$ and $N^*_{I,t}$ denote the number of domestic, imported and FDI goods that are available in the Home economy.

Optimal consumption and price indices

From the nested CES demand structure, the optimal consumption of domestic, imported and FDI baskets, as well as individual varieties, that are produced domestically by domestic firms, imported from foreign firms and produced domestically by affiliates of foreign firms can be derived as

$$C_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\omega}\alpha_1 C_t,$$

$$C_{X,t} = \left(\frac{P_{X,t}}{P_t}\right)^{-\omega}\alpha_2 C_t,$$

$$C_{I,t} = \left(\frac{P_{I,t}}{P_t}\right)^{-\omega}(1-\alpha_1-\alpha_2) C_t$$

$$c_{D,t}(\zeta) = V_{H,t}^{\sigma-1}\left(\frac{p_{D,t}(\zeta)}{P_{H,t}}\right)^{-\sigma} C_{H,t},$$
$$c_{X,t}(\vartheta) = V_{X,t}^{*\sigma-1}\left(\frac{p_{X,t}^{*}(\vartheta)}{P_{X,t}}\right)^{-\sigma} C_{X,t},$$
$$c_{I,t}(\varphi) = V_{I,t}^{*\sigma-1}\left(\frac{p_{I,t}^{*}(\varphi)}{P_{I,t}}\right)^{-\sigma} C_{I,t}.$$

$p_{X,t}^{*}(\vartheta)$ denotes the price of exported goods from foreign and $p_{I,t}^{*}$ is the price domestic affiliates of foreign firms charge in the domestic market, both are denominated in Home currency. Price indices are given by

$$P_t = \left[\alpha_1 P_{H,t}^{1-\omega} + \alpha_2 P_{X,t}^{1-\omega} + (1-\alpha_1-\alpha_2) P_{I,t}^{1-\omega}\right]^{\frac{1}{1-\omega}},$$
$$P_{H,t} = \frac{1}{V_{H,t}}\left(\int_{\zeta\in\Omega_t} p_{D,t}(\zeta)^{1-\sigma} d\zeta\right)^{\frac{1}{1-\sigma}},$$
$$P_{X,t} = \frac{1}{V_{X,t}^{*}}\left(\int_{\vartheta\in\Omega_t} p_{X,t}^{*}(\vartheta)^{1-\sigma} d\vartheta\right)^{\frac{1}{1-\sigma}},$$
$$P_{I,t} = \frac{1}{V_{I,t}^{*}}\left(\int_{\varphi\in\Omega_t} p_{I,t}^{*}(\varphi)^{1-\sigma} d\varphi\right)^{\frac{1}{1-\sigma}}.$$

Because the price indices are defined on a welfare basis, they fluctuate with changes in the extensive margin. The impact of extensive margins in price indices is greater, the higher the love for variety, ψ. Following Hamano (2015) we define the welfare-based consumer price index, P_t, as numeraire for the Home economy. Therefore, real prices are defined as

$$\varrho_{H,t} \equiv \frac{P_{H,t}}{P_t},$$
$$\varrho_{X,t} \equiv \frac{P_{X,t}}{P_t},$$
$$\varrho_{I,t} \equiv \frac{P_{I,t}}{P_t},$$
$$\rho_{D,t}(\zeta) \equiv \frac{p_{D,t}(\zeta)}{P_t},$$
$$\rho_{X,t}^{*}(\vartheta) \equiv \frac{p_{X,t}^{*}(\vartheta)}{P_t}, and$$
$$\rho_{I,t}^{*}(\varphi) \equiv \frac{p_{I,t}^{*}(\varphi)}{P_t}.$$

Similar expressions hold for foreign.

2.2.2 Firms

In the Home country, as well as in the Foreign country there is a continuum of firms. Each firm produces a unique variety of the consumption good with labor being the only production input. Firms are heterogeneous with respect to their individual idiosyncratic labor productivity level z. Furthermore, firms' labor productivity is dependent on an aggregate country-specific component Z_t.[6]

Entry into and exit from production

Every period there is a mass of new entrants $N_{E,t}$. New entrants are ex-ante identical and face an entry cost $f_{E,t}$ in terms of effective labor. In terms of domestic consumption these costs are $\frac{w_t f_{E,t}}{Z_{E,t}}$, where $Z_{E,t}$ denotes the aggregate labor productivity level in the entry sector. Firms finance these entry costs by issuing equity on the international financial markets. Upon entering, firms draw their idiosyncratic productivity z from a distribution $G(z)$ with support $[z_{min}, \infty)$. Once drawn, this idiosyncratic productivity remains constant over the firm's lifetime. There is a time-to-build lag, as firms only start producing one period after their entry decision. As there are no fixed costs of production, all firms that enter produce in any subsequent period until they are hit by an exit shock. This exit shock can occur every period with a fixed probability of δ and is independent of the idiosyncratic productivity parameter. The timing is such that the exit shock only hits at the very end of a given period, i.e. after the new entrants have decided to enter. This has two implications. First, a fraction δ of the new entrants will be forced to exit before they can actually start producing. The Home economy's law of motion for the number of producing firms in the current period is thus given by

$$N_{D,t} = (1-\delta)(N_{D,t-1} + N_{E,t-1}). \tag{2.7}$$

Second, households that invested in these entrants will immediately lose the value of their investment. Because households cannot foresee which individual firms are hit by the exogenous exit shock, it is optimal for them to hold the same amount of shares in every potentially producing firm of a given country. Therefore, it is without loss of generality to assume that all firms of a given country are owned by a mutual fund in which the individual households can invest. On aggregate firms will enter production as long as their expected profits from entering are at least zero. Because the average expected profit of the ex-ante identical firms is equal to their average share price $\tilde{v}^s$, firms will enter until

$$\tilde{v}^s = \frac{w_t f_{E,t}}{Z_{E,t}}. \tag{2.8}$$

For Foreign firms similar conditions hold.

Internationalization strategies

Firms can produce for the home market and engage in international activity, either through exports or foreign direct investment (FDI). The two internationalization strategies differ in their cost structures. Exporters produce their goods in their domestic plants, using domestic resources. They face a fixed period-by-period entry

[6]For an individual firm, z and Z_t affect its productivity independently of each other. Thus, for instance the production of a domestic firm with individual productivity z for the domestic market is $y_t = Z_t z l_t$, where l_t is the labor of this firm.

cost to the foreign market $f_{X,t}$, generally interpreted as a cost to set up and maintain a distribution network for their products. Furthermore, export firms face an iceberg type shipping cost $\tau_t \geq 1$. In contrast, FDI firms set up production plants or affiliates in the foreign country and produce using foreign labor input. Setting up and maintaining the foreign plant involves a period-by-period fixed cost $f_{I,t}$ which is assumed to be higher than the exporting fixed cost $f_{X,t}$. The reason is that these firms incur the same costs of maintaining a distribution network in addition to the costs of setting up their own plant. Therefore, on the one hand the fixed costs of FDI are higher than those of exporting, but on the other hand producing abroad saves FDI firms the variable shipping costs.

Production, pricing and profits

In order to evaluate whether or not it is useful for a firm with individual productivity z to engage in exporting or FDI, the firm weighs the different types of costs against the potential revenues these activities generate. To investigate the cost side it is useful not to look only at the production functions of the different plants[7], but also at the number of goods the firm is able to supply to the consumers at Home and Foreign for a given amount of hired labor. The latter differs from the former because of potential shipping costs. For each unit of domestic labor input, a Home firm can supply $Z_t z$ units of its unique variety to the domestic agents and $\frac{Z_t z}{\tau_t}$ units to the foreign agent. Furthermore, a foreign affiliate of a domestic firm can supply $Z_t^* z$ units of the Home firm's variety to the foreign agents for each unit of foreign labor input. The total supply functions of a Home firm for each production type are given by

$$y_{D,t}(z) = Z_t z l_{D,t}(z) \tag{2.9}$$

$$y_{X,t}(z) = \frac{Z_t z}{\tau_t} l_{X,t}(z) \quad \text{if firm } z \text{ exports} \tag{2.10}$$

$$y_{I,t}(z) = Z_t^* z l_{I,t}^*(z) \quad \text{if firm } z \text{ has an affiliate in the foreign country.} \tag{2.11}$$

On the revenue side, firms are monopolistically competitive and face an individual downward sloping demand function with constant elasticity σ in every market they serve. This induces the following profit maximizing real prices, where prices are denoted in terms of the destination country's consumption good:

$$\rho_{D,t}(z) = \frac{\sigma}{\sigma - 1} \frac{w_t}{Z_t z} \tag{2.12}$$

$$\rho_{X,t}(z) = \frac{\tau_t}{Q_t} \frac{\sigma}{\sigma - 1} \frac{w_t}{Z_t z} \quad \text{if firm } z \text{ exports} \tag{2.13}$$

$$\rho_{I,t}(z) = \frac{\sigma}{\sigma - 1} \frac{w_t^*}{Z_t^* z} \quad \text{if firm } z \text{ has an affiliate in the foreign country.} \tag{2.14}$$

Putting the revenue and cost sides together, we obtain the profits an individual firm with productivity z can generate from producing for the domestic market, from

[7] Generally a Home firm's production is $Z_t z l_t$ for a good produced at Home and $Z_t^* z l_t^*$ for goods produced by a potential affiliate.

producing for exporting and through its foreign affiliates, respectively. They are:

$$d_{D,t} = \frac{1}{\sigma} N_{D,t}^{\psi(\omega-1)-1} \rho_{D,t}(z)^{1-\omega} \alpha_1 C_t \tag{2.15}$$

$$d_{X,t} = \frac{1}{\sigma} Q_t N_{X,t}^{\psi(\omega-1)-1} \rho_{X,t}(z)^{1-\omega} \alpha_2^* C_t^* - \frac{w_t f_{X,t}}{Z_t} \quad \text{if firm } z \text{ exports} \tag{2.16}$$

$$d_{I,t} = Q_t \left[\frac{1}{\sigma} N_{I,t}^{\psi(\omega-1)-1} \rho_{I,t}(z)^{1-\omega} (1-\alpha_1-\alpha_2) C_t^* - \frac{w_t^* f_{I,t}^*}{Z_t^*} \right] \quad \text{if firm } z \text{ engages in FDI.} \tag{2.17}$$

In the profit equation for FDI firms $f_{I,t}^*$ is the FDI entry cost to the foreign market that the domestic firm has to pay. Total profits of a firm can be decomposed into profits generated by domestic sales, exports and FDI activities. Thus $d_t(z) = d_{D,t}(z)+d_{X,t}(z)+d_{I,t}(z)$. All profits are denominated in terms of Home consumption good. Similar equations apply to the foreign producers.

Firm averages

The heterogeneous firms and the variety they produce are completely characterized by the firm specific average productivity levels. Defining a specific distribution from which firms draw their individual labor productivity, we can define three distinct average productivity levels: The average productivity of all firms that produce in the Home economy $\tilde{z}_D$, the average productivity of Home exporters $\tilde{z}_{X,t}$ and the average productivity of Home FDI firms $\tilde{z}_{I,t}$. Generally, they are given by

$$\tilde{z}_{D,t} \equiv \left[\int_{z_{min}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}, \quad \tilde{z}_{X,t} \equiv \left[\frac{1}{G(z_{I,t}) - G(z_{X,t})} \int_{z_{X,t}}^{z_{I,t}} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}},$$

$$\tilde{z}_{I,t} \equiv \left[\frac{1}{1-G(z_{I,t})} \int_{z_{I,t}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}.$$

The variables $z_{X,t}$ and $z_{I,t}$ denote the productivity cutoff levels such that a given firm with productivity $z \geq z_{X,t}$ finds it profitable to export and a given firm with $z \geq z_{I,t}$ finds it profitable to engage in FDI. Due to the assumed cost structure it holds that $z_{I,t} > z_{X,t}$, such that there is a direct mapping from the drawn productivity to the production modes a firm engages in. The least productive firms only find production for the domestic market profitable, firms with intermediate productivity optimally serve the foreign market through exports and the firms with the highest firm-specific productivity engage in FDI. The cutoff levels vary depending on the economic situation in a given country and play a crucial role in the dynamics of the model economy. These levels are derived below using an assumption on the distribution of firm specific productivities. Following Melitz (2003) and Ghironi and Melitz (2005), we assume the firm specific productivity to be drawn from a Pareto distribution, which they show matches the actual distribution of firm-specific productivity well. The cumulative distribution function is given by

$$G(z) = 1 - (\frac{z_{min}}{z})^k, \tag{2.18}$$

where z denotes a specific cutoff productivity level and $k(>\sigma-1)$ is a shaping parameter. From the distributional assumption, the geometric productivity averages in terms of the productivity cutoff levels can be derived as

$$\tilde{z}_{D,t}=\nabla^{\frac{1}{\sigma-1}}z_{min},\quad \tilde{z}_{X,t}=\nabla^{\frac{1}{\sigma-1}}[\frac{z_{X,t}^{\sigma-1}z_{I,t}^{k}-z_{X,t}^{k}z_{I,t}^{\sigma-1}}{z_{I,t}^{k}-z_{X,t}^{k}}]^{\frac{1}{\sigma-1}}$$

$$\tilde{z}_{I,t}=\nabla^{\frac{1}{\sigma-1}}z_{I,t},\quad \text{with}\quad \nabla=\frac{k}{k-(\sigma-1)}.$$

Furthermore, the shares of exporters and FDI firms in the total number of domestic firms can be expressed as:

$$\frac{N_{X,t}}{N_{D,t}}=\frac{\tilde{z}_{X,t}^{-k}-\tilde{z}_{I,t}^{-k}}{(z_{min})^{-k}} \tag{2.19}$$

$$\frac{N_{I,t}}{N_{D,t}}=\frac{(\tilde{z}_{I,t})^{-k}\nabla^{\frac{k}{\theta-1}}}{(z_{min})^{-k}}. \tag{2.20}$$

Importantly, the distributional assumption allows us to calculate the cutoff levels of the individual productivities, for which firms break even when they consider to export or produce for the domestic economy only or when they consider to engage in FDI or to serve foreign markets only through exporting. Thus, firms with individual productivity equal to one of the cutoff levels are indifferent between exporting or not, or between setting up an affiliate or exporting, respectively. The productivity level of the marginal FDI firm is implicitly defined such that

$$z_{I,t}:\quad d_{I,t}=0\Leftrightarrow Q_t\left[\frac{1}{\sigma}N_{I,t}^{\psi(\omega-1)-1}\rho_{I,t}(z)^{1-\omega}(1-\alpha_1-\alpha_2)C_t^*\right]=Q_t\frac{w_t^*f_{I,t}^*}{Z_t^*} \tag{2.21}$$

This leads to a cutoff productivity of

$$z_{I,t}=\frac{1}{\sigma-1}N_{I,t}^{\frac{1-\psi(\omega-1)}{\omega-1}}\left(\frac{w_t^*\sigma}{Z_t^*}\right)^{\frac{\omega}{\omega-1}}\left(\frac{f_{I,t}^*}{(1-\alpha_1-\alpha_2)C_t^*}\right)^{\frac{1}{\omega-1}}.$$

Using this productivity cutoff level for FDI firms and keeping in mind that the definition of the average productivity level of all FDI firms is $\tilde{z}_{I,t}=\nabla^{\frac{1}{\sigma-1}}z_{I,t}$ we can determine the average profits of all domestic FDI firms from their FDI activity

$$\tilde{d}_{I,t}=Q_t\left(\nabla^{\frac{1-\omega}{1-\sigma}}-1\right)\frac{w_t^*}{Z_t^*}f_{I,t}^*.$$

The productivity level of the marginal export firm is such that

$$z_{X,t}:\quad d_{X,t}=0\Leftrightarrow\frac{1}{\sigma}Q_tN_{X,t}^{\psi(\omega-1)-1}\rho_{X,t}(z_{X,t})^{1-\omega}\alpha_2^*C_t^*=\frac{w_tf_{X,t}}{Z_t} \tag{2.22}$$

$$z_{X,t}=\left(\frac{\sigma f_{X,t}}{\alpha_2^*C_t^*}\right)^{\frac{1}{\omega-1}}\left(\frac{w_t}{Q_tZ_t}\right)^{\frac{\omega}{\omega-1}}\tau_t\frac{\sigma}{\sigma-1}N_{X,t}^{\frac{1-\psi(\omega-1)}{\omega-1}}.$$

Substituting in both the cutoff level of exporters $z_{X,t}$ and the cutoff level for FDI firms $z_{I,t}$ into the definition of the average productivity level of exporters yields

$$\tilde{z}_{X,t} = \nabla^{\frac{1}{\sigma-1}} \frac{\sigma^{\frac{\omega}{\omega-1}}}{(\sigma-1)} \left(\frac{1}{C_t^*}\right)^{\frac{1}{\omega-1}} N_{X,t}^{\frac{(1-\psi(\omega-1))}{\omega-1}} \left(\frac{w_t}{Q_t Z_t}\right)^{\frac{\omega}{\omega-1}} \tau_t \left[KK_t\right]^{\frac{1}{\sigma-1}}$$

where

$$KK_t = \frac{\left(\frac{f_{X,t}}{\alpha_2^*}\right)^{\frac{k-\sigma+1}{1-\omega}} - \left[(TOL_t)^{\omega} \tau_t^{1-\omega} \left(\frac{f_{I,t}^*}{(1-\alpha_1-\alpha_2)}\right) \left(\frac{N_{X,t}}{N_{I,t}}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k-\sigma+1}{1-\omega}}}{\left(\frac{f_{X,t}}{\alpha_2^*}\right)^{\frac{k}{1-\omega}} - \left[(TOL_t)^{\omega} \left(\frac{f_{I,t}^*}{1-\alpha_1^*-\alpha_2^*}\right) \tau_t^{1-\omega} \left(\frac{N_{X,t}}{N_{I,t}}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k}{1-\omega}}}.$$

with $TOL_t = \left(\frac{w_t}{Z_t Q_t}\right)^{-1} \left(\frac{w_t^*}{Z_t^*}\right)$ being the terms of labor. This implies average profits of exporting firms of

$$\tilde{d}_{X,t} = \left[\left(\nabla \left[\frac{\left(\frac{f_{X,t}}{\alpha_2^*}\right)^{\frac{k-\sigma+1}{1-\omega}} - \left[(TOL_t)^{\omega} \tau_t^{1-\omega} \left(\frac{f_{I,t}^*}{1-\alpha_1^*-\alpha_2^*}\right) \left(\frac{N_{X,t}}{N_{I,t}}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k-\sigma+1}{1-\omega}}}{\left(\frac{f_{X,t}}{\alpha_2^*}\right)^{\frac{k}{1-\omega}} - \left[(TOL_t)^{\omega} \left(\frac{f_{I,t}^*}{1-\alpha_1^*-\alpha_2^*}\right) \tau_t^{1-\omega} \left(\frac{N_{X,t}}{N_{I,t}}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k}{1-\omega}}}\right]\right)^{\frac{\omega-1}{\sigma-1}} \alpha_2^* - f_{X,t}\right] \frac{w_t}{Z_t}. \tag{2.23}$$

For Foreign an analogous expression holds. This completes the description of the firms.

2.2.3 The asset market structure and the household's budget constraints

The key to understanding the households' investment decisions in our model economy is the budget constraint of the Home and Foreign household. Its description gives us the opportunity to explain the asset market structure as well. The budget constraint of the Home representative household in terms of the Home consumption basket is given by

$$\begin{aligned} &C_t + s_{h,t+1}\tilde{v}_t^s(N_{D,t} + N_{E,t}) + s_{f,t+1}Q_t\tilde{v}_t^{s*}(N_{D,t}^* + N_{E,t}^*) + b_{h,t+1}v_t^b + b_{f,t+1}Q_t v_t^{b*} \\ &= w_t L_t + s_{h,t}N_{D,t}(\tilde{v}_t^s + \tilde{d}_t) + s_{f,t}N_{D,t}^*Q_t(\tilde{v}_t^{s*} + \tilde{d}_t^*) + b_{h,t}(v_t^b + d_t^b) + b_{f,t}Q_t(v_t^{b*} + d_t^{b*}). \end{aligned} \tag{2.24}$$

In essence, it has the usual interpretation that the Home household's consumption smoothing is restricted by its decision to split its income of the current period between consumption C_t and different means of savings to carry over consumption to future periods. w_t are real wages and L_t is the labor supply of the household. Thus, $w_t L_t$ gives the income from supplying labor. Q_t is the welfare based real exchange rate. It is defined as the welfare-based Foreign price index P_t^* converted to Home currency by the nominal exchange rate ϵ_t and divided the welfare-based Home price index P_t: $Q_t \equiv (\epsilon_t P_t^*)/P_t$. Therefore, Q_t is defined in terms of the Home consumption good. The Home household can invest in shares of the Home or the Foreign mutual fund. The Home mutual fund owns all the shares of the domestic new entrants $N_{E,t}$, as well as all the Home firms that produce in the current period $N_{D,t}$. The Foreign mutual fund owns all the shares of the foreign new entrants $N^*_{E,t}$ as well as all currently producing Foreign firms $N^*_{D,t}$. $s_{h,t+1}(s_{f,t+1})$ denote the shares in the mutual fund of Home (Foreign) firms the Home household buys in the current period. $\tilde{v}^s_t(\tilde{v}^{s*}_t)$ is the real price of a Home (Foreign) mutual fund share denominated in Home (Foreign) consumption goods. Furthermore, the Home household can invest in bonds issued by Home and Foreign. $b_{h,t+1}$ and $b_{f,t+1}$ denote the amount of Home and Foreign bond bought by the Home household, respectively. v^b_t and v^{b*}_t are the real prices of bonds issued in Home and Foreign. $\tilde{d}_t$ and $\tilde{d}^*_t$ are the average real dividends of Home (Foreign) firms and d^b_t and d^{b*}_t are the real dividends of bonds issued in Home and Foreign. For a bond indexed by the welfare-based price index we have $d^b_t = d^{b*}_t = 1$. For a bond indexed by the empirical CPI real dividends are given by $d^b_t = \frac{\hat{P}_{t+1}}{P_{t+1}}$ and $d^{b*}_t = \frac{\hat{P}^*_{t+1}}{P^*_{t+1}}$, where the empirical measure of CPI is denoted by $\hat{P}(\hat{P}^*)$. The interpretation of the pricing is that Home (Foreign) CPI indexed bonds give a nominal payoff of $\hat{P}(\hat{P}^*)$ units of Home (Foreign) currency next period. Since the rest of the variables in the budget constraint are deflated using the welfare-based price levels $P_t(P^*_t)$, the nominal payoff of $\hat{P}(\hat{P}^*)$ has to be divided by $P_t(P^*_t)$ to denote it in real terms. In Ghironi and Melitz (2005) the relation between the welfare-based and the empirical CPI measures is given by $P_t = N_t^{\frac{1}{1-\sigma}}\hat{P}_t$, with $N_t = N_{D,t} + N^*_{X,t}$, as the empirical measure usually does not account for the changes in the extensive margin, which influence the welfare-based CPI. The representative Foreign household maximizes its utility with respect to a symmetric real budget constraint denominated in Foreign consumption goods:

$$\begin{aligned} C^*_t &+ s^*_{f,t+1}\tilde{v}^{s*}_t(N^*_{D,t} + N^*_{E,t}) + s^*_{h,t+1}Q_t^{-1}\tilde{v}^s_t(N_{D,t} + N_{E,t}) + b^*_{h,t+1}Q_t^{-1}v^b_t + b^*_{f,t+1}v^{b*}_t \\ &= w^*_t L^*_t + s^*_{f,t}N^*_{D,t}(\tilde{v}^{s*}_t + \tilde{d}^*_t) + s^*_{h,t}N_{D,t}Q_t^{-1}(\tilde{v}^s_t + \tilde{d}_t) + b^*_{h,t}Q_t^{-1}(v^b_t + d^b_t) + b^*_{f,t}(v^{b*}_t + d^{b*}_t). \end{aligned}$$

Households' optimality conditions

Defining the returns on Home and Foreign equities as

$$r^s_{h,t} \equiv (1-\delta)\frac{\tilde{v}^s_t + \tilde{d}_t}{\tilde{v}^s_{t-1}}, \quad r^s_{f,t} \equiv (1-\delta)\frac{\tilde{v}^{s*}_t + \tilde{d}^*_t}{\tilde{v}^{s*}_{t-1}}\frac{Q_t}{Q_{t-1}}, \tag{2.25}$$

respectively, and the return on the domestic bond as

$$r^b_{h,t} \equiv \frac{v^b_t + d^b_t}{v^b_{t-1}}, \quad r^b_{f,t} \equiv \frac{v^{b*}_t + d^{b*}_t}{v^{b*}_{t-1}}\frac{Q_t}{Q_{t-1}}, \tag{2.26}$$

where the dividends paid by a regular bond are

$$d^b_t = 1, \quad d^{b*}_t = 1, \qquad \text{for welfare-based bonds} \tag{2.27}$$

$$d^b_t = \frac{\hat{P}_{t+1}}{P_{t+1}}, \quad d^{b*}_t = \frac{\hat{P}^*_{t+1}}{P^*_{t+1}}, \qquad \text{for CPI indexed bonds} \tag{2.28}$$

the first-order conditions of the household become

$$\chi L_t^{\frac{1}{\varphi}} = C_t^{-\gamma} w_t \tag{2.29a}$$

$$1 = \Upsilon(C_t) E_t\{(\frac{C_{t+1}}{C_t})^{-\gamma} r^s_{h,t+1}\} \tag{2.29b}$$

$$1 = \Upsilon(C_t) E_t\{(\frac{C_{t+1}}{C_t})^{-\gamma} r^s_{f,t+1}\} \tag{2.29c}$$

$$1 = \Upsilon(C_t) E_t\{(\frac{C_{t+1}}{C_t})^{-\gamma} r^b_{h,t+1}\} \tag{2.29d}$$

$$1 = \Upsilon(C_t) E_t\{(\frac{C_{t+1}}{C_t})^{-\gamma} r^b_{f,t+1}\} \tag{2.29e}$$

$$\begin{aligned} &C_t + s_{h,t+1}\tilde{v}_t^s(N_{D,t} + N_{E,t}) + s_{f,t+1}Q_t\tilde{v}_t^{s*}(N^*_{D,t} + N^*_{E,t}) + b_{h,t+1}v_t^b + b_{f,t+1}v_t^{b*} \\ &= w_tL_t + s_{h,t}N_{D,t}(\tilde{v}_t^s + \tilde{d}_t) + s_{f,t}N^*_{D,t}Q_t(\tilde{v}_t^{s*} + \tilde{d}^*_t) + b_{h,t}(v_t^b + d_t^b) + b_{f,t}(v_t^{b*} + d_t^{b*}). \end{aligned} \tag{2.29f}$$

Using the same definitions of returns on home and foreign equities as above the first-order conditions of the Foreign household are

$$\chi L_t^{*\frac{1}{\varphi}} = C_t^{*-\gamma} w^*_t \tag{2.30a}$$

$$1 = \Upsilon(C^*_t) E_t \left[\left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\gamma} r^s_{h,t+1} \frac{Q_t}{Q_{t+1}} \right] \tag{2.30b}$$

$$1 = \Upsilon(C^*_t) E_t \left[\left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\gamma} r^s_{f,t+1} \frac{Q_t}{Q_{t+1}} \right] \tag{2.30c}$$

$$1 = \Upsilon(C^*_t) E_t \left[\left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\gamma} r^b_{h,t+1} \frac{Q_t}{Q_{t+1}} \right] \tag{2.30d}$$

$$1 = \Upsilon(C^*_t) E_t \left[\left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\gamma} r^b_{f,t+1} \frac{Q_t}{Q_{t+1}} \right] \tag{2.30e}$$

$$\begin{aligned} &C^*_t + s^*_{f,t+1}\tilde{v}_t^{s*}(N^*_{D,t} + N^*_{E,t}) + s^*_{h,t+1}Q_t^{-1}\tilde{v}_t^s(N_{D,t} + N_{E,t}) + b^*_{h,t+1}Q_t^{-1}v_t^b + b^*_{f,t+1}v_t^{b*} \\ &= w^*_tL^*_t + s^*_{f,t}N^*_{D,t}(\tilde{v}_t^{s*} + \tilde{d}^*_t) + s^*_{h,t}N_{D,t}Q_t^{-1}(\tilde{v}_t^s + \tilde{d}_t) + b^*_{h,t}Q_t^{-1}(v_t^b + d_t^b) + b^*_{f,t}(v_t^{b*} + d_t^{b*}). \end{aligned} \tag{2.30f}$$

A detailed derivation of these conditions is given in the technical appendix **??**.

2.2.4 Market Clearing

In the model there are seven markets: Four asset markets, two labor markets and a good market. By Walras' law one of these market clearing conditions is implied by the other six and thus can be disregarded in the final system of equilibrium conditions.

Asset Markets

Following Hamano (2015) we assume that the equity supply of each individual firm is normalized to 1. Because households cannot foresee which individual firms are hit by the exogenous exit shock, they hold the same amount of shares in every potentially producing firm of a given country. Therefore, the home household's holdings in each home firm $s_{h,t+1}$ and the foreign household's holdings in each home firm $s^*_{h,t+1}$ have to sum to unity:

$$s_{h,t+1} + s^*_{h,t+1} = 1. \tag{2.31}$$

Analogously, the home household's holdings in each foreign firm $s_{f,t+1}$ and the foreign household's holdings in each foreign firm $s^*_{f,t+1}$ have to sum to unity:

$$s_{f,t+1} + s^*_{f,t+1} = 1. \tag{2.32}$$

Additionally, there are two international bond markets:

$$b_{h,t+1} + b^*_{h,t+1} = 0 \tag{2.33}$$

and

$$b_{f,t+1} + b^*_{f,t+1} = 0. \tag{2.34}$$

Labor market

There are three separate sources of labor demand for production in each country. Home firms demand domestic labor $l_{D,t}$ for production to serve the domestic market and $l_{X,t}$ to serve the foreign market through exports. Furthermore, foreign firms serving the Home market through their affiliates demand $l_{I,t}$ units of domestic labor. In addition to the use in production, firms hire labor in order to pay entry, export and FDI costs. Entering firms hire $\frac{N_{E,t} f_{E,t}}{Z_{E,t}}$ units of effective labor in total, domestic export firms $\frac{N_{X,t} f_{X,t}}{Z_t}$ and affiliates of foreign firms $\frac{N^*_{I,t} f_{I,t}}{Z_t}$.
Adding up all of these demand sources of the average firms and equating it to the labor supply of the representative household, the overall Home labor market clearing condition is

$$L_t = \frac{(\sigma-1)}{w_t}\left[N_{D,t}\tilde{d}_{D,t} + N_{X,t}\tilde{d}_{X,t} + N^*_{I,t}Q_t\tilde{d}^*_{I,t}\right] + \frac{N_{E,t} f_{E,t}}{Z_{E,t}} + \frac{\sigma}{Z_t}\left[N_{X,t} f_{X,t} + N^*_{I,t} f_{I,t}\right]. \tag{2.35}$$

For Foreign the condition is

$$L^*_t = \frac{(\sigma-1)}{w^*_t}\left[N^*_{D,t}\tilde{d}^*_{D,t} + N^*_{X,t}\tilde{d}^*_{X,t} + N_{I,t}Q_t^{-1}\tilde{d}_{I,t}\right] + \frac{N^*_{E,t} f^*_{E,t}}{Z^*_{E,t}} + \frac{\sigma}{Z^*_t}\left[N^*_{X,t} f^*_{X,t} + N_{I,t} f^*_{I,t}\right]. \tag{2.36}$$

Good Market

International good market clearing states that international consumption has to be equal to international labor and capital income, denoted in the Home consumption basket.

$$C_t + Q_t C^*_t = Y_t + Q_t Y^*_t \tag{2.37a}$$

where

$$Y_t = w_t L_t + N_{D,t}\tilde{d}_t - N_{E,t}\tilde{v}_t^s \tag{2.37b}$$
$$Y_t^* = w_t^* L_t^* + N_{D,t}^*\tilde{d}_t^* - N_{E,t}^*\tilde{v}_t^{s*}. \tag{2.37c}$$

2.2.5 The stochastic processes of production

The stochastic aggregate productivities and the productivities specific to the entry sector are assumed to be given by the following AR(1) processes:

$$log(Z_{t+1}) = \rho log(Z_t) + \epsilon_{t+1} \tag{2.38a}$$
$$log(Z_{t+1}^*) = \rho log(Z_t^*) + \epsilon_{t+1}^* \tag{2.38b}$$
$$log(Z_{E,t+1}) = \rho_E log(Z_{E,t}) + \epsilon_{E,t+1} \tag{2.38c}$$
$$log(Z_{E,t+1}^*) = \rho_E log(Z_{E,t}^*) + \epsilon_{E,t+1}^*. \tag{2.38d}$$

With 4 shocks and 4 assets that Home and Foreign households can invest in, we have complete markets in our model economy.

2.2.6 Net foreign asset dynamics

We can combine the two budget constraints of the households to obtain an expression for the net foreign asset dynamics of the Home economy:

$$\begin{aligned} & s_{h,t+1}\tilde{v}_t^s(N_{D,t}+N_{E,t}) + s_{f,t+1}Q_t\tilde{v}_t^{s*}(N_{D,t}^*+N_{E,t}^*) + b_{h,t+1}v_t^b + b_{f,t+1}Q_t v_t^{b*} \\ & \quad = s_{h,t}N_{D,t}(\tilde{v}_t^s+\tilde{d}_t) + s_{f,t}N_{D,t}^*Q_t(\tilde{v}_t^{s*}+\tilde{d}_t^*) + b_{h,t}(v_t^b+d_t^b) + b_{f,t}Q_t(v_t^{b*}+d_t^{b*}) \\ & +\frac{1}{2}\left(N_{E,t}\tilde{v}_t^s + Q_t N_{E,t}^*\tilde{v}_t^{s*}\right) - \frac{1}{2}\left(N_{D,t}\tilde{d}_t - Q_t N_{D,t}^*\tilde{d}_t^*\right) + \frac{1}{2}\left(w_t L_t - Q_t w_t^* L_t^*\right) - \frac{1}{2}\left(C_t - Q_t C_t^*\right). \end{aligned} \tag{2.39}$$

This condition can be interpreted as an equation for the dynamics of the Home country's financial wealth.

2.2.7 Equilibrium

The equilibrium conditions are given by four of the first-order conditions for the Home household (2.29a - 2.29e), the equivalent equations for the Foreign household (2.30a - 2.30e), the aggregate accounting constraint (2.39), the 16 optimality conditions of the firm and the 12 optimal pricing and dividend conditions, the 6 clearing conditions for the Home and Foreign labor and asset markets in (2.2.4) and the definitions of equity returns (2.25), bond returns (2.26), the two definitions of the dividends of bonds (2.27), the definition of the terms of labor, the income (2.37b), as well as the stochastic processes (2.38). As mentioned above, the goods market clearing condition (2.37) is left out due to Walras' law. All these conditions are summarized in section B.1. Theses are 58 equations, in 58 unknowns. The unknowns are C_t, C_t^*, L_t, L_t^* $s_{h,t}, s_{h,t}^*, s_{f,t}, s_{f,t}^*,$ $b_{h,t}, b_{f,t}, b_{h,t}^*, b_{f,t}^*$ $r_{h,t}^s, r_{f,t}^s,$

$r^b_{h,t}, r^b_{f,t}$ $\tilde{v}^s_t, \tilde{v}^{s*}_t$, v^b_t, v^{b*}_t, d^b_t, d^{b*}_t Y_t, Y^*_t, w_t, w^*_t, d_t, d^*_t, $N_{E,t}, N^*_{E,t}$, $\tilde{z}_{X,t}, \tilde{z}^*_{X,t}$, $\tilde{z}_{I,t}, \tilde{z}^*_{I,t}$, $N_{D,t}, N^*_{D,t}$, $N_{X,t}, N^*_{X,t}$, $N_{I,t}, N^*_{I,t}$, Q_t, TOL_t $\tilde{\rho}_{D,t}, \tilde{\rho}^*_{D,t}$, $\tilde{\rho}_{X,t}, \tilde{\rho}^*_{X,t}$, $\tilde{\rho}_{I,t}, \tilde{\rho}^*_{I,t}$, $\tilde{d}_{D,t}, \tilde{d}^*_{D,t}$, $\tilde{d}_{X,t}, \tilde{d}^*_{X,t}$ $\tilde{d}_{I,t}, \tilde{d}^*_{I,t}$, and $Z_t, Z^*_t, Z_{E,t}, Z^*_{E,t}$.

2.3 Portfolio intuition

Coeurdacier and Gourinchas (2011) derive general results for portfolio solutions in a complete markets economy like ours. We can use a linearized version of the real exchange rate and a static version of the household's budget constraint to obtain an intuition on how the household uses bonds and equity to hedge against exchange rate fluctuations and labor income risk.

2.3.1 Welfare-based and CPI-indexed real exchange rate

In general, the risk components in the welfare based real exchange rate in a model with a preference for variety of the Benassy (1996) type can be decomposed as

$$\hat{Q}_t = \hat{q}_t + \psi R_{v,t} \tag{2.40}$$

where ψ is the 'love of variety' parameter, $\hat{q}_t$ is the linearized CPI based real exchange rate and $R_{v,t}$ is the risk component associated with the varieties. The welfare-based real exchange rate in our model is

$$\begin{aligned} Q_t &= \frac{\epsilon_t P^*_t}{P_t} \\ &= \frac{\epsilon_t \left[\alpha_1 N^{*-\psi(1-\omega)}_{D,t} \tilde{p}^{*(1-\omega)}_{D,t} + \alpha_2 N^{-\psi(1-\omega)}_{X,t} \tilde{p}^{(1-\omega)}_{X,t} + (1-\alpha_1-\alpha_2) N^{-\psi(1-\omega)}_{I,t} \tilde{p}^{(1-\omega)}_{I,t}\right]^{\frac{1}{1-\omega}}}{\left[\alpha_1 N^{-\psi(1-\omega)}_{D,t} \tilde{p}^{(1-\omega)}_{D,t} + \alpha_2 N^{*-\psi(1-\omega)}_{X,t} \tilde{p}^{*(1-\omega)}_{X,t} + (1-\alpha_1-\alpha_2) N^{*-\psi(1-\omega)}_{I,t} \tilde{p}^{*(1-\omega)}_{I,t}\right]^{\frac{1}{1-\omega}}} \end{aligned}$$

Log-linearizing around the steady-state yields

$$\begin{aligned} \hat{Q}_t = (1-2S_{XD})\hat{TOL}_t - S_{XD}\hat{z}^R_X - (1-S_{DD}-S_{XD})\hat{z}^R_I + \psi(S_{DD}-S_{XD})\hat{N}^R_D \\ - \psi(1-S_{DD}-S_{XD})\hat{N}^R_X + \psi S_{XD}\left[\hat{N}^R_{D,t} - \hat{N}^R_{X,t}\right] \\ + \psi\left(1-S_{DD}-S_{XD}\right)\left[\hat{N}^R_{X,t} - \hat{N}^R_{I,t}\right], \end{aligned} \tag{2.41}$$

where $S_{DD} = \alpha_1 \bar{\rho}^{1-\omega}_{H,t}$, $S_{NX} = \alpha_2 \bar{\rho}^{1-\omega}_{X,t}$, $S_{NI} = (1-\alpha_1-\alpha_2)\bar{\rho}^{1-\omega}_{I,t} = (1-S_{DD}-S_{XD})$. The first three terms denote the effects of changes in the terms of labor, relative FDI cutoffs and relative export cutoffs on $\hat{Q}_t$, respectively. The last four terms represent the effect of changes in the relative numbers of domestic, export and FDI varieties on the welfare-based real exchange rate. The average terms of trade are related with the terms of labor as follows

$$TOT_t = \frac{\epsilon_t \tilde{p}_{X,t}}{\tilde{p}^*_{X,t}} = \frac{\tilde{z}^*_{X,t}}{\tilde{z}_{X,t}} TOL_t^{-1}.$$

Linearized this gives

$$\hat{TOT}_t = -\hat{TOL}_t - \hat{z}^R_{X,t}. \tag{2.42}$$

Therefore, the linearized real exchange rate can be rewritten as

$$\begin{aligned}\hat{Q}_t = (2S_{XD}-1)\hat{TOT}_t - (1-S_{XD})\hat{z}^R_{X,t} - (1-S_{DD}-S_{XD})\hat{z}^R_I + \psi(S_{DD}-S_{XD})\hat{N}^R_D \\ - \psi(1-S_{DD}-S_{XD})\hat{N}^R_X + \psi S_{XD}\left[\hat{N}^R_{D,t} - \hat{N}^R_{X,t}\right] \\ - \psi\left(1-S_{DD}-S_{XD}\right)\left[\hat{N}^R_{I,t} - \hat{N}^R_{X,t}\right],\end{aligned} \tag{2.43}$$

As for instance in Ghironi and Melitz (2005) and Hamano (2015), the welfare-based price indices in our model fully reflect changes in varieties and this affects optimal portfolios if bonds are indexed by the welfare-based price indices. As Broda and Weinstein (2004) point out, in reality the CPI's do not adjust for changes in varieties. Therefore a more realistic assumption is that agents base their decisions on empirical versions of CPI indices and the real exchange rate. To extract the effect of changes in the number of varieties from bonds and the real exchange rate, Hamano (2015) defines the price indices in a way such that the empirical real exchange rate does not reflect these changes any more. For our model it becomes

$$\hat{q}_t = (2S_{XD}-1)\hat{TOT}_t - (1-S_{XD})\hat{z}^R_{X,t} - (1-S_{DD}-S_{XD})\hat{z}^R_I. \tag{2.44}$$

2.3.2 Portfolios from static budget constraint and complete asset markets

We derive the intuition behind the optimal portfolios we get from a static version of the household's budget constraint. Expressing both the Home and Foreign budget constraints in terms of the Home consumption good, subtracting the Home from the Foreign constraint and log linearizing this equation yields

$$\hat{P}_t + \hat{C}_t - \left(\hat{P}^*_t + \hat{C}^*_t\right) = S_W\left(\hat{w}^R_t + \hat{l}^R_t\right) + (2s-1)\left[S_D\left(\hat{N}^R_{D,t} + \hat{\tilde{d}}^R_t\right) - S_I\left(\hat{N}^R_{E,t} + \hat{\tilde{v}}^{sR}_t\right)\right] + 2b'\hat{d}^{bR}_t, \tag{2.45}$$

with $S_W \equiv \frac{w}{C}$, $S_D \equiv \frac{N_D\tilde{d}}{C}$ and $S_I \equiv \frac{N_E\tilde{v}^s}{C}$ being the labor income, dividends and investments relative to consumption in the symmetric steady state, respectively. Bond holdings are multiplied by their dividend d^b and normalized by steady state consumption $b' = bd^b/C$. We use the following definitions $\hat{w}^R_t + \hat{l}^R_t \equiv \hat{w}_t + \hat{l}_t - (\hat{Q}_t + \hat{w}^*_t + \hat{l}^*_t)$, $\hat{N}^R_{D,t} + \hat{\tilde{d}}^R_t \equiv \hat{N}_{D,t} + \hat{\tilde{d}}_t - (\hat{N}^*_{D,t} + \hat{Q}_t + \hat{\tilde{d}}^*_t)$, $\hat{N}_{E,t} + \hat{\tilde{v}}^{sR}_t \equiv \hat{N}_{E,t} + \hat{\tilde{v}}^s_t - (\hat{N}^*_{E,t} + \hat{Q}_t + \hat{\tilde{v}}^{s*}_t)$ and $\hat{d}^{bR}_t \equiv \hat{d}^b_t - (\hat{Q}_t + \hat{d}^{b*}_t)$. The terms denote relative nominal labor income, relative average equity dividends, relative investment and relative bond dividends between Home and Foreign, respectively. Coeurdacier and Gourinchas (2011) find that for economies with complete asset markets, the portfolio solution of Devereux and Sutherland (2010) coincides with finding portfolios that replicate the complete market allocation, i.e. portfolios ensuring that the perfect risk-sharing condition

$$\hat{P}_t + \hat{C}_t - \left(\hat{P}^*_t + \hat{C}^*_t\right) = -\left(1 - \frac{1}{\gamma}\right)\hat{Q}_t \tag{2.46}$$

is satisfied for arbitrary realizations of the stochastic shocks driving the economy. It is important to stress that in this setup the static budget constraint (2.45) is not equivalent to the period-by-period budget constraint (2.24). The static budget constraint does not capture the period-by-period dynamics and thus we cannot deduce that if (2.45) holds, (2.24) holds as well. Coeurdacier and Gourinchas (2011) derive their results for a case where they are both equivalent. Although this is not the case here, we still rely on the static budget constraint as we find it useful to derive households' hedging motives from it, keeping in mind that there might be other factors influencing the solution to the dynamic model which we obtain by numerical procedures. Since our model is relatively complex, obtaining the portfolio intuition from the static budget constraint instead of deriving closed-form solutions of the portfolios simplifies our solution procedure. We proceed by substituting in the perfect risk-sharing condition under complete asset markets in the linearized budget constraint above. This yields

$$-\left(1-\frac{1}{\gamma}\right)\hat{Q}_t = S_W R_{w,t} + (2s-1)R_{e,t} + 2b' R_{b,t}, \tag{2.47}$$

with $R_{w,t} \equiv \left(\hat{w}_t^R + \hat{l}_t^R\right)$, $R_{e,t} \equiv \left[S_D\left(\hat{N}_{D,t}^R + \hat{\tilde{d}}_t^R\right) - S_I\left(\hat{N}_{E,t}^R + \hat{\tilde{v}}_t^{sR}\right)\right]$ and $R_{b,t} \equiv \hat{d}_t^{bR}$.

Portfolios

We postulate the following loadings of equities and bonds on the risk components in $\hat{Q}_t$:

$$\begin{aligned}
\hat{TOT}_t &= \phi_{TOT,b}R_{b,t} + \phi_{TOT,e}R_{e,t}\\
\hat{z}_{X,t}^R &= \phi_{ZX,b}R_{b,t} + \phi_{ZX,e}R_{e,t}\\
\hat{z}_{I,t}^R &= \phi_{ZI,b}R_{b,t} + \phi_{ZI,e}R_{e,t}\\
\hat{N}_{D,t}^R &= \phi_{ND,b}R_{b,t} + \phi_{ND,e}R_{e,t}\\
\hat{N}_{X,t}^R &= \phi_{NX,b}R_{b,t} + \phi_{NX,e}R_{e,t}\\
\hat{N}_{D,t}^R - \hat{N}_{X,t}^R &= \phi_{NDNX,b}R_{b,t} + \phi_{NDNX,e}R_{e,t}\\
\hat{N}_{I,t}^R - \hat{N}_{X,t}^R &= \phi_{NXNI,b}R_{b,t} + \phi_{NXNI,e}R_{e,t}.
\end{aligned}$$

Similarly, the loadings of the assets on labor income risk are

$$R_{w,t} = \phi_{w,b}R_{b,t} + \phi_{w,e}R_{e,t}.$$

Substituting in this set of relations into (2.47), we can solve for the optimal portfolio positions that replicate the complete market allocation. Thus, the portfolio allocations we find imply that (2.47) holds for all possible realizations of the relative shocks $\left(\hat{Z}_t, \hat{Z}_{E,t}\right)$. As in Coeurdacier and Gourinchas (2011) agents use bonds to hedge all risks deriving from the real exchange rate, while they use equity to hedge those risks orthogonal to the real exchange rate. Next we have to make some distinctions depending on the type of bonds that are traded in the model economy. If bonds perfectly load on the welfare-based real exchange rate and have returns $R_{b,t} = -\hat{Q}_t$ with dividends $d_t^b = d_t^* = 1$, from above we have

$$\hat{Q}_t = (2S_{DI}-1)\phi_{TOT,e} + S_{NI}\phi_{NDI,e} + (1-S_{DI})\phi_{NEX,e} + \psi(S_D-1)\phi_{IV,e} + \psi(S_I-1)\phi_{ID,e}. \tag{2.48}$$

Therefore, if bonds perfectly hedge all variations in the number of firms and their offered varieties, the optimal portfolios become those given in Coeurdacier and Gourinchas (2011)

$$s = \frac{1}{2}\left[1 - S_w \phi_{w,e}\right], \tag{2.49}$$

$$b' = \frac{1}{2}\left[\left(1 - \frac{1}{\gamma}\right) - S_w \phi_{w,b}\right]. \tag{2.50}$$

Home bias in equities arises only because the real exchange rate does not fully hedge against labor income risk. If the partial correlation between relative labor income and relative equity returns $\phi_{w,e}$ is negative it is optimal for home agents to hold a home biased share portfolio. This is a standard result in the literature (e.g.Heathcote and Perri (2013)). Note, that the structure of the model can influence the partial correlation $\phi_{w,e}$, as well as the steady state ratio $S_W = w/c$ and therefore, the home bias in a model with FDI can differ from that in a model without FDI if there is a systematic way in which FDI activity influences these two numbers. If bonds only hedge the CPI-based real exchange rate risk, i.e. $R_{b,t} = -\hat{q}_t$, the optimal portfolios become

$$s = \frac{1}{2}[1 - S_w \phi_{w,e} - \psi\left(1 - \frac{1}{\gamma}\right)((S_{DD} - S_{XD})\phi_{ND,e} \tag{2.51}$$

$$- (1 - S_{DD} - S_{XD})\phi_{NX,e} + S_{XD}\phi_{NDNX,e} - (1 - S_{DD} - S_{XD})\phi_{NINX,e})], \tag{2.52}$$

$$b' = \frac{1}{2}[\left(1 - \frac{1}{\gamma}\right) - S_w \phi_{w,b} - \psi\left(1 - \frac{1}{\gamma}\right)((S_{DD} - S_{XD})\phi_{ND,b} \tag{2.53}$$

$$- (1 - S_{DD} - S_{XD})\phi_{NX,b} + S_{XD}\phi_{NDNX,b} - (1 - S_{DD} - S_{XD})\phi_{NINX,b})]. \tag{2.54}$$

Portfolios without FDI from Hamano (2015) are given by

$$s = \frac{1}{2}\left[1 - S_w \phi_{w,e} - \psi\left(1 - \frac{1}{\gamma}\right)((2S_{ED} - 1)\phi_{ND,e} + (1 - S_{ED})\phi_{NDNX,e})\right], \tag{2.55}$$

$$b' = \frac{1}{2}\left[\left(1 - \frac{1}{\gamma}\right) - S_w \phi_{w,b} - \psi\left(1 - \frac{1}{\gamma}\right)((2S_{ED} - 1)\phi_{ND,b} + (1 - S_{ED})\phi_{NDNX,b})\right] \tag{2.56}$$

where $S_{ED} = \alpha \rho_H^{1-\omega}$ and α is the home bias in production in Hamano's model, which is $1 - \alpha_2$ in our model. An intuition is that the firms on average become less prone to the country specific shock if some of them engage in FDI. Therefore, the correlation could decrease contributing to a larger home bias in equity holdings.

2.4 Solution procedure and parametrization

Ghironi and Melitz (2005) show that in models in which heterogeneous firms are characterized in the way we do it here, despite the very rich heterogeneity, macroeconomic fluctuations only depend on the average characteristics of the average domestic producer, the average exporter and the average FDI firm and the fluctuations in the cutoff levels. Therefore the model with heterogeneous firms is observationally equivalent to an economy with a representative firm that has a domestic branch, an export branch and an FDI branch and is adjusting the relative sizes of these branches depending on the shocks to the economy. This feature is very convenient since aggregation becomes easy and we are able to solve this heterogeneous agent model by perturbation methods. Regular DSGE models are usually log-linearized around their non-stochastic steady-state and solved using an appropriate solution method (e.g. the method of undetermined coefficients, etc.). In models that include a country portfolio choice involving the possibility for the agents to invest in multiple assets, as in the model presented here, this approach becomes unfeasible. This is the case because up to a first-order approximation, the portfolio is indeterminate. In the non-stochastic steady-state, there is no uncertainty and portfolios with the same return do not pin down a unique portfolio choice. The same is true for a first-order approximation in which certainty equivalence holds and assets have to have the same expected payoff. The method developed by Devereux and Sutherland (2010, 2011) allows pinning down the steady-state portfolio choice uniquely by approximating the portfolio part of the model to the second order, while the rest of the model is approximated to the first order. Furthermore, the first-order dynamics of the portfolio choice can be pinned down using a third-order approximation of the portfolio part and a second-order approximation of the rest of the model. The parametrization of the model follows largely from Hamano (2015) and Contessi (2010) for the parts regarding foreign direct investment. The parameter governing relative risk aversion γ is set to 2. We interpret a period as a year and therefore set the discount factor β equal to 0.96. As is standard in this literature, the lower bound of the Pareto distribution determining the individual firm productivities is set to 1 and the time endowment of workers is also set to 1. The Frisch elasticity of labor supply is $\varphi = 2$ and χ is calibrated such that in steady state the households' labor supplies are equal to 1. Furthermore, we assume $k+1 > \sigma$ to ensure that the scaling factor $\triangledown = k/[k-(\sigma-1)]$ is positive. The shape parameter of the Pareto distribution k determines the dispersion of individual firm productivities. The elasticity of substitution among varieties σ is set to 3.8 and the elasticity of substitution between Home and Foreign goods ω is set to 2. This induces firms to charge an average price mark-up of 35.7 percent above the average marginal costs. In the presence of fixed entry costs to production this is not as high as it might seem. Ghironi and Melitz (2005) calibrate the exit shock δ to match the ratio of job destruction in the US, which is 10 percent. This gives them a value of $\delta = 0.025$. This parameter governs the ratio of entering to existing varieties $N_{E,t}/N_{D,t}$ in our model. The fixed entry cost to production is normalized to be $f_{E,t} = 1$. The fixed costs of international activity $f_{X,t}$ and $f_{I,t}$ are set relative to $f_{E,t}$. Following Contessi (2010) the share of FDI firms in the total number of firms is calibrated using the fixed entry cost to FDI activity $f_{I,t}$. Like him, we set $f_{I,}$ to 28 percent of $\Theta f_{E,t} = 0.0036$, which is the fixed cost of entering production of a new variety annualized. The fixed cost of exporting is set to about 10 percent of this annualized entry cost. Following Hamano (2015) we choose a preerence for variety parameter $\psi = 0.18$. Like Hamano (2015) we take the shock processes from Coeurdacier et al. (2010). For annual data from 1984 to 2004, they estimate productivity processes for the G7 countries. The AR(1) process for our 4 shocks in matrix notation is

$$Z_{t+1} = \Gamma Z_t + \epsilon_{t+1} \tag{2.57}$$

where $Z_t = [log(Z_t)\ log(Z_t^*)\ log(Z_{E,t})\ log(Z_{E,t}^*)]$ and $\epsilon_t = [\epsilon_t\ \epsilon_t^*\ \epsilon_{E,t}\ \ \epsilon_{E,t}^*]$. The matrix Γ and the variance-covariance matrix Σ of the innovations ϵ_t are assumed to be

$$\Gamma = \begin{bmatrix} 0.75 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 \\ 0 & 0 & 0.79 & 0 \\ 0 & 0 & 0 & 0.79 \end{bmatrix}$$

and

$$\Sigma = \begin{bmatrix} 0.0096 & 0.0043 & 0 & 0 \\ 0.0043 & 0.0096 & 0 & 0 \\ 0 & 0 & 0.0199 & 0.0038 \\ 0 & 0 & 0.0038 & 0.0199 \end{bmatrix}.$$ [8]

2.5 Optimal portfolios with different internationalization strategies

The results we obtain by applying the Devereux and Sutherland solution algorithm for our baseline calibration are given in Table 2.2.

Table 2.2 – Optimal Portfolios

	Welfare RER		Empirical RER	
	Without FDI	With FDI	Without FDI	With FDI
Equity	1.18	1.69	1.34	2.79
Bonds	0.74	0.15	0.45	0.05

The first two columns show the optimal equity and bond portfolios for our baseline model with bonds indexed by the welfare-based CPI, while the third and fourth column show the optimal portfolios in an economy with bonds indexed by the empirical CPI. The very first column shows the optimal bond and equity portfolio holdings of the Home household in a model in which firms can only tap international markets by exporting. The model produces a significant home bias in domestic equity holdings for all specifications. In fact, home households short foreign equity to take excessive long positions in domestic equity. With welfare-based bonds available, Home households optimally hold 118 percent of the shares of Home firms, while at the same time, they hold −18 percent of the shares of foreign firms, i.e. they increase their exposure to domestic equity by short-selling foreign equity. At the same time the Foreign household does the opposite trade and shorts home equity. Thus, this is a sustainable equilibrium outcome. The bond position is positive such that in order to hedge the real exchange rate risk, Home households save in domestic bonds and borrow abroad. Qualitatively, the same is the case with bonds based on the empirical CPI in column 3 which shows that the optimal equity bias increases to

[8] These variances and covariances are the same in relative magnitude as in Hamano (2015), but smaller in absolute magnitude to ensure sensible simulation results, i.e. that for instance consumption is not negative after shocks. Since asset markets are complete, this does not affect the portfolio solutions.

134 percent and bond positions decrease as the variety risk component drops out of the empirical real exchange rate and households have to hedge additional risk.[9] In our model with FDI, the optimal share Home households hold in the equity issued by domestic firms amounts to 169 percent and is thus higher than in the comparable benchmark model without FDI. Equity positions again increase with CPI-indexed bonds compared to welfare-indexed bonds and bond positions. Moreover, as before the Home household is more exposed to foreign bonds when these are CPI-based bonds. To explore possible sources of the high home bias we obtain, we take a closer look at the conditional correlations that enter the optimal portfolio choices that we derived from the static budget constraint. We calculate the conditional correlations using simulated data from our model. For this purpose, we simulate the model 10000 times for 500 periods, dropping the first 100 observations to ensure that the results are not sensible to the used initial conditions. The results are presented in Table 2.3.

Table 2.3 – Conditional correlations - Baseline Model

	$\phi_{w,\cdot}$	$\phi_{ND,\cdot}$	$\phi_{NX,\cdot}$	$\phi_{NDNX,\cdot}$	$\phi_{NINX,\cdot}$
FDI					
Equity	-0.5201	-0.0997	-0.0403	-0.0546	0.0667
Bonds	-0.5142	0.0405	-0.4251	0.3056	0.5042
No FDI					
Equity	-0.4822	-0.1445	-	-0.2463	-
Bonds	-0.5925	0.0068	-	0.0897	-

The entries in the above table give correlations of the relative wages, as well as the variety effects given in the portfolio solutions (2.51) with the relative return of one asset, conditional on the relative return of the other asset. The definition of the conditional returns is given by

$$\phi_{x,y} = \rho_{xy|z} = \frac{\rho_{xy} - \rho_{xz}\rho zy}{\sqrt{1-\rho_{xz}^2}\sqrt{1-\rho_{zy}^2}}. \quad (2.58)$$

For our baseline model with FDI x represents one of the relative differences $R_{w,t}$, $\hat{N}^R_{D,t}$, $\hat{N}^R_{X,t}$, $\hat{N}^R_{D,t} - \hat{N}^R_{X,t}$ and $\hat{N}^R_{I,t} - \hat{N}^R_{X,t}$. y represents $R_{e,t}$ or $R_{b,t}$, while z represents the other asset return, respectively. So for instance, $\phi_{w,e}$ represents the correlation between relative labor income and relative equity returns, conditional on relative bond returns. For the model without FDI, which is analogous to the model of Hamano (2015), x represents one of the relative differences $R_{w,t}$, $\hat{N}^R_{D,t}$, and $\hat{N}^R_{D,t} - \hat{N}^R_{X,t}$. Our portfolio equations in a model with FDI differ from the ones obtained by Hamano (2015) in a model without FDI in that they load on additional risk components and furthermore the weights given to these components. Therefore, they are only comparable to a limited degree. In the case where welfare-based bonds can be traded, the equity portfolios in both models only depend on the conditional

[9]Intuitively, there is risk related with the number of foreign varieties that are available in future periods, more varieties for instance imply higher consumption spending on these varieties in future periods. Since foreign goods are denominated in foreign currency exposure to foreign bonds reduces consumption spending risk in foreign currency and therefore hedges against fluctuations in the real exchange rate.

correlation between wages and equity returns, given the returns on bonds, $\phi_{w,e}$. As in Hamano (2015), the negative value of -0.48 is the main source of the high equity home bias in the model without FDI. The first column of table2.3 shows that the presence of FDI further lowers this correlation contributing to higher optimal equity holdings. For the case of CPI-based bonds, two sources of risk that equity positions aim to hedge against are qualitatively the same in both models, the number of domestic firms and the relative number of domestic firms and exporters. In both models the conditional correlations on these components are negative, contributing to home biased portfolios.[10] In addition, the optimal equity positions with FDI load on two additional terms, the number of exporters and the relative difference in the number of FDI firms and exporters. Since all the coefficients on the correlations are positive, a negative conditional correlation on these terms would suggest less home biased portfolios. The table shows that the conditional correlation on the number of exporters is negative, decreasing the optimal equity position, while the conditional correlation on the relative number of FDI firms and exporters is positive contributing to higher home equity position of the Home household. As both of these terms enter our portfolio solution with identical coefficients and the second correlation is higher than the first one, overall the presence of these additional risk channels contributes to higher home equity positions compared to the absence of these channels. From the above portfolio intuition we thus expect that the the home equity positions in our dynamic model are higher in a model of FDI compared to a model where FDI is not present and also higher in a model where CPI-based bonds are traded compared to one in which bonds are welfare based. This is consistent with the portfolio solution we obtain with our solution algorithm. Looking at the differences in bond portfolios, we see that the conditional correlation $\phi_{IV,b}$ and $\phi_{IV,b}$ are positive. This is likely to be the reason why agents hold less bonds in the economy with CPI based bonds. A shortfall of the model is the absolute magnitude of the portfolios we find and the fact that in our model the Home household wants to short foreign equity to gain additional exposure to domestic equity. Given that Hamano (2015) in a model without FDI also finds optimal Home equity shares above one and our findings for the baseline case without FDI are very similar to those in Hamano (2015), this does not come as a surprise. Our aim in this paper is to show that the presence of FDI raises the optimal equity home bias in a DSGE model with endogenous selection into internationalization modes of production, as is suggested in the empirical work by (Cai and Warnock, 2012). To make the theoretical model match the empirical data to a realistic extend the modeling framework has to be refined. Nonetheless, the direction in which portfolios move in our model in the presence of FDI is a valuable first step in this direction.

2.6 Conclusion

In this paper, we investigate two questions: How does the degree of internationalization of a country's firms influence the equity investment choices of its households, and how does it affect the widely observed equity home bias? Our rational is that in

[10]Since the coefficients on these terms are not directly comparable across models, we only compare them qualitatively not in terms of their magnitude.

addition to being prone to the economic conditions of their home country, internationally active firms also depend on the economic conditions of other countries they operate in. Thus, the shares of multinational firms provide a higher diversification benefit to investors than investing in a firm that is operating solely nationally. In a DSGE model that includes the endogenous choice of firms to become internationally active through either exports or foreign direct investment (FDI), we find that indeed the optimal equity holdings of agents are more strongly biased towards domestic firms than in a model with trade only. Our finding indicates that international diversification is not as bad as empirical measures of the equity home bias suggest and that the international activity of firms should be taken into account when calculating empirical measures of the equity home bias.

Chapter 3

Sovereign Bailouts: Why defaults are possible in a union after all

3.1 Introduction

Recently, the debate about the ongoing financial assistance measures for the European Monetary Union (EMU) countries which are experiencing troubles with their public finances in the European debt crisis has gained a new aspect. A report of the European Central Bank (HFCN (2013)) on household finances in the Eurosystem revealed that households in Germany have the lowest median wealth in the EMU, while at the same time households in Greece, Spain and Cyprus, have a median wealth that is well above the one in Germany. Therefore, the median household in the EMU member country carrying the largest burden from the financial assistance programmes provided by the EU and the ECB, has significantly less wealth than the median households in those countries that are receiving financial assistance or bailouts from the rest of the union (see Table C.1 in the appendix of this chapter). Although, the median alone is only a very restrictive measure of the households' wealth distribution in the EMU and it has been criticized that the survey only accounts for disposable wealth, some observers showed concerns about the effect that wealth inequality within and across EMU member countries might have on future financial assistance and the EU crisis policy. For example, the Financial Times wrote 'Poor Germans tire of bailing out eurozone'. [1] Thus, a concern is the effect of wealth inequality within Germany on its will to provide further financial assistance to troubled EMU member states. In this paper I show that wealth inequality in the bailout providing nation can indeed have a significant influence on the outcome of a bailout negotiation with a defaulting member state. In a multi-country model of sovereign default with bailout negotiations, the introduction of dispersely distributed wealth and the consequent dispersed holdings of government debt in the country providing a bailout induce the possibility that this nation refuses a bailout and the struggling member nation is forced to declare an outright default. An outcome that is not possible if one investigates a representative agent model.

[1] http://www.ft.com/intl/cms/s/0/2f89e5ee-930a-11e2-9593-00144feabdc0.html#axzz2U6ktrTCj

3.2 Literature Review

Structural empirical research on sovereign default started with Arellano (2008), who investigates endogenous external default in an incomplete market setting. She models a small open economy (given by a representative benevolent government) that trades obligations with risk-neutral international investors. In equilibrium, the bond price reflects the default probability of the country. Given the representative agent framework, distributional issues within the country or outside do not play any role. Countries that default are excluded from financial markets for a random number of periods and suffer a direct output cost. Default is only triggered by low realizations of the stochastic income of the economy. The distributional incentives of sovereign default within one country have been investigated by D'Erasmo and Mendoza (2016). They look at a single economy and argue that disperse holdings of the government's debt within the economy can trigger optimal default. Given an utilitarian social welfare function, the government optimally defaults if the repayment costs through lump-sum taxation on low income households that do not hold a large amount of government debt, is higher than the default costs encountered by high wealth households that hold a lot of obligations. Thus, the government optimally defaults if the debt holdings are very concentrated at high wealth households. Guembel and Sussman (2009) show that default can also be optimal when there are no default costs. They set up a two-period model with heterogeneous domestic households (three types), where debt can also be held by foreign agents. They model the default decision explicitly as a political process in which there is an election that decides whether the country defaults in the second period or not. The household taking this decision is the median voter. They find that if the debt is held by domestic and foreign agents, instead of exclusive foreign holdings, the median voter has incentives to repay even if there is no cost of default. Cooper (2012) builds a simple model of a monetary union in which a given member region can issue debt obligations that can be held by the representative agents in all the regions of the union. In the repayment period a stochastic income shock realizes for the debt issuing country. Given this realization and the fraction of debt held by domestic agents (distribution of debt across countries) the government decides whether to tax the domestic agents and to repay its debt or to default and to pay the default cost. Investigating the second period default decision the paper analyzes the role of fundamental as well as strategic uncertainty. Because the default threshold depends on the interest rate the country has to pay on bonds, the higher the probability agents attach to a default and the higher interest they demand, the more likely a default becomes. Moreover, Cooper (2012) uses his model to investigate debt guarantee and bailout policies in the monetary union. Because in his model the two regions are populated by representative agents, Cooper (2012) has to assume that there exists a planner who seeks to maximize the overall union welfare, in order to create an incentive for bailouts and incentives to hold regional debt for all agents in the union. If in his model the governments only cared about the welfare of their own citizens, the debt issuing region could default in all states of its economy and the Foreign government would always be willing to provide a bailout up to the fraction of debt that is held by the citizens of the other regions. If agents in the region providing a bailout anticipate that in the worst case their bond holdings will be taken of them by the government to finance a bailout for the debt issuing region, they will not hold debt of that region in equilibrium. In this paper, I combine some of the features of the above models. The basic two-period model structure is similar to the one used in Cooper (2012), but I investigate how his results change if the bailout providing nation is populated by heterogeneous household investors, similar to D'Erasmo and Mendoza (2016). Moreover, the countries in my model have a fundamental default incentive as in Arellano (2008) and government debt can be held in both countries such as in Guembel and Sussman (2009). This is the first paper which emphasizes that bailout incentives can depend on the distribution of bond holdings across heterogeneous household investors in the bailout providing nation and that household heterogeneity induces the possibility that a sovereign bailout to a struggling member country might be refused by other member countries.

3.3 The model

The model consists of two countries, Home and Foreign. Both countries are populated by a continuum of households. Each country has a government with a utilitarian social welfare function that only takes into account the welfare of households in their own country. The economy lasts for two periods $t = 0, 1$. The focus of this study is the effect of income inequality among households on bailout incentives between the two economies. I assume that the Home country can issue government debt to smooth a stochastic government spending shock, while the Foreign country's income stream is exogenously given. The crucial assumption is that households in Foreign are distinct in high initial income households and low initial income households and both can hold Home government debt to smooth their consumption. At the same time, households in Home are of one type only.

3.3.1 Households

There is a continuum of infinitesimally small households in Home and Foreign. Their mass is fixed to 1 in each country. Households obtain utility from consumption. Their discounted stream of utility from consumption in this two period model is

$$u(c_0) + \beta E_0 \left[u(c_1)\right] \tag{3.1}$$

where β is the discount factor and the utility function is of the standard CRRA type, given by

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}. \tag{3.2}$$

All households are endowed with an initial stock of income, which is the same for all Home households and differs between the two types of Foreign households. Furthermore, in both periods households in Home and Foreign receive an exogenous income (y_0, y_1) and (y_0^*, y_1^*), respectively.

Home households

Home households are identical. They are endowed with an initial income stream of y_0 in period $t = 0$[2] and y_1 in period $t = 1$. In addition, the Home household is assumed to have some initial bond holdings b_0. They have to pay a per capita lump-sum tax T_0 depending on how the Home government decides to finance first period government spending g_0 and how it chooses to smooth taxation over time. Note that for $T_0 < 0$ the Home household receives a transfer. Thus, the Home household's first period income is $y_0 + b_0 - T_0$, which it can spend on consumption c_0 or on newly issued government bonds b_1 at price q_0. This information is summarized in the Home household's budget constraint for the first period as follows

$$c_0 + q_0 b_1 \leq y_0 + b_0 - T_0. \tag{3.3}$$

The second period budget constraint is similar. Once the stochastic second period government spending shock g_1 has realized, the Home household takes the resulting taxation and repayment decision of the Home government as given and consumes what is left of the bond proceeds b_1 and its $t = 1$ period income y_1 after taxation T_1. A general form of the $t = 1$ budget constraint is

$$c_1 \leq y_1 + b_1 - T_1, \tag{3.4}$$

where the actual amounts of b_1 and T_1 depend on the government's decision to service its government debt, default or accept a bailout from Foreign. This is described in detail in the next section 3.3.2.

Foreign households

I assume that Foreign households do not face any uncertainty with respect to their own economy. To smooth their consumption stream they can invest in Home government bonds and are therefore prone to Home risk. The Foreign government is assumed to issue no government debt, it is only involved in bailout negotiations. An important assumption is that there are two types of Foreign households. The low income type starts period $t = 0$ with initial income y_0^{L*}, while the high income type starts with y_0^{H*}, where I assume that $y_0^{L*} < y_0^{H*}$. The fraction of low income households in Foreign is γ and the fraction of high income households in Foreign is $1 - \gamma$. The Foreign households' $t = 0$ budget constraints are

$$c_{0,i}^* + q_0 b_{1,i}^* \leq y_0^{i*} + b_{0,i}^* - T_0^* \tag{3.5}$$

for $i = L, H$. The Foreign government also finances government spending by levying lump-sum taxes on Foreign households and in this respect it cannot discriminate between high and low wealth households. Given the realization of the Home government spending shock and the resulting taxation decisions of the Foreign government, the $t = 1$ budget constraint of both Foreign household types is

$$c_{1,i}^* \leq y_1^* + b_1^{i*} - T_1^* \tag{3.6}$$

[2]In this two period model the initial income y_0 can also be seen as initial wealth. That is why I sometimes use the terms wealth and income, as well as wealth inequality and income inequality interchangeably in the model description. In general, wealth is a stock accumulated over time and income is the corresponding flow at a given point in time. Thus, in general these concepts cannot be used interchangeably.

for $i = L, H$, where again the actual amounts of b_1^{i*} and T_1^* depend on the government's decision to service its government debt, default or accept a bailout from Foreign. The next section will describe this in detail.

3.3.2 Government

The governments in both countries seek to maximize the welfare of the households within their own country. They do not take into account the welfare of households in the other country. I assume that governments maximize the welfare of all the types of households in their country in a utilitarian fashion, i.e. the utility that each individual household obtains from consumption is taken into consideration with equal weights. Thus, the objective of the Home government for each period is given by

$$u(c_0) + \beta u(c_1) \tag{3.7}$$

where c_0 and c_1 are the period $t = 0$ and $t = 1$ consumption levels of the Home representative household and $u(\cdot)$ is the instantaneous utility function which is assumed to be strictly increasing $u'(\cdot) > 0$ and strictly concave $u''(\cdot) < 0$. The objective of the Foreign government takes into account the utility of both low and high income households and is thus given by

$$\gamma u(c_0^{L*} + \beta u(c_1^{L*})) + (1 - \gamma)u(c_0^{H*} + \beta u(c_1^{H*})) \tag{3.8}$$

with foreign consumption levels for the low income household c_0^{L*}, c_1^{L*} and the high income household c_0^{H*}, c_1^{H*}, respectively. The fraction of low wealth agents in Foreign is γ and the instantaneous utility function $u(\cdot)$ is of the same form as the one for the Home household. For reasons of tractability, I assume that only the government of the Home country can issue a total amount of debt B_1 in the first period. This assumption makes us focus on the decision process concerning one country. The model can be extended to multiple debt issuing countries. Furthermore, the Home government is subject to government spending shocks in the second period and can impose a lump-sum tax on the Home household. In period $t = 0$ the government can finance the difference between tax income and government spending by issuing debt. In period $t = 1$ it has to run a balanced budget, thus it finances the spending shock by taxation and can decide to repay its debt or default upon the debt. Since, I assume a two period model, there is a fixed cost of defaulting to ensure some incentive to repay. The Home debt obligations can be bought and held by households in both countries. The revenues from issuing bonds are distributed lump-sum to the Home households in the first period. The motive for the government to issue debt is to smooth the consumption of the Home household. Therefore, Home's government bonds are the only means of savings that households in both countries have. The purpose of this paper is to show that international bond holdings create default incentives for the Home government and also incentives for the Foreign government to provide a bailout to Home. The second incentive is however limited by the dispersion of bond holdings within Foreign: If bond holdings are very concentrated at a small number of high income households, a default of the Home government leads to a more favorable consumption redistribution within Foreign than bailing out the Home government. The Foreign government will thus neglect a bailout to the Home government if income and bond holdings are very unequally distributed in Foreign. A crucial assumption for this results to hold is that the government can neither discriminate domestic and foreign households when it issues bonds nor when it repays. The repayment of the government bonds in the second period of the economy is not enforceable. Hence, the government chooses whether to repay its obligations or to default on them. If the state of the economy in the second period is such that it is welfare maximizing for the government to honor its obligations, it repays its creditors by taxing the second period income of the Home agents with a proportional tax. Otherwise, the Home government can default on its obligations. The assumption that the government cannot discriminate lenders rules out selective default. In the case that the Home government finds default optimal, the Foreign government can intervene and offer a bailout to the Home government, i.e. to pay a fraction of its outstanding debt in order to make them honor their remaining obligations. The Home government can accept this offer or decline it. If it declines, Home defaults and all domestic agents incur a default cost c_{def} proportional to their second period income. The Home and Foreign government budget constraints in period $t = 0$ are

$$T_0 = g_0 - qB_1 \tag{3.9}$$

and

$$T_0^* = g_0^*, \tag{3.10}$$

respectively. The Home government budget constraint in period $t = 1$ in case of repayment is

$$T_1 = g_1 + B_1. \tag{3.11}$$

In case of default it is

$$T_1 = \lambda y_1 \tag{3.12}$$

where it is assumed that the government defaults on the stock of debt B_1 the government spending shock g_1 and λy_1 is the output cost of default, which is proportional to the Home household's $t = 1$ income. If Home announces a default, but Foreign is willing to provide an acceptable bailout for Home, Home's government budget constraint is

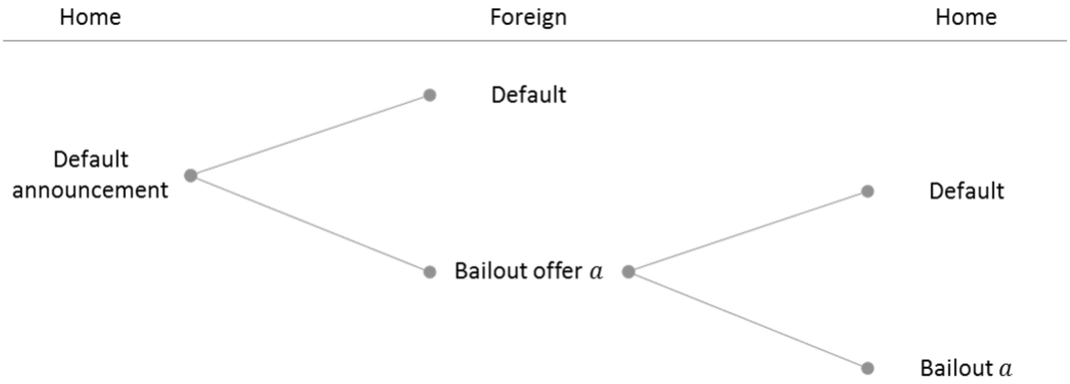
$$T_1 = g_1 + (1 - a)B \tag{3.13}$$

where a is the fraction of Home's debt the Foreign government is willing to provide as a bailout.

Bailout negotiations

The timing of the second period is described above. What is left to be specified is the exact negotiation between the two countries given a bailout decision. Figure 3.1 shows the assumed timing of the bailout negotiation. First, the Home government has to decide whether to repay or to default. Thereafter, the Foreign government decides whether to provide a bailout or not. The bailout offer made has to be accepted by the Home government, i.e. the Home government has to be at least as well off as under default.

Figure 3.1 – Timing of the bailout negotiation



3.3.3 Timing

The state of the economy in the second period is determined by the savings decisions the agents take in the first period and by the realizations of the second period government spending shock. In the first period agents make their savings decisions building expectations on the state of the economy in the second period and anticipating the outcome of the repayment and bailout decisions. The timing with which decisions are taken in the model is given by:

- Period 0:
  - Agents get to know the size of the exogenous government spending in Home ($t = 0$) and Foreign ($t = 0$ and $t = 1$)
  - Home government issues bonds and finances the remainder of the government spending shock by taxing households in a lump-sum fashion, while Foreign government finances g_0^* fully through taxation
  - Households decide on how much they want to save in government bonds, taking the prices and transfers as given
  - In equilibrium prices adjust such that the bond market is cleared
  - Agents consume
- Period 1:
  - The stochastic period $t = 1$ government spending shock realizes for Home
  - Given the distribution of bond holdings in the economy, the government decides to tax the agents with a proportional income tax and repay its debt or to default in the second period, taking into account that a fraction of aggregate income is lost
  - The Foreign government finances g_1^* through taxation
  - Given these decisions, the other member country can start a bailout negotiation
- Bailout negotiations

– In the subgame of bailout negotiations the Foreign country decides to intervene if the Home country prefers default to repayment. In this case both governments bargain over a fraction a of the home debt stock that the Foreign government is willing to pay in order to make the Home country pay its obligations. The bailout is financed by proportionally taxing the Foreign households' second period income.

3.4 Equilibrium

3.4.1 Households' saving problem

In the first period the different household types in Home and Foreign decide on their savings in Home government bonds. Because there is a continuum of households in both countries, the individual household is infinitesimally small and does not take into account its own influence on equilibrium market prices. Nonetheless, households form expectations about the state of the economy in the second period and the resulting default and bailout decisions the governments take. In equilibrium the expectations of all household types have to correspond to the actual state in the second period. The objective of all the household types in Home and Foreign is to smooth their consumption over the two periods of the economy, taking into account their income and wealth endowments in the first and second period, as well as the Home government's bond issuance decision and the consequences of its incentive to default. Thus, households decide on how much of their first period endowments to save in government bonds given their expectations about the state of the economy in the second period and the resulting repayment, bailout or default decision.

Home household

In period $t = 0$ the Home household chooses how much to save in Home government bonds, taking as given the total debt the Home government issues, B_1, the price of the debt q_0, its first period endowments and initial debt holding y_0 and b_0. It chooses its bond holdings into period $t = 1$, b_1

$$\begin{aligned} v(B_1, g_1, \gamma) = max_{b_1}\{u(y_0 + b_0 - q_0 b - T_0) + \beta E_{g_1}[R(B_1, g_1, \gamma)u(y_1 + b - T_1^R(g_1)) \\ + B(B_1, g_1, \gamma)u(y_1 - T_1^B(g_1)) + D(B_1, g_1, \gamma)u((1-\lambda)y_1 - T_1^D(g_1))]\} \end{aligned}$$

subject to the constraint that $b_1 \geq 0$. Here, R, B and D are indicator variables that take on the value 1 if the outcome of the bailout negotiation subgame is repayment, bailout or default, respectively, given the realized state of the economy in the second period (B_1, g_1, γ).

Foreign household

Foreign households have a similar optimization problem given by

$$v^*(B_1, g_1, \gamma) = max_{b_1^{i*}}\{u(y_0^* + b_0^{i*} - q_0 b_1^{i*} - T_0^*) + \beta E_{g_1}[R(B_1, g_1, \gamma)u(y_1^* + b_1^{i*} - T_1^{*R}(g_1)) \\ + B(B_1, g_1, \gamma)u(y_1^* - T_1^{*B}(g_1)) + D(B_1, g_1, \gamma)u(y_1^* - T_1^{*D}(g_1))]\}.$$

Market Clearing

Bond market clearing in each period is ensured if

$$B_t = b_t + \gamma b_t^{L*} + (1-\gamma)b_t^{H*} \tag{3.14}$$

for $t = 0, 1$ holds.

3.4.2 Government

Home government's default decision

In $t = 1$ the Home government chooses to default if it maximizes its social welfare function

$$max_{d\in(0,1)} \left[W_1^{d=0}(B_1), W_1^{d=1}\right]. \tag{3.15}$$

Substituting the government budget constraints, the welfare levels under repayment and default are

$$W_1^{d=0}(B_1, g_1) = u(y_1 - g_1 + b_1 - B_1) \tag{3.16}$$

and

$$W_1^{d=1}(g_1) = u((1-\lambda)y_1), \tag{3.17}$$

respectively. A difference of my model to the one of D'Erasmo and Mendoza (2016) is that I assume the consumption level in default not to depend on the second period government shock. Thus, if Home defaults it defaults on the debt stock and expenditure cost. At the same time I assume that the proportion of output lost in default is as high as 70 percent. The reason to make the welfare under default independent of the government spending shock is to ensure that there is only default at significantly high levels of government debt instead of defaults being optimal for large spending shocks, even with low levels of government debt stock.

Foreign government's bailout decision

If Home government chooses to default, the Foreign government will choose to make a given bailout offer a according to

$$max_{B\in(0,1)} \left[W_1^{*,B=0}(g_1), W_1^{*,B=1}(a, B_1, g_1)\right] \tag{3.18}$$

where the welfare values under bailout and non-bailout for the Foreign government are given by

$$W_1^{*,B=0}(g_1) = \gamma\left(u(y_1^{L*} - g_1^*)\right) + (1-\gamma)\left(u(y_1^{H*} - g_1^*)\right) \tag{3.19}$$

and

$$W_1^{*,B=1}(g_1) = \gamma \left(u(y_1^{L*} - g_1^* + b_1^{L*} - aB_1) \right) + (1-\gamma) \left(u(y_1^{H*} - g_1^* + b_1^{H*} - aB_1) \right). \tag{3.20}$$

Home's acceptance decision

Home's government will accept the Foreign government's bailout offer if

$$W_1^{B=1}(B_1, a) \geq W_1^{d=1}(g_1). \tag{3.21}$$

The welfare values of accepting and not accepting are given by

$$W_1^{B=1}(B_1, a) = u(y_1 - g_1 + b_1 - (1-a)B_1) \tag{3.22}$$

and

$$W_1^{d=1}(g_1) = u((1-\lambda)y_1). \tag{3.23}$$

As this is the last stage, the Foreign government will anticipate the Home government's acceptance decision and optimally provide the bailout such that the Home government is indifferent between accepting and not accepting. Thus, the optimal bailout size $\bar{a}$ will be such that

$$W_1^{B=1}(B_1, \bar{a}) = W_1^{d=1}(g_1). \tag{3.24}$$

3.4.3 Home government's debt issuance decision

In period $t = 0$, the Home government decides how much debt to issue taking into account the discounted welfare of the Home household only

$$max_{B_1} E_{g_1} \left[v(B_1, g_1, \gamma) \right]. \tag{3.25}$$

3.4.4 Equilibrium Definition

Definition: An equilibrium in the two period economy is given by bond demands of all the household types, bond prices and the second period decisions such that, given the total amount of debt issued B_1, and given the fractions of low wealth households in Foreign γ,

1. All household types choose their optimal bond holdings according to their maximization problems given their expectations about the bond holdings of the other household types, the bond price and the governments' second period decisions given their expectations,

2. the equilibrium price function clears the bond market in the first period, such that $b_1^L + b_1^H + b_1^{L*} + b_1^{H*} = B_1$,

3. the agents' expectations are consistent with the bond holdings of the other types of households and thus agents correctly anticipate the optimal repayment, bailout and default decisions of the two governments in the second period.

Following D'Erasmo and Mendoza (2016) I explicitly concentrate on equilibria in which the low wealth foreign household does not save in Home government bonds. This condition is checked for all equilibria solved for numerically. The bond market of the initial period is assumed to be cleared by definition of the initial bond holdings.

3.5 Intuition: A Bailout Model without heterogeneity

After having introduced the general model setup, I proceed by showing that heterogeneity within the bailout providing country is the crucial ingredient to arrive at the conclusions highlighted in this work. Therefore, I solve the second period bailout negotiation subgame in a world where Foreign households, as well as Home households are homogeneous. In this world economy I show that the Foreign government will always prefer to provide a bailout to an outright default of the Home government. This setup can be regarded as the special case of the general setup in which $\gamma = 1$. I will solve for the equilibrium of the economy by backward induction, starting with the bailout negotiations.

3.5.1 Backward induction: Home's acceptance decision

Taking as given that Home has announced to default in the second period given the state of the economy that has materialized (g_1, B_1) and Foreign has made a bailout offer $a \in [0,1]$, it is Home's choice to accept this offer. Its value from accepting the offer is

$$V_H^{acc} = u(y_1 - g_1 + b_1 - (1-a)B_1) \tag{3.26}$$

where y_1 is the Home agents period $t = 1$ income, b_1 is its bond holdings carried over from the first period and $(1-a)B_1$ are the taxes the government levies on the Home household in order to repay the outstanding debt obligations after a bailout of size aB. The Home government's value of declining the offer is

$$V_H^{dec} = u((1-\lambda)y_1). \tag{3.27}$$

where λ is the fraction of second period output that is lost in default. The Home government would only accept a bailout offer for which

$$V_H^{acc} \geq V_H^{dec}. \tag{3.28}$$

Therefore,

$$y_1 - g_1 + b_1 - (1-a)B_1 \geq (1-\lambda)y_1 \tag{3.29}$$

and

$$a \geq 1 + \frac{g_1 - b_1 - \lambda y_1}{B_1}. \tag{3.30}$$

This threshold decreases in the default costs, the Home bond holdings and also in the total debt stock B_1. It increases in the $t = 1$ government spending shock g_1. If the Home agent does not hold any of its own government's debt obligations and the default costs are zero, the Home government is not willing to pay a positive fraction of its debt and $a \geq 1 + \frac{g_1}{B_1} > 1$. On the other hand, if the Home agent holds all of the Home debt obligations, the threshold is $a \geq \frac{g_1 - \lambda y_1}{B_1}$ and can become negative if $g_1 > \lambda y_1$. In this case the government would repay in the first place. The level at which the Home government is indifferent between being bailed out by the Foreign government and defaulting is defined as

$$\bar{a} = 1 + \frac{g_1 - b_1 - \lambda y_1}{B_1}. \tag{3.31}$$

3.5.2 Backward induction: Foreign's bailout decision

The Foreign government is willing to provide a bailout to the Home government if its value from providing the bailout is higher than its value from not providing the bailout, i.e. if

$$V_F^{bailout} \geq V_F^{default} \tag{3.32}$$

$$u(y_1^* - g_1^* + b_1^* - aB_1) \geq u(y_1^* - g_1^*) \tag{3.33}$$

$$b_1^* \geq aB_1. \tag{3.34}$$

Therefore, the maximum bailout a_{max} Foreign is willing to provide is

$$a_{max} = \frac{b_1^*}{B_1} \tag{3.35}$$

which is the fraction of total debt held by agents in the Foreign country. This is already a key finding in this simplified version of the model: Without heterogeneity in Foreign, the Foreign government will always be willing to provide a bailout up to the fraction of total Home debt that is held by the Foreign representative household. It is not always the optimal amount for Foreign to offer to the Home government. As the Foreign government anticipates the optimal reaction function of the Home government in the subgame of Home's acceptance decision, the Foreign government's optimal strategy is to offer a bailout of size $\bar{a}$ to the Home government, i.e. the amount at which the Home government is indifferent between accepting the bailout and defaulting. The Foreign government will make this offer if

$$\bar{a} \leq a_{max}. \tag{3.36}$$

3.5.3 Backward induction: Home's default decision

In turn, Home anticipates Foreign's optimal behavior and its own optimal behavior from the point of its default decision onward. The Home government's value of repaying, bailout and default are given by

$$V_H^{repay} = u(y_1 - g_1 + b_1 - B_1), \tag{3.37}$$

$$V_H^{bailout} = u(y_1 - g_1 + b_1 - (1 - \bar{a})B_1), \tag{3.38}$$

and

$$V_H^{default} = u((1 - \lambda)y_2), \tag{3.39}$$

respectively. Because the optimal bailout offer of Foreign in the case that Home defaults will always set the Home government indifferent between being bailed out and defaulting, it holds that

$$V_H^{bailout} = V_H^{default}. \tag{3.40}$$

Thus, the Home government's decision to default or not reduces to a decision between two values. The Home country will prefer repayment to default (or being bailed out) if

$$V_H^{repay} \geq V_H^{default} \tag{3.41}$$

$$u(y_1 - g_1 + b_1 - B_1) \geq u((1-\lambda)y_1). \tag{3.42}$$

Thus, the Home government will repay if the Home household's holdings of government bonds are

$$b_1 \geq B_1 + g_1 - \lambda y_1. \tag{3.43}$$

The Home country would only default if it would also find it optimal to default in a world without bailouts.

3.6 Intuition: A Bailout Model with Heterogeneity

In this section, the bailout negotiation subgame is investigated for a world where there is household heterogeneity in the Foreign bailout providing country. In Home, for simplicity, there exists a representative household.

3.6.1 Backward induction: Home's acceptance decision

Again, taking as given that Home has announced to default in the second period given the state of the economy that has materialized (g_1, B_1, γ) and Foreign has made a bailout offer $a \in [0,1]$, Home takes the decision to accept or reject this offer according to

$$V_H^{acc} \geq V_H^{dec}. \tag{3.44}$$

Because there exists a representative Home household, the functional forms of V_H^{acc} and V_H^{dec} remain the same as in the previous section. Thus, the level at which the Home government is indifferent between being bailed out by the Foreign government and defaulting also remains

$$\bar{a} = 1 + \frac{g_1 - b_1 - \lambda y_1}{B_1}. \tag{3.45}$$

3.6.2 Backward induction: Foreign's bailout decision

Households in Foreign are now assumed to be heterogeneous in the way specified in the general model setup. Therefore, the Foreign government's bailout decision changes because the Foreign utilitarian government weighs the utility levels of low and high income households to take its bailout decision. The Foreign government is willing to provide a bailout to the Home government if its value from providing the bailout is higher than its value from not providing the bailout, i.e. if

$$V_F^{bailout} \geq V_F^{default} \tag{3.46}$$

$$\gamma u(y_1^{L*} - g_1^* + b_1^{L*} - aB_1) + (1-\gamma)u(y_1^{H*} - g_1^* + b_1^{H*} - aB_1) \geq u(y_1^* - g_1^*). \tag{3.47}$$

As I focus on equilibria in which $b_1^{L*} = 0$ and all household bond holdings cannot be negative, the respective market clearing conditions imply that $b_1^{H*} > b_1* \geq b_1^{L*}$. Here b_1* is the optimal level of bond holdings in a model without heterogeneity. Since the utility function is assumed to be strictly increasing and concave $\gamma u(y_1^{L*} - g_1^* + b_1^{L*} - aB_1) + (1-\gamma)u(y_1^{H*} - g_1^* + b_1^{H*} - aB_1) \leq u(y_1^* - g_1^* + b_1^* - aB_1)$. Therefore, the maximum bailout a_{max} Foreign is willing to provide is

$$a_{max} \leq \frac{b_1^*}{B_1}. \tag{3.48}$$

This result states that in an economy with wealth heterogeneity in the bailout providing country, this country is only willing to provide a bailout which is less or equal to the fraction of total debt held by its agents. This is the key finding in the model with heterogeneity: With heterogeneity in Foreign, the Foreign government will not be willing to provide a bailout up to the fraction of total Home debt that is held in total by the Foreign households. Again the share a_{max} is the maximum bailout level the Foreign government is willing to provide. It is not always the optimal amount for Foreign to offer a_{max} to the Home government. Anticipating the optimal reaction function of the Home government, the Foreign government's optimal strategy is to offer a bailout of size $\bar{a}$ to the Home government. The Foreign government will make this offer if

$$\bar{a} \leq a_{max}^{het} \leq a_{max}^{hom}. \tag{3.49}$$

This provides the main intuition of the paper. In the following, I investigate the optimal first period choices of the different agents and find a parameterization that illustrates under which circumstances bailouts and defaults are equilibrium outcomes of the model economy.

3.7 First period in a Bailout Model with Heterogeneity

Now, I turn to the optimal first period decisions of the households and the Home government in a model with household heterogeneity in the Foreign country. As outlined in the general model setup, only the Home government is assumed to be able to issue debt, while households in both countries can save using government bonds. Moreover, I assume log-utility and there are stochastic government expenditure shocks in the debt issuing Home country. For the Foreign country government expenditure is deterministic. The Home government spending shock in period $t = 1$ is drawn from known a set of realizations

$$\left[g_1^1 < g_1^2 < ... < g_1^M\right] \tag{3.50}$$

with transition probabilities

$$\Pi(g_1^j|g_0) \tag{3.51}$$

for $j = 1, ..., M$ with

$$\sum_{j=1}^{M} \Pi(g_1^j|g_0) = 1. \tag{3.52}$$

Furthermore, as D'Erasmo and Mendoza (2016) I concentrate on equilibria in which the low-type foreign household does not save in government bonds, i.e. $b_1^{L*} = 0$. This depends on the parameterization of the model and has to be checked in the theoretical and numerical exercises. In this model version all the possible $t = 1$ default and bailout scenarios are possible, so consumption levels will depend on the default and bailout negotiations.

Households

The preferences of the Home representative household are

$$ln(c_0)+\beta\left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)ln(c_1^R) + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)ln(c_1^B) + \sum_{j:D(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)ln(c_1^D)\right] \tag{3.53}$$

where the sums are taken over the second period states in which the anticipated outcome of the bailout negotiations result in repayment ($R(B_1, g_1^j, \gamma) = 1$), bailout ($B(B_1, g_1^j, \gamma) = 1$) and default ($D(B_1, g_1^j, \gamma) = 1$), respectively. To form expectations over the possible second period outcomes, the Home household weighs the consumption levels in each possible contingency by the probability of this contingency materializing. The period $t = 0$ and the $t = 1$ consumption levels under repayment, bailout and default are

$$c_0 = y_0 - \tau_0 + b_0 - qb_1 \tag{3.54}$$
$$c_1^R = y_1 - \tau_1^R + b_1 \tag{3.55}$$
$$c_1^B = y_1 - \tau_1^B + b_1 \tag{3.56}$$
$$c_1^D = (1 - \lambda)y_1. \tag{3.57}$$

The preferences of the two types of Foreign households are

$$ln(c_0^{i*})+\beta\left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)ln(c_1^{i*,R}) + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)ln(c_1^{i*,B}) + \sum_{j:D(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)ln(c_1^{i*,D})\right] \tag{3.58}$$

for $i = L, H$. With $b_1^{L*} = 0$ the budget constraints of the L-type are

$$c_0^{L*} = y_0^* - \tau_0^*$$
$$c_1^{L*,R} = y_1^* - \tau_1^{*,R}$$
$$c_1^{L*,B} = y_1^* - \tau_1^{*,B}$$
$$c_1^{L*,D} = y_1^* - \tau_1^{*,D}$$

and the ones for the H-type households are

$$\begin{aligned}
c_0^{H*} &= y_0^* - \tau_0^* + b_0^{H*} - qb_1^{H*} \\
c_1^{H*,R} &= y_1^* - \tau_1^{*,R} + b_1^{H*} \\
c_1^{H*,B} &= y_1^* - \tau_1^{*,B} + b_1^{H*} \\
c_1^{H*,D} &= y_1^* - \tau_1^{*,D} + b_1^{H*}.
\end{aligned}$$

All the three household types choose to save in the Home government bond by maximizing their expected utility over the two model periods. The resulting optimality condition (Euler condition) for the Home household yields

$$q = \beta \left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)\frac{c_0}{c_1^R} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)\frac{c_0}{c_1^B} \right]. \quad (3.59)$$

The Euler equation for the Foreign H-type household is

$$q = \beta \left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)\frac{c_0^{H*}}{c_1^{H*,R}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)\frac{c_0^{H*}}{c_1^{H*,B}} \right]. \quad (3.60)$$

Since I concentrate on equilibria in which the low-type foreign household does not save in government bonds, the parameterization has to be chosen such that the Foreign L-type household is 'credit constraint', i.e. given its income stream it would like to borrow, but bond holdings are restricted to $b_1^{L*} \geq 0$. This is the case if for a given equilibrium price

$$q > \beta \left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)\frac{c_0^{H*}}{c_1^{L*,R}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0)\frac{c_0^{H*}}{c_1^{L*,B}} \right] \quad (3.61)$$

holds, stating that the marginal cost of buying a unit of government bonds is strictly greater than the marginal benefit to shift consumption from $t = 0$ to $t = 1$.

Government

Home government's budget constraints for period $t = 0$ and for $t = 1$ under repayment and bailout are

$$\begin{aligned}
\tau_0 &= g_0 + B_0 - qB_1 \\
\tau_1^R &= g_1 + B_1, \\
\tau_1^B &= g_1 + (1-a)B_1,
\end{aligned}$$

respectively. The Foreign government cannot issue debt and therefore, its budget constraints are:

$$\begin{aligned}
\tau_0^* &= g_0^* \\
\tau_1^{*,R} &= g_1^*, \\
\tau_1^{*,B} &= g_1^* + aB_1, \\
\tau_1^{*,D} &= g_1^*.
\end{aligned}$$

Furthermore, the Home government chooses the optimal amount of debt it issues to maximize the Home household's welfare. Its objective function is

$$max_{B_1} ln(c_0) + \beta \left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) ln(c_1^R) + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) ln(c_1^B) + \sum_{j:D(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) ln(c_1^D) \right].$$

Bond market equilibrium

The bond market is in a competitive equilibrium if for a given amount of Home government debt, the following market clearing condition is fulfilled. Given that the Foreign L-type does not hold debt, this condition is

$$b_1 + (1-\gamma) b_1^{H*} = B_1. \tag{3.62}$$

Using the Euler conditions of the Home household and the H-type Foreign household together with the bond market equilibrium condition the market clearing bond price q_0 can be derived. Since the possibility exists that in equilibrium only one of the two household types holds all the debt supplied by the Home government, different cases arise for finding a solution: One in which only the Home household holds Home government debt, another in which both the Home and the H-type Foreign household hold Home government bonds and a third case in which only the H-type Foreign household holds Home government debt. In the following subsections I work through the implications of all three cases for the equilibrium bond price, optimal bond holdings by the different household types and the optimal bond issuance decision of the Home government. As the optimality conditions and optimal choices of the full model with household heterogeneity and Home government spending shocks is not always straightforward, I add some intuition of a simplified version of the model without uncertainty and commitment to repay by the Home government. The solution of this simplified model can be found in the appendix C.2.

Case 1: Only Home households holds bonds

If the equilibrium price is such that

$$\begin{aligned} q &= \beta \left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0}{c_1^R} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0}{c_1^B} \right] \\ &> \beta \left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0^{H*}}{c_1^{H*,R}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0^{H*}}{c_1^{H*,B}} \right], \end{aligned}$$

only the Home household will optimally hold bonds, and from market clearing the demand of the Home household would be $b_1 = B_1$. Therefore, the planner's problem is

$$\mathcal{L} = max_{B_1} ln(y_0 - g_0 - (1-\eta)B_0) + \beta[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) ln(y_1 - g_1)$$
$$+ \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) ln(y_1 - g_1 + aB_1) + \sum_{j:D(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) ln((1-\lambda)y_1)].$$

In this model with heterogeneity among households in Foreign, I have shown that the maximum bailout fraction Foreign is willing to provide is less than the fraction of Home government bonds held in total by Foreign households. Therefore Foreign is not willing to provide a positive bailout, if the Home household holds all the debt, i.e. $a^{max} \leq 0$. Therefore, there are no bailout states and the planner's problem becomes

$$\mathcal{L} = max_{B_1} ln(y_0 - g_0 - (1-\eta)B_0)$$
$$+ \beta \left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) ln(y_1 - g_1) + \sum_{j:D(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) ln((1-\lambda)y_1) \right]$$

This shows that the level of government debt only influences the Home agent's welfare through its influence on default and repayment probabilities, but that it does not influence intertemporal consumption. Furthermore, the possibility of default will make the equilibrium inconsistent. The Home household anticipates that it is taxed fully in period $t = 1$ to repay any debt issued by the Home government in period $t = 0$. Thus, it is only indifferent between holding the debt and not holding it, if there is full commitment to repay. Said differently, any positive default probability renders this case inconsistent. The Home government can only achieve this solution by not issuing any debt, i.e. $B_1 = 0$ is the optimal debt issuance decision in this case. In the simple model without uncertainty, the planner's problem becomes independent of Home government debt altogether if only the Home household holds bonds. In both cases, with and without uncertainty, the Home government cannot influence welfare at all with its debt issuance decision. Adding uncertainty therefore does not make a difference in case 1. This does not mean, however, that this case is generally not of interest. If we would add heterogeneity between households in Home, this case 1 would be similar to the one investigated by D'Erasmo and Mendoza (2016) and default would become a mean of redistribution between Home households, even if no Foreign agents hold debt.

Case 2: Both the Home and the Foreign L-type households hold Home government bonds

If the equilibrium price is such that

$$q = \beta \left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0}{c_1^R} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0}{c_1^B} \right]$$
$$= \beta \left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0^{H*}}{c_1^{H*,R}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0^{H*}}{c_1^{H*,B}} \right],$$

both the Home household and the Foreign H-type household want to hold Home government debt. The market clearing condition in the bond market for this case is given by $b_1 + (1-\gamma)b_1^{H*} = B_1$. Furthermore, I assume that Home households hold a fraction η of the initial debt stock B_0 and the Foreign H-type household holds the remainder, i.e. per capita it holds $\frac{1-\eta}{1-\gamma}B_0$. Using this in the price expression for the Home household, I obtain

$$q = \beta \frac{\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{y_0-g_0-(1-\eta)B_0}{y_1-g_1-(1-\gamma)b_1^{H*}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{y_0-g_0-(1-\eta)B_0}{y_1-g_1+aB_1-(1-\gamma)b_1^{H*}}}{1 - \sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{(1-\gamma)b_1^{H*}}{y_1-g_1-(1-\gamma)b_1^{H*}} - \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{(1-\gamma)b_1^{H*}}{y_1-g_1+aB_1-(1-\gamma)b_1^{H*}}}.$$

Note that the bond price depends on B_1 through the size of the bailouts in bailout states of the world, as well as through its influence on the second period bailout negotiation outcomes. From the price expression for the Foreign H-type household I obtain

$$q = \beta \frac{\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{y_0^*-g_0^*+\frac{1-\eta}{1-\gamma}B_0}{y_1^*-g_1^*+b_1^{H*}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{y_0^*-g_0^*+\frac{1-\eta}{1-\gamma}B_0}{y_1^*-g_1^*+aB_1+b_1^{H*}}}{1 + \sum_{j:R(B_1,g_1^j,\ldots)=1}^{M} \Pi(g_1^j|g_0) \frac{b_1^{H*}}{y_1^*-g_1^*+b_1^{H*}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{b_1^{H*}}{y_1^*-g_1^*+aB_1+b_1^{H*}}}.$$

Again the bond price depends on B_1 through the size of the bailouts in bailout states of the world, as well as through its influence on the second period bailout negotiation outcomes. In equilibrium,

$$\frac{\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{y_0-g_0-(1-\eta)B_0}{y_1-g_1-(1-\gamma)b_1^{H*}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{y_0-g_0-(1-\eta)B_0}{y_1-g_1+aB_1-(1-\gamma)b_1^{H*}}}{1 - \sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{(1-\gamma)b_1^{H*}}{y_1-g_1-(1-\gamma)b_1^{H*}} - \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{(1-\gamma)b_1^{H*}}{y_1-g_1+aB_1-(1-\gamma)b_1^{H*}}}$$
$$= \frac{\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{y_0^*-g_0^*+\frac{1-\eta}{1-\gamma}B_0}{y_1^*-g_1^*+b_1^{H*}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{y_0^*-g_0^*+\frac{1-\eta}{1-\gamma}B_0}{y_1^*-g_1^*+aB_1+b_1^{H*}}}{1 + \sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{b_1^{H*}}{y_1^*-g_1^*+b_1^{H*}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{b_1^{H*}}{y_1^*-g_1^*+aB_1+b_1^{H*}}}$$

This equation implicitly defines the optimal bond demand of Foreign H-type agents. In the numerical section, I investigate how changes in B_1 influence the equilibrium bond price for a suitable parameterization example. If there would be no default or bailout, the Foreign H-type's bond demand b_1^{H*} would be independent of B_1 and there would be a unique optimal level of savings the Foreign H-type household would like to hold in Home government bonds. With the possibility of default and bailout this level varies with the likelihood of default and bailout states, as well as with the total amount of Home debt and the bailout size in each state. To get a clearer picture of how the optimal bond demand and equilibrium pricing will look like, I present these expressions for the simple model without heterogeneity and uncertainty. The same steps for the simple model yield a closed form solution to optimal Foreign bond holdings

$$b_1^* = \frac{(y_0 - g_0 - (1-\eta)B_0)(y_1^* - g_1^*) - (y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0)(y_1 - g_1))}{(y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0)(1+\beta)(1-\eta) - (y_0 - g_0 - (1-\eta)B_0)(1+\beta)}. \tag{3.63}$$

One can see that the demand for Home government debt by the Foreign H-type household does not depend on the level of debt issued by the Home government B_1. It is solely determined by the parameters of the model. This demand pins down the equilibrium price of debt to

$$q = \beta \frac{y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0}{y_1^* - g_1^* + (1+\beta)b_1^{H*}} \tag{3.64}$$

which is then independent of B_1 as well. The bond demand of the Home household is derived from the market clearing condition as

$$b_1 = B_1 - (1-\gamma)b_1^{H*}. \tag{3.65}$$

Since the demand of the Foreign H-type household is independent of B_1, in the case that Home households and Foreign H-type households hold debt and bond holdings cannot be negative, the debt level issued by the government has to be at least $B_1 > (1-\gamma)b_1^{H*}$. Otherwise the Home household will not buy government bonds. For the case that $B_1 > (1-\gamma)b_1^{H*}$, the Home household will buy all residual government debt above $(1-\gamma)b_1^{H*}$. Therefore, the Home government will be indifferent between issuing $B_1 = (1-\gamma)b_1^{H*}$ and any other amount $B_1 > (1-\gamma)b_1^{H*}$. The intertemporal consumption levels of the Home household, and therefore its welfare, is not altered by issuing any $B_1 > (1-\gamma)b_1^{H*}$. This case plays a role, if welfare is maximized by an amount of debt larger or equal to the full demand for government bonds by the Foreign household. Interestingly, the potential welfare gain from issuing debt even under commitment in this model, is limited by the Foreign H-type household's demand for these bonds. In this case, the government can issue even more debt without influencing the Home household's welfare, as the Home household will just hold the residual government debt, keeping its consumption levels the same. Turning back to the full model, the presence of a government spending shock to the Home economy as well as default and bailout possibilities will lead to dependence of the Foreign bond demand on the Home government debt level. The higher the debt level B_1 and the higher therefore the possibility of default and bailout states, the lower the Foreign household will value the expected value from saving in Home bonds at a given price q_0. The overall effect on prices and demand is not clear from the analytical equation for the full model above and is investigated in more detail using a numerical example in the next section of the paper.

Case 3: Only Foreign H-type holds debt

The third case becomes relevant if the equilibrium price is such that

$$q = \beta \left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0^{H*}}{c_1^{H*,R}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0^{H*}}{c_1^{H*,B}} \right]$$
$$> \beta \left[\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0}{c_1^R} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{c_0}{c_1^B} \right].$$

In this case, only the Foreign H-type household is saving in Home government bonds. The market clearing condition in the bond market for this case is given by $(1-\gamma)b_1^{H*} = B_1$, so the bond demand of the Foreign H-type household is $b_1^{H*} = \frac{B_1}{(1-\gamma)}$. Using this in the price expression for the Foreign H-type household yields

$$q = \beta \frac{\sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0}{y_1^* - g_1^* + \frac{B_1}{(1-\gamma)}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0}{y_1^* - g_1^* + aB_1 + \frac{B_1}{(1-\gamma)}}}{1 + \sum_{j:R(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{\frac{B_1}{(1-\gamma)}}{y_1^* - g_1^* + \frac{B_1}{(1-\gamma)}} + \sum_{j:B(B_1,g_1^j,\gamma)=1}^{M} \Pi(g_1^j|g_0) \frac{\frac{B_1}{(1-\gamma)}}{y_1^* - g_1^* + aB_1 + \frac{B_1}{(1-\gamma)}}}$$

where the equilibrium bond price varies with the level of debt issued by the Home government in repayment as well as in bailout states. Again the numerical section provides an example of how the equilibrium price and optimal bond holdings react to changes in the government debt issued by the Home government. Again, to gain some baseline intuition in the simpler model using the price expression for the Foreign H-type household yields

$$q(B_1) = \beta \frac{y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0}{y_1^* - g_1^* + (1+\beta)\frac{B_1}{(1-\gamma)}}$$

where the equilibrium bond price varies with the level of debt issued by the Home government. In particular the first order condition of the equilibrium price with respect to B_1 is

$$q'(B_1) = \frac{-q(B_1)\frac{1+\beta}{1-\gamma}}{y_1^* - g_1^* + \frac{1+\beta}{(1-\gamma)}B_1}. \tag{3.66}$$

Given that the bond price in equilibrium is positive, it holds that

$$q'(B_1) < 0. \tag{3.67}$$

Therefore, in the simpler model the equilibrium price falls with the debt level B_1 in the case that only the Foreign H-type household holds government debt. As the presence of default and bailout states also depend on the government debt level B_1, it will add an additional layer of effects to the baseline effect in the simpler model. The overall effect is not clear here, but with modest probabilities of default and bailout states, the baseline effect will dominate the equilibrium price's dependence on the government debt stock B_1. The numerical section will shed more light on this for a specific example parameterization.

Table 3.1 – Parameterization

Parameter	Description	Value
β	discount rate	0.96
σ	relative risk aversion	1
γ	% of low wealth households in Foreign	[0.4,0.6,0.8]
λ	proportionate output loss in default	0.74
	Exogenous income process	
y_0	$t = 0$ Home household income	0.94
y_1	$t = 1$ Home household income	0.79
y_0^*	$t = 0$ Foreign household income	0.225
y_1^*	$t = 1$ Foreign household income	0.79
	Government spending process	
B_0	Initial debt of Home government	0.3
η	share of B_0 held by Home household	0.5
g_0	$t = 0$ Home government spending	0.5
g_0^*	$t = 0$ Foreign government spending	0.2
g_1^*	$t = 1$ Foreign government spending	0.3
g_1	$t = 1$ Home government spending	$[0.1, 0.5, 0.55, 0.58, 0.6]$
Π	g_1 probability distribution	$[0.9398, 0.02, 0.04, 0.0001, 0.0001]$

3.8 Numerical results

As the interpretation of the analytical expressions of the equilibrium bond price and the resulting equilibrium bond holdings for the different cases above is not straightforward, this section provides a numerical example and shows numerical comparative statics for the equilibrium outcomes of the model with heterogeneity and bailouts.

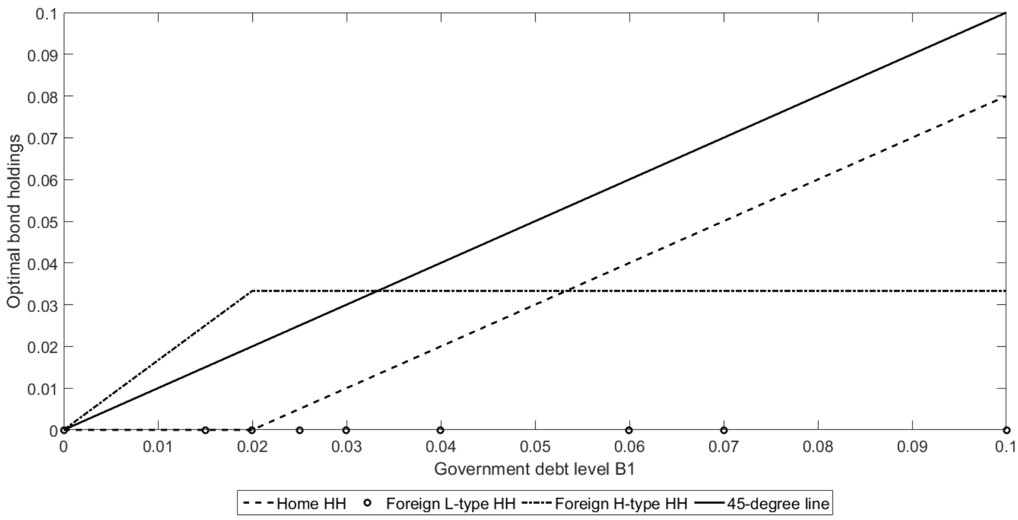
3.9 Parmeterization

In this paper I choose a parametrization for which all possible outcomes of the model happen in equilibrium to generate an example situation in which the Foreign government refuses a bailout. This is a feature that only arises due to the wealth heterogeneity in the Foreign bailout providing country. The chosen parametrization is specified in table 3.1. I start with the optimal bond holding decisions, which help to understand the optimal debt issuance decision of the Home government and equilibrium prices. Thereafter, I present the equilibrium default and bailout decisions of the second period.

3.9.1 Equilibrium Bond Holdings and Bond Price

For the parameterization chosen above, Figures 3.2 to 3.4 show the optimal equilibrium bond holdings of all three types of households and for three different values of the dispersion of wealth in the Foreign country γ. For $\gamma = 0.4$ in figure 3.2 it can be observed that the Foreign H-type household demands all the government bonds issued by the Home government for low values of government debt up to $B_1 = 0.02$ (blue line). For $B_1 = 0.02$ the H-type Foreign household is able to carry its optimal savings over to the next period for the given equilibrium price. From this point on all additional government debt issued by the Home government is bought by the Home household and the equilibrium price stays constant at the level determined by the Foreign H-type household's demand (purple line in figure 3.5). As I concentrate on equilibria in which the Foreign L-type household is credit constraint and does not hold any debt, it's optimal bond demand is zero for all debt levels (dots).

Figure 3.2 – Optimal bond holdings for $\gamma = 0.4$



For $\gamma = 0.6$ in figure 3.3 the debt level for which the Foreign H-type household can satisfy its optimal savings demand and the Home household starts to demand the residual is higher, at between approximately $B_1 = 0.06$ and $B_1 = 0.07$. From the equilibrium price figure, one can observe that the equilibrium price for the segment in which only the Foreign H-type demands bonds varies with the level of B_1, while as soon as the Home household starts demanding Home bonds as well, the demand of the Foreign H-type household determines a constant equilibrium price of Home bonds. This is in line with the findings in the theoretical section of this paper. Note, that for higher γ, the equilibrium bond price increases. This is surprising at first sight, as higher dispersion means that there are more states of bailout or outright default. At the same time, tough, the first period debt stock held by foreign households is constant while the share of high-types holding that debt is reduced. Therefore, the per-capita wealth of H-type households is increased, making them save more and thus they are willing to pay a higher price for a home bond. Due to the low probability of default and bailout states this second effect outweighs the first effect. From the theoretical section we could not anticipate the shape of how the equilibrium price depend on the government debt level, as the price function in the model with default and bailouts is not differentiable. Figure 3.5 shows that the equilibrium price for a given γ is falling in the government debt level, which is also due to the fact that the Home government is more likely to default on high debt levels, compared to low debt levels.

Figure 3.3 – Optimal bond holdings for $\gamma = 0.6$

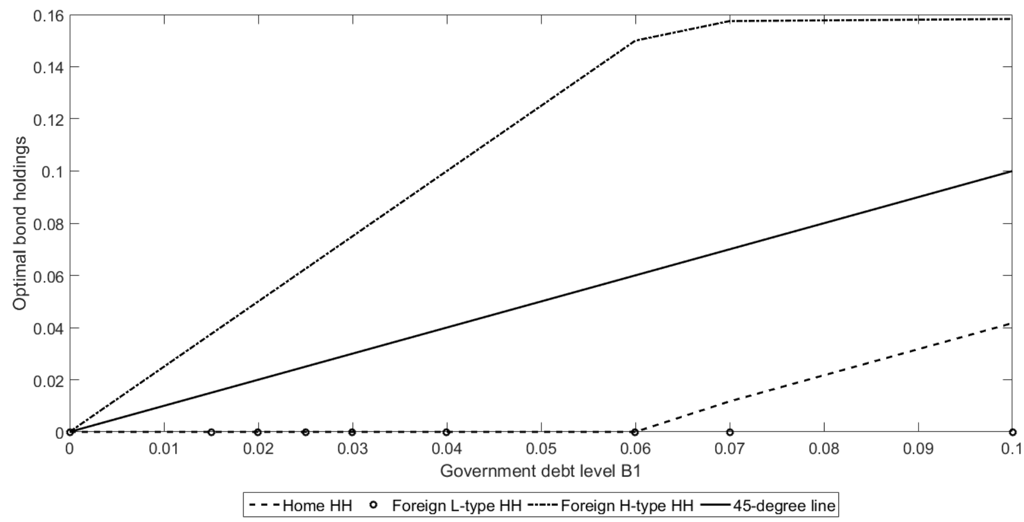
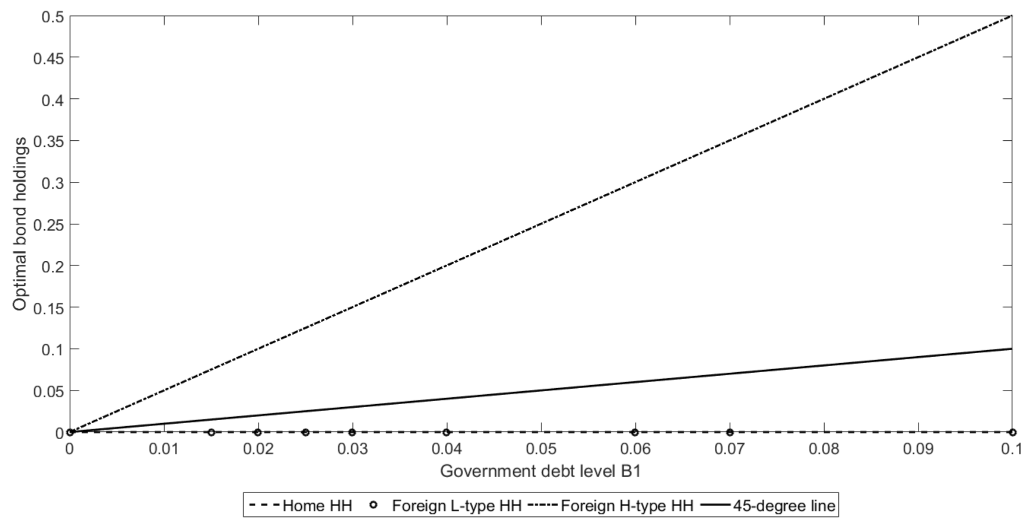


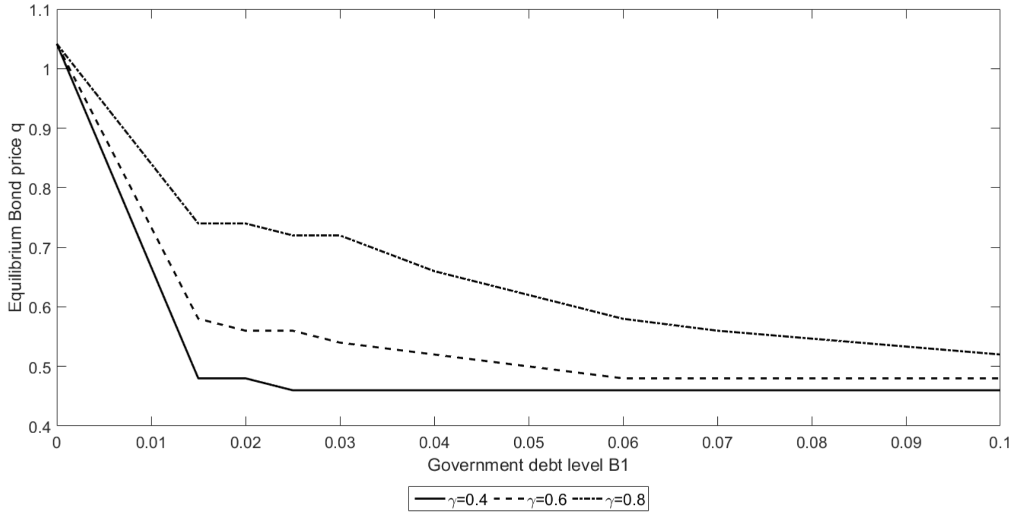
Figure 3.4 – Optimal bond holdings for $\gamma = 0.8$



For $\gamma = 0.8$ in figure 3.4 the debt level for which the Foreign H-type household can satisfy its optimal savings demand is higher than the specified range of government debt values investigated. The point to take away from the comparison of the different households' government bond demands for different dispersion values of wealth in the Foreign country is that the higher the dispersion, i.e. the lower the share of

Foreign households rich enough to be able to save in Home government debt, the higher the equilibrium bond holdings of the H-type Foreign households that satisfy market clearing and the higher the equilibrium price.

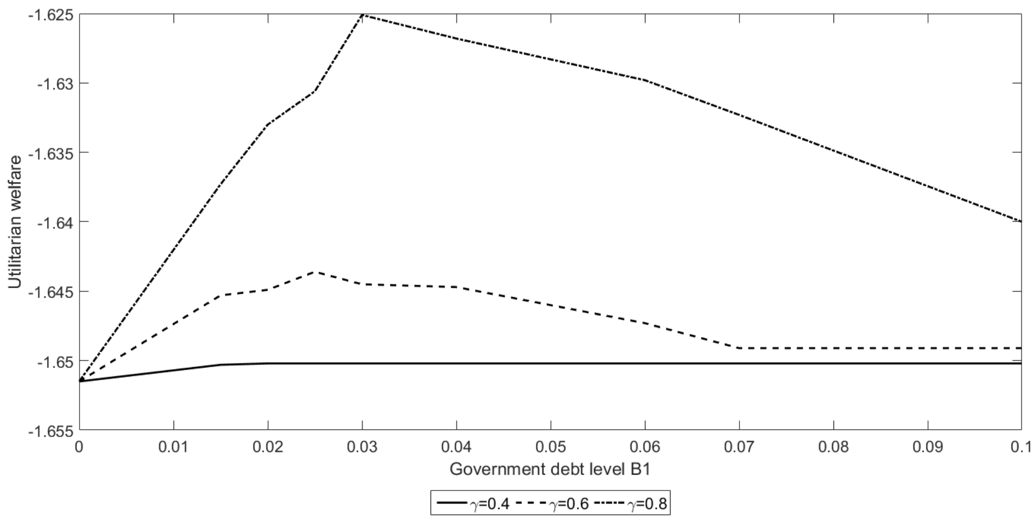
Figure 3.5 – Bond price vs. debt level



3.9.2 Optimal debt issuance depending on γ

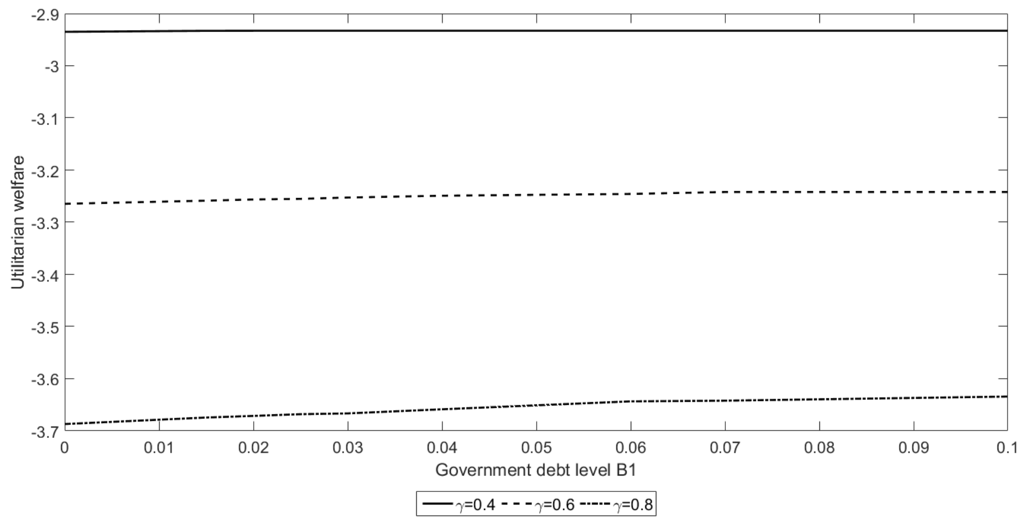
Figure **??** shows the welfare of the Home household according to the Home government's utilitarian welfare function. It shows that it is optimal for the Home household to issue a positive amount of debt. This amount is approximately $B_1 = 0.02$ for $\gamma = 0.4$, $B_1 = 0.025$ for $\gamma = 0.6$ and $B_1 = 0.03$ for $\gamma = 0.8$ as the welfare level peaks in these points for different values of γ. Looking at the full range of values for B_1 one can see that the welfare varies more for values of B_1 such that $B_1 \leq b_{1,max}^{H*}$, i.e. the point where the optimal demand of the Foreign H-type household is completely satisfied and the Home household starts buying its own debt. Since the probabilities on default and bailout states in the model are set to be low to show and highlight the possibility of a default outcome, the lines do not differ much from the deterministic case for which I explained the intuition above, once the Home household starts buying Home government bonds, the Home household's welfare is unaffected by the level of debt issued. While in the cases of default and bailouts, the welfare still varies slightly with the debt level.

Figure 3.6 – Home welfare



Foreign welfare does not vary significantly with the level of B_1. For the case of $\gamma = 0.8$ the welfare is improved with a higher level of B_1, since the Foreign H-type household is not able to satisfy its optimal demand. In equilibrium the Home government will only issue $B_1 = 0.03$ in this case. There are, however, significant welfare differences between the difference levels of γ. There are two drivers for this. First, with rising γ default and bailout states become more frequent, raising the chances that the Home government will not service its debt and lowering the Foreign H-type's welfare. Second, and more prominent, the weight of the H-type household in the Foreign government's utilitarian welfare function is reduced for higher γ contributing to lower levels of total welfare.

Figure 3.7 – Foreign welfare



3.9.3 Second period decisions

For the given parameterization and the optimal levels of indebtedness found above, the equilibrium repayment(light gray), bailout (gray) and default decisions (black), for each realization of the second period Home government spending shock and for the different levels of wealth dispersion in Foreign γ, are given in figures 3.8 to 3.10. They show that with increasing dispersion, the Home government will repay for less states of the world, while the number of bailout and default states increases. This is because a constant bailout size that sets the Home household indifferent between accepting the bailout and defaulting, has to be financed by a smaller portion of the Foreign households and these H-type households become less willing to provide the demanded bailout the fewer they are in numbers.

Figure 3.8 – Second period decisions in equilibrium, $\gamma = 0.4$

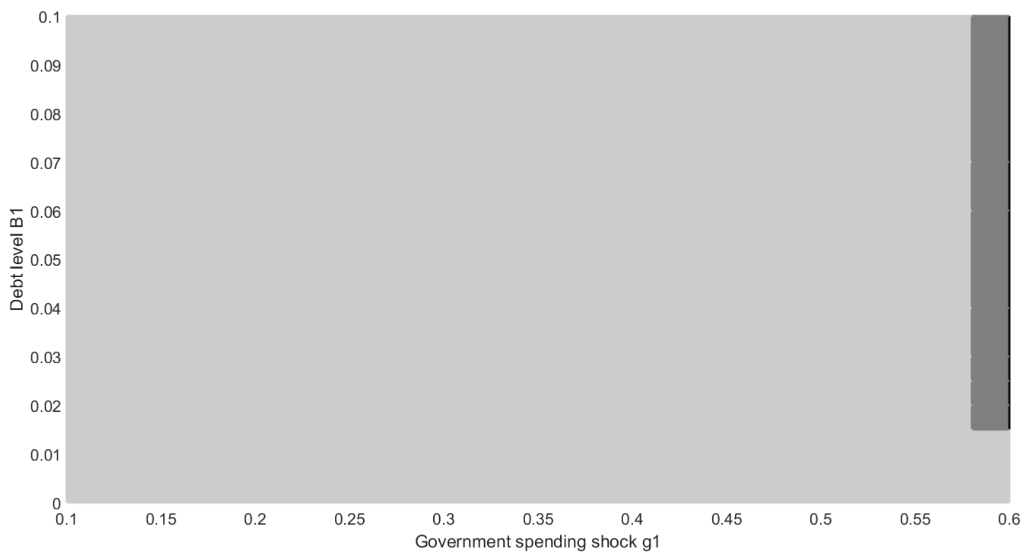


Figure 3.9 – Second period decisions in equilibrium, $\gamma = 0.6$

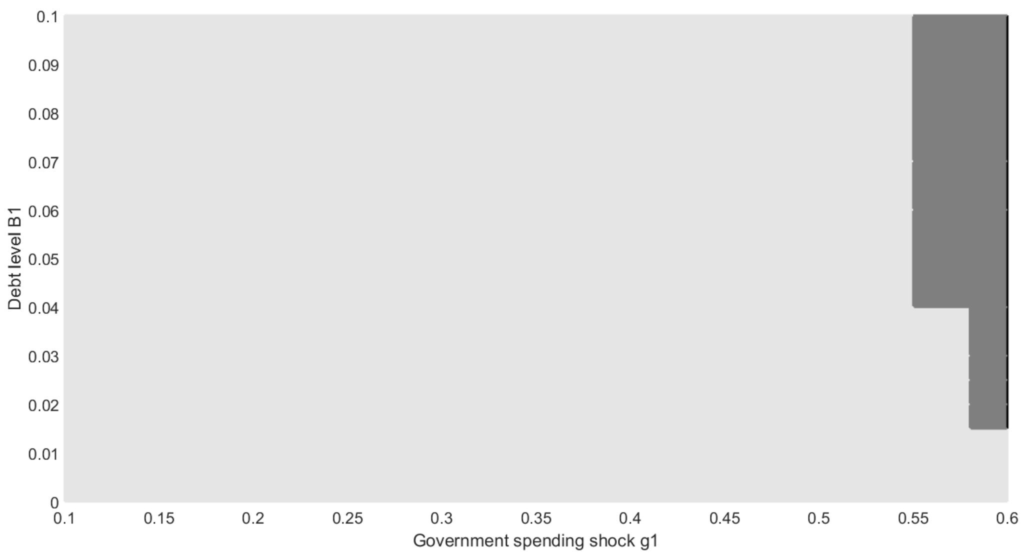
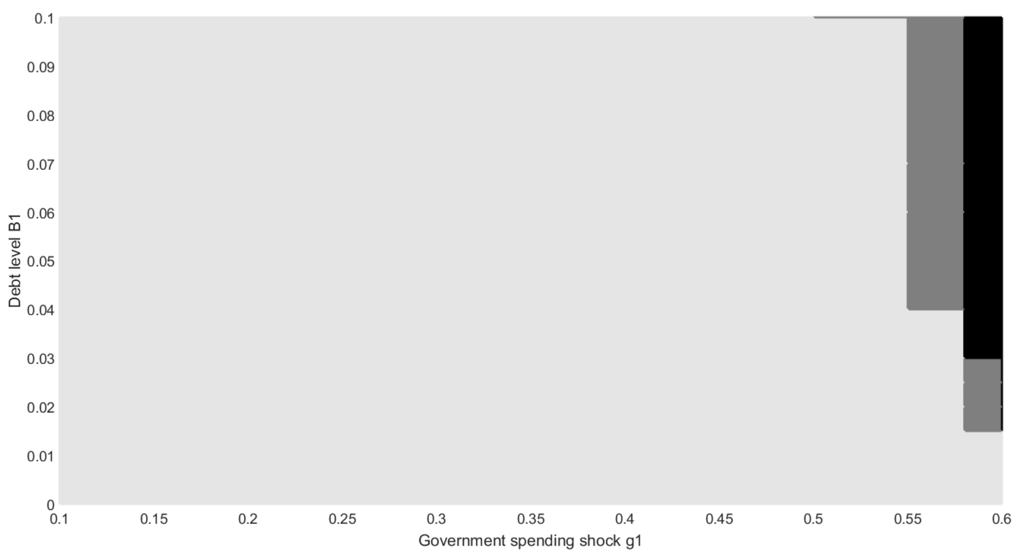


Figure 3.10 – Second period decisions in equilibrium, $\gamma = 0.8$



3.10 Discussion of Social Welfare Function

As for D'Erasmo and Mendoza (2016), the main result of this paper, namely that bailing out the Home country is not always optimal for Foreign even though households of the potentially helping Foreign country hold government bonds of the troubling Home country, depends on the assumption of a utilitarian government. This utilitarian government weighs the consumption levels of both types of agents in the Foreign country of this model equally. If one would instead assume a government that puts more weight on H-type households than on L-type households, the states of the world where outright default is optimal would decrease and in the extreme case that the Foreign government only values H-types' consumption, just as in a representative agent model, it would always be optimal for Foreign to provide a bailout to Home as soon as Foreign H-types hold a positive amount of Home government bonds. But as long as the Foreign government places a positive weight on L-type households and Home debt is held in Foreign, it can be optimal for Foreign to refuse a bailout to Home at least in some state states of the world.

3.11 Conclusion

In this paper I show how the default and bailout decisions of two countries that are interlinked through sovereign debt holdings of their households depend on the wealth distribution within the bailout providing country and the resulting dispersion in bond holdings of a specific country. I find that the willingness to bailout another country is shrinking in the dispersion of government bond holdings within the country providing the bailout. Thus, there are situations in which it is optimal for Foreign not to bail Home out although Foreign agents hold Home debt. The results of the paper have important policy implications for the current bailout policy in the EU debt crisis. They imply that the dispersion of bond holdings within and across EMU member countries play a significant role in determining the willingness of a country to provide a bailout for another country. My results indicate that there could arise situations in which a country might find it optimal not to provide a bailout to another country although its agents hold bonds of the distressed country. This finding is a major contribution to the literature on defaults and bailouts so far. It arises because of the distributional bailout incentives introduced in this paper.

Appendix A

Appendix Chapter 1

A.1 Country Groups

Table A.1 – Country Groups

Group	Included Countries
Full Sample	Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, United Kingdom, United States
EU-13	Austria, Belgium, Denmark, Finland, France, Germany, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, United Kingdom
G7	Canada, France, Germany, Italy, Japan, United Kingdom, United States
NAFTA	Canada, Mexico, United States
Oceania	Australia, New Zealand

A.2 Equilibrium conditions

A.2.1 Households

First, I will derive the first-order conditions of the Home household. The Lagrangian of the country 1 household's problem is given by

$$\mathcal{L} = max_{c_1(s^t),n_1(s^t),x_1(s^t),k_1(s^{t+1}),B_1(s^t),u_1(s^t)} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t)\{\frac{1}{\gamma}\left[c_1(s^t)^{\mu}(1-n_1(s^t))^{1-\mu}\right]^{\gamma}$$
$$+ \lambda_1(s^t)[q_1^a(s^t)\left(w_1(s^t)n_1(s^t) + u_1(s^t)r_1(s^t)k_1(s^t)\right) + q_1^a(s^t)\left(B_1(s^{t-1}) - \Phi(B_1(s^t))\right)$$
$$- c_1(s^t) - x_1(s^t) - q_1^a(s^t)Q(s^t)B_1(s^t) - \frac{\phi}{2}k_1(s^t)\left[\frac{x_1(s^t)}{k_1(s^t)} - \delta u_1(s^t)^{\eta}\right]^2]$$
$$+ \vartheta_1(s^t)[\left(1 - \delta u_1(s^t)^{\eta}\right)k_1(s^t) + x_1(s^t) - k_1(s^{t+1})]$$
$$+ \psi_1(s^t)\left[\Psi - u_1(s^t)\right]\}.$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial c_1(s^t)}: \quad \mu\frac{\left[c_1(s^t)^{\mu}(1-n_1(s^t))^{1-\mu}\right]^{\gamma}}{c_1(s^t)} = \lambda_1(s^t)$$
$$\frac{\partial \mathcal{L}}{\partial n_1(s^t)}: \quad (1-\mu)\frac{\left[c_1(s^t)^{\mu}(1-n_1(s^t))^{1-\mu}\right]^{\gamma}}{1-n_1(s^t)} = \lambda_1(s^t)q_1^a(s^t)w_1(s^t)$$
$$\frac{\partial \mathcal{L}}{\partial x_1(s^t)}: \quad \lambda_1(s^t) = \vartheta_1(s^t)\left[1 - \phi\left(\frac{x_1(s^t)}{k_1(s^t} - \delta u_1(s^t)^{\eta}\right)\right]$$
$$\frac{\partial \mathcal{L}}{\partial k_1(s^t)}: \quad \vartheta_1(s^t) = \beta\sum_{s^{t+1}}\Pi(s^{t+1}|s^t)\{\lambda_1(s^{t+1})[q_1^a(s^{t+1})r_1(s^{t+1})u_1(s^{t+1})$$
$$- \phi\left(\frac{1}{2}\left(\frac{x_1(s^{t+1})}{k_1(s^{t+1})} - \delta u_1(s^{t+1})^{\eta}\right) - \frac{x_1(s^{t+1})}{k_1(s^{t+1})}\right)\left(\frac{x_1(s^{t+1})}{k_1(s^{t+1})} - \delta u_1(s^{t+1})^{\eta}\right)]$$
$$+ \vartheta_1(s^{t+1})(1 - \delta u_1(s^{t+1})^{\eta})\}$$
$$\frac{\partial \mathcal{L}}{\partial B_1(s^t)}: \quad Q(s^t) = \beta\sum_{s^{t+1}}\Pi(s^{t+1}|s^t)\left[\frac{\lambda_1(s^{t+1})}{\lambda_1(s^t)}\frac{q_1^a(s^{t+1})}{q_1^a(s^t)}\right] - \Phi'\left(B_1(s^t)\right)$$
$$\frac{\partial \mathcal{L}}{\partial u_1(s^t)}: \quad \Psi_1(s^t) = \lambda_1(s^t)\left[q_1^a(s^t)r_1(s^t)k_1(s^t) + \eta\delta u_1(s^t)^{\eta-1}k_1(s^t)\phi\left(\frac{x_1(s^{t+1})}{k_1(s^{t+1})} - \delta u_1(s^{t+1})^{\eta}\right)\right]$$
$$- \vartheta_1(s^t)\eta\delta u_1(s^t)^{\eta-1}k_1(s^t).$$

For the Foreign household the Lagrangian is

$$\mathcal{L} = max_{c_2(s^t),n_2(s^t),x_2(s^t),k_2(s^{t+1}),B_2(s^t),u_2(s^t)} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t)\{\frac{1}{\gamma}\left[c_2(s^t)^{\mu}(1-n_2(s^t))^{1-\mu}\right]^{\gamma}$$
$$+ \lambda_2(s^t)[q_2^b(s^t)\left(w_2(s^t)n_2(s^t) + u_2(s^t)r_2(s^t)k_2(s^t)\right) + q_2^a(s^t)\left(B_2(s^{t-1}) - \Phi(B_2(s^t))\right)$$
$$- c_2(s^t) - x_2(s^t) - q_2^a(s^t)Q(s^t)B_2(s^t) - \frac{\phi}{2}k_2(s^t)\left[\frac{x_2(s^t)}{k_2(s^t)} - \delta u_2(s^t)^{\eta}\right]^2]$$
$$+ \vartheta_2(s^t)[\left(1 - \delta u_2(s^t)^{\eta}\right)k_2(s^t) + x_2(s^t) - k_2(s^{t+1})]$$
$$+ \psi_2(s^t)\left[\Psi - u_2(s^t)\right]\}.$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial c_2(s^t)}: \quad \mu \frac{[c_2(s^t)^\mu (1-n_2(s^t))^{1-\mu}]^\gamma}{c_2(s^t)} = \lambda_2(s^t) \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial n_2(s^t)}: \quad (1-\mu) \frac{[c_2(s^t)^\mu (1-n_2(s^t))^{1-\mu}]^\gamma}{1-n_2(s^t)} = \lambda_2(s^t) q_2^b(s^t) w_2(s^t) \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial x_2(s^t)}: \quad \lambda_2(s^t) = \vartheta_2(s^t) \left[1 - \phi \left(\frac{x_2(s^t)}{k_2(s^t} - \delta u_2(s^t)^\eta) \right) \right] \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial k_2(s^t)}: \quad \vartheta_2(s^t) = \beta \sum_{s^{t+1}} \Pi(s^{t+1}|s^t) \{ \lambda_2(s^{t+1}) [q_2^b(s^{t+1}) r_2(s^{t+1}) u_2(s^{t+1}) \tag{A.4}$$

$$-\phi \left(\frac{1}{2} \left(\frac{x_2(s^{t+1})}{k_2(s^{t+1})} - \delta u_2(s^{t+1})^\eta \right) - \frac{x_2(s^{t+1})}{k_2(s^{t+1})} \right) \left(\frac{x_2(s^{t+1})}{k_2(s^{t+1})} - \delta u_2(s^{t+1})^\eta \right)] \tag{A.5}$$

$$+ \vartheta_2(s^{t+1})(1 - \delta u_2(s^{t+1})^\eta) \} \tag{A.6}$$

$$\frac{\partial \mathcal{L}}{\partial B_2(s^t)}: \quad Q(s^t) = \beta \sum_{s^{t+1}} \Pi(s^{t+1}|s^t) \left[\frac{\lambda_2(s^{t+1})}{\lambda_2(s^t)} \frac{q_2^a(s^{t+1})}{q_2^a(s^t)} \right] - \Phi'\left(B_2(s^t)\right) \tag{A.7}$$

$$\frac{\partial \mathcal{L}}{\partial u_2(s^t)}: \quad \Psi_2(s^t) = \lambda_2(s^t) \left[q_2^b(s^t) r_2(s^t) k_2(s^t) + \eta \delta u_2(s^t)^{\eta-1} k_2(s^t) \phi \left(\frac{x_2(s^{t+1})}{k_2(s^{t+1})} - \delta u_2(s^{t+1})^\eta \right) \right] \tag{A.8}$$

$$- \vartheta_2(s^t) \eta \delta u_2(s^t)^{\eta-1} k_2(s^t). \tag{A.9}$$

In addition, to the first order condition, also the budget constraints and the laws of motion for capital are optimality conditions. Because of the occasionally binding capacity utilization constraint, two complementary slackness conditions form part of the optimality conditions as well. They are

$$\psi_i(s^t) \left[\Psi - u_i(s^t) \right] = 0 \tag{A.10}$$

for $i = 1, 2$.

Now, we turn to the optimality conditions of the firms.

A.2.2 Intermediate Firms

The intermediate firm's static maximization problem in country i after history s^t is given by

$$\mathcal{L} = max_{k_i(s^t), n_i(s^t)} \{ e^{z_i(s^t)} \left(u_i(s^t) k_i(s^t) \right)^\theta n_i(s^t)^{1-\theta} - w_i(s^t) n_i(s^t) - r_i(s^t) u_i(s^t) k_i(s^t) \}. \tag{A.11}$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial k_i(s^t)}: \quad e^{z_i(s^t)} \theta \left(u_i(s^t) k_i(s^t) \right)^{\theta-1} u_i(s^t) n_i(s^t)^{1-\theta} = r_i(s^t) u_i(s^t) \tag{A.12}$$

$$\frac{\partial \mathcal{L}}{\partial n_i(s^t)}: \quad e^{z_i(s^t)} (1-\theta) \left(u_i(s^t) k_i(s^t) \right)^\theta n_i(s^t)^{-\theta} = w_i(s^t). \tag{A.13}$$

This simplifies to:

$$\frac{\partial \mathcal{L}}{\partial k_i(s^t)}: \quad \theta \frac{y_i(s^t)}{k_i(s^t)} = r_i(s^t)u_i(s^t) \tag{A.14}$$

$$\frac{\partial \mathcal{L}}{\partial n_i(s^t)}: \quad (1-\theta)\frac{y_i(s^t)}{n_i(s^t)} = w_i(s^t) \tag{A.15}$$

for $i = 1, 2$, where $y_i(s^t) = F(z_i(s^t), k_i(s^t), n_i(s^t), u_i(s^t))$ $w_i(s^t)$ and $r_i(s^t)$ are denoted in local intermediary good.

A.2.3 Final good firms

The country 1's final good firm's maximization problem is

$$\mathcal{L} = max_{a_1(s^t),b_1(s^t)}\{[\omega_1 a_1(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\omega_1)b_1(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} - q_1^a(s^t)a_1(s^t) - q_1^b(s^t)b_1(s^t)\}. \tag{A.16}$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial a_1(s^t)}: \quad q_1^a(s^t) = \frac{\sigma}{\sigma-1}[\omega_1 a_1(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\omega_1)b_1(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}\omega_1 a_1(s^t)^{\frac{\sigma-1}{\sigma}-1} \tag{A.17}$$

$$\frac{\partial \mathcal{L}}{\partial b_1(s^t)}: \quad q_1^b(s^t) = \frac{\sigma}{\sigma-1}[\omega_1 a_1(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\omega_1)b_1(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}-1}\frac{\sigma-1}{\sigma}(1-\omega_1)b_1(s^t)^{\frac{\sigma-1}{\sigma}-1}. \tag{A.18}$$

They simplify to

$$q_1^a(s^t) = [\omega_1 a_1(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\omega_1)b_1(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}\omega_1 a_1(s^t)^{-\frac{1}{\sigma}} \tag{A.19}$$

$$q_1^b(s^t) = [\omega_1 a_1(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\omega_1)b_1(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}}(1-\omega_1)b_1(s^t)^{-\frac{1}{\sigma}}. \tag{A.20}$$

$$(q_1^a(s^t))^\sigma = [\omega_1 a_1(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\omega_1)b_1(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}\omega_1^\sigma a_1(s^t)^{-\frac{\sigma}{\sigma}} \tag{A.21}$$

$$(q_1^b(s^t))^\sigma = [\omega_1 a_1(s^t)^{\frac{\sigma-1}{\sigma}} + (1-\omega_1)b_1(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}(1-\omega_1)^\sigma b_1(s^t)^{-\frac{\sigma}{\sigma}}. \tag{A.22}$$

$$a_1(s^t) = G_1\omega_1^\sigma(q_1^a(s^t))^{-\sigma} \tag{A.23}$$

$$b_1(s^t) = G_1(1-\omega_1)^\sigma(q_1^b(s^t))^{-\sigma}. \tag{A.24}$$

For country 2's final good firm's maximization problem is

$$\mathcal{L} = max_{a_2(s^t),b_2(s^t)}\{[(1-\omega_1)a_i(s^t)^{\frac{\sigma-1}{\sigma}} + \omega_1 b_i(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} - q_2^a(s^t)a_2(s^t) - q_2^b(s^t)b_2(s^t)\}. \tag{A.25}$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial a_2(s^t)}: \quad q_2^a(s^t) = \frac{\sigma}{\sigma-1}[(1-\omega_1)a_i(s^t)^{\frac{\sigma-1}{\sigma}} + \omega_1 b_i(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}-1}(1-\omega_1)\frac{\sigma-1}{\sigma}a_i(s^t)^{\frac{\sigma-1}{\sigma}-1} \tag{A.26}$$

$$\frac{\partial \mathcal{L}}{\partial b_2(s^t)}: \quad q_2^b(s^t) = \frac{\sigma}{\sigma-1}[(1-\omega_1)a_i(s^t)^{\frac{\sigma-1}{\sigma}} + \omega_1 b_i(s^t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}\omega_1\frac{\sigma-1}{\sigma}b_i(s^t)^{\frac{\sigma-1}{\sigma}-1}. \tag{A.27}$$

And this simplifies to

$$a_2(s^t) = G_2\omega_1^\sigma(q_2^a(s^t))^{-\sigma} \quad (A.28)$$

$$b_2(s^t) = G_2(1-\omega_1)^\sigma(q_2^b(s^t))^{-\sigma}. \quad (A.29)$$

A.3 Data Sources

Table A.2 – Data Sources

Country	Variable	Source	Datastream ID (if available)
US	TFP	Fernald (2012)	-
	Capacity Utilization	Federal Reserve	USCAPUTLQ
	GDP	BEA	USGDP...D
Canada	Capacity Utilization	CANSIM	CNCAPUTLR
	GDP	OECD	CNOEXO03D
France	Capacity Utilization	OECD	FROBS076Q
	GDP	INSEE	FRGDP...D
Germany	Capacity Utilization	OECD	BDOBS076Q
	GDP	OECD	BDOEXO03D
Italy	Capacity Utilization	OECD	ITOBS076Q
	GDP	OECD	ITOEXO03D

A.4 Domestic US responses in specifications with Germany and Italy

Figure A.1 – US - Germany impulse response functions - US variables

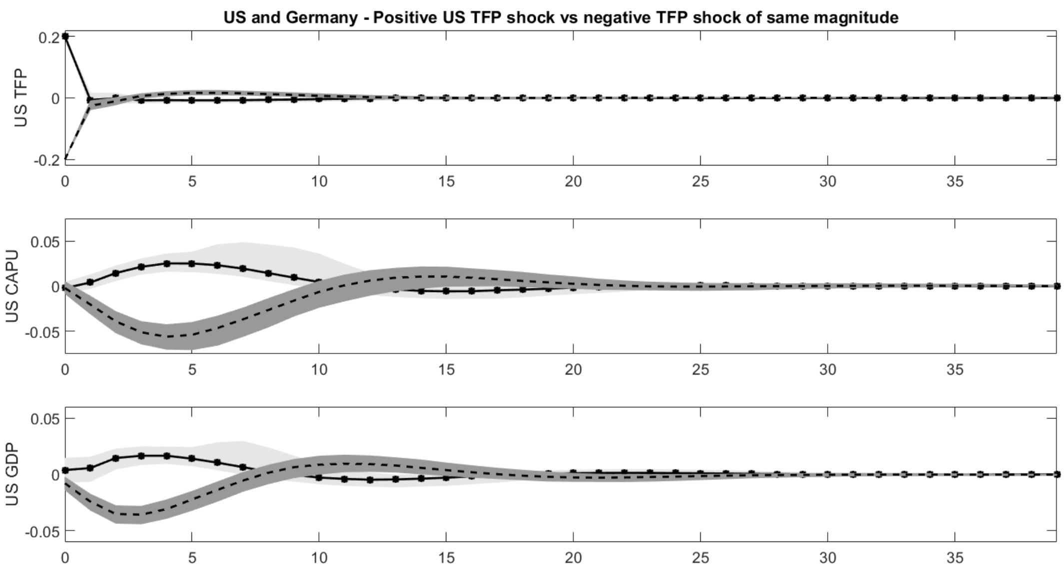
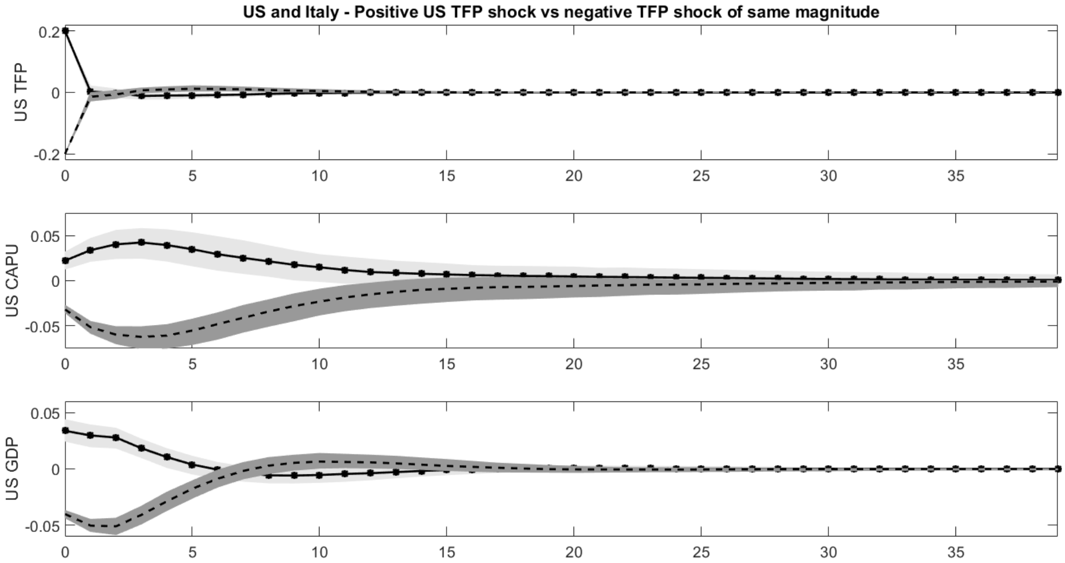


Figure A.2 – US - Italy impulse response functions - US variables



A.5 Results on omitted countries

US - France

For France the prior threshold estimate at $79.65 - 1.15$.

Figure A.3 – US - France impulse response functions - US variables

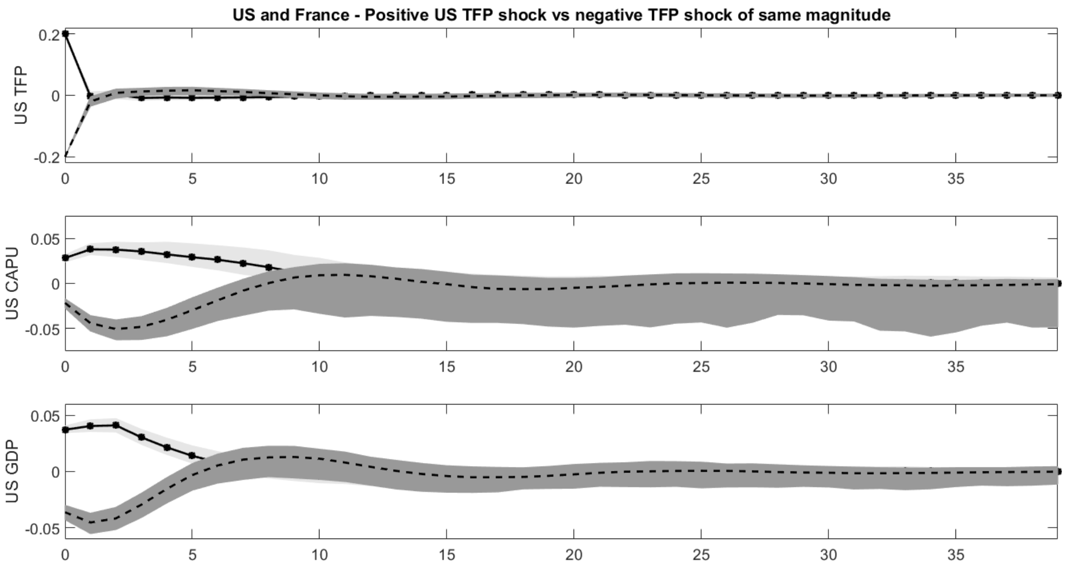
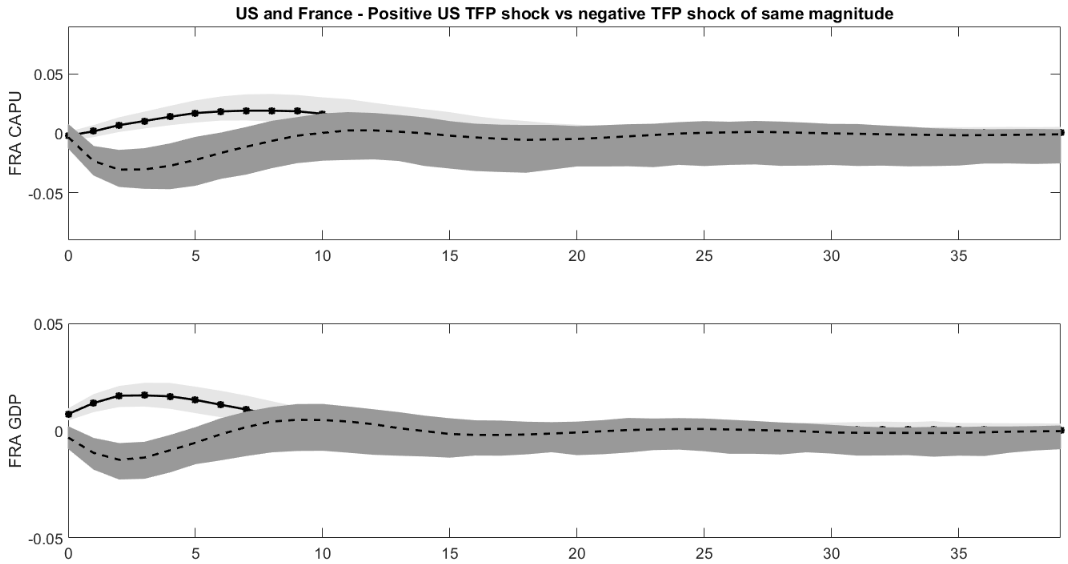


Figure A.4 – US - France impulse response functions - Canada variables



Appendix B

Appendix Chapter 2

B.1 The equation system

- Price indices:

$$\left[\alpha_1 \rho_{H,t}^{1-\omega} + \alpha_2 \rho_{X,t}^{1-\omega} + (1-\alpha_1-\alpha_2)\rho_{FDI,t}^{1-\omega}\right] = 1 \tag{B.1}$$

$$\left[\alpha_1 \rho_{H,t}^{*1-\omega} + \alpha_2 \rho_{X,t}^{*1-\omega} + (1-\alpha_1-\alpha_2)\rho_{FDI,t}^{*1-\omega}\right] = 1 \tag{B.2}$$

- Price indices - Domestic Components:

$$\rho_{H,t} = N_{D,t}^{-\psi} \tilde{\rho}_{D,t} \tag{B.3}$$

$$\rho_{H,t}^* = N_{D,t}^{*-\psi} \tilde{\rho}_{D,t}^* \tag{B.4}$$

- Price indices - Imported Components:

$$\rho_{X,t} = N_{X,t}^{*-\psi} \tilde{\rho}_{X,t}^* \tag{B.5}$$

$$\rho_{X,t}^* = N_{D,t}^{-\psi} \tilde{\rho}_{X,t} \tag{B.6}$$

- Price indices - FDI Components:

$$\rho_{FDI,t} = N_{I,t}^{*-\psi} \tilde{\rho}_{I,t}^* \tag{B.7}$$

$$\rho_{FDI,t}^* = N_{I,t}^{-\psi} \tilde{\rho}_{I,t} \tag{B.8}$$

- Firm level pricing - Domestically sold goods:

$$\tilde{\rho}_{D,t} = \frac{\sigma}{\sigma-1} \frac{w_t}{Z_t \tilde{z}_{D,t}} \tag{B.9}$$

$$\tilde{\rho}_{D,t}^* = \frac{\sigma}{\sigma-1} \frac{w_t^*}{Z_t^* \tilde{z}_{D,t}^*} \tag{B.10}$$

- Firm level pricing - Exported goods:

$$\tilde{\rho}_{X,t} = \frac{\tau_t}{Q_t} \frac{\sigma}{\sigma-1} \frac{w_t}{Z_t \tilde{z}_{X,t}} \tag{B.11}$$

$$\tilde{\rho}_{X,t}^* = \tau_t^* Q_t \frac{\sigma}{\sigma-1} \frac{w_t^*}{Z_t^* \tilde{z}_{X,t}^*} \tag{B.12}$$

- Firm level pricing - FDI goods sold by foreign affiliates:

$$\tilde{\rho}_{I,t} = \frac{\sigma}{\sigma - 1} \frac{w_t^*}{Z_t^* \tilde{z}_{I,t}} \tag{B.13}$$

$$\tilde{\rho}_{I,t}^* = \frac{\sigma}{\sigma - 1} \frac{w_t}{Z_t \tilde{z}_{I,t}}^* \tag{B.14}$$

- Total average profits:

$$\tilde{d}_t = \tilde{d}_{D,t} + \frac{N_{X,t}}{N_{D,t}} \tilde{d}_{X,t} + \frac{N_{I,t}}{N_{D,t}} \tilde{d}_{I,t} \tag{B.15}$$

$$\tilde{d}_t^* = \tilde{d}_{D,t}^* + \frac{N_{X,t}^*}{N_{D,t}^*} \tilde{d}_{X,t}^* + \frac{N_{I,t}^*}{N_{D,t}^*} \tilde{d}_{I,t}^* \tag{B.16}$$

- Average profits from domestically sold goods:

$$\tilde{d}_{D,t} = \frac{1}{\sigma} N_{D,t}^{\psi(\omega-1)-1} \tilde{\rho}_{D,t}^{1-\omega} \alpha_1 C_t \tag{B.17}$$

$$\tilde{d}_{D,t}^* = \frac{1}{\sigma} N_{D,t}^{*\psi(\omega-1)-1} \tilde{\rho}_{D,t}^{*1-\omega} \alpha_1 C_t^* \tag{B.18}$$

- Average profits from exported goods:

$$\tilde{d}_{X,t} = \frac{1}{\sigma} Q_t N_{X,t}^{\psi(\omega-1)-1} \tilde{\rho}_{X,t}^{1-\omega} \alpha_2^* C_t^* - \frac{w_t f_{X,t}}{Z_t} \tag{B.19}$$

$$\tilde{d}_{X,t}^* = \frac{1}{\sigma} \frac{1}{Q_t} N_{X,t}^{*\psi(\omega-1)-1} \tilde{\rho}_{X,t}^{*1-\omega} \alpha_2 C_t - \frac{w_t^* f_{X,t}^*}{Z_t^*} \tag{B.20}$$

- Average profits from FDI goods produced by foreign affiliate:

$$\tilde{d}_{I,t} = Q_t \left[\frac{1}{\sigma} N_{I,t}^{\psi(\omega-1)-1} \tilde{\rho}_{I,t}^{1-\omega} (1 - \alpha_1 - \alpha_2) C_t^* - \frac{w_t^* f_{I,t}^*}{Z_t^*} \right] \tag{B.21}$$

$$\tilde{d}_{I,t}^* = \frac{1}{Q_t} \left[\frac{1}{\sigma} N_{I,t}^{*\psi(\omega-1)-1} \tilde{\rho}_{I,t}^{*1-\omega} (1 - \alpha_1 - \alpha_2) C_t - \frac{w_t f_{I,t}}{Z_t} \right] \tag{B.22}$$

- Free-entry conditions:

$$\tilde{v}_t^s = w_t \frac{f_{E,t}}{Z_{E,t}} \tag{B.23}$$

$$\tilde{v}_t^{s*} = w_t^* \frac{f_{E,t}^*}{Z_{E,t}^*} \tag{B.24}$$

- Optimal labor supply:

$$\chi L_t^{\frac{1}{\psi}} = w_t C_t^{-\gamma} \tag{B.25}$$

$$\chi L_t^{*\frac{1}{\psi}} = w_t^* C_t^{*-\gamma} \tag{B.26}$$

- Labor market clearing:

$$L_t = \frac{(\sigma-1)}{w_t}\left[N_{D,t}\tilde{d}_{D,t} + N_{X,t}\tilde{d}_{X,t} + N^*_{I,t}Q_t\tilde{d}^*_{I,t}\right] + \frac{\sigma}{Z_t}\left[\frac{N_{E,t}f_{E,t}}{\sigma} + N_{X,t}f_{X,t} + N^*_{I,t}f_{I,t}\right] \tag{B.27}$$

$$L^*_t = \frac{(\sigma-1)}{w^*_t}\left[N^*_{D,t}\tilde{d}^*_{D,t} + N^*_{X,t}\tilde{d}^*_{X,t} + N_{I,t}Q_t^{-1}\tilde{d}_{I,t}\right] + \frac{\sigma}{Z^*_t}\left[\frac{N^*_{E,t}f^*_{E,t}}{\sigma} + N^*_{X,t}f^*_{X,t} + N_{I,t}f^*_{I,t}\right] \tag{B.28}$$

- Share of exporters:

$$\frac{N_{X,t}}{N_{D,t}} = (z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k}) \tag{B.29}$$

$$\frac{N^*_{X,t}}{N^*_{D,t}} = (z_{min})^{*k}(\tilde{z}_{X,t}^{*-k} - \tilde{z}_{I,t}^{*-k}) \tag{B.30}$$

- Share of FDI firms:

$$\frac{N_{I,t}}{N_{D,t}} = (z_{min})^k(\tilde{z}_{I,t})^{-k}\left(\frac{k}{k-(\sigma-1)}\right)^{\frac{k}{\sigma-1}} \tag{B.31}$$

$$\frac{N^*_{I,t}}{N^*_{D,t}} = (z_{min})^{*k}(\tilde{z}_{I,t})^{*-k}\left(\frac{k}{k-(\sigma-1)}\right)^{\frac{k}{\sigma-1}}, \tag{B.32}$$

- Zero-profit export cutoff:

$$\tilde{d}_{X,t} = \left[\left(\nabla\left[\frac{\left(\frac{f_{X,t}}{\alpha^*_2}\right)^{\frac{k-\sigma+1}{1-\omega}} - \left[(TOL_t)^{\omega}\,\tau_t^{1-\omega}\left(\frac{f^*_{I,t}}{1-\alpha^*_1-\alpha^*_2}\right)\left(\frac{N_{X,t}}{N_{I,t}}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k-\sigma+1}{1-\omega}}}{\left(\frac{f_{X,t}}{\alpha^*_2}\right)^{\frac{k}{1-\omega}} - \left[(TOL_t)^{\omega}\left(\frac{f^*_{I,t}}{1-\alpha^*_1-\alpha^*_2}\right)\tau_t^{1-\omega}\left(\frac{N_{X,t}}{N_{I,t}}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k}{1-\omega}}}\right]\right)^{\frac{\omega-1}{\sigma-1}}\alpha^*_2 - f_{X,t}\right]\frac{w_t}{Z_t} \tag{B.33}$$

$$\tilde{d}^*_{X,t} = \left[\left(\nabla\left[\frac{\left(\frac{f^*_{X,t}}{\alpha_2}\right)^{\frac{k-\sigma+1}{1-\omega}} - \left[(TOL_t)^{-\omega}\,\tau_t^{1-\omega}\left(\frac{f_{I,t}}{1-\alpha_1-\alpha_2}\right)\left(\frac{N^*_{X,t}}{N^*_{I,t}}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k-\sigma+1}{1-\omega}}}{\left(\frac{f^*_{X,t}}{\alpha_2}\right)^{\frac{k}{1-\omega}} - \left[(TOL_t)^{-\omega}\left(\frac{f_{I,t}}{1-\alpha_1-\alpha_2}\right)\tau_t^{1-\omega}\left(\frac{N^*_{X,t}}{N^*_{I,t}}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k}{1-\omega}}}\right]\right)^{\frac{\omega-1}{\sigma-1}}\alpha_2 - f^*_{X,t}\right]\frac{w^{*}_t{}^*}{Z_t} \tag{B.34}$$

- Zero-profit FDI cutoff:

$$\tilde{d}_{I,t} = Q_t\left(\nabla^{\frac{1-\omega}{1-\sigma}} - 1\right)\frac{w^*_t}{Z^*_t}f^*_{I,t} \tag{B.35}$$

$$\tilde{d}^*_{I,t} = \frac{1}{Q_t}\left(\nabla^{\frac{1-\omega}{1-\sigma}} - 1\right)\frac{w_t}{Z_t}f_{I,t} \tag{B.36}$$

- Law of motion of domestic firms:

$$N_{D,t+1} = (1-\delta)(N_{D,t} + N_{E,t}) \tag{B.37}$$

$$N^*_{D,t+1} = (1-\delta)(N^*_{D,t} + N^*_{E,t}) \tag{B.38}$$

- Terms of Labor:

$$TOL_t = \left(\frac{w_t}{Z_t Q_t}\right)^{-1}\left(\frac{w_t^*}{Z_t^*}\right) \tag{B.39}$$

- Definitions of returns:

$$r_{h,t}^s = (1-\delta)\frac{\tilde{v}_t^s + \tilde{d}_t}{\tilde{v}_{t-1}^s} \tag{B.40}$$

$$r_{f,t}^s = (1-\delta)\frac{\tilde{v}_t^{s*} + \tilde{d}_t^*}{\tilde{v}_{t-1}^{s*}}\frac{Q_t}{Q_{t-1}} \tag{B.41}$$

$$r_{h,t}^b = \frac{v_t^b + d_t^b}{v_{t-1}^b} \tag{B.42}$$

$$r_{f,t}^b = \frac{v_t^{b*} + d_t^{b*}}{v_{t-1}^{b*}}\frac{Q_t}{Q_{t-1}} \tag{B.43}$$

- Euler Home and Foreign:

$$C_t^{-\gamma-\nu}E_t\left[C_{t+1}^{-\gamma}\right] = C_t^{*-\gamma-\nu}E_t\left[*C_{t+1}^{-\gamma}\frac{Q_t}{Q_{t+1}}\right] \tag{B.44}$$

- Euler Home and Foreign:

$$1 = \bar{\beta}C_t^{\gamma-\nu}E_t\{(C_{t+1})^{-\gamma}r_{h,t+1}^s\} \tag{B.45}$$

$$1 = \bar{\beta}C_t^{\gamma-\nu}E_t\{(C_{t+1})^{-\gamma}r_{f,t+1}^s\} \tag{B.46}$$

$$1 = \bar{\beta}C_t^{\gamma-\nu}E_t\{(C_{t+1})^{-\gamma}r_{h,t+1}^b\} \tag{B.47}$$

$$1 = \bar{\beta}C_t^{\gamma-\nu}E_t\{(C_{t+1})^{-\gamma}r_{f,t+1}^b\} \tag{B.48}$$

- Expected Excess Returns:

$$r_{hx,t}^s = r_{h,t}^s - r_{h,t}^b \tag{B.49}$$

$$r_{fx,t}^s = r_{f,t}^s - r_{h,t}^b \tag{B.50}$$

$$r_{fx,t}^b = r_{f,t}^b - r_{h,t}^b \tag{B.51}$$

- Definitions of income:

$$Y_t = w_t L_t + N_{D,t}\tilde{d}_t - N_{E,t}\tilde{v}_t^s \tag{B.52}$$

$$Y_t^* = w_t^* L_t^* + N_{D,t}^*\tilde{d}_t^* - N_{E,t}^*\tilde{v}_t^{s*} \tag{B.53}$$

- Net exports:

$$NX_t = \frac{1}{2}\left[(Y_t - Q_t Y_t^*) - (C_t - Q_t C_t^*)\right] \tag{B.54}$$

- Net foreign assets:

$$NFA_{t+1} = NX_t + NFA_t r_{h,t}^b + a_{f,t-1}r_{fx,t}^s - a_{h,t-1}^{*s}r_{hx,t}^s + a_{f,t-1}^b r_{fx,t}^b \tag{B.55}$$

For the case of welfare indexed bonds bond dividends are $d_t^b = d_t^{*b} = 1$, the bond returns become:

$$r_{h,t}^b = \frac{v_t^b + 1}{v_{t-1}^b} \tag{B.56}$$

$$r_{f,t}^b = \frac{v_t^{b*} + 1}{v_{t-1}^{b*}} \frac{Q_t}{Q_{t-1}} \tag{B.57}$$

B.2 Steady state

I define the following shares:

$$S_{EDO} = \alpha_1 \bar{\rho_H}^{1-\omega} \tag{B.58}$$

$$S_{EXP} = \alpha_2 \bar{\rho_F}^{1-\omega} \tag{B.59}$$

$$S_{EDI} = (1 - \alpha_1 - \alpha_2) \rho \bar{_{FDI}}^{1-\omega}. \tag{B.60}$$

As well as the following ratios of steady state variables to consumption

$$S_D = \frac{\bar{N}_D \bar{d}}{\bar{C}} \tag{B.61}$$

$$S_{DD} = \frac{\bar{N}_D \bar{d}_D}{\bar{C}} \tag{B.62}$$

$$S_X = \frac{\bar{N}_X \bar{d}_X}{\bar{C}} \tag{B.63}$$

$$S_I = \frac{\bar{N}_I \bar{d}_I}{\bar{C}} \tag{B.64}$$

$$S_E = \frac{\bar{N}_E \bar{v}}{\bar{C}} \tag{B.65}$$

$$S_{FX} = \frac{\bar{N}_X \bar{w} f_X}{\bar{C}} \tag{B.66}$$

$$S_{FI} = \frac{\bar{N}_I \bar{w} f_I}{\bar{C}} \tag{B.67}$$

$$S_W = \frac{\bar{w}}{\bar{C}}. \tag{B.68}$$

B.2.1 Solving for the Steady State

We derive the symmetric steady state similarly to the technical appendix of Ghironi and Melitz (2005). First, we seek a way to pin down the average productivity of FDI firms, then we aim to find expressions for the other steady state values based on the average productivity levels. Since the steady state is symmetric all foreign variables are equal to their domestic counterpart, thus there are no asterisks. Furthermore, variables without a time subscript denote steady state values and in steady state $Z = Z_E = Q = TOL = 1$

As Ghironi and Melitz (2005) we fix the share of exporters to be $\frac{N_X}{N_D} = 0.21$ and to simplyfy the steady state derivation, we also fix the share of FDI firms to be $\frac{N_X}{N_D} = 0.10$.

B.2.2 Step 1: Deriving $\tilde{z}_X$ only in terms of $\tilde{z}_I$

The first expression on our way comes from the Euler equation for share holdings:

$$\begin{aligned}
C^{-\gamma} &= \Upsilon(C)C^{-\gamma}[r^b_{f,t+1}] \\
1 &= \Upsilon(C)[r^s_h] \\
1 &= \Upsilon(C)[(1-\delta)\frac{\tilde{v}^s+\tilde{d}}{\tilde{v}^s}] \\
\tilde{v}^s &= \Upsilon(C)(1-\delta)(\tilde{v}^s+\tilde{d}) \\
\tilde{v}^s - \Upsilon(C)(1-\delta)\tilde{v}^s &= \Upsilon(C)(1-\delta)\tilde{d} \\
\tilde{v}^s &= \frac{\Upsilon(C)(1-\delta)}{1-\Upsilon(C)(1-\delta)}\tilde{d}
\end{aligned}$$

Using the definition of total profits $\tilde{d} = \tilde{d}_D + \frac{N_X}{N_D}\tilde{d}_X + \frac{N_I}{N_D}\tilde{d}_I$, we get

$$\tilde{v}^s = \frac{\Upsilon(C)(1-\delta)}{1-\Upsilon(C)(1-\delta)}\left(\tilde{d}_D + \frac{N_X}{N_D}\tilde{d}_X + \frac{N_I}{N_D}\tilde{d}_I\right).$$

And combining this equation with the free entry condition $\tilde{v}^s = w_t f_E$ yields

$$\left(\tilde{d}_D + \frac{N_X}{N_D}\tilde{d}_X + \frac{N_I}{N_D}\tilde{d}_I\right) = \frac{1-\Upsilon(C)(1-\delta)}{\Upsilon(C)(1-\delta)}w_t f_E. \tag{B.69}$$

The second expression is derived from the zero profit export cutoff condition:

$$\tilde{d}_X = \left[\left(\nabla\left[\frac{\left(\frac{f_X}{\alpha_2}\right)^{\frac{k-\sigma+1}{1-\omega}} - \left[\tau^{1-\omega}\left(\frac{f_I}{1-\alpha_1-\alpha_2}\right)\left(\frac{N_X}{N_I}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k-\sigma+1}{1-\omega}}}{\left(\frac{f_X}{\alpha_2}\right)^{\frac{k}{1-\omega}} - \left[\left(\frac{f_I}{1-\alpha_1-\alpha_2}\right)\tau^{1-\omega}\left(\frac{N_X}{N_I}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k}{1-\omega}}}\right]\right)^{\frac{\omega-1}{\sigma-1}}\alpha_2 - f_X\right]w$$

which can be written as

$$\tilde{d}_X = Aw \tag{B.70}$$

with $A = \left[\left(\nabla\left[\frac{\left(\frac{f_X}{\alpha_2}\right)^{\frac{k-\sigma+1}{1-\omega}} - \left[\tau^{1-\omega}\left(\frac{f_I}{1-\alpha_1-\alpha_2}\right)\left(\frac{N_X}{N_I}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k-\sigma+1}{1-\omega}}}{\left(\frac{f_X}{\alpha_2}\right)^{\frac{k}{1-\omega}} - \left[\left(\frac{f_I}{1-\alpha_1-\alpha_2}\right)\tau^{1-\omega}\left(\frac{N_X}{N_I}\right)^{(\psi(\omega-1)-1)}\right]^{\frac{k}{1-\omega}}}\right]\right)^{\frac{\omega-1}{\sigma-1}}\alpha_2 - f_X\right]$. The steady state zero-profit FDI cutoff becomes

$$\begin{aligned}
\tilde{d}_I &= (\nabla - 1)\, w f_I \\
&= \left(\frac{k}{k-(\sigma-1)} - 1\right) w f_I \\
&= \left(\frac{k-k+(\sigma-1)}{k-(\sigma-1)}\right) w f_I
\end{aligned}$$

$$\tilde{d}_I = \left(\frac{\sigma-1}{k-(\sigma-1)}\right) w f_I. \tag{B.71}$$

From the steady state profits,

$$\begin{aligned}
\tilde{d}_D &= \frac{1}{\sigma} N_D^{\psi(\omega-1)-1} \tilde{\rho}_D^{1-\omega} \alpha_1 C \\
\tilde{d}_X &= \frac{1}{\sigma} N_X^{\psi(\omega-1)-1} \tilde{\rho}_X^{1-\omega} \alpha_2 C - w f_X \\
\tilde{d}_I &= \frac{1}{\sigma} N_I^{\psi(\omega-1)-1} \tilde{\rho}_I^{1-\omega} (1-\alpha_1-\alpha_2) C - w f_I
\end{aligned}$$

the second equation implies

$$\tilde{d}_X = \frac{1}{\sigma} N_D^{\psi(\omega-1)-1} \tilde{\rho}_D^{1-\omega} \left(\frac{N_X}{N_D}\right)^{\psi(\omega-1)-1} \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_D}\right)^{1-\omega} \alpha_2 C - w f_X.$$

As

$$\begin{aligned}
\left(\frac{N_X}{N_D}\right)^{\psi(\omega-1)-1} \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_D}\right)^{1-\omega} &= \frac{N_X^{\psi(\omega-1)-1} \tilde{\rho}_X^{1-\omega}}{N_D^{\psi(\omega-1)-1} \tilde{\rho}_D^{1-\omega}} \\
&= \frac{N_D}{N_X} \left(\frac{N_X^{-\psi} \tilde{\rho}_X}{N_D^{-\psi} \tilde{\rho}_D}\right)^{1-\omega} \\
&= \frac{N_D}{N_X} \left(\frac{\tilde{\rho}_F}{\tilde{\rho}_H}\right)^{1-\omega} \\
&= \frac{N_D}{N_X} \frac{\alpha_1}{\alpha_2} \frac{S_{EXP}}{S_{EDO}},
\end{aligned}$$

we have that

$$\begin{aligned}
\tilde{d}_X &= \frac{1}{\sigma} N_D^{\psi(\omega-1)-1} \tilde{\rho}_D^{1-\omega} \frac{N_D}{N_X} \frac{S_{EXP}}{S_{EDO}} \alpha_1 C - w f_X \\
&= \frac{S_{EXP}}{S_{EDO}} \frac{1}{\sigma} N_D^{\psi(\omega-1)-1} \tilde{\rho}_D^{1-\omega} \frac{N_D}{N_X} \alpha_1 C - w f_X \\
&= \frac{N_D}{N_X} \frac{S_{EXP}}{S_{EDO}} \tilde{d}_D - w f_X.
\end{aligned}$$

And using (B.70), we obtain

$$\tilde{d}_D = \frac{N_X}{N_D} \frac{S_{EDO}}{S_{EXP}} (A + f_X) w. \tag{B.72}$$

Similarly, the steady state equation for FDI profits implies

$$\tilde{d}_I = \frac{1}{\sigma} N_I^{\psi(\omega-1)-1} \tilde{\rho}_I^{1-\omega} (1-\alpha_1-\alpha_2)C - wf_I$$
$$= \left(\frac{N_I}{N_D}\right)^{\psi(\omega-1)-1} \left(\frac{\tilde{\rho}_I}{\tilde{\rho}_D}\right)^{1-\omega} \frac{1}{\sigma} N_D^{\psi(\omega-1)-1} \tilde{\rho}_D^{1-\omega} (1-\alpha_1-\alpha_2)C - wf_I.$$

As

$$\left(\frac{N_I}{N_D}\right)^{\psi(\omega-1)-1} \left(\frac{\tilde{\rho}_I}{\tilde{\rho}_D}\right)^{1-\omega} = \frac{N_I^{\psi(\omega-1)-1} \tilde{\rho}_I^{1-\omega}}{N_D^{\psi(\omega-1)-1} \tilde{\rho}_D^{1-\omega}}$$
$$= \frac{N_D}{N_I} \left(\frac{N_I^{-\psi} \tilde{\rho}_I}{N_D^{-\psi} \tilde{\rho}_D}\right)^{1-\omega}$$
$$= \frac{N_D}{N_I} \left(\frac{\tilde{\rho}_{FDI}}{\tilde{\rho}_H}\right)^{1-\omega}$$
$$= \frac{N_D}{N_I} \frac{\alpha_1}{1-\alpha_1-\alpha_2} \frac{S_{EDI}}{S_{EDO}}$$

we have that

$$\tilde{d}_I = \frac{N_D}{N_I} \frac{\alpha_1}{1-\alpha_1-\alpha_2} \frac{S_{EDI}}{S_{EDO}} \frac{1}{\sigma} N_D^{\psi(\omega-1)-1} \tilde{\rho}_D^{1-\omega} (1-\alpha_1-\alpha_2)C - wf_I$$
$$= \frac{N_D}{N_I} \frac{S_{EDI}}{S_{EDO}} \frac{1}{\sigma} N_D^{\psi(\omega-1)-1} \tilde{\rho}_D^{1-\omega} \alpha_1 C - wf_I$$
$$= \frac{N_D}{N_I} \frac{S_{EDI}}{S_{EDO}} \tilde{d}_D - wf_I.$$

And using (B.71), we obtain

$$\tilde{d}_D = \frac{N_I}{N_D} \frac{S_{EDO}}{S_{EDI}} \left(\frac{k}{k-(\sigma-1)}\right) wf_I. \tag{B.73}$$

Recalling the pricing equations in steady state

$$\tilde{\rho}_D = \frac{\sigma}{\sigma-1} \frac{w}{\tilde{z}_D} \tag{B.74}$$
$$\tilde{\rho}_X = \tau \frac{\sigma}{\sigma-1} \frac{w}{\tilde{z}_X} \tag{B.75}$$
$$\tilde{\rho}_I = \frac{\sigma}{\sigma-1} \frac{w}{\tilde{z}_I}, \tag{B.76}$$

the coefficient ratios can be written in terms of the average productivity levels $\tilde{z}_D$,

$\tilde{z}_X$ and $\tilde{z}_I$ as follows

$$\begin{aligned}\frac{S_{EDO}}{S_{EXP}} &= \frac{\alpha_1 N_D^{-\psi(1-\omega)}\tilde{\rho}_D^{1-\omega}}{\alpha_2 N_X^{-\psi(1-\omega)}\tilde{\rho}_X^{1-\omega}}\\ &= \frac{\alpha_1}{\alpha_2}\left(\frac{N_D}{N_X}\right)^{-\psi(1-\omega)}\left(\frac{\tilde{\rho}_D}{\tilde{\rho}_X}\right)^{1-\omega}\\ &= \frac{\alpha_1}{\alpha_2}\left(\frac{N_D}{N_X}\right)^{-\psi(1-\omega)}\left(\frac{\frac{\sigma}{\sigma-1}\frac{w}{\tilde{z}_D}}{\tau\frac{\sigma}{\sigma-1}\frac{w}{\tilde{z}_X}}\right)^{1-\omega}\\ &= \frac{\alpha_1}{\alpha_2}\left(\frac{N_D}{N_X}\right)^{-\psi(1-\omega)}\left(\frac{\tilde{z}_X}{\tau\tilde{z}_D}\right)^{1-\omega}\end{aligned}$$

$$\begin{aligned}\frac{S_{EDO}}{S_{EDI}} &= \frac{\alpha_1 N_D^{-\psi(1-\omega)}\tilde{\rho}_D^{1-\omega}}{(1-\alpha_1-\alpha_2) N_I^{-\psi(1-\omega)}\tilde{\rho}_I^{1-\omega}}\\ &= \frac{\alpha_1}{1-\alpha_1-\alpha_2}\left(\frac{N_D}{N_I}\right)^{-\psi(1-\omega)}\left(\frac{\tilde{\rho}_D}{\tilde{\rho}_I}\right)^{1-\omega}\\ &= \frac{\alpha_1}{1-\alpha_1-\alpha_2}\left(\frac{N_D}{N_I}\right)^{-\psi(1-\omega)}\left(\frac{\frac{\sigma}{\sigma-1}\frac{w}{\tilde{z}_D}}{\frac{\sigma}{\sigma-1}\frac{w}{\tilde{z}_I}}\right)^{1-\omega}\\ &= \frac{\alpha_1}{1-\alpha_1-\alpha_2}\left(\frac{N_D}{N_I}\right)^{-\psi(1-\omega)}\left(\frac{\tilde{z}_I}{\tilde{z}_D}\right)^{1-\omega}.\end{aligned}$$

Thus,

$$\tilde{d}_D = \frac{\alpha_1}{\alpha_2}\left(\frac{N_D}{N_X}\right)^{\psi(\omega-1)-1}\left(\frac{\tilde{z}_X}{\tau\tilde{z}_D}\right)^{1-\omega}(A+f_X)\,w \tag{B.77}$$

$$\tilde{d}_D = \frac{\alpha_1}{1-\alpha_1-\alpha_2}\left(\frac{N_D}{N_I}\right)^{\psi(\omega-1)-1}\left(\frac{\tilde{z}_I}{\tilde{z}_D}\right)^{1-\omega}\left(\frac{k}{k-(\sigma-1)}\right) w f_I. \tag{B.78}$$

Combining the two yields

$$\begin{aligned}&\frac{\alpha_1}{\alpha_2}\left(\frac{N_D}{N_X}\right)^{\psi(\omega-1)-1}\left(\frac{\tilde{z}_X}{\tau\tilde{z}_D}\right)^{1-\omega}(A+f_X)\,w\\ &\quad= \frac{\alpha_1}{1-\alpha_1-\alpha_2}\left(\frac{N_D}{N_I}\right)^{\psi(\omega-1)-1}\left(\frac{\tilde{z}_I}{\tilde{z}_D}\right)^{1-\omega}\left(\frac{k}{k-(\sigma-1)}\right) w f_I\end{aligned}$$

$$\tilde{z}_X = \tau\left(\frac{\alpha_2}{1-\alpha_1-\alpha_2}\frac{k}{k-(\sigma-1)}\right)^{\frac{1}{1-\omega}}\left(\frac{N_X}{N_I}\right)^{\frac{\psi(\omega-1)-1}{1-\omega}}\left(\frac{f_I}{A+f_X}\right)^{\frac{1}{1-\omega}}\tilde{z}_I$$

which directly relates $\tilde{z}_X$ to $\tilde{z}_I$ and depends only on parameters once $\frac{N_X}{N_D}$ and $\frac{N_I}{N_D}$ are fixed, as in our calibration.

B.2.3 Step 2: Deriving $\tilde{z}_I$ only in terms of parameters

The steady state share of exporting and FDI firms in the total number of domestic firms is:

$$\frac{N_{X,t}}{N_{D,t}} = (z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k}) \tag{B.79}$$

$$\frac{N_{I,t}}{N_{D,t}} = (z_{min})^k(\tilde{z}_{I,t})^{-k}\left(\frac{k}{k-(\sigma-1)}\right)^{\frac{k}{\sigma-1}}. \tag{B.80}$$

Using

$$\tilde{z}_D = \nabla^{\frac{1}{\sigma-1}} z_{min} = \left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{\sigma-1}} z_{min} \tag{B.81}$$

we derive an expression that implicitly defines $\tilde{z}_X$ in terms of parameters.

From

$$\left(\tilde{d}_D + \frac{N_X}{N_D}\tilde{d}_X + \frac{N_I}{N_D}\tilde{d}_I\right) = \frac{1-\Upsilon(C)(1-\delta)}{\Upsilon(C)(1-\delta)} w_t f_E$$

$$\left(\frac{\alpha_1}{1-\alpha_1-\alpha_2}\left(\frac{N_D}{N_I}\right)^{\psi(\omega-1)-1}\left(\frac{\tilde{z}_I}{\tilde{z}_D}\right)^{1-\omega}\left(\frac{k}{k-(\sigma-1)}\right) w f_I\right) + \left((z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k})\right) Aw$$
$$+\left((z_{min})^k(\tilde{z}_{I,t})^{-k}\left(\frac{k}{k-(\sigma-1)}\right)^{\frac{k}{\sigma-1}}\right)\left(\left(\frac{\sigma-1}{k-(\sigma-1)}\right) w f_I\right) = \frac{1-\Upsilon(C)(1-\delta)}{\Upsilon(C)(1-\delta)} w f_E$$

$$\left(\frac{\alpha_1}{1-\alpha_1-\alpha_2}\left(\frac{N_D}{N_I}\right)^{\psi(\omega-1)-1}\left(\frac{\tilde{z}_I}{\tilde{z}_D}\right)^{1-\omega}\nabla f_I\right) + \left((z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k})\right) A$$
$$+\left((z_{min})^k(\tilde{z}_{I,t})^{-k}\nabla^{\frac{k}{\sigma-1}}\right)\left((\nabla-1) f_I\right) = \frac{1-\Upsilon(C)(1-\delta)}{\Upsilon(C)(1-\delta)} f_E.$$

Using $\tilde{z}_D = \nabla^{\frac{1}{\sigma-1}} z_{min}$ and $\tilde{z}_X = \tau\left(\frac{\alpha_2}{1-\alpha_1-\alpha_2}\nabla\right)^{\frac{1}{1-\omega}}\left(\frac{N_X}{N_I}\right)^{\frac{\psi(\omega-1)-1}{1-\omega}}\left(\frac{f_I}{A+f_X}\right)^{\frac{1}{1-\omega}}\tilde{z}_I$ we obtain

$$\left(\frac{\alpha_1}{1-\alpha_1-\alpha_2}\left(\frac{N_D}{N_I}\right)^{\psi(\omega-1)-1}\left(\frac{\tilde{z}_I}{\nabla^{\frac{1}{\sigma-1}} z_{min}}\right)^{1-\omega}\nabla f_I\right)$$
$$+\left((z_{min})^k\left[\left(\tau\left(\frac{\alpha_2}{1-\alpha_1-\alpha_2}\nabla\right)^{\frac{1}{1-\omega}}\left(\frac{N_X}{N_I}\right)^{\frac{\psi(\omega-1)-1}{1-\omega}}\left(\frac{f_I}{A+f_X}\right)^{\frac{1}{1-\omega}}\tilde{z}_I\right)^{-k} - \tilde{z}_{I,t}^{-k}\right]\right) A$$
$$+\left((z_{min})^k(\tilde{z}_{I,t})^{-k}\nabla^{\frac{k}{\sigma-1}}\right)\left((\nabla-1) f_I\right) = \frac{1-\Upsilon(C)(1-\delta)}{\Upsilon(C)(1-\delta)} f_E.$$

which in essence determines $\tilde{z}_I$ in terms of parameters once $\frac{N_X}{N_D}$ and $\frac{N_I}{N_D}$ are fixed, as in our calibration.

B.2.4 Step 3: Solving for prices

The law of motion of the number of domestic firms in the symmetric steady state implies

$$N_E = \frac{\delta}{1-\delta} N_D. \tag{B.82}$$

From aggregating individual budget constraints in a symmetric steady state, we obtain

$$C + \tilde{v}^s N_E = wL + N_D \tilde{d}.$$

Using $\tilde{v}^s = wf_{E,t}$ this becomes

$$C + wf_{E,t} N_E = wL + N_D \tilde{d}.$$

From step 1 above we have

$$\begin{aligned} \tilde{d} &= \frac{1-\Upsilon(C)(1-\delta)}{\Upsilon(C)(1-\delta)} wf_E \\ C + wf_E N_E - wL &= \frac{1-\Upsilon(C)(1-\delta)}{\Upsilon(C)(1-\delta)} N_D wf_E. \end{aligned}$$

Using the law of motion

$$\begin{aligned} C + wf_E \frac{\delta}{1-\delta} N_D - wL &= \frac{1-\Upsilon(C)(1-\delta)}{\Upsilon(C)(1-\delta)} N_D wf_E \\ C &= \frac{1-\Upsilon(C)(1-\delta)}{\Upsilon(C)(1-\delta)} N_D wf_E - wf_{E,t} \frac{\delta}{1-\delta} N_D + wL \\ C &= \left(\frac{1-\Upsilon(C)(1-\delta)}{\Upsilon(C)(1-\delta)} - \frac{\delta}{1-\delta} \right) N_D wf_E + wL \\ C &= \left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)} \right) N_D wf_E + wL. \end{aligned}$$

In a final step this can be written as

$$\frac{C}{w} = L + \left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)} \right) N_D f_E. \tag{B.83}$$

Additionally, from the zero-profit FDI cutoff (B.71) and the steady state expression for FDI profits, we have

$$\begin{aligned} \tilde{d}_I &= \left(\frac{\sigma-1}{k-(\sigma-1)} \right) wf_I \\ \frac{1}{\sigma} N_I^{\psi(\omega-1)-1} \tilde{\rho}_I^{1-\omega} (1-\alpha_1-\alpha_2) C - wf_I &= \left(\frac{\sigma-1}{k-(\sigma-1)} \right) wf_I \\ \frac{1}{\sigma} N_I^{\psi(\omega-1)-1} \tilde{\rho}_I^{1-\omega} (1-\alpha_1-\alpha_2) C &= \left(1 + \frac{\sigma-1}{k-(\sigma-1)} \right) wf_I \\ \frac{C}{w} &= \frac{\sigma}{1-\alpha_1-\alpha_2} N_I^{1-\psi(\omega-1)} \tilde{\rho}_I^{\omega-1} \nabla f_I. \end{aligned}$$

Similarly, from the zero-profit export condition (B.70) and the steady state expression for export profits, we have

$$\begin{aligned}
\tilde{d}_X &= Aw \\
\frac{1}{\sigma} N_X^{\psi(\omega-1)-1} \tilde{\rho}_X^{1-\omega} \alpha_2 C - w f_X &= Aw \\
\frac{1}{\sigma} N_X^{\psi(\omega-1)-1} \tilde{\rho}_X^{1-\omega} \alpha_2 C &= (A + f_X)\, w \\
\frac{C}{w} &= \frac{\sigma}{\alpha_2} N_X^{1-\psi(\omega-1)} \tilde{\rho}_X^{\omega-1} (A + f_X)\,.
\end{aligned}$$

Now, from the price index equation in steady state

$$\alpha_1 \rho_H^{1-\omega} + \alpha_2 \rho_X^{1-\omega} + (1 - \alpha_1 - \alpha_2) \rho_{FDI}^{1-\omega} = 1$$

we write

$$\begin{aligned}
(1-\alpha_1-\alpha_2)(N_I^{-\psi}\tilde{\rho}_I)^{1-\omega} &= 1 - \alpha_1 (N_D^{-\psi}\tilde{\rho}_D)^{1-\omega} - \alpha_2 (N_X^{-\psi}\tilde{\rho}_X)^{1-\omega} \\
(1-\alpha_1-\alpha_2)(\left(\frac{N_I}{N_D}\right)^{-\psi} \tilde{\rho}_I)^{1-\omega} &= \left(\frac{1}{N_D^{-\psi(1-\omega)}}\right) - \alpha_1(\tilde{\rho}_D)^{1-\omega} - \alpha_2(\left(\frac{N_X}{N_D}\right)^{-\psi} \tilde{\rho}_X)^{1-\omega} \\
\left(\frac{N_I}{N_D}\right)^{-\psi(1-\omega)} \tilde{\rho}_I^{1-\omega} &= \frac{1}{(1-\alpha_1-\alpha_2)} \left(\left(\frac{1}{N_D^{-\psi(1-\omega)}}\right) - \alpha_1(\tilde{\rho}_D)^{1-\omega} - \alpha_2(\left(\frac{N_X}{N_D}\right)^{-\psi} \tilde{\rho}_X)^{1-\omega} \right) \\
\frac{\tilde{\rho}_I^{(\omega-1)}}{N_D^{-\psi(1-\omega)}} &= (1-\alpha_1-\alpha_2)\left(\frac{N_I}{N_D}\right)^{-\psi(1-\omega)} + \alpha_1 \left(\frac{\tilde{\rho}_D}{\tilde{\rho}_I}\right)^{1-\omega} + \alpha_2 \left(\frac{N_X}{N_D}\right)^{-\psi(1-\omega)} \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_I}\right)^{1-\omega}.
\end{aligned}$$

Using the ratios

$$\begin{aligned}
\frac{\tilde{\rho}_I}{\tilde{\rho}_D} &= \frac{\tilde{z}_D}{\tilde{z}_I} \\
\frac{\tilde{\rho}_X}{\tilde{\rho}_D} &= \tau \frac{\tilde{z}_D}{\tilde{z}_X} \\
\frac{\tilde{\rho}_I}{\tilde{\rho}_X} &= \frac{\tilde{z}_X}{\tau \tilde{z}_I}
\end{aligned}$$

as well as,

$$\begin{aligned}
\frac{N_{X,t}}{N_{D,t}} &= (z_{min})^k (\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k}) \\
\frac{N_{I,t}}{N_{D,t}} &= (z_{min})^k (\tilde{z}_{I,t})^{-k} \nabla^{\frac{k}{\sigma-1}},
\end{aligned}$$

we obtain

$$\begin{aligned}
\frac{\tilde{\rho}_I^{(\omega-1)}}{N_D^{-\psi(1-\omega)}} = (1-\alpha_1-\alpha_2) &\left((z_{min})^k (\tilde{z}_{I,t})^{-k} \nabla^{\frac{k}{\sigma-1}} \right)^{-\psi(1-\omega)} \\
&+ \alpha_1 \left(\frac{\tilde{z}_D}{\tilde{z}_I}\right)^{\omega-1} + \alpha_2 \left((z_{min})^k (\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k}) \right)^{-\psi(1-\omega)} \left(\frac{\tilde{z}_X}{\tau \tilde{z}_I}\right)^{\omega-1}. \quad \text{(B.84)}
\end{aligned}$$

As z_{min} is a parameter and we have already determined $\tilde{z}_D, \tilde{z}_X$ and $\tilde{z}_I$, we can summarize the right-hand side of the equation as

$$
\begin{aligned}
BB = (1-\alpha_1-\alpha_2)\left((z_{min})^k(\tilde{z}_{I,t})^{-k}\nabla^{\frac{k}{\sigma-1}}\right)^{-\psi(1-\omega)} \\
+\alpha_1\left(\frac{\tilde{z}_D}{\tilde{z}_I}\right)^{\omega-1}+\alpha_2\left((z_{min})^k(\tilde{z}_{X,t}^{-k}-\tilde{z}_{I,t}^{-k})\right)^{-\psi(1-\omega)}\left(\frac{\tilde{z}_X}{\tau\tilde{z}_I}\right)^{\omega-1}
\end{aligned}
$$

and simplify the whole equations to

$$
N_D=\left(\frac{\tilde{\rho}_I^{(\omega-1)}}{BB}\right)^{\frac{1}{-\psi(1-\omega)}}=\left(\frac{\tilde{\rho}_I^{\frac{(\omega-1)}{\psi(\omega-1)}}}{BB^{\frac{1}{-\psi(1-\omega)}}}\right)=\frac{\tilde{\rho}_I^{\frac{1}{\psi}}}{BB^{\frac{1}{\psi(\omega-1)}}}. \tag{B.85}
$$

From the equations above, we obtain

$$
L+\left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)}\right)N_D f_E=\frac{\sigma}{1-\alpha_1-\alpha_2}N_I^{1-\psi(\omega-1)}\tilde{\rho}_I^{\omega-1}\nabla f_I.
$$

With $N_D=\frac{\tilde{\rho}_I^{\frac{1}{\psi}}}{BB^{\frac{1}{\psi(\omega-1)}}}$ and $N_{I,t}=(z_{min})^k(\tilde{z}_{I,t})^{-k}\nabla^{\frac{k}{\sigma-1}}N_{D,t}$, we get

$$
\begin{aligned}
L+\left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)}\right)\left(\frac{\tilde{\rho}_I^{\frac{1}{\psi}}}{BB^{\frac{1}{\psi(\omega-1)}}}\right)f_E \\
=\frac{\sigma}{1-\alpha_1-\alpha_2}\left((z_{min})^k(\tilde{z}_{I,t})^{-k}\nabla^{\frac{k}{\sigma-1}}\frac{\tilde{\rho}_I^{\frac{1}{\psi}}}{BB^{\frac{1}{\psi(\omega-1)}}}\right)^{1-\psi(\omega-1)}\tilde{\rho}_I^{\omega-1}\nabla f_I. \\
L=\frac{\sigma}{1-\alpha_1-\alpha_2}\left((z_{min})^k(\tilde{z}_{I,t})^{-k}\nabla^{\frac{k}{\sigma-1}}\right)^{1-\psi(\omega-1)}BB^{\frac{\psi(\omega-1)-1}{\psi(\omega-1)}}\tilde{\rho}_I^{\frac{1}{\psi}}\nabla f_I \\
-\left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)}\right)BB^{-\frac{1}{\psi(\omega-1)}}\tilde{\rho}_I^{\frac{1}{\psi}}f_E.
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\rho}_I=[\frac{\sigma}{1-\alpha_1-\alpha_2}\left((z_{min})^k(\tilde{z}_{I,t})^{-k}\nabla^{\frac{k}{\sigma-1}}\right)^{1-\psi(\omega-1)}BB^{\frac{\psi(\omega-1)-1}{\psi(\omega-1)}}\nabla f_I \\
-\left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)}\right)BB^{-\frac{1}{\psi(\omega-1)}}f_E]^{-\psi}\left(\frac{1}{L}\right)^{-\psi}.
\end{aligned} \tag{B.86}
$$

This equation implicitly define $\tilde{\rho}_I$ in terms of parameters. Using a similar approach, we can derive an expression for $\tilde{\rho}_X$ in terms of parameters. Again starting from the price index equation in steady state

$$
\alpha_1\rho_H^{1-\omega}+\alpha_2\rho_X^{1-\omega}+(1-\alpha_1-\alpha_2)\rho_{FDI}^{1-\omega}=1
$$

we write

$$\alpha_2\left(N_X^{-\psi}\tilde{\rho}_X\right)^{1-\omega} = 1-\alpha_1\left(N_D^{-\psi}\tilde{\rho}_D\right)^{1-\omega} - (1-\alpha_1-\alpha_2)\left((N_I^{-\psi}\tilde{\rho}_I\right)^{1-\omega}$$

$$\alpha_2\left(\left(\frac{N_X}{N_D}\right)^{-\psi}\tilde{\rho}_X\right)^{1-\omega} = \left(\frac{1}{N_D^{-\psi(1-\omega)}}\right) - \alpha_1\left(\tilde{\rho}_D\right)^{1-\omega} - (1-\alpha_1-\alpha_2)\left(\left(\frac{N_I}{N_D}\right)^{-\psi}\tilde{\rho}_I\right)^{1-\omega}$$

$$\left(\left(\frac{N_X}{N_D}\right)^{-\psi}\tilde{\rho}_X\right)^{1-\omega} = \frac{1}{\alpha_2}\left[\left(\frac{1}{N_D^{-\psi(1-\omega)}}\right) - \alpha_1\left(\tilde{\rho}_D\right)^{1-\omega} - (1-\alpha_1-\alpha_2)\left(\left(\frac{N_I}{N_D}\right)^{-\psi}\tilde{\rho}_I\right)^{1-\omega}\right]$$

$$\left(\frac{N_X}{N_D}\right)^{-\psi(1-\omega)} = \frac{1}{\alpha_2}\left[\left(\frac{1}{\left(N_D^{-\psi}\tilde{\rho}_X\right)^{(1-\omega)}}\right) - \alpha_1\left(\frac{\tilde{\rho}_D}{\tilde{\rho}_X}\right)^{1-\omega} - (1-\alpha_1-\alpha_2)\left(\left(\frac{N_I}{N_D}\right)^{-\psi}\frac{\tilde{\rho}_I}{\tilde{\rho}_X}\right)^{1-\omega}\right]$$

$$\frac{\tilde{\rho}_X^{\omega-1}}{N_D^{-\psi(1-\omega)}} = \alpha_2\left(\frac{N_X}{N_D}\right)^{-\psi(1-\omega)} + \alpha_1\left(\frac{\tilde{\rho}_D}{\tilde{\rho}_X}\right)^{1-\omega} + (1-\alpha_1-\alpha_2)\left(\left(\frac{N_I}{N_D}\right)^{-\psi}\frac{\tilde{\rho}_I}{\tilde{\rho}_X}\right)^{1-\omega}$$

Again using the ratios

$$\frac{\tilde{\rho}_I}{\tilde{\rho}_D} = \frac{\tilde{z}_D}{\tilde{z}_I}$$
$$\frac{\tilde{\rho}_X}{\tilde{\rho}_D} = \tau\frac{\tilde{z}_D}{\tilde{z}_X}$$
$$\frac{\tilde{\rho}_I}{\tilde{\rho}_X} = \frac{\tilde{z}_X}{\tau\tilde{z}_I}$$

as well as,

$$\frac{N_{X,t}}{N_{D,t}} = (z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k})$$
$$\frac{N_{I,t}}{N_{D,t}} = (z_{min})^k(\tilde{z}_{I,t})^{-k}\nabla^{\frac{k}{\sigma-1}},$$

we obtain

$$\frac{\tilde{\rho}_X^{\omega-1}}{N_D^{-\psi(1-\omega)}} = \alpha_2\left((z_{min})^k(\tilde{z}_X^{-k} - \tilde{z}_I^{-k})\right)^{-\psi(1-\omega)} + \alpha_1\left(\frac{\tilde{z}_X}{\tau\tilde{z}_D}\right)^{1-\omega} + (1-\alpha_1-\alpha_2)\left(\left((z_{min})^k(\tilde{z}_I)^{-k}\nabla^{\frac{k}{\sigma-1}}\right)^{-\psi}\frac{\tilde{z}_X}{\tau\tilde{z}_I}\right)^{1-\omega} \quad \text{(B.87)}$$

As z_{min} is a parameter and we have already determined $\tilde{z}_D, \tilde{z}_X$ and $\tilde{z}_I$, we can summarize the right-hand side of the equation as

$$CC = \alpha_2\left((z_{min})^k(\tilde{z}_X^{-k} - \tilde{z}_I^{-k})\right)^{-\psi(1-\omega)} + \alpha_1\left(\frac{\tilde{z}_X}{\tau\tilde{z}_D}\right)^{1-\omega} + (1-\alpha_1-\alpha_2)\left(\left((z_{min})^k(\tilde{z}_I)^{-k}\nabla^{\frac{k}{\sigma-1}}\right)^{-\psi}\frac{\tilde{z}_X}{\tau\tilde{z}_I}\right)^{1-\omega}$$

and simplify the whole equations to

$$N_D = \left(\frac{\tilde{\rho}_X^{(\omega-1)}}{CC}\right)^{\frac{1}{-\psi(1-\omega)}} = \left(\frac{\tilde{\rho}_X^{\frac{(\omega-1)}{\psi(\omega-1)}}}{CC^{\frac{1}{-\psi(1-\omega)}}}\right) = \frac{\tilde{\rho}_X^{\frac{1}{\psi}}}{CC^{\frac{1}{\psi(\omega-1)}}}. \quad \text{(B.88)}$$

From the equations above, we obtain

$$L + \left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)}\right) N_D f_E = \frac{\sigma}{\alpha_2} N_X^{1-\psi(\omega-1)} \tilde{\rho}_X^{\omega-1} \left(A + f_X\right). \tag{B.89}$$

With $N_D = \left(\frac{\tilde{\rho}_X^{\frac{1}{\psi}}}{CC^{\frac{1}{\psi(\omega-1)}}}\right)$ and $N_{X,t} = \left((z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k})N_D\right)$, we get

$$\begin{aligned}
&L + \left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)}\right)\left(\frac{\tilde{\rho}_X^{\frac{1}{\psi}}}{CC^{\frac{1}{\psi(\omega-1)}}}\right) f_E \\
&= \frac{\sigma}{\alpha_2}\left((z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k})\left(\frac{\tilde{\rho}_X^{\frac{1}{\psi}}}{CC^{\frac{1}{\psi(\omega-1)}}}\right)\right)^{1-\psi(\omega-1)} \tilde{\rho}_X^{\omega-1}\left(A + f_X\right) \\
&= \frac{\sigma}{\alpha_2}\left((z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k})\right)^{1-\psi(\omega-1)}\left(\frac{\tilde{\rho}_X^{\frac{1-\psi(\omega-1)}{\psi}}}{CC^{\frac{1-\psi(\omega-1)}{\psi(\omega-1)}}}\right) \tilde{\rho}_X^{\omega-1}\left(A + f_X\right) \\
&= \frac{\sigma}{\alpha_2}\left((z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k})\right)^{1-\psi(\omega-1)}\left(\frac{\tilde{\rho}_X^{\frac{1}{\psi}}}{CC^{\frac{1-\psi(\omega-1)}{\psi(\omega-1)}}}\right)\left(A + f_X\right)
\end{aligned}$$

Which simplifies to

$$\begin{aligned}
&L = \\
&= \left[\frac{\sigma}{\alpha_2}\left((z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k})\right)^{1-\psi(\omega-1)}\left(\frac{\tilde{\rho}_X^{\frac{1}{\psi}}}{CC^{\frac{1-\psi(\omega-1)}{\psi(\omega-1)}}}\right)(A + f_X) - \left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)}\right)\left(\frac{\tilde{\rho}_X^{\frac{1}{\psi}}}{CC^{\frac{1}{\psi(\omega-1)}}}\right) f_E\right] \\
&= \left[\frac{\sigma}{\alpha_2}\left((z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k})\right)^{1-\psi(\omega-1)}\left(CC^{\frac{\psi(\omega-1)-1}{\psi(1-\omega)}}\right)(A + f_X) - \left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)}\right)\left(CC^{-\frac{1}{\psi(\omega-1)}}\right) f_E\right]\tilde{\rho}_X^{\frac{1}{\psi}}
\end{aligned} \tag{B.90}$$

or

$$\begin{aligned}
\tilde{\rho}_X = [&\frac{\sigma}{\alpha_2}\left((z_{min})^k(\tilde{z}_{X,t}^{-k} - \tilde{z}_{I,t}^{-k})\right)^{1-\psi(\omega-1)}\left(CC^{\frac{\psi(\omega-1)-1}{\psi(1-\omega)}}\right)(A + f_X) \\
&- \left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)}\right)\left(CC^{-\frac{1}{\psi(\omega-1)}}\right) f_E]^{-\psi}\left(\frac{1}{L}\right)^{-\psi}
\end{aligned} \tag{B.91}$$

This equation implicitly define $\tilde{\rho}_X$ in terms of parameters and variables we have solved for before.

B.2.5 Step 3: Solving for other steady state variables

We then have solutions for $\tilde{z}_D, \tilde{z}_X, \tilde{z}_I, \tilde{\rho}_X, \tilde{\rho}_I$ and can determine the rest of the steady state values.

$$N_D = \frac{\tilde{\rho}_X^{\frac{1}{\psi}}}{CC^{\frac{1}{\psi(\omega-1)}}} \tag{B.92}$$

$$N_X = \left((z_{min})^k(\tilde{z}_X^{-k} - \tilde{z}_I^{-k})N_D\right) \tag{B.93}$$

$$N_I = (z_{min})^k(\tilde{z}_I)^{-k}\nabla^{\frac{k}{\sigma-1}}N_D \tag{B.94}$$

$$\tilde{\rho}_D = \frac{\tilde{z}_I\tilde{\rho}_I}{\tilde{z}_D} \tag{B.95}$$

$$\tilde{\rho}_D = \frac{\tilde{z}_I\tilde{\rho}_I}{\tilde{z}_D} \tag{B.96}$$

From $\tilde{\rho}_X = \tau\frac{\sigma}{\sigma-1}\frac{w}{\tilde{z}_X}$, we have

$$w = \frac{\sigma-1}{\tau\sigma}\tilde{\rho}_X\tilde{z}_X. \tag{B.97}$$

$$C = \left[L + \left(\frac{1-\Upsilon(C)}{\Upsilon(C)(1-\delta)}\right)N_D f_E\right]w \tag{B.98}$$

$$N_E = \frac{\delta}{1-\delta}N_D \tag{B.99}$$

- Price indices - Domestic Components:

$$\rho_H = N_D^{-\psi}\tilde{\rho}_D \tag{B.100}$$

- Price indices - Imported Components:

$$\rho_X = N_X^{-\psi}\tilde{\rho}_X \tag{B.101}$$

- Price indices - FDI Components:

$$\rho_{FDI} = N_I^{-\psi}\tilde{\rho}_I \tag{B.102}$$

- Average profits from domestically sold goods:

$$\tilde{d}_D = \frac{1}{\sigma}N_D^{\psi(\omega-1)-1}\tilde{\rho}_D^{1-\omega}\alpha_1 C \tag{B.103}$$

- Average profits from exported goods:

$$\tilde{d}_X = \frac{1}{\sigma}N_X^{\psi(\omega-1)-1}\tilde{\rho}_X^{1-\omega}\alpha_2 C - wf_X \tag{B.104}$$

- Average profits from FDI goods produced by foreign affiliate:

$$\tilde{d}_I = \frac{1}{\sigma}N_I^{\psi(\omega-1)-1}\tilde{\rho}_I^{1-\omega}(1-\alpha_1-\alpha_2)C - wf_I \tag{B.105}$$

- Total average profits:

$$\tilde{d} = \tilde{d}_D + \frac{N_X}{N_D}\tilde{d}_X + \frac{N_I}{N_D}\tilde{d}_I \tag{B.106}$$

B.3 Log-linearized system

- Price indices:

$$\alpha_1 \mathsf{p}_{H,t} + \alpha_2 \mathsf{p}_{F,t} + (1-\alpha_1-\alpha_2)\mathsf{p}_{FDI,t} = 0 \tag{B.107}$$
$$\alpha_1 \mathsf{p}^*_{H,t} + \alpha_2 \mathsf{p}^*_{F,t} + (1-\alpha_1-\alpha_2)\mathsf{p}^*_{FDI,t} = 0 \tag{B.108}$$

- Price indices - Domestic Components:

$$\mathsf{p}_{H,t} = \tilde{\mathsf{p}}_{D,t} - \psi \mathsf{N}_{D,t} \tag{B.109}$$
$$\mathsf{p}^*_{H,t} = \tilde{\mathsf{p}}^*_{D,t} - \psi \mathsf{N}^*_{D,t} \tag{B.110}$$

- Price indices - Imported Components:

$$\mathsf{p}_{F,t} = \tilde{\mathsf{p}}^*_{X,t} - \psi \mathsf{N}^*_{X,t} \tag{B.111}$$
$$\mathsf{p}^*_{F,t} = \tilde{\mathsf{p}}_{X,t} - \psi \mathsf{N}_{X,t} \tag{B.112}$$

- Price indices - FDI Components:

$$\mathsf{p}_{FDI,t} = \tilde{\mathsf{p}}^*_{I,t} - \psi \mathsf{N}^*_{I,t} \tag{B.113}$$
$$\mathsf{p}^*_{FDI,t} = \tilde{\mathsf{p}}_{I,t} - \psi \mathsf{N}_{I,t} \tag{B.114}$$

- Firm level pricing - Domestically sold goods:

$$\tilde{\mathsf{p}}_{D,t} = \mathsf{w}_t - \mathsf{Z}_t \tag{B.115}$$
$$\tilde{\mathsf{p}}^*_{D,t} = \mathsf{w}^*_t - \mathsf{Z}^*_t \tag{B.116}$$

- Firm level pricing - Exported goods:

$$\tilde{\mathsf{p}}_{X,t} = \mathsf{w}_t - \mathsf{Z}_t - \mathsf{z}_{X,t} - \mathsf{Q}_t \tag{B.117}$$
$$\tilde{\mathsf{p}}^*_{X,t} = \mathsf{w}^*_t - \mathsf{Z}^*_t - \mathsf{z}^*_{X,t} + \mathsf{Q}_t \tag{B.118}$$

- Firm level pricing - FDI goods sold by foreign affiliates:

$$\tilde{\mathsf{p}}_{I,t} = \mathsf{w}^*_t - \mathsf{Z}^*_t - \mathsf{z}_{I,t} \tag{B.119}$$
$$\tilde{\mathsf{p}}^*_{I,t} = \mathsf{w}_t - \mathsf{Z}_t - \mathsf{z}^*_{I,t} \tag{B.120}$$

- Total average profits:

$$S_D \tilde{\mathsf{d}}_t = S_{DD}\tilde{\mathsf{d}}_{D,t} + (S_{DD} - S_D)\mathsf{N}_{D,t} + S_X\left(\mathsf{N}_{X,t} + \tilde{\mathsf{d}}_{X,t}\right) + S_I\left(\mathsf{N}_{I,t} + \tilde{\mathsf{d}}_{I,t}\right) \tag{B.121}$$
$$S_D \tilde{\mathsf{d}}^*_t = S_{DD}\tilde{\mathsf{d}}^*_{D,t} + (S_{DD} - S_D)\mathsf{N}^*_{D,t} + S_X\left(\mathsf{N}^*_{X,t} + \tilde{\mathsf{d}}^*_{X,t}\right) + S_I\left(\mathsf{N}^*_{I,t} + \tilde{\mathsf{d}}^*_{I,t}\right) \tag{B.122}$$

- Average profits from domestically sold goods:

$$\tilde{\mathsf{d}}_{D,t} = (\psi(\omega-1)-1)\mathsf{N}_{D,t} + (1-\omega)\tilde{\mathsf{p}}_{D,t} + \mathsf{C}_t \quad \text{(B.123)}$$
$$\tilde{\mathsf{d}}^*_{D,t} = (\psi(\omega-1)-1)\mathsf{N}^*_{D,t} + (1-\omega)\tilde{\mathsf{p}}^*_{D,t} + \mathsf{C}^*_t \quad \text{(B.124)}$$
$$\text{(B.125)}$$

- Average profits from exported goods:

$$\frac{S_X}{S_X+S_{FX}}\tilde{\mathsf{d}}_{X,t} + \frac{S_{FX}}{S_X+S_{FX}}(\mathsf{w}_t - \mathsf{z}_t) = \mathsf{Q}_t + (\psi(\omega-1)-1)\mathsf{N}_{X,t} + (1-\omega)\tilde{\mathsf{p}}_{X,t} + \mathsf{C}^*_t \quad \text{(B.126)}$$
$$\frac{S_X}{S_X+S_{FX}}\tilde{\mathsf{d}}^*_{X,t} + \frac{S_{FX}}{S_X+S_{FX}}(\mathsf{w}^*_t - \mathsf{z}^*_t) = -\mathsf{Q}_t + (\psi(\omega-1)-1)\mathsf{N}^*_{X,t} + (1-\omega)\tilde{\mathsf{p}}^*_{X,t} + \mathsf{C}_t \quad \text{(B.127)}$$

- Average profits from FDI goods produced by foreign affiliate:

$$\frac{S_I}{S_I+S_{FI}}\tilde{\mathsf{d}}_{I,t} + \frac{S_{FI}}{S_I+S_{FI}}(\mathsf{Q}_t + \mathsf{w}^*_t - \mathsf{z}^*_t) = \mathsf{Q}_t + (\psi(\omega-1)-1)\mathsf{N}_{I,t} + (1-\omega)\tilde{\mathsf{p}}_{I,t} + \mathsf{C}^*_t \quad \text{(B.128)}$$
$$\frac{S_I}{S_I+S_{FI}}\tilde{\mathsf{d}}^*_{I,t} + \frac{S_{FI}}{S_I+S_{FI}}(-\mathsf{Q}_t + \mathsf{w}_t - \mathsf{z}_t) = -\mathsf{Q}_t + (\psi(\omega-1)-1)\mathsf{N}^*_{I,t} + (1-\omega)\tilde{\mathsf{p}}^*_{I,t} + \mathsf{C}_t \quad \text{(B.129)}$$

- Free-entry conditions:

$$\tilde{\mathsf{v}}^s_t = \mathsf{w}_t - \mathsf{Z}_{E,t} \quad \text{(B.130)}$$
$$\tilde{\mathsf{v}}^{*s}_t = \mathsf{w}^*_t - \mathsf{Z}^*_{E,t} \quad \text{(B.131)}$$

- Optimal labor supply:

$$\mathsf{L}_t = \psi\left(\mathsf{w}_t - \gamma\mathsf{C}_t\right) \quad \text{(B.132)}$$
$$\mathsf{L}^*_t = \psi\left(\mathsf{w}^*_t - \gamma\mathsf{C}^*_t\right) \quad \text{(B.133)}$$

- Labor market clearing:

$$\begin{aligned}\bar{L}\mathsf{L}_t =& \frac{(\sigma-1)}{S_W}\left[S_{DD}(\mathsf{N}_{D,t}+\tilde{\mathsf{d}}_{D,t}) + S_X(\mathsf{N}_{X,t}+\tilde{\mathsf{d}}_{X,t}) + S_I(\mathsf{Q}_t+\mathsf{N}^*_{I,t}+\tilde{\mathsf{d}}^*_{I,t}) - S_D\mathsf{w}_t\right] \\ &+ \sigma\left[\frac{\bar{N}_E f_E}{\sigma}\mathsf{N}_{E,t} + f_{X,t}\bar{N}_X\mathsf{N}_{X,t} + f_{I,t}\bar{N}^*_I\mathsf{N}^*_{I,t} - (\frac{\bar{N}_{E,t}f_E}{\sigma} + f_X\bar{N}_X + f_I\bar{N}_I)\mathsf{Z}_t\right] \\ \bar{L}\mathsf{L}^*_t =& \frac{(\sigma-1)}{S_W}\left[S_{DD}(\mathsf{N}^*_{D,t}+\tilde{\mathsf{d}}^*_{D,t}) + S_X(\mathsf{N}^*_{X,t}+\tilde{\mathsf{d}}^*_{X,t}) + S_I(-\mathsf{Q}_t+\mathsf{N}_{I,t}+\tilde{\mathsf{d}}_{I,t}) - S_D\mathsf{w}^*_t\right] \\ &+ \sigma\left[\frac{\bar{N}_E f_E}{\sigma}\mathsf{N}^*_{E,t} + f_X\bar{N}_X\mathsf{N}^*_{X,t} + f_{I,t}\bar{N}_I\mathsf{N}_{I,t} - (\frac{\bar{N}_E f_E}{\sigma} + f_X\bar{N}_X + f_I\bar{N}_I)\mathsf{Z}^*_t\right]\end{aligned}$$

- Share of exporters:

$$\frac{\bar{N}_{X,t}}{\bar{N}_{D,t}}\left(\mathsf{N}_{X,t}-\mathsf{N}_{D,t}\right)=k(z_{min})^k(\bar{z}_{I,t}^{-k}\tilde{\mathsf{z}}_{I,t}-\bar{z}_{X,t}^{-k}\tilde{\mathsf{z}}_{X,t}) \tag{B.134}$$

$$\frac{\bar{N}_{X,t}}{\bar{N}_{D,t}}\left(\mathsf{N}^*_{X,t}-\mathsf{N}^*_{D,t}\right)=k(z_{min})^k(\bar{z}_{I,t}^{-k}\tilde{\mathsf{z}}^*_{I,t}-\bar{z}_{X,t}^{-k}\tilde{\mathsf{z}}^*_{X,t}) \tag{B.135}$$

- Share of FDI firms:

$$\mathsf{N}_{I,t}-\mathsf{N}_{D,t}=-k\tilde{\mathsf{z}}_{I,t} \tag{B.136}$$

$$\mathsf{N}^*_{I,t}-\mathsf{N}^*_{D,t}=-k\tilde{\mathsf{z}}^*_{I,t} \tag{B.137}$$

- Zero-profit export cutoff:

$$\frac{S_X}{S_X+S_{FX}}\left(\tilde{\mathsf{d}}_{X,t}-\mathsf{w}_t+\mathsf{z}_t\right)=\frac{\omega-1}{\sigma-1}\left(\bar{N}_X\bar{d}_X+\bar{N}_Xf_X\bar{w}\right)^{-\frac{\sigma-1}{\omega-1}}\left[(\psi(\omega-1)-1)\bar{M}M(\mathsf{N}_{I,t}-\mathsf{N}_{X,t})+\omega\bar{N}N\mathsf{TOL}_t\right]$$

$$\frac{S_X}{S_X+S_{FX}}\left(\tilde{\mathsf{d}}^*_{X,t}-\mathsf{w}^*_t+\mathsf{z}^*_t\right)=\frac{\omega-1}{\sigma-1}\left(\bar{N}_X\bar{d}_X+\bar{N}_Xf_X\bar{w}\right)^{-\frac{\sigma-1}{\omega-1}}\left[(\psi(\omega-1)-1)\bar{M}M(\mathsf{N}^*_{I,t}-\mathsf{N}^*_{X,t})-\omega\bar{N}N\mathsf{TOL}_t\right]$$

with

$$\bar{M}M=\frac{k\left[\bar{O}O^{\frac{1+k-\sigma}{1-\omega}}\left(\bar{P}P^{\frac{k}{1-\omega}}-\bar{O}O^{\frac{k}{1-\omega}}\right)+\nabla\bar{O}O^{\frac{k}{1-\omega}}\left(\bar{P}P^{\frac{1+k-\sigma}{1-\omega}}-\bar{O}O^{\frac{1+k-\sigma}{1-\omega}}\right)\right]}{(1-\omega)\left(\bar{P}P^{\frac{k}{1-\omega}}-\bar{O}O^{\frac{k}{1-\omega}}\right)^2}$$

$$\bar{N}N=\frac{k\left[-\bar{O}O^{\frac{1+k-\sigma}{1-\omega}}\left(\bar{P}P^{\frac{k}{1-\omega}}-\bar{O}O^{\frac{k}{1-\omega}}\right)+\nabla\bar{O}O^{\frac{k}{1-\omega}}\left(\bar{P}P^{\frac{1+k-\sigma}{1-\omega}}-\bar{O}O^{\frac{1+k-\sigma}{1-\omega}}\right)\right]}{(1-\omega)\left(\bar{P}P^{\frac{k}{1-\omega}}-\bar{O}O^{\frac{k}{1-\omega}}\right)^2}$$

$$\bar{O}O=\frac{f^*_{I,t}}{1-\alpha^*_1-\alpha^*_2}\left(\frac{\bar{N}_{X,t}}{\bar{N}_{I,t}}\right)^{(\psi(\omega-1)-1)}\tau^{1-\omega}$$

$$\bar{P}P=\frac{\bar{w}}{\alpha_2}$$

- Zero-profit FDI cutoff:

$$\tilde{\mathsf{d}}_{I,t}=\mathsf{Q}_t+\mathsf{w}^*_t-\mathsf{Z}^*_t \tag{B.138}$$

$$\tilde{\mathsf{d}}_{I,t}=-\mathsf{Q}_t+\mathsf{w}_t-\mathsf{Z}_t \tag{B.139}$$

- Law of motion of domestic firms:

$$\mathsf{N}_{D,t+1}=(1-\delta)\mathsf{N}_{D,t}+\delta\mathsf{N}_{E,t} \tag{B.140}$$

$$\mathsf{N}^*_{D,t+1}=(1-\delta)\mathsf{N}^*_{D,t}+\delta\mathsf{N}^*_{E,t} \tag{B.141}$$

- Terms of Labor:

$$\tilde{\mathsf{TOL}}_t=\mathsf{Q}_t-\mathsf{w}_t+\mathsf{w}^*_t+\mathsf{Z}_t-\mathsf{Z}^*_t \tag{B.142}$$

- Definitions of real returns:

$$\mathsf{r}^s_{h,t} = \beta(1-\delta)\tilde{\mathsf{v}}^s_t + (1-\beta(1-\delta))\tilde{\mathsf{d}}_t - \tilde{\mathsf{v}}^s_{t-1} \tag{B.143}$$
$$\mathsf{r}^s_{f,t} = \beta(1-\delta)\tilde{\mathsf{v}}^{*s}_t + (1-\beta(1-\delta))\tilde{\mathsf{d}}^*_t - \tilde{\mathsf{v}}^{*s}_{t-1} \tag{B.144}$$
$$\mathsf{r}^b_{h,t} = \beta\tilde{\mathsf{v}}^b_t + (1-\beta)\tilde{\mathsf{d}}^b_t - \tilde{\mathsf{v}}^b_{t-1} \tag{B.145}$$
$$\mathsf{r}^b_{f,t} = \beta\tilde{\mathsf{v}}^{*b}_t + (1-\beta)\tilde{\mathsf{d}}^{*b}_t - \tilde{\mathsf{v}}^{*b}_{t-1} \tag{B.146}$$

- Euler Home and Foreign:

$$(\mathsf{C}_t - \mathsf{C}^*_t) = \frac{\gamma}{\gamma-\nu} E_t\left[\mathsf{C}_{t+1} - \mathsf{C}^*_{t+1} + \mathsf{Q}_t - \mathsf{Q}_{t+1}\right] \tag{B.147}$$

- Euler Home and Foreign:

$$0 = (\gamma-\nu)\mathsf{C}_{t+1} - \gamma E_t\left[\mathsf{C}_{t+1} + \mathsf{r}^s_{h,t+1}\right] \tag{B.148}$$
$$0 = (\gamma-\nu)\mathsf{C}_{t+1} - \gamma E_t\left[\mathsf{C}_{t+1} + \mathsf{r}^s_{f,t+1}\right] \tag{B.149}$$
$$0 = (\gamma-\nu)\mathsf{C}_{t+1} - \gamma E_t\left[\mathsf{C}_{t+1} + \mathsf{r}^b_{h,t+1}\right] \tag{B.150}$$
$$0 = (\gamma-\nu)\mathsf{C}_{t+1} - \gamma E_t\left[\mathsf{C}_{t+1} + \mathsf{r}^b_{f,t+1}\right] \tag{B.151}$$

- Expected Excess Returns:

$$E_t\left[\mathsf{r}^s_{x,t+1}\right] = E_t\left[\beta(1-\delta)\mathsf{v}^s_{t+1} + (1-\beta(1-\delta))\mathsf{d}^s_{t+1} - \beta\mathsf{v}^b_{t+1} - (1-\beta)\mathsf{d}^b_{t+1}\right] - \mathsf{v}^s_t + \mathsf{v}^b_t$$
$$E_t\left[\mathsf{r}^{*s}_{x,t+1}\right] = E_t\left[\beta(1-\delta)\mathsf{v}^{*s}_{t+1} + (1-\beta(1-\delta))\mathsf{d}^{*s}_{t+1} - \beta\mathsf{v}^b_{t+1} - (1-\beta)\mathsf{d}^b_{t+1} + \mathsf{Q}_{t+1}\right] - \mathsf{v}^s_t + \mathsf{v}^b_t - \mathsf{Q}_t$$
$$E_t\left[\mathsf{r}^{*b}_{x,t+1}\right] = E_t\left[\beta\mathsf{v}^{*b}_{t+1} + (1-\beta)\mathsf{d}^{*b}_{t+1} - \beta\mathsf{v}^b_{t+1} - (1-\beta)\mathsf{d}^b_{t+1} + \mathsf{Q}_{t+1}\right] - \mathsf{v}^{*b}_t + \mathsf{v}^b_t - \mathsf{Q}_t$$

- Definitions of income:

$$\bar{Y}\mathsf{Y}_t = \bar{w}\bar{L}(\mathsf{w}_t + \mathsf{L}_t) + \bar{N}_D\bar{d}(\mathsf{N}_{D,t} + \tilde{\mathsf{d}}_t) - \bar{N}_E\bar{v}(\mathsf{N}_{E,t} + \tilde{\mathsf{v}}^s_t) \tag{B.152}$$
$$\bar{Y}\mathsf{Y}^*_t = \bar{w}\bar{L}(\mathsf{w}^*_t + \mathsf{L}^*_t) + \bar{N}_D\bar{d}(\mathsf{N}^*_{D,t} + \tilde{\mathsf{d}}^*_t) - \bar{N}_E\bar{v}(\mathsf{N}^*_{E,t} + \tilde{\mathsf{v}}^{*s}_t) \tag{B.153}$$

- Net exports:

$$\mathsf{NX}_t = \frac{1}{2}\left[\bar{Y}(\mathsf{Y}_t - \mathsf{Y}^*_t) - \bar{C}(\mathsf{C}_t - \mathsf{C}^*_t) + (\bar{C} - \bar{Y})\mathsf{Q}_t\right] \tag{B.154}$$

- Net foreign assets:

$$\mathsf{NFA}_{t+1} = \mathsf{NX}_t + \frac{1}{\beta}NFA_t + \tilde{a}^s_f\mathsf{r}^s_{fx,t+1} + \tilde{a}^{s*}_h\mathsf{r}^s_{hx,t+1} + \tilde{a}^b_f\mathsf{r}^b_{fx,t+1} \tag{B.155}$$

with

$$\tilde{a}^s_f = \tilde{a}^{*s}_h = \frac{a^s_f}{\beta\bar{C}} \quad \text{and} \quad \tilde{a}^b_f = -\tilde{a}^b_h = \frac{a^s_f}{\beta\bar{C}}. \tag{B.156}$$

For the case of welfare indexed bonds bond dividends are $d_t^b = d_t^{*b} = 1$, the bond returns become:

$$\mathsf{r}_{h,t}^b = \beta \mathsf{v}_t^b - \mathsf{v}_{t-1}^b \tag{B.157}$$

$$\mathsf{r}_{f,t}^b = \beta \mathsf{v}_t^{b*} - \mathsf{v}_{t-1}^{b*} + \mathsf{Q}_t - \mathsf{Q}_{t-1} \tag{B.158}$$

Expected Excess Returns:

$$\begin{aligned}
E_t\left[\mathsf{r}_{x,t+1}^s\right] &= \mathsf{r}_{h,t}^s - \mathsf{r}_{h,t}^b = \beta(1-\delta)E_t\left[\mathsf{v}_{t+1}^s + (1-\beta(1-\delta))\mathsf{d}_{t+1}^s - \beta\mathsf{v}_{t+1}^b\right] - \mathsf{v}_t^s + \mathsf{v}_t^b \\
E_t\left[\mathsf{r}_{x,t+1}^{*s}\right] &= \mathsf{r}_{f,t}^s - \mathsf{r}_{h,t}^b = E_t\left[\beta(1-\delta)\mathsf{v}_{t+1}^{*s} + (1-\beta(1-\delta))\mathsf{d}_{t+1}^{*s} - \beta\mathsf{v}_{t+1}^b + \mathsf{Q}_{t+1}\right] \\
&\quad - \mathsf{v}_t^s + \mathsf{v}_t^b - \mathsf{Q}_t \\
E_t\left[\mathsf{r}_{x,t+1}^{*b}\right] &= \mathsf{r}_{f,t}^b - \mathsf{r}_{h,t}^b = E_t\left[\beta\mathsf{v}_{t+1}^{*b} - \beta\mathsf{v}_{t+1}^b + \mathsf{Q}_{t+1}\right] - \mathsf{v}_t^{*b} + \mathsf{v}_t^b - \mathsf{Q}_t
\end{aligned}$$

Appendix C

Appendix Chapter 3

C.1 Net wealth distribution in the EMU

Figure C.1 – Net wealth distribution in surveyed EMU countries (ECB,2013)

Table 4.1 Net wealth by demographic and country characteristics

	Median Net Wealth (€1,000)	Mean Net Wealth (€1,000)	Share of Total Net Wealth (%)	Share of Households (%)
Country				
Belgium (2010)	206.2	338.6	5.0	3.4
S.E.	*(7.0)*	*(11.8)*		
Germany (2010)	51.4	195.2	24.3	28.7
S.E.	*(3.2)*	*(11.9)*		
Greece (2009)	101.9	147.8	1.9	3.0
S.E.	*(2.5)*	*(5.0)*		
Spain (2008)	182.7	291.4	15.6	12.3
S.E.	*(3.8)*	*(9.2)*		
France (2010)	115.8	233.4	20.4	20.2
S.E.	*(4.0)*	*(5.8)*		
Italy (2010)	173.5	275.2	20.6	17.2
S.E.	*(3.9)*	*(8.1)*		
Cyprus (2010)	266.9	670.9	0.6	0.2
S.E.	*(17.3)*	*(56.5)*		
Luxembourg (2010)	397.8	710.1	0.4	0.1
S.E.	*(17.1)*	*(58.2)*		
Malta (2010)	215.9	366.0	0.2	0.1
S.E.	*(11.1)*	*(51.8)*		
Netherlands (2009)	103.6	170.2	4.0	5.3
S.E.	*(8.1)*	*(6.2)*		
Austria (2010)	76.4	265.0	3.1	2.7
S.E.	*(11.0)*	*(47.9)*		
Portugal (2010)	75.2	152.9	1.9	2.8
S.E.	*(3.0)*	*(8.1)*		

C.2 The model without default and log-utility

Home country has a representative household while there are two types of households in Foreign. Only the Home government is assumed to be able to issue debt, while households in both countries can save using government bonds.

In this version of the model I assume a deterministic world with commitment by the Home government to repay its debt.

Furthermore, as D'Erasmo and Mendoza (2016) I concentrate on equilibria in which the low-type foreign household does not save in government bonds, i.e. $b_1^{L*} = 0$. This depends on the parameterization of the model and has to be checked in the theoretical and numerical exercises.

C.2.1 Households

The preferences of the Home representative household are given by

$$ln(c_0) + \beta ln(c_1) \tag{C.1}$$

with

$$\begin{aligned} c_0 &= y_0 - \tau_0 + b_0 - qb_1 \\ c_1 &= y_1 - \tau_1 + b_1. \end{aligned}$$

The preferences of the Foreign households are

$$ln(c_0^{i*}) + \beta ln(c_1^{i*}) \quad \text{for} \quad i = L, H. \tag{C.2}$$

With $b_1^{L*} = 0$ the budget constraints of the L-type are

$$\begin{aligned} c_0^* &= y_0^* - \tau_0^* \\ c_1^* &= y_1^* - \tau_1^* \end{aligned}$$

and the ones for the H-type households are

$$\begin{aligned} c_0^* &= y_0^* - \tau_0^* + b_0^{H*} - qb_1^{H*} \\ c_1^* &= y_1^* - \tau_1^* + b_1^{H*}. \end{aligned}$$

The savings problem of the Home household given an amount of government debt B_1 is

$$\mathcal{L} = max_{b_1} ln(y_0 - \tau_0 + b_0 - qb_1) + \beta ln(y_1 - \tau_1 + b_1).$$

The first-order condition is given by

$$\frac{\partial \mathcal{L}}{\partial b_1}: \qquad -\frac{q}{c_0} + \beta \frac{1}{c_1} = 0.$$

Thus, the Euler equation for the Home household is

$$q = \beta \frac{c_0}{c_1}. \tag{C.3}$$

The savings problem of the Foreign H-type household given an amount of government debt B_1 is

$$\mathcal{L} = max_{b_1^{H*}} ln(y_0^* - \tau_0^* + b_0^{H*} - q b_1^{H*}) + \beta ln(y_1^* - \tau_1^* + b_1^{H*}).$$

The first-order condition is given by

$$\frac{\partial \mathcal{L}}{\partial b_1^{H*}}: \qquad -\frac{q}{c_0^{H*}} + \beta \frac{1}{c_1^{H*}} = 0.$$

Thus, the Euler equation for the Foreign H-type household is

$$q = \beta \frac{c_0^{H*}}{c_1^{H*}}. \tag{C.4}$$

For the Foreign L-type household, the parameterization has to be chosen such that it is 'credit constraint', i.e. given its income stream it would like to borrow, but bond holdings are restricted to $b_1^{L*} \geq 0$. This is the case if for a given equilibrium price

$$q > \beta \frac{c_0^{H*}}{c_1^{H*}} \tag{C.5}$$

holds, stating that the marginal cost of buying a unit of government bonds is strictly greater than the marginal benefit to shift consumption from $t = 0$ to $t = 1$.

C.2.2 Government

Home government's budget constraints are

$$\begin{aligned} \tau_0 &= g_0 + B_0 - qB_1 \\ \tau_1 &= g_1 + B_1. \end{aligned}$$

The Foreign government cannot issue debt and therefore, its budget constraints are

$$\begin{aligned} \tau_0^* &= g_0^* \\ \tau_1^* &= g_1^*. \end{aligned}$$

C.2.3 Bond market equilibrium

The bond market is in a competitive equilibrium if for a given amount of Home government debt, the following market clearing condition is fulfilled

$$b_1 + \gamma b_1^{L*} + (1-\gamma) b_1^{H*} = B_1. \tag{C.6}$$

As I concentrate on equilibria in which the Foreign L-type does not hold debt, this reduces to

$$b_1 + (1-\gamma)b_1^{H*} = B_1. \tag{C.7}$$

Still there is the possibility that in equilibrium only one of the two households holds all the debt supplied by the Home government.

Starting with the Euler equation of the Home household and plugging in the budget constraints, the government budget constraints and the Home household's holdings of initial debt $b_0 = \eta B_0$, I obtain

$$q = \beta \frac{y_0 - g_0 - (1-\eta)B_0 + qB_1 - qb_1}{y_1 - g_1 - B_1 + b_1}. \tag{C.8}$$

For the Foreign H-type household a similar expression can be derived with $b_0^{H*} = \frac{(1-\eta)B_0}{1-\gamma}$

$$q = \beta \frac{y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0 - qb_1^{H*}}{y_1^* - g_1^* + b_1^{H*}}. \tag{C.9}$$

From these expressions, the bond prices are

$$q = \beta \frac{y_0 - g_0 - (1-\eta)B_0}{y_1 - g_1 - (1+\beta)B_1 + (1+\beta)b_1} \tag{C.10}$$

from the Home household's Euler equation and

$$q = \beta \frac{y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0}{y_1^* - g_1^* + (1+\beta)b_1^{H*}}. \tag{C.11}$$

for the Foreign H-type household.

Now there are different cases for equilibrium bond holdings:

Case 1: Only Home households hold bonds

If the equilibrium price is such that

$$q = \beta \frac{c_0}{c_1} > \beta \frac{c_0^{H*}}{c_1^{H*}}, \tag{C.12}$$

only the Home household will want to hold bonds, and from market clearing the demand of the Home household would be $b_1 = B_1$. Therefore, the planner's problem is

$$\begin{aligned}\mathcal{L} &= max_{B_1} ln(y_0 - g_0 - (1-\eta)B_0 + qB_1 - qb_1) + \beta ln(y_1 - g_1 - B_1 + b_1)\\ &= ln(y_0 - g_0 - (1-\eta)B_0) + \beta ln(y_1 - g_1)\end{aligned}$$

which shows that welfare is independent of B_1 and the government cannot influence intertemporal consumption by issuing debt. For the purpose of this paper, this case is not of interest.

Case 2: Both the Home and the Foreign L-type households hold Home government bonds

If the equilibrium price is such that

$$q = \beta \frac{c_0}{c_1} = \beta \frac{c_0^{H*}}{c_1^{H*}}, \tag{C.13}$$

both the Home household and the Foreign H-type household want to hold Home government debt. The market clearing condition in the bond market for this case is given by $b_1 + (1-\gamma)b_1^{H*} = B_1$.

Using this in the price expression for the Home household, I obtain

$$\begin{aligned} q &= \beta \frac{y_0 - g_0 - (1-\eta)B_0}{y_1 - g_1 - (1+\beta)B_1 + (1+\beta)b_1} \\ &= \beta \frac{y_0 - g_0 - (1-\eta)B_0}{y_1 - g_1 - (1+\beta)B_1 + (1+\beta)(B_1 - (1-\gamma)b_1^{H*})} \\ &= \beta \frac{y_0 - g_0 - (1-\eta)B_0}{y_1 - g_1 - (1+\beta)(1-\gamma)b_1^{H*}}. \end{aligned}$$

Note that this is independent of B_1 again. Since in this equilibrium the marginal benefit from holding Home government bonds is equal for Home and Foreign H-type household, it holds that

$$\frac{y_0 - g_0 - (1-\eta)B_0}{y_1 - g_1 - (1+\beta)(1-\gamma)b_1^{H*}} = \frac{y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0}{y_1^* - g_1^* + (1+\beta)b_1^{H*}}.$$

Solving for b_1^{H*} yields

$$b_1^* = \frac{(y_0 - g_0 - (1-\eta)B_0)(y_1^* - g_1^*) - (y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0)(y_1 - g_1))}{(y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0)(1+\beta)(1-\eta) - (y_0 - g_0 - (1-\eta)B_0)(1+\beta)}. \tag{C.14}$$

One can see that the demand for Home government debt by the Foreign H-type household does not depend on the level of debt issued by the Home government B_1. It is solely determined by the parameters of the model. This demand pins down the equilibrium price of debt to

$$q = \beta \frac{y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0}{y_1^* - g_1^* + (1+\beta)b_1^{H*}} \tag{C.15}$$

which is then independent of B_1 as well. The bond demand of the Home household is derived from the market clearing condition as

$$b_1 = B_1 - (1-\gamma)b_1^{H*}. \tag{C.16}$$

Since the demand of the Foreign H-type household is independent of B_1, in the case that Home households and Foreign H-type households hold debt and bondholdings cannot be negative, the debt level issued by the government has to be at least $B_1 > (1-\gamma)b_1^{H*}$. Otherwise the Home household will not buy government bonds. For the case that $B_1 > (1-\gamma)b_1^{H*}$, the Home household will buy all residual government debt above $(1-\gamma)b_1^{H*}$. Therefore, the Home government will be indifferent between issuing $B_1 = (1-\gamma)b_1^{H*}$ and any other amount $B_1 > (1-\gamma)b_1^{H*}$. The intertemporal consumption levels of the Home household, and therefore its welfare, is not altered by issuing any $B_1 > (1-\gamma)b_1^{H*}$.

This case plays a role, if welfare is maximized by an amount of debt larger or equal to the full demand for government bonds by the Foreign household. Interestingly, the potential welfare gain from issuing debt even under commitment in this model is limited by the Foreign H-type household's demand for these bonds. In this case, the government can issue even more debt, without influencing the Home household's welfare, as the Home household will just hold the residual government debt, keeping its consumption levels the same.

Case 3: Only Foreign H-type holds debt

If the equilibrium price is such that

$$q = \beta\frac{c_0^{H*}}{c_1^{H*}} > \beta\frac{c_0}{c_1}, \tag{C.17}$$

only the Foreign H-type household is saving in Home government bonds. The market clearing condition in the bond market for this case is given by $(1-\gamma)b_1^{H*} = B_1$, so the bond demand of the Foreign H-type household is $b_1^{H*} = \frac{B_1}{(1-\gamma)}$.

Using this in the price expression for the Foreign H-type household yields

$$\begin{aligned} q &= \beta\frac{y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0}{y_1^* - g_1^* + (1+\beta)b_1^{H*}} \\ q(B_1) &= \beta\frac{y_0^* - g_0^* + \frac{1-\eta}{1-\gamma}B_0}{y_1^* - g_1^* + (1+\beta)\frac{B_1}{(1-\gamma)}} \end{aligned}$$

where the equilibrium bond price varies with the level of debt issued by the Home government. In particular,

$$q'(B_1) = \frac{-q(B_1)\frac{1+\beta}{1-\gamma}}{y_1^* - g_1^* + \frac{1+\beta}{(1-\gamma)}B_1}. \tag{C.18}$$

Given that the bond price in equilibrium is positive, it holds that

$$q'(B_1) < 0. \tag{C.19}$$

In summary, in this deterministic model where I assume the Foreign L-type household to be borrowing constraint, there is an optimal amount of savings the Foreign H-type household would like to carry over from period $t = 0$ to $t = 1$, while at the same time, given our parameterization, there is an optimal amount of government debt the Home government issues in order to allow the Home household to borrow and shift consumption from $t = 1$ to $t = 0$. In the case that the debt level optimally issued by the Home government is lower than the amount the Foreign H-type desires optimally, i.e. the amount at which the Foreign governments demand is fully satisfied and the Home household starts saving in government bonds, only the Foreign H-type household will hold Home debt and the price will fall in the amount issued. If the Home government starts saving, case 2, the price will fully be determined by the demand of the Foreign H-type household, and at this price the Home household will buy all the debt in excess of the Foreign H-type household's demand. The Foreign H-type household's demand determines a maximum level of consumption that the Home government can transfer to the Home household in period $t = 0$, if it issues more than this amount there will be no additional welfare gains.

Optimal debt choice

The Home government chooses the optimal amount of debt it issues to maximize the Home household's welfare. Its objective function is

$$max_{B_1} ln(y_0 - g_0 - B_0 + qB_1 + b_0 - qb_1) + \beta ln(y_1 - g_1 - B_1 + b_1) \tag{C.20}$$

The first-order conditions depend on the different cases of which household types hold bonds. If $B_1 \geq (1-\gamma)b_1^{H*}$ both household want to save in government bonds, q is not dependent on B_1, but $b_1(B_1)$. It actually moves one-to-one with B_1, so the government is indifferent between issuing exactly $B_1 = (1-\gamma)b_1^{H*}$ or any greater amount.

If $B_1 < (1-\gamma)b_1^{H*}$, only the Foreign H-type household demands government bonds, the equilibrium bond price depends on B_1 and the maximization problem becomes

$$max_{B_1} ln(y_0 - g_0 - B_0 + q(B_1)B_1 + b_0) + \beta ln(y_1 - g_1 - B_1) \tag{C.21}$$

The first-order condition is

$$\frac{q'(B_1) + q(B_1)}{y_0 - g_0 - B_0 + qB_1 + b_0} = \frac{\beta}{y_1 - g_1 - B_1}. \tag{C.22}$$

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