# Internal Hierarchy and Stable Coalition Structures

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**Abstract.** *In deciding whether to join a coalition or not, an agent must consider both i) the expected power of the coalition and ii) her position in the vertical structure within the coalition. We establish the existence of a positive relationship between the degree of inequality in remuneration within coalitions and the number of coalitions to be formed endogenously in stable systems. We show that such coalitions can be mixed and balanced, rather than segregated, in terms of members' ability levels. In any stable system each coalition is of an efficient size and every agent is paid her marginal contribution.* (*JEL* Codes: C71, D71)

*Keywords:* Stable systems, Abilities, Cyclic partition, Non-segregation.

## 1 Introduction

Circumstances abound in which individual agents interact via the organisations they choose to belong to. This paper studies this type of situations and reports novel findings on the *competing teams/coalitions* (e.g. firms, political parties or gangs) that emerge endogenously, with an emphasis on how the structure and composition of teams are interrelated with the level of vertical inequality within teams.

Understanding what determines the number and composition of coalitions or teams has been a recurrent focus in many strands of literature (discussed below), but, to the best of our knowledge, no systematic work exists on the relationship between such *horizontal* segmentation incentives and the *vertical* structure within each endogenous team. An investigation of the interplay of these two dimensions, especially when the relevant agents are heterogeneous in their ability, can shed new lights on a number of interesting questions: As ability differentials among agents increase, should the number of rival teams increase or decrease, and should the endogenous teams become more or less segregated? Are there general connections between endogenous meritocracy and the degree of competition? This paper develops a cooperative game theoretic framework to address such questions in an institution-free environment, and provides some new insights.

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A key assumption of the model is that the relevant agents have observable, heterogeneous ability levels, and the total surplus of a team may depend both on its size and on the aggregate ability of its members. Given a distribution of agents' ability levels and a surplus function capturing technology available to teams, a partition of agents into coalitions/teams with imputation rules constitute a stable system if no profitable coalitional deviation is possible.

The main finding is that the forces of competition and selection lead to emergence of non-segregated rival teams that compensate members their respective maximal possible value to a team, which also implies a positive association between the number of rival teams formed and the internal inequality levels exhibited. Specifically, we show that (1) in every stable system, each team is efficient, every agent contributes her full potential to a team and is paid accordingly, and the rival teams are non-segregated (under mild conditions); (2) the more unequally shared is the surplus across ranks internally, i.e., the higher is the vertical inequality, the larger is the number of rival teams to be formed in a stable partition of the relevant agents; (3) in a heuristic class of environments in which teams are restricted to single-parameter imputation rules, a system is stable if and only if the agents are partitioned into *cyclic* teams (i.e., teams composed of equidistant agents in their ability ordering).

When teams compensate their members more unequally across their internal ranks, agents are motivated more strongly to be in a team in which they are ranked higher, either by moving to or forming another team. This insight underlies our main result that the number of teams in a stable system is positively related to the level of internal inequality they exhibit.

The finding that organisations tend to consist of members from widely dispersed ability levels (despite one-dimensional heterogeneity) contrasts starkly with the segregation outcomes that are prevalent in the literature on some other kinds of group formation, such as the important literature on clubs and jurisdictions providing local public goods. These studies (see e.g. Jehiel and Scotchmer, 2001, and references therein) differ from ours in that different jurisdictions provide different local public good quantities and internal division of surplus is not a key strategic variable. Moreover, typically agents are not differentiated in terms of ability.

The literatures on social classes (Akerlof, 1997), partnerships (Farrell and Scotchmer, 1988), hedonic games (Banerjee *et al.,* 2001; Bogomolnaia and Jackson, 2002; Le Breton *et al.,* 2008; Watts, 2007),[1] social status (Milchtaich and Winter, 2002), and organisation (e.g., Demange 2004, Garicano and Rossi-Hansberg, 2006, and an

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[1] In hedonic games agents' payoffs are determined by the composition of their own coalition only. In our game the agents' utility depends on an endogenously determined imputation rule in the coalition, as well as on the aggregate strength of the coalition, so it is not a proper hedonic game. Our model can be viewed as generalising Gamson games (see, e.g., Le Breton *et al.,* 2008): in this special class of hedonic games the total cake goes to the coalition that has more than half of the total talent, whereas our analysis includes settings where coalitions fight over market shares or power shares, with no magic value given to passing a 50% threshold.

earlier work on firm formation by Legros and Newman, 1996), are all broadly related, but our approach is distinguished from these studies in the following respect: vertically differentiated agents potentially face a dilemma between teaming up with more able agents for a more powerful team and teaming up with less able agents for a higher internal rank. Damiano *et al.* (2010) consider a similar tension but in a setting where agents decide which one to join from a *fixed* set of coalitions, motivated by contexts different from ours.[2] Watts (2007), on the other hand, analyzes two separate settings, one in which agents desire to team up with higher ability members (under the "average quality payoff"), and an opposite one in which they desire to team up with lower ability members (under the "big fish payoff").

Piccione and Razin (2011) study coalition formation in partition function games, in which an agent's social ranking is determined lexicographically, first by the "power relation" between the coalitions formed, then by her ability within the relevant coalition. The core is empty in this setting if the size dictates the power relation of coalitions. For this reason, they define a recursively stable solution concept, yielding existence and characterisation results in the spirit of our non-segregation results. A more recent paper, Barbera *et al.* (2014) show that meritocratic sharing norms in some coalitions can coexist with egalitarian norms in others.

The paper is organised as follows. Section 2 describes the model; section 3 provides a complete characterisation of stable systems, and the main result on the relationship between vertical inequality within teams and the number of endogenous teams; section 4 illustrates the additional cyclicity result that obtains when teams are restricted to single parameter imputation rules; section 5 concludes.

## 2  Model

Consider an economy with a population $\Omega = \{1, 2, \cdots, N\}$ consisting of $N(\leq \infty)$ agents, indexed according to their observable, heterogeneous ability (which could be, e.g., political ability, market ability, or criminal ability, depending on the application), denoted by $a_i > 0$ for agent $i \in \Omega$, so that $a_i > a_{i+1}$ for all $i$. For expositional clarity, in this note we assume a geometric distribution of abilities[3]:

$$a_i = a^{i-1} \ \text{ for all } \ i \in \Omega, \text{ where } \ a \in (0, 1).$$

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[2]In Damiano *et al.* (2010), agents of different abilities choose between two organisations of a fixed capacity of measure 1, when the agent's utility increases both in the average ability of the organisation (peer effect) and in her internal ranking (pecking order effect). If the value of each coalition is a function of the average ability of agents, they obtain some degree of segregation of ability types, with a larger overlap when the pecking order effect is stronger. Their results apply to very different contexts, such as students' choices among a fixed set of universities, rather than endogenous formation of organisations.

[3]The core results reported in this note extend to general distributions but the exposition is more complicated - see Morelli and Park (2014) for details.

Economic activity is assumed to take place through teams/coalitions where each team is a group of agents who perform a variety of tasks needed for production. We assume that there is a certain number $M(\leq N)$ of essential and complementary tasks, so that efficiency of a team $Z \subset \Omega$ improves in the number of its members, called its *size* and denoted by $|Z|$, until it reaches $M$ (as represented by the function $\zeta$ below).[4] Moreover, an agent's ability reflects her relative productivity in a team, so that the total surplus of a team of a given size increases linearly with the aggregate ability of its members, called its *power* and denoted by $p(Z)$, subject to a constraint that for a team to operate its power must exceed a certain minimum level, denoted by $P \geq 0$.

Formally, the total surplus $s(Z)$ of a team $Z \subset \Omega$ is

$$s(Z) \ := \ p(Z) \cdot \zeta(|Z|) \tag{1}$$

where $\zeta : \mathbb{N} \to (0, 1]$ is such that $\zeta(n-1) < \zeta(n)$ if $n \leq M$ and $\zeta(n) = 1$ if $n \geq M$, and

$$p(Z) \ = \ \begin{cases} \sum_{i \in Z} a^{i-1} & if \ \sum_{i \in Z} a^{i-1} \geq P \\ 0 & otherwise. \end{cases}$$

We say that a team $Z$ is *productive* if $p(Z) \geq P$ and *unproductive* otherwise; and that it is *efficient* if it is productive and $|Z| \geq M$, and *inefficient* otherwise.

The parameters $P$ and $M$ are meant to capture different technological and market characteristics of various sectors at different stages of development. In the extreme case of $M = 1$ and $P = 0$, there is no need to form a team because each agent's contribution is the same as a member of any team, as well as when standing alone. In the opposite extreme case of $M = N < \infty$, it seems intuitive that the grand coalition is likely to form as it is the unique efficient team. Below we focus on more interesting situations in which team-forming is desirable and multiple efficient teams may coexist in a system. In particular, we assume that

$$p(\Omega \setminus \{1\}) \geq P \quad \text{and} \quad M \leq N - 1. \tag{2}$$

An efficient team must include agent 1 if the first inequality fails, while it must include everyone if the second fails[5]: both preclude multiple efficient teams.

A population $\Omega$ of agents, their ability levels depicted by $a \in (0, 1)$, and the surplus function $s$, specify an "environment" in which teams get formed endogenously. Each endogenous team, $Z \subset \Omega$, adopts an *imputation rule,* denoted by $f : Z \to [0, 1]$, which specifies for each member $i \in Z$ a fraction $f(i)$ of the total

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[4]Note that it is implicitly assumed that agents are more productive when they specialise in one task rather than spreading their time across multiple tasks.

[5]Note that the second inequality holds if $M = N = \infty$, in which case all teams of infinite size are efficient as is the case in the illustration of the next section.

surplus $s(Z)$ to be allocated to that member, with $\sum_{i \in Z} f(i) = 1$. Thus, the payoff of agent $i \in Z$ is

$$u_i(Z, f) \;=\; f(i) \cdot s(Z). \tag{3}$$

A *system* is a pair $(\pi, \rho)$, where $\pi = \{Z_1, \cdots, Z_K\}$ is a partition of $\Omega$ into teams, and $\rho$ is a function that maps each team $Z_k \in \pi$ to an imputation rule of that team. We adopt the convention that teams in a system are labelled according to the order of ability of the most able member of each team, referred to as the "head" of the team. That is, $\min\{i|i \in Z_k\} < \min\{i|i \in Z_{k'}\}$ if $k < k'$.

A system $(\pi, \rho)$ is *stable* if there does not exist a deviation $D \subset \Omega$ that is *profitable* relative to the system $(\pi, \rho)$ in the sense that

$$u_i(D, f) \;\geq\; u_i(\pi(i), \rho(\pi(i))) \quad \forall i \in D \neq \emptyset$$

for some imputation rule $f$ that $D$ may adopt, where $\pi(i)$ is the team $Z_k \in \pi$ such that $i \in Z_k$, and the inequality is strict for some $i \in D$.

We are interested in understanding how the intra-team inequality in compensation relates to the structure and composition of endogenously formed teams. To facilitate exposition, we introduce the notion of the "rank" of members within a team: the agent paid the most within a team is ranked first, and so on. Then, an imputation rule of a team $Z \subset \Omega$ can be represented by a *rank-imputation rule* which is a vector

$$\mathbf{f} = (f_1, f_2, \cdots, f_{|Z|}) \in [0, 1]^{|Z|}$$

where $f_r$ is the fraction of the surplus allocated to the member ranked $r$-th, together with a "ranking rule" that assigns members of $Z$ to ranks 1 through $|Z|$. Note that $\sum_{r=1}^{|Z|} f_r = 1$. We compare intra-team inequality by comparing the ratios of the payoff each rank receives relative to the payoff received by the rank above, i.e., $f_{r+1}/f_r$ for $r = 1, 2, \cdots$.

## 3   Characterization of stable systems

Given an environment, define $L$ as the least able agent such that the team consisting of all agents from $L$ to $N$ is productive, that is,

$$L \;:=\; \max\{n \,|\, \sum_{i=n}^{N} a^{i-1} \geq P\}.$$

Note that $L \geq 2$ by (2). The set of stable systems is characterised as follows.

**Proposition 1** *A system is stable if and only if every team is efficient and every agent's payoff is equal to her ability.*

*Proof.* Consider a system where every team is efficient and every agent's payoff is equal to her ability. Then, no deviation may pay some of their members more than their abilities without reducing other members' payoffs below their abilities, because the total surplus of any team is at most the sum of its members' abilities by (1). Hence, such a system is stable.

Next, we prove the converse. In a system $(\pi, \rho)$ where one team, say $Z_\ell \in \pi$, is either unproductive or of a size less than $M$, the sum of the total surpluses of all teams in the system is strictly lower than the surplus of the grand coalition, $p(\Omega)$, because each team generates a surplus $s(Z_k) \leq \sum_{i \in Z_k} a^{i-1}$ by (1) where the inequality is strict if $p(Z_k) < P$ or $|Z_k| < M$. Thus, forming a grand coalition is a beneficial deviation because there is an imputation rule, for example, that rewards everyone in $Z_\ell$ more than and everyone else the same as in the system $(\pi, \rho)$. This proves that every team in a stable system must be efficient.

Moreover, with a view to reaching a contradiction, suppose that an agent, say $i$, receives a payoff less than her ability, $a^{i-1}$, in a system where every team is productive and of size $M$ or larger. Then, it would be clearly beneficial for agent $i$ to join another team, say $Z_k$, if it exists because she will increase the surplus of $Z_k$ by $a^{i-1}$ so that she can be paid more than before without reducing the payoff to any of the original members of $Z_k$. Consider the alternative case that there is no other team, i.e., the system consists of the grand coalition. Let $j$ denote an agent whose payoff exceeds her ability in the system, who must exist because the total surplus of the grand coalition is the sum of all agents' ability while agent $i$ is paid less than her ability. Then, the deviation $\Omega \setminus \{j\}$ would be profitable because, compared to the grand coalition, the total surplus is reduced by $a^{j-1}$ (as it remains to be efficient by (2)) while the total wage bill is reduced by more, so that an imputation rule exists that improves the payoff of everyone in $\Omega \setminus \{j\}$, strictly for some. This proves the converse. ∎

Thus, in a stable system every team must have an agent no less able than agent $L$ as its head in order to be productive, and must have an agent no more able than agent $M$ in order to be efficient. If $L < M$, therefore, any two teams are *non-segregated*: at least one of the teams has an agent whose ability is in-between the most and the least able members of the other team.

**Corollary 1** *If $L < M$, any two teams are non-segregated in every stable system.*

Having now characterised the set of stable systems, we move on to address the main question of the paper, namely, the relationship between vertical inequality within teams and the number and composition of teams. Consider two systems in a given environment, denoted by $F = (\pi_F, \rho_F)$ and $G = (\pi_G, \rho_G)$. We say that $F$ is less equal than $G$ if the fraction of each agent's payoff relative to that of the agent one rank above is uniformly lower in $F$ than in $G$. Formally, let $(f_1^k, f_2^k, \cdots) = \rho_F(Y_k)$

be the rank-imputation rule of a team $Y_k \in \pi_F$, and let $(g_1^\ell, g_2^\ell, \cdots) = \rho_G(Z_\ell)$ be the rank-imputation rule of a team $Z_\ell \in \pi_G$. Then, $F$ is "less equal" than $G$ if

$$\max_{\{(k,r)|Y_k \in \pi_F, 1 \le r \le |Y_k|-1\}} \frac{f_{r+1}^k}{f_r^k} \le \min_{\{(\ell,r)|Z_\ell \in \pi_G, 1 \le r \le |Z_\ell|-1\}} \frac{g_{r+1}^\ell}{g_r^\ell}, \tag{4}$$

and strictly so if the inequality in (4) is strict.

**Proposition 2** *Consider two stable systems $F = (\pi_F, \rho_F)$ and $G = (\pi_G, \rho_G)$ in an environment where $L < M$. If $F$ is (strictly) less equal than $G$, then $F$ has a (strictly) larger number of teams than $G$.*

*Proof.* As each agent's payoff is equal to her ability in any stable system by Proposition 1, $\frac{f_{r+1}^k}{f_r^k}$ and $\frac{g_{r+1}^\ell}{g_r^\ell}$ are of the form $a^\nu$ where $\nu \in \mathbb{N}$ is the distance between the two consecutively ranked agents in their ability ordering of $\Omega$. Let $a^{\nu_F}$ and $a^{\nu_G}$ denote the values on the LHS and RHS of (4). Then, the condition (4) implies that

(a) $a^{\nu_F} \le a^{\nu_G} \iff \nu_F \ge \nu_G$,

(b) any two consecutively ranked agents in $F$ are at least $\nu_F$ apart in their ability ordering of $\Omega$, and

(c) any two consecutively ranked agents in $G$ are at most $\nu_G$ apart in their ability ordering of $\Omega$.

In addition, recall from Proposition 1 that every team in $F$ or $G$

(d) must be productive, so its head is an agent in the set $\mathcal{L} = \{1, 2, \cdots, L\}$, and

(e) must be efficient, so contains an agent outside of $\mathcal{L}$.

Let $Y_1$ denote the team headed by agent 1 in $F$. Let $j$ denote the second-ranked agent in $Y_1$. As $j \ge 1 + \nu_F$ and no two agents between 2 and $1 + \nu_F$ belong to the same team by (b), there must be at least $\nu_F$ teams in $F$.

For the system $G$, let $h$ denote the least able agent who heads a team in $G$, where $h \le L$ by (d). If $h \le \nu_G$, it is clear that there may be at most $\nu_G$ teams in $G$. If $h > \nu_G$, consider the set of all agents between $h - \nu_G + 1$ and $h$, $\{h - \nu_G + 1, \cdots, h\}$. As any team headed by an agent more able than $h - \nu_G + 1$ must have a member in $\{h - \nu_G + 1, \cdots, h\}$ by (c) and (e), it follows that every team in $G$ has a member in $\{h - \nu_G + 1, \cdots, h\}$. Consequently, the number of teams in $G$ is at most the cardinality of $\{h - \nu_G + 1, \cdots, h\}$, i.e., $\nu_G$. The proposition follows from (a). ∎

In the environments considered above, stable systems always exist, admit a complete characterisation, and exhibit positive relationship between intra-team inequality and the number of teams in the system. Yet, it is difficult to envision the set of all stable systems and their interrelations, because teams of a large variety of sizes and imputation rules are viable in stable systems due to relatively general production technology and flexible intra-team negotiation of compensation that the model allows. For the sake of providing a clearer illustration, in the next section we present a heuristic class of more stringent environments where the set of all stable systems and their interrelations transpire more straightforwardly.

# 4   Single parameter imputation rules

We focus on environments in which countably infinite agents form teams whose efficiency improves with their size without bound, i.e., $M = N = \infty$. We also assume that $P = 0$. Hence, all efficient teams are of the same, infinite size, and all such teams are efficient. This simplifies the set of teams potentially viable in stable systems.

In addition, we postulate in this section that every team must adopt an imputation rule represented by a single parameter $\rho \in (0, 1)$, termed *imputation ratio,* which is the ratio of the surplus share of any team member relative to that of the member occupying the rank immediately above. That is, each team $Z$, including a deviation, is required to choose a rank-imputation rule such that $f_{r+1}/f_r$ is the same for all $r = 1, 2, \cdots, |Z| - 1$, which is the imputation ratio $\rho$. Note that a lower $\rho$ corresponds to greater internal inequality.

Then, denoting the rank of agent $i$ in a team $Z$ by $r_i(Z)$, the expected utility of agent $i$ in $Z$ is

$$u_i(Z, \rho) = \frac{s(Z) \cdot \rho^{r_i(Z)-1}}{1 + \rho + \cdots + \rho^{|Z|-1}} = \frac{s(Z)(1-\rho)\rho^{r_i(Z)-1}}{1 - \rho^{|Z|}} \tag{5}$$
$$= s(Z)(1-\rho)\rho^{r_i(Z)-1} \ \text{ if } \ |Z| = \infty.$$

Thus, every agent should decide which team to join not only on the basis of the team's power, $p(Z)$, but also on the basis of her expected rank in the team, $r_i(Z)$, and the vertical inequality, $\rho$.

We now represent a system as $(\pi, \vec{\rho})$ consisting of a partition $\pi = \{Z_1, \cdots, Z_K\}$ of $\Omega$ into $K$ teams, and a $K$-vector $\vec{\rho} = (\rho_1, \cdots, \rho_K)$ that specifies one imputation ratio $\rho_k \in (0, 1)$ for each team $Z_k \in \pi$. Such a system $(\pi, \vec{\rho})$ is *stable under single-parameter rule* if there does not exist a deviation $D \subset \Omega$ whose members are better off (some strictly) in the deviation under some imputation ratio.

Note that if a system $(\pi, \vec{\rho})$ is stable in the original sense (i.e., when deviations may adopt any imputation rule), it is stable under single-parameter rule as well. By Proposition 1, therefore, a system $(\pi, \vec{\rho})$ is stable under single-parameter rule if every team is efficient and every agent's payoff is equal to her ability. Below we identify an intuitive class of systems that exhibit these properties, which also consist of non-segregated rival teams; then, we verify that they are the only systems that are stable under single-parameter rule.

A team $Z$ is "$K$-cyclic" if it consists of every $K$-th agent starting from a certain agent $k$, i.e., $Z = \{k, k + K, k + 2K, k + 3K, \cdots\}$. Any such team is efficient as $p(Z) > 0$ and $|Z| = \infty$. A "$K$-cyclic partition" is $\pi_K^c = \{Z_1, \cdots, Z_K\}$ where each $Z_k$ is $K$-cyclic starting from agent $k$ for $k = 1, 2, \cdots, K$.

A "symmetric $K$-cyclic system" $(\pi_K^c, \vec{\rho})$ where $\pi_K^c$ is the $K$-cyclic partition and $\vec{\rho} = (a^K, \cdots, a^K)$, clearly delivers every agent a payoff that is equal to her ability

and thus, constitutes a stable system. In fact, the same conclusion holds so long as each team of a system is $K$-cyclic with an imputation ratio $\rho = a^K$ for some integer $K$, where the value of $K$ may vary across teams. We refer to such a system as a "generalised cyclic" system. For example, in the symmetric 4-cyclic system, if $Z_1$ and $Z_3$ merge to form a 2-cyclic team and adopt an imputation ratio of $a^2$, then the new system is a generalised cyclic system.

Having asserted that generalised cyclic systems are stable, we now establish that a system is stable under single-parameter rule if and only if it is of this kind. Therefore, organisations with varying norms of internal inequality may coexist. Moreover, the more unequal internal norms a system displays across the board, the larger is the number of rival organisations that have emerged in the system and the more widely dispersed are the agents' abilities within organisations.

**Proposition 3** *A system $(\pi, \vec{\rho})$ is stable under single-parameter rule if and only if it is a generalised cyclic system.*

*Proof.* It remains to show the "only if" part. To do this, we first show that agent $i$'s payoff is $a^{i-1}$ in any system $(\pi, \vec{\rho})$ that is stable (under single-parameter rule, which we omit below for brevity). With a view to reaching a contradiction, suppose to the contrary. Then, there exists an agent, say $j_1$, whose payoff is strictly less than $a^{j_1-1}$ in a stable system $(\pi, \vec{\rho})$. Find a sufficiently low $\rho' > 0$ such that $u' = (1 - \rho')a^{j_1-1}$ exceeds her payoff in $(\pi, \vec{\rho})$. For each $r = 2, 3, \cdots$, one can find an agent, say $j_r$, whose payoff in the system $(\pi, \vec{\rho})$ falls short of $u' \cdot (\rho')^{r-1}$, maintaining the feature that $j_r < j_{r+1}$. This is because there exists an agent $i$, where $i$ is arbitrarily large, whose payoff is arbitrarily low in the system either because her rank is arbitrarily low in an infinite team, or in the case that there is no team of an infinite size, because she is in a team of arbitrarily small power. Then, the deviation $D' = \{j_1, j_2, \cdots\}$ with the imputation ratio $\rho'$ is profitable because agent $j_r$ would have a payoff of $(1 - \rho')(\sum_{n=1}^{\infty} a^{j_n-1})(\rho')^{r-1} > u' \cdot (\rho')^{r-1}$. This proves that agent $i$ is paid $a^{i-1}$ in any stable system $(\pi, \vec{\rho})$.

Thus, every team $Z_k$ in a stable system must pay its members their abilities. For this to be the case, $s(Z_k)$ must be equal to the sum of its members abilities, which is possible only if $Z_k$ is efficient by (1), i.e., when $|Z_k| = \infty$ in the current environment. Moreover, to pay its members their abilities using an imputation ratio, $Z_k$ must be $K$-cyclic for some $K \in \mathbb{N}$. This completes the proof. ∎

# 5   Concluding Remarks

We have demonstrated that when agents with heterogeneous abilities need to form teams in order to produce surplus by performing a number of complementary tasks, the forces of competition and selection lead to emergence of non-segregated rival teams, and more of them if they exhibit higher levels of internal inequality.

Although our theoretical framework and results are not directly usable for normative analysis, it seems appropriate to mention that our results on the relationship between vertical inequality and number of competing organizations may also enter policy or regulation debates. For example, in relation to the recent discussion about the pros and cons of imposing less inequality in pay structures within certain kinds of firms, our analysis points to a possible implication of such a restriction that has not been noted, namely, that such a restriction could lead to greater concentration in the industry.

One limitation of our model is that the value of a team does not depend on the partition of the rest of the agents. While this limitation is not critical in some applications and contexts (e.g., production economies where the market shares tend to be proportional to rival firms' power), it is more limiting, for example, in political economy applications: in plurality rule elections it makes a big difference for a coalition expecting 30% of the votes whether the remaining 70% is divided into 7 small parties of 10% each or two parties of 35% each. An extension of the model in which the relative power of any coalition depends not only on the ability of its members but also on some other relevant dimension is in our future research agenda.

It would also be useful to analyse in the future the realistic extension in which abilities are more than one dimensional, for example, to study whether stable systems with more groups tend to have a different sorting of ability compositions relative to systems with less competition.

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